

Computer algebra independent integration tests

4-Trig-functions/4.7-Miscellaneous/4.7.7-Trig-functions

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3.241	$\int (2 \cos(c+dx) + 3 \sin(c+dx))^{5/2} dx$	969
3.242	$\int (2 \cos(c+dx) + 3 \sin(c+dx))^{3/2} dx$	972
3.243	$\int \sqrt{2 \cos(c+dx) + 3 \sin(c+dx)} dx$	975
3.244	$\int \frac{1}{\sqrt{2 \cos(c+dx)+3 \sin(c+dx)}} dx$	978
3.245	$\int \frac{1}{(2 \cos(c+dx)+3 \sin(c+dx))^{3/2}} dx$	981
3.246	$\int \frac{1}{(2 \cos(c+dx)+3 \sin(c+dx))^{5/2}} dx$	984
3.247	$\int \frac{1}{(2 \cos(c+dx)+3 \sin(c+dx))^{7/2}} dx$	987
3.248	$\int (a \cos(c+dx) + ia \sin(c+dx))^n dx$	990
3.249	$\int (a \cos(c+dx) + ia \sin(c+dx))^4 dx$	992
3.250	$\int (a \cos(c+dx) + ia \sin(c+dx))^3 dx$	994

3.251	$\int (a \cos(c + dx) + ia \sin(c + dx))^2 dx$	996
3.252	$\int (a \cos(c + dx) + ia \sin(c + dx)) dx$	998
3.253	$\int \frac{1}{a \cos(c+dx)+ia \sin(c+dx)} dx$	1000
3.254	$\int \frac{1}{(a \cos(c+dx)+ia \sin(c+dx))^2} dx$	1002
3.255	$\int \frac{1}{(a \cos(c+dx)+ia \sin(c+dx))^3} dx$	1004
3.256	$\int \frac{1}{(a \cos(c+dx)+ia \sin(c+dx))^4} dx$	1006
3.257	$\int (a \cos(c + dx) + ia \sin(c + dx))^{5/2} dx$	1008
3.258	$\int (a \cos(c + dx) + ia \sin(c + dx))^{3/2} dx$	1010
3.259	$\int \sqrt{a \cos(c + dx) + ia \sin(c + dx)} dx$	1012
3.260	$\int \frac{1}{\sqrt{a \cos(c+dx)+ia \sin(c+dx)}} dx$	1014
3.261	$\int \frac{1}{(a \cos(c+dx)+ia \sin(c+dx))^{3/2}} dx$	1016
3.262	$\int \frac{1}{(a \cos(c+dx)+ia \sin(c+dx))^{5/2}} dx$	1018
3.263	$\int (a \sec(x) + b \tan(x))^5 dx$	1020
3.264	$\int (a \sec(x) + b \tan(x))^4 dx$	1024
3.265	$\int (a \sec(x) + b \tan(x))^3 dx$	1027
3.266	$\int (a \sec(x) + b \tan(x))^2 dx$	1030
3.267	$\int (a \sec(x) + b \tan(x)) dx$	1032
3.268	$\int \frac{1}{a \sec(x)+b \tan(x)} dx$	1034
3.269	$\int \frac{1}{(a \sec(x)+b \tan(x))^2} dx$	1037
3.270	$\int \frac{1}{(a \sec(x)+b \tan(x))^3} dx$	1041
3.271	$\int \frac{1}{(a \sec(x)+b \tan(x))^4} dx$	1044
3.272	$\int \frac{1}{(a \sec(x)+b \tan(x))^5} dx$	1051
3.273	$\int (\sec(x) + \tan(x))^5 dx$	1055
3.274	$\int (\sec(x) + \tan(x))^4 dx$	1058
3.275	$\int (\sec(x) + \tan(x))^3 dx$	1061
3.276	$\int (\sec(x) + \tan(x))^2 dx$	1064
3.277	$\int (\sec(x) + \tan(x)) dx$	1067
3.278	$\int \frac{1}{\sec(x)+\tan(x)} dx$	1069
3.279	$\int \frac{1}{(\sec(x)+\tan(x))^2} dx$	1071
3.280	$\int \frac{1}{(\sec(x)+\tan(x))^3} dx$	1074
3.281	$\int \frac{1}{(\sec(x)+\tan(x))^4} dx$	1077
3.282	$\int \frac{1}{(\sec(x)+\tan(x))^5} dx$	1080
3.283	$\int (a \cot(x) + b \csc(x))^5 dx$	1083
3.284	$\int (a \cot(x) + b \csc(x))^4 dx$	1087
3.285	$\int (a \cot(x) + b \csc(x))^3 dx$	1090
3.286	$\int (a \cot(x) + b \csc(x))^2 dx$	1093
3.287	$\int (a \cot(x) + b \csc(x)) dx$	1095
3.288	$\int \frac{1}{a \cot(x)+b \csc(x)} dx$	1097
3.289	$\int \frac{1}{(a \cot(x)+b \csc(x))^2} dx$	1099
3.290	$\int \frac{1}{(a \cot(x)+b \csc(x))^3} dx$	1102
3.291	$\int \frac{1}{(a \cot(x)+b \csc(x))^4} dx$	1105
3.292	$\int \frac{1}{(a \cot(x)+b \csc(x))^5} dx$	1110
3.293	$\int (\cot(x) + \csc(x))^5 dx$	1113
3.294	$\int (\cot(x) + \csc(x))^4 dx$	1116

3.295	$\int (\cot(x) + \csc(x))^3 dx$	1119
3.296	$\int (\cot(x) + \csc(x))^2 dx$	1122
3.297	$\int (\cot(x) + \csc(x)) dx$	1125
3.298	$\int \frac{1}{\cot(x) + \csc(x)} dx$	1127
3.299	$\int \frac{1}{(\cot(x) + \csc(x))^2} dx$	1129
3.300	$\int \frac{1}{(\cot(x) + \csc(x))^3} dx$	1131
3.301	$\int \frac{1}{(\cot(x) + \csc(x))^4} dx$	1134
3.302	$\int \frac{1}{(\cot(x) + \csc(x))^5} dx$	1137
3.303	$\int (\csc(x) - \sin(x))^4 dx$	1140
3.304	$\int (\csc(x) - \sin(x))^3 dx$	1143
3.305	$\int (\csc(x) - \sin(x))^2 dx$	1146
3.306	$\int (\csc(x) - \sin(x)) dx$	1149
3.307	$\int \frac{1}{\csc(x) - \sin(x)} dx$	1151
3.308	$\int \frac{1}{(\csc(x) - \sin(x))^2} dx$	1153
3.309	$\int \frac{1}{(\csc(x) - \sin(x))^3} dx$	1155
3.310	$\int \frac{1}{(\csc(x) - \sin(x))^4} dx$	1158
3.311	$\int \frac{1}{(\csc(x) - \sin(x))^5} dx$	1160
3.312	$\int \frac{1}{(\csc(x) - \sin(x))^6} dx$	1163
3.313	$\int \frac{1}{(\csc(x) - \sin(x))^7} dx$	1165
3.314	$\int (\csc(x) - \sin(x))^{7/2} dx$	1168
3.315	$\int (\csc(x) - \sin(x))^{5/2} dx$	1171
3.316	$\int (\csc(x) - \sin(x))^{3/2} dx$	1174
3.317	$\int \sqrt{\csc(x) - \sin(x)} dx$	1177
3.318	$\int \frac{1}{\sqrt{\csc(x) - \sin(x)}} dx$	1180
3.319	$\int \frac{1}{(\csc(x) - \sin(x))^{3/2}} dx$	1184
3.320	$\int \frac{1}{(\csc(x) - \sin(x))^{5/2}} dx$	1188
3.321	$\int \frac{1}{(\csc(x) - \sin(x))^{7/2}} dx$	1193
3.322	$\int (-\cos(x) + \sec(x))^4 dx$	1198
3.323	$\int (-\cos(x) + \sec(x))^3 dx$	1201
3.324	$\int (-\cos(x) + \sec(x))^2 dx$	1204
3.325	$\int (-\cos(x) + \sec(x)) dx$	1207
3.326	$\int \frac{1}{-\cos(x) + \sec(x)} dx$	1209
3.327	$\int \frac{1}{(-\cos(x) + \sec(x))^2} dx$	1211
3.328	$\int \frac{1}{(-\cos(x) + \sec(x))^3} dx$	1213
3.329	$\int \frac{1}{(-\cos(x) + \sec(x))^4} dx$	1216
3.330	$\int \frac{1}{(-\cos(x) + \sec(x))^5} dx$	1218
3.331	$\int \frac{1}{(-\cos(x) + \sec(x))^6} dx$	1221
3.332	$\int \frac{1}{(-\cos(x) + \sec(x))^7} dx$	1223
3.333	$\int (-\cos(x) + \sec(x))^{7/2} dx$	1226
3.334	$\int (-\cos(x) + \sec(x))^{5/2} dx$	1229
3.335	$\int (-\cos(x) + \sec(x))^{3/2} dx$	1232
3.336	$\int \sqrt{-\cos(x) + \sec(x)} dx$	1235
3.337	$\int \frac{1}{\sqrt{-\cos(x) + \sec(x)}} dx$	1238

3.338	$\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}} dx$	1241
3.339	$\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}} dx$	1245
3.340	$\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}} dx$	1250
3.341	$\int (\sin(x) + \tan(x))^4 dx$	1255
3.342	$\int (\sin(x) + \tan(x))^3 dx$	1259
3.343	$\int (\sin(x) + \tan(x))^2 dx$	1262
3.344	$\int (\sin(x) + \tan(x)) dx$	1265
3.345	$\int \frac{1}{\sin(x)+\tan(x)} dx$	1267
3.346	$\int \frac{1}{(\sin(x)+\tan(x))^2} dx$	1270
3.347	$\int \frac{1}{(\sin(x)+\tan(x))^3} dx$	1273
3.348	$\int \frac{1}{(\sin(x)+\tan(x))^4} dx$	1276
3.349	$\int \frac{A+C \sin(x)}{b \cos(x)+c \sin(x)} dx$	1279
3.350	$\int \frac{A+C \sin(x)}{(b \cos(x)+c \sin(x))^2} dx$	1283
3.351	$\int \frac{A+C \sin(x)}{(b \cos(x)+c \sin(x))^3} dx$	1286
3.352	$\int \frac{A+B \cos(x)}{b \cos(x)+c \sin(x)} dx$	1290
3.353	$\int \frac{A+B \cos(x)}{(b \cos(x)+c \sin(x))^2} dx$	1294
3.354	$\int \frac{A+B \cos(x)}{(b \cos(x)+c \sin(x))^3} dx$	1297
3.355	$\int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^4 dx$	1301
3.356	$\int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^3 dx$	1305
3.357	$\int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^2 dx$	1309
3.358	$\int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right) dx$	1312
3.359	$\int \frac{1}{\sqrt{b^2+c^2}+b \cos(d+ex)+c \sin(d+ex)} dx$	1314
3.360	$\int \frac{1}{\left(\sqrt{b^2+c^2}+b \cos(d+ex)+c \sin(d+ex) \right)^2} dx$	1316
3.361	$\int \frac{1}{\left(\sqrt{b^2+c^2}+b \cos(d+ex)+c \sin(d+ex) \right)^3} dx$	1319
3.362	$\int \frac{1}{\left(\sqrt{b^2+c^2}+b \cos(d+ex)+c \sin(d+ex) \right)^4} dx$	1323
3.363	$\int (2a + 2a \cos(d + ex) + 2c \sin(d + ex))^3 dx$	1328
3.364	$\int (2a + 2a \cos(d + ex) + 2c \sin(d + ex))^2 dx$	1331
3.365	$\int (2a + 2a \cos(d + ex) + 2c \sin(d + ex)) dx$	1334
3.366	$\int \frac{1}{2a+2a \cos(d+ex)+2c \sin(d+ex)} dx$	1336
3.367	$\int \frac{1}{(2a+2a \cos(d+ex)+2c \sin(d+ex))^2} dx$	1339
3.368	$\int \frac{1}{(2a+2a \cos(d+ex)+2c \sin(d+ex))^3} dx$	1342
3.369	$\int \frac{1}{(2a+2a \cos(d+ex)+2c \sin(d+ex))^4} dx$	1346
3.370	$\int \frac{1}{2a+2a \cos(d+ex)+2a \sin(d+ex)} dx$	1350
3.371	$\int \frac{1}{(2a+2a \cos(d+ex)+2a \sin(d+ex))^2} dx$	1352
3.372	$\int \frac{1}{(2a+2a \cos(d+ex)+2a \sin(d+ex))^3} dx$	1355
3.373	$\int \frac{1}{(2a+2a \cos(d+ex)+2a \sin(d+ex))^4} dx$	1359
3.374	$\int (2a - 2a \cos(d + ex) + 2c \sin(d + ex))^3 dx$	1363
3.375	$\int (2a - 2a \cos(d + ex) + 2c \sin(d + ex))^2 dx$	1366

3.376	$\int (2a - 2a \cos(d + ex) + 2c \sin(d + ex)) dx$	1369
3.377	$\int \frac{1}{2a - 2a \cos(d + ex) + 2c \sin(d + ex)} dx$	1371
3.378	$\int \frac{1}{(2a - 2a \cos(d + ex) + 2c \sin(d + ex))^2} dx$	1374
3.379	$\int \frac{1}{(2a - 2a \cos(d + ex) + 2c \sin(d + ex))^3} dx$	1377
3.380	$\int \frac{1}{(2a - 2a \cos(d + ex) + 2c \sin(d + ex))^4} dx$	1381
3.381	$\int (2a + 2b \cos(d + ex) + 2a \sin(d + ex))^3 dx$	1385
3.382	$\int (2a + 2b \cos(d + ex) + 2a \sin(d + ex))^2 dx$	1388
3.383	$\int (2a + 2b \cos(d + ex) + 2a \sin(d + ex)) dx$	1391
3.384	$\int \frac{1}{2a + 2b \cos(d + ex) + 2a \sin(d + ex)} dx$	1393
3.385	$\int \frac{1}{(2a + 2b \cos(d + ex) + 2a \sin(d + ex))^2} dx$	1396
3.386	$\int \frac{1}{(2a + 2b \cos(d + ex) + 2a \sin(d + ex))^3} dx$	1399
3.387	$\int \frac{1}{(2a + 2b \cos(d + ex) + 2a \sin(d + ex))^4} dx$	1403
3.388	$\int (2a + 2b \cos(d + ex) - 2a \sin(d + ex))^3 dx$	1408
3.389	$\int (2a + 2b \cos(d + ex) - 2a \sin(d + ex))^2 dx$	1411
3.390	$\int (2a + 2b \cos(d + ex) - 2a \sin(d + ex)) dx$	1414
3.391	$\int \frac{1}{2a + 2b \cos(d + ex) - 2a \sin(d + ex)} dx$	1416
3.392	$\int \frac{1}{(2a + 2b \cos(d + ex) - 2a \sin(d + ex))^2} dx$	1419
3.393	$\int \frac{1}{(2a + 2b \cos(d + ex) - 2a \sin(d + ex))^3} dx$	1422
3.394	$\int \frac{1}{(2a + 2b \cos(d + ex) - 2a \sin(d + ex))^4} dx$	1426
3.395	$\int (a + b \cos(d + ex) + c \sin(d + ex))^4 dx$	1431
3.396	$\int (a + b \cos(d + ex) + c \sin(d + ex))^3 dx$	1435
3.397	$\int (a + b \cos(d + ex) + c \sin(d + ex))^2 dx$	1438
3.398	$\int (a + b \cos(d + ex) + c \sin(d + ex)) dx$	1441
3.399	$\int \frac{1}{a + b \cos(d + ex) + c \sin(d + ex)} dx$	1443
3.400	$\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^2} dx$	1447
3.401	$\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^3} dx$	1451
3.402	$\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^4} dx$	1457
3.403	$\int (2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2} dx$	1464
3.404	$\int (2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{3/2} dx$	1468
3.405	$\int \sqrt{2 + 3 \cos(d + ex) + 5 \sin(d + ex)} dx$	1472
3.406	$\int \frac{1}{\sqrt{2 + 3 \cos(d + ex) + 5 \sin(d + ex)}} dx$	1475
3.407	$\int \frac{1}{(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{3/2}} dx$	1478
3.408	$\int \frac{1}{(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2}} dx$	1481
3.409	$\int \frac{1}{(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{7/2}} dx$	1485
3.410	$\int (a + b \cos(d + ex) + c \sin(d + ex))^{5/2} dx$	1490
3.411	$\int (a + b \cos(d + ex) + c \sin(d + ex))^{3/2} dx$	1496
3.412	$\int \sqrt{a + b \cos(d + ex) + c \sin(d + ex)} dx$	1501
3.413	$\int \frac{1}{\sqrt{a + b \cos(d + ex) + c \sin(d + ex)}} dx$	1504
3.414	$\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^{3/2}} dx$	1507
3.415	$\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^{5/2}} dx$	1511
3.416	$\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^{7/2}} dx$	1517
3.417	$\int (5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2} dx$	1524
3.418	$\int (5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2} dx$	1527

3.419	$\int \sqrt{5 + 4 \cos(d + ex) + 3 \sin(d + ex)} dx$	1530
3.420	$\int \frac{1}{\sqrt{5+4 \cos(d+ex)+3 \sin(d+ex)}} dx$	1532
3.421	$\int \frac{1}{(5+4 \cos(d+ex)+3 \sin(d+ex))^{3/2}} dx$	1535
3.422	$\int \frac{1}{(5+4 \cos(d+ex)+3 \sin(d+ex))^{5/2}} dx$	1538
3.423	$\int (-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{7/2} dx$	1542
3.424	$\int (-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2} dx$	1545
3.425	$\int (-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2} dx$	1548
3.426	$\int \sqrt{-5 + 4 \cos(d + ex) + 3 \sin(d + ex)} dx$	1551
3.427	$\int \frac{1}{\sqrt{-5+4 \cos(d+ex)+3 \sin(d+ex)}} dx$	1553
3.428	$\int \frac{1}{(-5+4 \cos(d+ex)+3 \sin(d+ex))^{3/2}} dx$	1556
3.429	$\int \frac{1}{(-5+4 \cos(d+ex)+3 \sin(d+ex))^{5/2}} dx$	1559
3.430	$\int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{7/2} dx$	1563
3.431	$\int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{5/2} dx$	1566
3.432	$\int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{3/2} dx$	1569
3.433	$\int \sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)} dx$	1572
3.434	$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b \cos(d+ex)+c \sin(d+ex)}} dx$	1574
3.435	$\int \frac{1}{\left(\sqrt{b^2+c^2}+b \cos(d+ex)+c \sin(d+ex) \right)^{3/2}} dx$	1577
3.436	$\int \frac{1}{\left(\sqrt{b^2+c^2}+b \cos(d+ex)+c \sin(d+ex) \right)^{5/2}} dx$	1581
3.437	$\int \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{5/2} dx$	1585
3.438	$\int \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{3/2} dx$	1588
3.439	$\int \sqrt{-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)} dx$	1591
3.440	$\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b \cos(d+ex)+c \sin(d+ex)}} dx$	1593
3.441	$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b \cos(d+ex)+c \sin(d+ex) \right)^{3/2}} dx$	1596
3.442	$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b \cos(d+ex)+c \sin(d+ex) \right)^{5/2}} dx$	1600
3.443	$\int \frac{\sin(x)}{a+b \cos(x)+c \sin(x)} dx$	1604
3.444	$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)} dx$	1608
3.445	$\int \frac{1}{a+c \sec(x)+b \tan(x)} dx$	1611
3.446	$\int \frac{\sec(x)}{a+c \sec(x)+b \tan(x)} dx$	1615
3.447	$\int \frac{\sec^2(x)}{a+c \sec(x)+b \tan(x)} dx$	1618
3.448	$\int \frac{(a+b \sec(d+ex)+c \tan(d+ex))^{3/2}}{\sec^2(d+ex)} dx$	1623
3.449	$\int \frac{\sqrt{a+b \sec(d+ex)+c \tan(d+ex)}}{\sqrt{\sec(d+ex)}} dx$	1628
3.450	$\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b \sec(d+ex)+c \tan(d+ex)}} dx$	1632
3.451	$\int \frac{\sec^2(d+ex)}{(a+b \sec(d+ex)+c \tan(d+ex))^{3/2}} dx$	1635
3.452	$\int \frac{\sec^2(d+ex)}{(a+b \sec(d+ex)+c \tan(d+ex))^{5/2}} dx$	1639

3.453	$\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{3}{2}} dx$	1644
3.454	$\int \sqrt{\cos(d+ex)} \sqrt{a+b\sec(d+ex)+c\tan(d+ex)} dx$	1648
3.455	$\int \frac{1}{\sqrt{\cos(d+ex)} \sqrt{a+b\sec(d+ex)+c\tan(d+ex)}} dx$	1651
3.456	$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{3}{2}}} dx$	1655
3.457	$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{5}{2}}} dx$	1658
3.458	$\int \frac{1}{a+b\cot(x)+c\csc(x)} dx$	1662
3.459	$\int \frac{\csc(x)}{a+b\cot(x)+c\csc(x)} dx$	1666
3.460	$\int \frac{\csc^2(x)}{a+b\cot(x)+c\csc(x)} dx$	1669
3.461	$\int \frac{\csc(x)}{2+2\cot(x)+3\csc(x)} dx$	1673
3.462	$\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{\frac{3}{2}}}{\csc^{\frac{3}{2}}(d+ex)} dx$	1676
3.463	$\int \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}{\sqrt{\csc(d+ex)}} dx$	1681
3.464	$\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}} dx$	1685
3.465	$\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{\frac{3}{2}}} dx$	1688
3.466	$\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{\frac{5}{2}}} dx$	1692
3.467	$\int (a+c\cot(d+ex)+b\csc(d+ex))^{\frac{3}{2}} \sin^{\frac{3}{2}}(d+ex) dx$	1697
3.468	$\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)} \sqrt{\sin(d+ex)} dx$	1701
3.469	$\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)} \sqrt{\sin(d+ex)}} dx$	1704
3.470	$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{\frac{3}{2}} \sin^{\frac{3}{2}}(d+ex)} dx$	1708
3.471	$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{\frac{5}{2}} \sin^{\frac{5}{2}}(d+ex)} dx$	1711
3.472	$\int \frac{1}{\cos^2(x)+\sin^2(x)} dx$	1715
3.473	$\int \frac{1}{(\cos^2(x)+\sin^2(x))^2} dx$	1717
3.474	$\int \frac{1}{(\cos^2(x)+\sin^2(x))^3} dx$	1719
3.475	$\int \frac{1}{\cos^2(x)-\sin^2(x)} dx$	1721
3.476	$\int \frac{1}{(\cos^2(x)-\sin^2(x))^2} dx$	1723
3.477	$\int \frac{1}{(\cos^2(x)-\sin^2(x))^3} dx$	1725
3.478	$\int \frac{1}{\cos^2(x)+a^2\sin^2(x)} dx$	1728
3.479	$\int \frac{1}{b^2\cos^2(x)+\sin^2(x)} dx$	1735
3.480	$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)} dx$	1737
3.481	$\int \frac{1}{4\cos^2(1+2x)+3\sin^2(1+2x)} dx$	1739
3.482	$\int \frac{\sin^2(x)}{a\cos^2(x)+b\sin^2(x)} dx$	1741
3.483	$\int \frac{\cos^2(x)}{a\cos^2(x)+b\sin^2(x)} dx$	1744
3.484	$\int \frac{1}{\sec^2(x)+\tan^2(x)} dx$	1747
3.485	$\int \frac{1}{(\sec^2(x)+\tan^2(x))^2} dx$	1749
3.486	$\int \frac{1}{(\sec^2(x)+\tan^2(x))^3} dx$	1752

3.487	$\int \frac{1}{\sec^2(x) - \tan^2(x)} dx$	1755
3.488	$\int \frac{1}{(\sec^2(x) - \tan^2(x))^2} dx$	1757
3.489	$\int \frac{1}{(\sec^2(x) - \tan^2(x))^3} dx$	1759
3.490	$\int \frac{1}{\cot^2(x) + \csc^2(x)} dx$	1761
3.491	$\int \frac{1}{(\cot^2(x) + \csc^2(x))^2} dx$	1763
3.492	$\int \frac{1}{(\cot^2(x) + \csc^2(x))^3} dx$	1766
3.493	$\int \frac{1}{\cot^2(x) - \csc^2(x)} dx$	1769
3.494	$\int \frac{1}{(\cot^2(x) - \csc^2(x))^2} dx$	1771
3.495	$\int \frac{1}{(\cot^2(x) - \csc^2(x))^3} dx$	1773
3.496	$\int \frac{1}{a + b \cos^2(x) + c \sin^2(x)} dx$	1775
3.497	$\int \frac{x}{a + b \cos^2(x) + c \sin^2(x)} dx$	1777
3.498	$\int \frac{x^2}{a + b \cos^2(x) + c \sin^2(x)} dx$	1782
3.499	$\int (a + b \sin(d + ex)) (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^2 dx$	1788
3.500	$\int (a + b \sin(d + ex)) (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex)) dx$	1792
3.501	$\int \frac{a + b \sin(d + ex)}{b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex)} dx$	1795
3.502	$\int \frac{a + b \sin(d + ex)}{(b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^2} dx$	1798
3.503	$\int \frac{d + e \sin(x)}{a + b \sin(x) + c \sin^2(x)} dx$	1803
3.504	$\int (a + b \sin(d + ex)) (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2} dx$	1813
3.505	$\int (a + b \sin(d + ex)) \sqrt{b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex)} dx$	1817
3.506	$\int \frac{a + b \sin(d + ex)}{\sqrt{b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex)}} dx$	1820
3.507	$\int \frac{a + b \sin(d + ex)}{(b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2}} dx$	1824
3.508	$\int \frac{a + b \cos(x)}{b^2 + 2ab \cos(x) + a^2 \cos^2(x)} dx$	1829
3.509	$\int \frac{d + e \cos(x)}{a + b \cos(x) + c \cos^2(x)} dx$	1832
3.510	$\int (a + b \tan(d + ex)) (b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^2 dx$	1845
3.511	$\int (a + b \tan(d + ex)) (b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)) dx$	1849
3.512	$\int \frac{a + b \tan(d + ex)}{b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)} dx$	1852
3.513	$\int \frac{a + b \tan(d + ex)}{(b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^2} dx$	1856
3.514	$\int (a + b \tan(d + ex)) (b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^{3/2} dx$	1860
3.515	$\int (a + b \tan(d + ex)) \sqrt{b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)} dx$	1864
3.516	$\int \frac{a + b \tan(d + ex)}{\sqrt{b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)}} dx$	1867
3.517	$\int \frac{a + b \tan(d + ex)}{(b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^{3/2}} dx$	1870
3.518	$\int (a + b \sec(d + ex)) (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^2 dx$	1875
3.519	$\int (a + b \sec(d + ex)) (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)) dx$	1879
3.520	$\int \frac{a + b \sec(d + ex)}{b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)} dx$	1882
3.521	$\int \frac{a + b \sec(d + ex)}{(b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^2} dx$	1886

3.522	$\int (a + b \sec(d + ex)) (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2} dx$	1894
3.523	$\int (a + b \sec(d + ex)) \sqrt{b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)} dx$	1898
3.524	$\int \frac{a + b \sec(d + ex)}{\sqrt{b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)}} dx$	1901
3.525	$\int \frac{a + b \sec(d + ex)}{(b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}} dx$	1905
3.526	$\int \frac{\cos(x) - i \sin(x)}{\cos(x) + i \sin(x)} dx$	1910
3.527	$\int \frac{\cos(x) + i \sin(x)}{\cos(x) - i \sin(x)} dx$	1912
3.528	$\int \frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)} dx$	1914
3.529	$\int \frac{B \cos(x) + C \sin(x)}{b \cos(x) + c \sin(x)} dx$	1916
3.530	$\int \frac{B \cos(x) + C \sin(x)}{(b \cos(x) + c \sin(x))^2} dx$	1920
3.531	$\int \frac{B \cos(x) + C \sin(x)}{(b \cos(x) + c \sin(x))^3} dx$	1923
3.532	$\int \frac{A + B \cos(x) + C \sin(x)}{b \cos(x) + c \sin(x)} dx$	1926
3.533	$\int \frac{A + B \cos(x) + C \sin(x)}{(b \cos(x) + c \sin(x))^2} dx$	1930
3.534	$\int \frac{A + B \cos(x) + C \sin(x)}{(b \cos(x) + c \sin(x))^3} dx$	1933
3.535	$\int \frac{A + B \cos(x)}{a + b \cos(x) + c \sin(x)} dx$	1937
3.536	$\int \frac{A + B \cos(x)}{(a + b \cos(x) + c \sin(x))^2} dx$	1941
3.537	$\int \frac{A + B \cos(x)}{(a + b \cos(x) + c \sin(x))^3} dx$	1945
3.538	$\int \frac{A + B \cos(x)}{a + b \cos(x) + i b \sin(x)} dx$	1951
3.539	$\int \frac{A + B \cos(x)}{a + b \cos(x) - i b \sin(x)} dx$	1954
3.540	$\int \frac{A + C \sin(x)}{a + b \cos(x) + c \sin(x)} dx$	1957
3.541	$\int \frac{A + C \sin(x)}{(a + b \cos(x) + c \sin(x))^2} dx$	1961
3.542	$\int \frac{A + C \sin(x)}{(a + b \cos(x) + c \sin(x))^3} dx$	1965
3.543	$\int \frac{A + C \sin(x)}{a + b \cos(x) + i b \sin(x)} dx$	1971
3.544	$\int \frac{A + C \sin(x)}{a + b \cos(x) - i b \sin(x)} dx$	1974
3.545	$\int \frac{B \cos(x) + C \sin(x)}{a + b \cos(x) + c \sin(x)} dx$	1977
3.546	$\int \frac{B \cos(x) + C \sin(x)}{(a + b \cos(x) + c \sin(x))^2} dx$	1981
3.547	$\int \frac{B \cos(x) + C \sin(x)}{(a + b \cos(x) + c \sin(x))^3} dx$	1985
3.548	$\int \frac{B \cos(x) + C \sin(x)}{a + b \cos(x) + i b \sin(x)} dx$	1991
3.549	$\int \frac{B \cos(x) + C \sin(x)}{a + b \cos(x) - i b \sin(x)} dx$	1994
3.550	$\int \frac{A + B \cos(x) + C \sin(x)}{a + b \cos(x) + c \sin(x)} dx$	1997
3.551	$\int \frac{A + B \cos(x) + C \sin(x)}{(a + b \cos(x) + c \sin(x))^2} dx$	2002
3.552	$\int \frac{A + B \cos(x) + C \sin(x)}{(a + b \cos(x) + c \sin(x))^3} dx$	2006
3.553	$\int \frac{A + B \cos(x) + C \sin(x)}{a + b \cos(x) + i b \sin(x)} dx$	2012
3.554	$\int \frac{A + B \cos(x) + C \sin(x)}{a + b \cos(x) - i b \sin(x)} dx$	2015
3.555	$\int \frac{b^2 + c^2 + ab \cos(x) + ac \sin(x)}{(a + b \cos(x) + c \sin(x))^2} dx$	2018
3.556	$\int (a + b \cos(x) + c \sin(x))^{5/2} (d + be \cos(x) + ce \sin(x)) dx$	2020
3.557	$\int (a + b \cos(x) + c \sin(x))^{3/2} (d + be \cos(x) + ce \sin(x)) dx$	2025
3.558	$\int \sqrt{a + b \cos(x) + c \sin(x)} (d + be \cos(x) + ce \sin(x)) dx$	2029
3.559	$\int \frac{d + be \cos(x) + ce \sin(x)}{\sqrt{a + b \cos(x) + c \sin(x)}} dx$	2034
3.560	$\int \frac{d + be \cos(x) + ce \sin(x)}{(a + b \cos(x) + c \sin(x))^{3/2}} dx$	2038

3.561	$\int \frac{d+be \cos(x)+ce \sin(x)}{(a+b \cos(x)+c \sin(x))^{5/2}} dx$	2044
3.562	$\int \frac{A+B \cos(d+ex)+C \sin(d+ex)}{a+c \sin(d+ex)} dx$	2049
3.563	$\int \frac{A+B \cos(d+ex)+C \sin(d+ex)}{(a+c \sin(d+ex))^2} dx$	2054
3.564	$\int \frac{A+B \cos(d+ex)+C \sin(d+ex)}{(a+c \sin(d+ex))^3} dx$	2058
3.565	$\int \frac{A+B \cos(d+ex)+C \sin(d+ex)}{(a+c \sin(d+ex))^4} dx$	2063
3.566	$\int (a+b \cos(c+dx) \sin(c+dx))^m dx$	2069
3.567	$\int (a+b \cos(c+dx) \sin(c+dx))^3 dx$	2072
3.568	$\int (a+b \cos(c+dx) \sin(c+dx))^2 dx$	2075
3.569	$\int (a+b \cos(c+dx) \sin(c+dx)) dx$	2078
3.570	$\int \frac{1}{a+b \cos(c+dx) \sin(c+dx)} dx$	2080
3.571	$\int \frac{1}{(a+b \cos(c+dx) \sin(c+dx))^2} dx$	2083
3.572	$\int \frac{1}{(a+b \cos(c+dx) \sin(c+dx))^3} dx$	2087
3.573	$\int (a+b \cos(c+dx) \sin(c+dx))^{5/2} dx$	2091
3.574	$\int (a+b \cos(c+dx) \sin(c+dx))^{3/2} dx$	2095
3.575	$\int \sqrt{a+b \cos(c+dx) \sin(c+dx)} dx$	2099
3.576	$\int \frac{1}{\sqrt{a+b \cos(c+dx) \sin(c+dx)}} dx$	2102
3.577	$\int \frac{1}{(a+b \cos(c+dx) \sin(c+dx))^{3/2}} dx$	2105
3.578	$\int \frac{1}{(a+b \cos(c+dx) \sin(c+dx))^{5/2}} dx$	2108
3.579	$\int \frac{x^3}{a+b \cos(x) \sin(x)} dx$	2113
3.580	$\int \frac{x^2}{a+b \cos(x) \sin(x)} dx$	2119
3.581	$\int \frac{x}{a+b \cos(x) \sin(x)} dx$	2124
3.582	$\int \frac{1}{x(a+b \cos(x) \sin(x))} dx$	2129
3.583	$\int \frac{(bx)^{2-n} \sin^n(ax)}{(acx \cos(ax)-c \sin(ax))^2} dx$	2131
3.584	$\int \frac{(bx)^{2-n} \cos^n(ax)}{(c \cos(ax)+acx \sin(ax))^2} dx$	2133
3.585	$\int \frac{\sin^6(ax)}{x^4(ax \cos(ax)-\sin(ax))^2} dx$	2135
3.586	$\int \frac{\sin^5(ax)}{x^3(ax \cos(ax)-\sin(ax))^2} dx$	2142
3.587	$\int \frac{\sin^4(ax)}{x^2(ax \cos(ax)-\sin(ax))^2} dx$	2147
3.588	$\int \frac{\sin^3(ax)}{x(ax \cos(ax)-\sin(ax))^2} dx$	2151
3.589	$\int \frac{\sin^2(ax)}{(ax \cos(ax)-\sin(ax))^2} dx$	2154
3.590	$\int \frac{x \sin(ax)}{(ax \cos(ax)-\sin(ax))^2} dx$	2156
3.591	$\int \frac{x^2}{(ax \cos(ax)-\sin(ax))^2} dx$	2158
3.592	$\int \frac{x^3 \csc(ax)}{(ax \cos(ax)-\sin(ax))^2} dx$	2161
3.593	$\int \frac{x^4 \csc^2(ax)}{(ax \cos(ax)-\sin(ax))^2} dx$	2164
3.594	$\int \frac{\cos^6(ax)}{x^4(\cos(ax)+ax \sin(ax))^2} dx$	2168
3.595	$\int \frac{\cos^5(ax)}{x^3(\cos(ax)+ax \sin(ax))^2} dx$	2175
3.596	$\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax \sin(ax))^2} dx$	2180
3.597	$\int \frac{\cos^3(ax)}{x(\cos(ax)+ax \sin(ax))^2} dx$	2184
3.598	$\int \frac{\cos^2(ax)}{(\cos(ax)+ax \sin(ax))^2} dx$	2187
3.599	$\int \frac{x \cos(ax)}{(\cos(ax)+ax \sin(ax))^2} dx$	2189

3.600	$\int \frac{x^2}{(\cos(ax)+ax \sin(ax))^2} dx$	2191
3.601	$\int \frac{x^3 \sec(ax)}{(\cos(ax)+ax \sin(ax))^2} dx$	2194
3.602	$\int \frac{x^4 \sec^2(ax)}{(\cos(ax)+ax \sin(ax))^2} dx$	2197
3.603	$\int \sec^4(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx$	2201
3.604	$\int \sec^3(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx$	2204
3.605	$\int \sec^2(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx$	2207
3.606	$\int \sec(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx$	2210
3.607	$\int \sqrt{c \tan(a+bx) \tan(2(a+bx))} dx$	2213
3.608	$\int \cos(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx$	2216
3.609	$\int \cos^2(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx$	2220
3.610	$\int \cos^3(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx$	2224
3.611	$\int \sec^4(2(a+bx))(c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx$	2229
3.612	$\int \sec^3(2(a+bx))(c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx$	2233
3.613	$\int \sec^2(2(a+bx))(c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx$	2236
3.614	$\int \sec(2(a+bx))(c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx$	2239
3.615	$\int (c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx$	2242
3.616	$\int \cos(2(a+bx))(c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx$	2246
3.617	$\int \cos^2(2(a+bx))(c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx$	2250
3.618	$\int \cos^3(2(a+bx))(c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx$	2254
3.619	$\int \frac{\sec^4(2(a+bx))}{\sqrt{c \tan(a+bx) \tan(2(a+bx))}} dx$	2258
3.620	$\int \frac{\sec^3(2(a+bx))}{\sqrt{c \tan(a+bx) \tan(2(a+bx))}} dx$	2262
3.621	$\int \frac{\sec^2(2(a+bx))}{\sqrt{c \tan(a+bx) \tan(2(a+bx))}} dx$	2266
3.622	$\int \frac{\sec(2(a+bx))}{\sqrt{c \tan(a+bx) \tan(2(a+bx))}} dx$	2269
3.623	$\int \frac{1}{\sqrt{c \tan(a+bx) \tan(2(a+bx))}} dx$	2272
3.624	$\int \frac{\cos(2(a+bx))}{\sqrt{c \tan(a+bx) \tan(2(a+bx))}} dx$	2275
3.625	$\int \frac{\cos^2(2(a+bx))}{\sqrt{c \tan(a+bx) \tan(2(a+bx))}} dx$	2279
3.626	$\int \frac{\sec^4(2(a+bx))}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx$	2284
3.627	$\int \frac{\sec^3(2(a+bx))}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx$	2288
3.628	$\int \frac{\sec^2(2(a+bx))}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx$	2292
3.629	$\int \frac{\sec(2(a+bx))}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx$	2295
3.630	$\int \frac{1}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx$	2298
3.631	$\int \frac{\cos(2(a+bx))}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx$	2302
3.632	$\int \frac{\cos^2(2(a+bx))}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx$	2306
3.633	$\int \frac{\cot(x) \csc(x)}{\sqrt{\sin(2x)}} dx$	2311
3.634	$\int \frac{\csc^2(x) \sec(x)}{\sqrt{\sin(2x)}(-2+\tan(x))} dx$	2314
3.635	$\int \frac{\cos^2(x) \sin(x)}{(\sin^2(x)-\sin(2x)) \sin^2(2x)} dx$	2317
3.636	$\int \frac{\cos^3(x) \cos(2x)}{(\sin^2(x)-\sin(2x)) \sin^2(2x)} dx$	2321
3.637	$\int (b \sec(c+dx) + a \sin(c+dx))^n (a \cos(c+dx) + b \sec(c+dx) \tan(c+dx)) dx$	2325
3.638	$\int (b \sec(c+dx) + a \sin(c+dx))^3 (a \cos(c+dx) + b \sec(c+dx) \tan(c+dx)) dx$	2328
3.639	$\int (b \sec(c+dx) + a \sin(c+dx))^2 (a \cos(c+dx) + b \sec(c+dx) \tan(c+dx)) dx$	2331
3.640	$\int (b \sec(c+dx) + a \sin(c+dx)) (a \cos(c+dx) + b \sec(c+dx) \tan(c+dx)) dx$	2333

3.641	$\int \frac{a \cos(c+dx)+b \sec(c+dx) \tan(c+dx)}{b \sec(c+dx)+a \sin(c+dx)} dx$	2335
3.642	$\int \frac{a \cos(c+dx)+b \sec(c+dx) \tan(c+dx)}{(b \sec(c+dx)+a \sin(c+dx))^2} dx$	2338
3.643	$\int \frac{a \cos(c+dx)+b \sec(c+dx) \tan(c+dx)}{(b \sec(c+dx)+a \sin(c+dx))^3} dx$	2341
3.644	$\int F(c, d, \cos(a + bx), r, s) \sin(a + bx) dx$	2344
3.645	$\int \cos(a + bx)F(c, d, \sin(a + bx), r, s) dx$	2346
3.646	$\int F(c, d, \tan(a + bx), r, s) \sec^2(a + bx) dx$	2348
3.647	$\int \csc^2(a + bx)F(c, d, \cot(a + bx), r, s) dx$	2350
3.648	$\int \frac{\sin(x)}{a+b \cos(x)} dx$	2352
3.649	$\int (a + b \cos(x))^n \sin(x) dx$	2354
3.650	$\int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}} dx$	2356
3.651	$\int \cos(\cos(x)) \sin(x) dx$	2358
3.652	$\int \cos(x) \cos(\cos(x)) \sin(x) \sin(\cos(x)) dx$	2360
3.653	$\int \cos(\cos(x)) \sin(x) \sin^2(6 \cos(x)) dx$	2363
3.654	$\int \cos^3(x) (a + b \cos^2(x))^3 \sin(x) dx$	2366
3.655	$\int \sin(3x) \sin(\cos(3x)) dx$	2369
3.656	$\int e^{\cos(1+3x)} \cos(1 + 3x) \sin(1 + 3x) dx$	2371
3.657	$\int \frac{\cos^2(x) \sin(x)}{\sqrt{1-\cos^6(x)}} dx$	2373
3.658	$\int \frac{\sin^5(x)}{\sqrt{1-5 \cos(x)}} dx$	2376
3.659	$\int e^{n \cos(a+bx)} \sin(a + bx) dx$	2379
3.660	$\int e^{n \cos(ac+bcx)} \sin(c(a + bx)) dx$	2381
3.661	$\int e^{n \cos(c(a+bx))} \sin(ac + bcx) dx$	2383
3.662	$\int e^{n \cos(a+bx)} \tan(a + bx) dx$	2385
3.663	$\int e^{n \cos(ac+bcx)} \tan(c(a + bx)) dx$	2387
3.664	$\int e^{n \cos(c(a+bx))} \tan(ac + bcx) dx$	2389
3.665	$\int \frac{\cos(x)}{a+b \sin(x)} dx$	2391
3.666	$\int \cos(x)(a + b \sin(x))^n dx$	2393
3.667	$\int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}} dx$	2395
3.668	$\int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}} dx$	2397
3.669	$\int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}} dx$	2399
3.670	$\int \cos(x) \sqrt{1 + \csc(x)} dx$	2401
3.671	$\int \cos(x) \sqrt{4 - \sin^2(x)} dx$	2404
3.672	$\int \cos(x) \sin(x) \sqrt{1 + \sin^2(x)} dx$	2407
3.673	$\int \frac{\cos(x)}{\sqrt{2 \sin(x)+\sin^2(x)}} dx$	2409
3.674	$\int \cos(x) \cos(\sin(x)) dx$	2412
3.675	$\int \cos(x) \cos(\sin(x)) \cos(\sin(\sin(x))) dx$	2414
3.676	$\int \cos(x) \sec(\sin(x)) dx$	2416
3.677	$\int \cos(x) \sin^3(x) (a + b \sin^2(x))^3 dx$	2418
3.678	$\int e^{\sin(x)} \cos(x) \sin(x) dx$	2421
3.679	$\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}} dx$	2423
3.680	$\int \frac{e^{\sqrt{\sin(x)}} \cos(x)}{\sqrt{\sin(x)}} dx$	2426
3.681	$\int e^{4+\sin(x)} \cos(x) dx$	2428
3.682	$\int e^{\cos(x) \sin(x)} \cos(2x) dx$	2430

3.683	$\int e^{\cos(\frac{x}{2})\sin(\frac{x}{2})} \cos(x) dx$	2432
3.684	$\int e^{n \sin(a+bx)} \cos(a+bx) dx$	2434
3.685	$\int e^{n \sin(ac+bcx)} \cos(c(a+bx)) dx$	2436
3.686	$\int e^{n \sin(c(a+bx))} \cos(ac+bcx) dx$	2438
3.687	$\int e^{n \sin(a+bx)} \cot(a+bx) dx$	2440
3.688	$\int e^{n \sin(ac+bcx)} \cot(c(a+bx)) dx$	2442
3.689	$\int e^{n \sin(c(a+bx))} \cot(ac+bcx) dx$	2444
3.690	$\int \frac{\sec^2(x)}{a+b \tan(x)} dx$	2446
3.691	$\int \frac{\sec^2(x)}{1-\tan^2(x)} dx$	2448
3.692	$\int \frac{\sec^2(x)}{9+\tan^2(x)} dx$	2450
3.693	$\int \sec^2(x)(a+b \tan(x))^n dx$	2452
3.694	$\int \sec^2(x) \left(1 + \frac{1}{1+\tan^2(x)}\right) dx$	2454
3.695	$\int \frac{\sec^2(x)(2+\tan^2(x))}{1+\tan^2(x)} dx$	2456
3.696	$\int \frac{\sec^2(x)}{2+2 \tan(x)+\tan^2(x)} dx$	2458
3.697	$\int \frac{\sec^2(x)}{\tan^2(x)+\tan^3(x)} dx$	2460
3.698	$\int \frac{\sec^2(x)}{-\tan^2(x)+\tan^3(x)} dx$	2462
3.699	$\int \frac{\sec^2(x)}{3-4 \tan^3(x)} dx$	2464
3.700	$\int \frac{\sec^2(x)}{11-5 \tan(x)+5 \tan^2(x)} dx$	2468
3.701	$\int \frac{\sec^2(x)(a+b \tan(x))}{c+d \tan(x)} dx$	2471
3.702	$\int \frac{\sec^2(x)(a+b \tan(x))^2}{c+d \tan(x)} dx$	2474
3.703	$\int \frac{\sec^2(x)(a+b \tan(x))^3}{c+d \tan(x)} dx$	2477
3.704	$\int \frac{\sec^2(x) \tan^2(x)}{(2+\tan^3(x))^2} dx$	2480
3.705	$\int \sec^2(x) \tan^6(x) (1+\tan^2(x))^3 dx$	2482
3.706	$\int \frac{\sec^2(x)(2+\tan^2(x))}{1+\tan^3(x)} dx$	2485
3.707	$\int (1+\cos^2(x)) \sec^2(x) dx$	2488
3.708	$\int \frac{\sec^2(x)}{1+\sec^2(x)-3 \tan(x)} dx$	2490
3.709	$\int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}} dx$	2492
3.710	$\int \frac{\sec^2(x)}{\sqrt{1-4 \tan^2(x)}} dx$	2495
3.711	$\int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}} dx$	2498
3.712	$\int \sqrt{1-\cot^2(x)} \sec^2(x) dx$	2501
3.713	$\int \sec^2(x) \sqrt{1-\tan^2(x)} dx$	2504
3.714	$\int e^{\tan(x)} \sec^2(x) dx$	2507
3.715	$\int \sec^4(x) (-1+\sec^2(x))^2 \tan(x) dx$	2509
3.716	$\int \frac{\csc^2(x)}{a+b \cot(x)} dx$	2512
3.717	$\int (a+b \cot(x))^n \csc^2(x) dx$	2514
3.718	$\int \csc^2(x) (1+\sin^2(x)) dx$	2516
3.719	$\int \left(1 + \frac{1}{1+\cot^2(x)}\right) \csc^2(x) dx$	2518
3.720	$\int \frac{(a+b \cot(x)) \csc^2(x)}{c+d \cot(x)} dx$	2520

3.721	$\int \frac{(a+b \cot(x))^2 \csc^2(x)}{c+d \cot(x)} dx$	2523
3.722	$\int \frac{(a+b \cot(x))^3 \csc^2(x)}{c+d \cot(x)} dx$	2526
3.723	$\int e^{-\cot(x)} \csc^2(x) dx$	2529
3.724	$\int \frac{\sec(x) \tan(x)}{a+b \sec(x)} dx$	2531
3.725	$\int \frac{\sec(x) \tan(x)}{1+\sec^2(x)} dx$	2534
3.726	$\int \frac{\sec(x) \tan(x)}{9+4 \sec^2(x)} dx$	2536
3.727	$\int \frac{\sec(x) \tan(x)}{\sec(x)+\sec^2(x)} dx$	2538
3.728	$\int \frac{\sec(x) \tan(x)}{\sqrt{4+\sec^2(x)}} dx$	2540
3.729	$\int \frac{\sec(x) \tan(x)}{\sqrt{1+\cos^2(x)}} dx$	2543
3.730	$\int e^{\sec(x)} \sec(x) \tan(x) dx$	2545
3.731	$\int 2^{\sec(x)} \sec(x) \tan(x) dx$	2547
3.732	$\int \frac{\sec(2x) \tan(2x)}{(1+\sec(2x))^{3/2}} dx$	2549
3.733	$\int \sqrt{1+5 \cos^2(3x)} \sec(3x) \tan(3x) dx$	2551
3.734	$\int \frac{\sec(3x) \tan(3x)}{\sqrt{1+5 \cos^2(3x)}} dx$	2554
3.735	$\int \frac{\cot(x) \csc(x)}{a+b \csc(x)} dx$	2556
3.736	$\int 5^{\csc(3x)} \cot(3x) \csc(3x) dx$	2559
3.737	$\int \frac{\cot(x) \csc(x)}{1+\csc^2(x)} dx$	2561
3.738	$\int \frac{\cot(6x) \csc(6x)}{(5-11 \csc^2(6x))^2} dx$	2563
3.739	$\int \frac{\cot(x) \csc(x)}{\sqrt{1+\sin^2(x)}} dx$	2566
3.740	$\int \frac{\cot(5x) \csc^3(5x)}{\sqrt{1+\sin^2(5x)}} dx$	2568
3.741	$\int e^{n \sin(a+bx)} \sin(2a+2bx) dx$	2571
3.742	$\int e^{n \sin(a+bx)} \sin(2(a+bx)) dx$	2573
3.743	$\int e^{n \sin\left(\frac{a}{2}+\frac{bx}{2}\right)} \sin(a+bx) dx$	2575
3.744	$\int e^{n \sin\left(\frac{1}{2}(a+bx)\right)} \sin(a+bx) dx$	2578
3.745	$\int e^{n \cos(a+bx)} \sin(2a+2bx) dx$	2581
3.746	$\int e^{n \cos(a+bx)} \sin(2(a+bx)) dx$	2583
3.747	$\int e^{n \cos\left(\frac{a}{2}+\frac{bx}{2}\right)} \sin(a+bx) dx$	2585
3.748	$\int e^{n \cos\left(\frac{1}{2}(a+bx)\right)} \sin(a+bx) dx$	2588
3.749	$\int \csc(x) \log(\tan(x)) \sec(x) dx$	2591
3.750	$\int \csc(2x) \log(\tan(x)) dx$	2593
3.751	$\int e^{\cos^2(x)+\sin^2(x)} dx$	2595
3.752	$\int x \sec^2(x) dx$	2597
3.753	$\int x \cos^4(x^2) dx$	2599
3.754	$\int \sqrt{\cos(x)} \sin(x) dx$	2602
3.755	$\int e^{-2x} \tan(e^{-2x}) dx$	2604
3.756	$\int \frac{\sec(x) \sin(2x)}{1+\cos(x)} dx$	2606
3.757	$\int x \sec^2(3x) dx$	2608
3.758	$\int e^{-2\pi x} \cos(2\pi x) dx$	2610
3.759	$\int (\cos^{12}(x) \sin^{10}(x) - \cos^{10}(x) \sin^{12}(x)) dx$	2612
3.760	$\int x \cot(x^2) dx$	2615
3.761	$\int x \sec^2(x^2) dx$	2617

3.762	$\int \frac{\sin(8x)}{9+\sin^4(4x)} dx$	2619
3.763	$\int \frac{\cos(2x)}{8+\sin^2(2x)} dx$	2621
3.764	$\int x (\cos^3(x^2) - \sin^3(x^2)) dx$	2623
3.765	$\int \frac{\cos(x)\sin(x)}{1-\cos(x)} dx$	2626
3.766	$\int x \cos(x^2) dx$	2628
3.767	$\int x^2 \cos(4x^3) dx$	2630
3.768	$\int x^3 \cos(x^4) dx$	2632
3.769	$\int x \sin\left(\frac{x^2}{2}\right) dx$	2634
3.770	$\int x \sec(x^2) \tan(x^2) dx$	2636
3.771	$\int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2} dx$	2638
3.772	$\int x \tan(1+x^2) dx$	2641
3.773	$\int \sin(\pi(1+2x)) dx$	2643
3.774	$\int \frac{\cot(x)+\csc^2(x)}{1-\cos^2(x)} dx$	2645
3.775	$\int x^2 \cos(4x^3) \cos(5x^3) dx$	2647
3.776	$\int x^{14} \sin(x^3) dx$	2650
3.777	$\int e^{-3x^3} x^2 \sin(2x^3) dx$	2653
3.778	$\int 2x \cos(x^2) dx$	2655
3.779	$\int 3x^2 \cos(7+x^3) dx$	2657
3.780	$\int \left(\frac{1}{1+x^2} + \sin(x)\right) dx$	2659
3.781	$\int x \sin(1+x^2) dx$	2661
3.782	$\int x \cos(1+x^2) dx$	2663
3.783	$\int (1+x^2 \cos(x^3)) dx$	2665
3.784	$\int x^2 \sin(1+x^3) dx$	2667
3.785	$\int 12x^2 \cos(x^3) dx$	2669
3.786	$\int (1+x) \sin(1+x) dx$	2671
3.787	$\int x^5 \cos(x^3) dx$	2673
3.788	$\int e^{-3x} \cos(x) dx$	2675
3.789	$\int x^3 \sin(x^2) dx$	2677
3.790	$\int x^3 \cos(x^2) dx$	2679
3.791	$\int \cos(x) \cos(2 \sin(x)) dx$	2681
3.792	$\int \frac{\cos(x)\sin(x)}{1+\cos^2(x)} dx$	2683
3.793	$\int (1+\cos(x))(x+\sin(x))^3 dx$	2685
3.794	$\int (1+\cos(x)) \csc^2(x) dx$	2687
3.795	$\int \sin(x) \tan^2(x) dx$	2689
3.796	$\int e^{\sin(x)} \sec^2(x) (x \cos^3(x) - \sin(x)) dx$	2691
3.797	$\int x \csc^2(x) dx$	2693
3.798	$\int \cos(x) \sin\left(\frac{\pi}{6} + x\right) dx$	2695
3.799	$\int x \sin^3(x^2) dx$	2697
3.800	$\int \sin^2(x) \tan(x) dx$	2699
3.801	$\int \cos^2(x) \cot^3(x) dx$	2701
3.802	$\int \sec(x)(1-\sin(x)) dx$	2704
3.803	$\int (1+\cos(x)) \csc(x) dx$	2706
3.804	$\int \cos^2(x) (1-\tan^2(x)) dx$	2708
3.805	$\int \csc(2x)(\cos(x) + \sin(x)) dx$	2710
3.806	$\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)} dx$	2713

3.807	$\int \frac{\cos^2(x) \sin(x)}{5 + \cos^2(x)} dx$	2715
3.808	$\int \frac{\cos(x)}{\sin(x) + \sin^2(x)} dx$	2718
3.809	$\int \frac{\cos(x)}{\sin(x) + \sin^2(x)} dx$	2720
3.810	$\int \frac{1}{2 \sin(x) + \sin(2x)} dx$	2724
3.811	$\int (-3 + 4x + x^2) \sin(2x) dx$	2726
3.812	$\int e^{-3x} \cos(4x) dx$	2729
3.813	$\int \frac{\cos(x) \sin(x)}{\sqrt{1 + \sin(x)}} dx$	2731
3.814	$\int (x + 60 \cos^5(x) \sin^4(x)) dx$	2733
3.815	$\int \cos(x)(\sec(x) + \tan(x)) dx$	2735
3.816	$\int \cos(x) (\sec^3(x) + \tan(x)) dx$	2737
3.817	$\int \frac{1}{2} (-\cot(x) \csc(x) + \csc^2(x)) dx$	2739
3.818	$\int (-\csc^2(x) + \sin(2x)) dx$	2742
3.819	$\int (2 \cot(2x) - 3 \sin(3x)) dx$	2744
3.820	$\int x \sin(2x^2) dx$	2746
3.821	$\int -\cos(1-x) \sin(1-x) \sqrt{1 + \sin^2(1-x)} dx$	2748
3.822	$\int \frac{\cos(\frac{1}{x}) \sin(\frac{1}{x})}{x^2} dx$	2750
3.823	$\int \cos\left(\frac{1}{2}(1+3x)\right) \sin^3\left(\frac{1}{2}(1+3x)\right) dx$	2752
3.824	$\int 4x \tan(x^2) dx$	2754
3.825	$\int x \sec(5-x^2) dx$	2756
3.826	$\int \frac{\csc(\frac{1}{x})}{x^2} dx$	2758
3.827	$\int (\csc(x) - \sec(x))(\cos(x) + \sin(x)) dx$	2760
3.828	$\int (-\cos(3x) \sin(2x) + \cos(2x) \sin(3x)) dx$	2762
3.829	$\int 4x \sec^2(2x) dx$	2764
3.830	$\int 4 \sin^2(x) \tan^2(x) dx$	2766
3.831	$\int \cos^4(x) \cot^2(x) dx$	2769
3.832	$\int 16 \cos^2(x) \sin^2(x) dx$	2772
3.833	$\int 8 \cos^2(x) \sin^4(x) dx$	2774
3.834	$\int 35 \cos^3(x) \sin^4(x) dx$	2777
3.835	$\int 4 \cos^4(x) \sin^4(x) dx$	2779
3.836	$\int \frac{\cos(x)}{-\sin(x) + \sin^3(x)} dx$	2782
3.837	$\int (-1 + 2 \cos^2(x) + \cos(x) \sin(x)) dx$	2785
3.838	$\int (\cos^2(x) + \sin^2(x)) dx$	2787
3.839	$\int (-\cos^2(x) + \sin^2(x)) dx$	2789
3.840	$\int 2^{\sin(x)} \cos(x) dx$	2791
3.841	$\int (\tan^3(x) + \tan^5(x)) dx$	2793
3.842	$\int x \sec(x)(2 + x \tan(x)) dx$	2795
3.843	$\int \frac{\cot(\sqrt{x}) \csc(\sqrt{x})}{\sqrt{x}} dx$	2798
3.844	$\int \frac{\cos(\sqrt{x}) \sin(\sqrt{x})}{\sqrt{x}} dx$	2800
3.845	$\int \frac{\sec(\sqrt{x}) \tan(\sqrt{x})}{\sqrt{x}} dx$	2802
3.846	$\int \frac{\sin^2(x)}{a+b \sin(2x)} dx$	2804
3.847	$\int \frac{\cos^2(x)}{a+b \sin(2x)} dx$	2809
3.848	$\int \frac{\sin^2(x)}{a+b \cos(2x)} dx$	2813

3.849	$\int \frac{\cos^2(x)}{a+b \cos(2x)} dx$	2816
3.850	$\int \frac{\tan(c+dx)}{\sqrt{a \sin^2(c+dx)}} dx$	2820
3.851	$\int \frac{\cot(c+dx)}{\sqrt{a \cos^2(c+dx)}} dx$	2823
3.852	$\int \frac{x \cos(x^2)}{\sqrt{\sin(x^2)}} dx$	2826
3.853	$\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}} dx$	2828
3.854	$\int \frac{\cos^2(\log(x)) \sin^2(\log(x))}{x} dx$	2831
3.855	$\int \frac{\sin^3(x)}{\cos^3(x)+\sin^3(x)} dx$	2834
3.856	$\int \frac{\cos^3(x)}{\cos^3(x)+\sin^3(x)} dx$	2837
3.857	$\int \frac{\sec(x)}{-5+\cos^2(x)+4 \sin(x)} dx$	2840
3.858	$\int \frac{1}{\cos^2(x) \sqrt{3 \cos(x)+\sin(x)}} dx$	2843
3.859	$\int \frac{\csc(x) \sqrt{\cos(x)+\sin(x)}}{\cos^2(x)} dx$	2846
3.860	$\int \frac{\cos(x)+\sin(x)}{\sqrt{1+\sin(2x)}} dx$	2849
3.861	$\int \sec(x) \sqrt{\sec(x) + \tan(x)} dx$	2853
3.862	$\int \sec(x) \sqrt{4 + 3 \sec(x)} \tan(x) dx$	2856
3.863	$\int \sec(x) \sqrt{1 + \sec(x)} \tan^3(x) dx$	2858
3.864	$\int \cot^3(x) \csc(x) \sqrt{1 + \csc(x)} dx$	2861
3.865	$\int \sqrt{\csc(x)} (x \cos(x) - 4 \sec(x) \tan(x)) dx$	2864
3.866	$\int \cot(x) \sqrt{-1 + \csc^2(x)} (1 - \sin^2(x))^3 dx$	2867
3.867	$\int \cos(x) \sqrt{-1 + \csc^2(x)} (1 - \sin^2(x))^3 dx$	2871
3.868	$\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} dx$	2874
3.869	$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} dx$	2877
3.870	$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} dx$	2880
3.871	$\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx$	2884
3.872	$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx$	2887
3.873	$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx$	2891
3.874	$\int x \csc(x) \sec(x) \sqrt{a \sec^2(x)} dx$	2895
3.875	$\int x^2 \csc(x) \sec(x) \sqrt{a \sec^2(x)} dx$	2899
3.876	$\int x^3 \csc(x) \sec(x) \sqrt{a \sec^2(x)} dx$	2904
3.877	$\int x \csc(x) \sec(x) \sqrt{a \sec^4(x)} dx$	2909
3.878	$\int x^2 \csc(x) \sec(x) \sqrt{a \sec^4(x)} dx$	2913
3.879	$\int x^3 \csc(x) \sec(x) \sqrt{a \sec^4(x)} dx$	2918
3.880	$\int \sin(x) \sin(2x) \sin(3x) dx$	2924
3.881	$\int \cos(x) \cos(2x) \cos(3x) dx$	2926
3.882	$\int \cos(x) \sin(2x) \sin(3x) dx$	2928
3.883	$\int \cos(2x) \cos(3x) \sin(x) dx$	2930
3.884	$\int x \sin(x^2) dx$	2932
3.885	$\int (-\cos(x) + \sin(x))(\cos(x) + \sin(x))^5 dx$	2934
3.886	$\int 2x \sec^2(x) \tan(x) dx$	2936
3.887	$\int \frac{1+\cos^2(x)}{1+\cos(2x)} dx$	2939

3.888	$\int \frac{\sin(x)}{\cos^3(x) - \cos^5(x)} dx$	2941
3.889	$\int \sec(x) (5 - 11 \sec^5(x))^2 \tan(x) dx$	2944
3.890	$\int \sin^3(5x) \tan^3(5x) dx$	2946
3.891	$\int \sin^3(5x) \tan^4(5x) dx$	2949
3.892	$\int \sin^5(6x) \tan^3(6x) dx$	2951
3.893	$\int (-1 + \sec^2(2x))^3 \sin(2x) dx$	2954
3.894	$\int \sin(x) \tan^5(x) dx$	2956
3.895	$\int \cos^5(2x) \cot^4(2x) dx$	2959
3.896	$\int \cos(3x) (-1 + \csc^2(3x))^3 (1 - \sin^2(3x))^5 dx$	2961
3.897	$\int \cot(2x) (-1 + \csc^2(2x))^2 (1 - \sin^2(2x))^2 dx$	2964
3.898	$\int \cos(2x) (-1 + \csc^2(2x))^4 (1 - \sin^2(2x))^2 dx$	2967
3.899	$\int \cot(3x) (-1 + \csc^2(3x))^3 (1 - \sin^2(3x))^2 dx$	2970
3.900	$\int (1 + \cot^2(9x))^2 (1 + \tan^2(9x))^3 dx$	2973
3.901	$\int \frac{\cos(x)(9 - 7 \sin^3(x))^2}{1 - \sin^2(x)} dx$	2976
3.902	$\int \cos^4(2x) \cot^5(2x) dx$	2979
3.903	$\int \frac{\sec(x) \tan^2(x)}{4 + 3 \sec(x)} dx$	2982
3.904	$\int x \sec(1 + x) \tan(1 + x) dx$	2985
3.905	$\int \frac{\sin(2x)}{\sqrt{9 - \sin^2(x)}} dx$	2988
3.906	$\int \frac{\sin(2x)}{\sqrt{9 - \cos^4(x)}} dx$	2990
3.907	$\int \frac{\cos\left(\frac{1}{x}\right)}{x^5} dx$	2993
3.908	$\int \cos^3(1 + x) \sin^3(1 + x) dx$	2996
3.909	$\int (1 + 2x)^3 \sin^2(1 + 2x) dx$	2998
3.910	$\int \frac{-1 + \sec(x)}{1 - \tan(x)} dx$	3001
3.911	$\int x^2 \cos(3x) \cos(5x) dx$	3004
3.912	$\int \frac{\cos(x) + \sin(x)}{\sqrt{\cos(x)} \sqrt{\sin(x)}} dx$	3007
3.913	$\int \sec^2(x) (1 + \sin(x)) dx$	3011
3.914	$\int (10x^9 \cos(x^5 \log(x)) - x^{10} (x^4 + 5x^4 \log(x)) \sin(x^5 \log(x))) dx$	3013
3.915	$\int \cos^2\left(\frac{x}{2}\right) \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$	3015
3.916	$\int (2 + 3x)^2 \sin^3(x) dx$	3017
3.917	$\int \sec^{1+m}(x) \sin(x) dx$	3020
3.918	$\int \cos^n(a + bx) \sin^{-2-n}(a + bx) dx$	3022
3.919	$\int \frac{1}{\sec(x) + \sin(x) \tan(x)} dx$	3024
3.920	$\int (a + bx + cx^2) \sin(x) dx$	3026
3.921	$\int \frac{\sin(x^5)}{x} dx$	3028
3.922	$\int \frac{\sin(2^x)}{1 + 2^x} dx$	3030
3.923	$\int x \cos(2x^2) \sin^{\frac{3}{4}}(2x^2) dx$	3033
3.924	$\int x \sec^2(x^2) \tan^2(x^2) dx$	3035
3.925	$\int x^2 \cos^7(a + bx^3) \sin(a + bx^3) dx$	3037
3.926	$\int x^5 \cos^7(a + bx^3) \sin(a + bx^3) dx$	3039
3.927	$\int x^5 \sec^7(a + bx^3) \tan(a + bx^3) dx$	3043
3.928	$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2} dx$	3049
3.929	$\int 3x^2 \cos(x^3) dx$	3052

3.930	$\int (1 + 2x) \sec^2(1 + 2x) dx$	3054
3.931	$\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}} + \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}} + \frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b} \right) dx$	3057
3.932	$\int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}} dx$	3059
3.933	$\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)} dx$	3061
3.934	$\int \sin(c + dx) \left(a \sin^2(c + dx) + b \sin^3(c + dx) \right) dx$	3063
3.935	$\int \sin(c + dx) \left(a \sin^2(c + dx) + b \sin^3(c + dx) \right)^2 dx$	3066
3.936	$\int \sin(c + dx) \left(a \sin(c + dx) + b \sin^2(c + dx) + c \sin^3(c + dx) \right) dx$	3070
3.937	$\int \sin(c + dx) \left(a \sin(c + dx) + b \sin^2(c + dx) + c \sin^3(c + dx) \right)^2 dx$	3073
3.938	$\int \sin(c + dx) \left(a + \frac{b}{\sqrt{\sin(c+dx)}} + c \sin(c + dx) \right) dx$	3077
3.939	$\int \sin(c + dx) \left(a + \frac{b}{\sqrt{\sin(c+dx)}} + c \sin(c + dx) \right)^2 dx$	3080
3.940	$\int f^{a+bx} (\cos(c + dx) + i \sin(c + dx))^n dx$	3084
3.941	$\int f^{a+bx} (\cos(c + dx) - i \sin(c + dx))^n dx$	3087
3.942	$\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\cos^5(a+bx)+\sin^5(a+bx)} dx$	3090
3.943	$\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\cos^4(a+bx)+\sin^4(a+bx)} dx$	3093
3.944	$\int \frac{\cos^3(a+bx)-\sin^3(a+bx)}{\cos^3(a+bx)+\sin^3(a+bx)} dx$	3096
3.945	$\int \frac{\cos^2(a+bx)-\sin^2(a+bx)}{\cos^2(a+bx)+\sin^2(a+bx)} dx$	3099
3.946	$\int \frac{\cos(a+bx)-\sin(a+bx)}{\cos(a+bx)+\sin(a+bx)} dx$	3101
3.947	$\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)} dx$	3103
3.948	$\int \frac{-\csc^2(a+bx)+\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)} dx$	3106
3.949	$\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)} dx$	3108
3.950	$\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)} dx$	3111
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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [950]. This is test number [141].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric $2F1$ functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.37 (944)	% 0.63 (6)
Mathematica	% 98.74 (938)	% 1.26 (12)
Maple	% 95.58 (908)	% 4.42 (42)
Maxima	% 68.53 (651)	% 31.47 (299)
Fricas	% 89.58 (851)	% 10.42 (99)
Sympy	% 44.00 (418)	% 56.00 (532)
Giac	% 74.21 (705)	% 25.79 (245)
Mupad	% 73.68 (700)	% 26.32 (250)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

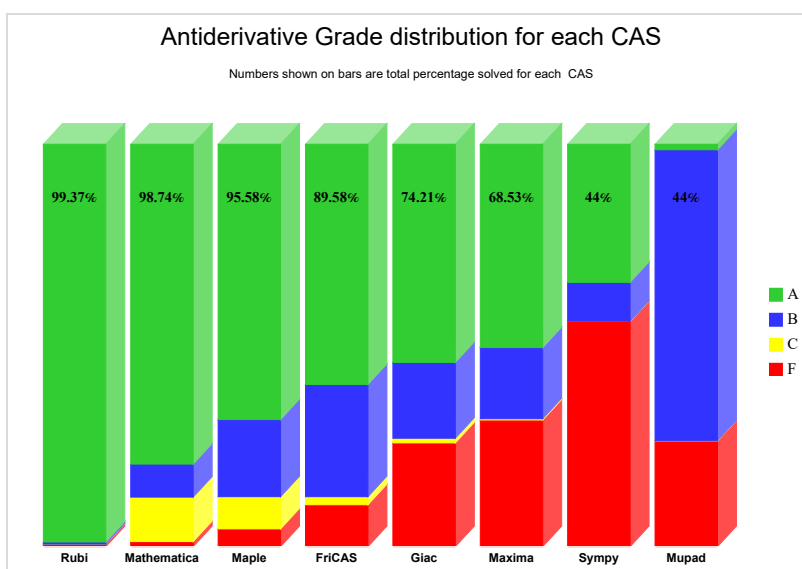
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

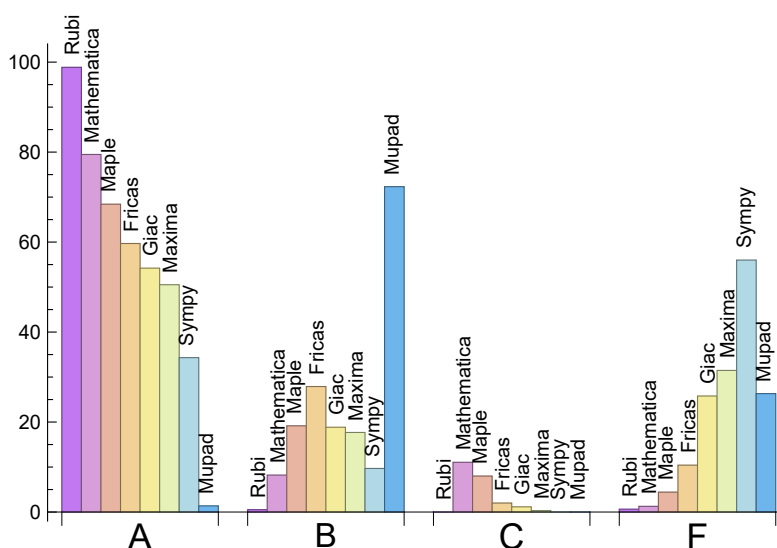
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.84	0.53	0.00	0.63
Mathematica	79.47	8.21	11.05	1.26
Maple	68.42	19.16	8.00	4.42
Maxima	50.53	17.68	0.32	31.47
Fricas	59.68	27.89	2.00	10.42
Sympy	34.32	9.68	0.00	56.00
Giac	54.21	18.84	1.16	25.79
Mupad	1.37	72.32	0.00	26.32

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure F .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	6	100.00 %	0.00 %	0.00 %
Mathematica	12	66.67 %	33.33 %	0.00 %
Maple	42	85.71 %	14.29 %	0.00 %
Maxima	299	62.54 %	5.02 %	32.44 %
Fricas	99	79.80 %	0.00 %	20.20 %
Sympy	532	65.04 %	34.59 %	0.38 %
Giac	245	71.43 %	15.10 %	13.47 %
Mupad	250	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

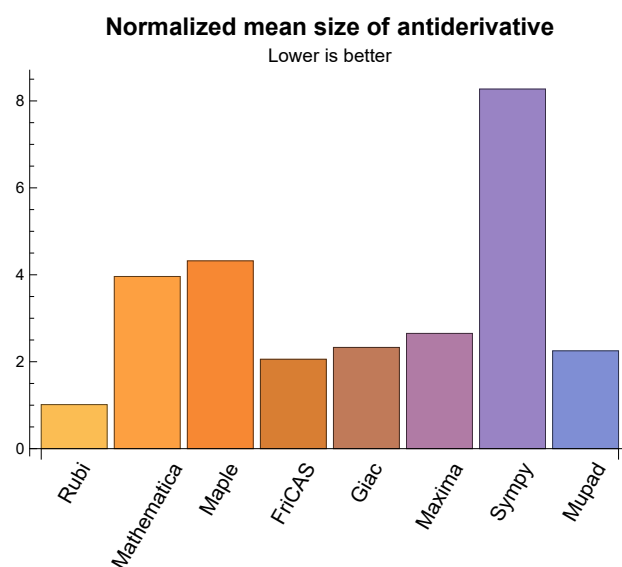
1.3 Performance

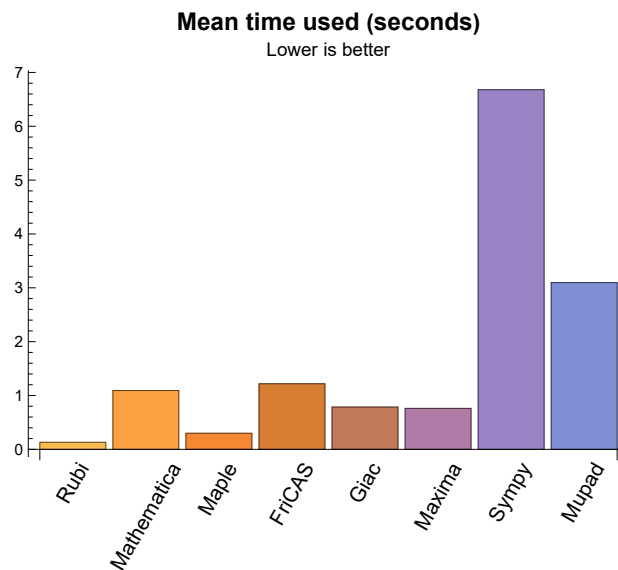
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	76.23	1.01	43.50	1.00
Mathematica	1.09	373.14	3.96	40.00	1.00
Maple	0.30	699.55	4.32	44.50	1.16
Maxima	0.76	113.26	2.65	29.00	1.00
Fricas	1.22	191.42	2.06	43.00	1.34
Sympy	6.68	233.72	8.27	35.00	1.37
Giac	0.79	154.70	2.33	37.00	1.10
Mupad	3.10	175.14	2.25	29.00	1.03

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{42, 43, 56, 57, 180, 582, 583, 584, 644, 645, 646, 647, 932}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {31, 32, 36, 37, 38, 46, 47, 51, 52, 87, 89, 93, 107, 108, 109, 160, 163, 240, 241, 242, 243, 244, 245, 246, 247, 271, 393, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 430, 431, 432, 433, 434, 437, 438, 439, 440, 448, 449, 450, 451, 452, 455, 462, 463, 464, 465, 466, 469, 497, 556, 557, 558, 559, 560, 561, 566, 588, 597, 630, 859}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

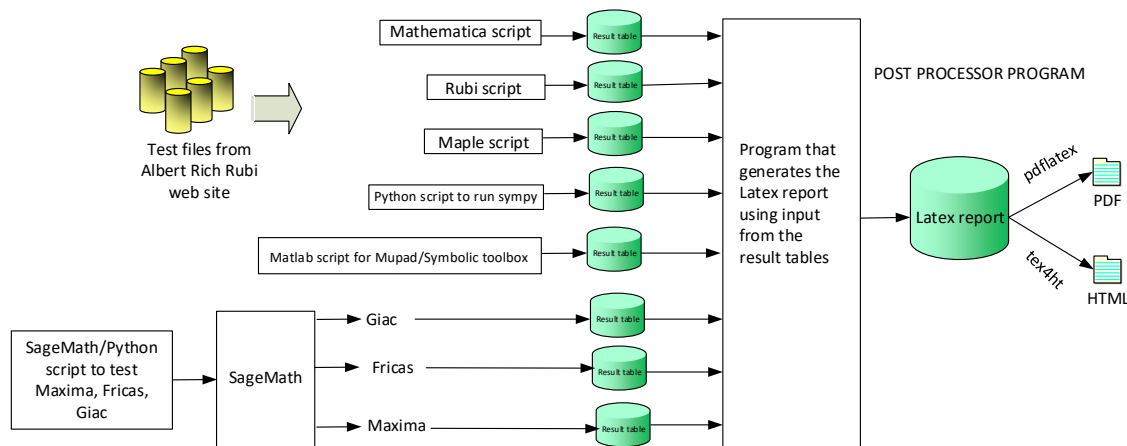
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
The following field present only in Rubi and Mathematica Tables
13. integer. 1 if result was verified or 0 if not verified.
The following fields present only in Rubi Tables
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870,

871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 913, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 932, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade: { 555, 759, 858, 860, 912 }

C grade: { }

F grade: { 796, 859, 914, 915, 931, 933 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 88, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 111, 113, 115, 116, 117, 118, 119, 120, 121, 122, 124, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 177, 178, 179, 180, 181, 183, 184, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 229, 230, 231, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 268, 270, 272, 273, 275, 276, 279, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 326, 327, 328, 330, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 344, 345, 346, 347, 348, 349, 350, 352, 353, 357, 358, 359, 360, 363, 364, 365, 367, 368, 371, 372, 373, 374, 375, 376, 377, 381, 382, 383, 385, 386, 388, 389, 390, 392, 393, 395, 396, 397, 398, 399, 400, 401, 417, 418, 419, 423, 424, 425, 426, 443, 444, 445, 446, 447, 458, 459, 460, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 504, 505, 506, 507, 508, 509, 515, 516, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 550, 551, 552, 553, 554, 555, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 582, 583, 584, 585, 586, 587, 589, 590, 591, 592, 593, 594, 595, 596, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 637, 639, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 674, 675, 676, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 782, 783, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 804, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 827, 828, 829, 830, 831, 832, 833, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 911, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 946, 947, 948, 949, 950 }

B grade: { 35, 90, 106, 108, 110, 114, 123, 127, 160, 163, 185, 189, 263, 267, 269, 271, 274, 277, 278, 280, 281, 287, 297, 306, 310, 312, 325, 329, 331, 341, 343, 361, 362, 366, 369, 370, 378, 380, 384, 387, 391, 394, 402, 461, 475, 497, 548, 549, 581, 638, 640, 654, 673, 677, 691, 705, 709, 710, 711, 712, 713, 728, 759, 760, 781, 784, 802, 803, 805, 806, 807, 826, 834, 861, 885, 904, 927, 945 }

C grade: { 31, 34, 36, 37, 38, 46, 51, 52, 62, 85, 87, 89, 105, 107, 109, 112, 125, 174, 175, 176, 182, 228, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 351, 354, 355, 356, 379, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 420, 421, 422, 427, 428, 429, 430, 431, 432, 433, 434, 437, 438, 439, 440, 448, 449, 450, 451, 452, 455, 462, 463, 464, 465, 466, 469, 503, 510, 511, 512, 513, 514, 517, 556, 557, 558, 559, 560, 561, 588, 597, 634, 635, 636, 657, 910, 912 }

F grade: { 435, 436, 441, 442, 453, 454, 456, 457, 467, 468, 470, 471 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 150, 151, 152, 159, 162, 180, 184, 185, 186, 187, 188, 189, 190, 191, 192, 201, 202, 203, 204, 206, 207, 208, 209, 211, 212, 213, 216, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 252, 253, 254, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 272, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 341, 342, 343, 344, 345, 346, 347, 348, 350, 351, 353, 354, 356, 357, 358, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 399, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 436, 437, 438, 442, 444, 446, 459, 460, 461, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 490, 491, 492, 496, 499, 500, 504, 505, 506, 510, 511, 514, 516, 518, 519, 520, 522, 523, 524, 526, 527, 528, 530, 531, 533, 534, 567, 568, 569, 570, 571, 576, 582, 583, 584, 590, 591, 593, 599, 600, 602, 603, 604, 605, 606, 611, 612, 613, 614, 637, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 723, 724, 725, 726, 727, 728, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 749, 750, 752, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 835, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 861, 862, 863, 866, 867, 868, 869, 870, 873, 874, 875, 876, 877, 879, 880, 881, 882, 883, 884, 886, 887, 889, 890, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 910, 911, 913, 915, 916, 917, 919, 920, 921, 922, 923, 924, 925, 928, 929, 930, 931, 932, 934, 935, 936, 937, 938, 939, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade: { 76, 78, 145, 146, 147, 148, 149, 160, 161, 163, 164, 193, 194, 195, 196, 197, 198, 199, 200, 205, 210, 214, 215, 219, 230, 249, 250, 251, 255, 271, 273, 274, 275, 291, 293, 294, 295, 333, 334, 335, 336, 337, 338, 339, 340, 349, 352, 355, 361, 362, 379, 380, 384, 385, 386, 387, 391, 392, 393, 394, 400, 401, 402, 410, 411, 412, 413, 414, 415, 416, 433, 434, 435, 439, 440, 441, 443, 445, 447, 458, 497, 498, 501, 502, 503, 507, 508, 509, 512, 513, 517, 521, 525, 529, 532, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 572, 573, 574, 575, 577, 578, 579, 580, 581, 589, 598, 607, 608, 609, 610, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 638, 639, 640, 670, 721, 722, 729, 759, 793, 795, 834, 836, 864, 871, 872, 885, 888, 891, 904, 926, 933, 940, 941 }

C grade: { 34, 139, 140, 141, 142, 153, 154, 155, 156, 157, 158, 319, 320, 321, 403, 404, 405, 406, 407, 408, 409, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 487, 488, 489, 493, 494, 495, 515, 588, 597, 633, 634, 635, 636, 709, 710, 711, 712, 713, 741, 742, 743, 744, 745, 746, 747, 748, 751, 796, 809, 859, 860, 878, 912, 914, 927 }

F grade: { 39, 40, 41, 53, 54, 55, 60, 79, 115, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 217, 218, 318, 566, 585, 586, 587, 592, 594, 595, 596, 601, 657, 865, 918 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 42, 44, 45, 48, 49, 50, 56, 58, 59, 60, 62, 66, 67, 68, 69, 70, 71, 72, 73, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 121, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 159, 162, 180, 184, 185, 186, 187, 188, 191, 192, 193, 194, 195, 196, 201, 202, 203, 204, 207, 208, 209, 212, 213, 214, }

220, 221, 222, 223, 224, 225, 226, 227, 229, 231, 252, 253, 254, 255, 256, 263, 264, 265, 266, 267, 274, 276, 277, 279, 283, 284, 285, 286, 287, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 310, 312, 322, 323, 324, 325, 327, 329, 331, 333, 341, 342, 343, 344, 345, 346, 347, 348, 350, 355, 356, 357, 358, 359, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 380, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 461, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 499, 500, 504, 505, 510, 511, 512, 514, 515, 516, 517, 518, 519, 522, 523, 528, 567, 568, 569, 582, 583, 584, 590, 599, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 723, 724, 725, 726, 727, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 745, 746, 749, 753, 754, 755, 756, 758, 759, 760, 763, 764, 765, 766, 767, 768, 769, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 798, 799, 800, 801, 802, 803, 806, 807, 808, 809, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 828, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 843, 844, 845, 852, 854, 857, 862, 863, 864, 867, 868, 869, 870, 871, 872, 873, 880, 881, 882, 883, 884, 885, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 908, 909, 910, 911, 913, 914, 916, 917, 920, 923, 924, 925, 926, 929, 932, 934, 935, 936, 937, 940, 941, 943, 945, 946, 948, 950 }

B grade: { 61, 64, 65, 74, 80, 81, 82, 85, 86, 90, 91, 92, 105, 110, 111, 112, 116, 117, 122, 123, 124, 125, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 157, 158, 189, 190, 219, 228, 230, 248, 249, 250, 251, 257, 258, 259, 260, 261, 262, 268, 270, 272, 273, 275, 278, 280, 281, 282, 288, 290, 292, 293, 294, 295, 307, 309, 311, 313, 314, 315, 316, 317, 326, 328, 330, 332, 334, 335, 336, 349, 351, 352, 353, 354, 377, 379, 384, 385, 386, 387, 391, 392, 393, 394, 444, 513, 529, 530, 531, 532, 533, 534, 589, 591, 593, 598, 600, 602, 607, 608, 609, 610, 615, 616, 654, 657, 670, 715, 722, 728, 750, 752, 757, 761, 770, 771, 796, 797, 804, 805, 810, 827, 829, 836, 841, 842, 850, 851, 853, 855, 856, 858, 859, 860, 866, 874, 876, 877, 878, 879, 886, 887, 888, 904, 915, 918, 919, 927, 928, 930, 933, 944, 947, 949 }

C grade: { 751, 907, 921 }

F grade: { 31, 32, 34, 36, 37, 38, 39, 40, 41, 43, 46, 47, 51, 52, 53, 54, 55, 57, 63, 75, 76, 77, 78, 79, 83, 84, 87, 88, 89, 93, 94, 106, 107, 108, 109, 113, 114, 115, 118, 119, 120, 126, 127, 153, 154, 155, 156, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 197, 198, 199, 200, 205, 206, 210, 211, 215, 216, 217, 218, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 269, 271, 289, 291, 318, 319, 320, 321, 337, 338, 339, 340, 360, 361, 362, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 497, 498, 501, 502, 503, 506, 507, 508, 509, 520, 521, 524, 525, 526, 527, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 585, 586, 587, 588, 592, 594, 595, 596, 597, 601, 603, 604, 605, 606, 611, 612, 613, 614, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 743, 744, 747, 748, 762, 846, 847, 848, 849, 861, 865, 875, 906, 912, 922, 931, 938, 939, 942 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 36, 37, 38, 42, 43, 44, 45, 48, 50, 51, 52, 56, 57, 58, 59, 61, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 86, 88, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 109, 111, 113, 117, 124, 126, 129, 130, 132, 134, 136, 138, 147, 148, 149, 150, 151, 152, 153, 155, 157, 158, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 220, 221, 222, 223, 224, 225, 227, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 268, 270, 273, 275, 276, 277, 278, 279, 280, 282, 284, 285, 286, 288, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 307, 309, 310, 311, 313, 314, 315, 316, 317, 321, 322, 323, 324, 326, 333, 334, 335, 336, 337, 341, 342, 344, 345, 346, 348, 355, 356, 357, 358, 359, 360, 363, 364, 365, 370, 371, 372, 373, 374, 375, 376, 381, 382, 383, 388, 389, 390, 395, 396, 397,

398, 417, 418, 419, 423, 424, 425, 426, 430, 431, 432, 433, 437, 438, 439, 444, 447, 461, 472, 473, 474, 476, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 499, 500, 501, 504, 505, 506, 507, 508, 510, 511, 512, 514, 515, 516, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 529, 532, 538, 539, 543, 544, 548, 549, 553, 554, 555, 562, 563, 567, 568, 569, 570, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 594, 595, 596, 597, 598, 599, 600, 603, 604, 605, 606, 607, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 635, 636, 637, 641, 642, 644, 645, 646, 647, 648, 649, 654, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 672, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 692, 693, 696, 700, 705, 706, 708, 715, 717, 723, 724, 725, 726, 727, 729, 730, 731, 734, 735, 736, 737, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 792, 794, 796, 799, 800, 802, 803, 804, 806, 807, 808, 809, 811, 812, 813, 814, 815, 817, 819, 820, 821, 822, 824, 827, 828, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 841, 842, 843, 844, 845, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 860, 861, 866, 867, 874, 880, 881, 882, 883, 884, 886, 887, 889, 890, 891, 892, 893, 894, 895, 896, 898, 900, 901, 903, 905, 907, 908, 909, 910, 911, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 925, 926, 927, 928, 929, 930, 933, 934, 935, 936, 937, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade: { 49, 62, 64, 65, 74, 80, 81, 82, 83, 84, 85, 87, 89, 90, 92, 93, 94, 105, 106, 108, 110, 112, 114, 116, 118, 119, 120, 121, 122, 123, 125, 127, 128, 131, 133, 135, 137, 139, 140, 141, 142, 143, 144, 145, 146, 154, 156, 159, 160, 162, 163, 195, 196, 197, 198, 214, 215, 216, 219, 226, 228, 229, 230, 231, 267, 269, 271, 272, 274, 281, 283, 287, 289, 291, 304, 306, 308, 312, 318, 319, 320, 325, 327, 328, 329, 330, 331, 332, 338, 339, 340, 343, 347, 349, 350, 351, 352, 353, 354, 361, 362, 366, 367, 368, 369, 377, 378, 379, 380, 384, 385, 386, 387, 391, 392, 393, 394, 399, 400, 401, 402, 420, 421, 422, 427, 428, 429, 434, 435, 436, 440, 441, 442, 443, 445, 446, 458, 459, 460, 475, 477, 478, 479, 480, 496, 497, 502, 503, 509, 513, 521, 530, 531, 533, 534, 535, 536, 537, 540, 541, 542, 545, 546, 547, 550, 551, 552, 564, 565, 571, 572, 581, 592, 593, 601, 602, 608, 616, 633, 634, 638, 639, 640, 643, 650, 651, 652, 653, 655, 657, 667, 668, 669, 670, 671, 673, 674, 675, 676, 677, 690, 691, 694, 695, 697, 698, 699, 701, 702, 703, 704, 707, 709, 710, 711, 712, 713, 714, 716, 718, 719, 720, 721, 722, 728, 732, 733, 738, 752, 759, 775, 791, 793, 795, 797, 798, 801, 805, 810, 816, 818, 823, 825, 826, 829, 836, 846, 847, 859, 862, 863, 864, 868, 871, 877, 885, 888, 897, 899, 902, 904, 906, 912, 913, 924 }

C grade: { 31, 32, 46, 47, 161, 164, 184, 498, 579, 580, 751, 869, 870, 872, 873, 875, 876, 878, 879 }

F grade: { 34, 39, 40, 41, 53, 54, 55, 60, 63, 79, 115, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 217, 218, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 556, 557, 558, 559, 560, 561, 566, 573, 574, 575, 576, 577, 578, 865, 931, 932, 938, 939 }

2.1.6 Sympy

A grade: { 1, 8, 15, 17, 18, 19, 22, 29, 30, 33, 35, 42, 43, 44, 45, 48, 49, 50, 56, 57, 58, 59, 62, 66, 67, 68, 69, 70, 71, 72, 73, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 124, 135, 136, 137, 138, 146, 180, 185, 186, 187, 188, 203, 204, 206, 219, 220, 221, 222, 223, 224, 225, 248, 249, 250, 251, 252, 253, 254, 255, 256, 264, 265, 266, 267, 268, 270, 272, 274, 276, 277, 284, 285, 286, 287, 294, 296, 303, 304, 305, 322, 323, 324, 341, 342, 343, 344, 349, 352, 355, 356, 357, 358, 363, 364, 365, 366, 370, 371, 372, 373, 374, 375, 376, 377, 381, 382, 383, 384, 388, 389, 390, 391, 395, 396, 397, 398, 399, 444, 481, 482, 483, 499, 500, 510, 511, 512, 526, 527, 528, 529, 532, 538, 539, 543, 544, 548, 549, 553, 554, 562, 567, 568, 569, 582, 589, 590, 598, 599, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 651, 652, 654, 655, 656, 659, 660, 661, 665, 666, 674, 675, 676, 677, 678, 679, 680, 681, 684, 685, 686, 701, 702, 703, 705, 706, 707, 714, 715, 718, 720, 721, 722, 723, 724, 725, 726, 728, 730, 731, 732, 735, 736, 737, 754, 755, 756, 758, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 795, 799, 800, 801, 806, 807, 808, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 823, 824, 825, 826, 827, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 842, 843, 844, 845, 846, 847, 852, 855, 856, 884, 887, 889, 890, 891, 892, 894, 895, 900, 901, 902, 905, 907, 908, 911, 913, 916, 920, 921, 923, 924, 925, 926, 929, 932, 933, 934, 935, 936, 937, 940, 941, 943, 944, 946 }

B grade: { 3, 4, 5, 10, 11, 12, 24, 25, 26, 80, 90, 92, 110, 121, 122, 123, 125, 128, 129, 130, 131, 132,

133, 134, 139, 140, 141, 142, 201, 202, 205, 207, 208, 210, 212, 213, 263, 273, 275, 278, 280, 282, 283, 293, 295, 297, 306, 325, 472, 473, 474, 475, 476, 477, 478, 591, 653, 672, 694, 695, 719, 727, 738, 751, 753, 759, 760, 775, 793, 798, 802, 803, 804, 805, 809, 821, 822, 828, 836, 841, 848, 849, 854, 862, 880, 881, 882, 883, 885, 888, 909, 945 }

C grade: { }

F grade: { 2, 6, 7, 9, 13, 14, 16, 20, 21, 23, 27, 28, 31, 32, 34, 36, 37, 38, 39, 40, 41, 46, 47, 51, 52, 53, 54, 55, 60, 61, 63, 64, 65, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 93, 94, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 209, 211, 214, 215, 216, 217, 218, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 257, 258, 259, 260, 261, 262, 269, 271, 279, 281, 288, 289, 290, 291, 292, 298, 299, 300, 301, 302, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 345, 346, 347, 348, 350, 351, 353, 354, 359, 360, 361, 362, 367, 368, 369, 378, 379, 380, 385, 386, 387, 392, 393, 394, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 479, 480, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 501, 502, 503, 504, 505, 506, 507, 508, 509, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 530, 531, 533, 534, 535, 536, 537, 540, 541, 542, 545, 546, 547, 550, 551, 552, 555, 556, 557, 558, 559, 560, 561, 563, 564, 565, 566, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 583, 584, 585, 586, 587, 588, 592, 593, 594, 595, 596, 597, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 650, 657, 658, 662, 663, 664, 667, 668, 669, 670, 671, 673, 682, 683, 687, 688, 689, 690, 691, 692, 693, 696, 697, 698, 699, 700, 704, 708, 709, 710, 711, 712, 713, 716, 717, 729, 733, 734, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 752, 757, 761, 762, 774, 796, 797, 810, 829, 850, 851, 853, 857, 858, 859, 860, 861, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 886, 893, 896, 897, 898, 899, 903, 904, 906, 910, 912, 914, 915, 917, 918, 919, 922, 927, 928, 930, 931, 938, 939, 942, 947, 948, 949, 950 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 42, 43, 44, 45, 48, 49, 50, 56, 57, 58, 59, 61, 62, 66, 67, 68, 69, 70, 71, 72, 73, 86, 88, 91, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 111, 113, 117, 119, 120, 124, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 150, 151, 152, 159, 180, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 220, 222, 223, 224, 225, 226, 227, 229, 231, 248, 252, 253, 254, 255, 256, 257, 258, 259, 260, 263, 264, 265, 266, 268, 269, 270, 272, 274, 276, 279, 281, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 306, 308, 310, 312, 322, 323, 324, 326, 327, 328, 329, 330, 331, 332, 344, 345, 346, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 361, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 399, 400, 443, 444, 445, 446, 447, 458, 459, 460, 461, 472, 473, 474, 476, 477, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 499, 500, 504, 505, 506, 520, 522, 523, 525, 526, 527, 529, 530, 531, 532, 533, 535, 536, 540, 541, 545, 546, 550, 551, 562, 563, 567, 568, 569, 570, 571, 572, 582, 583, 584, 589, 591, 598, 600, 640, 644, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 665, 666, 668, 669, 671, 672, 673, 674, 675, 677, 678, 679, 680, 681, 682, 684, 685, 686, 687, 690, 691, 692, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 708, 710, 711, 713, 714, 715, 716, 724, 725, 726, 727, 729, 730, 731, 734, 735, 736, 737, 738, 739, 740, 749, 753, 754, 755, 756, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 795, 798, 799, 800, 801, 802, 803, 804, 806, 807, 808, 810, 811, 812, 813, 814, 817, 818, 819, 820, 821, 822, 823, 824, 828, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 843, 844, 845, 846, 847, 851, 852, 853, 854, 855, 856, 857, 866, 867, 880, 881, 882, 883, 884, 887, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 901, 902, 903, 905, 906, 907, 908, 909, 911, 913, 916, 919, 920, 921, 922, 923, 924, 925, 926, 929, 932, 934, 935, 936, 937, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade: { 36, 37, 38, 51, 52, 64, 65, 75, 80, 81, 82, 83, 84, 85, 87, 89, 90, 92, 94, 110, 112, 114, 116, 118, 121, 122, 123, 125, 127, 143, 144, 145, 146, 157, 158, 162, 185, 196, 197, 198, 199, 200, 219, 221, 228, 230, 249, 250, 251, 261, 262, 267, 271, 273, 275, 277, 278, 280, 282, 284, 304, 307, 309, 311, 313, 325, 336, 338, 339, 340, 341, 342, 343, 347, 354, 362, 384, 385, 386, 387, 391, 392, 393, 394, 401, 402, 421, 422, 475, 478, 496, 501, 502, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 521, 524, 528, 534, 537, 538, 539, 542, 543, 544, 547, 548, 549, 552, 553, 554, 555, 564, 565, 590, 599, 638, 642, 650, 667, 670, 676, 683, 707, 718, 719, 720, 721, 722, 728, 732, 743, 744, 747, 748, 752, 757, 759, 775, 793, 796, 797, 805, 815, 816, 825, 826, 827, 829, 842, 848, 849, 850, 860, 861, 862, 863, 864, 885, 886, 888, 904, 910, 915, 927, 928, 930, 933 }

C grade: { 428, 429, 585, 586, 587, 588, 594, 595, 596, 597, 712 }

F grade: { 31, 32, 34, 39, 40, 41, 46, 47, 53, 54, 55, 60, 63, 74, 76, 77, 78, 79, 105, 106, 107, 108, 109, 115, 139, 140, 141, 142, 147, 148, 149, 153, 154, 155, 156, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 217, 218, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 314, 315, 316, 317, 318, 319, 320, 321, 333, 334, 335, 337, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 423, 424, 425, 426, 427, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 497, 498, 503, 556, 557, 558, 559, 560, 561, 566, 573, 574, 575, 576, 577, 578, 579, 580, 581, 592, 593, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 639, 641, 643, 662, 663, 664, 688, 689, 693, 709, 717, 723, 733, 741, 742, 745, 746, 750, 751, 809, 858, 859, 865, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 900, 912, 914, 917, 918, 931, 938, 939 }

2.1.8 Mupad

A grade: { 42, 43, 56, 57, 180, 582, 583, 584, 644, 645, 646, 647, 932 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 44, 45, 48, 49, 50, 58, 59, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 155, 159, 162, 165, 166, 167, 171, 172, 173, 183, 185, 186, 187, 188, 189, 190, 191, 192, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 317, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 336, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 419, 426, 443, 444, 445, 446, 447, 458, 459, 460, 461, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 499, 500, 501, 502, 503, 508, 509, 510, 511, 512, 513, 518, 519, 520, 521, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 562, 563, 564, 565, 567, 568, 569, 570, 571, 572, 589, 590, 598, 599, 603, 604, 605, 606, 611, 612, 613, 614, 633, 637, 638, 639, 640, 641, 642, 643, 648, 649, 650, 651, 652, 653, 654, 655, 656, 659, 660, 661, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 680, 681, 682, 683, 684, 685, 686, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 712, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 852, 854, 855, 856, 857, 858, 861, 862, 863, 864, 865, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 923, 924, 925, 926, 927, 928, 929, 930, 931,

933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

C grade: { }

F grade: { 31, 32, 34, 36, 37, 38, 39, 40, 41, 46, 47, 51, 52, 53, 54, 55, 60, 63, 79, 115, 148, 149, 151, 152, 153, 154, 156, 157, 158, 160, 161, 163, 164, 168, 169, 170, 174, 175, 176, 177, 178, 179, 181, 182, 184, 193, 194, 195, 196, 197, 198, 199, 217, 218, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 259, 260, 261, 262, 314, 315, 316, 318, 319, 320, 321, 333, 334, 335, 337, 338, 339, 340, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 497, 498, 504, 505, 506, 507, 514, 515, 516, 517, 522, 523, 524, 525, 556, 557, 558, 559, 560, 561, 566, 573, 574, 575, 576, 577, 578, 579, 580, 581, 585, 586, 587, 588, 591, 592, 593, 594, 595, 596, 597, 600, 601, 602, 607, 608, 609, 610, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 657, 658, 662, 663, 664, 679, 687, 688, 689, 709, 710, 711, 713, 850, 851, 853, 859, 860, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 921, 922 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	22	17	26	33	32	57	35
normalized size	1	1.00	0.50	0.39	0.59	0.75	0.73	1.30	0.80
time (sec)	N/A	0.040	0.036	0.075	0.411	0.653	0.263	0.150	2.582
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	22	17	26	33	0	57	16
normalized size	1	1.00	0.50	0.39	0.59	0.75	0.00	1.30	0.36
time (sec)	N/A	0.038	0.027	0.305	0.643	0.688	0.000	0.220	2.689
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	16	43	246	57	35
normalized size	1	1.00	0.46	0.35	0.33	0.90	5.12	1.19	0.73
time (sec)	N/A	0.022	0.046	0.168	0.890	0.647	6.818	0.128	2.486
Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	16	43	246	57	35
normalized size	1	1.00	0.46	0.35	0.33	0.90	5.12	1.19	0.73
time (sec)	N/A	0.020	0.057	0.099	0.411	0.731	7.115	0.146	2.380
Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	16	43	246	57	35
normalized size	1	1.00	0.46	0.35	0.33	0.90	5.12	1.19	0.73
time (sec)	N/A	0.026	0.025	0.227	0.565	0.735	6.995	0.157	2.376

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	16	43	0	16	16
normalized size	1	1.00	0.46	0.35	0.33	0.90	0.00	0.33	0.33
time (sec)	N/A	0.044	0.018	0.288	0.442	0.510	0.000	1.542	2.402
Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	16	43	0	57	16
normalized size	1	1.00	0.46	0.35	0.33	0.90	0.00	1.19	0.33
time (sec)	N/A	0.041	0.019	0.286	0.614	0.644	0.000	0.283	2.398
Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	54	74	42	39	16
normalized size	1	1.00	0.37	0.28	0.90	1.23	0.70	0.65	0.27
time (sec)	N/A	0.026	0.037	0.058	0.568	0.506	0.312	0.167	2.566
Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	54	74	0	39	16
normalized size	1	1.00	0.37	0.28	0.90	1.23	0.00	0.65	0.27
time (sec)	N/A	0.046	0.039	0.303	0.490	0.912	0.000	0.238	2.624
Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	34	86	1644	39	16
normalized size	1	1.00	0.37	0.28	0.57	1.43	27.40	0.65	0.27
time (sec)	N/A	0.020	0.058	0.148	0.409	0.654	18.471	0.162	2.453
Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	34	86	1644	39	16
normalized size	1	1.00	0.37	0.28	0.57	1.43	27.40	0.65	0.27
time (sec)	N/A	0.019	0.069	0.098	0.497	0.569	14.403	0.166	2.440

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	34	86	1644	39	16
normalized size	1	1.00	0.37	0.28	0.57	1.43	27.40	0.65	0.27
time (sec)	N/A	0.030	0.031	0.225	0.413	0.781	17.090	0.202	2.404
Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	34	86	0	39	16
normalized size	1	1.00	0.37	0.28	0.57	1.43	0.00	0.65	0.27
time (sec)	N/A	0.045	0.022	0.277	1.032	0.771	0.000	1.484	2.402
Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	34	86	0	39	16
normalized size	1	1.00	0.37	0.28	0.57	1.43	0.00	0.65	0.27
time (sec)	N/A	0.046	0.031	0.263	0.413	0.684	0.000	0.357	2.410
Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	22	18	27	31	34	57	36
normalized size	1	1.00	0.52	0.43	0.64	0.74	0.81	1.36	0.86
time (sec)	N/A	0.037	0.029	0.060	0.420	1.319	0.255	0.145	2.536
Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	22	18	27	31	0	57	17
normalized size	1	1.00	0.52	0.43	0.64	0.74	0.00	1.36	0.40
time (sec)	N/A	0.037	0.021	0.314	0.432	0.597	0.000	0.225	2.689
Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	17	43	76	57	36
normalized size	1	1.00	0.46	0.38	0.35	0.90	1.58	1.19	0.75
time (sec)	N/A	0.019	0.020	0.146	0.443	0.755	0.750	0.146	2.438

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	17	43	76	57	36
normalized size	1	1.00	0.46	0.38	0.35	0.90	1.58	1.19	0.75
time (sec)	N/A	0.016	0.040	0.095	0.443	0.685	0.648	0.145	2.367
Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	17	43	76	57	36
normalized size	1	1.00	0.46	0.38	0.35	0.90	1.58	1.19	0.75
time (sec)	N/A	0.027	0.020	0.228	0.444	0.508	0.798	0.158	2.367
Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	17	43	0	17	17
normalized size	1	1.00	0.46	0.38	0.35	0.90	0.00	0.35	0.35
time (sec)	N/A	0.042	0.018	0.269	1.181	0.609	0.000	1.232	2.371
Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	17	43	0	57	17
normalized size	1	1.00	0.46	0.38	0.35	0.90	0.00	1.19	0.35
time (sec)	N/A	0.045	0.018	0.276	0.745	0.554	0.000	0.327	2.380
Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	53	74	39	39	17
normalized size	1	1.00	0.36	0.30	0.87	1.21	0.64	0.64	0.28
time (sec)	N/A	0.030	0.026	0.059	0.433	0.541	0.308	0.164	2.501
Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	53	74	0	39	17
normalized size	1	1.00	0.36	0.30	0.87	1.21	0.00	0.64	0.28
time (sec)	N/A	0.046	0.034	0.313	0.526	0.549	0.000	0.240	2.713

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	32	85	1481	39	17
normalized size	1	1.00	0.36	0.30	0.52	1.39	24.28	0.64	0.28
time (sec)	N/A	0.020	0.064	0.147	0.501	0.587	14.067	0.187	2.515
Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	32	85	1481	39	17
normalized size	1	1.00	0.36	0.30	0.52	1.39	24.28	0.64	0.28
time (sec)	N/A	0.019	0.057	0.096	0.516	0.594	12.827	0.171	2.443
Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	32	85	1481	39	17
normalized size	1	1.00	0.36	0.30	0.52	1.39	24.28	0.64	0.28
time (sec)	N/A	0.031	0.029	0.229	0.536	0.575	16.478	0.206	2.418
Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	32	85	0	37	17
normalized size	1	1.00	0.36	0.30	0.52	1.39	0.00	0.61	0.28
time (sec)	N/A	0.043	0.021	0.277	1.342	0.536	0.000	1.294	2.400
Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	32	85	0	39	17
normalized size	1	1.00	0.36	0.30	0.52	1.39	0.00	0.64	0.28
time (sec)	N/A	0.047	0.034	0.302	1.182	0.538	0.000	0.418	2.334
Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	22	41	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.73	1.37	0.80	0.80
time (sec)	N/A	0.035	0.061	0.037	0.326	0.508	0.163	0.137	0.057

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	48	57	48	46	85	46	46
normalized size	1	1.00	0.86	1.02	0.86	0.82	1.52	0.82	0.82
time (sec)	N/A	0.067	0.090	0.039	0.517	0.487	0.304	0.137	2.308
Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	172	229	0	187	0	0	-1
normalized size	1	1.00	0.81	1.08	0.00	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.536	0.322	0.053	0.000	0.559	0.000	0.000	0.000
Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	238	320	0	434	0	0	-1
normalized size	1	1.00	0.88	1.18	0.00	1.60	0.00	0.00	-0.00
time (sec)	N/A	0.802	0.564	0.043	0.000	2.021	0.000	0.000	0.000
Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	10	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	1.00	0.80	0.80
time (sec)	N/A	0.018	0.023	0.007	0.309	0.588	0.258	0.125	2.419
Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	46	72	0	0	0	0	-1
normalized size	1	1.00	1.64	2.57	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.471	0.700	0.079	0.000	1.267	0.000	0.000	0.000
Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	26	12	17	22	10	11	11
normalized size	1	1.00	2.17	1.00	1.42	1.83	0.83	0.92	0.92
time (sec)	N/A	0.029	0.020	0.076	0.306	0.626	1.363	0.137	2.399

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	272	142	0	139	0	630	-1
normalized size	1	1.00	2.72	1.42	0.00	1.39	0.00	6.30	-0.01
time (sec)	N/A	0.164	5.353	0.036	0.000	1.086	0.000	9.107	0.000
Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	330	195	0	149	0	681	-1
normalized size	1	1.00	3.08	1.82	0.00	1.39	0.00	6.36	-0.01
time (sec)	N/A	0.192	7.270	0.071	0.000	0.608	0.000	62.661	0.000
Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	657	295	0	277	0	1239	-1
normalized size	1	1.00	3.39	1.52	0.00	1.43	0.00	6.39	-0.01
time (sec)	N/A	0.322	7.705	0.070	0.000	1.137	0.000	177.526	0.000
Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.111	0.106	0.576	0.000	2.026	0.000	0.000	0.000
Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.082	0.079	0.292	0.000	0.966	0.000	0.000	0.000
Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	0.045	0.070	0.000	0.949	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	5.908	0.179	0.000	1.145	0.000	0.000	0.000
Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.081	19.895	0.243	0.000	1.171	0.000	0.000	0.000
Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	26	25	24	23	41	24	24
normalized size	1	1.00	0.87	0.83	0.80	0.77	1.37	0.80	0.80
time (sec)	N/A	0.034	0.076	0.035	0.306	2.513	0.164	0.137	0.049
Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	51	57	46	44	85	46	48
normalized size	1	1.00	0.91	1.02	0.82	0.79	1.52	0.82	0.86
time (sec)	N/A	0.069	0.117	0.030	0.314	0.851	0.307	0.145	0.076
Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	172	229	0	189	0	0	-1
normalized size	1	1.00	0.81	1.08	0.00	0.89	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.336	0.048	0.000	1.123	0.000	0.000	0.000
Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	236	320	0	436	0	0	-1
normalized size	1	1.00	0.87	1.18	0.00	1.61	0.00	0.00	-0.00
time (sec)	N/A	0.562	0.582	0.050	0.000	1.036	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.136	0.037	0.045	0.350	1.626	0.432	0.141	2.260
Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	18	17	37	20	17	15
normalized size	1	1.00	1.00	0.82	0.77	1.68	0.91	0.77	0.68
time (sec)	N/A	0.189	0.055	0.079	0.411	0.941	0.684	0.119	2.316
Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	16	16	15	15	19	16
normalized size	1	1.00	0.83	0.67	0.67	0.62	0.62	0.79	0.67
time (sec)	N/A	0.493	0.173	0.072	0.306	1.705	5.494	0.149	2.349
Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	260	142	0	138	0	633	-1
normalized size	1	1.00	2.57	1.41	0.00	1.37	0.00	6.27	-0.01
time (sec)	N/A	0.133	5.510	0.095	0.000	4.118	0.000	12.269	0.000
Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	400	195	0	144	0	683	-1
normalized size	1	1.00	3.74	1.82	0.00	1.35	0.00	6.38	-0.01
time (sec)	N/A	0.161	6.135	0.136	0.000	2.197	0.000	68.629	0.000
Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.111	0.023	0.744	0.000	2.694	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	51	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.080	0.040	0.363	0.000	1.494	0.000	0.000	0.000
Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	0.007	0.079	0.000	1.773	0.000	0.000	0.000
Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	3.068	0.107	0.000	0.496	0.000	0.000	0.000
Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.084	11.878	0.250	0.000	1.129	0.000	0.000	0.000
Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	13	10	8	19
normalized size	1	1.00	1.00	0.89	0.78	1.44	1.11	0.89	2.11
time (sec)	N/A	0.010	0.013	0.004	0.302	1.996	0.171	0.143	3.179
Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	13	12	12	14	12	20
normalized size	1	1.00	1.12	0.81	0.75	0.75	0.88	0.75	1.25
time (sec)	N/A	0.018	0.037	0.008	0.537	1.972	0.178	0.122	2.584

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	0	80	0	0	0	-1
normalized size	1	1.00	1.00	0.00	1.14	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.023	0.105	0.414	0.858	0.000	0.000	0.000
Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	18	83	23	0	18	21
normalized size	1	1.00	0.95	0.95	4.37	1.21	0.00	0.95	1.11
time (sec)	N/A	0.018	0.744	0.152	0.376	1.661	0.000	8.595	2.511
Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	26	14	14	28	15	22	20
normalized size	1	1.00	1.62	0.88	0.88	1.75	0.94	1.38	1.25
time (sec)	N/A	0.018	0.049	0.007	0.481	1.222	0.169	0.129	2.549
Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	85	150	0	0	0	0	-1
normalized size	1	1.00	0.92	1.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	7.219	0.621	0.000	1.199	0.000	0.000	0.000
Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	48	6257	72	0	948	29
normalized size	1	1.00	1.00	1.37	178.77	2.06	0.00	27.09	0.83
time (sec)	N/A	0.033	0.037	0.169	108.038	0.942	0.000	1.153	0.089
Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	48	6257	72	0	948	29
normalized size	1	1.00	1.00	1.37	178.77	2.06	0.00	27.09	0.83
time (sec)	N/A	0.033	0.025	0.000	116.999	0.980	0.000	1.194	0.002

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	7	11	10	20	6	6
normalized size	1	1.00	1.00	0.47	0.73	0.67	1.33	0.40	0.40
time (sec)	N/A	0.009	0.006	0.049	0.307	2.012	0.452	0.122	0.027
Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	20	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	1.18	0.76	0.76
time (sec)	N/A	0.008	0.007	0.109	0.308	0.585	0.416	0.135	0.029
Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	18	20	13	13
normalized size	1	1.00	1.00	0.82	0.76	1.06	1.18	0.76	0.76
time (sec)	N/A	0.008	0.008	0.096	0.305	1.122	0.410	0.138	0.029
Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	28	26	78	29	64
normalized size	1	1.00	0.71	0.80	0.80	0.74	2.23	0.83	1.83
time (sec)	N/A	0.031	0.048	0.079	0.306	1.083	0.795	0.136	2.323
Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	9	20	11	9
normalized size	1	1.00	1.00	0.80	0.73	0.60	1.33	0.73	0.60
time (sec)	N/A	0.008	0.005	0.070	0.307	0.785	0.450	0.127	0.023
Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	20	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	1.18	0.76	0.76
time (sec)	N/A	0.008	0.006	0.102	0.538	0.830	0.415	0.122	0.025

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	17	20	13	17
normalized size	1	1.00	1.00	0.82	0.76	1.00	1.18	0.76	1.00
time (sec)	N/A	0.008	0.007	0.108	0.505	1.061	0.410	0.123	0.028
Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	28	28	24	37	29	37
normalized size	1	1.00	0.69	0.80	0.80	0.69	1.06	0.83	1.06
time (sec)	N/A	0.027	0.044	0.033	0.317	0.805	0.930	0.133	0.100
Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	141	38	0	0	17
normalized size	1	1.00	1.00	0.90	7.05	1.90	0.00	0.00	0.85
time (sec)	N/A	0.023	0.014	0.161	0.436	2.061	0.000	0.000	2.394
Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	21	38	0	39	0	364	26
normalized size	1	1.00	0.45	0.81	0.00	0.83	0.00	7.74	0.55
time (sec)	N/A	0.052	0.031	0.301	0.000	1.907	0.000	0.266	2.339
Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	69	115	0	101	0	0	103
normalized size	1	1.00	0.97	1.62	0.00	1.42	0.00	0.00	1.45
time (sec)	N/A	0.109	0.082	0.411	0.000	2.459	0.000	0.000	2.562
Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	100	84	0	136	0	0	107
normalized size	1	1.00	0.89	0.75	0.00	1.21	0.00	0.00	0.96
time (sec)	N/A	0.170	0.176	0.411	0.000	0.835	0.000	0.000	2.885

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	84	256	0	134	0	0	131
normalized size	1	1.00	0.94	2.88	0.00	1.51	0.00	0.00	1.47
time (sec)	N/A	0.271	0.150	0.615	0.000	1.793	0.000	0.000	3.113
Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	200	0	0	0	0	0	-1
normalized size	1	1.00	1.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.178	0.437	0.000	0.732	0.000	0.000	0.000
Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	12	37	19	19	19	10
normalized size	1	1.00	1.00	1.20	3.70	1.90	1.90	1.90	1.00
time (sec)	N/A	0.022	0.011	0.076	0.408	0.680	0.815	0.138	2.319
Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	127	36	0	34	16
normalized size	1	1.00	1.00	0.85	6.35	1.80	0.00	1.70	0.80
time (sec)	N/A	0.027	0.019	0.117	0.444	0.978	0.000	0.150	2.366
Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	30	173	52	0	50	29
normalized size	1	1.00	1.00	1.07	6.18	1.86	0.00	1.79	1.04
time (sec)	N/A	0.051	0.037	0.125	0.544	0.571	0.000	0.138	2.378
Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	76	70	0	127	0	111	119
normalized size	1	1.00	0.93	0.85	0.00	1.55	0.00	1.35	1.45
time (sec)	N/A	0.196	0.238	0.209	0.000	0.562	0.000	0.326	2.609

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	49	0	70	0	70	37
normalized size	1	1.00	1.00	1.29	0.00	1.84	0.00	1.84	0.97
time (sec)	N/A	0.082	0.070	0.158	0.000	0.869	0.000	0.168	2.500
Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	174	13	129	33	0	49	12
normalized size	1	1.00	11.60	0.87	8.60	2.20	0.00	3.27	0.80
time (sec)	N/A	0.015	0.415	0.134	0.427	0.602	0.000	0.171	2.289
Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	18	81	19	0	24	15
normalized size	1	1.00	0.81	0.86	3.86	0.90	0.00	1.14	0.71
time (sec)	N/A	0.027	0.009	0.188	0.418	0.876	0.000	0.142	2.274
Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	4845	54	0	121	0	133	112
normalized size	1	1.00	68.24	0.76	0.00	1.70	0.00	1.87	1.58
time (sec)	N/A	0.062	57.934	0.206	0.000	0.677	0.000	0.142	0.088
Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	43	0	72	0	67	47
normalized size	1	1.00	0.92	0.69	0.00	1.16	0.00	1.08	0.76
time (sec)	N/A	0.073	0.099	0.228	0.000	0.689	0.000	0.160	0.535
Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	627	80	0	153	0	182	118
normalized size	1	1.00	7.38	0.94	0.00	1.80	0.00	2.14	1.39
time (sec)	N/A	0.063	9.333	0.244	0.000	1.055	0.000	0.164	2.283

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	37	9	35	17	15	17	5
normalized size	1	1.00	5.29	1.29	5.00	2.43	2.14	2.43	0.71
time (sec)	N/A	0.012	0.006	0.080	0.421	2.742	0.851	0.139	0.109
Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	15	14	125	58	76	31	17
normalized size	1	1.00	0.33	0.31	2.78	1.29	1.69	0.69	0.38
time (sec)	N/A	0.040	0.019	0.205	0.441	1.090	1.869	0.180	2.793
Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	28	171	50	294	48	27
normalized size	1	1.00	1.00	1.08	6.58	1.92	11.31	1.85	1.04
time (sec)	N/A	0.025	0.021	0.227	0.433	1.972	7.364	0.136	2.433
Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	84	66	0	231	0	105	217
normalized size	1	1.00	0.51	0.40	0.00	1.40	0.00	0.64	1.32
time (sec)	N/A	0.141	0.106	0.287	0.000	0.933	0.000	0.282	2.592
Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	47	0	68	0	68	35
normalized size	1	1.00	0.83	1.31	0.00	1.89	0.00	1.89	0.97
time (sec)	N/A	0.046	0.036	0.263	0.000	1.240	0.000	0.151	2.460
Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	6	9	6	8	5	6	6
normalized size	1	1.00	0.75	1.12	0.75	1.00	0.62	0.75	0.75
time (sec)	N/A	0.030	0.011	0.102	0.309	0.829	0.981	0.133	2.252

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	6	6	7	6	6
normalized size	1	1.00	1.00	1.12	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.014	0.003	0.043	0.320	2.972	3.256	0.131	0.027
Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	7	11	6	22	6	6
normalized size	1	1.00	1.00	0.47	0.73	0.40	1.47	0.40	0.40
time (sec)	N/A	0.009	0.005	0.043	0.308	1.484	0.446	0.131	0.018
Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	22	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	1.29	0.76	0.76
time (sec)	N/A	0.008	0.006	0.070	0.307	0.900	0.416	0.137	0.026
Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	22	13	14
normalized size	1	1.00	1.00	0.82	0.76	0.76	1.29	0.76	0.82
time (sec)	N/A	0.008	0.005	0.058	0.313	0.554	0.409	0.123	0.024
Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	28	27	25	44	29	57
normalized size	1	1.00	0.74	0.80	0.77	0.71	1.26	0.83	1.63
time (sec)	N/A	0.030	0.047	0.018	0.316	0.434	0.777	0.139	2.285
Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	12	20	11	9
normalized size	1	1.00	1.00	0.80	0.73	0.80	1.33	0.73	0.60
time (sec)	N/A	0.009	0.005	0.066	0.305	1.586	0.448	0.135	0.025

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	7	20	13	7
normalized size	1	1.00	1.00	0.82	0.76	0.41	1.18	0.76	0.41
time (sec)	N/A	0.009	0.005	0.072	0.322	0.565	0.412	0.134	0.022
Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	18	20	13	13
normalized size	1	1.00	1.00	0.82	0.76	1.06	1.18	0.76	0.76
time (sec)	N/A	0.008	0.005	0.100	0.314	1.081	0.409	0.136	0.025
Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	28	25	56	29	39
normalized size	1	1.00	0.71	0.80	0.80	0.71	1.60	0.83	1.11
time (sec)	N/A	0.029	0.038	0.029	0.317	2.475	0.935	0.136	0.124
Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	183	18	133	38	0	0	42
normalized size	1	1.00	9.15	0.90	6.65	1.90	0.00	0.00	2.10
time (sec)	N/A	0.025	0.234	0.040	0.434	2.030	0.000	0.000	2.358
Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	48	19	0	38	0	0	42
normalized size	1	1.00	2.29	0.90	0.00	1.81	0.00	0.00	2.00
time (sec)	N/A	0.024	0.053	0.047	0.000	1.507	0.000	0.000	2.312
Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	6196	68	0	101	0	0	295
normalized size	1	1.00	87.27	0.96	0.00	1.42	0.00	0.00	4.15
time (sec)	N/A	0.083	59.547	0.078	0.000	0.945	0.000	0.000	2.446

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	215	72	0	129	0	0	407
normalized size	1	1.00	2.56	0.86	0.00	1.54	0.00	0.00	4.85
time (sec)	N/A	0.098	0.590	0.071	0.000	1.201	0.000	0.000	2.497
Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	679	104	0	134	0	0	787
normalized size	1	1.00	7.63	1.17	0.00	1.51	0.00	0.00	8.84
time (sec)	N/A	0.238	9.042	0.085	0.000	0.852	0.000	0.000	4.074
Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	25	14	37	21	19	19	20
normalized size	1	1.00	2.50	1.40	3.70	2.10	1.90	1.90	2.00
time (sec)	N/A	0.020	0.013	0.086	0.329	1.297	0.832	0.141	2.336
Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	47	36	131	39	0	39	39
normalized size	1	1.00	1.04	0.80	2.91	0.87	0.00	0.87	0.87
time (sec)	N/A	0.053	0.017	0.315	0.416	3.045	0.000	0.132	2.369
Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	73	30	165	53	0	50	67
normalized size	1	1.00	2.61	1.07	5.89	1.89	0.00	1.79	2.39
time (sec)	N/A	0.049	0.066	0.408	0.425	1.007	0.000	0.140	2.349
Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	133	82	0	137	0	117	611
normalized size	1	1.00	1.21	0.75	0.00	1.25	0.00	1.06	5.55
time (sec)	N/A	0.155	0.125	0.437	0.000	0.625	0.000	0.160	3.034

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	87	49	0	71	0	70	86
normalized size	1	1.00	2.29	1.29	0.00	1.87	0.00	1.84	2.26
time (sec)	N/A	0.071	0.090	0.555	0.000	0.691	0.000	0.175	2.435
Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	179	0	0	0	0	0	-1
normalized size	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.183	0.553	0.000	1.066	0.000	0.000	0.000
Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	13	137	33	0	31	12
normalized size	1	1.00	1.00	0.87	9.13	2.20	0.00	2.07	0.80
time (sec)	N/A	0.015	0.007	0.134	1.076	1.128	0.000	0.134	0.114
Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	15	13	76	53	0	31	16
normalized size	1	1.00	0.34	0.30	1.73	1.20	0.00	0.70	0.36
time (sec)	N/A	0.036	0.015	0.175	0.430	1.488	0.000	0.169	2.664
Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	67	54	0	121	0	99	95
normalized size	1	1.00	0.94	0.76	0.00	1.70	0.00	1.39	1.34
time (sec)	N/A	0.046	0.104	0.217	0.000	1.983	0.000	0.276	2.272
Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	84	68	0	231	0	105	217
normalized size	1	1.00	0.52	0.42	0.00	1.42	0.00	0.64	1.33
time (sec)	N/A	0.129	0.099	0.259	0.000	2.070	0.000	0.284	2.667

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	80	0	154	0	132	118
normalized size	1	1.00	0.95	0.94	0.00	1.81	0.00	1.55	1.39
time (sec)	N/A	0.061	0.083	0.264	0.000	4.125	0.000	0.245	2.292
Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	14	19	21	20	21	10
normalized size	1	1.00	1.00	1.40	1.90	2.10	2.00	2.10	1.00
time (sec)	N/A	0.018	0.006	0.117	0.313	0.680	1.053	0.141	2.247
Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	18	129	25	427	25	12
normalized size	1	1.00	1.00	1.29	9.21	1.79	30.50	1.79	0.86
time (sec)	N/A	0.020	0.007	0.139	0.478	1.472	5.350	0.126	0.025
Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	21	11	35	19	15	17	5
normalized size	1	1.00	3.00	1.57	5.00	2.71	2.14	2.43	0.71
time (sec)	N/A	0.011	0.003	0.080	0.315	2.121	0.875	0.123	0.027
Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	34	129	19	17	24	17
normalized size	1	1.00	1.00	1.62	6.14	0.90	0.81	1.14	0.81
time (sec)	N/A	0.026	0.008	0.223	0.426	1.274	1.332	0.140	0.103
Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	66	28	163	52	248	48	55
normalized size	1	1.00	2.54	1.08	6.27	2.00	9.54	1.85	2.12
time (sec)	N/A	0.026	0.059	0.141	0.442	0.829	6.139	0.135	2.307

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	80	0	72	0	67	51
normalized size	1	1.00	0.92	1.29	0.00	1.16	0.00	1.08	0.82
time (sec)	N/A	0.070	0.060	0.259	0.000	1.394	0.000	0.140	2.679
Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	83	47	0	70	0	68	74
normalized size	1	1.00	2.31	1.31	0.00	1.94	0.00	1.89	2.06
time (sec)	N/A	0.041	0.077	0.167	0.000	0.707	0.000	0.151	2.368
Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	57	63	25	57
normalized size	1	1.00	1.00	0.79	0.76	1.73	1.91	0.76	1.73
time (sec)	N/A	0.031	0.016	0.253	0.454	3.969	5.211	0.126	0.080
Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	39	71	25	78
normalized size	1	1.00	1.00	0.79	0.76	1.18	2.15	0.76	2.36
time (sec)	N/A	0.033	0.017	0.135	0.312	0.579	5.728	0.141	2.469
Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	32	48	19	25
normalized size	1	1.00	1.00	0.80	0.76	1.28	1.92	0.76	1.00
time (sec)	N/A	0.028	0.016	0.124	0.319	2.009	1.728	0.124	2.289
Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	42	42	17	17
normalized size	1	1.00	1.00	0.78	0.74	1.83	1.83	0.74	0.74
time (sec)	N/A	0.026	0.012	0.129	0.311	0.603	1.637	0.125	2.487

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	49	65	49	150
normalized size	1	1.00	1.00	0.79	0.76	1.48	1.97	1.48	4.55
time (sec)	N/A	0.031	0.014	0.141	0.561	0.612	5.187	0.140	2.698
Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	23	67	70	23	198
normalized size	1	1.00	1.00	0.77	0.74	2.16	2.26	0.74	6.39
time (sec)	N/A	0.030	0.016	0.450	0.727	1.289	5.272	0.140	3.179
Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	31	25	228	31	25
normalized size	1	1.00	1.00	0.78	0.76	0.61	5.56	0.76	0.61
time (sec)	N/A	0.043	0.018	0.171	0.355	0.987	18.542	0.125	2.271
Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	50	58	23	36
normalized size	1	1.00	0.96	0.89	0.85	1.85	2.15	0.85	1.33
time (sec)	N/A	0.024	0.047	0.053	0.307	0.623	0.742	0.124	2.476
Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	44	61	23	36
normalized size	1	1.00	0.96	0.89	0.85	1.63	2.26	0.85	1.33
time (sec)	N/A	0.025	0.034	0.046	0.703	0.820	0.734	0.137	2.461
Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	50	58	23	36
normalized size	1	1.00	0.96	0.89	0.85	1.85	2.15	0.85	1.33
time (sec)	N/A	0.017	0.025	0.045	0.564	0.558	0.724	0.124	2.275

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	44	58	23	36
normalized size	1	1.00	0.96	0.89	0.85	1.63	2.15	0.85	1.33
time (sec)	N/A	0.019	0.023	0.046	0.332	2.126	0.731	0.149	2.264
Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	31	173	371	145	7713	0	207
normalized size	1	1.00	0.79	4.44	9.51	3.72	197.77	0.00	5.31
time (sec)	N/A	0.066	0.518	0.136	0.653	0.635	6.576	0.000	4.989
Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	145	290	145	7720	0	196
normalized size	1	1.00	0.82	4.26	8.53	4.26	227.06	0.00	5.76
time (sec)	N/A	0.068	0.529	0.135	0.346	2.007	8.525	0.000	4.998
Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	31	177	549	118	7485	0	207
normalized size	1	1.00	0.79	4.54	14.08	3.03	191.92	0.00	5.31
time (sec)	N/A	0.032	0.510	0.189	0.928	1.526	24.277	0.000	4.815
Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	149	432	118	7499	0	200
normalized size	1	1.00	0.88	4.38	12.71	3.47	220.56	0.00	5.88
time (sec)	N/A	0.034	0.501	0.185	0.439	2.945	24.889	0.000	5.055
Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	55	349	107	0	171	249
normalized size	1	1.00	0.78	1.53	9.69	2.97	0.00	4.75	6.92
time (sec)	N/A	0.019	0.229	0.475	0.342	1.988	0.000	0.233	7.837

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	26	53	322	93	0	169	249
normalized size	1	1.00	0.79	1.61	9.76	2.82	0.00	5.12	7.55
time (sec)	N/A	0.018	0.235	0.460	0.357	0.650	0.000	0.217	7.728
Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	169	564	110	0	396	249
normalized size	1	1.00	0.78	4.69	15.67	3.06	0.00	11.00	6.92
time (sec)	N/A	0.018	0.243	0.490	0.354	0.754	0.000	0.238	7.774
Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	81	536	96	1838	397	249
normalized size	1	1.00	0.88	2.45	16.24	2.91	55.70	12.03	7.55
time (sec)	N/A	0.018	0.220	0.477	0.502	0.785	129.081	0.227	7.873
Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	177	57	22	0	0	20
normalized size	1	1.00	1.00	13.62	4.38	1.69	0.00	0.00	1.54
time (sec)	N/A	0.032	0.078	0.395	0.492	0.743	0.000	0.000	2.562
Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	587	57	26	0	0	-1
normalized size	1	1.00	0.74	18.94	1.84	0.84	0.00	0.00	-0.03
time (sec)	N/A	0.053	0.038	0.263	0.474	0.488	0.000	0.000	0.000
Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	29	324	82	38	0	0	-1
normalized size	1	1.00	0.58	6.48	1.64	0.76	0.00	0.00	-0.02
time (sec)	N/A	0.075	0.098	0.345	0.444	0.718	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	20	188	19	0	12	18
normalized size	1	1.00	1.00	1.54	14.46	1.46	0.00	0.92	1.38
time (sec)	N/A	0.038	0.068	0.254	0.624	0.664	0.000	0.156	2.653
Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	21	26	314	23	0	19	-1
normalized size	1	1.00	0.68	0.84	10.13	0.74	0.00	0.61	-0.03
time (sec)	N/A	0.068	0.036	0.248	0.542	0.728	0.000	0.148	0.000
Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	29	34	427	35	0	27	-1
normalized size	1	1.00	0.58	0.68	8.54	0.70	0.00	0.54	-0.02
time (sec)	N/A	0.094	0.098	0.303	0.627	0.599	0.000	0.143	0.000
Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	159	0	236	0	0	-1
normalized size	1	1.00	0.97	2.74	0.00	4.07	0.00	0.00	-0.02
time (sec)	N/A	0.078	0.139	0.524	0.000	1.557	0.000	0.000	0.000
Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	84	257	0	459	0	0	-1
normalized size	1	1.00	0.99	3.02	0.00	5.40	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.260	0.829	0.000	0.630	0.000	0.000	0.000
Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	58	154	0	227	0	0	132
normalized size	1	1.00	0.98	2.61	0.00	3.85	0.00	0.00	2.24
time (sec)	N/A	0.059	0.102	0.264	0.000	0.571	0.000	0.000	3.292

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	85	250	0	417	0	0	-1
normalized size	1	1.00	0.97	2.84	0.00	4.74	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.315	0.435	0.000	0.659	0.000	0.000	0.000
Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	86	250	80	0	322	-1
normalized size	1	1.00	0.96	1.72	5.00	1.60	0.00	6.44	-0.02
time (sec)	N/A	0.083	0.184	0.318	0.355	0.588	0.000	0.357	0.000
Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	87	250	81	0	322	-1
normalized size	1	1.00	0.96	1.74	5.00	1.62	0.00	6.44	-0.02
time (sec)	N/A	0.082	0.187	0.308	0.329	0.743	0.000	0.345	0.000
Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	23	205	0	40	24
normalized size	1	1.00	1.00	0.75	0.72	6.41	0.00	1.25	0.75
time (sec)	N/A	0.055	0.093	0.322	0.432	0.623	0.000	0.616	2.560
Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	512	1003	0	3344	0	0	-1
normalized size	1	1.00	2.43	4.75	0.00	15.85	0.00	0.00	-0.00
time (sec)	N/A	0.528	6.438	0.602	0.000	1.299	0.000	0.000	0.000
Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	294	1251	0	4644	0	0	-1
normalized size	1	1.00	0.87	3.71	0.00	13.78	0.00	0.00	-0.00
time (sec)	N/A	0.900	1.089	0.463	0.000	2.281	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	43	300	0	76	45
normalized size	1	1.00	1.00	0.85	1.08	7.50	0.00	1.90	1.12
time (sec)	N/A	0.605	0.255	0.362	1.069	0.750	0.000	0.972	2.597
Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	751	1670	0	4160	0	0	-1
normalized size	1	1.00	2.81	6.25	0.00	15.58	0.00	0.00	-0.00
time (sec)	N/A	0.719	4.226	0.605	0.000	1.383	0.000	0.000	0.000
Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	499	2061	0	5772	0	0	-1
normalized size	1	1.00	1.23	5.06	0.00	14.18	0.00	0.00	-0.00
time (sec)	N/A	1.075	2.214	0.547	0.000	1.285	0.000	0.000	0.000
Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	61	0	0	0	0	0	111
normalized size	1	1.00	0.39	0.00	0.00	0.00	0.00	0.00	0.72
time (sec)	N/A	0.199	0.512	0.402	0.000	0.000	0.000	0.000	3.053
Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	54	0	0	0	0	0	86
normalized size	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	0.73
time (sec)	N/A	0.176	0.333	0.176	0.000	0.000	0.000	0.000	2.808
Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	44	0	0	0	0	0	61
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.110	0.203	0.169	0.000	0.000	0.000	0.000	2.692

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	52	0	0	0	0	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.214	0.174	0.000	0.000	0.000	0.000	0.000
Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	65	0	0	0	0	0	-1
normalized size	1	1.00	0.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.243	0.183	0.000	0.000	0.000	0.000	0.000
Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	87	0	0	0	0	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.284	0.184	0.000	0.000	0.000	0.000	0.000
Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	113	0	0	0	0	0	216
normalized size	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	0.55
time (sec)	N/A	0.375	1.156	0.177	0.000	0.000	0.000	0.000	4.147
Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	95	0	0	0	0	0	159
normalized size	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	0.60
time (sec)	N/A	0.274	0.808	0.170	0.000	0.000	0.000	0.000	3.765
Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	73	0	0	0	0	0	123
normalized size	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	0.73
time (sec)	N/A	0.143	0.639	0.168	0.000	0.000	0.000	0.000	1.206

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	150	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.662	1.223	0.171	0.000	0.000	0.000	0.000	0.000
Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	231	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.665	1.438	0.171	0.000	0.000	0.000	0.000	0.000
Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	317	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.741	1.838	0.168	0.000	0.000	0.000	0.000	0.000
Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	767	767	247	0	0	0	0	0	-1
normalized size	1	1.00	0.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.351	2.658	0.208	0.000	0.000	0.000	0.000	0.000
Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	555	194	0	0	0	0	0	-1
normalized size	1	1.00	0.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.877	1.888	0.187	0.000	0.000	0.000	0.000	0.000
Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	154	0	0	0	0	0	-1
normalized size	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.511	1.220	0.354	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.646	3.544	0.188	0.000	0.000	0.000	0.000	0.000
Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	193	0	0	0	0	0	-1
normalized size	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.550	2.082	0.144	0.000	0.729	0.000	0.000	0.000
Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	154	0	0	0	0	0	-1
normalized size	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.155	1.556	0.150	0.000	0.000	0.000	0.000	0.000
Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	150	0	0	72	0	0	88
normalized size	1	1.00	0.88	0.00	0.00	0.42	0.00	0.00	0.51
time (sec)	N/A	0.973	0.603	0.146	0.000	0.556	0.000	0.000	3.316
Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	85	154	56	75	0	0	-1
normalized size	1	1.00	0.28	0.51	0.19	0.25	0.00	0.00	-0.00
time (sec)	N/A	0.438	0.090	0.145	0.494	0.771	0.000	0.000	0.000
Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	51	24	23	32	27	51	54
normalized size	1	1.00	2.83	1.33	1.28	1.78	1.50	2.83	3.00
time (sec)	N/A	0.098	0.015	0.099	0.307	0.665	2.168	0.175	2.476

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	67	52	54	60	61	100	403
normalized size	1	1.00	1.18	0.91	0.95	1.05	1.07	1.75	7.07
time (sec)	N/A	0.202	0.081	0.100	0.319	0.766	3.542	0.162	2.464
Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	80	77	84	77	92	125	431
normalized size	1	1.00	1.07	1.03	1.12	1.03	1.23	1.67	5.75
time (sec)	N/A	0.297	0.102	0.106	0.314	1.193	7.628	0.176	2.477
Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	97	103	118	89	116	149	460
normalized size	1	1.00	0.93	0.99	1.13	0.86	1.12	1.43	4.42
time (sec)	N/A	0.403	0.123	0.115	0.310	0.695	15.258	0.160	2.523
Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	71	46	63	47	0	46	25
normalized size	1	1.00	2.84	1.84	2.52	1.88	0.00	1.84	1.00
time (sec)	N/A	0.093	0.084	0.088	0.423	1.141	0.000	0.152	2.364
Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	76	71	93	85	0	77	50
normalized size	1	1.00	1.58	1.48	1.94	1.77	0.00	1.60	1.04
time (sec)	N/A	0.185	0.207	0.087	0.463	0.741	0.000	0.162	2.349
Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	88	95	119	122	0	102	92
normalized size	1	1.00	1.17	1.27	1.59	1.63	0.00	1.36	1.23
time (sec)	N/A	0.311	0.347	0.096	0.639	0.742	0.000	0.162	2.366

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	104	119	143	158	0	126	140
normalized size	1	1.00	1.08	1.24	1.49	1.65	0.00	1.31	1.46
time (sec)	N/A	0.414	0.750	0.096	0.771	0.568	0.000	0.155	2.340
Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	78	228	43	123	0	134	-1
normalized size	1	1.00	0.80	2.33	0.44	1.26	0.00	1.37	-0.01
time (sec)	N/A	0.432	0.158	0.382	1.119	0.730	0.000	0.210	0.000
Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	62	199	26	99	0	92	-1
normalized size	1	1.00	0.86	2.76	0.36	1.38	0.00	1.28	-0.01
time (sec)	N/A	0.294	0.101	0.310	1.105	0.656	0.000	0.189	0.000
Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	152	13	81	0	61	-1
normalized size	1	1.00	1.07	3.45	0.30	1.84	0.00	1.39	-0.02
time (sec)	N/A	0.160	0.036	0.303	0.470	0.779	0.000	0.180	0.000
Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	52	192	58	116	0	133	-1
normalized size	1	1.00	0.76	2.82	0.85	1.71	0.00	1.96	-0.01
time (sec)	N/A	0.195	0.048	0.348	1.053	0.606	0.000	0.702	0.000
Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	73	270	0	187	0	168	-1
normalized size	1	1.00	0.79	2.93	0.00	2.03	0.00	1.83	-0.01
time (sec)	N/A	0.336	0.370	0.345	0.000	0.698	0.000	0.764	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	95	322	0	234	0	199	-1
normalized size	1	1.00	0.79	2.68	0.00	1.95	0.00	1.66	-0.01
time (sec)	N/A	0.482	0.500	0.355	0.000	0.728	0.000	0.945	0.000
Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	80	0	35	0	283	-1
normalized size	1	1.00	1.00	3.20	0.00	1.40	0.00	11.32	-0.04
time (sec)	N/A	0.053	0.199	0.702	0.000	0.714	0.000	0.324	0.000
Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	91	0	36	0	397	68
normalized size	1	1.00	1.00	3.79	0.00	1.50	0.00	16.54	2.83
time (sec)	N/A	0.056	0.126	0.408	0.000	0.741	0.000	0.323	2.745
Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	15	41	8	8
normalized size	1	1.00	1.00	1.12	1.00	1.88	5.12	1.00	1.00
time (sec)	N/A	0.041	0.008	0.081	0.439	0.649	0.970	0.126	2.307
Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	16	15	35	248	49	26
normalized size	1	1.00	0.67	0.44	0.42	0.97	6.89	1.36	0.72
time (sec)	N/A	0.041	0.028	0.083	0.551	0.655	48.461	0.129	2.324
Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	10	15	12	16	8
normalized size	1	1.00	1.00	1.38	1.25	1.88	1.50	2.00	1.00
time (sec)	N/A	0.041	0.007	0.078	0.773	0.646	0.859	0.150	2.287

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	17	16	35	61	49	27
normalized size	1	1.00	0.62	0.46	0.43	0.95	1.65	1.32	0.73
time (sec)	N/A	0.038	0.031	0.075	0.423	1.143	2.660	0.130	2.286
Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	32	0	13	61	26	13
normalized size	1	1.00	1.00	2.29	0.00	0.93	4.36	1.86	0.93
time (sec)	N/A	0.128	0.010	0.098	0.000	0.635	56.758	0.147	2.474
Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	73	148	0	254	2608	110	2429
normalized size	1	1.00	0.70	1.41	0.00	2.42	24.84	1.05	23.13
time (sec)	N/A	0.260	0.152	0.108	0.000	1.017	111.367	0.138	4.043
Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	34	44	29	45	143	62	242
normalized size	1	1.00	0.60	0.77	0.51	0.79	2.51	1.09	4.25
time (sec)	N/A	0.134	0.085	0.128	0.423	1.328	10.667	0.139	2.427
Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	17	19	24	29	13
normalized size	1	1.00	1.00	1.33	1.13	1.27	1.60	1.93	0.87
time (sec)	N/A	0.089	0.013	0.109	0.420	0.612	1.178	0.145	2.309
Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	47	78	40	228	0	58	1987
normalized size	1	1.00	0.96	1.59	0.82	4.65	0.00	1.18	40.55
time (sec)	N/A	0.150	0.161	0.118	1.340	0.791	0.000	0.155	2.848

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	28	0	12	56	22	13
normalized size	1	1.00	1.00	2.15	0.00	0.92	4.31	1.69	1.00
time (sec)	N/A	0.119	0.011	0.115	0.000	0.680	103.318	0.145	2.461
Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	72	153	0	262	0	93	1646
normalized size	1	1.00	0.72	1.53	0.00	2.62	0.00	0.93	16.46
time (sec)	N/A	0.240	0.183	0.080	0.000	0.782	0.000	0.155	5.150
Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	31	42	28	45	520	62	249
normalized size	1	1.00	0.55	0.75	0.50	0.80	9.29	1.11	4.45
time (sec)	N/A	0.197	0.081	0.121	0.433	1.011	55.654	0.151	2.387
Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	14	18	51	23	12
normalized size	1	1.00	1.00	1.21	1.00	1.29	3.64	1.64	0.86
time (sec)	N/A	0.058	0.012	0.109	0.429	1.077	1.331	0.149	2.315
Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	47	84	42	255	0	70	1774
normalized size	1	1.00	0.96	1.71	0.86	5.20	0.00	1.43	36.20
time (sec)	N/A	0.161	0.151	0.125	0.429	2.032	0.000	0.143	2.946
Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	98	135	0	318	0	125	1302
normalized size	1	1.00	1.32	1.82	0.00	4.30	0.00	1.69	17.59
time (sec)	N/A	0.246	0.451	0.100	0.000	3.458	0.000	0.165	3.514

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	102	120	0	332	0	110	463
normalized size	1	1.00	1.42	1.67	0.00	4.61	0.00	1.53	6.43
time (sec)	N/A	0.238	0.530	0.106	0.000	3.955	0.000	0.170	2.832
Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	94	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.238	1.001	0.000	1.855	0.000	0.000	0.000
Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.171	1.055	0.000	1.177	0.000	0.000	0.000
Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	246	321	257	257	461	316	422
normalized size	1	1.00	1.94	2.53	2.02	2.02	3.63	2.49	3.32
time (sec)	N/A	0.078	1.022	0.414	0.330	1.010	6.556	0.664	6.158
Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	192	285	238	219	770	235	519
normalized size	1	1.00	1.19	1.77	1.48	1.36	4.78	1.46	3.22
time (sec)	N/A	0.079	0.691	0.335	0.330	1.184	4.623	0.315	4.117
Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	156	175	172	155	267	187	248
normalized size	1	1.00	1.66	1.86	1.83	1.65	2.84	1.99	2.64
time (sec)	N/A	0.045	0.455	0.267	0.748	0.863	2.217	0.292	2.720

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	107	153	136	121	381	122	320
normalized size	1	1.00	0.99	1.42	1.26	1.12	3.53	1.13	2.96
time (sec)	N/A	0.044	0.303	0.253	0.323	1.110	1.271	0.188	3.461
Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	81	75	84	77	117	91	104
normalized size	1	1.00	1.40	1.29	1.45	1.33	2.02	1.57	1.79
time (sec)	N/A	0.024	0.323	0.242	0.310	1.318	0.520	0.154	2.480
Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	52	70	68	52	128	50	63
normalized size	1	1.00	0.95	1.27	1.24	0.95	2.33	0.91	1.15
time (sec)	N/A	0.019	0.102	0.239	0.474	1.096	0.277	0.154	2.418
Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	46	25	24	23	31	24	38
normalized size	1	1.00	1.92	1.04	1.00	0.96	1.29	1.00	1.58
time (sec)	N/A	0.014	0.012	0.044	0.401	1.269	0.145	0.141	2.324
Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	43	80	131	0	74	39
normalized size	1	1.00	0.96	0.91	1.70	2.79	0.00	1.57	0.83
time (sec)	N/A	0.027	0.060	0.404	0.682	1.740	0.000	0.221	2.801
Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	21	21	57	0	20	47
normalized size	1	1.00	1.00	0.66	0.66	1.78	0.00	0.62	1.47
time (sec)	N/A	0.016	0.036	0.490	0.523	0.586	0.000	0.154	2.342

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	132	191	326	294	0	221	260
normalized size	1	1.00	1.28	1.85	3.17	2.85	0.00	2.15	2.52
time (sec)	N/A	0.057	0.294	0.534	0.463	1.375	0.000	0.230	4.555
Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	85	64	85	217	0	50	222
normalized size	1	1.00	0.87	0.65	0.87	2.21	0.00	0.51	2.27
time (sec)	N/A	0.042	0.290	0.559	0.338	0.674	0.000	0.153	3.116
Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	157	514	822	544	0	588	719
normalized size	1	1.00	1.01	3.29	5.27	3.49	0.00	3.77	4.61
time (sec)	N/A	0.086	1.147	0.604	1.038	1.126	0.000	0.309	6.093
Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	182	125	174	441	0	118	470
normalized size	1	1.00	1.21	0.83	1.15	2.92	0.00	0.78	3.11
time (sec)	N/A	0.069	0.523	0.644	0.790	2.052	0.000	0.221	5.151
Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	205	183	0	0	0	0	-1
normalized size	1	1.00	1.10	0.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	1.816	0.444	0.000	1.095	0.000	0.000	0.000
Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	256	246	0	0	0	0	-1
normalized size	1	1.00	1.95	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	1.600	0.372	0.000	0.929	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	143	163	0	0	0	0	-1
normalized size	1	1.00	1.09	1.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	1.290	0.372	0.000	0.811	0.000	0.000	0.000
Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	268	159	0	0	0	0	-1
normalized size	1	1.00	3.57	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	1.096	0.355	0.000	0.801	0.000	0.000	0.000
Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	92	121	0	0	0	0	-1
normalized size	1	1.00	1.23	1.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.188	0.251	0.000	0.871	0.000	0.000	0.000
Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	219	228	0	0	0	0	-1
normalized size	1	1.00	1.59	1.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	3.058	0.346	0.000	1.931	0.000	0.000	0.000
Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	145	178	0	0	0	0	-1
normalized size	1	1.00	1.02	1.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	1.684	0.335	0.000	1.884	0.000	0.000	0.000
Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	277	309	0	0	0	0	-1
normalized size	1	1.00	1.41	1.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	2.436	0.379	0.000	1.394	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	153	128	0	0	0	0	-1
normalized size	1	1.00	1.28	1.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.476	0.346	0.000	0.598	0.000	0.000	0.000
Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	199	174	0	0	0	0	-1
normalized size	1	1.00	2.65	2.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.820	0.372	0.000	0.971	0.000	0.000	0.000
Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	133	108	0	0	0	0	-1
normalized size	1	1.00	1.77	1.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.299	0.336	0.000	1.247	0.000	0.000	0.000
Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	184	112	0	0	0	0	-1
normalized size	1	1.00	6.81	4.15	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	0.811	0.333	0.000	1.187	0.000	0.000	0.000
Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	88	85	0	0	0	0	-1
normalized size	1	1.00	3.26	3.15	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.094	0.243	0.000	1.768	0.000	0.000	0.000
Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	190	162	0	0	0	0	-1
normalized size	1	1.00	2.60	2.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	1.032	0.336	0.000	1.090	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	157	118	0	0	0	0	-1
normalized size	1	1.00	2.09	1.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.706	0.332	0.000	0.505	0.000	0.000	0.000
Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	224	205	0	0	0	0	-1
normalized size	1	1.00	1.87	1.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	1.951	0.349	0.000	1.544	0.000	0.000	0.000
Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	31	59	23	44	23	-1
normalized size	1	1.00	0.97	0.97	1.84	0.72	1.38	0.72	-0.03
time (sec)	N/A	0.015	0.085	0.259	0.424	1.829	0.254	0.447	0.000
Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	151	132	17	37	52	84
normalized size	1	1.00	1.00	4.87	4.26	0.55	1.19	1.68	2.71
time (sec)	N/A	0.018	0.111	0.292	0.461	1.834	0.141	0.190	2.555
Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	76	83	17	37	52	66
normalized size	1	1.00	1.00	2.45	2.68	0.55	1.19	1.68	2.13
time (sec)	N/A	0.016	0.077	0.276	0.311	0.852	0.138	0.183	2.465
Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	73	69	17	37	52	44
normalized size	1	1.00	1.00	2.35	2.23	0.55	1.19	1.68	1.42
time (sec)	N/A	0.014	0.052	0.261	0.544	1.691	0.134	0.151	2.425

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	51	26	24	15	26	24	20
normalized size	1	1.00	1.96	1.00	0.92	0.58	1.00	0.92	0.77
time (sec)	N/A	0.014	0.012	0.002	1.120	1.932	0.123	0.150	2.389
Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	29	17	31	21	25
normalized size	1	1.00	1.00	0.79	1.00	0.59	1.07	0.72	0.86
time (sec)	N/A	0.015	0.032	0.402	0.323	0.917	0.142	0.135	2.389
Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	23	22	17	46	30	31
normalized size	1	1.00	1.00	0.74	0.71	0.55	1.48	0.97	1.00
time (sec)	N/A	0.016	0.040	0.398	0.328	0.935	0.144	0.153	2.412
Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	57	29	17	46	36	68
normalized size	1	1.00	1.00	1.84	0.94	0.55	1.48	1.16	2.19
time (sec)	N/A	0.015	0.045	0.429	0.628	1.621	0.145	0.146	2.465
Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	36	29	17	46	44	91
normalized size	1	1.00	1.00	1.16	0.94	0.55	1.48	1.42	2.94
time (sec)	N/A	0.015	0.047	0.420	0.566	1.329	0.150	0.165	2.558
Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	51	17	0	17	35
normalized size	1	1.00	0.97	0.85	1.55	0.52	0.00	0.52	1.06
time (sec)	N/A	0.016	0.030	0.363	0.693	1.469	0.000	0.567	0.432

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	51	17	0	17	33
normalized size	1	1.00	0.97	0.85	1.55	0.52	0.00	0.52	1.00
time (sec)	N/A	0.016	0.029	0.220	0.903	1.080	0.000	0.457	2.378
Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	28	51	17	0	25	-1
normalized size	1	1.00	0.97	0.90	1.65	0.55	0.00	0.81	-0.03
time (sec)	N/A	0.016	0.022	0.219	0.845	2.385	0.000	0.138	0.000
Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	28	51	17	0	37	-1
normalized size	1	1.00	0.97	0.90	1.65	0.55	0.00	1.19	-0.03
time (sec)	N/A	0.017	0.034	0.221	0.429	1.039	0.000	0.335	0.000
Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	51	17	0	65	-1
normalized size	1	1.00	0.97	0.85	1.55	0.52	0.00	1.97	-0.03
time (sec)	N/A	0.016	0.031	0.214	1.064	1.487	0.000	1.228	0.000
Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	51	17	0	67	-1
normalized size	1	1.00	0.97	0.85	1.55	0.52	0.00	2.03	-0.03
time (sec)	N/A	0.016	0.034	0.211	0.662	0.969	0.000	3.299	0.000
Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	303	199	204	166	308	178	272
normalized size	1	1.00	2.03	1.34	1.37	1.11	2.07	1.19	1.83
time (sec)	N/A	0.211	1.239	0.184	0.318	0.925	7.121	0.154	2.883

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	96	96	72	80	97	131	115
normalized size	1	1.00	0.96	0.96	0.72	0.80	0.97	1.31	1.15
time (sec)	N/A	0.196	0.197	0.089	1.049	0.839	4.224	0.160	2.532
Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	123	82	95	85	122	86	126
normalized size	1	1.00	1.64	1.09	1.27	1.13	1.63	1.15	1.68
time (sec)	N/A	0.136	0.572	0.082	0.353	2.072	4.936	0.142	2.503
Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	26	26	29	22	40	40
normalized size	1	1.00	0.93	0.96	0.96	1.07	0.81	1.48	1.48
time (sec)	N/A	0.054	0.047	0.049	0.410	1.622	1.254	0.161	2.380
Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	42	16	14	25	24	34	37
normalized size	1	1.00	3.50	1.33	1.17	2.08	2.00	2.83	3.08
time (sec)	N/A	0.007	0.005	0.003	0.303	1.495	0.091	0.129	2.493
Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	50	11	32	12	55
normalized size	1	1.00	1.00	1.09	4.55	1.00	2.91	1.09	5.00
time (sec)	N/A	0.034	0.007	0.118	0.504	1.020	0.418	0.130	3.792
Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	344	106	0	308	0	94	604
normalized size	1	1.00	5.21	1.61	0.00	4.67	0.00	1.42	9.15
time (sec)	N/A	0.133	2.021	0.163	0.000	1.058	0.000	0.168	2.806

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	40	57	201	83	503	43	106
normalized size	1	1.00	0.78	1.12	3.94	1.63	9.86	0.84	2.08
time (sec)	N/A	0.075	0.173	0.174	0.513	0.947	2.759	0.138	2.698
Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	2661	967	0	931	0	369	2782
normalized size	1	1.00	17.06	6.20	0.00	5.97	0.00	2.37	17.83
time (sec)	N/A	0.337	6.359	0.208	0.000	1.403	0.000	0.168	6.373
Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	86	130	483	217	1719	91	541
normalized size	1	1.00	0.85	1.29	4.78	2.15	17.02	0.90	5.36
time (sec)	N/A	0.116	0.338	0.211	0.507	1.058	15.190	0.139	3.837
Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	54	106	141	38	68	62	59
normalized size	1	1.00	1.80	3.53	4.70	1.27	2.27	2.07	1.97
time (sec)	N/A	0.050	0.106	0.118	0.344	0.957	7.127	0.141	2.438
Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	64	71	28	61	44	20	20
normalized size	1	1.00	2.13	2.37	0.93	2.03	1.47	0.67	0.67
time (sec)	N/A	0.102	0.133	0.095	0.440	0.911	4.096	0.155	2.373
Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	31	45	52	21	44	48	43
normalized size	1	1.00	1.72	2.50	2.89	1.17	2.44	2.67	2.39
time (sec)	N/A	0.044	0.023	0.096	0.329	0.907	4.807	0.172	2.395

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	15	14	28	10	14	14
normalized size	1	1.00	0.88	0.94	0.88	1.75	0.62	0.88	0.88
time (sec)	N/A	0.070	0.011	0.046	0.451	0.890	1.108	0.130	2.357
Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	9	38	13	10	9	20	31	19
normalized size	1	0.69	2.92	1.00	0.77	0.69	1.54	2.38	1.46
time (sec)	N/A	0.006	0.005	0.001	0.310	3.009	0.091	0.147	2.399
Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	16	6	31	5	17	22	21
normalized size	1	1.00	3.20	1.20	6.20	1.00	3.40	4.40	4.20
time (sec)	N/A	0.024	0.013	0.116	0.316	0.610	0.138	0.138	2.776
Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	27	15	28	25	0	14	14
normalized size	1	1.00	1.93	1.07	2.00	1.79	0.00	1.00	1.00
time (sec)	N/A	0.041	0.024	0.118	0.413	1.006	0.000	0.150	2.342
Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	34	17	64	20	301	45	41
normalized size	1	1.00	2.12	1.06	4.00	1.25	18.81	2.81	2.56
time (sec)	N/A	0.046	0.019	0.158	0.442	1.067	0.830	0.162	2.359
Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	62	23	64	63	0	20	19
normalized size	1	1.00	2.38	0.88	2.46	2.42	0.00	0.77	0.73
time (sec)	N/A	0.069	0.068	0.151	0.438	0.862	0.000	0.155	2.340

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	39	23	92	35	1059	64	61
normalized size	1	1.00	1.77	1.05	4.18	1.59	48.14	2.91	2.77
time (sec)	N/A	0.048	0.046	0.184	0.661	1.057	2.762	0.150	2.379
Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	143	204	188	292	308	169	174
normalized size	1	1.00	0.94	1.34	1.24	1.92	2.03	1.11	1.14
time (sec)	N/A	0.220	0.696	0.136	0.309	0.872	102.278	0.128	2.580
Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	95	93	80	95	97	215	127
normalized size	1	1.00	0.94	0.92	0.79	0.94	0.96	2.13	1.26
time (sec)	N/A	0.215	0.261	0.067	0.414	0.951	33.025	0.158	2.530
Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	79	87	87	128	124	86	82
normalized size	1	1.00	1.03	1.13	1.13	1.66	1.61	1.12	1.06
time (sec)	N/A	0.136	0.288	0.068	0.313	0.969	14.106	0.145	2.448
Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	29	29	28	31	52	30
normalized size	1	1.00	0.83	1.00	1.00	0.97	1.07	1.79	1.03
time (sec)	N/A	0.056	0.134	0.042	0.425	0.923	2.786	0.150	2.417
Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	25	16	15	27	24	15	27
normalized size	1	1.00	2.08	1.33	1.25	2.25	2.00	1.25	2.25
time (sec)	N/A	0.007	0.007	0.003	0.314	0.923	0.095	0.141	2.408

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	45	12	0	13	36
normalized size	1	1.00	1.00	1.08	3.75	1.00	0.00	1.08	3.00
time (sec)	N/A	0.035	0.016	0.108	0.404	1.204	0.000	0.121	3.283
Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	71	86	0	307	0	107	440
normalized size	1	1.00	1.06	1.28	0.00	4.58	0.00	1.60	6.57
time (sec)	N/A	0.117	0.269	0.122	0.000	1.045	0.000	0.137	3.044
Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	77	56	177	70	0	45	311
normalized size	1	1.00	1.54	1.12	3.54	1.40	0.00	0.90	6.22
time (sec)	N/A	0.079	0.112	0.119	0.428	0.844	0.000	0.126	2.770
Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	150	534	0	878	0	282	3068
normalized size	1	1.00	0.94	3.36	0.00	5.52	0.00	1.77	19.30
time (sec)	N/A	0.338	0.484	0.143	0.000	2.076	0.000	0.195	8.246
Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	138	132	497	166	0	93	538
normalized size	1	1.00	1.38	1.32	4.97	1.66	0.00	0.93	5.38
time (sec)	N/A	0.123	0.335	0.128	0.480	2.030	0.000	0.128	3.680
Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	32	105	125	37	68	22	34
normalized size	1	1.00	1.14	3.75	4.46	1.32	2.43	0.79	1.21
time (sec)	N/A	0.051	0.072	0.108	0.315	1.584	107.210	0.124	2.417

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	68	56	36	44	20	16
normalized size	1	1.00	1.00	2.27	1.87	1.20	1.47	0.67	0.53
time (sec)	N/A	0.101	0.044	0.088	0.418	1.702	35.042	0.131	2.413
Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	49	46	22	46	18	25
normalized size	1	1.00	1.00	2.45	2.30	1.10	2.30	0.90	1.25
time (sec)	N/A	0.048	0.039	0.081	0.309	1.675	14.638	0.146	2.385
Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	15	16	16	17	12	10
normalized size	1	1.00	0.75	0.94	1.00	1.00	1.06	0.75	0.62
time (sec)	N/A	0.069	0.022	0.043	0.406	1.157	2.703	0.158	2.404
Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	20	13	12	7	20	11	19
normalized size	1	1.00	2.22	1.44	1.33	0.78	2.22	1.22	2.11
time (sec)	N/A	0.005	0.004	0.001	0.304	1.030	0.096	0.151	2.409
Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	9	8	14	9	0	7	9
normalized size	1	1.00	1.29	1.14	2.00	1.29	0.00	1.00	1.29
time (sec)	N/A	0.027	0.011	0.108	0.406	0.915	0.000	0.132	2.857
Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	11	23	18	0	10	10
normalized size	1	1.00	0.86	0.79	1.64	1.29	0.00	0.71	0.71
time (sec)	N/A	0.041	0.012	0.109	0.406	1.096	0.000	0.144	2.407

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	15	28	21	0	14	18
normalized size	1	1.00	1.29	1.07	2.00	1.50	0.00	1.00	1.29
time (sec)	N/A	0.047	0.013	0.151	0.410	0.863	0.000	0.141	2.345
Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	17	35	40	0	16	16
normalized size	1	1.00	1.15	0.65	1.35	1.54	0.00	0.62	0.62
time (sec)	N/A	0.071	0.013	0.142	0.407	0.954	0.000	0.129	2.371
Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	32	25	39	38	0	22	26
normalized size	1	1.00	1.33	1.04	1.62	1.58	0.00	0.92	1.08
time (sec)	N/A	0.050	0.013	0.158	0.413	1.990	0.000	0.121	2.408
Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	39	36	51	44	39	59
normalized size	1	1.00	0.86	0.89	0.82	1.16	1.00	0.89	1.34
time (sec)	N/A	0.034	0.030	0.060	0.314	2.054	8.781	0.125	2.494
Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	61	32	37	57	42	99	75
normalized size	1	1.00	1.79	0.94	1.09	1.68	1.24	2.91	2.21
time (sec)	N/A	0.046	0.020	0.058	0.311	1.771	3.469	0.161	2.488
Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	15	16	20	15	23	21
normalized size	1	1.00	0.82	0.68	0.73	0.91	0.68	1.05	0.95
time (sec)	N/A	0.024	0.015	0.049	0.312	1.330	1.569	0.150	2.404

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	19	12	11	21	19	9	8
normalized size	1	1.00	2.38	1.50	1.38	2.62	2.38	1.12	1.00
time (sec)	N/A	0.005	0.004	0.001	0.318	0.944	0.089	0.148	0.021
Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	5	17	4	0	17	12
normalized size	1	1.00	1.00	2.50	8.50	2.00	0.00	8.50	6.00
time (sec)	N/A	0.019	0.004	0.099	0.308	2.197	0.000	0.138	2.462
Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	14	0	6	6
normalized size	1	1.00	1.00	0.88	0.75	1.75	0.00	0.75	0.75
time (sec)	N/A	0.014	0.003	0.106	0.326	1.696	0.000	0.130	2.406
Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	103	14	0	59	13
normalized size	1	1.00	1.00	0.82	6.06	0.82	0.00	3.47	0.76
time (sec)	N/A	0.038	0.019	0.116	0.311	1.203	0.000	0.132	2.547
Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	14	13	26	0	13	23
normalized size	1	1.00	2.18	0.82	0.76	1.53	0.00	0.76	1.35
time (sec)	N/A	0.018	0.017	0.113	0.311	1.541	0.000	0.122	2.566
Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	187	20	0	101	19
normalized size	1	1.00	1.00	0.80	7.48	0.80	0.00	4.04	0.76
time (sec)	N/A	0.040	0.015	0.122	0.358	0.969	0.000	0.152	2.944

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	57	20	19	40	0	19	33
normalized size	1	1.00	2.28	0.80	0.76	1.60	0.00	0.76	1.32
time (sec)	N/A	0.022	0.017	0.125	0.346	0.859	0.000	0.127	2.915
Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	271	26	0	143	25
normalized size	1	1.00	1.00	0.79	8.21	0.79	0.00	4.33	0.76
time (sec)	N/A	0.043	0.016	0.135	0.343	0.652	0.000	0.143	3.538
Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	37	40	578	44	0	0	-1
normalized size	1	1.00	0.51	0.55	7.92	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.080	0.313	0.572	0.998	0.000	0.000	0.000
Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	29	34	427	35	0	0	-1
normalized size	1	1.00	0.58	0.68	8.54	0.70	0.00	0.00	-0.02
time (sec)	N/A	0.112	0.077	0.282	0.556	1.719	0.000	0.000	0.000
Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	21	26	314	23	0	0	-1
normalized size	1	1.00	0.68	0.84	10.13	0.74	0.00	0.00	-0.03
time (sec)	N/A	0.081	0.036	0.238	0.524	0.911	0.000	0.000	0.000
Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	20	188	19	0	0	15
normalized size	1	1.00	1.00	1.54	14.46	1.46	0.00	0.00	1.15
time (sec)	N/A	0.049	0.025	0.234	0.508	0.770	0.000	0.000	2.483

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	44	0	0	124	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	2.07	0.00	0.00	-0.02
time (sec)	N/A	0.091	0.267	0.257	0.000	1.058	0.000	0.000	0.000
Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	60	450	0	152	0	0	-1
normalized size	1	1.00	0.75	5.62	0.00	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.151	0.549	0.000	1.802	0.000	0.000	0.000
Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	69	382	0	165	0	0	-1
normalized size	1	1.00	0.70	3.86	0.00	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.515	0.353	0.000	1.955	0.000	0.000	0.000
Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	74	487	0	167	0	0	-1
normalized size	1	1.00	0.63	4.13	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.263	0.387	0.000	1.124	0.000	0.000	0.000
Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	40	26	37	44	35	80
normalized size	1	1.00	0.86	0.91	0.59	0.84	1.00	0.80	1.82
time (sec)	N/A	0.031	0.032	0.059	0.322	1.854	8.968	0.135	2.573
Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	30	37	49	42	39	68
normalized size	1	1.00	1.12	0.88	1.09	1.44	1.24	1.15	2.00
time (sec)	N/A	0.042	0.010	0.059	0.319	0.942	3.738	0.146	2.464

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	16	13	12	22	14	18	49
normalized size	1	1.00	0.73	0.59	0.55	1.00	0.64	0.82	2.23
time (sec)	N/A	0.021	0.017	0.046	0.307	0.952	1.622	0.143	2.410
Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	37	12	11	21	19	29	14
normalized size	1	1.00	4.62	1.50	1.38	2.62	2.38	3.62	1.75
time (sec)	N/A	0.005	0.004	0.002	0.315	0.945	0.087	0.154	2.300
Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	7	21	6	0	6	6
normalized size	1	1.00	1.00	1.75	5.25	1.50	0.00	1.50	1.50
time (sec)	N/A	0.018	0.003	0.096	0.322	0.830	0.000	0.149	2.368
Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	18	0	6	6
normalized size	1	1.00	1.00	0.88	0.75	2.25	0.00	0.75	0.75
time (sec)	N/A	0.014	0.003	0.098	0.321	0.901	0.000	0.146	2.450
Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	73	28	0	14	14
normalized size	1	1.00	1.00	0.82	4.29	1.65	0.00	0.82	0.82
time (sec)	N/A	0.038	0.009	0.105	0.319	1.018	0.000	0.132	2.382
Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	14	14	39	0	14	16
normalized size	1	1.00	2.18	0.82	0.82	2.29	0.00	0.82	0.94
time (sec)	N/A	0.017	0.022	0.106	0.321	0.648	0.000	0.141	2.431

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	121	46	0	20	20
normalized size	1	1.00	1.00	0.80	4.84	1.84	0.00	0.80	0.80
time (sec)	N/A	0.041	0.011	0.109	0.335	1.384	0.000	0.143	2.421
Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	57	20	20	57	0	20	46
normalized size	1	1.00	2.28	0.80	0.80	2.28	0.00	0.80	1.84
time (sec)	N/A	0.021	0.019	0.110	0.319	1.471	0.000	0.124	2.446
Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	169	64	0	26	109
normalized size	1	1.00	1.00	0.79	5.12	1.94	0.00	0.79	3.30
time (sec)	N/A	0.042	0.013	0.113	0.334	2.268	0.000	0.149	2.506
Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	37	603	82	44	0	0	-1
normalized size	1	1.00	0.51	8.26	1.12	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.213	0.408	0.441	0.990	0.000	0.000	0.000
Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	29	321	82	38	0	0	-1
normalized size	1	1.00	0.58	6.42	1.64	0.76	0.00	0.00	-0.02
time (sec)	N/A	0.086	0.080	0.330	0.428	0.863	0.000	0.000	0.000
Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	584	57	26	0	0	-1
normalized size	1	1.00	0.74	18.84	1.84	0.84	0.00	0.00	-0.03
time (sec)	N/A	0.064	0.038	0.266	0.428	1.249	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	174	57	22	0	46	20
normalized size	1	1.00	1.00	13.38	4.38	1.69	0.00	3.54	1.54
time (sec)	N/A	0.041	0.026	0.313	0.434	1.260	0.000	0.205	2.424
Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	105	0	72	0	0	-1
normalized size	1	1.00	0.83	2.02	0.00	1.38	0.00	0.00	-0.02
time (sec)	N/A	0.078	0.250	0.281	0.000	1.005	0.000	0.000	0.000
Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	265	0	119	0	114	-1
normalized size	1	1.00	0.78	3.68	0.00	1.65	0.00	1.58	-0.01
time (sec)	N/A	0.094	0.175	0.314	0.000	0.974	0.000	0.366	0.000
Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	73	454	0	147	0	170	-1
normalized size	1	1.00	0.80	4.99	0.00	1.62	0.00	1.87	-0.01
time (sec)	N/A	0.121	0.649	0.320	0.000	1.024	0.000	0.405	0.000
Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	74	494	0	171	0	229	-1
normalized size	1	1.00	0.67	4.49	0.00	1.55	0.00	2.08	-0.01
time (sec)	N/A	0.140	0.358	0.358	0.000	1.630	0.000	0.449	0.000
Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	129	66	68	78	90	1375	88
normalized size	1	1.00	2.35	1.20	1.24	1.42	1.64	25.00	1.60
time (sec)	N/A	0.110	0.203	0.066	0.422	1.741	4.627	5.638	2.539

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	39	42	47	46	173	65
normalized size	1	1.00	1.05	1.03	1.11	1.24	1.21	4.55	1.71
time (sec)	N/A	0.050	0.039	0.069	0.317	2.016	5.985	0.451	2.450
Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	60	25	28	44	31	177	61
normalized size	1	1.00	2.40	1.00	1.12	1.76	1.24	7.08	2.44
time (sec)	N/A	0.063	0.095	0.046	0.409	0.971	1.663	0.202	2.423
Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	8	12	8	11	22
normalized size	1	1.00	1.00	1.10	0.80	1.20	0.80	1.10	2.20
time (sec)	N/A	0.005	0.003	0.003	0.309	1.435	0.051	0.141	2.399
Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	24	25	35	0	28	16
normalized size	1	1.00	1.46	1.00	1.04	1.46	0.00	1.17	0.67
time (sec)	N/A	0.058	0.014	0.106	0.308	1.897	0.000	0.144	2.467
Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	57	32	45	34	0	31	40
normalized size	1	1.00	1.73	0.97	1.36	1.03	0.00	0.94	1.21
time (sec)	N/A	0.127	0.015	0.115	0.310	1.448	0.000	0.140	2.393
Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	83	56	73	130	0	95	48
normalized size	1	1.00	1.38	0.93	1.22	2.17	0.00	1.58	0.80
time (sec)	N/A	0.072	0.017	0.126	0.314	1.821	0.000	0.156	2.386

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	129	64	97	78	0	65	87
normalized size	1	1.00	1.98	0.98	1.49	1.20	0.00	1.00	1.34
time (sec)	N/A	0.211	0.019	0.127	0.324	1.064	0.000	0.151	2.453
Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	68	150	153	144	641	131	695
normalized size	1	1.00	0.92	2.03	2.07	1.95	8.66	1.77	9.39
time (sec)	N/A	0.063	0.197	0.144	0.423	0.930	36.723	0.211	7.017
Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	82	108	146	200	0	130	105
normalized size	1	1.00	1.09	1.44	1.95	2.67	0.00	1.73	1.40
time (sec)	N/A	0.060	0.314	0.168	0.421	1.970	0.000	0.187	2.583
Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	132	177	338	279	0	199	227
normalized size	1	1.00	1.14	1.53	2.91	2.41	0.00	1.72	1.96
time (sec)	N/A	0.110	0.387	0.193	0.436	0.920	0.000	0.238	2.856
Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	67	150	153	143	678	131	692
normalized size	1	1.00	0.92	2.05	2.10	1.96	9.29	1.79	9.48
time (sec)	N/A	0.053	0.144	0.131	0.423	1.663	37.030	0.225	6.525
Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	82	109	156	201	0	132	126
normalized size	1	1.00	1.08	1.43	2.05	2.64	0.00	1.74	1.66
time (sec)	N/A	0.053	0.234	0.158	0.425	0.680	0.000	0.187	2.623

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	118	204	366	279	0	245	251
normalized size	1	1.00	1.02	1.76	3.16	2.41	0.00	2.11	2.16
time (sec)	N/A	0.108	0.337	0.181	0.443	0.932	0.000	0.246	2.879
Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	238	514	354	221	857	287	522
normalized size	1	1.00	0.97	2.09	1.44	0.90	3.48	1.17	2.12
time (sec)	N/A	0.169	1.419	0.302	0.328	1.064	3.038	0.347	7.319
Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	163	250	207	145	415	199	261
normalized size	1	1.00	0.92	1.40	1.16	0.81	2.33	1.12	1.47
time (sec)	N/A	0.102	0.598	0.251	0.322	0.848	1.375	0.235	7.324
Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	111	124	113	81	192	92	100
normalized size	1	1.00	0.96	1.07	0.97	0.70	1.66	0.79	0.86
time (sec)	N/A	0.058	0.212	0.220	0.307	0.945	0.454	0.168	3.075
Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	36	36	35	34	42	35	48
normalized size	1	1.00	0.97	0.97	0.95	0.92	1.14	0.95	1.30
time (sec)	N/A	0.015	0.033	0.039	0.306	0.920	0.142	0.123	2.672
Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	50	40	75	0	43	38
normalized size	1	1.00	1.00	1.02	0.82	1.53	0.00	0.88	0.78
time (sec)	N/A	0.036	0.099	0.289	0.307	1.640	0.000	0.160	2.824

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	98	233	0	192	0	160	274
normalized size	1	1.00	0.76	1.81	0.00	1.49	0.00	1.24	2.12
time (sec)	N/A	0.085	0.247	0.395	0.000	1.736	0.000	0.203	3.673
Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	420	496	0	490	0	346	592
normalized size	1	1.00	2.20	2.60	0.00	2.57	0.00	1.81	3.10
time (sec)	N/A	0.133	2.591	0.522	0.000	0.964	0.000	0.610	8.120
Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	533	823	0	739	0	599	1004
normalized size	1	1.00	2.06	3.18	0.00	2.85	0.00	2.31	3.88
time (sec)	N/A	0.189	2.030	0.710	0.000	1.850	0.000	2.525	12.306
Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	135	177	191	134	291	151	239
normalized size	1	1.00	0.86	1.13	1.22	0.85	1.85	0.96	1.52
time (sec)	N/A	0.143	0.413	0.256	0.321	1.990	0.756	0.183	2.575
Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	101	99	71	170	78	96
normalized size	1	1.00	1.14	1.25	1.22	0.88	2.10	0.96	1.19
time (sec)	N/A	0.050	0.144	0.219	0.305	1.423	0.314	0.163	3.213
Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	29	27	39	29	29
normalized size	1	1.00	1.83	1.03	1.00	0.93	1.34	1.00	1.00
time (sec)	N/A	0.016	0.015	0.001	0.303	0.883	0.140	0.139	2.428

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	57	23	29	60	63	23	22
normalized size	1	1.00	2.28	0.92	1.16	2.40	2.52	0.92	0.88
time (sec)	N/A	0.022	0.050	0.385	0.313	1.340	1.099	0.126	2.820
Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	115	91	90	154	0	86	79
normalized size	1	1.00	1.53	1.21	1.20	2.05	0.00	1.15	1.05
time (sec)	N/A	0.049	0.553	0.490	0.322	0.991	0.000	0.136	2.479
Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	186	211	190	433	0	171	162
normalized size	1	1.00	1.39	1.57	1.42	3.23	0.00	1.28	1.21
time (sec)	N/A	0.112	2.996	0.560	0.336	0.947	0.000	0.166	2.481
Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	492	378	307	791	0	304	260
normalized size	1	1.00	2.38	1.83	1.48	3.82	0.00	1.47	1.26
time (sec)	N/A	0.248	1.727	0.599	0.358	1.051	0.000	0.212	2.526
Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	50	21	28	31	36	21	20
normalized size	1	1.00	2.17	0.91	1.22	1.35	1.57	0.91	0.87
time (sec)	N/A	0.021	0.030	0.396	0.309	1.170	0.609	0.157	2.488
Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	93	60	80	100	168	68	59
normalized size	1	1.00	1.24	0.80	1.07	1.33	2.24	0.91	0.79
time (sec)	N/A	0.048	0.186	0.408	0.324	1.212	1.829	0.166	2.454

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	135	100	146	143	423	107	90
normalized size	1	1.00	1.10	0.81	1.19	1.16	3.44	0.87	0.73
time (sec)	N/A	0.107	0.571	0.447	0.340	1.764	6.486	0.165	2.446
Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	247	140	208	237	792	139	161
normalized size	1	1.00	1.47	0.83	1.24	1.41	4.71	0.83	0.96
time (sec)	N/A	0.186	0.984	0.456	0.346	0.952	23.777	0.201	2.441
Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	136	178	188	134	291	151	258
normalized size	1	1.00	0.87	1.13	1.20	0.85	1.85	0.96	1.64
time (sec)	N/A	0.134	0.434	0.250	0.329	1.947	0.764	0.187	3.214
Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	100	98	71	170	78	84
normalized size	1	1.00	1.14	1.23	1.21	0.88	2.10	0.96	1.04
time (sec)	N/A	0.047	0.145	0.223	0.323	0.525	0.320	0.161	2.504
Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	29	28	39	29	29
normalized size	1	1.00	1.83	1.03	1.00	0.97	1.34	1.00	1.00
time (sec)	N/A	0.014	0.012	0.001	0.311	0.843	0.146	0.149	2.429
Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	50	42	54	60	95	42	26
normalized size	1	1.00	2.00	1.68	2.16	2.40	3.80	1.68	1.04
time (sec)	N/A	0.021	0.151	0.421	0.315	0.905	1.300	0.158	2.617

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	229	110	137	162	0	115	91
normalized size	1	1.00	3.05	1.47	1.83	2.16	0.00	1.53	1.21
time (sec)	N/A	0.053	0.417	0.470	0.332	0.956	0.000	0.168	2.546
Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	350	272	265	438	0	239	186
normalized size	1	1.00	2.61	2.03	1.98	3.27	0.00	1.78	1.39
time (sec)	N/A	0.113	0.612	0.566	0.349	1.085	0.000	0.206	4.468
Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	494	416	382	796	0	363	301
normalized size	1	1.00	2.39	2.01	1.85	3.85	0.00	1.75	1.45
time (sec)	N/A	0.240	1.172	0.593	0.378	2.475	0.000	0.247	6.051
Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	135	177	191	127	291	151	292
normalized size	1	1.00	0.86	1.13	1.22	0.81	1.85	0.96	1.86
time (sec)	N/A	0.140	0.443	0.246	0.329	0.768	0.754	0.202	3.603
Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	101	99	72	170	79	127
normalized size	1	1.00	1.14	1.25	1.22	0.89	2.10	0.98	1.57
time (sec)	N/A	0.047	0.144	0.227	0.319	0.765	0.314	0.165	3.718
Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	29	27	39	29	29
normalized size	1	1.00	1.83	1.03	1.00	0.93	1.34	1.00	1.00
time (sec)	N/A	0.016	0.014	0.002	0.315	2.087	0.138	0.139	2.440

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	93	104	66	54	107	82	33
normalized size	1	1.00	2.82	3.15	2.00	1.64	3.24	2.48	1.00
time (sec)	N/A	0.022	0.067	0.413	0.321	1.752	1.691	0.202	2.830
Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	162	166	185	148	0	196	126
normalized size	1	1.00	1.95	2.00	2.23	1.78	0.00	2.36	1.52
time (sec)	N/A	0.050	0.552	0.534	0.337	1.250	0.000	0.218	2.721
Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	255	639	493	420	0	481	360
normalized size	1	1.00	1.80	4.50	3.47	2.96	0.00	3.39	2.54
time (sec)	N/A	0.112	2.434	0.572	0.348	0.857	0.000	0.241	6.445
Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	632	1069	963	729	0	1006	730
normalized size	1	1.00	2.94	4.97	4.48	3.39	0.00	4.68	3.40
time (sec)	N/A	0.244	3.014	0.586	0.393	1.089	0.000	0.306	7.217
Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	136	176	188	126	291	151	292
normalized size	1	1.00	0.87	1.12	1.20	0.80	1.85	0.96	1.86
time (sec)	N/A	0.136	0.436	0.253	0.320	0.925	0.758	0.182	3.439
Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	100	98	70	170	79	128
normalized size	1	1.00	1.14	1.23	1.21	0.86	2.10	0.98	1.58
time (sec)	N/A	0.047	0.149	0.233	0.324	0.902	0.316	0.142	3.743

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	29	26	39	29	29
normalized size	1	1.00	1.83	1.03	1.00	0.90	1.34	1.00	1.00
time (sec)	N/A	0.014	0.012	0.001	0.328	2.899	0.142	0.144	2.445
Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	96	61	62	57	109	82	32
normalized size	1	1.00	2.91	1.85	1.88	1.73	3.30	2.48	0.97
time (sec)	N/A	0.022	0.101	0.417	0.329	1.984	1.720	0.211	2.739
Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	166	178	182	154	0	198	126
normalized size	1	1.00	2.00	2.14	2.19	1.86	0.00	2.39	1.52
time (sec)	N/A	0.053	0.617	0.507	0.338	0.841	0.000	0.216	2.738
Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	261	687	491	423	0	481	361
normalized size	1	1.00	1.84	4.84	3.46	2.98	0.00	3.39	2.54
time (sec)	N/A	0.108	2.771	0.532	0.359	1.512	0.000	0.241	6.482
Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	636	1149	959	735	0	1006	731
normalized size	1	1.00	2.96	5.34	4.46	3.42	0.00	4.68	3.40
time (sec)	N/A	0.235	1.920	0.614	0.392	1.999	0.000	0.282	7.088
Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	237	335	330	255	682	286	376
normalized size	1	1.00	0.91	1.29	1.27	0.98	2.62	1.10	1.45
time (sec)	N/A	0.400	1.052	0.268	0.333	1.069	1.856	0.301	3.374

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	144	177	189	147	294	167	333
normalized size	1	1.00	0.85	1.04	1.11	0.86	1.73	0.98	1.96
time (sec)	N/A	0.186	0.433	0.243	0.331	2.942	0.755	0.199	3.700
Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	77	99	100	73	162	81	125
normalized size	1	1.00	0.85	1.09	1.10	0.80	1.78	0.89	1.37
time (sec)	N/A	0.046	0.173	0.224	0.311	1.516	0.307	0.150	3.784
Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	49	28	27	26	34	27	40
normalized size	1	1.00	1.81	1.04	1.00	0.96	1.26	1.00	1.48
time (sec)	N/A	0.016	0.012	0.001	0.311	0.887	0.139	0.147	2.506
Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	57	61	0	434	3179	91	75
normalized size	1	1.00	0.93	1.00	0.00	7.11	52.11	1.49	1.23
time (sec)	N/A	0.084	0.121	0.378	0.000	1.705	152.527	0.165	4.011
Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	116	424	0	819	0	222	195
normalized size	1	1.00	0.96	3.50	0.00	6.77	0.00	1.83	1.61
time (sec)	N/A	0.108	0.359	0.499	0.000	1.038	0.000	0.164	3.057
Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	200	3933	0	1947	0	892	700
normalized size	1	1.00	1.02	19.96	0.00	9.88	0.00	4.53	3.55
time (sec)	N/A	0.198	0.952	0.563	0.000	2.014	0.000	0.315	6.055

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	606	16909	0	4069	0	2671	1946
normalized size	1	1.00	2.08	57.91	0.00	13.93	0.00	9.15	6.66
time (sec)	N/A	0.376	2.113	0.658	0.000	1.632	0.000	0.576	4.806
Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	399	701	0	0	0	0	-1
normalized size	1	1.00	2.16	3.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	5.906	0.661	0.000	1.184	0.000	0.000	0.000
Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	349	686	0	0	0	0	-1
normalized size	1	1.00	2.51	4.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	3.420	0.458	0.000	0.868	0.000	0.000	0.000
Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	326	461	0	0	0	0	-1
normalized size	1	1.00	7.24	10.24	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	2.273	0.405	0.000	0.521	0.000	0.000	0.000
Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	128	152	0	0	0	0	-1
normalized size	1	1.00	2.84	3.38	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.258	0.314	0.000	2.261	0.000	0.000	0.000
Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	390	437	0	0	0	0	-1
normalized size	1	1.00	4.15	4.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	6.035	0.490	0.000	1.602	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	430	542	0	0	0	0	-1
normalized size	1	1.00	2.30	2.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	3.182	0.606	0.000	2.882	0.000	0.000	0.000
Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	436	571	0	0	0	0	-1
normalized size	1	1.00	1.87	2.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.260	3.838	0.626	0.000	0.835	0.000	0.000	0.000
Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	3767	2303	0	0	0	0	-1
normalized size	1	1.00	10.86	6.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.532	6.619	1.003	0.000	1.880	0.000	0.000	0.000
Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	2190	1516	0	0	0	0	-1
normalized size	1	1.00	7.74	5.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.282	6.264	0.708	0.000	0.856	0.000	0.000	0.000
Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	1408	720	0	0	0	0	-1
normalized size	1	1.00	13.04	6.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	6.231	0.460	0.000	0.751	0.000	0.000	0.000
Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	285	303	0	0	0	0	-1
normalized size	1	1.00	2.64	2.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.562	0.366	0.000	3.145	0.000	0.000	0.000

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	1540	2388	0	0	0	0	-1
normalized size	1	1.00	8.28	12.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	6.354	0.773	0.000	0.742	0.000	0.000	0.000
Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	2408	2967	0	0	0	0	-1
normalized size	1	1.00	6.30	7.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.363	6.409	2.135	0.000	2.965	0.000	0.000	0.000
Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	4116	3876	0	0	0	0	-1
normalized size	1	1.00	8.40	7.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.619	6.660	6.686	0.000	1.124	0.000	0.000	0.000
Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	130	74	0	101	0	0	-1
normalized size	1	1.00	0.94	0.53	0.00	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.595	0.367	0.000	1.086	0.000	0.000	0.000
Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	104	60	0	81	0	0	-1
normalized size	1	1.00	1.12	0.65	0.00	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.333	0.346	0.000	0.918	0.000	0.000	0.000
Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	75	50	0	61	0	0	39
normalized size	1	1.00	1.70	1.14	0.00	1.39	0.00	0.00	0.89
time (sec)	N/A	0.018	0.040	0.297	0.000	1.839	0.000	0.000	0.310

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	101	77	0	147	0	0	-1
normalized size	1	1.00	2.10	1.60	0.00	3.06	0.00	0.00	-0.02
time (sec)	N/A	0.065	0.105	0.254	0.000	1.171	0.000	0.000	0.000
Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	154	117	0	268	0	284	-1
normalized size	1	1.00	1.60	1.22	0.00	2.79	0.00	2.96	-0.01
time (sec)	N/A	0.053	0.295	0.313	0.000	0.926	0.000	0.744	0.000
Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	180	190	0	341	0	417	-1
normalized size	1	1.00	1.27	1.34	0.00	2.40	0.00	2.94	-0.01
time (sec)	N/A	0.077	0.405	0.316	0.000	2.933	0.000	0.974	0.000
Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	151	86	0	121	0	0	-1
normalized size	1	1.00	0.82	0.46	0.00	0.65	0.00	0.00	-0.01
time (sec)	N/A	0.094	1.818	0.283	0.000	0.846	0.000	0.000	0.000
Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	127	74	0	101	0	0	-1
normalized size	1	1.00	0.91	0.53	0.00	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.489	0.306	0.000	0.652	0.000	0.000	0.000
Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	103	60	0	80	0	0	-1
normalized size	1	1.00	1.11	0.65	0.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.223	0.306	0.000	0.728	0.000	0.000	0.000

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	75	50	0	59	0	0	39
normalized size	1	1.00	1.70	1.14	0.00	1.34	0.00	0.00	0.89
time (sec)	N/A	0.017	0.044	0.296	0.000	0.646	0.000	0.000	0.423
Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	99	77	0	88	0	0	-1
normalized size	1	1.00	2.02	1.57	0.00	1.80	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.093	0.329	0.000	2.112	0.000	0.000	0.000
Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	152	118	0	210	0	249	-1
normalized size	1	1.00	1.58	1.23	0.00	2.19	0.00	2.59	-0.01
time (sec)	N/A	0.052	0.313	0.369	0.000	1.034	0.000	0.529	0.000
Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	178	190	0	280	0	381	-1
normalized size	1	1.00	1.25	1.34	0.00	1.97	0.00	2.68	-0.01
time (sec)	N/A	0.075	0.379	0.338	0.000	0.893	0.000	0.805	0.000
Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	11888	306	0	268	0	0	-1
normalized size	1	1.00	46.08	1.19	0.00	1.04	0.00	0.00	-0.00
time (sec)	N/A	0.179	33.456	0.362	0.000	1.522	0.000	0.000	0.000
Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	11771	200	0	189	0	0	-1
normalized size	1	1.00	61.95	1.05	0.00	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.122	34.256	0.368	0.000	0.956	0.000	0.000	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	11679	126	0	125	0	0	-1
normalized size	1	1.00	92.69	1.00	0.00	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.075	33.015	0.369	0.000	0.680	0.000	0.000	0.000
Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	11586	113	0	80	0	0	-1
normalized size	1	1.00	210.65	2.05	0.00	1.45	0.00	0.00	-0.02
time (sec)	N/A	0.033	32.708	0.355	0.000	0.912	0.000	0.000	0.000
Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	63264	172	0	349	0	0	-1
normalized size	1	1.00	718.91	1.95	0.00	3.97	0.00	0.00	-0.01
time (sec)	N/A	0.120	33.827	0.378	0.000	2.776	0.000	0.000	0.000
Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	0	350	0	646	0	0	-1
normalized size	1	1.00	0.00	2.19	0.00	4.04	0.00	0.00	-0.01
time (sec)	N/A	0.133	180.015	0.410	0.000	1.819	0.000	0.000	0.000
Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	0	350	0	895	0	0	-1
normalized size	1	1.00	0.00	1.55	0.00	3.96	0.00	0.00	-0.00
time (sec)	N/A	0.186	180.032	0.402	0.000	2.526	0.000	0.000	0.000
Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	11602	204	0	192	0	0	-1
normalized size	1	1.00	59.19	1.04	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.134	34.189	0.375	0.000	1.122	0.000	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	11512	130	0	127	0	0	-1
normalized size	1	1.00	88.55	1.00	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.081	21.611	0.379	0.000	1.476	0.000	0.000	0.000
Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	11415	117	0	80	0	0	-1
normalized size	1	1.00	200.26	2.05	0.00	1.40	0.00	0.00	-0.02
time (sec)	N/A	0.038	21.948	0.355	0.000	2.892	0.000	0.000	0.000
Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	61904	175	0	107	0	0	-1
normalized size	1	1.00	680.26	1.92	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.098	34.470	0.302	0.000	1.146	0.000	0.000	0.000
Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	0	363	0	442	0	0	-1
normalized size	1	1.00	0.00	2.21	0.00	2.70	0.00	0.00	-0.01
time (sec)	N/A	0.126	180.001	0.362	0.000	0.859	0.000	0.000	0.000
Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	0	363	0	655	0	0	-1
normalized size	1	1.00	0.00	1.56	0.00	2.82	0.00	0.00	-0.00
time (sec)	N/A	0.170	180.061	0.454	0.000	1.197	0.000	0.000	0.000
Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	80	438	0	579	0	160	950
normalized size	1	1.00	0.79	4.34	0.00	5.73	0.00	1.58	9.41
time (sec)	N/A	0.096	0.217	0.116	0.000	1.207	0.000	0.163	11.444

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	30	22	25	41	11	22	25	34
normalized size	1	1.36	1.00	1.14	1.86	0.50	1.00	1.14	1.55
time (sec)	N/A	0.031	0.041	0.112	0.409	2.116	0.256	0.136	2.789
Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	79	414	0	553	0	158	988
normalized size	1	1.00	0.81	4.27	0.00	5.70	0.00	1.63	10.19
time (sec)	N/A	0.127	0.189	0.151	0.000	1.761	0.000	0.145	13.029
Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	53	0	349	0	73	47
normalized size	1	1.00	0.98	1.04	0.00	6.84	0.00	1.43	0.92
time (sec)	N/A	0.076	0.044	0.129	0.000	0.509	0.000	0.174	2.777
Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	120	430	0	663	0	161	977
normalized size	1	1.00	0.85	3.03	0.00	4.67	0.00	1.13	6.88
time (sec)	N/A	0.515	0.284	0.135	0.000	4.999	0.000	0.174	11.437
Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	2490	21186	0	0	0	0	-1
normalized size	1	1.00	6.71	57.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.447	6.455	4.408	0.000	0.879	0.000	0.000	0.000
Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	1580	12462	0	0	0	0	-1
normalized size	1	1.00	13.39	105.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	6.249	1.886	0.000	0.645	0.000	0.000	0.000

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	339	722	0	0	0	0	-1
normalized size	1	1.00	2.87	6.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.866	1.862	0.000	2.897	0.000	0.000	0.000
Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	1732	12574	0	0	0	0	-1
normalized size	1	1.00	7.22	52.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.216	6.402	1.220	0.000	1.837	0.000	0.000	0.000
Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	2708	64693	0	0	0	0	-1
normalized size	1	1.00	5.50	131.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.519	6.553	2.549	0.000	0.957	0.000	0.000	0.000
Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	0	20776	0	0	0	0	-1
normalized size	1	1.00	0.00	56.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	150.813	1.287	0.000	0.934	0.000	0.000	0.000
Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	0	12459	0	0	0	0	-1
normalized size	1	1.00	0.00	105.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	21.221	1.023	0.000	1.117	0.000	0.000	0.000
Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	506	714	0	0	0	0	-1
normalized size	1	1.00	4.29	6.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	2.901	1.527	0.000	0.892	0.000	0.000	0.000

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	0	12564	0	0	0	0	-1
normalized size	1	1.00	0.00	52.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	23.993	0.950	0.000	0.895	0.000	0.000	0.000
Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	0	64683	0	0	0	0	-1
normalized size	1	1.00	0.00	131.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.495	27.744	1.745	0.000	1.568	0.000	0.000	0.000
Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	80	446	0	555	0	158	965
normalized size	1	1.00	0.82	4.55	0.00	5.66	0.00	1.61	9.85
time (sec)	N/A	0.103	0.224	0.132	0.000	2.053	0.000	0.145	13.762
Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	53	0	349	0	73	47
normalized size	1	1.00	0.98	1.04	0.00	6.84	0.00	1.43	0.92
time (sec)	N/A	0.072	0.045	0.112	0.000	0.809	0.000	0.148	2.787
Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	104	184	0	669	0	142	531
normalized size	1	1.00	0.87	1.53	0.00	5.58	0.00	1.18	4.42
time (sec)	N/A	0.533	0.282	0.114	0.000	6.047	0.000	0.169	8.698
Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	51	10	14	24	0	22	9
normalized size	1	1.00	2.43	0.48	0.67	1.14	0.00	1.05	0.43
time (sec)	N/A	0.048	0.023	0.118	0.409	0.886	0.000	0.140	3.141

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	2490	20463	0	0	0	0	-1
normalized size	1	1.00	6.71	55.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.432	6.440	3.116	0.000	1.049	0.000	0.000	0.000
Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	1580	12367	0	0	0	0	-1
normalized size	1	1.00	13.39	104.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	6.242	1.757	0.000	1.232	0.000	0.000	0.000
Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	339	713	0	0	0	0	-1
normalized size	1	1.00	2.87	6.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.907	1.679	0.000	0.666	0.000	0.000	0.000
Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	1732	12236	0	0	0	0	-1
normalized size	1	1.00	7.22	50.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	6.400	1.122	0.000	1.098	0.000	0.000	0.000
Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	2708	62955	0	0	0	0	-1
normalized size	1	1.00	5.50	127.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.497	6.494	2.163	0.000	0.677	0.000	0.000	0.000
Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	0	20454	0	0	0	0	-1
normalized size	1	1.00	0.00	55.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.383	52.856	1.098	0.000	0.627	0.000	0.000	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	0	12365	0	0	0	0	-1
normalized size	1	1.00	0.00	104.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	12.339	1.056	0.000	1.297	0.000	0.000	0.000
Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	519	705	0	0	0	0	-1
normalized size	1	1.00	4.40	5.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	2.845	1.160	0.000	0.859	0.000	0.000	0.000
Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	0	12231	0	0	0	0	-1
normalized size	1	1.00	0.00	50.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.205	20.778	0.957	0.000	1.196	0.000	0.000	0.000
Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	0	62945	0	0	0	0	-1
normalized size	1	1.00	0.00	127.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.491	25.269	1.652	0.000	1.943	0.000	0.000	0.000
Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	10	1	1
normalized size	1	1.00	1.00	2.00	1.00	1.00	10.00	1.00	1.00
time (sec)	N/A	0.009	0.000	0.059	0.405	0.891	0.357	0.138	2.645
Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	22	1	1
normalized size	1	1.00	1.00	2.00	1.00	1.00	22.00	1.00	1.00
time (sec)	N/A	0.009	0.000	0.047	0.411	1.487	0.830	0.120	2.630

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	34	1	1
normalized size	1	1.00	1.00	2.00	1.00	1.00	34.00	1.00	1.00
time (sec)	N/A	0.009	0.000	0.049	0.411	0.986	2.096	0.127	2.594
Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	4	15	23	36	33	3
normalized size	1	1.00	2.09	0.36	1.36	2.09	3.27	3.00	0.27
time (sec)	N/A	0.015	0.007	0.098	0.321	0.850	0.345	0.131	2.905
Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	8	18	12	15	48	6	6
normalized size	1	1.00	0.62	1.38	0.92	1.15	3.69	0.46	0.46
time (sec)	N/A	0.023	0.003	0.105	0.304	1.306	1.336	0.119	2.633
Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	22	48	38	74	765	37	32
normalized size	1	1.00	0.69	1.50	1.19	2.31	23.91	1.16	1.00
time (sec)	N/A	0.027	0.006	0.115	0.311	0.764	3.476	0.134	2.651
Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	35	12007	20	9
normalized size	1	1.00	1.00	1.11	1.00	3.89	1334.11	2.22	1.00
time (sec)	N/A	0.018	0.032	0.116	0.405	0.686	22.401	0.146	2.834
Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	31	0	22	11
normalized size	1	1.00	1.00	1.09	1.00	2.82	0.00	2.00	1.00
time (sec)	N/A	0.020	0.031	0.109	0.411	2.044	0.000	0.133	2.832

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	43	0	26	15
normalized size	1	1.00	1.00	1.07	1.00	2.87	0.00	1.73	1.00
time (sec)	N/A	0.026	0.042	0.125	0.404	2.388	0.000	0.127	2.845
Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	25	18	17	43	87	61	36
normalized size	1	1.00	0.47	0.34	0.32	0.81	1.64	1.15	0.68
time (sec)	N/A	0.037	0.040	0.229	0.410	0.986	0.808	0.129	2.759
Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	36	38	35	182	241	48	51
normalized size	1	1.00	0.84	0.88	0.81	4.23	5.60	1.12	1.19
time (sec)	N/A	0.149	0.095	0.110	0.405	0.703	1.564	0.132	2.730
Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	36	36	35	181	267	48	48
normalized size	1	1.00	0.84	0.84	0.81	4.21	6.21	1.12	1.12
time (sec)	N/A	0.109	0.055	0.105	0.405	1.490	1.621	0.135	2.672
Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	19	16	15	35	0	15	15
normalized size	1	1.00	0.53	0.44	0.42	0.97	0.00	0.42	0.42
time (sec)	N/A	0.028	0.046	0.118	0.407	1.079	0.000	0.124	2.652
Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	42	27	27	68	0	27	27
normalized size	1	1.00	0.86	0.55	0.55	1.39	0.00	0.55	0.55
time (sec)	N/A	0.045	0.137	0.124	0.405	1.017	0.000	0.148	2.664

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	79	40	45	100	0	39	40
normalized size	1	1.00	1.07	0.54	0.61	1.35	0.00	0.53	0.54
time (sec)	N/A	0.055	0.180	0.131	0.415	2.123	0.000	0.149	2.698
Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	4	1	1	0	1	1
normalized size	1	1.00	1.00	4.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.012	0.001	0.062	0.428	0.764	0.000	0.139	2.746
Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	4	1	1	0	1	1
normalized size	1	1.00	1.00	4.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.013	0.001	0.062	0.408	0.842	0.000	0.149	2.581
Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	4	1	1	0	1	1
normalized size	1	1.00	1.00	4.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.012	0.000	0.066	0.406	1.529	0.000	0.142	2.571
Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	19	17	16	35	0	49	16
normalized size	1	1.00	0.51	0.46	0.43	0.95	0.00	1.32	0.43
time (sec)	N/A	0.031	0.042	0.159	0.410	1.321	0.000	0.141	2.713
Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	64	28	27	66	0	60	27
normalized size	1	1.00	1.36	0.60	0.57	1.40	0.00	1.28	0.57
time (sec)	N/A	0.040	0.105	0.184	0.404	1.074	0.000	0.136	2.688

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	39	42	98	0	69	43
normalized size	1	1.00	0.92	0.54	0.58	1.36	0.00	0.96	0.60
time (sec)	N/A	0.076	0.156	0.219	0.424	1.625	0.000	0.132	2.683
Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	6	3	3	0	3	3
normalized size	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.013	0.000	0.061	0.423	1.121	0.000	0.150	2.729
Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	4	1	1	0	1	1
normalized size	1	1.00	1.00	4.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.013	0.000	0.065	0.416	0.805	0.000	0.150	2.650
Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	6	3	3	0	3	3
normalized size	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.013	0.000	0.066	0.411	0.497	0.000	0.138	2.615
Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	26	259	0	61	43
normalized size	1	1.00	1.00	0.82	0.79	7.85	0.00	1.85	1.30
time (sec)	N/A	0.050	0.060	0.122	0.431	0.963	0.000	0.142	2.855
Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	507	820	0	2929	0	0	-1
normalized size	1	1.00	2.12	3.43	0.00	12.26	0.00	0.00	-0.00
time (sec)	N/A	0.492	3.019	0.299	0.000	1.347	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	258	1161	0	4357	0	0	-1
normalized size	1	1.00	0.71	3.18	0.00	11.94	0.00	0.00	-0.00
time (sec)	N/A	0.739	3.791	0.288	0.000	2.222	0.000	0.000	0.000
Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	149	255	246	150	566	158	456
normalized size	1	1.00	0.76	1.31	1.26	0.77	2.90	0.81	2.34
time (sec)	N/A	0.393	0.902	0.100	0.323	0.530	2.998	0.183	4.490
Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	77	115	112	76	204	79	88
normalized size	1	1.00	0.71	1.06	1.03	0.70	1.87	0.72	0.81
time (sec)	N/A	0.099	0.286	0.079	0.320	0.740	0.674	0.148	2.898
Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	52	0	23	0	52	39
normalized size	1	1.00	1.00	2.26	0.00	1.00	0.00	2.26	1.70
time (sec)	N/A	0.089	0.058	0.336	0.000	0.830	0.000	0.179	2.841
Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	140	1297	0	795	0	454	497
normalized size	1	1.00	0.89	8.26	0.00	5.06	0.00	2.89	3.17
time (sec)	N/A	0.415	0.961	0.332	0.000	1.981	0.000	0.253	6.063
Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	286	832	0	6695	0	0	10465
normalized size	1	1.00	1.18	3.44	0.00	27.67	0.00	0.00	43.24
time (sec)	N/A	0.940	0.765	0.178	0.000	18.385	0.000	0.000	17.109

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	140	269	556	112	0	239	-1
normalized size	1	1.00	0.42	0.81	1.68	0.34	0.00	0.72	-0.00
time (sec)	N/A	0.323	0.838	0.786	0.453	0.846	0.000	5.029	0.000
Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	70	107	187	43	0	98	-1
normalized size	1	1.00	0.38	0.58	1.01	0.23	0.00	0.53	-0.01
time (sec)	N/A	0.109	0.185	0.586	0.430	0.870	0.000	0.206	0.000
Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	85	176	0	204	0	208	-1
normalized size	1	1.00	0.62	1.28	0.00	1.49	0.00	1.52	-0.01
time (sec)	N/A	0.198	0.165	0.458	0.000	0.925	0.000	0.384	0.000
Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	144	741	0	527	0	479	-1
normalized size	1	1.00	0.60	3.10	0.00	2.21	0.00	2.00	-0.00
time (sec)	N/A	0.272	0.347	0.413	0.000	1.646	0.000	0.750	0.000
Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	33	0	11	0	32	24
normalized size	1	1.00	1.00	3.00	0.00	1.00	0.00	2.91	2.18
time (sec)	N/A	0.081	0.050	0.086	0.000	0.778	0.000	0.169	2.937
Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	241	2556	0	6697	0	5302	11781
normalized size	1	1.00	0.98	10.39	0.00	27.22	0.00	21.55	47.89
time (sec)	N/A	0.789	0.557	0.124	0.000	16.183	0.000	8.397	16.769

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	153	245	150	149	248	2326	205
normalized size	1	1.00	1.06	1.70	1.04	1.03	1.72	16.15	1.42
time (sec)	N/A	0.269	2.100	0.008	0.405	1.715	0.625	5.119	2.854
Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	88	117	74	74	122	709	105
normalized size	1	1.00	1.22	1.62	1.03	1.03	1.69	9.85	1.46
time (sec)	N/A	0.076	0.326	0.007	0.409	1.125	0.250	1.018	2.770
Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	187	222	161	191	1358	204	152
normalized size	1	1.00	1.85	2.20	1.59	1.89	13.45	2.02	1.50
time (sec)	N/A	0.258	2.338	0.164	0.412	0.722	1.623	0.894	3.089
Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	308	458	419	580	0	451	388
normalized size	1	1.00	1.56	2.32	2.13	2.94	0.00	2.29	1.97
time (sec)	N/A	0.535	5.409	0.176	0.432	2.021	0.000	1.938	3.261
Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	147	158	166	102	0	1751	-1
normalized size	1	1.00	0.52	0.56	0.58	0.36	0.00	6.17	-0.00
time (sec)	N/A	0.226	1.332	0.309	0.426	2.830	0.000	2.435	0.000
Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	58	75	65	38	0	395	-1
normalized size	1	1.00	0.48	0.61	0.53	0.31	0.00	3.24	-0.01
time (sec)	N/A	0.101	0.302	0.319	0.421	2.631	0.000	0.460	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	88	114	137	71	0	554	-1
normalized size	1	1.00	0.64	0.83	0.99	0.51	0.00	4.01	-0.01
time (sec)	N/A	0.188	0.684	0.227	0.424	0.725	0.000	3.068	0.000
Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	268	622	498	355	0	1571	-1
normalized size	1	1.00	0.85	1.97	1.58	1.12	0.00	4.97	-0.00
time (sec)	N/A	0.402	3.441	0.190	0.440	2.034	0.000	10.073	0.000
Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	130	246	299	198	0	470	323
normalized size	1	1.00	0.71	1.34	1.62	1.08	0.00	2.55	1.76
time (sec)	N/A	0.430	0.874	0.141	0.337	2.426	0.000	0.391	3.304
Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	110	126	125	0	191	160
normalized size	1	1.00	0.84	1.45	1.66	1.64	0.00	2.51	2.11
time (sec)	N/A	0.078	0.284	0.092	0.337	1.496	0.000	0.252	2.908
Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	97	163	0	279	0	145	444
normalized size	1	1.00	1.05	1.77	0.00	3.03	0.00	1.58	4.83
time (sec)	N/A	0.304	0.387	0.287	0.000	0.847	0.000	0.278	3.023
Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	276	1118	0	1335	0	490	5469
normalized size	1	1.00	1.20	4.86	0.00	5.80	0.00	2.13	23.78
time (sec)	N/A	0.833	1.529	0.348	0.000	2.110	0.000	0.486	11.473

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	128	387	440	162	0	652	-1
normalized size	1	1.00	0.36	1.08	1.23	0.45	0.00	1.82	-0.00
time (sec)	N/A	0.287	0.818	0.608	0.452	1.580	0.000	0.512	0.000
Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	67	211	164	85	0	224	-1
normalized size	1	1.00	0.39	1.22	0.95	0.49	0.00	1.29	-0.01
time (sec)	N/A	0.119	0.259	0.585	0.439	0.961	0.000	0.305	0.000
Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	92	157	0	184	0	1504	-1
normalized size	1	1.00	0.65	1.11	0.00	1.30	0.00	10.59	-0.01
time (sec)	N/A	0.213	0.368	0.665	0.000	1.064	0.000	4.827	0.000
Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	216	756	0	798	0	569	-1
normalized size	1	1.00	0.65	2.29	0.00	2.42	0.00	1.72	-0.00
time (sec)	N/A	0.566	1.004	0.614	0.000	1.086	0.000	5.124	0.000
Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	8	0	6	8	14	16
normalized size	1	1.00	1.12	0.47	0.00	0.35	0.47	0.82	0.94
time (sec)	N/A	0.040	0.005	0.196	0.000	0.703	0.077	0.141	2.760
Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	8	0	6	10	14	13
normalized size	1	1.00	1.12	0.47	0.00	0.35	0.59	0.82	0.76
time (sec)	N/A	0.037	0.006	0.174	0.000	0.652	0.076	0.140	2.745

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	11	7	16	32
normalized size	1	1.00	1.00	1.17	1.00	1.83	1.17	2.67	5.33
time (sec)	N/A	0.023	0.026	0.058	0.309	0.824	0.119	0.176	2.918
Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	39	111	181	59	371	77	1976
normalized size	1	1.00	0.83	2.36	3.85	1.26	7.89	1.64	42.04
time (sec)	N/A	0.041	0.142	0.135	0.422	0.882	0.877	0.167	12.819
Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	75	113	271	194	0	132	129
normalized size	1	1.00	1.01	1.53	3.66	2.62	0.00	1.78	1.74
time (sec)	N/A	0.068	0.241	0.173	0.426	1.627	0.000	0.236	3.007
Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	37	199	152	0	26	95
normalized size	1	1.00	0.97	0.56	3.02	2.30	0.00	0.39	1.44
time (sec)	N/A	0.057	0.189	0.188	0.340	0.540	0.000	0.202	2.823
Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	78	222	243	155	1042	148	1099
normalized size	1	1.00	0.93	2.64	2.89	1.85	12.40	1.76	13.08
time (sec)	N/A	0.058	0.253	0.137	0.416	0.772	38.589	0.248	9.406
Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	92	124	286	226	0	150	141
normalized size	1	1.00	1.08	1.46	3.36	2.66	0.00	1.76	1.66
time (sec)	N/A	0.057	0.275	0.168	0.424	3.023	0.000	0.234	3.051

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	122	218	451	311	0	270	264
normalized size	1	1.00	0.95	1.69	3.50	2.41	0.00	2.09	2.05
time (sec)	N/A	0.124	0.661	0.195	0.453	1.691	0.000	0.258	3.439
Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	95	544	0	625	0	178	1709
normalized size	1	1.00	0.83	4.73	0.00	5.43	0.00	1.55	14.86
time (sec)	N/A	0.130	0.286	0.124	0.000	1.784	0.000	0.180	25.561
Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	118	254	0	1277	0	209	205
normalized size	1	1.00	1.04	2.25	0.00	11.30	0.00	1.85	1.81
time (sec)	N/A	0.106	0.326	0.175	0.000	1.690	0.000	0.190	3.253
Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	326	1109	0	3402	0	1162	946
normalized size	1	1.00	1.63	5.54	0.00	17.01	0.00	5.81	4.73
time (sec)	N/A	0.255	0.937	0.210	0.000	2.159	0.000	0.546	6.845
Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	147	153	0	72	95	168	99
normalized size	1	1.00	1.75	1.82	0.00	0.86	1.13	2.00	1.18
time (sec)	N/A	0.045	0.222	0.183	0.000	0.756	0.576	0.162	4.333
Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	147	284	0	56	75	168	584
normalized size	1	1.00	1.75	3.38	0.00	0.67	0.89	2.00	6.95
time (sec)	N/A	0.042	0.191	0.185	0.000	2.144	0.578	0.155	8.386

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	96	542	0	625	0	177	1741
normalized size	1	1.00	0.83	4.67	0.00	5.39	0.00	1.53	15.01
time (sec)	N/A	0.108	0.288	0.129	0.000	2.587	0.000	0.185	24.863
Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	123	255	0	1301	0	206	204
normalized size	1	1.00	1.08	2.24	0.00	11.41	0.00	1.81	1.79
time (sec)	N/A	0.101	0.386	0.183	0.000	1.240	0.000	0.201	3.191
Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	361	1088	0	3513	0	1054	912
normalized size	1	1.00	1.80	5.44	0.00	17.56	0.00	5.27	4.56
time (sec)	N/A	0.249	0.925	0.217	0.000	1.891	0.000	0.520	6.734
Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	152	151	0	71	104	169	96
normalized size	1	1.00	1.79	1.78	0.00	0.84	1.22	1.99	1.13
time (sec)	N/A	0.046	0.293	0.184	0.000	0.968	0.798	0.155	4.354
Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	152	280	0	57	80	169	96
normalized size	1	1.00	1.79	3.29	0.00	0.67	0.94	1.99	1.13
time (sec)	N/A	0.046	0.258	0.175	0.000	0.550	0.744	0.176	4.343
Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	98	824	0	687	0	187	1864
normalized size	1	1.00	0.82	6.92	0.00	5.77	0.00	1.57	15.66
time (sec)	N/A	0.113	0.385	0.123	0.000	5.064	0.000	0.150	28.555

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	116	255	0	1316	0	205	202
normalized size	1	1.00	1.05	2.32	0.00	11.96	0.00	1.86	1.84
time (sec)	N/A	0.096	0.406	0.177	0.000	1.024	0.000	0.184	3.238
Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	311	881	0	3264	0	1034	923
normalized size	1	1.00	1.58	4.47	0.00	16.57	0.00	5.25	4.69
time (sec)	N/A	0.232	0.905	0.222	0.000	2.203	0.000	0.512	6.362
Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	87	195	212	0	78	116	178	118
normalized size	1	0.95	2.12	2.30	0.00	0.85	1.26	1.93	1.28
time (sec)	N/A	0.078	0.324	0.195	0.000	1.053	0.850	0.160	5.319
Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	85	195	388	0	68	102	178	118
normalized size	1	0.94	2.17	4.31	0.00	0.76	1.13	1.98	1.31
time (sec)	N/A	0.078	0.294	0.191	0.000	0.959	0.813	0.159	4.506
Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	110	954	0	711	0	199	2711
normalized size	1	1.00	0.84	7.28	0.00	5.43	0.00	1.52	20.69
time (sec)	N/A	0.126	0.386	0.132	0.000	1.845	0.000	0.184	55.105
Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	137	329	0	1556	0	241	227
normalized size	1	1.00	1.08	2.59	0.00	12.25	0.00	1.90	1.79
time (sec)	N/A	0.123	0.504	0.183	0.000	2.177	0.000	0.167	3.487

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	452	1422	0	4240	0	1506	1160
normalized size	1	1.00	1.91	6.00	0.00	17.89	0.00	6.35	4.89
time (sec)	N/A	0.277	1.281	0.226	0.000	1.620	0.000	0.543	8.215
Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	165	257	0	89	138	203	132
normalized size	1	1.00	1.57	2.45	0.00	0.85	1.31	1.93	1.26
time (sec)	N/A	0.074	0.445	0.185	0.000	1.460	1.373	0.148	6.927
Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	167	475	0	73	109	203	133
normalized size	1	1.00	1.62	4.61	0.00	0.71	1.06	1.97	1.29
time (sec)	N/A	0.073	0.451	0.194	0.000	1.091	1.271	0.151	6.858
Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	68	32	70	0	24	0	68	62
normalized size	1	2.83	1.33	2.92	0.00	1.00	0.00	2.83	2.58
time (sec)	N/A	0.068	0.094	0.189	0.000	0.893	0.000	0.179	3.031
Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	7823	3502	0	0	0	0	-1
normalized size	1	1.00	20.06	8.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.888	6.941	1.279	0.000	1.631	0.000	0.000	0.000
Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	5218	2238	0	0	0	0	-1
normalized size	1	1.00	17.75	7.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.555	6.584	1.011	0.000	0.996	0.000	0.000	0.000

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	3006	1460	0	0	0	0	-1
normalized size	1	1.00	13.13	6.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.332	6.370	0.774	0.000	1.179	0.000	0.000	0.000
Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	1319	777	0	0	0	0	-1
normalized size	1	1.00	7.33	4.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	6.305	0.654	0.000	0.961	0.000	0.000	0.000
Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	3176	2596	0	0	0	0	-1
normalized size	1	1.00	12.70	10.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.323	6.571	1.070	0.000	2.937	0.000	0.000	0.000
Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	5554	3164	0	0	0	0	-1
normalized size	1	1.00	14.69	8.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.561	6.952	3.886	0.000	0.954	0.000	0.000	0.000
Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	80	178	0	346	1110	141	1143
normalized size	1	1.00	0.95	2.12	0.00	4.12	13.21	1.68	13.61
time (sec)	N/A	0.152	0.254	0.253	0.000	0.996	29.516	0.180	9.625
Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	114	426	0	458	0	187	227
normalized size	1	1.00	0.97	3.61	0.00	3.88	0.00	1.58	1.92
time (sec)	N/A	0.157	0.433	0.395	0.000	1.364	0.000	0.198	3.327

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	174	1891	0	880	0	596	557
normalized size	1	1.00	0.94	10.22	0.00	4.76	0.00	3.22	3.01
time (sec)	N/A	0.246	0.914	0.428	0.000	1.170	0.000	0.265	5.272
Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	244	5051	0	1411	0	1340	1085
normalized size	1	1.00	0.95	19.58	0.00	5.47	0.00	5.19	4.21
time (sec)	N/A	0.405	2.637	0.463	0.000	0.787	0.000	0.307	6.218
Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	145	0	0	0	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.589	1.111	0.000	0.910	0.000	0.000	0.000
Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	75	106	80	97	190	75	125
normalized size	1	1.00	0.70	0.99	0.75	0.91	1.78	0.70	1.17
time (sec)	N/A	0.082	0.272	0.248	0.383	1.753	3.355	0.187	3.437
Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	48	69	48	63	129	46	78
normalized size	1	1.00	0.79	1.13	0.79	1.03	2.11	0.75	1.28
time (sec)	N/A	0.034	0.155	0.242	0.335	2.401	0.955	0.155	3.034
Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	38	19	18	22	26	18	22
normalized size	1	1.00	1.90	0.95	0.90	1.10	1.30	0.90	1.10
time (sec)	N/A	0.016	0.007	0.003	0.483	0.754	0.178	0.135	2.942

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	0	290	0	61	44
normalized size	1	1.00	1.00	0.94	0.00	6.04	0.00	1.27	0.92
time (sec)	N/A	0.066	0.076	0.313	0.000	0.836	0.000	0.163	3.114
Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	94	139	0	493	0	116	181
normalized size	1	1.00	0.99	1.46	0.00	5.19	0.00	1.22	1.91
time (sec)	N/A	0.109	0.408	0.445	0.000	1.446	0.000	0.177	3.050
Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	120	640	0	969	0	252	396
normalized size	1	1.00	0.81	4.30	0.00	6.50	0.00	1.69	2.66
time (sec)	N/A	0.177	0.930	0.488	0.000	1.063	0.000	0.190	3.874
Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	202	1138	0	0	0	0	-1
normalized size	1	1.00	0.76	4.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	1.848	0.521	0.000	2.714	0.000	0.000	0.000
Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	167	844	0	0	0	0	-1
normalized size	1	1.00	0.79	3.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.220	1.456	0.442	0.000	2.978	0.000	0.000	0.000
Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	312	0	0	0	0	-1
normalized size	1	1.00	0.99	4.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.108	0.407	0.000	2.013	0.000	0.000	0.000

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	165	0	0	0	0	-1
normalized size	1	1.00	0.92	2.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.134	0.426	0.000	2.005	0.000	0.000	0.000
Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	101	570	0	0	0	0	-1
normalized size	1	1.00	0.71	3.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.419	0.501	0.000	0.549	0.000	0.000	0.000
Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	201	1554	0	0	0	0	-1
normalized size	1	1.00	0.68	5.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.301	1.516	0.515	0.000	1.743	0.000	0.000	0.000
Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	340	2282	0	3324	0	0	-1
normalized size	1	1.00	0.74	4.95	0.00	7.21	0.00	0.00	-0.00
time (sec)	N/A	0.630	0.871	0.297	0.000	1.294	0.000	0.000	0.000
Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	256	1782	0	2506	0	0	-1
normalized size	1	1.00	0.75	5.24	0.00	7.37	0.00	0.00	-0.00
time (sec)	N/A	0.537	0.720	0.227	0.000	2.282	0.000	0.000	0.000
Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	788	1284	0	1688	0	0	-1
normalized size	1	1.00	3.50	5.71	0.00	7.50	0.00	0.00	-0.00
time (sec)	N/A	0.319	1.406	0.217	0.000	2.255	0.000	0.000	0.000

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.085	1.581	0.246	0.000	0.690	0.000	0.000	0.000
Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	5.465	2.193	0.000	0.964	0.000	0.000	0.000
Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	4.968	2.187	0.000	0.912	0.000	0.000	0.000
Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	198	0	0	186	0	7347	-1
normalized size	1	1.00	1.13	0.00	0.00	1.06	0.00	41.98	-0.01
time (sec)	N/A	0.297	1.447	180.000	0.000	1.074	0.000	1.202	0.000
Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	142	0	0	142	0	4175	-1
normalized size	1	1.00	1.08	0.00	0.00	1.08	0.00	31.87	-0.01
time (sec)	N/A	0.227	0.970	180.000	0.000	1.036	0.000	0.811	0.000
Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	0	0	77	0	1033	-1
normalized size	1	1.00	0.96	0.00	0.00	0.96	0.00	12.91	-0.01
time (sec)	N/A	0.131	0.821	180.000	0.000	1.034	0.000	0.477	0.000

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	A	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	242	108	0	45	0	496	-1
normalized size	1	1.00	4.32	1.93	0.00	0.80	0.00	8.86	-0.02
time (sec)	N/A	0.102	7.419	4.279	0.000	0.867	0.000	0.369	0.000
Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	77	114	24	20	39	24
normalized size	1	1.00	0.69	2.20	3.26	0.69	0.57	1.11	0.69
time (sec)	N/A	0.024	0.277	1.471	0.317	1.955	3.415	0.193	3.028
Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	21	19	42	23
normalized size	1	1.00	1.00	1.05	1.00	1.05	0.95	2.10	1.15
time (sec)	N/A	0.038	0.030	0.195	0.311	1.746	3.482	0.177	0.118
Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	54	100	34	112	53	-1
normalized size	1	1.00	0.91	1.54	2.86	0.97	3.20	1.51	-0.03
time (sec)	N/A	0.039	0.456	1.226	0.312	0.823	5.046	0.148	0.000
Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	157	0	0	295	0	0	-1
normalized size	1	1.00	1.51	0.00	0.00	2.84	0.00	0.00	-0.01
time (sec)	N/A	0.091	1.003	2.473	0.000	2.955	0.000	0.000	0.000
Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	102	172	608	406	0	0	-1
normalized size	1	1.00	0.80	1.35	4.79	3.20	0.00	0.00	-0.01
time (sec)	N/A	0.181	1.049	1.105	0.378	0.997	0.000	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	194	0	0	162	0	7279	-1
normalized size	1	1.00	1.10	0.00	0.00	0.92	0.00	41.36	-0.01
time (sec)	N/A	0.299	1.223	180.000	0.000	0.915	0.000	1.209	0.000
Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	136	0	0	185	0	3130	-1
normalized size	1	1.00	1.03	0.00	0.00	1.40	0.00	23.71	-0.01
time (sec)	N/A	0.226	0.795	180.000	0.000	0.921	0.000	0.691	0.000
Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	0	73	0	997	-1
normalized size	1	1.00	0.89	0.00	0.00	0.91	0.00	12.46	-0.01
time (sec)	N/A	0.128	0.677	180.000	0.000	0.900	0.000	0.489	0.000
Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	A	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	237	106	0	62	0	366	-1
normalized size	1	1.00	4.23	1.89	0.00	1.11	0.00	6.54	-0.02
time (sec)	N/A	0.093	7.403	5.142	0.000	0.951	0.000	0.353	0.000
Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	22	70	114	23	20	32	22
normalized size	1	1.00	0.65	2.06	3.35	0.68	0.59	0.94	0.65
time (sec)	N/A	0.022	0.214	1.787	0.328	3.572	3.003	0.167	0.163
Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	22	20	40	22
normalized size	1	1.00	1.00	1.05	1.00	1.16	1.05	2.11	1.16
time (sec)	N/A	0.056	0.019	0.238	0.305	0.861	3.007	0.177	0.093

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	53	100	36	0	52	-1
normalized size	1	1.00	0.94	1.61	3.03	1.09	0.00	1.58	-0.03
time (sec)	N/A	0.038	0.406	1.187	0.313	1.846	0.000	0.153	0.000
Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	176	0	0	290	0	0	-1
normalized size	1	1.00	1.60	0.00	0.00	2.64	0.00	0.00	-0.01
time (sec)	N/A	0.093	1.112	2.540	0.000	0.993	0.000	0.000	0.000
Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	130	141	381	378	0	0	-1
normalized size	1	1.00	1.05	1.14	3.07	3.05	0.00	0.00	-0.01
time (sec)	N/A	0.183	1.074	1.242	0.463	1.883	0.000	0.000	0.000
Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	64	98	0	106	0	0	463
normalized size	1	1.00	0.41	0.62	0.00	0.68	0.00	0.00	2.95
time (sec)	N/A	0.445	0.224	1.589	0.000	0.885	0.000	0.000	8.884
Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	62	88	0	84	0	0	148
normalized size	1	1.00	0.56	0.80	0.00	0.76	0.00	0.00	1.35
time (sec)	N/A	0.276	0.178	1.167	0.000	1.501	0.000	0.000	11.027
Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	44	78	0	64	0	0	129
normalized size	1	1.00	0.61	1.08	0.00	0.89	0.00	0.00	1.79
time (sec)	N/A	0.199	0.176	1.095	0.000	0.906	0.000	0.000	7.343

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	52	0	40	0	0	87
normalized size	1	1.00	0.91	1.58	0.00	1.21	0.00	0.00	2.64
time (sec)	N/A	0.065	0.082	0.971	0.000	0.427	0.000	0.000	3.677
Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	73	136	430	201	0	0	-1
normalized size	1	1.00	1.62	3.02	9.56	4.47	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.128	0.944	0.496	1.465	0.000	0.000	0.000
Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	92	391	1049	351	0	0	-1
normalized size	1	1.00	1.10	4.65	12.49	4.18	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.240	1.139	0.610	2.277	0.000	0.000	0.000
Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	105	657	1421	419	0	0	-1
normalized size	1	1.00	0.81	5.09	11.02	3.25	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.249	1.218	0.721	0.953	0.000	0.000	0.000
Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	116	933	2333	481	0	0	-1
normalized size	1	1.00	0.66	5.30	13.26	2.73	0.00	0.00	-0.01
time (sec)	N/A	0.288	0.294	1.171	1.131	2.224	0.000	0.000	0.000
Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	85	105	0	132	0	0	594
normalized size	1	1.00	0.41	0.50	0.00	0.63	0.00	0.00	2.86
time (sec)	N/A	0.528	0.347	1.119	0.000	1.047	0.000	0.000	10.183

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	73	95	0	111	0	0	479
normalized size	1	1.00	0.49	0.64	0.00	0.75	0.00	0.00	3.24
time (sec)	N/A	0.348	0.214	0.944	0.000	0.974	0.000	0.000	9.262
Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	59	85	0	88	0	0	149
normalized size	1	1.00	0.54	0.77	0.00	0.80	0.00	0.00	1.35
time (sec)	N/A	0.268	0.226	0.863	0.000	0.902	0.000	0.000	11.037
Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	51	61	0	67	0	0	158
normalized size	1	1.00	0.68	0.81	0.00	0.89	0.00	0.00	2.11
time (sec)	N/A	0.110	0.159	0.832	0.000	0.884	0.000	0.000	7.866
Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	86	253	1317	296	0	0	-1
normalized size	1	1.00	1.08	3.16	16.46	3.70	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.153	0.950	0.894	1.033	0.000	0.000	0.000
Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	93	518	1058	369	0	0	-1
normalized size	1	1.00	1.08	6.02	12.30	4.29	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.244	1.087	1.148	2.090	0.000	0.000	0.000
Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	105	792	0	437	0	0	-1
normalized size	1	1.00	0.79	5.95	0.00	3.29	0.00	0.00	-0.01
time (sec)	N/A	0.256	0.271	1.022	0.000	1.041	0.000	0.000	0.000

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	117	1078	0	503	0	0	-1
normalized size	1	1.00	0.64	5.92	0.00	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.312	0.230	1.112	0.000	0.728	0.000	0.000	0.000
Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	112	984	0	380	0	0	-1
normalized size	1	1.00	0.64	5.62	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.600	0.673	1.357	0.000	0.508	0.000	0.000	0.000
Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	89	677	0	294	0	0	-1
normalized size	1	1.00	0.69	5.25	0.00	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.359	0.391	1.208	0.000	2.195	0.000	0.000	0.000
Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	67	478	0	245	0	0	-1
normalized size	1	1.00	0.76	5.43	0.00	2.78	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.243	1.194	0.000	1.625	0.000	0.000	0.000
Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	64	236	0	146	0	0	-1
normalized size	1	1.00	1.16	4.29	0.00	2.65	0.00	0.00	-0.02
time (sec)	N/A	0.076	0.149	1.067	0.000	2.001	0.000	0.000	0.000
Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	94	301	0	309	0	0	-1
normalized size	1	1.00	0.94	3.01	0.00	3.09	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.313	1.007	0.000	1.904	0.000	0.000	0.000

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	1030	0	481	0	0	-1
normalized size	1	1.00	1.20	7.46	0.00	3.49	0.00	0.00	-0.01
time (sec)	N/A	0.279	2.321	1.171	0.000	0.963	0.000	0.000	0.000
Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	186	1835	0	569	0	0	-1
normalized size	1	1.00	1.02	10.08	0.00	3.13	0.00	0.00	-0.01
time (sec)	N/A	0.370	2.902	1.095	0.000	1.947	0.000	0.000	0.000
Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	100	1211	0	350	0	0	-1
normalized size	1	1.00	0.56	6.73	0.00	1.94	0.00	0.00	-0.01
time (sec)	N/A	0.513	1.304	1.129	0.000	1.155	0.000	0.000	0.000
Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	94	930	0	276	0	0	-1
normalized size	1	1.00	0.73	7.27	0.00	2.16	0.00	0.00	-0.01
time (sec)	N/A	0.306	0.603	1.148	0.000	1.998	0.000	0.000	0.000
Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	84	433	0	272	0	0	-1
normalized size	1	1.00	0.90	4.66	0.00	2.92	0.00	0.00	-0.01
time (sec)	N/A	0.236	0.588	1.041	0.000	1.026	0.000	0.000	0.000
Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	83	599	0	269	0	0	-1
normalized size	1	1.00	0.89	6.44	0.00	2.89	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.623	1.036	0.000	1.210	0.000	0.000	0.000

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	196	561	0	438	0	0	-1
normalized size	1	1.00	1.42	4.07	0.00	3.17	0.00	0.00	-0.01
time (sec)	N/A	0.143	3.700	1.015	0.000	0.665	0.000	0.000	0.000
Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	342	1157	0	528	0	0	-1
normalized size	1	1.00	1.92	6.50	0.00	2.97	0.00	0.00	-0.01
time (sec)	N/A	0.320	6.202	1.073	0.000	1.051	0.000	0.000	0.000
Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	356	1787	0	616	0	0	-1
normalized size	1	1.00	1.52	7.64	0.00	2.63	0.00	0.00	-0.00
time (sec)	N/A	0.497	6.207	1.024	0.000	0.975	0.000	0.000	0.000
Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	119	0	29	0	0	14
normalized size	1	1.00	1.00	7.44	0.00	1.81	0.00	0.00	0.88
time (sec)	N/A	0.087	0.031	0.213	0.000	1.869	0.000	0.000	3.099
Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	119	397	0	120	0	0	-1
normalized size	1	1.00	1.72	5.75	0.00	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.364	5.949	0.313	0.000	2.702	0.000	0.000	0.000
Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	183	397	0	120	0	0	-1
normalized size	1	1.00	2.32	5.03	0.00	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.567	4.955	0.280	0.000	1.040	0.000	0.000	0.000

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	184	761	0	136	0	0	-1
normalized size	1	1.00	1.94	8.01	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.576	15.721	0.383	0.000	0.469	0.000	0.000	0.000
Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	51	31	30	59	138	0	63
normalized size	1	1.00	1.70	1.03	1.00	1.97	4.60	0.00	2.10
time (sec)	N/A	0.059	1.218	0.461	0.305	3.048	74.067	0.000	5.540
Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	938	137	24	122	129	142	185
normalized size	1	1.00	36.08	5.27	0.92	4.69	4.96	5.46	7.12
time (sec)	N/A	0.044	6.538	0.608	0.308	0.686	35.718	1.954	3.566
Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	31	118	24	92	100	0	100
normalized size	1	1.00	1.19	4.54	0.92	3.54	3.85	0.00	3.85
time (sec)	N/A	0.043	1.224	0.556	0.310	0.982	11.161	0.000	3.223
Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	67	57	24	61	73	45	61
normalized size	1	1.00	2.58	2.19	0.92	2.35	2.81	1.73	2.35
time (sec)	N/A	0.028	0.037	0.460	0.549	0.976	3.457	2.671	3.177
Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	29	23	22	33	63	0	133
normalized size	1	1.00	1.32	1.05	1.00	1.50	2.86	0.00	6.05
time (sec)	N/A	0.048	0.463	0.519	0.318	1.134	7.479	0.000	4.859

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	27	25	24	29	49	108	47
normalized size	1	1.00	1.12	1.04	1.00	1.21	2.04	4.50	1.96
time (sec)	N/A	0.044	0.305	0.631	0.426	0.903	21.183	0.726	3.236
Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	29	25	24	63	80	0	291
normalized size	1	1.00	1.12	0.96	0.92	2.42	3.08	0.00	11.19
time (sec)	N/A	0.045	0.710	0.658	0.308	1.908	44.089	0.000	6.313
Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.013	0.045	0.053	0.000	0.858	0.000	0.000	0.000
Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.013	0.035	0.030	0.000	0.903	0.000	0.000	0.000
Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.016	0.081	0.056	0.000	0.864	0.000	0.000	0.000
Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	0.073	0.053	0.000	1.974	0.000	0.000	0.000

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	15	17	13	12
normalized size	1	1.00	1.00	1.08	1.00	1.25	1.42	1.08	1.00
time (sec)	N/A	0.022	0.018	0.033	0.310	0.734	0.327	0.131	0.057
Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	20	23	63	20	20
normalized size	1	1.00	0.95	1.05	1.00	1.15	3.15	1.00	1.00
time (sec)	N/A	0.025	0.031	0.032	0.380	0.628	2.018	0.139	3.150
Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	36	0	14	5
normalized size	1	1.00	1.00	1.20	1.00	7.20	0.00	2.80	1.00
time (sec)	N/A	0.023	0.021	0.059	1.414	0.958	0.000	0.125	3.078
Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	20	5	5	5
normalized size	1	1.00	1.00	1.20	1.00	4.00	1.00	1.00	1.00
time (sec)	N/A	0.009	2.566	0.007	0.545	0.789	0.436	0.140	0.088
Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	21	23	17	73	34	17	22
normalized size	1	1.00	0.75	0.82	0.61	2.61	1.21	0.61	0.79
time (sec)	N/A	0.026	1.476	0.010	0.586	0.999	5.567	0.140	3.046
Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	168	54	20	20
normalized size	1	1.00	1.00	0.81	0.77	6.46	2.08	0.77	0.77
time (sec)	N/A	0.046	4.503	0.155	0.326	1.921	47.646	0.152	3.134

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	137	40	103	39	46	39	39
normalized size	1	1.00	3.81	1.11	2.86	1.08	1.28	1.08	1.08
time (sec)	N/A	0.087	0.281	0.013	0.392	0.923	11.654	0.130	0.090
Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	22	7	7	7
normalized size	1	1.00	1.00	0.89	0.78	2.44	0.78	0.78	0.78
time (sec)	N/A	0.011	2.678	0.007	0.318	0.951	0.422	0.158	3.010
Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	24	26	17	17	26	17	17
normalized size	1	1.00	0.77	0.84	0.55	0.55	0.84	0.55	0.55
time (sec)	N/A	0.023	0.122	0.012	0.317	0.842	0.674	0.149	0.116
Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	162	0	18	29	0	7	-1
normalized size	1	1.00	18.00	0.00	2.00	3.22	0.00	0.78	-0.11
time (sec)	N/A	0.072	2.217	1.263	0.600	1.114	0.000	0.154	0.000
Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	59	49	51	34	0	75	-1
normalized size	1	1.00	0.83	0.69	0.72	0.48	0.00	1.06	-0.01
time (sec)	N/A	0.066	0.160	0.173	0.318	1.679	0.000	0.136	0.000
Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	17	17	39	17	17
normalized size	1	1.00	1.00	1.00	0.94	0.94	2.17	0.94	0.94
time (sec)	N/A	0.014	0.048	0.006	0.310	1.038	0.643	0.143	0.101

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	23	23	54	23	23
normalized size	1	1.00	1.00	1.04	1.00	1.00	2.35	1.00	1.00
time (sec)	N/A	0.015	0.235	0.057	0.371	1.402	9.836	0.152	3.160
Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	23	24	23	23	54	23	23
normalized size	1	1.00	0.96	1.00	0.96	0.96	2.25	0.96	0.96
time (sec)	N/A	0.014	0.042	0.033	0.321	0.897	2.226	0.148	3.023
Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	16	14	14	0	0	-1
normalized size	1	1.00	1.00	1.14	1.00	1.00	0.00	0.00	-0.07
time (sec)	N/A	0.022	0.040	0.033	0.374	1.490	0.000	0.000	0.000
Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	22	20	20	0	0	-1
normalized size	1	1.00	1.00	1.16	1.05	1.05	0.00	0.00	-0.05
time (sec)	N/A	0.022	0.061	0.074	0.385	0.892	0.000	0.000	0.000
Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	22	20	20	0	0	-1
normalized size	1	1.00	0.95	1.10	1.00	1.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	0.057	0.050	0.435	0.768	0.000	0.000	0.000
Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	14	12	11
normalized size	1	1.00	1.00	1.09	1.00	1.00	1.27	1.09	1.00
time (sec)	N/A	0.022	0.006	0.034	0.321	1.974	0.318	0.141	0.034

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	20	19	22	56	19	19
normalized size	1	1.00	0.95	1.05	1.00	1.16	2.95	1.00	1.00
time (sec)	N/A	0.022	0.020	0.026	0.321	0.571	1.877	0.148	3.135
Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	39	0	16	9
normalized size	1	1.00	1.00	1.33	1.00	13.00	0.00	5.33	3.00
time (sec)	N/A	0.023	0.008	0.057	0.414	1.190	0.000	0.128	0.021
Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	53	0	5	5
normalized size	1	1.00	1.00	0.86	0.71	7.57	0.00	0.71	0.71
time (sec)	N/A	0.025	0.008	0.076	0.418	3.019	0.000	0.154	2.979
Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	71	0	9	9
normalized size	1	1.00	1.00	0.77	0.69	5.46	0.00	0.69	0.69
time (sec)	N/A	0.026	0.028	0.084	0.417	0.596	0.000	0.224	2.984
Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	48	38	79	0	38	47
normalized size	1	1.00	1.00	2.29	1.81	3.76	0.00	1.81	2.24
time (sec)	N/A	0.024	0.015	0.079	0.328	0.988	0.000	0.150	3.095
Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	61	0	22	20
normalized size	1	1.00	1.00	0.82	0.79	2.18	0.00	0.79	0.71
time (sec)	N/A	0.026	0.018	0.072	0.463	1.075	0.000	0.141	2.965

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	12	27	10	10
normalized size	1	1.00	1.00	0.79	0.71	0.86	1.93	0.71	0.71
time (sec)	N/A	0.034	0.007	0.007	0.367	0.881	0.729	0.133	0.100
Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	40	17	20	35	0	20	14
normalized size	1	1.00	2.11	0.89	1.05	1.84	0.00	1.05	0.74
time (sec)	N/A	0.032	0.018	0.099	0.312	2.962	0.000	0.156	3.170
Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	17	3	3	3
normalized size	1	1.00	1.00	1.33	1.00	5.67	1.00	1.00	1.00
time (sec)	N/A	0.008	1.466	0.009	0.319	1.642	0.412	0.142	2.953
Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	41	5	4	4
normalized size	1	1.00	1.00	1.25	1.00	10.25	1.25	1.00	1.00
time (sec)	N/A	0.022	8.959	0.013	0.318	0.669	8.852	0.132	3.002
Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	8	47	10	29	21
normalized size	1	1.00	1.00	2.25	2.00	11.75	2.50	7.25	5.25
time (sec)	N/A	0.007	0.005	0.010	0.330	0.878	1.330	0.145	3.240
Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	128	40	39	103	44	39	73
normalized size	1	1.00	3.56	1.11	1.08	2.86	1.22	1.08	2.03
time (sec)	N/A	0.077	0.349	0.014	0.334	1.193	11.379	0.123	0.072

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	9	13	8	8	12	8	8
normalized size	1	1.00	0.64	0.93	0.57	0.57	0.86	0.57	0.57
time (sec)	N/A	0.016	0.010	0.004	0.330	0.861	0.636	0.138	2.915
Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	20	14	19	28	36	13	-1
normalized size	1	1.00	0.80	0.56	0.76	1.12	1.44	0.52	-0.04
time (sec)	N/A	0.049	0.020	0.157	0.322	0.892	2.069	0.130	0.000
Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	8	7	7
normalized size	1	1.00	1.00	0.80	0.70	0.70	0.80	0.70	0.70
time (sec)	N/A	0.027	0.010	0.019	0.319	1.184	0.461	0.149	3.002
Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	7	5	6
normalized size	1	1.00	1.00	1.00	0.83	0.83	1.17	0.83	1.00
time (sec)	N/A	0.010	0.009	0.028	0.315	0.940	0.540	0.121	2.908
Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	7	7	7	6	0	12	7
normalized size	1	1.00	0.70	0.70	0.70	0.60	0.00	1.20	0.70
time (sec)	N/A	0.012	0.030	0.079	0.950	0.787	0.000	0.148	2.993
Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	13	7	12	0	18	7
normalized size	1	1.00	1.00	1.30	0.70	1.20	0.00	1.80	0.70
time (sec)	N/A	0.011	0.009	0.079	0.328	0.718	0.000	0.145	2.952

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	16	16	36	16	16
normalized size	1	1.00	1.00	1.00	0.94	0.94	2.12	0.94	0.94
time (sec)	N/A	0.013	0.017	0.008	0.310	0.777	0.425	0.131	0.107
Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	23	22	22	51	22	22
normalized size	1	1.00	1.05	1.05	1.00	1.00	2.32	1.00	1.00
time (sec)	N/A	0.013	0.136	0.058	0.317	1.046	9.316	0.132	3.080
Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	22	22	51	22	22
normalized size	1	1.00	1.00	1.00	0.96	0.96	2.22	0.96	0.96
time (sec)	N/A	0.013	0.043	0.033	0.311	0.800	2.274	0.147	2.985
Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	13	13	0	13	-1
normalized size	1	1.00	1.00	1.31	1.00	1.00	0.00	1.00	-0.08
time (sec)	N/A	0.020	0.039	0.032	0.372	0.997	0.000	0.142	0.000
Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	23	19	19	0	0	-1
normalized size	1	1.00	1.00	1.28	1.06	1.06	0.00	0.00	-0.06
time (sec)	N/A	0.021	0.066	0.104	0.380	0.668	0.000	0.000	0.000
Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	23	19	19	0	0	-1
normalized size	1	1.00	0.95	1.21	1.00	1.00	0.00	0.00	-0.05
time (sec)	N/A	0.020	0.060	0.066	0.410	0.921	0.000	0.000	0.000

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	20	12	11	40	0	12	11
normalized size	1	1.00	1.82	1.09	1.00	3.64	0.00	1.09	1.00
time (sec)	N/A	0.034	0.056	0.073	0.311	1.825	0.000	0.122	3.026
Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	4	15	23	0	17	3
normalized size	1	1.00	2.09	0.36	1.36	2.09	0.00	1.55	0.27
time (sec)	N/A	0.031	0.006	0.099	0.313	0.569	0.000	0.156	3.082
Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	9	8	7	21	0	7	7
normalized size	1	1.00	0.33	0.30	0.26	0.78	0.00	0.26	0.26
time (sec)	N/A	0.029	0.024	0.102	0.409	0.548	0.000	0.152	2.882
Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	20	19	37	0	0	37
normalized size	1	1.00	0.95	1.05	1.00	1.95	0.00	0.00	1.95
time (sec)	N/A	0.035	0.189	0.079	0.314	1.058	0.000	0.000	3.563
Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	12	27	4	4
normalized size	1	1.00	1.00	1.25	1.00	3.00	6.75	1.00	1.00
time (sec)	N/A	0.043	0.005	0.115	0.413	0.891	0.743	0.127	2.937
Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	12	27	4	4
normalized size	1	1.00	1.00	1.25	1.00	3.00	6.75	1.00	1.00
time (sec)	N/A	0.063	0.003	0.114	0.499	0.888	0.736	0.124	2.997

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	6	5	35	0	5	5
normalized size	1	1.00	0.94	0.18	0.15	1.06	0.00	0.15	0.15
time (sec)	N/A	0.042	0.034	0.135	0.411	0.860	0.000	0.126	3.122
Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	15	16	18	17	36	0	19	16
normalized size	1	1.50	1.60	1.80	1.70	3.60	0.00	1.90	1.60
time (sec)	N/A	0.048	0.035	0.129	0.312	1.294	0.000	0.139	3.098
Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	15	16	16	15	36	0	17	14
normalized size	1	1.50	1.60	1.60	1.50	3.60	0.00	1.70	1.40
time (sec)	N/A	0.053	0.034	0.136	0.311	0.866	0.000	0.159	2.984
Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	74	80	89	441	0	61	75
normalized size	1	1.00	0.42	0.45	0.51	2.51	0.00	0.35	0.43
time (sec)	N/A	0.140	0.115	0.123	0.435	1.104	0.000	0.167	3.308
Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	22	18	17	48	0	17	17
normalized size	1	1.00	0.42	0.34	0.32	0.91	0.00	0.32	0.32
time (sec)	N/A	0.066	0.054	0.148	0.448	0.981	0.000	0.126	3.115
Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	54	35	28	71	29	29	27
normalized size	1	1.00	1.93	1.25	1.00	2.54	1.04	1.04	0.96
time (sec)	N/A	0.087	0.384	0.119	0.318	0.912	4.907	0.145	3.093

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	62	80	63	122	56	64	65
normalized size	1	1.00	1.17	1.51	1.19	2.30	1.06	1.21	1.23
time (sec)	N/A	0.138	0.592	0.133	0.353	1.627	7.628	0.149	2.984
Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	133	143	118	201	95	123	122
normalized size	1	1.00	1.71	1.83	1.51	2.58	1.22	1.58	1.56
time (sec)	N/A	0.150	0.960	0.162	0.524	2.111	10.567	0.148	2.955
Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	36	0	10	12
normalized size	1	1.00	1.00	0.92	0.83	3.00	0.00	0.83	1.00
time (sec)	N/A	0.076	0.035	0.129	0.318	0.851	0.000	0.149	2.930
Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	67	42	25	46	27	25	25
normalized size	1	1.00	2.03	1.27	0.76	1.39	0.82	0.76	0.76
time (sec)	N/A	0.092	0.025	0.064	0.394	0.861	19.284	0.139	2.922
Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	32	24	23	52	41	24	30
normalized size	1	1.00	0.70	0.52	0.50	1.13	0.89	0.52	0.65
time (sec)	N/A	0.089	0.221	0.204	0.410	2.513	9.252	0.158	2.965
Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	12	3	15	4
normalized size	1	1.00	1.00	1.25	1.00	3.00	0.75	3.75	1.00
time (sec)	N/A	0.018	0.002	0.076	0.459	0.408	5.384	0.193	2.882

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	29	14	13	29	0	15	9
normalized size	1	1.00	1.38	0.67	0.62	1.38	0.00	0.71	0.43
time (sec)	N/A	0.115	0.030	0.131	0.318	0.578	0.000	0.156	3.508
Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	43	103	8	25	0	0	-1
normalized size	1	1.00	4.78	11.44	0.89	2.78	0.00	0.00	-0.11
time (sec)	N/A	0.047	0.042	0.282	0.425	0.832	0.000	0.000	0.000
Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	47	171	7	45	0	7	-1
normalized size	1	1.00	5.22	19.00	0.78	5.00	0.00	0.78	-0.11
time (sec)	N/A	0.047	0.060	0.687	0.423	0.833	0.000	0.172	0.000
Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	46	171	16	67	0	17	-1
normalized size	1	1.00	3.29	12.21	1.14	4.79	0.00	1.21	-0.07
time (sec)	N/A	0.044	0.047	0.720	0.314	1.005	0.000	0.177	0.000
Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	52	223	30	78	0	142	19
normalized size	1	1.00	2.74	11.74	1.58	4.11	0.00	7.47	1.00
time (sec)	N/A	0.049	0.493	0.612	0.415	0.968	0.000	0.202	3.061
Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	63	492	20	72	0	20	-1
normalized size	1	1.00	2.42	18.92	0.77	2.77	0.00	0.77	-0.04
time (sec)	N/A	0.046	0.115	0.427	0.408	1.045	0.000	0.147	0.000

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	8	3	3	3
normalized size	1	1.00	1.00	1.00	0.75	2.00	0.75	0.75	0.75
time (sec)	N/A	0.012	0.059	0.048	0.317	0.518	0.952	0.132	3.102
Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	25	20	42	20	19	20	14
normalized size	1	1.00	1.47	1.18	2.47	1.18	1.12	1.18	0.82
time (sec)	N/A	0.067	0.017	0.046	0.312	0.806	6.969	0.150	2.923
Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	12	45	0	22	16
normalized size	1	1.00	1.67	1.08	1.00	3.75	0.00	1.83	1.33
time (sec)	N/A	0.041	0.059	0.073	0.332	1.439	0.000	0.167	3.012
Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	20	38	0	0	43
normalized size	1	1.00	0.95	1.05	1.00	1.90	0.00	0.00	2.15
time (sec)	N/A	0.041	0.205	0.080	0.317	0.653	0.000	0.000	3.186
Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	14	3	16	6
normalized size	1	1.00	1.00	1.17	1.33	2.33	0.50	2.67	1.00
time (sec)	N/A	0.016	0.002	0.064	0.409	0.672	4.214	0.147	2.935
Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	14	27	16	6
normalized size	1	1.00	1.00	1.17	1.33	2.33	4.50	2.67	1.00
time (sec)	N/A	0.047	0.005	0.088	0.409	0.660	0.699	0.141	2.942

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	56	56	46	76	31	68	35
normalized size	1	1.00	2.00	2.00	1.64	2.71	1.11	2.43	1.25
time (sec)	N/A	0.082	0.377	0.103	0.319	1.170	26.515	0.152	3.069
Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	62	119	92	182	58	139	92
normalized size	1	1.00	1.17	2.25	1.74	3.43	1.09	2.62	1.74
time (sec)	N/A	0.136	0.561	0.131	0.319	1.473	55.639	0.165	3.072
Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	135	202	161	320	97	232	141
normalized size	1	1.00	1.73	2.59	2.06	4.10	1.24	2.97	1.81
time (sec)	N/A	0.139	1.304	0.164	0.317	1.624	69.993	0.172	3.080
Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	9	5	0	7
normalized size	1	1.00	1.00	1.00	0.83	1.50	0.83	0.00	1.17
time (sec)	N/A	0.015	0.070	0.049	0.316	0.640	18.865	0.000	2.938
Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	20	20	12	11	19	14	22	48
normalized size	1	1.82	1.82	1.09	1.00	1.73	1.27	2.00	4.36
time (sec)	N/A	0.046	0.016	0.040	0.309	0.671	0.463	0.142	3.246
Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	4	5	5	3	5	7
normalized size	1	1.00	1.00	0.80	1.00	1.00	0.60	1.00	1.40
time (sec)	N/A	0.033	0.022	0.070	0.415	4.939	0.209	0.142	2.950

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	13
normalized size	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	1.18
time (sec)	N/A	0.035	0.026	0.059	0.438	0.967	0.233	0.132	3.044
Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	9	12	7	9	15	7	9
normalized size	1	1.00	1.29	1.71	1.00	1.29	2.14	1.00	1.29
time (sec)	N/A	0.032	0.005	0.082	0.310	0.845	0.216	0.145	3.072
Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	38	6	33	27	5	36	7
normalized size	1	1.00	7.60	1.20	6.60	5.40	1.00	7.20	1.40
time (sec)	N/A	0.045	0.028	0.099	0.313	0.946	0.727	0.150	3.102
Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	25	13	16	0	21	13
normalized size	1	1.00	1.00	1.92	1.00	1.23	0.00	1.62	1.00
time (sec)	N/A	0.079	0.017	0.109	0.420	0.668	0.000	0.150	0.106
Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	5	3	5	5
normalized size	1	1.00	1.00	1.00	0.75	1.25	0.75	1.25	1.25
time (sec)	N/A	0.022	0.008	0.020	0.331	0.922	0.614	0.140	3.093
Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	11	7	11	11
normalized size	1	1.00	1.00	1.11	1.00	1.22	0.78	1.22	1.22
time (sec)	N/A	0.022	0.007	0.022	0.313	0.473	0.651	0.146	3.123

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	20	11	10	29	12	37	18
normalized size	1	1.00	1.67	0.92	0.83	2.42	1.00	3.08	1.50
time (sec)	N/A	0.055	0.075	0.076	0.316	0.760	1.015	0.166	3.114
Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	65	35	122	0	0	36
normalized size	1	1.00	1.00	1.51	0.81	2.84	0.00	0.00	0.84
time (sec)	N/A	0.095	0.052	0.116	0.426	0.954	0.000	0.000	3.257
Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	34	20	20	0	34	18
normalized size	1	1.00	1.00	1.55	0.91	0.91	0.00	1.55	0.82
time (sec)	N/A	0.091	0.025	0.114	0.422	0.666	0.000	0.154	3.026
Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	20	20	13	12	20	17	22	31
normalized size	1	1.67	1.67	1.08	1.00	1.67	1.42	1.83	2.58
time (sec)	N/A	0.042	0.016	0.049	0.310	0.689	0.387	0.130	3.180
Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	14	12	14	14
normalized size	1	1.00	1.00	0.93	0.86	1.00	0.86	1.00	1.00
time (sec)	N/A	0.025	0.018	0.036	0.302	1.140	0.612	0.250	2.968
Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	6	3	3	5	3	26
normalized size	1	1.00	1.00	2.00	1.00	1.00	1.67	1.00	8.67
time (sec)	N/A	0.032	0.012	0.053	0.402	0.709	0.192	0.148	3.215

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	35	49	73	151	48	57
normalized size	1	1.00	0.95	0.81	1.14	1.70	3.51	1.12	1.33
time (sec)	N/A	0.057	0.659	0.080	0.402	0.641	1.715	0.244	3.148
Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	21	0	21	34
normalized size	1	1.00	1.00	1.07	1.00	1.50	0.00	1.50	2.43
time (sec)	N/A	0.077	0.018	0.164	0.400	1.069	0.000	0.151	3.114
Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	38	37	57	0	48	28
normalized size	1	1.00	0.65	0.88	0.86	1.33	0.00	1.12	0.65
time (sec)	N/A	0.106	0.055	0.237	0.401	0.684	0.000	0.507	3.144
Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	104	37	27	0	0	27
normalized size	1	1.00	0.65	2.42	0.86	0.63	0.00	0.00	0.63
time (sec)	N/A	0.037	0.058	0.144	0.339	0.600	0.000	0.000	3.178
Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	104	37	27	0	0	27
normalized size	1	1.00	0.65	2.42	0.86	0.63	0.00	0.00	0.63
time (sec)	N/A	0.035	0.028	0.000	0.352	0.649	0.000	0.000	0.002
Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	122	0	33	0	138	33
normalized size	1	1.00	0.56	1.91	0.00	0.52	0.00	2.16	0.52
time (sec)	N/A	0.036	0.064	0.130	0.000	0.982	0.000	0.231	3.177

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	122	0	33	0	138	33
normalized size	1	1.00	0.56	1.91	0.00	0.52	0.00	2.16	0.52
time (sec)	N/A	0.038	0.029	0.001	0.000	0.692	0.000	0.235	0.002
Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	105	37	27	0	0	27
normalized size	1	1.00	0.65	2.44	0.86	0.63	0.00	0.00	0.63
time (sec)	N/A	0.041	0.141	0.109	0.339	0.537	0.000	0.000	3.214
Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	105	37	27	0	0	27
normalized size	1	1.00	0.65	2.44	0.86	0.63	0.00	0.00	0.63
time (sec)	N/A	0.035	0.034	0.000	0.341	0.720	0.000	0.000	0.002
Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	123	0	33	0	195	33
normalized size	1	1.00	0.56	1.92	0.00	0.52	0.00	3.05	0.52
time (sec)	N/A	0.037	0.173	0.122	0.000	2.004	0.000	0.216	3.174
Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	123	0	33	0	195	33
normalized size	1	1.00	0.56	1.92	0.00	0.52	0.00	3.05	0.52
time (sec)	N/A	0.037	0.033	0.000	0.000	0.503	0.000	0.220	0.002
Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	12	0	7	27
normalized size	1	1.00	1.00	0.89	0.78	1.33	0.00	0.78	3.00
time (sec)	N/A	0.022	0.005	0.079	0.321	0.612	0.000	0.144	5.268

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	265	7	0	0	27
normalized size	1	1.00	1.00	0.89	29.44	0.78	0.00	0.00	3.00
time (sec)	N/A	0.020	0.010	0.062	0.456	0.510	0.000	0.000	3.527
Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	C	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	5	4	4	14	0	4
normalized size	1	1.00	1.00	1.67	1.33	1.33	4.67	0.00	1.33
time (sec)	N/A	0.009	0.000	0.060	0.426	0.549	0.142	0.000	0.026
Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	74	18	0	103	8
normalized size	1	1.00	1.00	1.12	9.25	2.25	0.00	12.88	1.00
time (sec)	N/A	0.018	0.004	0.024	0.423	0.528	0.000	0.157	0.024
Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	26	22	27	76	22	22
normalized size	1	1.00	0.82	0.76	0.65	0.79	2.24	0.65	0.65
time (sec)	N/A	0.022	0.017	0.069	0.318	2.298	0.956	0.133	2.973
Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	6	6	10	6	6
normalized size	1	1.00	1.00	0.70	0.60	0.60	1.00	0.60	0.60
time (sec)	N/A	0.012	0.003	0.017	0.318	0.588	0.250	0.137	0.067
Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	8	14	15	9	12
normalized size	1	1.00	1.00	0.82	0.73	1.27	1.36	0.82	1.09
time (sec)	N/A	0.012	0.008	0.049	0.314	1.376	0.312	0.132	3.536

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	9	8	7	9	8	7	7
normalized size	1	1.00	1.29	1.14	1.00	1.29	1.14	1.00	1.00
time (sec)	N/A	0.044	0.006	0.078	0.316	0.709	2.166	0.140	2.924
Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	74	28	0	103	15
normalized size	1	1.00	1.00	0.84	3.89	1.47	0.00	5.42	0.79
time (sec)	N/A	0.019	0.009	0.025	0.414	0.708	0.000	0.176	2.897
Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	26	31	26	29	32	27	25
normalized size	1	1.00	0.70	0.84	0.70	0.78	0.86	0.73	0.68
time (sec)	N/A	0.014	0.028	0.058	0.317	0.702	0.423	0.125	2.901
Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	129	49	176	8	39	236	37	49
normalized size	1	10.75	4.08	14.67	0.67	3.25	19.67	3.08	4.08
time (sec)	N/A	0.324	0.027	0.205	0.337	1.533	0.080	0.145	2.986
Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	19	8	7	13	19	8	19
normalized size	1	1.00	2.11	0.89	0.78	1.44	2.11	0.89	2.11
time (sec)	N/A	0.007	0.009	0.003	0.323	0.654	0.139	0.151	0.117
Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	35	12	0	6	14
normalized size	1	1.00	1.00	0.88	4.38	1.50	0.00	0.75	1.75
time (sec)	N/A	0.013	0.017	0.030	0.324	1.040	0.000	0.118	0.099

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	0	13	0	15	13
normalized size	1	1.00	1.00	0.80	0.00	0.87	0.00	1.00	0.87
time (sec)	N/A	0.029	0.015	0.172	0.000	0.657	0.000	0.657	2.965
Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	16	15	15	19	15	15
normalized size	1	1.00	0.87	0.70	0.65	0.65	0.83	0.65	0.65
time (sec)	N/A	0.022	0.012	0.040	0.429	1.577	0.266	0.148	0.074
Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	30	29	29	42	29	33
normalized size	1	1.00	1.00	0.81	0.78	0.78	1.14	0.78	0.89
time (sec)	N/A	0.034	0.023	0.171	0.328	1.963	0.516	0.125	2.973
Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	8	10	8	10	8
normalized size	1	1.00	1.20	0.90	0.80	1.00	0.80	1.00	0.80
time (sec)	N/A	0.033	0.019	0.040	0.321	1.375	0.168	0.145	2.918
Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.007	0.001	0.039	0.315	0.531	0.150	0.127	0.056
Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.012	0.003	0.039	0.317	0.765	0.279	0.123	0.059

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.009	0.002	0.039	0.314	0.674	0.477	0.138	0.067
Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.008	0.010	0.004	0.326	0.487	0.149	0.118	2.924
Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	56	8	5	8	8
normalized size	1	1.00	1.00	0.88	7.00	1.00	0.62	1.00	1.00
time (sec)	N/A	0.063	0.006	0.026	0.329	0.768	0.285	0.119	0.075
Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	11	67	13	7	10	10
normalized size	1	1.00	1.20	1.10	6.70	1.30	0.70	1.00	1.00
time (sec)	N/A	0.018	0.023	0.006	0.330	0.722	0.222	0.140	2.916
Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	15	12	10	13
normalized size	1	1.00	1.00	0.91	0.82	1.36	1.09	0.91	1.18
time (sec)	N/A	0.010	0.018	0.002	0.327	0.791	0.119	0.172	0.283
Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	12	12	10	13
normalized size	1	1.00	1.00	0.92	0.83	1.00	1.00	0.83	1.08
time (sec)	N/A	0.005	0.006	0.039	0.322	0.566	0.790	0.140	2.923

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	25	20	18	29	0	18	16
normalized size	1	1.00	1.19	0.95	0.86	1.38	0.00	0.86	0.76
time (sec)	N/A	0.060	0.017	0.150	0.326	0.675	0.000	0.153	3.001
Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	40	32	39	15
normalized size	1	1.00	1.00	0.84	0.79	2.11	1.68	2.05	0.79
time (sec)	N/A	0.037	0.009	0.230	0.331	0.502	5.251	0.141	3.027
Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	35	33	32	32	48	32	43
normalized size	1	1.00	0.74	0.70	0.68	0.68	1.02	0.68	0.91
time (sec)	N/A	0.064	0.031	0.029	0.327	0.711	71.975	0.124	3.004
Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	36	25	29	32	25	25
normalized size	1	1.00	0.80	1.03	0.71	0.83	0.91	0.71	0.71
time (sec)	N/A	0.158	0.044	0.026	0.334	0.753	1.817	0.148	2.966
Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
normalized size	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.007	0.001	0.001	0.325	0.786	0.152	0.136	2.924
Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6
normalized size	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.013	0.003	0.040	0.322	0.646	0.267	0.119	0.057

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	5	7	7
normalized size	1	1.00	1.00	1.14	1.00	1.00	0.71	1.00	1.00
time (sec)	N/A	0.004	0.007	0.001	0.427	0.841	0.080	0.120	0.055
Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	9	8	8	8	8	8
normalized size	1	1.00	2.10	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.009	0.013	0.004	0.324	0.713	0.153	0.123	0.042
Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.009	0.003	0.040	0.316	0.580	0.149	0.136	2.981
Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.010	0.003	0.001	0.325	0.623	0.264	0.142	0.046
Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	9	8	8	8	8	8
normalized size	1	1.00	2.10	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.012	0.013	0.004	0.324	0.958	0.261	0.139	0.047
Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6
normalized size	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.009	0.002	0.000	0.323	0.659	0.267	0.137	0.051

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	15	14	14
normalized size	1	1.00	1.00	1.07	1.00	1.00	1.07	1.00	1.00
time (sec)	N/A	0.011	0.028	0.028	0.324	0.527	0.154	0.138	2.945
Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
normalized size	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.017	0.007	0.053	0.327	0.587	1.515	0.137	2.965
Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	15	17	20	15	15
normalized size	1	1.00	0.70	0.78	0.65	0.74	0.87	0.65	0.65
time (sec)	N/A	0.009	0.013	0.025	0.327	0.693	0.402	0.135	0.021
Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
normalized size	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.017	0.002	0.005	0.327	0.529	0.483	0.140	2.959
Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
normalized size	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.017	0.006	0.039	0.327	1.524	0.480	0.137	0.050
Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	19	7	7	7
normalized size	1	1.00	1.00	0.89	0.78	2.11	0.78	0.78	0.78
time (sec)	N/A	0.010	1.384	0.033	0.328	0.793	0.419	0.136	0.072

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	11	10	9	17
normalized size	1	1.00	1.00	0.91	0.82	1.00	0.91	0.82	1.55
time (sec)	N/A	0.032	0.026	0.032	0.325	0.685	0.171	0.119	3.008
Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	65	8	45	36	61	8
normalized size	1	1.00	1.00	6.50	0.80	4.50	3.60	6.10	0.80
time (sec)	N/A	0.038	0.018	0.089	0.326	0.620	0.522	0.150	3.153
Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	12	13	10	8	8	6
normalized size	1	1.00	1.00	1.33	1.44	1.11	0.89	0.89	0.67
time (sec)	N/A	0.029	0.004	0.065	0.330	0.625	1.795	0.144	2.926
Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	20	7	11	7	7	7
normalized size	1	1.00	1.00	4.00	1.40	2.20	1.40	1.40	1.40
time (sec)	N/A	0.016	0.010	0.038	0.314	0.699	0.067	0.141	3.001
Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	B	A	F(-1)	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	13	30	88	14	0	794	14
normalized size	1	0.00	1.00	2.31	6.77	1.08	0.00	61.08	1.08
time (sec)	N/A	0.640	0.293	0.393	0.771	0.677	0.000	0.201	3.121
Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	104	20	0	52	9
normalized size	1	1.00	1.00	1.11	11.56	2.22	0.00	5.78	1.00
time (sec)	N/A	0.016	0.016	0.026	0.318	0.624	0.000	0.189	0.028

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	14	31	37	14	18
normalized size	1	1.00	1.00	0.75	0.70	1.55	1.85	0.70	0.90
time (sec)	N/A	0.017	0.011	0.146	0.306	1.736	0.460	0.156	0.028
Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	15	15	22	15	14
normalized size	1	1.00	1.00	0.79	0.79	0.79	1.16	0.79	0.74
time (sec)	N/A	0.015	0.010	0.023	0.315	0.676	0.480	0.136	2.952
Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	14	10	18	16
normalized size	1	1.00	1.00	0.93	1.14	1.00	0.71	1.29	1.14
time (sec)	N/A	0.015	0.005	0.039	0.305	0.799	0.072	0.125	2.950
Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	29	20	37	20	28	32
normalized size	1	1.00	0.91	1.32	0.91	1.68	0.91	1.27	1.45
time (sec)	N/A	0.033	0.023	0.059	0.318	0.603	0.083	0.128	2.971
Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	36	6	5	5	12	5	5
normalized size	1	1.00	7.20	1.20	1.00	1.00	2.40	1.00	1.00
time (sec)	N/A	0.015	0.007	0.053	0.314	1.437	2.044	0.143	2.938
Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	20	6	5	7	12	7	5
normalized size	1	1.00	2.86	0.86	0.71	1.00	1.71	1.00	0.71
time (sec)	N/A	0.016	0.006	0.050	0.314	0.741	1.888	0.130	2.907

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	8	6	11	5	14	9	6
normalized size	1	1.00	1.60	1.20	2.20	1.00	2.80	1.80	1.20
time (sec)	N/A	0.022	0.002	0.047	0.309	0.530	3.113	0.151	2.888
Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	61	20	69	35	32	29	24
normalized size	1	1.00	4.07	1.33	4.60	2.33	2.13	1.93	1.60
time (sec)	N/A	0.045	0.009	0.263	0.422	0.752	1.865	0.148	3.110
Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	26	12	11	15	12	15	11
normalized size	1	1.00	2.36	1.09	1.00	1.36	1.09	1.36	1.00
time (sec)	N/A	0.046	0.092	0.053	0.307	0.702	0.190	0.133	0.085
Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	82	18	17	17	19	17	17
normalized size	1	1.00	4.10	0.90	0.85	0.85	0.95	0.85	0.85
time (sec)	N/A	0.053	0.172	0.036	0.428	0.620	0.426	0.125	2.905
Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	13	10	12	9
normalized size	1	1.00	1.00	1.09	1.00	1.18	0.91	1.09	0.82
time (sec)	N/A	0.021	0.008	0.084	0.313	0.601	0.178	0.152	2.979
Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	1856	34	27	82	0	29
normalized size	1	1.00	1.00	71.38	1.31	1.04	3.15	0.00	1.12
time (sec)	N/A	0.049	0.043	0.750	0.425	1.390	1.067	0.000	3.075

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	39	24	220	35	0	28	16
normalized size	1	1.00	1.62	1.00	9.17	1.46	0.00	1.17	0.67
time (sec)	N/A	0.028	0.033	0.242	0.329	0.602	0.000	0.138	3.073
Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	35	38	26	39	26	34
normalized size	1	1.00	0.72	0.88	0.95	0.65	0.98	0.65	0.85
time (sec)	N/A	0.065	0.041	0.043	0.330	0.553	0.292	0.125	2.900
Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	19	21	26	19	19
normalized size	1	1.00	0.81	0.81	0.70	0.78	0.96	0.70	0.70
time (sec)	N/A	0.010	0.029	0.028	0.326	0.603	0.403	0.136	0.031
Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	31	18	17	12	26	17	12
normalized size	1	1.00	1.35	0.78	0.74	0.52	1.13	0.74	0.52
time (sec)	N/A	0.040	0.025	0.026	0.323	0.710	0.286	0.133	0.102
Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	46	41	24	36	27	24	24
normalized size	1	1.00	1.53	1.37	0.80	1.20	0.90	0.80	0.80
time (sec)	N/A	0.030	0.017	0.008	0.327	0.587	0.060	0.141	2.998
Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	3	14	6
normalized size	1	1.00	1.00	1.17	1.00	1.00	0.50	2.33	1.00
time (sec)	N/A	0.011	0.001	0.082	0.331	0.534	1.132	0.130	2.945

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	15	8	30	12
normalized size	1	1.00	1.00	1.14	1.00	2.14	1.14	4.29	1.71
time (sec)	N/A	0.038	0.004	0.095	0.328	0.736	4.809	0.135	2.967
Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	10	10	13	10	14	13	6
normalized size	1	1.00	0.77	0.77	1.00	0.77	1.08	1.00	0.46
time (sec)	N/A	0.014	0.004	0.036	0.321	1.083	0.062	0.149	2.930
Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	11	22	12	11	14
normalized size	1	1.00	1.00	0.91	1.00	2.00	1.09	1.00	1.27
time (sec)	N/A	0.008	0.006	0.032	0.326	0.587	0.056	0.127	2.918
Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	17	10	18	10	11	24
normalized size	1	1.00	1.00	1.70	1.00	1.80	1.00	1.10	2.40
time (sec)	N/A	0.008	0.010	0.027	0.320	0.915	0.056	0.146	3.083
Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.008	0.009	0.004	0.321	0.698	0.154	0.140	0.050
Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	14	32	12	12
normalized size	1	1.00	1.00	0.72	0.67	0.78	1.78	0.67	0.67
time (sec)	N/A	0.041	0.031	0.066	0.327	0.625	0.746	0.128	2.993

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	31	8	8
normalized size	1	1.00	1.00	0.90	0.80	0.80	3.10	0.80	0.80
time (sec)	N/A	0.012	0.006	0.003	0.326	0.602	1.285	0.135	2.919
Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	25	11	10	21	12	10	14
normalized size	1	1.00	1.56	0.69	0.62	1.31	0.75	0.62	0.88
time (sec)	N/A	0.016	0.021	0.037	0.328	0.750	0.463	0.205	0.079
Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	13	8	9	9
normalized size	1	1.00	1.00	1.14	1.00	1.86	1.14	1.29	1.29
time (sec)	N/A	0.007	0.006	0.003	0.326	0.607	0.119	0.138	0.074
Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	16	25	15	41	15
normalized size	1	1.00	1.00	1.31	1.23	1.92	1.15	3.15	1.15
time (sec)	N/A	0.012	0.017	0.003	0.324	0.686	1.032	0.159	3.526
Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	21	11	10	23	10	43	31
normalized size	1	1.00	4.20	2.20	2.00	4.60	2.00	8.60	6.20
time (sec)	N/A	0.009	0.014	0.003	0.326	0.647	1.257	0.137	3.681
Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	9	7	8	15	7	8	16	26
normalized size	1	1.29	1.00	1.14	2.14	1.00	1.14	2.29	3.71
time (sec)	N/A	0.045	0.007	0.118	0.327	0.617	2.311	0.152	3.254

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	20	4	4
normalized size	1	1.00	1.00	1.25	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.017	0.001	0.161	0.325	0.690	0.687	0.143	3.074
Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	21	14	74	27	0	81	13
normalized size	1	1.00	1.62	1.08	5.69	2.08	0.00	6.23	1.00
time (sec)	N/A	0.020	0.006	0.027	0.436	1.317	0.000	0.171	2.994
Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	28	20	22	20	20	18
normalized size	1	1.00	1.12	1.75	1.25	1.38	1.25	1.25	1.12
time (sec)	N/A	0.029	0.024	0.039	0.431	0.592	0.054	0.141	2.961
Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	34	35	28	36	31	26
normalized size	1	1.00	0.81	1.06	1.09	0.88	1.12	0.97	0.81
time (sec)	N/A	0.034	0.021	0.041	0.428	1.629	0.055	0.132	2.995
Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	19	10	19	12	10	18
normalized size	1	1.00	0.89	1.06	0.56	1.06	0.67	0.56	1.00
time (sec)	N/A	0.028	0.007	0.009	0.330	1.560	0.058	0.137	0.047
Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	29	18	25	32	22	24
normalized size	1	1.00	0.94	0.85	0.53	0.74	0.94	0.65	0.71
time (sec)	N/A	0.050	0.009	0.009	0.326	2.391	0.053	0.121	0.050

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	33	29	13	21	12	13	13
normalized size	1	1.00	2.54	2.23	1.00	1.62	0.92	1.00	1.00
time (sec)	N/A	0.024	0.010	0.009	0.324	0.468	0.055	0.129	0.039
Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	24	36	16	31	31	16	33
normalized size	1	1.00	0.52	0.78	0.35	0.67	0.67	0.35	0.72
time (sec)	N/A	0.056	0.007	0.011	0.326	1.967	0.064	0.139	0.041
Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	21	20	19	20	18	13
normalized size	1	1.00	1.00	2.33	2.22	2.11	2.22	2.00	1.44
time (sec)	N/A	0.028	0.005	0.076	0.323	0.573	0.324	0.125	3.008
Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	17	13	13	12	12	13	11
normalized size	1	1.00	1.21	0.93	0.93	0.86	0.86	0.93	0.79
time (sec)	N/A	0.017	0.005	0.024	0.324	0.712	0.054	0.143	2.970
Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
normalized size	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.010	0.000	0.016	0.332	1.351	0.055	0.137	2.934
Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	8	7	6	6	7	6	6
normalized size	1	1.00	1.33	1.17	1.00	1.00	1.17	1.00	1.00
time (sec)	N/A	0.011	0.002	0.001	0.323	0.412	0.056	0.120	2.936

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	9	9
normalized size	1	1.00	1.00	1.11	1.00	1.00	0.78	1.00	1.00
time (sec)	N/A	0.009	0.006	0.026	0.310	0.555	0.249	0.125	0.072
Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	35	6	22	6	6
normalized size	1	1.00	1.00	0.88	4.38	0.75	2.75	0.75	0.75
time (sec)	N/A	0.017	0.004	0.003	0.316	1.469	0.116	0.141	2.945
Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	9	51	8	5	26	8
normalized size	1	1.00	1.00	1.50	8.50	1.33	0.83	4.33	1.33
time (sec)	N/A	0.178	0.024	0.021	0.483	2.142	0.481	0.153	0.085
Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	8	8	8
normalized size	1	1.00	1.00	0.88	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.197	0.018	0.085	0.314	1.437	0.282	0.119	3.069
Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	12	9	8	8	8	6	8
normalized size	1	1.00	1.50	1.12	1.00	1.00	1.00	0.75	1.00
time (sec)	N/A	0.011	0.012	0.039	0.309	0.733	0.266	0.120	3.051
Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	8
normalized size	1	1.00	1.00	0.88	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.184	0.017	0.058	0.311	0.567	0.286	0.136	3.005

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	70	55	69	0	320	155	77	1108
normalized size	1	1.27	1.00	1.25	0.00	5.82	2.82	1.40	20.15
time (sec)	N/A	0.170	0.078	0.191	0.000	1.906	9.312	0.157	4.178
Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	70	54	69	0	322	155	77	1374
normalized size	1	1.27	0.98	1.25	0.00	5.85	2.82	1.40	24.98
time (sec)	N/A	0.133	0.060	0.172	0.000	1.277	9.453	0.158	3.438
Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	80	0	225	432	141	108
normalized size	1	1.00	0.92	1.54	0.00	4.33	8.31	2.71	2.08
time (sec)	N/A	0.125	0.087	0.134	0.000	1.220	35.268	0.172	3.434
Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	80	0	224	432	159	684
normalized size	1	1.00	0.96	1.54	0.00	4.31	8.31	3.06	13.15
time (sec)	N/A	0.094	0.054	0.131	0.000	1.110	35.327	0.148	3.319
Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	31	30	76	91	0	61	-1
normalized size	1	1.00	1.03	1.00	2.53	3.03	0.00	2.03	-0.03
time (sec)	N/A	0.035	0.038	0.146	0.426	0.894	0.000	0.345	0.000
Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	49	31	51	84	0	31	-1
normalized size	1	1.00	1.58	1.00	1.65	2.71	0.00	1.00	-0.03
time (sec)	N/A	0.032	0.062	0.159	0.322	1.213	0.000	0.247	0.000

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.013	0.003	0.020	0.310	2.081	0.292	0.138	3.185
Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	25	41	21	0	14	-1
normalized size	1	1.00	0.95	1.32	2.16	1.11	0.00	0.74	-0.05
time (sec)	N/A	0.030	0.014	0.448	0.425	1.326	0.000	0.158	0.000
Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	16	24	12	23	476	12	12
normalized size	1	1.00	0.55	0.83	0.41	0.79	16.41	0.41	0.41
time (sec)	N/A	0.056	0.017	0.042	0.302	0.415	20.216	0.127	3.201
Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	37	29	34	103	26	32	34	45
normalized size	1	1.28	1.00	1.17	3.55	0.90	1.10	1.17	1.55
time (sec)	N/A	0.133	0.096	0.140	0.411	2.014	0.366	0.167	3.309
Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	37	29	34	103	26	32	34	45
normalized size	1	1.28	1.00	1.17	3.55	0.90	1.10	1.17	1.55
time (sec)	N/A	0.091	0.073	0.136	0.420	1.010	0.370	0.165	3.192
Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	31	30	46	0	34	32
normalized size	1	1.00	0.86	0.70	0.68	1.05	0.00	0.77	0.73
time (sec)	N/A	0.056	0.077	0.161	0.314	1.168	0.000	0.121	0.066

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	88	19	16	145	15	0	0	15
normalized size	1	4.63	1.00	0.84	7.63	0.79	0.00	0.00	0.79
time (sec)	N/A	2.243	0.063	0.381	0.467	0.875	0.000	0.000	3.786
Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	B	B	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	0	68	917	518	96	0	0	-1
normalized size	1	0.00	1.55	20.84	11.77	2.18	0.00	0.00	-0.02
time (sec)	N/A	2.569	0.395	0.767	0.764	0.887	0.000	0.000	0.000
Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	C	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	72	17	12372	329	3	0	42	-1
normalized size	1	3.79	0.89	651.16	17.32	0.16	0.00	2.21	-0.05
time (sec)	N/A	1.707	0.013	0.417	0.495	1.389	0.000	0.168	0.000
Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	37	10	0	21	0	55	14
normalized size	1	1.00	2.85	0.77	0.00	1.62	0.00	4.23	1.08
time (sec)	N/A	0.146	0.042	0.144	0.000	0.893	0.000	0.323	0.292
Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	25	29	68	29
normalized size	1	1.00	1.00	0.79	0.71	1.79	2.07	4.86	2.07
time (sec)	N/A	0.044	0.052	0.063	0.324	1.256	0.679	0.160	3.231
Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	30	34	21	35	0	128	24
normalized size	1	1.00	1.20	1.36	0.84	1.40	0.00	5.12	0.96
time (sec)	N/A	0.085	0.191	0.168	0.333	0.868	0.000	0.169	3.334

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	18	38	21	44	0	128	24
normalized size	1	1.00	0.72	1.52	0.84	1.76	0.00	5.12	0.96
time (sec)	N/A	0.081	0.038	0.181	0.321	0.406	0.000	0.143	3.418
Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0	77
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	3.85
time (sec)	N/A	0.151	0.443	0.501	0.000	0.000	0.000	0.000	3.460
Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	84	40	54	136	34	0	97	-1
normalized size	1	1.11	0.53	0.71	1.79	0.45	0.00	1.28	-0.01
time (sec)	N/A	0.162	0.089	0.375	0.713	1.922	0.000	0.153	0.000
Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	55	65	86	41	0	44	-1
normalized size	1	1.00	0.68	0.80	1.06	0.51	0.00	0.54	-0.01
time (sec)	N/A	0.159	0.061	0.270	0.930	0.934	0.000	0.151	0.000
Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	69	98	79	124	0	0	-1
normalized size	1	1.00	0.91	1.29	1.04	1.63	0.00	0.00	-0.01
time (sec)	N/A	0.535	0.058	0.228	0.847	1.271	0.000	0.000	0.000
Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	99	132	107	227	0	0	-1
normalized size	1	1.00	0.77	1.03	0.84	1.77	0.00	0.00	-0.01
time (sec)	N/A	0.592	0.066	0.205	0.677	1.622	0.000	0.000	0.000

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	147	172	131	327	0	0	-1
normalized size	1	1.00	0.79	0.92	0.70	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.570	0.096	0.188	0.822	1.309	0.000	0.000	0.000
Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	50	147	83	138	0	0	-1
normalized size	1	1.00	0.62	1.81	1.02	1.70	0.00	0.00	-0.01
time (sec)	N/A	0.487	0.040	0.204	1.922	2.100	0.000	0.000	0.000
Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	75	183	113	248	0	0	-1
normalized size	1	1.00	0.69	1.68	1.04	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.574	0.060	0.185	1.098	2.498	0.000	0.000	0.000
Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	87	221	137	356	0	0	-1
normalized size	1	1.00	0.61	1.55	0.96	2.49	0.00	0.00	-0.01
time (sec)	N/A	0.610	0.068	0.190	0.554	2.051	0.000	0.000	0.000
Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	108	86	299	140	0	0	-1
normalized size	1	1.00	1.03	0.82	2.85	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.343	0.077	0.253	0.704	0.853	0.000	0.000	0.000
Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	174	200	0	337	0	0	-1
normalized size	1	1.00	0.77	0.89	0.00	1.50	0.00	0.00	-0.00
time (sec)	N/A	0.531	0.122	0.302	0.000	0.808	0.000	0.000	0.000

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	290	250	568	539	0	0	-1
normalized size	1	1.00	0.85	0.73	1.67	1.58	0.00	0.00	-0.00
time (sec)	N/A	0.629	0.412	0.418	0.476	1.147	0.000	0.000	0.000
Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	85	165	432	270	0	0	-1
normalized size	1	1.00	0.60	1.16	3.04	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.399	0.233	0.195	0.504	0.760	0.000	0.000	0.000
Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	138	254	653	550	0	0	-1
normalized size	1	1.00	0.63	1.15	2.97	2.50	0.00	0.00	-0.00
time (sec)	N/A	0.537	0.634	0.231	0.521	2.048	0.000	0.000	0.000
Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	191	324	870	736	0	0	-1
normalized size	1	1.00	0.54	0.91	2.44	2.07	0.00	0.00	-0.00
time (sec)	N/A	0.637	1.069	0.213	1.397	1.079	0.000	0.000	0.000
Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	17	114	13	14
normalized size	1	1.00	1.00	0.80	0.76	0.68	4.56	0.52	0.56
time (sec)	N/A	0.031	0.010	0.129	0.319	0.914	10.546	0.122	2.936
Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	25	116	22	22
normalized size	1	1.00	1.00	0.77	0.73	0.83	3.87	0.73	0.73
time (sec)	N/A	0.033	0.010	0.141	0.308	0.868	10.539	0.147	3.037

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	25	114	22	22
normalized size	1	1.00	1.00	0.77	0.73	0.83	3.80	0.73	0.73
time (sec)	N/A	0.033	0.008	0.093	0.316	0.711	10.409	0.132	3.008
Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	112	19	19
normalized size	1	1.00	1.00	0.80	0.76	0.76	4.48	0.76	0.76
time (sec)	N/A	0.031	0.008	0.085	0.312	0.880	10.452	0.131	3.178
Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.006	0.006	0.004	0.313	0.830	0.155	0.135	0.046
Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	97	9	34	46	19	20
normalized size	1	1.00	2.27	8.82	0.82	3.09	4.18	1.73	1.82
time (sec)	N/A	0.021	0.077	0.082	0.327	0.635	1.664	0.127	3.190
Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	18	12	133	15	0	52	16
normalized size	1	1.00	1.64	1.09	12.09	1.36	0.00	4.73	1.45
time (sec)	N/A	0.019	0.011	0.044	0.320	0.806	0.000	0.133	3.089
Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	18	13	7	8	8
normalized size	1	1.00	1.00	0.75	1.50	1.08	0.58	0.67	0.67
time (sec)	N/A	0.047	0.016	0.124	0.314	1.093	1.411	0.153	2.928

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	17	17	27	26	33	29	24	19
normalized size	1	1.42	1.42	2.25	2.17	2.75	2.42	2.00	1.58
time (sec)	N/A	0.039	0.014	0.076	0.306	0.930	1.492	0.143	0.087
Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	20	20	19	20	19
normalized size	1	1.00	1.00	0.95	1.05	1.05	1.00	1.05	1.00
time (sec)	N/A	0.040	0.013	0.049	0.307	1.067	24.474	0.139	3.672
Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	52	50	49	65	51	51	69
normalized size	1	1.00	1.18	1.14	1.11	1.48	1.16	1.16	1.57
time (sec)	N/A	0.038	0.044	0.108	0.311	0.920	0.112	0.299	3.112
Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	60	33	32	34	33	30
normalized size	1	1.00	0.95	1.62	0.89	0.86	0.92	0.89	0.81
time (sec)	N/A	0.033	0.029	0.111	0.307	1.847	0.086	0.757	3.099
Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	68	58	57	73	61	59	85
normalized size	1	1.00	1.26	1.07	1.06	1.35	1.13	1.09	1.57
time (sec)	N/A	0.041	0.096	0.121	0.308	0.933	0.109	0.423	7.214
Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	32	31	34	0	33	33
normalized size	1	1.00	1.00	0.86	0.84	0.92	0.00	0.89	0.89
time (sec)	N/A	0.039	0.027	0.096	0.305	0.410	0.000	0.145	2.944

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	42	46	46	49	49	42	69
normalized size	1	1.00	1.24	1.35	1.35	1.44	1.44	1.24	2.03
time (sec)	N/A	0.025	0.009	0.056	0.310	0.971	0.147	0.147	3.038
Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	68	41	52	42	41	42
normalized size	1	1.00	1.00	1.58	0.95	1.21	0.98	0.95	0.98
time (sec)	N/A	0.036	0.030	0.169	0.306	0.964	0.089	0.160	3.056
Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	72	73	92	0	73	74
normalized size	1	1.00	1.00	0.83	0.84	1.06	0.00	0.84	0.85
time (sec)	N/A	0.130	0.062	0.156	0.308	1.047	0.000	0.489	2.970
Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	44	79	0	52	71
normalized size	1	1.00	1.00	0.88	1.05	1.88	0.00	1.24	1.69
time (sec)	N/A	0.118	0.041	0.155	0.304	0.898	0.000	0.144	3.161
Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	57	84	0	57	57
normalized size	1	1.00	1.00	0.89	0.90	1.33	0.00	0.90	0.90
time (sec)	N/A	0.124	0.034	0.151	0.311	2.934	0.000	0.165	2.967
Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	49	52	103	0	60	84
normalized size	1	1.00	0.87	0.82	0.87	1.72	0.00	1.00	1.40
time (sec)	N/A	0.126	0.127	0.158	0.318	0.837	0.000	0.249	4.674

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	59	38	41	42	44	0	42
normalized size	1	1.00	1.26	0.81	0.87	0.89	0.94	0.00	0.89
time (sec)	N/A	0.102	0.049	0.137	0.326	0.626	5.196	0.000	5.378
Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	71	38	37	41	44	39	37
normalized size	1	1.00	1.65	0.88	0.86	0.95	1.02	0.91	0.86
time (sec)	N/A	0.116	0.022	0.062	0.321	1.946	2.946	6.001	0.077
Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	69	44	79	41	52	71
normalized size	1	1.00	1.00	1.64	1.05	1.88	0.98	1.24	1.69
time (sec)	N/A	0.041	0.029	0.109	0.335	1.956	0.105	0.147	3.051
Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	55	91	82	0	72	41
normalized size	1	1.00	0.85	0.74	1.23	1.11	0.00	0.97	0.55
time (sec)	N/A	0.246	0.083	0.072	1.075	0.834	0.000	0.228	3.137
Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	47	32	176	39	0	1179	34
normalized size	1	1.00	3.36	2.29	12.57	2.79	0.00	84.21	2.43
time (sec)	N/A	0.011	0.040	0.039	1.316	0.673	0.000	0.522	3.173
Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	10	12	12	10
normalized size	1	1.00	1.00	0.93	0.86	0.71	0.86	0.86	0.71
time (sec)	N/A	0.038	0.015	0.079	0.331	0.765	1.428	0.137	0.166

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	24	0	9	18
normalized size	1	1.00	1.00	0.91	0.00	2.18	0.00	0.82	1.64
time (sec)	N/A	0.054	0.016	0.113	0.000	1.313	0.000	0.148	3.142
Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	35	19	32	32	34	33
normalized size	1	1.00	0.94	1.03	0.56	0.94	0.94	1.00	0.97
time (sec)	N/A	0.049	0.028	0.059	0.367	0.870	3.681	0.144	2.998
Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	25	24	17	17	24	17	18
normalized size	1	1.00	1.19	1.14	0.81	0.81	1.14	0.81	0.86
time (sec)	N/A	0.030	0.013	0.075	0.559	0.817	1.780	0.145	0.069
Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	55	97	51	66	189	58	69
normalized size	1	1.00	0.56	0.98	0.52	0.67	1.91	0.59	0.70
time (sec)	N/A	0.060	0.229	0.042	0.398	1.090	1.298	0.127	3.063
Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	40	51	59	51	0	70	64
normalized size	1	1.00	1.08	1.38	1.59	1.38	0.00	1.89	1.73
time (sec)	N/A	0.090	0.059	0.108	0.583	1.409	0.000	0.205	3.128
Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	46	41	73	90	41	45
normalized size	1	1.00	0.86	0.81	0.72	1.28	1.58	0.72	0.79
time (sec)	N/A	0.073	0.095	0.140	0.325	1.130	6.019	0.120	3.048

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	243	68	134	0	85	0	0	51
normalized size	1	4.26	1.19	2.35	0.00	1.49	0.00	0.00	0.89
time (sec)	N/A	0.211	0.058	0.256	0.000	0.983	0.000	0.000	4.619
Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	8	7	17	7	10	10
normalized size	1	1.00	1.00	1.60	1.40	3.40	1.40	2.00	2.00
time (sec)	N/A	0.023	0.004	0.063	0.322	0.847	2.184	0.126	2.966
Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	A	A	F	F(-1)	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	11	30	11	11	0	0	11
normalized size	1	0.00	1.00	2.73	1.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.276	0.308	0.194	0.744	0.744	0.000	0.000	3.158
Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	B	A	F	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	24	22	74	27	0	93	38
normalized size	1	0.00	0.89	0.81	2.74	1.00	0.00	3.44	1.41
time (sec)	N/A	0.063	0.167	0.360	0.642	1.585	0.000	3.119	0.482
Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	50	62	66	50	100	51	65
normalized size	1	1.00	0.77	0.95	1.02	0.77	1.54	0.78	1.00
time (sec)	N/A	0.068	0.090	0.026	0.332	1.651	1.218	1.979	3.029
Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	10	14	0	0	10
normalized size	1	1.00	1.00	1.38	1.25	1.75	0.00	0.00	1.25
time (sec)	N/A	0.023	0.015	0.085	0.313	0.916	0.000	0.000	0.145

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	125	41	0	0	45
normalized size	1	1.00	1.00	0.00	3.91	1.28	0.00	0.00	1.41
time (sec)	N/A	0.040	0.079	0.368	0.444	0.954	0.000	0.000	3.430
Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	45	3	0	3	26
normalized size	1	1.00	1.00	1.33	15.00	1.00	0.00	1.00	8.67
time (sec)	N/A	0.030	0.018	0.152	0.334	1.419	0.000	0.118	3.190
Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	36	35	27	39	27	34
normalized size	1	1.00	0.91	1.03	1.00	0.77	1.11	0.77	0.97
time (sec)	N/A	0.065	0.042	0.032	0.323	0.588	0.325	0.124	0.061
Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	17	6	5	6	-1
normalized size	1	1.00	1.00	0.88	2.12	0.75	0.62	0.75	-0.12
time (sec)	N/A	0.007	0.002	0.026	0.361	0.731	0.638	0.131	0.000
Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	29	38	0	43	0	29	-1
normalized size	1	1.00	0.78	1.03	0.00	1.16	0.00	0.78	-0.03
time (sec)	N/A	0.173	0.071	0.034	0.000	0.851	0.000	0.126	0.000
Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	41
normalized size	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	2.93
time (sec)	N/A	0.013	0.006	0.019	0.325	0.872	84.149	0.163	3.143

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	15	8	20	7	8	19
normalized size	1	1.00	1.00	1.50	0.80	2.00	0.70	0.80	1.90
time (sec)	N/A	0.037	0.003	0.098	0.321	0.426	1.037	0.139	3.094
Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	27	15	56
normalized size	1	1.00	1.00	0.94	0.88	0.88	1.59	0.88	3.29
time (sec)	N/A	0.024	0.022	0.049	0.343	1.700	20.860	0.179	3.261
Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	120	403	126	85	241	126	147
normalized size	1	1.00	0.93	3.12	0.98	0.66	1.87	0.98	1.14
time (sec)	N/A	0.144	0.565	0.950	0.339	1.091	74.974	0.441	3.455
Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	352	160	3830	115	0	1455	730
normalized size	1	1.00	3.20	1.45	34.82	1.05	0.00	13.23	6.64
time (sec)	N/A	0.112	0.871	0.309	0.859	0.960	0.000	1.641	13.421
Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	36	12	0	20	14
normalized size	1	1.00	1.00	1.17	6.00	2.00	0.00	3.33	2.33
time (sec)	N/A	0.020	0.018	0.035	0.381	1.042	0.000	0.127	3.028
Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
normalized size	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.010	0.002	0.001	0.319	0.916	0.267	0.132	2.952

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	30	24	98	39	0	943	23
normalized size	1	1.00	1.11	0.89	3.63	1.44	0.00	34.93	0.85
time (sec)	N/A	0.023	0.015	0.041	0.451	0.827	0.000	0.374	0.095
Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	F	F(-2)	F	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	26	28	0	0	0	0	22
normalized size	1	0.00	1.00	1.08	0.00	0.00	0.00	0.00	0.85
time (sec)	N/A	0.811	0.448	1.042	0.000	0.000	0.000	0.000	3.475
Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.111	7.568	1.096	0.000	0.000	0.000	0.000	0.000
Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	B	A	A	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	0	9	57	82	10	10	83	10
normalized size	1	0.00	1.00	6.33	9.11	1.11	1.11	9.22	1.11
time (sec)	N/A	0.381	0.117	0.188	0.563	0.882	0.292	0.150	2.955
Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	76	60	57	60	150	62	111
normalized size	1	1.00	0.99	0.78	0.74	0.78	1.95	0.81	1.44
time (sec)	N/A	0.126	0.152	0.051	0.493	1.131	0.970	0.130	6.559
Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	134	125	131	123	326	143	210
normalized size	1	1.00	0.83	0.78	0.81	0.76	2.02	0.89	1.30
time (sec)	N/A	0.270	0.210	0.076	0.326	0.912	6.008	0.192	6.744

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	105	84	79	72	201	70	73
normalized size	1	1.00	1.18	0.94	0.89	0.81	2.26	0.79	0.82
time (sec)	N/A	0.109	0.151	0.076	0.460	0.879	1.034	0.153	3.183
Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	167	213	218	162	541	186	456
normalized size	1	1.00	0.58	0.74	0.76	0.56	1.88	0.65	1.58
time (sec)	N/A	0.400	0.449	0.086	0.404	1.962	6.813	0.178	4.517
Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	55	136	0	0	0	0	51
normalized size	1	1.00	0.90	2.23	0.00	0.00	0.00	0.00	0.84
time (sec)	N/A	0.289	0.179	0.338	0.000	1.780	0.000	0.000	3.247
Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	137	266	0	0	0	0	129
normalized size	1	1.00	0.93	1.80	0.00	0.00	0.00	0.00	0.87
time (sec)	N/A	0.239	0.281	0.325	0.000	1.460	0.000	0.000	6.684
Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	43	86	50	30	107	31	35
normalized size	1	1.00	1.26	2.53	1.47	0.88	3.15	0.91	1.03
time (sec)	N/A	0.097	0.100	0.437	0.807	0.630	6.708	0.616	3.456
Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	43	86	62	30	107	31	35
normalized size	1	1.00	1.19	2.39	1.72	0.83	2.97	0.86	0.97
time (sec)	N/A	0.097	0.089	0.393	0.492	0.932	6.665	0.834	3.351

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	73	184	0	150	0	128	226
normalized size	1	1.00	0.61	1.53	0.00	1.25	0.00	1.07	1.88
time (sec)	N/A	0.701	0.570	0.736	0.000	0.894	0.000	0.529	4.216
Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	25	108	58	74	122	48	23
normalized size	1	1.00	0.35	1.50	0.81	1.03	1.69	0.67	0.32
time (sec)	N/A	0.151	0.032	0.385	0.457	0.968	5.594	0.261	3.166
Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	42	56	154	42	76	52	105
normalized size	1	1.00	0.76	1.02	2.80	0.76	1.38	0.95	1.91
time (sec)	N/A	0.409	0.200	0.648	0.434	0.895	1.023	0.287	3.305
Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	33	17	22	16	32	14	14
normalized size	1	1.00	2.06	1.06	1.38	1.00	2.00	0.88	0.88
time (sec)	N/A	0.054	0.012	0.230	0.331	0.546	0.255	0.176	3.034
Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	22	31	29	50
normalized size	1	1.00	1.00	1.06	1.00	1.22	1.72	1.61	2.78
time (sec)	N/A	0.028	0.044	0.246	0.312	1.165	0.391	0.175	3.136
Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	32	70	22	0	29	50
normalized size	1	1.00	1.00	1.68	3.68	1.16	0.00	1.53	2.63
time (sec)	N/A	0.314	0.059	0.746	0.414	1.348	0.000	0.294	3.371

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	33	18	23	17	0	14	14
normalized size	1	1.00	1.94	1.06	1.35	1.00	0.00	0.82	0.82
time (sec)	N/A	0.175	0.015	0.342	0.311	1.407	0.000	0.243	3.054
Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	42	56	154	42	0	52	106
normalized size	1	1.00	0.78	1.04	2.85	0.78	0.00	0.96	1.96
time (sec)	N/A	0.532	0.235	0.827	0.438	0.935	0.000	0.390	3.233
Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	26	108	58	74	0	48	23
normalized size	1	1.00	0.36	1.50	0.81	1.03	0.00	0.67	0.32
time (sec)	N/A	1.396	0.024	0.511	0.428	1.155	0.000	0.386	3.089

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [341] had the largest ratio of [1.286]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	14	0.143
2	A	3	3	1.00	27	0.111
3	A	2	2	1.00	12	0.167
4	A	2	2	1.00	14	0.143
5	A	2	1	1.00	21	0.048
6	A	2	2	1.00	23	0.087
7	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	3	3	1.00	14	0.214
9	A	4	4	1.00	25	0.160
10	A	2	2	1.00	14	0.143
11	A	2	2	1.00	14	0.143
12	A	2	1	1.00	23	0.043
13	A	2	2	1.00	23	0.087
14	A	2	2	1.00	23	0.087
15	A	2	2	1.00	12	0.167
16	A	3	3	1.00	25	0.120
17	A	2	2	1.00	14	0.143
18	A	2	2	1.00	12	0.167
19	A	2	1	1.00	21	0.048
20	A	2	2	1.00	21	0.095
21	A	2	2	1.00	23	0.087
22	A	3	3	1.00	14	0.214
23	A	4	4	1.00	25	0.160
24	A	2	2	1.00	14	0.143
25	A	2	2	1.00	14	0.143
26	A	2	1	1.00	21	0.048
27	A	2	2	1.00	21	0.095
28	A	2	2	1.00	23	0.087
29	A	6	5	1.00	6	0.833
30	A	9	6	1.00	6	1.000
31	A	8	4	1.00	16	0.250
32	A	8	4	1.00	19	0.210
33	A	3	3	1.00	16	0.188
34	A	3	3	1.00	27	0.111
35	A	2	1	1.00	11	0.091
36	A	5	5	1.00	14	0.357
37	A	6	6	1.00	16	0.375
38	A	9	5	1.00	16	0.312
39	A	5	3	1.00	36	0.083
40	A	4	3	1.00	36	0.083
41	A	2	2	1.00	34	0.059
42	A	0	0	0.00	0	0.000
43	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	6	5	1.00	6	0.833
45	A	9	6	1.00	6	1.000
46	A	8	4	1.00	16	0.250
47	A	8	4	1.00	19	0.210
48	A	4	4	1.00	21	0.190
49	A	4	4	1.00	27	0.148
50	A	5	4	1.00	37	0.108
51	A	5	5	1.00	14	0.357
52	A	6	6	1.00	16	0.375
53	A	5	3	1.00	36	0.083
54	A	4	3	1.00	36	0.083
55	A	2	2	1.00	34	0.059
56	A	0	0	0.00	0	0.000
57	A	0	0	0.00	0	0.000
58	A	2	2	1.00	12	0.167
59	A	3	3	1.00	14	0.214
60	A	6	6	1.00	12	0.500
61	A	2	1	1.00	33	0.030
62	A	3	3	1.00	14	0.214
63	A	2	2	1.00	25	0.080
64	A	4	3	1.00	16	0.188
65	A	4	3	1.00	15	0.200
66	A	1	1	1.00	7	0.143
67	A	1	1	1.00	7	0.143
68	A	1	1	1.00	7	0.143
69	A	4	2	1.00	7	0.286
70	A	1	1	1.00	7	0.143
71	A	1	1	1.00	7	0.143
72	A	1	1	1.00	7	0.143
73	A	4	2	1.00	7	0.286
74	A	4	3	1.00	7	0.429
75	A	9	4	1.00	7	0.571
76	A	5	3	1.00	7	0.429
77	A	10	4	1.00	7	0.571
78	A	10	5	1.00	7	0.714
79	A	6	3	1.00	7	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	3	2	1.00	7	0.286
81	A	3	2	1.00	7	0.286
82	A	6	3	1.00	7	0.429
83	A	6	3	1.00	7	0.429
84	A	7	3	1.00	7	0.429
85	A	2	2	1.00	7	0.286
86	A	5	5	1.00	7	0.714
87	A	4	3	1.00	7	0.429
88	A	7	6	1.00	7	0.857
89	A	7	4	1.00	7	0.571
90	A	2	2	1.00	7	0.286
91	A	2	1	1.00	7	0.143
92	A	4	2	1.00	7	0.286
93	A	4	2	1.00	7	0.286
94	A	7	3	1.00	7	0.429
95	A	3	2	1.00	7	0.286
96	A	2	2	1.00	9	0.222
97	A	1	1	1.00	7	0.143
98	A	1	1	1.00	7	0.143
99	A	1	1	1.00	7	0.143
100	A	4	2	1.00	7	0.286
101	A	1	1	1.00	7	0.143
102	A	1	1	1.00	7	0.143
103	A	1	1	1.00	7	0.143
104	A	4	2	1.00	7	0.286
105	A	4	3	1.00	7	0.429
106	A	3	2	1.00	7	0.286
107	A	6	4	1.00	7	0.571
108	A	6	3	1.00	7	0.429
109	A	10	5	1.00	7	0.714
110	A	4	3	1.00	7	0.429
111	A	9	4	1.00	7	0.571
112	A	6	3	1.00	7	0.429
113	A	10	4	1.00	7	0.571
114	A	7	3	1.00	7	0.429
115	A	6	3	1.00	7	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	2	2	1.00	7	0.286
117	A	2	1	1.00	7	0.143
118	A	4	3	1.00	7	0.429
119	A	4	2	1.00	7	0.286
120	A	7	4	1.00	7	0.571
121	A	3	3	1.00	7	0.429
122	A	3	3	1.00	9	0.333
123	A	2	2	1.00	7	0.286
124	A	5	5	1.00	7	0.714
125	A	4	2	1.00	7	0.286
126	A	7	6	1.00	7	0.857
127	A	7	3	1.00	7	0.429
128	A	6	2	1.00	9	0.222
129	A	6	2	1.00	11	0.182
130	A	5	2	1.00	11	0.182
131	A	5	2	1.00	9	0.222
132	A	6	2	1.00	9	0.222
133	A	6	2	1.00	11	0.182
134	A	7	2	1.00	13	0.154
135	A	3	2	1.00	13	0.154
136	A	3	2	1.00	14	0.143
137	A	3	2	1.00	13	0.154
138	A	3	2	1.00	14	0.143
139	A	4	3	1.00	13	0.231
140	A	4	3	1.00	14	0.214
141	A	4	3	1.00	13	0.231
142	A	4	3	1.00	14	0.214
143	A	3	2	1.00	13	0.154
144	A	3	2	1.00	14	0.143
145	A	3	2	1.00	13	0.154
146	A	3	2	1.00	14	0.143
147	A	2	2	1.00	9	0.222
148	A	3	3	1.00	9	0.333
149	A	4	4	1.00	9	0.444
150	A	2	2	1.00	9	0.222
151	A	3	3	1.00	9	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	4	4	1.00	9	0.444
153	A	4	4	1.00	12	0.333
154	A	6	6	1.00	12	0.500
155	A	3	3	1.00	12	0.250
156	A	5	5	1.00	12	0.417
157	A	3	3	1.00	14	0.214
158	A	3	3	1.00	14	0.214
159	A	2	2	1.00	23	0.087
160	A	9	6	1.00	24	0.250
161	A	11	7	1.00	26	0.269
162	A	2	1	1.00	33	0.030
163	A	9	6	1.00	34	0.176
164	A	11	7	1.00	36	0.194
165	A	5	3	1.00	33	0.091
166	A	4	3	1.00	33	0.091
167	A	3	3	1.00	31	0.097
168	A	4	4	1.00	33	0.121
169	A	5	5	1.00	33	0.152
170	A	6	5	1.00	33	0.152
171	A	11	9	1.00	33	0.273
172	A	8	7	1.00	33	0.212
173	A	3	3	1.00	31	0.097
174	A	11	7	1.00	33	0.212
175	A	13	8	1.00	33	0.242
176	A	15	8	1.00	33	0.242
177	A	20	11	1.00	37	0.297
178	A	17	10	1.00	37	0.270
179	A	14	9	1.00	35	0.257
180	A	0	0	0.00	0	0.000
181	A	51	17	1.00	33	0.515
182	A	34	14	1.00	33	0.424
183	A	26	12	1.00	31	0.387
184	A	15	8	1.00	22	0.364
185	A	5	5	1.00	13	0.385
186	A	6	6	1.00	15	0.400
187	A	7	6	1.00	15	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
188	A	8	6	1.00	15	0.400
189	A	4	4	1.00	15	0.267
190	A	5	4	1.00	15	0.267
191	A	6	4	1.00	15	0.267
192	A	7	4	1.00	15	0.267
193	A	6	5	1.00	17	0.294
194	A	5	5	1.00	17	0.294
195	A	4	4	1.00	17	0.235
196	A	6	5	1.00	17	0.294
197	A	7	6	1.00	17	0.353
198	A	8	6	1.00	17	0.353
199	A	3	3	1.00	16	0.188
200	A	3	3	1.00	16	0.188
201	A	4	4	1.00	17	0.235
202	A	3	3	1.00	17	0.176
203	A	4	4	1.00	17	0.235
204	A	3	3	1.00	17	0.176
205	A	4	3	1.00	22	0.136
206	A	8	5	1.00	17	0.294
207	A	5	3	1.00	19	0.158
208	A	3	2	1.00	20	0.100
209	A	4	3	1.00	19	0.158
210	A	4	3	1.00	22	0.136
211	A	10	6	1.00	17	0.353
212	A	4	2	1.00	19	0.105
213	A	3	3	1.00	20	0.150
214	A	4	3	1.00	19	0.158
215	A	6	6	1.00	17	0.353
216	A	7	7	1.00	17	0.412
217	A	2	2	1.00	19	0.105
218	A	2	2	1.00	19	0.105
219	A	3	2	1.00	19	0.105
220	A	4	2	1.00	19	0.105
221	A	3	2	1.00	19	0.105
222	A	3	2	1.00	19	0.105
223	A	2	1	1.00	19	0.053

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	2	2	1.00	19	0.105
225	A	3	2	1.00	17	0.118
226	A	2	2	1.00	19	0.105
227	A	1	1	1.00	19	0.053
228	A	3	3	1.00	19	0.158
229	A	2	2	1.00	19	0.105
230	A	4	3	1.00	19	0.158
231	A	3	2	1.00	19	0.105
232	A	4	3	1.00	21	0.143
233	A	3	3	1.00	21	0.143
234	A	3	3	1.00	21	0.143
235	A	2	2	1.00	21	0.095
236	A	2	2	1.00	21	0.095
237	A	3	3	1.00	21	0.143
238	A	3	3	1.00	21	0.143
239	A	4	3	1.00	21	0.143
240	A	4	3	1.00	21	0.143
241	A	3	3	1.00	21	0.143
242	A	3	3	1.00	21	0.143
243	A	2	2	1.00	21	0.095
244	A	2	2	1.00	21	0.095
245	A	3	3	1.00	21	0.143
246	A	3	3	1.00	21	0.143
247	A	4	3	1.00	21	0.143
248	A	1	1	1.00	22	0.045
249	A	1	1	1.00	22	0.045
250	A	1	1	1.00	22	0.045
251	A	1	1	1.00	22	0.045
252	A	3	2	1.00	20	0.100
253	A	1	1	1.00	22	0.045
254	A	1	1	1.00	22	0.045
255	A	1	1	1.00	22	0.045
256	A	1	1	1.00	22	0.045
257	A	1	1	1.00	24	0.042
258	A	1	1	1.00	24	0.042
259	A	1	1	1.00	24	0.042

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
260	A	1	1	1.00	24	0.042
261	A	1	1	1.00	24	0.042
262	A	1	1	1.00	24	0.042
263	A	8	7	1.00	11	0.636
264	A	4	4	1.00	11	0.364
265	A	7	6	1.00	11	0.546
266	A	4	3	1.00	11	0.273
267	A	3	2	1.00	9	0.222
268	A	3	3	1.00	11	0.273
269	A	6	6	1.00	11	0.546
270	A	4	3	1.00	11	0.273
271	A	8	8	1.00	11	0.727
272	A	4	3	1.00	11	0.273
273	A	4	3	1.00	7	0.429
274	A	5	4	1.00	7	0.571
275	A	4	3	1.00	7	0.429
276	A	4	4	1.00	7	0.571
277	A	3	2	0.69	5	0.400
278	A	3	3	1.00	7	0.429
279	A	3	3	1.00	7	0.429
280	A	4	3	1.00	7	0.429
281	A	4	3	1.00	7	0.429
282	A	4	3	1.00	7	0.429
283	A	8	7	1.00	11	0.636
284	A	4	4	1.00	11	0.364
285	A	7	6	1.00	11	0.546
286	A	4	3	1.00	11	0.273
287	A	3	2	1.00	9	0.222
288	A	3	3	1.00	11	0.273
289	A	5	5	1.00	11	0.454
290	A	4	3	1.00	11	0.273
291	A	7	7	1.00	11	0.636
292	A	4	3	1.00	11	0.273
293	A	4	3	1.00	7	0.429
294	A	5	4	1.00	7	0.571
295	A	4	3	1.00	7	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
296	A	4	4	1.00	7	0.571
297	A	3	2	1.00	5	0.400
298	A	3	3	1.00	7	0.429
299	A	3	3	1.00	7	0.429
300	A	4	3	1.00	7	0.429
301	A	4	3	1.00	7	0.429
302	A	4	3	1.00	7	0.429
303	A	6	3	1.00	9	0.333
304	A	6	5	1.00	9	0.556
305	A	4	3	1.00	9	0.333
306	A	3	2	1.00	7	0.286
307	A	3	3	1.00	9	0.333
308	A	2	1	1.00	9	0.111
309	A	4	3	1.00	9	0.333
310	A	2	0	1.00	9	0.000
311	A	4	3	1.00	9	0.333
312	A	3	1	1.00	9	0.111
313	A	4	3	1.00	9	0.333
314	A	6	5	1.00	11	0.454
315	A	5	5	1.00	11	0.454
316	A	4	4	1.00	11	0.364
317	A	3	3	1.00	11	0.273
318	A	8	8	1.00	11	0.727
319	A	9	9	1.00	11	0.818
320	A	10	10	1.00	11	0.909
321	A	11	10	1.00	11	0.909
322	A	6	3	1.00	9	0.333
323	A	6	5	1.00	9	0.556
324	A	4	3	1.00	9	0.333
325	A	3	2	1.00	7	0.286
326	A	3	3	1.00	9	0.333
327	A	2	1	1.00	9	0.111
328	A	4	3	1.00	9	0.333
329	A	2	0	1.00	9	0.000
330	A	4	3	1.00	9	0.333
331	A	3	1	1.00	9	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
332	A	4	3	1.00	9	0.333
333	A	6	5	1.00	11	0.454
334	A	5	5	1.00	11	0.454
335	A	4	4	1.00	11	0.364
336	A	3	3	1.00	11	0.273
337	A	8	8	1.00	11	0.727
338	A	9	9	1.00	11	0.818
339	A	10	10	1.00	11	0.909
340	A	11	10	1.00	11	0.909
341	A	18	9	1.00	7	1.286
342	A	4	3	1.00	7	0.429
343	A	9	7	1.00	7	1.000
344	A	3	2	1.00	5	0.400
345	A	6	6	1.00	7	0.857
346	A	11	6	1.00	7	0.857
347	A	5	4	1.00	7	0.571
348	A	18	6	1.00	7	0.857
349	A	3	3	1.00	18	0.167
350	A	3	3	1.00	18	0.167
351	A	4	4	1.00	18	0.222
352	A	3	3	1.00	18	0.167
353	A	3	3	1.00	18	0.167
354	A	4	4	1.00	18	0.222
355	A	6	3	1.00	30	0.100
356	A	5	3	1.00	30	0.100
357	A	4	3	1.00	30	0.100
358	A	3	2	1.00	28	0.071
359	A	1	1	1.00	30	0.033
360	A	2	2	1.00	30	0.067
361	A	3	2	1.00	30	0.067
362	A	4	2	1.00	30	0.067
363	A	5	4	1.00	24	0.167
364	A	4	3	1.00	24	0.125
365	A	3	2	1.00	22	0.091
366	A	2	2	1.00	24	0.083
367	A	4	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
368	A	4	4	1.00	24	0.167
369	A	5	5	1.00	24	0.208
370	A	2	2	1.00	24	0.083
371	A	4	4	1.00	24	0.167
372	A	4	4	1.00	24	0.167
373	A	5	5	1.00	24	0.208
374	A	5	4	1.00	24	0.167
375	A	4	3	1.00	24	0.125
376	A	3	2	1.00	22	0.091
377	A	2	2	1.00	24	0.083
378	A	4	4	1.00	24	0.167
379	A	4	4	1.00	24	0.167
380	A	5	5	1.00	24	0.208
381	A	5	4	1.00	24	0.167
382	A	4	3	1.00	24	0.125
383	A	3	2	1.00	22	0.091
384	A	2	2	1.00	24	0.083
385	A	4	4	1.00	24	0.167
386	A	4	4	1.00	24	0.167
387	A	5	5	1.00	24	0.208
388	A	5	4	1.00	24	0.167
389	A	4	3	1.00	24	0.125
390	A	3	2	1.00	22	0.091
391	A	2	2	1.00	24	0.083
392	A	4	4	1.00	24	0.167
393	A	4	4	1.00	24	0.167
394	A	5	5	1.00	24	0.208
395	A	6	4	1.00	20	0.200
396	A	5	4	1.00	20	0.200
397	A	4	3	1.00	20	0.150
398	A	3	2	1.00	18	0.111
399	A	3	3	1.00	20	0.150
400	A	5	5	1.00	20	0.250
401	A	5	5	1.00	20	0.250
402	A	6	6	1.00	20	0.300
403	A	7	7	1.00	22	0.318

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
404	A	6	6	1.00	22	0.273
405	A	2	2	1.00	22	0.091
406	A	2	2	1.00	22	0.091
407	A	3	3	1.00	22	0.136
408	A	7	7	1.00	22	0.318
409	A	8	7	1.00	22	0.318
410	A	7	7	1.00	22	0.318
411	A	6	6	1.00	22	0.273
412	A	2	2	1.00	22	0.091
413	A	2	2	1.00	22	0.091
414	A	3	3	1.00	22	0.136
415	A	7	7	1.00	22	0.318
416	A	8	7	1.00	22	0.318
417	A	3	2	1.00	22	0.091
418	A	2	2	1.00	22	0.091
419	A	1	1	1.00	22	0.045
420	A	3	3	1.00	22	0.136
421	A	4	4	1.00	22	0.182
422	A	5	4	1.00	22	0.182
423	A	4	2	1.00	22	0.091
424	A	3	2	1.00	22	0.091
425	A	2	2	1.00	22	0.091
426	A	1	1	1.00	22	0.045
427	A	3	3	1.00	22	0.136
428	A	4	4	1.00	22	0.182
429	A	5	4	1.00	22	0.182
430	A	4	2	1.00	32	0.062
431	A	3	2	1.00	32	0.062
432	A	2	2	1.00	32	0.062
433	A	1	1	1.00	32	0.031
434	A	3	3	1.00	32	0.094
435	A	4	4	1.00	32	0.125
436	A	5	4	1.00	32	0.125
437	A	3	2	1.00	34	0.059
438	A	2	2	1.00	34	0.059
439	A	1	1	1.00	34	0.029

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
440	A	3	3	1.00	34	0.088
441	A	4	4	1.00	34	0.118
442	A	5	4	1.00	34	0.118
443	A	4	4	1.00	15	0.267
444	A	3	3	1.36	11	0.273
445	A	5	5	1.00	12	0.417
446	A	4	4	1.00	15	0.267
447	A	10	8	1.00	17	0.471
448	A	7	7	1.00	33	0.212
449	A	3	3	1.00	33	0.091
450	A	3	3	1.00	33	0.091
451	A	4	4	1.00	33	0.121
452	A	8	8	1.00	33	0.242
453	A	7	7	1.00	33	0.212
454	A	3	3	1.00	33	0.091
455	A	3	3	1.00	33	0.091
456	A	4	4	1.00	33	0.121
457	A	8	8	1.00	33	0.242
458	A	5	5	1.00	12	0.417
459	A	4	4	1.00	15	0.267
460	A	9	7	1.00	17	0.412
461	A	4	4	1.00	15	0.267
462	A	7	7	1.00	33	0.212
463	A	3	3	1.00	33	0.091
464	A	3	3	1.00	33	0.091
465	A	4	4	1.00	33	0.121
466	A	8	8	1.00	33	0.242
467	A	7	7	1.00	33	0.212
468	A	3	3	1.00	33	0.091
469	A	3	3	1.00	33	0.091
470	A	4	4	1.00	33	0.121
471	A	8	8	1.00	33	0.242
472	A	2	2	1.00	11	0.182
473	A	2	2	1.00	11	0.182
474	A	2	2	1.00	11	0.182
475	A	2	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
476	A	2	1	1.00	13	0.077
477	A	4	3	1.00	13	0.231
478	A	2	1	1.00	15	0.067
479	A	2	1	1.00	15	0.067
480	A	2	1	1.00	19	0.053
481	A	2	1	1.00	23	0.043
482	A	4	3	1.00	20	0.150
483	A	4	3	1.00	20	0.150
484	A	4	2	1.00	11	0.182
485	A	6	4	1.00	11	0.364
486	A	6	4	1.00	11	0.364
487	A	2	2	1.00	13	0.154
488	A	2	2	1.00	13	0.154
489	A	2	2	1.00	13	0.154
490	A	4	2	1.00	11	0.182
491	A	6	4	1.00	11	0.364
492	A	6	4	1.00	11	0.364
493	A	2	2	1.00	13	0.154
494	A	2	2	1.00	13	0.154
495	A	2	2	1.00	13	0.154
496	A	2	1	1.00	16	0.062
497	A	9	6	1.00	18	0.333
498	A	11	7	1.00	20	0.350
499	A	5	3	1.00	39	0.077
500	A	2	2	1.00	37	0.054
501	A	3	3	1.00	39	0.077
502	A	9	6	1.00	39	0.154
503	A	7	4	1.00	21	0.190
504	A	4	3	1.00	41	0.073
505	A	2	2	1.00	41	0.049
506	A	5	5	1.00	41	0.122
507	A	8	6	1.00	41	0.146
508	A	3	3	1.00	27	0.111
509	A	5	3	1.00	21	0.143
510	A	7	5	1.00	39	0.128
511	A	3	3	1.00	37	0.081

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
512	A	4	4	1.00	39	0.103
513	A	6	4	1.00	39	0.103
514	A	6	5	1.00	41	0.122
515	A	3	3	1.00	41	0.073
516	A	3	3	1.00	41	0.073
517	A	5	4	1.00	41	0.098
518	A	8	7	1.00	39	0.180
519	A	5	4	1.00	37	0.108
520	A	6	6	1.00	39	0.154
521	A	8	7	1.00	39	0.180
522	A	7	6	1.00	41	0.146
523	A	5	5	1.00	41	0.122
524	A	5	5	1.00	41	0.122
525	A	7	7	1.00	41	0.171
526	A	1	1	1.00	21	0.048
527	A	1	1	1.00	21	0.048
528	A	1	1	1.00	15	0.067
529	A	1	1	1.00	21	0.048
530	A	3	3	1.00	21	0.143
531	A	3	3	1.00	21	0.143
532	A	3	3	1.00	22	0.136
533	A	3	3	1.00	22	0.136
534	A	4	4	1.00	22	0.182
535	A	4	4	1.00	19	0.210
536	A	4	4	1.00	19	0.210
537	A	5	5	1.00	19	0.263
538	A	1	1	1.00	22	0.045
539	A	1	1	1.00	22	0.045
540	A	4	4	1.00	19	0.210
541	A	4	4	1.00	19	0.210
542	A	5	5	1.00	19	0.263
543	A	1	1	1.00	22	0.045
544	A	1	1	1.00	22	0.045
545	A	4	4	1.00	22	0.182
546	A	4	4	1.00	22	0.182
547	A	5	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
548	A	1	1	0.95	25	0.040
549	A	1	1	0.94	25	0.040
550	A	4	4	1.00	23	0.174
551	A	4	4	1.00	23	0.174
552	A	5	5	1.00	23	0.217
553	A	1	1	1.00	26	0.038
554	A	1	1	1.00	26	0.038
555	B	1	1	2.83	30	0.033
556	A	8	6	1.00	27	0.222
557	A	7	6	1.00	27	0.222
558	A	6	6	1.00	27	0.222
559	A	5	5	1.00	27	0.185
560	A	6	6	1.00	27	0.222
561	A	7	6	1.00	27	0.222
562	A	7	7	1.00	31	0.226
563	A	8	8	1.00	31	0.258
564	A	9	8	1.00	31	0.258
565	A	10	8	1.00	31	0.258
566	A	4	4	1.00	18	0.222
567	A	3	3	1.00	18	0.167
568	A	2	2	1.00	18	0.111
569	A	3	2	1.00	16	0.125
570	A	4	4	1.00	18	0.222
571	A	6	6	1.00	18	0.333
572	A	7	7	1.00	18	0.389
573	A	8	8	1.00	20	0.400
574	A	7	7	1.00	20	0.350
575	A	3	3	1.00	20	0.150
576	A	3	3	1.00	20	0.150
577	A	5	5	1.00	20	0.250
578	A	8	8	1.00	20	0.400
579	A	13	8	1.00	14	0.571
580	A	11	7	1.00	14	0.500
581	A	9	6	1.00	12	0.500
582	A	0	0	0.00	0	0.000
583	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
584	A	0	0	0.00	0	0.000
585	A	15	6	1.00	26	0.231
586	A	11	5	1.00	26	0.192
587	A	6	6	1.00	26	0.231
588	A	4	3	1.00	26	0.115
589	A	1	1	1.00	23	0.043
590	A	1	1	1.00	22	0.045
591	A	3	3	1.00	20	0.150
592	A	7	5	1.00	24	0.208
593	A	9	9	1.00	26	0.346
594	A	15	6	1.00	24	0.250
595	A	11	5	1.00	24	0.208
596	A	6	6	1.00	24	0.250
597	A	4	3	1.00	24	0.125
598	A	1	1	1.00	21	0.048
599	A	1	1	1.00	20	0.050
600	A	3	3	1.00	18	0.167
601	A	7	5	1.00	22	0.227
602	A	9	9	1.00	24	0.375
603	A	5	5	1.00	31	0.161
604	A	4	4	1.00	31	0.129
605	A	3	3	1.00	31	0.097
606	A	2	2	1.00	29	0.069
607	A	3	3	1.00	20	0.150
608	A	4	4	1.00	29	0.138
609	A	5	4	1.00	31	0.129
610	A	6	4	1.00	31	0.129
611	A	7	7	1.00	31	0.226
612	A	5	5	1.00	31	0.161
613	A	4	4	1.00	31	0.129
614	A	3	3	1.00	29	0.103
615	A	5	5	1.00	20	0.250
616	A	6	6	1.00	29	0.207
617	A	6	6	1.00	31	0.194
618	A	7	6	1.00	31	0.194
619	A	6	6	1.00	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
620	A	5	5	1.00	31	0.161
621	A	4	4	1.00	31	0.129
622	A	3	3	1.00	29	0.103
623	A	6	5	1.00	20	0.250
624	A	7	6	1.00	29	0.207
625	A	8	7	1.00	31	0.226
626	A	6	6	1.00	31	0.194
627	A	5	5	1.00	31	0.161
628	A	4	4	1.00	31	0.129
629	A	4	4	1.00	29	0.138
630	A	7	6	1.00	20	0.300
631	A	8	7	1.00	29	0.241
632	A	9	7	1.00	31	0.226
633	A	3	2	1.00	13	0.154
634	A	6	4	1.00	21	0.190
635	A	6	4	1.00	28	0.143
636	A	6	4	1.00	30	0.133
637	A	1	1	1.00	43	0.023
638	A	1	1	1.00	43	0.023
639	A	1	1	1.00	43	0.023
640	A	1	1	1.00	41	0.024
641	A	1	1	1.00	43	0.023
642	A	1	1	1.00	43	0.023
643	A	1	1	1.00	43	0.023
644	A	0	0	0.00	0	0.000
645	A	0	0	0.00	0	0.000
646	A	0	0	0.00	0	0.000
647	A	0	0	0.00	0	0.000
648	A	2	2	1.00	11	0.182
649	A	2	2	1.00	11	0.182
650	A	2	2	1.00	13	0.154
651	A	2	2	1.00	6	0.333
652	A	4	4	1.00	11	0.364
653	A	6	3	1.00	13	0.231
654	A	4	3	1.00	17	0.176
655	A	2	2	1.00	10	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	3	3	1.00	21	0.143
657	A	3	3	1.00	19	0.158
658	A	3	2	1.00	15	0.133
659	A	2	2	1.00	17	0.118
660	A	2	2	1.00	22	0.091
661	A	2	2	1.00	22	0.091
662	A	2	2	1.00	17	0.118
663	A	2	2	1.00	22	0.091
664	A	2	2	1.00	22	0.091
665	A	2	2	1.00	11	0.182
666	A	2	2	1.00	11	0.182
667	A	2	2	1.00	13	0.154
668	A	2	2	1.00	15	0.133
669	A	2	2	1.00	19	0.105
670	A	4	4	1.00	11	0.364
671	A	3	3	1.00	15	0.200
672	A	2	2	1.00	15	0.133
673	A	3	3	1.00	16	0.188
674	A	2	2	1.00	6	0.333
675	A	3	2	1.00	10	0.200
676	A	2	2	1.00	6	0.333
677	A	4	3	1.00	17	0.176
678	A	3	3	1.00	9	0.333
679	A	4	3	1.00	13	0.231
680	A	2	2	1.00	17	0.118
681	A	2	2	1.00	9	0.222
682	A	2	2	1.00	12	0.167
683	A	2	2	1.00	18	0.111
684	A	2	2	1.00	17	0.118
685	A	2	2	1.00	22	0.091
686	A	2	2	1.00	22	0.091
687	A	2	2	1.00	17	0.118
688	A	2	2	1.00	22	0.091
689	A	2	2	1.00	22	0.091
690	A	2	2	1.00	13	0.154
691	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
692	A	2	2	1.00	13	0.154
693	A	2	2	1.00	13	0.154
694	A	3	1	1.00	15	0.067
695	A	4	3	1.00	19	0.158
696	A	3	3	1.00	17	0.176
697	A	3	2	1.50	16	0.125
698	A	3	2	1.50	18	0.111
699	A	7	7	1.00	15	0.467
700	A	3	3	1.00	19	0.158
701	A	3	2	1.00	19	0.105
702	A	3	2	1.00	21	0.095
703	A	3	2	1.00	21	0.095
704	A	2	2	1.00	17	0.118
705	A	4	3	1.00	17	0.176
706	A	5	5	1.00	19	0.263
707	A	2	2	1.00	11	0.182
708	A	4	2	1.00	17	0.118
709	A	2	2	1.00	17	0.118
710	A	2	2	1.00	17	0.118
711	A	3	3	1.00	15	0.200
712	A	3	3	1.00	17	0.176
713	A	3	3	1.00	17	0.176
714	A	2	2	1.00	9	0.222
715	A	4	3	1.00	15	0.200
716	A	2	2	1.00	13	0.154
717	A	2	2	1.00	13	0.154
718	A	2	2	1.00	11	0.182
719	A	4	2	1.00	15	0.133
720	A	3	2	1.00	19	0.105
721	A	3	2	1.00	21	0.095
722	A	3	2	1.00	21	0.095
723	A	2	2	1.00	11	0.182
724	A	4	4	1.82	13	0.308
725	A	2	2	1.00	13	0.154
726	A	2	2	1.00	15	0.133
727	A	2	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
728	A	3	3	1.00	15	0.200
729	A	2	1	1.00	15	0.067
730	A	2	2	1.00	9	0.222
731	A	2	2	1.00	9	0.222
732	A	2	2	1.00	19	0.105
733	A	3	2	1.00	23	0.087
734	A	2	1	1.00	23	0.043
735	A	4	4	1.67	13	0.308
736	A	2	2	1.00	15	0.133
737	A	2	2	1.00	13	0.154
738	A	3	3	1.00	21	0.143
739	A	2	1	1.00	15	0.067
740	A	3	2	1.00	23	0.087
741	A	4	3	1.00	20	0.150
742	A	4	3	1.00	19	0.158
743	A	4	3	1.00	24	0.125
744	A	4	3	1.00	21	0.143
745	A	4	3	1.00	20	0.150
746	A	4	3	1.00	19	0.158
747	A	4	3	1.00	24	0.125
748	A	4	3	1.00	21	0.143
749	A	1	3	1.00	8	0.375
750	A	1	2	1.00	8	0.250
751	A	3	2	1.00	11	0.182
752	A	2	2	1.00	6	0.333
753	A	4	3	1.00	8	0.375
754	A	2	2	1.00	9	0.222
755	A	2	2	1.00	12	0.167
756	A	3	2	1.00	13	0.154
757	A	2	2	1.00	8	0.250
758	A	1	1	1.00	12	0.083
759	B	25	3	10.75	20	0.150
760	A	2	2	1.00	6	0.333
761	A	3	3	1.00	8	0.375
762	A	4	3	1.00	15	0.200
763	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
764	A	8	4	1.00	17	0.235
765	A	3	2	1.00	13	0.154
766	A	2	2	1.00	6	0.333
767	A	2	2	1.00	10	0.200
768	A	2	2	1.00	8	0.250
769	A	2	2	1.00	10	0.200
770	A	3	3	1.00	10	0.300
771	A	3	3	1.00	10	0.300
772	A	2	2	1.00	8	0.250
773	A	1	1	1.00	8	0.125
774	A	3	1	1.00	18	0.056
775	A	6	3	1.00	16	0.188
776	A	6	3	1.00	8	0.375
777	A	2	2	1.00	17	0.118
778	A	3	3	1.00	7	0.429
779	A	3	3	1.00	11	0.273
780	A	3	2	1.00	10	0.200
781	A	2	2	1.00	8	0.250
782	A	2	2	1.00	8	0.250
783	A	3	2	1.00	10	0.200
784	A	2	2	1.00	10	0.200
785	A	3	3	1.00	9	0.333
786	A	2	2	1.00	8	0.250
787	A	3	3	1.00	8	0.375
788	A	1	1	1.00	8	0.125
789	A	3	3	1.00	8	0.375
790	A	3	3	1.00	8	0.375
791	A	2	2	1.00	8	0.250
792	A	2	2	1.00	13	0.154
793	A	1	1	1.00	11	0.091
794	A	3	3	1.00	9	0.333
795	A	3	2	1.00	7	0.286
796	F	0	0	N/A	0	N/A
797	A	2	2	1.00	6	0.333
798	A	3	2	1.00	11	0.182
799	A	3	2	1.00	8	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
800	A	3	2	1.00	7	0.286
801	A	4	3	1.00	9	0.333
802	A	2	2	1.00	9	0.222
803	A	2	2	1.00	7	0.286
804	A	2	2	1.00	13	0.154
805	A	6	4	1.00	10	0.400
806	A	2	2	1.00	21	0.095
807	A	3	3	1.00	15	0.200
808	A	2	2	1.00	12	0.167
809	A	5	5	1.00	16	0.312
810	A	4	2	1.00	11	0.182
811	A	8	4	1.00	13	0.308
812	A	1	1	1.00	10	0.100
813	A	3	2	1.00	13	0.154
814	A	4	2	1.00	12	0.167
815	A	3	2	1.00	8	0.250
816	A	5	4	1.00	10	0.400
817	A	6	4	1.00	15	0.267
818	A	4	3	1.00	11	0.273
819	A	3	2	1.00	13	0.154
820	A	2	2	1.00	8	0.250
821	A	2	2	1.00	28	0.071
822	A	1	1	1.00	12	0.083
823	A	2	2	1.00	23	0.087
824	A	3	3	1.00	7	0.429
825	A	2	2	1.00	10	0.200
826	A	2	2	1.00	8	0.250
827	A	4	2	1.29	13	0.154
828	A	3	1	1.00	20	0.050
829	A	3	3	1.00	9	0.333
830	A	5	5	1.00	10	0.500
831	A	5	4	1.00	9	0.444
832	A	4	4	1.00	10	0.400
833	A	5	4	1.00	10	0.400
834	A	4	3	1.00	10	0.300
835	A	6	4	1.00	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
836	A	5	5	1.00	14	0.357
837	A	5	4	1.00	13	0.308
838	A	5	2	1.00	9	0.222
839	A	5	2	1.00	11	0.182
840	A	2	2	1.00	7	0.286
841	A	6	2	1.00	9	0.222
842	A	13	5	1.00	10	0.500
843	A	3	3	1.00	18	0.167
844	A	1	1	1.00	18	0.056
845	A	3	3	1.00	18	0.167
846	A	9	7	1.27	15	0.467
847	A	8	7	1.27	15	0.467
848	A	4	2	1.00	15	0.133
849	A	4	2	1.00	15	0.133
850	A	3	3	1.00	21	0.143
851	A	3	3	1.00	21	0.143
852	A	1	1	1.00	14	0.071
853	A	4	4	1.00	15	0.267
854	A	4	3	1.00	14	0.214
855	A	7	5	1.28	16	0.312
856	A	7	5	1.28	16	0.312
857	A	4	2	1.00	15	0.133
858	B	5	3	4.63	18	0.167
859	F	0	0	N/A	0	N/A
860	B	17	9	3.79	16	0.562
861	A	4	4	1.00	12	0.333
862	A	2	2	1.00	15	0.133
863	A	6	5	1.00	15	0.333
864	A	6	5	1.00	15	0.333
865	A	8	5	1.00	18	0.278
866	A	10	8	1.11	23	0.348
867	A	7	6	1.00	23	0.261
868	A	6	4	1.00	16	0.250
869	A	8	5	1.00	18	0.278
870	A	10	6	1.00	18	0.333
871	A	5	5	1.00	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
872	A	6	6	1.00	18	0.333
873	A	7	7	1.00	18	0.389
874	A	10	10	1.00	16	0.625
875	A	17	14	1.00	18	0.778
876	A	21	13	1.00	18	0.722
877	A	12	11	1.00	16	0.688
878	A	16	13	1.00	18	0.722
879	A	21	17	1.00	18	0.944
880	A	5	2	1.00	11	0.182
881	A	5	2	1.00	11	0.182
882	A	5	2	1.00	11	0.182
883	A	5	2	1.00	11	0.182
884	A	2	2	1.00	6	0.333
885	A	1	1	1.00	15	0.067
886	A	4	4	1.00	9	0.444
887	A	3	2	1.00	15	0.133
888	A	4	3	1.42	16	0.188
889	A	3	2	1.00	15	0.133
890	A	5	4	1.00	13	0.308
891	A	3	2	1.00	13	0.154
892	A	5	4	1.00	13	0.308
893	A	4	3	1.00	15	0.200
894	A	5	4	1.00	7	0.571
895	A	3	2	1.00	13	0.154
896	A	5	4	1.00	27	0.148
897	A	5	4	1.00	27	0.148
898	A	5	4	1.00	27	0.148
899	A	5	4	1.00	27	0.148
900	A	5	3	1.00	21	0.143
901	A	7	5	1.00	23	0.217
902	A	4	3	1.00	13	0.231
903	A	7	7	1.00	15	0.467
904	A	2	2	1.00	10	0.200
905	A	3	2	1.00	17	0.118
906	A	5	4	1.00	17	0.235
907	A	5	3	1.00	8	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
908	A	3	2	1.00	13	0.154
909	A	4	3	1.00	16	0.188
910	A	6	5	1.00	13	0.385
911	A	8	3	1.00	12	0.250
912	B	22	9	4.26	18	0.500
913	A	3	3	1.00	9	0.333
914	F	0	0	N/A	0	N/A
915	F	0	0	N/A	0	N/A
916	A	6	4	1.00	12	0.333
917	A	2	2	1.00	9	0.222
918	A	1	1	1.00	21	0.048
919	A	3	3	1.00	10	0.300
920	A	8	4	1.00	13	0.308
921	A	1	1	1.00	8	0.125
922	A	7	5	1.00	12	0.417
923	A	1	1	1.00	18	0.056
924	A	1	1	1.00	14	0.071
925	A	1	1	1.00	22	0.045
926	A	7	4	1.00	22	0.182
927	A	6	4	1.00	22	0.182
928	A	3	3	1.00	10	0.300
929	A	3	3	1.00	9	0.333
930	A	2	2	1.00	14	0.143
931	F	0	0	N/A	0	N/A
932	A	0	0	0.00	0	0.000
933	F	0	0	N/A	0	N/A
934	A	7	5	1.00	28	0.179
935	A	9	6	1.00	30	0.200
936	A	7	6	1.00	36	0.167
937	A	16	5	1.00	38	0.132
938	A	7	6	1.00	29	0.207
939	A	11	8	1.00	31	0.258
940	A	4	4	1.00	27	0.148
941	A	4	4	1.00	27	0.148
942	A	7	4	1.00	39	0.103
943	A	4	2	1.00	39	0.051

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
944	A	5	3	1.00	39	0.077
945	A	6	3	1.00	39	0.077
946	A	1	1	1.00	31	0.032
947	A	4	2	1.00	31	0.065
948	A	2	1	1.00	39	0.026
949	A	5	3	1.00	39	0.077
950	A	4	2	1.00	39	0.051

Chapter 3

Listing of integrals

$$3.1 \quad \int \frac{2}{3 - \cos(4 + 6x)} dx$$

Optimal. Leaf size=44

$$\frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(6x+4)}{-\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

[Out] 1/2*x*2^(1/2)+1/6*arctan(sin(4+6*x)/(3-cos(4+6*x)+2*2^(1/2)))*2^(1/2)

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {12, 2657}

$$\frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(6x+4)}{-\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[2/(3 - Cos[4 + 6*x]),x]

[Out] x/Sqrt[2] + ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] - Cos[4 + 6*x])]/(3*Sqrt[2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2657

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{q = Rt[a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*Sin[c + d*x])])]/(d*q), x] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] && PosQ[a]

Rubi steps

$$\begin{aligned} \int \frac{2}{3 - \cos(4 + 6x)} dx &= 2 \int \frac{1}{3 - \cos(4 + 6x)} dx \\ &= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}-\cos(4+6x)}\right)}{3\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.50

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[2/(3 - Cos[4 + 6*x]),x]

[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])

fricas [A] time = 0.65, size = 33, normalized size = 0.75

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(6x+4)-\sqrt{2}}{4\sin(6x+4)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(3-cos(4+6*x)),x, algorithm="fricas")

[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) - sqrt(2))/sin(6*x + 4))

giac [A] time = 0.15, size = 57, normalized size = 1.30

$$\frac{1}{6}\sqrt{2}\left(3x + \arctan\left(-\frac{\sqrt{2}\sin(6x+4)-2\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-2\cos(6x+4)+2}\right) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(3-cos(4+6*x)),x, algorithm="giac")

[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)

maple [A] time = 0.08, size = 17, normalized size = 0.39

$$\frac{\sqrt{2}\arctan\left(\sqrt{2}\tan(2+3x)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2/(3-cos(4+6*x)),x)

[Out] 1/6*2^(1/2)*arctan(2^(1/2)*tan(2+3*x))

maxima [A] time = 0.41, size = 26, normalized size = 0.59

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin(6x+4)}{\cos(6x+4)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(3-cos(4+6*x)),x, algorithm="maxima")

[Out] 1/6*sqrt(2)*arctan(sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))

mupad [B] time = 2.58, size = 35, normalized size = 0.80

$$\frac{\sqrt{2}(3x - \operatorname{atan}(\tan(3x+2)))}{6} + \frac{\sqrt{2}\operatorname{atan}(\sqrt{2}\tan(3x+2))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-2/(cos(6*x + 4) - 3),x)`

[Out] $(2^{(1/2)}*(3*x - \operatorname{atan}(\tan(3*x + 2))))/6 + (2^{(1/2)}*\operatorname{atan}(2^{(1/2)}*\tan(3*x + 2)))/6$

sympy [A] time = 0.26, size = 32, normalized size = 0.73

$$\frac{\sqrt{2} \left(\operatorname{atan}(\sqrt{2} \tan(3x + 2)) + \pi \left\lfloor \frac{3x - \frac{\pi}{2} + 2}{\pi} \right\rfloor \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2/(3-cos(4+6*x)),x)`

[Out] $\operatorname{sqrt}(2)*(\operatorname{atan}(\operatorname{sqrt}(2)*\tan(3*x + 2)) + \pi*\operatorname{floor}((3*x - \pi/2 + 2)/\pi))/6$

$$3.2 \quad \int \frac{2 \csc(4+6x)}{-\cot(4+6x)+3 \csc(4+6x)} dx$$

Optimal. Leaf size=44

$$\frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(6x+4)}{-\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

[Out] 1/2*x*2^(1/2)+1/6*arctan(sin(4+6*x)/(3-cos(4+6*x)+2*2^(1/2)))*2^(1/2)

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {12, 3166, 2657}

$$\frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(6x+4)}{-\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2*Csc[4 + 6*x])/(-Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]

[Out] x/Sqrt[2] + ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] - Cos[4 + 6*x])]/(3*Sqrt[2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2657

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{q = Rt[a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*Sin[c + d*x])])]/(d*q), x] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] && PosQ[a]

Rule 3166

Int[csc[(d_) + (e_)*(x_)]^(n_)*((a_) + csc[(d_) + (e_)*(x_)]*(b_) + cot[(d_) + (e_)*(x_)]*(c_))^(m_), x_Symbol] := Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned} \int \frac{2 \csc(4+6x)}{-\cot(4+6x)+3 \csc(4+6x)} dx &= 2 \int \frac{\csc(4+6x)}{-\cot(4+6x)+3 \csc(4+6x)} dx \\ &= 2 \int \frac{1}{3-\cos(4+6x)} dx \\ &= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}-\cos(4+6x)}\right)}{3\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.50

$$\frac{\tan^{-1}\left(\sqrt{2} \tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2*Csc[4 + 6*x])/(-Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]

[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])

fricas [A] time = 0.69, size = 33, normalized size = 0.75

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(6x+4)-\sqrt{2}}{4\sin(6x+4)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x, algorithm="fricas")

[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) - sqrt(2))/sin(6*x + 4))

giac [A] time = 0.22, size = 57, normalized size = 1.30

$$\frac{1}{6}\sqrt{2}\left(3x + \arctan\left(-\frac{\sqrt{2}\sin(6x+4)-2\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-2\cos(6x+4)+2}\right) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x, algorithm="giac")

[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)

maple [A] time = 0.30, size = 17, normalized size = 0.39

$$\frac{\sqrt{2}\arctan(\sqrt{2}\tan(2+3x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x)

[Out] 1/6*2^(1/2)*arctan(2^(1/2)*tan(2+3*x))

maxima [A] time = 0.64, size = 26, normalized size = 0.59

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin(6x+4)}{\cos(6x+4)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x, algorithm="maxima")

[Out] 1/6*sqrt(2)*arctan(sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))

mupad [B] time = 2.69, size = 16, normalized size = 0.36

$$\frac{\sqrt{2}\operatorname{atan}(\sqrt{2}\tan(3x+2))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-2/(sin(6*x + 4)*(cot(6*x + 4) - 3/sin(6*x + 4))),x)

[Out] (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)))/6

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-2 \int \frac{\csc(6x + 4)}{\cot(6x + 4) - 3 \csc(6x + 4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)), x)

[Out] -2*Integral(csc(6*x + 4)/(cot(6*x + 4) - 3*csc(6*x + 4)), x)

$$3.3 \quad \int \frac{1}{1+\sin^2(2+3x)} dx$$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

[Out] $1/2*x*2^{(1/2)}+1/6*\arctan(\cos(2+3*x)*\sin(2+3*x)/(1+\sin(2+3*x)^2+2^{(1/2)}))*2^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3181, 203}

$$\frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sin[2 + 3*x]^2)^(-1), x]

[Out] $x/\text{Sqrt}[2] + \text{ArcTan}[(\text{Cos}[2 + 3*x]*\text{Sin}[2 + 3*x])/(1 + \text{Sqrt}[2] + \text{Sin}[2 + 3*x]^2)]/(3*\text{Sqrt}[2])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3181

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)^2])^(-1), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{1+\sin^2(2+3x)} dx &= \frac{1}{3} \text{Subst}\left(\int \frac{1}{1+2x^2} dx, x, \tan(2+3x)\right) \\ &= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2} \tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sin[2 + 3*x]^2)^(-1), x]

[Out] $\text{ArcTan}[\text{Sqrt}[2]*\text{Tan}[2 + 3*x]]/(3*\text{Sqrt}[2])$

fricas [A] time = 0.65, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2-2\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sin(2+3*x)^2),x, algorithm="fricas")

[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x + 2)*sin(3*x + 2)))

giac [A] time = 0.13, size = 57, normalized size = 1.19

$$\frac{1}{6}\sqrt{2}\left(3x + \arctan\left(-\frac{\sqrt{2}\sin(6x+4)-2\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-2\cos(6x+4)+2}\right) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sin(2+3*x)^2),x, algorithm="giac")

[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)

maple [A] time = 0.17, size = 17, normalized size = 0.35

$$\frac{\sqrt{2}\arctan(\sqrt{2}\tan(2+3x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+sin(2+3*x)^2),x)

[Out] 1/6*2^(1/2)*arctan(2^(1/2)*tan(2+3*x))

maxima [A] time = 0.89, size = 16, normalized size = 0.33

$$\frac{1}{6}\sqrt{2}\arctan(\sqrt{2}\tan(3x+2))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sin(2+3*x)^2),x, algorithm="maxima")

[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))

mupad [B] time = 2.49, size = 35, normalized size = 0.73

$$\frac{\sqrt{2}(3x - \operatorname{atan}(\tan(3x+2)))}{6} + \frac{\sqrt{2}\operatorname{atan}(\sqrt{2}\tan(3x+2))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(3*x + 2)^2 + 1),x)

[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)))/6

sympy [B] time = 6.82, size = 246, normalized size = 5.12

$$\frac{47321\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left[\frac{3x-\frac{\pi}{2}+1}{\pi}\right]\right)}{83160\sqrt{2}+117606} + \frac{66922\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left[\frac{3x-\frac{\pi}{2}+1}{\pi}\right]\right)}{83160\sqrt{2}+117606} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(2+3*x)**2),x)
```

```
[Out] 47321*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2)))
+ pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 66922*sqrt(3
- 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))) + pi*floor((3*x/2 -
pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 8119*sqrt(2)*sqrt(2*sqrt(2) + 3)
*(atan(tan(3*x/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi
))/(83160*sqrt(2) + 117606) + 11482*sqrt(2*sqrt(2) + 3)*(atan(tan(3*x/2 + 1
)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) +
117606)
```

$$3.4 \quad \int \frac{1}{2 - \cos^2(2+3x)} dx$$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

[Out] $1/2*x*2^{(1/2)}+1/6*\arctan(\cos(2+3*x)*\sin(2+3*x)/(1+\sin(2+3*x)^2+2^{(1/2)}))*2^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3181, 203}

$$\frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 - Cos[2 + 3*x]^2)^(-1), x]

[Out] $x/\text{Sqrt}[2] + \text{ArcTan}[(\text{Cos}[2 + 3*x]*\text{Sin}[2 + 3*x])/((1 + \text{Sqrt}[2] + \text{Sin}[2 + 3*x]^2))]/(3*\text{Sqrt}[2])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3181

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2)^(-1), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{2 - \cos^2(2+3x)} dx &= -\left(\frac{1}{3} \text{Subst}\left(\int \frac{1}{2+x^2} dx, x, \cot(2+3x)\right)\right) \\ &= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2} \tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - Cos[2 + 3*x]^2)^(-1), x]

[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])

fricas [A] time = 0.73, size = 43, normalized size = 0.90

$$-\frac{1}{12} \sqrt{2} \arctan\left(\frac{3\sqrt{2} \cos(3x+2)^2 - 2\sqrt{2}}{4 \cos(3x+2) \sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-cos(2+3*x)^2), x, algorithm="fricas")

[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x + 2)*sin(3*x + 2)))

giac [A] time = 0.15, size = 57, normalized size = 1.19

$$\frac{1}{6} \sqrt{2} \left(3x + \arctan\left(-\frac{\sqrt{2} \sin(6x+4) - 2 \sin(6x+4)}{\sqrt{2} \cos(6x+4) + \sqrt{2} - 2 \cos(6x+4) + 2}\right) + 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-cos(2+3*x)^2), x, algorithm="giac")

[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)

maple [A] time = 0.10, size = 17, normalized size = 0.35

$$\frac{\sqrt{2} \arctan(\sqrt{2} \tan(2+3x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2-cos(2+3*x)^2), x)

[Out] 1/6*2^(1/2)*arctan(2^(1/2)*tan(2+3*x))

maxima [A] time = 0.41, size = 16, normalized size = 0.33

$$\frac{1}{6} \sqrt{2} \arctan\left(\sqrt{2} \tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-cos(2+3*x)^2), x, algorithm="maxima")

[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))

mupad [B] time = 2.38, size = 35, normalized size = 0.73

$$\frac{\sqrt{2} (3x - \operatorname{atan}(\tan(3x+2)))}{6} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2} \tan(3x+2))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(cos(3*x + 2)^2 - 2), x)

[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)))/6

sympy [B] time = 7.12, size = 246, normalized size = 5.12

$$\frac{47321\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left[\frac{3x-\frac{\pi}{2}+1}{\pi}\right]\right)}{83160\sqrt{2}+117606} + \frac{66922\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left[\frac{3x-\frac{\pi}{2}+1}{\pi}\right]\right)}{83160\sqrt{2}+117606} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-cos(2+3*x)**2),x)

[Out] 47321*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))))
 + pi*floor((3*x/2 - pi/2 + 1)/pi)/(83160*sqrt(2) + 117606) + 66922*sqrt(3
 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2)))) + pi*floor((3*x/2 -
 pi/2 + 1)/pi)/(83160*sqrt(2) + 117606) + 8119*sqrt(2)*sqrt(2*sqrt(2) + 3)
 *(atan(tan(3*x/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi
))/(83160*sqrt(2) + 117606) + 11482*sqrt(2*sqrt(2) + 3)*(atan(tan(3*x/2 + 1
)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) +
 117606)

$$3.5 \quad \int \frac{1}{\cos^2(2+3x)+2 \sin^2(2+3x)} dx$$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

[Out] $1/2*x*2^{(1/2)}+1/6*\arctan(\cos(2+3*x)*\sin(2+3*x)/(1+\sin(2+3*x)^2+2^{(1/2)}))*2^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {203}

$$\frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1), x]

[Out] $x/\text{Sqrt}[2] + \text{ArcTan}[(\text{Cos}[2 + 3*x]*\text{Sin}[2 + 3*x])/ (1 + \text{Sqrt}[2] + \text{Sin}[2 + 3*x]^2)] / (3*\text{Sqrt}[2])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\cos^2(2+3x)+2 \sin^2(2+3x)} dx &= \frac{1}{3} \text{Subst}\left(\int \frac{1}{1+2x^2} dx, x, \tan(2+3x)\right) \\ &= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2} \tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1), x]

[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])

fricas [A] time = 0.74, size = 43, normalized size = 0.90

$$-\frac{1}{12} \sqrt{2} \arctan\left(\frac{3\sqrt{2} \cos(3x+2)^2 - 2\sqrt{2}}{4 \cos(3x+2) \sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="fricas")

[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x + 2)*sin(3*x + 2)))

giac [A] time = 0.16, size = 57, normalized size = 1.19

$$\frac{1}{6} \sqrt{2} \left(3x + \arctan \left(\frac{\sqrt{2} \sin(6x + 4) - 2 \sin(6x + 4)}{\sqrt{2} \cos(6x + 4) + \sqrt{2} - 2 \cos(6x + 4) + 2} \right) + 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="giac")

[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)

maple [A] time = 0.23, size = 17, normalized size = 0.35

$$\frac{\sqrt{2} \arctan(\sqrt{2} \tan(2 + 3x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x)

[Out] 1/6*2^(1/2)*arctan(2^(1/2)*tan(2+3*x))

maxima [A] time = 0.57, size = 16, normalized size = 0.33

$$\frac{1}{6} \sqrt{2} \arctan(\sqrt{2} \tan(3x + 2))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="maxima")

[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))

mupad [B] time = 2.38, size = 35, normalized size = 0.73

$$\frac{\sqrt{2} (3x - \operatorname{atan}(\tan(3x + 2)))}{6} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2} \tan(3x + 2))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*sin(3*x + 2)^2 + cos(3*x + 2)^2),x)

[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)))/6

sympy [B] time = 6.99, size = 246, normalized size = 5.12

$$\frac{47321\sqrt{2}\sqrt{3-2\sqrt{2}} \left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right) + \pi \left\lfloor \frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi} \right\rfloor \right)}{83160\sqrt{2} + 117606} + \frac{66922\sqrt{3-2\sqrt{2}} \left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right) + \pi \left\lfloor \frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi} \right\rfloor \right)}{83160\sqrt{2} + 117606} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(2+3*x)**2+2*sin(2+3*x)**2),x)

[Out] 47321*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2)))) + pi*floor((3*x/2 - pi/2 + 1)/pi)/(83160*sqrt(2) + 117606) + 66922*sqrt(3

$$\begin{aligned}
& - 2\sqrt{2}) * (\operatorname{atan}(\tan(3x/2 + 1)/\sqrt{3 - 2\sqrt{2}})) + \pi * \operatorname{floor}((3x/2 - \\
& \pi/2 + 1)/\pi)) / (83160\sqrt{2} + 117606) + 8119\sqrt{2} * \sqrt{2\sqrt{2} + 3} \\
& * (\operatorname{atan}(\tan(3x/2 + 1)/\sqrt{2\sqrt{2} + 3}) + \pi * \operatorname{floor}((3x/2 - \pi/2 + 1)/\pi \\
&)) / (83160\sqrt{2} + 117606) + 11482\sqrt{2\sqrt{2} + 3} * (\operatorname{atan}(\tan(3x/2 + 1) \\
&)/\sqrt{2\sqrt{2} + 3}) + \pi * \operatorname{floor}((3x/2 - \pi/2 + 1)/\pi)) / (83160\sqrt{2} + \\
& 117606)
\end{aligned}$$

$$3.6 \quad \int \frac{\sec^2(2+3x)}{1+2 \tan^2(2+3x)} dx$$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

[Out] 1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2^(1/2)

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3675, 203}

$$\frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sec[2 + 3*x]^2/(1 + 2*Tan[2 + 3*x]^2), x]

[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^2)]/(3*Sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^n_)]^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(2+3x)}{1+2 \tan^2(2+3x)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{1+2x^2} dx, x, \tan(2+3x) \right) \\ &= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2} \tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2 + 3*x]^2/(1 + 2*Tan[2 + 3*x]^2), x]

[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])

fricas [A] time = 0.51, size = 43, normalized size = 0.90

$$-\frac{1}{12} \sqrt{2} \arctan\left(\frac{3\sqrt{2} \cos(3x+2)^2 - 2\sqrt{2}}{4 \cos(3x+2) \sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x, algorithm="fricas")

[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x + 2)*sin(3*x + 2)))

giac [A] time = 1.54, size = 16, normalized size = 0.33

$$\frac{1}{6} \sqrt{2} \arctan\left(\sqrt{2} \tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x, algorithm="giac")

[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))

maple [A] time = 0.29, size = 17, normalized size = 0.35

$$\frac{\sqrt{2} \arctan\left(\sqrt{2} \tan(2+3x)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x)

[Out] 1/6*2^(1/2)*arctan(2^(1/2)*tan(2+3*x))

maxima [A] time = 0.44, size = 16, normalized size = 0.33

$$\frac{1}{6} \sqrt{2} \arctan\left(\sqrt{2} \tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x, algorithm="maxima")

[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))

mupad [B] time = 2.40, size = 16, normalized size = 0.33

$$\frac{\sqrt{2} \operatorname{atan}\left(\sqrt{2} \tan(3x+2)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(3*x + 2)^2*(2*tan(3*x + 2)^2 + 1)),x)

[Out] (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)))/6

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(3x+2)}{2 \tan^2(3x+2) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2+3*x)**2/(1+2*tan(2+3*x)**2),x)

[Out] Integral(sec(3*x + 2)**2/(2*tan(3*x + 2)**2 + 1), x)

$$3.7 \quad \int \frac{\csc^2(2+3x)}{2+\cot^2(2+3x)} dx$$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

[Out] 1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2^(1/2)

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3675, 203}

$$\frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Csc[2 + 3*x]^2/(2 + Cot[2 + 3*x]^2), x]

[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^2)]/(3*Sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^n, x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2-1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\csc^2(2+3x)}{2+\cot^2(2+3x)} dx &= -\left(\frac{1}{3} \text{Subst}\left(\int \frac{1}{2+x^2} dx, x, \cot(2+3x)\right)\right) \\ &= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2} \tan(3x+2)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[2 + 3*x]^2/(2 + Cot[2 + 3*x]^2), x]

[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])

fricas [A] time = 0.64, size = 43, normalized size = 0.90

$$-\frac{1}{12} \sqrt{2} \arctan\left(\frac{3\sqrt{2} \cos(3x+2)^2 - 2\sqrt{2}}{4 \cos(3x+2) \sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2+3*x)^2/(2+cot(2+3*x)^2),x, algorithm="fricas")

[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x + 2)*sin(3*x + 2)))

giac [A] time = 0.28, size = 57, normalized size = 1.19

$$\frac{1}{6} \sqrt{2} \left(3x + \arctan\left(-\frac{\sqrt{2} \sin(6x+4) - 2 \sin(6x+4)}{\sqrt{2} \cos(6x+4) + \sqrt{2} - 2 \cos(6x+4) + 2}\right) + 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2+3*x)^2/(2+cot(2+3*x)^2),x, algorithm="giac")

[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)

maple [A] time = 0.29, size = 17, normalized size = 0.35

$$\frac{\sqrt{2} \arctan(\sqrt{2} \tan(2 + 3x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(2+3*x)^2/(2+cot(2+3*x)^2),x)

[Out] 1/6*2^(1/2)*arctan(2^(1/2)*tan(2+3*x))

maxima [A] time = 0.61, size = 16, normalized size = 0.33

$$\frac{1}{6} \sqrt{2} \arctan(\sqrt{2} \tan(3x + 2))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2+3*x)^2/(2+cot(2+3*x)^2),x, algorithm="maxima")

[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))

mupad [B] time = 2.40, size = 16, normalized size = 0.33

$$\frac{\sqrt{2} \operatorname{atan}(\sqrt{2} \tan(3x + 2))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(3*x + 2)^2*(cot(3*x + 2)^2 + 2)),x)

[Out] (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)))/6

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(3x + 2)}{\cot^2(3x + 2) + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)**2/(2+cot(2+3*x)**2),x)
```

```
[Out] Integral(csc(3*x + 2)**2/(cot(3*x + 2)**2 + 2), x)
```

$$3.8 \quad \int \frac{2}{1-3 \cos(4+6x)} dx$$

Optimal. Leaf size=60

$$\frac{\log(\cos(3x+2) - \sqrt{2} \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sqrt{2} \sin(3x+2) + \cos(3x+2))}{6\sqrt{2}}$$

[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {12, 2659, 207}

$$\frac{\log(\cos(3x+2) - \sqrt{2} \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sqrt{2} \sin(3x+2) + \cos(3x+2))}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[2/(1 - 3*Cos[4 + 6*x]),x]

[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{2}{1-3 \cos(4+6x)} dx &= 2 \int \frac{1}{1-3 \cos(4+6x)} dx \\ &= \frac{2}{3} \text{Subst} \left(\int \frac{1}{-2+4x^2} dx, x, \tan \left(\frac{1}{2}(4+6x) \right) \right) \\ &= \frac{\log(\cos(2+3x) - \sqrt{2} \sin(2+3x))}{6\sqrt{2}} - \frac{\log(\cos(2+3x) + \sqrt{2} \sin(2+3x))}{6\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.37

$$\frac{\tanh^{-1}(\sqrt{2} \tan(3x+2))}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[2/(1 - 3*Cos[4 + 6*x]),x]

[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]

fricas [A] time = 0.51, size = 74, normalized size = 1.23

$$\frac{1}{24} \sqrt{2} \log \left(-\frac{7 \cos(6x+4)^2 - 4(\sqrt{2} \cos(6x+4) - 3\sqrt{2}) \sin(6x+4) + 6 \cos(6x+4) - 17}{9 \cos(6x+4)^2 - 6 \cos(6x+4) + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(1-3*cos(4+6*x)),x, algorithm="fricas")

[Out] 1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 - 4*(sqrt(2)*cos(6*x + 4) - 3*sqrt(2))*sin(6*x + 4) + 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 - 6*cos(6*x + 4) + 1))

giac [A] time = 0.17, size = 39, normalized size = 0.65

$$\frac{1}{12} \sqrt{2} \log \left(\frac{|-2\sqrt{2} + 4 \tan(3x+2)|}{|2\sqrt{2} + 4 \tan(3x+2)|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(1-3*cos(4+6*x)),x, algorithm="giac")

[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x + 2)))

maple [A] time = 0.06, size = 17, normalized size = 0.28

$$\frac{\sqrt{2} \operatorname{arctanh}(\sqrt{2} \tan(2+3x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2/(1-3*cos(4+6*x)),x)

[Out] -1/6*2^(1/2)*arctanh(2^(1/2)*tan(2+3*x))

maxima [A] time = 0.57, size = 54, normalized size = 0.90

$$\frac{1}{12} \sqrt{2} \log \left(-\frac{\sqrt{2} - \frac{2 \sin(6x+4)}{\cos(6x+4)+1}}{\sqrt{2} + \frac{2 \sin(6x+4)}{\cos(6x+4)+1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(1-3*cos(4+6*x)),x, algorithm="maxima")

[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) + 2*sin(6*x + 4)/(cos(6*x + 4) + 1)))

mupad [B] time = 2.57, size = 16, normalized size = 0.27

$$\frac{\sqrt{2} \operatorname{atanh}(\sqrt{2} \tan(3x+2))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-2/(3*cos(6*x + 4) - 1),x)`

[Out] `-(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6`

sympy [A] time = 0.31, size = 42, normalized size = 0.70

$$\frac{\sqrt{2} \log\left(\tan(3x + 2) - \frac{\sqrt{2}}{2}\right)}{12} - \frac{\sqrt{2} \log\left(\tan(3x + 2) + \frac{\sqrt{2}}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2/(1-3*cos(4+6*x)),x)`

[Out] `sqrt(2)*log(tan(3*x + 2) - sqrt(2)/2)/12 - sqrt(2)*log(tan(3*x + 2) + sqrt(2)/2)/12`

$$3.9 \quad \int \frac{2 \csc(4+6x)}{-3 \cot(4+6x) + \csc(4+6x)} dx$$

Optimal. Leaf size=60

$$\frac{\log(\cos(3x+2) - \sqrt{2} \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sqrt{2} \sin(3x+2) + \cos(3x+2))}{6\sqrt{2}}$$

[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {12, 3166, 2659, 207}

$$\frac{\log(\cos(3x+2) - \sqrt{2} \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sqrt{2} \sin(3x+2) + \cos(3x+2))}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2*Csc[4 + 6*x])/(-3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]

[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3166

Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))^m, x_Symbol] := Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{2 \csc(4 + 6x)}{-3 \cot(4 + 6x) + \csc(4 + 6x)} dx &= 2 \int \frac{\csc(4 + 6x)}{-3 \cot(4 + 6x) + \csc(4 + 6x)} dx \\
&= 2 \int \frac{1}{1 - 3 \cos(4 + 6x)} dx \\
&= \frac{2}{3} \text{Subst} \left(\int \frac{1}{-2 + 4x^2} dx, x, \tan \left(\frac{1}{2}(4 + 6x) \right) \right) \\
&= \frac{\log(\cos(2 + 3x) - \sqrt{2} \sin(2 + 3x))}{6\sqrt{2}} - \frac{\log(\cos(2 + 3x) + \sqrt{2} \sin(2 + 3x))}{6\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}(\sqrt{2} \tan(3x + 2))}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2*Csc[4 + 6*x])/(-3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]

[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]

fricas [A] time = 0.91, size = 74, normalized size = 1.23

$$\frac{1}{24} \sqrt{2} \log \left(-\frac{7 \cos(6x + 4)^2 - 4(\sqrt{2} \cos(6x + 4) - 3\sqrt{2}) \sin(6x + 4) + 6 \cos(6x + 4) - 17}{9 \cos(6x + 4)^2 - 6 \cos(6x + 4) + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x, algorithm="fricas")

[Out] 1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 - 4*(sqrt(2)*cos(6*x + 4) - 3*sqrt(2))*sin(6*x + 4) + 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 - 6*cos(6*x + 4) + 1))

giac [A] time = 0.24, size = 39, normalized size = 0.65

$$\frac{1}{12} \sqrt{2} \log \left(\frac{|-2\sqrt{2} + 4 \tan(3x + 2)|}{|2\sqrt{2} + 4 \tan(3x + 2)|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x, algorithm="giac")

[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x + 2)))

maple [A] time = 0.30, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2} \operatorname{arctanh}(\sqrt{2} \tan(2 + 3x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x)

[Out] -1/6*2^(1/2)*arctanh(2^(1/2)*tan(2+3*x))

maxima [A] time = 0.49, size = 54, normalized size = 0.90

$$\frac{1}{12} \sqrt{2} \log \left(-\frac{\sqrt{2} - \frac{2 \sin(6x+4)}{\cos(6x+4)+1}}{\sqrt{2} + \frac{2 \sin(6x+4)}{\cos(6x+4)+1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x, algorithm="maxima")

[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) + 2*sin(6*x + 4)/(cos(6*x + 4) + 1)))

mupad [B] time = 2.62, size = 16, normalized size = 0.27

$$\frac{\sqrt{2} \operatorname{atanh}(\sqrt{2} \tan(3x + 2))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-2/(sin(6*x + 4)*(3*cot(6*x + 4) - 1/sin(6*x + 4))),x)

[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-2 \int \frac{\csc(6x + 4)}{3 \cot(6x + 4) - \csc(6x + 4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x)

[Out] -2*Integral(csc(6*x + 4)/(3*cot(6*x + 4) - csc(6*x + 4)), x)

$$3.10 \quad \int \frac{1}{-1+3 \sin^2(2+3x)} dx$$

Optimal. Leaf size=60

$$\frac{\log(\cos(3x+2) - \sqrt{2} \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sqrt{2} \sin(3x+2) + \cos(3x+2))}{6\sqrt{2}}$$

[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3181, 207}

$$\frac{\log(\cos(3x+2) - \sqrt{2} \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sqrt{2} \sin(3x+2) + \cos(3x+2))}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 3*Sin[2 + 3*x]^2)^(-1), x]

[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 3181

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)^2])^(-1), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{-1+3 \sin^2(2+3x)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{-1+2x^2} dx, x, \tan(2+3x) \right) \\ &= \frac{\log(\cos(2+3x) - \sqrt{2} \sin(2+3x))}{6\sqrt{2}} - \frac{\log(\cos(2+3x) + \sqrt{2} \sin(2+3x))}{6\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 22, normalized size = 0.37

$$\frac{\tanh^{-1}(\sqrt{2} \tan(3x+2))}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 3*Sin[2 + 3*x]^2)^(-1), x]

[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]

fricas [A] time = 0.65, size = 86, normalized size = 1.43

$$\frac{1}{24} \sqrt{2} \log \left(-\frac{7 \cos(3x+2)^4 - 4 \cos(3x+2)^2 - 4(\sqrt{2} \cos(3x+2)^3 - 2\sqrt{2} \cos(3x+2)) \sin(3x+2) - 4}{9 \cos(3x+2)^4 - 12 \cos(3x+2)^2 + 4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+3*sin(2+3*x)^2),x, algorithm="fricas")

[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*cos(3*x + 2)^2 + 4))

giac [A] time = 0.16, size = 39, normalized size = 0.65

$$\frac{1}{12} \sqrt{2} \log \left(\frac{|-2\sqrt{2} + 4 \tan(3x+2)|}{|2\sqrt{2} + 4 \tan(3x+2)|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+3*sin(2+3*x)^2),x, algorithm="giac")

[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x + 2)))

maple [A] time = 0.15, size = 17, normalized size = 0.28

$$\frac{\sqrt{2} \operatorname{arctanh}(\sqrt{2} \tan(2+3x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-1+3*sin(2+3*x)^2),x)

[Out] -1/6*2^(1/2)*arctanh(2^(1/2)*tan(2+3*x))

maxima [A] time = 0.41, size = 34, normalized size = 0.57

$$\frac{1}{12} \sqrt{2} \log \left(-\frac{\sqrt{2} - 2 \tan(3x+2)}{\sqrt{2} + 2 \tan(3x+2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+3*sin(2+3*x)^2),x, algorithm="maxima")

[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))

mupad [B] time = 2.45, size = 16, normalized size = 0.27

$$\frac{\sqrt{2} \operatorname{atanh}(\sqrt{2} \tan(3x+2))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*sin(3*x + 2)^2 - 1),x)

[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6

sympy [B] time = 18.47, size = 1644, normalized size = 27.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+3*sin(2+3*x)**2),x)

[Out]
$$\begin{aligned} & -1387702511766624\sqrt{5 - 2\sqrt{6}}\log(\tan(3x/2 + 1) - \sqrt{5 - 2\sqrt{6}}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}} \\ & \sqrt{2\sqrt{6} + 5}) - 566527178101133\sqrt{6}\sqrt{5 - 2\sqrt{6}}\log(\tan(3x/2 + 1) - \sqrt{5 - 2\sqrt{6}}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6})\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} \\ & + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) + 1376499295618884\sqrt{2\sqrt{6} + 5} \\ & \log(\tan(3x/2 + 1) - \sqrt{5 - 2\sqrt{6}}) /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6})\sqrt{5 - 2\sqrt{6}} \\ & \sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) + 561953484261121\sqrt{6}\sqrt{2\sqrt{6} + 5} \\ & \log(\tan(3x/2 + 1) - \sqrt{5 - 2\sqrt{6}}) /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6})\sqrt{5 - 2\sqrt{6}} \\ & \sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) - 1247944371758796\sqrt{2\sqrt{6} + 5} \\ & \log(\tan(3x/2 + 1) + \sqrt{5 - 2\sqrt{6}}) /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6})\sqrt{5 - 2\sqrt{6}} \\ & \sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) - 509471156364528\sqrt{6}\sqrt{2\sqrt{6} + 5} \\ & \log(\tan(3x/2 + 1) + \sqrt{5 - 2\sqrt{6}}) /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6})\sqrt{5 - 2\sqrt{6}} \\ & \sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) + 47005690897992\sqrt{6}\sqrt{5 - 2\sqrt{6}} \\ & \log(\tan(3x/2 + 1) + \sqrt{5 - 2\sqrt{6}}) /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6})\sqrt{5 - 2\sqrt{6}} \\ & \sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) + 115139957707068\sqrt{5 - 2\sqrt{6}} \\ & \log(\tan(3x/2 + 1) + \sqrt{5 - 2\sqrt{6}}) /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6})\sqrt{5 - 2\sqrt{6}} \\ & \sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) - 12353375735168316\sqrt{5 - 2\sqrt{6}} \\ & \log(\tan(3x/2 + 1) - \sqrt{2\sqrt{6} + 5}) /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6})\sqrt{5 - 2\sqrt{6}} \\ & \sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) - 5043244525340232\sqrt{6}\sqrt{5 - 2\sqrt{6}} \\ & \log(\tan(3x/2 + 1) - \sqrt{2\sqrt{6} + 5}) /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6})\sqrt{5 - 2\sqrt{6}} \\ & \sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) + 4748539075824\sqrt{6}\sqrt{2\sqrt{6} + 5} \\ & \log(\tan(3x/2 + 1) - \sqrt{2\sqrt{6} + 5}) /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6})\sqrt{5 - 2\sqrt{6}} \\ & \sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) + 11631497759436\sqrt{2\sqrt{6} + 5} \\ & \log(\tan(3x/2 + 1) - \sqrt{2\sqrt{6} + 5}) /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6})\sqrt{5 - 2\sqrt{6}} \\ & \sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) - 140186421619524\sqrt{2\sqrt{6} + 5} \\ & \log(\tan(3x/2 + 1) + \sqrt{2\sqrt{6} + 5}) /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6})\sqrt{5 - 2\sqrt{6}} \\ & \sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) - 57230866972417\sqrt{6}\sqrt{2\sqrt{6} + 5} \\ & \log(\tan(3x/2 + 1) + \sqrt{2\sqrt{6} + 5}) /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6})\sqrt{5 - 2\sqrt{6}} \\ & \sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) + 13625938289227872\sqrt{5 - 2\sqrt{6}} \\ & \log(\tan(3x/2 + 1) + \sqrt{2\sqrt{6} + 5}) /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6})\sqrt{5 - 2\sqrt{6}} \\ & \sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) + 5562766012543373\sqrt{6}\sqrt{5 - 2\sqrt{6}} \\ & \log(\tan(3x/2 + 1) + \sqrt{2\sqrt{6} + 5}) /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6})\sqrt{5 - 2\sqrt{6}} \\ & \sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \end{aligned}$$

$$3.11 \quad \int \frac{1}{2-3 \cos^2(2+3x)} dx$$

Optimal. Leaf size=60

$$\frac{\log(\cos(3x+2) - \sqrt{2} \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sqrt{2} \sin(3x+2) + \cos(3x+2))}{6\sqrt{2}}$$

[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3181, 206}

$$\frac{\log(\cos(3x+2) - \sqrt{2} \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sqrt{2} \sin(3x+2) + \cos(3x+2))}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 - 3*Cos[2 + 3*x]^2)^(-1), x]

[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3181

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2)^(-1), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{2-3 \cos^2(2+3x)} dx &= -\left(\frac{1}{3} \text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \cot(2+3x)\right)\right) \\ &= \frac{\log(\cos(2+3x) - \sqrt{2} \sin(2+3x))}{6\sqrt{2}} - \frac{\log(\cos(2+3x) + \sqrt{2} \sin(2+3x))}{6\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.07, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}(\sqrt{2} \tan(3x+2))}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - 3*Cos[2 + 3*x]^2)^(-1), x]

[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]

fricas [A] time = 0.57, size = 86, normalized size = 1.43

$$\frac{1}{24} \sqrt{2} \log \left(-\frac{7 \cos(3x+2)^4 - 4 \cos(3x+2)^2 - 4(\sqrt{2} \cos(3x+2)^3 - 2\sqrt{2} \cos(3x+2)) \sin(3x+2) - 4}{9 \cos(3x+2)^4 - 12 \cos(3x+2)^2 + 4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-3*cos(2+3*x)^2), x, algorithm="fricas")

[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*cos(3*x + 2)^2 + 4))

giac [A] time = 0.17, size = 39, normalized size = 0.65

$$\frac{1}{12} \sqrt{2} \log \left(\frac{|-2\sqrt{2} + 4 \tan(3x+2)|}{|2\sqrt{2} + 4 \tan(3x+2)|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-3*cos(2+3*x)^2), x, algorithm="giac")

[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x + 2)))

maple [A] time = 0.10, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2} \operatorname{arctanh}(\sqrt{2} \tan(2+3x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2-3*cos(2+3*x)^2), x)

[Out] -1/6*2^(1/2)*arctanh(2^(1/2)*tan(2+3*x))

maxima [A] time = 0.50, size = 34, normalized size = 0.57

$$\frac{1}{12} \sqrt{2} \log \left(-\frac{\sqrt{2} - 2 \tan(3x+2)}{\sqrt{2} + 2 \tan(3x+2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-3*cos(2+3*x)^2), x, algorithm="maxima")

[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))

mupad [B] time = 2.44, size = 16, normalized size = 0.27

$$-\frac{\sqrt{2} \operatorname{atanh}(\sqrt{2} \tan(3x+2))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(3*cos(3*x + 2)^2 - 2), x)

[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6

sympy [B] time = 14.40, size = 1644, normalized size = 27.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-3*cos(2+3*x)**2),x)

[Out]
$$\begin{aligned} & -1387702511766624\sqrt{5 - 2\sqrt{6}}\log(\tan(3x/2 + 1) - \sqrt{5 - 2\sqrt{6}}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}} \\ & \sqrt{2\sqrt{6} + 5}) - 566527178101133\sqrt{6}\sqrt{5 - 2\sqrt{6}}\log(\tan(3x/2 + 1) - \sqrt{5 - 2\sqrt{6}}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \\ & + 1376499295618884\sqrt{2\sqrt{6} + 5}\log(\tan(3x/2 + 1) - \sqrt{5 - 2\sqrt{6}}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \\ & + 561953484261121\sqrt{6}\sqrt{2\sqrt{6} + 5}\log(\tan(3x/2 + 1) - \sqrt{5 - 2\sqrt{6}}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \\ & - 1247944371758796\sqrt{2\sqrt{6} + 5}\log(\tan(3x/2 + 1) + \sqrt{5 - 2\sqrt{6}}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \\ & - 509471156364528\sqrt{6}\sqrt{2\sqrt{6} + 5}\log(\tan(3x/2 + 1) + \sqrt{5 - 2\sqrt{6}}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \\ & + 47005690897992\sqrt{6}\sqrt{5 - 2\sqrt{6}}\log(\tan(3x/2 + 1) + \sqrt{5 - 2\sqrt{6}}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \\ & + 115139957707068\sqrt{5 - 2\sqrt{6}}\log(\tan(3x/2 + 1) + \sqrt{5 - 2\sqrt{6}}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \\ & - 12353375735168316\sqrt{5 - 2\sqrt{6}}\log(\tan(3x/2 + 1) - \sqrt{2\sqrt{6} + 5}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \\ & - 5043244525340232\sqrt{6}\sqrt{5 - 2\sqrt{6}}\log(\tan(3x/2 + 1) - \sqrt{2\sqrt{6} + 5}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \\ & + 4748539075824\sqrt{6}\sqrt{2\sqrt{6} + 5}\log(\tan(3x/2 + 1) - \sqrt{2\sqrt{6} + 5}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \\ & + 11631497759436\sqrt{2\sqrt{6} + 5}\log(\tan(3x/2 + 1) - \sqrt{2\sqrt{6} + 5}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \\ & - 140186421619524\sqrt{2\sqrt{6} + 5}\log(\tan(3x/2 + 1) + \sqrt{2\sqrt{6} + 5}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \\ & - 57230866972417\sqrt{6}\sqrt{2\sqrt{6} + 5}\log(\tan(3x/2 + 1) + \sqrt{2\sqrt{6} + 5}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \\ & + 13625938289227872\sqrt{5 - 2\sqrt{6}}\log(\tan(3x/2 + 1) + \sqrt{2\sqrt{6} + 5}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \\ & + 5562766012543373\sqrt{6}\sqrt{5 - 2\sqrt{6}}\log(\tan(3x/2 + 1) + \sqrt{2\sqrt{6} + 5}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}) \\ & \sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \end{aligned}$$

$$3.12 \quad \int \frac{1}{-\cos^2(2+3x)+2\sin^2(2+3x)} dx$$

Optimal. Leaf size=60

$$\frac{\log(\cos(3x+2) - \sqrt{2}\sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sqrt{2}\sin(3x+2) + \cos(3x+2))}{6\sqrt{2}}$$

[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {207}

$$\frac{\log(\cos(3x+2) - \sqrt{2}\sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sqrt{2}\sin(3x+2) + \cos(3x+2))}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(-Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1), x]

[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{-\cos^2(2+3x)+2\sin^2(2+3x)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{-1+2x^2} dx, x, \tan(2+3x) \right) \\ &= \frac{\log(\cos(2+3x) - \sqrt{2}\sin(2+3x))}{6\sqrt{2}} - \frac{\log(\cos(2+3x) + \sqrt{2}\sin(2+3x))}{6\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.37

$$\frac{\tanh^{-1}(\sqrt{2}\tan(3x+2))}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1), x]

[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]

fricas [A] time = 0.78, size = 86, normalized size = 1.43

$$\frac{1}{24} \sqrt{2} \log \left(-\frac{7 \cos(3x+2)^4 - 4 \cos(3x+2)^2 - 4(\sqrt{2} \cos(3x+2)^3 - 2\sqrt{2} \cos(3x+2)) \sin(3x+2) - 4}{9 \cos(3x+2)^4 - 12 \cos(3x+2)^2 + 4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="fricas")

[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*cos(3*x + 2)^2 + 4))

giac [A] time = 0.20, size = 39, normalized size = 0.65

$$\frac{1}{12} \sqrt{2} \log \left(\frac{|-2\sqrt{2} + 4 \tan(3x + 2)|}{|2\sqrt{2} + 4 \tan(3x + 2)|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="giac")

[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x + 2)))

maple [A] time = 0.22, size = 17, normalized size = 0.28

$$\frac{\sqrt{2} \operatorname{arctanh}(\sqrt{2} \tan(2 + 3x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x)

[Out] -1/6*2^(1/2)*arctanh(2^(1/2)*tan(2+3*x))

maxima [A] time = 0.41, size = 34, normalized size = 0.57

$$\frac{1}{12} \sqrt{2} \log \left(-\frac{\sqrt{2} - 2 \tan(3x + 2)}{\sqrt{2} + 2 \tan(3x + 2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="maxima")

[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))

mupad [B] time = 2.40, size = 16, normalized size = 0.27

$$\frac{\sqrt{2} \operatorname{atanh}(\sqrt{2} \tan(3x + 2))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*sin(3*x + 2)^2 - cos(3*x + 2)^2),x)

[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6

sympy [B] time = 17.09, size = 1644, normalized size = 27.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(2+3*x)**2+2*sin(2+3*x)**2),x)

[Out] -1387702511766624*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6))*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) - 566527178101133*sqrt(6)*sqrt(5 - 2*sqrt(6))*1

$$\begin{aligned} & \log(\tan(3x/2 + 1) - \sqrt{5 - 2\sqrt{6}})/(-467972363532675 - 19104891739654 \\ & 8\sqrt{6} + 13665597568857156\sqrt{6}\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) + 137649929 \\ & 5618884\sqrt{2\sqrt{6} + 5}\log(\tan(3x/2 + 1) - \sqrt{5 - 2\sqrt{6}})/(-467 \\ & 972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) + 561953484261121\sqrt{6}\sqrt{2\sqrt{6} + 5}\log(\tan(3x \\ & /2 + 1) - \sqrt{5 - 2\sqrt{6}})/(-467972363532675 - 191048917396548\sqrt{6} \\ & + 13665597568857156\sqrt{6}\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473 \\ & 741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) - 1247944371758796\sqrt{q \\ & rt(2\sqrt{6} + 5)\log(\tan(3x/2 + 1) + \sqrt{5 - 2\sqrt{6}})/(-4679723635326 \\ & 75 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}\sqrt{5 - 2\sqrt{6}} \\ &)\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} \\ &) + 5)) - 509471156364528\sqrt{6}\sqrt{2\sqrt{6} + 5}\log(\tan(3x/2 + 1) + \\ & \sqrt{5 - 2\sqrt{6}})/(-467972363532675 - 191048917396548\sqrt{6} + 13665597 \\ & 568857156\sqrt{6}\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 334737410739183 \\ & 39\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) + 47005690897992\sqrt{6}\sqrt{5 \\ & - 2\sqrt{6}}\log(\tan(3x/2 + 1) + \sqrt{5 - 2\sqrt{6}})/(-467972363532675 - \\ & 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}\sqrt{5 - 2\sqrt{6}}\sqrt{q \\ & rt(2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \\ & + 115139957707068\sqrt{5 - 2\sqrt{6}}\log(\tan(3x/2 + 1) + \sqrt{5 - 2\sqrt{6}})/(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) - 12353375735168316\sqrt{5 - 2\sqrt{6}}\log \\ & (\tan(3x/2 + 1) - \sqrt{2\sqrt{6} + 5})/(-467972363532675 - 191048917396548\sqrt{6} \\ & + 13665597568857156\sqrt{6}\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) - 50432445253 \\ & 40232\sqrt{6}\sqrt{5 - 2\sqrt{6}}\log(\tan(3x/2 + 1) - \sqrt{2\sqrt{6} + 5}) \\ & /(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) + 4748539075824\sqrt{6}\sqrt{2\sqrt{6} + 5}\log(\tan \\ & (3x/2 + 1) - \sqrt{2\sqrt{6} + 5})/(-467972363532675 - 191048917396548\sqrt{6} \\ & + 13665597568857156\sqrt{6}\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 3 \\ & 3473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) + 11631497759436\sqrt{2\sqrt{6} + 5}\log(\tan(3x/2 + 1) - \sqrt{2\sqrt{6} + 5})/(-46797236353 \\ & 2675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}\sqrt{5 - 2\sqrt{6}} \\ &)\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} \\ &) + 5)) - 140186421619524\sqrt{2\sqrt{6} + 5}\log(\tan(3x/2 + 1) + \sqrt{2 \\ & *\sqrt{6} + 5})/(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857 \\ & 156\sqrt{6}\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{q \\ & rt(5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) - 57230866972417\sqrt{6}\sqrt{2\sqrt{6} \\ &) + 5}\log(\tan(3x/2 + 1) + \sqrt{2\sqrt{6} + 5})/(-467972363532675 - 19104 \\ & 8917396548\sqrt{6} + 13665597568857156\sqrt{6}\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} \\ &) + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) + \\ & 13625938289227872\sqrt{5 - 2\sqrt{6}}\log(\tan(3x/2 + 1) + \sqrt{2\sqrt{6} + 5})/(-467972363532675 - 191048917396548\sqrt{6} + 13665597568857156\sqrt{6}\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) + 5562766012543373\sqrt{6}\sqrt{5 - 2\sqrt{6}}\log \\ & (\tan(3x/2 + 1) + \sqrt{2\sqrt{6} + 5})/(-467972363532675 - 1910489173965 \\ & 48\sqrt{6} + 13665597568857156\sqrt{6}\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5} + 33473741073918339\sqrt{5 - 2\sqrt{6}}\sqrt{2\sqrt{6} + 5}) \end{aligned}$$

$$3.13 \quad \int \frac{\sec^2(2+3x)}{-1+2 \tan^2(2+3x)} dx$$

Optimal. Leaf size=60

$$\frac{\log(\cos(3x+2) - \sqrt{2} \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sqrt{2} \sin(3x+2) + \cos(3x+2))}{6\sqrt{2}}$$

[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3675, 207}

$$\frac{\log(\cos(3x+2) - \sqrt{2} \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sqrt{2} \sin(3x+2) + \cos(3x+2))}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sec[2 + 3*x]^2/(-1 + 2*Tan[2 + 3*x]^2), x]

[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2-1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(2+3x)}{-1+2 \tan^2(2+3x)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{-1+2x^2} dx, x, \tan(2+3x) \right) \\ &= \frac{\log(\cos(2+3x) - \sqrt{2} \sin(2+3x))}{6\sqrt{2}} - \frac{\log(\cos(2+3x) + \sqrt{2} \sin(2+3x))}{6\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.37

$$\frac{\tanh^{-1}(\sqrt{2} \tan(3x+2))}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2 + 3*x]^2/(-1 + 2*Tan[2 + 3*x]^2), x]

[Out] $-1/3 \cdot \text{ArcTanh}[\text{Sqrt}[2] \cdot \text{Tan}[2 + 3x]] / \text{Sqrt}[2]$

fricas [A] time = 0.77, size = 86, normalized size = 1.43

$$\frac{1}{24} \sqrt{2} \log \left(-\frac{7 \cos(3x+2)^4 - 4 \cos(3x+2)^2 - 4(\sqrt{2} \cos(3x+2)^3 - 2\sqrt{2} \cos(3x+2)) \sin(3x+2) - 4}{9 \cos(3x+2)^4 - 12 \cos(3x+2)^2 + 4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x, algorithm="fricas")`

[Out] $1/24 \cdot \text{sqrt}(2) \cdot \log(-7 \cdot \cos(3x+2)^4 - 4 \cdot \cos(3x+2)^2 - 4 \cdot (\text{sqrt}(2) \cdot \cos(3x+2)^3 - 2 \cdot \text{sqrt}(2) \cdot \cos(3x+2)) \cdot \sin(3x+2) - 4) / (9 \cdot \cos(3x+2)^4 - 12 \cdot \cos(3x+2)^2 + 4)$

giac [A] time = 1.48, size = 39, normalized size = 0.65

$$-\frac{1}{12} \sqrt{2} \log \left(\left| \frac{1}{2} \sqrt{2} + \tan(3x+2) \right| \right) + \frac{1}{12} \sqrt{2} \log \left(\left| -\frac{1}{2} \sqrt{2} + \tan(3x+2) \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x, algorithm="giac")`

[Out] $-1/12 \cdot \text{sqrt}(2) \cdot \log(\text{abs}(1/2 \cdot \text{sqrt}(2) + \tan(3x+2))) + 1/12 \cdot \text{sqrt}(2) \cdot \log(\text{abs}(-1/2 \cdot \text{sqrt}(2) + \tan(3x+2)))$

maple [A] time = 0.28, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2} \operatorname{arctanh}(\sqrt{2} \tan(2+3x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x)`

[Out] $-1/6 \cdot 2^{(1/2)} \cdot \operatorname{arctanh}(2^{(1/2)} \cdot \tan(2+3x))$

maxima [A] time = 1.03, size = 34, normalized size = 0.57

$$\frac{1}{12} \sqrt{2} \log \left(-\frac{\sqrt{2} - 2 \tan(3x+2)}{\sqrt{2} + 2 \tan(3x+2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x, algorithm="maxima")`

[Out] $1/12 \cdot \text{sqrt}(2) \cdot \log(-(\text{sqrt}(2) - 2 \cdot \tan(3x+2)) / (\text{sqrt}(2) + 2 \cdot \tan(3x+2)))$

mupad [B] time = 2.40, size = 16, normalized size = 0.27

$$-\frac{\sqrt{2} \operatorname{atanh}(\sqrt{2} \tan(3x+2))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cos(3*x+2)^2*(2*tan(3*x+2)^2-1)),x)`

[Out] $-(2^{(1/2)} \cdot \operatorname{atanh}(2^{(1/2)} \cdot \tan(3x+2))) / 6$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(3x+2)}{2 \tan^2(3x+2) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)**2/(-1+2*tan(2+3*x)**2), x)
```

```
[Out] Integral(sec(3*x + 2)**2/(2*tan(3*x + 2)**2 - 1), x)
```


$$3.14 \quad \int \frac{\csc^2(2+3x)}{2-\cot^2(2+3x)} dx$$

Optimal. Leaf size=60

$$\frac{\log(\cos(3x+2) - \sqrt{2} \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sqrt{2} \sin(3x+2) + \cos(3x+2))}{6\sqrt{2}}$$

[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3675, 206}

$$\frac{\log(\cos(3x+2) - \sqrt{2} \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sqrt{2} \sin(3x+2) + \cos(3x+2))}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Csc[2 + 3*x]^2/(2 - Cot[2 + 3*x]^2), x]

[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^n)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2-1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\csc^2(2+3x)}{2-\cot^2(2+3x)} dx &= -\left(\frac{1}{3} \text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \cot(2+3x)\right)\right) \\ &= \frac{\log(\cos(2+3x) - \sqrt{2} \sin(2+3x))}{6\sqrt{2}} - \frac{\log(\cos(2+3x) + \sqrt{2} \sin(2+3x))}{6\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.37

$$\frac{\tanh^{-1}(\sqrt{2} \tan(3x+2))}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[2 + 3*x]^2/(2 - Cot[2 + 3*x]^2), x]

[Out] $-1/3 \cdot \text{ArcTanh}[\text{Sqrt}[2] \cdot \text{Tan}[2 + 3x]] / \text{Sqrt}[2]$

fricas [A] time = 0.68, size = 86, normalized size = 1.43

$$\frac{1}{24} \sqrt{2} \log \left(-\frac{7 \cos(3x+2)^4 - 4 \cos(3x+2)^2 - 4(\sqrt{2} \cos(3x+2)^3 - 2\sqrt{2} \cos(3x+2)) \sin(3x+2) - 4}{9 \cos(3x+2)^4 - 12 \cos(3x+2)^2 + 4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(2+3*x)^2/(2-cot(2+3*x)^2),x, algorithm="fricas")`

[Out] $1/24 \cdot \text{sqrt}(2) \cdot \log(-7 \cdot \cos(3x+2)^4 - 4 \cdot \cos(3x+2)^2 - 4 \cdot (\text{sqrt}(2) \cdot \cos(3x+2)^3 - 2 \cdot \text{sqrt}(2) \cdot \cos(3x+2)) \cdot \sin(3x+2) - 4) / (9 \cdot \cos(3x+2)^4 - 12 \cdot \cos(3x+2)^2 + 4)$

giac [A] time = 0.36, size = 39, normalized size = 0.65

$$\frac{1}{12} \sqrt{2} \log \left(\frac{|-2\sqrt{2} + 4 \tan(3x+2)|}{|2\sqrt{2} + 4 \tan(3x+2)|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(2+3*x)^2/(2-cot(2+3*x)^2),x, algorithm="giac")`

[Out] $1/12 \cdot \text{sqrt}(2) \cdot \log(\text{abs}(-2 \cdot \text{sqrt}(2) + 4 \cdot \tan(3x+2)) / \text{abs}(2 \cdot \text{sqrt}(2) + 4 \cdot \tan(3x+2)))$

maple [A] time = 0.26, size = 17, normalized size = 0.28

$$\frac{\sqrt{2} \operatorname{arctanh}(\sqrt{2} \tan(2+3x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(2+3*x)^2/(2-cot(2+3*x)^2),x)`

[Out] $-1/6 \cdot 2^{(1/2)} \cdot \operatorname{arctanh}(2^{(1/2)} \cdot \tan(2+3x))$

maxima [A] time = 0.41, size = 34, normalized size = 0.57

$$\frac{1}{12} \sqrt{2} \log \left(-\frac{\sqrt{2} - 2 \tan(3x+2)}{\sqrt{2} + 2 \tan(3x+2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(2+3*x)^2/(2-cot(2+3*x)^2),x, algorithm="maxima")`

[Out] $1/12 \cdot \text{sqrt}(2) \cdot \log(-(\text{sqrt}(2) - 2 \cdot \tan(3x+2)) / (\text{sqrt}(2) + 2 \cdot \tan(3x+2)))$

mupad [B] time = 2.41, size = 16, normalized size = 0.27

$$\frac{\sqrt{2} \operatorname{atanh}(\sqrt{2} \tan(3x+2))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(sin(3*x+2)^2*(cot(3*x+2)^2-2)),x)`

[Out] $-(2^{(1/2)} \cdot \operatorname{atanh}(2^{(1/2)} \cdot \tan(3x+2))) / 6$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\csc^2(3x+2)}{\cot^2(3x+2)-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)**2/(2-cot(2+3*x)**2),x)
```

```
[Out] -Integral(csc(3*x + 2)**2/(cot(3*x + 2)**2 - 2), x)
```

$$3.15 \quad \int \frac{2}{3+\cos(4+6x)} dx$$

Optimal. Leaf size=42

$$\frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(6x+4)}{\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

[Out] 1/2*x*2^(1/2)-1/6*arctan(sin(4+6*x)/(3+cos(4+6*x)+2*2^(1/2)))*2^(1/2)

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {12, 2657}

$$\frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(6x+4)}{\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[2/(3 + Cos[4 + 6*x]), x]

[Out] x/Sqrt[2] - ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] + Cos[4 + 6*x])]/(3*Sqrt[2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2657

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{q = Rt[a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*Sin[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] && PosQ[a]

Rubi steps

$$\begin{aligned} \int \frac{2}{3 + \cos(4 + 6x)} dx &= 2 \int \frac{1}{3 + \cos(4 + 6x)} dx \\ &= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}+\cos(4+6x)}\right)}{3\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.52

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[2/(3 + Cos[4 + 6*x]), x]

[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])

fricas [A] time = 1.32, size = 31, normalized size = 0.74

$$-\frac{1}{12} \sqrt{2} \arctan\left(\frac{3\sqrt{2} \cos(6x+4) + \sqrt{2}}{4 \sin(6x+4)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(3+cos(4+6*x)),x, algorithm="fricas")

[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) + sqrt(2))/sin(6*x + 4))

giac [A] time = 0.14, size = 57, normalized size = 1.36

$$\frac{1}{6} \sqrt{2} \left(3x + \arctan \left(-\frac{\sqrt{2} \sin(6x + 4) - \sin(6x + 4)}{\sqrt{2} \cos(6x + 4) + \sqrt{2} - \cos(6x + 4) + 1} \right) + 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(3+cos(4+6*x)),x, algorithm="giac")

[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*cos(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)

maple [A] time = 0.06, size = 18, normalized size = 0.43

$$\frac{\sqrt{2} \arctan \left(\frac{\sqrt{2} \tan(2+3x)}{2} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2/(3+cos(4+6*x)),x)

[Out] 1/6*2^(1/2)*arctan(1/2*2^(1/2)*tan(2+3*x))

maxima [A] time = 0.42, size = 27, normalized size = 0.64

$$\frac{1}{6} \sqrt{2} \arctan \left(\frac{\sqrt{2} \sin(6x + 4)}{2(\cos(6x + 4) + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(3+cos(4+6*x)),x, algorithm="maxima")

[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))

mupad [B] time = 2.54, size = 36, normalized size = 0.86

$$\frac{\sqrt{2} (3x - \operatorname{atan}(\tan(3x + 2)))}{6} + \frac{\sqrt{2} \operatorname{atan} \left(\frac{\sqrt{2} \tan(3x+2)}{2} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2/(cos(6*x + 4) + 3),x)

[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan((2^(1/2)*tan(3*x + 2))/2))/6

sympy [A] time = 0.26, size = 34, normalized size = 0.81

$$\frac{\sqrt{2} \left(\operatorname{atan} \left(\frac{\sqrt{2} \tan(3x+2)}{2} \right) + \pi \left\lfloor \frac{3x - \frac{\pi}{2} + 2}{\pi} \right\rfloor \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(3+cos(4+6*x)),x)

[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x + 2)/2) + pi*floor((3*x - pi/2 + 2)/pi))/6

$$3.16 \quad \int \frac{2 \csc(4+6x)}{\cot(4+6x)+3 \csc(4+6x)} dx$$

Optimal. Leaf size=42

$$\frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(6x+4)}{\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

[Out] 1/2*x*2^(1/2)-1/6*arctan(sin(4+6*x)/(3+cos(4+6*x)+2*2^(1/2)))*2^(1/2)

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {12, 3166, 2657}

$$\frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(6x+4)}{\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2*Csc[4 + 6*x])/(Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]

[Out] x/Sqrt[2] - ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] + Cos[4 + 6*x])]/(3*Sqrt[2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2657

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{q = Rt[a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*Sin[c + d*x])])]/(d*q), x] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] && PosQ[a]

Rule 3166

Int[csc[(d_) + (e_)*(x_)]^(n_)*((a_) + csc[(d_) + (e_)*(x_)]*(b_) + cot[(d_) + (e_)*(x_)]*(c_))^(m_), x_Symbol] := Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned} \int \frac{2 \csc(4+6x)}{\cot(4+6x)+3 \csc(4+6x)} dx &= 2 \int \frac{\csc(4+6x)}{\cot(4+6x)+3 \csc(4+6x)} dx \\ &= 2 \int \frac{1}{3+\cos(4+6x)} dx \\ &= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}+\cos(4+6x)}\right)}{3\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.52

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2*Csc[4 + 6*x])/(Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]

[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])

fricas [A] time = 0.60, size = 31, normalized size = 0.74

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(6x+4)+\sqrt{2}}{4\sin(6x+4)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x, algorithm="fricas")

[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) + sqrt(2))/sin(6*x + 4))

giac [A] time = 0.23, size = 57, normalized size = 1.36

$$\frac{1}{6}\sqrt{2}\left(3x + \arctan\left(-\frac{\sqrt{2}\sin(6x+4) - \sin(6x+4)}{\sqrt{2}\cos(6x+4) + \sqrt{2} - \cos(6x+4) + 1}\right) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x, algorithm="giac")

[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*cos(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)

maple [A] time = 0.31, size = 18, normalized size = 0.43

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x)

[Out] 1/6*2^(1/2)*arctan(1/2*2^(1/2)*tan(2+3*x))

maxima [A] time = 0.43, size = 27, normalized size = 0.64

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin(6x+4)}{2(\cos(6x+4)+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x, algorithm="maxima")

[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))

mupad [B] time = 2.69, size = 17, normalized size = 0.40

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2/(sin(6*x + 4)*(cot(6*x + 4) + 3/sin(6*x + 4))),x)

[Out] (2^(1/2)*atan((2^(1/2)*tan(3*x + 2))/2))/6

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$2 \int \frac{\csc(6x + 4)}{\cot(6x + 4) + 3 \csc(6x + 4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)), x)

[Out] 2*Integral(csc(6*x + 4)/(cot(6*x + 4) + 3*csc(6*x + 4)), x)

$$3.17 \quad \int \frac{1}{2 - \sin^2(2+3x)} dx$$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

[Out] 1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2^(1/2)

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3181, 203}

$$\frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 - Sin[2 + 3*x]^2)^(-1), x]

[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^2)]/(3*Sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3181

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{2 - \sin^2(2 + 3x)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{2 + x^2} dx, x, \tan(2 + 3x) \right) \\ &= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - Sin[2 + 3*x]^2)^(-1), x]

[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])

fricas [A] time = 0.76, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2-\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-sin(2+3*x)^2),x, algorithm="fricas")

[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)*sin(3*x + 2)))

giac [A] time = 0.15, size = 57, normalized size = 1.19

$$\frac{1}{6}\sqrt{2}\left(3x + \arctan\left(-\frac{\sqrt{2}\sin(6x+4) - \sin(6x+4)}{\sqrt{2}\cos(6x+4) + \sqrt{2} - \cos(6x+4) + 1}\right) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-sin(2+3*x)^2),x, algorithm="giac")

[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*cos(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)

maple [A] time = 0.15, size = 18, normalized size = 0.38

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2-sin(2+3*x)^2),x)

[Out] 1/6*2^(1/2)*arctan(1/2*2^(1/2)*tan(2+3*x))

maxima [A] time = 0.44, size = 17, normalized size = 0.35

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2-sin(2+3*x)^2),x, algorithm="maxima")

[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))

mupad [B] time = 2.44, size = 36, normalized size = 0.75

$$\frac{\sqrt{2}(3x - \operatorname{atan}(\tan(3x+2)))}{6} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(sin(3*x + 2)^2 - 2),x)

[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan((2^(1/2)*tan(3*x + 2))/2))/6

sympy [A] time = 0.75, size = 76, normalized size = 1.58

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)-1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6} + \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)+1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-sin(2+3*x)**2),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) - 1) + pi*floor((3*x/2 - pi/2 + 1)/pi)
)/6 + sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) + 1) + pi*floor((3*x/2 - pi/2 +
1)/pi))/6
```

$$3.18 \quad \int \frac{1}{1+\cos^2(2+3x)} dx$$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

[Out] $1/2*x*2^{(1/2)}-1/6*\arctan(\cos(2+3*x)*\sin(2+3*x)/(1+\cos(2+3*x)^2+2^{(1/2)}))*2^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3181, 203}

$$\frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[2 + 3*x]^2)^(-1), x]

[Out] $x/\text{Sqrt}[2] - \text{ArcTan}[(\text{Cos}[2 + 3*x]*\text{Sin}[2 + 3*x])/((1 + \text{Sqrt}[2] + \text{Cos}[2 + 3*x]^2))]/(3*\text{Sqrt}[2])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3181

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{1+\cos^2(2+3x)} dx &= -\left(\frac{1}{3} \text{Subst}\left(\int \frac{1}{1+2x^2} dx, x, \cot(2+3x)\right)\right) \\ &= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Cos[2 + 3*x]^2)^(-1), x]

[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])

fricas [A] time = 0.68, size = 43, normalized size = 0.90

$$-\frac{1}{12} \sqrt{2} \arctan\left(\frac{3\sqrt{2} \cos(3x+2)^2 - \sqrt{2}}{4 \cos(3x+2) \sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+cos(2+3*x)^2),x, algorithm="fricas")

[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)*sin(3*x + 2)))

giac [A] time = 0.15, size = 57, normalized size = 1.19

$$\frac{1}{6} \sqrt{2} \left(3x + \arctan\left(-\frac{\sqrt{2} \sin(6x+4) - \sin(6x+4)}{\sqrt{2} \cos(6x+4) + \sqrt{2} - \cos(6x+4) + 1}\right) + 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+cos(2+3*x)^2),x, algorithm="giac")

[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*cos(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)

maple [A] time = 0.10, size = 18, normalized size = 0.38

$$\frac{\sqrt{2} \arctan\left(\frac{\sqrt{2} \tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+cos(2+3*x)^2),x)

[Out] 1/6*2^(1/2)*arctan(1/2*2^(1/2)*tan(2+3*x))

maxima [A] time = 0.44, size = 17, normalized size = 0.35

$$\frac{1}{6} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+cos(2+3*x)^2),x, algorithm="maxima")

[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))

mupad [B] time = 2.37, size = 36, normalized size = 0.75

$$\frac{\sqrt{2} (3x - \operatorname{atan}(\tan(3x+2)))}{6} + \frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} \tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(3*x + 2)^2 + 1),x)

[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan((2^(1/2)*tan(3*x + 2))/2))/6

sympy [A] time = 0.65, size = 76, normalized size = 1.58

$$\frac{\sqrt{2} \left(\operatorname{atan}\left(\sqrt{2} \tan\left(\frac{3x}{2} + 1\right) - 1\right) + \pi \left\lfloor \frac{\frac{3x}{2} - \frac{\pi}{2} + 1}{\pi} \right\rfloor \right)}{6} + \frac{\sqrt{2} \left(\operatorname{atan}\left(\sqrt{2} \tan\left(\frac{3x}{2} + 1\right) + 1\right) + \pi \left\lfloor \frac{\frac{3x}{2} - \frac{\pi}{2} + 1}{\pi} \right\rfloor \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2+3*x)**2),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) - 1) + pi*floor((3*x/2 - pi/2 + 1)/pi))/  
6 + sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) + 1) + pi*floor((3*x/2 - pi/2 +  
1)/pi))/6
```

$$3.19 \quad \int \frac{1}{2 \cos^2(2+3x) + \sin^2(2+3x)} dx$$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

[Out] 1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2^(1/2)

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {203}

$$\frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1), x]

[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^2)]/(3*Sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{2 \cos^2(2+3x) + \sin^2(2+3x)} dx &= \frac{1}{3} \text{Subst}\left(\int \frac{1}{2+x^2} dx, x, \tan(2+3x)\right) \\ &= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1), x]

[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])

fricas [A] time = 0.51, size = 43, normalized size = 0.90

$$-\frac{1}{12} \sqrt{2} \arctan\left(\frac{3\sqrt{2} \cos(3x+2)^2 - \sqrt{2}}{4 \cos(3x+2) \sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="fricas")

[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)*sin(3*x + 2)))

giac [A] time = 0.16, size = 57, normalized size = 1.19

$$\frac{1}{6} \sqrt{2} \left(3x + \arctan \left(-\frac{\sqrt{2} \sin(6x + 4) - \sin(6x + 4)}{\sqrt{2} \cos(6x + 4) + \sqrt{2} - \cos(6x + 4) + 1} \right) + 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="giac")

[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*cos(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)

maple [A] time = 0.23, size = 18, normalized size = 0.38

$$\frac{\sqrt{2} \arctan \left(\frac{\sqrt{2} \tan(2+3x)}{2} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x)

[Out] 1/6*2^(1/2)*arctan(1/2*2^(1/2)*tan(2+3*x))

maxima [A] time = 0.44, size = 17, normalized size = 0.35

$$\frac{1}{6} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} \tan(3x + 2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="maxima")

[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))

mupad [B] time = 2.37, size = 36, normalized size = 0.75

$$\frac{\sqrt{2} (3x - \operatorname{atan}(\tan(3x + 2)))}{6} + \frac{\sqrt{2} \operatorname{atan} \left(\frac{\sqrt{2} \tan(3x+2)}{2} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(3*x + 2)^2 + 2*cos(3*x + 2)^2),x)

[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan((2^(1/2)*tan(3*x + 2))/2))/6

sympy [A] time = 0.80, size = 76, normalized size = 1.58

$$\frac{\sqrt{2} \left(\operatorname{atan} \left(\sqrt{2} \tan \left(\frac{3x}{2} + 1 \right) - 1 \right) + \pi \left\lfloor \frac{\frac{3x}{2} - \frac{\pi}{2} + 1}{\pi} \right\rfloor \right)}{6} + \frac{\sqrt{2} \left(\operatorname{atan} \left(\sqrt{2} \tan \left(\frac{3x}{2} + 1 \right) + 1 \right) + \pi \left\lfloor \frac{\frac{3x}{2} - \frac{\pi}{2} + 1}{\pi} \right\rfloor \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(2+3*x)**2+sin(2+3*x)**2),x)

[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) - 1) + pi*floor((3*x/2 - pi/2 + 1)/pi))/6 + sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) + 1) + pi*floor((3*x/2 - pi/2 + 1)/pi))/6

$$3.20 \quad \int \frac{\sec^2(2+3x)}{2+\tan^2(2+3x)} dx$$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

[Out] $1/2*x*2^{(1/2)}-1/6*\arctan(\cos(2+3*x)*\sin(2+3*x)/(1+\cos(2+3*x)^2+2^{(1/2)}))*2^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3675, 203}

$$\frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sec[2 + 3*x]^2/(2 + Tan[2 + 3*x]^2), x]

[Out] $x/\text{Sqrt}[2] - \text{ArcTan}[(\text{Cos}[2 + 3*x]*\text{Sin}[2 + 3*x])/ (1 + \text{Sqrt}[2] + \text{Cos}[2 + 3*x]^2)] / (3*\text{Sqrt}[2])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(2+3x)}{2+\tan^2(2+3x)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{2+x^2} dx, x, \tan(2+3x) \right) \\ &= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2 + 3*x]^2/(2 + Tan[2 + 3*x]^2), x]

[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])

fricas [A] time = 0.61, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2-\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2+3*x)^2/(2+tan(2+3*x)^2),x, algorithm="fricas")

[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)*sin(3*x + 2)))

giac [A] time = 1.23, size = 17, normalized size = 0.35

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2+3*x)^2/(2+tan(2+3*x)^2),x, algorithm="giac")

[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))

maple [A] time = 0.27, size = 18, normalized size = 0.38

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2+3*x)^2/(2+tan(2+3*x)^2),x)

[Out] 1/6*2^(1/2)*arctan(1/2*2^(1/2)*tan(2+3*x))

maxima [A] time = 1.18, size = 17, normalized size = 0.35

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2+3*x)^2/(2+tan(2+3*x)^2),x, algorithm="maxima")

[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))

mupad [B] time = 2.37, size = 17, normalized size = 0.35

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(3*x + 2)^2*(tan(3*x + 2)^2 + 2)),x)

[Out] (2^(1/2)*atan((2^(1/2)*tan(3*x + 2))/2))/6

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(3x+2)}{\tan^2(3x+2)+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)**2/(2+tan(2+3*x)**2), x)
```

```
[Out] Integral(sec(3*x + 2)**2/(tan(3*x + 2)**2 + 2), x)
```

$$3.21 \quad \int \frac{\csc^2(2+3x)}{1+2 \cot^2(2+3x)} dx$$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

[Out] 1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2^(1/2)

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3675, 203}

$$\frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Csc[2 + 3*x]^2/(1 + 2*Cot[2 + 3*x]^2), x]

[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^2)]/(3*Sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2-1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\csc^2(2+3x)}{1+2 \cot^2(2+3x)} dx &= -\left(\frac{1}{3} \text{Subst}\left(\int \frac{1}{1+2x^2} dx, x, \cot(2+3x)\right)\right) \\ &= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[2 + 3*x]^2/(1 + 2*Cot[2 + 3*x]^2), x]

[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])

fricas [A] time = 0.55, size = 43, normalized size = 0.90

$$-\frac{1}{12} \sqrt{2} \arctan\left(\frac{3 \sqrt{2} \cos(3x + 2)^2 - \sqrt{2}}{4 \cos(3x + 2) \sin(3x + 2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x, algorithm="fricas")

[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)*sin(3*x + 2)))

giac [A] time = 0.33, size = 57, normalized size = 1.19

$$\frac{1}{6} \sqrt{2} \left(3x + \arctan\left(-\frac{\sqrt{2} \sin(6x + 4) - \sin(6x + 4)}{\sqrt{2} \cos(6x + 4) + \sqrt{2} - \cos(6x + 4) + 1}\right) + 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x, algorithm="giac")

[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*cos(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)

maple [A] time = 0.28, size = 18, normalized size = 0.38

$$\frac{\sqrt{2} \arctan\left(\frac{\sqrt{2} \tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x)

[Out] 1/6*2^(1/2)*arctan(1/2*2^(1/2)*tan(2+3*x))

maxima [A] time = 0.74, size = 17, normalized size = 0.35

$$\frac{1}{6} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \tan(3x + 2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x, algorithm="maxima")

[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))

mupad [B] time = 2.38, size = 17, normalized size = 0.35

$$\frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} \tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(3*x + 2)^2*(2*cot(3*x + 2)^2 + 1)),x)

[Out] (2^(1/2)*atan((2^(1/2)*tan(3*x + 2))/2))/6

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(3x + 2)}{2 \cot^2(3x + 2) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)**2/(1+2*cot(2+3*x)**2),x)
```

```
[Out] Integral(csc(3*x + 2)**2/(2*cot(3*x + 2)**2 + 1), x)
```

$$3.22 \quad \int -\frac{2}{1+3\cos(4+6x)} dx$$

Optimal. Leaf size=61

$$\frac{\log(\sqrt{2}\cos(3x+2) - \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sin(3x+2) + \sqrt{2}\cos(3x+2))}{6\sqrt{2}}$$

[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {12, 2659, 206}

$$\frac{\log(\sqrt{2}\cos(3x+2) - \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sin(3x+2) + \sqrt{2}\cos(3x+2))}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[-2/(1 + 3*Cos[4 + 6*x]), x]

[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 + 3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int -\frac{2}{1+3\cos(4+6x)} dx &= -\left(2 \int \frac{1}{1+3\cos(4+6x)} dx\right) \\ &= -\left(\frac{2}{3} \text{Subst}\left(\int \frac{1}{4-2x^2} dx, x, \tan\left(\frac{1}{2}(4+6x)\right)\right)\right) \\ &= \frac{\log(\sqrt{2}\cos(2+3x) - \sin(2+3x))}{6\sqrt{2}} - \frac{\log(\sqrt{2}\cos(2+3x) + \sin(2+3x))}{6\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[-2/(1 + 3*Cos[4 + 6*x]),x]

[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]

fricas [A] time = 0.54, size = 74, normalized size = 1.21

$$\frac{1}{24} \sqrt{2} \log \left(-\frac{7 \cos(6x+4)^2 + 4 \left(\sqrt{2} \cos(6x+4) + 3 \sqrt{2} \right) \sin(6x+4) - 6 \cos(6x+4) - 17}{9 \cos(6x+4)^2 + 6 \cos(6x+4) + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-2/(1+3*cos(4+6*x)),x, algorithm="fricas")

[Out] 1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 + 4*(sqrt(2)*cos(6*x + 4) + 3*sqrt(2))*sin(6*x + 4) - 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 + 6*cos(6*x + 4) + 1))

giac [A] time = 0.16, size = 39, normalized size = 0.64

$$\frac{1}{12} \sqrt{2} \log \left(\frac{\left| -2 \sqrt{2} + 2 \tan(3x+2) \right|}{\left| 2 \sqrt{2} + 2 \tan(3x+2) \right|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-2/(1+3*cos(4+6*x)),x, algorithm="giac")

[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x + 2)))

maple [A] time = 0.06, size = 18, normalized size = 0.30

$$\frac{\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{2} \tan(2+3x)}{2} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-2/(1+3*cos(4+6*x)),x)

[Out] -1/6*2^(1/2)*arctanh(1/2*2^(1/2)*tan(2+3*x))

maxima [A] time = 0.43, size = 53, normalized size = 0.87

$$\frac{1}{12} \sqrt{2} \log \left(-\frac{\sqrt{2} - \frac{\sin(6x+4)}{\cos(6x+4)+1}}{\sqrt{2} + \frac{\sin(6x+4)}{\cos(6x+4)+1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-2/(1+3*cos(4+6*x)),x, algorithm="maxima")

[Out] 1/12*sqrt(2)*log(-(sqrt(2) - sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) + sin(6*x + 4)/(cos(6*x + 4) + 1)))

mupad [B] time = 2.50, size = 17, normalized size = 0.28

$$\frac{\sqrt{2} \operatorname{atanh} \left(\frac{\sqrt{2} \tan(3x+2)}{2} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-2/(3*cos(6*x + 4) + 1),x)`

[Out] $-(2^{1/2} \operatorname{atanh}((2^{1/2} \tan(3x + 2))/2))/6$

sympy [A] time = 0.31, size = 39, normalized size = 0.64

$$\frac{\sqrt{2} \log(\tan(3x + 2) - \sqrt{2})}{12} - \frac{\sqrt{2} \log(\tan(3x + 2) + \sqrt{2})}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-2/(1+3*cos(4+6*x)),x)`

[Out] $\sqrt{2} \log(\tan(3x + 2) - \sqrt{2})/12 - \sqrt{2} \log(\tan(3x + 2) + \sqrt{2})/12$

$$3.23 \quad \int -\frac{2 \csc(4+6x)}{3 \cot(4+6x)+\csc(4+6x)} dx$$

Optimal. Leaf size=61

$$\frac{\log(\sqrt{2} \cos(3x+2) - \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sin(3x+2) + \sqrt{2} \cos(3x+2))}{6\sqrt{2}}$$

[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {12, 3166, 2659, 206}

$$\frac{\log(\sqrt{2} \cos(3x+2) - \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sin(3x+2) + \sqrt{2} \cos(3x+2))}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(-2*Csc[4 + 6*x])/(3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]

[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 + 3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3166

Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))^m_, x_Symbol] := Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int -\frac{2 \csc(4+6x)}{3 \cot(4+6x) + \csc(4+6x)} dx &= -\left(2 \int \frac{\csc(4+6x)}{3 \cot(4+6x) + \csc(4+6x)} dx\right) \\
&= -\left(2 \int \frac{1}{1+3 \cos(4+6x)} dx\right) \\
&= -\left(\frac{2}{3} \text{Subst}\left(\int \frac{1}{4-2x^2} dx, x, \tan\left(\frac{1}{2}(4+6x)\right)\right)\right) \\
&= \frac{\log(\sqrt{2} \cos(2+3x) - \sin(2+3x))}{6\sqrt{2}} - \frac{\log(\sqrt{2} \cos(2+3x) + \sin(2+3x))}{6\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.36

$$\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(-2*Csc[4 + 6*x])/(3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]

[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]

fricas [A] time = 0.55, size = 74, normalized size = 1.21

$$\frac{1}{24} \sqrt{2} \log\left(-\frac{7 \cos(6x+4)^2 + 4(\sqrt{2} \cos(6x+4) + 3\sqrt{2}) \sin(6x+4) - 6 \cos(6x+4) - 17}{9 \cos(6x+4)^2 + 6 \cos(6x+4) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x, algorithm="fricas")

[Out] 1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 + 4*(sqrt(2)*cos(6*x + 4) + 3*sqrt(2))*sin(6*x + 4) - 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 + 6*cos(6*x + 4) + 1))

giac [A] time = 0.24, size = 39, normalized size = 0.64

$$\frac{1}{12} \sqrt{2} \log\left(\frac{|-2\sqrt{2} + 2 \tan(3x+2)|}{|2\sqrt{2} + 2 \tan(3x+2)|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x, algorithm="giac")

[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x + 2)))

maple [A] time = 0.31, size = 18, normalized size = 0.30

$$-\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} \tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x)

[Out] -1/6*2^(1/2)*arctanh(1/2*2^(1/2)*tan(2+3*x))

maxima [A] time = 0.53, size = 53, normalized size = 0.87

$$\frac{1}{12} \sqrt{2} \log \left(-\frac{\sqrt{2} - \frac{\sin(6x+4)}{\cos(6x+4)+1}}{\sqrt{2} + \frac{\sin(6x+4)}{\cos(6x+4)+1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x, algorithm="maxima")

[Out] 1/12*sqrt(2)*log(-(sqrt(2) - sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) + sin(6*x + 4)/(cos(6*x + 4) + 1)))

mupad [B] time = 2.71, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2} \tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-2/(sin(6*x + 4)*(3*cot(6*x + 4) + 1/sin(6*x + 4))),x)

[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-2 \int \frac{\csc(6x+4)}{3 \cot(6x+4) + \csc(6x+4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x)

[Out] -2*Integral(csc(6*x + 4)/(3*cot(6*x + 4) + csc(6*x + 4)), x)

$$3.24 \quad \int \frac{1}{-2+3 \sin^2(2+3x)} dx$$

Optimal. Leaf size=61

$$\frac{\log(\sqrt{2} \cos(3x+2) - \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sin(3x+2) + \sqrt{2} \cos(3x+2))}{6\sqrt{2}}$$

[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3181, 207}

$$\frac{\log(\sqrt{2} \cos(3x+2) - \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sin(3x+2) + \sqrt{2} \cos(3x+2))}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(-2 + 3*Sin[2 + 3*x]^2)^(-1), x]

[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 + 3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 3181

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)^2])^(-1), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{-2+3 \sin^2(2+3x)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{-2+x^2} dx, x, \tan(2+3x) \right) \\ &= \frac{\log(\sqrt{2} \cos(2+3x) - \sin(2+3x))}{6\sqrt{2}} - \frac{\log(\sqrt{2} \cos(2+3x) + \sin(2+3x))}{6\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + 3*Sin[2 + 3*x]^2)^(-1), x]

[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]

fricas [A] time = 0.59, size = 85, normalized size = 1.39

$$\frac{1}{24} \sqrt{2} \log \left(-\frac{7 \cos(3x+2)^4 - 10 \cos(3x+2)^2 + 4(\sqrt{2} \cos(3x+2)^3 + \sqrt{2} \cos(3x+2)) \sin(3x+2) - 1}{9 \cos(3x+2)^4 - 6 \cos(3x+2)^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2+3*sin(2+3*x)^2),x, algorithm="fricas")

[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*cos(3*x + 2)^2 + 1))

giac [A] time = 0.19, size = 39, normalized size = 0.64

$$\frac{1}{12} \sqrt{2} \log \left(\frac{|-2\sqrt{2} + 2 \tan(3x+2)|}{|2\sqrt{2} + 2 \tan(3x+2)|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2+3*sin(2+3*x)^2),x, algorithm="giac")

[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x + 2)))

maple [A] time = 0.15, size = 18, normalized size = 0.30

$$\frac{\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{2} \tan(2+3x)}{2} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-2+3*sin(2+3*x)^2),x)

[Out] -1/6*2^(1/2)*arctanh(1/2*2^(1/2)*tan(2+3*x))

maxima [A] time = 0.50, size = 32, normalized size = 0.52

$$\frac{1}{12} \sqrt{2} \log \left(-\frac{\sqrt{2} - \tan(3x+2)}{\sqrt{2} + \tan(3x+2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2+3*sin(2+3*x)^2),x, algorithm="maxima")

[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))

mupad [B] time = 2.51, size = 17, normalized size = 0.28

$$\frac{\sqrt{2} \operatorname{atanh} \left(\frac{\sqrt{2} \tan(3x+2)}{2} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*sin(3*x + 2)^2 - 2),x)

[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6

sympy [B] time = 14.07, size = 1481, normalized size = 24.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2+3*sin(2+3*x)**2),x)

[Out]
$$\begin{aligned} & -4988289\sqrt{3}\sqrt{\sqrt{3} + 2}\log(\tan(3x/2 + 1) - \sqrt{2 - \sqrt{3}}) / \\ & (-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} - 2003742\sqrt{3} + 3470583 \\ & + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) - 136929\sqrt{2 - \sqrt{3}} \\ & \sqrt{3}\log(\tan(3x/2 + 1) - \sqrt{2 - \sqrt{3}}) / (-39175383\sqrt{2 - \sqrt{3}} \\ &)\sqrt{\sqrt{3} + 2} - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}} \\ & \sqrt{\sqrt{3} + 2}) + 79056\sqrt{3}\sqrt{2 - \sqrt{3}}\log(\tan(3x/ \\ & 2 + 1) - \sqrt{2 - \sqrt{3}}) / (-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} \\ & - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) \\ & + 8639970\sqrt{\sqrt{3} + 2}\log(\tan(3x/2 + 1) - \sqrt{2 - \sqrt{3}}) \\ &) / (-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} - 2003742\sqrt{3} + 3470583 \\ & + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) - 11281635\sqrt{\sqrt{3} + 2} \\ & \log(\tan(3x/2 + 1) + \sqrt{2 - \sqrt{3}}) / (-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} \\ & - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) \\ & - 487723\sqrt{3}\sqrt{2 - \sqrt{3}}\log(\tan(3x/2 + 1) + \sqrt{2 - \sqrt{3}}) / (-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} \\ & - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) \\ & + 844761\sqrt{2 - \sqrt{3}}\log(\tan(3x/2 + 1) + \sqrt{2 - \sqrt{3}}) / (-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} \\ & - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) \\ & + 6513455\sqrt{\sqrt{3} + 2}\log(\tan(3x/2 + 1) + \sqrt{2 - \sqrt{3}}) / (-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} \\ & - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) \\ & - 1820207\sqrt{3}\sqrt{\sqrt{3} + 2}\log(\tan(3x/2 + 1) - \sqrt{\sqrt{3} + 2}) / (-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} \\ & - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) \\ & - 3022905\sqrt{2 - \sqrt{3}}\log(\tan(3x/2 + 1) - \sqrt{\sqrt{3} + 2}) / (-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} \\ & - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) \\ & + 1745275\sqrt{3}\sqrt{2 - \sqrt{3}}\log(\tan(3x/2 + 1) - \sqrt{\sqrt{3} + 2}) / (-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} \\ & - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) \\ & + 3152691\sqrt{\sqrt{3} + 2}\log(\tan(3x/2 + 1) - \sqrt{\sqrt{3} + 2}) / (-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} \\ & - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) \\ & - 1336608\sqrt{3}\sqrt{2 - \sqrt{3}}\log(\tan(3x/2 + 1) + \sqrt{\sqrt{3} + 2}) / (-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} \\ & - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) \\ & - 511026\sqrt{\sqrt{3} + 2}\log(\tan(3x/2 + 1) + \sqrt{\sqrt{3} + 2}) / (-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} \\ & - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) \\ & + 295041\sqrt{3}\sqrt{\sqrt{3} + 2}\log(\tan(3x/2 + 1) + \sqrt{\sqrt{3} + 2}) / (-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} \\ & - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) \\ & + 2315073\sqrt{2 - \sqrt{3}}\log(\tan(3x/2 + 1) + \sqrt{\sqrt{3} + 2}) / (-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} \\ & - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) \\ & \sqrt{\sqrt{3} + 2}) \end{aligned}$$

$$3.25 \quad \int \frac{1}{1-3\cos^2(2+3x)} dx$$

Optimal. Leaf size=61

$$\frac{\log(\sqrt{2}\cos(3x+2) - \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sin(3x+2) + \sqrt{2}\cos(3x+2))}{6\sqrt{2}}$$

[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3181, 206}

$$\frac{\log(\sqrt{2}\cos(3x+2) - \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sin(3x+2) + \sqrt{2}\cos(3x+2))}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - 3*Cos[2 + 3*x]^2)^(-1), x]

[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 + 3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3181

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{1-3\cos^2(2+3x)} dx &= -\left(\frac{1}{3} \text{Subst}\left(\int \frac{1}{1-2x^2} dx, x, \cot(2+3x)\right)\right) \\ &= \frac{\log(\sqrt{2}\cos(2+3x) - \sin(2+3x))}{6\sqrt{2}} - \frac{\log(\sqrt{2}\cos(2+3x) + \sin(2+3x))}{6\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 3*Cos[2 + 3*x]^2)^(-1), x]

[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]

fricas [A] time = 0.59, size = 85, normalized size = 1.39

$$\frac{1}{24} \sqrt{2} \log \left(-\frac{7 \cos(3x+2)^4 - 10 \cos(3x+2)^2 + 4(\sqrt{2} \cos(3x+2)^3 + \sqrt{2} \cos(3x+2)) \sin(3x+2) - 1}{9 \cos(3x+2)^4 - 6 \cos(3x+2)^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-3*cos(2+3*x)^2), x, algorithm="fricas")

[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*cos(3*x + 2)^2 + 1))

giac [A] time = 0.17, size = 39, normalized size = 0.64

$$\frac{1}{12} \sqrt{2} \log \left(\frac{|-2\sqrt{2} + 2 \tan(3x+2)|}{|2\sqrt{2} + 2 \tan(3x+2)|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-3*cos(2+3*x)^2), x, algorithm="giac")

[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x + 2)))

maple [A] time = 0.10, size = 18, normalized size = 0.30

$$-\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} \tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-3*cos(2+3*x)^2), x)

[Out] -1/6*2^(1/2)*arctanh(1/2*2^(1/2)*tan(2+3*x))

maxima [A] time = 0.52, size = 32, normalized size = 0.52

$$\frac{1}{12} \sqrt{2} \log \left(-\frac{\sqrt{2} - \tan(3x+2)}{\sqrt{2} + \tan(3x+2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-3*cos(2+3*x)^2), x, algorithm="maxima")

[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))

mupad [B] time = 2.44, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2} \tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(3*cos(3*x + 2)^2 - 1), x)

[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6

sympy [B] time = 12.83, size = 1481, normalized size = 24.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-3*cos(2+3*x)**2),x)

[Out]
$$\begin{aligned} & -4988289\sqrt{3}\sqrt{\sqrt{3} + 2}\log(\tan(3x/2 + 1) - \sqrt{2 - \sqrt{3}})/ \\ & (-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} - 2003742\sqrt{3} + 3470583 \\ & + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) - 136929\sqrt{2 - \sqrt{3}} \\ & \sqrt{3}\log(\tan(3x/2 + 1) - \sqrt{2 - \sqrt{3}})/(-39175383\sqrt{2 - \sqrt{3}} \\ &)\sqrt{\sqrt{3} + 2} - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}} \\ & \sqrt{\sqrt{3} + 2}) + 79056\sqrt{3}\sqrt{2 - \sqrt{3}}\log(\tan(3x/ \\ & 2 + 1) - \sqrt{2 - \sqrt{3}})/(-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} \\ & - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} \\ & (3) + 2}) + 8639970\sqrt{\sqrt{3} + 2}\log(\tan(3x/2 + 1) - \sqrt{2 - \sqrt{3}}) \\ &)/(-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} - 2003742\sqrt{3} + 347058 \\ & 3 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) - 11281635\sqrt{\sqrt{3} \\ & (3) + 2}\log(\tan(3x/2 + 1) + \sqrt{2 - \sqrt{3}})/(-39175383\sqrt{2 - \sqrt{3}} \\ &)\sqrt{\sqrt{3} + 2} - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}} \\ & \sqrt{\sqrt{3} + 2}) - 487723\sqrt{3}\sqrt{2 - \sqrt{3}}\log(\tan \\ & (3x/2 + 1) + \sqrt{2 - \sqrt{3}})/(-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} \\ & (3) + 2} - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} \\ & (3) + 2}) + 844761\sqrt{2 - \sqrt{3}}\log(\tan(3x/2 + 1) + \sqrt{2 - \sqrt{3}}) \\ &)/(-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} - 2003742\sqrt{3} + 34 \\ & 70583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) + 6513455\sqrt{\sqrt{3} \\ & (3) + 2}\log(\tan(3x/2 + 1) + \sqrt{2 - \sqrt{3}})/(-39175383\sqrt{2 - \sqrt{3}} \\ &)\sqrt{\sqrt{3} + 2} - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}} \\ & \sqrt{\sqrt{3} + 2}) - 1820207\sqrt{3}\sqrt{\sqrt{3} \\ & (3) + 2}\log(\tan(3x/2 + 1) - \sqrt{\sqrt{3} + 2})/(-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} \\ & (3) + 2} - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}} \\ & \sqrt{\sqrt{3} + 2}) - 3022905\sqrt{2 - \sqrt{3}}\log(\tan(3x/2 + 1) - \\ & \sqrt{\sqrt{3} + 2})/(-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} - 2003742 \\ & \sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) \\ & + 1745275\sqrt{3}\sqrt{2 - \sqrt{3}}\log(\tan(3x/2 + 1) - \sqrt{\sqrt{3} + 2}) \\ &)/(-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} - 2003742\sqrt{3} + 3470583 \\ & + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) + 3152691\sqrt{\sqrt{3} \\ & (3) + 2}\log(\tan(3x/2 + 1) - \sqrt{\sqrt{3} + 2})/(-39175383\sqrt{2 - \sqrt{3}} \\ &)\sqrt{\sqrt{3} + 2} - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}} \\ & \sqrt{\sqrt{3} + 2}) - 1336608\sqrt{3}\sqrt{2 - \sqrt{3}}\log(\tan(\\ & 3x/2 + 1) + \sqrt{\sqrt{3} + 2})/(-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} \\ & - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} \\ & (3) + 2}) - 511026\sqrt{\sqrt{3} + 2}\log(\tan(3x/2 + 1) + \sqrt{\sqrt{3} + 2}) \\ &)/(-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2} - 2003742\sqrt{3} + 347 \\ & 0583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} + 2}) + 295041\sqrt{\sqrt{3} \\ & (3) + 2}\log(\tan(3x/2 + 1) + \sqrt{\sqrt{3} + 2})/(-39175383\sqrt{2 - \sqrt{3}} \\ &)\sqrt{\sqrt{3} + 2} - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}} \\ & \sqrt{\sqrt{3} + 2}) + 2315073\sqrt{2 - \sqrt{3}}\log(\tan(\\ & 3x/2 + 1) + \sqrt{\sqrt{3} + 2})/(-39175383\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} \\ & (3) + 2} - 2003742\sqrt{3} + 3470583 + 22617918\sqrt{3}\sqrt{2 - \sqrt{3}}\sqrt{\sqrt{3} \\ & (3) + 2}) \end{aligned}$$

$$3.26 \quad \int \frac{1}{-2 \cos^2(2+3x) + \sin^2(2+3x)} dx$$

Optimal. Leaf size=61

$$\frac{\log(\sqrt{2} \cos(3x+2) - \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sin(3x+2) + \sqrt{2} \cos(3x+2))}{6\sqrt{2}}$$

[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {207}

$$\frac{\log(\sqrt{2} \cos(3x+2) - \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sin(3x+2) + \sqrt{2} \cos(3x+2))}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(-2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1), x]

[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 + 3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{-2 \cos^2(2+3x) + \sin^2(2+3x)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{-2 + x^2} dx, x, \tan(2+3x) \right) \\ &= \frac{\log(\sqrt{2} \cos(2+3x) - \sin(2+3x))}{6\sqrt{2}} - \frac{\log(\sqrt{2} \cos(2+3x) + \sin(2+3x))}{6\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.36

$$\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(-2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1), x]

[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]

fricas [A] time = 0.57, size = 85, normalized size = 1.39

$$\frac{1}{24} \sqrt{2} \log \left(-\frac{7 \cos(3x+2)^4 - 10 \cos(3x+2)^2 + 4(\sqrt{2} \cos(3x+2)^3 + \sqrt{2} \cos(3x+2)) \sin(3x+2) - 1}{9 \cos(3x+2)^4 - 6 \cos(3x+2)^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="fricas")

[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*cos(3*x + 2)^2 + 1))

giac [A] time = 0.21, size = 39, normalized size = 0.64

$$\frac{1}{12} \sqrt{2} \log \left(\frac{|-2\sqrt{2} + 2 \tan(3x + 2)|}{|2\sqrt{2} + 2 \tan(3x + 2)|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="giac")

[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x + 2)))

maple [A] time = 0.23, size = 18, normalized size = 0.30

$$-\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} \tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x)

[Out] -1/6*2^(1/2)*arctanh(1/2*2^(1/2)*tan(2+3*x))

maxima [A] time = 0.54, size = 32, normalized size = 0.52

$$\frac{1}{12} \sqrt{2} \log \left(-\frac{\sqrt{2} - \tan(3x + 2)}{\sqrt{2} + \tan(3x + 2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="maxima")

[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))

mupad [B] time = 2.42, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2} \tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(3*x + 2)^2 - 2*cos(3*x + 2)^2),x)

[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6

sympy [B] time = 16.48, size = 1481, normalized size = 24.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2*cos(2+3*x)**2+sin(2+3*x)**2),x)

[Out] -4988289*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-39175383*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 2003742*sqrt(3) + 3470583 + 22617918*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 136929*sqrt(2 - s

$$\begin{aligned}
& \text{qrt}(3)) * \log(\tan(3*x/2 + 1) - \text{sqrt}(2 - \text{sqrt}(3))) / (-39175383 * \text{sqrt}(2 - \text{sqrt}(3)) \\
&) * \text{sqrt}(\text{sqrt}(3) + 2) - 2003742 * \text{sqrt}(3) + 3470583 + 22617918 * \text{sqrt}(3) * \text{sqrt}(2 - \\
& \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2)) + 79056 * \text{sqrt}(3) * \text{sqrt}(2 - \text{sqrt}(3)) * \log(\tan(3*x/ \\
& 2 + 1) - \text{sqrt}(2 - \text{sqrt}(3))) / (-39175383 * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2) \\
& - 2003742 * \text{sqrt}(3) + 3470583 + 22617918 * \text{sqrt}(3) * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(\\
& 3) + 2)) + 8639970 * \text{sqrt}(\text{sqrt}(3) + 2) * \log(\tan(3*x/2 + 1) - \text{sqrt}(2 - \text{sqrt}(3)) \\
&)) / (-39175383 * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2) - 2003742 * \text{sqrt}(3) + 347058 \\
& 3 + 22617918 * \text{sqrt}(3) * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2)) - 11281635 * \text{sqrt}(s \\
& \text{qrt}(3) + 2) * \log(\tan(3*x/2 + 1) + \text{sqrt}(2 - \text{sqrt}(3))) / (-39175383 * \text{sqrt}(2 - \text{sqr} \\
& \text{t}(3)) * \text{sqrt}(\text{sqrt}(3) + 2) - 2003742 * \text{sqrt}(3) + 3470583 + 22617918 * \text{sqrt}(3) * \text{sqrt} \\
& (2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2)) - 487723 * \text{sqrt}(3) * \text{sqrt}(2 - \text{sqrt}(3)) * \log(\tan \\
& (3*x/2 + 1) + \text{sqrt}(2 - \text{sqrt}(3))) / (-39175383 * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) \\
& + 2) - 2003742 * \text{sqrt}(3) + 3470583 + 22617918 * \text{sqrt}(3) * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt} \\
& (\text{sqrt}(3) + 2)) + 844761 * \text{sqrt}(2 - \text{sqrt}(3)) * \log(\tan(3*x/2 + 1) + \text{sqrt}(2 - \text{sqrt} \\
& (3))) / (-39175383 * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2) - 2003742 * \text{sqrt}(3) + 34 \\
& 70583 + 22617918 * \text{sqrt}(3) * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2)) + 6513455 * \text{sqr} \\
& \text{t}(3) * \text{sqrt}(\text{sqrt}(3) + 2) * \log(\tan(3*x/2 + 1) + \text{sqrt}(2 - \text{sqrt}(3))) / (-39175383 * \text{s} \\
& \text{qrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2) - 2003742 * \text{sqrt}(3) + 3470583 + 22617918 * \text{s} \\
& \text{qrt}(3) * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2)) - 1820207 * \text{sqrt}(3) * \text{sqrt}(\text{sqrt}(3) \\
& + 2) * \log(\tan(3*x/2 + 1) - \text{sqrt}(\text{sqrt}(3) + 2)) / (-39175383 * \text{sqrt}(2 - \text{sqrt}(3)) * \text{s} \\
& \text{qrt}(\text{sqrt}(3) + 2) - 2003742 * \text{sqrt}(3) + 3470583 + 22617918 * \text{sqrt}(3) * \text{sqrt}(2 - \text{sqr} \\
& \text{t}(3)) * \text{sqrt}(\text{sqrt}(3) + 2)) - 3022905 * \text{sqrt}(2 - \text{sqrt}(3)) * \log(\tan(3*x/2 + 1) - \\
& \text{sqrt}(\text{sqrt}(3) + 2)) / (-39175383 * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2) - 2003742 \\
& * \text{sqrt}(3) + 3470583 + 22617918 * \text{sqrt}(3) * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2)) \\
& + 1745275 * \text{sqrt}(3) * \text{sqrt}(2 - \text{sqrt}(3)) * \log(\tan(3*x/2 + 1) - \text{sqrt}(\text{sqrt}(3) + 2)) \\
& / (-39175383 * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2) - 2003742 * \text{sqrt}(3) + 3470583 \\
& + 22617918 * \text{sqrt}(3) * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2)) + 3152691 * \text{sqrt}(sqr \\
& \text{t}(3) + 2) * \log(\tan(3*x/2 + 1) - \text{sqrt}(\text{sqrt}(3) + 2)) / (-39175383 * \text{sqrt}(2 - \text{sqrt}(\\
& 3)) * \text{sqrt}(\text{sqrt}(3) + 2) - 2003742 * \text{sqrt}(3) + 3470583 + 22617918 * \text{sqrt}(3) * \text{sqrt}(2 \\
& - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2)) - 1336608 * \text{sqrt}(3) * \text{sqrt}(2 - \text{sqrt}(3)) * \log(\tan(\\
& 3*x/2 + 1) + \text{sqrt}(\text{sqrt}(3) + 2)) / (-39175383 * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + \\
& 2) - 2003742 * \text{sqrt}(3) + 3470583 + 22617918 * \text{sqrt}(3) * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(s \\
& \text{qrt}(3) + 2)) - 511026 * \text{sqrt}(\text{sqrt}(3) + 2) * \log(\tan(3*x/2 + 1) + \text{sqrt}(\text{sqrt}(3) + \\
& 2)) / (-39175383 * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2) - 2003742 * \text{sqrt}(3) + 347 \\
& 0583 + 22617918 * \text{sqrt}(3) * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2)) + 295041 * \text{sqrt}(\\
& 3) * \text{sqrt}(\text{sqrt}(3) + 2) * \log(\tan(3*x/2 + 1) + \text{sqrt}(\text{sqrt}(3) + 2)) / (-39175383 * \text{sqr} \\
& \text{t}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2) - 2003742 * \text{sqrt}(3) + 3470583 + 22617918 * \text{sqr} \\
& \text{t}(3) * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) + 2)) + 2315073 * \text{sqrt}(2 - \text{sqrt}(3)) * \log(t \\
& \text{an}(3*x/2 + 1) + \text{sqrt}(\text{sqrt}(3) + 2)) / (-39175383 * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqrt}(\text{sqrt}(3) \\
&) + 2) - 2003742 * \text{sqrt}(3) + 3470583 + 22617918 * \text{sqrt}(3) * \text{sqrt}(2 - \text{sqrt}(3)) * \text{sqr} \\
& \text{t}(\text{sqrt}(3) + 2))
\end{aligned}$$

$$3.27 \quad \int \frac{\sec^2(2+3x)}{-2+\tan^2(2+3x)} dx$$

Optimal. Leaf size=61

$$\frac{\log(\sqrt{2} \cos(3x+2) - \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sin(3x+2) + \sqrt{2} \cos(3x+2))}{6\sqrt{2}}$$

[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3675, 207}

$$\frac{\log(\sqrt{2} \cos(3x+2) - \sin(3x+2))}{6\sqrt{2}} - \frac{\log(\sin(3x+2) + \sqrt{2} \cos(3x+2))}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sec[2 + 3*x]^2/(-2 + Tan[2 + 3*x]^2), x]

[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 + 3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(-n_)]^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2-1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(2+3x)}{-2+\tan^2(2+3x)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{-2+x^2} dx, x, \tan(2+3x) \right) \\ &= \frac{\log(\sqrt{2} \cos(2+3x) - \sin(2+3x))}{6\sqrt{2}} - \frac{\log(\sqrt{2} \cos(2+3x) + \sin(2+3x))}{6\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2 + 3*x]^2/(-2 + Tan[2 + 3*x]^2), x]

[Out] $-1/3 \cdot \text{ArcTanh}[\text{Tan}[2 + 3x]/\text{Sqrt}[2]]/\text{Sqrt}[2]$

fricas [A] time = 0.54, size = 85, normalized size = 1.39

$$\frac{1}{24} \sqrt{2} \log \left(-\frac{7 \cos(3x+2)^4 - 10 \cos(3x+2)^2 + 4(\sqrt{2} \cos(3x+2)^3 + \sqrt{2} \cos(3x+2)) \sin(3x+2) - 1}{9 \cos(3x+2)^4 - 6 \cos(3x+2)^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x, algorithm="fricas")`

[Out] $1/24 \cdot \text{sqrt}(2) \cdot \log(-7 \cdot \cos(3x+2)^4 - 10 \cdot \cos(3x+2)^2 + 4 \cdot (\text{sqrt}(2) \cdot \cos(3x+2)^3 + \text{sqrt}(2) \cdot \cos(3x+2)) \cdot \sin(3x+2) - 1) / (9 \cdot \cos(3x+2)^4 - 6 \cdot \cos(3x+2)^2 + 1)$

giac [A] time = 1.29, size = 37, normalized size = 0.61

$$-\frac{1}{12} \sqrt{2} \log \left(\left| \sqrt{2} + \tan(3x+2) \right| \right) + \frac{1}{12} \sqrt{2} \log \left(\left| -\sqrt{2} + \tan(3x+2) \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x, algorithm="giac")`

[Out] $-1/12 \cdot \text{sqrt}(2) \cdot \log(\text{abs}(\text{sqrt}(2) + \tan(3x+2))) + 1/12 \cdot \text{sqrt}(2) \cdot \log(\text{abs}(-\text{sqrt}(2) + \tan(3x+2)))$

maple [A] time = 0.28, size = 18, normalized size = 0.30

$$-\frac{\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{2} \tan(2+3x)}{2} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x)`

[Out] $-1/6 \cdot 2^{(1/2)} \cdot \operatorname{arctanh}(1/2 \cdot 2^{(1/2)} \cdot \tan(2+3x))$

maxima [A] time = 1.34, size = 32, normalized size = 0.52

$$\frac{1}{12} \sqrt{2} \log \left(-\frac{\sqrt{2} - \tan(3x+2)}{\sqrt{2} + \tan(3x+2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x, algorithm="maxima")`

[Out] $1/12 \cdot \text{sqrt}(2) \cdot \log(-(\text{sqrt}(2) - \tan(3x+2))/(\text{sqrt}(2) + \tan(3x+2)))$

mupad [B] time = 2.40, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2} \operatorname{atanh} \left(\frac{\sqrt{2} \tan(3x+2)}{2} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cos(3*x+2)^2*(tan(3*x+2)^2-2)),x)`

[Out] $-(2^{(1/2)} \cdot \operatorname{atanh}((2^{(1/2)} \cdot \tan(3x+2))/2))/6$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(3x+2)}{\tan^2(3x+2)-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)**2/(-2+tan(2+3*x)**2), x)
```

```
[Out] Integral(sec(3*x + 2)**2/(tan(3*x + 2)**2 - 2), x)
```


$$3.28 \quad \int \frac{\csc^2(2+3x)}{1-2 \cot^2(2+3x)} dx$$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2} \cos(3x+2) - \sin(3x+2)\right)}{6\sqrt{2}} - \frac{\log\left(\sin(3x+2) + \sqrt{2} \cos(3x+2)\right)}{6\sqrt{2}}$$

[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3675, 206}

$$\frac{\log\left(\sqrt{2} \cos(3x+2) - \sin(3x+2)\right)}{6\sqrt{2}} - \frac{\log\left(\sin(3x+2) + \sqrt{2} \cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Csc[2 + 3*x]^2/(1 - 2*Cot[2 + 3*x]^2), x]

[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 + 3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\csc^2(2+3x)}{1-2 \cot^2(2+3x)} dx &= -\left(\frac{1}{3} \text{Subst}\left(\int \frac{1}{1-2x^2} dx, x, \cot(2+3x)\right)\right) \\ &= \frac{\log\left(\sqrt{2} \cos(2+3x) - \sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2} \cos(2+3x) + \sin(2+3x)\right)}{6\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[2 + 3*x]^2/(1 - 2*Cot[2 + 3*x]^2), x]

[Out] $-1/3 \cdot \text{ArcTanh}[\text{Tan}[2 + 3x]/\text{Sqrt}[2]]/\text{Sqrt}[2]$

fricas [A] time = 0.54, size = 85, normalized size = 1.39

$$\frac{1}{24} \sqrt{2} \log \left(-\frac{7 \cos(3x+2)^4 - 10 \cos(3x+2)^2 + 4(\sqrt{2} \cos(3x+2)^3 + \sqrt{2} \cos(3x+2)) \sin(3x+2) - 1}{9 \cos(3x+2)^4 - 6 \cos(3x+2)^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x, algorithm="fricas")`

[Out] $1/24 \cdot \text{sqrt}(2) \cdot \log(-7 \cdot \cos(3x+2)^4 - 10 \cdot \cos(3x+2)^2 + 4 \cdot (\text{sqrt}(2) \cdot \cos(3x+2)^3 + \text{sqrt}(2) \cdot \cos(3x+2)) \cdot \sin(3x+2) - 1) / (9 \cdot \cos(3x+2)^4 - 6 \cdot \cos(3x+2)^2 + 1)$

giac [A] time = 0.42, size = 39, normalized size = 0.64

$$\frac{1}{12} \sqrt{2} \log \left(\frac{|-2\sqrt{2} + 2 \tan(3x+2)|}{|2\sqrt{2} + 2 \tan(3x+2)|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x, algorithm="giac")`

[Out] $1/12 \cdot \text{sqrt}(2) \cdot \log(\text{abs}(-2 \cdot \text{sqrt}(2) + 2 \cdot \tan(3x+2)) / \text{abs}(2 \cdot \text{sqrt}(2) + 2 \cdot \tan(3x+2)))$

maple [A] time = 0.30, size = 18, normalized size = 0.30

$$\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} \tan(2+3x)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x)`

[Out] $-1/6 \cdot 2^{(1/2)} \cdot \operatorname{arctanh}(1/2 \cdot 2^{(1/2)} \cdot \tan(2+3x))$

maxima [A] time = 1.18, size = 32, normalized size = 0.52

$$\frac{1}{12} \sqrt{2} \log \left(-\frac{\sqrt{2} - \tan(3x+2)}{\sqrt{2} + \tan(3x+2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x, algorithm="maxima")`

[Out] $1/12 \cdot \text{sqrt}(2) \cdot \log(-(\text{sqrt}(2) - \tan(3x+2)) / (\text{sqrt}(2) + \tan(3x+2)))$

mupad [B] time = 2.33, size = 17, normalized size = 0.28

$$\frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2} \tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(sin(3*x+2)^2*(2*cot(3*x+2)^2-1)),x)`

[Out] $-(2^{(1/2)} \cdot \operatorname{atanh}((2^{(1/2)} \cdot \tan(3x+2))/2)) / 6$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\csc^2(3x+2)}{2\cot^2(3x+2)-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2+3*x)**2/(1-2*cot(2+3*x)**2), x)

[Out] -Integral(csc(3*x + 2)**2/(2*cot(3*x + 2)**2 - 1), x)

3.29 $\int (x + \sin(x))^2 dx$

Optimal. Leaf size=30

$$\frac{x^3}{3} + \frac{x}{2} + 2 \sin(x) - 2x \cos(x) - \frac{1}{2} \sin(x) \cos(x)$$

[Out] 1/2*x+1/3*x^3-2*x*cos(x)+2*sin(x)-1/2*cos(x)*sin(x)

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6742, 3296, 2637, 2635, 8}

$$\frac{x^3}{3} + \frac{x}{2} + 2 \sin(x) - 2x \cos(x) - \frac{1}{2} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[(x + Sin[x])^2,x]

[Out] x/2 + x^3/3 - 2*x*cos[x] + 2*sin[x] - (Cos[x]*Sin[x])/2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*cos[c + d*x])*(b*sin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int (x + \sin(x))^2 dx &= \int (x^2 + 2x \sin(x) + \sin^2(x)) dx \\ &= \frac{x^3}{3} + 2 \int x \sin(x) dx + \int \sin^2(x) dx \\ &= \frac{x^3}{3} - 2x \cos(x) - \frac{1}{2} \cos(x) \sin(x) + \frac{\int 1 dx}{2} + 2 \int \cos(x) dx \\ &= \frac{x}{2} + \frac{x^3}{3} - 2x \cos(x) + 2 \sin(x) - \frac{1}{2} \cos(x) \sin(x) \end{aligned}$$

Mathematica [A] time = 0.06, size = 30, normalized size = 1.00

$$\frac{1}{6}x(2x^2 + 3) + 2 \sin(x) - \frac{1}{4} \sin(2x) - 2x \cos(x)$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sin[x])^2,x]

[Out] (x*(3 + 2*x^2))/6 - 2*x*Cos[x] + 2*Sin[x] - Sin[2*x]/4

fricas [A] time = 0.51, size = 22, normalized size = 0.73

$$\frac{1}{3}x^3 - 2x \cos(x) - \frac{1}{2}(\cos(x) - 4) \sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+sin(x))^2,x, algorithm="fricas")

[Out] 1/3*x^3 - 2*x*cos(x) - 1/2*(cos(x) - 4)*sin(x) + 1/2*x

giac [A] time = 0.14, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3 - 2x \cos(x) + \frac{1}{2}x - \frac{1}{4} \sin(2x) + 2 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+sin(x))^2,x, algorithm="giac")

[Out] 1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\frac{x}{2} + \frac{x^3}{3} - 2x \cos(x) + 2 \sin(x) - \frac{\cos(x) \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+sin(x))^2,x)

[Out] 1/2*x+1/3*x^3-2*x*cos(x)+2*sin(x)-1/2*cos(x)*sin(x)

maxima [A] time = 0.33, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3 - 2x \cos(x) + \frac{1}{2}x - \frac{1}{4} \sin(2x) + 2 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+sin(x))^2,x, algorithm="maxima")

[Out] 1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)

mupad [B] time = 0.06, size = 24, normalized size = 0.80

$$\frac{x}{2} + 2 \sin(x) - \frac{\cos(x) \sin(x)}{2} - 2x \cos(x) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + sin(x))^2,x)

[Out] x/2 + 2*sin(x) - (cos(x)*sin(x))/2 - 2*x*cos(x) + x^3/3

sympy [A] time = 0.16, size = 41, normalized size = 1.37

$$\frac{x^3}{3} + \frac{x \sin^2(x)}{2} + \frac{x \cos^2(x)}{2} - 2x \cos(x) - \frac{\sin(x) \cos(x)}{2} + 2 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+sin(x))**2,x)

[Out] x**3/3 + x*sin(x)**2/2 + x*cos(x)**2/2 - 2*x*cos(x) - sin(x)*cos(x)/2 + 2*sin(x)

3.30 $\int (x + \sin(x))^3 dx$

Optimal. Leaf size=56

$$\frac{x^4}{4} + \frac{3x^2}{4} - 3x^2 \cos(x) + \frac{3 \sin^2(x)}{4} + 6x \sin(x) + \frac{\cos^3(x)}{3} + 5 \cos(x) - \frac{3}{2}x \sin(x) \cos(x)$$

[Out] $3/4*x^2+1/4*x^4+5*\cos(x)-3*x^2*\cos(x)+1/3*\cos(x)^3+6*x*\sin(x)-3/2*x*\cos(x)*\sin(x)+3/4*\sin(x)^2$

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6742, 3296, 2638, 3310, 30, 2633}

$$\frac{x^4}{4} + \frac{3x^2}{4} - 3x^2 \cos(x) + \frac{3 \sin^2(x)}{4} + 6x \sin(x) + \frac{\cos^3(x)}{3} + 5 \cos(x) - \frac{3}{2}x \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[(x + Sin[x])^3,x]

[Out] $(3*x^2)/4 + x^4/4 + 5*\cos[x] - 3*x^2*\cos[x] + \cos[x]^3/3 + 6*x*\sin[x] - (3*x*\cos[x]*\sin[x])/2 + (3*\sin[x]^2)/4$

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2633

Int[sin[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2638

Int[sin[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + (f_)*(x_)], x_Symbol] := -Simp[((c + d*x)^m * Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1) * Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3310

Int[((c_) + (d_)*(x_))*((b_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(d*(b*Ssin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Ssin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Ssin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int (x + \sin(x))^3 dx &= \int (x^3 + 3x^2 \sin(x) + 3x \sin^2(x) + \sin^3(x)) dx \\
&= \frac{x^4}{4} + 3 \int x^2 \sin(x) dx + 3 \int x \sin^2(x) dx + \int \sin^3(x) dx \\
&= \frac{x^4}{4} - 3x^2 \cos(x) - \frac{3}{2} x \cos(x) \sin(x) + \frac{3 \sin^2(x)}{4} + \frac{3 \int x dx}{2} + 6 \int x \cos(x) dx - \text{Subst} \left(\int (1 - \sin^2(x)) \sin(x) dx \right) \\
&= \frac{3x^2}{4} + \frac{x^4}{4} - \cos(x) - 3x^2 \cos(x) + \frac{\cos^3(x)}{3} + 6x \sin(x) - \frac{3}{2} x \cos(x) \sin(x) + \frac{3 \sin^2(x)}{4} - 6 \int \sin^3(x) dx \\
&= \frac{3x^2}{4} + \frac{x^4}{4} + 5 \cos(x) - 3x^2 \cos(x) + \frac{\cos^3(x)}{3} + 6x \sin(x) - \frac{3}{2} x \cos(x) \sin(x) + \frac{3 \sin^2(x)}{4}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 48, normalized size = 0.86

$$\frac{1}{24} (6x(x^3 + 3x + 24 \sin(x) - 3 \sin(2x)) - 18(4x^2 - 7) \cos(x) - 9 \cos(2x) + 2 \cos(3x))$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sin[x])^3, x]

[Out] (-18*(-7 + 4*x^2)*Cos[x] - 9*Cos[2*x] + 2*Cos[3*x] + 6*x*(3*x + x^3 + 24*Sin[x] - 3*Sin[2*x]))/24

fricas [A] time = 0.49, size = 46, normalized size = 0.82

$$\frac{1}{4} x^4 + \frac{1}{3} \cos(x)^3 + \frac{3}{4} x^2 - (3x^2 - 5) \cos(x) - \frac{3}{4} \cos(x)^2 - \frac{3}{2} (x \cos(x) - 4x) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+sin(x))^3,x, algorithm="fricas")

[Out] 1/4*x^4 + 1/3*cos(x)^3 + 3/4*x^2 - (3*x^2 - 5)*cos(x) - 3/4*cos(x)^2 - 3/2*(x*cos(x) - 4*x)*sin(x)

giac [A] time = 0.14, size = 46, normalized size = 0.82

$$\frac{1}{4} x^4 + \frac{3}{4} x^2 - \frac{3}{4} (4x^2 - 7) \cos(x) - \frac{3}{4} x \sin(2x) + 6x \sin(x) + \frac{1}{12} \cos(3x) - \frac{3}{8} \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+sin(x))^3,x, algorithm="giac")

[Out] 1/4*x^4 + 3/4*x^2 - 3/4*(4*x^2 - 7)*cos(x) - 3/4*x*sin(2*x) + 6*x*sin(x) + 1/12*cos(3*x) - 3/8*cos(2*x)

maple [A] time = 0.04, size = 57, normalized size = 1.02

$$-\frac{(2 + \sin^2(x)) \cos(x)}{3} + 3x \left(-\frac{\cos(x) \sin(x)}{2} + \frac{x}{2} \right) - \frac{3x^2}{4} + \frac{3(\sin^2(x))}{4} - 3x^2 \cos(x) + 6 \cos(x) + 6x \sin(x) + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+sin(x))^3,x)

[Out] -1/3*(2+sin(x)^2)*cos(x)+3*x*(-1/2*cos(x)*sin(x)+1/2*x)-3/4*x^2+3/4*sin(x)^2-3*x^2*cos(x)+6*cos(x)+6*x*sin(x)+1/4*x^4

maxima [A] time = 0.52, size = 48, normalized size = 0.86

$$\frac{1}{4}x^4 + \frac{1}{3}\cos(x)^3 + \frac{3}{4}x^2 - 3(x^2 - 2)\cos(x) - \frac{3}{4}x\sin(2x) + 6x\sin(x) - \frac{3}{8}\cos(2x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+sin(x))^3,x, algorithm="maxima")

[Out] 1/4*x^4 + 1/3*cos(x)^3 + 3/4*x^2 - 3*(x^2 - 2)*cos(x) - 3/4*x*sin(2*x) + 6*x*sin(x) - 3/8*cos(2*x) - cos(x)

mupad [B] time = 2.31, size = 46, normalized size = 0.82

$$5\cos(x) - 3x^2\cos(x) - \frac{3\cos(x)^2}{4} + \frac{\cos(x)^3}{3} + 6x\sin(x) + \frac{3x^2}{4} + \frac{x^4}{4} - \frac{3x\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + sin(x))^3,x)

[Out] 5*cos(x) - 3*x^2*cos(x) - (3*cos(x)^2)/4 + cos(x)^3/3 + 6*x*sin(x) + (3*x^2)/4 + x^4/4 - (3*x*cos(x)*sin(x))/2

sympy [A] time = 0.30, size = 85, normalized size = 1.52

$$\frac{x^4}{4} + \frac{3x^2\sin^2(x)}{4} + \frac{3x^2\cos^2(x)}{4} - 3x^2\cos(x) - \frac{3x\sin(x)\cos(x)}{2} + 6x\sin(x) - \sin^2(x)\cos(x) + \frac{3\sin^2(x)}{4} - \frac{2\cos^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+sin(x))**3,x)

[Out] x**4/4 + 3*x**2*sin(x)**2/4 + 3*x**2*cos(x)**2/4 - 3*x**2*cos(x) - 3*x*sin(x)*cos(x)/2 + 6*x*sin(x) - sin(x)**2*cos(x) + 3*sin(x)**2/4 - 2*cos(x)**3/3 + 6*cos(x)

3.31 $\int \frac{\sin(a+bx)}{c+dx^2} dx$

Optimal. Leaf size=213

$$\frac{\sin\left(a - \frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{Ci}\left(xb + \frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\sin\left(a + \frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}} - bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cos\left(a + \frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}} - bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cos\left(a - \frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}$$

[Out] $1/2*\cos(a+b*(-c)^{(1/2)}/d^{(1/2)})*Si(b*x-b*(-c)^{(1/2)}/d^{(1/2)})/(-c)^{(1/2)}/d^{(1/2)}-1/2*\cos(a-b*(-c)^{(1/2)}/d^{(1/2)})*Si(b*x+b*(-c)^{(1/2)}/d^{(1/2)})/(-c)^{(1/2)}/d^{(1/2)}-1/2*Ci(b*x+b*(-c)^{(1/2)}/d^{(1/2)})*\sin(a-b*(-c)^{(1/2)}/d^{(1/2)})/(-c)^{(1/2)}/d^{(1/2)}+1/2*Ci(-b*x+b*(-c)^{(1/2)}/d^{(1/2)})*\sin(a+b*(-c)^{(1/2)}/d^{(1/2)})/(-c)^{(1/2)}/d^{(1/2)}$

Rubi [A] time = 0.54, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3333, 3303, 3299, 3302}

$$\frac{\sin\left(a - \frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}} + bx\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\sin\left(a + \frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}} - bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cos\left(a + \frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}} - bx\right)}{2\sqrt{-c}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b*x]/(c + d*x^2), x]

[Out] $-(\text{CosIntegral}[(b*\text{Sqrt}[-c])/ \text{Sqrt}[d] + b*x]*\text{Sin}[a - (b*\text{Sqrt}[-c])/ \text{Sqrt}[d]])/(2*\text{Sqrt}[-c]*\text{Sqrt}[d]) + (\text{CosIntegral}[(b*\text{Sqrt}[-c])/ \text{Sqrt}[d] - b*x]*\text{Sin}[a + (b*\text{Sqrt}[-c])/ \text{Sqrt}[d]])/(2*\text{Sqrt}[-c]*\text{Sqrt}[d]) - (\text{Cos}[a + (b*\text{Sqrt}[-c])/ \text{Sqrt}[d]]*\text{SinIntegral}[(b*\text{Sqrt}[-c])/ \text{Sqrt}[d] - b*x])/(2*\text{Sqrt}[-c]*\text{Sqrt}[d]) - (\text{Cos}[a - (b*\text{Sqrt}[-c])/ \text{Sqrt}[d]]*\text{SinIntegral}[(b*\text{Sqrt}[-c])/ \text{Sqrt}[d] + b*x])/(2*\text{Sqrt}[-c]*\text{Sqrt}[d])$

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3333

Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)}{c+dx^2} dx &= \int \left(\frac{\sqrt{-c} \sin(a+bx)}{2c(\sqrt{-c}-\sqrt{d}x)} + \frac{\sqrt{-c} \sin(a+bx)}{2c(\sqrt{-c}+\sqrt{d}x)} \right) dx \\
&= -\frac{\int \frac{\sin(a+bx)}{\sqrt{-c}-\sqrt{d}x} dx}{2\sqrt{-c}} - \frac{\int \frac{\sin(a+bx)}{\sqrt{-c}+\sqrt{d}x} dx}{2\sqrt{-c}} \\
&= -\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right) \int \frac{\sin\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}x} dx}{2\sqrt{-c}} + \frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right) \int \frac{\sin\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{\sqrt{-c}-\sqrt{d}x} dx}{2\sqrt{-c}} - \frac{\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right) \int \frac{1}{\sqrt{-c}+\sqrt{d}x} dx}{2\sqrt{-c}} \\
&= -\frac{\text{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right) \sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\text{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right) \sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}
\end{aligned}$$

Mathematica [C] time = 0.32, size = 172, normalized size = 0.81

$$\frac{i\left(\sin\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Ci}\left(b\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)\right) - \sin\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Ci}\left(b\left(x-\frac{i\sqrt{c}}{\sqrt{d}}\right)\right) + \cos\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Si}\left(b\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)\right) + \cos\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Si}\left(b\left(x-\frac{i\sqrt{c}}{\sqrt{d}}\right)\right)}{2\sqrt{c}\sqrt{d}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sin[a + b*x]/(c + d*x^2), x]

[Out] ((I/2)*(CosIntegral[b*((I*Sqrt[c])/Sqrt[d] + x)]*Sin[a - (I*b*Sqrt[c])/Sqrt[d]] - CosIntegral[b*((-I)*Sqrt[c])/Sqrt[d] + x])*Sin[a + (I*b*Sqrt[c])/Sqrt[d]] + Cos[a - (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[b*((I*Sqrt[c])/Sqrt[d] + x)] + Cos[a + (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(I*b*Sqrt[c])/Sqrt[d] - b*x))/(Sqrt[c]*Sqrt[d])

fricas [C] time = 0.56, size = 187, normalized size = 0.88

$$\frac{\sqrt{\frac{b^2c}{d}} \text{Ei}\left(ibx - \sqrt{\frac{b^2c}{d}}\right) e^{\left(ia + \sqrt{\frac{b^2c}{d}}\right)} - \sqrt{\frac{b^2c}{d}} \text{Ei}\left(ibx + \sqrt{\frac{b^2c}{d}}\right) e^{\left(ia - \sqrt{\frac{b^2c}{d}}\right)} + \sqrt{\frac{b^2c}{d}} \text{Ei}\left(-ibx - \sqrt{\frac{b^2c}{d}}\right) e^{\left(-ia + \sqrt{\frac{b^2c}{d}}\right)} - \sqrt{\frac{b^2c}{d}} \text{Ei}\left(-ibx + \sqrt{\frac{b^2c}{d}}\right) e^{\left(-ia - \sqrt{\frac{b^2c}{d}}\right)}}{4bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)/(d*x^2+c), x, algorithm="fricas")

[Out] 1/4*(sqrt(b^2*c/d)*Ei(I*b*x - sqrt(b^2*c/d))*e^(I*a + sqrt(b^2*c/d)) - sqrt(b^2*c/d)*Ei(I*b*x + sqrt(b^2*c/d))*e^(I*a - sqrt(b^2*c/d)) + sqrt(b^2*c/d)*Ei(-I*b*x - sqrt(b^2*c/d))*e^(-I*a + sqrt(b^2*c/d)) - sqrt(b^2*c/d)*Ei(-I*b*x + sqrt(b^2*c/d))*e^(-I*a - sqrt(b^2*c/d)))/(b*c)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)}{dx^2+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)/(d*x^2+c), x, algorithm="giac")

[Out] integrate(sin(b*x + a)/(d*x^2 + c), x)

maple [A] time = 0.05, size = 229, normalized size = 1.08

$$b \left(\frac{\text{Si}\left(bx+a-\frac{b\sqrt{-cd}+da}{d}\right) \cos\left(\frac{b\sqrt{-cd}+da}{d}\right) + \text{Ci}\left(bx+a-\frac{b\sqrt{-cd}+da}{d}\right) \sin\left(\frac{b\sqrt{-cd}+da}{d}\right)}{2\left(\frac{b\sqrt{-cd}+da}{d}-a\right)d} + \frac{\text{Si}\left(bx+a+\frac{b\sqrt{-cd}-da}{d}\right) \cos\left(\frac{b\sqrt{-cd}-da}{d}\right) + \text{Ci}\left(bx+a+\frac{b\sqrt{-cd}-da}{d}\right) \sin\left(\frac{b\sqrt{-cd}-da}{d}\right)}{2\left(\frac{b\sqrt{-cd}-da}{d}+a\right)d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(b*x+a)/(d*x^2+c),x)`

[Out] $b \cdot \frac{1}{2} / ((b \cdot (-c \cdot d)^{(1/2)} + d \cdot a) / d - a) / d * (\text{Si}(b \cdot x + a - (b \cdot (-c \cdot d)^{(1/2)} + d \cdot a) / d) * \cos((b \cdot (-c \cdot d)^{(1/2)} + d \cdot a) / d) + \text{Ci}(b \cdot x + a - (b \cdot (-c \cdot d)^{(1/2)} + d \cdot a) / d) * \sin((b \cdot (-c \cdot d)^{(1/2)} + d \cdot a) / d)) + 1/2 / ((- (b \cdot (-c \cdot d)^{(1/2)} - d \cdot a) / d - a) / d * (\text{Si}(b \cdot x + a + (b \cdot (-c \cdot d)^{(1/2)} - d \cdot a) / d) * \cos((b \cdot (-c \cdot d)^{(1/2)} - d \cdot a) / d) - \text{Ci}(b \cdot x + a + (b \cdot (-c \cdot d)^{(1/2)} - d \cdot a) / d) * \sin((b \cdot (-c \cdot d)^{(1/2)} - d \cdot a) / d)))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx + a)}{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(b*x+a)/(d*x^2+c),x, algorithm="maxima")`

[Out] `integrate(sin(b*x + a)/(d*x^2 + c), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(a + bx)}{d x^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(a + b*x)/(c + d*x^2),x)`

[Out] `int(sin(a + b*x)/(c + d*x^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a + bx)}{c + dx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(b*x+a)/(d*x**2+c),x)`

[Out] `Integral(sin(a + b*x)/(c + d*x**2), x)`

$$3.32 \quad \int \frac{\sin(a+bx)}{c+dx+ex^2} dx$$

Optimal. Leaf size=271

$$\frac{\sin\left(a - \frac{b(d-\sqrt{d^2-4ce})}{2e}\right) \text{Ci}\left(\frac{b(d-\sqrt{d^2-4ce})}{2e} + bx\right)}{\sqrt{d^2-4ce}} - \frac{\sin\left(a - \frac{b(\sqrt{d^2-4ce}+d)}{2e}\right) \text{Ci}\left(\frac{b(d+\sqrt{d^2-4ce})}{2e} + bx\right)}{\sqrt{d^2-4ce}} + \frac{\cos\left(a - \frac{b(d-\sqrt{d^2-4ce})}{2e}\right)}{\sqrt{d^2-4ce}}$$

[Out] $\cos(a-1/2*b*(d-(-4*c*e+d^2)^{(1/2)})/e)*\text{Si}(b*x+1/2*b*(d-(-4*c*e+d^2)^{(1/2)})/e)/(-4*c*e+d^2)^{(1/2)} - \cos(a-1/2*b*(d+(-4*c*e+d^2)^{(1/2)})/e)*\text{Si}(b*x+1/2*b*(d+(-4*c*e+d^2)^{(1/2)})/e)/(-4*c*e+d^2)^{(1/2)} + \text{Ci}(b*x+1/2*b*(d-(-4*c*e+d^2)^{(1/2)})/e)*\sin(a-1/2*b*(d-(-4*c*e+d^2)^{(1/2)})/e)/(-4*c*e+d^2)^{(1/2)} - \text{Ci}(b*x+1/2*b*(d+(-4*c*e+d^2)^{(1/2)})/e)*\sin(a-1/2*b*(d+(-4*c*e+d^2)^{(1/2)})/e)/(-4*c*e+d^2)^{(1/2)}$

Rubi [A] time = 0.80, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, number of rules / integrand size = 0.210, Rules used = {6728, 3303, 3299, 3302}

$$\frac{\sin\left(a - \frac{b(d-\sqrt{d^2-4ce})}{2e}\right) \text{CosIntegral}\left(\frac{b(d-\sqrt{d^2-4ce})}{2e} + bx\right)}{\sqrt{d^2-4ce}} - \frac{\sin\left(a - \frac{b(\sqrt{d^2-4ce}+d)}{2e}\right) \text{CosIntegral}\left(\frac{b(\sqrt{d^2-4ce}+d)}{2e} + bx\right)}{\sqrt{d^2-4ce}}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b*x]/(c + d*x + e*x^2), x]

[Out] $(\text{CosIntegral}[(b*(d - \text{Sqrt}[d^2 - 4*c*e]))/(2*e) + b*x]*\text{Sin}[a - (b*(d - \text{Sqrt}[d^2 - 4*c*e]))/(2*e)])/(\text{Sqrt}[d^2 - 4*c*e]) - (\text{CosIntegral}[(b*(d + \text{Sqrt}[d^2 - 4*c*e]))/(2*e) + b*x]*\text{Sin}[a - (b*(d + \text{Sqrt}[d^2 - 4*c*e]))/(2*e)])/(\text{Sqrt}[d^2 - 4*c*e]) + (\text{Cos}[a - (b*(d - \text{Sqrt}[d^2 - 4*c*e]))/(2*e) + b*x])/(\text{Sqrt}[d^2 - 4*c*e]) - (\text{Cos}[a - (b*(d + \text{Sqrt}[d^2 - 4*c*e]))/(2*e) + b*x])/(\text{Sqrt}[d^2 - 4*c*e])$

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)}{c+dx+ex^2} dx &= \int \left(\frac{2e \sin(a+bx)}{\sqrt{d^2-4ce} (d-\sqrt{d^2-4ce}+2ex)} - \frac{2e \sin(a+bx)}{\sqrt{d^2-4ce} (d+\sqrt{d^2-4ce}+2ex)} \right) dx \\
&= \frac{(2e) \int \frac{\sin(a+bx)}{d-\sqrt{d^2-4ce}+2ex} dx}{\sqrt{d^2-4ce}} - \frac{(2e) \int \frac{\sin(a+bx)}{d+\sqrt{d^2-4ce}+2ex} dx}{\sqrt{d^2-4ce}} \\
&= \frac{\left(2e \cos \left(a - \frac{b(d-\sqrt{d^2-4ce})}{2e} \right) \right) \int \frac{\sin \left(\frac{b(d-\sqrt{d^2-4ce})}{2e} + bx \right)}{d-\sqrt{d^2-4ce}+2ex} dx}{\sqrt{d^2-4ce}} - \frac{\left(2e \cos \left(a - \frac{b(d+\sqrt{d^2-4ce})}{2e} \right) \right) \int \frac{\sin \left(\frac{b(d+\sqrt{d^2-4ce})}{2e} + bx \right)}{d+\sqrt{d^2-4ce}+2ex} dx}{\sqrt{d^2-4ce}} \\
&= \frac{\text{Ci} \left(\frac{b(d-\sqrt{d^2-4ce})}{2e} + bx \right) \sin \left(a - \frac{b(d-\sqrt{d^2-4ce})}{2e} \right)}{\sqrt{d^2-4ce}} - \frac{\text{Ci} \left(\frac{b(d+\sqrt{d^2-4ce})}{2e} + bx \right) \sin \left(a - \frac{b(d+\sqrt{d^2-4ce})}{2e} \right)}{\sqrt{d^2-4ce}}
\end{aligned}$$

Mathematica [A] time = 0.56, size = 238, normalized size = 0.88

$$\frac{\sin \left(a + \frac{b(\sqrt{d^2-4ce}-d)}{2e} \right) \text{Ci} \left(\frac{b(d+2ex-\sqrt{d^2-4ce})}{2e} \right) - \sin \left(a - \frac{b(\sqrt{d^2-4ce}+d)}{2e} \right) \text{Ci} \left(\frac{b(d+2ex+\sqrt{d^2-4ce})}{2e} \right) - \cos \left(a + \frac{b(\sqrt{d^2-4ce}-d)}{2e} \right)}{\sqrt{d^2-4ce}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sin[a + b*x]/(c + d*x + e*x^2), x]

[Out] (CosIntegral[(b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)]*Sin[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)] - CosIntegral[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)]*Sin[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)] - Cos[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e) - b*x] - Cos[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e]))/Sqrt[d^2 - 4*c*e]

fricas [C] time = 2.02, size = 434, normalized size = 1.60

$$e^{\sqrt{-\frac{b^2d^2-4b^2ce}{e^2}}} \text{Ei} \left(\frac{-2ibex - ibd - e\sqrt{-\frac{b^2d^2-4b^2ce}{e^2}}}{2e} \right) e^{\left(\frac{ibd - 2iae + e\sqrt{-\frac{b^2d^2-4b^2ce}{e^2}}}{2e} \right)} - e^{\sqrt{-\frac{b^2d^2-4b^2ce}{e^2}}} \text{Ei} \left(\frac{-2ibex - ibd + e\sqrt{-\frac{b^2d^2-4b^2ce}{e^2}}}{2e} \right) e^{\left(\frac{ibd + 2iae + e\sqrt{-\frac{b^2d^2-4b^2ce}{e^2}}}{2e} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)/(e*x^2+d*x+c), x, algorithm="fricas")

[Out] -1/2*(e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(-2*I*b*e*x - I*b*d - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(I*b*d - 2*I*a*e + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(-2*I*b*e*x - I*b*d + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(I*b*d - 2*I*a*e - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*I*b*e*x + I*b*d - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(-I*b*d + 2*I*a*e + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*I*b*e*x + I*b*d + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)

$b^2*c*e)/e^2))/e)*e^{(1/2*(-I*b*d + 2*I*a*e - e*\sqrt{-(b^2*d^2 - 4*b^2*c*e)/e^2}))/e))/(b*d^2 - 4*b*c*e)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx + a)}{ex^2 + dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)/(e*x^2+d*x+c), x, algorithm="giac")

[Out] integrate(sin(b*x + a)/(e*x^2 + d*x + c), x)

maple [A] time = 0.04, size = 320, normalized size = 1.18

$$b \left(\frac{\operatorname{Si}\left(bx + a - \frac{2ae - db + \sqrt{-4b^2ce + b^2d^2}}{2e}\right) \cos\left(\frac{2ae - db + \sqrt{-4b^2ce + b^2d^2}}{2e}\right) + \operatorname{Ci}\left(bx + a - \frac{2ae - db + \sqrt{-4b^2ce + b^2d^2}}{2e}\right) \sin\left(\frac{2ae - db + \sqrt{-4b^2ce + b^2d^2}}{2e}\right)}{\sqrt{-4b^2ce + b^2d^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(b*x+a)/(e*x^2+d*x+c), x)

[Out] $b*(1/(-4*b^2*c*e+b^2*d^2)^{(1/2)}*(\operatorname{Si}(b*x+a-1/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*d^2)^{(1/2)})))*\cos(1/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*d^2)^{(1/2)}))+\operatorname{Ci}(b*x+a-1/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*d^2)^{(1/2)}))*\sin(1/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*d^2)^{(1/2)})))-1/(-4*b^2*c*e+b^2*d^2)^{(1/2)}*(\operatorname{Si}(b*x+a+1/2*(-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^{(1/2)}))/e)*\cos(1/2*(-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^{(1/2)}))/e)-\operatorname{Ci}(b*x+a+1/2*(-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^{(1/2)}))/e)*\sin(1/2*(-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^{(1/2)}))/e))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx + a)}{ex^2 + dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)/(e*x^2+d*x+c), x, algorithm="maxima")

[Out] integrate(sin(b*x + a)/(e*x^2 + d*x + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(a + bx)}{ex^2 + dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a + b*x)/(c + d*x + e*x^2), x)

[Out] int(sin(a + b*x)/(c + d*x + e*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a + bx)}{c + dx + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)/(e*x**2+d*x+c), x)

[Out] Integral(sin(a + b*x)/(c + d*x + e*x**2), x)

$$3.33 \quad \int \frac{\sin(\sqrt{-7+x})}{\sqrt{-7+x}} dx$$

Optimal. Leaf size=10

$$-2 \cos(\sqrt{x-7})$$

[Out] -2*cos((-7+x)^(1/2))

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3431, 15, 2638}

$$-2 \cos(\sqrt{x-7})$$

Antiderivative was successfully verified.

[In] Int[Sin[Sqrt[-7 + x]]/Sqrt[-7 + x],x]

[Out] -2*Cos[Sqrt[-7 + x]]

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3431

Int[((g_.) + (h_.)*(x_)^(m_.))*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/(n*f), Subst[Int[ExpandIntegrand[(a + b*SIN[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p, 0] && IntegerQ[1/n]

Rubi steps

$$\begin{aligned} \int \frac{\sin(\sqrt{-7+x})}{\sqrt{-7+x}} dx &= 2 \operatorname{Subst} \left(\int \frac{x \sin(x)}{\sqrt{x^2}} dx, x, \sqrt{-7+x} \right) \\ &= 2 \operatorname{Subst} \left(\int \sin(x) dx, x, \sqrt{-7+x} \right) \\ &= -2 \cos(\sqrt{-7+x}) \end{aligned}$$

Mathematica [A] time = 0.02, size = 10, normalized size = 1.00

$$-2 \cos(\sqrt{x-7})$$

Antiderivative was successfully verified.

[In] Integrate[Sin[Sqrt[-7 + x]]/Sqrt[-7 + x],x]

[Out] -2*Cos[Sqrt[-7 + x]]

fricas [A] time = 0.59, size = 8, normalized size = 0.80

$$-2 \cos(\sqrt{x-7})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((-7+x)^(1/2))/(-7+x)^(1/2),x, algorithm="fricas")

[Out] -2*cos(sqrt(x - 7))

giac [A] time = 0.12, size = 8, normalized size = 0.80

$$-2 \cos(\sqrt{x-7})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((-7+x)^(1/2))/(-7+x)^(1/2),x, algorithm="giac")

[Out] -2*cos(sqrt(x - 7))

maple [A] time = 0.01, size = 9, normalized size = 0.90

$$-2 \cos(\sqrt{-7+x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin((-7+x)^(1/2))/(-7+x)^(1/2),x)

[Out] -2*cos((-7+x)^(1/2))

maxima [A] time = 0.31, size = 8, normalized size = 0.80

$$-2 \cos(\sqrt{x-7})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((-7+x)^(1/2))/(-7+x)^(1/2),x, algorithm="maxima")

[Out] -2*cos(sqrt(x - 7))

mupad [B] time = 2.42, size = 8, normalized size = 0.80

$$-2 \cos(\sqrt{x-7})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin((x - 7)^(1/2))/(x - 7)^(1/2),x)

[Out] -2*cos((x - 7)^(1/2))

sympy [A] time = 0.26, size = 10, normalized size = 1.00

$$-2 \cos(\sqrt{x-7})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((-7+x)**(1/2))/(-7+x)**(1/2),x)

[Out] -2*cos(sqrt(x - 7))

$$3.34 \quad \int \frac{\sqrt{b-\frac{a}{x^2}} \sin(x)}{\sqrt{a-bx^2}} dx$$

Optimal. Leaf size=28

$$\frac{x\text{Si}(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

[Out] x*Si(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)

Rubi [A] time = 0.47, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6721, 23, 3299}

$$\frac{x\text{Si}(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b - a/x^2]*Sin[x])/Sqrt[a - b*x^2],x]

[Out] (Sqrt[b - a/x^2]*x*SinIntegral[x])/Sqrt[a - b*x^2]

Rule 23

Int[(u_.)*((a_.) + (b_.)*(v_))^(m_)*((c_.) + (d_.)*(v_))^(n_), x_Symbol] :> Dist[(a + b*v)^m/(c + d*v)^n, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] || GtQ[b/d, 0])

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 6721

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(b^IntPart[p]*(a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), Int[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b-\frac{a}{x^2}} \sin(x)}{\sqrt{a-bx^2}} dx &= \frac{\left(\sqrt{b-\frac{a}{x^2}} x\right) \int \frac{\sqrt{1-\frac{bx^2}{a}} \sin(x)}{x\sqrt{a-bx^2}} dx}{\sqrt{1-\frac{bx^2}{a}}} \\ &= \frac{\left(\sqrt{b-\frac{a}{x^2}} x\right) \int \frac{\sin(x)}{x} dx}{\sqrt{a-bx^2}} \\ &= \frac{\sqrt{b-\frac{a}{x^2}} x\text{Si}(x)}{\sqrt{a-bx^2}} \end{aligned}$$

Mathematica [C] time = 0.70, size = 46, normalized size = 1.64

$$\frac{ix(\text{Ei}(-ix) - \text{Ei}(ix))\sqrt{b - \frac{a}{x^2}}}{2\sqrt{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b - a/x^2]*Sin[x])/Sqrt[a - b*x^2],x]

[Out] ((I/2)*Sqrt[b - a/x^2]*x*(ExpIntegralEi[(-I)*x] - ExpIntegralEi[I*x]))/Sqrt[a - b*x^2]

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-bx^2 + a}\sqrt{\frac{bx^2 - a}{x^2}}\sin(x)}{bx^2 - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-b*x^2 + a)*sqrt((b*x^2 - a)/x^2)*sin(x)/(b*x^2 - a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b - \frac{a}{x^2}} \sin(x)}{\sqrt{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b - a/x^2)*sin(x)/sqrt(-b*x^2 + a), x)

maple [C] time = 0.08, size = 72, normalized size = 2.57

$$-\frac{\sqrt{-\frac{bx^2+a}{x^2}}(bx^2-a)x\sqrt{\frac{-bx^2+a}{bx^2-a}}\left(-i\text{Si}(x) + \frac{i\pi\text{csgn}(x)}{2}\right)}{(-bx^2+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)

[Out] -(-(-b*x^2+a)/x^2)^(1/2)*(b*x^2-a)/(-b*x^2+a)^(3/2)*x*(1/(b*x^2-a)*(-b*x^2+a))^(1/2)*(-I*Si(x)+1/2*I*Pi*csgn(x))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b - \frac{a}{x^2}} \sin(x)}{\sqrt{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b - a/x^2)*sin(x)/sqrt(-b*x^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin(x) \sqrt{b - \frac{a}{x^2}}}{\sqrt{a - b x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x)*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2), x)

[Out] int((sin(x)*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x^2} + b} \sin(x)}{\sqrt{a - b x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2), x)

[Out] Integral(sqrt(-a/x**2 + b)*sin(x)/sqrt(a - b*x**2), x)

$$3.35 \quad \int \frac{1}{x(1+\sin(\log(x)))} dx$$

Optimal. Leaf size=12

$$-\frac{\cos(\log(x))}{\sin(\log(x)) + 1}$$

[Out] `-cos(ln(x))/(1+sin(ln(x)))`

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2648}

$$-\frac{\cos(\log(x))}{\sin(\log(x)) + 1}$$

Antiderivative was successfully verified.

[In] `Int[1/(x*(1 + Sin[Log[x]])),x]`

[Out] `-(Cos[Log[x]]/(1 + Sin[Log[x]]))`

Rule 2648

`Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

Rubi steps

$$\begin{aligned} \int \frac{1}{x(1 + \sin(\log(x)))} dx &= \text{Subst} \left(\int \frac{1}{1 + \sin(x)} dx, x, \log(x) \right) \\ &= -\frac{\cos(\log(x))}{1 + \sin(\log(x))} \end{aligned}$$

Mathematica [B] time = 0.02, size = 26, normalized size = 2.17

$$\frac{2 \sin\left(\frac{\log(x)}{2}\right)}{\sin\left(\frac{\log(x)}{2}\right) + \cos\left(\frac{\log(x)}{2}\right)}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x*(1 + Sin[Log[x]])),x]`

[Out] `(2*Sin[Log[x]/2])/(Cos[Log[x]/2] + Sin[Log[x]/2])`

fricas [A] time = 0.63, size = 22, normalized size = 1.83

$$-\frac{\cos(\log(x)) - \sin(\log(x)) + 1}{\cos(\log(x)) + \sin(\log(x)) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+sin(log(x))),x, algorithm="fricas")`

[Out] `-(cos(log(x)) - sin(log(x)) + 1)/(cos(log(x)) + sin(log(x)) + 1)`

giac [A] time = 0.14, size = 11, normalized size = 0.92

$$-\frac{2}{\tan\left(\frac{1}{2}\log(x)\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(1+sin(log(x))),x, algorithm="giac")

[Out] -2/(tan(1/2*log(x)) + 1)

maple [A] time = 0.08, size = 12, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{\ln(x)}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(1+sin(ln(x))),x)

[Out] -2/(tan(1/2*ln(x))+1)

maxima [A] time = 0.31, size = 17, normalized size = 1.42

$$-\frac{2}{\frac{\sin(\log(x))}{\cos(\log(x))+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(1+sin(log(x))),x, algorithm="maxima")

[Out] -2/(sin(log(x))/(cos(log(x)) + 1) + 1)

mupad [B] time = 2.40, size = 11, normalized size = 0.92

$$-\frac{2}{\tan\left(\frac{\ln(x)}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(sin(log(x)) + 1)),x)

[Out] -2/(tan(log(x)/2) + 1)

sympy [A] time = 1.36, size = 10, normalized size = 0.83

$$-\frac{2}{\tan\left(\frac{\log(x)}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(1+sin(ln(x))),x)

[Out] -2/(tan(log(x)/2) + 1)

3.36 $\int \sin\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=100

$$\frac{\cos\left(\frac{b}{d}\right)(bc-ad)\text{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{\sin\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d}$$

[Out] $(-a*d+b*c)*\text{Ci}((-a*d+b*c)/d/(d*x+c))*\cos(b/d)/d^2+(-a*d+b*c)*\text{Si}((-a*d+b*c)/d/(d*x+c))*\sin(b/d)/d^2+(d*x+c)*\sin((b*x+a)/(d*x+c))/d$

Rubi [A] time = 0.16, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4563, 3297, 3303, 3299, 3302}

$$\frac{\cos\left(\frac{b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{\sin\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sin[(a + b*x)/(c + d*x)], x]

[Out] $((b*c - a*d)*\text{Cos}[b/d]*\text{CosIntegral}[(b*c - a*d)/(d*(c + d*x))])/d^2 + ((c + d*x)*\text{Sin}[(a + b*x)/(c + d*x)])/d + ((b*c - a*d)*\text{Sin}[b/d]*\text{SinIntegral}[(b*c - a*d)/(d*(c + d*x))])/d^2$

Rule 3297

Int[((c_.) + (d_.)*(x_.))^(m_)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 4563

Int[Sin[((e_.)*((a_.) + (b_.)*(x_.)))/((c_.) + (d_.)*(x_.))]^(n_.), x_Symbol] :> -Dist[d^(-1), Subst[Int[Sin[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x], x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned}
\int \sin\left(\frac{a+bx}{c+dx}\right) dx &= \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{b}{d} - \frac{(bc-ad)x}{d}\right)}{x^2} dx, x, \frac{1}{c+dx}\right)}{d} \\
&= \frac{(c+dx) \sin\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad) \text{Subst}\left(\int \frac{\cos\left(\frac{b}{d} - \frac{(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2} \\
&= \frac{(c+dx) \sin\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left((bc-ad) \cos\left(\frac{b}{d}\right)\right) \text{Subst}\left(\int \frac{\cos\left(\frac{(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{(bc-ad) \sin\left(\frac{b}{d}\right)}{d} \\
&= \frac{(bc-ad) \cos\left(\frac{b}{d}\right) \text{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{(c+dx) \sin\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad) \sin\left(\frac{b}{d}\right) \text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}
\end{aligned}$$

Mathematica [C] time = 5.35, size = 272, normalized size = 2.72

$$\frac{2 \cos\left(\frac{b}{d}\right) (bc-ad) \text{Ci}\left(\frac{ad-bc}{d(c+dx)}\right) + d \exp\left(-\frac{i(ad+2bc+bdx)}{d(c+dx)}\right) \left(ic \left(e^{\frac{2ibc}{d(c+dx)}} - e^{2i\left(\frac{a}{c+dx} + \frac{b}{d}\right)} \right) + dx \sin\left(\frac{b}{d}\right) \left(e^{i\left(\frac{2a}{c+dx} + \frac{b}{d}\right)} + e^{\frac{ib(3c+dx)}{d(c+dx)}} \right) \right)}{2d^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sin[(a + b*x)/(c + d*x)], x]

[Out] (2*(b*c - a*d)*Cos[b/d]*CosIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + (d*(I*c*(E^(((2*I)*b*c)/(d*(c + d*x))) - E^(((2*I)*(b/d + a/(c + d*x)))))) + d*(E^((I*b*(3*c + d*x))/(d*(c + d*x))) + E^(I*(b/d + (2*a)/(c + d*x))))*x*Sin[b/d] + 2*d*E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x)))*x*Cos[b/d]*Sin[(-(b*c) + a*d)/(d*(c + d*x))])/E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x))) - 2*(b*c - a*d)*Sin[b/d]*SinIntegral[(-(b*c) + a*d)/(d*(c + d*x))])/(2*d^2)

fricas [A] time = 1.09, size = 139, normalized size = 1.39

$$\frac{2(bc-ad) \sin\left(\frac{b}{d}\right) \text{Si}\left(-\frac{bc-ad}{d^2x+cd}\right) - \left((bc-ad) \text{Ci}\left(\frac{bc-ad}{d^2x+cd}\right) + (bc-ad) \text{Ci}\left(-\frac{bc-ad}{d^2x+cd}\right)\right) \cos\left(\frac{b}{d}\right) - 2(d^2x+cd) \sin\left(\frac{bx+a}{dx+c}\right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((b*x+a)/(d*x+c)), x, algorithm="fricas")

[Out] -1/2*(2*(b*c - a*d)*sin(b/d)*sin_integral(-(b*c - a*d)/(d^2*x + c*d)) - ((b*c - a*d)*cos_integral((b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral(-(b*c - a*d)/(d^2*x + c*d)))*cos(b/d) - 2*(d^2*x + c*d)*sin((b*x + a)/(d*x + c)))/d^2

giac [B] time = 9.11, size = 630, normalized size = 6.30

$$\left(b^3 c^2 \cos\left(\frac{b}{d}\right) \text{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right) - 2ab^2cd \cos\left(\frac{b}{d}\right) \text{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right) - \frac{(bx+a)b^2c^2d \cos\left(\frac{b}{d}\right) \text{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)}{dx+c} + a^2bd^2 \cos\left(\frac{b}{d}\right) \text{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((b*x+a)/(d*x+c)), x, algorithm="giac")


```
[Out] (b^3*c^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) - 2*a*b^2*c*d*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) - (b*x + a)*b^2*c^2*d*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + a^2*b*d^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) + 2*(b*x + a)*a*b*c*d^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - (b*x + a)*a^2*d^3*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + b^3*c^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) - 2*a*b^2*c*d*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) - (b*x + a)*b^2*c^2*d*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + a^2*b*d^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) + 2*(b*x + a)*a*b*c*d^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - (b*x + a)*a^2*d^3*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + b^2*c^2*d*sin((b*x + a)/(d*x + c)) - 2*a*b*c*d^2*sin((b*x + a)/(d*x + c)) + a^2*d^3*sin((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

maple [A] time = 0.04, size = 142, normalized size = 1.42

$$-(da - cb) \left(-\frac{\sin\left(\frac{b}{d} + \frac{da - cb}{d(dx + c)}\right)}{\left(\left(\frac{b}{d} + \frac{da - cb}{d(dx + c)}\right)d - b\right)d} + \frac{-\frac{\text{Si}\left(\frac{da - cb}{d(dx + c)}\right)\sin\left(\frac{b}{d}\right)}{d} + \frac{\text{Ci}\left(\frac{da - cb}{d(dx + c)}\right)\cos\left(\frac{b}{d}\right)}{d}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((b*x+a)/(d*x+c)),x)
```

```
[Out] -(a*d-b*c)*(-sin(b/d+(a*d-b*c)/d/(d*x+c))/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/d + (-Si((a*d-b*c)/d/(d*x+c))*sin(b/d)/d+Ci((a*d-b*c)/d/(d*x+c))*cos(b/d)/d)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\frac{bx + a}{dx + c}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sin((b*x + a)/(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(\frac{a + bx}{c + dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((a + b*x)/(c + d*x)),x)
```

```
[Out] int(sin((a + b*x)/(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\frac{a + bx}{c + dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c)),x)
```

```
[Out] Integral(sin((a + b*x)/(c + d*x)), x)
```

3.37 $\int \sin^2\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=107

$$\frac{\sin\left(\frac{2b}{d}\right)(bc-ad)\text{Ci}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2} - \frac{\cos\left(\frac{2b}{d}\right)(bc-ad)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

[Out] $-(a*d+b*c)*\cos(2*b/d)*\text{Si}(2*(-a*d+b*c)/d/(d*x+c))/d^2+(a*d+b*c)*\text{Ci}(2*(-a*d+b*c)/d/(d*x+c))*\sin(2*b/d)/d^2+(d*x+c)*\sin((b*x+a)/(d*x+c))^2/d$

Rubi [A] time = 0.19, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4563, 3313, 12, 3303, 3299, 3302}

$$\frac{\sin\left(\frac{2b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2} - \frac{\cos\left(\frac{2b}{d}\right)(bc-ad)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sin[(a + b*x)/(c + d*x)]^2, x]

[Out] $((b*c - a*d)*\text{CosIntegral}[(2*(b*c - a*d))/(d*(c + d*x)])*\text{Sin}[(2*b)/d])/d^2 + ((c + d*x)*\text{Sin}[(a + b*x)/(c + d*x)]^2)/d - ((b*c - a*d)*\text{Cos}[(2*b)/d]*\text{SinIntegral}[(2*(b*c - a*d))/(d*(c + d*x)])/d^2$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3313

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m + 1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n - 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

Rule 4563

```
Int[Sin[((e_.)*(a_.) + (b_.)*(x_))]/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
:> -Dist[d^(-1), Subst[Int[Sin[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rubi steps

$$\int \sin^2\left(\frac{a+bx}{c+dx}\right) dx = -\frac{\text{Subst}\left(\int \frac{\sin^2\left(\frac{b}{d} - \frac{(bc-ad)x}{d}\right)}{x^2} dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx) \sin^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(2(bc-ad)) \text{Subst}\left(\int \frac{\sin\left(\frac{2b}{d} - \frac{2(bc-ad)x}{d}\right)}{2x} dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx) \sin^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad) \text{Subst}\left(\int \frac{\sin\left(\frac{2b}{d} - \frac{2(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx) \sin^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left((bc-ad) \cos\left(\frac{2b}{d}\right)\right) \text{Subst}\left(\int \frac{\sin\left(\frac{2(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{(bc-ad)}{d^2}$$

$$= \frac{(bc-ad) \text{Ci}\left(\frac{2(bc-ad)}{d(c+dx)}\right) \sin\left(\frac{2b}{d}\right)}{d^2} + \frac{(c+dx) \sin^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad) \cos\left(\frac{2b}{d}\right) \text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

Mathematica [C] time = 7.27, size = 330, normalized size = 3.08

$$8 \sin\left(\frac{2b}{d}\right) (bc-ad) \text{Ci}\left(\frac{2(ad-bc)}{d(c+dx)}\right) - d \exp\left(-\frac{2i(ad+2bc+bdx)}{d(c+dx)}\right) \left(dx \left(-4 \exp\left(\frac{2i(ad+2bc+bdx)}{d(c+dx)}\right) + e^{4i\left(\frac{a}{c+dx} + \frac{b}{d}\right)} + e^{\frac{4ia}{c+dx}} + e^{\frac{4ib}{d}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[(a + b*x)/(c + d*x)]^2, x]
```

```
[Out] (8*(b*c - a*d)*CosIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*Sin[(2*b)/d] -
(d*(2*c*(E^(((4*I)*b*c)/(d*(c + d*x)))) + E^(((4*I)*(b/d + a/(c + d*x)))))) +
d*(E^(((4*I)*a)/(c + d*x)) + E^(((4*I)*b*c)/(d*(c + d*x)))) + E^(((4*I)*b*(2
*c + d*x))/(d*(c + d*x))) - 4*E^(((2*I)*(2*b*c + a*d + b*d*x))/(d*(c + d*x)
)) + E^(((4*I)*(b/d + a/(c + d*x))))*x - 4*d*E^(((2*I)*(2*b*c + a*d + b*d*x)
))/(d*(c + d*x))*x*Sin[(2*b)/d]*Sin[(2*(-(b*c) + a*d))/(d*(c + d*x))])/E^((
((2*I)*(2*b*c + a*d + b*d*x))/(d*(c + d*x))) + 8*(b*c - a*d)*Cos[(2*b)/d]*S
inIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))])/(8*d^2)
```

fricas [A] time = 0.61, size = 149, normalized size = 1.39

$$\frac{2d^2x - 2(d^2x + cd) \cos\left(\frac{bx+a}{dx+c}\right)^2 + 2(bc-ad) \cos\left(\frac{2b}{d}\right) \text{Si}\left(-\frac{2(bc-ad)}{d^2x+cd}\right) + (bc-ad) \text{Ci}\left(\frac{2(bc-ad)}{d^2x+cd}\right) + (bc-ad) \text{Ci}\left(\frac{2(bc-ad)}{d^2x+cd}\right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^2, x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*x - 2*(d^2*x + c*d)*cos((b*x + a)/(d*x + c))^2 + 2*(b*c - a*d)*c
os(2*b/d)*sin_integral(-2*(b*c - a*d)/(d^2*x + c*d)) + ((b*c - a*d)*cos_int
```

egral(2*(b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral(-2*(b*c - a*d)/(d^2*x + c*d))*sin(2*b/d)/d^2

giac [B] time = 62.66, size = 681, normalized size = 6.36

$$\left(2b^3c^2 \operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right) \sin\left(\frac{2b}{d}\right) - 4ab^2cd \operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right) \sin\left(\frac{2b}{d}\right) - \frac{2(bx+a)b^2c^2d \operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right) \sin\left(\frac{2b}{d}\right)}{dx+c} + 2a^2ba \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((b*x+a)/(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{1}{2} * (2 * b^3 * c^2 * \cos_integral(-2 * (b - (b * x + a) * d / (d * x + c)) / d) * \sin(2 * b / d) - 4 * a * b^2 * c * d * \cos_integral(-2 * (b - (b * x + a) * d / (d * x + c)) / d) * \sin(2 * b / d) - 2 * (b * x + a) * b^2 * c^2 * d * \cos_integral(-2 * (b - (b * x + a) * d / (d * x + c)) / d) * \sin(2 * b / d) / (d * x + c) + 2 * a^2 * b * d^2 * \cos_integral(-2 * (b - (b * x + a) * d / (d * x + c)) / d) * \sin(2 * b / d) + 4 * (b * x + a) * a * b * c * d^2 * \cos_integral(-2 * (b - (b * x + a) * d / (d * x + c)) / d) * \sin(2 * b / d) / (d * x + c) - 2 * (b * x + a) * a^2 * d^3 * \cos_integral(-2 * (b - (b * x + a) * d / (d * x + c)) / d) * \sin(2 * b / d) / (d * x + c) - 2 * b^3 * c^2 * \cos(2 * b / d) * \sin_integral(2 * (b - (b * x + a) * d / (d * x + c)) / d) + 4 * a * b^2 * c * d * \cos(2 * b / d) * \sin_integral(2 * (b - (b * x + a) * d / (d * x + c)) / d) + 2 * (b * x + a) * b^2 * c^2 * d * \cos(2 * b / d) * \sin_integral(2 * (b - (b * x + a) * d / (d * x + c)) / d) / (d * x + c) - 2 * a^2 * b * d^2 * \cos(2 * b / d) * \sin_integral(2 * (b - (b * x + a) * d / (d * x + c)) / d) - 4 * (b * x + a) * a * b * c * d^2 * \cos(2 * b / d) * \sin_integral(2 * (b - (b * x + a) * d / (d * x + c)) / d) / (d * x + c) + 2 * (b * x + a) * a^2 * d^3 * \cos(2 * b / d) * \sin_integral(2 * (b - (b * x + a) * d / (d * x + c)) / d) / (d * x + c) - b^2 * c^2 * d * \cos(2 * (b * x + a) / (d * x + c)) + 2 * a * b * c * d^2 * \cos(2 * (b * x + a) / (d * x + c)) - a^2 * d^3 * \cos(2 * (b * x + a) / (d * x + c)) + b^2 * c^2 * d - 2 * a * b * c * d^2 + a^2 * d^3) * (b * c / (b * c - a * d)^2 - a * d / (b * c - a * d)^2) / (b * d^2 - (b * x + a) * d^3 / (d * x + c))$

maple [A] time = 0.07, size = 195, normalized size = 1.82

$$\frac{(da - cb) \left(\frac{d}{2 \left(\frac{b}{d} + \frac{da - cb}{d(dx+c)} \right)^{d-b}} - \frac{d^2 \left(\frac{2 \cos\left(\frac{2da-2cb}{d(dx+c)} + \frac{2b}{d}\right)}{\left(\frac{b}{d} + \frac{da-cb}{d(dx+c)}\right)^{d-b}} - \frac{2 \left(\frac{2 \operatorname{Si}\left(\frac{2da-2cb}{d(dx+c)}\right) \cos\left(\frac{2b}{d}\right) + 2 \operatorname{Ci}\left(\frac{2da-2cb}{d(dx+c)}\right) \sin\left(\frac{2b}{d}\right) \right)}{d} \right)}{4} \right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin((b*x+a)/(d*x+c))^2,x)

[Out] $-1/d^2 * (a*d - b*c) * (-1/2 * d / ((b/d + (a*d - b*c)/d) / (d*x+c)) * d - b) - 1/4 * d^2 * (-2 * \cos(2 * (a*d - b*c) / d / (d*x+c) + 2 * b/d) / ((b/d + (a*d - b*c)/d) / (d*x+c)) * d - b) / d - 2 * (2 * \operatorname{Si}(2 * (a*d - b*c) / d / (d*x+c)) * \cos(2 * b/d) / d + 2 * \operatorname{Ci}(2 * (a*d - b*c) / d / (d*x+c)) * \sin(2 * b/d) / d) / d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} x - \frac{1}{2} \int \cos\left(\frac{2(bx+a)}{dx+c}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((b*x+a)/(d*x+c))^2,x, algorithm="maxima")

[Out] $1/2*x - 1/2*\text{integrate}(\cos(2*(b*x + a)/(d*x + c)), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(\frac{a + b x}{c + d x}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin((a + b*x)/(c + d*x))^2, x)`

[Out] `int(sin((a + b*x)/(c + d*x))^2, x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin((b*x+a)/(d*x+c))**2, x)`

[Out] Timed out

3.38 $\int \sin^3\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=194

$$\frac{3 \cos\left(\frac{b}{d}\right)(bc-ad)\text{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} - \frac{3 \cos\left(\frac{3b}{d}\right)(bc-ad)\text{Ci}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2} + \frac{3 \sin\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} - \frac{3 \sin\left(\frac{3b}{d}\right)(bc-ad)\text{Si}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}$$

[Out] $3/4*(-a*d+b*c)*\text{Ci}((-a*d+b*c)/d/(d*x+c))*\cos(b/d)/d^2-3/4*(-a*d+b*c)*\text{Ci}(3*(-a*d+b*c)/d/(d*x+c))*\cos(3*b/d)/d^2+3/4*(-a*d+b*c)*\text{Si}((-a*d+b*c)/d/(d*x+c))*\sin(b/d)/d^2-3/4*(-a*d+b*c)*\text{Si}(3*(-a*d+b*c)/d/(d*x+c))*\sin(3*b/d)/d^2+(d*x+c)*\sin((b*x+a)/(d*x+c))^3/d$

Rubi [A] time = 0.32, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {4563, 3313, 3303, 3299, 3302}

$$\frac{3 \cos\left(\frac{b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} - \frac{3 \cos\left(\frac{3b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2} + \frac{3 \sin\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} - \frac{3 \sin\left(\frac{3b}{d}\right)(bc-ad)\text{Si}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sin}[(a + b*x)/(c + d*x)]^3, x]$

[Out] $(3*(b*c - a*d)*\text{Cos}[b/d]*\text{CosIntegral}[(b*c - a*d)/(d*(c + d*x))])/(4*d^2) - (3*(b*c - a*d)*\text{Cos}[(3*b)/d]*\text{CosIntegral}[(3*(b*c - a*d)/(d*(c + d*x))])/(4*d^2) + ((c + d*x)*\text{Sin}[(a + b*x)/(c + d*x)]^3)/d + (3*(b*c - a*d)*\text{Sin}[b/d]*\text{SinIntegral}[(b*c - a*d)/(d*(c + d*x))])/(4*d^2) - (3*(b*c - a*d)*\text{Sin}[(3*b)/d]*\text{SinIntegral}[(3*(b*c - a*d)/(d*(c + d*x))])/(4*d^2)$

Rule 3299

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Simp}[\text{SinIntegral}[e + f*x]/d, x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{EqQ}[d*e - c*f, 0]$

Rule 3302

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Simp}[\text{CosIntegral}[e - \text{Pi}/2 + f*x]/d, x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{EqQ}[d*(e - \text{Pi}/2) - c*f, 0]$

Rule 3303

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/(c + d*x), x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{NeQ}[d*e - c*f, 0]$

Rule 3313

$\text{Int}[(c_. + (d_.)*(x_.))^(m_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] \rightarrow \text{Simp}[(c + d*x)^(m + 1)*\text{Sin}[e + f*x]^n/(d*(m + 1)), x] - \text{Dist}[(f*n)/(d*(m + 1)), \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^(m + 1), \text{Cos}[e + f*x]*\text{Sin}[e + f*x]^(n - 1), x], x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x\} \ \&\& \ \text{IGtQ}[n, 1] \ \&\& \ \text{GeQ}[m, -2] \ \&\& \ \text{LtQ}[m, -1]$

Rule 4563

$\text{Int}[\text{Sin}[(e_.)*((a_.) + (b_.)*(x_.))]/((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] \rightarrow -\text{Dist}[d^(-1), \text{Subst}[\text{Int}[\text{Sin}[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x], x]$

, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \sin^3\left(\frac{a+bx}{c+dx}\right) dx &= -\frac{\text{Subst}\left(\int \frac{\sin^3\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2} dx, x, \frac{1}{c+dx}\right)}{d} \\ &= \frac{(c+dx) \sin^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(3(bc-ad)) \text{Subst}\left(\int \left(-\frac{\cos\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{4x} + \frac{\cos\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{4x}\right) dx, x, \frac{1}{c+dx}\right)}{d^2} \\ &= \frac{(c+dx) \sin^3\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(3(bc-ad)) \text{Subst}\left(\int \frac{\cos\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{4d^2} + \frac{(3(bc-ad)) \text{Subst}\left(\int \frac{\cos\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{4d^2} \\ &= \frac{(c+dx) \sin^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left(3(bc-ad) \cos\left(\frac{b}{d}\right)\right) \text{Subst}\left(\int \frac{\cos\left(\frac{(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{4d^2} - \frac{(3(bc-ad)) \text{Subst}\left(\int \frac{\cos\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{4d^2} \\ &= \frac{3(bc-ad) \cos\left(\frac{b}{d}\right) \text{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} - \frac{3(bc-ad) \cos\left(\frac{3b}{d}\right) \text{Ci}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx) \sin^3\left(\frac{a+bx}{c+dx}\right)}{d} \end{aligned}$$

Mathematica [C] time = 7.71, size = 657, normalized size = 3.39

$$\frac{3(acd - bc^2) \left(\frac{i \left(1 + e^{\frac{2ib}{d}} \right) \left(e^{\frac{2ibc}{d(c+dx)}} - e^{\frac{2ia}{c+dx}} \right) \exp\left(-\frac{i(ad+2bc+bdx)}{d(c+dx)}\right)}{4(bc-ad)} - \frac{i \left(-1 + e^{\frac{2ib}{d}} \right) \left(e^{\frac{2ia}{c+dx}} + e^{\frac{2ibc}{d(c+dx)}} \right) \exp\left(-\frac{i(ad+2bc+bdx)}{d(c+dx)}\right)}{4(bc-ad)} \right)}{4d} + \dots$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sin[(a + b*x)/(c + d*x)]^3, x]

[Out] (-3*(-(b*c^2) + a*c*d)*(((I/4)*(1 + E^(((2*I)*b)/d)))*(-E^(((2*I)*a)/(c + d*x)) + E^(((2*I)*b*c)/(d*(c + d*x)))))/((b*c - a*d)*E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x)))) - ((I/4)*(-1 + E^(((2*I)*b)/d))*(E^(((2*I)*a)/(c + d*x)) + E^(((2*I)*b*c)/(d*(c + d*x)))))/((b*c - a*d)*E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x)))))/(4*d) + (3*(-(b*c^2) + a*c*d)*(((I/12)*(1 + E^(((6*I)*b)/d)))*(-E^(((6*I)*a)/(c + d*x)) + E^(((6*I)*b*c)/(d*(c + d*x)))))/((b*c - a*d)*E^(((3*I)*(2*b*c + a*d + b*d*x))/(d*(c + d*x)))) - ((I/12)*(-1 + E^(((6*I)*b)/d))*(E^(((6*I)*a)/(c + d*x)) + E^(((6*I)*b*c)/(d*(c + d*x)))))/((b*c - a*d)*E^(((3*I)*(2*b*c + a*d + b*d*x))/(d*(c + d*x)))))/(4*d) + (3*x*Cos[(-b*c) + a*d]/(d*(c + d*x)))*Sin[b/d]/4 - (x*Cos[(3*(-b*c) + a*d)]/(d*(c + d*x)))*Sin[(3*b)/d]/4 + (3*x*Cos[b/d]*Sin[(-b*c) + a*d]/(d*(c + d*x)))/4 - (x*Cos[(3*b)/d]*Sin[(3*(-b*c) + a*d)]/(d*(c + d*x)))/4 + (3*(-b*c) + a*d)*(-Cos[b/d]*CosIntegral[(-b*c) + a*d]/(d*(c + d*x))) + Cos[(3*b)/d]*CosIntegral[(3*(-b*c) + a*d)]/(d*(c + d*x)) + Sin[b/d]*SinIntegral[(-b*c) + a*d]/(d*(c + d*x)) - Sin[(3*b)/d]*SinIntegral[(3*(-b*c) + a*d)]/(d*(c + d*x)))/(4*d^2)

fricas [A] time = 1.14, size = 277, normalized size = 1.43

$$6(bc - ad) \sin\left(\frac{b}{d}\right) \text{Si}\left(-\frac{bc-ad}{d^2x+cd}\right) - 6(bc - ad) \sin\left(\frac{3b}{d}\right) \text{Si}\left(-\frac{3(bc-ad)}{d^2x+cd}\right) + 3\left((bc - ad) \text{Ci}\left(\frac{3(bc-ad)}{d^2x+cd}\right) + (bc - ad) \text{Ci}\left(\frac{bc-ad}{d^2x+cd}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((b*x+a)/(d*x+c))^3,x, algorithm="fricas")

[Out] $-1/8*(6*(b*c - a*d)*\sin(b/d)*\sin_integral(-(b*c - a*d)/(d^2*x + c*d)) - 6*(b*c - a*d)*\sin(3*b/d)*\sin_integral(-3*(b*c - a*d)/(d^2*x + c*d)) + 3*((b*c - a*d)*\cos_integral(3*(b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*\cos_integral(-3*(b*c - a*d)/(d^2*x + c*d)))*\cos(3*b/d) - 3*((b*c - a*d)*\cos_integral((b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*\cos_integral(-(b*c - a*d)/(d^2*x + c*d)))*\cos(b/d) - 8*(d^2*x - (d^2*x + c*d)*\cos((b*x + a)/(d*x + c))^2 + c*d)*\sin((b*x + a)/(d*x + c))/d^2$

giac [B] time = 177.53, size = 1239, normalized size = 6.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((b*x+a)/(d*x+c))^3,x, algorithm="giac")

[Out] $1/4*(3*b^3*c^2*\cos(b/d)*\cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) - 6*a*b^2*c*d*\cos(b/d)*\cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) - 3*(b*x + a)*b^2*c^2*d*\cos(b/d)*\cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + 3*a^2*b*d^2*\cos(b/d)*\cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) + 6*(b*x + a)*a*b*c*d^2*\cos(b/d)*\cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 3*(b*x + a)*a^2*d^3*\cos(b/d)*\cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 3*b^3*c^2*\cos(3*b/d)*\cos_integral(-3*(b - (b*x + a)*d/(d*x + c))/d) + 6*a*b^2*c*d*\cos(3*b/d)*\cos_integral(-3*(b - (b*x + a)*d/(d*x + c))/d) + 3*(b*x + a)*b^2*c^2*d*\cos(3*b/d)*\cos_integral(-3*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 3*a^2*b*d^2*\cos(3*b/d)*\cos_integral(-3*(b - (b*x + a)*d/(d*x + c))/d) - 6*(b*x + a)*a*b*c*d^2*\cos(3*b/d)*\cos_integral(-3*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + 3*(b*x + a)*a^2*d^3*\cos(3*b/d)*\cos_integral(-3*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 3*b^3*c^2*\sin(3*b/d)*\sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d) + 6*a*b^2*c*d*\sin(3*b/d)*\sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d) + 3*(b*x + a)*b^2*c^2*d*\sin(3*b/d)*\sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 3*a^2*b*d^2*\sin(3*b/d)*\sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d) - 6*(b*x + a)*a*b*c*d^2*\sin(3*b/d)*\sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + 3*(b*x + a)*a^2*d^3*\sin(3*b/d)*\sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + 3*b^3*c^2*\sin(b/d)*\sin_integral((b - (b*x + a)*d/(d*x + c))/d) - 6*a*b^2*c*d*\sin(b/d)*\sin_integral((b - (b*x + a)*d/(d*x + c))/d) - 3*(b*x + a)*b^2*c^2*d*\sin(b/d)*\sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + 3*a^2*b*d^2*\sin(b/d)*\sin_integral((b - (b*x + a)*d/(d*x + c))/d) + 6*(b*x + a)*a*b*c*d^2*\sin(b/d)*\sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 3*(b*x + a)*a^2*d^3*\sin(b/d)*\sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - b^2*c^2*d*\sin(3*(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*\sin(3*(b*x + a)/(d*x + c)) - a^2*d^3*\sin(3*(b*x + a)/(d*x + c)) + 3*b^2*c^2*d*\sin((b*x + a)/(d*x + c)) - 6*a*b*c*d^2*\sin((b*x + a)/(d*x + c)) + 3*a^2*d^3*\sin((b*x + a)/(d*x + c))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))$

maple [A] time = 0.07, size = 295, normalized size = 1.52

$$(da - cb) \left[\frac{d^2 \left(\frac{3 \sin\left(\frac{3da-3cb}{d(dx+c)} + \frac{3b}{d}\right)}{\left(\frac{b}{d} + \frac{da-cb}{d(dx+c)}\right)^{d-b} d} + \frac{9 \operatorname{Si}\left(\frac{3da-3cb}{d(dx+c)}\right) \sin\left(\frac{3b}{d}\right)}{d} + \frac{9 \operatorname{Ci}\left(\frac{3da-3cb}{d(dx+c)}\right) \cos\left(\frac{3b}{d}\right)}{d} \right)}{12} + \frac{3d^2 \left(\frac{\sin\left(\frac{b}{d} + \frac{da-cb}{d(dx+c)}\right)}{\left(\frac{b}{d} + \frac{da-cb}{d(dx+c)}\right)^{d-b} d} + \frac{\operatorname{Si}\left(\frac{da-cb}{d(dx+c)}\right) \sin\left(\frac{b}{d}\right)}{d} + \frac{\operatorname{Ci}\left(\frac{da-cb}{d(dx+c)}\right)}{d} \right)}{4} \right] d^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin((b*x+a)/(d*x+c))^3,x)

[Out]
$$-1/d^2*(a*d-b*c)*(-1/12*d^2*(-3*\sin(3*(a*d-b*c)/d/(d*x+c)+3*b/d)/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/d+3*(-3*Si(3*(a*d-b*c)/d/(d*x+c))*\sin(3*b/d)/d+3*Ci(3*(a*d-b*c)/d/(d*x+c))*\cos(3*b/d)/d)/d)+3/4*d^2*(-\sin(b/d+(a*d-b*c)/d/(d*x+c)))/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/d+(-Si((a*d-b*c)/d/(d*x+c))*\sin(b/d)/d+Ci((a*d-b*c)/d/(d*x+c))*\cos(b/d)/d)/d)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(\frac{bx+a}{dx+c}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((b*x+a)/(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(sin((b*x + a)/(d*x + c))^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(\frac{a+bx}{c+dx}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin((a + b*x)/(c + d*x))^3,x)

[Out] int(sin((a + b*x)/(c + d*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((b*x+a)/(d*x+c))**3,x)

[Out] Timed out

$$3.39 \quad \int \frac{\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Optimal. Leaf size=58

$$\frac{\text{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a} - \frac{3\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

[Out] $-3/4*\text{Si}((-a*x+1)^{(1/2)}/(a*x+1)^{(1/2))}/a+1/4*\text{Si}(3*(-a*x+1)^{(1/2)}/(a*x+1)^{(1/2))}/a$

Rubi [A] time = 0.11, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6681, 3312, 3299}

$$\frac{\text{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a} - \frac{3\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

[In] Int[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2), x]

[Out] $(-3*\text{SinIntegral}[\text{Sqrt}[1 - a*x]/\text{Sqrt}[1 + a*x]])/(4*a) + \text{SinIntegral}[(3*\text{Sqrt}[1 - a*x])/\text{Sqrt}[1 + a*x]]/(4*a)$

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3312

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 6681

Int[((a_.) + (b_.)*(F_))(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] := Dist[(2*e*g)/(C*(e*f - d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx &= -\frac{\text{Subst}\left(\int \frac{\sin^3(x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\ &= -\frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4x} - \frac{\sin(3x)}{4x}\right) dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\ &= \frac{\text{Subst}\left(\int \frac{\sin(3x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{3\text{Subst}\left(\int \frac{\sin(x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} \\ &= -\frac{3\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} + \frac{\text{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} \end{aligned}$$

Mathematica [A] time = 0.11, size = 53, normalized size = 0.91

$$\frac{\operatorname{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right) - 3\operatorname{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]

[Out] (-3*SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + SinIntegral[(3*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(4*a)

fricas [F] time = 2.03, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2 - 1\right)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="fricas")

[Out] integral((cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="giac")

[Out] integrate(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\sin^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)

[Out] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="maxima")

[Out] -integrate(sin(sqrt(-a*x + 1))/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)

[Out] -int(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sin^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1), x)

[Out] -Integral(sin(sqrt(-a*x + 1))/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)

$$3.40 \quad \int \frac{\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Optimal. Leaf size=58

$$\frac{\text{Ci}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

[Out] 1/2*Ci(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6681, 3312, 3302}

$$\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

[In] Int[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]

[Out] CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(2*a)

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 3312

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 6681

Int[((a_.) + (b_.)*(F_))(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)])^(n_)/((A_.) + (C_.)*(x_)^2), x_Symbol] := Dist[(2*e*g)/(C*(e*f - d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx &= -\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cos(2x)}{2x}\right) dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} \\
&= \frac{\text{Ci}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 57, normalized size = 0.98

$$\frac{\text{Ci}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a} - \frac{\log(1-ax)}{4a} + \frac{\log(ax+1)}{4a}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]

[Out] CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[1 - a*x]/(4*a) + Log[1 + a*x]/(4*a)

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2 - 1}{a^2x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1), x, algorithm="fricas")

[Out] integral((cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - 1)/(a^2*x^2 - 1), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sin(sqrt(-a*x+1)/sqrt(a*x+1))^2/(a^2*x^2-1), x, algorithm="giac")

[Out] integrate(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)

[Out] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a \int \frac{\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1} dx + a \int \frac{\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{(a^2x^2-1)\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2 + (a^2x^2-1)\sin\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2} dx + \log(ax+1) - \log(ax-1)$$

$$4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="maxima")

[Out] 1/4*(4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x) + 4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/((a^2*x^2 - 1)*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^2*x^2 - 1)*sin(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x) + log(a*x + 1) - log(a*x - 1))/a

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$- \int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1),x)

[Out] -int(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\sin^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)

[Out] -Integral(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)

$$3.41 \quad \int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Optimal. Leaf size=26

$$-\frac{\operatorname{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

[Out] -Si((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {6681, 3299}

$$-\frac{\operatorname{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]

[Out] -(SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 6681

Int[((a_.) + (b_.)*(F_))(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)])^n_/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f - d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx &= -\frac{\operatorname{Subst}\left(\int \frac{\sin(x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\ &= -\frac{\operatorname{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \end{aligned}$$

Mathematica [A] time = 0.05, size = 26, normalized size = 1.00

$$-\frac{\operatorname{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]

[Out] -(SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{\sin \left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}} \right)}{a^2x^2 - 1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fricas")

[Out] integral(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sin \left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}} \right)}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="giac")

[Out] integrate(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sin \left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}} \right)}{-a^2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)

[Out] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sin \left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}} \right)}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="maxima")

[Out] -integrate(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{\sin \left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}} \right)}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)

[Out] -int(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1), x)

[Out] -Integral(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(-a**2*x**2 + 1), x)

$$3.42 \quad \int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Optimal. Leaf size=40

$$\text{Int}\left(\frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)}, x\right)$$

[Out] Unintegrable(csc((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a*x+1)/(a*x+1), x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]

[Out] -(Defer[Subst][Defer[Int][Csc[x]/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)

Rubi steps

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx = -\frac{\text{Subst}\left(\int \frac{\csc(x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 5.91, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]

[Out] Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]

fricas [A] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{(a^2x^2 - 1) \sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)), x, algorithm="fricas")

[Out] integral(-1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(a^2x^2 - 1) \sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="giac")

[Out] integrate(-1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)

maple [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{(-a^2x^2 + 1) \sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)

[Out] int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{1}{(a^2x^2 - 1) \sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="maxima")

[Out] -integrate(1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$- \int \frac{1}{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) (a^2x^2 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)

[Out] -int(1/(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{1}{a^2x^2 \sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right) - \sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a**2*x**2+1)/sin((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)

[Out] -Integral(1/(a**2*x**2*sin(sqrt(-a*x + 1)/sqrt(a*x + 1)) - sin(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)

$$3.43 \quad \int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Optimal. Leaf size=42

$$\text{Int}\left(\frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)}, x\right)$$

[Out] Unintegrable(csc((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a*x+1)/(a*x+1), x)

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]

[Out] -(Defer[Subst][Defer[Int][Csc[x]^2/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)

Rubi steps

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx = -\frac{\text{Subst}\left(\int \frac{\csc^2(x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 19.89, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]

[Out] Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]

fricas [A] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{a^2x^2 - (a^2x^2 - 1)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="fricas")

[Out] integral(-1/(a^2*x^2 - (a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - 1), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(a^2x^2 - 1) \sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="giac")

[Out] integrate(-1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{(-a^2x^2 + 1) \sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)

[Out] int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="maxima")

[Out] Timed out

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)^2 (a^2x^2 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)),x)

[Out] -int(1/(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2x^2 \sin^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right) - \sin^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a**2*x**2+1)/sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)

[Out] -Integral(1/(a**2*x**2*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))**2 - sin(sqrt(-a*x + 1)/sqrt(a*x + 1))**2), x)

3.44 $\int (x + \cos(x))^2 dx$

Optimal. Leaf size=30

$$\frac{x^3}{3} + \frac{x}{2} + 2x \sin(x) + 2 \cos(x) + \frac{1}{2} \sin(x) \cos(x)$$

[Out] 1/2*x+1/3*x^3+2*cos(x)+2*x*sin(x)+1/2*cos(x)*sin(x)

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6742, 3296, 2638, 2635, 8}

$$\frac{x^3}{3} + \frac{x}{2} + 2x \sin(x) + 2 \cos(x) + \frac{1}{2} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[(x + Cos[x])^2,x]

[Out] x/2 + x^3/3 + 2*Cos[x] + 2*x*Sin[x] + (Cos[x]*Sin[x])/2

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x] * (b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_)^(m_.))*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]

Rubi steps

$$\begin{aligned} \int (x + \cos(x))^2 dx &= \int (x^2 + 2x \cos(x) + \cos^2(x)) dx \\ &= \frac{x^3}{3} + 2 \int x \cos(x) dx + \int \cos^2(x) dx \\ &= \frac{x^3}{3} + 2x \sin(x) + \frac{1}{2} \cos(x) \sin(x) + \frac{\int 1 dx}{2} - 2 \int \sin(x) dx \\ &= \frac{x}{2} + \frac{x^3}{3} + 2 \cos(x) + 2x \sin(x) + \frac{1}{2} \cos(x) \sin(x) \end{aligned}$$

Mathematica [A] time = 0.08, size = 26, normalized size = 0.87

$$\frac{1}{6} \left(x \left(2x^2 + 12 \sin(x) + 3 \right) + 3(\sin(x) + 4) \cos(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x + Cos[x])^2, x]

[Out] (3*Cos[x]*(4 + Sin[x]) + x*(3 + 2*x^2 + 12*Sin[x]))/6

fricas [A] time = 2.51, size = 23, normalized size = 0.77

$$\frac{1}{3} x^3 + \frac{1}{2} (4x + \cos(x)) \sin(x) + \frac{1}{2} x + 2 \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+cos(x))^2,x, algorithm="fricas")

[Out] 1/3*x^3 + 1/2*(4*x + cos(x))*sin(x) + 1/2*x + 2*cos(x)

giac [A] time = 0.14, size = 24, normalized size = 0.80

$$\frac{1}{3} x^3 + 2x \sin(x) + \frac{1}{2} x + 2 \cos(x) + \frac{1}{4} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+cos(x))^2,x, algorithm="giac")

[Out] 1/3*x^3 + 2*x*sin(x) + 1/2*x + 2*cos(x) + 1/4*sin(2*x)

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\frac{x}{2} + \frac{x^3}{3} + 2 \cos(x) + 2x \sin(x) + \frac{\cos(x) \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+cos(x))^2,x)

[Out] 1/2*x+1/3*x^3+2*cos(x)+2*x*sin(x)+1/2*cos(x)*sin(x)

maxima [A] time = 0.31, size = 24, normalized size = 0.80

$$\frac{1}{3} x^3 + 2x \sin(x) + \frac{1}{2} x + 2 \cos(x) + \frac{1}{4} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+cos(x))^2,x, algorithm="maxima")

[Out] 1/3*x^3 + 2*x*sin(x) + 1/2*x + 2*cos(x) + 1/4*sin(2*x)

mupad [B] time = 0.05, size = 24, normalized size = 0.80

$$\frac{x}{2} + 2 \cos(x) + \frac{\cos(x) \sin(x)}{2} + 2x \sin(x) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + cos(x))^2,x)

[Out] x/2 + 2*cos(x) + (cos(x)*sin(x))/2 + 2*x*sin(x) + x^3/3

sympy [A] time = 0.16, size = 41, normalized size = 1.37

$$\frac{x^3}{3} + \frac{x \sin^2(x)}{2} + 2x \sin(x) + \frac{x \cos^2(x)}{2} + \frac{\sin(x) \cos(x)}{2} + 2 \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+cos(x))**2,x)

[Out] x**3/3 + x*sin(x)**2/2 + 2*x*sin(x) + x*cos(x)**2/2 + sin(x)*cos(x)/2 + 2*cos(x)

3.45 $\int (x + \cos(x))^3 dx$

Optimal. Leaf size=56

$$\frac{x^4}{4} + \frac{3x^2}{4} + 3x^2 \sin(x) - \frac{\sin^3(x)}{3} - 5 \sin(x) + \frac{3 \cos^2(x)}{4} + 6x \cos(x) + \frac{3}{2} x \sin(x) \cos(x)$$

[Out] $3/4*x^2+1/4*x^4+6*x*\cos(x)+3/4*\cos(x)^2-5*\sin(x)+3*x^2*\sin(x)+3/2*x*\cos(x)*\sin(x)-1/3*\sin(x)^3$

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6742, 3296, 2637, 3310, 30, 2633}

$$\frac{x^4}{4} + \frac{3x^2}{4} + 3x^2 \sin(x) - \frac{\sin^3(x)}{3} - 5 \sin(x) + \frac{3 \cos^2(x)}{4} + 6x \cos(x) + \frac{3}{2} x \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[(x + Cos[x])^3,x]

[Out] $(3*x^2)/4 + x^4/4 + 6*x*\cos[x] + (3*\cos[x]^2)/4 - 5*\sin[x] + 3*x^2*\sin[x] + (3*x*\cos[x]*\sin[x])/2 - \sin[x]^3/3$

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3310

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(d*(b*Ssin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Ssin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Ssin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]

Rubi steps

$$\begin{aligned}
\int (x + \cos(x))^3 dx &= \int (x^3 + 3x^2 \cos(x) + 3x \cos^2(x) + \cos^3(x)) dx \\
&= \frac{x^4}{4} + 3 \int x^2 \cos(x) dx + 3 \int x \cos^2(x) dx + \int \cos^3(x) dx \\
&= \frac{x^4}{4} + \frac{3 \cos^2(x)}{4} + 3x^2 \sin(x) + \frac{3}{2} x \cos(x) \sin(x) + \frac{3 \int x dx}{2} - 6 \int x \sin(x) dx - \text{Subst} \left(\int \right) \\
&= \frac{3x^2}{4} + \frac{x^4}{4} + 6x \cos(x) + \frac{3 \cos^2(x)}{4} + \sin(x) + 3x^2 \sin(x) + \frac{3}{2} x \cos(x) \sin(x) - \frac{\sin^3(x)}{3} - \dots \\
&= \frac{3x^2}{4} + \frac{x^4}{4} + 6x \cos(x) + \frac{3 \cos^2(x)}{4} - 5 \sin(x) + 3x^2 \sin(x) + \frac{3}{2} x \cos(x) \sin(x) - \frac{\sin^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 51, normalized size = 0.91

$$\frac{1}{12} (3x^4 + 9x^2 + 9(4x^2 - 7) \sin(x) + 9x \sin(2x) + \sin(3x)) + 6x \cos(x) + \frac{3}{8} \cos(2x)$$

Antiderivative was successfully verified.

[In] Integrate[(x + Cos[x])^3, x]

[Out] 6*x*Cos[x] + (3*Cos[2*x])/8 + (9*x^2 + 3*x^4 + 9*(-7 + 4*x^2)*Sin[x] + 9*x*Sin[2*x] + Sin[3*x])/12

fricas [A] time = 0.85, size = 44, normalized size = 0.79

$$\frac{1}{4} x^4 + \frac{3}{4} x^2 + 6x \cos(x) + \frac{3}{4} \cos(x)^2 + \frac{1}{6} (18x^2 + 9x \cos(x) + 2 \cos(x)^2 - 32) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+cos(x))^3,x, algorithm="fricas")

[Out] 1/4*x^4 + 3/4*x^2 + 6*x*cos(x) + 3/4*cos(x)^2 + 1/6*(18*x^2 + 9*x*cos(x) + 2*cos(x)^2 - 32)*sin(x)

giac [A] time = 0.15, size = 46, normalized size = 0.82

$$\frac{1}{4} x^4 + \frac{3}{4} x^2 + 6x \cos(x) + \frac{3}{4} x \sin(2x) + \frac{3}{4} (4x^2 - 7) \sin(x) + \frac{3}{8} \cos(2x) + \frac{1}{12} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+cos(x))^3,x, algorithm="giac")

[Out] 1/4*x^4 + 3/4*x^2 + 6*x*cos(x) + 3/4*x*sin(2*x) + 3/4*(4*x^2 - 7)*sin(x) + 3/8*cos(2*x) + 1/12*sin(3*x)

maple [A] time = 0.03, size = 57, normalized size = 1.02

$$\frac{(2 + \cos^2(x)) \sin(x)}{3} + 3x \left(\frac{\cos(x) \sin(x)}{2} + \frac{x}{2} \right) - \frac{3x^2}{4} - \frac{3(\sin^2(x))}{4} + 3x^2 \sin(x) - 6 \sin(x) + 6x \cos(x) + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+cos(x))^3,x)

[Out] 1/3*(2+cos(x)^2)*sin(x)+3*x*(1/2*cos(x)*sin(x)+1/2*x)-3/4*x^2-3/4*sin(x)^2+3*x^2*sin(x)-6*sin(x)+6*x*cos(x)+1/4*x^4

maxima [A] time = 0.31, size = 46, normalized size = 0.82

$$\frac{1}{4}x^4 - \frac{1}{3}\sin(x)^3 + \frac{3}{4}x^2 + 6x\cos(x) + \frac{3}{4}x\sin(2x) + 3(x^2 - 2)\sin(x) + \frac{3}{8}\cos(2x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+cos(x))^3,x, algorithm="maxima")

[Out] 1/4*x^4 - 1/3*sin(x)^3 + 3/4*x^2 + 6*x*cos(x) + 3/4*x*sin(2*x) + 3*(x^2 - 2)*sin(x) + 3/8*cos(2*x) + sin(x)

mupad [B] time = 0.08, size = 48, normalized size = 0.86

$$3x^2\sin(x) - \frac{16\sin(x)}{3} + \frac{3\cos(x)^2}{4} + \frac{\cos(x)^2\sin(x)}{3} + 6x\cos(x) + \frac{3x^2}{4} + \frac{x^4}{4} + \frac{3x\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + cos(x))^3,x)

[Out] 3*x^2*sin(x) - (16*sin(x))/3 + (3*cos(x)^2)/4 + (cos(x)^2*sin(x))/3 + 6*x*cos(x) + (3*x^2)/4 + x^4/4 + (3*x*cos(x)*sin(x))/2

sympy [A] time = 0.31, size = 85, normalized size = 1.52

$$\frac{x^4}{4} + \frac{3x^2\sin^2(x)}{4} + 3x^2\sin(x) + \frac{3x^2\cos^2(x)}{4} + \frac{3x\sin(x)\cos(x)}{2} + 6x\cos(x) + \frac{2\sin^3(x)}{3} - \frac{3\sin^2(x)}{4} + \sin(x)\cos^2(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+cos(x))**3,x)

[Out] x**4/4 + 3*x**2*sin(x)**2/4 + 3*x**2*sin(x) + 3*x**2*cos(x)**2/4 + 3*x*sin(x)*cos(x)/2 + 6*x*cos(x) + 2*sin(x)**3/3 - 3*sin(x)**2/4 + sin(x)*cos(x)**2 - 6*sin(x)

$$3.46 \quad \int \frac{\cos(a+bx)}{c+dx^2} dx$$

Optimal. Leaf size=213

$$\frac{\cos\left(a + \frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}} - bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cos\left(a - \frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{Ci}\left(xb + \frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\sin\left(a + \frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}} - bx\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\sin\left(a - \frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{Si}\left(xb + \frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}$$

[Out] $-1/2*\text{Ci}(b*x+b*(-c)^{(1/2)}/d^{(1/2)})*\cos(a-b*(-c)^{(1/2)}/d^{(1/2)})/(-c)^{(1/2)}/d^{(1/2)}+1/2*\text{Ci}(-b*x+b*(-c)^{(1/2)}/d^{(1/2)})*\cos(a+b*(-c)^{(1/2)}/d^{(1/2)})/(-c)^{(1/2)}/d^{(1/2)}+1/2*\text{Si}(b*x+b*(-c)^{(1/2)}/d^{(1/2)})*\sin(a-b*(-c)^{(1/2)}/d^{(1/2)})/(-c)^{(1/2)}/d^{(1/2)}-1/2*\text{Si}(b*x-b*(-c)^{(1/2)}/d^{(1/2)})*\sin(a+b*(-c)^{(1/2)}/d^{(1/2)})/(-c)^{(1/2)}/d^{(1/2)}$

Rubi [A] time = 0.31, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3334, 3303, 3299, 3302}

$$\frac{\cos\left(a + \frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}} - bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cos\left(a - \frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}} + bx\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\sin\left(a + \frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]/(c + d*x^2), x]

[Out] $(\text{Cos}[a + (b*\text{Sqrt}[-c])/ \text{Sqrt}[d]]*\text{CosIntegral}[(b*\text{Sqrt}[-c])/ \text{Sqrt}[d] - b*x])/(2*\text{Sqrt}[-c]*\text{Sqrt}[d]) - (\text{Cos}[a - (b*\text{Sqrt}[-c])/ \text{Sqrt}[d]]*\text{CosIntegral}[(b*\text{Sqrt}[-c])/ \text{Sqrt}[d] + b*x])/(2*\text{Sqrt}[-c]*\text{Sqrt}[d]) + (\text{Sin}[a + (b*\text{Sqrt}[-c])/ \text{Sqrt}[d]]*\text{SinIntegral}[(b*\text{Sqrt}[-c])/ \text{Sqrt}[d] - b*x])/(2*\text{Sqrt}[-c]*\text{Sqrt}[d]) + (\text{Sin}[a - (b*\text{Sqrt}[-c])/ \text{Sqrt}[d]]*\text{SinIntegral}[(b*\text{Sqrt}[-c])/ \text{Sqrt}[d] + b*x])/(2*\text{Sqrt}[-c]*\text{Sqrt}[d])$

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3334

Int[Cos[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)}{c+dx^2} dx &= \int \left(\frac{\sqrt{-c} \cos(a+bx)}{2c(\sqrt{-c}-\sqrt{d}x)} + \frac{\sqrt{-c} \cos(a+bx)}{2c(\sqrt{-c}+\sqrt{d}x)} \right) dx \\
&= -\frac{\int \frac{\cos(a+bx)}{\sqrt{-c}-\sqrt{d}x} dx}{2\sqrt{-c}} - \frac{\int \frac{\cos(a+bx)}{\sqrt{-c}+\sqrt{d}x} dx}{2\sqrt{-c}} \\
&= -\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right) \int \frac{\cos\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}x} dx}{2\sqrt{-c}} - \frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right) \int \frac{\cos\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{\sqrt{-c}-\sqrt{d}x} dx}{2\sqrt{-c}} + \frac{\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right) \int \frac{\sin\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}x} dx}{2\sqrt{-c}} \\
&= \frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}
\end{aligned}$$

Mathematica [C] time = 0.34, size = 172, normalized size = 0.81

$$\frac{i \left(\cos\left(a + \frac{ib\sqrt{c}}{\sqrt{d}}\right) \text{Ci}\left(b\left(x - \frac{i\sqrt{c}}{\sqrt{d}}\right)\right) - \cos\left(a - \frac{ib\sqrt{c}}{\sqrt{d}}\right) \text{Ci}\left(b\left(x + \frac{i\sqrt{c}}{\sqrt{d}}\right)\right) + \sin\left(a - \frac{ib\sqrt{c}}{\sqrt{d}}\right) \text{Si}\left(b\left(x + \frac{i\sqrt{c}}{\sqrt{d}}\right)\right) + \sin\left(a + \frac{ib\sqrt{c}}{\sqrt{d}}\right) \text{Si}\left(b\left(x - \frac{i\sqrt{c}}{\sqrt{d}}\right)\right) \right)}{2\sqrt{c}\sqrt{d}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[a + b*x]/(c + d*x^2), x]

[Out] $((-1/2*I)*(\text{Cos}[a + (I*b*\text{Sqrt}[c])/ \text{Sqrt}[d]]*\text{CosIntegral}[b*((-I)*\text{Sqrt}[c])/ \text{Sqrt}[d] + x]) - \text{Cos}[a - (I*b*\text{Sqrt}[c])/ \text{Sqrt}[d]]*\text{CosIntegral}[b*((I*\text{Sqrt}[c])/ \text{Sqrt}[d] + x)] + \text{Sin}[a - (I*b*\text{Sqrt}[c])/ \text{Sqrt}[d]]*\text{SinIntegral}[b*((I*\text{Sqrt}[c])/ \text{Sqrt}[d] + x)] + \text{Sin}[a + (I*b*\text{Sqrt}[c])/ \text{Sqrt}[d]]*\text{SinIntegral}[(I*b*\text{Sqrt}[c])/ \text{Sqrt}[d] - b*x]))/(\text{Sqrt}[c]*\text{Sqrt}[d])$

fricas [C] time = 1.12, size = 189, normalized size = 0.89

$$\frac{2i\sqrt{\frac{b^2c}{d}} \text{Ei}\left(ibx - \sqrt{\frac{b^2c}{d}}\right) e^{\left(ia + \sqrt{\frac{b^2c}{d}}\right)} - 2i\sqrt{\frac{b^2c}{d}} \text{Ei}\left(ibx + \sqrt{\frac{b^2c}{d}}\right) e^{\left(ia - \sqrt{\frac{b^2c}{d}}\right)} - 2i\sqrt{\frac{b^2c}{d}} \text{Ei}\left(-ibx - \sqrt{\frac{b^2c}{d}}\right) e^{\left(-ia + \sqrt{\frac{b^2c}{d}}\right)} + 2i\sqrt{\frac{b^2c}{d}} \text{Ei}\left(-ibx + \sqrt{\frac{b^2c}{d}}\right) e^{\left(-ia - \sqrt{\frac{b^2c}{d}}\right)}}{8bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)/(d*x^2+c), x, algorithm="fricas")

[Out] $1/8*(2*I*\text{sqrt}(b^2*c/d)*\text{Ei}(I*b*x - \text{sqrt}(b^2*c/d))*e^{(I*a + \text{sqrt}(b^2*c/d))} - 2*I*\text{sqrt}(b^2*c/d)*\text{Ei}(I*b*x + \text{sqrt}(b^2*c/d))*e^{(I*a - \text{sqrt}(b^2*c/d))} - 2*I*\text{sqrt}(b^2*c/d)*\text{Ei}(-I*b*x - \text{sqrt}(b^2*c/d))*e^{(-I*a + \text{sqrt}(b^2*c/d))} + 2*I*\text{sqrt}(b^2*c/d)*\text{Ei}(-I*b*x + \text{sqrt}(b^2*c/d))*e^{(-I*a - \text{sqrt}(b^2*c/d))})/(b*c)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)}{dx^2+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)/(d*x^2+c), x, algorithm="giac")

[Out] integrate(cos(b*x + a)/(d*x^2 + c), x)

maple [A] time = 0.05, size = 229, normalized size = 1.08

$$b \left(\frac{-\text{Si}\left(bx + a - \frac{b\sqrt{-cd}+da}{d}\right) \sin\left(\frac{b\sqrt{-cd}+da}{d}\right) + \text{Ci}\left(bx + a - \frac{b\sqrt{-cd}+da}{d}\right) \cos\left(\frac{b\sqrt{-cd}+da}{d}\right)}{2d\left(\frac{b\sqrt{-cd}+da}{d} - a\right)} + \frac{\text{Si}\left(bx + a + \frac{b\sqrt{-cd}-da}{d}\right) \sin\left(\frac{b\sqrt{-cd}-da}{d}\right) - \text{Ci}\left(bx + a + \frac{b\sqrt{-cd}-da}{d}\right) \cos\left(\frac{b\sqrt{-cd}-da}{d}\right)}{2d\left(\frac{b\sqrt{-cd}-da}{d} + a\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x+a)/(d*x^2+c),x)`

[Out] $b*(1/2/d/((b*(-c*d)^{(1/2)+d*a)/d-a)*(-\text{Si}(b*x+a-(b*(-c*d)^{(1/2)+d*a)/d})*\sin((b*(-c*d)^{(1/2)+d*a)/d)+\text{Ci}(b*x+a-(b*(-c*d)^{(1/2)+d*a)/d})*\cos((b*(-c*d)^{(1/2)+d*a)/d))+1/2/d/(-(b*(-c*d)^{(1/2)-d*a)/d-a)*(\text{Si}(b*x+a+(b*(-c*d)^{(1/2)-d*a)/d})*\sin((b*(-c*d)^{(1/2)-d*a)/d)+\text{Ci}(b*x+a+(b*(-c*d)^{(1/2)-d*a)/d})*\cos((b*(-c*d)^{(1/2)-d*a)/d))))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx + a)}{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)/(d*x^2+c),x, algorithm="maxima")`

[Out] `integrate(cos(b*x + a)/(d*x^2 + c), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a + bx)}{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b*x)/(c + d*x^2),x)`

[Out] `int(cos(a + b*x)/(c + d*x^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a + bx)}{c + dx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)/(d*x**2+c),x)`

[Out] `Integral(cos(a + b*x)/(c + d*x**2), x)`

$$3.47 \quad \int \frac{\cos(a+bx)}{c+dx+ex^2} dx$$

Optimal. Leaf size=271

$$\frac{\cos\left(a - \frac{b(d-\sqrt{d^2-4ce})}{2e}\right) \operatorname{Ci}\left(\frac{b(d-\sqrt{d^2-4ce})}{2e} + bx\right)}{\sqrt{d^2-4ce}} - \frac{\cos\left(a - \frac{b(\sqrt{d^2-4ce}+d)}{2e}\right) \operatorname{Ci}\left(\frac{b(d+\sqrt{d^2-4ce})}{2e} + bx\right)}{\sqrt{d^2-4ce}} - \frac{\sin\left(a - \frac{b(d-\sqrt{d^2-4ce})}{2e}\right)}{\sqrt{d^2-4ce}}$$

[Out] Ci(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)*cos(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)-Ci(b*x+1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*cos(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)-Si(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)*sin(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)+Si(b*x+1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*sin(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)

Rubi [A] time = 0.56, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, number of rules / integrand size = 0.210, Rules used = {6728, 3303, 3299, 3302}

$$\frac{\cos\left(a - \frac{b(d-\sqrt{d^2-4ce})}{2e}\right) \operatorname{CosIntegral}\left(\frac{b(d-\sqrt{d^2-4ce})}{2e} + bx\right)}{\sqrt{d^2-4ce}} - \frac{\cos\left(a - \frac{b(\sqrt{d^2-4ce}+d)}{2e}\right) \operatorname{CosIntegral}\left(\frac{b(\sqrt{d^2-4ce}+d)}{2e} + bx\right)}{\sqrt{d^2-4ce}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]/(c + d*x + e*x^2), x]

[Out] (Cos[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIntegral[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cos[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Sin[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] + (Sin[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 6728

Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(2*n_.)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n))], x}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)}{c+dx+ex^2} dx &= \int \left(\frac{2e \cos(a+bx)}{\sqrt{d^2-4ce} (d-\sqrt{d^2-4ce}+2ex)} - \frac{2e \cos(a+bx)}{\sqrt{d^2-4ce} (d+\sqrt{d^2-4ce}+2ex)} \right) dx \\
&= \frac{(2e) \int \frac{\cos(a+bx)}{d-\sqrt{d^2-4ce}+2ex} dx}{\sqrt{d^2-4ce}} - \frac{(2e) \int \frac{\cos(a+bx)}{d+\sqrt{d^2-4ce}+2ex} dx}{\sqrt{d^2-4ce}} \\
&= \frac{\left(2e \cos \left(a - \frac{b(d-\sqrt{d^2-4ce})}{2e} \right) \right) \int \frac{\cos \left(\frac{b(d-\sqrt{d^2-4ce})}{2e} + bx \right)}{d-\sqrt{d^2-4ce}+2ex} dx}{\sqrt{d^2-4ce}} - \frac{\left(2e \cos \left(a - \frac{b(d+\sqrt{d^2-4ce})}{2e} \right) \right) \int \frac{\cos \left(\frac{b(d+\sqrt{d^2-4ce})}{2e} + bx \right)}{d+\sqrt{d^2-4ce}+2ex} dx}{\sqrt{d^2-4ce}} \\
&= \frac{\cos \left(a - \frac{b(d-\sqrt{d^2-4ce})}{2e} \right) \text{Ci} \left(\frac{b(d-\sqrt{d^2-4ce})}{2e} + bx \right)}{\sqrt{d^2-4ce}} - \frac{\cos \left(a - \frac{b(d+\sqrt{d^2-4ce})}{2e} \right) \text{Ci} \left(\frac{b(d+\sqrt{d^2-4ce})}{2e} + bx \right)}{\sqrt{d^2-4ce}} + \dots
\end{aligned}$$

Mathematica [A] time = 0.58, size = 236, normalized size = 0.87

$$\frac{\cos \left(a + \frac{b(\sqrt{d^2-4ce}-d)}{2e} \right) \text{Ci} \left(\frac{b(d+2ex-\sqrt{d^2-4ce})}{2e} \right) - \cos \left(a - \frac{b(\sqrt{d^2-4ce}+d)}{2e} \right) \text{Ci} \left(\frac{b(d+2ex+\sqrt{d^2-4ce})}{2e} \right) + \sin \left(a + \frac{b(\sqrt{d^2-4ce}-d)}{2e} \right) \text{Si} \left(\frac{b(d+2ex-\sqrt{d^2-4ce})}{2e} \right) - \sin \left(a - \frac{b(\sqrt{d^2-4ce}+d)}{2e} \right) \text{Si} \left(\frac{b(d+2ex+\sqrt{d^2-4ce})}{2e} \right)}{\sqrt{d^2-4ce}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[a + b*x]/(c + d*x + e*x^2), x]

[Out] (Cos[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIntegral[(b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)] - Cos[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIntegral[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)] + Sin[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e) - b*x] + Sin[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)])/Sqrt[d^2 - 4*c*e]

fricas [C] time = 1.04, size = 436, normalized size = 1.61

$$-i e \sqrt{-\frac{b^2 d^2 - 4 b^2 c e}{e^2}} \text{Ei} \left(\frac{-2i b e x - i b d - e \sqrt{-\frac{b^2 d^2 - 4 b^2 c e}{e^2}}}{2e} \right) e^{\left(\frac{i b d - 2i a e + e \sqrt{-\frac{b^2 d^2 - 4 b^2 c e}{e^2}}}{2e} \right)} + i e \sqrt{-\frac{b^2 d^2 - 4 b^2 c e}{e^2}} \text{Ei} \left(\frac{-2i b e x - i b d + e \sqrt{-\frac{b^2 d^2 - 4 b^2 c e}{e^2}}}{2e} \right) e^{\left(\frac{i b d - 2i a e - e \sqrt{-\frac{b^2 d^2 - 4 b^2 c e}{e^2}}}{2e} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)/(e*x^2+d*x+c), x, algorithm="fricas")

[Out] -1/2*(-I*e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(-2*I*b*e*x - I*b*d - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(I*b*d - 2*I*a*e + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) + I*e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(-2*I*b*e*x - I*b*d + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(I*b*d - 2*I*a*e - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) + I*e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*I*b*e*x + I*b*d - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(-I*b*d + 2*I*a*e + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) - I*e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*I*b*e*x + I*b*d + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(-I*b*d + 2*I*a*e - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)

$(d^2 - 4b^2ce)/e^2)/e)^{1/2}(-Ibd + 2Iae - e\sqrt{-(b^2d^2 - 4b^2ce)/e^2})/e)/(bd^2 - 4b^2ce)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx + a)}{ex^2 + dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")

[Out] integrate(cos(b*x + a)/(e*x^2 + d*x + c), x)

maple [A] time = 0.05, size = 320, normalized size = 1.18

$$b \left(\frac{-\operatorname{Si}\left(bx + a - \frac{2ae - db + \sqrt{-4b^2ce + b^2d^2}}{2e}\right) \sin\left(\frac{2ae - db + \sqrt{-4b^2ce + b^2d^2}}{2e}\right) + \operatorname{Ci}\left(bx + a - \frac{2ae - db + \sqrt{-4b^2ce + b^2d^2}}{2e}\right) \cos\left(\frac{2ae - db + \sqrt{-4b^2ce + b^2d^2}}{2e}\right)}{\sqrt{-4b^2ce + b^2d^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)/(e*x^2+d*x+c),x)

[Out] $b \cdot (1/(-4b^2ce + b^2d^2)^{1/2}) \cdot (-\operatorname{Si}(bx + a - 1/2/e \cdot (2ae - db + (-4b^2ce + b^2d^2)^{1/2})) \cdot \sin(1/2/e \cdot (2ae - db + (-4b^2ce + b^2d^2)^{1/2})) + \operatorname{Ci}(bx + a - 1/2/e \cdot (2ae - db + (-4b^2ce + b^2d^2)^{1/2})) \cdot \cos(1/2/e \cdot (2ae - db + (-4b^2ce + b^2d^2)^{1/2}))) - 1/(-4b^2ce + b^2d^2)^{1/2} \cdot (\operatorname{Si}(bx + a + 1/2 \cdot (-2ae + db + (-4b^2ce + b^2d^2)^{1/2}))/e \cdot \sin(1/2 \cdot (-2ae + db + (-4b^2ce + b^2d^2)^{1/2}))/e + \operatorname{Ci}(bx + a + 1/2 \cdot (-2ae + db + (-4b^2ce + b^2d^2)^{1/2}))/e \cdot \cos(1/2 \cdot (-2ae + db + (-4b^2ce + b^2d^2)^{1/2}))/e))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx + a)}{ex^2 + dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")

[Out] integrate(cos(b*x + a)/(e*x^2 + d*x + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a + bx)}{ex^2 + dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b*x)/(c + d*x + e*x^2),x)

[Out] int(cos(a + b*x)/(c + d*x + e*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a + bx)}{c + dx + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)/(e*x**2+d*x+c),x)

[Out] Integral(cos(a + b*x)/(c + d*x + e*x**2), x)

$$3.48 \quad \int \frac{x \cos(\sqrt{1+x^2})}{\sqrt{1+x^2}} dx$$

Optimal. Leaf size=10

$$\sin(\sqrt{x^2+1})$$

[Out] sin((x^2+1)^(1/2))

Rubi [A] time = 0.14, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6715, 3432, 15, 2637}

$$\sin(\sqrt{x^2+1})$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]

[Out] Sin[Sqrt[1 + x^2]]

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3432

Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_)^(n_))]*(b_.))^(p_.)*((g_.) + (h_.)*(x_)^(m_.)), x_Symbol] := Dist[1/(n*f), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p, 0] && IntegerQ[1/n]

Rule 6715

Int[(u_)*(x_)^(m_.), x_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionQ[fQ[x^(m + 1), u, x]

Rubi steps

$$\begin{aligned} \int \frac{x \cos(\sqrt{1+x^2})}{\sqrt{1+x^2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\cos(\sqrt{1+x})}{\sqrt{1+x}} dx, x, x^2 \right) \\ &= \text{Subst} \left(\int \frac{x \cos(x)}{\sqrt{x^2}} dx, x, \sqrt{1+x^2} \right) \\ &= 1 \text{Subst} \left(\int \cos(x) dx, x, \sqrt{1+x^2} \right) \\ &= \sin(\sqrt{1+x^2}) \end{aligned}$$

Mathematica [A] time = 0.04, size = 10, normalized size = 1.00

$$\sin\left(\sqrt{x^2 + 1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x*Cos[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]

[Out] Sin[Sqrt[1 + x^2]]

fricas [A] time = 1.63, size = 8, normalized size = 0.80

$$\sin\left(\sqrt{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="fricas")

[Out] sin(sqrt(x^2 + 1))

giac [A] time = 0.14, size = 8, normalized size = 0.80

$$\sin\left(\sqrt{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="giac")

[Out] sin(sqrt(x^2 + 1))

maple [A] time = 0.04, size = 9, normalized size = 0.90

$$\sin\left(\sqrt{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x)

[Out] sin((x^2+1)^(1/2))

maxima [A] time = 0.35, size = 8, normalized size = 0.80

$$\sin\left(\sqrt{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="maxima")

[Out] sin(sqrt(x^2 + 1))

mupad [B] time = 2.26, size = 8, normalized size = 0.80

$$\sin\left(\sqrt{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*cos((x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2),x)

[Out] sin((x^2 + 1)^(1/2))

sympy [A] time = 0.43, size = 8, normalized size = 0.80

$$\sin\left(\sqrt{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((x**2+1)**(1/2))/(x**2+1)**(1/2), x)
```

```
[Out] sin(sqrt(x**2 + 1))
```

$$3.49 \quad \int \frac{x \cos\left(\sqrt{3} \sqrt{2+x^2}\right)}{\sqrt{2+x^2}} dx$$

Optimal. Leaf size=22

$$\frac{\sin\left(\sqrt{3} \sqrt{x^2 + 2}\right)}{\sqrt{3}}$$

[Out] 1/3*sin(3^(1/2)*(x^2+2)^(1/2))*3^(1/2)

Rubi [A] time = 0.19, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6715, 3432, 15, 2637}

$$\frac{\sin\left(\sqrt{3} \sqrt{x^2 + 2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[Sqrt[3]*Sqrt[2 + x^2]])/Sqrt[2 + x^2], x]

[Out] Sin[Sqrt[3]*Sqrt[2 + x^2]]/Sqrt[3]

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3432

Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])*(b_.))^(p_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Dist[1/(n*f), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p, 0] && IntegerQ[1/n]

Rule 6715

Int[(u_)*(x_)^(m_.), x_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

Rubi steps

$$\begin{aligned}
\int \frac{x \cos(\sqrt{3} \sqrt{2+x^2})}{\sqrt{2+x^2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{\cos(\sqrt{3} \sqrt{2+x})}{\sqrt{2+x}} dx, x, x^2 \right) \\
&= \text{Subst} \left(\int \frac{x \cos(\sqrt{3} x)}{\sqrt{x^2}} dx, x, \sqrt{2+x^2} \right) \\
&= 1 \text{Subst} \left(\int \cos(\sqrt{3} x) dx, x, \sqrt{2+x^2} \right) \\
&= \frac{\sin(\sqrt{3} \sqrt{2+x^2})}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 22, normalized size = 1.00

$$\frac{\sin(\sqrt{3} \sqrt{x^2+2})}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Cos[Sqrt[3]*Sqrt[2 + x^2]])/Sqrt[2 + x^2],x]

[Out] Sin[Sqrt[3]*Sqrt[2 + x^2]]/Sqrt[3]

fricas [B] time = 0.94, size = 37, normalized size = 1.68

$$\frac{2\sqrt{3} \tan\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+2}\right)}{3\left(\tan\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x, algorithm="fricas")

[Out] 2/3*sqrt(3)*tan(1/2*sqrt(3)*sqrt(x^2 + 2))/(tan(1/2*sqrt(3)*sqrt(x^2 + 2))^2 + 1)

giac [A] time = 0.12, size = 17, normalized size = 0.77

$$\frac{1}{3}\sqrt{3} \sin(\sqrt{3} \sqrt{x^2+2})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x, algorithm="giac")

[Out] 1/3*sqrt(3)*sin(sqrt(3)*sqrt(x^2 + 2))

maple [A] time = 0.08, size = 18, normalized size = 0.82

$$\frac{\sin(\sqrt{3} \sqrt{x^2+2})\sqrt{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x)

[Out] 1/3*sin(3^(1/2)*(x^2+2)^(1/2))*3^(1/2)

maxima [A] time = 0.41, size = 17, normalized size = 0.77

$$\frac{1}{3} \sqrt{3} \sin\left(\sqrt{3} \sqrt{x^2 + 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*sin(sqrt(3)*sqrt(x^2 + 2))

mupad [B] time = 2.32, size = 15, normalized size = 0.68

$$\frac{\sqrt{3} \sin\left(\sqrt{3x^2 + 6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*cos(3^(1/2)*(x^2 + 2)^(1/2)))/(x^2 + 2)^(1/2),x)

[Out] (3^(1/2)*sin((3*x^2 + 6)^(1/2)))/3

sympy [A] time = 0.68, size = 20, normalized size = 0.91

$$\frac{\sqrt{3} \sin\left(\sqrt{3} \sqrt{x^2 + 2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(3**(1/2)*(x**2+2)**(1/2))/(x**2+2)**(1/2),x)

[Out] sqrt(3)*sin(sqrt(3)*sqrt(x**2 + 2))/3

$$3.50 \quad \int \frac{(-1+2x) \cos\left(\sqrt{6+3(-1+2x)^2}\right)}{\sqrt{6+3(-1+2x)^2}} dx$$

Optimal. Leaf size=24

$$\frac{1}{6} \sin\left(\sqrt{3} \sqrt{(2x-1)^2+2}\right)$$

[Out] 1/6*sin(3^(1/2)*(2+(-1+2*x)^2)^(1/2))

Rubi [A] time = 0.49, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6715, 3432, 15, 2637}

$$\frac{1}{6} \sin\left(\sqrt{3} \sqrt{(2x-1)^2+2}\right)$$

Antiderivative was successfully verified.

[In] Int[((-1 + 2*x)*Cos[Sqrt[6 + 3*(-1 + 2*x)^2]])/Sqrt[6 + 3*(-1 + 2*x)^2], x]

[Out] Sin[Sqrt[3]*Sqrt[2 + (-1 + 2*x)^2]]/6

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3432

Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_)^(n_))]*(b_.))^(p_.)*((g_.) + (h_.)*(x_)^(m_.)), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p, 0] && IntegerQ[1/n]

Rule 6715

Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

Rubi steps

$$\begin{aligned}
\int \frac{(-1+2x) \cos(\sqrt{6+3(-1+2x)^2})}{\sqrt{6+3(-1+2x)^2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x \cos(\sqrt{6+3x^2})}{\sqrt{6+3x^2}} dx, x, -1+2x \right) \\
&= \frac{1}{4} \text{Subst} \left(\int \frac{\cos(\sqrt{6+3x})}{\sqrt{6+3x}} dx, x, (-1+2x)^2 \right) \\
&= \frac{1}{6} \text{Subst} \left(\int \frac{x \cos(x)}{\sqrt{x^2}} dx, x, \sqrt{3} \sqrt{2+(-1+2x)^2} \right) \\
&= \frac{1}{6} \text{Subst} \left(\int \cos(x) dx, x, \sqrt{3} \sqrt{2+(-1+2x)^2} \right) \\
&= \frac{1}{6} \sin \left(\sqrt{3} \sqrt{2+(-1+2x)^2} \right)
\end{aligned}$$

Mathematica [A] time = 0.17, size = 20, normalized size = 0.83

$$\frac{1}{6} \sin \left(\sqrt{3(1-2x)^2 + 6} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((-1+2*x)*Cos[Sqrt[6+3*(-1+2*x)^2]])/Sqrt[6+3*(-1+2*x)^2],x]

[Out] Sin[Sqrt[6+3*(1-2*x)^2]]/6

fricas [A] time = 1.70, size = 15, normalized size = 0.62

$$\frac{1}{6} \sin \left(\sqrt{12x^2 - 12x + 9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x, algorithm="fricas")

[Out] 1/6*sin(sqrt(12*x^2 - 12*x + 9))

giac [A] time = 0.15, size = 19, normalized size = 0.79

$$\frac{1}{6} \sin \left(\sqrt{3} \sqrt{4x^2 - 4x + 3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x, algorithm="giac")

[Out] 1/6*sin(sqrt(3)*sqrt(4*x^2 - 4*x + 3))

maple [A] time = 0.07, size = 16, normalized size = 0.67

$$\frac{\sin \left(\sqrt{12x^2 - 12x + 9} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x)

[Out] 1/6*sin((12*x^2-12*x+9)^(1/2))

maxima [A] time = 0.31, size = 16, normalized size = 0.67

$$\frac{1}{6} \sin\left(\sqrt{3(2x-1)^2+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x, algorithm="maxima")

[Out] 1/6*sin(sqrt(3*(2*x - 1)^2 + 6))

mupad [B] time = 2.35, size = 16, normalized size = 0.67

$$\frac{\sin\left(\sqrt{3(2x-1)^2+6}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos((3*(2*x - 1)^2 + 6)^(1/2))*(2*x - 1))/(3*(2*x - 1)^2 + 6)^(1/2),x)

[Out] sin((3*(2*x - 1)^2 + 6)^(1/2))/6

sympy [A] time = 5.49, size = 15, normalized size = 0.62

$$\frac{\sin\left(\sqrt{3(2x-1)^2+6}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+2*x)*cos((6+3*(-1+2*x)**2)**(1/2))/(6+3*(-1+2*x)**2)**(1/2),x)

[Out] sin(sqrt(3*(2*x - 1)**2 + 6))/6

3.51 $\int \cos\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=101

$$-\frac{\sin\left(\frac{b}{d}\right)(bc-ad)\text{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{\cos\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d}$$

[Out] $(d*x+c)*\cos((b*x+a)/(d*x+c))/d+(-a*d+b*c)*\cos(b/d)*\text{Si}((-a*d+b*c)/d/(d*x+c))/d^2-(-a*d+b*c)*\text{Ci}((-a*d+b*c)/d/(d*x+c))*\sin(b/d)/d^2$

Rubi [A] time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4564, 3297, 3303, 3299, 3302}

$$-\frac{\sin\left(\frac{b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{\cos\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cos[(a + b*x)/(c + d*x)], x]

[Out] $((c + d*x)*\text{Cos}[(a + b*x)/(c + d*x)]/d - ((b*c - a*d)*\text{CosIntegral}[(b*c - a*d)/(d*(c + d*x))]*\text{Sin}[b/d])/d^2 + ((b*c - a*d)*\text{Cos}[b/d]*\text{SinIntegral}[(b*c - a*d)/(d*(c + d*x))])/d^2$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 4564

Int[Cos[((e_.)*(a_.) + (b_.)*(x_))]/((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> -Dist[d^(-1), Subst[Int[Cos[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x], x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d, 0]

Rubi steps

$$\int \cos\left(\frac{a+bx}{c+dx}\right) dx = \frac{\text{Subst}\left(\int \frac{\cos\left(\frac{b}{d} - \frac{(bc-ad)x}{d}\right)}{x^2} dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx) \cos\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad) \text{Subst}\left(\int \frac{\sin\left(\frac{b}{d} - \frac{(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx) \cos\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left((bc-ad) \cos\left(\frac{b}{d}\right)\right) \text{Subst}\left(\int \frac{\sin\left(\frac{(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2} - \frac{\left((bc-ad) \sin\left(\frac{b}{d}\right)\right) \text{Subst}\left(\int \frac{\cos\left(\frac{(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx) \cos\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad) \text{Ci}\left(\frac{bc-ad}{d(c+dx)}\right) \sin\left(\frac{b}{d}\right)}{d^2} + \frac{(bc-ad) \cos\left(\frac{b}{d}\right) \text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

Mathematica [C] time = 5.51, size = 260, normalized size = 2.57

$$\frac{-4 \sin\left(\frac{b}{d}\right) (bc-ad) \text{Ci}\left(\frac{ad-bc}{d(c+dx)}\right) + d \exp\left(-\frac{i(ad+2bc+bdx)}{d(c+dx)}\right) \left(2c \left(e^{2i\left(\frac{a}{c+dx} + \frac{b}{d}\right)} + e^{\frac{2ibc}{d(c+dx)}}\right) + dx \left(1 + e^{\frac{2ib}{d}}\right) \left(e^{\frac{2ia}{c+dx}} + e^{\frac{2ibc}{d(c+dx)}}\right)\right)}{4d^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[(a + b*x)/(c + d*x)], x]

[Out] $(-4*(b*c - a*d)*\text{CosIntegral}[(-(b*c) + a*d)/(d*(c + d*x))]*\text{Sin}[b/d] + (d*(2*c*(E^(((2*I)*b*c)/(d*(c + d*x))) + E^(((2*I)*(b/d + a/(c + d*x)))))) + d*(1 + E^(((2*I)*b)/d))*E^(((2*I)*a)/(c + d*x)) + E^(((2*I)*b*c)/(d*(c + d*x))))*x - 4*d*E^(((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x)))*x*\text{Sin}[b/d]*\text{Sin}[(-(b*c) + a*d)/(d*(c + d*x))])/E^(((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x))) - 4*(b*c - a*d)*\text{Cos}[b/d]*\text{SinIntegral}[(-(b*c) + a*d)/(d*(c + d*x))])/ (4*d^2)$

fricas [A] time = 4.12, size = 138, normalized size = 1.37

$$\frac{2(bc-ad) \cos\left(\frac{b}{d}\right) \text{Si}\left(-\frac{bc-ad}{d^2x+cd}\right) - 2(d^2x+cd) \cos\left(\frac{bx+a}{dx+c}\right) + \left((bc-ad) \text{Ci}\left(\frac{bc-ad}{d^2x+cd}\right) + (bc-ad) \text{Ci}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos((b*x+a)/(d*x+c)), x, algorithm="fricas")

[Out] $-1/2*(2*(b*c - a*d)*\cos(b/d)*\text{sin_integral}(-(b*c - a*d)/(d^2*x + c*d)) - 2*(d^2*x + c*d)*\cos((b*x + a)/(d*x + c)) + ((b*c - a*d)*\cos_integral((b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*\cos_integral(-(b*c - a*d)/(d^2*x + c*d)))*\text{sin}(b/d)/d^2$

giac [B] time = 12.27, size = 633, normalized size = 6.27

$$\frac{\left(b^3c^2 \text{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right) \sin\left(\frac{b}{d}\right) - 2ab^2cd \text{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right) \sin\left(\frac{b}{d}\right) - \frac{(bx+a)b^2c^2d \text{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right) \sin\left(\frac{b}{d}\right)}{dx+c} + a^2bd^2 \text{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right) \sin\left(\frac{b}{d}\right)\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos((b*x+a)/(d*x+c)), x, algorithm="giac")

```
[Out] -(b^3*c^2*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d) - 2*a*b^2*c
*d*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d) - (b*x + a)*b^2*c^
2*d*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d)/(d*x + c) + a^2*b
*d^2*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d) + 2*(b*x + a)*a*
b*c*d^2*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d)/(d*x + c) - (
b*x + a)*a^2*d^3*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d)/(d*x
+ c) - b^3*c^2*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) + 2*a*
b^2*c*d*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) + (b*x + a)*b^
2*c^2*d*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - a^
2*b*d^2*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) - 2*(b*x + a)*
a*b*c*d^2*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) +
(b*x + a)*a^2*d^3*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x
+ c) - b^2*c^2*d*cos((b*x + a)/(d*x + c)) + 2*a*b*c*d^2*cos((b*x + a)/(d*x
+ c)) - a^2*d^3*cos((b*x + a)/(d*x + c))*(b*c/(b*c - a*d)^2 - a*d/(b*c -
a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

maple [A] time = 0.10, size = 142, normalized size = 1.41

$$-(da - cb) \left(\frac{\cos\left(\frac{b}{d} + \frac{da-cb}{d(dx+c)}\right)}{\left(\left(\frac{b}{d} + \frac{da-cb}{d(dx+c)}\right)d - b\right)d} - \frac{\frac{\text{Si}\left(\frac{da-cb}{d(dx+c)}\right)\cos\left(\frac{b}{d}\right)}{d} + \frac{\text{Ci}\left(\frac{da-cb}{d(dx+c)}\right)\sin\left(\frac{b}{d}\right)}{d}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((b*x+a)/(d*x+c)),x)
```

```
[Out] -(a*d-b*c)*(-cos(b/d+(a*d-b*c)/d/(d*x+c))/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/d
-(Si((a*d-b*c)/d/(d*x+c))*cos(b/d)/d+Ci((a*d-b*c)/d/(d*x+c))*sin(b/d)/d)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{bx + a}{dx + c}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos((b*x + a)/(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(\frac{a + bx}{c + dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((a + b*x)/(c + d*x)),x)
```

```
[Out] int(cos((a + b*x)/(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(\frac{a + bx}{c + dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c)),x)
```

```
[Out] Integral(cos((a + b*x)/(c + d*x)), x)
```

3.52 $\int \cos^2\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=107

$$-\frac{\sin\left(\frac{2b}{d}\right)(bc-ad)\text{Ci}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2} + \frac{\cos\left(\frac{2b}{d}\right)(bc-ad)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

[Out] $(d*x+c)*\cos((b*x+a)/(d*x+c))^2/d+(-a*d+b*c)*\cos(2*b/d)*\text{Si}(2*(-a*d+b*c)/d/(d*x+c))/d^2-(-a*d+b*c)*\text{Ci}(2*(-a*d+b*c)/d/(d*x+c))*\sin(2*b/d)/d^2$

Rubi [A] time = 0.16, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4564, 3313, 12, 3303, 3299, 3302}

$$-\frac{\sin\left(\frac{2b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2} + \frac{\cos\left(\frac{2b}{d}\right)(bc-ad)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

[In] `Int[Cos[(a + b*x)/(c + d*x)]^2, x]`

[Out] $((c + d*x)*\text{Cos}[(a + b*x)/(c + d*x)]^2)/d - ((b*c - a*d)*\text{CosIntegral}[(2*(b*c - a*d))/(d*(c + d*x))]*\text{Sin}[(2*b)/d])/d^2 + ((b*c - a*d)*\text{Cos}[(2*b)/d]*\text{SinIntegral}[(2*(b*c - a*d))/(d*(c + d*x))])/d^2$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

Rule 3299

`Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]`

Rule 3302

`Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]`

Rule 3303

`Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]`

Rule 3313

`Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m + 1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n - 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]`

Rule 4564

```
Int[Cos[((e._)*((a._) + (b._)*(x_)))/((c._) + (d._)*(x_))]^(n_), x_Symbol]
:> -Dist[d^(-1), Subst[Int[Cos[(b*e)/d - (e*(b*c - a*d)*x)/d]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rubi steps

$$\begin{aligned} \int \cos^2\left(\frac{a+bx}{c+dx}\right) dx &= \frac{\text{Subst}\left(\int \frac{\cos^2\left(\frac{b}{d} - \frac{(bc-ad)x}{d}\right)}{x^2} dx, x, \frac{1}{c+dx}\right)}{d} \\ &= \frac{(c+dx) \cos^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(2(bc-ad)) \text{Subst}\left(\int -\frac{\sin\left(\frac{2b}{d} - \frac{2(bc-ad)x}{d}\right)}{2x} dx, x, \frac{1}{c+dx}\right)}{d^2} \\ &= \frac{(c+dx) \cos^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad) \text{Subst}\left(\int \frac{\sin\left(\frac{2b}{d} - \frac{2(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2} \\ &= \frac{(c+dx) \cos^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left((bc-ad) \cos\left(\frac{2b}{d}\right)\right) \text{Subst}\left(\int \frac{\sin\left(\frac{2(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2} - \frac{(bc-ad)}{d^2} \\ &= \frac{(c+dx) \cos^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad) \text{Ci}\left(\frac{2(bc-ad)}{d(c+dx)}\right) \sin\left(\frac{2b}{d}\right)}{d^2} + \frac{(bc-ad) \cos\left(\frac{2b}{d}\right) \text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2} \end{aligned}$$

Mathematica [C] time = 6.14, size = 400, normalized size = 3.74

$$\frac{(acd - bc^2) \left(\frac{\left(-1 + e^{\frac{4ib}{d}} \right) \left(e^{\frac{4ibc}{d(c+dx)}} - e^{\frac{4ia}{c+dx}} \right) \exp\left(-\frac{2i(ad+2bc+bdx)}{d(c+dx)} \right)}{8(bc-ad)} - \frac{\left(1 + e^{\frac{4ib}{d}} \right) \left(e^{\frac{4ia}{c+dx}} + e^{\frac{4ibc}{d(c+dx)}} \right) \exp\left(-\frac{2i(ad+2bc+bdx)}{d(c+dx)} \right)}{8(bc-ad)} \right)}{d} + \frac{2ad \sin\left(\frac{2b}{d}\right) \text{Ci}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[(a + b*x)/(c + d*x)]^2, x]
```

```
[Out] (((-b*c^2) + a*c*d)*(((1 - E^(((4*I)*b)/d))*(-E^(((4*I)*a)/(c + d*x)) + E^
(((4*I)*b*c)/(d*(c + d*x)))))/(8*(b*c - a*d)*E^(((2*I)*(2*b*c + a*d + b*d*x)
))/(d*(c + d*x)))) - ((1 + E^(((4*I)*b)/d))*(-E^(((4*I)*a)/(c + d*x)) + E^((
(4*I)*b*c)/(d*(c + d*x)))))/(8*(b*c - a*d)*E^(((2*I)*(2*b*c + a*d + b*d*x)
)/(d*(c + d*x)))))/d + (x*Cos[(2*b)/d]*Cos[(2*(-b*c) + a*d))/(d*(c + d*x)
])/2 - (x*Sin[(2*b)/d]*Sin[(2*(-b*c) + a*d))/(d*(c + d*x)))/2 + (d^2*x -
2*b*c*CosIntegral[(2*(-b*c) + a*d)/(d*(c + d*x))]*Sin[(2*b)/d] + 2*a*d*Co
sIntegral[(2*(-b*c) + a*d)/(d*(c + d*x))]*Sin[(2*b)/d] - 2*b*c*Cos[(2*b)/
d]*SinIntegral[(2*(-b*c) + a*d)/(d*(c + d*x))] + 2*a*d*Cos[(2*b)/d]*SinIn
tegral[(2*(-b*c) + a*d)/(d*(c + d*x))]/(2*d^2)
```

fricas [A] time = 2.20, size = 144, normalized size = 1.35

$$\frac{2(d^2x + cd) \cos\left(\frac{bx+a}{dx+c}\right)^2 - 2(bc-ad) \cos\left(\frac{2b}{d}\right) \text{Si}\left(-\frac{2(bc-ad)}{d^2x+cd}\right) - \left((bc-ad) \text{Ci}\left(\frac{2(bc-ad)}{d^2x+cd}\right) + (bc-ad) \text{Ci}\left(-\frac{2(bc-ad)}{d^2x+cd}\right)\right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```


[Out] $1/2*(2*(d^2*x + c*d)*\cos((b*x + a)/(d*x + c))^2 - 2*(b*c - a*d)*\cos(2*b/d)*\sin_integral(-2*(b*c - a*d)/(d^2*x + c*d)) - ((b*c - a*d)*\cos_integral(2*(b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*\cos_integral(-2*(b*c - a*d)/(d^2*x + c*d)))*\sin(2*b/d))/d^2$

giac [B] time = 68.63, size = 683, normalized size = 6.38

$$\left(2b^3c^2 \operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right) \sin\left(\frac{2b}{d}\right) - 4ab^2cd \operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right) \sin\left(\frac{2b}{d}\right) - \frac{2(bx+a)b^2c^2d \operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right) \sin\left(\frac{2b}{d}\right)}{dx+c} + 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos((b*x+a)/(d*x+c))^2,x, algorithm="giac")`

[Out] $-1/2*(2*b^3*c^2*\cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*\sin(2*b/d) - 4*a*b^2*c*d*\cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*\sin(2*b/d) - 2*(b*x + a)*b^2*c^2*d*\cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*\sin(2*b/d)/(d*x + c) + 2*a^2*b*d^2*\cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*\sin(2*b/d) + 4*(b*x + a)*a*b*c*d^2*\cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*\sin(2*b/d)/(d*x + c) - 2*(b*x + a)*a^2*d^3*\cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*\sin(2*b/d)/(d*x + c) - 2*b^3*c^2*\cos(2*b/d)*\sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d) + 4*a*b^2*c*d*\cos(2*b/d)*\sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d) + 2*(b*x + a)*b^2*c^2*d*\cos(2*b/d)*\sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 2*a^2*b*d^2*\cos(2*b/d)*\sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d) - 4*(b*x + a)*a*b*c*d^2*\cos(2*b/d)*\sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + 2*(b*x + a)*a^2*d^3*\cos(2*b/d)*\sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - b^2*c^2*d*\cos(2*(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*\cos(2*(b*x + a)/(d*x + c)) - a^2*d^3*\cos(2*(b*x + a)/(d*x + c)) - b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))$

maple [A] time = 0.14, size = 195, normalized size = 1.82

$$\frac{(da - cb) \left(\frac{d^2 \left(\frac{2 \cos\left(\frac{2da-2cb}{d(dx+c)} + \frac{2b}{d}\right)}{\left(\frac{b}{d} + \frac{da-cb}{d(dx+c)}\right)^{d-b}} \right)}{4} - \frac{2 \operatorname{Si}\left(\frac{2da-2cb}{d(dx+c)}\right) \cos\left(\frac{2b}{d}\right) + 2 \operatorname{Ci}\left(\frac{2da-2cb}{d(dx+c)}\right) \sin\left(\frac{2b}{d}\right)}{d} \right)}{2 \left(\frac{b}{d} + \frac{da-cb}{d(dx+c)}\right)^{d-b}} - \frac{d}{2 \left(\frac{b}{d} + \frac{da-cb}{d(dx+c)}\right)^{d-b}} \right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos((b*x+a)/(d*x+c))^2,x)`

[Out] $-1/d^2*(a*d-b*c)*(1/4*d^2*(-2*\cos(2*(a*d-b*c)/d/(d*x+c)+2*b/d)/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/d-2*(2*\operatorname{Si}(2*(a*d-b*c)/d/(d*x+c))*\cos(2*b/d)/d+2*\operatorname{Ci}(2*(a*d-b*c)/d/(d*x+c))*\sin(2*b/d)/d)/d-1/2*d/((b/d+(a*d-b*c)/d/(d*x+c))*d-b))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x + \frac{1}{2} \int \cos\left(\frac{2(bx+a)}{dx+c}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos((b*x+a)/(d*x+c))^2,x, algorithm="maxima")

[Out] 1/2*x + 1/2*integrate(cos(2*(b*x + a)/(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(\frac{a + bx}{c + dx}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos((a + b*x)/(c + d*x))^2,x)

[Out] int(cos((a + b*x)/(c + d*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos((b*x+a)/(d*x+c))**2,x)

[Out] Timed out

$$3.53 \quad \int \frac{\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Optimal. Leaf size=58

$$-\frac{3\text{Ci}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a} - \frac{\text{Ci}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

[Out] $-3/4*\text{Ci}((-a*x+1)^{(1/2)}/(a*x+1)^{(1/2)})/a-1/4*\text{Ci}(3*(-a*x+1)^{(1/2)}/(a*x+1)^{(1/2)})/a$

Rubi [A] time = 0.11, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6681, 3312, 3302}

$$-\frac{3\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a} - \frac{\text{CosIntegral}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

[In] `Int[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]`

[Out] $(-3*\text{CosIntegral}[\text{Sqrt}[1 - a*x]/\text{Sqrt}[1 + a*x]])/(4*a) - \text{CosIntegral}[(3*\text{Sqrt}[1 - a*x])/\text{Sqrt}[1 + a*x]]/(4*a)$

Rule 3302

`Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]`

Rule 3312

`Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))`

Rule 6681

`Int[((a_.) + (b_.)*(F_))(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)])^(n_)/((A_.) + (C_.)*(x_)^2), x_Symbol] := Dist[(2*e*g)/(C*(e*f - d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] && IGtQ[n, 0]`

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx &= -\frac{\text{Subst}\left(\int \frac{\cos^3(x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4x} + \frac{\cos(3x)}{4x}\right) dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\text{Subst}\left(\int \frac{\cos(3x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{3\text{Subst}\left(\int \frac{\cos(x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} \\
&= -\frac{3\text{Ci}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{\text{Ci}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.91

$$-\frac{3\text{Ci}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) + \text{Ci}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2), x]

[Out] -1/4*(3*CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + CosIntegral[(3*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a

fricas [F] time = 2.69, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1), x, algorithm="fricas")

[Out] integral(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1), x, algorithm="giac")

[Out] integrate(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\cos^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)`

[Out] `int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="maxima")`

[Out] `-integrate(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1),x)`

[Out] `-int(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cos^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)`

[Out] `-Integral(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)`

$$3.54 \quad \int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Optimal. Leaf size=58

$$-\frac{\text{Ci}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

[Out] $-1/2*\text{Ci}(2*(-a*x+1)^{(1/2)}/(a*x+1)^{(1/2)})/a-1/2*\ln((-a*x+1)^{(1/2)}/(a*x+1)^{(1/2)})/a$

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6681, 3312, 3302}

$$-\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

[In] `Int[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]`

[Out] $-\text{CosIntegral}[(2*\text{Sqrt}[1 - a*x])/ \text{Sqrt}[1 + a*x]]/(2*a) - \text{Log}[\text{Sqrt}[1 - a*x]/ \text{Sqrt}[1 + a*x]]/(2*a)$

Rule 3302

`Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]`

Rule 3312

`Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))`

Rule 6681

`Int[((a_.) + (b_.)*(F_))(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] := Dist[(2*e*g)/(C*(e*f - d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] && IGtQ[n, 0]`

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx &= -\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x} + \frac{\cos(2x)}{2x}\right) dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} \\
&= -\frac{\text{Ci}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.88

$$-\frac{\text{Ci}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right) + \log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]

[Out] -1/2*(CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/a

fricas [F] time = 1.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1), x, algorithm="fricas")

[Out] integral(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1), x, algorithm="giac")

[Out] integrate(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)`

[Out] `int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a \int \frac{\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1} dx + a \int \frac{\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{(a^2x^2-1)\cos\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2 + (a^2x^2-1)\sin\left(\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2} dx - \log(ax+1) + \log(ax-1)$$

$$4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="maxima")`

[Out] `-1/4*(4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x) + 4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/((a^2*x^2 - 1)*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^2*x^2 - 1)*sin(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x) - log(a*x + 1) + log(a*x - 1))/a`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$- \int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1),x)`

[Out] `-int(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\cos^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)`

[Out] `-Integral(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)`

$$3.55 \quad \int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Optimal. Leaf size=26

$$-\frac{\text{Ci}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

[Out] -Ci((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {6681, 3302}

$$-\frac{\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]

[Out] -(CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 6681

Int[((a_.) + (b_.)*(F_))(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] := Dist[(2*e*g)/(C*(e*f - d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx &= -\frac{\text{Subst}\left(\int \frac{\cos(x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\ &= -\frac{\text{Ci}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$-\frac{\text{Ci}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]

[Out] -(CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)

fricas [F] time = 1.77, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2 - 1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fricas")

[Out] integral(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="giac")

[Out] integrate(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)

[Out] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="maxima")

[Out] -integrate(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)

[Out] -int(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1), x)

[Out] -Integral(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(-a**2*x**2 + 1), x)

$$3.56 \quad \int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Optimal. Leaf size=40

$$\text{Int} \left(\frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)}, x \right)$$

[Out] Unintegrable(sec((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a*x+1)/(a*x+1), x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]

[Out] -(Defer[Subst][Defer[Int][Sec[x]/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)

Rubi steps

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx = -\frac{\text{Subst}\left(\int \frac{\sec(x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 3.07, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]

[Out] Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{1}{(a^2x^2 - 1) \cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))), x, algorithm="fricas")

[Out] integral(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(a^2x^2 - 1) \cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="giac")

[Out] integrate(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)

maple [A] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{(-a^2x^2 + 1) \cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)

[Out] int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{1}{(a^2x^2 - 1) \cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="maxima")

[Out] -integrate(1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$- \int \frac{1}{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right) (a^2x^2 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)

[Out] -int(1/(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{1}{a^2x^2 \cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right) - \cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a**2*x**2+1)/cos((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)

[Out] -Integral(1/(a**2*x**2*cos(sqrt(-a*x + 1)/sqrt(a*x + 1)) - cos(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)

$$3.57 \quad \int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Optimal. Leaf size=42

$$\text{Int}\left(\frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{(1-ax)(ax+1)}, x\right)$$

[Out] Unintegrable(sec((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a*x+1)/(a*x+1), x)

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]

[Out] -(Defer[Subst][Defer[Int][Sec[x]^2/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)

Rubi steps

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx = -\frac{\text{Subst}\left(\int \frac{\sec^2(x)}{x} dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 11.88, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]

[Out] Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]

fricas [A] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{(a^2x^2 - 1)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="fricas")

[Out] integral(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(a^2x^2 - 1)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="giac")

[Out] integrate(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{(-a^2x^2 + 1) \cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)

[Out] int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="maxima")

[Out] Timed out

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)^2 (a^2x^2 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)),x)

[Out] -int(1/(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2x^2 \cos^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right) - \cos^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a**2*x**2+1)/cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)

[Out] -Integral(1/(a**2*x**2*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))**2 - cos(sqrt(-a*x + 1)/sqrt(a*x + 1))**2), x)

$$3.58 \quad \int \frac{\tan(\sqrt{x})}{\sqrt{x}} dx$$

Optimal. Leaf size=9

$$-2 \log(\cos(\sqrt{x}))$$

[Out] -2*ln(cos(x^(1/2)))

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3747, 3475}

$$-2 \log(\cos(\sqrt{x}))$$

Antiderivative was successfully verified.

[In] Int[Tan[Sqrt[x]]/Sqrt[x],x]

[Out] -2*Log[Cos[Sqrt[x]]]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3747

Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\tan(\sqrt{x})}{\sqrt{x}} dx &= 2 \text{Subst} \left(\int \tan(x) dx, x, \sqrt{x} \right) \\ &= -2 \log(\cos(\sqrt{x})) \end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$-2 \log(\cos(\sqrt{x}))$$

Antiderivative was successfully verified.

[In] Integrate[Tan[Sqrt[x]]/Sqrt[x],x]

[Out] -2*Log[Cos[Sqrt[x]]]

fricas [A] time = 2.00, size = 13, normalized size = 1.44

$$-\log\left(\frac{1}{\tan(\sqrt{x})^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x^(1/2))/x^(1/2),x, algorithm="fricas")

[Out] $-\log(1/(\tan(\sqrt{x})^2 + 1))$

giac [A] time = 0.14, size = 8, normalized size = 0.89

$$-2 \log(|\cos(\sqrt{x})|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x^(1/2))/x^(1/2),x, algorithm="giac")`

[Out] $-2*\log(\text{abs}(\cos(\sqrt{x})))$

maple [A] time = 0.00, size = 8, normalized size = 0.89

$$-2 \ln(\cos(\sqrt{x}))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x^(1/2))/x^(1/2),x)`

[Out] $-2*\ln(\cos(x^(1/2)))$

maxima [A] time = 0.30, size = 7, normalized size = 0.78

$$2 \log(\sec(\sqrt{x}))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x^(1/2))/x^(1/2),x, algorithm="maxima")`

[Out] $2*\log(\sec(\sqrt{x}))$

mupad [B] time = 3.18, size = 19, normalized size = 2.11

$$-2 \ln(e^{\sqrt{x} 2i} + 1) + \sqrt{x} 2i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(x^(1/2))/x^(1/2),x)`

[Out] $x^(1/2)*2i - 2*\log(\exp(x^(1/2)*2i) + 1)$

sympy [A] time = 0.17, size = 10, normalized size = 1.11

$$\log(\tan^2(\sqrt{x}) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(x**(1/2))/x**(1/2),x)`

[Out] $\log(\tan(\sqrt{x})**2 + 1)$

$$3.59 \quad \int \frac{\tan^2(\sqrt{x})}{\sqrt{x}} dx$$

Optimal. Leaf size=16

$$2 \tan(\sqrt{x}) - 2\sqrt{x}$$

[Out] $-2*x^{(1/2)}+2*\tan(x^{(1/2)})$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3747, 3473, 8}

$$2 \tan(\sqrt{x}) - 2\sqrt{x}$$

Antiderivative was successfully verified.

[In] Int[Tan[Sqrt[x]]^2/Sqrt[x], x]

[Out] $-2*\text{Sqrt}[x] + 2*\text{Tan}[\text{Sqrt}[x]]$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3747

Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\tan^2(\sqrt{x})}{\sqrt{x}} dx &= 2 \text{Subst} \left(\int \tan^2(x) dx, x, \sqrt{x} \right) \\ &= 2 \tan(\sqrt{x}) - 2 \text{Subst} \left(\int 1 dx, x, \sqrt{x} \right) \\ &= -2\sqrt{x} + 2 \tan(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.04, size = 18, normalized size = 1.12

$$2 \tan(\sqrt{x}) - 2 \tan^{-1}(\tan(\sqrt{x}))$$

Antiderivative was successfully verified.

[In] Integrate[Tan[Sqrt[x]]^2/Sqrt[x], x]

[Out] $-2*\text{ArcTan}[\text{Tan}[\text{Sqrt}[x]]] + 2*\text{Tan}[\text{Sqrt}[x]]$

fricas [A] time = 1.97, size = 12, normalized size = 0.75

$$-2 \sqrt{x} + 2 \tan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x^(1/2))^2/x^(1/2),x, algorithm="fricas")

[Out] -2*sqrt(x) + 2*tan(sqrt(x))

giac [A] time = 0.12, size = 12, normalized size = 0.75

$$-2\sqrt{x} + 2 \tan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x^(1/2))^2/x^(1/2),x, algorithm="giac")

[Out] -2*sqrt(x) + 2*tan(sqrt(x))

maple [A] time = 0.01, size = 13, normalized size = 0.81

$$-2\sqrt{x} + 2 \tan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x^(1/2))^2/x^(1/2),x)

[Out] -2*x^(1/2)+2*tan(x^(1/2))

maxima [A] time = 0.54, size = 12, normalized size = 0.75

$$-2\sqrt{x} + 2 \tan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x^(1/2))^2/x^(1/2),x, algorithm="maxima")

[Out] -2*sqrt(x) + 2*tan(sqrt(x))

mupad [B] time = 2.58, size = 20, normalized size = 1.25

$$-2\sqrt{x} + \frac{4i}{e^{\sqrt{x}2i} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x^(1/2))^2/x^(1/2),x)

[Out] 4i/(exp(x^(1/2)*2i) + 1) - 2*x^(1/2)

sympy [A] time = 0.18, size = 14, normalized size = 0.88

$$-2\sqrt{x} + 2 \tan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x**(1/2))**2/x**(1/2),x)

[Out] -2*sqrt(x) + 2*tan(sqrt(x))

3.60 $\int \sqrt{x} \tan(\sqrt{x}) dx$

Optimal. Leaf size=70

$$2i\sqrt{x} \operatorname{Li}_2(-e^{2i\sqrt{x}}) - \operatorname{Li}_3(-e^{2i\sqrt{x}}) + \frac{2}{3}ix^{3/2} - 2x \log(1 + e^{2i\sqrt{x}})$$

[Out] $\frac{2}{3}I*x^{(3/2)} - 2*x*\ln(1 + \exp(2*I*x^{(1/2)})) - \operatorname{polylog}(3, -\exp(2*I*x^{(1/2)})) + 2*I*\operatorname{polylog}(2, -\exp(2*I*x^{(1/2)}))*x^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3747, 3719, 2190, 2531, 2282, 6589}

$$2i\sqrt{x} \operatorname{PolyLog}(2, -e^{2i\sqrt{x}}) - \operatorname{PolyLog}(3, -e^{2i\sqrt{x}}) + \frac{2}{3}ix^{3/2} - 2x \log(1 + e^{2i\sqrt{x}})$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]*Tan[Sqrt[x]], x]

[Out] $((2*I)/3)*x^{(3/2)} - 2*x*\log[1 + E^{((2*I)*\sqrt{x})}] + (2*I)*\sqrt{x}*\operatorname{PolyLog}[2, -E^{((2*I)*\sqrt{x})}] - \operatorname{PolyLog}[3, -E^{((2*I)*\sqrt{x})}]$

Rule 2190

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)] / ((a_) + (b_)*(F_)^((g_)*(e_) + (f_)*(x_)))^(n_), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]) / (b*f*g*n*Log[F]), x] - Dist[(d*m) / (b*f*g*n*Log[F]), Int[(c + d*x)^(m-1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 2531

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)]) / (b*c*n*Log[F]), x] + Dist[(g*m) / (b*c*n*Log[F]), Int[(f + g*x)^(m-1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 3719

Int[(((c_) + (d_)*(x_))^(m_))*tan[(e_) + (f_)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m+1)) / (d*(m+1)), x] - Dist[2*I, Int[(((c + d*x)^m*E^(2*I*(e + f*x))) / (1 + E^(2*I*(e + f*x))))], x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 3747

Int[(x_)^(m_)*((a_) + (b_)*Tan[(c_) + (d_)*(x_)^(n_)])^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n] - 1)*(a + b*Tan[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +

1)/n], 0] && IntegerQ[p]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned}
 \int \sqrt{x} \tan(\sqrt{x}) dx &= 2 \operatorname{Subst}\left(\int x^2 \tan(x) dx, x, \sqrt{x}\right) \\
 &= \frac{2}{3}ix^{3/2} - 4i \operatorname{Subst}\left(\int \frac{e^{2ix}x^2}{1 + e^{2ix}} dx, x, \sqrt{x}\right) \\
 &= \frac{2}{3}ix^{3/2} - 2x \log(1 + e^{2i\sqrt{x}}) + 4 \operatorname{Subst}\left(\int x \log(1 + e^{2ix}) dx, x, \sqrt{x}\right) \\
 &= \frac{2}{3}ix^{3/2} - 2x \log(1 + e^{2i\sqrt{x}}) + 2i\sqrt{x} \operatorname{Li}_2(-e^{2i\sqrt{x}}) - 2i \operatorname{Subst}\left(\int \operatorname{Li}_2(-e^{2ix}) dx, x, \sqrt{x}\right) \\
 &= \frac{2}{3}ix^{3/2} - 2x \log(1 + e^{2i\sqrt{x}}) + 2i\sqrt{x} \operatorname{Li}_2(-e^{2i\sqrt{x}}) - \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2(-x)}{x} dx, x, e^{2i\sqrt{x}}\right) \\
 &= \frac{2}{3}ix^{3/2} - 2x \log(1 + e^{2i\sqrt{x}}) + 2i\sqrt{x} \operatorname{Li}_2(-e^{2i\sqrt{x}}) - \operatorname{Li}_3(-e^{2i\sqrt{x}})
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.00

$$2i\sqrt{x} \operatorname{Li}_2(-e^{2i\sqrt{x}}) - \operatorname{Li}_3(-e^{2i\sqrt{x}}) + \frac{2}{3}ix^{3/2} - 2x \log(1 + e^{2i\sqrt{x}})$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]*Tan[Sqrt[x]], x]

[Out] ((2*I)/3)*x^(3/2) - 2*x*Log[1 + E^((2*I)*Sqrt[x])] + (2*I)*Sqrt[x]*PolyLog[2, -E^((2*I)*Sqrt[x])] - PolyLog[3, -E^((2*I)*Sqrt[x])]

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\operatorname{integral}(\sqrt{x} \tan(\sqrt{x}), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*tan(x^(1/2)), x, algorithm="fricas")

[Out] integral(sqrt(x)*tan(sqrt(x)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \tan(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*tan(x^(1/2)), x, algorithm="giac")

[Out] integrate(sqrt(x)*tan(sqrt(x)), x)

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \sqrt{x} \tan(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*tan(x^(1/2)),x)`

[Out] `int(x^(1/2)*tan(x^(1/2)),x)`

maxima [A] time = 0.41, size = 80, normalized size = 1.14

$-2ix \arctan(\sin(2\sqrt{x}), \cos(2\sqrt{x}) + 1) - x \log(\cos(2\sqrt{x})^2 + \sin(2\sqrt{x})^2 + 2\cos(2\sqrt{x}) + 1) + \frac{2}{3}ix^{\frac{3}{2}} + 2i\sqrt{x}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)*tan(x^(1/2)),x, algorithm="maxima")`

[Out] `-2*I*x*arctan2(sin(2*sqrt(x)), cos(2*sqrt(x)) + 1) - x*log(cos(2*sqrt(x))^2 + sin(2*sqrt(x))^2 + 2*cos(2*sqrt(x)) + 1) + 2/3*I*x^(3/2) + 2*I*sqrt(x)*dilog(-e^(2*I*sqrt(x))) - polylog(3, -e^(2*I*sqrt(x)))`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x} \tan(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*tan(x^(1/2)),x)`

[Out] `int(x^(1/2)*tan(x^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \tan(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)*tan(x**(1/2)),x)`

[Out] `Integral(sqrt(x)*tan(sqrt(x)), x)`

$$3.61 \quad \int \left(\frac{b \tan(a+bx+cx^2)}{2c} + x \tan(a+bx+cx^2) \right) dx$$

Optimal. Leaf size=19

$$-\frac{\log(\cos(a+bx+cx^2))}{2c}$$

[Out] -1/2*ln(cos(c*x^2+b*x+a))/c

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {3763}

$$-\frac{\log(\cos(a+bx+cx^2))}{2c}$$

Antiderivative was successfully verified.

[In] Int[(b*Tan[a + b*x + c*x^2])/(2*c) + x*Tan[a + b*x + c*x^2], x]

[Out] -Log[Cos[a + b*x + c*x^2]]/(2*c)

Rule 3763

Int[((d_.) + (e_.)*(x_.))*Tan[(a_.) + (b_.)*(x_.) + (c_.)*(x_)^2], x_Symbol]
 :> -Simp[(e*Log[Cos[a + b*x + c*x^2]])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c),
 Int[Tan[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0]

Rubi steps

$$\int \left(\frac{b \tan(a+bx+cx^2)}{2c} + x \tan(a+bx+cx^2) \right) dx = \frac{b \int \tan(a+bx+cx^2) dx}{2c} + \int x \tan(a+bx+cx^2) dx$$

$$= -\frac{\log(\cos(a+bx+cx^2))}{2c}$$

Mathematica [A] time = 0.74, size = 18, normalized size = 0.95

$$-\frac{\log(\cos(a+x(b+cx)))}{2c}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Tan[a + b*x + c*x^2])/(2*c) + x*Tan[a + b*x + c*x^2], x]

[Out] -1/2*Log[Cos[a + x*(b + c*x)]]/c

fricas [A] time = 1.66, size = 23, normalized size = 1.21

$$-\frac{\log\left(\frac{1}{\tan(cx^2+bx+a)^2+1}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a), x, algorithm="fricas")

[Out] $-1/4*\log(1/(\tan(cx^2 + bx + a)^2 + 1))/c$

giac [A] time = 8.60, size = 18, normalized size = 0.95

$$-\frac{\log\left(\left|\cos\left(cx^2 + bx + a\right)\right|\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x, algorithm="giac")`

[Out] $-1/2*\log(\text{abs}(\cos(cx^2 + bx + a)))/c$

maple [A] time = 0.15, size = 18, normalized size = 0.95

$$-\frac{\ln\left(\cos\left(cx^2 + bx + a\right)\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x)`

[Out] $-1/2*\ln(\cos(cx^2+b*x+a))/c$

maxima [B] time = 0.38, size = 83, normalized size = 4.37

$$\frac{\log\left(\cos\left(2cx^2\right)^2 + 2\cos\left(2cx^2\right)\cos\left(2bx + 2a\right) + \cos\left(2bx + 2a\right)^2 + \sin\left(2cx^2\right)^2 - 2\sin\left(2cx^2\right)\sin\left(2bx + 2a\right)\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x, algorithm="maxima")`

[Out] $-1/4*\log(\cos(2cx^2)^2 + 2*\cos(2cx^2)*\cos(2bx + 2a) + \cos(2bx + 2a)^2 + \sin(2cx^2)^2 - 2*\sin(2cx^2)*\sin(2bx + 2a) + \sin(2bx + 2a)^2)/c$

mupad [B] time = 2.51, size = 21, normalized size = 1.11

$$\frac{\ln\left(\tan\left(cx^2 + bx + a\right)^2 + 1\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*tan(a + b*x + c*x^2) + (b*tan(a + b*x + c*x^2)))/(2*c),x)`

[Out] $\log(\tan(a + b*x + c*x^2)^2 + 1)/(4*c)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int b \tan(a + bx + cx^2) dx + \int 2cx \tan(a + bx + cx^2) dx}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/2*b*tan(c*x**2+b*x+a)/c+x*tan(c*x**2+b*x+a),x)`

[Out] $(\text{Integral}(b*\tan(a + b*x + c*x**2), x) + \text{Integral}(2*c*x*\tan(a + b*x + c*x**2), x))/(2*c)$

$$3.62 \quad \int \frac{\cot^2(\sqrt{x})}{\sqrt{x}} dx$$

Optimal. Leaf size=16

$$-2\sqrt{x} - 2 \cot(\sqrt{x})$$

[Out] -2*cot(x^(1/2))-2*x^(1/2)

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3748, 3473, 8}

$$-2\sqrt{x} - 2 \cot(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Cot[Sqrt[x]]^2/Sqrt[x], x]

[Out] -2*Sqrt[x] - 2*Cot[Sqrt[x]]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3748

Int[((a_.) + Cot[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cot[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\cot^2(\sqrt{x})}{\sqrt{x}} dx &= 2 \text{Subst} \left(\int \cot^2(x) dx, x, \sqrt{x} \right) \\ &= -2 \cot(\sqrt{x}) - 2 \text{Subst} \left(\int 1 dx, x, \sqrt{x} \right) \\ &= -2\sqrt{x} - 2 \cot(\sqrt{x}) \end{aligned}$$

Mathematica [C] time = 0.05, size = 26, normalized size = 1.62

$$-2 \cot(\sqrt{x}) {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; -\tan^2(\sqrt{x}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[Sqrt[x]]^2/Sqrt[x], x]

[Out] -2*Cot[Sqrt[x]]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[Sqrt[x]]^2]

fricas [B] time = 1.22, size = 28, normalized size = 1.75

$$\frac{2(\sqrt{x} \sin(2\sqrt{x}) + \cos(2\sqrt{x}) + 1)}{\sin(2\sqrt{x})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x^(1/2))^2/x^(1/2),x, algorithm="fricas")

[Out] -2*(sqrt(x)*sin(2*sqrt(x)) + cos(2*sqrt(x)) + 1)/sin(2*sqrt(x))

giac [A] time = 0.13, size = 22, normalized size = 1.38

$$-2\sqrt{x} - \frac{1}{\tan\left(\frac{1}{2}\sqrt{x}\right)} + \tan\left(\frac{1}{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x^(1/2))^2/x^(1/2),x, algorithm="giac")

[Out] -2*sqrt(x) - 1/tan(1/2*sqrt(x)) + tan(1/2*sqrt(x))

maple [A] time = 0.01, size = 14, normalized size = 0.88

$$-2 \cot(\sqrt{x}) + \pi - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x^(1/2))^2/x^(1/2),x)

[Out] -2*cot(x^(1/2))+Pi-2*x^(1/2)

maxima [A] time = 0.48, size = 14, normalized size = 0.88

$$-2\sqrt{x} - \frac{2}{\tan(\sqrt{x})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x^(1/2))^2/x^(1/2),x, algorithm="maxima")

[Out] -2*sqrt(x) - 2/tan(sqrt(x))

mupad [B] time = 2.55, size = 20, normalized size = 1.25

$$-2\sqrt{x} - \frac{4i}{e^{\sqrt{x}2i} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x^(1/2))^2/x^(1/2),x)

[Out] - 4i/(exp(x^(1/2)*2i) - 1) - 2*x^(1/2)

sympy [A] time = 0.17, size = 15, normalized size = 0.94

$$-2\sqrt{x} - 2 \cot(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x**(1/2))**2/x**(1/2),x)

[Out] -2*sqrt(x) - 2*cot(sqrt(x))

$$3.63 \quad \int \frac{\sqrt{a+b \sec(c+dx)}}{1+\cos(c+dx)} dx$$

Optimal. Leaf size=92

$$\frac{\sqrt{\frac{1}{\sec(c+dx)+1}} \sqrt{a+b \sec(c+dx)} E\left(\sin^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right) \middle| \frac{a-b}{a+b}\right)}{d \sqrt{\frac{a+b \sec(c+dx)}{(a+b)(\sec(c+dx)+1)}}$$

[Out] EllipticE(tan(d*x+c)/(1+sec(d*x+c)), ((a-b)/(a+b))^(1/2))*(1/(1+sec(d*x+c)))^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((a+b*sec(d*x+c))/(a+b)/(1+sec(d*x+c)))^(1/2)

Rubi [A] time = 0.16, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2829, 3968}

$$\frac{\sqrt{\frac{1}{\sec(c+dx)+1}} \sqrt{a+b \sec(c+dx)} E\left(\sin^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right) \middle| \frac{a-b}{a+b}\right)}{d \sqrt{\frac{a+b \sec(c+dx)}{(a+b)(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sec[c + d*x]]/(1 + Cos[c + d*x]), x]

[Out] (EllipticE[ArcSin[Tan[c + d*x]/(1 + Sec[c + d*x])], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[a + b*Sec[c + d*x]]/(d*Sqrt[(a + b*Sec[c + d*x])]/((a + b)*(1 + Sec[c + d*x]))])

Rule 2829

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] := Int[((b + a*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/Csc[e + f*x]^m, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !IntegerQ[n] && IntegerQ[m]

Rule 3968

Int[(csc[(e_.) + (f_.)*(x_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]/(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] := -Simp[(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c/(c + d*Csc[e + f*x])]*EllipticE[ArcSin[(c*Cot[e + f*x])/(c + d*Csc[e + f*x])], -((b*c - a*d)/(b*c + a*d))]/(d*f*Sqrt[(c*d*(a + b*Csc[e + f*x]))/(b*c + a*d)*(c + d*Csc[e + f*x])]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a+b \sec(c+dx)}}{1+\cos(c+dx)} dx &= \int \frac{\sec(c+dx)\sqrt{a+b \sec(c+dx)}}{1+\sec(c+dx)} dx \\ &= \frac{E\left(\sin^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right) \middle| \frac{a-b}{a+b}\right) \sqrt{\frac{1}{1+\sec(c+dx)}} \sqrt{a+b \sec(c+dx)}}{d \sqrt{\frac{a+b \sec(c+dx)}{(a+b)(1+\sec(c+dx))}}} \end{aligned}$$

Mathematica [A] time = 7.22, size = 85, normalized size = 0.92

$$\frac{\sqrt{\frac{1}{\sec(c+dx)+1}} \sqrt{a+b \sec(c+dx)} E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)}{d \sqrt{\frac{a \cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/(1 + Cos[c + d*x]),x]

[Out] (EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[a + b*Sec[c + d*x]]/(d*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]))

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b \sec(dx+c)+a}}{\cos(dx+c)+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(d*x + c) + a)/(cos(d*x + c) + 1), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec(dx+c)+a}}{\cos(dx+c)+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(d*x + c) + a)/(cos(d*x + c) + 1), x)

maple [A] time = 0.62, size = 150, normalized size = 1.63

$$\frac{\text{EllipticE}\left(\frac{\cos(dx+c)-1}{\sin(dx+c)}, \sqrt{\frac{a-b}{a+b}}\right) \sqrt{\frac{a \cos(dx+c)+b}{(1+\cos(dx+c))(a+b)}} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} (\cos(dx+c)-1) \sqrt{\frac{a \cos(dx+c)+b}{\cos(dx+c)}} (1+\cos(dx+c))}{d (a \cos(dx+c)+b) \sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x)

[Out] -1/d*EllipticE((cos(d*x+c)-1)/sin(d*x+c), ((a-b)/(a+b))^(1/2))*((a*cos(d*x+c)+b)/(1+cos(d*x+c)))/(a+b)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)-1)*((a*cos(d*x+c)+b)/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2/(a*cos(d*x+c)+b)/sin(d*x+c)^2*(-a-b)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec(dx+c)+a}}{\cos(dx+c)+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(sqrt(b*sec(d*x + c) + a)/(cos(d*x + c) + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{b}{\cos(c+dx)}}}{\cos(c+dx) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d*x))^(1/2)/(cos(c + d*x) + 1), x)

[Out] int((a + b/cos(c + d*x))^(1/2)/(cos(c + d*x) + 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec(c + dx)}}{\cos(c + dx) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(1/2)/(1+cos(d*x+c)), x)

[Out] Integral(sqrt(a + b*sec(c + d*x))/(cos(c + d*x) + 1), x)

3.64 $\int \sec(a + bx) \sec(2a + 2bx) dx$

Optimal. Leaf size=35

$$\frac{\sqrt{2} \tanh^{-1}(\sqrt{2} \sin(a + bx))}{b} - \frac{\tanh^{-1}(\sin(a + bx))}{b}$$

[Out] $-\operatorname{arctanh}(\sin(b*x+a))/b + \operatorname{arctanh}(\sin(b*x+a)*2^{(1/2)})*2^{(1/2)}/b$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4364, 1093, 207}

$$\frac{\sqrt{2} \tanh^{-1}(\sqrt{2} \sin(a + bx))}{b} - \frac{\tanh^{-1}(\sin(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sec}[a + b*x]*\operatorname{Sec}[2*a + 2*b*x], x]$

[Out] $-(\operatorname{ArcTanh}[\operatorname{Sin}[a + b*x]]/b) + (\operatorname{Sqrt}[2]*\operatorname{ArcTanh}[\operatorname{Sqrt}[2]*\operatorname{Sin}[a + b*x]])/b$

Rule 207

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Rt}[-a, 2]]/(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2]), x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{LtQ}[a, 0] \ || \ \operatorname{GtQ}[b, 0])$

Rule 1093

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^{-1}, x_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[b^2 - 4*a*c, 2]\}, \operatorname{Dist}[c/q, \operatorname{Int}[1/(b/2 - q/2 + c*x^2), x], x] - \operatorname{Dist}[c/q, \operatorname{Int}[1/(b/2 + q/2 + c*x^2), x], x] /; \operatorname{FreeQ}\{a, b, c, x\} \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \ \&\& \operatorname{PosQ}[b^2 - 4*a*c]$

Rule 4364

$\operatorname{Int}[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^{(n_)}, x_Symbol] \rightarrow \operatorname{With}\{d = \operatorname{FreeFactors}[\operatorname{Sin}[c*(a + b*x)], x]\}, \operatorname{Dist}[d/(b*c), \operatorname{Subst}[\operatorname{Int}[\operatorname{SubstFor}[(1 - d^2*x^2)^{(n-1)/2}, \operatorname{Sin}[c*(a + b*x)]/d, u, x], x], x, \operatorname{Sin}[c*(a + b*x)]/d], x] /; \operatorname{FunctionOfQ}[\operatorname{Sin}[c*(a + b*x)]/d, u, x] /; \operatorname{FreeQ}\{a, b, c, x\} \ \&\& \operatorname{IntegerQ}[(n-1)/2] \ \&\& \operatorname{NonsumQ}[u] \ \&\& (\operatorname{EqQ}[F, \operatorname{Cos}] \ || \ \operatorname{EqQ}[F, \operatorname{cos}])$

Rubi steps

$$\begin{aligned} \int \sec(a + bx) \sec(2a + 2bx) dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{1-3x^2+2x^4} dx, x, \sin(a + bx)\right)}{b} \\ &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{-2+2x^2} dx, x, \sin(a + bx)\right)}{b} - \frac{2 \operatorname{Subst}\left(\int \frac{1}{-1+2x^2} dx, x, \sin(a + bx)\right)}{b} \\ &= -\frac{\tanh^{-1}(\sin(a + bx))}{b} + \frac{\sqrt{2} \tanh^{-1}(\sqrt{2} \sin(a + bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.04, size = 35, normalized size = 1.00

$$\frac{\sqrt{2} \tanh^{-1}(\sqrt{2} \sin(a + bx))}{b} - \frac{\tanh^{-1}(\sin(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b*x]*Sec[2*a + 2*b*x], x]

[Out] -(ArcTanh[Sin[a + b*x]]/b) + (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b

fricas [B] time = 0.94, size = 72, normalized size = 2.06

$$\frac{\sqrt{2} \log\left(-\frac{2 \cos(bx+a)^2 - 2\sqrt{2} \sin(bx+a) - 3}{2 \cos(bx+a)^2 - 1}\right) - \log(\sin(bx+a) + 1) + \log(-\sin(bx+a) + 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sec(2*b*x+2*a), x, algorithm="fricas")

[Out] 1/2*(sqrt(2)*log(-(2*cos(b*x + a)^2 - 2*sqrt(2)*sin(b*x + a) - 3)/(2*cos(b*x + a)^2 - 1)) - log(sin(b*x + a) + 1) + log(-sin(b*x + a) + 1))/b

giac [B] time = 1.15, size = 948, normalized size = 27.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sec(2*b*x+2*a), x, algorithm="giac")

[Out] 1/2*(sqrt(2)*log(abs(2*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 2*tan(1/2*a)^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12*tan(1/2*a)^5 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 30*tan(1/2*a)^4 + 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^3 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 30*tan(1/2*a)^2 - 2*sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*tan(1/2*a) + 2)/abs(2*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 2*tan(1/2*a)^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12*tan(1/2*a)^5 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 30*tan(1/2*a)^4 + 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^3 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 30*tan(1/2*a)^2 + 2*sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*tan(1/2*a) + 2)) + sqrt(2)*log(abs(2*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 + 2*tan(1/2*a)^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12*tan(1/2*a)^5 + 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 30*tan(1/2*a)^4 + 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^3 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a) + 30*tan(1/2*a)^2 - 2*sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*tan(1/2*a) - 2)/abs(2*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 + 2*tan(1/2*a)^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12*tan(1/2*a)^5 + 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 30*tan(1/2*a)^4 + 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^3 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a) + 30*tan(1/2*a)^2 + 2*sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*tan(1/2*a) - 2)) - 2*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a) - 1)) + 2*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))/b

maple [A] time = 0.17, size = 48, normalized size = 1.37

$$\frac{\ln(\sin(bx+a) - 1)}{2b} + \frac{\operatorname{arctanh}(\sin(bx+a)\sqrt{2})\sqrt{2}}{b} - \frac{\ln(1 + \sin(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)*sec(2*b*x+2*a),x)

[Out] 1/2/b*ln(sin(b*x+a)-1)+arctanh(sin(b*x+a)*2^(1/2))*2^(1/2)/b-1/2/b*ln(1+sin(b*x+a))

maxima [B] time = 108.04, size = 6257, normalized size = 178.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="maxima")

[Out]
$$-1/8*(2*(\sqrt{2}*\cos(3*a)*\cos(3/4*\arctan2(\sin(4*a), \cos(4*a)))) - \sqrt{2}*\cos(a)*\cos(1/4*\arctan2(\sin(4*a), \cos(4*a))) + \sqrt{2}*\sin(3*a)*\sin(3/4*\arctan2(\sin(4*a), \cos(4*a))) - \sqrt{2}*\sin(a)*\sin(1/4*\arctan2(\sin(4*a), \cos(4*a))) * \arctan2(\sqrt{2}*\cos(1/4*\arctan2(\sin(4*a), \cos(4*a))))*\sin(b*x) + \sqrt{2}*\cos(b*x)*\sin(1/4*\arctan2(\sin(4*a), \cos(4*a))) + \cos(1/2*\arctan2(\sin(4*a), \cos(4*a)))*\sin(2*b*x) + \cos(2*b*x)*\sin(1/2*\arctan2(\sin(4*a), \cos(4*a))), \sqrt{2}*\cos(b*x)*\cos(1/4*\arctan2(\sin(4*a), \cos(4*a))) - \sqrt{2}*\sin(b*x)*\sin(1/4*\arctan2(\sin(4*a), \cos(4*a))) + \cos(2*b*x)*\cos(1/2*\arctan2(\sin(4*a), \cos(4*a))) - \sin(2*b*x)*\sin(1/2*\arctan2(\sin(4*a), \cos(4*a))) + 1) - 2*(\sqrt{2}*\cos(3*a)*\cos(3/4*\arctan2(\sin(4*a), \cos(4*a))) - \sqrt{2}*\cos(a)*\cos(1/4*\arctan2(\sin(4*a), \cos(4*a))) + \sqrt{2}*\sin(3*a)*\sin(3/4*\arctan2(\sin(4*a), \cos(4*a))) - \sqrt{2}*\sin(a)*\sin(1/4*\arctan2(\sin(4*a), \cos(4*a))))*\arctan2(-\sqrt{2}*\cos(1/4*\arctan2(\sin(4*a), \cos(4*a))))*\sin(b*x) - \sqrt{2}*\cos(b*x)*\sin(1/4*\arctan2(\sin(4*a), \cos(4*a))) + \cos(1/2*\arctan2(\sin(4*a), \cos(4*a)))*\sin(2*b*x) + \cos(2*b*x)*\sin(1/2*\arctan2(\sin(4*a), \cos(4*a))), -\sqrt{2}*\cos(b*x)*\cos(1/4*\arctan2(\sin(4*a), \cos(4*a))) + \sqrt{2}*\sin(b*x)*\sin(1/4*\arctan2(\sin(4*a), \cos(4*a))) + \cos(2*b*x)*\cos(1/2*\arctan2(\sin(4*a), \cos(4*a))) - \sin(2*b*x)*\sin(1/2*\arctan2(\sin(4*a), \cos(4*a))) + 1) - 2*((\sqrt{2}*\cos(3*a)*\cos(1/2*\arctan2(\sin(4*a), \cos(4*a))) + \sqrt{2}*\sin(3*a)*\sin(1/2*\arctan2(\sin(4*a), \cos(4*a))) + \sqrt{2}*\cos(a)*\cos(1/2*\pi + 1/4*\arctan2(\sin(4*a), \cos(4*a))) + (\sqrt{2}*\cos(1/2*\arctan2(\sin(4*a), \cos(4*a))))*\sin(3*a) - \sqrt{2}*\cos(3*a)*\sin(1/2*\arctan2(\sin(4*a), \cos(4*a))) + \sqrt{2}*\sin(a))*\sin(1/2*\pi + 1/4*\arctan2(\sin(4*a), \cos(4*a))))*\arctan2(-2*((\sqrt{2}*\cos(1/2*\arctan2(\sin(4*a), \cos(4*a))))*\sin(1/2*\pi + 1/4*\arctan2(\sin(4*a), \cos(4*a))) - \sqrt{2}*\cos(1/2*\pi + 1/4*\arctan2(\sin(4*a), \cos(4*a))))*\sin(1/2*\arctan2(\sin(4*a), \cos(4*a))))*\cos(b*x) + \cos(1/4*\arctan2(\sin(4*a), \cos(4*a)))*\sin(1/2*\pi + 1/4*\arctan2(\sin(4*a), \cos(4*a))) - (\sqrt{2}*\cos(1/2*\pi + 1/4*\arctan2(\sin(4*a), \cos(4*a))))*\cos(1/2*\arctan2(\sin(4*a), \cos(4*a))) + \sqrt{2}*\sin(1/2*\pi + 1/4*\arctan2(\sin(4*a), \cos(4*a)))*\sin(1/2*\arctan2(\sin(4*a), \cos(4*a))))*\sin(b*x) - \cos(1/2*\pi + 1/4*\arctan2(\sin(4*a), \cos(4*a)))*\sin(1/4*\arctan2(\sin(4*a), \cos(4*a))))/(2*(\cos(1/2*\arctan2(\sin(4*a), \cos(4*a))))^2 + \sin(1/2*\arctan2(\sin(4*a), \cos(4*a))))^2*\cos(b*x)^2 + 2*(\cos(1/2*\arctan2(\sin(4*a), \cos(4*a))))^2 + \sin(1/2*\arctan2(\sin(4*a), \cos(4*a))))^2*\sin(b*x)^2 + \cos(1/2*\pi + 1/4*\arctan2(\sin(4*a), \cos(4*a))))^2 + 2*(\sqrt{2}*\cos(1/2*\pi + 1/4*\arctan2(\sin(4*a), \cos(4*a))))*\cos(1/2*\arctan2(\sin(4*a), \cos(4*a))) + \sqrt{2}*\cos(1/2*\arctan2(\sin(4*a), \cos(4*a)))*\cos(1/4*\arctan2(\sin(4*a), \cos(4*a))) + \sqrt{2}*\sin(1/2*\pi + 1/4*\arctan2(\sin(4*a), \cos(4*a)))*\sin(1/2*\arctan2(\sin(4*a), \cos(4*a))) + \sqrt{2}*\sin(1/2*\arctan2(\sin(4*a), \cos(4*a)))*\sin(1/4*\arctan2(\sin(4*a), \cos(4*a))))*\cos(b*x) + 2*\cos(1/2*\pi + 1/4*\arctan2(\sin(4*a), \cos(4*a)))*\cos(1/4*\arctan2(\sin(4*a), \cos(4*a))) + \cos(1/4*\arctan2(\sin(4*a), \cos(4*a)))^2 + \sin(1/2*\pi + 1/4*\arctan2(\sin(4*a), \cos(4*a))))^2 + 2*(\sqrt{2}*\cos(1/2*\arctan2(\sin(4*a), \cos(4*a))))*\sin(1/2*\pi + 1/4*\arctan2(\sin(4*a), \cos(4*a))) - \sqrt{2}*\cos(1/2*\pi + 1/4*\arctan2(\sin(4*a), \cos(4*a)))*\sin(1/2*\arctan2(\sin(4*a), \cos(4*a))) - \sqrt{2}*\cos(1/4*\arctan2(\sin(4*a), \cos(4*a)))*\sin(1/2*\arctan2(\sin(4*a), \cos(4*a))) + \sqrt{2}*\cos(1/2*\arctan2(\sin(4*a), \cos(4*a)))*\sin(1/4*\arctan2(\sin(4*a), \cos(4*a))))*\sin(b*x) + 2*\sin(1/2*\pi + 1/4*\arctan2(\sin(4*a), \cos(4*a)))*\sin(1/4*\arctan2(\sin(4*a), \cos(4*a))) + \sin(1/4*\arctan2(\sin(4*a), \cos(4*a)))^2), (2*(\cos(1/2*\arctan2(\sin(4*a), \cos(4*a))))^2 + \sin(1/2*\arctan2$$


```
*a), cos(4*a))))*sin(b*x) + sin(1/4*arctan2(sin(4*a), cos(4*a)))^2)) - 4*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2))/b
```

mupad [B] time = 0.09, size = 29, normalized size = 0.83

$$\frac{\operatorname{atanh}(\sin(a + bx)) - \sqrt{2} \operatorname{atanh}(\sqrt{2} \sin(a + bx))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*cos(2*a + 2*b*x)),x)
```

```
[Out] -(atanh(sin(a + b*x)) - 2^(1/2)*atanh(2^(1/2)*sin(a + b*x)))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec(a + bx) \sec(2a + 2bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x)
```

```
[Out] Integral(sec(a + b*x)*sec(2*a + 2*b*x), x)
```

3.65 $\int \sec(a + bx) \sec(2(a + bx)) dx$

Optimal. Leaf size=35

$$\frac{\sqrt{2} \tanh^{-1}(\sqrt{2} \sin(a + bx))}{b} - \frac{\tanh^{-1}(\sin(a + bx))}{b}$$

[Out] $-\text{arctanh}(\sin(b*x+a))/b + \text{arctanh}(\sin(b*x+a)*2^{(1/2)})*2^{(1/2)}/b$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4364, 1093, 207}

$$\frac{\sqrt{2} \tanh^{-1}(\sqrt{2} \sin(a + bx))}{b} - \frac{\tanh^{-1}(\sin(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b*x]*Sec[2*(a + b*x)], x]

[Out] $-(\text{ArcTanh}[\text{Sin}[a + b*x]]/b) + (\text{Sqrt}[2]*\text{ArcTanh}[\text{Sqrt}[2]*\text{Sin}[a + b*x]])/b$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 4364

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned} \int \sec(a + bx) \sec(2(a + bx)) dx &= \frac{\text{Subst}\left(\int \frac{1}{1-3x^2+2x^4} dx, x, \sin(a + bx)\right)}{b} \\ &= \frac{2 \text{Subst}\left(\int \frac{1}{-2+2x^2} dx, x, \sin(a + bx)\right)}{b} - \frac{2 \text{Subst}\left(\int \frac{1}{-1+2x^2} dx, x, \sin(a + bx)\right)}{b} \\ &= -\frac{\tanh^{-1}(\sin(a + bx))}{b} + \frac{\sqrt{2} \tanh^{-1}(\sqrt{2} \sin(a + bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 1.00

$$\frac{\sqrt{2} \tanh^{-1}(\sqrt{2} \sin(a + bx))}{b} - \frac{\tanh^{-1}(\sin(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b*x]*Sec[2*(a + b*x)],x]

[Out] -(ArcTanh[Sin[a + b*x]]/b) + (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b

fricas [B] time = 0.98, size = 72, normalized size = 2.06

$$\frac{\sqrt{2} \log\left(-\frac{2 \cos(bx+a)^2 - 2\sqrt{2} \sin(bx+a) - 3}{2 \cos(bx+a)^2 - 1}\right) - \log(\sin(bx+a) + 1) + \log(-\sin(bx+a) + 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="fricas")

[Out] 1/2*(sqrt(2)*log(-(2*cos(b*x + a)^2 - 2*sqrt(2)*sin(b*x + a) - 3)/(2*cos(b*x + a)^2 - 1)) - log(sin(b*x + a) + 1) + log(-sin(b*x + a) + 1))/b

giac [B] time = 1.19, size = 948, normalized size = 27.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="giac")

[Out] 1/2*(sqrt(2)*log(abs(2*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 2*tan(1/2*a)^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12*tan(1/2*a)^5 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 30*tan(1/2*a)^4 + 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^3 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 30*tan(1/2*a)^2 - 2*sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*tan(1/2*a) + 2)/abs(2*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 2*tan(1/2*a)^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12*tan(1/2*a)^5 - 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 30*tan(1/2*a)^4 + 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^3 + 12*tan(1/2*b*x + 2*a)*tan(1/2*a) - 30*tan(1/2*a)^2 + 2*sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*tan(1/2*a) + 2)) + sqrt(2)*log(abs(2*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 + 2*tan(1/2*a)^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12*tan(1/2*a)^5 + 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 30*tan(1/2*a)^4 + 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^3 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a) + 30*tan(1/2*a)^2 - 2*sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*tan(1/2*a) - 2)/abs(2*tan(1/2*b*x + 2*a)*tan(1/2*a)^6 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 + 2*tan(1/2*a)^6 - 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^4 + 12*tan(1/2*a)^5 + 40*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 30*tan(1/2*a)^4 + 30*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 40*tan(1/2*a)^3 - 12*tan(1/2*b*x + 2*a)*tan(1/2*a) + 30*tan(1/2*a)^2 + 2*sqrt(2)*(tan(1/2*a)^6 + 3*tan(1/2*a)^4 + 3*tan(1/2*a)^2 + 1) - 2*tan(1/2*b*x + 2*a) + 12*tan(1/2*a) - 2)) - 2*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - tan(1/2*b*x + 2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a) - 1)) + 2*log(abs(tan(1/2*b*x + 2*a)*tan(1/2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 + tan(1/2*b*x + 2*a)^3 - 3*tan(1/2*b*x + 2*a)*tan(1/2*a) + 3*tan(1/2*a)^2 + tan(1/2*b*x + 2*a) - 3*tan(1/2*a) - 1)))/b

maple [A] time = 0.00, size = 48, normalized size = 1.37

$$\frac{\ln(\sin(bx+a)-1)}{2b} + \frac{\operatorname{arctanh}\left(\sin(bx+a)\sqrt{2}\right)\sqrt{2}}{b} - \frac{\ln(1+\sin(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sec(2*b*x+2*a), x)
```

```
[Out] 1/2/b*ln(sin(b*x+a)-1)+arctanh(sin(b*x+a)*2^(1/2))*2^(1/2)/b-1/2/b*ln(1+sin
(b*x+a))
```

```
maxima [B] time = 117.00, size = 6257, normalized size = 178.77
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a), x, algorithm="maxima")
```

```
[Out] -1/8*(2*(sqrt(2)*cos(3*a)*cos(3/4*arctan2(sin(4*a), cos(4*a)))) - sqrt(2)*co
s(a)*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(3/4*arctan
2(sin(4*a), cos(4*a))) - sqrt(2)*sin(a)*sin(1/4*arctan2(sin(4*a), cos(4*a))
))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(b*x) + sqrt(2)*
cos(b*x)*sin(1/4*arctan2(sin(4*a), cos(4*a))) + cos(1/2*arctan2(sin(4*a), c
os(4*a)))*sin(2*b*x) + cos(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))), sqr
t(2)*cos(b*x)*cos(1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*sin(b*x)*sin(1
/4*arctan2(sin(4*a), cos(4*a))) + cos(2*b*x)*cos(1/2*arctan2(sin(4*a), cos(
4*a))) - sin(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + 1) - 2*(sqrt(2)*
cos(3*a)*cos(3/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(a)*cos(1/4*arct
an2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(3/4*arctan2(sin(4*a), cos(4
*a))) - sqrt(2)*sin(a)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*arctan2(-sqrt(
2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(b*x) - sqrt(2)*cos(b*x)*sin(1/4
*arctan2(sin(4*a), cos(4*a))) + cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(2*
b*x) + cos(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))), -sqrt(2)*cos(b*x)*c
os(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(b*x)*sin(1/4*arctan2(sin(
4*a), cos(4*a))) + cos(2*b*x)*cos(1/2*arctan2(sin(4*a), cos(4*a))) - sin(2*
b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + 1) - 2*((sqrt(2)*cos(3*a)*cos(1
/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(1/2*arctan2(sin(4*a)
, cos(4*a))) + sqrt(2)*cos(a)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))
) + (sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(3*a) - sqrt(2)*cos(3*
a)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(a)*sin(1/2*pi + 1/4*
arctan2(sin(4*a), cos(4*a))))*arctan2(-2*((sqrt(2)*cos(1/2*arctan2(sin(4*a)
, cos(4*a)))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/
2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))
))*cos(b*x) + cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*pi + 1/4*arctan2
(sin(4*a), cos(4*a))) - (sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)
))*cos(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi + 1/4*arctan2
(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))))*sin(b*x) - cos(
1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a)
))))/(2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a),
cos(4*a)))^2)*cos(b*x)^2 + 2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin
(1/2*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 + cos(1/2*pi + 1/4*arctan2(
sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(
4*a)))*cos(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(1/2*arctan2(sin(4
*a), cos(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi +
1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sq
rt(2)*sin(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*
a))))*cos(b*x) + 2*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*cos(1/4*ar
ctan2(sin(4*a), cos(4*a))) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 + sin(1
/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*arctan2(sin
(4*a), cos(4*a)))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*c
os(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(
4*a))) - sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4
*a), cos(4*a))) + sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arct
an2(sin(4*a), cos(4*a))))*sin(b*x) + 2*sin(1/2*pi + 1/4*arctan2(sin(4*a), c
os(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))) + sin(1/4*arctan2(sin(4*a),
cos(4*a)))^2), (2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2
```



```
*a), cos(4*a))))*sin(b*x) + sin(1/4*arctan2(sin(4*a), cos(4*a)))^2)) - 4*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2))/b
```

mupad [B] time = 0.00, size = 29, normalized size = 0.83

$$\frac{\operatorname{atanh}(\sin(a + bx)) - \sqrt{2} \operatorname{atanh}(\sqrt{2} \sin(a + bx))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*cos(2*a + 2*b*x)), x)
```

```
[Out] -(atanh(sin(a + b*x)) - 2^(1/2)*atanh(2^(1/2)*sin(a + b*x)))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec(a + bx) \sec(2a + 2bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a), x)
```

```
[Out] Integral(sec(a + b*x)*sec(2*a + 2*b*x), x)
```

3.66 $\int \sin(x) \sin(2x) dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2} - \frac{1}{6} \sin(3x)$$

[Out] 1/2*sin(x)-1/6*sin(3*x)

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4282}

$$\frac{\sin(x)}{2} - \frac{1}{6} \sin(3x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Sin[2*x],x]

[Out] Sin[x]/2 - Sin[3*x]/6

Rule 4282

Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\int \sin(x) \sin(2x) dx = \frac{\sin(x)}{2} - \frac{1}{6} \sin(3x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin(x)}{2} - \frac{1}{6} \sin(3x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]*Sin[2*x],x]

[Out] Sin[x]/2 - Sin[3*x]/6

fricas [A] time = 2.01, size = 10, normalized size = 0.67

$$-\frac{2}{3} (\cos(x)^2 - 1) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(2*x),x, algorithm="fricas")

[Out] -2/3*(cos(x)^2 - 1)*sin(x)

giac [A] time = 0.12, size = 6, normalized size = 0.40

$$\frac{2}{3} \sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(2*x),x, algorithm="giac")

[Out] $2/3*\sin(x)^3$

maple [A] time = 0.05, size = 7, normalized size = 0.47

$$\frac{2(\sin^3(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)*sin(2*x),x)`

[Out] $2/3*\sin(x)^3$

maxima [A] time = 0.31, size = 11, normalized size = 0.73

$$-\frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)*sin(2*x),x, algorithm="maxima")`

[Out] $-1/6*\sin(3*x) + 1/2*\sin(x)$

mupad [B] time = 0.03, size = 6, normalized size = 0.40

$$\frac{2\sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(2*x)*sin(x),x)`

[Out] $(2*\sin(x)^3)/3$

sympy [A] time = 0.45, size = 20, normalized size = 1.33

$$-\frac{2\sin(x)\cos(2x)}{3} + \frac{\sin(2x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)*sin(2*x),x)`

[Out] $-2*\sin(x)*\cos(2*x)/3 + \sin(2*x)*\cos(x)/3$

3.67 $\int \sin(x) \sin(3x) dx$

Optimal. Leaf size=17

$$\frac{1}{4} \sin(2x) - \frac{1}{8} \sin(4x)$$

[Out] 1/4*sin(2*x)-1/8*sin(4*x)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4282}

$$\frac{1}{4} \sin(2x) - \frac{1}{8} \sin(4x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Sin[3*x],x]

[Out] Sin[2*x]/4 - Sin[4*x]/8

Rule 4282

Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\int \sin(x) \sin(3x) dx = \frac{1}{4} \sin(2x) - \frac{1}{8} \sin(4x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{4} \sin(2x) - \frac{1}{8} \sin(4x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]*Sin[3*x],x]

[Out] Sin[2*x]/4 - Sin[4*x]/8

fricas [A] time = 0.59, size = 13, normalized size = 0.76

$$-(\cos(x)^3 - \cos(x)) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(3*x),x, algorithm="fricas")

[Out] -(cos(x)^3 - cos(x))*sin(x)

giac [A] time = 0.14, size = 13, normalized size = 0.76

$$-\frac{1}{8} \sin(4x) + \frac{1}{4} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(3*x),x, algorithm="giac")

[Out] $-1/8*\sin(4*x) + 1/4*\sin(2*x)$

maple [A] time = 0.11, size = 14, normalized size = 0.82

$$\frac{\sin(2x)}{4} - \frac{\sin(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)*sin(3*x),x)`

[Out] $1/4*\sin(2*x)-1/8*\sin(4*x)$

maxima [A] time = 0.31, size = 13, normalized size = 0.76

$$-\frac{1}{8} \sin(4x) + \frac{1}{4} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)*sin(3*x),x, algorithm="maxima")`

[Out] $-1/8*\sin(4*x) + 1/4*\sin(2*x)$

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$\frac{\sin(2x)}{4} - \frac{\sin(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(3*x)*sin(x),x)`

[Out] $\sin(2*x)/4 - \sin(4*x)/8$

sympy [A] time = 0.42, size = 20, normalized size = 1.18

$$-\frac{3 \sin(x) \cos(3x)}{8} + \frac{\sin(3x) \cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)*sin(3*x),x)`

[Out] $-3*\sin(x)*\cos(3*x)/8 + \sin(3*x)*\cos(x)/8$

3.68 $\int \sin(x) \sin(4x) dx$

Optimal. Leaf size=17

$$\frac{1}{6} \sin(3x) - \frac{1}{10} \sin(5x)$$

[Out] 1/6*sin(3*x)-1/10*sin(5*x)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4282}

$$\frac{1}{6} \sin(3x) - \frac{1}{10} \sin(5x)$$

Antiderivative was successfully verified.

[In] Int [Sin [x] *Sin [4*x] , x]

[Out] Sin [3*x] /6 - Sin [5*x] /10

Rule 4282

Int [sin [(a_.) + (b_.)*(x_.)]*sin [(c_.) + (d_.)*(x_.)], x_Symbol] :> Simp [Sin [a - c + (b - d)*x] / (2*(b - d)), x] - Simp [Sin [a + c + (b + d)*x] / (2*(b + d)), x] /; FreeQ [{a, b, c, d}, x] && NeQ [b^2 - d^2, 0]

Rubi steps

$$\int \sin(x) \sin(4x) dx = \frac{1}{6} \sin(3x) - \frac{1}{10} \sin(5x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6} \sin(3x) - \frac{1}{10} \sin(5x)$$

Antiderivative was successfully verified.

[In] Integrate [Sin [x] *Sin [4*x] , x]

[Out] Sin [3*x] /6 - Sin [5*x] /10

fricas [A] time = 1.12, size = 18, normalized size = 1.06

$$-\frac{4}{15} (6 \cos(x)^4 - 7 \cos(x)^2 + 1) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate (sin(x)*sin(4*x), x, algorithm="fricas")

[Out] -4/15*(6*cos(x)^4 - 7*cos(x)^2 + 1)*sin(x)

giac [A] time = 0.14, size = 13, normalized size = 0.76

$$-\frac{8}{5} \sin(x)^5 + \frac{4}{3} \sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(4*x),x, algorithm="giac")

[Out] $-8/5*\sin(x)^5 + 4/3*\sin(x)^3$

maple [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{\sin(3x)}{6} - \frac{\sin(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*sin(4*x),x)

[Out] $1/6*\sin(3*x)-1/10*\sin(5*x)$

maxima [A] time = 0.31, size = 13, normalized size = 0.76

$$-\frac{1}{10} \sin(5x) + \frac{1}{6} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(4*x),x, algorithm="maxima")

[Out] $-1/10*\sin(5*x) + 1/6*\sin(3*x)$

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$\frac{\sin(3x)}{6} - \frac{\sin(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(4*x)*sin(x),x)

[Out] $\sin(3*x)/6 - \sin(5*x)/10$

sympy [A] time = 0.41, size = 20, normalized size = 1.18

$$-\frac{4 \sin(x) \cos(4x)}{15} + \frac{\sin(4x) \cos(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(4*x),x)

[Out] $-4*\sin(x)*\cos(4*x)/15 + \sin(4*x)*\cos(x)/15$

3.69 $\int \sin(x) \sin(mx) dx$

Optimal. Leaf size=35

$$\frac{\sin((1-m)x)}{2(1-m)} - \frac{\sin((m+1)x)}{2(m+1)}$$

[Out] 1/2*sin((1-m)*x)/(1-m)-1/2*sin((1+m)*x)/(1+m)

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4569, 2637}

$$\frac{\sin((1-m)x)}{2(1-m)} - \frac{\sin((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Sin[m*x],x]

[Out] Sin[(1-m)*x]/(2*(1-m)) - Sin[(1+m)*x]/(2*(1+m))

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4569

Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] := Int[ExpandTrigReduce[Sin[v]^p * Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \sin(x) \sin(mx) dx &= \int \left(\frac{1}{2} \cos((1-m)x) - \frac{1}{2} \cos((1+m)x) \right) dx \\ &= \frac{1}{2} \int \cos((1-m)x) dx - \frac{1}{2} \int \cos((1+m)x) dx \\ &= \frac{\sin((1-m)x)}{2(1-m)} - \frac{\sin((1+m)x)}{2(1+m)} \end{aligned}$$

Mathematica [A] time = 0.05, size = 25, normalized size = 0.71

$$\frac{\cos(x) \sin(mx) - m \sin(x) \cos(mx)}{m^2 - 1}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]*Sin[m*x],x]

[Out] (-(m*Cos[m*x]*Sin[x]) + Cos[x]*Sin[m*x])/(-1 + m^2)

fricas [A] time = 1.08, size = 26, normalized size = 0.74

$$-\frac{m \cos(mx) \sin(x) - \cos(x) \sin(mx)}{m^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(m*x),x, algorithm="fricas")

[Out] $-(m \cos(m x) \sin(x) - \cos(x) \sin(m x)) / (m^2 - 1)$

giac [A] time = 0.14, size = 29, normalized size = 0.83

$$-\frac{\sin(mx+x)}{2(m+1)} + \frac{\sin(mx-x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(m*x),x, algorithm="giac")

[Out] $-1/2 \sin(m x + x) / (m + 1) + 1/2 \sin(m x - x) / (m - 1)$

maple [A] time = 0.08, size = 28, normalized size = 0.80

$$\frac{\sin((-1+m)x)}{-2+2m} - \frac{\sin((1+m)x)}{2(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*sin(m*x),x)

[Out] $1/2 / (-1+m) \sin((-1+m)x) - 1/2 \sin((1+m)x) / (1+m)$

maxima [A] time = 0.31, size = 28, normalized size = 0.80

$$-\frac{\sin((m+1)x)}{2(m+1)} - \frac{\sin(-(m-1)x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(m*x),x, algorithm="maxima")

[Out] $-1/2 \sin((m+1)x) / (m+1) - 1/2 \sin(-(m-1)x) / (m-1)$

mupad [B] time = 2.32, size = 64, normalized size = 1.83

$$\left\{ \begin{array}{ll} \frac{x}{2} - \frac{\sin(2x)}{4} & \text{if } m = 1 \\ \frac{\sin(2x)}{4} - \frac{x}{2} & \text{if } m = -1 \\ \frac{\sin(x(m-1))}{2m-2} - \frac{\sin(x(m+1))}{2m+2} & \text{if } m \neq -1 \wedge m \neq 1 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(m*x)*sin(x),x)

[Out] $\text{piecewise}(m == 1, x/2 - \sin(2*x)/4, m == -1, -x/2 + \sin(2*x)/4, m \neq -1 \ \& \ m \neq 1, \sin(x*(m-1))/(2*m-2) - \sin(x*(m+1))/(2*m+2))$

sympy [A] time = 0.79, size = 78, normalized size = 2.23

$$\left\{ \begin{array}{ll} -\frac{x \sin^2(x)}{2} - \frac{x \cos^2(x)}{2} + \frac{\sin(x) \cos(x)}{2} & \text{for } m = -1 \\ \frac{x \sin^2(x)}{2} + \frac{x \cos^2(x)}{2} - \frac{\sin(x) \cos(x)}{2} & \text{for } m = 1 \\ -\frac{m \sin(x) \cos(mx)}{m^2-1} + \frac{\sin(mx) \cos(x)}{m^2-1} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(m*x),x)
```

```
[Out] Piecewise((-x*sin(x)**2/2 - x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, -1)), (x  
*sin(x)**2/2 + x*cos(x)**2/2 - sin(x)*cos(x)/2, Eq(m, 1)), (-m*sin(x)*cos(m  
*x)/(m**2 - 1) + sin(m*x)*cos(x)/(m**2 - 1), True))
```

3.70 $\int \cos(2x) \sin(x) dx$

Optimal. Leaf size=15

$$\frac{\cos(x)}{2} - \frac{1}{6} \cos(3x)$$

[Out] 1/2*cos(x)-1/6*cos(3*x)

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4284}

$$\frac{\cos(x)}{2} - \frac{1}{6} \cos(3x)$$

Antiderivative was successfully verified.

[In] Int[Cos[2*x]*Sin[x],x]

[Out] Cos[x]/2 - Cos[3*x]/6

Rule 4284

Int[cos[(c_.) + (d_.)*(x_.)]*sin[(a_.) + (b_.)*(x_.)], x_Symbol] :> -Simp[Cos[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\int \cos(2x) \sin(x) dx = \frac{\cos(x)}{2} - \frac{1}{6} \cos(3x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\cos(x)}{2} - \frac{1}{6} \cos(3x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[2*x]*Sin[x],x]

[Out] Cos[x]/2 - Cos[3*x]/6

fricas [A] time = 0.78, size = 9, normalized size = 0.60

$$-\frac{2}{3} \cos(x)^3 + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*sin(x),x, algorithm="fricas")

[Out] -2/3*cos(x)^3 + cos(x)

giac [A] time = 0.13, size = 11, normalized size = 0.73

$$-\frac{1}{6} \cos(3x) + \frac{1}{2} \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*sin(x),x, algorithm="giac")

[Out] $-1/6*\cos(3*x) + 1/2*\cos(x)$

maple [A] time = 0.07, size = 12, normalized size = 0.80

$$\frac{\cos(x)}{2} - \frac{\cos(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(2*x)*sin(x),x)`

[Out] $1/2*\cos(x)-1/6*\cos(3*x)$

maxima [A] time = 0.31, size = 11, normalized size = 0.73

$$-\frac{1}{6} \cos(3x) + \frac{1}{2} \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(2*x)*sin(x),x, algorithm="maxima")`

[Out] $-1/6*\cos(3*x) + 1/2*\cos(x)$

mupad [B] time = 0.02, size = 9, normalized size = 0.60

$$\cos(x) - \frac{2 \cos(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(2*x)*sin(x),x)`

[Out] $\cos(x) - (2*\cos(x)^3)/3$

sympy [A] time = 0.45, size = 20, normalized size = 1.33

$$\frac{2 \sin(x) \sin(2x)}{3} + \frac{\cos(x) \cos(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(2*x)*sin(x),x)`

[Out] $2*\sin(x)*\sin(2*x)/3 + \cos(x)*\cos(2*x)/3$

3.71 $\int \cos(3x) \sin(x) dx$

Optimal. Leaf size=17

$$\frac{1}{4} \cos(2x) - \frac{1}{8} \cos(4x)$$

[Out] 1/4*cos(2*x)-1/8*cos(4*x)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4284}

$$\frac{1}{4} \cos(2x) - \frac{1}{8} \cos(4x)$$

Antiderivative was successfully verified.

[In] Int[Cos[3*x]*Sin[x],x]

[Out] Cos[2*x]/4 - Cos[4*x]/8

Rule 4284

Int[cos[(c_.) + (d_.)*(x_.)]*sin[(a_.) + (b_.)*(x_.)], x_Symbol] := -Simp[Cos[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\int \cos(3x) \sin(x) dx = \frac{1}{4} \cos(2x) - \frac{1}{8} \cos(4x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\cos^2(x)}{2} - \frac{1}{8} \cos(4x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[3*x]*Sin[x],x]

[Out] Cos[x]^2/2 - Cos[4*x]/8

fricas [A] time = 0.83, size = 13, normalized size = 0.76

$$-\cos(x)^4 + \frac{3}{2} \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)*sin(x),x, algorithm="fricas")

[Out] -cos(x)^4 + 3/2*cos(x)^2

giac [A] time = 0.12, size = 13, normalized size = 0.76

$$-\sin(x)^4 + \frac{1}{2} \sin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)*sin(x),x, algorithm="giac")

[Out] $-\sin(x)^4 + 1/2*\sin(x)^2$

maple [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{\cos(2x)}{4} - \frac{\cos(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(3*x)*sin(x),x)

[Out] $1/4*\cos(2*x) - 1/8*\cos(4*x)$

maxima [A] time = 0.54, size = 13, normalized size = 0.76

$$-\frac{1}{8} \cos(4x) + \frac{1}{4} \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)*sin(x),x, algorithm="maxima")

[Out] $-1/8*\cos(4*x) + 1/4*\cos(2*x)$

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{3 \cos(x)^2}{2} - \cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(3*x)*sin(x),x)

[Out] $(3*\cos(x)^2)/2 - \cos(x)^4$

sympy [A] time = 0.41, size = 20, normalized size = 1.18

$$\frac{3 \sin(x) \sin(3x)}{8} + \frac{\cos(x) \cos(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)*sin(x),x)

[Out] $3*\sin(x)*\sin(3*x)/8 + \cos(x)*\cos(3*x)/8$

3.72 $\int \cos(4x) \sin(x) dx$

Optimal. Leaf size=17

$$\frac{1}{6} \cos(3x) - \frac{1}{10} \cos(5x)$$

[Out] 1/6*cos(3*x)-1/10*cos(5*x)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4284}

$$\frac{1}{6} \cos(3x) - \frac{1}{10} \cos(5x)$$

Antiderivative was successfully verified.

[In] Int[Cos[4*x]*Sin[x],x]

[Out] Cos[3*x]/6 - Cos[5*x]/10

Rule 4284

Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\int \cos(4x) \sin(x) dx = \frac{1}{6} \cos(3x) - \frac{1}{10} \cos(5x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6} \cos(3x) - \frac{1}{10} \cos(5x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[4*x]*Sin[x],x]

[Out] Cos[3*x]/6 - Cos[5*x]/10

fricas [A] time = 1.06, size = 17, normalized size = 1.00

$$-\frac{8}{5} \cos(x)^5 + \frac{8}{3} \cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(4*x)*sin(x),x, algorithm="fricas")

[Out] -8/5*cos(x)^5 + 8/3*cos(x)^3 - cos(x)

giac [A] time = 0.12, size = 13, normalized size = 0.76

$$-\frac{1}{10} \cos(5x) + \frac{1}{6} \cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(4*x)*sin(x),x, algorithm="giac")

[Out] -1/10*cos(5*x) + 1/6*cos(3*x)

maple [A] time = 0.11, size = 14, normalized size = 0.82

$$\frac{\cos(3x)}{6} - \frac{\cos(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(4*x)*sin(x),x)

[Out] 1/6*cos(3*x)-1/10*cos(5*x)

maxima [A] time = 0.51, size = 13, normalized size = 0.76

$$-\frac{1}{10} \cos(5x) + \frac{1}{6} \cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(4*x)*sin(x),x, algorithm="maxima")

[Out] -1/10*cos(5*x) + 1/6*cos(3*x)

mupad [B] time = 0.03, size = 17, normalized size = 1.00

$$-\frac{8 \cos(x)^5}{5} + \frac{8 \cos(x)^3}{3} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(4*x)*sin(x),x)

[Out] (8*cos(x)^3)/3 - cos(x) - (8*cos(x)^5)/5

sympy [A] time = 0.41, size = 20, normalized size = 1.18

$$\frac{4 \sin(x) \sin(4x)}{15} + \frac{\cos(x) \cos(4x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(4*x)*sin(x),x)

[Out] 4*sin(x)*sin(4*x)/15 + cos(x)*cos(4*x)/15

3.73 $\int \cos(mx) \sin(x) dx$

Optimal. Leaf size=35

$$-\frac{\cos((1-m)x)}{2(1-m)} - \frac{\cos((m+1)x)}{2(m+1)}$$

[Out] $-1/2*\cos((1-m)*x)/(1-m)-1/2*\cos((1+m)*x)/(1+m)$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4574, 2638}

$$-\frac{\cos((1-m)x)}{2(1-m)} - \frac{\cos((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

[In] Int[Cos[m*x]*Sin[x],x]

[Out] $-\text{Cos}[(1-m)*x]/(2*(1-m)) - \text{Cos}[(1+m)*x]/(2*(1+m))$

Rule 2638

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4574

Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p *Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))

Rubi steps

$$\begin{aligned} \int \cos(mx) \sin(x) dx &= \int \left(\frac{1}{2} \sin((1-m)x) + \frac{1}{2} \sin((1+m)x) \right) dx \\ &= \frac{1}{2} \int \sin((1-m)x) dx + \frac{1}{2} \int \sin((1+m)x) dx \\ &= -\frac{\cos((1-m)x)}{2(1-m)} - \frac{\cos((1+m)x)}{2(1+m)} \end{aligned}$$

Mathematica [A] time = 0.04, size = 24, normalized size = 0.69

$$\frac{m \sin(x) \sin(mx) + \cos(x) \cos(mx)}{m^2 - 1}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[m*x]*Sin[x],x]

[Out] $(\text{Cos}[x]*\text{Cos}[m*x] + m*\text{Sin}[x]*\text{Sin}[m*x])/(-1 + m^2)$

fricas [A] time = 0.80, size = 24, normalized size = 0.69

$$\frac{m \sin(mx) \sin(x) + \cos(mx) \cos(x)}{m^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(m*x)*sin(x),x, algorithm="fricas")

[Out] (m*sin(m*x)*sin(x) + cos(m*x)*cos(x))/(m^2 - 1)

giac [A] time = 0.13, size = 29, normalized size = 0.83

$$-\frac{\cos(mx+x)}{2(m+1)} + \frac{\cos(mx-x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(m*x)*sin(x),x, algorithm="giac")

[Out] -1/2*cos(m*x + x)/(m + 1) + 1/2*cos(m*x - x)/(m - 1)

maple [A] time = 0.03, size = 28, normalized size = 0.80

$$\frac{\cos((-1+m)x)}{-2+2m} - \frac{\cos((1+m)x)}{2(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(m*x)*sin(x),x)

[Out] 1/2*cos((-1+m)*x)/(-1+m)-1/2*cos((1+m)*x)/(1+m)

maxima [A] time = 0.32, size = 28, normalized size = 0.80

$$-\frac{\cos((m+1)x)}{2(m+1)} + \frac{\cos(-(m-1)x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(m*x)*sin(x),x, algorithm="maxima")

[Out] -1/2*cos((m + 1)*x)/(m + 1) + 1/2*cos(-(m - 1)*x)/(m - 1)

mupad [B] time = 0.10, size = 37, normalized size = 1.06

$$\begin{cases} \frac{\sin(x)^2}{2} & \text{if } m = -1 \vee m = 1 \\ \frac{\cos(x(m-1))}{2m-2} - \frac{\cos(x(m+1))}{2m+2} & \text{if } m \neq -1 \wedge m \neq 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(m*x)*sin(x),x)

[Out] piecewise(m == -1 | m == 1, sin(x)^2/2, m ~= -1 & m ~= 1, cos(x*(m - 1))/(2*m - 2) - cos(x*(m + 1))/(2*m + 2))

sympy [A] time = 0.93, size = 37, normalized size = 1.06

$$\begin{cases} \frac{\sin^2(x)}{2} & \text{for } m = -1 \vee m = 1 \\ \frac{m \sin(x) \sin(mx)}{m^2-1} + \frac{\cos(x) \cos(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(m*x)*sin(x),x)

[Out] Piecewise((sin(x)**2/2, Eq(m, -1) | Eq(m, 1)), (m*sin(x)*sin(m*x)/(m**2 - 1) + cos(x)*cos(m*x)/(m**2 - 1), True))

3.74 $\int \sin(x) \tan(2x) dx$

Optimal. Leaf size=20

$$\frac{\tanh^{-1}(\sqrt{2} \sin(x))}{\sqrt{2}} - \sin(x)$$

[Out] $-\sin(x) + 1/2 \cdot \arctanh(\sin(x) \cdot 2^{(1/2)}) \cdot 2^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {12, 321, 206}

$$\frac{\tanh^{-1}(\sqrt{2} \sin(x))}{\sqrt{2}} - \sin(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Tan[2*x],x]

[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2] - Sin[x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^p, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned} \int \sin(x) \tan(2x) dx &= \text{Subst} \left(\int \frac{2x^2}{1-2x^2} dx, x, \sin(x) \right) \\ &= 2 \text{Subst} \left(\int \frac{x^2}{1-2x^2} dx, x, \sin(x) \right) \\ &= -\sin(x) + \text{Subst} \left(\int \frac{1}{1-2x^2} dx, x, \sin(x) \right) \\ &= \frac{\tanh^{-1}(\sqrt{2} \sin(x))}{\sqrt{2}} - \sin(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{\tanh^{-1}(\sqrt{2} \sin(x))}{\sqrt{2}} - \sin(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]*Tan[2*x],x]

[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2] - Sin[x]

fricas [B] time = 2.06, size = 38, normalized size = 1.90

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2-2\sqrt{2}\sin(x)-3}{2\cos(x)^2-1}\right)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(2*x),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - sin(x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \tan(2x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(2*x),x, algorithm="giac")

[Out] integrate(sin(x)*tan(2*x), x)

maple [A] time = 0.16, size = 18, normalized size = 0.90

$$-\sin(x) + \frac{\operatorname{arctanh}(\sin(x)\sqrt{2})\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*tan(2*x),x)

[Out] -sin(x)+1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)

maxima [B] time = 0.44, size = 141, normalized size = 7.05

$$\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)-\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(2*x),x, algorithm="maxima")

[Out] 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - sin(x)

mupad [B] time = 2.39, size = 17, normalized size = 0.85

$$\frac{\sqrt{2}\operatorname{atanh}(\sqrt{2}\sin(x))}{2}-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(2*x)*sin(x),x)

```
[Out] (2^(1/2)*atanh(2^(1/2)*sin(x)))/2 - sin(x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \sin(x) \tan(2x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(2*x), x)
```

```
[Out] Integral(sin(x)*tan(2*x), x)
```

3.75 $\int \sin(x) \tan(3x) dx$

Optimal. Leaf size=47

$$-\sin(x) - \frac{1}{6} \log(1 - 2 \sin(x)) - \frac{1}{6} \log(1 - \sin(x)) + \frac{1}{6} \log(\sin(x) + 1) + \frac{1}{6} \log(2 \sin(x) + 1)$$

[Out] $-1/6*\ln(1-2*\sin(x))-1/6*\ln(1-\sin(x))+1/6*\ln(1+\sin(x))+1/6*\ln(1+2*\sin(x))-\sin(x)$

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1279, 1161, 616, 31}

$$-\sin(x) - \frac{1}{6} \log(1 - 2 \sin(x)) - \frac{1}{6} \log(1 - \sin(x)) + \frac{1}{6} \log(\sin(x) + 1) + \frac{1}{6} \log(2 \sin(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Tan[3*x],x]

[Out] $-\text{Log}[1 - 2*\text{Sin}[x]]/6 - \text{Log}[1 - \text{Sin}[x]]/6 + \text{Log}[1 + \text{Sin}[x]]/6 + \text{Log}[1 + 2*\text{Sin}[x]]/6 - \text{Sin}[x]$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 1161

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 1279

Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p + 3))*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rubi steps

$$\begin{aligned}
\int \sin(x) \tan(3x) dx &= \text{Subst} \left(\int \frac{x^2(3-4x^2)}{1-5x^2+4x^4} dx, x, \sin(x) \right) \\
&= -\sin(x) - \frac{1}{4} \text{Subst} \left(\int \frac{-4+8x^2}{1-5x^2+4x^4} dx, x, \sin(x) \right) \\
&= -\sin(x) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2} dx, x, \sin(x) \right) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2} dx, x, \sin(x) \right) \\
&= -\sin(x) - \frac{1}{6} \text{Subst} \left(\int \frac{1}{-1+x} dx, x, \sin(x) \right) - \frac{1}{6} \text{Subst} \left(\int \frac{1}{-\frac{1}{2}+x} dx, x, \sin(x) \right) + \frac{1}{6} \text{Subst} \left(\int \frac{1}{-1-x} dx, x, \sin(x) \right) + \frac{1}{6} \text{Subst} \left(\int \frac{1}{-\frac{1}{2}-x} dx, x, \sin(x) \right) \\
&= -\frac{1}{6} \log(1-2\sin(x)) - \frac{1}{6} \log(1-\sin(x)) + \frac{1}{6} \log(1+\sin(x)) + \frac{1}{6} \log(1+2\sin(x)) - \sin(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 21, normalized size = 0.45

$$-\sin(x) + \frac{1}{3} \tanh^{-1}(\sin(x)) + \frac{1}{3} \tanh^{-1}(2\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]*Tan[3*x],x]

[Out] ArcTanh[Sin[x]]/3 + ArcTanh[2*Sin[x]]/3 - Sin[x]

fricas [A] time = 1.91, size = 39, normalized size = 0.83

$$\frac{1}{6} \log(2\sin(x) + 1) + \frac{1}{6} \log(\sin(x) + 1) - \frac{1}{6} \log(-\sin(x) + 1) - \frac{1}{6} \log(-2\sin(x) + 1) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(3*x),x, algorithm="fricas")

[Out] 1/6*log(2*sin(x) + 1) + 1/6*log(sin(x) + 1) - 1/6*log(-sin(x) + 1) - 1/6*log(-2*sin(x) + 1) - sin(x)

giac [B] time = 0.27, size = 364, normalized size = 7.74

$$\log \left(\frac{\tan\left(\frac{1}{2}x\right)^4 + 8\tan\left(\frac{1}{2}x\right)^3 + 18\tan\left(\frac{1}{2}x\right)^2 + 8\tan\left(\frac{1}{2}x\right) + 1}{\tan\left(\frac{1}{2}x\right)^4 + 2\tan\left(\frac{1}{2}x\right)^2 + 1} \right) \tan\left(\frac{1}{2}x\right)^2 - \log \left(\frac{\tan\left(\frac{1}{2}x\right)^4 - 8\tan\left(\frac{1}{2}x\right)^3 + 18\tan\left(\frac{1}{2}x\right)^2 - 8\tan\left(\frac{1}{2}x\right) + 1}{\tan\left(\frac{1}{2}x\right)^4 + 2\tan\left(\frac{1}{2}x\right)^2 + 1} \right) \tan\left(\frac{1}{2}x\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(3*x),x, algorithm="giac")

[Out] 1/12*(log((tan(1/2*x)^4 + 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 + 8*tan(1/2*x) + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - log((tan(1/2*x)^4 - 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 - 8*tan(1/2*x) + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 2*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - 2*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + log((tan(1/2*x)^4 + 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 + 8*tan(1/2*x) + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)) - log((tan(1/2*x)^4 - 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 - 8*tan(1/2*x) + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)) + 2*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1))

$$\frac{1}{(\tan(1/2*x)^2 + 1)} - \frac{2*\log(2*(\tan(1/2*x)^2 - 2*\tan(1/2*x) + 1))}{(\tan(1/2*x)^2 + 1)} - \frac{24*\tan(1/2*x)}{(\tan(1/2*x)^2 + 1)}$$

maple [A] time = 0.30, size = 38, normalized size = 0.81

$$-\frac{\ln(-1 + 2 \sin(x))}{6} + \frac{\ln(1 + 2 \sin(x))}{6} - \frac{\ln(\sin(x) - 1)}{6} + \frac{\ln(1 + \sin(x))}{6} - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*tan(3*x),x)

[Out] -1/6*ln(-1+2*sin(x))+1/6*ln(1+2*sin(x))-1/6*ln(sin(x)-1)+1/6*ln(1+sin(x))-sin(x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\cos(3x) + \cos(x)) \cos(4x) - (\cos(2x) - 1) \cos(3x) - \cos(2x) \cos(x) + (\sin(3x) + \sin(x)) \sin(4x) - \sin(3x) \sin(2x) - \sin(2x) \sin(x) + \cos(x)}{3(2(\cos(2x) - 1) \cos(4x) - \cos(4x)^2 - \cos(2x)^2 - \sin(4x)^2 + 2 \sin(4x) \sin(2x) - \sin(2x)^2 + 2 \cos(2x) - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(3*x),x, algorithm="maxima")

[Out] integrate(-1/3*((cos(3*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - cos(2*x)*cos(x) + (sin(3*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*x)*sin(x) + cos(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/6*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/6*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - sin(x)

mupad [B] time = 2.34, size = 26, normalized size = 0.55

$$\frac{2 \operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{3} + \frac{\operatorname{atanh}(2 \sin(x))}{3} - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(3*x)*sin(x),x)

[Out] (2*atanh(sin(x/2)/cos(x/2)))/3 + atanh(2*sin(x))/3 - sin(x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \tan(3x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(3*x),x)

[Out] Integral(sin(x)*tan(3*x), x)

3.76 $\int \sin(x) \tan(4x) dx$

Optimal. Leaf size=71

$$-\sin(x) + \frac{1}{4}\sqrt{2-\sqrt{2}} \tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{4}\sqrt{2+\sqrt{2}} \tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)$$

[Out] $-\sin(x) + 1/4 \cdot \operatorname{arctanh}(2 \sin(x) / (2 - 2^{1/2})^{1/2}) \cdot (2 - 2^{1/2})^{1/2} + 1/4 \cdot \operatorname{arctanh}(2 \sin(x) / (2 + 2^{1/2})^{1/2}) \cdot (2 + 2^{1/2})^{1/2}$

Rubi [A] time = 0.11, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1279, 1166, 207}

$$-\sin(x) + \frac{1}{4}\sqrt{2-\sqrt{2}} \tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{4}\sqrt{2+\sqrt{2}} \tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Tan[4*x],x]

[Out] $(\sqrt{2 - \sqrt{2}} \cdot \operatorname{ArcTanh}[(2 \sin(x)) / \sqrt{2 - \sqrt{2}}]) / 4 + (\sqrt{2 + \sqrt{2}} \cdot \operatorname{ArcTanh}[(2 \sin(x)) / \sqrt{2 + \sqrt{2}}]) / 4 - \sin(x)$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1279

Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m-1)*(a + b*x^2 + c*x^4)^(p+1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m-2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m-1) + (b*e*(m+2*p+1) - c*d*(m+4*p+3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rubi steps

$$\begin{aligned}
\int \sin(x) \tan(4x) dx &= \text{Subst} \left(\int \frac{x^2 (4 - 8x^2)}{1 - 8x^2 + 8x^4} dx, x, \sin(x) \right) \\
&= -\sin(x) - \frac{1}{8} \text{Subst} \left(\int \frac{-8 + 32x^2}{1 - 8x^2 + 8x^4} dx, x, \sin(x) \right) \\
&= -\sin(x) - (2 - \sqrt{2}) \text{Subst} \left(\int \frac{1}{-4 + 2\sqrt{2} + 8x^2} dx, x, \sin(x) \right) - (2 + \sqrt{2}) \text{Subst} \left(\int \frac{1}{-4 - 2\sqrt{2} + 8x^2} dx, x, \sin(x) \right) \\
&= \frac{1}{4} \sqrt{2 - \sqrt{2}} \tanh^{-1} \left(\frac{2 \sin(x)}{\sqrt{2 - \sqrt{2}}} \right) + \frac{1}{4} \sqrt{2 + \sqrt{2}} \tanh^{-1} \left(\frac{2 \sin(x)}{\sqrt{2 + \sqrt{2}}} \right) - \sin(x)
\end{aligned}$$

Mathematica [A] time = 0.08, size = 69, normalized size = 0.97

$$\frac{1}{4} \left(-4 \sin(x) + \sqrt{2 - \sqrt{2}} \tanh^{-1} \left(\frac{2 \sin(x)}{\sqrt{2 - \sqrt{2}}} \right) + \sqrt{2 + \sqrt{2}} \tanh^{-1} \left(\frac{2 \sin(x)}{\sqrt{2 + \sqrt{2}}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]*Tan[4*x],x]

[Out] (Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]] + Sqrt[2 + Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]] - 4*Sin[x])/4

fricas [A] time = 2.46, size = 101, normalized size = 1.42

$$\frac{1}{8} \sqrt{\sqrt{2} + 2} \log \left(\sqrt{\sqrt{2} + 2} + 2 \sin(x) \right) - \frac{1}{8} \sqrt{\sqrt{2} + 2} \log \left(\sqrt{\sqrt{2} + 2} - 2 \sin(x) \right) + \frac{1}{8} \sqrt{-\sqrt{2} + 2} \log \left(\sqrt{-\sqrt{2} + 2} + 2 \sin(x) \right) - \frac{1}{8} \sqrt{-\sqrt{2} + 2} \log \left(\sqrt{-\sqrt{2} + 2} - 2 \sin(x) \right) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(4*x),x, algorithm="fricas")

[Out] 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) + 2*sin(x)) - 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) - 2*sin(x)) + 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2) + 2*sin(x)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2) - 2*sin(x)) - sin(x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \tan(4x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(4*x),x, algorithm="giac")

[Out] integrate(sin(x)*tan(4*x), x)

maple [B] time = 0.41, size = 115, normalized size = 1.62

$$\frac{(\sqrt{2} - 2) \sqrt{2} \operatorname{arctanh} \left(\frac{2 \sin(x)}{\sqrt{2 - \sqrt{2}}} \right) + \sqrt{2 + \sqrt{2}} \sqrt{2} \operatorname{arctanh} \left(\frac{2 \sin(x)}{\sqrt{2 + \sqrt{2}}} \right) - \sin(x) + \frac{\sqrt{2} \operatorname{arctanh} \left(\frac{2 \sin(x)}{\sqrt{2 - \sqrt{2}}} \right) \sqrt{2} \operatorname{arctanh} \left(\frac{2 \sin(x)}{\sqrt{2 + \sqrt{2}}} \right)}{4\sqrt{2 - \sqrt{2}}} - \frac{\sqrt{2} \operatorname{arctanh} \left(\frac{2 \sin(x)}{\sqrt{2 - \sqrt{2}}} \right) \sqrt{2} \operatorname{arctanh} \left(\frac{2 \sin(x)}{\sqrt{2 + \sqrt{2}}} \right)}{4\sqrt{2 + \sqrt{2}}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*tan(4*x),x)

[Out] $\frac{1}{4}*(2^{(1/2)}-2)*2^{(1/2)}/(2-2^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2*\sin(x)/(2-2^{(1/2)})^{(1/2)})+1/4*(2+2^{(1/2)})^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(2*\sin(x)/(2+2^{(1/2)})^{(1/2)})-\sin(x)+1/4*2^{(1/2)}/(2-2^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2*\sin(x)/(2-2^{(1/2)})^{(1/2)})-1/4*2^{(1/2)}/(2+2^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2*\sin(x)/(2+2^{(1/2)})^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\cos(7x) + \cos(x)) \cos(8x) + (\sin(7x) + \sin(x)) \sin(8x) + \cos(7x) + \cos(x)}{\cos(8x)^2 + \sin(8x)^2 + 2 \cos(8x) + 1} dx - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(4*x),x, algorithm="maxima")

[Out] integrate(((cos(7*x) + cos(x))*cos(8*x) + (sin(7*x) + sin(x))*sin(8*x) + cos(7*x) + cos(x))/(cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x) + 1), x) - sin(x)

mupad [B] time = 2.56, size = 103, normalized size = 1.45

$$\frac{\operatorname{atanh}\left(\frac{34 \sin(x) \sqrt{\sqrt{2}+2} + 24 \sqrt{2} \sin(x) \sqrt{\sqrt{2}+2}}{41 \sqrt{2}+58}\right) \sqrt{\sqrt{2}+2}}{4} - \sin(x) - \frac{\operatorname{atanh}\left(\frac{34 \sin(x) \sqrt{2-\sqrt{2}} - 24 \sqrt{2} \sin(x) \sqrt{2-\sqrt{2}}}{41 \sqrt{2}-58}\right) \sqrt{2-\sqrt{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(4*x)*sin(x),x)

[Out] $(\operatorname{atanh}((34*\sin(x)*(2^{(1/2)}+2)^{(1/2)}+24*2^{(1/2)}*\sin(x)*(2^{(1/2)}+2)^{(1/2)})/(41*2^{(1/2)}+58))*(2^{(1/2)}+2)^{(1/2)})/4 - \sin(x) - (\operatorname{atanh}((34*\sin(x)*(2-2^{(1/2)})^{(1/2)}-24*2^{(1/2)}*\sin(x)*(2-2^{(1/2)})^{(1/2)})/(41*2^{(1/2)}-58))*(2-2^{(1/2)})^{(1/2)})/4$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \tan(4x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(4*x),x)

[Out] Integral(sin(x)*tan(4*x), x)

3.77 $\int \sin(x) \tan(5x) dx$

Optimal. Leaf size=112

$$-\sin(x) - \frac{1}{20}(1 - \sqrt{5}) \log(-4 \sin(x) - \sqrt{5} + 1) - \frac{1}{20}(1 + \sqrt{5}) \log(-4 \sin(x) + \sqrt{5} + 1) + \frac{1}{20}(1 - \sqrt{5}) \log(4 \sin(x) + \sqrt{5} - 1) + \frac{1}{20}(1 + \sqrt{5}) \log(4 \sin(x) - \sqrt{5} - 1)$$

```
[Out] 1/5*arctanh(sin(x))-sin(x)-1/20*ln(1-4*sin(x)-5^(1/2))*(-5^(1/2)+1)+1/20*ln(1+4*sin(x)-5^(1/2))*(-5^(1/2)+1)-1/20*ln(1-4*sin(x)+5^(1/2))*(5^(1/2)+1)+1/20*ln(1+4*sin(x)+5^(1/2))*(5^(1/2)+1)
```

Rubi [A] time = 0.17, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2075, 207, 632, 31}

$$-\sin(x) - \frac{1}{20}(1 - \sqrt{5}) \log(-4 \sin(x) - \sqrt{5} + 1) - \frac{1}{20}(1 + \sqrt{5}) \log(-4 \sin(x) + \sqrt{5} + 1) + \frac{1}{20}(1 - \sqrt{5}) \log(4 \sin(x) + \sqrt{5} - 1) + \frac{1}{20}(1 + \sqrt{5}) \log(4 \sin(x) - \sqrt{5} - 1)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[5*x],x]
```

```
[Out] ArcTanh[Sin[x]]/5 - ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Sin[x]])/20 - ((1 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Sin[x]])/20 + ((1 - Sqrt[5])*Log[1 - Sqrt[5] + 4*Sin[x]])/20 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Sin[x]])/20 - Sin[x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x] /; PolyQ[Qm, x] && PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin(x) \tan(5x) dx &= \text{Subst} \left(\int \frac{x^2 (5 - 20x^2 + 16x^4)}{1 - 13x^2 + 28x^4 - 16x^6} dx, x, \sin(x) \right) \\
&= \text{Subst} \left(\int \left(-1 - \frac{1}{5(-1+x^2)} - \frac{2(1+x)}{5(-1-2x+4x^2)} + \frac{2(-1+x)}{5(-1+2x+4x^2)} \right) dx, x, \sin(x) \right) \\
&= -\sin(x) - \frac{1}{5} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \sin(x) \right) - \frac{2}{5} \text{Subst} \left(\int \frac{1+x}{-1-2x+4x^2} dx, x, \sin(x) \right) \\
&= \frac{1}{5} \tanh^{-1}(\sin(x)) - \sin(x) + \frac{1}{5} (1 - \sqrt{5}) \text{Subst} \left(\int \frac{1}{1 - \sqrt{5} + 4x} dx, x, \sin(x) \right) - \frac{1}{5} (1 - \sqrt{5}) \text{Subst} \left(\int \frac{1}{1 + \sqrt{5} + 4x} dx, x, \sin(x) \right) \\
&= \frac{1}{5} \tanh^{-1}(\sin(x)) - \frac{1}{20} (1 - \sqrt{5}) \log(1 - \sqrt{5} - 4\sin(x)) - \frac{1}{20} (1 + \sqrt{5}) \log(1 + \sqrt{5} - 4\sin(x))
\end{aligned}$$

Mathematica [A] time = 0.18, size = 100, normalized size = 0.89

$$\frac{1}{20} (-20 \sin(x) + (\sqrt{5} - 1) \log(-4 \sin(x) - \sqrt{5} + 1) - (1 + \sqrt{5}) \log(-4 \sin(x) + \sqrt{5} + 1) - (\sqrt{5} - 1) \log(4 \sin(x) - \sqrt{5} + 1) + (1 + \sqrt{5}) \log(4 \sin(x) + \sqrt{5} + 1))$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]*Tan[5*x], x]

[Out] (4*ArcTanh[Sin[x]] + (-1 + Sqrt[5])*Log[1 - Sqrt[5] - 4*Sin[x]] - (1 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Sin[x]] - (-1 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Sin[x]] + (1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Sin[x]] - 20*Sin[x])/20

fricas [A] time = 0.84, size = 136, normalized size = 1.21

$$\frac{1}{20} \sqrt{5} \log\left(\frac{8 \cos(x)^2 - 4(\sqrt{5} - 1) \sin(x) + \sqrt{5} - 11}{4 \cos(x)^2 + 2 \sin(x) - 3}\right) + \frac{1}{20} \sqrt{5} \log\left(-\frac{8 \cos(x)^2 - 4(\sqrt{5} + 1) \sin(x) - \sqrt{5} - 11}{4 \cos(x)^2 - 2 \sin(x) - 3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(5*x), x, algorithm="fricas")

[Out] 1/20*sqrt(5)*log((8*cos(x)^2 - 4*(sqrt(5) - 1)*sin(x) + sqrt(5) - 11)/(4*cos(x)^2 + 2*sin(x) - 3)) + 1/20*sqrt(5)*log(-(8*cos(x)^2 - 4*(sqrt(5) + 1)*sin(x) - sqrt(5) - 11)/(4*cos(x)^2 - 2*sin(x) - 3)) - 1/20*log(4*cos(x)^2 + 2*sin(x) - 3) + 1/20*log(4*cos(x)^2 - 2*sin(x) - 3) + 1/10*log(sin(x) + 1) - 1/10*log(-sin(x) + 1) - sin(x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \tan(5x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(5*x), x, algorithm="giac")

[Out] integrate(sin(x)*tan(5*x), x)

maple [A] time = 0.41, size = 84, normalized size = 0.75

$$\frac{\ln(4(\sin^2(x)) + 2 \sin(x) - 1)}{20} + \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(8 \sin(x) + 2) \sqrt{5}}{10}\right)}{10} - \frac{\ln(\sin(x) - 1)}{10} - \frac{\ln(4(\sin^2(x)) - 2 \sin(x) - 1)}{20} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*tan(5*x),x)

[Out] $\frac{1}{20} \ln(4 \sin(x)^2 + 2 \sin(x) - 1) + \frac{1}{10} 5^{1/2} \operatorname{arctanh}\left(\frac{1}{10}(8 \sin(x) + 2) 5^{1/2}\right) - \frac{1}{10} \ln(\sin(x) - 1) - \frac{1}{20} \ln(4 \sin(x)^2 - 2 \sin(x) - 1) + \frac{1}{10} 5^{1/2} \operatorname{arctanh}\left(\frac{1}{10}(8 \sin(x) - 2) 5^{1/2}\right) + \frac{1}{10} \ln(1 + \sin(x)) - \sin(x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3 \cos(7x) - \cos(5x) - \cos(3x) + 3 \cos(x)) \cos(8x) - 3(\cos(6x) - \cos(4x) + \cos(2x) - 1) \cos(7x) + (3 \cos(6x) - \cos(4x) + \cos(2x) - 1) \cos(5x) - (\cos(3x) - 3 \cos(x)) \cos(6x) - (\cos(4x) - \cos(2x) + 1) \cos(5x) - (\cos(3x) - 3 \cos(x)) \cos(4x) + (\cos(2x) - 1) \cos(3x) - 3 \cos(2x) \cos(x) + (3 \sin(7x) - \sin(5x) - \sin(3x) + 3 \sin(x)) \sin(8x) - 3(\sin(6x) - \sin(4x) + \sin(2x)) \sin(7x) + (\sin(5x) + \sin(3x) - 3 \sin(x)) \sin(6x) - (\sin(4x) - \sin(2x)) \sin(5x) - (\sin(3x) - 3 \sin(x)) \sin(4x) + \sin(3x) \sin(2x) - 3 \sin(2x) \sin(x) + 3 \cos(x)}{5(2(\cos(6x) - \cos(4x) + \cos(2x) - 1) \cos(8x) - \cos(8x)^2 + 2(\cos(4x) - \cos(2x) + 1) \cos(6x) - \cos(6x)^2 + 2(\cos(2x) - 1) \cos(4x) - \cos(4x)^2 - \cos(2x)^2 + 2(\sin(6x) - \sin(4x) + \sin(2x)) \sin(8x) - \sin(8x)^2 + 2(\sin(4x) - \sin(2x)) \sin(6x) - \sin(6x)^2 - \sin(4x)^2 + 2 \sin(4x) \sin(2x) - \sin(2x)^2 + 2 \cos(2x) - 1), x) + \frac{1}{10} \log(\cos(x)^2 + \sin(x)^2 + 2 \sin(x) + 1) - \frac{1}{10} \log(\cos(x)^2 + \sin(x)^2 - 2 \sin(x) + 1) - \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(5*x),x, algorithm="maxima")

[Out] integrate(-1/5*((3*cos(7*x) - cos(5*x) - cos(3*x) + 3*cos(x))*cos(8*x) - 3*(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(7*x) + (cos(5*x) + cos(3*x) - 3*cos(x))*cos(6*x) - (cos(4*x) - cos(2*x) + 1)*cos(5*x) - (cos(3*x) - 3*cos(x))*cos(4*x) + (cos(2*x) - 1)*cos(3*x) - 3*cos(2*x)*cos(x) + (3*sin(7*x) - sin(5*x) - sin(3*x) + 3*sin(x))*sin(8*x) - 3*(sin(6*x) - sin(4*x) + sin(2*x))*sin(7*x) + (sin(5*x) + sin(3*x) - 3*sin(x))*sin(6*x) - (sin(4*x) - sin(2*x))*sin(5*x) - (sin(3*x) - 3*sin(x))*sin(4*x) + sin(3*x)*sin(2*x) - 3*sin(2*x)*sin(x) + 3*cos(x))/(2*(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 2*(cos(4*x) - cos(2*x) + 1)*cos(6*x) - cos(6*x)^2 + 2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 + 2*(sin(6*x) - sin(4*x) + sin(2*x))*sin(8*x) - sin(8*x)^2 + 2*(sin(4*x) - sin(2*x))*sin(6*x) - sin(6*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/10*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/10*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - sin(x)

mupad [B] time = 2.89, size = 107, normalized size = 0.96

$$\frac{2 \operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{5} + \frac{\operatorname{atan}\left(\frac{\sin(x)1042i - \sqrt{5} \sin(x)466i}{377\sqrt{5} - 843}\right)1i}{10} - \frac{\operatorname{atanh}\left(\sin(x) - \sqrt{5} \sin(x)\right)}{10 \sin(x)} - \frac{\sqrt{5} \operatorname{atanh}\left(\sin(x) - \sqrt{5} \sin(x)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(5*x)*sin(x),x)

[Out] $\frac{\operatorname{atan}\left(\frac{\sin(x)1042i - 5^{1/2} \sin(x)466i}{(377 \cdot 5^{1/2} - 843)}\right)1i}{10} - \operatorname{atanh}\left(\frac{\sin(x) - 5^{1/2} \sin(x)}{10}\right) + \frac{(2 \operatorname{atanh}\left(\frac{\sin(x/2)}{\cos(x/2)}\right))}{5} - \sin(x) - \frac{(5^{1/2} \operatorname{atanh}\left(\sin(x) - 5^{1/2} \sin(x)\right))}{10} - \frac{(5^{1/2} \operatorname{atan}\left(\frac{\sin(x)1042i - 5^{1/2} \sin(x)466i}{(377 \cdot 5^{1/2} - 843)}\right)1i)}{10}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \tan(5x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(5*x),x)

[Out] Integral(sin(x)*tan(5*x), x)

3.78 $\int \sin(x) \tan(6x) dx$

Optimal. Leaf size=89

$$-\sin(x) + \frac{\tanh^{-1}(\sqrt{2} \sin(x))}{3\sqrt{2}} + \frac{1}{6}\sqrt{2-\sqrt{3}} \tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{6}\sqrt{2+\sqrt{3}} \tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)$$

[Out] $-\sin(x) + 1/6 \cdot \operatorname{arctanh}(\sin(x) \cdot 2^{(1/2)}) \cdot 2^{(1/2)} + 1/6 \cdot \operatorname{arctanh}(2 \cdot \sin(x) / (1/2 \cdot 6^{(1/2)} - 1/2 \cdot 2^{(1/2)})) \cdot (1/2 \cdot 6^{(1/2)} - 1/2 \cdot 2^{(1/2)}) + 1/6 \cdot \operatorname{arctanh}(2 \cdot \sin(x) / (1/2 \cdot 6^{(1/2)} + 1/2 \cdot 2^{(1/2)})) \cdot (1/2 \cdot 6^{(1/2)} + 1/2 \cdot 2^{(1/2)})$

Rubi [A] time = 0.27, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {12, 6742, 2073, 207, 1166}

$$-\sin(x) + \frac{\tanh^{-1}(\sqrt{2} \sin(x))}{3\sqrt{2}} + \frac{1}{6}\sqrt{2-\sqrt{3}} \tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{6}\sqrt{2+\sqrt{3}} \tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Tan[6*x],x]

[Out] $\operatorname{ArcTanh}[\operatorname{Sqrt}[2] \cdot \operatorname{Sin}[x]] / (3 \cdot \operatorname{Sqrt}[2]) + (\operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]] \cdot \operatorname{ArcTanh}[(2 \cdot \operatorname{Sin}[x]) / \operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]]) / 6 + (\operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]] \cdot \operatorname{ArcTanh}[(2 \cdot \operatorname{Sin}[x]) / \operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]]]) / 6 - \operatorname{Sin}[x]$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 2073

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] := With[{PP = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \sin(x) \tan(6x) dx &= \text{Subst} \left(\int \frac{2x^2 (3 - 16x^2 + 16x^4)}{1 - 18x^2 + 48x^4 - 32x^6} dx, x, \sin(x) \right) \\
&= 2 \text{Subst} \left(\int \frac{x^2 (3 - 16x^2 + 16x^4)}{1 - 18x^2 + 48x^4 - 32x^6} dx, x, \sin(x) \right) \\
&= 2 \text{Subst} \left(\int \left(-\frac{1}{2} + \frac{1 - 12x^2 + 16x^4}{2(1 - 18x^2 + 48x^4 - 32x^6)} \right) dx, x, \sin(x) \right) \\
&= -\sin(x) + \text{Subst} \left(\int \frac{1 - 12x^2 + 16x^4}{1 - 18x^2 + 48x^4 - 32x^6} dx, x, \sin(x) \right) \\
&= -\sin(x) + \text{Subst} \left(\int \left(-\frac{1}{3(-1 + 2x^2)} - \frac{2(-1 + 8x^2)}{3(1 - 16x^2 + 16x^4)} \right) dx, x, \sin(x) \right) \\
&= -\sin(x) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{-1 + 2x^2} dx, x, \sin(x) \right) - \frac{2}{3} \text{Subst} \left(\int \frac{-1 + 8x^2}{1 - 16x^2 + 16x^4} dx, x, \sin(x) \right) \\
&= \frac{\tanh^{-1}(\sqrt{2} \sin(x))}{3\sqrt{2}} - \sin(x) - \frac{1}{3} (4(2 - \sqrt{3})) \text{Subst} \left(\int \frac{1}{-8 + 4\sqrt{3} + 16x^2} dx, x, \sin(x) \right) \\
&= \frac{\tanh^{-1}(\sqrt{2} \sin(x))}{3\sqrt{2}} + \frac{1}{6} \sqrt{2 - \sqrt{3}} \tanh^{-1} \left(\frac{2 \sin(x)}{\sqrt{2 - \sqrt{3}}} \right) + \frac{1}{6} \sqrt{2 + \sqrt{3}} \tanh^{-1} \left(\frac{2 \sin(x)}{\sqrt{2 + \sqrt{3}}} \right)
\end{aligned}$$

Mathematica [A] time = 0.15, size = 84, normalized size = 0.94

$$\frac{1}{6} \left(-6 \sin(x) + \sqrt{2} \tanh^{-1}(\sqrt{2} \sin(x)) + \sqrt{2 - \sqrt{3}} \tanh^{-1} \left(\frac{2 \sin(x)}{\sqrt{2 - \sqrt{3}}} \right) + \sqrt{2 + \sqrt{3}} \tanh^{-1} \left(\frac{2 \sin(x)}{\sqrt{2 + \sqrt{3}}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]*Tan[6*x],x]

[Out] (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[x]] + Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[3]]] + Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[3]]] - 6*Sin[x])/6

fricas [A] time = 1.79, size = 134, normalized size = 1.51

$$\frac{1}{12} \sqrt{\sqrt{3} + 2} \log \left(\sqrt{\sqrt{3} + 2} + 2 \sin(x) \right) - \frac{1}{12} \sqrt{\sqrt{3} + 2} \log \left(\sqrt{\sqrt{3} + 2} - 2 \sin(x) \right) + \frac{1}{12} \sqrt{-\sqrt{3} + 2} \log \left(\sqrt{-\sqrt{3} + 2} + 2 \sin(x) \right) - \frac{1}{12} \sqrt{-\sqrt{3} + 2} \log \left(\sqrt{-\sqrt{3} + 2} - 2 \sin(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(6*x),x, algorithm="fricas")

[Out] 1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2) + 2*sin(x)) - 1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2) - 2*sin(x)) + 1/12*sqrt(-sqrt(3) + 2)*log(sqrt(-sqrt(3) + 2) + 2*sin(x)) - 1/12*sqrt(-sqrt(3) + 2)*log(sqrt(-sqrt(3) + 2) - 2*sin(x)) + 1/12*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - sin(x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \tan(6x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(6*x),x, algorithm="giac")

[Out] integrate(sin(x)*tan(6*x), x)

maple [B] time = 0.62, size = 256, normalized size = 2.88

$$\frac{(-3 + 2\sqrt{3})\sqrt{3} \operatorname{arctanh}\left(\frac{8\sin(x)}{2\sqrt{6}-2\sqrt{2}}\right)}{6\sqrt{6}-6\sqrt{2}} + \frac{(3 + 2\sqrt{3})\sqrt{3} \operatorname{arctanh}\left(\frac{8\sin(x)}{2\sqrt{6}+2\sqrt{2}}\right)}{6\sqrt{6}+6\sqrt{2}} + \frac{\operatorname{arctanh}(\sin(x)\sqrt{2})\sqrt{2}}{6} - \frac{4 \operatorname{arctanh}(\sin(x))}{3(2\sqrt{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*tan(6*x),x)

[Out] $\frac{1}{3}(-3+2\sqrt{3})\sqrt{3} \operatorname{arctanh}\left(\frac{8\sin(x)}{2\sqrt{6}-2\sqrt{2}}\right) + \frac{1}{3}(3+2\sqrt{3})\sqrt{3} \operatorname{arctanh}\left(\frac{8\sin(x)}{2\sqrt{6}+2\sqrt{2}}\right) + \frac{1}{6} \operatorname{arctanh}(\sin(x)\sqrt{2})\sqrt{2} - \frac{4}{3} \operatorname{arctanh}(\sin(x))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{24}\sqrt{2} \log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{24}\sqrt{2} \log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(6*x),x, algorithm="maxima")

[Out] $\frac{1}{24}\sqrt{2} \log(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2) - \frac{1}{24}\sqrt{2} \log(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2) + \frac{1}{24}\sqrt{2} \log(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2) - \frac{1}{24}\sqrt{2} \log(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2) + \int (-1/3*((2\cos(7*x) - \cos(5*x) - \cos(3*x) + 2\cos(x))*\cos(8*x) - 2*(\cos(4*x) - 1)*\cos(7*x) + (\cos(4*x) - 1)*\cos(5*x) + (\cos(3*x) - 2\cos(x))*\cos(4*x) + (2\sin(7*x) - \sin(5*x) - \sin(3*x) + 2\sin(x))*\sin(8*x) + (\sin(3*x) - 2\sin(x))*\sin(4*x) - 2\sin(7*x)*\sin(4*x) + \sin(5*x)*\sin(4*x) - \cos(3*x) + 2\cos(x))/(2*(\cos(4*x) - 1)*\cos(8*x) - \cos(8*x)^2 - \cos(4*x)^2 - \sin(8*x)^2 + 2\sin(8*x)*\sin(4*x) - \sin(4*x)^2 + 2\cos(4*x) - 1), x) - \sin(x)$

mupad [B] time = 3.11, size = 131, normalized size = 1.47

$$\frac{\sqrt{2} \operatorname{atanh}(\sqrt{2} \sin(x))}{6} - \frac{\sqrt{2} \operatorname{atanh}(\sqrt{2} \sin(x) - \sqrt{6} \sin(x))}{12} - \frac{\sqrt{6} \operatorname{atanh}(\sqrt{2} \sin(x) - \sqrt{6} \sin(x))}{12} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(6*x)*sin(x),x)

[Out] $(2^{1/2} \operatorname{atan}((2^{1/2} \sin(x) * 102818i - 6^{1/2} \sin(x) * 59362i) / (40545 * 2^{1/2} * 6^{1/2} - 140452)) * 1i) / 12 - \sin(x) - (6^{1/2} \operatorname{atan}((2^{1/2} \sin(x) * 102818i - 6^{1/2} \sin(x) * 59362i) / (40545 * 2^{1/2} * 6^{1/2} - 140452)) * 1i) / 12 + (2^{1/2} \operatorname{atanh}(2^{1/2} \sin(x))) / 6 - (2^{1/2} \operatorname{atanh}(2^{1/2} \sin(x) - 6^{1/2} \sin(x))) / 12 - (6^{1/2} \operatorname{atanh}(2^{1/2} \sin(x) - 6^{1/2} \sin(x))) / 12$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \tan(6x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(6*x),x)
```

```
[Out] Integral(sin(x)*tan(6*x), x)
```

3.79 $\int \sin(x) \tan(nx) dx$

Optimal. Leaf size=105

$$-ie^{-ix} {}_2F_1\left(1, -\frac{1}{2n}; 1 - \frac{1}{2n}; -e^{2inx}\right) - ie^{ix} {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -e^{2inx}\right) + \frac{1}{2}ie^{-ix} + \frac{1}{2}ie^{ix}$$

[Out] $1/2*I/\exp(I*x)+1/2*I*\exp(I*x)-I*\text{hypergeom}([1, -1/2/n], [1-1/2/n], -\exp(2*I*n*x))/\exp(I*x)-I*\exp(I*x)*\text{hypergeom}([1, 1/2/n], [1+1/2/n], -\exp(2*I*n*x))$

Rubi [A] time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4557, 2194, 2251}

$$-ie^{-ix} {}_2F_1\left(1, -\frac{1}{2n}; 1 - \frac{1}{2n}; -e^{2inx}\right) - ie^{ix} {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -e^{2inx}\right) + \frac{1}{2}ie^{-ix} + \frac{1}{2}ie^{ix}$$

Antiderivative was successfully verified.

[In] Int [Sin [x] *Tan [n*x] , x]

[Out] $(I/2)/E^{(I*x)} + (I/2)*E^{(I*x)} - (I*\text{Hypergeometric2F1}[1, -1/(2*n), 1 - 1/(2*n), -E^{((2*I)*n*x)}])/E^{(I*x)} - I*E^{(I*x)}*\text{Hypergeometric2F1}[1, 1/(2*n), (2 + n^{(-1)})/2, -E^{((2*I)*n*x)}]$

Rule 2194

Int [((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 2251

Int [((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b*F^(e*(c + d*x)))/a])])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 4557

Int [Sin [(a_.) + (b_.)*(x_)]*Tan [(c_.) + (d_.)*(x_)], x_Symbol] := Int [1/(E^(I*(a + b*x))*2) - E^(I*(a + b*x))/2 - 1/(E^(I*(a + b*x))*(1 + E^(2*I*(c + d*x)))) + E^(I*(a + b*x))/(1 + E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \int \sin(x) \tan(nx) dx &= \int \left(\frac{e^{-ix}}{2} - \frac{e^{ix}}{2} - \frac{e^{-ix}}{1 + e^{2inx}} + \frac{e^{ix}}{1 + e^{2inx}} \right) dx \\ &= \frac{1}{2} \int e^{-ix} dx - \frac{1}{2} \int e^{ix} dx - \int \frac{e^{-ix}}{1 + e^{2inx}} dx + \int \frac{e^{ix}}{1 + e^{2inx}} dx \\ &= \frac{1}{2}ie^{-ix} + \frac{1}{2}ie^{ix} - ie^{-ix} {}_2F_1\left(1, -\frac{1}{2n}; 1 - \frac{1}{2n}; -e^{2inx}\right) - ie^{ix} {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); -e^{2inx}\right) \end{aligned}$$

Mathematica [A] time = 0.18, size = 200, normalized size = 1.90

$$\frac{ie^{-2ix} \left((2n+1)e^{i(2nx+x)} {}_2F_1\left(1, 1 - \frac{1}{2n}; 2 - \frac{1}{2n}; -e^{2inx}\right) + (2n-1) \left((2n+1)e^{ix} \left({}_2F_1\left(1, -\frac{1}{2n}; 1 - \frac{1}{2n}; -e^{2inx}\right) + e^{2ix} \right) \right) \right)}{2(4n^2 - 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]*Tan[n*x],x]

[Out] $((-1/2*I)*(E^{I*(x + 2*n*x)}*(1 + 2*n)*\text{Hypergeometric2F1}[1, 1 - 1/(2*n), 2 - 1/(2*n), -E^{((2*I)*n*x)}] + (-1 + 2*n)*(-E^{I*(3 + 2*n)*x})*\text{Hypergeometric2F1}[1, 1 + 1/(2*n), 2 + 1/(2*n), -E^{((2*I)*n*x)}]) + E^{I*x}*(1 + 2*n)*(\text{Hypergeometric2F1}[1, -1/2*1/n, 1 - 1/(2*n), -E^{((2*I)*n*x)}] + E^{((2*I)*x)}*\text{Hypergeometric2F1}[1, 1/(2*n), 1 + 1/(2*n), -E^{((2*I)*n*x)}])))/E^{((2*I)*x)}*(-1 + 4*n^2))$

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}(\sin(x) \tan(nx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(n*x),x, algorithm="fricas")

[Out] integral(sin(x)*tan(n*x), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \tan(nx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(n*x),x, algorithm="giac")

[Out] integrate(sin(x)*tan(n*x), x)

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \sin(x) \tan(nx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*tan(n*x),x)

[Out] int(sin(x)*tan(n*x),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \tan(nx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(n*x),x, algorithm="maxima")

[Out] integrate(sin(x)*tan(n*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(nx) \sin(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(n*x)*sin(x),x)

[Out] int(tan(n*x)*sin(x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \tan(nx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(n*x), x)

[Out] Integral(sin(x)*tan(n*x), x)

3.80 $\int \cot(2x) \sin(x) dx$

Optimal. Leaf size=10

$$\sin(x) - \frac{1}{2} \tanh^{-1}(\sin(x))$$

[Out] $-1/2*\operatorname{arctanh}(\sin(x))+\sin(x)$

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {388, 206}

$$\sin(x) - \frac{1}{2} \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cot}[2*x]*\operatorname{Sin}[x], x]$

[Out] $-\operatorname{ArcTanh}[\operatorname{Sin}[x]]/2 + \operatorname{Sin}[x]$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{Gt}Q[a, 0] \ || \ \operatorname{Lt}Q[b, 0])$

Rule 388

$\operatorname{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)}), x_Symbol] \rightarrow \operatorname{Simp}[(d*x*(a + b*x^n)^{(p + 1)})/(b*(n*(p + 1) + 1)), x] - \operatorname{Dist}[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), \operatorname{Int}[(a + b*x^n)^p, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{NeQ}[n*(p + 1) + 1, 0]$

Rubi steps

$$\begin{aligned} \int \cot(2x) \sin(x) dx &= \operatorname{Subst} \left(\int \frac{1 - 2x^2}{2 - 2x^2} dx, x, \sin(x) \right) \\ &= \sin(x) - \operatorname{Subst} \left(\int \frac{1}{2 - 2x^2} dx, x, \sin(x) \right) \\ &= -\frac{1}{2} \tanh^{-1}(\sin(x)) + \sin(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\sin(x) - \frac{1}{2} \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Integrate}[\operatorname{Cot}[2*x]*\operatorname{Sin}[x], x]$

[Out] $-1/2*\operatorname{ArcTanh}[\operatorname{Sin}[x]] + \operatorname{Sin}[x]$

fricas [B] time = 0.68, size = 19, normalized size = 1.90

$$-\frac{1}{4} \log(\sin(x) + 1) + \frac{1}{4} \log(-\sin(x) + 1) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(2*x)*sin(x),x, algorithm="fricas")

[Out] $-1/4*\log(\sin(x) + 1) + 1/4*\log(-\sin(x) + 1) + \sin(x)$

giac [B] time = 0.14, size = 19, normalized size = 1.90

$$-\frac{1}{4} \log(\sin(x) + 1) + \frac{1}{4} \log(-\sin(x) + 1) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(2*x)*sin(x),x, algorithm="giac")

[Out] $-1/4*\log(\sin(x) + 1) + 1/4*\log(-\sin(x) + 1) + \sin(x)$

maple [A] time = 0.08, size = 12, normalized size = 1.20

$$\sin(x) - \frac{\ln(\sec(x) + \tan(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(2*x)*sin(x),x)

[Out] $\sin(x) - 1/2*\ln(\sec(x) + \tan(x))$

maxima [B] time = 0.41, size = 37, normalized size = 3.70

$$-\frac{1}{4} \log(\cos(x)^2 + \sin(x)^2 + 2 \sin(x) + 1) + \frac{1}{4} \log(\cos(x)^2 + \sin(x)^2 - 2 \sin(x) + 1) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(2*x)*sin(x),x, algorithm="maxima")

[Out] $-1/4*\log(\cos(x)^2 + \sin(x)^2 + 2*\sin(x) + 1) + 1/4*\log(\cos(x)^2 + \sin(x)^2 - 2*\sin(x) + 1) + \sin(x)$

mupad [B] time = 2.32, size = 10, normalized size = 1.00

$$\sin(x) - \operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(2*x)*sin(x),x)

[Out] $\sin(x) - \operatorname{atanh}(\tan(x/2))$

sympy [B] time = 0.82, size = 19, normalized size = 1.90

$$\frac{\log(\sin(x) - 1)}{4} - \frac{\log(\sin(x) + 1)}{4} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(2*x)*sin(x),x)

[Out] $\log(\sin(x) - 1)/4 - \log(\sin(x) + 1)/4 + \sin(x)$

3.81 $\int \cot(3x) \sin(x) dx$

Optimal. Leaf size=20

$$\sin(x) - \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] $\sin(x) - 1/3 * \operatorname{arctanh}(2/3 * \sin(x) * 3^{(1/2)}) * 3^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {388, 206}

$$\sin(x) - \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] `Int[Cot[3*x]*Sin[x],x]`

[Out] $-(\operatorname{ArcTanh}[(2*\sin[x])/Sqrt[3]]/Sqrt[3]) + \sin[x]$

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 388

`Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]`

Rubi steps

$$\begin{aligned} \int \cot(3x) \sin(x) dx &= \operatorname{Subst}\left(\int \frac{1 - 4x^2}{3 - 4x^2} dx, x, \sin(x)\right) \\ &= \sin(x) - 2 \operatorname{Subst}\left(\int \frac{1}{3 - 4x^2} dx, x, \sin(x)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \sin(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$\sin(x) - \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] `Integrate[Cot[3*x]*Sin[x],x]`

[Out] $-(\operatorname{ArcTanh}[(2*\sin[x])/Sqrt[3]]/Sqrt[3]) + \sin[x]$

fricas [B] time = 0.98, size = 36, normalized size = 1.80

$$\frac{1}{6} \sqrt{3} \log\left(-\frac{4 \cos(x)^2 + 4 \sqrt{3} \sin(x) - 7}{4 \cos(x)^2 - 1}\right) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(3*x)*sin(x),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) + sin(x)

giac [B] time = 0.15, size = 34, normalized size = 1.70

$$\frac{1}{6} \sqrt{3} \log\left(\frac{|-4 \sqrt{3} + 8 \sin(x)|}{|4 \sqrt{3} + 8 \sin(x)|}\right) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(3*x)*sin(x),x, algorithm="giac")

[Out] 1/6*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) + sin(x)

maple [A] time = 0.12, size = 17, normalized size = 0.85

$$\sin(x) - \frac{\operatorname{arctanh}\left(\frac{2 \sin(x) \sqrt{3}}{3}\right) \sqrt{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(3*x)*sin(x),x)

[Out] sin(x)-1/3*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)

maxima [B] time = 0.44, size = 127, normalized size = 6.35

$$-\frac{1}{12} \sqrt{3} \log\left(\frac{4}{3} \cos(x)^2 + \frac{4}{3} \sin(x)^2 + \frac{4}{3} \sqrt{3} \sin(x) + \frac{4}{3} \cos(x) + \frac{4}{3}\right) - \frac{1}{12} \sqrt{3} \log\left(\frac{4}{3} \cos(x)^2 + \frac{4}{3} \sin(x)^2 + \frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(3*x)*sin(x),x, algorithm="maxima")

[Out] -1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) - 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 - 4/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) + sin(x)

mupad [B] time = 2.37, size = 16, normalized size = 0.80

$$\sin(x) - \frac{\sqrt{3} \operatorname{atanh}\left(\frac{2 \sqrt{3} \sin(x)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(3*x)*sin(x),x)

[Out] sin(x) - (3^(1/2)*atanh((2*3^(1/2)*sin(x))/3))/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \cot(3x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(3*x)*sin(x),x)`

[Out] `Integral(sin(x)*cot(3*x), x)`

3.82 $\int \cot(4x) \sin(x) dx$

Optimal. Leaf size=28

$$\sin(x) - \frac{1}{4} \tanh^{-1}(\sin(x)) - \frac{\tanh^{-1}(\sqrt{2} \sin(x))}{2\sqrt{2}}$$

[Out] -1/4*arctanh(sin(x))+sin(x)-1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1676, 1166, 207}

$$\sin(x) - \frac{1}{4} \tanh^{-1}(\sin(x)) - \frac{\tanh^{-1}(\sqrt{2} \sin(x))}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Cot[4*x]*Sin[x],x]

[Out] -ArcTanh[Sin[x]]/4 - ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + Sin[x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int(((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1676

Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1

Rubi steps

$$\begin{aligned} \int \cot(4x) \sin(x) dx &= \text{Subst} \left(\int \frac{1 - 8x^2 + 8x^4}{4 - 12x^2 + 8x^4} dx, x, \sin(x) \right) \\ &= \text{Subst} \left(\int \left(1 - \frac{3 - 4x^2}{4 - 12x^2 + 8x^4} \right) dx, x, \sin(x) \right) \\ &= \sin(x) - \text{Subst} \left(\int \frac{3 - 4x^2}{4 - 12x^2 + 8x^4} dx, x, \sin(x) \right) \\ &= \sin(x) + 2 \text{Subst} \left(\int \frac{1}{-8 + 8x^2} dx, x, \sin(x) \right) + 2 \text{Subst} \left(\int \frac{1}{-4 + 8x^2} dx, x, \sin(x) \right) \\ &= -\frac{1}{4} \tanh^{-1}(\sin(x)) - \frac{\tanh^{-1}(\sqrt{2} \sin(x))}{2\sqrt{2}} + \sin(x) \end{aligned}$$

Mathematica [A] time = 0.04, size = 28, normalized size = 1.00

$$\sin(x) - \frac{1}{4} \tanh^{-1}(\sin(x)) - \frac{\tanh^{-1}(\sqrt{2} \sin(x))}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[4*x]*Sin[x],x]

[Out] -1/4*ArcTanh[Sin[x]] - ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + Sin[x]

fricas [B] time = 0.57, size = 52, normalized size = 1.86

$$\frac{1}{8} \sqrt{2} \log\left(-\frac{2 \cos(x)^2 + 2 \sqrt{2} \sin(x) - 3}{2 \cos(x)^2 - 1}\right) - \frac{1}{8} \log(\sin(x) + 1) + \frac{1}{8} \log(-\sin(x) + 1) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(4*x)*sin(x),x, algorithm="fricas")

[Out] 1/8*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1) + sin(x)

giac [B] time = 0.14, size = 50, normalized size = 1.79

$$\frac{1}{8} \sqrt{2} \log\left(\frac{|-2\sqrt{2} + 4 \sin(x)|}{|2\sqrt{2} + 4 \sin(x)|}\right) - \frac{1}{8} \log(\sin(x) + 1) + \frac{1}{8} \log(-\sin(x) + 1) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(4*x)*sin(x),x, algorithm="giac")

[Out] 1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1) + sin(x)

maple [A] time = 0.12, size = 30, normalized size = 1.07

$$\sin(x) + \frac{\ln(\sin(x) - 1)}{8} - \frac{\operatorname{arctanh}(\sin(x)\sqrt{2})\sqrt{2}}{4} - \frac{\ln(1 + \sin(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(4*x)*sin(x),x)

[Out] sin(x)+1/8*ln(sin(x)-1)-1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)-1/8*ln(1+sin(x))

maxima [B] time = 0.54, size = 173, normalized size = 6.18

$$-\frac{1}{16} \sqrt{2} \log\left(2 \cos(x)^2 + 2 \sin(x)^2 + 2 \sqrt{2} \cos(x) + 2 \sqrt{2} \sin(x) + 2\right) + \frac{1}{16} \sqrt{2} \log\left(2 \cos(x)^2 + 2 \sin(x)^2 + 2 \sqrt{2} \cos(x) - 2 \sqrt{2} \sin(x) + 2\right) - \frac{1}{16} \sqrt{2} \log\left(2 \cos(x)^2 + 2 \sin(x)^2 - 2 \sqrt{2} \cos(x) + 2 \sqrt{2} \sin(x) + 2\right) + \frac{1}{16} \sqrt{2} \log\left(2 \cos(x)^2 + 2 \sin(x)^2 - 2 \sqrt{2} \cos(x) - 2 \sqrt{2} \sin(x) + 2\right) - \frac{1}{8} \log(\cos(x)^2 + \sin(x)^2 + 2 \sin(x) + 1) + \frac{1}{8} \log(\cos(x)^2 + \sin(x)^2 - 2 \sin(x) + 1) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(4*x)*sin(x),x, algorithm="maxima")

[Out] -1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + sin(x)

mupad [B] time = 2.38, size = 29, normalized size = 1.04

$$\sin(x) - \frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2} - \frac{\sqrt{2} \operatorname{atanh}\left(\sqrt{2} \sin(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(4*x)*sin(x),x)`

[Out] `sin(x) - atanh(sin(x/2)/cos(x/2))/2 - (2^(1/2)*atanh(2^(1/2)*sin(x)))/4`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \cot(4x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(4*x)*sin(x),x)`

[Out] `Integral(sin(x)*cot(4*x), x)`

3.83 $\int \cot(5x) \sin(x) dx$

Optimal. Leaf size=82

$$\sin(x) - \frac{1}{5} \sqrt{\frac{1}{2}(5 + \sqrt{5})} \tanh^{-1} \left(2 \sqrt{\frac{2}{5 + \sqrt{5}}} \sin(x) \right) - \frac{1}{5} \sqrt{\frac{1}{2}(5 - \sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{5}} (5 + \sqrt{5}) \sin(x) \right)$$

[Out] $\sin(x) - 1/10 * \operatorname{arctanh}(1/5 * \sin(x) * (50 + 10 * 5^{(1/2)})^{(1/2)}) * (10 - 2 * 5^{(1/2)})^{(1/2)} - 1/10 * \operatorname{arctanh}(2 * \sin(x) * 2^{(1/2)} / (5 + 5^{(1/2)})^{(1/2)}) * (10 + 2 * 5^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.20, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1676, 1166, 207}

$$\sin(x) - \frac{1}{5} \sqrt{\frac{1}{2}(5 + \sqrt{5})} \tanh^{-1} \left(2 \sqrt{\frac{2}{5 + \sqrt{5}}} \sin(x) \right) - \frac{1}{5} \sqrt{\frac{1}{2}(5 - \sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{5}} (5 + \sqrt{5}) \sin(x) \right)$$

Antiderivative was successfully verified.

[In] Int[Cot[5*x]*Sin[x],x]

[Out] $-(\operatorname{Sqrt}[(5 + \operatorname{Sqrt}[5])/2] * \operatorname{ArcTanh}[2 * \operatorname{Sqrt}[2/(5 + \operatorname{Sqrt}[5])] * \operatorname{Sin}[x]])/5 - (\operatorname{Sqrt}[(5 - \operatorname{Sqrt}[5])/2] * \operatorname{ArcTanh}[\operatorname{Sqrt}[(2 * (5 + \operatorname{Sqrt}[5]))/5] * \operatorname{Sin}[x]])/5 + \operatorname{Sin}[x]$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1676

Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1

Rubi steps

$$\begin{aligned} \int \cot(5x) \sin(x) dx &= \operatorname{Subst} \left(\int \frac{1 - 12x^2 + 16x^4}{5 - 20x^2 + 16x^4} dx, x, \sin(x) \right) \\ &= \operatorname{Subst} \left(\int \left(1 - \frac{4(1 - 2x^2)}{5 - 20x^2 + 16x^4} \right) dx, x, \sin(x) \right) \\ &= \sin(x) - 4 \operatorname{Subst} \left(\int \frac{1 - 2x^2}{5 - 20x^2 + 16x^4} dx, x, \sin(x) \right) \\ &= \sin(x) + \frac{1}{5} (4(5 - \sqrt{5})) \operatorname{Subst} \left(\int \frac{1}{-10 + 2\sqrt{5} + 16x^2} dx, x, \sin(x) \right) + \frac{1}{5} (4(5 + \sqrt{5})) \operatorname{Subst} \left(\int \frac{1}{-10 - 2\sqrt{5} + 16x^2} dx, x, \sin(x) \right) \\ &= -\frac{1}{5} \sqrt{\frac{1}{2}(5 + \sqrt{5})} \tanh^{-1} \left(2 \sqrt{\frac{2}{5 + \sqrt{5}}} \sin(x) \right) - \frac{1}{5} \sqrt{\frac{1}{2}(5 - \sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{5}} (5 + \sqrt{5}) \sin(x) \right) \end{aligned}$$

Mathematica [A] time = 0.24, size = 76, normalized size = 0.93

$$\frac{1}{10} \left(10 \sin(x) - \sqrt{10 - 2\sqrt{5}} \tanh^{-1} \left(\sqrt{2 + \frac{2}{\sqrt{5}}} \sin(x) \right) - \sqrt{2(5 + \sqrt{5})} \tanh^{-1} \left(2\sqrt{\frac{2}{5 + \sqrt{5}}} \sin(x) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[5*x]*Sin[x], x]

[Out] $(-\text{Sqrt}[10 - 2*\text{Sqrt}[5]]*\text{ArcTanh}[\text{Sqrt}[2 + 2/\text{Sqrt}[5]]*\text{Sin}[x]]) - \text{Sqrt}[2*(5 + \text{Sqrt}[5])] * \text{ArcTanh}[2*\text{Sqrt}[2/(5 + \text{Sqrt}[5])] * \text{Sin}[x]] + 10*\text{Sin}[x])/10$

fricas [B] time = 0.56, size = 127, normalized size = 1.55

$$-\frac{1}{20} \sqrt{2} \sqrt{\sqrt{5} + 5} \log \left(\sqrt{2} \sqrt{\sqrt{5} + 5} + 4 \sin(x) \right) + \frac{1}{20} \sqrt{2} \sqrt{\sqrt{5} + 5} \log \left(\sqrt{2} \sqrt{\sqrt{5} + 5} - 4 \sin(x) \right) - \frac{1}{20} \sqrt{2} \sqrt{\sqrt{5} + 5} \log \left(\sqrt{2} \sqrt{\sqrt{5} + 5} + 4 \sin(x) \right) + \frac{1}{20} \sqrt{2} \sqrt{\sqrt{5} + 5} \log \left(\sqrt{2} \sqrt{\sqrt{5} + 5} - 4 \sin(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(5*x)*sin(x), x, algorithm="fricas")

[Out] $-1/20*\text{sqrt}(2)*\text{sqrt}(\text{sqrt}(5) + 5)*\log(\text{sqrt}(2)*\text{sqrt}(\text{sqrt}(5) + 5) + 4*\sin(x)) + 1/20*\text{sqrt}(2)*\text{sqrt}(\text{sqrt}(5) + 5)*\log(\text{sqrt}(2)*\text{sqrt}(\text{sqrt}(5) + 5) - 4*\sin(x)) - 1/20*\text{sqrt}(2)*\text{sqrt}(-\text{sqrt}(5) + 5)*\log(\text{sqrt}(2)*\text{sqrt}(-\text{sqrt}(5) + 5) + 4*\sin(x)) + 1/20*\text{sqrt}(2)*\text{sqrt}(-\text{sqrt}(5) + 5)*\log(\text{sqrt}(2)*\text{sqrt}(-\text{sqrt}(5) + 5) - 4*\sin(x)) + \sin(x)$

giac [B] time = 0.33, size = 111, normalized size = 1.35

$$-\frac{1}{20} \sqrt{2} \sqrt{5} + 10 \log \left(\left(\frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\sqrt{5} + 5} + \sin(x) \right) \right) + \frac{1}{20} \sqrt{2} \sqrt{5} + 10 \log \left(\left(-\frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\sqrt{5} + 5} + \sin(x) \right) \right) - \frac{1}{20} \sqrt{2} \sqrt{5} + 10 \log \left(\left(\frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\sqrt{5} + 5} + \sin(x) \right) \right) + \frac{1}{20} \sqrt{2} \sqrt{5} + 10 \log \left(\left(-\frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\sqrt{5} + 5} + \sin(x) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(5*x)*sin(x), x, algorithm="giac")

[Out] $-1/20*\text{sqrt}(2*\text{sqrt}(5) + 10)*\log(\text{abs}(1/2*\text{sqrt}(1/2)*\text{sqrt}(\text{sqrt}(5) + 5) + \sin(x))) + 1/20*\text{sqrt}(2*\text{sqrt}(5) + 10)*\log(\text{abs}(-1/2*\text{sqrt}(1/2)*\text{sqrt}(\text{sqrt}(5) + 5) + \sin(x))) - 1/20*\text{sqrt}(-2*\text{sqrt}(5) + 10)*\log(\text{abs}(\text{sqrt}(-1/8*\text{sqrt}(5) + 5/8) + \sin(x))) + 1/20*\text{sqrt}(-2*\text{sqrt}(5) + 10)*\log(\text{abs}(-\text{sqrt}(-1/8*\text{sqrt}(5) + 5/8) + \sin(x))) + \sin(x)$

maple [A] time = 0.21, size = 70, normalized size = 0.85

$$\sin(x) - \frac{(\sqrt{5} - 1) \sqrt{5} \operatorname{arctanh} \left(\frac{4 \sin(x)}{\sqrt{10 - 2\sqrt{5}}} \right)}{5\sqrt{10 - 2\sqrt{5}}} - \frac{(\sqrt{5} + 1) \sqrt{5} \operatorname{arctanh} \left(\frac{4 \sin(x)}{\sqrt{10 + 2\sqrt{5}}} \right)}{5\sqrt{10 + 2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(5*x)*sin(x), x)

[Out] $\sin(x) - 1/5*(5^{(1/2)} - 1)*5^{(1/2)}/((10 - 2*5^{(1/2)})^{(1/2)}) * \operatorname{arctanh}(4*\sin(x)/((10 - 2*5^{(1/2)})^{(1/2)})) - 1/5*(5^{(1/2)} + 1)*5^{(1/2)}/((10 + 2*5^{(1/2)})^{(1/2)}) * \operatorname{arctanh}(4*\sin(x)/((10 + 2*5^{(1/2)})^{(1/2)}))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(5*x)*sin(x),x, algorithm="maxima")

[Out] -integrate(1/2*((cos(3*x) + cos(2*x) + cos(x))*cos(4*x) + (2*cos(2*x) + 2*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + (2*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + (sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + 2*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + cos(x))/(2*(cos(3*x) + cos(2*x) + cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 2*(cos(2*x) + cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + 2*(sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + sin(4*x)^2 + 2*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1), x) - integrate(-1/2*((cos(3*x) - cos(2*x) + cos(x))*cos(4*x) + (2*cos(2*x) - 2*cos(x) + 1)*cos(3*x) - cos(3*x)^2 + (2*cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + (sin(3*x) - sin(2*x) + sin(x))*sin(4*x) + 2*(sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2 + cos(x))/(2*(cos(3*x) - cos(2*x) + cos(x) - 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x) - cos(x) + 1)*cos(3*x) - cos(3*x)^2 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + 2*(sin(3*x) - sin(2*x) + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2 + 2*cos(x) - 1), x) + sin(x)

mupad [B] time = 2.61, size = 119, normalized size = 1.45

$$\sin(x) \frac{\sqrt{2} \operatorname{atanh}\left(\frac{\frac{25\sqrt{2}\sin(x)\sqrt{\sqrt{5}+5}}{2} + \frac{11\sqrt{2}\sqrt{5}\sin(x)\sqrt{\sqrt{5}+5}}{2}}{20\sqrt{5}+45}}{\sqrt{\sqrt{5}+5}}\right) + \sqrt{2} \operatorname{atanh}\left(\frac{\frac{25\sqrt{2}\sin(x)\sqrt{\sqrt{5}-5}}{2} - \frac{11\sqrt{2}\sqrt{5}\sin(x)\sqrt{\sqrt{5}-5}}{2}}{20\sqrt{5}-45}}{\sqrt{\sqrt{5}-5}}\right)}{10} + \frac{\sqrt{2} \operatorname{atanh}\left(\frac{\frac{25\sqrt{2}\sin(x)\sqrt{\sqrt{5}+5}}{2} + \frac{11\sqrt{2}\sqrt{5}\sin(x)\sqrt{\sqrt{5}+5}}{2}}{20\sqrt{5}+45}}{\sqrt{\sqrt{5}+5}}\right) + \sqrt{2} \operatorname{atanh}\left(\frac{\frac{25\sqrt{2}\sin(x)\sqrt{\sqrt{5}-5}}{2} - \frac{11\sqrt{2}\sqrt{5}\sin(x)\sqrt{\sqrt{5}-5}}{2}}{20\sqrt{5}-45}}{\sqrt{\sqrt{5}-5}}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(5*x)*sin(x),x)

[Out] sin(x) - (2^(1/2)*atanh(((25*2^(1/2)*sin(x)*(5^(1/2) + 5)^(1/2))/2 + (11*2^(1/2)*5^(1/2)*sin(x)*(5^(1/2) + 5)^(1/2))/2)/(20*5^(1/2) + 45))*(5^(1/2) + 5)^(1/2))/10 + (2^(1/2)*atanh(((25*2^(1/2)*sin(x)*(5 - 5^(1/2))^(1/2))/2 - (11*2^(1/2)*5^(1/2)*sin(x)*(5 - 5^(1/2))^(1/2))/2)/(20*5^(1/2) - 45))*(5 - 5^(1/2))^(1/2))/10

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \cot(5x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(5*x)*sin(x),x)

[Out] Integral(sin(x)*cot(5*x), x)

3.84 $\int \cot(6x) \sin(x) dx$

Optimal. Leaf size=38

$$\sin(x) - \frac{1}{6} \tanh^{-1}(\sin(x)) - \frac{1}{6} \tanh^{-1}(2 \sin(x)) - \frac{\tanh^{-1}\left(\frac{2 \sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

[Out] $-1/6*\operatorname{arctanh}(\sin(x))-1/6*\operatorname{arctanh}(2*\sin(x))+\sin(x)-1/6*\operatorname{arctanh}(2/3*\sin(x)*3^{(1/2)})*3^{(1/2)}$

Rubi [A] time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {12, 2073, 207}

$$\sin(x) - \frac{1}{6} \tanh^{-1}(\sin(x)) - \frac{1}{6} \tanh^{-1}(2 \sin(x)) - \frac{\tanh^{-1}\left(\frac{2 \sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Cot[6*x]*Sin[x],x]

[Out] $-\operatorname{ArcTanh}[\operatorname{Sin}[x]]/6 - \operatorname{ArcTanh}[2*\operatorname{Sin}[x]]/6 - \operatorname{ArcTanh}[(2*\operatorname{Sin}[x])/Sqrt[3]]/(2*Sqrt[3]) + \operatorname{Sin}[x]$

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2073

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \cot(6x) \sin(x) dx &= \operatorname{Subst} \left(\int \frac{1 - 18x^2 + 48x^4 - 32x^6}{2(3 - 19x^2 + 32x^4 - 16x^6)} dx, x, \sin(x) \right) \\ &= \frac{1}{2} \operatorname{Subst} \left(\int \frac{1 - 18x^2 + 48x^4 - 32x^6}{3 - 19x^2 + 32x^4 - 16x^6} dx, x, \sin(x) \right) \\ &= \frac{1}{2} \operatorname{Subst} \left(\int \left(2 + \frac{1}{3(-1 + x^2)} + \frac{2}{-3 + 4x^2} + \frac{2}{3(-1 + 4x^2)} \right) dx, x, \sin(x) \right) \\ &= \sin(x) + \frac{1}{6} \operatorname{Subst} \left(\int \frac{1}{-1 + x^2} dx, x, \sin(x) \right) + \frac{1}{3} \operatorname{Subst} \left(\int \frac{1}{-1 + 4x^2} dx, x, \sin(x) \right) + \operatorname{Subst} \left(\int \frac{1}{3 - 19x^2 + 32x^4 - 16x^6} dx, x, \sin(x) \right) \\ &= -\frac{1}{6} \tanh^{-1}(\sin(x)) - \frac{1}{6} \tanh^{-1}(2 \sin(x)) - \frac{\tanh^{-1}\left(\frac{2 \sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}} + \sin(x) \end{aligned}$$

Mathematica [A] time = 0.07, size = 38, normalized size = 1.00

$$\sin(x) - \frac{1}{6} \tanh^{-1}(\sin(x)) - \frac{1}{6} \tanh^{-1}(2 \sin(x)) - \frac{\tanh^{-1}\left(\frac{2 \sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[6*x]*Sin[x],x]

[Out] -1/6*ArcTanh[Sin[x]] - ArcTanh[2*Sin[x]]/6 - ArcTanh[(2*Sin[x])/Sqrt[3]]/(2*Sqrt[3]) + Sin[x]

fricas [B] time = 0.87, size = 70, normalized size = 1.84

$$\frac{1}{12} \sqrt{3} \log\left(-\frac{4 \cos(x)^2 + 4 \sqrt{3} \sin(x) - 7}{4 \cos(x)^2 - 1}\right) - \frac{1}{12} \log(2 \sin(x) + 1) - \frac{1}{12} \log(\sin(x) + 1) + \frac{1}{12} \log(-\sin(x) + 1) + \frac{1}{12} \log(-2 \sin(x) + 1) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(6*x)*sin(x),x, algorithm="fricas")

[Out] 1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) - 1/12*log(2*sin(x) + 1) - 1/12*log(sin(x) + 1) + 1/12*log(-sin(x) + 1) + 1/12*log(-2*sin(x) + 1) + sin(x)

giac [B] time = 0.17, size = 70, normalized size = 1.84

$$\frac{1}{12} \sqrt{3} \log\left(\frac{|-4 \sqrt{3} + 8 \sin(x)|}{|4 \sqrt{3} + 8 \sin(x)|}\right) - \frac{1}{12} \log(\sin(x) + 1) + \frac{1}{12} \log(-\sin(x) + 1) - \frac{1}{12} \log(|2 \sin(x) + 1|) + \frac{1}{12} \log(|2 \sin(x) - 1|) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(6*x)*sin(x),x, algorithm="giac")

[Out] 1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) - 1/12*log(sin(x) + 1) + 1/12*log(-sin(x) + 1) - 1/12*log(abs(2*sin(x) + 1)) + 1/12*log(abs(2*sin(x) - 1)) + sin(x)

maple [A] time = 0.16, size = 49, normalized size = 1.29

$$\sin(x) + \frac{\ln(-1 + 2 \sin(x))}{12} - \frac{\ln(1 + 2 \sin(x))}{12} - \frac{\operatorname{arctanh}\left(\frac{2 \sin(x) \sqrt{3}}{3}\right) \sqrt{3}}{6} + \frac{\ln(\sin(x) - 1)}{12} - \frac{\ln(1 + \sin(x))}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(6*x)*sin(x),x)

[Out] sin(x)+1/12*ln(-1+2*sin(x))-1/12*ln(1+2*sin(x))-1/6*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)+1/12*ln(sin(x)-1)-1/12*ln(1+sin(x))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{24} \sqrt{3} \log\left(\frac{4}{3} \cos(x)^2 + \frac{4}{3} \sin(x)^2 + \frac{4}{3} \sqrt{3} \sin(x) + \frac{4}{3} \cos(x) + \frac{4}{3}\right) - \frac{1}{24} \sqrt{3} \log\left(\frac{4}{3} \cos(x)^2 + \frac{4}{3} \sin(x)^2 + \frac{4}{3} \sqrt{3} \sin(x) + \frac{4}{3} \cos(x) + \frac{4}{3}\right) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(6*x)*sin(x),x, algorithm="maxima")

[Out] -1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) - 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + sin(x)

$n(x) - 4/3*\cos(x) + 4/3) + 1/24*\sqrt{3}*\log(4/3*\cos(x)^2 + 4/3*\sin(x)^2 - 4/3*\sqrt{3}*\sin(x) + 4/3*\cos(x) + 4/3) + 1/24*\sqrt{3}*\log(4/3*\cos(x)^2 + 4/3*\sin(x)^2 - 4/3*\sqrt{3}*\sin(x) - 4/3*\cos(x) + 4/3) - \text{integrate}(-1/6*((\cos(3*x) + \cos(x))*\cos(4*x) - (\cos(2*x) - 1)*\cos(3*x) - \cos(2*x)*\cos(x) + (\sin(3*x) + \sin(x))*\sin(4*x) - \sin(3*x)*\sin(2*x) - \sin(2*x)*\sin(x) + \cos(x))/(2*(\cos(2*x) - 1)*\cos(4*x) - \cos(4*x)^2 - \cos(2*x)^2 - \sin(4*x)^2 + 2*\sin(4*x)*\sin(2*x) - \sin(2*x)^2 + 2*\cos(2*x) - 1), x) - 1/12*\log(\cos(x)^2 + \sin(x)^2 + 2*\sin(x) + 1) + 1/12*\log(\cos(x)^2 + \sin(x)^2 - 2*\sin(x) + 1) + \sin(x)$

mupad [B] time = 2.50, size = 37, normalized size = 0.97

$$\sin(x) - \frac{\operatorname{atanh}(2 \sin(x))}{6} - \frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{3} - \frac{\sqrt{3} \operatorname{atanh}\left(\frac{2\sqrt{3} \sin(x)}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(6*x)*sin(x),x)

[Out] $\sin(x) - \operatorname{atanh}(2*\sin(x))/6 - \operatorname{atanh}(\sin(x/2)/\cos(x/2))/3 - (3^{(1/2)}*\operatorname{atanh}((2*3^{(1/2)}*\sin(x))/3))/6$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \cot(6x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(6*x)*sin(x),x)

[Out] Integral(sin(x)*cot(6*x), x)

3.85 $\int \sec(2x) \sin(x) dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}(\sqrt{2} \cos(x))}{\sqrt{2}}$$

[Out] 1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4357, 207}

$$\frac{\tanh^{-1}(\sqrt{2} \cos(x))}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sec[2*x]*Sin[x],x]

[Out] ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 4357

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned} \int \sec(2x) \sin(x) dx &= -\text{Subst} \left(\int \frac{1}{-1 + 2x^2} dx, x, \cos(x) \right) \\ &= \frac{\tanh^{-1}(\sqrt{2} \cos(x))}{\sqrt{2}} \end{aligned}$$

Mathematica [C] time = 0.41, size = 174, normalized size = 11.60

$$\frac{4 \tanh^{-1} \left(\tan \left(\frac{x}{2} \right) + \sqrt{2} \right) - \log \left(-\sqrt{2} \sin(x) - \sqrt{2} \cos(x) + 2 \right) + \log \left(-\sqrt{2} \sin(x) + \sqrt{2} \cos(x) + 2 \right) + 2i \tan^{-1} \left(\frac{\sqrt{2} \cos(x)}{1 + \sqrt{2} \sin(x)} \right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*x]*Sin[x],x]

[Out] ((2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*Cos[x/2] - Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((-1 + Sqrt[2])*Cos[x/2] - Sin[x/2])] + 4*ArcTanh[Sqrt[2] + Tan[x/2]] - Log[2 - Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]] + Log[2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/(4*Sqrt[2])

fricas [B] time = 0.60, size = 33, normalized size = 2.20

$$\frac{1}{4} \sqrt{2} \log \left(-\frac{2 \cos(x)^2 + 2 \sqrt{2} \cos(x) + 1}{2 \cos(x)^2 - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*x)*sin(x),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))

giac [B] time = 0.17, size = 49, normalized size = 3.27

$$\frac{1}{4} \sqrt{2} \log \left(\frac{\left| -4 \sqrt{2} - \frac{2(\cos(x)-1)}{\cos(x)+1} - 6 \right|}{\left| 4 \sqrt{2} - \frac{2(\cos(x)-1)}{\cos(x)+1} - 6 \right|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*x)*sin(x),x, algorithm="giac")

[Out] 1/4*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6))

maple [A] time = 0.13, size = 13, normalized size = 0.87

$$\frac{\operatorname{arctanh}(\cos(x)\sqrt{2})\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2*x)*sin(x),x)

[Out] 1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)

maxima [B] time = 0.43, size = 129, normalized size = 8.60

$$\frac{1}{8} \sqrt{2} \log \left(2 \sqrt{2} \sin(2x) \sin(x) + 2 \left(\sqrt{2} \cos(x) + 1 \right) \cos(2x) + \cos(2x)^2 + 2 \cos(x)^2 + \sin(2x)^2 + 2 \sin(x)^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*x)*sin(x),x, algorithm="maxima")

[Out] 1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 1) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 1)

mupad [B] time = 2.29, size = 12, normalized size = 0.80

$$\frac{\sqrt{2} \operatorname{atanh}(\sqrt{2} \cos(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/cos(2*x),x)

[Out] (2^(1/2)*atanh(2^(1/2)*cos(x)))/2

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \sec(2x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*sec(2*x), x)
```


3.86 $\int \sec(3x) \sin(x) dx$

Optimal. Leaf size=21

$$\frac{1}{3} \log(\cos(x)) - \frac{1}{6} \log(3 - 4 \cos^2(x))$$

[Out] 1/3*ln(cos(x))-1/6*ln(3-4*cos(x)^2)

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {4357, 266, 36, 29, 31}

$$\frac{1}{3} \log(\cos(x)) - \frac{1}{6} \log(3 - 4 \cos^2(x))$$

Antiderivative was successfully verified.

[In] Int[Sec[3*x]*Sin[x],x]

[Out] Log[Cos[x]]/3 - Log[3 - 4*Cos[x]^2]/6

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4357

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned}
\int \sec(3x) \sin(x) dx &= -\text{Subst} \left(\int \frac{1}{x(-3+4x^2)} dx, x, \cos(x) \right) \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{x(-3+4x)} dx, x, \cos^2(x) \right) \right) \\
&= \frac{1}{6} \text{Subst} \left(\int \frac{1}{x} dx, x, \cos^2(x) \right) - \frac{2}{3} \text{Subst} \left(\int \frac{1}{-3+4x} dx, x, \cos^2(x) \right) \\
&= \frac{1}{3} \log(\cos(x)) - \frac{1}{6} \log(3-4\cos^2(x))
\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.81

$$-\frac{1}{3} \tanh^{-1} \left(\frac{1}{3} (8 \sin^2(x) - 5) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[3*x]*Sin[x],x]

[Out] -1/3*ArcTanh[(-5 + 8*Sin[x]^2)/3]

fricas [A] time = 0.88, size = 19, normalized size = 0.90

$$-\frac{1}{6} \log(4 \cos(x)^2 - 3) + \frac{1}{3} \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(3*x)*sin(x),x, algorithm="fricas")

[Out] -1/6*log(4*cos(x)^2 - 3) + 1/3*log(-cos(x))

giac [A] time = 0.14, size = 24, normalized size = 1.14

$$\frac{1}{6} \log(-\sin(x)^2 + 1) - \frac{1}{6} \log(|4 \sin(x)^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(3*x)*sin(x),x, algorithm="giac")

[Out] 1/6*log(-sin(x)^2 + 1) - 1/6*log(abs(4*sin(x)^2 - 1))

maple [A] time = 0.19, size = 18, normalized size = 0.86

$$-\frac{\ln(4(\cos^2(x)) - 3)}{6} + \frac{\ln(\cos(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(3*x)*sin(x),x)

[Out] -1/6*ln(4*cos(x)^2-3)+1/3*ln(cos(x))

maxima [B] time = 0.42, size = 81, normalized size = 3.86

$$-\frac{1}{12} \log(-2(\cos(2x) - 1) \cos(4x) + \cos(4x)^2 + \cos(2x)^2 + \sin(4x)^2 - 2 \sin(4x) \sin(2x) + \sin(2x)^2 - 2 \cos(2x) \sin(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(3*x)*sin(x),x, algorithm="maxima")

[Out] $-1/12*\log(-2*(\cos(2*x) - 1)*\cos(4*x) + \cos(4*x)^2 + \cos(2*x)^2 + \sin(4*x)^2 - 2*\sin(4*x)*\sin(2*x) + \sin(2*x)^2 - 2*\cos(2*x) + 1) + 1/6*\log(\cos(2*x)^2 + \sin(2*x)^2 + 2*\cos(2*x) + 1)$

mupad [B] time = 2.27, size = 15, normalized size = 0.71

$$\frac{\ln(\cos(x))}{3} - \frac{\ln\left(\cos(x)^2 - \frac{3}{4}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/cos(3*x),x)

[Out] $\log(\cos(x))/3 - \log(\cos(x)^2 - 3/4)/6$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \sec(3x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(3*x)*sin(x),x)

[Out] Integral(sin(x)*sec(3*x), x)

3.87 $\int \sec(4x) \sin(x) dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2}(2+\sqrt{2})} - \frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2}(2-\sqrt{2})}$$

[Out] $-1/2*\operatorname{arctanh}(2*\cos(x)/(2-2^{(1/2)})^{(1/2)})/(4-2*2^{(1/2)})^{(1/2)}+1/2*\operatorname{arctanh}(2*\cos(x)/(2+2^{(1/2)})^{(1/2)})/(4+2*2^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4357, 1093, 207}

$$\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2}(2+\sqrt{2})} - \frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2}(2-\sqrt{2})}$$

Antiderivative was successfully verified.

[In] Int[Sec[4*x]*Sin[x],x]

[Out] $-\operatorname{ArcTanh}[(2*\cos[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) + \operatorname{ArcTanh}[(2*\cos[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 4357

Int[(u_)*(F_) [(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned} \int \sec(4x) \sin(x) dx &= -\operatorname{Subst}\left(\int \frac{1}{1-8x^2+8x^4} dx, x, \cos(x)\right) \\ &= -\left(\sqrt{2} \operatorname{Subst}\left(\int \frac{1}{-4-2\sqrt{2}+8x^2} dx, x, \cos(x)\right)\right) + \sqrt{2} \operatorname{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2} dx, x, \right. \\ &\quad \left. \frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2}(2-\sqrt{2})} + \frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2}(2+\sqrt{2})}\right) \end{aligned}$$

Mathematica [C] time = 57.93, size = 4845, normalized size = 68.24

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[4*x]*Sin[x],x]

[Out]
$$\begin{aligned} &((-2*(-1)^{(3/8)}*(1 + \sqrt{2}))*x - (2*(-1)^{(1/4)}*(-2 - (1 - I)*(-1)^{(5/8)} + (-1)^{(5/8)}*\sqrt{2}))*\text{ArcTan}[(-\text{Cos}[x] + (1 + \sqrt{2})*\text{Sin}[x])/(2*(-1)^{(3/8)} + \text{Cos}[x] - \sqrt{2}*\text{Cos}[x] + \text{Sin}[x])]/((-1 + I) + 2*(-1)^{(3/8)} + \sqrt{2}) - \\ &(2*(1 - I)^{(3/2)}*2^{(1/4)}*((-3 - I) + 2*(-1)^{(5/8)} + (2 + I)*\sqrt{2} - (2 + 2*I)*(-1)^{(3/8)}*\sqrt{2} + 2*(-1)^{(5/8)}*\sqrt{2}))*\text{ArcTan}[\frac{((1 + I) + I*\sqrt{2} + ((-1 + I) + 2*(-1)^{(3/8)} + \sqrt{2}))*\text{Tan}[x/2]}{(\sqrt{1 - I})*2^{(3/4)}}]/((-1 + I) + 2*(-1)^{(3/8)} + \sqrt{2}) + 2*(-1)^{(3/8)}*\text{Log}[\text{Sec}[x/2]^2] + ((-1)^{(3/4)}*(-2 - (1 - I)*(-1)^{(5/8)} + (-1)^{(5/8)}*\sqrt{2}))*\text{Log}[-(\text{Sec}[x/2]^4*(-2 + (1 - I)*\sqrt{2} + 2*(-1)^{(3/8)}*(-1 + \sqrt{2}))*\text{Cos}[x] + \sqrt{2}*\text{Cos}[2*x] - 2*(-1)^{(3/8)}*\text{Sin}[x] + \sqrt{2}*\text{Sin}[2*x])]]/((-1 + I) + 2*(-1)^{(3/8)} + \sqrt{2})))*((-1/2 - I/2)/(((-1 + I) + \sqrt{1 - I})*\sqrt{1 + I})*(-((-1 - I)^{(3/2)}*(1 - I)^{(1/4)}*(1 + I)^{(1/4)}) - (1 + I)*\text{Cos}[x] + I*\sqrt{1 - I})*\sqrt{1 + I}*\text{Cos}[x] + (1 - I)*\text{Sin}[x] + \sqrt{1 - I})*\sqrt{1 + I}*\text{Sin}[x]) - \text{Sin}[x]/(\sqrt{-1 - I}*(1 - I)^{(1/4)}*(1 + I)^{(1/4)}*((-1 + I) + \sqrt{1 - I})*\sqrt{1 + I})*(-((-1 - I)^{(3/2)}*(1 - I)^{(1/4)}*(1 + I)^{(1/4)}) - (1 + I)*\text{Cos}[x] + I*\sqrt{1 - I})*\sqrt{1 + I}*\text{Cos}[x] + (1 - I)*\text{Sin}[x] + \sqrt{1 - I})*\sqrt{1 + I}*\text{Sin}[x]) - ((I/2)*\sqrt{-1 - I}*(1 - I)^{(1/4)}*(1 + I)^{(1/4)}*\text{Sin}[x])/(((-1 + I) + \sqrt{1 - I})*\sqrt{1 + I})*(-((-1 - I)^{(3/2)}*(1 - I)^{(1/4)}*(1 + I)^{(1/4)}) - (1 + I)*\text{Cos}[x] + I*\sqrt{1 - I})*\sqrt{1 + I}*\text{Cos}[x] + (1 - I)*\text{Sin}[x] + \sqrt{1 - I})*\sqrt{1 + I}*\text{Sin}[x])))/(-2*(-1)^{(3/8)}*(1 + \sqrt{2}) - (2*(-1)^{(1/4)}*(-2 - (1 - I)*(-1)^{(5/8)} + (-1)^{(5/8)}*\sqrt{2}))*((1 + \sqrt{2}))*\text{Cos}[x] + \text{Sin}[x])/(2*(-1)^{(3/8)} + \text{Cos}[x] - \sqrt{2}*\text{Cos}[x] + \text{Sin}[x]) - ((\text{Cos}[x] - \text{Sin}[x] + \sqrt{2})*\text{Sin}[x])*(-\text{Cos}[x] + (1 + \sqrt{2}))*\text{Sin}[x])/(2*(-1)^{(3/8)} + \text{Cos}[x] - \sqrt{2}*\text{Cos}[x] + \text{Sin}[x])^2)/(((-1 + I) + 2*(-1)^{(3/8)} + \sqrt{2}))*((1 + (-\text{Cos}[x] + (1 + \sqrt{2}))*\text{Sin}[x])^2/(2*(-1)^{(3/8)} + \text{Cos}[x] - \sqrt{2}*\text{Cos}[x] + \text{Sin}[x])^2) + 2*(-1)^{(3/8)}*\text{Tan}[x/2] - ((-1)^{(3/4)}*(-2 - (1 - I)*(-1)^{(5/8)} + (-1)^{(5/8)}*\sqrt{2}))*\text{Cos}[x/2]^4*(-(\text{Sec}[x/2]^4*(-2*(-1)^{(3/8)}*\text{Cos}[x] + 2*\sqrt{2})*\text{Cos}[2*x] - 2*(-1)^{(3/8)}*(-1 + \sqrt{2}))*\text{Sin}[x] - 2*\sqrt{2}*\text{Sin}[2*x]) - 2*\text{Sec}[x/2]^4*(-2 + (1 - I)*\sqrt{2} + 2*(-1)^{(3/8)}*(-1 + \sqrt{2}))*\text{Cos}[x] + \sqrt{2}*\text{Cos}[2*x] - 2*(-1)^{(3/8)}*\text{Sin}[x] + \sqrt{2}*\text{Sin}[2*x])* \text{Tan}[x/2]))/(((-1 + I) + 2*(-1)^{(3/8)} + \sqrt{2}))*(-2 + (1 - I)*\sqrt{2} + 2*(-1)^{(3/8)}*(-1 + \sqrt{2}))*\text{Cos}[x] + \sqrt{2}*\text{Cos}[2*x] - 2*(-1)^{(3/8)}*\text{Sin}[x] + \sqrt{2}*\text{Sin}[2*x]) - ((1 - I)*((-3 - I) + 2*(-1)^{(5/8)} + (2 + I)*\sqrt{2} - (2 + 2*I)*(-1)^{(3/8)}*\sqrt{2} + 2*(-1)^{(5/8)}*\sqrt{2}))*\text{Sec}[x/2]^2/(\sqrt{2}*(1 + ((1/4 + I/4))*((1 + I) + I*\sqrt{2} + ((-1 + I) + 2*(-1)^{(3/8)} + \sqrt{2}))*\text{Tan}[x/2])^2/\sqrt{2})) + ((-4*\sqrt{-1 - I}*(-1 + \sqrt{2}))*\text{ArcTanh}[\frac{(-I)*((1 + I) + \sqrt{2}) + ((1 + I) + 2*(-1)^{(5/8)} - \sqrt{2}))*\text{Tan}[x/2]}{(\sqrt{-1 - I})*2^{(3/4)}}] + (-1)^{(1/8)}*2^{(1/4)}*(2*\text{ArcTan}[\frac{(\text{Cos}[x] + (1 + \sqrt{2}))*\text{Sin}[x]}{2*(-1)^{(5/8)} + (-1 + \sqrt{2}))*\text{Cos}[x] + \text{Sin}[x]}] - I*(2*(1 + \sqrt{2}))*x + 2*\text{Log}[\text{Sec}[x/2]^2] - \text{Log}[\text{Sec}[x/2]^4*(2 - (1 + I)*\sqrt{2} + 2*(-1)^{(5/8)}*(-1 + \sqrt{2}))*\text{Cos}[x] - \sqrt{2}*\text{Cos}[2*x] + 2*(-1)^{(5/8)}*\text{Sin}[x] + \sqrt{2}*\text{Sin}[2*x])]))*(2 + I*\sqrt{-1 + I})*2^{(1/4)}*((1 + I) + \sqrt{2}))*\text{Sin}[x])/ (2^{(1/4)}*(4*\sqrt{-1 + I})*2^{(1/4)}*(-1 - I) + \sqrt{2}) - 8*(-1 + \sqrt{2}))*\text{Cos}[x] - 8*\text{Sin}[x])*((2*(-1)^{(1/8)}*(-2 - (1 + I)*\sqrt{2} + (-1)^{(1/8)}*((1 + I) + I*\sqrt{2}))*\text{Cos}[x] + (2*I)*(1 + \sqrt{2}))*\text{Cos}[2*x] + (-1)^{(1/8)}*\text{Sin}[x] - (-1)^{(5/8)}*\text{Sin}[x] + 3*(-1)^{(1/8)}*\sqrt{2}*\text{Sin}[x] - (2*I)*\text{Sin}[2*x]))/(2 - (1 + I)*\sqrt{2} + 2*(-1)^{(5/8)}*(-1 + \sqrt{2}))*\text{Cos}[x] - \sqrt{2}*\text{Cos}[2*x] + 2*(-1)^{(5/8)}*\text{Sin}[x] + \sqrt{2}*\text{Sin}[2*x]) - (((1 + I) + 2*(-1)^{(5/8)} - \sqrt{2}))*(-1 + \sqrt{2}))*\text{Sec}[x/2]^2/(1 + ((1/4 - I/4))*((1 + I) + \sqrt{2} + ((-1 - I) - 2*(-1)^{(5/8)} + \sqrt{2}))*\text{Tan}[x/2])^2/\sqrt{2})) + ((-2*(-1)^{(3/8)}*\sqrt{2}*(1 + (-1)^{(1/4)})*x + (2*(-2*I + 2*(-1)^{(3/4)} + 2*(-1)^{(1/8)}*\sqrt{2} - (-1)^{(3/8)}*\sqrt{2} + (-1)^{(7/8)}*\sqrt{2}))*\text{ArcTan}[\text{Cos}[x]/(-((-1)^{(1/8)}*\sqrt{2}) + (-1)^{(3/4)}*\text{Cos}[x] + (1 + (-1)^{(1/8)}*\sqrt{2}))*\text{Sin}[x]]]))$$

$$\begin{aligned}
& ((1/4)) * \sin[x]) / (-I + (-1)^{3/4} + (-1)^{1/8} * \sqrt{2}) - ((4 + 4I) * (-1)^{5/8} * ((3 - 3I) - (2 - 2I) * \sqrt{2} + (-1)^{1/8} * \sqrt{2} - (-1)^{3/8} * \sqrt{2} \\
& + (1 - I) * (-1)^{5/8} * \sqrt{2} + (1 + I) * (-1)^{7/8} * \sqrt{2}) * \operatorname{ArcTanh}[(1/2 + I/2) * (-1)^{5/8} * (-1 - (-1)^{1/4} + (-I + (-1)^{3/4} + (-1)^{1/8} * \sqrt{2})) \\
& * \tan[x/2]]) / (-I + (-1)^{3/4} + (-1)^{1/8} * \sqrt{2}) - 2 * (-1)^{7/8} * \sqrt{2} * \\
& (-1 + (-1)^{1/4}) * \log[\sec[x/2]^2] - ((-1 + (-1)^{1/4}) * (2 - (-1)^{3/8} * \sqrt{2} \\
& + (-1)^{5/8} * \sqrt{2})) * \log[(1/4 + I/4) * \sec[x/2]^4 * ((2 - 2I) + 6 * \sqrt{2} \\
& - (4 - 4I) * (-1)^{7/8} * \sqrt{2} * \cos[x] - 2 * ((1 + I) + \sqrt{2}) * \cos[2x] - (4 - 4I) * (-1)^{1/8} * \sqrt{2} * \sin[x] - (4 - 4I) * (-1)^{3/8} * \sqrt{2} * \sin[x] - \\
& (2 - 2I) * \sin[2x] + (2I) * \sqrt{2} * \sin[2x])]) / (-I + (-1)^{3/4} + (-1)^{1/8} \\
& * \sqrt{2})) * (I / (\sqrt{1 - I} * ((-1 + I) + \sqrt{1 - I}) * \sqrt{1 + I})^2 * (\sqrt{-1 - I} * (1 - I)^{3/4} * (1 + I)^{1/4} + \sqrt{1 - I} * \cos[x] - \sqrt{1 + I} * \cos[x] \\
& + I * \sqrt{1 - I} * \sin[x] + I * \sqrt{1 + I} * \sin[x])) + 1 / (\sqrt{1 + I} * ((-1 + I) \\
& + \sqrt{1 - I}) * \sqrt{1 + I})^2 * (\sqrt{-1 - I} * (1 - I)^{3/4} * (1 + I)^{1/4} + \sqrt{1 - I} * \cos[x] - \sqrt{1 + I} * \cos[x] + I * \sqrt{1 - I} * \sin[x] + I * \sqrt{1 + I} * \sin[x])) - (2 * \sin[x]) / (\sqrt{-1 - I} * (1 - I)^{1/4} * (1 + I)^{3/4} * ((-1 + I) \\
& + \sqrt{1 - I}) * \sqrt{1 + I})^2 * (\sqrt{-1 - I} * (1 - I)^{3/4} * (1 + I)^{1/4} + \sqrt{1 - I} * \cos[x] - \sqrt{1 + I} * \cos[x] + I * \sqrt{1 - I} * \sin[x] + I * \sqrt{1 + I} * \sin[x])))) / (-2 * (-1)^{3/8} * \sqrt{2} * (1 + (-1)^{1/4}) + (2 * (-2I + 2 * (-1)^{3/4} + 2 * (-1)^{1/8} * \sqrt{2} - (-1)^{3/8} * \sqrt{2} + (-1)^{7/8} * \sqrt{2})) * (- \\
& (\cos[x] * ((1 + (-1)^{1/4}) * \cos[x] - (-1)^{3/4} * \sin[x])) / (-((-1)^{1/8} * \sqrt{2} \\
&] + (-1)^{3/4} * \cos[x] + (1 + (-1)^{1/4}) * \sin[x])^2) - \sin[x] / (-((-1)^{1/8} \\
& * \sqrt{2}) + (-1)^{3/4} * \cos[x] + (1 + (-1)^{1/4}) * \sin[x])) / ((-I + (-1)^{3/4} \\
&) + (-1)^{1/8} * \sqrt{2}) * (1 + \cos[x]^2 / (-((-1)^{1/8} * \sqrt{2}) + (-1)^{3/4} * \cos[x] + (1 + (-1)^{1/4}) * \sin[x])^2) - 2 * (-1)^{7/8} * \sqrt{2} * (-1 + (-1)^{1/4}) * \tan[x/2] - ((2 - 2I) * (-1 + (-1)^{1/4}) * (2 - (-1)^{3/8} * \sqrt{2} + (-1)^{5/8} * \sqrt{2})) * \cos[x/2]^4 * ((1/4 + I/4) * \sec[x/2]^4 * ((-4 + 4I) * (-1)^{1/8} * \sqrt{2} * \cos[x] - (4 - 4I) * (-1)^{3/8} * \sqrt{2} * \cos[x] - (4 - 4I) * \cos[2x] + (4 * I) * \sqrt{2} * \cos[2x] + (4 - 4I) * (-1)^{7/8} * \sqrt{2} * \sin[x] + 4 * ((1 + I) + \sqrt{2}) * \sin[2x]) + (1/2 + I/2) * \sec[x/2]^4 * ((2 - 2I) + 6 * \sqrt{2} - (4 - 4I) * (-1)^{7/8} * \sqrt{2} * \cos[x] - 2 * ((1 + I) + \sqrt{2}) * \cos[2x] - (4 - 4I) * (-1)^{1/8} * \sqrt{2} * \sin[x] - (4 - 4I) * (-1)^{3/8} * \sqrt{2} * \sin[x] - (2 - 2I) * \sin[2x] + (2I) * \sqrt{2} * \sin[2x]) * \tan[x/2])) / ((-I + (-1)^{3/4} + (-1)^{1/8} * \sqrt{2})) * ((2 - 2I) + 6 * \sqrt{2} - (4 - 4I) * (-1)^{7/8} * \sqrt{2} * \cos[x] - 2 * ((1 + I) + \sqrt{2}) * \cos[2x] - (4 - 4I) * (-1)^{1/8} * \sqrt{2} * \sin[x] - (4 - 4I) * (-1)^{3/8} * \sqrt{2} * \sin[x] - (2 - 2I) * \sin[2x] + (2I) * \sqrt{2} * \sin[2x])]) + (2 * (-1)^{3/4} * ((3 - 3I) - (2 - 2I) * \sqrt{2} + (-1)^{1/8} * \sqrt{2} - (-1)^{3/8} * \sqrt{2} + (1 - I) * (-1)^{5/8} * \sqrt{2} + (1 + I) * (-1)^{7/8} * \sqrt{2}) * \sec[x/2]^2 / (1 + ((-1)^{3/4} * (-1 - (-1)^{1/4} + (-I + (-1)^{3/4} + (-1)^{1/8} * \sqrt{2})) * \tan[x/2])^2) / 2) + ((2 * ((-1)^{1/8} + (-1)^{3/8}) * x - (2 * (-1)^{7/8} * (2 - \sqrt{2}) - (-1)^{3/8} * \sqrt{2} + (-1)^{5/8} * \sqrt{2})) * \operatorname{ArcTan}[\cos[x] / (-((-1)^{1/8} * \sqrt{2}) + (-1)^{3/4} * \cos[x] - (1 + (-1)^{1/4}) * \sin[x])]) / (-I + (-1)^{3/4} + (-1)^{1/8} * \sqrt{2}) - ((4 + 4I) * (-1)^{5/8} * (3I + (-1)^{1/8} - (-1)^{3/8} - (1 + I) * (-1)^{5/8} - (2I) * \sqrt{2} + (1 + I) * (-1)^{5/8} * \sqrt{2})) * \operatorname{ArcTanh}[(1/2 + I/2) * (-1)^{5/8} * (1 + (-1)^{1/4} + (-I + (-1)^{3/4} + (-1)^{1/8} * \sqrt{2})) * \tan[x/2]]) / (-I + (-1)^{3/4} + (-1)^{1/8} * \sqrt{2}) + 2 * (-1)^{3/8} * (-I + (-1)^{1/4}) * \log[\sec[x/2]^2] + ((-1)^{3/8} * (2 - \sqrt{2}) - (-1)^{3/8} * \sqrt{2} + (-1)^{5/8} * \sqrt{2})) * \log[(1/4 + I/4) * \sec[x/2]^4 * ((2 - 2I) + 6 * \sqrt{2} - (4 - 4I) * (-1)^{7/8} * \sqrt{2} * \cos[x] - 2 * ((1 + I) + \sqrt{2}) * \cos[2x] + (4 - 4I) * (-1)^{1/8} * ((1 + I) + \sqrt{2}) * \sin[x] + (2 - 2I) * \sin[2x] - (2I) * \sqrt{2} * \sin[2x])]) / (-I + (-1)^{3/4} + (-1)^{1/8} * \sqrt{2})) * (1 / (\sqrt{1 - I} * ((-1 - I) + \sqrt{1 - I}) * \sqrt{1 + I})^2 * (-\sqrt{-1 + I} * (1 - I)^{1/4} * (1 + I)^{3/4}) + \sqrt{1 - I} * \cos[x] - \sqrt{1 + I} * \cos[x] - I * \sqrt{1 - I} * \sin[x] - I * \sqrt{1 + I} * \sin[x])) - I / (\sqrt{1 + I} * ((-1 - I) + \sqrt{1 - I}) * \sqrt{1 + I})^2 * (-\sqrt{-1 + I} * (1 - I)^{1/4} * (1 + I)^{3/4}) + \sqrt{1 - I} * \cos[x] - \sqrt{1 + I} * \cos[x] - I * \sqrt{1 - I} * \sin[x] - I * \sqrt{1 + I} * \sin[x])) + (2 * \sin[x]) / (\sqrt{-1 + I} * (1 - I)^{3/4} * (1 + I)^{1/4} * ((-1 - I) + \sqrt{1 - I}) * \sqrt{1 + I})^2 * (-\sqrt{-1 + I} * (1 - I)^{1/4} * (1 + I)^{3/4}) + \sqrt{1 - I} * \cos[x] - \sqrt{1 + I} * \cos[x] - I * \sqrt{1 - I} * \sin[x] - I * \sqrt{1 + I} * \sin[x]
\end{aligned}$$

$$\frac{I*\sin[x])})/(2*((-1)^{(1/8)} + (-1)^{(3/8)}) - (2*(-1)^{(7/8)}*(2 - \text{Sqrt}[2] - (-1)^{(3/8)}*\text{Sqrt}[2] + (-1)^{(5/8)}*\text{Sqrt}[2]))*(-((\text{Cos}[x]*(-((1 + (-1)^{(1/4)})*\text{Cos}[x]) - (-1)^{(3/4)}*\text{Sin}[x]))/(-((-1)^{(1/8)}*\text{Sqrt}[2]) + (-1)^{(3/4)}*\text{Cos}[x] - (1 + (-1)^{(1/4)})*\text{Sin}[x])^2) - \text{Sin}[x]/(-((-1)^{(1/8)}*\text{Sqrt}[2]) + (-1)^{(3/4)}*\text{Cos}[x] - (1 + (-1)^{(1/4)})*\text{Sin}[x])))/((-I + (-1)^{(3/4)} + (-1)^{(1/8)}*\text{Sqrt}[2]))*(1 + \text{Cos}[x]^2/(-((-1)^{(1/8)}*\text{Sqrt}[2]) + (-1)^{(3/4)}*\text{Cos}[x] - (1 + (-1)^{(1/4)})*\text{Sin}[x])^2)) + 2*(-1)^{(3/8)}*(-I + (-1)^{(1/4)})*\text{Tan}[x/2] + ((2 - 2*I)*(-1)^{(3/8)}*(2 - \text{Sqrt}[2] - (-1)^{(3/8)}*\text{Sqrt}[2] + (-1)^{(5/8)}*\text{Sqrt}[2])* \text{Cos}[x/2]^4*((1/4 + I/4)*\text{Sec}[x/2]^4*((4 - 4*I)*(-1)^{(1/8)}*((1 + I) + \text{Sqrt}[2])* \text{Cos}[x] + (4 - 4*I)*\text{Cos}[2*x] - (4*I)*\text{Sqrt}[2]*\text{Cos}[2*x] + (4 - 4*I)*(-1)^{(7/8)}*\text{Sqrt}[2]*\text{Sin}[x] + 4*((1 + I) + \text{Sqrt}[2])* \text{Sin}[2*x])) + (1/2 + I/2)*\text{Sec}[x/2]^4*((2 - 2*I) + 6*\text{Sqrt}[2] - (4 - 4*I)*(-1)^{(7/8)}*\text{Sqrt}[2]*\text{Cos}[x] - 2*((1 + I) + \text{Sqrt}[2])* \text{Cos}[2*x] + (4 - 4*I)*(-1)^{(1/8)}*((1 + I) + \text{Sqrt}[2])* \text{Sin}[x] + (2 - 2*I)*\text{Sin}[2*x] - (2*I)*\text{Sqrt}[2]*\text{Sin}[2*x])* \text{Tan}[x/2])))/((-I + (-1)^{(3/4)} + (-1)^{(1/8)}*\text{Sqrt}[2]))*((2 - 2*I) + 6*\text{Sqrt}[2] - (4 - 4*I)*(-1)^{(7/8)}*\text{Sqrt}[2]*\text{Cos}[x] - 2*((1 + I) + \text{Sqrt}[2])* \text{Cos}[2*x] + (4 - 4*I)*(-1)^{(1/8)}*((1 + I) + \text{Sqrt}[2])* \text{Sin}[x] + (2 - 2*I)*\text{Sin}[2*x] - (2*I)*\text{Sqrt}[2]*\text{Sin}[2*x])) + (2*(-1)^{(3/4)}*(3*I + (-1)^{(1/8)} - (-1)^{(3/8)} - (1 + I)*(-1)^{(5/8)} - (2*I)*\text{Sqrt}[2] + (1 + I)*(-1)^{(5/8)}*\text{Sqrt}[2])* \text{Sec}[x/2]^2)/(1 + ((-1)^{(3/4)}*(1 + (-1)^{(1/4)} + (-I + (-1)^{(3/4)} + (-1)^{(1/8)}*\text{Sqrt}[2])* \text{Tan}[x/2])^2)/2))$$

fricas [B] time = 0.68, size = 121, normalized size = 1.70

$$-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)+2\cos(x)\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)-2\cos(x)\right)+\frac{1}{8}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(4*x)*sin(x),x, algorithm="fricas")

[Out]
$$-1/8*\text{sqrt}(\text{sqrt}(2) + 2)*\log(\text{sqrt}(\text{sqrt}(2) + 2)*(\text{sqrt}(2) - 1) + 2*\cos(x)) + 1/8*\text{sqrt}(\text{sqrt}(2) + 2)*\log(\text{sqrt}(\text{sqrt}(2) + 2)*(\text{sqrt}(2) - 1) - 2*\cos(x)) + 1/8*\text{sqrt}(-\text{sqrt}(2) + 2)*\log((\text{sqrt}(2) + 1)*\text{sqrt}(-\text{sqrt}(2) + 2) + 2*\cos(x)) - 1/8*\text{sqrt}(-\text{sqrt}(2) + 2)*\log((\text{sqrt}(2) + 1)*\text{sqrt}(-\text{sqrt}(2) + 2) - 2*\cos(x))$$

giac [B] time = 0.14, size = 133, normalized size = 1.87

$$\frac{2.16139547686000 \log\left(-\frac{\cos(x)-1}{\cos(x)+1} - 0.0395661298966000\right)}{\frac{140(\cos(x)-1)}{\cos(x)+1} + 28.1312524456150} - \frac{4.18450863968000 \log\left(-\frac{\cos(x)-1}{\cos(x)+1} - 0.4464626}{\frac{140(\cos(x)-1)}{\cos(x)+1} + 44.3876588494000}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(4*x)*sin(x),x, algorithm="giac")

[Out]
$$-2.16139547686000*\log(-(\cos(x) - 1)/(\cos(x) + 1) - 0.0395661298966000)/(140*(\cos(x) - 1)/(\cos(x) + 1) + 28.1312524456150) - 4.18450863968000*\log(-(\cos(x) - 1)/(\cos(x) + 1) - 0.446462692172000)/(140*(\cos(x) - 1)/(\cos(x) + 1) + 44.3876588494000) - 20.9929814212000*\log(-(\cos(x) - 1)/(\cos(x) + 1) - 2.23982880884000)/(140*(\cos(x) - 1)/(\cos(x) + 1) + 404.466590643000) - 1380.66111446200*\log(-(\cos(x) - 1)/(\cos(x) + 1) - 25.2741423691000)/(140*(\cos(x) - 1)/(\cos(x) + 1) - 10892.9855019000)$$

maple [A] time = 0.21, size = 54, normalized size = 0.76

$$-\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2}-\sqrt{2}}\right)}{4\sqrt{2}-\sqrt{2}} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2}+\sqrt{2}}\right)}{4\sqrt{2}+\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(4*x)*sin(x),x)`

[Out] $-1/4*2^{(1/2)}/(2-2^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2*\cos(x)/(2-2^{(1/2)})^{(1/2)})+1/4*2^{(1/2)}/(2+2^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2*\cos(x)/(2+2^{(1/2)})^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec(4x) \sin(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(4*x)*sin(x),x, algorithm="maxima")`

[Out] `integrate(sec(4*x)*sin(x), x)`

mupad [B] time = 0.09, size = 112, normalized size = 1.58

$$\frac{\operatorname{atanh}\left(\frac{\cos(x)\sqrt{2-\sqrt{2}}}{64\left(\frac{\sqrt{2}}{128}-\frac{1}{64}\right)} - \frac{\sqrt{2}\cos(x)\sqrt{2-\sqrt{2}}}{64\left(\frac{\sqrt{2}}{128}-\frac{1}{64}\right)}\right)\sqrt{2-\sqrt{2}}}{4} - \frac{\operatorname{atanh}\left(\frac{\cos(x)\sqrt{\sqrt{2}+2}}{64\left(\frac{\sqrt{2}}{128}+\frac{1}{64}\right)} + \frac{\sqrt{2}\cos(x)\sqrt{\sqrt{2}+2}}{64\left(\frac{\sqrt{2}}{128}+\frac{1}{64}\right)}\right)\sqrt{\sqrt{2}+2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/cos(4*x),x)`

[Out] $(\operatorname{atanh}((\cos(x)*(2-2^{(1/2)})^{(1/2)})/(64*(2^{(1/2)}/128-1/64)) - (2^{(1/2)}*\cos(x)*(2-2^{(1/2)})^{(1/2)})/(64*(2^{(1/2)}/128-1/64)))*(2-2^{(1/2)})^{(1/2)})/4 - (\operatorname{atanh}((\cos(x)*(2^{(1/2)}+2)^{(1/2)})/(64*(2^{(1/2)}/128+1/64)) + (2^{(1/2)}*\cos(x)*(2^{(1/2)}+2)^{(1/2)})/(64*(2^{(1/2)}/128+1/64)))*(2^{(1/2)}+2)^{(1/2)})/4$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \sec(4x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(4*x)*sin(x),x)`

[Out] `Integral(sin(x)*sec(4*x), x)`

3.88 $\int \sec(5x) \sin(x) dx$

Optimal. Leaf size=62

$$\frac{1}{20} (1 + \sqrt{5}) \log(-8 \cos^2(x) - \sqrt{5} + 5) + \frac{1}{20} (1 - \sqrt{5}) \log(-8 \cos^2(x) + \sqrt{5} + 5) - \frac{1}{5} \log(\cos(x))$$

[Out] -1/5*ln(cos(x))+1/20*ln(5-8*cos(x)^2+5^(1/2))*(-5^(1/2)+1)+1/20*ln(5-8*cos(x)^2-5^(1/2))*(5^(1/2)+1)

Rubi [A] time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {4357, 1114, 705, 29, 632, 31}

$$\frac{1}{20} (1 + \sqrt{5}) \log(-8 \cos^2(x) - \sqrt{5} + 5) + \frac{1}{20} (1 - \sqrt{5}) \log(-8 \cos^2(x) + \sqrt{5} + 5) - \frac{1}{5} \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Sec[5*x]*Sin[x],x]

[Out] -Log[Cos[x]]/5 + ((1 + Sqrt[5])*Log[5 - Sqrt[5] - 8*Cos[x]^2])/20 + ((1 - Sqrt[5])*Log[5 + Sqrt[5] - 8*Cos[x]^2])/20

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 705

Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]

Rule 1114

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 4357

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned}
\int \sec(5x) \sin(x) dx &= -\text{Subst} \left(\int \frac{1}{x(5-20x^2+16x^4)} dx, x, \cos(x) \right) \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{x(5-20x+16x^2)} dx, x, \cos^2(x) \right) \right) \\
&= -\left(\frac{1}{10} \text{Subst} \left(\int \frac{1}{x} dx, x, \cos^2(x) \right) \right) - \frac{1}{10} \text{Subst} \left(\int \frac{20-16x}{5-20x+16x^2} dx, x, \cos^2(x) \right) \\
&= -\frac{1}{5} \log(\cos(x)) + \frac{1}{5} (4(1-\sqrt{5})) \text{Subst} \left(\int \frac{1}{-10-2\sqrt{5}+16x} dx, x, \cos^2(x) \right) + \frac{1}{5} (4(1+\sqrt{5})) \text{Subst} \left(\int \frac{1}{-10+2\sqrt{5}+16x} dx, x, \cos^2(x) \right) \\
&= -\frac{1}{5} \log(\cos(x)) + \frac{1}{20} (1+\sqrt{5}) \log(5-\sqrt{5}-8\cos^2(x)) + \frac{1}{20} (1-\sqrt{5}) \log(5+\sqrt{5}-8\cos^2(x))
\end{aligned}$$

Mathematica [A] time = 0.10, size = 57, normalized size = 0.92

$$\frac{1}{20} (-4 \log(\cos(x)) - (\sqrt{5}-1) \log(4 \cos(2x) - \sqrt{5}-1) + (1+\sqrt{5}) \log(4 \cos(2x) + \sqrt{5}-1))$$

Antiderivative was successfully verified.

[In] Integrate[Sec[5*x]*Sin[x],x]

[Out] (-4*Log[Cos[x]] - (-1 + Sqrt[5])*Log[-1 - Sqrt[5] + 4*Cos[2*x]] + (1 + Sqrt[5])*Log[-1 + Sqrt[5] + 4*Cos[2*x]])/20

fricas [A] time = 0.69, size = 72, normalized size = 1.16

$$\frac{1}{20} \sqrt{5} \log \left(\frac{32 \cos(x)^4 + 8(\sqrt{5}-5) \cos(x)^2 - 5\sqrt{5} + 15}{16 \cos(x)^4 - 20 \cos(x)^2 + 5} \right) + \frac{1}{20} \log(16 \cos(x)^4 - 20 \cos(x)^2 + 5) - \frac{1}{5} \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(5*x)*sin(x),x, algorithm="fricas")

[Out] 1/20*sqrt(5)*log((32*cos(x)^4 + 8*(sqrt(5) - 5)*cos(x)^2 - 5*sqrt(5) + 15)/(16*cos(x)^4 - 20*cos(x)^2 + 5)) + 1/20*log(16*cos(x)^4 - 20*cos(x)^2 + 5) - 1/5*log(-cos(x))

giac [A] time = 0.16, size = 67, normalized size = 1.08

$$\frac{1}{20} \sqrt{5} \log \left(\frac{|32 \sin(x)^2 - 4\sqrt{5} - 12|}{|32 \sin(x)^2 + 4\sqrt{5} - 12|} \right) - \frac{1}{10} \log(-\sin(x)^2 + 1) + \frac{1}{20} \log(|16 \sin(x)^4 - 12 \sin(x)^2 + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(5*x)*sin(x),x, algorithm="giac")

[Out] 1/20*sqrt(5)*log(abs(32*sin(x)^2 - 4*sqrt(5) - 12)/abs(32*sin(x)^2 + 4*sqrt(5) - 12)) - 1/10*log(-sin(x)^2 + 1) + 1/20*log(abs(16*sin(x)^4 - 12*sin(x)^2 + 1))

maple [A] time = 0.23, size = 43, normalized size = 0.69

$$\frac{\ln(16(\cos^4(x)) - 20(\cos^2(x)) + 5)}{20} + \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(32(\cos^2(x))-20)\sqrt{5}}{20}\right)}{10} - \frac{\ln(\cos(x))}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(5*x)*sin(x),x)

[Out] 1/20*ln(16*cos(x)^4-20*cos(x)^2+5)+1/10*5^(1/2)*arctanh(1/20*(32*cos(x)^2-20)*5^(1/2))-1/5*ln(cos(x))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(5*x)*sin(x),x, algorithm="maxima")

[Out] 1/5*integrate(-(cos(4*x)*sin(8*x) - cos(8*x)*sin(4*x) + cos(3/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x) + cos(1/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x) - cos(4*x)*sin(3/2*arctan2(sin(4*x), cos(4*x))) - cos(4*x)*sin(1/2*arctan2(sin(4*x), cos(4*x))) - sin(4*x))/(2*(cos(4*x) + 1)*cos(8*x) + cos(8*x)^2 + cos(4*x)^2 - 2*(cos(8*x) + cos(4*x) - cos(1/2*arctan2(sin(4*x), cos(4*x)))) + 1)*cos(3/2*arctan2(sin(4*x), cos(4*x))) + cos(3/2*arctan2(sin(4*x), cos(4*x)))^2 - 2*(cos(8*x) + cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x), cos(4*x))) + cos(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(8*x)^2 + 2*sin(8*x)*sin(4*x) + sin(4*x)^2 - 2*(sin(8*x) + sin(4*x) - sin(1/2*arctan2(sin(4*x), cos(4*x)))))*sin(3/2*arctan2(sin(4*x), cos(4*x))) + sin(3/2*arctan2(sin(4*x), cos(4*x)))^2 - 2*(sin(8*x) + sin(4*x))*sin(1/2*arctan2(sin(4*x), cos(4*x))) + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2 + 2*cos(4*x) + 1), x) + 4/5*integrate(-(cos(2*x)*sin(8*x) - cos(2*x)*sin(6*x) + cos(2*x)*sin(4*x) - cos(8*x)*sin(2*x) + cos(6*x)*sin(2*x) - cos(4*x)*sin(2*x) - sin(2*x))/(2*(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 2*(cos(4*x) - cos(2*x) + 1)*cos(6*x) - cos(6*x)^2 + 2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 + 2*(sin(6*x) - sin(4*x) + sin(2*x))*sin(8*x) - sin(8*x)^2 + 2*(sin(4*x) - sin(2*x))*sin(6*x) - sin(6*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) - 2/5*integrate(-(cos(4/3*arctan2(sin(6*x), cos(6*x)))*sin(6*x) + cos(2/3*arctan2(sin(6*x), cos(6*x)))*sin(6*x) - cos(1/3*arctan2(sin(6*x), cos(6*x)))*sin(6*x) - cos(6*x)*sin(4/3*arctan2(sin(6*x), cos(6*x))) - cos(6*x)*sin(2/3*arctan2(sin(6*x), cos(6*x))) + cos(6*x)*sin(1/3*arctan2(sin(6*x), cos(6*x))) + sin(6*x))/(cos(6*x)^2 - 2*(cos(6*x) - cos(2/3*arctan2(sin(6*x), cos(6*x))) + cos(1/3*arctan2(sin(6*x), cos(6*x)))) - 1)*cos(4/3*arctan2(sin(6*x), cos(6*x))) + cos(4/3*arctan2(sin(6*x), cos(6*x)))^2 - 2*(cos(6*x) + cos(1/3*arctan2(sin(6*x), cos(6*x))) - 1)*cos(2/3*arctan2(sin(6*x), cos(6*x))) + cos(2/3*arctan2(sin(6*x), cos(6*x)))^2 + 2*(cos(6*x) - 1)*cos(1/3*arctan2(sin(6*x), cos(6*x))) + cos(1/3*arctan2(sin(6*x), cos(6*x)))^2 + sin(6*x)^2 - 2*(sin(6*x) - sin(2/3*arctan2(sin(6*x), cos(6*x)))) + sin(1/3*arctan2(sin(6*x), cos(6*x)))))*sin(4/3*arctan2(sin(6*x), cos(6*x))) + sin(4/3*arctan2(sin(6*x), cos(6*x)))^2 - 2*(sin(6*x) + sin(1/3*arctan2(sin(6*x), cos(6*x)))))*sin(2/3*arctan2(sin(6*x), cos(6*x))) + sin(2/3*arctan2(sin(6*x), cos(6*x)))^2 + 2*sin(6*x)*sin(1/3*arctan2(sin(6*x), cos(6*x))) + sin(1/3*arctan2(sin(6*x), cos(6*x)))^2 - 2*cos(6*x) + 1), x) - 2/5*integrate(-(sin(8*x) - sin(6*x) + sin(4*x) - sin(2*x))/(2*(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 2*(cos(4*x) - cos(2*x) + 1)*cos(6*x) - cos(6*x)^2 + 2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 + 2*(sin(6*x) - sin(4*x) + sin(2*x))*sin(8*x) - sin(8*x)^2 + 2*(sin(4*x) - sin(2*x))*sin(6*x) - sin(6*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) - 1/10*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)

mupad [B] time = 0.54, size = 47, normalized size = 0.76

$$\ln\left(\cos(x)^2 + \frac{\sqrt{5}}{8} - \frac{5}{8}\right)\left(\frac{\sqrt{5}}{20} + \frac{1}{20}\right) - \ln\left(\cos(x)^2 - \frac{\sqrt{5}}{8} - \frac{5}{8}\right)\left(\frac{\sqrt{5}}{20} - \frac{1}{20}\right) - \frac{\ln(\cos(x))}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(5*x),x)
```

```
[Out] log(cos(x)^2 + 5^(1/2)/8 - 5/8)*(5^(1/2)/20 + 1/20) - log(cos(x)^2 - 5^(1/2)
)/8 - 5/8)*(5^(1/2)/20 - 1/20) - log(cos(x))/5
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \sin(x) \sec(5x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(5*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*sec(5*x), x)
```

3.89 $\int \sec(6x) \sin(x) dx$

Optimal. Leaf size=85

$$-\frac{\tanh^{-1}(\sqrt{2} \cos(x))}{3\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}} + \frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

[Out] $-1/6*\operatorname{arctanh}(\cos(x)*2^{(1/2)})*2^{(1/2)}+1/6*\operatorname{arctanh}(2*\cos(x)/(1/2*6^{(1/2)}-1/2*2^{(1/2)}))/(1/2*6^{(1/2)}-1/2*2^{(1/2)})+1/6*\operatorname{arctanh}(2*\cos(x)/(1/2*6^{(1/2)}+1/2*2^{(1/2)}))/(1/2*6^{(1/2)}+1/2*2^{(1/2)})$

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4357, 2057, 207, 1166}

$$-\frac{\tanh^{-1}(\sqrt{2} \cos(x))}{3\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}} + \frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Int[Sec[6*x]*Sin[x],x]

[Out] $-\operatorname{ArcTanh}[\operatorname{Sqrt}[2]*\operatorname{Cos}[x]]/(3*\operatorname{Sqrt}[2]) + \operatorname{ArcTanh}[(2*\operatorname{Cos}[x])/ \operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]]/(6*\operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]) + \operatorname{ArcTanh}[(2*\operatorname{Cos}[x])/ \operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]]]/(6*\operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]])$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int(((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 2057

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(u /. x -> x^2)^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x^2] && ILtQ[p, 0]

Rule 4357

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned}
\int \sec(6x) \sin(x) dx &= -\text{Subst} \left(\int \frac{1}{-1 + 18x^2 - 48x^4 + 32x^6} dx, x, \cos(x) \right) \\
&= -\text{Subst} \left(\int \left(-\frac{1}{3(-1 + 2x^2)} + \frac{4(-1 + 2x^2)}{3(1 - 16x^2 + 16x^4)} \right) dx, x, \cos(x) \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \frac{1}{-1 + 2x^2} dx, x, \cos(x) \right) - \frac{4}{3} \text{Subst} \left(\int \frac{-1 + 2x^2}{1 - 16x^2 + 16x^4} dx, x, \cos(x) \right) \\
&= -\frac{\tanh^{-1}(\sqrt{2} \cos(x))}{3\sqrt{2}} - \frac{4}{3} \text{Subst} \left(\int \frac{1}{-8 - 4\sqrt{3} + 16x^2} dx, x, \cos(x) \right) - \frac{4}{3} \text{Subst} \left(\int \frac{1}{-8 + 4\sqrt{3} + 16x^2} dx, x, \cos(x) \right) \\
&= -\frac{\tanh^{-1}(\sqrt{2} \cos(x))}{3\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}} + \frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}
\end{aligned}$$

Mathematica [C] time = 9.33, size = 627, normalized size = 7.38

$$\frac{1}{24} \left((-4 - 4i)(-1)^{3/4} \tanh^{-1} \left(\frac{\tan\left(\frac{x}{2}\right) - 1}{\sqrt{2}} \right) - (4 - 4i)\sqrt[4]{-1} \tanh^{-1} \left(\frac{\tan\left(\frac{x}{2}\right) + 1}{\sqrt{2}} \right) + \frac{2(1 + \sqrt{2}) \left(x - \log(\sec^2\left(\frac{x}{2}\right)) \right)}{\dots} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[6*x]*Sin[x],x]

[Out] $((-4 - 4I)(-1)^{3/4} \text{ArcTanh}[(1 + \text{Tan}[x/2])/\text{Sqrt}[2]] - (4 - 4I)(-1)^{1/4} \text{ArcTanh}[(1 + \text{Tan}[x/2])/\text{Sqrt}[2]] + (2*(1 + \text{Sqrt}[2])*(x + 2*\text{Sqrt}[3]*\text{ArcTanh}[(2 + (2 + \text{Sqrt}[2])* \text{Tan}[x/2])/\text{Sqrt}[6]] - \text{Log}[\text{Sec}[x/2]^2 + \text{Log}[-(\text{Sec}[x/2]^2*(\text{Sqrt}[2] - 2*\text{Cos}[x] + 2*\text{Sin}[x]))])])/(2 + \text{Sqrt}[2]) - \text{Sqrt}[2]*(x - 2*\text{Sqrt}[3]*\text{ArcTanh}[(\text{Sqrt}[2] + (-1 + \text{Sqrt}[2])* \text{Tan}[x/2])/\text{Sqrt}[3]] - \text{Log}[\text{Sec}[x/2]^2 + \text{Log}[\text{Sec}[x/2]^2*(1 + \text{Sqrt}[2]*\text{Cos}[x] - \text{Sqrt}[2]*\text{Sin}[x])]) + (2*(2*(\text{Sqrt}[2] + \text{Sqrt}[3])* \text{ArcTanh}[(2 + (2 + \text{Sqrt}[6])* \text{Tan}[x/2])/\text{Sqrt}[2]] + (3 + \text{Sqrt}[6])*(x - \text{Log}[\text{Sec}[x/2]^2 + \text{Log}[-(\text{Sec}[x/2]^2*(\text{Sqrt}[6] - 2*\text{Cos}[x] + 2*\text{Sin}[x]))])])*(1 + \text{Sqrt}[6]*\text{Sin}[x])*(3 + \text{Sqrt}[6] - (2 + \text{Sqrt}[6])* \text{Cos}[x] + (2 + \text{Sqrt}[6])* \text{Sin}[x])))/((12 + 5*\text{Sqrt}[6])* \text{Cos}[2*x] + 2*\text{Cos}[x]*(5 + 2*\text{Sqrt}[6] + 5*\text{Sqrt}[6]* \text{Sin}[x]) - 2*(12 + 5*\text{Sqrt}[6] + 4*(5 + 2*\text{Sqrt}[6])* \text{Sin}[x] - 6*\text{Sin}[2*x])) + ((-2*(-2 + \text{Sqrt}[6])* \text{ArcTanh}[\text{Sqrt}[2] + (\text{Sqrt}[2] - \text{Sqrt}[3])* \text{Tan}[x/2]] + (3*\text{Sqrt}[2] - 2*\text{Sqrt}[3])*(x - \text{Log}[\text{Sec}[x/2]^2 + \text{Log}[-(\text{Sec}[x/2]^2*(\text{Sqrt}[3] + \text{Sqrt}[2]* \text{Cos}[x] - \text{Sqrt}[2]* \text{Sin}[x]))])])*(\text{Sqrt}[2] - 2*\text{Sqrt}[3]* \text{Sin}[x])*(-3 + \text{Sqrt}[6] - (-2 + \text{Sqrt}[6])* \text{Cos}[x] + (-2 + \text{Sqrt}[6])* \text{Sin}[x])))/((-12 + 5*\text{Sqrt}[6])* \text{Cos}[2*x] + 2*\text{Cos}[x]*(-5 + 2*\text{Sqrt}[6] + 5*\text{Sqrt}[6]* \text{Sin}[x]) - 2*(-12 + 5*\text{Sqrt}[6] + 4*(-5 + 2*\text{Sqrt}[6])* \text{Sin}[x] + 6*\text{Sin}[2*x])))/24$

fricas [B] time = 1.05, size = 153, normalized size = 1.80

$$-\frac{1}{12} \sqrt{\sqrt{3} + 2} \log\left(\sqrt{\sqrt{3} + 2}(\sqrt{3} - 2) + 2 \cos(x)\right) + \frac{1}{12} \sqrt{\sqrt{3} + 2} \log\left(\sqrt{\sqrt{3} + 2}(\sqrt{3} - 2) - 2 \cos(x)\right) + \frac{1}{12} \sqrt{-\sqrt{3} + 2} \log\left(\sqrt{-\sqrt{3} + 2}(\sqrt{3} - 2) + 2 \cos(x)\right) + \frac{1}{12} \sqrt{-\sqrt{3} + 2} \log\left(\sqrt{-\sqrt{3} + 2}(\sqrt{3} - 2) - 2 \cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(6*x)*sin(x),x, algorithm="fricas")

[Out] $-1/12*\text{sqrt}(\text{sqrt}(3) + 2)*\log(\text{sqrt}(\text{sqrt}(3) + 2)*(\text{sqrt}(3) - 2) + 2*\cos(x)) + 1/12*\text{sqrt}(\text{sqrt}(3) + 2)*\log(\text{sqrt}(\text{sqrt}(3) + 2)*(\text{sqrt}(3) - 2) - 2*\cos(x)) + 1/12*\text{sqrt}(-\text{sqrt}(3) + 2)*\log((\text{sqrt}(3) + 2)*\text{sqrt}(-\text{sqrt}(3) + 2) + 2*\cos(x)) - 1/12*\text{sqrt}(-\text{sqrt}(3) + 2)*\log((\text{sqrt}(3) + 2)*\text{sqrt}(-\text{sqrt}(3) + 2) - 2*\cos(x))$

$2*\sqrt{-\sqrt{3} + 2}*\log((\sqrt{3} + 2)*\sqrt{-\sqrt{3} + 2} - 2*\cos(x)) + 1/1$
 $2*\sqrt{2}*\log((2*\cos(x)^2 - 2*\sqrt{2}*\cos(x) + 1)/(2*\cos(x)^2 - 1))$

giac [B] time = 0.16, size = 182, normalized size = 2.14

$$-\frac{1}{12}\sqrt{2}\log\left(\frac{\left| -4\sqrt{2} - \frac{2(\cos(x)-1)}{\cos(x)+1} - 6 \right|}{\left| 4\sqrt{2} - \frac{2(\cos(x)-1)}{\cos(x)+1} - 6 \right|}\right) - \frac{2.39014968180000 \log\left(-\frac{\cos(x)-1}{\cos(x)+1} - 0.0173323801210000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1} + 60.0540532247402} + \frac{5.82951931426000 \log\left(-\frac{\cos(x)-1}{\cos(x)+1} - 0.588790706481000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1} + 121.584934401100} + \frac{16.8155413244667 \log\left(-\frac{\cos(x)-1}{\cos(x)+1} - 1.69839637242000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1} + 559.622604171000} - \frac{7956.25491093333 \log\left(-\frac{\cos(x)-1}{\cos(x)+1} - 57.6954805410000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1} - 168981.261592000}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(6*x)*sin(x),x, algorithm="giac")

[Out] $-1/12*\sqrt{2}*\log(\text{abs}(-4*\sqrt{2} - 2*(\cos(x) - 1)/(\cos(x) + 1) - 6)/\text{abs}(4*\sqrt{2} - 2*(\cos(x) - 1)/(\cos(x) + 1) - 6)) - 2.39014968180000*\log(-(\cos(x) - 1)/(\cos(x) + 1) - 0.0173323801210000)/(268*(\cos(x) - 1)/(\cos(x) + 1) + 60.0540532247402) + 5.82951931426000*\log(-(\cos(x) - 1)/(\cos(x) + 1) - 0.588790706481000)/(268*(\cos(x) - 1)/(\cos(x) + 1) + 121.584934401100) + 16.8155413244667*\log(-(\cos(x) - 1)/(\cos(x) + 1) - 1.69839637242000)/(268*(\cos(x) - 1)/(\cos(x) + 1) + 559.622604171000) - 7956.25491093333*\log(-(\cos(x) - 1)/(\cos(x) + 1) - 57.6954805410000)/(268*(\cos(x) - 1)/(\cos(x) + 1) - 168981.261592000)$

maple [A] time = 0.24, size = 80, normalized size = 0.94

$$\frac{2 \operatorname{arctanh}\left(\frac{8 \cos(x)}{2\sqrt{6}-2\sqrt{2}}\right)}{3(2\sqrt{6}-2\sqrt{2})} + \frac{2 \operatorname{arctanh}\left(\frac{8 \cos(x)}{2\sqrt{6}+2\sqrt{2}}\right)}{3(2\sqrt{6}+2\sqrt{2})} - \frac{\operatorname{arctanh}(\cos(x)\sqrt{2})\sqrt{2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(6*x)*sin(x),x)

[Out] $2/3/(2*6^{(1/2)}-2*2^{(1/2)})*\operatorname{arctanh}(8*\cos(x)/(2*6^{(1/2)}-2*2^{(1/2)}))+2/3/(2*6^{(1/2)}+2*2^{(1/2)})*\operatorname{arctanh}(8*\cos(x)/(2*6^{(1/2)}+2*2^{(1/2)}))-1/6*\operatorname{arctanh}(\cos(x)*2^{(1/2)})*2^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{24}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x) + 2\left(\sqrt{2}\cos(x) + 1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(6*x)*sin(x),x, algorithm="maxima")

[Out] $-1/24*\sqrt{2}*\log(2*\sqrt{2}*\sin(2*x)*\sin(x) + 2*(\sqrt{2}*\cos(x) + 1)*\cos(2*x) + \cos(2*x)^2 + 2*\cos(x)^2 + \sin(2*x)^2 + 2*\sin(x)^2 + 2*\sqrt{2}*\cos(x) + 1) + 1/24*\sqrt{2}*\log(-2*\sqrt{2}*\sin(2*x)*\sin(x) - 2*(\sqrt{2}*\cos(x) - 1)*\cos(2*x) + \cos(2*x)^2 + 2*\cos(x)^2 + \sin(2*x)^2 + 2*\sin(x)^2 - 2*\sqrt{2}*\cos(x) + 1) - \text{integrate}(1/3*((\sin(7*x) - \sin(5*x) + \sin(3*x) - \sin(x))*\cos(8*x) - (\sin(3*x) - \sin(x))*\cos(4*x) - (\cos(7*x) - \cos(5*x) + \cos(3*x) - \cos(x))*\sin(8*x) - (\cos(4*x) - 1)*\sin(7*x) + (\cos(4*x) - 1)*\sin(5*x) + (\cos(3*x) - \cos(x))*\sin(4*x) + \cos(7*x)*\sin(4*x) - \cos(5*x)*\sin(4*x) + \sin(3*x) - \sin(x))/(2*(\cos(4*x) - 1)*\cos(8*x) - \cos(8*x)^2 - \cos(4*x)^2 - \sin(8*x)^2 + 2*\sin(8*x)*\sin(4*x) - \sin(4*x)^2 + 2*\cos(4*x) - 1), x)$

mupad [B] time = 2.28, size = 118, normalized size = 1.39

$$\operatorname{atanh}\left(\frac{5\sqrt{2}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304} + \frac{1}{1048576}\right)} + \frac{3\sqrt{6}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304} + \frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12} + \frac{\sqrt{6}}{12}\right) - \operatorname{atanh}\left(\frac{5\sqrt{2}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304} + \frac{1}{1048576}\right)} - \frac{3\sqrt{6}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304} + \frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12} - \frac{\sqrt{6}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/cos(6*x),x)`

[Out] $\operatorname{atanh}\left(\frac{5\sqrt{2}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304} + \frac{1}{1048576}\right)} + \frac{3\sqrt{6}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304} + \frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12} + \frac{\sqrt{6}}{12}\right) - \operatorname{atanh}\left(\frac{5\sqrt{2}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304} - \frac{1}{1048576}\right)} - \frac{3\sqrt{6}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304} - \frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12} - \frac{\sqrt{6}}{12}\right) - \frac{\sqrt{2}\operatorname{atanh}\left(\frac{2\sqrt{2}\cos(x)}{6}\right)}{6}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \sec(6x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(6*x)*sin(x),x)`

[Out] `Integral(sin(x)*sec(6*x), x)`

3.90 $\int \csc(2x) \sin(x) dx$

Optimal. Leaf size=7

$$\frac{1}{2} \tanh^{-1}(\sin(x))$$

[Out] 1/2*arctanh(sin(x))

Rubi [A] time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4288, 3770}

$$\frac{1}{2} \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Csc[2*x]*Sin[x],x]

[Out] ArcTanh[Sin[x]]/2

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :-> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4288

Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)^(p_.), x_Symbol] :-> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*SIn[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \csc(2x) \sin(x) dx &= \frac{1}{2} \int \sec(x) dx \\ &= \frac{1}{2} \tanh^{-1}(\sin(x)) \end{aligned}$$

Mathematica [B] time = 0.01, size = 37, normalized size = 5.29

$$\frac{1}{2} \left(\log \left(\sin \left(\frac{x}{2} \right) + \cos \left(\frac{x}{2} \right) \right) - \log \left(\cos \left(\frac{x}{2} \right) - \sin \left(\frac{x}{2} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[2*x]*Sin[x],x]

[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])/2

fricas [B] time = 2.74, size = 17, normalized size = 2.43

$$\frac{1}{4} \log(\sin(x) + 1) - \frac{1}{4} \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2*x)*sin(x),x, algorithm="fricas")

[Out] $\frac{1}{4}\log(\sin(x) + 1) - \frac{1}{4}\log(-\sin(x) + 1)$

giac [B] time = 0.14, size = 17, normalized size = 2.43

$$\frac{1}{4} \log(\sin(x) + 1) - \frac{1}{4} \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(2*x)*sin(x),x, algorithm="giac")`

[Out] $\frac{1}{4}\log(\sin(x) + 1) - \frac{1}{4}\log(-\sin(x) + 1)$

maple [A] time = 0.08, size = 9, normalized size = 1.29

$$\frac{\ln(\sec(x) + \tan(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(2*x)*sin(x),x)`

[Out] $\frac{1}{2}\ln(\sec(x) + \tan(x))$

maxima [B] time = 0.42, size = 35, normalized size = 5.00

$$\frac{1}{4} \log(\cos(x)^2 + \sin(x)^2 + 2 \sin(x) + 1) - \frac{1}{4} \log(\cos(x)^2 + \sin(x)^2 - 2 \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(2*x)*sin(x),x, algorithm="maxima")`

[Out] $\frac{1}{4}\log(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1) - \frac{1}{4}\log(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1)$

mupad [B] time = 0.11, size = 5, normalized size = 0.71

$$\frac{\operatorname{atanh}(\sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/sin(2*x),x)`

[Out] $\operatorname{atanh}(\sin(x))/2$

sympy [B] time = 0.85, size = 15, normalized size = 2.14

$$-\frac{\log(\sin(x) - 1)}{4} + \frac{\log(\sin(x) + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(2*x)*sin(x),x)`

[Out] $-\log(\sin(x) - 1)/4 + \log(\sin(x) + 1)/4$

3.91 $\int \csc(3x) \sin(x) dx$

Optimal. Leaf size=45

$$\frac{\log(\sin(x) + \sqrt{3} \cos(x))}{2\sqrt{3}} - \frac{\log(\sqrt{3} \cos(x) - \sin(x))}{2\sqrt{3}}$$

[Out] $-1/6*\ln(-\sin(x)+\cos(x)*3^{(1/2)})*3^{(1/2)}+1/6*\ln(\sin(x)+\cos(x)*3^{(1/2)})*3^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {206}

$$\frac{\log(\sin(x) + \sqrt{3} \cos(x))}{2\sqrt{3}} - \frac{\log(\sqrt{3} \cos(x) - \sin(x))}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Csc[3*x]*Sin[x],x]

[Out] $-\text{Log}[\text{Sqrt}[3]*\text{Cos}[x] - \text{Sin}[x]]/(2*\text{Sqrt}[3]) + \text{Log}[\text{Sqrt}[3]*\text{Cos}[x] + \text{Sin}[x]]/(2*\text{Sqrt}[3])$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \csc(3x) \sin(x) dx &= \text{Subst}\left(\int \frac{1}{3-x^2} dx, x, \tan(x)\right) \\ &= -\frac{\log(\sqrt{3} \cos(x) - \sin(x))}{2\sqrt{3}} + \frac{\log(\sqrt{3} \cos(x) + \sin(x))}{2\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 0.33

$$\frac{\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[3*x]*Sin[x],x]

[Out] ArcTanh[Tan[x]/Sqrt[3]]/Sqrt[3]

fricas [A] time = 1.09, size = 58, normalized size = 1.29

$$\frac{1}{12} \sqrt{3} \log\left(-\frac{8 \cos(x)^4 - 16 \cos(x)^2 - 4(2\sqrt{3} \cos(x)^3 + \sqrt{3} \cos(x)) \sin(x) - 1}{16 \cos(x)^4 - 8 \cos(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(3*x)*sin(x),x, algorithm="fricas")

[Out] $\frac{1}{12}\sqrt{3}\log(-8\cos(x)^4 - 16\cos(x)^2 - 4(2\sqrt{3}\cos(x))^3 + \sqrt{3}(3\cos(x))\sin(x) - 1)/(16\cos(x)^4 - 8\cos(x)^2 + 1)$

giac [A] time = 0.18, size = 31, normalized size = 0.69

$$-\frac{1}{6}\sqrt{3}\log\left(\frac{|-2\sqrt{3} + 2\tan(x)|}{|2\sqrt{3} + 2\tan(x)|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(3*x)*sin(x),x, algorithm="giac")`

[Out] $-1/6\sqrt{3}\log(\text{abs}(-2\sqrt{3} + 2\tan(x))/\text{abs}(2\sqrt{3} + 2\tan(x)))$

maple [A] time = 0.20, size = 14, normalized size = 0.31

$$\frac{\sqrt{3}\operatorname{arctanh}\left(\frac{\tan(x)\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(3*x)*sin(x),x)`

[Out] $1/3\cdot 3^{(1/2)}\cdot\operatorname{arctanh}(1/3\tan(x)\cdot 3^{(1/2)})$

maxima [B] time = 0.44, size = 125, normalized size = 2.78

$$-\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2 + \frac{4}{3}\sin(x)^2 + \frac{4}{3}\sqrt{3}\sin(x) + \frac{4}{3}\cos(x) + \frac{4}{3}\right) + \frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2 + \frac{4}{3}\sin(x)^2 + \frac{4}{3}\sqrt{3}\sin(x) + \frac{4}{3}\cos(x) + \frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(3*x)*sin(x),x, algorithm="maxima")`

[Out] $-1/12\sqrt{3}\log(4/3\cos(x)^2 + 4/3\sin(x)^2 + 4/3\sqrt{3}\sin(x) + 4/3\cos(x) + 4/3) + 1/12\sqrt{3}\log(4/3\cos(x)^2 + 4/3\sin(x)^2 + 4/3\sqrt{3}\sin(x) - 4/3\cos(x) + 4/3) + 1/12\sqrt{3}\log(4/3\cos(x)^2 + 4/3\sin(x)^2 - 4/3\sqrt{3}\sin(x) + 4/3\cos(x) + 4/3) - 1/12\sqrt{3}\log(4/3\cos(x)^2 + 4/3\sin(x)^2 - 4/3\sqrt{3}\sin(x) - 4/3\cos(x) + 4/3)$

mupad [B] time = 2.79, size = 17, normalized size = 0.38

$$\frac{\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}\sin(x)}{3\cos(x)}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/sin(3*x),x)`

[Out] $(3^{(1/2)}\cdot\operatorname{atanh}((3^{(1/2)}\sin(x))/(3\cos(x))))/3$

sympy [A] time = 1.87, size = 76, normalized size = 1.69

$$\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right) - \sqrt{3}\right)}{6} - \frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right) - \frac{\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right) + \frac{\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right) + \sqrt{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(3*x)*sin(x),x)`

[Out] $\sqrt{3}\log(\tan(x/2) - \sqrt{3})/6 - \sqrt{3}\log(\tan(x/2) - \sqrt{3}/3)/6 + \sqrt{3}\log(\tan(x/2) + \sqrt{3}/3)/6 - \sqrt{3}\log(\tan(x/2) + \sqrt{3})/6$

3.92 $\int \csc(4x) \sin(x) dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}(\sqrt{2} \sin(x))}{2\sqrt{2}} - \frac{1}{4} \tanh^{-1}(\sin(x))$$

[Out] $-1/4*\operatorname{arctanh}(\sin(x))+1/4*\operatorname{arctanh}(\sin(x)*2^{(1/2)})*2^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1093, 207}

$$\frac{\tanh^{-1}(\sqrt{2} \sin(x))}{2\sqrt{2}} - \frac{1}{4} \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Csc[4*x]*Sin[x],x]

[Out] $-\operatorname{ArcTanh}[\operatorname{Sin}[x]]/4 + \operatorname{ArcTanh}[\operatorname{Sqrt}[2]*\operatorname{Sin}[x]]/(2*\operatorname{Sqrt}[2])$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \csc(4x) \sin(x) dx &= \operatorname{Subst} \left(\int \frac{1}{4 - 12x^2 + 8x^4} dx, x, \sin(x) \right) \\ &= 2 \operatorname{Subst} \left(\int \frac{1}{-8 + 8x^2} dx, x, \sin(x) \right) - 2 \operatorname{Subst} \left(\int \frac{1}{-4 + 8x^2} dx, x, \sin(x) \right) \\ &= -\frac{1}{4} \tanh^{-1}(\sin(x)) + \frac{\tanh^{-1}(\sqrt{2} \sin(x))}{2\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 1.00

$$\frac{\tanh^{-1}(\sqrt{2} \sin(x))}{2\sqrt{2}} - \frac{1}{4} \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Csc[4*x]*Sin[x],x]

[Out] $-1/4*\operatorname{ArcTanh}[\operatorname{Sin}[x]] + \operatorname{ArcTanh}[\operatorname{Sqrt}[2]*\operatorname{Sin}[x]]/(2*\operatorname{Sqrt}[2])$

fricas [B] time = 1.97, size = 50, normalized size = 1.92

$$\frac{1}{8} \sqrt{2} \log \left(-\frac{2 \cos(x)^2 - 2\sqrt{2} \sin(x) - 3}{2 \cos(x)^2 - 1} \right) - \frac{1}{8} \log(\sin(x) + 1) + \frac{1}{8} \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(4*x)*sin(x),x, algorithm="fricas")

[Out] 1/8*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)

giac [B] time = 0.14, size = 48, normalized size = 1.85

$$-\frac{1}{8} \sqrt{2} \log \left(\frac{|-2\sqrt{2} + 4 \sin(x)|}{|2\sqrt{2} + 4 \sin(x)|} \right) - \frac{1}{8} \log(\sin(x) + 1) + \frac{1}{8} \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(4*x)*sin(x),x, algorithm="giac")

[Out] -1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)

maple [A] time = 0.23, size = 28, normalized size = 1.08

$$\frac{\ln(\sin(x) - 1)}{8} + \frac{\operatorname{arctanh}(\sin(x)\sqrt{2})\sqrt{2}}{4} - \frac{\ln(1 + \sin(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(4*x)*sin(x),x)

[Out] 1/8*ln(sin(x)-1)+1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)-1/8*ln(1+sin(x))

maxima [B] time = 0.43, size = 171, normalized size = 6.58

$$\frac{1}{16} \sqrt{2} \log \left(2 \cos(x)^2 + 2 \sin(x)^2 + 2\sqrt{2} \cos(x) + 2\sqrt{2} \sin(x) + 2 \right) - \frac{1}{16} \sqrt{2} \log \left(2 \cos(x)^2 + 2 \sin(x)^2 + 2\sqrt{2} \cos(x) - 2\sqrt{2} \sin(x) + 2 \right) + \frac{1}{16} \sqrt{2} \log \left(2 \cos(x)^2 + 2 \sin(x)^2 - 2\sqrt{2} \cos(x) + 2\sqrt{2} \sin(x) + 2 \right) - \frac{1}{16} \sqrt{2} \log \left(2 \cos(x)^2 + 2 \sin(x)^2 - 2\sqrt{2} \cos(x) - 2\sqrt{2} \sin(x) + 2 \right) - \frac{1}{8} \log(\cos(x)^2 + \sin(x)^2 + 2 \sin(x) + 1) + \frac{1}{8} \log(\cos(x)^2 + \sin(x)^2 - 2 \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(4*x)*sin(x),x, algorithm="maxima")

[Out] 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)

mupad [B] time = 2.43, size = 27, normalized size = 1.04

$$\frac{\sqrt{2} \operatorname{atanh}(\sqrt{2} \sin(x))}{4} - \frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/sin(4*x),x)

[Out] (2^(1/2)*atanh(2^(1/2)*sin(x)))/4 - atanh(sin(x/2)/cos(x/2))/2

sympy [B] time = 7.36, size = 294, normalized size = 11.31

$$\frac{27720\sqrt{2} \log\left(\tan\left(\frac{x}{2}\right) - 1\right)}{110880\sqrt{2} + 156808} + \frac{39202 \log\left(\tan\left(\frac{x}{2}\right) - 1\right)}{110880\sqrt{2} + 156808} - \frac{39202 \log\left(\tan\left(\frac{x}{2}\right) + 1\right)}{110880\sqrt{2} + 156808} - \frac{27720\sqrt{2} \log\left(\tan\left(\frac{x}{2}\right) + 1\right)}{110880\sqrt{2} + 156808} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(4*x)*sin(x),x)

[Out] 27720*sqrt(2)*log(tan(x/2) - 1)/(110880*sqrt(2) + 156808) + 39202*log(tan(x/2) - 1)/(110880*sqrt(2) + 156808) - 39202*log(tan(x/2) + 1)/(110880*sqrt(2) + 156808) - 27720*sqrt(2)*log(tan(x/2) + 1)/(110880*sqrt(2) + 156808) + 27720*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(x/2) + 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2) + 1 + sqrt(2))/(110880*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2) - 1)/(110880*sqrt(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) - 1)/(110880*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(2) + 156808)

3.93 $\int \csc(5x) \sin(x) dx$

Optimal. Leaf size=165

$$-\frac{1}{10}\sqrt{\frac{1}{2}(5-\sqrt{5})} \log\left(\sqrt{5-2\sqrt{5}} \cos(x) - \sin(x)\right) + \frac{1}{10}\sqrt{\frac{1}{2}(5+\sqrt{5})} \log\left(\sqrt{5+2\sqrt{5}} \cos(x) - \sin(x)\right) + \frac{1}{10}\sqrt{\frac{1}{2}(5-\sqrt{5})} \log\left(\sqrt{5-2\sqrt{5}} \cos(x) + \sin(x)\right) - \frac{1}{10}\sqrt{\frac{1}{2}(5+\sqrt{5})} \log\left(\sqrt{5+2\sqrt{5}} \cos(x) + \sin(x)\right)$$

[Out] $-1/20*\ln(-\sin(x)+\cos(x)*(5-2*5^{(1/2)})^{(1/2)})*(10-2*5^{(1/2)})^{(1/2)}+1/20*\ln(\sin(x)+\cos(x)*(5-2*5^{(1/2)})^{(1/2)})*(10-2*5^{(1/2)})^{(1/2)}+1/20*\ln(-\sin(x)+\cos(x)*(5+2*5^{(1/2)})^{(1/2)})*(10+2*5^{(1/2)})^{(1/2)}-1/20*\ln(\sin(x)+\cos(x)*(5+2*5^{(1/2)})^{(1/2)})*(10+2*5^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.14, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1166, 207}

$$-\frac{1}{10}\sqrt{\frac{1}{2}(5-\sqrt{5})} \log\left(\sqrt{5-2\sqrt{5}} \cos(x) - \sin(x)\right) + \frac{1}{10}\sqrt{\frac{1}{2}(5+\sqrt{5})} \log\left(\sqrt{5+2\sqrt{5}} \cos(x) - \sin(x)\right) + \frac{1}{10}\sqrt{\frac{1}{2}(5-\sqrt{5})} \log\left(\sqrt{5-2\sqrt{5}} \cos(x) + \sin(x)\right) - \frac{1}{10}\sqrt{\frac{1}{2}(5+\sqrt{5})} \log\left(\sqrt{5+2\sqrt{5}} \cos(x) + \sin(x)\right)$$

Antiderivative was successfully verified.

[In] Int[Csc[5*x]*Sin[x], x]

[Out] $-(\text{Sqrt}[(5 - \text{Sqrt}[5])/2]*\text{Log}[\text{Sqrt}[5 - 2*\text{Sqrt}[5]]*\text{Cos}[x] - \text{Sin}[x]])/10 + (\text{Sqrt}[(5 + \text{Sqrt}[5])/2]*\text{Log}[\text{Sqrt}[5 + 2*\text{Sqrt}[5]]*\text{Cos}[x] - \text{Sin}[x]])/10 + (\text{Sqrt}[(5 - \text{Sqrt}[5])/2]*\text{Log}[\text{Sqrt}[5 - 2*\text{Sqrt}[5]]*\text{Cos}[x] + \text{Sin}[x]])/10 - (\text{Sqrt}[(5 + \text{Sqrt}[5])/2]*\text{Log}[\text{Sqrt}[5 + 2*\text{Sqrt}[5]]*\text{Cos}[x] + \text{Sin}[x]])/10$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \csc(5x) \sin(x) dx &= \text{Subst}\left(\int \frac{1+x^2}{5-10x^2+x^4} dx, x, \tan(x)\right) \\ &= \frac{1}{10}(5-3\sqrt{5}) \text{Subst}\left(\int \frac{1}{-5+2\sqrt{5}+x^2} dx, x, \tan(x)\right) + \frac{1}{10}(5+3\sqrt{5}) \text{Subst}\left(\int \frac{1}{-5-2\sqrt{5}+x^2} dx, x, \tan(x)\right) \\ &= -\frac{1}{10}\sqrt{\frac{1}{2}(5-\sqrt{5})} \log\left(\sqrt{5-2\sqrt{5}} \cos(x) - \sin(x)\right) + \frac{1}{10}\sqrt{\frac{1}{2}(5+\sqrt{5})} \log\left(\sqrt{5+2\sqrt{5}} \cos(x) - \sin(x)\right) \\ &\quad + \frac{1}{10}\sqrt{\frac{1}{2}(5-\sqrt{5})} \log\left(\sqrt{5-2\sqrt{5}} \cos(x) + \sin(x)\right) - \frac{1}{10}\sqrt{\frac{1}{2}(5+\sqrt{5})} \log\left(\sqrt{5+2\sqrt{5}} \cos(x) + \sin(x)\right) \end{aligned}$$

Mathematica [A] time = 0.11, size = 84, normalized size = 0.51

$$\frac{\sqrt{5+\sqrt{5}} \tanh^{-1}\left(\frac{(\sqrt{5}-3)\tan(x)}{\sqrt{10-2\sqrt{5}}}\right) + \sqrt{5-\sqrt{5}} \tanh^{-1}\left(\frac{(3+\sqrt{5})\tan(x)}{\sqrt{2(5+\sqrt{5})}}\right)}{5\sqrt{2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[5*x]*Sin[x],x]

[Out] (Sqrt[5 + Sqrt[5]]*ArcTanh[((-3 + Sqrt[5])*Tan[x])/Sqrt[10 - 2*Sqrt[5]]] + Sqrt[5 - Sqrt[5]]*ArcTanh[((3 + Sqrt[5])*Tan[x])/Sqrt[2*(5 + Sqrt[5])]])/(5*Sqrt[2])

fricas [B] time = 0.93, size = 231, normalized size = 1.40

$$-\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+5}\cos(x)\sin(x)+2\left(\sqrt{5}+1\right)\cos(x)^2-\sqrt{5}+3\right)+\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(5*x)*sin(x),x, algorithm="fricas")

[Out] -1/40*sqrt(2)*sqrt(sqrt(5)+5)*log((sqrt(5)*sqrt(2)-sqrt(2))*sqrt(sqrt(5)+5)*cos(x)*sin(x)+2*(sqrt(5)+1)*cos(x)^2-sqrt(5)+3)+1/40*sqrt(2)*sqrt(sqrt(5)+5)*log(-(sqrt(5)*sqrt(2)-sqrt(2))*sqrt(sqrt(5)+5)*cos(x)*sin(x)+2*(sqrt(5)+1)*cos(x)^2-sqrt(5)+3)-1/40*sqrt(2)*sqrt(-sqrt(5)+5)*log((sqrt(5)*sqrt(2)+sqrt(2))*sqrt(-sqrt(5)+5)*cos(x)*sin(x)+2*(sqrt(5)-1)*cos(x)^2-sqrt(5)-3)+1/40*sqrt(2)*sqrt(-sqrt(5)+5)*log(-(sqrt(5)*sqrt(2)+sqrt(2))*sqrt(-sqrt(5)+5)*cos(x)*sin(x)+2*(sqrt(5)-1)*cos(x)^2-sqrt(5)-3)

giac [A] time = 0.28, size = 105, normalized size = 0.64

$$-\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+10}\log\left(\left|\sqrt{2}\sqrt{\sqrt{5}+5}+\tan(x)\right|\right)+\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+10}\log\left(\left|-\sqrt{2}\sqrt{\sqrt{5}+5}+\tan(x)\right|\right)+\frac{1}{20}\sqrt{-2}\sqrt{\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(5*x)*sin(x),x, algorithm="giac")

[Out] -1/20*sqrt(2*sqrt(5)+10)*log(abs(sqrt(2*sqrt(5)+5)+tan(x))) + 1/20*sqrt(2*sqrt(5)+10)*log(abs(-sqrt(2*sqrt(5)+5)+tan(x))) + 1/20*sqrt(-2*sqrt(5)+10)*log(abs(sqrt(-2*sqrt(5)+5)+tan(x))) - 1/20*sqrt(-2*sqrt(5)+10)*log(abs(-sqrt(-2*sqrt(5)+5)+tan(x)))

maple [A] time = 0.29, size = 66, normalized size = 0.40

$$\frac{(3+\sqrt{5})\sqrt{5}\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{5+2\sqrt{5}}}\right)}{10\sqrt{5+2\sqrt{5}}}-\frac{(\sqrt{5}-3)\sqrt{5}\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{5-2\sqrt{5}}}\right)}{10\sqrt{5-2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(5*x)*sin(x),x)

[Out] -1/10*(3+5^(1/2))*5^(1/2)/(5+2*5^(1/2))^(1/2)*arctanh(tan(x)/(5+2*5^(1/2))^(1/2))-1/10*(5^(1/2)-3)*5^(1/2)/(5-2*5^(1/2))^(1/2)*arctanh(tan(x)/(5-2*5^(1/2))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(5x)\sin(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(5*x)*sin(x),x, algorithm="maxima")

[Out] integrate(csc(5*x)*sin(x), x)

mupad [B] time = 2.59, size = 217, normalized size = 1.32

$$\sqrt{2} \operatorname{atanh} \left(\frac{34359738368 \sqrt{2} \tan\left(\frac{x}{2}\right) \sqrt{\sqrt{5}+5}}{1953125 \left(\frac{90194313216 \sqrt{5}}{1953125} - \frac{90194313216 \sqrt{5} \tan\left(\frac{x}{2}\right)^2}{1953125} - \frac{201863462912 \tan\left(\frac{x}{2}\right)^2}{1953125} + \frac{201863462912}{1953125} \right)} - \frac{77309411328 \sqrt{2}}{9765625 \left(\frac{90194313216 \sqrt{5}}{1953125} - \frac{90194313216 \sqrt{5} \tan\left(\frac{x}{2}\right)^2}{1953125} \right)} \right) \frac{1}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/sin(5*x), x)

[Out] $(2^{(1/2)} \operatorname{atanh}(- (34359738368 * 2^{(1/2)} * \tan(x/2) * (5^{(1/2)} + 5)^{(1/2)}) / (1953125 * ((90194313216 * 5^{(1/2)}) / 1953125 - (90194313216 * 5^{(1/2)} * \tan(x/2)^2) / 1953125 - (201863462912 * \tan(x/2)^2) / 1953125 + 201863462912 / 1953125)) - (77309411328 * 2^{(1/2)} * 5^{(1/2)} * \tan(x/2) * (5^{(1/2)} + 5)^{(1/2)}) / (9765625 * ((90194313216 * 5^{(1/2)}) / 1953125 - (90194313216 * 5^{(1/2)} * \tan(x/2)^2) / 1953125 - (201863462912 * \tan(x/2)^2) / 1953125 + 201863462912 / 1953125))) * (5^{(1/2)} + 5)^{(1/2)}) / 10 - (2^{(1/2)} \operatorname{atanh}((77309411328 * 2^{(1/2)} * 5^{(1/2)} * \tan(x/2) * (5 - 5^{(1/2)})^{(1/2)}) / (9765625 * ((90194313216 * 5^{(1/2)}) / 1953125 - (90194313216 * 5^{(1/2)} * \tan(x/2)^2) / 1953125 + (201863462912 * \tan(x/2)^2) / 1953125 - 201863462912 / 1953125)) - (34359738368 * 2^{(1/2)} * \tan(x/2) * (5 - 5^{(1/2)})^{(1/2)}) / (1953125 * ((90194313216 * 5^{(1/2)}) / 1953125 - (90194313216 * 5^{(1/2)} * \tan(x/2)^2) / 1953125 + (201863462912 * \tan(x/2)^2) / 1953125 - 201863462912 / 1953125))) * (5 - 5^{(1/2)})^{(1/2)}) / 10$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \operatorname{csc}(5x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(5*x)*sin(x), x)

[Out] Integral(sin(x)*csc(5*x), x)

3.94 $\int \csc(6x) \sin(x) dx$

Optimal. Leaf size=36

$$\frac{1}{6} \tanh^{-1}(\sin(x)) + \frac{1}{6} \tanh^{-1}(2 \sin(x)) - \frac{\tanh^{-1}\left(\frac{2 \sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

[Out] 1/6*arctanh(sin(x))+1/6*arctanh(2*sin(x))-1/6*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {12, 2057, 207}

$$\frac{1}{6} \tanh^{-1}(\sin(x)) + \frac{1}{6} \tanh^{-1}(2 \sin(x)) - \frac{\tanh^{-1}\left(\frac{2 \sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Csc[6*x]*Sin[x],x]

[Out] ArcTanh[Sin[x]]/6 + ArcTanh[2*SIN[x]]/6 - ArcTanh[(2*SIN[x])/Sqrt[3]]/(2*Sqrt[3])

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2057

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(u /. x -> x^2)^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x^2] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \csc(6x) \sin(x) dx &= \text{Subst} \left(\int \frac{1}{2(3 - 19x^2 + 32x^4 - 16x^6)} dx, x, \sin(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{3 - 19x^2 + 32x^4 - 16x^6} dx, x, \sin(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(-\frac{1}{3(-1 + x^2)} + \frac{2}{-3 + 4x^2} - \frac{2}{3(-1 + 4x^2)} \right) dx, x, \sin(x) \right) \\ &= -\left(\frac{1}{6} \text{Subst} \left(\int \frac{1}{-1 + x^2} dx, x, \sin(x) \right) \right) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{-1 + 4x^2} dx, x, \sin(x) \right) + \text{Subst} \left(\int \frac{1}{-1 + 4x^2} dx, x, \sin(x) \right) \\ &= \frac{1}{6} \tanh^{-1}(\sin(x)) + \frac{1}{6} \tanh^{-1}(2 \sin(x)) - \frac{\tanh^{-1}\left(\frac{2 \sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 30, normalized size = 0.83

$$\frac{1}{6} \left(\tanh^{-1}(\sin(x)) + \tanh^{-1}(2 \sin(x)) - \sqrt{3} \tanh^{-1} \left(\frac{2 \sin(x)}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[6*x]*Sin[x],x]

[Out] (ArcTanh[Sin[x]] + ArcTanh[2*Sin[x]] - Sqrt[3]*ArcTanh[(2*Sin[x])/Sqrt[3]])/6

fricas [B] time = 1.24, size = 68, normalized size = 1.89

$$\frac{1}{12} \sqrt{3} \log \left(-\frac{4 \cos(x)^2 + 4 \sqrt{3} \sin(x) - 7}{4 \cos(x)^2 - 1} \right) + \frac{1}{12} \log(2 \sin(x) + 1) + \frac{1}{12} \log(\sin(x) + 1) - \frac{1}{12} \log(-\sin(x) + 1) - \frac{1}{12} \log(-2 \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(6*x)*sin(x),x, algorithm="fricas")

[Out] 1/12*sqrt(3)*log(-4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) + 1/12*log(2*sin(x) + 1) + 1/12*log(sin(x) + 1) - 1/12*log(-sin(x) + 1) - 1/12*log(-2*sin(x) + 1)

giac [B] time = 0.15, size = 68, normalized size = 1.89

$$\frac{1}{12} \sqrt{3} \log \left(\frac{|-4 \sqrt{3} + 8 \sin(x)|}{|4 \sqrt{3} + 8 \sin(x)|} \right) + \frac{1}{12} \log(\sin(x) + 1) - \frac{1}{12} \log(-\sin(x) + 1) + \frac{1}{12} \log(|2 \sin(x) + 1|) - \frac{1}{12} \log(|-2 \sin(x) + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(6*x)*sin(x),x, algorithm="giac")

[Out] 1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) + 1/12*log(sin(x) + 1) - 1/12*log(-sin(x) + 1) + 1/12*log(abs(2*sin(x) + 1)) - 1/12*log(abs(2*sin(x) - 1))

maple [A] time = 0.26, size = 47, normalized size = 1.31

$$-\frac{\ln(-1 + 2 \sin(x))}{12} + \frac{\ln(1 + 2 \sin(x))}{12} - \frac{\operatorname{arctanh} \left(\frac{2 \sin(x) \sqrt{3}}{3} \right) \sqrt{3}}{6} - \frac{\ln(\sin(x) - 1)}{12} + \frac{\ln(1 + \sin(x))}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(6*x)*sin(x),x)

[Out] -1/12*ln(-1+2*sin(x))+1/12*ln(1+2*sin(x))-1/6*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)-1/12*ln(sin(x)-1)+1/12*ln(1+sin(x))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{24} \sqrt{3} \log \left(\frac{4}{3} \cos(x)^2 + \frac{4}{3} \sin(x)^2 + \frac{4}{3} \sqrt{3} \sin(x) + \frac{4}{3} \cos(x) + \frac{4}{3} \right) - \frac{1}{24} \sqrt{3} \log \left(\frac{4}{3} \cos(x)^2 + \frac{4}{3} \sin(x)^2 + \frac{4}{3} \sqrt{3} \sin(x) + \frac{4}{3} \cos(x) + \frac{4}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(6*x)*sin(x),x, algorithm="maxima")

[Out] -1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) - 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3)

```
n(x) - 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 - 4/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) + integrate(-1/6*((cos(3*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - cos(2*x)*cos(x) + (sin(3*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*x)*sin(x) + cos(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/12*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/12*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

mupad [B] time = 2.46, size = 35, normalized size = 0.97

$$\frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{3} + \frac{\operatorname{atanh}(2 \sin(x))}{6} - \frac{\sqrt{3} \operatorname{atanh}\left(\frac{2\sqrt{3} \sin(x)}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/sin(6*x),x)

[Out] atanh(sin(x/2)/cos(x/2))/3 + atanh(2*sin(x))/6 - (3^(1/2)*atanh((2*3^(1/2)*sin(x))/3))/6

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \csc(6x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(6*x)*sin(x),x)

[Out] Integral(sin(x)*csc(6*x), x)

3.95 $\int \csc(x) \sin(3x) dx$

Optimal. Leaf size=8

$$x + 2 \sin(x) \cos(x)$$

[Out] x+2*cos(x)*sin(x)

Rubi [A] time = 0.03, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {385, 203}

$$x + 2 \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Csc[x]*Sin[3*x],x]

[Out] x + 2*Cos[x]*Sin[x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rubi steps

$$\begin{aligned} \int \csc(x) \sin(3x) dx &= \text{Subst} \left(\int \frac{3 - x^2}{(1 + x^2)^2} dx, x, \tan(x) \right) \\ &= 2 \cos(x) \sin(x) + \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \tan(x) \right) \\ &= x + 2 \cos(x) \sin(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 6, normalized size = 0.75

$$x + \sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]*Sin[3*x],x]

[Out] x + Sin[2*x]

fricas [A] time = 0.83, size = 8, normalized size = 1.00

$$2 \cos(x) \sin(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)*sin(3*x),x, algorithm="fricas")

[Out] $2*\cos(x)*\sin(x) + x$

giac [A] time = 0.13, size = 6, normalized size = 0.75

$$x + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)*sin(3*x),x, algorithm="giac")`

[Out] $x + \sin(2*x)$

maple [A] time = 0.10, size = 9, normalized size = 1.12

$$x + 2 \cos(x) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(x)*sin(3*x),x)`

[Out] $x+2*\cos(x)*\sin(x)$

maxima [A] time = 0.31, size = 6, normalized size = 0.75

$$x + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)*sin(3*x),x, algorithm="maxima")`

[Out] $x + \sin(2*x)$

mupad [B] time = 2.25, size = 6, normalized size = 0.75

$$x + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(3*x)/sin(x),x)`

[Out] $x + \sin(2*x)$

sympy [A] time = 0.98, size = 5, normalized size = 0.62

$$x + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)*sin(3*x),x)`

[Out] $x + \sin(2*x)$

3.96 $\int \csc(3x) \sin(6x) dx$

Optimal. Leaf size=8

$$\frac{2}{3} \sin(3x)$$

[Out] 2/3*sin(3*x)

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4288, 2637}

$$\frac{2}{3} \sin(3x)$$

Antiderivative was successfully verified.

[In] Int[Csc[3*x]*Sin[6*x],x]

[Out] (2*Sin[3*x])/3

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 4288

Int[((f_.)*sin[(a_.) + (b_.)*(x_.)])^(n_.)*sin[(c_.) + (d_.)*(x_.)]^(p_.), x_Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Ssin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \csc(3x) \sin(6x) dx &= 2 \int \cos(3x) dx \\ &= \frac{2}{3} \sin(3x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{2}{3} \sin(3x)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[3*x]*Sin[6*x],x]

[Out] (2*Sin[3*x])/3

fricas [A] time = 2.97, size = 6, normalized size = 0.75

$$\frac{2}{3} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(3*x)*sin(6*x),x, algorithm="fricas")

[Out] 2/3*sin(3*x)

giac [A] time = 0.13, size = 6, normalized size = 0.75

$$\frac{2}{3} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(3*x)*sin(6*x),x, algorithm="giac")

[Out] 2/3*sin(3*x)

maple [A] time = 0.04, size = 9, normalized size = 1.12

$$\frac{2}{3 \csc(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(3*x)*sin(6*x),x)

[Out] 2/3/csc(3*x)

maxima [A] time = 0.32, size = 6, normalized size = 0.75

$$\frac{2}{3} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(3*x)*sin(6*x),x, algorithm="maxima")

[Out] 2/3*sin(3*x)

mupad [B] time = 0.03, size = 6, normalized size = 0.75

$$\frac{2 \sin(3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(6*x)/sin(3*x),x)

[Out] (2*sin(3*x))/3

sympy [A] time = 3.26, size = 7, normalized size = 0.88

$$\frac{2 \sin(3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(3*x)*sin(6*x),x)

[Out] 2*sin(3*x)/3

3.97 $\int \cos(x) \sin(2x) dx$

Optimal. Leaf size=15

$$-\frac{\cos(x)}{2} - \frac{1}{6} \cos(3x)$$

[Out] $-1/2*\cos(x)-1/6*\cos(3*x)$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4284}

$$-\frac{\cos(x)}{2} - \frac{1}{6} \cos(3x)$$

Antiderivative was successfully verified.

[In] `Int[Cos[x]*Sin[2*x],x]`

[Out] $-\text{Cos}[x]/2 - \text{Cos}[3*x]/6$

Rule 4284

`Int[cos[(c_.) + (d_.)*(x_.)]*sin[(a_.) + (b_.)*(x_.)], x_Symbol] := -Simp[Cos[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]`

Rubi steps

$$\int \cos(x) \sin(2x) dx = -\frac{\cos(x)}{2} - \frac{1}{6} \cos(3x)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{\cos(x)}{2} - \frac{1}{6} \cos(3x)$$

Antiderivative was successfully verified.

[In] `Integrate[Cos[x]*Sin[2*x],x]`

[Out] $-1/2*\text{Cos}[x] - \text{Cos}[3*x]/6$

fricas [A] time = 1.48, size = 6, normalized size = 0.40

$$-\frac{2}{3} \cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*sin(2*x),x, algorithm="fricas")`

[Out] $-2/3*\cos(x)^3$

giac [A] time = 0.13, size = 6, normalized size = 0.40

$$-\frac{2}{3} \cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*sin(2*x),x, algorithm="giac")`

[Out] $-2/3*\cos(x)^3$

maple [A] time = 0.04, size = 7, normalized size = 0.47

$$-\frac{2(\cos^3(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*sin(2*x),x)`

[Out] $-2/3*\cos(x)^3$

maxima [A] time = 0.31, size = 11, normalized size = 0.73

$$-\frac{1}{6}\cos(3x) - \frac{1}{2}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*sin(2*x),x, algorithm="maxima")`

[Out] $-1/6*\cos(3*x) - 1/2*\cos(x)$

mupad [B] time = 0.02, size = 6, normalized size = 0.40

$$-\frac{2\cos(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(2*x)*cos(x),x)`

[Out] $-(2*\cos(x)^3)/3$

sympy [A] time = 0.45, size = 22, normalized size = 1.47

$$-\frac{\sin(x)\sin(2x)}{3} - \frac{2\cos(x)\cos(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*sin(2*x),x)`

[Out] $-\sin(x)*\sin(2*x)/3 - 2*\cos(x)*\cos(2*x)/3$

3.98 $\int \cos(x) \sin(3x) dx$

Optimal. Leaf size=17

$$-\frac{1}{4} \cos(2x) - \frac{1}{8} \cos(4x)$$

[Out] -1/4*cos(2*x)-1/8*cos(4*x)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4284}

$$-\frac{1}{4} \cos(2x) - \frac{1}{8} \cos(4x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[3*x],x]

[Out] -Cos[2*x]/4 - Cos[4*x]/8

Rule 4284

Int[cos[(c_.) + (d_.)*(x_.)]*sin[(a_.) + (b_.)*(x_.)], x_Symbol] :> -Simp[Cos[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\int \cos(x) \sin(3x) dx = -\frac{1}{4} \cos(2x) - \frac{1}{8} \cos(4x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{1}{2} \cos^2(x) - \frac{1}{8} \cos(4x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sin[3*x],x]

[Out] -1/2*Cos[x]^2 - Cos[4*x]/8

fricas [A] time = 0.90, size = 13, normalized size = 0.76

$$-\cos(x)^4 + \frac{1}{2} \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(3*x),x, algorithm="fricas")

[Out] -cos(x)^4 + 1/2*cos(x)^2

giac [A] time = 0.14, size = 13, normalized size = 0.76

$$-\cos(x)^4 + \frac{1}{2} \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(3*x),x, algorithm="giac")

[Out] $-\cos(x)^4 + 1/2*\cos(x)^2$

maple [A] time = 0.07, size = 14, normalized size = 0.82

$$-\left(\cos^4(x)\right) + \frac{\left(\cos^2(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(3*x),x)

[Out] $-\cos(x)^4+1/2*\cos(x)^2$

maxima [A] time = 0.31, size = 13, normalized size = 0.76

$$-\frac{1}{8} \cos(4x) - \frac{1}{4} \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(3*x),x, algorithm="maxima")

[Out] $-1/8*\cos(4*x) - 1/4*\cos(2*x)$

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$\frac{\cos(x)^2}{2} - \cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(3*x)*cos(x),x)

[Out] $\cos(x)^2/2 - \cos(x)^4$

sympy [A] time = 0.42, size = 22, normalized size = 1.29

$$\frac{\sin(x) \sin(3x)}{8} - \frac{3 \cos(x) \cos(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(3*x),x)

[Out] $-\sin(x)*\sin(3*x)/8 - 3*\cos(x)*\cos(3*x)/8$

3.99 $\int \cos(x) \sin(4x) dx$

Optimal. Leaf size=17

$$-\frac{1}{6} \cos(3x) - \frac{1}{10} \cos(5x)$$

[Out] -1/6*cos(3*x)-1/10*cos(5*x)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4284}

$$-\frac{1}{6} \cos(3x) - \frac{1}{10} \cos(5x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[4*x],x]

[Out] -Cos[3*x]/6 - Cos[5*x]/10

Rule 4284

Int[cos[(c_.) + (d_.)*(x_.)]*sin[(a_.) + (b_.)*(x_.)], x_Symbol] :> -Simp[Cos[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\int \cos(x) \sin(4x) dx = -\frac{1}{6} \cos(3x) - \frac{1}{10} \cos(5x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{1}{6} \cos(3x) - \frac{1}{10} \cos(5x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sin[4*x],x]

[Out] -1/6*Cos[3*x] - Cos[5*x]/10

fricas [A] time = 0.55, size = 13, normalized size = 0.76

$$-\frac{8}{5} \cos(x)^5 + \frac{4}{3} \cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(4*x),x, algorithm="fricas")

[Out] -8/5*cos(x)^5 + 4/3*cos(x)^3

giac [A] time = 0.12, size = 13, normalized size = 0.76

$$-\frac{8}{5} \cos(x)^5 + \frac{4}{3} \cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(4*x),x, algorithm="giac")

[Out] $-8/5*\cos(x)^5 + 4/3*\cos(x)^3$

maple [A] time = 0.06, size = 14, normalized size = 0.82

$$-\frac{8(\cos^5(x))}{5} + \frac{4(\cos^3(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(4*x),x)

[Out] $-8/5*\cos(x)^5+4/3*\cos(x)^3$

maxima [A] time = 0.31, size = 13, normalized size = 0.76

$$-\frac{1}{10} \cos(5x) - \frac{1}{6} \cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(4*x),x, algorithm="maxima")

[Out] $-1/10*\cos(5*x) - 1/6*\cos(3*x)$

mupad [B] time = 0.02, size = 14, normalized size = 0.82

$$\frac{4 \cos(x)^3 (6 \cos(x)^2 - 5)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(4*x)*cos(x),x)

[Out] $-(4*\cos(x)^3*(6*\cos(x)^2 - 5))/15$

sympy [A] time = 0.41, size = 22, normalized size = 1.29

$$-\frac{\sin(x) \sin(4x)}{15} - \frac{4 \cos(x) \cos(4x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(4*x),x)

[Out] $-\sin(x)*\sin(4*x)/15 - 4*\cos(x)*\cos(4*x)/15$

3.100 $\int \cos(x) \sin(mx) dx$

Optimal. Leaf size=35

$$\frac{\cos((1-m)x)}{2(1-m)} - \frac{\cos((m+1)x)}{2(m+1)}$$

[Out] 1/2*cos((1-m)*x)/(1-m)-1/2*cos((1+m)*x)/(1+m)

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4574, 2638}

$$\frac{\cos((1-m)x)}{2(1-m)} - \frac{\cos((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[m*x],x]

[Out] Cos[(1-m)*x]/(2*(1-m)) - Cos[(1+m)*x]/(2*(1+m))

Rule 2638

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4574

Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] := Int[ExpandTrigReduce[Sin[v]^p * Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))

Rubi steps

$$\begin{aligned} \int \cos(x) \sin(mx) dx &= \int \left(-\frac{1}{2} \sin((1-m)x) + \frac{1}{2} \sin((1+m)x) \right) dx \\ &= -\left(\frac{1}{2} \int \sin((1-m)x) dx \right) + \frac{1}{2} \int \sin((1+m)x) dx \\ &= \frac{\cos((1-m)x)}{2(1-m)} - \frac{\cos((1+m)x)}{2(1+m)} \end{aligned}$$

Mathematica [A] time = 0.05, size = 26, normalized size = 0.74

$$\frac{\sin(x) \sin(mx) + m \cos(x) \cos(mx)}{1 - m^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sin[m*x],x]

[Out] (m*Cos[x]*Cos[m*x] + Sin[x]*Sin[m*x])/(1 - m^2)

fricas [A] time = 0.43, size = 25, normalized size = 0.71

$$-\frac{m \cos(mx) \cos(x) + \sin(mx) \sin(x)}{m^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(m*x),x, algorithm="fricas")

[Out] $-(m*\cos(m*x)*\cos(x) + \sin(m*x)*\sin(x))/(m^2 - 1)$

giac [A] time = 0.14, size = 29, normalized size = 0.83

$$-\frac{\cos(mx+x)}{2(m+1)} - \frac{\cos(mx-x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(m*x),x, algorithm="giac")

[Out] $-1/2*\cos(m*x+x)/(m+1) - 1/2*\cos(m*x-x)/(m-1)$

maple [A] time = 0.02, size = 28, normalized size = 0.80

$$-\frac{\cos((-1+m)x)}{2(-1+m)} - \frac{\cos((1+m)x)}{2(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(m*x),x)

[Out] $-1/2*\cos((-1+m)*x)/(-1+m) - 1/2*\cos((1+m)*x)/(1+m)$

maxima [A] time = 0.32, size = 27, normalized size = 0.77

$$-\frac{\cos((m+1)x)}{2(m+1)} - \frac{\cos((m-1)x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(m*x),x, algorithm="maxima")

[Out] $-1/2*\cos((m+1)*x)/(m+1) - 1/2*\cos((m-1)*x)/(m-1)$

mupad [B] time = 2.29, size = 57, normalized size = 1.63

$$\left\{ \begin{array}{ll} \frac{\sin(x)^2}{2} & \text{if } m = 1 \\ \frac{\cos(x)^2}{2} & \text{if } m = -1 \\ -\frac{\cos(x(m-1))}{2m-2} - \frac{\cos(x(m+1))}{2m+2} & \text{if } m \neq -1 \wedge m \neq 1 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(m*x)*cos(x),x)

[Out] $\text{piecewise}(m == 1, \sin(x)^2/2, m == -1, \cos(x)^2/2, m \neq -1 \ \& \ m \neq 1, -\cos(x*(m-1))/(2*m-2) - \cos(x*(m+1))/(2*m+2))$

sympy [A] time = 0.78, size = 44, normalized size = 1.26

$$\left\{ \begin{array}{ll} -\frac{\sin^2(x)}{2} & \text{for } m = -1 \\ \frac{\sin^2(x)}{2} & \text{for } m = 1 \\ -\frac{m \cos(x) \cos(mx)}{m^2-1} - \frac{\sin(x) \sin(mx)}{m^2-1} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(m*x),x)
```

```
[Out] Piecewise((-sin(x)**2/2, Eq(m, -1)), (sin(x)**2/2, Eq(m, 1)), (-m*cos(x)*cos(m*x)/(m**2 - 1) - sin(x)*sin(m*x)/(m**2 - 1), True))
```

3.101 $\int \cos(x) \cos(2x) dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2} + \frac{1}{6} \sin(3x)$$

[Out] 1/2*sin(x)+1/6*sin(3*x)

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4283}

$$\frac{\sin(x)}{2} + \frac{1}{6} \sin(3x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[2*x],x]

[Out] Sin[x]/2 + Sin[3*x]/6

Rule 4283

Int[cos[(a_.) + (b_.)*(x_.)]*cos[(c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[Sin[a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\int \cos(x) \cos(2x) dx = \frac{\sin(x)}{2} + \frac{1}{6} \sin(3x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin(x)}{2} + \frac{1}{6} \sin(3x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cos[2*x],x]

[Out] Sin[x]/2 + Sin[3*x]/6

fricas [A] time = 1.59, size = 12, normalized size = 0.80

$$\frac{1}{3} (2 \cos(x)^2 + 1) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*x),x, algorithm="fricas")

[Out] 1/3*(2*cos(x)^2 + 1)*sin(x)

giac [A] time = 0.13, size = 11, normalized size = 0.73

$$\frac{1}{6} \sin(3x) + \frac{1}{2} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*x),x, algorithm="giac")

[Out] $1/6*\sin(3*x) + 1/2*\sin(x)$

maple [A] time = 0.07, size = 12, normalized size = 0.80

$$\frac{\sin(x)}{2} + \frac{\sin(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*cos(2*x),x)`

[Out] $1/2*\sin(x)+1/6*\sin(3*x)$

maxima [A] time = 0.30, size = 11, normalized size = 0.73

$$\frac{1}{6} \sin(3x) + \frac{1}{2} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(2*x),x, algorithm="maxima")`

[Out] $1/6*\sin(3*x) + 1/2*\sin(x)$

mupad [B] time = 0.02, size = 9, normalized size = 0.60

$$\sin(x) - \frac{2 \sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(2*x)*cos(x),x)`

[Out] $\sin(x) - (2*\sin(x)^3)/3$

sympy [A] time = 0.45, size = 20, normalized size = 1.33

$$-\frac{\sin(x) \cos(2x)}{3} + \frac{2 \sin(2x) \cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(2*x),x)`

[Out] $-\sin(x)*\cos(2*x)/3 + 2*\sin(2*x)*\cos(x)/3$

3.102 $\int \cos(x) \cos(3x) dx$

Optimal. Leaf size=17

$$\frac{1}{4} \sin(2x) + \frac{1}{8} \sin(4x)$$

[Out] 1/4*sin(2*x)+1/8*sin(4*x)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4283}

$$\frac{1}{4} \sin(2x) + \frac{1}{8} \sin(4x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[3*x],x]

[Out] Sin[2*x]/4 + Sin[4*x]/8

Rule 4283

Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\int \cos(x) \cos(3x) dx = \frac{1}{4} \sin(2x) + \frac{1}{8} \sin(4x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{4} \sin(2x) + \frac{1}{8} \sin(4x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cos[3*x],x]

[Out] Sin[2*x]/4 + Sin[4*x]/8

fricas [A] time = 0.57, size = 7, normalized size = 0.41

$$\cos(x)^3 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(3*x),x, algorithm="fricas")

[Out] cos(x)^3*sin(x)

giac [A] time = 0.13, size = 13, normalized size = 0.76

$$\frac{1}{8} \sin(4x) + \frac{1}{4} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(3*x),x, algorithm="giac")

[Out] $1/8*\sin(4*x) + 1/4*\sin(2*x)$

maple [A] time = 0.07, size = 14, normalized size = 0.82

$$\frac{\sin(2x)}{4} + \frac{\sin(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*cos(3*x),x)`

[Out] $1/4*\sin(2*x)+1/8*\sin(4*x)$

maxima [A] time = 0.32, size = 13, normalized size = 0.76

$$\frac{1}{8} \sin(4x) + \frac{1}{4} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(3*x),x, algorithm="maxima")`

[Out] $1/8*\sin(4*x) + 1/4*\sin(2*x)$

mupad [B] time = 0.02, size = 7, normalized size = 0.41

$$\cos(x)^3 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(3*x)*cos(x),x)`

[Out] $\cos(x)^3*\sin(x)$

sympy [A] time = 0.41, size = 20, normalized size = 1.18

$$-\frac{\sin(x) \cos(3x)}{8} + \frac{3 \sin(3x) \cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(3*x),x)`

[Out] $-\sin(x)*\cos(3*x)/8 + 3*\sin(3*x)*\cos(x)/8$

3.103 $\int \cos(x) \cos(4x) dx$

Optimal. Leaf size=17

$$\frac{1}{6} \sin(3x) + \frac{1}{10} \sin(5x)$$

[Out] 1/6*sin(3*x)+1/10*sin(5*x)

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4283}

$$\frac{1}{6} \sin(3x) + \frac{1}{10} \sin(5x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[4*x],x]

[Out] Sin[3*x]/6 + Sin[5*x]/10

Rule 4283

Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\int \cos(x) \cos(4x) dx = \frac{1}{6} \sin(3x) + \frac{1}{10} \sin(5x)$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6} \sin(3x) + \frac{1}{10} \sin(5x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cos[4*x],x]

[Out] Sin[3*x]/6 + Sin[5*x]/10

fricas [A] time = 1.08, size = 18, normalized size = 1.06

$$\frac{1}{15} (24 \cos(x)^4 - 8 \cos(x)^2 - 1) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(4*x),x, algorithm="fricas")

[Out] 1/15*(24*cos(x)^4 - 8*cos(x)^2 - 1)*sin(x)

giac [A] time = 0.14, size = 13, normalized size = 0.76

$$\frac{1}{10} \sin(5x) + \frac{1}{6} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(4*x),x, algorithm="giac")

[Out] 1/10*sin(5*x) + 1/6*sin(3*x)

maple [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{\sin(3x)}{6} + \frac{\sin(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*cos(4*x),x)

[Out] 1/6*sin(3*x)+1/10*sin(5*x)

maxima [A] time = 0.31, size = 13, normalized size = 0.76

$$\frac{1}{10} \sin(5x) + \frac{1}{6} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(4*x),x, algorithm="maxima")

[Out] 1/10*sin(5*x) + 1/6*sin(3*x)

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{\sin(3x)}{6} + \frac{\sin(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(4*x)*cos(x),x)

[Out] sin(3*x)/6 + sin(5*x)/10

sympy [A] time = 0.41, size = 20, normalized size = 1.18

$$-\frac{\sin(x) \cos(4x)}{15} + \frac{4 \sin(4x) \cos(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(4*x),x)

[Out] -sin(x)*cos(4*x)/15 + 4*sin(4*x)*cos(x)/15

3.104 $\int \cos(x) \cos(mx) dx$

Optimal. Leaf size=35

$$\frac{\sin((1-m)x)}{2(1-m)} + \frac{\sin((m+1)x)}{2(m+1)}$$

[Out] 1/2*sin((1-m)*x)/(1-m)+1/2*sin((1+m)*x)/(1+m)

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4570, 2637}

$$\frac{\sin((1-m)x)}{2(1-m)} + \frac{\sin((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[m*x],x]

[Out] Sin[(1-m)*x]/(2*(1-m)) + Sin[(1+m)*x]/(2*(1+m))

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4570

Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p *Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \cos(x) \cos(mx) dx &= \int \left(\frac{1}{2} \cos((1-m)x) + \frac{1}{2} \cos((1+m)x) \right) dx \\ &= \frac{1}{2} \int \cos((1-m)x) dx + \frac{1}{2} \int \cos((1+m)x) dx \\ &= \frac{\sin((1-m)x)}{2(1-m)} + \frac{\sin((1+m)x)}{2(1+m)} \end{aligned}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 0.71

$$\frac{m \cos(x) \sin(mx) - \sin(x) \cos(mx)}{m^2 - 1}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cos[m*x],x]

[Out] (-(Cos[m*x]*Sin[x]) + m*Cos[x]*Sin[m*x])/(-1 + m^2)

fricas [A] time = 2.47, size = 25, normalized size = 0.71

$$\frac{m \cos(x) \sin(mx) - \cos(mx) \sin(x)}{m^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(m*x),x, algorithm="fricas")

[Out] (m*cos(x)*sin(m*x) - cos(m*x)*sin(x))/(m^2 - 1)

giac [A] time = 0.14, size = 29, normalized size = 0.83

$$\frac{\sin(mx+x)}{2(m+1)} + \frac{\sin(mx-x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(m*x),x, algorithm="giac")

[Out] 1/2*sin(m*x + x)/(m + 1) + 1/2*sin(m*x - x)/(m - 1)

maple [A] time = 0.03, size = 28, normalized size = 0.80

$$\frac{\sin((-1+m)x)}{-2+2m} + \frac{\sin((1+m)x)}{2+2m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*cos(m*x),x)

[Out] 1/2/(-1+m)*sin((-1+m)*x)+1/2*sin((1+m)*x)/(1+m)

maxima [A] time = 0.32, size = 28, normalized size = 0.80

$$\frac{\sin((m+1)x)}{2(m+1)} - \frac{\sin(-(m-1)x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(m*x),x, algorithm="maxima")

[Out] 1/2*sin((m + 1)*x)/(m + 1) - 1/2*sin(-(m - 1)*x)/(m - 1)

mupad [B] time = 0.12, size = 39, normalized size = 1.11

$$\begin{cases} \frac{x}{2} + \frac{\sin(2x)}{4} & \text{if } m = -1 \vee m = 1 \\ \frac{\sin(x(m-1))}{2m-2} + \frac{\sin(x(m+1))}{2m+2} & \text{if } m \neq -1 \wedge m \neq 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(m*x)*cos(x),x)

[Out] piecewise(m == -1 | m == 1, x/2 + sin(2*x)/4, m ~= -1 & m ~= 1, sin(x*(m - 1))/(2*m - 2) + sin(x*(m + 1))/(2*m + 2))

sympy [A] time = 0.94, size = 56, normalized size = 1.60

$$\begin{cases} \frac{x \sin^2(x)}{2} + \frac{x \cos^2(x)}{2} + \frac{\sin(x) \cos(x)}{2} & \text{for } m = -1 \vee m = 1 \\ \frac{m \sin(mx) \cos(x)}{m^2-1} - \frac{\sin(x) \cos(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(m*x),x)

[Out] Piecewise((x*sin(x)**2/2 + x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, -1) | Eq(m, 1)), (m*sin(m*x)*cos(x)/(m**2 - 1) - sin(x)*cos(m*x)/(m**2 - 1), True))

3.105 $\int \cos(x) \tan(2x) dx$

Optimal. Leaf size=20

$$\frac{\tanh^{-1}(\sqrt{2} \cos(x))}{\sqrt{2}} - \cos(x)$$

[Out] $-\cos(x) + 1/2 \cdot \operatorname{arctanh}(\cos(x) \cdot 2^{(1/2)}) \cdot 2^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {12, 321, 207}

$$\frac{\tanh^{-1}(\sqrt{2} \cos(x))}{\sqrt{2}} - \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[x] \cdot \text{Tan}[2 \cdot x], x]$

[Out] $\text{ArcTanh}[\text{Sqrt}[2] \cdot \text{Cos}[x]] / \text{Sqrt}[2] - \text{Cos}[x]$

Rule 12

$\text{Int}[(a_)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$ FreeQ[a, x] && !MatchQ[u, (b_)(v_)] /; FreeQ[b, x]

Rule 207

$\text{Int}[(a_ + (b_)(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[(\text{Rt}[b, 2] \cdot x) / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] \cdot \text{Rt}[b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 321

$\text{Int}[(c_)(x_)^m((a_ + (b_)(x_)^n)^p), x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}(c \cdot x)^{m-n+1}(a + b \cdot x^n)^{p+1}) / (b \cdot (m + n \cdot p + 1)), x] - \text{Dist}[(a \cdot c^{(n-1)}(m - n + 1)) / (b \cdot (m + n \cdot p + 1)), \text{Int}[(c \cdot x)^{m-n}(a + b \cdot x^n)^p, x], x] /;$ FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n \cdot p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned} \int \cos(x) \tan(2x) dx &= -\text{Subst} \left(\int \frac{2x^2}{-1 + 2x^2} dx, x, \cos(x) \right) \\ &= - \left(2 \text{Subst} \left(\int \frac{x^2}{-1 + 2x^2} dx, x, \cos(x) \right) \right) \\ &= -\cos(x) - \text{Subst} \left(\int \frac{1}{-1 + 2x^2} dx, x, \cos(x) \right) \\ &= \frac{\tanh^{-1}(\sqrt{2} \cos(x))}{\sqrt{2}} - \cos(x) \end{aligned}$$

Mathematica [C] time = 0.23, size = 183, normalized size = 9.15

$$\frac{-4\sqrt{2} \cos(x) + 4 \tanh^{-1} \left(\tan \left(\frac{x}{2} \right) + \sqrt{2} \right) - \log \left(-\sqrt{2} \sin(x) - \sqrt{2} \cos(x) + 2 \right) + \log \left(-\sqrt{2} \sin(x) + \sqrt{2} \cos(x) \right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Tan[2*x],x]

[Out] ((2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*Cos[x/2] - Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((-1 + Sqrt[2])*Cos[x/2] - Sin[x/2])] + 4*ArcTanh[Sqrt[2] + Tan[x/2]] - 4*Sqrt[2]*Cos[x] - Log[2 - Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]] + Log[2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/(4*Sqrt[2])

fricas [B] time = 2.03, size = 38, normalized size = 1.90

$$\frac{1}{4} \sqrt{2} \log\left(-\frac{2 \cos(x)^2 + 2 \sqrt{2} \cos(x) + 1}{2 \cos(x)^2 - 1}\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(2*x),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) - cos(x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \tan(2x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(2*x),x, algorithm="giac")

[Out] integrate(cos(x)*tan(2*x), x)

maple [A] time = 0.04, size = 18, normalized size = 0.90

$$-\cos(x) + \frac{\operatorname{arctanh}(\cos(x)\sqrt{2})\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*tan(2*x),x)

[Out] -cos(x)+1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)

maxima [B] time = 0.43, size = 133, normalized size = 6.65

$$\frac{1}{8} \sqrt{2} \log\left(2 \sqrt{2} \sin(2x) \sin(x) + 2\left(\sqrt{2} \cos(x) + 1\right) \cos(2x) + \cos(2x)^2 + 2 \cos(x)^2 + \sin(2x)^2 + 2 \sin(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(2*x),x, algorithm="maxima")

[Out] 1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 1) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 1) - cos(x)

mupad [B] time = 2.36, size = 42, normalized size = 2.10

$$-\frac{\sqrt{2} \operatorname{atanh}\left(\frac{8 \sqrt{2} \tan\left(\frac{x}{2}\right)^2}{12 \tan\left(\frac{x}{2}\right)^2 - 4}\right)}{2} - \frac{2}{\tan\left(\frac{x}{2}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(2*x)*cos(x),x)`

[Out] $-\frac{\sqrt{2} \operatorname{atanh}\left(\frac{8\sqrt{2}\tan^2\left(\frac{x}{2}\right)}{12\tan^2\left(\frac{x}{2}\right)-4}\right)}{2} - \frac{2}{\tan^2\left(\frac{x}{2}\right)+1}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \tan(2x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*tan(2*x),x)`

[Out] `Integral(cos(x)*tan(2*x), x)`

3.106 $\int \cos(x) \tan(3x) dx$

Optimal. Leaf size=21

$$\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}} - \cos(x)$$

[Out] $-\cos(x) + 1/3 \cdot \operatorname{arctanh}(2/3 \cdot \cos(x) \cdot 3^{(1/2)}) \cdot 3^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {388, 206}

$$\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}} - \cos(x)$$

Antiderivative was successfully verified.

[In] `Int[Cos[x]*Tan[3*x],x]`

[Out] `ArcTanh[(2*Cos[x])/Sqrt[3]]/Sqrt[3] - Cos[x]`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 388

`Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]`

Rubi steps

$$\begin{aligned} \int \cos(x) \tan(3x) dx &= -\operatorname{Subst}\left(\int \frac{1-4x^2}{3-4x^2} dx, x, \cos(x)\right) \\ &= -\cos(x) + 2 \operatorname{Subst}\left(\int \frac{1}{3-4x^2} dx, x, \cos(x)\right) \\ &= \frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}} - \cos(x) \end{aligned}$$

Mathematica [B] time = 0.05, size = 48, normalized size = 2.29

$$-\cos(x) - \frac{\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-2}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+2}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] `Integrate[Cos[x]*Tan[3*x],x]`

[Out] `-(ArcTanh[(-2 + Tan[x/2])/Sqrt[3]]/Sqrt[3]) + ArcTanh[(2 + Tan[x/2])/Sqrt[3]]/Sqrt[3] - Cos[x]`

fricas [B] time = 1.51, size = 38, normalized size = 1.81

$$\frac{1}{6} \sqrt{3} \log\left(-\frac{4 \cos(x)^2 + 4 \sqrt{3} \cos(x) + 3}{4 \cos(x)^2 - 3}\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(3*x),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) - cos(x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \tan(3x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(3*x),x, algorithm="giac")

[Out] integrate(cos(x)*tan(3*x), x)

maple [A] time = 0.05, size = 19, normalized size = 0.90

$$-\cos(x) + \frac{\operatorname{arctanh}\left(\frac{2\cos(x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*tan(3*x),x)

[Out] -cos(x)+1/3*arctanh(2/3*cos(x)*3^(1/2))*3^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\cos(x) - \int \frac{(\sin(3x) - \sin(x)) \cos(4x) - (\cos(3x) - \cos(x)) \sin(4x) - (\cos(2x) - 1) \sin(3x) + \cos(3x) \sin(4x)}{2(\cos(2x) - 1) \cos(4x) - \cos(4x)^2 - \cos(2x)^2 - \sin(4x)^2 + 2 \sin(4x) \sin(2x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(3*x),x, algorithm="maxima")

[Out] -cos(x) - integrate(((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*sin(4*x) - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + cos(2*x)*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x)

mupad [B] time = 2.31, size = 42, normalized size = 2.00

$$\frac{\sqrt{3} \operatorname{atanh}\left(\frac{32 \sqrt{3} \tan\left(\frac{x}{2}\right)^2}{3 \left(\frac{56 \tan\left(\frac{x}{2}\right)^2}{3} - \frac{8}{3}\right)}\right)}{3} - \frac{2}{\tan\left(\frac{x}{2}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(3*x)*cos(x),x)

[Out] - (3^(1/2)*atanh((32*3^(1/2)*tan(x/2)²)/(3*((56*tan(x/2)²)/3 - 8/3)))/3
 - 2/(tan(x/2)² + 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \tan(3x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(3*x), x)

[Out] Integral(cos(x)*tan(3*x), x)

3.107 $\int \cos(x) \tan(4x) dx$

Optimal. Leaf size=71

$$-\cos(x) + \frac{1}{4}\sqrt{2-\sqrt{2}} \tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{4}\sqrt{2+\sqrt{2}} \tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)$$

[Out] $-\cos(x) + 1/4 \cdot \operatorname{arctanh}(2 \cdot \cos(x) / (2 - 2^{1/2}))^{1/2} \cdot (2 - 2^{1/2})^{1/2} + 1/4 \cdot \operatorname{arctanh}(2 \cdot \cos(x) / (2 + 2^{1/2}))^{1/2} \cdot (2 + 2^{1/2})^{1/2}$

Rubi [A] time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {12, 1279, 1166, 207}

$$-\cos(x) + \frac{1}{4}\sqrt{2-\sqrt{2}} \tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{4}\sqrt{2+\sqrt{2}} \tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Tan[4*x], x]

[Out] $(\sqrt{2 - \sqrt{2}} \cdot \operatorname{ArcTanh}[(2 \cdot \cos(x)) / \sqrt{2 - \sqrt{2}}]) / 4 + (\sqrt{2 + \sqrt{2}} \cdot \operatorname{ArcTanh}[(2 \cdot \cos(x)) / \sqrt{2 + \sqrt{2}}]) / 4 - \cos(x)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1279

Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e*f*(f*x)^(m-1)*(a + b*x^2 + c*x^4)^(p+1))/(c*(m+4*p+3)), x] - Dist[f^2/(c*(m+4*p+3)), Int[(f*x)^(m-2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m-1) + (b*e*(m+2*p+1) - c*d*(m+4*p+3))*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 1] && NeQ[m+4*p+3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rubi steps

$$\begin{aligned}
\int \cos(x) \tan(4x) dx &= -\text{Subst} \left(\int \frac{4x^2(-1+2x^2)}{1-8x^2+8x^4} dx, x, \cos(x) \right) \\
&= - \left(4 \text{Subst} \left(\int \frac{x^2(-1+2x^2)}{1-8x^2+8x^4} dx, x, \cos(x) \right) \right) \\
&= -\cos(x) + \frac{1}{2} \text{Subst} \left(\int \frac{2-8x^2}{1-8x^2+8x^4} dx, x, \cos(x) \right) \\
&= -\cos(x) + (-2 + \sqrt{2}) \text{Subst} \left(\int \frac{1}{-4 + 2\sqrt{2} + 8x^2} dx, x, \cos(x) \right) - (2 + \sqrt{2}) \text{Subst} \left(\int \frac{1}{-4 - 2\sqrt{2} + 8x^2} dx, x, \cos(x) \right) \\
&= \frac{1}{4} \sqrt{2 - \sqrt{2}} \tanh^{-1} \left(\frac{2 \cos(x)}{\sqrt{2 - \sqrt{2}}} \right) + \frac{1}{4} \sqrt{2 + \sqrt{2}} \tanh^{-1} \left(\frac{2 \cos(x)}{\sqrt{2 + \sqrt{2}}} \right) - \cos(x)
\end{aligned}$$

Mathematica [C] time = 59.55, size = 6196, normalized size = 87.27

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[x]*Tan[4*x],x]

[Out] Result too large to show

fricas [A] time = 0.94, size = 101, normalized size = 1.42

$$\frac{1}{8} \sqrt{\sqrt{2} + 2} \log \left(\sqrt{\sqrt{2} + 2} + 2 \cos(x) \right) - \frac{1}{8} \sqrt{\sqrt{2} + 2} \log \left(\sqrt{\sqrt{2} + 2} - 2 \cos(x) \right) + \frac{1}{8} \sqrt{-\sqrt{2} + 2} \log \left(\sqrt{-\sqrt{2} + 2} + 2 \cos(x) \right) - \frac{1}{8} \sqrt{-\sqrt{2} + 2} \log \left(\sqrt{-\sqrt{2} + 2} - 2 \cos(x) \right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(4*x),x, algorithm="fricas")

[Out] 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) + 2*cos(x)) - 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) - 2*cos(x)) + 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2) + 2*cos(x)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2) - 2*cos(x)) - cos(x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \tan(4x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(4*x),x, algorithm="giac")

[Out] integrate(cos(x)*tan(4*x), x)

maple [A] time = 0.08, size = 68, normalized size = 0.96

$$-\cos(x) + \frac{(\sqrt{2} - 1) \sqrt{2} \operatorname{arctanh} \left(\frac{2 \cos(x)}{\sqrt{2 - \sqrt{2}}} \right)}{4 \sqrt{2 - \sqrt{2}}} + \frac{(1 + \sqrt{2}) \sqrt{2} \operatorname{arctanh} \left(\frac{2 \cos(x)}{\sqrt{2 + \sqrt{2}}} \right)}{4 \sqrt{2 + \sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*tan(4*x),x)

[Out] $-\cos(x)+1/4*(2^{(1/2)}-1)*2^{(1/2)}/(2-2^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2*\cos(x)/(2-2^{(1/2)}))^{(1/2)}+1/4*(1+2^{(1/2)})*2^{(1/2)}/(2+2^{(1/2)})^{(1/2)}*\operatorname{arctanh}(2*\cos(x)/(2+2^{(1/2)}))^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\cos(x) - \int \frac{(\sin(7x) - \sin(x)) \cos(8x) - (\cos(7x) - \cos(x)) \sin(8x) + \sin(7x) - \sin(x)}{\cos(8x)^2 + \sin(8x)^2 + 2 \cos(8x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(4*x),x, algorithm="maxima")

[Out] $-\cos(x) - \operatorname{integrate}(-((\sin(7*x) - \sin(x))*\cos(8*x) - (\cos(7*x) - \cos(x))*\sin(8*x) + \sin(7*x) - \sin(x))/(\cos(8*x)^2 + \sin(8*x)^2 + 2*\cos(8*x) + 1), x)$

mupad [B] time = 2.45, size = 295, normalized size = 4.15

$$\operatorname{atanh}\left(\frac{219747975168 \tan\left(\frac{x}{2}\right)^2 \sqrt{2-\sqrt{2}}}{6098518016 \sqrt{2}-254015438848 \sqrt{2} \tan\left(\frac{x}{2}\right)^2+386664497152 \tan\left(\frac{x}{2}\right)^2-20887633920} - \frac{15971909632}{6098518016 \sqrt{2}-254015438848 \sqrt{2} \tan\left(\frac{x}{2}\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(4*x)*cos(x),x)

[Out] $-(\operatorname{atanh}((219747975168*\tan(x/2)^2*(2-2^{(1/2)})^{(1/2)})/(6098518016*2^{(1/2)}-254015438848*2^{(1/2)}*\tan(x/2)^2+386664497152*\tan(x/2)^2-20887633920)) - (15971909632*(2-2^{(1/2)})^{(1/2)})/(6098518016*2^{(1/2)}-254015438848*2^{(1/2)}*\tan(x/2)^2+386664497152*\tan(x/2)^2-20887633920)) - (130056978432*2^{(1/2)}*\tan(x/2)^2*(2-2^{(1/2)})^{(1/2)})/(6098518016*2^{(1/2)}-254015438848*2^{(1/2)}*\tan(x/2)^2+386664497152*\tan(x/2)^2-20887633920))*(2-2^{(1/2)})^{(1/2)}/4 - 2/(\tan(x/2)^2+1) - (\operatorname{atanh}((15971909632*(2^{(1/2)}+2)^{(1/2)})/(6098518016*2^{(1/2)}-254015438848*2^{(1/2)}*\tan(x/2)^2-386664497152*\tan(x/2)^2+20887633920)) - (219747975168*\tan(x/2)^2*(2^{(1/2)}+2)^{(1/2)})/(6098518016*2^{(1/2)}-254015438848*2^{(1/2)}*\tan(x/2)^2-386664497152*\tan(x/2)^2+20887633920)) - (130056978432*2^{(1/2)}*\tan(x/2)^2*(2^{(1/2)}+2)^{(1/2)})/(6098518016*2^{(1/2)}-254015438848*2^{(1/2)}*\tan(x/2)^2-386664497152*\tan(x/2)^2+20887633920))*(2^{(1/2)}+2)^{(1/2)}/4$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \tan(4x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(4*x),x)

[Out] Integral(cos(x)*tan(4*x), x)

3.108 $\int \cos(x) \tan(5x) dx$

Optimal. Leaf size=84

$$-\cos(x) + \frac{1}{5} \sqrt{\frac{1}{2}(5 + \sqrt{5})} \tanh^{-1} \left(2 \sqrt{\frac{2}{5 + \sqrt{5}}} \cos(x) \right) + \frac{1}{5} \sqrt{\frac{1}{2}(5 - \sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{5}(5 + \sqrt{5})} \cos(x) \right)$$

[Out] $-\cos(x) + 1/10 \cdot \operatorname{arctanh}(1/5 \cdot \cos(x) \cdot (50 + 10 \cdot 5^{(1/2)})^{(1/2)}) \cdot (10 - 2 \cdot 5^{(1/2)})^{(1/2)} + 1/10 \cdot \operatorname{arctanh}(2 \cdot \cos(x) \cdot 2^{(1/2)} / (5 + 5^{(1/2)})^{(1/2)}) \cdot (10 + 2 \cdot 5^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1676, 1166, 207}

$$-\cos(x) + \frac{1}{5} \sqrt{\frac{1}{2}(5 + \sqrt{5})} \tanh^{-1} \left(2 \sqrt{\frac{2}{5 + \sqrt{5}}} \cos(x) \right) + \frac{1}{5} \sqrt{\frac{1}{2}(5 - \sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{5}(5 + \sqrt{5})} \cos(x) \right)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Tan[5*x],x]

[Out] $(\operatorname{Sqrt}[(5 + \operatorname{Sqrt}[5])/2] \cdot \operatorname{ArcTanh}[2 \cdot \operatorname{Sqrt}[2/(5 + \operatorname{Sqrt}[5])] \cdot \operatorname{Cos}[x]])/5 + (\operatorname{Sqrt}[(5 - \operatorname{Sqrt}[5])/2] \cdot \operatorname{ArcTanh}[\operatorname{Sqrt}[(2 \cdot (5 + \operatorname{Sqrt}[5]))/5] \cdot \operatorname{Cos}[x]])/5 - \operatorname{Cos}[x]$

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1676

Int[(Pq_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1

Rubi steps

$$\begin{aligned} \int \cos(x) \tan(5x) dx &= -\operatorname{Subst} \left(\int \frac{1 - 12x^2 + 16x^4}{5 - 20x^2 + 16x^4} dx, x, \cos(x) \right) \\ &= -\operatorname{Subst} \left(\int \left(1 - \frac{4(1 - 2x^2)}{5 - 20x^2 + 16x^4} \right) dx, x, \cos(x) \right) \\ &= -\cos(x) + 4 \operatorname{Subst} \left(\int \frac{1 - 2x^2}{5 - 20x^2 + 16x^4} dx, x, \cos(x) \right) \\ &= -\cos(x) - \frac{1}{5} (4(5 - \sqrt{5})) \operatorname{Subst} \left(\int \frac{1}{-10 + 2\sqrt{5} + 16x^2} dx, x, \cos(x) \right) - \frac{1}{5} (4(5 + \sqrt{5})) \operatorname{Subst} \left(\int \frac{1}{-10 - 2\sqrt{5} + 16x^2} dx, x, \cos(x) \right) \\ &= \frac{1}{5} \sqrt{\frac{1}{2}(5 + \sqrt{5})} \tanh^{-1} \left(2 \sqrt{\frac{2}{5 + \sqrt{5}}} \cos(x) \right) + \frac{1}{5} \sqrt{\frac{1}{2}(5 - \sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{5}(5 + \sqrt{5})} \cos(x) \right) \end{aligned}$$

Mathematica [B] time = 0.59, size = 215, normalized size = 2.56

$$-\cos(x) + \frac{(1 + \sqrt{5}) \tanh^{-1}\left(\frac{4 - (\sqrt{5} - 1)\tan\left(\frac{x}{2}\right)}{\sqrt{2(5 + \sqrt{5})}}\right)}{\sqrt{10(5 + \sqrt{5})}} + \frac{(1 + \sqrt{5}) \tanh^{-1}\left(\frac{(\sqrt{5} - 1)\tan\left(\frac{x}{2}\right) + 4}{\sqrt{2(5 + \sqrt{5})}}\right)}{\sqrt{10(5 + \sqrt{5})}} + \frac{(\sqrt{5} - 1) \tanh^{-1}\left(\frac{4 - (1 + \sqrt{5})\tan\left(\frac{x}{2}\right)}{\sqrt{10 - 2\sqrt{5}}}\right)}{\sqrt{50 - 10\sqrt{5}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[x]*Tan[5*x], x]

[Out] ((1 + Sqrt[5])*ArcTanh[(4 - (-1 + Sqrt[5])*Tan[x/2])/Sqrt[2*(5 + Sqrt[5])]])/Sqrt[10*(5 + Sqrt[5])] + ((1 + Sqrt[5])*ArcTanh[(4 + (-1 + Sqrt[5])*Tan[x/2])/Sqrt[2*(5 + Sqrt[5])]])/Sqrt[10*(5 + Sqrt[5])] + ((-1 + Sqrt[5])*ArcTanh[(4 - (1 + Sqrt[5])*Tan[x/2])/Sqrt[10 - 2*Sqrt[5]])]/Sqrt[50 - 10*Sqrt[5]] + ((-1 + Sqrt[5])*ArcTanh[(4 + (1 + Sqrt[5])*Tan[x/2])/Sqrt[10 - 2*Sqrt[5]])]/Sqrt[50 - 10*Sqrt[5]] - Cos[x]

fricas [B] time = 1.20, size = 129, normalized size = 1.54

$$\frac{1}{20} \sqrt{2} \sqrt{\sqrt{5} + 5} \log\left(\sqrt{2} \sqrt{\sqrt{5} + 5} + 4 \cos(x)\right) - \frac{1}{20} \sqrt{2} \sqrt{\sqrt{5} + 5} \log\left(\sqrt{2} \sqrt{\sqrt{5} + 5} - 4 \cos(x)\right) + \frac{1}{20} \sqrt{2} \sqrt{\sqrt{5} - 5} \log\left(\sqrt{2} \sqrt{\sqrt{5} - 5} + 4 \cos(x)\right) - \frac{1}{20} \sqrt{2} \sqrt{\sqrt{5} - 5} \log\left(\sqrt{2} \sqrt{\sqrt{5} - 5} - 4 \cos(x)\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(5*x), x, algorithm="fricas")

[Out] 1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) + 4*cos(x)) - 1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) - 4*cos(x)) + 1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) + 4*cos(x)) - 1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) - 4*cos(x)) - cos(x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \tan(5x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(5*x), x, algorithm="giac")

[Out] integrate(cos(x)*tan(5*x), x)

maple [A] time = 0.07, size = 72, normalized size = 0.86

$$-\cos(x) + \frac{(\sqrt{5} - 1) \sqrt{5} \operatorname{arctanh}\left(\frac{4 \cos(x)}{\sqrt{10 - 2\sqrt{5}}}\right)}{5\sqrt{10 - 2\sqrt{5}}} + \frac{(\sqrt{5} + 1) \sqrt{5} \operatorname{arctanh}\left(\frac{4 \cos(x)}{\sqrt{10 + 2\sqrt{5}}}\right)}{5\sqrt{10 + 2\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*tan(5*x), x)

[Out] -cos(x) + 1/5*(5^(1/2) - 1)*5^(1/2)/(10 - 2*5^(1/2))^(1/2)*arctanh(4*cos(x)/(10 - 2*5^(1/2))^(1/2)) + 1/5*(5^(1/2) + 1)*5^(1/2)/(10 + 2*5^(1/2))^(1/2)*arctanh(4*cos(x)/(10 + 2*5^(1/2))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\cos(x) - \frac{\int (\sin(7x) - \sin(5x) + \sin(3x) - \sin(x)) \cos(8x) + (\sin(6x) - \sin(4x) + \sin(2x)) \cos(7x) + (\sin(x) - \sin(3x) + \sin(5x) - \sin(7x)) \cos(6x) dx}{2(\cos(6x) - \cos(8x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(5*x),x, algorithm="maxima")

[Out] $-\cos(x) - \text{integrate}(((\sin(7x) - \sin(5x) + \sin(3x) - \sin(x))\cos(8x) + (\sin(6x) - \sin(4x) + \sin(2x))\cos(7x) + (\sin(5x) - \sin(3x) + \sin(x))\cos(6x) + (\sin(4x) - \sin(2x))\cos(5x) + (\sin(3x) - \sin(x))\cos(4x) - (\cos(7x) - \cos(5x) + \cos(3x) - \cos(x))\sin(8x) - (\cos(6x) - \cos(4x) + \cos(2x) - 1)\sin(7x) - (\cos(5x) - \cos(3x) + \cos(x))\sin(6x) - (\cos(4x) - \cos(2x) + 1)\sin(5x) - (\cos(3x) - \cos(x))\sin(4x) - (\cos(2x) - 1)\sin(3x) + \cos(3x)\sin(2x) - \cos(x)\sin(2x) + \cos(2x)\sin(x) - \sin(x))/(2(\cos(6x) - \cos(4x) + \cos(2x) - 1)\cos(8x) - \cos(8x)^2 + 2(\cos(4x) - \cos(2x) + 1)\cos(6x) - \cos(6x)^2 + 2(\cos(2x) - 1)\cos(4x) - \cos(4x)^2 - \cos(2x)^2 + 2(\sin(6x) - \sin(4x) + \sin(2x))\sin(8x) - \sin(8x)^2 + 2(\sin(4x) - \sin(2x))\sin(6x) - \sin(6x)^2 - \sin(4x)^2 + 2\sin(4x)\sin(2x) - \sin(2x)^2 + 2\cos(2x) - 1), x)$

mupad [B] time = 2.50, size = 407, normalized size = 4.85

$$\sqrt{2} \operatorname{atanh} \left(\frac{18032420192256 \sqrt{2} \tan\left(\frac{x}{2}\right)^2 \sqrt{\sqrt{5}+5}}{8851927597056 \sqrt{5} - \frac{676375744741376 \sqrt{5} \tan\left(\frac{x}{2}\right)^2}{25} - \frac{333433343574016 \tan\left(\frac{x}{2}\right)^2}{5} + 2398739234816} - \frac{867583393792 \sqrt{5}}{25} - \frac{676375744741376 \sqrt{5} \tan\left(\frac{x}{2}\right)^2}{25} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(5*x)*cos(x),x)

[Out] $(2^{(1/2)} \operatorname{atanh}((18032420192256 \cdot 2^{(1/2)} \cdot \tan(x/2)^2 \cdot (5^{(1/2)} + 5)^{(1/2)}) / ((8851927597056 \cdot 5^{(1/2)}) / 25 - (676375744741376 \cdot 5^{(1/2)} \cdot \tan(x/2)^2) / 25 - (333433343574016 \cdot \tan(x/2)^2) / 5 + 2398739234816) - (867583393792 \cdot 2^{(1/2)} \cdot 5^{(1/2)} \cdot (5^{(1/2)} + 5)^{(1/2)}) / (25 \cdot ((8851927597056 \cdot 5^{(1/2)}) / 25 - (676375744741376 \cdot 5^{(1/2)} \cdot \tan(x/2)^2) / 25 - (333433343574016 \cdot \tan(x/2)^2) / 5 + 2398739234816)) - (3805341024256 \cdot 2^{(1/2)} \cdot (5^{(1/2)} + 5)^{(1/2)}) / (5 \cdot ((8851927597056 \cdot 5^{(1/2)}) / 25 - (676375744741376 \cdot 5^{(1/2)} \cdot \tan(x/2)^2) / 25 - (333433343574016 \cdot \tan(x/2)^2) / 5 + 2398739234816)) + (6886980059136 \cdot 2^{(1/2)} \cdot 5^{(1/2)} \cdot \tan(x/2)^2 \cdot (5^{(1/2)} + 5)^{(1/2)}) / ((8851927597056 \cdot 5^{(1/2)}) / 25 - (676375744741376 \cdot 5^{(1/2)} \cdot \tan(x/2)^2) / 25 - (333433343574016 \cdot \tan(x/2)^2) / 5 + 2398739234816)) \cdot (5^{(1/2)} + 5)^{(1/2)}) / 10 - (2^{(1/2)} \operatorname{atanh}((867583393792 \cdot 2^{(1/2)} \cdot 5^{(1/2)} \cdot (5 - 5^{(1/2)})^{(1/2)}) / (25 \cdot ((8851927597056 \cdot 5^{(1/2)}) / 25 - (676375744741376 \cdot 5^{(1/2)} \cdot \tan(x/2)^2) / 25 + (333433343574016 \cdot \tan(x/2)^2) / 5 - 2398739234816)) - (3805341024256 \cdot 2^{(1/2)} \cdot (5 - 5^{(1/2)})^{(1/2)}) / (5 \cdot ((8851927597056 \cdot 5^{(1/2)}) / 25 - (676375744741376 \cdot 5^{(1/2)} \cdot \tan(x/2)^2) / 25 + (333433343574016 \cdot \tan(x/2)^2) / 5 - 2398739234816)) + (18032420192256 \cdot 2^{(1/2)} \cdot \tan(x/2)^2 \cdot (5 - 5^{(1/2)})^{(1/2)}) / ((8851927597056 \cdot 5^{(1/2)}) / 25 - (676375744741376 \cdot 5^{(1/2)} \cdot \tan(x/2)^2) / 25 + (333433343574016 \cdot \tan(x/2)^2) / 5 - 2398739234816) - (6886980059136 \cdot 2^{(1/2)} \cdot 5^{(1/2)} \cdot \tan(x/2)^2 \cdot (5 - 5^{(1/2)})^{(1/2)}) / ((8851927597056 \cdot 5^{(1/2)}) / 25 - (676375744741376 \cdot 5^{(1/2)} \cdot \tan(x/2)^2) / 25 + (333433343574016 \cdot \tan(x/2)^2) / 5 - 2398739234816)) \cdot (5 - 5^{(1/2)})^{(1/2)}) / 10 - 2 / (\tan(x/2)^2 + 1)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \tan(5x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(5*x),x)

[Out] Integral(cos(x)*tan(5*x), x)

3.109 $\int \cos(x) \tan(6x) dx$

Optimal. Leaf size=89

$$-\cos(x) + \frac{\tanh^{-1}(\sqrt{2} \cos(x))}{3\sqrt{2}} + \frac{1}{6} \sqrt{2-\sqrt{3}} \tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{6} \sqrt{2+\sqrt{3}} \tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)$$

[Out] $-\cos(x) + 1/6 \cdot \operatorname{arctanh}(\cos(x) \cdot 2^{(1/2)}) \cdot 2^{(1/2)} + 1/6 \cdot \operatorname{arctanh}(2 \cdot \cos(x) / (1/2 \cdot 6^{(1/2)} - 1/2 \cdot 2^{(1/2)})) \cdot (1/2 \cdot 6^{(1/2)} - 1/2 \cdot 2^{(1/2)}) + 1/6 \cdot \operatorname{arctanh}(2 \cdot \cos(x) / (1/2 \cdot 6^{(1/2)} + 1/2 \cdot 2^{(1/2)})) \cdot (1/2 \cdot 6^{(1/2)} + 1/2 \cdot 2^{(1/2)})$

Rubi [A] time = 0.24, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {12, 6742, 2073, 207, 1166}

$$-\cos(x) + \frac{\tanh^{-1}(\sqrt{2} \cos(x))}{3\sqrt{2}} + \frac{1}{6} \sqrt{2-\sqrt{3}} \tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{6} \sqrt{2+\sqrt{3}} \tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Tan[6*x], x]

[Out] $\operatorname{ArcTanh}[\operatorname{Sqrt}[2] \cdot \operatorname{Cos}[x]] / (3 \cdot \operatorname{Sqrt}[2]) + (\operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]] \cdot \operatorname{ArcTanh}[(2 \cdot \operatorname{Cos}[x]) / \operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]]) / 6 + (\operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]] \cdot \operatorname{ArcTanh}[(2 \cdot \operatorname{Cos}[x]) / \operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]]]) / 6 - \operatorname{Cos}[x]$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 2073

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] := With[{PP = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \cos(x) \tan(6x) dx &= -\text{Subst} \left(\int \frac{2x^2(-3 + 16x^2 - 16x^4)}{1 - 18x^2 + 48x^4 - 32x^6} dx, x, \cos(x) \right) \\
&= - \left(2 \text{Subst} \left(\int \frac{x^2(-3 + 16x^2 - 16x^4)}{1 - 18x^2 + 48x^4 - 32x^6} dx, x, \cos(x) \right) \right) \\
&= - \left(2 \text{Subst} \left(\int \left(\frac{1}{2} - \frac{1 - 12x^2 + 16x^4}{2(1 - 18x^2 + 48x^4 - 32x^6)} \right) dx, x, \cos(x) \right) \right) \\
&= -\cos(x) + \text{Subst} \left(\int \frac{1 - 12x^2 + 16x^4}{1 - 18x^2 + 48x^4 - 32x^6} dx, x, \cos(x) \right) \\
&= -\cos(x) + \text{Subst} \left(\int \left(-\frac{1}{3(-1 + 2x^2)} - \frac{2(-1 + 8x^2)}{3(1 - 16x^2 + 16x^4)} \right) dx, x, \cos(x) \right) \\
&= -\cos(x) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{-1 + 2x^2} dx, x, \cos(x) \right) - \frac{2}{3} \text{Subst} \left(\int \frac{-1 + 8x^2}{1 - 16x^2 + 16x^4} dx, x, \cos(x) \right) \\
&= \frac{\tanh^{-1}(\sqrt{2} \cos(x))}{3\sqrt{2}} - \cos(x) - \frac{1}{3} (4(2 - \sqrt{3})) \text{Subst} \left(\int \frac{1}{-8 + 4\sqrt{3} + 16x^2} dx, x, \cos(x) \right) \\
&= \frac{\tanh^{-1}(\sqrt{2} \cos(x))}{3\sqrt{2}} + \frac{1}{6} \sqrt{2 - \sqrt{3}} \tanh^{-1} \left(\frac{2 \cos(x)}{\sqrt{2 - \sqrt{3}}} \right) + \frac{1}{6} \sqrt{2 + \sqrt{3}} \tanh^{-1} \left(\frac{2 \cos(x)}{\sqrt{2 + \sqrt{3}}} \right)
\end{aligned}$$

Mathematica [C] time = 9.04, size = 679, normalized size = 7.63

$$-\cos(x) + \left(-\frac{1}{6} - \frac{i}{6} \right) \sqrt[4]{-1} \tan^{-1} \left(\left(\frac{1}{2} + \frac{i}{2} \right) \sqrt[4]{-1} \sec \left(\frac{x}{2} \right) \left(\sin \left(\frac{x}{2} \right) + \cos \left(\frac{x}{2} \right) \right) \right) + \left(\frac{1}{6} + \frac{i}{6} \right) (-1)^{3/4} \tanh^{-1} \left(\left(\frac{1}{2} + \frac{i}{2} \right) \sqrt[4]{-1} \sec \left(\frac{x}{2} \right) \left(\sin \left(\frac{x}{2} \right) + \cos \left(\frac{x}{2} \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[x]*Tan[6*x],x]

[Out] $(-1/6 - I/6)*(-1)^{(1/4)}*ArcTan[(1/2 + I/2)*(-1)^{(1/4)}*Sec[x/2]*(Cos[x/2] + Sin[x/2])] + (1/6 + I/6)*(-1)^{(3/4)}*ArcTanh[(1/2 + I/2)*(-1)^{(3/4)}*Sec[x/2]*(Cos[x/2] - Sin[x/2])] - Cos[x] - ((1 + Sqrt[2])*(x - 2*Sqrt[3]*ArcTanh[(2 + (2 + Sqrt[2])*Tan[x/2])/Sqrt[6]] - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[2] - 2*Cos[x] + 2*Sin[x]))]))/(12*(2 + Sqrt[2])) + (x + 2*Sqrt[3]*ArcTanh[(Sqrt[2] + (-1 + Sqrt[2])*Tan[x/2])/Sqrt[3]] - Log[Sec[x/2]^2] + Log[Sec[x/2]^2*(1 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x])])/(12*Sqrt[2]) - ((2*(-2 + Sqrt[6])*ArcTanh[Sqrt[2] + (Sqrt[2] - Sqrt[3])*Tan[x/2]] + (3*Sqrt[2] - 2*Sqrt[3])*(x - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[3] + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]))]))*(Sqrt[2] - Sqrt[3]*Sin[x])*(-3 + Sqrt[6] - (-2 + Sqrt[6])*Cos[x] + (-2 + Sqrt[6])*Sin[x]))/(12*(-36 + 15*Sqrt[6] + (20 - 8*Sqrt[6])*Cos[x] + (12 - 5*Sqrt[6])*Cos[2*x] - 50*Sin[x] + 20*Sqrt[6]*Sin[x] + 12*Sin[2*x] - 5*Sqrt[6]*Sin[2*x])) + ((-2*(Sqrt[2] + Sqrt[3])*ArcTanh[(2 + (2 + Sqrt[6])*Tan[x/2])/Sqrt[2]] + (3 + Sqrt[6])*(x - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[6] - 2*Cos[x] + 2*Sin[x]))]))*(2 + Sqrt[6]*Sin[x])*(3 + Sqrt[6] - (2 + Sqrt[6])*Cos[x] + (2 + Sqrt[6])*Sin[x]))/(12*(-36 - 15*Sqrt[6] + 4*(5 + 2*Sqrt[6])*Cos[x] + (12 + 5*Sqrt[6])*Cos[2*x] - 50*Sin[x] - 20*Sqrt[6]*Sin[x] + 12*Sin[2*x] + 5*Sqrt[6]*Sin[2*x]))$

fricas [A] time = 0.85, size = 134, normalized size = 1.51

$$\frac{1}{12} \sqrt{\sqrt{3} + 2} \log \left(\sqrt{\sqrt{3} + 2} + 2 \cos(x) \right) - \frac{1}{12} \sqrt{\sqrt{3} + 2} \log \left(\sqrt{\sqrt{3} + 2} - 2 \cos(x) \right) + \frac{1}{12} \sqrt{-\sqrt{3} + 2} \log \left(\sqrt{-\sqrt{3} + 2} + 2 \cos(x) \right) - \frac{1}{12} \sqrt{-\sqrt{3} + 2} \log \left(\sqrt{-\sqrt{3} + 2} - 2 \cos(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(6*x),x, algorithm="fricas")

[Out] 1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2) + 2*cos(x)) - 1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2) - 2*cos(x)) + 1/12*sqrt(-sqrt(3) + 2)*log(sqrt(-sqrt(3) + 2) + 2*cos(x)) - 1/12*sqrt(-sqrt(3) + 2)*log(sqrt(-sqrt(3) + 2) - 2*cos(x)) + 1/12*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) - cos(x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \tan(6x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(6*x),x, algorithm="giac")

[Out] integrate(cos(x)*tan(6*x), x)

maple [A] time = 0.08, size = 104, normalized size = 1.17

$$-\cos(x) + \frac{2(-3 + 2\sqrt{3})\sqrt{3} \operatorname{arctanh}\left(\frac{8\cos(x)}{2\sqrt{6} - 2\sqrt{2}}\right)}{9(2\sqrt{6} - 2\sqrt{2})} + \frac{2(3 + 2\sqrt{3})\sqrt{3} \operatorname{arctanh}\left(\frac{8\cos(x)}{2\sqrt{6} + 2\sqrt{2}}\right)}{9(2\sqrt{6} + 2\sqrt{2})} + \frac{\operatorname{arctanh}(\cos(x)\sqrt{2})}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*tan(6*x),x)

[Out] -cos(x) + 2/9*(-3 + 2*3^(1/2))*3^(1/2)/(2*6^(1/2) - 2*2^(1/2))*arctanh(8*cos(x)/(2*6^(1/2) - 2*2^(1/2))) + 2/9*(3 + 2*3^(1/2))*3^(1/2)/(2*6^(1/2) + 2*2^(1/2))*arctanh(8*cos(x)/(2*6^(1/2) + 2*2^(1/2))) + 1/6*arctanh(cos(x)*2^(1/2))*2^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{24} \sqrt{2} \log\left(2\sqrt{2} \sin(2x) \sin(x) + 2\left(\sqrt{2} \cos(x) + 1\right) \cos(2x) + \cos(2x)^2 + 2 \cos(x)^2 + \sin(2x)^2 + 2 \sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(6*x),x, algorithm="maxima")

[Out] 1/24*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 1) - 1/24*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 1) - cos(x) - integrate(1/3*((2*sin(7*x) + sin(5*x) - sin(3*x) - 2*sin(x))*cos(8*x) + (sin(3*x) + 2*sin(x))*cos(4*x) - (2*cos(7*x) + cos(5*x) - cos(3*x) - 2*cos(x))*sin(8*x) - 2*(cos(4*x) - 1)*sin(7*x) - (cos(4*x) - 1)*sin(5*x) - (cos(3*x) + 2*cos(x))*sin(4*x) + 2*cos(7*x)*sin(4*x) + cos(5*x)*sin(4*x) - sin(3*x) - 2*sin(x))/(2*(cos(4*x) - 1)*cos(8*x) - cos(8*x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(4*x) - sin(4*x)^2 + 2*cos(4*x) - 1), x)

mupad [B] time = 4.07, size = 787, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(6*x)*cos(x),x)

```
[Out] (6^(1/2)*(atan((2^(1/2)*321030945816576i)/(213254896304333030400*tan(x/2)^4
- 129275829262795438080*tan(x/2)^2 + 2176593611144037376) + (6^(1/2)*88840
5273481134080i)/(213254896304333030400*tan(x/2)^4 - 129275829262795438080*t
an(x/2)^2 + 2176593611144037376) - (2^(1/2)*tan(x/2)^2*18711054724802560i)/
(213254896304333030400*tan(x/2)^4 - 129275829262795438080*tan(x/2)^2 + 2176
593611144037376) + (2^(1/2)*tan(x/2)^4*10905601889064960i)/(213254896304333
030400*tan(x/2)^4 - 129275829262795438080*tan(x/2)^2 + 2176593611144037376)
- (6^(1/2)*tan(x/2)^2*52765833462352287744i)/(213254896304333030400*tan(x/
2)^4 - 129275829262795438080*tan(x/2)^2 + 2176593611144037376) + (6^(1/2)*t
an(x/2)^4*87054650497106012160i)/(213254896304333030400*tan(x/2)^4 - 129275
829262795438080*tan(x/2)^2 + 2176593611144037376)) + atan((2^(1/2)*14433255
04589801788190484332544i)/(589232404262260650654553866240*2^(1/2)*6^(1/2) +
119129717169909888440949339586560*tan(x/2)^2 - 343672717269879599464668620
39040*2^(1/2)*6^(1/2)*tan(x/2)^2 - 2087090309450798997834557292544) - (6^(1
/2)*852047139771204346616741888000i)/(589232404262260650654553866240*2^(1/2
)*6^(1/2) + 119129717169909888440949339586560*tan(x/2)^2 - 3436727172698795
9946466862039040*2^(1/2)*6^(1/2)*tan(x/2)^2 - 20870903094507989978345572925
44) - (2^(1/2)*tan(x/2)^2*84182283571305304543568582410240i)/(5892324042622
60650654553866240*2^(1/2)*6^(1/2) + 119129717169909888440949339586560*tan(x
/2)^2 - 34367271726987959946466862039040*2^(1/2)*6^(1/2)*tan(x/2)^2 - 20870
90309450798997834557292544) + (6^(1/2)*tan(x/2)^2*4863450107523648650487342
4060416i)/(589232404262260650654553866240*2^(1/2)*6^(1/2) + 119129717169909
888440949339586560*tan(x/2)^2 - 34367271726987959946466862039040*2^(1/2)*6^
(1/2)*tan(x/2)^2 - 2087090309450798997834557292544))*i)/12 - 2/(tan(x/2)^
2 + 1) - (2^(1/2)*(2*atan((2^(1/2)*2276803846003180334341033033728i)/(18766
876017666378997952094928896*tan(x/2)^2 - 3219886877884552553529320931328) -
(2^(1/2)*tan(x/2)^2*13270185293778646110081740963840i)/(187668760176663789
97952094928896*tan(x/2)^2 - 3219886877884552553529320931328)) - atan((2^(1/
2)*321030945816576i)/(213254896304333030400*tan(x/2)^4 - 129275829262795438
080*tan(x/2)^2 + 2176593611144037376) + (6^(1/2)*888405273481134080i)/(2132
54896304333030400*tan(x/2)^4 - 129275829262795438080*tan(x/2)^2 + 217659361
1144037376) - (2^(1/2)*tan(x/2)^2*18711054724802560i)/(21325489630433303040
0*tan(x/2)^4 - 129275829262795438080*tan(x/2)^2 + 2176593611144037376) + (2
^(1/2)*tan(x/2)^4*10905601889064960i)/(213254896304333030400*tan(x/2)^4 - 1
29275829262795438080*tan(x/2)^2 + 2176593611144037376) - (6^(1/2)*tan(x/2)^
2*52765833462352287744i)/(213254896304333030400*tan(x/2)^4 - 12927582926279
5438080*tan(x/2)^2 + 2176593611144037376) + (6^(1/2)*tan(x/2)^4*87054650497
106012160i)/(213254896304333030400*tan(x/2)^4 - 129275829262795438080*tan(x
/2)^2 + 2176593611144037376)) + atan((2^(1/2)*14433255045898017881904843325
44i)/(589232404262260650654553866240*2^(1/2)*6^(1/2) + 11912971716990988844
0949339586560*tan(x/2)^2 - 34367271726987959946466862039040*2^(1/2)*6^(1/2)
*tan(x/2)^2 - 2087090309450798997834557292544) - (6^(1/2)*85204713977120434
6616741888000i)/(589232404262260650654553866240*2^(1/2)*6^(1/2) + 119129717
169909888440949339586560*tan(x/2)^2 - 34367271726987959946466862039040*2^(1
/2)*6^(1/2)*tan(x/2)^2 - 2087090309450798997834557292544) - (2^(1/2)*tan(x/
2)^2*84182283571305304543568582410240i)/(589232404262260650654553866240*2^(
1/2)*6^(1/2) + 119129717169909888440949339586560*tan(x/2)^2 - 3436727172698
7959946466862039040*2^(1/2)*6^(1/2)*tan(x/2)^2 - 20870903094507989978345572
92544) + (6^(1/2)*tan(x/2)^2*48634501075236486504873424060416i)/(5892324042
62260650654553866240*2^(1/2)*6^(1/2) + 119129717169909888440949339586560*ta
n(x/2)^2 - 34367271726987959946466862039040*2^(1/2)*6^(1/2)*tan(x/2)^2 - 20
87090309450798997834557292544))*i)/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \tan(6x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*tan(6*x),x)

```
[Out] Integral(cos(x)*tan(6*x), x)
```

3.110 $\int \cos(x) \cot(2x) dx$

Optimal. Leaf size=10

$$\cos(x) - \frac{1}{2} \tanh^{-1}(\cos(x))$$

[Out] $-1/2*\operatorname{arctanh}(\cos(x))+\cos(x)$

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {12, 388, 206}

$$\cos(x) - \frac{1}{2} \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cos}[x]*\operatorname{Cot}[2*x], x]$

[Out] $-\operatorname{ArcTanh}[\operatorname{Cos}[x]]/2 + \operatorname{Cos}[x]$

Rule 12

$\operatorname{Int}[(a_)*(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[u, (b_)*(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 206

$\operatorname{Int}[(a_)+(b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 388

$\operatorname{Int}[(a_)+(b_.)*(x_)^{(n_)})^{(p_)*((c_)+(d_.)*(x_)^{(n_)})}, x_Symbol] \rightarrow \operatorname{Simp}[(d*x*(a+b*x^n)^{(p+1)})/(b*(n*(p+1)+1)), x] - \operatorname{Dist}[(a*d-b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), \operatorname{Int}[(a+b*x^n)^p, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \operatorname{NeQ}[b*c-a*d, 0] \ \&\& \ \operatorname{NeQ}[n*(p+1)+1, 0]$

Rubi steps

$$\begin{aligned} \int \cos(x) \cot(2x) dx &= -\operatorname{Subst}\left(\int \frac{-1+2x^2}{2(1-x^2)} dx, x, \cos(x)\right) \\ &= -\left(\frac{1}{2} \operatorname{Subst}\left(\int \frac{-1+2x^2}{1-x^2} dx, x, \cos(x)\right)\right) \\ &= \cos(x) - \frac{1}{2} \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cos(x)\right) \\ &= -\frac{1}{2} \tanh^{-1}(\cos(x)) + \cos(x) \end{aligned}$$

Mathematica [B] time = 0.01, size = 25, normalized size = 2.50

$$\cos(x) + \frac{1}{2} \log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{2} \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cot[2*x],x]

[Out] Cos[x] - Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2

fricas [B] time = 1.30, size = 21, normalized size = 2.10

$$\cos(x) - \frac{1}{4} \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + \frac{1}{4} \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(2*x),x, algorithm="fricas")

[Out] cos(x) - 1/4*log(1/2*cos(x) + 1/2) + 1/4*log(-1/2*cos(x) + 1/2)

giac [B] time = 0.14, size = 19, normalized size = 1.90

$$\cos(x) - \frac{1}{4} \log(\cos(x) + 1) + \frac{1}{4} \log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(2*x),x, algorithm="giac")

[Out] cos(x) - 1/4*log(cos(x) + 1) + 1/4*log(-cos(x) + 1)

maple [A] time = 0.09, size = 14, normalized size = 1.40

$$\cos(x) + \frac{\ln(\csc(x) - \cot(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*cot(2*x),x)

[Out] cos(x)+1/2*ln(csc(x)-cot(x))

maxima [B] time = 0.33, size = 37, normalized size = 3.70

$$\cos(x) - \frac{1}{4} \log(\cos(x)^2 + \sin(x)^2 + 2 \cos(x) + 1) + \frac{1}{4} \log(\cos(x)^2 + \sin(x)^2 - 2 \cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(2*x),x, algorithm="maxima")

[Out] cos(x) - 1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)

mupad [B] time = 2.34, size = 20, normalized size = 2.00

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2} + \frac{2}{\tan\left(\frac{x}{2}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(2*x)*cos(x),x)

[Out] log(tan(x/2))/2 + 2/(tan(x/2)^2 + 1)

sympy [B] time = 0.83, size = 19, normalized size = 1.90

$$\frac{\log(\cos(x) - 1)}{4} - \frac{\log(\cos(x) + 1)}{4} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(2*x),x)

[Out] log(cos(x) - 1)/4 - log(cos(x) + 1)/4 + cos(x)

3.111 $\int \cos(x) \cot(3x) dx$

Optimal. Leaf size=45

$$\cos(x) + \frac{1}{6} \log(1 - 2 \cos(x)) + \frac{1}{6} \log(1 - \cos(x)) - \frac{1}{6} \log(\cos(x) + 1) - \frac{1}{6} \log(2 \cos(x) + 1)$$

[Out] $\cos(x) + 1/6 * \ln(1 - 2 * \cos(x)) + 1/6 * \ln(1 - \cos(x)) - 1/6 * \ln(1 + \cos(x)) - 1/6 * \ln(1 + 2 * \cos(x))$

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1279, 1161, 616, 31}

$$\cos(x) + \frac{1}{6} \log(1 - 2 \cos(x)) + \frac{1}{6} \log(1 - \cos(x)) - \frac{1}{6} \log(\cos(x) + 1) - \frac{1}{6} \log(2 \cos(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cot[3*x],x]

[Out] $\text{Cos}[x] + \text{Log}[1 - 2 * \text{Cos}[x]]/6 + \text{Log}[1 - \text{Cos}[x]]/6 - \text{Log}[1 + \text{Cos}[x]]/6 - \text{Log}[1 + 2 * \text{Cos}[x]]/6$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 1161

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))

Rule 1279

Int[((f_.)*(x_)^m)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p + 3))*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rubi steps

$$\begin{aligned}
\int \cos(x) \cot(3x) dx &= -\text{Subst} \left(\int \frac{x^2(3-4x^2)}{1-5x^2+4x^4} dx, x, \cos(x) \right) \\
&= \cos(x) + \frac{1}{4} \text{Subst} \left(\int \frac{-4+8x^2}{1-5x^2+4x^4} dx, x, \cos(x) \right) \\
&= \cos(x) + \frac{1}{4} \text{Subst} \left(\int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2} dx, x, \cos(x) \right) + \frac{1}{4} \text{Subst} \left(\int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2} dx, x, \cos(x) \right) \\
&= \cos(x) + \frac{1}{6} \text{Subst} \left(\int \frac{1}{-1+x} dx, x, \cos(x) \right) + \frac{1}{6} \text{Subst} \left(\int \frac{1}{-\frac{1}{2}+x} dx, x, \cos(x) \right) - \frac{1}{6} \text{Subst} \left(\int \frac{1}{1+x} dx, x, \cos(x) \right) \\
&= \cos(x) + \frac{1}{6} \log(1-2\cos(x)) + \frac{1}{6} \log(1-\cos(x)) - \frac{1}{6} \log(1+\cos(x)) - \frac{1}{6} \log(1+2\cos(x))
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.04

$$\cos(x) + \frac{1}{3} \log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{3} \log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{1}{6} \log(1-2\cos(x)) - \frac{1}{6} \log(2\cos(x)+1)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cot[3*x],x]

[Out] Cos[x] - Log[Cos[x/2]]/3 + Log[1 - 2*Cos[x]]/6 - Log[1 + 2*Cos[x]]/6 + Log[Sin[x/2]]/3

fricas [A] time = 3.04, size = 39, normalized size = 0.87

$$\cos(x) - \frac{1}{6} \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + \frac{1}{6} \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) + \frac{1}{6} \log(-2\cos(x)+1) - \frac{1}{6} \log(-2\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(3*x),x, algorithm="fricas")

[Out] cos(x) - 1/6*log(1/2*cos(x) + 1/2) + 1/6*log(-1/2*cos(x) + 1/2) + 1/6*log(-2*cos(x) + 1) - 1/6*log(-2*cos(x) - 1)

giac [A] time = 0.13, size = 39, normalized size = 0.87

$$\cos(x) - \frac{1}{6} \log(\cos(x)+1) + \frac{1}{6} \log(-\cos(x)+1) - \frac{1}{6} \log(|2\cos(x)+1|) + \frac{1}{6} \log(|2\cos(x)-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(3*x),x, algorithm="giac")

[Out] cos(x) - 1/6*log(cos(x) + 1) + 1/6*log(-cos(x) + 1) - 1/6*log(abs(2*cos(x) + 1)) + 1/6*log(abs(2*cos(x) - 1))

maple [A] time = 0.32, size = 36, normalized size = 0.80

$$\frac{\ln(2\cos(x)-1)}{6} - \frac{\ln(1+2\cos(x))}{6} + \frac{\ln(-1+\cos(x))}{6} - \frac{\ln(1+\cos(x))}{6} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*cot(3*x),x)

[Out] $\frac{1}{6}\ln(2\cos(x)-1)-\frac{1}{6}\ln(1+2\cos(x))+\frac{1}{6}\ln(-1+\cos(x))-\frac{1}{6}\ln(1+\cos(x))+\cos(x)$

maxima [B] time = 0.42, size = 131, normalized size = 2.91

$\cos(x)-\frac{1}{12}\log\left(2(\cos(x)+1)\cos(2x)+\cos(2x)^2+\cos(x)^2+\sin(2x)^2+2\sin(2x)\sin(x)+\sin(x)^2+2\cos(x)\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cot(3*x),x, algorithm="maxima")`

[Out] $\cos(x) - \frac{1}{12}\log(2(\cos(x) + 1)\cos(2x) + \cos(2x)^2 + \cos(x)^2 + \sin(2x)^2 + 2\sin(2x)\sin(x) + \sin(x)^2 + 2\cos(x) + 1) + \frac{1}{12}\log(-2(\cos(x) - 1)\cos(2x) + \cos(2x)^2 + \cos(x)^2 + \sin(2x)^2 - 2\sin(2x)\sin(x) + \sin(x)^2 - 2\cos(x) + 1) - \frac{1}{6}\log(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1) + \frac{1}{6}\log(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1)$

mupad [B] time = 2.37, size = 39, normalized size = 0.87

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{3} + \frac{\operatorname{atanh}\left(\frac{8}{183\left(\frac{488\tan\left(\frac{x}{2}\right)^2}{243} - \frac{56}{81}\right)} + \frac{121}{122}\right)}{3} + \frac{2}{\tan\left(\frac{x}{2}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(3*x)*cos(x),x)`

[Out] $\log(\tan(x/2))/3 + \operatorname{atanh}(8/(183*((488*\tan(x/2)^2)/243 - 56/81))) + 121/122)/3 + 2/(\tan(x/2)^2 + 1)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \cot(3x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cot(3*x),x)`

[Out] `Integral(cos(x)*cot(3*x), x)`

3.112 $\int \cos(x) \cot(4x) dx$

Optimal. Leaf size=28

$$\cos(x) - \frac{1}{4} \tanh^{-1}(\cos(x)) - \frac{\tanh^{-1}(\sqrt{2} \cos(x))}{2\sqrt{2}}$$

[Out] $-1/4*\operatorname{arctanh}(\cos(x))+\cos(x)-1/4*\operatorname{arctanh}(\cos(x)*2^{(1/2)})*2^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1676, 1166, 207}

$$\cos(x) - \frac{1}{4} \tanh^{-1}(\cos(x)) - \frac{\tanh^{-1}(\sqrt{2} \cos(x))}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cot[4*x],x]

[Out] $-\operatorname{ArcTanh}[\operatorname{Cos}[x]]/4 - \operatorname{ArcTanh}[\operatorname{Sqrt}[2]*\operatorname{Cos}[x]]/(2*\operatorname{Sqrt}[2]) + \operatorname{Cos}[x]$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1676

Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1

Rubi steps

$$\begin{aligned} \int \cos(x) \cot(4x) dx &= -\operatorname{Subst}\left(\int \frac{-1 + 8x^2 - 8x^4}{4 - 12x^2 + 8x^4} dx, x, \cos(x)\right) \\ &= -\operatorname{Subst}\left(\int \left(-1 + \frac{3 - 4x^2}{4 - 12x^2 + 8x^4}\right) dx, x, \cos(x)\right) \\ &= \cos(x) - \operatorname{Subst}\left(\int \frac{3 - 4x^2}{4 - 12x^2 + 8x^4} dx, x, \cos(x)\right) \\ &= \cos(x) + 2\operatorname{Subst}\left(\int \frac{1}{-8 + 8x^2} dx, x, \cos(x)\right) + 2\operatorname{Subst}\left(\int \frac{1}{-4 + 8x^2} dx, x, \cos(x)\right) \\ &= -\frac{1}{4} \tanh^{-1}(\cos(x)) - \frac{\tanh^{-1}(\sqrt{2} \cos(x))}{2\sqrt{2}} + \cos(x) \end{aligned}$$

Mathematica [C] time = 0.07, size = 73, normalized size = 2.61

$$\frac{1}{4} \left(4 \cos(x) + \log \left(\sin \left(\frac{x}{2} \right) \right) - \log \left(\cos \left(\frac{x}{2} \right) \right) + (-1 - i)(-1)^{3/4} \tanh^{-1} \left(\frac{\tan \left(\frac{x}{2} \right) - 1}{\sqrt{2}} \right) - (1 - i)\sqrt[4]{-1} \tanh^{-1} \left(\frac{\tan \left(\frac{x}{2} \right)}{\sqrt{2}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cot[4*x],x]

[Out] $((-1 - I)*(-1)^{(3/4)}*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] - (1 - I)*(-1)^{(1/4)}*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] + 4*Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]])/4$

fricas [B] time = 1.01, size = 53, normalized size = 1.89

$$\frac{1}{8} \sqrt{2} \log \left(\frac{2 \cos(x)^2 - 2 \sqrt{2} \cos(x) + 1}{2 \cos(x)^2 - 1} \right) + \cos(x) - \frac{1}{8} \log \left(\frac{1}{2} \cos(x) + \frac{1}{2} \right) + \frac{1}{8} \log \left(-\frac{1}{2} \cos(x) + \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(4*x),x, algorithm="fricas")

[Out] $1/8*\sqrt{2}*\log((2*\cos(x)^2 - 2*\sqrt{2}*\cos(x) + 1)/(2*\cos(x)^2 - 1)) + \cos(x) - 1/8*\log(1/2*\cos(x) + 1/2) + 1/8*\log(-1/2*\cos(x) + 1/2)$

giac [B] time = 0.14, size = 50, normalized size = 1.79

$$\frac{1}{8} \sqrt{2} \log \left(\frac{|-2 \sqrt{2} + 4 \cos(x)|}{|2 \sqrt{2} + 4 \cos(x)|} \right) + \cos(x) - \frac{1}{8} \log(\cos(x) + 1) + \frac{1}{8} \log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(4*x),x, algorithm="giac")

[Out] $1/8*\sqrt{2}*\log(\text{abs}(-2*\sqrt{2} + 4*\cos(x))/\text{abs}(2*\sqrt{2} + 4*\cos(x))) + \cos(x) - 1/8*\log(\cos(x) + 1) + 1/8*\log(-\cos(x) + 1)$

maple [A] time = 0.41, size = 30, normalized size = 1.07

$$\frac{\ln(-1 + \cos(x))}{8} - \frac{\operatorname{arctanh}(\cos(x)\sqrt{2})\sqrt{2}}{4} - \frac{\ln(1 + \cos(x))}{8} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*cot(4*x),x)

[Out] $1/8*\ln(-1+\cos(x))-1/4*\operatorname{arctanh}(\cos(x)*2^{(1/2)})*2^{(1/2)}-1/8*\ln(1+\cos(x))+\cos(x)$

maxima [B] time = 0.42, size = 165, normalized size = 5.89

$$-\frac{1}{16} \sqrt{2} \log \left(2 \sqrt{2} \sin(2x) \sin(x) + 2 \left(\sqrt{2} \cos(x) + 1 \right) \cos(2x) + \cos(2x)^2 + 2 \cos(x)^2 + \sin(2x)^2 + 2 \sin(x)^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(4*x),x, algorithm="maxima")

[Out] $-1/16*\sqrt{2}*\log(2*\sqrt{2}*\sin(2*x)*\sin(x) + 2*(\sqrt{2}*\cos(x) + 1)*\cos(2*x) + \cos(2*x)^2 + 2*\cos(x)^2 + \sin(2*x)^2 + 2*\sin(x)^2 + 2*\sqrt{2}*\cos(x) + 1) + 1/16*\sqrt{2}*\log(-2*\sqrt{2}*\sin(2*x)*\sin(x) - 2*(\sqrt{2}*\cos(x) - 1)*\cos(2*x) + \cos(2*x)^2 + 2*\cos(x)^2 + \sin(2*x)^2 + 2*\sin(x)^2 - 2*\sqrt{2}*\cos(x) - 1)$

$s(x) + 1) + \cos(x) - 1/8 \cdot \log(\cos(x)^2 + \sin(x)^2 + 2 \cdot \cos(x) + 1) + 1/8 \cdot \log(\cos(x)^2 + \sin(x)^2 - 2 \cdot \cos(x) + 1)$

mupad [B] time = 2.35, size = 67, normalized size = 2.39

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4} - \frac{\sqrt{2} \operatorname{atanh}\left(\frac{7\sqrt{2}}{8\left(\frac{29\tan\left(\frac{x}{2}\right)^2}{4} - \frac{5}{4}\right)} - \frac{41\sqrt{2}\tan\left(\frac{x}{2}\right)^2}{8\left(\frac{29\tan\left(\frac{x}{2}\right)^2}{4} - \frac{5}{4}\right)}\right)}{4} + \frac{2}{\tan\left(\frac{x}{2}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(4*x)*cos(x),x)`

[Out] $\log(\tan(x/2))/4 - (2^{(1/2)} \cdot \operatorname{atanh}((7 \cdot 2^{(1/2)}) / (8 \cdot ((29 \cdot \tan(x/2)^2) / 4 - 5/4))) - (41 \cdot 2^{(1/2)} \cdot \tan(x/2)^2) / (8 \cdot ((29 \cdot \tan(x/2)^2) / 4 - 5/4))) / 4 + 2 / (\tan(x/2)^2 + 1)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \cot(4x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cot(4*x),x)`

[Out] `Integral(cos(x)*cot(4*x), x)`

3.113 $\int \cos(x) \cot(5x) dx$

Optimal. Leaf size=110

$$\cos(x) + \frac{1}{20}(1 - \sqrt{5}) \log(-4 \cos(x) - \sqrt{5} + 1) + \frac{1}{20}(1 + \sqrt{5}) \log(-4 \cos(x) + \sqrt{5} + 1) - \frac{1}{20}(1 - \sqrt{5}) \log(4 \cos(x) + \sqrt{5} - 1) - \frac{1}{20}(1 + \sqrt{5}) \log(4 \cos(x) - \sqrt{5} - 1)$$

[Out] $-1/5 \operatorname{arctanh}(\cos(x)) + \cos(x) + 1/20 \ln(1 - 4 \cos(x) - 5^{1/2}) * (-5^{1/2} + 1) - 1/20 \ln(1 + 4 \cos(x) - 5^{1/2}) * (-5^{1/2} + 1) + 1/20 \ln(1 - 4 \cos(x) + 5^{1/2}) * (5^{1/2} + 1) - 1/20 \ln(1 + 4 \cos(x) + 5^{1/2}) * (5^{1/2} + 1)$

Rubi [A] time = 0.16, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2075, 207, 632, 31}

$$\cos(x) + \frac{1}{20}(1 - \sqrt{5}) \log(-4 \cos(x) - \sqrt{5} + 1) + \frac{1}{20}(1 + \sqrt{5}) \log(-4 \cos(x) + \sqrt{5} + 1) - \frac{1}{20}(1 - \sqrt{5}) \log(4 \cos(x) + \sqrt{5} - 1) - \frac{1}{20}(1 + \sqrt{5}) \log(4 \cos(x) - \sqrt{5} - 1)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cot[5*x],x]

[Out] $-\operatorname{ArcTanh}[\cos(x)]/5 + \cos(x) + ((1 - \sqrt{5}) \log[1 - \sqrt{5} - 4 \cos(x)])/20 + ((1 + \sqrt{5}) \log[1 + \sqrt{5} - 4 \cos(x)])/20 - ((1 - \sqrt{5}) \log[1 - \sqrt{5} + 4 \cos(x)])/20 - ((1 + \sqrt{5}) \log[1 + \sqrt{5} + 4 \cos(x)])/20$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 2075

Int[(P_)^(p_)*(Qm_), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x] /; PolyQ[Qm, x] && PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \cos(x) \cot(5x) dx &= -\text{Subst} \left(\int \frac{x^2(5 - 20x^2 + 16x^4)}{1 - 13x^2 + 28x^4 - 16x^6} dx, x, \cos(x) \right) \\
&= -\text{Subst} \left(\int \left(-1 - \frac{1}{5(-1 + x^2)} - \frac{2(1 + x)}{5(-1 - 2x + 4x^2)} + \frac{2(-1 + x)}{5(-1 + 2x + 4x^2)} \right) dx, x, \cos(x) \right) \\
&= \cos(x) + \frac{1}{5} \text{Subst} \left(\int \frac{1}{-1 + x^2} dx, x, \cos(x) \right) + \frac{2}{5} \text{Subst} \left(\int \frac{1 + x}{-1 - 2x + 4x^2} dx, x, \cos(x) \right) \\
&= -\frac{1}{5} \tanh^{-1}(\cos(x)) + \cos(x) - \frac{1}{5} (1 - \sqrt{5}) \text{Subst} \left(\int \frac{1}{1 - \sqrt{5} + 4x} dx, x, \cos(x) \right) + \frac{1}{5} (1 + \sqrt{5}) \text{Subst} \left(\int \frac{1}{1 + \sqrt{5} + 4x} dx, x, \cos(x) \right) \\
&= -\frac{1}{5} \tanh^{-1}(\cos(x)) + \cos(x) + \frac{1}{20} (1 - \sqrt{5}) \log(1 - \sqrt{5} - 4\cos(x)) + \frac{1}{20} (1 + \sqrt{5}) \log(1 + \sqrt{5} + 4\cos(x))
\end{aligned}$$

Mathematica [A] time = 0.12, size = 133, normalized size = 1.21

$$\frac{1}{100} \left(100 \cos(x) + 20 \log \left(\sin \left(\frac{x}{2} \right) \right) - 20 \log \left(\cos \left(\frac{x}{2} \right) \right) + \sqrt{5} (\sqrt{5} - 5) \log(-4 \cos(x) - \sqrt{5} + 1) + \sqrt{5} (5 + \sqrt{5}) \log(-4 \cos(x) + \sqrt{5} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cot[5*x],x]

[Out] (100*Cos[x] - 20*Log[Cos[x/2]] + Sqrt[5]*(-5 + Sqrt[5])*Log[1 - Sqrt[5] - 4*Cos[x]] + Sqrt[5]*(5 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cos[x]] - Sqrt[5]*(-5 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cos[x]] - Sqrt[5]*(5 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Cos[x]] + 20*Log[Sin[x/2]])/100

fricas [A] time = 0.62, size = 137, normalized size = 1.25

$$\frac{1}{20} \sqrt{5} \log \left(-\frac{4(\sqrt{5} - 1)\cos(x) - 8\cos(x)^2 + \sqrt{5} - 3}{4\cos(x)^2 + 2\cos(x) - 1} \right) + \frac{1}{20} \sqrt{5} \log \left(-\frac{4(\sqrt{5} + 1)\cos(x) - 8\cos(x)^2 - \sqrt{5} - 3}{4\cos(x)^2 - 2\cos(x) - 1} \right) + \cos(x) - \frac{1}{10} \log(\cos(x) + 1) + \frac{1}{10} \log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(5*x),x, algorithm="fricas")

[Out] 1/20*sqrt(5)*log(-(4*(sqrt(5) - 1)*cos(x) - 8*cos(x)^2 + sqrt(5) - 3)/(4*cos(x)^2 + 2*cos(x) - 1)) + 1/20*sqrt(5)*log(-(4*(sqrt(5) + 1)*cos(x) - 8*cos(x)^2 - sqrt(5) - 3)/(4*cos(x)^2 - 2*cos(x) - 1)) + cos(x) - 1/20*log(4*cos(x)^2 + 2*cos(x) - 1) + 1/20*log(4*cos(x)^2 - 2*cos(x) - 1) - 1/10*log(1/2*cos(x) + 1/2) + 1/10*log(-1/2*cos(x) + 1/2)

giac [A] time = 0.16, size = 117, normalized size = 1.06

$$\frac{1}{20} \sqrt{5} \log \left(\frac{|-2\sqrt{5} + 8\cos(x) + 2|}{|2\sqrt{5} + 8\cos(x) + 2|} \right) + \frac{1}{20} \sqrt{5} \log \left(\frac{|-2\sqrt{5} + 8\cos(x) - 2|}{|2\sqrt{5} + 8\cos(x) - 2|} \right) + \cos(x) - \frac{1}{10} \log(\cos(x) + 1) + \frac{1}{10} \log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(5*x),x, algorithm="giac")

[Out] 1/20*sqrt(5)*log(abs(-2*sqrt(5) + 8*cos(x) + 2)/abs(2*sqrt(5) + 8*cos(x) + 2)) + 1/20*sqrt(5)*log(abs(-2*sqrt(5) + 8*cos(x) - 2)/abs(2*sqrt(5) + 8*cos(x) - 2)) + cos(x) - 1/10*log(cos(x) + 1) + 1/10*log(-cos(x) + 1) - 1/20*log(abs(4*cos(x)^2 + 2*cos(x) - 1)) + 1/20*log(abs(4*cos(x)^2 - 2*cos(x) - 1))

maple [A] time = 0.44, size = 82, normalized size = 0.75

$$\frac{\ln\left(4\left(\cos^2(x)\right)+2\cos(x)-1\right)}{20} - \frac{\sqrt{5}\operatorname{arctanh}\left(\frac{(8\cos(x)+2)\sqrt{5}}{10}\right)}{10} + \frac{\ln(-1+\cos(x))}{10} + \frac{\ln\left(4\left(\cos^2(x)\right)-2\cos(x)-1\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*cot(5*x),x)

[Out] -1/20*ln(4*cos(x)^2+2*cos(x)-1)-1/10*5^(1/2)*arctanh(1/10*(8*cos(x)+2)*5^(1/2))+1/10*ln(-1+cos(x))+1/20*ln(4*cos(x)^2-2*cos(x)-1)-1/10*5^(1/2)*arctanh(1/10*(8*cos(x)-2)*5^(1/2))-1/10*ln(1+cos(x))+cos(x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(5*x),x, algorithm="maxima")

[Out] cos(x) + 1/10*integrate(-(cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) + cos(3/2*arctan2(sin(2*x), cos(2*x))*sin(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x))*sin(2*x) - cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) - cos(2*x)*sin(1/2*arctan2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 - 2*(cos(4*x) + cos(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x), cos(2*x)))^2 - 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), cos(2*x))) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*(sin(4*x) + sin(2*x) - sin(1/2*arctan2(sin(2*x), cos(2*x))))*sin(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(2*x), cos(2*x)))^2 - 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*x)))) + sin(1/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) + 1/10*integrate((cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) - cos(3/2*arctan2(sin(2*x), cos(2*x))*sin(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x))*sin(2*x) + cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) + cos(2*x)*sin(1/2*arctan2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + 2*(cos(4*x) + cos(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), cos(2*x))) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*x) + sin(2*x)^2 + 2*(sin(4*x) + sin(2*x) + sin(1/2*arctan2(sin(2*x), cos(2*x))))*sin(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*x)))) + sin(1/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) - 1/10*integrate((cos(x)*sin(4*x) + cos(x)*sin(3*x) + cos(x)*sin(2*x) - cos(4*x)*sin(x) - cos(3*x)*sin(x) - cos(2*x)*sin(x) - sin(x))/(2*(cos(3*x) + cos(2*x) + cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 2*(cos(2*x) + cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + 2*(sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + sin(4*x)^2 + 2*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1), x) - 1/10*integrate(-(cos(x)*sin(4*x) - cos(x)*sin(3*x) + cos(x)*sin(2*x) - cos(4*x)*sin(x) + cos(3*x)*sin(x) - cos(2*x)*sin(x) - sin(x))/(2*(cos(3*x) - cos(2*x) + cos(x) - 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x) - cos(x) + 1)*cos(3*x) - cos(3*x)^2 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + 2*(sin(3*x) - sin(2*x) + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2 + 2*cos(x) - 1), x) + 3/10*integrate(-(cos(4/3*arctan2(sin(3*x), cos(3*x))*sin(3*x) + cos(2/3*arctan2(sin(3*x), cos(3*x))*sin(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x))*sin(3*x) - cos(3*x)*sin(4/3*arctan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(

```

2/3*arctan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(1/3*arctan2(sin(3*x), cos(3
*x))) + sin(3*x))/(cos(3*x)^2 + 2*(cos(3*x) + cos(2/3*arctan2(sin(3*x), cos
(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*x))) + 1)*cos(4/3*arctan2(sin(3*x
), cos(3*x))) + cos(4/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) + cos(
1/3*arctan2(sin(3*x), cos(3*x))) + 1)*cos(2/3*arctan2(sin(3*x), cos(3*x)))
+ cos(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) + 1)*cos(1/3*arctan2
(sin(3*x), cos(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*x)))^2 + sin(3*x)^2
+ 2*(sin(3*x) + sin(2/3*arctan2(sin(3*x), cos(3*x)))) + sin(1/3*arctan2(sin
(3*x), cos(3*x))))*sin(4/3*arctan2(sin(3*x), cos(3*x))) + sin(4/3*arctan2(s
in(3*x), cos(3*x)))^2 + 2*(sin(3*x) + sin(1/3*arctan2(sin(3*x), cos(3*x))))
*sin(2/3*arctan2(sin(3*x), cos(3*x))) + sin(2/3*arctan2(sin(3*x), cos(3*x))
)^2 + 2*sin(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(1/3*arctan2(sin
(3*x), cos(3*x)))^2 + 2*cos(3*x) + 1), x) + 3/10*integrate(-(cos(4/3*arctan
2(sin(3*x), cos(3*x))) * sin(3*x) + cos(2/3*arctan2(sin(3*x), cos(3*x))) * sin(
3*x) - cos(1/3*arctan2(sin(3*x), cos(3*x))) * sin(3*x) - cos(3*x) * sin(4/3*arc
tan2(sin(3*x), cos(3*x))) - cos(3*x) * sin(2/3*arctan2(sin(3*x), cos(3*x)))) +
cos(3*x) * sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(3*x))/(cos(3*x)^2 - 2*
(cos(3*x) - cos(2/3*arctan2(sin(3*x), cos(3*x)))) + cos(1/3*arctan2(sin(3*x)
, cos(3*x))) - 1)*cos(4/3*arctan2(sin(3*x), cos(3*x))) + cos(4/3*arctan2(si
n(3*x), cos(3*x)))^2 - 2*(cos(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x)))) -
1)*cos(2/3*arctan2(sin(3*x), cos(3*x))) + cos(2/3*arctan2(sin(3*x), cos(3*
x)))^2 + 2*(cos(3*x) - 1)*cos(1/3*arctan2(sin(3*x), cos(3*x))) + cos(1/3*ar
ctan2(sin(3*x), cos(3*x)))^2 + sin(3*x)^2 - 2*(sin(3*x) - sin(2/3*arctan2(s
in(3*x), cos(3*x))) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(4/3*arctan2
(sin(3*x), cos(3*x))) + sin(4/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*(sin(3*x
) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(2/3*arctan2(sin(3*x), cos(3*x
))) + sin(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*sin(3*x)*sin(1/3*arctan2(s
in(3*x), cos(3*x))) + sin(1/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*cos(3*x) +
1), x) + 1/5*integrate((sin(4*x) + sin(3*x) + sin(2*x) + sin(x))/(2*(cos(3
*x) + cos(2*x) + cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 2*(cos(2*x) + cos(x) +
1)*cos(3*x) + cos(3*x)^2 + 2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2
+ 2*(sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + sin(4*x)^2 + 2*(sin(2*x) + s
in(x))*sin(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 +
2*cos(x) + 1), x) - 1/5*integrate(-(sin(4*x) - sin(3*x) + sin(2*x) - sin(x)
))/(2*(cos(3*x) - cos(2*x) + cos(x) - 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x)
- cos(x) + 1)*cos(3*x) - cos(3*x)^2 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2
- cos(x)^2 + 2*(sin(3*x) - sin(2*x) + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(s
in(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) -
sin(x)^2 + 2*cos(x) - 1), x) - 1/10*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
+ 1/10*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)

```

mupad [B] time = 3.03, size = 611, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(5*x)*cos(x), x)

```

[Out] (atan((tan(x/2)^2*4813499234516992i)/(1220703125*((213485644414976*5^(1/2))
/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 - (495922908
5483008*tan(x/2)^2)/1220703125 + 110872433262592/244140625)) - 954873235374
08i/(244140625*((213485644414976*5^(1/2))/1220703125 - (2152646198689792*5^
(1/2)*tan(x/2)^2)/1220703125 - (4959229085483008*tan(x/2)^2)/1220703125 + 1
10872433262592/244140625)) - (5^(1/2)*247887795585024i)/(1220703125*((21348
5644414976*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220
703125 - (4959229085483008*tan(x/2)^2)/1220703125 + 110872433262592/2441406
25)) + (5^(1/2)*tan(x/2)^2*2217818569310208i)/(1220703125*((213485644414976
*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 - (
4959229085483008*tan(x/2)^2)/1220703125 + 110872433262592/244140625))) * 1i)/
10 + (atan(95487323537408i/(244140625*((213485644414976*5^(1/2))/1220703125

```

$$\begin{aligned}
& - (2152646198689792*5^{(1/2)}*\tan(x/2)^2)/1220703125 + (4959229085483008*\tan \\
& (x/2)^2)/1220703125 - 110872433262592/244140625)) - (5^{(1/2)}*24788779558502 \\
& 4i)/(1220703125*((213485644414976*5^{(1/2)})/1220703125 - (2152646198689792*5 \\
& ^{(1/2)}*\tan(x/2)^2)/1220703125 + (4959229085483008*\tan(x/2)^2)/1220703125 - \\
& 110872433262592/244140625)) - (\tan(x/2)^2*4813499234516992i)/(1220703125*((\\
& 213485644414976*5^{(1/2)})/1220703125 - (2152646198689792*5^{(1/2)}*\tan(x/2)^2) \\
& /1220703125 + (4959229085483008*\tan(x/2)^2)/1220703125 - 110872433262592/24 \\
& 4140625)) + (5^{(1/2)}*\tan(x/2)^2*2217818569310208i)/(1220703125*((2134856444 \\
& 14976*5^{(1/2)})/1220703125 - (2152646198689792*5^{(1/2)}*\tan(x/2)^2)/122070312 \\
& 5 + (4959229085483008*\tan(x/2)^2)/1220703125 - 110872433262592/244140625))) \\
& *i)/10 + \log(\tan(x/2))/5 + 2/(\tan(x/2)^2 + 1) + (5^{(1/2)}*(\operatorname{atan}((\tan(x/2)^2 \\
& *4813499234516992i)/(1220703125*((213485644414976*5^{(1/2)})/1220703125 - (21 \\
& 52646198689792*5^{(1/2)}*\tan(x/2)^2)/1220703125 - (4959229085483008*\tan(x/2)^ \\
& 2)/1220703125 + 110872433262592/244140625)) - 95487323537408i/(244140625*((\\
& 213485644414976*5^{(1/2)})/1220703125 - (2152646198689792*5^{(1/2)}*\tan(x/2)^2) \\
& /1220703125 - (4959229085483008*\tan(x/2)^2)/1220703125 + 110872433262592/24 \\
& 4140625)) - (5^{(1/2)}*247887795585024i)/(1220703125*((213485644414976*5^{(1/2) \\
& })/1220703125 - (2152646198689792*5^{(1/2)}*\tan(x/2)^2)/1220703125 - (4959229 \\
& 085483008*\tan(x/2)^2)/1220703125 + 110872433262592/244140625)) + (5^{(1/2)}*t \\
& \operatorname{an}(x/2)^2*2217818569310208i)/(1220703125*((213485644414976*5^{(1/2)})/1220703 \\
& 125 - (2152646198689792*5^{(1/2)}*\tan(x/2)^2)/1220703125 - (4959229085483008* \\
& \tan(x/2)^2)/1220703125 + 110872433262592/244140625))) - \operatorname{atan}(95487323537408 \\
& i/(244140625*((213485644414976*5^{(1/2)})/1220703125 - (2152646198689792*5^{(1 \\
& /2)}*\tan(x/2)^2)/1220703125 + (4959229085483008*\tan(x/2)^2)/1220703125 - 110 \\
& 872433262592/244140625)) - (5^{(1/2)}*247887795585024i)/(1220703125*((2134856 \\
& 44414976*5^{(1/2)})/1220703125 - (2152646198689792*5^{(1/2)}*\tan(x/2)^2)/122070 \\
& 3125 + (4959229085483008*\tan(x/2)^2)/1220703125 - 110872433262592/244140625 \\
&)) - (\tan(x/2)^2*4813499234516992i)/(1220703125*((213485644414976*5^{(1/2)})/ \\
& 1220703125 - (2152646198689792*5^{(1/2)}*\tan(x/2)^2)/1220703125 + (4959229085 \\
& 483008*\tan(x/2)^2)/1220703125 - 110872433262592/244140625)) + (5^{(1/2)}*\tan(\\
& x/2)^2*2217818569310208i)/(1220703125*((213485644414976*5^{(1/2)})/1220703125 \\
& - (2152646198689792*5^{(1/2)}*\tan(x/2)^2)/1220703125 + (4959229085483008*\tan \\
& (x/2)^2)/1220703125 - 110872433262592/244140625))))*i)/10
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \cot(5x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(5*x), x)

[Out] Integral(cos(x)*cot(5*x), x)

3.114 $\int \cos(x) \cot(6x) dx$

Optimal. Leaf size=38

$$\cos(x) - \frac{1}{6} \tanh^{-1}(\cos(x)) - \frac{1}{6} \tanh^{-1}(2 \cos(x)) - \frac{\tanh^{-1}\left(\frac{2 \cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

[Out] $-1/6*\operatorname{arctanh}(\cos(x))-1/6*\operatorname{arctanh}(2*\cos(x))+\cos(x)-1/6*\operatorname{arctanh}(2/3*\cos(x)*3^{(1/2)})*3^{(1/2)}$

Rubi [A] time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {12, 2073, 207}

$$\cos(x) - \frac{1}{6} \tanh^{-1}(\cos(x)) - \frac{1}{6} \tanh^{-1}(2 \cos(x)) - \frac{\tanh^{-1}\left(\frac{2 \cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cot[6*x],x]

[Out] $-\operatorname{ArcTanh}[\cos(x)]/6 - \operatorname{ArcTanh}[2*\cos(x)]/6 - \operatorname{ArcTanh}[(2*\cos(x))/\operatorname{Sqrt}[3]]/(2*\operatorname{Sqrt}[3]) + \cos(x)$

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2073

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \cos(x) \cot(6x) dx &= -\operatorname{Subst}\left(\int \frac{-1 + 18x^2 - 48x^4 + 32x^6}{2(3 - 19x^2 + 32x^4 - 16x^6)} dx, x, \cos(x)\right) \\ &= -\left(\frac{1}{2} \operatorname{Subst}\left(\int \frac{-1 + 18x^2 - 48x^4 + 32x^6}{3 - 19x^2 + 32x^4 - 16x^6} dx, x, \cos(x)\right)\right) \\ &= -\left(\frac{1}{2} \operatorname{Subst}\left(\int \left(-2 - \frac{1}{3(-1 + x^2)} - \frac{2}{-3 + 4x^2} - \frac{2}{3(-1 + 4x^2)}\right) dx, x, \cos(x)\right)\right) \\ &= \cos(x) + \frac{1}{6} \operatorname{Subst}\left(\int \frac{1}{-1 + x^2} dx, x, \cos(x)\right) + \frac{1}{3} \operatorname{Subst}\left(\int \frac{1}{-1 + 4x^2} dx, x, \cos(x)\right) + S \\ &= -\frac{1}{6} \tanh^{-1}(\cos(x)) - \frac{1}{6} \tanh^{-1}(2 \cos(x)) - \frac{\tanh^{-1}\left(\frac{2 \cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}} + \cos(x) \end{aligned}$$

Mathematica [B] time = 0.09, size = 87, normalized size = 2.29

$$\frac{1}{12} \left(12 \cos(x) + 2 \log \left(\sin \left(\frac{x}{2} \right) \right) - 2 \log \left(\cos \left(\frac{x}{2} \right) \right) + \log(1 - 2 \cos(x)) - \log(2 \cos(x) + 1) + 2\sqrt{3} \tanh^{-1} \left(\frac{\tan \left(\frac{x}{2} \right)}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cot[6*x],x]

[Out] (2*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] - 2*Sqrt[3]*ArcTanh[(2 + Tan[x/2])/Sqrt[3]] + 12*Cos[x] - 2*Log[Cos[x/2]] + Log[1 - 2*Cos[x]] - Log[1 + 2*Cos[x]] + 2*Log[Sin[x/2]])/12

fricas [B] time = 0.69, size = 71, normalized size = 1.87

$$\frac{1}{12} \sqrt{3} \log \left(\frac{4 \cos(x)^2 - 4\sqrt{3} \cos(x) + 3}{4 \cos(x)^2 - 3} \right) + \cos(x) - \frac{1}{12} \log \left(\frac{1}{2} \cos(x) + \frac{1}{2} \right) + \frac{1}{12} \log \left(-\frac{1}{2} \cos(x) + \frac{1}{2} \right) + \frac{1}{12} \log(-2 \cos(x) + 1) - \frac{1}{12} \log(-2 \cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(6*x),x, algorithm="fricas")

[Out] 1/12*sqrt(3)*log((4*cos(x)^2 - 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) + cos(x) - 1/12*log(1/2*cos(x) + 1/2) + 1/12*log(-1/2*cos(x) + 1/2) + 1/12*log(-2*cos(x) + 1) - 1/12*log(-2*cos(x) - 1)

giac [B] time = 0.17, size = 70, normalized size = 1.84

$$\frac{1}{12} \sqrt{3} \log \left(\frac{|-4\sqrt{3} + 8 \cos(x)|}{|4\sqrt{3} + 8 \cos(x)|} \right) + \cos(x) - \frac{1}{12} \log(\cos(x) + 1) + \frac{1}{12} \log(-\cos(x) + 1) - \frac{1}{12} \log(|2 \cos(x) + 1|) + \frac{1}{12} \log(|2 \cos(x) - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(6*x),x, algorithm="giac")

[Out] 1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*cos(x))/abs(4*sqrt(3) + 8*cos(x))) + cos(x) - 1/12*log(cos(x) + 1) + 1/12*log(-cos(x) + 1) - 1/12*log(abs(2*cos(x) + 1)) + 1/12*log(abs(2*cos(x) - 1))

maple [A] time = 0.56, size = 49, normalized size = 1.29

$$\frac{\ln(2 \cos(x) - 1)}{12} - \frac{\ln(1 + 2 \cos(x))}{12} - \frac{\operatorname{arctanh} \left(\frac{2 \cos(x) \sqrt{3}}{3} \right) \sqrt{3}}{6} + \frac{\ln(-1 + \cos(x))}{12} - \frac{\ln(1 + \cos(x))}{12} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*cot(6*x),x)

[Out] 1/12*ln(2*cos(x)-1)-1/12*ln(1+2*cos(x))-1/6*arctanh(2/3*cos(x)*3^(1/2))*3^(1/2)+1/12*ln(-1+cos(x))-1/12*ln(1+cos(x))+cos(x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\cos(x) + \int \frac{(\sin(3x) - \sin(x)) \cos(4x) - (\cos(3x) - \cos(x)) \sin(4x) - (\cos(2x) - 1) \sin(3x) + \cos(3x) \sin(2x)}{2(2(\cos(2x) - 1) \cos(4x) - \cos(4x)^2 - \cos(2x)^2 - \sin(4x)^2 + 2 \sin(4x) \sin(2x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(6*x),x, algorithm="maxima")

```
[Out] cos(x) + integrate(1/2*((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*
sin(4*x) - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) +
cos(2*x)*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)
)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) -
1/24*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2*
sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/24*log(-2*(cos(x) - 1)*cos(2
*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2 - 2
*cos(x) + 1) - 1/12*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/12*log(cos(
x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

mupad [B] time = 2.44, size = 86, normalized size = 2.26

$$\frac{\operatorname{atanh}\left(\frac{1073741824}{10761687\left(\frac{427973089951744\tan\left(\frac{x}{2}\right)^2}{14348907} - \frac{47552804159488}{4782969}\right)} + \frac{797161}{797162}\right) + \ln\left(\tan\left(\frac{x}{2}\right)\right)}{6} + \frac{\sqrt{3}\operatorname{atanh}\left(\frac{303181204553728}{4782969\left(\frac{7314051205955584\tan\left(\frac{x}{2}\right)^2}{4782969} - 525125250187264/4782969\right)}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(6*x)*cos(x), x)
```

```
[Out] atanh(1073741824/(10761687*((427973089951744*tan(x/2)^2)/14348907 - 4755280
4159488/4782969)) + 797161/797162)/6 + log(tan(x/2))/6 - (3^(1/2)*atanh((30
3181204553728*3^(1/2))/(4782969*((7314051205955584*tan(x/2)^2)/4782969 - 52
5125250187264/4782969)) - (4222769432625152*3^(1/2)*tan(x/2)^2)/(4782969*((
7314051205955584*tan(x/2)^2)/4782969 - 525125250187264/4782969))))/6 + 2/(t
an(x/2)^2 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \cot(6x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(6*x), x)
```

```
[Out] Integral(cos(x)*cot(6*x), x)
```

3.115 $\int \cos(x) \cot(nx) dx$

Optimal. Leaf size=92

$$e^{-ix} {}_2F_1\left(1, -\frac{1}{2n}; 1 - \frac{1}{2n}; e^{2inx}\right) - e^{ix} {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); e^{2inx}\right) - \frac{e^{-ix}}{2} + \frac{e^{ix}}{2}$$

[Out] $-1/2/\exp(I*x)+1/2*\exp(I*x)+\text{hypergeom}([1, -1/2/n], [1-1/2/n], \exp(2*I*n*x))/\exp(I*x)-\exp(I*x)*\text{hypergeom}([1, 1/2/n], [1+1/2/n], \exp(2*I*n*x))$

Rubi [A] time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4558, 2194, 2251}

$$e^{-ix} {}_2F_1\left(1, -\frac{1}{2n}; 1 - \frac{1}{2n}; e^{2inx}\right) - e^{ix} {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); e^{2inx}\right) - \frac{e^{-ix}}{2} + \frac{e^{ix}}{2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cot[n*x], x]

[Out] $-1/(2*E^{(I*x)}) + E^{(I*x)}/2 + \text{Hypergeometric2F1}[1, -1/(2*n), 1 - 1/(2*n), E^{((2*I)*n*x)}/E^{(I*x)} - E^{(I*x)*\text{Hypergeometric2F1}[1, 1/(2*n), (2 + n^{(-1)})/2, E^{((2*I)*n*x)}]$

Rule 2194

Int[((F_)^((c_)*(a_) + (b_)*(x_)))^(n_), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 2251

Int[((a_) + (b_)*(F_)^((e_)*((c_) + (d_)*(x_)))^(p_)*(G_)^((h_)*((f_) + (g_)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b*F^(e*(c + d*x)))/a])]/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 4558

Int[Cos[(a_) + (b_)*(x_)]*Cot[(c_) + (d_)*(x_)], x_Symbol] :> Int[I/(E^(I*(a + b*x))^2 + (I*E^(I*(a + b*x)))/2 - I/(E^(I*(a + b*x))*(1 - E^(2*I*(c + d*x)))) - (I*E^(I*(a + b*x)))/(1 - E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \int \cos(x) \cot(nx) dx &= \int \left(\frac{1}{2}ie^{-ix} + \frac{1}{2}ie^{ix} - \frac{ie^{-ix}}{1 - e^{2inx}} - \frac{ie^{ix}}{1 - e^{2inx}} \right) dx \\ &= \frac{1}{2}i \int e^{-ix} dx + \frac{1}{2}i \int e^{ix} dx - i \int \frac{e^{-ix}}{1 - e^{2inx}} dx - i \int \frac{e^{ix}}{1 - e^{2inx}} dx \\ &= -\frac{1}{2}e^{-ix} + \frac{e^{ix}}{2} + e^{-ix} {}_2F_1\left(1, -\frac{1}{2n}; 1 - \frac{1}{2n}; e^{2inx}\right) - e^{ix} {}_2F_1\left(1, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); e^{2inx}\right) \end{aligned}$$

Mathematica [A] time = 0.18, size = 179, normalized size = 1.95

$$\frac{1}{2}e^{-2ix} \left(-\frac{e^{i(2nx+x)} {}_2F_1\left(1, 1 - \frac{1}{2n}; 2 - \frac{1}{2n}; e^{2inx}\right)}{2n-1} - \frac{e^{i(2n+3)x} {}_2F_1\left(1, 1 + \frac{1}{2n}; 2 + \frac{1}{2n}; e^{2inx}\right)}{2n+1} \right) + e^{ix} {}_2F_1\left(1, -\frac{1}{2n}; 1 - \frac{1}{2n}; e^{2inx}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cot[n*x],x]

[Out]
$$\frac{-\left(\frac{E^{I(x+2nx)} \operatorname{Hypergeometric2F1}\left[1, 1 - \frac{1}{2n}, 2 - \frac{1}{2n}, E^{(2I)nx}\right]}{-1 + 2n}\right) - \left(\frac{E^{I(3+2n)x} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{1}{2n}, 2 + \frac{1}{2n}, E^{(2I)nx}\right]}{1 + 2n}\right) + E^{Ix} \operatorname{Hypergeometric2F1}\left[1, -\frac{1}{2n}, 1 - \frac{1}{2n}, E^{(2I)nx}\right] - E^{(3I)x} \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2n}, 1 + \frac{1}{2n}, E^{(2I)nx}\right]}{2E^{(2I)x}}$$

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}(\cos(x) \cot(nx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(n*x),x, algorithm="fricas")

[Out] integral(cos(x)*cot(n*x), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \cot(nx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(n*x),x, algorithm="giac")

[Out] integrate(cos(x)*cot(n*x), x)

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \cos(x) \cot(nx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*cot(n*x),x)

[Out] int(cos(x)*cot(n*x),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \cot(nx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cot(n*x),x, algorithm="maxima")

[Out] integrate(cos(x)*cot(n*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(nx) \cos(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(n*x)*cos(x),x)

[Out] int(cot(n*x)*cos(x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \cot(nx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(n*x),x)
```

```
[Out] Integral(cos(x)*cot(n*x), x)
```

3.116 $\int \cos(x) \sec(2x) dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}(\sqrt{2} \sin(x))}{\sqrt{2}}$$

[Out] 1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4356, 206}

$$\frac{\tanh^{-1}(\sqrt{2} \sin(x))}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sec[2*x],x]

[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 4356

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned} \int \cos(x) \sec(2x) dx &= \text{Subst} \left(\int \frac{1}{1-2x^2} dx, x, \sin(x) \right) \\ &= \frac{\tanh^{-1}(\sqrt{2} \sin(x))}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\tanh^{-1}(\sqrt{2} \sin(x))}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sec[2*x],x]

[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2]

fricas [B] time = 1.13, size = 33, normalized size = 2.20

$$\frac{1}{4} \sqrt{2} \log \left(-\frac{2 \cos(x)^2 - 2 \sqrt{2} \sin(x) - 3}{2 \cos(x)^2 - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(2*x),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1))

giac [B] time = 0.13, size = 31, normalized size = 2.07

$$\frac{1}{4}\sqrt{2}\log\left(\left|\frac{1}{2}\sqrt{2} + \sin(x)\right|\right) - \frac{1}{4}\sqrt{2}\log\left(\left|-\frac{1}{2}\sqrt{2} + \sin(x)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(2*x),x, algorithm="giac")

[Out] 1/4*sqrt(2)*log(abs(1/2*sqrt(2) + sin(x))) - 1/4*sqrt(2)*log(abs(-1/2*sqrt(2) + sin(x)))

maple [A] time = 0.13, size = 13, normalized size = 0.87

$$\frac{\operatorname{arctanh}(\sin(x)\sqrt{2})\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sec(2*x),x)

[Out] 1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)

maxima [B] time = 1.08, size = 137, normalized size = 9.13

$$\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(2*x),x, algorithm="maxima")

[Out] 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2)

mupad [B] time = 0.11, size = 12, normalized size = 0.80

$$\frac{\sqrt{2}\operatorname{atanh}(\sqrt{2}\sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/cos(2*x),x)

[Out] (2^(1/2)*atanh(2^(1/2)*sin(x)))/2

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \sec(2x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(2*x),x)

[Out] Integral(cos(x)*sec(2*x), x)

3.117 $\int \cos(x) \sec(3x) dx$

Optimal. Leaf size=44

$$\frac{\log(\sqrt{3} \sin(x) + \cos(x))}{2\sqrt{3}} - \frac{\log(\cos(x) - \sqrt{3} \sin(x))}{2\sqrt{3}}$$

[Out] $-1/6*\ln(\cos(x)-\sin(x)*3^{(1/2)})*3^{(1/2)}+1/6*\ln(\cos(x)+\sin(x)*3^{(1/2)})*3^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {206}

$$\frac{\log(\sqrt{3} \sin(x) + \cos(x))}{2\sqrt{3}} - \frac{\log(\cos(x) - \sqrt{3} \sin(x))}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sec[3*x],x]

[Out] $-\text{Log}[\text{Cos}[x] - \text{Sqrt}[3]*\text{Sin}[x]]/(2*\text{Sqrt}[3]) + \text{Log}[\text{Cos}[x] + \text{Sqrt}[3]*\text{Sin}[x]]/(2*\text{Sqrt}[3])$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \cos(x) \sec(3x) dx &= \text{Subst}\left(\int \frac{1}{1-3x^2} dx, x, \tan(x)\right) \\ &= -\frac{\log(\cos(x) - \sqrt{3} \sin(x))}{2\sqrt{3}} + \frac{\log(\cos(x) + \sqrt{3} \sin(x))}{2\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 0.34

$$\frac{\tanh^{-1}(\sqrt{3} \tan(x))}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sec[3*x],x]

[Out] ArcTanh[Sqrt[3]*Tan[x]]/Sqrt[3]

fricas [A] time = 1.49, size = 53, normalized size = 1.20

$$\frac{1}{12} \sqrt{3} \log\left(-\frac{8 \cos(x)^4 + 4(2\sqrt{3} \cos(x)^3 - 3\sqrt{3} \cos(x)) \sin(x) - 9}{16 \cos(x)^4 - 24 \cos(x)^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(3*x),x, algorithm="fricas")

[Out] $\frac{1}{12}\sqrt{3}\log(-8\cos(x)^4 + 4(2\sqrt{3})\cos(x)^3 - 3\sqrt{3}\cos(x))\sin(x) - 9)/(16\cos(x)^4 - 24\cos(x)^2 + 9)$

giac [A] time = 0.17, size = 31, normalized size = 0.70

$$-\frac{1}{6}\sqrt{3}\log\left(\frac{|-2\sqrt{3} + 6\tan(x)|}{|2\sqrt{3} + 6\tan(x)|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*sec(3*x),x, algorithm="giac")`

[Out] $-1/6\sqrt{3}\log(\text{abs}(-2\sqrt{3} + 6\tan(x))/\text{abs}(2\sqrt{3} + 6\tan(x)))$

maple [A] time = 0.18, size = 13, normalized size = 0.30

$$\frac{\sqrt{3}\operatorname{arctanh}(\tan(x)\sqrt{3})}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*sec(3*x),x)`

[Out] $1/3\sqrt{3}\operatorname{arctanh}(\tan(x)\sqrt{3})$

maxima [B] time = 0.43, size = 76, normalized size = 1.73

$$\frac{1}{12}\sqrt{3}\left(\log\left(\frac{4}{3}\cos(2x)^2 + \frac{4}{3}\sin(2x)^2 + \frac{4}{3}\sqrt{3}\sin(2x) - \frac{4}{3}\cos(2x) + \frac{4}{3}\right) - \log\left(\frac{4}{3}\cos(2x)^2 + \frac{4}{3}\sin(2x)^2 - \frac{4}{3}\sqrt{3}\sin(2x) - \frac{4}{3}\cos(2x) + \frac{4}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*sec(3*x),x, algorithm="maxima")`

[Out] $\frac{1}{12}\sqrt{3}(\log(4/3\cos(2x)^2 + 4/3\sin(2x)^2 + 4/3\sqrt{3}\sin(2x) - 4/3\cos(2x) + 4/3) - \log(4/3\cos(2x)^2 + 4/3\sin(2x)^2 - 4/3\sqrt{3}\sin(2x) - 4/3\cos(2x) + 4/3))$

mupad [B] time = 2.66, size = 16, normalized size = 0.36

$$\frac{\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}\sin(x)}{\cos(x)}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)/cos(3*x),x)`

[Out] $(\sqrt{3}\operatorname{atanh}(\sqrt{3}\sin(x)/\cos(x)))/3$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\sec(3x)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*sec(3*x),x)`

[Out] `Integral(cos(x)*sec(3*x), x)`

3.118 $\int \cos(x) \sec(4x) dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2}(2-\sqrt{2})} - \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2}(2+\sqrt{2})}$$

[Out] $1/2*\operatorname{arctanh}(2*\sin(x)/(2-2^{(1/2)})^{(1/2)})/(4-2*2^{(1/2)})^{(1/2)}-1/2*\operatorname{arctanh}(2*\sin(x)/(2+2^{(1/2)})^{(1/2)})/(4+2*2^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4356, 1093, 207}

$$\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2}(2-\sqrt{2})} - \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2}(2+\sqrt{2})}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sec[4*x],x]

[Out] ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 4356

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned} \int \cos(x) \sec(4x) dx &= \operatorname{Subst}\left(\int \frac{1}{1-8x^2+8x^4} dx, x, \sin(x)\right) \\ &= \sqrt{2} \operatorname{Subst}\left(\int \frac{1}{-4-2\sqrt{2}+8x^2} dx, x, \sin(x)\right) - \sqrt{2} \operatorname{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2} dx, x, \sin(x)\right) \\ &= \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2}(2-\sqrt{2})} - \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2}(2+\sqrt{2})} \end{aligned}$$

Mathematica [A] time = 0.10, size = 67, normalized size = 0.94

$$\frac{1}{4}\sqrt{2+\sqrt{2}} \tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2}(2+\sqrt{2})}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sec[4*x],x]

[Out] (Sqrt[2 + Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]])/4 - ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])

fricas [B] time = 1.98, size = 121, normalized size = 1.70

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}(\sqrt{2}-1)+2\sin(x)\right) - \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}(\sqrt{2}-1)-2\sin(x)\right) - \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}(\sqrt{2}+1)+2\sin(x)\right) + \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}(\sqrt{2}+1)-2\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(4*x),x, algorithm="fricas")

[Out] 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) + 2*sin(x)) - 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) - 2*sin(x)) - 1/8*sqrt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) + 2*sin(x)) + 1/8*sqrt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 2*sin(x))

giac [B] time = 0.28, size = 99, normalized size = 1.39

$$-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left|\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin(x)\right|\right) + \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left|-\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin(x)\right|\right) + \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\left|\sqrt{\sqrt{2}+2}(\sqrt{2}-1)+2\sin(x)\right|\right) - \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\left|\sqrt{\sqrt{2}+2}(\sqrt{2}-1)-2\sin(x)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(4*x),x, algorithm="giac")

[Out] -1/8*sqrt(-sqrt(2) + 2)*log(abs(1/2*sqrt(sqrt(2) + 2) + sin(x))) + 1/8*sqrt(-sqrt(2) + 2)*log(abs(-1/2*sqrt(sqrt(2) + 2) + sin(x))) + 1/8*sqrt(sqrt(2) + 2)*log(abs(sqrt(-1/4*sqrt(2) + 1/2) + sin(x))) - 1/8*sqrt(sqrt(2) + 2)*log(abs(-sqrt(-1/4*sqrt(2) + 1/2) + sin(x)))

maple [A] time = 0.22, size = 54, normalized size = 0.76

$$\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sec(4*x),x)

[Out] 1/4*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))-1/4*2^(1/2)/(2+2^(1/2))^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \sec(4x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(4*x),x, algorithm="maxima")

[Out] integrate(cos(x)*sec(4*x), x)

mupad [B] time = 2.27, size = 95, normalized size = 1.34

$$\frac{\operatorname{atanh}\left(\frac{2 \sin(x) \sqrt{\sqrt{2}+2} + 2 \sqrt{2} \sin(x) \sqrt{\sqrt{2}+2}}{\sqrt{2}+2}\right) \sqrt{\sqrt{2}+2}}{4} - \frac{\operatorname{atanh}\left(\frac{2 \sin(x) \sqrt{2-\sqrt{2}} - 2 \sqrt{2} \sin(x) \sqrt{2-\sqrt{2}}}{\sqrt{2}-2}\right) \sqrt{2-\sqrt{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/cos(4*x),x)

[Out] (atanh((2*sin(x)*(2^(1/2) + 2)^(1/2) + 2*2^(1/2)*sin(x)*(2^(1/2) + 2)^(1/2))/(2^(1/2) + 2))*(2^(1/2) + 2)^(1/2))/4 - (atanh((2*sin(x)*(2 - 2^(1/2))^(1/2) - 2*2^(1/2)*sin(x)*(2 - 2^(1/2))^(1/2))/(2^(1/2) - 2))*(2 - 2^(1/2))^(1/2))/4

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \sec(4x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(4*x),x)

[Out] Integral(cos(x)*sec(4*x), x)

3.119 $\int \cos(x) \sec(5x) dx$

Optimal. Leaf size=163

$$\frac{1}{10} \sqrt{\frac{1}{2}(5-\sqrt{5})} \log\left(\cos(x) - \sqrt{5-2\sqrt{5}} \sin(x)\right) - \frac{1}{10} \sqrt{\frac{1}{2}(5-\sqrt{5})} \log\left(\sqrt{5-2\sqrt{5}} \sin(x) + \cos(x)\right) - \frac{1}{10} \sqrt{\frac{1}{2}(5-\sqrt{5})} \log\left(\cos(x) + \sqrt{5-2\sqrt{5}} \sin(x)\right) + \frac{1}{10} \sqrt{\frac{1}{2}(5-\sqrt{5})} \log\left(\cos(x) - \sqrt{5-2\sqrt{5}} \sin(x)\right)$$

[Out] 1/20*ln(cos(x)-sin(x)*(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-1/20*ln(cos(x)+sin(x)*(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-1/20*ln(cos(x)-sin(x)*(5+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)+1/20*ln(cos(x)+sin(x)*(5+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)

Rubi [A] time = 0.13, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1166, 207}

$$\frac{1}{10} \sqrt{\frac{1}{2}(5-\sqrt{5})} \log\left(\cos(x) - \sqrt{5-2\sqrt{5}} \sin(x)\right) - \frac{1}{10} \sqrt{\frac{1}{2}(5-\sqrt{5})} \log\left(\sqrt{5-2\sqrt{5}} \sin(x) + \cos(x)\right) - \frac{1}{10} \sqrt{\frac{1}{2}(5-\sqrt{5})} \log\left(\cos(x) + \sqrt{5-2\sqrt{5}} \sin(x)\right) + \frac{1}{10} \sqrt{\frac{1}{2}(5-\sqrt{5})} \log\left(\cos(x) - \sqrt{5-2\sqrt{5}} \sin(x)\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sec[5*x], x]

[Out] (Sqrt[(5 - Sqrt[5])/2]*Log[Cos[x] - Sqrt[5 - 2*Sqrt[5]]*Sin[x]])/10 - (Sqrt[(5 - Sqrt[5])/2]*Log[Cos[x] + Sqrt[5 - 2*Sqrt[5]]*Sin[x]])/10 - (Sqrt[(5 + Sqrt[5])/2]*Log[Cos[x] - Sqrt[5 + 2*Sqrt[5]]*Sin[x]])/10 + (Sqrt[(5 + Sqrt[5])/2]*Log[Cos[x] + Sqrt[5 + 2*Sqrt[5]]*Sin[x]])/10

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \cos(x) \sec(5x) dx &= \text{Subst} \left(\int \frac{1+x^2}{1-10x^2+5x^4} dx, x, \tan(x) \right) \\ &= \frac{1}{2} (1-\sqrt{5}) \text{Subst} \left(\int \frac{1}{-5+2\sqrt{5}+5x^2} dx, x, \tan(x) \right) + \frac{1}{2} (1+\sqrt{5}) \text{Subst} \left(\int \frac{1}{-5-2\sqrt{5}+5x^2} dx, x, \tan(x) \right) \\ &= \frac{1}{10} \sqrt{\frac{1}{2}(5-\sqrt{5})} \log\left(\cos(x) - \sqrt{5-2\sqrt{5}} \sin(x)\right) - \frac{1}{10} \sqrt{\frac{1}{2}(5-\sqrt{5})} \log\left(\cos(x) + \sqrt{5-2\sqrt{5}} \sin(x)\right) \\ &\quad - \frac{1}{10} \sqrt{\frac{1}{2}(5+\sqrt{5})} \log\left(\cos(x) - \sqrt{5+2\sqrt{5}} \sin(x)\right) + \frac{1}{10} \sqrt{\frac{1}{2}(5+\sqrt{5})} \log\left(\cos(x) + \sqrt{5+2\sqrt{5}} \sin(x)\right) \end{aligned}$$

Mathematica [A] time = 0.10, size = 84, normalized size = 0.52

$$\frac{\sqrt{5+\sqrt{5}} \tanh^{-1}\left(\frac{(5+\sqrt{5})\tan(x)}{\sqrt{10-2\sqrt{5}}}\right) + \sqrt{5-\sqrt{5}} \tanh^{-1}\left(\frac{(\sqrt{5}-5)\tan(x)}{\sqrt{2(5+\sqrt{5})}}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sec[5*x],x]

[Out] (Sqrt[5 + Sqrt[5]]*ArcTanh[((5 + Sqrt[5])*Tan[x])/Sqrt[10 - 2*Sqrt[5]]] + Sqrt[5 - Sqrt[5]]*ArcTanh[((-5 + Sqrt[5])*Tan[x])/Sqrt[2*(5 + Sqrt[5])]])/(5*Sqrt[2])

fricas [B] time = 2.07, size = 231, normalized size = 1.42

$$-\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+5}\cos(x)\sin(x)+2\left(\sqrt{5}+1\right)\cos(x)^2-\sqrt{5}-5\right)+\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(5*x),x, algorithm="fricas")

[Out] -1/40*sqrt(2)*sqrt(sqrt(5)+5)*log((sqrt(5)*sqrt(2)-sqrt(2))*sqrt(sqrt(5)+5)*cos(x)*sin(x)+2*(sqrt(5)+1)*cos(x)^2-sqrt(5)-5)+1/40*sqrt(2)*sqrt(sqrt(5)+5)*log(-(sqrt(5)*sqrt(2)-sqrt(2))*sqrt(sqrt(5)+5)*cos(x)*sin(x)+2*(sqrt(5)+1)*cos(x)^2-sqrt(5)-5)-1/40*sqrt(2)*sqrt(-sqrt(5)+5)*log((sqrt(5)*sqrt(2)+sqrt(2))*sqrt(-sqrt(5)+5)*cos(x)*sin(x)+2*(sqrt(5)-1)*cos(x)^2-sqrt(5)+5)+1/40*sqrt(2)*sqrt(-sqrt(5)+5)*log(-(sqrt(5)*sqrt(2)+sqrt(2))*sqrt(-sqrt(5)+5)*cos(x)*sin(x)+2*(sqrt(5)-1)*cos(x)^2-sqrt(5)+5)

giac [A] time = 0.28, size = 105, normalized size = 0.64

$$-\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|\sqrt{\frac{2}{5}}\sqrt{5}+1+\tan(x)\right|\right)+\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|-\sqrt{\frac{2}{5}}\sqrt{5}+1+\tan(x)\right|\right)+\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(5*x),x, algorithm="giac")

[Out] -1/20*sqrt(-2*sqrt(5)+10)*log(abs(sqrt(2/5)*sqrt(5)+1+tan(x)))+1/20*sqrt(-2*sqrt(5)+10)*log(abs(-sqrt(2/5)*sqrt(5)+1+tan(x)))+1/20*sqrt(2)*sqrt(sqrt(5)+5)*log(abs(sqrt(-2/5)*sqrt(5)+1+tan(x)))-1/20*sqrt(2)*sqrt(sqrt(5)+5)*log(abs(-sqrt(-2/5)*sqrt(5)+1+tan(x)))

maple [A] time = 0.26, size = 68, normalized size = 0.42

$$\frac{(5+\sqrt{5})\sqrt{5}\operatorname{arctanh}\left(\frac{5\tan(x)}{\sqrt{25+10\sqrt{5}}}\right)}{10\sqrt{25+10\sqrt{5}}}-\frac{\sqrt{5}(\sqrt{5}-5)\operatorname{arctanh}\left(\frac{5\tan(x)}{\sqrt{25-10\sqrt{5}}}\right)}{10\sqrt{25-10\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sec(5*x),x)

[Out] -1/10*(5+5^(1/2))*5^(1/2)/(25+10*5^(1/2))^(1/2)*arctanh(5*tan(x)/(25+10*5^(1/2))^(1/2))-1/10*5^(1/2)*(5^(1/2)-5)/(25-10*5^(1/2))^(1/2)*arctanh(5*tan(x)/(25-10*5^(1/2))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \sec(5x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(5*x),x, algorithm="maxima")

[Out] integrate(cos(x)*sec(5*x), x)

mupad [B] time = 2.67, size = 217, normalized size = 1.33

$$\sqrt{2} \operatorname{atanh} \left(\frac{34359738368 \sqrt{2} \tan\left(\frac{x}{2}\right) \sqrt{5-\sqrt{5}}}{5 \left(\frac{124554051584 \sqrt{5}}{25} - \frac{124554051584 \sqrt{5} \tan\left(\frac{x}{2}\right)^2}{25} - \frac{55834574848 \tan\left(\frac{x}{2}\right)^2}{5} + \frac{55834574848}{5} \right)} - \frac{77309411328 \sqrt{2} \sqrt{5} \tan\left(\frac{x}{2}\right)}{25 \left(\frac{124554051584 \sqrt{5}}{25} - \frac{124554051584 \sqrt{5} \tan\left(\frac{x}{2}\right)^2}{25} - \frac{55834574848 \tan\left(\frac{x}{2}\right)^2}{5} + \frac{55834574848}{5} \right)} \right) \frac{1}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/cos(5*x), x)

[Out] $(2^{(1/2)} \operatorname{atanh}(- (34359738368 * 2^{(1/2)} * \tan(x/2) * (5 - 5^{(1/2)})^{(1/2)}) / (5 * ((124554051584 * 5^{(1/2)}) / 25 - (124554051584 * 5^{(1/2)} * \tan(x/2)^2) / 25 - (55834574848 * \tan(x/2)^2) / 5 + 55834574848 / 5)) - (77309411328 * 2^{(1/2)} * 5^{(1/2)} * \tan(x/2) * (5 - 5^{(1/2)})^{(1/2)}) / (25 * ((124554051584 * 5^{(1/2)}) / 25 - (124554051584 * 5^{(1/2)} * \tan(x/2)^2) / 25 - (55834574848 * \tan(x/2)^2) / 5 + 55834574848 / 5))) * (5 - 5^{(1/2)})^{(1/2)}) / 10 - (2^{(1/2)} \operatorname{atanh}((77309411328 * 2^{(1/2)} * 5^{(1/2)} * \tan(x/2) * (5^{(1/2)} + 5)^{(1/2)}) / (25 * ((124554051584 * 5^{(1/2)}) / 25 - (124554051584 * 5^{(1/2)} * \tan(x/2)^2) / 25 + (55834574848 * \tan(x/2)^2) / 5 - 55834574848 / 5)) - (34359738368 * 2^{(1/2)} * \tan(x/2) * (5^{(1/2)} + 5)^{(1/2)}) / (5 * ((124554051584 * 5^{(1/2)}) / 25 - (124554051584 * 5^{(1/2)} * \tan(x/2)^2) / 25 + (55834574848 * \tan(x/2)^2) / 5 - 55834574848 / 5))) * (5^{(1/2)} + 5)^{(1/2)}) / 10$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \sec(5x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(5*x), x)

[Out] Integral(cos(x)*sec(5*x), x)

3.120 $\int \cos(x) \sec(6x) dx$

Optimal. Leaf size=85

$$-\frac{\tanh^{-1}(\sqrt{2} \sin(x))}{3\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}} + \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

[Out] $-1/6*\operatorname{arctanh}(\sin(x)*2^{(1/2)})*2^{(1/2)}+1/6*\operatorname{arctanh}(2*\sin(x)/(1/2*6^{(1/2)}-1/2*2^{(1/2)}))/(1/2*6^{(1/2)}-1/2*2^{(1/2)})+1/6*\operatorname{arctanh}(2*\sin(x)/(1/2*6^{(1/2)}+1/2*2^{(1/2)}))/(1/2*6^{(1/2)}+1/2*2^{(1/2)})$

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4356, 2057, 207, 1166}

$$-\frac{\tanh^{-1}(\sqrt{2} \sin(x))}{3\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}} + \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sec[6*x],x]

[Out] $-\operatorname{ArcTanh}[\operatorname{Sqrt}[2]*\operatorname{Sin}[x]]/(3*\operatorname{Sqrt}[2]) + \operatorname{ArcTanh}[(2*\operatorname{Sin}[x])/ \operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]]/(6*\operatorname{Sqrt}[2 - \operatorname{Sqrt}[3]]) + \operatorname{ArcTanh}[(2*\operatorname{Sin}[x])/ \operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]]]/(6*\operatorname{Sqrt}[2 + \operatorname{Sqrt}[3]])$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int(((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 2057

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(u /. x -> x^2)^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x^2] && ILtQ[p, 0]

Rule 4356

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned}
\int \cos(x) \sec(6x) dx &= \text{Subst} \left(\int \frac{1}{1 - 18x^2 + 48x^4 - 32x^6} dx, x, \sin(x) \right) \\
&= \text{Subst} \left(\int \left(\frac{1}{3(-1 + 2x^2)} - \frac{4(-1 + 2x^2)}{3(1 - 16x^2 + 16x^4)} \right) dx, x, \sin(x) \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \frac{1}{-1 + 2x^2} dx, x, \sin(x) \right) - \frac{4}{3} \text{Subst} \left(\int \frac{-1 + 2x^2}{1 - 16x^2 + 16x^4} dx, x, \sin(x) \right) \\
&= -\frac{\tanh^{-1}(\sqrt{2} \sin(x))}{3\sqrt{2}} - \frac{4}{3} \text{Subst} \left(\int \frac{1}{-8 - 4\sqrt{3} + 16x^2} dx, x, \sin(x) \right) - \frac{4}{3} \text{Subst} \left(\int \frac{1}{-8 + 4\sqrt{3} + 16x^2} dx, x, \sin(x) \right) \\
&= -\frac{\tanh^{-1}(\sqrt{2} \sin(x))}{3\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{2 \sin(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}} + \frac{\tanh^{-1}\left(\frac{2 \sin(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 81, normalized size = 0.95

$$\frac{1}{6} \left(-\sqrt{2} \tanh^{-1}(\sqrt{2} \sin(x)) + \sqrt{2 + \sqrt{3}} \tanh^{-1}\left(\frac{2 \sin(x)}{\sqrt{2 - \sqrt{3}}}\right) + \sqrt{2 - \sqrt{3}} \tanh^{-1}\left(\frac{2 \sin(x)}{\sqrt{2 + \sqrt{3}}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sec[6*x],x]

[Out] (-(Sqrt[2]*ArcTanh[Sqrt[2]*Sin[x]]) + Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[3]]] + Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[3]]])/6

fricas [B] time = 4.13, size = 154, normalized size = 1.81

$$-\frac{1}{12} \sqrt{\sqrt{3} + 2} \log\left(\sqrt{\sqrt{3} + 2}(\sqrt{3} - 2) + 2 \sin(x)\right) + \frac{1}{12} \sqrt{\sqrt{3} + 2} \log\left(\sqrt{\sqrt{3} + 2}(\sqrt{3} - 2) - 2 \sin(x)\right) + \frac{1}{12} \sqrt{2 - \sqrt{3}} \log\left(\frac{2 \sin(x)}{\sqrt{2 - \sqrt{3}}}\right) + \frac{1}{12} \sqrt{2 + \sqrt{3}} \log\left(\frac{2 \sin(x)}{\sqrt{2 + \sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(6*x),x, algorithm="fricas")

[Out] -1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + 2*sin(x)) + 1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) - 2*sin(x)) + 1/12*sqrt(2 - sqrt(3))*log((sqrt(3) + 2)*sqrt(2 - sqrt(3)) + 2*sin(x)) - 1/12*sqrt(2 - sqrt(3))*log((sqrt(3) + 2)*sqrt(2 - sqrt(3)) - 2*sin(x)) + 1/12*sqrt(2 + sqrt(3))*log((sqrt(3) + 2)*sqrt(2 + sqrt(3)) + 2*sin(x)) - 1/12*sqrt(2 + sqrt(3))*log((sqrt(3) + 2)*sqrt(2 + sqrt(3)) - 2*sin(x))

giac [A] time = 0.25, size = 132, normalized size = 1.55

$$\frac{1}{24} (\sqrt{6} - \sqrt{2}) \log\left(\left|\frac{1}{4} \sqrt{6} + \frac{1}{4} \sqrt{2} + \sin(x)\right|\right) + \frac{1}{24} (\sqrt{6} + \sqrt{2}) \log\left(\left|\frac{1}{4} \sqrt{6} - \frac{1}{4} \sqrt{2} + \sin(x)\right|\right) - \frac{1}{24} (\sqrt{6} + \sqrt{2}) \log\left(\left|\frac{1}{4} \sqrt{6} + \frac{1}{4} \sqrt{2} + \sin(x)\right|\right) + \frac{1}{24} (\sqrt{6} - \sqrt{2}) \log\left(\left|\frac{1}{4} \sqrt{6} - \frac{1}{4} \sqrt{2} + \sin(x)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(6*x),x, algorithm="giac")

[Out] 1/24*(sqrt(6) - sqrt(2))*log(abs(1/4*sqrt(6) + 1/4*sqrt(2) + sin(x))) + 1/24*(sqrt(6) + sqrt(2))*log(abs(1/4*sqrt(6) - 1/4*sqrt(2) + sin(x))) - 1/24*(sqrt(6) + sqrt(2))*log(abs(-1/4*sqrt(6) + 1/4*sqrt(2) + sin(x))) - 1/24*(sqrt(6) - sqrt(2))*log(abs(1/4*sqrt(6) - 1/4*sqrt(2) + sin(x)))

rt(6) - sqrt(2))*log(abs(-1/4*sqrt(6) - 1/4*sqrt(2) + sin(x))) + 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x)))

maple [A] time = 0.26, size = 80, normalized size = 0.94

$$\frac{2 \operatorname{arctanh}\left(\frac{8 \sin(x)}{2\sqrt{6}-2\sqrt{2}}\right)}{3(2\sqrt{6}-2\sqrt{2})} + \frac{2 \operatorname{arctanh}\left(\frac{8 \sin(x)}{2\sqrt{6}+2\sqrt{2}}\right)}{3(2\sqrt{6}+2\sqrt{2})} - \frac{\operatorname{arctanh}(\sin(x)\sqrt{2})\sqrt{2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sec(6*x),x)

[Out] 2/3/(2*6^(1/2)-2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)-2*2^(1/2)))+2/3/(2*6^(1/2)+2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)+2*2^(1/2)))-1/6*arctanh(sin(x)*2^(1/2))*2^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{24}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)+\frac{1}{24}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(6*x),x, algorithm="maxima")

[Out] -1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + integrate(-1/3*((cos(7*x) + cos(5*x) + cos(3*x) + cos(x))*cos(8*x) - (cos(4*x) - 1)*cos(7*x) - (cos(4*x) - 1)*cos(5*x) - (cos(3*x) + cos(x))*cos(4*x) + (sin(7*x) + sin(5*x) + sin(3*x) + sin(x))*sin(8*x) - (sin(3*x) + sin(x))*sin(4*x) - sin(7*x)*sin(4*x) - sin(5*x)*sin(4*x) + cos(3*x) + cos(x))/(2*(cos(4*x) - 1)*cos(8*x) - cos(8*x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(4*x) - sin(4*x)^2 + 2*cos(4*x) - 1), x)

mupad [B] time = 2.29, size = 118, normalized size = 1.39

$$\operatorname{atanh}\left(\frac{5\sqrt{2}\sin(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}+\frac{1}{1048576}\right)}+\frac{3\sqrt{6}\sin(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}+\frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12}+\frac{\sqrt{6}}{12}\right)-\operatorname{atanh}\left(\frac{5\sqrt{2}\sin(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/cos(6*x),x)

[Out] atanh((5*2^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576))) + (3*6^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576)))*(2^(1/2)/12 + 6^(1/2)/12) - atanh((5*2^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 - 1/1048576))) - (3*6^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 - 1/1048576)))*(2^(1/2)/12 - 6^(1/2)/12) - (2^(1/2)*atanh(2^(1/2)*sin(x)))/6

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \sec(6x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(6*x),x)

[Out] Integral(cos(x)*sec(6*x), x)

3.121 $\int \cos(2x) \sec(x) dx$

Optimal. Leaf size=10

$$2 \sin(x) - \tanh^{-1}(\sin(x))$$

[Out] -arctanh(sin(x))+2*sin(x)

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4364, 388, 206}

$$2 \sin(x) - \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[2*x]*Sec[x],x]

[Out] -ArcTanh[Sin[x]] + 2*Sin[x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)], Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 4364

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^2)^(n - 1)/2, Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned} \int \cos(2x) \sec(x) dx &= \text{Subst} \left(\int \frac{1 - 2x^2}{1 - x^2} dx, x, \sin(x) \right) \\ &= 2 \sin(x) - \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \sin(x) \right) \\ &= -\tanh^{-1}(\sin(x)) + 2 \sin(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$2 \sin(x) - \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[2*x]*Sec[x],x]

[Out] -ArcTanh[Sin[x]] + 2*Sin[x]

fricas [B] time = 0.68, size = 21, normalized size = 2.10

$$-\frac{1}{2} \log(\sin(x) + 1) + \frac{1}{2} \log(-\sin(x) + 1) + 2 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*sec(x),x, algorithm="fricas")

[Out] -1/2*log(sin(x) + 1) + 1/2*log(-sin(x) + 1) + 2*sin(x)

giac [B] time = 0.14, size = 21, normalized size = 2.10

$$-\frac{1}{2} \log(\sin(x) + 1) + \frac{1}{2} \log(-\sin(x) + 1) + 2 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*sec(x),x, algorithm="giac")

[Out] -1/2*log(sin(x) + 1) + 1/2*log(-sin(x) + 1) + 2*sin(x)

maple [A] time = 0.12, size = 14, normalized size = 1.40

$$2 \sin(x) - \ln(\sec(x) + \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)*sec(x),x)

[Out] 2*sin(x)-ln(sec(x)+tan(x))

maxima [A] time = 0.31, size = 19, normalized size = 1.90

$$-\frac{1}{2} \log(\sin(x) + 1) + \frac{1}{2} \log(\sin(x) - 1) + 2 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*sec(x),x, algorithm="maxima")

[Out] -1/2*log(sin(x) + 1) + 1/2*log(sin(x) - 1) + 2*sin(x)

mupad [B] time = 2.25, size = 10, normalized size = 1.00

$$2 \sin(x) - \operatorname{atanh}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)/cos(x),x)

[Out] 2*sin(x) - atanh(sin(x))

sympy [B] time = 1.05, size = 20, normalized size = 2.00

$$\frac{\log(\sin(x) - 1)}{2} - \frac{\log(\sin(x) + 1)}{2} + 2 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*sec(x),x)

[Out] log(sin(x) - 1)/2 - log(sin(x) + 1)/2 + 2*sin(x)

3.122 $\int \cos(4x) \sec(2x) dx$

Optimal. Leaf size=14

$$\sin(2x) - \frac{1}{2} \tanh^{-1}(\sin(2x))$$

[Out] $-1/2*\operatorname{arctanh}(\sin(2*x))+\sin(2*x)$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4364, 388, 206}

$$\sin(2x) - \frac{1}{2} \tanh^{-1}(\sin(2x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cos}[4*x]*\operatorname{Sec}[2*x], x]$

[Out] $-\operatorname{ArcTanh}[\operatorname{Sin}[2*x]]/2 + \operatorname{Sin}[2*x]$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ \|\ \operatorname{LtQ}[b, 0])$

Rule 388

$\operatorname{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)}), x_Symbol] \rightarrow \operatorname{Simp}[(d*x*(a + b*x^n)^{(p + 1)})/(b*(n*(p + 1) + 1)), x] - \operatorname{Dist}[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), \operatorname{Int}[(a + b*x^n)^p, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, n\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{NeQ}[n*(p + 1) + 1, 0]$

Rule 4364

$\operatorname{Int}[(u_)*(F_)[(c_)*((a_ + (b_)*(x_))]^{(n_)}, x_Symbol] \rightarrow \operatorname{With}\{d = \operatorname{FreeFactors}[\operatorname{Sin}[c*(a + b*x)], x]\}, \operatorname{Dist}[d/(b*c), \operatorname{Subst}[\operatorname{Int}[\operatorname{SubstFor}[(1 - d^2*x^2)^{(n - 1)/2}, \operatorname{Sin}[c*(a + b*x)]/d, u, x], x], x, \operatorname{Sin}[c*(a + b*x)]/d], x] /;$ $\operatorname{FunctionOfQ}[\operatorname{Sin}[c*(a + b*x)]/d, u, x] /;$ $\operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{IntegerQ}[(n - 1)/2] \ \&\& \operatorname{NonsumQ}[u] \ \&\& (\operatorname{EqQ}[F, \operatorname{Cos}] \ \|\ \operatorname{EqQ}[F, \operatorname{cos}])$

Rubi steps

$$\begin{aligned} \int \cos(4x) \sec(2x) dx &= \frac{1}{2} \operatorname{Subst} \left(\int \frac{1 - 2x^2}{1 - x^2} dx, x, \sin(2x) \right) \\ &= \sin(2x) - \frac{1}{2} \operatorname{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \sin(2x) \right) \\ &= -\frac{1}{2} \tanh^{-1}(\sin(2x)) + \sin(2x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\sin(2x) - \frac{1}{2} \tanh^{-1}(\sin(2x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[4*x]*Sec[2*x],x]

[Out] $-1/2*\text{ArcTanh}[\text{Sin}[2*x]] + \text{Sin}[2*x]$

fricas [B] time = 1.47, size = 25, normalized size = 1.79

$$-\frac{1}{4} \log(\sin(2x) + 1) + \frac{1}{4} \log(-\sin(2x) + 1) + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(4*x)*sec(2*x),x, algorithm="fricas")

[Out] $-1/4*\log(\sin(2*x) + 1) + 1/4*\log(-\sin(2*x) + 1) + \sin(2*x)$

giac [B] time = 0.13, size = 25, normalized size = 1.79

$$-\frac{1}{4} \log(\sin(2x) + 1) + \frac{1}{4} \log(-\sin(2x) + 1) + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(4*x)*sec(2*x),x, algorithm="giac")

[Out] $-1/4*\log(\sin(2*x) + 1) + 1/4*\log(-\sin(2*x) + 1) + \sin(2*x)$

maple [A] time = 0.14, size = 18, normalized size = 1.29

$$-\frac{\ln(\sec(2x) + \tan(2x))}{2} + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(4*x)*sec(2*x),x)

[Out] $-1/2*\ln(\sec(2*x)+\tan(2*x))+\sin(2*x)$

maxima [B] time = 0.48, size = 129, normalized size = 9.21

$$\frac{1}{4} \log\left(2 \cos(x)^2 + 2 \sin(x)^2 + 2\sqrt{2} \cos(x) + 2\sqrt{2} \sin(x) + 2\right) - \frac{1}{4} \log\left(2 \cos(x)^2 + 2 \sin(x)^2 + 2\sqrt{2} \cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(4*x)*sec(2*x),x, algorithm="maxima")

[Out] $1/4*\log(2*\cos(x)^2 + 2*\sin(x)^2 + 2*\sqrt{2}*\cos(x) + 2*\sqrt{2}*\sin(x) + 2) - 1/4*\log(2*\cos(x)^2 + 2*\sin(x)^2 + 2*\sqrt{2}*\cos(x) - 2*\sqrt{2}*\sin(x) + 2) - 1/4*\log(2*\cos(x)^2 + 2*\sin(x)^2 - 2*\sqrt{2}*\cos(x) + 2*\sqrt{2}*\sin(x) + 2) + 1/4*\log(2*\cos(x)^2 + 2*\sin(x)^2 - 2*\sqrt{2}*\cos(x) - 2*\sqrt{2}*\sin(x) + 2) + \sin(2*x)$

mupad [B] time = 0.02, size = 12, normalized size = 0.86

$$\sin(2x) - \frac{\text{atanh}(\sin(2x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(4*x)/cos(2*x),x)

[Out] $\sin(2*x) - \text{atanh}(\sin(2*x))/2$

sympy [B] time = 5.35, size = 427, normalized size = 30.50

$$-4x + \frac{32x \tan^4\left(\frac{x}{2}\right)}{8 \tan^4\left(\frac{x}{2}\right) + 16 \tan^2\left(\frac{x}{2}\right) + 8} + \frac{64x \tan^2\left(\frac{x}{2}\right)}{8 \tan^4\left(\frac{x}{2}\right) + 16 \tan^2\left(\frac{x}{2}\right) + 8} + \frac{32x}{8 \tan^4\left(\frac{x}{2}\right) + 16 \tan^2\left(\frac{x}{2}\right) + 8} - \frac{3 \log\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(4*x)*sec(2*x),x)

[Out] $-4x + \frac{32x \tan(x/2)^4}{(8 \tan(x/2)^4 + 16 \tan(x/2)^2 + 8)} + \frac{64x \tan(x/2)^2}{(8 \tan(x/2)^4 + 16 \tan(x/2)^2 + 8)} + \frac{32x}{(8 \tan(x/2)^4 + 16 \tan(x/2)^2 + 8)} - \frac{3 \log(\tan(x/2)^2 - 2 \tan(x/2) - 1)}{2} + \frac{3 \log(\tan(x/2)^2 + 2 \tan(x/2) - 1)}{2} + \frac{8 \log(\tan(x/2)^2 - 2 \tan(x/2) - 1) \tan(x/2)^4}{(8 \tan(x/2)^4 + 16 \tan(x/2)^2 + 8)} + \frac{16 \log(\tan(x/2)^2 - 2 \tan(x/2) - 1) \tan(x/2)^2}{(8 \tan(x/2)^4 + 16 \tan(x/2)^2 + 8)} + \frac{8 \log(\tan(x/2)^2 - 2 \tan(x/2) - 1)}{(8 \tan(x/2)^4 + 16 \tan(x/2)^2 + 8)} - \frac{8 \log(\tan(x/2)^2 + 2 \tan(x/2) - 1) \tan(x/2)^4}{(8 \tan(x/2)^4 + 16 \tan(x/2)^2 + 8)} - \frac{16 \log(\tan(x/2)^2 + 2 \tan(x/2) - 1) \tan(x/2)^2}{(8 \tan(x/2)^4 + 16 \tan(x/2)^2 + 8)} - \frac{8 \log(\tan(x/2)^2 + 2 \tan(x/2) - 1)}{(8 \tan(x/2)^4 + 16 \tan(x/2)^2 + 8)} - \frac{32 \tan(x/2)^3}{(8 \tan(x/2)^4 + 16 \tan(x/2)^2 + 8)} + \frac{32 \tan(x/2)}{(8 \tan(x/2)^4 + 16 \tan(x/2)^2 + 8)}$

3.123 $\int \cos(x) \csc(2x) dx$

Optimal. Leaf size=7

$$-\frac{1}{2} \tanh^{-1}(\cos(x))$$

[Out] -1/2*arctanh(cos(x))

Rubi [A] time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4287, 3770}

$$-\frac{1}{2} \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Csc[2*x],x]

[Out] -ArcTanh[Cos[x]]/2

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4287

Int[(cos[(a_.) + (b_.)*(x_)])*(e_.)^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_Symbol] := Dist[2^p/e^p, Int[(e*cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \cos(x) \csc(2x) dx &= \frac{1}{2} \int \csc(x) dx \\ &= -\frac{1}{2} \tanh^{-1}(\cos(x)) \end{aligned}$$

Mathematica [B] time = 0.00, size = 21, normalized size = 3.00

$$\frac{1}{2} \left(\log \left(\sin \left(\frac{x}{2} \right) \right) - \log \left(\cos \left(\frac{x}{2} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Csc[2*x],x]

[Out] (-Log[Cos[x/2]] + Log[Sin[x/2]])/2

fricas [B] time = 2.12, size = 19, normalized size = 2.71

$$-\frac{1}{4} \log \left(\frac{1}{2} \cos(x) + \frac{1}{2} \right) + \frac{1}{4} \log \left(-\frac{1}{2} \cos(x) + \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(2*x),x, algorithm="fricas")

[Out] $-1/4*\log(1/2*\cos(x) + 1/2) + 1/4*\log(-1/2*\cos(x) + 1/2)$

giac [B] time = 0.12, size = 17, normalized size = 2.43

$$-\frac{1}{4} \log(\cos(x) + 1) + \frac{1}{4} \log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*csc(2*x),x, algorithm="giac")`

[Out] $-1/4*\log(\cos(x) + 1) + 1/4*\log(-\cos(x) + 1)$

maple [A] time = 0.08, size = 11, normalized size = 1.57

$$\frac{\ln(\csc(x) - \cot(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*csc(2*x),x)`

[Out] $1/2*\ln(\csc(x) - \cot(x))$

maxima [B] time = 0.32, size = 35, normalized size = 5.00

$$-\frac{1}{4} \log(\cos(x)^2 + \sin(x)^2 + 2 \cos(x) + 1) + \frac{1}{4} \log(\cos(x)^2 + \sin(x)^2 - 2 \cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*csc(2*x),x, algorithm="maxima")`

[Out] $-1/4*\log(\cos(x)^2 + \sin(x)^2 + 2*\cos(x) + 1) + 1/4*\log(\cos(x)^2 + \sin(x)^2 - 2*\cos(x) + 1)$

mupad [B] time = 0.03, size = 5, normalized size = 0.71

$$-\frac{\operatorname{atanh}(\cos(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)/sin(2*x),x)`

[Out] $-\operatorname{atanh}(\cos(x))/2$

sympy [B] time = 0.87, size = 15, normalized size = 2.14

$$\frac{\log(\cos(x) - 1)}{4} - \frac{\log(\cos(x) + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*csc(2*x),x)`

[Out] $\log(\cos(x) - 1)/4 - \log(\cos(x) + 1)/4$

3.124 $\int \cos(x) \csc(3x) dx$

Optimal. Leaf size=21

$$\frac{1}{3} \log(\sin(x)) - \frac{1}{6} \log(3 - 4 \sin^2(x))$$

[Out] 1/3*ln(sin(x))-1/6*ln(3-4*sin(x)^2)

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {4356, 266, 36, 31, 29}

$$\frac{1}{3} \log(\sin(x)) - \frac{1}{6} \log(3 - 4 \sin^2(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Csc[3*x],x]

[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4356

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned}
\int \cos(x) \csc(3x) dx &= \text{Subst} \left(\int \frac{1}{x(3-4x^2)} dx, x, \sin(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(3-4x)x} dx, x, \sin^2(x) \right) \\
&= \frac{1}{6} \text{Subst} \left(\int \frac{1}{x} dx, x, \sin^2(x) \right) + \frac{2}{3} \text{Subst} \left(\int \frac{1}{3-4x} dx, x, \sin^2(x) \right) \\
&= \frac{1}{3} \log(\sin(x)) - \frac{1}{6} \log(3-4\sin^2(x))
\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{1}{3} \log(\sin(x)) - \frac{1}{6} \log(3-4\sin^2(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Csc[3*x],x]

[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6

fricas [A] time = 1.27, size = 19, normalized size = 0.90

$$-\frac{1}{6} \log(4 \cos(x)^2 - 1) + \frac{1}{3} \log\left(\frac{1}{2} \sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(3*x),x, algorithm="fricas")

[Out] -1/6*log(4*cos(x)^2 - 1) + 1/3*log(1/2*sin(x))

giac [A] time = 0.14, size = 24, normalized size = 1.14

$$\frac{1}{6} \log(-\cos(x)^2 + 1) - \frac{1}{6} \log(|4 \cos(x)^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(3*x),x, algorithm="giac")

[Out] 1/6*log(-cos(x)^2 + 1) - 1/6*log(abs(4*cos(x)^2 - 1))

maple [A] time = 0.22, size = 34, normalized size = 1.62

$$-\frac{\ln(2 \cos(x) - 1)}{6} - \frac{\ln(1 + 2 \cos(x))}{6} + \frac{\ln(-1 + \cos(x))}{6} + \frac{\ln(1 + \cos(x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*csc(3*x),x)

[Out] -1/6*ln(2*cos(x)-1)-1/6*ln(1+2*cos(x))+1/6*ln(-1+cos(x))+1/6*ln(1+cos(x))

maxima [B] time = 0.43, size = 129, normalized size = 6.14

$$-\frac{1}{12} \log(2(\cos(x) + 1) \cos(2x) + \cos(2x)^2 + \cos(x)^2 + \sin(2x)^2 + 2 \sin(2x) \sin(x) + \sin(x)^2 + 2 \cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(3*x),x, algorithm="maxima")

[Out] $-1/12*\log(2*(\cos(x) + 1)*\cos(2*x) + \cos(2*x)^2 + \cos(x)^2 + \sin(2*x)^2 + 2*\sin(2*x)*\sin(x) + \sin(x)^2 + 2*\cos(x) + 1) - 1/12*\log(-2*(\cos(x) - 1)*\cos(2*x) + \cos(2*x)^2 + \cos(x)^2 + \sin(2*x)^2 - 2*\sin(2*x)*\sin(x) + \sin(x)^2 - 2*\cos(x) + 1) + 1/6*\log(\cos(x)^2 + \sin(x)^2 + 2*\cos(x) + 1) + 1/6*\log(\cos(x)^2 + \sin(x)^2 - 2*\cos(x) + 1)$

mupad [B] time = 0.10, size = 17, normalized size = 0.81

$$\frac{\ln(\sin(x))}{3} - \frac{\ln\left(\frac{1}{4} - \cos(x)^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/sin(3*x),x)

[Out] $\log(\sin(x))/3 - \log(1/4 - \cos(x)^2)/6$

sympy [A] time = 1.33, size = 17, normalized size = 0.81

$$-\frac{\log(4\sin^2(x) - 3)}{6} + \frac{\log(\sin(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(3*x),x)

[Out] $-\log(4*\sin(x)**2 - 3)/6 + \log(\sin(x))/3$

3.125 $\int \cos(x) \csc(4x) dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}(\sqrt{2} \cos(x))}{2\sqrt{2}} - \frac{1}{4} \tanh^{-1}(\cos(x))$$

[Out] $-1/4*\operatorname{arctanh}(\cos(x))+1/4*\operatorname{arctanh}(\cos(x)*2^{(1/2)})*2^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1093, 206}

$$\frac{\tanh^{-1}(\sqrt{2} \cos(x))}{2\sqrt{2}} - \frac{1}{4} \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cos}[x]*\operatorname{Csc}[4*x], x]$

[Out] $-\operatorname{ArcTanh}[\operatorname{Cos}[x]]/4 + \operatorname{ArcTanh}[\operatorname{Sqrt}[2]*\operatorname{Cos}[x]]/(2*\operatorname{Sqrt}[2])$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 1093

$\operatorname{Int}[(a_ + (b_)*(x_)^2 + (c_)*(x_)^4)^{-1}, x_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[b^2 - 4*a*c, 2]\}, \operatorname{Dist}[c/q, \operatorname{Int}[1/(b/2 - q/2 + c*x^2), x], x] - \operatorname{Dist}[c/q, \operatorname{Int}[1/(b/2 + q/2 + c*x^2), x], x] /; \operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \ \&\& \operatorname{PosQ}[b^2 - 4*a*c]$

Rubi steps

$$\begin{aligned} \int \cos(x) \csc(4x) dx &= -\operatorname{Subst}\left(\int \frac{1}{-4 + 12x^2 - 8x^4} dx, x, \cos(x)\right) \\ &= 2 \operatorname{Subst}\left(\int \frac{1}{4 - 8x^2} dx, x, \cos(x)\right) - 2 \operatorname{Subst}\left(\int \frac{1}{8 - 8x^2} dx, x, \cos(x)\right) \\ &= -\frac{1}{4} \tanh^{-1}(\cos(x)) + \frac{\tanh^{-1}(\sqrt{2} \cos(x))}{2\sqrt{2}} \end{aligned}$$

Mathematica [C] time = 0.06, size = 66, normalized size = 2.54

$$\frac{1}{4} \left(\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + (1+i)(-1)^{3/4} \tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right) - 1}{\sqrt{2}}\right) + \sqrt{2} \tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right) + 1}{\sqrt{2}}\right) \right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Integrate}[\operatorname{Cos}[x]*\operatorname{Csc}[4*x], x]$

[Out] $((1 + I)*(-1)^{(3/4)}*\operatorname{ArcTanh}[(-1 + \operatorname{Tan}[x/2])/ \operatorname{Sqrt}[2]] + \operatorname{Sqrt}[2]*\operatorname{ArcTanh}[(1 + \operatorname{Tan}[x/2])/ \operatorname{Sqrt}[2]] - \operatorname{Log}[\operatorname{Cos}[x/2]] + \operatorname{Log}[\operatorname{Sin}[x/2]])/4$

fricas [B] time = 0.83, size = 52, normalized size = 2.00

$$\frac{1}{8} \sqrt{2} \log \left(-\frac{2 \cos(x)^2 + 2 \sqrt{2} \cos(x) + 1}{2 \cos(x)^2 - 1} \right) - \frac{1}{8} \log \left(\frac{1}{2} \cos(x) + \frac{1}{2} \right) + \frac{1}{8} \log \left(-\frac{1}{2} \cos(x) + \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(4*x),x, algorithm="fricas")

[Out] 1/8*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) - 1/8*log(1/2*cos(x) + 1/2) + 1/8*log(-1/2*cos(x) + 1/2)

giac [B] time = 0.13, size = 48, normalized size = 1.85

$$-\frac{1}{8} \sqrt{2} \log \left(\frac{|-2 \sqrt{2} + 4 \cos(x)|}{|2 \sqrt{2} + 4 \cos(x)|} \right) - \frac{1}{8} \log(\cos(x) + 1) + \frac{1}{8} \log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(4*x),x, algorithm="giac")

[Out] -1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*cos(x))/abs(2*sqrt(2) + 4*cos(x))) - 1/8*log(cos(x) + 1) + 1/8*log(-cos(x) + 1)

maple [A] time = 0.14, size = 28, normalized size = 1.08

$$\frac{\ln(-1 + \cos(x))}{8} + \frac{\operatorname{arctanh}(\cos(x)\sqrt{2})\sqrt{2}}{4} - \frac{\ln(1 + \cos(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*csc(4*x),x)

[Out] 1/8*ln(-1+cos(x))+1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)-1/8*ln(1+cos(x))

maxima [B] time = 0.44, size = 163, normalized size = 6.27

$$\frac{1}{16} \sqrt{2} \log \left(2 \sqrt{2} \sin(2x) \sin(x) + 2 \left(\sqrt{2} \cos(x) + 1 \right) \cos(2x) + \cos(2x)^2 + 2 \cos(x)^2 + \sin(2x)^2 + 2 \sin(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(4*x),x, algorithm="maxima")

[Out] 1/16*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 1) - 1/16*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 1) - 1/8*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)

mupad [B] time = 2.31, size = 55, normalized size = 2.12

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4} + \frac{\sqrt{2} \operatorname{atanh}\left(\frac{41\sqrt{2}}{8\left(\frac{169\tan\left(\frac{x}{2}\right)^2}{4} - \frac{29}{4}\right)} - \frac{239\sqrt{2}\tan\left(\frac{x}{2}\right)^2}{8\left(\frac{169\tan\left(\frac{x}{2}\right)^2}{4} - \frac{29}{4}\right)}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)/sin(4*x),x)`

[Out] $\log(\tan(x/2))/4 + (2^{(1/2)}*\operatorname{atanh}((41*2^{(1/2)}))/(8*((169*\tan(x/2)^2)/4 - 29/4))) - (239*2^{(1/2)}*\tan(x/2)^2)/(8*((169*\tan(x/2)^2)/4 - 29/4)))/4$

sympy [B] time = 6.14, size = 248, normalized size = 9.54

$$\frac{19601\sqrt{2} \log\left(\tan\left(\frac{x}{2}\right) - 1 + \sqrt{2}\right)}{110880\sqrt{2} + 156808} - \frac{27720 \log\left(\tan\left(\frac{x}{2}\right) - 1 + \sqrt{2}\right)}{110880\sqrt{2} + 156808} + \frac{27720 \log\left(\tan\left(\frac{x}{2}\right) + 1 + \sqrt{2}\right)}{110880\sqrt{2} + 156808} + \frac{19601\sqrt{2} \log\left(\tan\left(\frac{x}{2}\right) + 1 + \sqrt{2}\right)}{110880\sqrt{2} + 156808}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*csc(4*x),x)`

[Out] $-19601*\sqrt{2}*\log(\tan(x/2) - 1 + \sqrt{2})/(110880*\sqrt{2} + 156808) - 27720*\log(\tan(x/2) - 1 + \sqrt{2})/(110880*\sqrt{2} + 156808) + 27720*\log(\tan(x/2) + 1 + \sqrt{2})/(110880*\sqrt{2} + 156808) + 19601*\sqrt{2}*\log(\tan(x/2) + 1 + \sqrt{2})/(110880*\sqrt{2} + 156808) + 27720*\log(\tan(x/2) - \sqrt{2} - 1)/(110880*\sqrt{2} + 156808) + 19601*\sqrt{2}*\log(\tan(x/2) - \sqrt{2} - 1)/(110880*\sqrt{2} + 156808) - 19601*\sqrt{2}*\log(\tan(x/2) - \sqrt{2} + 1)/(110880*\sqrt{2} + 156808) - 27720*\log(\tan(x/2) - \sqrt{2} + 1)/(110880*\sqrt{2} + 156808) + 27720*\sqrt{2}*\log(\tan(x/2))/(110880*\sqrt{2} + 156808) + 39202*\log(\tan(x/2))/(110880*\sqrt{2} + 156808)$

3.126 $\int \cos(x) \csc(5x) dx$

Optimal. Leaf size=62

$$-\frac{1}{20}(1 + \sqrt{5}) \log(-8 \sin^2(x) - \sqrt{5} + 5) - \frac{1}{20}(1 - \sqrt{5}) \log(-8 \sin^2(x) + \sqrt{5} + 5) + \frac{1}{5} \log(\sin(x))$$

[Out] 1/5*ln(sin(x))-1/20*ln(5-8*sin(x)^2+5^(1/2))*(-5^(1/2)+1)-1/20*ln(5-8*sin(x)^2-5^(1/2))*(5^(1/2)+1)

Rubi [A] time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {4356, 1114, 705, 29, 632, 31}

$$-\frac{1}{20}(1 + \sqrt{5}) \log(-8 \sin^2(x) - \sqrt{5} + 5) - \frac{1}{20}(1 - \sqrt{5}) \log(-8 \sin^2(x) + \sqrt{5} + 5) + \frac{1}{5} \log(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Csc[5*x], x]

[Out] Log[Sin[x]]/5 - ((1 + Sqrt[5])*Log[5 - Sqrt[5] - 8*Sin[x]^2])/20 - ((1 - Sqrt[5])*Log[5 + Sqrt[5] - 8*Sin[x]^2])/20

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 705

Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]

Rule 1114

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 4356

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned}
\int \cos(x) \csc(5x) dx &= \text{Subst} \left(\int \frac{1}{x(5 - 20x^2 + 16x^4)} dx, x, \sin(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(5 - 20x + 16x^2)} dx, x, \sin^2(x) \right) \\
&= \frac{1}{10} \text{Subst} \left(\int \frac{1}{x} dx, x, \sin^2(x) \right) + \frac{1}{10} \text{Subst} \left(\int \frac{20 - 16x}{5 - 20x + 16x^2} dx, x, \sin^2(x) \right) \\
&= \frac{1}{5} \log(\sin(x)) - \frac{1}{5} (4(1 - \sqrt{5})) \text{Subst} \left(\int \frac{1}{-10 - 2\sqrt{5} + 16x} dx, x, \sin^2(x) \right) - \frac{1}{5} (4(1 + \sqrt{5})) \text{Subst} \left(\int \frac{1}{-10 + 2\sqrt{5} + 16x} dx, x, \sin^2(x) \right) \\
&= \frac{1}{5} \log(\sin(x)) - \frac{1}{20} (1 + \sqrt{5}) \log(5 - \sqrt{5} - 8 \sin^2(x)) - \frac{1}{20} (1 - \sqrt{5}) \log(5 + \sqrt{5} - 8 \sin^2(x))
\end{aligned}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.92

$$\frac{1}{20} (4 \log(\sin(x)) - ((1 + \sqrt{5}) \log(4 \cos(2x) - \sqrt{5} + 1)) + (\sqrt{5} - 1) \log(4 \cos(2x) + \sqrt{5} + 1))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Csc[5*x],x]

[Out] (-((1 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cos[2*x]]) + (-1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Cos[2*x]] + 4*Log[Sin[x]])/20

fricas [A] time = 1.39, size = 72, normalized size = 1.16

$$\frac{1}{20} \sqrt{5} \log \left(\frac{32 \cos(x)^4 + 8(\sqrt{5} - 3) \cos(x)^2 - 3\sqrt{5} + 7}{16 \cos(x)^4 - 12 \cos(x)^2 + 1} \right) - \frac{1}{20} \log(16 \cos(x)^4 - 12 \cos(x)^2 + 1) + \frac{1}{5} \log \left(\frac{1}{2} \sin(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(5*x),x, algorithm="fricas")

[Out] 1/20*sqrt(5)*log((32*cos(x)^4 + 8*(sqrt(5) - 3)*cos(x)^2 - 3*sqrt(5) + 7)/(16*cos(x)^4 - 12*cos(x)^2 + 1)) - 1/20*log(16*cos(x)^4 - 12*cos(x)^2 + 1) + 1/5*log(1/2*sin(x))

giac [A] time = 0.14, size = 67, normalized size = 1.08

$$-\frac{1}{20} \sqrt{5} \log \left(\frac{|32 \cos(x)^2 - 4\sqrt{5} - 12|}{|32 \cos(x)^2 + 4\sqrt{5} - 12|} \right) + \frac{1}{10} \log(-\cos(x)^2 + 1) - \frac{1}{20} \log(|16 \cos(x)^4 - 12 \cos(x)^2 + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(5*x),x, algorithm="giac")

[Out] -1/20*sqrt(5)*log(abs(32*cos(x)^2 - 4*sqrt(5) - 12)/abs(32*cos(x)^2 + 4*sqrt(5) - 12)) + 1/10*log(-cos(x)^2 + 1) - 1/20*log(abs(16*cos(x)^4 - 12*cos(x)^2 + 1))

maple [A] time = 0.26, size = 80, normalized size = 1.29

$$\frac{\ln(4(\cos^2(x)) + 2 \cos(x) - 1)}{20} - \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(8 \cos(x) + 2)\sqrt{5}}{10}\right)}{10} + \frac{\ln(-1 + \cos(x))}{10} - \frac{\ln(4(\cos^2(x)) - 2 \cos(x) - 1)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(5*x),x)
```

```
[Out] -1/20*ln(4*cos(x)^2+2*cos(x)-1)-1/10*5^(1/2)*arctanh(1/10*(8*cos(x)+2)*5^(1/2))+1/10*ln(-1+cos(x))-1/20*ln(4*cos(x)^2-2*cos(x)-1)+1/10*5^(1/2)*arctanh(1/10*(8*cos(x)-2)*5^(1/2))+1/10*ln(1+cos(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(5*x),x, algorithm="maxima")
```

```
[Out] -1/10*integrate(-(cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) + cos(3/2*arctan2(sin(2*x), cos(2*x))))*sin(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x))))*sin(2*x) - cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) - cos(2*x)*sin(1/2*arctan2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 - 2*(cos(4*x) + cos(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x)))) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x), cos(2*x)))^2 - 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), cos(2*x))) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*(sin(4*x) + sin(2*x) - sin(1/2*arctan2(sin(2*x), cos(2*x))))*sin(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(2*x), cos(2*x)))^2 - 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*x))) + sin(1/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) + 1/10*integrate((cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) - cos(3/2*arctan2(sin(2*x), cos(2*x))))*sin(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x))))*sin(2*x) + cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) + cos(2*x)*sin(1/2*arctan2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + 2*(cos(4*x) + cos(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x)))) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), cos(2*x))) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*x) + sin(2*x)^2 + 2*(sin(4*x) + sin(2*x) + sin(1/2*arctan2(sin(2*x), cos(2*x))))*sin(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*x))) + sin(1/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) - 1/10*integrate((cos(x)*sin(4*x) + cos(x)*sin(3*x) + cos(x)*sin(2*x) - cos(4*x)*sin(x) - cos(3*x)*sin(x) - cos(2*x)*sin(x) - sin(x))/(2*(cos(3*x) + cos(2*x) + cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 2*(cos(2*x) + cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + 2*(sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + sin(4*x)^2 + 2*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1), x) + 1/10*integrate(-(cos(x)*sin(4*x) - cos(x)*sin(3*x) + cos(x)*sin(2*x) - cos(4*x)*sin(x) + cos(3*x)*sin(x) - cos(2*x)*sin(x) - sin(x))/(2*(cos(3*x) - cos(2*x) + cos(x) - 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x) - cos(x) + 1)*cos(3*x) - cos(3*x)^2 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + 2*(sin(3*x) - sin(2*x) + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2 + 2*cos(x) - 1), x) + 3/10*integrate(-(cos(4/3*arctan2(sin(3*x), cos(3*x))))*sin(3*x) + cos(2/3*arctan2(sin(3*x), cos(3*x))))*sin(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x))))*sin(3*x) - cos(3*x)*sin(4/3*arctan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(2/3*arctan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(3*x))/(cos(3*x)^2 + 2*(cos(3*x) + cos(2/3*arctan2(sin(3*x), cos(3*x)))) + cos(1/3*arctan2(sin(3*x), cos(3*x))) + 1)*cos(4/3*arctan2(sin(3*x), cos(3*x))) + cos(4/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x)))) + 1)*cos(2/3*arctan2(sin(3*x), cos(3*x))) + cos(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) + 1)*cos(1/3*arctan2(sin(3*x), cos(3*x)))
```

), cos(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*x)))^2 + sin(3*x)^2 + 2*(sin(3*x) + sin(2/3*arctan2(sin(3*x), cos(3*x)))) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(4/3*arctan2(sin(3*x), cos(3*x))) + sin(4/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(sin(3*x) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(2/3*arctan2(sin(3*x), cos(3*x))) + sin(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*sin(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(1/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*cos(3*x) + 1), x) - 3/10*integrate(-(cos(4/3*arctan2(sin(3*x), cos(3*x))))*sin(3*x) + cos(2/3*arctan2(sin(3*x), cos(3*x))))*sin(3*x) - cos(1/3*arctan2(sin(3*x), cos(3*x))))*sin(3*x) - cos(3*x)*sin(4/3*arctan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(2/3*arctan2(sin(3*x), cos(3*x))) + cos(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(3*x))/(cos(3*x)^2 - 2*(cos(3*x) - cos(2/3*arctan2(sin(3*x), cos(3*x)))) + cos(1/3*arctan2(sin(3*x), cos(3*x)))) - 1)*cos(4/3*arctan2(sin(3*x), cos(3*x))) + cos(4/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*(cos(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x)))) - 1)*cos(2/3*arctan2(sin(3*x), cos(3*x))) + cos(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) - 1)*cos(1/3*arctan2(sin(3*x), cos(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*x)))^2 + sin(3*x)^2 - 2*(sin(3*x) - sin(2/3*arctan2(sin(3*x), cos(3*x)))) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(4/3*arctan2(sin(3*x), cos(3*x))) + sin(4/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*(sin(3*x) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(2/3*arctan2(sin(3*x), cos(3*x))) + sin(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*sin(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(1/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*cos(3*x) + 1), x) + 1/5*integrate((sin(4*x) + sin(3*x) + sin(2*x) + sin(x))/(2*(cos(3*x) + cos(2*x) + cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 2*(cos(2*x) + cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + 2*(sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + sin(4*x)^2 + 2*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1), x) + 1/5*integrate(-(sin(4*x) - sin(3*x) + sin(2*x) - sin(x))/(2*(cos(3*x) - cos(2*x) + cos(x) - 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x) - cos(x) + 1)*cos(3*x) - cos(3*x)^2 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + 2*(sin(3*x) - sin(2*x) + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2 + 2*cos(x) - 1), x) + 1/10*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/10*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)

mupad [B] time = 2.68, size = 51, normalized size = 0.82

$$\frac{\ln(\sin(x))}{5} + \ln\left(-\cos(x)^2 - \frac{\sqrt{5}}{8} + \frac{3}{8}\right)\left(\frac{\sqrt{5}}{20} - \frac{1}{20}\right) - \ln\left(-\cos(x)^2 + \frac{\sqrt{5}}{8} + \frac{3}{8}\right)\left(\frac{\sqrt{5}}{20} + \frac{1}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/sin(5*x), x)

[Out] log(sin(x))/5 + log(3/8 - 5^(1/2)/8 - cos(x)^2)*(5^(1/2)/20 - 1/20) - log(5^(1/2)/8 - cos(x)^2 + 3/8)*(5^(1/2)/20 + 1/20)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \csc(5x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(5*x), x)

[Out] Integral(cos(x)*csc(5*x), x)

3.127 $\int \cos(x) \csc(6x) dx$

Optimal. Leaf size=36

$$-\frac{1}{6} \tanh^{-1}(\cos(x)) - \frac{1}{6} \tanh^{-1}(2 \cos(x)) + \frac{\tanh^{-1}\left(\frac{2 \cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

[Out] $-1/6*\operatorname{arctanh}(\cos(x))-1/6*\operatorname{arctanh}(2*\cos(x))+1/6*\operatorname{arctanh}(2/3*\cos(x)*3^{(1/2)})*3^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {12, 2057, 207}

$$-\frac{1}{6} \tanh^{-1}(\cos(x)) - \frac{1}{6} \tanh^{-1}(2 \cos(x)) + \frac{\tanh^{-1}\left(\frac{2 \cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Csc[6*x],x]

[Out] $-\operatorname{ArcTanh}[\cos(x)]/6 - \operatorname{ArcTanh}[2*\cos(x)]/6 + \operatorname{ArcTanh}[(2*\cos(x))/\operatorname{Sqrt}[3]]/(2*\operatorname{Sqrt}[3])$

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2057

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(u /. x -> x^2)^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x^2] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \cos(x) \csc(6x) dx &= -\operatorname{Subst}\left(\int \frac{1}{2(3-19x^2+32x^4-16x^6)} dx, x, \cos(x)\right) \\ &= -\left(\frac{1}{2} \operatorname{Subst}\left(\int \frac{1}{3-19x^2+32x^4-16x^6} dx, x, \cos(x)\right)\right) \\ &= -\left(\frac{1}{2} \operatorname{Subst}\left(\int \left(-\frac{1}{3(-1+x^2)} + \frac{2}{-3+4x^2} - \frac{2}{3(-1+4x^2)}\right) dx, x, \cos(x)\right)\right) \\ &= \frac{1}{6} \operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \cos(x)\right) + \frac{1}{3} \operatorname{Subst}\left(\int \frac{1}{-1+4x^2} dx, x, \cos(x)\right) - \operatorname{Subst}\left(\int \frac{1}{-1+4x^2} dx, x, \cos(x)\right) \\ &= -\frac{1}{6} \tanh^{-1}(\cos(x)) - \frac{1}{6} \tanh^{-1}(2 \cos(x)) + \frac{\tanh^{-1}\left(\frac{2 \cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}} \end{aligned}$$

Mathematica [B] time = 0.08, size = 83, normalized size = 2.31

$$\frac{1}{12} \left(2 \log \left(\sin \left(\frac{x}{2} \right) \right) - 2 \log \left(\cos \left(\frac{x}{2} \right) \right) + \log(1 - 2 \cos(x)) - \log(2 \cos(x) + 1) - 2\sqrt{3} \tanh^{-1} \left(\frac{\tan \left(\frac{x}{2} \right) - 2}{\sqrt{3}} \right) + 2\sqrt{3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Csc[6*x],x]

[Out] (-2*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] + 2*Sqrt[3]*ArcTanh[(2 + Tan[x/2])/Sqrt[3]] - 2*Log[Cos[x/2]] + Log[1 - 2*Cos[x]] - Log[1 + 2*Cos[x]] + 2*Log[Sin[x/2]])/12

fricas [B] time = 0.71, size = 70, normalized size = 1.94

$$\frac{1}{12} \sqrt{3} \log \left(-\frac{4 \cos(x)^2 + 4 \sqrt{3} \cos(x) + 3}{4 \cos(x)^2 - 3} \right) - \frac{1}{12} \log \left(\frac{1}{2} \cos(x) + \frac{1}{2} \right) + \frac{1}{12} \log \left(-\frac{1}{2} \cos(x) + \frac{1}{2} \right) + \frac{1}{12} \log(-2 \cos(x) + 1) - \frac{1}{12} \log(-2 \cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(6*x),x, algorithm="fricas")

[Out] 1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) - 1/12*log(1/2*cos(x) + 1/2) + 1/12*log(-1/2*cos(x) + 1/2) + 1/12*log(-2*cos(x) + 1) - 1/12*log(-2*cos(x) - 1)

giac [B] time = 0.15, size = 68, normalized size = 1.89

$$-\frac{1}{12} \sqrt{3} \log \left(\frac{|-4 \sqrt{3} + 8 \cos(x)|}{|4 \sqrt{3} + 8 \cos(x)|} \right) - \frac{1}{12} \log(\cos(x) + 1) + \frac{1}{12} \log(-\cos(x) + 1) - \frac{1}{12} \log(|2 \cos(x) + 1|) + \frac{1}{12} \log(|-2 \cos(x) - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(6*x),x, algorithm="giac")

[Out] -1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*cos(x))/abs(4*sqrt(3) + 8*cos(x))) - 1/12*log(cos(x) + 1) + 1/12*log(-cos(x) + 1) - 1/12*log(abs(2*cos(x) + 1)) + 1/12*log(abs(-2*cos(x) - 1))

maple [A] time = 0.17, size = 47, normalized size = 1.31

$$\frac{\ln(2 \cos(x) - 1)}{12} - \frac{\ln(1 + 2 \cos(x))}{12} + \frac{\operatorname{arctanh} \left(\frac{2 \cos(x) \sqrt{3}}{3} \right) \sqrt{3}}{6} + \frac{\ln(-1 + \cos(x))}{12} - \frac{\ln(1 + \cos(x))}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*csc(6*x),x)

[Out] 1/12*ln(2*cos(x)-1)-1/12*ln(1+2*cos(x))+1/6*arctanh(2/3*cos(x)*3^(1/2))*3^(1/2)+1/12*ln(-1+cos(x))-1/12*ln(1+cos(x))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(\sin(3x) - \sin(x)) \cos(4x) - (\cos(3x) - \cos(x)) \sin(4x) - (\cos(2x) - 1) \sin(3x) + \cos(3x) \sin(2x) - \cos(2x) \sin(x)}{2 \left((\cos(2x) - 1) \cos(4x) - \cos(4x)^2 - \cos(2x)^2 - \sin(4x)^2 + 2 \sin(4x) \sin(2x) - \sin(2x)^2 \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(6*x),x, algorithm="maxima")

```
[Out] -integrate(1/2*((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*sin(4*x)
- (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + cos(2*x)
*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - si
n(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) - 1/24*lo
g(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2*sin(2*x)
*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/24*log(-2*(cos(x) - 1)*cos(2*x) + co
s(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2 - 2*cos(x)
+ 1) - 1/12*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/12*log(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1)
```

mupad [B] time = 2.37, size = 74, normalized size = 2.06

$$\frac{\operatorname{atanh}\left(\frac{1073741824}{10761687\left(\frac{427973089951744 \tan\left(\frac{x}{2}\right)^2}{14348907} - \frac{47552804159488}{4782969}\right)} + \frac{797161}{797162}\right) \ln\left(\tan\left(\frac{x}{2}\right)\right) + \sqrt{3} \operatorname{atanh}\left(\frac{42227694326251}{4782969\left(\frac{101871591633190912 \tan\left(\frac{x}{2}\right)}{4782969}\right)}\right)}{6} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{6} + \frac{\sqrt{3} \operatorname{atanh}\left(\frac{42227694326251}{4782969\left(\frac{101871591633190912 \tan\left(\frac{x}{2}\right)}{4782969}\right)}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(6*x), x)
```

```
[Out] atanh(1073741824/(10761687*((427973089951744*tan(x/2)^2)/14348907 - 4755280
4159488/4782969)) + 797161/797162)/6 + log(tan(x/2))/6 + (3^(1/2)*atanh((42
22769432625152*3^(1/2))/(4782969*((101871591633190912*tan(x/2)^2)/4782969 -
7314051205955584/4782969)) - (19605196950732800*3^(1/2)*tan(x/2)^2)/(15943
23*((101871591633190912*tan(x/2)^2)/4782969 - 7314051205955584/4782969))))/
6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \csc(6x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(6*x), x)
```

```
[Out] Integral(cos(x)*csc(6*x), x)
```

3.128 $\int \cos^3(6x) \sin(x) dx$

Optimal. Leaf size=33

$$\frac{3}{40} \cos(5x) - \frac{3}{56} \cos(7x) + \frac{1}{136} \cos(17x) - \frac{1}{152} \cos(19x)$$

[Out] 3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)-1/152*cos(19*x)

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4354, 2638}

$$\frac{3}{40} \cos(5x) - \frac{3}{56} \cos(7x) + \frac{1}{136} \cos(17x) - \frac{1}{152} \cos(19x)$$

Antiderivative was successfully verified.

[In] Int[Cos[6*x]^3*Sin[x],x]

[Out] (3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 - Cos[19*x]/152

Rule 2638

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4354

Int[(F_)[(a_.) + (b_.)*(x_.)]^(p_.)*(G_)[(c_.) + (d_.)*(x_.)]^(q_.), x_Symbol] := Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \cos^3(6x) \sin(x) dx &= \int \left(-\frac{3}{8} \sin(5x) + \frac{3}{8} \sin(7x) - \frac{1}{8} \sin(17x) + \frac{1}{8} \sin(19x) \right) dx \\ &= -\left(\frac{1}{8} \int \sin(17x) dx \right) + \frac{1}{8} \int \sin(19x) dx - \frac{3}{8} \int \sin(5x) dx + \frac{3}{8} \int \sin(7x) dx \\ &= \frac{3}{40} \cos(5x) - \frac{3}{56} \cos(7x) + \frac{1}{136} \cos(17x) - \frac{1}{152} \cos(19x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{3}{40} \cos(5x) - \frac{3}{56} \cos(7x) + \frac{1}{136} \cos(17x) - \frac{1}{152} \cos(19x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[6*x]^3*Sin[x],x]

[Out] (3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 - Cos[19*x]/152

fricas [B] time = 3.97, size = 57, normalized size = 1.73

$$-\frac{32768}{19} \cos(x)^{19} + \frac{147456}{17} \cos(x)^{17} - 18432 \cos(x)^{15} + 21504 \cos(x)^{13} - 14976 \cos(x)^{11} + 6336 \cos(x)^9 - \frac{11112}{7} \cos(x)^7 + \frac{11112}{7} \cos(x)^5 - \frac{11112}{7} \cos(x)^3 + \frac{11112}{7} \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(6*x)^3*sin(x),x, algorithm="fricas")

[Out] $-32768/19*\cos(x)^{19} + 147456/17*\cos(x)^{17} - 18432*\cos(x)^{15} + 21504*\cos(x)^{13} - 14976*\cos(x)^{11} + 6336*\cos(x)^9 - 11112/7*\cos(x)^7 + 1116/5*\cos(x)^5 - 18*\cos(x)^3 + \cos(x)$

giac [A] time = 0.13, size = 25, normalized size = 0.76

$$-\frac{1}{152} \cos(19x) + \frac{1}{136} \cos(17x) - \frac{3}{56} \cos(7x) + \frac{3}{40} \cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(6*x)^3*sin(x),x, algorithm="giac")

[Out] $-1/152*\cos(19*x) + 1/136*\cos(17*x) - 3/56*\cos(7*x) + 3/40*\cos(5*x)$

maple [A] time = 0.25, size = 26, normalized size = 0.79

$$\frac{3 \cos(5x)}{40} - \frac{3 \cos(7x)}{56} + \frac{\cos(17x)}{136} - \frac{\cos(19x)}{152}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(6*x)^3*sin(x),x)

[Out] $3/40*\cos(5*x) - 3/56*\cos(7*x) + 1/136*\cos(17*x) - 1/152*\cos(19*x)$

maxima [A] time = 0.45, size = 25, normalized size = 0.76

$$-\frac{1}{152} \cos(19x) + \frac{1}{136} \cos(17x) - \frac{3}{56} \cos(7x) + \frac{3}{40} \cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(6*x)^3*sin(x),x, algorithm="maxima")

[Out] $-1/152*\cos(19*x) + 1/136*\cos(17*x) - 3/56*\cos(7*x) + 3/40*\cos(5*x)$

mupad [B] time = 0.08, size = 57, normalized size = 1.73

$$-\frac{32768 \cos(x)^{19}}{19} + \frac{147456 \cos(x)^{17}}{17} - 18432 \cos(x)^{15} + 21504 \cos(x)^{13} - 14976 \cos(x)^{11} + 6336 \cos(x)^9 - \frac{11112 \cos(x)^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(6*x)^3*sin(x),x)

[Out] $\cos(x) - 18*\cos(x)^3 + (1116*\cos(x)^5)/5 - (11112*\cos(x)^7)/7 + 6336*\cos(x)^9 - 14976*\cos(x)^{11} + 21504*\cos(x)^{13} - 18432*\cos(x)^{15} + (147456*\cos(x)^{17})/7 - (32768*\cos(x)^{19})/19$

sympy [B] time = 5.21, size = 63, normalized size = 1.91

$$\frac{1296 \sin(x) \sin^3(6x)}{11305} + \frac{1926 \sin(x) \sin(6x) \cos^2(6x)}{11305} + \frac{216 \sin^2(6x) \cos(x) \cos(6x)}{11305} + \frac{251 \cos(x) \cos^3(6x)}{11305}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(6*x)**3*sin(x),x)

[Out] $1296*\sin(x)*\sin(6*x)**3/11305 + 1926*\sin(x)*\sin(6*x)*\cos(6*x)**2/11305 + 216*\sin(6*x)**2*\cos(x)*\cos(6*x)/11305 + 251*\cos(x)*\cos(6*x)**3/11305$

3.129 $\int \cos^3(6x) \sin(9x) dx$

Optimal. Leaf size=33

$$-\frac{1}{8} \cos(3x) + \frac{1}{72} \cos(9x) - \frac{1}{40} \cos(15x) - \frac{1}{216} \cos(27x)$$

[Out] $-1/8*\cos(3*x)+1/72*\cos(9*x)-1/40*\cos(15*x)-1/216*\cos(27*x)$

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4354, 2638}

$$-\frac{1}{8} \cos(3x) + \frac{1}{72} \cos(9x) - \frac{1}{40} \cos(15x) - \frac{1}{216} \cos(27x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[6*x]^3*\text{Sin}[9*x], x]$

[Out] $-\text{Cos}[3*x]/8 + \text{Cos}[9*x]/72 - \text{Cos}[15*x]/40 - \text{Cos}[27*x]/216$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 4354

$\text{Int}[(F_)[(a_.) + (b_.)*(x_.)]^{(p_.)}*(G_)[(c_.) + (d_.)*(x_.)]^{(q_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[\text{ActivateTrig}[F[a + b*x]^p*G[c + d*x]^q], x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ (\text{EqQ}[F, \sin] \ || \ \text{EqQ}[F, \cos]) \ \&\& \ (\text{EqQ}[G, \sin] \ || \ \text{EqQ}[G, \cos]) \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{IGtQ}[q, 0]$

Rubi steps

$$\begin{aligned} \int \cos^3(6x) \sin(9x) dx &= \int \left(\frac{3}{8} \sin(3x) - \frac{1}{8} \sin(9x) + \frac{3}{8} \sin(15x) + \frac{1}{8} \sin(27x) \right) dx \\ &= -\left(\frac{1}{8} \int \sin(9x) dx \right) + \frac{1}{8} \int \sin(27x) dx + \frac{3}{8} \int \sin(3x) dx + \frac{3}{8} \int \sin(15x) dx \\ &= -\frac{1}{8} \cos(3x) + \frac{1}{72} \cos(9x) - \frac{1}{40} \cos(15x) - \frac{1}{216} \cos(27x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$-\frac{1}{8} \cos(3x) + \frac{1}{72} \cos(9x) - \frac{1}{40} \cos(15x) - \frac{1}{216} \cos(27x)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Cos}[6*x]^3*\text{Sin}[9*x], x]$

[Out] $-1/8*\text{Cos}[3*x] + \text{Cos}[9*x]/72 - \text{Cos}[15*x]/40 - \text{Cos}[27*x]/216$

fricas [A] time = 0.58, size = 39, normalized size = 1.18

$$-\frac{32}{27} \cos(3x)^9 + \frac{8}{3} \cos(3x)^7 - \frac{12}{5} \cos(3x)^5 + \frac{10}{9} \cos(3x)^3 - \frac{1}{3} \cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(6*x)^3*sin(9*x),x, algorithm="fricas")

[Out] $-32/27*\cos(3*x)^9 + 8/3*\cos(3*x)^7 - 12/5*\cos(3*x)^5 + 10/9*\cos(3*x)^3 - 1/3*\cos(3*x)$

giac [A] time = 0.14, size = 25, normalized size = 0.76

$$-\frac{1}{216} \cos(27x) - \frac{1}{40} \cos(15x) + \frac{1}{72} \cos(9x) - \frac{1}{8} \cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(6*x)^3*sin(9*x),x, algorithm="giac")

[Out] $-1/216*\cos(27*x) - 1/40*\cos(15*x) + 1/72*\cos(9*x) - 1/8*\cos(3*x)$

maple [A] time = 0.14, size = 26, normalized size = 0.79

$$-\frac{\cos(3x)}{8} + \frac{\cos(9x)}{72} - \frac{\cos(15x)}{40} - \frac{\cos(27x)}{216}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(6*x)^3*sin(9*x),x)

[Out] $-1/8*\cos(3*x)+1/72*\cos(9*x)-1/40*\cos(15*x)-1/216*\cos(27*x)$

maxima [A] time = 0.31, size = 25, normalized size = 0.76

$$-\frac{1}{216} \cos(27x) - \frac{1}{40} \cos(15x) + \frac{1}{72} \cos(9x) - \frac{1}{8} \cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(6*x)^3*sin(9*x),x, algorithm="maxima")

[Out] $-1/216*\cos(27*x) - 1/40*\cos(15*x) + 1/72*\cos(9*x) - 1/8*\cos(3*x)$

mupad [B] time = 2.47, size = 78, normalized size = 2.36

$$\frac{2 \left(135 \tan\left(\frac{3x}{2}\right)^{16} - 900 \tan\left(\frac{3x}{2}\right)^{14} + 5640 \tan\left(\frac{3x}{2}\right)^{12} - 13140 \tan\left(\frac{3x}{2}\right)^{10} + 15534 \tan\left(\frac{3x}{2}\right)^8 - 4044 \tan\left(\frac{3x}{2}\right) \right)}{135 \left(\tan\left(\frac{3x}{2}\right)^2 + 1 \right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(6*x)^3*sin(9*x),x)

[Out] $-(2*(36*\tan((3*x)/2)^2 + 1584*\tan((3*x)/2)^4 - 4044*\tan((3*x)/2)^6 + 15534*\tan((3*x)/2)^8 - 13140*\tan((3*x)/2)^{10} + 5640*\tan((3*x)/2)^{12} - 900*\tan((3*x)/2)^{14} + 135*\tan((3*x)/2)^{16} + 19))/(135*(\tan((3*x)/2)^2 + 1)^9)$

sympy [B] time = 5.73, size = 71, normalized size = 2.15

$$\frac{16 \sin^3(6x) \sin(9x)}{135} - \frac{8 \sin^2(6x) \cos(6x) \cos(9x)}{45} - \frac{2 \sin(6x) \sin(9x) \cos^2(6x)}{45} - \frac{19 \cos^3(6x) \cos(9x)}{135}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(6*x)**3*sin(9*x),x)

[Out] $-16*\sin(6*x)**3*\sin(9*x)/135 - 8*\sin(6*x)**2*\cos(6*x)*\cos(9*x)/45 - 2*\sin(6*x)*\sin(9*x)*\cos(6*x)**2/45 - 19*\cos(6*x)**3*\cos(9*x)/135$

3.130 $\int \cos(2x) \sin^2(6x) dx$

Optimal. Leaf size=25

$$\frac{1}{4} \sin(2x) - \frac{1}{40} \sin(10x) - \frac{1}{56} \sin(14x)$$

[Out] 1/4*sin(2*x)-1/40*sin(10*x)-1/56*sin(14*x)

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4354, 2637}

$$\frac{1}{4} \sin(2x) - \frac{1}{40} \sin(10x) - \frac{1}{56} \sin(14x)$$

Antiderivative was successfully verified.

[In] Int[Cos[2*x]*Sin[6*x]^2,x]

[Out] Sin[2*x]/4 - Sin[10*x]/40 - Sin[14*x]/56

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4354

Int[(F_)[(a_.) + (b_.)*(x_.)]^(p_.)*(G_)[(c_.) + (d_.)*(x_.)]^(q_.), x_Symbol] := Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \cos(2x) \sin^2(6x) dx &= \int \left(\frac{1}{2} \cos(2x) - \frac{1}{4} \cos(10x) - \frac{1}{4} \cos(14x) \right) dx \\ &= -\left(\frac{1}{4} \int \cos(10x) dx \right) - \frac{1}{4} \int \cos(14x) dx + \frac{1}{2} \int \cos(2x) dx \\ &= \frac{1}{4} \sin(2x) - \frac{1}{40} \sin(10x) - \frac{1}{56} \sin(14x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{1}{4} \sin(2x) - \frac{1}{40} \sin(10x) - \frac{1}{56} \sin(14x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[2*x]*Sin[6*x]^2,x]

[Out] Sin[2*x]/4 - Sin[10*x]/40 - Sin[14*x]/56

fricas [A] time = 2.01, size = 32, normalized size = 1.28

$$-\frac{1}{70} (80 \cos(2x)^6 - 72 \cos(2x)^4 + 9 \cos(2x)^2 - 17) \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*sin(6*x)^2,x, algorithm="fricas")

[Out] -1/70*(80*cos(2*x)^6 - 72*cos(2*x)^4 + 9*cos(2*x)^2 - 17)*sin(2*x)

giac [A] time = 0.12, size = 19, normalized size = 0.76

$$-\frac{1}{56} \sin(14x) - \frac{1}{40} \sin(10x) + \frac{1}{4} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*sin(6*x)^2,x, algorithm="giac")

[Out] -1/56*sin(14*x) - 1/40*sin(10*x) + 1/4*sin(2*x)

maple [A] time = 0.12, size = 20, normalized size = 0.80

$$\frac{\sin(2x)}{4} - \frac{\sin(10x)}{40} - \frac{\sin(14x)}{56}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)*sin(6*x)^2,x)

[Out] 1/4*sin(2*x)-1/40*sin(10*x)-1/56*sin(14*x)

maxima [A] time = 0.32, size = 19, normalized size = 0.76

$$-\frac{1}{56} \sin(14x) - \frac{1}{40} \sin(10x) + \frac{1}{4} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*sin(6*x)^2,x, algorithm="maxima")

[Out] -1/56*sin(14*x) - 1/40*sin(10*x) + 1/4*sin(2*x)

mupad [B] time = 2.29, size = 25, normalized size = 1.00

$$\frac{8 \sin(2x)^7}{7} - \frac{12 \sin(2x)^5}{5} + \frac{3 \sin(2x)^3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)*sin(6*x)^2,x)

[Out] (3*sin(2*x)^3)/2 - (12*sin(2*x)^5)/5 + (8*sin(2*x)^7)/7

sympy [B] time = 1.73, size = 48, normalized size = 1.92

$$\frac{17 \sin(2x) \sin^2(6x)}{70} + \frac{9 \sin(2x) \cos^2(6x)}{35} - \frac{3 \sin(6x) \cos(2x) \cos(6x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*sin(6*x)**2,x)

[Out] 17*sin(2*x)*sin(6*x)**2/70 + 9*sin(2*x)*cos(6*x)**2/35 - 3*sin(6*x)*cos(2*x)*cos(6*x)/35

3.131 $\int \cos(x) \sin^2(6x) dx$

Optimal. Leaf size=23

$$\frac{\sin(x)}{2} - \frac{1}{44} \sin(11x) - \frac{1}{52} \sin(13x)$$

[Out] 1/2*sin(x)-1/44*sin(11*x)-1/52*sin(13*x)

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4354, 2637}

$$\frac{\sin(x)}{2} - \frac{1}{44} \sin(11x) - \frac{1}{52} \sin(13x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[6*x]^2,x]

[Out] Sin[x]/2 - Sin[11*x]/44 - Sin[13*x]/52

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 4354

Int[(F_)[(a_.) + (b_.)*(x_.)]^(p_.)*(G_)[(c_.) + (d_.)*(x_.)]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /;
FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \cos(x) \sin^2(6x) dx &= \int \left(\frac{\cos(x)}{2} - \frac{1}{4} \cos(11x) - \frac{1}{4} \cos(13x) \right) dx \\ &= -\left(\frac{1}{4} \int \cos(11x) dx \right) - \frac{1}{4} \int \cos(13x) dx + \frac{1}{2} \int \cos(x) dx \\ &= \frac{\sin(x)}{2} - \frac{1}{44} \sin(11x) - \frac{1}{52} \sin(13x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\sin(x)}{2} - \frac{1}{44} \sin(11x) - \frac{1}{52} \sin(13x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sin[6*x]^2,x]

[Out] Sin[x]/2 - Sin[11*x]/44 - Sin[13*x]/52

fricas [B] time = 0.60, size = 42, normalized size = 1.83

$$-\frac{4}{143} (2816 \cos(x)^{12} - 6912 \cos(x)^{10} + 6048 \cos(x)^8 - 2240 \cos(x)^6 + 315 \cos(x)^4 - 9 \cos(x)^2 - 18) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(6*x)^2,x, algorithm="fricas")

[Out] $-4/143*(2816*\cos(x)^{12} - 6912*\cos(x)^{10} + 6048*\cos(x)^8 - 2240*\cos(x)^6 + 315*\cos(x)^4 - 9*\cos(x)^2 - 18)*\sin(x)$

giac [A] time = 0.13, size = 17, normalized size = 0.74

$$-\frac{1}{52} \sin(13x) - \frac{1}{44} \sin(11x) + \frac{1}{2} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(6*x)^2,x, algorithm="giac")

[Out] $-1/52*\sin(13*x) - 1/44*\sin(11*x) + 1/2*\sin(x)$

maple [A] time = 0.13, size = 18, normalized size = 0.78

$$\frac{\sin(x)}{2} - \frac{\sin(11x)}{44} - \frac{\sin(13x)}{52}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(6*x)^2,x)

[Out] $1/2*\sin(x)-1/44*\sin(11*x)-1/52*\sin(13*x)$

maxima [A] time = 0.31, size = 17, normalized size = 0.74

$$-\frac{1}{52} \sin(13x) - \frac{1}{44} \sin(11x) + \frac{1}{2} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(6*x)^2,x, algorithm="maxima")

[Out] $-1/52*\sin(13*x) - 1/44*\sin(11*x) + 1/2*\sin(x)$

mupad [B] time = 2.49, size = 17, normalized size = 0.74

$$\frac{\sin(x)}{2} - \frac{\sin(13x)}{52} - \frac{\sin(11x)}{44}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(6*x)^2*cos(x),x)

[Out] $\sin(x)/2 - \sin(13*x)/52 - \sin(11*x)/44$

sympy [B] time = 1.64, size = 42, normalized size = 1.83

$$\frac{71 \sin(x) \sin^2(6x)}{143} + \frac{72 \sin(x) \cos^2(6x)}{143} - \frac{12 \sin(6x) \cos(x) \cos(6x)}{143}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(6*x)**2,x)

[Out] $71*\sin(x)*\sin(6*x)**2/143 + 72*\sin(x)*\cos(6*x)**2/143 - 12*\sin(6*x)*\cos(x)*\cos(6*x)/143$

3.132 $\int \cos(x) \sin^3(6x) dx$

Optimal. Leaf size=33

$$-\frac{3}{40} \cos(5x) - \frac{3}{56} \cos(7x) + \frac{1}{136} \cos(17x) + \frac{1}{152} \cos(19x)$$

[Out] -3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)+1/152*cos(19*x)

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4354, 2638}

$$-\frac{3}{40} \cos(5x) - \frac{3}{56} \cos(7x) + \frac{1}{136} \cos(17x) + \frac{1}{152} \cos(19x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[6*x]^3,x]

[Out] (-3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 + Cos[19*x]/152

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4354

Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol] := Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \cos(x) \sin^3(6x) dx &= \int \left(\frac{3}{8} \sin(5x) + \frac{3}{8} \sin(7x) - \frac{1}{8} \sin(17x) - \frac{1}{8} \sin(19x) \right) dx \\ &= -\left(\frac{1}{8} \int \sin(17x) dx \right) - \frac{1}{8} \int \sin(19x) dx + \frac{3}{8} \int \sin(5x) dx + \frac{3}{8} \int \sin(7x) dx \\ &= -\frac{3}{40} \cos(5x) - \frac{3}{56} \cos(7x) + \frac{1}{136} \cos(17x) + \frac{1}{152} \cos(19x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{3}{40} \cos(5x) - \frac{3}{56} \cos(7x) + \frac{1}{136} \cos(17x) + \frac{1}{152} \cos(19x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sin[6*x]^3,x]

[Out] (-3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 + Cos[19*x]/152

fricas [A] time = 0.61, size = 49, normalized size = 1.48

$$\frac{32768}{19} \cos(x)^{19} - \frac{131072}{17} \cos(x)^{17} + 14336 \cos(x)^{15} - 14336 \cos(x)^{13} + 8320 \cos(x)^{11} - 2816 \cos(x)^9 + \frac{3672}{7} \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(6*x)^3,x, algorithm="fricas")

[Out] 32768/19*cos(x)^19 - 131072/17*cos(x)^17 + 14336*cos(x)^15 - 14336*cos(x)^13 + 8320*cos(x)^11 - 2816*cos(x)^9 + 3672/7*cos(x)^7 - 216/5*cos(x)^5

giac [A] time = 0.14, size = 49, normalized size = 1.48

$$\frac{32768}{19} \cos(x)^{19} - \frac{131072}{17} \cos(x)^{17} + 14336 \cos(x)^{15} - 14336 \cos(x)^{13} + 8320 \cos(x)^{11} - 2816 \cos(x)^9 + \frac{3672}{7} \cos(x)^7 - \frac{216}{5} \cos(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(6*x)^3,x, algorithm="giac")

[Out] 32768/19*cos(x)^19 - 131072/17*cos(x)^17 + 14336*cos(x)^15 - 14336*cos(x)^13 + 8320*cos(x)^11 - 2816*cos(x)^9 + 3672/7*cos(x)^7 - 216/5*cos(x)^5

maple [A] time = 0.14, size = 26, normalized size = 0.79

$$-\frac{3 \cos(5x)}{40} - \frac{3 \cos(7x)}{56} + \frac{\cos(17x)}{136} + \frac{\cos(19x)}{152}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(6*x)^3,x)

[Out] -3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)+1/152*cos(19*x)

maxima [A] time = 0.56, size = 25, normalized size = 0.76

$$\frac{1}{152} \cos(19x) + \frac{1}{136} \cos(17x) - \frac{3}{56} \cos(7x) - \frac{3}{40} \cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(6*x)^3,x, algorithm="maxima")

[Out] 1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) - 3/40*cos(5*x)

mupad [B] time = 2.70, size = 150, normalized size = 4.55

$$32 \left(305235 \tan\left(\frac{x}{2}\right)^{34} - 9665775 \tan\left(\frac{x}{2}\right)^{32} + 153838440 \tan\left(\frac{x}{2}\right)^{30} - 1348695544 \tan\left(\frac{x}{2}\right)^{28} + 7083812484 \tan\left(\frac{x}{2}\right)^{26} - 1348695544 \tan\left(\frac{x}{2}\right)^{24} + 153838440 \tan\left(\frac{x}{2}\right)^{22} - 9665775 \tan\left(\frac{x}{2}\right)^{20} + 305235 \tan\left(\frac{x}{2}\right)^{18} - 1348695544 \tan\left(\frac{x}{2}\right)^{16} + 153838440 \tan\left(\frac{x}{2}\right)^{14} - 9665775 \tan\left(\frac{x}{2}\right)^{12} + 305235 \tan\left(\frac{x}{2}\right)^{10} - 1348695544 \tan\left(\frac{x}{2}\right)^8 + 153838440 \tan\left(\frac{x}{2}\right)^6 - 9665775 \tan\left(\frac{x}{2}\right)^4 + 305235 \tan\left(\frac{x}{2}\right)^2 + 81 \right) / (11305 \tan^2\left(\frac{x}{2}\right) + 1)^{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(6*x)^3*cos(x),x)

[Out] -(32*(1539*tan(x/2)^2 - 291384*tan(x/2)^4 + 9744264*tan(x/2)^6 - 153524484*tan(x/2)^8 + 1349637412*tan(x/2)^10 - 7081614792*tan(x/2)^12 + 23582909592*tan(x/2)^14 - 51607368282*tan(x/2)^16 + 75935973762*tan(x/2)^18 - 75928491144*tan(x/2)^20 + 51613490424*tan(x/2)^22 - 23578828164*tan(x/2)^24 + 7083812484*tan(x/2)^26 - 1348695544*tan(x/2)^28 + 153838440*tan(x/2)^30 - 9665775*tan(x/2)^32 + 305235*tan(x/2)^34 + 81))/(11305*(tan(x/2)^2 + 1)^19)

sympy [B] time = 5.19, size = 65, normalized size = 1.97

$$\frac{251 \sin(x) \sin^3(6x)}{11305} - \frac{216 \sin(x) \sin(6x) \cos^2(6x)}{11305} - \frac{1926 \sin^2(6x) \cos(x) \cos(6x)}{11305} - \frac{1296 \cos(x) \cos^3(6x)}{11305}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(6*x)**3,x)

[Out] -251*sin(x)*sin(6*x)**3/11305 - 216*sin(x)*sin(6*x)*cos(6*x)**2/11305 - 1926*sin(6*x)**2*cos(x)*cos(6*x)/11305 - 1296*cos(x)*cos(6*x)**3/11305

3.133 $\int \cos(7x) \sin^3(6x) dx$

Optimal. Leaf size=31

$$\frac{3 \cos(x)}{8} + \frac{1}{88} \cos(11x) - \frac{3}{104} \cos(13x) + \frac{1}{200} \cos(25x)$$

[Out] 3/8*cos(x)+1/88*cos(11*x)-3/104*cos(13*x)+1/200*cos(25*x)

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4354, 2638}

$$\frac{3 \cos(x)}{8} + \frac{1}{88} \cos(11x) - \frac{3}{104} \cos(13x) + \frac{1}{200} \cos(25x)$$

Antiderivative was successfully verified.

[In] Int[Cos[7*x]*Sin[6*x]^3,x]

[Out] (3*Cos[x])/8 + Cos[11*x]/88 - (3*Cos[13*x])/104 + Cos[25*x]/200

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4354

Int[(F_)[(a_.) + (b_.)*(x_)^(p_.)]*(G_)[(c_.) + (d_.)*(x_)^(q_.), x_Symbol] := Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \cos(7x) \sin^3(6x) dx &= \int \left(-\frac{3 \sin(x)}{8} - \frac{1}{8} \sin(11x) + \frac{3}{8} \sin(13x) - \frac{1}{8} \sin(25x) \right) dx \\ &= -\left(\frac{1}{8} \int \sin(11x) dx \right) - \frac{1}{8} \int \sin(25x) dx - \frac{3}{8} \int \sin(x) dx + \frac{3}{8} \int \sin(13x) dx \\ &= \frac{3 \cos(x)}{8} + \frac{1}{88} \cos(11x) - \frac{3}{104} \cos(13x) + \frac{1}{200} \cos(25x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.00

$$\frac{3 \cos(x)}{8} + \frac{1}{88} \cos(11x) - \frac{3}{104} \cos(13x) + \frac{1}{200} \cos(25x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[7*x]*Sin[6*x]^3,x]

[Out] (3*Cos[x])/8 + Cos[11*x]/88 - (3*Cos[13*x])/104 + Cos[25*x]/200

fricas [B] time = 1.29, size = 67, normalized size = 2.16

$$\frac{2097152}{25} \cos(x)^{25} - 524288 \cos(x)^{23} + 1441792 \cos(x)^{21} - 2293760 \cos(x)^{19} + 2334720 \cos(x)^{17} - \frac{7938048}{5} \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(7*x)*sin(6*x)^3,x, algorithm="fricas")

[Out] 2097152/25*cos(x)^25 - 524288*cos(x)^23 + 1441792*cos(x)^21 - 2293760*cos(x)^19 + 2334720*cos(x)^17 - 7938048/5*cos(x)^15 + 9503232/13*cos(x)^13 - 2484992/11*cos(x)^11 + 45248*cos(x)^9 - 5400*cos(x)^7 + 1512/5*cos(x)^5

giac [A] time = 0.14, size = 23, normalized size = 0.74

$$\frac{1}{200} \cos(25x) - \frac{3}{104} \cos(13x) + \frac{1}{88} \cos(11x) + \frac{3}{8} \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(7*x)*sin(6*x)^3,x, algorithm="giac")

[Out] 1/200*cos(25*x) - 3/104*cos(13*x) + 1/88*cos(11*x) + 3/8*cos(x)

maple [A] time = 0.45, size = 24, normalized size = 0.77

$$\frac{3 \cos(x)}{8} + \frac{\cos(11x)}{88} - \frac{3 \cos(13x)}{104} + \frac{\cos(25x)}{200}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(7*x)*sin(6*x)^3,x)

[Out] 3/8*cos(x)+1/88*cos(11*x)-3/104*cos(13*x)+1/200*cos(25*x)

maxima [A] time = 0.73, size = 23, normalized size = 0.74

$$\frac{1}{200} \cos(25x) - \frac{3}{104} \cos(13x) + \frac{1}{88} \cos(11x) + \frac{3}{8} \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(7*x)*sin(6*x)^3,x, algorithm="maxima")

[Out] 1/200*cos(25*x) - 3/104*cos(13*x) + 1/88*cos(11*x) + 3/8*cos(x)

mupad [B] time = 3.18, size = 198, normalized size = 6.39

$$32 \left(-96525 \tan\left(\frac{x}{2}\right)^{46} + 8655075 \tan\left(\frac{x}{2}\right)^{44} - 300482325 \tan\left(\frac{x}{2}\right)^{42} + 5743927475 \tan\left(\frac{x}{2}\right)^{40} - 67792485475 \tan\left(\frac{x}{2}\right)^{38} + \dots \right) / (3575 * (\tan(x/2)^2 + 1)^{25})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(7*x)*sin(6*x)^3,x)

[Out] (32*(2025*tan(x/2)^2 + 120825*tan(x/2)^4 - 8468775*tan(x/2)^6 + 301506975*tan(x/2)^8 - 5739623945*tan(x/2)^10 + 67806830575*tan(x/2)^12 - 523829476225*tan(x/2)^14 + 2750536240650*tan(x/2)^16 - 10084340561350*tan(x/2)^18 + 26326043727610*tan(x/2)^20 - 49575456537350*tan(x/2)^22 + 67896209197950*tan(x/2)^24 - 67895787973650*tan(x/2)^26 + 49575817586750*tan(x/2)^28 - 26325778958050*tan(x/2)^30 + 10084506042325*tan(x/2)^32 - 2750448633075*tan(x/2)^34 + 523868412925*tan(x/2)^36 - 67792485475*tan(x/2)^38 + 5743927475*tan(x/2)^40 - 300482325*tan(x/2)^42 + 8655075*tan(x/2)^44 - 96525*tan(x/2)^46 + 81) / (3575*(tan(x/2)^2 + 1)^25)

sympy [B] time = 5.27, size = 70, normalized size = 2.26

$$\frac{1421 \sin^3(6x) \sin(7x)}{3575} + \frac{1062 \sin^2(6x) \cos(6x) \cos(7x)}{3575} + \frac{1512 \sin(6x) \sin(7x) \cos^2(6x)}{3575} + \frac{1296 \cos^3(6x) \cos(7x)}{3575}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(7*x)*sin(6*x)**3,x)
```

```
[Out] 1421*sin(6*x)**3*sin(7*x)/3575 + 1062*sin(6*x)**2*cos(6*x)*cos(7*x)/3575 +  
1512*sin(6*x)*sin(7*x)*cos(6*x)**2/3575 + 1296*cos(6*x)**3*cos(7*x)/3575
```

3.134 $\int \cos^2(3x) \sin^3(2x) dx$

Optimal. Leaf size=41

$$-\frac{3}{16} \cos(2x) + \frac{3}{64} \cos(4x) + \frac{1}{48} \cos(6x) - \frac{3}{128} \cos(8x) + \frac{1}{192} \cos(12x)$$

[Out] $-3/16*\cos(2*x)+3/64*\cos(4*x)+1/48*\cos(6*x)-3/128*\cos(8*x)+1/192*\cos(12*x)$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4354, 2638}

$$-\frac{3}{16} \cos(2x) + \frac{3}{64} \cos(4x) + \frac{1}{48} \cos(6x) - \frac{3}{128} \cos(8x) + \frac{1}{192} \cos(12x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[3*x]^2*\text{Sin}[2*x]^3,x]$

[Out] $(-3*\text{Cos}[2*x])/16 + (3*\text{Cos}[4*x])/64 + \text{Cos}[6*x]/48 - (3*\text{Cos}[8*x])/128 + \text{Cos}[12*x]/192$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 4354

$\text{Int}[(F_)[(a_.) + (b_.)*(x_.)]^{(p_.)}*(G_)[(c_.) + (d_.)*(x_.)]^{(q_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[\text{ActivateTrig}[F[a + b*x]^p*G[c + d*x]^q], x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& (\text{EqQ}[F, \sin] \parallel \text{EqQ}[F, \cos]) \&\& (\text{EqQ}[G, \sin] \parallel \text{EqQ}[G, \cos]) \&\& \text{IGtQ}[p, 0] \&\& \text{IGtQ}[q, 0]$

Rubi steps

$$\begin{aligned} \int \cos^2(3x) \sin^3(2x) dx &= \int \left(\frac{3}{8} \sin(2x) - \frac{3}{16} \sin(4x) - \frac{1}{8} \sin(6x) + \frac{3}{16} \sin(8x) - \frac{1}{16} \sin(12x) \right) dx \\ &= -\left(\frac{1}{16} \int \sin(12x) dx \right) - \frac{1}{8} \int \sin(6x) dx - \frac{3}{16} \int \sin(4x) dx + \frac{3}{16} \int \sin(8x) dx + \frac{3}{8} \int \sin(2x) dx \\ &= -\frac{3}{16} \cos(2x) + \frac{3}{64} \cos(4x) + \frac{1}{48} \cos(6x) - \frac{3}{128} \cos(8x) + \frac{1}{192} \cos(12x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.00

$$-\frac{3}{16} \cos(2x) + \frac{3}{64} \cos(4x) + \frac{1}{48} \cos(6x) - \frac{3}{128} \cos(8x) + \frac{1}{192} \cos(12x)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Cos}[3*x]^2*\text{Sin}[2*x]^3,x]$

[Out] $(-3*\text{Cos}[2*x])/16 + (3*\text{Cos}[4*x])/64 + \text{Cos}[6*x]/48 - (3*\text{Cos}[8*x])/128 + \text{Cos}[12*x]/192$

fricas [A] time = 0.99, size = 25, normalized size = 0.61

$$\frac{32}{3} \cos(x)^{12} - 32 \cos(x)^{10} + 33 \cos(x)^8 - 12 \cos(x)^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="fricas")

[Out] 32/3*cos(x)^12 - 32*cos(x)^10 + 33*cos(x)^8 - 12*cos(x)^6

giac [A] time = 0.12, size = 31, normalized size = 0.76

$$\frac{1}{192} \cos(12x) - \frac{3}{128} \cos(8x) + \frac{1}{48} \cos(6x) + \frac{3}{64} \cos(4x) - \frac{3}{16} \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="giac")

[Out] 1/192*cos(12*x) - 3/128*cos(8*x) + 1/48*cos(6*x) + 3/64*cos(4*x) - 3/16*cos(2*x)

maple [A] time = 0.17, size = 32, normalized size = 0.78

$$-\frac{3 \cos(2x)}{16} + \frac{3 \cos(4x)}{64} + \frac{\cos(6x)}{48} - \frac{3 \cos(8x)}{128} + \frac{\cos(12x)}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(3*x)^2*sin(2*x)^3,x)

[Out] -3/16*cos(2*x)+3/64*cos(4*x)+1/48*cos(6*x)-3/128*cos(8*x)+1/192*cos(12*x)

maxima [A] time = 0.35, size = 31, normalized size = 0.76

$$\frac{1}{192} \cos(12x) - \frac{3}{128} \cos(8x) + \frac{1}{48} \cos(6x) + \frac{3}{64} \cos(4x) - \frac{3}{16} \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="maxima")

[Out] 1/192*cos(12*x) - 3/128*cos(8*x) + 1/48*cos(6*x) + 3/64*cos(4*x) - 3/16*cos(2*x)

mupad [B] time = 2.27, size = 25, normalized size = 0.61

$$\frac{32 \cos(x)^{12}}{3} - 32 \cos(x)^{10} + 33 \cos(x)^8 - 12 \cos(x)^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(3*x)^2*sin(2*x)^3,x)

[Out] 33*cos(x)^8 - 12*cos(x)^6 - 32*cos(x)^10 + (32*cos(x)^12)/3

sympy [B] time = 18.54, size = 228, normalized size = 5.56

$$-\frac{x \sin^3(2x) \sin^2(3x)}{16} + \frac{x \sin^3(2x) \cos^2(3x)}{16} - \frac{3x \sin^2(2x) \sin(3x) \cos(2x) \cos(3x)}{8} + \frac{3x \sin(2x) \sin^2(3x) \cos^2(2x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)**2*sin(2*x)**3,x)

[Out] -x*sin(2*x)**3*sin(3*x)**2/16 + x*sin(2*x)**3*cos(3*x)**2/16 - 3*x*sin(2*x)**2*sin(3*x)*cos(2*x)*cos(3*x)/8 + 3*x*sin(2*x)*sin(3*x)**2*cos(2*x)**2/16 - 3*x*sin(2*x)*cos(2*x)**2*cos(3*x)**2/16 + x*sin(3*x)*cos(2*x)**3*cos(3*x)/8 + 5*sin(2*x)**3*sin(3*x)*cos(3*x)/16 - sin(2*x)**2*sin(3*x)**2*cos(2*x)/2 - 3*sin(2*x)*sin(3*x)*cos(2*x)**2*cos(3*x)/8 - 11*sin(3*x)**2*cos(2*x)**3/96 - 7*cos(2*x)**3*cos(3*x)**2/32

3.135 $\int \sin(a + bx) \sin(c + bx) dx$

Optimal. Leaf size=27

$$\frac{1}{2}x \cos(a - c) - \frac{\sin(a + 2bx + c)}{4b}$$

[Out] 1/2*x*cos(a-c)-1/4*sin(2*b*x+a+c)/b

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4569, 2637}

$$\frac{1}{2}x \cos(a - c) - \frac{\sin(a + 2bx + c)}{4b}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b*x]*Sin[c + b*x],x]

[Out] (x*Cos[a - c])/2 - Sin[a + c + 2*b*x]/(4*b)

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4569

Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] := Int[ExpandTrigReduce[Sin[v]^p * Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \sin(a + bx) \sin(c + bx) dx &= \int \left(\frac{1}{2} \cos(a - c) - \frac{1}{2} \cos(a + c + 2bx) \right) dx \\ &= \frac{1}{2}x \cos(a - c) - \frac{1}{2} \int \cos(a + c + 2bx) dx \\ &= \frac{1}{2}x \cos(a - c) - \frac{\sin(a + c + 2bx)}{4b} \end{aligned}$$

Mathematica [A] time = 0.05, size = 26, normalized size = 0.96

$$\frac{\sin(a + 2bx + c) - 2bx \cos(a - c)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a + b*x]*Sin[c + b*x],x]

[Out] -1/4*(-2*b*x*Cos[a - c] + Sin[a + c + 2*b*x])/b

fricas [B] time = 0.62, size = 50, normalized size = 1.85

$$\frac{bx \cos(-a + c) - \cos(bx + c) \cos(-a + c) \sin(bx + c) + \cos(bx + c)^2 \sin(-a + c)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="fricas")

[Out] $\frac{1}{2}*(b*x*\cos(-a + c) - \cos(b*x + c)*\cos(-a + c)*\sin(b*x + c) + \cos(b*x + c)^2*\sin(-a + c))/b$

giac [A] time = 0.12, size = 23, normalized size = 0.85

$$\frac{1}{2}x \cos(a - c) - \frac{\sin(2bx + a + c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="giac")

[Out] $\frac{1}{2}*x*\cos(a - c) - \frac{1}{4}*\sin(2*b*x + a + c)/b$

maple [A] time = 0.05, size = 24, normalized size = 0.89

$$\frac{x \cos(a - c)}{2} - \frac{\sin(2bx + a + c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(b*x+a)*sin(b*x+c),x)

[Out] $\frac{1}{2}*x*\cos(a-c)-\frac{1}{4}*\sin(2*b*x+a+c)/b$

maxima [A] time = 0.31, size = 23, normalized size = 0.85

$$\frac{1}{2}x \cos(-a + c) - \frac{\sin(2bx + a + c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="maxima")

[Out] $\frac{1}{2}*x*\cos(-a + c) - \frac{1}{4}*\sin(2*b*x + a + c)/b$

mupad [B] time = 2.48, size = 36, normalized size = 1.33

$$\begin{cases} x \sin(a) \sin(c) & \text{if } b = 0 \\ \frac{x \cos(a-c)}{2} - \frac{\sin(a+c+2bx)}{4b} & \text{if } b \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a + b*x)*sin(c + b*x),x)

[Out] $\text{piecewise}(b == 0, x*\sin(a)*\sin(c), b \neq 0, (x*\cos(a - c))/2 - \sin(a + c + 2*b*x)/(4*b))$

sympy [A] time = 0.74, size = 58, normalized size = 2.15

$$\begin{cases} \frac{x \sin(a+bx) \sin(bx+c)}{2} + \frac{x \cos(a+bx) \cos(bx+c)}{2} - \frac{\sin(bx+c) \cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x \sin(a) \sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)*sin(b*x+c),x)

[Out] $\text{Piecewise}((x*\sin(a + b*x)*\sin(b*x + c))/2 + x*\cos(a + b*x)*\cos(b*x + c)/2 - \sin(b*x + c)*\cos(a + b*x)/(2*b), \text{Ne}(b, 0)), (x*\sin(a)*\sin(c), \text{True}))$

3.136 $\int \sin(c - bx) \sin(a + bx) dx$

Optimal. Leaf size=27

$$\frac{\sin(a + 2bx - c)}{4b} - \frac{1}{2}x \cos(a + c)$$

[Out] $-1/2*x*\cos(a+c)+1/4*\sin(2*b*x+a-c)/b$

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4569, 2637}

$$\frac{\sin(a + 2bx - c)}{4b} - \frac{1}{2}x \cos(a + c)$$

Antiderivative was successfully verified.

[In] Int[Sin[c - b*x]*Sin[a + b*x],x]

[Out] $-(x*\cos[a + c])/2 + \sin[a - c + 2*b*x]/(4*b)$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4569

Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] := Int[ExpandTrigReduce[Sin[v]^p * Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \sin(c - bx) \sin(a + bx) dx &= \int \left(-\frac{1}{2} \cos(a + c) + \frac{1}{2} \cos(a - c + 2bx) \right) dx \\ &= -\frac{1}{2}x \cos(a + c) + \frac{1}{2} \int \cos(a - c + 2bx) dx \\ &= -\frac{1}{2}x \cos(a + c) + \frac{\sin(a - c + 2bx)}{4b} \end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.96

$$\frac{\sin(a + 2bx - c) - 2bx \cos(a + c)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c - b*x]*Sin[a + b*x],x]

[Out] $(-2*b*x*\cos[a + c] + \sin[a - c + 2*b*x])/(4*b)$

fricas [A] time = 0.82, size = 44, normalized size = 1.63

$$\frac{bx \cos(a + c) - \cos(bx + a) \cos(a + c) \sin(bx + a) + \cos(bx + a)^2 \sin(a + c)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="fricas")

[Out] $-1/2*(b*x*\cos(a + c) - \cos(b*x + a)*\cos(a + c)*\sin(b*x + a) + \cos(b*x + a)^2*\sin(a + c))/b$

giac [A] time = 0.14, size = 23, normalized size = 0.85

$$-\frac{1}{2}x \cos(a + c) + \frac{\sin(2bx + a - c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="giac")

[Out] $-1/2*x*\cos(a + c) + 1/4*\sin(2*b*x + a - c)/b$

maple [A] time = 0.05, size = 24, normalized size = 0.89

$$-\frac{x \cos(a + c)}{2} + \frac{\sin(2bx + a - c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-sin(b*x-c)*sin(b*x+a),x)

[Out] $-1/2*x*\cos(a+c)+1/4*\sin(2*b*x+a-c)/b$

maxima [A] time = 0.70, size = 23, normalized size = 0.85

$$-\frac{1}{2}x \cos(a + c) + \frac{\sin(2bx + a - c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="maxima")

[Out] $-1/2*x*\cos(a + c) + 1/4*\sin(2*b*x + a - c)/b$

mupad [B] time = 2.46, size = 36, normalized size = 1.33

$$\begin{cases} x \sin(a) \sin(c) & \text{if } b = 0 \\ \frac{\sin(a-c+2bx)}{4b} - \frac{x \cos(a+c)}{2} & \text{if } b \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a + b*x)*sin(c - b*x),x)

[Out] $\text{piecewise}(b == 0, x*\sin(a)*\sin(c), b \neq 0, \sin(a - c + 2*b*x)/(4*b) - (x*\cos(a + c))/2)$

sympy [A] time = 0.73, size = 61, normalized size = 2.26

$$-\begin{cases} \frac{x \sin(a+bx) \sin(bx-c)}{2} + \frac{x \cos(a+bx) \cos(bx-c)}{2} - \frac{\sin(bx-c) \cos(a+bx)}{2b} & \text{for } b \neq 0 \\ -x \sin(a) \sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sin(b*x-c)*sin(b*x+a),x)

[Out] $-\text{Piecewise}((x*\sin(a + b*x)*\sin(b*x - c)/2 + x*\cos(a + b*x)*\cos(b*x - c)/2 - \sin(b*x - c)*\cos(a + b*x)/(2*b), \text{Ne}(b, 0)), (-x*\sin(a)*\sin(c), \text{True}))$

3.137 $\int \cos(a + bx) \cos(c + bx) dx$

Optimal. Leaf size=27

$$\frac{\sin(a + 2bx + c)}{4b} + \frac{1}{2}x \cos(a - c)$$

[Out] 1/2*x*cos(a-c)+1/4*sin(2*b*x+a+c)/b

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4570, 2637}

$$\frac{\sin(a + 2bx + c)}{4b} + \frac{1}{2}x \cos(a - c)$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]*Cos[c + b*x],x]

[Out] (x*cos[a - c])/2 + Sin[a + c + 2*b*x]/(4*b)

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4570

Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] := Int[ExpandTrigReduce[Cos[v]^p * Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \cos(a + bx) \cos(c + bx) dx &= \int \left(\frac{1}{2} \cos(a - c) + \frac{1}{2} \cos(a + c + 2bx) \right) dx \\ &= \frac{1}{2}x \cos(a - c) + \frac{1}{2} \int \cos(a + c + 2bx) dx \\ &= \frac{1}{2}x \cos(a - c) + \frac{\sin(a + c + 2bx)}{4b} \end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{\sin(a + 2bx + c) + 2bx \cos(a - c)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b*x]*Cos[c + b*x],x]

[Out] (2*b*x*cos[a - c] + Sin[a + c + 2*b*x])/4*b

fricas [B] time = 0.56, size = 50, normalized size = 1.85

$$\frac{bx \cos(-a + c) + \cos(bx + c) \cos(-a + c) \sin(bx + c) - \cos(bx + c)^2 \sin(-a + c)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="fricas")

[Out] $\frac{1}{2}*(b*x*\cos(-a + c) + \cos(b*x + c)*\cos(-a + c)*\sin(b*x + c) - \cos(b*x + c)^2*\sin(-a + c))/b$

giac [A] time = 0.12, size = 23, normalized size = 0.85

$$\frac{1}{2}x \cos(a - c) + \frac{\sin(2bx + a + c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="giac")

[Out] $\frac{1}{2}*x*\cos(a - c) + \frac{1}{4}*\sin(2*b*x + a + c)/b$

maple [A] time = 0.04, size = 24, normalized size = 0.89

$$\frac{x \cos(a - c)}{2} + \frac{\sin(2bx + a + c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*cos(b*x+c),x)

[Out] $\frac{1}{2}*x*\cos(a-c)+\frac{1}{4}*\sin(2*b*x+a+c)/b$

maxima [A] time = 0.56, size = 23, normalized size = 0.85

$$\frac{1}{2}x \cos(-a + c) + \frac{\sin(2bx + a + c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="maxima")

[Out] $\frac{1}{2}*x*\cos(-a + c) + \frac{1}{4}*\sin(2*b*x + a + c)/b$

mupad [B] time = 2.27, size = 36, normalized size = 1.33

$$\begin{cases} x \cos(a) \cos(c) & \text{if } b = 0 \\ \frac{x \cos(a-c)}{2} + \frac{\sin(a+c+2bx)}{4b} & \text{if } b \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b*x)*cos(c + b*x),x)

[Out] $\text{piecewise}(b == 0, x*\cos(a)*\cos(c), b \neq 0, (x*\cos(a - c))/2 + \sin(a + c + 2*b*x)/(4*b))$

sympy [A] time = 0.72, size = 58, normalized size = 2.15

$$\begin{cases} \frac{x \sin(a+bx) \sin(bx+c)}{2} + \frac{x \cos(a+bx) \cos(bx+c)}{2} + \frac{\sin(a+bx) \cos(bx+c)}{2b} & \text{for } b \neq 0 \\ x \cos(a) \cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*cos(b*x+c),x)

[Out] $\text{Piecewise}((x*\sin(a + b*x)*\sin(b*x + c))/2 + x*\cos(a + b*x)*\cos(b*x + c)/2 + \sin(a + b*x)*\cos(b*x + c)/(2*b), \text{Ne}(b, 0)), (x*\cos(a)*\cos(c), \text{True}))$

3.138 $\int \cos(c - bx) \cos(a + bx) dx$

Optimal. Leaf size=27

$$\frac{\sin(a + 2bx - c)}{4b} + \frac{1}{2}x \cos(a + c)$$

[Out] 1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4570, 2637}

$$\frac{\sin(a + 2bx - c)}{4b} + \frac{1}{2}x \cos(a + c)$$

Antiderivative was successfully verified.

[In] Int[Cos[c - b*x]*Cos[a + b*x], x]

[Out] (x*cos[a + c])/2 + Sin[a - c + 2*b*x]/(4*b)

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4570

Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] := Int[ExpandTrigReduce[Cos[v]^p * Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \cos(c - bx) \cos(a + bx) dx &= \int \left(\frac{1}{2} \cos(a + c) + \frac{1}{2} \cos(a - c + 2bx) \right) dx \\ &= \frac{1}{2}x \cos(a + c) + \frac{1}{2} \int \cos(a - c + 2bx) dx \\ &= \frac{1}{2}x \cos(a + c) + \frac{\sin(a - c + 2bx)}{4b} \end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{\sin(a + 2bx - c) + 2bx \cos(a + c)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c - b*x]*Cos[a + b*x], x]

[Out] (2*b*x*cos[a + c] + Sin[a - c + 2*b*x])/ (4*b)

fricas [A] time = 2.13, size = 44, normalized size = 1.63

$$\frac{bx \cos(a + c) + \cos(bx + a) \cos(a + c) \sin(bx + a) - \cos(bx + a)^2 \sin(a + c)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="fricas")

[Out] 1/2*(b*x*cos(a + c) + cos(b*x + a)*cos(a + c)*sin(b*x + a) - cos(b*x + a)^2 *sin(a + c))/b

giac [A] time = 0.15, size = 23, normalized size = 0.85

$$\frac{1}{2} x \cos(a + c) + \frac{\sin(2bx + a - c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="giac")

[Out] 1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b

maple [A] time = 0.05, size = 24, normalized size = 0.89

$$\frac{x \cos(a + c)}{2} + \frac{\sin(2bx + a - c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x-c)*cos(b*x+a),x)

[Out] 1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b

maxima [A] time = 0.33, size = 23, normalized size = 0.85

$$\frac{1}{2} x \cos(a + c) + \frac{\sin(2bx + a - c)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="maxima")

[Out] 1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b

mupad [B] time = 2.26, size = 36, normalized size = 1.33

$$\begin{cases} x \cos(a) \cos(c) & \text{if } b = 0 \\ \frac{\sin(a-c+2bx)}{4b} + \frac{x \cos(a+c)}{2} & \text{if } b \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b*x)*cos(c - b*x),x)

[Out] piecewise(b == 0, x*cos(a)*cos(c), b ~= 0, sin(a - c + 2*b*x)/(4*b) + (x*cos(a + c))/2)

sympy [A] time = 0.73, size = 58, normalized size = 2.15

$$\begin{cases} \frac{x \sin(a+bx) \sin(bx-c)}{2} + \frac{x \cos(a+bx) \cos(bx-c)}{2} + \frac{\sin(bx-c) \cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x \cos(a) \cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x-c)*cos(b*x+a),x)

[Out] Piecewise((x*sin(a + b*x)*sin(b*x - c)/2 + x*cos(a + b*x)*cos(b*x - c)/2 + sin(b*x - c)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*cos(a)*cos(c), True))

3.139 $\int \tan(a + bx) \tan(c + bx) dx$

Optimal. Leaf size=39

$$-\frac{\cot(a-c)\log(\cos(a+bx))}{b} + \frac{\cot(a-c)\log(\cos(bx+c))}{b} - x$$

[Out] $-x - \cot(a-c) \ln(\cos(bx+a))/b + \cot(a-c) \ln(\cos(bx+c))/b$

Rubi [A] time = 0.07, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4612, 4610, 3475}

$$-\frac{\cot(a-c)\log(\cos(a+bx))}{b} + \frac{\cot(a-c)\log(\cos(bx+c))}{b} - x$$

Antiderivative was successfully verified.

[In] Int[Tan[a + b*x]*Tan[c + b*x], x]

[Out] $-x - (\cot[a - c] \log[\cos[a + b*x]])/b + (\cot[a - c] \log[\cos[c + b*x]])/b$

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4610

Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_.) + (d_.)*(x_)], x_Symbol] :> -Dist[Csc[(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]

Rule 4612

Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[(b*x)/d, x] + Dist[(b*cos[(b*c - a*d)/d])/d, Int[Sec[a + b*x]*Sec[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \tan(a + bx) \tan(c + bx) dx &= -x + \cos(a - c) \int \sec(a + bx) \sec(c + bx) dx \\ &= -x + \cot(a - c) \int \tan(a + bx) dx - \cot(a - c) \int \tan(c + bx) dx \\ &= -x - \frac{\cot(a - c) \log(\cos(a + bx))}{b} + \frac{\cot(a - c) \log(\cos(c + bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.52, size = 31, normalized size = 0.79

$$\frac{\cot(a-c)(\log(\cos(bx+c)) - \log(\cos(a+bx)))}{b} - x$$

Antiderivative was successfully verified.

[In] Integrate[Tan[a + b*x]*Tan[c + b*x], x]

[Out] $-x + (\cot[a - c] * (-\log[\cos[a + b*x]] + \log[\cos[c + b*x]]))/b$

fricas [B] time = 0.63, size = 145, normalized size = 3.72

$$\frac{2bx \sin(-2a + 2c) - (\cos(-2a + 2c) + 1) \log\left(-\frac{(\cos(-2a+2c)-1) \tan(bx+c)^2 - 2 \sin(-2a+2c) \tan(bx+c) - \cos(-2a+2c)-1}{(\cos(-2a+2c)+1) \tan(bx+c)^2 + \cos(-2a+2c)+1}\right) + (\cos(-2a+2c) + 1) \log(1/(\tan(bx+c)^2 + 1))}{2b \sin(-2a + 2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="fricas")

[Out]
$$-1/2*(2*b*x*\sin(-2*a + 2*c) - (\cos(-2*a + 2*c) + 1)*\log(-((\cos(-2*a + 2*c) - 1)*\tan(b*x + c)^2 - 2*\sin(-2*a + 2*c)*\tan(b*x + c) - \cos(-2*a + 2*c) - 1)/((\cos(-2*a + 2*c) + 1)*\tan(b*x + c)^2 + \cos(-2*a + 2*c) + 1)) + (\cos(-2*a + 2*c) + 1)*\log(1/(\tan(b*x + c)^2 + 1)))/(b*\sin(-2*a + 2*c))$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: $(2*\pi/x/2) > (-2*\pi/x/2) 2/b*(-1/2*b*x+(2*\tan(a/2)^2*\tan(c/2)^3-2*\tan(a/2)^2*\tan(c/2)+8*\tan(a/2)*\tan(c/2)^2-2*\tan(c/2)^3+2*\tan(c/2)^2)/ (8*\tan(a/2)^2*\tan(c/2)^2-8*\tan(a/2)*\tan(c/2)^3+8*\tan(a/2)*\tan(c/2)-8*\tan(c/2)^2)*\ln(\text{abs}(2*\tan(b*x)*\tan(c/2)+\tan(c/2)^2-1))+ (2*\tan(a/2)^3*\tan(c/2)^2-2*\tan(a/2)^3+8*\tan(a/2)^2*\tan(c/2)-2*\tan(a/2)*\tan(c/2)^2+2*\tan(a/2))/ (-8*\tan(a/2)^3*\tan(c/2)+8*\tan(a/2)^2*\tan(c/2)^2-8*\tan(a/2)^2+8*\tan(a/2)*\tan(c/2))*\ln(\text{abs}(2*\tan(b*x)*\tan(a/2)+\tan(a/2)^2-1))$

maple [C] time = 0.14, size = 173, normalized size = 4.44

$$-x \frac{i \ln(1 + e^{2i(bx+a)}) e^{2ia}}{b(e^{2ia} - e^{2ic})} - \frac{i \ln(1 + e^{2i(bx+a)}) e^{2ic}}{b(e^{2ia} - e^{2ic})} + \frac{i \ln(e^{2i(bx+a)} + e^{2i(a-c)}) e^{2ia}}{b(e^{2ia} - e^{2ic})} + \frac{i \ln(e^{2i(bx+a)} + e^{2i(a-c)}) e^{2ic}}{b(e^{2ia} - e^{2ic})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(b*x+a)*tan(b*x+c),x)

[Out]
$$-x - I/b/(\exp(2*I*a) - \exp(2*I*c)) * \ln(1 + \exp(2*I*(b*x+a))) * \exp(2*I*a) - I/b/(\exp(2*I*a) - \exp(2*I*c)) * \ln(1 + \exp(2*I*(b*x+a))) * \exp(2*I*c) + I/b/(\exp(2*I*a) - \exp(2*I*c)) * \ln(\exp(2*I*(b*x+a)) + \exp(2*I*(a-c))) * \exp(2*I*a) + I/b/(\exp(2*I*a) - \exp(2*I*c)) * \ln(\exp(2*I*(b*x+a)) + \exp(2*I*(a-c))) * \exp(2*I*c)$$

maxima [B] time = 0.65, size = 371, normalized size = 9.51

$$\frac{(2b \cos(2a) \cos(2c) - b \cos(2c)^2 + 2b \sin(2a) \sin(2c) - b \sin(2c)^2 - (\cos(2a)^2 + \sin(2a)^2)b)x + (\cos(2a) \sin(2c) - \sin(2a) \cos(2c))}{2b \sin(2a) \sin(2c) - b \sin(2c)^2 - (\cos(2a)^2 + \sin(2a)^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="maxima")

[Out]
$$-((2*b*\cos(2*a)*\cos(2*c) - b*\cos(2*c)^2 + 2*b*\sin(2*a)*\sin(2*c) - b*\sin(2*c)^2 - (\cos(2*a)^2 + \sin(2*a)^2)*b)*x + (\cos(2*a)^2 - \cos(2*c)^2 + \sin(2*a)^2 - \sin(2*c)^2)*\arctan2(\sin(2*b*x) - \sin(2*a), \cos(2*b*x) + \cos(2*a)) - (\cos(2*a)^2 - \cos(2*c)^2 + \sin(2*a)^2 - \sin(2*c)^2)*\arctan2(\sin(2*b*x) - \sin(2*c), \cos(2*b*x) + \cos(2*c)) - (\cos(2*c)*\sin(2*a) - \cos(2*a)*\sin(2*c))*\log(\cos(2*b*x)^2 + 2*\cos(2*b*x)*\cos(2*a) + \cos(2*a)^2 + \sin(2*b*x)^2 - 2*\sin(2*b*x)*\sin(2*a) + \sin(2*a)^2)$$

$x) \cdot \sin(2a) + \sin(2a)^2) + (\cos(2c) \cdot \sin(2a) - \cos(2a) \cdot \sin(2c)) \cdot \log(\cos(2bx)^2 + 2 \cos(2bx) \cos(2c) + \cos(2c)^2 + \sin(2bx)^2 - 2 \sin(2bx) \sin(2c) + \sin(2c)^2) / (2b \cos(2a) \cos(2c) - b \cos(2c)^2 + 2b \sin(2a) \sin(2c) - b \sin(2c)^2 - (\cos(2a)^2 + \sin(2a)^2) \cdot b)$

mupad [B] time = 4.99, size = 207, normalized size = 5.31

$$\frac{\frac{x}{2} + x \left(\sin(a-c)^2 - \frac{1}{2} \right)}{\sin(a-c)^2} - \frac{\frac{\sin(2a-2c) \ln(\sin(2a-2c)^2 - \sin(a+bx)^2) + \sin(3a-2c+bx)^2 + \sin(4a-4c) + \sin(6a-4c+2bx) - \sin(2a+2c)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(a + b*x)*tan(c + b*x), x)

[Out] $-(x/2 + x(\sin(a-c)^2 - 1/2))/\sin(a-c)^2 - ((\sin(2a-2c) \log(\sin(4a-4c) + \sin(6a-4c+2bx) - \sin(2a+2bx) + \sin(2a-2c)^2) - \sin(a+bx)^2 + \sin(3a-2c+bx)^2)/2 - (\sin(2a-2c) \log(\sin(4a-4c) + \sin(4a-2c+2bx) - \sin(2c+2bx) + \sin(2a-2c)^2) - \sin(c+bx)^2 + \sin(2a-c+bx)^2)/2) / (b \sin(a-c)^2)$

sympy [B] time = 6.58, size = 7713, normalized size = 197.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(b*x+a)*tan(b*x+c), x)

[Out] Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (b*x*tan(c)**4*tan(b*x)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) - b*x*tan(c)**3/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) - b*x*tan(c)**2*tan(b*x)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) + b*x*tan(c)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) + 2*log(tan(b*x) - 1/tan(c))*tan(c)**3*tan(b*x)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) - 2*log(tan(b*x) - 1/tan(c))*tan(c)**2/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) + log(tan(b*x)**2 + 1)*tan(c)**2/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) - tan(c)**2/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) - 1/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)), Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi) + pi*floor(c/pi - 1/2))), (0, Eq(b, 0)), (-2*b*x*tan(a)/(2*b*tan(a)**3 + 2*b*tan(a)) - 2*log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**3 + 2*b*tan(a)) - log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*tan(a)**3 + 2*b*tan(a)), Eq(c, 0)), (-2*b*x*tan(c)/(2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(tan(b*x) - 1/tan(c))/(2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(c)**3 + 2*b*tan(c)), Eq(a, 0)), (2*b*x*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c)) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*b*x*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c)) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*b*x*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c)) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*b*

$$\begin{aligned}
& x \tan(c) / (2b \tan(a)^3 \tan(c)^2 + 2b \tan(a)^3 - 2b \tan(a)^2 \tan(c)^3 \\
& - 2b \tan(a)^2 \tan(c) + 2b \tan(a) \tan(c)^2 + 2b \tan(a) - 2b \tan(c)^3 \\
& - 2b \tan(c)) - 2 \log(\tan(bx) - 1/\tan(a)) \tan(c)^2 / (2b \tan(a)^3 \tan(c) \\
& **2 + 2b \tan(a)^3 - 2b \tan(a)^2 \tan(c)^3 - 2b \tan(a)^2 \tan(c) + 2b \tan(a) \tan(c)^2 \\
& + 2b \tan(a) - 2b \tan(c)^3 - 2b \tan(c)) - 2 \log(\tan(bx) - 1/\tan(a)) / (2b \tan(a)^3 \tan(c)^2 \\
& + 2b \tan(a)^3 - 2b \tan(a)^2 \tan(c)^3 - 2b \tan(a)^2 \tan(c) + 2b \tan(a) \tan(c)^2 \\
& + 2b \tan(a) - 2b \tan(c)^3 - 2b \tan(c)) + 2 \log(\tan(bx) - 1/\tan(c)) \tan(a)^2 / (2b \tan(a)^3 \tan(c)^2 \\
& + 2b \tan(a)^3 - 2b \tan(a)^2 \tan(c)^3 - 2b \tan(a)^2 \tan(c) + 2b \tan(a) \tan(c)^2 \\
& + 2b \tan(a) - 2b \tan(c)^3 - 2b \tan(c)) + 2 \log(\tan(bx) - 1/\tan(c)) / (2b \tan(a)^3 \tan(c)^2 \\
& + 2b \tan(a)^3 - 2b \tan(a)^2 \tan(c)^3 - 2b \tan(a)^2 \tan(c) + 2b \tan(a) \tan(c)^2 \\
& + 2b \tan(a) - 2b \tan(c)^3 - 2b \tan(c)) + \log(\tan(bx)^2 + 1) \tan(a)^2 / (2b \tan(a)^3 \tan(c)^2 \\
& + 2b \tan(a)^3 - 2b \tan(a)^2 \tan(c)^3 - 2b \tan(a)^2 \tan(c) + 2b \tan(a) \tan(c)^2 \\
& + 2b \tan(a) - 2b \tan(c)^3 - 2b \tan(c)), True)) + Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), \\
& (-4bx \tan(c)^2 \tan(bx) / (2b \tan(c)^5 \tan(bx) - 2b \tan(c)^4 + 4b \tan(c)^3 \tan(bx) \\
& - 4b \tan(c)^2 + 2b \tan(c) \tan(bx) - 2b) + 4bx \tan(c) / (2b \tan(c)^5 \tan(bx) - 2b \tan(c)^4 \\
& + 4b \tan(c)^3 \tan(bx) - 4b \tan(c)^2 + 2b \tan(c) \tan(bx) - 2b) + 2 \log(\tan(bx) - 1/\tan(c)) \\
& \tan(c)^3 \tan(bx) / (2b \tan(c)^5 \tan(bx) - 2b \tan(c)^4 + 4b \tan(c)^3 \tan(bx) \\
& - 4b \tan(c)^2 + 2b \tan(c) \tan(bx) - 2b) - 2 \log(\tan(bx) - 1/\tan(c)) \tan(c)^2 / (2b \tan(c)^5 \tan(bx) \\
& - 2b \tan(c)^4 + 4b \tan(c)^3 \tan(bx) - 4b \tan(c)^2 + 2b \tan(c) \tan(bx) - 2b) - 2 \log(\tan(bx) - 1/\tan(c)) \\
& \tan(c) \tan(bx) / (2b \tan(c)^5 \tan(bx) - 2b \tan(c)^4 + 4b \tan(c)^3 \tan(bx) - 4b \tan(c)^2 \\
& + 2b \tan(c) \tan(bx) - 2b) + 2 \log(\tan(bx) - 1/\tan(c)) / (2b \tan(c)^5 \tan(bx) - 2b \tan(c)^4 \\
& + 4b \tan(c)^3 \tan(bx) - 4b \tan(c)^2 + 2b \tan(c) \tan(bx) - 2b) - \log(\tan(bx)^2 + 1) \tan(c)^3 \tan(bx) / (2b \tan(c)^5 \tan(bx) \\
& - 2b \tan(c)^4 + 4b \tan(c)^3 \tan(bx) - 4b \tan(c)^2 + 2b \tan(c) \tan(bx) - 2b) + \log(\tan(bx)^2 + 1) \tan(c)^2 / (2b \tan(c)^5 \tan(bx) \\
& - 2b \tan(c)^4 + 4b \tan(c)^3 \tan(bx) - 4b \tan(c)^2 + 2b \tan(c) \tan(bx) - 2b) + \log(\tan(bx)^2 + 1) \tan(c) \tan(bx) / (2b \tan(c)^5 \tan(bx) \\
& - 2b \tan(c)^4 + 4b \tan(c)^3 \tan(bx) - 4b \tan(c)^2 + 2b \tan(c) \tan(bx) - 2b) - \log(\tan(bx)^2 + 1) / (2b \tan(c)^5 \tan(bx) \\
& - 2b \tan(c)^4 + 4b \tan(c)^3 \tan(bx) - 4b \tan(c)^2 + 2b \tan(c) \tan(bx) - 2b) - 2 \tan(c)^2 / (2b \tan(c)^5 \tan(bx) \\
& - 2b \tan(c)^4 + 4b \tan(c)^3 \tan(bx) - 4b \tan(c)^2 + 2b \tan(c) \tan(bx) - 2b) - \\
& 2 / (2b \tan(c)^5 \tan(bx) - 2b \tan(c)^4 + 4b \tan(c)^3 \tan(bx) - 4b \tan(c)^2 + 2b \tan(c) \tan(bx) - 2b), \\
& Eq(a, \operatorname{atan}(\tan(c)) + \pi \operatorname{floor}((c - \pi/2)/\pi) + \pi \operatorname{floor}(c/\pi - 1/2))), (0, Eq(b, 0)), (-2bx \tan(a) / (2b \tan(a)^2 \\
& + 2b) - 2 \log(\tan(bx) - 1/\tan(a)) / (2b \tan(a)^2 + 2b) + \log(\tan(bx)^2 + 1) / (2b \tan(a)^2 + 2b), \\
& Eq(c, 0)), (-2bx \tan(c) / (2b \tan(c)^2 + 2b) - 2 \log(\tan(bx) - 1/\tan(c)) / (2b \tan(c)^2 + 2b) \\
& + \log(\tan(bx)^2 + 1) / (2b \tan(c)^2 + 2b), Eq(a, 0)), (-2bx \tan(a)^2 / (2b \tan(a)^3 \tan(c)^2 \\
& + 2b \tan(a)^3 - 2b \tan(a)^2 \tan(c)^3 - 2b \tan(a)^2 \tan(c) + 2b \tan(a) \tan(c)^2 + 2b \tan(a) \\
& - 2b \tan(c)^3 - 2b \tan(c)) - 2 \log(\tan(bx) - 1/\tan(a)) \tan(a) \tan(c)^2 / (2b \tan(a)^3 \tan(c)^2 \\
& + 2b \tan(a)^3 - 2b \tan(a)^2 \tan(c)^3 - 2b \tan(a)^2 \tan(c) + 2b \tan(a) \tan(c)^2 + 2b \tan(a) \\
& - 2b \tan(c)^3 - 2b \tan(c)) - 2 \log(\tan(bx) - 1/\tan(a)) \tan(a) / (2b \tan(a)^3 \tan(c)^2 + 2b \tan(a)^3 \\
& - 2b \tan(a)^2 \tan(c)^3 - 2b \tan(a)^2 \tan(c) + 2b \tan(a) \tan(c)^2 + 2b \tan(a) - 2b \tan(c)^3 - 2b \\
& \tan(c)) + 2 \log(\tan(bx) - 1/\tan(c)) \tan(c) / (2b \tan(a)^3 \tan(c)^2 + 2b
\end{aligned}$$


```

*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*
tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)/(2*b*tan
(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*
tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) -
log(tan(b*x)**2 + 1)*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*
tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan
(a) - 2*b*tan(c)**3 - 2*b*tan(c)), True))*tan(c) + Piecewise((x, Eq(a, 0) &
Eq(b, 0) & Eq(c, 0)), (-b*x*tan(c)**3*tan(b*x)/(b*tan(c)**5*tan(b*x) - b*t
an(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*tan(b*x) - b)
+ b*x*tan(c)**2/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x)
) - 2*b*tan(c)**2 + b*tan(c)*tan(b*x) - b) + b*x*tan(c)*tan(b*x)/(b*tan(c)*
**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2*b*tan(c)**2 + b*tan(
c)*tan(b*x) - b) - b*x/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*
tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*tan(b*x) - b) - 2*log(tan(b*x) - 1/tan(
c))*tan(c)**2*tan(b*x)/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*
tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*tan(b*x) - b) + 2*log(tan(b*x) - 1/tan(
c))*tan(c)/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2
*b*tan(c)**2 + b*tan(c)*tan(b*x) - b) + log(tan(b*x)**2 + 1)*tan(c)**2*tan(
b*x)/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2*b*tan
(c)**2 + b*tan(c)*tan(b*x) - b) - log(tan(b*x)**2 + 1)*tan(c)/(b*tan(c)**5*
tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*
tan(b*x) - b) - tan(c)**3/(b*tan(c)**5*tan(b*x) - b*tan(c)**4 + 2*b*tan(c)*
**3*tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*tan(b*x) - b) - tan(c)/(b*tan(c)**5*
tan(b*x) - b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) - 2*b*tan(c)**2 + b*tan(c)*
tan(b*x) - b), Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi) + pi*floor(c/pi
- 1/2))), (x, Eq(b, 0)), (2*b*x/(2*b*tan(a)**2 + 2*b) - 2*log(tan(b*x) - 1
/tan(a))*tan(a)/(2*b*tan(a)**2 + 2*b) + log(tan(b*x)**2 + 1)*tan(a)/(2*b*ta
n(a)**2 + 2*b), Eq(c, 0)), (2*b*x/(2*b*tan(c)**2 + 2*b) - 2*log(tan(b*x) -
1/tan(c))*tan(c)/(2*b*tan(c)**2 + 2*b) + log(tan(b*x)**2 + 1)*tan(c)/(2*b*t
an(c)**2 + 2*b), Eq(a, 0)), (-2*b*x*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)*
**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*t
an(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*b*x*tan(a)*t
an(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3
- 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3
- 2*b*tan(c)) + 2*b*x*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*
b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*t
an(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*b*x*tan(c)/(2*b*tan(a)**3*tan(c)**2
+ 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan
(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(b*x) -
1/tan(a))*tan(a)**2*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2
*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*
tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(b*x) - 1/tan(a))*tan(a)**2
/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*t
an(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*t
an(c)) + 2*log(tan(b*x) - 1/tan(c))*tan(a)**2*tan(c)**2/(2*b*tan(a)**3*tan(
c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*
b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(b
*x) - 1/tan(c))*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*ta
n(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a)
) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*tan(a)
)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*ta
n(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - lo
g(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b
*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*ta
n(a) - 2*b*tan(c)**3 - 2*b*tan(c)), True))*tan(a)*tan(c)

```

3.140 $\int \tan(c - bx) \tan(a + bx) dx$

Optimal. Leaf size=34

$$-\frac{\cot(a+c)\log(\cos(c-bx))}{b} + \frac{\cot(a+c)\log(\cos(a+bx))}{b} + x$$

[Out] $x - \cot(a+c) \cdot \ln(\cos(b \cdot x - c)) / b + \cot(a+c) \cdot \ln(\cos(b \cdot x + a)) / b$

Rubi [A] time = 0.07, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4612, 4610, 3475}

$$-\frac{\cot(a+c)\log(\cos(c-bx))}{b} + \frac{\cot(a+c)\log(\cos(a+bx))}{b} + x$$

Antiderivative was successfully verified.

[In] Int[Tan[c - b*x]*Tan[a + b*x], x]

[Out] $x - (\cot[a + c] \cdot \log[\cos[c - b \cdot x]]) / b + (\cot[a + c] \cdot \log[\cos[a + b \cdot x]]) / b$

Rule 3475

Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4610

Int[Sec[(a_.) + (b_.)*(x_.)]*Sec[(c_.) + (d_.)*(x_.)], x_Symbol] := -Dist[Csc[(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]

Rule 4612

Int[Tan[(a_.) + (b_.)*(x_.)]*Tan[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[(b*x)/d, x] + Dist[(b*cos[(b*c - a*d)/d])/d, Int[Sec[a + b*x]*Sec[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \tan(c - bx) \tan(a + bx) dx &= x - \cos(a + c) \int \sec(c - bx) \sec(a + bx) dx \\ &= x - \cot(a + c) \int \tan(c - bx) dx - \cot(a + c) \int \tan(a + bx) dx \\ &= x - \frac{\cot(a + c) \log(\cos(c - bx))}{b} + \frac{\cot(a + c) \log(\cos(a + bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.53, size = 28, normalized size = 0.82

$$\frac{\cot(a+c)(\log(\cos(a+bx)) - \log(\cos(c-bx)))}{b} + x$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c - b*x]*Tan[a + b*x], x]

[Out] $x + (\cot[a + c] \cdot (-\log[\cos[c - b \cdot x]] + \log[\cos[a + b \cdot x]])) / b$

fricas [B] time = 2.01, size = 145, normalized size = 4.26

$$\frac{2bx \sin(2a+2c) - (\cos(2a+2c) + 1) \log\left(-\frac{(\cos(2a+2c)-1) \tan(bx+a)^2 - 2 \sin(2a+2c) \tan(bx+a) - \cos(2a+2c) - 1}{(\cos(2a+2c)+1) \tan(bx+a)^2 + \cos(2a+2c) + 1}\right) + (\cos(2a+2c) + 1) \log(1/(\tan(bx+a)^2 + 1))}{2b \sin(2a+2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="fricas")

[Out] 1/2*(2*b*x*sin(2*a + 2*c) - (cos(2*a + 2*c) + 1)*log(-((cos(2*a + 2*c) - 1)*tan(b*x + a)^2 - 2*sin(2*a + 2*c)*tan(b*x + a) - cos(2*a + 2*c) - 1)/((cos(2*a + 2*c) + 1)*tan(b*x + a)^2 + cos(2*a + 2*c) + 1)) + (cos(2*a + 2*c) + 1)*log(1/(tan(b*x + a)^2 + 1)))/(b*sin(2*a + 2*c))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)-2/b*(-1/2*b*x+(-2*tan(a/2)^2*tan(c/2)^3+2*tan(a/2)^2*tan(c/2)+8*tan(a/2)*tan(c/2)^2+2*tan(c/2)^3-2*tan(c/2)^2)/((8*tan(a/2)^2*tan(c/2)^2+8*tan(a/2)*tan(c/2)^3-8*tan(a/2)*tan(c/2)-8*tan(c/2)^2)*ln(abs(2*tan(b*x)*tan(c/2)-tan(c/2)^2+1))+(-2*tan(a/2)^3*tan(c/2)^2+2*tan(a/2)^3+8*tan(a/2)^2*tan(c/2)+2*tan(a/2)*tan(c/2)^2-2*tan(a/2))/(-8*tan(a/2)^3*tan(c/2)-8*tan(a/2)^2*tan(c/2)^2+8*tan(a/2)^2+8*tan(a/2)*tan(c/2))*ln(abs(2*tan(b*x)*tan(a/2)+tan(a/2)^2-1)))

maple [C] time = 0.14, size = 145, normalized size = 4.26

$$x + \frac{i \ln(1 + e^{2i(bx+a)}) e^{2i(a+c)}}{b(e^{2i(a+c)} - 1)} + \frac{i \ln(1 + e^{2i(bx+a)})}{b(e^{2i(a+c)} - 1)} - \frac{i \ln(e^{2i(a+c)} + e^{2i(bx+a)}) e^{2i(a+c)}}{b(e^{2i(a+c)} - 1)} - \frac{i \ln(e^{2i(a+c)} + e^{2i(bx+a)})}{b(e^{2i(a+c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-tan(b*x-c)*tan(b*x+a),x)

[Out] x+I/b/(exp(2*I*(a+c))-1)*ln(1+exp(2*I*(b*x+a)))*exp(2*I*(a+c))+I/b/(exp(2*I*(a+c))-1)*ln(1+exp(2*I*(b*x+a)))-I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(a+c))+exp(2*I*(b*x+a)))*exp(2*I*(a+c))-I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(a+c))+exp(2*I*(b*x+a)))

maxima [B] time = 0.35, size = 290, normalized size = 8.53

$$\frac{(b \cos(2a+2c)^2 + b \sin(2a+2c)^2 - 2b \cos(2a+2c) + b)x - (\cos(2a+2c)^2 + \sin(2a+2c)^2 - 1) \arctan(\sin(2bx) - \sin(2a), \cos(2bx) + \cos(2a)) + (\cos(2a+2c)^2 + \sin(2a+2c)^2 - 1) \arctan(\sin(2bx) + \sin(2c), \cos(2bx) + \cos(2c)) + \log(\cos(2bx)^2 + 2\cos(2bx)*\cos(2a) + \cos(2a)^2 + \sin(2bx)^2 - 2\sin(2bx)*\sin(2a) + \sin(2a)^2)*\sin(2a+2c) - \log(\cos(2bx)^2 + 2\cos(2bx)*\cos(2c) + \cos(2c)^2 + \sin(2bx)^2 - 2\sin(2bx)*\sin(2c) + \sin(2c)^2) \arctan(\sin(2bx) - \sin(2c), \cos(2bx) + \cos(2c))}{2b \sin(2a+2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="maxima")

[Out] ((b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)*x - (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) + (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(2*b*x) + sin(2*c), cos(2*b*x) + cos(2*c)) + log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2)*sin(2*a + 2*c) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2) \arctan2(sin(2*b*x) - sin(2*c), cos(2*b*x) + cos(2*c)))/2b \sin(2a+2c)

$$\frac{\sin(2bx)^2 + 2\sin(2bx)\sin(2c) + \sin(2c)^2 \sin(2a + 2c)}{(b\cos(2a + 2c))^2 + b\sin(2a + 2c)^2 - 2b\cos(2a + 2c) + b}$$

mupad [B] time = 5.00, size = 196, normalized size = 5.76

$$\frac{\frac{x}{2} + x \left(\sin(a+c)^2 - \frac{1}{2} \right)}{\sin(a+c)^2} + \frac{\sin(2a+2c) \ln(\sin(2a+2c)^{2i} \sin(a+bx)^{2i} \sin(3a+2c+bx)^{2i} \sin(4a+4c) + \sin(6a+4c+2bx) - \sin(2a+2c))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(a + b*x)*tan(c - b*x), x)

[Out] (x/2 + x*(sin(a + c)^2 - 1/2))/sin(a + c)^2 + ((sin(2*a + 2*c)*log(sin(4*a + 4*c) + sin(6*a + 4*c + 2*b*x) - sin(2*a + 2*b*x) + sin(2*a + 2*c)^2*2i - sin(a + b*x)^2*2i + sin(3*a + 2*c + b*x)^2*2i))/2 - (sin(2*a + 2*c)*log(sin(4*a + 4*c) + sin(4*a + 2*c + 2*b*x) + sin(2*c - 2*b*x) + sin(2*a + c + b*x)^2*2i + sin(2*a + 2*c)^2*2i - sin(c - b*x)^2*2i))/2)/(b*sin(a + c)^2)

sympy [B] time = 8.52, size = 7720, normalized size = 227.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-tan(b*x-c)*tan(b*x+a), x)

[Out] Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (2*b*x*tan(a)/(2*b*tan(a)**2 + 2*b) + 2*log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**2 + 2*b) - log(tan(b*x)**2 + 1)/(2*b*tan(a)**2 + 2*b), Eq(c, 0)), (-4*b*x*tan(c)**2*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 4*b*x*tan(c)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 2*log(tan(b*x) + 1/tan(c))*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 2*log(tan(b*x) + 1/tan(c))*tan(c)**2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) + 2*log(tan(b*x) + 1/tan(c))*tan(c)*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) + log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) + log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - log(tan(b*x)**2 + 1)*tan(c)*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - log(tan(b*x)**2 + 1)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 2*tan(c)**2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b), Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi - 1/2))), (0, Eq(b, 0)), (-2*b*x*tan(c)/(2*b*tan(c)**2 + 2*b) + 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c)**2 + 2*b) - log(tan(b*x)**2 + 1)/(2*b*tan(c)**2 + 2*b), Eq(a, 0)), (2*b*x*tan(a)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*b*x*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(a))*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c))

```

2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*
b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(a))*tan(a)/(2*b*tan(a)**
3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c
) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log
(tan(b*x) + 1/tan(c))*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a
)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**
2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*t
an(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 +
2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 +
2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**
2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*ta
n(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2
+ 1)*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a
)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) +
2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)/(2*b*tan(a)**3*t
an(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) +
2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(
b*x)**2 + 1)*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**
2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*
b*tan(c)**3 + 2*b*tan(c)), True))*tan(a) - Piecewise((0, Eq(a, 0) & Eq(b, 0
) & Eq(c, 0)), (2*b*x*tan(a)/(2*b*tan(a)**2 + 2*b) + 2*log(tan(b*x) - 1/tan
(a))/(2*b*tan(a)**2 + 2*b) - log(tan(b*x)**2 + 1)/(2*b*tan(a)**2 + 2*b), Eq
(c, 0)), (-4*b*x*tan(c)**2*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4
+ 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 4*
b*x*tan(c)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x)
+ 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 2*log(tan(b*x) + 1/tan(c))*
tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*
tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 2*log(tan(b*x) + 1/
tan(c))*tan(c)**2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*t
an(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) + 2*log(tan(b*x) + 1/t
an(c))*tan(c)*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)
**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) + 2*log(tan(b*x)
+ 1/tan(c))/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x)
) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) + log(tan(b*x)**2 + 1)*tan(c
)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b
*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) + log(tan(b*x)**2 + 1)*tan
(c)**2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4
*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - log(tan(b*x)**2 + 1)*tan(c)*tan
(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*
b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - log(tan(b*x)**2 + 1)/(2*b*tan(c)
**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b
*tan(c)*tan(b*x) + 2*b) - 2*tan(c)**2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)
**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) -
2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*ta
n(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b), Eq(a, -atan(tan(c)) - pi*floor((c - p
i/2)/pi) - pi*floor(c/pi - 1/2))), (0, Eq(b, 0)), (-2*b*x*tan(c)/(2*b*tan(c)
)**2 + 2*b) + 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c)**2 + 2*b) - log(tan(b*
x)**2 + 1)/(2*b*tan(c)**2 + 2*b), Eq(a, 0)), (2*b*x*tan(a)**2/(2*b*tan(a)**
3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c
) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*b*x
*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**
3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**
3 + 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(a))*tan(a)*tan(c)**2/(2*b*tan(a)**
3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c
) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*lo
g(tan(b*x) - 1/tan(a))*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*
b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*t
an(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*tan(a)**2*
tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 +

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2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 +
2*b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*tan(c)/(2*b*tan(a)**3*tan(c)**2 +
2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)
)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 +
1)*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**
2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*
b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)*tan(c)**2/(2*b*tan(
a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*t
an(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 1
og(tan(b*x)**2 + 1)*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*t
an(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(
a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)/(2*b*tan(a)*
**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(
c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)), True)
)*tan(c) + Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (2*b*x*tan(a)/(2*
b*tan(a)**3 + 2*b*tan(a)) + 2*log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**3 + 2*b
*tan(a)) + log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*tan(a)**3 + 2*b*tan(a)), Eq(
c, 0)), (-b*x*tan(c)**4*tan(b*x)/(b*tan(c)**6*tan(b*x) + b*tan(c)**5 + 2*b*
tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) + b*tan(c)) - b*x
*tan(c)**3/(b*tan(c)**6*tan(b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2
*b*tan(c)**3 + b*tan(c)**2*tan(b*x) + b*tan(c)) + b*x*tan(c)**2*tan(b*x)/(b
*tan(c)**6*tan(b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3
+ b*tan(c)**2*tan(b*x) + b*tan(c)) + b*x*tan(c)/(b*tan(c)**6*tan(b*x) + b*t
an(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) +
b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*tan(c)**3*tan(b*x)/(b*tan(c)**6*tan(
b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*t
an(b*x) + b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*tan(c)**2/(b*tan(c)**6*tan(
b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*
tan(b*x) + b*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(b*tan(c)**6
*tan(b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)
**2*tan(b*x) + b*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)**2/(b*tan(c)**6*tan(
b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*t
an(b*x) + b*tan(c)) + tan(c)**2/(b*tan(c)**6*tan(b*x) + b*tan(c)**5 + 2*b*t
an(c)**4*tan(b*x) + 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) + b*tan(c)) + 1/(b
*tan(c)**6*tan(b*x) + b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3
+ b*tan(c)**2*tan(b*x) + b*tan(c)), Eq(a, -atan(tan(c)) - pi*floor((c - pi/
2)/pi) - pi*floor(c/pi - 1/2))), (0, Eq(b, 0)), (2*b*x*tan(c)/(2*b*tan(c)**
3 + 2*b*tan(c)) - 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c)**3 + 2*b*tan(c)) -
log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(c)**3 + 2*b*tan(c)), Eq(a, 0)), (2
*b*x*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)
**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) +
2*b*tan(c)**3 + 2*b*tan(c)) + 2*b*x*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)*
**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*t
an(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*b*x*tan(a)/(
2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(
a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(
c)) + 2*b*x*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**
2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*
b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(a))*tan(c)**2/(2*b*tan(a)
)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*t
an(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*
log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(
a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a)
+ 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(tan(b*x) + 1/tan(c))*tan(a)**2/(2*b*
tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)*
**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c))
- 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*
b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*t
an(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*t

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an(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**
2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c))
- log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 +
2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*
b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)), True)) - Piecewise((-x, Eq(a, 0) &
Eq(b, 0) & Eq(c, 0)), (-2*b*x/(2*b*tan(a)**2 + 2*b) + 2*log(tan(b*x) - 1/tan
(a))*tan(a)/(2*b*tan(a)**2 + 2*b) - log(tan(b*x)**2 + 1)*tan(a)/(2*b*tan(a)
)**2 + 2*b), Eq(c, 0)), (b*x*tan(c)**3*tan(b*x)/(b*tan(c)**5*tan(b*x) + b*t
an(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*tan(b*x) + b)
+ b*x*tan(c)**2/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x)
) + 2*b*tan(c)**2 + b*tan(c)*tan(b*x) + b) - b*x*tan(c)*tan(b*x)/(b*tan(c)*
**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2*b*tan(c)**2 + b*tan(
c)*tan(b*x) + b) - b*x/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*
tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*tan(b*x) + b) - 2*log(tan(b*x) + 1/tan(
c))*tan(c)**2*tan(b*x)/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*
tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*tan(b*x) + b) - 2*log(tan(b*x) + 1/tan(
c))*tan(c)/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2
*b*tan(c)**2 + b*tan(c)*tan(b*x) + b) + log(tan(b*x)**2 + 1)*tan(c)**2*tan(
b*x)/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2*b*tan
(c)**2 + b*tan(c)*tan(b*x) + b) + log(tan(b*x)**2 + 1)*tan(c)/(b*tan(c)**5*
tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*
tan(b*x) + b) + tan(c)**3/(b*tan(c)**5*tan(b*x) + b*tan(c)**4 + 2*b*tan(c)*
**3*tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*tan(b*x) + b) + tan(c)/(b*tan(c)**5*
tan(b*x) + b*tan(c)**4 + 2*b*tan(c)**3*tan(b*x) + 2*b*tan(c)**2 + b*tan(c)*
tan(b*x) + b), Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi
- 1/2))), (-x, Eq(b, 0)), (-2*b*x/(2*b*tan(c)**2 + 2*b) - 2*log(tan(b*x)
+ 1/tan(c))*tan(c)/(2*b*tan(c)**2 + 2*b) + log(tan(b*x)**2 + 1)*tan(c)/(2*b
*tan(c)**2 + 2*b), Eq(a, 0)), (-2*b*x*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)
)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b
*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*b*x*tan(a)
*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)*
**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)
**3 + 2*b*tan(c)) - 2*b*x*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 +
2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b
*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*b*x*tan(c)/(2*b*tan(a)**3*tan(c)*
**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*t
an(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x)
- 1/tan(a))*tan(a)**2*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 +
2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*
b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*tan(a)*
**2*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(
c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(
c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*tan(
a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*
tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) +
log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2
*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*
tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)), True))*tan(a)*tan(c)

```

3.141 $\int \cot(a + bx) \cot(c + bx) dx$

Optimal. Leaf size=39

$$-\frac{\cot(a-c) \log(\sin(a+bx))}{b} + \frac{\cot(a-c) \log(\sin(bx+c))}{b} - x$$

[Out] $-x - \cot(a-c) \ln(\sin(bx+a))/b + \cot(a-c) \ln(\sin(bx+c))/b$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4613, 4611, 3475}

$$-\frac{\cot(a-c) \log(\sin(a+bx))}{b} + \frac{\cot(a-c) \log(\sin(bx+c))}{b} - x$$

Antiderivative was successfully verified.

[In] Int[Cot[a + b*x]*Cot[c + b*x],x]

[Out] $-x - (\cot[a - c] \log[\sin[a + b*x]])/b + (\cot[a - c] \log[\sin[c + b*x]])/b$

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4611

Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Csc[(b*c - a*d)/b], Int[Cot[a + b*x], x], x] - Dist[Csc[(b*c - a*d)/d], Int[Cot[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]

Rule 4613

Int[Cot[(a_.) + (b_.)*(x_)]*Cot[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[(b*x)/d, x] + Dist[Cos[(b*c - a*d)/d], Int[Csc[a + b*x]*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \cot(a + bx) \cot(c + bx) dx &= -x + \cos(a - c) \int \csc(a + bx) \csc(c + bx) dx \\ &= -x - \cot(a - c) \int \cot(a + bx) dx + \cot(a - c) \int \cot(c + bx) dx \\ &= -x - \frac{\cot(a - c) \log(\sin(a + bx))}{b} + \frac{\cot(a - c) \log(\sin(c + bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.51, size = 31, normalized size = 0.79

$$\frac{\cot(a-c)(\log(\sin(bx+c)) - \log(\sin(a+bx)))}{b} - x$$

Antiderivative was successfully verified.

[In] Integrate[Cot[a + b*x]*Cot[c + b*x],x]

[Out] $-x + (\cot[a - c] * (-\log[\sin[a + b*x]] + \log[\sin[c + b*x]]))/b$

fricas [B] time = 1.53, size = 118, normalized size = 3.03

$$\frac{2bx \sin(-2a + 2c) - (\cos(-2a + 2c) + 1) \log\left(-\frac{\cos(2bx+2c)\cos(-2a+2c)+\sin(2bx+2c)\sin(-2a+2c)-1}{\cos(-2a+2c)+1}\right) + (\cos(-2a + 2c) + 1) \log(-1/2\cos(2bx + 2c) + 1/2)}{2b \sin(-2a + 2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="fricas")

[Out] -1/2*(2*b*x*sin(-2*a + 2*c) - (cos(-2*a + 2*c) + 1)*log(-(cos(2*b*x + 2*c)*cos(-2*a + 2*c) + sin(2*b*x + 2*c)*sin(-2*a + 2*c) - 1)/(cos(-2*a + 2*c) + 1)) + (cos(-2*a + 2*c) + 1)*log(-1/2*cos(2*b*x + 2*c) + 1/2))/(b*sin(-2*a + 2*c))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2/b*(-1/2*b*x+(tan(a/2)^2*tan(c/2)^4-2*tan(a/2)^2*tan(c/2)^2+tan(a/2)^2+4*tan(a/2)*tan(c/2)^3-4*tan(a/2)*tan(c/2)-tan(c/2)^4+2*tan(c/2)^2-1)/(4*tan(a/2)^2*tan(c/2)^3-4*tan(a/2)^2*tan(c/2)-4*tan(a/2)*tan(c/2)^4+8*tan(a/2)*tan(c/2)^2-4*tan(a/2)-4*tan(c/2)^3+4*tan(c/2))*ln(abs(tan(b*x)*tan(c/2)^2-tan(b*x)-2*tan(c/2)))+(tan(a/2)^4*tan(c/2)^2-tan(a/2)^4+4*tan(a/2)^3*tan(c/2)-2*tan(a/2)^2*tan(c/2)^2+2*tan(a/2)^2-4*tan(a/2)*tan(c/2)+tan(c/2)^2-1)/(-4*tan(a/2)^4*tan(c/2)+4*tan(a/2)^3*tan(c/2)^2-4*tan(a/2)^3+8*tan(a/2)^2*tan(c/2)-4*tan(a/2)*tan(c/2)^2+4*tan(a/2)-4*tan(c/2))*ln(abs(tan(b*x)*tan(a/2)^2-tan(b*x)-2*tan(a/2)))

maple [C] time = 0.19, size = 177, normalized size = 4.54

$$-x - \frac{i \ln(e^{2i(bx+a)} - 1) e^{2ia}}{b(e^{2ia} - e^{2ic})} - \frac{i \ln(e^{2i(bx+a)} - 1) e^{2ic}}{b(e^{2ia} - e^{2ic})} + \frac{i \ln(e^{2i(bx+a)} - e^{2i(a-c)}) e^{2ia}}{b(e^{2ia} - e^{2ic})} + \frac{i \ln(e^{2i(bx+a)} - e^{2i(a-c)}) e^{2ic}}{b(e^{2ia} - e^{2ic})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(b*x+a)*cot(b*x+c),x)

[Out] -x-I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))-1)*exp(2*I*a)-I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))-1)*exp(2*I*c)+I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*exp(2*I*a)+I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*exp(2*I*c)

maxima [B] time = 0.93, size = 549, normalized size = 14.08

$$\frac{(2b \cos(2a) \cos(2c) - b \cos(2c)^2 + 2b \sin(2a) \sin(2c) - b \sin(2c)^2 - (\cos(2a)^2 + \sin(2a)^2)b)x + (\cos(2a) \cos(2c) - \cos(2a) \sin(2c) + \sin(2a) \cos(2c) - \sin(2a) \sin(2c))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="maxima")

[Out] -((2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)*x + (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + (cos(2*a)^2

$$\begin{aligned}
& -\cos(2c)^2 + \sin(2a)^2 - \sin(2c)^2 \cdot \arctan_2(\sin(bx) - \sin(a), \cos(bx) \\
& + \cos(a)) - (\cos(2a)^2 - \cos(2c)^2 + \sin(2a)^2 - \sin(2c)^2) \cdot \arctan_2(\sin(bx) + \sin(c), \cos(bx) - \cos(c)) \\
& - (\cos(2a)^2 - \cos(2c)^2 + \sin(2a)^2 - \sin(2c)^2) \cdot \arctan_2(\sin(bx) - \sin(c), \cos(bx) + \cos(c)) \\
& - (\cos(2c) \cdot \sin(2a) - \cos(2a) \cdot \sin(2c)) \cdot \log(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2) \\
& - (\cos(2c) \cdot \sin(2a) - \cos(2a) \cdot \sin(2c)) \cdot \log(\cos(bx)^2 - 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 + 2\sin(bx)\sin(a) + \sin(a)^2) \\
& + (\cos(2c) \cdot \sin(2a) - \cos(2a) \cdot \sin(2c)) \cdot \log(\cos(bx)^2 + 2\cos(bx)\cos(c) + \cos(c)^2 + \sin(bx)^2 - 2\sin(bx)\sin(c) + \sin(c)^2) \\
& + (\cos(2c) \cdot \sin(2a) - \cos(2a) \cdot \sin(2c)) \cdot \log(\cos(bx)^2 - 2\cos(bx)\cos(c) + \cos(c)^2 + \sin(bx)^2 + 2\sin(bx)\sin(c) + \sin(c)^2) \\
&) / (2b\cos(2a)\cos(2c) - b\cos(2c)^2 + 2b\sin(2a)\sin(2c) - b\sin(2c)^2 - (\cos(2a)^2 + \sin(2a)^2)b)
\end{aligned}$$

mupad [B] time = 4.82, size = 207, normalized size = 5.31

$$\frac{\frac{x}{2} + x \left(\sin(a-c)^2 - \frac{1}{2} \right)}{\sin(a-c)^2} - \frac{\sin(2a-2c) \ln(\sin(2a-2c)^{2i} + \sin(a+bx)^{2i} - \sin(3a-2c+bx)^{2i} + \sin(4a-4c) - \sin(6a-4c+2bx) + \sin(2a+2c))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(a + b*x)*cot(c + b*x), x)

[Out] $-(x/2 + x(\sin(a-c)^2 - 1/2))/\sin(a-c)^2 - ((\sin(2a-2c) \cdot \log(\sin(4a-4c) - \sin(6a-4c+2bx)) + \sin(2a+2bx) + \sin(2a-2c)^{2i} + \sin(a+bx)^{2i} - \sin(3a-2c+bx)^{2i}))/2 - (\sin(2a-2c) \cdot \log(\sin(4a-4c) - \sin(4a-2c+2bx) + \sin(2c+2bx) + \sin(2a-2c)^{2i} + \sin(c+bx)^{2i} - \sin(2a-c+bx)^{2i}))/2)/(b \cdot \sin(a-c)^2)$

sympy [B] time = 24.28, size = 7485, normalized size = 191.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+a)*cot(b*x+c), x)

[Out] Piecewise((x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b, 0) & Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi) + pi*floor(c/pi - 1/2))), (-b*x*tan(c)**5/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - b*x*tan(c)**4*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) + b*x*tan(c)**3/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) + b*x*tan(c)**2*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - 2*log(tan(c) + tan(b*x))*tan(c)**4/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - 2*log(tan(c) + tan(b*x))*tan(c)**3*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) + log(tan(b*x)**2 + 1)*tan(c)**4/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) + log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - tan(c)**6/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - tan(c)**4/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x))), Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi) + pi*floor(c/pi - 1/2))), (x/(cot(a)*cot(c) + zoo*cot(a) + zoo*cot(c) + zoo), Eq(b, 0)), (-2*b*x*tan(a)**3*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(a))

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n(c)**3 - 2*b*tan(c)) + 2*b*x*tan(a)**2*tan(c)**3/(2*b*tan(a)**3*tan(c)**2
+ 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(
a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*b*x*tan(a)**2*t
an(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 -
2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 -
2*b*tan(c)) - 2*b*x*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)*
*3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2
+ 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(a) + tan(b*x))*tan(a
)**3*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan
(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan
(c)**3 - 2*b*tan(c)) - 2*log(tan(a) + tan(b*x))*tan(a)**3*tan(c)/(2*b*tan(a
)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan
(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*
log(tan(c) + tan(b*x))*tan(a)**3*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*t
an(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(
c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(c) + tan(b*x))
*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*
tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*
tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)**3*tan(c)/(2*b*tan(a
)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan
(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - log
(tan(b*x)**2 + 1)*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3
- 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 +
2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)), True)) + Piecewise((zoo*x/(zoo*cot
(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b, 0) & Eq(a, atan(tan(c)) + p
i*floor((c - pi/2)/pi) + pi*floor(c/pi - 1/2))), (b*x*tan(c)**5/(b*tan(c)**
5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c
) + b*tan(b*x)) + b*x*tan(c)**4*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*tan(b*x
) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - b*x*t
an(c)**3/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**
2*tan(b*x) + b*tan(c) + b*tan(b*x)) - b*x*tan(c)**2*tan(b*x)/(b*tan(c)**5 +
b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) +
b*tan(b*x)) + 2*log(tan(c) + tan(b*x))*tan(c)**4/(b*tan(c)**5 + b*tan(c)**
4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)
) + 2*log(tan(c) + tan(b*x))*tan(c)**3*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*
tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x))
- log(tan(b*x)**2 + 1)*tan(c)**4/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*
tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - log(tan(b*x)*
*2 + 1)*tan(c)**3*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)
**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - tan(c)**4/(b*tan(c)
**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan
(c) + b*tan(b*x)) - tan(c)**2/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan
(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)), Eq(a, atan(tan(c)
) + pi*floor((c - pi/2)/pi) + pi*floor(c/pi - 1/2))), (zoo*x/(cot(a)*cot(c)
+ zoo*cot(a) + zoo*cot(c) + zoo), Eq(b, 0)), (2*b*x*tan(a)**3*tan(c)**2/(2
*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(
a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(
c)) - 2*b*x*tan(a)**2*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 -
2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b
*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*b*x*tan(a)**2*tan(c)/(2*b*tan(a)*
*3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(
c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*b*
x*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2
*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b
*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(a) + tan(b*x))*tan(a)*tan(c)**3/(2*b*t
an(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**
2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c))
- 2*log(tan(a) + tan(b*x))*tan(a)*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan
(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)

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**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(c) + tan(b*x))*t
an(a)**3*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*ta
n(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*ta
n(c)**3 - 2*b*tan(c)) + 2*log(tan(c) + tan(b*x))*tan(a)*tan(c)/(2*b*tan(a)*
**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(
c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - log(
tan(b*x)**2 + 1)*tan(a)**3*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3
- 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2
*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*tan(c
)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2
*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2
*b*tan(c)), True))*cot(a)*cot(c) - Piecewise((zoo*x/(zoo*cot(c) + zoo + cot
(c)/tan(c) + zoo/tan(c)), Eq(b, 0) & Eq(a, atan(tan(c)) + pi*floor((c - pi/
2)/pi) + pi*floor(c/pi - 1/2))), (4*b*x*tan(c)**4/(2*b*tan(c)**5 + 2*b*tan(
c)**4*tan(b*x) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*
tan(b*x)) + 4*b*x*tan(c)**3*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x
) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) - 2
*log(tan(c) + tan(b*x))*tan(c)**5/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) +
4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) - 2*lo
g(tan(c) + tan(b*x))*tan(c)**4*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(
b*x) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x))
+ 2*log(tan(c) + tan(b*x))*tan(c)**3/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x
) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + 2
*log(tan(c) + tan(b*x))*tan(c)**2*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*t
an(b*x) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x
)) + log(tan(b*x)**2 + 1)*tan(c)**5/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x)
+ 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + lo
g(tan(b*x)**2 + 1)*tan(c)**4*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*
x) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) -
log(tan(b*x)**2 + 1)*tan(c)**3/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*
b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) - log(tan
(b*x)**2 + 1)*tan(c)**2*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) +
4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + 2*tan
(c)**5/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*b*tan(c)**3 + 4*b*tan(c)
**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + 2*tan(c)**3/(2*b*tan(c)**5 + 2
*b*tan(c)**4*tan(b*x) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c)
+ 2*b*tan(b*x)), Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi) + pi*floor(c/
pi - 1/2))), (zoo*x/(cot(a)*cot(c) + zoo*cot(a) + zoo*cot(c) + zoo), Eq(b,
0)), (2*b*x*tan(a)**3*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b
*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*ta
n(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*b*x*tan(a)*tan(c)**3/(2*b*tan(a)**3*
tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c)
+ 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(t
an(a) + tan(b*x))*tan(a)**2*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)
**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2
+ 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(a) + tan(b*x))*tan(
a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c
)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c
)**3 - 2*b*tan(c)) - 2*log(tan(c) + tan(b*x))*tan(a)**3*tan(c)**2/(2*b*tan(
a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*t
an(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2
*log(tan(c) + tan(b*x))*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan
(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)
**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a
)**3*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan
(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan
(c)**3 - 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**2*tan(c)**3/(2*b*tan(a)
**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan
(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - log

```

```

(tan(b*x)**2 + 1)*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3
- 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 +
2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*tan(
c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 -
2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 -
2*b*tan(c)), True))*cot(a) - Piecewise((zoo*x/(zoo*cot(c) + zoo + cot(c)/ta
n(c) + zoo/tan(c)), Eq(b, 0) & Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi)
+ pi*floor(c/pi - 1/2))), (4*b*x*tan(c)**4/(2*b*tan(c)**5 + 2*b*tan(c)**4*
tan(b*x) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*
x)) + 4*b*x*tan(c)**3*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*
b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) - 2*log(t
an(c) + tan(b*x))*tan(c)**5/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*b*t
an(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) - 2*log(tan(
c) + tan(b*x))*tan(c)**4*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) +
4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + 2*lo
g(tan(c) + tan(b*x))*tan(c)**3/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*
b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + 2*log(t
an(c) + tan(b*x))*tan(c)**2*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x)
) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + l
og(tan(b*x)**2 + 1)*tan(c)**5/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*b
*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + log(tan(
b*x)**2 + 1)*tan(c)**4*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4
*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) - log(ta
n(b*x)**2 + 1)*tan(c)**3/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*b*tan(
c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) - log(tan(b*x)*
**2 + 1)*tan(c)**2*tan(b*x)/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*b*ta
n(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + 2*tan(c)**5
/(2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) + 4*b*tan(c)**3 + 4*b*tan(c)**2*ta
n(b*x) + 2*b*tan(c) + 2*b*tan(b*x)) + 2*tan(c)**3/(2*b*tan(c)**5 + 2*b*tan(
c)**4*tan(b*x) + 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) + 2*b*tan(c) + 2*b*
tan(b*x)), Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi) + pi*floor(c/pi - 1
/2))), (zoo*x/(cot(a)*cot(c) + zoo*cot(a) + zoo*cot(c) + zoo), Eq(b, 0)), (
2*b*x*tan(a)**3*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)
)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) -
2*b*tan(c)**3 - 2*b*tan(c)) - 2*b*x*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)
**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*
tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(a)
+ tan(b*x))*tan(a)**2*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 -
2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b
*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(a) + tan(b*x))*tan(a)**2*
tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 -
2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 -
2*b*tan(c)) - 2*log(tan(c) + tan(b*x))*tan(a)**3*tan(c)**2/(2*b*tan(a)**3*
tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c)
+ 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(t
an(c) + tan(b*x))*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3
- 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 +
2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)**3*t
an(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3
- 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3
- 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**2*tan(c)**3/(2*b*tan(a)**3*ta
n(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) +
2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - log(tan(b
*x)**2 + 1)*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b
*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*ta
n(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*tan(c)**2/
(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*ta
n(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*ta
n(c)), True))*cot(c)

```


3.142 $\int \cot(c - bx) \cot(a + bx) dx$

Optimal. Leaf size=34

$$-\frac{\cot(a+c)\log(\sin(c-bx))}{b} + \frac{\cot(a+c)\log(\sin(a+bx))}{b} + x$$

[Out] $x - \cot(a+c) \cdot \ln(-\sin(b \cdot x - c)) / b + \cot(a+c) \cdot \ln(\sin(b \cdot x + a)) / b$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4613, 4611, 3475}

$$-\frac{\cot(a+c)\log(\sin(c-bx))}{b} + \frac{\cot(a+c)\log(\sin(a+bx))}{b} + x$$

Antiderivative was successfully verified.

[In] Int[Cot[c - b*x]*Cot[a + b*x], x]

[Out] $x - (\text{Cot}[a + c] \cdot \text{Log}[\text{Sin}[c - b \cdot x]]) / b + (\text{Cot}[a + c] \cdot \text{Log}[\text{Sin}[a + b \cdot x]]) / b$

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4611

Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Csc[(b*c - a*d)/b], Int[Cot[a + b*x], x], x] - Dist[Csc[(b*c - a*d)/d], Int[Cot[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]

Rule 4613

Int[Cot[(a_.) + (b_.)*(x_)]*Cot[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[(b*x)/d, x] + Dist[Cos[(b*c - a*d)/d], Int[Csc[a + b*x]*Csc[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \cot(c - bx) \cot(a + bx) dx &= x + \cos(a + c) \int \csc(c - bx) \csc(a + bx) dx \\ &= x + \cot(a + c) \int \cot(c - bx) dx + \cot(a + c) \int \cot(a + bx) dx \\ &= x - \frac{\cot(a + c) \log(\sin(c - bx))}{b} + \frac{\cot(a + c) \log(\sin(a + bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.50, size = 30, normalized size = 0.88

$$\frac{\cot(a+c)(\log(-\sin(a+bx)) - \log(\sin(c-bx)))}{b} + x$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c - b*x]*Cot[a + b*x], x]

[Out] $x + (\text{Cot}[a + c] \cdot (-\text{Log}[\text{Sin}[c - b \cdot x]] + \text{Log}[-\text{Sin}[a + b \cdot x]])) / b$

fricas [B] time = 2.94, size = 118, normalized size = 3.47

$$\frac{2bx \sin(2a + 2c) - (\cos(2a + 2c) + 1) \log\left(-\frac{\cos(2bx+2a)\cos(2a+2c)+\sin(2bx+2a)\sin(2a+2c)-1}{\cos(2a+2c)+1}\right) + (\cos(2a + 2c) + 1)}{2b \sin(2a + 2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="fricas")

[Out] 1/2*(2*b*x*sin(2*a + 2*c) - (cos(2*a + 2*c) + 1)*log(-(cos(2*b*x + 2*a)*cos(2*a + 2*c) + sin(2*b*x + 2*a)*sin(2*a + 2*c) - 1)/(cos(2*a + 2*c) + 1)) + (cos(2*a + 2*c) + 1)*log(-1/2*cos(2*b*x + 2*a) + 1/2))/(b*sin(2*a + 2*c))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)-2/b*(-1/2*b*x+(tan(a/2)^2*tan(c/2)^4-2*tan(a/2)^2*tan(c/2)^2+tan(a/2)^2-4*tan(a/2)*tan(c/2)^3+4*tan(a/2)*tan(c/2)-tan(c/2)^4+2*tan(c/2)^2-1)/(-4*tan(a/2)^2*tan(c/2)^3+4*tan(a/2)^2*tan(c/2)-4*tan(a/2)*tan(c/2)^4+8*tan(a/2)*tan(c/2)^2-4*tan(a/2)+4*tan(c/2)^3-4*tan(c/2))*ln(abs(tan(b*x)*tan(c/2)^2-tan(b*x)+2*tan(c/2)))+(tan(a/2)^4*tan(c/2)^2-tan(a/2)^4-4*tan(a/2)^3*tan(c/2)-2*tan(a/2)^2*tan(c/2)^2+2*tan(a/2)^2+4*tan(a/2)*tan(c/2)+tan(c/2)^2-1)/(4*tan(a/2)^4*tan(c/2)+4*tan(a/2)^3*tan(c/2)^2-4*tan(a/2)^3-8*tan(a/2)^2*tan(c/2)-4*tan(a/2)*tan(c/2)^2+4*tan(a/2)+4*tan(c/2))*ln(abs(tan(b*x)*tan(a/2)^2-tan(b*x)-2*tan(a/2)))

maple [C] time = 0.18, size = 149, normalized size = 4.38

$$x - \frac{i \ln(-e^{2i(a+c)} + e^{2i(bx+a)}) e^{2i(a+c)}}{b(e^{2i(a+c)} - 1)} - \frac{i \ln(-e^{2i(a+c)} + e^{2i(bx+a)})}{b(e^{2i(a+c)} - 1)} + \frac{i \ln(e^{2i(bx+a)} - 1) e^{2i(a+c)}}{b(e^{2i(a+c)} - 1)} + \frac{i \ln(e^{2i(bx+a)} - 1)}{b(e^{2i(a+c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-cot(b*x-c)*cot(b*x+a),x)

[Out] x-I/b/(exp(2*I*(a+c))-1)*ln(-exp(2*I*(a+c))+exp(2*I*(b*x+a)))*exp(2*I*(a+c))-I/b/(exp(2*I*(a+c))-1)*ln(-exp(2*I*(a+c))+exp(2*I*(b*x+a)))+I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(b*x+a))-1)*exp(2*I*(a+c))+I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(b*x+a))-1)

maxima [B] time = 0.44, size = 432, normalized size = 12.71

$$\frac{(b \cos(2a + 2c)^2 + b \sin(2a + 2c)^2 - 2b \cos(2a + 2c) + b)x - (\cos(2a + 2c)^2 + \sin(2a + 2c)^2 - 1) \arctan(s$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="maxima")

[Out] ((b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)*x - (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) - (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(b*x) - sin(a), cos(b*x) + cos(a)) + (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(b*x) + sin(c), cos(b*x) + cos(c)) + (cos(2*a + 2*c)^2 + sin(2*a + 2*c


```

*b*tan(c)**3 + 2*b*tan(c)) - 2*b*x*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**
2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*ta
n(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(a) +
tan(b*x))*tan(a)**3*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*
b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*t
an(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(a) + tan(b*x))*tan(a)**3*ta
n(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2
*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2
*b*tan(c)) - 2*log(-tan(c) + tan(b*x))*tan(a)**3*tan(c)**3/(2*b*tan(a)**3*t
an(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) +
2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(-t
an(c) + tan(b*x))*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3
+ 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 +
2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**3*t
an(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 +
2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 +
2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**
2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*ta
n(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)), True)) + Piecewi
se((zoo*x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b, 0) & Eq(a,
-atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi - 1/2))), (-b*x*ta
n(c)**5/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**
2*tan(b*x) - b*tan(c) + b*tan(b*x)) + b*x*tan(c)**4*tan(b*x)/(-b*tan(c)**5
+ b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c)
+ b*tan(b*x)) + b*x*tan(c)**3/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*ta
n(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - b*x*tan(c)**2*t
an(b*x)/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**
2*tan(b*x) - b*tan(c) + b*tan(b*x)) + 2*log(-tan(c) + tan(b*x))*tan(c)**4/(
-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x)
) - b*tan(c) + b*tan(b*x)) - 2*log(-tan(c) + tan(b*x))*tan(c)**3*tan(b*x)/(
-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x)
) - b*tan(c) + b*tan(b*x)) - log(tan(b*x)**2 + 1)*tan(c)**4/(-b*tan(c)**5 +
b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) +
b*tan(b*x)) + log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(-b*tan(c)**5 + b*ta
n(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*ta
n(b*x)) - tan(c)**4/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 +
2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - tan(c)**2/(-b*tan(c)**5 +
b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) +
b*tan(b*x)), Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi
- 1/2))), (zoo*x/(-cot(a)*cot(c) + zoo*cot(a) + zoo*cot(c) + zoo), Eq(b, 0
)), (2*b*x*tan(a)**3*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2
*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*
tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*b*x*tan(a)**2*tan(c)**3/(2*b*tan(a)
)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*ta
n(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*
b*x*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)*
**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) +
2*b*tan(c)**3 + 2*b*tan(c)) + 2*b*x*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**
2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*ta
n(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(a) +
tan(b*x))*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*ta
n(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(
a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(a) + tan(b*x))*tan(a)*tan(c)/(
2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan
(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan
(c)) - 2*log(-tan(c) + tan(b*x))*tan(a)**3*tan(c)/(2*b*tan(a)**3*tan(c)**2
+ 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(
a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(-tan(c) + t
an(b*x))*tan(a)*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)

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)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) +
  2*b*tan(c)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)**3*tan(c)/(2*b*t
an(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**
2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c))
- log(tan(b*x)**2 + 1)*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(
a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)*
*2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)), True))*cot(a)*cot(c) + Piece
wise((zoo*x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b, 0) & Eq(
a, -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi - 1/2))), (4*b*x
*tan(c)**4/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*t
an(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 4*b*x*tan(c)**3*tan(b*x)/(
-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)**2*tan
(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) + 2*log(-tan(c) + tan(b*x))*tan(c)**5/(
-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)**2*tan
(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 2*log(-tan(c) + tan(b*x))*tan(c)**4*tan
(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)
**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 2*log(-tan(c) + tan(b*x))*tan(c)
**3/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)*
*2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) + 2*log(-tan(c) + tan(b*x))*tan(c)
**2*tan(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b
*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - log(tan(b*x)**2 + 1)*tan
(c)**5/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)
**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) + log(tan(b*x)**2 + 1)*tan(c)**4
*tan(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan
(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) + log(tan(b*x)**2 + 1)*tan(c)
**3/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)**
2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - log(tan(b*x)**2 + 1)*tan(c)**2*tan
(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)
)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 2*tan(c)**5/(-2*b*tan(c)**5 +
2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c)
) + 2*b*tan(b*x)) - 2*tan(c)**3/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) -
4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)), Eq(a,
-atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi - 1/2))), (zoo*x/(
-cot(a)*cot(c) + zoo*cot(a) + zoo*cot(c) + zoo), Eq(b, 0)), (-2*b*x*tan(a)**
3*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3
+ 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3
+ 2*b*tan(c)) + 2*b*x*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(
a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)*
*2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(tan(a) + tan(b*x))*tan
(a)**2*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*
tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*
tan(c)**3 + 2*b*tan(c)) - 2*log(tan(a) + tan(b*x))*tan(a)**2*tan(c)/(2*b*tan
(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2
*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) -
  2*log(-tan(c) + tan(b*x))*tan(a)**3*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2
*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*
tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(-tan(c) + tan(
b*x))*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)
)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) +
  2*b*tan(c)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)**3*tan(c)**2/(2*
b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)
)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)
)) + log(tan(b*x)**2 + 1)*tan(a)**2*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*
b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*t
an(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*
tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*t
an(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*t
an(c)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*tan(c)**2/(2*b*tan(a)*
*3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(

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c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)), True)
)*cot(a) - Piecewise((zoo*x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c))
, Eq(b, 0) & Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi
- 1/2))), (4*b*x*tan(c)**4/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*t
an(c)**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 4*b*x*tan(
c)**3*tan(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4
*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) + 2*log(-tan(c) + tan(b*
x))*tan(c)**5/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*
b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 2*log(-tan(c) + tan(b*x
))*tan(c)**4*tan(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)
**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 2*log(-tan(c) +
tan(b*x))*tan(c)**3/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)*
**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) + 2*log(-tan(c) +
tan(b*x))*tan(c)**2*tan(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b
*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - log(tan(
b*x)**2 + 1)*tan(c)**5/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)
**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) + log(tan(b*x)**
2 + 1)*tan(c)**4*tan(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*ta
n(c)**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) + log(tan(b*x)
)**2 + 1)*tan(c)**3/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**
3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - log(tan(b*x)**2 +
1)*tan(c)**2*tan(b*x)/(-2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)
)**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 2*tan(c)**5/(-
2*b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(
b*x) - 2*b*tan(c) + 2*b*tan(b*x)) - 2*tan(c)**3/(-2*b*tan(c)**5 + 2*b*tan(c)
)**4*tan(b*x) - 4*b*tan(c)**3 + 4*b*tan(c)**2*tan(b*x) - 2*b*tan(c) + 2*b*t
an(b*x)), Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi) - pi*floor(c/pi - 1
/2))), (zoo*x/(-cot(a)*cot(c) + zoo*cot(a) + zoo*cot(c) + zoo), Eq(b, 0)),
(-2*b*x*tan(a)**3*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan
(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a)
+ 2*b*tan(c)**3 + 2*b*tan(c)) + 2*b*x*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(
c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*
b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(tan(a)
+ tan(b*x))*tan(a)**2*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3
+ 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2
*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(tan(a) + tan(b*x))*tan(a)**
2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3
+ 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3
+ 2*b*tan(c)) - 2*log(-tan(c) + tan(b*x))*tan(a)**3*tan(c)**2/(2*b*tan(a)*
**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(
c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*lo
g(-tan(c) + tan(b*x))*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)
)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**
2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*
**3*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(
c))**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)
)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)**2*tan(c)**3/(2*b*tan(a)**
3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c)
) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + log(t
an(b*x)**2 + 1)*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 +
2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*
b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*tan(c)
)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*
b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*
b*tan(c)), True))*cot(c)

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3.143 $\int \sec(a + bx) \sec(c + bx) dx$

Optimal. Leaf size=36

$$\frac{\csc(a - c) \log(\cos(bx + c))}{b} - \frac{\csc(a - c) \log(\cos(a + bx))}{b}$$

[Out] $-\csc(a-c)*\ln(\cos(b*x+a))/b+\csc(a-c)*\ln(\cos(b*x+c))/b$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4610, 3475}

$$\frac{\csc(a - c) \log(\cos(bx + c))}{b} - \frac{\csc(a - c) \log(\cos(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b*x]*Sec[c + b*x], x]

[Out] $-\left(\frac{\csc[a - c] \log[\cos[a + b*x]]}{b}\right) + \left(\frac{\csc[a - c] \log[\cos[c + b*x]]}{b}\right)$

Rule 3475

Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4610

Int[Sec[(a_.) + (b_.)*(x_.)]*Sec[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Dist[Csc[(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \sec(a + bx) \sec(c + bx) dx &= \csc(a - c) \int \tan(a + bx) dx - \csc(a - c) \int \tan(c + bx) dx \\ &= -\frac{\csc(a - c) \log(\cos(a + bx))}{b} + \frac{\csc(a - c) \log(\cos(c + bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.23, size = 28, normalized size = 0.78

$$-\frac{\csc(a - c)(\log(\cos(a + bx)) - \log(\cos(bx + c)))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b*x]*Sec[c + b*x], x]

[Out] $-\left(\frac{\csc[a - c](\log[\cos[a + b*x]] - \log[\cos[c + b*x]])}{b}\right)$

fricas [B] time = 1.99, size = 107, normalized size = 2.97

$$\frac{\log(\cos(bx + c)^2) - \log\left(\frac{4(2 \cos(bx+c) \cos(-a+c) \sin(bx+c) \sin(-a+c) + (2 \cos(-a+c)^2 - 1) \cos(bx+c)^2 - \cos(-a+c)^2 + 1)}{\cos(-a+c)^2 + 2 \cos(-a+c) + 1}\right)}{2b \sin(-a + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="fricas")

[Out] $-1/2*(\log(\cos(b*x + c)^2) - \log(4*(2*\cos(b*x + c)*\cos(-a + c)*\sin(b*x + c)*\sin(-a + c) + (2*\cos(-a + c)^2 - 1)*\cos(b*x + c)^2 - \cos(-a + c)^2 + 1)/(\cos(-a + c)^2 + 2*\cos(-a + c) + 1)))/(b*\sin(-a + c))$

giac [B] time = 0.23, size = 171, normalized size = 4.75

$$\frac{\left(\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 + \tan\left(\frac{1}{2}c\right)^2 + 1\right) \log\left(\left|2 \tan(bx + a) \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right) - 2 \tan(bx + a) \tan\left(\frac{1}{2}c\right)\right|\right)}{2 \left(\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 + \tan\left(\frac{1}{2}c\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="giac")

[Out] $1/2*(\tan(1/2*a)^2*\tan(1/2*c)^2 + \tan(1/2*a)^2 + \tan(1/2*c)^2 + 1)*\log(\text{abs}(2*\tan(b*x + a)*\tan(1/2*a)^2*\tan(1/2*c) - 2*\tan(b*x + a)*\tan(1/2*a)*\tan(1/2*c)^2 + \tan(1/2*a)^2*\tan(1/2*c)^2 + 2*\tan(b*x + a)*\tan(1/2*a) - \tan(1/2*a)^2 - 2*\tan(b*x + a)*\tan(1/2*c) + 4*\tan(1/2*a)*\tan(1/2*c) - \tan(1/2*c)^2 + 1))/((\tan(1/2*a)^2*\tan(1/2*c) - \tan(1/2*a)*\tan(1/2*c)^2 + \tan(1/2*a) - \tan(1/2*c))*b)$

maple [A] time = 0.48, size = 55, normalized size = 1.53

$$\frac{\ln(-\tan(bx + a) \cos(a) \sin(c) + \tan(bx + a) \sin(a) \cos(c) + \cos(a) \cos(c) + \sin(a) \sin(c))}{b(\cos(a) \sin(c) - \sin(a) \cos(c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)*sec(b*x+c),x)

[Out] $-1/b/(\cos(a)*\sin(c)-\sin(a)*\cos(c))*\ln(-\tan(b*x+a)*\cos(a)*\sin(c)+\tan(b*x+a)*\sin(a)*\cos(c)+\cos(a)*\cos(c)+\sin(a)*\sin(c))$

maxima [B] time = 0.34, size = 349, normalized size = 9.69

$$\frac{2((\cos(2a) - \cos(2c)) \cos(a + c) + (\sin(2a) - \sin(2c)) \sin(a + c)) \arctan(\sin(2bx) - \sin(2a), \cos(2bx) + \cos(2a))}{b(e^{a2i-c2i} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="maxima")

[Out] $-(2*((\cos(2*a) - \cos(2*c))*\cos(a + c) + (\sin(2*a) - \sin(2*c))*\sin(a + c))*\arctan2(\sin(2*b*x) - \sin(2*a), \cos(2*b*x) + \cos(2*a)) - 2*((\cos(2*a) - \cos(2*c))*\cos(a + c) + (\sin(2*a) - \sin(2*c))*\sin(a + c))*\arctan2(\sin(2*b*x) - \sin(2*c), \cos(2*b*x) + \cos(2*c)) - ((\sin(2*a) - \sin(2*c))*\cos(a + c) - (\cos(2*a) - \cos(2*c))*\sin(a + c))*\log(\cos(2*b*x)^2 + 2*\cos(2*b*x)*\cos(2*a) + \cos(2*a)^2 + \sin(2*b*x)^2 - 2*\sin(2*b*x)*\sin(2*a) + \sin(2*a)^2) + ((\sin(2*a) - \sin(2*c))*\cos(a + c) - (\cos(2*a) - \cos(2*c))*\sin(a + c))*\log(\cos(2*b*x)^2 + 2*\cos(2*b*x)*\cos(2*c) + \cos(2*c)^2 + \sin(2*b*x)^2 - 2*\sin(2*b*x)*\sin(2*c) + \sin(2*c)^2))/(2*b*\cos(2*a)*\cos(2*c) - b*\cos(2*c)^2 + 2*b*\sin(2*a)*\sin(2*c) - b*\sin(2*c)^2 - (\cos(2*a)^2 + \sin(2*a)^2)*b)$

mupad [B] time = 7.84, size = 249, normalized size = 6.92

$$\frac{2\sqrt{-e^{a2i-c2i}} \left(\ln\left(-\frac{2\sqrt{-e^{a2i-c2i}}(4be^{a2i}e^{-c2i}+2be^{a2i}e^{bx2i}+2be^{a4i}e^{-c2i}e^{bx2i})}{b(e^{a2i}e^{-c2i}-1)} + e^{a1i}e^{a2i}e^{-c1i}e^{bx2i}4i\right) - \ln\left(-\frac{2\sqrt{-e^{a2i}e^{-c2i}}}{b(e^{a2i-c2i}-1)}\right) \right)}{b(e^{a2i-c2i}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*cos(c + b*x)),x)
```

```
[Out] (2*(-exp(a*2i - c*2i))^(1/2)*(log(exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*x*2i)
)*4i - (2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp
(a*2i)*exp(b*x*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*2i)))/(b*(exp(a*2i)*e
xp(-c*2i) - 1))) - log(exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*x*2i)*4i - (2*(
-exp(a*2i)*exp(-c*2i))^(1/2)*(4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp(a*2i)*exp(
b*x*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*2i)))/(b - b*exp(a*2i)*exp(-c*2i
)))))/(b*(exp(a*2i - c*2i) - 1))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \sec(a + bx) \sec(bx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(b*x+c),x)
```

```
[Out] Integral(sec(a + b*x)*sec(b*x + c), x)
```

3.144 $\int \sec(c - bx) \sec(a + bx) dx$

Optimal. Leaf size=33

$$\frac{\csc(a+c)\log(\cos(c-bx))}{b} - \frac{\csc(a+c)\log(\cos(a+bx))}{b}$$

[Out] $\csc(a+c)*\ln(\cos(b*x-c))/b - \csc(a+c)*\ln(\cos(b*x+a))/b$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4610, 3475}

$$\frac{\csc(a+c)\log(\cos(c-bx))}{b} - \frac{\csc(a+c)\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[c - b*x]*Sec[a + b*x], x]

[Out] (Csc[a + c]*Log[Cos[c - b*x]])/b - (Csc[a + c]*Log[Cos[a + b*x]])/b

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4610

Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_.) + (d_.)*(x_)], x_Symbol] := -Dist[Csc[(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \sec(c - bx) \sec(a + bx) dx &= \csc(a + c) \int \tan(c - bx) dx + \csc(a + c) \int \tan(a + bx) dx \\ &= \frac{\csc(a + c) \log(\cos(c - bx))}{b} - \frac{\csc(a + c) \log(\cos(a + bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.23, size = 26, normalized size = 0.79

$$\frac{\csc(a+c)(\log(\cos(c-bx)) - \log(\cos(a+bx)))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c - b*x]*Sec[a + b*x], x]

[Out] (Csc[a + c]*(Log[Cos[c - b*x]] - Log[Cos[a + b*x]]))/b

fricas [B] time = 0.65, size = 93, normalized size = 2.82

$$\frac{\log(\cos(bx+a)^2) - \log\left(\frac{4(2\cos(bx+a)\cos(a+c)\sin(bx+a)\sin(a+c) + (2\cos(a+c)^2 - 1)\cos(bx+a)^2 - \cos(a+c)^2 + 1)}{\cos(a+c)^2 + 2\cos(a+c) + 1}\right)}{2b\sin(a+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="fricas")

[Out] $-1/2*(\log(\cos(b*x + a)^2) - \log(4*(2*\cos(b*x + a)*\cos(a + c)*\sin(b*x + a)*\sin(a + c) + (2*\cos(a + c)^2 - 1)*\cos(b*x + a)^2 - \cos(a + c)^2 + 1)/(\cos(a + c)^2 + 2*\cos(a + c) + 1)))/(b*\sin(a + c))$

giac [B] time = 0.22, size = 169, normalized size = 5.12

$$\frac{\left(\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 + \tan\left(\frac{1}{2}c\right)^2 + 1\right) \log\left(\left|2 \tan(bx + a) \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right) + 2 \tan(bx + a) \tan\left(\frac{1}{2}c\right) + 2 \tan(bx + a) \tan\left(\frac{1}{2}a\right) + 2 \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right) + 1\right|\right)}{2 \left(\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 + \tan\left(\frac{1}{2}c\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="giac")

[Out] $-1/2*(\tan(1/2*a)^2*\tan(1/2*c)^2 + \tan(1/2*a)^2 + \tan(1/2*c)^2 + 1)*\log(\text{abs}(2*\tan(b*x + a)*\tan(1/2*a)^2*\tan(1/2*c) + 2*\tan(b*x + a)*\tan(1/2*a)*\tan(1/2*c)^2 - \tan(1/2*a)^2*\tan(1/2*c)^2 - 2*\tan(b*x + a)*\tan(1/2*a) + \tan(1/2*a)^2 - 2*\tan(b*x + a)*\tan(1/2*c) + 4*\tan(1/2*a)*\tan(1/2*c) + \tan(1/2*c)^2 - 1))/((\tan(1/2*a)^2*\tan(1/2*c) + \tan(1/2*a)*\tan(1/2*c)^2 - \tan(1/2*a) - \tan(1/2*c))*b)$

maple [A] time = 0.46, size = 53, normalized size = 1.61

$$\frac{\ln(\tan(bx + a) \cos(a) \sin(c) + \tan(bx + a) \sin(a) \cos(c) + \cos(a) \cos(c) - \sin(a) \sin(c))}{b(\sin(a) \cos(c) + \cos(a) \sin(c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x-c)*sec(b*x+a),x)

[Out] $1/b/(\sin(a)*\cos(c)+\cos(a)*\sin(c))*\ln(\tan(b*x+a)*\cos(a)*\sin(c)+\tan(b*x+a)*\sin(a)*\cos(c)+\cos(a)*\cos(c)-\sin(a)*\sin(c))$

maxima [B] time = 0.36, size = 322, normalized size = 9.76

$$\frac{2(\cos(2a + 2c)\cos(a + c) + \sin(2a + 2c)\sin(a + c) - \cos(a + c)) \arctan(\sin(2bx) - \sin(2a), \cos(2bx) - \cos(2a))}{b(\sin(a) \cos(c) + \cos(a) \sin(c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="maxima")

[Out] $(2*(\cos(2*a + 2*c)*\cos(a + c) + \sin(2*a + 2*c)*\sin(a + c) - \cos(a + c))*\arctan2(\sin(2*b*x) - \sin(2*a), \cos(2*b*x) + \cos(2*a)) - 2*(\cos(2*a + 2*c)*\cos(a + c) + \sin(2*a + 2*c)*\sin(a + c) - \cos(a + c))*\arctan2(\sin(2*b*x) + \sin(2*c), \cos(2*b*x) + \cos(2*c)) - (\cos(a + c)*\sin(2*a + 2*c) - \cos(2*a + 2*c)*\sin(a + c) + \sin(a + c))*\log(\cos(2*b*x)^2 + 2*\cos(2*b*x)*\cos(2*a) + \cos(2*a)^2 + \sin(2*b*x)^2 - 2*\sin(2*b*x)*\sin(2*a) + \sin(2*a)^2) + (\cos(a + c)*\sin(2*a + 2*c) - \cos(2*a + 2*c)*\sin(a + c) + \sin(a + c))*\log(\cos(2*b*x)^2 + 2*\cos(2*b*x)*\cos(2*c) + \cos(2*c)^2 + \sin(2*b*x)^2 + 2*\sin(2*b*x)*\sin(2*c) + \sin(2*c)^2))/(b*\cos(2*a + 2*c)^2 + b*\sin(2*a + 2*c)^2 - 2*b*\cos(2*a + 2*c) + b)$

mupad [B] time = 7.73, size = 249, normalized size = 7.55

$$\frac{2\sqrt{-e^{a+2i+c}2i}}{b(e^{a+2i+c}2i - 1)} \left(\ln\left(-\frac{2\sqrt{-e^{a+2i+c}2i}}{b(e^{a+2i+c}2i - 1)} (4be^{a+2i}e^{c2i} + 2be^{a+2i}e^{bx2i} + 2be^{a+4i}e^{c2i}e^{bx2i}) + e^{a+1i}e^{a+2i}e^{c+1i}e^{bx2i}4i\right) - \ln\left(-\frac{2\sqrt{-e^{a+2i+c}2i}}{b(e^{a+2i+c}2i - 1)}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cos(a + b*x)*cos(c - b*x)),x)`

[Out] $(2*(-\exp(a*2i + c*2i))^{1/2}*(\log(\exp(a*1i)*\exp(a*2i)*\exp(c*1i)*\exp(b*x*2i)*4i - (2*(-\exp(a*2i)*\exp(c*2i))^{1/2}*(4*b*\exp(a*2i)*\exp(c*2i) + 2*b*\exp(a*2i)*\exp(b*x*2i) + 2*b*\exp(a*4i)*\exp(c*2i)*\exp(b*x*2i))))/(b*(\exp(a*2i)*\exp(c*2i) - 1))) - \log(\exp(a*1i)*\exp(a*2i)*\exp(c*1i)*\exp(b*x*2i)*4i - (2*(-\exp(a*2i)*\exp(c*2i))^{1/2}*(4*b*\exp(a*2i)*\exp(c*2i) + 2*b*\exp(a*2i)*\exp(b*x*2i) + 2*b*\exp(a*4i)*\exp(c*2i)*\exp(b*x*2i)))/(b - b*\exp(a*2i)*\exp(c*2i))))/(b*(\exp(a*2i + c*2i) - 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec(a + bx) \sec(bx - c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(b*x-c)*sec(b*x+a),x)`

[Out] `Integral(sec(a + b*x)*sec(b*x - c), x)`

3.145 $\int \csc(a + bx) \csc(c + bx) dx$

Optimal. Leaf size=36

$$\frac{\csc(a - c) \log(\sin(bx + c))}{b} - \frac{\csc(a - c) \log(\sin(a + bx))}{b}$$

[Out] $-\csc(a-c)*\ln(\sin(b*x+a))/b+\csc(a-c)*\ln(\sin(b*x+c))/b$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4611, 3475}

$$\frac{\csc(a - c) \log(\sin(bx + c))}{b} - \frac{\csc(a - c) \log(\sin(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b*x]*Csc[c + b*x], x]

[Out] $-\left(\frac{\text{Csc}[a - c] \text{Log}[\text{Sin}[a + b*x]]}{b}\right) + \left(\frac{\text{Csc}[a - c] \text{Log}[\text{Sin}[c + b*x]]}{b}\right)$

Rule 3475

Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4611

Int[Csc[(a_.) + (b_.)*(x_.)]*Csc[(c_.) + (d_.)*(x_.)], x_Symbol] :> Dist[Csc[(b*c - a*d)/b], Int[Cot[a + b*x], x], x] - Dist[Csc[(b*c - a*d)/d], Int[Cot[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \csc(a + bx) \csc(c + bx) dx &= -(\csc(a - c) \int \cot(a + bx) dx) + \csc(a - c) \int \cot(c + bx) dx \\ &= -\frac{\csc(a - c) \log(\sin(a + bx))}{b} + \frac{\csc(a - c) \log(\sin(c + bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.24, size = 28, normalized size = 0.78

$$-\frac{\csc(a - c)(\log(\sin(a + bx)) - \log(\sin(bx + c)))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[a + b*x]*Csc[c + b*x], x]

[Out] $-\left(\frac{\text{Csc}[a - c] (\text{Log}[\text{Sin}[a + b*x]] - \text{Log}[\text{Sin}[c + b*x]])}{b}\right)$

fricas [B] time = 0.75, size = 110, normalized size = 3.06

$$\frac{\log\left(-\frac{1}{4} \cos(bx + c)^2 + \frac{1}{4}\right) - \log\left(-\frac{2 \cos(bx+c) \cos(-a+c) \sin(bx+c) \sin(-a+c) + (2 \cos(-a+c)^2 - 1) \cos(bx+c)^2 - \cos(-a+c)^2}{\cos(-a+c)^2 + 2 \cos(-a+c) + 1}\right)}{2 b \sin(-a + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="fricas")

[Out] $-1/2*(\log(-1/4*\cos(b*x + c)^2 + 1/4) - \log(-(2*\cos(b*x + c)*\cos(-a + c)*\sin(b*x + c)*\sin(-a + c) + (2*\cos(-a + c)^2 - 1)*\cos(b*x + c)^2 - \cos(-a + c)^2)/(\cos(-a + c)^2 + 2*\cos(-a + c) + 1)))/(b*\sin(-a + c))$

giac [B] time = 0.24, size = 396, normalized size = 11.00

$$\frac{\left(\tan\left(\frac{1}{2}a\right)^4 \tan\left(\frac{1}{2}c\right)^4 + 4 \tan\left(\frac{1}{2}a\right)^3 \tan\left(\frac{1}{2}c\right)^3 - \tan\left(\frac{1}{2}a\right)^4 + 4 \tan\left(\frac{1}{2}a\right)^3 \tan\left(\frac{1}{2}c\right) + 4 \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^3 - \tan\left(\frac{1}{2}c\right)^4 + 4 \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right) + 1\right) \log\left(\left|\tan(bx+a)\right|\right)}{\tan\left(\frac{1}{2}a\right)^4 \tan\left(\frac{1}{2}c\right)^3 - \tan\left(\frac{1}{2}a\right)^3 \tan\left(\frac{1}{2}c\right)^4 - \tan\left(\frac{1}{2}a\right)^4 \tan\left(\frac{1}{2}c\right) + 6 \tan\left(\frac{1}{2}a\right)^3 \tan\left(\frac{1}{2}c\right)^2 - 6 \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^3 + \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^4 - \tan\left(\frac{1}{2}a\right)^3 + 6 \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right) - 6 \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}c\right)^3 + \tan\left(\frac{1}{2}a\right) - \tan\left(\frac{1}{2}c\right) - (\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 + \tan\left(\frac{1}{2}c\right)^2 + 1) \log(\tan(bx+a)) / (\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right) - \tan\left(\frac{1}{2}c\right))} / b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="giac")

[Out] $1/2*((\tan(1/2*a)^4*\tan(1/2*c)^4 + 4*\tan(1/2*a)^3*\tan(1/2*c)^3 - \tan(1/2*a)^4 + 4*\tan(1/2*a)^3*\tan(1/2*c) + 4*\tan(1/2*a)*\tan(1/2*c)^3 - \tan(1/2*c)^4 + 4*\tan(1/2*a)*\tan(1/2*c) + 1)*\log(\text{abs}(\tan(b*x + a)*\tan(1/2*a)^2*\tan(1/2*c)^2 - \tan(b*x + a)*\tan(1/2*a)^2 + 4*\tan(b*x + a)*\tan(1/2*a)*\tan(1/2*c) - 2*\tan(1/2*a)^2*\tan(1/2*c) - \tan(b*x + a)*\tan(1/2*c)^2 + 2*\tan(1/2*a)*\tan(1/2*c)^2 + \tan(b*x + a) - 2*\tan(1/2*a) + 2*\tan(1/2*c)))/(\tan(1/2*a)^4*\tan(1/2*c)^3 - \tan(1/2*a)^3*\tan(1/2*c)^4 - \tan(1/2*a)^4*\tan(1/2*c) + 6*\tan(1/2*a)^3*\tan(1/2*c)^2 - 6*\tan(1/2*a)^2*\tan(1/2*c)^3 + \tan(1/2*a)*\tan(1/2*c)^4 - \tan(1/2*a)^3 + 6*\tan(1/2*a)^2*\tan(1/2*c) - 6*\tan(1/2*a)*\tan(1/2*c)^2 + \tan(1/2*c)^3 + \tan(1/2*a) - \tan(1/2*c)) - (\tan(1/2*a)^2*\tan(1/2*c)^2 + \tan(1/2*a)^2 + \tan(1/2*c)^2 + 1)*\log(\text{abs}(\tan(b*x + a)))/(\tan(1/2*a)^2*\tan(1/2*c) - \tan(1/2*a)*\tan(1/2*c)^2 + \tan(1/2*a) - \tan(1/2*c)))/b$

maple [B] time = 0.49, size = 169, normalized size = 4.69

$$\frac{\ln(\tan(bx+a)\cos(a)\cos(c) + \tan(bx+a)\sin(a)\sin(c) + \cos(a)\sin(c) - \sin(a)\cos(c))\cos(a)\cos(c) \ln(\tan(bx+a)\cos(a)\cos(c) + \tan(bx+a)\sin(a)\sin(c) + \cos(a)\sin(c) - \sin(a)\cos(c))}{b(\cos(a)\sin(c) - \sin(a)\cos(c))(\cos(a)\cos(c) + \sin(a)\sin(c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)*csc(b*x+c),x)

[Out] $-1/b/(\cos(a)*\sin(c)-\sin(a)*\cos(c))/(\cos(a)*\cos(c)+\sin(a)*\sin(c))*\ln(\tan(b*x+a)*\cos(a)*\cos(c)+\tan(b*x+a)*\sin(a)*\sin(c)+\cos(a)*\sin(c)-\sin(a)*\cos(c))*\cos(a)*\cos(c)-1/b/(\cos(a)*\sin(c)-\sin(a)*\cos(c))/(\cos(a)*\cos(c)+\sin(a)*\sin(c))*\ln(\tan(b*x+a)*\cos(a)*\cos(c)+\tan(b*x+a)*\sin(a)*\sin(c)+\cos(a)*\sin(c)-\sin(a)*\cos(c))*\sin(a)*\sin(c)+1/b/(\cos(a)*\sin(c)-\sin(a)*\cos(c))*\ln(\tan(b*x+a))$

maxima [B] time = 0.35, size = 564, normalized size = 15.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="maxima")

[Out] $-(2*((\cos(2*a) - \cos(2*c))*\cos(a + c) + (\sin(2*a) - \sin(2*c))*\sin(a + c))*\arctan2(\sin(b*x) + \sin(a), \cos(b*x) - \cos(a)) + 2*((\cos(2*a) - \cos(2*c))*\cos(a + c) + (\sin(2*a) - \sin(2*c))*\sin(a + c))*\arctan2(\sin(b*x) - \sin(a), \cos(b*x) + \cos(a)) - 2*((\cos(2*a) - \cos(2*c))*\cos(a + c) + (\sin(2*a) - \sin(2*c))*\sin(a + c))*\arctan2(\sin(b*x) + \sin(c), \cos(b*x) - \cos(c)) - 2*((\cos(2*a) - \cos(2*c))*\cos(a + c) + (\sin(2*a) - \sin(2*c))*\sin(a + c))*\arctan2(\sin(b*x) - \sin(c), \cos(b*x) + \cos(c)) - ((\sin(2*a) - \sin(2*c))*\cos(a + c) - (\cos(2*a) - \cos(2*c))*\sin(a + c))*\log(\cos(b*x)^2 + 2*\cos(b*x)*\cos(a) + \cos(a)^2 + \sin(b*x)^2 - 2*\sin(b*x)*\sin(a) + \sin(a)^2) - ((\sin(2*a) - \sin(2*c))*\cos(a + c) - (\cos(2*a) - \cos(2*c))*\sin(a + c))$

$$c) - (\cos(2a) - \cos(2c)) \sin(a + c) \log(\cos(bx)^2 - 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 + 2\sin(bx)\sin(a) + \sin(a)^2) + ((\sin(2a) - \sin(2c)) \cos(a + c) - (\cos(2a) - \cos(2c)) \sin(a + c)) \log(\cos(bx)^2 + 2\cos(bx)\cos(c) + \cos(c)^2 + \sin(bx)^2 - 2\sin(bx)\sin(c) + \sin(c)^2) + ((\sin(2a) - \sin(2c)) \cos(a + c) - (\cos(2a) - \cos(2c)) \sin(a + c)) \log(\cos(bx)^2 - 2\cos(bx)\cos(c) + \cos(c)^2 + \sin(bx)^2 + 2\sin(bx)\sin(c) + \sin(c)^2) / (2b\cos(2a)\cos(2c) - b\cos(2c)^2 + 2b\sin(2a)\sin(2c) - b\sin(2c)^2 - (\cos(2a)^2 + \sin(2a)^2)b)$$

mupad [B] time = 7.77, size = 249, normalized size = 6.92

$$\frac{2\sqrt{-e^{a2i-c2i}} \left(\ln \left(\frac{2\sqrt{-e^{a2i}e^{-c2i}} (-4be^{a2i}e^{-c2i} + 2be^{a2i}e^{bx2i} + 2be^{a4i}e^{-c2i}e^{bx2i})}{b(e^{a2i}e^{-c2i}-1)} - e^{a1i}e^{a2i}e^{-c1i}e^{bx2i}4i \right) - \ln \left(\frac{2\sqrt{-e^{a2i}e^{-c2i}}}{b(e^{a2i-c2i}-1)} \right) \right)}{b(e^{a2i-c2i}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(a + b*x)*sin(c + b*x)), x)

[Out] $(2*(-\exp(a*2i - c*2i))^{(1/2)} * (\log((2*(-\exp(a*2i)*\exp(-c*2i))^{(1/2)} * (2*b*\exp(a*2i)*\exp(b*x*2i) - 4*b*\exp(a*2i)*\exp(-c*2i) + 2*b*\exp(a*4i)*\exp(-c*2i)*\exp(b*x*2i))) / (b*(\exp(a*2i)*\exp(-c*2i) - 1)) - \exp(a*1i)*\exp(a*2i)*\exp(-c*1i)*\exp(b*x*2i)*4i) - \log((2*(-\exp(a*2i)*\exp(-c*2i))^{(1/2)} * (2*b*\exp(a*2i)*\exp(b*x*2i) - 4*b*\exp(a*2i)*\exp(-c*2i) + 2*b*\exp(a*4i)*\exp(-c*2i)*\exp(b*x*2i))) / (b - b*\exp(a*2i)*\exp(-c*2i)) - \exp(a*1i)*\exp(a*2i)*\exp(-c*1i)*\exp(b*x*2i)*4i))) / (b*(\exp(a*2i - c*2i) - 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(a + bx) \csc(bx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*csc(b*x+c), x)

[Out] Integral(csc(a + b*x)*csc(b*x + c), x)

3.146 $\int \csc(c - bx) \csc(a + bx) dx$

Optimal. Leaf size=33

$$\frac{\csc(a + c) \log(\sin(a + bx))}{b} - \frac{\csc(a + c) \log(\sin(c - bx))}{b}$$

[Out] $-\csc(a+c)*\ln(-\sin(b*x-c))/b+\csc(a+c)*\ln(\sin(b*x+a))/b$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4611, 3475}

$$\frac{\csc(a + c) \log(\sin(a + bx))}{b} - \frac{\csc(a + c) \log(\sin(c - bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Csc[c - b*x]*Csc[a + b*x], x]

[Out] $-\left(\frac{\text{Csc}[a + c] \cdot \text{Log}[\text{Sin}[c - b*x]]}{b}\right) + \left(\frac{\text{Csc}[a + c] \cdot \text{Log}[\text{Sin}[a + b*x]]}{b}\right)$

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4611

Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[Csc[(b*c - a*d)/b], Int[Cot[a + b*x], x], x] - Dist[Csc[(b*c - a*d)/d], Int[Cot[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \csc(c - bx) \csc(a + bx) dx &= \csc(a + c) \int \cot(c - bx) dx + \csc(a + c) \int \cot(a + bx) dx \\ &= -\frac{\csc(a + c) \log(\sin(c - bx))}{b} + \frac{\csc(a + c) \log(\sin(a + bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.22, size = 29, normalized size = 0.88

$$-\frac{\csc(a + c)(\log(\sin(c - bx)) - \log(-\sin(a + bx)))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c - b*x]*Csc[a + b*x], x]

[Out] $-\left(\frac{\text{Csc}[a + c] \cdot (\text{Log}[\text{Sin}[c - b*x]] - \text{Log}[-\text{Sin}[a + b*x]])}{b}\right)$

fricas [B] time = 0.78, size = 96, normalized size = 2.91

$$\frac{\log\left(-\frac{1}{4} \cos(bx + a)^2 + \frac{1}{4}\right) - \log\left(-\frac{2 \cos(bx+a) \cos(a+c) \sin(bx+a) \sin(a+c) + (2 \cos(a+c)^2 - 1) \cos(bx+a)^2 - \cos(a+c)^2}{\cos(a+c)^2 + 2 \cos(a+c) + 1}\right)}{2b \sin(a + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="fricas")

[Out] $\frac{1}{2} * (\log(-1/4 * \cos(b*x + a)^2 + 1/4) - \log(-(2 * \cos(b*x + a) * \cos(a + c) * \sin(b*x + a) * \sin(a + c) + (2 * \cos(a + c)^2 - 1) * \cos(b*x + a)^2 - \cos(a + c)^2) / (\cos(a + c)^2 + 2 * \cos(a + c) + 1))) / (b * \sin(a + c))$

giac [B] time = 0.23, size = 397, normalized size = 12.03

$$\frac{\left(\tan\left(\frac{1}{2}a\right)^4 \tan\left(\frac{1}{2}c\right)^4 - 4 \tan\left(\frac{1}{2}a\right)^3 \tan\left(\frac{1}{2}c\right)^3 - \tan\left(\frac{1}{2}a\right)^4 - 4 \tan\left(\frac{1}{2}a\right)^3 \tan\left(\frac{1}{2}c\right) - 4 \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^3 - \tan\left(\frac{1}{2}c\right)^4 - 4 \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right) + 1\right) \log\left(\left|\tan\left(\frac{1}{2}a\right)\right| + \left|\tan\left(\frac{1}{2}c\right)\right|\right)}{\tan\left(\frac{1}{2}a\right)^4 \tan\left(\frac{1}{2}c\right)^3 + \tan\left(\frac{1}{2}a\right)^3 \tan\left(\frac{1}{2}c\right)^4 - \tan\left(\frac{1}{2}a\right)^4 \tan\left(\frac{1}{2}c\right) - 6 \tan\left(\frac{1}{2}a\right)^3 \tan\left(\frac{1}{2}c\right)^2 - 6 \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^3 + \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^4 + \tan\left(\frac{1}{2}a\right)^3 \tan\left(\frac{1}{2}c\right) - 6 \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 - 6 \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^3 + \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^4 + \tan\left(\frac{1}{2}a\right)^3 \tan\left(\frac{1}{2}c\right) + 6 \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right) + 6 \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}c\right)^3 - \tan\left(\frac{1}{2}a\right) - \tan\left(\frac{1}{2}c\right)) - (\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 + \tan\left(\frac{1}{2}c\right)^2 + 1) * \log(\text{abs}(\tan(b*x + a))) / (\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right) + \tan\left(\frac{1}{2}a\right) * \tan\left(\frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}a\right) - \tan\left(\frac{1}{2}c\right))} / b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="giac")

[Out] $\frac{1}{2} * ((\tan(1/2*a)^4 * \tan(1/2*c)^4 - 4 * \tan(1/2*a)^3 * \tan(1/2*c)^3 - \tan(1/2*a)^4 - 4 * \tan(1/2*a)^3 * \tan(1/2*c) - 4 * \tan(1/2*a) * \tan(1/2*c)^3 - \tan(1/2*c)^4 - 4 * \tan(1/2*a) * \tan(1/2*c) + 1) * \log(\text{abs}(\tan(b*x + a) * \tan(1/2*a)^2 * \tan(1/2*c)^2 - \tan(b*x + a) * \tan(1/2*a)^2 - 4 * \tan(b*x + a) * \tan(1/2*a) * \tan(1/2*c) + 2 * \tan(1/2*a)^2 * \tan(1/2*c) - \tan(b*x + a) * \tan(1/2*c)^2 + 2 * \tan(1/2*a) * \tan(1/2*c)^2 + \tan(b*x + a) - 2 * \tan(1/2*a) - 2 * \tan(1/2*c))) / (\tan(1/2*a)^4 * \tan(1/2*c)^3 + \tan(1/2*a)^3 * \tan(1/2*c)^4 - \tan(1/2*a)^4 * \tan(1/2*c) - 6 * \tan(1/2*a)^3 * \tan(1/2*c)^2 - 6 * \tan(1/2*a)^2 * \tan(1/2*c)^3 - \tan(1/2*a) * \tan(1/2*c)^4 + \tan(1/2*a)^3 + 6 * \tan(1/2*a)^2 * \tan(1/2*c) + 6 * \tan(1/2*a) * \tan(1/2*c)^2 + \tan(1/2*c)^3 - \tan(1/2*a) - \tan(1/2*c)) - (\tan(1/2*a)^2 * \tan(1/2*c)^2 + \tan(1/2*a)^2 + \tan(1/2*c)^2 + 1) * \log(\text{abs}(\tan(b*x + a))) / (\tan(1/2*a)^2 * \tan(1/2*c) + \tan(1/2*a) * \tan(1/2*c)^2 - \tan(1/2*a) - \tan(1/2*c)) / b$

maple [B] time = 0.48, size = 81, normalized size = 2.45

$$\frac{\ln(\tan(bx + a) \cos(a) \cos(c) - \tan(bx + a) \sin(a) \sin(c) - \cos(a) \sin(c) - \sin(a) \cos(c))}{b(\sin(a) \cos(c) + \cos(a) \sin(c))} + \frac{\ln(\tan(bx + a) \cos(a) \cos(c) - \tan(bx + a) \sin(a) \sin(c) - \cos(a) \sin(c) - \sin(a) \cos(c))}{b(\sin(a) \cos(c) + \cos(a) \sin(c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-csc(b*x-c)*csc(b*x+a),x)

[Out] $-1/b / (\sin(a) * \cos(c) + \cos(a) * \sin(c)) * \ln(\tan(b*x+a) * \cos(a) * \cos(c) - \tan(b*x+a) * \sin(a) * \sin(c) - \cos(a) * \sin(c) - \sin(a) * \cos(c)) + 1/b / (\sin(a) * \cos(c) + \cos(a) * \sin(c)) * \ln(\tan(b*x+a))$

maxima [B] time = 0.50, size = 536, normalized size = 16.24

$$\frac{2(\cos(2a + 2c) \cos(a + c) + \sin(2a + 2c) \sin(a + c) - \cos(a + c)) \arctan(\sin(bx) + \sin(a), \cos(bx) - \cos(a))}{b(\sin(a) \cos(c) + \cos(a) \sin(c))} + \frac{2(\cos(2a + 2c) \cos(a + c) + \sin(2a + 2c) \sin(a + c) - \cos(a + c)) \arctan(\sin(bx) - \sin(a), \cos(bx) + \cos(a))}{b(\sin(a) \cos(c) + \cos(a) \sin(c))} - \frac{2(\cos(2a + 2c) \cos(a + c) + \sin(2a + 2c) \sin(a + c) - \cos(a + c)) \arctan(\sin(bx) + \sin(c), \cos(bx) + \cos(c))}{b(\sin(a) \cos(c) + \cos(a) \sin(c))} - \frac{2(\cos(2a + 2c) \cos(a + c) + \sin(2a + 2c) \sin(a + c) - \cos(a + c)) \arctan(\sin(bx) - \sin(c), \cos(bx) - \cos(c))}{b(\sin(a) \cos(c) + \cos(a) \sin(c))} - (\cos(a + c) * \sin(2a + 2c) - \cos(2a + 2c) * \sin(a + c) + \sin(a + c)) * \log(\cos(b*x)^2 + 2 * \cos(b*x) * \cos(a) + \cos(a)^2 + \sin(b*x)^2 - 2 * \sin(b*x) * \sin(a) + \sin(a)^2) - (\cos(a + c) * \sin(2a + 2c) - \cos(2a + 2c) * \sin(a + c) + \sin(a + c)) * \log(\cos(b*x)^2 - 2 * \cos(b*x) * \cos(a) + \cos(a)^2 + \sin(b*x)^2 - 2 * \sin(b*x) * \sin(a) + \sin(a)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="maxima")

[Out] $-(2 * (\cos(2*a + 2*c) * \cos(a + c) + \sin(2*a + 2*c) * \sin(a + c) - \cos(a + c)) * \arctan(\sin(b*x) + \sin(a), \cos(b*x) - \cos(a)) + 2 * (\cos(2*a + 2*c) * \cos(a + c) + \sin(2*a + 2*c) * \sin(a + c) - \cos(a + c)) * \arctan(\sin(b*x) - \sin(a), \cos(b*x) + \cos(a)) - 2 * (\cos(2*a + 2*c) * \cos(a + c) + \sin(2*a + 2*c) * \sin(a + c) - \cos(a + c)) * \arctan(\sin(b*x) + \sin(c), \cos(b*x) + \cos(c)) - 2 * (\cos(2*a + 2*c) * \cos(a + c) + \sin(2*a + 2*c) * \sin(a + c) - \cos(a + c)) * \arctan(\sin(b*x) - \sin(c), \cos(b*x) - \cos(c)) - (\cos(a + c) * \sin(2*a + 2*c) - \cos(2*a + 2*c) * \sin(a + c) + \sin(a + c)) * \log(\cos(b*x)^2 + 2 * \cos(b*x) * \cos(a) + \cos(a)^2 + \sin(b*x)^2 - 2 * \sin(b*x) * \sin(a) + \sin(a)^2) - (\cos(a + c) * \sin(2*a + 2*c) - \cos(2*a + 2*c) * \sin(a + c) + \sin(a + c)) * \log(\cos(b*x)^2 - 2 * \cos(b*x) * \cos(a) + \cos(a)^2 + \sin(b*x)^2 - 2 * \sin(b*x) * \sin(a) + \sin(a)^2)) / b$

$$\frac{\begin{aligned} & a^2 + \sin(bx)^2 + 2\sin(bx)\sin(a) + \sin(a)^2 + (\cos(a+c)\sin(2a+2c) - \cos(2a+2c)\sin(a+c) + \sin(a+c))\log(\cos(bx)^2 + 2\cos(bx)\cos(c) + \cos(c)^2 + \sin(bx)^2 + 2\sin(bx)\sin(c) + \sin(c)^2) + (\cos(a+c) \\ & * \sin(2a+2c) - \cos(2a+2c)\sin(a+c) + \sin(a+c))\log(\cos(bx)^2 - 2\cos(bx)\cos(c) + \cos(c)^2 + \sin(bx)^2 - 2\sin(bx)\sin(c) + \sin(c)^2) \\ & / (b\cos(2a+2c)^2 + b\sin(2a+2c)^2 - 2b\cos(2a+2c) + b) \end{aligned}}$$

mupad [B] time = 7.87, size = 249, normalized size = 7.55

$$\frac{2\sqrt{-e^{a2i+c2i}} \left(\ln \left(\frac{2\sqrt{-e^{a2i}e^{c2i}} (-4be^{a2i}e^{c2i} + 2be^{a2i}e^{bx2i} + 2be^{a4i}e^{c2i}e^{bx2i})}{b(e^{a2i}e^{c2i}-1)} + e^{a1i}e^{a2i}e^{c1i}e^{bx2i}4i \right) - \ln \left(\frac{2\sqrt{-e^{a2i}e^{c2i}} (-4b}{b(e^{a2i+c2i}-1)} \right) \right)}{b(e^{a2i+c2i}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(a + b*x)*sin(c - b*x)),x)

[Out] $(2*(-\exp(a*2i + c*2i))^{(1/2)} * (\log((2*(-\exp(a*2i)*\exp(c*2i))^{(1/2)} * (2*b*\exp(a*2i)*\exp(b*x*2i) - 4*b*\exp(a*2i)*\exp(c*2i) + 2*b*\exp(a*4i)*\exp(c*2i)*\exp(b*x*2i)))) / (b*(\exp(a*2i)*\exp(c*2i) - 1)) + \exp(a*1i)*\exp(a*2i)*\exp(c*1i)*\exp(b*x*2i)*4i) - \log((2*(-\exp(a*2i)*\exp(c*2i))^{(1/2)} * (2*b*\exp(a*2i)*\exp(b*x*2i) - 4*b*\exp(a*2i)*\exp(c*2i) + 2*b*\exp(a*4i)*\exp(c*2i)*\exp(b*x*2i)))) / (b - b*\exp(a*2i)*\exp(c*2i)) + \exp(a*1i)*\exp(a*2i)*\exp(c*1i)*\exp(b*x*2i)*4i)) / (b*(\exp(a*2i + c*2i) - 1))$

sympy [A] time = 129.08, size = 1838, normalized size = 55.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-csc(b*x-c)*csc(b*x+a),x)

[Out] Piecewise((-tan(c/2)**4*tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)) - 2*tan(c/2)**2*tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)) - tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)), Eq(a, 2*atan(1/tan(c/2))), (tan(c/2)**4*tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)) + 2*tan(c/2)**2*tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)) + tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)), Eq(a, -2*atan(tan(c/2)) - 2*pi*floor((c/2 - pi/2)/pi) - 2*pi*floor(c/(2*pi) - 1/2))), (x/(sin(a)*sin(c)), Eq(b, 0)), (log(tan(a/2) + tan(b*x/2))*tan(a/2)/(2*b) + log(tan(a/2) + tan(b*x/2))/(2*b*tan(a/2)) + log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)/(2*b) + log(tan(b*x/2) - 1/tan(a/2))/(2*b*tan(a/2)) - log(tan(b*x/2))*tan(a/2)/(2*b) - log(tan(b*x/2))/(2*b*tan(a/2)), Eq(c, 0)), (-log(-tan(c/2) + tan(b*x/2))*tan(c/2)/(2*b) - log(-tan(c/2) + tan(b*x/2))/(2*b*tan(c/2)) - log(tan(b*x/2) + 1/tan(c/2))*tan(c/2)/(2*b) - log(tan(b*x/2) + 1/tan(c/2))/(2*b*tan(c/2)) + log(tan(b*x/2))*tan(c/2)/(2*b) + log(tan(b*x/2))/(2*b*tan(c/2)), Eq(a, 0)), (-log(tan(a/2) + tan(b*x/2))*tan(a/2)**2*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) - log(tan(a/2) + tan(b*x/2))*tan(a/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) - log(tan(a/2) + tan(b*x/2))*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) - log(tan(a/2) + tan(b*x/2))/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) + log(-tan(c/2) + tan(b*x/2))*tan(a/2)**2*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) + log(-tan(c/2) + tan(b*x/2))*tan(a/2)**2*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) + log(-tan(c/2) + tan(b*x/2))

```

*x/2))*tan(a/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2
*b*tan(a/2) - 2*b*tan(c/2)) + log(-tan(c/2) + tan(b*x/2))*tan(c/2)**2/(2*b*
tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/
2)) + log(-tan(c/2) + tan(b*x/2))/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*
tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) - log(tan(b*x/2) - 1/tan(a/2))*t
an(a/2)**2*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2
- 2*b*tan(a/2) - 2*b*tan(c/2)) - log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)**2/
(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*t
an(c/2)) - log(tan(b*x/2) - 1/tan(a/2))*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/
2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) - log(tan(b*x/
2) - 1/tan(a/2))/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b
*tan(a/2) - 2*b*tan(c/2)) + log(tan(b*x/2) + 1/tan(c/2))*tan(a/2)**2*tan(c/
2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) -
2*b*tan(c/2)) + log(tan(b*x/2) + 1/tan(c/2))*tan(a/2)**2/(2*b*tan(a/2)**2*
tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) + log(ta
n(b*x/2) + 1/tan(c/2))*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)
*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) + log(tan(b*x/2) + 1/tan(c/2))/
(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*t
an(c/2)), True))

```

3.147 $\int \sqrt{\sin(x) \tan(x)} dx$

Optimal. Leaf size=13

$$-2 \cot(x) \sqrt{\sin(x) \tan(x)}$$

[Out] $-2 * \cot(x) * (\sin(x) * \tan(x))^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4400, 2589}

$$-2 \cot(x) \sqrt{\sin(x) \tan(x)}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[Sin[x]*Tan[x]],x]`

[Out] $-2 * \text{Cot}[x] * \text{Sqrt}[\text{Sin}[x] * \text{Tan}[x]]$

Rule 2589

`Int[((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]`

Rule 4400

`Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] := With[{uu = ActivateTrig[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPart[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x], x] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])`

Rubi steps

$$\begin{aligned} \int \sqrt{\sin(x) \tan(x)} dx &= \frac{\sqrt{\sin(x) \tan(x)} \int \sqrt{\sin(x)} \sqrt{\tan(x)} dx}{\sqrt{\sin(x)} \sqrt{\tan(x)}} \\ &= -2 \cot(x) \sqrt{\sin(x) \tan(x)} \end{aligned}$$

Mathematica [A] time = 0.08, size = 13, normalized size = 1.00

$$-2 \cot(x) \sqrt{\sin(x) \tan(x)}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[Sin[x]*Tan[x]],x]`

[Out] $-2 * \text{Cot}[x] * \text{Sqrt}[\text{Sin}[x] * \text{Tan}[x]]$

fricas [A] time = 0.74, size = 22, normalized size = 1.69

$$-\frac{2 \sqrt{-\frac{\cos(x)^2 - 1}{\cos(x)}} \cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)*tan(x))^(1/2),x, algorithm="fricas")

[Out] -2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/sin(x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin(x) \tan(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)*tan(x))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(sin(x)*tan(x)), x)

maple [B] time = 0.40, size = 177, normalized size = 13.62

$$\frac{(-1 + \cos(x)) \left(4 \cos(x) \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} + 4 \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} + \ln \left(-\frac{2 \left(2(\cos^2(x)) \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} - (\cos^2(x)) + 2 \cos(x) - 2 \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} \right)}{\sin(x)^2} \right)}{4 \sin(x)^3 \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x)*tan(x))^(1/2),x)

[Out] 1/4*(-1+cos(x))*(4*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)+4*(-cos(x)/(1+cos(x))^2)^(1/2)+ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)-ln(-2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2))*cos(x)*(-(-1+cos(x)^2)/cos(x))^(1/2)/sin(x)^3/(-cos(x)/(1+cos(x))^2)^(1/2)*4^(1/2)

maxima [B] time = 0.49, size = 57, normalized size = 4.38

$$\frac{2 \left(\frac{\sin(x)^2}{(\cos(x)+1)^2} - 1 \right)}{\sqrt{\frac{\sin(x)}{\cos(x)+1} + 1} \sqrt{-\frac{\sin(x)}{\cos(x)+1} + 1} \sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)*tan(x))^(1/2),x, algorithm="maxima")

[Out] 2*(sin(x)^2/(cos(x) + 1)^2 - 1)/(sqrt(sin(x)/(cos(x) + 1) + 1)*sqrt(-sin(x)/(cos(x) + 1) + 1)*sqrt(sin(x)^2/(cos(x) + 1)^2 + 1))

mupad [B] time = 2.56, size = 20, normalized size = 1.54

$$-\frac{2 \sin(x)}{\sqrt{\frac{1}{\cos(x)}} \sqrt{1 - \cos(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x)*tan(x))^(1/2),x)

[Out] -(2*sin(x))/((1/cos(x))^(1/2)*(1 - cos(x)^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin(x) \tan(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(x)*tan(x)), x)
```

3.148 $\int (\sin(x) \tan(x))^{3/2} dx$

Optimal. Leaf size=31

$$\frac{8}{3} \csc(x) \sqrt{\sin(x) \tan(x)} - \frac{2}{3} \sin(x) \sqrt{\sin(x) \tan(x)}$$

[Out] $8/3*\csc(x)*(\sin(x)*\tan(x))^{(1/2)}-2/3*\sin(x)*(\sin(x)*\tan(x))^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4400, 2598, 2589}

$$\frac{8}{3} \csc(x) \sqrt{\sin(x) \tan(x)} - \frac{2}{3} \sin(x) \sqrt{\sin(x) \tan(x)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sin}[x]*\text{Tan}[x])^{(3/2)}, x]$

[Out] $(8*\text{Csc}[x]*\text{Sqrt}[\text{Sin}[x]*\text{Tan}[x]])/3 - (2*\text{Sin}[x]*\text{Sqrt}[\text{Sin}[x]*\text{Tan}[x]])/3$

Rule 2589

$\text{Int}[(a_* \sin[e_* + (f_*)*(x_*)])^{(m_*)} * ((b_*) \tan[e_* + (f_*)*(x_*)])^{(n_*)}, x_Symbol] :> -\text{Simp}[(b*(a*\text{Sin}[e + f*x])^m * (b*\text{Tan}[e + f*x])^{(n-1)}) / (f*m), x] /;$ $\text{FreeQ}\{a, b, e, f, m, n\}, x\} \ \&\& \ \text{EqQ}[m + n - 1, 0]$

Rule 2598

$\text{Int}[(a_* \sin[e_* + (f_*)*(x_*)])^{(m_*)} * ((b_*) \tan[e_* + (f_*)*(x_*)])^{(n_*)}, x_Symbol] :> -\text{Simp}[(b*(a*\text{Sin}[e + f*x])^m * (b*\text{Tan}[e + f*x])^{(n-1)}) / (f*m), x] + \text{Dist}[(a^2*(m+n-1))/m, \text{Int}[(a*\text{Sin}[e + f*x])^{(m-2)} * (b*\text{Tan}[e + f*x])^n, x], x] /;$ $\text{FreeQ}\{a, b, e, f, n\}, x\} \ \&\& \ (\text{GtQ}[m, 1] \ || \ (\text{EqQ}[m, 1] \ \& \ \text{EqQ}[n, 1/2])) \ \&\& \ \text{IntegersQ}[2*m, 2*n]$

Rule 4400

$\text{Int}[(u_*) * ((v_*)^{(m_*)} * (w_*)^{(n_*)})^{(p_*)}, x_Symbol] :> \text{With}\{\{uu = \text{ActivateTrig}[u], vv = \text{ActivateTrig}[v], ww = \text{ActivateTrig}[w]\}, \text{Dist}[(vv^m * ww^n)^{\text{FracPart}[p]}] / (vv^{(m*\text{FracPart}[p])} * ww^{(n*\text{FracPart}[p])}), \text{Int}[uu * vv^{(m*p)} * ww^{(n*p)}, x], x] /;$ $\text{FreeQ}\{m, n, p\}, x\} \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ (!\text{InertTrigFreeQ}[v] \ || \ !\text{InertTrigFreeQ}[w])$

Rubi steps

$$\begin{aligned} \int (\sin(x) \tan(x))^{3/2} dx &= \frac{\sqrt{\sin(x) \tan(x)} \int \sin^2(x) \tan^2(x) dx}{\sqrt{\sin(x)} \sqrt{\tan(x)}} \\ &= -\frac{2}{3} \sin(x) \sqrt{\sin(x) \tan(x)} + \frac{(4\sqrt{\sin(x) \tan(x)}) \int \frac{\tan^2(x)}{\sqrt{\sin(x)}} dx}{3\sqrt{\sin(x)} \sqrt{\tan(x)}} \\ &= \frac{8}{3} \csc(x) \sqrt{\sin(x) \tan(x)} - \frac{2}{3} \sin(x) \sqrt{\sin(x) \tan(x)} \end{aligned}$$

Mathematica [A] time = 0.04, size = 23, normalized size = 0.74

$$\frac{2}{3} \sin(x) (4 \csc^2(x) - 1) \sqrt{\sin(x) \tan(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sin[x]*Tan[x])^(3/2),x]

[Out] (2*(-1 + 4*Csc[x]^2)*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3

fricas [A] time = 0.49, size = 26, normalized size = 0.84

$$\frac{2(\cos(x)^2 + 3)\sqrt{-\frac{\cos(x)^2 - 1}{\cos(x)}}}{3 \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)*tan(x))^(3/2),x, algorithm="fricas")

[Out] 2/3*(cos(x)^2 + 3)*sqrt(-(cos(x)^2 - 1)/cos(x))/sin(x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\sin(x) \tan(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)*tan(x))^(3/2),x, algorithm="giac")

[Out] integrate((sin(x)*tan(x))^(3/2), x)

maple [B] time = 0.26, size = 587, normalized size = 18.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x)*tan(x))^(3/2),x)

[Out]
$$\begin{aligned} & -1/12*(-1+\cos(x))^2*(3*\cos(x)^3*\ln(-(2*\cos(x))^2*(-\cos(x)/(1+\cos(x))^2))^{1/2}) \\ & -\cos(x)^2+2*\cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{1/2}-1)/\sin(x)^2*(-\cos(x)/(1 \\ & +\cos(x))^2)^{3/2}-3*\cos(x)^3*\ln(-2*(2*\cos(x))^2*(-\cos(x)/(1+\cos(x))^2)^{1/2} \\ & -\cos(x)^2+2*\cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{1/2}-1)/\sin(x)^2*(-\cos(x)/(1+ \\ & \cos(x))^2)^{3/2}+9*\ln(-2*(2*\cos(x))^2*(-\cos(x)/(1+\cos(x))^2)^{1/2}-\cos(x)^2+2* \\ & \cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{1/2}-1)/\sin(x)^2*\cos(x)^2*(-\cos(x)/(1+\cos \\ & (x))^2)^{3/2}-9*\ln(-2*(2*\cos(x))^2*(-\cos(x)/(1+\cos(x))^2)^{1/2}-\cos(x)^2+2* \\ & \cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{1/2}-1)/\sin(x)^2*\cos(x)^2*(-\cos(x)/(1+\cos \\ & (x))^2)^{3/2}+9*\cos(x)*\ln(-2*(2*\cos(x))^2*(-\cos(x)/(1+\cos(x))^2)^{1/2}-\cos(x)^2 \\ & +2*\cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{1/2}-1)/\sin(x)^2*(-\cos(x)/(1+\cos(x))^2 \\ &)^{3/2}-9*\cos(x)*\ln(-2*(2*\cos(x))^2*(-\cos(x)/(1+\cos(x))^2)^{1/2}-\cos(x)^2+2* \\ & \cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{1/2}-1)/\sin(x)^2*(-\cos(x)/(1+\cos(x))^2)^{3/2} \\ & +3*\ln(-2*(2*\cos(x))^2*(-\cos(x)/(1+\cos(x))^2)^{1/2}-\cos(x)^2+2*\cos(x)-2*(- \\ & \cos(x)/(1+\cos(x))^2)^{1/2}-1)/\sin(x)^2*(-\cos(x)/(1+\cos(x))^2)^{3/2}-3*\ln(-2 \\ & *(2*\cos(x))^2*(-\cos(x)/(1+\cos(x))^2)^{1/2}-\cos(x)^2+2*\cos(x)-2*(-\cos(x)/(1+ \\ & \cos(x))^2)^{1/2}-1)/\sin(x)^2*(-\cos(x)/(1+\cos(x))^2)^{3/2}-4*\cos(x)^3-12*\cos \\ & (x))*(1+\cos(x))^2*(-(-1+\cos(x)^2)/\cos(x))^{3/2}/\sin(x)^{7*4^{1/2}} \end{aligned}$$

maxima [B] time = 0.47, size = 57, normalized size = 1.84

$$\frac{8\left(\frac{\sin(x)^6}{(\cos(x)+1)^6} - 1\right)}{3\left(\frac{\sin(x)}{\cos(x)+1} + 1\right)^{\frac{3}{2}}\left(-\frac{\sin(x)}{\cos(x)+1} + 1\right)^{\frac{3}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((sin(x)*tan(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -8/3*(sin(x)^6/(cos(x) + 1)^6 - 1)/((sin(x)/(cos(x) + 1) + 1)^(3/2)*(-sin(x)
)/(cos(x) + 1) + 1)^(3/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int (\sin(x) \tan(x))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*tan(x))^(3/2),x)
```

```
[Out] int((sin(x)*tan(x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\sin(x) \tan(x))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))**(3/2),x)
```

```
[Out] Integral((sin(x)*tan(x))**(3/2), x)
```

3.149 $\int (\sin(x) \tan(x))^{5/2} dx$

Optimal. Leaf size=50

$$-\frac{2}{5} \sin^2(x) \tan(x) \sqrt{\sin(x) \tan(x)} + \frac{16}{15} \tan(x) \sqrt{\sin(x) \tan(x)} + \frac{64}{15} \cot(x) \sqrt{\sin(x) \tan(x)}$$

[Out] $64/15*\cot(x)*(\sin(x)*\tan(x))^{(1/2)}+16/15*(\sin(x)*\tan(x))^{(1/2)}*\tan(x)-2/5*\sin(x)^2*(\sin(x)*\tan(x))^{(1/2)}*\tan(x)$

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4400, 2598, 2594, 2589}

$$-\frac{2}{5} \sin^2(x) \tan(x) \sqrt{\sin(x) \tan(x)} + \frac{16}{15} \tan(x) \sqrt{\sin(x) \tan(x)} + \frac{64}{15} \cot(x) \sqrt{\sin(x) \tan(x)}$$

Antiderivative was successfully verified.

[In] Int[(Sin[x]*Tan[x])^(5/2),x]

[Out] $(64*\cot[x]*\text{Sqrt}[\sin[x]*\tan[x]])/15 + (16*\tan[x]*\text{Sqrt}[\sin[x]*\tan[x]])/15 - (2*\sin[x]^2*\tan[x]*\text{Sqrt}[\sin[x]*\tan[x]])/5$

Rule 2589

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> -Simp[(b*(a*SIN[e + f*x])^m*(b*TAN[e + f*x])^(n - 1))/(f*(m + n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]

Rule 2594

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[(b*(a*SIN[e + f*x])^m*(b*TAN[e + f*x])^(n - 1))/(f*(m + n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*SIN[e + f*x])^m*(b*TAN[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && IntegersQ[2*m, 2*n] && !(GtQ[m, 1] && !IntegerQ[(m - 1)/2])

Rule 2598

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> -Simp[(b*(a*SIN[e + f*x])^m*(b*TAN[e + f*x])^(n - 1))/(f*(m + n - 1)), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*SIN[e + f*x])^(m - 2)*(b*TAN[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]

Rule 4400

Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^p, x_Symbol] :> With[{uu = ActivateTrig[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPart[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x], x] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])

Rubi steps

$$\begin{aligned}
\int (\sin(x) \tan(x))^{5/2} dx &= \frac{\sqrt{\sin(x) \tan(x)} \int \sin^{\frac{5}{2}}(x) \tan^{\frac{5}{2}}(x) dx}{\sqrt{\sin(x)} \sqrt{\tan(x)}} \\
&= -\frac{2}{5} \sin^2(x) \tan(x) \sqrt{\sin(x) \tan(x)} + \frac{(8\sqrt{\sin(x) \tan(x)}) \int \sqrt{\sin(x)} \tan^{\frac{5}{2}}(x) dx}{5\sqrt{\sin(x)} \sqrt{\tan(x)}} \\
&= \frac{16}{15} \tan(x) \sqrt{\sin(x) \tan(x)} - \frac{2}{5} \sin^2(x) \tan(x) \sqrt{\sin(x) \tan(x)} - \frac{(32\sqrt{\sin(x) \tan(x)}) \int}{15\sqrt{\sin(x)}} \\
&= \frac{64}{15} \cot(x) \sqrt{\sin(x) \tan(x)} + \frac{16}{15} \tan(x) \sqrt{\sin(x) \tan(x)} - \frac{2}{5} \sin^2(x) \tan(x) \sqrt{\sin(x) \tan(x)}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 29, normalized size = 0.58

$$\frac{2}{15} \tan(x) \sqrt{\sin(x) \tan(x)} (3 \cos^2(x) + 32 \cot^2(x) + 5)$$

Antiderivative was successfully verified.

[In] Integrate[(Sin[x]*Tan[x])^(5/2), x]

[Out] (2*(5 + 3*Cos[x]^2 + 32*Cot[x]^2)*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15

fricas [A] time = 0.72, size = 38, normalized size = 0.76

$$\frac{2(3 \cos(x)^4 - 30 \cos(x)^2 - 5) \sqrt{-\frac{\cos(x)^2 - 1}{\cos(x)}}}{15 \cos(x) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)*tan(x))^(5/2), x, algorithm="fricas")

[Out] -2/15*(3*cos(x)^4 - 30*cos(x)^2 - 5)*sqrt(-(cos(x)^2 - 1)/cos(x))/(cos(x)*sin(x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\sin(x) \tan(x))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)*tan(x))^(5/2), x, algorithm="giac")

[Out] integrate((sin(x)*tan(x))^(5/2), x)

maple [B] time = 0.34, size = 324, normalized size = 6.48

$$\frac{(-1 + \cos(x))^2 \left(6 (\cos^4(x)) - 15 (\cos^2(x)) \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} \ln \left(-\frac{2(\cos^2(x)) \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} - (\cos^2(x)) + 2 \cos(x) - 2 \sqrt{-\frac{\cos(x)}{(1+\cos(x))}}}{\sin(x)^2} \right) \right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x)*tan(x))^(5/2), x)

[Out] -1/30*(-1+cos(x))^2*(6*cos(x)^4-15*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(-2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+

$\cos(x)^2)^{(1/2)-1}/\sin(x)^2+15*\cos(x)^2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)*\ln(-2*(2*\cos(x)^2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)-\cos(x)^2+2*\cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)-1)/\sin(x)^2)-15*\cos(x)*(-\cos(x)/(1+\cos(x))^2)^{(1/2)*\ln(-2*\cos(x)^2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)-\cos(x)^2+2*\cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)-1)/\sin(x)^2+15*\cos(x)*(-\cos(x)/(1+\cos(x))^2)^{(1/2)*\ln(-2*(2*\cos(x)^2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)-\cos(x)^2+2*\cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)-1)/\sin(x)^2)-60*\cos(x)^2-10)*\cos(x)*(1+\cos(x))^2*(-(-1+\cos(x))^2)/\cos(x))^{(5/2)}/\sin(x)^9*4^{(1/2)}$

maxima [B] time = 0.44, size = 82, normalized size = 1.64

$$\frac{32 \left(\frac{5 \sin(x)^4}{(\cos(x)+1)^4} - \frac{5 \sin(x)^6}{(\cos(x)+1)^6} + \frac{2 \sin(x)^{10}}{(\cos(x)+1)^{10}} - 2 \right)}{15 \left(\frac{\sin(x)}{\cos(x)+1} + 1 \right)^{\frac{5}{2}} \left(-\frac{\sin(x)}{\cos(x)+1} + 1 \right)^{\frac{5}{2}} \left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1 \right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)*tan(x))^(5/2),x, algorithm="maxima")

[Out] -32/15*(5*sin(x)^4/(cos(x) + 1)^4 - 5*sin(x)^6/(cos(x) + 1)^6 + 2*sin(x)^10/(cos(x) + 1)^10 - 2)/((sin(x)/(cos(x) + 1) + 1)^(5/2)*(-sin(x)/(cos(x) + 1) + 1)^(5/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(5/2))

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (\sin(x) \tan(x))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x)*tan(x))^(5/2),x)

[Out] int((sin(x)*tan(x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)*tan(x))**(5/2),x)

[Out] Timed out

3.150 $\int \sqrt{\cos(x) \cot(x)} dx$

Optimal. Leaf size=13

$$2 \tan(x) \sqrt{\cos(x) \cot(x)}$$

[Out] $2 * (\cos(x) * \cot(x))^{(1/2)} * \tan(x)$

Rubi [A] time = 0.04, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4400, 2589}

$$2 \tan(x) \sqrt{\cos(x) \cot(x)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[x]*Cot[x]],x]

[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]

Rule 2589

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]

Rule 4400

Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPart[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x], x] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])

Rubi steps

$$\begin{aligned} \int \sqrt{\cos(x) \cot(x)} dx &= \frac{\sqrt{\cos(x) \cot(x)} \int \sqrt{\cos(x)} \sqrt{\cot(x)} dx}{\sqrt{\cos(x)} \sqrt{\cot(x)}} \\ &= 2 \sqrt{\cos(x) \cot(x)} \tan(x) \end{aligned}$$

Mathematica [A] time = 0.07, size = 13, normalized size = 1.00

$$2 \tan(x) \sqrt{\cos(x) \cot(x)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[x]*Cot[x]],x]

[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]

fricas [A] time = 0.66, size = 19, normalized size = 1.46

$$\frac{2 \sqrt{\frac{\cos(x)^2}{\sin(x)}} \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)*cot(x))^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(cos(x)^2/sin(x))*sin(x)/cos(x)

giac [A] time = 0.16, size = 12, normalized size = 0.92

$$2 \operatorname{sgn}(\cos(x)) \operatorname{sgn}(\sin(x)) \sqrt{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)*cot(x))^(1/2),x, algorithm="giac")

[Out] 2*sgn(cos(x))*sgn(sin(x))*sqrt(sin(x))

maple [A] time = 0.25, size = 20, normalized size = 1.54

$$\frac{2 \sin(x) \sqrt{\frac{\cos^2(x)}{\sin(x)}}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x)*cot(x))^(1/2),x)

[Out] 2*sin(x)*(cos(x)^2/sin(x))^(1/2)/cos(x)

maxima [B] time = 0.62, size = 188, normalized size = 14.46

$$\left(\left(\cos\left(\frac{3}{2}x\right) - \cos\left(\frac{1}{2}x\right) + \sin\left(\frac{3}{2}x\right) + \sin\left(\frac{1}{2}x\right) \right) \cos\left(\frac{1}{2} \arctan(\sin(x), \cos(x) - 1)\right) - \left(\cos\left(\frac{3}{2}x\right) - \cos\left(\frac{1}{2}x\right) - \sin\left(\frac{3}{2}x\right) - \sin\left(\frac{1}{2}x\right) \right) \right) \cos\left(\frac{1}{2} \arctan(\sin(x), \cos(x) + 1)\right) - \left(\cos\left(\frac{3}{2}x\right) - \cos\left(\frac{1}{2}x\right) - \sin\left(\frac{3}{2}x\right) - \sin\left(\frac{1}{2}x\right) \right) \cos\left(\frac{1}{2} \arctan(\sin(x), \cos(x) - 1)\right) + \left(\cos\left(\frac{3}{2}x\right) - \cos\left(\frac{1}{2}x\right) + \sin\left(\frac{3}{2}x\right) + \sin\left(\frac{1}{2}x\right) \right) \sin\left(\frac{1}{2} \arctan(\sin(x), \cos(x) - 1)\right) - \left(\cos\left(\frac{3}{2}x\right) - \cos\left(\frac{1}{2}x\right) + \sin\left(\frac{3}{2}x\right) + \sin\left(\frac{1}{2}x\right) \right) \sin\left(\frac{1}{2} \arctan(\sin(x), \cos(x) + 1)\right) \right) / \left((\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1)^{1/4} (\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1)^{1/4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)*cot(x))^(1/2),x, algorithm="maxima")

[Out] (((cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*cos(1/2*arctan2(sin(x), cos(x) - 1)) - (cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*sin(1/2*arctan2(sin(x), cos(x) - 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - ((cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*cos(1/2*arctan2(sin(x), cos(x) - 1)) + (cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*sin(1/2*arctan2(sin(x), cos(x) - 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4))

mupad [B] time = 2.65, size = 18, normalized size = 1.38

$$\frac{2 |\cos(x)| \sin(x)^{3/2}}{|\sin(x)| \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x)*cot(x))^(1/2),x)

[Out] (2*abs(cos(x))*sin(x)^(3/2))/(abs(sin(x))*cos(x))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cos(x) \cot(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)*cot(x))**(1/2),x)

[Out] Integral(sqrt(cos(x)*cot(x)), x)

3.151 $\int (\cos(x) \cot(x))^{3/2} dx$

Optimal. Leaf size=31

$$\frac{2}{3} \cos(x) \sqrt{\cos(x) \cot(x)} - \frac{8}{3} \sec(x) \sqrt{\cos(x) \cot(x)}$$

[Out] $2/3*\cos(x)*(\cos(x)*\cot(x))^{(1/2)}-8/3*\sec(x)*(\cos(x)*\cot(x))^{(1/2)}$

Rubi [A] time = 0.07, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4400, 2598, 2589}

$$\frac{2}{3} \cos(x) \sqrt{\cos(x) \cot(x)} - \frac{8}{3} \sec(x) \sqrt{\cos(x) \cot(x)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[x]*\text{Cot}[x])^{(3/2)}, x]$

[Out] $(2*\text{Cos}[x]*\text{Sqrt}[\text{Cos}[x]*\text{Cot}[x]])/3 - (8*\text{Sqrt}[\text{Cos}[x]*\text{Cot}[x]]*\text{Sec}[x])/3$

Rule 2589

$\text{Int}[(a_* \sin[e_*] + (f_*)(x_*))^{(m_*)} * ((b_*) \tan[e_*] + (f_*)(x_*))^{(n_*)}, x_Symbol] :> -\text{Simp}[(b*(a*\text{Sin}[e + f*x])^m*(b*\text{Tan}[e + f*x])^{(n - 1)})/(f*m), x] /; \text{FreeQ}\{a, b, e, f, m, n\}, x] \ \&\& \ \text{EqQ}[m + n - 1, 0]$

Rule 2598

$\text{Int}[(a_* \sin[e_*] + (f_*)(x_*))^{(m_*)} * ((b_*) \tan[e_*] + (f_*)(x_*))^{(n_*)}, x_Symbol] :> -\text{Simp}[(b*(a*\text{Sin}[e + f*x])^m*(b*\text{Tan}[e + f*x])^{(n - 1)})/(f*m), x] + \text{Dist}[(a^2*(m + n - 1))/m, \text{Int}[(a*\text{Sin}[e + f*x])^{(m - 2)}*(b*\text{Tan}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, e, f, n\}, x] \ \&\& \ (\text{GtQ}[m, 1] \ || \ (\text{EqQ}[m, 1] \ \& \ \text{EqQ}[n, 1/2])) \ \&\& \ \text{IntegersQ}[2*m, 2*n]$

Rule 4400

$\text{Int}[(u_*)^{(v_*)} * ((w_*)^{(n_*)})^{(p_*)}, x_Symbol] :> \text{With}\{\{uu = \text{ActivateTrig}[u], vv = \text{ActivateTrig}[v], ww = \text{ActivateTrig}[w]\}, \text{Dist}[(vv^m*ww^n)^{\text{FracPart}[p]} / (vv^{(m*\text{FracPart}[p])} * ww^{(n*\text{FracPart}[p])}), \text{Int}[uu*vv^{(m*p)}*ww^{(n*p)}, x], x] /; \text{FreeQ}\{m, n, p\}, x] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ (!\text{InertTrigFreeQ}[v] \ || \ !\text{InertTrigFreeQ}[w])$

Rubi steps

$$\begin{aligned} \int (\cos(x) \cot(x))^{3/2} dx &= \frac{\sqrt{\cos(x) \cot(x)} \int \cos^3(x) \cot^3(x) dx}{\sqrt{\cos(x)} \sqrt{\cot(x)}} \\ &= \frac{2}{3} \cos(x) \sqrt{\cos(x) \cot(x)} + \frac{(4\sqrt{\cos(x) \cot(x)}) \int \frac{\cot^3(x)}{\sqrt{\cos(x)}} dx}{3\sqrt{\cos(x)} \sqrt{\cot(x)}} \\ &= \frac{2}{3} \cos(x) \sqrt{\cos(x) \cot(x)} - \frac{8}{3} \sqrt{\cos(x) \cot(x)} \sec(x) \end{aligned}$$

Mathematica [A] time = 0.04, size = 21, normalized size = 0.68

$$\frac{2}{3} (\cos^2(x) - 4) \sec(x) \sqrt{\cos(x) \cot(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]*Cot[x])^(3/2),x]

[Out] (2*(-4 + Cos[x]^2)*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3

fricas [A] time = 0.73, size = 23, normalized size = 0.74

$$\frac{2(\cos(x)^2 - 4)\sqrt{\frac{\cos(x)^2}{\sin(x)}}}{3 \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)*cot(x))^(3/2),x, algorithm="fricas")

[Out] 2/3*(cos(x)^2 - 4)*sqrt(cos(x)^2/sin(x))/cos(x)

giac [A] time = 0.15, size = 19, normalized size = 0.61

$$-\frac{2}{3}\left(\sin(x)^{\frac{3}{2}} + \frac{3}{\sqrt{\sin(x)}}\right)\operatorname{sgn}(\cos(x))\operatorname{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)*cot(x))^(3/2),x, algorithm="giac")

[Out] -2/3*(sin(x)^(3/2) + 3/sqrt(sin(x)))*sgn(cos(x))*sgn(sin(x))

maple [A] time = 0.25, size = 26, normalized size = 0.84

$$\frac{2(\cos^2(x) - 4)\left(\frac{\cos^2(x)}{\sin(x)}\right)^{\frac{3}{2}}\sin(x)}{3 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x)*cot(x))^(3/2),x)

[Out] 2/3*(cos(x)^2-4)*(cos(x)^2/sin(x))^(3/2)*sin(x)/cos(x)^3

maxima [B] time = 0.54, size = 314, normalized size = 10.13

$$\frac{(\cos(x)^2 + \sin(x)^2 + 2 \cos(x) + 1)^{\frac{1}{4}}(\cos(x)^2 + \sin(x)^2 - 2 \cos(x) + 1)^{\frac{1}{4}}\left(\left(\cos\left(\frac{9}{2}x\right) - 15 \cos\left(\frac{5}{2}x\right) - \cos\left(\frac{3}{2}x\right) + \sin\left(\frac{9}{2}x\right) + 15 \sin\left(\frac{5}{2}x\right) - \sin\left(\frac{3}{2}x\right) - 15 \sin\left(\frac{1}{2}x\right)\right) \cos\left(\frac{3}{2} \arctan2(\sin(x), \cos(x) - 1)\right) + \left(\cos\left(\frac{9}{2}x\right) - 15 \cos\left(\frac{5}{2}x\right) - \cos\left(\frac{3}{2}x\right) + 15 \cos\left(\frac{1}{2}x\right) + \sin\left(\frac{9}{2}x\right) - 15 \sin\left(\frac{5}{2}x\right) + \sin\left(\frac{3}{2}x\right) + 15 \sin\left(\frac{1}{2}x\right)\right) \sin\left(\frac{3}{2} \arctan2(\sin(x), \cos(x) - 1)\right) \cos\left(\frac{3}{2} \arctan2(\sin(x), \cos(x) + 1)\right) + \left(\cos\left(\frac{9}{2}x\right) - 15 \cos\left(\frac{5}{2}x\right) - \cos\left(\frac{3}{2}x\right) + 15 \cos\left(\frac{1}{2}x\right) + \sin\left(\frac{9}{2}x\right) - 15 \sin\left(\frac{5}{2}x\right) + \sin\left(\frac{3}{2}x\right) + 15 \sin\left(\frac{1}{2}x\right)\right) \cos\left(\frac{3}{2} \arctan2(\sin(x), \cos(x) - 1)\right) - \left(\cos\left(\frac{9}{2}x\right) - 15 \cos\left(\frac{5}{2}x\right) - \cos\left(\frac{3}{2}x\right) + 15 \cos\left(\frac{1}{2}x\right) - \sin\left(\frac{9}{2}x\right) + 15 \sin\left(\frac{5}{2}x\right) - \sin\left(\frac{3}{2}x\right) - 15 \sin\left(\frac{1}{2}x\right)\right) \sin\left(\frac{3}{2} \arctan2(\sin(x), \cos(x) - 1)\right) \sin\left(\frac{3}{2} \arctan2(\sin(x), \cos(x) + 1)\right) \right)}{(\cos(x)^4 + \sin(x)^4 + 2(\cos(x)^2 + 1)\sin(x)^2 - 2\cos(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)*cot(x))^(3/2),x, algorithm="maxima")

[Out] 1/6*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4)*(((cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) - sin(9/2*x) + 15*sin(5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*cos(3/2*arctan2(sin(x), cos(x) - 1)) + (cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) + sin(9/2*x) - 15*sin(5/2*x) + sin(3/2*x) + 15*sin(1/2*x))*sin(3/2*arctan2(sin(x), cos(x) - 1)))*cos(3/2*arctan2(sin(x), cos(x) + 1)) + ((cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) + sin(9/2*x) - 15*sin(5/2*x) + sin(3/2*x) + 15*sin(1/2*x))*cos(3/2*arctan2(sin(x), cos(x) - 1)) - (cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) - sin(9/2*x) + 15*sin(5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*sin(3/2*arctan2(sin(x), cos(x) - 1)))*sin(3/2*arctan2(sin(x), cos(x) + 1)))/(cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 + 1)*sin(x)^2 - 2*cos(x)^2 + 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int (\cos(x) \cot(x))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(x)*cot(x))^(3/2), x)`

[Out] `int((cos(x)*cot(x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\cos(x) \cot(x))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((cos(x)*cot(x))**(3/2), x)`

[Out] `Integral((cos(x)*cot(x))**(3/2), x)`

3.152 $\int (\cos(x) \cot(x))^{5/2} dx$

Optimal. Leaf size=50

$$\frac{2}{5} \cos^2(x) \cot(x) \sqrt{\cos(x) \cot(x)} - \frac{16}{15} \cot(x) \sqrt{\cos(x) \cot(x)} - \frac{64}{15} \tan(x) \sqrt{\cos(x) \cot(x)}$$

[Out] $-16/15*\cot(x)*(\cos(x)*\cot(x))^{(1/2)}+2/5*\cos(x)^2*\cot(x)*(\cos(x)*\cot(x))^{(1/2)}-64/15*(\cos(x)*\cot(x))^{(1/2)}*\tan(x)$

Rubi [A] time = 0.09, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4400, 2598, 2594, 2589}

$$\frac{2}{5} \cos^2(x) \cot(x) \sqrt{\cos(x) \cot(x)} - \frac{16}{15} \cot(x) \sqrt{\cos(x) \cot(x)} - \frac{64}{15} \tan(x) \sqrt{\cos(x) \cot(x)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Cot[x])^(5/2), x]

[Out] $(-16*\text{Cot}[x]*\text{Sqrt}[\text{Cos}[x]*\text{Cot}[x]])/15 + (2*\text{Cos}[x]^2*\text{Cot}[x]*\text{Sqrt}[\text{Cos}[x]*\text{Cot}[x]])/5 - (64*\text{Sqrt}[\text{Cos}[x]*\text{Cot}[x]]*\text{Tan}[x])/15$

Rule 2589

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]

Rule 2594

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && IntegersQ[2*m, 2*n] && !(GtQ[m, 1] && !IntegerQ[(m - 1)/2])

Rule 2598

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]

Rule 4400

Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^p, x_Symbol] :> With[{uu = ActivateTrig[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPart[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x], x] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])

Rubi steps

$$\begin{aligned}
\int (\cos(x) \cot(x))^{5/2} dx &= \frac{\sqrt{\cos(x) \cot(x)} \int \cos^{\frac{5}{2}}(x) \cot^{\frac{5}{2}}(x) dx}{\sqrt{\cos(x)} \sqrt{\cot(x)}} \\
&= \frac{2}{5} \cos^2(x) \cot(x) \sqrt{\cos(x) \cot(x)} + \frac{(8\sqrt{\cos(x) \cot(x)}) \int \sqrt{\cos(x)} \cot^{\frac{5}{2}}(x) dx}{5\sqrt{\cos(x)} \sqrt{\cot(x)}} \\
&= -\frac{16}{15} \cot(x) \sqrt{\cos(x) \cot(x)} + \frac{2}{5} \cos^2(x) \cot(x) \sqrt{\cos(x) \cot(x)} - \frac{(32\sqrt{\cos(x) \cot(x)})}{15\sqrt{\cos(x)}} \\
&= -\frac{16}{15} \cot(x) \sqrt{\cos(x) \cot(x)} + \frac{2}{5} \cos^2(x) \cot(x) \sqrt{\cos(x) \cot(x)} - \frac{64}{15} \sqrt{\cos(x) \cot(x)} \tan(x)
\end{aligned}$$

Mathematica [A] time = 0.10, size = 29, normalized size = 0.58

$$-\frac{2}{15} \tan(x) \sqrt{\cos(x) \cot(x)} (3 \cos^2(x) + 5 \cot^2(x) + 32)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]*Cot[x])^(5/2), x]

[Out] (-2*Sqrt[Cos[x]*Cot[x]]*(32 + 3*Cos[x]^2 + 5*Cot[x]^2)*Tan[x])/15

fricas [A] time = 0.60, size = 35, normalized size = 0.70

$$\frac{2(3 \cos(x)^4 + 24 \cos(x)^2 - 32) \sqrt{\frac{\cos(x)^2}{\sin(x)}}}{15 \cos(x) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)*cot(x))^(5/2), x, algorithm="fricas")

[Out] 2/15*(3*cos(x)^4 + 24*cos(x)^2 - 32)*sqrt(cos(x)^2/sin(x))/(cos(x)*sin(x))

giac [A] time = 0.14, size = 27, normalized size = 0.54

$$\frac{2}{15} \left(3 \sin(x)^{\frac{5}{2}} - 30 \sqrt{\sin(x)} - \frac{5}{\sin(x)^{\frac{3}{2}}} \right) \operatorname{sgn}(\cos(x)) \operatorname{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)*cot(x))^(5/2), x, algorithm="giac")

[Out] 2/15*(3*sin(x)^(5/2) - 30*sqrt(sin(x)) - 5/sin(x)^(3/2))*sgn(cos(x))*sgn(sin(x))

maple [A] time = 0.30, size = 34, normalized size = 0.68

$$\frac{2(3(\cos^4(x)) + 24(\cos^2(x)) - 32) \left(\frac{\cos^2(x)}{\sin(x)}\right)^{\frac{5}{2}} \sin(x)}{15 \cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x)*cot(x))^(5/2), x)

[Out] 2/15*(3*cos(x)^4+24*cos(x)^2-32)*(cos(x)^2/sin(x))^(5/2)*sin(x)/cos(x)^5

maxima [B] time = 0.63, size = 427, normalized size = 8.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)*cot(x))^(5/2),x, algorithm="maxima")

[Out]
$$-1/60 * (((3*\cos(15/2*x) + 105*\cos(11/2*x) - 410*\cos(7/2*x) - 3*\cos(5/2*x) + 410*\cos(3/2*x) - 105*\cos(1/2*x) + 3*\sin(15/2*x) + 105*\sin(11/2*x) - 410*\sin(7/2*x) + 3*\sin(5/2*x) + 410*\sin(3/2*x) + 105*\sin(1/2*x)) * \cos(5/2 * \arctan2(\sin(x), \cos(x) - 1)) - (3*\cos(15/2*x) + 105*\cos(11/2*x) - 410*\cos(7/2*x) - 3*\cos(5/2*x) + 410*\cos(3/2*x) - 105*\cos(1/2*x) - 3*\sin(15/2*x) - 105*\sin(11/2*x) + 410*\sin(7/2*x) - 3*\sin(5/2*x) - 410*\sin(3/2*x) - 105*\sin(1/2*x)) * \sin(5/2 * \arctan2(\sin(x), \cos(x) - 1))) * \cos(5/2 * \arctan2(\sin(x), \cos(x) + 1)) - ((3*\cos(15/2*x) + 105*\cos(11/2*x) - 410*\cos(7/2*x) - 3*\cos(5/2*x) + 410*\cos(3/2*x) - 105*\cos(1/2*x) - 3*\sin(15/2*x) - 105*\sin(11/2*x) + 410*\sin(7/2*x) - 3*\sin(5/2*x) - 410*\sin(3/2*x) - 105*\sin(1/2*x)) * \cos(5/2 * \arctan2(\sin(x), \cos(x) - 1)) + (3*\cos(15/2*x) + 105*\cos(11/2*x) - 410*\cos(7/2*x) - 3*\cos(5/2*x) + 410*\cos(3/2*x) - 105*\cos(1/2*x) + 3*\sin(15/2*x) + 105*\sin(11/2*x) - 410*\sin(7/2*x) + 3*\sin(5/2*x) + 410*\sin(3/2*x) + 105*\sin(1/2*x)) * \sin(5/2 * \arctan2(\sin(x), \cos(x) - 1))) * \sin(5/2 * \arctan2(\sin(x), \cos(x) + 1))) / ((\cos(x)^4 + \sin(x)^4 + 2 * (\cos(x)^2 + 1) * \sin(x)^2 - 2 * \cos(x)^2 + 1) * (\cos(x)^2 + \sin(x)^2 + 2 * \cos(x) + 1)^{1/4} * (\cos(x)^2 + \sin(x)^2 - 2 * \cos(x) + 1)^{1/4})$$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (\cos(x) \cot(x))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x)*cot(x))^(5/2),x)

[Out] int((cos(x)*cot(x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)*cot(x))**(5/2),x)

[Out] Timed out

$$3.153 \quad \int \frac{x \cos(x)}{(a+b \sin(x))^2} dx$$

Optimal. Leaf size=58

$$\frac{2 \tan^{-1} \left(\frac{a \tan(\frac{x}{2}) + b}{\sqrt{a^2 - b^2}} \right)}{b \sqrt{a^2 - b^2}} - \frac{x}{b(a + b \sin(x))}$$

[Out] $-x/b/(a+b*\sin(x))+2*\arctan((b+a*\tan(1/2*x))/(a^2-b^2)^{(1/2)})/b/(a^2-b^2)^{(1/2)}$

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4422, 2660, 618, 204}

$$\frac{2 \tan^{-1} \left(\frac{a \tan(\frac{x}{2}) + b}{\sqrt{a^2 - b^2}} \right)}{b \sqrt{a^2 - b^2}} - \frac{x}{b(a + b \sin(x))}$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[x])/(a + b*Sin[x])^2,x]

[Out] $(2*\text{ArcTan}[(b + a*\text{Tan}[x/2])/ \text{Sqrt}[a^2 - b^2]])/(b*\text{Sqrt}[a^2 - b^2]) - x/(b*(a + b*\text{Sin}[x]))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 4422

Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^m*(a + b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{x \cos(x)}{(a + b \sin(x))^2} dx &= -\frac{x}{b(a + b \sin(x))} + \frac{\int \frac{1}{a + b \sin(x)} dx}{b} \\
&= -\frac{x}{b(a + b \sin(x))} + \frac{2 \operatorname{Subst}\left(\int \frac{1}{a + 2bx + ax^2} dx, x, \tan\left(\frac{x}{2}\right)\right)}{b} \\
&= -\frac{x}{b(a + b \sin(x))} - \frac{4 \operatorname{Subst}\left(\int \frac{1}{-4(a^2 - b^2) - x^2} dx, x, 2b + 2a \tan\left(\frac{x}{2}\right)\right)}{b} \\
&= \frac{2 \tan^{-1}\left(\frac{b + a \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2}}\right)}{b\sqrt{a^2 - b^2}} - \frac{x}{b(a + b \sin(x))}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 56, normalized size = 0.97

$$\frac{2 \tan^{-1}\left(\frac{a \tan\left(\frac{x}{2}\right) + b}{\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}} - \frac{x}{a + b \sin(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Cos[x])/(a + b*Sin[x])^2,x]

[Out] ((2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - x/(a + b*Sin[x]))/b

fricas [A] time = 1.56, size = 236, normalized size = 4.07

$$\left[\frac{\sqrt{-a^2 + b^2} (b \sin(x) + a) \log\left(\frac{(2a^2 - b^2) \cos(x)^2 - 2ab \sin(x) - a^2 - b^2 + 2(a \cos(x) \sin(x) + b \cos(x)) \sqrt{-a^2 + b^2}}{b^2 \cos(x)^2 - 2ab \sin(x) - a^2 - b^2}\right) + 2(a^2 - b^2)x \sqrt{a^2 - b^2}}{2(a^3b - ab^3 + (a^2b^2 - b^4) \sin(x))}, - \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x)/(a+b*sin(x))^2,x, algorithm="fricas")

[Out] [-1/2*(sqrt(-a^2 + b^2)*(b*sin(x) + a)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a^2 - b^2)*x)/(a^3*b - a*b^3 + (a^2*b^2 - b^4)*sin(x)), -(sqrt(a^2 - b^2)*(b*sin(x) + a)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) + (a^2 - b^2)*x)/(a^3*b - a*b^3 + (a^2*b^2 - b^4)*sin(x))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cos(x)}{(b \sin(x) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x)/(a+b*sin(x))^2,x, algorithm="giac")

[Out] integrate(x*cos(x)/(b*sin(x) + a)^2, x)

maple [C] time = 0.52, size = 159, normalized size = 2.74

$$-\frac{2ix e^{ix}}{b(b e^{2ix} - b + 2ia e^{ix})} - \frac{\ln\left(e^{ix} + \frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2} b}\right)}{\sqrt{-a^2+b^2} b} + \frac{\ln\left(e^{ix} + \frac{ia\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2} b}\right)}{\sqrt{-a^2+b^2} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(x)/(a+b*sin(x))^2,x)

[Out] $-2*I*x*\exp(I*x)/b/(b*\exp(2*I*x)-b+2*I*a*\exp(I*x))-1/(-a^2+b^2)^{(1/2)}/b*\ln(\exp(I*x)+(I*a*(-a^2+b^2)^{(1/2)}-a^2+b^2)/(-a^2+b^2)^{(1/2)}/b)+1/(-a^2+b^2)^{(1/2)}/b*\ln(\exp(I*x)+(I*a*(-a^2+b^2)^{(1/2)}+a^2-b^2)/(-a^2+b^2)^{(1/2)}/b)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x)/(a+b*sin(x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more details)Is 4*b^2-4*a^2 positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x \cos(x)}{(a + b \sin(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*cos(x))/(a + b*sin(x))^2,x)

[Out] int((x*cos(x))/(a + b*sin(x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x)/(a+b*sin(x))**2,x)

[Out] Timed out

$$3.154 \quad \int \frac{x \cos(x)}{(a+b \sin(x))^3} dx$$

Optimal. Leaf size=85

$$\frac{a \tan^{-1}\left(\frac{a \tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}} + \frac{\cos(x)}{2(a^2-b^2)(a+b \sin(x))} - \frac{x}{2b(a+b \sin(x))^2}$$

[Out] a*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b/(a^2-b^2)^(3/2)-1/2*x/b/(a+b*sin(x))^2+1/2*cos(x)/(a^2-b^2)/(a+b*sin(x))

Rubi [A] time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4422, 2664, 12, 2660, 618, 204}

$$\frac{a \tan^{-1}\left(\frac{a \tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b(a^2-b^2)^{3/2}} + \frac{\cos(x)}{2(a^2-b^2)(a+b \sin(x))} - \frac{x}{2b(a+b \sin(x))^2}$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[x])/(a + b*Sin[x])^3,x]

[Out] (a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]]/(b*(a^2 - b^2)^(3/2)) - x/(2*b*(a + b*Sin[x])^2) + Cos[x]/(2*(a^2 - b^2)*(a + b*Sin[x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2664

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[((e + f*x)^(m*(a + b*SIN[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*SIN[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x \cos(x)}{(a + b \sin(x))^3} dx &= -\frac{x}{2b(a + b \sin(x))^2} + \frac{\int \frac{1}{(a+b \sin(x))^2} dx}{2b} \\ &= -\frac{x}{2b(a + b \sin(x))^2} + \frac{\cos(x)}{2(a^2 - b^2)(a + b \sin(x))} + \frac{\int \frac{a}{a+b \sin(x)} dx}{2b(a^2 - b^2)} \\ &= -\frac{x}{2b(a + b \sin(x))^2} + \frac{\cos(x)}{2(a^2 - b^2)(a + b \sin(x))} + \frac{a \int \frac{1}{a+b \sin(x)} dx}{2b(a^2 - b^2)} \\ &= -\frac{x}{2b(a + b \sin(x))^2} + \frac{\cos(x)}{2(a^2 - b^2)(a + b \sin(x))} + \frac{a \operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2} dx, x, \tan\left(\frac{x}{2}\right)\right)}{b(a^2 - b^2)} \\ &= -\frac{x}{2b(a + b \sin(x))^2} + \frac{\cos(x)}{2(a^2 - b^2)(a + b \sin(x))} - \frac{(2a) \operatorname{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2} dx, x, 2b + \frac{a \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b(a^2 - b^2)} \\ &= \frac{a \tan^{-1}\left(\frac{b+a \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b(a^2 - b^2)^{3/2}} - \frac{x}{2b(a + b \sin(x))^2} + \frac{\cos(x)}{2(a^2 - b^2)(a + b \sin(x))} \end{aligned}$$

Mathematica [A] time = 0.26, size = 84, normalized size = 0.99

$$\frac{a \tan^{-1}\left(\frac{a \tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b(a^2 - b^2)^{3/2}} + \frac{\cos(x)(a+b \sin(x))}{2(a-b)(a+b)} - \frac{x}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Cos[x])/(a + b*SIN[x])^3,x]

[Out] (a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]]/(b*(a^2 - b^2)^(3/2)) + (-x/b) + (Cos[x]*(a + b*SIN[x]))/((a - b)*(a + b)))/(2*(a + b*SIN[x])^2)

fricas [B] time = 0.63, size = 459, normalized size = 5.40

$$\frac{2(a^2b^2 - b^4) \cos(x) \sin(x) - (ab^2 \cos(x)^2 - 2a^2b \sin(x) - a^3 - ab^2) \sqrt{-a^2 + b^2} \log\left(-\frac{(2a^2-b^2) \cos(x)^2 - 2ab \sin(x) - a^3}{b^2 \cos(x)}\right)}{4(a^6b - a^4b^3 - a^2b^5 + b^7 - (a^4b^3 - 2a^2b^5 + b^7) \cos(x)^2 + 2ab^2 \sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x)/(a+b*sin(x))^3,x, algorithm="fricas")

[Out] [1/4*(2*(a^2*b^2 - b^4)*cos(x)*sin(x) - (a*b^2*cos(x)^2 - 2*a^2*b*sin(x) - a^3 - a*b^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) -

$$\frac{a^2 - b^2 - 2(a \cos(x) \sin(x) + b \cos(x)) \sqrt{-a^2 + b^2}}{(b^2 \cos(x)^2 - 2ab \sin(x) - a^2 - b^2)} - \frac{2(a^4 - 2a^2b^2 + b^4)x + 2(a^3b - ab^3) \cos(x)}{(a^6b - a^4b^3 - a^2b^5 + b^7 - (a^4b^3 - 2a^2b^5 + b^7) \cos(x)^2 + 2(a^5b^2 - 2a^3b^4 + ab^6) \sin(x))} + \frac{1}{2} \frac{((a^2b^2 - b^4) \cos(x) \sin(x) + (ab^2 \cos(x)^2 - 2a^2b \sin(x) - a^3 - ab^2) \sqrt{a^2 - b^2}) \arctan(-a \sin(x) + b) / (\sqrt{a^2 - b^2} \cos(x)) - (a^4 - 2a^2b^2 + b^4)x + (a^3b - ab^3) \cos(x)}{(a^6b - a^4b^3 - a^2b^5 + b^7 - (a^4b^3 - 2a^2b^5 + b^7) \cos(x)^2 + 2(a^5b^2 - 2a^3b^4 + ab^6) \sin(x))}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cos(x)}{(b \sin(x) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x)/(a+b*sin(x))^3,x, algorithm="giac")

[Out] integrate(x*cos(x)/(b*sin(x) + a)^3, x)

maple [C] time = 0.83, size = 257, normalized size = 3.02

$$\frac{2ia^2e^{2ix} + ib^2e^{2ix} + 2xa^2e^{2ix} + ba e^{3ix} - 2b^2x e^{2ix} - ib^2 - 3ab e^{ix}}{(b e^{2ix} - b + 2ia e^{ix})^2 (a^2 - b^2) b} - \frac{a \ln \left(e^{ix} + \frac{ia \sqrt{-a^2 + b^2} - a^2 + b^2}{\sqrt{-a^2 + b^2} b} \right)}{2\sqrt{-a^2 + b^2} (a + b) (a - b) b} + \frac{a \ln \left(e^{ix} + \frac{ia \sqrt{-a^2 + b^2} - a^2 + b^2}{\sqrt{-a^2 + b^2} b} \right)}{2\sqrt{-a^2 + b^2} (a - b) b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(x)/(a+b*sin(x))^3,x)

[Out] $(2Ia^2 \exp(2Ix) + Ib^2 \exp(2Ix) + 2xa^2 \exp(2Ix) + ba e^{3ix} - 2b^2x e^{2ix} - ib^2 - 3ab e^{ix}) / (b \exp(2Ix) - b + 2Ia \exp(Ix))^2 / (a^2 - b^2) / b - 1/2 / (-a^2 + b^2)^{(1/2)} * a / (a + b) / (a - b) / b * \ln(\exp(Ix) + (Ia * (-a^2 + b^2)^{(1/2)} - a^2 + b^2) / (-a^2 + b^2)^{(1/2)} / b) + 1/2 / (-a^2 + b^2)^{(1/2)} * a / (a + b) / (a - b) / b * \ln(\exp(Ix) + (Ia * (-a^2 + b^2)^{(1/2)} + a^2 - b^2) / (-a^2 + b^2)^{(1/2)} / b)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x)/(a+b*sin(x))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more details)Is 4*b^2-4*a^2 positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \cos(x)}{(a + b \sin(x))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*cos(x))/(a + b*sin(x))^3,x)

[Out] int((x*cos(x))/(a + b*sin(x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

$$3.155 \quad \int \frac{x \sin(x)}{(a+b \cos(x))^2} dx$$

Optimal. Leaf size=59

$$\frac{x}{b(a+b \cos(x))} - \frac{2 \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}$$

[Out] $x/b/(a+b*\cos(x))-2*\arctan((a-b)^{(1/2)}*\tan(1/2*x)/(a+b)^{(1/2}))/b/(a-b)^{(1/2)}/(a+b)^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4423, 2659, 205}

$$\frac{x}{b(a+b \cos(x))} - \frac{2 \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[(x*Sin[x])/(a + b*Cos[x])^2,x]

[Out] $(-2*\text{ArcTan}[(\text{Sqrt}[a - b]*\text{Tan}[x/2])/\text{Sqrt}[a + b]])/(\text{Sqrt}[a - b]*b*\text{Sqrt}[a + b]) + x/(b*(a + b*\text{Cos}[x]))$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 4423

Int[(Cos[(c_.) + (d_.)*(x_)])*(b_.) + (a_)^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[((e + f*x)^m*(a + b*Cos[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Cos[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rubi steps

$$\begin{aligned} \int \frac{x \sin(x)}{(a+b \cos(x))^2} dx &= \frac{x}{b(a+b \cos(x))} - \frac{\int \frac{1}{a+b \cos(x)} dx}{b} \\ &= \frac{x}{b(a+b \cos(x))} - \frac{2 \text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{x}{2}\right)\right)}{b} \\ &= -\frac{2 \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b} b \sqrt{a+b}} + \frac{x}{b(a+b \cos(x))} \end{aligned}$$

Mathematica [A] time = 0.10, size = 58, normalized size = 0.98

$$\frac{2 \tanh^{-1} \left(\frac{(a-b) \tan\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}} \right)}{b\sqrt{b^2-a^2}} + \frac{x}{b(a+b \cos(x))}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sin[x])/(a + b*Cos[x])^2,x]

[Out] (2*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(b*Sqrt[-a^2 + b^2]) + x/(b*(a + b*Cos[x]))

fricas [A] time = 0.57, size = 227, normalized size = 3.85

$$\left[\frac{\sqrt{-a^2 + b^2} (b \cos(x) + a) \log \left(\frac{2ab \cos(x) + (2a^2 - b^2) \cos(x)^2 - 2\sqrt{-a^2 + b^2} (a \cos(x) + b) \sin(x) - a^2 + 2b^2}{b^2 \cos(x)^2 + 2ab \cos(x) + a^2} \right) - 2(a^2 - b^2)x \sqrt{a^2 - b^2}}{2(a^3b - ab^3 + (a^2b^2 - b^4) \cos(x))}, - \frac{\sqrt{a^2 - b^2} (b \cos(x) + a) \arctan \left(\frac{(a \cos(x) + b) \sin(x)}{\sqrt{a^2 - b^2}} \right) - (a^2 - b^2)x}{2(a^3b - ab^3 + (a^2b^2 - b^4) \cos(x))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)/(a+b*cos(x))^2,x, algorithm="fricas")

[Out] [-1/2*(sqrt(-a^2 + b^2)*(b*cos(x) + a)*log((2*a*b*cos(x) + (2*a^2 - b^2)*cos(x)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*b*cos(x) + a^2)) - 2*(a^2 - b^2)*x)/(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cos(x)), -(sqrt(a^2 - b^2)*(b*cos(x) + a)*arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x))) - (a^2 - b^2)*x)/(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cos(x))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(x)}{(b \cos(x) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)/(a+b*cos(x))^2,x, algorithm="giac")

[Out] integrate(x*sin(x)/(b*cos(x) + a)^2, x)

maple [C] time = 0.26, size = 154, normalized size = 2.61

$$\frac{2x e^{ix}}{b(b e^{2ix} + 2a e^{ix} + b)} - \frac{i \ln \left(e^{ix} + \frac{a\sqrt{a^2-b^2} + a^2 - b^2}{\sqrt{a^2-b^2} b} \right)}{\sqrt{a^2-b^2} b} + \frac{i \ln \left(e^{ix} + \frac{a\sqrt{a^2-b^2} - a^2 + b^2}{\sqrt{a^2-b^2} b} \right)}{\sqrt{a^2-b^2} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(x)/(a+b*cos(x))^2,x)

[Out] 2*x*exp(I*x)/b/(b*exp(2*I*x)+2*a*exp(I*x)+b)-I/(a^2-b^2)^(1/2)/b*ln(exp(I*x)+(a*(a^2-b^2)^(1/2)+a^2-b^2)/(a^2-b^2)^(1/2)/b)+I/(a^2-b^2)^(1/2)/b*ln(exp(I*x)+(a*(a^2-b^2)^(1/2)-a^2+b^2)/(a^2-b^2)^(1/2)/b)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)/(a+b*cos(x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 3.29, size = 132, normalized size = 2.24

$$\frac{2xe^{x1i}}{b(2ae^{x1i} + 2be^{x1i}\cos(x))} + \frac{\ln\left(2e^{x1i} - \frac{(b+ae^{x1i})2i}{\sqrt{a+b}\sqrt{b-a}}\right)}{b\sqrt{a+b}\sqrt{b-a}} - \frac{\ln\left(2e^{x1i} + \frac{(b+ae^{x1i})2i}{\sqrt{a+b}\sqrt{b-a}}\right)}{b\sqrt{a+b}\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*sin(x))/(a + b*cos(x))^2,x)

[Out] (2*x*exp(x*1i))/(b*(2*a*exp(x*1i) + 2*b*exp(x*1i)*cos(x))) + log(2*exp(x*1i) - ((b + a*exp(x*1i))*2i)/((a + b)^(1/2)*(b - a)^(1/2)))/(b*(a + b)^(1/2)*(b - a)^(1/2)) - log(2*exp(x*1i) + ((b + a*exp(x*1i))*2i)/((a + b)^(1/2)*(b - a)^(1/2)))/(b*(a + b)^(1/2)*(b - a)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(x)}{(a + b \cos(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)/(a+b*cos(x))**2,x)

[Out] Integral(x*sin(x)/(a + b*cos(x))**2, x)

$$3.156 \quad \int \frac{x \sin(x)}{(a+b \cos(x))^3} dx$$

Optimal. Leaf size=88

$$\frac{\sin(x)}{2(a^2 - b^2)(a + b \cos(x))} + \frac{x}{2b(a + b \cos(x))^2} - \frac{a \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b(a-b)^{3/2}(a+b)^{3/2}}$$

[Out] $-a \arctan((a-b)^{1/2} \tan(1/2*x)/(a+b)^{1/2})/(a-b)^{3/2}/b/(a+b)^{3/2} + 1/2 * x/b/(a+b*\cos(x))^2 + 1/2 * \sin(x)/(a^2-b^2)/(a+b*\cos(x))$

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {4423, 2664, 12, 2659, 205}

$$\frac{\sin(x)}{2(a^2 - b^2)(a + b \cos(x))} + \frac{x}{2b(a + b \cos(x))^2} - \frac{a \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x*Sin[x])/(a + b*Cos[x])^3,x]

[Out] $-((a*\text{ArcTan}[(\text{Sqrt}[a - b]*\text{Tan}[x/2])/\text{Sqrt}[a + b]])/((a - b)^{3/2}*b*(a + b)^{3/2})) + x/(2*b*(a + b*\text{Cos}[x])^2) + \text{Sin}[x]/(2*(a^2 - b^2)*(a + b*\text{Cos}[x]))$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2664

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 4423

Int[(Cos[(c_.) + (d_.)*(x_)])*(b_.) + (a_)^(n_.)*((e_.) + (f_.)*(x_)^(m_.))*Sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[((e + f*x)^m*(a + b*Cos[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Cos[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{x \sin(x)}{(a + b \cos(x))^3} dx &= \frac{x}{2b(a + b \cos(x))^2} - \frac{\int \frac{1}{(a+b \cos(x))^2} dx}{2b} \\
&= \frac{x}{2b(a + b \cos(x))^2} + \frac{\sin(x)}{2(a^2 - b^2)(a + b \cos(x))} - \frac{\int \frac{a}{a+b \cos(x)} dx}{2b(a^2 - b^2)} \\
&= \frac{x}{2b(a + b \cos(x))^2} + \frac{\sin(x)}{2(a^2 - b^2)(a + b \cos(x))} - \frac{a \int \frac{1}{a+b \cos(x)} dx}{2b(a^2 - b^2)} \\
&= \frac{x}{2b(a + b \cos(x))^2} + \frac{\sin(x)}{2(a^2 - b^2)(a + b \cos(x))} - \frac{a \operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{x}{2}\right)\right)}{b(a^2 - b^2)} \\
&= -\frac{a \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b(a+b)^{3/2}} + \frac{x}{2b(a + b \cos(x))^2} + \frac{\sin(x)}{2(a^2 - b^2)(a + b \cos(x))}
\end{aligned}$$

Mathematica [A] time = 0.32, size = 85, normalized size = 0.97

$$\frac{\frac{\sin(x)(a+b \cos(x))}{(a-b)(a+b)} + \frac{x}{b}}{2(a + b \cos(x))^2} - \frac{a \tanh^{-1}\left(\frac{(a-b) \tan\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{b(b^2 - a^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sin[x])/(a + b*Cos[x])^3,x]

[Out] -((a*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(b*(-a^2 + b^2)^(3/2))) + (x/b + ((a + b*Cos[x])*Sin[x])/((a - b)*(a + b)))/(2*(a + b*Cos[x])^2)

fricas [B] time = 0.66, size = 417, normalized size = 4.74

$$\left[\frac{(ab^2 \cos(x)^2 + 2a^2b \cos(x) + a^3)\sqrt{-a^2 + b^2} \log\left(\frac{2ab \cos(x) + (2a^2 - b^2) \cos(x)^2 + 2\sqrt{-a^2 + b^2}(a \cos(x) + b) \sin(x) - a^2 + 2b^2}{b^2 \cos(x)^2 + 2ab \cos(x) + a^2}\right) + 2(a^4}{4(a^6b - 2a^4b^3 + a^2b^5 + (a^4b^3 - 2a^2b^5 + b^7) \cos(x)^2 + 2(a^5b^2 - 2a^3b^4 + a^2b^6) \cos(x))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)/(a+b*cos(x))^3,x, algorithm="fricas")

[Out] [1/4*((a*b^2*cos(x)^2 + 2*a^2*b*cos(x) + a^3)*sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b^2)*cos(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*b*cos(x) + a^2)) + 2*(a^4 - 2*a^2*b^2 + b^4)*x + 2*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cos(x))*sin(x))/(a^6*b - 2*a^4*b^3 + a^2*b^5 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(x)), -1/2*((a*b^2*cos(x)^2 + 2*a^2*b*cos(x) + a^3)*sqrt(a^2 - b^2)*arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x))) - (a^4 - 2*a^2*b^2 + b^4)*x - (a^3*b - a*b^3 + (a^2*b^2 - b^4)*cos(x))*sin(x))/(a^6*b - 2*a^4*b^3 + a^2*b^5 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(x))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(x)}{(b \cos(x) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)/(a+b*cos(x))^3,x, algorithm="giac")

[Out] integrate(x*sin(x)/(b*cos(x) + a)^3, x)

maple [C] time = 0.44, size = 250, normalized size = 2.84

$$\frac{i(-2ia^2xe^{2ix} + 2ib^2xe^{2ix} + ba e^{3ix} + 2a^2e^{2ix} + b^2e^{2ix} + 3ab e^{ix} + b^2)}{b(b e^{2ix} + 2a e^{ix} + b)^2(a^2 - b^2)} - \frac{ia \ln\left(e^{ix} + \frac{a\sqrt{a^2-b^2} + a^2 - b^2}{\sqrt{a^2-b^2} b}\right)}{2\sqrt{a^2 - b^2} (a + b)(a - b)b} + \frac{ia \ln\left(e^{ix} - \frac{a\sqrt{a^2-b^2} + a^2 - b^2}{\sqrt{a^2-b^2} b}\right)}{2\sqrt{a^2 - b^2} (a + b)(a - b)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(x)/(a+b*cos(x))^3,x)

[Out] I*(-2*I*a^2*x*exp(2*I*x)+2*I*b^2*x*exp(2*I*x)+b*a*exp(3*I*x)+2*a^2*exp(2*I*x)+b^2*exp(2*I*x)+3*a*b*exp(I*x)+b^2)/b/(b*exp(2*I*x)+2*a*exp(I*x)+b)^2/(a^2-b^2)-1/2*I/(a^2-b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(I*x)+(a*(a^2-b^2)^(1/2)+a^2-b^2)/(a^2-b^2)^(1/2)/b)+1/2*I/(a^2-b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(I*x)+(a*(a^2-b^2)^(1/2)-a^2+b^2)/(a^2-b^2)^(1/2)/b)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)/(a+b*cos(x))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \sin(x)}{(a + b \cos(x))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*sin(x))/(a + b*cos(x))^3,x)

[Out] int((x*sin(x))/(a + b*cos(x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x)/(a+b*cos(x))**3,x)

[Out] Timed out

$$3.157 \quad \int \frac{x \sec^2(x)}{(a+b \tan(x))^2} dx$$

Optimal. Leaf size=50

$$\frac{ax}{b(a^2 + b^2)} + \frac{\log(a \cos(x) + b \sin(x))}{a^2 + b^2} - \frac{x}{b(a + b \tan(x))}$$

[Out] a*x/b/(a^2+b^2)+ln(a*cos(x)+b*sin(x))/(a^2+b^2)-x/b/(a+b*tan(x))

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4424, 3484, 3530}

$$\frac{ax}{b(a^2 + b^2)} + \frac{\log(a \cos(x) + b \sin(x))}{a^2 + b^2} - \frac{x}{b(a + b \tan(x))}$$

Antiderivative was successfully verified.

[In] Int[(x*Sec[x]^2)/(a + b*Tan[x])^2,x]

[Out] (a*x)/(b*(a^2 + b^2)) + Log[a*Cos[x] + b*Sin[x]]/(a^2 + b^2) - x/(b*(a + b*Tan[x]))

Rule 3484

Int[((a_) + (b_)*tan[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 3530

Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]

Rule 4424

Int[((e_) + (f_)*(x_))^(m_)*Sec[(c_) + (d_)*(x_)]^2*((a_) + (b_)*Tan[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Simp[((e + f*x)^m*(a + b*Tan[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rubi steps

$$\begin{aligned} \int \frac{x \sec^2(x)}{(a + b \tan(x))^2} dx &= -\frac{x}{b(a + b \tan(x))} + \frac{\int \frac{1}{a + b \tan(x)} dx}{b} \\ &= \frac{ax}{b(a^2 + b^2)} - \frac{x}{b(a + b \tan(x))} + \frac{\int \frac{b - a \tan(x)}{a + b \tan(x)} dx}{a^2 + b^2} \\ &= \frac{ax}{b(a^2 + b^2)} + \frac{\log(a \cos(x) + b \sin(x))}{a^2 + b^2} - \frac{x}{b(a + b \tan(x))} \end{aligned}$$

Mathematica [A] time = 0.18, size = 48, normalized size = 0.96

$$\frac{a \log(a \cos(x) + b \sin(x)) - bx}{a^3 + ab^2} + \frac{x \sin(x)}{a^2 \cos(x) + ab \sin(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sec[x]^2)/(a + b*Tan[x])^2,x]

[Out] $(-(b*x) + a*\text{Log}[a*\text{Cos}[x] + b*\text{Sin}[x]])/(a^3 + a*b^2) + (x*\text{Sin}[x])/(a^2*\text{Cos}[x] + a*b*\text{Sin}[x])$

fricas [A] time = 0.59, size = 80, normalized size = 1.60

$$\frac{2bx \cos(x) - 2ax \sin(x) - (a \cos(x) + b \sin(x)) \log(2ab \cos(x) \sin(x) + (a^2 - b^2) \cos(x)^2 + b^2)}{2((a^3 + ab^2) \cos(x) + (a^2b + b^3) \sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x)^2/(a+b*tan(x))^2,x, algorithm="fricas")

[Out] $-1/2*(2*b*x*\cos(x) - 2*a*x*\sin(x) - (a*\cos(x) + b*\sin(x))*\log(2*a*b*\cos(x)*\sin(x) + (a^2 - b^2)*\cos(x)^2 + b^2))/((a^3 + a*b^2)*\cos(x) + (a^2*b + b^3)*\sin(x))$

giac [B] time = 0.36, size = 322, normalized size = 6.44

$$\frac{2bx \tan\left(\frac{1}{2}x\right)^2 - a \log\left(\frac{4\left(a^2 \tan\left(\frac{1}{2}x\right)^4 - 4ab \tan\left(\frac{1}{2}x\right)^3 - 2a^2 \tan\left(\frac{1}{2}x\right)^2 + 4b^2 \tan\left(\frac{1}{2}x\right)^2 + 4ab \tan\left(\frac{1}{2}x\right) + a^2\right)}{\tan\left(\frac{1}{2}x\right)^4 + 2 \tan\left(\frac{1}{2}x\right)^2 + 1}\right) \tan\left(\frac{1}{2}x\right)^2 + 4ax \tan\left(\frac{1}{2}x\right)}{2\left(a^3 \tan\left(\frac{1}{2}x\right) + (a^2b + b^3)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x)^2/(a+b*tan(x))^2,x, algorithm="giac")

[Out] $-1/2*(2*b*x*\tan(1/2*x)^2 - a*\log(4*(a^2*\tan(1/2*x)^4 - 4*a*b*\tan(1/2*x)^3 - 2*a^2*\tan(1/2*x)^2 + 4*b^2*\tan(1/2*x)^2 + 4*a*b*\tan(1/2*x) + a^2)/(\tan(1/2*x)^4 + 2*\tan(1/2*x)^2 + 1))*\tan(1/2*x)^2 + 4*a*x*\tan(1/2*x) + 2*b*\log(4*(a^2*\tan(1/2*x)^4 - 4*a*b*\tan(1/2*x)^3 - 2*a^2*\tan(1/2*x)^2 + 4*b^2*\tan(1/2*x)^2 + 4*a*b*\tan(1/2*x) + a^2)/(\tan(1/2*x)^4 + 2*\tan(1/2*x)^2 + 1))*\tan(1/2*x) - 2*b*x + a*\log(4*(a^2*\tan(1/2*x)^4 - 4*a*b*\tan(1/2*x)^3 - 2*a^2*\tan(1/2*x)^2 + 4*b^2*\tan(1/2*x)^2 + 4*a*b*\tan(1/2*x) + a^2)/(\tan(1/2*x)^4 + 2*\tan(1/2*x)^2 + 1)))/(a^3*\tan(1/2*x)^2 + a*b^2*\tan(1/2*x)^2 - 2*a^2*b*\tan(1/2*x) - 2*b^3*\tan(1/2*x) - a^3 - a*b^2)$

maple [C] time = 0.32, size = 86, normalized size = 1.72

$$-\frac{2ix}{a^2 + b^2} + \frac{2ix}{(-ib e^{2ix} + a e^{2ix} + ib + a)(-ib + a)} + \frac{\ln\left(e^{2ix} - \frac{ib+a}{ib-a}\right)}{a^2 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sec(x)^2/(a+b*tan(x))^2,x)

[Out] $-2*I/(a^2+b^2)*x+2*I*x/(-I*b*\exp(2*I*x)+a*\exp(2*I*x)+I*b+a)/(-I*b+a)+1/(a^2+b^2)*\ln(\exp(2*I*x)-(I*b+a)/(I*b-a))$

maxima [B] time = 0.35, size = 250, normalized size = 5.00

$$\frac{8 abx \cos(2x) - 4(a^2 - b^2)x \sin(2x) - ((a^2 + b^2) \cos(2x))^2 + 4 ab \sin(2x) + (a^2 + b^2) \sin(2x)^2 + a^2 + b^2 + 2}{2(a^4 + 2a^2b^2 + b^4 + (a^4 + 2a^2b^2 + b^4) \cos(2x)^2 + (a^4 + 2a^2b^2 + b^4) \sin(2x)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x)^2/(a+b*tan(x))^2,x, algorithm="maxima")

[Out] -1/2*(8*a*b*x*cos(2*x) - 4*(a^2 - b^2)*x*sin(2*x) - ((a^2 + b^2)*cos(2*x))^2 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2 + a^2 + b^2 + 2*(a^2 - b^2)*cos(2*x))*log(((a^2 + b^2)*cos(2*x)^2 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2 + a^2 + b^2 + 2*(a^2 - b^2)*cos(2*x))/(a^2 + b^2)))/(a^4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cos(2*x)^2 + (a^4 + 2*a^2*b^2 + b^4)*sin(2*x)^2 + 2*(a^4 - b^4)*cos(2*x) + 4*(a^3*b + a*b^3)*sin(2*x))

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\cos(x)^2 (a + b \tan(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(cos(x)^2*(a + b*tan(x))^2),x)

[Out] int(x/(cos(x)^2*(a + b*tan(x))^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sec^2(x)}{(a + b \tan(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x)**2/(a+b*tan(x))**2,x)

[Out] Integral(x*sec(x)**2/(a + b*tan(x))**2, x)

$$3.158 \quad \int \frac{x \csc^2(x)}{(a+b \cot(x))^2} dx$$

Optimal. Leaf size=50

$$-\frac{ax}{b(a^2+b^2)} + \frac{\log(a \sin(x) + b \cos(x))}{a^2+b^2} + \frac{x}{b(a+b \cot(x))}$$

[Out] $-a*x/b/(a^2+b^2)+x/b/(a+b*\cot(x))+\ln(b*\cos(x)+a*\sin(x))/(a^2+b^2)$

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4425, 3484, 3530}

$$-\frac{ax}{b(a^2+b^2)} + \frac{\log(a \sin(x) + b \cos(x))}{a^2+b^2} + \frac{x}{b(a+b \cot(x))}$$

Antiderivative was successfully verified.

[In] Int[(x*Csc[x]^2)/(a + b*Cot[x])^2,x]

[Out] $-((a*x)/(b*(a^2 + b^2))) + x/(b*(a + b*\cot[x])) + \text{Log}[b*\cos[x] + a*\sin[x]]/(a^2 + b^2)$

Rule 3484

Int[((a_) + (b_.)*tan[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 3530

Int[((c_) + (d_.)*tan[(e_) + (f_)*(x_)])/((a_) + (b_.)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]

Rule 4425

Int[Csc[(c_) + (d_)*(x_)]^2*(Cot[(c_) + (d_)*(x_)]*(b_) + (a_))^(n_)*((e_) + (f_)*(x_))^(m_), x_Symbol] := -Simp[((e + f*x)^m*(a + b*Cot[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Cot[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rubi steps

$$\begin{aligned} \int \frac{x \csc^2(x)}{(a+b \cot(x))^2} dx &= \frac{x}{b(a+b \cot(x))} - \frac{\int \frac{1}{a+b \cot(x)} dx}{b} \\ &= -\frac{ax}{b(a^2+b^2)} + \frac{x}{b(a+b \cot(x))} + \frac{\int \frac{-b+a \cot(x)}{a+b \cot(x)} dx}{a^2+b^2} \\ &= -\frac{ax}{b(a^2+b^2)} + \frac{x}{b(a+b \cot(x))} + \frac{\log(b \cos(x) + a \sin(x))}{a^2+b^2} \end{aligned}$$

Mathematica [A] time = 0.19, size = 48, normalized size = 0.96

$$\frac{b \log(a \sin(x) + b \cos(x)) - ax}{a^2 b + b^3} + \frac{x \sin(x)}{ab \sin(x) + b^2 \cos(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Csc[x]^2)/(a + b*Cot[x])^2,x]

[Out] $(-(a*x) + b*\text{Log}[b*\text{Cos}[x] + a*\text{Sin}[x]])/(a^2*b + b^3) + (x*\text{Sin}[x])/(b^2*\text{Cos}[x] + a*b*\text{Sin}[x])$

fricas [A] time = 0.74, size = 81, normalized size = 1.62

$$\frac{2ax \cos(x) - 2bx \sin(x) - (b \cos(x) + a \sin(x)) \log(2ab \cos(x) \sin(x) - (a^2 - b^2) \cos(x)^2 + a^2)}{2((a^2b + b^3) \cos(x) + (a^3 + ab^2) \sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)^2/(a+b*cot(x))^2,x, algorithm="fricas")

[Out] $-1/2*(2*a*x*\cos(x) - 2*b*x*\sin(x) - (b*\cos(x) + a*\sin(x))*\log(2*a*b*\cos(x)*\sin(x) - (a^2 - b^2)*\cos(x)^2 + a^2))/((a^2*b + b^3)*\cos(x) + (a^3 + a*b^2)*\sin(x))$

giac [B] time = 0.34, size = 322, normalized size = 6.44

$$\frac{2ax \tan\left(\frac{1}{2}x\right)^2 - b \log\left(\frac{4\left(b^2 \tan\left(\frac{1}{2}x\right)^4 - 4ab \tan\left(\frac{1}{2}x\right)^3 + 4a^2 \tan\left(\frac{1}{2}x\right)^2 - 2b^2 \tan\left(\frac{1}{2}x\right) + 4ab \tan\left(\frac{1}{2}x\right) + b^2\right)}{\tan\left(\frac{1}{2}x\right)^4 + 2 \tan\left(\frac{1}{2}x\right)^2 + 1}\right) \tan\left(\frac{1}{2}x\right)^2 + 4bx \tan\left(\frac{1}{2}x\right)}{2\left(a^2b \tan\left(\frac{1}{2}x\right) + \dots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)^2/(a+b*cot(x))^2,x, algorithm="giac")

[Out] $-1/2*(2*a*x*\tan(1/2*x)^2 - b*\log(4*(b^2*\tan(1/2*x)^4 - 4*a*b*\tan(1/2*x)^3 + 4*a^2*\tan(1/2*x)^2 - 2*b^2*\tan(1/2*x) + 4*a*b*\tan(1/2*x) + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 4*b*x*\tan(1/2*x) + 2*a*\log(4*(b^2*\tan(1/2*x)^4 - 4*a*b*\tan(1/2*x)^3 + 4*a^2*\tan(1/2*x)^2 - 2*b^2*\tan(1/2*x) + 4*a*b*\tan(1/2*x) + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x) - 2*a*x + b*\log(4*(b^2*\tan(1/2*x)^4 - 4*a*b*\tan(1/2*x)^3 + 4*a^2*\tan(1/2*x)^2 - 2*b^2*\tan(1/2*x) + 4*a*b*\tan(1/2*x) + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/(a^2*b*tan(1/2*x)^2 + b^3*tan(1/2*x)^2 - 2*a^3*tan(1/2*x) - 2*a*b^2*tan(1/2*x) - a^2*b - b^3)$

maple [C] time = 0.31, size = 87, normalized size = 1.74

$$-\frac{2ix}{a^2 + b^2} - \frac{2ix}{(ib e^{2ix} + a e^{2ix} + ib - a)(ib + a)} + \frac{\ln\left(e^{2ix} + \frac{ib-a}{ib+a}\right)}{a^2 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*csc(x)^2/(a+b*cot(x))^2,x)

[Out] $-2*I/(a^2+b^2)*x - 2*I*x/(I*b*\exp(2*I*x) + a*\exp(2*I*x) + I*b - a)/(I*b + a) + 1/(a^2 + b^2)*\ln(\exp(2*I*x) + (I*b - a)/(I*b + a))$

maxima [B] time = 0.33, size = 250, normalized size = 5.00

$$\frac{8 abx \cos(2x) + 4(a^2 - b^2)x \sin(2x) - ((a^2 + b^2) \cos(2x))^2 + 4 ab \sin(2x) + (a^2 + b^2) \sin(2x)^2 + a^2 + b^2}{2(a^4 + 2a^2b^2 + b^4 + (a^4 + 2a^2b^2 + b^4) \cos(2x)^2 + (a^4 + 2a^2b^2 + b^4) \sin(2x)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)^2/(a+b*cot(x))^2,x, algorithm="maxima")

[Out] -1/2*(8*a*b*x*cos(2*x) + 4*(a^2 - b^2)*x*sin(2*x) - ((a^2 + b^2)*cos(2*x))^2 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*x))*log(((a^2 + b^2)*cos(2*x)^2 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*x))/(a^2 + b^2)))/(a^4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cos(2*x)^2 + (a^4 + 2*a^2*b^2 + b^4)*sin(2*x)^2 - 2*(a^4 - b^4)*cos(2*x) + 4*(a^3*b + a*b^3)*sin(2*x))

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sin(x)^2 (a + b \cot(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(sin(x)^2*(a + b*cot(x))^2),x)

[Out] int(x/(sin(x)^2*(a + b*cot(x))^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \csc^2(x)}{(a + b \cot(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)**2/(a+b*cot(x))**2,x)

[Out] Integral(x*csc(x)**2/(a + b*cot(x))**2, x)

$$3.159 \quad \int \frac{\sec^2(c+dx)}{a+b \tan^2(c+dx)} dx$$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{\sqrt{b} \tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} d}$$

[Out] arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/d/a^(1/2)/b^(1/2)

Rubi [A] time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3675, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b} \tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2), x]

[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c+dx)}{a+b \tan^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tan(c+dx)\right)}{d} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b} \tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} d} \end{aligned}$$

Mathematica [A] time = 0.09, size = 32, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b} \tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2), x]

[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)

fricas [B] time = 0.62, size = 205, normalized size = 6.41

$$\left[\frac{\sqrt{-ab} \log\left(\frac{(a^2+6ab+b^2)\cos(dx+c)^4-2(3ab+b^2)\cos(dx+c)^2+4((a+b)\cos(dx+c)^3-b\cos(dx+c))\sqrt{-ab}\sin(dx+c)+b^2}{(a^2-2ab+b^2)\cos(dx+c)^4+2(ab-b^2)\cos(dx+c)^2+b^2}\right)}{4abd}, \sqrt{ab} \arctan\left(\frac{b \tan(dx+c)}{\sqrt{ab}}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")

[Out] [-1/4*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*x + c)^2 + b^2))/(a*b*d), -1/2*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))/(a*b*d)]

giac [A] time = 0.62, size = 40, normalized size = 1.25

$$\frac{\pi \left\lfloor \frac{dx+c}{\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(b) + \arctan\left(\frac{b \tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")

[Out] (pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))/(sqrt(a*b)*d)

maple [A] time = 0.32, size = 24, normalized size = 0.75

$$\frac{\arctan\left(\frac{\tan(dx+c)b}{\sqrt{ab}}\right)}{d\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x)

[Out] 1/d/(a*b)^(1/2)*arctan(tan(d*x+c)*b/(a*b)^(1/2))

maxima [A] time = 0.43, size = 23, normalized size = 0.72

$$\frac{\arctan\left(\frac{b \tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")

[Out] arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*d)

mupad [B] time = 2.56, size = 24, normalized size = 0.75

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b} \tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)),x)

[Out] $\text{atan}((b^{1/2} \cdot \tan(c + d \cdot x)) / a^{1/2}) / (a^{1/2} \cdot b^{1/2} \cdot d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c + dx)}{a + b \tan^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)**2), x)`

[Out] `Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)`

$$3.160 \quad \int \frac{x \sec^2(c+dx)}{a+b \tan^2(c+dx)} dx$$

Optimal. Leaf size=211

$$-\frac{\operatorname{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{4\sqrt{a}\sqrt{b}d^2} + \frac{\operatorname{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}+\sqrt{b})^2}\right)}{4\sqrt{a}\sqrt{b}d^2} - \frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}+\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d}$$

[Out] $-1/2*I*x*\ln(1+(a-b)*\exp(2*I*(d*x+c))/(a^{(1/2)}-b^{(1/2)})^2)/d/a^{(1/2)}/b^{(1/2)}$
 $+1/2*I*x*\ln(1+(a-b)*\exp(2*I*(d*x+c))/(a^{(1/2)}+b^{(1/2)})^2)/d/a^{(1/2)}/b^{(1/2)}$
 $-1/4*\operatorname{polylog}(2, -(a-b)*\exp(2*I*(d*x+c))/(a^{(1/2)}-b^{(1/2)})^2)/d^2/a^{(1/2)}/b^{(1/2)}$
 $+1/4*\operatorname{polylog}(2, -(a-b)*\exp(2*I*(d*x+c))/(a^{(1/2)}+b^{(1/2)})^2)/d^2/a^{(1/2)}/b^{(1/2)}$

Rubi [A] time = 0.53, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4588, 3321, 2264, 2190, 2279, 2391}

$$-\frac{\operatorname{PolyLog}\left(2, -\frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{4\sqrt{a}\sqrt{b}d^2} + \frac{\operatorname{PolyLog}\left(2, -\frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}+\sqrt{b})^2}\right)}{4\sqrt{a}\sqrt{b}d^2} - \frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}+\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x*\operatorname{Sec}[c + d*x]^2)/(a + b*\operatorname{Tan}[c + d*x]^2), x]$

[Out] $((-I/2)*x*\operatorname{Log}[1 + ((a - b)*E^{((2*I)*(c + d*x))})/(\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b])^2])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[b]*d) + ((I/2)*x*\operatorname{Log}[1 + ((a - b)*E^{((2*I)*(c + d*x))})/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b])^2])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[b]*d) - \operatorname{PolyLog}[2, -(((a - b)*E^{((2*I)*(c + d*x))})/(\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b])^2)]/(4*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[b]*d^2) + \operatorname{PolyLog}[2, -(((a - b)*E^{((2*I)*(c + d*x))})/(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b])^2)]/(4*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[b]*d^2)$

Rule 2190

$\operatorname{Int}[(\operatorname{F}_-)^{((g_-)*(e_-) + (f_-)*(x_-))}^{(n_-)} * ((c_-) + (d_-)*(x_-))^{(m_-)}] / ((a_-) + (b_-)*(\operatorname{F}_-)^{((g_-)*(e_-) + (f_-)*(x_-))}^{(n_-)}), x_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^m * \operatorname{Log}[1 + (b*(F^{(g*(e + f*x)))})^n/a] / (b*f*g*n * \operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m) / (b*f*g*n * \operatorname{Log}[F]), \operatorname{Int}[(c + d*x)^{(m-1)} * \operatorname{Log}[1 + (b*(F^{(g*(e + f*x)))})^n/a], x], x] /;$ $\operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \ \&\& \ \operatorname{IGtQ}[m, 0]$

Rule 2264

$\operatorname{Int}[(\operatorname{F}_-)^{(u_-)} * ((f_-) + (g_-)*(x_-))^{(m_-)}] / ((a_-) + (b_-)*(\operatorname{F}_-)^{(u_-)} + (c_-)*(\operatorname{F}_-)^{(v_-)}), x_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[b^2 - 4*a*c, 2]\}, \operatorname{Dist}[(2*c)/q, \operatorname{Int}[(f + g*x)^m * F^u / (b - q + 2*c * F^u), x], x] - \operatorname{Dist}[(2*c)/q, \operatorname{Int}[(f + g*x)^m * F^u / (b + q + 2*c * F^u), x], x] /;$ $\operatorname{FreeQ}\{F, a, b, c, f, g\}, x\} \ \&\& \ \operatorname{EqQ}[v, 2*u] \ \&\& \ \operatorname{LinearQ}[u, x] \ \&\& \ \operatorname{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \operatorname{IGtQ}[m, 0]$

Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_-) + (b_-)*(\operatorname{F}_-)^{(e_-)*((c_-) + (d_-)*(x_-))}^{(n_-)}], x_Symbol] \rightarrow \operatorname{Dist}[1/(d*e*n * \operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /;$ $\operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x\} \ \&\& \ \operatorname{GtQ}[a, 0]$

Rule 2391

$\operatorname{Int}[\operatorname{Log}[(c_-)*((d_-) + (e_-)*(x_-))^{(n_-)}] / (x_-), x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c*e*x^n)]/n, x] /;$ $\operatorname{FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \operatorname{EqQ}[c*d, 1]$

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4588

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_.) + (c_.)*Tan[(d_.) + (e_.)*(x_)]^2), x_Symbol] := Dist[2, Int[(f + g*x)^m/(b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{b, c, d, e, f, g}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x \sec^2(c + dx)}{a + b \tan^2(c + dx)} dx = 2 \int \frac{x}{a + b + (a - b) \cos(2c + 2dx)} dx$$

$$= 4 \int \frac{e^{i(2c+2dx)} x}{a - b + 2(a + b)e^{i(2c+2dx)} + (a - b)e^{2i(2c+2dx)}} dx$$

$$= \frac{(2(a - b)) \int \frac{e^{i(2c+2dx)} x}{-4\sqrt{a}\sqrt{b} + 2(a+b) + 2(a-b)e^{i(2c+2dx)}} dx}{\sqrt{a}\sqrt{b}} - \frac{(2(a - b)) \int \frac{e^{i(2c+2dx)} x}{4\sqrt{a}\sqrt{b} + 2(a+b) + 2(a-b)e^{i(2c+2dx)}} dx}{\sqrt{a}\sqrt{b}}$$

$$= -\frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}+\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{i \int \log\left(1 + \frac{2(a-b)e^{i(2c+2dx)}}{-4\sqrt{a}\sqrt{b} + 2(a+b)}\right) dx}{2\sqrt{a}\sqrt{b}d}$$

$$= -\frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}+\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{\text{Subst}\left(\int \frac{\log\left(1 + \frac{2(a-b)x}{-4\sqrt{a}\sqrt{b} + 2(a+b)}\right)}{x} dx\right)}{4\sqrt{a}\sqrt{b}d^2}$$

$$= -\frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}+\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d} - \frac{\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{4\sqrt{a}\sqrt{b}d^2} + \frac{\text{Li}_2\left(-\frac{(a-b)}{(\sqrt{a}+\sqrt{b})^2}\right)}{4\sqrt{a}\sqrt{b}d^2}$$

Mathematica [B] time = 6.44, size = 512, normalized size = 2.43

$$x \left(-\sqrt{a} \text{Li}_2\left(\frac{\sqrt{b}(1-i \tan(c+dx))}{i\sqrt{-a}+\sqrt{b}}\right) - \sqrt{a} \text{Li}_2\left(\frac{\sqrt{b}(i \tan(c+dx)+1)}{i\sqrt{-a}+\sqrt{b}}\right) + \sqrt{a} \text{Li}_2\left(-\frac{\sqrt{b}(\tan(c+dx)-i)}{\sqrt{-a}+i\sqrt{b}}\right) + \sqrt{a} \text{Li}_2\left(\frac{\sqrt{b}(\tan(c+dx)+i)}{\sqrt{-a}+i\sqrt{b}}\right) + 4i \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2), x]

[Out] (x*((4*I)*Sqrt[-a]*c*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] - Sqrt[a]*Log[1 + I*Tan[c + d*x]]*Log[(Sqrt[-a] - Sqrt[b]*Tan[c + d*x])/(Sqrt[-a] - I*Sqrt[b])] + Sqrt[a]*Log[1 - I*Tan[c + d*x]]*Log[(Sqrt[-a] - Sqrt[b]*Tan[c + d*x])/(Sqrt[-a] + I*Sqrt[b])] - Sqrt[a]*Log[1 - I*Tan[c + d*x]]*Log[(Sqrt[-a] + Sqrt[b]*Tan[c + d*x])/(Sqrt[-a] - I*Sqrt[b])] + Sqrt[a]*Log[1 + I*Tan[c + d*x]]*Log[(Sqrt[-a] + Sqrt[b]*Tan[c + d*x])/(Sqrt[-a] + I*Sqrt[b])] - Sqrt[a]*PolyLog[2, (Sqrt[b]*(1 - I*Tan[c + d*x]))/(I*Sqrt[-a] + Sqrt[b])] - Sqrt[a]*PolyLog[2, (Sqrt[b]*(1 + I*Tan[c + d*x]))/(I*Sqrt[-a] + Sqrt[b])] + Sqrt[a]*PolyLog[2, (Sqrt[b]*(1 - I*Tan[c + d*x]))/(I*Sqrt[-a] - Sqrt[b])] - Sqrt[a]*PolyLog[2, (Sqrt[b]*(1 + I*Tan[c + d*x]))/(I*Sqrt[-a] - Sqrt[b])])

```
rt[a]*PolyLog[2, -((Sqrt[b]*(-I + Tan[c + d*x]))/(Sqrt[-a] + I*Sqrt[b]))] +
  Sqrt[a]*PolyLog[2, (Sqrt[b]*(I + Tan[c + d*x]))/(Sqrt[-a] + I*Sqrt[b]))]/
  (2*Sqrt[-a^2]*Sqrt[b]*d*((2*I)*c + Log[1 - I*Tan[c + d*x]] - Log[1 + I*Tan[
  c + d*x]]))
```

fricas [B] time = 1.30, size = 3344, normalized size = 15.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(-4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2*(a - b)*s
sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d
*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2*(a -
b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*cos(d*x + c) - 2*I*s
in(d*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2*(
a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*cos(d*x + c) + 2
*I*sin(d*x + c)) - 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-
(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*cos(d*x + c)
- 2*I*sin(d*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sq
rt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*cos(d*x +
c) + 2*I*sin(d*x + c)) - 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2
*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*cos(d*
x + c) - 2*I*sin(d*x + c)) - 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*lo
g(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*cos
(d*x + c) + 2*I*sin(d*x + c)) + 4*I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c
*log(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*
cos(d*x + c) - 2*I*sin(d*x + c)) + 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*
dilog(1/2*((2*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a -
b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))
)*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a +
2*b)/(a - b) + 1) + 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*
(a + b)*cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x +
c) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a
- b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b) +
1) + 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a + b)*cos(d*x
+ c) + (-2*I*a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (I*a - I*
b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(
a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b) + 1) + 4*(a - b)
*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b)*cos(d*x + c) - (-2*I*
a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c)
))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b +
b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b) + 1) - 4*(a - b)*sqrt(a*b/(a^2
- 2*a*b + b^2))*dilog(1/2*((2*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d
*x + c) + 4*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2
- 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a
- b)) - 2*a + 2*b)/(a - b) + 1) - 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*
dilog(-1/2*((2*(a + b)*cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a
- b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))
)*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2
*b)/(a - b) + 1) - 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a
+ b)*cos(d*x + c) + (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c)
) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a -
b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*b)/(a - b) + 1
) - 4*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b)*cos(d*x
+ c) - (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (-I*a + I*
b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a
^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(a - b) + 1) + 4*(I*(a -
b)*d*x + I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b)*co
```

s(dx + c) + (2*I*a + 2*I*b)*sin(dx + c) - 4*((a - b)*cos(dx + c) - (-I*a + I*b)*sin(dx + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(-I*(a - b)*dx - I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b)*cos(dx + c) - (2*I*a + 2*I*b)*sin(dx + c) - 4*((a - b)*cos(dx + c) + (-I*a + I*b)*sin(dx + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*(a - b)*dx - I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b)*cos(dx + c) + (-2*I*a - 2*I*b)*sin(dx + c) - 4*((a - b)*cos(dx + c) - (I*a - I*b)*sin(dx + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(I*(a - b)*dx + I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b)*cos(dx + c) - (-2*I*a - 2*I*b)*sin(dx + c) - 4*((a - b)*cos(dx + c) + (I*a - I*b)*sin(dx + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*(a - b)*dx - I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b)*cos(dx + c) + (2*I*a + 2*I*b)*sin(dx + c) + 4*((a - b)*cos(dx + c) + (I*a - I*b)*sin(dx + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(I*(a - b)*dx + I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b)*cos(dx + c) - (2*I*a + 2*I*b)*sin(dx + c) + 4*((a - b)*cos(dx + c) - (I*a - I*b)*sin(dx + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(I*(a - b)*dx + I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2*(a + b)*cos(dx + c) + (-2*I*a - 2*I*b)*sin(dx + c) + 4*((a - b)*cos(dx + c) + (-I*a + I*b)*sin(dx + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(-I*(a - b)*dx - I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/2*((2*(a + b)*cos(dx + c) - (-2*I*a - 2*I*b)*sin(dx + c) + 4*((a - b)*cos(dx + c) - (-I*a + I*b)*sin(dx + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(a - b))) / (a*b*d^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sec(dx + c)^2}{b \tan(dx + c)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(dx+c)^2/(a+b*tan(dx+c)^2),x, algorithm="giac")

[Out] integrate(x*sec(dx + c)^2/(b*tan(dx + c)^2 + a), x)

maple [B] time = 0.60, size = 1003, normalized size = 4.75

$$\frac{acx}{d\sqrt{ab}(-2\sqrt{ab}-a-b)} - \frac{bcx}{d\sqrt{ab}(-2\sqrt{ab}-a-b)} - \frac{i \ln\left(1 - \frac{(a-b)e^{2i(dx+c)}}{-2\sqrt{ab}-a-b}\right)ax}{2d\sqrt{ab}(-2\sqrt{ab}-a-b)} - \frac{\text{polylog}\left(2, \frac{(a-b)e^{2i(dx+c)}}{-2\sqrt{ab}-a-b}\right)}{2d^2(-2\sqrt{ab}-a-b)} - \text{polylog}\left(2, \frac{(a-b)e^{2i(dx+c)}}{-2\sqrt{ab}-a-b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sec(dx+c)^2/(a+b*tan(dx+c)^2),x)

[Out] -1/d/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*c*x-1/d/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*c*x-1/2*I/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(dx+c)))/(-2*(a*b)^(1/2)-a-b)*a*c-1/4/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*polylog(2,(a-b)*exp(2*I*(dx+c)))/(-2*(a*b)^(1/2)-a-b)*b-1/2/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*c^2-1/2/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*c^2-1/4/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*polylog(2,(a-b)*exp(2*I*(dx+c)))/(-2*(a*b)^(1/2)-a-b)*a-I/d^2*c/(a*b)^(1/2)*arctanh(1/4*(2*(a-b)*exp(2*I*(d

$$\begin{aligned} & *x+c)) + 2*a+2*b)/(a*b)^{(1/2)} - 1/2*I/d^2/(a*b)^{(1/2)} * \ln(1-(a-b)*\exp(2*I*(d*x+c))) / (2*(a*b)^{(1/2)} - a-b) * c - I/d^2 / (-2*(a*b)^{(1/2)} - a-b) * \ln(1-(a-b)*\exp(2*I*(d*x+c))) / (-2*(a*b)^{(1/2)} - a-b) * c - 1/2*I/d / (a*b)^{(1/2)} * \ln(1-(a-b)*\exp(2*I*(d*x+c))) / (2*(a*b)^{(1/2)} - a-b) * x - I/d / (-2*(a*b)^{(1/2)} - a-b) * \ln(1-(a-b)*\exp(2*I*(d*x+c))) / (-2*(a*b)^{(1/2)} - a-b) * x - 1/2/d^2 / (a*b)^{(1/2)} * c^2 - 1/d^2 / (-2*(a*b)^{(1/2)} - a-b) * c^2 - 1/2/d^2 / (-2*(a*b)^{(1/2)} - a-b) * \text{polylog}(2, (a-b)*\exp(2*I*(d*x+c))) / (-2*(a*b)^{(1/2)} - a-b) - 1/4/d^2 / (a*b)^{(1/2)} * \text{polylog}(2, (a-b)*\exp(2*I*(d*x+c))) / (2*(a*b)^{(1/2)} - a-b) - 1/2 / (a*b)^{(1/2)} / (-2*(a*b)^{(1/2)} - a-b) * a*x^2 - 1/2 / (a*b)^{(1/2)} / (-2*(a*b)^{(1/2)} - a-b) * b*x^2 - 1/d / (a*b)^{(1/2)} * c*x - 2/d / (-2*(a*b)^{(1/2)} - a-b) * c*x - 1/2*I/d / (a*b)^{(1/2)} / (-2*(a*b)^{(1/2)} - a-b) * \ln(1-(a-b)*\exp(2*I*(d*x+c))) / (-2*(a*b)^{(1/2)} - a-b) * b*x - 1/2*I/d^2 / (a*b)^{(1/2)} / (-2*(a*b)^{(1/2)} - a-b) * \ln(1-(a-b)*\exp(2*I*(d*x+c))) / (-2*(a*b)^{(1/2)} - a-b) * b*c - 1/2*I/d / (a*b)^{(1/2)} / (-2*(a*b)^{(1/2)} - a-b) * \ln(1-(a-b)*\exp(2*I*(d*x+c))) / (-2*(a*b)^{(1/2)} - a-b) * a*x - 1/2 / (a*b)^{(1/2)} * x^2 - 1 / (-2*(a*b)^{(1/2)} - a-b) * x^2 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sec(dx + c)^2}{b \tan(dx + c)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")

[Out] integrate(x*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\cos(c + dx)^2 (b \tan(c + dx)^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)),x)

[Out] int(x/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sec^2(c + dx)}{a + b \tan^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)

[Out] Integral(x*sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)

$$3.161 \quad \int \frac{x^2 \sec^2(c+dx)}{a+b \tan^2(c+dx)} dx$$

Optimal. Leaf size=337

$$\frac{i\text{Li}_3\left(-\frac{(\sqrt{a}-\sqrt{b})e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right)}{4\sqrt{a}\sqrt{b}d^3} - \frac{i\text{Li}_3\left(-\frac{(\sqrt{a}+\sqrt{b})e^{2i(c+dx)}}{\sqrt{a}-\sqrt{b}}\right)}{4\sqrt{a}\sqrt{b}d^3} - \frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}+\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d^2} - \frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})}\right)}{2\sqrt{a}\sqrt{b}d}$$

[Out] $-1/2*I*x^2*\ln(1+(a-b)*\exp(2*I*(d*x+c)))/(a^{(1/2)}-b^{(1/2)})^2/d/a^{(1/2)}/b^{(1/2)}+1/2*I*x^2*\ln(1+(a-b)*\exp(2*I*(d*x+c)))/(a^{(1/2)}+b^{(1/2)})^2/d/a^{(1/2)}/b^{(1/2)}-1/2*x*\text{polylog}(2,-(a-b)*\exp(2*I*(d*x+c)))/(a^{(1/2)}-b^{(1/2)})^2/d^2/a^{(1/2)}/b^{(1/2)}+1/2*x*\text{polylog}(2,-(a-b)*\exp(2*I*(d*x+c)))/(a^{(1/2)}+b^{(1/2)})^2/d^2/a^{(1/2)}/b^{(1/2)}+1/4*I*\text{polylog}(3,-\exp(2*I*(d*x+c))*(a^{(1/2)}-b^{(1/2)}))/(a^{(1/2)}+b^{(1/2)})/d^3/a^{(1/2)}/b^{(1/2)}-1/4*I*\text{polylog}(3,-\exp(2*I*(d*x+c))*(a^{(1/2)}+b^{(1/2)}))/(a^{(1/2)}-b^{(1/2)})/d^3/a^{(1/2)}/b^{(1/2)}$

Rubi [A] time = 0.90, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4588, 3321, 2264, 2190, 2531, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(2, -\frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{x\text{PolyLog}\left(2, -\frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}+\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{i\text{PolyLog}\left(3, -\frac{(\sqrt{a}-\sqrt{b})e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right)}{4\sqrt{a}\sqrt{b}d^3} - \frac{i\text{PolyLog}\left(3, -\frac{(\sqrt{a}+\sqrt{b})e^{2i(c+dx)}}{\sqrt{a}-\sqrt{b}}\right)}{4\sqrt{a}\sqrt{b}d^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^2*\text{Sec}[c + d*x]^2)/(a + b*\text{Tan}[c + d*x]^2), x]$

[Out] $((-I/2)*x^2*\text{Log}[1 + ((a - b)*E^{((2*I)*(c + d*x))})/(\text{Sqrt}[a] - \text{Sqrt}[b])^2])/(\text{Sqrt}[a]*\text{Sqrt}[b]*d) + ((I/2)*x^2*\text{Log}[1 + ((a - b)*E^{((2*I)*(c + d*x))})/(\text{Sqrt}[a] + \text{Sqrt}[b])^2])/(\text{Sqrt}[a]*\text{Sqrt}[b]*d) - (x*\text{PolyLog}[2, -(((a - b)*E^{((2*I)*(c + d*x))})/(\text{Sqrt}[a] - \text{Sqrt}[b])^2)])/(2*\text{Sqrt}[a]*\text{Sqrt}[b]*d^2) + (x*\text{PolyLog}[2, -(((a - b)*E^{((2*I)*(c + d*x))})/(\text{Sqrt}[a] + \text{Sqrt}[b])^2)])/(2*\text{Sqrt}[a]*\text{Sqrt}[b]*d^2) + ((I/4)*\text{PolyLog}[3, -(((\text{Sqrt}[a] - \text{Sqrt}[b])*E^{((2*I)*(c + d*x))})/(\text{Sqrt}[a] + \text{Sqrt}[b]))])/(\text{Sqrt}[a]*\text{Sqrt}[b]*d^3) - ((I/4)*\text{PolyLog}[3, -(((\text{Sqrt}[a] + \text{Sqrt}[b])*E^{((2*I)*(c + d*x))})/(\text{Sqrt}[a] - \text{Sqrt}[b]))])/(\text{Sqrt}[a]*\text{Sqrt}[b]*d^3)$

Rule 2190

$\text{Int}[(((F_)^{((g_)*(e_) + (f_)*(x_))})^{(n_)*((c_) + (d_)*(x_))^{(m_))})/((a_) + (b_)*((F_)^{((g_)*(e_) + (f_)*(x_))})^{(n_)}), x_Symbol] :> \text{Simp} [((c + d*x)^m*\text{Log}[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /;$ FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2264

$\text{Int}[((F_)^{(u_)*((f_) + (g_)*(x_))^{(m_))})/((a_) + (b_)*(F_)^{(u_)} + (c_)* (F_)^{(v_)}), x_Symbol] :> \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/q, \text{Int}[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - \text{Dist}[(2*c)/q, \text{Int}[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /;$ FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

Rule 2282

$\text{Int}[u_, x_Symbol] :> \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /;$ FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^{(n_)})^{(m_)} /; FreeQ[


```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*x_)))]^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_)^(m_.))/((a_.) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4588

```
Int[(((f_.) + (g_.)*(x_)^(m_.))*Sec[(d_.) + (e_.)*(x_)]^2)/((b_.) + (c_.)*Ta
n[(d_.) + (e_.)*(x_)]^2), x_Symbol] := Dist[2, Int[(f + g*x)^m/(b + c + (b
- c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{b, c, d, e, f, g}, x] && IGtQ[m, 0
]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \sec^2(c + dx)}{a + b \tan^2(c + dx)} dx &= 2 \int \frac{x^2}{a + b + (a - b) \cos(2c + 2dx)} dx \\
&= 4 \int \frac{e^{i(2c+2dx)} x^2}{a - b + 2(a + b)e^{i(2c+2dx)} + (a - b)e^{2i(2c+2dx)}} dx \\
&= \frac{(2(a - b)) \int \frac{e^{i(2c+2dx)} x^2}{-4\sqrt{a}\sqrt{b} + 2(a+b) + 2(a-b)e^{i(2c+2dx)}} dx}{\sqrt{a}\sqrt{b}} - \frac{(2(a - b)) \int \frac{e^{i(2c+2dx)} x^2}{4\sqrt{a}\sqrt{b} + 2(a+b) + 2(a-b)e^{i(2c+2dx)}} dx}{\sqrt{a}\sqrt{b}} \\
&= -\frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}+\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{i \int x \log\left(1 + \frac{2(a-b)e^{i(2c+2dx)}}{-4\sqrt{a}\sqrt{b} + 2(a+b)}\right)}{\sqrt{a}\sqrt{b}d} \\
&= -\frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}+\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d} - \frac{x \operatorname{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{x \operatorname{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}+\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d^2} \\
&= -\frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}+\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d} - \frac{x \operatorname{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{x \operatorname{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}+\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d^2} \\
&= -\frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}+\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d} - \frac{x \operatorname{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}-\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{x \operatorname{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{(\sqrt{a}+\sqrt{b})^2}\right)}{2\sqrt{a}\sqrt{b}d^2}
\end{aligned}$$

Mathematica [A] time = 1.09, size = 294, normalized size = 0.87

$$\frac{i\left(2d^2x^2 \log\left(1 + \frac{(\sqrt{a}-\sqrt{b})e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right) - 2d^2x^2 \log\left(1 + \frac{(\sqrt{a}+\sqrt{b})e^{2i(c+dx)}}{\sqrt{a}-\sqrt{b}}\right) - 2idx \operatorname{Li}_2\left(\frac{(\sqrt{b}-\sqrt{a})e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right) + 2idx \operatorname{Li}_2\left(-\frac{(\sqrt{a}+\sqrt{b})e^{2i(c+dx)}}{\sqrt{a}-\sqrt{b}}\right)\right)}{4\sqrt{a}\sqrt{b}d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2), x]

[Out] ((I/4)*(2*d^2*x^2*Log[1 + ((Sqrt[a] - Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])]) - 2*d^2*x^2*Log[1 + ((Sqrt[a] + Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b])]) - (2*I)*d*x*PolyLog[2, ((-Sqrt[a] + Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])] + (2*I)*d*x*PolyLog[2, -(((Sqrt[a] + Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b]))] + PolyLog[3, ((-Sqrt[a] + Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])] - PolyLog[3, -(((Sqrt[a] + Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b]))]))/(Sqrt[a]*Sqrt[b]*d^3)

fricas [C] time = 2.28, size = 4644, normalized size = 13.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2), x, algorithm="fricas")

[Out] 1/16*(8*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog(1/2*((2*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b) + 1) + 8*(a

$$\begin{aligned}
& - b) \sqrt{a*b/(a^2 - 2*a*b + b^2)} * d*x * \operatorname{dilog}(-1/2*((2*(a + b)*\cos(d*x + c) \\
& - (2*I*a + 2*I*b)*\sin(d*x + c) - 4*((a - b)*\cos(d*x + c) + (-I*a + I*b)*\sin \\
& (d*x + c)) * \sqrt{a*b/(a^2 - 2*a*b + b^2)}) * \sqrt{-(2*(a - b)*\sqrt{a*b/(a^2 - \\
& 2*a*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b) + 1) + 8*(a - b) * \sqrt{ \\
& a*b/(a^2 - 2*a*b + b^2)} * d*x * \operatorname{dilog}(1/2*((2*(a + b)*\cos(d*x + c) + (-2*I*a - \\
& 2*I*b)*\sin(d*x + c) - 4*((a - b)*\cos(d*x + c) - (I*a - I*b)*\sin(d*x + c)) * \\
& \sqrt{a*b/(a^2 - 2*a*b + b^2)}) * \sqrt{-(2*(a - b)*\sqrt{a*b/(a^2 - 2*a*b + b^2} \\
&)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b) + 1) + 8*(a - b) * \sqrt{a*b/(a^2 - \\
& 2*a*b + b^2)} * d*x * \operatorname{dilog}(-1/2*((2*(a + b)*\cos(d*x + c) - (-2*I*a - 2*I*b)*\sin \\
& (d*x + c) - 4*((a - b)*\cos(d*x + c) + (I*a - I*b)*\sin(d*x + c)) * \sqrt{a*b/(\\
& a^2 - 2*a*b + b^2)}) * \sqrt{-(2*(a - b)*\sqrt{a*b/(a^2 - 2*a*b + b^2)) + a + b \\
&)/(a - b)) + 2*a - 2*b)/(a - b) + 1) - 8*(a - b) * \sqrt{a*b/(a^2 - 2*a*b + b^ \\
& 2)} * d*x * \operatorname{dilog}(1/2*((2*(a + b)*\cos(d*x + c) + (2*I*a + 2*I*b)*\sin(d*x + c) + \\
& 4*((a - b)*\cos(d*x + c) + (I*a - I*b)*\sin(d*x + c)) * \sqrt{a*b/(a^2 - 2*a*b \\
& + b^2)}) * \sqrt{((2*(a - b)*\sqrt{a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - \\
& 2*a + 2*b)/(a - b) + 1) - 8*(a - b) * \sqrt{a*b/(a^2 - 2*a*b + b^2)} * d*x * \operatorname{dilog} \\
& (-1/2*((2*(a + b)*\cos(d*x + c) - (2*I*a + 2*I*b)*\sin(d*x + c) + 4*((a - b)* \\
& \cos(d*x + c) - (I*a - I*b)*\sin(d*x + c)) * \sqrt{a*b/(a^2 - 2*a*b + b^2)}) * \sqrt{ \\
& t((2*(a - b)*\sqrt{a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(\\
& a - b) + 1) - 8*(a - b) * \sqrt{a*b/(a^2 - 2*a*b + b^2)} * d*x * \operatorname{dilog}(1/2*((2*(a \\
& + b)*\cos(d*x + c) + (-2*I*a - 2*I*b)*\sin(d*x + c) + 4*((a - b)*\cos(d*x + c) \\
& + (-I*a + I*b)*\sin(d*x + c)) * \sqrt{a*b/(a^2 - 2*a*b + b^2)}) * \sqrt{((2*(a - b) \\
&) * \sqrt{a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*b)/(a - b) + 1) \\
& - 8*(a - b) * \sqrt{a*b/(a^2 - 2*a*b + b^2)} * d*x * \operatorname{dilog}(-1/2*((2*(a + b)*\cos(d \\
& *x + c) - (-2*I*a - 2*I*b)*\sin(d*x + c) + 4*((a - b)*\cos(d*x + c) - (-I*a + \\
& I*b)*\sin(d*x + c)) * \sqrt{a*b/(a^2 - 2*a*b + b^2)}) * \sqrt{((2*(a - b)*\sqrt{a*b \\
& / (a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(a - b) + 1) + 4*I*(a \\
& - b) * \sqrt{a*b/(a^2 - 2*a*b + b^2)} * c^2 * \log(2*\sqrt{-(2*(a - b)*\sqrt{a*b/(a^2 \\
& - 2*a*b + b^2)) + a + b)/(a - b)) + 2*\cos(d*x + c) + 2*I*\sin(d*x + c)) - 4 \\
& *I*(a - b) * \sqrt{a*b/(a^2 - 2*a*b + b^2)} * c^2 * \log(2*\sqrt{-(2*(a - b)*\sqrt{a* \\
& b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*\cos(d*x + c) - 2*I*\sin(d*x + c \\
&)) - 4*I*(a - b) * \sqrt{a*b/(a^2 - 2*a*b + b^2)} * c^2 * \log(2*\sqrt{-(2*(a - b)*\sqrt{ \\
& a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*\cos(d*x + c) + 2*I*\sin(d \\
& *x + c)) + 4*I*(a - b) * \sqrt{a*b/(a^2 - 2*a*b + b^2)} * c^2 * \log(2*\sqrt{-(2*(a \\
& - b)*\sqrt{a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*\cos(d*x + c) - 2*I \\
& *\sin(d*x + c)) - 4*I*(a - b) * \sqrt{a*b/(a^2 - 2*a*b + b^2)} * c^2 * \log(2*\sqrt{((\\
& 2*(a - b)*\sqrt{a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*\cos(d*x + c) \\
& + 2*I*\sin(d*x + c)) + 4*I*(a - b) * \sqrt{a*b/(a^2 - 2*a*b + b^2)} * c^2 * \log(2*\sqrt{ \\
& a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*\cos(d*x + c) + 2*I*\sin(d*x \\
& + c) - 2*I*\sin(d*x + c)) + 4*I*(a - b) * \sqrt{a*b/(a^2 - 2*a*b + b^2)} * c^2 * \log \\
& (2*\sqrt{((2*(a - b)*\sqrt{a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*\cos \\
& (d*x + c) + 2*I*\sin(d*x + c)) - 4*I*(a - b) * \sqrt{a*b/(a^2 - 2*a*b + b^2)} * c \\
& ^2 * \log(2*\sqrt{((2*(a - b)*\sqrt{a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - \\
& 2*\cos(d*x + c) - 2*I*\sin(d*x + c)) + 4*(I*(a - b)*d^2*x^2 - I*(a - b)*c^2) * \\
& \sqrt{a*b/(a^2 - 2*a*b + b^2)} * \log(-1/2*((2*(a + b)*\cos(d*x + c) + (2*I*a + \\
& 2*I*b)*\sin(d*x + c) - 4*((a - b)*\cos(d*x + c) - (-I*a + I*b)*\sin(d*x + c)) * \\
& \sqrt{a*b/(a^2 - 2*a*b + b^2)}) * \sqrt{-(2*(a - b)*\sqrt{a*b/(a^2 - 2*a*b + b^2} \\
&)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(-I*(a - b)*d^2*x^2 + I*(a - \\
& b)*c^2) * \sqrt{a*b/(a^2 - 2*a*b + b^2)} * \log(1/2*((2*(a + b)*\cos(d*x + c) - (\\
& 2*I*a + 2*I*b)*\sin(d*x + c) - 4*((a - b)*\cos(d*x + c) + (-I*a + I*b)*\sin(d* \\
& x + c)) * \sqrt{a*b/(a^2 - 2*a*b + b^2)}) * \sqrt{-(2*(a - b)*\sqrt{a*b/(a^2 - 2*a \\
& *b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*(a - b)*d^2*x^2 \\
& + I*(a - b)*c^2) * \sqrt{a*b/(a^2 - 2*a*b + b^2)} * \log(-1/2*((2*(a + b)*\cos(d*x \\
& + c) + (-2*I*a - 2*I*b)*\sin(d*x + c) - 4*((a - b)*\cos(d*x + c) - (I*a - I* \\
& b)*\sin(d*x + c)) * \sqrt{a*b/(a^2 - 2*a*b + b^2)}) * \sqrt{-(2*(a - b)*\sqrt{a*b/(\\
& a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*a + 2*b)/(a - b)) + 4*(I*(a - b)* \\
& d^2*x^2 - I*(a - b)*c^2) * \sqrt{a*b/(a^2 - 2*a*b + b^2)} * \log(1/2*((2*(a + b)* \\
& \cos(d*x + c) - (-2*I*a - 2*I*b)*\sin(d*x + c) - 4*((a - b)*\cos(d*x + c) + (I \\
& *a - I*b)*\sin(d*x + c)) * \sqrt{a*b/(a^2 - 2*a*b + b^2)}) * \sqrt{-(2*(a - b)*\sqrt{
\end{aligned}$$

```

t(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*a - 2*b)/(a - b)) + 4*(-I*
(a - b)*d^2*x^2 + I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-1/2*((2
*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x +
c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a -
b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*b)/(a - b)) +
4*(I*(a - b)*d^2*x^2 - I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(1/
2*((2*(a + b)*cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(
d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2
*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*a - 2*b)/(a -
b)) + 4*(I*(a - b)*d^2*x^2 - I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*l
og(-1/2*((2*(a + b)*cos(d*x + c) + (-2*I*a - 2*I*b)*sin(d*x + c) + 4*((a -
b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))
*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*a + 2*
b)/(a - b)) + 4*(-I*(a - b)*d^2*x^2 + I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b
+ b^2))*log(1/2*((2*(a + b)*cos(d*x + c) - (-2*I*a - 2*I*b)*sin(d*x + c) +
4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b
+ b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) +
2*a - 2*b)/(a - b)) + 4*(2*I*a - 2*I*b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*polyl
og(3, -1/2*(2*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a -
b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))
)*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b))/(a - b)
+ 4*(-2*I*a + 2*I*b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*polylog(3, 1/2*(2*(a +
b)*cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) +
(-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)
*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b))/(a - b)) + 4*(-2*I*a + 2*I
b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*polylog(3, -1/2*(2*(a + b)*cos(d*x + c) +
(-2*I*a - 2*I*b)*sin(d*x + c) - 4*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(
d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2
*a*b + b^2)) + a + b)/(a - b))/(a - b)) + 4*(2*I*a - 2*I*b)*sqrt(a*b/(a^2 -
2*a*b + b^2))*polylog(3, 1/2*(2*(a + b)*cos(d*x + c) - (-2*I*a - 2*I*b)*sin
(d*x + c) - 4*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a
^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)
/(a - b))/(a - b)) + 4*(-2*I*a + 2*I*b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*polyl
og(3, -1/2*(2*(a + b)*cos(d*x + c) + (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a -
b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))
*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b))/(a - b)) +
4*(2*I*a - 2*I*b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*polylog(3, 1/2*(2*(a + b)*
cos(d*x + c) - (2*I*a + 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (I*
a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(
a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b))/(a - b)) + 4*(2*I*a - 2*I*b)*sq
r t(a*b/(a^2 - 2*a*b + b^2))*polylog(3, -1/2*(2*(a + b)*cos(d*x + c) + (-2*I*
a - 2*I*b)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x +
c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b +
b^2)) - a - b)/(a - b))/(a - b)) + 4*(-2*I*a + 2*I*b)*sqrt(a*b/(a^2 - 2*a*b
+ b^2))*polylog(3, 1/2*(2*(a + b)*cos(d*x + c) - (-2*I*a - 2*I*b)*sin(d*x
+ c) + 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 -
2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a -
b)))/(a - b)))/(a*b*d^3)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sec(dx + c)^2}{b \tan(dx + c)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")

[Out] integrate(x^2*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)

maple [B] time = 0.46, size = 1251, normalized size = 3.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x)

[Out] $I/d^3*c^2/(a*b)^{(1/2)}*\operatorname{arctanh}(1/4*(2*(a-b)*\exp(2*I*(d*x+c))+2*a+2*b)/(a*b)^{(1/2)})-I/d/(-2*(a*b)^{(1/2)}-a-b)*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(-2*(a*b)^{(1/2)}-a-b))*x^2+2/3/d^3/(a*b)^{(1/2)}/(-2*(a*b)^{(1/2)}-a-b)*a*c^3+2/3/d^3/(a*b)^{(1/2)}/(-2*(a*b)^{(1/2)}-a-b)*b*c^3+I/d^3/(-2*(a*b)^{(1/2)}-a-b)*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(-2*(a*b)^{(1/2)}-a-b))*c^2+1/2*I/d^3/(a*b)^{(1/2)}*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(2*(a*b)^{(1/2)}-a-b))*c^2-1/2*I/d/(a*b)^{(1/2)}*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(2*(a*b)^{(1/2)}-a-b))*x^2+2/3/d^3/(a*b)^{(1/2)}*c^3+4/3/d^3/(-2*(a*b)^{(1/2)}-a-b)*c^3+1/d^2/(a*b)^{(1/2)}*c^2*x+2/d^2/(-2*(a*b)^{(1/2)}-a-b)*c^2*x-1/2/d^2/(a*b)^{(1/2)}*\operatorname{polylog}(2,(a-b)*\exp(2*I*(d*x+c)))/(2*(a*b)^{(1/2)}-a-b))*x-1/d^2/(-2*(a*b)^{(1/2)}-a-b)*\operatorname{polylog}(2,(a-b)*\exp(2*I*(d*x+c)))/(-2*(a*b)^{(1/2)}-a-b))*x-1/3/(a*b)^{(1/2)}/(-2*(a*b)^{(1/2)}-a-b)*b*x^3-1/3/(a*b)^{(1/2)}/(-2*(a*b)^{(1/2)}-a-b)*a*x^3-1/4*I/d^3/(a*b)^{(1/2)}*\operatorname{polylog}(3,(a-b)*\exp(2*I*(d*x+c)))/(2*(a*b)^{(1/2)}-a-b))-1/2*I/d^3/(-2*(a*b)^{(1/2)}-a-b)*\operatorname{polylog}(3,(a-b)*\exp(2*I*(d*x+c)))/(-2*(a*b)^{(1/2)}-a-b))+1/2*I/d^3/(a*b)^{(1/2)}/(-2*(a*b)^{(1/2)}-a-b)*b*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(-2*(a*b)^{(1/2)}-a-b))*c^2-1/2*I/d/(a*b)^{(1/2)}/(-2*(a*b)^{(1/2)}-a-b)*a*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(-2*(a*b)^{(1/2)}-a-b))*x^2-1/2*I/d/(a*b)^{(1/2)}/(-2*(a*b)^{(1/2)}-a-b)*b*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(-2*(a*b)^{(1/2)}-a-b))*x^2-1/3/(a*b)^{(1/2)}*x^3-2/3/(-2*(a*b)^{(1/2)}-a-b)*x^3+1/d^2/(a*b)^{(1/2)}/(-2*(a*b)^{(1/2)}-a-b)*a*c^2*x+1/d^2/(a*b)^{(1/2)}/(-2*(a*b)^{(1/2)}-a-b)*b*c^2*x-1/2/d^2/(a*b)^{(1/2)}/(-2*(a*b)^{(1/2)}-a-b)*b*\operatorname{polylog}(2,(a-b)*\exp(2*I*(d*x+c)))/(-2*(a*b)^{(1/2)}-a-b))*x-1/2/d^2/(a*b)^{(1/2)}/(-2*(a*b)^{(1/2)}-a-b)*a*\operatorname{polylog}(2,(a-b)*\exp(2*I*(d*x+c)))/(-2*(a*b)^{(1/2)}-a-b))*x-1/4*I/d^3/(a*b)^{(1/2)}/(-2*(a*b)^{(1/2)}-a-b)*a*\operatorname{polylog}(3,(a-b)*\exp(2*I*(d*x+c)))/(-2*(a*b)^{(1/2)}-a-b))-1/4*I/d^3/(a*b)^{(1/2)}/(-2*(a*b)^{(1/2)}-a-b)*b*\operatorname{polylog}(3,(a-b)*\exp(2*I*(d*x+c)))/(-2*(a*b)^{(1/2)}-a-b))+1/2*I/d^3/(a*b)^{(1/2)}/(-2*(a*b)^{(1/2)}-a-b)*a*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(-2*(a*b)^{(1/2)}-a-b))*c^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sec(dx+c)^2}{b \tan(dx+c)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")

[Out] integrate(x^2*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\cos(c+dx)^2 (b \tan(c+dx)^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(cos(c+d*x)^2*(a+b*tan(c+d*x)^2)),x)

[Out] int(x^2/(cos(c+d*x)^2*(a+b*tan(c+d*x)^2)),x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sec^2(c+dx)}{a+b \tan^2(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sec(d*x+c)**2/(a+b*tan(d*x+c)**2), x)
```

```
[Out] Integral(x**2*sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)
```

$$3.162 \quad \int \frac{\sec^2(c+dx)}{a+c \sec^2(c+dx)+b \tan^2(c+dx)} dx$$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+c} \tan(c+dx)}{\sqrt{a+c}}\right)}{d\sqrt{a+c} \sqrt{b+c}}$$

[Out] arctan((b+c)^(1/2)*tan(d*x+c)/(a+c)^(1/2))/d/(a+c)^(1/2)/(b+c)^(1/2)

Rubi [A] time = 0.61, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+c} \tan(c+dx)}{\sqrt{a+c}}\right)}{d\sqrt{a+c} \sqrt{b+c}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^2/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2), x]

[Out] ArcTan[(Sqrt[b + c]*Tan[c + d*x])/Sqrt[a + c]]/(Sqrt[a + c]*Sqrt[b + c]*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(c+dx)}{a+c \sec^2(c+dx)+b \tan^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+c+(b+c)x^2} dx, x, \tan(c+dx)\right)}{d} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b+c} \tan(c+dx)}{\sqrt{a+c}}\right)}{\sqrt{a+c} \sqrt{b+c} d} \end{aligned}$$

Mathematica [A] time = 0.25, size = 40, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b+c} \tan(c+dx)}{\sqrt{a+c}}\right)}{d\sqrt{a+c} \sqrt{b+c}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^2/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2), x]

[Out] ArcTan[(Sqrt[b + c]*Tan[c + d*x])/Sqrt[a + c]]/(Sqrt[a + c]*Sqrt[b + c]*d)

fricas [B] time = 0.75, size = 300, normalized size = 7.50

$$\left[\frac{\sqrt{-ab - (a+b)c - c^2} \log\left(\frac{(a^2+6ab+b^2+8(a+b)c+8c^2) \cos(dx+c)^4 - 2(3ab+b^2+(3a+5b)c+4c^2) \cos(dx+c)^2 + 4((a+b+2c) \cos(dx+c))^3}{(a^2-2ab+b^2) \cos(dx+c)^4 + 2(ab-b^2+(a-b)c) \cos(dx+c)^2 + b}\right)}{4(ab + (a+b)c + c^2)d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="fricas")

[Out] [-1/4*sqrt(-a*b - (a + b)*c - c^2)*log(((a^2 + 6*a*b + b^2 + 8*(a + b)*c + 8*c^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2 + (3*a + 5*b)*c + 4*c^2)*cos(d*x + c)^2 + 4*((a + b + 2*c)*cos(d*x + c)^3 - (b + c)*cos(d*x + c))*sqrt(-a*b - (a + b)*c - c^2)*sin(d*x + c) + b^2 + 2*b*c + c^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2 + (a - b)*c)*cos(d*x + c)^2 + b^2 + 2*b*c + c^2))/((a*b + (a + b)*c + c^2)*d), -1/2*arctan(1/2*((a + b + 2*c)*cos(d*x + c)^2 - b - c)/(sqrt(a*b + (a + b)*c + c^2)*cos(d*x + c)*sin(d*x + c)))/(sqrt(a*b + (a + b)*c + c^2)*d)]

giac [B] time = 0.97, size = 76, normalized size = 1.90

$$\frac{\pi \left[\frac{dx+c}{\pi} + \frac{1}{2} \right] \operatorname{sgn}(2b+2c) + \arctan\left(\frac{b \tan(dx+c) + c \tan(dx+c)}{\sqrt{ab+ac+bc+c^2}}\right)}{\sqrt{ab+ac+bc+c^2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="giac")

[Out] (pi*floor((d*x + c)/pi + 1/2)*sgn(2*b + 2*c) + arctan((b*tan(d*x + c) + c*tan(d*x + c))/sqrt(a*b + a*c + b*c + c^2)))/(sqrt(a*b + a*c + b*c + c^2)*d)

maple [A] time = 0.36, size = 34, normalized size = 0.85

$$\frac{\arctan\left(\frac{(b+c) \tan(dx+c)}{\sqrt{(a+c)(b+c)}}\right)}{d \sqrt{(a+c)(b+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x)

[Out] 1/d/((a+c)*(b+c))^(1/2)*arctan((b+c)*tan(d*x+c)/((a+c)*(b+c))^(1/2))

maxima [A] time = 1.07, size = 43, normalized size = 1.08

$$\frac{\arctan\left(\frac{(b+c) \tan(dx+c)}{\sqrt{ab+(a+b)c+c^2}}\right)}{\sqrt{ab+(a+b)c+c^2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="maxima")

[Out] arctan((b + c)*tan(d*x + c)/sqrt(a*b + (a + b)*c + c^2))/(sqrt(a*b + (a + b)*c + c^2)*d)

mupad [B] time = 2.60, size = 45, normalized size = 1.12

$$\frac{\operatorname{atan}\left(\frac{\tan(c+dx)(b+c)}{\sqrt{ab+ac+bc+c^2}}\right)}{d \sqrt{ab+ac+bc+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)),x)

[Out] atan((tan(c + d*x)*(b + c))/(a*b + a*c + b*c + c^2)^(1/2))/(d*(a*b + a*c + b*c + c^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c + dx)}{a + b \tan^2(c + dx) + c \sec^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+c*sec(d*x+c)**2+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2 + c*sec(c + d*x)**2), x)
```

$$3.163 \quad \int \frac{x \sec^2(c+dx)}{a+c \sec^2(c+dx)+b \tan^2(c+dx)} dx$$

Optimal. Leaf size=267

$$\frac{\operatorname{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{4d^2\sqrt{a+c}\sqrt{b+c}} + \frac{\operatorname{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2(c+\sqrt{a+c}\sqrt{b+c})}\right)}{4d^2\sqrt{a+c}\sqrt{b+c}} - \frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{2d\sqrt{a+c}\sqrt{b+c}} + \frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{2(\sqrt{a+c}\sqrt{b+c}+a+b+2c)}\right)}{2d\sqrt{a+c}\sqrt{b+c}}$$

[Out] $-1/2*I*x*\ln(1+(a-b)*\exp(2*I*(d*x+c))/(a+b+2*c-2*(a+c)^{(1/2)}*(b+c)^{(1/2)}))/d$
 $/((a+c)^{(1/2)}*(b+c)^{(1/2)}+1/2*I*x*\ln(1+(a-b)*\exp(2*I*(d*x+c))/(a+b+2*c+2*(a+c)^{(1/2)}*(b+c)^{(1/2)}))/d$
 $/(a+c)^{(1/2)}*(b+c)^{(1/2)}-1/4*\operatorname{polylog}(2,-(a-b)*\exp(2*I*(d*x+c))/(a+b+2*c-2*(a+c)^{(1/2)}*(b+c)^{(1/2)}))/d^2$
 $/(a+c)^{(1/2)}*(b+c)^{(1/2)}+1/4*\operatorname{polylog}(2,-(a-b)*\exp(2*I*(d*x+c))/(a+b+2*c+2*(a+c)^{(1/2)}*(b+c)^{(1/2)}))/d^2$
 $/(a+c)^{(1/2)}*(b+c)^{(1/2)}$

Rubi [A] time = 0.72, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 34, number of rules / integrand size = 0.176, Rules used = {4589, 3321, 2264, 2190, 2279, 2391}

$$\frac{\operatorname{PolyLog}\left(2, -\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{4d^2\sqrt{a+c}\sqrt{b+c}} + \frac{\operatorname{PolyLog}\left(2, -\frac{(a-b)e^{2i(c+dx)}}{2(\sqrt{a+c}\sqrt{b+c}+c)+a+b}\right)}{4d^2\sqrt{a+c}\sqrt{b+c}} - \frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{2d\sqrt{a+c}\sqrt{b+c}} + \frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{2(\sqrt{a+c}\sqrt{b+c}+c)+a+b}\right)}{2d\sqrt{a+c}\sqrt{b+c}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x*\operatorname{Sec}[c+d*x]^2)/(a+c*\operatorname{Sec}[c+d*x]^2+b*\operatorname{Tan}[c+d*x]^2), x]$

[Out] $((-1/2)*x*\operatorname{Log}[1+((a-b)*E^{((2*I)*(c+d*x))})/(a+b+2*c-2*\operatorname{Sqrt}[a+c]*\operatorname{Sqrt}[b+c])])/(d*\operatorname{Sqrt}[a+c]*\operatorname{Sqrt}[b+c]) + ((1/2)*x*\operatorname{Log}[1+((a-b)*E^{((2*I)*(c+d*x))})/(a+b+2*(c+\operatorname{Sqrt}[a+c]*\operatorname{Sqrt}[b+c])])]/(d*\operatorname{Sqrt}[a+c]*\operatorname{Sqrt}[b+c]) - \operatorname{PolyLog}[2, -((a-b)*E^{((2*I)*(c+d*x))})/(a+b+2*c-2*\operatorname{Sqrt}[a+c]*\operatorname{Sqrt}[b+c])])/(4*d*\operatorname{Sqrt}[a+c]*\operatorname{Sqrt}[b+c]*d^2) + \operatorname{PolyLog}[2, -((a-b)*E^{((2*I)*(c+d*x))})/(a+b+2*(c+\operatorname{Sqrt}[a+c]*\operatorname{Sqrt}[b+c])])]/(4*d*\operatorname{Sqrt}[a+c]*\operatorname{Sqrt}[b+c]*d^2)$

Rule 2190

$\operatorname{Int}[(((F_)^{((g_*)*((e_*)+(f_*)*(x_)))})^{(n_*)*((c_*)+(d_*)*(x_))^{(m_*)})}/((a_*)+(b_*)*((F_)^{((g_*)*((e_*)+(f_*)*(x_)))})^{(n_*)}), x_Symbol] :> \operatorname{Simp}[(c+d*x)^m*\operatorname{Log}[1+(b*(F^{(g*(e+f*x)))})^n/a]/(b*f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(b*f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c+d*x)^{(m-1)}*\operatorname{Log}[1+(b*(F^{(g*(e+f*x)))})^n/a], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \operatorname{IGtQ}[m, 0]$

Rule 2264

$\operatorname{Int}[((F_)^{(u_*)*((f_*)+(g_*)*(x_))^{(m_*)})}/((a_*)+(b_*)*(F_)^{(u_*)}+(c_*)*(F_)^{(v_*)}), x_Symbol] :> \operatorname{With}\{q = \operatorname{Rt}[b^2-4*a*c, 2]\}, \operatorname{Dist}[(2*c)/q, \operatorname{Int}[(f+g*x)^m*(F^u)/(b-q+2*c*(F^u)), x], x] - \operatorname{Dist}[(2*c)/q, \operatorname{Int}[(f+g*x)^m*(F^u)/(b+q+2*c*(F^u)), x], x] /; \operatorname{FreeQ}\{F, a, b, c, f, g\}, x] \&\& \operatorname{EqQ}[v, 2*u] \&\& \operatorname{LinearQ}[u, x] \&\& \operatorname{NeQ}[b^2-4*a*c, 0] \&\& \operatorname{IGtQ}[m, 0]$

Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_*)+(b_*)*((F_)^{((e_*)*((c_*)+(d_*)*(x_)))})^{(n_*)}], x_Symbol] :> \operatorname{Dist}[1/(d*e*n*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a+b*x]/x], x, (F^{(e*(c+d*x))})^n], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \operatorname{GtQ}[a, 0]$

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 3321

Int[((c_.) + (d_.)*(x_)^(m_.))/((a_.) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 4589

Int[(((f_.) + (g_.)*(x_)^(m_.))*Sec[(d_.) + (e_.)*(x_)^2]/((b_.) + (a_.)*Sec[(d_.) + (e_.)*(x_)^2 + (c_.)*Tan[(d_.) + (e_.)*(x_)^2]), x_Symbol] := Dist[2, Int[(f + g*x)^m/(2*a + b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c, 0]

Rubi steps

$$\begin{aligned} \int \frac{x \sec^2(c + dx)}{a + c \sec^2(c + dx) + b \tan^2(c + dx)} dx &= 2 \int \frac{x}{a + b + 2c + (a - b) \cos(2c + 2dx)} dx \\ &= 4 \int \frac{e^{i(2c+2dx)} x}{a - b + 2(a + b + 2c)e^{i(2c+2dx)} + (a - b)e^{2i(2c+2dx)}} dx \\ &= \frac{(2(a - b)) \int \frac{e^{i(2c+2dx)} x}{-4\sqrt{a+c}\sqrt{b+c} + 2(a+b+2c) + 2(a-b)e^{i(2c+2dx)}} dx}{\sqrt{a+c}\sqrt{b+c}} - \frac{(2(a - b)) \int \frac{e^{i(2c+2dx)} x}{4\sqrt{a+c}\sqrt{b+c} + 2(a+b+2c) + 2(a-b)e^{i(2c+2dx)}} dx}{\sqrt{a+c}\sqrt{b+c}} \\ &= -\frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}d} + \frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{a+b+2(c+\sqrt{a+c}\sqrt{b+c})}\right)}{2\sqrt{a+c}\sqrt{b+c}d} \\ &= -\frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}d} + \frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{a+b+2(c+\sqrt{a+c}\sqrt{b+c})}\right)}{2\sqrt{a+c}\sqrt{b+c}d} \\ &= -\frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}d} + \frac{ix \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{a+b+2(c+\sqrt{a+c}\sqrt{b+c})}\right)}{2\sqrt{a+c}\sqrt{b+c}d} \end{aligned}$$

Mathematica [B] time = 4.23, size = 751, normalized size = 2.81

$$x(\sqrt{a+c} - \sqrt{-b-c} \tan(c+dx))(\sqrt{a+c} + \sqrt{-b-c} \tan(c+dx)) \left(i\sqrt{b+c} \operatorname{Li}_2\left(\frac{\sqrt{-b-c}(1-i \tan(c+dx))}{\sqrt{-b-c}-i\sqrt{a+c}}\right) - i\sqrt{b+c} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2), x]

[Out] (x*(4*Sqrt[-b - c]*c*ArcTan[(Sqrt[b + c]*Tan[c + d*x])/Sqrt[a + c]] - I*Sqrt[b + c]*Log[1 + I*Tan[c + d*x]]*Log[(I*(Sqrt[a + c] - Sqrt[-b - c]*Tan[c + d*x]))/(Sqrt[-b - c] + I*Sqrt[a + c])] + I*Sqrt[b + c]*Log[1 - I*Tan[c + d*x]]*Log[(I*(-Sqrt[a + c] + Sqrt[-b - c]*Tan[c + d*x]))/(Sqrt[-b - c] - I*S

$$\begin{aligned} & \text{qrt}[a + c]] + I*\text{Sqrt}[b + c]*\text{Log}[1 + I*\text{Tan}[c + d*x]]*\text{Log}[((-I)*(\text{Sqrt}[a + c] \\ & + \text{Sqrt}[-b - c]*\text{Tan}[c + d*x]))/(\text{Sqrt}[-b - c] - I*\text{Sqrt}[a + c])] - I*\text{Sqrt}[b + \\ & c]*\text{Log}[1 - I*\text{Tan}[c + d*x]]*\text{Log}[(I*(\text{Sqrt}[a + c] + \text{Sqrt}[-b - c]*\text{Tan}[c + d*x] \\ &))/(\text{Sqrt}[-b - c] + I*\text{Sqrt}[a + c])] + I*\text{Sqrt}[b + c]*\text{PolyLog}[2, (\text{Sqrt}[-b - c] \\ & *(1 - I*\text{Tan}[c + d*x]))/(\text{Sqrt}[-b - c] - I*\text{Sqrt}[a + c])] - I*\text{Sqrt}[b + c]*\text{Poly} \\ & \text{Log}[2, (\text{Sqrt}[-b - c]*(1 - I*\text{Tan}[c + d*x]))/(\text{Sqrt}[-b - c] + I*\text{Sqrt}[a + c])] \\ & + I*\text{Sqrt}[b + c]*\text{PolyLog}[2, (\text{Sqrt}[-b - c]*(1 + I*\text{Tan}[c + d*x]))/(\text{Sqrt}[-b - c] \\ & - I*\text{Sqrt}[a + c])] - I*\text{Sqrt}[b + c]*\text{PolyLog}[2, (\text{Sqrt}[-b - c]*(1 + I*\text{Tan}[c + \\ & d*x]))/(\text{Sqrt}[-b - c] + I*\text{Sqrt}[a + c])]*(\text{Sqrt}[a + c] - \text{Sqrt}[-b - c]*\text{Tan}[c \\ & + d*x])*(\text{Sqrt}[a + c] + \text{Sqrt}[-b - c]*\text{Tan}[c + d*x])/(2*\text{Sqrt}[a + c]*\text{Sqrt}[-(b \\ & + c)^2]*d*(2*c - I*\text{Log}[1 - I*\text{Tan}[c + d*x]] + I*\text{Log}[1 + I*\text{Tan}[c + d*x]])*(a \\ & + c*\text{Sec}[c + d*x]^2 + b*\text{Tan}[c + d*x]^2)) \end{aligned}$$

fricas [B] time = 1.38, size = 4160, normalized size = 15.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(d*x+c)^2/(a*c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/16*(-4*I*(a - b)*c*\text{sqrt}((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*\text{log}(\\ & 2*\text{sqrt}(-(2*(a - b)*\text{sqrt}((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + \\ & b + 2*c)/(a - b)) + 2*\text{cos}(d*x + c) + 2*I*\text{sin}(d*x + c)) + 4*I*(a - b)*c*\text{sqrt} \\ & ((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*\text{log}(2*\text{sqrt}(-(2*(a - b)*\text{sqrt}((\\ & a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*\text{cos} \\ & (d*x + c) - 2*I*\text{sin}(d*x + c)) + 4*I*(a - b)*c*\text{sqrt}((a*b + (a + b)*c + c^2)/ \\ & (a^2 - 2*a*b + b^2))*\text{log}(2*\text{sqrt}(-(2*(a - b)*\text{sqrt}((a*b + (a + b)*c + c^2)/(a \\ & ^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*\text{cos}(d*x + c) + 2*I*\text{sin}(d*x + \\ & c)) - 4*I*(a - b)*c*\text{sqrt}((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*\text{log}(\\ & 2*\text{sqrt}(-(2*(a - b)*\text{sqrt}((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + \\ & b + 2*c)/(a - b)) - 2*\text{cos}(d*x + c) - 2*I*\text{sin}(d*x + c)) + 4*I*(a - b)*c*\text{sqrt} \\ & ((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*\text{log}(2*\text{sqrt}((2*(a - b)*\text{sqrt}((a \\ & *b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*\text{cos}(\\ & d*x + c) + 2*I*\text{sin}(d*x + c)) - 4*I*(a - b)*c*\text{sqrt}((a*b + (a + b)*c + c^2)/(\\ & a^2 - 2*a*b + b^2))*\text{log}(2*\text{sqrt}((2*(a - b)*\text{sqrt}((a*b + (a + b)*c + c^2)/(a^2 \\ & - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*\text{cos}(d*x + c) - 2*I*\text{sin}(d*x + c \\ &)) - 4*I*(a - b)*c*\text{sqrt}((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*\text{log}(2* \\ & \text{sqrt}((2*(a - b)*\text{sqrt}((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - \\ & 2*c)/(a - b)) - 2*\text{cos}(d*x + c) + 2*I*\text{sin}(d*x + c)) + 4*I*(a - b)*c*\text{sqrt}((a \\ & *b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*\text{log}(2*\text{sqrt}((2*(a - b)*\text{sqrt}((a*b \\ & + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) - 2*\text{cos}(d*x \\ & + c) - 2*I*\text{sin}(d*x + c)) + 4*(a - b)*\text{sqrt}((a*b + (a + b)*c + c^2)/(a^2 - 2 \\ & *a*b + b^2))*\text{dilog}(1/2*((2*(a + b + 2*c)*\text{cos}(d*x + c) + (2*I*a + 2*I*b + 4* \\ & I*c)*\text{sin}(d*x + c) - 4*((a - b)*\text{cos}(d*x + c) - (-I*a + I*b)*\text{sin}(d*x + c))*\text{sq} \\ & \text{rt}((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*\text{sqrt}(-(2*(a - b)*\text{sqrt}((a*b \\ & + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*a + 2* \\ & b)/(a - b) + 1) + 4*(a - b)*\text{sqrt}((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2 \\ &))*\text{dilog}(-1/2*((2*(a + b + 2*c)*\text{cos}(d*x + c) - (2*I*a + 2*I*b + 4*I*c)*\text{sin} \\ & (d*x + c) - 4*((a - b)*\text{cos}(d*x + c) + (-I*a + I*b)*\text{sin}(d*x + c))*\text{sqrt}((a*b + \\ & (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*\text{sqrt}(-(2*(a - b)*\text{sqrt}((a*b + (a + b) \\ &)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*a - 2*b)/(a - b \\ &) + 1) + 4*(a - b)*\text{sqrt}((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*\text{dilog} \\ & (1/2*((2*(a + b + 2*c)*\text{cos}(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*\text{sin}(d*x + c) \\ & - 4*((a - b)*\text{cos}(d*x + c) - (I*a - I*b)*\text{sin}(d*x + c))*\text{sqrt}((a*b + (a + b)*c \\ & + c^2)/(a^2 - 2*a*b + b^2)))*\text{sqrt}(-(2*(a - b)*\text{sqrt}((a*b + (a + b)*c + c^2) \\ & / (a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*a + 2*b)/(a - b) + 1) + 4 \\ & *(a - b)*\text{sqrt}((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*\text{dilog}(-1/2*((2*(\\ & a + b + 2*c)*\text{cos}(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*\text{sin}(d*x + c) - 4*((a - \\ & b)*\text{cos}(d*x + c) + (I*a - I*b)*\text{sin}(d*x + c))*\text{sqrt}((a*b + (a + b)*c + c^2)/(\end{aligned}$$

$$\begin{aligned}
& a^2 - 2ab + b^2)) \sqrt{-(2(a-b)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} + a+b+2c)/(a-b)} + 2a-2b)/(a-b) + 1 - 4(a-b)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \\
& \sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \operatorname{dilog}(1/2((2(a+b+2c)\cos(dx+c) + (2Ia+2Ib+4Ic)\sin(dx+c) + 4((a-b)\cos(dx+c) + (Ia-Ib)\sin(dx+c)))\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \\
& \sqrt{(2(a-b)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} - a-b-2c)/(a-b) - 2a+2b)/(a-b) + 1 - 4(a-b)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \operatorname{dilog}(-1/2((2(a+b+2c)\cos(dx+c) - (2Ia+2Ib+4Ic)\sin(dx+c) + 4((a-b)\cos(dx+c) - (Ia-Ib)\sin(dx+c)))\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \\
& \sqrt{(2(a-b)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} - a-b-2c)/(a-b) + 2a-2b)/(a-b) + 1 - 4(a-b)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \operatorname{dilog}(1/2((2(a+b+2c)\cos(dx+c) + (-2Ia-2Ib-4Ic)\sin(dx+c) + 4((a-b)\cos(dx+c) + (-Ia+Ib)\sin(dx+c)))\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \\
& \sqrt{(2(a-b)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} - a-b-2c)/(a-b) - 2a+2b)/(a-b) + 1 - 4(a-b)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \operatorname{dilog}(-1/2((2(a+b+2c)\cos(dx+c) - (-2Ia-2Ib-4Ic)\sin(dx+c) + 4((a-b)\cos(dx+c) - (-Ia+Ib)\sin(dx+c)))\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \\
& \sqrt{(2(a-b)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} - a-b-2c)/(a-b) + 2a-2b)/(a-b) + 1 + 4(I(a-b)dx + I(a-b)c)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \log(-1/2((2(a+b+2c)\cos(dx+c) + (2Ia+2Ib+4Ic)\sin(dx+c) - 4((a-b)\cos(dx+c) - (-Ia+Ib)\sin(dx+c)))\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \\
& \sqrt{-(2(a-b)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} + a+b+2c)/(a-b) - 2a+2b)/(a-b) + 4(-I(a-b)dx - I(a-b)c)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \log(1/2((2(a+b+2c)\cos(dx+c) - (2Ia+2Ib+4Ic)\sin(dx+c) - 4((a-b)\cos(dx+c) + (-Ia+Ib)\sin(dx+c)))\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \\
& \sqrt{-(2(a-b)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} + a+b+2c)/(a-b) + 2a-2b)/(a-b) + 4(-I(a-b)dx - I(a-b)c)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \log(-1/2((2(a+b+2c)\cos(dx+c) + (-2Ia-2Ib-4Ic)\sin(dx+c) - 4((a-b)\cos(dx+c) - (Ia-Ib)\sin(dx+c)))\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \\
& \sqrt{-(2(a-b)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} + a+b+2c)/(a-b) - 2a+2b)/(a-b) + 4(I(a-b)dx + I(a-b)c)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \log(1/2((2(a+b+2c)\cos(dx+c) - (-2Ia-2Ib-4Ic)\sin(dx+c) - 4((a-b)\cos(dx+c) + (Ia-Ib)\sin(dx+c)))\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \\
& \sqrt{-(2(a-b)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} + a+b+2c)/(a-b) + 2a-2b)/(a-b) + 4(-I(a-b)dx - I(a-b)c)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \log(-1/2((2(a+b+2c)\cos(dx+c) + (2Ia+2Ib+4Ic)\sin(dx+c) + 4((a-b)\cos(dx+c) + (Ia-Ib)\sin(dx+c)))\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \\
& \sqrt{(2(a-b)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} - a-b-2c)/(a-b) - 2a+2b)/(a-b) + 4(I(a-b)dx + I(a-b)c)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \log(1/2((2(a+b+2c)\cos(dx+c) - (2Ia+2Ib+4Ic)\sin(dx+c) + 4((a-b)\cos(dx+c) - (Ia-Ib)\sin(dx+c)))\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \\
& \sqrt{(2(a-b)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} - a-b-2c)/(a-b) + 2a-2b)/(a-b) + 4(I(a-b)dx + I(a-b)c)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \log(-1/2((2(a+b+2c)\cos(dx+c) + (-2Ia-2Ib-4Ic)\sin(dx+c) + 4((a-b)\cos(dx+c) + (-Ia+Ib)\sin(dx+c)))\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \\
& \sqrt{(2(a-b)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} - a-b-2c)/(a-b) - 2a+2b)/(a-b) + 4(-I(a-b)dx - I(a-b)c)\sqrt{(ab+(a+b)c+c^2)/(a^2-2ab+b^2)} \\
& \log(1/2((2(a+b+2c)\cos(dx+c) - (-2Ia-2Ib-4Ic)\sin(dx+c)
\end{aligned}$$

+ c) + 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*a - 2*b)/(a - b))/((a*b + (a + b)*c + c^2)*d^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sec(dx + c)^2}{c \sec(dx + c)^2 + b \tan(dx + c)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="giac")

[Out] integrate(x*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)

maple [B] time = 0.60, size = 1670, normalized size = 6.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x)

[Out]
$$\begin{aligned} & -1/d/((a+c)*(b+c))^{1/2}/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c)*a*c*x-1/d/((a+c)*(b+c))^{1/2}/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c)*b*c*x-I/d^2/((a+c)*(b+c))^{1/2}/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c)*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*c^2-1/2*I/d^2/((a+c)*(b+c))^{1/2}/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c)*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*b*c-1/2*I/d/((a+c)*(b+c))^{1/2}/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c)*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*a*x-1/2*I/d/((a+c)*(b+c))^{1/2}/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c)*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*b*x-1/2*I/d^2/((a+c)*(b+c))^{1/2}*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(2*((a+c)*(b+c))^{1/2}-a-b-2*c))*c-I/d^2/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c)*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*c-1/2*I/d/((a+c)*(b+c))^{1/2}*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(2*((a+c)*(b+c))^{1/2}-a-b-2*c))*x-I/d/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c)*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*x-I/d^2*c/(a*b+a*c+b*c+c^2)^{(1/2)}*\operatorname{arctanh}(1/4*(2*(a-b)*\exp(2*I*(d*x+c))+2*a+2*b+4*c)/(a*b+a*c+b*c+c^2)^{(1/2)})-1/d/((a+c)*(b+c))^{1/2}*c*x-2/d/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c)*c*x-1/d^2/((a+c)*(b+c))^{1/2}/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*c^3-1/2/((a+c)*(b+c))^{1/2}/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*b*x^2-1/((a+c)*(b+c))^{1/2}/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*c*x^2-1/2/((a+c)*(b+c))^{1/2}/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*a*x^2-I/d/((a+c)*(b+c))^{1/2}/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*\ln(1-(a-b)*\exp(2*I*(d*x+c)))/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*c*x-1/2*I/d^2/((a+c)*(b+c))^{1/2}/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*a*c^2-1/2/d^2/((a+c)*(b+c))^{1/2}/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*b*c^2-1/4/d^2/((a+c)*(b+c))^{1/2}/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*polylog(2,(a-b)*\exp(2*I*(d*x+c)))/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*a-1/4/d^2/((a+c)*(b+c))^{1/2}/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*polylog(2,(a-b)*\exp(2*I*(d*x+c)))/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*b-1/2/d^2/((a+c)*(b+c))^{1/2}/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*polylog(2,(a-b)*\exp(2*I*(d*x+c)))/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*c-1/4/d^2/((a+c)*(b+c))^{1/2}*polylog(2,(a-b)*\exp(2*I*(d*x+c)))/(2*((a+c)*(b+c))^{1/2}-a-b-2*c))-1/2/d^2/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*polylog(2,(a-b)*\exp(2*I*(d*x+c)))/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))-1/2/d^2/((a+c)*(b+c))^{1/2}*c^2-1/d^2/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*c^2-1/2/((a+c)*(b+c))^{1/2}*x^2-1/(-2*((a+c)*(b+c))^{1/2}-a-b-2*c))*x^2 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sec(dx + c)^2}{c \sec(dx + c)^2 + b \tan(dx + c)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="maxima")

[Out] integrate(x*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\cos(c + dx)^2 \left(a + \frac{c}{\cos(c+dx)^2} + b \tan(c + dx)^2 \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)),x)

[Out] int(x/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sec^2(c + dx)}{a + b \tan^2(c + dx) + c \sec^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(d*x+c)**2/(a+c*sec(d*x+c)**2+b*tan(d*x+c)**2),x)

[Out] Integral(x*sec(c + d*x)**2/(a + b*tan(c + d*x)**2 + c*sec(c + d*x)**2), x)

$$3.164 \quad \int \frac{x^2 \sec^2(c+dx)}{a+c \sec^2(c+dx)+b \tan^2(c+dx)} dx$$

Optimal. Leaf size=407

$$-\frac{i\text{Li}_3\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{4d^3\sqrt{a+c}\sqrt{b+c}} + \frac{i\text{Li}_3\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2(c+\sqrt{a+c}\sqrt{b+c})}\right)}{4d^3\sqrt{a+c}\sqrt{b+c}} - \frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2d^2\sqrt{a+c}\sqrt{b+c}} + \frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2(c+\sqrt{a+c}\sqrt{b+c})}\right)}{2d^2\sqrt{a+c}\sqrt{b+c}}$$

[Out] $-1/2*I*x^2*\ln(1+(a-b)*\exp(2*I*(d*x+c))/(a+b+2*c-2*(a+c)^{(1/2)}*(b+c)^{(1/2)}))/d/(a+c)^{(1/2)}/(b+c)^{(1/2)}+1/2*I*x^2*\ln(1+(a-b)*\exp(2*I*(d*x+c))/(a+b+2*c+2*(a+c)^{(1/2)}*(b+c)^{(1/2)}))/d/(a+c)^{(1/2)}/(b+c)^{(1/2)}-1/2*x*\text{polylog}(2,-(a-b)*\exp(2*I*(d*x+c))/(a+b+2*c-2*(a+c)^{(1/2)}*(b+c)^{(1/2)}))/d^2/(a+c)^{(1/2)}/(b+c)^{(1/2)}+1/2*x*\text{polylog}(2,-(a-b)*\exp(2*I*(d*x+c))/(a+b+2*c+2*(a+c)^{(1/2)}*(b+c)^{(1/2)}))/d^2/(a+c)^{(1/2)}/(b+c)^{(1/2)}-1/4*I*\text{polylog}(3,-(a-b)*\exp(2*I*(d*x+c))/(a+b+2*c-2*(a+c)^{(1/2)}*(b+c)^{(1/2)}))/d^3/(a+c)^{(1/2)}/(b+c)^{(1/2)}+1/4*I*\text{polylog}(3,-(a-b)*\exp(2*I*(d*x+c))/(a+b+2*c+2*(a+c)^{(1/2)}*(b+c)^{(1/2)}))/d^3/(a+c)^{(1/2)}/(b+c)^{(1/2)}$

Rubi [A] time = 1.08, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4589, 3321, 2264, 2190, 2531, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{2d^2\sqrt{a+c}\sqrt{b+c}} + \frac{x\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{2(\sqrt{a+c}\sqrt{b+c}+c)+a+b}\right)}{2d^2\sqrt{a+c}\sqrt{b+c}} - \frac{i\text{PolyLog}\left(3,-\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{4d^3\sqrt{a+c}\sqrt{b+c}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]

[Out] $((-I/2)*x^2*\text{Log}[1 + ((a - b)*E^{((2*I)*(c + d*x))})/(a + b + 2*c - 2*\text{Sqrt}[a + c]*\text{Sqrt}[b + c])])/(\text{Sqrt}[a + c]*\text{Sqrt}[b + c]*d) + ((I/2)*x^2*\text{Log}[1 + ((a - b)*E^{((2*I)*(c + d*x))})/(a + b + 2*(c + \text{Sqrt}[a + c]*\text{Sqrt}[b + c])])])/(\text{Sqrt}[a + c]*\text{Sqrt}[b + c]*d) - (x*\text{PolyLog}[2, -(((a - b)*E^{((2*I)*(c + d*x))})/(a + b + 2*c - 2*\text{Sqrt}[a + c]*\text{Sqrt}[b + c])])]/(2*\text{Sqrt}[a + c]*\text{Sqrt}[b + c]*d^2) + (x*\text{PolyLog}[2, -(((a - b)*E^{((2*I)*(c + d*x))})/(a + b + 2*(c + \text{Sqrt}[a + c]*\text{Sqrt}[b + c])])])]/(2*\text{Sqrt}[a + c]*\text{Sqrt}[b + c]*d^2) - ((I/4)*\text{PolyLog}[3, -(((a - b)*E^{((2*I)*(c + d*x))})/(a + b + 2*c - 2*\text{Sqrt}[a + c]*\text{Sqrt}[b + c])])])/(\text{Sqrt}[a + c]*\text{Sqrt}[b + c]*d^3) + ((I/4)*\text{PolyLog}[3, -(((a - b)*E^{((2*I)*(c + d*x))})/(a + b + 2*(c + \text{Sqrt}[a + c]*\text{Sqrt}[b + c])])])])/(\text{Sqrt}[a + c]*\text{Sqrt}[b + c]*d^3)$

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2264

Int[(((F_)^(u_.)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_.) + (c_.)*(F_)^(v_.)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

Rule 2282


```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4589

```
Int((((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_.) + (a_.)*Sec[(d_.) + (e_.)*(x_)]^2 + (c_.)*Tan[(d_.) + (e_.)*(x_)]^2), x_Symbol] := Dist[2, Int[(f + g*x)^m/(2*a + b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \sec^2(c + dx)}{a + c \sec^2(c + dx) + b \tan^2(c + dx)} dx &= 2 \int \frac{x^2}{a + b + 2c + (a - b) \cos(2c + 2dx)} dx \\
&= 4 \int \frac{e^{i(2c+2dx)} x^2}{a - b + 2(a + b + 2c)e^{i(2c+2dx)} + (a - b)e^{2i(2c+2dx)}} dx \\
&= \frac{(2(a - b)) \int \frac{e^{i(2c+2dx)} x^2}{-4\sqrt{a+c} \sqrt{b+c} + 2(a+b+2c) + 2(a-b)e^{i(2c+2dx)}} dx}{\sqrt{a+c} \sqrt{b+c}} - \frac{(2(a - b)) \int \frac{e^{i(2c+2dx)} x^2}{4\sqrt{a+c} \sqrt{b+c}} dx}{\sqrt{a+c} \sqrt{b+c}} \\
&= -\frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c} \sqrt{b+c}}\right)}{2\sqrt{a+c} \sqrt{b+c} d} + \frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{a+b+2(c+\sqrt{a+c} \sqrt{b+c})}\right)}{2\sqrt{a+c} \sqrt{b+c} d} \\
&= -\frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c} \sqrt{b+c}}\right)}{2\sqrt{a+c} \sqrt{b+c} d} + \frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{a+b+2(c+\sqrt{a+c} \sqrt{b+c})}\right)}{2\sqrt{a+c} \sqrt{b+c} d} \\
&= -\frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c} \sqrt{b+c}}\right)}{2\sqrt{a+c} \sqrt{b+c} d} + \frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{a+b+2(c+\sqrt{a+c} \sqrt{b+c})}\right)}{2\sqrt{a+c} \sqrt{b+c} d} \\
&= -\frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c} \sqrt{b+c}}\right)}{2\sqrt{a+c} \sqrt{b+c} d} + \frac{ix^2 \log\left(1 + \frac{(a-b)e^{2i(c+dx)}}{a+b+2(c+\sqrt{a+c} \sqrt{b+c})}\right)}{2\sqrt{a+c} \sqrt{b+c} d}
\end{aligned}$$

Mathematica [A] time = 2.21, size = 499, normalized size = 1.23

$$ie^{2ic} \left(2d^2 x^2 \log\left(1 + \frac{(a-b)e^{2i(2c+dx)}}{-2\sqrt{e^{4ic}(a+c)(b+c)} + ae^{2ic} + be^{2ic} + 2ce^{2ic}}\right) - 2d^2 x^2 \log\left(1 + \frac{(a-b)e^{2i(2c+dx)}}{2\sqrt{e^{4ic}(a+c)(b+c)} + ae^{2ic} + be^{2ic} + 2ce^{2ic}}\right) - 2idx \operatorname{Li}_2\left(\frac{(a-b)e^{2i(2c+dx)}}{-2\sqrt{e^{4ic}(a+c)(b+c)} + ae^{2ic} + be^{2ic} + 2ce^{2ic}}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2), x]
[Out] ((-1/4*I)*E^((2*I)*c)*(2*d^2*x^2*Log[1 + ((a - b)*E^((2*I)*(2*c + d*x)))/(a
+E^((2*I)*c) + b*E^((2*I)*c) + 2*c*E^((2*I)*c) - 2*Sqrt[(a + c)*(b + c)*E^((
(4*I)*c)]]] - 2*d^2*x^2*Log[1 + ((a - b)*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)
*c) + b*E^((2*I)*c) + 2*c*E^((2*I)*c) + 2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)
])] - (2*I)*d*x*PolyLog[2, ((-a + b)*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c) +
b*E^((2*I)*c) + 2*c*E^((2*I)*c) - 2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)]]] +
(2*I)*d*x*PolyLog[2, ((-a + b)*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c) + b*E^
((2*I)*c) + 2*c*E^((2*I)*c) + 2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)]]] + PolyL
og[3, ((-a + b)*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c) + b*E^((2*I)*c) + 2*c
*E^((2*I)*c) - 2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)]]] - PolyLog[3, ((-a + b)
*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c) + b*E^((2*I)*c) + 2*c*E^((2*I)*c) +
2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)]])))/(d^3*Sqrt[(a + c)*(b + c)*E^((4*I)*
c)])
```

fricas [C] time = 1.28, size = 5772, normalized size = 14.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2), x, algorithm="
fricas")
```

```
[Out] 1/16*(8*(a - b)*d*x*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog
(1/2*((2*(a + b + 2*c)*cos(d*x + c) + (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c)
- 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*
c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2
)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*a + 2*b)/(a - b) + 1) +
8*(a - b)*d*x*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(-1/2*
((2*(a + b + 2*c)*cos(d*x + c) - (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) - 4*(
(a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c
^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^
2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*a - 2*b)/(a - b) + 1) + 8*(a
- b)*d*x*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a
+ b + 2*c)*cos(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a -
b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a
^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*
a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*a + 2*b)/(a - b) + 1) + 8*(a - b)*d
*x*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b
+ 2*c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a - b)*co
s(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 -
2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b +
b^2)) + a + b + 2*c)/(a - b)) + 2*a - 2*b)/(a - b) + 1) - 8*(a - b)*d*x*sq
rt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a + b + 2*c)
*cos(d*x + c) + (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x +
c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b +
b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) -
a - b - 2*c)/(a - b)) - 2*a + 2*b)/(a - b) + 1) - 8*(a - b)*d*x*sqrt((a*b
+ (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b + 2*c)*cos(d*
x + c) - (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (
I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))
*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b
- 2*c)/(a - b)) + 2*a - 2*b)/(a - b) + 1) - 8*(a - b)*d*x*sqrt((a*b + (a +
b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(1/2*((2*(a + b + 2*c)*cos(d*x + c) +
(-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) + (-I*a +
I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(
(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)
/(a - b)) - 2*a + 2*b)/(a - b) + 1) - 8*(a - b)*d*x*sqrt((a*b + (a + b)*c +
c^2)/(a^2 - 2*a*b + b^2))*dilog(-1/2*((2*(a + b + 2*c)*cos(d*x + c) - (-2*
I*a - 2*I*b - 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (-I*a + I*b)*
sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a
- b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a -
b)) + 2*a - 2*b)/(a - b) + 1) + 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^
2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)
)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d*
x + c)) - 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))
*log(2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) +
a + b + 2*c)/(a - b)) + 2*cos(d*x + c) - 2*I*sin(d*x + c)) - 4*I*(a - b)*c
^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)
*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b))
- 2*cos(d*x + c) + 2*I*sin(d*x + c)) + 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*
c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c
+ c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*cos(d*x + c) - 2*I*
sin(d*x + c)) - 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b +
b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^
2)) - a - b - 2*c)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d*x + c)) + 4*I*(a -
b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a
- b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a -
b)) + 2*cos(d*x + c) - 2*I*sin(d*x + c)) + 4*I*(a - b)*c^2*sqrt((a*b + (a
+ b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)
*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) - 2*cos(d*x + c) + 2
*I*sin(d*x + c)) - 4*I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*
```



```

*c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) - 4*((a - b)*cos(d
*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a
*b + b^2))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^
2)) + a + b + 2*c)/(a - b))/(a - b)) + 4*(-2*I*a + 2*I*b)*sqrt((a*b + (a +
b)*c + c^2)/(a^2 - 2*a*b + b^2))*polylog(3, -1/2*(2*(a + b + 2*c)*cos(d*x +
c) + (2*I*a + 2*I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) + (I*a
- I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*sq
rt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2
*c)/(a - b))/(a - b)) + 4*(2*I*a - 2*I*b)*sqrt((a*b + (a + b)*c + c^2)/(a^2
- 2*a*b + b^2))*polylog(3, 1/2*(2*(a + b + 2*c)*cos(d*x + c) - (2*I*a + 2*
I*b + 4*I*c)*sin(d*x + c) + 4*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x +
c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*sqrt((2*(a - b)*sq
rt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b))/(a -
b)) + 4*(2*I*a - 2*I*b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*
polylog(3, -1/2*(2*(a + b + 2*c)*cos(d*x + c) + (-2*I*a - 2*I*b - 4*I*c)*si
n(d*x + c) + 4*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a*b
+ (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*sqrt((2*(a - b)*sqrt((a*b + (a +
b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b))/(a - b)) + 4*(-2*I
*a + 2*I*b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*polylog(3, 1/
2*(2*(a + b + 2*c)*cos(d*x + c) - (-2*I*a - 2*I*b - 4*I*c)*sin(d*x + c) + 4
*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c +
c^2)/(a^2 - 2*a*b + b^2))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a
^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b))/(a - b)))/(a*b + (a + b)*c + c^
2)*d^3)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sec(dx + c)^2}{c \sec(dx + c)^2 + b \tan(dx + c)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] integrate(x^2*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)
```

maple [B] time = 0.55, size = 2061, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x)
```

```
[Out] -I/d/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c*ln(1-(a-b)*exp(
2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x^2-1/2*I/d/((a+c)*(b+c))^(1
/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*ln(1-(a-b)*exp(2*I*(d*x+c)))/(-2*((a
+c)*(b+c))^(1/2)-a-b-2*c))*x^2-1/2*I/d/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))
^(1/2)-a-b-2*c)*a*ln(1-(a-b)*exp(2*I*(d*x+c)))/(-2*((a+c)*(b+c))^(1/2)-a-b-2
*c))*x^2+1/2*I/d^3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*ln
(1-(a-b)*exp(2*I*(d*x+c)))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c^2+1/2*I/d^3/
((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*ln(1-(a-b)*exp(2*I*(
d*x+c)))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c^2-1/2/d^2/((a+c)*(b+c))^(1/2)/
(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*polylog(2, (a-b)*exp(2*I*(d*x+c)))/(-2*((a
+c)*(b+c))^(1/2)-a-b-2*c))*x-1/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/
2)-a-b-2*c)*c*polylog(2, (a-b)*exp(2*I*(d*x+c)))/(-2*((a+c)*(b+c))^(1/2)-a-b-
2*c))*x-1/2/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*poly
log(2, (a-b)*exp(2*I*(d*x+c)))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x+1/d^2/((a
+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*c^2*x+1/d^2/((a+c)*(b+c)
)^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*c^2*x-1/4*I/d^3/((a+c)*(b+c))^(1
```

$$\frac{1/2}{(-2*((a+c)*(b+c))^{1/2}-a-b-2*c)} * a * \text{polylog}(3, (a-b) * \exp(2*I*(d*x+c))) / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) - 1/4*I/d^3 / ((a+c)*(b+c))^{1/2} / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * b * \text{polylog}(3, (a-b) * \exp(2*I*(d*x+c))) / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) + 2/d^2 * c^3 / ((a+c)*(b+c))^{1/2} / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * x + 2/3/d^3 / ((a+c)*(b+c))^{1/2} / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * a * c^3 + 2/3/d^3 / ((a+c)*(b+c))^{1/2} / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * b * c^3 + I/d^3 * c^2 / (a * b + a * c + b * c + c^2)^{1/2} * \text{arctanh}(1/4 * (2 * (a-b) * \exp(2*I*(d*x+c))) + 2*a + 2*b + 4*c) / (a * b + a * c + b * c + c^2)^{1/2} - I/d / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * \ln(1 - (a-b) * \exp(2*I*(d*x+c))) / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * x^2 - 1/2 * I/d / ((a+c)*(b+c))^{1/2} * \ln(1 - (a-b) * \exp(2*I*(d*x+c))) / (2*((a+c)*(b+c))^{1/2}-a-b-2*c) * x^2 + 1/2 * I/d^3 * c^2 / ((a+c)*(b+c))^{1/2} * \ln(1 - (a-b) * \exp(2*I*(d*x+c))) / (2*((a+c)*(b+c))^{1/2}-a-b-2*c) + I/d^3 * c^2 / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * \ln(1 - (a-b) * \exp(2*I*(d*x+c))) / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) - 1/3 / ((a+c)*(b+c))^{1/2} / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * a * x^3 - 1/3 / ((a+c)*(b+c))^{1/2} / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * b * x^3 - 2/3 / ((a+c)*(b+c))^{1/2} / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * c * x^3 - 1/2/d^2 / ((a+c)*(b+c))^{1/2} * \text{polylog}(2, (a-b) * \exp(2*I*(d*x+c))) / (2*((a+c)*(b+c))^{1/2}-a-b-2*c) * x - 1/d^2 / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * \text{polylog}(2, (a-b) * \exp(2*I*(d*x+c))) / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * x + 4/3/d^3 * c^4 / ((a+c)*(b+c))^{1/2} / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) + 1/d^2 * c^2 / ((a+c)*(b+c))^{1/2} * x + 2/d^2 * c^2 / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * x - 1/4 * I/d^3 / ((a+c)*(b+c))^{1/2} * \text{polylog}(3, (a-b) * \exp(2*I*(d*x+c))) / (2*((a+c)*(b+c))^{1/2}-a-b-2*c) - 1/2 * I/d^3 / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * \text{polylog}(3, (a-b) * \exp(2*I*(d*x+c))) / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) + 2/3/d^3 * c^3 / ((a+c)*(b+c))^{1/2} + 4/3/d^3 * c^3 / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) - 1/2 * I/d^3 / ((a+c)*(b+c))^{1/2} / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * c * \text{polylog}(3, (a-b) * \exp(2*I*(d*x+c))) / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) + I/d^3 / ((a+c)*(b+c))^{1/2} / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * c^3 * \ln(1 - (a-b) * \exp(2*I*(d*x+c))) / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) - 1/3 / ((a+c)*(b+c))^{1/2} * x^3 - 2/3 / (-2*((a+c)*(b+c))^{1/2}-a-b-2*c) * x^3$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sec(dx + c)^2}{c \sec(dx + c)^2 + b \tan(dx + c)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="maxima")

[Out] integrate(x^2*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\cos(c + dx)^2 \left(a + \frac{c}{\cos(c+dx)^2} + b \tan(c + dx)^2 \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)),x)

[Out] int(x^2/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sec^2(c + dx)}{a + b \tan^2(c + dx) + c \sec^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*sec(d*x+c)**2/(a+c*sec(d*x+c)**2+b*tan(d*x+c)**2),x)

[Out] Integral(x**2*sec(c + d*x)**2/(a + b*tan(c + d*x)**2 + c*sec(c + d*x)**2), x)

3.165 $\int x^3 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} dx$

Optimal. Leaf size=155

$$\frac{6\sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{f^4} - \frac{6x \tan(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{f^3} + \frac{3x^2 \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{f^2}$$

```
[Out] -6*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^4+3*x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-6*x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/f^3+x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.20, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4604, 3296, 2638}

$$\frac{3x^2 \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{f^2} - \frac{6\sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{f^4} - \frac{6x \tan(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (-6*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^4 + (3*x^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (6*x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f^3 + (x^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_.) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n-m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && IntegerQ[n - m, 0]
```

Rubi steps

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)

[Out] int(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a \sin(fx + e) + a} \sqrt{c \sin(fx + e) + c} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x^3, x)

mupad [B] time = 3.05, size = 111, normalized size = 0.72

$$\frac{\sqrt{-a(\sin(e + fx) - 1)} \sqrt{c(\sin(e + fx) + 1)} (6 \cos(2e + 2fx) - 3f^2 x^2 + 6fx \sin(2e + 2fx) - 3f^2)}{f^4 (\cos(2e + 2fx) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2),x)

[Out] -((-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(6*cos(2*e + 2*f*x) - 3*f^2*x^2 + 6*f*x*sin(2*e + 2*f*x) - 3*f^2*x^2*cos(2*e + 2*f*x) - f^3*x^3*sin(2*e + 2*f*x) + 6)))/(f^4*(cos(2*e + 2*f*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{c(\sin(e + fx) + 1)} \sqrt{-a(\sin(e + fx) - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2),x)

[Out] Integral(x**3*sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1)), x)

3.166 $\int x^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} dx$

Optimal. Leaf size=118

$$\frac{2 \tan(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{f^3} + \frac{2x \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{f^2} + \frac{x^2 \tan(e + fx) \sqrt{a - a \sin(e + fx)}}{f}$$

[Out] $2*x*(a-a*\sin(f*x+e))^{(1/2)}*(c+c*\sin(f*x+e))^{(1/2)}/f^2-2*(a-a*\sin(f*x+e))^{(1/2)}*(c+c*\sin(f*x+e))^{(1/2)}*\tan(f*x+e)/f^3+x^2*(a-a*\sin(f*x+e))^{(1/2)}*(c+c*\sin(f*x+e))^{(1/2)}*\tan(f*x+e)/f$

Rubi [A] time = 0.18, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4604, 3296, 2637}

$$\frac{2x \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{f^2} - \frac{2 \tan(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{f^3} + \frac{x^2 \tan(e + fx) \sqrt{a - a \sin(e + fx)}}{f}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]

[Out] $(2*x*\text{Sqrt}[a - a*\text{Sin}[e + f*x]]*\text{Sqrt}[c + c*\text{Sin}[e + f*x]])/f^2 - (2*\text{Sqrt}[a - a*\text{Sin}[e + f*x]]*\text{Sqrt}[c + c*\text{Sin}[e + f*x]]*\text{Tan}[e + f*x])/f^3 + (x^2*\text{Sqrt}[a - a*\text{Sin}[e + f*x]]*\text{Sqrt}[c + c*\text{Sin}[e + f*x]]*\text{Tan}[e + f*x])/f$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 4604

Int[((g_.) + (h_.)*(x_))^(p_.)*((a_.) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n-m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && IntegerQ[n - m, 0]

Rubi steps

$$\begin{aligned} \int x^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} dx &= (\sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}) \int x^2 \cos(e + fx) dx \\ &= \frac{x^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} \tan(e + fx)}{f} - \frac{(2 \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)})}{f} \int x dx \\ &= \frac{2x \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{f^2} + \frac{x^2 \sqrt{a - a \sin(e + fx)}}{f} \\ &= \frac{2x \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{f^2} - \frac{2 \sqrt{a - a \sin(e + fx)}}{f} \end{aligned}$$

Mathematica [A] time = 0.33, size = 54, normalized size = 0.46

$$\frac{\left(\left(f^2 x^2 - 2\right) \tan(e + f x) + 2 f x\right) \sqrt{a - a \sin(e + f x)} \sqrt{c(\sin(e + f x) + 1)}}{f^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]

[Out] (Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(2*f*x + (-2 + f^2*x^2)*Tan[e + f*x]))/f^3

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-2*f^3*(-2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1)))/(-2*f^3)^2-4*f^4*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(f*x+exp(1)))/(-2*f^3)^2

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)

[Out] int(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a \sin(fx + e) + a} \sqrt{c \sin(fx + e) + c} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x^2, x)

mupad [B] time = 2.81, size = 86, normalized size = 0.73

$$\frac{\sqrt{-a(\sin(e+fx)-1)}\sqrt{c(\sin(e+fx)+1)}\left(2fx-2\sin(2e+2fx)+2fx\left(2\cos(e+fx)^2-1\right)+f^2x^2\right)}{2f^3\cos(e+fx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2),x)

[Out] ((-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(2*f*x - 2*sin(2*e + 2*f*x) + 2*f*x*(2*cos(e + f*x)^2 - 1) + f^2*x^2*sin(2*e + 2*f*x)))/(2*f^3*cos(e + f*x)^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{c(\sin(e+fx)+1)} \sqrt{-a(\sin(e+fx)-1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2),x)

[Out] Integral(x**2*sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1)), x)

3.167 $\int x\sqrt{a - a\sin(e + fx)}\sqrt{c + c\sin(e + fx)} dx$

Optimal. Leaf size=74

$$\frac{\sqrt{a - a\sin(e + fx)}\sqrt{c\sin(e + fx) + c}}{f^2} + \frac{x \tan(e + fx)\sqrt{a - a\sin(e + fx)}\sqrt{c\sin(e + fx) + c}}{f}$$

[Out] (a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2+x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/f

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4604, 3296, 2638}

$$\frac{\sqrt{a - a\sin(e + fx)}\sqrt{c\sin(e + fx) + c}}{f^2} + \frac{x \tan(e + fx)\sqrt{a - a\sin(e + fx)}\sqrt{c\sin(e + fx) + c}}{f}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]

[Out] (Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 + (x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 4604

Int[((g_.) + (h_.)*(x_))^(p_.)*((a_.) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && IntegerQ[n - m, 0]

Rubi steps

$$\begin{aligned} \int x\sqrt{a - a\sin(e + fx)}\sqrt{c + c\sin(e + fx)} dx &= (\sec(e + fx)\sqrt{a - a\sin(e + fx)}\sqrt{c + c\sin(e + fx)}) \int x \cos(e + fx) dx \\ &= \frac{x\sqrt{a - a\sin(e + fx)}\sqrt{c + c\sin(e + fx)} \tan(e + fx)}{f} - \frac{(\sec(e + fx)\sqrt{a - a\sin(e + fx)}\sqrt{c + c\sin(e + fx)})}{f} \\ &= \frac{\sqrt{a - a\sin(e + fx)}\sqrt{c + c\sin(e + fx)}}{f^2} + \frac{x\sqrt{a - a\sin(e + fx)}}{f} \end{aligned}$$

Mathematica [A] time = 0.20, size = 44, normalized size = 0.59

$$\frac{(fx \tan(e + fx) + 1)\sqrt{a - a\sin(e + fx)}\sqrt{c(\sin(e + fx) + 1)}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[c*(1 + Sin[e + f*x])] * Sqrt[a - a*Sin[e + f*x]] * (1 + f*x*Tan[e + f*x])) / f^2
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-1/2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(f*x+exp(1))/f^2-1/2*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/f)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int x \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a \sin(fx + e) + a} \sqrt{c \sin(fx + e) + c} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x, x)
```

mupad [B] time = 2.69, size = 61, normalized size = 0.82

$$\frac{\sqrt{-a (\sin(e + fx) - 1)} \left(2 \cos(e + fx)^2 + fx \sin(2e + 2fx) \right) \sqrt{c (\sin(e + fx) + 1)}}{2 f^2 \cos(e + fx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2),x)
```

```
[Out] ((-a*(sin(e + f*x) - 1))^(1/2)*(2*cos(e + f*x)^2 + f*x*sin(2*e + 2*f*x))*(c*(sin(e + f*x) + 1))^(1/2))/(2*f^2*cos(e + f*x)^2)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int x \sqrt{c(\sin(e + fx) + 1)} \sqrt{-a(\sin(e + fx) - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(x*sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1)), x)
```

$$3.168 \quad \int \frac{\sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)}}{x} dx$$

Optimal. Leaf size=86

$$\cos(e)\text{Ci}(fx) \sec(e+fx)\sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} - \sin(e)\text{Si}(fx) \sec(e+fx)\sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)}$$

[Out] Ci(f*x)*cos(e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-sec(f*x+e)*Si(f*x)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.18, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4604, 3303, 3299, 3302}

$$\cos(e)\text{CosIntegral}(fx) \sec(e+fx)\sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} - \sin(e)\text{Si}(fx) \sec(e+fx)\sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x,x]

[Out] Cos[e]*CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]] - Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[f*x]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 4604

Int[((g_.) + (h_.)*(x_))^(p_.)*((a_.) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && IntegerQ[n - m, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)}}{x} dx &= \left(\sec(e+fx) \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} \right) \int \frac{\cos(e+fx)}{x} dx \\ &= \left(\cos(e) \sec(e+fx) \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} \right) \int \frac{\cos(e+fx)}{x} dx \\ &= \cos(e)\text{Ci}(fx) \sec(e+fx) \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} - \sin(e)\text{Si}(fx) \sec(e+fx) \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} \end{aligned}$$

Mathematica [A] time = 0.21, size = 52, normalized size = 0.60

$$\sec(e + fx)\sqrt{a - a \sin(e + fx)}\sqrt{c(\sin(e + fx) + 1)}(\cos(e)\text{Ci}(fx) - \sin(e)\text{Si}(fx))$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x,x]

[Out] Sec[e + f*x]*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(Cos[e]*CosIntegral[f*x] - Sin[e]*SinIntegral[f*x])

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))-sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))+4*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))+2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(1/2*exp(1))-2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(1/2*exp(1))+sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(1/2*exp(1))^2+sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(1/2*exp(1))^2)/(4*tan(1/2*exp(1))^2+4)

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x)

[Out] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a \sin(fx + e) + a} \sqrt{c \sin(fx + e) + c}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x,x)

[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c(\sin(e + fx) + 1)} \sqrt{-a(\sin(e + fx) - 1)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2)/x,x)

[Out] Integral(sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1))/x, x)

$$3.169 \quad \int \frac{\sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)}}{x^2} dx$$

Optimal. Leaf size=123

$$-f \sin(e) \operatorname{Ci}(fx) \sec(e+fx) \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} - f \cos(e) \operatorname{Si}(fx) \sec(e+fx) \sqrt{a-a \sin(e+fx)}$$

```
[Out] -(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x-f*cos(e)*sec(f*x+e)*Si(f*x)
)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-f*Ci(f*x)*sec(f*x+e)*sin(e)
*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4604, 3297, 3303, 3299, 3302}

$$-f \sin(e) \operatorname{CosIntegral}(fx) \sec(e+fx) \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} - f \cos(e) \operatorname{Si}(fx) \sec(e+fx) \sqrt{a-a \sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^2,x]
```

```
[Out] -((Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x) - f*CosIntegral[f*x]
*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]] -
f*Cos[e]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Sin
Integral[f*x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_.) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_.) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[(a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
```


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 n(1/2*f*x)^2+2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))
 -1/4*pi))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-8*sign(sin(1/2*(f*x+exp(1))-1/4*
 pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/2*f*x)+2*f*x*
 Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi
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))*im(Ci(f*x))-f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(
 1))-1/4*pi))*im(Ci(-f*x))-2*f*x*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*
 sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2+2*f*x*Si(f*x)*sign(sin
 (1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*f*x)^
 2-f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
 *im(Ci(f*x))*tan(1/2*exp(1))^2+f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(
 cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(f*x))*tan(1/2*f*x)^2+f*x*sign(sin(1/2*(
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 2*exp(1))^2-f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1)
))-1/4*pi))*im(Ci(-f*x))*tan(1/2*f*x)^2+2*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*
 pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(f*x))*tan(1/2*exp(1))+2*f*x*s
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 -f*x))*tan(1/2*exp(1))-2*f*x*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sig
 n(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2-f*x*sign(s
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 *tan(1/2*exp(1))^2*tan(1/2*f*x)^2+f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*si
 gn(cos(1/2*(f*x+exp(1))-1/4*pi))*im(Ci(-f*x))*tan(1/2*exp(1))^2*tan(1/2*f*x
)^2+2*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*
 pi))*re(Ci(f*x))*tan(1/2*exp(1))*tan(1/2*f*x)^2+2*f*x*sign(sin(1/2*(f*x+exp
 (1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(1/2*exp(1
))*tan(1/2*f*x)^2)/(4*x*tan(1/2*exp(1))^2*tan(1/2*f*x)^2+4*x*tan(1/2*exp(1
))^2+4*x*tan(1/2*f*x)^2+4*x)

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x)

[Out] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a \sin(fx + e) + a} \sqrt{c \sin(fx + e) + c}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x^2,x)

[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c(\sin(e + fx) + 1)} \sqrt{-a(\sin(e + fx) - 1)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2)/x**2,x)

[Out] Integral(sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1))/x**2, x)

$$3.170 \quad \int \frac{\sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)}}{x^3} dx$$

Optimal. Leaf size=176

$$-\frac{1}{2}f^2 \cos(e) \text{Ci}(fx) \sec(e+fx) \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} + \frac{1}{2}f^2 \sin(e) \text{Si}(fx) \sec(e+fx) \sqrt{a-a \sin(e+fx)}$$

[Out] $-1/2*(a-a*\sin(f*x+e))^{(1/2)}*(c+c*\sin(f*x+e))^{(1/2)}/x^2-1/2*f^2*\text{Ci}(f*x)*\cos(e)*\sec(f*x+e)*(a-a*\sin(f*x+e))^{(1/2)}*(c+c*\sin(f*x+e))^{(1/2)}+1/2*f^2*\sec(f*x+e)*\text{Si}(f*x)*\sin(e)*(a-a*\sin(f*x+e))^{(1/2)}*(c+c*\sin(f*x+e))^{(1/2)}+1/2*f*(a-a*\sin(f*x+e))^{(1/2)}*(c+c*\sin(f*x+e))^{(1/2)}*\tan(f*x+e)/x$

Rubi [A] time = 0.23, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4604, 3297, 3303, 3299, 3302}

$$-\frac{1}{2}f^2 \cos(e) \text{CosIntegral}(fx) \sec(e+fx) \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} + \frac{1}{2}f^2 \sin(e) \text{Si}(fx) \sec(e+fx) \sqrt{a-a \sin(e+fx)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^3,x]

[Out] $-(\text{Sqrt}[a - a*\text{Sin}[e + f*x]]*\text{Sqrt}[c + c*\text{Sin}[e + f*x]])/(2*x^2) - (f^2*\text{Cos}[e]*\text{CosIntegral}[f*x]*\text{Sec}[e + f*x]*\text{Sqrt}[a - a*\text{Sin}[e + f*x]]*\text{Sqrt}[c + c*\text{Sin}[e + f*x]])/2 + (f^2*\text{Sec}[e + f*x]*\text{Sin}[e]*\text{Sqrt}[a - a*\text{Sin}[e + f*x]]*\text{Sqrt}[c + c*\text{Sin}[e + f*x]]*\text{SinIntegral}[f*x])/2 + (f*\text{Sqrt}[a - a*\text{Sin}[e + f*x]]*\text{Sqrt}[c + c*\text{Sin}[e + f*x]]*\text{Tan}[e + f*x])/(2*x)$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 4604

Int[((g_.) + (h_.)*(x_))^(p_.)*((a_.) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c

```
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{x^3} dx &= \left(\sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} \right) \int \frac{\cos(e + fx)}{x^3} \\
&= -\frac{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{2x^2} - \frac{1}{2} \left(f \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} \right) \\
&= -\frac{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{2x^2} + \frac{f \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{2x^2} \\
&= -\frac{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{2x^2} + \frac{f \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{2x^2} \\
&= -\frac{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{2x^2} - \frac{1}{2} f^2 \cos(e) \text{Ci}(fx) \sec(e)
\end{aligned}$$

Mathematica [A] time = 0.28, size = 87, normalized size = 0.49

$$\frac{\sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c(\sin(e + fx) + 1)} \left(-f^2 x^2 \cos(e) \text{Ci}(fx) + f^2 x^2 \sin(e) \text{Si}(fx) + fx \sin(e + fx) - c \right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^3,x]
[Out] (Sec[e + f*x]*Sqrt[c*(1 + Sin[e + f*x]])*Sqrt[a - a*Sin[e + f*x]]*(-Cos[e +
f*x] - f^2*x^2*Cos[e]*CosIntegral[f*x] + f*x*Sin[e + f*x] + f^2*x^2*Sin[e]
*SinIntegral[f*x]))/(2*x^2)
fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x, algorithm="f
ricas")
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (has polynomial part)
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x, algorithm="g
iac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
```


maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x)

[Out] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a \sin(fx + e) + a} \sqrt{c \sin(fx + e) + c}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x^3,x)

[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c(\sin(e + fx) + 1)} \sqrt{-a(\sin(e + fx) - 1)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2)/x**3,x)

[Out] Integral(sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1))/x**3, x)

3.171 $\int x^3 \sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{3/2} dx$

Optimal. Leaf size=393

$$\frac{3c \sin(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{8f^4} - \frac{6c \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{f^4} - \frac{3c \sin(e + fx)}{f^3}$$

```
[Out] 1/2*x^3*sec(f*x+e)*(c+c*sin(f*x+e))^(5/2)*(a-a*sin(f*x+e))^(1/2)/c/f-6*c*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^4+3*c*x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2+3/8*c*x*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^3-3/4*c*x^3*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f-3/8*c*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^4+3/4*c*x^2*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-6*c*x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/f^3-3/4*c*x*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/f^3
```

Rubi [A] time = 0.38, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4604, 4422, 3317, 3296, 2638, 3311, 30, 2635, 8}

$$\frac{3cx^2 \sin(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{4f^2} + \frac{3cx^2 \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{f^2} - \frac{3c \sin(e + fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-6*c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^4 + (3*c*x^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 + (3*c*x*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(8*f^3) - (3*c*x^3*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f) - (3*c*Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(8*f^4) + (3*c*x^2*Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f^2) + (x^3*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(5/2))/(2*c*f) - (6*c*x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f^3 - (3*c*x*Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/(4*f^3)
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
Simp[(d*m*(c + d*x)^(m - 1)*(b*SIN[e + f*x])^n)/(f^2*n^2), x] + (Dist[
(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*SIN[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*SIN[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*SIN[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.),
x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*SIN[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 4422

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] := Simp[((e + f*x)^m*(a + b*SIN[c + d*x
])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m
- 1)*(a + b*SIN[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_.) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)
((c_.) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*SIN[e + f*x])^FracPart[m]*(c + d*SIN[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*SIN[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

3.172 $\int x^2 \sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{3/2} dx$

Optimal. Leaf size=265

$$\frac{c \sin(e + fx) \tan(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{4f^3} - \frac{2c \tan(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{f^3}$$

[Out] $\frac{1}{2}x^2 \sec(fx+e) (c+c\sin(fx+e))^{5/2} (a-a\sin(fx+e))^{1/2} / c/f + 2cx (a-a\sin(fx+e))^{1/2} (c+c\sin(fx+e))^{1/2} / f^2 - 3/4cx^2 \sec(fx+e) (a-a\sin(fx+e))^{1/2} (c+c\sin(fx+e))^{1/2} / f + 1/2cx \sin(fx+e) (a-a\sin(fx+e))^{1/2} (c+c\sin(fx+e))^{1/2} / f^2 - 2c (a-a\sin(fx+e))^{1/2} (c+c\sin(fx+e))^{1/2} \tan(fx+e) / f^3 - 1/4c \sin(fx+e) (a-a\sin(fx+e))^{1/2} (c+c\sin(fx+e))^{1/2} \tan(fx+e) / f^3$

Rubi [A] time = 0.27, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4604, 4422, 3317, 3296, 2637, 3310, 30}

$$\frac{2cx \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{f^2} + \frac{cx \sin(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{2f^2} - \frac{c \sin(e + fx)}{f^3}$$

Antiderivative was successfully verified.

[In] `Int[x^2*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]`

[Out] $(2cx \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}) / f^2 - (3cx^2 \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}) / (4f) + (cx \sin(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}) / (2f^2) + (x^2 \sec(e + fx) \sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{5/2}) / (2cf) - (2c \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c} \tan(e + fx)) / f^3 - (c \sin(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c} \tan(e + fx)) / (4f^3)$

Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

Rule 2637

`Int[sin[Pi/2 + (c_) + (d_)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rule 3296

`Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + (f_)*(x_)], x_Symbol] := -Simp[(c + d*x)^m * Cos[e + f*x] / f, x] + Dist[(d*m) / f, Int[(c + d*x)^(m - 1) * Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rule 3310

`Int[((c_) + (d_)*(x_))*((b_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(d*(b*Sin[e + f*x])^n) / (f^2*n^2), x] + (Dist[(b^2*(n - 1)) / n, Int[(c + d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1)) / (f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]`

Rule 3317

`Int[((c_) + (d_)*(x_))^(m_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],`

`x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])`

Rule 4422

`Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] := Simp[((e + f*x)^m*(a + b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]`

Rule 4604

`Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && IntegerQ[n - m, 0]`

Rubi steps

$$\begin{aligned} \int x^2 \sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{3/2} dx &= \left(\sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} \right) \int x^2 \cos(e + fx) \sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{5/2} dx \\ &= \frac{x^2 \sec(e + fx) \sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{5/2}}{2cf} - \frac{(c + c \sin(e + fx))^{5/2} \int x^2 \cos(e + fx) \sqrt{a - a \sin(e + fx)} dx}{2cf} \\ &= -\frac{cx^2 \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{2f} + \frac{x^2 \sec(e + fx) \sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{5/2}}{2cf} \\ &= \frac{2cx \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{f^2} - \frac{cx^2 \sec(e + fx) \sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{5/2}}{2cf} \\ &= \frac{2cx \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{f^2} - \frac{3cx^2 \sec(e + fx) \sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{5/2}}{2cf} \end{aligned}$$

Mathematica [A] time = 0.81, size = 95, normalized size = 0.36

$$\frac{c \sqrt{a - a \sin(e + fx)} \sqrt{c(\sin(e + fx) + 1)} \left(8(f^2 x^2 - 2) \tan(e + fx) - (2f^2 x^2 - 1) \cos(2(e + fx)) \sec(e + fx) + 4f \right)}{8f^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]

[Out] (c*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(16*f*x - (-1 + 2*f^2*x^2)*Cos[2*(e + f*x)]*Sec[e + f*x] + 4*f*x*Sin[e + f*x] + 8*(-2 + f^2*x^2)*Tan[e + f*x]))/(8*f^3)

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-2*f^3*(-2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sign(f*x+exp(1))/(-2*f^3)^2+32*f^3*(-2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+4*c*f^2*x^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(2*f*x+2*exp(1))/(-32*f^3)^2-128*c*f^4*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(2*f*x+2*exp(1))/(-32*f^3)^2-4*c*f^4*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(f*x+exp(1))/(-2*f^3)^2)
```

```
maple [F] time = 0.17, size = 0, normalized size = 0.00
```

$$\int x^2 (c + c \sin(fx + e))^{\frac{3}{2}} \sqrt{a - a \sin(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

```
maxima [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \sqrt{-a \sin(fx + e) + a} (c \sin(fx + e) + c)^{\frac{3}{2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)*x^2, x)
```

```
mupad [B] time = 3.77, size = 159, normalized size = 0.60
```

$$c \sqrt{-a (\sin(e + fx) - 1)} \sqrt{c (\sin(e + fx) + 1)} (\cos(e + fx) + \cos(3e + 3fx) - 16 \sin(2e + 2fx) + 16$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2),x)
```

```
[Out] (c*(-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(cos(e + f*x)
+ cos(3*e + 3*f*x) - 16*sin(2*e + 2*f*x) + 16*f*x + 16*f*x*cos(2*e + 2*f*x
) + 2*f*x*sin(3*e + 3*f*x) - 2*f^2*x^2*cos(e + f*x) + 2*f*x*sin(e + f*x) -
2*f^2*x^2*cos(3*e + 3*f*x) + 8*f^2*x^2*sin(2*e + 2*f*x)))/(8*f^3*(cos(2*e +
2*f*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (c(\sin(e + fx) + 1))^{\frac{3}{2}} \sqrt{-a(\sin(e + fx) - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**2*(c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1)), x)
```

3.173 $\int x \sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{3/2} dx$

Optimal. Leaf size=168

$$\frac{c \sin(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{4f^2} + \frac{c \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{f^2} + \frac{x \sec(e + fx) \sqrt{a - a \sin(e + fx)}}{f}$$

[Out] 1/2*x*sec(f*x+e)*(c+c*sin(f*x+e))^(5/2)*(a-a*sin(f*x+e))^(1/2)/c/f+c*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-3/4*c*x*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f+1/4*c*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2

Rubi [A] time = 0.14, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4604, 4422, 2644}

$$\frac{c \sin(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{4f^2} + \frac{c \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}{f^2} + \frac{x \sec(e + fx) \sqrt{a - a \sin(e + fx)}}{f}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]

[Out] (c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (3*c*x*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f) + (c*Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f^2) + (x*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(5/2))/(2*c*f)

Rule 2644

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^2, x_Symbol] := Simp[((2*a^2 + b^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*Sin[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]

Rule 4422

Int[Cos[(c_) + (d_)*(x_)]*((e_) + (f_)*(x_))^(m_)*((a_) + (b_)*Sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[((e + f*x)^m*(a + b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^(m - 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 4604

Int[((g_) + (h_)*(x_))^(p_)*((a_) + (b_)*Sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*Sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && IntegerQ[n - m, 0]

Rubi steps

$$\begin{aligned} \int x\sqrt{a - a\sin(e + fx)}(c + c\sin(e + fx))^{3/2} dx &= \left(\sec(e + fx)\sqrt{a - a\sin(e + fx)}\sqrt{c + c\sin(e + fx)}\right) \int x \cos(e - \\ &= \frac{x \sec(e + fx)\sqrt{a - a\sin(e + fx)}(c + c\sin(e + fx))^{5/2}}{2cf} - \frac{(\sec(e + fx)\sqrt{a - a\sin(e + fx)})^{5/2}}{2cf} \\ &= \frac{c\sqrt{a - a\sin(e + fx)}\sqrt{c + c\sin(e + fx)}}{f^2} - \frac{3cx \sec(e + fx)\sqrt{a - a\sin(e + fx)}}{f^2} \end{aligned}$$

Mathematica [A] time = 0.64, size = 73, normalized size = 0.43

$$\frac{c\sqrt{a - a\sin(e + fx)}\sqrt{c(\sin(e + fx) + 1)}(\sin(e + fx) + 4fx \tan(e + fx) - fx \cos(2(e + fx)) \sec(e + fx) + 4)}{4f^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]

[Out] (c*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(4 - f*x*Cos[2*(e + f*x)]*Sec[e + f*x] + Sin[e + f*x] + 4*f*x*Tan[e + f*x]))/(4*f^2)

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-1/16*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(2*f*x+2*exp(1))/f^2-1/2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(f*x+exp(1))/f^2-1/2*c*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/f+1/8*c*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*exp(1))/f)

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int x(c + c\sin(fx + e))^{3/2} \sqrt{a - a\sin(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)`

[Out] `int(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a \sin(fx + e) + a} (c \sin(fx + e) + c)^{\frac{3}{2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)*x, x)`

mupad [B] time = 1.21, size = 123, normalized size = 0.73

$$\frac{c \sqrt{-a (\sin(e + fx) - 1)} \sqrt{c (\sin(e + fx) + 1)} \left(-16 \sin(e + fx)^2 + \sin(e + fx) + \sin(3e + 3fx) + 8f \right)}{8f^2 \left(2 \sin(e + fx)^2 - 2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2),x)`

[Out] `-(c*(-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(sin(e + f*x) + sin(3*e + 3*f*x) - 16*sin(e + f*x)^2 + 8*f*x*sin(2*e + 2*f*x) + 2*f*x*(2*sin(e/2 + (f*x)/2)^2 - 1) + 2*f*x*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1) + 16))/(8*f^2*(2*sin(e + f*x)^2 - 2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (c (\sin(e + fx) + 1))^{\frac{3}{2}} \sqrt{-a (\sin(e + fx) - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2),x)`

[Out] `Integral(x*(c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1)), x)`

$$3.174 \quad \int \frac{\sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{3/2}}{x} dx$$

Optimal. Leaf size=186

$$\frac{1}{2}c \sin(2e) \text{Ci}(2fx) \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c} + c \cos(e) \text{Ci}(fx) \sec(e + fx) \sqrt{a - a \sin(e + fx)}$$

```
[Out] c*Ci(f*x)*cos(e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)+1/2*c*cos(2*e)*sec(f*x+e)*Si(2*f*x)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-c*sec(f*x+e)*Si(f*x)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)+1/2*c*Ci(2*f*x)*sec(f*x+e)*sin(2*e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4604, 6741, 12, 6742, 3303, 3299, 3302}

$$\frac{1}{2}c \sin(2e) \text{CosIntegral}(2fx) \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c} + c \cos(e) \text{CosIntegral}(fx) \sec(e + fx)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x]))^(3/2))/x,x]
```

```
[Out] c*Cos[e]*CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]] + (c*CosIntegral[2*f*x]*Sec[e + f*x]*Sin[2*e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/2 - c*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[f*x] + (c*Cos[2*e]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[2*f*x])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)]^(m_)*((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] := Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
```

EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && IntegerQ[n - m, 0]

Rule 6741

Int[u_, x_Symbol] :=> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

Rule 6742

Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{3/2}}{x} dx &= \left(\sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} \right) \int \frac{\cos(e + fx)}{x} dx \\ &= \left(\sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} \right) \int \frac{c \cos(e + fx)}{x} dx \\ &= \left(c \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} \right) \int \frac{\cos(e + fx)}{x} dx \\ &= \left(c \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} \right) \int \left(\frac{\cos(e + fx)}{x} \right) dx \\ &= \frac{1}{2} \left(c \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} \right) \int \frac{\sin(2(e + fx))}{x} dx \\ &= \left(c \cos(e) \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} \right) \int \frac{\sin(2(e + fx))}{x} dx \\ &= c \cos(e) \operatorname{Ci}(fx) \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} \end{aligned}$$

Mathematica [C] time = 1.22, size = 150, normalized size = 0.81

$$\frac{c e^{-i(e-fx)} \sqrt{-i c e^{-i(e+fx)} (e^{i(e+fx)} + i)^2} (2e^{ie} \operatorname{Ei}(-ifx) + 2e^{3ie} \operatorname{Ei}(ifx) + i (\operatorname{Ei}(-2ifx) - e^{4ie} \operatorname{Ei}(2ifx))) \sqrt{a - a \sin(e + fx)}}{2\sqrt{2} (1 + e^{2i(e+fx)})}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x,x]

[Out] (c*Sqrt[(-I)*c*(I + E^(I*(e + f*x)))^2]/E^(I*(e + f*x))]*(2*E^(I*e)*ExpIntegralEi[(-I)*f*x] + 2*E^((3*I)*e)*ExpIntegralEi[I*f*x] + I*(ExpIntegralEi[(-2*I)*f*x] - E^((4*I)*e)*ExpIntegralEi[(2*I)*f*x]))*Sqrt[a - a*Sin[e + f*x]]/(2*Sqrt[2]*E^(I*(e - f*x))*(1 + E^((2*I)*(e + f*x))))

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

$$\begin{aligned}
& p(1) - 1/4\pi) * \operatorname{im}(\operatorname{Ci}(-f*x)) * \tan(1/2 * \exp(1)) + c * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{im}(\operatorname{Ci}(-2*f*x)) * \tan(1/2 * \exp(1)) \\
& - c * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{im}(\operatorname{Ci}(-2*f*x)) * \tan(\exp(1)) \\
& + 2 * c * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{re}(\operatorname{Ci}(f*x)) * \tan(1/2 * \exp(1)) \\
& - 2 * c * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{re}(\operatorname{Ci}(f*x)) * \tan(\exp(1)) \\
& - 2 * c * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{re}(\operatorname{Ci}(2*f*x)) * \tan(\exp(1)) \\
& + 2 * c * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{re}(\operatorname{Ci}(-f*x)) * \tan(1/2 * \exp(1)) \\
& - 2 * c * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{re}(\operatorname{Ci}(-f*x)) * \tan(\exp(1)) \\
& - 2 * c * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{re}(\operatorname{Ci}(-2*f*x)) * \tan(\exp(1)) \\
& + 8 * c * \operatorname{Si}(f*x) * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1)) * \tan(\exp(1)) \\
& + 2 * c * \operatorname{Si}(2*f*x) * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1)) \\
& + 2 * \tan(\exp(1)) + 4 * c * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{im}(\operatorname{Ci}(f*x)) * \tan(1/2 * \exp(1)) * \tan(\exp(1)) \\
& + c * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{im}(\operatorname{Ci}(2*f*x)) * \tan(1/2 * \exp(1)) \\
& + 2 * \tan(\exp(1)) - 4 * c * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{im}(\operatorname{Ci}(-f*x)) * \tan(1/2 * \exp(1)) * \tan(\exp(1)) \\
& - c * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{im}(\operatorname{Ci}(-2*f*x)) * \tan(1/2 * \exp(1)) \\
& + 2 * \tan(\exp(1)) + 2 * c * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{re}(\operatorname{Ci}(f*x)) * \tan(1/2 * \exp(1)) \\
& + 2 * \tan(\exp(1)) - 2 * c * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{re}(\operatorname{Ci}(2*f*x)) * \tan(1/2 * \exp(1)) \\
& + 2 * \tan(\exp(1)) + 2 * c * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{re}(\operatorname{Ci}(-f*x)) * \tan(1/2 * \exp(1)) \\
& + 2 * \tan(\exp(1)) - 2 * c * \operatorname{sign}(\sin(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{sign}(\cos(1/2 * (f*x + \exp(1)) - 1/4 * \pi)) * \operatorname{re}(\operatorname{Ci}(-2*f*x)) * \tan(1/2 * \exp(1)) \\
& + 2 * \tan(\exp(1)) / (8 * \tan(1/2 * \exp(1)) + 2 * \tan(\exp(1)) + 8 * \tan(1/2 * \exp(1)) + 8 * \tan(\exp(1)) + 8)
\end{aligned}$$

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(c + c \sin(fx + e))^{3/2} \sqrt{a - a \sin(fx + e)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x)

[Out] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a \sin(fx + e) + a} (c \sin(fx + e) + c)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x,x)`

[Out] `int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(\sin(e + fx) + 1))^{\frac{3}{2}} \sqrt{-a(\sin(e + fx) - 1)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2)/x,x)`

[Out] `Integral((c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1))/x, x)`

$$3.175 \quad \int \frac{\sqrt{a-a \sin(e+fx)} (c+c \sin(e+fx))^{3/2}}{x^2} dx$$

Optimal. Leaf size=273

$$-cf \sin(e) \operatorname{Ci}(fx) \sec(e+fx) \sqrt{a-a \sin(e+fx)} \sqrt{c \sin(e+fx)+c} + cf \cos(2e) \operatorname{Ci}(2fx) \sec(e+fx) \sqrt{a-a \sin(e+fx)}$$

```
[Out] -c*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x+c*f*Ci(2*f*x)*cos(2*e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-c*f*cos(e)*sec(f*x+e)*Si(f*x)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-c*f*Ci(f*x)*sec(f*x+e)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-c*f*sec(f*x+e)*Si(2*f*x)*sin(2*e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-1/2*c*sec(f*x+e)*sin(2*f*x+2*e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x
```

Rubi [A] time = 0.66, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4604, 6741, 12, 6742, 3297, 3303, 3299, 3302}

$$-cf \sin(e) \operatorname{CosIntegral}(fx) \sec(e+fx) \sqrt{a-a \sin(e+fx)} \sqrt{c \sin(e+fx)+c} + cf \cos(2e) \operatorname{CosIntegral}(2fx) \sec(e+fx) \sqrt{a-a \sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^2,x]
```

```
[Out] -((c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x) + c*f*Cos[2*e]*CosIntegral[2*f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]] - c*f*CosIntegral[f*x]*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]] - (c*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Sin[2*e + 2*f*x])/(2*x) - c*f*Cos[e]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[f*x] - c*f*Sec[e + f*x]*Sin[2*e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[2*f*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_.) + (b_.)*Sin[(e_.) + (f_.)*(x_)]^(m_)*
((c_.) + (d_.)*Sin[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] := Dist[(a^IntPart[m
]*c^IntPart[m]*(a + b*SIN[e + f*x])^FracPart[m]*(c + d*SIN[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*COS[e + f*x]^(2*m)*(c
+ d*SIN[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{3/2}}{x^2} dx &= (\sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}) \int \frac{\cos(e + fx)}{x^2} dx \\ &= (\sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}) \int \frac{c \cos(e + fx)}{x^2} dx \\ &= (c \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}) \int \frac{\cos(e + fx)}{x^2} dx \\ &= (c \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}) \int \left(\frac{\cos(e + fx)}{x^2} \right) dx \\ &= \frac{1}{2} (c \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}) \int \frac{\sin(2(e + fx))}{x^2} dx \\ &= -\frac{c \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{x} - \frac{c \sec(e + fx) \sqrt{a - a \sin(e + fx)}}{x} \\ &= -\frac{c \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{x} - \frac{c \sec(e + fx) \sqrt{a - a \sin(e + fx)}}{x} \\ &= -\frac{c \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{x} + cf \cos(2e) \text{Ci}(2fx) \end{aligned}$$

Mathematica [C] time = 1.44, size = 231, normalized size = 0.85

$$\frac{ce^{-i(e+fx)} \sqrt{-ice^{-i(e+fx)} (e^{i(e+fx)} + i)^2} (-2ifxe^{i(e+2fx)} \text{Ei}(-ifx) + 2ifxe^{3ie+2ifx} \text{Ei}(ifx) + 2fxe^{2i(2e+fx)} \text{Ei}(2ifx) - 2e^{i(e+fx)})}{2\sqrt{2}x(1 + e^{2i(e+fx)})}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - aSin[e + f*x]]*(c + cSin[e + f*x])^(3/2))/x^2,x]
```

```
[Out] (c*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2]/E^(I*(e + f*x))]*(-I - 2*E^(I*(e +
f*x)) - 2*E^((3*I)*(e + f*x)) + I*E^((4*I)*(e + f*x)) - (2*I)*E^(I*(e + 2*
```



```

^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(f*x)^2*tan(1/2*exp(1)
))^2*tan(exp(1))^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f
*x+exp(1))-1/4*pi))*re(Ci(2*f*x))*tan(f*x)^2*tan(1/2*f*x)^2*tan(exp(1))^2+c
*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*
re(Ci(2*f*x))*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+2*c*f*x*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x)
))*tan(f*x)^2*tan(1/2*exp(1))*tan(1/2*f*x)^2+2*c*f*x*sign(sin(1/2*(f*x+exp(1)
))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(f*x)^2*tan(
1/2*exp(1))*tan(exp(1))^2+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(c
os(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-f*x))*tan(1/2*exp(1))*tan(1/2*f*x)^2*ta
n(exp(1))^2-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))*re(Ci(-2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2+c*f
*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re
(Ci(-2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(exp(1))^2+c*f*x*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x))*tan
(f*x)^2*tan(1/2*f*x)^2*tan(exp(1))^2+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci(-2*f*x))*tan(1/2*exp(1))^2*tan(
1/2*f*x)^2*tan(exp(1))^2-2*c*f*x*Si(f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*
x)^2*tan(exp(1))^2+4*c*f*x*Si(2*f*x)*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^
2*tan(exp(1))-c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp
(1))-1/4*pi))*im(Ci(f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(e
xp(1))^2+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1)
))-1/4*pi))*im(Ci(2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(e
xp(1))+c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1
/4*pi))*im(Ci(-f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1)
)^2-2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/
4*pi))*im(Ci(-2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1)
))+2*c*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))*re(Ci(f*x))*tan(f*x)^2*tan(1/2*exp(1))*tan(1/2*f*x)^2*tan(exp(1))^2+c
*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*
re(Ci(2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+2*c
*f*x*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*
re(Ci(-f*x))*tan(f*x)^2*tan(1/2*exp(1))*tan(1/2*f*x)^2*tan(exp(1))^2+c*f*x*
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*re(Ci
(-2*f*x))*tan(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2)/(4*x*t
an(f*x)^2*tan(1/2*exp(1))^2*tan(1/2*f*x)^2*tan(exp(1))^2+4*x*tan(f*x)^2*tan
(1/2*exp(1))^2*tan(1/2*f*x)^2+4*x*tan(f*x)^2*tan(1/2*exp(1))^2*tan(exp(1))^
2+4*x*tan(f*x)^2*tan(1/2*f*x)^2*tan(exp(1))^2+4*x*tan(1/2*exp(1))^2*tan(1/2
*f*x)^2*tan(exp(1))^2+4*x*tan(f*x)^2*tan(1/2*exp(1))^2+4*x*tan(f*x)^2*tan(1
/2*f*x)^2+4*x*tan(1/2*exp(1))^2*tan(exp(1))^2+4*x*tan(1/2*f*x)^2*tan(exp(1))^2+4*x*t
an(f*x)^2+4*x*tan(1/2*exp(1))^2+4*x*tan(1/2*f*x)^2+4*x*tan(exp(1))^2+4*x

```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(c + c \sin(fx + e))^{3/2} \sqrt{a - a \sin(fx + e)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x)

[Out] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a \sin(fx + e) + a} (c \sin(fx + e) + c)^{3/2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a - a \sin(e + f x)} (c + c \sin(e + f x))^{3/2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x^2,x)

[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(\sin(e + f x) + 1))^{3/2} \sqrt{-a(\sin(e + f x) - 1)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2)/x**2,x)

[Out] Integral((c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1))/x**2, x)

$$3.176 \quad \int \frac{\sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{3/2}}{x^3} dx$$

Optimal. Leaf size=385

$$-cf^2 \sin(2e) \text{Ci}(2fx) \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c} - \frac{1}{2} cf^2 \cos(e) \text{Ci}(fx) \sec(e + fx) \sqrt{a - a \sin(e + fx)}$$

```
[Out] -1/2*c*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2-1/2*c*f^2*Ci(f*x)*
cos(e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-1/2*c*f*cos
(2*f*x+2*e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x-c*f^
2*cos(2*e)*sec(f*x+e)*Si(2*f*x)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/
2)+1/2*c*f^2*sec(f*x+e)*Si(f*x)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+
e))^(1/2)-c*f^2*Ci(2*f*x)*sec(f*x+e)*sin(2*e)*(a-a*sin(f*x+e))^(1/2)*(c+c*s
in(f*x+e))^(1/2)-1/4*c*sec(f*x+e)*sin(2*f*x+2*e)*(a-a*sin(f*x+e))^(1/2)*(c+
c*sin(f*x+e))^(1/2)/x^2+1/2*c*f*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/
2)*tan(f*x+e)/x
```

Rubi [A] time = 0.74, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4604, 6741, 12, 6742, 3297, 3303, 3299, 3302}

$$-cf^2 \sin(2e) \text{CosIntegral}(2fx) \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c} - \frac{1}{2} cf^2 \cos(e) \text{CosIntegral}(fx) \sec(e + fx) \sqrt{a - a \sin(e + fx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x]))^(3/2))/x^3,x]
```

```
[Out] -(c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(2*x^2) - (c*f*Cos[2
*e + 2*f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
/(2*x) - (c*f^2*Cos[e]*CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x
]]*Sqrt[c + c*Sin[e + f*x]])/2 - c*f^2*CosIntegral[2*f*x]*Sec[e + f*x]*Sin[
2*e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]] - (c*Sec[e + f*x]*Sq
rt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Sin[2*e + 2*f*x])/(4*x^2) +
(c*f^2*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x
]]*SinIntegral[f*x])/2 - c*f^2*Cos[2*e]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x
]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[2*f*x] + (c*f*Sqrt[a - a*Sin[e + f*
x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/(2*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

$c*f, 0]$

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_.) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_) *
((c_.) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[(a^IntPart[m]
*c^IntPart[m]*(a + b*SIN[e + f*x])^FracPart[m]*(c + d*SIN[e + f*x])^FracPa
rt[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*SIN[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{3/2}}{x^3} dx &= \left(\sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} \right) \int \frac{\cos(e + fx)}{x^3} dx \\
&= \left(\sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} \right) \int \frac{c \cos(e + fx)}{x^3} dx \\
&= \left(c \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} \right) \int \frac{\cos(e + fx)}{x^3} dx \\
&= \left(c \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} \right) \int \left(\frac{\cos(e + fx)}{x^3} \right) dx \\
&= \frac{1}{2} \left(c \sec(e + fx) \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} \right) \int \frac{\sin(e + fx)}{x^2} dx \\
&= -\frac{c \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{2x^2} - \frac{c \sec(e + fx) \sqrt{a - a \sin(e + fx)}}{2x} \\
&= -\frac{c \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{2x^2} - \frac{cf \cos(2e + 2fx)}{2x^2} \\
&= -\frac{c \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{2x^2} - \frac{cf \cos(2e + 2fx)}{2x^2} \\
&= -\frac{c \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{2x^2} - \frac{cf \cos(2e + 2fx)}{2x^2}
\end{aligned}$$

Mathematica [C] time = 1.84, size = 317, normalized size = 0.82

$$\frac{c^2 e^{-2i(e+fx)} \left(e^{i(e+fx)} + i \right) \left(2if^2 x^2 e^{i(e+2fx)} \text{Ei}(-ifx) + 2if^2 x^2 e^{3ie+2ifx} \text{Ei}(ifx) + 4f^2 x^2 e^{2i(2e+fx)} \text{Ei}(2ifx) + 2fx e^{i(e+fx)} \right)}{4\sqrt{2} x^2 \left(e^{i(e+fx)} - i \right)}$$

[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a - a \sin(e + f x)} (c + c \sin(e + f x))^{3/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x^3,x)

[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(\sin(e + f x) + 1))^{3/2} \sqrt{-a(\sin(e + f x) - 1)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2)/x**3,x)

[Out] Integral((c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1))/x**3, x)

$$3.177 \quad \int \frac{(g+hx)^3 \sqrt{a-a \sin(e+fx)}}{\sqrt{c+c \sin(e+fx)}} dx$$

Optimal. Leaf size=767

$$\frac{6iah^3 \text{Li}_4(-ie^{i(e+fx)}) \cos(e+fx)}{f^4 \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} + c} + \frac{6iah^3 \text{Li}_4(ie^{i(e+fx)}) \cos(e+fx)}{f^4 \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} + c} + \frac{3iah^3 \text{Li}_4(-e^{2i(e+fx)})}{4f^4 \sqrt{a-a \sin(e+fx)}}$$

[Out] $-1/4 * I * a * (h * x + g)^4 * \cos(f * x + e) / h / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} - 2 * I * a * (h * x + g)^3 * \arctan(\exp(I * (f * x + e))) * \cos(f * x + e) / f / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} + a * (h * x + g)^3 * \cos(f * x + e) * \ln(1 + \exp(2 * I * (f * x + e))) / f / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} + 3 * I * a * h * (h * x + g)^2 * \cos(f * x + e) * \text{polylog}(2, -I * \exp(I * (f * x + e))) / f^2 / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} - 3 * I * a * h * (h * x + g)^2 * \cos(f * x + e) * \text{polylog}(2, I * \exp(I * (f * x + e))) / f^2 / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} - 3/2 * I * a * h * (h * x + g)^2 * \cos(f * x + e) * \text{polylog}(2, -\exp(2 * I * (f * x + e))) / f^2 / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} - 6 * a * h^2 * (h * x + g) * \cos(f * x + e) * \text{polylog}(3, -I * \exp(I * (f * x + e))) / f^3 / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} + 6 * a * h^2 * (h * x + g) * \cos(f * x + e) * \text{polylog}(3, I * \exp(I * (f * x + e))) / f^3 / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} + 3/2 * a * h^2 * (h * x + g) * \cos(f * x + e) * \text{polylog}(3, -\exp(2 * I * (f * x + e))) / f^3 / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} - 6 * I * a * h^3 * \cos(f * x + e) * \text{polylog}(4, -I * \exp(I * (f * x + e))) / f^4 / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} + 6 * I * a * h^3 * \cos(f * x + e) * \text{polylog}(4, I * \exp(I * (f * x + e))) / f^4 / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} + 3/4 * I * a * h^3 * \cos(f * x + e) * \text{polylog}(4, -\exp(2 * I * (f * x + e))) / f^4 / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2}$

Rubi [A] time = 1.35, antiderivative size = 767, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {4604, 6741, 12, 6742, 4181, 2531, 6609, 2282, 6589, 3719, 2190}

$$\frac{6ah^2(g+hx) \cos(e+fx) \text{PolyLog}(3, -ie^{i(e+fx)})}{f^3 \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} + c} + \frac{6ah^2(g+hx) \cos(e+fx) \text{PolyLog}(3, ie^{i(e+fx)})}{f^3 \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} + c} + \frac{3ah^2(g+hx)}{2f^3 \sqrt{a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(g + hx)^3 \text{Sqrt}[a - a \text{Sin}[e + fx]] / \text{Sqrt}[c + c \text{Sin}[e + fx]], x]$

[Out] $((-I/4) * a * (g + hx)^4 * \text{Cos}[e + fx]) / (h * \text{Sqrt}[a - a \text{Sin}[e + fx]] * \text{Sqrt}[c + c \text{Sin}[e + fx]]) - ((2 * I) * a * (g + hx)^3 * \text{ArcTan}[E^{(I * (e + fx))}] * \text{Cos}[e + fx]) / (f * \text{Sqrt}[a - a \text{Sin}[e + fx]] * \text{Sqrt}[c + c \text{Sin}[e + fx]]) + (a * (g + hx)^3 * \text{Cos}[e + fx] * \text{Log}[1 + E^{((2 * I) * (e + fx))}]) / (f * \text{Sqrt}[a - a \text{Sin}[e + fx]] * \text{Sqrt}[c + c \text{Sin}[e + fx]]) + ((3 * I) * a * h * (g + hx)^2 * \text{Cos}[e + fx] * \text{PolyLog}[2, (-I) * E^{(I * (e + fx))}]) / (f^2 * \text{Sqrt}[a - a \text{Sin}[e + fx]] * \text{Sqrt}[c + c \text{Sin}[e + fx]]) - ((3 * I) * a * h * (g + hx)^2 * \text{Cos}[e + fx] * \text{PolyLog}[2, I * E^{(I * (e + fx))}]) / (f^2 * \text{Sqrt}[a - a \text{Sin}[e + fx]] * \text{Sqrt}[c + c \text{Sin}[e + fx]]) - (((3 * I) / 2) * a * h * (g + hx)^2 * \text{Cos}[e + fx] * \text{PolyLog}[2, -E^{((2 * I) * (e + fx))}]) / (f^2 * \text{Sqrt}[a - a \text{Sin}[e + fx]] * \text{Sqrt}[c + c \text{Sin}[e + fx]]) - (6 * a * h^2 * (g + hx) * \text{Cos}[e + fx] * \text{PolyLog}[3, (-I) * E^{(I * (e + fx))}]) / (f^3 * \text{Sqrt}[a - a \text{Sin}[e + fx]] * \text{Sqrt}[c + c \text{Sin}[e + fx]]) + (6 * a * h^2 * (g + hx) * \text{Cos}[e + fx] * \text{PolyLog}[3, I * E^{(I * (e + fx))}]) / (f^3 * \text{Sqrt}[a - a \text{Sin}[e + fx]] * \text{Sqrt}[c + c \text{Sin}[e + fx]]) + (3 * a * h^2 * (g + hx) * \text{Cos}[e + fx] * \text{PolyLog}[3, -E^{((2 * I) * (e + fx))}]) / (2 * f^3 * \text{Sqrt}[a - a \text{Sin}[e + fx]] * \text{Sqrt}[c + c \text{Sin}[e + fx]]) - ((6 * I) * a * h^3 * \text{Cos}[e + fx] * \text{PolyLog}[4, (-I) * E^{(I * (e + fx))}]) / (f^4 * \text{Sqrt}[a - a \text{Sin}[e + fx]] * \text{Sqrt}[c + c \text{Sin}[e + fx]]) + ((6 * I) * a * h^3 * \text{Cos}[e + fx] * \text{PolyLog}[4, I * E^{(I * (e + fx))}]) / (f^4 * \text{Sqrt}[a - a \text{Sin}[e + fx]] * \text{Sqrt}[c + c \text{Sin}[e + fx]]) + (((3 * I) / 4) * a * h^3 * \text{Cos}[e + fx] * \text{PolyLog}[4, -E^{((2 * I) * (e + fx))}]) / (f^4 * \text{Sqrt}[a - a \text{Sin}[e + fx]] * \text{Sqrt}[c + c \text{Sin}[e + fx]])$

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + (f_)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_) + Pi*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4604

```
Int[((g_) + (h_)*(x_))^(p_)*((a_) + (b_)*Sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*Sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*SIN[e + f*x])^FracPart[m]*(c + d*SIN[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c + d*SIN[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && IGtQ[n - m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```


Mathematica [A] time = 2.66, size = 247, normalized size = 0.32

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) e^{-\frac{1}{2}i(e+fx)} \left(e^{i(e+fx)} + i\right) \sqrt{a - a \sin(e + fx)} \left(\frac{24h(f^2(g+hx)^2 \text{Li}_2(-ie^{-i(e+fx)}) - 2h(if(g+hx) \text{Li}_3(-ie^{-i(e+fx)}) + h \text{Li}_4(-ie^{-i(e+fx)}))}{f^4}}\right)}{\sqrt{2} \sqrt{-ice^{-i(e+fx)} \left(e^{i(e+fx)} + i\right)^2} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[((g + h*x)^3*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]

[Out] ((1/4 + I/4)*(I + E^(I*(e + f*x)))*((g + h*x)^4/h - ((8*I)*(g + h*x)^3*Log[1 + I/E^(I*(e + f*x))])/f + (24*h*(f^2*(g + h*x)^2*PolyLog[2, (-I)/E^(I*(e + f*x))] - 2*h*(I*f*(g + h*x)*PolyLog[3, (-I)/E^(I*(e + f*x))] + h*PolyLog[4, (-I)/E^(I*(e + f*x))]))/f^4)*Sqrt[a - a*Sin[e + f*x]]/(Sqrt[2]*E^((I/2)*(e + f*x))*Sqrt[(-I)*c*(I + E^(I*(e + f*x)))^2]/E^(I*(e + f*x))]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(hx + g)^3 \sqrt{-a \sin(fx + e) + a}}{\sqrt{c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((h*x + g)^3*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x)

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(hx + g)^3 \sqrt{a - a \sin(fx + e)}}{\sqrt{c + c \sin(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)

[Out] int((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(hx + g)^3 \sqrt{-a \sin(fx + e) + a}}{\sqrt{c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((h*x + g)^3*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g + hx)^3 \sqrt{a - a \sin(e + fx)}}{\sqrt{c + c \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g + h*x)^3*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2),x)

[Out] int(((g + h*x)^3*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a(\sin(e + fx) - 1)}(g + hx)^3}{\sqrt{c(\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)**3*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(1/2),x)

[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))*(g + h*x)**3/sqrt(c*(sin(e + f*x) + 1)), x)

$$3.178 \quad \int \frac{(g+hx)^2 \sqrt{a-a \sin(e+fx)}}{\sqrt{c+c \sin(e+fx)}} dx$$

Optimal. Leaf size=555

$$\frac{2ah^2 \text{Li}_3(-ie^{i(e+fx)}) \cos(e+fx)}{f^3 \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} + c} + \frac{2ah^2 \text{Li}_3(ie^{i(e+fx)}) \cos(e+fx)}{f^3 \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} + c} + \frac{ah^2 \text{Li}_3(-e^{2i(e+fx)}) \cos(e+fx)}{2f^3 \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} + c}$$

[Out] $-1/3 * I * a * (h * x + g)^3 * \cos(f * x + e) / h / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} - 2 * I * a * (h * x + g)^2 * \arctan(\exp(I * (f * x + e))) * \cos(f * x + e) / f / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} + a * (h * x + g)^2 * \cos(f * x + e) * \ln(1 + \exp(2 * I * (f * x + e))) / f / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} + 2 * I * a * h * (h * x + g) * \cos(f * x + e) * \text{polylog}(2, -I * \exp(I * (f * x + e))) / f^2 / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} - 2 * I * a * h * (h * x + g) * \cos(f * x + e) * \text{polylog}(2, I * \exp(I * (f * x + e))) / f^2 / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} - I * a * h * (h * x + g) * \cos(f * x + e) * \text{polylog}(2, -\exp(2 * I * (f * x + e))) / f^2 / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} - 2 * a * h^2 * \cos(f * x + e) * \text{polylog}(3, -I * \exp(I * (f * x + e))) / f^3 / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} + 2 * a * h^2 * \cos(f * x + e) * \text{polylog}(3, I * \exp(I * (f * x + e))) / f^3 / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2} + 1/2 * a * h^2 * \cos(f * x + e) * \text{polylog}(3, -\exp(2 * I * (f * x + e))) / f^3 / (a - a * \sin(f * x + e))^{1/2} / (c + c * \sin(f * x + e))^{1/2}$

Rubi [A] time = 0.88, antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {4604, 6741, 12, 6742, 4181, 2531, 2282, 6589, 3719, 2190}

$$\frac{2iah(g+hx) \cos(e+fx) \text{PolyLog}(2, -ie^{i(e+fx)})}{f^2 \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} + c} - \frac{2iah(g+hx) \cos(e+fx) \text{PolyLog}(2, ie^{i(e+fx)})}{f^2 \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} + c} - \frac{iah(g+hx) \cos(e+fx)}{f^2 \sqrt{a-a \sin(e+fx)} \sqrt{c+c \sin(e+fx)} + c}$$

Antiderivative was successfully verified.

[In] Int[((g + h*x)^2*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]

[Out] $((-I/3) * a * (g + h * x)^3 * \cos[e + f * x]) / (h * \text{Sqrt}[a - a * \sin[e + f * x]] * \text{Sqrt}[c + c * \sin[e + f * x]]) - ((2 * I) * a * (g + h * x)^2 * \arctan[E^{i * (e + f * x)}] * \cos[e + f * x]) / (f * \text{Sqrt}[a - a * \sin[e + f * x]] * \text{Sqrt}[c + c * \sin[e + f * x]]) + (a * (g + h * x)^2 * \cos[e + f * x] * \log[1 + E^{(2 * I) * (e + f * x)}]) / (f * \text{Sqrt}[a - a * \sin[e + f * x]] * \text{Sqrt}[c + c * \sin[e + f * x]]) + ((2 * I) * a * h * (g + h * x) * \cos[e + f * x] * \text{polylog}[2, (-I) * E^{i * (e + f * x)}]) / (f^2 * \text{Sqrt}[a - a * \sin[e + f * x]] * \text{Sqrt}[c + c * \sin[e + f * x]]) - ((2 * I) * a * h * (g + h * x) * \cos[e + f * x] * \text{polylog}[2, I * E^{i * (e + f * x)}]) / (f^2 * \text{Sqrt}[a - a * \sin[e + f * x]] * \text{Sqrt}[c + c * \sin[e + f * x]]) - (I * a * h * (g + h * x) * \cos[e + f * x] * \text{polylog}[2, -E^{(2 * I) * (e + f * x)}]) / (f^2 * \text{Sqrt}[a - a * \sin[e + f * x]] * \text{Sqrt}[c + c * \sin[e + f * x]]) - (2 * a * h^2 * \cos[e + f * x] * \text{polylog}[3, (-I) * E^{i * (e + f * x)}]) / (f^3 * \text{Sqrt}[a - a * \sin[e + f * x]] * \text{Sqrt}[c + c * \sin[e + f * x]]) + (2 * a * h^2 * \cos[e + f * x] * \text{polylog}[3, I * E^{i * (e + f * x)}]) / (f^3 * \text{Sqrt}[a - a * \sin[e + f * x]] * \text{Sqrt}[c + c * \sin[e + f * x]]) + (a * h^2 * \cos[e + f * x] * \text{polylog}[3, -E^{(2 * I) * (e + f * x)}]) / (2 * f^3 * \text{Sqrt}[a - a * \sin[e + f * x]] * \text{Sqrt}[c + c * \sin[e + f * x]])$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2190

Int[(((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[(((c + d*x)^m * Log[1 + (b*(F^(g*(e + f*x)))^n)/a]) / (b*f*g*n * Log[F]), x] - Dist[(d*m) / (b*f*g*n * Log[F]), Int[(c + d*x)^(m - 1) * Log[1 + (b*(F^(g*(e + f*x)))^n)], x]

)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))^*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 2531

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*x_)))]^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] :=> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 3719

Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + (f_)*(x_)], x_Symbol] :=> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 4181

Int[csc[(e_) + Pi*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] :=> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 4604

Int[((g_) + (h_)*(x_))^(p_)*((a_) + (b_)*Sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*Sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :=> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*SIN[e + f*x])^FracPart[m]*(c + d*SIN[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*cos[e + f*x]^(2*m)*(c + d*SIN[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && IntegerQ[n - m, 0]

Rule 6589

Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_Symbol] :=> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rule 6741

Int[u_, x_Symbol] :=> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

Rule 6742

Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]

Rubi steps

$$\begin{aligned}
\int \frac{(g+hx)^2 \sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}} dx &= \frac{\cos(e+fx) \int (g+hx)^2 \sec(e+fx)(a-a\sin(e+fx)) dx}{\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= \frac{\cos(e+fx) \int a(g+hx)^2 \sec(e+fx)(1-\sin(e+fx)) dx}{\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= \frac{(a\cos(e+fx)) \int (g+hx)^2 \sec(e+fx)(1-\sin(e+fx)) dx}{\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= \frac{(a\cos(e+fx)) \int ((g+hx)^2 \sec(e+fx) - (g+hx)^2 \tan(e+fx)) dx}{\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= \frac{(a\cos(e+fx)) \int (g+hx)^2 \sec(e+fx) dx}{\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} - \frac{(a\cos(e+fx)) \int (g+hx)^2 \tan(e+fx) dx}{\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= -\frac{ia(g+hx)^3 \cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2 \tan^{-1}(e^{i(e+fx)}) \cos(e+fx)}{f\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= -\frac{ia(g+hx)^3 \cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2 \tan^{-1}(e^{i(e+fx)}) \cos(e+fx)}{f\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= -\frac{ia(g+hx)^3 \cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2 \tan^{-1}(e^{i(e+fx)}) \cos(e+fx)}{f\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= -\frac{ia(g+hx)^3 \cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2 \tan^{-1}(e^{i(e+fx)}) \cos(e+fx)}{f\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= -\frac{ia(g+hx)^3 \cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2 \tan^{-1}(e^{i(e+fx)}) \cos(e+fx)}{f\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 1.89, size = 194, normalized size = 0.35

$$\frac{\sqrt{2} (e^{i(e+fx)} + i) \sqrt{a-a\sin(e+fx)} (f^2(g+hx)^2 (f(g+hx) - 6ih \log(1 + ie^{-i(e+fx)})) + 12fh^2(g+hx) \text{Li}_2(-ie^{-i(e+fx)}))}{3f^3h (e^{i(e+fx)} - i) \sqrt{-ice^{-i(e+fx)} (e^{i(e+fx)} + i)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((g+h*x)^2*Sqrt[a-a*Sin[e+f*x]])/Sqrt[c+c*Sin[e+f*x]],x]

[Out] (Sqrt[2]*(I+E^(I*(e+f*x)))*(f^2*(g+h*x)^2*(f*(g+h*x)-(6*I)*h*Log[1+I/E^(I*(e+f*x))]) + 12*f*h^2*(g+h*x)*PolyLog[2,(-I)/E^(I*(e+f*x))]) - (12*I)*h^3*PolyLog[3,(-I)/E^(I*(e+f*x))]*Sqrt[a-a*Sin[e+f*x]])/(3*(-I+E^(I*(e+f*x)))*Sqrt[((-I)*c*(I+E^(I*(e+f*x)))^2)/E^(I*(e+f*x))])*f^3*h

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(hx + g)^2 \sqrt{-a \sin(fx + e) + a}}{\sqrt{c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((h*x + g)^2*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x)

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(hx + g)^2 \sqrt{a - a \sin(fx + e)}}{\sqrt{c + c \sin(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)

[Out] int((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(hx + g)^2 \sqrt{-a \sin(fx + e) + a}}{\sqrt{c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((h*x + g)^2*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g + hx)^2 \sqrt{a - a \sin(e + fx)}}{\sqrt{c + c \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g + h*x)^2*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2),x)

[Out] int(((g + h*x)^2*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a(\sin(e + fx) - 1)}(g + hx)^2}{\sqrt{c(\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))*(g + h*x)**2/sqrt(c*(sin(e + f*x) + 1)), x)
```

$$3.179 \quad \int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}} dx$$

Optimal. Leaf size=355

$$\frac{iahLi_2(-ie^{i(e+fx)})\cos(e+fx)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}} - \frac{iahLi_2(ie^{i(e+fx)})\cos(e+fx)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}} - \frac{iahLi_2(-e^{2i(e+fx)})}{2f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

[Out] $-1/2*I*a*(h*x+g)^2*\cos(f*x+e)/h/(a-a*\sin(f*x+e))^{(1/2)}/(c+c*\sin(f*x+e))^{(1/2)}-2*I*a*(h*x+g)*\arctan(\exp(I*(f*x+e)))*\cos(f*x+e)/f/(a-a*\sin(f*x+e))^{(1/2)}/(c+c*\sin(f*x+e))^{(1/2)}+a*(h*x+g)*\cos(f*x+e)*\ln(1+\exp(2*I*(f*x+e)))/f/(a-a*\sin(f*x+e))^{(1/2)}/(c+c*\sin(f*x+e))^{(1/2)}+I*a*h*\cos(f*x+e)*\text{polylog}(2,-I*\exp(I*(f*x+e)))/f^2/(a-a*\sin(f*x+e))^{(1/2)}/(c+c*\sin(f*x+e))^{(1/2)}-I*a*h*\cos(f*x+e)*\text{polylog}(2,I*\exp(I*(f*x+e)))/f^2/(a-a*\sin(f*x+e))^{(1/2)}/(c+c*\sin(f*x+e))^{(1/2)}-1/2*I*a*h*\cos(f*x+e)*\text{polylog}(2,-\exp(2*I*(f*x+e)))/f^2/(a-a*\sin(f*x+e))^{(1/2)}/(c+c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.51, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4604, 6741, 12, 6742, 4181, 2279, 2391, 3719, 2190}

$$\frac{iah\cos(e+fx)\text{PolyLog}(2,-ie^{i(e+fx)})}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}} - \frac{iah\cos(e+fx)\text{PolyLog}(2,ie^{i(e+fx)})}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}} - \frac{iah\cos(e+fx)\text{PolyLog}(2,-e^{2i(e+fx)})}{2f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

Antiderivative was successfully verified.

[In] Int[((g + h*x)*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]], x]

[Out] $((-I/2)*a*(g+h*x)^2*\cos[e+f*x])/(h*\sqrt{a-a*\sin[e+f*x]}*\sqrt{c+c*\sin[e+f*x]}) - ((2*I)*a*(g+h*x)*\arctan[E^{I*(e+f*x)}]*\cos[e+f*x])/(f*\sqrt{a-a*\sin[e+f*x]}*\sqrt{c+c*\sin[e+f*x]}) + (a*(g+h*x)*\cos[e+f*x]*\log[1+E^{(2*I)*(e+f*x)}])/(f*\sqrt{a-a*\sin[e+f*x]}*\sqrt{c+c*\sin[e+f*x]}) + (I*a*h*\cos[e+f*x]*\text{PolyLog}[2,(-I)*E^{I*(e+f*x)}])/(f^2*\sqrt{a-a*\sin[e+f*x]}*\sqrt{c+c*\sin[e+f*x]}) - (I*a*h*\cos[e+f*x]*\text{PolyLog}[2,I*E^{I*(e+f*x)}])/(f^2*\sqrt{a-a*\sin[e+f*x]}*\sqrt{c+c*\sin[e+f*x]}) - ((I/2)*a*h*\cos[e+f*x]*\text{PolyLog}[2,-E^{(2*I)*(e+f*x)}])/(f^2*\sqrt{a-a*\sin[e+f*x]}*\sqrt{c+c*\sin[e+f*x]})$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2190

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^((n_)*((c_) + (d_)*(x_))^(m_)))/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^((n_))), x_Symbol] := Simp[(((c + d*x)^m*log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*log[F]), x] - Dist[(d*m)/(b*f*g*n*log[F]), Int[(c + d*x)^(m-1)*log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^((n_)))]], x_Symbol] := Dist[1/(d*e*n*log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

```
Int[Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_)^(m_.))*tan[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_)^(m_.)), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_)^(p_.))*((a_.) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*SIN[e + f*x])^FracPart[m]*(c + d*SIN[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c + d*SIN[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && IGtQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
```

Rubi steps

$$\begin{aligned}
\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}} dx &= \frac{\cos(e+fx) \int (g+hx) \sec(e+fx)(a-a\sin(e+fx)) dx}{\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= \frac{\cos(e+fx) \int a(g+hx) \sec(e+fx)(1-\sin(e+fx)) dx}{\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= \frac{(a\cos(e+fx)) \int (g+hx) \sec(e+fx)(1-\sin(e+fx)) dx}{\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= \frac{(a\cos(e+fx)) \int ((g+hx) \sec(e+fx) - (g+hx) \tan(e+fx)) dx}{\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= \frac{(a\cos(e+fx)) \int (g+hx) \sec(e+fx) dx}{\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} - \frac{(a\cos(e+fx)) \int (g+hx) \tan(e+fx) dx}{\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= -\frac{ia(g+hx)^2 \cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx) \tan^{-1}(e^{i(e+fx)})}{f\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= -\frac{ia(g+hx)^2 \cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx) \tan^{-1}(e^{i(e+fx)})}{f\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= -\frac{ia(g+hx)^2 \cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx) \tan^{-1}(e^{i(e+fx)})}{f\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} \\
&= -\frac{ia(g+hx)^2 \cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx) \tan^{-1}(e^{i(e+fx)})}{f\sqrt{a-a\sin(e+fx)} \sqrt{c+c\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 1.22, size = 154, normalized size = 0.43

$$\frac{(e^{i(e+fx)} + i) \sqrt{a - a \sin(e + fx)} (f (fx(2g + hx) - 4i(g + hx) \log(1 + ie^{-i(e+fx)})) + 4h \operatorname{Li}_2(-ie^{-i(e+fx)}))}{\sqrt{2} f^2 (e^{i(e+fx)} - i) \sqrt{-ice^{-i(e+fx)} (e^{i(e+fx)} + i)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((g + h*x)*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]

[Out] ((I + E^(I*(e + f*x)))*(f*(f*x*(2*g + h*x) - (4*I)*(g + h*x)*Log[1 + I/E^(I*(e + f*x))]) + 4*h*PolyLog[2, (-I)/E^(I*(e + f*x))])*Sqrt[a - a*Sin[e + f*x])/(Sqrt[2]*(-I + E^(I*(e + f*x)))*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*f^2)

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(hx+g)\sqrt{-a\sin(fx+e)+a}}{\sqrt{c\sin(fx+e)+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((h*x + g)*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x)

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(hx + g) \sqrt{a - a \sin(fx + e)}}{\sqrt{c + c \sin(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)

[Out] int((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(hx + g) \sqrt{-a \sin(fx + e) + a}}{\sqrt{c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((h*x + g)*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g + hx) \sqrt{a - a \sin(e + fx)}}{\sqrt{c + c \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((g + h*x)*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2),x)

[Out] int(((g + h*x)*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a(\sin(e + fx) - 1)}(g + hx)}{\sqrt{c(\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(1/2),x)

[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))*(g + h*x)/sqrt(c*(sin(e + f*x) + 1)), x)

$$3.180 \quad \int \frac{\sqrt{a - a \sin(e + fx)}}{(g + hx)\sqrt{c + c \sin(e + fx)}} dx$$

Optimal. Leaf size=110

$$\frac{a \cos(e + fx) \operatorname{Int}\left(\frac{\sec(e + fx)}{g + hx}, x\right)}{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} - \frac{a \cos(e + fx) \operatorname{Int}\left(\frac{\tan(e + fx)}{g + hx}, x\right)}{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}$$

[Out] $a \cos(f*x+e) * \operatorname{Unintegrable}(\sec(f*x+e)/(h*x+g), x) / (a - a * \sin(f*x+e))^{(1/2)} / (c + c * \sin(f*x+e))^{(1/2)} - a \cos(f*x+e) * \operatorname{Unintegrable}(\tan(f*x+e)/(h*x+g), x) / (a - a * \sin(f*x+e))^{(1/2)} / (c + c * \sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.65, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{a - a \sin(e + fx)}}{(g + hx)\sqrt{c + c \sin(e + fx)}} dx$$

Verification is Not applicable to the result.

[In] $\operatorname{Int}[\operatorname{Sqrt}[a - a * \operatorname{Sin}[e + f*x]] / ((g + h*x) * \operatorname{Sqrt}[c + c * \operatorname{Sin}[e + f*x]])], x]$

[Out] $(a * \operatorname{Cos}[e + f*x] * \operatorname{Defer}[\operatorname{Int}][\operatorname{Sec}[e + f*x] / (g + h*x), x]) / (\operatorname{Sqrt}[a - a * \operatorname{Sin}[e + f*x]] * \operatorname{Sqrt}[c + c * \operatorname{Sin}[e + f*x]]) - (a * \operatorname{Cos}[e + f*x] * \operatorname{Defer}[\operatorname{Int}][\operatorname{Tan}[e + f*x] / (g + h*x), x]) / (\operatorname{Sqrt}[a - a * \operatorname{Sin}[e + f*x]] * \operatorname{Sqrt}[c + c * \operatorname{Sin}[e + f*x]])$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a - a \sin(e + fx)}}{(g + hx)\sqrt{c + c \sin(e + fx)}} dx &= \frac{\cos(e + fx) \int \frac{\sec(e + fx)(a - a \sin(e + fx))}{g + hx} dx}{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\ &= \frac{\cos(e + fx) \int \frac{a \sec(e + fx)(1 - \sin(e + fx))}{g + hx} dx}{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\ &= \frac{(a \cos(e + fx)) \int \frac{\sec(e + fx)(1 - \sin(e + fx))}{g + hx} dx}{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\ &= \frac{(a \cos(e + fx)) \int \left(\frac{\sec(e + fx)}{g + hx} - \frac{\tan(e + fx)}{g + hx} \right) dx}{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\ &= \frac{(a \cos(e + fx)) \int \frac{\sec(e + fx)}{g + hx} dx}{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} - \frac{(a \cos(e + fx)) \int \frac{\tan(e + fx)}{g + hx} dx}{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 3.54, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a - a \sin(e + fx)}}{(g + hx)\sqrt{c + c \sin(e + fx)}} dx$$

Verification is Not applicable to the result.

[In] $\operatorname{Integrate}[\operatorname{Sqrt}[a - a * \operatorname{Sin}[e + f*x]] / ((g + h*x) * \operatorname{Sqrt}[c + c * \operatorname{Sin}[e + f*x]])], x]$

[Out] $\operatorname{Integrate}[\operatorname{Sqrt}[a - a * \operatorname{Sin}[e + f*x]] / ((g + h*x) * \operatorname{Sqrt}[c + c * \operatorname{Sin}[e + f*x]])], x]$

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a \sin(fx + e) + a}}{(hx + g)\sqrt{c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-a*sin(f*x + e) + a)/((h*x + g)*sqrt(c*sin(f*x + e) + c)), x)

maple [A] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a - a \sin(fx + e)}}{(hx + g)\sqrt{c + c \sin(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x)

[Out] int((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a \sin(fx + e) + a}}{(hx + g)\sqrt{c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-a*sin(f*x + e) + a)/((h*x + g)*sqrt(c*sin(f*x + e) + c)), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a - a \sin(e + fx)}}{(g + hx)\sqrt{c + c \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - a*sin(e + f*x))^(1/2)/((g + h*x)*(c + c*sin(e + f*x))^(1/2)),x)

[Out] `int((a - a*sin(e + f*x))^(1/2)/((g + h*x)*(c + c*sin(e + f*x))^(1/2)), x)`
sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a(\sin(e + fx) - 1)}}{\sqrt{c(\sin(e + fx) + 1)}(g + hx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a-a*sin(f*x+e))**(1/2)/(h*x+g)/(c+c*sin(f*x+e))**(1/2),x)`

[Out] `Integral(sqrt(-a*(sin(e + f*x) - 1))/(sqrt(c*(sin(e + f*x) + 1))*(g + h*x)), x)`

$$3.181 \quad \int \frac{x^3 \sqrt{a - a \sin(e + fx)}}{(c + c \sin(e + fx))^{3/2}} dx$$

Optimal. Leaf size=536

$$\frac{6ia \operatorname{Li}_2(-ie^{i(e+fx)}) \cos(e+fx)}{cf^4 \sqrt{a-a \sin(e+fx)} \sqrt{c \sin(e+fx)+c}} - \frac{6ia \operatorname{Li}_2(ie^{i(e+fx)}) \cos(e+fx)}{cf^4 \sqrt{a-a \sin(e+fx)} \sqrt{c \sin(e+fx)+c}} - \frac{3ia \operatorname{Li}_2(-e^{2i(e+fx)}) \cos(e+fx)}{cf^4 \sqrt{a-a \sin(e+fx)} \sqrt{c \sin(e+fx)+c}}$$

[Out] $-3*a*x^2/c/f^2/(a-a*\sin(f*x+e))^{(1/2)}/(c+c*\sin(f*x+e))^{(1/2)}-3*I*a*x^2*\cos(f*x+e)/c/f^2/(a-a*\sin(f*x+e))^{(1/2)}/(c+c*\sin(f*x+e))^{(1/2)}-12*I*a*x*\arctan(\exp(I*(f*x+e)))*\cos(f*x+e)/c/f^3/(a-a*\sin(f*x+e))^{(1/2)}/(c+c*\sin(f*x+e))^{(1/2)}+6*a*x*\cos(f*x+e)*\ln(1+\exp(2*I*(f*x+e)))/c/f^3/(a-a*\sin(f*x+e))^{(1/2)}/(c+c*\sin(f*x+e))^{(1/2)}+6*I*a*\cos(f*x+e)*\operatorname{polylog}(2,-I*\exp(I*(f*x+e)))/c/f^4/(a-a*\sin(f*x+e))^{(1/2)}/(c+c*\sin(f*x+e))^{(1/2)}-6*I*a*\cos(f*x+e)*\operatorname{polylog}(2,I*\exp(I*(f*x+e)))/c/f^4/(a-a*\sin(f*x+e))^{(1/2)}/(c+c*\sin(f*x+e))^{(1/2)}-3*I*a*\cos(f*x+e)*\operatorname{polylog}(2,-\exp(2*I*(f*x+e)))/c/f^4/(a-a*\sin(f*x+e))^{(1/2)}/(c+c*\sin(f*x+e))^{(1/2)}-a*x^3*\sec(f*x+e)/c/f/(a-a*\sin(f*x+e))^{(1/2)}/(c+c*\sin(f*x+e))^{(1/2)}+3*a*x^2*\sin(f*x+e)/c/f^2/(a-a*\sin(f*x+e))^{(1/2)}/(c+c*\sin(f*x+e))^{(1/2)}+a*x^3*\tan(f*x+e)/c/f/(a-a*\sin(f*x+e))^{(1/2)}/(c+c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 3.55, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 51, number of rules used = 17, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {4604, 6741, 12, 6742, 4186, 4181, 2279, 2391, 2531, 6609, 2282, 6589, 3757, 4184, 3719, 2190, 4413}

$$\frac{6ia \cos(e+fx) \operatorname{PolyLog}(2, -ie^{i(e+fx)})}{cf^4 \sqrt{a-a \sin(e+fx)} \sqrt{c \sin(e+fx)+c}} - \frac{6ia \cos(e+fx) \operatorname{PolyLog}(2, ie^{i(e+fx)})}{cf^4 \sqrt{a-a \sin(e+fx)} \sqrt{c \sin(e+fx)+c}} - \frac{3ia \cos(e+fx) \operatorname{PolyLog}(2, -e^{2i(e+fx)})}{cf^4 \sqrt{a-a \sin(e+fx)} \sqrt{c \sin(e+fx)+c}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^3 \sqrt{a - a \sin[e + f*x]})/(c + c \sin[e + f*x])^{(3/2)}, x]$

[Out] $(-3*a*x^2)/(c*f^2*\sqrt{a - a*\sin[e + f*x]}*\sqrt{c + c*\sin[e + f*x]}) - ((3*I)*a*x^2*\cos[e + f*x])/(c*f^2*\sqrt{a - a*\sin[e + f*x]}*\sqrt{c + c*\sin[e + f*x]}) - ((12*I)*a*x*\operatorname{ArcTan}[E^{(I*(e + f*x))}]*\cos[e + f*x])/(c*f^3*\sqrt{a - a*\sin[e + f*x]}*\sqrt{c + c*\sin[e + f*x]}) + (6*a*x*\cos[e + f*x]*\log[1 + E^{((2*I)*(e + f*x))}])/(c*f^3*\sqrt{a - a*\sin[e + f*x]}*\sqrt{c + c*\sin[e + f*x]}) + ((6*I)*a*\cos[e + f*x]*\operatorname{PolyLog}[2, (-I)*E^{(I*(e + f*x))}])/(c*f^4*\sqrt{a - a*\sin[e + f*x]}*\sqrt{c + c*\sin[e + f*x]}) - ((6*I)*a*\cos[e + f*x]*\operatorname{PolyLog}[2, I*E^{(I*(e + f*x))}])/(c*f^4*\sqrt{a - a*\sin[e + f*x]}*\sqrt{c + c*\sin[e + f*x]}) - ((3*I)*a*\cos[e + f*x]*\operatorname{PolyLog}[2, -E^{((2*I)*(e + f*x))}])/(c*f^4*\sqrt{a - a*\sin[e + f*x]}*\sqrt{c + c*\sin[e + f*x]}) - (a*x^3*\sec[e + f*x])/(c*f*\sqrt{a - a*\sin[e + f*x]}*\sqrt{c + c*\sin[e + f*x]}) + (3*a*x^2*\sin[e + f*x])/(c*f^2*\sqrt{a - a*\sin[e + f*x]}*\sqrt{c + c*\sin[e + f*x]}) + (a*x^3*\tan[e + f*x])/(c*f*\sqrt{a - a*\sin[e + f*x]}*\sqrt{c + c*\sin[e + f*x]})$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 2190

$\operatorname{Int}[(((F_)^((g_)*((e_.) + (f_.)*(x_))))^{(n_.)}*((c_.) + (d_.)*(x_))^{(m_.)})/((a_.) + (b_.)*((F_)^((g_)*((e_.) + (f_.)*(x_))))^{(n_.)}), x_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^m*\log[1 + (b*(F^{(g*(e + f*x))))^n]/a]/(b*f*g^n*\log[F]), x] - \operatorname{Dist}[(d*m)/(b*f*g^n*\log[F]), \operatorname{Int}[(c + d*x)^{(m-1)}*\log[1 + (b*(F^{(g*(e + f*x))))^n]/a], x], x] /; \operatorname{FreeQ}[\{F, a, b, c, d, e, f, g, n\}, x] \ \&\& \ \operatorname{IGtQ}[m, 0]$

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_)^(m_.))*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
+ f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
- Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_)^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x)
)], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_)^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_)^(m_.), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
```

$(c + dx)^m (b \operatorname{Csc}[e + fx])^{n-2}, x, x] - \operatorname{Simp}[(b^2 d^m (c + dx)^{m-1} (b \operatorname{Csc}[e + fx])^{n-2}) / (f^2 (n-1)(n-2)), x] /;$ FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

Rule 4413

$\operatorname{Int}[(c + dx)^m \operatorname{Sec}[a + bx] \operatorname{Tan}[a + bx]^p, x_Symbol] := -\operatorname{Int}[(c + dx)^m \operatorname{Sec}[a + bx] \operatorname{Tan}[a + bx]^{p-2}, x] + \operatorname{Int}[(c + dx)^m \operatorname{Sec}[a + bx]^3 \operatorname{Tan}[a + bx]^{p-2}, x] /;$ FreeQ[{a, b, c, d, m}, x] && IGtQ[p/2, 0]

Rule 4604

$\operatorname{Int}[(g + hx)^p ((a + b \operatorname{Sin}[e + fx])^m ((c + d \operatorname{Sin}[e + fx])^n) / \operatorname{Cos}[e + fx]^{2m}), x_Symbol] := \operatorname{Dist}[(a^{\operatorname{IntPart}[m]} c^{\operatorname{IntPart}[m]} (a + b \operatorname{Sin}[e + fx])^{\operatorname{FracPart}[m]} (c + d \operatorname{Sin}[e + fx])^{\operatorname{FracPart}[m]}) / \operatorname{Cos}[e + fx]^{2 \operatorname{FracPart}[m]}, \operatorname{Int}[(g + hx)^p \operatorname{Cos}[e + fx]^{2m} (c + d \operatorname{Sin}[e + fx])^{n-m}], x, x] /;$ FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && IntegerQ[n - m, 0]

Rule 6589

$\operatorname{Int}[\operatorname{PolyLog}[n, (c + dx)^p] / ((d + ex)^n), x_Symbol] := \operatorname{Simp}[\operatorname{PolyLog}[n + 1, c(a + bx)^p] / (e^p), x] /;$ FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rule 6609

$\operatorname{Int}[(e + fx)^m \operatorname{PolyLog}[n, (d + ex)^p] / ((c + dx)^p), x_Symbol] := \operatorname{Simp}[(e + fx)^m \operatorname{PolyLog}[n + 1, d(F^{c(a + bx)})^p] / (b^m c^p \operatorname{Log}[F]), x] - \operatorname{Dist}[(f^m) / (b^m c^p \operatorname{Log}[F]), \operatorname{Int}[(e + fx)^{m-1} \operatorname{PolyLog}[n + 1, d(F^{c(a + bx)})^p], x, x] /;$ FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rule 6741

$\operatorname{Int}[u, x_Symbol] := \operatorname{With}[\{v = \operatorname{NormalizeIntegrand}[u, x]\}, \operatorname{Int}[v, x] /; v \neq u]$

Rule 6742

$\operatorname{Int}[u, x_Symbol] := \operatorname{With}[\{v = \operatorname{ExpandIntegrand}[u, x]\}, \operatorname{Int}[v, x] /; \operatorname{SumQ}[v]$

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \sqrt{a - a \sin(e + fx)}}{(c + c \sin(e + fx))^{3/2}} dx &= \frac{\cos(e + fx) \int x^3 \sec^3(e + fx)(a - a \sin(e + fx))^2 dx}{ac \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
&= \frac{\cos(e + fx) \int a^2 x^3 \sec^3(e + fx)(1 - \sin(e + fx))^2 dx}{ac \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
&= \frac{(a \cos(e + fx)) \int x^3 \sec^3(e + fx)(1 - \sin(e + fx))^2 dx}{c \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
&= \frac{(a \cos(e + fx)) \int (x^3 \sec^3(e + fx) - 2x^3 \sec^2(e + fx) \tan(e + fx) + x^3 \sec(e + fx) \tan^2(e + fx)) dx}{c \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
&= \frac{(a \cos(e + fx)) \int x^3 \sec^3(e + fx) dx}{c \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} + \frac{(a \cos(e + fx)) \int x^3 \sec(e + fx) \tan^2(e + fx) dx}{c \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
&= -\frac{3ax^2}{2cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} - \frac{ax^3 \sec(e + fx)}{cf \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
&= -\frac{3ax^2}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} - \frac{6iax \tan^{-1}(e^{i(e+fx)}) \cos(e + fx)}{cf^3 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
&= -\frac{3ax^2}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} - \frac{3iax^2 \cos(e + fx)}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
&= -\frac{3ax^2}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} - \frac{3iax^2 \cos(e + fx)}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
&= -\frac{3ax^2}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} - \frac{3iax^2 \cos(e + fx)}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
&= -\frac{3ax^2}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} - \frac{3iax^2 \cos(e + fx)}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
&= -\frac{3ax^2}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} - \frac{3iax^2 \cos(e + fx)}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}
\end{aligned}$$

Mathematica [A] time = 2.08, size = 193, normalized size = 0.36

$$\frac{\sqrt{a - a \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(fx \left(-12 \log(1 - ie^{i(e+fx)}) + 3fx \cos(e + fx) + 3i \right) \right)}{f^4 (c(\sin(e + fx) + 1))^{3/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]

[Out] -(((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[a - a*Sin[e + f*x]]*((12*I)*PolyLog[2, I*E^(I*(e + f*x))]*(1 + Sin[e + f*x]) + f*x*((3*I)*f*x + f^2*x^2 + 3*f*x*Cos[e + f*x] - 12*Log[1 - I*E^(I*(e + f*x))] + (3*I)*(f*x + (4*I)*Log[1 - I*E^(I*(e + f*x))])*Sin[e + f*x])))/(f^4*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(c*(1 + Sin[e + f*x]))^(3/2)))

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{\sqrt{-a \sin(fx + e) + a} \sqrt{c \sin(fx + e) + cx^3}}{c^2 \cos(fx + e)^2 - 2c^2 \sin(fx + e) - 2c^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] integral(-sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x^3/(c^2*cos(f*x + e)^2 - 2*c^2*sin(f*x + e) - 2*c^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a \sin(fx + e) + a} x^3}{(c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x^3/(c*sin(f*x + e) + c)^(3/2), x)

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{a - a \sin(fx + e)}}{(c + c \sin(fx + e))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)

[Out] int(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a \sin(fx + e) + a} x^3}{(c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x^3/(c*sin(f*x + e) + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \sqrt{a - a \sin(e + fx)}}{(c + c \sin(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2),x)

[Out] int((x^3*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{-a (\sin(e + fx) - 1)}}{(c (\sin(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**3*sqrt(-a*(sin(e + f*x) - 1))/(c*(sin(e + f*x) + 1))**(3/2), x)
```

$$3.182 \quad \int \frac{x^2 \sqrt{a - a \sin(e + fx)}}{(c + c \sin(e + fx))^{3/2}} dx$$

Optimal. Leaf size=280

$$\frac{2a \cos(e + fx) \log(\cos(e + fx))}{cf^3 \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}} + \frac{2a \cos(e + fx) \tanh^{-1}(\sin(e + fx))}{cf^3 \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}} + \frac{2ax \sin(e + fx)}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}$$

[Out] $-2*a*x/c/f^2/(a-a*\sin(f*x+e))^{(1/2)/(c+c*\sin(f*x+e))^{(1/2)+2*a*\operatorname{arctanh}(\sin(f*x+e))*\cos(f*x+e)/c/f^3/(a-a*\sin(f*x+e))^{(1/2)/(c+c*\sin(f*x+e))^{(1/2)+2*a*\cos(f*x+e)*\ln(\cos(f*x+e))/c/f^3/(a-a*\sin(f*x+e))^{(1/2)/(c+c*\sin(f*x+e))^{(1/2)-a*x^2*\sec(f*x+e)/c/f/(a-a*\sin(f*x+e))^{(1/2)/(c+c*\sin(f*x+e))^{(1/2)+2*a*x*\sin(f*x+e)/c/f^2/(a-a*\sin(f*x+e))^{(1/2)/(c+c*\sin(f*x+e))^{(1/2)+a*x^2*\tan(f*x+e)/c/f/(a-a*\sin(f*x+e))^{(1/2)/(c+c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 2.16, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {4604, 6741, 12, 6742, 4186, 3770, 4181, 2531, 2282, 6589, 3757, 4184, 3475, 4413}

$$\frac{2ax \sin(e + fx)}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}} - \frac{2ax}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}} + \frac{2a \cos(e + fx) \log(\cos(e + fx))}{cf^3 \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}}$$

Antiderivative was successfully verified.

[In] `Int[(x^2*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]`

[Out] $(-2*a*x)/(c*f^2*\sqrt{a - a*\sin[e + f*x]}*\sqrt{c + c*\sin[e + f*x]}) + (2*a*\operatorname{ArcTanh}[\sin[e + f*x]]*\cos[e + f*x])/(c*f^3*\sqrt{a - a*\sin[e + f*x]}*\sqrt{c + c*\sin[e + f*x]}) + (2*a*\cos[e + f*x]*\log[\cos[e + f*x]])/(c*f^3*\sqrt{a - a*\sin[e + f*x]}*\sqrt{c + c*\sin[e + f*x]}) - (a*x^2*\sec[e + f*x])/(c*f*\sqrt{a - a*\sin[e + f*x]}*\sqrt{c + c*\sin[e + f*x]}) + (2*a*x*\sin[e + f*x])/(c*f^2*\sqrt{a - a*\sin[e + f*x]}*\sqrt{c + c*\sin[e + f*x]}) + (a*x^2*\tan[e + f*x])/(c*f*\sqrt{a - a*\sin[e + f*x]}*\sqrt{c + c*\sin[e + f*x]})$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 2282

`Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

Rule 2531

`Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))]*(f_.) + (g_.)*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

Rule 3475

`Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3757

Int[(x_)^(m_)*Sec[(a_) + (b_)*(x_)^(n_)]^(p_)*Tan[(a_) + (b_)*(x_)^(n_)]^(q_), x_Symbol] := Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x] - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4181

Int[csc[(e_) + Pi*(k_) + (f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 4184

Int[csc[(e_) + (f_)*(x_)]^2*((c_) + (d_)*(x_))^(m_), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 4186

Int[(csc[(e_) + (f_)*(x_)]*(b_))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

Rule 4413

Int[((c_) + (d_)*(x_))^(m_)*Sec[(a_) + (b_)*(x_)]*Tan[(a_) + (b_)*(x_)]^(p_), x_Symbol] := -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p/2, 0]

Rule 4604

Int[((g_) + (h_)*(x_))^(p_)*((a_) + (b_)*Sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*Sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*SIN[e + f*x])^FracPart[m]*(c + d*SIN[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c + d*SIN[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && IGtQ[n - m, 0]

Rule 6589

Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rule 6741

Int[u_, x_Symbol] :=> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

Rule 6742

Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2 \sqrt{a - a \sin(e + fx)}}{(c + c \sin(e + fx))^{3/2}} dx &= \frac{\cos(e + fx) \int x^2 \sec^3(e + fx)(a - a \sin(e + fx))^2 dx}{ac \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
 &= \frac{\cos(e + fx) \int a^2 x^2 \sec^3(e + fx)(1 - \sin(e + fx))^2 dx}{ac \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
 &= \frac{(a \cos(e + fx)) \int x^2 \sec^3(e + fx)(1 - \sin(e + fx))^2 dx}{c \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
 &= \frac{(a \cos(e + fx)) \int (x^2 \sec^3(e + fx) - 2x^2 \sec^2(e + fx) \tan(e + fx) + x^2 \sec(e + fx)) dx}{c \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
 &= \frac{(a \cos(e + fx)) \int x^2 \sec^3(e + fx) dx}{c \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} + \frac{(a \cos(e + fx)) \int x^2 \sec(e + fx) \tan^2(e + fx) dx}{c \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
 &= -\frac{ax}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} - \frac{ax^2 \sec(e + fx)}{cf \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
 &= -\frac{2ax}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} + \frac{iax^2 \tan^{-1}(e^{i(e+fx)}) \cos(e + fx)}{cf \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
 &= -\frac{2ax}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} + \frac{2a \tanh^{-1}(\sin(e + fx)) \cos(e + fx)}{cf^3 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
 &= -\frac{2ax}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} + \frac{2a \tanh^{-1}(\sin(e + fx)) \cos(e + fx)}{cf^3 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
 &= -\frac{2ax}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} + \frac{2a \tanh^{-1}(\sin(e + fx)) \cos(e + fx)}{cf^3 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} \\
 &= -\frac{2ax}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}} + \frac{2a \tanh^{-1}(\sin(e + fx)) \cos(e + fx)}{cf^3 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}
 \end{aligned}$$

Mathematica [C] time = 1.56, size = 154, normalized size = 0.55

$$\frac{\sqrt{a - a \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(-4 \log\left(e^{i(e+fx)} + i\right) + 2fx \cos(e + fx) + (2ifx - 4 \log\left(e^{i(e+fx)} + i\right)) \right)}{f^3 (c(\sin(e + fx) + 1))^{3/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]

[Out] -(((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[a - a*Sin[e + f*x]]*((2*I)*f*x + f^2*x^2 + 2*f*x*Cos[e + f*x] - 4*Log[I + E^(I*(e + f*x))]) + ((2*I)*f*x

$-4 \cdot \text{Log}[I + E^{(I \cdot (e + f \cdot x))}] \cdot \text{Sin}[e + f \cdot x]) / (f^3 \cdot (\text{Cos}[(e + f \cdot x)/2] - \text{Sin}[(e + f \cdot x)/2]) \cdot (c \cdot (1 + \text{Sin}[e + f \cdot x]))^{(3/2)})$

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a \sin(fx + e) + ax^2}}{(c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x^2/(c*sin(f*x + e) + c)^(3/2), x)

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{a - a \sin(fx + e)}}{(c + c \sin(fx + e))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)

[Out] int(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a \sin(fx + e) + ax^2}}{(c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x^2/(c*sin(f*x + e) + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sqrt{a - a \sin(e + fx)}}{(c + c \sin(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2),x)

[Out] `int((x^2*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{-a(\sin(e + fx) - 1)}}{(c(\sin(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(3/2),x)`

[Out] `Integral(x**2*sqrt(-a*(sin(e + f*x) - 1))/(c*(sin(e + f*x) + 1))**(3/2), x)`

$$3.183 \quad \int \frac{x \sqrt{a - a \sin(e + fx)}}{(c + c \sin(e + fx))^{3/2}} dx$$

Optimal. Leaf size=171

$$\frac{a \sin(e + fx)}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}} - \frac{a}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}} + \frac{ax \tan(e + fx)}{cf \sqrt{a - a \sin(e + fx)}}$$

[Out] -a/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-a*x*sec(f*x+e)/c/f/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+a*sin(f*x+e)/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+a*x*tan(f*x+e)/c/f/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.97, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {4604, 6741, 12, 6742, 4185, 4181, 2279, 2391, 3757, 3767, 8, 4413}

$$\frac{a \sin(e + fx)}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}} - \frac{a}{cf^2 \sqrt{a - a \sin(e + fx)} \sqrt{c \sin(e + fx) + c}} + \frac{ax \tan(e + fx)}{cf \sqrt{a - a \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(x*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]

[Out] -(a/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])) - (a*x*Sec[e + f*x])/(c*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*Sin[e + f*x])/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*x*Tan[e + f*x])/(c*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 3757

Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(n_.)]^(q_.), x_Symbol] := Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x] - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)^(n_.)], x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,

d}, x] && IGtQ[n/2, 0]

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Dist
[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.)), x_Symbol]
:> -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x]
+ (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x],
x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*Sec[(a_.) + (b_.)*(x_.)]*Tan[(a_.) + (b_.)*(x
_.)]^(p_.), x_Symbol]
:> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4604

```
Int[((g_.) + (h_.)*(x_.))^(p_.)*((a_.) + (b_.)*Sin[(e_.) + (f_.)*(x_.)])^(m_)
*((c_.) + (d_.)*Sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol]
:> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sine + f*x))^FracPart[m]*(c + d*Sine
+ f*x))^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Cos[e + f*x]
^(2*m)*(c + d*Sine + f*x)^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6741

```
Int[u_, x_Symbol]
:> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

Rule 6742

```
Int[u_, x_Symbol]
:> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}} dx &= \frac{\cos(e+fx) \int x \sec^3(e+fx)(a-a\sin(e+fx))^2 dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\
&= \frac{\cos(e+fx) \int a^2x \sec^3(e+fx)(1-\sin(e+fx))^2 dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\
&= \frac{(a\cos(e+fx)) \int x \sec^3(e+fx)(1-\sin(e+fx))^2 dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\
&= \frac{(a\cos(e+fx)) \int (x \sec^3(e+fx) - 2x \sec^2(e+fx) \tan(e+fx) + x \sec(e+fx) \tan^2(e+fx)) dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\
&= \frac{(a\cos(e+fx)) \int x \sec^3(e+fx) dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{(a\cos(e+fx)) \int x \sec(e+fx) \tan^2(e+fx) dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\
&= -\frac{a}{2cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax \sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\
&= -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{iax \tan^{-1}(e^{i(e+fx)}) \cos(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\
&= -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax \sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\
&= -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ia \cos(e+fx) \text{Li}_2(-ie^{i(e+fx)})}{2cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\
&= -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax \sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.60, size = 150, normalized size = 0.88

$$\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c(\sin(e+fx)+1)}\left(fx \sin\left(\frac{e}{2}\right) - \sin\left(\frac{e}{2}+fx\right) + \cos\left(\frac{e}{2}\right)(fx-1) + \cos\left(\frac{e}{2}+fx\right) + \sin\left(\frac{e}{2}+fx\right)\right)}{c^2f^2\left(\sin\left(\frac{e}{2}\right) + \cos\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]

[Out] -((((-1 + f*x)*Cos[e/2] + Cos[e/2 + f*x] + Sin[e/2] + f*x*Sin[e/2] - Sin[e/2 + f*x])*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]])/(c^2*f^2*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)

fricas [A] time = 0.56, size = 72, normalized size = 0.42

$$-\frac{(fx + \cos(fx + e))\sqrt{-a\sin(fx + e) + a}\sqrt{c\sin(fx + e) + c}}{c^2f^2\cos(fx + e)\sin(fx + e) + c^2f^2\cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] -(f*x + cos(f*x + e))*sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/(c^2*f^2*cos(f*x + e)*sin(f*x + e) + c^2*f^2*cos(f*x + e))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a \sin(fx + e) + ax}}{(c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x/(c*sin(f*x + e) + c)^(3/2), x)

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x \sqrt{a - a \sin(fx + e)}}{(c + c \sin(fx + e))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)

[Out] int(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a \sin(fx + e) + ax}}{(c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x/(c*sin(f*x + e) + c)^(3/2), x)

mupad [B] time = 3.32, size = 88, normalized size = 0.51

$$\frac{\sqrt{-a (\sin(e + fx) - 1)} (\cos(2e + 2fx) + 2fx \cos(e + fx) + 1 - \cos(e + fx) 2i - \sin(2e + 2fx) 1i)}{c f^2 (\cos(2e + 2fx) + 1) \sqrt{c (\sin(e + fx) + 1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2),x)

[Out] -((-a*(sin(e + f*x) - 1))^(1/2)*(cos(2*e + 2*f*x) - cos(e + f*x)*2i - sin(2*e + 2*f*x)*1i + 2*f*x*cos(e + f*x) + 1))/(c*f^2*(cos(2*e + 2*f*x) + 1)*(c*(sin(e + f*x) + 1))^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sqrt{-a (\sin(e + fx) - 1)}}{(c (\sin(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(3/2),x)

[Out] Integral(x*sqrt(-a*(sin(e + f*x) - 1))/(c*(sin(e + f*x) + 1))**(3/2), x)

$$3.184 \quad \int \frac{z^2 \sqrt{1+\cos(z)}}{\sqrt{1-\cos(z)}} dz$$

Optimal. Leaf size=300

$$\frac{2iz\text{Li}_2(-e^{iz})\sin(z)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}} - \frac{2iz\text{Li}_2(e^{iz})\sin(z)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}} - \frac{iz\text{Li}_2(e^{2iz})\sin(z)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}} - \frac{2\text{Li}_3(-e^{iz})\sin(z)}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}} + \sqrt{\dots}$$

```
[Out] -1/3*I*z^3*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)-2*z^2*arctanh(exp(I*z))
*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)+z^2*ln(1-exp(2*I*z))*sin(z)/(1-co
s(z))^(1/2)/(1+cos(z))^(1/2)+2*I*z*polylog(2,-exp(I*z))*sin(z)/(1-cos(z))^(
1/2)/(1+cos(z))^(1/2)-2*I*z*polylog(2,exp(I*z))*sin(z)/(1-cos(z))^(1/2)/(1+
cos(z))^(1/2)-I*z*polylog(2,exp(2*I*z))*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(
1/2)-2*polylog(3,-exp(I*z))*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)+2*pol
ylog(3,exp(I*z))*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)+1/2*polylog(3,exp
(2*I*z))*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4605, 6742, 3717, 2190, 2531, 2282, 6589, 4183}

$$\frac{2iz \sin(z) \text{PolyLog}(2, -e^{iz})}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}} - \frac{2iz \sin(z) \text{PolyLog}(2, e^{iz})}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}} - \frac{iz \sin(z) \text{PolyLog}(2, e^{2iz})}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}} - \frac{2 \sin(z) \text{PolyLog}(3, -e^{iz})}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(z^2*Sqrt[1 + Cos[z]])/Sqrt[1 - Cos[z]],z]
```

```
[Out] ((-I/3)*z^3*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - (2*z^2*ArcTanh[E^
(I*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + (z^2*Log[1 - E^((2*I)*
z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + ((2*I)*z*PolyLog[2, -E^(I
*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - ((2*I)*z*PolyLog[2, E^(I
*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - (I*z*PolyLog[2, E^((2*I)
*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - (2*PolyLog[3, -E^(I*z)]*
Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + (2*PolyLog[3, E^(I*z)]*Sin[z]
)/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + (PolyLog[3, E^((2*I)*z)]*Sin[z])/(2
*Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]])
```

Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
```

, g, n}, x] && GtQ[m, 0]

Rule 3717

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_.)], x_Symbol]
:= Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^(m)*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]
```

Rule 4605

```
Int[(Cos[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(Cos[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.)*((g_.) + (h_.)*(x_.))^(p_.), x_Symbol] := Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Cos[e + f*x])^FracPart[m]*(c + d*Cos[e + f*x])^FracPart[m])/Sin[e + f*x]^(2*FracPart[m]), Int[(g + h*x)^p*Sin[e + f*x]^(2*m)*(c + d*Cos[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && IGtQ[n - m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{z^2 \sqrt{1 + \cos(z)}}{\sqrt{1 - \cos(z)}} dz &= \frac{\sin(z) \int z^2 (1 + \cos(z)) \csc(z) dz}{\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} \\
&= \frac{\sin(z) \int (z^2 \cot(z) + z^2 \csc(z)) dz}{\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} \\
&= \frac{\sin(z) \int z^2 \cot(z) dz}{\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} + \frac{\sin(z) \int z^2 \csc(z) dz}{\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} \\
&= -\frac{iz^3 \sin(z)}{3\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} - \frac{2z^2 \tanh^{-1}(e^{iz}) \sin(z)}{\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} - \frac{(2i \sin(z)) \int \frac{e^{2iz} z^2}{1 - e^{2iz}} dz}{\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} \\
&= -\frac{iz^3 \sin(z)}{3\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} - \frac{2z^2 \tanh^{-1}(e^{iz}) \sin(z)}{\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} + \frac{z^2 \log(1 - e^{2iz}) \sin(z)}{\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} + \dots \\
&= -\frac{iz^3 \sin(z)}{3\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} - \frac{2z^2 \tanh^{-1}(e^{iz}) \sin(z)}{\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} + \frac{z^2 \log(1 - e^{2iz}) \sin(z)}{\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} + \dots \\
&= -\frac{iz^3 \sin(z)}{3\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} - \frac{2z^2 \tanh^{-1}(e^{iz}) \sin(z)}{\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} + \frac{z^2 \log(1 - e^{2iz}) \sin(z)}{\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} + \dots \\
&= -\frac{iz^3 \sin(z)}{3\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} - \frac{2z^2 \tanh^{-1}(e^{iz}) \sin(z)}{\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} + \frac{z^2 \log(1 - e^{2iz}) \sin(z)}{\sqrt{1 - \cos(z)} \sqrt{1 + \cos(z)}} + \dots
\end{aligned}$$

Mathematica [A] time = 0.09, size = 85, normalized size = 0.28

$$\frac{\sqrt{\cos(z) + 1} \tan\left(\frac{z}{2}\right) \left(12iz \operatorname{Li}_2(e^{-iz}) + 12\operatorname{Li}_3(e^{-iz}) + iz^3 + 6z^2 \log(1 - e^{-iz}) - i\pi^3\right)}{3\sqrt{1 - \cos(z)}}$$

Antiderivative was successfully verified.

[In] Integrate[(z^2*Sqrt[1 + Cos[z]])/Sqrt[1 - Cos[z]],z]

[Out] (Sqrt[1 + Cos[z]]*((-I)*Pi^3 + I*z^3 + 6*z^2*Log[1 - E^((-I)*z)] + (12*I)*z*PolyLog[2, E^((-I)*z)] + 12*PolyLog[3, E^((-I)*z)]*Tan[z/2])/(3*Sqrt[1 - Cos[z]])

fricas [C] time = 0.77, size = 75, normalized size = 0.25

$$z^2 \log(-\cos(z) + i \sin(z) + 1) + z^2 \log(-\cos(z) - i \sin(z) + 1) - 2iz \operatorname{Li}_2(\cos(z) + i \sin(z)) + 2iz \operatorname{Li}_2(\cos(z) - i \sin(z))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z, algorithm="fricas")

[Out] z^2*log(-cos(z) + I*sin(z) + 1) + z^2*log(-cos(z) - I*sin(z) + 1) - 2*I*z*dilog(cos(z) + I*sin(z)) + 2*I*z*dilog(cos(z) - I*sin(z)) + 2*polylog(3, cos(z) + I*sin(z)) + 2*polylog(3, cos(z) - I*sin(z))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{z^2 \sqrt{\cos(z) + 1}}{\sqrt{-\cos(z) + 1}} dz$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z, algorithm="giac")

[Out] integrate(z^2*sqrt(cos(z) + 1)/sqrt(-cos(z) + 1), z)

maple [A] time = 0.14, size = 154, normalized size = 0.51

$$\frac{(e^{iz} - 1) \sqrt{(e^{iz} + 1)^2 e^{-iz}} z^3}{3 \sqrt{-(e^{iz} - 1)^2 e^{-iz}} (e^{iz} + 1)} + \frac{2i (e^{iz} - 1) \sqrt{(e^{iz} + 1)^2 e^{-iz}} \left(\frac{iz^3}{3} - z^2 \ln(1 - e^{iz}) + 2iz \operatorname{polylog}(2, e^{iz}) - 2 \operatorname{polylog}(3, e^{iz}) \right)}{\sqrt{-(e^{iz} - 1)^2 e^{-iz}} (e^{iz} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2), z)

[Out] 1/3/(-(exp(I*z)-1)^2*exp(-I*z))^(1/2)*(exp(I*z)-1)*((exp(I*z)+1)^2*exp(-I*z))^(1/2)/(exp(I*z)+1)*z^3+2*I/(-(exp(I*z)-1)^2*exp(-I*z))^(1/2)*(exp(I*z)-1)*((exp(I*z)+1)^2*exp(-I*z))^(1/2)/(exp(I*z)+1)*(1/3*I*z^3-z^2*ln(1-exp(I*z)))+2*I*z*polylog(2,exp(I*z))-2*polylog(3,exp(I*z)))

maxima [A] time = 0.49, size = 56, normalized size = 0.19

$$\frac{1}{3}iz^3 + 2iz^2 \arctan(\sin(z), -\cos(z) + 1) - z^2 \log(\cos(z)^2 + \sin(z)^2 - 2\cos(z) + 1) + 4iz \operatorname{Li}_2(e^{iz}) - 4\operatorname{Li}_3(e^{iz})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2), z, algorithm="maxima")

[Out] 1/3*I*z^3 + 2*I*z^2*arctan2(sin(z), -cos(z) + 1) - z^2*log(cos(z)^2 + sin(z)^2 - 2*cos(z) + 1) + 4*I*z*dilog(e^(I*z)) - 4*polylog(3, e^(I*z))

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{z^2 \sqrt{\cos(z) + 1}}{\sqrt{1 - \cos(z)}} dz$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((z^2*(cos(z) + 1)^(1/2))/(1 - cos(z))^(1/2), z)

[Out] int((z^2*(cos(z) + 1)^(1/2))/(1 - cos(z))^(1/2), z)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{z^2 \sqrt{\cos(z) + 1}}{\sqrt{1 - \cos(z)}} dz$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(z**2*(1+cos(z))**(1/2)/(1-cos(z))**(1/2), z)

[Out] Integral(z**2*sqrt(cos(z) + 1)/sqrt(1 - cos(z)), z)

3.185 $\int (a + a \cos(x))(A + B \sec(x)) dx$

Optimal. Leaf size=18

$$ax(A + B) + aA \sin(x) + aB \tanh^{-1}(\sin(x))$$

[Out] a*(A+B)*x+a*B*arctanh(sin(x))+a*A*sin(x)

Rubi [A] time = 0.10, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2828, 2968, 3023, 2735, 3770}

$$ax(A + B) + aA \sin(x) + aB \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[x])*(A + B*Sec[x]),x]

[Out] a*(A + B)*x + a*B*ArcTanh[Sin[x]] + a*A*Sin[x]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2828

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ[n]

Rule 2968

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (a + a \cos(x))(A + B \sec(x)) dx &= \int (a + a \cos(x))(B + A \cos(x)) \sec(x) dx \\
&= \int (aB + (aA + aB) \cos(x) + aA \cos^2(x)) \sec(x) dx \\
&= aA \sin(x) + \int (aB + a(A + B) \cos(x)) \sec(x) dx \\
&= a(A + B)x + aA \sin(x) + (aB) \int \sec(x) dx \\
&= a(A + B)x + aB \tanh^{-1}(\sin(x)) + aA \sin(x)
\end{aligned}$$

Mathematica [B] time = 0.01, size = 51, normalized size = 2.83

$$aAx + aA \sin(x) + aBx - aB \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + aB \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Cos[x])*(A + B*Sec[x]),x]

[Out] a*A*x + a*B*x - a*B*Log[Cos[x/2] - Sin[x/2]] + a*B*Log[Cos[x/2] + Sin[x/2]] + a*A*Sin[x]

fricas [A] time = 0.67, size = 32, normalized size = 1.78

$$(A + B)ax + \frac{1}{2}Ba \log(\sin(x) + 1) - \frac{1}{2}Ba \log(-\sin(x) + 1) + Aa \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))*(A+B*sec(x)),x, algorithm="fricas")

[Out] (A + B)*a*x + 1/2*B*a*log(sin(x) + 1) - 1/2*B*a*log(-sin(x) + 1) + A*a*sin(x)

giac [B] time = 0.18, size = 51, normalized size = 2.83

$$Ba \log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right) - Ba \log\left(\left|\tan\left(\frac{1}{2}x\right) - 1\right|\right) + (Aa + Ba)x + \frac{2Aa \tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))*(A+B*sec(x)),x, algorithm="giac")

[Out] B*a*log(abs(tan(1/2*x) + 1)) - B*a*log(abs(tan(1/2*x) - 1)) + (A*a + B*a)*x + 2*A*a*tan(1/2*x)/(tan(1/2*x)^2 + 1)

maple [A] time = 0.10, size = 24, normalized size = 1.33

$$aA \sin(x) + Bax + aAx + Ba \ln(\sec(x) + \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(x))*(A+B*sec(x)),x)

[Out] a*A*sin(x)+B*a*x+a*A*x+B*a*ln(sec(x)+tan(x))

maxima [A] time = 0.31, size = 23, normalized size = 1.28

$$Aax + Bax + Ba \log(\sec(x) + \tan(x)) + Aa \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))*(A+B*sec(x)),x, algorithm="maxima")

[Out] A*a*x + B*a*x + B*a*log(sec(x) + tan(x)) + A*a*sin(x)

mupad [B] time = 2.48, size = 54, normalized size = 3.00

$$2 A a \operatorname{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right) + 2 B a \operatorname{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right) + 2 B a \operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right) + A a \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*cos(x))*(A + B/cos(x)),x)

[Out] 2*A*a*atan(sin(x/2)/cos(x/2)) + 2*B*a*atan(sin(x/2)/cos(x/2)) + 2*B*a*atanh(sin(x/2)/cos(x/2)) + A*a*sin(x)

sympy [A] time = 2.17, size = 27, normalized size = 1.50

$$Aax + Aa \sin(x) + Bax + Ba \log(\tan(x) + \sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))*(A+B*sec(x)),x)

[Out] A*a*x + A*a*sin(x) + B*a*x + B*a*log(tan(x) + sec(x))

3.186 $\int (a + a \cos(x))^2 (A + B \sec(x)) dx$

Optimal. Leaf size=57

$$\frac{1}{2}a^2x(3A + 4B) + \frac{1}{2}a^2(3A + 2B)\sin(x) + \frac{1}{2}A\sin(x)(a^2\cos(x) + a^2) + a^2B\tanh^{-1}(\sin(x))$$

[Out] $1/2*a^2*(3*A+4*B)*x+a^2*B*\operatorname{arctanh}(\sin(x))+1/2*a^2*(3*A+2*B)*\sin(x)+1/2*A*(a^2+a^2*\cos(x))*\sin(x)$

Rubi [A] time = 0.20, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2828, 2976, 2968, 3023, 2735, 3770}

$$\frac{1}{2}a^2x(3A + 4B) + \frac{1}{2}a^2(3A + 2B)\sin(x) + \frac{1}{2}A\sin(x)(a^2\cos(x) + a^2) + a^2B\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[x])^2*(A + B*\operatorname{Sec}[x]),x]$

[Out] $(a^2*(3*A + 4*B)*x)/2 + a^2*B*\operatorname{ArcTanh}[\operatorname{Sin}[x]] + (a^2*(3*A + 2*B)*\operatorname{Sin}[x])/2 + (A*(a^2 + a^2*\operatorname{Cos}[x])*\operatorname{Sin}[x])/2$

Rule 2735

$\operatorname{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]) / ((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \operatorname{Simp}[(b*x)/d, x] - \operatorname{Dist}[(b*c - a*d)/d, \operatorname{Int}[1/(c + d*\sin[e + f*x]), x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f\}, x\} \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0]$

Rule 2828

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)])*(d_.) + (c_.)]^{(n_.)} * ((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}, x_Symbol] \rightarrow \operatorname{Int}[(a + b*\sin[e + f*x])^m * (d + c*\sin[e + f*x])^n / \sin[e + f*x]^n, x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x\} \ \&\& \ \operatorname{IntegerQ}[n]$

Rule 2968

$\operatorname{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)} * ((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)]) * ((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \operatorname{Int}[(a + b*\sin[e + f*x])^m * (A*c + (B*c + A*d)*\sin[e + f*x] + B*d*\sin[e + f*x]^2), x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x\} \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0]$

Rule 2976

$\operatorname{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)} * ((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)]) * ((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*B*\operatorname{Cos}[e + f*x] * (a + b*\sin[e + f*x])^{(m-1)} * (c + d*\sin[e + f*x])^{(n+1)}) / (d*f*(m+n+1)), x] + \operatorname{Dist}[1/(d*(m+n+1)), \operatorname{Int}[(a + b*\sin[e + f*x])^{(m-1)} * (c + d*\sin[e + f*x])^n * \operatorname{Simp}[a*A*d*(m+n+1) + B*(a*c*(m-1) + b*d*(n+1)) + (A*b*d*(m+n+1) - B*(b*c*m - a*d*(2*m+n))]*\sin[e + f*x], x], x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x\} \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \ \operatorname{NeQ}[c^2 - d^2, 0] \ \&\& \ \operatorname{GtQ}[m, 1/2] \ \&\& \ \operatorname{!LtQ}[n, -1] \ \&\& \ \operatorname{IntegerQ}[2*m] \ \&\& \ (\operatorname{IntegerQ}[2*n] \ || \ \operatorname{EqQ}[c, 0])$

Rule 3023

$\operatorname{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)} * ((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)]) + (C_.)*\sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] \rightarrow -\operatorname{Simp}[(C*\operatorname{Cos}[e + f*x] * (a + b*\sin[e + f*x])^{(m+1)}) / (b*f*(m+2)), x] + \operatorname{Dist}[1/(b*(m +$

2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int (a + a \cos(x))^2 (A + B \sec(x)) dx &= \int (a + a \cos(x))^2 (B + A \cos(x)) \sec(x) dx \\
 &= \frac{1}{2} A (a^2 + a^2 \cos(x)) \sin(x) + \frac{1}{2} \int (a + a \cos(x)) (2aB + a(3A + 2B) \cos(x)) dx \\
 &= \frac{1}{2} A (a^2 + a^2 \cos(x)) \sin(x) + \frac{1}{2} \int (2a^2 B + (2a^2 B + a^2(3A + 2B)) \cos(x)) dx \\
 &= \frac{1}{2} a^2 (3A + 2B) \sin(x) + \frac{1}{2} A (a^2 + a^2 \cos(x)) \sin(x) + \frac{1}{2} \int (2a^2 B + a^2(3A + 2B)) \cos(x) dx \\
 &= \frac{1}{2} a^2 (3A + 4B)x + \frac{1}{2} a^2 (3A + 2B) \sin(x) + \frac{1}{2} A (a^2 + a^2 \cos(x)) \sin(x) + \frac{1}{2} \int (2a^2 B + a^2(3A + 2B)) \cos(x) dx \\
 &= \frac{1}{2} a^2 (3A + 4B)x + a^2 B \tanh^{-1}(\sin(x)) + \frac{1}{2} a^2 (3A + 2B) \sin(x) + \frac{1}{2} A (a^2 + a^2 \cos(x)) \sin(x)
 \end{aligned}$$

Mathematica [A] time = 0.08, size = 67, normalized size = 1.18

$$\frac{1}{4} a^2 \left(4(2A + B) \sin(x) + 6Ax + A \sin(2x) + 8Bx - 4B \log \left(\cos \left(\frac{x}{2} \right) - \sin \left(\frac{x}{2} \right) \right) + 4B \log \left(\sin \left(\frac{x}{2} \right) + \cos \left(\frac{x}{2} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Cos[x])^2*(A + B*Sec[x]), x]

[Out] (a^2*(6*A*x + 8*B*x - 4*B*Log[Cos[x/2] - Sin[x/2]] + 4*B*Log[Cos[x/2] + Sin[x/2]] + 4*(2*A + B)*Sin[x] + A*Sin[2*x]))/4

fricas [A] time = 0.77, size = 60, normalized size = 1.05

$$\frac{1}{2} (3A + 4B) a^2 x + \frac{1}{2} B a^2 \log(\sin(x) + 1) - \frac{1}{2} B a^2 \log(-\sin(x) + 1) + \frac{1}{2} (A a^2 \cos(x) + 2(2A + B) a^2) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^2*(A+B*sec(x)), x, algorithm="fricas")

[Out] 1/2*(3*A + 4*B)*a^2*x + 1/2*B*a^2*log(sin(x) + 1) - 1/2*B*a^2*log(-sin(x) + 1) + 1/2*(A*a^2*cos(x) + 2*(2*A + B)*a^2)*sin(x)

giac [A] time = 0.16, size = 100, normalized size = 1.75

$$B a^2 \log \left(\left| \tan \left(\frac{1}{2} x \right) + 1 \right| \right) - B a^2 \log \left(\left| \tan \left(\frac{1}{2} x \right) - 1 \right| \right) + \frac{1}{2} (3 A a^2 + 4 B a^2) x + \frac{3 A a^2 \tan \left(\frac{1}{2} x \right)^3 + 2 B a^2 \tan \left(\frac{1}{2} x \right)^3}{\left(\tan \left(\frac{1}{2} x \right) \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^2*(A+B*sec(x)), x, algorithm="giac")

[Out] $B*a^2*\log(\text{abs}(\tan(1/2*x) + 1)) - B*a^2*\log(\text{abs}(\tan(1/2*x) - 1)) + 1/2*(3*A*a^2 + 4*B*a^2)*x + (3*A*a^2*\tan(1/2*x)^3 + 2*B*a^2*\tan(1/2*x)^3 + 5*A*a^2*\tan(1/2*x) + 2*B*a^2*\tan(1/2*x))/(\tan(1/2*x)^2 + 1)^2$

maple [A] time = 0.10, size = 52, normalized size = 0.91

$$\frac{a^2 A \sin(x) \cos(x)}{2} + \frac{3a^2 Ax}{2} + a^2 B \sin(x) + 2a^2 A \sin(x) + 2a^2 Bx + a^2 B \ln(\sec(x) + \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*cos(x))^2*(A+B*sec(x)),x)`

[Out] $1/2*a^2*A*\sin(x)*\cos(x)+3/2*a^2*A*x+a^2*B*\sin(x)+2*a^2*A*\sin(x)+2*a^2*B*x+a^2*B*\ln(\sec(x)+\tan(x))$

maxima [A] time = 0.32, size = 54, normalized size = 0.95

$$\frac{1}{4} Aa^2(2x + \sin(2x)) + Aa^2x + 2Ba^2x + Ba^2 \log(\sec(x) + \tan(x)) + 2Aa^2 \sin(x) + Ba^2 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cos(x))^2*(A+B*sec(x)),x, algorithm="maxima")`

[Out] $1/4*A*a^2*(2*x + \sin(2*x)) + A*a^2*x + 2*B*a^2*x + B*a^2*\log(\sec(x) + \tan(x)) + 2*A*a^2*\sin(x) + B*a^2*\sin(x)$

mupad [B] time = 2.46, size = 403, normalized size = 7.07

$$\frac{(3Aa^2 + 2Ba^2) \tan\left(\frac{x}{2}\right)^3 + (5Aa^2 + 2Ba^2) \tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^4 + 2 \tan\left(\frac{x}{2}\right)^2 + 1} + a^2 \operatorname{atan}\left(\frac{216A^3a^6 \tan\left(\frac{x}{2}\right)}{216A^3a^6 + 864A^2Ba^6 + 1248AB^2a^6 + 640B^3a^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cos(x))^2*(A + B/cos(x)),x)`

[Out] $(\tan(x/2)^3*(3*A*a^2 + 2*B*a^2) + \tan(x/2)*(5*A*a^2 + 2*B*a^2))/((2*\tan(x/2)^2 + \tan(x/2)^4 + 1) + a^2*\operatorname{atan}((216*A^3*a^6*\tan(x/2))/(216*A^3*a^6 + 640*B^3*a^6 + 1248*A*B^2*a^6 + 864*A^2*B*a^6) + (640*B^3*a^6*\tan(x/2))/(216*A^3*a^6 + 640*B^3*a^6 + 1248*A*B^2*a^6 + 864*A^2*B*a^6) + (1248*A*B^2*a^6*\tan(x/2))/(216*A^3*a^6 + 640*B^3*a^6 + 1248*A*B^2*a^6 + 864*A^2*B*a^6) + (864*A^2*B*a^6*\tan(x/2))/(216*A^3*a^6 + 640*B^3*a^6 + 1248*A*B^2*a^6 + 864*A^2*B*a^6))*(3*A + 4*B) + 2*B*a^2*\operatorname{atanh}((320*B^3*a^6*\tan(x/2))/(320*B^3*a^6 + 384*A*B^2*a^6 + 144*A^2*B*a^6) + (384*A*B^2*a^6*\tan(x/2))/(320*B^3*a^6 + 384*A*B^2*a^6 + 144*A^2*B*a^6) + (144*A^2*B*a^6*\tan(x/2))/(320*B^3*a^6 + 384*A*B^2*a^6 + 144*A^2*B*a^6))$

sympy [A] time = 3.54, size = 61, normalized size = 1.07

$$\frac{3Aa^2x}{2} + 2Aa^2 \sin(x) + \frac{Aa^2 \sin(2x)}{4} + 2Ba^2x + Ba^2 \log(\tan(x) + \sec(x)) + Ba^2 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cos(x))**2*(A+B*sec(x)),x)`

[Out] $3*A*a**2*x/2 + 2*A*a**2*\sin(x) + A*a**2*\sin(2*x)/4 + 2*B*a**2*x + B*a**2*\log(\tan(x) + \sec(x)) + B*a**2*\sin(x)$

3.187 $\int (a + a \cos(x))^3 (A + B \sec(x)) dx$

Optimal. Leaf size=75

$$\frac{1}{2}a^3x(5A+7B)+\frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{6}(5A+3B)\sin(x)\left(a^3\cos(x)+a^3\right)+a^3B\tanh^{-1}(\sin(x))+\frac{1}{3}aA\sin(x)(a\cos(x)+a^3)$$

[Out] 1/2*a^3*(5*A+7*B)*x+a^3*B*arctanh(sin(x))+5/2*a^3*(A+B)*sin(x)+1/3*a*A*(a+a*cos(x))^2*sin(x)+1/6*(5*A+3*B)*(a^3+a^3*cos(x))*sin(x)

Rubi [A] time = 0.30, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2828, 2976, 2968, 3023, 2735, 3770}

$$\frac{1}{2}a^3x(5A+7B)+\frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{6}(5A+3B)\sin(x)\left(a^3\cos(x)+a^3\right)+a^3B\tanh^{-1}(\sin(x))+\frac{1}{3}aA\sin(x)(a\cos(x)+a^3)$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[x])^3*(A + B*Sec[x]),x]

[Out] (a^3*(5*A + 7*B)*x)/2 + a^3*B*ArcTanh[Sin[x]] + (5*a^3*(A + B)*Sin[x])/2 + (a*A*(a + a*Cos[x])^2*Ssin[x])/3 + ((5*A + 3*B)*(a^3 + a^3*Cos[x])*Sin[x])/6

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2828

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ[n]

Rule 2968

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 2976

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> -Simp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m-1)*(c + d*Sin[e + f*x])^(n+1))/(d*f*(m+n+1)), x] + Dist[1/(d*(m+n+1)), Int[(a + b*Sin[e + f*x])^(m-1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m+n+1) + B*(a*c*(m-1) + b*d*(n+1)) + (A*b*d*(m+n+1) - B*(b*c*m - a*d*(2*m+n)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m +

2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (a + a \cos(x))^3 (A + B \sec(x)) dx &= \int (a + a \cos(x))^3 (B + A \cos(x)) \sec(x) dx \\ &= \frac{1}{3} a A (a + a \cos(x))^2 \sin(x) + \frac{1}{3} \int (a + a \cos(x))^2 (3aB + a(5A + 3B) \cos(x)) dx \\ &= \frac{1}{3} a A (a + a \cos(x))^2 \sin(x) + \frac{1}{6} (5A + 3B) (a^3 + a^3 \cos(x)) \sin(x) + \frac{1}{6} \int (a + a \cos(x))^2 (3aB + a(5A + 3B) \cos(x)) dx \\ &= \frac{1}{3} a A (a + a \cos(x))^2 \sin(x) + \frac{1}{6} (5A + 3B) (a^3 + a^3 \cos(x)) \sin(x) + \frac{1}{6} \int (6a^2 B + 5a^2 A \cos(x)) dx \\ &= \frac{5}{2} a^3 (A + B) \sin(x) + \frac{1}{3} a A (a + a \cos(x))^2 \sin(x) + \frac{1}{6} (5A + 3B) (a^3 + a^3 \cos(x)) \sin(x) \\ &= \frac{1}{2} a^3 (5A + 7B)x + \frac{5}{2} a^3 (A + B) \sin(x) + \frac{1}{3} a A (a + a \cos(x))^2 \sin(x) + \frac{1}{6} (5A + 3B) (a^3 + a^3 \cos(x)) \sin(x) \\ &= \frac{1}{2} a^3 (5A + 7B)x + a^3 B \tanh^{-1}(\sin(x)) + \frac{5}{2} a^3 (A + B) \sin(x) + \frac{1}{3} a A (a + a \cos(x))^2 \sin(x) \end{aligned}$$

Mathematica [A] time = 0.10, size = 80, normalized size = 1.07

$$\frac{1}{12} a^3 \left(9(5A + 4B) \sin(x) + 3(3A + B) \sin(2x) + 30Ax + A \sin(3x) + 42Bx - 12B \log \left(\cos \left(\frac{x}{2} \right) - \sin \left(\frac{x}{2} \right) \right) + 12B \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Cos[x])^3*(A + B*Sec[x]), x]

[Out] (a^3*(30*A*x + 42*B*x - 12*B*Log[Cos[x/2] - Sin[x/2]] + 12*B*Log[Cos[x/2] + Sin[x/2]] + 9*(5*A + 4*B)*Sin[x] + 3*(3*A + B)*Sin[2*x] + A*Ssin[3*x]))/12

fricas [A] time = 1.19, size = 77, normalized size = 1.03

$$\frac{1}{2} (5A + 7B) a^3 x + \frac{1}{2} B a^3 \log(\sin(x) + 1) - \frac{1}{2} B a^3 \log(-\sin(x) + 1) + \frac{1}{6} (2Aa^3 \cos(x)^2 + 3(3A + B)a^3 \cos(x) + 2(11A + 9B)a^3) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^3*(A+B*sec(x)), x, algorithm="fricas")

[Out] 1/2*(5*A + 7*B)*a^3*x + 1/2*B*a^3*log(sin(x) + 1) - 1/2*B*a^3*log(-sin(x) + 1) + 1/6*(2*A*a^3*cos(x)^2 + 3*(3*A + B)*a^3*cos(x) + 2*(11*A + 9*B)*a^3)*sin(x)

giac [A] time = 0.18, size = 125, normalized size = 1.67

$$Ba^3 \log \left(\left| \tan \left(\frac{1}{2} x \right) + 1 \right| \right) - Ba^3 \log \left(\left| \tan \left(\frac{1}{2} x \right) - 1 \right| \right) + \frac{1}{2} (5Aa^3 + 7Ba^3)x + \frac{15Aa^3 \tan \left(\frac{1}{2} x \right)^5 + 15Ba^3 \tan \left(\frac{1}{2} x \right)^5}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^3*(A+B*sec(x)),x, algorithm="giac")

[Out] $Ba^3 \log(\abs{\tan(1/2*x) + 1}) - Ba^3 \log(\abs{\tan(1/2*x) - 1}) + 1/2*(5Aa^3 + 7Ba^3)*x + 1/3*(15Aa^3*\tan(1/2*x)^5 + 15Ba^3*\tan(1/2*x)^5 + 40Aa^3*\tan(1/2*x)^3 + 36Ba^3*\tan(1/2*x)^3 + 33Aa^3*\tan(1/2*x) + 21Ba^3*\tan(1/2*x))/(\tan(1/2*x)^2 + 1)^3$

maple [A] time = 0.11, size = 77, normalized size = 1.03

$$\frac{Aa^3(2 + \cos^2(x))\sin(x)}{3} + \frac{Ba^3\sin(x)\cos(x)}{2} + \frac{7Ba^3x}{2} + \frac{3Aa^3\sin(x)\cos(x)}{2} + \frac{5Aa^3x}{2} + 3Ba^3\sin(x) + 3Aa^3\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(x))^3*(A+B*sec(x)),x)

[Out] $1/3Aa^3*(2+\cos(x)^2)*\sin(x) + 1/2Ba^3*\sin(x)*\cos(x) + 7/2Ba^3*x + 3/2Aa^3*\sin(x)*\cos(x) + 5/2Aa^3*x + 3Ba^3*\sin(x) + 3Aa^3*\sin(x) + Ba^3*\ln(\sec(x) + \tan(x))$

maxima [A] time = 0.31, size = 84, normalized size = 1.12

$$-\frac{1}{3}(\sin(x)^3 - 3\sin(x))Aa^3 + \frac{3}{4}Aa^3(2x + \sin(2x)) + \frac{1}{4}Ba^3(2x + \sin(2x)) + Aa^3x + 3Ba^3x + Ba^3 \log(\sec(x) + \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^3*(A+B*sec(x)),x, algorithm="maxima")

[Out] $-1/3*(\sin(x)^3 - 3*\sin(x))*Aa^3 + 3/4Aa^3*(2*x + \sin(2*x)) + 1/4Ba^3*(2*x + \sin(2*x)) + Aa^3*x + 3Ba^3*x + Ba^3*\log(\sec(x) + \tan(x)) + 3Aa^3*\sin(x) + 3Ba^3*\sin(x)$

mupad [B] time = 2.48, size = 431, normalized size = 5.75

$$\frac{(5Aa^3 + 5Ba^3)\tan\left(\frac{x}{2}\right)^5 + \left(\frac{40Aa^3}{3} + 12Ba^3\right)\tan\left(\frac{x}{2}\right)^3 + (11Aa^3 + 7Ba^3)\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^6 + 3\tan\left(\frac{x}{2}\right)^4 + 3\tan\left(\frac{x}{2}\right)^2 + 1} + a^3 \operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{1000A^3a^9 + 400A^2Ba^9 + 400A^2Ba^9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*cos(x))^3*(A + B/cos(x)),x)

[Out] $(\tan(x/2)^5*(5Aa^3 + 5Ba^3) + \tan(x/2)^3*((40Aa^3)/3 + 12Ba^3) + \tan(x/2)*(11Aa^3 + 7Ba^3))/((3*\tan(x/2)^2 + 3*\tan(x/2)^4 + \tan(x/2)^6 + 1) + a^3*\operatorname{atan}((1000A^3a^9*\tan(x/2))/(1000A^3a^9 + 2968B^3a^9 + 6040A^2Ba^9 + 4200A^2Ba^9) + (2968B^3a^9*\tan(x/2))/(1000A^3a^9 + 2968B^3a^9 + 6040A^2Ba^9 + 4200A^2Ba^9) + (6040A^2Ba^9*\tan(x/2))/(1000A^3a^9 + 2968B^3a^9 + 6040A^2Ba^9 + 4200A^2Ba^9) + (4200A^2Ba^9*\tan(x/2))/(1000A^3a^9 + 2968B^3a^9 + 6040A^2Ba^9 + 4200A^2Ba^9))*(5A + 7B) + 2Ba^3*\operatorname{atanh}((848B^3a^9*\tan(x/2))/(848B^3a^9 + 1120A^2Ba^9 + 400A^2Ba^9) + (1120A^2Ba^9*\tan(x/2))/(848B^3a^9 + 1120A^2Ba^9 + 400A^2Ba^9) + (400A^2Ba^9*\tan(x/2))/(848B^3a^9 + 1120A^2Ba^9 + 400A^2Ba^9))$

sympy [A] time = 7.63, size = 92, normalized size = 1.23

$$\frac{5Aa^3x}{2} - \frac{Aa^3\sin^3(x)}{3} + 4Aa^3\sin(x) + \frac{3Aa^3\sin(2x)}{4} + \frac{7Ba^3x}{2} + Ba^3 \log(\tan(x) + \sec(x)) + \frac{Ba^3\sin(x)\cos(x)}{2} + 3Ba^3\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^3*(A+B*sec(x)),x)

```
[Out] 5*A*a**3*x/2 - A*a**3*sin(x)**3/3 + 4*A*a**3*sin(x) + 3*A*a**3*sin(2*x)/4 +  
7*B*a**3*x/2 + B*a**3*log(tan(x) + sec(x)) + B*a**3*sin(x)*cos(x)/2 + 3*B*  
a**3*sin(x)
```


3.188 $\int (a + a \cos(x))^4 (A + B \sec(x)) dx$

Optimal. Leaf size=104

$$\frac{1}{8}a^4x(35A+48B)+\frac{5}{8}a^4(7A+8B)\sin(x)+\frac{1}{24}(35A+32B)\sin(x)(a^4\cos(x)+a^4)+a^4B\tanh^{-1}(\sin(x))+\frac{1}{12}(7A+4B)(a^2+a^2\cos(x))^2\sin(x)+\frac{1}{24}(35A+32B)(a^4+a^4\cos(x))\sin(x)$$

[Out] 1/8*a^4*(35*A+48*B)*x+a^4*B*arctanh(sin(x))+5/8*a^4*(7*A+8*B)*sin(x)+1/4*a*A*(a+a*cos(x))^3*sin(x)+1/12*(7*A+4*B)*(a^2+a^2*cos(x))^2*sin(x)+1/24*(35*A+32*B)*(a^4+a^4*cos(x))*sin(x)

Rubi [A] time = 0.40, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2828, 2976, 2968, 3023, 2735, 3770}

$$\frac{1}{8}a^4x(35A+48B)+\frac{5}{8}a^4(7A+8B)\sin(x)+\frac{1}{12}(7A+4B)\sin(x)(a^2\cos(x)+a^2)^2+\frac{1}{24}(35A+32B)\sin(x)(a^4\cos(x)+a^4)$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[x])^4*(A + B*Sec[x]),x]

[Out] (a^4*(35*A + 48*B)*x)/8 + a^4*B*ArcTanh[Sin[x]] + (5*a^4*(7*A + 8*B)*Sin[x])/8 + (a*A*(a + a*Cos[x])^3*Ssin[x])/4 + ((7*A + 4*B)*(a^2 + a^2*Cos[x])^2*Ssin[x])/12 + ((35*A + 32*B)*(a^4 + a^4*Cos[x])*Sin[x])/24

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2828

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ[n]

Rule 2968

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 2976

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m-1)*(c + d*Sin[e + f*x])^(n+1))/(d*f*(m+n+1)), x] + Dist[1/(d*(m+n+1)), Int[(a + b*Sin[e + f*x])^(m-1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m+n+1) + B*(a*c*(m-1) + b*d*(n+1)) + (A*b*d*(m+n+1) - B*(b*c*m - a*d*(2*m+n)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int (a + a \cos(x))^4 (A + B \sec(x)) dx &= \int (a + a \cos(x))^4 (B + A \cos(x)) \sec(x) dx \\
 &= \frac{1}{4} a A (a + a \cos(x))^3 \sin(x) + \frac{1}{4} \int (a + a \cos(x))^3 (4aB + a(7A + 4B) \cos(x)) dx \\
 &= \frac{1}{4} a A (a + a \cos(x))^3 \sin(x) + \frac{1}{12} (7A + 4B) (a^2 + a^2 \cos(x))^2 \sin(x) + \frac{1}{12} \int (a + a \cos(x))^2 (4aB + a(7A + 4B) \cos(x)) dx \\
 &= \frac{1}{4} a A (a + a \cos(x))^3 \sin(x) + \frac{1}{12} (7A + 4B) (a^2 + a^2 \cos(x))^2 \sin(x) + \frac{1}{24} (35A + 48B) a^2 \sin(x) + \frac{1}{24} \int (a + a \cos(x)) (4aB + a(7A + 4B) \cos(x)) dx \\
 &= \frac{1}{4} a A (a + a \cos(x))^3 \sin(x) + \frac{1}{12} (7A + 4B) (a^2 + a^2 \cos(x))^2 \sin(x) + \frac{1}{24} (35A + 48B) a^2 \sin(x) + \frac{1}{24} (4aB + a(7A + 4B) \cos(x)) dx \\
 &= \frac{5}{8} a^4 (7A + 8B) \sin(x) + \frac{1}{4} a A (a + a \cos(x))^3 \sin(x) + \frac{1}{12} (7A + 4B) (a^2 + a^2 \cos(x))^2 \sin(x) + \frac{1}{24} (4aB + a(7A + 4B) \cos(x)) dx \\
 &= \frac{1}{8} a^4 (35A + 48B) x + \frac{5}{8} a^4 (7A + 8B) \sin(x) + \frac{1}{4} a A (a + a \cos(x))^3 \sin(x) + \frac{1}{12} (7A + 4B) (a^2 + a^2 \cos(x))^2 \sin(x) + \frac{1}{24} (4aB + a(7A + 4B) \cos(x)) dx \\
 &= \frac{1}{8} a^4 (35A + 48B) x + a^4 B \tanh^{-1}(\sin(x)) + \frac{5}{8} a^4 (7A + 8B) \sin(x) + \frac{1}{4} a A (a + a \cos(x))^3 \sin(x) + \frac{1}{12} (7A + 4B) (a^2 + a^2 \cos(x))^2 \sin(x) + \frac{1}{24} (4aB + a(7A + 4B) \cos(x)) dx
 \end{aligned}$$

Mathematica [A] time = 0.12, size = 97, normalized size = 0.93

$$\frac{1}{96} a^4 \left(24(28A + 27B) \sin(x) + 24(7A + 4B) \sin(2x) + 420Ax + 32A \sin(3x) + 3A \sin(4x) + 576Bx + 8B \sin(3x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^4*(A + B*Sec[x]), x]
```

```
[Out] (a^4*(420*A*x + 576*B*x - 96*B*Log[Cos[x/2] - Sin[x/2]] + 96*B*Log[Cos[x/2] + Sin[x/2]] + 24*(28*A + 27*B)*Sin[x] + 24*(7*A + 4*B)*Sin[2*x] + 32*A*Sine[3*x] + 8*B*Sine[3*x] + 3*A*Sine[4*x]))/96
```

fricas [A] time = 0.70, size = 89, normalized size = 0.86

$$\frac{1}{8} (35A + 48B) a^4 x + \frac{1}{2} B a^4 \log(\sin(x) + 1) - \frac{1}{2} B a^4 \log(-\sin(x) + 1) + \frac{1}{24} (6Aa^4 \cos(x)^3 + 8(4A + B)a^4 \cos(x)^2 + 12Aa^4 \cos(x) + 160(A + B)a^4) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^4*(A+B*sec(x)), x, algorithm="fricas")
```

```
[Out] 1/8*(35*A + 48*B)*a^4*x + 1/2*B*a^4*log(sin(x) + 1) - 1/2*B*a^4*log(-sin(x) + 1) + 1/24*(6*A*a^4*cos(x)^3 + 8*(4*A + B)*a^4*cos(x)^2 + 3*(27*A + 16*B)*a^4*cos(x) + 160*(A + B)*a^4)*sin(x)
```

giac [A] time = 0.16, size = 149, normalized size = 1.43

$$B a^4 \log\left(\left|\tan\left(\frac{1}{2} x\right) + 1\right|\right) - B a^4 \log\left(\left|\tan\left(\frac{1}{2} x\right) - 1\right|\right) + \frac{1}{8} (35 A a^4 + 48 B a^4) x + \frac{105 A a^4 \tan\left(\frac{1}{2} x\right)^7 + 120 B a^4 \tan\left(\frac{1}{2} x\right)^6 + 105 A a^4 \tan\left(\frac{1}{2} x\right)^5 + 120 B a^4 \tan\left(\frac{1}{2} x\right)^4 + 105 A a^4 \tan\left(\frac{1}{2} x\right)^3 + 120 B a^4 \tan\left(\frac{1}{2} x\right)^2 + 105 A a^4 \tan\left(\frac{1}{2} x\right) + 120 B a^4}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^4*(A+B*sec(x)),x, algorithm="giac")

[Out] $B*a^4*\log(\text{abs}(\tan(1/2*x) + 1)) - B*a^4*\log(\text{abs}(\tan(1/2*x) - 1)) + 1/8*(35*A*a^4 + 48*B*a^4)*x + 1/12*(105*A*a^4*\tan(1/2*x)^7 + 120*B*a^4*\tan(1/2*x)^7 + 385*A*a^4*\tan(1/2*x)^5 + 424*B*a^4*\tan(1/2*x)^5 + 511*A*a^4*\tan(1/2*x)^3 + 520*B*a^4*\tan(1/2*x)^3 + 279*A*a^4*\tan(1/2*x) + 216*B*a^4*\tan(1/2*x))/(\tan(1/2*x)^2 + 1)^4$

maple [A] time = 0.12, size = 103, normalized size = 0.99

$$\frac{A a^4 \sin(x) (\cos^3(x))}{4} + \frac{27 A a^4 \sin(x) \cos(x)}{8} + \frac{35 A a^4 x}{8} + \frac{B a^4 (2 + \cos^2(x)) \sin(x)}{3} + \frac{4 A a^4 (2 + \cos^2(x)) \sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(x))^4*(A+B*sec(x)),x)

[Out] $1/4*A*a^4*\sin(x)*\cos(x)^3+27/8*A*a^4*\sin(x)*\cos(x)+35/8*A*a^4*x+1/3*B*a^4*(2+\cos(x)^2)*\sin(x)+4/3*A*a^4*(2+\cos(x)^2)*\sin(x)+2*B*a^4*\sin(x)*\cos(x)+6*B*a^4*x+6*B*a^4*\sin(x)+4*A*a^4*\sin(x)+B*a^4*\ln(\sec(x)+\tan(x))$

maxima [A] time = 0.31, size = 118, normalized size = 1.13

$$-\frac{4}{3}(\sin(x)^3 - 3 \sin(x))Aa^4 - \frac{1}{3}(\sin(x)^3 - 3 \sin(x))Ba^4 + \frac{1}{32}Aa^4(12x + \sin(4x) + 8 \sin(2x)) + \frac{3}{2}Aa^4(2x + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^4*(A+B*sec(x)),x, algorithm="maxima")

[Out] $-4/3*(\sin(x)^3 - 3*\sin(x))*A*a^4 - 1/3*(\sin(x)^3 - 3*\sin(x))*B*a^4 + 1/32*A*a^4*(12*x + \sin(4*x) + 8*\sin(2*x)) + 3/2*A*a^4*(2*x + \sin(2*x)) + B*a^4*(2*x + \sin(2*x)) + A*a^4*x + 4*B*a^4*x + B*a^4*\log(\sec(x) + \tan(x)) + 4*A*a^4*\sin(x) + 6*B*a^4*\sin(x)$

mupad [B] time = 2.52, size = 460, normalized size = 4.42

$$\frac{\left(\frac{35 A a^4}{4} + 10 B a^4\right) \tan\left(\frac{x}{2}\right)^7 + \left(\frac{385 A a^4}{12} + \frac{106 B a^4}{3}\right) \tan\left(\frac{x}{2}\right)^5 + \left(\frac{511 A a^4}{12} + \frac{130 B a^4}{3}\right) \tan\left(\frac{x}{2}\right)^3 + \left(\frac{93 A a^4}{4} + 18 B a^4\right) \tan\left(\frac{x}{2}\right) + 4 A a^4 x + 4 B a^4 x + B a^4 \log(\sec(x) + \tan(x)) + 4 A a^4 \sin(x) + 6 B a^4 \sin(x)}{\tan\left(\frac{x}{2}\right)^8 + 4 \tan\left(\frac{x}{2}\right)^6 + 6 \tan\left(\frac{x}{2}\right)^4 + 4 \tan\left(\frac{x}{2}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*cos(x))^4*(A + B/cos(x)),x)

[Out] $(\tan(x/2)^7*((35*A*a^4)/4 + 10*B*a^4) + \tan(x/2)^5*((385*A*a^4)/12 + (106*B*a^4)/3) + \tan(x/2)^3*((511*A*a^4)/12 + (130*B*a^4)/3) + \tan(x/2)*((93*A*a^4)/4 + 18*B*a^4))/(4*\tan(x/2)^2 + 6*\tan(x/2)^4 + 4*\tan(x/2)^6 + \tan(x/2)^8 + 1) + (a^4*atan((42875*A^3*a^12*\tan(x/2))/(8*((42875*A^3*a^12)/8 + 14208*B^3*a^12 + 30520*A*B^2*a^12 + 22050*A^2*B*a^12))) + (14208*B^3*a^12*\tan(x/2)))/((42875*A^3*a^12)/8 + 14208*B^3*a^12 + 30520*A*B^2*a^12 + 22050*A^2*B*a^12) + (30520*A*B^2*a^12*\tan(x/2))/((42875*A^3*a^12)/8 + 14208*B^3*a^12 + 30520*A*B^2*a^12 + 22050*A^2*B*a^12) + (22050*A^2*B*a^12*\tan(x/2))/((42875*A^3*a^12)/8 + 14208*B^3*a^12 + 30520*A*B^2*a^12 + 22050*A^2*B*a^12))*((35*A + 48*B))/4 + 2*B*a^4*atanh((2368*B^3*a^12*\tan(x/2))/(2368*B^3*a^12 + 3360*A*B^2*a^12 + 1225*A^2*B*a^12) + (3360*A*B^2*a^12*\tan(x/2))/(2368*B^3*a^12 + 3360*A*B^2*a^12 + 1225*A^2*B*a^12) + (1225*A^2*B*a^12*\tan(x/2))/(2368*B^3*a^12 + 3360*A*B^2*a^12 + 1225*A^2*B*a^12))$

sympy [A] time = 15.26, size = 116, normalized size = 1.12

$$\frac{35Aa^4x}{8} - \frac{4Aa^4 \sin^3(x)}{3} + 8Aa^4 \sin(x) + \frac{7Aa^4 \sin(2x)}{4} + \frac{Aa^4 \sin(4x)}{32} + 6Ba^4x + Ba^4 \log(\tan(x) + \sec(x)) - \frac{Ba^4 \sin^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))**4*(A+B*sec(x)),x)

[Out] 35*A*a**4*x/8 - 4*A*a**4*sin(x)**3/3 + 8*A*a**4*sin(x) + 7*A*a**4*sin(2*x)/4 + A*a**4*sin(4*x)/32 + 6*B*a**4*x + B*a**4*log(tan(x) + sec(x)) - B*a**4*sin(x)**3/3 + 2*B*a**4*sin(x)*cos(x) + 7*B*a**4*sin(x)

$$3.189 \quad \int \frac{A+B \sec(x)}{a+a \cos(x)} dx$$

Optimal. Leaf size=25

$$\frac{(A-B) \sin(x)}{a \cos(x) + a} + \frac{B \tanh^{-1}(\sin(x))}{a}$$

[Out] B*arctanh(sin(x))/a+(A-B)*sin(x)/(a+a*cos(x))

Rubi [A] time = 0.09, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2828, 2978, 12, 3770}

$$\frac{(A-B) \sin(x)}{a \cos(x) + a} + \frac{B \tanh^{-1}(\sin(x))}{a}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[x])/(a + a*cos[x]), x]

[Out] (B*ArcTanh[Sin[x]])/a + ((A - B)*Sin[x])/(a + a*cos[x])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2828

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ[n]

Rule 2978

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(x)}{a + a \cos(x)} dx &= \int \frac{(B + A \cos(x)) \sec(x)}{a + a \cos(x)} dx \\
&= \frac{(A - B) \sin(x)}{a + a \cos(x)} + \frac{\int aB \sec(x) dx}{a^2} \\
&= \frac{(A - B) \sin(x)}{a + a \cos(x)} + \frac{B \int \sec(x) dx}{a} \\
&= \frac{B \tanh^{-1}(\sin(x))}{a} + \frac{(A - B) \sin(x)}{a + a \cos(x)}
\end{aligned}$$

Mathematica [B] time = 0.08, size = 71, normalized size = 2.84

$$\frac{2 \cos\left(\frac{x}{2}\right) \left((B - A) \sin\left(\frac{x}{2}\right) + B \cos\left(\frac{x}{2}\right) \left(\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right) \right) \right)}{a(\cos(x) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x]),x]

[Out] (-2*Cos[x/2]*(B*Cos[x/2]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) + (-A + B)*Sin[x/2]))/(a*(1 + Cos[x]))

fricas [A] time = 1.14, size = 47, normalized size = 1.88

$$\frac{(B \cos(x) + B) \log(\sin(x) + 1) - (B \cos(x) + B) \log(-\sin(x) + 1) + 2(A - B) \sin(x))}{2(a \cos(x) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x)),x, algorithm="fricas")

[Out] 1/2*((B*cos(x) + B)*log(sin(x) + 1) - (B*cos(x) + B)*log(-sin(x) + 1) + 2*(A - B)*sin(x))/(a*cos(x) + a)

giac [A] time = 0.15, size = 46, normalized size = 1.84

$$\frac{B \log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right)}{a} - \frac{B \log\left(\left|\tan\left(\frac{1}{2}x\right) - 1\right|\right)}{a} + \frac{A \tan\left(\frac{1}{2}x\right) - B \tan\left(\frac{1}{2}x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x)),x, algorithm="giac")

[Out] B*log(abs(tan(1/2*x) + 1))/a - B*log(abs(tan(1/2*x) - 1))/a + (A*tan(1/2*x) - B*tan(1/2*x))/a

maple [A] time = 0.09, size = 46, normalized size = 1.84

$$\frac{A \tan\left(\frac{x}{2}\right)}{a} - \frac{B \tan\left(\frac{x}{2}\right)}{a} - \frac{B \ln\left(\tan\left(\frac{x}{2}\right) - 1\right)}{a} + \frac{B \ln\left(1 + \tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(x))/(a+a*cos(x)),x)

[Out] 1/a*A*tan(1/2*x)-1/a*B*tan(1/2*x)-1/a*B*ln(tan(1/2*x)-1)+1/a*B*ln(1+tan(1/2*x))

maxima [B] time = 0.42, size = 63, normalized size = 2.52

$$B \left(\frac{\log\left(\frac{\sin(x)}{\cos(x)+1} + 1\right)}{a} - \frac{\log\left(\frac{\sin(x)}{\cos(x)+1} - 1\right)}{a} - \frac{\sin(x)}{a(\cos(x)+1)} \right) + \frac{A \sin(x)}{a(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x)),x, algorithm="maxima")

[Out] B*(log(sin(x)/(cos(x) + 1) + 1)/a - log(sin(x)/(cos(x) + 1) - 1)/a - sin(x)/(a*(cos(x) + 1))) + A*sin(x)/(a*(cos(x) + 1))

mupad [B] time = 2.36, size = 25, normalized size = 1.00

$$\frac{2 B \operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a} + \frac{\tan\left(\frac{x}{2}\right) (A - B)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(x))/(a + a*cos(x)),x)

[Out] (2*B*atanh(tan(x/2)))/a + (tan(x/2)*(A - B))/a

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos(x)+1} dx + \int \frac{B \sec(x)}{\cos(x)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x)),x)

[Out] (Integral(A/(cos(x) + 1), x) + Integral(B*sec(x)/(cos(x) + 1), x))/a

$$3.190 \quad \int \frac{A+B \sec(x)}{(a+a \cos(x))^2} dx$$

Optimal. Leaf size=48

$$\frac{(A-4B) \sin(x)}{3a^2(\cos(x)+1)} + \frac{B \tanh^{-1}(\sin(x))}{a^2} + \frac{(A-B) \sin(x)}{3(a \cos(x)+a)^2}$$

[Out] B*arctanh(sin(x))/a^2+1/3*(A-4*B)*sin(x)/a^2/(1+cos(x))+1/3*(A-B)*sin(x)/(a+a*cos(x))^2

Rubi [A] time = 0.18, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2828, 2978, 12, 3770}

$$\frac{(A-4B) \sin(x)}{3a^2(\cos(x)+1)} + \frac{B \tanh^{-1}(\sin(x))}{a^2} + \frac{(A-B) \sin(x)}{3(a \cos(x)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^2,x]

[Out] (B*ArcTanh[Sin[x]])/a^2 + ((A - 4*B)*Sin[x])/(3*a^2*(1 + Cos[x])) + ((A - B)*Sin[x])/(3*(a + a*Cos[x])^2)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2828

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ[n]

Rule 2978

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(x)}{(a + a \cos(x))^2} dx &= \int \frac{(B + A \cos(x)) \sec(x)}{(a + a \cos(x))^2} dx \\
&= \frac{(A - B) \sin(x)}{3(a + a \cos(x))^2} + \frac{\int \frac{(3aB + a(A-B) \cos(x)) \sec(x)}{a + a \cos(x)} dx}{3a^2} \\
&= \frac{(A - 4B) \sin(x)}{3a^2(1 + \cos(x))} + \frac{(A - B) \sin(x)}{3(a + a \cos(x))^2} + \frac{\int 3a^2 B \sec(x) dx}{3a^4} \\
&= \frac{(A - 4B) \sin(x)}{3a^2(1 + \cos(x))} + \frac{(A - B) \sin(x)}{3(a + a \cos(x))^2} + \frac{B \int \sec(x) dx}{a^2} \\
&= \frac{B \tanh^{-1}(\sin(x))}{a^2} + \frac{(A - 4B) \sin(x)}{3a^2(1 + \cos(x))} + \frac{(A - B) \sin(x)}{3(a + a \cos(x))^2}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 76, normalized size = 1.58

$$\frac{\sin(x)((A - 4B) \cos(x) + 2A - 5B) - 12B \cos^4\left(\frac{x}{2}\right) \left(\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)\right)}{3a^2(\cos(x) + 1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^2,x]

[Out] (-12*B*Cos[x/2]^4*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]])) + (2*A - 5*B + (A - 4*B)*Cos[x])*Sin[x]/(3*a^2*(1 + Cos[x])^2)

fricas [A] time = 0.74, size = 85, normalized size = 1.77

$$\frac{3(B \cos(x)^2 + 2B \cos(x) + B) \log(\sin(x) + 1) - 3(B \cos(x)^2 + 2B \cos(x) + B) \log(-\sin(x) + 1) + 2((A - 4B) \cos(x) + 2A - 5B) \sin(x)}{6(a^2 \cos(x)^2 + 2a^2 \cos(x) + a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))^2,x, algorithm="fricas")

[Out] 1/6*(3*(B*cos(x)^2 + 2*B*cos(x) + B)*log(sin(x) + 1) - 3*(B*cos(x)^2 + 2*B*cos(x) + B)*log(-sin(x) + 1) + 2*((A - 4*B)*cos(x) + 2*A - 5*B)*sin(x))/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2)

giac [A] time = 0.16, size = 77, normalized size = 1.60

$$\frac{B \log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right)}{a^2} - \frac{B \log\left(\left|\tan\left(\frac{1}{2}x\right) - 1\right|\right)}{a^2} + \frac{Aa^4 \tan\left(\frac{1}{2}x\right)^3 - Ba^4 \tan\left(\frac{1}{2}x\right)^3 + 3Aa^4 \tan\left(\frac{1}{2}x\right) - 9Ba^4 \tan\left(\frac{1}{2}x\right)}{6a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))^2,x, algorithm="giac")

[Out] B*log(abs(tan(1/2*x) + 1))/a^2 - B*log(abs(tan(1/2*x) - 1))/a^2 + 1/6*(A*a^4*tan(1/2*x)^3 - B*a^4*tan(1/2*x)^3 + 3*A*a^4*tan(1/2*x) - 9*B*a^4*tan(1/2*x))/a^6

maple [A] time = 0.09, size = 71, normalized size = 1.48

$$\frac{\left(\tan^3\left(\frac{x}{2}\right)\right) A}{6a^2} - \frac{\left(\tan^3\left(\frac{x}{2}\right)\right) B}{6a^2} + \frac{A \tan\left(\frac{x}{2}\right)}{2a^2} - \frac{3B \tan\left(\frac{x}{2}\right)}{2a^2} - \frac{B \ln\left(\tan\left(\frac{x}{2}\right) - 1\right)}{a^2} + \frac{B \ln\left(1 + \tan\left(\frac{x}{2}\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(x))/(a+a*cos(x))^2,x)

[Out] $1/6/a^2*\tan(1/2*x)^3*A-1/6/a^2*\tan(1/2*x)^3*B+1/2/a^2*A*\tan(1/2*x)-3/2/a^2*B*\tan(1/2*x)-1/a^2*B*\ln(\tan(1/2*x)-1)+1/a^2*B*\ln(1+\tan(1/2*x))$

maxima [B] time = 0.46, size = 93, normalized size = 1.94

$$-\frac{1}{6}B\left(\frac{\frac{9\sin(x)}{\cos(x)+1} + \frac{\sin(x)^3}{(\cos(x)+1)^3}}{a^2} - \frac{6\log\left(\frac{\sin(x)}{\cos(x)+1} + 1\right)}{a^2} + \frac{6\log\left(\frac{\sin(x)}{\cos(x)+1} - 1\right)}{a^2}\right) + \frac{A\left(\frac{3\sin(x)}{\cos(x)+1} + \frac{\sin(x)^3}{(\cos(x)+1)^3}\right)}{6a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))^2,x, algorithm="maxima")

[Out] $-1/6*B*((9*\sin(x)/(\cos(x) + 1) + \sin(x)^3/(\cos(x) + 1)^3)/a^2 - 6*\log(\sin(x)/(\cos(x) + 1) + 1)/a^2 + 6*\log(\sin(x)/(\cos(x) + 1) - 1)/a^2) + 1/6*A*(3*\sin(x)/(\cos(x) + 1) + \sin(x)^3/(\cos(x) + 1)^3)/a^2$

mupad [B] time = 2.35, size = 50, normalized size = 1.04

$$\tan\left(\frac{x}{2}\right)\left(\frac{A-B}{2a^2} - \frac{B}{a^2}\right) + \frac{\tan\left(\frac{x}{2}\right)^3(A-B)}{6a^2} + \frac{2B\operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(x))/(a + a*cos(x))^2,x)

[Out] $\tan(x/2)*((A - B)/(2*a^2) - B/a^2) + (\tan(x/2)^3*(A - B))/(6*a^2) + (2*B*atanh(\tan(x/2)))/a^2$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos^2(x)+2\cos(x)+1} dx + \int \frac{B\sec(x)}{\cos^2(x)+2\cos(x)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))**2,x)

[Out] $(\operatorname{Integral}(A/(\cos(x)**2 + 2*\cos(x) + 1), x) + \operatorname{Integral}(B*\sec(x)/(\cos(x)**2 + 2*\cos(x) + 1), x))/a**2$

$$3.191 \quad \int \frac{A+B \sec(x)}{(a+a \cos(x))^3} dx$$

Optimal. Leaf size=75

$$\frac{2(A-11B) \sin(x)}{15(a^3 \cos(x) + a^3)} + \frac{B \tanh^{-1}(\sin(x))}{a^3} + \frac{(2A-7B) \sin(x)}{15a(a \cos(x) + a)^2} + \frac{(A-B) \sin(x)}{5(a \cos(x) + a)^3}$$

[Out] B*arctanh(sin(x))/a^3+1/5*(A-B)*sin(x)/(a+a*cos(x))^3+1/15*(2*A-7*B)*sin(x)/a/(a+a*cos(x))^2+2/15*(A-11*B)*sin(x)/(a^3+a^3*cos(x))

Rubi [A] time = 0.31, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2828, 2978, 12, 3770}

$$\frac{2(A-11B) \sin(x)}{15(a^3 \cos(x) + a^3)} + \frac{B \tanh^{-1}(\sin(x))}{a^3} + \frac{(2A-7B) \sin(x)}{15a(a \cos(x) + a)^2} + \frac{(A-B) \sin(x)}{5(a \cos(x) + a)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^3,x]

[Out] (B*ArcTanh[Sin[x]])/a^3 + ((A - B)*Sin[x])/(5*(a + a*Cos[x])^3) + ((2*A - 7*B)*Sin[x])/(15*a*(a + a*Cos[x])^2) + (2*(A - 11*B)*Sin[x])/(15*(a^3 + a^3*Cos[x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2828

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] := Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ[n]

Rule 2978

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(x)}{(a + a \cos(x))^3} dx &= \int \frac{(B + A \cos(x)) \sec(x)}{(a + a \cos(x))^3} dx \\
&= \frac{(A - B) \sin(x)}{5(a + a \cos(x))^3} + \frac{\int \frac{(5aB + 2a(A - B) \cos(x)) \sec(x)}{(a + a \cos(x))^2} dx}{5a^2} \\
&= \frac{(A - B) \sin(x)}{5(a + a \cos(x))^3} + \frac{(2A - 7B) \sin(x)}{15a(a + a \cos(x))^2} + \frac{\int \frac{(15a^2B + a^2(2A - 7B) \cos(x)) \sec(x)}{a + a \cos(x)} dx}{15a^4} \\
&= \frac{(A - B) \sin(x)}{5(a + a \cos(x))^3} + \frac{(2A - 7B) \sin(x)}{15a(a + a \cos(x))^2} + \frac{2(A - 11B) \sin(x)}{15(a^3 + a^3 \cos(x))} + \frac{\int 15a^3B \sec(x) dx}{15a^6} \\
&= \frac{(A - B) \sin(x)}{5(a + a \cos(x))^3} + \frac{(2A - 7B) \sin(x)}{15a(a + a \cos(x))^2} + \frac{2(A - 11B) \sin(x)}{15(a^3 + a^3 \cos(x))} + \frac{B \int \sec(x) dx}{a^3} \\
&= \frac{B \tanh^{-1}(\sin(x))}{a^3} + \frac{(A - B) \sin(x)}{5(a + a \cos(x))^3} + \frac{(2A - 7B) \sin(x)}{15a(a + a \cos(x))^2} + \frac{2(A - 11B) \sin(x)}{15(a^3 + a^3 \cos(x))}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 88, normalized size = 1.17

$$\frac{\sin(x)((6A - 51B) \cos(x) + (A - 11B) \cos(2x) + 8A - 43B) - 120B \cos^6\left(\frac{x}{2}\right) \left(\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{15a^3(\cos(x) + 1)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^3,x]

[Out] (-120*B*Cos[x/2]^6*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]])) + (8*A - 43*B + (6*A - 51*B)*Cos[x] + (A - 11*B)*Cos[2*x])*Sin[x]/(15*a^3*(1 + Cos[x])^3)

fricas [A] time = 0.74, size = 122, normalized size = 1.63

$$\frac{15 \left(B \cos(x)^3 + 3 B \cos(x)^2 + 3 B \cos(x) + B \right) \log(\sin(x) + 1) - 15 \left(B \cos(x)^3 + 3 B \cos(x)^2 + 3 B \cos(x) + B \right) \log(-\sin(x) + 1)}{30 \left(a^3 \cos(x)^3 + 3 a^3 \cos(x)^2 + 3 a^3 \cos(x) + a^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))^3,x, algorithm="fricas")

[Out] 1/30*(15*(B*cos(x)^3 + 3*B*cos(x)^2 + 3*B*cos(x) + B)*log(sin(x) + 1) - 15*(B*cos(x)^3 + 3*B*cos(x)^2 + 3*B*cos(x) + B)*log(-sin(x) + 1) + 2*(2*(A - 1*B)*cos(x)^2 + 3*(2*A - 17*B)*cos(x) + 7*A - 32*B)*sin(x))/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 + 3*a^3*cos(x) + a^3)

giac [A] time = 0.16, size = 102, normalized size = 1.36

$$\frac{B \log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right)}{a^3} - \frac{B \log\left(\left|\tan\left(\frac{1}{2}x\right) - 1\right|\right)}{a^3} + \frac{3Aa^{12} \tan\left(\frac{1}{2}x\right)^5 - 3Ba^{12} \tan\left(\frac{1}{2}x\right)^5 + 10Aa^{12} \tan\left(\frac{1}{2}x\right)^3 - 20Ba^{12} \tan\left(\frac{1}{2}x\right)^3 + 15Aa^{12} \tan\left(\frac{1}{2}x\right) - 105Ba^{12} \tan\left(\frac{1}{2}x\right)}{60a^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))^3,x, algorithm="giac")

[Out] B*log(abs(tan(1/2*x) + 1))/a^3 - B*log(abs(tan(1/2*x) - 1))/a^3 + 1/60*(3*A*a^12*tan(1/2*x)^5 - 3*B*a^12*tan(1/2*x)^5 + 10*A*a^12*tan(1/2*x)^3 - 20*B*a^12*tan(1/2*x)^3 + 15*A*a^12*tan(1/2*x) - 105*B*a^12*tan(1/2*x))/a^15

maple [A] time = 0.10, size = 95, normalized size = 1.27

$$\frac{\left(\tan^3\left(\frac{x}{2}\right)\right) A}{6a^3} - \frac{\left(\tan^3\left(\frac{x}{2}\right)\right) B}{3a^3} - \frac{B \ln\left(\tan\left(\frac{x}{2}\right) - 1\right)}{a^3} + \frac{A \tan\left(\frac{x}{2}\right)}{4a^3} - \frac{7B \tan\left(\frac{x}{2}\right)}{4a^3} + \frac{B \ln\left(1 + \tan\left(\frac{x}{2}\right)\right)}{a^3} + \frac{\left(\tan^5\left(\frac{x}{2}\right)\right) A}{20a^3} - \frac{\left(\tan^5\left(\frac{x}{2}\right)\right) B}{20a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(x))/(a+a*cos(x))^3,x)

[Out] 1/6/a^3*tan(1/2*x)^3*A-1/3/a^3*tan(1/2*x)^3*B-1/a^3*B*ln(tan(1/2*x)-1)+1/4/a^3*A*tan(1/2*x)-7/4/a^3*B*tan(1/2*x)+1/a^3*B*ln(1+tan(1/2*x))+1/20/a^3*tan(1/2*x)^5*A-1/20/a^3*tan(1/2*x)^5*B

maxima [A] time = 0.64, size = 119, normalized size = 1.59

$$-\frac{1}{60} B \left(\frac{\frac{105 \sin(x)}{\cos(x)+1} + \frac{20 \sin(x)^3}{(\cos(x)+1)^3} + \frac{3 \sin(x)^5}{(\cos(x)+1)^5}}{a^3} - \frac{60 \log\left(\frac{\sin(x)}{\cos(x)+1} + 1\right)}{a^3} + \frac{60 \log\left(\frac{\sin(x)}{\cos(x)+1} - 1\right)}{a^3} \right) + \frac{A \left(\frac{15 \sin(x)}{\cos(x)+1} + \frac{10 \sin(x)^3}{(\cos(x)+1)^3} \right)}{60 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))^3,x, algorithm="maxima")

[Out] -1/60*B*((105*sin(x)/(cos(x) + 1) + 20*sin(x)^3/(cos(x) + 1)^3 + 3*sin(x)^5/(cos(x) + 1)^5)/a^3 - 60*log(sin(x)/(cos(x) + 1) + 1)/a^3 + 60*log(sin(x)/(cos(x) + 1) - 1)/a^3) + 1/60*A*(15*sin(x)/(cos(x) + 1) + 10*sin(x)^3/(cos(x) + 1)^3 + 3*sin(x)^5/(cos(x) + 1)^5)/a^3

mupad [B] time = 2.37, size = 92, normalized size = 1.23

$$\tan\left(\frac{x}{2}\right)^3 \left(\frac{A-B}{12a^3} + \frac{A-3B}{12a^3} \right) + \tan\left(\frac{x}{2}\right) \left(\frac{A-B}{4a^3} + \frac{A-3B}{4a^3} - \frac{A+3B}{4a^3} \right) + \frac{\tan\left(\frac{x}{2}\right)^5 (A-B)}{20a^3} + \frac{2B \operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(x))/(a + a*cos(x))^3,x)

[Out] tan(x/2)^3*((A - B)/(12*a^3) + (A - 3*B)/(12*a^3)) + tan(x/2)*((A - B)/(4*a^3) + (A - 3*B)/(4*a^3) - (A + 3*B)/(4*a^3)) + (tan(x/2)^5*(A - B))/(20*a^3) + (2*B*atanh(tan(x/2)))/a^3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos^3(x)+3\cos^2(x)+3\cos(x)+1} dx + \int \frac{B \sec(x)}{\cos^3(x)+3\cos^2(x)+3\cos(x)+1} dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))**3,x)

[Out] (Integral(A/(cos(x)**3 + 3*cos(x)**2 + 3*cos(x) + 1), x) + Integral(B*sec(x)/(cos(x)**3 + 3*cos(x)**2 + 3*cos(x) + 1), x))/a**3

$$3.192 \quad \int \frac{A+B \sec(x)}{(a+a \cos(x))^4} dx$$

Optimal. Leaf size=96

$$\frac{2(3A - 80B) \sin(x)}{105a^4(\cos(x) + 1)} + \frac{(6A - 55B) \sin(x)}{105a^4(\cos(x) + 1)^2} + \frac{B \tanh^{-1}(\sin(x))}{a^4} + \frac{(3A - 10B) \sin(x)}{35a(a \cos(x) + a)^3} + \frac{(A - B) \sin(x)}{7(a \cos(x) + a)^4}$$

[Out] B*arctanh(sin(x))/a^4+1/105*(6*A-55*B)*sin(x)/a^4/(1+cos(x))^2+2/105*(3*A-80*B)*sin(x)/a^4/(1+cos(x))+1/7*(A-B)*sin(x)/(a+a*cos(x))^4+1/35*(3*A-10*B)*sin(x)/a/(a+a*cos(x))^3

Rubi [A] time = 0.41, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2828, 2978, 12, 3770}

$$\frac{2(3A - 80B) \sin(x)}{105a^4(\cos(x) + 1)} + \frac{(6A - 55B) \sin(x)}{105a^4(\cos(x) + 1)^2} + \frac{B \tanh^{-1}(\sin(x))}{a^4} + \frac{(3A - 10B) \sin(x)}{35a(a \cos(x) + a)^3} + \frac{(A - B) \sin(x)}{7(a \cos(x) + a)^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^4,x]

[Out] (B*ArcTanh[Sin[x]])/a^4 + ((6*A - 55*B)*Sin[x])/(105*a^4*(1 + Cos[x])^2) + (2*(3*A - 80*B)*Sin[x])/(105*a^4*(1 + Cos[x])) + ((A - B)*Sin[x])/(7*(a + a*Cos[x])^4) + ((3*A - 10*B)*Sin[x])/(35*a*(a + a*Cos[x])^3)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2828

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ[n]

Rule 2978

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(x)}{(a + a \cos(x))^4} dx &= \int \frac{(B + A \cos(x)) \sec(x)}{(a + a \cos(x))^4} dx \\
&= \frac{(A - B) \sin(x)}{7(a + a \cos(x))^4} + \frac{\int \frac{(7aB + 3a(A - B) \cos(x)) \sec(x)}{(a + a \cos(x))^3} dx}{7a^2} \\
&= \frac{(A - B) \sin(x)}{7(a + a \cos(x))^4} + \frac{(3A - 10B) \sin(x)}{35a(a + a \cos(x))^3} + \frac{\int \frac{(35a^2B + 2a^2(3A - 10B) \cos(x)) \sec(x)}{(a + a \cos(x))^2} dx}{35a^4} \\
&= \frac{(6A - 55B) \sin(x)}{105a^4(1 + \cos(x))^2} + \frac{(A - B) \sin(x)}{7(a + a \cos(x))^4} + \frac{(3A - 10B) \sin(x)}{35a(a + a \cos(x))^3} + \frac{\int \frac{(105a^3B + a^3(6A - 55B) \cos(x)) \sec(x)}{a + a \cos(x)} dx}{105a^6} \\
&= \frac{(6A - 55B) \sin(x)}{105a^4(1 + \cos(x))^2} + \frac{(A - B) \sin(x)}{7(a + a \cos(x))^4} + \frac{(3A - 10B) \sin(x)}{35a(a + a \cos(x))^3} + \frac{2(3A - 80B) \sin(x)}{105(a^4 + a^4 \cos(x))} + \\
&= \frac{(6A - 55B) \sin(x)}{105a^4(1 + \cos(x))^2} + \frac{(A - B) \sin(x)}{7(a + a \cos(x))^4} + \frac{(3A - 10B) \sin(x)}{35a(a + a \cos(x))^3} + \frac{2(3A - 80B) \sin(x)}{105(a^4 + a^4 \cos(x))} + \\
&= \frac{B \tanh^{-1}(\sin(x))}{a^4} + \frac{(6A - 55B) \sin(x)}{105a^4(1 + \cos(x))^2} + \frac{(A - B) \sin(x)}{7(a + a \cos(x))^4} + \frac{(3A - 10B) \sin(x)}{35a(a + a \cos(x))^3} + \frac{2(3A - 80B) \sin(x)}{105(a^4 + a^4 \cos(x))}
\end{aligned}$$

Mathematica [A] time = 0.75, size = 104, normalized size = 1.08

$$\frac{\sin(x)((87A - 1480B) \cos(x) + (24A - 535B) \cos(2x) + 3A \cos(3x) + 96A - 80B \cos(3x) - 1055B) - 3360B \cos(x)}{210a^4(\cos(x) + 1)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^4,x]

[Out] (-3360*B*Cos[x/2]^8*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) + (96*A - 1055*B + (87*A - 1480*B)*Cos[x] + (24*A - 535*B)*Cos[2*x] + 3*A*Cos[3*x] - 80*B*Cos[3*x])*Sin[x])/(210*a^4*(1 + Cos[x])^4)

fricas [A] time = 0.57, size = 158, normalized size = 1.65

$$\frac{105(B \cos(x)^4 + 4B \cos(x)^3 + 6B \cos(x)^2 + 4B \cos(x) + B) \log(\sin(x) + 1) - 105(B \cos(x)^4 + 4B \cos(x)^3 + 6B \cos(x)^2 + 4B \cos(x) + B) \log(-\sin(x) + 1) + 2*(2*(3A - 80B)*\cos(x)^3 + (24A - 535B)*\cos(x)^2 + (39A - 620B)*\cos(x) + 36A - 260B)*\sin(x)}{210(a^4 \cos(x) + a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))^4,x, algorithm="fricas")

[Out] 1/210*(105*(B*cos(x)^4 + 4*B*cos(x)^3 + 6*B*cos(x)^2 + 4*B*cos(x) + B)*log(sin(x) + 1) - 105*(B*cos(x)^4 + 4*B*cos(x)^3 + 6*B*cos(x)^2 + 4*B*cos(x) + B)*log(-sin(x) + 1) + 2*(2*(3*A - 80*B)*cos(x)^3 + (24*A - 535*B)*cos(x)^2 + (39*A - 620*B)*cos(x) + 36*A - 260*B)*sin(x))/(a^4*cos(x)^4 + 4*a^4*cos(x)^3 + 6*a^4*cos(x)^2 + 4*a^4*cos(x) + a^4)

giac [A] time = 0.15, size = 126, normalized size = 1.31

$$\frac{B \log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right)}{a^4} - \frac{B \log\left(\left|\tan\left(\frac{1}{2}x\right) - 1\right|\right)}{a^4} + \frac{15Aa^{24} \tan\left(\frac{1}{2}x\right)^7 - 15Ba^{24} \tan\left(\frac{1}{2}x\right)^7 + 63Aa^{24} \tan\left(\frac{1}{2}x\right)^5}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))^4,x, algorithm="giac")

[Out] $B \cdot \log(\abs{\tan(1/2*x) + 1})/a^4 - B \cdot \log(\abs{\tan(1/2*x) - 1})/a^4 + 1/840 \cdot (15 * A * a^{24} * \tan(1/2*x)^7 - 15 * B * a^{24} * \tan(1/2*x)^7 + 63 * A * a^{24} * \tan(1/2*x)^5 - 10 * B * a^{24} * \tan(1/2*x)^5 + 105 * A * a^{24} * \tan(1/2*x)^3 - 385 * B * a^{24} * \tan(1/2*x)^3 + 105 * A * a^{24} * \tan(1/2*x) - 1575 * B * a^{24} * \tan(1/2*x))/a^{28}$

maple [A] time = 0.10, size = 119, normalized size = 1.24

$$\frac{\left(\tan^7\left(\frac{x}{2}\right)\right)A}{56a^4} - \frac{\left(\tan^7\left(\frac{x}{2}\right)\right)B}{56a^4} + \frac{\left(\tan^3\left(\frac{x}{2}\right)\right)A}{8a^4} - \frac{11\left(\tan^3\left(\frac{x}{2}\right)\right)B}{24a^4} - \frac{B \ln\left(\tan\left(\frac{x}{2}\right) - 1\right)}{a^4} + \frac{A \tan\left(\frac{x}{2}\right)}{8a^4} - \frac{15B \tan\left(\frac{x}{2}\right)}{8a^4} + \frac{B \ln\left(1 + \tan\left(\frac{x}{2}\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sec(x))/(a+a*cos(x))^4,x)`

[Out] $1/56/a^4 * \tan(1/2*x)^7 * A - 1/56/a^4 * \tan(1/2*x)^7 * B + 1/8/a^4 * \tan(1/2*x)^3 * A - 11/24/a^4 * \tan(1/2*x)^3 * B - 1/a^4 * B * \ln(\tan(1/2*x) - 1) + 1/8/a^4 * A * \tan(1/2*x) - 15/8/a^4 * B * \tan(1/2*x) + 1/a^4 * B * \ln(1 + \tan(1/2*x)) + 3/40/a^4 * \tan(1/2*x)^5 * A - 1/8/a^4 * \tan(1/2*x)^5 * B$

maxima [A] time = 0.77, size = 143, normalized size = 1.49

$$-\frac{1}{168} B \left(\frac{\frac{315 \sin(x)}{\cos(x)+1} + \frac{77 \sin(x)^3}{(\cos(x)+1)^3} + \frac{21 \sin(x)^5}{(\cos(x)+1)^5} + \frac{3 \sin(x)^7}{(\cos(x)+1)^7}}{a^4} - \frac{168 \log\left(\frac{\sin(x)}{\cos(x)+1} + 1\right)}{a^4} + \frac{168 \log\left(\frac{\sin(x)}{\cos(x)+1} - 1\right)}{a^4} \right) + \frac{A \left(\frac{35 \sin(x)}{\cos(x)+1} + 1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(x))/(a+a*cos(x))^4,x, algorithm="maxima")`

[Out] $-1/168 * B * \left(\frac{315 * \sin(x)}{\cos(x) + 1} + \frac{77 * \sin(x)^3}{(\cos(x) + 1)^3} + \frac{21 * \sin(x)^5}{(\cos(x) + 1)^5} + \frac{3 * \sin(x)^7}{(\cos(x) + 1)^7} \right) / a^4 - 168 * \log(\sin(x) / (\cos(x) + 1) + 1) / a^4 + 168 * \log(\sin(x) / (\cos(x) + 1) - 1) / a^4 + 1/280 * A * \left(\frac{35 * \sin(x)}{\cos(x) + 1} + 1 \right) / a^4 + 35 * \sin(x)^3 / (\cos(x) + 1)^3 + 21 * \sin(x)^5 / (\cos(x) + 1)^5 + 5 * \sin(x)^7 / (\cos(x) + 1)^7 / a^4$

mupad [B] time = 2.34, size = 140, normalized size = 1.46

$$\tan\left(\frac{x}{2}\right) \left(\frac{A - B}{8a^4} - \frac{3B}{4a^4} + \frac{2A - 4B}{8a^4} - \frac{2A + 4B}{8a^4} \right) + \tan\left(\frac{x}{2}\right)^5 \left(\frac{A - B}{40a^4} + \frac{2A - 4B}{40a^4} \right) + \tan\left(\frac{x}{2}\right)^3 \left(\frac{A - B}{24a^4} - \frac{B}{4a^4} + \frac{2A - 4B}{24a^4} \right) + \frac{B \operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B/cos(x))/(a + a*cos(x))^4,x)`

[Out] $\tan(x/2) * \left(\frac{A - B}{8 * a^4} - \frac{3 * B}{4 * a^4} + \frac{2 * A - 4 * B}{8 * a^4} - \frac{2 * A + 4 * B}{8 * a^4} \right) + \tan(x/2)^5 * \left(\frac{A - B}{40 * a^4} + \frac{2 * A - 4 * B}{40 * a^4} \right) + \tan(x/2)^3 * \left(\frac{A - B}{24 * a^4} - \frac{B}{4 * a^4} + \frac{2 * A - 4 * B}{24 * a^4} \right) + \frac{B * \operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a^4}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos^4(x) + 4 \cos^3(x) + 6 \cos^2(x) + 4 \cos(x) + 1} dx + \int \frac{B \sec(x)}{\cos^4(x) + 4 \cos^3(x) + 6 \cos^2(x) + 4 \cos(x) + 1} dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(x))/(a+a*cos(x))**4,x)`

[Out] $\left(\operatorname{Integral}\left(\frac{A}{\cos(x)^4 + 4 \cos(x)^3 + 6 \cos(x)^2 + 4 \cos(x) + 1}, x \right) + \operatorname{Integral}\left(\frac{B \sec(x)}{\cos(x)^4 + 4 \cos(x)^3 + 6 \cos(x)^2 + 4 \cos(x) + 1}, x \right) \right) / a^4$

3.193 $\int (a + a \cos(x))^{5/2} (A + B \sec(x)) dx$

Optimal. Leaf size=98

$$2a^{5/2}B \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{a \cos(x) + a}}\right) + \frac{2a^3(32A + 35B) \sin(x)}{15\sqrt{a \cos(x) + a}} + \frac{2}{15}a^2(8A+5B) \sin(x)\sqrt{a \cos(x) + a} + \frac{2}{5}aA \sin(x)(a \cos(x) + a)^{3/2}$$

[Out] 2*a^(5/2)*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))+2/5*a*A*(a+a*cos(x))^(3/2)*sin(x)+2/15*a^3*(32*A+35*B)*sin(x)/(a+a*cos(x))^(1/2)+2/15*a^2*(8*A+5*B)*sin(x)*(a+a*cos(x))^(1/2)

Rubi [A] time = 0.43, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2828, 2976, 2981, 2773, 206}

$$\frac{2a^3(32A + 35B) \sin(x)}{15\sqrt{a \cos(x) + a}} + \frac{2}{15}a^2(8A+5B) \sin(x)\sqrt{a \cos(x) + a} + 2a^{5/2}B \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{a \cos(x) + a}}\right) + \frac{2}{5}aA \sin(x)(a \cos(x) + a)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[x])^(5/2)*(A + B*Sec[x]),x]

[Out] 2*a^(5/2)*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]] + (2*a^3*(32*A + 35*B)*Sin[x])/(15*Sqrt[a + a*Cos[x]]) + (2*a^2*(8*A + 5*B)*Sqrt[a + a*Cos[x]]*Sin[x])/15 + (2*a*A*(a + a*Cos[x])^(3/2)*Sin[x])/5

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2773

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Ssin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2828

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_)^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Ssin[e + f*x])^m*(d + c*Ssin[e + f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ[n]

Rule 2976

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*B*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m-1)*(c + d*Ssin[e + f*x])^(n+1))/(d*f*(m+n+1)), x] + Dist[1/(d*(m+n+1)), Int[(a + b*Ssin[e + f*x])^(m-1)*(c + d*Ssin[e + f*x])^n*Simp[a*A*d*(m+n+1) + B*(a*c*(m-1) + b*d*(n+1)) + (A*b*d*(m+n+1) - B*(b*c*m - a*d*(2*m+n)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 2981

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(d*f*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int (a + a \cos(x))^{5/2} (A + B \sec(x)) dx &= \int (a + a \cos(x))^{5/2} (B + A \cos(x)) \sec(x) dx \\ &= \frac{2}{5} a A (a + a \cos(x))^{3/2} \sin(x) + \frac{2}{5} \int (a + a \cos(x))^{3/2} \left(\frac{5aB}{2} + \frac{1}{2} a(8A + 5B) \right) dx \\ &= \frac{2}{15} a^2 (8A + 5B) \sqrt{a + a \cos(x)} \sin(x) + \frac{2}{5} a A (a + a \cos(x))^{3/2} \sin(x) + \frac{4}{15} \int (a + a \cos(x))^{3/2} dx \\ &= \frac{2a^3(32A + 35B) \sin(x)}{15\sqrt{a + a \cos(x)}} + \frac{2}{15} a^2 (8A + 5B) \sqrt{a + a \cos(x)} \sin(x) + \frac{2}{5} a A (a + a \cos(x))^{3/2} \sin(x) \\ &= \frac{2a^3(32A + 35B) \sin(x)}{15\sqrt{a + a \cos(x)}} + \frac{2}{15} a^2 (8A + 5B) \sqrt{a + a \cos(x)} \sin(x) + \frac{2}{5} a A (a + a \cos(x))^{3/2} \sin(x) \\ &= 2a^{5/2} B \tanh^{-1} \left(\frac{\sqrt{a} \sin(x)}{\sqrt{a + a \cos(x)}} \right) + \frac{2a^3(32A + 35B) \sin(x)}{15\sqrt{a + a \cos(x)}} + \frac{2}{15} a^2 (8A + 5B) \sqrt{a + a \cos(x)} \sin(x) \end{aligned}$$

Mathematica [A] time = 0.16, size = 78, normalized size = 0.80

$$\frac{1}{30} a^2 \sec\left(\frac{x}{2}\right) \sqrt{a(\cos(x) + 1)} \left(2 \sin\left(\frac{x}{2}\right) (2(14A + 5B) \cos(x) + 3A \cos(2x) + 89A + 80B) + 30\sqrt{2} B \tanh^{-1}\left(\sqrt{2} \sin\left(\frac{x}{2}\right)\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^(5/2)*(A + B*Sec[x]), x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(30*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]] + 2*(89*A + 80*B + 2*(14*A + 5*B)*Cos[x] + 3*A*Cos[2*x])*Sin[x/2]))/30
```

fricas [A] time = 0.73, size = 123, normalized size = 1.26

$$\frac{15 (Ba^2 \cos(x) + Ba^2) \sqrt{a} \log\left(\frac{a \cos(x)^3 - 7a \cos(x)^2 - 4\sqrt{a} \cos(x) + a \sqrt{a} (\cos(x) - 2) \sin(x) + 8a}{\cos(x)^3 + \cos(x)^2}\right) + 4 (3Aa^2 \cos(x)^2 + (14A + 5B)a^2) \sqrt{a} \sin(x)}{30 (\cos(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(5/2)*(A+B*sec(x)), x, algorithm="fricas")
```

```
[Out] 1/30*(15*(B*a^2*cos(x) + B*a^2)*sqrt(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)) + 4*(3*A*a^2*cos(x)^2 + (14*A + 5*B)*a^2*cos(x) + (43*A + 40*B)*a^2)*sqrt(a*cos(x) + a)*sin(x))/(cos(x) + 1)
```

giac [A] time = 0.21, size = 134, normalized size = 1.37

$$\frac{1}{30} \sqrt{2} \left(48 A a^2 \operatorname{sgn}\left(\cos\left(\frac{1}{2} x\right)\right) \sin\left(\frac{1}{2} x\right)^5 - 160 A a^2 \operatorname{sgn}\left(\cos\left(\frac{1}{2} x\right)\right) \sin\left(\frac{1}{2} x\right)^3 - 40 B a^2 \operatorname{sgn}\left(\cos\left(\frac{1}{2} x\right)\right) \sin\left(\frac{1}{2} x\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^(5/2)*(A+B*sec(x)),x, algorithm="giac")

[Out] $\frac{1}{30}\sqrt{2}*(48*A*a^2*\text{sgn}(\cos(1/2*x))*\sin(1/2*x)^5 - 160*A*a^2*\text{sgn}(\cos(1/2*x))*\sin(1/2*x)^3 - 40*B*a^2*\text{sgn}(\cos(1/2*x))*\sin(1/2*x)^3 - 15*\sqrt{2}*B*a^2*\log(\text{abs}(-2*\sqrt{2} + 4*\sin(1/2*x))/\text{abs}(2*\sqrt{2} + 4*\sin(1/2*x)))*\text{sgn}(\cos(1/2*x)) + 240*A*a^2*\text{sgn}(\cos(1/2*x))*\sin(1/2*x) + 180*B*a^2*\text{sgn}(\cos(1/2*x))*\sin(1/2*x))*\sqrt{a}$

maple [B] time = 0.38, size = 228, normalized size = 2.33

$$a^{\frac{3}{2}} \cos\left(\frac{x}{2}\right) \sqrt{a \left(\sin^2\left(\frac{x}{2}\right)\right)} \left(24A\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{x}{2}\right)\right)} \sqrt{a} \left(\sin^4\left(\frac{x}{2}\right)\right) - 20\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{x}{2}\right)\right)} \sqrt{a} (4A + B) \left(\sin^2\left(\frac{x}{2}\right)\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(x))^(5/2)*(A+B*sec(x)),x)

[Out] $\frac{1}{15}a^{(3/2)}*\cos(1/2*x)*(a*\sin(1/2*x)^2)^{(1/2)}*(24*A*2^{(1/2)}*(a*\sin(1/2*x)^2)^{(1/2)}*a^{(1/2)}*\sin(1/2*x)^4 - 20*2^{(1/2)}*(a*\sin(1/2*x)^2)^{(1/2)}*a^{(1/2)}*(4*A+B)*\sin(1/2*x)^2 + 120*A*2^{(1/2)}*(a*\sin(1/2*x)^2)^{(1/2)}*a^{(1/2)} + 90*B*2^{(1/2)}*(a*\sin(1/2*x)^2)^{(1/2)}*a^{(1/2)} + 15*B*\ln(4/(2*\cos(1/2*x)+2^{(1/2)}))*(a*2^{(1/2)}*\cos(1/2*x)+a^{(1/2)}*2^{(1/2)}*(a*\sin(1/2*x)^2)^{(1/2)}+2*a))*a + 15*B*\ln(-4/(-2*\cos(1/2*x)+2^{(1/2)}))*(a^{(1/2)}*2^{(1/2)}*(a*\sin(1/2*x)^2)^{(1/2)}-a*2^{(1/2)}*\cos(1/2*x)+2*a))*a)/\sin(1/2*x)/(\cos(1/2*x)^2*a)^{(1/2)}$

maxima [A] time = 1.12, size = 43, normalized size = 0.44

$$\frac{1}{30} \left(3\sqrt{2} a^2 \sin\left(\frac{5}{2}x\right) + 25\sqrt{2} a^2 \sin\left(\frac{3}{2}x\right) + 150\sqrt{2} a^2 \sin\left(\frac{1}{2}x\right) \right) A\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^(5/2)*(A+B*sec(x)),x, algorithm="maxima")

[Out] $\frac{1}{30}*(3*\sqrt{2}*a^2*\sin(5/2*x) + 25*\sqrt{2}*a^2*\sin(3/2*x) + 150*\sqrt{2}*a^2*\sin(1/2*x))*A*\sqrt{a}$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a \cos(x))^{5/2} \left(A + \frac{B}{\cos(x)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*cos(x))^(5/2)*(A + B/cos(x)),x)

[Out] int((a + a*cos(x))^(5/2)*(A + B/cos(x)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))**(5/2)*(A+B*sec(x)),x)

[Out] Timed out

3.194 $\int (a + a \cos(x))^{3/2} (A + B \sec(x)) dx$

Optimal. Leaf size=72

$$2a^{3/2}B \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{a \cos(x) + a}}\right) + \frac{2a^2(4A + 3B) \sin(x)}{3\sqrt{a \cos(x) + a}} + \frac{2}{3}aA \sin(x)\sqrt{a \cos(x) + a}$$

[Out] $2*a^{(3/2)}*B*\operatorname{arctanh}(\sin(x)*a^{(1/2)}/(a+a*\cos(x))^{(1/2)})+2/3*a^2*(4*A+3*B)*\sin(x)/(a+a*\cos(x))^{(1/2)}+2/3*a*A*\sin(x)*(a+a*\cos(x))^{(1/2)}$

Rubi [A] time = 0.29, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2828, 2976, 2981, 2773, 206}

$$\frac{2a^2(4A + 3B) \sin(x)}{3\sqrt{a \cos(x) + a}} + 2a^{3/2}B \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{a \cos(x) + a}}\right) + \frac{2}{3}aA \sin(x)\sqrt{a \cos(x) + a}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[x])^{(3/2)}*(A + B*\operatorname{Sec}[x]),x]$

[Out] $2*a^{(3/2)}*B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[x])/(\operatorname{Sqrt}[a + a*\operatorname{Cos}[x]])] + (2*a^2*(4*A + 3*B)*\operatorname{Sin}[x])/(3*\operatorname{Sqrt}[a + a*\operatorname{Cos}[x]]) + (2*a*A*\operatorname{Sqrt}[a + a*\operatorname{Cos}[x]]*\operatorname{Sin}[x])/3$

Rule 206

$\operatorname{Int}[(a + (b*x^2)^{-1}), x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/(\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2])), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 2773

$\operatorname{Int}[\operatorname{Sqrt}[(a + (b*\sin(e + f*x)) + (c + (d*\sin(e + f*x)) + (f*x)))], x_Symbol] \rightarrow \operatorname{Dist}[(-2*b)/f, \operatorname{Subst}[\operatorname{Int}[1/(b*c + a*d - d*x^2), x], x, (b*\cos[e + f*x])/(\operatorname{Sqrt}[a + b*\sin[e + f*x]])], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{NeQ}[c^2 - d^2, 0]$

Rule 2828

$\operatorname{Int}[(\operatorname{csc}(e + f*x) + (d + (c + (a + (b*\sin(e + f*x)) + (f*x)))^m)^n), x_Symbol] \rightarrow \operatorname{Int}[(a + b*\sin[e + f*x])^m*(d + c*\sin[e + f*x])^n/\operatorname{Sin}[e + f*x]^n, x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x] \ \&\& \operatorname{IntegerQ}[n]$

Rule 2976

$\operatorname{Int}[(a + (b*\sin(e + f*x) + (c + (d*\sin(e + f*x) + (f*x)))^m)^n), x_Symbol] \rightarrow -\operatorname{Simp}[(b*B*\cos[e + f*x]*(a + b*\sin[e + f*x])^{m-1}*(c + d*\sin[e + f*x])^{n+1})/(d*f*(m+n+1)), x] + \operatorname{Dist}[1/(d*(m+n+1)), \operatorname{Int}[(a + b*\sin[e + f*x])^{m-1}*(c + d*\sin[e + f*x])^n*\operatorname{Simp}[a*A*d*(m+n+1) + B*(a*c*(m-1) + b*d*(n+1)) + (A*b*d*(m+n+1) - B*(b*c*m - a*d*(2*m+n))]*\operatorname{Sin}[e + f*x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{NeQ}[c^2 - d^2, 0] \ \&\& \operatorname{GtQ}[m, 1/2] \ \&\& \operatorname{!LtQ}[n, -1] \ \&\& \operatorname{IntegerQ}[2*m] \ \&\& (\operatorname{IntegerQ}[2*n] \ || \ \operatorname{EqQ}[c, 0])$

Rule 2981

$\operatorname{Int}[\operatorname{Sqrt}[(a + (b*\sin(e + f*x) + (c + (d*\sin(e + f*x) + (f*x)))^n)], x_Symbol] \rightarrow \operatorname{Simp}$

$$\begin{aligned} & [(-2*b*B*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^{(n + 1)})/(d*f*(2*n + 3)*\text{Sqrt}[a + \\ & b*\text{Sin}[e + f*x]]), x] + \text{Dist}[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b \\ & *d*(2*n + 3)), \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^n, x], x] \\ & /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - \\ & b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& !\text{LtQ}[n, -1] \end{aligned}$$

Rubi steps

$$\begin{aligned} \int (a + a \cos(x))^{3/2} (A + B \sec(x)) dx &= \int (a + a \cos(x))^{3/2} (B + A \cos(x)) \sec(x) dx \\ &= \frac{2}{3} a A \sqrt{a + a \cos(x)} \sin(x) + \frac{2}{3} \int \sqrt{a + a \cos(x)} \left(\frac{3aB}{2} + \frac{1}{2} a(4A + 3B) \right) dx \\ &= \frac{2a^2(4A + 3B) \sin(x)}{3\sqrt{a + a \cos(x)}} + \frac{2}{3} a A \sqrt{a + a \cos(x)} \sin(x) + (aB) \int \sqrt{a + a \cos(x)} dx \\ &= \frac{2a^2(4A + 3B) \sin(x)}{3\sqrt{a + a \cos(x)}} + \frac{2}{3} a A \sqrt{a + a \cos(x)} \sin(x) - (2a^2B) \text{Subst} \left(\int \frac{1}{\sqrt{a + a \cos(x)}} dx \right) \\ &= 2a^{3/2} B \tanh^{-1} \left(\frac{\sqrt{a} \sin(x)}{\sqrt{a + a \cos(x)}} \right) + \frac{2a^2(4A + 3B) \sin(x)}{3\sqrt{a + a \cos(x)}} + \frac{2}{3} a A \sqrt{a + a \cos(x)} \sin(x) \end{aligned}$$

Mathematica [A] time = 0.10, size = 62, normalized size = 0.86

$$\frac{1}{3} a \sec\left(\frac{x}{2}\right) \sqrt{a(\cos(x) + 1)} \left(2 \sin\left(\frac{x}{2}\right) (A \cos(x) + 5A + 3B) + 3\sqrt{2} B \tanh^{-1}\left(\sqrt{2} \sin\left(\frac{x}{2}\right)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Cos[x])^(3/2)*(A + B*Sec[x]), x]

[Out] (a*Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(3*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]] + 2*(5*A + 3*B + A*Cos[x])*Sin[x/2]))/3

fricas [A] time = 0.66, size = 99, normalized size = 1.38

$$\frac{3(Ba \cos(x) + Ba)\sqrt{a} \log\left(\frac{a \cos(x)^3 - 7a \cos(x)^2 - 4\sqrt{a} \cos(x) + a \sqrt{a} (\cos(x) - 2) \sin(x) + 8a}{\cos(x)^3 + \cos(x)^2}\right) + 4(Aa \cos(x) + (5A + 3B)a)\sqrt{a}}{6(\cos(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^(3/2)*(A+B*sec(x)),x, algorithm="fricas")

[Out] 1/6*(3*(B*a*cos(x) + B*a)*sqrt(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a)*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)) + 4*(A*a*cos(x) + (5*A + 3*B)*a)*sqrt(a*cos(x) + a)*sin(x)/(cos(x) + 1)

giac [A] time = 0.19, size = 92, normalized size = 1.28

$$-\frac{1}{6} \sqrt{2} \left(8 A a \operatorname{sgn}\left(\cos\left(\frac{1}{2} x\right)\right) \sin\left(\frac{1}{2} x\right)^3 + 3 \sqrt{2} B a \log\left(\frac{\left| -2 \sqrt{2} + 4 \sin\left(\frac{1}{2} x\right) \right|}{\left| 2 \sqrt{2} + 4 \sin\left(\frac{1}{2} x\right) \right|}\right) \operatorname{sgn}\left(\cos\left(\frac{1}{2} x\right)\right) - 24 A a \operatorname{sgn}\left(\cos\left(\frac{1}{2} x\right)\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^(3/2)*(A+B*sec(x)),x, algorithm="giac")

[Out] -1/6*sqrt(2)*(8*A*a*sgn(cos(1/2*x))*sin(1/2*x)^3 + 3*sqrt(2)*B*a*log(abs(-2*sqrt(2) + 4*sin(1/2*x))/abs(2*sqrt(2) + 4*sin(1/2*x)))*sgn(cos(1/2*x)) - 24*A*a*sgn(cos(1/2*x)))

$4Aa \operatorname{sgn}(\cos(1/2x)) \sin(1/2x) - 12B \operatorname{sgn}(\cos(1/2x)) \sin(1/2x) \sqrt{a}$

maple [B] time = 0.31, size = 199, normalized size = 2.76

$$\frac{\sqrt{a} \cos\left(\frac{x}{2}\right) \sqrt{a \left(\sin^2\left(\frac{x}{2}\right)\right)} \left(-4A\sqrt{a} \sqrt{2} \sqrt{a \left(\sin^2\left(\frac{x}{2}\right)\right)} \left(\sin^2\left(\frac{x}{2}\right)\right) + 12A\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{x}{2}\right)\right)} \sqrt{a} + 6B\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{x}{2}\right)\right)} \right)}{3 \sin\left(\frac{x}{2}\right) \sqrt{\cos^2\left(\frac{x}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*cos(x))^(3/2)*(A+B*sec(x)),x)`

[Out] $\frac{1}{3} a^{1/2} \cos(1/2x) (a \sin(1/2x)^2)^{1/2} (-4A a^{1/2} 2^{1/2} (a \sin(1/2x)^2)^{1/2} \sin(1/2x)^2 + 12A 2^{1/2} (a \sin(1/2x)^2)^{1/2} a^{1/2} + 6B 2^{1/2} (a \sin(1/2x)^2)^{1/2} a^{1/2} + 3B \ln(4/(2 \cos(1/2x) + 2^{1/2}))) (a 2^{1/2} \cos(1/2x) + a^{1/2} 2^{1/2} (a \sin(1/2x)^2)^{1/2} + 2a) a + 3B \ln(4/(-2 \cos(1/2x) + 2^{1/2}))) (a^{1/2} 2^{1/2} (a \sin(1/2x)^2)^{1/2} - a 2^{1/2} \cos(1/2x) + 2a) a) / \sin(1/2x) / (\cos(1/2x)^{2a})^{1/2}$

maxima [A] time = 1.11, size = 26, normalized size = 0.36

$$\frac{1}{3} \left(\sqrt{2} a \sin\left(\frac{3}{2}x\right) + 9 \sqrt{2} a \sin\left(\frac{1}{2}x\right) \right) A \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cos(x))^(3/2)*(A+B*sec(x)),x, algorithm="maxima")`

[Out] $\frac{1}{3} (\sqrt{2} a \sin(3/2x) + 9 \sqrt{2} a \sin(1/2x)) A \sqrt{a}$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a \cos(x))^{3/2} \left(A + \frac{B}{\cos(x)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cos(x))^(3/2)*(A + B/cos(x)),x)`

[Out] `int((a + a*cos(x))^(3/2)*(A + B/cos(x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\cos(x) + 1))^{3/2} (A + B \sec(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cos(x))**(3/2)*(A+B*sec(x)),x)`

[Out] `Integral((a*(cos(x) + 1))**(3/2)*(A + B*sec(x)), x)`

3.195 $\int \sqrt{a + a \cos(x)} (A + B \sec(x)) dx$

Optimal. Leaf size=44

$$\frac{2aA \sin(x)}{\sqrt{a \cos(x) + a}} + 2\sqrt{a} B \tanh^{-1} \left(\frac{\sqrt{a} \sin(x)}{\sqrt{a \cos(x) + a}} \right)$$

[Out] $2*B*\operatorname{arctanh}(\sin(x)*a^{(1/2)}/(a+a*\cos(x))^{(1/2)})*a^{(1/2)}+2*a*A*\sin(x)/(a+a*\cos(x))^{(1/2)}$

Rubi [A] time = 0.16, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2828, 2981, 2773, 206}

$$\frac{2aA \sin(x)}{\sqrt{a \cos(x) + a}} + 2\sqrt{a} B \tanh^{-1} \left(\frac{\sqrt{a} \sin(x)}{\sqrt{a \cos(x) + a}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Cos[x]]*(A + B*Sec[x]),x]

[Out] $2*\operatorname{Sqrt}[a]*B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\sin[x])/\operatorname{Sqrt}[a + a*\cos[x]]] + (2*a*A*\sin[x])/\operatorname{Sqrt}[a + a*\cos[x]]$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2773

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*cos[e + f*x])/Sqrt[a + b*sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2828

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_)^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[((a + b*sin[e + f*x])^m*(d + c*sin[e + f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ[n]

Rule 2981

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^n, x_Symbol] := Simp[(-2*b*B*cos[e + f*x]*(c + d*sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a + b*sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b*d*(2*n + 3)), Int[Sqrt[a + b*sin[e + f*x]]*(c + d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \sqrt{a + a \cos(x)} (A + B \sec(x)) dx &= \int \sqrt{a + a \cos(x)} (B + A \cos(x)) \sec(x) dx \\
&= \frac{2aA \sin(x)}{\sqrt{a + a \cos(x)}} + B \int \sqrt{a + a \cos(x)} \sec(x) dx \\
&= \frac{2aA \sin(x)}{\sqrt{a + a \cos(x)}} - (2aB) \operatorname{Subst} \left(\int \frac{1}{a - x^2} dx, x, -\frac{a \sin(x)}{\sqrt{a + a \cos(x)}} \right) \\
&= 2\sqrt{a} B \tanh^{-1} \left(\frac{\sqrt{a} \sin(x)}{\sqrt{a + a \cos(x)}} \right) + \frac{2aA \sin(x)}{\sqrt{a + a \cos(x)}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 1.07

$$\sec\left(\frac{x}{2}\right) \sqrt{a(\cos(x) + 1)} \left(2A \sin\left(\frac{x}{2}\right) + \sqrt{2} B \tanh^{-1}\left(\sqrt{2} \sin\left(\frac{x}{2}\right)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Cos[x]]*(A + B*Sec[x]), x]

[Out] Sqrt[a*(1 + Cos[x])] * Sec[x/2] * (Sqrt[2] * B * ArcTanh[Sqrt[2] * Sin[x/2]] + 2 * A * Sin[x/2])

fricas [B] time = 0.78, size = 81, normalized size = 1.84

$$\frac{(B \cos(x) + B) \sqrt{a} \log\left(\frac{a \cos(x)^3 - 7a \cos(x)^2 - 4\sqrt{a \cos(x) + a} \sqrt{a} (\cos(x) - 2) \sin(x) + 8a}{\cos(x)^3 + \cos(x)^2}\right) + 4 \sqrt{a \cos(x) + a} A \sin(x)}{2(\cos(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^(1/2)*(A+B*sec(x)), x, algorithm="fricas")

[Out] 1/2*((B*cos(x) + B)*sqrt(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)) + 4*sqrt(a*cos(x) + a)*A*sin(x))/(cos(x) + 1)

giac [A] time = 0.18, size = 61, normalized size = 1.39

$$-\frac{1}{2} \sqrt{2} \left(\sqrt{2} B \log \left(\frac{|-2\sqrt{2} + 4 \sin\left(\frac{1}{2}x\right)|}{|2\sqrt{2} + 4 \sin\left(\frac{1}{2}x\right)|} \right) \operatorname{sgn} \left(\cos\left(\frac{1}{2}x\right) \right) - 4 A \operatorname{sgn} \left(\cos\left(\frac{1}{2}x\right) \right) \sin\left(\frac{1}{2}x\right) \right) \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^(1/2)*(A+B*sec(x)), x, algorithm="giac")

[Out] -1/2*sqrt(2)*(sqrt(2)*B*log(abs(-2*sqrt(2) + 4*sin(1/2*x))/abs(2*sqrt(2) + 4*sin(1/2*x))))*sgn(cos(1/2*x)) - 4*A*sgn(cos(1/2*x))*sin(1/2*x)*sqrt(a)

maple [B] time = 0.30, size = 152, normalized size = 3.45

$$\frac{\cos\left(\frac{x}{2}\right) \sqrt{a \left(\sin^2\left(\frac{x}{2}\right)\right)} \left(2A\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{x}{2}\right)\right)} \sqrt{a} + B \ln \left(\frac{4a\sqrt{2} \cos\left(\frac{x}{2}\right) + 4\sqrt{a} \sqrt{2} \sqrt{a \left(\sin^2\left(\frac{x}{2}\right)\right)} + 8a}{2 \cos\left(\frac{x}{2}\right) + \sqrt{2}} \right) a + B \ln \left(-\frac{4\left(\sqrt{a} \sqrt{2} \sqrt{a} \right)}{\dots} \right) \right)}{\sqrt{a} \sin\left(\frac{x}{2}\right) \sqrt{\left(\cos^2\left(\frac{x}{2}\right)\right) a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(x))^(1/2)*(A+B*sec(x)), x)


```
[Out] 1/a^(1/2)*cos(1/2*x)*(a*sin(1/2*x)^2)^(1/2)*(2*A*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1/2)+B*ln(4/(2*cos(1/2*x)+2^(1/2)))*(a*2^(1/2)*cos(1/2*x)+a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)+2*a))*a+B*ln(-4/(-2*cos(1/2*x)+2^(1/2)))*(a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)-a*2^(1/2)*cos(1/2*x)+2*a))*a/sin(1/2*x)/(cos(1/2*x)^2*a)^(1/2)
```

maxima [A] time = 0.47, size = 13, normalized size = 0.30

$$2\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*A*sqrt(a)*sin(1/2*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a + a \cos(x)} \left(A + \frac{B}{\cos(x)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^(1/2)*(A + B/cos(x)),x)
```

```
[Out] int((a + a*cos(x))^(1/2)*(A + B/cos(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\cos(x) + 1)} (A + B \sec(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(1/2)*(A+B*sec(x)),x)
```

```
[Out] Integral(sqrt(a*(cos(x) + 1))*(A + B*sec(x)), x)
```

$$3.196 \quad \int \frac{A+B \sec(x)}{\sqrt{a+a \cos(x)}} dx$$

Optimal. Leaf size=68

$$\frac{\sqrt{2}(A-B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{2} \sqrt{a \cos(x)+a}}\right)}{\sqrt{a}} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{a \cos(x)+a}}\right)}{\sqrt{a}}$$

[Out] $2*B*\operatorname{arctanh}(\sin(x)*a^{(1/2)}/(a+a*\cos(x))^{(1/2)})/a^{(1/2)}+(A-B)*\operatorname{arctanh}(1/2*\sin(x)*a^{(1/2)}*2^{(1/2)}/(a+a*\cos(x))^{(1/2)})*2^{(1/2)}/a^{(1/2)}$

Rubi [A] time = 0.20, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2828, 2985, 2649, 206, 2773}

$$\frac{\sqrt{2}(A-B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{2} \sqrt{a \cos(x)+a}}\right)}{\sqrt{a}} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{a \cos(x)+a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[x])/Sqrt[a + a*Cos[x]],x]

[Out] $(2*B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[x])/(\operatorname{Sqrt}[a + a*\operatorname{Cos}[x]])]/\operatorname{Sqrt}[a] + (\operatorname{Sqrt}[2]*(A - B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Cos}[x]])])/\operatorname{Sqrt}[a]$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2773

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2828

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_)^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ[n]

Rule 2985

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b

$$^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$$

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(x)}{\sqrt{a + a \cos(x)}} dx &= \int \frac{(B + A \cos(x)) \sec(x)}{\sqrt{a + a \cos(x)}} dx \\ &= \frac{B \int \sqrt{a + a \cos(x)} \sec(x) dx}{a} - (-A + B) \int \frac{1}{\sqrt{a + a \cos(x)}} dx \\ &= - \left((2(A - B)) \text{Subst} \left(\int \frac{1}{2a - x^2} dx, x, -\frac{a \sin(x)}{\sqrt{a + a \cos(x)}} \right) \right) - (2B) \text{Subst} \left(\int \frac{1}{a - x^2} dx, x, \frac{a \sin(x)}{\sqrt{a + a \cos(x)}} \right) \\ &= \frac{2B \tanh^{-1} \left(\frac{\sqrt{a} \sin(x)}{\sqrt{a + a \cos(x)}} \right)}{\sqrt{a}} + \frac{\sqrt{2} (A - B) \tanh^{-1} \left(\frac{\sqrt{a} \sin(x)}{\sqrt{2} \sqrt{a + a \cos(x)}} \right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 52, normalized size = 0.76

$$\frac{2 \cos\left(\frac{x}{2}\right) \left((A - B) \tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right) + \sqrt{2} B \tanh^{-1}\left(\sqrt{2} \sin\left(\frac{x}{2}\right)\right) \right)}{\sqrt{a}(\cos(x) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[x])/Sqrt[a + a*Cos[x]], x]

[Out] (2*((A - B)*ArcTanh[Sin[x/2]] + Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]])*Cos[x/2])/Sqrt[a*(1 + Cos[x])]

fricas [B] time = 0.61, size = 116, normalized size = 1.71

$$\frac{\sqrt{2} (A - B) \sqrt{a} \log \left(-\frac{\cos(x)^2 + \frac{2\sqrt{2}\sqrt{a\cos(x)+a}\sin(x)}{\sqrt{a}} - 2\cos(x) - 3}{\cos(x)^2 + 2\cos(x) + 1} \right) - B\sqrt{a} \log \left(\frac{a\cos(x)^3 - 7a\cos(x)^2 - 4\sqrt{a\cos(x)+a}\sqrt{a}(\cos(x)-2)\sin(x) + 8a}{\cos(x)^3 + \cos(x)^2} \right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))^(1/2), x, algorithm="fricas")

[Out] -1/2*(sqrt(2)*(A - B)*sqrt(a)*log(-(cos(x)^2 + 2*sqrt(2)*sqrt(a*cos(x) + a)*sin(x)/sqrt(a) - 2*cos(x) - 3)/(cos(x)^2 + 2*cos(x) + 1)) - B*sqrt(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)))/a

giac [B] time = 0.70, size = 133, normalized size = 1.96

$$\frac{\sqrt{2} (A\sqrt{a} - B\sqrt{a}) \log \left(\left(\sqrt{a} \tan\left(\frac{1}{2}x\right) - \sqrt{a \tan\left(\frac{1}{2}x\right)^2 + a} \right)^2 \right)}{2a} + \frac{B \log \left(\left(\sqrt{a} \tan\left(\frac{1}{2}x\right) - \sqrt{a \tan\left(\frac{1}{2}x\right)^2 + a} \right)^2 \right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))^(1/2), x, algorithm="giac")

[Out] -1/2*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))^2)/a + B*log(abs((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))^2 - a*(2*sqrt(2) + 3)))/sqrt(a) - B*log(abs((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))^2 + a*(2*sqrt(2) - 3)))/sqrt(a)

maple [B] time = 0.35, size = 192, normalized size = 2.82

$$\frac{\cos\left(\frac{x}{2}\right)\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)\left(\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)+4a}}{\cos\left(\frac{x}{2}\right)}\right)A-\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)+4a}}{\cos\left(\frac{x}{2}\right)}\right)B+B\ln\left(\frac{4a\sqrt{2}\cos\left(\frac{x}{2}\right)+4\sqrt{a}\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}}{2\cos\left(\frac{x}{2}\right)+\sqrt{a}\sin\left(\frac{x}{2}\right)}\right)}{\sqrt{a}\sin\left(\frac{x}{2}\right)\sqrt{\left(\cos^2\left(\frac{x}{2}\right)\right)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(x))/(a+a*cos(x))^(1/2),x)

[Out] cos(1/2*x)*(a*sin(1/2*x)^2)^(1/2)*(2^(1/2)*ln(4/cos(1/2*x)*(a^(1/2)*(a*sin(1/2*x)^2)^(1/2)+a))*A-2^(1/2)*ln(4/cos(1/2*x)*(a^(1/2)*(a*sin(1/2*x)^2)^(1/2)+a))*B+B*ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1/2)*cos(1/2*x)+a^(1/2)*2^(1/2))*(a*sin(1/2*x)^2)^(1/2)+2*a))+B*ln(-4/(-2*cos(1/2*x)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)-a*2^(1/2)*cos(1/2*x)+2*a)))/a^(1/2)/sin(1/2*x)/(cos(1/2*x)^2*a)^(1/2)

maxima [A] time = 1.05, size = 58, normalized size = 0.85

$$\frac{\left(\sqrt{2}\log\left(\cos\left(\frac{1}{2}x\right)^2+\sin\left(\frac{1}{2}x\right)^2+2\sin\left(\frac{1}{2}x\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}x\right)^2+\sin\left(\frac{1}{2}x\right)^2-2\sin\left(\frac{1}{2}x\right)+1\right)\right)A}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))^(1/2),x, algorithm="maxima")

[Out] 1/2*(sqrt(2)*log(cos(1/2*x)^2 + sin(1/2*x)^2 + 2*sin(1/2*x) + 1) - sqrt(2)*log(cos(1/2*x)^2 + sin(1/2*x)^2 - 2*sin(1/2*x) + 1))*A/sqrt(a)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(x)}}{\sqrt{a + a \cos(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(x))/(a + a*cos(x))^(1/2),x)

[Out] int((A + B/cos(x))/(a + a*cos(x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(x)}{\sqrt{a(\cos(x) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))**(1/2),x)

[Out] Integral((A + B*sec(x))/sqrt(a*(cos(x) + 1)), x)

$$3.197 \quad \int \frac{A+B \sec(x)}{(a+a \cos(x))^{3/2}} dx$$

Optimal. Leaf size=92

$$\frac{(A-5B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{2} \sqrt{a \cos(x)+a}}\right)}{2\sqrt{2} a^{3/2}} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{a \cos(x)+a}}\right)}{a^{3/2}} + \frac{(A-B) \sin(x)}{2(a \cos(x)+a)^{3/2}}$$

[Out] $2*B*\operatorname{arctanh}(\sin(x)*a^{(1/2)}/(a+a*\cos(x))^{(1/2)})/a^{(3/2)}+1/2*(A-B)*\sin(x)/(a+a*\cos(x))^{(3/2)}+1/4*(A-5*B)*\operatorname{arctanh}(1/2*\sin(x)*a^{(1/2)}*2^{(1/2)}/(a+a*\cos(x))^{(1/2)})/a^{(3/2)}*2^{(1/2)}$

Rubi [A] time = 0.34, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2828, 2978, 2985, 2649, 206, 2773}

$$\frac{(A-5B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{2} \sqrt{a \cos(x)+a}}\right)}{2\sqrt{2} a^{3/2}} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{a \cos(x)+a}}\right)}{a^{3/2}} + \frac{(A-B) \sin(x)}{2(a \cos(x)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A+B*\operatorname{Sec}[x])/(a+a*\operatorname{Cos}[x])^{(3/2)},x]$

[Out] $(2*B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[x])/\operatorname{Sqrt}[a+a*\operatorname{Cos}[x]])]/a^{(3/2)}+((A-5*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Cos}[x]])])/(2*\operatorname{Sqrt}[2]*a^{(3/2)})+((A-B)*\operatorname{Sin}[x])/(2*(a+a*\operatorname{Cos}[x])^{(3/2)})$

Rule 206

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 2649

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_+ + (b_+)*\sin[(c_+ + (d_+)*(x_+)])], x_Symbol] \rightarrow \operatorname{Dist}[-2/d, \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, (b*\operatorname{Cos}[c + d*x])/\operatorname{Sqrt}[a + b*\operatorname{Sin}[c + d*x]]], x] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 2773

$\operatorname{Int}[\operatorname{Sqrt}[(a_+ + (b_+)*\sin[(e_+ + (f_+)*(x_+)])]/((c_+ + (d_+)*\sin[(e_+ + (f_+)*(x_+)])), x_Symbol] \rightarrow \operatorname{Dist}[(-2*b)/f, \operatorname{Subst}[\operatorname{Int}[1/(b*c + a*d - d*x^2), x], x, (b*\operatorname{Cos}[e + f*x])/\operatorname{Sqrt}[a + b*\operatorname{Sin}[e + f*x]]], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{EqQ}[a^2 - b^2, 0] \&\& \operatorname{NeQ}[c^2 - d^2, 0]$

Rule 2828

$\operatorname{Int}[(\operatorname{csc}[(e_+ + (f_+)*(x_+)]*(d_+ + (c_+))^{(n_+)}*((a_+ + (b_+)*\sin[(e_+ + (f_+)*(x_+)]))^{(m_+)}), x_Symbol] \rightarrow \operatorname{Int}[(a + b*\operatorname{Sin}[e + f*x])^m*(d + c*\operatorname{Sin}[e + f*x])^n/\operatorname{Sin}[e + f*x]^n, x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f, m\}, x] \&\& \operatorname{IntegerQ}[n]$

Rule 2978

$\operatorname{Int}[(a_+ + (b_+)*\sin[(e_+ + (f_+)*(x_+)]))^{(m_+)}*((A_+ + (B_+)*\sin[(e_+ + (f_+)*(x_+)]))^{(n_+)}), x_Symbol] \rightarrow \operatorname{Simp}[(b*(A*b - a*B)*\operatorname{Cos}[e + f*x]*(a + b*\operatorname{Sin}[e + f*x])^m*(c + d*\operatorname{Sin}[e + f*x])^{(n+1)})/(a*f*(2*m+1)*(b*c - a*d)), x] + \operatorname{Dist}[1/(a*(2*m+1)*(b*c - a*d)),$

```
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \sec(x)}{(a + a \cos(x))^{3/2}} dx &= \int \frac{(B + A \cos(x)) \sec(x)}{(a + a \cos(x))^{3/2}} dx \\ &= \frac{(A - B) \sin(x)}{2(a + a \cos(x))^{3/2}} + \frac{\int \frac{(2aB + \frac{1}{2}a(A - B) \cos(x)) \sec(x)}{\sqrt{a + a \cos(x)}} dx}{2a^2} \\ &= \frac{(A - B) \sin(x)}{2(a + a \cos(x))^{3/2}} + \frac{(A - 5B) \int \frac{1}{\sqrt{a + a \cos(x)}} dx}{4a} + \frac{B \int \sqrt{a + a \cos(x)} \sec(x) dx}{a^2} \\ &= \frac{(A - B) \sin(x)}{2(a + a \cos(x))^{3/2}} - \frac{(A - 5B) \operatorname{Subst}\left(\int \frac{1}{2a - x^2} dx, x, -\frac{a \sin(x)}{\sqrt{a + a \cos(x)}}\right)}{2a} - \frac{(2B) \operatorname{Subst}\left(\int \frac{1}{a - x^2} dx, x, \frac{a \sin(x)}{\sqrt{a + a \cos(x)}}\right)}{a} \\ &= \frac{2B \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{a + a \cos(x)}}\right)}{a^{3/2}} + \frac{(A - 5B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{2} \sqrt{a + a \cos(x)}}\right)}{2\sqrt{2} a^{3/2}} + \frac{(A - B) \sin(x)}{2(a + a \cos(x))^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.37, size = 73, normalized size = 0.79

$$\frac{\frac{1}{2}(A - B) \sin(x) + (A - 5B) \cos^3\left(\frac{x}{2}\right) \tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right) + 4\sqrt{2} B \cos^3\left(\frac{x}{2}\right) \tanh^{-1}\left(\sqrt{2} \sin\left(\frac{x}{2}\right)\right)}{(a(\cos(x) + 1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^(3/2), x]
```

```
[Out] ((A - 5*B)*ArcTanh[Sin[x/2]]*Cos[x/2]^3 + 4*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]]*Cos[x/2]^3 + ((A - B)*Sin[x])/2)/(a*(1 + Cos[x]))^(3/2)
```

fricas [B] time = 0.70, size = 187, normalized size = 2.03

$$\frac{\sqrt{2} \left((A - 5B) \cos(x)^2 + 2(A - 5B) \cos(x) + A - 5B \right) \sqrt{a} \log\left(-\frac{a \cos(x)^2 + 2\sqrt{2} \sqrt{a \cos(x) + a} \sqrt{a} \sin(x) - 2a \cos(x) - 3a}{\cos(x)^2 + 2 \cos(x) + 1} \right) - 4 \sqrt{2} \sqrt{a} \cos(x)}{8(a^2 \cos(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(3/2), x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(2))*((A - 5*B)*cos(x)^2 + 2*(A - 5*B)*cos(x) + A - 5*B)*sqrt(a)*log(-(a*cos(x))^2 + 2*sqrt(2)*sqrt(a*cos(x) + a)*sqrt(a)*sin(x) - 2*a*cos(x))
```

$$- 3a)/(\cos(x)^2 + 2\cos(x) + 1)) - 4*(B*\cos(x)^2 + 2*B*\cos(x) + B)*\sqrt{a} \\ * \log((a*\cos(x)^3 - 7*a*\cos(x)^2 - 4*\sqrt{a*\cos(x) + a})*\sqrt{a}*(\cos(x) - 2) \\ * \sin(x) + 8*a)/(\cos(x)^3 + \cos(x)^2)) - 4*\sqrt{a*\cos(x) + a}*(A - B)*\sin(x) \\)/(a^2*\cos(x)^2 + 2*a^2*\cos(x) + a^2)$$

giac [B] time = 0.76, size = 168, normalized size = 1.83

$$\frac{\sqrt{2}(A\sqrt{a} - 5B\sqrt{a}) \log\left(\left(\sqrt{a} \tan\left(\frac{1}{2}x\right) - \sqrt{a \tan^2\left(\frac{1}{2}x\right) + a}\right)^2\right)}{8a^2} + \frac{B \log\left(\left(\sqrt{a} \tan\left(\frac{1}{2}x\right) - \sqrt{a \tan^2\left(\frac{1}{2}x\right) + a}\right)^2\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))^(3/2),x, algorithm="giac")

[Out] -1/8*sqrt(2)*(A*sqrt(a) - 5*B*sqrt(a))*log((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))^2)/a^2 + B*log(abs((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a^(3/2) - B*log(abs((sqrt(a)*tan(1/2*x) - sqrt(a*tan(1/2*x)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^(3/2) + 1/4*sqrt(a*tan(1/2*x)^2 + a)*(sqrt(2)*A*a - sqrt(2)*B*a)*tan(1/2*x)/a^3

maple [B] time = 0.34, size = 270, normalized size = 2.93

$$\frac{\sqrt{a \left(\sin^2\left(\frac{x}{2}\right)\right)} \left(A\sqrt{2} \ln\left(\frac{4\sqrt{a} \sqrt{a \left(\sin^2\left(\frac{x}{2}\right)\right) + 4a}}{\cos\left(\frac{x}{2}\right)}\right) \left(\cos^2\left(\frac{x}{2}\right)\right) a - 5B\sqrt{2} \ln\left(\frac{4\sqrt{a} \sqrt{a \left(\sin^2\left(\frac{x}{2}\right)\right) + 4a}}{\cos\left(\frac{x}{2}\right)}\right) \left(\cos^2\left(\frac{x}{2}\right)\right) a + 4B \ln\left(\frac{4\sqrt{a} \sqrt{a \left(\sin^2\left(\frac{x}{2}\right)\right) + 4a}}{\cos\left(\frac{x}{2}\right)}\right) \left(\cos^2\left(\frac{x}{2}\right)\right) a \right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(x))/(a+a*cos(x))^(3/2),x)

[Out] 1/4/a^(5/2)/cos(1/2*x)*(a*sin(1/2*x)^2)^(1/2)*(A*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*x)^2)^(1/2)+2*a)/cos(1/2*x))*cos(1/2*x)^2*a-5*B*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*x)^2)^(1/2)+2*a)/cos(1/2*x))*cos(1/2*x)^2*a+4*B*ln(4/(2*cos(1/2*x)+2^(1/2)))*(a*2^(1/2)*cos(1/2*x)+a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)+2*a))*cos(1/2*x)^2*a+4*B*ln(-4*(a*2^(1/2)*cos(1/2*x)-a^(1/2)*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)-2*a)/(2*cos(1/2*x)-2^(1/2)))*cos(1/2*x)^2*a+A*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1/2)-B*2^(1/2)*(a*sin(1/2*x)^2)^(1/2)*a^(1/2))/sin(1/2*x)/(cos(1/2*x)^2*a)^(1/2)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(x)}}{(a + a \cos(x))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(x))/(a + a*cos(x))^(3/2),x)

[Out] `int((A + B/cos(x))/(a + a*cos(x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(x)}{(a(\cos(x) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sec(x))/(a+a*cos(x))**(3/2), x)`

[Out] `Integral((A + B*sec(x))/(a*(cos(x) + 1))**(3/2), x)`

$$3.198 \quad \int \frac{A+B \sec(x)}{(a+a \cos(x))^{5/2}} dx$$

Optimal. Leaf size=120

$$\frac{(3A - 43B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{2} \sqrt{a \cos(x)+a}}\right)}{16\sqrt{2} a^{5/2}} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{a \cos(x)+a}}\right)}{a^{5/2}} + \frac{(3A - 11B) \sin(x)}{16a(a \cos(x) + a)^{3/2}} + \frac{(A - B) \sin(x)}{4(a \cos(x) + a)^{5/2}}$$

[Out] $2*B*\operatorname{arctanh}(\sin(x)*a^{(1/2)}/(a+a*\cos(x))^{(1/2)})/a^{(5/2)}+1/4*(A-B)*\sin(x)/(a+a*\cos(x))^{(5/2)}+1/16*(3*A-11*B)*\sin(x)/a/(a+a*\cos(x))^{(3/2)}+1/32*(3*A-43*B)*\operatorname{arctanh}(1/2*\sin(x)*a^{(1/2)}*2^{(1/2)}/(a+a*\cos(x))^{(1/2)})/a^{(5/2)}*2^{(1/2)}$

Rubi [A] time = 0.48, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2828, 2978, 2985, 2649, 206, 2773}

$$\frac{(3A - 43B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{2} \sqrt{a \cos(x)+a}}\right)}{16\sqrt{2} a^{5/2}} + \frac{2B \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{a \cos(x)+a}}\right)}{a^{5/2}} + \frac{(3A - 11B) \sin(x)}{16a(a \cos(x) + a)^{3/2}} + \frac{(A - B) \sin(x)}{4(a \cos(x) + a)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^(5/2), x]

[Out] $(2*B*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[x])/(\operatorname{Sqrt}[a + a*\operatorname{Cos}[x]])])/a^{(5/2)} + ((3*A - 43*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Cos}[x]])])/(16*\operatorname{Sqrt}[2]*a^{(5/2)}) + ((A - B)*\operatorname{Sin}[x])/(4*(a + a*\operatorname{Cos}[x])^{(5/2)}) + ((3*A - 11*B)*\operatorname{Sin}[x])/(16*a*(a + a*\operatorname{Cos}[x])^{(3/2)})$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Ssin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2773

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Ssin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2828

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.) + (c_)^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[((a + b*Ssin[e + f*x])^m*(d + c*Ssin[e + f*x])^n)/Sin[e + f*x]^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ[n]

Rule 2978

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*(c + d*Ssin[e + f*x])^(

```

n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

```

Rule 2985

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])/(Sqrt[(a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_.)]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sec(x)}{(a + a \cos(x))^{5/2}} dx &= \int \frac{(B + A \cos(x)) \sec(x)}{(a + a \cos(x))^{5/2}} dx \\
&= \frac{(A - B) \sin(x)}{4(a + a \cos(x))^{5/2}} + \frac{\int \frac{(4aB + \frac{3}{2}a(A - B) \cos(x)) \sec(x)}{(a + a \cos(x))^{3/2}} dx}{4a^2} \\
&= \frac{(A - B) \sin(x)}{4(a + a \cos(x))^{5/2}} + \frac{(3A - 11B) \sin(x)}{16a(a + a \cos(x))^{3/2}} + \frac{\int \frac{(8a^2B + \frac{1}{4}a^2(3A - 11B) \cos(x)) \sec(x)}{\sqrt{a + a \cos(x)}} dx}{8a^4} \\
&= \frac{(A - B) \sin(x)}{4(a + a \cos(x))^{5/2}} + \frac{(3A - 11B) \sin(x)}{16a(a + a \cos(x))^{3/2}} + \frac{(3A - 43B) \int \frac{1}{\sqrt{a + a \cos(x)}} dx}{32a^2} + \frac{B \int \sqrt{a + a \cos(x)} dx}{16a^2} \\
&= \frac{(A - B) \sin(x)}{4(a + a \cos(x))^{5/2}} + \frac{(3A - 11B) \sin(x)}{16a(a + a \cos(x))^{3/2}} - \frac{(3A - 43B) \operatorname{Subst}\left(\int \frac{1}{2a - x^2} dx, x, -\frac{a \sin(x)}{\sqrt{a + a \cos(x)}}\right)}{16a^2} \\
&= \frac{2B \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{a + a \cos(x)}}\right)}{a^{5/2}} + \frac{(3A - 43B) \tanh^{-1}\left(\frac{\sqrt{a} \sin(x)}{\sqrt{2} \sqrt{a + a \cos(x)}}\right)}{16\sqrt{2} a^{5/2}} + \frac{(A - B) \sin(x)}{4(a + a \cos(x))^{5/2}} + \frac{(3A - 43B) \sin(x)}{16a(a + a \cos(x))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.50, size = 95, normalized size = 0.79

$$\frac{\tan\left(\frac{x}{2}\right) (3A \cos(x) + 7A - 11B \cos(x) - 15B) + 2(3A - 43B) \cos^3\left(\frac{x}{2}\right) \tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right) + 64\sqrt{2} B \cos^3\left(\frac{x}{2}\right) \tanh^{-1}\left(\frac{\sin(x)}{\sqrt{2} \sqrt{a + a \cos(x)}}\right)}{16a(a(\cos(x) + 1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^(5/2), x]
```

```
[Out] (2*(3*A - 43*B)*ArcTanh[Sin[x/2]]*Cos[x/2]^3 + 64*Sqrt[2]*B*ArcTanh[Sqrt[2]
*Sin[x/2]]*Cos[x/2]^3 + (7*A - 15*B + 3*A*Cos[x] - 11*B*Cos[x])*Tan[x/2])/
(16*a*(a*(1 + Cos[x]))^(3/2))
```

fricas [B] time = 0.73, size = 234, normalized size = 1.95

$$\frac{\sqrt{2} \left((3A - 43B) \cos(x)^3 + 3(3A - 43B) \cos(x)^2 + 3(3A - 43B) \cos(x) + 3A - 43B \right) \sqrt{a} \log\left(-\frac{a \cos(x)^2 + 2\sqrt{2} \sqrt{a} \sin(x) + a}{a(\cos(x) + 1)}\right)}{16a(a(\cos(x) + 1))^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))^(5/2),x, algorithm="fricas")

[Out]
$$-1/64*\sqrt{2}*((3*A - 43*B)*\cos(x)^3 + 3*(3*A - 43*B)*\cos(x)^2 + 3*(3*A - 43*B)*\cos(x) + 3*A - 43*B)*\sqrt{a}*\log(-(a*\cos(x)^2 + 2*\sqrt{2}*\sqrt{a*\cos(x) + a})*\sqrt{a}*\sin(x) - 2*a*\cos(x) - 3*a)/(\cos(x)^2 + 2*\cos(x) + 1)) - 32*(B*\cos(x)^3 + 3*B*\cos(x)^2 + 3*B*\cos(x) + B)*\sqrt{a}*\log((a*\cos(x)^3 - 7*a*\cos(x)^2 - 4*\sqrt{a*\cos(x) + a})*\sqrt{a}*(\cos(x) - 2)*\sin(x) + 8*a)/(\cos(x)^3 + \cos(x)^2)) - 4*((3*A - 11*B)*\cos(x) + 7*A - 15*B)*\sqrt{a*\cos(x) + a}*\sin(x)/(a^3*\cos(x)^3 + 3*a^3*\cos(x)^2 + 3*a^3*\cos(x) + a^3)$$

giac [B] time = 0.95, size = 199, normalized size = 1.66

$$\frac{1}{32} \sqrt{a \tan\left(\frac{1}{2}x\right)^2 + a} \left(\frac{2\sqrt{2}(Aa^5 - Ba^5) \tan\left(\frac{1}{2}x\right)^2}{a^8} + \frac{\sqrt{2}(5Aa^5 - 13Ba^5)}{a^8} \right) \tan\left(\frac{1}{2}x\right) - \frac{\sqrt{2}(3A\sqrt{a} - 43B\sqrt{a})}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))^(5/2),x, algorithm="giac")

[Out]
$$1/32*\sqrt{a*\tan(1/2*x)^2 + a}*(2*\sqrt{2}*(A*a^5 - B*a^5)*\tan(1/2*x)^2/a^8 + \sqrt{2}*(5*A*a^5 - 13*B*a^5)/a^8)*\tan(1/2*x) - 1/64*\sqrt{2}*(3*A*\sqrt{a} - 43*B*\sqrt{a})*\log((\sqrt{a}*\tan(1/2*x) - \sqrt{a*\tan(1/2*x)^2 + a})^2/a^3 + B*\log(\text{abs}((\sqrt{a}*\tan(1/2*x) - \sqrt{a*\tan(1/2*x)^2 + a})^2 - a*(2*\sqrt{2} + 3)))/a^{(5/2)} - B*\log(\text{abs}((\sqrt{a}*\tan(1/2*x) - \sqrt{a*\tan(1/2*x)^2 + a})^2 + a*(2*\sqrt{2} - 3)))/a^{(5/2)})$$

maple [B] time = 0.36, size = 322, normalized size = 2.68

$$\sqrt{a \left(\sin^2\left(\frac{x}{2}\right)\right)} \left(3A\sqrt{2} \ln\left(\frac{4\sqrt{a} \sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right) + 4a}}{\cos\left(\frac{x}{2}\right)}\right) \left(\cos^4\left(\frac{x}{2}\right)\right) a - 43B\sqrt{2} \ln\left(\frac{4\sqrt{a} \sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right) + 4a}}{\cos\left(\frac{x}{2}\right)}\right) a \left(\cos^4\left(\frac{x}{2}\right)\right) + 32 \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(x))/(a+a*cos(x))^(5/2),x)

[Out]
$$1/32/a^{(7/2)}/\cos(1/2*x)^3*(a*\sin(1/2*x)^2)^{(1/2)}*(3*A*2^{(1/2)}*\ln(2*(2*a^{(1/2)}*(a*\sin(1/2*x)^2)^{(1/2)}+2*a)/\cos(1/2*x))*\cos(1/2*x)^4*a-43*B*2^{(1/2)}*\ln(2*(2*a^{(1/2)}*(a*\sin(1/2*x)^2)^{(1/2)}+2*a)/\cos(1/2*x))*a*\cos(1/2*x)^4+32*B*\ln(4/(2*\cos(1/2*x)+2^{(1/2)}))*(a*2^{(1/2)}*\cos(1/2*x)+a^{(1/2)}*2^{(1/2)}*(a*\sin(1/2*x)^2)^{(1/2)}+2*a)*a*\cos(1/2*x)^4+32*B*\ln(-4*(a*2^{(1/2)}*\cos(1/2*x)-a^{(1/2)}*2^{(1/2)}*(a*\sin(1/2*x)^2)^{(1/2)}-2*a)/(2*\cos(1/2*x)-2^{(1/2)}))*a*\cos(1/2*x)^4+3*A*a^{(1/2)}*2^{(1/2)}*(a*\sin(1/2*x)^2)^{(1/2)}*\cos(1/2*x)^2-11*B*a^{(1/2)}*2^{(1/2)}*(a*\sin(1/2*x)^2)^{(1/2)}*\cos(1/2*x)^2+2*A*2^{(1/2)}*(a*\sin(1/2*x)^2)^{(1/2)}*a^{(1/2)}-2*B*2^{(1/2)}*(a*\sin(1/2*x)^2)^{(1/2)}*a^{(1/2)})/\sin(1/2*x)/(\cos(1/2*x)^2*a)^{(1/2)}$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(x)}}{(a + a \cos(x))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(x))/(a + a*cos(x))^(5/2), x)

[Out] int((A + B/cos(x))/(a + a*cos(x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec(x)}{(a(\cos(x) + 1))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(x))/(a+a*cos(x))**(5/2), x)

[Out] Integral((A + B*sec(x))/(a*(cos(x) + 1))**(5/2), x)

$$3.199 \quad \int \frac{x(b+a \sin(x))}{(a+b \sin(x))^2} dx$$

Optimal. Leaf size=25

$$\frac{\log(a + b \sin(x))}{b} - \frac{x \cos(x)}{a + b \sin(x)}$$

[Out] $\ln(a+b*\sin(x))/b-x*\cos(x)/(a+b*\sin(x))$

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4592, 2668, 31}

$$\frac{\log(a + b \sin(x))}{b} - \frac{x \cos(x)}{a + b \sin(x)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*(b + a*\text{Sin}[x]))/(a + b*\text{Sin}[x])^2, x]$

[Out] $\text{Log}[a + b*\text{Sin}[x]]/b - (x*\text{Cos}[x])/(a + b*\text{Sin}[x])$

Rule 31

$\text{Int}[(a_ + (b_)*(x_))^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rule 2668

$\text{Int}[\cos[(e_ + (f_)*(x_))]^{(p_)}*((a_ + (b_)*\sin[(e_ + (f_)*(x_))])^{(m_)}), x_Symbol] \rightarrow \text{Dist}[1/(b^p*f), \text{Subst}[\text{Int}[(a + x)^m*(b^2 - x^2)^{(p-1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x] \&\& \text{IntegerQ}[(p-1)/2] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 4592

$\text{Int}[(e_ + (f_)*(x_))*((A_ + (B_)*\text{Sin}[(c_ + (d_)*(x_))]))/((a_ + (b_)*\text{Sin}[(c_ + (d_)*(x_))])^2), x_Symbol] \rightarrow -\text{Simp}[(B*(e + f*x)*\text{Cos}[c + d*x])/(a*d*(a + b*\text{Sin}[c + d*x])), x] + \text{Dist}[(B*f)/(a*d), \text{Int}[\text{Cos}[c + d*x]/(a + b*\text{Sin}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{EqQ}[a*A - b*B, 0]$

Rubi steps

$$\begin{aligned} \int \frac{x(b+a \sin(x))}{(a+b \sin(x))^2} dx &= -\frac{x \cos(x)}{a+b \sin(x)} + \int \frac{\cos(x)}{a+b \sin(x)} dx \\ &= -\frac{x \cos(x)}{a+b \sin(x)} + \frac{\text{Subst}\left(\int \frac{1}{a+x} dx, x, b \sin(x)\right)}{b} \\ &= \frac{\log(a+b \sin(x))}{b} - \frac{x \cos(x)}{a+b \sin(x)} \end{aligned}$$

Mathematica [A] time = 0.20, size = 25, normalized size = 1.00

$$\frac{\log(a + b \sin(x))}{b} - \frac{x \cos(x)}{a + b \sin(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(b + a*Sin[x]))/(a + b*Sin[x])^2,x]

[Out] Log[a + b*Sin[x]]/b - (x*Cos[x])/(a + b*Sin[x])

fricas [A] time = 0.71, size = 35, normalized size = 1.40

$$\frac{bx \cos(x) - (b \sin(x) + a) \log(b \sin(x) + a)}{b^2 \sin(x) + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b+a*sin(x))/(a+b*sin(x))^2,x, algorithm="fricas")

[Out] -(b*x*cos(x) - (b*sin(x) + a)*log(b*sin(x) + a))/(b^2*sin(x) + a*b)

giac [B] time = 0.32, size = 283, normalized size = 11.32

$$4bx \tan\left(\frac{1}{2}x\right)^2 + a \log\left(\frac{4\left(a^2 \tan\left(\frac{1}{2}x\right)^4 + 4ab \tan\left(\frac{1}{2}x\right)^3 + 2a^2 \tan\left(\frac{1}{2}x\right)^2 + 4b^2 \tan\left(\frac{1}{2}x\right) + 4ab \tan\left(\frac{1}{2}x\right) + a^2\right)}{\tan\left(\frac{1}{2}x\right)^4 + 2 \tan\left(\frac{1}{2}x\right)^2 + 1}\right) \tan\left(\frac{1}{2}x\right)^2 + 2b \log\left(\frac{4\left(a^2 \tan\left(\frac{1}{2}x\right)^4 + 4ab \tan\left(\frac{1}{2}x\right)^3 + 2a^2 \tan\left(\frac{1}{2}x\right)^2 + 4b^2 \tan\left(\frac{1}{2}x\right) + 4ab \tan\left(\frac{1}{2}x\right) + a^2\right)}{\tan\left(\frac{1}{2}x\right)^4 + 2 \tan\left(\frac{1}{2}x\right)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b+a*sin(x))/(a+b*sin(x))^2,x, algorithm="giac")

[Out] 1/2*(4*b*x*tan(1/2*x)^2 + a*log(4*(a^2*tan(1/2*x)^4 + 4*a*b*tan(1/2*x)^3 + 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x) + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 2*b*log(4*(a^2*tan(1/2*x)^4 + 4*a*b*tan(1/2*x)^3 + 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x) + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x) - 4*b*x + a*log(4*(a^2*tan(1/2*x)^4 + 4*a*b*tan(1/2*x)^3 + 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x) + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/(a*b*tan(1/2*x)^2 + 2*b^2*tan(1/2*x) + a*b)

maple [B] time = 0.70, size = 80, normalized size = 3.20

$$\frac{x \left(\tan^4\left(\frac{x}{2}\right)\right) - x}{\left(1 + \tan^2\left(\frac{x}{2}\right)\right) \left(a \left(\tan^2\left(\frac{x}{2}\right)\right) + 2b \tan\left(\frac{x}{2}\right) + a\right)} + \frac{\ln\left(a \left(\tan^2\left(\frac{x}{2}\right)\right) + 2b \tan\left(\frac{x}{2}\right) + a\right)}{b} - \frac{\ln\left(1 + \tan^2\left(\frac{x}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b+a*sin(x))/(a+b*sin(x))^2,x)

[Out] (x*tan(1/2*x)^4-x)/(1+tan(1/2*x)^2)/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)+1/b*ln(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)-1/b*ln(1+tan(1/2*x)^2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b+a*sin(x))/(a+b*sin(x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more details)Is 4*b^2-4*a^2 positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x (b + a \sin(x))}{(a + b \sin(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(b + a*sin(x)))/(a + b*sin(x))^2,x)

[Out] int((x*(b + a*sin(x)))/(a + b*sin(x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b+a*sin(x))/(a+b*sin(x))**2,x)

[Out] Timed out

$$3.200 \quad \int \frac{x(b+a \cos(x))}{(a+b \cos(x))^2} dx$$

Optimal. Leaf size=24

$$\frac{\log(a+b \cos(x))}{b} + \frac{x \sin(x)}{a+b \cos(x)}$$

[Out] $\ln(a+b*\cos(x))/b+x*\sin(x)/(a+b*\cos(x))$

Rubi [A] time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4593, 2668, 31}

$$\frac{\log(a+b \cos(x))}{b} + \frac{x \sin(x)}{a+b \cos(x)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x*(b + a*\text{Cos}[x]))/(a + b*\text{Cos}[x])^2, x]$

[Out] $\text{Log}[a + b*\text{Cos}[x]]/b + (x*\text{Sin}[x])/(a + b*\text{Cos}[x])$

Rule 31

$\text{Int}[(a_ + (b_)*(x_))^{(-1)}, x_Symbol] := \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rule 2668

$\text{Int}[\cos[(e_ + (f_)*(x_))]^{(p_)}*((a_ + (b_)*\sin[(e_ + (f_)*(x_))])^{(m_)}), x_Symbol] := \text{Dist}[1/(b^p*f), \text{Subst}[\text{Int}[(a + x)^m*(b^2 - x^2)^{(p-1)/2}, x], x, b*\sin[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x] \&\& \text{IntegerQ}[(p-1)/2] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 4593

$\text{Int}[(\cos[(c_ + (d_)*(x_)]*(B_ + (A_))*((e_ + (f_)*(x_)))/(\cos[(c_ + (d_)*(x_)]*(b_ + (a_))^2, x_Symbol] := \text{Simp}[(B*(e + f*x)*\sin[c + d*x])/(a*d*(a + b*\cos[c + d*x])), x] - \text{Dist}[(B*f)/(a*d), \text{Int}[\sin[c + d*x]/(a + b*\cos[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{EqQ}[a*A - b*B, 0]$

Rubi steps

$$\begin{aligned} \int \frac{x(b+a \cos(x))}{(a+b \cos(x))^2} dx &= \frac{x \sin(x)}{a+b \cos(x)} - \int \frac{\sin(x)}{a+b \cos(x)} dx \\ &= \frac{x \sin(x)}{a+b \cos(x)} + \frac{\text{Subst}\left(\int \frac{1}{a+x} dx, x, b \cos(x)\right)}{b} \\ &= \frac{\log(a+b \cos(x))}{b} + \frac{x \sin(x)}{a+b \cos(x)} \end{aligned}$$

Mathematica [A] time = 0.13, size = 24, normalized size = 1.00

$$\frac{\log(a+b \cos(x))}{b} + \frac{x \sin(x)}{a+b \cos(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(b + a*cos[x]))/(a + b*cos[x])^2,x]

[Out] Log[a + b*cos[x]]/b + (x*sin[x])/(a + b*cos[x])

fricas [A] time = 0.74, size = 36, normalized size = 1.50

$$\frac{bx \sin(x) + (b \cos(x) + a) \log(-b \cos(x) - a)}{b^2 \cos(x) + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b+a*cos(x))/(a+b*cos(x))^2,x, algorithm="fricas")

[Out] (b*x*sin(x) + (b*cos(x) + a)*log(-b*cos(x) - a))/(b^2*cos(x) + a*b)

giac [B] time = 0.32, size = 397, normalized size = 16.54

$$a \log \left(\frac{4 \left(a^2 \tan\left(\frac{1}{2}x\right)^4 - 2ab \tan\left(\frac{1}{2}x\right)^4 + b^2 \tan\left(\frac{1}{2}x\right)^4 + 2a^2 \tan\left(\frac{1}{2}x\right)^2 - 2b^2 \tan\left(\frac{1}{2}x\right)^2 + a^2 + 2ab + b^2 \right)}{\tan\left(\frac{1}{2}x\right)^4 + 2 \tan\left(\frac{1}{2}x\right)^2 + 1} \right) \tan\left(\frac{1}{2}x\right)^2 - b \log \left(\frac{4 \left(a^2 \tan\left(\frac{1}{2}x\right)^4 - 2ab \tan\left(\frac{1}{2}x\right)^4 + b^2 \tan\left(\frac{1}{2}x\right)^4 + 2a^2 \tan\left(\frac{1}{2}x\right)^2 - 2b^2 \tan\left(\frac{1}{2}x\right)^2 + a^2 + 2ab + b^2 \right)}{\tan\left(\frac{1}{2}x\right)^4 + 2 \tan\left(\frac{1}{2}x\right)^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b+a*cos(x))/(a+b*cos(x))^2,x, algorithm="giac")

[Out] 1/2*(a*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1/2*x)^4 + b^2*tan(1/2*x)^4 + 2*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2 + 2*a*b + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - b*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1/2*x)^4 + b^2*tan(1/2*x)^4 + 2*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2 + 2*a*b + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 8*b*x*tan(1/2*x) + a*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1/2*x)^4 + b^2*tan(1/2*x)^4 + 2*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2 + 2*a*b + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)) + b*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1/2*x)^4 + b^2*tan(1/2*x)^4 + 2*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2 + 2*a*b + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/(a*b*tan(1/2*x)^2 - b^2*tan(1/2*x)^2 + a*b + b^2)

maple [B] time = 0.41, size = 91, normalized size = 3.79

$$\frac{2x \tan\left(\frac{x}{2}\right) + 2x \left(\tan^3\left(\frac{x}{2}\right)\right)}{\left(1 + \tan^2\left(\frac{x}{2}\right)\right) \left(a \left(\tan^2\left(\frac{x}{2}\right)\right) - b \left(\tan^2\left(\frac{x}{2}\right)\right) + a + b\right)} + \frac{\ln\left(a \left(\tan^2\left(\frac{x}{2}\right)\right) - b \left(\tan^2\left(\frac{x}{2}\right)\right) + a + b\right)}{b} - \frac{\ln\left(1 + \tan^2\left(\frac{x}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b+a*cos(x))/(a+b*cos(x))^2,x)

[Out] (2*x*tan(1/2*x)+2*x*tan(1/2*x)^3)/(1+tan(1/2*x)^2)/(a*tan(1/2*x)^2-b*tan(1/2*x)^2+a+b)+1/b*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+a+b)-1/b*ln(1+tan(1/2*x)^2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b+a*cos(x))/(a+b*cos(x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details) Is $4a^2-4b^2$ positive or negative?

mupad [B] time = 2.74, size = 68, normalized size = 2.83

$$\frac{\ln(b + 2ae^{x1i} + be^{x2i})}{b} - \frac{x2i}{b} + \frac{x2i + \frac{axe^{x1i}2i}{b}}{b + 2ae^{x1i} + be^{x2i}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(b + a*cos(x)))/(a + b*cos(x))^2,x)

[Out] $\log(b + 2a\exp(x1i) + b\exp(x2i))/b - (x2i)/b + (x2i + (a*x*\exp(x1i)*2i)/b)/(b + 2a*\exp(x1i) + b*\exp(x2i))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b+a*cos(x))/(a+b*cos(x))**2,x)

[Out] Timed out

$$3.201 \quad \int \frac{1+\sin^2(x)}{1-\sin^2(x)} dx$$

Optimal. Leaf size=8

$$2 \tan(x) - x$$

[Out] -x+2*tan(x)

Rubi [A] time = 0.04, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3171, 3175, 3767, 8}

$$2 \tan(x) - x$$

Antiderivative was successfully verified.

[In] Int[(1 + Sin[x]^2)/(1 - Sin[x]^2), x]

[Out] -x + 2*Tan[x]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3171

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^2)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a + b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]

Rule 3175

Int[(u_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2)^(p_), x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1 + \sin^2(x)}{1 - \sin^2(x)} dx &= -x + 2 \int \frac{1}{1 - \sin^2(x)} dx \\ &= -x + 2 \int \sec^2(x) dx \\ &= -x - 2 \text{Subst}\left(\int 1 dx, x, -\tan(x)\right) \\ &= -x + 2 \tan(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$2 \tan(x) - x$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sin[x]^2)/(1 - Sin[x]^2),x]

[Out] -x + 2*Tan[x]

fricas [A] time = 0.65, size = 15, normalized size = 1.88

$$\frac{x \cos(x) - 2 \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sin(x)^2)/(1-sin(x)^2),x, algorithm="fricas")

[Out] -(x*cos(x) - 2*sin(x))/cos(x)

giac [A] time = 0.13, size = 8, normalized size = 1.00

$$-x + 2 \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sin(x)^2)/(1-sin(x)^2),x, algorithm="giac")

[Out] -x + 2*tan(x)

maple [A] time = 0.08, size = 9, normalized size = 1.12

$$-x + 2 \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+sin(x)^2)/(1-sin(x)^2),x)

[Out] -x+2*tan(x)

maxima [A] time = 0.44, size = 8, normalized size = 1.00

$$-x + 2 \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sin(x)^2)/(1-sin(x)^2),x, algorithm="maxima")

[Out] -x + 2*tan(x)

mupad [B] time = 2.31, size = 8, normalized size = 1.00

$$2 \tan(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(sin(x)^2 + 1)/(sin(x)^2 - 1),x)

[Out] 2*tan(x) - x

sympy [B] time = 0.97, size = 41, normalized size = 5.12

$$-\frac{x \tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) - 1} + \frac{x}{\tan^2\left(\frac{x}{2}\right) - 1} - \frac{4 \tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sin(x)**2)/(1-sin(x)**2),x)

[Out] -x*tan(x/2)**2/(tan(x/2)**2 - 1) + x/(tan(x/2)**2 - 1) - 4*tan(x/2)/(tan(x/2)**2 - 1)

$$3.202 \quad \int \frac{1 - \sin^2(x)}{1 + \sin^2(x)} dx$$

Optimal. Leaf size=36

$$\sqrt{2}x - x + \sqrt{2} \tan^{-1} \left(\frac{\sin(x) \cos(x)}{\sin^2(x) + \sqrt{2} + 1} \right)$$

[Out] $-x + x \cdot 2^{(1/2)} + \arctan(\cos(x) \cdot \sin(x) / (1 + \sin(x)^2 + 2^{(1/2)})) \cdot 2^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3171, 3181, 203}

$$\sqrt{2}x - x + \sqrt{2} \tan^{-1} \left(\frac{\sin(x) \cos(x)}{\sin^2(x) + \sqrt{2} + 1} \right)$$

Antiderivative was successfully verified.

[In] Int[(1 - Sin[x]^2)/(1 + Sin[x]^2), x]

[Out] $-x + \text{Sqrt}[2] \cdot x + \text{Sqrt}[2] \cdot \text{ArcTan}[(\text{Cos}[x] \cdot \text{Sin}[x]) / (1 + \text{Sqrt}[2] + \text{Sin}[x]^2)]$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3171

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^2)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a + b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]

Rule 3181

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2)^(-1), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rubi steps

$$\begin{aligned} \int \frac{1 - \sin^2(x)}{1 + \sin^2(x)} dx &= -x + 2 \int \frac{1}{1 + \sin^2(x)} dx \\ &= -x + 2 \text{Subst} \left(\int \frac{1}{1 + 2x^2} dx, x, \tan(x) \right) \\ &= -x + \sqrt{2}x + \sqrt{2} \tan^{-1} \left(\frac{\cos(x) \sin(x)}{1 + \sqrt{2} + \sin^2(x)} \right) \end{aligned}$$

Mathematica [A] time = 0.03, size = 24, normalized size = 0.67

$$-2 \left(\frac{x}{2} - \frac{\tan^{-1}(\sqrt{2} \tan(x))}{\sqrt{2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sin[x]^2)/(1 + Sin[x]^2),x]

[Out] -2*(x/2 - ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2])

fricas [A] time = 0.65, size = 35, normalized size = 0.97

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sin(x)^2)/(1+sin(x)^2),x, algorithm="fricas")

[Out] -1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) - x

giac [A] time = 0.13, size = 49, normalized size = 1.36

$$\sqrt{2}\left(x + \arctan\left(-\frac{\sqrt{2}\sin(2x) - 2\sin(2x)}{\sqrt{2}\cos(2x) + \sqrt{2} - 2\cos(2x) + 2}\right)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sin(x)^2)/(1+sin(x)^2),x, algorithm="giac")

[Out] sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - 2*cos(2*x) + 2))) - x

maple [A] time = 0.08, size = 16, normalized size = 0.44

$$\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-sin(x)^2)/(1+sin(x)^2),x)

[Out] 2^(1/2)*arctan(2^(1/2)*tan(x))-x

maxima [A] time = 0.55, size = 15, normalized size = 0.42

$$\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sin(x)^2)/(1+sin(x)^2),x, algorithm="maxima")

[Out] sqrt(2)*arctan(sqrt(2)*tan(x)) - x

mupad [B] time = 2.32, size = 26, normalized size = 0.72

$$\sqrt{2}(x - \operatorname{atan}(\tan(x))) - x + \sqrt{2}\operatorname{atan}\left(\sqrt{2}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(sin(x)^2 - 1)/(sin(x)^2 + 1),x)

[Out] 2^(1/2)*(x - atan(tan(x))) - x + 2^(1/2)*atan(2^(1/2)*tan(x))

sympy [B] time = 48.46, size = 248, normalized size = 6.89

$$\frac{22619537x}{15994428\sqrt{2} + 22619537} - \frac{15994428\sqrt{2}x}{15994428\sqrt{2} + 22619537} + \frac{54608393\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right) + \pi\left[\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right]\right)}{15994428\sqrt{2} + 22619537}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sin(x)**2)/(1+sin(x)**2),x)

[Out]
$$\begin{aligned} & -22619537*x/(15994428*\sqrt{2} + 22619537) - 15994428*\sqrt{2}*x/(15994428*\sqrt{2} + 22619537) + 54608393*\sqrt{2}*\sqrt{3 - 2*\sqrt{2}}*(\operatorname{atan}(\tan(x/2)/\sqrt{3 - 2*\sqrt{2}})) + \pi*\operatorname{floor}((x/2 - \pi/2)/\pi)/(15994428*\sqrt{2} + 22619537) \\ & + 77227930*\sqrt{3 - 2*\sqrt{2}}*(\operatorname{atan}(\tan(x/2)/\sqrt{3 - 2*\sqrt{2}})) + \pi*\operatorname{floor}((x/2 - \pi/2)/\pi)/(15994428*\sqrt{2} + 22619537) + 9369319*\sqrt{2}*\sqrt{2*\sqrt{2} + 3}*(\operatorname{atan}(\tan(x/2)/\sqrt{2*\sqrt{2} + 3})) + \pi*\operatorname{floor}((x/2 - \pi/2)/\pi)/(15994428*\sqrt{2} + 22619537) + 13250218*\sqrt{2*\sqrt{2} + 3}*(\operatorname{atan}(\tan(x/2)/\sqrt{2*\sqrt{2} + 3})) + \pi*\operatorname{floor}((x/2 - \pi/2)/\pi)/(15994428*\sqrt{2} + 22619537) \end{aligned}$$

$$3.203 \quad \int \frac{1+\cos^2(x)}{1-\cos^2(x)} dx$$

Optimal. Leaf size=8

$$-x - 2 \cot(x)$$

[Out] -x-2*cot(x)

Rubi [A] time = 0.04, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3171, 3175, 3767, 8}

$$-x - 2 \cot(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x]^2)/(1 - Cos[x]^2), x]

[Out] -x - 2*Cot[x]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3171

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]^2)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a + b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]

Rule 3175

Int[(u_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_), x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1 + \cos^2(x)}{1 - \cos^2(x)} dx &= -x + 2 \int \frac{1}{1 - \cos^2(x)} dx \\ &= -x + 2 \int \csc^2(x) dx \\ &= -x - 2 \operatorname{Subst}\left(\int 1 dx, x, \cot(x)\right) \\ &= -x - 2 \cot(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$-x - 2 \cot(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Cos[x]^2)/(1 - Cos[x]^2), x]

[Out] -x - 2*Cot[x]

fricas [A] time = 0.65, size = 15, normalized size = 1.88

$$-\frac{x \sin(x) + 2 \cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)^2)/(1-cos(x)^2), x, algorithm="fricas")

[Out] -(x*sin(x) + 2*cos(x))/sin(x)

giac [A] time = 0.15, size = 16, normalized size = 2.00

$$-x - \frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)^2)/(1-cos(x)^2), x, algorithm="giac")

[Out] -x - 1/tan(1/2*x) + tan(1/2*x)

maple [A] time = 0.08, size = 11, normalized size = 1.38

$$-\frac{2}{\tan(x)} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+cos(x)^2)/(1-cos(x)^2), x)

[Out] -2/tan(x)-x

maxima [A] time = 0.77, size = 10, normalized size = 1.25

$$-x - \frac{2}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)^2)/(1-cos(x)^2), x, algorithm="maxima")

[Out] -x - 2/tan(x)

mupad [B] time = 2.29, size = 8, normalized size = 1.00

$$-x - 2 \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(cos(x)^2 + 1)/(cos(x)^2 - 1), x)

[Out] - x - 2*cot(x)

sympy [A] time = 0.86, size = 12, normalized size = 1.50

$$-x + \tan\left(\frac{x}{2}\right) - \frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)**2)/(1-cos(x)**2), x)

[Out] -x + tan(x/2) - 1/tan(x/2)

$$3.204 \quad \int \frac{1 - \cos^2(x)}{1 + \cos^2(x)} dx$$

Optimal. Leaf size=37

$$\sqrt{2}x - x - \sqrt{2} \tan^{-1} \left(\frac{\sin(x) \cos(x)}{\cos^2(x) + \sqrt{2} + 1} \right)$$

[Out] $-x + x \cdot 2^{(1/2)} - \arctan(\cos(x) \cdot \sin(x) / (1 + \cos(x)^2 + 2^{(1/2)})) \cdot 2^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3171, 3181, 203}

$$\sqrt{2}x - x - \sqrt{2} \tan^{-1} \left(\frac{\sin(x) \cos(x)}{\cos^2(x) + \sqrt{2} + 1} \right)$$

Antiderivative was successfully verified.

[In] Int[(1 - Cos[x]^2)/(1 + Cos[x]^2), x]

[Out] $-x + \text{Sqrt}[2] \cdot x - \text{Sqrt}[2] \cdot \text{ArcTan}[(\text{Cos}[x] \cdot \text{Sin}[x]) / (1 + \text{Sqrt}[2] + \text{Cos}[x]^2)]$

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3171

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)^2])/((a_) + (b_)*sin[(e_) + (f_)*(x_)^2]), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a + b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]

Rule 3181

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)^2])^(-1), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rubi steps

$$\begin{aligned} \int \frac{1 - \cos^2(x)}{1 + \cos^2(x)} dx &= -x + 2 \int \frac{1}{1 + \cos^2(x)} dx \\ &= -x - 2 \text{Subst} \left(\int \frac{1}{1 + 2x^2} dx, x, \cot(x) \right) \\ &= -x + \sqrt{2}x - \sqrt{2} \tan^{-1} \left(\frac{\cos(x) \sin(x)}{1 + \sqrt{2} + \cos^2(x)} \right) \end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.62

$$2 \left(\frac{\tan^{-1} \left(\frac{\tan(x)}{\sqrt{2}} \right)}{\sqrt{2}} - \frac{x}{2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Cos[x]^2)/(1 + Cos[x]^2), x]

[Out] 2*(-1/2*x + ArcTan[Tan[x]/Sqrt[2]]/Sqrt[2])

fricas [A] time = 1.14, size = 35, normalized size = 0.95

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2 - \sqrt{2}}{4\cos(x)\sin(x)}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-cos(x)^2)/(1+cos(x)^2), x, algorithm="fricas")

[Out] -1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) - x

giac [A] time = 0.13, size = 49, normalized size = 1.32

$$\sqrt{2}\left(x + \arctan\left(-\frac{\sqrt{2}\sin(2x) - \sin(2x)}{\sqrt{2}\cos(2x) + \sqrt{2} - \cos(2x) + 1}\right)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-cos(x)^2)/(1+cos(x)^2), x, algorithm="giac")

[Out] sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - cos(2*x) + 1))) - x

maple [A] time = 0.08, size = 17, normalized size = 0.46

$$\sqrt{2}\arctan\left(\frac{\sqrt{2}\tan(x)}{2}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-cos(x)^2)/(1+cos(x)^2), x)

[Out] 2^(1/2)*arctan(1/2*2^(1/2)*tan(x))-x

maxima [A] time = 0.42, size = 16, normalized size = 0.43

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-cos(x)^2)/(1+cos(x)^2), x, algorithm="maxima")

[Out] sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - x

mupad [B] time = 2.29, size = 27, normalized size = 0.73

$$\sqrt{2}(x - \operatorname{atan}(\tan(x))) - x + \sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(cos(x)^2 - 1)/(cos(x)^2 + 1), x)

[Out] 2^(1/2)*(x - atan(tan(x))) - x + 2^(1/2)*atan((2^(1/2)*tan(x))/2)

sympy [A] time = 2.66, size = 61, normalized size = 1.65

$$-x + \sqrt{2} \left(\operatorname{atan} \left(\sqrt{2} \tan \left(\frac{x}{2} \right) - 1 \right) + \pi \left\lfloor \frac{\frac{x}{2} - \frac{\pi}{2}}{\pi} \right\rfloor \right) + \sqrt{2} \left(\operatorname{atan} \left(\sqrt{2} \tan \left(\frac{x}{2} \right) + 1 \right) + \pi \left\lfloor \frac{\frac{x}{2} - \frac{\pi}{2}}{\pi} \right\rfloor \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-cos(x)**2)/(1+cos(x)**2),x)

[Out] -x + sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi)) + sqrt(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))

$$3.205 \quad \int \frac{-1 + \frac{c^2}{d^2} + \sin^2(x)}{c + d \cos(x)} dx$$

Optimal. Leaf size=14

$$\frac{cx}{d^2} - \frac{\sin(x)}{d}$$

[Out] c*x/d^2-sin(x)/d

Rubi [A] time = 0.13, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4397, 3016, 2637}

$$\frac{cx}{d^2} - \frac{\sin(x)}{d}$$

Antiderivative was successfully verified.

[In] Int[(-1 + c^2/d^2 + Sin[x]^2)/(c + d*Cos[x]),x]

[Out] (c*x)/d^2 - Sin[x]/d

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 3016

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[-a + b*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A*b^2 + a^2*C, 0]

Rule 4397

Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \frac{-1 + \frac{c^2}{d^2} + \sin^2(x)}{c + d \cos(x)} dx &= \int \frac{\frac{c^2}{d^2} - \cos^2(x)}{c + d \cos(x)} dx \\ &= -\frac{\int (-c + d \cos(x)) dx}{d^2} \\ &= \frac{cx}{d^2} - \frac{\int \cos(x) dx}{d} \\ &= \frac{cx}{d^2} - \frac{\sin(x)}{d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{cx}{d^2} - \frac{\sin(x)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + c^2/d^2 + Sin[x]^2)/(c + d*Cos[x]),x]

[Out] $(c*x)/d^2 - \text{Sin}[x]/d$

fricas [A] time = 0.63, size = 13, normalized size = 0.93

$$\frac{cx - d \sin(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x, algorithm="fricas")`

[Out] $(c*x - d*\sin(x))/d^2$

giac [A] time = 0.15, size = 26, normalized size = 1.86

$$\frac{cx}{d^2} - \frac{2 \tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x, algorithm="giac")`

[Out] $c*x/d^2 - 2*\tan(1/2*x)/((\tan(1/2*x)^2 + 1)*d)$

maple [B] time = 0.10, size = 32, normalized size = 2.29

$$-\frac{2 \tan\left(\frac{x}{2}\right)}{d\left(1 + \tan^2\left(\frac{x}{2}\right)\right)} + \frac{2c \arctan\left(\tan\left(\frac{x}{2}\right)\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x)`

[Out] $-2/d*\tan(1/2*x)/(1+\tan(1/2*x)^2)+2/d^2*c*\arctan(\tan(1/2*x))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see 'assume?' for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 2.47, size = 13, normalized size = 0.93

$$\frac{cx - d \sin(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((sin(x)^2 + c^2/d^2 - 1)/(c + d*cos(x)),x)`

[Out] $(c*x - d*\sin(x))/d^2$

sympy [B] time = 56.76, size = 61, normalized size = 4.36

$$\frac{cx \tan^2\left(\frac{x}{2}\right)}{d^2 \tan^2\left(\frac{x}{2}\right) + d^2} + \frac{cx}{d^2 \tan^2\left(\frac{x}{2}\right) + d^2} - \frac{2d \tan\left(\frac{x}{2}\right)}{d^2 \tan^2\left(\frac{x}{2}\right) + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c**2/d**2+sin(x)**2)/(c+d*cos(x)),x)
```

```
[Out] c*x*tan(x/2)**2/(d**2*tan(x/2)**2 + d**2) + c*x/(d**2*tan(x/2)**2 + d**2) -  
2*d*tan(x/2)/(d**2*tan(x/2)**2 + d**2)
```

$$3.206 \quad \int \frac{a+b \sin^2(x)}{c+d \cos(x)} dx$$

Optimal. Leaf size=105

$$\frac{2a \tan^{-1}\left(\frac{\sqrt{c-d} \tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d} \sqrt{c+d}} + \frac{bcx}{d^2} - \frac{2b\sqrt{c-d} \sqrt{c+d} \tan^{-1}\left(\frac{\sqrt{c-d} \tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{d^2} - \frac{b \sin(x)}{d}$$

[Out] $b*c*x/d^2 - b*\sin(x)/d + 2*a*\arctan((c-d)^{(1/2)}*\tan(1/2*x)/(c+d)^{(1/2)})/(c-d)^{(1/2)}/(c+d)^{(1/2)} - 2*b*\arctan((c-d)^{(1/2)}*\tan(1/2*x)/(c+d)^{(1/2)})*(c-d)^{(1/2)}/(c+d)^{(1/2)}/d^2$

Rubi [A] time = 0.26, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {4401, 2659, 205, 2695, 2735}

$$\frac{2a \tan^{-1}\left(\frac{\sqrt{c-d} \tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d} \sqrt{c+d}} + \frac{bcx}{d^2} - \frac{2b\sqrt{c-d} \sqrt{c+d} \tan^{-1}\left(\frac{\sqrt{c-d} \tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{d^2} - \frac{b \sin(x)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[x]^2)/(c + d*Cos[x]), x]

[Out] $(b*c*x)/d^2 + (2*a*\text{ArcTan}[(\text{Sqrt}[c - d]*\text{Tan}[x/2])/\text{Sqrt}[c + d]])/(\text{Sqrt}[c - d]*\text{Sqrt}[c + d]) - (2*b*\text{Sqrt}[c - d]*\text{Sqrt}[c + d]*\text{ArcTan}[(\text{Sqrt}[c - d]*\text{Tan}[x/2])/\text{Sqrt}[c + d]])/d^2 - (b*\text{Sin}[x])/d$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2695

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && IntegersQ[2*m, 2*p]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 4401

Int[u_, x_Symbol] := With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /; !InertTrigFreeQ[u]

Rubi steps

$$\begin{aligned}
\int \frac{a + b \sin^2(x)}{c + d \cos(x)} dx &= \int \left(\frac{a}{c + d \cos(x)} + \frac{b \sin^2(x)}{c + d \cos(x)} \right) dx \\
&= a \int \frac{1}{c + d \cos(x)} dx + b \int \frac{\sin^2(x)}{c + d \cos(x)} dx \\
&= -\frac{b \sin(x)}{d} + (2a) \text{Subst} \left(\int \frac{1}{c + d + (c - d)x^2} dx, x, \tan\left(\frac{x}{2}\right) \right) - \frac{b \int \frac{-d - c \cos(x)}{c + d \cos(x)} dx}{d} \\
&= \frac{bcx}{d^2} + \frac{2a \tan^{-1} \left(\frac{\sqrt{c-d} \tan\left(\frac{x}{2}\right)}{\sqrt{c+d}} \right)}{\sqrt{c-d} \sqrt{c+d}} - \frac{b \sin(x)}{d} + \frac{(b(-c^2 + d^2)) \int \frac{1}{c + d \cos(x)} dx}{d^2} \\
&= \frac{bcx}{d^2} + \frac{2a \tan^{-1} \left(\frac{\sqrt{c-d} \tan\left(\frac{x}{2}\right)}{\sqrt{c+d}} \right)}{\sqrt{c-d} \sqrt{c+d}} - \frac{b \sin(x)}{d} + \frac{(2b(-c^2 + d^2)) \text{Subst} \left(\int \frac{1}{c + d + (c - d)x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{d^2} \\
&= \frac{bcx}{d^2} + \frac{2a \tan^{-1} \left(\frac{\sqrt{c-d} \tan\left(\frac{x}{2}\right)}{\sqrt{c+d}} \right)}{\sqrt{c-d} \sqrt{c+d}} - \frac{2b\sqrt{c-d} \sqrt{c+d} \tan^{-1} \left(\frac{\sqrt{c-d} \tan\left(\frac{x}{2}\right)}{\sqrt{c+d}} \right)}{d^2} - \frac{b \sin(x)}{d}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 73, normalized size = 0.70

$$\frac{2(ad^2 + b(d^2 - c^2)) \tanh^{-1} \left(\frac{(c-d) \tan\left(\frac{x}{2}\right)}{\sqrt{d^2 - c^2}} \right)}{\sqrt{d^2 - c^2}} + bcx - bd \sin(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[x]^2)/(c + d*Cos[x]), x]

[Out] (b*c*x - (2*(a*d^2 + b*(-c^2 + d^2))*ArcTanh[((c - d)*Tan[x/2])/Sqrt[-c^2 + d^2]])/Sqrt[-c^2 + d^2] - b*d*Sin[x])/d^2

fricas [A] time = 1.02, size = 254, normalized size = 2.42

$$\left[\frac{(bc^2 - (a + b)d^2) \sqrt{-c^2 + d^2} \log \left(\frac{2cd \cos(x) + (2c^2 - d^2) \cos(x)^2 + 2\sqrt{-c^2 + d^2} (c \cos(x) + d) \sin(x) - c^2 + 2d^2}{d^2 \cos(x)^2 + 2cd \cos(x) + c^2} \right) + 2(bc^3 - bcd^2)x - 2}{2(c^2d^2 - d^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)), x, algorithm="fricas")

[Out] [1/2*((b*c^2 - (a + b)*d^2)*sqrt(-c^2 + d^2)*log((2*c*d*cos(x) + (2*c^2 - d^2)*cos(x)^2 + 2*sqrt(-c^2 + d^2)*(c*cos(x) + d)*sin(x) - c^2 + 2*d^2)/(d^2*cos(x)^2 + 2*c*d*cos(x) + c^2)) + 2*(b*c^3 - b*c*d^2)*x - 2*(b*c^2*d - b*d^3)*sin(x))/(c^2*d^2 - d^4), -((b*c^2 - (a + b)*d^2)*sqrt(c^2 - d^2)*arctan(-(c*cos(x) + d)/(sqrt(c^2 - d^2)*sin(x))) - (b*c^3 - b*c*d^2)*x + (b*c^2*d - b*d^3)*sin(x))/(c^2*d^2 - d^4)]

giac [A] time = 0.14, size = 110, normalized size = 1.05

$$\frac{bcx}{d^2} - \frac{2b \tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)d} + \frac{2(bc^2 - ad^2 - bd^2) \left(\pi \left[\frac{x}{2\pi} + \frac{1}{2} \right] \text{sgn}(-2c + 2d) + \arctan\left(\frac{c \tan\left(\frac{1}{2}x\right) - d \tan\left(\frac{1}{2}x\right)}{\sqrt{c^2 - d^2}} \right) \right)}{\sqrt{c^2 - d^2} d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)),x, algorithm="giac")

[Out] $b*c*x/d^2 - 2*b*\tan(1/2*x)/((\tan(1/2*x)^2 + 1)*d) + 2*(b*c^2 - a*d^2 - b*d^2)*(pi*\text{floor}(1/2*x/pi + 1/2)*\text{sgn}(-2*c + 2*d) + \arctan(-(c*\tan(1/2*x) - d*\tan(1/2*x))/\sqrt{c^2 - d^2}))/(\sqrt{c^2 - d^2}*d^2)$

maple [A] time = 0.11, size = 148, normalized size = 1.41

$$\frac{2 \arctan\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right) a}{\sqrt{(c+d)(c-d)}} - \frac{2 \arctan\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right) c^2 b}{d^2 \sqrt{(c+d)(c-d)}} + \frac{2 \arctan\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right) b}{\sqrt{(c+d)(c-d)}} - \frac{2b \tan\left(\frac{x}{2}\right)}{d\left(1 + \tan^2\left(\frac{x}{2}\right)\right)} + \frac{2bc \arctan\left(\tan\left(\frac{x}{2}\right)\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(x)^2)/(c+d*cos(x)),x)

[Out] $2/((c+d)*(c-d))^{1/2}*\arctan((c-d)*\tan(1/2*x)/((c+d)*(c-d))^{1/2})*a-2/d^2/((c+d)*(c-d))^{1/2}*\arctan((c-d)*\tan(1/2*x)/((c+d)*(c-d))^{1/2})*c^2*b+2/((c+d)*(c-d))^{1/2}*\arctan((c-d)*\tan(1/2*x)/((c+d)*(c-d))^{1/2})*b-2*b/d*\tan(1/2*x)/(1+\tan(1/2*x)^2)+2*b/d^2*c*\arctan(\tan(1/2*x))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see 'assume?' for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 4.04, size = 2429, normalized size = 23.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*sin(x)^2)/(c + d*cos(x)),x)

[Out] $(b*c^2*d*\sin(x))/(d^4 - c^2*d^2) - (b*d^3*\sin(x))/(d^4 - c^2*d^2) - (2*b*c^3*\text{atan}(\sin(x/2)/\cos(x/2)))/(d^4 - c^2*d^2) - (a*d^2*\text{atan}((a^2*d^7*\sin(x/2)*(d^2 - c^2)^{1/2}*1i - b^2*c^5*\sin(x/2)*(d^2 - c^2)^{3/2}*2i - b^2*c^7*\sin(x/2)*(d^2 - c^2)^{1/2}*2i + b^2*d^7*\sin(x/2)*(d^2 - c^2)^{1/2}*1i - a^2*c*d^4*\sin(x/2)*(d^2 - c^2)^{3/2}*2i + a^2*c*d^6*\sin(x/2)*(d^2 - c^2)^{1/2}*1i - b^2*c*d^4*\sin(x/2)*(d^2 - c^2)^{3/2}*2i + b^2*c*d^6*\sin(x/2)*(d^2 - c^2)^{1/2}*1i - a^2*c^2*d^5*\sin(x/2)*(d^2 - c^2)^{1/2}*1i - a^2*c^3*d^4*\sin(x/2)*(d^2 - c^2)^{1/2}*1i - b^2*c^2*d^5*\sin(x/2)*(d^2 - c^2)^{1/2}*2i + b^2*c^3*d^2*\sin(x/2)*(d^2 - c^2)^{3/2}*4i - b^2*c^3*d^4*\sin(x/2)*(d^2 - c^2)^{1/2}*4i + b^2*c^4*d^3*\sin(x/2)*(d^2 - c^2)^{1/2}*1i + b^2*c^5*d^2*\sin(x/2)*(d^2 - c^2)^{1/2}*5i + a*b*d^7*\sin(x/2)*(d^2 - c^2)^{1/2}*2i - a*b*c*d^4*\sin(x/2)*(d^2 - c^2)^{3/2}*4i + a*b*c*d^6*\sin(x/2)*(d^2 - c^2)^{1/2}*2i - a*b*c^2*d^5*\sin(x/2)*(d^2 - c^2)^{1/2}*4i + a*b*c^3*d^2*\sin(x/2)*(d^2 - c^2)^{3/2}*4i - a*b*c^3*d^4*\sin(x/2)*(d^2 - c^2)^{1/2}*4i + a*b*c^4*d^3*\sin(x/2)*(d^2 - c^2)^{1/2}*2i + a*b*c^5*d^2*\sin(x/2)*(d^2 - c^2)^{1/2}*2i)/(a^2*d^8*\cos(x/2) + b^2*d^8*\cos(x/2) + 2*a*b*d^8*\cos(x/2) - 2*a^2*c^2*d^6*\cos(x/2) + a^2*c^4*d^4*\cos(x/2) - 3*b^2*c^2*d^6*\cos(x/2) + 3*b^2*c^4*d^4*\cos(x/2) - b^2*c^6*d^2*\cos(x/2) - 6*a*b*c^2*d^6*\cos(x/2) + 6*a*b*c^4*d^4*\cos(x/2) - 2*a*b*c$

$$\begin{aligned} & \left(\frac{d^6 \cos(x/2)}{d^2} \right) \cdot \frac{(d^2 - c^2)^{1/2} \cdot 2i}{(d^4 - c^2 d^2)} + (b \cdot c^2 \cdot \operatorname{atan}((a^2 d^7 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 1i - b^2 \cdot c^5 \sin(x/2) \cdot (d^2 - c^2)^{3/2} \cdot 2i - \\ & b^2 \cdot c^7 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 2i + b^2 \cdot d^7 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 1i - a^2 \cdot c \cdot d^4 \sin(x/2) \cdot (d^2 - c^2)^{3/2} \cdot 2i + a^2 \cdot c \cdot d^6 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 1i - \\ & b^2 \cdot c \cdot d^4 \sin(x/2) \cdot (d^2 - c^2)^{3/2} \cdot 2i + b^2 \cdot c \cdot d^6 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 1i - a^2 \cdot c^2 \cdot d^5 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 1i - a^2 \cdot c^3 \cdot d^4 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 1i - \\ & b^2 \cdot c^2 \cdot d^5 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 2i + b^2 \cdot c^3 \cdot d^2 \sin(x/2) \cdot (d^2 - c^2)^{3/2} \cdot 4i - b^2 \cdot c^3 \cdot d^4 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 4i + b^2 \cdot c^4 \cdot d^3 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 1i + \\ & b^2 \cdot c^5 \cdot d^2 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 5i + a \cdot b \cdot d^7 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 2i - a \cdot b \cdot c \cdot d^4 \sin(x/2) \cdot (d^2 - c^2)^{3/2} \cdot 4i + a \cdot b \cdot c \cdot d^6 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 2i - \\ & a \cdot b \cdot c^2 \cdot d^5 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 4i + a \cdot b \cdot c^3 \cdot d^2 \sin(x/2) \cdot (d^2 - c^2)^{3/2} \cdot 4i - a \cdot b \cdot c^3 \cdot d^4 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 4i + a \cdot b \cdot c^4 \cdot d^3 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 2i + \\ & a \cdot b \cdot c^5 \cdot d^2 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 2i) / (a^2 \cdot d^8 \cos(x/2) + b^2 \cdot d^8 \cos(x/2) + 2 \cdot a \cdot b \cdot d^8 \cos(x/2) - 2 \cdot a^2 \cdot c^2 \cdot d^6 \cos(x/2) + a^2 \cdot c^4 \cdot d^4 \cos(x/2) - 3 \cdot b^2 \cdot c^2 \cdot d^6 \cos(x/2) + 3 \cdot b^2 \cdot c^4 \cdot d^4 \cos(x/2) - b^2 \cdot c^6 \cdot d^2 \cos(x/2) - 6 \cdot a \cdot b \cdot c^2 \cdot d^6 \cos(x/2) + 6 \cdot a \cdot b \cdot c^4 \cdot d^4 \cos(x/2) - 2 \cdot a \cdot b \cdot c^6 \cdot d^2 \cos(x/2)) \cdot (d^2 - c^2)^{1/2} \cdot 2i) / (d^4 - c^2 d^2) - (b \cdot d^2 \cdot \operatorname{atan}((a^2 d^7 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 1i - b^2 \cdot c^5 \sin(x/2) \cdot (d^2 - c^2)^{3/2} \cdot 2i - b^2 \cdot c^7 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 2i + b^2 \cdot d^7 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 1i - a^2 \cdot c \cdot d^4 \sin(x/2) \cdot (d^2 - c^2)^{3/2} \cdot 2i + a^2 \cdot c \cdot d^6 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 1i - b^2 \cdot c \cdot d^4 \sin(x/2) \cdot (d^2 - c^2)^{3/2} \cdot 2i + b^2 \cdot c \cdot d^6 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 1i - a^2 \cdot c^2 \cdot d^5 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 1i - a^2 \cdot c^3 \cdot d^4 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 1i - b^2 \cdot c^2 \cdot d^5 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 2i + b^2 \cdot c^3 \cdot d^2 \sin(x/2) \cdot (d^2 - c^2)^{3/2} \cdot 4i - b^2 \cdot c^3 \cdot d^4 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 4i + b^2 \cdot c^4 \cdot d^3 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 1i + b^2 \cdot c^5 \cdot d^2 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 5i + a \cdot b \cdot d^7 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 2i - a \cdot b \cdot c \cdot d^4 \sin(x/2) \cdot (d^2 - c^2)^{3/2} \cdot 4i + a \cdot b \cdot c \cdot d^6 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 2i - a \cdot b \cdot c^2 \cdot d^5 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 4i + a \cdot b \cdot c^3 \cdot d^2 \sin(x/2) \cdot (d^2 - c^2)^{3/2} \cdot 4i - a \cdot b \cdot c^3 \cdot d^4 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 4i + a \cdot b \cdot c^4 \cdot d^3 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 2i + a \cdot b \cdot c^5 \cdot d^2 \sin(x/2) \cdot (d^2 - c^2)^{1/2} \cdot 2i) / (a^2 \cdot d^8 \cos(x/2) + b^2 \cdot d^8 \cos(x/2) + 2 \cdot a \cdot b \cdot d^8 \cos(x/2) - 2 \cdot a^2 \cdot c^2 \cdot d^6 \cos(x/2) + a^2 \cdot c^4 \cdot d^4 \cos(x/2) - 3 \cdot b^2 \cdot c^2 \cdot d^6 \cos(x/2) + 3 \cdot b^2 \cdot c^4 \cdot d^4 \cos(x/2) - b^2 \cdot c^6 \cdot d^2 \cos(x/2) - 6 \cdot a \cdot b \cdot c^2 \cdot d^6 \cos(x/2) + 6 \cdot a \cdot b \cdot c^4 \cdot d^4 \cos(x/2) - 2 \cdot a \cdot b \cdot c^6 \cdot d^2 \cos(x/2)) \cdot (d^2 - c^2)^{1/2} \cdot 2i) / (d^4 - c^2 d^2) + (2 \cdot b \cdot c \cdot d^2 \cdot \operatorname{atan}(\sin(x/2) / \cos(x/2))) / (d^4 - c^2 d^2) \end{aligned}$$

sympy [A] time = 111.37, size = 2608, normalized size = 24.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(x)**2)/(c+d*cos(x)),x)

[Out] Piecewise((zoo*(-a*log(tan(x/2) - 1)*tan(x/2)**2/(tan(x/2)**2 + 1) - a*log(tan(x/2) - 1)/(tan(x/2)**2 + 1) + a*log(tan(x/2) + 1)*tan(x/2)**2/(tan(x/2)**2 + 1) + a*log(tan(x/2) + 1)/(tan(x/2)**2 + 1) - b*log(tan(x/2) - 1)*tan(x/2)**2/(tan(x/2)**2 + 1) - b*log(tan(x/2) - 1)/(tan(x/2)**2 + 1) + b*log(tan(x/2) + 1)*tan(x/2)**2/(tan(x/2)**2 + 1) + b*log(tan(x/2) + 1)/(tan(x/2)**2 + 1) - 2*b*tan(x/2)/(tan(x/2)**2 + 1)), Eq(c, 0) & Eq(d, 0)), (a*tan(x/2)**3/(d*tan(x/2)**2 + d) + a*tan(x/2)/(d*tan(x/2)**2 + d) + b*x*tan(x/2)**2/(d*tan(x/2)**2 + d) + b*x/(d*tan(x/2)**2 + d) - 2*b*tan(x/2)/(d*tan(x/2)**2 + d), Eq(c, d)), (a*tan(x/2)**2/(d*tan(x/2)**3 + d*tan(x/2)) + a/(d*tan(x/2)**3 + d*tan(x/2)) - b*x*tan(x/2)**3/(d*tan(x/2)**3 + d*tan(x/2)) - b*x*tan(x/2)/(d*tan(x/2)**3 + d*tan(x/2)) - 2*b*tan(x/2)**2/(d*tan(x/2)**3 + d*tan(x/2)), Eq(c, -d)), ((a*x + b*x*sin(x)**2/2 + b*x*cos(x)**2/2 - b*sin(x)*cos(x)/2)/c, Eq(d, 0)), (a*d**2*log(-sqrt(-c/(c - d) - d/(c - d))) + tan(x/2))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) + a*d**2*log(-sqrt(-c/(c - d) - d/(c - d))

$$3.207 \quad \int \frac{a+b \sin^2(x)}{c+c \cos^2(x)} dx$$

Optimal. Leaf size=57

$$\frac{x(a+2b)}{\sqrt{2}c} - \frac{(a+2b) \tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}c} - \frac{bx}{c}$$

[Out] $-b*x/c+1/2*(a+2*b)*x/c*2^{(1/2)}-1/2*(a+2*b)*\arctan(\cos(x)*\sin(x)/(1+\cos(x)^2+2^{(1/2)}))/c*2^{(1/2)}$

Rubi [A] time = 0.13, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {12, 1166, 203}

$$\frac{x(a+2b)}{\sqrt{2}c} - \frac{(a+2b) \tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}c} - \frac{bx}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[x]^2)/(c + c*Cos[x]^2), x]

[Out] $-((b*x)/c) + ((a + 2*b)*x)/(Sqrt[2]*c) - ((a + 2*b)*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2))]/(Sqrt[2]*c)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{a+b \sin^2(x)}{c+c \cos^2(x)} dx &= \text{Subst}\left(\int \frac{a+(a+b)x^2}{c(2+3x^2+x^4)} dx, x, \tan(x)\right) \\ &= \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{2+3x^2+x^4} dx, x, \tan(x)\right)}{c} \\ &= -\frac{b \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \tan(x)\right)}{c} + \frac{(a+2b) \text{Subst}\left(\int \frac{1}{2+x^2} dx, x, \tan(x)\right)}{c} \\ &= -\frac{bx}{c} + \frac{(a+2b)x}{\sqrt{2}c} - \frac{(a+2b) \tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}c} \end{aligned}$$

Mathematica [A] time = 0.08, size = 34, normalized size = 0.60

$$-\frac{(-a-2b)\tan^{-1}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}c} - \frac{bx}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[x]^2)/(c + c*Cos[x]^2), x]

[Out] -((b*x)/c) - ((-a - 2*b)*ArcTan[Tan[x]/Sqrt[2]])/(Sqrt[2]*c)

fricas [A] time = 1.33, size = 45, normalized size = 0.79

$$\frac{\sqrt{2}(a+2b)\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)+4bx}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(x)^2)/(c+c*cos(x)^2), x, algorithm="fricas")

[Out] -1/4*(sqrt(2)*(a + 2*b)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) + 4*b*x)/c

giac [A] time = 0.14, size = 62, normalized size = 1.09

$$\frac{\sqrt{2}(a+2b)\left(x + \arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)}{2c} - \frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(x)^2)/(c+c*cos(x)^2), x, algorithm="giac")

[Out] 1/2*sqrt(2)*(a + 2*b)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - cos(2*x) + 1)))/c - b*x/c

maple [A] time = 0.13, size = 44, normalized size = 0.77

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\tan(x)}{2}\right)a}{2c} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\tan(x)}{2}\right)b}{c} - \frac{b\arctan(\tan(x))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(x)^2)/(c+c*cos(x)^2), x)

[Out] 1/2/c*2^(1/2)*arctan(1/2*2^(1/2)*tan(x))*a+1/c*2^(1/2)*arctan(1/2*2^(1/2)*tan(x))*b-1/c*b*arctan(tan(x))

maxima [A] time = 0.42, size = 29, normalized size = 0.51

$$\frac{\sqrt{2}(a+2b)\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)}{2c} - \frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(x)^2)/(c+c*cos(x)^2), x, algorithm="maxima")

[Out] 1/2*sqrt(2)*(a + 2*b)*arctan(1/2*sqrt(2)*tan(x))/c - b*x/c

mupad [B] time = 2.43, size = 242, normalized size = 4.25

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}a^3\tan(x)}{2(a^3+6a^2b+10ab^2+4b^3)} + \frac{2\sqrt{2}b^3\tan(x)}{a^3+6a^2b+10ab^2+4b^3} + \frac{5\sqrt{2}ab^2\tan(x)}{a^3+6a^2b+10ab^2+4b^3} + \frac{3\sqrt{2}a^2b\tan(x)}{a^3+6a^2b+10ab^2+4b^3}\right)(a+2b) - b\operatorname{atan}\left(\tan(x)\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*sin(x)^2)/(c + c*cos(x)^2),x)`

[Out] $(2^{(1/2)}*\operatorname{atan}((2^{(1/2)}*a^3*\tan(x))/(2*(10*a*b^2 + 6*a^2*b + a^3 + 4*b^3))) + (2*2^{(1/2)}*b^3*\tan(x))/(10*a*b^2 + 6*a^2*b + a^3 + 4*b^3) + (5*2^{(1/2)}*a*b^2*\tan(x))/(10*a*b^2 + 6*a^2*b + a^3 + 4*b^3) + (3*2^{(1/2)}*a^2*b*\tan(x))/(10*a*b^2 + 6*a^2*b + a^3 + 4*b^3))*(a + 2*b)/(2*c) - (b*\operatorname{atan}((4*b^3*\tan(x))/(8*a*b^2 + 2*a^2*b + 4*b^3) + (8*a*b^2*\tan(x))/(8*a*b^2 + 2*a^2*b + 4*b^3) + (2*a^2*b*\tan(x))/(8*a*b^2 + 2*a^2*b + 4*b^3)))/c$

sympy [B] time = 10.67, size = 143, normalized size = 2.51

$$\frac{\sqrt{2} a \left(\operatorname{atan} \left(\sqrt{2} \tan \left(\frac{x}{2} \right) - 1 \right) + \pi \left\lfloor \frac{\frac{x}{2} - \frac{\pi}{2}}{\pi} \right\rfloor \right)}{2c} + \frac{\sqrt{2} a \left(\operatorname{atan} \left(\sqrt{2} \tan \left(\frac{x}{2} \right) + 1 \right) + \pi \left\lfloor \frac{\frac{x}{2} - \frac{\pi}{2}}{\pi} \right\rfloor \right)}{2c} - \frac{bx}{c} + \frac{\sqrt{2} b \left(\operatorname{atan} \left(\sqrt{2} \tan \left(\frac{x}{2} \right) - 1 \right) + \pi \left\lfloor \frac{\frac{x}{2} - \frac{\pi}{2}}{\pi} \right\rfloor \right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sin(x)**2)/(c+c*cos(x)**2),x)`

[Out] $\operatorname{sqrt}(2)*a*(\operatorname{atan}(\operatorname{sqrt}(2)*\tan(x/2) - 1) + \pi*\operatorname{floor}((x/2 - \pi/2)/\pi))/(2*c) + \operatorname{sqrt}(2)*a*(\operatorname{atan}(\operatorname{sqrt}(2)*\tan(x/2) + 1) + \pi*\operatorname{floor}((x/2 - \pi/2)/\pi))/(2*c) - b*x/c + \operatorname{sqrt}(2)*b*(\operatorname{atan}(\operatorname{sqrt}(2)*\tan(x/2) - 1) + \pi*\operatorname{floor}((x/2 - \pi/2)/\pi))/c + \operatorname{sqrt}(2)*b*(\operatorname{atan}(\operatorname{sqrt}(2)*\tan(x/2) + 1) + \pi*\operatorname{floor}((x/2 - \pi/2)/\pi))/c$

$$3.208 \quad \int \frac{a+b \sin^2(x)}{c-c \cos^2(x)} dx$$

Optimal. Leaf size=15

$$\frac{bx}{c} - \frac{a \cot(x)}{c}$$

[Out] b*x/c-a*cot(x)/c

Rubi [A] time = 0.09, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {453, 205}

$$\frac{bx}{c} - \frac{a \cot(x)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[x]^2)/(c - c*Cos[x]^2),x]

[Out] (b*x)/c - (a*Cot[x])/c

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 453

Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m+1)*(a + b*x^n)^(p+1)/(a*e^(m+1)), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{a+b \sin^2(x)}{c-c \cos^2(x)} dx &= \text{Subst} \left(\int \frac{a+(a+b)x^2}{x^2(c+cx^2)} dx, x, \tan(x) \right) \\ &= -\frac{a \cot(x)}{c} + b \text{Subst} \left(\int \frac{1}{c+cx^2} dx, x, \tan(x) \right) \\ &= \frac{bx}{c} - \frac{a \cot(x)}{c} \end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{bx}{c} - \frac{a \cot(x)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[x]^2)/(c - c*Cos[x]^2),x]

[Out] (b*x)/c - (a*Cot[x])/c

fricas [A] time = 0.61, size = 19, normalized size = 1.27

$$\frac{bx \sin(x) - a \cos(x)}{c \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(x)^2)/(c-c*cos(x)^2),x, algorithm="fricas")

[Out] (b*x*sin(x) - a*cos(x))/(c*sin(x))

giac [A] time = 0.15, size = 29, normalized size = 1.93

$$\frac{bx}{c} + \frac{a \tan\left(\frac{1}{2}x\right)}{2c} - \frac{a}{2c \tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(x)^2)/(c-c*cos(x)^2),x, algorithm="giac")

[Out] b*x/c + 1/2*a*tan(1/2*x)/c - 1/2*a/(c*tan(1/2*x))

maple [A] time = 0.11, size = 20, normalized size = 1.33

$$-\frac{a}{c \tan(x)} + \frac{b \arctan(\tan(x))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(x)^2)/(c-c*cos(x)^2),x)

[Out] -1/c*a/tan(x)+1/c*b*arctan(tan(x))

maxima [A] time = 0.42, size = 17, normalized size = 1.13

$$\frac{bx}{c} - \frac{a}{c \tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(x)^2)/(c-c*cos(x)^2),x, algorithm="maxima")

[Out] b*x/c - a/(c*tan(x))

mupad [B] time = 2.31, size = 13, normalized size = 0.87

$$\frac{bx - a \cot(x)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*sin(x)^2)/(c - c*cos(x)^2),x)

[Out] (b*x - a*cot(x))/c

sympy [B] time = 1.18, size = 24, normalized size = 1.60

$$\frac{a \tan\left(\frac{x}{2}\right)}{2c} - \frac{a}{2c \tan\left(\frac{x}{2}\right)} + \frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(x)**2)/(c-c*cos(x)**2),x)

[Out] a*tan(x/2)/(2*c) - a/(2*c*tan(x/2)) + b*x/c

$$3.209 \quad \int \frac{a+b \sin^2(x)}{c+d \cos^2(x)} dx$$

Optimal. Leaf size=49

$$\frac{(ad + b(c + d)) \tan^{-1} \left(\frac{\sqrt{c} \tan(x)}{\sqrt{c+d}} \right)}{\sqrt{c} d \sqrt{c+d}} - \frac{bx}{d}$$

[Out] $-b*x/d+(a*d+b*(c+d))*\arctan(c^{(1/2)}*\tan(x)/(c+d)^{(1/2)})/d/c^{(1/2)/(c+d)^{(1/2)}}$

Rubi [A] time = 0.15, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {522, 203, 205}

$$\frac{(ad + b(c + d)) \tan^{-1} \left(\frac{\sqrt{c} \tan(x)}{\sqrt{c+d}} \right)}{\sqrt{c} d \sqrt{c+d}} - \frac{bx}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[x]^2)/(c + d*Cos[x]^2), x]

[Out] $-(b*x)/d + ((a*d + b*(c + d))*\text{ArcTan}[(\text{Sqrt}[c]*\text{Tan}[x])/\text{Sqrt}[c + d]])/(\text{Sqrt}[c]*d*\text{Sqrt}[c + d])$

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{a + b \sin^2(x)}{c + d \cos^2(x)} dx &= \text{Subst} \left(\int \frac{a + (a + b)x^2}{(1 + x^2)(c + d + cx^2)} dx, x, \tan(x) \right) \\ &= -\frac{b \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right)}{d} + \frac{(ad + b(c + d)) \text{Subst} \left(\int \frac{1}{c+d+cx^2} dx, x, \tan(x) \right)}{d} \\ &= -\frac{bx}{d} + \frac{(ad + b(c + d)) \tan^{-1} \left(\frac{\sqrt{c} \tan(x)}{\sqrt{c+d}} \right)}{\sqrt{c} d \sqrt{c+d}} \end{aligned}$$

Mathematica [A] time = 0.16, size = 47, normalized size = 0.96

$$\frac{\frac{(ad+b(c+d)) \tan^{-1}\left(\frac{\sqrt{c} \tan(x)}{\sqrt{c+d}}\right) - bx}{\sqrt{c} \sqrt{c+d}}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[x]^2)/(c + d*Cos[x]^2), x]

[Out] (- (b*x) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c]*Tan[x])/Sqrt[c + d]])/(Sqrt[c]*Sqrt[c + d])/d

fricas [A] time = 0.79, size = 228, normalized size = 4.65

$$\left[\frac{(bc + (a + b)d)\sqrt{-c^2 - cd} \log\left(\frac{(8c^2 + 8cd + d^2)\cos(x)^4 - 2(4c^2 + 3cd)\cos(x)^2 + 4((2c + d)\cos(x)^3 - c\cos(x))\sqrt{-c^2 - cd}\sin(x) + c^2}{d^2\cos(x)^4 + 2cd\cos(x)^2 + c^2}\right) + 4}{4(c^2d + cd^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)^2), x, algorithm="fricas")

[Out] [-1/4*((b*c + (a + b)*d)*sqrt(-c^2 - c*d)*log(((8*c^2 + 8*c*d + d^2)*cos(x)^4 - 2*(4*c^2 + 3*c*d)*cos(x)^2 + 4*((2*c + d)*cos(x)^3 - c*cos(x))*sqrt(-c^2 - c*d)*sin(x) + c^2)/(d^2*cos(x)^4 + 2*c*d*cos(x)^2 + c^2)) + 4*(b*c^2 + b*c*d)*x)/(c^2*d + c*d^2), -1/2*((b*c + (a + b)*d)*sqrt(c^2 + c*d)*arctan(1/2*((2*c + d)*cos(x)^2 - c)/(sqrt(c^2 + c*d)*cos(x)*sin(x))) + 2*(b*c^2 + b*c*d)*x)/(c^2*d + c*d^2)]

giac [A] time = 0.15, size = 58, normalized size = 1.18

$$-\frac{bx}{d} + \frac{\left(\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(c) + \arctan\left(\frac{c \tan(x)}{\sqrt{c^2 + cd}}\right)\right)(bc + ad + bd)}{\sqrt{c^2 + cd} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)^2), x, algorithm="giac")

[Out] -b*x/d + (pi*floor(x/pi + 1/2)*sgn(c) + arctan(c*tan(x)/sqrt(c^2 + c*d)))*(b*c + a*d + b*d)/(sqrt(c^2 + c*d)*d)

maple [A] time = 0.12, size = 78, normalized size = 1.59

$$\frac{\arctan\left(\frac{c \tan(x)}{\sqrt{(c+d)c}}\right) a}{\sqrt{(c+d)c}} + \frac{\arctan\left(\frac{c \tan(x)}{\sqrt{(c+d)c}}\right) cb}{d\sqrt{(c+d)c}} + \frac{\arctan\left(\frac{c \tan(x)}{\sqrt{(c+d)c}}\right) b}{\sqrt{(c+d)c}} - \frac{b \arctan(\tan(x))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(x)^2)/(c+d*cos(x)^2), x)

[Out] 1/((c+d)*c)^(1/2)*arctan(c*tan(x)/((c+d)*c)^(1/2))*a+1/d/((c+d)*c)^(1/2)*arctan(c*tan(x)/((c+d)*c)^(1/2))*c*b+1/((c+d)*c)^(1/2)*arctan(c*tan(x)/((c+d)*c)^(1/2))*b-b/d*arctan(tan(x))

maxima [A] time = 1.34, size = 40, normalized size = 0.82

$$-\frac{bx}{d} + \frac{(bc + (a + b)d) \arctan\left(\frac{c \tan(x)}{\sqrt{(c+d)c}}\right)}{\sqrt{(c+d)c} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -b*x/d + (b*c + (a + b)*d)*arctan(c*tan(x)/sqrt((c + d)*c))/(sqrt((c + d)*c)*d)
```

mupad [B] time = 2.85, size = 1987, normalized size = 40.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x)^2)/(c + d*cos(x)^2),x)
```

```
[Out] - (b*c^2*x)/(c*d^2 + c^2*d) - (a*d*atan((a^2*d^3*tan(x)*(- c*d - c^2)^(3/2)
*1i + b^2*c^3*tan(x)*(- c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(- c*d - c^2)^(
1/2)*2i + b^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(- c*d
- c^2)^(3/2)*2i + a^2*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c*d^2*tan(x)
)*(- c*d - c^2)^(3/2)*4i + b^2*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c^
2*d*tan(x)*(- c*d - c^2)^(3/2)*5i + b^2*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*6i
+ a^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*2i + a^2*c^3*d^2*tan(x)*(- c*d -
c^2)^(1/2)*1i + b^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*4i + b^2*c^3*d^2*tan
(x)*(- c*d - c^2)^(1/2)*7i + a*b*d^3*tan(x)*(- c*d - c^2)^(3/2)*2i + a*b*c*
d^2*tan(x)*(- c*d - c^2)^(3/2)*6i + a*b*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*2i
+ a*b*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*4i + a*b*c^4*d*tan(x)*(- c*d - c^2)
^(1/2)*2i + a*b*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*6i + a*b*c^3*d^2*tan(x)*
(- c*d - c^2)^(1/2)*6i)/(b^2*c^5*d + a^2*c^2*d^4 + 2*a^2*c^3*d^3 + a^2*c^4*
d^2 + b^2*c^2*d^4 + 3*b^2*c^3*d^3 + 3*b^2*c^4*d^2 + 2*a*b*c^5*d + 2*a*b*c^2
*d^4 + 6*a*b*c^3*d^3 + 6*a*b*c^4*d^2))*(- c*d - c^2)^(1/2)*1i)/(c*d^2 + c^2
*d) - (b*c*atan((a^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + b^2*c^3*tan(x)*(-
c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(- c*d - c^2)^(1/2)*2i + b^2*d^3*tan(x)
)*(- c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*2i + a^2*c*
d^4*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*4i
+ b^2*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c^2*d*tan(x)*(- c*d - c^2)
^(3/2)*5i + b^2*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*6i + a^2*c^2*d^3*tan(x)*(-
c*d - c^2)^(1/2)*2i + a^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c^2*
d^3*tan(x)*(- c*d - c^2)^(1/2)*4i + b^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*
7i + a*b*d^3*tan(x)*(- c*d - c^2)^(3/2)*2i + a*b*c*d^2*tan(x)*(- c*d - c^2)
^(3/2)*6i + a*b*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d*tan(x)*(- c
*d - c^2)^(3/2)*4i + a*b*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d^3*
tan(x)*(- c*d - c^2)^(1/2)*6i + a*b*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*6i)/
(b^2*c^5*d + a^2*c^2*d^4 + 2*a^2*c^3*d^3 + a^2*c^4*d^2 + b^2*c^2*d^4 + 3*b^
2*c^3*d^3 + 3*b^2*c^4*d^2 + 2*a*b*c^5*d + 2*a*b*c^2*d^4 + 6*a*b*c^3*d^3 + 6
*a*b*c^4*d^2))*(- c*d - c^2)^(1/2)*1i)/(c*d^2 + c^2*d) - (b*d*atan((a^2*d^3
*tan(x)*(- c*d - c^2)^(3/2)*1i + b^2*c^3*tan(x)*(- c*d - c^2)^(3/2)*2i + b^
2*c^5*tan(x)*(- c*d - c^2)^(1/2)*2i + b^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i
+ a^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*2i + a^2*c*d^4*tan(x)*(- c*d - c^2)
^(1/2)*1i + b^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*4i + b^2*c*d^4*tan(x)*(- c
*d - c^2)^(1/2)*1i + b^2*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*5i + b^2*c^4*d*ta
n(x)*(- c*d - c^2)^(1/2)*6i + a^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*2i + a
^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c^2*d^3*tan(x)*(- c*d - c^2)
^(1/2)*4i + b^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*7i + a*b*d^3*tan(x)*(- c
*d - c^2)^(3/2)*2i + a*b*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*6i + a*b*c*d^4*ta
n(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*4i + a*b
*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d^3*tan(x)*(- c*d - c^2)^(1/
2)*6i + a*b*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*6i)/(b^2*c^5*d + a^2*c^2*d^4
+ 2*a^2*c^3*d^3 + a^2*c^4*d^2 + b^2*c^2*d^4 + 3*b^2*c^3*d^3 + 3*b^2*c^4*d^
2 + 2*a*b*c^5*d + 2*a*b*c^2*d^4 + 6*a*b*c^3*d^3 + 6*a*b*c^4*d^2))*(- c*d -
c^2)^(1/2)*1i)/(c*d^2 + c^2*d) - (b*c*d*x)/(c*d^2 + c^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(x)**2)/(c+d*cos(x)**2), x)

[Out] Timed out

$$3.210 \quad \int \frac{-1 + \frac{c^2}{d^2} + \cos^2(x)}{c + d \sin(x)} dx$$

Optimal. Leaf size=13

$$\frac{cx}{d^2} + \frac{\cos(x)}{d}$$

[Out] c*x/d^2+cos(x)/d

Rubi [A] time = 0.12, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4397, 3016, 2638}

$$\frac{cx}{d^2} + \frac{\cos(x)}{d}$$

Antiderivative was successfully verified.

[In] Int[(-1 + c^2/d^2 + Cos[x]^2)/(c + d*Sin[x]),x]

[Out] (c*x)/d^2 + Cos[x]/d

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3016

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A*b^2 + a^2*C, 0]

Rule 4397

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \frac{-1 + \frac{c^2}{d^2} + \cos^2(x)}{c + d \sin(x)} dx &= \int \frac{\frac{c^2}{d^2} - \sin^2(x)}{c + d \sin(x)} dx \\ &= -\frac{\int (-c + d \sin(x)) dx}{d^2} \\ &= \frac{cx}{d^2} - \frac{\int \sin(x) dx}{d} \\ &= \frac{cx}{d^2} + \frac{\cos(x)}{d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{cx}{d^2} + \frac{\cos(x)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + c^2/d^2 + Cos[x]^2)/(c + d*Sin[x]),x]

[Out] $(c*x)/d^2 + \text{Cos}[x]/d$

fricas [A] time = 0.68, size = 12, normalized size = 0.92

$$\frac{cx + d \cos(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x, algorithm="fricas")`

[Out] $(c*x + d*\cos(x))/d^2$

giac [A] time = 0.15, size = 22, normalized size = 1.69

$$\frac{cx}{d^2} + \frac{2}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x, algorithm="giac")`

[Out] $c*x/d^2 + 2/((\tan(1/2*x)^2 + 1)*d)$

maple [B] time = 0.12, size = 28, normalized size = 2.15

$$\frac{2}{d\left(1 + \tan^2\left(\frac{x}{2}\right)\right)} + \frac{2c \arctan\left(\tan\left(\frac{x}{2}\right)\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x)`

[Out] $2/d/(1+\tan(1/2*x)^2)+2/d^2*c*\arctan(\tan(1/2*x))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 2.46, size = 13, normalized size = 1.00

$$\frac{\cos(x)}{d} + \frac{cx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(x)^2 + c^2/d^2 - 1)/(c + d*sin(x)),x)`

[Out] $\cos(x)/d + (c*x)/d^2$

sympy [B] time = 103.32, size = 56, normalized size = 4.31

$$\frac{cx \tan^2\left(\frac{x}{2}\right)}{d^2 \tan^2\left(\frac{x}{2}\right) + d^2} + \frac{cx}{d^2 \tan^2\left(\frac{x}{2}\right) + d^2} + \frac{2d}{d^2 \tan^2\left(\frac{x}{2}\right) + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c**2/d**2+cos(x)**2)/(c+d*sin(x)),x)
```

```
[Out] c*x*tan(x/2)**2/(d**2*tan(x/2)**2 + d**2) + c*x/(d**2*tan(x/2)**2 + d**2) +  
2*d/(d**2*tan(x/2)**2 + d**2)
```


$$3.211 \quad \int \frac{a+b \cos^2(x)}{c+d \sin(x)} dx$$

Optimal. Leaf size=100

$$\frac{2a \tan^{-1}\left(\frac{c \tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} - \frac{2b\sqrt{c^2-d^2} \tan^{-1}\left(\frac{c \tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2} + \frac{bcx}{d^2} + \frac{b \cos(x)}{d}$$

[Out] $b*c*x/d^2+b*\cos(x)/d+2*a*\arctan((d+c*\tan(1/2*x))/(c^2-d^2)^{(1/2)})/(c^2-d^2)^{(1/2)}-2*b*\arctan((d+c*\tan(1/2*x))/(c^2-d^2)^{(1/2)})*(c^2-d^2)^{(1/2)}/d^2$

Rubi [A] time = 0.24, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4401, 2660, 618, 204, 2695, 2735}

$$\frac{2a \tan^{-1}\left(\frac{c \tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} - \frac{2b\sqrt{c^2-d^2} \tan^{-1}\left(\frac{c \tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2} + \frac{bcx}{d^2} + \frac{b \cos(x)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[x]^2)/(c + d*Sin[x]), x]

[Out] $(b*c*x)/d^2 + (2*a*\text{ArcTan}[(d + c*\text{Tan}[x/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] - (2*b*Sqrt[c^2 - d^2]*\text{ArcTan}[(d + c*\text{Tan}[x/2])/Sqrt[c^2 - d^2]])/d^2 + (b*\text{Cos}[x])/d$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2695

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && IntegersQ[2*m, 2*p]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*

$\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 4401

$\text{Int}[u_, x_Symbol] :> \text{With}[\{v = \text{ExpandTrig}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]] /; \text{!InertTrigFreeQ}[u]$

Rubi steps

$$\begin{aligned} \int \frac{a + b \cos^2(x)}{c + d \sin(x)} dx &= \int \left(\frac{a}{c + d \sin(x)} + \frac{b \cos^2(x)}{c + d \sin(x)} \right) dx \\ &= a \int \frac{1}{c + d \sin(x)} dx + b \int \frac{\cos^2(x)}{c + d \sin(x)} dx \\ &= \frac{b \cos(x)}{d} + (2a) \text{Subst} \left(\int \frac{1}{c + 2dx + cx^2} dx, x, \tan\left(\frac{x}{2}\right) \right) + \frac{b \int \frac{d+c \sin(x)}{c+d \sin(x)} dx}{d} \\ &= \frac{bcx}{d^2} + \frac{b \cos(x)}{d} - (4a) \text{Subst} \left(\int \frac{1}{-4(c^2 - d^2) - x^2} dx, x, 2d + 2c \tan\left(\frac{x}{2}\right) \right) - \frac{(b(c^2 - d^2)) \int}{d^2} \\ &= \frac{bcx}{d^2} + \frac{2a \tan^{-1} \left(\frac{d+c \tan(\frac{x}{2})}{\sqrt{c^2-d^2}} \right)}{\sqrt{c^2-d^2}} + \frac{b \cos(x)}{d} - \frac{(2b(c^2 - d^2)) \text{Subst} \left(\int \frac{1}{c+2dx+cx^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{d^2} \\ &= \frac{bcx}{d^2} + \frac{2a \tan^{-1} \left(\frac{d+c \tan(\frac{x}{2})}{\sqrt{c^2-d^2}} \right)}{\sqrt{c^2-d^2}} + \frac{b \cos(x)}{d} + \frac{(4b(c^2 - d^2)) \text{Subst} \left(\int \frac{1}{-4(c^2-d^2)-x^2} dx, x, 2d + 2c \tan\left(\frac{x}{2}\right) \right)}{d^2} \\ &= \frac{bcx}{d^2} + \frac{2a \tan^{-1} \left(\frac{d+c \tan(\frac{x}{2})}{\sqrt{c^2-d^2}} \right)}{\sqrt{c^2-d^2}} - \frac{2b\sqrt{c^2-d^2} \tan^{-1} \left(\frac{d+c \tan(\frac{x}{2})}{\sqrt{c^2-d^2}} \right)}{d^2} + \frac{b \cos(x)}{d} \end{aligned}$$

Mathematica [A] time = 0.18, size = 72, normalized size = 0.72

$$\frac{2(ad^2 + b(d^2 - c^2)) \tan^{-1} \left(\frac{c \tan(\frac{x}{2}) + d}{\sqrt{c^2 - d^2}} \right) + b(cx + d \cos(x))}{d^2 \sqrt{c^2 - d^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[x]^2)/(c + d*Sin[x]),x]

[Out] ((2*(a*d^2 + b*(-c^2 + d^2))*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] + b*(c*x + d*Cos[x]))/d^2

fricas [A] time = 0.78, size = 262, normalized size = 2.62

$$\left[\frac{(bc^2 - (a + b)d^2)\sqrt{-c^2 + d^2} \log \left(\frac{(2c^2 - d^2) \cos(x)^2 - 2cd \sin(x) - c^2 - d^2 + 2(c \cos(x) \sin(x) + d \cos(x))\sqrt{-c^2 + d^2}}{d^2 \cos(x)^2 - 2cd \sin(x) - c^2 - d^2} \right) + 2(bc^3 - bcd^2)x + \dots}{2(c^2d^2 - d^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)),x, algorithm="fricas")

[Out] [1/2*((b*c^2 - (a + b)*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(x)^2 - 2*c*d*sin(x) - c^2 - d^2 + 2*(c*cos(x)*sin(x) + d*cos(x))*sqrt(-c^2 + d^2))

$$\frac{1}{(d^2 \cos(x)^2 - 2cd \sin(x) - c^2 - d^2)} + \frac{2(bc^3 - bcd^2)x + 2(bc^2d - bd^3)\cos(x)}{(c^2d^2 - d^4)} + \frac{((b^2c^2 - (a+b)d^2)\sqrt{c^2 - d^2})\arctan\left(\frac{c \sin(x) + d}{\sqrt{c^2 - d^2}\cos(x)}\right) + (b^2c^3 - bcd^2)x + (b^2c^2d - bd^3)\cos(x)}{(c^2d^2 - d^4)}$$

giac [A] time = 0.15, size = 93, normalized size = 0.93

$$\frac{bcx}{d^2} - \frac{2(bc^2 - ad^2 - bd^2) \left(\pi \left[\frac{x}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(c) + \arctan\left(\frac{c \tan\left(\frac{1}{2}x\right) + d}{\sqrt{c^2 - d^2}}\right) \right)}{\sqrt{c^2 - d^2} d^2} + \frac{2b}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)),x, algorithm="giac")

[Out] $b^2cx/d^2 - 2(b^2c^2 - a^2d^2 - b^2d^2) \cdot (\pi \cdot \text{floor}(1/2 \cdot x/\pi + 1/2) \cdot \text{sgn}(c) + \arctan((c \cdot \tan(1/2 \cdot x) + d)/\sqrt{c^2 - d^2}))/(\sqrt{c^2 - d^2} \cdot d^2) + 2b/((\tan(1/2 \cdot x)^2 + 1) \cdot d)$

maple [A] time = 0.08, size = 153, normalized size = 1.53

$$\frac{2 \arctan\left(\frac{2c \tan\left(\frac{x}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right) a}{\sqrt{c^2 - d^2}} - \frac{2 \arctan\left(\frac{2c \tan\left(\frac{x}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right) c^2 b}{d^2 \sqrt{c^2 - d^2}} + \frac{2 \arctan\left(\frac{2c \tan\left(\frac{x}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right) b}{\sqrt{c^2 - d^2}} + \frac{2b}{d \left(1 + \tan^2\left(\frac{x}{2}\right)\right)} + \frac{2bc \arctan\left(\frac{x}{2}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(x)^2)/(c+d*sin(x)),x)

[Out] $2/(c^2-d^2)^{(1/2)} \cdot \arctan(1/2 \cdot (2c \cdot \tan(1/2 \cdot x) + 2d)/(c^2-d^2)^{(1/2)}) \cdot a - 2/d^2 \cdot (c^2-d^2)^{(1/2)} \cdot \arctan(1/2 \cdot (2c \cdot \tan(1/2 \cdot x) + 2d)/(c^2-d^2)^{(1/2)}) \cdot c^2 \cdot b + 2/(c^2-d^2)^{(1/2)} \cdot \arctan(1/2 \cdot (2c \cdot \tan(1/2 \cdot x) + 2d)/(c^2-d^2)^{(1/2)}) \cdot b + 2 \cdot b/d \cdot (1 + \tan(1/2 \cdot x)^2) + 2 \cdot b/d^2 \cdot c \cdot \arctan(\tan(1/2 \cdot x))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more details) Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 5.15, size = 1646, normalized size = 16.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(x)^2)/(c + d*sin(x)),x)

[Out] $(2b)/(d \cdot (\tan(x/2)^2 + 1)) - (\operatorname{atan}\left(\left(\left(-c + d\right) \cdot \left(c - d\right)\right)^{(1/2)} \cdot \left(\left(32b^2c^4\right)/d - \left(32 \tan(x/2) \cdot \left(a^2cd^5 + b^2cd^5 + 2b^2c^5d - 4b^2c^3d^3 + 2a^2b^2cd^5 - 2a^2b^2c^3d^3\right)\right)/d^3 + \left(\left(-c + d\right) \cdot \left(c - d\right)\right)^{(1/2)} \cdot \left(32a^2cd^2 + \left(32 \tan(x/2) \cdot \left(2a^2cd^6 - 2b^2c^3d^4 + 2b^2cd^6\right)\right)/d^3 + \left(\left(-c + d\right) \cdot \left(c - d\right)\right)^{(1/2)} \cdot \left(32c^2d^3 + \left(32 \tan(x/2) \cdot \left(3cd^7 - 2c^3d^5\right)\right)/d^3\right) \cdot \left(a^2d^2 - b\right)\right)$

$$\begin{aligned}
& *c^2 + b*d^2)) / (d^4 - c^2*d^2)) * (a*d^2 - b*c^2 + b*d^2)) / (d^4 - c^2*d^2)) * (\\
& a*d^2 - b*c^2 + b*d^2) * i) / (d^4 - c^2*d^2) - ((-(c + d) * (c - d))^{1/2} * ((32 \\
& * \tan(x/2) * (a^2*c*d^5 + b^2*c*d^5 + 2*b^2*c^5*d - 4*b^2*c^3*d^3 + 2*a*b*c*d^5 \\
& - 2*a*b*c^3*d^3)) / d^3 - (32*b^2*c^4) / d + ((-(c + d) * (c - d))^{1/2} * (32*a* \\
& c^2*d^2 + (32*\tan(x/2) * (2*a*c*d^6 - 2*b*c^3*d^4 + 2*b*c*d^6)) / d^3 - ((-(c + \\
& d) * (c - d))^{1/2} * (32*c^2*d^3 + (32*\tan(x/2) * (3*c*d^7 - 2*c^3*d^5)) / d^3) * (\\
& a*d^2 - b*c^2 + b*d^2)) / (d^4 - c^2*d^2)) * (a*d^2 - b*c^2 + b*d^2)) / (d^4 - c^ \\
& 2*d^2)) * (a*d^2 - b*c^2 + b*d^2) * i) / (d^4 - c^2*d^2)) / ((64*(b^3*c^2*d^2 - a \\
& b^2*c^4 - b^3*c^4 + 2*a*b^2*c^2*d^2 + a^2*b*c^2*d^2)) / d^2 + (64*\tan(x/2) * (2 \\
& * b^3*c^3*d^2 - 2*b^3*c^5 + 2*a*b^2*c^3*d^2)) / d^3 + ((-(c + d) * (c - d))^{1/2} \\
&) * ((32*b^2*c^4) / d - (32*\tan(x/2) * (a^2*c*d^5 + b^2*c*d^5 + 2*b^2*c^5*d - 4*b \\
& ^2*c^3*d^3 + 2*a*b*c*d^5 - 2*a*b*c^3*d^3)) / d^3 + ((-(c + d) * (c - d))^{1/2} * \\
& (32*a*c^2*d^2 + (32*\tan(x/2) * (2*a*c*d^6 - 2*b*c^3*d^4 + 2*b*c*d^6)) / d^3 + (\\
& -(c + d) * (c - d))^{1/2} * (32*c^2*d^3 + (32*\tan(x/2) * (3*c*d^7 - 2*c^3*d^5)) / \\
& d^3) * (a*d^2 - b*c^2 + b*d^2)) / (d^4 - c^2*d^2)) * (a*d^2 - b*c^2 + b*d^2)) / (d^ \\
& 4 - c^2*d^2)) * (a*d^2 - b*c^2 + b*d^2)) / (d^4 - c^2*d^2) + ((-(c + d) * (c - d) \\
&)^{1/2} * ((32*\tan(x/2) * (a^2*c*d^5 + b^2*c*d^5 + 2*b^2*c^5*d - 4*b^2*c^3*d^3 \\
& + 2*a*b*c*d^5 - 2*a*b*c^3*d^3)) / d^3 - (32*b^2*c^4) / d + ((-(c + d) * (c - d))^{ \\
& 1/2} * (32*a*c^2*d^2 + (32*\tan(x/2) * (2*a*c*d^6 - 2*b*c^3*d^4 + 2*b*c*d^6)) / d \\
& ^3 - ((-(c + d) * (c - d))^{1/2} * (32*c^2*d^3 + (32*\tan(x/2) * (3*c*d^7 - 2*c^3* \\
& d^5)) / d^3) * (a*d^2 - b*c^2 + b*d^2)) / (d^4 - c^2*d^2)) * (a*d^2 - b*c^2 + b*d^2 \\
&)) / (d^4 - c^2*d^2)) * (a*d^2 - b*c^2 + b*d^2)) / (d^4 - c^2*d^2))) * (-(c + d) * (c \\
& - d))^{1/2} * (a*d^2 - b*c^2 + b*d^2) * 2i) / (d^4 - c^2*d^2) + (2*b*c*atan((64* \\
& b^3*c^2*\tan(x/2)) / (64*b^3*c^2 + 128*a*b^2*c^2 + 64*a^2*b*c^2 - (64*b^3*c^4) \\
& / d^2 - (128*a*b^2*c^4) / d^2) - (64*b^3*c^4*\tan(x/2)) / (64*b^3*c^2*d^2 - 128*a \\
& * b^2*c^4 - 64*b^3*c^4 + 128*a*b^2*c^2*d^2 + 64*a^2*b*c^2*d^2) - (128*a*b^2* \\
& c^4*\tan(x/2)) / (64*b^3*c^2*d^2 - 128*a*b^2*c^4 - 64*b^3*c^4 + 128*a*b^2*c^2* \\
& d^2 + 64*a^2*b*c^2*d^2) + (128*a*b^2*c^2*\tan(x/2)) / (64*b^3*c^2 + 128*a*b^2* \\
& c^2 + 64*a^2*b*c^2 - (64*b^3*c^4) / d^2 - (128*a*b^2*c^4) / d^2) + (64*a^2*b*c^ \\
& 2*\tan(x/2)) / (64*b^3*c^2 + 128*a*b^2*c^2 + 64*a^2*b*c^2 - (64*b^3*c^4) / d^2 - \\
& (128*a*b^2*c^4) / d^2))) / d^2
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)**2)/(c+d*sin(x)),x)

[Out] Timed out

$$3.212 \quad \int \frac{a+b \cos^2(x)}{c+c \sin^2(x)} dx$$

Optimal. Leaf size=56

$$\frac{x(a+2b)}{\sqrt{2}c} + \frac{(a+2b) \tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}c} - \frac{bx}{c}$$

[Out] $-b*x/c+1/2*(a+2*b)*x/c*2^{(1/2)}+1/2*(a+2*b)*\arctan(\cos(x)*\sin(x)/(1+\sin(x)^2+2^{(1/2)}))/c*2^{(1/2)}$

Rubi [A] time = 0.20, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1166, 205}

$$\frac{x(a+2b)}{\sqrt{2}c} + \frac{(a+2b) \tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}c} - \frac{bx}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[x]^2)/(c + c*Sin[x]^2), x]

[Out] $-(b*x)/c + ((a + 2*b)*x)/(Sqrt[2]*c) + ((a + 2*b)*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2))]/(Sqrt[2]*c)$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{a+b \cos^2(x)}{c+c \sin^2(x)} dx &= \text{Subst}\left(\int \frac{a+b+ax^2}{c+3cx^2+2cx^4} dx, x, \tan(x)\right) \\ &= -\left((2b) \text{Subst}\left(\int \frac{1}{2c+2cx^2} dx, x, \tan(x)\right)\right) + (a+2b) \text{Subst}\left(\int \frac{1}{c+2cx^2} dx, x, \tan(x)\right) \\ &= -\frac{bx}{c} + \frac{(a+2b)x}{\sqrt{2}c} + \frac{(a+2b) \tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}c} \end{aligned}$$

Mathematica [A] time = 0.08, size = 31, normalized size = 0.55

$$\frac{(a+2b) \tan^{-1}\left(\sqrt{2} \tan(x)\right)}{\sqrt{2}c} - \frac{bx}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*cos[x]^2)/(c + c*sin[x]^2),x]

[Out] -((b*x)/c) + ((a + 2*b)*ArcTan[Sqrt[2]*Tan[x]])/(Sqrt[2]*c)

fricas [A] time = 1.01, size = 45, normalized size = 0.80

$$\frac{\sqrt{2}(a+2b)\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)+4bx}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)^2)/(c+c*sin(x)^2),x, algorithm="fricas")

[Out] -1/4*(sqrt(2)*(a + 2*b)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) + 4*b*x)/c

giac [A] time = 0.15, size = 62, normalized size = 1.11

$$\frac{\sqrt{2}(a+2b)\left(x + \arctan\left(-\frac{\sqrt{2}\sin(2x)-2\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-2\cos(2x)+2}\right)\right)}{2c} - \frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)^2)/(c+c*sin(x)^2),x, algorithm="giac")

[Out] 1/2*sqrt(2)*(a + 2*b)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - 2*cos(2*x) + 2)))/c - b*x/c

maple [A] time = 0.12, size = 42, normalized size = 0.75

$$\frac{\sqrt{2}\arctan(\sqrt{2}\tan(x))a}{2c} + \frac{\sqrt{2}\arctan(\sqrt{2}\tan(x))b}{c} - \frac{b\arctan(\tan(x))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(x)^2)/(c+c*sin(x)^2),x)

[Out] 1/2/c*2^(1/2)*arctan(2^(1/2)*tan(x))*a+1/c*2^(1/2)*arctan(2^(1/2)*tan(x))*b-1/c*b*arctan(tan(x))

maxima [A] time = 0.43, size = 28, normalized size = 0.50

$$\frac{\sqrt{2}(a+2b)\arctan(\sqrt{2}\tan(x))}{2c} - \frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)^2)/(c+c*sin(x)^2),x, algorithm="maxima")

[Out] 1/2*sqrt(2)*(a + 2*b)*arctan(sqrt(2)*tan(x))/c - b*x/c

mupad [B] time = 2.39, size = 249, normalized size = 4.45

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{4\sqrt{2}a^3\tan(x)}{4a^3+24a^2b+40ab^2+16b^3} + \frac{16\sqrt{2}b^3\tan(x)}{4a^3+24a^2b+40ab^2+16b^3} + \frac{40\sqrt{2}ab^2\tan(x)}{4a^3+24a^2b+40ab^2+16b^3} + \frac{24\sqrt{2}a^2b\tan(x)}{4a^3+24a^2b+40ab^2+16b^3}\right)(a+2b)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(x)^2)/(c + c*sin(x)^2),x)

[Out] (2^(1/2)*atan((4*2^(1/2)*a^3*tan(x))/(40*a*b^2 + 24*a^2*b + 4*a^3 + 16*b^3) + (16*2^(1/2)*b^3*tan(x))/(40*a*b^2 + 24*a^2*b + 4*a^3 + 16*b^3) + (40*2^(1/2)*

$$\frac{1}{2} * a * b^2 * \tan(x) / (40 * a * b^2 + 24 * a^2 * b + 4 * a^3 + 16 * b^3) + (24 * 2^{(1/2)} * a^2 * b * \tan(x) / (40 * a * b^2 + 24 * a^2 * b + 4 * a^3 + 16 * b^3)) * (a + 2 * b) / (2 * c) - (b * \operatorname{atan}((8 * b^3 * \tan(x)) / (16 * a * b^2 + 4 * a^2 * b + 8 * b^3) + (16 * a * b^2 * \tan(x)) / (16 * a * b^2 + 4 * a^2 * b + 8 * b^3) + (4 * a^2 * b * \tan(x)) / (16 * a * b^2 + 4 * a^2 * b + 8 * b^3))) / c$$

sympy [B] time = 55.65, size = 520, normalized size = 9.29

$$\frac{54608393\sqrt{2}a\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left[\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right]\right)}{31988856\sqrt{2}c+45239074c}+\frac{77227930a\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left[\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right]\right)}{31988856\sqrt{2}c+45239074c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)**2)/(c+c*sin(x)**2),x)

[Out] 54608393*sqrt(2)*a*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2)))) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c) + 77227930*a*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2)))) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c) + 9369319*sqrt(2)*a*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c) + 13250218*a*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c) - 45239074*b*x/(31988856*sqrt(2)*c + 45239074*c) - 31988856*sqrt(2)*b*x/(31988856*sqrt(2)*c + 45239074*c) + 109216786*sqrt(2)*b*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2)))) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c) + 154455860*b*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2)))) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c) + 18738638*sqrt(2)*b*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c) + 26500436*b*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c)

$$3.213 \quad \int \frac{a+b \cos^2(x)}{c-c \sin^2(x)} dx$$

Optimal. Leaf size=14

$$\frac{a \tan(x)}{c} + \frac{bx}{c}$$

[Out] b*x/c+a*tan(x)/c

Rubi [A] time = 0.06, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3175, 3012, 8}

$$\frac{a \tan(x)}{c} + \frac{bx}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[x]^2)/(c - c*Sin[x]^2),x]

[Out] (b*x)/c + (a*Tan[x])/c

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3012

Int[((b_)*sin[(e_.) + (f_)*(x_)]^(m_))*((A_) + (C_)*sin[(e_.) + (f_)*(x_)]^2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3175

Int[(u_)*((a_) + (b_)*sin[(e_.) + (f_)*(x_)]^2)^(p_), x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{a+b \cos^2(x)}{c-c \sin^2(x)} dx &= \frac{\int (a+b \cos^2(x)) \sec^2(x) dx}{c} \\ &= \frac{a \tan(x)}{c} + \frac{b \int 1 dx}{c} \\ &= \frac{bx}{c} + \frac{a \tan(x)}{c} \end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{a \tan(x)}{c} + \frac{bx}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[x]^2)/(c - c*Sin[x]^2),x]

[Out] (b*x)/c + (a*Tan[x])/c

fricas [A] time = 1.08, size = 18, normalized size = 1.29

$$\frac{bx \cos(x) + a \sin(x)}{c \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)^2)/(c-c*sin(x)^2),x, algorithm="fricas")

[Out] (b*x*cos(x) + a*sin(x))/(c*cos(x))

giac [A] time = 0.15, size = 23, normalized size = 1.64

$$\frac{b \arctan\left(\frac{|c| \tan(x)}{c}\right)}{|c|} + \frac{a \tan(x)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)^2)/(c-c*sin(x)^2),x, algorithm="giac")

[Out] b*arctan(abs(c)*tan(x)/c)/abs(c) + a*tan(x)/c

maple [A] time = 0.11, size = 17, normalized size = 1.21

$$\frac{a \tan(x)}{c} + \frac{b \arctan(\tan(x))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(x)^2)/(c-c*sin(x)^2),x)

[Out] a*tan(x)/c+1/c*b*arctan(tan(x))

maxima [A] time = 0.43, size = 14, normalized size = 1.00

$$\frac{bx}{c} + \frac{a \tan(x)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)^2)/(c-c*sin(x)^2),x, algorithm="maxima")

[Out] b*x/c + a*tan(x)/c

mupad [B] time = 2.31, size = 12, normalized size = 0.86

$$\frac{bx + a \tan(x)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(x)^2)/(c - c*sin(x)^2),x)

[Out] (b*x + a*tan(x))/c

sympy [B] time = 1.33, size = 51, normalized size = 3.64

$$-\frac{2a \tan\left(\frac{x}{2}\right)}{c \tan^2\left(\frac{x}{2}\right) - c} + \frac{bx \tan^2\left(\frac{x}{2}\right)}{c \tan^2\left(\frac{x}{2}\right) - c} - \frac{bx}{c \tan^2\left(\frac{x}{2}\right) - c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)**2)/(c-c*sin(x)**2),x)

[Out] -2*a*tan(x/2)/(c*tan(x/2)**2 - c) + b*x*tan(x/2)**2/(c*tan(x/2)**2 - c) - b*x/(c*tan(x/2)**2 - c)

$$3.214 \quad \int \frac{a+b \cos^2(x)}{c+d \sin^2(x)} dx$$

Optimal. Leaf size=49

$$\frac{(ad + b(c + d)) \tan^{-1} \left(\frac{\sqrt{c+d} \tan(x)}{\sqrt{c}} \right)}{\sqrt{c} d \sqrt{c + d}} - \frac{bx}{d}$$

[Out] $-b*x/d+(a*d+b*(c+d))*\arctan((c+d)^{(1/2)}*\tan(x)/c^{(1/2)})/d/c^{(1/2)}/(c+d)^{(1/2)}$

Rubi [A] time = 0.16, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {522, 203, 205}

$$\frac{(ad + b(c + d)) \tan^{-1} \left(\frac{\sqrt{c+d} \tan(x)}{\sqrt{c}} \right)}{\sqrt{c} d \sqrt{c + d}} - \frac{bx}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[x]^2)/(c + d*Sin[x]^2),x]

[Out] $-((b*x)/d) + ((a*d + b*(c + d))*\text{ArcTan}[(\text{Sqrt}[c + d]*\text{Tan}[x])/\text{Sqrt}[c]])/(\text{Sqrt}[c]*d*\text{Sqrt}[c + d])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{a + b \cos^2(x)}{c + d \sin^2(x)} dx &= \text{Subst} \left(\int \frac{a + b + ax^2}{(1 + x^2)(c + (c + d)x^2)} dx, x, \tan(x) \right) \\ &= -\frac{b \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right)}{d} + \frac{(-ac + (a + b)(c + d)) \text{Subst} \left(\int \frac{1}{c+(c+d)x^2} dx, x, \tan(x) \right)}{d} \\ &= -\frac{bx}{d} + \frac{(ad + b(c + d)) \tan^{-1} \left(\frac{\sqrt{c+d} \tan(x)}{\sqrt{c}} \right)}{\sqrt{c} d \sqrt{c + d}} \end{aligned}$$

Mathematica [A] time = 0.15, size = 47, normalized size = 0.96

$$\frac{(ad+b(c+d)) \tan^{-1}\left(\frac{\sqrt{c+d} \tan(x)}{\sqrt{c}}\right) - bx}{\sqrt{c} \sqrt{c+d}} - \frac{bx}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[x]^2)/(c + d*Sin[x]^2), x]

[Out] $(- (b*x) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c + d]*Tan[x])/Sqrt[c]])/(Sqrt[c]*Sqrt[c + d]))/d$

fricas [B] time = 2.03, size = 255, normalized size = 5.20

$$\left[\frac{(bc + (a + b)d)\sqrt{-c^2 - cd} \log\left(\frac{(8c^2 + 8cd + d^2)\cos(x)^4 - 2(4c^2 + 5cd + d^2)\cos(x)^2 + 4((2c + d)\cos(x)^3 - (c + d)\cos(x))\sqrt{-c^2 - cd}\sin(x) + c^2}{d^2\cos(x)^4 - 2(cd + d^2)\cos(x)^2 + c^2 + 2cd + d^2}\right)}{4(c^2d + cd^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)^2), x, algorithm="fricas")

[Out] $[-1/4*((b*c + (a + b)*d)*sqrt(-c^2 - c*d)*log(((8*c^2 + 8*c*d + d^2)*cos(x)^4 - 2*(4*c^2 + 5*c*d + d^2)*cos(x)^2 + 4*((2*c + d)*cos(x)^3 - (c + d)*cos(x))*sqrt(-c^2 - c*d)*sin(x) + c^2 + 2*c*d + d^2)/(d^2*cos(x)^4 - 2*(c*d + d^2)*cos(x)^2 + c^2 + 2*c*d + d^2)) + 4*(b*c^2 + b*c*d)*x/(c^2*d + c*d^2), -1/2*((b*c + (a + b)*d)*sqrt(c^2 + c*d)*arctan(1/2*((2*c + d)*cos(x)^2 - c - d)/(sqrt(c^2 + c*d)*cos(x)*sin(x))) + 2*(b*c^2 + b*c*d)*x/(c^2*d + c*d^2)]$

giac [A] time = 0.14, size = 70, normalized size = 1.43

$$-\frac{bx}{d} + \frac{\left(\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(2c + 2d) + \arctan\left(\frac{c \tan(x) + d \tan(x)}{\sqrt{c^2 + cd}}\right)\right)(bc + ad + bd)}{\sqrt{c^2 + cd} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)^2), x, algorithm="giac")

[Out] $-b*x/d + (\pi*\text{floor}(x/\pi + 1/2)*\text{sgn}(2*c + 2*d) + \arctan((c*\tan(x) + d*\tan(x))/\sqrt{c^2 + c*d}))* (b*c + a*d + b*d)/(\sqrt{c^2 + c*d}*d)$

maple [B] time = 0.12, size = 84, normalized size = 1.71

$$\frac{\arctan\left(\frac{(c+d)\tan(x)}{\sqrt{(c+d)c}}\right)a}{\sqrt{(c+d)c}} + \frac{\arctan\left(\frac{(c+d)\tan(x)}{\sqrt{(c+d)c}}\right)cb}{d\sqrt{(c+d)c}} + \frac{\arctan\left(\frac{(c+d)\tan(x)}{\sqrt{(c+d)c}}\right)b}{\sqrt{(c+d)c}} - \frac{b \arctan(\tan(x))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(x)^2)/(c+d*sin(x)^2), x)

[Out] $1/((c+d)*c)^{(1/2)}*\arctan((c+d)*\tan(x)/((c+d)*c)^{(1/2)})*a+1/d/((c+d)*c)^{(1/2)}*\arctan((c+d)*\tan(x)/((c+d)*c)^{(1/2)})*c*b+1/((c+d)*c)^{(1/2)}*\arctan((c+d)*\tan(x)/((c+d)*c)^{(1/2)})*b-b/d*\arctan(\tan(x))$

maxima [A] time = 0.43, size = 42, normalized size = 0.86

$$-\frac{bx}{d} + \frac{(bc + (a + b)d) \arctan\left(\frac{(c+d)\tan(x)}{\sqrt{(c+d)c}}\right)}{\sqrt{(c+d)c} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)^2),x, algorithm="maxima")

[Out] -b*x/d + (b*c + (a + b)*d)*arctan((c + d)*tan(x)/sqrt((c + d)*c))/(sqrt((c + d)*c)*d)

mupad [B] time = 2.95, size = 1774, normalized size = 36.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(x)^2)/(c + d*sin(x)^2),x)

[Out] - (b*c^2*x)/(c*d^2 + c^2*d) - (a*d*atan((a^2*d^3*tan(x)*(-c*d - c^2)^(3/2)*1i + b^2*c^3*tan(x)*(-c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(-c*d - c^2)^(1/2)*2i + b^2*d^3*tan(x)*(-c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(-c*d - c^2)^(3/2)*2i + b^2*c*d^2*tan(x)*(-c*d - c^2)^(3/2)*4i + b^2*c^2*d*tan(x)*(-c*d - c^2)^(3/2)*5i + b^2*c^4*d*tan(x)*(-c*d - c^2)^(1/2)*6i + a^2*c^2*d^3*tan(x)*(-c*d - c^2)^(1/2)*1i + a^2*c^3*d^2*tan(x)*(-c*d - c^2)^(1/2)*1i + b^2*c^2*d^3*tan(x)*(-c*d - c^2)^(1/2)*2i + b^2*c^3*d^2*tan(x)*(-c*d - c^2)^(1/2)*6i + a*b*d^3*tan(x)*(-c*d - c^2)^(3/2)*2i + a*b*c*d^2*tan(x)*(-c*d - c^2)^(3/2)*6i + a*b*c^2*d*tan(x)*(-c*d - c^2)^(3/2)*4i + a*b*c^4*d*tan(x)*(-c*d - c^2)^(1/2)*2i + a*b*c^2*d^3*tan(x)*(-c*d - c^2)^(1/2)*2i + a*b*c^3*d^2*tan(x)*(-c*d - c^2)^(1/2)*4i)/(b^2*c^5*d + a^2*c^2*d^4 + 2*a^2*c^3*d^3 + a^2*c^4*d^2 + b^2*c^2*d^4 + 3*b^2*c^3*d^3 + 3*b^2*c^4*d^2 + 2*a*b*c^5*d + 2*a*b*c^2*d^4 + 6*a*b*c^3*d^3 + 6*a*b*c^4*d^2))*(-c*d - c^2)^(1/2)*1i)/(c*d^2 + c^2*d) - (b*c*atan((a^2*d^3*tan(x)*(-c*d - c^2)^(3/2)*1i + b^2*c^3*tan(x)*(-c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(-c*d - c^2)^(1/2)*2i + b^2*d^3*tan(x)*(-c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(-c*d - c^2)^(3/2)*2i + b^2*c*d^2*tan(x)*(-c*d - c^2)^(3/2)*4i + b^2*c^2*d*tan(x)*(-c*d - c^2)^(3/2)*5i + b^2*c^4*d*tan(x)*(-c*d - c^2)^(1/2)*6i + a^2*c^2*d^3*tan(x)*(-c*d - c^2)^(1/2)*1i + a^2*c^3*d^2*tan(x)*(-c*d - c^2)^(1/2)*1i + b^2*c^2*d^3*tan(x)*(-c*d - c^2)^(1/2)*2i + b^2*c^3*d^2*tan(x)*(-c*d - c^2)^(1/2)*6i + a*b*d^3*tan(x)*(-c*d - c^2)^(3/2)*2i + a*b*c*d^2*tan(x)*(-c*d - c^2)^(3/2)*6i + a*b*c^2*d*tan(x)*(-c*d - c^2)^(3/2)*4i + a*b*c^4*d*tan(x)*(-c*d - c^2)^(1/2)*2i + a*b*c^2*d^3*tan(x)*(-c*d - c^2)^(1/2)*2i + a*b*c^3*d^2*tan(x)*(-c*d - c^2)^(1/2)*4i)/(b^2*c^5*d + a^2*c^2*d^4 + 2*a^2*c^3*d^3 + a^2*c^4*d^2 + b^2*c^2*d^4 + 3*b^2*c^3*d^3 + 3*b^2*c^4*d^2 + 2*a*b*c^5*d + 2*a*b*c^2*d^4 + 6*a*b*c^3*d^3 + 6*a*b*c^4*d^2))*(-c*d - c^2)^(1/2)*1i)/(c*d^2 + c^2*d) - (b*c*d*x)/(c*d^2 + c^2*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)/(c+d*sin(x)**2),x)
```

```
[Out] Timed out
```

$$3.215 \quad \int \frac{a+b \sec^2(x)}{c+d \cos(x)} dx$$

Optimal. Leaf size=74

$$\frac{2(ac^2 + bd^2) \tan^{-1}\left(\frac{\sqrt{c-d} \tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c^2 \sqrt{c-d} \sqrt{c+d}} - \frac{bd \tanh^{-1}(\sin(x))}{c^2} + \frac{b \tan(x)}{c}$$

[Out] $-b*d*\operatorname{arctanh}(\sin(x))/c^2+2*(a*c^2+b*d^2)*\operatorname{arctan}((c-d)^{(1/2)}*\tan(1/2*x)/(c+d)^{(1/2}))/c^2/(c-d)^{(1/2)/(c+d)^{(1/2)+b*\tan(x)/c}$

Rubi [A] time = 0.25, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {4234, 3056, 3001, 3770, 2659, 205}

$$\frac{2(ac^2 + bd^2) \tan^{-1}\left(\frac{\sqrt{c-d} \tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c^2 \sqrt{c-d} \sqrt{c+d}} - \frac{bd \tanh^{-1}(\sin(x))}{c^2} + \frac{b \tan(x)}{c}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{Sec}[x]^2)/(c + d*\operatorname{Cos}[x]), x]$

[Out] $(2*(a*c^2 + b*d^2)*\operatorname{ArcTan}[(\operatorname{Sqrt}[c - d]*\operatorname{Tan}[x/2])/(\operatorname{Sqrt}[c + d])]/(c^2*\operatorname{Sqrt}[c - d]*\operatorname{Sqrt}[c + d]) - (b*d*\operatorname{ArcTanh}[\operatorname{Sin}[x]])/c^2 + (b*\operatorname{Tan}[x])/c$

Rule 205

$\operatorname{Int}[(a + b*x^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]*\operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]])/a, x] /;$ $\operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{PosQ}[a/b]$

Rule 2659

$\operatorname{Int}[(a + b*\sin[\pi/2 + (c + d)*x])^{-1}, x_Symbol] \rightarrow \operatorname{With}\{e = \operatorname{FreeFactors}[\operatorname{Tan}[(c + d*x)/2], x], \operatorname{Dist}[(2*e)/d, \operatorname{Subst}[\operatorname{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \operatorname{Tan}[(c + d*x)/2]/e], x]\} /;$ $\operatorname{FreeQ}\{a, b, c, d, x\} \ \&\& \ \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 3001

$\operatorname{Int}[(A + B*\sin[e + f*x])/(C + D*\sin[e + f*x]), x_Symbol] \rightarrow \operatorname{Dist}[(A*B - a*B)/(b*c - a*d), \operatorname{Int}[1/(a + b*\sin[e + f*x]), x], x] + \operatorname{Dist}[(B*c - A*d)/(b*c - a*d), \operatorname{Int}[1/(c + d*\sin[e + f*x]), x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{NeQ}[a^2 - b^2, 0] \ \&\& \ \operatorname{NeQ}[c^2 - d^2, 0]$

Rule 3056

$\operatorname{Int}[(A + B*\sin[e + f*x])^m*(C + D*\sin[e + f*x])^n, x_Symbol] \rightarrow -\operatorname{Simp}[(A*b^2 + a^2*C)*\operatorname{Cos}[e + f*x]*(a + b*\sin[e + f*x])^{m+1}*(c + d*\sin[e + f*x])^{n+1}/(f*(m+1)*(b*c - a*d)*(a^2 - b^2)), x] + \operatorname{Dist}[1/((m+1)*(b*c - a*d)*(a^2 - b^2)), \operatorname{Int}[(a + b*\sin[e + f*x])^{m+1}*(c + d*\sin[e + f*x])^n*\operatorname{Simp}[a*(m+1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m+n+2) - (c*(A*b^2 + a^2*C) + b*(m+1)*(b*c - a*d)*(A + C))*\sin[e + f*x] - d*(A*b^2 + a^2*C)*(m+n+3)*\sin[e + f*x]^2, x], x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, A, C, n\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{NeQ}[a^2 - b^2, 0] \ \&\& \ \operatorname{NeQ}[c^2 - d^2, 0] \ \&\& \ \operatorname{LtQ}[m, -1] \ \&\& \ ((\operatorname{EqQ}[a, 0] \ \&\& \ \operatorname{IntegerQ}[m] \ \&\& \ !\operatorname{IntegerQ}[n]) \ || \ !(\operatorname{IntegerQ}[2*n] \ \&\& \ \operatorname{LtQ}[n, -1] \ \&\& \ ((\operatorname{IntegerQ}[n] \ \&\& \ !\operatorname{IntegerQ}[m]) \ || \ \operatorname{EqQ}[a, 0])$

)))

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 4234

```
Int[(u_)*((A_) + (C_)*sec[(a_.) + (b_.)*(x_)^2], x_Symbol] := Int[(Active
teTrig[u]*(C + A*Cos[a + b*x]^2))/Cos[a + b*x]^2, x] /; FreeQ[{a, b, A, C},
x] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec^2(x)}{c + d \cos(x)} dx &= \int \frac{(b + a \cos^2(x)) \sec^2(x)}{c + d \cos(x)} dx \\
&= \frac{b \tan(x)}{c} + \frac{\int \frac{(-bd+ac \cos(x)) \sec(x)}{c+d \cos(x)} dx}{c} \\
&= \frac{b \tan(x)}{c} - \frac{(bd) \int \sec(x) dx}{c^2} + \left(a + \frac{bd^2}{c^2}\right) \int \frac{1}{c + d \cos(x)} dx \\
&= -\frac{bd \tanh^{-1}(\sin(x))}{c^2} + \frac{b \tan(x)}{c} + \left(2 \left(a + \frac{bd^2}{c^2}\right)\right) \text{Subst} \left(\int \frac{1}{c + d + (c - d)x^2} dx, x, \tan\left(\frac{x}{2}\right) \right) \\
&= \frac{2 \left(a + \frac{bd^2}{c^2}\right) \tan^{-1} \left(\frac{\sqrt{c-d} \tan\left(\frac{x}{2}\right)}{\sqrt{c+d}} \right)}{\sqrt{c-d} \sqrt{c+d}} - \frac{bd \tanh^{-1}(\sin(x))}{c^2} + \frac{b \tan(x)}{c}
\end{aligned}$$

Mathematica [A] time = 0.45, size = 98, normalized size = 1.32

$$\frac{2(ac^2+bd^2) \tanh^{-1} \left(\frac{(c-d) \tan\left(\frac{x}{2}\right)}{\sqrt{d^2-c^2}} \right)}{\sqrt{d^2-c^2}} + bc \tan(x) + bd \left(\log \left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right) \right) - \log \left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right) \right) \right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[x]^2)/(c + d*Cos[x]), x]
```

```
[Out] ((-2*(a*c^2 + b*d^2)*ArcTanh[((c - d)*Tan[x/2])/Sqrt[-c^2 + d^2]])/Sqrt[-c^
2 + d^2] + b*d*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) + b*c*
Tan[x])/c^2
```

fricas [B] time = 3.46, size = 318, normalized size = 4.30

$$\left[\frac{(ac^2 + bd^2) \sqrt{-c^2 + d^2} \cos(x) \log \left(\frac{2cd \cos(x) + (2c^2 - d^2) \cos(x)^2 + 2\sqrt{-c^2 + d^2} (c \cos(x) + d) \sin(x) - c^2 + 2d^2}{d^2 \cos(x)^2 + 2cd \cos(x) + c^2} \right) + (bc^2d - bd^3) \cos(x)}{2(c^4 - c^2d^2) \cos(x)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(x)^2)/(c+d*cos(x)), x, algorithm="fricas")
```

```
[Out] [-1/2*((a*c^2 + b*d^2)*sqrt(-c^2 + d^2)*cos(x)*log((2*c*d*cos(x) + (2*c^2 -
d^2)*cos(x)^2 + 2*sqrt(-c^2 + d^2)*(c*cos(x) + d)*sin(x) - c^2 + 2*d^2)/(d
```

$$\begin{aligned} &^2*\cos(x)^2 + 2*c*d*\cos(x) + c^2)) + (b*c^2*d - b*d^3)*\cos(x)*\log(\sin(x) + \\ &1) - (b*c^2*d - b*d^3)*\cos(x)*\log(-\sin(x) + 1) - 2*(b*c^3 - b*c*d^2)*\sin(x) \\ &)/((c^4 - c^2*d^2)*\cos(x)), 1/2*(2*(a*c^2 + b*d^2)*\sqrt{c^2 - d^2}*\arctan(- \\ &(c*\cos(x) + d)/(\sqrt{c^2 - d^2}*\sin(x)))*\cos(x) - (b*c^2*d - b*d^3)*\cos(x)* \\ &\log(\sin(x) + 1) + (b*c^2*d - b*d^3)*\cos(x)*\log(-\sin(x) + 1) + 2*(b*c^3 - b* \\ &c*d^2)*\sin(x))/((c^4 - c^2*d^2)*\cos(x)) \end{aligned}$$

giac [A] time = 0.17, size = 125, normalized size = 1.69

$$\frac{bd \log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right)}{c^2} + \frac{bd \log\left(\left|\tan\left(\frac{1}{2}x\right) - 1\right|\right)}{c^2} - \frac{2b \tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2 - 1\right)c} - \frac{2(ac^2 + bd^2)\left(\pi\left\lfloor\frac{x}{2\pi} + \frac{1}{2}\right\rfloor \operatorname{sgn}(-2c + 2d)\right)}{\sqrt{c^2 - d^2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(x)^2)/(c+d*cos(x)),x, algorithm="giac")

[Out] -b*d*log(abs(tan(1/2*x) + 1))/c^2 + b*d*log(abs(tan(1/2*x) - 1))/c^2 - 2*b*tan(1/2*x)/((tan(1/2*x)^2 - 1)*c) - 2*(a*c^2 + b*d^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*x) - d*tan(1/2*x))/sqrt(c^2 - d^2)))/sqrt(c^2 - d^2)*c^2

maple [B] time = 0.10, size = 135, normalized size = 1.82

$$\frac{2 \arctan\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)a}{\sqrt{(c+d)(c-d)}} + \frac{2 \arctan\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)b d^2}{c^2\sqrt{(c+d)(c-d)}} - \frac{b}{c\left(\tan\left(\frac{x}{2}\right) - 1\right)} + \frac{db \ln\left(\tan\left(\frac{x}{2}\right) - 1\right)}{c^2} - \frac{b}{c\left(1 + \tan\left(\frac{x}{2}\right)\right)} - \frac{db \ln\left(\tan\left(\frac{x}{2}\right) + 1\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(x)^2)/(c+d*cos(x)),x)

[Out] 2/((c+d)*(c-d))^(1/2)*arctan((c-d)*tan(1/2*x)/((c+d)*(c-d))^(1/2))*a+2/c^2/((c+d)*(c-d))^(1/2)*arctan((c-d)*tan(1/2*x)/((c+d)*(c-d))^(1/2))*b*d^2-b/c/(tan(1/2*x)-1)+d*b/c^2*ln(tan(1/2*x)-1)-b/c/(1+tan(1/2*x))-d*b/c^2*ln(1+tan(1/2*x))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(x)^2)/(c+d*cos(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 3.51, size = 1302, normalized size = 17.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(x)^2)/(c + d*cos(x)),x)

[Out] (b*c^3*sin(x))/(c^4*cos(x) - c^2*d^2*cos(x)) - (b*c*d^2*sin(x))/(c^4*cos(x) - c^2*d^2*cos(x)) + (2*b*d^3*atanh(sin(x)/2)/cos(x/2))*cos(x)/(c^4*cos(x) - c^2*d^2*cos(x))


```

- c^2*d^2*cos(x)) + (a*c^2*atan((a^2*c^7*sin(x/2)*(d^2 - c^2)^(1/2)*1i + b^
2*d^5*sin(x/2)*(d^2 - c^2)^(3/2)*2i - b^2*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*2i
+ a^2*c^4*d*sin(x/2)*(d^2 - c^2)^(3/2)*2i + a^2*c^6*d*sin(x/2)*(d^2 - c^2)
^(1/2)*1i - a^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c^5*d^2*sin(x/2
)*(d^2 - c^2)^(1/2)*1i + b^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*3i - b^2*c^
3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2
)*1i + b^2*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*1i + a*b*c^2*d^3*sin(x/2)*(d^
2 - c^2)^(3/2)*4i - a*b*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*2i - a*b*c^3*d^4
*sin(x/2)*(d^2 - c^2)^(1/2)*2i + a*b*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*2i
+ a*b*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*2i)/(a^2*c^8*cos(x/2) + a^2*c^4*d^
4*cos(x/2) - 2*a^2*c^6*d^2*cos(x/2) + b^2*c^2*d^6*cos(x/2) - 2*b^2*c^4*d^4*
cos(x/2) + b^2*c^6*d^2*cos(x/2) + 2*a*b*c^2*d^6*cos(x/2) - 4*a*b*c^4*d^4*co
s(x/2) + 2*a*b*c^6*d^2*cos(x/2))) *cos(x)*(d^2 - c^2)^(1/2)*2i)/(c^4*cos(x)
- c^2*d^2*cos(x)) + (b*d^2*atan((a^2*c^7*sin(x/2)*(d^2 - c^2)^(1/2)*1i + b^
2*d^5*sin(x/2)*(d^2 - c^2)^(3/2)*2i - b^2*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*2i
+ a^2*c^4*d*sin(x/2)*(d^2 - c^2)^(3/2)*2i + a^2*c^6*d*sin(x/2)*(d^2 - c^2)
^(1/2)*1i - a^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c^5*d^2*sin(x/2
)*(d^2 - c^2)^(1/2)*1i + b^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*3i - b^2*c^
3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2
)*1i + b^2*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*1i + a*b*c^2*d^3*sin(x/2)*(d^
2 - c^2)^(3/2)*4i - a*b*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*2i - a*b*c^3*d^4
*sin(x/2)*(d^2 - c^2)^(1/2)*2i + a*b*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*2i
+ a*b*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*2i)/(a^2*c^8*cos(x/2) + a^2*c^4*d^
4*cos(x/2) - 2*a^2*c^6*d^2*cos(x/2) + b^2*c^2*d^6*cos(x/2) - 2*b^2*c^4*d^4*
cos(x/2) + b^2*c^6*d^2*cos(x/2) + 2*a*b*c^2*d^6*cos(x/2) - 4*a*b*c^4*d^4*co
s(x/2) + 2*a*b*c^6*d^2*cos(x/2))) *cos(x)*(d^2 - c^2)^(1/2)*2i)/(c^4*cos(x)
- c^2*d^2*cos(x)) - (2*b*c^2*d*atanh(sin(x/2)/cos(x/2))*cos(x))/(c^4*cos(x)
- c^2*d^2*cos(x))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec^2(x)}{c + d \cos(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(x)**2)/(c+d*cos(x)),x)

[Out] Integral((a + b*sec(x)**2)/(c + d*cos(x)), x)

$$3.216 \quad \int \frac{a+b \csc^2(x)}{c+d \sin(x)} dx$$

Optimal. Leaf size=72

$$\frac{2(ac^2 + bd^2) \tan^{-1}\left(\frac{c \tan\left(\frac{x}{2}\right) + d}{\sqrt{c^2 - d^2}}\right)}{c^2 \sqrt{c^2 - d^2}} + \frac{bd \tanh^{-1}(\cos(x))}{c^2} - \frac{b \cot(x)}{c}$$

[Out] b*d*arctanh(cos(x))/c^2-b*cot(x)/c+2*(a*c^2+b*d^2)*arctan((d+c*tan(1/2*x))/(c^2-d^2)^(1/2))/c^2/(c^2-d^2)^(1/2)

Rubi [A] time = 0.24, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4233, 3056, 3001, 3770, 2660, 618, 204}

$$\frac{2(ac^2 + bd^2) \tan^{-1}\left(\frac{c \tan\left(\frac{x}{2}\right) + d}{\sqrt{c^2 - d^2}}\right)}{c^2 \sqrt{c^2 - d^2}} + \frac{bd \tanh^{-1}(\cos(x))}{c^2} - \frac{b \cot(x)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Csc[x]^2)/(c + d*Sin[x]),x]

[Out] (2*(a*c^2 + b*d^2)*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]]/(c^2*Sqrt[c^2 - d^2]) + (b*d*ArcTanh[Cos[x]])/c^2 - (b*Cot[x])/c

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3001

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3056

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/(m +

```

1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))

```

Rule 3770

```

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]

```

Rule 4233

```

Int[(csc[(a_.) + (b_.)*(x_.)]^2*(C_.) + (A_.))*(u_), x_Symbol] := Int[(Activa
teTrig[u]*(C + A*Sin[a + b*x]^2))/Sin[a + b*x]^2, x] /; FreeQ[{a, b, A, C},
x] && KnownSineIntegrandQ[u, x]

```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \csc^2(x)}{c + d \sin(x)} dx &= \int \frac{\csc^2(x) (b + a \sin^2(x))}{c + d \sin(x)} dx \\
&= -\frac{b \cot(x)}{c} + \frac{\int \frac{\csc(x)(-bd+ac \sin(x))}{c+d \sin(x)} dx}{c} \\
&= -\frac{b \cot(x)}{c} - \frac{(bd) \int \csc(x) dx}{c^2} + \left(a + \frac{bd^2}{c^2}\right) \int \frac{1}{c + d \sin(x)} dx \\
&= \frac{bd \tanh^{-1}(\cos(x))}{c^2} - \frac{b \cot(x)}{c} + \left(2 \left(a + \frac{bd^2}{c^2}\right)\right) \text{Subst} \left(\int \frac{1}{c + 2dx + cx^2} dx, x, \tan\left(\frac{x}{2}\right) \right) \\
&= \frac{bd \tanh^{-1}(\cos(x))}{c^2} - \frac{b \cot(x)}{c} - \left(4 \left(a + \frac{bd^2}{c^2}\right)\right) \text{Subst} \left(\int \frac{1}{-4(c^2 - d^2) - x^2} dx, x, 2d + 2c \tan\left(\frac{x}{2}\right) \right) \\
&= \frac{2 \left(a + \frac{bd^2}{c^2}\right) \tan^{-1} \left(\frac{d+c \tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}} \right)}{\sqrt{c^2-d^2}} + \frac{bd \tanh^{-1}(\cos(x))}{c^2} - \frac{b \cot(x)}{c}
\end{aligned}$$

Mathematica [A] time = 0.53, size = 102, normalized size = 1.42

$$\frac{\csc\left(\frac{x}{2}\right) \sec\left(\frac{x}{2}\right) \left(\frac{2 \sin(x)(ac^2+bd^2) \tan^{-1}\left(\frac{c \tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} - b \left(c \cos(x) + d \sin(x) \left(\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) \right) \right) \right)}{2c^2}$$

Antiderivative was successfully verified.

```

[In] Integrate[(a + b*Csc[x]^2)/(c + d*Sin[x]), x]

```

```

[Out] (Csc[x/2]*Sec[x/2]*((2*(a*c^2 + b*d^2)*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]]*Sin[x])/Sqrt[c^2 - d^2] - b*(c*Cos[x] + d*(-Log[Cos[x/2]] + Log[Sin[x/2]])*Sin[x])))/(2*c^2)

```

fricas [B] time = 3.95, size = 332, normalized size = 4.61

$$\left[\frac{(ac^2 + bd^2)\sqrt{-c^2 + d^2} \log\left(\frac{(2c^2 - d^2)\cos(x)^2 - 2cd\sin(x) - c^2 - d^2 + 2(c\cos(x)\sin(x) + d\cos(x))\sqrt{-c^2 + d^2}}{d^2\cos(x)^2 - 2cd\sin(x) - c^2 - d^2}\right) \sin(x) - (bc^2d - bd^3) \log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x)\right) + (bc^2d - bd^3) \log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x)\right) + 2*(bc^3 - b*c*d^2)*\cos(x)}{2(c^4 - c^2d^2)\sin(x)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(x)^2)/(c+d*sin(x)),x, algorithm="fricas")

[Out] [-1/2*((a*c^2 + b*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(x)^2 - 2*c*d*sin(x) - c^2 - d^2 + 2*(c*cos(x)*sin(x) + d*cos(x))*sqrt(-c^2 + d^2))/(d^2*cos(x)^2 - 2*c*d*sin(x) - c^2 - d^2))*sin(x) - (b*c^2*d - b*d^3)*log(1/2*cos(x) + 1/2)*sin(x) + (b*c^2*d - b*d^3)*log(-1/2*cos(x) + 1/2)*sin(x) + 2*(b*c^3 - b*c*d^2)*cos(x))/((c^4 - c^2*d^2)*sin(x)), -1/2*(2*(a*c^2 + b*d^2)*sqrt(c^2 - d^2)*arctan(-(c*sin(x) + d)/(sqrt(c^2 - d^2)*cos(x)))*sin(x) - (b*c^2*d - b*d^3)*log(1/2*cos(x) + 1/2)*sin(x) + (b*c^2*d - b*d^3)*log(-1/2*cos(x) + 1/2)*sin(x) + 2*(b*c^3 - b*c*d^2)*cos(x))/((c^4 - c^2*d^2)*sin(x))]

giac [A] time = 0.17, size = 110, normalized size = 1.53

$$-\frac{bd \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{c^2} + \frac{b \tan\left(\frac{1}{2}x\right)}{2c} + \frac{2(ac^2 + bd^2)\left(\pi\left[\frac{x}{2\pi} + \frac{1}{2}\right] \operatorname{sgn}(c) + \arctan\left(\frac{c \tan\left(\frac{1}{2}x\right) + d}{\sqrt{c^2 - d^2}}\right)\right)}{\sqrt{c^2 - d^2} c^2} + \frac{2bd \tan\left(\frac{1}{2}x\right) - bc}{2c^2 \tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(x)^2)/(c+d*sin(x)),x, algorithm="giac")

[Out] -b*d*log(abs(tan(1/2*x)))/c^2 + 1/2*b*tan(1/2*x)/c + 2*(a*c^2 + b*d^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*x) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*c^2) + 1/2*(2*b*d*tan(1/2*x) - b*c)/(c^2*tan(1/2*x))

maple [A] time = 0.11, size = 120, normalized size = 1.67

$$\frac{b \tan\left(\frac{x}{2}\right)}{2c} - \frac{b}{2c \tan\left(\frac{x}{2}\right)} - \frac{db \ln\left(\tan\left(\frac{x}{2}\right)\right)}{c^2} + \frac{2 \arctan\left(\frac{2c \tan\left(\frac{x}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right) a}{\sqrt{c^2 - d^2}} + \frac{2 \arctan\left(\frac{2c \tan\left(\frac{x}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right) b d^2}{c^2 \sqrt{c^2 - d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*csc(x)^2)/(c+d*sin(x)),x)

[Out] 1/2*b/c*tan(1/2*x)-1/2*b/c/tan(1/2*x)-1/c^2*d*b*ln(tan(1/2*x))+2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*x)+2*d)/(c^2-d^2)^(1/2))*a+2/c^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*x)+2*d)/(c^2-d^2)^(1/2))*b*d^2

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(x)^2)/(c+d*sin(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 2.83, size = 463, normalized size = 6.43

$$b d^3 \ln\left(\tan\left(\frac{x}{2}\right)\right) - b c^2 d \ln\left(\tan\left(\frac{x}{2}\right)\right) + a c^2 \operatorname{atan}\left(\frac{a c^3 \sqrt{d^2-c^2} \operatorname{atan}\left(\frac{x}{2}\right) \sqrt{d^2-c^2} + b c d^2 \sqrt{d^2-c^2} + 2 i + a c^2 d \tan\left(\frac{x}{2}\right) \sqrt{d^2-c^2}}{4 b d^4 \tan\left(\frac{x}{2}\right) - a c^4 \tan\left(\frac{x}{2}\right) + a c^3 d + 2 b c d^3 - b c^3 d + 2 a c^2 d^2 \tan\left(\frac{x}{2}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/sin(x)^2)/(c + d*sin(x)),x)

[Out] (b*d^3*log(tan(x/2)) + a*c^2*atan((a*c^3*(d^2 - c^2)^(1/2)*1i + b*d^3*tan(x/2)*(d^2 - c^2)^(1/2)*4i + b*c*d^2*(d^2 - c^2)^(1/2)*2i + a*c^2*d*tan(x/2)*(d^2 - c^2)^(1/2)*2i - b*c^2*d*tan(x/2)*(d^2 - c^2)^(1/2)*1i)/(4*b*d^4*tan(x/2) - a*c^4*tan(x/2) + a*c^3*d + 2*b*c*d^3 - b*c^3*d + 2*a*c^2*d^2*tan(x/2) - 3*b*c^2*d^2*tan(x/2)))*(d^2 - c^2)^(1/2)*2i + b*d^2*atan((a*c^3*(d^2 - c^2)^(1/2)*1i + b*d^3*tan(x/2)*(d^2 - c^2)^(1/2)*4i + b*c*d^2*(d^2 - c^2)^(1/2)*2i + a*c^2*d*tan(x/2)*(d^2 - c^2)^(1/2)*2i - b*c^2*d*tan(x/2)*(d^2 - c^2)^(1/2)*1i)/(4*b*d^4*tan(x/2) - a*c^4*tan(x/2) + a*c^3*d + 2*b*c*d^3 - b*c^3*d + 2*a*c^2*d^2*tan(x/2) - 3*b*c^2*d^2*tan(x/2)))*(d^2 - c^2)^(1/2)*2i - b*c^2*d*log(tan(x/2)))/(c^4 - c^2*d^2) - (b*c^3 - b*c*d^2)/(c^4*tan(x) - c^2*d^2*tan(x))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \csc^2(x)}{c + d \sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(x)**2)/(c+d*sin(x)),x)

[Out] Integral((a + b*csc(x)**2)/(c + d*sin(x)), x)

3.217 $\int (a \cos(c + dx) + b \sin(c + dx))^n dx$

Optimal. Leaf size=136

$$\frac{\sin\left(-\tan^{-1}(a, b) + c + dx\right) (a \cos(c + dx) + b \sin(c + dx))^n \left(\frac{a \cos(c + dx) + b \sin(c + dx)}{\sqrt{a^2 + b^2}}\right)^{-n} \cos^{n+1}\left(-\tan^{-1}(a, b) + c + dx\right)}{d(n+1)\sqrt{\sin^2\left(-\tan^{-1}(a, b) + c + dx\right)}}$$

[Out] $-\cos(c+d*x-\arctan(a,b))^{(1+n)}*\text{hypergeom}([1/2, 1/2+1/2*n], [3/2+1/2*n], \cos(c+d*x-\arctan(a,b))^2)*(a*\cos(d*x+c)+b*\sin(d*x+c))^n*\sin(c+d*x-\arctan(a,b))/d/(1+n)/(((a*\cos(d*x+c)+b*\sin(d*x+c))/(a^2+b^2)^{(1/2)})^n)/(\sin(c+d*x-\arctan(a,b))^2)^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3078, 2643}

$$\frac{\sin\left(-\tan^{-1}(a, b) + c + dx\right) (a \cos(c + dx) + b \sin(c + dx))^n \left(\frac{a \cos(c + dx) + b \sin(c + dx)}{\sqrt{a^2 + b^2}}\right)^{-n} \cos^{n+1}\left(-\tan^{-1}(a, b) + c + dx\right)}{d(n+1)\sqrt{\sin^2\left(-\tan^{-1}(a, b) + c + dx\right)}}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x]

[Out] $-\left(\left(\cos[c + d*x - \text{ArcTan}[a, b]]\right)^{(1+n)}*\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(1+n)}{2}, \frac{(3+n)}{2}, \cos[c + d*x - \text{ArcTan}[a, b]]^2\right]*(a*\cos[c + d*x] + b*\sin[c + d*x])^n*\sin[c + d*x - \text{ArcTan}[a, b]]\right)/\left(d*(1+n)*\left((a*\cos[c + d*x] + b*\sin[c + d*x])/\sqrt{a^2 + b^2}\right)^n*\sqrt{\sin[c + d*x - \text{ArcTan}[a, b]]^2}\right)$

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3078

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 + b^2, 0] || EqQ[a^2 + b^2, 0])

Rubi steps

$$\int (a \cos(c + dx) + b \sin(c + dx))^n dx = \left((a \cos(c + dx) + b \sin(c + dx))^n \left(\frac{a \cos(c + dx) + b \sin(c + dx)}{\sqrt{a^2 + b^2}} \right)^{-n} \right) \int \cos^{1+n}\left(c + dx - \tan^{-1}(a, b)\right) {}_2F_1\left(\frac{1}{2}, \frac{1+n}{2}; \frac{3+n}{2}; \cos^2\left(c + dx - \tan^{-1}(a, b)\right)\right) dx$$

Mathematica [A] time = 0.24, size = 94, normalized size = 0.69

$$\frac{\sin\left(2\left(\tan^{-1}\left(\frac{a}{b}\right) + c + dx\right)\right) \sin^2\left(\tan^{-1}\left(\frac{a}{b}\right) + c + dx\right)^{-\frac{n}{2}-\frac{1}{2}} (a \cos(c + dx) + b \sin(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3}{2}; \cos^2\left(\tan^{-1}\left(\frac{a}{b}\right) + c + dx\right)\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a*cos[c + d*x] + b*sin[c + d*x])^n,x]

[Out] $-1/2 * (\text{Hypergeometric2F1}[1/2, (1 - n)/2, 3/2, \cos[c + d*x + \text{ArcTan}[a/b]]^2] * (a * \cos[c + d*x] + b * \sin[c + d*x])^n * (\sin[c + d*x + \text{ArcTan}[a/b]]^2)^{-1/2 - n/2} * \sin[2 * (c + d*x + \text{ArcTan}[a/b])]) / d$

fricas [F] time = 1.86, size = 0, normalized size = 0.00

$$\int (a \cos(dx + c) + b \sin(dx + c))^n, x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^n,x, algorithm="fricas")

[Out] integral((a*cos(d*x + c) + b*sin(d*x + c))^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cos(dx + c) + b \sin(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^n,x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^n, x)

maple [F] time = 1.00, size = 0, normalized size = 0.00

$$\int (a \cos(dx + c) + b \sin(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+b*sin(d*x+c))^n,x)

[Out] int((a*cos(d*x+c)+b*sin(d*x+c))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cos(dx + c) + b \sin(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^n,x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a \cos(c + dx) + b \sin(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(c + d*x) + b*sin(c + d*x))^n,x)

[Out] int((a*cos(c + d*x) + b*sin(c + d*x))^n, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^n,x)

[Out] Timed out

3.218 $\int (2 \cos(c + dx) + 3 \sin(c + dx))^n dx$

Optimal. Leaf size=95

$$\frac{13^{n/2} \sin\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right) \cos^{n+1}\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right) {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{d(n+1) \sqrt{\sin^2\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right)}}$$

[Out] $-13^{(1/2*n)} * \cos(c+d*x-\arctan(3/2))^{(1+n)} * \text{hypergeom}([1/2, 1/2+1/2*n], [3/2+1/2*n], \cos(c+d*x-\arctan(3/2))^2) * \sin(c+d*x-\arctan(3/2))/d/(1+n)/(\sin(c+d*x-\arctan(3/2))^2)^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3077, 2643}

$$\frac{13^{n/2} \sin\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right) \cos^{n+1}\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right) {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{d(n+1) \sqrt{\sin^2\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right)}}$$

Antiderivative was successfully verified.

[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^n,x]

[Out] $-((13^{(n/2)} * \text{Cos}[c + d*x - \text{ArcTan}[3/2]]^{(1+n)} * \text{Hypergeometric2F1}[1/2, (1+n)/2, (3+n)/2, \text{Cos}[c + d*x - \text{ArcTan}[3/2]]^2] * \text{Sin}[c + d*x - \text{ArcTan}[3/2]])/(d*(1+n)*\text{Sqrt}[\text{Sin}[c + d*x - \text{ArcTan}[3/2]]^2]))$

Rule 2643

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3077

Int[(cos[(c_.) + (d_)*(x_)]*(a_.) + (b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned} \int (2 \cos(c + dx) + 3 \sin(c + dx))^n dx &= 13^{n/2} \int \cos^n\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right) dx \\ &= \frac{13^{n/2} \cos^{1+n}\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right) {}_2F_1\left(\frac{1}{2}, \frac{1+n}{2}; \frac{3+n}{2}; \cos^2\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{d(1+n) \sqrt{\sin^2\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right)}} \end{aligned}$$

Mathematica [A] time = 0.17, size = 88, normalized size = 0.93

$$\frac{\sin\left(2\left(c + dx + \tan^{-1}\left(\frac{2}{3}\right)\right)\right) \sin^2\left(c + dx + \tan^{-1}\left(\frac{2}{3}\right)\right)^{-\frac{n}{2}-\frac{1}{2}} (3 \sin(c + dx) + 2 \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1-n}{2}; \frac{3}{2}; \cos^2\left(c + dx + \tan^{-1}\left(\frac{2}{3}\right)\right)\right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^n,x]

[Out] $-1/2*(\text{Hypergeometric2F1}[1/2, (1 - n)/2, 3/2, \text{Cos}[c + d*x + \text{ArcTan}[2/3]]^2] * (2*\text{Cos}[c + d*x] + 3*\text{Sin}[c + d*x])^n * (\text{Sin}[c + d*x + \text{ArcTan}[2/3]]^2)^{-1/2 - n/2} * \text{Sin}[2*(c + d*x + \text{ArcTan}[2/3])]) / d$

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}((2 \cos(dx + c) + 3 \sin(dx + c))^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^n,x, algorithm="fricas")

[Out] integral((2*cos(d*x + c) + 3*sin(d*x + c))^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2 \cos(dx + c) + 3 \sin(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^n,x, algorithm="giac")

[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^n, x)

maple [F] time = 1.06, size = 0, normalized size = 0.00

$$\int (2 \cos(dx + c) + 3 \sin(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*cos(d*x+c)+3*sin(d*x+c))^n,x)

[Out] int((2*cos(d*x+c)+3*sin(d*x+c))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2 \cos(dx + c) + 3 \sin(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^n,x, algorithm="maxima")

[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2 \cos(c + dx) + 3 \sin(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*cos(c + d*x) + 3*sin(c + d*x))^n,x)

[Out] int((2*cos(c + d*x) + 3*sin(c + d*x))^n, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 \sin(c + dx) + 2 \cos(c + dx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**n,x)

[Out] Integral((3*sin(c + d*x) + 2*cos(c + d*x))**n, x)

3.219 $\int (a \cos(c + dx) + b \sin(c + dx))^7 dx$

Optimal. Leaf size=127

$$\frac{3(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))^5}{5d} + \frac{(a^2 + b^2)^2(b \cos(c + dx) - a \sin(c + dx))^3}{d} - \frac{(a^2 + b^2)^3(b \cos(c + dx) - a \sin(c + dx))}{d}$$

[Out] $-(a^2+b^2)^3*(b*\cos(d*x+c)-a*\sin(d*x+c))/d+(a^2+b^2)^2*(b*\cos(d*x+c)-a*\sin(d*x+c))^3/d-3/5*(a^2+b^2)*(b*\cos(d*x+c)-a*\sin(d*x+c))^5/d+1/7*(b*\cos(d*x+c)-a*\sin(d*x+c))^7/d$

Rubi [A] time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3072, 194}

$$\frac{3(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))^5}{5d} + \frac{(a^2 + b^2)^2(b \cos(c + dx) - a \sin(c + dx))^3}{d} - \frac{(a^2 + b^2)^3(b \cos(c + dx) - a \sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^7,x]

[Out] $-\left(\frac{(a^2 + b^2)^3(b \cos[c + d*x] - a \sin[c + d*x])}{d} + \frac{(a^2 + b^2)^2(b \cos[c + d*x] - a \sin[c + d*x])^3}{d} - \frac{3(a^2 + b^2)(b \cos[c + d*x] - a \sin[c + d*x])^5}{5d} + \frac{(b \cos[c + d*x] - a \sin[c + d*x])^7}{7d}\right)$

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3072

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned} \int (a \cos(c + dx) + b \sin(c + dx))^7 dx &= -\frac{\text{Subst}\left(\int (a^2 + b^2 - x^2)^3 dx, x, b \cos(c + dx) - a \sin(c + dx)\right)}{d} \\ &= -\frac{\text{Subst}\left(\int \left(a^6 \left(1 + \frac{3a^4b^2 + 3a^2b^4 + b^6}{a^6}\right) - 3a^4 \left(1 + \frac{2a^2b^2 + b^4}{a^4}\right)x^2 + 3a^2 \left(1 + \frac{b^2}{a^2}\right)x^4 - b^0 x^6\right) dx, x, b \cos(c + dx) - a \sin(c + dx)\right)}{d} \\ &= -\frac{(a^2 + b^2)^3(b \cos(c + dx) - a \sin(c + dx))}{d} + \frac{(a^2 + b^2)^2(b \cos(c + dx) - a \sin(c + dx))^3}{d} - \frac{3(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))^5}{5d} + \frac{(b \cos(c + dx) - a \sin(c + dx))^7}{7d} \end{aligned}$$

Mathematica [A] time = 1.02, size = 246, normalized size = 1.94

$$\frac{1225a(a^2 + b^2)^3 \sin(c + dx) + 245a(a^2 - 3b^2)(a^2 + b^2)^2 \sin(3(c + dx)) - 1225b(a^2 + b^2)^3 \cos(c + dx) + 245b(b^2 - 3a^2) \cos(3(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^7,x]

```
[Out] (-1225*b*(a^2 + b^2)^3*cos[c + d*x] + 245*b*(-3*a^2 + b^2)*(a^2 + b^2)^2*cos[3*(c + d*x)] - 49*b*(5*a^6 - 5*a^4*b^2 - 9*a^2*b^4 + b^6)*cos[5*(c + d*x)] + 5*b*(-7*a^6 + 35*a^4*b^2 - 21*a^2*b^4 + b^6)*cos[7*(c + d*x)] + 1225*a*(a^2 + b^2)^3*sin[c + d*x] + 245*a*(a^2 - 3*b^2)*(a^2 + b^2)^2*sin[3*(c + d*x)] + 49*a*(a^6 - 9*a^4*b^2 - 5*a^2*b^4 + 5*b^6)*sin[5*(c + d*x)] + 5*a*(a^6 - 21*a^4*b^2 + 35*a^2*b^4 - 7*b^6)*sin[7*(c + d*x)]/(2240*d)
```

fricas [B] time = 1.01, size = 257, normalized size = 2.02

$$\frac{35b^7 \cos(dx + c) + 5(7a^6b - 35a^4b^3 + 21a^2b^5 - b^7) \cos(dx + c)^7 + 7(35a^4b^3 - 42a^2b^5 + 3b^7) \cos(dx + c)^5}{448d} - \frac{7(5a^6b - 5a^4b^3 - 9a^2b^5 + b^7) \cos(5dx + 5c)}{320d} - \frac{7(3a^6b - 3a^4b^3 - 9a^2b^5 + b^7) \cos(7dx + 7c)}{448d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^7,x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^7*cos(d*x + c) + 5*(7*a^6*b - 35*a^4*b^3 + 21*a^2*b^5 - b^7)*cos(d*x + c)^7 + 7*(35*a^4*b^3 - 42*a^2*b^5 + 3*b^7)*cos(d*x + c)^5 + 35*(7*a^2*b^5 - b^7)*cos(d*x + c)^3 - (16*a^7 + 56*a^5*b^2 + 70*a^3*b^4 + 35*a*b^6 + 5*(a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^6 + (6*a^7 + 21*a^5*b^2 - 280*a^3*b^4 + 105*a*b^6)*cos(d*x + c)^4 + (8*a^7 + 28*a^5*b^2 + 35*a^3*b^4 - 105*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/d
```

giac [B] time = 0.66, size = 316, normalized size = 2.49

$$\frac{(7a^6b - 35a^4b^3 + 21a^2b^5 - b^7) \cos(7dx + 7c)}{448d} - \frac{7(5a^6b - 5a^4b^3 - 9a^2b^5 + b^7) \cos(5dx + 5c)}{320d} - \frac{7(3a^6b - 3a^4b^3 - 9a^2b^5 + b^7) \cos(7dx + 7c)}{448d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^7,x, algorithm="giac")
```

```
[Out] -1/448*(7*a^6*b - 35*a^4*b^3 + 21*a^2*b^5 - b^7)*cos(7*d*x + 7*c)/d - 7/320*(5*a^6*b - 5*a^4*b^3 - 9*a^2*b^5 + b^7)*cos(5*d*x + 5*c)/d - 7/64*(3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*cos(3*d*x + 3*c)/d - 35/64*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)/d + 1/448*(a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b^6)*sin(7*d*x + 7*c)/d + 7/320*(a^7 - 9*a^5*b^2 - 5*a^3*b^4 + 5*a*b^6)*sin(5*d*x + 5*c)/d + 7/64*(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*sin(3*d*x + 3*c)/d + 35/64*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sin(d*x + c)/d
```

maple [B] time = 0.41, size = 321, normalized size = 2.53

$$\frac{b^7 \left(\frac{16}{5} + \sin^6(dx+c) + \frac{6(\sin^4(dx+c))}{5} + \frac{8(\sin^2(dx+c))}{5} \right) \cos(dx+c)}{7} + a b^6 \left(\sin^7(dx+c) \right) + 21 a^2 b^5 \left(-\frac{(\sin^4(dx+c))(\cos^3(dx+c))}{7} - \frac{4(\sin^2(dx+c))(\cos^2(dx+c))}{7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^7,x)
```

```
[Out] 1/d*(-1/7*b^7*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c)+a*b^6*sin(d*x+c)^7+21*a^2*b^5*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*cos(d*x+c)^3-8/105*cos(d*x+c)^3)+35*a^3*b^4*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*(2+cos(d*x+c)^2)*sin(d*x+c))+35*a^4*b^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+21*a^5*b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-a^6*b*cos(d*x+c)^7+1/7*a^7*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [B] time = 0.33, size = 257, normalized size = 2.02

$$\frac{35 a^6 b \cos(dx + c)^7 - 35 a b^6 \sin(dx + c)^7 + (5 \sin(dx + c)^7 - 21 \sin(dx + c)^5 + 35 \sin(dx + c)^3 - 35 \sin(dx + c)) a^7 - 7(15 \sin(dx + c)^7 - 42 \sin(dx + c)^5 + 35 \sin(dx + c)^3) a^5 b^2 - 35(5 \cos(dx + c)^7 - 7 \cos(dx + c)^5) a^4 b^3 + 35(5 \sin(dx + c)^7 - 7 \sin(dx + c)^5) a^3 b^4 + 7(15 \cos(dx + c)^7 - 42 \cos(dx + c)^5 + 35 \cos(dx + c)^3) a^2 b^5 - (5 \cos(dx + c)^7 - 21 \cos(dx + c)^5 + 35 \cos(dx + c)^3 - 35 \cos(dx + c)) b^7}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^7,x, algorithm="maxima")

[Out]
$$\frac{-1/35*(35*a^6*b*\cos(d*x + c)^7 - 35*a*b^6*\sin(d*x + c)^7 + (5*\sin(d*x + c)^7 - 21*\sin(d*x + c)^5 + 35*\sin(d*x + c)^3 - 35*\sin(d*x + c))*a^7 - 7*(15*\sin(d*x + c)^7 - 42*\sin(d*x + c)^5 + 35*\sin(d*x + c)^3)*a^5*b^2 - 35*(5*\cos(d*x + c)^7 - 7*\cos(d*x + c)^5)*a^4*b^3 + 35*(5*\sin(d*x + c)^7 - 7*\sin(d*x + c)^5)*a^3*b^4 + 7*(15*\cos(d*x + c)^7 - 42*\cos(d*x + c)^5 + 35*\cos(d*x + c)^3)*a^2*b^5 - (5*\cos(d*x + c)^7 - 21*\cos(d*x + c)^5 + 35*\cos(d*x + c)^3 - 35*\cos(d*x + c))*b^7}{d}$$

mupad [B] time = 6.16, size = 422, normalized size = 3.32

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 (70 a^6 b - 140 a^4 b^3 + 224 a^2 b^5) - 2 a^7 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{13} - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 \left(\frac{424 a^7}{35} + \frac{912 a^5 b^2}{5} - 192 a^3\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(c + d*x) + b*sin(c + d*x))^7,x)

[Out]
$$\begin{aligned} & -(\tan(c/2 + (d*x)/2)^8*(70*a^6*b + 224*a^2*b^5 - 140*a^4*b^3) - 2*a^7*\tan(c/2 + (d*x)/2)^{13} - \tan(c/2 + (d*x)/2)^7*(128*a*b^6 + (424*a^7)/35 - 192*a^3*b^4 + (912*a^5*b^2)/5) + \tan(c/2 + (d*x)/2)^4*(42*a^6*b + (96*b^7)/5 + (336*a^2*b^5)/5 - 56*a^4*b^3) + 2*a^6*b + (32*b^7)/35 - \tan(c/2 + (d*x)/2)^5*((86*a^7)/5 + 224*a^3*b^4 - (224*a^5*b^2)/5) - \tan(c/2 + (d*x)/2)^9*((86*a^7)/5 + 224*a^3*b^4 - (224*a^5*b^2)/5) + \tan(c/2 + (d*x)/2)^2*((32*b^7)/5 + (112*a^2*b^5)/5 + 28*a^4*b^3) + \tan(c/2 + (d*x)/2)^6*(32*b^7 - 112*a^2*b^5 + 280*a^4*b^3) + (16*a^2*b^5)/5 + 4*a^4*b^3 - \tan(c/2 + (d*x)/2)^3*(4*a^7 + 56*a^5*b^2) - \tan(c/2 + (d*x)/2)^{11}*(4*a^7 + 56*a^5*b^2) - 2*a^7*\tan(c/2 + (d*x)/2) + 140*a^4*b^3*\tan(c/2 + (d*x)/2)^{10} + 14*a^6*b*\tan(c/2 + (d*x)/2)^{12})/(d*(\tan(c/2 + (d*x)/2)^2 + 1)^7) \end{aligned}$$

sympy [A] time = 6.56, size = 461, normalized size = 3.63

$$\left\{ \begin{array}{l} \frac{16a^7 \sin^7(c+dx)}{35d} + \frac{8a^7 \sin^5(c+dx) \cos^2(c+dx)}{5d} + \frac{2a^7 \sin^3(c+dx) \cos^4(c+dx)}{d} + \frac{a^7 \sin(c+dx) \cos^6(c+dx)}{d} - \frac{a^6 b \cos^7(c+dx)}{d} + \frac{8a^5 b^2 \sin^7(c+dx)}{5d} \\ x(a \cos(c) + b \sin(c))^7 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**7,x)

[Out]
$$\text{Piecewise}((16*a**7*\sin(c + d*x)**7/(35*d) + 8*a**7*\sin(c + d*x)**5*\cos(c + d*x)**2/(5*d) + 2*a**7*\sin(c + d*x)**3*\cos(c + d*x)**4/d + a**7*\sin(c + d*x)*\cos(c + d*x)**6/d - a**6*b*\cos(c + d*x)**7/d + 8*a**5*b**2*\sin(c + d*x)**7/(5*d) + 28*a**5*b**2*\sin(c + d*x)**5*\cos(c + d*x)**2/(5*d) + 7*a**5*b**2*\sin(c + d*x)**3*\cos(c + d*x)**4/d - 7*a**4*b**3*\sin(c + d*x)**2*\cos(c + d*x)**5/d - 2*a**4*b**3*\cos(c + d*x)**7/d + 2*a**3*b**4*\sin(c + d*x)**7/d + 7*a**3*b**4*\sin(c + d*x)**5*\cos(c + d*x)**2/d - 7*a**2*b**5*\sin(c + d*x)**4*\cos(c + d*x)**3/d - 28*a**2*b**5*\sin(c + d*x)**2*\cos(c + d*x)**5/(5*d) - 8*a**2*b**5*\cos(c + d*x)**7/(5*d) + a*b**6*\sin(c + d*x)**7/d - b**7*\sin(c + d*x)**6*\cos(c + d*x)/d - 2*b**7*\sin(c + d*x)**4*\cos(c + d*x)**3/d - 8*b**7*\sin(c + d*x)**2*\cos(c + d*x)**5/(5*d) - 16*b**7*\cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**7, True))$$

3.220 $\int (a \cos(c + dx) + b \sin(c + dx))^6 dx$

Optimal. Leaf size=161

$$\frac{5(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))^3}{24d} - \frac{5(a^2 + b^2)^2(b \cos(c + dx) - a \sin(c + dx))}{24d}$$

[Out] $5/16*(a^2+b^2)^3*x-5/16*(a^2+b^2)^2*(b*\cos(d*x+c)-a*\sin(d*x+c))*(a*\cos(d*x+c)+b*\sin(d*x+c))/d-5/24*(a^2+b^2)*(b*\cos(d*x+c)-a*\sin(d*x+c))*(a*\cos(d*x+c)+b*\sin(d*x+c))^3/d-1/6*(b*\cos(d*x+c)-a*\sin(d*x+c))*(a*\cos(d*x+c)+b*\sin(d*x+c))^5/d$

Rubi [A] time = 0.08, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3073, 8}

$$\frac{5(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))^3}{24d} - \frac{5(a^2 + b^2)^2(b \cos(c + dx) - a \sin(c + dx))}{24d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^6, x]$

[Out] $(5*(a^2 + b^2)^3*x)/16 - (5*(a^2 + b^2)^2*(b*\text{Cos}[c + d*x] - a*\text{Sin}[c + d*x])*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]))/(16*d) - (5*(a^2 + b^2)*(b*\text{Cos}[c + d*x] - a*\text{Sin}[c + d*x])*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^3)/(24*d) - ((b*\text{Cos}[c + d*x] - a*\text{Sin}[c + d*x])*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^5)/(6*d)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 3073

$\text{Int}[(\cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*\sin[(c_.) + (d_.)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x] - a*\text{Sin}[c + d*x])*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^{(n-1)}]/(d*n), x] + \text{Dist}[(n-1)*(a^2 + b^2)/n, \text{Int}[(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{IntegerQ}[(n-1)/2] \&\& \text{GtQ}[n, 1]$

Rubi steps

$$\begin{aligned} \int (a \cos(c + dx) + b \sin(c + dx))^6 dx &= -\frac{(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))^5}{6d} + \frac{1}{6} \left(5 \frac{(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))^3}{24d} \right. \\ &= -\frac{5(a^2 + b^2)^2(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))^2}{16d} \\ &= \frac{5}{16}(a^2 + b^2)^3 x - \frac{5(a^2 + b^2)^2(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))}{16d} \end{aligned}$$

Mathematica [A] time = 0.69, size = 192, normalized size = 1.19

$$-36ab(a^4 - b^4)\cos(4(c + dx)) + 60(a^2 + b^2)^3(c + dx) + 45(a^2 - b^2)(a^2 + b^2)^2\sin(2(c + dx)) - 90ab(a^2 + b^2)$$

Antiderivative was successfully verified.

[In] Integrate[(a*cos[c + d*x] + b*sin[c + d*x])^6,x]

[Out] (60*(a^2 + b^2)^3*(c + d*x) - 90*a*b*(a^2 + b^2)^2*cos[2*(c + d*x)] - 36*a*b*(a^4 - b^4)*cos[4*(c + d*x)] - 2*a*b*(3*a^4 - 10*a^2*b^2 + 3*b^4)*cos[6*(c + d*x)] + 45*(a^2 - b^2)*(a^2 + b^2)^2*sin[2*(c + d*x)] + 9*(a^6 - 5*a^4*b^2 - 5*a^2*b^4 + b^6)*sin[4*(c + d*x)] + (a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)*sin[6*(c + d*x)]/(192*d)

fricas [A] time = 1.18, size = 219, normalized size = 1.36

$$\frac{144 ab^5 \cos(dx + c)^2 + 16(3a^5b - 10a^3b^3 + 3ab^5) \cos(dx + c)^6 + 48(5a^3b^3 - 3ab^5) \cos(dx + c)^4 - 15(a^6 + 3a^4b^2 + 3a^2b^4 + b^6)}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="fricas")

[Out] -1/48*(144*a*b^5*cos(d*x + c)^2 + 16*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^6 + 48*(5*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^4 - 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d*x - (8*(a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)*cos(d*x + c)^5 + 2*(5*a^6 + 15*a^4*b^2 - 105*a^2*b^4 + 13*b^6)*cos(d*x + c)^3 + 3*(5*a^6 + 15*a^4*b^2 + 15*a^2*b^4 - 11*b^6)*cos(d*x + c))*sin(d*x + c))/d

giac [A] time = 0.32, size = 235, normalized size = 1.46

$$\frac{5}{16} (a^6 + 3a^4b^2 + 3a^2b^4 + b^6)x - \frac{(3a^5b - 10a^3b^3 + 3ab^5) \cos(6dx + 6c)}{96d} - \frac{3(a^5b - ab^5) \cos(4dx + 4c)}{16d} - \frac{15(a^5b - ab^5) \cos(2dx + 2c)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="giac")

[Out] 5/16*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x - 1/96*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*cos(6*d*x + 6*c)/d - 3/16*(a^5*b - a*b^5)*cos(4*d*x + 4*c)/d - 15/32*(a^5*b + 2*a^3*b^3 + a*b^5)*cos(2*d*x + 2*c)/d + 1/192*(a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)*sin(6*d*x + 6*c)/d + 3/64*(a^6 - 5*a^4*b^2 - 5*a^2*b^4 + b^6)*sin(4*d*x + 4*c)/d + 15/64*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*sin(2*d*x + 2*c)/d

maple [A] time = 0.34, size = 285, normalized size = 1.77

$$b^6 \left(\frac{\left(\sin^5(dx+c) + \frac{5 \sin^3(dx+c)}{4} + \frac{15 \sin(dx+c)}{8} \right) \cos(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right) + a b^5 \left(\sin^6(dx+c) + 15a^2b^4 \left(-\frac{(\sin^3(dx+c))(\cos^3(dx+c))}{6} - \frac{(\sin^3(dx+c))(\cos(dx+c))}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+b*sin(d*x+c))^6,x)

[Out] 1/d*(b^6*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/16*d*x+5/16*c)+a*b^5*sin(d*x+c)^6+15*a^2*b^4*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d*x+c)*cos(d*x+c)^3+1/16*sin(d*x+c)*cos(d*x+c)+1/16*d*x+1/16*c)+20*a^3*b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+15*a^4*b^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)-a^5*b*cos(d*x+c)^6+a^6*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))

maxima [A] time = 0.33, size = 238, normalized size = 1.48

$$192 a^5 b \cos(dx + c)^6 - 192 ab^5 \sin(dx + c)^6 + (4 \sin(2 dx + 2c)^3 - 60 dx - 60 c - 9 \sin(4 dx + 4c) - 48 \sin(2 dx + 2c)) a^6 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="maxima")

[Out] -1/192*(192*a^5*b*cos(d*x + c)^6 - 192*a*b^5*sin(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^6 - 15*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^4*b^2 + 320*(2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*a^3*b^3 + 15*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^2*b^4 - (4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b^6)/d

mupad [B] time = 4.12, size = 519, normalized size = 3.22

$$\frac{5 \operatorname{atan}\left(\frac{5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) (a^2 + b^2)^3}{8\left(\frac{5a^6}{8} + \frac{15a^4b^2}{8} + \frac{15a^2b^4}{8} + \frac{5b^6}{8}\right)}\right) (a^2 + b^2)^3}{8d} - \frac{5 \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}\right) (a^2 + b^2)^3}{8d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 (40a^5b^6 - 15a^4b^2 + 320a^3b^3 - 15a^2b^4 + 60ab^5 - b^6)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(c + d*x) + b*sin(c + d*x))^6,x)

[Out] (5*atan((5*tan(c/2 + (d*x)/2)*(a^2 + b^2)^3)/(8*((5*a^6)/8 + (5*b^6)/8 + (15*a^2*b^4)/8 + (15*a^4*b^2)/8)))*(a^2 + b^2)^3)/(8*d) - (5*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)*(a^2 + b^2)^3)/(8*d) + (tan(c/2 + (d*x)/2)^6*(64*a*b^5 + 40*a^5*b - (160*a^3*b^3)/3) - tan(c/2 + (d*x)/2)*((5*b^6)/8 - (11*a^6)/8 + (15*a^2*b^4)/8 + (15*a^4*b^2)/8) + tan(c/2 + (d*x)/2)^11*((5*b^6)/8 - (11*a^6)/8 + (15*a^2*b^4)/8 + (15*a^4*b^2)/8) - tan(c/2 + (d*x)/2)^3*((5*a^6)/24 + (85*b^6)/24 + (85*a^2*b^4)/8 - (235*a^4*b^2)/8) + tan(c/2 + (d*x)/2)^9*((5*a^6)/24 + (85*b^6)/24 + (85*a^2*b^4)/8 - (235*a^4*b^2)/8) + tan(c/2 + (d*x)/2)^5*((15*a^6)/4 - (33*b^6)/4 + (285*a^2*b^4)/4 - (195*a^4*b^2)/4) - tan(c/2 + (d*x)/2)^7*((15*a^6)/4 - (33*b^6)/4 + (285*a^2*b^4)/4 - (195*a^4*b^2)/4) + 80*a^3*b^3*tan(c/2 + (d*x)/2)^4 + 80*a^3*b^3*tan(c/2 + (d*x)/2)^8 + 12*a^5*b*tan(c/2 + (d*x)/2)^2 + 12*a^5*b*tan(c/2 + (d*x)/2)^10)/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))

sympy [A] time = 4.62, size = 770, normalized size = 4.78

$$\left\{ \begin{array}{l} \frac{5a^6x \sin^6(c+dx)}{16} + \frac{15a^6x \sin^4(c+dx) \cos^2(c+dx)}{16} + \frac{15a^6x \sin^2(c+dx) \cos^4(c+dx)}{16} + \frac{5a^6x \cos^6(c+dx)}{16} + \frac{5a^6 \sin^5(c+dx) \cos(c+dx)}{16d} + \frac{5a^6}{16d} \\ x (a \cos(c) + b \sin(c))^6 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**6,x)

[Out] Piecewise((5*a**6*x*sin(c + d*x)**6/16 + 15*a**6*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*a**6*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**6*x*cos(c + d*x)**6/16 + 5*a**6*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**6*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**6*sin(c + d*x)*cos(c + d*x)**5/(16*d) - a**5*b*cos(c + d*x)**6/d + 15*a**4*b**2*x*sin(c + d*x)**6/16 + 45*a**4*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 45*a**4*b**2*x*sin(c + d*x)**2*c

```

os(c + d*x)**4/16 + 15*a**4*b**2*x*cos(c + d*x)**6/16 + 15*a**4*b**2*sin(c
+ d*x)**5*cos(c + d*x)/(16*d) + 5*a**4*b**2*sin(c + d*x)**3*cos(c + d*x)**3
/(2*d) - 15*a**4*b**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 5*a**3*b**3*sin
(c + d*x)**6/(3*d) + 5*a**3*b**3*sin(c + d*x)**4*cos(c + d*x)**2/d + 15*a**
2*b**4*x*sin(c + d*x)**6/16 + 45*a**2*b**4*x*sin(c + d*x)**4*cos(c + d*x)**
2/16 + 45*a**2*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 15*a**2*b**4*x*c
os(c + d*x)**6/16 + 15*a**2*b**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 5*a*
*2*b**4*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 15*a**2*b**4*sin(c + d*x)*c
os(c + d*x)**5/(16*d) + a*b**5*sin(c + d*x)**6/d + 5*b**6*x*sin(c + d*x)**6
/16 + 15*b**6*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*b**6*x*sin(c + d*x)
**2*cos(c + d*x)**4/16 + 5*b**6*x*cos(c + d*x)**6/16 - 11*b**6*sin(c + d*x)
**5*cos(c + d*x)/(16*d) - 5*b**6*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - 5*
b**6*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c
))**6, True))

```


3.221 $\int (a \cos(c + dx) + b \sin(c + dx))^5 dx$

Optimal. Leaf size=94

$$\frac{2(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))^3}{3d} - \frac{(a^2 + b^2)^2(b \cos(c + dx) - a \sin(c + dx))}{d} - \frac{(b \cos(c + dx) - a \sin(c + dx))^5}{5d}$$

[Out] $-(a^2+b^2)^2*(b*\cos(d*x+c)-a*\sin(d*x+c))/d+2/3*(a^2+b^2)*(b*\cos(d*x+c)-a*\sin(d*x+c))^3/d-1/5*(b*\cos(d*x+c)-a*\sin(d*x+c))^5/d$

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3072, 194}

$$\frac{2(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))^3}{3d} - \frac{(a^2 + b^2)^2(b \cos(c + dx) - a \sin(c + dx))}{d} - \frac{(b \cos(c + dx) - a \sin(c + dx))^5}{5d}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]

[Out] $-(((a^2 + b^2)^2*(b*\cos[c + d*x] - a*\sin[c + d*x]))/d) + (2*(a^2 + b^2)*(b*\cos[c + d*x] - a*\sin[c + d*x])^3)/(3*d) - (b*\cos[c + d*x] - a*\sin[c + d*x])^5/(5*d)$

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3072

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned} \int (a \cos(c + dx) + b \sin(c + dx))^5 dx &= -\frac{\text{Subst}\left(\int (a^2 + b^2 - x^2)^2 dx, x, b \cos(c + dx) - a \sin(c + dx)\right)}{d} \\ &= -\frac{\text{Subst}\left(\int \left(a^4 \left(1 + \frac{2a^2b^2 + b^4}{a^4}\right) - 2a^2 \left(1 + \frac{b^2}{a^2}\right)x^2 + x^4\right) dx, x, b \cos(c + dx) - a \sin(c + dx)\right)}{d} \\ &= -\frac{(a^2 + b^2)^2 (b \cos(c + dx) - a \sin(c + dx))}{d} + \frac{2(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))^3}{3d} \end{aligned}$$

Mathematica [A] time = 0.46, size = 156, normalized size = 1.66

$$\frac{150a(a^2 + b^2)^2 \sin(c + dx) - 150b(a^2 + b^2)^2 \cos(c + dx) + 25a(a^4 - 2a^2b^2 - 3b^4) \sin(3(c + dx)) + 3a(a^4 - 10a^2b^2 - 5b^4) \cos(3(c + dx))}{240d}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]

[Out] $(-150*b*(a^2 + b^2)^2*\text{Cos}[c + d*x] + 25*b*(-3*a^4 - 2*a^2*b^2 + b^4)*\text{Cos}[3*(c + d*x)] - 3*b*(5*a^4 - 10*a^2*b^2 + b^4)*\text{Cos}[5*(c + d*x)] + 150*a*(a^2 + b^2)^2*\text{Sin}[c + d*x] + 25*a*(a^4 - 2*a^2*b^2 - 3*b^4)*\text{Sin}[3*(c + d*x)] + 3*a*(a^4 - 10*a^2*b^2 + 5*b^4)*\text{Sin}[5*(c + d*x)])/(240*d)$

fricas [A] time = 0.86, size = 155, normalized size = 1.65

$$\frac{15 b^5 \cos(dx + c) + 3(5 a^4 b - 10 a^2 b^3 + b^5) \cos(dx + c)^5 + 10(5 a^2 b^3 - b^5) \cos(dx + c)^3 - (8 a^5 + 20 a^3 b^2 + 15 a b^4) \cos(dx + c)}{15 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")`

[Out] $-1/15*(15*b^5*\text{cos}(d*x + c) + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*\text{cos}(d*x + c)^5 + 10*(5*a^2*b^3 - b^5)*\text{cos}(d*x + c)^3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4 + 3*(a^5 - 10*a^3*b^2 + 5*a*b^4)*\text{cos}(d*x + c)^4 + 2*(2*a^5 + 5*a^3*b^2 - 15*a*b^4)*\text{cos}(d*x + c)^2)*\text{sin}(d*x + c))/d$

giac [B] time = 0.29, size = 187, normalized size = 1.99

$$\frac{(5 a^4 b - 10 a^2 b^3 + b^5) \cos(5 dx + 5 c)}{80 d} - \frac{5(3 a^4 b + 2 a^2 b^3 - b^5) \cos(3 dx + 3 c)}{48 d} - \frac{5(a^4 b + 2 a^2 b^3 + b^5) \cos(dx + c)}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")`

[Out] $-1/80*(5*a^4*b - 10*a^2*b^3 + b^5)*\text{cos}(5*d*x + 5*c)/d - 5/48*(3*a^4*b + 2*a^2*b^3 - b^5)*\text{cos}(3*d*x + 3*c)/d - 5/8*(a^4*b + 2*a^2*b^3 + b^5)*\text{cos}(d*x + c)/d + 1/80*(a^5 - 10*a^3*b^2 + 5*a*b^4)*\text{sin}(5*d*x + 5*c)/d + 5/48*(a^5 - 2*a^3*b^2 - 3*a*b^4)*\text{sin}(3*d*x + 3*c)/d + 5/8*(a^5 + 2*a^3*b^2 + a*b^4)*\text{sin}(d*x + c)/d$

maple [A] time = 0.27, size = 175, normalized size = 1.86

$$\frac{b^5 \left(\frac{8}{3} + \sin^4(dx+c) + \frac{4(\sin^2(dx+c))}{3} \right) \cos(dx+c)}{5} + a b^4 (\sin^5(dx+c)) + 10 a^2 b^3 \left(-\frac{(\sin^2(dx+c))(\cos^3(dx+c))}{5} - \frac{2(\cos^3(dx+c))}{15} \right) + 10 a^3 b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*cos(d*x+c)+b*sin(d*x+c))^5,x)`

[Out] $1/d*(-1/5*b^5*(8/3+\text{sin}(d*x+c)^4+4/3*\text{sin}(d*x+c)^2)*\text{cos}(d*x+c)+a*b^4*\text{sin}(d*x+c)^5+10*a^2*b^3*(-1/5*\text{sin}(d*x+c)^2*\text{cos}(d*x+c)^3-2/15*\text{cos}(d*x+c)^3)+10*a^3*b^2*(-1/5*\text{sin}(d*x+c)*\text{cos}(d*x+c)^4+1/15*(2+\text{cos}(d*x+c)^2)*\text{sin}(d*x+c))-a^4*b*\text{cos}(d*x+c)^5+1/5*a^5*(8/3+\text{cos}(d*x+c)^4+4/3*\text{cos}(d*x+c)^2)*\text{sin}(d*x+c))$

maxima [A] time = 0.75, size = 172, normalized size = 1.83

$$-\frac{a^4 b \cos(dx + c)^5}{d} + \frac{a b^4 \sin(dx + c)^5}{d} + \frac{(3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c)) a^5}{15 d} - \frac{2(3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c)) b^5}{15 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")`

[Out] $-a^4*b*\text{cos}(d*x + c)^5/d + a*b^4*\text{sin}(d*x + c)^5/d + 1/15*(3*\text{sin}(d*x + c)^5 - 10*\text{sin}(d*x + c)^3 + 15*\text{sin}(d*x + c))*a^5/d - 2/3*(3*\text{sin}(d*x + c)^5 - 5*\text{sin}(d*x + c)^3)*a^3*b^2/d + 2/3*(3*\text{cos}(d*x + c)^5 - 5*\text{cos}(d*x + c)^3)*a^2*b^3/d - 1/15*(3*\text{cos}(d*x + c)^5 - 10*\text{cos}(d*x + c)^3 + 15*\text{cos}(d*x + c))*b^5/d$

mupad [B] time = 2.72, size = 248, normalized size = 2.64

$$2 \left(\frac{3 \sin(c+dx) a^5 \cos(c+dx)^4}{2} + 2 \sin(c+dx) a^5 \cos(c+dx)^2 + 4 \sin(c+dx) a^5 - \frac{15 a^4 b \cos(c+dx)^5}{2} - 15 \sin(c+dx) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5,x)

[Out] (2*(4*a^5*sin(c + d*x) - (15*b^5*cos(c + d*x))/2 + 5*b^5*cos(c + d*x)^3 - (3*b^5*cos(c + d*x)^5)/2 - (15*a^4*b*cos(c + d*x)^5)/2 + 2*a^5*cos(c + d*x)^2*sin(c + d*x) + (3*a^5*cos(c + d*x)^4*sin(c + d*x))/2 + 10*a^3*b^2*sin(c + d*x) - 25*a^2*b^3*cos(c + d*x)^3 + 15*a^2*b^3*cos(c + d*x)^5 + (15*a*b^4*sin(c + d*x))/2 + 5*a^3*b^2*cos(c + d*x)^2*sin(c + d*x) - 15*a^3*b^2*cos(c + d*x)^4*sin(c + d*x) - 15*a*b^4*cos(c + d*x)^2*sin(c + d*x) + (15*a*b^4*cos(c + d*x)^4*sin(c + d*x))/2))/(15*d)

sympy [A] time = 2.22, size = 267, normalized size = 2.84

$$\left\{ \begin{array}{l} \frac{8a^5 \sin^5(c+dx)}{15d} + \frac{4a^5 \sin^3(c+dx) \cos^2(c+dx)}{3d} + \frac{a^5 \sin(c+dx) \cos^4(c+dx)}{d} - \frac{a^4 b \cos^5(c+dx)}{d} + \frac{4a^3 b^2 \sin^5(c+dx)}{3d} + \frac{10a^3 b^2 \sin^3(c+dx) \cos^2(c+dx)}{3d} \\ x (a \cos(c) + b \sin(c))^5 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**5,x)

[Out] Piecewise((8*a**5*sin(c + d*x)**5/(15*d) + 4*a**5*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + a**5*sin(c + d*x)*cos(c + d*x)**4/d - a**4*b*cos(c + d*x)**5/d + 4*a**3*b**2*sin(c + d*x)**5/(3*d) + 10*a**3*b**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - 10*a**2*b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 4*a**2*b**3*cos(c + d*x)**5/(3*d) + a*b**4*sin(c + d*x)**5/d - b**5*sin(c + d*x)**4*cos(c + d*x)/d - 4*b**5*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 8*b**5*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5, True))

3.222 $\int (a \cos(c + dx) + b \sin(c + dx))^4 dx$

Optimal. Leaf size=108

$$\frac{3(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))}{8d} + \frac{3}{8}x(a^2 + b^2)^2 - \frac{(b \cos(c + dx) - a \sin(c + dx))^3}{4d}$$

[Out] 3/8*(a^2+b^2)^2*x-3/8*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))/d-1/4*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^3/d

Rubi [A] time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3073, 8}

$$\frac{3(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))}{8d} + \frac{3}{8}x(a^2 + b^2)^2 - \frac{(b \cos(c + dx) - a \sin(c + dx))^3}{4d}$$

Antiderivative was successfully verified.

[In] Int[(a*cos[c + d*x] + b*sin[c + d*x])^4,x]

[Out] (3*(a^2 + b^2)^2*x)/8 - (3*(a^2 + b^2)*(b*cos[c + d*x] - a*sin[c + d*x])*(a*cos[c + d*x] + b*sin[c + d*x]))/(8*d) - ((b*cos[c + d*x] - a*sin[c + d*x])*(a*cos[c + d*x] + b*sin[c + d*x])^3)/(4*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3073

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[((b*cos[c + d*x] - a*sin[c + d*x])*(a*cos[c + d*x] + b*sin[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*cos[c + d*x] + b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[(n - 1)/2] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int (a \cos(c + dx) + b \sin(c + dx))^4 dx &= -\frac{(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))^3}{4d} + \frac{1}{4} \left(3(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx)) \right. \\ &= -\frac{3(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))}{8d} \\ &= \frac{3}{8}(a^2 + b^2)^2 x - \frac{3(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))^3}{8d} \end{aligned}$$

Mathematica [A] time = 0.30, size = 107, normalized size = 0.99

$$\frac{8(a^4 - b^4) \sin(2(c + dx)) + 12(a^2 + b^2)^2 (c + dx) - 16ab(a^2 + b^2) \cos(2(c + dx)) - 4ab(a^2 - b^2) \cos(4(c + dx))}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[(a*cos[c + d*x] + b*sin[c + d*x])^4,x]

[Out] $(12*(a^2 + b^2)^2*(c + d*x) - 16*a*b*(a^2 + b^2)*\cos[2*(c + d*x)] - 4*a*b*(a^2 - b^2)*\cos[4*(c + d*x)] + 8*(a^4 - b^4)*\sin[2*(c + d*x)] + (a^4 - 6*a^2*b^2 + b^4)*\sin[4*(c + d*x)])/(32*d)$

fricas [A] time = 1.11, size = 121, normalized size = 1.12

$$\frac{16ab^3 \cos(dx + c)^2 + 8(a^3b - ab^3) \cos(dx + c)^4 - 3(a^4 + 2a^2b^2 + b^4)dx - (2(a^4 - 6a^2b^2 + b^4) \cos(dx + c))}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")

[Out] $-1/8*(16*a*b^3*\cos(d*x + c)^2 + 8*(a^3*b - a*b^3)*\cos(d*x + c)^4 - 3*(a^4 + 2*a^2*b^2 + b^4)*d*x - (2*(a^4 - 6*a^2*b^2 + b^4)*\cos(d*x + c)^3 + (3*a^4 + 6*a^2*b^2 - 5*b^4)*\cos(d*x + c))*\sin(d*x + c))/d$

giac [A] time = 0.19, size = 122, normalized size = 1.13

$$\frac{3}{8}(a^4 + 2a^2b^2 + b^4)x - \frac{(a^3b - ab^3) \cos(4dx + 4c)}{8d} - \frac{(a^3b + ab^3) \cos(2dx + 2c)}{2d} + \frac{(a^4 - 6a^2b^2 + b^4) \sin(4dx + 4c)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")

[Out] $3/8*(a^4 + 2*a^2*b^2 + b^4)*x - 1/8*(a^3*b - a*b^3)*\cos(4*d*x + 4*c)/d - 1/2*(a^3*b + a*b^3)*\cos(2*d*x + 2*c)/d + 1/32*(a^4 - 6*a^2*b^2 + b^4)*\sin(4*d*x + 4*c)/d + 1/4*(a^4 - b^4)*\sin(2*d*x + 2*c)/d$

maple [A] time = 0.25, size = 153, normalized size = 1.42

$$\frac{b^4 \left(-\frac{(\sin^3(dx+c) + \frac{3\sin(dx+c)}{2})\cos(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + ab^3 (\sin^4(dx+c)) + 6a^2b^2 \left(-\frac{\sin(dx+c)(\cos^3(dx+c))}{4} + \frac{\sin(dx+c)\cos(dx+c)}{8} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+b*sin(d*x+c))^4,x)

[Out] $1/d*(b^4*(-1/4*(\sin(d*x+c)^3+3/2*\sin(d*x+c))*\cos(d*x+c)+3/8*d*x+3/8*c)+a*b^3*\sin(d*x+c)^4+6*a^2*b^2*(-1/4*\sin(d*x+c)*\cos(d*x+c)^3+1/8*\sin(d*x+c)*\cos(d*x+c)+1/8*d*x+1/8*c)-\cos(d*x+c)^4*a^3*b+a^4*(1/4*(\cos(d*x+c)^3+3/2*\cos(d*x+c))*\sin(d*x+c)+3/8*d*x+3/8*c))$

maxima [A] time = 0.32, size = 136, normalized size = 1.26

$$\frac{a^3b \cos(dx + c)^4}{d} + \frac{ab^3 \sin(dx + c)^4}{d} + \frac{(12dx + 12c + \sin(4dx + 4c) + 8 \sin(2dx + 2c))a^4}{32d} + \frac{3(4dx + 4c - \sin(4dx + 4c) + 8 \sin(2dx + 2c))b^4}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")

[Out] $-a^3*b*\cos(d*x + c)^4/d + a*b^3*\sin(d*x + c)^4/d + 1/32*(12*d*x + 12*c + \sin(4*d*x + 4*c) + 8*\sin(2*d*x + 2*c))*a^4/d + 3/16*(4*d*x + 4*c - \sin(4*d*x + 4*c))*a^2*b^2/d + 1/32*(12*d*x + 12*c + \sin(4*d*x + 4*c) - 8*\sin(2*d*x + 2*c))*b^4/d$

mupad [B] time = 3.46, size = 320, normalized size = 2.96

$$\frac{3 \operatorname{atan} \left(\frac{3 \tan \left(\frac{c}{2} + \frac{dx}{2} \right) (a^2 + b^2)^2}{4 \left(\frac{3a^4}{4} + \frac{3a^2b^2}{2} + \frac{3b^4}{4} \right)} \right) (a^2 + b^2)^2}{4d} + \frac{\tan \left(\frac{c}{2} + \frac{dx}{2} \right)^7 \left(-\frac{5a^4}{4} + \frac{3a^2b^2}{2} + \frac{3b^4}{4} \right) - \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^3 \left(\frac{3a^4}{4} - \frac{21a^2b^2}{2} + \frac{3b^4}{4} \right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*cos(c + d*x) + b*sin(c + d*x))^4,x)`

[Out] $(3*\operatorname{atan}((3*\tan(c/2 + (d*x)/2)*(a^2 + b^2)^2)/(4*((3*a^4)/4 + (3*b^4)/4 + (3*a^2*b^2)/2)))*(a^2 + b^2)^2)/(4*d) + (\tan(c/2 + (d*x)/2)^7*((3*b^4)/4 - (5*a^4)/4 + (3*a^2*b^2)/2) - \tan(c/2 + (d*x)/2)^3*((3*a^4)/4 + (11*b^4)/4 - (21*a^2*b^2)/2) + \tan(c/2 + (d*x)/2)^5*((3*a^4)/4 + (11*b^4)/4 - (21*a^2*b^2)/2) - \tan(c/2 + (d*x)/2)*((3*b^4)/4 - (5*a^4)/4 + (3*a^2*b^2)/2) + 8*a^3*b*\tan(c/2 + (d*x)/2)^2 + 16*a*b^3*\tan(c/2 + (d*x)/2)^4 + 8*a^3*b*\tan(c/2 + (d*x)/2)^6)/(d*(4*\tan(c/2 + (d*x)/2)^2 + 6*\tan(c/2 + (d*x)/2)^4 + 4*\tan(c/2 + (d*x)/2)^6 + \tan(c/2 + (d*x)/2)^8 + 1)) - (3*(\operatorname{atan}(\tan(c/2 + (d*x)/2)) - (d*x)/2)*(a^2 + b^2)^2)/(4*d)$

sympy [A] time = 1.27, size = 381, normalized size = 3.53

$$\left\{ \begin{array}{l} \frac{3a^4x\sin^4(c+dx)}{8} + \frac{3a^4x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3a^4x\cos^4(c+dx)}{8} + \frac{3a^4\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a^4\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{a^3b\cos^4(c+dx)}{d} \\ x(a\cos(c) + b\sin(c))^4 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*cos(d*x+c)+b*sin(d*x+c))**4,x)`

[Out] `Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*cos(c + d*x)**2*cos(c + d*x)**2/4 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) - a**3*b*cos(c + d*x)**4/d + 3*a**2*b**2*x*sin(c + d*x)**4/4 + 3*a**2*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 3*a**2*b**2*x*cos(c + d*x)**4/4 + 3*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) - 3*a**2*b**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + a*b**3*sin(c + d*x)**4/d + 3*b**4*x*sin(c + d*x)**4/8 + 3*b**4*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b**4*x*cos(c + d*x)**4/8 - 5*b**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**4, True))`

3.223 $\int (a \cos(c + dx) + b \sin(c + dx))^3 dx$

Optimal. Leaf size=58

$$\frac{(b \cos(c + dx) - a \sin(c + dx))^3}{3d} - \frac{(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))}{d}$$

[Out] $-(a^2+b^2)*(b*\cos(d*x+c)-a*\sin(d*x+c))/d+1/3*(b*\cos(d*x+c)-a*\sin(d*x+c))^3/d$

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {3072}

$$\frac{(b \cos(c + dx) - a \sin(c + dx))^3}{3d} - \frac{(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]

[Out] $-(((a^2 + b^2)*(b*\cos[c + d*x] - a*\sin[c + d*x]))/d) + (b*\cos[c + d*x] - a*\sin[c + d*x])^3/(3*d)$

Rule 3072

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned} \int (a \cos(c + dx) + b \sin(c + dx))^3 dx &= -\frac{\text{Subst}\left(\int (a^2 + b^2 - x^2) dx, x, b \cos(c + dx) - a \sin(c + dx)\right)}{d} \\ &= -\frac{(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))}{d} + \frac{(b \cos(c + dx) - a \sin(c + dx))^3}{3d} \end{aligned}$$

Mathematica [A] time = 0.32, size = 81, normalized size = 1.40

$$\frac{(b^3 - 3a^2b) \cos(3(c + dx)) - 9b(a^2 + b^2) \cos(c + dx) + 2a \sin(c + dx) \left((a^2 - 3b^2) \cos(2(c + dx)) + 5a^2 + 3b^2 \right)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]

[Out] $(-9*b*(a^2 + b^2)*\cos[c + d*x] + (-3*a^2*b + b^3)*\cos[3*(c + d*x)] + 2*a*(5*a^2 + 3*b^2 + (a^2 - 3*b^2)*\cos[2*(c + d*x)])*\sin[c + d*x])/(12*d)$

fricas [A] time = 1.32, size = 77, normalized size = 1.33

$$\frac{3b^3 \cos(dx + c) + (3a^2b - b^3) \cos(dx + c)^3 - (2a^3 + 3ab^2 + (a^3 - 3ab^2) \cos(dx + c)^2) \sin(dx + c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out] $-1/3*(3*b^3*\cos(d*x + c) + (3*a^2*b - b^3)*\cos(d*x + c)^3 - (2*a^3 + 3*a*b^2 + (a^3 - 3*a*b^2)*\cos(d*x + c)^2)*\sin(d*x + c))/d$

giac [A] time = 0.15, size = 91, normalized size = 1.57

$$-\frac{(3a^2b - b^3)\cos(3dx + 3c)}{12d} - \frac{3(a^2b + b^3)\cos(dx + c)}{4d} + \frac{(a^3 - 3ab^2)\sin(3dx + 3c)}{12d} + \frac{3(a^3 + ab^2)\sin(dx + c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] $-1/12*(3*a^2*b - b^3)*\cos(3*d*x + 3*c)/d - 3/4*(a^2*b + b^3)*\cos(d*x + c)/d + 1/12*(a^3 - 3*a*b^2)*\sin(3*d*x + 3*c)/d + 3/4*(a^3 + a*b^2)*\sin(d*x + c)/d$

maple [A] time = 0.24, size = 75, normalized size = 1.29

$$\frac{-\frac{b^3(2+\sin^2(dx+c))\cos(dx+c)}{3} + ab^2(\sin^3(dx+c)) - a^2b(\cos^3(dx+c)) + \frac{a^3(2+\cos^2(dx+c))\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+b*sin(d*x+c))^3,x)

[Out] $1/d*(-1/3*b^3*(2+\sin(d*x+c)^2)*\cos(d*x+c)+a*b^2*\sin(d*x+c)^3-a^2*b*\cos(d*x+c)^3+1/3*a^3*(2+\cos(d*x+c)^2)*\sin(d*x+c))$

maxima [A] time = 0.31, size = 84, normalized size = 1.45

$$-\frac{a^2b\cos(dx+c)^3}{d} + \frac{ab^2\sin(dx+c)^3}{d} - \frac{(\sin(dx+c)^3 - 3\sin(dx+c))a^3}{3d} + \frac{(\cos(dx+c)^3 - 3\cos(dx+c))b^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")

[Out] $-a^2*b*\cos(d*x + c)^3/d + a*b^2*\sin(d*x + c)^3/d - 1/3*(\sin(d*x + c)^3 - 3*\sin(d*x + c))*a^3/d + 1/3*(\cos(d*x + c)^3 - 3*\cos(d*x + c))*b^3/d$

mupad [B] time = 2.48, size = 104, normalized size = 1.79

$$\frac{\frac{\sin(c+dx)a^3\cos(c+dx)^2}{3} + \frac{2\sin(c+dx)a^3}{3} - a^2b\cos(c+dx)^3 - \sin(c+dx)ab^2\cos(c+dx)^2 + \sin(c+dx)ab^2 + \frac{b^3\cos(c+dx)^2}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3,x)

[Out] $((2*a^3*\sin(c + d*x))/3 - b^3*\cos(c + d*x) + (b^3*\cos(c + d*x)^3)/3 - a^2*b*\cos(c + d*x)^3 + (a^3*\cos(c + d*x)^2*\sin(c + d*x))/3 + a*b^2*\sin(c + d*x) - a*b^2*\cos(c + d*x)^2*\sin(c + d*x))/d$

sympy [A] time = 0.52, size = 117, normalized size = 2.02

$$\begin{cases} \frac{2a^3\sin^3(c+dx)}{3d} + \frac{a^3\sin(c+dx)\cos^2(c+dx)}{d} - \frac{a^2b\cos^3(c+dx)}{d} + \frac{ab^2\sin^3(c+dx)}{d} - \frac{b^3\sin^2(c+dx)\cos(c+dx)}{d} - \frac{2b^3\cos^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a\cos(c) + b\sin(c))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2/d - a**2*b*cos(c + d*x)**3/d + a*b**2*sin(c + d*x)**3/d - b**3*sin(c + d*x)**2*cos(c + d*x)/d - 2*b**3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3, True))
```

3.224 $\int (a \cos(c + dx) + b \sin(c + dx))^2 dx$

Optimal. Leaf size=55

$$\frac{1}{2}x(a^2 + b^2) - \frac{(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))}{2d}$$

[Out] $1/2*(a^2+b^2)*x-1/2*(b*\cos(d*x+c)-a*\sin(d*x+c))*(a*\cos(d*x+c)+b*\sin(d*x+c))/d$

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3073, 8}

$$\frac{1}{2}x(a^2 + b^2) - \frac{(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]

[Out] $((a^2 + b^2)*x)/2 - ((b*\cos[c + d*x] - a*\sin[c + d*x])*(a*\cos[c + d*x] + b*\sin[c + d*x]))/(2*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3073

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[(n - 1)/2] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int (a \cos(c + dx) + b \sin(c + dx))^2 dx &= -\frac{(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))}{2d} + \frac{1}{2}(a^2 + b^2)x \\ &= \frac{1}{2}(a^2 + b^2)x - \frac{(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))}{2d} \end{aligned}$$

Mathematica [A] time = 0.10, size = 52, normalized size = 0.95

$$\frac{2(a^2 + b^2)(c + dx) + (a^2 - b^2)\sin(2(c + dx)) - 2ab \cos(2(c + dx))}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]

[Out] $(2*(a^2 + b^2)*(c + d*x) - 2*a*b*\cos[2*(c + d*x)] + (a^2 - b^2)*\sin[2*(c + d*x)])/(4*d)$

fricas [A] time = 1.10, size = 52, normalized size = 0.95

$$\frac{2ab \cos(dx + c)^2 - (a^2 + b^2)dx - (a^2 - b^2) \cos(dx + c) \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] $-1/2*(2*a*b*\cos(d*x + c)^2 - (a^2 + b^2)*d*x - (a^2 - b^2)*\cos(d*x + c)*\sin(d*x + c))/d$

giac [A] time = 0.15, size = 50, normalized size = 0.91

$$\frac{1}{2}(a^2 + b^2)x - \frac{ab \cos(2dx + 2c)}{2d} + \frac{(a^2 - b^2) \sin(2dx + 2c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] $1/2*(a^2 + b^2)*x - 1/2*a*b*\cos(2*d*x + 2*c)/d + 1/4*(a^2 - b^2)*\sin(2*d*x + 2*c)/d$

maple [A] time = 0.24, size = 70, normalized size = 1.27

$$\frac{b^2 \left(-\frac{\sin(dx+c)\cos(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) - (\cos^2(dx+c))ab + a^2 \left(\frac{\sin(dx+c)\cos(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+b*sin(d*x+c))^2,x)

[Out] $1/d*(b^2*(-1/2*\sin(d*x+c)*\cos(d*x+c)+1/2*d*x+1/2*c)-\cos(d*x+c)^2*a*b+a^2*(1/2*\sin(d*x+c)*\cos(d*x+c)+1/2*d*x+1/2*c))$

maxima [A] time = 0.47, size = 68, normalized size = 1.24

$$-\frac{ab \cos(dx+c)^2}{d} + \frac{(2dx+2c+\sin(2dx+2c))a^2}{4d} + \frac{(2dx+2c-\sin(2dx+2c))b^2}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] $-a*b*\cos(d*x + c)^2/d + 1/4*(2*d*x + 2*c + \sin(2*d*x + 2*c))*a^2/d + 1/4*(2*d*x + 2*c - \sin(2*d*x + 2*c))*b^2/d$

mupad [B] time = 2.42, size = 63, normalized size = 1.15

$$\frac{a^2 x}{2} + \frac{b^2 x}{2} + \frac{a^2 \sin(2c + 2dx)}{4d} - \frac{b^2 \sin(2c + 2dx)}{4d} - \frac{ab \cos(2c + 2dx)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2,x)

[Out] $(a^2*x)/2 + (b^2*x)/2 + (a^2*\sin(2*c + 2*d*x))/(4*d) - (b^2*\sin(2*c + 2*d*x))/(4*d) - (a*b*\cos(2*c + 2*d*x))/(2*d)$

sympy [A] time = 0.28, size = 128, normalized size = 2.33

$$\left\{ \begin{array}{l} \frac{a^2 x \sin^2(c+dx)}{2} + \frac{a^2 x \cos^2(c+dx)}{2} + \frac{a^2 \sin(c+dx) \cos(c+dx)}{2d} - \frac{ab \cos^2(c+dx)}{d} + \frac{b^2 x \sin^2(c+dx)}{2} + \frac{b^2 x \cos^2(c+dx)}{2} - \frac{b^2 \sin(c+dx) \cos(c+dx)}{2d} \\ x(a \cos(c) + b \sin(c))^2 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(c + d*x)*cos(c + d*x)/(2*d) - a*b*cos(c + d*x)**2/d + b**2*x*sin(c + d*x)**2/2 + b**2*x*cos(c + d*x)**2/2 - b**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**2, True))
```

3.225 $\int (a \cos(c + dx) + b \sin(c + dx)) dx$

Optimal. Leaf size=24

$$\frac{a \sin(c + dx)}{d} - \frac{b \cos(c + dx)}{d}$$

[Out] $-b \cos(dx+c)/d + a \sin(dx+c)/d$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2637, 2638}

$$\frac{a \sin(c + dx)}{d} - \frac{b \cos(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] `Int[a*Cos[c + d*x] + b*Sin[c + d*x],x]`

[Out] $-(b \cos[c + d*x])/d + (a \sin[c + d*x])/d$

Rule 2637

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 2638

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \int (a \cos(c + dx) + b \sin(c + dx)) dx &= a \int \cos(c + dx) dx + b \int \sin(c + dx) dx \\ &= -\frac{b \cos(c + dx)}{d} + \frac{a \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.92

$$\frac{a \sin(c) \cos(dx)}{d} + \frac{a \cos(c) \sin(dx)}{d} + \frac{b \sin(c) \sin(dx)}{d} - \frac{b \cos(c) \cos(dx)}{d}$$

Antiderivative was successfully verified.

[In] `Integrate[a*Cos[c + d*x] + b*Sin[c + d*x],x]`

[Out] $-(b \cos[c] \cos[d*x])/d + (a \cos[d*x] \sin[c])/d + (a \cos[c] \sin[d*x])/d + (b \sin[c] \sin[d*x])/d$

fricas [A] time = 1.27, size = 23, normalized size = 0.96

$$-\frac{b \cos(dx + c) - a \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="fricas")`

[Out] $-(b \cos(dx + c) - a \sin(dx + c))/d$

giac [A] time = 0.14, size = 24, normalized size = 1.00

$$-\frac{b \cos(dx + c)}{d} + \frac{a \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="giac")

[Out] -b*cos(d*x + c)/d + a*sin(d*x + c)/d

maple [A] time = 0.04, size = 25, normalized size = 1.04

$$-\frac{b \cos(dx + c)}{d} + \frac{a \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a*cos(d*x+c)+b*sin(d*x+c),x)

[Out] -b*cos(d*x+c)/d+a*sin(d*x+c)/d

maxima [A] time = 0.40, size = 24, normalized size = 1.00

$$-\frac{b \cos(dx + c)}{d} + \frac{a \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="maxima")

[Out] -b*cos(d*x + c)/d + a*sin(d*x + c)/d

mupad [B] time = 2.32, size = 38, normalized size = 1.58

$$-\frac{2 \cos\left(\frac{c}{2} + \frac{dx}{2}\right) \left(b \cos\left(\frac{c}{2} + \frac{dx}{2}\right) - a \sin\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a*cos(c + d*x) + b*sin(c + d*x),x)

[Out] -(2*cos(c/2 + (d*x)/2)*(b*cos(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2)))/d

sympy [A] time = 0.15, size = 31, normalized size = 1.29

$$a \left(\begin{cases} \frac{\sin(c+dx)}{d} & \text{for } d \neq 0 \\ x \cos(c) & \text{otherwise} \end{cases} \right) + b \left(\begin{cases} -\frac{\cos(c+dx)}{d} & \text{for } d \neq 0 \\ x \sin(c) & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x)

[Out] a*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True)) + b*Piecewise((-cos(c + d*x)/d, Ne(d, 0)), (x*sin(c), True))

$$3.226 \quad \int \frac{1}{a \cos(c+dx)+b \sin(c+dx)} dx$$

Optimal. Leaf size=47

$$\frac{\tanh^{-1}\left(\frac{b \cos(c+dx)-a \sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

[Out] $-\operatorname{arctanh}((b \cos(d*x+c)-a \sin(d*x+c))/(a^2+b^2)^{(1/2)})/d/(a^2+b^2)^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3074, 206}

$$\frac{\tanh^{-1}\left(\frac{b \cos(c+dx)-a \sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a \cos[c + d*x] + b \sin[c + d*x])^{-1}, x]$

[Out] $-(\operatorname{ArcTanh}[(b \cos[c + d*x] - a \sin[c + d*x])/\operatorname{Sqrt}[a^2 + b^2]])/(\operatorname{Sqrt}[a^2 + b^2]*d)$

Rule 206

$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1 \cdot \operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 3074

$\operatorname{Int}[(\cos[(c + d \cdot x)] + (d \cdot x) \sin[(c + d \cdot x)])^{-1}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{-1}, \operatorname{Subst}[\operatorname{Int}[1/(a^2 + b^2 - x^2), x], x, b \cos[c + d \cdot x] - a \sin[c + d \cdot x]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{NeQ}[a^2 + b^2, 0]$

Rubi steps

$$\int \frac{1}{a \cos(c+dx)+b \sin(c+dx)} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{a^2+b^2-x^2} dx, x, b \cos(c+dx) - a \sin(c+dx)\right)}{d}$$

$$= \frac{\tanh^{-1}\left(\frac{b \cos(c+dx)-a \sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2} d}$$

Mathematica [A] time = 0.06, size = 45, normalized size = 0.96

$$\frac{2 \tanh^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Integrate}[(a \cos[c + d*x] + b \sin[c + d*x])^{-1}, x]$

[Out] $(2 \operatorname{ArcTanh}[(-b + a \operatorname{Tan}[(c + d*x)/2])/\operatorname{Sqrt}[a^2 + b^2]])/(\operatorname{Sqrt}[a^2 + b^2]*d)$

fricas [B] time = 1.74, size = 131, normalized size = 2.79

$$\frac{\log\left(\frac{-2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2a^2-b^2+2\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)}{2\sqrt{a^2+b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))/(sqrt(a^2 + b^2)*d)

giac [A] time = 0.22, size = 74, normalized size = 1.57

$$\frac{\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")

[Out] -log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)

maple [A] time = 0.40, size = 43, normalized size = 0.91

$$\frac{2\operatorname{arctanh}\left(\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c)),x)

[Out] 2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tan(1/2*d*x+1/2*c)*a-2*b)/(a^2+b^2)^(1/2))

maxima [A] time = 0.68, size = 80, normalized size = 1.70

$$\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")

[Out] -log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)

mupad [B] time = 2.80, size = 39, normalized size = 0.83

$$\frac{2\operatorname{atanh}\left(\frac{b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] -(2*atanh((b - a*tan(c/2 + (d*x)/2))/(a^2 + b^2)^(1/2)))/(d*(a^2 + b^2)^(1/2))
```

```
sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

$$3.227 \quad \int \frac{1}{(a \cos(c+dx)+b \sin(c+dx))^2} dx$$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)}{ad(a \cos(c+dx)+b \sin(c+dx))}$$

[Out] sin(d*x+c)/a/d/(a*cos(d*x+c)+b*sin(d*x+c))

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {3075}

$$\frac{\sin(c+dx)}{ad(a \cos(c+dx)+b \sin(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]

[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))

Rule 3075

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{1}{(a \cos(c+dx)+b \sin(c+dx))^2} dx = \frac{\sin(c+dx)}{ad(a \cos(c+dx)+b \sin(c+dx))}$$

Mathematica [A] time = 0.04, size = 32, normalized size = 1.00

$$\frac{\sin(c+dx)}{ad(a \cos(c+dx)+b \sin(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]

[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))

fricas [A] time = 0.59, size = 57, normalized size = 1.78

$$-\frac{b \cos(dx+c) - a \sin(dx+c)}{(a^3 + ab^2)d \cos(dx+c) + (a^2b + b^3)d \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/((a^3 + a*b^2)*d*cos(d*x + c) + (a^2*b + b^3)*d*sin(d*x + c))

giac [A] time = 0.15, size = 20, normalized size = 0.62

$$-\frac{1}{(b \tan(dx+c) + a)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/((b*tan(d*x + c) + a)*b*d)

maple [A] time = 0.49, size = 21, normalized size = 0.66

$$-\frac{1}{db(a + b \tan(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)

[Out] -1/d/b/(a+b*tan(d*x+c))

maxima [A] time = 0.52, size = 21, normalized size = 0.66

$$-\frac{1}{(b^2 \tan(dx + c) + ab)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/((b^2*tan(d*x + c) + a*b)*d)

mupad [B] time = 2.34, size = 47, normalized size = 1.47

$$\frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{ad \left(-a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 2b \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)

[Out] (2*tan(c/2 + (d*x)/2))/(a*d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cos(c + dx) + b \sin(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)

[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**(-2), x)

$$3.228 \quad \int \frac{1}{(a \cos(c+dx)+b \sin(c+dx))^3} dx$$

Optimal. Leaf size=103

$$\frac{b \cos(c+dx) - a \sin(c+dx)}{2d(a^2 + b^2)(a \cos(c+dx) + b \sin(c+dx))^2} - \frac{\tanh^{-1}\left(\frac{b \cos(c+dx) - a \sin(c+dx)}{\sqrt{a^2 + b^2}}\right)}{2d(a^2 + b^2)^{3/2}}$$

[Out] $-1/2*\operatorname{arctanh}((b*\cos(d*x+c)-a*\sin(d*x+c))/(a^2+b^2)^{(1/2)))/(a^2+b^2)^{(3/2)}/d$
 $+1/2*(-b*\cos(d*x+c)+a*\sin(d*x+c))/(a^2+b^2)/d/(a*\cos(d*x+c)+b*\sin(d*x+c))^2$

Rubi [A] time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3076, 3074, 206}

$$\frac{b \cos(c+dx) - a \sin(c+dx)}{2d(a^2 + b^2)(a \cos(c+dx) + b \sin(c+dx))^2} - \frac{\tanh^{-1}\left(\frac{b \cos(c+dx) - a \sin(c+dx)}{\sqrt{a^2 + b^2}}\right)}{2d(a^2 + b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3), x]

[Out] $-\operatorname{ArcTanh}[(b*\cos[c + d*x] - a*\sin[c + d*x])/ \operatorname{Sqrt}[a^2 + b^2]] / (2*(a^2 + b^2)^{(3/2)*d}) - (b*\cos[c + d*x] - a*\sin[c + d*x]) / (2*(a^2 + b^2)*d*(a*\cos[c + d*x] + b*\sin[c + d*x])^2$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3074

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 3076

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a \cos(c+dx) + b \sin(c+dx))^3} dx &= -\frac{b \cos(c+dx) - a \sin(c+dx)}{2(a^2 + b^2) d(a \cos(c+dx) + b \sin(c+dx))^2} + \frac{\int \frac{1}{a \cos(c+dx) + b \sin(c+dx)} dx}{2(a^2 + b^2)} \\ &= -\frac{b \cos(c+dx) - a \sin(c+dx)}{2(a^2 + b^2) d(a \cos(c+dx) + b \sin(c+dx))^2} - \frac{\text{Subst}\left(\int \frac{1}{a^2 + b^2 - x^2} dx\right)}{2(a^2 + b^2)} \\ &= -\frac{\tanh^{-1}\left(\frac{b \cos(c+dx) - a \sin(c+dx)}{\sqrt{a^2 + b^2}}\right)}{2(a^2 + b^2)^{3/2} d} - \frac{b \cos(c+dx) - a \sin(c+dx)}{2(a^2 + b^2) d(a \cos(c+dx) + b \sin(c+dx))} \end{aligned}$$

Mathematica [C] time = 0.29, size = 132, normalized size = 1.28

$$\frac{(a^2 + b^2)(a \sin(c+dx) - b \cos(c+dx)) + 2\sqrt{a^2 + b^2}(a \cos(c+dx) + b \sin(c+dx))^2 \tanh^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) - b}{\sqrt{a^2 + b^2}}\right)}{2d(a - ib)^2(a + ib)^2(a \cos(c+dx) + b \sin(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]

[Out] ((a^2 + b^2)*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]) + 2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(2*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)

fricas [B] time = 1.38, size = 294, normalized size = 2.85

$$\frac{(2ab \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 + b^2) \sqrt{a^2 + b^2} \log\left(-\frac{2ab \cos(dx+c) \sin(dx+c) + (a^2 - b^2) \cos(dx+c)^2 + b^2}{2ab \cos(dx+c) \sin(dx+c)}\right)}{4\left((a^6 + a^4b^2 - a^2b^4 - b^6)d \cos(dx+c)^2 + 2(a^5b + 2a^3b^3 + ab^5)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/4*((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 2*(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)*d)

giac [B] time = 0.23, size = 221, normalized size = 2.15

$$\frac{\log\left(\frac{2a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 2b - 2\sqrt{a^2 + b^2}}{2a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 2b + 2\sqrt{a^2 + b^2}}\right)}{(a^2 + b^2)^{\frac{3}{2}}} - \frac{2\left(a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 2ab^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + a^2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 2b^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{(a^4 + a^2b^2)\left(a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 2b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - a\right)^2} - \frac{2d}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/2*(log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2)))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^3*

$$\tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*\tan(1/2*d*x + 1/2*c)^2 + a^2*b*\tan(1/2*d*x + 1/2*c) - 2*b^3*\tan(1/2*d*x + 1/2*c)^2 + a^3*\tan(1/2*d*x + 1/2*c) - 2*a*b^2*\tan(1/2*d*x + 1/2*c) - a^2*b)/((a^4 + a^2*b^2)*(a*\tan(1/2*d*x + 1/2*c)^2 - 2*b*\tan(1/2*d*x + 1/2*c) - a)^2)/d$$

maple [A] time = 0.53, size = 191, normalized size = 1.85

$$\frac{2 \left(\frac{(a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^2+b^2)a} - \frac{b(a^2-2b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^2+b^2)a^2} - \frac{(a^2-2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a(a^2+b^2)} + \frac{b}{2a^2+2b^2} \right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2} + \frac{\operatorname{arctanh}\left(\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a-2b}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)

[Out] 1/d*(-2*(-1/2*(a^2+2*b^2)/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(a^2-2*b^2)/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*(a^2-2*b^2)/a/(a^2+b^2)*tan(1/2*d*x+1/2*c)+1/2*b/(a^2+b^2))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*tan(1/2*d*x+1/2*c)*a-2*b)/(a^2+b^2)^(1/2)))

maxima [B] time = 0.46, size = 326, normalized size = 3.17

$$\frac{2 \left(a^2 b - \frac{(a^3-2ab^2)\sin(dx+c)}{\cos(dx+c)+1} - \frac{(a^2b-2b^3)\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{(a^3+2ab^2)\sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{a^6+a^4b^2+\frac{4(a^5b+a^3b^3)\sin(dx+c)}{\cos(dx+c)+1} - \frac{2(a^6-a^4b^2-2a^2b^4)\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{4(a^5b+a^3b^3)\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{(a^6+a^4b^2)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -1/2*(2*(a^2*b - (a^3 - 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) - (a^2*b - 2*b^3)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (a^3 + 2*a*b^2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^6 + a^4*b^2 + 4*(a^5*b + a^3*b^3)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(a^6 - a^4*b^2 - 2*a^2*b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(a^5*b + a^3*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (a^6 + a^4*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2))/d

mupad [B] time = 4.55, size = 260, normalized size = 2.52

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2-2b^2)}{a(a^2+b^2)} - \frac{b}{a^2+b^2} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a^2+2b^2)}{a(a^2+b^2)} + \frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a^2-2b^2)}{a^2(a^2+b^2)}}{d \left(a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 (2a^2 - 4b^2) + a^2 - 4ab \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + 4ab \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \right)} + \frac{\operatorname{atanh}\left(\frac{2a \tan\left(\frac{c}{2} + \frac{dx}{2}\right) - b}{a^2 + b^2}\right)}{d(a^2 + b^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)

[Out] ((tan(c/2 + (d*x)/2)*(a^2 - 2*b^2))/(a*(a^2 + b^2)) - b/(a^2 + b^2) + (tan(c/2 + (d*x)/2)^3*(a^2 + 2*b^2))/(a*(a^2 + b^2)) + (b*tan(c/2 + (d*x)/2)^2*(a^2 - 2*b^2))/(a^2*(a^2 + b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2)) + atanh(((2*a*tan(c/2 + (d*x)/2) - (2*a^2*b + 2*b^3))/(a^2 + b^2))*(a^2/2 + b^2/2))/(a^2 + b^2)^(3/2))/(d*(a^2 + b^2)^(3/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)

[Out] Timed out

$$3.229 \quad \int \frac{1}{(a \cos(c+dx)+b \sin(c+dx))^4} dx$$

Optimal. Leaf size=98

$$\frac{2 \sin(c+dx)}{3ad(a^2+b^2)(a \cos(c+dx)+b \sin(c+dx))} - \frac{b \cos(c+dx) - a \sin(c+dx)}{3d(a^2+b^2)(a \cos(c+dx)+b \sin(c+dx))^3}$$

[Out] 1/3*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^3+2/3*sin(d*x+c)/a/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3076, 3075}

$$\frac{2 \sin(c+dx)}{3ad(a^2+b^2)(a \cos(c+dx)+b \sin(c+dx))} - \frac{b \cos(c+dx) - a \sin(c+dx)}{3d(a^2+b^2)(a \cos(c+dx)+b \sin(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Int[(a*cos[c + d*x] + b*sin[c + d*x])^(-4), x]

[Out] -(b*cos[c + d*x] - a*sin[c + d*x])/(3*(a^2 + b^2)*d*(a*cos[c + d*x] + b*sin[c + d*x])^3) + (2*sin[c + d*x])/(3*a*(a^2 + b^2)*d*(a*cos[c + d*x] + b*sin[c + d*x]))

Rule 3075

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(-2), x_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*cos[c + d*x] + b*sin[c + d*x])), x] / ; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 3076

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] :> Simp[((b*cos[c + d*x] - a*sin[c + d*x])*(a*cos[c + d*x] + b*sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^2 + b^2)), Int[(a*cos[c + d*x] + b*sin[c + d*x])^(n + 2), x], x] / ; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a \cos(c+dx)+b \sin(c+dx))^4} dx &= -\frac{b \cos(c+dx) - a \sin(c+dx)}{3(a^2+b^2)d(a \cos(c+dx)+b \sin(c+dx))^3} + \frac{2 \int \frac{1}{(a \cos(c+dx)+b \sin(c+dx))}}{3(a^2+b^2)} \\ &= -\frac{b \cos(c+dx) - a \sin(c+dx)}{3(a^2+b^2)d(a \cos(c+dx)+b \sin(c+dx))^3} + \frac{2 \sin(c+dx)}{3a(a^2+b^2)d(a \cos(c+dx)+b \sin(c+dx))^3} \end{aligned}$$

Mathematica [A] time = 0.29, size = 85, normalized size = 0.87

$$\frac{\sin(c+dx) \left((a^2 - b^2) \cos(2(c+dx)) + 2a^2 + b^2 \right) - ab \cos(3(c+dx))}{3ad(a^2+b^2)(a \cos(c+dx)+b \sin(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*cos[c + d*x] + b*sin[c + d*x])^(-4), x]

[Out] $(-(a*b*\cos[3*(c + d*x)]) + (2*a^2 + b^2 + (a^2 - b^2)*\cos[2*(c + d*x)])*\sin[c + d*x]) / (3*a*(a^2 + b^2)*d*(a*\cos[c + d*x] + b*\sin[c + d*x])^3)$

fricas [B] time = 0.67, size = 217, normalized size = 2.21

$$\frac{2(3a^2b - b^3)\cos(dx + c)^3 - 3(a^2b - b^3)\cos(dx + c) - (a^3 + 3ab^2 + 2(a^3 - 3ab^2))\sin(dx + c)}{3((a^7 - a^5b^2 - 5a^3b^4 - 3ab^6)d\cos(dx + c)^3 + 3(a^5b^2 + 2a^3b^4 + ab^6)d\cos(dx + c) + ((3a^6b + 5a^4b^3 + a^2b^5) + (3a^4b^3 + 5a^2b^5 + b^7)d)\sin(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")`

[Out] $-1/3*(2*(3*a^2*b - b^3)*\cos(d*x + c)^3 - 3*(a^2*b - b^3)*\cos(d*x + c) - (a^3 + 3*a*b^2 + 2*(a^3 - 3*a*b^2)*\cos(d*x + c)^2)*\sin(d*x + c)) / ((a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*d*\cos(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d*\cos(d*x + c) + ((3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*d*\cos(d*x + c)^2 + (3*a^4*b^3 + 5*a^2*b^5 + b^7)*d)*\sin(d*x + c))$

giac [A] time = 0.15, size = 50, normalized size = 0.51

$$\frac{3b^2 \tan(dx + c)^2 + 3ab \tan(dx + c) + a^2 + b^2}{3(b \tan(dx + c) + a)^3 b^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")`

[Out] $-1/3*(3*b^2*\tan(d*x + c)^2 + 3*a*b*\tan(d*x + c) + a^2 + b^2) / ((b*\tan(d*x + c) + a)^3*b^3*d)$

maple [A] time = 0.56, size = 64, normalized size = 0.65

$$\frac{\frac{a}{b^3(a+b \tan(dx+c))^2} - \frac{a^2+b^2}{3b^3(a+b \tan(dx+c))^3} - \frac{1}{b^3(a+b \tan(dx+c))}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)`

[Out] $1/d*(a/b^3/(a+b*\tan(d*x+c))^2 - 1/3*(a^2+b^2)/b^3/(a+b*\tan(d*x+c))^3 - 1/b^3/(a+b*\tan(d*x+c)))$

maxima [A] time = 0.34, size = 85, normalized size = 0.87

$$\frac{3b^2 \tan(dx + c)^2 + 3ab \tan(dx + c) + a^2 + b^2}{3(b^6 \tan(dx + c)^3 + 3ab^5 \tan(dx + c)^2 + 3a^2b^4 \tan(dx + c) + a^3b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")`

[Out] $-1/3*(3*b^2*\tan(d*x + c)^2 + 3*a*b*\tan(d*x + c) + a^2 + b^2) / ((b^6*\tan(d*x + c)^3 + 3*a*b^5*\tan(d*x + c)^2 + 3*a^2*b^4*\tan(d*x + c) + a^3*b^3)*d)$

mupad [B] time = 3.12, size = 222, normalized size = 2.27

$$\frac{\frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{a} + \frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{a} - \frac{4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (a^2 - 2b^2)}{3a^3} + \frac{4b \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{a^2}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 (12ab^2 - 3a^3) - a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 (12ab^2 - 3a^3) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (12a^2b^2 - 3a^3) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^5)/a + (2*tan(c/2 + (d*x)/2))/a - (4*tan(c/2 + (d*x)/2)^3*(a^2 - 2*b^2))/(3*a^3) + (4*b*tan(c/2 + (d*x)/2)^2)/a^2 - (4*b*tan(c/2 + (d*x)/2)^4)/a^2)/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*tan(c/2 + (d*x)/2)^5))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

$$3.230 \quad \int \frac{1}{(a \cos(c+dx)+b \sin(c+dx))^5} dx$$

Optimal. Leaf size=156

$$\frac{3(b \cos(c+dx) - a \sin(c+dx))}{8d(a^2 + b^2)^2 (a \cos(c+dx) + b \sin(c+dx))^2} - \frac{b \cos(c+dx) - a \sin(c+dx)}{4d(a^2 + b^2)(a \cos(c+dx) + b \sin(c+dx))^4} - \frac{3 \tanh^{-1}\left(\frac{b \cos(c+dx) - a \sin(c+dx)}{a \cos(c+dx) + b \sin(c+dx)}\right)}{8d(a^2 + b^2)}$$

[Out] $-3/8*\operatorname{arctanh}((b*\cos(d*x+c)-a*\sin(d*x+c))/(a^2+b^2)^{(1/2)})/(a^2+b^2)^{(5/2)}/d$
 $+1/4*(-b*\cos(d*x+c)+a*\sin(d*x+c))/(a^2+b^2)/d/(a*\cos(d*x+c)+b*\sin(d*x+c))^4$
 $-3/8*(b*\cos(d*x+c)-a*\sin(d*x+c))/(a^2+b^2)^2/d/(a*\cos(d*x+c)+b*\sin(d*x+c))^2$

Rubi [A] time = 0.09, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3076, 3074, 206}

$$\frac{3(b \cos(c+dx) - a \sin(c+dx))}{8d(a^2 + b^2)^2 (a \cos(c+dx) + b \sin(c+dx))^2} - \frac{b \cos(c+dx) - a \sin(c+dx)}{4d(a^2 + b^2)(a \cos(c+dx) + b \sin(c+dx))^4} - \frac{3 \tanh^{-1}\left(\frac{b \cos(c+dx) - a \sin(c+dx)}{a \cos(c+dx) + b \sin(c+dx)}\right)}{8d(a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a*\operatorname{Cos}[c + d*x] + b*\operatorname{Sin}[c + d*x])^{(-5)}, x]$

[Out] $(-3*\operatorname{ArcTanh}[(b*\operatorname{Cos}[c + d*x] - a*\operatorname{Sin}[c + d*x])/ \operatorname{Sqrt}[a^2 + b^2]])/(8*(a^2 + b^2)^{(5/2)*d} - (b*\operatorname{Cos}[c + d*x] - a*\operatorname{Sin}[c + d*x])/(4*(a^2 + b^2)*d*(a*\operatorname{Cos}[c + d*x] + b*\operatorname{Sin}[c + d*x])^4) - (3*(b*\operatorname{Cos}[c + d*x] - a*\operatorname{Sin}[c + d*x]))/(8*(a^2 + b^2)^2*d*(a*\operatorname{Cos}[c + d*x] + b*\operatorname{Sin}[c + d*x])^2)$

Rule 206

$\operatorname{Int}[(a_.) + (b_.)*(x_)^{(-1)}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 3074

$\operatorname{Int}[(\operatorname{cos}[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*\operatorname{sin}[(c_.) + (d_.)*(x_)])^{(-1)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[1/(a^2 + b^2 - x^2), x], x, b*\operatorname{Cos}[c + d*x] - a*\operatorname{Sin}[c + d*x]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{NeQ}[a^2 + b^2, 0]$

Rule 3076

$\operatorname{Int}[(\operatorname{cos}[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*\operatorname{sin}[(c_.) + (d_.)*(x_)])^{(n_)}, x_Symbol] \rightarrow \operatorname{Simp}[(b*\operatorname{Cos}[c + d*x] - a*\operatorname{Sin}[c + d*x])*(a*\operatorname{Cos}[c + d*x] + b*\operatorname{Sin}[c + d*x])^{(n+1)})/(d*(n+1)*(a^2 + b^2)), x] + \operatorname{Dist}[(n+2)/((n+1)*(a^2 + b^2)), \operatorname{Int}[(a*\operatorname{Cos}[c + d*x] + b*\operatorname{Sin}[c + d*x])^{(n+2)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{NeQ}[a^2 + b^2, 0] \ \&\& \operatorname{LtQ}[n, -1] \ \&\& \operatorname{NeQ}[n, -2]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a \cos(c+dx) + b \sin(c+dx))^5} dx &= -\frac{b \cos(c+dx) - a \sin(c+dx)}{4(a^2 + b^2)d(a \cos(c+dx) + b \sin(c+dx))^4} + \frac{3 \int \frac{1}{(a \cos(c+dx) + b \sin(c+dx))}}{4(a^2 + b^2)} \\
&= -\frac{b \cos(c+dx) - a \sin(c+dx)}{4(a^2 + b^2)d(a \cos(c+dx) + b \sin(c+dx))^4} - \frac{3(b \cos(c+dx) - a \sin(c+dx))}{8(a^2 + b^2)^2 d(a \cos(c+dx) + b \sin(c+dx))^3} \\
&= -\frac{b \cos(c+dx) - a \sin(c+dx)}{4(a^2 + b^2)d(a \cos(c+dx) + b \sin(c+dx))^4} - \frac{3(b \cos(c+dx) - a \sin(c+dx))}{8(a^2 + b^2)^2 d(a \cos(c+dx) + b \sin(c+dx))^3} \\
&= -\frac{3 \tanh^{-1}\left(\frac{b \cos(c+dx) - a \sin(c+dx)}{\sqrt{a^2 + b^2}}\right)}{8(a^2 + b^2)^{5/2} d} - \frac{b \cos(c+dx) - a \sin(c+dx)}{4(a^2 + b^2)d(a \cos(c+dx) + b \sin(c+dx))^4}
\end{aligned}$$

Mathematica [A] time = 1.15, size = 157, normalized size = 1.01

$$\frac{6 \tanh^{-1}\left(\frac{a \tan\left(\frac{1}{2}(c+dx)\right) - b}{\sqrt{a^2 + b^2}}\right)}{(a^2 + b^2)^{5/2}} + \frac{(3b^3 - 9a^2b) \cos(3(c+dx)) - 11b(a^2 + b^2) \cos(c+dx) + 2a \sin(c+dx)(3(a^2 - 3b^2) \cos(2(c+dx)) + 7a^2 + b^2)}{4(a^2 + b^2)^2 (a \cos(c+dx) + b \sin(c+dx))^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5), x]

[Out] ((6*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2])^(5/2) + (-11*b*(a^2 + b^2)*Cos[c + d*x] + (-9*a^2*b + 3*b^3)*Cos[3*(c + d*x)] + 2*a*(7*a^2 + b^2 + 3*(a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]/(4*(a^2 + b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)/(8*d)

fricas [B] time = 1.13, size = 544, normalized size = 3.49

$$\frac{6(3a^4b + 2a^2b^3 - b^5) \cos(dx + c)^3 - 3((a^4 - 6a^2b^2 + b^4) \cos(dx + c)^4 + b^4 + 2(3a^2b^2 - b^4) \cos(dx + c)^2 + 4b^4)}{16((a^{10} - 3a^8b^2 - 14a^6b^4 - 14a^4b^6 - 3a^2b^8 + b^{10})d \cos(dx + c) + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")

[Out] -1/16*(6*(3*a^4*b + 2*a^2*b^3 - b^5)*cos(d*x + c)^3 - 3*((a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^4 + b^4 + 2*(3*a^2*b^2 - b^4)*cos(d*x + c)^2 + 4*(a*b^3*cos(d*x + c) + (a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 2*(4*a^4*b - a^2*b^3 - 5*b^5)*cos(d*x + c) - 2*(2*a^5 + 7*a^3*b^2 + 5*a*b^4 + 3*(a^5 - 2*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^10 - 3*a^8*b^2 - 14*a^6*b^4 - 14*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)^4 + 2*(3*a^8*b^2 + 8*a^6*b^4 + 6*a^4*b^6 - b^10)*d*cos(d*x + c)^2 + (a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10)*d + 4*((a^9*b + 2*a^7*b^3 - 2*a^3*b^7 - a*b^9)*d*cos(d*x + c)^3 + (a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*d*cos(d*x + c))*sin(d*x + c))

giac [B] time = 0.31, size = 588, normalized size = 3.77

$$\frac{3 \log\left(\frac{\left|2a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2b - 2\sqrt{a^2 + b^2}\right|}{\left|2a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2b + 2\sqrt{a^2 + b^2}\right|}\right)}{(a^4 + 2a^2b^2 + b^4)\sqrt{a^2 + b^2}} - \frac{2\left(5a^7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 16a^5b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 8a^3b^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 3a^6b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 - 48a^4b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 - 24a^2b^5 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 3a^7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 36a^5b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 56a^3b^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 32ab^6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 15a^6b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 114a^4b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 8a^2b^5 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 16b^7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 3a^7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 84a^5b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 56a^3b^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 32ab^6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 23a^6b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 64a^4b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 24a^2b^5 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 5a^7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 24a^5b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 8a^3b^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 5a^6b - 2a^4b^3\right)}{(a^8 + 2a^6b^2 + a^4b^4)(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 2b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - a)^4)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")

[Out] -1/8*(3*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(5*a^7*tan(1/2*d*x + 1/2*c)^7 + 16*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 8*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 3*a^6*b*tan(1/2*d*x + 1/2*c)^6 - 48*a^4*b^3*tan(1/2*d*x + 1/2*c)^6 - 24*a^2*b^5*tan(1/2*d*x + 1/2*c)^6 + 3*a^7*tan(1/2*d*x + 1/2*c)^5 - 36*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 56*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 32*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 15*a^6*b*tan(1/2*d*x + 1/2*c)^4 + 114*a^4*b^3*tan(1/2*d*x + 1/2*c)^4 + 8*a^2*b^5*tan(1/2*d*x + 1/2*c)^4 - 16*b^7*tan(1/2*d*x + 1/2*c)^4 + 3*a^7*tan(1/2*d*x + 1/2*c)^3 + 84*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 56*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 32*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 23*a^6*b*tan(1/2*d*x + 1/2*c)^2 - 64*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*b^5*tan(1/2*d*x + 1/2*c)^2 + 5*a^7*tan(1/2*d*x + 1/2*c) - 24*a^5*b^2*tan(1/2*d*x + 1/2*c) - 8*a^3*b^4*tan(1/2*d*x + 1/2*c) - 5*a^6*b - 2*a^4*b^3)/((a^8 + 2*a^6*b^2 + a^4*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^4))/d

maple [B] time = 0.60, size = 514, normalized size = 3.29

$$\frac{2\left(-\frac{(5a^4+16a^2b^2+8b^4)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a(a^4+2a^2b^2+b^4)}+\frac{3b(a^4+16a^2b^2+8b^4)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2(a^4+2a^2b^2+b^4)}-\frac{(3a^6-36a^4b^2+56a^2b^4+32b^6)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^3(a^4+2a^2b^2+b^4)}+\frac{b(15a^6-114a^4b^2-8a^2b^4+16b^6)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^4(a^4+2a^2b^2+b^4)}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x)

[Out] 1/d*(-2*(-1/8*(5*a^4+16*a^2*b^2+8*b^4)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^7+3/8*b*(a^4+16*a^2*b^2+8*b^4)/a^2/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^6-1/8/a^3*(3*a^6-36*a^4*b^2+56*a^2*b^4+32*b^6)/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^5+1/8/a^4*b*(15*a^6-114*a^4*b^2-8*a^2*b^4+16*b^6)/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^4-1/8/a^3*(3*a^6+84*a^4*b^2-56*a^2*b^4-32*b^6)/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^3-1/8*b*(23*a^4-64*a^2*b^2-24*b^4)/a^2/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^2-1/8*(5*a^4-24*a^2*b^2-8*b^4)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)+1/8*b*(5*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)^4+3/4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*tan(1/2*d*x+1/2*c)*a-2*b)/(a^2+b^2)^(1/2)))

maxima [B] time = 1.04, size = 822, normalized size = 5.27

$$\frac{2\left(5a^6b+2a^4b^3-\frac{(5a^7-24a^5b^2-8a^3b^4)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(23a^6b-64a^4b^3-24a^2b^5)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(3a^7+84a^5b^2-56a^3b^4-32ab^6)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^{12}+2a^{10}b^2+a^8b^4+\frac{8(a^{11}b+2a^9b^3+a^7b^5)\sin(dx+c)}{\cos(dx+c)+1}-\frac{4(a^{12}-4a^{10}b^2-11a^8b^4-6a^6b^6)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8(3a^{11}b+2a^9b^3-5a^7b^5-4a^5b^7)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2(3a^{12}-18a^{10}b^2+12a^8b^4-6a^6b^6)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")

[Out]
$$-1/8*(2*(5*a^6*b + 2*a^4*b^3 - (5*a^7 - 24*a^5*b^2 - 8*a^3*b^4)*\sin(d*x + c))/(\cos(d*x + c) + 1) - (23*a^6*b - 64*a^4*b^3 - 24*a^2*b^5)*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 - (3*a^7 + 84*a^5*b^2 - 56*a^3*b^4 - 32*a*b^6)*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + (15*a^6*b - 114*a^4*b^3 - 8*a^2*b^5 + 16*b^7)*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 - (3*a^7 - 36*a^5*b^2 + 56*a^3*b^4 + 32*a*b^6)*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5 + 3*(a^6*b + 16*a^4*b^3 + 8*a^2*b^5)*\sin(d*x + c)^6/(\cos(d*x + c) + 1)^6 - (5*a^7 + 16*a^5*b^2 + 8*a^3*b^4)*\sin(d*x + c)^7/(\cos(d*x + c) + 1)^7)/(a^12 + 2*a^10*b^2 + a^8*b^4 + 8*(a^11*b + 2*a^9*b^3 + a^7*b^5)*\sin(d*x + c)/(\cos(d*x + c) + 1) - 4*(a^12 - 4*a^10*b^2 - 11*a^8*b^4 - 6*a^6*b^6)*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 - 8*(3*a^11*b + 2*a^9*b^3 - 5*a^7*b^5 - 4*a^5*b^7)*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 2*(3*a^12 - 18*a^10*b^2 - 37*a^8*b^4 - 8*a^6*b^6 + 8*a^4*b^8)*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + 8*(3*a^11*b + 2*a^9*b^3 - 5*a^7*b^5 - 4*a^5*b^7)*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5 - 4*(a^12 - 4*a^10*b^2 - 11*a^8*b^4 - 6*a^6*b^6)*\sin(d*x + c)^6/(\cos(d*x + c) + 1)^6 - 8*(a^11*b + 2*a^9*b^3 + a^7*b^5)*\sin(d*x + c)^7/(\cos(d*x + c) + 1)^7 + (a^12 + 2*a^10*b^2 + a^8*b^4)*\sin(d*x + c)^8/(\cos(d*x + c) + 1)^8 + 3*\log((b - a*\sin(d*x + c))/(\cos(d*x + c) + 1) + \sqrt{a^2 + b^2})/(b - a*\sin(d*x + c)/(\cos(d*x + c) + 1) - \sqrt{a^2 + b^2}))/((a^4 + 2*a^2*b^2 + b^4)*\sqrt{a^2 + b^2}))/d$$

mupad [B] time = 6.09, size = 719, normalized size = 4.61

$$\frac{\frac{5a^2b+2b^3}{4(a^4+2a^2b^2+b^4)} + \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(a^4b+16a^2b^3+8b^5)}{4a^2(a^4+2a^2b^2+b^4)} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(-23a^4b+64a^2b^3+24b^5)}{4a^2(a^4+2a^2b^2+b^4)} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(-5a^4+24a^2b^2+8b^4)}{4a(a^4+2a^2b^2+b^4)} - \dots}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(6a^4-48a^2b^2+16b^4) + a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 + a^4 - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(4a^4-24a^2b^2) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(4a^4-24a^2b^2) - \dots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^5,x)

[Out]
$$\left(\operatorname{atan}\left(\frac{a^4b+16a^2b^3+8b^5}{a^4+b^4+2a^2b^2} - a^5\tan\left(\frac{c}{2}+\frac{d*x}{2}\right)\right) - a^4b^4\tan\left(\frac{c}{2}+\frac{d*x}{2}\right) - a^3b^2\tan\left(\frac{c}{2}+\frac{d*x}{2}\right)\right)/\left(a^2+b^2\right)^{5/2} + \dots$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**5,x)

[Out] Timed out

$$3.231 \quad \int \frac{1}{(a \cos(c+dx)+b \sin(c+dx))^6} dx$$

Optimal. Leaf size=151

$$\frac{8 \sin(c+dx)}{15ad(a^2+b^2)^2(a \cos(c+dx)+b \sin(c+dx))} - \frac{4(b \cos(c+dx)-a \sin(c+dx))}{15d(a^2+b^2)^2(a \cos(c+dx)+b \sin(c+dx))^3} - \frac{b \cos(c+dx)}{5d(a^2+b^2)^2(a \cos(c+dx)+b \sin(c+dx))^5}$$

[Out] 1/5*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^5-
4/15*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c))^3+
8/15*sin(d*x+c)/a/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c))

Rubi [A] time = 0.07, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, number of rules / integrand size = 0.105, Rules used = {3076, 3075}

$$\frac{8 \sin(c+dx)}{15ad(a^2+b^2)^2(a \cos(c+dx)+b \sin(c+dx))} - \frac{4(b \cos(c+dx)-a \sin(c+dx))}{15d(a^2+b^2)^2(a \cos(c+dx)+b \sin(c+dx))^3} - \frac{b \cos(c+dx)}{5d(a^2+b^2)^2(a \cos(c+dx)+b \sin(c+dx))^5}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-6),x]

[Out] -(b*Cos[c + d*x] - a*Sin[c + d*x])/(5*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) - (4*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(15*(a^2 + b^2)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (8*Sin[c + d*x])/(15*a*(a^2 + b^2)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))

Rule 3075

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 3076

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n+1))/(d*(n+1)*(a^2 + b^2)), x] + Dist[(n+2)/((n+1)*(a^2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n+2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a \cos(c+dx)+b \sin(c+dx))^6} dx &= -\frac{b \cos(c+dx)-a \sin(c+dx)}{5(a^2+b^2)d(a \cos(c+dx)+b \sin(c+dx))^5} + \frac{4 \int \frac{1}{(a \cos(c+dx)+b \sin(c+dx))^5} dx}{5(a^2+b^2)} \\ &= -\frac{b \cos(c+dx)-a \sin(c+dx)}{5(a^2+b^2)d(a \cos(c+dx)+b \sin(c+dx))^5} - \frac{4(b \cos(c+dx)-a \sin(c+dx))}{15(a^2+b^2)^2d(a \cos(c+dx)+b \sin(c+dx))^3} \\ &= -\frac{b \cos(c+dx)-a \sin(c+dx)}{5(a^2+b^2)d(a \cos(c+dx)+b \sin(c+dx))^5} - \frac{4(b \cos(c+dx)-a \sin(c+dx))}{15(a^2+b^2)^2d(a \cos(c+dx)+b \sin(c+dx))^3} \end{aligned}$$

Mathematica [A] time = 0.52, size = 182, normalized size = 1.21

$$\frac{10a^4 \sin(c+dx) + 5a^4 \sin(3(c+dx)) + a^4 \sin(5(c+dx)) + (4ab^3 - 4a^3b) \cos(5(c+dx)) + 20a^2b^2 \sin(c+dx)}{30ad(a^2+b^2)^2(a \cos(c+dx)+b \sin(c+dx))^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*cos[c + d*x] + b*sin[c + d*x])^(-6),x]
```

```
[Out] (-10*a*b*(a^2 + b^2)*Cos[3*(c + d*x)] + (-4*a^3*b + 4*a*b^3)*Cos[5*(c + d*x)] + 10*a^4*Sin[c + d*x] + 20*a^2*b^2*Sin[c + d*x] + 10*b^4*Sin[c + d*x] + 5*a^4*Sin[3*(c + d*x)] - 5*b^4*Sin[3*(c + d*x)] + a^4*Sin[5*(c + d*x)] - 6*a^2*b^2*Sin[5*(c + d*x)] + b^4*Sin[5*(c + d*x)]/(30*a*(a^2 + b^2)^2*d*(a*cos[c + d*x] + b*sin[c + d*x])^5)
```

```
fricas [B] time = 2.05, size = 441, normalized size = 2.92
```

$$\frac{8(5a^4b - 10a^2b^3 + b^5)\cos(dx + c)^5 - 20(a^4b - 6a^2b^3 + b^5)d\cos(dx + c)^5 + 10(a^9b^2 + 2a^7b^4 - 2a^3b^8 - ab^{10})d\cos(dx + c)^5}{15((a^{11} - 7a^9b^2 - 22a^7b^4 - 14a^5b^6 + 5a^3b^8 + 5ab^{10})d\cos(dx + c)^5 + 10(a^9b^2 + 2a^7b^4 - 2a^3b^8 - ab^{10})d\cos(dx + c)^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^5 - 20*(a^4*b - 6*a^2*b^3 + b^5)*cos(d*x + c)^3 - 5*(a^4*b + 6*a^2*b^3 - 3*b^5)*cos(d*x + c) - (3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 8*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 + 4*(a^5 + 10*a^3*b^2 - 15*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^11 - 7*a^9*b^2 - 22*a^7*b^4 - 14*a^5*b^6 + 5*a^3*b^8 + 5*a*b^10)*d*cos(d*x + c)^5 + 10*(a^9*b^2 + 2*a^7*b^4 - 2*a^3*b^8 - a*b^10)*d*cos(d*x + c)^3 + 5*(a^7*b^4 + 3*a^5*b^6 + 3*a^3*b^8 + a*b^10)*d*cos(d*x + c) + ((5*a^10*b + 5*a^8*b^3 - 14*a^6*b^5 - 22*a^4*b^7 - 7*a^2*b^9 + b^11)*d*cos(d*x + c)^4 + 2*(5*a^8*b^3 + 14*a^6*b^5 + 12*a^4*b^7 + 2*a^2*b^9 - b^11)*d*cos(d*x + c)^2 + (a^6*b^5 + 3*a^4*b^7 + 3*a^2*b^9 + b^11)*d)*sin(d*x + c))
```

```
giac [A] time = 0.22, size = 118, normalized size = 0.78
```

$$\frac{15b^4 \tan(dx + c)^4 + 30ab^3 \tan(dx + c)^3 + 30a^2b^2 \tan(dx + c)^2 + 10b^4 \tan(dx + c)^2 + 15a^3b \tan(dx + c) + 5a^4}{15(b \tan(dx + c) + a)^5 b^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*b^4*tan(d*x + c)^4 + 30*a*b^3*tan(d*x + c)^3 + 30*a^2*b^2*tan(d*x + c)^2 + 10*b^4*tan(d*x + c)^2 + 15*a^3*b*tan(d*x + c) + 5*a*b^3*tan(d*x + c) + 3*a^4 + a^2*b^2 + 3*b^4)/((b*tan(d*x + c) + a)^5*b^5*d)
```

```
maple [A] time = 0.64, size = 125, normalized size = 0.83
```

$$\frac{-\frac{a^4+2a^2b^2+b^4}{5b^5(a+b \tan(dx+c))^5} + \frac{a(a^2+b^2)}{b^5(a+b \tan(dx+c))^4} - \frac{6a^2+2b^2}{3b^5(a+b \tan(dx+c))^3} + \frac{2a}{b^5(a+b \tan(dx+c))^2} - \frac{1}{b^5(a+b \tan(dx+c))}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x)
```

```
[Out] 1/d*(-1/5*(a^4+2*a^2*b^2+b^4)/b^5/(a+b*tan(d*x+c))^5+a*(a^2+b^2)/b^5/(a+b*tan(d*x+c))^4-1/3*(6*a^2+2*b^2)/b^5/(a+b*tan(d*x+c))^3+2*a/b^5/(a+b*tan(d*x+c))^2-1/b^5/(a+b*tan(d*x+c)))
```

```
maxima [A] time = 0.79, size = 174, normalized size = 1.15
```

$$\frac{15b^4 \tan(dx + c)^4 + 30ab^3 \tan(dx + c)^3 + 3a^4 + a^2b^2 + 3b^4 + 10(3a^2b^2 + b^4)\tan(dx + c)^2 + 5(3a^3b + ab^3)\tan(dx + c)}{15(b^{10} \tan(dx + c)^5 + 5ab^9 \tan(dx + c)^4 + 10a^2b^8 \tan(dx + c)^3 + 10a^3b^7 \tan(dx + c)^2 + 5a^4b^6 \tan(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="maxima")

[Out]
$$\frac{-1/15*(15*b^4*\tan(d*x + c)^4 + 30*a*b^3*\tan(d*x + c)^3 + 3*a^4 + a^2*b^2 + 3*b^4 + 10*(3*a^2*b^2 + b^4)*\tan(d*x + c)^2 + 5*(3*a^3*b + a*b^3)*\tan(d*x + c))}{(b^{10}*\tan(d*x + c)^5 + 5*a*b^9*\tan(d*x + c)^4 + 10*a^2*b^8*\tan(d*x + c)^3 + 10*a^3*b^7*\tan(d*x + c)^2 + 5*a^4*b^6*\tan(d*x + c) + a^5*b^5)*d}$$

mupad [B] time = 5.15, size = 470, normalized size = 3.11

$$\frac{\frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9}{a} + \frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{a}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 (10 a^5 - 120 a^3 b^2 + 80 a b^4) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (40 a^4 b - 80 a^2 b^3) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 (40 a^4 b - 80 a^2 b^3) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 (29 a^4 + 24 b^4 - 112 a^2 b^2) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 (80 a^4 b^4 + 10 a^5 - 120 a^3 b^2) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (40 a^4 b^4 - 80 a^2 b^3) - a^5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 (80 a^4 b^4 + 10 a^5 - 120 a^3 b^2) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 (60 a^4 b^4 + 32 b^5 - 160 a^2 b^3) + a^5 - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 (5 a^5 - 40 a^3 b^2) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 (5 a^5 - 40 a^3 b^2) + 10 a^4 b \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 10 a^4 b \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^6,x)

[Out]
$$\left(\frac{(2*\tan(c/2 + (d*x)/2)^9)/a + (2*\tan(c/2 + (d*x)/2))/a - (8*\tan(c/2 + (d*x)/2)^4*(7*a^2*b - 6*b^3))/(3*a^4) + (8*\tan(c/2 + (d*x)/2)^6*(7*a^2*b - 6*b^3))/(3*a^4) - (8*\tan(c/2 + (d*x)/2)^3*(a^2 - 6*b^2))/(3*a^3) - (8*\tan(c/2 + (d*x)/2)^7*(a^2 - 6*b^2))/(3*a^3) + (8*b*\tan(c/2 + (d*x)/2)^2)/a^2 - (8*b*\tan(c/2 + (d*x)/2)^8)/a^2 + (4*\tan(c/2 + (d*x)/2)^5*(29*a^4 + 24*b^4 - 112*a^2*b^2))/(15*a^5) \right) / (d*(\tan(c/2 + (d*x)/2)^4*(80*a*b^4 + 10*a^5 - 120*a^3*b^2) - \tan(c/2 + (d*x)/2)^3*(40*a^4*b - 80*a^2*b^3) - \tan(c/2 + (d*x)/2)^7*(40*a^4*b - 80*a^2*b^3) - a^5*\tan(c/2 + (d*x)/2)^{10} - \tan(c/2 + (d*x)/2)^6*(80*a*b^4 + 10*a^5 - 120*a^3*b^2) + \tan(c/2 + (d*x)/2)^5*(60*a^4*b + 32*b^5 - 160*a^2*b^3) + a^5 - \tan(c/2 + (d*x)/2)^2*(5*a^5 - 40*a^3*b^2) + \tan(c/2 + (d*x)/2)^8*(5*a^5 - 40*a^3*b^2) + 10*a^4*b*\tan(c/2 + (d*x)/2) + 10*a^4*b*\tan(c/2 + (d*x)/2)^9)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**6,x)

[Out] Timed out

3.232 $\int (a \cos(c + dx) + b \sin(c + dx))^{7/2} dx$

Optimal. Leaf size=186

$$\frac{10(a^2 + b^2)^2 \sqrt{\frac{a \cos(c+dx) + b \sin(c+dx)}{\sqrt{a^2 + b^2}}} F\left(\frac{1}{2}(c + dx - \tan^{-1}(a, b)) \middle| 2\right)}{21d \sqrt{a \cos(c + dx) + b \sin(c + dx)}} - \frac{10(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx)) \sqrt{a^2 + b^2}}{21d}$$

```
[Out] -2/7*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^(5/2)/d-10/21*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^(1/2)/d+10/21*(a^2+b^2)^2*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*arctan(a,b))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*((a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(1/2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3073, 3078, 2641}

$$\frac{10(a^2 + b^2)^2 \sqrt{\frac{a \cos(c+dx) + b \sin(c+dx)}{\sqrt{a^2 + b^2}}} F\left(\frac{1}{2}(c + dx - \tan^{-1}(a, b)) \middle| 2\right)}{21d \sqrt{a \cos(c + dx) + b \sin(c + dx)}} - \frac{10(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx)) \sqrt{a^2 + b^2}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(7/2), x]
```

```
[Out] (-10*(a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x])*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]/(21*d) - (2*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(5/2))/(7*d) + (10*(a^2 + b^2)^2*EllipticF[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(21*d*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3078

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 + b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned} \int (a \cos(c + dx) + b \sin(c + dx))^{7/2} dx &= -\frac{2(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))^{5/2}}{7d} + \\ &= -\frac{10(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))\sqrt{a \cos(c + dx) + b \sin(c + dx)}}{21d} \\ &= -\frac{10(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))\sqrt{a \cos(c + dx) + b \sin(c + dx)}}{21d} \\ &= -\frac{10(a^2 + b^2)(b \cos(c + dx) - a \sin(c + dx))\sqrt{a \cos(c + dx) + b \sin(c + dx)}}{21d} \end{aligned}$$

Mathematica [C] time = 1.82, size = 205, normalized size = 1.10

$$\frac{20(a^2+b^2)^2 \tan(\tan^{-1}(\frac{a}{b})+c+dx) \sqrt{\cos^2(\tan^{-1}(\frac{a}{b})+c+dx)} {}_2F_1\left(\frac{1}{4}, \frac{5}{4}; \sin^2(c+dx+\tan^{-1}(\frac{a}{b}))\right)}{\sqrt{b\sqrt{\frac{a^2}{b^2}+1} \sin(\tan^{-1}(\frac{a}{b})+c+dx)}} + \sqrt{a \cos(c + dx) + b \sin(c + dx)} \left((3b^3 \right.$$

Antiderivative was successfully verified.

[In] Integrate[(a*cos[c + d*x] + b*sin[c + d*x])^(7/2), x]

[Out] (Sqrt[a*cos[c + d*x] + b*sin[c + d*x]]*(-23*b*(a^2 + b^2)*Cos[c + d*x] + (-9*a^2*b + 3*b^3)*Cos[3*(c + d*x)] + 2*a*(13*a^2 + 7*b^2 + 3*(a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]) + (20*(a^2 + b^2)^2*Sqrt[Cos[c + d*x + ArcTan[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[a/b]]^2]*Tan[c + d*x + ArcTan[a/b]])/Sqrt[Sqrt[1 + a^2/b^2]*b*sin[c + d*x + ArcTan[a/b]]])/(42*d)

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(3ab^2 \cos(dx + c) + (a^3 - 3ab^2) \cos(dx + c)^3 + (b^3 + (3a^2b - b^3) \cos(dx + c)^2) \sin(dx + c)\right) \sqrt{a \cos(dx + c) + b \sin(dx + c)}\right), x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(7/2), x, algorithm="fricas")

[Out] integral((3*a*b^2*cos(d*x + c) + (a^3 - 3*a*b^2)*cos(d*x + c)^3 + (b^3 + (3*a^2*b - b^3)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cos(dx + c) + b \sin(dx + c))^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(7/2), x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(7/2), x)

maple [A] time = 0.44, size = 183, normalized size = 0.98

$$\frac{(a^2 + b^2)^2 \left(6 \left(\sin^5(dx + c - \arctan(-a, b)) \right) + 5 \sqrt{1 + \sin(dx + c - \arctan(-a, b))} \sqrt{-2 \sin(dx + c - \arctan(-a, b))} \right)}{21 \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x)`

[Out] $\frac{1}{21}(a^2+b^2)^2(6\sin(d*x+c-\arctan(-a,b))^5+5(1+\sin(d*x+c-\arctan(-a,b)))^{1/2}(-2\sin(d*x+c-\arctan(-a,b))+2)^{1/2}(-\sin(d*x+c-\arctan(-a,b)))^{1/2})\text{EllipticF}((1+\sin(d*x+c-\arctan(-a,b)))^{1/2},1/2\sqrt{2})+4\sin(d*x+c-\arctan(-a,b))^3-10\sin(d*x+c-\arctan(-a,b)))/\cos(d*x+c-\arctan(-a,b))/(\sin(d*x+c-\arctan(-a,b))\sqrt{a^2+b^2})^{1/2}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cos(dx + c) + b \sin(dx + c))^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="maxima")`

[Out] `integrate((a*cos(d*x + c) + b*sin(d*x + c))^(7/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a \cos(c + dx) + b \sin(c + dx))^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*cos(c + d*x) + b*sin(c + d*x))^(7/2),x)`

[Out] `int((a*cos(c + d*x) + b*sin(c + d*x))^(7/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*cos(d*x+c)+b*sin(d*x+c))**(7/2),x)`

[Out] Timed out

3.233 $\int (a \cos(c + dx) + b \sin(c + dx))^{5/2} dx$

Optimal. Leaf size=131

$$\frac{6(a^2 + b^2) \sqrt{a \cos(c + dx) + b \sin(c + dx)} E\left(\frac{1}{2}(c + dx - \tan^{-1}(a, b))\right) \sqrt{2(b \cos(c + dx) - a \sin(c + dx))}}{5d \sqrt{\frac{a \cos(c + dx) + b \sin(c + dx)}{\sqrt{a^2 + b^2}}}} \frac{(a \cos(c + dx) + b \sin(c + dx))^{3/2}}{5d}$$

[Out] $-2/5*(b*\cos(d*x+c)-a*\sin(d*x+c))*(a*\cos(d*x+c)+b*\sin(d*x+c))^{3/2}/d+6/5*(a^2+b^2)*(\cos(1/2*c+1/2*d*x-1/2*\arctan(a,b))^2)^{1/2}/\cos(1/2*c+1/2*d*x-1/2*\arctan(a,b))*\text{EllipticE}(\sin(1/2*c+1/2*d*x-1/2*\arctan(a,b)),2^{1/2})*(a*\cos(d*x+c)+b*\sin(d*x+c))^{1/2}/d/((a*\cos(d*x+c)+b*\sin(d*x+c))/(a^2+b^2)^{1/2})^{1/2}$

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3073, 3078, 2639}

$$\frac{6(a^2 + b^2) \sqrt{a \cos(c + dx) + b \sin(c + dx)} E\left(\frac{1}{2}(c + dx - \tan^{-1}(a, b))\right) \sqrt{2(b \cos(c + dx) - a \sin(c + dx))}}{5d \sqrt{\frac{a \cos(c + dx) + b \sin(c + dx)}{\sqrt{a^2 + b^2}}}} \frac{(a \cos(c + dx) + b \sin(c + dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(5/2), x]

[Out] $(-2*(b*\cos[c + d*x] - a*\sin[c + d*x])*(a*\cos[c + d*x] + b*\sin[c + d*x])^{3/2})/(5*d) + (6*(a^2 + b^2)*\text{EllipticE}[(c + d*x - \text{ArcTan}[a, b])/2, 2]*\text{Sqrt}[a*\cos[c + d*x] + b*\sin[c + d*x]])/(5*d*\text{Sqrt}[(a*\cos[c + d*x] + b*\sin[c + d*x])/\text{Sqrt}[a^2 + b^2]])$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3073

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[(n - 1)/2] && GtQ[n, 1]

Rule 3078

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 + b^2, 0] || EqQ[a^2 + b^2, 0])

Rubi steps

$$\begin{aligned} \int (a \cos(c + dx) + b \sin(c + dx))^{5/2} dx &= -\frac{2(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))^{3/2}}{5d} + \frac{1}{5} \left(\frac{3}{5} \right) \\ &= -\frac{2(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))^{3/2}}{5d} + \frac{3}{5} \left(\frac{3}{5} \right) \\ &= -\frac{2(b \cos(c + dx) - a \sin(c + dx))(a \cos(c + dx) + b \sin(c + dx))^{3/2}}{5d} + \frac{6}{5} \left(\frac{3}{5} \right) \end{aligned}$$

Mathematica [C] time = 1.60, size = 256, normalized size = 1.95

$$\sqrt{a \cos(c + dx) + b \sin(c + dx)} \left(b(a^2 - b^2) \sin(2(c + dx)) + 6a(a^2 + b^2) - 2ab^2 \cos(2(c + dx)) \right) - \frac{3(a^2 + b^2)^2 \cos(-\arctan(b/a))}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(5/2), x]

[Out] (Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]*(6*a*(a^2 + b^2) - 2*a*b^2*Cos[2*(c + d*x)] + b*(a^2 - b^2)*Sin[2*(c + d*x)]) - (3*(a^2 + b^2)^2*Cos[c + d*x - ArcTan[b/a]]*(b*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d*x - ArcTan[b/a]]^2]*Sin[c + d*x - ArcTan[b/a]] + Sqrt[Sin[c + d*x - ArcTan[b/a]]^2]*(2*a*Cos[c + d*x - ArcTan[b/a]] - b*Sin[c + d*x - ArcTan[b/a]])))/((a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]])^(3/2)*Sqrt[Sin[c + d*x - ArcTan[b/a]]^2]))/(5*b*d)

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral} \left((2ab \cos(dx + c) \sin(dx + c) + (a^2 - b^2) \cos(dx + c)^2 + b^2) \sqrt{a \cos(dx + c) + b \sin(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(5/2), x, algorithm="fricas")

[Out] integral((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cos(dx + c) + b \sin(dx + c))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(5/2), x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(5/2), x)

maple [A] time = 0.37, size = 246, normalized size = 1.88

$$(a^2 + b^2)^{3/2} \left(6\sqrt{1 + \sin(dx + c - \arctan(-a, b))} \sqrt{-2 \sin(dx + c - \arctan(-a, b)) + 2} \sqrt{-\sin(dx + c - \arctan(-a, b))} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x)`

[Out]
$$-1/5*(a^2+b^2)^{(3/2)}*(6*(1+\sin(d*x+c-\arctan(-a,b)))^{(1/2)}*(-2*\sin(d*x+c-\arctan(-a,b))+2)^{(1/2)}*(-\sin(d*x+c-\arctan(-a,b)))^{(1/2)}*\text{EllipticE}((1+\sin(d*x+c-\arctan(-a,b)))^{(1/2)},1/2*2^{(1/2)})-3*(1+\sin(d*x+c-\arctan(-a,b)))^{(1/2)}*(-2*\sin(d*x+c-\arctan(-a,b))+2)^{(1/2)}*(-\sin(d*x+c-\arctan(-a,b)))^{(1/2)}*\text{EllipticF}((1+\sin(d*x+c-\arctan(-a,b)))^{(1/2)},1/2*2^{(1/2)})-2*\sin(d*x+c-\arctan(-a,b))^{(1/2)}+2*\sin(d*x+c-\arctan(-a,b))^2)/\cos(d*x+c-\arctan(-a,b))/(\sin(d*x+c-\arctan(-a,b)))*(a^2+b^2)^{(1/2))^{(1/2)}/d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cos(dx + c) + b \sin(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate((a*cos(d*x + c) + b*sin(d*x + c))^(5/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a \cos(c + dx) + b \sin(c + dx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*cos(c + d*x) + b*sin(c + d*x))^(5/2),x)`

[Out] `int((a*cos(c + d*x) + b*sin(c + d*x))^(5/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*cos(d*x+c)+b*sin(d*x+c))**(5/2),x)`

[Out] Timed out

3.234 $\int (a \cos(c + dx) + b \sin(c + dx))^{3/2} dx$

Optimal. Leaf size=131

$$\frac{2(a^2 + b^2) \sqrt{\frac{a \cos(c+dx) + b \sin(c+dx)}{\sqrt{a^2+b^2}}} F\left(\frac{1}{2} \left(c + dx - \tan^{-1}\left(\frac{a}{b}\right)\right) \middle| 2\right)}{3d \sqrt{a \cos(c + dx) + b \sin(c + dx)}} - \frac{2(b \cos(c + dx) - a \sin(c + dx)) \sqrt{a \cos(c + dx) + b \sin(c + dx)}}{3d}$$

[Out] $-2/3*(b*\cos(d*x+c)-a*\sin(d*x+c))*(a*\cos(d*x+c)+b*\sin(d*x+c))^{(1/2)}/d+2/3*(a^2+b^2)*(\cos(1/2*c+1/2*d*x-1/2*\arctan(a,b))^{(1/2)})^{(1/2)}/\cos(1/2*c+1/2*d*x-1/2*\arctan(a,b))*\text{EllipticF}(\sin(1/2*c+1/2*d*x-1/2*\arctan(a,b)),2^{(1/2)})*((a*\cos(d*x+c)+b*\sin(d*x+c))/(a^2+b^2)^{(1/2)})^{(1/2)}/d/(a*\cos(d*x+c)+b*\sin(d*x+c))^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3073, 3078, 2641}

$$\frac{2(a^2 + b^2) \sqrt{\frac{a \cos(c+dx) + b \sin(c+dx)}{\sqrt{a^2+b^2}}} F\left(\frac{1}{2} \left(c + dx - \tan^{-1}\left(\frac{a}{b}\right)\right) \middle| 2\right)}{3d \sqrt{a \cos(c + dx) + b \sin(c + dx)}} - \frac{2(b \cos(c + dx) - a \sin(c + dx)) \sqrt{a \cos(c + dx) + b \sin(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] `Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(3/2), x]`

[Out] $(-2*(b*\text{Cos}[c + d*x] - a*\text{Sin}[c + d*x])* \text{Sqrt}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/(3*d) + (2*(a^2 + b^2)* \text{EllipticF}[(c + d*x - \text{ArcTan}[a, b])/2, 2]* \text{Sqrt}[(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])/ \text{Sqrt}[a^2 + b^2]])/(3*d*\text{Sqrt}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])$

Rule 2641

`Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]`

Rule 3073

`Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[(n - 1)/2] && GtQ[n, 1]`

Rule 3078

`Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 + b^2, 0] || EqQ[a^2 + b^2, 0])`

Rubi steps

$$\int (a \cos(c + dx) + b \sin(c + dx))^{3/2} dx = -\frac{2(b \cos(c + dx) - a \sin(c + dx))\sqrt{a \cos(c + dx) + b \sin(c + dx)}}{3d} + \frac{1}{3} \left(\frac{2(b \cos(c + dx) - a \sin(c + dx))\sqrt{a \cos(c + dx) + b \sin(c + dx)}}{3d} + \frac{2(b \cos(c + dx) - a \sin(c + dx))\sqrt{a \cos(c + dx) + b \sin(c + dx)}}{3d} + \dots \right)$$

Mathematica [C] time = 1.29, size = 143, normalized size = 1.09

$$2 \left[\frac{(a^2 + b^2) \tan(\tan^{-1}(\frac{a}{b}) + c + dx) \sqrt{\cos^2(\tan^{-1}(\frac{a}{b}) + c + dx)} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \sin^2(c + dx + \tan^{-1}(\frac{a}{b}))\right)}{\sqrt{b \sqrt{\frac{a^2}{b^2} + 1} \sin(\tan^{-1}(\frac{a}{b}) + c + dx)}} + \sqrt{a \cos(c + dx) + b \sin(c + dx)} (a \sin(\dots)) \right] / (3d)$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(3/2), x]

[Out] (2*((-b*Cos[c + d*x]) + a*Sin[c + d*x])*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]) + ((a^2 + b^2)*Sqrt[Cos[c + d*x + ArcTan[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[a/b]]^2]*Tan[c + d*x + ArcTan[a/b]])/Sqrt[Sqrt[1 + a^2/b^2]*b*Sin[c + d*x + ArcTan[a/b]]])/(3*d)

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left((a \cos(dx + c) + b \sin(dx + c))^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(3/2), x, algorithm="fricas")

[Out] integral((a*cos(d*x + c) + b*sin(d*x + c))^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cos(dx + c) + b \sin(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(3/2), x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(3/2), x)

maple [A] time = 0.37, size = 163, normalized size = 1.24

$$\frac{(a^2 + b^2) \left(\sqrt{1 + \sin(dx + c - \arctan(-a, b))} \sqrt{-2 \sin(dx + c - \arctan(-a, b)) + 2} \sqrt{-\sin(dx + c - \arctan(-a, b))} \right)}{3 \cos(dx + c - \arctan(-a, b))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+b*sin(d*x+c))^(3/2), x)

[Out] $\frac{1}{3}(a^2+b^2)*((1+\sin(dx+c-\arctan(-a,b)))^{1/2}*(-2*\sin(dx+c-\arctan(-a,b))+2)^{1/2}*(-\sin(dx+c-\arctan(-a,b)))^{1/2}*\text{EllipticF}((1+\sin(dx+c-\arctan(-a,b)))^{1/2},1/2*2^{1/2}))+2*\sin(dx+c-\arctan(-a,b))^3-2*\sin(dx+c-\arctan(-a,b)))/\cos(dx+c-\arctan(-a,b))/(\sin(dx+c-\arctan(-a,b))*(a^2+b^2)^{1/2})^{1/2}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cos(dx + c) + b \sin(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*cos(dx+c)+b*sin(dx+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate((a*cos(dx + c) + b*sin(dx + c))^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a \cos(c + dx) + b \sin(c + dx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*cos(c + dx) + b*sin(c + dx))^(3/2),x)`

[Out] `int((a*cos(c + dx) + b*sin(c + dx))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cos(c + dx) + b \sin(c + dx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*cos(dx+c)+b*sin(dx+c))**(3/2),x)`

[Out] `Integral((a*cos(c + dx) + b*sin(c + dx))**(3/2), x)`

3.235 $\int \sqrt{a \cos(c + dx) + b \sin(c + dx)} dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{a \cos(c + dx) + b \sin(c + dx)} E\left(\frac{1}{2}(c + dx - \tan^{-1}(a, b))\right) \Big|_2}{d \sqrt{\frac{a \cos(c+dx)+b \sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

[Out] $2*(\cos(1/2*c+1/2*d*x-1/2*\arctan(a,b))^2)^{(1/2)}/\cos(1/2*c+1/2*d*x-1/2*\arctan(a,b))*\text{EllipticE}(\sin(1/2*c+1/2*d*x-1/2*\arctan(a,b)),2^{(1/2)})*(a*\cos(d*x+c)+b*\sin(d*x+c))^{(1/2)}/d/((a*\cos(d*x+c)+b*\sin(d*x+c))/(a^2+b^2)^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3078, 2639}

$$\frac{2\sqrt{a \cos(c + dx) + b \sin(c + dx)} E\left(\frac{1}{2}(c + dx - \tan^{-1}(a, b))\right) \Big|_2}{d \sqrt{\frac{a \cos(c+dx)+b \sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]

[Out] $(2*\text{EllipticE}[(c + d*x - \text{ArcTan}[a, b])/2, 2]*\text{Sqrt}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/(d*\text{Sqrt}[(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])/ \text{Sqrt}[a^2 + b^2]])$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3078

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 + b^2, 0] || EqQ[a^2 + b^2, 0])

Rubi steps

$$\begin{aligned} \int \sqrt{a \cos(c + dx) + b \sin(c + dx)} dx &= \frac{\sqrt{a \cos(c + dx) + b \sin(c + dx)} \int \sqrt{\cos(c + dx - \tan^{-1}(a, b))} dx}{\sqrt{\frac{a \cos(c+dx)+b \sin(c+dx)}{\sqrt{a^2+b^2}}}} \\ &= \frac{2E\left(\frac{1}{2}(c + dx - \tan^{-1}(a, b))\right) \Big|_2 \sqrt{a \cos(c + dx) + b \sin(c + dx)}}{d \sqrt{\frac{a \cos(c+dx)+b \sin(c+dx)}{\sqrt{a^2+b^2}}}} \end{aligned}$$

Mathematica [C] time = 1.10, size = 268, normalized size = 3.57

$$\cos\left(-\tan^{-1}\left(\frac{b}{a}\right) + c + dx\right) \left(\sqrt{\sin^2\left(-\tan^{-1}\left(\frac{b}{a}\right) + c + dx\right)}\right) \left(b(a^2 + b^2) \sin\left(-\tan^{-1}\left(\frac{b}{a}\right) + c + dx\right) - 2a(a^2 + b^2)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*cos[c + d*x] + b*sin[c + d*x]], x]

[Out] (Cos[c + d*x - ArcTan[b/a]]*(-(b*(a^2 + b^2)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d*x - ArcTan[b/a]]^2]*Sin[c + d*x - ArcTan[b/a]]) + Sqrt[Sin[c + d*x - ArcTan[b/a]]^2]*(-2*a*(a^2 + b^2)*Cos[c + d*x - ArcTan[b/a]] + 2*a^2*Sqrt[1 + b^2/a^2]*Sqrt[a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]])*Sqrt[a*cos[c + d*x] + b*sin[c + d*x]] + b*(a^2 + b^2)*Sin[c + d*x - ArcTan[b/a]]))/(b*d*(a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]])^(3/2)*Sqrt[Sin[c + d*x - ArcTan[b/a]]^2])

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a \cos(dx + c) + b \sin(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \cos(dx + c) + b \sin(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)

maple [A] time = 0.36, size = 159, normalized size = 2.12

$$\frac{\sqrt{a^2 + b^2} \sqrt{1 + \sin(dx + c - \arctan(-a, b))} \sqrt{-2 \sin(dx + c - \arctan(-a, b)) + 2} \sqrt{-\sin(dx + c - \arctan(-a, b))}}{\cos(dx + c - \arctan(-a, b)) \sqrt{\sin(dx + c - \arctan(-a, b))}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+b*sin(d*x+c))^(1/2), x)

[Out] -(a^2+b^2)^(1/2)*(1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*(2*EllipticE((1+sin(d*x+c-arctan(-a,b)))^(1/2), 1/2*2^(1/2))-EllipticF((1+sin(d*x+c-arctan(-a,b)))^(1/2), 1/2*2^(1/2)))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/2))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \cos(dx + c) + b \sin(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a \cos(c + dx) + b \sin(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^(1/2), x)
```

```
[Out] int((a*cos(c + d*x) + b*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \cos(c + dx) + b \sin(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**(1/2), x)
```

```
[Out] Integral(sqrt(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

$$3.236 \quad \int \frac{1}{\sqrt{a \cos(c+dx)+b \sin(c+dx)}} dx$$

Optimal. Leaf size=75

$$\frac{2\sqrt{\frac{a \cos(c+dx)+b \sin(c+dx)}{\sqrt{a^2+b^2}}} F\left(\frac{1}{2}(c+dx - \tan^{-1}(a,b))\middle|2\right)}{d\sqrt{a \cos(c+dx)+b \sin(c+dx)}}$$

[Out] $2*(\cos(1/2*c+1/2*d*x-1/2*\arctan(a,b))^2)^{(1/2)}/\cos(1/2*c+1/2*d*x-1/2*\arctan(a,b))*\text{EllipticF}(\sin(1/2*c+1/2*d*x-1/2*\arctan(a,b)),2^{(1/2)})*((a*\cos(d*x+c)+b*\sin(d*x+c))/(\sqrt{a^2+b^2})^{(1/2)})^{(1/2)}/d/(a*\cos(d*x+c)+b*\sin(d*x+c))^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3078, 2641}

$$\frac{2\sqrt{\frac{a \cos(c+dx)+b \sin(c+dx)}{\sqrt{a^2+b^2}}} F\left(\frac{1}{2}(c+dx - \tan^{-1}(a,b))\middle|2\right)}{d\sqrt{a \cos(c+dx)+b \sin(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]

[Out] $(2*\text{EllipticF}[(c+d*x - \text{ArcTan}[a,b])/2,2]*\text{Sqrt}[(a*\text{Cos}[c+d*x]+b*\text{Sin}[c+d*x])/(\text{Sqrt}[a^2+b^2])])/(d*\text{Sqrt}[a*\text{Cos}[c+d*x]+b*\text{Sin}[c+d*x]])$

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3078

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 + b^2, 0] || EqQ[a^2 + b^2, 0])

Rubi steps

$$\int \frac{1}{\sqrt{a \cos(c+dx)+b \sin(c+dx)}} dx = \frac{\sqrt{\frac{a \cos(c+dx)+b \sin(c+dx)}{\sqrt{a^2+b^2}}} \int \frac{1}{\sqrt{\cos(c+dx-\tan^{-1}(a,b))}} dx}{\sqrt{a \cos(c+dx)+b \sin(c+dx)}}$$

$$= \frac{2F\left(\frac{1}{2}(c+dx - \tan^{-1}(a,b))\middle|2\right) \sqrt{\frac{a \cos(c+dx)+b \sin(c+dx)}{\sqrt{a^2+b^2}}}}{d\sqrt{a \cos(c+dx)+b \sin(c+dx)}}$$

Mathematica [C] time = 0.19, size = 92, normalized size = 1.23

$$\frac{2 \tan\left(\tan^{-1}\left(\frac{a}{b}\right) + c + dx\right) \sqrt{\cos^2\left(\tan^{-1}\left(\frac{a}{b}\right) + c + dx\right)} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \sin^2\left(c + dx + \tan^{-1}\left(\frac{a}{b}\right)\right)\right)}{d\sqrt{b\sqrt{\frac{a^2}{b^2}+1} \sin\left(\tan^{-1}\left(\frac{a}{b}\right) + c + dx\right)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a*cos[c + d*x] + b*sin[c + d*x]],x]

[Out] (2*Sqrt[Cos[c + d*x + ArcTan[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[a/b]]^2]*Tan[c + d*x + ArcTan[a/b]])/(d*Sqrt[Sqrt[1 + a^2/b^2]*b*sin[c + d*x + ArcTan[a/b]]])

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{a \cos(dx + c) + b \sin(dx + c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(1/sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \cos(dx + c) + b \sin(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)

maple [A] time = 0.25, size = 121, normalized size = 1.61

$$\frac{\sqrt{1 + \sin(dx + c - \arctan(-a, b))} \sqrt{-2 \sin(dx + c - \arctan(-a, b)) + 2} \sqrt{-\sin(dx + c - \arctan(-a, b))} \text{EllipticE}\left(\arctan\left(\frac{\sqrt{1 + \sin(dx + c - \arctan(-a, b))}}{\cos(dx + c - \arctan(-a, b))}\right), \sqrt{\sin(dx + c - \arctan(-a, b))} \sqrt{a^2 + b^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x)

[Out] (1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*EllipticF((1+sin(d*x+c-arctan(-a,b)))^(1/2),1/2*2^(1/2))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/2))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \cos(dx + c) + b \sin(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a \cos(c + dx) + b \sin(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(1/2), x)`

[Out] `int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \cos(c + dx) + b \sin(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(1/2), x)`

[Out] `Integral(1/sqrt(a*cos(c + d*x) + b*sin(c + d*x)), x)`

$$3.237 \quad \int \frac{1}{(a \cos(c+dx)+b \sin(c+dx))^{3/2}} dx$$

Optimal. Leaf size=138

$$\frac{2\sqrt{a \cos(c+dx)+b \sin(c+dx)} E\left(\frac{1}{2}(c+dx - \tan^{-1}(a,b))\right) \Big|_2}{d(a^2+b^2) \sqrt{\frac{a \cos(c+dx)+b \sin(c+dx)}{\sqrt{a^2+b^2}}}} - \frac{2(b \cos(c+dx) - a \sin(c+dx))}{d(a^2+b^2) \sqrt{a \cos(c+dx)+b \sin(c+dx)}}$$

[Out] $-2*(b*\cos(d*x+c)-a*\sin(d*x+c))/(a^2+b^2)/d/(a*\cos(d*x+c)+b*\sin(d*x+c))^{1/2}$
 $-2*(\cos(1/2*c+1/2*d*x-1/2*\arctan(a,b))^2)^{1/2}/\cos(1/2*c+1/2*d*x-1/2*\arctan(a,b))*\text{EllipticE}(\sin(1/2*c+1/2*d*x-1/2*\arctan(a,b)),2^{1/2})*(a*\cos(d*x+c)+b*\sin(d*x+c))^{1/2}/(a^2+b^2)/d/((a*\cos(d*x+c)+b*\sin(d*x+c))/(a^2+b^2))^{1/2}$

Rubi [A] time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3076, 3078, 2639}

$$\frac{2\sqrt{a \cos(c+dx)+b \sin(c+dx)} E\left(\frac{1}{2}(c+dx - \tan^{-1}(a,b))\right) \Big|_2}{d(a^2+b^2) \sqrt{\frac{a \cos(c+dx)+b \sin(c+dx)}{\sqrt{a^2+b^2}}}} - \frac{2(b \cos(c+dx) - a \sin(c+dx))}{d(a^2+b^2) \sqrt{a \cos(c+dx)+b \sin(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3/2), x]

[Out] $(-2*(b*\cos[c + d*x] - a*\sin[c + d*x]))/((a^2 + b^2)*d*\text{Sqrt}[a*\cos[c + d*x] + b*\sin[c + d*x]]) - (2*\text{EllipticE}[(c + d*x - \text{ArcTan}[a, b])/2, 2]*\text{Sqrt}[a*\cos[c + d*x] + b*\sin[c + d*x]])/((a^2 + b^2)*d*\text{Sqrt}[(a*\cos[c + d*x] + b*\sin[c + d*x])/\text{Sqrt}[a^2 + b^2]])$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3076

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]

Rule 3078

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 + b^2, 0] || EqQ[a^2 + b^2, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(a \cos(c + dx) + b \sin(c + dx))^{3/2}} dx &= -\frac{2(b \cos(c + dx) - a \sin(c + dx))}{(a^2 + b^2) d \sqrt{a \cos(c + dx) + b \sin(c + dx)}} - \frac{\int \sqrt{a \cos(c + dx) + b \sin(c + dx)}}{a^2 + b^2} \\ &= -\frac{2(b \cos(c + dx) - a \sin(c + dx))}{(a^2 + b^2) d \sqrt{a \cos(c + dx) + b \sin(c + dx)}} - \frac{\sqrt{a \cos(c + dx) + b \sin(c + dx)}}{(a^2 + b^2)} \\ &= -\frac{2(b \cos(c + dx) - a \sin(c + dx))}{(a^2 + b^2) d \sqrt{a \cos(c + dx) + b \sin(c + dx)}} - \frac{2E\left(\frac{1}{2}(c + dx - \tan^{-1}(a/b))\right)}{(a^2 + b^2)} \end{aligned}$$

Mathematica [C] time = 3.06, size = 219, normalized size = 1.59

$$\frac{\tan\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\sqrt{a\sqrt{\frac{b^2}{a^2}+1}\cos\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)} {}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(c+dx-\tan^{-1}\left(\frac{b}{a}\right)\right)\right)}{\sqrt{\sin^2\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)}} - \tan\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\sqrt{a\sqrt{\frac{b^2}{a^2}}}$$

$$d(a^2 + b^2)$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3/2), x]

[Out] ((-2*b*Cos[c + d*x])/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]] + (2*a*Sin[c + d*x])/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]] - Sqrt[a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]])*Tan[c + d*x - ArcTan[b/a]] + (Sqrt[a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]])*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d*x - ArcTan[b/a]]^2]*Tan[c + d*x - ArcTan[b/a]])/Sqrt[Sin[c + d*x - ArcTan[b/a]]^2])/((a^2 + b^2)*d)

fricas [F] time = 1.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a \cos(dx + c) + b \sin(dx + c)}}{2ab \cos(dx + c) \sin(dx + c) + (a^2 - b^2) \cos(dx + c)^2 + b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(a*cos(d*x + c) + b*sin(d*x + c))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cos(dx + c) + b \sin(dx + c))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2), x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(3/2), x)

maple [A] time = 0.35, size = 228, normalized size = 1.65

$$2\sqrt{1 + \sin(dx + c - \arctan(-a, b))} \sqrt{-2 \sin(dx + c - \arctan(-a, b)) + 2} \sqrt{-\sin(dx + c - \arctan(-a, b))} \text{Elliptic}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x)`

[Out] $(2*(1+\sin(d*x+c-\arctan(-a,b)))^{1/2}*(-2*\sin(d*x+c-\arctan(-a,b))+2)^{1/2}*(-\sin(d*x+c-\arctan(-a,b)))^{1/2}*\text{EllipticE}((1+\sin(d*x+c-\arctan(-a,b)))^{1/2},1/2*2^{1/2})-(1+\sin(d*x+c-\arctan(-a,b)))^{1/2}*(-2*\sin(d*x+c-\arctan(-a,b))+2)^{1/2}*(-\sin(d*x+c-\arctan(-a,b)))^{1/2}*\text{EllipticF}((1+\sin(d*x+c-\arctan(-a,b)))^{1/2},1/2*2^{1/2})-2*\cos(d*x+c-\arctan(-a,b))^2/(a^2+b^2)^{1/2}/\cos(d*x+c-\arctan(-a,b))/(\sin(d*x+c-\arctan(-a,b))*(a^2+b^2)^{1/2}))^{1/2}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cos(dx + c) + b \sin(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate((a*cos(d*x + c) + b*sin(d*x + c))^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a \cos(c + dx) + b \sin(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(3/2),x)`

[Out] `int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cos(c + dx) + b \sin(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(3/2),x)`

[Out] `Integral((a*cos(c + d*x) + b*sin(c + d*x))**(-3/2), x)`

$$3.238 \quad \int \frac{1}{(a \cos(c+dx)+b \sin(c+dx))^{5/2}} dx$$

Optimal. Leaf size=142

$$\frac{2\sqrt{\frac{a \cos(c+dx)+b \sin(c+dx)}{\sqrt{a^2+b^2}}} F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{a}{b}\right)\right)\middle|2\right)}{3d\left(a^2+b^2\right)\sqrt{a \cos(c+dx)+b \sin(c+dx)}} - \frac{2(b \cos(c+dx)-a \sin(c+dx))}{3d\left(a^2+b^2\right)\left(a \cos(c+dx)+b \sin(c+dx)\right)^{3/2}}$$

[Out] $-2/3*(b*\cos(d*x+c)-a*\sin(d*x+c))/(a^2+b^2)/d/(a*\cos(d*x+c)+b*\sin(d*x+c))^{(3/2)+2/3*(\cos(1/2*c+1/2*d*x-1/2*\arctan(a,b))^2)^{(1/2)}/\cos(1/2*c+1/2*d*x-1/2*\arctan(a,b))*\text{EllipticF}(\sin(1/2*c+1/2*d*x-1/2*\arctan(a,b)),2^{(1/2)})*((a*\cos(d*x+c)+b*\sin(d*x+c))/(a^2+b^2)^{(1/2)})^{(1/2)}/(a^2+b^2)/d/(a*\cos(d*x+c)+b*\sin(d*x+c))^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3076, 3078, 2641}

$$\frac{2\sqrt{\frac{a \cos(c+dx)+b \sin(c+dx)}{\sqrt{a^2+b^2}}} F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{a}{b}\right)\right)\middle|2\right)}{3d\left(a^2+b^2\right)\sqrt{a \cos(c+dx)+b \sin(c+dx)}} - \frac{2(b \cos(c+dx)-a \sin(c+dx))}{3d\left(a^2+b^2\right)\left(a \cos(c+dx)+b \sin(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5/2), x]

[Out] $(-2*(b*\text{Cos}[c + d*x] - a*\text{Sin}[c + d*x]))/(3*(a^2 + b^2)*d*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^{(3/2)}) + (2*\text{EllipticF}[(c + d*x - \text{ArcTan}[a, b])/2, 2]*\text{Sqrt}[(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])/ \text{Sqrt}[a^2 + b^2]])/(3*(a^2 + b^2)*d*\text{Sqrt}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])$

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3076

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]

Rule 3078

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 + b^2, 0] || EqQ[a^2 + b^2, 0])

Rubi steps

$$\int \frac{1}{(a \cos(c + dx) + b \sin(c + dx))^{5/2}} dx = -\frac{2(b \cos(c + dx) - a \sin(c + dx))}{3(a^2 + b^2)d(a \cos(c + dx) + b \sin(c + dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a \cos(c + dx) + b \sin(c + dx)}}}{3(a^2 + b^2)}$$

$$= -\frac{2(b \cos(c + dx) - a \sin(c + dx))}{3(a^2 + b^2)d(a \cos(c + dx) + b \sin(c + dx))^{3/2}} + \frac{\sqrt{\frac{a \cos(c + dx) + b \sin(c + dx)}{\sqrt{a^2 + b^2}}}}{3(a^2 + b^2)\sqrt{a^2 + b^2}}$$

$$= -\frac{2(b \cos(c + dx) - a \sin(c + dx))}{3(a^2 + b^2)d(a \cos(c + dx) + b \sin(c + dx))^{3/2}} + \frac{2F\left(\frac{1}{2}(c + dx - \arctan(-a/b))\right)}{3(a^2 + b^2)d}$$

Mathematica [C] time = 1.68, size = 145, normalized size = 1.02

$$\frac{2 \left(\frac{\tan(\tan^{-1}(\frac{a}{b}) + c + dx) \sqrt{\cos^2(\tan^{-1}(\frac{a}{b}) + c + dx)} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \sin^2(c + dx + \tan^{-1}(\frac{a}{b}))\right)}{\sqrt{b \sqrt{\frac{a^2}{b^2} + 1} \sin(\tan^{-1}(\frac{a}{b}) + c + dx)}} + \frac{a \sin(c + dx) - b \cos(c + dx)}{(a \cos(c + dx) + b \sin(c + dx))^{3/2}} \right)}{3d(a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5/2), x]

[Out] (2*((-(b*Cos[c + d*x]) + a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x])^(3/2) + (Sqrt[Cos[c + d*x + ArcTan[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[a/b]]^2]*Tan[c + d*x + ArcTan[a/b]])/Sqrt[Sqrt[1 + a^2/b^2]*b*Sin[c + d*x + ArcTan[a/b]]]))/(3*(a^2 + b^2)*d)

fricas [F] time = 1.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a \cos(dx + c) + b \sin(dx + c)}}{3ab^2 \cos(dx + c) + (a^3 - 3ab^2) \cos(dx + c)^3 + (b^3 + (3a^2b - b^3) \cos(dx + c)^2) \sin(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2), x, algorithm="fricas")

[Out] integral(sqrt(a*cos(d*x + c) + b*sin(d*x + c))/(3*a*b^2*cos(d*x + c) + (a^3 - 3*a*b^2)*cos(d*x + c)^3 + (b^3 + (3*a^2*b - b^3)*cos(d*x + c)^2)*sin(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cos(dx + c) + b \sin(dx + c))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2), x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(5/2), x)

maple [A] time = 0.34, size = 178, normalized size = 1.25

$$\sqrt{1 + \sin(dx + c - \arctan(-a/b))} \sqrt{-2 \sin(dx + c - \arctan(-a/b)) + 2} \sqrt{-\sin(dx + c - \arctan(-a/b))} \text{EllipticE}\left(\frac{dx + c - \arctan(-a/b)}{2}, \frac{1}{2}\right)$$

$$3 \sin(dx + c - \arctan(-a/b)) (a^2 + b^2) \cos(dx + c - \arctan(-a/b))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x)`

[Out] `1/3/sin(d*x+c-arctan(-a,b))/(a^2+b^2)*((1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*EllipticF((1+sin(d*x+c-arctan(-a,b)))^(1/2),1/2*2^(1/2))*sin(d*x+c-arctan(-a,b))-2*cos(d*x+c-arctan(-a,b))^2/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b)))*(a^2+b^2)^(1/2))^(1/2)/d`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cos(dx + c) + b \sin(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate((a*cos(d*x + c) + b*sin(d*x + c))^(5/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a \cos(c + dx) + b \sin(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(5/2),x)`

[Out] `int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(5/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x)`

[Out] Timed out

$$3.239 \quad \int \frac{1}{(a \cos(c+dx)+b \sin(c+dx))^{7/2}} dx$$

Optimal. Leaf size=197

$$\frac{6\sqrt{a \cos(c+dx)+b \sin(c+dx)} E\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|2\right)}{5d(a^2+b^2)^2 \sqrt{\frac{a \cos(c+dx)+b \sin(c+dx)}{\sqrt{a^2+b^2}}}} - \frac{6(b \cos(c+dx)-a \sin(c+dx))}{5d(a^2+b^2)^2 \sqrt{a \cos(c+dx)+b \sin(c+dx)}}$$

[Out] $-2/5*(b*\cos(d*x+c)-a*\sin(d*x+c))/(a^2+b^2)/d/(a*\cos(d*x+c)+b*\sin(d*x+c))^{(5/2)}-6/5*(b*\cos(d*x+c)-a*\sin(d*x+c))/(a^2+b^2)^2/d/(a*\cos(d*x+c)+b*\sin(d*x+c))^{(1/2)}-6/5*(\cos(1/2*c+1/2*d*x-1/2*\arctan(a,b))^2)^{(1/2)}/\cos(1/2*c+1/2*d*x-1/2*\arctan(a,b))*\text{EllipticE}(\sin(1/2*c+1/2*d*x-1/2*\arctan(a,b)),2^{(1/2)})*(a*\cos(d*x+c)+b*\sin(d*x+c))^{(1/2)}/(a^2+b^2)^2/d/((a*\cos(d*x+c)+b*\sin(d*x+c))/(a^2+b^2)^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3076, 3078, 2639}

$$\frac{6\sqrt{a \cos(c+dx)+b \sin(c+dx)} E\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|2\right)}{5d(a^2+b^2)^2 \sqrt{\frac{a \cos(c+dx)+b \sin(c+dx)}{\sqrt{a^2+b^2}}}} - \frac{6(b \cos(c+dx)-a \sin(c+dx))}{5d(a^2+b^2)^2 \sqrt{a \cos(c+dx)+b \sin(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-7/2), x]

[Out] $(-2*(b*\text{Cos}[c + d*x] - a*\text{Sin}[c + d*x]))/(5*(a^2 + b^2)*d*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^{(5/2)}) - (6*(b*\text{Cos}[c + d*x] - a*\text{Sin}[c + d*x]))/(5*(a^2 + b^2)^2*d*\text{Sqrt}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]]) - (6*\text{EllipticE}[(c + d*x - \text{ArcTan}[a, b])/2, 2]*\text{Sqrt}[a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x]])/(5*(a^2 + b^2)^2*d*\text{Sqrt}[(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])/ \text{Sqrt}[a^2 + b^2]])$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3076

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]

Rule 3078

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 + b^2, 0] || EqQ[a^2 + b^2, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a \cos(c + dx) + b \sin(c + dx))^{7/2}} dx &= -\frac{2(b \cos(c + dx) - a \sin(c + dx))}{5(a^2 + b^2) d(a \cos(c + dx) + b \sin(c + dx))^{5/2}} + \frac{3 \int \frac{1}{(a \cos(c + dx) + b \sin(c + dx))^{5/2}} dx}{5(a^2 + b^2)} \\
&= -\frac{2(b \cos(c + dx) - a \sin(c + dx))}{5(a^2 + b^2) d(a \cos(c + dx) + b \sin(c + dx))^{5/2}} - \frac{6(b \cos(c + dx) - a \sin(c + dx))}{5(a^2 + b^2)^2 d \sqrt{a \cos(c + dx) + b \sin(c + dx)}} \\
&= -\frac{2(b \cos(c + dx) - a \sin(c + dx))}{5(a^2 + b^2) d(a \cos(c + dx) + b \sin(c + dx))^{5/2}} - \frac{6(b \cos(c + dx) - a \sin(c + dx))}{5(a^2 + b^2)^2 d \sqrt{a \cos(c + dx) + b \sin(c + dx)}} \\
&= -\frac{2(b \cos(c + dx) - a \sin(c + dx))}{5(a^2 + b^2) d(a \cos(c + dx) + b \sin(c + dx))^{5/2}} - \frac{6(b \cos(c + dx) - a \sin(c + dx))}{5(a^2 + b^2)^2 d \sqrt{a \cos(c + dx) + b \sin(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 2.44, size = 277, normalized size = 1.41

$$\frac{\cos\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\left(3b \sin\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4}; \frac{3}{4}; \cos^2\left(c+dx-\tan^{-1}\left(\frac{b}{a}\right)\right)\right)-3\sqrt{\sin^2\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)}\left(b \sin\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)-2a \cos\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\right)\right)}{\sqrt{\sin^2\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)}\left(a\sqrt{\frac{b^2}{a^2}+1} \cos\left(-\tan^{-1}\left(\frac{b}{a}\right)+c+dx\right)\right)^{3/2}}$$

$$5bd(a^2 + b^2)$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-7/2), x]

[Out] ((-2*(3*a^2*Cos[c + d*x]^3 - a*b*Sin[c + d*x] + 6*a*b*Cos[c + d*x]^2*Sin[c + d*x] + b^2*Cos[c + d*x]*(1 + 3*Sin[c + d*x]^2)))/(a*Cos[c + d*x] + b*Sin[c + d*x])^(5/2) + (Cos[c + d*x - ArcTan[b/a]]*(3*b*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d*x - ArcTan[b/a]]^2]*Sin[c + d*x - ArcTan[b/a]] - 3*sqrt[Sin[c + d*x - ArcTan[b/a]]^2]*(-2*a*Cos[c + d*x - ArcTan[b/a]] + b*Sin[c + d*x - ArcTan[b/a]])))/((a*sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]])^(3/2)*sqrt[Sin[c + d*x - ArcTan[b/a]]^2]))/(5*b*(a^2 + b^2)*d)

fricas [F] time = 1.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a \cos(dx + c) + b \sin(dx + c)}}{(a^4 - 6a^2b^2 + b^4) \cos(dx + c)^4 + b^4 + 2(3a^2b^2 - b^4) \cos(dx + c)^2 + 4(ab^3 \cos(dx + c) + (a^3b - ab^3) \sin(dx + c))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2), x, algorithm="fricas")

[Out] integral(sqrt(a*cos(d*x + c) + b*sin(d*x + c))/((a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^4 + b^4 + 2*(3*a^2*b^2 - b^4)*cos(d*x + c)^2 + 4*(a*b^3*cos(d*x + c) + (a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cos(dx + c) + b \sin(dx + c))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2), x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-7/2), x)

maple [A] time = 0.38, size = 309, normalized size = 1.57

$$\sqrt{a^2 + b^2} \left(6\sqrt{1 + \sin(dx + c - \arctan(-a, b))} \sqrt{-2 \sin(dx + c - \arctan(-a, b))} + 2 \sqrt{-\sin(dx + c - \arctan(-a, b))} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2), x)

[Out] 1/5/sin(d*x+c-arctan(-a,b))²*(a²+b²)^(1/2)*(6*(1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*sin(d*x+c-arctan(-a,b))²*EllipticE((1+sin(d*x+c-arctan(-a,b)))^(1/2), 1/2*2^(1/2))-3*(1+sin(d*x+c-arctan(-a,b)))^(1/2)*(-2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*(-sin(d*x+c-arctan(-a,b)))^(1/2)*sin(d*x+c-arctan(-a,b))²*EllipticF((1+sin(d*x+c-arctan(-a,b)))^(1/2), 1/2*2^(1/2))+6*sin(d*x+c-arctan(-a,b))⁴-4*sin(d*x+c-arctan(-a,b))²-2)/(a⁴+2*a²*b²+b⁴)/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a²+b²)^(1/2))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cos(dx + c) + b \sin(dx + c))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2), x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a \cos(c + dx) + b \sin(c + dx))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(7/2), x)

[Out] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2), x)

[Out] Timed out

3.240 $\int (2 \cos(c + dx) + 3 \sin(c + dx))^{7/2} dx$

Optimal. Leaf size=120

$$\frac{2(3 \cos(c + dx) - 2 \sin(c + dx))(3 \sin(c + dx) + 2 \cos(c + dx))^{5/2}}{7d} - \frac{130(3 \cos(c + dx) - 2 \sin(c + dx))\sqrt{3 \sin(c + dx)}}{21d}$$

[Out] 130/21*13^(3/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*arctan(3/2))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d-2/7*(3*cos(d*x+c)-2*sin(d*x+c))*(2*cos(d*x+c)+3*sin(d*x+c))^(5/2)/d-130/21*(3*cos(d*x+c)-2*sin(d*x+c))*(2*cos(d*x+c)+3*sin(d*x+c))^(1/2)/d

Rubi [A] time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3073, 3077, 2641}

$$\frac{2(3 \cos(c + dx) - 2 \sin(c + dx))(3 \sin(c + dx) + 2 \cos(c + dx))^{5/2}}{7d} - \frac{130(3 \cos(c + dx) - 2 \sin(c + dx))\sqrt{3 \sin(c + dx)}}{21d}$$

Antiderivative was successfully verified.

[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(7/2), x]

[Out] (130*13^(3/4)*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(21*d) - (130*(3*Cos[c + d*x] - 2*Sin[c + d*x])*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]])/(21*d) - (2*(3*Cos[c + d*x] - 2*Sin[c + d*x])*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2))/(7*d)

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3073

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[(n - 1)/2] && GtQ[n, 1]

Rule 3077

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned} \int (2 \cos(c + dx) + 3 \sin(c + dx))^{7/2} dx &= -\frac{2(3 \cos(c + dx) - 2 \sin(c + dx))(2 \cos(c + dx) + 3 \sin(c + dx))^{5/2}}{7d} + \\ &= -\frac{130(3 \cos(c + dx) - 2 \sin(c + dx))\sqrt{2 \cos(c + dx) + 3 \sin(c + dx)}}{21d} \\ &= -\frac{130(3 \cos(c + dx) - 2 \sin(c + dx))\sqrt{2 \cos(c + dx) + 3 \sin(c + dx)}}{21d} \\ &= \frac{130 \cdot 13^{3/4} F\left(\frac{1}{2}\left(c + dx - \tan^{-1}\left(\frac{2}{3}\right)\right) \middle| 2\right)}{21d} - \frac{130(3 \cos(c + dx) - 2 \sin(c + dx))\sqrt{2 \cos(c + dx) + 3 \sin(c + dx)}}{21d} \end{aligned}$$

Mathematica [C] time = 0.48, size = 153, normalized size = 1.28

$$260 \cdot 13^{3/4} \sqrt{-\left(\sin\left(c + dx + \tan^{-1}\left(\frac{2}{3}\right)\right) - 1\right) \sin\left(c + dx + \tan^{-1}\left(\frac{2}{3}\right)\right)} \sqrt{\sin\left(c + dx + \tan^{-1}\left(\frac{2}{3}\right)\right) + 1} \sec\left(c + dx + \tan^{-1}\left(\frac{2}{3}\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(7/2), x]

[Out] (- (Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]]*(897*Cos[c + d*x] + 27*Cos[3*(c + d*x)] - 598*Sin[c + d*x] + 138*Sin[3*(c + d*x)])) + 260*13^(3/4)*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3]]^2]*Sec[c + d*x + ArcTan[2/3]]*Sqrt[-((-1 + Sin[c + d*x + ArcTan[2/3]])*Sin[c + d*x + ArcTan[2/3]])]*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]])]/(42*d)

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(46 \cos(dx + c)\right)^3 - 9\left(\cos(dx + c)^2 + 3\right) \sin(dx + c) - 54 \cos(dx + c)\right) \sqrt{2 \cos(dx + c) + 3 \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(7/2), x, algorithm="fricas")

[Out] integral(- (46*cos(d*x + c))^3 - 9*(cos(d*x + c)^2 + 3)*sin(d*x + c) - 54*cos(d*x + c))*sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2 \cos(dx + c) + 3 \sin(dx + c))^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(7/2), x, algorithm="giac")

[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(7/2), x)

maple [A] time = 0.35, size = 128, normalized size = 1.07

$$\frac{338 \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right) \cos^4\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)}{7} + \frac{845 \sqrt{1 + \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)} \sqrt{-2 \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right) + 2} \sqrt{-\sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)}}{21} \cos\left(dx + c + \arctan\left(\frac{2}{3}\right)\right) \sqrt{\sqrt{13} \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x)

[Out] (338/7*sin(d*x+c+arctan(2/3))*cos(d*x+c+arctan(2/3))^4+845/21*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((1+sin(d*x+c+arctan(2/3)))^(1/2),1/2*2^(1/2))-2704/21*cos(d*x+c+arctan(2/3))^2*sin(d*x+c+arctan(2/3)))/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2 \cos(dx + c) + 3 \sin(dx + c))^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="maxima")

[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2 \cos(c + dx) + 3 \sin(c + dx))^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*cos(c + d*x) + 3*sin(c + d*x))^(7/2),x)

[Out] int((2*cos(c + d*x) + 3*sin(c + d*x))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**(7/2),x)

[Out] Timed out

3.241 $\int (2 \cos(c + dx) + 3 \sin(c + dx))^{5/2} dx$

Optimal. Leaf size=75

$$\frac{78\sqrt[4]{13} E\left(\frac{1}{2}\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right)\middle| 2\right)}{5d} - \frac{2(3 \cos(c + dx) - 2 \sin(c + dx))(3 \sin(c + dx) + 2 \cos(c + dx))^{3/2}}{5d}$$

[Out] 78/5*13^(1/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*arctan(3/2))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d-2/5*(3*cos(d*x+c)-2*sin(d*x+c))*(2*cos(d*x+c)+3*sin(d*x+c))^(3/2)/d

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3073, 3077, 2639}

$$\frac{78\sqrt[4]{13} E\left(\frac{1}{2}\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right)\middle| 2\right)}{5d} - \frac{2(3 \cos(c + dx) - 2 \sin(c + dx))(3 \sin(c + dx) + 2 \cos(c + dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2), x]

[Out] (78*13^(1/4)*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/(5*d) - (2*(3*Cos[c + d*x] - 2*Sin[c + d*x])*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))/(5*d)

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3073

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n - 1)/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[(n - 1)/2] && GtQ[n, 1]

Rule 3077

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned} \int (2 \cos(c + dx) + 3 \sin(c + dx))^{5/2} dx &= -\frac{2(3 \cos(c + dx) - 2 \sin(c + dx))(2 \cos(c + dx) + 3 \sin(c + dx))^{3/2}}{5d} + \\ &= -\frac{2(3 \cos(c + dx) - 2 \sin(c + dx))(2 \cos(c + dx) + 3 \sin(c + dx))^{3/2}}{5d} + \\ &= \frac{78\sqrt[4]{13} E\left(\frac{1}{2}\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right)\middle| 2\right)}{5d} - \frac{2(3 \cos(c + dx) - 2 \sin(c + dx))(3 \sin(c + dx) + 2 \cos(c + dx))^{3/2}}{5d} \end{aligned}$$

Mathematica [C] time = 0.82, size = 199, normalized size = 2.65

$$\frac{39\sqrt[4]{13} \sin\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4}; \frac{3}{4}; \cos^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{\sqrt{-\left(\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)-1\right) \cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)} \sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)+1}} + \sqrt{3 \sin(c+dx) + 2 \cos(c+dx)} (-5 \sin(2(c+dx)))$$

$$5d$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2), x]

[Out] (Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]]*(52 - 12*Cos[2*(c + d*x)] - 5*Sin[2*(c + d*x)]) - (13*13^(1/4)*(4*Cos[c + d*x - ArcTan[3/2]] - 3*Sin[c + d*x - ArcTan[3/2]]))/Sqrt[Cos[c + d*x - ArcTan[3/2]]] - (39*13^(1/4)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d*x - ArcTan[3/2]]^2*Sin[c + d*x - ArcTan[3/2]]]/(Sqrt[-((-1 + Cos[c + d*x - ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/2]])]*Sqrt[1 + Cos[c + d*x - ArcTan[3/2]]])))/(5*d)

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(5 \cos(dx+c)^2 - 12 \cos(dx+c) \sin(dx+c) - 9\right) \sqrt{2 \cos(dx+c) + 3 \sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(5/2), x, algorithm="fricas")

[Out] integral(-(5*cos(d*x + c))^2 - 12*cos(d*x + c)*sin(d*x + c) - 9)*sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2 \cos(dx+c) + 3 \sin(dx+c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(5/2), x, algorithm="giac")

[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(5/2), x)

maple [A] time = 0.37, size = 174, normalized size = 2.32

$$13\sqrt{13} \left(6\sqrt{1 + \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)} \sqrt{-2 \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)} + 2 \sqrt{-\sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*cos(d*x+c)+3*sin(d*x+c))^(5/2), x)

[Out] -13/5*13^(1/2)*(6*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticE((1+sin(d*x+c+arctan(2/3)))^(1/2), 1/2*2^(1/2))-3*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((1+sin(d*x+c+arctan(2/3)))^(1/2), 1/2*2^(1/2))-2*sin(d*x+c+arctan(2/3))^4+2*sin(d*x+c+arctan(2/3))^2)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2 \cos(dx+c) + 3 \sin(dx+c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2 \cos(c + dx) + 3 \sin(c + dx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(c + d*x) + 3*sin(c + d*x))^(5/2),x)
```

```
[Out] int((2*cos(c + d*x) + 3*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.242 $\int (2 \cos(c + dx) + 3 \sin(c + dx))^{3/2} dx$

Optimal. Leaf size=75

$$\frac{2 \cdot 13^{3/4} F\left(\frac{1}{2}\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right) \middle| 2\right)}{3d} - \frac{2(3 \cos(c + dx) - 2 \sin(c + dx))\sqrt{3 \sin(c + dx) + 2 \cos(c + dx)}}{3d}$$

[Out] $2/3 \cdot 13^{3/4} \cdot (\cos(1/2 \cdot c + 1/2 \cdot d \cdot x - 1/2 \cdot \arctan(3/2))^2)^{1/2} / \cos(1/2 \cdot c + 1/2 \cdot d \cdot x - 1/2 \cdot \arctan(3/2)) \cdot \text{EllipticF}(\sin(1/2 \cdot c + 1/2 \cdot d \cdot x - 1/2 \cdot \arctan(3/2)), 2^{1/2}) / d - 2(3 \cos(d \cdot x + c) - 2 \sin(d \cdot x + c)) \cdot (2 \cos(d \cdot x + c) + 3 \sin(d \cdot x + c))^{1/2} / d$

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3073, 3077, 2641}

$$\frac{2 \cdot 13^{3/4} F\left(\frac{1}{2}\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right) \middle| 2\right)}{3d} - \frac{2(3 \cos(c + dx) - 2 \sin(c + dx))\sqrt{3 \sin(c + dx) + 2 \cos(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2 \cdot \text{Cos}[c + d \cdot x] + 3 \cdot \text{Sin}[c + d \cdot x])^{3/2}, x]$

[Out] $(2 \cdot 13^{3/4} \cdot \text{EllipticF}[(c + d \cdot x - \text{ArcTan}[3/2])/2, 2]) / (3 \cdot d) - (2 \cdot (3 \cdot \text{Cos}[c + d \cdot x] - 2 \cdot \text{Sin}[c + d \cdot x]) \cdot \text{Sqrt}[2 \cdot \text{Cos}[c + d \cdot x] + 3 \cdot \text{Sin}[c + d \cdot x]]) / (3 \cdot d)$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.) \cdot (x_)]], x_Symbol] \rightarrow \text{Simp}[(2 \cdot \text{EllipticF}[(1 \cdot (c - \text{Pi}/2 + d \cdot x))/2, 2]) / d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3073

$\text{Int}[(\cos[(c_.) + (d_.) \cdot (x_)] \cdot (a_.) + (b_.) \cdot \sin[(c_.) + (d_.) \cdot (x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b \cdot \text{Cos}[c + d \cdot x] - a \cdot \text{Sin}[c + d \cdot x]) \cdot (a \cdot \text{Cos}[c + d \cdot x] + b \cdot \text{Sin}[c + d \cdot x])^{(n-1)} / (d \cdot n), x] + \text{Dist}[(n-1) \cdot (a^2 + b^2) / n, \text{Int}[(a \cdot \text{Cos}[c + d \cdot x] + b \cdot \text{Sin}[c + d \cdot x])^{(n-2)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ !\text{IntegerQ}[(n-1)/2] \ \&\& \ \text{GtQ}[n, 1]$

Rule 3077

$\text{Int}[(\cos[(c_.) + (d_.) \cdot (x_)] \cdot (a_.) + (b_.) \cdot \sin[(c_.) + (d_.) \cdot (x_)])^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(a^2 + b^2)^{(n/2)}, \text{Int}[\text{Cos}[c + d \cdot x - \text{ArcTan}[a, b]]^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ !(\text{GeQ}[n, 1] \ || \ \text{LeQ}[n, -1]) \ \&\& \ \text{GtQ}[a^2 + b^2, 0]$

Rubi steps

$$\begin{aligned} \int (2 \cos(c + dx) + 3 \sin(c + dx))^{3/2} dx &= -\frac{2(3 \cos(c + dx) - 2 \sin(c + dx))\sqrt{2 \cos(c + dx) + 3 \sin(c + dx)}}{3d} + \frac{13}{3} \\ &= -\frac{2(3 \cos(c + dx) - 2 \sin(c + dx))\sqrt{2 \cos(c + dx) + 3 \sin(c + dx)}}{3d} + \frac{1}{3} \cdot 13 \\ &= \frac{2 \cdot 13^{3/4} F\left(\frac{1}{2}\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right) \middle| 2\right)}{3d} - \frac{2(3 \cos(c + dx) - 2 \sin(c + dx))\sqrt{3 \sin(c + dx) + 2 \cos(c + dx)}}{3d} \end{aligned}$$

Mathematica [C] time = 0.30, size = 133, normalized size = 1.77

$$2 \cdot 13^{3/4} \sqrt{-\left(\sin\left(c + dx + \tan^{-1}\left(\frac{2}{3}\right)\right) - 1\right) \sin\left(c + dx + \tan^{-1}\left(\frac{2}{3}\right)\right)} \sqrt{\sin\left(c + dx + \tan^{-1}\left(\frac{2}{3}\right)\right) + 1} \sec\left(c + dx + \tan^{-1}\left(\frac{2}{3}\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2), x]

[Out] (2*(-3*Cos[c + d*x] + 2*Sin[c + d*x])*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]] + 2*13^(3/4)*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3]]]^2)*Sec[c + d*x + ArcTan[2/3]]*Sqrt[-((-1 + Sin[c + d*x + ArcTan[2/3]])*Sin[c + d*x + ArcTan[2/3]])]*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]])]/(3*d)

fricas [F] time = 1.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(2 \cos(dx + c) + 3 \sin(dx + c)\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(3/2), x, algorithm="fricas")

[Out] integral((2*cos(d*x + c) + 3*sin(d*x + c))^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2 \cos(dx + c) + 3 \sin(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(3/2), x, algorithm="giac")

[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(3/2), x)

maple [A] time = 0.34, size = 108, normalized size = 1.44

$$\frac{13 \sqrt{1 + \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)} \sqrt{-2 \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right) + 2} \sqrt{-\sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)} \text{EllipticF}\left(\sqrt{1 + \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)}, \frac{\sqrt{2}}{2}\right) - 26 \left(\cos\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)\right)^{\frac{3}{2}}}{3 \cos\left(dx + c + \arctan\left(\frac{2}{3}\right)\right) \sqrt{\sqrt{13} \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*cos(d*x+c)+3*sin(d*x+c))^(3/2), x)

[Out] (13/3*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((1+sin(d*x+c+arctan(2/3)))^(1/2), 1/2*2^(1/2))-26/3*cos(d*x+c+arctan(2/3))^2*sin(d*x+c+arctan(2/3)))/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2 \cos(dx + c) + 3 \sin(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (2 \cos(c + dx) + 3 \sin(c + dx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*cos(c + d*x) + 3*sin(c + d*x))^(3/2), x)

[Out] int((2*cos(c + d*x) + 3*sin(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**(3/2), x)

[Out] Timed out

3.243 $\int \sqrt{2 \cos(c + dx) + 3 \sin(c + dx)} dx$

Optimal. Leaf size=27

$$\frac{2\sqrt[4]{13} E\left(\frac{1}{2}\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right) \middle| 2\right)}{d}$$

[Out] $2*13^{(1/4)}*(\cos(1/2*c+1/2*d*x-1/2*\arctan(3/2))^2)^{(1/2)}/\cos(1/2*c+1/2*d*x-1/2*\arctan(3/2))*\text{EllipticE}(\sin(1/2*c+1/2*d*x-1/2*\arctan(3/2)),2^{(1/2)})/d$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3077, 2639}

$$\frac{2\sqrt[4]{13} E\left(\frac{1}{2}\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right) \middle| 2\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]

[Out] $(2*13^{(1/4)}*\text{EllipticE}[(c + d*x - \text{ArcTan}[3/2])/2, 2])/d$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3077

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{2 \cos(c + dx) + 3 \sin(c + dx)} dx &= \sqrt[4]{13} \int \sqrt{\cos\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right)} dx \\ &= \frac{2\sqrt[4]{13} E\left(\frac{1}{2}\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right) \middle| 2\right)}{d} \end{aligned}$$

Mathematica [C] time = 0.81, size = 184, normalized size = 6.81

$$\frac{3\sqrt[4]{13} \sin\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4}; \frac{3}{4}; \cos^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{\sqrt{-\left(\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)-1\right)\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)+1}} + 4\sqrt{3 \sin(c + dx) + 2 \cos(c + dx)} - 4\sqrt[4]{13} \sqrt{}}{3d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]

[Out] $(-4*13^{(1/4)}*\text{Sqrt}[\text{Cos}[c + d*x - \text{ArcTan}[3/2]]] + 4*\text{Sqrt}[2*\text{Cos}[c + d*x] + 3*\text{Sin}[c + d*x]] + (3*13^{(1/4)}*\text{Sin}[c + d*x - \text{ArcTan}[3/2]]))/\text{Sqrt}[\text{Cos}[c + d*x - A$

`rcTan[3/2]]] - (3*13^(1/4)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/(Sqrt[-((-1 + Cos[c + d*x - ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/2]])]*Sqrt[1 + Cos[c + d*x - ArcTan[3/2]])])/(3*d)`

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{2 \cos(dx + c) + 3 \sin(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2 \cos(dx + c) + 3 \sin(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)`

maple [A] time = 0.33, size = 112, normalized size = 4.15

$$\frac{\sqrt{13} \sqrt{1 + \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)} \sqrt{-2 \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right) + 2} \sqrt{-\sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)} \left(2 \text{EllipticE}\left(\arctan\left(\frac{2}{3}\right)\right) - \text{EllipticF}\left(\arctan\left(\frac{2}{3}\right)\right)\right)}{\cos\left(dx + c + \arctan\left(\frac{2}{3}\right)\right) \sqrt{\sqrt{13} \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x)`

[Out] `-13^(1/2)*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*(2*EllipticE((1+sin(d*x+c+arctan(2/3)))^(1/2),1/2*2^(1/2))-EllipticF((1+sin(d*x+c+arctan(2/3)))^(1/2),1/2*2^(1/2)))/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2 \cos(dx + c) + 3 \sin(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{2 \cos(c + dx) + 3 \sin(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*cos(c + d*x) + 3*sin(c + d*x))^(1/2),x)`

[Out] `int((2*cos(c + d*x) + 3*sin(c + d*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3 \sin(c + dx) + 2 \cos(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(3*sin(c + d*x) + 2*cos(c + d*x)), x)
```

$$3.244 \quad \int \frac{1}{\sqrt{2 \cos(c+dx)+3 \sin(c+dx)}} dx$$

Optimal. Leaf size=27

$$\frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{\sqrt[4]{13}d}$$

[Out] $2/13*13^{(3/4)}*(\cos(1/2*c+1/2*d*x-1/2*\arctan(3/2))^2)^{(1/2)}/\cos(1/2*c+1/2*d*x-1/2*\arctan(3/2))*\text{EllipticF}(\sin(1/2*c+1/2*d*x-1/2*\arctan(3/2)),2^{(1/2)})/d$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3077, 2641}

$$\frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{\sqrt[4]{13}d}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]

[Out] (2*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(13^(1/4)*d)

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3077

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{2 \cos(c+dx)+3 \sin(c+dx)}} dx &= \frac{\int \frac{1}{\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}} dx}{\sqrt[4]{13}} \\ &= \frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{\sqrt[4]{13}d} \end{aligned}$$

Mathematica [C] time = 0.09, size = 88, normalized size = 3.26

$$\frac{2\sqrt{-\left(\left(\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)-1\right)\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)\right)}\sqrt{\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)+1}\sec\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)}{\sqrt[4]{13}d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]

[Out] (2*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3]]^2]*Sec[c + d*x + ArcTan[2/3]]*Sqrt[-((-1 + Sin[c + d*x + ArcTan[2/3]])*Sin[c + d*x + ArcTan[2/3]])]*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]])/(13^(1/4)*d)

fricas [F] time = 1.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2 \cos(dx + c) + 3 \sin(dx + c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(1/sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2 \cos(dx + c) + 3 \sin(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)

maple [A] time = 0.24, size = 85, normalized size = 3.15

$$\frac{\sqrt{1 + \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)} \sqrt{-2 \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right) + 2} \sqrt{-\sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)} \text{EllipticF}}{\cos\left(dx + c + \arctan\left(\frac{2}{3}\right)\right) \sqrt{\sqrt{13} \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x)

[Out] (1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((1+sin(d*x+c+arctan(2/3)))^(1/2),1/2*2^(1/2))/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2 \cos(dx + c) + 3 \sin(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{2 \cos(c + dx) + 3 \sin(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(1/2),x)

[Out] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3 \sin(c + dx) + 2 \cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*sin(c + d*x) + 2*cos(c + d*x)), x)
```


$$3.245 \quad \int \frac{1}{(2 \cos(c+dx)+3 \sin(c+dx))^{3/2}} dx$$

Optimal. Leaf size=73

$$\frac{2(3 \cos(c+dx) - 2 \sin(c+dx))}{13d\sqrt{3 \sin(c+dx) + 2 \cos(c+dx)}} - \frac{2E\left(\frac{1}{2}\left(c+dx - \tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{13^{3/4}d}$$

[Out] $-2/13*13^{(1/4)}*(\cos(1/2*c+1/2*d*x-1/2*\arctan(3/2))^{(1/2)})^{(1/2)}/\cos(1/2*c+1/2*d*x-1/2*\arctan(3/2))*\text{EllipticE}(\sin(1/2*c+1/2*d*x-1/2*\arctan(3/2)),2^{(1/2)})/d - 2/13*(3*\cos(d*x+c)-2*\sin(d*x+c))/d/(2*\cos(d*x+c)+3*\sin(d*x+c))^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3076, 3077, 2639}

$$\frac{2(3 \cos(c+dx) - 2 \sin(c+dx))}{13d\sqrt{3 \sin(c+dx) + 2 \cos(c+dx)}} - \frac{2E\left(\frac{1}{2}\left(c+dx - \tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{13^{3/4}d}$$

Antiderivative was successfully verified.

[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-3/2), x]

[Out] $(-2*\text{EllipticE}[(c + d*x - \text{ArcTan}[3/2])/2, 2])/(13^{(3/4)}*d) - (2*(3*\text{Cos}[c + d*x] - 2*\text{Sin}[c + d*x]))/(13*d*\text{Sqrt}[2*\text{Cos}[c + d*x] + 3*\text{Sin}[c + d*x]])$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3076

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[((b*cos[c + d*x] - a*sin[c + d*x])*(a*cos[c + d*x] + b*sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^2 + b^2)), Int[(a*cos[c + d*x] + b*sin[c + d*x])^(n + 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]

Rule 3077

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(2 \cos(c+dx)+3 \sin(c+dx))^{3/2}} dx &= -\frac{2(3 \cos(c+dx) - 2 \sin(c+dx))}{13d\sqrt{2 \cos(c+dx) + 3 \sin(c+dx)}} - \frac{1}{13} \int \sqrt{2 \cos(c+dx) + 3 \sin(c+dx)} dx \\ &= -\frac{2(3 \cos(c+dx) - 2 \sin(c+dx))}{13d\sqrt{2 \cos(c+dx) + 3 \sin(c+dx)}} - \frac{\int \sqrt{\cos\left(c+dx - \tan^{-1}\left(\frac{3}{2}\right)\right)} dx}{13^{3/4}} \\ &= -\frac{2E\left(\frac{1}{2}\left(c+dx - \tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{13^{3/4}d} - \frac{2(3 \cos(c+dx) - 2 \sin(c+dx))}{13d\sqrt{2 \cos(c+dx) + 3 \sin(c+dx)}} \end{aligned}$$

Mathematica [C] time = 1.03, size = 190, normalized size = 2.60

$$\frac{3 \sin\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4}; \frac{3}{4}; \cos^2\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{13^{3/4} \sqrt{-\left(\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)-1\right) \cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)} \sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)+1}} - \frac{2 \cos(c+dx)}{\sqrt{3} \sin(c+dx)+2 \cos(c+dx)} + \frac{4 \sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}}{13^{3/4}}$$

$$3d$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-3/2), x]

[Out] ((4*sqrt[Cos[c + d*x - ArcTan[3/2]]])/13^(3/4) - (2*Cos[c + d*x])/sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]] - (3*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*sqrt[Cos[c + d*x - ArcTan[3/2]]]) + (3*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d*x - ArcTan[3/2]]^2*Sin[c + d*x - ArcTan[3/2]]]/(13^(3/4)*sqrt[-(-1 + Cos[c + d*x - ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/2]]])*sqrt[1 + Cos[c + d*x - ArcTan[3/2]]])/(3*d)

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{2} \cos(dx+c) + 3 \sin(dx+c)}{5 \cos(dx+c)^2 - 12 \cos(dx+c) \sin(dx+c) - 9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2), x, algorithm="fricas")

[Out] integral(-sqrt(2*cos(d*x + c) + 3*sin(d*x + c))/(5*cos(d*x + c)^2 - 12*cos(d*x + c)*sin(d*x + c) - 9), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2 \cos(dx+c) + 3 \sin(dx+c))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2), x, algorithm="giac")

[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(3/2), x)

maple [A] time = 0.34, size = 162, normalized size = 2.22

$$\sqrt{13} \left(2 \sqrt{1 + \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)} \sqrt{-2 \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right) + 2} \sqrt{-\sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)} \right) \text{Ellip}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2), x)

[Out] 1/13*13^(1/2)*(2*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticE((1+sin(d*x+c+arctan(2/3)))^(1/2), 1/2*2^(1/2))-(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((1+sin(d*x+c+arctan(2/3)))^(1/2), 1/2*2^(1/2))-2*cos(d*x+c+arctan(2/3))^2/cos(d*x+c+arctan(2/3)))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2 \cos(dx+c) + 3 \sin(dx+c))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(2 \cos(c + dx) + 3 \sin(c + dx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(3/2),x)

[Out] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(3/2),x)

[Out] Timed out

$$3.246 \quad \int \frac{1}{(2 \cos(c+dx)+3 \sin(c+dx))^{5/2}} dx$$

Optimal. Leaf size=75

$$\frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{39\sqrt[4]{13}d} - \frac{2(3 \cos(c+dx) - 2 \sin(c+dx))}{39d(3 \sin(c+dx) + 2 \cos(c+dx))^{3/2}}$$

[Out] $2/507*13^{(3/4)}*(\cos(1/2*c+1/2*d*x-1/2*\arctan(3/2))^2)^{(1/2)}/\cos(1/2*c+1/2*d*x-1/2*\arctan(3/2))*\text{EllipticF}(\sin(1/2*c+1/2*d*x-1/2*\arctan(3/2)),2^{(1/2)})/d-2/39*(3*\cos(d*x+c)-2*\sin(d*x+c))/d/(2*\cos(d*x+c)+3*\sin(d*x+c))^{(3/2)}$

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3076, 3077, 2641}

$$\frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{39\sqrt[4]{13}d} - \frac{2(3 \cos(c+dx) - 2 \sin(c+dx))}{39d(3 \sin(c+dx) + 2 \cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-5/2),x]

[Out] (2*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(39*13^(1/4)*d) - (2*(3*Cos[c + d*x] - 2*Sin[c + d*x]))/(39*d*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3076

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]

Rule 3077

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(2 \cos(c+dx) + 3 \sin(c+dx))^{5/2}} dx &= -\frac{2(3 \cos(c+dx) - 2 \sin(c+dx))}{39d(2 \cos(c+dx) + 3 \sin(c+dx))^{3/2}} + \frac{1}{39} \int \frac{1}{\sqrt{2 \cos(c+dx) + 3 \sin(c+dx)}} dx \\ &= -\frac{2(3 \cos(c+dx) - 2 \sin(c+dx))}{39d(2 \cos(c+dx) + 3 \sin(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{\cos(c+dx-\tan^{-1}(\frac{3}{2}))}} dx}{39\sqrt[4]{13}} \\ &= \frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{39\sqrt[4]{13}d} - \frac{2(3 \cos(c+dx) - 2 \sin(c+dx))}{39d(2 \cos(c+dx) + 3 \sin(c+dx))^{3/2}} \end{aligned}$$

Mathematica [C] time = 0.71, size = 157, normalized size = 2.09

$$\frac{\sqrt{2}13^{3/4}\sqrt{\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)+1}\left(3\sin(c+dx)+2\cos(c+dx)\right)^{3/2}\sec\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)\sqrt{2\sin\left(c+dx+\tan^{-1}\left(\frac{2}{3}\right)\right)}}{507d(3\sin(c+dx)+2\cos(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-5/2), x]

[Out] (-78*Cos[c + d*x] + 52*Sin[c + d*x] + Sqrt[2]*13^(3/4)*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3]]^2]*Sec[c + d*x + ArcTan[2/3]]*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2)*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]]*Sqrt[-1 + Cos[2*(c + d*x + ArcTan[2/3])] + 2*Sin[c + d*x + ArcTan[2/3]]])/(507*d*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{2}\cos(dx+c)+3\sin(dx+c)}{46\cos(dx+c)^3-9(\cos(dx+c)^2+3)\sin(dx+c)-54\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2), x, algorithm="fricas")

[Out] integral(-sqrt(2*cos(d*x + c) + 3*sin(d*x + c))/(46*cos(d*x + c)^3 - 9*(cos(d*x + c)^2 + 3)*sin(d*x + c) - 54*cos(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2\cos(dx+c)+3\sin(dx+c))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2), x, algorithm="giac")

[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(5/2), x)

maple [A] time = 0.33, size = 118, normalized size = 1.57

$$\frac{\sqrt{1+\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\text{EllipticF}\left(\frac{2}{3}, \frac{1}{2}\right)}{39\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2), x)

[Out] 1/39/sin(d*x+c+arctan(2/3))*((1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((1+sin(d*x+c+arctan(2/3)))^(1/2), 1/2*2^(1/2))*sin(d*x+c+arctan(2/3))-2*cos(d*x+c+arctan(2/3))^2)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2\cos(dx+c)+3\sin(dx+c))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(2 \cos(c + dx) + 3 \sin(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(5/2),x)

[Out] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(5/2),x)

[Out] Timed out

$$3.247 \quad \int \frac{1}{(2 \cos(c+dx)+3 \sin(c+dx))^{7/2}} dx$$

Optimal. Leaf size=120

$$\frac{6(3 \cos(c+dx) - 2 \sin(c+dx))}{845d\sqrt{3 \sin(c+dx) + 2 \cos(c+dx)}} - \frac{2(3 \cos(c+dx) - 2 \sin(c+dx))}{65d(3 \sin(c+dx) + 2 \cos(c+dx))^{5/2}} - \frac{6E\left(\frac{1}{2}\left(c+dx - \tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{65 \cdot 13^{3/4}d}$$

[Out] $-6/845 \cdot 13^{1/4} \cdot (\cos(1/2 \cdot c + 1/2 \cdot d \cdot x - 1/2 \cdot \arctan(3/2))^2)^{1/2} / \cos(1/2 \cdot c + 1/2 \cdot d \cdot x - 1/2 \cdot \arctan(3/2)) \cdot \text{EllipticE}(\sin(1/2 \cdot c + 1/2 \cdot d \cdot x - 1/2 \cdot \arctan(3/2)), 2^{1/2}) / d - 2/65 \cdot (3 \cdot \cos(d \cdot x + c) - 2 \cdot \sin(d \cdot x + c)) / d / (2 \cdot \cos(d \cdot x + c) + 3 \cdot \sin(d \cdot x + c))^{5/2} - 6/845 \cdot (3 \cdot \cos(d \cdot x + c) - 2 \cdot \sin(d \cdot x + c)) / d / (2 \cdot \cos(d \cdot x + c) + 3 \cdot \sin(d \cdot x + c))^{1/2}$

Rubi [A] time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3076, 3077, 2639}

$$\frac{6(3 \cos(c+dx) - 2 \sin(c+dx))}{845d\sqrt{3 \sin(c+dx) + 2 \cos(c+dx)}} - \frac{2(3 \cos(c+dx) - 2 \sin(c+dx))}{65d(3 \sin(c+dx) + 2 \cos(c+dx))^{5/2}} - \frac{6E\left(\frac{1}{2}\left(c+dx - \tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{65 \cdot 13^{3/4}d}$$

Antiderivative was successfully verified.

[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-7/2), x]

[Out] $(-6 \cdot \text{EllipticE}[(c + d \cdot x - \text{ArcTan}[3/2])/2, 2]) / (65 \cdot 13^{3/4} \cdot d) - (2 \cdot (3 \cdot \text{Cos}[c + d \cdot x] - 2 \cdot \text{Sin}[c + d \cdot x])) / (65 \cdot d \cdot (2 \cdot \text{Cos}[c + d \cdot x] + 3 \cdot \text{Sin}[c + d \cdot x])^{5/2}) - (6 \cdot (3 \cdot \text{Cos}[c + d \cdot x] - 2 \cdot \text{Sin}[c + d \cdot x])) / (845 \cdot d \cdot \text{Sqrt}[2 \cdot \text{Cos}[c + d \cdot x] + 3 \cdot \text{Sin}[c + d \cdot x]])$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 3076

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]

Rule 3077

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2 \cos(c + dx) + 3 \sin(c + dx))^{7/2}} dx &= -\frac{2(3 \cos(c + dx) - 2 \sin(c + dx))}{65d(2 \cos(c + dx) + 3 \sin(c + dx))^{5/2}} + \frac{3}{65} \int \frac{1}{(2 \cos(c + dx) + 3 \sin(c + dx))^{5/2}} dx \\
&= -\frac{2(3 \cos(c + dx) - 2 \sin(c + dx))}{65d(2 \cos(c + dx) + 3 \sin(c + dx))^{5/2}} - \frac{6(3 \cos(c + dx) - 2 \sin(c + dx))}{845d\sqrt{2 \cos(c + dx) + 3 \sin(c + dx)}} \\
&= -\frac{2(3 \cos(c + dx) - 2 \sin(c + dx))}{65d(2 \cos(c + dx) + 3 \sin(c + dx))^{5/2}} - \frac{6(3 \cos(c + dx) - 2 \sin(c + dx))}{845d\sqrt{2 \cos(c + dx) + 3 \sin(c + dx)}} \\
&= -\frac{6E\left(\frac{1}{2}\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right)\middle| 2\right)}{65 \cdot 13^{3/4}d} - \frac{2(3 \cos(c + dx) - 2 \sin(c + dx))}{65d(2 \cos(c + dx) + 3 \sin(c + dx))^{5/2}}
\end{aligned}$$

Mathematica [C] time = 1.95, size = 224, normalized size = 1.87

$$\frac{3 \sin\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4}; \frac{3}{4}; \cos^2\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right)\right)}{13^{3/4} \sqrt{-\left(\cos\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right) - 1\right) \cos\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right)} \sqrt{\cos\left(c + dx - \tan^{-1}\left(\frac{3}{2}\right)\right) + 1}} + \frac{-4(\sin(c + dx) + 3 \sin(3(c + dx))) - 33 \cos(c + dx) + 5 \cos(3(c + dx))}{2(3 \sin(c + dx) + 2 \cos(c + dx))^{5/2}}$$

$65d$

Warning: Unable to verify antiderivative.

[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-7/2), x]

[Out] ((4*Sqrt[Cos[c + d*x - ArcTan[3/2]]])/13^(3/4) + (-33*Cos[c + d*x] + 5*Cos[3*(c + d*x)] - 4*(Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(2*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2)) - (3*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqrt[Cos[c + d*x - ArcTan[3/2]]]) + (3*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqrt[-((-1 + Cos[c + d*x - ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/2]])]*Sqrt[1 + Cos[c + d*x - ArcTan[3/2]]]))/(65*d)

fricas [F] time = 1.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{2 \cos(dx + c) + 3 \sin(dx + c)}}{119 \cos(dx + c)^4 - 54 \cos(dx + c)^2 + 24(5 \cos(dx + c)^3 - 9 \cos(dx + c)) \sin(dx + c) - 81}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2), x, algorithm="fricas")

[Out] integral(-sqrt(2*cos(d*x + c) + 3*sin(d*x + c))/(119*cos(d*x + c)^4 - 54*cos(d*x + c)^2 + 24*(5*cos(d*x + c)^3 - 9*cos(d*x + c))*sin(d*x + c) - 81), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2 \cos(dx + c) + 3 \sin(dx + c))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2), x, algorithm="giac")

[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(7/2), x)

maple [A] time = 0.35, size = 205, normalized size = 1.71

$$\sqrt{13} \left(6\sqrt{1 + \sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)} \sqrt{-2\sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right) + 2} \sqrt{-\sin\left(dx + c + \arctan\left(\frac{2}{3}\right)\right)} \right) \left(\dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x)

[Out] 1/845*13^(1/2)/sin(d*x+c+arctan(2/3))^2*(6*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*sin(d*x+c+arctan(2/3))^2*EllipticE((1+sin(d*x+c+arctan(2/3)))^(1/2),1/2*2^(1/2))-3*(1+sin(d*x+c+arctan(2/3)))^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*sin(d*x+c+arctan(2/3))^2*EllipticF((1+sin(d*x+c+arctan(2/3)))^(1/2),1/2*2^(1/2))+6*sin(d*x+c+arctan(2/3))^4-4*sin(d*x+c+arctan(2/3))^2-2)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2 \cos(dx + c) + 3 \sin(dx + c))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="maxima")

[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(2 \cos(c + dx) + 3 \sin(c + dx))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(7/2),x)

[Out] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(7/2),x)

[Out] Timed out

3.248 $\int (a \cos(c + dx) + ia \sin(c + dx))^n dx$

Optimal. Leaf size=32

$$\frac{i(a \cos(c + dx) + ia \sin(c + dx))^n}{dn}$$

[Out] $-I*(a*\cos(d*x+c)+I*a*\sin(d*x+c))^n/d/n$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3071}

$$\frac{i(a \cos(c + dx) + ia \sin(c + dx))^n}{dn}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^n, x]$

[Out] $((-I)*(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^n)/(d*n)$

Rule 3071

$\text{Int}[(\cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*\sin[(c_.) + (d_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(a*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^n)/(b*d*n), x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int (a \cos(c + dx) + ia \sin(c + dx))^n dx = -\frac{i(a \cos(c + dx) + ia \sin(c + dx))^n}{dn}$$

Mathematica [A] time = 0.09, size = 31, normalized size = 0.97

$$\frac{i(a(\cos(c + dx) + i \sin(c + dx)))^n}{dn}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^n, x]$

[Out] $((-I)*(a*(\text{Cos}[c + d*x] + I*\text{Sin}[c + d*x]))^n)/(d*n)$

fricas [A] time = 1.83, size = 23, normalized size = 0.72

$$\frac{i e^{(idnx + icn + n \log(a))}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a*\cos(d*x+c)+I*a*\sin(d*x+c))^n, x, \text{algorithm}="fricas")$

[Out] $-I*e^{(I*d*n*x + I*c*n + n*\log(a))}/(d*n)$

giac [A] time = 0.45, size = 23, normalized size = 0.72

$$\frac{i e^{(idnx + icn + n \log(a))}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x, algorithm="giac")

[Out] -I*e^(I*d*n*x + I*c*n + n*log(a))/(d*n)

maple [A] time = 0.26, size = 31, normalized size = 0.97

$$\frac{i(a \cos(dx + c) + ia \sin(dx + c))^n}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x)

[Out] -I*(a*cos(d*x+c)+I*a*sin(d*x+c))^n/d/n

maxima [B] time = 0.42, size = 59, normalized size = 1.84

$$\frac{ia^n e^{\left(-n \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)+n \log\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)\right)}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x, algorithm="maxima")

[Out] -I*a^n*e^(-n*log(sin(d*x + c)/(cos(d*x + c) + 1) + I) + n*log(-sin(d*x + c)/(cos(d*x + c) + 1) + I))/(d*n)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int (a \cos(c + dx) + a \sin(c + dx) 1i)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n,x)

[Out] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n, x)

sympy [A] time = 0.25, size = 44, normalized size = 1.38

$$\begin{cases} x & \text{for } n = 0 \wedge (d = 0 \vee n = 0) \\ x(ia \sin(c) + a \cos(c))^n & \text{for } d = 0 \\ -\frac{i(ia \sin(c+dx)+a \cos(c+dx))^n}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))*n,x)

[Out] Piecewise((x, Eq(n, 0) & (Eq(d, 0) | Eq(n, 0))), (x*(I*a*sin(c) + a*cos(c))*n, Eq(d, 0)), (-I*(I*a*sin(c + d*x) + a*cos(c + d*x))*n/(d*n), True))

3.249 $\int (a \cos(c + dx) + ia \sin(c + dx))^4 dx$

Optimal. Leaf size=31

$$-\frac{i(a \cos(c + dx) + ia \sin(c + dx))^4}{4d}$$

[Out] $-1/4*I*(a*\cos(d*x+c)+I*a*\sin(d*x+c))^4/d$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3071}

$$-\frac{i(a \cos(c + dx) + ia \sin(c + dx))^4}{4d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^4, x]$

[Out] $((-I/4)*(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^4)/d$

Rule 3071

$\text{Int}[(\cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*\sin[(c_.) + (d_.)*(x_)])^{(n_)}, x_Symbol] :> \text{Simp}[(a*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^n)/(b*d*n), x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int (a \cos(c + dx) + ia \sin(c + dx))^4 dx = -\frac{i(a \cos(c + dx) + ia \sin(c + dx))^4}{4d}$$

Mathematica [A] time = 0.11, size = 31, normalized size = 1.00

$$-\frac{i(a \cos(c + dx) + ia \sin(c + dx))^4}{4d}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^4, x]$

[Out] $((-1/4*I)*(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^4)/d$

fricas [A] time = 1.83, size = 17, normalized size = 0.55

$$-\frac{i a^4 e^{(4i dx + 4i c)}}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a*\cos(d*x+c)+I*a*\sin(d*x+c))^4, x, \text{algorithm}="fricas")$

[Out] $-1/4*I*a^4*e^{(4*I*d*x + 4*I*c)}/d$

giac [B] time = 0.19, size = 52, normalized size = 1.68

$$-\frac{i a^4 e^{(4i dx + 4i c)}}{8 d} - \frac{i a^4 e^{(-4i dx - 4i c)}}{8 d} + \frac{a^4 \sin(4 dx + 4 c)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="giac")

[Out] $-1/8*I*a^4*e^{(4*I*d*x + 4*I*c)}/d - 1/8*I*a^4*e^{(-4*I*d*x - 4*I*c)}/d + 1/4*a^4*\sin(4*d*x + 4*c)/d$

maple [B] time = 0.29, size = 151, normalized size = 4.87

$$\frac{a^4 \left(-\frac{\left(\sin^3(dx+c) + \frac{3\sin(dx+c)}{2} \right) \cos(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) - ia^4 \left(\sin^4(dx+c) \right) - 6a^4 \left(-\frac{\sin(dx+c)\cos^3(dx+c)}{4} + \frac{\sin(dx+c)\cos(dx+c)}{8} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x)

[Out] $1/d*(a^4*(-1/4*(\sin(d*x+c)^3+3/2*\sin(d*x+c))*\cos(d*x+c)+3/8*d*x+3/8*c)-I*a^4*\sin(d*x+c)^4-6*a^4*(-1/4*\sin(d*x+c)*\cos(d*x+c)^3+1/8*\sin(d*x+c)*\cos(d*x+c)+1/8*d*x+1/8*c)-I*a^4*\cos(d*x+c)^4+a^4*(1/4*(\cos(d*x+c)^3+3/2*\cos(d*x+c))*\sin(d*x+c)+3/8*d*x+3/8*c))$

maxima [B] time = 0.46, size = 132, normalized size = 4.26

$$\frac{ia^4 \cos(dx+c)^4}{d} - \frac{ia^4 \sin(dx+c)^4}{d} + \frac{(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))a^4}{32d} + \frac{(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))a^4}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="maxima")

[Out] $-I*a^4*\cos(d*x+c)^4/d - I*a^4*\sin(d*x+c)^4/d + 1/32*(12*d*x+12*c+\sin(4*d*x+4*c)+8*\sin(2*d*x+2*c))*a^4/d + 1/32*(12*d*x+12*c+\sin(4*d*x+4*c)-8*\sin(2*d*x+2*c))*a^4/d - 3/16*(4*d*x+4*c-\sin(4*d*x+4*c))*a^4/d$

mupad [B] time = 2.55, size = 84, normalized size = 2.71

$$\frac{2a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1 \right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 4i - 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - \tan\left(\frac{c}{2} + \frac{dx}{2}\right) 4i + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(c+d*x)+a*sin(c+d*x)*1i)^4,x)

[Out] $-(2*a^4*\tan(c/2+(d*x)/2)*(\tan(c/2+(d*x)/2)^2-1))/(d*(\tan(c/2+(d*x)/2)^3*4i-6*\tan(c/2+(d*x)/2)^2-\tan(c/2+(d*x)/2)*4i+\tan(c/2+(d*x)/2)^4+1))$

sympy [A] time = 0.14, size = 37, normalized size = 1.19

$$\begin{cases} -\frac{ia^4 e^{Aic} e^{Aidx}}{4d} & \text{for } 4d \neq 0 \\ a^4 x e^{Aic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x)

[Out] $\text{Piecewise}((-I*a**4*\exp(4*I*c)*\exp(4*I*d*x)/(4*d), \text{Ne}(4*d, 0)), (a**4*x*\exp(4*I*c), \text{True}))$

3.250 $\int (a \cos(c + dx) + ia \sin(c + dx))^3 dx$

Optimal. Leaf size=31

$$-\frac{i(a \cos(c + dx) + ia \sin(c + dx))^3}{3d}$$

[Out] $-1/3*I*(a*\cos(d*x+c)+I*a*\sin(d*x+c))^3/d$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3071}

$$-\frac{i(a \cos(c + dx) + ia \sin(c + dx))^3}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^3, x]$

[Out] $((-I/3)*(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^3)/d$

Rule 3071

$\text{Int}[(\cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*\sin[(c_.) + (d_.)*(x_)])^{(n_)}, x_Symbol] :> \text{Simp}[(a*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^n)/(b*d*n), x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int (a \cos(c + dx) + ia \sin(c + dx))^3 dx = -\frac{i(a \cos(c + dx) + ia \sin(c + dx))^3}{3d}$$

Mathematica [A] time = 0.08, size = 31, normalized size = 1.00

$$-\frac{i(a \cos(c + dx) + ia \sin(c + dx))^3}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^3, x]$

[Out] $((-1/3*I)*(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^3)/d$

fricas [A] time = 0.85, size = 17, normalized size = 0.55

$$-\frac{ia^3e^{(3idx+3ic)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a*\cos(d*x+c)+I*a*\sin(d*x+c))^3, x, \text{algorithm}="fricas")$

[Out] $-1/3*I*a^3*e^{(3*I*d*x + 3*I*c)}/d$

giac [B] time = 0.18, size = 52, normalized size = 1.68

$$-\frac{ia^3e^{(3idx+3ic)}}{6d} - \frac{ia^3e^{(-3idx-3ic)}}{6d} + \frac{a^3 \sin(3dx + 3c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")

[Out] $-1/6*I*a^3*e^{(3*I*d*x + 3*I*c)/d} - 1/6*I*a^3*e^{(-3*I*d*x - 3*I*c)/d} + 1/3*a^3*\sin(3*d*x + 3*c)/d$

maple [B] time = 0.28, size = 76, normalized size = 2.45

$$\frac{ia^3(2+\sin^2(dx+c))\cos(dx+c)}{3} - a^3(\sin^3(dx+c)) - ia^3(\cos^3(dx+c)) + \frac{a^3(2+\cos^2(dx+c))\sin(dx+c)}{3}$$

$$d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)

[Out] $1/d*(1/3*I*a^3*(2+\sin(d*x+c)^2)*\cos(d*x+c)-a^3*\sin(d*x+c)^3-I*a^3*\cos(d*x+c)^3+1/3*a^3*(2+\cos(d*x+c)^2)*\sin(d*x+c))$

maxima [B] time = 0.31, size = 83, normalized size = 2.68

$$\frac{ia^3 \cos(dx+c)^3}{d} - \frac{a^3 \sin(dx+c)^3}{d} - \frac{i(\cos(dx+c)^3 - 3 \cos(dx+c))a^3}{3d} - \frac{(\sin(dx+c)^3 - 3 \sin(dx+c))a^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] $-I*a^3*\cos(d*x + c)^3/d - a^3*\sin(d*x + c)^3/d - 1/3*I*(\cos(d*x + c)^3 - 3*\cos(d*x + c))*a^3/d - 1/3*(\sin(d*x + c)^3 - 3*\sin(d*x + c))*a^3/d$

mupad [B] time = 2.47, size = 66, normalized size = 2.13

$$\frac{2a^3 \left(3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1 \right)}{3d \left(-\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 3i + 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 1i \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)

[Out] $-(2*a^3*(3*\tan(c/2 + (d*x)/2)^2 - 1))/(3*d*(3*\tan(c/2 + (d*x)/2) - \tan(c/2 + (d*x)/2)^2*3i - \tan(c/2 + (d*x)/2)^3 + 1i))$

sympy [A] time = 0.14, size = 37, normalized size = 1.19

$$\begin{cases} -\frac{ia^3 e^{3ic} e^{3idx}}{3d} & \text{for } 3d \neq 0 \\ a^3 x e^{3ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))*3,x)

[Out] $\text{Piecewise}((-I*a**3*\exp(3*I*c)*\exp(3*I*d*x)/(3*d), \text{Ne}(3*d, 0)), (a**3*x*\exp(3*I*c), \text{True}))$

3.251 $\int (a \cos(c + dx) + ia \sin(c + dx))^2 dx$

Optimal. Leaf size=31

$$-\frac{i(a \cos(c + dx) + ia \sin(c + dx))^2}{2d}$$

[Out] $-1/2*I*(a*\cos(d*x+c)+I*a*\sin(d*x+c))^2/d$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3071}

$$-\frac{i(a \cos(c + dx) + ia \sin(c + dx))^2}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^2, x]$

[Out] $((-I/2)*(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^2)/d$

Rule 3071

$\text{Int}[(\cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*\sin[(c_.) + (d_.)*(x_)])^{(n_.)}, x$
 $_Symbol] :> \text{Simp}[(a*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^n)/(b*d*n), x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int (a \cos(c + dx) + ia \sin(c + dx))^2 dx = -\frac{i(a \cos(c + dx) + ia \sin(c + dx))^2}{2d}$$

Mathematica [A] time = 0.05, size = 31, normalized size = 1.00

$$-\frac{i(a \cos(c + dx) + ia \sin(c + dx))^2}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^2, x]$

[Out] $((-1/2*I)*(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^2)/d$

fricas [A] time = 1.69, size = 17, normalized size = 0.55

$$-\frac{i a^2 e^{(2i dx + 2ic)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a*\cos(d*x+c)+I*a*\sin(d*x+c))^2, x, \text{algorithm}="fricas")$

[Out] $-1/2*I*a^2*e^{(2*I*d*x + 2*I*c)}/d$

giac [B] time = 0.15, size = 52, normalized size = 1.68

$$-\frac{i a^2 e^{(2i dx + 2ic)}}{4d} - \frac{i a^2 e^{(-2i dx - 2ic)}}{4d} + \frac{a^2 \sin(2 dx + 2 c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")

[Out] $-1/4*I*a^2*e^{(2*I*d*x + 2*I*c)}/d - 1/4*I*a^2*e^{(-2*I*d*x - 2*I*c)}/d + 1/2*a^2*\sin(2*d*x + 2*c)/d$

maple [B] time = 0.26, size = 73, normalized size = 2.35

$$\frac{-a^2 \left(-\frac{\sin(dx+c)\cos(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) - ia^2 \left(\cos^2(dx+c) \right) + a^2 \left(\frac{\sin(dx+c)\cos(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)

[Out] $1/d*(-a^2*(-1/2*\sin(d*x+c)*\cos(d*x+c)+1/2*d*x+1/2*c)-I*a^2*\cos(d*x+c)^2+a^2*(1/2*\sin(d*x+c)*\cos(d*x+c)+1/2*d*x+1/2*c))$

maxima [B] time = 0.54, size = 69, normalized size = 2.23

$$-\frac{ia^2 \cos(dx+c)^2}{d} + \frac{(2dx+2c+\sin(2dx+2c))a^2}{4d} - \frac{(2dx+2c-\sin(2dx+2c))a^2}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] $-I*a^2*\cos(d*x + c)^2/d + 1/4*(2*d*x + 2*c + \sin(2*d*x + 2*c))*a^2/d - 1/4*(2*d*x + 2*c - \sin(2*d*x + 2*c))*a^2/d$

mupad [B] time = 2.42, size = 44, normalized size = 1.42

$$\frac{2a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + \tan\left(\frac{c}{2} + \frac{dx}{2}\right) 2i - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)

[Out] $-(2*a^2*\tan(c/2 + (d*x)/2))/(d*(\tan(c/2 + (d*x)/2)*2i + \tan(c/2 + (d*x)/2)^2 - 1))$

sympy [A] time = 0.13, size = 37, normalized size = 1.19

$$\begin{cases} -\frac{ia^2e^{2ic}e^{2idx}}{2d} & \text{for } 2d \neq 0 \\ a^2xe^{2ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)

[Out] $\text{Piecewise}((-I*a**2*\exp(2*I*c)*\exp(2*I*d*x)/(2*d), \text{Ne}(2*d, 0)), (a**2*x*\exp(2*I*c), \text{True}))$

3.252 $\int (a \cos(c + dx) + ia \sin(c + dx)) dx$

Optimal. Leaf size=26

$$\frac{a \sin(c + dx)}{d} - \frac{ia \cos(c + dx)}{d}$$

[Out] $-I*a*\cos(d*x+c)/d+a*\sin(d*x+c)/d$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2637, 2638}

$$\frac{a \sin(c + dx)}{d} - \frac{ia \cos(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] `Int[a*Cos[c + d*x] + I*a*Sin[c + d*x],x]`

[Out] $((-I)*a*\cos[c + d*x])/d + (a*\sin[c + d*x])/d$

Rule 2637

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 2638

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \int (a \cos(c + dx) + ia \sin(c + dx)) dx &= (ia) \int \sin(c + dx) dx + a \int \cos(c + dx) dx \\ &= -\frac{ia \cos(c + dx)}{d} + \frac{a \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.96

$$\frac{ia \sin(c) \sin(dx)}{d} - \frac{ia \cos(c) \cos(dx)}{d} + \frac{a \sin(c) \cos(dx)}{d} + \frac{a \cos(c) \sin(dx)}{d}$$

Antiderivative was successfully verified.

[In] `Integrate[a*Cos[c + d*x] + I*a*Sin[c + d*x],x]`

[Out] $((-I)*a*\cos[c]*\cos[d*x])/d + (a*\cos[d*x]*\sin[c])/d + (a*\cos[c]*\sin[d*x])/d + (I*a*\sin[c]*\sin[d*x])/d$

fricas [A] time = 1.93, size = 15, normalized size = 0.58

$$-\frac{iae^{(idx+ic)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x, algorithm="fricas")`

[Out] $-I*a*e^{(I*d*x + I*c)}/d$

giac [A] time = 0.15, size = 24, normalized size = 0.92

$$-\frac{ia \cos(dx + c)}{d} + \frac{a \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x, algorithm="giac")

[Out] -I*a*cos(d*x + c)/d + a*sin(d*x + c)/d

maple [A] time = 0.00, size = 26, normalized size = 1.00

$$-\frac{ia \cos(dx + c)}{d} + \frac{a \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a*cos(d*x+c)+I*a*sin(d*x+c),x)

[Out] -I*a*cos(d*x+c)/d+a*sin(d*x+c)/d

maxima [A] time = 1.12, size = 24, normalized size = 0.92

$$-\frac{ia \cos(dx + c)}{d} + \frac{a \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x, algorithm="maxima")

[Out] -I*a*cos(d*x + c)/d + a*sin(d*x + c)/d

mupad [B] time = 2.39, size = 20, normalized size = 0.77

$$\frac{2a}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 1i \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a*cos(c + d*x) + a*sin(c + d*x)*1i,x)

[Out] (2*a)/(d*(tan(c/2 + (d*x)/2) + 1i))

sympy [A] time = 0.12, size = 26, normalized size = 1.00

$$\begin{cases} -\frac{iae^{ic}e^{idx}}{d} & \text{for } d \neq 0 \\ axe^{ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x)

[Out] Piecewise((-I*a*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (a*x*exp(I*c), True))

$$3.253 \quad \int \frac{1}{a \cos(c+dx)+ia \sin(c+dx)} dx$$

Optimal. Leaf size=29

$$\frac{i}{d(a \cos(c+dx) + ia \sin(c+dx))}$$

[Out] I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3071}

$$\frac{i}{d(a \cos(c+dx) + ia \sin(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]

[Out] I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))

Rule 3071

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{1}{a \cos(c+dx) + ia \sin(c+dx)} dx = \frac{i}{d(a \cos(c+dx) + ia \sin(c+dx))}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.00

$$\frac{i}{d(a \cos(c+dx) + ia \sin(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]

[Out] I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))

fricas [A] time = 0.92, size = 17, normalized size = 0.59

$$\frac{i e^{(-idx-ic)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")

[Out] I*e^(-I*d*x - I*c)/(a*d)

giac [A] time = 0.13, size = 21, normalized size = 0.72

$$\frac{2}{ad \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - i \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")

[Out] 2/(a*d*(tan(1/2*d*x + 1/2*c) - I))

maple [A] time = 0.40, size = 23, normalized size = 0.79

$$\frac{2}{da \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - i \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)

[Out] 2/d/a/(tan(1/2*d*x+1/2*c)-I)

maxima [A] time = 0.32, size = 29, normalized size = 1.00

$$\frac{2}{\left(-i a + \frac{a \sin(dx+c)}{\cos(dx+c)+1} \right) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")

[Out] 2/((-I*a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)

mupad [B] time = 2.39, size = 25, normalized size = 0.86

$$\frac{2i}{a d \left(1 + \tan \left(\frac{c}{2} + \frac{dx}{2} \right) i \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)

[Out] 2i/(a*d*(tan(c/2 + (d*x)/2)*1i + 1))

sympy [A] time = 0.14, size = 31, normalized size = 1.07

$$\begin{cases} \frac{ie^{-ic}e^{-idx}}{ad} & \text{for } ade^{ic} \neq 0 \\ \frac{xe^{-ic}}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)

[Out] Piecewise((I*exp(-I*c)*exp(-I*d*x)/(a*d), Ne(a*d*exp(I*c), 0)), (x*exp(-I*c)/a, True))

$$3.254 \quad \int \frac{1}{(a \cos(c+dx)+ia \sin(c+dx))^2} dx$$

Optimal. Leaf size=31

$$\frac{i}{2d(a \cos(c+dx) + ia \sin(c+dx))^2}$$

[Out] 1/2*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^2

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3071}

$$\frac{i}{2d(a \cos(c+dx) + ia \sin(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]

[Out] (I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)

Rule 3071

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{1}{(a \cos(c+dx) + ia \sin(c+dx))^2} dx = \frac{i}{2d(a \cos(c+dx) + ia \sin(c+dx))^2}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 1.00

$$\frac{i}{2d(a \cos(c+dx) + ia \sin(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]

[Out] (I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)

fricas [A] time = 0.93, size = 17, normalized size = 0.55

$$\frac{i e^{(-2i dx - 2ic)}}{2 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] 1/2*I*e^(-2*I*d*x - 2*I*c)/(a^2*d)

giac [A] time = 0.15, size = 30, normalized size = 0.97

$$-\frac{2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^2 d \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")

[Out] -2*tan(1/2*d*x + 1/2*c)/(a^2*d*(tan(1/2*d*x + 1/2*c) - I)^2)

maple [A] time = 0.40, size = 23, normalized size = 0.74

$$\frac{i}{d a^2 (1 + i \tan(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)

[Out] I/d/a^2/(1+I*tan(d*x+c))

maxima [A] time = 0.33, size = 22, normalized size = 0.71

$$\frac{1}{(a^2 \tan(dx + c) - i a^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/((a^2*tan(d*x + c) - I*a^2)*d)

mupad [B] time = 2.41, size = 31, normalized size = 1.00

$$-\frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{a^2 d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right) - i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)

[Out] -(2*tan(c/2 + (d*x)/2))/(a^2*d*(tan(c/2 + (d*x)/2) - 1i)^2)

sympy [A] time = 0.14, size = 46, normalized size = 1.48

$$\begin{cases} \frac{ie^{-2ic}e^{-2idx}}{2a^2d} & \text{for } 2a^2de^{2ic} \neq 0 \\ \frac{xe^{-2ic}}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)

[Out] Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(2*a**2*d), Ne(2*a**2*d*exp(2*I*c), 0)), (x*exp(-2*I*c)/a**2, True))

$$3.255 \quad \int \frac{1}{(a \cos(c+dx)+ia \sin(c+dx))^3} dx$$

Optimal. Leaf size=31

$$\frac{i}{3d(a \cos(c+dx) + ia \sin(c+dx))^3}$$

[Out] 1/3*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^3

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3071}

$$\frac{i}{3d(a \cos(c+dx) + ia \sin(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]

[Out] (I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)

Rule 3071

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{1}{(a \cos(c+dx) + ia \sin(c+dx))^3} dx = \frac{i}{3d(a \cos(c+dx) + ia \sin(c+dx))^3}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 1.00

$$\frac{i}{3d(a \cos(c+dx) + ia \sin(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]

[Out] (I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)

fricas [A] time = 1.62, size = 17, normalized size = 0.55

$$\frac{i e^{(-3i dx - 3i c)}}{3 a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/3*I*e^(-3*I*d*x - 3*I*c)/(a^3*d)

giac [A] time = 0.15, size = 36, normalized size = 1.16

$$\frac{2 \left(3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right)}{3 a^3 d \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - i \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")

[Out] $\frac{2}{3} \cdot (3 \cdot \tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^2 - 1) / (a^3 \cdot d \cdot (\tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c) - I)^3)$

maple [B] time = 0.43, size = 57, normalized size = 1.84

$$\frac{\frac{2}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - i} - \frac{8}{3\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - i\right)^3} + \frac{4i}{\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - i\right)^2}}{d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)

[Out] $\frac{2}{d/a^3} \cdot (1/(\tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c) - I) - 4/3/(\tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c) - I)^3 + 2 \cdot I/(\tan(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c) - I)^2)$

maxima [A] time = 0.63, size = 29, normalized size = 0.94

$$\frac{i \cos(3 dx + 3 c) + \sin(3 dx + 3 c)}{3 a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] $\frac{1}{3} \cdot (I \cdot \cos(3 \cdot d \cdot x + 3 \cdot c) + \sin(3 \cdot d \cdot x + 3 \cdot c)) / (a^3 \cdot d)$

mupad [B] time = 2.46, size = 68, normalized size = 2.19

$$\frac{2 \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 3i - i \right)}{3 a^3 d \left(-\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 1i - 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + \tan\left(\frac{c}{2} + \frac{dx}{2}\right) 3i + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)

[Out] $-(2 \cdot (\tan(c/2 + (d \cdot x)/2)^2 \cdot 3i - 1i)) / (3 \cdot a^3 \cdot d \cdot (\tan(c/2 + (d \cdot x)/2) \cdot 3i - 3 \cdot \tan(c/2 + (d \cdot x)/2)^2 - \tan(c/2 + (d \cdot x)/2)^3 \cdot 1i + 1))$

sympy [A] time = 0.14, size = 46, normalized size = 1.48

$$\begin{cases} \frac{ie^{-3ic}e^{-3idx}}{3a^3d} & \text{for } 3a^3de^{3ic} \neq 0 \\ \frac{xe^{-3ic}}{a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)

[Out] Piecewise((I*exp(-3*I*c)*exp(-3*I*d*x)/(3*a**3*d), Ne(3*a**3*d*exp(3*I*c), 0)), (x*exp(-3*I*c)/a**3, True))

$$3.256 \quad \int \frac{1}{(a \cos(c+dx)+ia \sin(c+dx))^4} dx$$

Optimal. Leaf size=31

$$\frac{i}{4d(a \cos(c+dx) + ia \sin(c+dx))^4}$$

[Out] 1/4*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^4

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3071}

$$\frac{i}{4d(a \cos(c+dx) + ia \sin(c+dx))^4}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-4),x]

[Out] (I/4)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)

Rule 3071

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{1}{(a \cos(c+dx) + ia \sin(c+dx))^4} dx = \frac{i}{4d(a \cos(c+dx) + ia \sin(c+dx))^4}$$

Mathematica [A] time = 0.05, size = 31, normalized size = 1.00

$$\frac{i}{4d(a \cos(c+dx) + ia \sin(c+dx))^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-4),x]

[Out] (I/4)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)

fricas [A] time = 1.33, size = 17, normalized size = 0.55

$$\frac{i e^{(-4i dx - 4ic)}}{4 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="fricas")

[Out] 1/4*I*e^(-4*I*d*x - 4*I*c)/(a^4*d)

giac [A] time = 0.16, size = 44, normalized size = 1.42

$$\frac{2 \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 - \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{a^4 d \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - i \right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="giac")

[Out] $-2*(\tan(1/2*d*x + 1/2*c)^3 - \tan(1/2*d*x + 1/2*c))/(a^4*d*(\tan(1/2*d*x + 1/2*c) - I)^4)$

maple [A] time = 0.42, size = 36, normalized size = 1.16

$$\frac{-\frac{i}{(\tan(dx+c)-i)^2} - \frac{1}{\tan(dx+c)-i}}{d a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x)

[Out] $1/d/a^4*(-I/(\tan(d*x+c)-I)^2-1/(\tan(d*x+c)-I))$

maxima [A] time = 0.57, size = 29, normalized size = 0.94

$$\frac{i \cos(4 dx + 4 c) + \sin(4 dx + 4 c)}{4 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="maxima")

[Out] $1/4*(I*\cos(4*d*x + 4*c) + \sin(4*d*x + 4*c))/(a^4*d)$

mupad [B] time = 2.56, size = 91, normalized size = 2.94

$$\frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 1i - i\right)}{a^4 d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 1i + 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 6i - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 1i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^4,x)

[Out] $-(2*\tan(c/2 + (d*x)/2)*(\tan(c/2 + (d*x)/2)^2*1i - 1i))/(a^4*d*(4*\tan(c/2 + (d*x)/2)^3 - \tan(c/2 + (d*x)/2)^2*6i - 4*\tan(c/2 + (d*x)/2) + \tan(c/2 + (d*x)/2)^4*1i + 1i))$

sympy [A] time = 0.15, size = 46, normalized size = 1.48

$$\begin{cases} \frac{ie^{-4ic}e^{-4idx}}{4a^4d} & \text{for } 4a^4de^{4ic} \neq 0 \\ \frac{xe^{-4ic}}{a^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**4,x)

[Out] $\text{Piecewise}((I*\exp(-4*I*c)*\exp(-4*I*d*x)/(4*a**4*d), \text{Ne}(4*a**4*d*\exp(4*I*c), 0)), (x*\exp(-4*I*c)/a**4, \text{True}))$

$$3.257 \quad \int (a \cos(c + dx) + ia \sin(c + dx))^{5/2} dx$$

Optimal. Leaf size=33

$$-\frac{2i(a \cos(c + dx) + ia \sin(c + dx))^{5/2}}{5d}$$

[Out] $-2/5*I*(a*\cos(d*x+c)+I*a*\sin(d*x+c))^{(5/2)}/d$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {3071}

$$-\frac{2i(a \cos(c + dx) + ia \sin(c + dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^{(5/2)}, x]$

[Out] $(((-2*I)/5)*(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^{(5/2)})/d$

Rule 3071

$\text{Int}[(\cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*\sin[(c_.) + (d_.)*(x_.)])^{(n_.)}, x_Symbol] :> \text{Simp}[(a*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^n)/(b*d*n), x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int (a \cos(c + dx) + ia \sin(c + dx))^{5/2} dx = -\frac{2i(a \cos(c + dx) + ia \sin(c + dx))^{5/2}}{5d}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.97

$$-\frac{2i(a(\cos(c + dx) + i \sin(c + dx)))^{5/2}}{5d}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^{(5/2)}, x]$

[Out] $(((-2*I)/5)*(a*(\text{Cos}[c + d*x] + I*\text{Sin}[c + d*x]))^{(5/2)})/d$

fricas [A] time = 1.47, size = 17, normalized size = 0.52

$$-\frac{2i a^{\frac{5}{2}} e^{\left(\frac{5}{2}i dx + \frac{5}{2}ic\right)}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a*\cos(d*x+c)+I*a*\sin(d*x+c))^{(5/2)}, x, \text{algorithm}="fricas")$

[Out] $-2/5*I*a^{(5/2)}*e^{(5/2*I*d*x + 5/2*I*c)}/d$

giac [A] time = 0.57, size = 17, normalized size = 0.52

$$-\frac{2i a^{\frac{5}{2}} e^{\left(\frac{5}{2}i dx + \frac{5}{2}ic\right)}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] $-2/5*I*a^{5/2}*e^{(5/2*I*d*x + 5/2*I*c)}/d$

maple [A] time = 0.36, size = 28, normalized size = 0.85

$$\frac{2i(a\cos(dx+c) + ia\sin(dx+c))^{5/2}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x)

[Out] $-2/5*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^{5/2}/d$

maxima [B] time = 0.69, size = 51, normalized size = 1.55

$$\frac{2i a^2 \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + i \right)^{5/2}}{5d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + i \right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="maxima")

[Out] $-2/5*I*a^{5/2}*(-\sin(d*x + c)/(\cos(d*x + c) + 1) + I)^{5/2}/(d*(\sin(d*x + c)/(\cos(d*x + c) + 1) + I)^{5/2})$

mupad [B] time = 0.43, size = 35, normalized size = 1.06

$$\frac{a^2 e^{c 2i} e^{d x 2i} \sqrt{a e^{c 1i} e^{d x 1i}} 2i}{5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^(5/2),x)

[Out] $-(a^2*\exp(c*2i)*\exp(d*x*2i)*(a*\exp(c*1i)*\exp(d*x*1i))^{1/2}*2i)/(5*d)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**(5/2),x)

[Out] Timed out

3.258 $\int (a \cos(c + dx) + ia \sin(c + dx))^{3/2} dx$

Optimal. Leaf size=33

$$\frac{2i(a \cos(c + dx) + ia \sin(c + dx))^{3/2}}{3d}$$

[Out] $-2/3*I*(a*\cos(d*x+c)+I*a*\sin(d*x+c))^{(3/2)}/d$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {3071}

$$\frac{2i(a \cos(c + dx) + ia \sin(c + dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^{(3/2)}, x]$

[Out] $(((-2*I)/3)*(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^{(3/2)})/d$

Rule 3071

$\text{Int}[(\cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*\sin[(c_.) + (d_.)*(x_.)])^{(n_.)}, x_Symbol] :> \text{Simp}[(a*(a*\text{Cos}[c + d*x] + b*\text{Sin}[c + d*x])^n)/(b*d*n), x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\int (a \cos(c + dx) + ia \sin(c + dx))^{3/2} dx = -\frac{2i(a \cos(c + dx) + ia \sin(c + dx))^{3/2}}{3d}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.97

$$\frac{2i(a(\cos(c + dx) + i \sin(c + dx)))^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x])^{(3/2)}, x]$

[Out] $(((-2*I)/3)*(a*(\text{Cos}[c + d*x] + I*\text{Sin}[c + d*x]))^{(3/2)})/d$

fricas [A] time = 1.08, size = 17, normalized size = 0.52

$$\frac{2i a^{\frac{3}{2}} e^{\left(\frac{3}{2}i dx + \frac{3}{2}ic\right)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a*\cos(d*x+c)+I*a*\sin(d*x+c))^{(3/2)}, x, \text{algorithm}="fricas")$

[Out] $-2/3*I*a^{(3/2)}*e^{(3/2*I*d*x + 3/2*I*c)}/d$

giac [A] time = 0.46, size = 17, normalized size = 0.52

$$\frac{2i a^{\frac{3}{2}} e^{\left(\frac{3}{2}i dx + \frac{3}{2}ic\right)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="giac")

[Out] $-2/3*I*a^{(3/2)}*e^{(3/2*I*d*x + 3/2*I*c)}/d$

maple [A] time = 0.22, size = 28, normalized size = 0.85

$$\frac{2i(a\cos(dx+c)+ia\sin(dx+c))^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x)

[Out] $-2/3*I*(a*\cos(d*x+c)+I*a*\sin(d*x+c))^{(3/2)}/d$

maxima [B] time = 0.90, size = 51, normalized size = 1.55

$$\frac{2i a^{\frac{3}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + i \right)^{\frac{3}{2}}}{3d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + i \right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="maxima")

[Out] $-2/3*I*a^{(3/2)}*(-\sin(d*x + c)/(\cos(d*x + c) + 1) + I)^{(3/2)}/(d*(\sin(d*x + c)/(\cos(d*x + c) + 1) + I)^{(3/2)})$

mupad [B] time = 2.38, size = 33, normalized size = 1.00

$$\frac{a e^{c1i} e^{dx1i} \sqrt{a e^{c1i} e^{dx1i}} 2i}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^(3/2),x)

[Out] $-(a*\exp(c*1i)*\exp(d*x*1i)*(a*\exp(c*1i)*\exp(d*x*1i))^{(1/2)}*2i)/(3*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ia\sin(c+dx) + a\cos(c+dx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**(3/2),x)

[Out] Integral((I*a*sin(c + d*x) + a*cos(c + d*x))**(3/2), x)

$$3.259 \quad \int \sqrt{a \cos(c + dx) + ia \sin(c + dx)} dx$$

Optimal. Leaf size=31

$$\frac{2i\sqrt{a \cos(c + dx) + ia \sin(c + dx)}}{d}$$

[Out] $-2*I*(a*\cos(d*x+c)+I*a*\sin(d*x+c))^{(1/2)}/d$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {3071}

$$\frac{2i\sqrt{a \cos(c + dx) + ia \sin(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]

[Out] $((-2*I)*\text{Sqrt}[a*\text{Cos}[c + d*x] + I*a*\text{Sin}[c + d*x]])/d$

Rule 3071

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \sqrt{a \cos(c + dx) + ia \sin(c + dx)} dx = -\frac{2i\sqrt{a \cos(c + dx) + ia \sin(c + dx)}}{d}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.97

$$\frac{2i\sqrt{a(\cos(c + dx) + i \sin(c + dx))}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]

[Out] $((-2*I)*\text{Sqrt}[a*(\text{Cos}[c + d*x] + I*\text{Sin}[c + d*x]))/d$

fricas [A] time = 2.39, size = 17, normalized size = 0.55

$$\frac{2i\sqrt{a}e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="fricas")

[Out] $-2*I*\text{sqrt}(a)*e^{(1/2*I*d*x + 1/2*I*c)}/d$

giac [A] time = 0.14, size = 25, normalized size = 0.81

$$\frac{2i\sqrt{a \cos(dx + c) + ia \sin(dx + c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="giac")

[Out] -2*I*sqrt(a*cos(d*x + c) + I*a*sin(d*x + c))/d

maple [A] time = 0.22, size = 28, normalized size = 0.90

$$-\frac{2i\sqrt{a\cos(dx+c)+ia\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x)

[Out] -2*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2)/d

maxima [B] time = 0.84, size = 51, normalized size = 1.65

$$-\frac{2i\sqrt{a}\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}{d\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -2*I*sqrt(a)*sqrt(-sin(d*x + c)/(cos(d*x + c) + 1) + I)/(d*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + I))

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a\cos(c+dx)+a\sin(c+dx)} 1i dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^(1/2),x)

[Out] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ia\sin(c+dx)+a\cos(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**(1/2),x)

[Out] Integral(sqrt(I*a*sin(c + d*x) + a*cos(c + d*x)), x)

$$3.260 \quad \int \frac{1}{\sqrt{a \cos(c+dx)+ia \sin(c+dx)}} dx$$

Optimal. Leaf size=31

$$\frac{2i}{d\sqrt{a \cos(c+dx)+ia \sin(c+dx)}}$$

[Out] $2*I/d/(a*\cos(d*x+c)+I*a*\sin(d*x+c))^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {3071}

$$\frac{2i}{d\sqrt{a \cos(c+dx)+ia \sin(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]

[Out] (2*I)/(d*Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]])

Rule 3071

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{1}{\sqrt{a \cos(c+dx)+ia \sin(c+dx)}} dx = \frac{2i}{d\sqrt{a \cos(c+dx)+ia \sin(c+dx)}}$$

Mathematica [A] time = 0.03, size = 30, normalized size = 0.97

$$\frac{2i}{d\sqrt{a(\cos(c+dx)+i \sin(c+dx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]

[Out] (2*I)/(d*Sqrt[a*(Cos[c + d*x] + I*Sin[c + d*x])])

fricas [A] time = 1.04, size = 17, normalized size = 0.55

$$\frac{2i e^{\left(-\frac{1}{2}i dx - \frac{1}{2}i c\right)}}{\sqrt{a} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="fricas")

[Out] $2*I*e^{(-1/2*I*d*x - 1/2*I*c)}/(\text{sqrt}(a)*d)$

giac [A] time = 0.33, size = 37, normalized size = 1.19

$$\frac{2i}{d \sqrt{\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - ia}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="giac")

[Out] 2*I/(d*sqrt(-(a*tan(1/2*d*x + 1/2*c) - I*a)/(tan(1/2*d*x + 1/2*c) + I)))

maple [A] time = 0.22, size = 28, normalized size = 0.90

$$\frac{2i}{d\sqrt{a\cos(dx+c)+ia\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x)

[Out] 2*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2)

maxima [B] time = 0.43, size = 51, normalized size = 1.65

$$\frac{2i\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}{\sqrt{a}d\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 2*I*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + I)/(sqrt(a)*d*sqrt(-sin(d*x + c)/(cos(d*x + c) + 1) + I))

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{a\cos(c+dx)+a\sin(c+dx)}i} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(1/2),x)

[Out] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ia\sin(c+dx)+a\cos(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**(1/2),x)

[Out] Integral(1/sqrt(I*a*sin(c + d*x) + a*cos(c + d*x)), x)

$$3.261 \quad \int \frac{1}{(a \cos(c+dx)+ia \sin(c+dx))^{3/2}} dx$$

Optimal. Leaf size=33

$$\frac{2i}{3d(a \cos(c+dx) + ia \sin(c+dx))^{3/2}}$$

[Out] 2/3*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2)

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {3071}

$$\frac{2i}{3d(a \cos(c+dx) + ia \sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3/2), x]

[Out] ((2*I)/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2))

Rule 3071

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{1}{(a \cos(c+dx) + ia \sin(c+dx))^{3/2}} dx = \frac{2i}{3d(a \cos(c+dx) + ia \sin(c+dx))^{3/2}}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.97

$$\frac{2i}{3d(a(\cos(c+dx) + i \sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3/2), x]

[Out] ((2*I)/3)/(d*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(3/2))

fricas [A] time = 1.49, size = 17, normalized size = 0.52

$$\frac{2i e^{\left(-\frac{3}{2}i dx - \frac{3}{2}ic\right)}}{3 a^{\frac{3}{2}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2), x, algorithm="fricas")

[Out] 2/3*I*e^(-3/2*I*d*x - 3/2*I*c)/(a^(3/2)*d)

giac [B] time = 1.23, size = 65, normalized size = 1.97

$$\frac{2i \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + i \right)}{3 \left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - ia \right) d \sqrt{-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - ia}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="giac")

[Out] $-2/3*I*(\tan(1/2*d*x + 1/2*c) + I)/((a*\tan(1/2*d*x + 1/2*c) - I*a)*d*\sqrt{-(a*\tan(1/2*d*x + 1/2*c) - I*a)/(\tan(1/2*d*x + 1/2*c) + I)})$

maple [A] time = 0.21, size = 28, normalized size = 0.85

$$\frac{2i}{3d(a\cos(dx+c) + ia\sin(dx+c))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x)

[Out] $2/3*I/d/(a*\cos(d*x+c)+I*a*\sin(d*x+c))^{3/2}$

maxima [B] time = 1.06, size = 51, normalized size = 1.55

$$\frac{2i\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + i\right)^{\frac{3}{2}}}{3a^{\frac{3}{2}}d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + i\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="maxima")

[Out] $2/3*I*(\sin(d*x + c)/(\cos(d*x + c) + 1) + I)^{3/2}/(a^{3/2}*d*(-\sin(d*x + c)/(\cos(d*x + c) + 1) + I)^{3/2})$

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a\cos(c+dx) + a\sin(c+dx)1i)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(3/2),x)

[Out] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\sin(c+dx) + a\cos(c+dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**(3/2),x)

[Out] Integral((I*a*sin(c + d*x) + a*cos(c + d*x))**(-3/2), x)

$$3.262 \quad \int \frac{1}{(a \cos(c+dx)+ia \sin(c+dx))^{5/2}} dx$$

Optimal. Leaf size=33

$$\frac{2i}{5d(a \cos(c+dx) + ia \sin(c+dx))^{5/2}}$$

[Out] 2/5*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2)

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {3071}

$$\frac{2i}{5d(a \cos(c+dx) + ia \sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-5/2), x]

[Out] ((2*I)/5)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2))

Rule 3071

Int[(cos[(c_.) + (d_.)*(x_.)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\int \frac{1}{(a \cos(c+dx) + ia \sin(c+dx))^{5/2}} dx = \frac{2i}{5d(a \cos(c+dx) + ia \sin(c+dx))^{5/2}}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.97

$$\frac{2i}{5d(a(\cos(c+dx) + i \sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-5/2), x]

[Out] ((2*I)/5)/(d*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(5/2))

fricas [A] time = 0.97, size = 17, normalized size = 0.52

$$\frac{2i e^{\left(-\frac{5}{2}i dx - \frac{5}{2}ic\right)}}{5 a^{\frac{5}{2}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2), x, algorithm="fricas")

[Out] 2/5*I*e^(-5/2*I*d*x - 5/2*I*c)/(a^(5/2)*d)

giac [B] time = 3.30, size = 67, normalized size = 2.03

$$\frac{2i \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + i \right)^2}{5 \left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - i a \right)^2 d \sqrt{-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - i a}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] 2/5*I*(tan(1/2*d*x + 1/2*c) + I)^2/((a*tan(1/2*d*x + 1/2*c) - I*a)^2*d*sqrt(-(a*tan(1/2*d*x + 1/2*c) - I*a)/(tan(1/2*d*x + 1/2*c) + I)))

maple [A] time = 0.21, size = 28, normalized size = 0.85

$$\frac{2i}{5d(a \cos(dx + c) + ia \sin(dx + c))^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x)

[Out] 2/5*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2)

maxima [B] time = 0.66, size = 51, normalized size = 1.55

$$\frac{2i \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + i \right)^{\frac{5}{2}}}{5 a^{\frac{5}{2}} d \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + i \right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 2/5*I*(sin(d*x + c)/(cos(d*x + c) + 1) + I)^(5/2)/(a^(5/2)*d*(-sin(d*x + c)/(cos(d*x + c) + 1) + I)^(5/2))

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a \cos(c + dx) + a \sin(c + dx) 1i)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(5/2),x)

[Out] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia \sin(c + dx) + a \cos(c + dx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**(5/2),x)

[Out] Integral((I*a*sin(c + d*x) + a*cos(c + d*x))**(-5/2), x)

3.263 $\int (a \sec(x) + b \tan(x))^5 dx$

Optimal. Leaf size=149

$$-\frac{1}{16}(a+b)^3(3a^2-9ab+8b^2)\log(1-\sin(x))+\frac{1}{16}(a-b)^3(3a^2+9ab+8b^2)\log(\sin(x)+1)+\frac{1}{8}\sec^2(x)(a+b\sin(x))^2$$

[Out] -1/16*(a+b)^3*(3*a^2-9*a*b+8*b^2)*ln(1-sin(x))+1/16*(a-b)^3*(3*a^2+9*a*b+8*b^2)*ln(1+sin(x))-1/8*a*(7-3*a^2/b^2)*b^4*sin(x)+1/4*sec(x)^4*(b+a*sin(x))*(a+b*sin(x))^4+1/8*sec(x)^2*(a+b*sin(x))^2*(2*b*(a^2-2*b^2)+a*(3*a^2-5*b^2)*sin(x))

Rubi [A] time = 0.21, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {4391, 2668, 739, 819, 774, 633, 31}

$$-\frac{1}{8}ab^4\left(7-\frac{3a^2}{b^2}\right)\sin(x)-\frac{1}{16}(a+b)^3(3a^2-9ab+8b^2)\log(1-\sin(x))+\frac{1}{16}(a-b)^3(3a^2+9ab+8b^2)\log(\sin(x)+1)+$$

Antiderivative was successfully verified.

[In] Int[(a*Sec[x] + b*Tan[x])^5, x]

[Out] -((a + b)^3*(3*a^2 - 9*a*b + 8*b^2)*Log[1 - Sin[x]])/16 + ((a - b)^3*(3*a^2 + 9*a*b + 8*b^2)*Log[1 + Sin[x]])/16 - (a*(7 - (3*a^2)/b^2)*b^4*Sin[x])/8 + (Sec[x]^4*(b + a*Sin[x])*(a + b*Sin[x])^4)/4 + (Sec[x]^2*(a + b*Sin[x])^2*(2*b*(a^2 - 2*b^2) + a*(3*a^2 - 5*b^2)*Sin[x]))/8

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 633

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 739

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 774

Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]

Rule 819

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2


```
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a,
c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_.)]^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m
_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_.)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_.)]^(n_.))^p, x_Symbol] := Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int (a \sec(x) + b \tan(x))^5 dx &= \int \sec^5(x)(a + b \sin(x))^5 dx \\
&= b^5 \operatorname{Subst} \left(\int \frac{(a+x)^5}{(b^2-x^2)^3} dx, x, b \sin(x) \right) \\
&= \frac{1}{4} \sec^4(x)(b + a \sin(x))(a + b \sin(x))^4 - \frac{1}{4} b^3 \operatorname{Subst} \left(\int \frac{(a+x)^3(-3a^2+4b^2+ax)}{(b^2-x^2)^2} dx, x, b \sin(x) \right) \\
&= \frac{1}{4} \sec^4(x)(b + a \sin(x))(a + b \sin(x))^4 + \frac{1}{8} \sec^2(x)(a + b \sin(x))^2 (2b(a^2 - 2b^2) + a^2) \\
&= \frac{1}{8} ab^2 (3a^2 - 7b^2) \sin(x) + \frac{1}{4} \sec^4(x)(b + a \sin(x))(a + b \sin(x))^4 + \frac{1}{8} \sec^2(x)(a + b \sin(x))^2 (2b(a^2 - 2b^2) + a^2) \\
&= \frac{1}{8} ab^2 (3a^2 - 7b^2) \sin(x) + \frac{1}{4} \sec^4(x)(b + a \sin(x))(a + b \sin(x))^4 + \frac{1}{8} \sec^2(x)(a + b \sin(x))^2 (2b(a^2 - 2b^2) + a^2) \\
&= -\frac{1}{16} (a+b)^3 (3a^2 - 9ab + 8b^2) \log(1 - \sin(x)) + \frac{1}{16} (a-b)^3 (3a^2 + 9ab + 8b^2) \log(\sin(x) + 1) + 4(b^2 - a^2) \sec^2(x)
\end{aligned}$$

Mathematica [B] time = 1.24, size = 303, normalized size = 2.03

$$\frac{(a^2 - b^2)^2 ((a+b)^3 (3a^2 - 9ab + 8b^2) \log(1 - \sin(x)) - (a-b)^3 (3a^2 + 9ab + 8b^2) \log(\sin(x) + 1)) + 4(b^2 - a^2) \sec^2(x)}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^5, x]
```

```
[Out] -1/16*((a^2 - b^2)^2*((a + b)^3*(3*a^2 - 9*a*b + 8*b^2)*Log[1 - Sin[x]] - (a - b)^3*(3*a^2 + 9*a*b + 8*b^2)*Log[1 + Sin[x]]) - 10*a*b^2*(9*a^6 - 6*a^4*b^2 + 8*a^2*b^4 - 3*b^6)*Sin[x] + 8*b^3*(-15*a^6 - 4*a^4*b^2 - 2*a^2*b^4 + b^6)*Sin[x]^2 - 10*a*b^4*(9*a^4 + 8*a^2*b^2 - b^4)*Sin[x]^3 + 4*b^5*(-9*a^4 - 12*a^2*b^2 + b^4)*Sin[x]^4 - 2*a*b^6*(3*a^2 + 5*b^2)*Sin[x]^5 + 4*(-a^2 + b^2)*Sec[x]^4*(-b + a*Sin[x])*(a + b*Sin[x])^6 + 2*Sec[x]^2*(a + b*Sin[x])^6*(6*a^2*b + 2*b^3 - 3*a^3*Sin[x] - 5*a*b^2*Sin[x]))/(a^2 - b^2)^2
```

fricas [A] time = 0.92, size = 166, normalized size = 1.11

$$\frac{(3a^5 - 10a^3b^2 + 15ab^4 - 8b^5)\cos(x)^4\log(\sin(x) + 1) - (3a^5 - 10a^3b^2 + 15ab^4 + 8b^5)\cos(x)^4\log(-\sin(x) + 1)}{\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sec(x)+b*tan(x))^5,x, algorithm="fricas")

[Out] 1/16*((3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cos(x)^4*log(sin(x) + 1) - (3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cos(x)^4*log(-sin(x) + 1) + 20*a^4*b + 40*a^2*b^3 + 4*b^5 - 16*(5*a^2*b^3 + b^5)*cos(x)^2 + 2*(2*a^5 + 20*a^3*b^2 + 10*a*b^4 + (3*a^5 - 10*a^3*b^2 - 25*a*b^4)*cos(x)^2)*sin(x))/cos(x)^4

giac [A] time = 0.15, size = 178, normalized size = 1.19

$$\frac{1}{16} (3a^5 - 10a^3b^2 + 15ab^4 - 8b^5) \log(\sin(x) + 1) - \frac{1}{16} (3a^5 - 10a^3b^2 + 15ab^4 + 8b^5) \log(-\sin(x) + 1) + \frac{6b^5 \sin(x)}{\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sec(x)+b*tan(x))^5,x, algorithm="giac")

[Out] 1/16*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*log(sin(x) + 1) - 1/16*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*log(-sin(x) + 1) + 1/8*(6*b^5*sin(x)^4 - 3*a^5*sin(x)^3 + 10*a^3*b^2*sin(x)^3 + 25*a*b^4*sin(x)^3 + 40*a^2*b^3*sin(x)^2 - 4*b^5*sin(x)^2 + 5*a^5*sin(x) + 10*a^3*b^2*sin(x) - 15*a*b^4*sin(x) + 10*a^4*b - 20*a^2*b^3)/(sin(x)^2 - 1)^2

maple [A] time = 0.18, size = 199, normalized size = 1.34

$$\frac{a^5 \tan(x) (\sec^3(x))}{4} + \frac{3a^5 \sec(x) \tan(x)}{8} + \frac{3a^5 \ln(\sec(x) + \tan(x))}{8} + \frac{5a^4 b}{4 \cos(x)^4} + \frac{5a^3 b^2 (\sin^3(x))}{2 \cos(x)^4} + \frac{5a^3 b^2 (\sin^3(x))}{4 \cos(x)^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*sec(x)+b*tan(x))^5,x)

[Out] 1/4*a^5*tan(x)*sec(x)^3+3/8*a^5*sec(x)*tan(x)+3/8*a^5*ln(sec(x)+tan(x))+5/4*a^4*b/cos(x)^4+5/2*a^3*b^2*sin(x)^3/cos(x)^4+5/4*a^3*b^2*sin(x)^3/cos(x)^2+5/4*a^3*b^2*sin(x)-5/4*a^3*b^2*ln(sec(x)+tan(x))+5/2*a^2*b^3*sin(x)^4/cos(x)^4+5/4*a*b^4*sin(x)^5/cos(x)^4-5/8*a*b^4*sin(x)^5/cos(x)^2-5/8*a*b^4*sin(x)^3-15/8*a*b^4*sin(x)+15/8*a*b^4*ln(sec(x)+tan(x))+1/4*b^5*tan(x)^4-1/2*b^5*tan(x)^2-b^5*ln(cos(x))

maxima [A] time = 0.32, size = 204, normalized size = 1.37

$$\frac{5}{2} a^2 b^3 \tan(x)^4 + \frac{5}{16} ab^4 \left(\frac{2(5 \sin(x)^3 - 3 \sin(x))}{\sin(x)^4 - 2 \sin(x)^2 + 1} + 3 \log(\sin(x) + 1) - 3 \log(\sin(x) - 1) \right) - \frac{1}{16} a^5 \left(\frac{2(3 \sin(x)^3 - 5 \sin(x))}{\sin(x)^4 - 2 \sin(x)^2 + 1} + 3 \log(\sin(x) + 1) - 3 \log(\sin(x) - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sec(x)+b*tan(x))^5,x, algorithm="maxima")

[Out] 5/2*a^2*b^3*tan(x)^4 + 5/16*a*b^4*(2*(5*sin(x)^3 - 3*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 3*log(sin(x) + 1) - 3*log(sin(x) - 1)) - 1/16*a^5*(2*(3*sin(x)^3 - 5*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) - 3*log(sin(x) + 1) + 3*log(sin(x) - 1)) + 5/8*a^3*b^2*(2*(sin(x)^3 + sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) - log(sin(x) + 1) + log(sin(x) - 1)) + 1/4*b^5*((4*sin(x)^2 - 3)/(sin(x)^4 - 2*sin(x)^2 + 1) - 2*log(sin(x)^2 - 1)) + 5/4*a^4*b/(sin(x)^2 - 1)^2

mupad [B] time = 2.88, size = 272, normalized size = 1.83

$$\frac{\left(\frac{5a^5}{4} + \frac{5a^3b^2}{2} - \frac{15ab^4}{4}\right) \tan\left(\frac{x}{2}\right)^7 + (10a^4b - 2b^5) \tan\left(\frac{x}{2}\right)^6 + \left(\frac{3a^5}{4} + \frac{35a^3b^2}{2} + \frac{55ab^4}{4}\right) \tan\left(\frac{x}{2}\right)^5 + (40a^2b^3 + 8b^5) \tan\left(\frac{x}{2}\right)^4}{\tan\left(\frac{x}{2}\right)^8 - 4 \tan\left(\frac{x}{2}\right)^6 + 6 \tan\left(\frac{x}{2}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*tan(x) + a/cos(x))^5,x)

[Out] $(\tan(x/2)^2(10a^4b - 2b^5) + \tan(x/2)^6(10a^4b - 2b^5) + \tan(x/2)^4(8b^5 + 40a^2b^3) + \tan(x/2)((5a^5)/4 - (15a^3b^2)/4 + (5a^3b^2)/2) + \tan(x/2)^7((5a^5)/4 - (15a^3b^2)/4 + (5a^3b^2)/2) + \tan(x/2)^3((55a^5b^4)/4 + (3a^5)/4 + (35a^3b^2)/2) + \tan(x/2)^5((55a^5b^4)/4 + (3a^5)/4 + (35a^3b^2)/2))/(6\tan(x/2)^4 - 4\tan(x/2)^2 - 4\tan(x/2)^6 + \tan(x/2)^8 + 1) + b^5\log(\tan(x/2)^2 + 1) - (\log(\tan(x/2) - 1)(a + b)^3(3a^2 - 9ab + 8b^2))/8 + \log(\tan(x/2) + 1)(a - b)^3((9ab)/8 + (3a^2)/8 + b^2))$

sympy [B] time = 7.12, size = 308, normalized size = 2.07

$$-\frac{3a^5 \log(\sin(x) - 1)}{16} + \frac{3a^5 \log(\sin(x) + 1)}{16} - \frac{3a^5 \sin^3(x)}{8 \sin^4(x) - 16 \sin^2(x) + 8} + \frac{5a^5 \sin(x)}{8 \sin^4(x) - 16 \sin^2(x) + 8} + \frac{5a^4 b \sec(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sec(x)+b*tan(x))**5,x)

[Out] $-3a**5*\log(\sin(x) - 1)/16 + 3a**5*\log(\sin(x) + 1)/16 - 3a**5*\sin(x)**3/(8*\sin(x)**4 - 16*\sin(x)**2 + 8) + 5a**5*\sin(x)/(8*\sin(x)**4 - 16*\sin(x)**2 + 8) + 5a**4*b*\sec(x)**4/4 + 5a**3*b**2*\log(\sin(x) - 1)/8 - 5a**3*b**2*\log(\sin(x) + 1)/8 + 10a**3*b**2*\sin(x)**3/(8*\sin(x)**4 - 16*\sin(x)**2 + 8) + 10a**3*b**2*\sin(x)/(8*\sin(x)**4 - 16*\sin(x)**2 + 8) + 5a**2*b**3*\tan(x)**4/2 - 15a*b**4*\log(\sin(x) - 1)/16 + 15a*b**4*\log(\sin(x) + 1)/16 + 25a*b**4*\sin(x)**3/(8*\sin(x)**4 - 16*\sin(x)**2 + 8) - 15a*b**4*\sin(x)/(8*\sin(x)**4 - 16*\sin(x)**2 + 8) + b**5*\log(\sec(x)**2)/2 + b**5*\sec(x)**4/4 - b**5*\sec(x)**2$

3.264 $\int (a \sec(x) + b \tan(x))^4 dx$

Optimal. Leaf size=100

$$\frac{4}{3}ab(a^2 - 2b^2)\cos(x) + \frac{1}{3}b^2(2a^2 - 3b^2)\sin(x)\cos(x) - \frac{1}{3}\sec(x)(a+b\sin(x))^2(ab - (2a^2 - 3b^2)\sin(x)) + \frac{1}{3}\sec^3(x)$$

[Out] b^4*x+4/3*a*b*(a^2-2*b^2)*cos(x)+1/3*b^2*(2*a^2-3*b^2)*cos(x)*sin(x)+1/3*sec(x)^3*(b+a*sin(x))*(a+b*sin(x))^3-1/3*sec(x)*(a+b*sin(x))^2*(a*b-(2*a^2-3*b^2)*sin(x))

Rubi [A] time = 0.20, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4391, 2691, 2861, 2734}

$$\frac{4}{3}ab(a^2 - 2b^2)\cos(x) + \frac{1}{3}b^2(2a^2 - 3b^2)\sin(x)\cos(x) - \frac{1}{3}\sec(x)(a+b\sin(x))^2(ab - (2a^2 - 3b^2)\sin(x)) + \frac{1}{3}\sec^3(x)$$

Antiderivative was successfully verified.

[In] Int[(a*Sec[x] + b*Tan[x])^4,x]

[Out] b^4*x + (4*a*b*(a^2 - 2*b^2)*Cos[x])/3 + (b^2*(2*a^2 - 3*b^2)*Cos[x]*Sin[x])/3 + (Sec[x]^3*(b + a*SIN[x])*(a + b*SIN[x])^3)/3 - (Sec[x]*(a + b*SIN[x])^2*(a*b - (2*a^2 - 3*b^2)*Sin[x]))/3

Rule 2691

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^ (p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^ (m_.), x_Symbol] :> -Simp[((g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^(m - 1)*(b + a*sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*cos[e + f*x])^(p + 2)*(a + b*sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(p + 2) + a*b*(m + p + 1)*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p] || IntegerQ[m])

Rule 2734

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^ (n_.), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*cos[e + f*x])/f, x] - Simp[(b*d*cos[e + f*x]*sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2861

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^ (p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^ (m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[((g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m*(d + c*sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*cos[e + f*x])^(p + 2)*(a + b*sin[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[p, -1] && IntegerQ[2*m] && !(EqQ[m, 1] && NeQ[c^2 - d^2, 0]) && SimplifierQ[c + d*x, a + b*x]

Rule 4391

Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_.)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x_.)]^(n_.))^ (p_.), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a*sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned}
\int (a \sec(x) + b \tan(x))^4 dx &= \int \sec^4(x)(a + b \sin(x))^4 dx \\
&= \frac{1}{3} \sec^3(x)(b + a \sin(x))(a + b \sin(x))^3 - \frac{1}{3} \int \sec^2(x)(a + b \sin(x))^2 (-2a^2 + 3b^2 - 2ab \sin(x)) dx \\
&= \frac{1}{3} \sec^3(x)(b + a \sin(x))(a + b \sin(x))^3 - \frac{1}{3} \sec(x)(a + b \sin(x))^2 (ab - (2a^2 - 3b^2) \sin(x)) \\
&= b^4 x + \frac{4}{3} ab (a^2 - 2b^2) \cos(x) + \frac{1}{3} b^2 (2a^2 - 3b^2) \cos(x) \sin(x) + \frac{1}{3} \sec^3(x)(b + a \sin(x))^3
\end{aligned}$$

Mathematica [A] time = 0.20, size = 96, normalized size = 0.96

$$\frac{1}{12} \sec^3(x) (6a^4 \sin(x) + 2a^4 \sin(3x) + 16a^3 b + 18a^2 b^2 \sin(x) - 6a^2 b^2 \sin(3x) - 24ab^3 \cos(2x) - 8ab^3 - 4b^4 \sin(3x))$$

Antiderivative was successfully verified.

[In] Integrate[(a*Sec[x] + b*Tan[x])^4,x]

[Out] (Sec[x]^3*(16*a^3*b - 8*a*b^3 + 9*b^4*x*Cos[x] - 24*a*b^3*Cos[2*x] + 3*b^4*x*Cos[3*x] + 6*a^4*Sin[x] + 18*a^2*b^2*Sin[x] + 2*a^4*Sin[3*x] - 6*a^2*b^2*Sin[3*x] - 4*b^4*Sin[3*x]))/12

fricas [A] time = 0.84, size = 80, normalized size = 0.80

$$\frac{3b^4x \cos(x)^3 - 12ab^3 \cos(x)^2 + 4a^3b + 4ab^3 + (a^4 + 6a^2b^2 + b^4 + 2(a^4 - 3a^2b^2 - 2b^4) \cos(x)^2) \sin(x)}{3 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sec(x)+b*tan(x))^4,x, algorithm="fricas")

[Out] 1/3*(3*b^4*x*cos(x)^3 - 12*a*b^3*cos(x)^2 + 4*a^3*b + 4*a*b^3 + (a^4 + 6*a^2*b^2 + b^4 + 2*(a^4 - 3*a^2*b^2 - 2*b^4)*cos(x)^2)*sin(x))/cos(x)^3

giac [A] time = 0.16, size = 131, normalized size = 1.31

$$b^4 x - \frac{2 \left(3a^4 \tan\left(\frac{1}{2}x\right)^5 - 3b^4 \tan\left(\frac{1}{2}x\right)^5 + 12a^3b \tan\left(\frac{1}{2}x\right)^4 - 2a^4 \tan\left(\frac{1}{2}x\right)^3 + 24a^2b^2 \tan\left(\frac{1}{2}x\right)^3 + 10b^4 \tan\left(\frac{1}{2}x\right)^3 \right)}{3 \left(\tan\left(\frac{1}{2}x\right)^2 - 1 \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sec(x)+b*tan(x))^4,x, algorithm="giac")

[Out] b^4*x - 2/3*(3*a^4*tan(1/2*x)^5 - 3*b^4*tan(1/2*x)^5 + 12*a^3*b*tan(1/2*x)^4 - 2*a^4*tan(1/2*x)^3 + 24*a^2*b^2*tan(1/2*x)^3 + 10*b^4*tan(1/2*x)^3 + 24*a*b^3*tan(1/2*x)^2 + 3*a^4*tan(1/2*x) - 3*b^4*tan(1/2*x) + 4*a^3*b - 8*a*b^3)/tan(1/2*x)^2 - 1)^3

maple [A] time = 0.09, size = 96, normalized size = 0.96

$$-a^4 \left(-\frac{2}{3} - \frac{(\sec^2(x))}{3} \right) \tan(x) + \frac{4a^3b}{3 \cos(x)^3} + \frac{2a^2b^2 (\sin^3(x))}{\cos(x)^3} + 4ab^3 \left(\frac{\sin^4(x)}{3 \cos(x)^3} - \frac{\sin^4(x)}{3 \cos(x)} - \frac{(2 + \sin^2(x)) \cos(x)}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*sec(x)+b*tan(x))^4,x)

[Out] $-a^4*(-2/3-1/3*\sec(x)^2)*\tan(x)+4/3*a^3*b/\cos(x)^3+2*a^2*b^2*\sin(x)^3/\cos(x)^3+4*a*b^3*(1/3*\sin(x)^4/\cos(x)^3-1/3*\sin(x)^4/\cos(x)-1/3*(2+\sin(x)^2)*\cos(x))+b^4*(1/3*\tan(x)^3-\tan(x)+x)$

maxima [A] time = 1.05, size = 72, normalized size = 0.72

$$2a^2b^2 \tan(x)^3 + \frac{1}{3} (\tan(x)^3 + 3 \tan(x))a^4 + \frac{1}{3} (\tan(x)^3 + 3x - 3 \tan(x))b^4 - \frac{4(3 \cos(x)^2 - 1)ab^3}{3 \cos(x)^3} + \frac{4a^3b}{3 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sec(x)+b*tan(x))^4,x, algorithm="maxima")

[Out] $2*a^2*b^2*\tan(x)^3 + 1/3*(\tan(x)^3 + 3*\tan(x))*a^4 + 1/3*(\tan(x)^3 + 3*x - 3*\tan(x))*b^4 - 4/3*(3*\cos(x)^2 - 1)*a*b^3/\cos(x)^3 + 4/3*a^3*b/\cos(x)^3$

mupad [B] time = 2.53, size = 115, normalized size = 1.15

$$b^4 x - \frac{\tan\left(\frac{x}{2}\right) (2a^4 - 2b^4) - \frac{16ab^3}{3} + \frac{8a^3b}{3} + \tan\left(\frac{x}{2}\right)^3 \left(-\frac{4a^4}{3} + 16a^2b^2 + \frac{20b^4}{3}\right) + \tan\left(\frac{x}{2}\right)^5 (2a^4 - 2b^4) + 16ab^3 \tan\left(\frac{x}{2}\right)}{\left(\tan\left(\frac{x}{2}\right)^2 - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*tan(x) + a/cos(x))^4,x)

[Out] $b^4*x - (\tan(x/2)*(2*a^4 - 2*b^4) - (16*a*b^3)/3 + (8*a^3*b)/3 + \tan(x/2)^3*((20*b^4)/3 - (4*a^4)/3 + 16*a^2*b^2) + \tan(x/2)^5*(2*a^4 - 2*b^4) + 16*a*b^3*\tan(x/2)^2 + 8*a^3*b*\tan(x/2)^4)/(\tan(x/2)^2 - 1)^3$

sympy [A] time = 4.22, size = 97, normalized size = 0.97

$$\frac{a^4 \tan^3(x)}{3} + a^4 \tan(x) + \frac{4a^3b \sec^3(x)}{3} + 2a^2b^2 \tan^3(x) + \frac{4ab^3 \sec^3(x)}{3} - 4ab^3 \sec(x) + b^4x + \frac{b^4 \sin^3(x)}{3 \cos^3(x)} - \frac{b^4 \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sec(x)+b*tan(x))**4,x)

[Out] $a**4*\tan(x)**3/3 + a**4*\tan(x) + 4*a**3*b*\sec(x)**3/3 + 2*a**2*b**2*\tan(x)**3 + 4*a*b**3*\sec(x)**3/3 - 4*a*b**3*\sec(x) + b**4*x + b**4*\sin(x)**3/(3*\cos(x)**3) - b**4*\sin(x)/\cos(x)$

3.265 $\int (a \sec(x) + b \tan(x))^3 dx$

Optimal. Leaf size=75

$$\frac{1}{2}ab^2 \sin(x) + \frac{1}{4}(a+2b)(a-b)^2 \log(\sin(x)+1) - \frac{1}{4}(a-2b)(a+b)^2 \log(1-\sin(x)) + \frac{1}{2} \sec^2(x)(a \sin(x)+b)(a+b \sin(x))$$

[Out] $-1/4*(a-2*b)*(a+b)^2*\ln(1-\sin(x))+1/4*(a-b)^2*(a+2*b)*\ln(1+\sin(x))+1/2*a*b^2*\sin(x)+1/2*\sec(x)^2*(b+a*\sin(x))*(a+b*\sin(x))^2$

Rubi [A] time = 0.14, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {4391, 2668, 739, 774, 633, 31}

$$\frac{1}{2}ab^2 \sin(x) + \frac{1}{4}(a+2b)(a-b)^2 \log(\sin(x)+1) - \frac{1}{4}(a-2b)(a+b)^2 \log(1-\sin(x)) + \frac{1}{2} \sec^2(x)(a \sin(x)+b)(a+b \sin(x))$$

Antiderivative was successfully verified.

[In] Int[(a*Sec[x] + b*Tan[x])^3,x]

[Out] $-((a - 2*b)*(a + b)^2*\text{Log}[1 - \text{Sin}[x]])/4 + ((a - b)^2*(a + 2*b)*\text{Log}[1 + \text{Sin}[x]])/4 + (a*b^2*\text{Sin}[x])/2 + (\text{Sec}[x]^2*(b + a*\text{Sin}[x])*(a + b*\text{Sin}[x])^2)/2$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 633

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 739

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 774

Int((((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x]/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]

Rule 2668

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

Rule 4391

Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a

*Sin[c + d*x]^n]^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned}
 \int (a \sec(x) + b \tan(x))^3 dx &= \int \sec^3(x)(a + b \sin(x))^3 dx \\
 &= b^3 \operatorname{Subst} \left(\int \frac{(a+x)^3}{(b^2-x^2)^2} dx, x, b \sin(x) \right) \\
 &= \frac{1}{2} \sec^2(x)(b+a \sin(x))(a+b \sin(x))^2 - \frac{1}{2} b \operatorname{Subst} \left(\int \frac{(a+x)(-a^2+2b^2+ax)}{b^2-x^2} dx, x \right) \\
 &= \frac{1}{2} ab^2 \sin(x) + \frac{1}{2} \sec^2(x)(b+a \sin(x))(a+b \sin(x))^2 + \frac{1}{2} b \operatorname{Subst} \left(\int \frac{-ab^2-a(-a^2-x^2)}{b^2-x^2} dx, x \right) \\
 &= \frac{1}{2} ab^2 \sin(x) + \frac{1}{2} \sec^2(x)(b+a \sin(x))(a+b \sin(x))^2 + \frac{1}{4} ((a-2b)(a+b)^2) \operatorname{Subst} \left(\int \frac{1}{1-x} dx, x \right) \\
 &= -\frac{1}{4}(a-2b)(a+b)^2 \log(1-\sin(x)) + \frac{1}{4}(a-b)^2(a+2b) \log(1+\sin(x)) + \frac{1}{2} ab^2 \sin(x)
 \end{aligned}$$

Mathematica [A] time = 0.57, size = 123, normalized size = 1.64

$$\frac{2a^4b \sec^2(x) + (a^2 - b^2) \left((a - 2b)(a + b)^2 \log(1 - \sin(x)) - (a - b)^2(a + 2b) \log(\sin(x) + 1) \right) + (-8a^4b + 4a^2b^3 + 2b^5) \tan(x)}{4(b^2 - a^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Sec[x] + b*Tan[x])^3, x]

[Out] ((a^2 - b^2)*((a - 2*b)*(a + b)^2*Log[1 - Sin[x]] - (a - b)^2*(a + 2*b)*Log[1 + Sin[x]]) + 2*a^4*b*Sec[x]^2 - 2*a*(a^4 + 2*a^2*b^2 - 3*b^4)*Sec[x]*Tan[x] + (-8*a^4*b + 4*a^2*b^3 + 2*b^5)*Tan[x]^2)/(4*(-a^2 + b^2))

fricas [A] time = 2.07, size = 85, normalized size = 1.13

$$\frac{(a^3 - 3ab^2 + 2b^3) \cos(x)^2 \log(\sin(x) + 1) - (a^3 - 3ab^2 - 2b^3) \cos(x)^2 \log(-\sin(x) + 1) + 6a^2b + 2b^3 + 2(a^3 - 3ab^2 + 2b^3) \sin(x)}{4 \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sec(x)+b*tan(x))^3,x, algorithm="fricas")

[Out] 1/4*((a^3 - 3*a*b^2 + 2*b^3)*cos(x)^2*log(sin(x) + 1) - (a^3 - 3*a*b^2 - 2*b^3)*cos(x)^2*log(-sin(x) + 1) + 6*a^2*b + 2*b^3 + 2*(a^3 + 3*a*b^2)*sin(x))/cos(x)^2

giac [A] time = 0.14, size = 86, normalized size = 1.15

$$\frac{1}{4} (a^3 - 3ab^2 + 2b^3) \log(\sin(x) + 1) - \frac{1}{4} (a^3 - 3ab^2 - 2b^3) \log(-\sin(x) + 1) - \frac{b^3 \sin(x)^2 + a^3 \sin(x) + 3ab^2 \sin(x)}{2(\sin(x)^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sec(x)+b*tan(x))^3,x, algorithm="giac")

[Out] 1/4*(a^3 - 3*a*b^2 + 2*b^3)*log(sin(x) + 1) - 1/4*(a^3 - 3*a*b^2 - 2*b^3)*log(-sin(x) + 1) - 1/2*(b^3*sin(x)^2 + a^3*sin(x) + 3*a*b^2*sin(x) + 3*a^2*b)/(sin(x)^2 - 1)

maple [A] time = 0.08, size = 82, normalized size = 1.09

$$\frac{a^3 \sec(x) \tan(x)}{2} + \frac{a^3 \ln(\sec(x) + \tan(x))}{2} + \frac{3a^2 b}{2 \cos(x)^2} + \frac{3a b^2 (\sin^3(x))}{2 \cos(x)^2} + \frac{3a b^2 \sin(x)}{2} - \frac{3a b^2 \ln(\sec(x) + \tan(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*sec(x)+b*tan(x))^3,x)

[Out] 1/2*a^3*sec(x)*tan(x)+1/2*a^3*ln(sec(x)+tan(x))+3/2*a^2*b/cos(x)^2+3/2*a*b^2*sin(x)^3/cos(x)^2+3/2*a*b^2*sin(x)-3/2*a*b^2*ln(sec(x)+tan(x))+1/2*b^3*tan(x)^2+b^3*ln(cos(x))

maxima [A] time = 0.35, size = 95, normalized size = 1.27

$$\frac{3}{2} a^2 b \tan(x)^2 - \frac{3}{4} a b^2 \left(\frac{2 \sin(x)}{\sin(x)^2 - 1} + \log(\sin(x) + 1) - \log(\sin(x) - 1) \right) - \frac{1}{4} a^3 \left(\frac{2 \sin(x)}{\sin(x)^2 - 1} - \log(\sin(x) + 1) + \log(\sin(x) - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sec(x)+b*tan(x))^3,x, algorithm="maxima")

[Out] 3/2*a^2*b*tan(x)^2 - 3/4*a*b^2*(2*sin(x)/(sin(x)^2 - 1) + log(sin(x) + 1) - log(sin(x) - 1)) - 1/4*a^3*(2*sin(x)/(sin(x)^2 - 1) - log(sin(x) + 1) + log(sin(x) - 1)) - 1/2*b^3*(1/(sin(x)^2 - 1) - log(sin(x)^2 - 1))

mupad [B] time = 2.50, size = 126, normalized size = 1.68

$$\frac{(a^3 + 3 a b^2) \tan\left(\frac{x}{2}\right)^3 + (6 a^2 b + 2 b^3) \tan\left(\frac{x}{2}\right)^2 + (a^3 + 3 a b^2) \tan\left(\frac{x}{2}\right) - b^3 \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - \frac{\ln\left(\tan\left(\frac{x}{2}\right) - 1\right)}{\tan\left(\frac{x}{2}\right)^4 - 2 \tan\left(\frac{x}{2}\right)^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*tan(x) + a/cos(x))^3,x)

[Out] (tan(x/2)^2*(6*a^2*b + 2*b^3) + tan(x/2)*(3*a*b^2 + a^3) + tan(x/2)^3*(3*a*b^2 + a^3))/(tan(x/2)^4 - 2*tan(x/2)^2 + 1) - b^3*log(tan(x/2)^2 + 1) - (log(tan(x/2) - 1)*(a + b)^2*(a - 2*b))/2 + (log(tan(x/2) + 1)*(a - b)^2*(a + 2*b))/2

sympy [A] time = 4.94, size = 122, normalized size = 1.63

$$-\frac{a^3 \log(\sin(x) - 1)}{4} + \frac{a^3 \log(\sin(x) + 1)}{4} - \frac{a^3 \sin(x)}{2 \sin^2(x) - 2} + \frac{3a^2 b \sec^2(x)}{2} + \frac{3ab^2 \log(\sin(x) - 1)}{4} - \frac{3ab^2 \log(\sin(x) + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sec(x)+b*tan(x))**3,x)

[Out] -a**3*log(sin(x) - 1)/4 + a**3*log(sin(x) + 1)/4 - a**3*sin(x)/(2*sin(x)**2 - 2) + 3*a**2*b*sec(x)**2/2 + 3*a*b**2*log(sin(x) - 1)/4 - 3*a*b**2*log(sin(x) + 1)/4 - 3*a*b**2*sin(x)/(2*sin(x)**2 - 2) - b**3*log(sec(x)**2)/2 + b**3*sec(x)**2/2

3.266 $\int (a \sec(x) + b \tan(x))^2 dx$

Optimal. Leaf size=27

$$ab \cos(x) + \sec(x)(a \sin(x) + b)(a + b \sin(x)) + b^2(-x)$$

[Out] $-b^2x + a*b*\cos(x) + \sec(x)*(b+a*\sin(x))*(a+b*\sin(x))$

Rubi [A] time = 0.05, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4391, 2691, 2638}

$$ab \cos(x) + \sec(x)(a \sin(x) + b)(a + b \sin(x)) + b^2(-x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*\text{Sec}[x] + b*\text{Tan}[x])^2, x]$

[Out] $-(b^2*x) + a*b*\text{Cos}[x] + \text{Sec}[x]*(b + a*\text{Sin}[x])*(a + b*\text{Sin}[x])$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2691

$\text{Int}[(\cos[(e_.) + (f_.)*(x_.)]*(g_.))^p * ((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] \rightarrow -\text{Simp}[(g*\text{Cos}[e + f*x])^{p+1} * (a + b*\text{Sin}[e + f*x])^{m-1} * (b + a*\text{Sin}[e + f*x]) / (f*g*(p+1)), x] + \text{Dist}[1/(g^2*(p+1)), \text{Int}[(g*\text{Cos}[e + f*x])^{p+2} * (a + b*\text{Sin}[e + f*x])^{m-2} * (b^2*(m-1) + a^2*(p+2) + a*b*(m+p+1)*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, e, f, g\}, x \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[m, 1] \&\& \text{LtQ}[p, -1] \&\& (\text{IntegersQ}[2*m, 2*p] || \text{IntegerQ}[m])$

Rule 4391

$\text{Int}[(u_.)*((b_.)*\sec[(c_.) + (d_.)*(x_.)]^{n_.}) + (a_.)*\tan[(c_.) + (d_.)*(x_.)]^{n_.})^p, x_Symbol] \rightarrow \text{Int}[\text{ActivateTrig}[u]*\text{Sec}[c + d*x]^{n*p} * (b + a*\text{Sin}[c + d*x]^n)^p, x] /; \text{FreeQ}\{a, b, c, d\}, x \&\& \text{IntegersQ}[n, p]$

Rubi steps

$$\begin{aligned} \int (a \sec(x) + b \tan(x))^2 dx &= \int \sec^2(x)(a + b \sin(x))^2 dx \\ &= \sec(x)(b + a \sin(x))(a + b \sin(x)) - \int (b^2 + ab \sin(x)) dx \\ &= -b^2x + \sec(x)(b + a \sin(x))(a + b \sin(x)) - (ab) \int \sin(x) dx \\ &= -b^2x + ab \cos(x) + \sec(x)(b + a \sin(x))(a + b \sin(x)) \end{aligned}$$

Mathematica [A] time = 0.05, size = 25, normalized size = 0.93

$$(a^2 + b^2) \tan(x) + 2ab \sec(x) + b^2 (-\tan^{-1}(\tan(x)))$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(a*\text{Sec}[x] + b*\text{Tan}[x])^2, x]$

[Out] $-(b^2 \operatorname{ArcTan}[\tan(x)]) + 2ab \operatorname{Sec}(x) + (a^2 + b^2) \tan(x)$

fricas [A] time = 1.62, size = 29, normalized size = 1.07

$$\frac{b^2 x \cos(x) - 2ab - (a^2 + b^2) \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*sec(x)+b*tan(x))^2,x, algorithm="fricas")`

[Out] $-(b^2 x \cos(x) - 2ab - (a^2 + b^2) \sin(x)) / \cos(x)$

giac [A] time = 0.16, size = 40, normalized size = 1.48

$$-b^2 x - \frac{2 \left(a^2 \tan\left(\frac{1}{2}x\right) + b^2 \tan\left(\frac{1}{2}x\right) + 2ab \right)}{\tan\left(\frac{1}{2}x\right)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*sec(x)+b*tan(x))^2,x, algorithm="giac")`

[Out] $-b^2 x - 2(a^2 \tan(1/2*x) + b^2 \tan(1/2*x) + 2ab) / (\tan(1/2*x)^2 - 1)$

maple [A] time = 0.05, size = 26, normalized size = 0.96

$$a^2 \tan(x) + \frac{2ab}{\cos(x)} + b^2 (\tan(x) - x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*sec(x)+b*tan(x))^2,x)`

[Out] $a^2 \tan(x) + 2ab / \cos(x) + b^2 (\tan(x) - x)$

maxima [A] time = 0.41, size = 26, normalized size = 0.96

$$-b^2(x - \tan(x)) + a^2 \tan(x) + \frac{2ab}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*sec(x)+b*tan(x))^2,x, algorithm="maxima")`

[Out] $-b^2(x - \tan(x)) + a^2 \tan(x) + 2ab / \cos(x)$

mupad [B] time = 2.38, size = 40, normalized size = 1.48

$$-\frac{4ab + \tan\left(\frac{x}{2}\right) (2a^2 + 2b^2)}{\tan\left(\frac{x}{2}\right)^2 - 1} - b^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*tan(x) + a/cos(x))^2,x)`

[Out] $-(4ab + \tan(x/2)(2a^2 + 2b^2)) / (\tan(x/2)^2 - 1) - b^2 x$

sympy [A] time = 1.25, size = 22, normalized size = 0.81

$$a^2 \tan(x) + 2ab \sec(x) + b^2 (-x + \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*sec(x)+b*tan(x))**2,x)`

[Out] $a^2 \tan(x) + 2ab \sec(x) + b^2 (-x + \tan(x))$

3.267 $\int (a \sec(x) + b \tan(x)) dx$

Optimal. Leaf size=12

$$a \tanh^{-1}(\sin(x)) - b \log(\cos(x))$$

[Out] a*arctanh(sin(x))-b*ln(cos(x))

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3770, 3475}

$$a \tanh^{-1}(\sin(x)) - b \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[a*Sec[x] + b*Tan[x], x]

[Out] a*ArcTanh[Sin[x]] - b*Log[Cos[x]]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (a \sec(x) + b \tan(x)) dx &= a \int \sec(x) dx + b \int \tan(x) dx \\ &= a \tanh^{-1}(\sin(x)) - b \log(\cos(x)) \end{aligned}$$

Mathematica [B] time = 0.01, size = 42, normalized size = 3.50

$$-a \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + a \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right) - b \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[a*Sec[x] + b*Tan[x], x]

[Out] -(b*Log[Cos[x]]) - a*Log[Cos[x/2] - Sin[x/2]] + a*Log[Cos[x/2] + Sin[x/2]]

fricas [B] time = 1.49, size = 25, normalized size = 2.08

$$\frac{1}{2} (a - b) \log(\sin(x) + 1) - \frac{1}{2} (a + b) \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*sec(x)+b*tan(x), x, algorithm="fricas")

[Out] 1/2*(a - b)*log(sin(x) + 1) - 1/2*(a + b)*log(-sin(x) + 1)

giac [B] time = 0.13, size = 34, normalized size = 2.83

$$\frac{1}{4} a \left(\log\left(\left|\frac{1}{\sin(x)} + \sin(x) + 2\right|\right) - \log\left(\left|\frac{1}{\sin(x)} + \sin(x) - 2\right|\right) \right) - b \log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*sec(x)+b*tan(x),x, algorithm="giac")

[Out] $\frac{1}{4}a*(\log(\text{abs}(1/\sin(x) + \sin(x) + 2)) - \log(\text{abs}(1/\sin(x) + \sin(x) - 2))) - b*\log(\text{abs}(\cos(x)))$

maple [A] time = 0.00, size = 16, normalized size = 1.33

$$a \ln(\sec(x) + \tan(x)) - b \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a*sec(x)+b*tan(x),x)

[Out] $a*\ln(\sec(x)+\tan(x))-b*\ln(\cos(x))$

maxima [A] time = 0.30, size = 14, normalized size = 1.17

$$a \log(\sec(x) + \tan(x)) + b \log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*sec(x)+b*tan(x),x, algorithm="maxima")

[Out] $a*\log(\sec(x) + \tan(x)) + b*\log(\sec(x))$

mupad [B] time = 2.49, size = 37, normalized size = 3.08

$$b \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - \ln\left(\tan\left(\frac{x}{2}\right) - 1\right) (a + b) + \ln\left(\tan\left(\frac{x}{2}\right) + 1\right) (a - b)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(b*tan(x) + a/cos(x),x)

[Out] $b*\log(\tan(x/2)^2 + 1) - \log(\tan(x/2) - 1)*(a + b) + \log(\tan(x/2) + 1)*(a - b)$

sympy [A] time = 0.09, size = 24, normalized size = 2.00

$$a \left(-\frac{\log(\sin(x) - 1)}{2} + \frac{\log(\sin(x) + 1)}{2} \right) - b \log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*sec(x)+b*tan(x),x)

[Out] $a*(-\log(\sin(x) - 1)/2 + \log(\sin(x) + 1)/2) - b*\log(\cos(x))$

$$3.268 \quad \int \frac{1}{a \sec(x) + b \tan(x)} dx$$

Optimal. Leaf size=11

$$\frac{\log(a + b \sin(x))}{b}$$

[Out] ln(a+b*sin(x))/b

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3159, 2668, 31}

$$\frac{\log(a + b \sin(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[(a*Sec[x] + b*Tan[x])^(-1), x]

[Out] Log[a + b*Sin[x]]/b

Rule 31

Int[((a_) + (b_.)*(x_))^(p_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2668

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

Rule 3159

Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]), x] /; FreeQ[{a, b, c, d, e}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{a \sec(x) + b \tan(x)} dx &= \int \frac{\cos(x)}{a + b \sin(x)} dx \\ &= \frac{\text{Subst}\left(\int \frac{1}{a+x} dx, x, b \sin(x)\right)}{b} \\ &= \frac{\log(a + b \sin(x))}{b} \end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{\log(a + b \sin(x))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Sec[x] + b*Tan[x])^(-1), x]

[Out] Log[a + b*Sin[x]]/b

fricas [A] time = 1.02, size = 11, normalized size = 1.00

$$\frac{\log(b \sin(x) + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sec(x)+b*tan(x)),x, algorithm="fricas")

[Out] log(b*sin(x) + a)/b

giac [A] time = 0.13, size = 12, normalized size = 1.09

$$\frac{\log(|b \sin(x) + a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sec(x)+b*tan(x)),x, algorithm="giac")

[Out] log(abs(b*sin(x) + a))/b

maple [A] time = 0.12, size = 12, normalized size = 1.09

$$\frac{\ln(a + b \sin(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*sec(x)+b*tan(x)),x)

[Out] ln(a+b*sin(x))/b

maxima [B] time = 0.50, size = 50, normalized size = 4.55

$$\frac{\log\left(a + \frac{2b \sin(x)}{\cos(x)+1} + \frac{a \sin(x)^2}{(\cos(x)+1)^2}\right)}{b} - \frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sec(x)+b*tan(x)),x, algorithm="maxima")

[Out] log(a + 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/b - log(sin(x)^2/(cos(x) + 1)^2 + 1)/b

mupad [B] time = 3.79, size = 55, normalized size = 5.00

$$\frac{2 \operatorname{atanh}\left(\frac{b\left(2a^3 \sin(x) + \frac{5a^2b}{2} - b^3 - \frac{a^2b \cos(2x)}{2}\right)}{(2a^2 + \sin(x)ab - b^2)^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*tan(x) + a/cos(x)),x)

[Out] (2*atanh((b*(2*a^3*sin(x) + (5*a^2*b)/2 - b^3 - (a^2*b*cos(2*x))/2))/(2*a^2 - b^2 + a*b*sin(x))^2))/b

sympy [A] time = 0.42, size = 32, normalized size = 2.91

$$\begin{cases} \frac{\log\left(\frac{a \sec(x)}{b} + \tan(x)\right)}{b} - \frac{\log(\tan^2(x)+1)}{2b} & \text{for } b \neq 0 \\ \frac{\tan(x)}{a \sec(x)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x)),x)
```

```
[Out] Piecewise((log(a*sec(x)/b + tan(x))/b - log(tan(x)**2 + 1)/(2*b), Ne(b, 0))  
, (tan(x)/(a*sec(x)), True))
```


$$3.269 \quad \int \frac{1}{(a \sec(x) + b \tan(x))^2} dx$$

Optimal. Leaf size=66

$$\frac{2a \tan^{-1}\left(\frac{a \tan\left(\frac{x}{2}\right) + b}{\sqrt{a^2 - b^2}}\right)}{b^2 \sqrt{a^2 - b^2}} - \frac{\cos(x)}{b(a + b \sin(x))} - \frac{x}{b^2}$$

[Out] $-x/b^2 - \cos(x)/b/(a+b*\sin(x)) + 2*a*\arctan((b+a*\tan(1/2*x))/(a^2-b^2)^{(1/2)})/b^2/(a^2-b^2)^{(1/2)}$

Rubi [A] time = 0.13, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {4391, 2693, 2735, 2660, 618, 204}

$$\frac{2a \tan^{-1}\left(\frac{a \tan\left(\frac{x}{2}\right) + b}{\sqrt{a^2 - b^2}}\right)}{b^2 \sqrt{a^2 - b^2}} - \frac{\cos(x)}{b(a + b \sin(x))} - \frac{x}{b^2}$$

Antiderivative was successfully verified.

[In] Int[(a*Sec[x] + b*Tan[x])^(-2), x]

[Out] $-(x/b^2) + (2*a*\text{ArcTan}[(b + a*\text{Tan}[x/2])/ \text{Sqrt}[a^2 - b^2]])/(b^2*\text{Sqrt}[a^2 - b^2]) - \text{Cos}[x]/(b*(a + b*\text{Sin}[x]))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2693

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && IntegersQ[2*m, 2*p]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 4391

Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_.)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x_.)]^(n_.))^p, x_Symbol] := Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(a \sec(x) + b \tan(x))^2} dx &= \int \frac{\cos^2(x)}{(a + b \sin(x))^2} dx \\
 &= -\frac{\cos(x)}{b(a + b \sin(x))} - \frac{\int \frac{\sin(x)}{a + b \sin(x)} dx}{b} \\
 &= -\frac{x}{b^2} - \frac{\cos(x)}{b(a + b \sin(x))} + \frac{a \int \frac{1}{a + b \sin(x)} dx}{b^2} \\
 &= -\frac{x}{b^2} - \frac{\cos(x)}{b(a + b \sin(x))} + \frac{(2a) \operatorname{Subst}\left(\int \frac{1}{a + 2bx + ax^2} dx, x, \tan\left(\frac{x}{2}\right)\right)}{b^2} \\
 &= -\frac{x}{b^2} - \frac{\cos(x)}{b(a + b \sin(x))} - \frac{(4a) \operatorname{Subst}\left(\int \frac{1}{-4(a^2 - b^2) - x^2} dx, x, 2b + 2a \tan\left(\frac{x}{2}\right)\right)}{b^2} \\
 &= -\frac{x}{b^2} + \frac{2a \tan^{-1}\left(\frac{b + a \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2}}\right)}{b^2 \sqrt{a^2 - b^2}} - \frac{\cos(x)}{b(a + b \sin(x))}
 \end{aligned}$$

Mathematica [B] time = 2.02, size = 344, normalized size = 5.21

$$(\sin(x) + 1) \cos(x) \left(2a(a - b) \sqrt{1 - \sin(x)} (a + b \sin(x)) \tanh^{-1} \left(\frac{\sqrt{a-b} \sqrt{-\frac{b(\sin(x)+1)}{a-b}}}{\sqrt{a+b} \sqrt{-\frac{b(\sin(x)-1)}{a+b}}} \right) + \sqrt{a+b} \left(-(b-a) \sqrt{\frac{b-b \sin(x)}{a+b}} \right) \right)$$

$$(a - b)^{5/2} (a + b)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Sec[x] + b*Tan[x])^(-2), x]

[Out] (Cos[x]*(1 + Sin[x])*(2*a*(a - b)*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[x]))/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[x]))/(a + b))])]*Sqrt[1 - Sin[x]]*(a + b*Sin[x]) + Sqrt[a + b]*(-2*a*Sqrt[a - b]*ArcTanh[Sqrt[(b*(1 + Sin[x]))/(-a + b)]/Sqrt[(b - b*Sin[x])/(a + b)]]*Sqrt[1 - Sin[x]]*(a + b*Sin[x]) - (-a + b)*Sqrt[(b - b*Sin[x])/(a + b)]*(Sqrt[a - b]*(a + b)*Sqrt[1 - Sin[x]]*Sqrt[-((b*(1 + Sin[x]))/(a - b))]) + 2*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[x]))/(a - b))])/(Sqrt[2]*Sqrt[b])]*(a + b*Sin[x])))/(a - b)^(5/2)*(a + b)^(3/2)*Sqrt[1 - Sin[x]]*(-((b*(1 + Sin[x]))/(a - b)))^(3/2)*Sqrt[(b - b*Sin[x])/(a + b)]*(a + b*Sin[x]))

fricas [B] time = 1.06, size = 308, normalized size = 4.67

$$\left[\frac{2(a^2b - b^3)x \sin(x) + (ab \sin(x) + a^2)\sqrt{-a^2 + b^2} \log\left(\frac{(2a^2 - b^2)\cos(x)^2 - 2ab \sin(x) - a^2 - b^2 + 2(a \cos(x) \sin(x) + b \cos(x))\sqrt{-a^2 + b^2}}{b^2 \cos(x)^2 - 2ab \sin(x) - a^2 - b^2}\right)}{2(a^3b^2 - ab^4 + (a^2b^3 - b^5)\sin(x))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sec(x)+b*tan(x))^2,x, algorithm="fricas")

[Out]
$$\left[-\frac{1}{2} \cdot (2 \cdot (a^2 \cdot b - b^3) \cdot x \cdot \sin(x) + (a \cdot b \cdot \sin(x) + a^2) \cdot \sqrt{-a^2 + b^2}) \cdot \log\left(\frac{(2 \cdot a^2 - b^2) \cdot \cos(x)^2 - 2 \cdot a \cdot b \cdot \sin(x) - a^2 - b^2 + 2 \cdot (a \cdot \cos(x) \cdot \sin(x) + b \cdot \cos(x)) \cdot \sqrt{-a^2 + b^2}}{(b^2 \cdot \cos(x)^2 - 2 \cdot a \cdot b \cdot \sin(x) - a^2 - b^2)}\right) + 2 \cdot (a^3 - a \cdot b^2) \cdot x + 2 \cdot (a^2 \cdot b - b^3) \cdot \cos(x) \right] / (a^3 \cdot b^2 - a \cdot b^4 + (a^2 \cdot b^3 - b^5) \cdot \sin(x)), -\left((a^2 \cdot b - b^3) \cdot x \cdot \sin(x) + (a \cdot b \cdot \sin(x) + a^2) \cdot \sqrt{a^2 - b^2} \right) \cdot \arctan\left(\frac{-(a \cdot \sin(x) + b)}{\sqrt{a^2 - b^2} \cdot \cos(x)}\right) + (a^3 - a \cdot b^2) \cdot x + (a^2 \cdot b - b^3) \cdot \cos(x) \right] / (a^3 \cdot b^2 - a \cdot b^4 + (a^2 \cdot b^3 - b^5) \cdot \sin(x))]$$

giac [A] time = 0.17, size = 94, normalized size = 1.42

$$\frac{2 \left(\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(a) + \arctan\left(\frac{a \tan\left(\frac{1}{2}x\right) + b}{\sqrt{a^2 - b^2}}\right) \right) a}{\sqrt{a^2 - b^2} b^2} - \frac{x}{b^2} - \frac{2 \left(b \tan\left(\frac{1}{2}x\right) + a \right)}{\left(a \tan\left(\frac{1}{2}x\right)^2 + 2 b \tan\left(\frac{1}{2}x\right) + a \right) a b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sec(x)+b*tan(x))^2,x, algorithm="giac")

[Out]
$$2 \cdot (\pi \cdot \operatorname{floor}(1/2 \cdot x / \pi + 1/2) \cdot \operatorname{sgn}(a) + \arctan((a \cdot \tan(1/2 \cdot x) + b) / \sqrt{a^2 - b^2})) \cdot a / (\sqrt{a^2 - b^2} \cdot b^2) - x / b^2 - 2 \cdot (b \cdot \tan(1/2 \cdot x) + a) / ((a \cdot \tan(1/2 \cdot x)^2 + 2 \cdot b \cdot \tan(1/2 \cdot x) + a) \cdot a \cdot b)$$

maple [A] time = 0.16, size = 106, normalized size = 1.61

$$\frac{2 \tan\left(\frac{x}{2}\right)}{\left(a \left(\tan^2\left(\frac{x}{2}\right) + 2b \tan\left(\frac{x}{2}\right) + a\right) a - b \left(a \left(\tan^2\left(\frac{x}{2}\right) + 2b \tan\left(\frac{x}{2}\right) + a\right) a\right)} + \frac{2a \arctan\left(\frac{2a \tan\left(\frac{x}{2}\right) + 2b}{2\sqrt{a^2 - b^2}}\right)}{b^2 \sqrt{a^2 - b^2}} - \frac{2 \arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*sec(x)+b*tan(x))^2,x)

[Out]
$$-2 / (a \cdot \tan(1/2 \cdot x)^2 + 2 \cdot b \cdot \tan(1/2 \cdot x) + a) / a \cdot \tan(1/2 \cdot x) - 2 / b / (a \cdot \tan(1/2 \cdot x)^2 + 2 \cdot b \cdot \tan(1/2 \cdot x) + a) + 2 / b^2 \cdot a / (a^2 - b^2)^{(1/2)} \cdot \arctan(1/2 \cdot (2 \cdot a \cdot \tan(1/2 \cdot x) + 2 \cdot b) / (a^2 - b^2)^{(1/2)}) - 2 / b^2 \cdot \arctan(\tan(1/2 \cdot x))$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sec(x)+b*tan(x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more details) Is 4*b^2-4*a^2 positive or negative?

mupad [B] time = 2.81, size = 604, normalized size = 9.15

$$\frac{x}{b^2} \frac{2 \tan\left(\frac{x}{2}\right)}{a} + \frac{2}{b} \frac{a \operatorname{atan}\left(\frac{32 a^2 b^3 + \frac{32 \tan\left(\frac{x}{2}\right) (2 a b^3 - 2 a^3 b)}{b^3} + \frac{a \left(32 a b^2 + 64 a^2 b \tan\left(\frac{x}{2}\right) + \frac{32 a^2 b^3 + \frac{32 \tan\left(\frac{x}{2}\right) (3 a b^7 - 2 a^3 b^5)}{b^3}}{b^2 \sqrt{b^2 - a^2}}\right)}{b^2 \sqrt{b^2 - a^2}}\right)}{b^2 \sqrt{b^2 - a^2}}}{a \operatorname{atan}\left(\frac{32 a^2 b^3 + \frac{32 \tan\left(\frac{x}{2}\right) (2 a b^3 - 2 a^3 b)}{b^3} + \frac{a \left(32 a b^2 + 64 a^2 b \tan\left(\frac{x}{2}\right) + \frac{32 a^2 b^3 + \frac{32 \tan\left(\frac{x}{2}\right) (3 a b^7 - 2 a^3 b^5)}{b^3}}{b^2 \sqrt{b^2 - a^2}}\right)}{b^2 \sqrt{b^2 - a^2}}\right)}{b^2 \sqrt{b^2 - a^2}} + \frac{128 a^2 \tan\left(\frac{x}{2}\right)}{b^3} + \frac{a \operatorname{atan}\left(\frac{32 a^2 b^3 + \frac{32 \tan\left(\frac{x}{2}\right) (2 a b^3 - 2 a^3 b)}{b^3} + \frac{a \left(32 a b^2 + 64 a^2 b \tan\left(\frac{x}{2}\right) + \frac{32 a^2 b^3 + \frac{32 \tan\left(\frac{x}{2}\right) (3 a b^7 - 2 a^3 b^5)}{b^3}}{b^2 \sqrt{b^2 - a^2}}\right)}{b^2 \sqrt{b^2 - a^2}}\right)}{b^2 \sqrt{b^2 - a^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(x) + a/cos(x))^2,x)
```

```
[Out] - x/b^2 - ((2*tan(x/2))/a + 2/b)/(a + 2*b*tan(x/2) + a*tan(x/2)^2) - (a*atan(((a*((32*a^2)/b + (32*tan(x/2)*(2*a*b^3 - 2*a^3*b))/b^3 + (a*(32*a*b^2 + 64*a^2*b*tan(x/2) + (a*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3)))/(b^2*(b^2 - a^2)^(1/2)))))/(b^2*(b^2 - a^2)^(1/2)))*1i)/(b^2*(b^2 - a^2)^(1/2)) + (a*((32*a^2)/b + (32*tan(x/2)*(2*a*b^3 - 2*a^3*b))/b^3 - (a*(32*a*b^2 + 64*a^2*b*tan(x/2) - (a*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3)))/(b^2*(b^2 - a^2)^(1/2)))))/(b^2*(b^2 - a^2)^(1/2)))*1i)/(b^2*(b^2 - a^2)^(1/2)))/((128*a^2*tan(x/2))/b^3 + (a*((32*a^2)/b + (32*tan(x/2)*(2*a*b^3 - 2*a^3*b))/b^3 + (a*(32*a*b^2 + 64*a^2*b*tan(x/2) + (a*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3)))/(b^2*(b^2 - a^2)^(1/2)))))/(b^2*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)) - (a*((32*a^2)/b + (32*tan(x/2)*(2*a*b^3 - 2*a^3*b))/b^3 - (a*(32*a*b^2 + 64*a^2*b*tan(x/2) - (a*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3)))/(b^2*(b^2 - a^2)^(1/2)))))/(b^2*(b^2 - a^2)^(1/2)))/((128*a^2*tan(x/2))/b^3 + (a*((32*a^2)/b + (32*tan(x/2)*(2*a*b^3 - 2*a^3*b))/b^3 + (a*(32*a*b^2 + 64*a^2*b*tan(x/2) + (a*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3)))/(b^2*(b^2 - a^2)^(1/2)))))/(b^2*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)))*2i)/(b^2*(b^2 - a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \sec(x) + b \tan(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))**2,x)
```

```
[Out] Integral((a*sec(x) + b*tan(x))**(-2), x)
```

$$3.270 \quad \int \frac{1}{(a \sec(x) + b \tan(x))^3} dx$$

Optimal. Leaf size=51

$$\frac{a^2 - b^2}{2b^3(a + b \sin(x))^2} - \frac{2a}{b^3(a + b \sin(x))} - \frac{\log(a + b \sin(x))}{b^3}$$

[Out] $-\ln(a+b*\sin(x))/b^3+1/2*(a^2-b^2)/b^3/(a+b*\sin(x))^2-2*a/b^3/(a+b*\sin(x))$

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4391, 2668, 697}

$$\frac{a^2 - b^2}{2b^3(a + b \sin(x))^2} - \frac{2a}{b^3(a + b \sin(x))} - \frac{\log(a + b \sin(x))}{b^3}$$

Antiderivative was successfully verified.

[In] Int[(a*Sec[x] + b*Tan[x])^(-3), x]

[Out] $-(\text{Log}[a + b*\text{Sin}[x]]/b^3) + (a^2 - b^2)/(2*b^3*(a + b*\text{Sin}[x])^2) - (2*a)/(b^3*(a + b*\text{Sin}[x]))$

Rule 697

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rule 2668

Int[cos[(e_) + (f_)*(x_)]^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*SIN[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

Rule 4391

Int[(u_)*((b_)*sec[(c_) + (d_)*(x_)]^(n_) + (a_)*tan[(c_) + (d_)*(x_)])^(p_), x_Symbol] := Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a*SIN[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a \sec(x) + b \tan(x))^3} dx &= \int \frac{\cos^3(x)}{(a + b \sin(x))^3} dx \\ &= \frac{\text{Subst}\left(\int \frac{b^2 - x^2}{(a+x)^3} dx, x, b \sin(x)\right)}{b^3} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{-a-x} + \frac{-a^2+b^2}{(a+x)^3} + \frac{2a}{(a+x)^2}\right) dx, x, b \sin(x)\right)}{b^3} \\ &= -\frac{\log(a + b \sin(x))}{b^3} + \frac{a^2 - b^2}{2b^3(a + b \sin(x))^2} - \frac{2a}{b^3(a + b \sin(x))} \end{aligned}$$

Mathematica [A] time = 0.17, size = 40, normalized size = 0.78

$$-\frac{\frac{3a^2+4ab \sin(x)+b^2}{2(a+b \sin(x))^2} + \log(a + b \sin(x))}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Sec[x] + b*Tan[x])^(-3),x]

[Out] -((Log[a + b*Sin[x]] + (3*a^2 + b^2 + 4*a*b*Sin[x])/(2*(a + b*Sin[x])^2))/b^3)

fricas [A] time = 0.95, size = 83, normalized size = 1.63

$$\frac{4ab\sin(x) + 3a^2 + b^2 - 2(b^2\cos(x)^2 - 2ab\sin(x) - a^2 - b^2)\log(b\sin(x) + a)}{2(b^5\cos(x)^2 - 2ab^4\sin(x) - a^2b^3 - b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sec(x)+b*tan(x))^3,x, algorithm="fricas")

[Out] 1/2*(4*a*b*sin(x) + 3*a^2 + b^2 - 2*(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)*log(b*sin(x) + a))/(b^5*cos(x)^2 - 2*a*b^4*sin(x) - a^2*b^3 - b^5)

giac [A] time = 0.14, size = 43, normalized size = 0.84

$$-\frac{\log(|b\sin(x) + a|)}{b^3} + \frac{3b\sin(x)^2 + 2a\sin(x) - b}{2(b\sin(x) + a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sec(x)+b*tan(x))^3,x, algorithm="giac")

[Out] -log(abs(b*sin(x) + a))/b^3 + 1/2*(3*b*sin(x)^2 + 2*a*sin(x) - b)/((b*sin(x) + a)^2*b^2)

maple [A] time = 0.17, size = 57, normalized size = 1.12

$$-\frac{2a}{b^3(a + b\sin(x))} - \frac{\ln(a + b\sin(x))}{b^3} + \frac{a^2}{2b^3(a + b\sin(x))^2} - \frac{1}{2b(a + b\sin(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*sec(x)+b*tan(x))^3,x)

[Out] -2*a/b^3/(a+b*sin(x))-ln(a+b*sin(x))/b^3+1/2/b^3/(a+b*sin(x))^2*a^2-1/2/b/(a+b*sin(x))^2

maxima [B] time = 0.51, size = 201, normalized size = 3.94

$$\frac{2\left(\frac{(a^3+ab^2)\sin(x)}{\cos(x)+1} + \frac{(3a^2b+b^3)\sin(x)^2}{(\cos(x)+1)^2} + \frac{(a^3+ab^2)\sin(x)^3}{(\cos(x)+1)^3}\right)}{a^4b^2 + \frac{4a^3b^3\sin(x)}{\cos(x)+1} + \frac{4a^3b^3\sin(x)^3}{(\cos(x)+1)^3} + \frac{a^4b^2\sin(x)^4}{(\cos(x)+1)^4} + \frac{2(a^4b^2+2a^2b^4)\sin(x)^2}{(\cos(x)+1)^2}} - \frac{\log\left(a + \frac{2b\sin(x)}{\cos(x)+1} + \frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^3} + \frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sec(x)+b*tan(x))^3,x, algorithm="maxima")

[Out] 2*((a^3 + a*b^2)*sin(x)/(cos(x) + 1) + (3*a^2*b + b^3)*sin(x)^2/(cos(x) + 1)^2 + (a^3 + a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a^4*b^2 + 4*a^3*b^3*sin(x)/(cos(x) + 1) + 4*a^3*b^3*sin(x)^3/(cos(x) + 1)^3 + a^4*b^2*sin(x)^4/(cos(x) + 1)^4 + 2*(a^4*b^2 + 2*a^2*b^4)*sin(x)^2/(cos(x) + 1)^2) - log(a + 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/b^3 + log(sin(x)^2/(cos(x) + 1)^2 + 1)/b^3

mupad [B] time = 2.70, size = 106, normalized size = 2.08

$$\frac{2 a^3 b \sin(x) + 3 a^2 b^2 \sin(x)^2 + 2 a b^3 \sin(x) + b^4 \sin(x)^2}{2 a^4 b^3 + 4 a^3 b^4 \sin(x) + 2 a^2 b^5 \sin(x)^2} - \frac{2 \operatorname{atanh}\left(\frac{b^2 + a \sin(x) b}{2 a^2 + \sin(x) a b - b^2}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*tan(x) + a/cos(x))^3,x)

[Out] (b^4*sin(x)^2 + 3*a^2*b^2*sin(x)^2 + 2*a*b^3*sin(x) + 2*a^3*b*sin(x))/(2*a^4*b^3 + 2*a^2*b^5*sin(x)^2 + 4*a^3*b^4*sin(x)) - (2*atanh((b^2 + a*b*sin(x))/(2*a^2 - b^2 + a*b*sin(x))))/b^3

sympy [A] time = 2.76, size = 503, normalized size = 9.86

$$\left\{ \begin{array}{l} -\frac{2a^2 \log\left(\frac{a \sec(x)}{b} + \tan(x)\right) \sec^2(x)}{2a^2 b^3 \sec^2(x) + 4ab^4 \tan(x) \sec(x) + 2b^5 \tan^2(x)} + \frac{a^2 \log(\tan^2(x)+1) \sec^2(x)}{2a^2 b^3 \sec^2(x) + 4ab^4 \tan(x) \sec(x) + 2b^5 \tan^2(x)} - \frac{a^2 \sec^2(x)}{2a^2 b^3 \sec^2(x) + 4ab^4 \tan(x) \sec(x) + 2b^5 \tan^2(x)} \\ \frac{2 \tan^3(x) + \tan(x)}{3 \sec^3(x) + \sec^3(x)} \\ a^3 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sec(x)+b*tan(x))**3,x)

[Out] Piecewise((-2*a**2*log(a*sec(x)/b + tan(x))*sec(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) + a**2*log(tan(x)**2 + 1)*sec(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) - a**2*sec(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) - 4*a*b*log(a*sec(x)/b + tan(x))*tan(x)*sec(x)/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) + 2*a*b*log(tan(x)**2 + 1)*tan(x)*sec(x)/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) - 2*b**2*log(a*sec(x)/b + tan(x))*tan(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) + b**2*log(tan(x)**2 + 1)*tan(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) + b**2*tan(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) - b**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2), Ne(b, 0)), ((2*tan(x)**3/(3*sec(x)**3) + tan(x)/sec(x)**3)/a**3, True))

$$3.271 \quad \int \frac{1}{(a \sec(x) + b \tan(x))^4} dx$$

Optimal. Leaf size=156

$$\frac{a \cos^3(x)}{2b(a^2 - b^2)(a + b \sin(x))^2} - \frac{a(2a^2 - 3b^2) \tan^{-1}\left(\frac{a \tan(\frac{x}{2}) + b}{\sqrt{a^2 - b^2}}\right)}{b^4(a^2 - b^2)^{3/2}} + \frac{\cos(x)(2(a^2 - b^2) + ab \sin(x))}{2b^3(a^2 - b^2)(a + b \sin(x))} - \frac{\cos^3(x)}{3b(a + b \sin(x))^3}$$

[Out] $x/b^4 - a*(2*a^2 - 3*b^2)*\arctan((b + a*\tan(1/2*x))/(\sqrt{a^2 - b^2}))/b^4/(a^2 - b^2)^{(3/2)} - 1/3*\cos(x)^3/b/(a + b*\sin(x))^3 + 1/2*a*\cos(x)^3/b/(a^2 - b^2)/(a + b*\sin(x))^2 + 1/2*\cos(x)*(2*a^2 - 2*b^2 + a*b*\sin(x))/b^3/(a^2 - b^2)/(a + b*\sin(x))$

Rubi [A] time = 0.34, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {4391, 2693, 2864, 2863, 2735, 2660, 618, 204}

$$-\frac{a(2a^2 - 3b^2) \tan^{-1}\left(\frac{a \tan(\frac{x}{2}) + b}{\sqrt{a^2 - b^2}}\right)}{b^4(a^2 - b^2)^{3/2}} + \frac{a \cos^3(x)}{2b(a^2 - b^2)(a + b \sin(x))^2} + \frac{\cos(x)(2(a^2 - b^2) + ab \sin(x))}{2b^3(a^2 - b^2)(a + b \sin(x))} - \frac{\cos^3(x)}{3b(a + b \sin(x))^3}$$

Antiderivative was successfully verified.

[In] Int[(a*Sec[x] + b*Tan[x])^(-4), x]

[Out] $x/b^4 - (a*(2*a^2 - 3*b^2)*\text{ArcTan}[(b + a*\text{Tan}[x/2])/(\sqrt{a^2 - b^2})])/(b^4*(a^2 - b^2)^{(3/2)}) - \text{Cos}[x]^3/(3*b*(a + b*\text{Sin}[x])^3) + (a*\text{Cos}[x]^3)/(2*b*(a^2 - b^2)*(a + b*\text{Sin}[x])^2) + (\text{Cos}[x]*(2*(a^2 - b^2) + a*b*\text{Sin}[x]))/(2*b^3*(a^2 - b^2)*(a + b*\text{Sin}[x]))$

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2693

Int[(cos[(e_) + (f_)*(x_)])*(g_)^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && IntegerQ[2*m, 2*p]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2863

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.))^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]

Rule 2864

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.))^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

Rule 4391

Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)])^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x_)])^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a \sec(x) + b \tan(x))^4} dx &= \int \frac{\cos^4(x)}{(a + b \sin(x))^4} dx \\
&= -\frac{\cos^3(x)}{3b(a + b \sin(x))^3} - \frac{\int \frac{\cos^2(x) \sin(x)}{(a + b \sin(x))^3} dx}{b} \\
&= -\frac{\cos^3(x)}{3b(a + b \sin(x))^3} + \frac{a \cos^3(x)}{2b(a^2 - b^2)(a + b \sin(x))^2} + \frac{\int \frac{\cos^2(x)(2b + a \sin(x))}{(a + b \sin(x))^2} dx}{2b(a^2 - b^2)} \\
&= -\frac{\cos^3(x)}{3b(a + b \sin(x))^3} + \frac{a \cos^3(x)}{2b(a^2 - b^2)(a + b \sin(x))^2} + \frac{\cos(x)(2(a^2 - b^2) + ab \sin(x))}{2b^3(a^2 - b^2)(a + b \sin(x))} \\
&= \frac{x}{b^4} - \frac{\cos^3(x)}{3b(a + b \sin(x))^3} + \frac{a \cos^3(x)}{2b(a^2 - b^2)(a + b \sin(x))^2} + \frac{\cos(x)(2(a^2 - b^2) + ab \sin(x))}{2b^3(a^2 - b^2)(a + b \sin(x))} \\
&= \frac{x}{b^4} - \frac{\cos^3(x)}{3b(a + b \sin(x))^3} + \frac{a \cos^3(x)}{2b(a^2 - b^2)(a + b \sin(x))^2} + \frac{\cos(x)(2(a^2 - b^2) + ab \sin(x))}{2b^3(a^2 - b^2)(a + b \sin(x))} \\
&= \frac{x}{b^4} - \frac{\cos^3(x)}{3b(a + b \sin(x))^3} + \frac{a \cos^3(x)}{2b(a^2 - b^2)(a + b \sin(x))^2} + \frac{\cos(x)(2(a^2 - b^2) + ab \sin(x))}{2b^3(a^2 - b^2)(a + b \sin(x))} \\
&= \frac{x}{b^4} - \frac{a(2a^2 - 3b^2) \tan^{-1}\left(\frac{b + a \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2}}\right)}{b^4(a^2 - b^2)^{3/2}} - \frac{\cos^3(x)}{3b(a + b \sin(x))^3} + \frac{a \cos^3(x)}{2b(a^2 - b^2)(a + b \sin(x))}
\end{aligned}$$

Mathematica [B] time = 6.36, size = 2661, normalized size = 17.06

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a*Sec[x] + b*Tan[x])^(-4), x]

[Out] (Sec[x]*(a + b*Sin[x])^4*(-1/3*(b*(-b/(a - b)) - (b*Sin[x])/(a - b))^(5/2) * (b/(a + b) - (b*Sin[x])/(a + b))^(5/2))/(((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(a + b))*(a + b*Sin[x])^3) - ((a*b^3*(-b/(a - b)) - (b*Sin[x])/(a - b))^(5/2)*(b/(a + b) - (b*Sin[x])/(a + b))^(5/2))/(2*(a^2 - b^2))*((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(a + b))*(a + b*Sin[x])^2) - (-((((-3*a^2*b^5)/((a - b)^2*(a + b)^2) + (2*b^5*(3*a^2 - 2*b^2))/((a - b)^2*(a + b)^2)) * (-b/(a - b)) - (b*Sin[x])/(a - b))^(5/2)*(b/(a + b) - (b*Sin[x])/(a + b))^(5/2))/(((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(a + b))*(a + b*Sin[x]))) - ((16*sqrt[2]*b^6*(3*a^2 - 4*b^2)*(-b/(a - b)) - (b*Sin[x])/(a - b))^(5/2)*sqrt[b/(a + b) - (b*Sin[x])/(a + b)]*(1 + ((a - b)*(-b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(5/2)*((5*(1/(2*(1 + ((a - b)*(-b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(-1)))/8 - (15*b^3*((a - b)*(-b/(a - b)) - (b*Sin[x])/(a - b)))/b - ((a - b)^2*(-b/(a - b)) - (b*Sin[x])/(a - b))^2/(3*b^2) - (sqrt[2]*sqrt[a - b]*ArcSinh[(sqrt[a - b]*sqrt[-b/(a - b) - (b*Sin[x])/(a - b)])/(sqrt[2]*sqrt[b])]*sqrt[-b/(a - b) - (b*Sin[x])/(a - b)])/(sqrt[b]*sqrt[1 + ((a - b)*(-b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b)])))/(32*(a - b)^3*(-b/(a - b) - (b*Sin[x])/(a - b))^3*(1 + ((a - b)*(-b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^2))/((5*(a - b)^2*(a + b)^4*sqrt[((a + b)*(b/(a + b) - (b*Sin[x])/(a + b)))/b]) + ((-((a*b^7*(6*a^2 - 7*b^2))/((a - b)^3*(a + b)^3)) + (4*a*b^7*(3*a^2 - 4*b^2))/((a - b)^3*(a + b)^3))*((4*sqrt[2]*(-b/(a - b)) - (b*Sin[x])/(a - b))^(3/2)*sqrt[b/(a + b)

$$\begin{aligned}
& - (b \sin[x]) / (a + b) \cdot (1 + ((a - b) \cdot (-b / (a - b)) - (b \sin[x]) / (a - b))) / (2 \\
& * b)^{5/2} \cdot ((3 / (4 \cdot (1 + ((a - b) \cdot (-b / (a - b)) - (b \sin[x]) / (a - b)))) / (2 * b)) \\
& ^2 + (1 + ((a - b) \cdot (-b / (a - b)) - (b \sin[x]) / (a - b))) / (2 * b))^{-1} / 2 + (\\
& 3 * b^2 \cdot ((a - b) \cdot (-b / (a - b)) - (b \sin[x]) / (a - b))) / b - (\text{Sqrt}[2] * \text{Sqrt}[a - \\
& b] * \text{ArcSinh}[(\text{Sqrt}[a - b] * \text{Sqrt}[-b / (a - b)] - (b \sin[x]) / (a - b)]) / (\text{Sqrt}[2] * \text{S} \\
& \text{qrt}[b])] * \text{Sqrt}[-b / (a - b)] - (b \sin[x]) / (a - b)) / (\text{Sqrt}[b] * \text{Sqrt}[1 + ((a - b) \\
&) \cdot (-b / (a - b)) - (b \sin[x]) / (a - b)) / (2 * b)])) / (8 * (a - b)^2 \cdot (-b / (a - b)) \\
& - (b \sin[x]) / (a - b))^2 \cdot (1 + ((a - b) \cdot (-b / (a - b)) - (b \sin[x]) / (a - b))) \\
& / (2 * b)^2)) / (3 * (a + b) * \text{Sqrt}[(a + b) \cdot (b / (a + b)) - (b \sin[x]) / (a + b)) / b]) \\
& - (((-((a * b) / (a - b)) + b^2 / (a - b)) * (-(((a * b) / (a - b)) + b^2 / (a - b)) * (\\
& -(((a * b) / (a + b)) - b^2 / (a + b)) * ((2 * \text{Sqrt}[a - b] * \text{ArcTanh}[(\text{Sqrt}[a - b] * \text{S} \\
& \text{qrt}[-b / (a - b)] - (b \sin[x]) / (a - b)]) / (\text{Sqrt}[a + b] * \text{Sqrt}[b / (a + b) - (b \sin \\
& [x]) / (a + b)])) / (b * \text{Sqrt}[a + b]) - (2 * \text{Sqrt}[-((a * b) / (a + b)) - b^2 / (a + b)] * \\
& \text{ArcTanh}[(\text{Sqrt}[-((a * b) / (a + b)) - b^2 / (a + b)] * \text{Sqrt}[-b / (a - b)] - (b \sin[x] \\
&) / (a - b)) / (\text{Sqrt}[-((a * b) / (a - b)) + b^2 / (a - b)] * \text{Sqrt}[b / (a + b) - (b \sin[x] \\
&) / (a + b)])) / (b * \text{Sqrt}[-((a * b) / (a - b)) + b^2 / (a - b)])) / b) + (2 * \text{Sqrt}[2] * (\\
& a - b) * \text{Sqrt}[-b / (a - b)] - (b \sin[x]) / (a - b)) * \text{Sqrt}[b / (a + b) - (b \sin[x]) / \\
& (a + b)] * (1 + ((a - b) \cdot (-b / (a - b)) - (b \sin[x]) / (a - b))) / (2 * b))^{3/2} \cdot ((\\
& \text{Sqrt}[b] * \text{ArcSinh}[(\text{Sqrt}[a - b] * \text{Sqrt}[-b / (a - b)] - (b \sin[x]) / (a - b)]) / (\text{Sqrt} \\
& [2] * \text{Sqrt}[b])] / (\text{Sqrt}[2] * \text{Sqrt}[a - b] * \text{Sqrt}[-b / (a - b)] - (b \sin[x]) / (a - b)] \\
& * (1 + ((a - b) \cdot (-b / (a - b)) - (b \sin[x]) / (a - b))) / (2 * b))^{3/2} + 1 / (2 * (1 \\
& + ((a - b) \cdot (-b / (a - b)) - (b \sin[x]) / (a - b))) / (2 * b))) / (b * (a + b) * \text{Sqrt} \\
& ((a + b) \cdot (b / (a + b)) - (b \sin[x]) / (a + b)) / b))) / b) + (4 * \text{Sqrt}[2] * \text{Sqrt}[-b / (\\
& a - b)] - (b \sin[x]) / (a - b)) * \text{Sqrt}[b / (a + b) - (b \sin[x]) / (a + b)] * (1 + ((a \\
& - b) \cdot (-b / (a - b)) - (b \sin[x]) / (a - b))) / (2 * b))^{5/2} \cdot ((3 * \text{Sqrt}[b] * \text{ArcSinh} \\
& [(\text{Sqrt}[a - b] * \text{Sqrt}[-b / (a - b)] - (b \sin[x]) / (a - b))] / (\text{Sqrt}[2] * \text{Sqrt}[b])) / \\
& (4 * \text{Sqrt}[2] * \text{Sqrt}[a - b] * \text{Sqrt}[-b / (a - b)] - (b \sin[x]) / (a - b)) * (1 + ((a - b) \\
&) \cdot (-b / (a - b)) - (b \sin[x]) / (a - b))) / (2 * b))^{5/2} + (3 / (2 * (1 + ((a - b) \cdot \\
& (-b / (a - b)) - (b \sin[x]) / (a - b))) / (2 * b))^2 + (1 + ((a - b) \cdot (-b / (a - b)) \\
&) - (b \sin[x]) / (a - b))) / (2 * b))^{-1} / 4) / ((a + b) * \text{Sqrt}[(a + b) \cdot (b / (a + b) \\
& - (b \sin[x]) / (a + b)) / b])) / b) / (((a * b) / (a - b) - b^2 / (a - b)) * ((a * b) / \\
& (a + b) + b^2 / (a + b))) / (2 * ((a * b) / (a - b) - b^2 / (a - b)) * ((a * b) / (a + b) + \\
& b^2 / (a + b))) / (3 * ((a * b) / (a - b) - b^2 / (a - b)) * ((a * b) / (a + b) + b^2 / (a + b \\
&)))) / ((1 - (a + b * \sin[x]) / (a - b))^{3/2} * (1 - (a + b * \sin[x]) / (a + b))^{3/2} \\
&) * (a * \text{Sec}[x] + b * \text{Tan}[x])^4)
\end{aligned}$$

fricas [B] time = 1.40, size = 931, normalized size = 5.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sec(x)+b*tan(x))^4,x, algorithm="fricas")

[Out] [-1/12*(36*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x)^2 + 2*(11*a^4*b^3 - 19*a^2*b^5 + 8*b^7)*cos(x)^3 + 3*(2*a^6 + 3*a^4*b^2 - 9*a^2*b^4 - 3*(2*a^4*b^2 - 3*a^2*b^4)*cos(x)^2 + (6*a^5*b - 7*a^3*b^3 - 3*a*b^5 - (2*a^3*b^3 - 3*a*b^5)*cos(x)^2)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 12*(a^7 + a^5*b^2 - 5*a^3*b^4 + 3*a*b^6)*x - 12*(a^6*b - 2*a^2*b^5 + b^7)*cos(x) + 6*(2*(a^4*b^3 - 2*a^2*b^5 + b^7)*x*cos(x)^2 - 2*(3*a^6*b - 5*a^4*b^3 + a^2*b^5 + b^7)*x - (5*a^5*b^2 - 8*a^3*b^4 + 3*a*b^6)*cos(x))*sin(x))/(a^7*b^4 + a^5*b^6 - 5*a^3*b^8 + 3*a*b^10 - 3*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*cos(x)^2 + (3*a^6*b^5 - 5*a^4*b^7 + a^2*b^9 + b^11 - (a^4*b^7 - 2*a^2*b^9 + b^11)*cos(x)^2)*sin(x)), -1/6*(18*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x)^2 + (11*a^4*b^3 - 19*a^2*b^5 + 8*b^7)*cos(x)^3 - 3*(2*a^6 + 3*a^4*b^2 - 9*a^2*b^4 - 3*(2*a^4*b^2 - 3*a^2*b^4)*cos(x)^2 + (6*a^5*b - 7*a^3*b^3 - 3*a*b^5 - (2*a^3*b^3 - 3*a*b^5)*cos(x)^2)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - 6*(a^7 + a^5*b^2 - 5*a^3*b^4 + 3*a*b^6)*x - 6*(a^6*b - 2*a^2*b^5 + b^7)*cos(x) + 3*(2*(a^4*b^3 - 2*a^2*b^5 + b^7)*x*cos(x)^2 - 2*(3*a^6*b - 5*a^4*b^3

+ a²*b⁵ + b⁷)*x - (5*a⁵*b² - 8*a³*b⁴ + 3*a*b⁶)*cos(x))*sin(x))/(a⁷*b⁴ + a⁵*b⁶ - 5*a³*b⁸ + 3*a*b¹⁰ - 3*(a⁵*b⁶ - 2*a³*b⁸ + a*b¹⁰)*cos(x)² + (3*a⁶*b⁵ - 5*a⁴*b⁷ + a²*b⁹ + b¹¹ - (a⁴*b⁷ - 2*a²*b⁹ + b¹¹)*cos(x)²)*sin(x))]

giac [B] time = 0.17, size = 369, normalized size = 2.37

$$\frac{(2a^3 - 3ab^2) \left(\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(a) + \arctan \left(\frac{a \tan\left(\frac{1}{2}x\right) + b}{\sqrt{a^2 - b^2}} \right) \right)}{(a^2b^4 - b^6)\sqrt{a^2 - b^2}} + \frac{3a^6b \tan\left(\frac{1}{2}x\right)^5 - 6a^4b^3 \tan\left(\frac{1}{2}x\right)^5 + 6a^2b^5 \tan\left(\frac{1}{2}x\right)^5}{(a^2b^4 - b^6)\sqrt{a^2 - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sec(x)+b*tan(x))^4,x, algorithm="giac")

[Out] -(2*a³ - 3*a*b²)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a² - b²)))/((a²*b⁴ - b⁶)*sqrt(a² - b²)) + 1/3*(3*a⁶*b*tan(1/2*x)⁵ - 6*a⁴*b³*tan(1/2*x)⁵ + 6*a²*b⁵*tan(1/2*x)⁵ + 6*a⁷*tan(1/2*x)⁴ + 9*a⁵*b²*tan(1/2*x)⁴ - 12*a³*b⁴*tan(1/2*x)⁴ + 12*a*b⁶*tan(1/2*x)⁴ + 36*a⁶*b*tan(1/2*x)³ - 6*a⁴*b³*tan(1/2*x)³ - 8*a²*b⁵*tan(1/2*x)³ + 8*b⁷*tan(1/2*x)³ + 12*a⁷*tan(1/2*x)² + 48*a⁵*b²*tan(1/2*x)² - 42*a³*b⁴*tan(1/2*x)² + 12*a*b⁶*tan(1/2*x)² + 33*a⁶*b*tan(1/2*x) - 24*a⁴*b³*tan(1/2*x) + 6*a²*b⁵*tan(1/2*x) + 6*a⁷ - 5*a⁵*b² + 2*a³*b⁴)/(a⁵*b³ - a³*b⁵)*(a*tan(1/2*x)² + 2*b*tan(1/2*x) + a)³ + x/b⁴

maple [B] time = 0.21, size = 967, normalized size = 6.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*sec(x)+b*tan(x))^4,x)

[Out] 1/b²/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³*a³/(a²-b²)*tan(1/2*x)⁵-2/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³*a/(a²-b²)*tan(1/2*x)⁵+2*b²/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³/a/(a²-b²)*tan(1/2*x)⁵+2/b³/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³/(a²-b²)*a⁴*tan(1/2*x)⁴+3/b/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³/(a²-b²)*a²*tan(1/2*x)⁴-4*b/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³/(a²-b²)*tan(1/2*x)⁴+4*b³/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³/(a²-b²)/a²*tan(1/2*x)⁴+12/b²/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³*a³/(a²-b²)*tan(1/2*x)³-2/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³*a/(a²-b²)*tan(1/2*x)³-8/3*b²/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³/a/(a²-b²)*tan(1/2*x)³+8/3*b⁴/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³/a³/(a²-b²)*tan(1/2*x)³+4/b³/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³/(a²-b²)*a⁴*tan(1/2*x)²+16/b/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³/(a²-b²)*tan(1/2*x)²+4*b³/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³/(a²-b²)/a²*tan(1/2*x)²+11/b²/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³*a³/(a²-b²)*tan(1/2*x)-8/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³*a/(a²-b²)*tan(1/2*x)+2*b²/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³/a/(a²-b²)*tan(1/2*x)+2/b³/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³/(a²-b²)*a⁴-5/3/b/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³/(a²-b²)*a²+2/3*b/(a*tan(1/2*x)²+2*b*tan(1/2*x)+a)³/(a²-b²)-2/b⁴*a³/(a²-b²)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a²-b²)^(1/2))+3/b²*a/(a²-b²)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a²-b²)^(1/2))+2/b⁴*arctan(tan(1/2*x))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 6.37, size = 2782, normalized size = 17.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(x) + a/cos(x))^4,x)
```

```
[Out] ((6*a^4 + 2*b^4 - 5*a^2*b^2)/(3*b^3*(a^2 - b^2)) + (tan(x/2)*(11*a^4 + 2*b^
4 - 8*a^2*b^2))/(a*b^2*(a^2 - b^2)) + (tan(x/2)^5*(a^4 + 2*b^4 - 2*a^2*b^2)
)/(a*b^2*(a^2 - b^2)) + (tan(x/2)^4*(2*a^6 + 4*b^6 - 4*a^2*b^4 + 3*a^4*b^2)
)/(a^2*b^3*(a^2 - b^2)) + (2*tan(x/2)^2*(2*a^6 + 2*b^6 - 7*a^2*b^4 + 8*a^4*
b^2))/(a^2*b^3*(a^2 - b^2)) + (2*tan(x/2)^3*(3*a^2 + 2*b^2)*(6*a^4 + 2*b^4
- 5*a^2*b^2))/(3*a^3*b^2*(a^2 - b^2)))/(tan(x/2)^2*(12*a*b^2 + 3*a^3) + tan
(x/2)^4*(12*a*b^2 + 3*a^3) + tan(x/2)^3*(12*a^2*b + 8*b^3) + a^3 + a^3*tan(
x/2)^6 + 6*a^2*b*tan(x/2)^5 + 6*a^2*b*tan(x/2)) + (2*atan((48*a^3*b^3*tan(x
/2))/((176*a^3*b^15)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (160*a^5*b^13)/(b^12 -
2*a^2*b^10 + a^4*b^8) + (48*a^7*b^11)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (64*
a*b^17)/(b^12 - 2*a^2*b^10 + a^4*b^8)) - (64*a*b^5*tan(x/2))/((176*a^3*b^15
)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (160*a^5*b^13)/(b^12 - 2*a^2*b^10 + a^4*b
^8) + (48*a^7*b^11)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (64*a*b^17)/(b^12 - 2*a
^2*b^10 + a^4*b^8))))/b^4 + (a*atan(((a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)
^3)^(1/2))*((8*(4*a^2*b^7 - 8*a^4*b^5 + 4*a^6*b^3))/(b^12 - 2*a^2*b^10 + a^4
*b^8) + (8*tan(x/2)*(8*a*b^9 - 29*a^3*b^7 + 28*a^5*b^5 - 8*a^7*b^3))/(b^13
- 2*a^2*b^11 + a^4*b^9) - (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))*
(8*(4*a*b^12 - 6*a^3*b^10 + 2*a^5*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*
tan(x/2)*(12*a^2*b^12 - 20*a^4*b^10 + 8*a^6*b^8))/(b^13 - 2*a^2*b^11 + a^4*
b^9) - (a*((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 +
a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11)
)/(b^13 - 2*a^2*b^11 + a^4*b^9))*((2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/
2)))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))))/(2*(b^10 - 3*a^2*b^8 + 3
*a^4*b^6 - a^6*b^4))*1i)/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)) + (a
*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))*((8*(4*a^2*b^7 - 8*a^4*b^5 +
4*a^6*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(8*a*b^9 - 29*a^3*b
^7 + 28*a^5*b^5 - 8*a^7*b^3))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (a*(2*a^2 - 3
*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))*((8*(4*a*b^12 - 6*a^3*b^10 + 2*a^5*b^8)
)/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a^2*b^12 - 20*a^4*b^10 + 8
*a^6*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (a*((8*(4*a^2*b^15 - 8*a^4*b^13
+ 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*
a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*((2*a^2
- 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 -
a^6*b^4)))))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*1i)/(2*(b^10 - 3*
a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/((16*(2*a^5 - 3*a^3*b^2))/(b^12 - 2*a^2*b^
10 + a^4*b^8) + (16*tan(x/2)*(8*a^6 + 12*a^2*b^4 - 20*a^4*b^2))/(b^13 - 2*a
^2*b^11 + a^4*b^9) - (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))*((8*(4
*a^2*b^7 - 8*a^4*b^5 + 4*a^6*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x
/2)*(8*a*b^9 - 29*a^3*b^7 + 28*a^5*b^5 - 8*a^7*b^3))/(b^13 - 2*a^2*b^11 + a
^4*b^9) - (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))*((8*(4*a*b^12 - 6
*a^3*b^10 + 2*a^5*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a^2
*b^12 - 20*a^4*b^10 + 8*a^6*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9) - (a*((8*(4
*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*ta
n(x/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*
b^11 + a^4*b^9))*((2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)))/(2*(b^10 - 3
*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b
```

```

^4))))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)) + (a*(2*a^2 - 3*b^2)*(-
(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^7 - 8*a^4*b^5 + 4*a^6*b^3))/(b^12 -
2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(8*a*b^9 - 29*a^3*b^7 + 28*a^5*b^5 - 8
*a^7*b^3)))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(
a - b)^3)^(1/2)*((8*(4*a*b^12 - 6*a^3*b^10 + 2*a^5*b^8))/(b^12 - 2*a^2*b^10
+ a^4*b^8) + (8*tan(x/2)*(12*a^2*b^12 - 20*a^4*b^10 + 8*a^6*b^8)))/(b^13 -
2*a^2*b^11 + a^4*b^9) + (a*((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^1
2 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b
^13 - 8*a^7*b^11)))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(2*a^2 - 3*b^2)*(-(a + b)
^3*(a - b)^3)^(1/2))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))))/(2*(b^1
0 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 -
a^6*b^4))))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*1i)/(b^10 - 3*a^2*
b^8 + 3*a^4*b^6 - a^6*b^4)

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \sec(x) + b \tan(x))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sec(x)+b*tan(x))**4,x)

[Out] Integral((a*sec(x) + b*tan(x))**(-4), x)

$$3.272 \quad \int \frac{1}{(a \sec(x) + b \tan(x))^5} dx$$

Optimal. Leaf size=101

$$-\frac{(a^2 - b^2)^2}{4b^5(a + b \sin(x))^4} + \frac{4a(a^2 - b^2)}{3b^5(a + b \sin(x))^3} - \frac{3a^2 - b^2}{b^5(a + b \sin(x))^2} + \frac{4a}{b^5(a + b \sin(x))} + \frac{\log(a + b \sin(x))}{b^5}$$

[Out] $\ln(a + b \sin(x)) / b^5 - 1/4 * (a^2 - b^2)^2 / b^5 / (a + b \sin(x))^4 + 4/3 * a * (a^2 - b^2) / b^5 / (a + b \sin(x))^3 + (-3 * a^2 + b^2) / b^5 / (a + b \sin(x))^2 + 4 * a / b^5 / (a + b \sin(x))$

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4391, 2668, 697}

$$-\frac{(a^2 - b^2)^2}{4b^5(a + b \sin(x))^4} + \frac{4a(a^2 - b^2)}{3b^5(a + b \sin(x))^3} - \frac{3a^2 - b^2}{b^5(a + b \sin(x))^2} + \frac{4a}{b^5(a + b \sin(x))} + \frac{\log(a + b \sin(x))}{b^5}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a * \text{Sec}[x] + b * \text{Tan}[x])^{-5}, x]$

[Out] $\text{Log}[a + b * \text{Sin}[x]] / b^5 - (a^2 - b^2)^2 / (4 * b^5 * (a + b * \text{Sin}[x])^4) + (4 * a * (a^2 - b^2)) / (3 * b^5 * (a + b * \text{Sin}[x])^3) - (3 * a^2 - b^2) / (b^5 * (a + b * \text{Sin}[x])^2) + (4 * a) / (b^5 * (a + b * \text{Sin}[x]))$

Rule 697

$\text{Int}[(d + (e \cdot x)^m) \cdot ((a + (c \cdot x)^2)^p), x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(d + e \cdot x)^m \cdot (a + c \cdot x^2)^p, x], x] /;$ $\text{FreeQ}\{a, c, d, e, m, x\} \ \&\& \ \text{NeQ}[c \cdot d^2 + a \cdot e^2, 0] \ \&\& \ \text{IGtQ}[p, 0]$

Rule 2668

$\text{Int}[\cos[(e \cdot x) + (f \cdot x)]^p \cdot ((a + (b \cdot x) \cdot \sin[(e \cdot x) + (f \cdot x)])^m), x_Symbol] := \text{Dist}[1 / (b^p \cdot f), \text{Subst}[\text{Int}[(a + x)^m \cdot (b^2 - x^2)^{(p-1)/2}, x], x, b \cdot \text{Sin}[e + f \cdot x]], x] /;$ $\text{FreeQ}\{a, b, e, f, m, x\} \ \&\& \ \text{IntegerQ}[(p-1)/2] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 4391

$\text{Int}[(u + (b \cdot x) \cdot \sec[(c + d \cdot x)])^n + (a + (d \cdot x) \cdot \tan[(c + d \cdot x)])^n]^p, x_Symbol] := \text{Int}[\text{ActivateTrig}[u] \cdot \text{Sec}[c + d \cdot x]^{(n \cdot p)} \cdot (b + a \cdot \text{Sin}[c + d \cdot x])^n]^p, x] /;$ $\text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{IntegersQ}[n, p]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(a \sec(x) + b \tan(x))^5} dx &= \int \frac{\cos^5(x)}{(a + b \sin(x))^5} dx \\ &= \frac{\text{Subst}\left(\int \frac{(b^2 - x^2)^2}{(a+x)^5} dx, x, b \sin(x)\right)}{b^5} \\ &= \frac{\text{Subst}\left(\int \left(\frac{(a^2 - b^2)^2}{(a+x)^5} - \frac{4(a^3 - ab^2)}{(a+x)^4} + \frac{2(3a^2 - b^2)}{(a+x)^3} - \frac{4a}{(a+x)^2} + \frac{1}{a+x}\right) dx, x, b \sin(x)\right)}{b^5} \\ &= \frac{\log(a + b \sin(x))}{b^5} - \frac{(a^2 - b^2)^2}{4b^5(a + b \sin(x))^4} + \frac{4a(a^2 - b^2)}{3b^5(a + b \sin(x))^3} - \frac{3a^2 - b^2}{b^5(a + b \sin(x))^2} \end{aligned}$$

Mathematica [A] time = 0.34, size = 86, normalized size = 0.85

$$\frac{25a^4 + 12b^2(9a^2 + b^2) \sin^2(x) + 8ab(11a^2 + b^2) \sin(x) + 2a^2b^2 + 48ab^3 \sin^3(x) - 3b^4}{12(a+b \sin(x))^4} + \frac{\log(a + b \sin(x))}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^(-5),x]
[Out] (Log[a + b*Sin[x]] + (25*a^4 + 2*a^2*b^2 - 3*b^4 + 8*a*b*(11*a^2 + b^2)*Sin[x] + 12*b^2*(9*a^2 + b^2)*Sin[x]^2 + 48*a*b^3*Sin[x]^3)/(12*(a + b*Sin[x]^4))/b^5
```

fricas [B] time = 1.06, size = 217, normalized size = 2.15

$$\frac{25 a^4 + 110 a^2 b^2 + 9 b^4 - 12 (9 a^2 b^2 + b^4) \cos(x)^2 + 12 (b^4 \cos(x)^4 + a^4 + 6 a^2 b^2 + b^4 - 2 (3 a^2 b^2 + b^4) \cos(x)^2 - 4 b^4 \cos(x)^4 + a^4 + 6 a^2 b^2 + b^4 - 2 (3 a^2 b^2 + b^4) \cos(x)^2 - 4 (a^2 b^3 \cos(x)^2 - a^3 b - a^2 b^3) \sin(x) \log(b \sin(x) + a) - 8 (6 a^2 b^3 \cos(x)^2 - 11 a^3 b - 7 a^2 b^3) \sin(x)) / (b^9 \cos(x)^4 + a^4 b^5 + 6 a^2 b^7 + b^9 - 2 (3 a^2 b^7 + b^9) \cos(x)^2 - 4 (a^2 b^8 \cos(x)^2 - a^3 b^6 - a^2 b^8) \sin(x))}{12 (b^9 \cos(x)^4 + a^4 b^5 + 6 a^2 b^7 + b^9 - 2 (3 a^2 b^7 + b^9) \cos(x)^2 - 4 (a^2 b^8 \cos(x)^2 - a^3 b^6 - a^2 b^8) \sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^5,x, algorithm="fricas")
[Out] 1/12*(25*a^4 + 110*a^2*b^2 + 9*b^4 - 12*(9*a^2*b^2 + b^4)*cos(x)^2 + 12*(b^4*cos(x)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*cos(x)^2 - 4*(a*b^3*cos(x)^2 - a^3*b - a*b^3)*sin(x))*log(b*sin(x) + a) - 8*(6*a*b^3*cos(x)^2 - 11*a^3*b - 7*a*b^3)*sin(x))/(b^9*cos(x)^4 + a^4*b^5 + 6*a^2*b^7 + b^9 - 2*(3*a^2*b^7 + b^9)*cos(x)^2 - 4*(a*b^8*cos(x)^2 - a^3*b^6 - a*b^8)*sin(x))
```

giac [A] time = 0.14, size = 91, normalized size = 0.90

$$\frac{\log(|b \sin(x) + a|)}{b^5} - \frac{25 b^3 \sin(x)^4 + 52 a b^2 \sin(x)^3 + 42 a^2 b \sin(x)^2 - 12 b^3 \sin(x)^2 + 12 a^3 \sin(x) - 8 a b^2 \sin(x) - 12 a^4}{12 (b \sin(x) + a)^4 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^5,x, algorithm="giac")
[Out] log(abs(b*sin(x) + a))/b^5 - 1/12*(25*b^3*sin(x)^4 + 52*a*b^2*sin(x)^3 + 42*a^2*b*sin(x)^2 - 12*b^3*sin(x)^2 + 12*a^3*sin(x) - 8*a*b^2*sin(x) - 2*a^2*b + 3*b^3)/((b*sin(x) + a)^4*b^4)
```

maple [A] time = 0.21, size = 130, normalized size = 1.29

$$\frac{4a}{b^5 (a + b \sin(x))} - \frac{a^4}{4b^5 (a + b \sin(x))^4} + \frac{a^2}{2b^3 (a + b \sin(x))^4} - \frac{1}{4b (a + b \sin(x))^4} + \frac{4a^3}{3b^5 (a + b \sin(x))^3} - \frac{4a}{3b^3 (a + b \sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)+b*tan(x))^5,x)
[Out] 4*a/b^5/(a+b*sin(x))-1/4/b^5/(a+b*sin(x))^4*a^4+1/2/b^3/(a+b*sin(x))^4*a^2-1/4/b/(a+b*sin(x))^4+4/3*a^3/b^5/(a+b*sin(x))^3-4/3*a/b^3/(a+b*sin(x))^3+ln(a+b*sin(x))/b^5-3/b^5/(a+b*sin(x))^2*a^2+1/b^3/(a+b*sin(x))^2
```

maxima [B] time = 0.51, size = 483, normalized size = 4.78

$$\frac{2 \left(\frac{3(a^7 - a^3 b^4) \sin(x)}{\cos(x) + 1} + \frac{3(7 a^6 b - 3 a^2 b^5) \sin(x)^2}{(\cos(x) + 1)^2} + \frac{(9 a^7 + 52 a^5 b^2 - a^3 b^4 - 12 ab^6) \sin(x)^3}{(\cos(x) + 1)^3} + \frac{2(21 a^6 b + 25 a^4 b^3 - 7 a^2 b^5 - 3 b^7) \sin(x)^4}{(\cos(x) + 1)^4} + \frac{(9 a^7 + 52 a^5 b^2 - a^3 b^4 - 12 ab^6) \sin(x)^5}{(\cos(x) + 1)^5} \right)}{3 \left(a^8 b^4 + \frac{8 a^7 b^5 \sin(x)}{\cos(x) + 1} + \frac{8 a^7 b^5 \sin(x)^7}{(\cos(x) + 1)^7} + \frac{a^8 b^4 \sin(x)^8}{(\cos(x) + 1)^8} + \frac{4(a^8 b^4 + 6 a^6 b^6) \sin(x)^2}{(\cos(x) + 1)^2} + \frac{8(3 a^7 b^5 + 4 a^5 b^7) \sin(x)^3}{(\cos(x) + 1)^3} + \frac{2(3 a^8 b^4 + 24 a^6 b^6 + 8 a^4 b^8) \sin(x)^4}{(\cos(x) + 1)^4} + \frac{4(a^8 b^4 + 6 a^6 b^6) \sin(x)^5}{(\cos(x) + 1)^5} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sec(x)+b*tan(x))^5,x, algorithm="maxima")

[Out]
$$-2/3*(3*(a^7 - a^3*b^4)*\sin(x)/(\cos(x) + 1) + 3*(7*a^6*b - 3*a^2*b^5)*\sin(x)^2/(\cos(x) + 1)^2 + (9*a^7 + 52*a^5*b^2 - a^3*b^4 - 12*a*b^6)*\sin(x)^3/(\cos(x) + 1)^3 + 2*(21*a^6*b + 25*a^4*b^3 - 7*a^2*b^5 - 3*b^7)*\sin(x)^4/(\cos(x) + 1)^4 + (9*a^7 + 52*a^5*b^2 - a^3*b^4 - 12*a*b^6)*\sin(x)^5/(\cos(x) + 1)^5 + 3*(7*a^6*b - 3*a^2*b^5)*\sin(x)^6/(\cos(x) + 1)^6 + 3*(a^7 - a^3*b^4)*\sin(x)^7/(\cos(x) + 1)^7)/(a^8*b^4 + 8*a^7*b^5*\sin(x)/(\cos(x) + 1) + 8*a^7*b^5*\sin(x)^2/(\cos(x) + 1)^2 + a^8*b^4*\sin(x)^3/(\cos(x) + 1)^3 + 2*(3*a^8*b^4 + 24*a^6*b^6 + 8*a^4*b^8)*\sin(x)^4/(\cos(x) + 1)^4 + 8*(3*a^7*b^5 + 4*a^5*b^7)*\sin(x)^5/(\cos(x) + 1)^5 + 4*(a^8*b^4 + 6*a^6*b^6)*\sin(x)^6/(\cos(x) + 1)^6) + \log(a + 2*b*\sin(x)/(\cos(x) + 1) + a*\sin(x)^2/(\cos(x) + 1)^2)/b^5 - \log(\sin(x)^2/(\cos(x) + 1)^2 + 1)/b^5$$

mupad [B] time = 3.84, size = 541, normalized size = 5.36

$$2 \operatorname{atanh} \left(\frac{16a}{\frac{32a^3}{b^2} - 16a \tan\left(\frac{x}{2}\right)^2 - 16a + \frac{32a^2 \tan\left(\frac{x}{2}\right)}{b} + \frac{32a^3 \tan\left(\frac{x}{2}\right)^2}{b^2}} + \frac{16a \tan\left(\frac{x}{2}\right)^2}{\frac{32a^3}{b^2} - 16a \tan\left(\frac{x}{2}\right)^2 - 16a + \frac{32a^2 \tan\left(\frac{x}{2}\right)}{b} + \frac{32a^3 \tan\left(\frac{x}{2}\right)^2}{b^2}} + \frac{16a \tan\left(\frac{x}{2}\right)^2}{32a^2 \tan\left(\frac{x}{2}\right) - 16ab} \right) / b^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*tan(x) + a/cos(x))^5,x)

[Out]
$$(2*\operatorname{atanh}((16*a)/((32*a^3)/b^2 - 16*a*\tan(x/2)^2 - 16*a + (32*a^2*\tan(x/2))/b + (32*a^3*\tan(x/2)^2)/b^2) + (16*a*\tan(x/2)^2)/((32*a^3)/b^2 - 16*a*\tan(x/2)^2 - 16*a + (32*a^2*\tan(x/2))/b + (32*a^3*\tan(x/2)^2)/b^2) + (32*a^2*\tan(x/2))/(32*a^2*\tan(x/2) - 16*a*b + (32*a^3)/b + (32*a^3*\tan(x/2)^2)/b - 16*a*b*\tan(x/2)^2))/b^5 - ((2*\tan(x/2)^2*(7*a^4 - 3*b^4))/(a^2*b^3) + (2*\tan(x/2)^6*(7*a^4 - 3*b^4))/(a^2*b^3) + (2*\tan(x/2)*(a^4 - b^4))/(a*b^4) + (2*\tan(x/2)^7*(a^4 - b^4))/(a*b^4) + (4*\tan(x/2)^4*(21*a^6 - 3*b^6 - 7*a^2*b^4 + 25*a^4*b^2))/(3*a^4*b^3) + (2*\tan(x/2)^3*(9*a^6 - 12*b^6 - a^2*b^4 + 52*a^4*b^2))/(3*a^3*b^4) + (2*\tan(x/2)^5*(9*a^6 - 12*b^6 - a^2*b^4 + 52*a^4*b^2))/(3*a^3*b^4))/(\tan(x/2)^2*(4*a^4 + 24*a^2*b^2) + \tan(x/2)^6*(4*a^4 + 24*a^2*b^2) + \tan(x/2)^3*(32*a*b^3 + 24*a^3*b) + \tan(x/2)^5*(32*a*b^3 + 24*a^3*b) + \tan(x/2)^4*(6*a^4 + 16*b^4 + 48*a^2*b^2) + a^4 + a^4*\tan(x/2)^8 + 8*a^3*b*\tan(x/2)^7 + 8*a^3*b*\tan(x/2))$$

sympy [A] time = 15.19, size = 1719, normalized size = 17.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sec(x)+b*tan(x))**5,x)

[Out]
$$\operatorname{Piecewise}((36*a**4*\log(a*\sec(x)/b + \tan(x))*\sec(x)**4/(36*a**4*b**5*\sec(x)**4 + 144*a**3*b**6*\tan(x)*\sec(x)**3 + 216*a**2*b**7*\tan(x)**2*\sec(x)**2 + 144*a*b**8*\tan(x)**3*\sec(x) + 36*b**9*\tan(x)**4) - 18*a**4*\log(\tan(x)**2 + 1)*\sec(x)**4/(36*a**4*b**5*\sec(x)**4 + 144*a**3*b**6*\tan(x)*\sec(x)**3 + 216*a**2*b**7*\tan(x)**2*\sec(x)**2 + 144*a*b**8*\tan(x)**3*\sec(x) + 36*b**9*\tan(x)**4) + 20*a**4*\sec(x)**4/(36*a**4*b**5*\sec(x)**4 + 144*a**3*b**6*\tan(x)*\sec(x)**3 + 216*a**2*b**7*\tan(x)**2*\sec(x)**2 + 144*a*b**8*\tan(x)**3*\sec(x) + 36*b**9*\tan(x)**4) + 144*a**3*b*\log(a*\sec(x)/b + \tan(x))*\tan(x)*\sec(x)**3/(36*a**4*b**5*\sec(x)**4 + 144*a**3*b**6*\tan(x)*\sec(x)**3 + 216*a**2*b**7*\tan(x)**2*\sec(x)**2 + 144*a*b**8*\tan(x)**3*\sec(x) + 36*b**9*\tan(x)**4) - 72*a$$

```

**3*b*log(tan(x)**2 + 1)*tan(x)*sec(x)**3/(36*a**4*b**5*sec(x)**4 + 144*a**
3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*ta
n(x)**3*sec(x) + 36*b**9*tan(x)**4) + 44*a**3*b*tan(x)*sec(x)**3/(36*a**4*b
**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*se
c(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) + 216*a**2*b**2*
log(a*sec(x)/b + tan(x))*tan(x)**2*sec(x)**2/(36*a**4*b**5*sec(x)**4 + 144*
a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8
*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) - 108*a**2*b**2*log(tan(x)**2 + 1)*t
an(x)**2*sec(x)**2/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3
+ 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**
9*tan(x)**4) + 6*a**2*b**2*sec(x)**2/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**
6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)*
**3*sec(x) + 36*b**9*tan(x)**4) + 144*a*b**3*log(a*sec(x)/b + tan(x))*tan(x)
**3*sec(x)/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a
**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)
**4) - 72*a*b**3*log(tan(x)**2 + 1)*tan(x)**3*sec(x)/(36*a**4*b**5*sec(x)**
4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 14
4*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) - 52*a*b**3*tan(x)**3*sec(x)
/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*t
an(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) + 24*
a*b**3*tan(x)*sec(x)/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)*
**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b
**9*tan(x)**4) + 36*b**4*log(a*sec(x)/b + tan(x))*tan(x)**4/(36*a**4*b**5*s
ec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)*
**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) - 18*b**4*log(tan(x)*
**2 + 1)*tan(x)**4/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3
+ 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9
*tan(x)**4) - 28*b**4*tan(x)**4/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan
(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*se
c(x) + 36*b**9*tan(x)**4) + 18*b**4*tan(x)**2/(36*a**4*b**5*sec(x)**4 + 144
*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**
8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) - 9*b**4/(36*a**4*b**5*sec(x)**4 +
144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*
b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4), Ne(b, 0)), ((8*tan(x)**5/(15*se
c(x)**5) + 4*tan(x)**3/(3*sec(x)**5) + tan(x)/sec(x)**5)/a**5, True))

```

3.273 $\int (\sec(x) + \tan(x))^5 dx$

Optimal. Leaf size=30

$$-\frac{4}{1 - \sin(x)} + \frac{2}{(1 - \sin(x))^2} - \log(1 - \sin(x))$$

[Out] $-\ln(1 - \sin(x)) + 2/(1 - \sin(x))^2 - 4/(1 - \sin(x))$

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4391, 2667, 43}

$$-\frac{4}{1 - \sin(x)} + \frac{2}{(1 - \sin(x))^2} - \log(1 - \sin(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[x] + \text{Tan}[x])^5, x]$

[Out] $-\text{Log}[1 - \text{Sin}[x]] + 2/(1 - \text{Sin}[x])^2 - 4/(1 - \text{Sin}[x])$

Rule 43

$\text{Int}[(a_. + (b_.)(x_.))^{(m_.)}((c_. + (d_.)(x_.))^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) || \text{LtQ}[9*m + 5*(n + 1), 0] || \text{GtQ}[m + n + 2, 0])$

Rule 2667

$\text{Int}[\cos[(e_. + (f_.)(x_.))^{(p_.)}((a_. + (b_.)\sin[(e_. + (f_.)(x_.))^{(m_.)}), x_Symbol] \rightarrow \text{Dist}[1/(b^p*f), \text{Subst}[\text{Int}[(a + x)^{(m + (p - 1)/2)}*(a - x)^{((p - 1)/2)}, x], x, b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x] \&\& \text{IntegerQ}[(p - 1)/2] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& (\text{GeQ}[p, -1] || !\text{IntegerQ}[m + 1/2])$

Rule 4391

$\text{Int}[(u_.)((b_.)\sec[(c_. + (d_.)(x_.))^{(n_.)} + (a_.)\tan[(c_. + (d_.)(x_.))^{(n_.)}])^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ActivateTrig}[u]*\text{Sec}[c + d*x]^{(n*p)}*(b + a*\text{Sin}[c + d*x]^n)^p, x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{IntegersQ}[n, p]$

Rubi steps

$$\begin{aligned} \int (\sec(x) + \tan(x))^5 dx &= \int \sec^5(x)(1 + \sin(x))^5 dx \\ &= \text{Subst}\left(\int \frac{(1+x)^2}{(1-x)^3} dx, x, \sin(x)\right) \\ &= \text{Subst}\left(\int \left(\frac{1}{1-x} - \frac{4}{(-1+x)^3} - \frac{4}{(-1+x)^2}\right) dx, x, \sin(x)\right) \\ &= -\log(1 - \sin(x)) + \frac{2}{(1 - \sin(x))^2} - \frac{4}{1 - \sin(x)} \end{aligned}$$

Mathematica [A] time = 0.11, size = 54, normalized size = 1.80

$$\frac{11 \tan^4(x)}{4} - \frac{\tan^2(x)}{2} + \frac{5 \sec^4(x)}{4} + \tanh^{-1}(\sin(x)) - \log(\cos(x)) - \tan(x) \sec^3(x) + 5 \tan^3(x) \sec(x) + \tan(x) \sec(x)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x] + Tan[x])^5,x]

[Out] ArcTanh[Sin[x]] - Log[Cos[x]] + (5*Sec[x]^4)/4 + Sec[x]*Tan[x] - Sec[x]^3*Tan[x] - Tan[x]^2/2 + 5*Sec[x]*Tan[x]^3 + (11*Tan[x]^4)/4

fricas [A] time = 0.96, size = 38, normalized size = 1.27

$$\frac{(\cos(x)^2 + 2 \sin(x) - 2) \log(-\sin(x) + 1) + 4 \sin(x) - 2}{\cos(x)^2 + 2 \sin(x) - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)+tan(x))^5,x, algorithm="fricas")

[Out] -((cos(x)^2 + 2*sin(x) - 2)*log(-sin(x) + 1) + 4*sin(x) - 2)/(cos(x)^2 + 2*sin(x) - 2)

giac [B] time = 0.14, size = 62, normalized size = 2.07

$$\frac{25 \tan\left(\frac{1}{2}x\right)^4 - 100 \tan\left(\frac{1}{2}x\right)^3 + 198 \tan\left(\frac{1}{2}x\right)^2 - 100 \tan\left(\frac{1}{2}x\right) + 25}{6 \left(\tan\left(\frac{1}{2}x\right) - 1\right)^4} + \log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) - 2 \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)+tan(x))^5,x, algorithm="giac")

[Out] 1/6*(25*tan(1/2*x)^4 - 100*tan(1/2*x)^3 + 198*tan(1/2*x)^2 - 100*tan(1/2*x) + 25)/(tan(1/2*x) - 1)^4 + log(tan(1/2*x)^2 + 1) - 2*log(abs(tan(1/2*x) - 1))

maple [B] time = 0.12, size = 106, normalized size = 3.53

$$-\left(-\frac{(\sec^3(x))}{4} - \frac{3 \sec(x)}{8}\right) \tan(x) + \ln(\sec(x) + \tan(x)) + \frac{5}{4 \cos(x)^4} + \frac{5(\sin^3(x))}{2 \cos(x)^4} + \frac{5(\sin^3(x))}{4 \cos(x)^2} - \frac{5 \sin(x)}{8} + \frac{5(\sin^4(x))}{2 \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sec(x)+tan(x))^5,x)

[Out] -(-1/4*sec(x)^3-3/8*sec(x))*tan(x)+ln(sec(x)+tan(x))+5/4/cos(x)^4+5/2*sin(x)^3/cos(x)^4+5/4*sin(x)^3/cos(x)^2-5/8*sin(x)+5/2*sin(x)^4/cos(x)^4+5/4*sin(x)^5/cos(x)^4-5/8*sin(x)^5/cos(x)^2-5/8*sin(x)^3+1/4*tan(x)^4-1/2*tan(x)^2-ln(cos(x))

maxima [B] time = 0.34, size = 141, normalized size = 4.70

$$\frac{5}{2} \tan(x)^4 + \frac{5(5 \sin(x)^3 - 3 \sin(x))}{8(\sin(x)^4 - 2 \sin(x)^2 + 1)} - \frac{3 \sin(x)^3 - 5 \sin(x)}{8(\sin(x)^4 - 2 \sin(x)^2 + 1)} + \frac{5(\sin(x)^3 + \sin(x))}{4(\sin(x)^4 - 2 \sin(x)^2 + 1)} + \frac{4 \sin(x)}{4(\sin(x)^4 - 2 \sin(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)+tan(x))^5,x, algorithm="maxima")

[Out] 5/2*tan(x)^4 + 5/8*(5*sin(x)^3 - 3*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) - 1/8*(3*sin(x)^3 - 5*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 5/4*(sin(x)^3 + sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 1/4*(4*sin(x)^2 - 3)/(sin(x)^4 - 2*sin(x)^2 + 1) + 5/4/(sin(x)^2 - 1)^2 - 1/2*log(sin(x)^2 - 1) + 1/2*log(sin(x) + 1) - 1/2*log(sin(x) - 1)

mupad [B] time = 2.44, size = 59, normalized size = 1.97

$$\ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - 2 \ln\left(\tan\left(\frac{x}{2}\right) - 1\right) + \frac{8 \tan\left(\frac{x}{2}\right)^2}{\tan\left(\frac{x}{2}\right)^4 - 4 \tan\left(\frac{x}{2}\right)^3 + 6 \tan\left(\frac{x}{2}\right)^2 - 4 \tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(x) + 1/cos(x))^5, x)

[Out] log(tan(x/2)^2 + 1) - 2*log(tan(x/2) - 1) + (8*tan(x/2)^2)/(6*tan(x/2)^2 - 4*tan(x/2) - 4*tan(x/2)^3 + tan(x/2)^4 + 1)

sympy [B] time = 7.13, size = 68, normalized size = 2.27

$$-\frac{\log(\sin(x) - 1)}{2} + \frac{\log(\sin(x) + 1)}{2} + \frac{\log(\sec^2(x))}{2} + \frac{5 \tan^4(x)}{2} + \frac{3 \sec^4(x)}{2} - \sec^2(x) + \frac{32 \sin^3(x)}{8 \sin^4(x) - 16 \sin^2(x) + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)+tan(x))**5, x)

[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2 + log(sec(x)**2)/2 + 5*tan(x)**4/2 + 3*sec(x)**4/2 - sec(x)**2 + 32*sin(x)**3/(8*sin(x)**4 - 16*sin(x)**2 + 8)

3.274 $\int (\sec(x) + \tan(x))^4 dx$

Optimal. Leaf size=30

$$x + \frac{2 \cos^3(x)}{3(1 - \sin(x))^3} - \frac{2 \cos(x)}{1 - \sin(x)}$$

[Out] x+2/3*cos(x)^3/(1-sin(x))^3-2*cos(x)/(1-sin(x))

Rubi [A] time = 0.10, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4391, 2670, 2680, 8}

$$x + \frac{2 \cos^3(x)}{3(1 - \sin(x))^3} - \frac{2 \cos(x)}{1 - \sin(x)}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x] + Tan[x])^4,x]

[Out] x + (2*Cos[x]^3)/(3*(1 - Sin[x])^3) - (2*Cos[x])/(1 - Sin[x])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2670

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m + p + 1, 0] && !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]

Rule 4391

Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_.)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x_.)]^(n_.))^p, x_Symbol] := Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned}
\int (\sec(x) + \tan(x))^4 dx &= \int \sec^4(x)(1 + \sin(x))^4 dx \\
&= \int \frac{\cos^4(x)}{(1 - \sin(x))^4} dx \\
&= \frac{2 \cos^3(x)}{3(1 - \sin(x))^3} - \int \frac{\cos^2(x)}{(1 - \sin(x))^2} dx \\
&= \frac{2 \cos^3(x)}{3(1 - \sin(x))^3} - \frac{2 \cos(x)}{1 - \sin(x)} + \int 1 dx \\
&= x + \frac{2 \cos^3(x)}{3(1 - \sin(x))^3} - \frac{2 \cos(x)}{1 - \sin(x)}
\end{aligned}$$

Mathematica [B] time = 0.13, size = 64, normalized size = 2.13

$$\frac{-3(3x + 8) \cos\left(\frac{x}{2}\right) + (3x + 16) \cos\left(\frac{3x}{2}\right) + 6 \sin\left(\frac{x}{2}\right) (2x + x \cos(x) + 4)}{6 \left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x] + Tan[x])^4, x]

[Out] -1/6*(-3*(8 + 3*x)*Cos[x/2] + (16 + 3*x)*Cos[(3*x)/2] + 6*(4 + 2*x + x*Cos[x])*Sin[x/2))/(Cos[x/2] - Sin[x/2])^3

fricas [B] time = 0.91, size = 61, normalized size = 2.03

$$\frac{(3x + 8) \cos(x)^2 - (3x - 4) \cos(x) + ((3x - 8) \cos(x) + 6x - 4) \sin(x) - 6x - 4}{3(\cos(x)^2 + (\cos(x) + 2) \sin(x) - \cos(x) - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)+tan(x))^4,x, algorithm="fricas")

[Out] 1/3*((3*x + 8)*cos(x)^2 - (3*x - 4)*cos(x) + ((3*x - 8)*cos(x) + 6*x - 4)*sin(x) - 6*x - 4)/(cos(x)^2 + (cos(x) + 2)*sin(x) - cos(x) - 2)

giac [A] time = 0.15, size = 20, normalized size = 0.67

$$x - \frac{8 \left(3 \tan\left(\frac{1}{2} x\right) - 1\right)}{3 \left(\tan\left(\frac{1}{2} x\right) - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)+tan(x))^4,x, algorithm="giac")

[Out] x - 8/3*(3*tan(1/2*x) - 1)/(tan(1/2*x) - 1)^3

maple [B] time = 0.10, size = 71, normalized size = 2.37

$$-\left(\frac{2}{3} - \frac{\sec^2(x)}{3}\right) \tan(x) + \frac{4}{3 \cos(x)^3} + \frac{2(\sin^3(x))}{\cos(x)^3} + \frac{4(\sin^4(x))}{3 \cos(x)^3} - \frac{4(\sin^4(x))}{3 \cos(x)} - \frac{4(2 + \sin^2(x)) \cos(x)}{3} + \frac{(\tan^3(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sec(x)+tan(x))^4,x)

[Out] $-(-2/3-1/3*\sec(x)^2)*\tan(x)+4/3/\cos(x)^3+2*\sin(x)^3/\cos(x)^3+4/3*\sin(x)^4/\cos(x)^3-4/3*\sin(x)^4/\cos(x)-4/3*(2+\sin(x)^2)*\cos(x)+1/3*\tan(x)^3-\tan(x)+x$

maxima [A] time = 0.44, size = 28, normalized size = 0.93

$$\frac{8}{3} \tan(x)^3 + x - \frac{4(3 \cos(x)^2 - 1)}{3 \cos(x)^3} + \frac{4}{3 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)+tan(x))^4,x, algorithm="maxima")

[Out] $8/3*\tan(x)^3 + x - 4/3*(3*\cos(x)^2 - 1)/\cos(x)^3 + 4/3/\cos(x)^3$

mupad [B] time = 2.37, size = 20, normalized size = 0.67

$$x - \frac{8 \tan\left(\frac{x}{2}\right) - \frac{8}{3}}{\left(\tan\left(\frac{x}{2}\right) - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(x) + 1/cos(x))^4,x)

[Out] $x - (8*\tan(x/2) - 8/3)/(\tan(x/2) - 1)^3$

sympy [A] time = 4.10, size = 44, normalized size = 1.47

$$x + \frac{\sin^3(x)}{3 \cos^3(x)} - \frac{\sin(x)}{\cos(x)} + \frac{7 \tan^3(x)}{3} + \tan(x) + \frac{8 \sec^3(x)}{3} - 4 \sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)+tan(x))**4,x)

[Out] $x + \sin(x)**3/(3*\cos(x)**3) - \sin(x)/\cos(x) + 7*\tan(x)**3/3 + \tan(x) + 8*\sec(x)**3/3 - 4*\sec(x)$

3.275 $\int (\sec(x) + \tan(x))^3 dx$

Optimal. Leaf size=18

$$\frac{2}{1 - \sin(x)} + \log(1 - \sin(x))$$

[Out] $\ln(1 - \sin(x)) + 2/(1 - \sin(x))$

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4391, 2667, 43}

$$\frac{2}{1 - \sin(x)} + \log(1 - \sin(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[x] + \text{Tan}[x])^3, x]$

[Out] $\text{Log}[1 - \text{Sin}[x]] + 2/(1 - \text{Sin}[x])$

Rule 43

$\text{Int}[(a_. + (b_.)(x_.))^{(m_.)}((c_. + (d_.)(x_.))^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) || \text{LtQ}[9*m + 5*(n + 1), 0] || \text{GtQ}[m + n + 2, 0])$

Rule 2667

$\text{Int}[\cos[(e_. + (f_.)(x_.))^{(p_.)}((a_. + (b_.)\sin[(e_. + (f_.)(x_.))^{(m_.)}), x_Symbol] := \text{Dist}[1/(b^p*f), \text{Subst}[\text{Int}[(a + x)^{(m + (p - 1)/2)}*(a - x)^{((p - 1)/2)}, x], x, b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x] \&\& \text{IntegerQ}[(p - 1)/2] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& (\text{GeQ}[p, -1] || !\text{IntegerQ}[m + 1/2])$

Rule 4391

$\text{Int}[(u_.)((b_.)\sec[(c_. + (d_.)(x_.))^{(n_.)} + (a_.)\tan[(c_. + (d_.)(x_.))^{(n_.)}])^{(p_.)}, x_Symbol] := \text{Int}[\text{ActivateTrig}[u]*\text{Sec}[c + d*x]^{(n*p)}*(b + a*\text{Sin}[c + d*x]^n)^p, x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{IntegersQ}[n, p]$

Rubi steps

$$\begin{aligned} \int (\sec(x) + \tan(x))^3 dx &= \int \sec^3(x)(1 + \sin(x))^3 dx \\ &= \text{Subst}\left(\int \frac{1+x}{(1-x)^2} dx, x, \sin(x)\right) \\ &= \text{Subst}\left(\int \left(\frac{2}{(-1+x)^2} + \frac{1}{-1+x}\right) dx, x, \sin(x)\right) \\ &= \log(1 - \sin(x)) + \frac{2}{1 - \sin(x)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.72

$$\frac{\tan^2(x)}{2} + \frac{3 \sec^2(x)}{2} - \tanh^{-1}(\sin(x)) + \log(\cos(x)) + 2 \tan(x) \sec(x)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x] + Tan[x])^3,x]

[Out] -ArcTanh[Sin[x]] + Log[Cos[x]] + (3*Sec[x]^2)/2 + 2*Sec[x]*Tan[x] + Tan[x]^2/2

fricas [A] time = 0.91, size = 21, normalized size = 1.17

$$\frac{(\sin(x) - 1) \log(-\sin(x) + 1) - 2}{\sin(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)+tan(x))^3,x, algorithm="fricas")

[Out] ((sin(x) - 1)*log(-sin(x) + 1) - 2)/(sin(x) - 1)

giac [B] time = 0.17, size = 48, normalized size = 2.67

$$-\frac{3 \tan\left(\frac{1}{2}x\right)^2 - 10 \tan\left(\frac{1}{2}x\right) + 3}{\left(\tan\left(\frac{1}{2}x\right) - 1\right)^2} - \log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) + 2 \log\left(\left|\tan\left(\frac{1}{2}x\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)+tan(x))^3,x, algorithm="giac")

[Out] -(3*tan(1/2*x)^2 - 10*tan(1/2*x) + 3)/(tan(1/2*x) - 1)^2 - log(tan(1/2*x)^2 + 1) + 2*log(abs(tan(1/2*x) - 1))

maple [B] time = 0.10, size = 45, normalized size = 2.50

$$\frac{\sec(x) \tan(x)}{2} - \ln(\sec(x) + \tan(x)) + \frac{3}{2 \cos(x)^2} + \frac{3 \sin^3(x)}{2 \cos(x)^2} + \frac{3 \sin(x)}{2} + \frac{\tan^2(x)}{2} + \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sec(x)+tan(x))^3,x)

[Out] 1/2*sec(x)*tan(x)-ln(sec(x)+tan(x))+3/2/cos(x)^2+3/2*sin(x)^3/cos(x)^2+3/2*sin(x)+1/2*tan(x)^2+ln(cos(x))

maxima [B] time = 0.33, size = 52, normalized size = 2.89

$$\frac{3}{2} \tan(x)^2 - \frac{2 \sin(x)}{\sin(x)^2 - 1} - \frac{1}{2(\sin(x)^2 - 1)} + \frac{1}{2} \log(\sin(x)^2 - 1) - \frac{1}{2} \log(\sin(x) + 1) + \frac{1}{2} \log(\sin(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)+tan(x))^3,x, algorithm="maxima")

[Out] 3/2*tan(x)^2 - 2*sin(x)/(sin(x)^2 - 1) - 1/2/(sin(x)^2 - 1) + 1/2*log(sin(x)^2 - 1) - 1/2*log(sin(x) + 1) + 1/2*log(sin(x) - 1)

mupad [B] time = 2.40, size = 43, normalized size = 2.39

$$2 \ln\left(\tan\left(\frac{x}{2}\right) - 1\right) - \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) + \frac{4 \tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2 - 2 \tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((tan(x) + 1/cos(x))^3,x)`

[Out] $2\log(\tan(x/2) - 1) - \log(\tan(x/2)^2 + 1) + (4\tan(x/2))/(\tan(x/2)^2 - 2\tan(x/2) + 1)$

sympy [B] time = 4.81, size = 44, normalized size = 2.44

$$\frac{\log(\sin(x) - 1)}{2} - \frac{\log(\sin(x) + 1)}{2} - \frac{\log(\sec^2(x))}{2} + 2\sec^2(x) - \frac{4\sin(x)}{2\sin^2(x) - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((sec(x)+tan(x))**3,x)`

[Out] $\log(\sin(x) - 1)/2 - \log(\sin(x) + 1)/2 - \log(\sec(x)**2)/2 + 2*\sec(x)**2 - 4*\sin(x)/(2*\sin(x)**2 - 2)$

3.276 $\int (\sec(x) + \tan(x))^2 dx$

Optimal. Leaf size=16

$$\frac{2 \cos(x)}{1 - \sin(x)} - x$$

[Out] $-x+2*\cos(x)/(1-\sin(x))$

Rubi [A] time = 0.07, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4391, 2670, 2680, 8}

$$\frac{2 \cos(x)}{1 - \sin(x)} - x$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[x] + \text{Tan}[x])^2, x]$

[Out] $-x + (2*\text{Cos}[x])/(1 - \text{Sin}[x])$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2670

$\text{Int}[(\cos[(e_.) + (f_.)*(x_.)]*(g_.))^{(p_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}, x_Symbol] \rightarrow \text{Dist}[(a/g)^{(2*m)}, \text{Int}[(g*\cos[e + f*x])^{(2*m + p)} / (a - b*\sin[e + f*x])^m, x], x] /; \text{FreeQ}[\{a, b, e, f, g\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{LtQ}[p, -1] \&\& \text{GeQ}[2*m + p, 0]$

Rule 2680

$\text{Int}[(\cos[(e_.) + (f_.)*(x_.)]*(g_.))^{(p_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(2*g*(g*\cos[e + f*x])^{(p - 1)}*(a + b*\sin[e + f*x])^{(m + 1)}) / (b*f*(2*m + p + 1)), x] + \text{Dist}[(g^2*(p - 1)) / (b^2*(2*m + p + 1)), \text{Int}[(g*\cos[e + f*x])^{(p - 2)}*(a + b*\sin[e + f*x])^{(m + 2)}, x], x] /; \text{FreeQ}[\{a, b, e, f, g\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LeQ}[m, -2] \&\& \text{GtQ}[p, 1] \&\& \text{NeQ}[2*m + p + 1, 0] \&\& !\text{ILtQ}[m + p + 1, 0] \&\& \text{IntegersQ}[2*m, 2*p]$

Rule 4391

$\text{Int}[(u_.)*((b_.)*\sec[(c_.) + (d_.)*(x_.)]^{(n_.)} + (a_.)*\tan[(c_.) + (d_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ActivateTrig}[u]*\text{Sec}[c + d*x]^{(n*p)}*(b + a*\sin[c + d*x]^n)^p, x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{IntegersQ}[n, p]$

Rubi steps

$$\begin{aligned} \int (\sec(x) + \tan(x))^2 dx &= \int \sec^2(x)(1 + \sin(x))^2 dx \\ &= \int \frac{\cos^2(x)}{(1 - \sin(x))^2} dx \\ &= \frac{2 \cos(x)}{1 - \sin(x)} - \int 1 dx \\ &= -x + \frac{2 \cos(x)}{1 - \sin(x)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.88

$$-\tan^{-1}(\tan(x)) + 2 \tan(x) + 2 \sec(x)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x] + Tan[x])^2,x]

[Out] -ArcTan[Tan[x]] + 2*Sec[x] + 2*Tan[x]

fricas [A] time = 0.89, size = 28, normalized size = 1.75

$$\frac{(x-2)\cos(x) - (x+2)\sin(x) + x - 2}{\cos(x) - \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)+tan(x))^2,x, algorithm="fricas")

[Out] -((x-2)*cos(x) - (x+2)*sin(x) + x - 2)/(cos(x) - sin(x) + 1)

giac [A] time = 0.13, size = 14, normalized size = 0.88

$$-x - \frac{4}{\tan\left(\frac{1}{2}x\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)+tan(x))^2,x, algorithm="giac")

[Out] -x - 4/(tan(1/2*x) - 1)

maple [A] time = 0.05, size = 15, normalized size = 0.94

$$2 \tan(x) + \frac{2}{\cos(x)} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sec(x)+tan(x))^2,x)

[Out] 2*tan(x)+2/cos(x)-x

maxima [A] time = 0.45, size = 14, normalized size = 0.88

$$-x + \frac{2}{\cos(x)} + 2 \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sec(x)+tan(x))^2,x, algorithm="maxima")

[Out] -x + 2/cos(x) + 2*tan(x)

mupad [B] time = 2.36, size = 14, normalized size = 0.88

$$-x - \frac{4}{\tan\left(\frac{x}{2}\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(x) + 1/cos(x))^2,x)

[Out] $-x - 4/(\tan(x/2) - 1)$

sympy [A] time = 1.11, size = 10, normalized size = 0.62

$$-x + 2 \tan(x) + 2 \sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((sec(x)+tan(x))**2,x)`

[Out] $-x + 2*\tan(x) + 2*\sec(x)$

3.277 $\int (\sec(x) + \tan(x)) dx$

Optimal. Leaf size=13

$$-2 \log \left(\cos \left(\frac{1}{4}(2x + \pi) \right) \right)$$

[Out] $-2 \ln(\cos(1/4 \pi + 1/2 x))$

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3770, 3475}

$$\tanh^{-1}(\sin(x)) - \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Sec[x] + Tan[x], x]

[Out] ArcTanh[Sin[x]] - Log[Cos[x]]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (\sec(x) + \tan(x)) dx &= \int \sec(x) dx + \int \tan(x) dx \\ &= \tanh^{-1}(\sin(x)) - \log(\cos(x)) \end{aligned}$$

Mathematica [B] time = 0.00, size = 38, normalized size = 2.92

$$-\log(\cos(x)) - \log \left(\cos \left(\frac{x}{2} \right) - \sin \left(\frac{x}{2} \right) \right) + \log \left(\sin \left(\frac{x}{2} \right) + \cos \left(\frac{x}{2} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x] + Tan[x], x]

[Out] -Log[Cos[x]] - Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]

fricas [A] time = 3.01, size = 9, normalized size = 0.69

$$-\log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)+tan(x), x, algorithm="fricas")

[Out] -log(-sin(x) + 1)

giac [B] time = 0.15, size = 31, normalized size = 2.38

$$\frac{1}{4} \log \left(\left| \frac{1}{\sin(x)} + \sin(x) + 2 \right| \right) - \frac{1}{4} \log \left(\left| \frac{1}{\sin(x)} + \sin(x) - 2 \right| \right) - \log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)+tan(x),x, algorithm="giac")

[Out] $\frac{1}{4} \log(\text{abs}(1/\sin(x) + \sin(x) + 2)) - \frac{1}{4} \log(\text{abs}(1/\sin(x) + \sin(x) - 2)) - \log(\text{abs}(\cos(x)))$

maple [A] time = 0.00, size = 13, normalized size = 1.00

$$\ln(\sec(x) + \tan(x)) - \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)+tan(x),x)

[Out] $\ln(\sec(x) + \tan(x)) - \ln(\cos(x))$

maxima [A] time = 0.31, size = 10, normalized size = 0.77

$$\log(\sec(x) + \tan(x)) + \log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)+tan(x),x, algorithm="maxima")

[Out] $\log(\sec(x) + \tan(x)) + \log(\sec(x))$

mupad [B] time = 2.40, size = 19, normalized size = 1.46

$$\ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - 2 \ln\left(\tan\left(\frac{x}{2}\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x) + 1/cos(x),x)

[Out] $\log(\tan(x/2)^2 + 1) - 2 \log(\tan(x/2) - 1)$

sympy [A] time = 0.09, size = 20, normalized size = 1.54

$$-\frac{\log(\sin(x) - 1)}{2} + \frac{\log(\sin(x) + 1)}{2} - \log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)+tan(x),x)

[Out] $-\log(\sin(x) - 1)/2 + \log(\sin(x) + 1)/2 - \log(\cos(x))$

$$3.278 \quad \int \frac{1}{\sec(x)+\tan(x)} dx$$

Optimal. Leaf size=5

$$\log(\sin(x) + 1)$$

[Out] ln(1+sin(x))

Rubi [A] time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3159, 2667, 31}

$$\log(\sin(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[(Sec[x] + Tan[x])^(-1), x]

[Out] Log[1 + Sin[x]]

Rule 31

Int[((a_) + (b_.)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 3159

Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]), x] /; FreeQ[{a, b, c, d, e}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sec(x) + \tan(x)} dx &= \int \frac{\cos(x)}{1 + \sin(x)} dx \\ &= \text{Subst} \left(\int \frac{1}{1 + x} dx, x, \sin(x) \right) \\ &= \log(1 + \sin(x)) \end{aligned}$$

Mathematica [B] time = 0.01, size = 16, normalized size = 3.20

$$2 \log \left(\sin \left(\frac{x}{2} \right) + \cos \left(\frac{x}{2} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x] + Tan[x])^(-1), x]

[Out] 2*Log[Cos[x/2] + Sin[x/2]]

fricas [A] time = 0.61, size = 5, normalized size = 1.00

$$\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x)),x, algorithm="fricas")

[Out] log(sin(x) + 1)

giac [B] time = 0.14, size = 22, normalized size = 4.40

$$-\log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) + 2\log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x)),x, algorithm="giac")

[Out] -log(tan(1/2*x)^2 + 1) + 2*log(abs(tan(1/2*x) + 1))

maple [A] time = 0.12, size = 6, normalized size = 1.20

$$\ln(1 + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sec(x)+tan(x)),x)

[Out] ln(1+sin(x))

maxima [B] time = 0.32, size = 31, normalized size = 6.20

$$2\log\left(\frac{\sin(x)}{\cos(x)+1} + 1\right) - \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x)),x, algorithm="maxima")

[Out] 2*log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)

mupad [B] time = 2.78, size = 21, normalized size = 4.20

$$2\ln\left(\tan\left(\frac{x}{2}\right) + 1\right) - \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(tan(x) + 1/cos(x)),x)

[Out] 2*log(tan(x/2) + 1) - log(tan(x/2)^2 + 1)

sympy [B] time = 0.14, size = 17, normalized size = 3.40

$$\log(\tan(x) + \sec(x)) - \frac{\log(\tan^2(x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x)),x)

[Out] log(tan(x) + sec(x)) - log(tan(x)**2 + 1)/2

$$3.279 \quad \int \frac{1}{(\sec(x) + \tan(x))^2} dx$$

Optimal. Leaf size=14

$$-x - \frac{2 \cos(x)}{\sin(x) + 1}$$

[Out] $-x - 2 \cos(x) / (1 + \sin(x))$

Rubi [A] time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4391, 2680, 8}

$$-x - \frac{2 \cos(x)}{\sin(x) + 1}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[x] + \text{Tan}[x])^{-2}, x]$

[Out] $-x - (2 \cos[x]) / (1 + \sin[x])$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2680

$\text{Int}[(\cos[e_] + (f_)*(x_)]*(g_))^{(p_)*((a_) + (b_)*\sin[e_] + (f_)*(x_))^{(m_)}, x_Symbol] \rightarrow \text{Simp}[(2*g*(g*\cos[e + f*x])^{(p-1)}*(a + b*\sin[e + f*x])^{(m+1)}) / (b*f*(2*m + p + 1)), x] + \text{Dist}[(g^{2*(p-1)}) / (b^{2*(2*m + p + 1)}), \text{Int}[(g*\cos[e + f*x])^{(p-2)}*(a + b*\sin[e + f*x])^{(m+2)}, x], x] /; \text{FreeQ}\{a, b, e, f, g\}, x \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LeQ}[m, -2] \&\& \text{GtQ}[p, 1] \&\& \text{NeQ}[2*m + p + 1, 0] \&\& !\text{LtQ}[m + p + 1, 0] \&\& \text{IntegersQ}[2*m, 2*p]$

Rule 4391

$\text{Int}[(u_)*((b_)*\sec[(c_) + (d_)*(x_)]^{(n_)} + (a_)*\tan[(c_) + (d_)*(x_)]^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Int}[\text{ActivateTrig}[u]*\text{Sec}[c + d*x]^{(n*p)}*(b + a*\sin[c + d*x]^n)^p, x] /; \text{FreeQ}\{a, b, c, d\}, x \&\& \text{IntegersQ}[n, p]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(\sec(x) + \tan(x))^2} dx &= \int \frac{\cos^2(x)}{(1 + \sin(x))^2} dx \\ &= -\frac{2 \cos(x)}{1 + \sin(x)} - \int 1 dx \\ &= -x - \frac{2 \cos(x)}{1 + \sin(x)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.93

$$\frac{4 \sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)} - x$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x] + Tan[x])^(-2), x]

[Out] $-x + (4*\sin[x/2]) / (\cos[x/2] + \sin[x/2])$

fricas [A] time = 1.01, size = 25, normalized size = 1.79

$$\frac{(x + 2) \cos(x) + (x - 2) \sin(x) + x + 2}{\cos(x) + \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x))^2,x, algorithm="fricas")

[Out] $-((x + 2)*\cos(x) + (x - 2)*\sin(x) + x + 2) / (\cos(x) + \sin(x) + 1)$

giac [A] time = 0.15, size = 14, normalized size = 1.00

$$-x - \frac{4}{\tan\left(\frac{1}{2}x\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x))^2,x, algorithm="giac")

[Out] $-x - 4 / (\tan(1/2*x) + 1)$

maple [A] time = 0.12, size = 15, normalized size = 1.07

$$-\frac{4}{1 + \tan\left(\frac{x}{2}\right)} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sec(x)+tan(x))^2,x)

[Out] $-4 / (1 + \tan(1/2*x)) - x$

maxima [A] time = 0.41, size = 28, normalized size = 2.00

$$-\frac{4}{\frac{\sin(x)}{\cos(x)+1} + 1} - 2 \arctan\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x))^2,x, algorithm="maxima")

[Out] $-4 / (\sin(x) / (\cos(x) + 1) + 1) - 2 * \arctan(\sin(x) / (\cos(x) + 1))$

mupad [B] time = 2.34, size = 14, normalized size = 1.00

$$-x - \frac{4}{\tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(tan(x) + 1/cos(x))^2,x)

[Out] $-x - 4 / (\tan(x/2) + 1)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\tan(x) + \sec(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))**2,x)
```

```
[Out] Integral((tan(x) + sec(x))**(-2), x)
```

$$3.280 \quad \int \frac{1}{(\sec(x) + \tan(x))^3} dx$$

Optimal. Leaf size=16

$$-\frac{2}{\sin(x) + 1} - \log(\sin(x) + 1)$$

[Out] $-\ln(1 + \sin(x)) - 2/(1 + \sin(x))$

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4391, 2667, 43}

$$-\frac{2}{\sin(x) + 1} - \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[x] + \text{Tan}[x])^{-3}, x]$

[Out] $-\text{Log}[1 + \text{Sin}[x]] - 2/(1 + \text{Sin}[x])$

Rule 43

$\text{Int}[(a_.) + (b_.)(x_)^{(m_.)} * ((c_.) + (d_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2667

$\text{Int}[\cos[(e_.) + (f_.)(x_)]^{(p_.)} * ((a_.) + (b_.)\sin[(e_.) + (f_.)(x_)])^{(m_.)}], x_Symbol] \rightarrow \text{Dist}[1/(b^p * f), \text{Subst}[\text{Int}[(a + x)^{m + (p - 1)/2} * (a - x)^{(p - 1)/2}, x], x, b * \text{Sin}[e + f * x]], x] /;$ FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 4391

$\text{Int}[(u_.) * ((b_.)\sec[(c_.) + (d_.)(x_)]^{(n_.)} + (a_.)\tan[(c_.) + (d_.)(x_)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ActivateTrig}[u] * \text{Sec}[c + d * x]^{(n * p)} * (b + a * \text{Sin}[c + d * x]^n)^p, x] /;$ FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned} \int \frac{1}{(\sec(x) + \tan(x))^3} dx &= \int \frac{\cos^3(x)}{(1 + \sin(x))^3} dx \\ &= \text{Subst} \left(\int \frac{1 - x}{(1 + x)^2} dx, x, \sin(x) \right) \\ &= \text{Subst} \left(\int \left(\frac{1}{-1 - x} + \frac{2}{(1 + x)^2} \right) dx, x, \sin(x) \right) \\ &= -\log(1 + \sin(x)) - \frac{2}{1 + \sin(x)} \end{aligned}$$

Mathematica [B] time = 0.02, size = 34, normalized size = 2.12

$$-\frac{2}{\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)^2} - 2 \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x] + Tan[x])^(-3), x]

[Out] -2*Log[Cos[x/2] + Sin[x/2]] - 2/(Cos[x/2] + Sin[x/2])^2

fricas [A] time = 1.07, size = 20, normalized size = 1.25

$$-\frac{(\sin(x) + 1) \log(\sin(x) + 1) + 2}{\sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x))^3,x, algorithm="fricas")

[Out] -((sin(x) + 1)*log(sin(x) + 1) + 2)/(sin(x) + 1)

giac [B] time = 0.16, size = 45, normalized size = 2.81

$$\frac{3 \tan\left(\frac{1}{2}x\right)^2 + 10 \tan\left(\frac{1}{2}x\right) + 3}{\left(\tan\left(\frac{1}{2}x\right) + 1\right)^2} + \log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) - 2 \log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x))^3,x, algorithm="giac")

[Out] (3*tan(1/2*x)^2 + 10*tan(1/2*x) + 3)/(tan(1/2*x) + 1)^2 + log(tan(1/2*x)^2 + 1) - 2*log(abs(tan(1/2*x) + 1))

maple [A] time = 0.16, size = 17, normalized size = 1.06

$$-\ln(1 + \sin(x)) - \frac{2}{1 + \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sec(x)+tan(x))^3,x)

[Out] -ln(1+sin(x))-2/(1+sin(x))

maxima [B] time = 0.44, size = 64, normalized size = 4.00

$$\frac{4 \sin(x)}{\left(\frac{2 \sin(x)}{\cos(x)+1} + \frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)(\cos(x) + 1)} - 2 \log\left(\frac{\sin(x)}{\cos(x) + 1} + 1\right) + \log\left(\frac{\sin(x)^2}{(\cos(x) + 1)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x))^3,x, algorithm="maxima")

[Out] 4*sin(x)/((2*sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) - 2*log(sin(x)/(cos(x) + 1) + 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)

mupad [B] time = 2.36, size = 41, normalized size = 2.56

$$\ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - 2 \ln\left(\tan\left(\frac{x}{2}\right) + 1\right) + \frac{4 \tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2 + 2 \tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(tan(x) + 1/cos(x))^3,x)

[Out] $\log(\tan(x/2)^2 + 1) - 2\log(\tan(x/2) + 1) + (4\tan(x/2))/(2\tan(x/2) + \tan(x/2)^2 + 1)$

sympy [B] time = 0.83, size = 301, normalized size = 18.81

$$\frac{2 \log(\tan(x) + \sec(x)) \tan^2(x)}{2 \tan^2(x) + 4 \tan(x) \sec(x) + 2 \sec^2(x)} - \frac{4 \log(\tan(x) + \sec(x)) \tan(x) \sec(x)}{2 \tan^2(x) + 4 \tan(x) \sec(x) + 2 \sec^2(x)} - \frac{2 \log(\tan(x) + \sec(x))}{2 \tan^2(x) + 4 \tan(x) \sec(x) + 2 \sec^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x))**3,x)

[Out] $-2\log(\tan(x) + \sec(x))\tan(x)**2/(2\tan(x)**2 + 4\tan(x)*\sec(x) + 2*\sec(x)**2) - 4\log(\tan(x) + \sec(x))\tan(x)*\sec(x)/(2\tan(x)**2 + 4\tan(x)*\sec(x) + 2*\sec(x)**2) - 2\log(\tan(x) + \sec(x))*\sec(x)**2/(2\tan(x)**2 + 4\tan(x)*\sec(x) + 2*\sec(x)**2) + \log(\tan(x)**2 + 1)*\tan(x)**2/(2\tan(x)**2 + 4\tan(x)*\sec(x) + 2*\sec(x)**2) + 2\log(\tan(x)**2 + 1)*\tan(x)*\sec(x)/(2\tan(x)**2 + 4\tan(x)*\sec(x) + 2*\sec(x)**2) + \log(\tan(x)**2 + 1)*\sec(x)**2/(2\tan(x)**2 + 4\tan(x)*\sec(x) + 2*\sec(x)**2) + \tan(x)**2/(2\tan(x)**2 + 4\tan(x)*\sec(x) + 2*\sec(x)**2) - \sec(x)**2/(2\tan(x)**2 + 4\tan(x)*\sec(x) + 2*\sec(x)**2) - 1/(2\tan(x)**2 + 4\tan(x)*\sec(x) + 2*\sec(x)**2)$

$$3.281 \quad \int \frac{1}{(\sec(x) + \tan(x))^4} dx$$

Optimal. Leaf size=26

$$x - \frac{2 \cos^3(x)}{3(\sin(x) + 1)^3} + \frac{2 \cos(x)}{\sin(x) + 1}$$

[Out] $x - 2/3 * \cos(x)^3 / (1 + \sin(x))^3 + 2 * \cos(x) / (1 + \sin(x))$

Rubi [A] time = 0.07, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4391, 2680, 8}

$$x - \frac{2 \cos^3(x)}{3(\sin(x) + 1)^3} + \frac{2 \cos(x)}{\sin(x) + 1}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[x] + \text{Tan}[x])^{-4}, x]$

[Out] $x - (2 * \text{Cos}[x]^3) / (3 * (1 + \text{Sin}[x])^3) + (2 * \text{Cos}[x]) / (1 + \text{Sin}[x])$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a * x, x] /; \text{FreeQ}[a, x]$

Rule 2680

$\text{Int}[(\cos[(e_.) + (f_.)(x_.)] * (g_.))^{(p_.)} * ((a_.) + (b_.) * \sin[(e_.) + (f_.)(x_.)])^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(2 * g * (g * \cos[e + f * x])^{(p - 1)} * (a + b * \sin[e + f * x])^{(m + 1)}) / (b * f * (2 * m + p + 1)), x] + \text{Dist}[(g^{2 * (p - 1)}) / (b^{2 * (2 * m + p + 1)}), \text{Int}[(g * \cos[e + f * x])^{(p - 2)} * (a + b * \sin[e + f * x])^{(m + 2)}, x], x] /; \text{FreeQ}\{a, b, e, f, g\}, x \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LeQ}[m, -2] \&\& \text{GtQ}[p, 1] \&\& \text{NeQ}[2 * m + p + 1, 0] \&\& !\text{ILtQ}[m + p + 1, 0] \&\& \text{IntegersQ}[2 * m, 2 * p]$

Rule 4391

$\text{Int}[(u_.) * ((b_.) * \sec[(c_.) + (d_.)(x_.)]^{(n_.)} + (a_.) * \tan[(c_.) + (d_.)(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ActivateTrig}[u] * \text{Sec}[c + d * x]^{(n * p)} * (b + a * \sin[c + d * x]^n)^p, x] /; \text{FreeQ}\{a, b, c, d\}, x \&\& \text{IntegersQ}[n, p]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(\sec(x) + \tan(x))^4} dx &= \int \frac{\cos^4(x)}{(1 + \sin(x))^4} dx \\ &= -\frac{2 \cos^3(x)}{3(1 + \sin(x))^3} - \int \frac{\cos^2(x)}{(1 + \sin(x))^2} dx \\ &= -\frac{2 \cos^3(x)}{3(1 + \sin(x))^3} + \frac{2 \cos(x)}{1 + \sin(x)} + \int 1 dx \\ &= x - \frac{2 \cos^3(x)}{3(1 + \sin(x))^3} + \frac{2 \cos(x)}{1 + \sin(x)} \end{aligned}$$

Mathematica [B] time = 0.07, size = 62, normalized size = 2.38

$$\frac{3(3x - 8) \cos\left(\frac{x}{2}\right) + (16 - 3x) \cos\left(\frac{3x}{2}\right) + 6 \sin\left(\frac{x}{2}\right) (2x + x \cos(x) - 4)}{6 \left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x] + Tan[x])^(-4), x]

[Out] (3*(-8 + 3*x)*Cos[x/2] + (16 - 3*x)*Cos[(3*x)/2] + 6*(-4 + 2*x + x*Cos[x])*Sin[x/2])/(6*(Cos[x/2] + Sin[x/2])^3)

fricas [B] time = 0.86, size = 63, normalized size = 2.42

$$\frac{(3x - 8)\cos(x)^2 - (3x + 4)\cos(x) - ((3x + 8)\cos(x) + 6x + 4)\sin(x) - 6x + 4}{3(\cos(x)^2 - (\cos(x) + 2)\sin(x) - \cos(x) - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x))^4,x, algorithm="fricas")

[Out] 1/3*((3*x - 8)*cos(x)^2 - (3*x + 4)*cos(x) - ((3*x + 8)*cos(x) + 6*x + 4)*sin(x) - 6*x + 4)/(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)

giac [A] time = 0.16, size = 20, normalized size = 0.77

$$x + \frac{8\left(3\tan\left(\frac{1}{2}x\right) + 1\right)}{3\left(\tan\left(\frac{1}{2}x\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x))^4,x, algorithm="giac")

[Out] x + 8/3*(3*tan(1/2*x) + 1)/(tan(1/2*x) + 1)^3

maple [A] time = 0.15, size = 23, normalized size = 0.88

$$-\frac{16}{3\left(1 + \tan\left(\frac{x}{2}\right)\right)^3} + \frac{8}{\left(1 + \tan\left(\frac{x}{2}\right)\right)^2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sec(x)+tan(x))^4,x)

[Out] -16/3/(1+tan(1/2*x))^3+8/(1+tan(1/2*x))^2+x

maxima [B] time = 0.44, size = 64, normalized size = 2.46

$$\frac{8\left(\frac{3\sin(x)}{\cos(x)+1} + 1\right)}{3\left(\frac{3\sin(x)}{\cos(x)+1} + \frac{3\sin(x)^2}{(\cos(x)+1)^2} + \frac{\sin(x)^3}{(\cos(x)+1)^3} + 1\right)} + 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x))^4,x, algorithm="maxima")

[Out] 8/3*(3*sin(x)/(cos(x) + 1) + 1)/(3*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(cos(x) + 1)^2 + sin(x)^3/(cos(x) + 1)^3 + 1) + 2*arctan(sin(x)/(cos(x) + 1))

mupad [B] time = 2.34, size = 19, normalized size = 0.73

$$x + \frac{8\tan\left(\frac{x}{2}\right) + \frac{8}{3}}{\left(\tan\left(\frac{x}{2}\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(tan(x) + 1/cos(x))^4,x)`

[Out] `x + (8*tan(x/2) + 8/3)/(tan(x/2) + 1)^3`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\tan(x) + \sec(x))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sec(x)+tan(x))**4,x)`

[Out] `Integral((tan(x) + sec(x))**(-4), x)`

$$3.282 \quad \int \frac{1}{(\sec(x) + \tan(x))^5} dx$$

Optimal. Leaf size=22

$$\frac{4}{\sin(x) + 1} - \frac{2}{(\sin(x) + 1)^2} + \log(\sin(x) + 1)$$

[Out] $\ln(1 + \sin(x)) - 2/(1 + \sin(x))^2 + 4/(1 + \sin(x))$

Rubi [A] time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4391, 2667, 43}

$$\frac{4}{\sin(x) + 1} - \frac{2}{(\sin(x) + 1)^2} + \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sec}[x] + \text{Tan}[x])^{-5}, x]$

[Out] $\text{Log}[1 + \text{Sin}[x]] - 2/(1 + \text{Sin}[x])^2 + 4/(1 + \text{Sin}[x])$

Rule 43

$\text{Int}[(a_. + (b_.)(x_.))^{(m_.)}((c_.) + (d_.)(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

Rule 2667

$\text{Int}[\cos[(e_.) + (f_.)(x_.)]^{(p_.)}((a_.) + (b_.)\sin[(e_.) + (f_.)(x_.)])^{(m_.)}, x_Symbol] \rightarrow \text{Dist}[1/(b^p*f), \text{Subst}[\text{Int}[(a + x)^{(m + (p - 1)/2)}*(a - x)^{(p - 1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x \ \&\& \ \text{IntegerQ}[(p - 1)/2] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ (\text{GeQ}[p, -1] \ || \ !\text{IntegerQ}[m + 1/2])$

Rule 4391

$\text{Int}[(u_.)*((b_.)\sec[(c_.) + (d_.)(x_.)]^{(n_.)} + (a_.)\tan[(c_.) + (d_.)(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ActivateTrig}[u]*\text{Sec}[c + d*x]^{(n*p)}*(b + a*\text{Sin}[c + d*x]^n)^p, x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{IntegersQ}[n, p]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(\sec(x) + \tan(x))^5} dx &= \int \frac{\cos^5(x)}{(1 + \sin(x))^5} dx \\ &= \text{Subst} \left(\int \frac{(1-x)^2}{(1+x)^3} dx, x, \sin(x) \right) \\ &= \text{Subst} \left(\int \left(\frac{4}{(1+x)^3} - \frac{4}{(1+x)^2} + \frac{1}{1+x} \right) dx, x, \sin(x) \right) \\ &= \log(1 + \sin(x)) - \frac{2}{(1 + \sin(x))^2} + \frac{4}{1 + \sin(x)} \end{aligned}$$

Mathematica [A] time = 0.05, size = 39, normalized size = 1.77

$$\frac{4 \sin(x) + 2}{\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)^4} + 2 \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x] + Tan[x])^(-5), x]

[Out] 2*Log[Cos[x/2] + Sin[x/2]] + (2 + 4*Sin[x])/(Cos[x/2] + Sin[x/2])^4

fricas [A] time = 1.06, size = 35, normalized size = 1.59

$$\frac{(\cos(x)^2 - 2 \sin(x) - 2) \log(\sin(x) + 1) - 4 \sin(x) - 2}{\cos(x)^2 - 2 \sin(x) - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x))^5,x, algorithm="fricas")

[Out] ((cos(x)^2 - 2*sin(x) - 2)*log(sin(x) + 1) - 4*sin(x) - 2)/(cos(x)^2 - 2*sin(x) - 2)

giac [B] time = 0.15, size = 64, normalized size = 2.91

$$\frac{25 \tan\left(\frac{1}{2}x\right)^4 + 100 \tan\left(\frac{1}{2}x\right)^3 + 198 \tan\left(\frac{1}{2}x\right)^2 + 100 \tan\left(\frac{1}{2}x\right) + 25}{6 \left(\tan\left(\frac{1}{2}x\right) + 1\right)^4} - \log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) + 2 \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x))^5,x, algorithm="giac")

[Out] -1/6*(25*tan(1/2*x)^4 + 100*tan(1/2*x)^3 + 198*tan(1/2*x)^2 + 100*tan(1/2*x) + 25)/(tan(1/2*x) + 1)^4 - log(tan(1/2*x)^2 + 1) + 2*log(abs(tan(1/2*x) + 1))

maple [A] time = 0.18, size = 23, normalized size = 1.05

$$\ln(1 + \sin(x)) - \frac{2}{(1 + \sin(x))^2} + \frac{4}{1 + \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sec(x)+tan(x))^5,x)

[Out] ln(1+sin(x))-2/(1+sin(x))^2+4/(1+sin(x))

maxima [B] time = 0.66, size = 92, normalized size = 4.18

$$\frac{8 \sin(x)^2}{\left(\frac{4 \sin(x)}{\cos(x)+1} + \frac{6 \sin(x)^2}{(\cos(x)+1)^2} + \frac{4 \sin(x)^3}{(\cos(x)+1)^3} + \frac{\sin(x)^4}{(\cos(x)+1)^4} + 1\right)(\cos(x) + 1)^2} + 2 \log\left(\frac{\sin(x)}{\cos(x) + 1} + 1\right) - \log\left(\frac{\sin(x)^2}{(\cos(x) + 1)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+tan(x))^5,x, algorithm="maxima")

[Out] -8*sin(x)^2/((4*sin(x)/(cos(x) + 1) + 6*sin(x)^2/(cos(x) + 1)^2 + 4*sin(x)^3/(cos(x) + 1)^3 + sin(x)^4/(cos(x) + 1)^4 + 1)*(cos(x) + 1)^2) + 2*log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)

mupad [B] time = 2.38, size = 61, normalized size = 2.77

$$2 \ln\left(\tan\left(\frac{x}{2}\right) + 1\right) - \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - \frac{8 \tan\left(\frac{x}{2}\right)^2}{\tan\left(\frac{x}{2}\right)^4 + 4 \tan\left(\frac{x}{2}\right)^3 + 6 \tan\left(\frac{x}{2}\right)^2 + 4 \tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(tan(x) + 1/cos(x))^5,x)`

[Out] $2*\log(\tan(x/2) + 1) - \log(\tan(x/2)^2 + 1) - (8*\tan(x/2)^2)/(4*\tan(x/2) + 6*\tan(x/2)^2 + 4*\tan(x/2)^3 + \tan(x/2)^4 + 1)$

sympy [B] time = 2.76, size = 1059, normalized size = 48.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sec(x)+tan(x))**5,x)`

[Out] $36*\log(\tan(x) + \sec(x))*\tan(x)**4/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) + 144*\log(\tan(x) + \sec(x))*\tan(x)**3*\sec(x)/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) + 216*\log(\tan(x) + \sec(x))*\tan(x)**2*\sec(x)**2/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) + 144*\log(\tan(x) + \sec(x))*\tan(x)*\sec(x)**3/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) + 36*\log(\tan(x) + \sec(x))*\sec(x)**4/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) - 18*\log(\tan(x)**2 + 1)*\tan(x)**4/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) - 72*\log(\tan(x)**2 + 1)*\tan(x)**3*\sec(x)/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) - 108*\log(\tan(x)**2 + 1)*\tan(x)**2*\sec(x)**2/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) - 72*\log(\tan(x)**2 + 1)*\tan(x)*\sec(x)**3/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) - 18*\log(\tan(x)**2 + 1)*\sec(x)**4/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) - 28*\tan(x)**4/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) - 52*\tan(x)**3*\sec(x)/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) + 18*\tan(x)**2/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) + 44*\tan(x)*\sec(x)**3/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) + 24*\tan(x)*\sec(x)/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) + 20*\sec(x)**4/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) + 6*\sec(x)**2/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4) - 9/(36*\tan(x)**4 + 144*\tan(x)**3*\sec(x) + 216*\tan(x)**2*\sec(x)**2 + 144*\tan(x)*\sec(x)**3 + 36*\sec(x)**4)$

3.283 $\int (a \cot(x) + b \csc(x))^5 dx$

Optimal. Leaf size=152

$$\frac{1}{8}a^2b(7a^2 - 3b^2)\cos(x) + \frac{1}{16}(a+b)^3(8a^2 - 9ab + 3b^2)\log(1-\cos(x)) + \frac{1}{16}(a-b)^3(8a^2 + 9ab + 3b^2)\log(\cos(x)) +$$

[Out] 1/8*a^2*b*(7*a^2-3*b^2)*cos(x)+1/8*(b+a*cos(x))^2*(2*a*(2*a^2-b^2)+b*(5*a^2-3*b^2)*cos(x))*csc(x)^2-1/4*(b+a*cos(x))^4*(a+b*cos(x))*csc(x)^4+1/16*(a+b)^3*(8*a^2-9*a*b+3*b^2)*ln(1-cos(x))+1/16*(a-b)^3*(8*a^2+9*a*b+3*b^2)*ln(1+cos(x))

Rubi [A] time = 0.22, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {4392, 2668, 739, 819, 774, 633, 31}

$$\frac{1}{8}a^2b(7a^2 - 3b^2)\cos(x) + \frac{1}{16}(a+b)^3(8a^2 - 9ab + 3b^2)\log(1-\cos(x)) + \frac{1}{16}(a-b)^3(8a^2 + 9ab + 3b^2)\log(\cos(x)) +$$

Antiderivative was successfully verified.

[In] Int[(a*Cot[x] + b*Csc[x])^5,x]

[Out] (a^2*b*(7*a^2 - 3*b^2)*Cos[x])/8 + ((b + a*Cos[x])^2*(2*a*(2*a^2 - b^2) + b*(5*a^2 - 3*b^2)*Cos[x])*Csc[x]^2)/8 - ((b + a*Cos[x])^4*(a + b*Cos[x])*Csc[x]^4)/4 + ((a + b)^3*(8*a^2 - 9*a*b + 3*b^2)*Log[1 - Cos[x]])/16 + ((a - b)^3*(8*a^2 + 9*a*b + 3*b^2)*Log[1 + Cos[x]])/16

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 633

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 739

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 774

Int[(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]

Rule 819

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2

```
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a,
c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!LtQ[m + 2*p + 3, 0])
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_.)]^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_.)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_.)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int (a \cot(x) + b \csc(x))^5 dx &= \int (b + a \cos(x))^5 \csc^5(x) dx \\
&= - \left(a^5 \operatorname{Subst} \left(\int \frac{(b+x)^5}{(a^2-x^2)^3} dx, x, a \cos(x) \right) \right) \\
&= -\frac{1}{4} (b + a \cos(x))^4 (a + b \cos(x)) \csc^4(x) + \frac{1}{4} a^3 \operatorname{Subst} \left(\int \frac{(b+x)^3 (4a^2 - 3b^2 + bx)}{(a^2-x^2)^2} dx, x, a \cos(x) \right) \\
&= \frac{1}{8} (b + a \cos(x))^2 (2a(2a^2 - b^2) + b(5a^2 - 3b^2) \cos(x)) \csc^2(x) - \frac{1}{4} (b + a \cos(x))^4 (a + b \cos(x)) \csc^4(x) \\
&= \frac{1}{8} a^2 b (7a^2 - 3b^2) \cos(x) + \frac{1}{8} (b + a \cos(x))^2 (2a(2a^2 - b^2) + b(5a^2 - 3b^2) \cos(x)) \csc^2(x) \\
&= \frac{1}{8} a^2 b (7a^2 - 3b^2) \cos(x) + \frac{1}{8} (b + a \cos(x))^2 (2a(2a^2 - b^2) + b(5a^2 - 3b^2) \cos(x)) \csc^2(x) \\
&= \frac{1}{8} a^2 b (7a^2 - 3b^2) \cos(x) + \frac{1}{8} (b + a \cos(x))^2 (2a(2a^2 - b^2) + b(5a^2 - 3b^2) \cos(x)) \csc^2(x)
\end{aligned}$$

Mathematica [A] time = 0.70, size = 143, normalized size = 0.94

$$\frac{1}{64} \left(8(a+b)^3 (8a^2 - 9ab + 3b^2) \log \left(\sin \left(\frac{x}{2} \right) \right) + 8(8a^2 + 9ab + 3b^2) (a-b)^3 \log \left(\cos \left(\frac{x}{2} \right) \right) - (a+b)^5 \csc^4 \left(\frac{x}{2} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^5, x]
```

```
[Out] (2*(7*a - 3*b)*(a + b)^4*Csc[x/2]^2 - (a + b)^5*Csc[x/2]^4 + 8*(a - b)^3*(8
*a^2 + 9*a*b + 3*b^2)*Log[Cos[x/2]] + 8*(a + b)^3*(8*a^2 - 9*a*b + 3*b^2)*L
og[Sin[x/2]] + 2*(a - b)^4*(7*a + 3*b)*Sec[x/2]^2 - (a - b)^5*Sec[x/2]^4)/4
```

fricas [B] time = 0.87, size = 292, normalized size = 1.92

$$12 a^5 + 40 a^3 b^2 - 20 a b^4 - 2 (25 a^4 b + 10 a^2 b^3 - 3 b^5) \cos(x)^3 - 16 (a^5 + 5 a^3 b^2) \cos(x)^2 + 10 (3 a^4 b - 2 a^2 b^3 - b^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cot(x)+b*csc(x))^5,x, algorithm="fricas")

[Out] $\frac{1}{16}(12a^5 + 40a^3b^2 - 20ab^4 - 2(25a^4b + 10a^2b^3 - 3b^5)\cos(x)^3 - 16(a^5 + 5a^3b^2)\cos(x)^2 + 10(3a^4b - 2a^2b^3 - b^5)\cos(x) + (8a^5 - 15a^4b + 10a^2b^3 - 3b^5 + (8a^5 - 15a^4b + 10a^2b^3 - 3b^5)\cos(x)^4 - 2(8a^5 - 15a^4b + 10a^2b^3 - 3b^5)\cos(x)^2) \log(1/2\cos(x) + 1/2) + (8a^5 + 15a^4b - 10a^2b^3 + 3b^5 + (8a^5 + 15a^4b - 10a^2b^3 + 3b^5)\cos(x)^4 - 2(8a^5 + 15a^4b - 10a^2b^3 + 3b^5)\cos(x)^2) \log(-1/2\cos(x) + 1/2) / (\cos(x)^4 - 2\cos(x)^2 + 1)$

giac [A] time = 0.13, size = 169, normalized size = 1.11

$$\frac{1}{16} (8a^5 - 15a^4b + 10a^2b^3 - 3b^5) \log(\cos(x) + 1) + \frac{1}{16} (8a^5 + 15a^4b - 10a^2b^3 + 3b^5) \log(-\cos(x) + 1) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cot(x)+b*csc(x))^5,x, algorithm="giac")

[Out] $\frac{1}{16}(8a^5 - 15a^4b + 10a^2b^3 - 3b^5)\log(\cos(x) + 1) + \frac{1}{16}(8a^5 + 15a^4b - 10a^2b^3 + 3b^5)\log(-\cos(x) + 1) + \frac{1}{8}(6a^5 + 20a^3b^2 - 10ab^4 - (25a^4b + 10a^2b^3 - 3b^5)\cos(x)^3 - 8(a^5 + 5a^3b^2)\cos(x)^2 + 5(3a^4b - 2a^2b^3 - b^5)\cos(x)) / ((\cos(x) + 1)^2(\cos(x) - 1)^2)$

maple [A] time = 0.14, size = 204, normalized size = 1.34

$$-\frac{a^5(\cot^4(x))}{4} + \frac{a^5(\cot^2(x))}{2} + a^5 \ln(\sin(x)) - \frac{5a^4b(\cos^5(x))}{4\sin(x)^4} + \frac{5a^4b(\cos^5(x))}{8\sin(x)^2} + \frac{5(\cos^3(x))a^4b}{8} + \frac{15a^4b\cos(x)}{8} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cot(x)+b*csc(x))^5,x)

[Out] $-1/4a^5\cot(x)^4 + 1/2a^5\cot(x)^2 + a^5\ln(\sin(x)) - 5/4a^4b/\sin(x)^4\cos(x)^5 + 5/8a^4b/\sin(x)^2\cos(x)^5 + 5/8\cos(x)^3a^4b + 15/8a^4b\cos(x) + 15/8a^4b\ln(\csc(x) - \cot(x)) - 5/2a^3b^2/\sin(x)^4\cos(x)^4 - 5/2a^2b^3/\sin(x)^4\cos(x)^3 - 5/4a^2b^3/\sin(x)^2\cos(x)^3 - 5/4\cos(x)a^2b^3 - 5/4a^2b^3\ln(\csc(x) - \cot(x)) - 5/4ab^4/\sin(x)^4 - 1/4b^5\cot(x)\csc(x)^3 - 3/8b^5\csc(x)\cot(x) + 3/8b^5\ln(\csc(x) - \cot(x))$

maxima [A] time = 0.31, size = 188, normalized size = 1.24

$$-\frac{5}{2}a^3b^2\cot(x)^4 - \frac{5}{16}a^4b\left(\frac{2(5\cos(x)^3 - 3\cos(x))}{\cos(x)^4 - 2\cos(x)^2 + 1} + 3\log(\cos(x) + 1) - 3\log(\cos(x) - 1)\right) + \frac{1}{16}b^5\left(\frac{2(3\cos(x)^3 - 3\cos(x))}{\cos(x)^4 - 2\cos(x)^2 + 1} + 3\log(\cos(x) + 1) - 3\log(\cos(x) - 1)\right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cot(x)+b*csc(x))^5,x, algorithm="maxima")

[Out] $-5/2a^3b^2\cot(x)^4 - 5/16a^4b(2(5\cos(x)^3 - 3\cos(x)) / (\cos(x)^4 - 2\cos(x)^2 + 1) + 3\log(\cos(x) + 1) - 3\log(\cos(x) - 1)) + 1/16b^5(2(3\cos(x)^3 - 3\cos(x)) / (\cos(x)^4 - 2\cos(x)^2 + 1) - 3\log(\cos(x) + 1) + 3\log(\cos(x) - 1)) - 5/8a^2b^3(2(\cos(x)^3 + \cos(x)) / (\cos(x)^4 - 2\cos(x)^2 + 1) - \log(\cos(x) + 1) + \log(\cos(x) - 1)) + 1/4a^5((4\sin(x)^2 - 1) / \sin(x)^4 + 2\log(\sin(x)^2)) - 5/4ab^4/\sin(x)^4$

mupad [B] time = 2.58, size = 174, normalized size = 1.14

$$\tan\left(\frac{x}{2}\right)^2 \left(\frac{5(a+b)(a-b)^4}{32} + \frac{(a-b)^5}{32}\right) - \frac{\frac{5ab^4}{4} + \frac{5a^4b}{4} - \tan\left(\frac{x}{2}\right)^2(3a^5 + 10a^4b + 10a^3b^2 - 5ab^4 - 2b^5) + \dots}{16\tan\left(\frac{x}{2}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b/sin(x) + a*cot(x))^5,x)`

[Out] $\tan(x/2)^2 \cdot ((5(a+b)(a-b)^4)/32 + (a-b)^5/32) - ((5ab^4)/4 + (5a^4b)/4 - \tan(x/2)^2 \cdot (10a^4b - 5ab^4 + 3a^5 - 2b^5 + 10a^3b^2) + a^5/4 + b^5/4 + (5a^2b^3)/2 + (5a^3b^2)/2) / (16 \tan(x/2)^4) - a^5 \log(\tan(x/2)^2 + 1) + \log(\tan(x/2)) \cdot ((15a^4b)/8 + a^5 + (3b^5)/8 - (5a^2b^3)/4) - (\tan(x/2)^4 \cdot (a-b)^5) / 64$

sympy [B] time = 102.28, size = 308, normalized size = 2.03

$$-\frac{a^5 \log(\csc^2(x))}{2} - \frac{a^5 \csc^4(x)}{4} + a^5 \csc^2(x) + \frac{15a^4b \log(\cos(x) - 1)}{16} - \frac{15a^4b \log(\cos(x) + 1)}{16} - \frac{25a^4b \cos^3(x)}{8 \cos^4(x) - 16 \cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*cot(x)+b*csc(x))**5,x)`

[Out] $-a^{**5} \log(\csc(x)**2) / 2 - a^{**5} \csc(x)**4 / 4 + a^{**5} \csc(x)**2 + 15a^{**4}b \log(\cos(x) - 1) / 16 - 15a^{**4}b \log(\cos(x) + 1) / 16 - 25a^{**4}b \cos(x)**3 / (8 \cos(x)**4 - 16 \cos(x)**2 + 8) + 15a^{**4}b \cos(x) / (8 \cos(x)**4 - 16 \cos(x)**2 + 8) - 5a^{**3}b**2 \cot(x)**4 / 2 - 5a^{**2}b**3 \log(\cos(x) - 1) / 8 + 5a^{**2}b**3 \log(\cos(x) + 1) / 8 - 10a^{**2}b**3 \cos(x)**3 / (8 \cos(x)**4 - 16 \cos(x)**2 + 8) - 10a^{**2}b**3 \cos(x) / (8 \cos(x)**4 - 16 \cos(x)**2 + 8) - 5ab^{**4} \csc(x)**4 / 4 + 3b^{**5} \log(\cos(x) - 1) / 16 - 3b^{**5} \log(\cos(x) + 1) / 16 + 3b^{**5} \cos(x)**3 / (8 \cos(x)**4 - 16 \cos(x)**2 + 8) - 5b^{**5} \cos(x) / (8 \cos(x)**4 - 16 \cos(x)**2 + 8)$

3.284 $\int (a \cot(x) + b \csc(x))^4 dx$

Optimal. Leaf size=101

$$a^4x + \frac{4}{3}ab(2a^2 - b^2)\sin(x) + \frac{1}{3}a^2(3a^2 - 2b^2)\sin(x)\cos(x) + \frac{1}{3}\csc(x)(a\cos(x) + b)^2((3a^2 - 2b^2)\cos(x) + ab) - \frac{1}{3}$$

[Out] $a^4x + \frac{1}{3}(b + a\cos(x))^2(a*b + (3*a^2 - 2*b^2)*\cos(x))*\csc(x) - \frac{1}{3}(b + a\cos(x))^3*(a + b*\cos(x))*\csc(x)^3 + \frac{4}{3}a*b*(2*a^2 - b^2)*\sin(x) + \frac{1}{3}a^2*(3*a^2 - 2*b^2)*\cos(x)*\sin(x)$

Rubi [A] time = 0.22, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4392, 2691, 2861, 2734}

$$\frac{4}{3}ab(2a^2 - b^2)\sin(x) + \frac{1}{3}a^2(3a^2 - 2b^2)\sin(x)\cos(x) + \frac{1}{3}\csc(x)(a\cos(x) + b)^2((3a^2 - 2b^2)\cos(x) + ab) + a^4x - \frac{1}{3}$$

Antiderivative was successfully verified.

[In] Int[(a*Cot[x] + b*Csc[x])^4, x]

[Out] $a^4x + ((b + a\cos[x])^2(a*b + (3*a^2 - 2*b^2)*\cos[x])*Csc[x])/3 - ((b + a\cos[x])^3(a + b*\cos[x])*Csc[x]^3)/3 + (4*a*b*(2*a^2 - b^2)*\sin[x])/3 + (a^2*(3*a^2 - 2*b^2)*\cos[x]*\sin[x])/3$

Rule 2691

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] := -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p] || IntegerQ[m])

Rule 2734

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]*(x_.)), x_Symbol] := Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2861

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]*(x_.)), x_Symbol] := -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*(d + c*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[p, -1] && IntegerQ[2*m] && !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && SimplifierQ[c + d*x, a + b*x])

Rule 4392

Int[(cot[(c_.) + (d_.)*(x_.)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_.)]^(n_.)*(b_.))^(p_.)*(u_.), x_Symbol] := Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned}
\int (a \cot(x) + b \csc(x))^4 dx &= \int (b + a \cos(x))^4 \csc^4(x) dx \\
&= -\frac{1}{3}(b + a \cos(x))^3(a + b \cos(x)) \csc^3(x) - \frac{1}{3} \int (b + a \cos(x))^2 (3a^2 - 2b^2 + ab \cos(x)) \csc^3(x) dx \\
&= \frac{1}{3}(b + a \cos(x))^2 (ab + (3a^2 - 2b^2) \cos(x)) \csc(x) - \frac{1}{3}(b + a \cos(x))^3(a + b \cos(x)) \csc(x) \\
&= a^4 x + \frac{1}{3}(b + a \cos(x))^2 (ab + (3a^2 - 2b^2) \cos(x)) \csc(x) - \frac{1}{3}(b + a \cos(x))^3(a + b \cos(x)) \csc(x)
\end{aligned}$$

Mathematica [A] time = 0.26, size = 95, normalized size = 0.94

$$-\frac{1}{12} \csc^3(x) (-9a^4 x \sin(x) + 3a^4 x \sin(3x) + 4a^4 \cos(3x) + 24a^3 b \cos(2x) - 8a^3 b + 6a^2 b^2 \cos(3x) + 6b^2 (3a^2 + b^2) \sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cot[x] + b*Csc[x])^4,x]

[Out] -1/12*(Csc[x]^3*(-8*a^3*b + 16*a*b^3 + 6*b^2*(3*a^2 + b^2)*Cos[x] + 24*a^3*b*Cos[2*x] + 4*a^4*Cos[3*x] + 6*a^2*b^2*Cos[3*x] - 2*b^4*Cos[3*x] - 9*a^4*x*Sin[x] + 3*a^4*x*Sin[3*x]))

fricas [A] time = 0.95, size = 95, normalized size = 0.94

$$\frac{12 a^3 b \cos(x)^2 - 8 a^3 b + 4 a b^3 + 2 (2 a^4 + 3 a^2 b^2 - b^4) \cos(x)^3 - 3 (a^4 - b^4) \cos(x) + 3 (a^4 x \cos(x)^2 - a^4 x) \sin(x)}{3 (\cos(x)^2 - 1) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cot(x)+b*csc(x))^4,x, algorithm="fricas")

[Out] 1/3*(12*a^3*b*cos(x)^2 - 8*a^3*b + 4*a*b^3 + 2*(2*a^4 + 3*a^2*b^2 - b^4)*cos(x)^3 - 3*(a^4 - b^4)*cos(x) + 3*(a^4*x*cos(x)^2 - a^4*x)*sin(x))/((cos(x)^2 - 1)*sin(x))

giac [B] time = 0.16, size = 215, normalized size = 2.13

$$\frac{1}{24} a^4 \tan\left(\frac{1}{2} x\right)^3 - \frac{1}{6} a^3 b \tan\left(\frac{1}{2} x\right)^3 + \frac{1}{4} a^2 b^2 \tan\left(\frac{1}{2} x\right)^3 - \frac{1}{6} a b^3 \tan\left(\frac{1}{2} x\right)^3 + \frac{1}{24} b^4 \tan\left(\frac{1}{2} x\right)^3 + a^4 x - \frac{5}{8} a^4 \tan\left(\frac{1}{2} x\right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cot(x)+b*csc(x))^4,x, algorithm="giac")

[Out] 1/24*a^4*tan(1/2*x)^3 - 1/6*a^3*b*tan(1/2*x)^3 + 1/4*a^2*b^2*tan(1/2*x)^3 - 1/6*a*b^3*tan(1/2*x)^3 + 1/24*b^4*tan(1/2*x)^3 + a^4*x - 5/8*a^4*tan(1/2*x) + 3/2*a^3*b*tan(1/2*x) - 3/4*a^2*b^2*tan(1/2*x) - 1/2*a*b^3*tan(1/2*x) + 3/8*b^4*tan(1/2*x) + 1/24*(15*a^4*tan(1/2*x)^2 + 36*a^3*b*tan(1/2*x)^2 + 18*a^2*b^2*tan(1/2*x)^2 - 12*a*b^3*tan(1/2*x)^2 - 9*b^4*tan(1/2*x)^2 - a^4 - 4*a^3*b - 6*a^2*b^2 - 4*a*b^3 - b^4)/tan(1/2*x)^3

maple [A] time = 0.07, size = 93, normalized size = 0.92

$$a^4 \left(-\frac{\cot^3(x)}{3} + \cot(x) + x \right) + 4a^3 b \left(-\frac{\cos^4(x)}{3 \sin(x)^3} + \frac{\cos^4(x)}{3 \sin(x)} + \frac{(2 + \cos^2(x)) \sin(x)}{3} \right) - \frac{2a^2 b^2 (\cos^3(x))}{\sin(x)^3} - \frac{4a b^3}{3 \sin(x)^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cot(x)+b*csc(x))^4,x)

[Out] $a^4*(-1/3*\cot(x)^3+\cot(x)+x)+4*a^3*b*(-1/3/\sin(x)^3*\cos(x)^4+1/3/\sin(x)*\cos(x)^4+1/3*(2+\cos(x)^2)*\sin(x))-2*a^2*b^2/\sin(x)^3*\cos(x)^3-4/3*a*b^3/\sin(x)^3+b^4*(-2/3-1/3*csc(x)^2)*\cot(x)$

maxima [A] time = 0.41, size = 80, normalized size = 0.79

$$-2a^2b^2 \cot(x)^3 + \frac{1}{3}a^4 \left(3x + \frac{3 \tan(x)^2 - 1}{\tan(x)^3} \right) + \frac{4(3 \sin(x)^2 - 1)a^3b}{3 \sin(x)^3} - \frac{(3 \tan(x)^2 + 1)b^4}{3 \tan(x)^3} - \frac{4ab^3}{3 \sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cot(x)+b*csc(x))^4,x, algorithm="maxima")

[Out] $-2*a^2*b^2*\cot(x)^3 + 1/3*a^4*(3*x + (3*\tan(x)^2 - 1)/\tan(x)^3) + 4/3*(3*\sin(x)^2 - 1)*a^3*b/\sin(x)^3 - 1/3*(3*\tan(x)^2 + 1)*b^4/\tan(x)^3 - 4/3*a*b^3/\sin(x)^3$

mupad [B] time = 2.53, size = 127, normalized size = 1.26

$$a^4x - \frac{\frac{4ab^3}{3} + \frac{4a^3b}{3} - \tan\left(\frac{x}{2}\right)^2 (5a^4 + 12a^3b + 6a^2b^2 - 4ab^3 - 3b^4) + \frac{a^4}{3} + \frac{b^4}{3} + 2a^2b^2}{8 \tan\left(\frac{x}{2}\right)^3} - \tan\left(\frac{x}{2}\right) \left(\frac{(a+b)(a-b)}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b/sin(x) + a*cot(x))^4,x)

[Out] $a^4*x - ((4*a*b^3)/3 + (4*a^3*b)/3 - \tan(x/2)^2*(12*a^3*b - 4*a*b^3 + 5*a^4 - 3*b^4 + 6*a^2*b^2) + a^4/3 + b^4/3 + 2*a^2*b^2)/(8*\tan(x/2)^3) - \tan(x/2)*(((a+b)*(a-b)^3)/2 + (a-b)^4/8) + (\tan(x/2)^3*(a-b)^4)/24$

sympy [A] time = 33.02, size = 97, normalized size = 0.96

$$a^4x + \frac{a^4 \cos(x)}{\sin(x)} - \frac{a^4 \cos^3(x)}{3 \sin^3(x)} - \frac{4a^3b \csc^3(x)}{3} + 4a^3b \csc(x) - 2a^2b^2 \cot^3(x) - \frac{4ab^3 \csc^3(x)}{3} - \frac{b^4 \cot^3(x)}{3} - b^4 \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cot(x)+b*csc(x))**4,x)

[Out] $a**4*x + a**4*\cos(x)/\sin(x) - a**4*\cos(x)**3/(3*\sin(x)**3) - 4*a**3*b*csc(x)**3/3 + 4*a**3*b*csc(x) - 2*a**2*b**2*cot(x)**3 - 4*a*b**3*csc(x)**3/3 - b**4*cot(x)**3/3 - b**4*cot(x)$

3.285 $\int (a \cot(x) + b \csc(x))^3 dx$

Optimal. Leaf size=77

$$-\frac{1}{2}a^2b \cos(x) - \frac{1}{4}(2a-b)(a+b)^2 \log(1-\cos(x)) - \frac{1}{4}(a-b)^2(2a+b) \log(\cos(x)+1) - \frac{1}{2} \csc^2(x)(a \cos(x)+b)^2(a+b \cos(x))$$

[Out] $-1/2*a^2*b*\cos(x) - 1/2*(b+a*\cos(x))^2*(a+b*\cos(x))*\csc(x)^2 - 1/4*(2*a-b)*(a+b)^2*\ln(1-\cos(x)) - 1/4*(a-b)^2*(2*a+b)*\ln(1+\cos(x))$

Rubi [A] time = 0.14, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {4392, 2668, 739, 774, 633, 31}

$$-\frac{1}{2}a^2b \cos(x) - \frac{1}{4}(2a-b)(a+b)^2 \log(1-\cos(x)) - \frac{1}{4}(a-b)^2(2a+b) \log(\cos(x)+1) - \frac{1}{2} \csc^2(x)(a \cos(x)+b)^2(a+b \cos(x))$$

Antiderivative was successfully verified.

[In] Int[(a*Cot[x] + b*Csc[x])^3, x]

[Out] $-(a^2*b*\cos[x])/2 - ((b + a*\cos[x])^2*(a + b*\cos[x])*Csc[x]^2)/2 - ((2*a - b)*(a + b)^2*\log[1 - \cos[x]])/4 - ((a - b)^2*(2*a + b)*\log[1 + \cos[x]])/4$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 633

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 739

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 774

Int[(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]

Rule 2668

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

Rule 4392

Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b_.))^(p_)*(u_.), x_Symbol] := Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a

*Cos[c + d*x]^n]^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned}
 \int (a \cot(x) + b \csc(x))^3 dx &= \int (b + a \cos(x))^3 \csc^3(x) dx \\
 &= - \left(a^3 \operatorname{Subst} \left(\int \frac{(b+x)^3}{(a^2-x^2)^2} dx, x, a \cos(x) \right) \right) \\
 &= -\frac{1}{2} (b + a \cos(x))^2 (a + b \cos(x)) \csc^2(x) + \frac{1}{2} a \operatorname{Subst} \left(\int \frac{(b+x)(2a^2 - b^2 + bx)}{a^2 - x^2} dx, x, a \cos(x) \right) \\
 &= -\frac{1}{2} a^2 b \cos(x) - \frac{1}{2} (b + a \cos(x))^2 (a + b \cos(x)) \csc^2(x) - \frac{1}{2} a \operatorname{Subst} \left(\int \frac{-a^2 b - b^2 x}{a^2 - x^2} dx, x, a \cos(x) \right) \\
 &= -\frac{1}{2} a^2 b \cos(x) - \frac{1}{2} (b + a \cos(x))^2 (a + b \cos(x)) \csc^2(x) + \frac{1}{4} ((2a - b)(a + b)^2) \operatorname{Subst} \left(\int \frac{1}{a^2 - x^2} dx, x, a \cos(x) \right) \\
 &= -\frac{1}{2} a^2 b \cos(x) - \frac{1}{2} (b + a \cos(x))^2 (a + b \cos(x)) \csc^2(x) - \frac{1}{4} (2a - b)(a + b)^2 \log \left(\frac{a + \cos(x)}{a - \cos(x)} \right)
 \end{aligned}$$

Mathematica [A] time = 0.29, size = 79, normalized size = 1.03

$$\frac{1}{8} \left(-(a+b)^3 \csc^2\left(\frac{x}{2}\right) + (a-b)^3 \left(-\sec^2\left(\frac{x}{2}\right) \right) - 4(2a-b)(a+b)^2 \log\left(\sin\left(\frac{x}{2}\right)\right) - 4(2a+b)(a-b)^2 \log\left(\cos\left(\frac{x}{2}\right)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cot[x] + b*Csc[x])^3,x]

[Out] (-(a + b)^3 Csc[x/2]^2) - 4*(a - b)^2*(2*a + b)*Log[Cos[x/2]] - 4*(2*a - b)*(a + b)^2*Log[Sin[x/2]] - (a - b)^3*Sec[x/2]^2)/8

fricas [A] time = 0.97, size = 128, normalized size = 1.66

$$\frac{2a^3 + 6ab^2 + 2(3a^2b + b^3)\cos(x) + (2a^3 - 3a^2b + b^3 - (2a^3 - 3a^2b + b^3)\cos(x)^2)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + (2a^3 - 3a^2b + b^3)\cos(x)^2\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)}{4(\cos(x)^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cot(x)+b*csc(x))^3,x, algorithm="fricas")

[Out] 1/4*(2*a^3 + 6*a*b^2 + 2*(3*a^2*b + b^3)*cos(x) + (2*a^3 - 3*a^2*b + b^3 - (2*a^3 - 3*a^2*b + b^3)*cos(x)^2)*log(1/2*cos(x) + 1/2) + (2*a^3 + 3*a^2*b - b^3 - (2*a^3 + 3*a^2*b - b^3)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(cos(x)^2 - 1)

giac [A] time = 0.15, size = 86, normalized size = 1.12

$$-\frac{1}{4} (2a^3 - 3a^2b + b^3) \log(\cos(x) + 1) - \frac{1}{4} (2a^3 + 3a^2b - b^3) \log(-\cos(x) + 1) + \frac{a^3 + 3ab^2 + (3a^2b + b^3)\cos(x)}{2(\cos(x) + 1)(\cos(x) - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cot(x)+b*csc(x))^3,x, algorithm="giac")

[Out] -1/4*(2*a^3 - 3*a^2*b + b^3)*log(cos(x) + 1) - 1/4*(2*a^3 + 3*a^2*b - b^3)*log(-cos(x) + 1) + 1/2*(a^3 + 3*a*b^2 + (3*a^2*b + b^3)*cos(x))/(cos(x) + 1)*(cos(x) - 1)

maple [A] time = 0.07, size = 87, normalized size = 1.13

$$-\frac{a^3 \cot^2(x)}{2} - a^3 \ln(\sin(x)) - \frac{3a^2b \cos^3(x)}{2 \sin(x)^2} - \frac{3a^2b \cos(x)}{2} - \frac{3a^2b \ln(\csc(x) - \cot(x))}{2} - \frac{3ab^2}{2 \sin(x)^2} - \frac{b^3 \csc(x) \cot(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cot(x)+b*csc(x))^3,x)

[Out] $-\frac{1}{2}a^3 \cot(x)^2 - a^3 \ln(\sin(x)) - \frac{3}{2}a^2b/\sin(x)^2 \cos(x)^3 - \frac{3}{2}a^2b \cos(x) - \frac{3}{2}a^2b \ln(\csc(x) - \cot(x)) - \frac{3}{2}a^2b^2/\sin(x)^2 - \frac{1}{2}b^3 \csc(x) \cot(x) + \frac{1}{2}b^3 \ln(\csc(x) - \cot(x))$

maxima [A] time = 0.31, size = 87, normalized size = 1.13

$$-\frac{3}{2}ab^2 \cot(x)^2 + \frac{3}{4}a^2b \left(\frac{2 \cos(x)}{\cos(x)^2 - 1} + \log(\cos(x) + 1) - \log(\cos(x) - 1) \right) + \frac{1}{4}b^3 \left(\frac{2 \cos(x)}{\cos(x)^2 - 1} - \log(\cos(x) + 1) + \log(\cos(x) - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cot(x)+b*csc(x))^3,x, algorithm="maxima")

[Out] $-\frac{3}{2}a^2b^2 \cot(x)^2 + \frac{3}{4}a^2b \left(\frac{2 \cos(x)}{\cos(x)^2 - 1} + \log(\cos(x) + 1) - \log(\cos(x) - 1) \right) + \frac{1}{4}b^3 \left(\frac{2 \cos(x)}{\cos(x)^2 - 1} - \log(\cos(x) + 1) + \log(\cos(x) - 1) \right) - \frac{1}{2}a^3 \left(\frac{1}{\sin(x)^2} + \log(\sin(x)^2) \right)$

mupad [B] time = 2.45, size = 82, normalized size = 1.06

$$a^3 \ln \left(\tan \left(\frac{x}{2} \right)^2 + 1 \right) - \frac{\frac{a^3}{8} + \frac{3a^2b}{8} + \frac{3ab^2}{8} + \frac{b^3}{8}}{\tan \left(\frac{x}{2} \right)^2} - \ln \left(\tan \left(\frac{x}{2} \right) \right) \left(a^3 + \frac{3a^2b}{2} - \frac{b^3}{2} \right) - \frac{\tan \left(\frac{x}{2} \right)^2 (a-b)^3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b/sin(x) + a*cot(x))^3,x)

[Out] $\frac{a^3 \log(\tan(x/2)^2 + 1) - ((3a^2b)/8 + (3a^2b)/8 + a^3/8 + b^3/8)/\tan(x/2)^2 - \log(\tan(x/2)) * ((3a^2b)/2 + a^3 - b^3/2) - (\tan(x/2)^2 * (a-b)^3)/8}{8}$

sympy [A] time = 14.11, size = 124, normalized size = 1.61

$$\frac{a^3 \log(-\csc^2(x))}{2} - \frac{a^3 \csc^2(x)}{2} - \frac{3a^2b \log(\cos(x) - 1)}{4} + \frac{3a^2b \log(\cos(x) + 1)}{4} + \frac{3a^2b \cos(x)}{2 \cos^2(x) - 2} - \frac{3ab^2 \csc^2(x)}{2} + \frac{b^3 \log(\cos(x) - 1)}{4} - \frac{b^3 \log(\cos(x) + 1)}{4} + \frac{b^3 \cos(x)}{2 \cos^2(x) - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cot(x)+b*csc(x))**3,x)

[Out] $a^3 \log(-\csc(x)^2)/2 - a^3 \csc(x)^2/2 - \frac{3a^2b \log(\cos(x) - 1)}{4} + \frac{3a^2b \log(\cos(x) + 1)}{4} + \frac{3a^2b \cos(x)}{2 \cos^2(x) - 2} - \frac{3ab^2 \csc^2(x)}{2} + \frac{b^3 \log(\cos(x) - 1)}{4} - \frac{b^3 \log(\cos(x) + 1)}{4} + \frac{b^3 \cos(x)}{2 \cos^2(x) - 2}$

3.286 $\int (a \cot(x) + b \csc(x))^2 dx$

Optimal. Leaf size=29

$$a^2(-x) - ab \sin(x) - \csc(x)(a \cos(x) + b)(a + b \cos(x))$$

[Out] $-a^2*x - (b+a*\cos(x))*(a+b*\cos(x))*\csc(x) - a*b*\sin(x)$

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4392, 2691, 2637}

$$a^2(-x) - ab \sin(x) - \csc(x)(a \cos(x) + b)(a + b \cos(x))$$

Antiderivative was successfully verified.

[In] Int[(a*Cot[x] + b*Csc[x])^2,x]

[Out] $-(a^2*x) - (b + a*\cos[x])*(a + b*\cos[x])*Csc[x] - a*b*\sin[x]$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2691

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)]^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := -Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*(b + a*Sin[e + f*x]))/(f*g*(p + 1)), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p] || IntegerQ[m])

Rule 4392

Int[(cot[(c_.) + (d_.)*(x_)])^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)])^(n_.)*(b_.)]^(p_.)*(u_.), x_Symbol] := Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned} \int (a \cot(x) + b \csc(x))^2 dx &= \int (b + a \cos(x))^2 \csc^2(x) dx \\ &= -(b + a \cos(x))(a + b \cos(x)) \csc(x) - \int (a^2 + ab \cos(x)) dx \\ &= -a^2x - (b + a \cos(x))(a + b \cos(x)) \csc(x) - (ab) \int \cos(x) dx \\ &= -a^2x - (b + a \cos(x))(a + b \cos(x)) \csc(x) - ab \sin(x) \end{aligned}$$

Mathematica [A] time = 0.13, size = 24, normalized size = 0.83

$$-\left((a^2 + b^2) \cot(x)\right) - a(ax + 2b \csc(x))$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cot[x] + b*Csc[x])^2,x]

[Out] $-\left((a^2 + b^2)\cot(x)\right) - a(a x + 2b\operatorname{Csc}[x])$

fricas [A] time = 0.92, size = 28, normalized size = 0.97

$$-\frac{a^2 x \sin(x) + 2ab + (a^2 + b^2)\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*cot(x)+b*csc(x))^2,x, algorithm="fricas")`

[Out] $-(a^2 x \sin(x) + 2ab + (a^2 + b^2)\cos(x))/\sin(x)$

giac [A] time = 0.15, size = 52, normalized size = 1.79

$$-a^2 x + \frac{1}{2} a^2 \tan\left(\frac{1}{2} x\right) - ab \tan\left(\frac{1}{2} x\right) + \frac{1}{2} b^2 \tan\left(\frac{1}{2} x\right) - \frac{a^2 + 2ab + b^2}{2 \tan\left(\frac{1}{2} x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*cot(x)+b*csc(x))^2,x, algorithm="giac")`

[Out] $-a^2 x + 1/2 a^2 \tan(1/2 x) - a b \tan(1/2 x) + 1/2 b^2 \tan(1/2 x) - 1/2 (a^2 + 2ab + b^2)/\tan(1/2 x)$

maple [A] time = 0.04, size = 29, normalized size = 1.00

$$a^2(-\cot(x) - x) - \frac{2ab}{\sin(x)} - b^2 \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*cot(x)+b*csc(x))^2,x)`

[Out] $a^2(-\cot(x) - x) - 2ab/\sin(x) - b^2 \cot(x)$

maxima [A] time = 0.42, size = 29, normalized size = 1.00

$$-a^2\left(x + \frac{1}{\tan(x)}\right) - \frac{2ab}{\sin(x)} - \frac{b^2}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*cot(x)+b*csc(x))^2,x, algorithm="maxima")`

[Out] $-a^2(x + 1/\tan(x)) - 2ab/\sin(x) - b^2/\tan(x)$

mupad [B] time = 2.42, size = 30, normalized size = 1.03

$$-\frac{\cos(x) a^2 + 2ab + \cos(x) b^2}{\sin(x)} - a^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b/sin(x) + a*cot(x))^2,x)`

[Out] $-(2ab + a^2 \cos(x) + b^2 \cos(x))/\sin(x) - a^2 x$

sympy [A] time = 2.79, size = 31, normalized size = 1.07

$$-a^2 x - \frac{a^2 \cos(x)}{\sin(x)} - 2ab \operatorname{csc}(x) - b^2 \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*cot(x)+b*csc(x))**2,x)`

[Out] $-a**2*x - a**2*cos(x)/sin(x) - 2*a*b*csc(x) - b**2*cot(x)$

3.287 $\int (a \cot(x) + b \csc(x)) dx$

Optimal. Leaf size=12

$$a \log(\sin(x)) - b \tanh^{-1}(\cos(x))$$

[Out] $-b \cdot \operatorname{arctanh}(\cos(x)) + a \cdot \ln(\sin(x))$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3475, 3770}

$$a \log(\sin(x)) - b \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[a \cdot \operatorname{Cot}[x] + b \cdot \operatorname{Csc}[x], x]$

[Out] $-(b \cdot \operatorname{ArcTanh}[\operatorname{Cos}[x]]) + a \cdot \operatorname{Log}[\operatorname{Sin}[x]]$

Rule 3475

$\operatorname{Int}[\tan[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[\operatorname{Cos}[c + d * x], x]]/d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 3770

$\operatorname{Int}[\csc[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d * x]]/d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int (a \cot(x) + b \csc(x)) dx &= a \int \cot(x) dx + b \int \csc(x) dx \\ &= -b \tanh^{-1}(\cos(x)) + a \log(\sin(x)) \end{aligned}$$

Mathematica [B] time = 0.01, size = 25, normalized size = 2.08

$$a \log(\sin(x)) + b \log\left(\sin\left(\frac{x}{2}\right)\right) - b \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Integrate}[a \cdot \operatorname{Cot}[x] + b \cdot \operatorname{Csc}[x], x]$

[Out] $-(b \cdot \operatorname{Log}[\operatorname{Cos}[x/2]]) + b \cdot \operatorname{Log}[\operatorname{Sin}[x/2]] + a \cdot \operatorname{Log}[\operatorname{Sin}[x]]$

fricas [B] time = 0.92, size = 27, normalized size = 2.25

$$\frac{1}{2}(a - b) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + \frac{1}{2}(a + b) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(a \cdot \cot(x) + b \cdot \csc(x), x, \operatorname{algorithm}="fricas")$

[Out] $1/2 * (a - b) * \log(1/2 * \cos(x) + 1/2) + 1/2 * (a + b) * \log(-1/2 * \cos(x) + 1/2)$

giac [A] time = 0.14, size = 15, normalized size = 1.25

$$a \log(|\sin(x)|) + b \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*cot(x)+b*csc(x),x, algorithm="giac")

[Out] a*log(abs(sin(x))) + b*log(abs(tan(1/2*x)))

maple [A] time = 0.00, size = 16, normalized size = 1.33

$$a \ln(\sin(x)) - b \ln(\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a*cot(x)+b*csc(x),x)

[Out] a*ln(sin(x))-b*ln(cot(x)+csc(x))

maxima [A] time = 0.31, size = 15, normalized size = 1.25

$$-b \log(\cot(x) + \csc(x)) + a \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*cot(x)+b*csc(x),x, algorithm="maxima")

[Out] -b*log(cot(x) + csc(x)) + a*log(sin(x))

mupad [B] time = 2.41, size = 27, normalized size = 2.25

$$a \ln\left(\tan\left(\frac{x}{2}\right)\right) - a \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) + b \ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(b/sin(x) + a*cot(x),x)

[Out] a*log(tan(x/2)) - a*log(tan(x/2)^2 + 1) + b*log(tan(x/2))

sympy [A] time = 0.10, size = 24, normalized size = 2.00

$$a \log(\sin(x)) + b \left(\frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*cot(x)+b*csc(x),x)

[Out] a*log(sin(x)) + b*(log(cos(x) - 1)/2 - log(cos(x) + 1)/2)

$$3.288 \quad \int \frac{1}{a \cot(x) + b \csc(x)} dx$$

Optimal. Leaf size=12

$$\frac{\log(a \cos(x) + b)}{a}$$

[Out] $-\ln(b+a*\cos(x))/a$

Rubi [A] time = 0.04, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3160, 2668, 31}

$$\frac{\log(a \cos(x) + b)}{a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*\text{Cot}[x] + b*\text{Csc}[x])^{-1}, x]$

[Out] $-(\text{Log}[b + a*\text{Cos}[x]])/a$

Rule 31

$\text{Int}[(a_ + (b_)*(x_))^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 2668

$\text{Int}[\cos[(e_ + (f_)*(x_))]^{(p_)} * ((a_ + (b_)*\sin[(e_ + (f_)*(x_))])^{(m_)}), x_Symbol] \rightarrow \text{Dist}[1/(b^p*f), \text{Subst}[\text{Int}[(a + x)^m*(b^2 - x^2)^{(p-1)/2}], x], x, b*\text{Sin}[e + f*x], x] /; \text{FreeQ}[\{a, b, e, f, m\}, x] \&\& \text{IntegerQ}[(p-1)/2] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 3160

$\text{Int}[(a_ + \csc[(d_ + (e_)*(x_))]*(b_ + \cot[(d_ + (e_)*(x_))]*(c_))]^{-1}), x_Symbol] \rightarrow \text{Int}[\text{Sin}[d + e*x]/(b + a*\text{Sin}[d + e*x] + c*\text{Cos}[d + e*x]), x] /; \text{FreeQ}[\{a, b, c, d, e\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{1}{a \cot(x) + b \csc(x)} dx &= \int \frac{\sin(x)}{b + a \cos(x)} dx \\ &= -\frac{\text{Subst}\left(\int \frac{1}{b+x} dx, x, a \cos(x)\right)}{a} \\ &= -\frac{\log(b + a \cos(x))}{a} \end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 1.00

$$\frac{\log(a \cos(x) + b)}{a}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(a*\text{Cot}[x] + b*\text{Csc}[x])^{-1}, x]$

[Out] $-(\text{Log}[b + a*\text{Cos}[x]])/a$

fricas [A] time = 1.20, size = 12, normalized size = 1.00

$$\frac{\log(a \cos(x) + b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cot(x)+b*csc(x)),x, algorithm="fricas")

[Out] -log(a*cos(x) + b)/a

giac [A] time = 0.12, size = 13, normalized size = 1.08

$$\frac{\log(|a \cos(x) + b|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cot(x)+b*csc(x)),x, algorithm="giac")

[Out] -log(abs(a*cos(x) + b))/a

maple [A] time = 0.11, size = 13, normalized size = 1.08

$$\frac{\ln(b + a \cos(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cot(x)+b*csc(x)),x)

[Out] -ln(b+a*cos(x))/a

maxima [B] time = 0.40, size = 45, normalized size = 3.75

$$-\frac{\log\left(a + b - \frac{(a-b)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a} + \frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cot(x)+b*csc(x)),x, algorithm="maxima")

[Out] -log(a + b - (a - b)*sin(x)^2/(cos(x) + 1)^2)/a + log(sin(x)^2/(cos(x) + 1)^2 + 1)/a

mupad [B] time = 3.28, size = 36, normalized size = 3.00

$$\frac{\operatorname{atan}\left(\frac{a \sin\left(\frac{x}{2}\right)^2}{-1i a \sin\left(\frac{x}{2}\right)^2 + a 1i + b 1i}\right) 2i}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b/sin(x) + a*cot(x)),x)

[Out] (atan((a*sin(x/2)^2)/(a*1i + b*1i - a*sin(x/2)^2*1i))*2i)/a

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a \cot(x) + b \csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cot(x)+b*csc(x)),x)

[Out] Integral(1/(a*cot(x) + b*csc(x)), x)

$$3.289 \quad \int \frac{1}{(a \cot(x) + b \csc(x))^2} dx$$

Optimal. Leaf size=67

$$\frac{2b \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2 \sqrt{a-b} \sqrt{a+b}} - \frac{x}{a^2} + \frac{\sin(x)}{a(a \cos(x) + b)}$$

[Out] $-x/a^2 + \sin(x)/a/(b+a*\cos(x)) + 2*b*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*x)/(a+b)^{(1/2)})/a^2/(a-b)^{(1/2)/(a+b)^{(1/2)}$

Rubi [A] time = 0.12, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {4392, 2693, 2735, 2659, 208}

$$\frac{2b \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2 \sqrt{a-b} \sqrt{a+b}} - \frac{x}{a^2} + \frac{\sin(x)}{a(a \cos(x) + b)}$$

Antiderivative was successfully verified.

[In] Int[(a*Cot[x] + b*Csc[x])^(-2), x]

[Out] $-(x/a^2) + (2*b*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a-b]*\operatorname{Tan}[x/2])/\operatorname{Sqrt}[a+b]])/(a^2*\operatorname{Sqrt}[a-b]*\operatorname{Sqrt}[a+b]) + \operatorname{Sin}[x]/(a*(b+a*\operatorname{Cos}[x]))$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2693

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+1)), x] + Dist[(g^2*(p-1))/(b*(m+1)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && IntegersQ[2*m, 2*p]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 4392

Int[(cot[(c_.) + (d_.)*(x_)])^(n_)*(a_.) + csc[(c_.) + (d_.)*(x_)])^(n_)*(b_.)]^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a \cot(x) + b \csc(x))^2} dx &= \int \frac{\sin^2(x)}{(b + a \cos(x))^2} dx \\
&= \frac{\sin(x)}{a(b + a \cos(x))} - \frac{\int \frac{\cos(x)}{b + a \cos(x)} dx}{a} \\
&= -\frac{x}{a^2} + \frac{\sin(x)}{a(b + a \cos(x))} + \frac{b \int \frac{1}{b + a \cos(x)} dx}{a^2} \\
&= -\frac{x}{a^2} + \frac{\sin(x)}{a(b + a \cos(x))} + \frac{(2b) \text{Subst} \left(\int \frac{1}{a + b + (-a + b)x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{a^2} \\
&= -\frac{x}{a^2} + \frac{2b \tanh^{-1} \left(\frac{\sqrt{a-b} \tan\left(\frac{x}{2}\right)}{\sqrt{a+b}} \right)}{a^2 \sqrt{a-b} \sqrt{a+b}} + \frac{\sin(x)}{a(b + a \cos(x))}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 71, normalized size = 1.06

$$\frac{2b \tanh^{-1} \left(\frac{(b-a) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2}} \right)}{\sqrt{a^2 - b^2}} + \frac{-a \sin(x) + ax \cos(x) + bx}{a \cos(x) + b}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cot[x] + b*Csc[x])^(-2), x]

[Out] -(((2*b*ArcTanh[(-a + b)*Tan[x/2]]/Sqrt[a^2 - b^2]))/Sqrt[a^2 - b^2] + (b*x + a*x*Cos[x] - a*Sin[x])/(b + a*Cos[x]))/a^2

fricas [B] time = 1.05, size = 307, normalized size = 4.58

$$\left[\frac{2(a^3 - ab^2)x \cos(x) - (ab \cos(x) + b^2)\sqrt{a^2 - b^2} \log\left(\frac{2ab \cos(x) - (a^2 - 2b^2)\cos(x)^2 + 2\sqrt{a^2 - b^2}(b \cos(x) + a)\sin(x) + 2a^2 - b^2}{a^2 \cos(x)^2 + 2ab \cos(x) + b^2}\right) + 2}{2(a^4b - a^2b^3 + (a^5 - a^3b^2)\cos(x))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cot(x)+b*csc(x))^2,x, algorithm="fricas")

[Out] [-1/2*(2*(a^3 - a*b^2)*x*cos(x) - (a*b*cos(x) + b^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(x) - (a^2 - 2*b^2)*cos(x)^2 + 2*sqrt(a^2 - b^2)*(b*cos(x) + a)*sin(x) + 2*a^2 - b^2)/(a^2*cos(x)^2 + 2*a*b*cos(x) + b^2)) + 2*(a^2*b - b^3)*x - 2*(a^3 - a*b^2)*sin(x))/(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*cos(x)), -((a^3 - a*b^2)*x*cos(x) - (a*b*cos(x) + b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(x) + a)/((a^2 - b^2)*sin(x)))) + (a^2*b - b^3)*x - (a^3 - a*b^2)*sin(x))/(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*cos(x))]

giac [A] time = 0.14, size = 107, normalized size = 1.60

$$\frac{2 \left(\pi \left[\frac{x}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a + 2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2}x\right) - b \tan\left(\frac{1}{2}x\right)}{\sqrt{-a^2 + b^2}} \right) \right) b}{\sqrt{-a^2 + b^2} a^2} - \frac{x}{a^2} - \frac{2 \tan\left(\frac{1}{2}x\right)}{\left(a \tan\left(\frac{1}{2}x\right)^2 - b \tan\left(\frac{1}{2}x\right)^2 - a - b \right) a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^2,x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x))/sqrt(-a^2 + b^2)))*b/(sqrt(-a^2 + b^2)*a^2) - x/a^2 - 2*tan(1/2*x)/((a*tan(1/2*x)^2 - b*tan(1/2*x)^2 - a - b)*a)
```

maple [A] time = 0.12, size = 86, normalized size = 1.28

$$-\frac{2 \tan\left(\frac{x}{2}\right)}{a\left(a\left(\tan^2\left(\frac{x}{2}\right)\right) - b\left(\tan^2\left(\frac{x}{2}\right)\right) - a - b\right)} + \frac{2b \operatorname{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{a^2\sqrt{(a+b)(a-b)}} - \frac{2 \operatorname{arctan}\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)+b*csc(x))^2,x)
```

```
[Out] -2/a*tan(1/2*x)/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)+2/a^2*b/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*x)*(a-b)/((a+b)*(a-b))^(1/2))-2/a^2*arctan(tan(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.04, size = 440, normalized size = 6.57

$$a^3 \sin(x) + b^2 \left(-a \sin(x) + \operatorname{atan} \left(\frac{-a^5 \sin\left(\frac{x}{2}\right) \sqrt{a^2-b^2} \operatorname{Im} + b^3 \sin\left(\frac{x}{2}\right) (a^2-b^2)^{3/2} \operatorname{Im} + b^5 \sin\left(\frac{x}{2}\right) \sqrt{a^2-b^2} \operatorname{Im} + a^4 b \sin\left(\frac{x}{2}\right) \sqrt{a^2-b^2} \operatorname{Im} - a^2 b^3}{\cos\left(\frac{x}{2}\right) a^6 - 2 \cos\left(\frac{x}{2}\right) a^4 b^2 + \cos\left(\frac{x}{2}\right) a^2 b^4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sin(x) + a*cot(x))^2,x)
```

```
[Out] (a^3*sin(x) + b^2*(atan((b^3*sin(x/2)*(a^2 - b^2)^(3/2)*2i - a^5*sin(x/2)*(a^2 - b^2)^(1/2)*1i + b^5*sin(x/2)*(a^2 - b^2)^(1/2)*2i + a^4*b*sin(x/2)*(a^2 - b^2)^(1/2)*1i - a^2*b^3*sin(x/2)*(a^2 - b^2)^(1/2)*3i + a^3*b^2*sin(x/2)*(a^2 - b^2)^(1/2)*1i)/(a^6*cos(x/2) + a^2*b^4*cos(x/2) - 2*a^4*b^2*cos(x/2))))*(a^2 - b^2)^(1/2)*2i - a*sin(x)) + a*b*atan((b^3*sin(x/2)*(a^2 - b^2)^(3/2)*2i - a^5*sin(x/2)*(a^2 - b^2)^(1/2)*1i + b^5*sin(x/2)*(a^2 - b^2)^(1/2)*2i + a^4*b*sin(x/2)*(a^2 - b^2)^(1/2)*1i - a^2*b^3*sin(x/2)*(a^2 - b^2)^(1/2)*3i + a^3*b^2*sin(x/2)*(a^2 - b^2)^(1/2)*1i)/(a^6*cos(x/2) + a^2*b^4*cos(x/2) - 2*a^4*b^2*cos(x/2)))*cos(x)*(a^2 - b^2)^(1/2)*2i)/(a^4*b - a^2*b^3 + a^5*cos(x) - a^3*b^2*cos(x)) - (2*atan(sin(x/2)/cos(x/2)))/a^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cot(x) + b \csc(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))**2,x)
```

```
[Out] Integral((a*cot(x) + b*csc(x))**(-2), x)
```

$$3.290 \quad \int \frac{1}{(a \cot(x) + b \csc(x))^3} dx$$

Optimal. Leaf size=50

$$\frac{2b}{a^3(a \cos(x) + b)} + \frac{\log(a \cos(x) + b)}{a^3} + \frac{a^2 - b^2}{2a^3(a \cos(x) + b)^2}$$

[Out] $1/2*(a^2-b^2)/a^3/(b+a*\cos(x))^2+2*b/a^3/(b+a*\cos(x))+\ln(b+a*\cos(x))/a^3$

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4392, 2668, 697}

$$\frac{a^2 - b^2}{2a^3(a \cos(x) + b)^2} + \frac{2b}{a^3(a \cos(x) + b)} + \frac{\log(a \cos(x) + b)}{a^3}$$

Antiderivative was successfully verified.

[In] Int[(a*Cot[x] + b*Csc[x])^(-3), x]

[Out] $(a^2 - b^2)/(2*a^3*(b + a*\cos[x])^2) + (2*b)/(a^3*(b + a*\cos[x])) + \text{Log}[b + a*\cos[x]]/a^3$

Rule 697

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rule 2668

Int[cos[(e_) + (f_)*(x_)]^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

Rule 4392

Int[(cot[(c_) + (d_)*(x_)]^(n_)*(a_) + csc[(c_) + (d_)*(x_)]^(n_)*(b_))^(p_)*(u_), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a \cot(x) + b \csc(x))^3} dx &= \int \frac{\sin^3(x)}{(b + a \cos(x))^3} dx \\ &= -\frac{\text{Subst}\left(\int \frac{a^2 - x^2}{(b+x)^3} dx, x, a \cos(x)\right)}{a^3} \\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{-b-x} + \frac{a^2 - b^2}{(b+x)^3} + \frac{2b}{(b+x)^2}\right) dx, x, a \cos(x)\right)}{a^3} \\ &= \frac{a^2 - b^2}{2a^3(b + a \cos(x))^2} + \frac{2b}{a^3(b + a \cos(x))} + \frac{\log(b + a \cos(x))}{a^3} \end{aligned}$$

Mathematica [A] time = 0.11, size = 77, normalized size = 1.54

$$\frac{a^2 \cos(2x) \log(a \cos(x) + b) + a^2 \log(a \cos(x) + b) + a^2 + 2b^2 \log(a \cos(x) + b) + 4ab \cos(x)(\log(a \cos(x) + b) + \frac{1}{a \cos(x)})}{2a^3(a \cos(x) + b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cot[x] + b*Csc[x])^(-3), x]

[Out] (a^2 + 3*b^2 + a^2*Log[b + a*Cos[x]] + 2*b^2*Log[b + a*Cos[x]] + a^2*Cos[2*x]*Log[b + a*Cos[x]] + 4*a*b*Cos[x]*(1 + Log[b + a*Cos[x]]))/(2*a^3*(b + a*Cos[x])^2)

fricas [A] time = 0.84, size = 70, normalized size = 1.40

$$\frac{4 ab \cos(x) + a^2 + 3 b^2 + 2 (a^2 \cos(x)^2 + 2 ab \cos(x) + b^2) \log(a \cos(x) + b)}{2 (a^5 \cos(x)^2 + 2 a^4 b \cos(x) + a^3 b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cot(x)+b*csc(x))^3,x, algorithm="fricas")

[Out] 1/2*(4*a*b*cos(x) + a^2 + 3*b^2 + 2*(a^2*cos(x)^2 + 2*a*b*cos(x) + b^2)*log(a*cos(x) + b))/(a^5*cos(x)^2 + 2*a^4*b*cos(x) + a^3*b^2)

giac [A] time = 0.13, size = 45, normalized size = 0.90

$$\frac{\log(|a \cos(x) + b|)}{a^3} + \frac{4 b \cos(x) + \frac{a^2+3b^2}{a}}{2 (a \cos(x) + b)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cot(x)+b*csc(x))^3,x, algorithm="giac")

[Out] log(abs(a*cos(x) + b))/a^3 + 1/2*(4*b*cos(x) + (a^2 + 3*b^2)/a)/((a*cos(x) + b)^2*a^2)

maple [A] time = 0.12, size = 56, normalized size = 1.12

$$\frac{\ln(b + a \cos(x))}{a^3} + \frac{1}{2a(b + a \cos(x))^2} - \frac{b^2}{2a^3(b + a \cos(x))^2} + \frac{2b}{a^3(b + a \cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cot(x)+b*csc(x))^3,x)

[Out] ln(b+a*cos(x))/a^3+1/2/a/(b+a*cos(x))^2-1/2/a^3/(b+a*cos(x))^2*b^2+2*b/a^3/(b+a*cos(x))

maxima [B] time = 0.43, size = 177, normalized size = 3.54

$$\frac{2 \left(ab + b^2 + \frac{(a^2 - 2ab + b^2) \sin(x)^2}{(\cos(x) + 1)^2} \right)}{a^5 + a^4 b - a^3 b^2 - a^2 b^3 - \frac{2(a^5 - a^4 b - a^3 b^2 + a^2 b^3) \sin(x)^2}{(\cos(x) + 1)^2} + \frac{(a^5 - 3a^4 b + 3a^3 b^2 - a^2 b^3) \sin(x)^4}{(\cos(x) + 1)^4}} + \frac{\log\left(a + b - \frac{(a-b) \sin(x)^2}{(\cos(x) + 1)^2}\right)}{a^3} - \frac{\log\left(\frac{a-b}{\cos(x) + 1}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cot(x)+b*csc(x))^3,x, algorithm="maxima")

[Out] 2*(a*b + b^2 + (a^2 - 2*a*b + b^2)*sin(x)^2/(cos(x) + 1)^2)/(a^5 + a^4*b - a^3*b^2 - a^2*b^3 - 2*(a^5 - a^4*b - a^3*b^2 + a^2*b^3)*sin(x)^2/(cos(x) + 1)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sin(x)^4/(cos(x) + 1)^4) + log(a + b - (a - b)*sin(x)^2/(cos(x) + 1)^2)/a^3 - log(sin(x)^2/(cos(x) + 1)^2 + 1)/a^3

mupad [B] time = 2.77, size = 311, normalized size = 6.22

$$\frac{\frac{2(b^2+ab)}{a^2(a-b)} + \frac{2 \tan\left(\frac{x}{2}\right)^2 (a-b)}{a^2}}{\tan\left(\frac{x}{2}\right)^4 (a^2 - 2ab + b^2) + 2ab - \tan\left(\frac{x}{2}\right)^2 (2a^2 - 2b^2) + a^2 + b^2} \cdot 2 \operatorname{atanh}\left(\frac{32 \tan\left(\frac{x}{2}\right)^2}{\frac{32b^3}{a^3} - \frac{32b^2}{a^2} - \frac{32b}{a} + \frac{32b \tan\left(\frac{x}{2}\right)^2}{a} - \frac{64b^2 \tan\left(\frac{x}{2}\right)^2}{a^2} + \dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b/sin(x) + a*cot(x))^3,x)`

[Out] $((2*(a*b + b^2))/(a^2*(a - b)) + (2*\tan(x/2)^2*(a - b)/a^2)/(\tan(x/2)^4*(a^2 - 2*a*b + b^2) + 2*a*b - \tan(x/2)^2*(2*a^2 - 2*b^2) + a^2 + b^2) - (2*\operatorname{atanh}((32*\tan(x/2)^2)/((32*b^3)/a^3 - (32*b^2)/a^2 - (32*b)/a + (32*b*\tan(x/2)^2)/a - (64*b^2*\tan(x/2)^2)/a^2 + (32*b^3*\tan(x/2)^2)/a^3 + 32) - (64*b*\tan(x/2)^2)/(32*a - 32*b + 32*b*\tan(x/2)^2 - (32*b^2)/a + (32*b^3)/a^2 - (64*b^2*\tan(x/2)^2)/a + (32*b^3*\tan(x/2)^2)/a^2) + (32*b^2*\tan(x/2)^2)/(32*a^2 - 32*a*b - 32*b^2 - 64*b^2*\tan(x/2)^2 + (32*b^3)/a + (32*b^3*\tan(x/2)^2)/a + 32*a*b*\tan(x/2)^2))/a^3$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cot(x) + b \csc(x))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*cot(x)+b*csc(x))**3,x)`

[Out] `Integral((a*cot(x) + b*csc(x))**(-3), x)`

$$3.291 \quad \int \frac{1}{(a \cot(x) + b \csc(x))^4} dx$$

Optimal. Leaf size=159

$$\frac{x}{a^4} + \frac{b \sin^3(x)}{2a(a^2 - b^2)(a \cos(x) + b)^2} - \frac{b(3a^2 - 2b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}} - \frac{\sin(x)(2(a^2 - b^2) - ab \cos(x))}{2a^3(a^2 - b^2)(a \cos(x) + b)} + \frac{\sin(x)}{3a(a \cos(x) + b)}$$

[Out] $x/a^4 - b*(3*a^2 - 2*b^2)*\operatorname{arctanh}((a-b)^{(1/2)}*\tan(1/2*x)/(a+b)^{(1/2)})/a^4/(a-b)^{(3/2)/(a+b)^{(3/2)} - 1/2*(2*a^2 - 2*b^2 - a*b*\cos(x))*\sin(x)/a^3/(a^2 - b^2)/(b+a*\cos(x)) + 1/3*\sin(x)^3/a/(b+a*\cos(x))^3 + 1/2*b*\sin(x)^3/a/(a^2 - b^2)/(b+a*\cos(x))^2$

Rubi [A] time = 0.34, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {4392, 2693, 2864, 2863, 2735, 2659, 208}

$$\frac{b \sin^3(x)}{2a(a^2 - b^2)(a \cos(x) + b)^2} - \frac{\sin(x)(2(a^2 - b^2) - ab \cos(x))}{2a^3(a^2 - b^2)(a \cos(x) + b)} - \frac{b(3a^2 - 2b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}} + \frac{x}{a^4} + \frac{\sin(x)}{3a(a \cos(x) + b)}$$

Antiderivative was successfully verified.

[In] Int[(a*Cot[x] + b*Csc[x])^(-4), x]

[Out] $x/a^4 - (b*(3*a^2 - 2*b^2)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a - b]*\operatorname{Tan}[x/2]/\operatorname{Sqrt}[a + b]])/(a^4*(a - b)^{(3/2)*(a + b)^{(3/2)}) - ((2*(a^2 - b^2) - a*b*\cos[x])*Sin[x])/(2*a^3*(a^2 - b^2)*(b + a*\cos[x])) + Sin[x]^3/(3*a*(b + a*\cos[x])^3) + (b*\sin[x]^3)/(2*a*(a^2 - b^2)*(b + a*\cos[x])^2)$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2693

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && IntegersQ[2*m, 2*p]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2863

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*p + b*d*(m + 1)*Sin[e + f*x]))/(b^2*f*(m + 1)*(m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(m + 1)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_.)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_.)]^(n_.)*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(a \cot(x) + b \csc(x))^4} dx &= \int \frac{\sin^4(x)}{(b + a \cos(x))^4} dx \\
 &= \frac{\sin^3(x)}{3a(b + a \cos(x))^3} - \frac{\int \frac{\cos(x) \sin^2(x)}{(b + a \cos(x))^3} dx}{a} \\
 &= \frac{\sin^3(x)}{3a(b + a \cos(x))^3} + \frac{b \sin^3(x)}{2a(a^2 - b^2)(b + a \cos(x))^2} - \frac{\int \frac{(2a + b \cos(x)) \sin^2(x)}{(b + a \cos(x))^2} dx}{2a(a^2 - b^2)} \\
 &= -\frac{(2(a^2 - b^2) - ab \cos(x)) \sin(x)}{2a^3(a^2 - b^2)(b + a \cos(x))} + \frac{\sin^3(x)}{3a(b + a \cos(x))^3} + \frac{b \sin^3(x)}{2a(a^2 - b^2)(b + a \cos(x))} \\
 &= \frac{x}{a^4} - \frac{(2(a^2 - b^2) - ab \cos(x)) \sin(x)}{2a^3(a^2 - b^2)(b + a \cos(x))} + \frac{\sin^3(x)}{3a(b + a \cos(x))^3} + \frac{b \sin^3(x)}{2a(a^2 - b^2)(b + a \cos(x))} \\
 &= \frac{x}{a^4} - \frac{(2(a^2 - b^2) - ab \cos(x)) \sin(x)}{2a^3(a^2 - b^2)(b + a \cos(x))} + \frac{\sin^3(x)}{3a(b + a \cos(x))^3} + \frac{b \sin^3(x)}{2a(a^2 - b^2)(b + a \cos(x))} \\
 &= \frac{x}{a^4} - \frac{b(3a^2 - 2b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}} - \frac{(2(a^2 - b^2) - ab \cos(x)) \sin(x)}{2a^3(a^2 - b^2)(b + a \cos(x))} + \frac{\sin^3(x)}{3a(b + a \cos(x))^3}
 \end{aligned}$$

Mathematica [A] time = 0.48, size = 150, normalized size = 0.94

$$\sin(x) \left(\frac{a(8a^2 - 11b^2)(a \cos(x) + b)^2}{(a-b)(a+b)} - \frac{6b(2b^2 - 3a^2) \csc(x)(a \cos(x) + b)^3 \tanh^{-1}\left(\frac{(b-a) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2}}\right)}{(a^2 - b^2)^{3/2}} + 2a(a^2 - b^2) + 7ab(a \cos(x) + b) + 6x \right)$$

$$6a^4(a \cos(x) + b)^3$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cot[x] + b*Csc[x])^(-4), x]

[Out] $((2*a*(a^2 - b^2) + 7*a*b*(b + a*\cos[x]) - (a*(8*a^2 - 11*b^2)*(b + a*\cos[x]))^2)/((a - b)*(a + b)) + 6*x*(b + a*\cos[x])^3*\text{Csc}[x] - (6*b*(-3*a^2 + 2*b^2)*\text{ArcTanh}[\frac{(-a + b)*\tan[x/2]}{\sqrt{a^2 - b^2}}]*(b + a*\cos[x])^3*\text{Csc}[x])/(a^2 - b^2)^{(3/2)}*\sin[x])/(6*a^4*(b + a*\cos[x])^3)$

fricas [B] time = 2.08, size = 878, normalized size = 5.52

$$\frac{12(a^7 - 2a^5b^2 + a^3b^4)x \cos(x)^3 + 36(a^6b - 2a^4b^3 + a^2b^5)x \cos(x)^2 + 36(a^5b^2 - 2a^3b^4 + ab^6)x \cos(x) + 3(\dots)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cot(x)+b*csc(x))^4,x, algorithm="fricas")

[Out] $[1/12*(12*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*\cos(x)^3 + 36*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x*\cos(x)^2 + 36*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*\cos(x) + 3*(3*a^2*b^4 - 2*b^6 + (3*a^5*b - 2*a^3*b^3)*\cos(x)^3 + 3*(3*a^4*b^2 - 2*a^2*b^4)*\cos(x)^2 + 3*(3*a^3*b^3 - 2*a*b^5)*\cos(x))*\sqrt{a^2 - b^2}*\log((2*a*b*\cos(x) - (a^2 - 2*b^2)*\cos(x)^2 - 2*\sqrt{a^2 - b^2}*(b*\cos(x) + a)*\sin(x) + 2*a^2 - b^2)/(a^2*\cos(x)^2 + 2*a*b*\cos(x) + b^2)) + 12*(a^4*b^3 - 2*a^2*b^5 + b^7)*x + 2*(2*a^7 - 7*a^5*b^2 + 11*a^3*b^4 - 6*a*b^6 - (8*a^7 - 19*a^5*b^2 + 11*a^3*b^4)*\cos(x)^2 - 3*(3*a^6*b - 8*a^4*b^3 + 5*a^2*b^5)*\cos(x))*\sin(x)]/(a^8*b^3 - 2*a^6*b^5 + a^4*b^7 + (a^11 - 2*a^9*b^2 + a^7*b^4)*\cos(x)^3 + 3*(a^10*b - 2*a^8*b^3 + a^6*b^5)*\cos(x)^2 + 3*(a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*\cos(x)), 1/6*(6*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*\cos(x)^3 + 18*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x*\cos(x)^2 + 18*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*\cos(x) - 3*(3*a^2*b^4 - 2*b^6 + (3*a^5*b - 2*a^3*b^3)*\cos(x)^3 + 3*(3*a^4*b^2 - 2*a^2*b^4)*\cos(x)^2 + 3*(3*a^3*b^3 - 2*a*b^5)*\cos(x))*\sqrt{-a^2 + b^2}*\arctan(-\sqrt{-a^2 + b^2}*(b*\cos(x) + a)/((a^2 - b^2)*\sin(x))) + 6*(a^4*b^3 - 2*a^2*b^5 + b^7)*x + (2*a^7 - 7*a^5*b^2 + 11*a^3*b^4 - 6*a*b^6 - (8*a^7 - 19*a^5*b^2 + 11*a^3*b^4)*\cos(x)^2 - 3*(3*a^6*b - 8*a^4*b^3 + 5*a^2*b^5)*\cos(x))*\sin(x)]/(a^8*b^3 - 2*a^6*b^5 + a^4*b^7 + (a^11 - 2*a^9*b^2 + a^7*b^4)*\cos(x)^3 + 3*(a^10*b - 2*a^8*b^3 + a^6*b^5)*\cos(x)^2 + 3*(a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*\cos(x))]$

giac [A] time = 0.20, size = 282, normalized size = 1.77

$$\frac{(3a^2b - 2b^3) \left(\pi \left[\frac{x}{2\pi} + \frac{1}{2} \right] \text{sgn}(-2a + 2b) + \arctan \left(-\frac{a \tan(\frac{1}{2}x) - b \tan(\frac{1}{2}x)}{\sqrt{-a^2 + b^2}} \right) \right)}{(a^6 - a^4b^2) \sqrt{-a^2 + b^2}} + \frac{6a^4 \tan\left(\frac{1}{2}x\right)^5 - 9a^3b \tan\left(\frac{1}{2}x\right)^5}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cot(x)+b*csc(x))^4,x, algorithm="giac")

[Out] $-(3*a^2*b - 2*b^3)*(pi*\text{floor}(1/2*x/pi + 1/2)*\text{sgn}(-2*a + 2*b) + \arctan(-(a*\tan(1/2*x) - b*\tan(1/2*x))/\sqrt{-a^2 + b^2}))/((a^6 - a^4*b^2)*\sqrt{-a^2 + b^2}) + 1/3*(6*a^4*\tan(1/2*x)^5 - 9*a^3*b*\tan(1/2*x)^5 - 6*a^2*b^2*\tan(1/2*x)^5 + 15*a*b^3*\tan(1/2*x)^5 - 6*b^4*\tan(1/2*x)^5 - 20*a^4*\tan(1/2*x)^3 + 32*a^2*b^2*\tan(1/2*x)^3 - 12*b^4*\tan(1/2*x)^3 + 6*a^4*\tan(1/2*x) + 9*a^3*b*\tan(1/2*x) - 6*a^2*b^2*\tan(1/2*x) - 15*a*b^3*\tan(1/2*x) - 6*b^4*\tan(1/2*x))/((a^5 - a^3*b^2)*(a*\tan(1/2*x)^2 - b*\tan(1/2*x)^2 - a - b)^3) + x/a^4$

maple [B] time = 0.14, size = 534, normalized size = 3.36

$$\frac{2 \left(\tan^5\left(\frac{x}{2}\right)\right)}{\left(a \left(\tan^2\left(\frac{x}{2}\right)\right) - b \left(\tan^2\left(\frac{x}{2}\right)\right) - a - b\right)^3 (a + b)} - \frac{\left(\tan^5\left(\frac{x}{2}\right)\right) b}{a \left(a \left(\tan^2\left(\frac{x}{2}\right)\right) - b \left(\tan^2\left(\frac{x}{2}\right)\right) - a - b\right)^3 (a + b)} - \frac{3 \left(a \left(\tan^2\left(\frac{x}{2}\right)\right) - b \left(\tan^2\left(\frac{x}{2}\right)\right) - a - b\right)^3}{a^2 \left(a \left(\tan^2\left(\frac{x}{2}\right)\right) - b \left(\tan^2\left(\frac{x}{2}\right)\right) - a - b\right)^3 (a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cot(x)+b*csc(x))^4,x)

[Out] 2/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a+b)*tan(1/2*x)^5-1/a/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a+b)*tan(1/2*x)^5*b-3/a^2/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a+b)*tan(1/2*x)^5*b^2+2/a^3/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a+b)*tan(1/2*x)^5*b^3-20/3/a/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3*tan(1/2*x)^3+4/a^3/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3*tan(1/2*x)^3*b^2+2/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a-b)*tan(1/2*x)+1/a/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a-b)*tan(1/2*x)*b-3/a^2/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a-b)*tan(1/2*x)*b^2-2/a^3/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3/(a-b)*tan(1/2*x)*b^3-3/a^2*b/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*x)*(a-b)/((a+b)*(a-b))^(1/2))+2/a^4*b^3/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*x)*(a-b)/((a+b)*(a-b))^(1/2))+2/a^4*arctan(tan(1/2*x))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cot(x)+b*csc(x))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 8.25, size = 3068, normalized size = 19.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b/sin(x) + a*cot(x))^4,x)

[Out] (2*atan(((((((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6*a^11*b^2)))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (tan(x/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)*8i)/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*1i)/a^4 + (8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2))/a^4 - (((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6*a^11*b^2)))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (tan(x/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)*8i)/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*1i)/a^4 - (8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2))/a^4)/((16*(6*a^4*b - 2*a*b^4 + 4*b^5 - 10*a^2*b^3 + 3*a^3*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6*a^11*b^2)))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (tan(x/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)*8i)/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*1i)/a^4 + (8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2))*1i)/a^4 + (((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6*a^11*b^2)))/(a

$$\begin{aligned}
& ^{11}b + a^{12} - a^9b^3 - a^{10}b^2) + (\tan(x/2)*(8a^{13}b - 8a^8b^6 + 8a^9b^5 + 16a^{10}b^4 - 16a^{11}b^3 - 8a^{12}b^2)*8i)/(a^4*(a^8b + a^9 - a^6b^3 - a^7b^2))*1i)/a^4 - (8*\tan(x/2)*(4a^6 - 8a^5b - 8a*b^5 + 8b^6 - 16a^2b^4 + 16a^3b^3 + 5a^4b^2))/(a^8b + a^9 - a^6b^3 - a^7b^2))*1i)/a^4)))/a^4 + ((4*\tan(x/2)^3*(5a^2 - 3b^2))/(3a^3) - (\tan(x/2)*(a + b)*(ab - 2a^2 + 2b^2))/(a^3b - a^4) + (\tan(x/2)^5*(3a*b^2 + a^2b - 2a^3 - 2b^3))/(a^3*(a + b)))/(3a*b^2 - \tan(x/2)^6*(3a*b^2 - 3a^2b + a^3 - b^3) + 3a^2b + \tan(x/2)^2*(3a*b^2 - 3a^2b - 3a^3 + 3b^3) - \tan(x/2)^4*(3a*b^2 + 3a^2b - 3a^3 - 3b^3) + a^3 + b^3) + (b*\operatorname{atan}(((b*(3a^2 - 2b^2))*((a + b)^3*(a - b)^3)^{(1/2))*((8*\tan(x/2)*(4a^6 - 8a^5b - 8a*b^5 + 8b^6 - 16a^2b^4 + 16a^3b^3 + 5a^4b^2))/(a^8b + a^9 - a^6b^3 - a^7b^2) + (b*((8*(6a^{12}b - 4a^{13} + 4a^8b^5 - 2a^9b^4 - 10a^{10}b^3 + 6a^{11}b^2)))/(a^{11}b + a^{12} - a^9b^3 - a^{10}b^2) - (4b*\tan(x/2)*(3a^2 - 2b^2))*((a + b)^3*(a - b)^3)^{(1/2))*((8a^{13}b - 8a^8b^6 + 8a^9b^5 + 16a^{10}b^4 - 16a^{11}b^3 - 8a^{12}b^2)))/((a^8b + a^9 - a^6b^3 - a^7b^2)*(a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2)))*((a + b)^3*(a - b)^3)^{(1/2)))/(2*(a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2))*1i)/(2*(a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2)) + (b*(3a^2 - 2b^2))*((a + b)^3*(a - b)^3)^{(1/2))*((8*\tan(x/2)*(4a^6 - 8a^5b - 8a*b^5 + 8b^6 - 16a^2b^4 + 16a^3b^3 + 5a^4b^2))/(a^8b + a^9 - a^6b^3 - a^7b^2) - (b*((8*(6a^{12}b - 4a^{13} + 4a^8b^5 - 2a^9b^4 - 10a^{10}b^3 + 6a^{11}b^2)))/(a^{11}b + a^{12} - a^9b^3 - a^{10}b^2) + (4b*\tan(x/2)*(3a^2 - 2b^2))*((a + b)^3*(a - b)^3)^{(1/2))*((8a^{13}b - 8a^8b^6 + 8a^9b^5 + 16a^{10}b^4 - 16a^{11}b^3 - 8a^{12}b^2)))/((a^8b + a^9 - a^6b^3 - a^7b^2)*(a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2)))*((a + b)^3*(a - b)^3)^{(1/2)))/(2*(a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2)))*1i)/(2*(a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2)))/((16*(6a^4b - 2a*b^4 + 4b^5 - 10a^2b^3 + 3a^3b^2))/(a^{11}b + a^{12} - a^9b^3 - a^{10}b^2) + (b*(3a^2 - 2b^2))*((a + b)^3*(a - b)^3)^{(1/2))*((8*\tan(x/2)*(4a^6 - 8a^5b - 8a*b^5 + 8b^6 - 16a^2b^4 + 16a^3b^3 + 5a^4b^2))/(a^8b + a^9 - a^6b^3 - a^7b^2) + (b*((8*(6a^{12}b - 4a^{13} + 4a^8b^5 - 2a^9b^4 - 10a^{10}b^3 + 6a^{11}b^2)))/(a^{11}b + a^{12} - a^9b^3 - a^{10}b^2) - (4b*\tan(x/2)*(3a^2 - 2b^2))*((a + b)^3*(a - b)^3)^{(1/2))*((8a^{13}b - 8a^8b^6 + 8a^9b^5 + 16a^{10}b^4 - 16a^{11}b^3 - 8a^{12}b^2)))/((a^8b + a^9 - a^6b^3 - a^7b^2)*(a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2)))*((a + b)^3*(a - b)^3)^{(1/2)))/(2*(a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2)) - (b*(3a^2 - 2b^2))*((a + b)^3*(a - b)^3)^{(1/2))*((8*\tan(x/2)*(4a^6 - 8a^5b - 8a*b^5 + 8b^6 - 16a^2b^4 + 16a^3b^3 + 5a^4b^2))/(a^8b + a^9 - a^6b^3 - a^7b^2) - (b*((8*(6a^{12}b - 4a^{13} + 4a^8b^5 - 2a^9b^4 - 10a^{10}b^3 + 6a^{11}b^2)))/(a^{11}b + a^{12} - a^9b^3 - a^{10}b^2) + (4b*\tan(x/2)*(3a^2 - 2b^2))*((a + b)^3*(a - b)^3)^{(1/2))*((8a^{13}b - 8a^8b^6 + 8a^9b^5 + 16a^{10}b^4 - 16a^{11}b^3 - 8a^{12}b^2)))/((a^8b + a^9 - a^6b^3 - a^7b^2)*(a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2)))*((a + b)^3*(a - b)^3)^{(1/2)))/(2*(a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2)))*1i)/(a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cot(x)+b*csc(x))**4,x)

[Out] Timed out

$$3.292 \quad \int \frac{1}{(a \cot(x) + b \csc(x))^5} dx$$

Optimal. Leaf size=100

$$\frac{4b}{a^5(a \cos(x) + b)} - \frac{\log(a \cos(x) + b)}{a^5} + \frac{(a^2 - b^2)^2}{4a^5(a \cos(x) + b)^4} + \frac{4b(a^2 - b^2)}{3a^5(a \cos(x) + b)^3} - \frac{a^2 - 3b^2}{a^5(a \cos(x) + b)^2}$$

[Out] $1/4*(a^2-b^2)^2/a^5/(b+a*\cos(x))^4+4/3*b*(a^2-b^2)/a^5/(b+a*\cos(x))^3+(-a^2+3*b^2)/a^5/(b+a*\cos(x))^2-4*b/a^5/(b+a*\cos(x))-ln(b+a*\cos(x))/a^5$

Rubi [A] time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, number of rules / integrand size = 0.273, Rules used = {4392, 2668, 697}

$$\frac{(a^2 - b^2)^2}{4a^5(a \cos(x) + b)^4} + \frac{4b(a^2 - b^2)}{3a^5(a \cos(x) + b)^3} - \frac{a^2 - 3b^2}{a^5(a \cos(x) + b)^2} - \frac{4b}{a^5(a \cos(x) + b)} - \frac{\log(a \cos(x) + b)}{a^5}$$

Antiderivative was successfully verified.

[In] Int[(a*Cot[x] + b*Csc[x])^(-5), x]

[Out] $(a^2 - b^2)^2/(4*a^5*(b + a*\cos[x])^4) + (4*b*(a^2 - b^2))/(3*a^5*(b + a*\cos[x])^3) - (a^2 - 3*b^2)/(a^5*(b + a*\cos[x])^2) - (4*b)/(a^5*(b + a*\cos[x])) - \text{Log}[b + a*\cos[x]]/a^5$

Rule 697

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rule 2668

Int[cos[(e_) + (f_)*(x_)]^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

Rule 4392

Int[(cot[(c_) + (d_)*(x_)]^(n_)*(a_) + csc[(c_) + (d_)*(x_)]^(n_)*(b_))^(p_)*(u_), x_Symbol] := Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a*cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a \cot(x) + b \csc(x))^5} dx &= \int \frac{\sin^5(x)}{(b + a \cos(x))^5} dx \\ &= \frac{\text{Subst}\left(\int \frac{(a^2 - x^2)^2}{(b+x)^5} dx, x, a \cos(x)\right)}{a^5} \\ &= \frac{\text{Subst}\left(\int \left(\frac{(a^2 - b^2)^2}{(b+x)^5} - \frac{4b(-a^2 + b^2)}{(b+x)^4} - \frac{2(a^2 - 3b^2)}{(b+x)^3} - \frac{4b}{(b+x)^2} + \frac{1}{b+x}\right) dx, x, a \cos(x)\right)}{a^5} \\ &= \frac{(a^2 - b^2)^2}{4a^5(b + a \cos(x))^4} + \frac{4b(a^2 - b^2)}{3a^5(b + a \cos(x))^3} - \frac{a^2 - 3b^2}{a^5(b + a \cos(x))^2} - \frac{4b}{a^5(b + a \cos(x))} - \frac{1}{a^5} \end{aligned}$$

Mathematica [A] time = 0.34, size = 138, normalized size = 1.38

$$\frac{12a^4 \cos^4(x) \log(a \cos(x) + b) - 3a^4 + 48a^3b \cos^3(x)(\log(a \cos(x) + b) + 1) + 12a^2 \cos^2(x) (a^2 + 6b^2 \log(a \cos(x) + b)) - 12a^5(a \cos(x) + b)}{12a^5(a \cos(x) + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cot[x] + b*Csc[x])^(-5), x]

[Out]
$$-1/12*(-3*a^4 + 2*a^2*b^2 + 25*b^4 + 12*b^4*\text{Log}[b + a*\text{Cos}[x]] + 12*a^4*\text{Cos}[x]^4*\text{Log}[b + a*\text{Cos}[x]] + 48*a^3*b*\text{Cos}[x]^3*(1 + \text{Log}[b + a*\text{Cos}[x]]) + 12*a^2*\text{Cos}[x]^2*(a^2 + 9*b^2 + 6*b^2*\text{Log}[b + a*\text{Cos}[x]]) + 8*a*b*\text{Cos}[x]*(a^2 + 11*b^2 + 6*b^2*\text{Log}[b + a*\text{Cos}[x]])))/(a^5*(b + a*\text{Cos}[x])^4)$$

fricas [A] time = 2.03, size = 166, normalized size = 1.66

$$\frac{48a^3b \cos(x)^3 - 3a^4 + 2a^2b^2 + 25b^4 + 12(a^4 + 9a^2b^2) \cos(x)^2 + 8(a^3b + 11ab^3) \cos(x) + 12(a^4 \cos(x)^4 + 4a^6b^3)}{12(a^9 \cos(x)^4 + 4a^8b \cos(x)^3 + 6a^7b^2 \cos(x)^2 + 4a^6b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cot(x)+b*csc(x))^5,x, algorithm="fricas")

[Out]
$$-1/12*(48*a^3*b*\cos(x)^3 - 3*a^4 + 2*a^2*b^2 + 25*b^4 + 12*(a^4 + 9*a^2*b^2)*\cos(x)^2 + 8*(a^3*b + 11*a*b^3)*\cos(x) + 12*(a^4*\cos(x)^4 + 4*a^3*b*\cos(x)^3 + 6*a^2*b^2*\cos(x)^2 + 4*a*b^3*\cos(x) + b^4)*\log(a*\cos(x) + b))/(a^9*\cos(x)^4 + 4*a^8*b*\cos(x)^3 + 6*a^7*b^2*\cos(x)^2 + 4*a^6*b^3*\cos(x) + a^5*b^4)$$

giac [A] time = 0.13, size = 93, normalized size = 0.93

$$\frac{\log(|a \cos(x) + b|)}{a^5} - \frac{48a^2b \cos(x)^3 + 12(a^3 + 9ab^2) \cos(x)^2 + 8(a^2b + 11b^3) \cos(x) - \frac{3a^4 - 2a^2b^2 - 25b^4}{a}}{12(a \cos(x) + b)^4 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cot(x)+b*csc(x))^5,x, algorithm="giac")

[Out]
$$-\log(\text{abs}(a*\cos(x) + b))/a^5 - 1/12*(48*a^2*b*\cos(x)^3 + 12*(a^3 + 9*a*b^2)*\cos(x)^2 + 8*(a^2*b + 11*b^3)*\cos(x) - (3*a^4 - 2*a^2*b^2 - 25*b^4)/a)/((a*\cos(x) + b)^4*a^4)$$

maple [A] time = 0.13, size = 132, normalized size = 1.32

$$\frac{4b}{3a^3(b+a \cos(x))^3} - \frac{4b^3}{3a^5(b+a \cos(x))^3} - \frac{4b}{a^5(b+a \cos(x))} - \frac{\ln(b+a \cos(x))}{a^5} - \frac{1}{a^3(b+a \cos(x))^2} + \frac{3b^2}{a^5(b+a \cos(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*cot(x)+b*csc(x))^5,x)

[Out]
$$4/3*b/a^3/(b+a*\cos(x))^3 - 4/3*b^3/a^5/(b+a*\cos(x))^3 - 4*b/a^5/(b+a*\cos(x)) - \ln(b+a*\cos(x))/a^5 - 1/a^3/(b+a*\cos(x))^2 + 3/a^5/(b+a*\cos(x))^2*b^2 + 1/4/a/(b+a*\cos(x))^4 - 1/2/a^3/(b+a*\cos(x))^4*b^2 + 1/4/a^5/(b+a*\cos(x))^4*b^4$$

maxima [B] time = 0.48, size = 497, normalized size = 4.97

$$\frac{2 \left(5a^4b + 10a^3b^2 + 2a^2b^3 - 6ab^4 - 3b^5 + \frac{(3a^5 - 17a^4b - 6a^3b^2 + 26a^2b^3 + 3ab^4 - 9b^5) \sin(x)^2}{(\cos(x)+1)^2} \right)}{3 \left(a^{10} + 2a^9b - a^8b^2 - 4a^7b^3 - a^6b^4 + 2a^5b^5 + a^4b^6 - \frac{4(a^{10} - 3a^8b^2 + 3a^6b^4 - a^4b^6) \sin(x)^2}{(\cos(x)+1)^2} + \frac{6(a^{10} - 2a^9b - a^8b^2 + 4a^7b^3 - a^6b^4) \sin(x)^2}{(\cos(x)+1)^2} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cot(x)+b*csc(x))^5,x, algorithm="maxima")

[Out]
$$-2/3*(5*a^4*b + 10*a^3*b^2 + 2*a^2*b^3 - 6*a*b^4 - 3*b^5 + (3*a^5 - 17*a^4*b - 6*a^3*b^2 + 26*a^2*b^3 + 3*a*b^4 - 9*b^5)*\sin(x)^2/(\cos(x) + 1)^2 - 3*(4*a^5 - 13*a^4*b + 12*a^3*b^2 + 2*a^2*b^3 - 8*a*b^4 + 3*b^5)*\sin(x)^4/(\cos(x) + 1)^4 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*\sin(x)^6/(\cos(x) + 1)^6)/(a^{10} + 2*a^9*b - a^8*b^2 - 4*a^7*b^3 - a^6*b^4 + 2*a^5*b^5 + a^4*b^6 - 4*(a^{10} - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*\sin(x)^2/(\cos(x) + 1)^2 + 6*(a^{10} - 2*a^9*b - a^8*b^2 + 4*a^7*b^3 - a^6*b^4 - 2*a^5*b^5 + a^4*b^6)*\sin(x)^4/(\cos(x) + 1)^4 - 4*(a^{10} - 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 + 4*a^5*b^5 - a^4*b^6)*\sin(x)^6/(\cos(x) + 1)^6 + (a^{10} - 6*a^9*b + 15*a^8*b^2 - 20*a^7*b^3 + 15*a^6*b^4 - 6*a^5*b^5 + a^4*b^6)*\sin(x)^8/(\cos(x) + 1)^8) - \log(a + b - (a - b)*\sin(x)^2/(\cos(x) + 1)^2)/a^5 + \log(\sin(x)^2/(\cos(x) + 1)^2 + 1)/a^5$$

mupad [B] time = 3.68, size = 538, normalized size = 5.38

$$2 \operatorname{atanh} \left(\frac{\frac{32 \tan\left(\frac{x}{2}\right)^2}{\frac{32b^3}{a^3} - \frac{32b^2}{a^2} + \frac{32b}{a} + \frac{32b \tan\left(\frac{x}{2}\right)^2}{a} - \frac{64b^2 \tan\left(\frac{x}{2}\right)^2}{a^2} + \frac{32b^3 \tan\left(\frac{x}{2}\right)^2}{a^3} + 32}{\frac{32a - 32b + 32b \tan\left(\frac{x}{2}\right)^2}{a} - \frac{32b^2}{a} + \frac{32b^3}{a^2} - \frac{64b^2 \tan\left(\frac{x}{2}\right)^2}{a} + \frac{32b^3 \tan\left(\frac{x}{2}\right)^2}{a^2} + 32} \right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b/sin(x) + a*cot(x))^5,x)

[Out]
$$(2*\operatorname{atanh}((32*\tan(x/2)^2)/((32*b^3)/a^3 - (32*b^2)/a^2 - (32*b)/a + (32*b*\tan(x/2)^2)/a - (64*b^2*\tan(x/2)^2)/a^2 + (32*b^3*\tan(x/2)^2)/a^3 + 32) - (64*b*\tan(x/2)^2)/(32*a - 32*b + 32*b*\tan(x/2)^2 - (32*b^2)/a + (32*b^3)/a^2 - (64*b^2*\tan(x/2)^2)/a + (32*b^3*\tan(x/2)^2)/a^2) + (32*b^2*\tan(x/2)^2)/(32*a^2 - 32*a*b - 32*b^2 - 64*b^2*\tan(x/2)^2 + (32*b^3)/a + (32*b^3*\tan(x/2)^2)/a + 32*a*b*\tan(x/2)^2))/a^5 - ((2*\tan(x/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/a^4 + (2*(5*a^4*b - 6*a*b^4 - 3*b^5 + 2*a^2*b^3 + 10*a^3*b^2))/(3*a^4*(a - b)^2) + (2*\tan(x/2)^4*(2*a*b^2 + 5*a^2*b - 4*a^3 - 3*b^3))/a^4 + (2*\tan(x/2)^2*(6*a*b^3 - 14*a^3*b + 3*a^4 + 9*b^4 - 20*a^2*b^2))/(3*a^4*(a - b))) / (4*a*b^3 + 4*a^3*b + \tan(x/2)^4*(6*a^4 + 6*b^4 - 12*a^2*b^2) + \tan(x/2)^2*(8*a*b^3 - 8*a^3*b - 4*a^4 + 4*b^4) - \tan(x/2)^6*(8*a*b^3 - 8*a^3*b + 4*a^4 - 4*b^4) + a^4 + b^4 + \tan(x/2)^8*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2) + 6*a^2*b^2)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*cot(x)+b*csc(x))**5,x)

[Out] Timed out

3.293 $\int (\cot(x) + \csc(x))^5 dx$

Optimal. Leaf size=28

$$\frac{4}{1 - \cos(x)} - \frac{2}{(1 - \cos(x))^2} + \log(1 - \cos(x))$$

[Out] $-2/(1 - \cos(x))^2 + 4/(1 - \cos(x)) + \ln(1 - \cos(x))$

Rubi [A] time = 0.05, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4392, 2667, 43}

$$\frac{4}{1 - \cos(x)} - \frac{2}{(1 - \cos(x))^2} + \log(1 - \cos(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cot}[x] + \text{Csc}[x])^5, x]$

[Out] $-2/(1 - \text{Cos}[x])^2 + 4/(1 - \text{Cos}[x]) + \text{Log}[1 - \text{Cos}[x]]$

Rule 43

$\text{Int}[(a_. + (b_.)(x_.))^{(m_.)}((c_. + (d_.)(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) || \text{LtQ}[9*m + 5*(n + 1), 0] || \text{GtQ}[m + n + 2, 0])$

Rule 2667

$\text{Int}[\cos[(e_. + (f_.)(x_.))]^{(p_.)}((a_. + (b_.)\sin[(e_. + (f_.)(x_.))])^{(m_.)}, x_Symbol] \rightarrow \text{Dist}[1/(b^p*f), \text{Subst}[\text{Int}[(a + x)^{m + (p - 1)/2}*(a - x)^{(p - 1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x] \&\& \text{IntegerQ}[(p - 1)/2] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& (\text{GeQ}[p, -1] || !\text{IntegerQ}[m + 1/2])$

Rule 4392

$\text{Int}[(\cot[(c_. + (d_.)(x_.))]^{(n_.)}(a_. + \csc[(c_. + (d_.)(x_.))]^{(n_.)}(b_.))^{(p_.)}(u_.), x_Symbol] \rightarrow \text{Int}[\text{ActivateTrig}[u]*\text{Csc}[c + d*x]^{(n*p)}*(b + a*\text{Cos}[c + d*x]^n)^p, x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{IntegersQ}[n, p]$

Rubi steps

$$\begin{aligned} \int (\cot(x) + \csc(x))^5 dx &= \int (1 + \cos(x))^5 \csc^5(x) dx \\ &= -\text{Subst}\left(\int \frac{(1+x)^2}{(1-x)^3} dx, x, \cos(x)\right) \\ &= -\text{Subst}\left(\int \left(\frac{1}{1-x} - \frac{4}{(-1+x)^3} - \frac{4}{(-1+x)^2}\right) dx, x, \cos(x)\right) \\ &= -\frac{2}{(1 - \cos(x))^2} + \frac{4}{1 - \cos(x)} + \log(1 - \cos(x)) \end{aligned}$$

Mathematica [A] time = 0.07, size = 32, normalized size = 1.14

$$-\frac{1}{2} \csc^4\left(\frac{x}{2}\right) + 2 \csc^2\left(\frac{x}{2}\right) + 2 \log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x] + Csc[x])^5,x]

[Out] 2*Csc[x/2]^2 - Csc[x/2]^4/2 + 2*Log[Sin[x/2]]

fricas [A] time = 1.58, size = 37, normalized size = 1.32

$$\frac{(\cos(x)^2 - 2 \cos(x) + 1) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) - 4 \cos(x) + 2}{\cos(x)^2 - 2 \cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x))^5,x, algorithm="fricas")

[Out] ((cos(x)^2 - 2*cos(x) + 1)*log(-1/2*cos(x) + 1/2) - 4*cos(x) + 2)/(cos(x)^2 - 2*cos(x) + 1)

giac [A] time = 0.12, size = 22, normalized size = 0.79

$$-\frac{2(2 \cos(x) - 1)}{(\cos(x) - 1)^2} + \log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x))^5,x, algorithm="giac")

[Out] -2*(2*cos(x) - 1)/(cos(x) - 1)^2 + log(-cos(x) + 1)

maple [B] time = 0.11, size = 105, normalized size = 3.75

$$-\frac{(\cot^4(x))}{4} + \frac{(\cot^2(x))}{2} + \ln(\sin(x)) - \frac{5(\cos^5(x))}{4 \sin(x)^4} + \frac{5(\cos^5(x))}{8 \sin(x)^2} + \frac{5(\cos^3(x))}{8} + \frac{5 \cos(x)}{8} + \ln(\csc(x) - \cot(x)) - \frac{5(\cos^5(x))}{2 \sin(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(x)+csc(x))^5,x)

[Out] -1/4*cot(x)^4+1/2*cot(x)^2+ln(sin(x))-5/4/sin(x)^4*cos(x)^5+5/8/sin(x)^2*cos(x)^5+5/8*cos(x)^3+5/8*cos(x)+ln(csc(x)-cot(x))-5/2/sin(x)^4*cos(x)^4-5/2/sin(x)^4*cos(x)^3-5/4/sin(x)^2*cos(x)^3-5/4/sin(x)^4+(-1/4*csc(x)^3-3/8*csc(x))*cot(x)

maxima [B] time = 0.31, size = 125, normalized size = 4.46

$$-\frac{5}{2} \cot(x)^4 - \frac{5(5 \cos(x)^3 - 3 \cos(x))}{8(\cos(x)^4 - 2 \cos(x)^2 + 1)} + \frac{3 \cos(x)^3 - 5 \cos(x)}{8(\cos(x)^4 - 2 \cos(x)^2 + 1)} - \frac{5(\cos(x)^3 + \cos(x))}{4(\cos(x)^4 - 2 \cos(x)^2 + 1)} + \frac{4 \sin(x)^2 - 1}{4 \sin(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x))^5,x, algorithm="maxima")

[Out] -5/2*cot(x)^4 - 5/8*(5*cos(x)^3 - 3*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) + 1/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) - 5/4*(cos(x)^3 + cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) + 1/4*(4*sin(x)^2 - 1)/sin(x)^4 - 5/4/sin(x)^4 + 1/2*log(sin(x)^2) - 1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)

mupad [B] time = 2.42, size = 34, normalized size = 1.21

$$2 \ln\left(\tan\left(\frac{x}{2}\right)\right) - \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) + \frac{\tan\left(\frac{x}{2}\right)^2 - \frac{1}{2}}{\tan\left(\frac{x}{2}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cot(x) + 1/sin(x))^5,x)`

[Out] $2*\log(\tan(x/2)) - \log(\tan(x/2)^2 + 1) + (\tan(x/2)^2 - 1/2)/\tan(x/2)^4$

sympy [B] time = 107.21, size = 68, normalized size = 2.43

$$\frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2} - \frac{\log(\csc^2(x))}{2} - \frac{5 \cot^4(x)}{2} - \frac{3 \csc^4(x)}{2} + \csc^2(x) - \frac{32 \cos^3(x)}{8 \cos^4(x) - 16 \cos^2(x) + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((cot(x)+csc(x))**5,x)`

[Out] $\log(\cos(x) - 1)/2 - \log(\cos(x) + 1)/2 - \log(\csc(x)**2)/2 - 5*\cot(x)**4/2 - 3*\csc(x)**4/2 + \csc(x)**2 - 32*\cos(x)**3/(8*\cos(x)**4 - 16*\cos(x)**2 + 8)$

3.294 $\int (\cot(x) + \csc(x))^4 dx$

Optimal. Leaf size=30

$$x - \frac{2 \sin^3(x)}{3(1 - \cos(x))^3} + \frac{2 \sin(x)}{1 - \cos(x)}$$

[Out] $x + 2 \sin(x) / (1 - \cos(x)) - 2/3 \sin(x)^3 / (1 - \cos(x))^3$

Rubi [A] time = 0.10, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4392, 2670, 2680, 8}

$$x - \frac{2 \sin^3(x)}{3(1 - \cos(x))^3} + \frac{2 \sin(x)}{1 - \cos(x)}$$

Antiderivative was successfully verified.

[In] Int[(Cot[x] + Csc[x])^4, x]

[Out] $x + (2 \sin[x]) / (1 - \cos[x]) - (2 \sin[x]^3) / (3(1 - \cos[x])^3)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2670

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := Dist[(a/g)^(2*m), Int[(g*cos[e + f*x])^(2*m + p)/(a - b*sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := Simp[(2*g*(g*cos[e + f*x])^(p - 1)*(a + b*sin[e + f*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p + 1)), Int[(g*cos[e + f*x])^(p - 2)*(a + b*sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m + p + 1, 0] && !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]

Rule 4392

Int[(cot[(c_.) + (d_.)*(x_.)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_.)]^(n_.)*(b_.))^p*(u_.), x_Symbol] := Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a*cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned}
\int (\cot(x) + \csc(x))^4 dx &= \int (1 + \cos(x))^4 \csc^4(x) dx \\
&= \int \frac{\sin^4(x)}{(1 - \cos(x))^4} dx \\
&= -\frac{2 \sin^3(x)}{3(1 - \cos(x))^3} - \int \frac{\sin^2(x)}{(1 - \cos(x))^2} dx \\
&= \frac{2 \sin(x)}{1 - \cos(x)} - \frac{2 \sin^3(x)}{3(1 - \cos(x))^3} + \int 1 dx \\
&= x + \frac{2 \sin(x)}{1 - \cos(x)} - \frac{2 \sin^3(x)}{3(1 - \cos(x))^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 30, normalized size = 1.00

$$x + \frac{8}{3} \cot\left(\frac{x}{2}\right) - \frac{2}{3} \cot\left(\frac{x}{2}\right) \csc^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x] + Csc[x])^4, x]

[Out] x + (8*Cot[x/2])/3 - (2*Cot[x/2]*Csc[x/2]^2)/3

fricas [A] time = 1.70, size = 36, normalized size = 1.20

$$\frac{8 \cos(x)^2 + 3(x \cos(x) - x) \sin(x) + 4 \cos(x) - 4}{3(\cos(x) - 1) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x))^4,x, algorithm="fricas")

[Out] 1/3*(8*cos(x)^2 + 3*(x*cos(x) - x)*sin(x) + 4*cos(x) - 4)/((cos(x) - 1)*sin(x))

giac [A] time = 0.13, size = 20, normalized size = 0.67

$$x + \frac{2 \left(3 \tan\left(\frac{1}{2}x\right)^2 - 1 \right)}{3 \tan\left(\frac{1}{2}x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x))^4,x, algorithm="giac")

[Out] x + 2/3*(3*tan(1/2*x)^2 - 1)/tan(1/2*x)^3

maple [B] time = 0.09, size = 68, normalized size = 2.27

$$-\frac{(\cot^3(x))}{3} + \cot(x) + x - \frac{4(\cos^4(x))}{3 \sin(x)^3} + \frac{4(\cos^4(x))}{3 \sin(x)} + \frac{4(2 + \cos^2(x)) \sin(x)}{3} - \frac{2(\cos^3(x))}{\sin(x)^3} - \frac{4}{3 \sin(x)^3} + \left(-\frac{2}{3} - \frac{(\csc^2(x))}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(x)+csc(x))^4,x)

[Out] -1/3*cot(x)^3+cot(x)+x-4/3/sin(x)^3*cos(x)^4+4/3/sin(x)*cos(x)^4+4/3*(2+cos(x)^2)*sin(x)-2/sin(x)^3*cos(x)^3-4/3/sin(x)^3+(-2/3-1/3*csc(x)^2)*cot(x)

maxima [B] time = 0.42, size = 56, normalized size = 1.87

$$-2 \cot(x)^3 + x + \frac{4(3 \sin(x)^2 - 1)}{3 \sin(x)^3} - \frac{3 \tan(x)^2 + 1}{3 \tan(x)^3} + \frac{3 \tan(x)^2 - 1}{3 \tan(x)^3} - \frac{4}{3 \sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x))^4,x, algorithm="maxima")

[Out] -2*cot(x)^3 + x + 4/3*(3*sin(x)^2 - 1)/sin(x)^3 - 1/3*(3*tan(x)^2 + 1)/tan(x)^3 + 1/3*(3*tan(x)^2 - 1)/tan(x)^3 - 4/3/sin(x)^3

mupad [B] time = 2.41, size = 16, normalized size = 0.53

$$-\frac{2 \cot\left(\frac{x}{2}\right)^3}{3} + 2 \cot\left(\frac{x}{2}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(x) + 1/sin(x))^4,x)

[Out] x + 2*cot(x/2) - (2*cot(x/2)^3)/3

sympy [A] time = 35.04, size = 44, normalized size = 1.47

$$x - \frac{7 \cot^3(x)}{3} - \cot(x) - \frac{8 \csc^3(x)}{3} + 4 \csc(x) + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3 \sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x))**4,x)

[Out] x - 7*cot(x)**3/3 - cot(x) - 8*csc(x)**3/3 + 4*csc(x) + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)

3.295 $\int (\cot(x) + \csc(x))^3 dx$

Optimal. Leaf size=20

$$-\frac{2}{1 - \cos(x)} - \log(1 - \cos(x))$$

[Out] $-2/(1 - \cos(x)) - \ln(1 - \cos(x))$

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4392, 2667, 43}

$$-\frac{2}{1 - \cos(x)} - \log(1 - \cos(x))$$

Antiderivative was successfully verified.

[In] Int[(Cot[x] + Csc[x])^3, x]

[Out] $-2/(1 - \text{Cos}[x]) - \text{Log}[1 - \text{Cos}[x]]$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2667

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 4392

Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b_.))^(p_.)*(u_.), x_Symbol] := Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned} \int (\cot(x) + \csc(x))^3 dx &= \int (1 + \cos(x))^3 \csc^3(x) dx \\ &= -\text{Subst} \left(\int \frac{1+x}{(1-x)^2} dx, x, \cos(x) \right) \\ &= -\text{Subst} \left(\int \left(\frac{2}{(-1+x)^2} + \frac{1}{-1+x} \right) dx, x, \cos(x) \right) \\ &= -\frac{2}{1 - \cos(x)} - \log(1 - \cos(x)) \end{aligned}$$

Mathematica [A] time = 0.04, size = 20, normalized size = 1.00

$$-\csc^2\left(\frac{x}{2}\right) - 2 \log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x] + Csc[x])^3,x]

[Out] -Csc[x/2]^2 - 2*Log[Sin[x/2]]

fricas [A] time = 1.68, size = 22, normalized size = 1.10

$$-\frac{(\cos(x) - 1) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) - 2}{\cos(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x))^3,x, algorithm="fricas")

[Out] -((cos(x) - 1)*log(-1/2*cos(x) + 1/2) - 2)/(cos(x) - 1)

giac [A] time = 0.15, size = 18, normalized size = 0.90

$$\frac{2}{\cos(x) - 1} - \log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x))^3,x, algorithm="giac")

[Out] 2/(cos(x) - 1) - log(-cos(x) + 1)

maple [B] time = 0.08, size = 49, normalized size = 2.45

$$-\frac{(\cot^2(x))}{2} - \ln(\sin(x)) - \frac{3(\cos^3(x))}{2\sin(x)^2} - \frac{3\cos(x)}{2} - \ln(\csc(x) - \cot(x)) - \frac{3}{2\sin(x)^2} - \frac{\cot(x)\csc(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(x)+csc(x))^3,x)

[Out] -1/2*cot(x)^2 - ln(sin(x)) - 3/2/sin(x)^2*cos(x)^3 - 3/2*cos(x) - ln(csc(x) - cot(x)) - 3/2/sin(x)^2 - 1/2*cot(x)*csc(x)

maxima [B] time = 0.31, size = 46, normalized size = 2.30

$$-\frac{3}{2} \cot(x)^2 + \frac{2 \cos(x)}{\cos(x)^2 - 1} - \frac{1}{2 \sin(x)^2} - \frac{1}{2} \log(\sin(x)^2) + \frac{1}{2} \log(\cos(x) + 1) - \frac{1}{2} \log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x))^3,x, algorithm="maxima")

[Out] -3/2*cot(x)^2 + 2*cos(x)/(cos(x)^2 - 1) - 1/2/sin(x)^2 - 1/2*log(sin(x)^2) + 1/2*log(cos(x) + 1) - 1/2*log(cos(x) - 1)

mupad [B] time = 2.39, size = 25, normalized size = 1.25

$$\ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - 2 \ln\left(\tan\left(\frac{x}{2}\right)\right) - \frac{1}{\tan\left(\frac{x}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(x) + 1/sin(x))^3,x)

[Out] log(tan(x/2)^2 + 1) - 2*log(tan(x/2)) - 1/tan(x/2)^2

sympy [B] time = 14.64, size = 46, normalized size = 2.30

$$-\frac{\log(\cos(x) - 1)}{2} + \frac{\log(\cos(x) + 1)}{2} + \frac{\log(-\csc^2(x))}{2} - 2 \csc^2(x) + \frac{4 \cos(x)}{2 \cos^2(x) - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x))**3,x)

[Out] -log(cos(x) - 1)/2 + log(cos(x) + 1)/2 + log(-csc(x)**2)/2 - 2*csc(x)**2 + 4*cos(x)/(2*cos(x)**2 - 2)

3.296 $\int (\cot(x) + \csc(x))^2 dx$

Optimal. Leaf size=16

$$-x - \frac{2 \sin(x)}{1 - \cos(x)}$$

[Out] $-x - 2 \sin(x) / (1 - \cos(x))$

Rubi [A] time = 0.07, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4392, 2670, 2680, 8}

$$-x - \frac{2 \sin(x)}{1 - \cos(x)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cot}[x] + \text{Csc}[x])^2, x]$

[Out] $-x - (2 \sin[x]) / (1 - \cos[x])$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2670

$\text{Int}[(\cos[(e_.) + (f_.)*(x_.)]*(g_.))^{(p_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}, x_Symbol] \rightarrow \text{Dist}[(a/g)^{(2*m)}, \text{Int}[(g*\cos[e + f*x])^{(2*m + p)} / (a - b*\sin[e + f*x])^m, x], x] /; \text{FreeQ}[\{a, b, e, f, g\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{LtQ}[p, -1] \&\& \text{GeQ}[2*m + p, 0]$

Rule 2680

$\text{Int}[(\cos[(e_.) + (f_.)*(x_.)]*(g_.))^{(p_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(2*g*(g*\cos[e + f*x])^{(p - 1)}*(a + b*\sin[e + f*x])^{(m + 1)}) / (b*f*(2*m + p + 1)), x] + \text{Dist}[(g^{2*(p - 1)}) / (b^{2*(2*m + p + 1)}), \text{Int}[(g*\cos[e + f*x])^{(p - 2)}*(a + b*\sin[e + f*x])^{(m + 2)}, x], x] /; \text{FreeQ}[\{a, b, e, f, g\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LeQ}[m, -2] \&\& \text{GtQ}[p, 1] \&\& \text{NeQ}[2*m + p + 1, 0] \&\& !\text{ILtQ}[m + p + 1, 0] \&\& \text{IntegersQ}[2*m, 2*p]$

Rule 4392

$\text{Int}[(\cot[(c_.) + (d_.)*(x_.)]^{(n_.)}*(a_.) + \csc[(c_.) + (d_.)*(x_.)]^{(n_.)}*(b_.))^{(p_.)}*(u_.), x_Symbol] \rightarrow \text{Int}[\text{ActivateTrig}[u]*\text{Csc}[c + d*x]^{(n*p)}*(b + a*\cos[c + d*x]^n)^p, x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{IntegersQ}[n, p]$

Rubi steps

$$\begin{aligned} \int (\cot(x) + \csc(x))^2 dx &= \int (1 + \cos(x))^2 \csc^2(x) dx \\ &= \int \frac{\sin^2(x)}{(1 - \cos(x))^2} dx \\ &= -\frac{2 \sin(x)}{1 - \cos(x)} - \int 1 dx \\ &= -x - \frac{2 \sin(x)}{1 - \cos(x)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 0.75

$$-x - 2 \cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x] + Csc[x])^2,x]

[Out] -x - 2*Cot[x/2]

fricas [A] time = 1.16, size = 16, normalized size = 1.00

$$\frac{x \sin(x) + 2 \cos(x) + 2}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x))^2,x, algorithm="fricas")

[Out] -(x*sin(x) + 2*cos(x) + 2)/sin(x)

giac [A] time = 0.16, size = 12, normalized size = 0.75

$$-x - \frac{2}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x))^2,x, algorithm="giac")

[Out] -x - 2/tan(1/2*x)

maple [A] time = 0.04, size = 15, normalized size = 0.94

$$-2 \cot(x) - x - \frac{2}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(x)+csc(x))^2,x)

[Out] -2*cot(x)-x-2/sin(x)

maxima [A] time = 0.41, size = 16, normalized size = 1.00

$$-x - \frac{2}{\sin(x)} - \frac{2}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x))^2,x, algorithm="maxima")

[Out] -x - 2/sin(x) - 2/tan(x)

mupad [B] time = 2.40, size = 10, normalized size = 0.62

$$-x - 2 \cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(x) + 1/sin(x))^2,x)

[Out] $-x - 2\cot(x/2)$

sympy [A] time = 2.70, size = 17, normalized size = 1.06

$$-x - \cot(x) - 2 \csc(x) - \frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((cot(x)+csc(x))**2,x)`

[Out] $-x - \cot(x) - 2\csc(x) - \cos(x)/\sin(x)$

3.297 $\int (\cot(x) + \csc(x)) dx$

Optimal. Leaf size=9

$$\log(\sin(x)) - \tanh^{-1}(\cos(x))$$

[Out] $-\operatorname{arctanh}(\cos(x)) + \ln(\sin(x))$

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3475, 3770}

$$\log(\sin(x)) - \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\cot[x] + \operatorname{Csc}[x], x]$

[Out] $-\operatorname{ArcTanh}[\cos[x]] + \operatorname{Log}[\sin[x]]$

Rule 3475

$\operatorname{Int}[\tan[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[\cos[c + d*x], x]]/d, x] /; \operatorname{FreeQ}\{c, d, x\}$

Rule 3770

$\operatorname{Int}[\csc[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\cos[c + d*x]]/d, x] /; \operatorname{FreeQ}\{c, d, x\}$

Rubi steps

$$\begin{aligned} \int (\cot(x) + \csc(x)) dx &= \int \cot(x) dx + \int \csc(x) dx \\ &= -\tanh^{-1}(\cos(x)) + \log(\sin(x)) \end{aligned}$$

Mathematica [B] time = 0.00, size = 20, normalized size = 2.22

$$\log\left(\sin\left(\frac{x}{2}\right)\right) + \log(\sin(x)) - \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Integrate}[\cot[x] + \operatorname{Csc}[x], x]$

[Out] $-\operatorname{Log}[\cos[x/2]] + \operatorname{Log}[\sin[x/2]] + \operatorname{Log}[\sin[x]]$

fricas [A] time = 1.03, size = 7, normalized size = 0.78

$$\log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(\cot(x) + \csc(x), x, \text{algorithm}="fricas")$

[Out] $\log(-1/2*\cos(x) + 1/2)$

giac [A] time = 0.15, size = 11, normalized size = 1.22

$$\log(|\sin(x)|) + \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)+csc(x),x, algorithm="giac")

[Out] log(abs(sin(x))) + log(abs(tan(1/2*x)))

maple [A] time = 0.00, size = 13, normalized size = 1.44

$$\ln(\sin(x)) - \ln(\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)+csc(x),x)

[Out] ln(sin(x))-ln(cot(x)+csc(x))

maxima [A] time = 0.30, size = 12, normalized size = 1.33

$$-\log(\cot(x) + \csc(x)) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)+csc(x),x, algorithm="maxima")

[Out] -log(cot(x) + csc(x)) + log(sin(x))

mupad [B] time = 2.41, size = 19, normalized size = 2.11

$$2 \ln\left(\tan\left(\frac{x}{2}\right)\right) - \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x) + 1/sin(x),x)

[Out] 2*log(tan(x/2)) - log(tan(x/2)^2 + 1)

sympy [B] time = 0.10, size = 20, normalized size = 2.22

$$\frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2} + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)+csc(x),x)

[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2 + log(sin(x))

$$3.298 \quad \int \frac{1}{\cot(x) + \csc(x)} dx$$

Optimal. Leaf size=7

$$-\log(\cos(x) + 1)$$

[Out] -ln(1+cos(x))

Rubi [A] time = 0.03, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3160, 2667, 31}

$$-\log(\cos(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[(Cot[x] + Csc[x])^(-1), x]

[Out] -Log[1 + Cos[x]]

Rule 31

Int[((a_) + (b_.)*(x_))^(p-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 3160

Int[((a_.) + csc[(d_.) + (e_.)*(x_)])*(b_.) + cot[(d_.) + (e_.)*(x_)])*(c_.)^(p-1), x_Symbol] := Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]), x] /; FreeQ[{a, b, c, d, e}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\cot(x) + \csc(x)} dx &= \int \frac{\sin(x)}{1 + \cos(x)} dx \\ &= -\text{Subst} \left(\int \frac{1}{1+x} dx, x, \cos(x) \right) \\ &= -\log(1 + \cos(x)) \end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.29

$$-2 \log \left(\cos \left(\frac{x}{2} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x] + Csc[x])^(-1), x]

[Out] -2*Log[Cos[x/2]]

fricas [A] time = 0.91, size = 9, normalized size = 1.29

$$-\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)+csc(x)),x, algorithm="fricas")

[Out] -log(1/2*cos(x) + 1/2)

giac [A] time = 0.13, size = 7, normalized size = 1.00

$$-\log(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)+csc(x)),x, algorithm="giac")

[Out] -log(cos(x) + 1)

maple [A] time = 0.11, size = 8, normalized size = 1.14

$$-\ln(1 + \cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x)+csc(x)),x)

[Out] -ln(1+cos(x))

maxima [A] time = 0.41, size = 14, normalized size = 2.00

$$\log\left(\frac{\sin(x)^2}{(\cos(x) + 1)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)+csc(x)),x, algorithm="maxima")

[Out] log(sin(x)^2/(cos(x) + 1)^2 + 1)

mupad [B] time = 2.86, size = 9, normalized size = 1.29

$$\ln\left(\tan\left(\frac{x}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x) + 1/sin(x)),x)

[Out] log(tan(x/2)^2 + 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cot(x) + \csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)+csc(x)),x)

[Out] Integral(1/(cot(x) + csc(x)), x)

$$3.299 \quad \int \frac{1}{(\cot(x) + \csc(x))^2} dx$$

Optimal. Leaf size=14

$$\frac{2 \sin(x)}{\cos(x) + 1} - x$$

[Out] $-x + 2 \sin(x) / (1 + \cos(x))$

Rubi [A] time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4392, 2680, 8}

$$\frac{2 \sin(x)}{\cos(x) + 1} - x$$

Antiderivative was successfully verified.

[In] Int[(Cot[x] + Csc[x])^(-2), x]

[Out] $-x + (2 \sin[x]) / (1 + \cos[x])$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^ (p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^ (m_.), x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m + p + 1, 0] && !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]

Rule 4392

Int[(cot[(c_.) + (d_.)*(x_.)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_.)]^(n_.)*(b_.))^ (p_.)*(u_.), x_Symbol] := Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned} \int \frac{1}{(\cot(x) + \csc(x))^2} dx &= \int \frac{\sin^2(x)}{(1 + \cos(x))^2} dx \\ &= \frac{2 \sin(x)}{1 + \cos(x)} - \int 1 dx \\ &= -x + \frac{2 \sin(x)}{1 + \cos(x)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 0.86

$$2 \tan\left(\frac{x}{2}\right) - x$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x] + Csc[x])^(-2), x]

[Out] $-x + 2*\text{Tan}[x/2]$

fricas [A] time = 1.10, size = 18, normalized size = 1.29

$$\frac{x \cos(x) + x - 2 \sin(x)}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(cot(x)+csc(x))^2,x, algorithm="fricas")`

[Out] $-(x*\cos(x) + x - 2*\sin(x))/(\cos(x) + 1)$

giac [A] time = 0.14, size = 10, normalized size = 0.71

$$-x + 2 \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(cot(x)+csc(x))^2,x, algorithm="giac")`

[Out] $-x + 2*\tan(1/2*x)$

maple [A] time = 0.11, size = 11, normalized size = 0.79

$$2 \tan\left(\frac{x}{2}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cot(x)+csc(x))^2,x)`

[Out] $2*\tan(1/2*x)-x$

maxima [A] time = 0.41, size = 23, normalized size = 1.64

$$\frac{2 \sin(x)}{\cos(x) + 1} - 2 \arctan\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(cot(x)+csc(x))^2,x, algorithm="maxima")`

[Out] $2*\sin(x)/(\cos(x) + 1) - 2*\arctan(\sin(x)/(\cos(x) + 1))$

mupad [B] time = 2.41, size = 10, normalized size = 0.71

$$2 \tan\left(\frac{x}{2}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cot(x) + 1/sin(x))^2,x)`

[Out] $2*\tan(x/2) - x$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\cot(x) + \csc(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(cot(x)+csc(x))**2,x)`

[Out] `Integral((cot(x) + csc(x))**(-2), x)`

$$3.300 \quad \int \frac{1}{(\cot(x) + \csc(x))^3} dx$$

Optimal. Leaf size=14

$$\frac{2}{\cos(x) + 1} + \log(\cos(x) + 1)$$

[Out] 2/(1+cos(x))+ln(1+cos(x))

Rubi [A] time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4392, 2667, 43}

$$\frac{2}{\cos(x) + 1} + \log(\cos(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[(Cot[x] + Csc[x])^(-3), x]

[Out] 2/(1 + Cos[x]) + Log[1 + Cos[x]]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 2667

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1)/2, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])]

Rule 4392

Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b_.))^(p_.)*(u_.), x_Symbol] := Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegerQ[n, p]

Rubi steps

$$\begin{aligned} \int \frac{1}{(\cot(x) + \csc(x))^3} dx &= \int \frac{\sin^3(x)}{(1 + \cos(x))^3} dx \\ &= -\text{Subst} \left(\int \frac{1-x}{(1+x)^2} dx, x, \cos(x) \right) \\ &= -\text{Subst} \left(\int \left(\frac{1}{-1-x} + \frac{2}{(1+x)^2} \right) dx, x, \cos(x) \right) \\ &= \frac{2}{1 + \cos(x)} + \log(1 + \cos(x)) \end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.29

$$\sec^2\left(\frac{x}{2}\right) + 2 \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x] + Csc[x])^(-3), x]

[Out] 2*Log[Cos[x/2]] + Sec[x/2]^2

fricas [A] time = 0.86, size = 21, normalized size = 1.50

$$\frac{(\cos(x) + 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 2}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)+csc(x))^3,x, algorithm="fricas")

[Out] ((cos(x) + 1)*log(1/2*cos(x) + 1/2) + 2)/(cos(x) + 1)

giac [A] time = 0.14, size = 14, normalized size = 1.00

$$\frac{2}{\cos(x) + 1} + \log(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)+csc(x))^3,x, algorithm="giac")

[Out] 2/(cos(x) + 1) + log(cos(x) + 1)

maple [A] time = 0.15, size = 15, normalized size = 1.07

$$\frac{2}{1 + \cos(x)} + \ln(1 + \cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x)+csc(x))^3,x)

[Out] 2/(1+cos(x))+ln(1+cos(x))

maxima [A] time = 0.41, size = 28, normalized size = 2.00

$$\frac{\sin(x)^2}{(\cos(x) + 1)^2} - \log\left(\frac{\sin(x)^2}{(\cos(x) + 1)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)+csc(x))^3,x, algorithm="maxima")

[Out] sin(x)^2/(cos(x) + 1)^2 - log(sin(x)^2/(cos(x) + 1)^2 + 1)

mupad [B] time = 2.35, size = 18, normalized size = 1.29

$$\tan\left(\frac{x}{2}\right)^2 - \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x) + 1/sin(x))^3,x)

[Out] tan(x/2)^2 - log(tan(x/2)^2 + 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\cot(x) + \csc(x))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))**3,x)
```

```
[Out] Integral((cot(x) + csc(x))**(-3), x)
```

$$3.301 \quad \int \frac{1}{(\cot(x) + \csc(x))^4} dx$$

Optimal. Leaf size=26

$$x + \frac{2 \sin^3(x)}{3(\cos(x) + 1)^3} - \frac{2 \sin(x)}{\cos(x) + 1}$$

[Out] $x - 2 \sin(x) / (1 + \cos(x)) + 2/3 \sin(x)^3 / (1 + \cos(x))^3$

Rubi [A] time = 0.07, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4392, 2680, 8}

$$x + \frac{2 \sin^3(x)}{3(\cos(x) + 1)^3} - \frac{2 \sin(x)}{\cos(x) + 1}$$

Antiderivative was successfully verified.

[In] `Int[(Cot[x] + Csc[x])^(-4), x]`

[Out] `x - (2*Sin[x])/(1 + Cos[x]) + (2*Sin[x]^3)/(3*(1 + Cos[x])^3)`

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2680

`Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m + p + 1, 0] && !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]`

Rule 4392

`Int[(cot[(c_.) + (d_.)*(x_.)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_.)]^(n_.)*(b_.))^(p_.)*(u_.), x_Symbol] := Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]`

Rubi steps

$$\begin{aligned} \int \frac{1}{(\cot(x) + \csc(x))^4} dx &= \int \frac{\sin^4(x)}{(1 + \cos(x))^4} dx \\ &= \frac{2 \sin^3(x)}{3(1 + \cos(x))^3} - \int \frac{\sin^2(x)}{(1 + \cos(x))^2} dx \\ &= -\frac{2 \sin(x)}{1 + \cos(x)} + \frac{2 \sin^3(x)}{3(1 + \cos(x))^3} + \int 1 dx \\ &= x - \frac{2 \sin(x)}{1 + \cos(x)} + \frac{2 \sin^3(x)}{3(1 + \cos(x))^3} \end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.15

$$x - \frac{8}{3} \tan\left(\frac{x}{2}\right) + \frac{2}{3} \tan\left(\frac{x}{2}\right) \sec^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x] + Csc[x])^(-4), x]

[Out] x - (8*Tan[x/2])/3 + (2*Sec[x/2]^2*Tan[x/2])/3

fricas [A] time = 0.95, size = 40, normalized size = 1.54

$$\frac{3x \cos(x)^2 + 6x \cos(x) - 4(2 \cos(x) + 1) \sin(x) + 3x}{3(\cos(x)^2 + 2 \cos(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)+csc(x))^4,x, algorithm="fricas")

[Out] 1/3*(3*x*cos(x)^2 + 6*x*cos(x) - 4*(2*cos(x) + 1)*sin(x) + 3*x)/(cos(x)^2 + 2*cos(x) + 1)

giac [A] time = 0.13, size = 16, normalized size = 0.62

$$\frac{2}{3} \tan\left(\frac{1}{2}x\right)^3 + x - 2 \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)+csc(x))^4,x, algorithm="giac")

[Out] 2/3*tan(1/2*x)^3 + x - 2*tan(1/2*x)

maple [A] time = 0.14, size = 17, normalized size = 0.65

$$\frac{2\left(\tan^3\left(\frac{x}{2}\right)\right)}{3} - 2 \tan\left(\frac{x}{2}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x)+csc(x))^4,x)

[Out] 2/3*tan(1/2*x)^3-2*tan(1/2*x)+x

maxima [A] time = 0.41, size = 35, normalized size = 1.35

$$-\frac{2 \sin(x)}{\cos(x) + 1} + \frac{2 \sin(x)^3}{3(\cos(x) + 1)^3} + 2 \arctan\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)+csc(x))^4,x, algorithm="maxima")

[Out] -2*sin(x)/(cos(x) + 1) + 2/3*sin(x)^3/(cos(x) + 1)^3 + 2*arctan(sin(x)/(cos(x) + 1))

mupad [B] time = 2.37, size = 16, normalized size = 0.62

$$\frac{2 \tan\left(\frac{x}{2}\right)^3}{3} - 2 \tan\left(\frac{x}{2}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x) + 1/sin(x))^4,x)

[Out] x - 2*tan(x/2) + (2*tan(x/2)^3)/3

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)+csc(x))**4,x)

[Out] Timed out

$$3.302 \quad \int \frac{1}{(\cot(x) + \csc(x))^5} dx$$

Optimal. Leaf size=24

$$-\frac{4}{\cos(x) + 1} + \frac{2}{(\cos(x) + 1)^2} - \log(\cos(x) + 1)$$

[Out] 2/(1+cos(x))^2-4/(1+cos(x))-ln(1+cos(x))

Rubi [A] time = 0.05, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4392, 2667, 43}

$$-\frac{4}{\cos(x) + 1} + \frac{2}{(\cos(x) + 1)^2} - \log(\cos(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[(Cot[x] + Csc[x])^(-5), x]

[Out] 2/(1 + Cos[x])^2 - 4/(1 + Cos[x]) - Log[1 + Cos[x]]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2667

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 4392

Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b_.))^(p_.)*(u_.), x_Symbol] := Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + *Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]

Rubi steps

$$\begin{aligned} \int \frac{1}{(\cot(x) + \csc(x))^5} dx &= \int \frac{\sin^5(x)}{(1 + \cos(x))^5} dx \\ &= -\text{Subst} \left(\int \frac{(1-x)^2}{(1+x)^3} dx, x, \cos(x) \right) \\ &= -\text{Subst} \left(\int \left(\frac{4}{(1+x)^3} - \frac{4}{(1+x)^2} + \frac{1}{1+x} \right) dx, x, \cos(x) \right) \\ &= \frac{2}{(1 + \cos(x))^2} - \frac{4}{1 + \cos(x)} - \log(1 + \cos(x)) \end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.33

$$\frac{1}{2} \sec^4\left(\frac{x}{2}\right) - 2 \sec^2\left(\frac{x}{2}\right) - 2 \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x] + Csc[x])^(-5), x]

[Out] -2*Log[Cos[x/2]] - 2*Sec[x/2]^2 + Sec[x/2]^4/2

fricas [A] time = 1.99, size = 38, normalized size = 1.58

$$\frac{(\cos(x)^2 + 2 \cos(x) + 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 4 \cos(x) + 2}{\cos(x)^2 + 2 \cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)+csc(x))^5,x, algorithm="fricas")

[Out] -((cos(x)^2 + 2*cos(x) + 1)*log(1/2*cos(x) + 1/2) + 4*cos(x) + 2)/(cos(x)^2 + 2*cos(x) + 1)

giac [A] time = 0.12, size = 22, normalized size = 0.92

$$-\frac{2(2 \cos(x) + 1)}{(\cos(x) + 1)^2} - \log(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)+csc(x))^5,x, algorithm="giac")

[Out] -2*(2*cos(x) + 1)/(cos(x) + 1)^2 - log(cos(x) + 1)

maple [A] time = 0.16, size = 25, normalized size = 1.04

$$\frac{2}{(1 + \cos(x))^2} - \frac{4}{1 + \cos(x)} - \ln(1 + \cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x)+csc(x))^5,x)

[Out] 2/(1+cos(x))^2-4/(1+cos(x))-ln(1+cos(x))

maxima [A] time = 0.41, size = 39, normalized size = 1.62

$$-\frac{\sin(x)^2}{(\cos(x) + 1)^2} + \frac{\sin(x)^4}{2(\cos(x) + 1)^4} + \log\left(\frac{\sin(x)^2}{(\cos(x) + 1)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)+csc(x))^5,x, algorithm="maxima")

[Out] -sin(x)^2/(cos(x) + 1)^2 + 1/2*sin(x)^4/(cos(x) + 1)^4 + log(sin(x)^2/(cos(x) + 1)^2 + 1)

mupad [B] time = 2.41, size = 26, normalized size = 1.08

$$\ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - \tan\left(\frac{x}{2}\right)^2 + \frac{\tan\left(\frac{x}{2}\right)^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x) + 1/sin(x))^5,x)

[Out] log(tan(x/2)^2 + 1) - tan(x/2)^2 + tan(x/2)^4/2

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)+csc(x))**5,x)

[Out] Timed out

3.303 $\int (\csc(x) - \sin(x))^4 dx$

Optimal. Leaf size=44

$$\frac{35x}{8} - \frac{35 \cot^3(x)}{24} + \frac{35 \cot(x)}{8} + \frac{1}{4} \cos^4(x) \cot^3(x) + \frac{7}{8} \cos^2(x) \cot^3(x)$$

[Out] 35/8*x+35/8*cot(x)-35/24*cot(x)^3+7/8*cos(x)^2*cot(x)^3+1/4*cos(x)^4*cot(x)^3

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {290, 325, 203}

$$\frac{35x}{8} - \frac{35 \cot^3(x)}{24} + \frac{35 \cot(x)}{8} + \frac{1}{4} \cos^4(x) \cot^3(x) + \frac{7}{8} \cos^2(x) \cot^3(x)$$

Antiderivative was successfully verified.

[In] Int[(Csc[x] - Sin[x])^4,x]

[Out] (35*x)/8 + (35*Cot[x])/8 - (35*Cot[x]^3)/24 + (7*Cos[x]^2*Cot[x]^3)/8 + (Cos[x]^4*Cot[x]^3)/4

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 325

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned}
\int (\csc(x) - \sin(x))^4 dx &= \text{Subst} \left(\int \frac{1}{x^4 (1+x^2)^3} dx, x, \tan(x) \right) \\
&= \frac{1}{4} \cos^4(x) \cot^3(x) + \frac{7}{4} \text{Subst} \left(\int \frac{1}{x^4 (1+x^2)^2} dx, x, \tan(x) \right) \\
&= \frac{7}{8} \cos^2(x) \cot^3(x) + \frac{1}{4} \cos^4(x) \cot^3(x) + \frac{35}{8} \text{Subst} \left(\int \frac{1}{x^4 (1+x^2)} dx, x, \tan(x) \right) \\
&= -\frac{35}{24} \cot^3(x) + \frac{7}{8} \cos^2(x) \cot^3(x) + \frac{1}{4} \cos^4(x) \cot^3(x) - \frac{35}{8} \text{Subst} \left(\int \frac{1}{x^2 (1+x^2)} dx, x, \tan(x) \right) \\
&= \frac{35 \cot(x)}{8} - \frac{35 \cot^3(x)}{24} + \frac{7}{8} \cos^2(x) \cot^3(x) + \frac{1}{4} \cos^4(x) \cot^3(x) + \frac{35}{8} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right) \\
&= \frac{35x}{8} + \frac{35 \cot(x)}{8} - \frac{35 \cot^3(x)}{24} + \frac{7}{8} \cos^2(x) \cot^3(x) + \frac{1}{4} \cos^4(x) \cot^3(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.86

$$\frac{35x}{8} + \frac{3}{4} \sin(2x) + \frac{1}{32} \sin(4x) + \frac{10 \cot(x)}{3} - \frac{1}{3} \cot(x) \csc^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^4,x]

[Out] (35*x)/8 + (10*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3 + (3*Sin[2*x])/4 + Sin[4*x]/32

fricas [A] time = 2.05, size = 51, normalized size = 1.16

$$-\frac{6 \cos(x)^7 + 21 \cos(x)^5 - 140 \cos(x)^3 - 105 (x \cos(x)^2 - x) \sin(x) + 105 \cos(x)}{24 (\cos(x)^2 - 1) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^4,x, algorithm="fricas")

[Out] -1/24*(6*cos(x)^7 + 21*cos(x)^5 - 140*cos(x)^3 - 105*(x*cos(x)^2 - x)*sin(x) + 105*cos(x))/((cos(x)^2 - 1)*sin(x))

giac [A] time = 0.13, size = 39, normalized size = 0.89

$$\frac{35}{8} x + \frac{11 \tan(x)^3 + 13 \tan(x)}{8 (\tan(x)^2 + 1)^2} + \frac{9 \tan(x)^2 - 1}{3 \tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^4,x, algorithm="giac")

[Out] 35/8*x + 1/8*(11*tan(x)^3 + 13*tan(x))/(tan(x)^2 + 1)^2 + 1/3*(9*tan(x)^2 - 1)/tan(x)^3

maple [A] time = 0.06, size = 39, normalized size = 0.89

$$-\frac{\left(\sin^3(x) + \frac{3 \sin(x)}{2}\right) \cos(x)}{4} + \frac{35x}{8} + 2 \cos(x) \sin(x) + 4 \cot(x) + \left(-\frac{2}{3} - \frac{\left(\csc^2(x)\right)}{3}\right) \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((csc(x)-sin(x))^4,x)`

[Out] $-1/4*(\sin(x)^3+3/2*\sin(x))*\cos(x)+35/8*x+2*\cos(x)*\sin(x)+4*\cot(x)+(-2/3-1/3)*\csc(x)^2*\cot(x)$

maxima [A] time = 0.31, size = 36, normalized size = 0.82

$$\frac{35}{8}x + \frac{4}{\tan(x)} - \frac{3 \tan(x)^2 + 1}{3 \tan(x)^3} + \frac{1}{32} \sin(4x) + \frac{3}{4} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((csc(x)-sin(x))^4,x, algorithm="maxima")`

[Out] $35/8*x + 4/\tan(x) - 1/3*(3*\tan(x)^2 + 1)/\tan(x)^3 + 1/32*\sin(4*x) + 3/4*\sin(2*x)$

mupad [B] time = 2.49, size = 59, normalized size = 1.34

$$\frac{\frac{\cos(x)^7}{4} + \frac{7 \cos(x)^5}{8} - \frac{35 \cos(x)^3}{6} + \frac{35 \cos(x)}{8}}{\sin(x) - \cos(x)^2 \sin(x)} - \frac{\frac{35x}{8} - \frac{35x \cos(x)^2}{8}}{\cos(x)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((sin(x) - 1/sin(x))^4,x)`

[Out] $((35*\cos(x))/8 - (35*\cos(x)^3)/6 + (7*\cos(x)^5)/8 + \cos(x)^7/4)/(\sin(x) - \cos(x)^2*\sin(x)) - ((35*x)/8 - (35*x*\cos(x)^2)/8)/(\cos(x)^2 - 1)$

sympy [A] time = 8.78, size = 44, normalized size = 1.00

$$\frac{35x}{8} + 2 \sin(x) \cos(x) - \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32} - \frac{\cot^3(x)}{3} - \cot(x) + \frac{4 \cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((csc(x)-sin(x))**4,x)`

[Out] $35*x/8 + 2*\sin(x)*\cos(x) - \sin(2*x)/4 + \sin(4*x)/32 - \cot(x)**3/3 - \cot(x) + 4*\cos(x)/\sin(x)$

3.304 $\int (\csc(x) - \sin(x))^3 dx$

Optimal. Leaf size=34

$$-\frac{5 \cos^3(x)}{6} - \frac{5 \cos(x)}{2} - \frac{1}{2} \cos^3(x) \cot^2(x) + \frac{5}{2} \tanh^{-1}(\cos(x))$$

[Out] 5/2*arctanh(cos(x))-5/2*cos(x)-5/6*cos(x)^3-1/2*cos(x)^3*cot(x)^2

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4397, 2592, 288, 302, 206}

$$-\frac{5 \cos^3(x)}{6} - \frac{5 \cos(x)}{2} - \frac{1}{2} \cos^3(x) \cot^2(x) + \frac{5}{2} \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[(Csc[x] - Sin[x])^3,x]

[Out] (5*ArcTanh[Cos[x]])/2 - (5*Cos[x])/2 - (5*Cos[x]^3)/6 - (Cos[x]^3*Cot[x]^2)/2

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*n*(p+1)), x] - Dist[(c^n*(m-n+1))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !IntegerQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a+b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n-1]

Rule 2592

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)^(n_.), x_Symbol] := With[{ff = FreeFactors[Sin[e+f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m+n)/(a^2-ff^2*x^2)^((n+1)/2), x], x, (a*Sin[e+f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n+1)/2]

Rule 4397

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned}
\int (\csc(x) - \sin(x))^3 dx &= \int \cos^3(x) \cot^3(x) dx \\
&= -\text{Subst} \left(\int \frac{x^6}{(1-x^2)^2} dx, x, \cos(x) \right) \\
&= -\frac{1}{2} \cos^3(x) \cot^2(x) + \frac{5}{2} \text{Subst} \left(\int \frac{x^4}{1-x^2} dx, x, \cos(x) \right) \\
&= -\frac{1}{2} \cos^3(x) \cot^2(x) + \frac{5}{2} \text{Subst} \left(\int \left(-1 - x^2 + \frac{1}{1-x^2} \right) dx, x, \cos(x) \right) \\
&= -\frac{5 \cos(x)}{2} - \frac{5 \cos^3(x)}{6} - \frac{1}{2} \cos^3(x) \cot^2(x) + \frac{5}{2} \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \cos(x) \right) \\
&= \frac{5}{2} \tanh^{-1}(\cos(x)) - \frac{5 \cos(x)}{2} - \frac{5 \cos^3(x)}{6} - \frac{1}{2} \cos^3(x) \cot^2(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 1.79

$$-\frac{9 \cos(x)}{4} - \frac{1}{12} \cos(3x) - \frac{1}{8} \csc^2\left(\frac{x}{2}\right) + \frac{1}{8} \sec^2\left(\frac{x}{2}\right) - \frac{5}{2} \log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{5}{2} \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^3, x]

[Out] (-9*Cos[x])/4 - Cos[3*x]/12 - Csc[x/2]^2/8 + (5*Log[Cos[x/2]])/2 - (5*Log[Sin[x/2]])/2 + Sec[x/2]^2/8

fricas [B] time = 1.77, size = 57, normalized size = 1.68

$$\frac{4 \cos(x)^5 + 20 \cos(x)^3 - 15 (\cos(x)^2 - 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 15 (\cos(x)^2 - 1) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) - 30 \cos(x)}{12 (\cos(x)^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^3,x, algorithm="fricas")

[Out] -1/12*(4*cos(x)^5 + 20*cos(x)^3 - 15*(cos(x)^2 - 1)*log(1/2*cos(x) + 1/2) + 15*(cos(x)^2 - 1)*log(-1/2*cos(x) + 1/2) - 30*cos(x))/(cos(x)^2 - 1)

giac [B] time = 0.16, size = 99, normalized size = 2.91

$$\frac{\left(\frac{10(\cos(x)-1)}{\cos(x)+1} + 1\right)(\cos(x)+1)}{8(\cos(x)-1)} - \frac{\cos(x)-1}{8(\cos(x)+1)} - \frac{2\left(\frac{12(\cos(x)-1)}{\cos(x)+1} - \frac{9(\cos(x)-1)^2}{(\cos(x)+1)^2} - 7\right)}{3\left(\frac{\cos(x)-1}{\cos(x)+1} - 1\right)^3} - \frac{5}{4} \log\left(\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^3,x, algorithm="giac")

[Out] 1/8*(10*(cos(x) - 1)/(cos(x) + 1) + 1)*(cos(x) + 1)/(cos(x) - 1) - 1/8*(cos(x) - 1)/(cos(x) + 1) - 2/3*(12*(cos(x) - 1)/(cos(x) + 1) - 9*(cos(x) - 1)^2/(cos(x) + 1)^2 - 7)/((cos(x) - 1)/(cos(x) + 1) - 1)^3 - 5/4*log(-(cos(x) - 1)/(cos(x) + 1))

maple [A] time = 0.06, size = 32, normalized size = 0.94

$$\frac{(2 + \sin^2(x)) \cos(x)}{3} - 3 \cos(x) - \frac{5 \ln(\csc(x) - \cot(x))}{2} - \frac{\cot(x) \csc(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((csc(x)-sin(x))^3,x)`

[Out] `1/3*(2+sin(x)^2)*cos(x)-3*cos(x)-5/2*ln(csc(x)-cot(x))-1/2*cot(x)*csc(x)`

maxima [A] time = 0.31, size = 37, normalized size = 1.09

$$-\frac{1}{3} \cos(x)^3 + \frac{\cos(x)}{2(\cos(x)^2 - 1)} - 2 \cos(x) + \frac{5}{4} \log(\cos(x) + 1) - \frac{5}{4} \log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((csc(x)-sin(x))^3,x, algorithm="maxima")`

[Out] `-1/3*cos(x)^3 + 1/2*cos(x)/(cos(x)^2 - 1) - 2*cos(x) + 5/4*log(cos(x) + 1) - 5/4*log(cos(x) - 1)`

mupad [B] time = 2.49, size = 75, normalized size = 2.21

$$\frac{\tan\left(\frac{x}{2}\right)^2}{8} - \frac{\frac{49 \tan\left(\frac{x}{2}\right)^6}{8} + \frac{67 \tan\left(\frac{x}{2}\right)^4}{8} + \frac{121 \tan\left(\frac{x}{2}\right)^2}{24} + \frac{1}{8}}{\tan\left(\frac{x}{2}\right)^8 + 3 \tan\left(\frac{x}{2}\right)^6 + 3 \tan\left(\frac{x}{2}\right)^4 + \tan\left(\frac{x}{2}\right)^2} - \frac{5 \ln\left(\tan\left(\frac{x}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(sin(x) - 1/sin(x))^3,x)`

[Out] `tan(x/2)^2/8 - ((121*tan(x/2)^2)/24 + (67*tan(x/2)^4)/8 + (49*tan(x/2)^6)/8 + 1/8)/(tan(x/2)^2 + 3*tan(x/2)^4 + 3*tan(x/2)^6 + tan(x/2)^8) - (5*log(tan(x/2)))/2`

sympy [A] time = 3.47, size = 42, normalized size = 1.24

$$-\frac{5 \log(\cos(x) - 1)}{4} + \frac{5 \log(\cos(x) + 1)}{4} - \frac{\cos^3(x)}{3} - 2 \cos(x) + \frac{\cos(x)}{2 \cos^2(x) - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((csc(x)-sin(x))**3,x)`

[Out] `-5*log(cos(x) - 1)/4 + 5*log(cos(x) + 1)/4 - cos(x)**3/3 - 2*cos(x) + cos(x)/(2*cos(x)**2 - 2)`

3.305 $\int (\csc(x) - \sin(x))^2 dx$

Optimal. Leaf size=22

$$-\frac{3x}{2} - \frac{3 \cot(x)}{2} + \frac{1}{2} \cos^2(x) \cot(x)$$

[Out] $-3/2*x-3/2*\cot(x)+1/2*\cos(x)^2*\cot(x)$

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {290, 325, 203}

$$-\frac{3x}{2} - \frac{3 \cot(x)}{2} + \frac{1}{2} \cos^2(x) \cot(x)$$

Antiderivative was successfully verified.

[In] Int[(Csc[x] - Sin[x])^2,x]

[Out] $(-3*x)/2 - (3*\cot[x])/2 + (\cos[x]^2*\cot[x])/2$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 290

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m+n*(p+1)+1)/(a*n*(p+1)), Int[(c*x)^m*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 325

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned} \int (\csc(x) - \sin(x))^2 dx &= \text{Subst} \left(\int \frac{1}{x^2(1+x^2)^2} dx, x, \tan(x) \right) \\ &= \frac{1}{2} \cos^2(x) \cot(x) + \frac{3}{2} \text{Subst} \left(\int \frac{1}{x^2(1+x^2)} dx, x, \tan(x) \right) \\ &= -\frac{3 \cot(x)}{2} + \frac{1}{2} \cos^2(x) \cot(x) - \frac{3}{2} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right) \\ &= -\frac{3x}{2} - \frac{3 \cot(x)}{2} + \frac{1}{2} \cos^2(x) \cot(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 0.82

$$-\frac{3x}{2} - \frac{1}{4} \sin(2x) - \cot(x)$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^2,x]

[Out] (-3*x)/2 - Cot[x] - Sin[2*x]/4

fricas [A] time = 1.33, size = 20, normalized size = 0.91

$$\frac{\cos(x)^3 - 3x \sin(x) - 3 \cos(x)}{2 \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^2,x, algorithm="fricas")

[Out] 1/2*(cos(x)^3 - 3*x*sin(x) - 3*cos(x))/sin(x)

giac [A] time = 0.15, size = 23, normalized size = 1.05

$$-\frac{3}{2}x - \frac{3 \tan(x)^2 + 2}{2(\tan(x)^3 + \tan(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^2,x, algorithm="giac")

[Out] -3/2*x - 1/2*(3*tan(x)^2 + 2)/(tan(x)^3 + tan(x))

maple [A] time = 0.05, size = 15, normalized size = 0.68

$$-\frac{\cos(x) \sin(x)}{2} - \frac{3x}{2} - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((csc(x)-sin(x))^2,x)

[Out] -1/2*cos(x)*sin(x)-3/2*x-cot(x)

maxima [A] time = 0.31, size = 16, normalized size = 0.73

$$-\frac{3}{2}x - \frac{1}{\tan(x)} - \frac{1}{4} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^2,x, algorithm="maxima")

[Out] -3/2*x - 1/tan(x) - 1/4*sin(2*x)

mupad [B] time = 2.40, size = 21, normalized size = 0.95

$$-\frac{3x}{2} - \frac{\frac{3 \cos(x)}{2} - \frac{\cos(x)^3}{2}}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x) - 1/sin(x))^2,x)

[Out] - (3*x)/2 - ((3*cos(x))/2 - cos(x)^3/2)/sin(x)

sympy [A] time = 1.57, size = 15, normalized size = 0.68

$$-\frac{3x}{2} - \frac{\sin(2x)}{4} - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**2,x)
```

```
[Out] -3*x/2 - sin(2*x)/4 - cot(x)
```


3.306 $\int (\csc(x) - \sin(x)) dx$

Optimal. Leaf size=8

$$\cos(x) - \tanh^{-1}(\cos(x))$$

[Out] $-\operatorname{arctanh}(\cos(x)) + \cos(x)$

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3770, 2638}

$$\cos(x) - \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csc}[x] - \operatorname{Sin}[x], x]$

[Out] $-\operatorname{ArcTanh}[\operatorname{Cos}[x]] + \operatorname{Cos}[x]$

Rule 2638

$\operatorname{Int}[\sin[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{Cos}[c + d*x]/d, x] /; \operatorname{FreeQ}[\{c, d\}, x]$

Rule 3770

$\operatorname{Int}[\csc[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \operatorname{FreeQ}[\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int (\csc(x) - \sin(x)) dx &= \int \csc(x) dx - \int \sin(x) dx \\ &= -\tanh^{-1}(\cos(x)) + \cos(x) \end{aligned}$$

Mathematica [B] time = 0.00, size = 19, normalized size = 2.38

$$\cos(x) + \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Integrate}[\operatorname{Csc}[x] - \operatorname{Sin}[x], x]$

[Out] $\operatorname{Cos}[x] - \operatorname{Log}[\operatorname{Cos}[x/2]] + \operatorname{Log}[\operatorname{Sin}[x/2]]$

fricas [B] time = 0.94, size = 21, normalized size = 2.62

$$\cos(x) - \frac{1}{2} \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + \frac{1}{2} \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(\csc(x) - \sin(x), x, \operatorname{algorithm} = \text{"fricas"})$

[Out] $\cos(x) - 1/2 * \log(1/2 * \cos(x) + 1/2) + 1/2 * \log(-1/2 * \cos(x) + 1/2)$

giac [A] time = 0.15, size = 9, normalized size = 1.12

$$\cos(x) + \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)-sin(x),x, algorithm="giac")

[Out] cos(x) + log(abs(tan(1/2*x)))

maple [A] time = 0.00, size = 12, normalized size = 1.50

$$\cos(x) - \ln(\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)-sin(x),x)

[Out] cos(x)-ln(cot(x)+csc(x))

maxima [A] time = 0.32, size = 11, normalized size = 1.38

$$\cos(x) - \log(\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)-sin(x),x, algorithm="maxima")

[Out] cos(x) - log(cot(x) + csc(x))

mupad [B] time = 0.02, size = 8, normalized size = 1.00

$$\ln\left(\tan\left(\frac{x}{2}\right)\right) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sin(x) - sin(x),x)

[Out] log(tan(x/2)) + cos(x)

sympy [B] time = 0.09, size = 19, normalized size = 2.38

$$\frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)-sin(x),x)

[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2 + cos(x)

$$3.307 \quad \int \frac{1}{\csc(x) - \sin(x)} dx$$

Optimal. Leaf size=2

$\sec(x)$

[Out] $\sec(x)$

Rubi [A] time = 0.02, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4397, 2606, 8}

$\sec(x)$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Csc}[x] - \text{Sin}[x])^{-1}, x]$

[Out] $\text{Sec}[x]$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2606

$\text{Int}[(a_)*\sec[(e_.) + (f_.)*(x_)]^{(m_.)}*((b_.)*\tan[(e_.) + (f_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[a/f, \text{Subst}[\text{Int}[(a*x)^{(m-1)}*(-1+x^2)^{(n-1)/2}, x], x, \text{Sec}[e+f*x]], x] /; \text{FreeQ}\{a, e, f, m\}, x] \&\& \text{IntegerQ}[(n-1)/2] \&\& \text{!(IntegerQ}[m/2] \&\& \text{LtQ}[0, m, n+1])$

Rule 4397

$\text{Int}[u_, x_Symbol] \rightarrow \text{Int}[\text{TrigSimplify}[u], x] /; \text{TrigSimplifyQ}[u]$

Rubi steps

$$\begin{aligned} \int \frac{1}{\csc(x) - \sin(x)} dx &= \int \sec(x) \tan(x) dx \\ &= \text{Subst}\left(\int 1 dx, x, \sec(x)\right) \\ &= \sec(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$\sec(x)$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(\text{Csc}[x] - \text{Sin}[x])^{-1}, x]$

[Out] $\text{Sec}[x]$

fricas [A] time = 2.20, size = 4, normalized size = 2.00

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(\csc(x) - \sin(x)), x, \text{algorithm} = \text{"fricas"})$

[Out] $1/\cos(x)$

giac [B] time = 0.14, size = 17, normalized size = 8.50

$$\frac{2}{\frac{\cos(x)-1}{\cos(x)+1} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(csc(x)-sin(x)),x, algorithm="giac")`

[Out] $2/((\cos(x) - 1)/(\cos(x) + 1) + 1)$

maple [A] time = 0.10, size = 5, normalized size = 2.50

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(csc(x)-sin(x)),x)`

[Out] $1/\cos(x)$

maxima [B] time = 0.31, size = 17, normalized size = 8.50

$$-\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(csc(x)-sin(x)),x, algorithm="maxima")`

[Out] $-2/(\sin(x)^2/(\cos(x) + 1)^2 - 1)$

mupad [B] time = 2.46, size = 12, normalized size = 6.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(sin(x) - 1/sin(x)),x)`

[Out] $-2/(\tan(x/2)^2 - 1)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{-\sin(x) + \csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(csc(x)-sin(x)),x)`

[Out] `Integral(1/(-sin(x) + csc(x)), x)`

$$3.308 \quad \int \frac{1}{(\csc(x) - \sin(x))^2} dx$$

Optimal. Leaf size=8

$$\frac{\tan^3(x)}{3}$$

[Out] 1/3*tan(x)^3

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {30}

$$\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[(Csc[x] - Sin[x])^(-2), x]

[Out] Tan[x]^3/3

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{1}{(\csc(x) - \sin(x))^2} dx = \text{Subst}\left(\int x^2 dx, x, \tan(x)\right) = \frac{\tan^3(x)}{3}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^(-2), x]

[Out] Tan[x]^3/3

fricas [B] time = 1.70, size = 14, normalized size = 1.75

$$\frac{(\cos(x)^2 - 1) \sin(x)}{3 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^2,x, algorithm="fricas")

[Out] -1/3*(cos(x)^2 - 1)*sin(x)/cos(x)^3

giac [A] time = 0.13, size = 6, normalized size = 0.75

$$\frac{1}{3} \tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^2,x, algorithm="giac")

[Out] 1/3*tan(x)^3

maple [A] time = 0.11, size = 7, normalized size = 0.88

$$\frac{(\tan^3(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(csc(x)-sin(x))^2,x)

[Out] 1/3*tan(x)^3

maxima [A] time = 0.33, size = 6, normalized size = 0.75

$$\frac{1}{3} \tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^2,x, algorithm="maxima")

[Out] 1/3*tan(x)^3

mupad [B] time = 2.41, size = 6, normalized size = 0.75

$$\frac{\tan(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x) - 1/sin(x))^2,x)

[Out] tan(x)^3/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\sin(x) + \csc(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))**2,x)

[Out] Integral((-sin(x) + csc(x))**(-2), x)

$$3.309 \quad \int \frac{1}{(\csc(x) - \sin(x))^3} dx$$

Optimal. Leaf size=17

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

[Out] $-1/3*\sec(x)^3+1/5*\sec(x)^5$

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4397, 2606, 14}

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[(Csc[x] - Sin[x])^(-3), x]

[Out] -Sec[x]^3/3 + Sec[x]^5/5

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 2606

Int[((a_)*sec[(e_)+(f_)*(x_)]^(m_))*((b_)*tan[(e_)+(f_)*(x_)]^(n_)), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m-1)*(-1+x^2)^((n-1)/2), x], x, Sec[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n+1])

Rule 4397

Int[u, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \frac{1}{(\csc(x) - \sin(x))^3} dx &= \int \sec^3(x) \tan^3(x) dx \\ &= \text{Subst} \left(\int x^2 (-1 + x^2) dx, x, \sec(x) \right) \\ &= \text{Subst} \left(\int (-x^2 + x^4) dx, x, \sec(x) \right) \\ &= -\frac{1}{3} \sec^3(x) + \frac{\sec^5(x)}{5} \end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^(-3), x]

[Out] $-1/3*\text{Sec}[x]^3 + \text{Sec}[x]^5/5$

fricas [A] time = 1.20, size = 14, normalized size = 0.82

$$-\frac{5 \cos(x)^2 - 3}{15 \cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(csc(x)-sin(x))^3,x, algorithm="fricas")`

[Out] $-1/15*(5*\cos(x)^2 - 3)/\cos(x)^5$

giac [B] time = 0.13, size = 59, normalized size = 3.47

$$-\frac{4 \left(\frac{5(\cos(x)-1)}{\cos(x)+1} - \frac{5(\cos(x)-1)^2}{(\cos(x)+1)^2} + \frac{15(\cos(x)-1)^3}{(\cos(x)+1)^3} + 1 \right)}{15 \left(\frac{\cos(x)-1}{\cos(x)+1} + 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(csc(x)-sin(x))^3,x, algorithm="giac")`

[Out] $-4/15*(5*(\cos(x) - 1)/(\cos(x) + 1) - 5*(\cos(x) - 1)^2/(\cos(x) + 1)^2 + 15*(\cos(x) - 1)^3/(\cos(x) + 1)^3 + 1)/((\cos(x) - 1)/(\cos(x) + 1) + 1)^5$

maple [A] time = 0.12, size = 14, normalized size = 0.82

$$-\frac{1}{3 \cos(x)^3} + \frac{1}{5 \cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(csc(x)-sin(x))^3,x)`

[Out] $-1/3/\cos(x)^3+1/5/\cos(x)^5$

maxima [B] time = 0.31, size = 103, normalized size = 6.06

$$-\frac{4 \left(\frac{5 \sin(x)^2}{(\cos(x)+1)^2} + \frac{5 \sin(x)^4}{(\cos(x)+1)^4} + \frac{15 \sin(x)^6}{(\cos(x)+1)^6} - 1 \right)}{15 \left(\frac{5 \sin(x)^2}{(\cos(x)+1)^2} - \frac{10 \sin(x)^4}{(\cos(x)+1)^4} + \frac{10 \sin(x)^6}{(\cos(x)+1)^6} - \frac{5 \sin(x)^8}{(\cos(x)+1)^8} + \frac{\sin(x)^{10}}{(\cos(x)+1)^{10}} - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(csc(x)-sin(x))^3,x, algorithm="maxima")`

[Out] $-4/15*(5*\sin(x)^2/(\cos(x) + 1)^2 + 5*\sin(x)^4/(\cos(x) + 1)^4 + 15*\sin(x)^6/(\cos(x) + 1)^6 - 1)/(5*\sin(x)^2/(\cos(x) + 1)^2 - 10*\sin(x)^4/(\cos(x) + 1)^4 + 10*\sin(x)^6/(\cos(x) + 1)^6 - 5*\sin(x)^8/(\cos(x) + 1)^8 + \sin(x)^{10}/(\cos(x) + 1)^{10} - 1)$

mupad [B] time = 2.55, size = 13, normalized size = 0.76

$$\frac{1}{5 \cos(x)^5} - \frac{1}{3 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(sin(x) - 1/sin(x))^3,x)`

[Out] $1/(5*\cos(x)^5) - 1/(3*\cos(x)^3)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\sin(x) + \csc(x))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))**3,x)

[Out] Integral((-sin(x) + csc(x))**(-3), x)

$$3.310 \quad \int \frac{1}{(\csc(x) - \sin(x))^4} dx$$

Optimal. Leaf size=17

$$\frac{\tan^7(x)}{7} + \frac{\tan^5(x)}{5}$$

[Out] 1/5*tan(x)^5+1/7*tan(x)^7

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{\tan^7(x)}{7} + \frac{\tan^5(x)}{5}$$

Antiderivative was successfully verified.

[In] Int[(Csc[x] - Sin[x])^(-4), x]

[Out] Tan[x]^5/5 + Tan[x]^7/7

Rubi steps

$$\begin{aligned} \int \frac{1}{(\csc(x) - \sin(x))^4} dx &= \text{Subst} \left(\int (x^4 + x^6) dx, x, \tan(x) \right) \\ &= \frac{\tan^5(x)}{5} + \frac{\tan^7(x)}{7} \end{aligned}$$

Mathematica [B] time = 0.02, size = 37, normalized size = 2.18

$$\frac{2 \tan(x)}{35} + \frac{1}{7} \tan(x) \sec^6(x) - \frac{8}{35} \tan(x) \sec^4(x) + \frac{1}{35} \tan(x) \sec^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^(-4), x]

[Out] (2*Tan[x])/35 + (Sec[x]^2*Tan[x])/35 - (8*Sec[x]^4*Tan[x])/35 + (Sec[x]^6*Tan[x])/7

fricas [A] time = 1.54, size = 26, normalized size = 1.53

$$\frac{(2 \cos(x)^6 + \cos(x)^4 - 8 \cos(x)^2 + 5) \sin(x)}{35 \cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^4,x, algorithm="fricas")

[Out] 1/35*(2*cos(x)^6 + cos(x)^4 - 8*cos(x)^2 + 5)*sin(x)/cos(x)^7

giac [A] time = 0.12, size = 13, normalized size = 0.76

$$\frac{1}{7} \tan(x)^7 + \frac{1}{5} \tan(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^4,x, algorithm="giac")

[Out] 1/7*tan(x)^7 + 1/5*tan(x)^5

maple [A] time = 0.11, size = 14, normalized size = 0.82

$$\frac{(\tan^5(x))}{5} + \frac{(\tan^7(x))}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(csc(x)-sin(x))^4,x)

[Out] 1/5*tan(x)^5+1/7*tan(x)^7

maxima [A] time = 0.31, size = 13, normalized size = 0.76

$$\frac{1}{7} \tan(x)^7 + \frac{1}{5} \tan(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^4,x, algorithm="maxima")

[Out] 1/7*tan(x)^7 + 1/5*tan(x)^5

mupad [B] time = 2.57, size = 23, normalized size = 1.35

$$\frac{2 \cos(x)^2 \sin(x)^5 + 5 \sin(x)^5}{35 \cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x) - 1/sin(x))^4,x)

[Out] (5*sin(x)^5 + 2*cos(x)^2*sin(x)^5)/(35*cos(x)^7)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\sin(x) + \csc(x))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))**4,x)

[Out] Integral((-sin(x) + csc(x))**(-4), x)

$$3.311 \quad \int \frac{1}{(\csc(x) - \sin(x))^5} dx$$

Optimal. Leaf size=25

$$\frac{\sec^9(x)}{9} - \frac{2 \sec^7(x)}{7} + \frac{\sec^5(x)}{5}$$

[Out] $1/5*\sec(x)^5-2/7*\sec(x)^7+1/9*\sec(x)^9$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4397, 2606, 270}

$$\frac{\sec^9(x)}{9} - \frac{2 \sec^7(x)}{7} + \frac{\sec^5(x)}{5}$$

Antiderivative was successfully verified.

[In] Int[(Csc[x] - Sin[x])^(-5), x]

[Out] Sec[x]^5/5 - (2*Sec[x]^7)/7 + Sec[x]^9/9

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 4397

Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \frac{1}{(\csc(x) - \sin(x))^5} dx &= \int \sec^5(x) \tan^5(x) dx \\ &= \text{Subst} \left(\int x^4 (-1 + x^2)^2 dx, x, \sec(x) \right) \\ &= \text{Subst} \left(\int (x^4 - 2x^6 + x^8) dx, x, \sec(x) \right) \\ &= \frac{\sec^5(x)}{5} - \frac{2 \sec^7(x)}{7} + \frac{\sec^9(x)}{9} \end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{\sec^9(x)}{9} - \frac{2 \sec^7(x)}{7} + \frac{\sec^5(x)}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^(-5), x]

[Out] $\sec(x)^5/5 - (2*\sec(x)^7)/7 + \sec(x)^9/9$

fricas [A] time = 0.97, size = 20, normalized size = 0.80

$$\frac{63 \cos(x)^4 - 90 \cos(x)^2 + 35}{315 \cos(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(csc(x)-sin(x))^5,x, algorithm="fricas")`

[Out] $1/315*(63*\cos(x)^4 - 90*\cos(x)^2 + 35)/\cos(x)^9$

giac [B] time = 0.15, size = 101, normalized size = 4.04

$$\frac{16 \left(\frac{9(\cos(x)-1)}{\cos(x)+1} + \frac{36(\cos(x)-1)^2}{(\cos(x)+1)^2} - \frac{126(\cos(x)-1)^3}{(\cos(x)+1)^3} + \frac{441(\cos(x)-1)^4}{(\cos(x)+1)^4} - \frac{315(\cos(x)-1)^5}{(\cos(x)+1)^5} + \frac{210(\cos(x)-1)^6}{(\cos(x)+1)^6} + 1 \right)}{315 \left(\frac{\cos(x)-1}{\cos(x)+1} + 1 \right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(csc(x)-sin(x))^5,x, algorithm="giac")`

[Out] $16/315*(9*(\cos(x) - 1)/(\cos(x) + 1) + 36*(\cos(x) - 1)^2/(\cos(x) + 1)^2 - 126*(\cos(x) - 1)^3/(\cos(x) + 1)^3 + 441*(\cos(x) - 1)^4/(\cos(x) + 1)^4 - 315*(\cos(x) - 1)^5/(\cos(x) + 1)^5 + 210*(\cos(x) - 1)^6/(\cos(x) + 1)^6 + 1)/((\cos(x) - 1)/(\cos(x) + 1) + 1)^9$

maple [A] time = 0.12, size = 20, normalized size = 0.80

$$-\frac{2}{7 \cos(x)^7} + \frac{1}{9 \cos(x)^9} + \frac{1}{5 \cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(csc(x)-sin(x))^5,x)`

[Out] $-2/7/\cos(x)^7+1/9/\cos(x)^9+1/5/\cos(x)^5$

maxima [B] time = 0.36, size = 187, normalized size = 7.48

$$\frac{16 \left(\frac{9 \sin(x)^2}{(\cos(x)+1)^2} - \frac{36 \sin(x)^4}{(\cos(x)+1)^4} - \frac{126 \sin(x)^6}{(\cos(x)+1)^6} - \frac{441 \sin(x)^8}{(\cos(x)+1)^8} - \frac{315 \sin(x)^{10}}{(\cos(x)+1)^{10}} - \frac{210 \sin(x)^{12}}{(\cos(x)+1)^{12}} - 1 \right)}{315 \left(\frac{9 \sin(x)^2}{(\cos(x)+1)^2} - \frac{36 \sin(x)^4}{(\cos(x)+1)^4} + \frac{84 \sin(x)^6}{(\cos(x)+1)^6} - \frac{126 \sin(x)^8}{(\cos(x)+1)^8} + \frac{126 \sin(x)^{10}}{(\cos(x)+1)^{10}} - \frac{84 \sin(x)^{12}}{(\cos(x)+1)^{12}} + \frac{36 \sin(x)^{14}}{(\cos(x)+1)^{14}} - \frac{9 \sin(x)^{16}}{(\cos(x)+1)^{16}} + \frac{\sin(x)^{18}}{(\cos(x)+1)^{18}} - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(csc(x)-sin(x))^5,x, algorithm="maxima")`

[Out] $16/315*(9*\sin(x)^2/(\cos(x) + 1)^2 - 36*\sin(x)^4/(\cos(x) + 1)^4 - 126*\sin(x)^6/(\cos(x) + 1)^6 - 441*\sin(x)^8/(\cos(x) + 1)^8 - 315*\sin(x)^{10}/(\cos(x) + 1)^{10} - 210*\sin(x)^{12}/(\cos(x) + 1)^{12} - 1)/(9*\sin(x)^2/(\cos(x) + 1)^2 - 36*\sin(x)^4/(\cos(x) + 1)^4 + 84*\sin(x)^6/(\cos(x) + 1)^6 - 126*\sin(x)^8/(\cos(x) + 1)^8 + 126*\sin(x)^{10}/(\cos(x) + 1)^{10} - 84*\sin(x)^{12}/(\cos(x) + 1)^{12} + 36*\sin(x)^{14}/(\cos(x) + 1)^{14} - 9*\sin(x)^{16}/(\cos(x) + 1)^{16} + \sin(x)^{18}/(\cos(x) + 1)^{18} - 1)$

mupad [B] time = 2.94, size = 19, normalized size = 0.76

$$\frac{1}{5 \cos(x)^5} - \frac{2}{7 \cos(x)^7} + \frac{1}{9 \cos(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(sin(x) - 1/sin(x))^5,x)`

[Out] `1/(5*cos(x)^5) - 2/(7*cos(x)^7) + 1/(9*cos(x)^9)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\sin(x) + \csc(x))^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(csc(x)-sin(x))**5,x)`

[Out] `Integral((-sin(x) + csc(x))**(-5), x)`

$$3.312 \quad \int \frac{1}{(\csc(x) - \sin(x))^6} dx$$

Optimal. Leaf size=25

$$\frac{\tan^{11}(x)}{11} + \frac{2 \tan^9(x)}{9} + \frac{\tan^7(x)}{7}$$

[Out] 1/7*tan(x)^7+2/9*tan(x)^9+1/11*tan(x)^11

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {270}

$$\frac{\tan^{11}(x)}{11} + \frac{2 \tan^9(x)}{9} + \frac{\tan^7(x)}{7}$$

Antiderivative was successfully verified.

[In] Int[(Csc[x] - Sin[x])^(-6), x]

[Out] Tan[x]^7/7 + (2*Tan[x]^9)/9 + Tan[x]^11/11

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(\csc(x) - \sin(x))^6} dx &= \text{Subst} \left(\int x^6 (1 + x^2)^2 dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int (x^6 + 2x^8 + x^{10}) dx, x, \tan(x) \right) \\ &= \frac{\tan^7(x)}{7} + \frac{2 \tan^9(x)}{9} + \frac{\tan^{11}(x)}{11} \end{aligned}$$

Mathematica [B] time = 0.02, size = 57, normalized size = 2.28

$$-\frac{8 \tan(x)}{693} + \frac{1}{11} \tan(x) \sec^{10}(x) - \frac{23}{99} \tan(x) \sec^8(x) + \frac{113}{693} \tan(x) \sec^6(x) - \frac{1}{231} \tan(x) \sec^4(x) - \frac{4}{693} \tan(x) \sec^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^(-6), x]

[Out] (-8*Tan[x])/693 - (4*Sec[x]^2*Tan[x])/693 - (Sec[x]^4*Tan[x])/231 + (113*Sec[x]^6*Tan[x])/693 - (23*Sec[x]^8*Tan[x])/99 + (Sec[x]^10*Tan[x])/11

fricas [B] time = 0.86, size = 40, normalized size = 1.60

$$\frac{(8 \cos(x)^{10} + 4 \cos(x)^8 + 3 \cos(x)^6 - 113 \cos(x)^4 + 161 \cos(x)^2 - 63) \sin(x)}{693 \cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^6,x, algorithm="fricas")

[Out] -1/693*(8*cos(x)^10 + 4*cos(x)^8 + 3*cos(x)^6 - 113*cos(x)^4 + 161*cos(x)^2 - 63)*sin(x)/cos(x)^11

giac [A] time = 0.13, size = 19, normalized size = 0.76

$$\frac{1}{11} \tan(x)^{11} + \frac{2}{9} \tan(x)^9 + \frac{1}{7} \tan(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^6,x, algorithm="giac")

[Out] 1/11*tan(x)^11 + 2/9*tan(x)^9 + 1/7*tan(x)^7

maple [A] time = 0.12, size = 20, normalized size = 0.80

$$\frac{(\tan^7(x))}{7} + \frac{2(\tan^9(x))}{9} + \frac{(\tan^{11}(x))}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(csc(x)-sin(x))^6,x)

[Out] 1/7*tan(x)^7+2/9*tan(x)^9+1/11*tan(x)^11

maxima [A] time = 0.35, size = 19, normalized size = 0.76

$$\frac{1}{11} \tan(x)^{11} + \frac{2}{9} \tan(x)^9 + \frac{1}{7} \tan(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^6,x, algorithm="maxima")

[Out] 1/11*tan(x)^11 + 2/9*tan(x)^9 + 1/7*tan(x)^7

mupad [B] time = 2.91, size = 33, normalized size = 1.32

$$\frac{8 \cos(x)^4 \sin(x)^7 + 28 \cos(x)^2 \sin(x)^7 + 63 \sin(x)^7}{693 \cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x) - 1/sin(x))^6,x)

[Out] (63*sin(x)^7 + 28*cos(x)^2*sin(x)^7 + 8*cos(x)^4*sin(x)^7)/(693*cos(x)^11)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\sin(x) + \csc(x))^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))**6,x)

[Out] Integral((-sin(x) + csc(x))**(-6), x)

$$3.313 \quad \int \frac{1}{(\csc(x) - \sin(x))^7} dx$$

Optimal. Leaf size=33

$$\frac{\sec^{13}(x)}{13} - \frac{3 \sec^{11}(x)}{11} + \frac{\sec^9(x)}{3} - \frac{\sec^7(x)}{7}$$

[Out] $-1/7*\sec(x)^7+1/3*\sec(x)^9-3/11*\sec(x)^{11}+1/13*\sec(x)^{13}$

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4397, 2606, 270}

$$\frac{\sec^{13}(x)}{13} - \frac{3 \sec^{11}(x)}{11} + \frac{\sec^9(x)}{3} - \frac{\sec^7(x)}{7}$$

Antiderivative was successfully verified.

[In] Int[(Csc[x] - Sin[x])^(-7), x]

[Out] $-\text{Sec}[x]^7/7 + \text{Sec}[x]^9/3 - (3*\text{Sec}[x]^11)/11 + \text{Sec}[x]^13/13$

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 4397

Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \frac{1}{(\csc(x) - \sin(x))^7} dx &= \int \sec^7(x) \tan^7(x) dx \\ &= \text{Subst} \left(\int x^6 (-1 + x^2)^3 dx, x, \sec(x) \right) \\ &= \text{Subst} \left(\int (-x^6 + 3x^8 - 3x^{10} + x^{12}) dx, x, \sec(x) \right) \\ &= -\frac{1}{7} \sec^7(x) + \frac{\sec^9(x)}{3} - \frac{3 \sec^{11}(x)}{11} + \frac{\sec^{13}(x)}{13} \end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{\sec^{13}(x)}{13} - \frac{3 \sec^{11}(x)}{11} + \frac{\sec^9(x)}{3} - \frac{\sec^7(x)}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^(-7), x]

[Out] $-1/7*\text{Sec}[x]^7 + \text{Sec}[x]^9/3 - (3*\text{Sec}[x]^11)/11 + \text{Sec}[x]^13/13$

fricas [A] time = 0.65, size = 26, normalized size = 0.79

$$-\frac{429 \cos(x)^6 - 1001 \cos(x)^4 + 819 \cos(x)^2 - 231}{3003 \cos(x)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(csc(x)-sin(x))^7,x, algorithm="fricas")`

[Out] $-1/3003*(429*\cos(x)^6 - 1001*\cos(x)^4 + 819*\cos(x)^2 - 231)/\cos(x)^{13}$

giac [B] time = 0.14, size = 143, normalized size = 4.33

$$\frac{32 \left(\frac{13(\cos(x)-1)}{\cos(x)+1} + \frac{78(\cos(x)-1)^2}{(\cos(x)+1)^2} + \frac{286(\cos(x)-1)^3}{(\cos(x)+1)^3} - \frac{2288(\cos(x)-1)^4}{(\cos(x)+1)^4} + \frac{10296(\cos(x)-1)^5}{(\cos(x)+1)^5} - \frac{16302(\cos(x)-1)^6}{(\cos(x)+1)^6} + \frac{18018(\cos(x)-1)^7}{(\cos(x)+1)^7} \right)}{3003 \left(\frac{\cos(x)-1}{\cos(x)+1} + 1 \right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(csc(x)-sin(x))^7,x, algorithm="giac")`

[Out] $-32/3003*(13*(\cos(x) - 1)/(\cos(x) + 1) + 78*(\cos(x) - 1)^2/(\cos(x) + 1)^2 + 286*(\cos(x) - 1)^3/(\cos(x) + 1)^3 - 2288*(\cos(x) - 1)^4/(\cos(x) + 1)^4 + 10296*(\cos(x) - 1)^5/(\cos(x) + 1)^5 - 16302*(\cos(x) - 1)^6/(\cos(x) + 1)^6 + 18018*(\cos(x) - 1)^7/(\cos(x) + 1)^7 - 9009*(\cos(x) - 1)^8/(\cos(x) + 1)^8 + 3003*(\cos(x) - 1)^9/(\cos(x) + 1)^9 + 1)/((\cos(x) - 1)/(\cos(x) + 1) + 1)^{13}$

maple [A] time = 0.14, size = 26, normalized size = 0.79

$$\frac{1}{13 \cos(x)^{13}} - \frac{1}{7 \cos(x)^7} + \frac{1}{3 \cos(x)^9} - \frac{3}{11 \cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(csc(x)-sin(x))^7,x)`

[Out] $1/13/\cos(x)^{13}-1/7/\cos(x)^7+1/3/\cos(x)^9-3/11/\cos(x)^{11}$

maxima [B] time = 0.34, size = 271, normalized size = 8.21

$$\frac{32 \left(\frac{13 \sin(x)^2}{(\cos(x)+1)^2} - \frac{78 \sin(x)^4}{(\cos(x)+1)^4} + \frac{286 \sin(x)^6}{(\cos(x)+1)^6} + \frac{2288 \sin(x)^8}{(\cos(x)+1)^8} + \frac{10296 \sin(x)^{10}}{(\cos(x)+1)^{10}} + \frac{16302 \sin(x)^{12}}{(\cos(x)+1)^{12}} + \frac{18018 \sin(x)^{14}}{(\cos(x)+1)^{14}} \right)}{3003 \left(\frac{13 \sin(x)^2}{(\cos(x)+1)^2} - \frac{78 \sin(x)^4}{(\cos(x)+1)^4} + \frac{286 \sin(x)^6}{(\cos(x)+1)^6} - \frac{715 \sin(x)^8}{(\cos(x)+1)^8} + \frac{1287 \sin(x)^{10}}{(\cos(x)+1)^{10}} - \frac{1716 \sin(x)^{12}}{(\cos(x)+1)^{12}} + \frac{1716 \sin(x)^{14}}{(\cos(x)+1)^{14}} - \frac{1287 \sin(x)^{16}}{(\cos(x)+1)^{16}} + \frac{715 \sin(x)^{18}}{(\cos(x)+1)^{18}} - \frac{286 \sin(x)^{20}}{(\cos(x)+1)^{20}} + \frac{78 \sin(x)^{22}}{(\cos(x)+1)^{22}} - \frac{13 \sin(x)^{24}}{(\cos(x)+1)^{24}} + \sin(x)^{26}/(\cos(x) + 1)^{26} - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(csc(x)-sin(x))^7,x, algorithm="maxima")`

[Out] $-32/3003*(13*\sin(x)^2/(\cos(x) + 1)^2 - 78*\sin(x)^4/(\cos(x) + 1)^4 + 286*\sin(x)^6/(\cos(x) + 1)^6 + 2288*\sin(x)^8/(\cos(x) + 1)^8 + 10296*\sin(x)^{10}/(\cos(x) + 1)^{10} + 16302*\sin(x)^{12}/(\cos(x) + 1)^{12} + 18018*\sin(x)^{14}/(\cos(x) + 1)^{14} + 9009*\sin(x)^{16}/(\cos(x) + 1)^{16} + 3003*\sin(x)^{18}/(\cos(x) + 1)^{18} - 1)/(13*\sin(x)^2/(\cos(x) + 1)^2 - 78*\sin(x)^4/(\cos(x) + 1)^4 + 286*\sin(x)^6/(\cos(x) + 1)^6 - 715*\sin(x)^8/(\cos(x) + 1)^8 + 1287*\sin(x)^{10}/(\cos(x) + 1)^{10} - 1716*\sin(x)^{12}/(\cos(x) + 1)^{12} + 1716*\sin(x)^{14}/(\cos(x) + 1)^{14} - 1287*\sin(x)^{16}/(\cos(x) + 1)^{16} + 715*\sin(x)^{18}/(\cos(x) + 1)^{18} - 286*\sin(x)^{20}/(\cos(x) + 1)^{20} + 78*\sin(x)^{22}/(\cos(x) + 1)^{22} - 13*\sin(x)^{24}/(\cos(x) + 1)^{24} + \sin(x)^{26}/(\cos(x) + 1)^{26} - 1)$

mupad [B] time = 3.54, size = 25, normalized size = 0.76

$$\frac{1}{3 \cos(x)^9} - \frac{1}{7 \cos(x)^7} - \frac{3}{11 \cos(x)^{11}} + \frac{1}{13 \cos(x)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(sin(x) - 1/sin(x))^7,x)

[Out] 1/(3*cos(x)^9) - 1/(7*cos(x)^7) - 3/(11*cos(x)^11) + 1/(13*cos(x)^13)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))**7,x)

[Out] Timed out

3.314 $\int (\csc(x) - \sin(x))^{7/2} dx$

Optimal. Leaf size=73

$$\frac{2}{7} \cos^3(x) \cot^2(x) \sqrt{\cos(x) \cot(x)} + \frac{8}{7} \cos(x) \cot^2(x) \sqrt{\cos(x) \cot(x)} - \frac{64}{35} \cot(x) \csc(x) \sqrt{\cos(x) \cot(x)} + \frac{256}{35} \sec(x) \sqrt{\cos(x) \cot(x)}$$

[Out] $8/7*\cos(x)*\cot(x)^2*(\cos(x)*\cot(x))^{(1/2)}+2/7*\cos(x)^3*\cot(x)^2*(\cos(x)*\cot(x))^{(1/2)}-64/35*\cot(x)*\csc(x)*(\cos(x)*\cot(x))^{(1/2)}+256/35*\sec(x)*(\cos(x)*\cot(x))^{(1/2)}$

Rubi [A] time = 0.15, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {4397, 4400, 2598, 2594, 2589}

$$\frac{2}{7} \cos^3(x) \cot^2(x) \sqrt{\cos(x) \cot(x)} + \frac{8}{7} \cos(x) \cot^2(x) \sqrt{\cos(x) \cot(x)} - \frac{64}{35} \cot(x) \csc(x) \sqrt{\cos(x) \cot(x)} + \frac{256}{35} \sec(x) \sqrt{\cos(x) \cot(x)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Csc}[x] - \text{Sin}[x])^{(7/2)}, x]$

[Out] $(8*\text{Cos}[x]*\text{Cot}[x]^2*\text{Sqrt}[\text{Cos}[x]*\text{Cot}[x]])/7 + (2*\text{Cos}[x]^3*\text{Cot}[x]^2*\text{Sqrt}[\text{Cos}[x]*\text{Cot}[x]])/7 - (64*\text{Cot}[x]*\text{Sqrt}[\text{Cos}[x]*\text{Cot}[x]]*\text{Csc}[x])/35 + (256*\text{Sqrt}[\text{Cos}[x]*\text{Cot}[x]]*\text{Sec}[x])/35$

Rule 2589

$\text{Int}[(a_* \sin[e_*] + (f_*)(x_*))^{(m_*)} * ((b_*) \tan[e_*] + (f_*)(x_*))^{(n_*)}, x_Symbol] \rightarrow -\text{Simp}[(b*(a*\text{Sin}[e + f*x])^m*(b*\text{Tan}[e + f*x])^{(n-1)})/(f*m), x] /;$ FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]

Rule 2594

$\text{Int}[(a_* \sin[e_*] + (f_*)(x_*))^{(m_*)} * ((b_*) \tan[e_*] + (f_*)(x_*))^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*(a*\text{Sin}[e + f*x])^m*(b*\text{Tan}[e + f*x])^{(n-1)})/(f*(n-1)), x] - \text{Dist}[(b^2*(m+n-1))/(n-1), \text{Int}[(a*\text{Sin}[e + f*x])^m*(b*\text{Tan}[e + f*x])^{(n-2)}, x], x] /;$ FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && IntegersQ[2*m, 2*n] && !(GtQ[m, 1] && !IntegerQ[(m-1)/2])

Rule 2598

$\text{Int}[(a_* \sin[e_*] + (f_*)(x_*))^{(m_*)} * ((b_*) \tan[e_*] + (f_*)(x_*))^{(n_*)}, x_Symbol] \rightarrow -\text{Simp}[(b*(a*\text{Sin}[e + f*x])^m*(b*\text{Tan}[e + f*x])^{(n-1)})/(f*m), x] + \text{Dist}[(a^2*(m+n-1))/m, \text{Int}[(a*\text{Sin}[e + f*x])^{(m-2)}*(b*\text{Tan}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]

Rule 4397

$\text{Int}[u_*, x_Symbol] \rightarrow \text{Int}[\text{TrigSimplify}[u], x] /;$ TrigSimplifyQ[u]

Rule 4400

$\text{Int}[(u_*) * ((v_*)^{(m_*)} * (w_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{With}\{\{uu = \text{ActivateTrig}[u], vv = \text{ActivateTrig}[v], ww = \text{ActivateTrig}[w]\}, \text{Dist}[(vv^m*ww^n)^{\text{FracPart}[p]} / (vv^{(m*\text{FracPart}[p])} * ww^{(n*\text{FracPart}[p])}), \text{Int}[uu*vv^{(m*p)}*ww^{(n*p)}, x], x] /;$ FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])

Rubi steps

$$\begin{aligned}
\int (\csc(x) - \sin(x))^{7/2} dx &= \int (\cos(x) \cot(x))^{7/2} dx \\
&= \frac{\sqrt{\cos(x) \cot(x)} \int \cos^{\frac{7}{2}}(x) \cot^{\frac{7}{2}}(x) dx}{\sqrt{\cos(x)} \sqrt{\cot(x)}} \\
&= \frac{2}{7} \cos^3(x) \cot^2(x) \sqrt{\cos(x) \cot(x)} + \frac{(12\sqrt{\cos(x) \cot(x)}) \int \cos^{\frac{3}{2}}(x) \cot^{\frac{7}{2}}(x) dx}{7\sqrt{\cos(x)} \sqrt{\cot(x)}} \\
&= \frac{8}{7} \cos(x) \cot^2(x) \sqrt{\cos(x) \cot(x)} + \frac{2}{7} \cos^3(x) \cot^2(x) \sqrt{\cos(x) \cot(x)} + \frac{(32\sqrt{\cos(x) \cot(x)}) \int \cos^{\frac{1}{2}}(x) \cot^{\frac{7}{2}}(x) dx}{7\sqrt{\cos(x)} \sqrt{\cot(x)}} \\
&= \frac{8}{7} \cos(x) \cot^2(x) \sqrt{\cos(x) \cot(x)} + \frac{2}{7} \cos^3(x) \cot^2(x) \sqrt{\cos(x) \cot(x)} - \frac{64}{35} \cot(x) \sqrt{\cos(x) \cot(x)} \\
&= \frac{8}{7} \cos(x) \cot^2(x) \sqrt{\cos(x) \cot(x)} + \frac{2}{7} \cos^3(x) \cot^2(x) \sqrt{\cos(x) \cot(x)} - \frac{64}{35} \cot(x) \sqrt{\cos(x) \cot(x)}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 37, normalized size = 0.51

$$-\frac{1}{70} \sec(x) \sqrt{\cos(x) \cot(x)} (115 \cos^2(x) + 5 \cos(3x) \cos(x) + 28 \cot^2(x) - 512)$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^(7/2), x]

[Out] -1/70*(Sqrt[Cos[x]*Cot[x]]*(-512 + 115*Cos[x]^2 + 5*Cos[x]*Cos[3*x] + 28*Cot[x]^2)*Sec[x])

fricas [A] time = 1.00, size = 44, normalized size = 0.60

$$\frac{2(5 \cos(x)^6 + 20 \cos(x)^4 - 160 \cos(x)^2 + 128) \sqrt{\frac{\cos(x)^2}{\sin(x)}}}{35(\cos(x)^3 - \cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^(7/2), x, algorithm="fricas")

[Out] -2/35*(5*cos(x)^6 + 20*cos(x)^4 - 160*cos(x)^2 + 128)*sqrt(cos(x)^2/sin(x))/(cos(x)^3 - cos(x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\csc(x) - \sin(x))^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^(7/2), x, algorithm="giac")

[Out] integrate((csc(x) - sin(x))^(7/2), x)

maple [A] time = 0.31, size = 40, normalized size = 0.55

$$\frac{2(5(\cos^6(x)) + 20(\cos^4(x)) - 160(\cos^2(x)) + 128) \sin(x) \left(\frac{\cos^2(x)}{\sin(x)}\right)^{\frac{7}{2}}}{35 \cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((csc(x)-sin(x))^(7/2),x)`

[Out] $2/35*(5*\cos(x)^6+20*\cos(x)^4-160*\cos(x)^2+128)*\sin(x)*(\cos(x)^2/\sin(x))^{(7/2)}/\cos(x)^7$

maxima [B] time = 0.57, size = 578, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((csc(x)-sin(x))^(7/2),x, algorithm="maxima")`

[Out] $-1/280*(\cos(x)^2 + \sin(x)^2 + 2*\cos(x) + 1)^{(1/4)}*(\cos(x)^2 + \sin(x)^2 - 2*\cos(x) + 1)^{(1/4)}*((5*\cos(21/2*x) + 105*\cos(17/2*x) - 2275*\cos(13/2*x) + 5817*\cos(9/2*x) - 5*\cos(7/2*x) - 5817*\cos(5/2*x) - 105*\cos(3/2*x) + 2275*\cos(1/2*x) - 5*\sin(21/2*x) - 105*\sin(17/2*x) + 2275*\sin(13/2*x) - 5817*\sin(9/2*x) - 5*\sin(7/2*x) + 5817*\sin(5/2*x) - 105*\sin(3/2*x) - 2275*\sin(1/2*x))*\cos(7/2*\arctan2(\sin(x), \cos(x) - 1)) + (5*\cos(21/2*x) + 105*\cos(17/2*x) - 2275*\cos(13/2*x) + 5817*\cos(9/2*x) - 5*\cos(7/2*x) - 5817*\cos(5/2*x) - 105*\cos(3/2*x) + 2275*\cos(1/2*x) + 5*\sin(21/2*x) + 105*\sin(17/2*x) - 2275*\sin(13/2*x) + 5817*\sin(9/2*x) + 5*\sin(7/2*x) - 5817*\sin(5/2*x) + 105*\sin(3/2*x) + 2275*\sin(1/2*x))*\sin(7/2*\arctan2(\sin(x), \cos(x) - 1)))*\cos(7/2*\arctan2(\sin(x), \cos(x) + 1)) + ((5*\cos(21/2*x) + 105*\cos(17/2*x) - 2275*\cos(13/2*x) + 5817*\cos(9/2*x) - 5*\cos(7/2*x) - 5817*\cos(5/2*x) - 105*\cos(3/2*x) + 2275*\cos(1/2*x) + 5*\sin(21/2*x) + 105*\sin(17/2*x) - 2275*\sin(13/2*x) + 5817*\sin(9/2*x) + 5*\sin(7/2*x) - 5817*\sin(5/2*x) + 105*\sin(3/2*x) + 2275*\sin(1/2*x))*\cos(7/2*\arctan2(\sin(x), \cos(x) - 1)) - (5*\cos(21/2*x) + 105*\cos(17/2*x) - 2275*\cos(13/2*x) + 5817*\cos(9/2*x) - 5*\cos(7/2*x) - 5817*\cos(5/2*x) - 105*\cos(3/2*x) + 2275*\cos(1/2*x) - 5*\sin(21/2*x) - 105*\sin(17/2*x) + 2275*\sin(13/2*x) - 5817*\sin(9/2*x) - 5*\sin(7/2*x) + 5817*\sin(5/2*x) - 105*\sin(3/2*x) - 2275*\sin(1/2*x))*\sin(7/2*\arctan2(\sin(x), \cos(x) - 1)))*\sin(7/2*\arctan2(\sin(x), \cos(x) + 1)))/(\cos(x)^8 + \sin(x)^8 + 4*(\cos(x)^2 + 1)*\sin(x)^6 - 4*\cos(x)^6 + 2*(3*\cos(x)^4 + 2*\cos(x)^2 + 3)*\sin(x)^4 + 6*\cos(x)^4 + 4*(\cos(x)^6 - \cos(x)^4 - \cos(x)^2 + 1)*\sin(x)^2 - 4*\cos(x)^2 + 1)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\sin(x)} - \sin(x) \right)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/sin(x) - sin(x))^(7/2),x)`

[Out] `int((1/sin(x) - sin(x))^(7/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((csc(x)-sin(x))**(7/2),x)`

[Out] Timed out

3.315 $\int (\csc(x) - \sin(x))^{5/2} dx$

Optimal. Leaf size=50

$$\frac{2}{5} \cos^2(x) \cot(x) \sqrt{\cos(x) \cot(x)} - \frac{16}{15} \cot(x) \sqrt{\cos(x) \cot(x)} - \frac{64}{15} \tan(x) \sqrt{\cos(x) \cot(x)}$$

[Out] $-16/15*\cot(x)*(\cos(x)*\cot(x))^{(1/2)}+2/5*\cos(x)^2*\cot(x)*(\cos(x)*\cot(x))^{(1/2)}-64/15*(\cos(x)*\cot(x))^{(1/2)}*\tan(x)$

Rubi [A] time = 0.11, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {4397, 4400, 2598, 2594, 2589}

$$\frac{2}{5} \cos^2(x) \cot(x) \sqrt{\cos(x) \cot(x)} - \frac{16}{15} \cot(x) \sqrt{\cos(x) \cot(x)} - \frac{64}{15} \tan(x) \sqrt{\cos(x) \cot(x)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Csc}[x] - \text{Sin}[x])^{(5/2)}, x]$

[Out] $(-16*\text{Cot}[x]*\text{Sqrt}[\text{Cos}[x]*\text{Cot}[x]])/15 + (2*\text{Cos}[x]^2*\text{Cot}[x]*\text{Sqrt}[\text{Cos}[x]*\text{Cot}[x]])/5 - (64*\text{Sqrt}[\text{Cos}[x]*\text{Cot}[x]]*\text{Tan}[x])/15$

Rule 2589

$\text{Int}[(a_*\sin[e_*] + (f_*)(x_*))^{(m_*)}*((b_*)\tan[e_*] + (f_*)(x_*))^{(n_*)}, x_Symbol] :> -\text{Simp}[(b*(a*\text{Sin}[e + f*x])^m*(b*\text{Tan}[e + f*x])^{(n - 1)})/(f*m), x] /; \text{FreeQ}\{a, b, e, f, m, n\}, x] \ \&\& \ \text{EqQ}[m + n - 1, 0]$

Rule 2594

$\text{Int}[(a_*\sin[e_*] + (f_*)(x_*))^{(m_*)}*((b_*)\tan[e_*] + (f_*)(x_*))^{(n_*)}, x_Symbol] :> \text{Simp}[(b*(a*\text{Sin}[e + f*x])^m*(b*\text{Tan}[e + f*x])^{(n - 1)})/(f*(n - 1)), x] - \text{Dist}[(b^2*(m + n - 1))/(n - 1), \text{Int}[(a*\text{Sin}[e + f*x])^m*(b*\text{Tan}[e + f*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{a, b, e, f, m\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegersQ}[2*m, 2*n] \ \&\& \ !(\text{GtQ}[m, 1] \ \&\& \ !\text{IntegerQ}[(m - 1)/2])$

Rule 2598

$\text{Int}[(a_*\sin[e_*] + (f_*)(x_*))^{(m_*)}*((b_*)\tan[e_*] + (f_*)(x_*))^{(n_*)}, x_Symbol] :> -\text{Simp}[(b*(a*\text{Sin}[e + f*x])^m*(b*\text{Tan}[e + f*x])^{(n - 1)})/(f*m), x] + \text{Dist}[(a^2*(m + n - 1))/m, \text{Int}[(a*\text{Sin}[e + f*x])^{(m - 2)}*(b*\text{Tan}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, e, f, n\}, x] \ \&\& \ (\text{GtQ}[m, 1] \ || \ (\text{EqQ}[m, 1] \ \&\& \ \text{EqQ}[n, 1/2])) \ \&\& \ \text{IntegersQ}[2*m, 2*n]$

Rule 4397

$\text{Int}[u_, x_Symbol] :> \text{Int}[\text{TrigSimplify}[u], x] /; \text{TrigSimplifyQ}[u]$

Rule 4400

$\text{Int}[(u_)*((v_)^{(m_*)}*(w_)^{(n_*)})^{(p_*)}, x_Symbol] :> \text{With}\{\text{uu} = \text{ActivateTrig}[u], \text{vv} = \text{ActivateTrig}[v], \text{ww} = \text{ActivateTrig}[w]\}, \text{Dist}[(\text{vv}^m*\text{ww}^n)^{\text{FracPart}[p]}/(\text{vv}^{(m*\text{FracPart}[p])}*\text{ww}^{(n*\text{FracPart}[p])}), \text{Int}[\text{uu}*\text{vv}^{(m*p)}*\text{ww}^{(n*p)}, x], x] /; \text{FreeQ}\{m, n, p\}, x] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ (!\text{InertTrigFreeQ}[v] \ || \ !\text{InertTrigFreeQ}[w])$

Rubi steps

$$\begin{aligned}
\int (\csc(x) - \sin(x))^{5/2} dx &= \int (\cos(x) \cot(x))^{5/2} dx \\
&= \frac{\sqrt{\cos(x) \cot(x)} \int \cos^5(x) \cot^5(x) dx}{\sqrt{\cos(x)} \sqrt{\cot(x)}} \\
&= \frac{2}{5} \cos^2(x) \cot(x) \sqrt{\cos(x) \cot(x)} + \frac{(8\sqrt{\cos(x) \cot(x)}) \int \sqrt{\cos(x)} \cot^5(x) dx}{5\sqrt{\cos(x)} \sqrt{\cot(x)}} \\
&= -\frac{16}{15} \cot(x) \sqrt{\cos(x) \cot(x)} + \frac{2}{5} \cos^2(x) \cot(x) \sqrt{\cos(x) \cot(x)} - \frac{(32\sqrt{\cos(x) \cot(x)}) \int \sqrt{\cos(x)} \cot^3(x) dx}{15\sqrt{\cos(x)}} \\
&= -\frac{16}{15} \cot(x) \sqrt{\cos(x) \cot(x)} + \frac{2}{5} \cos^2(x) \cot(x) \sqrt{\cos(x) \cot(x)} - \frac{64}{15} \sqrt{\cos(x) \cot(x)} \tan(x)
\end{aligned}$$

Mathematica [A] time = 0.08, size = 29, normalized size = 0.58

$$-\frac{2}{15} \tan(x) \sqrt{\cos(x) \cot(x)} (3 \cos^2(x) + 5 \cot^2(x) + 32)$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^(5/2), x]

[Out] (-2*Sqrt[Cos[x]*Cot[x]]*(32 + 3*Cos[x]^2 + 5*Cot[x]^2)*Tan[x])/15

fricas [A] time = 1.72, size = 35, normalized size = 0.70

$$\frac{2(3 \cos(x)^4 + 24 \cos(x)^2 - 32) \sqrt{\frac{\cos(x)^2}{\sin(x)}}}{15 \cos(x) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^(5/2), x, algorithm="fricas")

[Out] 2/15*(3*cos(x)^4 + 24*cos(x)^2 - 32)*sqrt(cos(x)^2/sin(x))/(cos(x)*sin(x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\csc(x) - \sin(x))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^(5/2), x, algorithm="giac")

[Out] integrate((csc(x) - sin(x))^(5/2), x)

maple [A] time = 0.28, size = 34, normalized size = 0.68

$$\frac{2(3(\cos^4(x)) + 24(\cos^2(x)) - 32) \left(\frac{\cos^2(x)}{\sin(x)}\right)^{5/2} \sin(x)}{15 \cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((csc(x)-sin(x))^(5/2), x)

[Out] 2/15*(3*cos(x)^4+24*cos(x)^2-32)*(cos(x)^2/sin(x))^(5/2)*sin(x)/cos(x)^5

maxima [B] time = 0.56, size = 427, normalized size = 8.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^(5/2),x, algorithm="maxima")

[Out]
$$-1/60 * ((3 * \cos(15/2 * x) + 105 * \cos(11/2 * x) - 410 * \cos(7/2 * x) - 3 * \cos(5/2 * x) + 410 * \cos(3/2 * x) - 105 * \cos(1/2 * x) + 3 * \sin(15/2 * x) + 105 * \sin(11/2 * x) - 410 * \sin(7/2 * x) + 3 * \sin(5/2 * x) + 410 * \sin(3/2 * x) + 105 * \sin(1/2 * x)) * \cos(5/2 * \arctan2(\sin(x), \cos(x) - 1)) - (3 * \cos(15/2 * x) + 105 * \cos(11/2 * x) - 410 * \cos(7/2 * x) - 3 * \cos(5/2 * x) + 410 * \cos(3/2 * x) - 105 * \cos(1/2 * x) - 3 * \sin(15/2 * x) - 105 * \sin(11/2 * x) + 410 * \sin(7/2 * x) - 3 * \sin(5/2 * x) - 410 * \sin(3/2 * x) - 105 * \sin(1/2 * x)) * \sin(5/2 * \arctan2(\sin(x), \cos(x) - 1))) * \cos(5/2 * \arctan2(\sin(x), \cos(x) + 1)) - ((3 * \cos(15/2 * x) + 105 * \cos(11/2 * x) - 410 * \cos(7/2 * x) - 3 * \cos(5/2 * x) + 410 * \cos(3/2 * x) - 105 * \cos(1/2 * x) - 3 * \sin(15/2 * x) - 105 * \sin(11/2 * x) + 410 * \sin(7/2 * x) - 3 * \sin(5/2 * x) - 410 * \sin(3/2 * x) - 105 * \sin(1/2 * x)) * \cos(5/2 * \arctan2(\sin(x), \cos(x) - 1)) + (3 * \cos(15/2 * x) + 105 * \cos(11/2 * x) - 410 * \cos(7/2 * x) - 3 * \cos(5/2 * x) + 410 * \cos(3/2 * x) - 105 * \cos(1/2 * x) + 3 * \sin(15/2 * x) + 105 * \sin(11/2 * x) - 410 * \sin(7/2 * x) + 3 * \sin(5/2 * x) + 410 * \sin(3/2 * x) + 105 * \sin(1/2 * x)) * \sin(5/2 * \arctan2(\sin(x), \cos(x) - 1))) * \sin(5/2 * \arctan2(\sin(x), \cos(x) + 1))) / ((\cos(x)^4 + \sin(x)^4 + 2 * (\cos(x)^2 + 1) * \sin(x)^2 - 2 * \cos(x)^2 + 1) * (\cos(x)^2 + \sin(x)^2 - 2 * \cos(x) + 1)^{1/4}) * (\cos(x)^2 + \sin(x)^2 - 2 * \cos(x) + 1)^{1/4})$$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\sin(x)} - \sin(x) \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/sin(x) - sin(x))^(5/2),x)

[Out] int((1/sin(x) - sin(x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))**(5/2),x)

[Out] Timed out

3.316 $\int (\csc(x) - \sin(x))^{3/2} dx$

Optimal. Leaf size=31

$$\frac{2}{3} \cos(x) \sqrt{\cos(x) \cot(x)} - \frac{8}{3} \sec(x) \sqrt{\cos(x) \cot(x)}$$

[Out] $2/3*\cos(x)*(\cos(x)*\cot(x))^{(1/2)}-8/3*\sec(x)*(\cos(x)*\cot(x))^{(1/2)}$

Rubi [A] time = 0.08, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4397, 4400, 2598, 2589}

$$\frac{2}{3} \cos(x) \sqrt{\cos(x) \cot(x)} - \frac{8}{3} \sec(x) \sqrt{\cos(x) \cot(x)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Csc}[x] - \text{Sin}[x])^{(3/2)}, x]$

[Out] $(2*\text{Cos}[x]*\text{Sqrt}[\text{Cos}[x]*\text{Cot}[x]])/3 - (8*\text{Sqrt}[\text{Cos}[x]*\text{Cot}[x]]*\text{Sec}[x])/3$

Rule 2589

$\text{Int}[(a_*)*\sin[(e_*) + (f_*)(x_*)]^{(m_*)}*((b_*)*\tan[(e_*) + (f_*)(x_*)]^{(n_*)}, x_Symbol] :> -\text{Simp}[(b*(a*\sin[e + f*x])^m*(b*\tan[e + f*x])^{(n-1)})/(f*m), x] /; \text{FreeQ}\{a, b, e, f, m, n\}, x] \&\& \text{EqQ}[m + n - 1, 0]$

Rule 2598

$\text{Int}[(a_*)*\sin[(e_*) + (f_*)(x_*)]^{(m_*)}*((b_*)*\tan[(e_*) + (f_*)(x_*)]^{(n_*)}, x_Symbol] :> -\text{Simp}[(b*(a*\sin[e + f*x])^m*(b*\tan[e + f*x])^{(n-1)})/(f*m), x] + \text{Dist}[(a^2*(m+n-1))/m, \text{Int}[(a*\sin[e + f*x])^{(m-2)}*(b*\tan[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, e, f, n\}, x] \&\& (\text{GtQ}[m, 1] \|\| (\text{EqQ}[m, 1] \& \& \text{EqQ}[n, 1/2])) \&\& \text{IntegersQ}[2*m, 2*n]$

Rule 4397

$\text{Int}[u_, x_Symbol] :> \text{Int}[\text{TrigSimplify}[u], x] /; \text{TrigSimplifyQ}[u]$

Rule 4400

$\text{Int}[(u_*)*((v_*)^{(m_*)}*(w_*)^{(n_*)})^{(p_*)}, x_Symbol] :> \text{With}\{\{uu = \text{ActivateTrig}[u], vv = \text{ActivateTrig}[v], ww = \text{ActivateTrig}[w]\}, \text{Dist}[(vv^m*ww^n)^{\text{FracPart}[p]}/(vv^{(m*\text{FracPart}[p])}*ww^{(n*\text{FracPart}[p])}), \text{Int}[uu*vv^{(m*p)}*ww^{(n*p)}, x], x] /; \text{FreeQ}\{m, n, p\}, x] \&\& !\text{IntegerQ}[p] \&\& (!\text{InertTrigFreeQ}[v] \|\| !\text{InertTrigFreeQ}[w])$

Rubi steps

$$\begin{aligned} \int (\csc(x) - \sin(x))^{3/2} dx &= \int (\cos(x) \cot(x))^{3/2} dx \\ &= \frac{\sqrt{\cos(x) \cot(x)} \int \cos^{\frac{3}{2}}(x) \cot^{\frac{3}{2}}(x) dx}{\sqrt{\cos(x)} \sqrt{\cot(x)}} \\ &= \frac{2}{3} \cos(x) \sqrt{\cos(x) \cot(x)} + \frac{(4\sqrt{\cos(x) \cot(x)}) \int \frac{\cot^{\frac{3}{2}}(x)}{\sqrt{\cos(x)}} dx}{3\sqrt{\cos(x)} \sqrt{\cot(x)}} \\ &= \frac{2}{3} \cos(x) \sqrt{\cos(x) \cot(x)} - \frac{8}{3} \sqrt{\cos(x) \cot(x)} \sec(x) \end{aligned}$$

Mathematica [A] time = 0.04, size = 21, normalized size = 0.68

$$\frac{2}{3} (\cos^2(x) - 4) \sec(x) \sqrt{\cos(x) \cot(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^(3/2), x]

[Out] (2*(-4 + Cos[x]^2)*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3

fricas [A] time = 0.91, size = 23, normalized size = 0.74

$$\frac{2 (\cos(x)^2 - 4) \sqrt{\frac{\cos(x)^2}{\sin(x)}}}{3 \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^(3/2), x, algorithm="fricas")

[Out] 2/3*(cos(x)^2 - 4)*sqrt(cos(x)^2/sin(x))/cos(x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\csc(x) - \sin(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^(3/2), x, algorithm="giac")

[Out] integrate((csc(x) - sin(x))^(3/2), x)

maple [A] time = 0.24, size = 26, normalized size = 0.84

$$\frac{2 (\cos^2(x) - 4) \left(\frac{\cos^2(x)}{\sin(x)}\right)^{\frac{3}{2}} \sin(x)}{3 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((csc(x)-sin(x))^(3/2), x)

[Out] 2/3*(cos(x)^2-4)*(cos(x)^2/sin(x))^(3/2)*sin(x)/cos(x)^3

maxima [B] time = 0.52, size = 314, normalized size = 10.13

$$\frac{(\cos(x)^2 + \sin(x)^2 + 2 \cos(x) + 1)^{\frac{1}{4}} (\cos(x)^2 + \sin(x)^2 - 2 \cos(x) + 1)^{\frac{1}{4}} \left(\left(\cos\left(\frac{9}{2}x\right) - 15 \cos\left(\frac{5}{2}x\right) - \cos\left(\frac{3}{2}x\right) \right) \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^(3/2), x, algorithm="maxima")

[Out] 1/6*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4)*(((cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) - sin(9/2*x) + 15*sin(5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*cos(3/2*arctan2(sin(x), cos(x) - 1)) + (cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) + sin(9/2*x) - 15*sin(5/2*x) + sin(3/2*x) + 15*sin(1/2*x))*sin(3/2*arctan2(sin(x), cos(x) - 1)))*cos(3/2*arctan2(sin(x), cos(x) + 1)) + ((cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) + sin(9/2*x) - 15*sin(5/2*x)

) + sin(3/2*x) + 15*sin(1/2*x))*cos(3/2*arctan2(sin(x), cos(x) - 1)) - (cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) - sin(9/2*x) + 15*sin(5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*sin(3/2*arctan2(sin(x), cos(x) - 1)))*sin(3/2*arctan2(sin(x), cos(x) + 1)))/(cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 + 1)*sin(x)^2 - 2*cos(x)^2 + 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(\frac{1}{\sin(x)} - \sin(x) \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/sin(x) - sin(x))^(3/2), x)

[Out] int((1/sin(x) - sin(x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\sin(x) + \csc(x))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))**(3/2), x)

[Out] Integral((-sin(x) + csc(x))**(3/2), x)

3.317 $\int \sqrt{\csc(x) - \sin(x)} dx$

Optimal. Leaf size=13

$$2 \tan(x) \sqrt{\cos(x) \cot(x)}$$

[Out] 2*(cos(x)*cot(x))^(1/2)*tan(x)

Rubi [A] time = 0.05, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4397, 4400, 2589}

$$2 \tan(x) \sqrt{\cos(x) \cot(x)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Csc[x] - Sin[x]], x]

[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]

Rule 2589

Int[((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]

Rule 4397

Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rule 4400

Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^p, x_Symbol] :> With[{uu = ActivateTrig[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPart[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x], x] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])

Rubi steps

$$\begin{aligned} \int \sqrt{\csc(x) - \sin(x)} dx &= \int \sqrt{\cos(x) \cot(x)} dx \\ &= \frac{\sqrt{\cos(x) \cot(x)} \int \sqrt{\cos(x)} \sqrt{\cot(x)} dx}{\sqrt{\cos(x)} \sqrt{\cot(x)}} \\ &= 2\sqrt{\cos(x) \cot(x)} \tan(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 13, normalized size = 1.00

$$2 \tan(x) \sqrt{\cos(x) \cot(x)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Csc[x] - Sin[x]], x]

[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]

fricas [A] time = 0.77, size = 19, normalized size = 1.46

$$\frac{2 \sqrt{\frac{\cos(x)^2}{\sin(x)}} \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(cos(x)^2/sin(x))*sin(x)/cos(x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\csc(x) - \sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(csc(x) - sin(x)), x)

maple [A] time = 0.23, size = 20, normalized size = 1.54

$$\frac{2 \sin(x) \sqrt{\frac{\cos^2(x)}{\sin(x)}}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((csc(x)-sin(x))^(1/2),x)

[Out] 2*sin(x)*(cos(x)^2/sin(x))^(1/2)/cos(x)

maxima [B] time = 0.51, size = 188, normalized size = 14.46

$$\frac{\left(\left(\cos\left(\frac{3}{2}x\right) - \cos\left(\frac{1}{2}x\right) + \sin\left(\frac{3}{2}x\right) + \sin\left(\frac{1}{2}x\right) \right) \cos\left(\frac{1}{2} \arctan(\sin(x), \cos(x) - 1)\right) - \left(\cos\left(\frac{3}{2}x\right) - \cos\left(\frac{1}{2}x\right) - \sin\left(\frac{3}{2}x\right) - \sin\left(\frac{1}{2}x\right) \right) \right)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="maxima")

[Out] (((cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*cos(1/2*arctan2(sin(x), cos(x) - 1)) - (cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*sin(1/2*arctan2(sin(x), cos(x) - 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - ((cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*cos(1/2*arctan2(sin(x), cos(x) - 1)) + (cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*sin(1/2*arctan2(sin(x), cos(x) - 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4))

mupad [B] time = 2.48, size = 15, normalized size = 1.15

$$\frac{2 |\cos(x)|}{\cos(x) \sqrt{\frac{1}{\sin(x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/sin(x) - sin(x))^(1/2),x)

```
[Out] (2*abs(cos(x)))/(cos(x)*(1/sin(x))^(1/2))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \sqrt{-\sin(x) + \csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**(1/2),x)
```

```
[Out] Integral(sqrt(-sin(x) + csc(x)), x)
```

$$3.318 \quad \int \frac{1}{\sqrt{\csc(x) - \sin(x)}} dx$$

Optimal. Leaf size=60

$$\frac{\cos(x) \tan^{-1}(\sqrt{-\sin(x)})}{\sqrt{-\sin(x)} \sqrt{\cos(x) \cot(x)}} - \frac{\cos(x) \tanh^{-1}(\sqrt{-\sin(x)})}{\sqrt{-\sin(x)} \sqrt{\cos(x) \cot(x)}}$$

[Out] arctan((-sin(x))^(1/2))*cos(x)/(cos(x)*cot(x))^(1/2)/(-sin(x))^(1/2)-arctanh((-sin(x))^(1/2))*cos(x)/(cos(x)*cot(x))^(1/2)/(-sin(x))^(1/2)

Rubi [A] time = 0.09, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {4397, 4400, 2601, 2564, 329, 298, 203, 206}

$$\frac{\cos(x) \tan^{-1}(\sqrt{-\sin(x)})}{\sqrt{-\sin(x)} \sqrt{\cos(x) \cot(x)}} - \frac{\cos(x) \tanh^{-1}(\sqrt{-\sin(x)})}{\sqrt{-\sin(x)} \sqrt{\cos(x) \cot(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[Csc[x] - Sin[x]],x]

[Out] (ArcTan[Sqrt[-Sin[x]]]*Cos[x])/(Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]]) - (ArcTanh[Sqrt[-Sin[x]]]*Cos[x])/(Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2564

Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && LtQ[0, m, n])

Rule 2601


```
Int[((a_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.))*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x]^n)/(a*Sin[e + f*x])^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1)])) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4397

```
Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_.)^(m_.)*(w_.)^(n_.))^(p_), x_Symbol] := With[{uu = ActivateTrig[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPart[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x], x] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt{\csc(x) - \sin(x)}} dx &= \int \frac{1}{\sqrt{\cos(x) \cot(x)}} dx \\
 &= \frac{(\sqrt{\cos(x)} \sqrt{\cot(x)}) \int \frac{1}{\sqrt{\cos(x)} \sqrt{\cot(x)}} dx}{\sqrt{\cos(x) \cot(x)}} \\
 &= \frac{\cos(x) \int \sec(x) \sqrt{-\sin(x)} dx}{\sqrt{\cos(x) \cot(x)} \sqrt{-\sin(x)}} \\
 &= -\frac{\cos(x) \operatorname{Subst}\left(\int \frac{\sqrt{x}}{1-x^2} dx, x, -\sin(x)\right)}{\sqrt{\cos(x) \cot(x)} \sqrt{-\sin(x)}} \\
 &= -\frac{(2 \cos(x)) \operatorname{Subst}\left(\int \frac{x^2}{1-x^4} dx, x, \sqrt{-\sin(x)}\right)}{\sqrt{\cos(x) \cot(x)} \sqrt{-\sin(x)}} \\
 &= -\frac{\cos(x) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{-\sin(x)}\right)}{\sqrt{\cos(x) \cot(x)} \sqrt{-\sin(x)}} + \frac{\cos(x) \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{-\sin(x)}\right)}{\sqrt{\cos(x) \cot(x)} \sqrt{-\sin(x)}} \\
 &= \frac{\tan^{-1}\left(\sqrt{-\sin(x)}\right) \cos(x)}{\sqrt{\cos(x) \cot(x)} \sqrt{-\sin(x)}} - \frac{\tanh^{-1}\left(\sqrt{-\sin(x)}\right) \cos(x)}{\sqrt{\cos(x) \cot(x)} \sqrt{-\sin(x)}}
 \end{aligned}$$

Mathematica [A] time = 0.27, size = 44, normalized size = 0.73

$$\frac{\sin(x) \tan(x) \sqrt{\cos(x) \cot(x)} \left(\tan^{-1}\left(\sqrt[4]{\sin^2(x)}\right) - \tanh^{-1}\left(\sqrt[4]{\sin^2(x)}\right) \right)}{\sin^2(x)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Csc[x] - Sin[x]], x]
```

```
[Out] -(((ArcTan[(Sin[x]^2)^(1/4)] - ArcTanh[(Sin[x]^2)^(1/4)])*Sqrt[Cos[x]*Cot[x]]*Sin[x]*Tan[x])/(Sin[x]^2)^(3/4))
```

fricas [B] time = 1.06, size = 124, normalized size = 2.07

$$\frac{1}{2} \arctan \left(\frac{2 \sqrt{\frac{\cos(x)^2}{\sin(x)}} \sin(x)}{\cos(x) \sin(x) - \cos(x)} \right) + \frac{1}{4} \log \left(\frac{\cos(x)^3 - 5 \cos(x)^2 - (\cos(x)^2 + 6 \cos(x) + 4) \sin(x) + 4 (\cos(x)^2 - (\cos(x) + 1) \sin(x) - 1) \sqrt{\cos(x)^2 / \sin(x)} - 2 \cos(x) + 4)}{\cos(x)^3 + 3 \cos(x)^2 - (\cos(x)^2 - 2 \cos(x) - 4) \sin(x) - 2 \cos(x) - 4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^(1/2),x, algorithm="fricas")

[Out] 1/2*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x))) + 1/4*log((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) + 4)*sin(x) + 4*(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) - 2*cos(x) + 4)/(cos(x)^3 + 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\csc(x) - \sin(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(csc(x) - sin(x)), x)

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\csc(x) - \sin(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(csc(x)-sin(x))^(1/2),x)

[Out] int(1/(csc(x)-sin(x))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\csc(x) - \sin(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(csc(x) - sin(x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{1}{\sin(x)} - \sin(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/sin(x) - sin(x))^(1/2),x)

[Out] int(1/(1/sin(x) - sin(x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\sin(x) + \csc(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-sin(x) + csc(x)), x)
```

$$3.319 \quad \int \frac{1}{(\csc(x) - \sin(x))^{3/2}} dx$$

Optimal. Leaf size=80

$$\frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} + \frac{\sqrt{-\sin(x)}\cot(x)\tan^{-1}(\sqrt{-\sin(x)})}{4\sqrt{\cos(x)\cot(x)}} + \frac{\sqrt{-\sin(x)}\cot(x)\tanh^{-1}(\sqrt{-\sin(x)})}{4\sqrt{\cos(x)\cot(x)}}$$

[Out] $1/2*\sec(x)/(\cos(x)*\cot(x))^{(1/2)}+1/4*\arctan((-\sin(x))^{(1/2)})*\cot(x)*(-\sin(x))^{(1/2)}/(\cos(x)*\cot(x))^{(1/2)}+1/4*\operatorname{arctanh}((-\sin(x))^{(1/2)})*\cot(x)*(-\sin(x))^{(1/2)}/(\cos(x)*\cot(x))^{(1/2)}$

Rubi [A] time = 0.12, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {4397, 4400, 2597, 2601, 2564, 329, 212, 206, 203}

$$\frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} + \frac{\sqrt{-\sin(x)}\cot(x)\tan^{-1}(\sqrt{-\sin(x)})}{4\sqrt{\cos(x)\cot(x)}} + \frac{\sqrt{-\sin(x)}\cot(x)\tanh^{-1}(\sqrt{-\sin(x)})}{4\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

[In] Int[(Csc[x] - Sin[x])^(-3/2), x]

[Out] Sec[x]/(2*Sqrt[Cos[x]*Cot[x]]) + (ArcTan[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/(4*Sqrt[Cos[x]*Cot[x]]) + (ArcTanh[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/(4*Sqrt[Cos[x]*Cot[x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2564

Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && LtQ[0, m, n])

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] && !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x]^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1)])) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4397

```
Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_.)^(m_.)*(w_.)^(n_.))^(p_), x_Symbol] := With[{uu = ActivateTrig[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPart[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x], x] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(\csc(x) - \sin(x))^{3/2}} dx &= \int \frac{1}{(\cos(x) \cot(x))^{3/2}} dx \\
 &= \frac{(\sqrt{\cos(x)} \sqrt{\cot(x)}) \int \frac{1}{\cos^2(x) \cot^2(x)} dx}{\sqrt{\cos(x) \cot(x)}} \\
 &= \frac{\sec(x)}{2\sqrt{\cos(x) \cot(x)}} - \frac{(\sqrt{\cos(x)} \sqrt{\cot(x)}) \int \frac{\sqrt{\cot(x)}}{\cos^2(x)} dx}{4\sqrt{\cos(x) \cot(x)}} \\
 &= \frac{\sec(x)}{2\sqrt{\cos(x) \cot(x)}} - \frac{(\cot(x) \sqrt{-\sin(x)}) \int \frac{\sec(x)}{\sqrt{-\sin(x)}} dx}{4\sqrt{\cos(x) \cot(x)}} \\
 &= \frac{\sec(x)}{2\sqrt{\cos(x) \cot(x)}} + \frac{(\cot(x) \sqrt{-\sin(x)}) \text{Subst} \left(\int \frac{1}{\sqrt{x}(1-x^2)} dx, x, -\sin(x) \right)}{4\sqrt{\cos(x) \cot(x)}} \\
 &= \frac{\sec(x)}{2\sqrt{\cos(x) \cot(x)}} + \frac{(\cot(x) \sqrt{-\sin(x)}) \text{Subst} \left(\int \frac{1}{1-x^4} dx, x, \sqrt{-\sin(x)} \right)}{2\sqrt{\cos(x) \cot(x)}} \\
 &= \frac{\sec(x)}{2\sqrt{\cos(x) \cot(x)}} + \frac{(\cot(x) \sqrt{-\sin(x)}) \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sqrt{-\sin(x)} \right)}{4\sqrt{\cos(x) \cot(x)}} + \frac{(\cot(x) \sqrt{-\sin(x)})}{4\sqrt{\cos(x) \cot(x)}} \\
 &= \frac{\sec(x)}{2\sqrt{\cos(x) \cot(x)}} + \frac{\tan^{-1}(\sqrt{-\sin(x)}) \cot(x) \sqrt{-\sin(x)}}{4\sqrt{\cos(x) \cot(x)}} + \frac{\tanh^{-1}(\sqrt{-\sin(x)}) \cot(x) \sqrt{-\sin(x)}}{4\sqrt{\cos(x) \cot(x)}}
 \end{aligned}$$

Mathematica [A] time = 0.15, size = 60, normalized size = 0.75

$$\frac{2\sqrt[4]{\sin^2(x)} \sec(x) + \cos(x) \left(-\tan^{-1} \left(\sqrt[4]{\sin^2(x)} \right) \right) - \cos(x) \tanh^{-1} \left(\sqrt[4]{\sin^2(x)} \right)}{4\sqrt[4]{\sin^2(x)} \sqrt{\cos(x) \cot(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^(-3/2), x]

[Out] $-(\text{ArcTan}[(\text{Sin}[x]^2)^{(1/4)}] * \text{Cos}[x]) - \text{ArcTanh}[(\text{Sin}[x]^2)^{(1/4)}] * \text{Cos}[x] + 2 * \text{Sec}[x] * (\text{Sin}[x]^2)^{(1/4)} / (4 * \text{Sqrt}[\text{Cos}[x] * \text{Cot}[x]]) * (\text{Sin}[x]^2)^{(1/4)})$

fricas [B] time = 1.80, size = 152, normalized size = 1.90

$$\frac{2 \arctan \left(\frac{2 \sqrt{\frac{\cos(x)^2}{\sin(x)}} \sin(x)}{\cos(x) \sin(x) - \cos(x)} \right) \cos(x)^3 + \cos(x)^3 \log \left(\frac{\cos(x)^3 - 5 \cos(x)^2 - (\cos(x)^2 + 6 \cos(x) + 4) \sin(x) - 4 (\cos(x)^2 - (\cos(x) + 1) \sin(x) - 1)}{\cos(x)^3 + 3 \cos(x)^2 - (\cos(x)^2 - 2 \cos(x) - 4) \sin(x) - 2 \cos(x) - 4} \right)}{16 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^(3/2),x, algorithm="fricas")

[Out] $1/16 * (2 * \arctan(2 * \text{sqrt}(\cos(x)^2 / \sin(x)) * \sin(x) / (\cos(x) * \sin(x) - \cos(x))) * \cos(x)^3 + \cos(x)^3 * \log((\cos(x)^3 - 5 * \cos(x)^2 - (\cos(x)^2 + 6 * \cos(x) + 4) * \sin(x) - 4 * (\cos(x)^2 - (\cos(x) + 1) * \sin(x) - 1) * \text{sqrt}(\cos(x)^2 / \sin(x)) - 2 * \cos(x) + 4) / (\cos(x)^3 + 3 * \cos(x)^2 - (\cos(x)^2 - 2 * \cos(x) - 4) * \sin(x) - 2 * \cos(x) - 4)) + 8 * \text{sqrt}(\cos(x)^2 / \sin(x)) * \sin(x)) / \cos(x)^3$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\csc(x) - \sin(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^(3/2),x, algorithm="giac")

[Out] integrate((csc(x) - sin(x))^-3/2, x)

maple [C] time = 0.55, size = 450, normalized size = 5.62

$$\frac{(-1 + \cos(x)) \left(i \sin(x) (\cos^2(x)) \text{EllipticPi} \left(\sqrt{\frac{i \cos(x) + \sin(x) - i}{\sin(x)}}, \frac{1}{2} - \frac{i}{2}, \frac{\sqrt{2}}{2} \right) \sqrt{\frac{-i \cos(x) + \sin(x) + i}{\sin(x)}} \sqrt{\frac{i \cos(x) + \sin(x) - i}{\sin(x)}} \sqrt{-\frac{i}{\sin(x)}} \right)}{16 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(csc(x)-sin(x))^(3/2),x)

[Out] $1/8 * (-1 + \cos(x)) * (I * \sin(x) * \cos(x)^2 * \text{EllipticPi}(((I * \cos(x) + \sin(x) - I) / \sin(x))^{\frac{1}{2}}, 1/2 - 1/2 * I, 1/2 * 2^{\frac{1}{2}})) * ((-I * \cos(x) + \sin(x) + I) / \sin(x))^{\frac{1}{2}} * ((I * \cos(x) + \sin(x) - I) / \sin(x))^{\frac{1}{2}} * (-I * (-1 + \cos(x)) / \sin(x))^{\frac{1}{2}} + I * \sin(x) * \cos(x)^2 * \text{EllipticPi}(((I * \cos(x) + \sin(x) - I) / \sin(x))^{\frac{1}{2}}, 1/2 + 1/2 * I, 1/2 * 2^{\frac{1}{2}})) * ((-I * \cos(x) + \sin(x) + I) / \sin(x))^{\frac{1}{2}} * ((I * \cos(x) + \sin(x) - I) / \sin(x))^{\frac{1}{2}} * (-I * (-1 + \cos(x)) / \sin(x))^{\frac{1}{2}} - 2 * I * \sin(x) * \cos(x)^2 * \text{EllipticF}(((I * \cos(x) + \sin(x) - I) / \sin(x))^{\frac{1}{2}}, 1/2 * 2^{\frac{1}{2}})) * ((-I * \cos(x) + \sin(x) + I) / \sin(x))^{\frac{1}{2}} * ((I * \cos(x) + \sin(x) - I) / \sin(x))^{\frac{1}{2}} * (-I * (-1 + \cos(x)) / \sin(x))^{\frac{1}{2}} - \sin(x) * \cos(x)^2 * \text{EllipticPi}(((I * \cos(x) + \sin(x) - I) / \sin(x))^{\frac{1}{2}}, 1/2 - 1/2 * I, 1/2 * 2^{\frac{1}{2}})) * ((-I * \cos(x) + \sin(x) + I) / \sin(x))^{\frac{1}{2}} * ((I * \cos(x) + \sin(x) - I) / \sin(x))^{\frac{1}{2}} * (-I * (-1 + \cos(x)) / \sin(x))^{\frac{1}{2}}$

+I)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)+sin(x)*cos(x)^2*EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)+2*cos(x)*2^(1/2)-2*2^(1/2))*cos(x)*(1+cos(x))^2/(cos(x)^2/sin(x))^(3/2)/sin(x)^5*2^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\csc(x) - \sin(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^(3/2),x, algorithm="maxima")

[Out] integrate((csc(x) - sin(x))^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\sin(x)} - \sin(x)\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/sin(x) - sin(x))^(3/2),x)

[Out] int(1/(1/sin(x) - sin(x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\sin(x) + \csc(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))**(3/2),x)

[Out] Integral((-sin(x) + csc(x))**(-3/2), x)

$$3.320 \quad \int \frac{1}{(\csc(x) - \sin(x))^{5/2}} dx$$

Optimal. Leaf size=99

$$-\frac{3 \tan(x)}{16\sqrt{\cos(x) \cot(x)}} - \frac{3 \cos(x) \tan^{-1}(\sqrt{-\sin(x)})}{32\sqrt{-\sin(x)}\sqrt{\cos(x) \cot(x)}} + \frac{3 \cos(x) \tanh^{-1}(\sqrt{-\sin(x)})}{32\sqrt{-\sin(x)}\sqrt{\cos(x) \cot(x)}} + \frac{\tan(x) \sec^2(x)}{4\sqrt{\cos(x) \cot(x)}}$$

[Out] $-3/32*\arctan((- \sin(x))^{(1/2)})*\cos(x)/(\cos(x)*\cot(x))^{(1/2)}/(- \sin(x))^{(1/2)}+3/32*\operatorname{arctanh}((- \sin(x))^{(1/2)})*\cos(x)/(\cos(x)*\cot(x))^{(1/2)}/(- \sin(x))^{(1/2)}-3/16*\tan(x)/(\cos(x)*\cot(x))^{(1/2)}+1/4*\sec(x)^2*\tan(x)/(\cos(x)*\cot(x))^{(1/2)}$

Rubi [A] time = 0.15, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {4397, 4400, 2597, 2599, 2601, 2564, 329, 298, 203, 206}

$$-\frac{3 \tan(x)}{16\sqrt{\cos(x) \cot(x)}} - \frac{3 \cos(x) \tan^{-1}(\sqrt{-\sin(x)})}{32\sqrt{-\sin(x)}\sqrt{\cos(x) \cot(x)}} + \frac{3 \cos(x) \tanh^{-1}(\sqrt{-\sin(x)})}{32\sqrt{-\sin(x)}\sqrt{\cos(x) \cot(x)}} + \frac{\tan(x) \sec^2(x)}{4\sqrt{\cos(x) \cot(x)}}$$

Antiderivative was successfully verified.

[In] Int[(Csc[x] - Sin[x])^(-5/2), x]

[Out] $(-3*\operatorname{ArcTan}[\operatorname{Sqrt}[-\operatorname{Sin}[x]]]*\operatorname{Cos}[x])/(32*\operatorname{Sqrt}[\operatorname{Cos}[x]*\operatorname{Cot}[x]]*\operatorname{Sqrt}[-\operatorname{Sin}[x]]) + (3*\operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sin}[x]]]*\operatorname{Cos}[x])/(32*\operatorname{Sqrt}[\operatorname{Cos}[x]*\operatorname{Cot}[x]]*\operatorname{Sqrt}[-\operatorname{Sin}[x]]) - (3*\operatorname{Tan}[x])/(16*\operatorname{Sqrt}[\operatorname{Cos}[x]*\operatorname{Cot}[x]]) + (\operatorname{Sec}[x]^2*\operatorname{Tan}[x])/(4*\operatorname{Sqrt}[\operatorname{Cos}[x]*\operatorname{Cot}[x]])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2564

Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && LtQ[0, m, n])

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] && !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1))/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x]^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1)])) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4397

```
Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] := With[{uu = ActivateTrig[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPart[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x], x] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(\csc(x) - \sin(x))^{5/2}} dx &= \int \frac{1}{(\cos(x) \cot(x))^{5/2}} dx \\
&= \frac{\int \frac{1}{\cos^2(x) \cot^2(x)} dx}{\sqrt{\cos(x) \cot(x)}} \\
&= \frac{\sec^2(x) \tan(x)}{4\sqrt{\cos(x) \cot(x)}} - \frac{(3\sqrt{\cos(x)} \sqrt{\cot(x)}) \int \frac{1}{\cos^2(x) \sqrt{\cot(x)}} dx}{8\sqrt{\cos(x) \cot(x)}} \\
&= -\frac{3 \tan(x)}{16\sqrt{\cos(x) \cot(x)}} + \frac{\sec^2(x) \tan(x)}{4\sqrt{\cos(x) \cot(x)}} - \frac{(3\sqrt{\cos(x)} \sqrt{\cot(x)}) \int \frac{1}{\sqrt{\cos(x)} \sqrt{\cot(x)}} dx}{32\sqrt{\cos(x) \cot(x)}} \\
&= -\frac{3 \tan(x)}{16\sqrt{\cos(x) \cot(x)}} + \frac{\sec^2(x) \tan(x)}{4\sqrt{\cos(x) \cot(x)}} - \frac{(3 \cos(x)) \int \sec(x) \sqrt{-\sin(x)} dx}{32\sqrt{\cos(x) \cot(x)} \sqrt{-\sin(x)}} \\
&= -\frac{3 \tan(x)}{16\sqrt{\cos(x) \cot(x)}} + \frac{\sec^2(x) \tan(x)}{4\sqrt{\cos(x) \cot(x)}} + \frac{(3 \cos(x)) \text{Subst} \left(\int \frac{\sqrt{x}}{1-x^2} dx, x, -\sin(x) \right)}{32\sqrt{\cos(x) \cot(x)} \sqrt{-\sin(x)}} \\
&= -\frac{3 \tan(x)}{16\sqrt{\cos(x) \cot(x)}} + \frac{\sec^2(x) \tan(x)}{4\sqrt{\cos(x) \cot(x)}} + \frac{(3 \cos(x)) \text{Subst} \left(\int \frac{x^2}{1-x^4} dx, x, \sqrt{-\sin(x)} \right)}{16\sqrt{\cos(x) \cot(x)} \sqrt{-\sin(x)}} \\
&= -\frac{3 \tan(x)}{16\sqrt{\cos(x) \cot(x)}} + \frac{\sec^2(x) \tan(x)}{4\sqrt{\cos(x) \cot(x)}} + \frac{(3 \cos(x)) \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sqrt{-\sin(x)} \right)}{32\sqrt{\cos(x) \cot(x)} \sqrt{-\sin(x)}} \\
&= -\frac{3 \tan^{-1}(\sqrt{-\sin(x)}) \cos(x)}{32\sqrt{\cos(x) \cot(x)} \sqrt{-\sin(x)}} + \frac{3 \tanh^{-1}(\sqrt{-\sin(x)}) \cos(x)}{32\sqrt{\cos(x) \cot(x)} \sqrt{-\sin(x)}} - \frac{3 \tan(x)}{16\sqrt{\cos(x) \cot(x)}} +
\end{aligned}$$

Mathematica [A] time = 0.51, size = 69, normalized size = 0.70

$$\frac{\sin(x) \tan(x) \sqrt{\cos(x) \cot(x)} \left(-3 \tan^{-1} \left(\sqrt[4]{\sin^2(x)} \right) + 3 \tanh^{-1} \left(\sqrt[4]{\sin^2(x)} \right) + \sin^2(x)^{3/4} (3 \cos(2x) - 5) \sec^4(x) \right)}{32 \sin^2(x)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^(-5/2), x]

[Out] -1/32*(Sqrt[Cos[x]*Cot[x]]*Sin[x]*(-3*ArcTan[(Sin[x]^2)^(1/4)] + 3*ArcTanh[(Sin[x]^2)^(1/4)] + (-5 + 3*Cos[2*x])*Sec[x]^4*(Sin[x]^2)^(3/4))*Tan[x])/(Sin[x]^2)^(3/4)

fricas [B] time = 1.95, size = 165, normalized size = 1.67

$$\frac{6 \arctan \left(\frac{2 \sqrt{\frac{\cos(x)^2}{\sin(x)}} \sin(x)}{\cos(x) \sin(x) - \cos(x)} \right) \cos(x)^5 - 3 \cos(x)^5 \log \left(\frac{\cos(x)^3 - 5 \cos(x)^2 - (\cos(x)^2 + 6 \cos(x) + 4) \sin(x) - 4 (\cos(x)^2 - (\cos(x) + 1) \sin(x) - 1) \sqrt{\cos(x)^2 / \sin(x)} - 2 \cos(x) + 4}{\cos(x)^3 + 3 \cos(x)^2 - (\cos(x)^2 - 2 \cos(x) - 4) \sin(x) - 2 \cos(x)} \right)}{128 \cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^(5/2), x, algorithm="fricas")

[Out] -1/128*(6*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x)))*cos(x)^5 - 3*cos(x)^5*log((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) + 4)*sin(x) - 4*(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) - 2*cos(x) + 4)/(cos(x)^3 + 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*c

$\cos(x) - 4) - 8*(3*\cos(x)^4 - 7*\cos(x)^2 + 4)*\sqrt{\cos(x)^2/\sin(x)})/\cos(x)^5$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\csc(x) - \sin(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^(5/2),x, algorithm="giac")

[Out] integrate((csc(x) - sin(x))^(5/2), x)

maple [C] time = 0.35, size = 382, normalized size = 3.86

$$(-1 + \cos(x)) \left(3i (\cos^4(x)) \operatorname{EllipticPi} \left(\sqrt{\frac{i \cos(x) + \sin(x) - i}{\sin(x)}}, \frac{1}{2} - \frac{i}{2}, \frac{\sqrt{2}}{2} \right) \sqrt{\frac{-i \cos(x) + \sin(x) + i}{\sin(x)}} \sqrt{\frac{i \cos(x) + \sin(x) - i}{\sin(x)}} \sqrt{\frac{-i \cos(x) + \sin(x) + i}{\sin(x)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(csc(x)-sin(x))^(5/2),x)

[Out] $-1/64*(-1+\cos(x))*(3*I*\cos(x)^4*\operatorname{EllipticPi}(((I*\cos(x)+\sin(x)-I)/\sin(x))^{1/2}, 1/2-1/2*I, 1/2*2^{1/2}))*((-I*\cos(x)+\sin(x)+I)/\sin(x))^{1/2}*((I*\cos(x)+\sin(x)-I)/\sin(x))^{1/2}*(-I*(-1+\cos(x))/\sin(x))^{1/2}-3*I*\cos(x)^4*\operatorname{EllipticPi}(((I*\cos(x)+\sin(x)-I)/\sin(x))^{1/2}, 1/2+1/2*I, 1/2*2^{1/2}))*((-I*\cos(x)+\sin(x)+I)/\sin(x))^{1/2}*((I*\cos(x)+\sin(x)-I)/\sin(x))^{1/2}*(-I*(-1+\cos(x))/\sin(x))^{1/2}-3*\cos(x)^4*\operatorname{EllipticPi}(((I*\cos(x)+\sin(x)-I)/\sin(x))^{1/2}, 1/2-1/2*I, 1/2*2^{1/2}))*((-I*\cos(x)+\sin(x)+I)/\sin(x))^{1/2}*((I*\cos(x)+\sin(x)-I)/\sin(x))^{1/2}*(-I*(-1+\cos(x))/\sin(x))^{1/2}-3*\cos(x)^4*\operatorname{EllipticPi}(((I*\cos(x)+\sin(x)-I)/\sin(x))^{1/2}, 1/2+1/2*I, 1/2*2^{1/2}))*((-I*\cos(x)+\sin(x)+I)/\sin(x))^{1/2}*((I*\cos(x)+\sin(x)-I)/\sin(x))^{1/2}*(-I*(-1+\cos(x))/\sin(x))^{1/2}+6*\cos(x)^3*2^{1/2}-6*\cos(x)^2*2^{1/2}-8*\cos(x)*2^{1/2}+8*2^{1/2}))*\cos(x)*(1+\cos(x))^2/\sin(x)^5/(\cos(x)^2/\sin(x))^{5/2}*2^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\csc(x) - \sin(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^(5/2),x, algorithm="maxima")

[Out] integrate((csc(x) - sin(x))^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\sin(x)} - \sin(x)\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/sin(x) - sin(x))^(5/2),x)

[Out] int(1/(1/sin(x) - sin(x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\sin(x) + \csc(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))**(5/2),x)

[Out] Integral((-sin(x) + csc(x))**(-5/2), x)

$$3.321 \quad \int \frac{1}{(\csc(x) - \sin(x))^{7/2}} dx$$

Optimal. Leaf size=118

$$-\frac{5 \sec^3(x)}{48\sqrt{\cos(x) \cot(x)}} + \frac{5 \sec(x)}{192\sqrt{\cos(x) \cot(x)}} - \frac{5\sqrt{-\sin(x)} \cot(x) \tan^{-1}(\sqrt{-\sin(x)})}{128\sqrt{\cos(x) \cot(x)}} - \frac{5\sqrt{-\sin(x)} \cot(x) \tanh^{-1}(\sqrt{-\sin(x)})}{128\sqrt{\cos(x) \cot(x)}}$$

[Out] 5/192*sec(x)/(cos(x)*cot(x))^(1/2)-5/48*sec(x)^3/(cos(x)*cot(x))^(1/2)-5/128*arctan((-sin(x))^(1/2))*cot(x)*(-sin(x))^(1/2)/(cos(x)*cot(x))^(1/2)-5/128*arctanh((-sin(x))^(1/2))*cot(x)*(-sin(x))^(1/2)/(cos(x)*cot(x))^(1/2)+1/6*sec(x)^3*tan(x)^2/(cos(x)*cot(x))^(1/2)

Rubi [A] time = 0.18, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {4397, 4400, 2597, 2599, 2601, 2564, 329, 212, 206, 203}

$$-\frac{5 \sec^3(x)}{48\sqrt{\cos(x) \cot(x)}} + \frac{5 \sec(x)}{192\sqrt{\cos(x) \cot(x)}} - \frac{5\sqrt{-\sin(x)} \cot(x) \tan^{-1}(\sqrt{-\sin(x)})}{128\sqrt{\cos(x) \cot(x)}} - \frac{5\sqrt{-\sin(x)} \cot(x) \tanh^{-1}(\sqrt{-\sin(x)})}{128\sqrt{\cos(x) \cot(x)}}$$

Antiderivative was successfully verified.

[In] Int[(Csc[x] - Sin[x])^(-7/2), x]

[Out] (5*Sec[x])/(192*Sqrt[Cos[x]*Cot[x]]) - (5*Sec[x]^3)/(48*Sqrt[Cos[x]*Cot[x]]) - (5*ArcTan[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/(128*Sqrt[Cos[x]*Cot[x]]) - (5*ArcTanh[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/(128*Sqrt[Cos[x]*Cot[x]]) + (Sec[x]^3*Tan[x]^2)/(6*Sqrt[Cos[x]*Cot[x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2564

Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^(n/2), x], x, a*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In

tegerQ[(m - 1)/2] && LtQ[0, m, n])

Rule 2597

Int[((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] && !(EqQ[n, -3/2] && EqQ[m, 1])

Rule 2599

Int[((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1))/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]

Rule 2601

Int[((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n]/(a*Sin[e + f*x])^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1)])) || IntegersQ[m - 1/2, n - 1/2])

Rule 4397

Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rule 4400

Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^p, x_Symbol] :> With[{uu = ActivateTrig[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPart[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x], x] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])

Rubi steps

$$\begin{aligned}
\int \frac{1}{(\csc(x) - \sin(x))^{7/2}} dx &= \int \frac{1}{(\cos(x) \cot(x))^{7/2}} dx \\
&= \frac{(\sqrt{\cos(x)} \sqrt{\cot(x)}) \int \frac{1}{\cos^2(x) \cot^2(x)} dx}{\sqrt{\cos(x) \cot(x)}} \\
&= \frac{\sec^3(x) \tan^2(x)}{6\sqrt{\cos(x) \cot(x)}} - \frac{(5\sqrt{\cos(x)} \sqrt{\cot(x)}) \int \frac{1}{\cos^2(x) \cot^2(x)} dx}{12\sqrt{\cos(x) \cot(x)}} \\
&= -\frac{5 \sec^3(x)}{48\sqrt{\cos(x) \cot(x)}} + \frac{\sec^3(x) \tan^2(x)}{6\sqrt{\cos(x) \cot(x)}} + \frac{(5\sqrt{\cos(x)} \sqrt{\cot(x)}) \int \frac{\sqrt{\cot(x)}}{\cos^2(x)} dx}{96\sqrt{\cos(x) \cot(x)}} \\
&= \frac{5 \sec(x)}{192\sqrt{\cos(x) \cot(x)}} - \frac{5 \sec^3(x)}{48\sqrt{\cos(x) \cot(x)}} + \frac{\sec^3(x) \tan^2(x)}{6\sqrt{\cos(x) \cot(x)}} + \frac{(5\sqrt{\cos(x)} \sqrt{\cot(x)})}{128\sqrt{\cos(x) \cot(x)}} \\
&= \frac{5 \sec(x)}{192\sqrt{\cos(x) \cot(x)}} - \frac{5 \sec^3(x)}{48\sqrt{\cos(x) \cot(x)}} + \frac{\sec^3(x) \tan^2(x)}{6\sqrt{\cos(x) \cot(x)}} + \frac{(5 \cot(x) \sqrt{-\sin(x)})}{128\sqrt{\cos(x) \cot(x)}} \\
&= \frac{5 \sec(x)}{192\sqrt{\cos(x) \cot(x)}} - \frac{5 \sec^3(x)}{48\sqrt{\cos(x) \cot(x)}} + \frac{\sec^3(x) \tan^2(x)}{6\sqrt{\cos(x) \cot(x)}} - \frac{(5 \cot(x) \sqrt{-\sin(x)})}{128\sqrt{\cos(x) \cot(x)}} \\
&= \frac{5 \sec(x)}{192\sqrt{\cos(x) \cot(x)}} - \frac{5 \sec^3(x)}{48\sqrt{\cos(x) \cot(x)}} + \frac{\sec^3(x) \tan^2(x)}{6\sqrt{\cos(x) \cot(x)}} - \frac{(5 \cot(x) \sqrt{-\sin(x)})}{6\sqrt{\cos(x) \cot(x)}} \\
&= \frac{5 \sec(x)}{192\sqrt{\cos(x) \cot(x)}} - \frac{5 \sec^3(x)}{48\sqrt{\cos(x) \cot(x)}} + \frac{\sec^3(x) \tan^2(x)}{6\sqrt{\cos(x) \cot(x)}} - \frac{(5 \cot(x) \sqrt{-\sin(x)})}{12\sqrt{\cos(x) \cot(x)}} \\
&= \frac{5 \sec(x)}{192\sqrt{\cos(x) \cot(x)}} - \frac{5 \sec^3(x)}{48\sqrt{\cos(x) \cot(x)}} - \frac{5 \tan^{-1}(\sqrt{-\sin(x)}) \cot(x) \sqrt{-\sin(x)}}{128\sqrt{\cos(x) \cot(x)}}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 74, normalized size = 0.63

$$\frac{2\sqrt[4]{\sin^2(x)} \sec(x) (32 \sec^4(x) - 52 \sec^2(x) + 5) + 15 \cos(x) \tan^{-1}\left(\sqrt[4]{\sin^2(x)}\right) + 15 \cos(x) \tanh^{-1}\left(\sqrt[4]{\sin^2(x)}\right)}{384\sqrt[4]{\sin^2(x)} \sqrt{\cos(x) \cot(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^(-7/2), x]

[Out] (15*ArcTan[(Sin[x]^2)^(1/4)]*Cos[x] + 15*ArcTanh[(Sin[x]^2)^(1/4)]*Cos[x] + 2*Sec[x]*(5 - 52*Sec[x]^2 + 32*Sec[x]^4)*(Sin[x]^2)^(1/4))/(384*Sqrt[Cos[x]*Cot[x]]*(Sin[x]^2)^(1/4))

fricas [A] time = 1.12, size = 167, normalized size = 1.42

$$\frac{30 \arctan\left(\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}} \sin(x)}{\cos(x) \sin(x) - \cos(x)}\right) \cos(x)^7 - 15 \cos(x)^7 \log\left(\frac{\cos(x)^3 - 5 \cos(x)^2 - (\cos(x)^2 + 6 \cos(x) + 4) \sin(x) + 4(\cos(x)^2 - (\cos(x) + 1) \sin(x))}{\cos(x)^3 + 3 \cos(x)^2 - (\cos(x)^2 - 2 \cos(x) - 4) \sin(x)}\right)}{1536 \cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(csc(x)-sin(x))^(7/2), x, algorithm="fricas")

```
[Out] -1/1536*(30*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x)))
*cos(x)^7 - 15*cos(x)^7*log((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) +
4)*sin(x) + 4*(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) -
2*cos(x) + 4)/(cos(x)^3 + 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) -
2*cos(x) - 4)) - 8*(5*cos(x)^4 - 52*cos(x)^2 + 32)*sqrt(cos(x)^2/sin(x))*si
n(x))/cos(x)^7
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\csc(x) - \sin(x))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((csc(x) - sin(x))^(7/2), x)
```

maple [C] time = 0.39, size = 487, normalized size = 4.13

$$(-1 + \cos(x)) \left(-15i \sin(x) (\cos^6(x)) \sqrt{\frac{i \cos(x) + \sin(x) - i}{\sin(x)}} \sqrt{\frac{-i \cos(x) + \sin(x) + i}{\sin(x)}} \sqrt{\frac{i(-1 + \cos(x))}{\sin(x)}} \operatorname{EllipticPi} \left(\sqrt{\frac{i \cos(x) + \sin(x) - i}{\sin(x)}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^(7/2),x)
```

```
[Out] 1/768*(-1+cos(x))*(-15*I*sin(x)*cos(x)^6*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)
*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*Elliptic
Pi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-15*I*sin(x)*co
s(x)^6*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)
*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),
1/2+1/2*I,1/2*2^(1/2))+30*I*sin(x)*cos(x)^6*((I*cos(x)+sin(x)-I)/sin(x)
)^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*E
llipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))+15*sin(x)*cos(x)^6
*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I
*(-1+cos(x))/sin(x))^(1/2)*EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/
2-1/2*I,1/2*2^(1/2))-15*sin(x)*cos(x)^6*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)
*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticP
i(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+10*2^(1/2)*cos(
x)^5-10*2^(1/2)*cos(x)^4-104*cos(x)^3*2^(1/2)+104*cos(x)^2*2^(1/2)+64*cos(x)
)*2^(1/2)-64*2^(1/2))*cos(x)*(1+cos(x))^2/(cos(x)^2/sin(x))^(7/2)/sin(x)^7*
2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\csc(x) - \sin(x))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((csc(x) - sin(x))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\sin(x)} - \sin(x)\right)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sin(x) - sin(x))^(7/2), x)
```

```
[Out] int(1/(1/sin(x) - sin(x))^(7/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**(7/2), x)
```

```
[Out] Timed out
```

3.322 $\int (-\cos(x) + \sec(x))^4 dx$

Optimal. Leaf size=44

$$\frac{35x}{8} + \frac{35 \tan^3(x)}{24} - \frac{35 \tan(x)}{8} - \frac{1}{4} \sin^4(x) \tan^3(x) - \frac{7}{8} \sin^2(x) \tan^3(x)$$

[Out] 35/8*x-35/8*tan(x)+35/24*tan(x)^3-7/8*sin(x)^2*tan(x)^3-1/4*sin(x)^4*tan(x)^3

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {288, 302, 203}

$$\frac{35x}{8} + \frac{35 \tan^3(x)}{24} - \frac{35 \tan(x)}{8} - \frac{1}{4} \sin^4(x) \tan^3(x) - \frac{7}{8} \sin^2(x) \tan^3(x)$$

Antiderivative was successfully verified.

[In] Int[(-Cos[x] + Sec[x])^4, x]

[Out] (35*x)/8 - (35*Tan[x])/8 + (35*Tan[x]^3)/24 - (7*Sin[x]^2*Tan[x]^3)/8 - (Sin[x]^4*Tan[x]^3)/4

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 288

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rubi steps

$$\begin{aligned}
\int (-\cos(x) + \sec(x))^4 dx &= \text{Subst} \left(\int \frac{x^8}{(1+x^2)^3} dx, x, \tan(x) \right) \\
&= -\frac{1}{4} \sin^4(x) \tan^3(x) + \frac{7}{4} \text{Subst} \left(\int \frac{x^6}{(1+x^2)^2} dx, x, \tan(x) \right) \\
&= -\frac{7}{8} \sin^2(x) \tan^3(x) - \frac{1}{4} \sin^4(x) \tan^3(x) + \frac{35}{8} \text{Subst} \left(\int \frac{x^4}{1+x^2} dx, x, \tan(x) \right) \\
&= -\frac{7}{8} \sin^2(x) \tan^3(x) - \frac{1}{4} \sin^4(x) \tan^3(x) + \frac{35}{8} \text{Subst} \left(\int \left(-1 + x^2 + \frac{1}{1+x^2} \right) dx, x, \right. \\
&= -\frac{35 \tan(x)}{8} + \frac{35 \tan^3(x)}{24} - \frac{7}{8} \sin^2(x) \tan^3(x) - \frac{1}{4} \sin^4(x) \tan^3(x) + \frac{35}{8} \text{Subst} \left(\int \right. \\
&= \frac{35x}{8} - \frac{35 \tan(x)}{8} + \frac{35 \tan^3(x)}{24} - \frac{7}{8} \sin^2(x) \tan^3(x) - \frac{1}{4} \sin^4(x) \tan^3(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.86

$$\frac{35x}{8} - \frac{3}{4} \sin(2x) + \frac{1}{32} \sin(4x) - \frac{10 \tan(x)}{3} + \frac{1}{3} \tan(x) \sec^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sec[x])^4, x]

[Out] (35*x)/8 - (3*Sin[2*x])/4 + Sin[4*x]/32 - (10*Tan[x])/3 + (Sec[x]^2*Tan[x])/3

fricas [A] time = 1.85, size = 37, normalized size = 0.84

$$\frac{105 x \cos(x)^3 + (6 \cos(x)^6 - 39 \cos(x)^4 - 80 \cos(x)^2 + 8) \sin(x)}{24 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^4,x, algorithm="fricas")

[Out] 1/24*(105*x*cos(x)^3 + (6*cos(x)^6 - 39*cos(x)^4 - 80*cos(x)^2 + 8)*sin(x))/cos(x)^3

giac [A] time = 0.14, size = 35, normalized size = 0.80

$$\frac{1}{3} \tan(x)^3 + \frac{35}{8} x - \frac{13 \tan(x)^3 + 11 \tan(x)}{8 (\tan(x)^2 + 1)^2} - 3 \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^4,x, algorithm="giac")

[Out] 1/3*tan(x)^3 + 35/8*x - 1/8*(13*tan(x)^3 + 11*tan(x))/(tan(x)^2 + 1)^2 - 3*tan(x)

maple [A] time = 0.06, size = 40, normalized size = 0.91

$$-\left(-\frac{2}{3} - \frac{\sec^2(x)}{3} \right) \tan(x) - 4 \tan(x) + \frac{35x}{8} - 2 \cos(x) \sin(x) + \frac{\left(\cos^3(x) + \frac{3 \cos(x)}{2} \right) \sin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-cos(x)+sec(x))^4,x)`

[Out] $-(-2/3-1/3*\sec(x)^2)*\tan(x)-4*\tan(x)+35/8*x-2*\cos(x)*\sin(x)+1/4*(\cos(x)^3+3/2*\cos(x))*\sin(x)$

maxima [A] time = 0.32, size = 26, normalized size = 0.59

$$\frac{1}{3} \tan(x)^3 + \frac{35}{8} x + \frac{1}{32} \sin(4x) - \frac{3}{4} \sin(2x) - 3 \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-cos(x)+sec(x))^4,x, algorithm="maxima")`

[Out] $1/3*\tan(x)^3 + 35/8*x + 1/32*\sin(4*x) - 3/4*\sin(2*x) - 3*\tan(x)$

mupad [B] time = 2.57, size = 80, normalized size = 1.82

$$\frac{35x}{8} + \frac{\frac{35 \tan\left(\frac{x}{2}\right)^{13}}{4} + \frac{35 \tan\left(\frac{x}{2}\right)^{11}}{6} - \frac{329 \tan\left(\frac{x}{2}\right)^9}{12} - 17 \tan\left(\frac{x}{2}\right)^7 - \frac{329 \tan\left(\frac{x}{2}\right)^5}{12} + \frac{35 \tan\left(\frac{x}{2}\right)^3}{6} + \frac{35 \tan\left(\frac{x}{2}\right)}{4}}{\left(\tan\left(\frac{x}{2}\right)^2 - 1\right)^3 \left(\tan\left(\frac{x}{2}\right)^2 + 1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(x) - 1/cos(x))^4,x)`

[Out] $(35*x)/8 + ((35*\tan(x/2))/4 + (35*\tan(x/2)^3)/6 - (329*\tan(x/2)^5)/12 - 17*\tan(x/2)^7 - (329*\tan(x/2)^9)/12 + (35*\tan(x/2)^11)/6 + (35*\tan(x/2)^13)/4) / ((\tan(x/2)^2 - 1)^3 * (\tan(x/2)^2 + 1)^4)$

sympy [A] time = 8.97, size = 44, normalized size = 1.00

$$\frac{35x}{8} - 2 \sin(x) \cos(x) - \frac{4 \sin(x)}{\cos(x)} + \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32} + \frac{\tan^3(x)}{3} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-cos(x)+sec(x))**4,x)`

[Out] $35*x/8 - 2*\sin(x)*\cos(x) - 4*\sin(x)/\cos(x) + \sin(2*x)/4 + \sin(4*x)/32 + \tan(x)**3/3 + \tan(x)$

3.323 $\int(-\cos(x) + \sec(x))^3 dx$

Optimal. Leaf size=34

$$\frac{5 \sin^3(x)}{6} + \frac{5 \sin(x)}{2} + \frac{1}{2} \sin^3(x) \tan^2(x) - \frac{5}{2} \tanh^{-1}(\sin(x))$$

[Out] $-5/2*\operatorname{arctanh}(\sin(x))+5/2*\sin(x)+5/6*\sin(x)^3+1/2*\sin(x)^3*\tan(x)^2$

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4397, 2592, 288, 302, 206}

$$\frac{5 \sin^3(x)}{6} + \frac{5 \sin(x)}{2} + \frac{1}{2} \sin^3(x) \tan^2(x) - \frac{5}{2} \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(-\operatorname{Cos}[x] + \operatorname{Sec}[x])^3, x]$

[Out] $(-5*\operatorname{ArcTanh}[\operatorname{Sin}[x]])/2 + (5*\operatorname{Sin}[x])/2 + (5*\operatorname{Sin}[x]^3)/6 + (\operatorname{Sin}[x]^3*\operatorname{Tan}[x]^2)/2$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 288

$\operatorname{Int}[(c_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow \operatorname{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*n*(p+1)), x] - \operatorname{Dist}[(c^n*(m-n+1))/(b*n*(p+1)), \operatorname{Int}[(c*x)^{(m-n)}*(a + b*x^n)^{(p+1)}, x], x] /;$ $\operatorname{FreeQ}\{a, b, c\}, x \ \&\& \ \operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{LtQ}[p, -1] \ \&\& \ \operatorname{GtQ}[m+1, n] \ \&\& \ !\operatorname{LtQ}[(m+n*(p+1)+1)/n, 0] \ \&\& \ \operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 302

$\operatorname{Int}[(x_)^{(m_)} / ((a_ + (b_)*(x_)^{(n_)}), x_Symbol] \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[x^m, a + b*x^n, x], x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{IGtQ}[m, 0] \ \&\& \ \operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{GtQ}[m, 2*n-1]$

Rule 2592

$\operatorname{Int}[(a_)*\sin[(e_ + (f_)*(x_))]^{(m_)}*\tan[(e_ + (f_)*(x_))]^{(n_)}, x_Symbol] \rightarrow \operatorname{With}\{\operatorname{ff} = \operatorname{FreeFactors}[\operatorname{Sin}[e + f*x], x]\}, \operatorname{Dist}[\operatorname{ff}/f, \operatorname{Subst}[\operatorname{Int}[(\operatorname{ff}*x)^{(m+n)} / (a^2 - \operatorname{ff}^2*x^2)^{((n+1)/2)}, x], x, (a*\operatorname{Sin}[e + f*x])/ff], x] /;$ $\operatorname{FreeQ}\{a, e, f, m\}, x \ \&\& \ \operatorname{IntegerQ}[(n+1)/2]$

Rule 4397

$\operatorname{Int}[u, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{TrigSimplify}[u], x] /;$ $\operatorname{TrigSimplifyQ}[u]$

Rubi steps

$$\begin{aligned}
\int (-\cos(x) + \sec(x))^3 dx &= \int \sin^3(x) \tan^3(x) dx \\
&= \text{Subst} \left(\int \frac{x^6}{(1-x^2)^2} dx, x, \sin(x) \right) \\
&= \frac{1}{2} \sin^3(x) \tan^2(x) - \frac{5}{2} \text{Subst} \left(\int \frac{x^4}{1-x^2} dx, x, \sin(x) \right) \\
&= \frac{1}{2} \sin^3(x) \tan^2(x) - \frac{5}{2} \text{Subst} \left(\int \left(-1 - x^2 + \frac{1}{1-x^2} \right) dx, x, \sin(x) \right) \\
&= \frac{5 \sin(x)}{2} + \frac{5 \sin^3(x)}{6} + \frac{1}{2} \sin^3(x) \tan^2(x) - \frac{5}{2} \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sin(x) \right) \\
&= -\frac{5}{2} \tanh^{-1}(\sin(x)) + \frac{5 \sin(x)}{2} + \frac{5 \sin^3(x)}{6} + \frac{1}{2} \sin^3(x) \tan^2(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.12

$$-\frac{1}{3} \sin^3(x) \tan^2(x) - \frac{5}{3} \sin(x) \tan^2(x) - \frac{5}{2} \tanh^{-1}(\sin(x)) + \frac{5}{2} \tan(x) \sec(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sec[x])^3, x]

[Out] (-5*ArcTanh[Sin[x]])/2 + (5*Sec[x]*Tan[x])/2 - (5*Sin[x]*Tan[x]^2)/3 - (Sin[x]^3*Tan[x]^2)/3

fricas [A] time = 0.94, size = 49, normalized size = 1.44

$$\frac{15 \cos(x)^2 \log(\sin(x) + 1) - 15 \cos(x)^2 \log(-\sin(x) + 1) + 2(2 \cos(x)^4 - 14 \cos(x)^2 - 3) \sin(x)}{12 \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^3,x, algorithm="fricas")

[Out] -1/12*(15*cos(x)^2*log(sin(x) + 1) - 15*cos(x)^2*log(-sin(x) + 1) + 2*(2*cos(x)^4 - 14*cos(x)^2 - 3)*sin(x))/cos(x)^2

giac [A] time = 0.15, size = 39, normalized size = 1.15

$$\frac{1}{3} \sin(x)^3 - \frac{\sin(x)}{2(\sin(x)^2 - 1)} - \frac{5}{4} \log(\sin(x) + 1) + \frac{5}{4} \log(-\sin(x) + 1) + 2 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^3,x, algorithm="giac")

[Out] 1/3*sin(x)^3 - 1/2*sin(x)/(sin(x)^2 - 1) - 5/4*log(sin(x) + 1) + 5/4*log(-sin(x) + 1) + 2*sin(x)

maple [A] time = 0.06, size = 30, normalized size = 0.88

$$\frac{\sec(x) \tan(x)}{2} - \frac{5 \ln(\sec(x) + \tan(x))}{2} + 3 \sin(x) - \frac{(2 + \cos^2(x)) \sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-cos(x)+sec(x))^3,x)

[Out] 1/2*sec(x)*tan(x)-5/2*ln(sec(x)+tan(x))+3*sin(x)-1/3*(2+cos(x)^2)*sin(x)

maxima [A] time = 0.32, size = 37, normalized size = 1.09

$$\frac{1}{3} \sin(x)^3 - \frac{\sin(x)}{2(\sin(x)^2 - 1)} - \frac{5}{4} \log(\sin(x) + 1) + \frac{5}{4} \log(\sin(x) - 1) + 2 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^3,x, algorithm="maxima")

[Out] 1/3*sin(x)^3 - 1/2*sin(x)/(sin(x)^2 - 1) - 5/4*log(sin(x) + 1) + 5/4*log(sin(x) - 1) + 2*sin(x)

mupad [B] time = 2.46, size = 68, normalized size = 2.00

$$\frac{5 \tan\left(\frac{x}{2}\right)^9 + \frac{20 \tan\left(\frac{x}{2}\right)^7}{3} - \frac{22 \tan\left(\frac{x}{2}\right)^5}{3} + \frac{20 \tan\left(\frac{x}{2}\right)^3}{3} + 5 \tan\left(\frac{x}{2}\right)}{\left(\tan\left(\frac{x}{2}\right)^2 - 1\right)^2 \left(\tan\left(\frac{x}{2}\right)^2 + 1\right)^3} - 5 \operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(cos(x) - 1/cos(x))^3,x)

[Out] (5*tan(x/2) + (20*tan(x/2)^3)/3 - (22*tan(x/2)^5)/3 + (20*tan(x/2)^7)/3 + 5*tan(x/2)^9)/((tan(x/2)^2 - 1)^2*(tan(x/2)^2 + 1)^3) - 5*atanh(tan(x/2))

sympy [A] time = 3.74, size = 42, normalized size = 1.24

$$\frac{5 \log(\sin(x) - 1)}{4} - \frac{5 \log(\sin(x) + 1)}{4} + \frac{\sin^3(x)}{3} + 2 \sin(x) - \frac{\sin(x)}{2 \sin^2(x) - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))**3,x)

[Out] 5*log(sin(x) - 1)/4 - 5*log(sin(x) + 1)/4 + sin(x)**3/3 + 2*sin(x) - sin(x)/(2*sin(x)**2 - 2)

3.324 $\int (-\cos(x) + \sec(x))^2 dx$

Optimal. Leaf size=22

$$-\frac{3x}{2} + \frac{3 \tan(x)}{2} - \frac{1}{2} \sin^2(x) \tan(x)$$

[Out] $-3/2*x+3/2*\tan(x)-1/2*\sin(x)^2*\tan(x)$

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {288, 321, 203}

$$-\frac{3x}{2} + \frac{3 \tan(x)}{2} - \frac{1}{2} \sin^2(x) \tan(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-\text{Cos}[x] + \text{Sec}[x])^2, x]$

[Out] $(-3*x)/2 + (3*\text{Tan}[x])/2 - (\text{Sin}[x]^2*\text{Tan}[x])/2$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[\text{Rt}[b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 288

$\text{Int}[(c_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*n*(p+1)), x] - \text{Dist}[(c^{(n*(m-n+1))})/(b*n*(p+1)), \text{Int}[(c*x)^{(m-n)}*(a + b*x^n)^{(p+1)}, x], x] /;$ FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !ILtQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

$\text{Int}[(c_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^{(n*(m-n+1))})/(b*(m+n*p+1)), \text{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned} \int (-\cos(x) + \sec(x))^2 dx &= \text{Subst} \left(\int \frac{x^4}{(1+x^2)^2} dx, x, \tan(x) \right) \\ &= -\frac{1}{2} \sin^2(x) \tan(x) + \frac{3}{2} \text{Subst} \left(\int \frac{x^2}{1+x^2} dx, x, \tan(x) \right) \\ &= \frac{3 \tan(x)}{2} - \frac{1}{2} \sin^2(x) \tan(x) - \frac{3}{2} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right) \\ &= -\frac{3x}{2} + \frac{3 \tan(x)}{2} - \frac{1}{2} \sin^2(x) \tan(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 0.73

$$-\frac{3x}{2} + \frac{1}{4} \sin(2x) + \tan(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sec[x])^2,x]

[Out] (-3*x)/2 + Sin[2*x]/4 + Tan[x]

fricas [A] time = 0.95, size = 22, normalized size = 1.00

$$\frac{3x \cos(x) - (\cos(x)^2 + 2) \sin(x)}{2 \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^2,x, algorithm="fricas")

[Out] -1/2*(3*x*cos(x) - (cos(x)^2 + 2)*sin(x))/cos(x)

giac [A] time = 0.14, size = 18, normalized size = 0.82

$$-\frac{3}{2}x + \frac{\tan(x)}{2(\tan(x)^2 + 1)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^2,x, algorithm="giac")

[Out] -3/2*x + 1/2*tan(x)/(tan(x)^2 + 1) + tan(x)

maple [A] time = 0.05, size = 13, normalized size = 0.59

$$\tan(x) - \frac{3x}{2} + \frac{\cos(x) \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-cos(x)+sec(x))^2,x)

[Out] tan(x)-3/2*x+1/2*cos(x)*sin(x)

maxima [A] time = 0.31, size = 12, normalized size = 0.55

$$-\frac{3}{2}x + \frac{1}{4} \sin(2x) + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^2,x, algorithm="maxima")

[Out] -3/2*x + 1/4*sin(2*x) + tan(x)

mupad [B] time = 2.41, size = 49, normalized size = 2.23

$$-\frac{3x}{2} - \frac{3 \tan\left(\frac{x}{2}\right)^5 + 2 \tan\left(\frac{x}{2}\right)^3 + 3 \tan\left(\frac{x}{2}\right)}{\left(\tan\left(\frac{x}{2}\right)^2 - 1\right) \left(\tan\left(\frac{x}{2}\right)^2 + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x) - 1/cos(x))^2,x)

[Out] - (3*x)/2 - (3*tan(x/2) + 2*tan(x/2)^3 + 3*tan(x/2)^5)/((tan(x/2)^2 - 1)*(tan(x/2)^2 + 1)^2)

sympy [A] time = 1.62, size = 14, normalized size = 0.64

$$-\frac{3x}{2} + \frac{\sin(2x)}{4} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))**2,x)

[Out] -3*x/2 + sin(2*x)/4 + tan(x)

3.325 $\int(-\cos(x) + \sec(x)) dx$

Optimal. Leaf size=8

$$\tanh^{-1}(\sin(x)) - \sin(x)$$

[Out] arctanh(sin(x))-sin(x)

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2637, 3770}

$$\tanh^{-1}(\sin(x)) - \sin(x)$$

Antiderivative was successfully verified.

[In] Int[-Cos[x] + Sec[x],x]

[Out] ArcTanh[Sin[x]] - Sin[x]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int(-\cos(x) + \sec(x)) dx &= -\int \cos(x) dx + \int \sec(x) dx \\ &= \tanh^{-1}(\sin(x)) - \sin(x) \end{aligned}$$

Mathematica [B] time = 0.00, size = 37, normalized size = 4.62

$$-\sin(x) - \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[-Cos[x] + Sec[x],x]

[Out] -Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] - Sin[x]

fricas [B] time = 0.95, size = 21, normalized size = 2.62

$$\frac{1}{2} \log(\sin(x) + 1) - \frac{1}{2} \log(-\sin(x) + 1) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-cos(x)+sec(x),x, algorithm="fricas")

[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)

giac [B] time = 0.15, size = 29, normalized size = 3.62

$$\frac{1}{4} \log\left(\left|\frac{1}{\sin(x)} + \sin(x) + 2\right|\right) - \frac{1}{4} \log\left(\left|\frac{1}{\sin(x)} + \sin(x) - 2\right|\right) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-cos(x)+sec(x),x, algorithm="giac")

[Out] $\frac{1}{4} \log(\text{abs}(1/\sin(x) + \sin(x) + 2)) - \frac{1}{4} \log(\text{abs}(1/\sin(x) + \sin(x) - 2)) - \sin(x)$

maple [A] time = 0.00, size = 12, normalized size = 1.50

$$-\sin(x) + \ln(\sec(x) + \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-cos(x)+sec(x),x)

[Out] $-\sin(x) + \ln(\sec(x) + \tan(x))$

maxima [A] time = 0.32, size = 11, normalized size = 1.38

$$\log(\sec(x) + \tan(x)) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-cos(x)+sec(x),x, algorithm="maxima")

[Out] $\log(\sec(x) + \tan(x)) - \sin(x)$

mupad [B] time = 2.30, size = 14, normalized size = 1.75

$$\ln\left(\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(x) - cos(x),x)

[Out] $\log(\tan(x/2 + \pi/4)) - \sin(x)$

sympy [B] time = 0.09, size = 19, normalized size = 2.38

$$-\frac{\log(\sin(x) - 1)}{2} + \frac{\log(\sin(x) + 1)}{2} - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-cos(x)+sec(x),x)

[Out] $-\log(\sin(x) - 1)/2 + \log(\sin(x) + 1)/2 - \sin(x)$

$$3.326 \quad \int \frac{1}{-\cos(x) + \sec(x)} dx$$

Optimal. Leaf size=4

$$-\csc(x)$$

[Out] -csc(x)

Rubi [A] time = 0.02, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4397, 2606, 8}

$$-\csc(x)$$

Antiderivative was successfully verified.

[In] Int[(-Cos[x] + Sec[x])^(-1), x]

[Out] -Csc[x]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m-1)*(-1+x^2)^((n-1)/2), x], x, Sec[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n+1])

Rule 4397

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \frac{1}{-\cos(x) + \sec(x)} dx &= \int \cot(x) \csc(x) dx \\ &= -\text{Subst}\left(\int 1 dx, x, \csc(x)\right) \\ &= -\csc(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$-\csc(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sec[x])^(-1), x]

[Out] -Csc[x]

fricas [A] time = 0.83, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x)), x, algorithm="fricas")

[Out] $-1/\sin(x)$

giac [A] time = 0.15, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x)),x, algorithm="giac")`

[Out] $-1/\sin(x)$

maple [A] time = 0.10, size = 7, normalized size = 1.75

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-cos(x)+sec(x)),x)`

[Out] $-1/\sin(x)$

maxima [B] time = 0.32, size = 21, normalized size = 5.25

$$-\frac{\cos(x) + 1}{2 \sin(x)} - \frac{\sin(x)}{2(\cos(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x)),x, algorithm="maxima")`

[Out] $-1/2*(\cos(x) + 1)/\sin(x) - 1/2*\sin(x)/(\cos(x) + 1)$

mupad [B] time = 2.37, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(cos(x) - 1/cos(x)),x)`

[Out] $-1/\sin(x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\cos(x) - \sec(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x)),x)`

[Out] $-\text{Integral}(1/(\cos(x) - \sec(x)), x)$

$$3.327 \quad \int \frac{1}{(-\cos(x) + \sec(x))^2} dx$$

Optimal. Leaf size=8

$$-\frac{1}{3} \cot^3(x)$$

[Out] -1/3*cot(x)^3

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {30}

$$-\frac{1}{3} \cot^3(x)$$

Antiderivative was successfully verified.

[In] Int[(-Cos[x] + Sec[x])^(-2), x]

[Out] -Cot[x]^3/3

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{(-\cos(x) + \sec(x))^2} dx &= \text{Subst} \left(\int \frac{1}{x^4} dx, x, \tan(x) \right) \\ &= -\frac{1}{3} \cot^3(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$-\frac{1}{3} \cot^3(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sec[x])^(-2), x]

[Out] -1/3*Cot[x]^3

fricas [B] time = 0.90, size = 18, normalized size = 2.25

$$\frac{\cos(x)^3}{3(\cos(x)^2 - 1)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^2,x, algorithm="fricas")

[Out] 1/3*cos(x)^3/((cos(x)^2 - 1)*sin(x))

giac [A] time = 0.15, size = 6, normalized size = 0.75

$$-\frac{1}{3 \tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^2,x, algorithm="giac")

[Out] -1/3/tan(x)^3

maple [A] time = 0.10, size = 7, normalized size = 0.88

$$-\frac{1}{3 \tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-cos(x)+sec(x))^2,x)

[Out] -1/3/tan(x)^3

maxima [A] time = 0.32, size = 6, normalized size = 0.75

$$-\frac{1}{3 \tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^2,x, algorithm="maxima")

[Out] -1/3/tan(x)^3

mupad [B] time = 2.45, size = 6, normalized size = 0.75

$$-\frac{\cot(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x) - 1/cos(x))^2,x)

[Out] -cot(x)^3/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\cos(x) + \sec(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))**2,x)

[Out] Integral((-cos(x) + sec(x))**(-2), x)

$$3.328 \quad \int \frac{1}{(-\cos(x) + \sec(x))^3} dx$$

Optimal. Leaf size=17

$$\frac{\csc^3(x)}{3} - \frac{\csc^5(x)}{5}$$

[Out] 1/3*csc(x)^3-1/5*csc(x)^5

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4397, 2606, 14}

$$\frac{\csc^3(x)}{3} - \frac{\csc^5(x)}{5}$$

Antiderivative was successfully verified.

[In] Int[(-Cos[x] + Sec[x])^(-3), x]

[Out] Csc[x]^3/3 - Csc[x]^5/5

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 2606

Int[((a_)*sec[(e_)] + (f_)*(x_))]^(m_)*((b_)*tan[(e_)] + (f_)*(x_))]^(n_), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m-1)*(-1+x^2)^((n-1)/2), x], x, Sec[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n+1])

Rule 4397

Int[u, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \frac{1}{(-\cos(x) + \sec(x))^3} dx &= \int \cot^3(x) \csc^3(x) dx \\ &= -\text{Subst} \left(\int x^2 (-1 + x^2) dx, x, \csc(x) \right) \\ &= -\text{Subst} \left(\int (-x^2 + x^4) dx, x, \csc(x) \right) \\ &= \frac{\csc^3(x)}{3} - \frac{\csc^5(x)}{5} \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\csc^3(x)}{3} - \frac{\csc^5(x)}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sec[x])^(-3), x]

[Out] $\text{Csc}[x]^3/3 - \text{Csc}[x]^5/5$

fricas [B] time = 1.02, size = 28, normalized size = 1.65

$$\frac{5 \cos(x)^2 - 2}{15 (\cos(x)^4 - 2 \cos(x)^2 + 1) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x))^3,x, algorithm="fricas")`

[Out] $-1/15*(5*\cos(x)^2 - 2)/((\cos(x)^4 - 2*\cos(x)^2 + 1)*\sin(x))$

giac [A] time = 0.13, size = 14, normalized size = 0.82

$$\frac{5 \sin(x)^2 - 3}{15 \sin(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x))^3,x, algorithm="giac")`

[Out] $1/15*(5*\sin(x)^2 - 3)/\sin(x)^5$

maple [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{1}{3 \sin(x)^3} - \frac{1}{5 \sin(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-cos(x)+sec(x))^3,x)`

[Out] $1/3/\sin(x)^3 - 1/5/\sin(x)^5$

maxima [B] time = 0.32, size = 73, normalized size = 4.29

$$\frac{\left(\frac{5 \sin(x)^2}{(\cos(x)+1)^2} + \frac{30 \sin(x)^4}{(\cos(x)+1)^4} - 3\right)(\cos(x) + 1)^5}{480 \sin(x)^5} + \frac{\sin(x)}{16 (\cos(x) + 1)} + \frac{\sin(x)^3}{96 (\cos(x) + 1)^3} - \frac{\sin(x)^5}{160 (\cos(x) + 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x))^3,x, algorithm="maxima")`

[Out] $1/480*(5*\sin(x)^2/(\cos(x) + 1)^2 + 30*\sin(x)^4/(\cos(x) + 1)^4 - 3)*(\cos(x) + 1)^5/\sin(x)^5 + 1/16*\sin(x)/(\cos(x) + 1) + 1/96*\sin(x)^3/(\cos(x) + 1)^3 - 1/160*\sin(x)^5/(\cos(x) + 1)^5$

mupad [B] time = 2.38, size = 14, normalized size = 0.82

$$\frac{5 \sin(x)^2 - 3}{15 \sin(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(cos(x) - 1/cos(x))^3,x)`

[Out] $(5*\sin(x)^2 - 3)/(15*\sin(x)^5)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\cos^3(x) - 3 \cos^2(x) \sec(x) + 3 \cos(x) \sec^2(x) - \sec^3(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**3,x)
```

```
[Out] -Integral(1/(cos(x)**3 - 3*cos(x)**2*sec(x) + 3*cos(x)*sec(x)**2 - sec(x)**3), x)
```

$$3.329 \quad \int \frac{1}{(-\cos(x) + \sec(x))^4} dx$$

Optimal. Leaf size=17

$$-\frac{1}{7} \cot^7(x) - \frac{\cot^5(x)}{5}$$

[Out] $-1/5*\cot(x)^5-1/7*\cot(x)^7$

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$-\frac{1}{7} \cot^7(x) - \frac{\cot^5(x)}{5}$$

Antiderivative was successfully verified.

[In] Int[(-Cos[x] + Sec[x])^(-4), x]

[Out] $-\text{Cot}[x]^5/5 - \text{Cot}[x]^7/7$

Rubi steps

$$\begin{aligned} \int \frac{1}{(-\cos(x) + \sec(x))^4} dx &= \text{Subst} \left(\int \left(\frac{1}{x^8} + \frac{1}{x^6} \right) dx, x, \tan(x) \right) \\ &= -\frac{1}{5} \cot^5(x) - \frac{\cot^7(x)}{7} \end{aligned}$$

Mathematica [B] time = 0.02, size = 37, normalized size = 2.18

$$-\frac{2 \cot(x)}{35} - \frac{1}{7} \cot(x) \csc^6(x) + \frac{8}{35} \cot(x) \csc^4(x) - \frac{1}{35} \cot(x) \csc^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sec[x])^(-4), x]

[Out] $(-2*\text{Cot}[x])/35 - (\text{Cot}[x]*\text{Csc}[x]^2)/35 + (8*\text{Cot}[x]*\text{Csc}[x]^4)/35 - (\text{Cot}[x]*\text{Csc}[x]^6)/7$

fricas [B] time = 0.65, size = 39, normalized size = 2.29

$$\frac{2 \cos(x)^7 - 7 \cos(x)^5}{35 (\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^4,x, algorithm="fricas")

[Out] $-1/35*(2*\cos(x)^7 - 7*\cos(x)^5)/((\cos(x)^6 - 3*\cos(x)^4 + 3*\cos(x)^2 - 1)*\sin(x))$

giac [A] time = 0.14, size = 14, normalized size = 0.82

$$-\frac{7 \tan(x)^2 + 5}{35 \tan(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^4,x, algorithm="giac")

[Out] $-1/35*(7*\tan(x)^2 + 5)/\tan(x)^7$

maple [A] time = 0.11, size = 14, normalized size = 0.82

$$-\frac{1}{7 \tan(x)^7} - \frac{1}{5 \tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-cos(x)+sec(x))^4,x)`

[Out] $-1/7/\tan(x)^7-1/5/\tan(x)^5$

maxima [A] time = 0.32, size = 14, normalized size = 0.82

$$-\frac{7 \tan(x)^2 + 5}{35 \tan(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x))^4,x, algorithm="maxima")`

[Out] $-1/35*(7*\tan(x)^2 + 5)/\tan(x)^7$

mupad [B] time = 2.43, size = 16, normalized size = 0.94

$$\frac{\cos(x)^5 (\cos(2x) - 6)}{35 \sin(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cos(x) - 1/cos(x))^4,x)`

[Out] $(\cos(x)^5*(\cos(2*x) - 6))/(35*\sin(x)^7)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\cos(x) + \sec(x))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x))**4,x)`

[Out] `Integral((-cos(x) + sec(x))**(-4), x)`

$$3.330 \quad \int \frac{1}{(-\cos(x) + \sec(x))^5} dx$$

Optimal. Leaf size=25

$$-\frac{1}{9} \csc^9(x) + \frac{2 \csc^7(x)}{7} - \frac{\csc^5(x)}{5}$$

[Out] $-1/5*\csc(x)^5+2/7*\csc(x)^7-1/9*\csc(x)^9$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4397, 2606, 270}

$$-\frac{1}{9} \csc^9(x) + \frac{2 \csc^7(x)}{7} - \frac{\csc^5(x)}{5}$$

Antiderivative was successfully verified.

[In] Int[(-Cos[x] + Sec[x])^(-5), x]

[Out] $-\text{Csc}[x]^5/5 + (2*\text{Csc}[x]^7)/7 - \text{Csc}[x]^9/9$

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m-1)*(-1+x^2)^((n-1)/2), x], x, Sec[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n+1])

Rule 4397

Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \frac{1}{(-\cos(x) + \sec(x))^5} dx &= \int \cot^5(x) \csc^5(x) dx \\ &= -\text{Subst}\left(\int x^4 (-1+x^2)^2 dx, x, \csc(x)\right) \\ &= -\text{Subst}\left(\int (x^4 - 2x^6 + x^8) dx, x, \csc(x)\right) \\ &= -\frac{1}{5} \csc^5(x) + \frac{2 \csc^7(x)}{7} - \frac{\csc^9(x)}{9} \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{9} \csc^9(x) + \frac{2 \csc^7(x)}{7} - \frac{\csc^5(x)}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sec[x])^(-5), x]

[Out] $-1/5*\text{Csc}[x]^5 + (2*\text{Csc}[x]^7)/7 - \text{Csc}[x]^9/9$

fricas [B] time = 1.38, size = 46, normalized size = 1.84

$$\frac{63 \cos(x)^4 - 36 \cos(x)^2 + 8}{315 (\cos(x)^8 - 4 \cos(x)^6 + 6 \cos(x)^4 - 4 \cos(x)^2 + 1) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x))^5,x, algorithm="fricas")`

[Out] $-1/315*(63*\cos(x)^4 - 36*\cos(x)^2 + 8)/((\cos(x)^8 - 4*\cos(x)^6 + 6*\cos(x)^4 - 4*\cos(x)^2 + 1)*\sin(x))$

giac [A] time = 0.14, size = 20, normalized size = 0.80

$$-\frac{63 \sin(x)^4 - 90 \sin(x)^2 + 35}{315 \sin(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x))^5,x, algorithm="giac")`

[Out] $-1/315*(63*\sin(x)^4 - 90*\sin(x)^2 + 35)/\sin(x)^9$

maple [A] time = 0.11, size = 20, normalized size = 0.80

$$-\frac{1}{9 \sin(x)^9} + \frac{2}{7 \sin(x)^7} - \frac{1}{5 \sin(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-cos(x)+sec(x))^5,x)`

[Out] $-1/9/\sin(x)^9+2/7/\sin(x)^7-1/5/\sin(x)^5$

maxima [B] time = 0.33, size = 121, normalized size = 4.84

$$\frac{\left(\frac{45 \sin(x)^2}{(\cos(x)+1)^2} + \frac{252 \sin(x)^4}{(\cos(x)+1)^4} - \frac{420 \sin(x)^6}{(\cos(x)+1)^6} - \frac{1890 \sin(x)^8}{(\cos(x)+1)^8} - 35\right)(\cos(x)+1)^9}{161280 \sin(x)^9} - \frac{3 \sin(x)}{256 (\cos(x)+1)} - \frac{\sin(x)^3}{384 (\cos(x)+1)^3} + \frac{1}{640}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x))^5,x, algorithm="maxima")`

[Out] $1/161280*(45*\sin(x)^2/(\cos(x)+1)^2 + 252*\sin(x)^4/(\cos(x)+1)^4 - 420*\sin(x)^6/(\cos(x)+1)^6 - 1890*\sin(x)^8/(\cos(x)+1)^8 - 35)*(\cos(x)+1)^9/\sin(x)^9 - 3/256*\sin(x)/(\cos(x)+1) - 1/384*\sin(x)^3/(\cos(x)+1)^3 + 1/640*\sin(x)^5/(\cos(x)+1)^5 + 1/3584*\sin(x)^7/(\cos(x)+1)^7 - 1/4608*\sin(x)^9/(\cos(x)+1)^9$

mupad [B] time = 2.42, size = 20, normalized size = 0.80

$$\frac{63 \sin(x)^4 - 90 \sin(x)^2 + 35}{315 \sin(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(cos(x) - 1/cos(x))^5,x)`

[Out] $-(63*\sin(x)^4 - 90*\sin(x)^2 + 35)/(315*\sin(x)^9)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))**5,x)

[Out] Timed out

$$3.331 \quad \int \frac{1}{(-\cos(x) + \sec(x))^6} dx$$

Optimal. Leaf size=25

$$-\frac{1}{11} \cot^{11}(x) - \frac{2 \cot^9(x)}{9} - \frac{\cot^7(x)}{7}$$

[Out] $-1/7*\cot(x)^7-2/9*\cot(x)^9-1/11*\cot(x)^{11}$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {270}

$$-\frac{1}{11} \cot^{11}(x) - \frac{2 \cot^9(x)}{9} - \frac{\cot^7(x)}{7}$$

Antiderivative was successfully verified.

[In] Int[(-Cos[x] + Sec[x])^(-6), x]

[Out] -Cot[x]^7/7 - (2*Cot[x]^9)/9 - Cot[x]^11/11

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(-\cos(x) + \sec(x))^6} dx &= \text{Subst} \left(\int \frac{(1+x^2)^2}{x^{12}} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(\frac{1}{x^{12}} + \frac{2}{x^{10}} + \frac{1}{x^8} \right) dx, x, \tan(x) \right) \\ &= -\frac{1}{7} \cot^7(x) - \frac{2 \cot^9(x)}{9} - \frac{\cot^{11}(x)}{11} \end{aligned}$$

Mathematica [B] time = 0.02, size = 57, normalized size = 2.28

$$\frac{8 \cot(x)}{693} - \frac{1}{11} \cot(x) \csc^{10}(x) + \frac{23}{99} \cot(x) \csc^8(x) - \frac{113}{693} \cot(x) \csc^6(x) + \frac{1}{231} \cot(x) \csc^4(x) + \frac{4}{693} \cot(x) \csc^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sec[x])^(-6), x]

[Out] (8*Cot[x])/693 + (4*Cot[x]*Csc[x]^2)/693 + (Cot[x]*Csc[x]^4)/231 - (113*Cot[x]*Csc[x]^6)/693 + (23*Cot[x]*Csc[x]^8)/99 - (Cot[x]*Csc[x]^10)/11

fricas [B] time = 1.47, size = 57, normalized size = 2.28

$$\frac{8 \cos(x)^{11} - 44 \cos(x)^9 + 99 \cos(x)^7}{693 (\cos(x)^{10} - 5 \cos(x)^8 + 10 \cos(x)^6 - 10 \cos(x)^4 + 5 \cos(x)^2 - 1) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^6,x, algorithm="fricas")

[Out] $1/693*(8*\cos(x)^{11} - 44*\cos(x)^9 + 99*\cos(x)^7)/((\cos(x)^{10} - 5*\cos(x)^8 + 10*\cos(x)^6 - 10*\cos(x)^4 + 5*\cos(x)^2 - 1)*\sin(x))$

giac [A] time = 0.12, size = 20, normalized size = 0.80

$$-\frac{99 \tan(x)^4 + 154 \tan(x)^2 + 63}{693 \tan(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x))^6,x, algorithm="giac")`

[Out] $-1/693*(99*\tan(x)^4 + 154*\tan(x)^2 + 63)/\tan(x)^{11}$

maple [A] time = 0.11, size = 20, normalized size = 0.80

$$-\frac{2}{9 \tan(x)^9} - \frac{1}{7 \tan(x)^7} - \frac{1}{11 \tan(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-cos(x)+sec(x))^6,x)`

[Out] $-2/9/\tan(x)^9-1/7/\tan(x)^7-1/11/\tan(x)^{11}$

maxima [A] time = 0.32, size = 20, normalized size = 0.80

$$-\frac{99 \tan(x)^4 + 154 \tan(x)^2 + 63}{693 \tan(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x))^6,x, algorithm="maxima")`

[Out] $-1/693*(99*\tan(x)^4 + 154*\tan(x)^2 + 63)/\tan(x)^{11}$

mupad [B] time = 2.45, size = 46, normalized size = 1.84

$$-\frac{80 \cos(x)^7 - 18 \cos(x)^7 (2 \cos(x)^2 - 1) + \cos(x)^7 (2 (2 \cos(x)^2 - 1)^2 - 1)}{693 \sin(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cos(x) - 1/cos(x))^6,x)`

[Out] $-(80*\cos(x)^7 - 18*\cos(x)^7*(2*\cos(x)^2 - 1) + \cos(x)^7*(2*(2*\cos(x)^2 - 1)^2 - 1))/(693*\sin(x)^{11})$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x))**6,x)`

[Out] Timed out

$$3.332 \quad \int \frac{1}{(-\cos(x) + \sec(x))^7} dx$$

Optimal. Leaf size=33

$$-\frac{1}{13} \csc^{13}(x) + \frac{3 \csc^{11}(x)}{11} - \frac{\csc^9(x)}{3} + \frac{\csc^7(x)}{7}$$

[Out] 1/7*csc(x)^7-1/3*csc(x)^9+3/11*csc(x)^11-1/13*csc(x)^13

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4397, 2606, 270}

$$-\frac{1}{13} \csc^{13}(x) + \frac{3 \csc^{11}(x)}{11} - \frac{\csc^9(x)}{3} + \frac{\csc^7(x)}{7}$$

Antiderivative was successfully verified.

[In] Int[(-Cos[x] + Sec[x])^(-7), x]

[Out] Csc[x]^7/7 - Csc[x]^9/3 + (3*Csc[x]^11)/11 - Csc[x]^13/13

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m-1)*(-1+x^2)^((n-1)/2), x], x, Sec[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n+1])

Rule 4397

Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \frac{1}{(-\cos(x) + \sec(x))^7} dx &= \int \cot^7(x) \csc^7(x) dx \\ &= -\text{Subst} \left(\int x^6 (-1 + x^2)^3 dx, x, \csc(x) \right) \\ &= -\text{Subst} \left(\int (-x^6 + 3x^8 - 3x^{10} + x^{12}) dx, x, \csc(x) \right) \\ &= \frac{\csc^7(x)}{7} - \frac{\csc^9(x)}{3} + \frac{3 \csc^{11}(x)}{11} - \frac{\csc^{13}(x)}{13} \end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{1}{13} \csc^{13}(x) + \frac{3 \csc^{11}(x)}{11} - \frac{\csc^9(x)}{3} + \frac{\csc^7(x)}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sec[x])^(-7), x]

[Out] $\text{Csc}[x]^7/7 - \text{Csc}[x]^9/3 + (3*\text{Csc}[x]^11)/11 - \text{Csc}[x]^13/13$

fricas [B] time = 2.27, size = 64, normalized size = 1.94

$$\frac{429 \cos(x)^6 - 286 \cos(x)^4 + 104 \cos(x)^2 - 16}{3003 (\cos(x)^{12} - 6 \cos(x)^{10} + 15 \cos(x)^8 - 20 \cos(x)^6 + 15 \cos(x)^4 - 6 \cos(x)^2 + 1) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x))^7,x, algorithm="fricas")`

[Out] $-1/3003*(429*\cos(x)^6 - 286*\cos(x)^4 + 104*\cos(x)^2 - 16)/((\cos(x)^{12} - 6*\cos(x)^{10} + 15*\cos(x)^8 - 20*\cos(x)^6 + 15*\cos(x)^4 - 6*\cos(x)^2 + 1)*\sin(x))$

giac [A] time = 0.15, size = 26, normalized size = 0.79

$$\frac{429 \sin(x)^6 - 1001 \sin(x)^4 + 819 \sin(x)^2 - 231}{3003 \sin(x)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x))^7,x, algorithm="giac")`

[Out] $1/3003*(429*\sin(x)^6 - 1001*\sin(x)^4 + 819*\sin(x)^2 - 231)/\sin(x)^{13}$

maple [A] time = 0.11, size = 26, normalized size = 0.79

$$-\frac{1}{13 \sin(x)^{13}} - \frac{1}{3 \sin(x)^9} + \frac{1}{7 \sin(x)^7} + \frac{3}{11 \sin(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-cos(x)+sec(x))^7,x)`

[Out] $-1/13/\sin(x)^{13}-1/3/\sin(x)^9+1/7/\sin(x)^7+3/11/\sin(x)^{11}$

maxima [B] time = 0.33, size = 169, normalized size = 5.12

$$\frac{\left(\frac{273 \sin(x)^2}{(\cos(x)+1)^2} + \frac{2002 \sin(x)^4}{(\cos(x)+1)^4} - \frac{2574 \sin(x)^6}{(\cos(x)+1)^6} - \frac{9009 \sin(x)^8}{(\cos(x)+1)^8} + \frac{15015 \sin(x)^{10}}{(\cos(x)+1)^{10}} + \frac{60060 \sin(x)^{12}}{(\cos(x)+1)^{12}} - 231\right)(\cos(x) + 1)^{13}}{24600576 \sin(x)^{13}} + \frac{5 \sin(x)}{2048 (\cos(x) + 1)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-cos(x)+sec(x))^7,x, algorithm="maxima")`

[Out] $1/24600576*(273*\sin(x)^2/(\cos(x) + 1)^2 + 2002*\sin(x)^4/(\cos(x) + 1)^4 - 2574*\sin(x)^6/(\cos(x) + 1)^6 - 9009*\sin(x)^8/(\cos(x) + 1)^8 + 15015*\sin(x)^{10}/(\cos(x) + 1)^{10} + 60060*\sin(x)^{12}/(\cos(x) + 1)^{12} - 231)*(\cos(x) + 1)^{13}/\sin(x)^{13} + 5/2048*\sin(x)/(\cos(x) + 1) + 5/8192*\sin(x)^3/(\cos(x) + 1)^3 - 3/8192*\sin(x)^5/(\cos(x) + 1)^5 - 3/28672*\sin(x)^7/(\cos(x) + 1)^7 + 1/12288*\sin(x)^9/(\cos(x) + 1)^9 + 1/90112*\sin(x)^{11}/(\cos(x) + 1)^{11} - 1/106496*\sin(x)^{13}/(\cos(x) + 1)^{13})$

mupad [B] time = 2.51, size = 109, normalized size = 3.30

$$-\frac{\cot\left(\frac{x}{2}\right)^{13}}{106496} + \frac{\cot\left(\frac{x}{2}\right)^{11}}{90112} + \frac{\cot\left(\frac{x}{2}\right)^9}{12288} - \frac{3 \cot\left(\frac{x}{2}\right)^7}{28672} - \frac{3 \cot\left(\frac{x}{2}\right)^5}{8192} + \frac{5 \cot\left(\frac{x}{2}\right)^3}{8192} + \frac{5 \cot\left(\frac{x}{2}\right)}{2048} - \frac{\tan\left(\frac{x}{2}\right)^{13}}{106496} + \frac{\tan\left(\frac{x}{2}\right)^{11}}{90112} + \frac{\tan\left(\frac{x}{2}\right)^9}{12288}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(cos(x) - 1/cos(x))^7,x)`

```
[Out] (5*cot(x/2))/2048 + (5*tan(x/2))/2048 + (5*cot(x/2)^3)/8192 - (3*cot(x/2)^5)/8192 - (3*cot(x/2)^7)/28672 + cot(x/2)^9/12288 + cot(x/2)^11/90112 - cot(x/2)^13/106496 + (5*tan(x/2)^3)/8192 - (3*tan(x/2)^5)/8192 - (3*tan(x/2)^7)/28672 + tan(x/2)^9/12288 + tan(x/2)^11/90112 - tan(x/2)^13/106496
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**7,x)
```

```
[Out] Timed out
```

3.333 $\int (-\cos(x) + \sec(x))^{7/2} dx$

Optimal. Leaf size=73

$$-\frac{2}{7} \sin^3(x) \tan^2(x) \sqrt{\sin(x) \tan(x)} - \frac{8}{7} \sin(x) \tan^2(x) \sqrt{\sin(x) \tan(x)} - \frac{256}{35} \csc(x) \sqrt{\sin(x) \tan(x)} + \frac{64}{35} \tan(x) \sec(x)$$

[Out] $-256/35*\csc(x)*(\sin(x)*\tan(x))^{(1/2)}+64/35*\sec(x)*(\sin(x)*\tan(x))^{(1/2)}*\tan(x)-8/7*\sin(x)*(\sin(x)*\tan(x))^{(1/2)}*\tan(x)^2-2/7*\sin(x)^3*(\sin(x)*\tan(x))^{(1/2)}*\tan(x)^2$

Rubi [A] time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {4397, 4400, 2598, 2594, 2589}

$$-\frac{2}{7} \sin^3(x) \tan^2(x) \sqrt{\sin(x) \tan(x)} - \frac{8}{7} \sin(x) \tan^2(x) \sqrt{\sin(x) \tan(x)} - \frac{256}{35} \csc(x) \sqrt{\sin(x) \tan(x)} + \frac{64}{35} \tan(x) \sec(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-\text{Cos}[x] + \text{Sec}[x])^{(7/2)}, x]$

[Out] $(-256*\text{Csc}[x]*\text{Sqrt}[\text{Sin}[x]*\text{Tan}[x]])/35 + (64*\text{Sec}[x]*\text{Tan}[x]*\text{Sqrt}[\text{Sin}[x]*\text{Tan}[x]])/35 - (8*\text{Sin}[x]*\text{Tan}[x]^2*\text{Sqrt}[\text{Sin}[x]*\text{Tan}[x]])/7 - (2*\text{Sin}[x]^3*\text{Tan}[x]^2*\text{Sqrt}[\text{Sin}[x]*\text{Tan}[x]])/7$

Rule 2589

$\text{Int}[(a_* \sin[e_*] + (f_*)*(x_*))^{(m_*)}*((b_*)*\tan[e_*] + (f_*)*(x_*))^{(n_*)}, x_Symbol] \rightarrow -\text{Simp}[(b*(a*\text{Sin}[e + f*x])^m*(b*\text{Tan}[e + f*x])^{(n-1)})/(f*m), x] /;$ FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]

Rule 2594

$\text{Int}[(a_* \sin[e_*] + (f_*)*(x_*))^{(m_*)}*((b_*)*\tan[e_*] + (f_*)*(x_*))^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*(a*\text{Sin}[e + f*x])^m*(b*\text{Tan}[e + f*x])^{(n-1)})/(f*(n-1)), x] - \text{Dist}[(b^2*(m+n-1))/(n-1), \text{Int}[(a*\text{Sin}[e + f*x])^m*(b*\text{Tan}[e + f*x])^{(n-2)}, x], x] /;$ FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && IntegersQ[2*m, 2*n] && !(GtQ[m, 1] && !IntegerQ[(m-1)/2])

Rule 2598

$\text{Int}[(a_* \sin[e_*] + (f_*)*(x_*))^{(m_*)}*((b_*)*\tan[e_*] + (f_*)*(x_*))^{(n_*)}, x_Symbol] \rightarrow -\text{Simp}[(b*(a*\text{Sin}[e + f*x])^m*(b*\text{Tan}[e + f*x])^{(n-1)})/(f*m), x] + \text{Dist}[(a^2*(m+n-1))/m, \text{Int}[(a*\text{Sin}[e + f*x])^{(m-2)}*(b*\text{Tan}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]

Rule 4397

$\text{Int}[u_*, x_Symbol] \rightarrow \text{Int}[\text{TrigSimplify}[u], x] /;$ TrigSimplifyQ[u]

Rule 4400

$\text{Int}[(u_*)*((v_*)^{(m_*)}*(w_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{With}[\{uu = \text{ActivateTrig}[u], vv = \text{ActivateTrig}[v], ww = \text{ActivateTrig}[w]\}, \text{Dist}[(vv^m*ww^n)^{\text{FracPart}[p]}/(vv^{(m*\text{FracPart}[p])}*ww^{(n*\text{FracPart}[p])}), \text{Int}[uu*vv^{(m*p)}*ww^{(n*p)}, x], x] /;$ FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])

Rubi steps

$$\begin{aligned}
\int (-\cos(x) + \sec(x))^{7/2} dx &= \int (\sin(x) \tan(x))^{7/2} dx \\
&= \frac{\sqrt{\sin(x) \tan(x)} \int \sin^{7/2}(x) \tan^{7/2}(x) dx}{\sqrt{\sin(x)} \sqrt{\tan(x)}} \\
&= -\frac{2}{7} \sin^3(x) \tan^2(x) \sqrt{\sin(x) \tan(x)} + \frac{(12\sqrt{\sin(x) \tan(x)}) \int \sin^{3/2}(x) \tan^{7/2}(x) dx}{7\sqrt{\sin(x)} \sqrt{\tan(x)}} \\
&= -\frac{8}{7} \sin(x) \tan^2(x) \sqrt{\sin(x) \tan(x)} - \frac{2}{7} \sin^3(x) \tan^2(x) \sqrt{\sin(x) \tan(x)} + \frac{(32\sqrt{\sin(x) \tan(x)}) \int \sin^{1/2}(x) \tan^{7/2}(x) dx}{7\sqrt{\sin(x)} \sqrt{\tan(x)}} \\
&= \frac{64}{35} \sec(x) \tan(x) \sqrt{\sin(x) \tan(x)} - \frac{8}{7} \sin(x) \tan^2(x) \sqrt{\sin(x) \tan(x)} - \frac{2}{7} \sin^3(x) \tan^2(x) \sqrt{\sin(x) \tan(x)} \\
&= -\frac{256}{35} \csc(x) \sqrt{\sin(x) \tan(x)} + \frac{64}{35} \sec(x) \tan(x) \sqrt{\sin(x) \tan(x)} - \frac{8}{7} \sin(x) \tan^2(x) \sqrt{\sin(x) \tan(x)}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 37, normalized size = 0.51

$$\frac{1}{70} \sec(x) \sqrt{\sin(x) \tan(x)} (28 \tan(x) - 512 \cot(x) - 5(\sin(3x) - 23 \sin(x)) \cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sec[x])^(7/2), x]

[Out] (Sec[x]*Sqrt[Sin[x]*Tan[x]]*(-512*Cot[x] - 5*Cos[x]*(-23*Sin[x] + Sin[3*x]) + 28*Tan[x]))/70

fricas [A] time = 0.99, size = 44, normalized size = 0.60

$$\frac{2(5 \cos(x)^6 - 35 \cos(x)^4 - 105 \cos(x)^2 + 7) \sqrt{-\frac{\cos(x)^2 - 1}{\cos(x)}}}{35 \cos(x)^2 \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^(7/2), x, algorithm="fricas")

[Out] 2/35*(5*cos(x)^6 - 35*cos(x)^4 - 105*cos(x)^2 + 7)*sqrt(-(cos(x)^2 - 1)/cos(x))/(cos(x)^2*sin(x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\cos(x) + \sec(x))^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^(7/2), x, algorithm="giac")

[Out] integrate((-cos(x) + sec(x))^(7/2), x)

maple [B] time = 0.41, size = 603, normalized size = 8.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-cos(x)+sec(x))^(7/2),x)

[Out] $\frac{1}{70}(-1+\cos(x))^2(-105\cos(x)^4(-\cos(x)/(1+\cos(x))^2)^{3/2}\ln(-2(2\cos(x)^2(-\cos(x)/(1+\cos(x))^2)^{1/2}-\cos(x)^2+2\cos(x)-2(-\cos(x)/(1+\cos(x))^2)^{1/2}-1)/\sin(x)^2)+105\cos(x)^4(-\cos(x)/(1+\cos(x))^2)^{3/2}\ln(-(2\cos(x)^2(-\cos(x)/(1+\cos(x))^2)^{1/2}-\cos(x)^2+2\cos(x)-2(-\cos(x)/(1+\cos(x))^2)^{1/2}-1)/\sin(x)^2)-315\cos(x)^3\ln(-2(2\cos(x)^2(-\cos(x)/(1+\cos(x))^2)^{1/2}-\cos(x)^2+2\cos(x)-2(-\cos(x)/(1+\cos(x))^2)^{1/2}-1)/\sin(x)^2)*(-\cos(x)/(1+\cos(x))^2)^{3/2}+315\cos(x)^3\ln(-(2\cos(x)^2(-\cos(x)/(1+\cos(x))^2)^{1/2}-\cos(x)^2+2\cos(x)-2(-\cos(x)/(1+\cos(x))^2)^{1/2}-1)/\sin(x)^2)*(-\cos(x)/(1+\cos(x))^2)^{3/2}+20\cos(x)^6-315\ln(-2(2\cos(x)^2(-\cos(x)/(1+\cos(x))^2)^{1/2}-\cos(x)^2+2\cos(x)-2(-\cos(x)/(1+\cos(x))^2)^{1/2}-1)/\sin(x)^2)*\cos(x)^2(-\cos(x)/(1+\cos(x))^2)^{3/2}+315\ln(-2(2\cos(x)^2(-\cos(x)/(1+\cos(x))^2)^{1/2}-\cos(x)^2+2\cos(x)-2(-\cos(x)/(1+\cos(x))^2)^{1/2}-1)/\sin(x)^2)*\cos(x)^2(-\cos(x)/(1+\cos(x))^2)^{3/2}-105\cos(x)\ln(-2(2\cos(x)^2(-\cos(x)/(1+\cos(x))^2)^{1/2}-\cos(x)^2+2\cos(x)-2(-\cos(x)/(1+\cos(x))^2)^{1/2}-1)/\sin(x)^2)*(-\cos(x)/(1+\cos(x))^2)^{3/2}+105\cos(x)\ln(-2(2\cos(x)^2(-\cos(x)/(1+\cos(x))^2)^{1/2}-\cos(x)^2+2\cos(x)-2(-\cos(x)/(1+\cos(x))^2)^{1/2}-1)/\sin(x)^2)*(-\cos(x)/(1+\cos(x))^2)^{3/2}-140\cos(x)^4-420\cos(x)^2+28)\cos(x)*(1+\cos(x))^2(-(-1+\cos(x)^2)/\cos(x))^{7/2}/\sin(x)^{11}}$

maxima [A] time = 0.44, size = 82, normalized size = 1.12

$$\frac{128 \left(\frac{7 \sin(x)^4}{(\cos(x)+1)^4} - \frac{7 \sin(x)^{10}}{(\cos(x)+1)^{10}} + \frac{2 \sin(x)^{14}}{(\cos(x)+1)^{14}} - 2 \right)}{35 \left(\frac{\sin(x)}{\cos(x)+1} + 1 \right)^{\frac{7}{2}} \left(-\frac{\sin(x)}{\cos(x)+1} + 1 \right)^{\frac{7}{2}} \left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1 \right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^(7/2),x, algorithm="maxima")

[Out] $128/35*(7*\sin(x)^4/(\cos(x) + 1)^4 - 7*\sin(x)^{10}/(\cos(x) + 1)^{10} + 2*\sin(x)^{14}/(\cos(x) + 1)^{14} - 2)/((\sin(x)/(\cos(x) + 1) + 1)^{7/2}*(-\sin(x)/(\cos(x) + 1) + 1)^{7/2}*(\sin(x)^2/(\cos(x) + 1)^2 + 1)^{7/2})$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos(x)} - \cos(x) \right)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/cos(x) - cos(x))^(7/2),x)

[Out] int((1/cos(x) - cos(x))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))**(7/2),x)

[Out] Timed out

3.334 $\int (-\cos(x) + \sec(x))^{5/2} dx$

Optimal. Leaf size=50

$$-\frac{2}{5} \sin^2(x) \tan(x) \sqrt{\sin(x) \tan(x)} + \frac{16}{15} \tan(x) \sqrt{\sin(x) \tan(x)} + \frac{64}{15} \cot(x) \sqrt{\sin(x) \tan(x)}$$

[Out] 64/15*cot(x)*(sin(x)*tan(x))^(1/2)+16/15*(sin(x)*tan(x))^(1/2)*tan(x)-2/5*sin(x)^2*(sin(x)*tan(x))^(1/2)*tan(x)

Rubi [A] time = 0.09, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {4397, 4400, 2598, 2594, 2589}

$$-\frac{2}{5} \sin^2(x) \tan(x) \sqrt{\sin(x) \tan(x)} + \frac{16}{15} \tan(x) \sqrt{\sin(x) \tan(x)} + \frac{64}{15} \cot(x) \sqrt{\sin(x) \tan(x)}$$

Antiderivative was successfully verified.

[In] Int[(-Cos[x] + Sec[x])^(5/2), x]

[Out] (64*Cot[x]*Sqrt[Sin[x]*Tan[x]])/15 + (16*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15 - (2*Sin[x]^2*Tan[x]*Sqrt[Sin[x]*Tan[x]])/5

Rule 2589

Int[((a_)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]

Rule 2594

Int[((a_)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] - Dist[(b^2*(m + n - 1))/(n - 1), Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && IntegerQ[2*m, 2*n] && !(GtQ[m, 1] && !IntegerQ[(m - 1)/2])

Rule 2598

Int[((a_)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*m), x] + Dist[(a^2*(m + n - 1))/m, Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && IntegerQ[2*m, 2*n]

Rule 4397

Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rule 4400

Int[(u_)*((v_)^(m_)*(w_)^(n_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPart[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x], x] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])

Rubi steps

$$\begin{aligned}
\int (-\cos(x) + \sec(x))^{5/2} dx &= \int (\sin(x) \tan(x))^{5/2} dx \\
&= \frac{\sqrt{\sin(x) \tan(x)} \int \sin^5(x) \tan^5(x) dx}{\sqrt{\sin(x)} \sqrt{\tan(x)}} \\
&= -\frac{2}{5} \sin^2(x) \tan(x) \sqrt{\sin(x) \tan(x)} + \frac{(8\sqrt{\sin(x) \tan(x)}) \int \sqrt{\sin(x)} \tan^5(x) dx}{5\sqrt{\sin(x)} \sqrt{\tan(x)}} \\
&= \frac{16}{15} \tan(x) \sqrt{\sin(x) \tan(x)} - \frac{2}{5} \sin^2(x) \tan(x) \sqrt{\sin(x) \tan(x)} - \frac{(32\sqrt{\sin(x) \tan(x)})}{15\sqrt{\sin(x)}} \\
&= \frac{64}{15} \cot(x) \sqrt{\sin(x) \tan(x)} + \frac{16}{15} \tan(x) \sqrt{\sin(x) \tan(x)} - \frac{2}{5} \sin^2(x) \tan(x) \sqrt{\sin(x) \tan(x)}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 29, normalized size = 0.58

$$\frac{2}{15} \tan(x) \sqrt{\sin(x) \tan(x)} (3 \cos^2(x) + 32 \cot^2(x) + 5)$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sec[x])^(5/2), x]

[Out] (2*(5 + 3*Cos[x]^2 + 32*Cot[x]^2)*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15

fricas [A] time = 0.86, size = 38, normalized size = 0.76

$$\frac{2(3 \cos(x)^4 - 30 \cos(x)^2 - 5) \sqrt{-\frac{\cos(x)^2 - 1}{\cos(x)}}}{15 \cos(x) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^(5/2), x, algorithm="fricas")

[Out] -2/15*(3*cos(x)^4 - 30*cos(x)^2 - 5)*sqrt(-(cos(x)^2 - 1)/cos(x))/(cos(x)*sin(x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\cos(x) + \sec(x))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^(5/2), x, algorithm="giac")

[Out] integrate((-cos(x) + sec(x))^(5/2), x)

maple [B] time = 0.33, size = 321, normalized size = 6.42

$$(-1 + \cos(x))^2 \left(6(\cos^4(x)) - 15(\cos^2(x)) \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} \ln \left(-\frac{2(\cos^2(x)) \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} - (\cos^2(x)) + 2\cos(x) - 2 \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}}{\sin(x)^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-cos(x)+sec(x))^(5/2), x)

```
[Out] -1/15*(-1+cos(x))^2*(6*cos(x)^4-15*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)*ln
(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+
cos(x))^2)^(1/2)-1)/sin(x)^2)+15*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(-
2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+
cos(x))^2)^(1/2)-1)/sin(x)^2)-15*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(-2
*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(
x))^2)^(1/2)-1)/sin(x)^2)+15*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(-2*(2*c
os(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x)
)^2)^(1/2)-1)/sin(x)^2)-60*cos(x)^2-10)*cos(x)*(1+cos(x))^2*(-(-1+cos(x))^2
/cos(x))^(5/2)/sin(x)^9
```

maxima [B] time = 0.43, size = 82, normalized size = 1.64

$$-\frac{32 \left(\frac{5 \sin(x)^4}{(\cos(x)+1)^4} - \frac{5 \sin(x)^6}{(\cos(x)+1)^6} + \frac{2 \sin(x)^{10}}{(\cos(x)+1)^{10}} - 2 \right)}{15 \left(\frac{\sin(x)}{\cos(x)+1} + 1 \right)^{\frac{5}{2}} \left(-\frac{\sin(x)}{\cos(x)+1} + 1 \right)^{\frac{5}{2}} \left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1 \right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -32/15*(5*sin(x)^4/(cos(x) + 1)^4 - 5*sin(x)^6/(cos(x) + 1)^6 + 2*sin(x)^10
/(cos(x) + 1)^10 - 2)/((sin(x)/(cos(x) + 1) + 1)^(5/2)*(-sin(x)/(cos(x) + 1
) + 1)^(5/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\cos(x)} - \cos(x) \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x) - cos(x))^(5/2),x)
```

```
[Out] int((1/cos(x) - cos(x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**(5/2),x)
```

```
[Out] Timed out
```

3.335 $\int (-\cos(x) + \sec(x))^{3/2} dx$

Optimal. Leaf size=31

$$\frac{8}{3} \csc(x) \sqrt{\sin(x) \tan(x)} - \frac{2}{3} \sin(x) \sqrt{\sin(x) \tan(x)}$$

[Out] $8/3*\csc(x)*(\sin(x)*\tan(x))^{(1/2)}-2/3*\sin(x)*(\sin(x)*\tan(x))^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4397, 4400, 2598, 2589}

$$\frac{8}{3} \csc(x) \sqrt{\sin(x) \tan(x)} - \frac{2}{3} \sin(x) \sqrt{\sin(x) \tan(x)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-\text{Cos}[x] + \text{Sec}[x])^{(3/2)}, x]$

[Out] $(8*\text{Csc}[x]*\text{Sqrt}[\text{Sin}[x]*\text{Tan}[x]])/3 - (2*\text{Sin}[x]*\text{Sqrt}[\text{Sin}[x]*\text{Tan}[x]])/3$

Rule 2589

$\text{Int}[(a_*)*\sin[(e_*) + (f_*)(x_*)]^{(m_*)}*((b_*)*\tan[(e_*) + (f_*)(x_*)]^{(n_*)}, x_Symbol] :> -\text{Simp}[(b*(a*\sin[e + f*x])^m*(b*\tan[e + f*x])^{(n-1)})/(f*m), x] /; \text{FreeQ}\{a, b, e, f, m, n\}, x] \&\& \text{EqQ}[m + n - 1, 0]$

Rule 2598

$\text{Int}[(a_*)*\sin[(e_*) + (f_*)(x_*)]^{(m_*)}*((b_*)*\tan[(e_*) + (f_*)(x_*)]^{(n_*)}, x_Symbol] :> -\text{Simp}[(b*(a*\sin[e + f*x])^m*(b*\tan[e + f*x])^{(n-1)})/(f*m), x] + \text{Dist}[(a^2*(m + n - 1))/m, \text{Int}[(a*\sin[e + f*x])^{(m-2)}*(b*\tan[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, e, f, n\}, x] \&\& (\text{GtQ}[m, 1] \|\| (\text{EqQ}[m, 1] \& \& \text{EqQ}[n, 1/2])) \&\& \text{IntegersQ}[2*m, 2*n]$

Rule 4397

$\text{Int}[u_, x_Symbol] :> \text{Int}[\text{TrigSimplify}[u], x] /; \text{TrigSimplifyQ}[u]$

Rule 4400

$\text{Int}[(u_*)*((v_*)^{(m_*)}*(w_*)^{(n_*)})^{(p_*)}, x_Symbol] :> \text{With}\{\{uu = \text{ActivateTrig}[u], vv = \text{ActivateTrig}[v], ww = \text{ActivateTrig}[w]\}, \text{Dist}[(vv^m*ww^n)^{\text{FracPart}[p]}/(vv^{(m*\text{FracPart}[p])}*ww^{(n*\text{FracPart}[p])}), \text{Int}[uu*vv^{(m*p)}*ww^{(n*p)}, x], x]] /; \text{FreeQ}\{m, n, p\}, x] \&\& !\text{IntegerQ}[p] \&\& (!\text{InertTrigFreeQ}[v] \|\| !\text{InertTrigFreeQ}[w])$

Rubi steps

$$\begin{aligned} \int (-\cos(x) + \sec(x))^{3/2} dx &= \int (\sin(x) \tan(x))^{3/2} dx \\ &= \frac{\sqrt{\sin(x) \tan(x)} \int \sin^{\frac{3}{2}}(x) \tan^{\frac{3}{2}}(x) dx}{\sqrt{\sin(x)} \sqrt{\tan(x)}} \\ &= -\frac{2}{3} \sin(x) \sqrt{\sin(x) \tan(x)} + \frac{(4\sqrt{\sin(x) \tan(x)}) \int \frac{\tan^{\frac{3}{2}}(x)}{\sqrt{\sin(x)}} dx}{3\sqrt{\sin(x)} \sqrt{\tan(x)}} \\ &= \frac{8}{3} \csc(x) \sqrt{\sin(x) \tan(x)} - \frac{2}{3} \sin(x) \sqrt{\sin(x) \tan(x)} \end{aligned}$$

Mathematica [A] time = 0.04, size = 23, normalized size = 0.74

$$\frac{2}{3} \sin(x) \left(4 \csc^2(x) - 1\right) \sqrt{\sin(x) \tan(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sec[x])^(3/2), x]

[Out] (2*(-1 + 4*Csc[x]^2)*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3

fricas [A] time = 1.25, size = 26, normalized size = 0.84

$$\frac{2 \left(\cos(x)^2 + 3\right) \sqrt{-\frac{\cos(x)^2 - 1}{\cos(x)}}}{3 \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^(3/2), x, algorithm="fricas")

[Out] 2/3*(cos(x)^2 + 3)*sqrt(-(cos(x)^2 - 1)/cos(x))/sin(x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\cos(x) + \sec(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^(3/2), x, algorithm="giac")

[Out] integrate((-cos(x) + sec(x))^(3/2), x)

maple [B] time = 0.27, size = 584, normalized size = 18.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-cos(x)+sec(x))^(3/2), x)

[Out] $\frac{1}{6}(-1+\cos(x))^2(3\cos(x)^3\ln(-2*(2*\cos(x))^2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-\cos(x)^2+2*\cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-1)/\sin(x)^2)*(-\cos(x)/(1+\cos(x))^2)^{(3/2)}-3*\cos(x)^3\ln(-(2*\cos(x))^2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-\cos(x)^2+2*\cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-1)/\sin(x)^2)*(-\cos(x)/(1+\cos(x))^2)^{(3/2)}+9*\ln(-2*(2*\cos(x))^2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-\cos(x)^2+2*\cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-1)/\sin(x)^2)*\cos(x)^2*(-\cos(x)/(1+\cos(x))^2)^{(3/2)}-9*\ln(-(2*\cos(x))^2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-\cos(x)^2+2*\cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-1)/\sin(x)^2)*\cos(x)^2*(-\cos(x)/(1+\cos(x))^2)^{(3/2)}+9*\cos(x)*\ln(-2*(2*\cos(x))^2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-\cos(x)^2+2*\cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-1)/\sin(x)^2)*(-\cos(x)/(1+\cos(x))^2)^{(3/2)}-9*\cos(x)*\ln(-(2*\cos(x))^2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-\cos(x)^2+2*\cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-1)/\sin(x)^2)*(-\cos(x)/(1+\cos(x))^2)^{(3/2)}+3*\ln(-2*(2*\cos(x))^2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-\cos(x)^2+2*\cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-1)/\sin(x)^2)*(-\cos(x)/(1+\cos(x))^2)^{(3/2)}-3*\ln(-(2*\cos(x))^2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-\cos(x)^2+2*\cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-1)/\sin(x)^2)*(-\cos(x)/(1+\cos(x))^2)^{(3/2)}+4*\cos(x)^3+12*\cos(x))*(-1+\cos(x))^2*(-(-1+\cos(x))^2/\cos(x))^{(3/2)}/\sin(x)^7$

maxima [B] time = 0.43, size = 57, normalized size = 1.84

$$\frac{8 \left(\frac{\sin(x)^6}{(\cos(x)+1)^6} - 1 \right)}{3 \left(\frac{\sin(x)}{\cos(x)+1} + 1 \right)^{\frac{3}{2}} \left(-\frac{\sin(x)}{\cos(x)+1} + 1 \right)^{\frac{3}{2}} \left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1 \right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^(3/2),x, algorithm="maxima")

[Out] -8/3*(sin(x)^6/(cos(x) + 1)^6 - 1)/((sin(x)/(cos(x) + 1) + 1)^(3/2)*(-sin(x))/(cos(x) + 1) + 1)^(3/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(3/2))

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(\frac{1}{\cos(x)} - \cos(x) \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/cos(x) - cos(x))^(3/2),x)

[Out] int((1/cos(x) - cos(x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\cos(x) + \sec(x))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))**(3/2),x)

[Out] Integral((-cos(x) + sec(x))**(3/2), x)

3.336 $\int \sqrt{-\cos(x) + \sec(x)} dx$

Optimal. Leaf size=13

$$-2 \cot(x) \sqrt{\sin(x) \tan(x)}$$

[Out] $-2*\cot(x)*(sin(x)*tan(x))^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4397, 4400, 2589}

$$-2 \cot(x) \sqrt{\sin(x) \tan(x)}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[-Cos[x] + Sec[x]], x]`

[Out] $-2*\cot(x)*\sqrt{\sin(x)*\tan(x)}$

Rule 2589

`Int[((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> -Simp[(b*(a*SIN[e + f*x])^m*(b*TAN[e + f*x])^(n - 1))/(f*m), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]`

Rule 4397

`Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]`

Rule 4400

`Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^p, x_Symbol] :> With[{uu = ActivateTrig[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPart[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x], x] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])`

Rubi steps

$$\begin{aligned} \int \sqrt{-\cos(x) + \sec(x)} dx &= \int \sqrt{\sin(x) \tan(x)} dx \\ &= \frac{\sqrt{\sin(x) \tan(x)} \int \sqrt{\sin(x)} \sqrt{\tan(x)} dx}{\sqrt{\sin(x)} \sqrt{\tan(x)}} \\ &= -2 \cot(x) \sqrt{\sin(x) \tan(x)} \end{aligned}$$

Mathematica [A] time = 0.03, size = 13, normalized size = 1.00

$$-2 \cot(x) \sqrt{\sin(x) \tan(x)}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[-Cos[x] + Sec[x]], x]`

[Out] $-2*\cot(x)*\sqrt{\sin(x)*\tan(x)}$

fricas [A] time = 1.26, size = 22, normalized size = 1.69

$$\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^(1/2),x, algorithm="fricas")

[Out] -2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/sin(x)

giac [B] time = 0.20, size = 46, normalized size = 3.54

$$\frac{4\operatorname{sgn}\left(-\tan\left(\frac{1}{2}x\right)^3 - \tan\left(\frac{1}{2}x\right)\right)\operatorname{sgn}(\cos(x))}{\frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^4 + 1} - 1}{\tan\left(\frac{1}{2}x\right)^2} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^(1/2),x, algorithm="giac")

[Out] -4*sgn(-tan(1/2*x)^3 - tan(1/2*x))*sgn(cos(x))/((sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 - 1)

maple [B] time = 0.31, size = 174, normalized size = 13.38

$$\frac{(-1 + \cos(x)) \left(4 \cos(x) \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} + 4 \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} + \ln \left(-\frac{2 \left(2 \cos^2(x) \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} - (\cos^2(x) + 2 \cos(x) - 2 \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} \right)}{\sin(x)^2} \right)}{2 \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} \sin(x)^3} \right)}{2 \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} \sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-cos(x)+sec(x))^(1/2),x)

[Out] 1/2*(-1+cos(x))*(4*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)+4*(-cos(x)/(1+cos(x))^2)^(1/2)+ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)-ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2))*cos(x)*(-(-1+cos(x)^2)/cos(x))^(1/2)/(-cos(x)/(1+cos(x))^2)^(1/2)/sin(x)^3

maxima [B] time = 0.43, size = 57, normalized size = 4.38

$$\frac{2\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} - 1\right)}{\sqrt{\frac{\sin(x)}{\cos(x)+1} + 1} \sqrt{-\frac{\sin(x)}{\cos(x)+1} + 1} \sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^(1/2),x, algorithm="maxima")

[Out] 2*(sin(x)^2/(cos(x) + 1)^2 - 1)/(sqrt(sin(x)/(cos(x) + 1) + 1)*sqrt(-sin(x)/(cos(x) + 1) + 1)*sqrt(sin(x)^2/(cos(x) + 1)^2 + 1))

mupad [B] time = 2.42, size = 20, normalized size = 1.54

$$-\frac{2 \sin(x)}{\sqrt{\frac{1}{\cos(x)}} \sqrt{1 - \cos(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/cos(x) - cos(x))^(1/2), x)`

[Out] `-(2*sin(x))/((1/cos(x))^(1/2)*(1 - cos(x)^2)^(1/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\cos(x) + \sec(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-cos(x)+sec(x))**(1/2), x)`

[Out] `Integral(sqrt(-cos(x) + sec(x)), x)`

$$3.337 \quad \int \frac{1}{\sqrt{-\cos(x)+\sec(x)}} dx$$

Optimal. Leaf size=52

$$\frac{\sin(x) \tan^{-1}(\sqrt{\cos(x)})}{\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} - \frac{\sin(x) \tanh^{-1}(\sqrt{\cos(x)})}{\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}}$$

[Out] arctan(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)-arctanh(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {4397, 4400, 2601, 2565, 329, 298, 203, 206}

$$\frac{\sin(x) \tan^{-1}(\sqrt{\cos(x)})}{\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} - \frac{\sin(x) \tanh^{-1}(\sqrt{\cos(x)})}{\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-Cos[x] + Sec[x]],x]

[Out] (ArcTan[Sqrt[Cos[x]]*Sin[x]]/(Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]]) - (ArcTanh[Sqrt[Cos[x]]*Sin[x]]/(Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]]))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2565

Int[(cos[(e_.) + (f_.)*(x_)])*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^(n/2), x], x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])

Rule 2601

```
Int[((a_)*sin[(e_)+(f_)*(x_)]^(m_))*((b_)*tan[(e_)+(f_)*(x_)]^(n_), x_Symbol] := Dist[(Cos[e+f*x]^n*(b*Tan[e+f*x]^n)/(a*Sin[e+f*x]^n, Int[(a*Sin[e+f*x]^(m+n)/Cos[e+f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1)])) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4397

```
Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_)*((v_)^(m_)*(w_)^(n_))^(p_), x_Symbol] := With[{uu = ActivateTrig[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPart[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x], x] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt{-\cos(x) + \sec(x)}} dx &= \int \frac{1}{\sqrt{\sin(x) \tan(x)}} dx \\
 &= \frac{(\sqrt{\sin(x)} \sqrt{\tan(x)}) \int \frac{1}{\sqrt{\sin(x)} \sqrt{\tan(x)}} dx}{\sqrt{\sin(x) \tan(x)}} \\
 &= \frac{\sin(x) \int \sqrt{\cos(x)} \csc(x) dx}{\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} \\
 &= -\frac{\sin(x) \operatorname{Subst}\left(\int \frac{\sqrt{x}}{1-x^2} dx, x, \cos(x)\right)}{\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} \\
 &= -\frac{(2 \sin(x)) \operatorname{Subst}\left(\int \frac{x^2}{1-x^4} dx, x, \sqrt{\cos(x)}\right)}{\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} \\
 &= -\frac{\sin(x) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\cos(x)}\right)}{\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} + \frac{\sin(x) \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{\cos(x)}\right)}{\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} \\
 &= \frac{\tan^{-1}(\sqrt{\cos(x)}) \sin(x)}{\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} - \frac{\tanh^{-1}(\sqrt{\cos(x)}) \sin(x)}{\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}}
 \end{aligned}$$

Mathematica [A] time = 0.25, size = 43, normalized size = 0.83

$$\frac{\cos(x) \cot(x) \sqrt{\sin(x) \tan(x)} \left(\tan^{-1}\left(\sqrt[4]{\cos^2(x)}\right) - \tanh^{-1}\left(\sqrt[4]{\cos^2(x)}\right) \right)}{\cos^2(x)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Cos[x] + Sec[x]], x]
```

```
[Out] ((ArcTan[(Cos[x]^2)^(1/4)] - ArcTanh[(Cos[x]^2)^(1/4)])*Cos[x]*Cot[x]*Sqrt[Sin[x]*Tan[x]])/(Cos[x]^2)^(3/4)
```

fricas [A] time = 1.00, size = 72, normalized size = 1.38

$$-\frac{1}{2} \arctan\left(\frac{2\sqrt{\frac{\cos(x)^2-1}{\cos(x)}} \cos(x)}{(\cos(x)-1)\sin(x)}\right) + \frac{1}{2} \log\left(\frac{(\cos(x)+1)\sin(x) - 2\sqrt{\frac{\cos(x)^2-1}{\cos(x)}} \cos(x)}{(\cos(x)-1)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^(1/2),x, algorithm="fricas")

[Out] $-1/2*\arctan(2*\sqrt{-(\cos(x)^2 - 1)/\cos(x)}*\cos(x)/((\cos(x) - 1)*\sin(x))) + 1/2*\log(((\cos(x) + 1)*\sin(x) - 2*\sqrt{-(\cos(x)^2 - 1)/\cos(x)}*\cos(x))/((\cos(x) - 1)*\sin(x)))$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(x) + \sec(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(-cos(x) + sec(x)), x)

maple [B] time = 0.28, size = 105, normalized size = 2.02

$$\frac{\left(\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}} \right) + \ln\left(-\frac{2(\cos^2(x))\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} - (\cos^2(x) + 2\cos(x) - 2\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} - 1)}{\sin(x)^2} \right) \right) (1 + \cos(x)) \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}}{2 \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-cos(x)+sec(x))^(1/2),x)

[Out] $-1/2*(\arctan(1/2/(-\cos(x)/(1+\cos(x))^2))^{(1/2)} + \ln(-2*\cos(x)^2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)} - \cos(x)^2 + 2*\cos(x) - 2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)} - 1)/\sin(x)^2))*(1+\cos(x))*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}*((1-\cos(x)^2)/\cos(x))^{(1/2)}/\sin(x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(x) + \sec(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(-cos(x) + sec(x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{1}{\cos(x)} - \cos(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/cos(x) - cos(x))^(1/2),x)

[Out] int(1/(1/cos(x) - cos(x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\cos(x) + \sec(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))**(1/2),x)

[Out] Integral(1/sqrt(-cos(x) + sec(x)), x)

$$3.338 \quad \int \frac{1}{(-\cos(x) + \sec(x))^{3/2}} dx$$

Optimal. Leaf size=72

$$\frac{\sin(x) \tan^{-1}(\sqrt{\cos(x)})}{4\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} - \frac{\csc(x)}{2\sqrt{\sin(x) \tan(x)}} + \frac{\sin(x) \tanh^{-1}(\sqrt{\cos(x)})}{4\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}}$$

[Out] $-1/2*\csc(x)/(\sin(x)*\tan(x))^{(1/2)}+1/4*\arctan(\cos(x)^{(1/2)})*\sin(x)/\cos(x)^{(1/2)}/(\sin(x)*\tan(x))^{(1/2)}+1/4*\operatorname{arctanh}(\cos(x)^{(1/2)})*\sin(x)/\cos(x)^{(1/2)}/(\sin(x)*\tan(x))^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {4397, 4400, 2597, 2601, 2565, 329, 212, 206, 203}

$$\frac{\sin(x) \tan^{-1}(\sqrt{\cos(x)})}{4\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} - \frac{\csc(x)}{2\sqrt{\sin(x) \tan(x)}} + \frac{\sin(x) \tanh^{-1}(\sqrt{\cos(x)})}{4\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}}$$

Antiderivative was successfully verified.

[In] Int[(-Cos[x] + Sec[x])^(-3/2), x]

[Out] $-\operatorname{Csc}[x]/(2*\operatorname{Sqrt}[\operatorname{Sin}[x]*\operatorname{Tan}[x]]) + (\operatorname{ArcTan}[\operatorname{Sqrt}[\operatorname{Cos}[x]]]*\operatorname{Sin}[x])/(4*\operatorname{Sqrt}[\operatorname{Cos}[x]*\operatorname{Sqrt}[\operatorname{Sin}[x]*\operatorname{Tan}[x]]) + (\operatorname{ArcTanh}[\operatorname{Sqrt}[\operatorname{Cos}[x]]]*\operatorname{Sin}[x])/(4*\operatorname{Sqrt}[\operatorname{Cos}[x]*\operatorname{Sqrt}[\operatorname{Sin}[x]*\operatorname{Tan}[x]])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2565

Int[(cos[(e_.) + (f_.)*(x_)])*(a_.)^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(
n_), x_Symbol] := Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(
m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] && !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(
n_), x_Symbol] := Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x])
^m, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)])) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4397

```
Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^p, x_Symbol] := With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-\cos(x) + \sec(x))^{3/2}} dx &= \int \frac{1}{(\sin(x) \tan(x))^{3/2}} dx \\
&= \frac{(\sqrt{\sin(x)} \sqrt{\tan(x)}) \int \frac{1}{\sin^2(x) \tan^2(x)} dx}{\sqrt{\sin(x) \tan(x)}} \\
&= -\frac{\csc(x)}{2\sqrt{\sin(x) \tan(x)}} - \frac{(\sqrt{\sin(x)} \sqrt{\tan(x)}) \int \frac{\sqrt{\tan(x)}}{\sin^2(x)} dx}{4\sqrt{\sin(x) \tan(x)}} \\
&= -\frac{\csc(x)}{2\sqrt{\sin(x) \tan(x)}} - \frac{\sin(x) \int \frac{\csc(x)}{\sqrt{\cos(x)}} dx}{4\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} \\
&= -\frac{\csc(x)}{2\sqrt{\sin(x) \tan(x)}} + \frac{\sin(x) \operatorname{Subst}\left(\int \frac{1}{\sqrt{x}(1-x^2)} dx, x, \cos(x)\right)}{4\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} \\
&= -\frac{\csc(x)}{2\sqrt{\sin(x) \tan(x)}} + \frac{\sin(x) \operatorname{Subst}\left(\int \frac{1}{1-x^4} dx, x, \sqrt{\cos(x)}\right)}{2\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} \\
&= -\frac{\csc(x)}{2\sqrt{\sin(x) \tan(x)}} + \frac{\sin(x) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\cos(x)}\right)}{4\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} + \frac{\sin(x) \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{\cos(x)}\right)}{4\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} \\
&= -\frac{\csc(x)}{2\sqrt{\sin(x) \tan(x)}} + \frac{\tan^{-1}(\sqrt{\cos(x)}) \sin(x)}{4\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} + \frac{\tanh^{-1}(\sqrt{\cos(x)}) \sin(x)}{4\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 56, normalized size = 0.78

$$\frac{\cot(x)\sqrt{\sin(x)\tan(x)}\left(\tan^{-1}\left(\sqrt[4]{\cos^2(x)}\right)-2\sqrt[4]{\cos^2(x)}\csc^2(x)+\tanh^{-1}\left(\sqrt[4]{\cos^2(x)}\right)\right)}{4\sqrt[4]{\cos^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sec[x])^(-3/2), x]

[Out] (Cot[x]*(ArcTan[(Cos[x]^2)^(1/4)] + ArcTanh[(Cos[x]^2)^(1/4)] - 2*(Cos[x]^2)^(1/4)*Csc[x]^2)*Sqrt[Sin[x]*Tan[x]])/(4*(Cos[x]^2)^(1/4))

fricas [B] time = 0.97, size = 119, normalized size = 1.65

$$\frac{(\cos(x)^2 - 1) \arctan\left(\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}} \cos(x)}{(\cos(x)-1)\sin(x)}\right) \sin(x) - (\cos(x)^2 - 1) \log\left(\frac{(\cos(x)+1)\sin(x)+2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}} \cos(x)}{(\cos(x)-1)\sin(x)}\right) \sin(x)}{8(\cos(x)^2 - 1)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^(3/2), x, algorithm="fricas")

[Out] -1/8*((cos(x)^2 - 1)*arctan(2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/((cos(x) - 1)*sin(x)))*sin(x) - (cos(x)^2 - 1)*log(((cos(x) + 1)*sin(x) + 2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x))/((cos(x) - 1)*sin(x)))*sin(x) - 4*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/((cos(x)^2 - 1)*sin(x)))

giac [B] time = 0.37, size = 114, normalized size = 1.58

$$\frac{\frac{\tan\left(\frac{1}{2}x\right)^2}{\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1-1}} - 2\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1} - \frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1-1}}{\tan\left(\frac{1}{2}x\right)^2} - 2\arcsin\left(\tan\left(\frac{1}{2}x\right)^2\right) - 2\log\left(-\frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1-1}}{\tan\left(\frac{1}{2}x\right)^2}\right)}{16\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^4-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^(3/2), x, algorithm="giac")

[Out] -1/16*(tan(1/2*x)^2/(sqrt(-tan(1/2*x)^4 + 1) - 1) - 2*sqrt(-tan(1/2*x)^4 + 1) - (sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 - 2*arcsin(tan(1/2*x)^2) - 2*log(-(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2))/(sgn(tan(1/2*x)^4 - 1)*sgn(tan(1/2*x)))

maple [B] time = 0.31, size = 265, normalized size = 3.68

$$(-1 + \cos(x)) \left(8(\cos^2(x)) \left(-\frac{\cos(x)}{(1+\cos(x))^2} \right)^{\frac{3}{2}} + 16\cos(x) \left(-\frac{\cos(x)}{(1+\cos(x))^2} \right)^{\frac{3}{2}} - (\cos^2(x)) \ln \left(-\frac{2(\cos^2(x))\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} - (\cos(x))}{(1+\cos(x))^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-cos(x)+sec(x))^(3/2), x)

[Out] -1/8*(-1+cos(x))*(8*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)+16*cos(x)*(-cos(x)/(1+cos(x))^2)^(3/2)-cos(x)^2*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2))

$$-\cos(x)^2+2*\cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-1/\sin(x)^2+\cos(x)^2*\arctan(1/2/(-\cos(x)/(1+\cos(x))^2)^{(1/2)})+8*(-\cos(x)/(1+\cos(x))^2)^{(3/2)}+4*\cos(x)*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-4*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}+\ln(-2*\cos(x)^2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-\cos(x)^2+2*\cos(x)-2*(-\cos(x)/(1+\cos(x))^2)^{(1/2)}-1)/\sin(x)^2-\arctan(1/2/(-\cos(x)/(1+\cos(x))^2)^{(1/2)}))/(-(-1+\cos(x)^2)/\cos(x))^{(3/2)}/\cos(x)/\sin(x)/(-\cos(x)/(1+\cos(x))^2)^{(1/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\cos(x) + \sec(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^(3/2),x, algorithm="maxima")

[Out] integrate((-cos(x) + sec(x))^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(x)} - \cos(x)\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/cos(x) - cos(x))^(3/2),x)

[Out] int(1/(1/cos(x) - cos(x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\cos(x) + \sec(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))**(3/2),x)

[Out] Integral((-cos(x) + sec(x))**(3/2), x)

$$3.339 \quad \int \frac{1}{(-\cos(x) + \sec(x))^{5/2}} dx$$

Optimal. Leaf size=91

$$\frac{3 \sin(x) \tan^{-1}(\sqrt{\cos(x)})}{32\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} + \frac{3 \cot(x)}{16\sqrt{\sin(x) \tan(x)}} + \frac{3 \sin(x) \tanh^{-1}(\sqrt{\cos(x)})}{32\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} - \frac{\cot(x) \csc^2(x)}{4\sqrt{\sin(x) \tan(x)}}$$

[Out] 3/16*cot(x)/(sin(x)*tan(x))^(1/2)-1/4*cot(x)*csc(x)^2/(sin(x)*tan(x))^(1/2)
-3/32*arctan(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)+3/32*arctanh(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)

Rubi [A] time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {4397, 4400, 2597, 2599, 2601, 2565, 329, 298, 203, 206}

$$\frac{3 \sin(x) \tan^{-1}(\sqrt{\cos(x)})}{32\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} + \frac{3 \cot(x)}{16\sqrt{\sin(x) \tan(x)}} + \frac{3 \sin(x) \tanh^{-1}(\sqrt{\cos(x)})}{32\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} - \frac{\cot(x) \csc^2(x)}{4\sqrt{\sin(x) \tan(x)}}$$

Antiderivative was successfully verified.

[In] Int[(-Cos[x] + Sec[x])^(-5/2), x]

[Out] (3*Cot[x])/(16*Sqrt[Sin[x]*Tan[x]]) - (Cot[x]*Csc[x]^2)/(4*Sqrt[Sin[x]*Tan[x]]) - (3*ArcTan[Sqrt[Cos[x]]*Sin[x])/(32*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]]) + (3*ArcTanh[Sqrt[Cos[x]]*Sin[x])/(32*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2565

Int[(cos[(e_.) + (f_.)*(x_)])*(a_.)^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])

Rule 2597

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] && !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2599

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1))/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2601

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x]^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1)])) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4397

```
Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^p, x_Symbol] := With[{uu = ActivateTrig[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPart[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x], x] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-\cos(x) + \sec(x))^{5/2}} dx &= \int \frac{1}{(\sin(x) \tan(x))^{5/2}} dx \\
&= \frac{\left(\sqrt{\sin(x)} \sqrt{\tan(x)}\right) \int \frac{1}{\sin^2(x) \tan^2(x)} dx}{\sqrt{\sin(x) \tan(x)}} \\
&= \frac{\cot(x) \csc^2(x)}{4\sqrt{\sin(x) \tan(x)}} - \frac{\left(3\sqrt{\sin(x)} \sqrt{\tan(x)}\right) \int \frac{1}{\sin^2(x) \sqrt{\tan(x)}} dx}{8\sqrt{\sin(x) \tan(x)}} \\
&= \frac{3 \cot(x)}{16\sqrt{\sin(x) \tan(x)}} - \frac{\cot(x) \csc^2(x)}{4\sqrt{\sin(x) \tan(x)}} - \frac{\left(3\sqrt{\sin(x)} \sqrt{\tan(x)}\right) \int \frac{1}{\sqrt{\sin(x)} \sqrt{\tan(x)}} dx}{32\sqrt{\sin(x) \tan(x)}} \\
&= \frac{3 \cot(x)}{16\sqrt{\sin(x) \tan(x)}} - \frac{\cot(x) \csc^2(x)}{4\sqrt{\sin(x) \tan(x)}} - \frac{(3 \sin(x)) \int \sqrt{\cos(x)} \csc(x) dx}{32\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} \\
&= \frac{3 \cot(x)}{16\sqrt{\sin(x) \tan(x)}} - \frac{\cot(x) \csc^2(x)}{4\sqrt{\sin(x) \tan(x)}} + \frac{(3 \sin(x)) \text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2} dx, x, \cos(x)\right)}{32\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} \\
&= \frac{3 \cot(x)}{16\sqrt{\sin(x) \tan(x)}} - \frac{\cot(x) \csc^2(x)}{4\sqrt{\sin(x) \tan(x)}} + \frac{(3 \sin(x)) \text{Subst}\left(\int \frac{x^2}{1-x^4} dx, x, \sqrt{\cos(x)}\right)}{16\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} \\
&= \frac{3 \cot(x)}{16\sqrt{\sin(x) \tan(x)}} - \frac{\cot(x) \csc^2(x)}{4\sqrt{\sin(x) \tan(x)}} + \frac{(3 \sin(x)) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\cos(x)}\right)}{32\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} \\
&= \frac{3 \cot(x)}{16\sqrt{\sin(x) \tan(x)}} - \frac{\cot(x) \csc^2(x)}{4\sqrt{\sin(x) \tan(x)}} - \frac{3 \tan^{-1}\left(\sqrt{\cos(x)}\right) \sin(x)}{32\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} + \frac{3 \tanh^{-1}}{32\sqrt{\cos(x)}}
\end{aligned}$$

Mathematica [A] time = 0.65, size = 73, normalized size = 0.80

$$\frac{\cot(x)\sqrt{\sin(x) \tan(x)} \left(3 \cos(x) \tan^{-1}\left(\sqrt[4]{\cos^2(x)}\right) - 3 \cos(x) \tanh^{-1}\left(\sqrt[4]{\cos^2(x)}\right) + \cos^2(x)^{3/4}(3 \cos(2x) + 5)\right)}{32 \cos^2(x)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sec[x])^(-5/2), x]

[Out] -1/32*(Cot[x]*(3*ArcTan[(Cos[x]^2)^(1/4)]*Cos[x] - 3*ArcTanh[(Cos[x]^2)^(1/4)]*Cos[x] + (Cos[x]^2)^(3/4)*(5 + 3*Cos[2*x])*Cot[x]*Csc[x]^3)*Sqrt[Sin[x]*Tan[x]])/(Cos[x]^2)^(3/4)

fricas [B] time = 1.02, size = 147, normalized size = 1.62

$$\frac{3 \left(\cos(x)^4 - 2 \cos(x)^2 + 1\right) \arctan\left(\frac{2 \sqrt{-\frac{\cos(x)^2 - 1}{\cos(x)}} \cos(x)}{(\cos(x) - 1) \sin(x)}\right) \sin(x) + 3 \left(\cos(x)^4 - 2 \cos(x)^2 + 1\right) \log\left(\frac{(\cos(x) + 1) \sin(x) + 2 \sqrt{-\frac{\cos(x)^2 - 1}{\cos(x)}} \cos(x)}{(\cos(x) - 1) \sin(x)}\right)}{64 \left(\cos(x)^4 - 2 \cos(x)^2 + 1\right) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^(5/2), x, algorithm="fricas")

[Out] 1/64*(3*(cos(x)^4 - 2*cos(x)^2 + 1)*arctan(2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/((cos(x) - 1)*sin(x)))*sin(x) + 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(((cos(x) + 1)*sin(x) + 2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x))/((cos(x) - 1)*sin(x)))*sin(x) - 4*(3*cos(x)^4 + cos(x)^2)*sqrt(-(cos(x)^2 - 1)/cos(x)))/((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x))

giac [B] time = 0.40, size = 170, normalized size = 1.87

$$\frac{\left(\frac{4 \left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4 + 1} - 1 \right)}{\tan\left(\frac{1}{2}x\right)^2} + 1 \right) \tan\left(\frac{1}{2}x\right)^4}{\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4 + 1} - 1 \right)^2} - 4 \sqrt{-\tan\left(\frac{1}{2}x\right)^4 + 1} \left(\tan\left(\frac{1}{2}x\right)^2 - 2 \right) - \frac{4 \left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4 + 1} - 1 \right)}{\tan\left(\frac{1}{2}x\right)^2} - \frac{\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4 + 1} - 1 \right)^2}{\tan\left(\frac{1}{2}x\right)^4} - 1}{256 \operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^4 - 1\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^(5/2),x, algorithm="giac")

[Out] 1/256*((4*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 + 1)*tan(1/2*x)^4/(sqrt(-tan(1/2*x)^4 + 1) - 1)^2 - 4*sqrt(-tan(1/2*x)^4 + 1)*(tan(1/2*x)^2 - 2) - 4*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 - (sqrt(-tan(1/2*x)^4 + 1) - 1)^2/tan(1/2*x)^4 - 12*arcsin(tan(1/2*x)^2) + 12*log(-sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2))/(sgn(tan(1/2*x)^4 - 1)*sgn(tan(1/2*x)))

maple [B] time = 0.32, size = 454, normalized size = 4.99

$$\frac{\left(24 \left(\cos^3(x) \right) \left(-\frac{\cos(x)}{(1+\cos(x))^2} \right)^{\frac{3}{2}} + 40 \left(\cos^2(x) \right) \left(-\frac{\cos(x)}{(1+\cos(x))^2} \right)^{\frac{3}{2}} - 12 \left(\cos^3(x) \right) \sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}} - 3 \left(\cos^3(x) \right) \ln \left(-\frac{2(\cos^2(x) + \cos(x) + 1)}{\cos(x) + 1} \right) \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-cos(x)+sec(x))^(5/2),x)

[Out] 1/64*(24*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(3/2)+40*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)-12*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(1/2)-3*cos(x)^3*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)-3*cos(x)^3*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))+8*cos(x)*(-cos(x)/(1+cos(x))^2)^(3/2)+24*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)+3*cos(x)^2*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+3*cos(x)^2*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))-8*(-cos(x)/(1+cos(x))^2)^(3/2)-12*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)+3*cos(x)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+3*cos(x)*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))-3*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)-3*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2)))*sin(x)/((-1+cos(x)^2)/cos(x))^(5/2)/cos(x)^2/(-cos(x)/(1+cos(x))^2)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\cos(x) + \sec(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^(5/2),x, algorithm="maxima")

[Out] integrate((-cos(x) + sec(x))^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(x)} - \cos(x)\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/cos(x) - cos(x))^(5/2), x)

[Out] int(1/(1/cos(x) - cos(x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\cos(x) + \sec(x))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))**(5/2), x)

[Out] Integral((-cos(x) + sec(x))**(-5/2), x)

$$3.340 \quad \int \frac{1}{(-\cos(x) + \sec(x))^{7/2}} dx$$

Optimal. Leaf size=110

$$-\frac{5 \sin(x) \tan^{-1}(\sqrt{\cos(x)})}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} + \frac{5 \csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{5 \csc(x)}{192\sqrt{\sin(x)\tan(x)}} - \frac{5 \sin(x) \tanh^{-1}(\sqrt{\cos(x)})}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x) \csc(x)}{6\sqrt{\sin(x)\tan(x)}}$$

[Out] -5/192*csc(x)/(sin(x)*tan(x))^(1/2)+5/48*csc(x)^3/(sin(x)*tan(x))^(1/2)-1/6*cot(x)^2*csc(x)^3/(sin(x)*tan(x))^(1/2)-5/128*arctan(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)-5/128*arctanh(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)

Rubi [A] time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {4397, 4400, 2597, 2599, 2601, 2565, 329, 212, 206, 203}

$$-\frac{5 \sin(x) \tan^{-1}(\sqrt{\cos(x)})}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} + \frac{5 \csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{5 \csc(x)}{192\sqrt{\sin(x)\tan(x)}} - \frac{5 \sin(x) \tanh^{-1}(\sqrt{\cos(x)})}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x) \csc(x)}{6\sqrt{\sin(x)\tan(x)}}$$

Antiderivative was successfully verified.

[In] Int[(-Cos[x] + Sec[x])^(-7/2), x]

[Out] (-5*Csc[x])/(192*Sqrt[Sin[x]*Tan[x]]) + (5*Csc[x]^3)/(48*Sqrt[Sin[x]*Tan[x]]) - (Cot[x]^2*Csc[x]^3)/(6*Sqrt[Sin[x]*Tan[x]]) - (5*ArcTan[Sqrt[Cos[x]]]*Sin[x])/(128*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]]) - (5*ArcTanh[Sqrt[Cos[x]]]*Sin[x])/(128*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k), x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2565

Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&

!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])

Rule 2597

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[((a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 1))/(b*f*(m + n + 1)), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && NeQ[m + n + 1, 0] && IntegerQ[2*m, 2*n] && !(EqQ[n, -3/2] && EqQ[m, 1])

Rule 2599

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^(n - 1))/(a^2*f*(m + n + 1)), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e + f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] && NeQ[m + n + 1, 0] && IntegerQ[2*m, 2*n]

Rule 2601

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[(Cos[e + f*x]^n*(b*Tan[e + f*x])^n)/(a*Sin[e + f*x]^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1)])) || IntegerQ[m - 1/2, n - 1/2])

Rule 4397

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rule 4400

Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^p, x_Symbol] := With[{uu = ActivateTrig[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPart[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x], x] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-\cos(x) + \sec(x))^{7/2}} dx &= \int \frac{1}{(\sin(x) \tan(x))^{7/2}} dx \\
&= \frac{\int \frac{1}{\sin^2(x) \tan^2(x)} dx}{\sqrt{\sin(x) \tan(x)}} \\
&= -\frac{\cot^2(x) \csc^3(x)}{6\sqrt{\sin(x) \tan(x)}} - \frac{(5\sqrt{\sin(x)} \sqrt{\tan(x)}) \int \frac{1}{\sin^2(x) \tan^2(x)} dx}{12\sqrt{\sin(x) \tan(x)}} \\
&= \frac{5 \csc^3(x)}{48\sqrt{\sin(x) \tan(x)}} - \frac{\cot^2(x) \csc^3(x)}{6\sqrt{\sin(x) \tan(x)}} + \frac{(5\sqrt{\sin(x)} \sqrt{\tan(x)}) \int \frac{\sqrt{\tan(x)}}{\sin^2(x)} dx}{96\sqrt{\sin(x) \tan(x)}} \\
&= -\frac{5 \csc(x)}{192\sqrt{\sin(x) \tan(x)}} + \frac{5 \csc^3(x)}{48\sqrt{\sin(x) \tan(x)}} - \frac{\cot^2(x) \csc^3(x)}{6\sqrt{\sin(x) \tan(x)}} + \frac{(5\sqrt{\sin(x)} \sqrt{\tan(x)}) \int \frac{\csc^2(x)}{\sqrt{\cos(x)}} dx}{128\sqrt{\sin(x) \tan(x)}} \\
&= -\frac{5 \csc(x)}{192\sqrt{\sin(x) \tan(x)}} + \frac{5 \csc^3(x)}{48\sqrt{\sin(x) \tan(x)}} - \frac{\cot^2(x) \csc^3(x)}{6\sqrt{\sin(x) \tan(x)}} + \frac{(5 \sin(x)) \int \frac{\csc^2(x)}{\sqrt{\cos(x)}} dx}{128\sqrt{\cos(x)} \sqrt{\sin(x)}} \\
&= -\frac{5 \csc(x)}{192\sqrt{\sin(x) \tan(x)}} + \frac{5 \csc^3(x)}{48\sqrt{\sin(x) \tan(x)}} - \frac{\cot^2(x) \csc^3(x)}{6\sqrt{\sin(x) \tan(x)}} - \frac{(5 \sin(x)) \text{Subst} \left(\int \frac{\csc^2(x)}{\sqrt{\cos(x)}} dx \right)}{128\sqrt{\cos(x)}} \\
&= -\frac{5 \csc(x)}{192\sqrt{\sin(x) \tan(x)}} + \frac{5 \csc^3(x)}{48\sqrt{\sin(x) \tan(x)}} - \frac{\cot^2(x) \csc^3(x)}{6\sqrt{\sin(x) \tan(x)}} - \frac{(5 \sin(x)) \text{Subst} \left(\int \frac{\csc^2(x)}{\sqrt{\cos(x)}} dx \right)}{64\sqrt{\cos(x)}} \\
&= -\frac{5 \csc(x)}{192\sqrt{\sin(x) \tan(x)}} + \frac{5 \csc^3(x)}{48\sqrt{\sin(x) \tan(x)}} - \frac{\cot^2(x) \csc^3(x)}{6\sqrt{\sin(x) \tan(x)}} - \frac{(5 \sin(x)) \text{Subst} \left(\int \frac{\csc^2(x)}{\sqrt{\cos(x)}} dx \right)}{128\sqrt{\cos(x)}} \\
&= -\frac{5 \csc(x)}{192\sqrt{\sin(x) \tan(x)}} + \frac{5 \csc^3(x)}{48\sqrt{\sin(x) \tan(x)}} - \frac{\cot^2(x) \csc^3(x)}{6\sqrt{\sin(x) \tan(x)}} - \frac{5 \tan^{-1}(\sqrt{\cos(x)})}{128\sqrt{\cos(x)} \sqrt{\sin(x)}}
\end{aligned}$$

Mathematica [A] time = 0.36, size = 74, normalized size = 0.67

$$\frac{\cot(x) \sqrt{\sin(x) \tan(x)} \left(15 \tan^{-1} \left(\sqrt[4]{\cos^2(x)} \right) + 2 \sqrt[4]{\cos^2(x)} \left(32 \csc^4(x) - 52 \csc^2(x) + 5 \right) \csc^2(x) + 15 \tanh^{-1} \left(\sqrt[4]{\cos^2(x)} \right) \right)}{384 \sqrt[4]{\cos^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sec[x])^(-7/2), x]

[Out] -1/384*(Cot[x]*(15*ArcTan[(Cos[x]^2)^(1/4)] + 15*ArcTanh[(Cos[x]^2)^(1/4)] + 2*(Cos[x]^2)^(1/4)*Csc[x]^2*(5 - 52*Csc[x]^2 + 32*Csc[x]^4))*Sqrt[Sin[x]*Tan[x]])/(Cos[x]^2)^(1/4)

fricas [B] time = 1.63, size = 171, normalized size = 1.55

$$15 \left(\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1 \right) \arctan \left(\frac{2 \sqrt{\frac{-\cos(x)^2 - 1}{\cos(x)}} \cos(x)}{(\cos(x) - 1) \sin(x)} \right) \sin(x) + 15 \left(\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1 \right)$$

$$768 \left(\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^(7/2),x, algorithm="fricas")


```
[Out] 1/768*(15*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*arctan(2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/((cos(x) - 1)*sin(x)))*sin(x) + 15*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*log(((cos(x) + 1)*sin(x) - 2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x))/((cos(x) - 1)*sin(x)))*sin(x) + 4*(5*cos(x)^5 + 42*cos(x)^3 - 15*cos(x))*sqrt(-(cos(x)^2 - 1)/cos(x))/((cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*sin(x))
```

giac [B] time = 0.45, size = 229, normalized size = 2.08

$$\frac{\left(\frac{3 \left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4 + 1} - 1 \right)^2}{\tan\left(\frac{1}{2}x\right)^2} - \frac{27 \left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4 + 1} - 1 \right)^2}{\tan\left(\frac{1}{2}x\right)^4} + 1 \right) \tan\left(\frac{1}{2}x\right)^6}{\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4 + 1} - 1 \right)^3} - 4 \sqrt{-\tan\left(\frac{1}{2}x\right)^4 + 1} \left(\left(2 \tan\left(\frac{1}{2}x\right)^2 - 3 \right) \tan\left(\frac{1}{2}x\right)^2 - 14 \right) + 3072 \operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(7/2),x, algorithm="giac")
```

```
[Out] -1/3072*((3*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 - 27*(sqrt(-tan(1/2*x)^4 + 1) - 1)^2/tan(1/2*x)^4 + 1)*tan(1/2*x)^6/(sqrt(-tan(1/2*x)^4 + 1) - 1)^3 - 4*sqrt(-tan(1/2*x)^4 + 1)*((2*tan(1/2*x)^2 - 3)*tan(1/2*x)^2 - 14) + 27*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 - 3*(sqrt(-tan(1/2*x)^4 + 1) - 1)^2/tan(1/2*x)^4 - (sqrt(-tan(1/2*x)^4 + 1) - 1)^3/tan(1/2*x)^6 + 60*arcsin(tan(1/2*x)^2) + 60*log(-(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2))/sgn(tan(1/2*x)^4 - 1)*sgn(tan(1/2*x))
```

maple [B] time = 0.36, size = 494, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^(7/2),x)
```

```
[Out] 1/768*(56*cos(x)^4*(-cos(x)/(1+cos(x))^2)^(3/2)-16*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(3/2)-15*cos(x)^4*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+15*cos(x)^4*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))-192*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)+76*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(1/2)+30*cos(x)^3*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)-30*cos(x)^3*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))+16*cos(x)*(-cos(x)/(1+cos(x))^2)^(3/2)-148*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)+136*(-cos(x)/(1+cos(x))^2)^(3/2)+196*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)-30*cos(x)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+30*cos(x)*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))-60*(-cos(x)/(1+cos(x))^2)^(1/2)+15*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)-15*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2)))*sin(x)^3/(-1+cos(x))/(-1+cos(x)^2)/cos(x))^(7/2)/cos(x)^3/(-cos(x)/(1+cos(x))^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\cos(x) + \sec(x))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^(7/2),x, algorithm="maxima")

[Out] integrate((-cos(x) + sec(x))^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(x)} - \cos(x)\right)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/cos(x) - cos(x))^(7/2),x)

[Out] int(1/(1/cos(x) - cos(x))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))**(7/2),x)

[Out] Timed out

3.341 $\int (\sin(x) + \tan(x))^4 dx$

Optimal. Leaf size=55

$$-\frac{61x}{8} - \frac{4\sin^3(x)}{3} + \frac{\tan^3(x)}{3} + 5\tan(x) - 2\tanh^{-1}(\sin(x)) + \frac{1}{4}\sin(x)\cos^3(x) + \frac{19}{8}\sin(x)\cos(x) + 2\tan(x)\sec(x)$$

[Out] $-61/8*x - 2*\operatorname{arctanh}(\sin(x)) + 19/8*\cos(x)*\sin(x) + 1/4*\cos(x)^3*\sin(x) - 4/3*\sin(x)^3 + 5*\tan(x) + 2*\sec(x)*\tan(x) + 1/3*\tan(x)^3$

Rubi [A] time = 0.11, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.286$, Rules used = {4397, 2709, 2637, 2635, 8, 2633, 3770, 3767, 3768}

$$-\frac{61x}{8} - \frac{4\sin^3(x)}{3} + \frac{\tan^3(x)}{3} + 5\tan(x) - 2\tanh^{-1}(\sin(x)) + \frac{1}{4}\sin(x)\cos^3(x) + \frac{19}{8}\sin(x)\cos(x) + 2\tan(x)\sec(x)$$

Antiderivative was successfully verified.

[In] Int[(Sin[x] + Tan[x])^4,x]

[Out] $(-61*x)/8 - 2*\operatorname{ArcTanh}[\sin[x]] + (19*\cos[x]*\sin[x])/8 + (\cos[x]^3*\sin[x])/4 - (4*\sin[x]^3)/3 + 5*\tan[x] + 2*\sec[x]*\tan[x] + \tan[x]^3/3$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*cos[c + d*x])*(b*sin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2709

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_), x_Symbol] := Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x])^p*(a + b*sin[e + f*x])^(m - p/2)/(a - b*sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m, p/2] && (LtQ[p, 0] || GtQ[m - p/2, 0])

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 4397

```
Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\int (\sin(x) + \tan(x))^4 dx &= \int (1 + \cos(x))^4 \tan^4(x) dx \\
&= \int (-10 - 4 \cos(x) + 4 \cos^2(x) + 4 \cos^3(x) + \cos^4(x) - 4 \sec(x) + 4 \sec^2(x) + 4 \sec^3(x)) dx \\
&= -10x - 4 \int \cos(x) dx + 4 \int \cos^2(x) dx + 4 \int \cos^3(x) dx - 4 \int \sec(x) dx + 4 \int \sec^2(x) dx \\
&= -10x - 4 \tanh^{-1}(\sin(x)) - 4 \sin(x) + 2 \cos(x) \sin(x) + \frac{1}{4} \cos^3(x) \sin(x) + 2 \sec(x) \tan(x) \\
&= -8x - 2 \tanh^{-1}(\sin(x)) + \frac{19}{8} \cos(x) \sin(x) + \frac{1}{4} \cos^3(x) \sin(x) - \frac{4 \sin^3(x)}{3} + 5 \tan(x) + \\
&= -\frac{61x}{8} - 2 \tanh^{-1}(\sin(x)) + \frac{19}{8} \cos(x) \sin(x) + \frac{1}{4} \cos^3(x) \sin(x) - \frac{4 \sin^3(x)}{3} + 5 \tan(x)
\end{aligned}$$

Mathematica [B] time = 0.20, size = 129, normalized size = 2.35

$$\frac{1}{768} \sec^3(x) \left(1395 \sin(x) + 672 \sin(2x) + 1265 \sin(3x) + 129 \sin(5x) + 32 \sin(6x) + 3 \sin(7x) - 72 \cos(x) \left(61x - \right. \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^4, x]
```

```
[Out] (Sec[x]^3*(-72*Cos[x]*(61*x - 16*Log[Cos[x/2] - Sin[x/2]] + 16*Log[Cos[x/2]
+ Sin[x/2]]) - 24*Cos[3*x]*(61*x - 16*Log[Cos[x/2] - Sin[x/2]] + 16*Log[Co
s[x/2] + Sin[x/2]]) + 1395*Sin[x] + 672*Sin[2*x] + 1265*Sin[3*x] + 129*Sin[
5*x] + 32*Sin[6*x] + 3*Sin[7*x]))/768
```

fricas [A] time = 1.74, size = 78, normalized size = 1.42

$$\frac{183 x \cos(x)^3 + 24 \cos(x)^3 \log(\sin(x) + 1) - 24 \cos(x)^3 \log(-\sin(x) + 1) - (6 \cos(x)^6 + 32 \cos(x)^5 + 57 \cos(x)^4 - 32 \cos(x)^3 + 112 \cos(x)^2 + 48 \cos(x) + 8) \sin(x)}{24 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^4,x, algorithm="fricas")
```

```
[Out] -1/24*(183*x*cos(x)^3 + 24*cos(x)^3*log(sin(x) + 1) - 24*cos(x)^3*log(-sin(
x) + 1) - (6*cos(x)^6 + 32*cos(x)^5 + 57*cos(x)^4 - 32*cos(x)^3 + 112*cos(x)
)^2 + 48*cos(x) + 8)*sin(x))/cos(x)^3
```

giac [B] time = 5.64, size = 1375, normalized size = 25.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)+tan(x))^4,x, algorithm="giac")

[Out] $\frac{1}{24} \cdot (8 \cdot \tan(1/2 \cdot x)^{10} \cdot \tan(x)^5 - 183 \cdot x \cdot \tan(1/2 \cdot x)^{10} \cdot \tan(x)^2 - 24 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 + 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^{10} \cdot \tan(x)^2 + 24 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 - 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^{10} \cdot \tan(x)^2 + 128 \cdot \tan(1/2 \cdot x)^{10} \cdot \tan(x)^3 + 8 \cdot \tan(1/2 \cdot x)^8 \cdot \tan(x)^5 - 183 \cdot x \cdot \tan(1/2 \cdot x)^{10} - 24 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 + 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^{10} + 24 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 - 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^{10} + 180 \cdot \tan(1/2 \cdot x)^{10} \cdot \tan(x) - 183 \cdot x \cdot \tan(1/2 \cdot x)^8 \cdot \tan(x)^2 - 24 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 + 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^8 \cdot \tan(x)^2 + 24 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 - 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^8 \cdot \tan(x)^2 + 96 \cdot \tan(1/2 \cdot x)^9 \cdot \tan(x)^2 + 128 \cdot \tan(1/2 \cdot x)^8 \cdot \tan(x)^3 - 16 \cdot \tan(1/2 \cdot x)^6 \cdot \tan(x)^5 - 183 \cdot x \cdot \tan(1/2 \cdot x)^8 - 24 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 + 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^8 + 24 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 - 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^8 + 96 \cdot \tan(1/2 \cdot x)^9 + 180 \cdot \tan(1/2 \cdot x)^8 \cdot \tan(x) + 366 \cdot x \cdot \tan(1/2 \cdot x)^6 \cdot \tan(x)^2 + 48 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 + 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^6 \cdot \tan(x)^2 - 48 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 - 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^6 \cdot \tan(x)^2 + 128 \cdot \tan(1/2 \cdot x)^7 \cdot \tan(x)^2 - 256 \cdot \tan(1/2 \cdot x)^6 \cdot \tan(x)^3 - 16 \cdot \tan(1/2 \cdot x)^4 \cdot \tan(x)^5 + 366 \cdot x \cdot \tan(1/2 \cdot x)^6 + 48 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 + 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^6 - 48 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 - 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^6 + 128 \cdot \tan(1/2 \cdot x)^7 - 360 \cdot \tan(1/2 \cdot x)^6 \cdot \tan(x) + 366 \cdot x \cdot \tan(1/2 \cdot x)^4 \cdot \tan(x)^2 + 48 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 + 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^4 \cdot \tan(x)^2 - 48 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 - 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^4 \cdot \tan(x)^2 + 1088 \cdot \tan(1/2 \cdot x)^5 \cdot \tan(x)^2 - 256 \cdot \tan(1/2 \cdot x)^4 \cdot \tan(x)^3 + 8 \cdot \tan(1/2 \cdot x)^2 \cdot \tan(x)^5 + 366 \cdot x \cdot \tan(1/2 \cdot x)^4 + 48 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 + 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^4 - 48 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 - 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^4 + 1088 \cdot \tan(1/2 \cdot x)^5 - 360 \cdot \tan(1/2 \cdot x)^4 \cdot \tan(x) - 183 \cdot x \cdot \tan(1/2 \cdot x)^2 \cdot \tan(x)^2 - 24 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 + 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^2 \cdot \tan(x)^2 + 24 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 - 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^2 \cdot \tan(x)^2 + 128 \cdot \tan(1/2 \cdot x)^3 \cdot \tan(x)^2 + 128 \cdot \tan(1/2 \cdot x)^2 \cdot \tan(x)^3 + 8 \cdot \tan(x)^5 - 183 \cdot x \cdot \tan(1/2 \cdot x)^2 - 24 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 + 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^2 + 24 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 - 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(1/2 \cdot x)^2 + 128 \cdot \tan(1/2 \cdot x)^3 + 180 \cdot \tan(1/2 \cdot x)^2 \cdot \tan(x) - 183 \cdot x \cdot \tan(x)^2 - 24 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 + 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(x)^2 + 24 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 - 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) \cdot \tan(x)^2 + 96 \cdot \tan(1/2 \cdot x) \cdot \tan(x)^2 + 128 \cdot \tan(x)^3 - 183 \cdot x - 24 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 + 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) + 24 \cdot \log(2 \cdot (\tan(1/2 \cdot x)^2 - 2 \cdot \tan(1/2 \cdot x) + 1) / (\tan(1/2 \cdot x)^2 + 1)) + 96 \cdot \tan(1/2 \cdot x) + 180 \cdot \tan(x)) / (\tan(1/2 \cdot x)^{10} \cdot \tan(x)^2 + \tan(1/2 \cdot x)^{10} + \tan(1/2 \cdot x)^8 \cdot \tan(x)^2 + \tan(1/2 \cdot x)^8 - 2 \cdot \tan(1/2 \cdot x)^6 \cdot \tan(x)^2 - 2 \cdot \tan(1/2 \cdot x)^6 - 2 \cdot \tan(1/2 \cdot x)^4 \cdot \tan(x)^2 - 2 \cdot \tan(1/2 \cdot x)^4 + \tan(1/2 \cdot x)^2 \cdot \tan(x)^2 + \tan(1/2 \cdot x)^2 + \tan(x)^2 + 1) + 1/32 \cdot \sin(4 \cdot x)$

maple [A] time = 0.07, size = 66, normalized size = 1.20

$$\frac{23 \left(\sin^3(x) + \frac{3 \sin(x)}{2} \right) \cos(x)}{4} - \frac{61x}{8} + \frac{2 \left(\sin^3(x) \right)}{3} + 2 \sin(x) - 2 \ln(\sec(x) + \tan(x)) + \frac{6 \left(\sin^5(x) \right)}{\cos(x)} + \frac{2 \left(\sin^5(x) \right)}{\cos(x)^2} + \left(\tan(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x)+tan(x))^4,x)

[Out] $23/4*(\sin(x)^3+3/2*\sin(x))*\cos(x)-61/8*x+2/3*\sin(x)^3+2*\sin(x)-2*\ln(\sec(x)+\tan(x))+6*\sin(x)^5/\cos(x)+2*\sin(x)^5/\cos(x)^2+1/3*\tan(x)^3-\tan(x)$

maxima [A] time = 0.42, size = 68, normalized size = 1.24

$$-\frac{4}{3}\sin(x)^3 + \frac{1}{3}\tan(x)^3 - \frac{61}{8}x - \frac{2\sin(x)}{\sin(x)^2 - 1} + \frac{3\tan(x)}{\tan(x)^2 + 1} - \log(\sin(x) + 1) + \log(\sin(x) - 1) + \frac{1}{32}\sin(4x) - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)+tan(x))^4,x, algorithm="maxima")

[Out] $-4/3*\sin(x)^3 + 1/3*\tan(x)^3 - 61/8*x - 2*\sin(x)/(\sin(x)^2 - 1) + 3*\tan(x)/(\tan(x)^2 + 1) - \log(\sin(x) + 1) + \log(\sin(x) - 1) + 1/32*\sin(4*x) - 1/4*\sin(2*x) + 5*\tan(x)$

mupad [B] time = 2.54, size = 88, normalized size = 1.60

$$-\frac{61x}{8} - 4 \operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right) - \frac{\frac{45 \tan\left(\frac{x}{2}\right)^{13}}{4} + \frac{29 \tan\left(\frac{x}{2}\right)^{11}}{6} - \frac{455 \tan\left(\frac{x}{2}\right)^9}{12} - 15 \tan\left(\frac{x}{2}\right)^7 + \frac{179 \tan\left(\frac{x}{2}\right)^5}{4} + \frac{31 \tan\left(\frac{x}{2}\right)^3}{2} + \frac{77 \tan\left(\frac{x}{2}\right)}{4}}{\left(\tan\left(\frac{x}{2}\right)^2 - 1\right)^3 \left(\tan\left(\frac{x}{2}\right)^2 + 1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x) + tan(x))^4,x)

[Out] $-(61*x)/8 - 4*\operatorname{atanh}(\tan(x/2)) - ((77*\tan(x/2))/4 + (31*\tan(x/2)^3)/2 + (179*\tan(x/2)^5)/4 - 15*\tan(x/2)^7 - (455*\tan(x/2)^9)/12 + (29*\tan(x/2)^{11})/6 + (45*\tan(x/2)^{13})/4)/((\tan(x/2)^2 - 1)^3*(\tan(x/2)^2 + 1)^4)$

sympy [A] time = 4.63, size = 90, normalized size = 1.64

$$-\frac{61x}{8} + \log(\sin(x) - 1) - \log(\sin(x) + 1) - \frac{4\sin^3(x)}{3} + \frac{6\sin^3(x)}{\cos(x)} + \frac{\sin^3(x)}{3\cos^3(x)} + 9\sin(x)\cos(x) - \frac{\sin(x)}{\cos(x)} - \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)+tan(x))**4,x)

[Out] $-61*x/8 + \log(\sin(x) - 1) - \log(\sin(x) + 1) - 4*\sin(x)**3/3 + 6*\sin(x)**3/\cos(x) + \sin(x)**3/(3*\cos(x)**3) + 9*\sin(x)*\cos(x) - \sin(x)/\cos(x) - \sin(2*x)/4 + \sin(4*x)/32 - 4*\sin(x)/(2*\sin(x)**2 - 2)$

3.342 $\int (\sin(x) + \tan(x))^3 dx$

Optimal. Leaf size=38

$$\frac{\cos^3(x)}{3} + \frac{3 \cos^2(x)}{2} + 2 \cos(x) + \frac{\sec^2(x)}{2} + 3 \sec(x) - 2 \log(\cos(x))$$

[Out] 2*cos(x)+3/2*cos(x)^2+1/3*cos(x)^3-2*ln(cos(x))+3*sec(x)+1/2*sec(x)^2

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4397, 2707, 75}

$$\frac{\cos^3(x)}{3} + \frac{3 \cos^2(x)}{2} + 2 \cos(x) + \frac{\sec^2(x)}{2} + 3 \sec(x) - 2 \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[(Sin[x] + Tan[x])^3,x]

[Out] 2*Cos[x] + (3*Cos[x]^2)/2 + Cos[x]^3/3 - 2*Log[Cos[x]] + 3*Sec[x] + Sec[x]^2/2

Rule 75

Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p + 2, 0] && GtQ[n + 2*p, 0])

Rule 2707

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 4397

Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int (\sin(x) + \tan(x))^3 dx &= \int (1 + \cos(x))^3 \tan^3(x) dx \\ &= -\text{Subst} \left(\int \frac{(1-x)(1+x)^4}{x^3} dx, x, \cos(x) \right) \\ &= -\text{Subst} \left(\int \left(-2 + \frac{1}{x^3} + \frac{3}{x^2} + \frac{2}{x} - 3x - x^2 \right) dx, x, \cos(x) \right) \\ &= 2 \cos(x) + \frac{3 \cos^2(x)}{2} + \frac{\cos^3(x)}{3} - 2 \log(\cos(x)) + 3 \sec(x) + \frac{\sec^2(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 1.05

$$\frac{9 \cos(x)}{4} + \frac{3}{4} \cos(2x) + \frac{1}{12} \cos(3x) + \frac{\sec^2(x)}{2} + 3 \sec(x) - 2 \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[(Sin[x] + Tan[x])^3,x]

[Out] (9*cos[x])/4 + (3*cos[2*x])/4 + Cos[3*x]/12 - 2*Log[Cos[x]] + 3*Sec[x] + Sec[x]^2/2

fricas [A] time = 2.02, size = 47, normalized size = 1.24

$$\frac{4 \cos(x)^5 + 18 \cos(x)^4 + 24 \cos(x)^3 - 24 \cos(x)^2 \log(-\cos(x)) - 9 \cos(x)^2 + 36 \cos(x) + 6}{12 \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)+tan(x))^3,x, algorithm="fricas")

[Out] 1/12*(4*cos(x)^5 + 18*cos(x)^4 + 24*cos(x)^3 - 24*cos(x)^2*log(-cos(x)) - 9*cos(x)^2 + 36*cos(x) + 6)/cos(x)^2

giac [B] time = 0.45, size = 173, normalized size = 4.55

$$\frac{\tan\left(\frac{1}{2}x\right)^4 \tan(x)^4 - 2 \log\left(\frac{4}{\tan(x)^2+1}\right) \tan\left(\frac{1}{2}x\right)^4 \tan(x)^2 - 10 \tan\left(\frac{1}{2}x\right)^4 \tan(x)^2 - 2 \log\left(\frac{4}{\tan(x)^2+1}\right) \tan\left(\frac{1}{2}x\right)^4 - 8}{2 \left(\tan\left(\frac{1}{2}x\right)^4 \tan(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)+tan(x))^3,x, algorithm="giac")

[Out] 1/2*(tan(1/2*x)^4*tan(x)^4 - 2*log(4/(tan(x)^2 + 1))*tan(1/2*x)^4*tan(x)^2 - 10*tan(1/2*x)^4*tan(x)^2 - 2*log(4/(tan(x)^2 + 1))*tan(1/2*x)^4 - 8*tan(1/2*x)^4 - 3*tan(1/2*x)^2*tan(x)^2 - tan(x)^4 + 2*log(4/(tan(x)^2 + 1))*tan(x)^2 - 3*tan(1/2*x)^2 - 11*tan(x)^2 + 2*log(4/(tan(x)^2 + 1)) - 13)/(tan(1/2*x)^4*tan(x)^2 + tan(1/2*x)^4 - tan(x)^2 - 1) + 1/12*cos(3*x)

maple [A] time = 0.07, size = 39, normalized size = 1.03

$$\frac{8(2 + \sin^2(x)) \cos(x)}{3} - \frac{3(\sin^2(x))}{2} - 2 \ln(\cos(x)) + \frac{3(\sin^4(x))}{\cos(x)} + \frac{(\tan^2(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x)+tan(x))^3,x)

[Out] 8/3*(2+sin(x)^2)*cos(x)-3/2*sin(x)^2-2*ln(cos(x))+3*sin(x)^4/cos(x)+1/2*tan(x)^2

maxima [A] time = 0.32, size = 42, normalized size = 1.11

$$\frac{1}{3} \cos(x)^3 - \frac{3}{2} \sin(x)^2 - \frac{1}{2(\sin(x)^2 - 1)} + \frac{3}{\cos(x)} + 2 \cos(x) - \log(\sin(x)^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)+tan(x))^3,x, algorithm="maxima")

[Out] 1/3*cos(x)^3 - 3/2*sin(x)^2 - 1/2/(sin(x)^2 - 1) + 3/cos(x) + 2*cos(x) - log(sin(x)^2 - 1)

mupad [B] time = 2.45, size = 65, normalized size = 1.71

$$4 \operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right) + \frac{-4 \tan\left(\frac{x}{2}\right)^8 - 4 \tan\left(\frac{x}{2}\right)^6 + \frac{20 \tan\left(\frac{x}{2}\right)^4}{3} + \frac{20 \tan\left(\frac{x}{2}\right)^2}{3} + \frac{32}{3}}{\left(\tan\left(\frac{x}{2}\right)^2 - 1\right)^2 \left(\tan\left(\frac{x}{2}\right)^2 + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((sin(x) + tan(x))^3,x)`

[Out] $4*\operatorname{atanh}(\tan(x/2)^2) + ((20*\tan(x/2)^2)/3 + (20*\tan(x/2)^4)/3 - 4*\tan(x/2)^6 - 4*\tan(x/2)^8 + 32/3)/((\tan(x/2)^2 - 1)^2*(\tan(x/2)^2 + 1)^3)$

sympy [A] time = 5.99, size = 46, normalized size = 1.21

$$-3 \log(\cos(x)) - \frac{\log(\sec^2(x))}{2} + \frac{\cos^3(x)}{3} + \frac{3 \cos^2(x)}{2} + 2 \cos(x) + \frac{\sec^2(x)}{2} + \frac{3}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((sin(x)+tan(x))**3,x)`

[Out] $-3*\log(\cos(x)) - \log(\sec(x)**2)/2 + \cos(x)**3/3 + 3*\cos(x)**2/2 + 2*\cos(x) + \sec(x)**2/2 + 3/\cos(x)$

3.343 $\int (\sin(x) + \tan(x))^2 dx$

Optimal. Leaf size=25

$$-\frac{x}{2} - 2 \sin(x) + \tan(x) + 2 \tanh^{-1}(\sin(x)) - \frac{1}{2} \sin(x) \cos(x)$$

[Out] $-1/2*x+2*\operatorname{arctanh}(\sin(x))-2*\sin(x)-1/2*\cos(x)*\sin(x)+\tan(x)$

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {4397, 2709, 2637, 2635, 8, 3770, 3767}

$$-\frac{x}{2} - 2 \sin(x) + \tan(x) + 2 \tanh^{-1}(\sin(x)) - \frac{1}{2} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sin}[x] + \operatorname{Tan}[x])^2, x]$

[Out] $-x/2 + 2*\operatorname{ArcTanh}[\operatorname{Sin}[x]] - 2*\operatorname{Sin}[x] - (\operatorname{Cos}[x]*\operatorname{Sin}[x])/2 + \operatorname{Tan}[x]$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 2635

$\operatorname{Int}[(b_*\sin[(c_*) + (d_*)*(x_*)])^{(n_*)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*\operatorname{Cos}[c + d*x])*(b*\operatorname{Sin}[c + d*x])^{(n-1)}]/(d*n), x] + \operatorname{Dist}[(b^2*(n-1))/n, \operatorname{Int}[(b*\operatorname{Sin}[c + d*x])^{(n-2)}, x], x] /; \operatorname{FreeQ}[\{b, c, d\}, x] \ \&\& \operatorname{GtQ}[n, 1] \ \&\& \operatorname{IntegerQ}[2*n]$

Rule 2637

$\operatorname{Int}[\sin[\operatorname{Pi}/2 + (c_*) + (d_*)*(x_*)], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Sin}[c + d*x]/d, x] /; \operatorname{FreeQ}[\{c, d\}, x]$

Rule 2709

$\operatorname{Int}[(a_*) + (b_*)\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*\tan[(e_*) + (f_*)*(x_*)]^{(p_*)}, x_Symbol] \rightarrow \operatorname{Dist}[a^p, \operatorname{Int}[\operatorname{ExpandIntegrand}[(\operatorname{Sin}[e + f*x])^p*(a + b*\operatorname{Sin}[e + f*x])^{(m-p/2)}]/(a - b*\operatorname{Sin}[e + f*x])^{(p/2)}, x], x] /; \operatorname{FreeQ}[\{a, b, e, f\}, x] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{IntegersQ}[m, p/2] \ \&\& (\operatorname{LtQ}[p, 0] \ || \ \operatorname{GtQ}[m - p/2, 0])$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2-1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\& \operatorname{IGtQ}[n/2, 0]$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_*) + (d_*)*(x_*)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \operatorname{FreeQ}[\{c, d\}, x]$

Rule 4397

$\operatorname{Int}[u_, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{TrigSimplify}[u], x] /; \operatorname{TrigSimplifyQ}[u]$

Rubi steps

$$\begin{aligned}
\int (\sin(x) + \tan(x))^2 dx &= \int (1 + \cos(x))^2 \tan^2(x) dx \\
&= \int (-2 \cos(x) - \cos^2(x) + 2 \sec(x) + \sec^2(x)) dx \\
&= -2 \int \cos(x) dx + 2 \int \sec(x) dx - \int \cos^2(x) dx + \int \sec^2(x) dx \\
&= 2 \tanh^{-1}(\sin(x)) - 2 \sin(x) - \frac{1}{2} \cos(x) \sin(x) - \frac{\int 1 dx}{2} - \text{Subst}\left(\int 1 dx, x, -\tan(x)\right) \\
&= -\frac{x}{2} + 2 \tanh^{-1}(\sin(x)) - 2 \sin(x) - \frac{1}{2} \cos(x) \sin(x) + \tan(x)
\end{aligned}$$

Mathematica [B] time = 0.09, size = 60, normalized size = 2.40

$$-\frac{x}{2} - 2 \sin(x) + \frac{7 \tan(x)}{8} - \frac{1}{8} \sin(3x) \sec(x) - 2 \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + 2 \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sin[x] + Tan[x])^2, x]

[Out] -1/2*x - 2*Log[Cos[x/2] - Sin[x/2]] + 2*Log[Cos[x/2] + Sin[x/2]] - 2*Sin[x] - (Sec[x]*Sin[3*x])/8 + (7*Tan[x])/8

fricas [B] time = 0.97, size = 44, normalized size = 1.76

$$\frac{x \cos(x) - 2 \cos(x) \log(\sin(x) + 1) + 2 \cos(x) \log(-\sin(x) + 1) + (\cos(x)^2 + 4 \cos(x) - 2) \sin(x)}{2 \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)+tan(x))^2,x, algorithm="fricas")

[Out] -1/2*(x*cos(x) - 2*cos(x)*log(sin(x) + 1) + 2*cos(x)*log(-sin(x) + 1) + (cos(x)^2 + 4*cos(x) - 2)*sin(x))/cos(x)

giac [B] time = 0.20, size = 177, normalized size = 7.08

$$\frac{\frac{1}{2} x - \frac{x \tan\left(\frac{1}{2} x\right)^2 - \log\left(\frac{2\left(\tan\left(\frac{1}{2} x\right)^2 + 2 \tan\left(\frac{1}{2} x\right) + 1\right)}{\tan\left(\frac{1}{2} x\right)^2 + 1}\right) \tan\left(\frac{1}{2} x\right)^2 + \log\left(\frac{2\left(\tan\left(\frac{1}{2} x\right)^2 - 2 \tan\left(\frac{1}{2} x\right) + 1\right)}{\tan\left(\frac{1}{2} x\right)^2 + 1}\right) \tan\left(\frac{1}{2} x\right)^2 - \tan\left(\frac{1}{2} x\right)^2}{\tan\left(\frac{1}{2} x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)+tan(x))^2,x, algorithm="giac")

[Out] 1/2*x - (x*tan(1/2*x)^2 - log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - tan(1/2*x)^2*tan(x) + x - log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1)) + log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1)) + 4*tan(1/2*x) - tan(x))/(tan(1/2*x)^2 + 1) - 1/4*asin(2*x)

maple [A] time = 0.05, size = 25, normalized size = 1.00

$$-\frac{\cos(x) \sin(x)}{2} - \frac{x}{2} - 2 \sin(x) + 2 \ln(\sec(x) + \tan(x)) + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((sin(x)+tan(x))^2,x)`

[Out] `-1/2*cos(x)*sin(x)-1/2*x-2*sin(x)+2*ln(sec(x)+tan(x))+tan(x)`

maxima [A] time = 0.41, size = 28, normalized size = 1.12

$$-\frac{1}{2}x + \log(\sin(x) + 1) - \log(\sin(x) - 1) - \frac{1}{4}\sin(2x) - 2\sin(x) + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((sin(x)+tan(x))^2,x, algorithm="maxima")`

[Out] `-1/2*x + log(sin(x) + 1) - log(sin(x) - 1) - 1/4*sin(2*x) - 2*sin(x) + tan(x)`

mupad [B] time = 2.42, size = 61, normalized size = 2.44

$$4 \operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right) - \frac{x}{2} + \frac{5 \tan\left(\frac{x}{2}\right)^5 + 6 \tan\left(\frac{x}{2}\right)^3 - 3 \tan\left(\frac{x}{2}\right)}{-\tan\left(\frac{x}{2}\right)^6 - \tan\left(\frac{x}{2}\right)^4 + \tan\left(\frac{x}{2}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((sin(x) + tan(x))^2,x)`

[Out] `4*atanh(tan(x/2)) - x/2 + (6*tan(x/2)^3 - 3*tan(x/2) + 5*tan(x/2)^5)/(tan(x/2)^2 - tan(x/2)^4 - tan(x/2)^6 + 1)`

sympy [A] time = 1.66, size = 31, normalized size = 1.24

$$-\frac{x}{2} - \log(\sin(x) - 1) + \log(\sin(x) + 1) - 2\sin(x) - \frac{\sin(2x)}{4} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((sin(x)+tan(x))**2,x)`

[Out] `-x/2 - log(sin(x) - 1) + log(sin(x) + 1) - 2*sin(x) - sin(2*x)/4 + tan(x)`

3.344 $\int (\sin(x) + \tan(x)) dx$

Optimal. Leaf size=10

$$-\cos(x) - \log(\cos(x))$$

[Out] $-\cos(x) - \ln(\cos(x))$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2638, 3475}

$$-\cos(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Sin[x] + Tan[x], x]

[Out] -Cos[x] - Log[Cos[x]]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (\sin(x) + \tan(x)) dx &= \int \sin(x) dx + \int \tan(x) dx \\ &= -\cos(x) - \log(\cos(x)) \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\cos(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x] + Tan[x], x]

[Out] -Cos[x] - Log[Cos[x]]

fricas [A] time = 1.43, size = 12, normalized size = 1.20

$$-\cos(x) - \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)+tan(x), x, algorithm="fricas")

[Out] $-\cos(x) - \log(-\cos(x))$

giac [A] time = 0.14, size = 11, normalized size = 1.10

$$-\cos(x) - \log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)+tan(x),x, algorithm="giac")

[Out] -cos(x) - log(abs(cos(x)))

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$-\cos(x) - \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)+tan(x),x)

[Out] -cos(x)-ln(cos(x))

maxima [A] time = 0.31, size = 8, normalized size = 0.80

$$-\cos(x) + \log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)+tan(x),x, algorithm="maxima")

[Out] -cos(x) + log(sec(x))

mupad [B] time = 2.40, size = 22, normalized size = 2.20

$$2 \operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right) - \frac{2}{\tan\left(\frac{x}{2}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x) + tan(x),x)

[Out] 2*atanh(tan(x/2)^2) - 2/(tan(x/2)^2 + 1)

sympy [A] time = 0.05, size = 8, normalized size = 0.80

$$-\log(\cos(x)) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)+tan(x),x)

[Out] -log(cos(x)) - cos(x)

$$3.345 \quad \int \frac{1}{\sin(x)+\tan(x)} dx$$

Optimal. Leaf size=24

$$-\frac{1}{2} \csc^2(x) - \frac{1}{2} \tanh^{-1}(\cos(x)) + \frac{1}{2} \cot(x) \csc(x)$$

[Out] -1/2*arctanh(cos(x))+1/2*cot(x)*csc(x)-1/2*csc(x)^2

Rubi [A] time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {4397, 2706, 2606, 30, 2611, 3770}

$$-\frac{1}{2} \csc^2(x) - \frac{1}{2} \tanh^{-1}(\cos(x)) + \frac{1}{2} \cot(x) \csc(x)$$

Antiderivative was successfully verified.

[In] Int[(Sin[x] + Tan[x])^(-1), x]

[Out] -ArcTanh[Cos[x]]/2 + (Cot[x]*Csc[x])/2 - Csc[x]^2/2

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2606

Int[((a_)*sec[(e_) + (f_)*(x_)])^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 2611

Int[((a_)*sec[(e_) + (f_)*(x_)])^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]

Rule 2706

Int[((g_)*tan[(e_) + (f_)*(x_)])^(p_)/((a_) + (b_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x] - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4397

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sin(x) + \tan(x)} dx &= \int \frac{\cot(x)}{1 + \cos(x)} dx \\
&= - \int \cot^2(x) \csc(x) dx + \int \cot(x) \csc^2(x) dx \\
&= \frac{1}{2} \cot(x) \csc(x) + \frac{1}{2} \int \csc(x) dx - \text{Subst}\left(\int x dx, x, \csc(x)\right) \\
&= -\frac{1}{2} \tanh^{-1}(\cos(x)) + \frac{1}{2} \cot(x) \csc(x) - \frac{\csc^2(x)}{2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.46

$$-\frac{1}{4} \sec^2\left(\frac{x}{2}\right) + \frac{1}{2} \log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{2} \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sin[x] + Tan[x])^(-1), x]

[Out] -1/2*Log[Cos[x/2]] + Log[Sin[x/2]]/2 - Sec[x/2]^2/4

fricas [A] time = 1.90, size = 35, normalized size = 1.46

$$-\frac{(\cos(x) + 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) - (\cos(x) + 1) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 2}{4(\cos(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sin(x)+tan(x)), x, algorithm="fricas")

[Out] -1/4*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) + 1/2) + 2)/(cos(x) + 1)

giac [A] time = 0.14, size = 28, normalized size = 1.17

$$\frac{\cos(x) - 1}{4(\cos(x) + 1)} + \frac{1}{4} \log\left(-\frac{\cos(x) - 1}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sin(x)+tan(x)), x, algorithm="giac")

[Out] 1/4*(cos(x) - 1)/(cos(x) + 1) + 1/4*log(-(cos(x) - 1)/(cos(x) + 1))

maple [A] time = 0.11, size = 24, normalized size = 1.00

$$\frac{\ln(-1 + \cos(x))}{4} - \frac{1}{2(1 + \cos(x))} - \frac{\ln(1 + \cos(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x)+tan(x)), x)

[Out] 1/4*ln(-1+cos(x))-1/2/(1+cos(x))-1/4*ln(1+cos(x))

maxima [A] time = 0.31, size = 25, normalized size = 1.04

$$-\frac{\sin(x)^2}{4(\cos(x) + 1)^2} + \frac{1}{2} \log\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sin(x)+tan(x)),x, algorithm="maxima")

[Out] $-1/4*\sin(x)^2/(\cos(x) + 1)^2 + 1/2*\log(\sin(x)/(\cos(x) + 1))$

mupad [B] time = 2.47, size = 16, normalized size = 0.67

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2} - \frac{\tan\left(\frac{x}{2}\right)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x) + tan(x)),x)

[Out] $\log(\tan(x/2))/2 - \tan(x/2)^2/4$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(x) + \tan(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sin(x)+tan(x)),x)

[Out] Integral(1/(sin(x) + tan(x)), x)

$$3.346 \quad \int \frac{1}{(\sin(x)+\tan(x))^2} dx$$

Optimal. Leaf size=33

$$-\frac{2}{5} \cot^5(x) - \frac{\cot^3(x)}{3} + \frac{2 \csc^5(x)}{5} - \frac{2 \csc^3(x)}{3}$$

[Out] $-1/3*\cot(x)^3-2/5*\cot(x)^5-2/3*\csc(x)^3+2/5*\csc(x)^5$

Rubi [A] time = 0.13, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {4397, 2711, 2607, 30, 2606, 14}

$$-\frac{2}{5} \cot^5(x) - \frac{\cot^3(x)}{3} + \frac{2 \csc^5(x)}{5} - \frac{2 \csc^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[(Sin[x] + Tan[x])^(-2), x]

[Out] $-\text{Cot}[x]^3/3 - (2*\text{Cot}[x]^5)/5 - (2*\text{Csc}[x]^3)/3 + (2*\text{Csc}[x]^5)/5$

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 30

Int[(x_)^m, x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2606

Int[((a_)*sec[(e_) + (f_)*(x_)])^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 2607

Int[sec[(e_) + (f_)*(x_)]^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 2711

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((g_)*tan[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]

Rule 4397

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(\sin(x) + \tan(x))^2} dx &= \int \frac{\cot^2(x)}{(1 + \cos(x))^2} dx \\
&= \int (\cot^4(x) \csc^2(x) - 2 \cot^3(x) \csc^3(x) + \cot^2(x) \csc^4(x)) dx \\
&= -\left(2 \int \cot^3(x) \csc^3(x) dx\right) + \int \cot^4(x) \csc^2(x) dx + \int \cot^2(x) \csc^4(x) dx \\
&= 2 \operatorname{Subst}\left(\int x^2(-1+x^2) dx, x, \csc(x)\right) + \operatorname{Subst}\left(\int x^4 dx, x, -\cot(x)\right) + \operatorname{Subst}\left(\int x^2 dx, x, \csc(x)\right) \\
&= -\frac{1}{5} \cot^5(x) + 2 \operatorname{Subst}\left(\int (-x^2 + x^4) dx, x, \csc(x)\right) + \operatorname{Subst}\left(\int (x^2 + x^4) dx, x, -\cot(x)\right) \\
&= -\frac{1}{3} \cot^3(x) - \frac{2 \cot^5(x)}{5} - \frac{2 \csc^3(x)}{3} + \frac{2 \csc^5(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.73

$$-\frac{7}{120} \tan\left(\frac{x}{2}\right) - \frac{1}{8} \cot\left(\frac{x}{2}\right) + \frac{1}{40} \tan\left(\frac{x}{2}\right) \sec^4\left(\frac{x}{2}\right) - \frac{11}{120} \tan\left(\frac{x}{2}\right) \sec^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sin[x] + Tan[x])^(-2), x]

[Out] -1/8*Cot[x/2] - (7*Tan[x/2])/120 - (11*Sec[x/2]^2*Tan[x/2])/120 + (Sec[x/2]^4*Tan[x/2])/40

fricas [A] time = 1.45, size = 34, normalized size = 1.03

$$\frac{\cos(x)^3 + 2 \cos(x)^2 + 8 \cos(x) + 4}{15 (\cos(x)^2 + 2 \cos(x) + 1) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sin(x)+tan(x))^2,x, algorithm="fricas")

[Out] -1/15*(cos(x)^3 + 2*cos(x)^2 + 8*cos(x) + 4)/((cos(x)^2 + 2*cos(x) + 1)*sin(x))

giac [A] time = 0.14, size = 31, normalized size = 0.94

$$\frac{1}{40} \tan\left(\frac{1}{2}x\right)^5 - \frac{1}{24} \tan\left(\frac{1}{2}x\right)^3 - \frac{1}{8 \tan\left(\frac{1}{2}x\right)} - \frac{1}{8} \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sin(x)+tan(x))^2,x, algorithm="giac")

[Out] 1/40*tan(1/2*x)^5 - 1/24*tan(1/2*x)^3 - 1/8/tan(1/2*x) - 1/8*tan(1/2*x)

maple [A] time = 0.12, size = 32, normalized size = 0.97

$$\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)}{40} - \frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{24} - \frac{\tan\left(\frac{x}{2}\right)}{8} - \frac{1}{8 \tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x)+tan(x))^2,x)

[Out] $1/40*\tan(1/2*x)^5-1/24*\tan(1/2*x)^3-1/8*\tan(1/2*x)-1/8/\tan(1/2*x)$

maxima [A] time = 0.31, size = 45, normalized size = 1.36

$$-\frac{\cos(x)+1}{8\sin(x)} - \frac{\sin(x)}{8(\cos(x)+1)} - \frac{\sin(x)^3}{24(\cos(x)+1)^3} + \frac{\sin(x)^5}{40(\cos(x)+1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sin(x)+tan(x))^2,x, algorithm="maxima")`

[Out] $-1/8*(\cos(x)+1)/\sin(x) - 1/8*\sin(x)/(\cos(x)+1) - 1/24*\sin(x)^3/(\cos(x)+1)^3 + 1/40*\sin(x)^5/(\cos(x)+1)^5$

mupad [B] time = 2.39, size = 40, normalized size = 1.21

$$-\frac{8\cos\left(\frac{x}{2}\right)^6 - 4\cos\left(\frac{x}{2}\right)^4 + 14\cos\left(\frac{x}{2}\right)^2 - 3}{120\cos\left(\frac{x}{2}\right)^5\sin\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sin(x) + tan(x))^2,x)`

[Out] $-(14*\cos(x/2)^2 - 4*\cos(x/2)^4 + 8*\cos(x/2)^6 - 3)/(120*\cos(x/2)^5*\sin(x/2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\sin(x) + \tan(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sin(x)+tan(x))**2,x)`

[Out] `Integral((sin(x) + tan(x))**(-2), x)`

$$3.347 \quad \int \frac{1}{(\sin(x)+\tan(x))^3} dx$$

Optimal. Leaf size=60

$$-\frac{1}{32(1-\cos(x))} - \frac{1}{16(\cos(x)+1)} - \frac{3}{32(\cos(x)+1)^2} + \frac{1}{6(\cos(x)+1)^3} - \frac{1}{16(\cos(x)+1)^4} + \frac{1}{32} \tanh^{-1}(\cos(x))$$

[Out] 1/32*arctanh(cos(x))-1/32/(1-cos(x))-1/16/(1+cos(x))^4+1/6/(1+cos(x))^3-3/32/(1+cos(x))^2-1/16/(1+cos(x))

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4397, 2707, 88, 207}

$$-\frac{1}{32(1-\cos(x))} - \frac{1}{16(\cos(x)+1)} - \frac{3}{32(\cos(x)+1)^2} + \frac{1}{6(\cos(x)+1)^3} - \frac{1}{16(\cos(x)+1)^4} + \frac{1}{32} \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[(Sin[x] + Tan[x])^(-3), x]

[Out] ArcTanh[Cos[x]]/32 - 1/(32*(1 - Cos[x])) - 1/(16*(1 + Cos[x])^4) + 1/(6*(1 + Cos[x])^3) - 3/(32*(1 + Cos[x])^2) - 1/(16*(1 + Cos[x]))

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2707

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p_.), x_Symbol] := Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^(p + 1)/2], x], x, b*Sin[e + f*x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

Rule 4397

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(\sin(x) + \tan(x))^3} dx &= \int \frac{\cot^3(x)}{(1 + \cos(x))^3} dx \\
&= -\text{Subst} \left(\int \frac{x^3}{(1-x)^2(1+x)^5} dx, x, \cos(x) \right) \\
&= -\text{Subst} \left(\int \left(\frac{1}{32(-1+x)^2} - \frac{1}{4(1+x)^5} + \frac{1}{2(1+x)^4} - \frac{3}{16(1+x)^3} - \frac{1}{16(1+x)^2} + \frac{1}{32(-1+x)} \right) dx, x, \cos(x) \right) \\
&= -\frac{1}{32(1-\cos(x))} - \frac{1}{16(1+\cos(x))^4} + \frac{1}{6(1+\cos(x))^3} - \frac{3}{32(1+\cos(x))^2} - \frac{1}{16(1+\cos(x))} \\
&= \frac{1}{32} \tanh^{-1}(\cos(x)) - \frac{1}{32(1-\cos(x))} - \frac{1}{16(1+\cos(x))^4} + \frac{1}{6(1+\cos(x))^3} - \frac{3}{32(1+\cos(x))^2} - \frac{1}{16(1+\cos(x))}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.38

$$-\frac{1}{64} \csc^2\left(\frac{x}{2}\right) - \frac{1}{256} \sec^8\left(\frac{x}{2}\right) + \frac{1}{48} \sec^6\left(\frac{x}{2}\right) - \frac{3}{128} \sec^4\left(\frac{x}{2}\right) - \frac{1}{32} \sec^2\left(\frac{x}{2}\right) - \frac{1}{32} \log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{1}{32} \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sin[x] + Tan[x])^(-3), x]

[Out] -1/64*Csc[x/2]^2 + Log[Cos[x/2]]/32 - Log[Sin[x/2]]/32 - Sec[x/2]^2/32 - (3*Sec[x/2]^4)/128 + Sec[x/2]^6/48 - Sec[x/2]^8/256

fricas [B] time = 1.82, size = 130, normalized size = 2.17

$$\frac{6 \cos(x)^4 + 18 \cos(x)^3 - 50 \cos(x)^2 - 3(\cos(x)^5 + 3 \cos(x)^4 + 2 \cos(x)^3 - 2 \cos(x)^2 - 3 \cos(x) - 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 3(\cos(x)^5 + 3 \cos(x)^4 + 2 \cos(x)^3 - 2 \cos(x)^2 - 3 \cos(x) - 1) \log\left(\frac{1}{2} \cos(x) - \frac{1}{2}\right)}{192(\cos(x)^5 + 3 \cos(x)^4 + 2 \cos(x)^3 - 2 \cos(x)^2 - 3 \cos(x) - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sin(x)+tan(x))^3,x, algorithm="fricas")

[Out] -1/192*(6*cos(x)^4 + 18*cos(x)^3 - 50*cos(x)^2 - 3*(cos(x)^5 + 3*cos(x)^4 + 2*cos(x)^3 - 2*cos(x)^2 - 3*cos(x) - 1)*log(1/2*cos(x) + 1/2) + 3*(cos(x)^5 + 3*cos(x)^4 + 2*cos(x)^3 - 2*cos(x)^2 - 3*cos(x) - 1)*log(-1/2*cos(x) + 1/2) - 54*cos(x) - 16)/(cos(x)^5 + 3*cos(x)^4 + 2*cos(x)^3 - 2*cos(x)^2 - 3*cos(x) - 1)

giac [B] time = 0.16, size = 95, normalized size = 1.58

$$\frac{\left(\frac{\cos(x)-1}{\cos(x)+1} + 1\right)(\cos(x)+1)}{64(\cos(x)-1)} + \frac{\cos(x)-1}{32(\cos(x)+1)} + \frac{(\cos(x)-1)^2}{64(\cos(x)+1)^2} - \frac{(\cos(x)-1)^3}{192(\cos(x)+1)^3} - \frac{(\cos(x)-1)^4}{256(\cos(x)+1)^4} - \frac{1}{64} \log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sin(x)+tan(x))^3,x, algorithm="giac")

[Out] 1/64*((cos(x) - 1)/(cos(x) + 1) + 1)*(cos(x) + 1)/(cos(x) - 1) + 1/32*(cos(x) - 1)/(cos(x) + 1) + 1/64*(cos(x) - 1)^2/(cos(x) + 1)^2 - 1/192*(cos(x) - 1)^3/(cos(x) + 1)^3 - 1/256*(cos(x) - 1)^4/(cos(x) + 1)^4 - 1/64*log(-(cos(x) - 1)/(cos(x) + 1))

maple [A] time = 0.13, size = 56, normalized size = 0.93

$$\frac{1}{-32 + 32 \cos(x)} - \frac{\ln(-1 + \cos(x))}{64} - \frac{1}{16(1 + \cos(x))^4} + \frac{1}{6(1 + \cos(x))^3} - \frac{3}{32(1 + \cos(x))^2} - \frac{1}{16(1 + \cos(x))} + \frac{\ln(1 + \cos(x))}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x)+tan(x))^3,x)

[Out] 1/32/(-1+cos(x))-1/64*ln(-1+cos(x))-1/16/(1+cos(x))^4+1/6/(1+cos(x))^3-3/32/(1+cos(x))^2-1/16/(1+cos(x))+1/64*ln(1+cos(x))

maxima [A] time = 0.31, size = 73, normalized size = 1.22

$$\frac{(\cos(x) + 1)^2}{64 \sin(x)^2} - \frac{\sin(x)^2}{32 (\cos(x) + 1)^2} + \frac{\sin(x)^4}{64 (\cos(x) + 1)^4} + \frac{\sin(x)^6}{192 (\cos(x) + 1)^6} - \frac{\sin(x)^8}{256 (\cos(x) + 1)^8} - \frac{1}{32} \log\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sin(x)+tan(x))^3,x, algorithm="maxima")

[Out] -1/64*(cos(x) + 1)^2/sin(x)^2 - 1/32*sin(x)^2/(cos(x) + 1)^2 + 1/64*sin(x)^4/(cos(x) + 1)^4 + 1/192*sin(x)^6/(cos(x) + 1)^6 - 1/256*sin(x)^8/(cos(x) + 1)^8 - 1/32*log(sin(x)/(cos(x) + 1))

mupad [B] time = 2.39, size = 48, normalized size = 0.80

$$\frac{\tan\left(\frac{x}{2}\right)^4}{64} - \frac{1}{64 \tan\left(\frac{x}{2}\right)^2} - \frac{\tan\left(\frac{x}{2}\right)^2}{32} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{32} + \frac{\tan\left(\frac{x}{2}\right)^6}{192} - \frac{\tan\left(\frac{x}{2}\right)^8}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x) + tan(x))^3,x)

[Out] tan(x/2)^4/64 - 1/(64*tan(x/2)^2) - tan(x/2)^2/32 - log(tan(x/2))/32 + tan(x/2)^6/192 - tan(x/2)^8/256

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\sin(x) + \tan(x))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sin(x)+tan(x))**3,x)

[Out] Integral((sin(x) + tan(x))**(-3), x)

$$3.348 \quad \int \frac{1}{(\sin(x)+\tan(x))^4} dx$$

Optimal. Leaf size=65

$$-\frac{8}{11} \cot^{11}(x) - \frac{16 \cot^9(x)}{9} - \frac{9 \cot^7(x)}{7} - \frac{\cot^5(x)}{5} + \frac{8 \csc^{11}(x)}{11} - \frac{20 \csc^9(x)}{9} + \frac{16 \csc^7(x)}{7} - \frac{4 \csc^5(x)}{5}$$

[Out] -1/5*cot(x)^5-9/7*cot(x)^7-16/9*cot(x)^9-8/11*cot(x)^11-4/5*csc(x)^5+16/7*csc(x)^7-20/9*csc(x)^9+8/11*csc(x)^11

Rubi [A] time = 0.21, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {4397, 2711, 2607, 14, 2606, 270}

$$-\frac{8}{11} \cot^{11}(x) - \frac{16 \cot^9(x)}{9} - \frac{9 \cot^7(x)}{7} - \frac{\cot^5(x)}{5} + \frac{8 \csc^{11}(x)}{11} - \frac{20 \csc^9(x)}{9} + \frac{16 \csc^7(x)}{7} - \frac{4 \csc^5(x)}{5}$$

Antiderivative was successfully verified.

[In] Int[(Sin[x] + Tan[x])^(-4), x]

[Out] -Cot[x]^5/5 - (9*Cot[x]^7)/7 - (16*Cot[x]^9)/9 - (8*Cot[x]^11)/11 - (4*Csc[x]^5)/5 + (16*Csc[x]^7)/7 - (20*Csc[x]^9)/9 + (8*Csc[x]^11)/11

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 270

Int[((c_)*(x_))^(m_)*((a_ + (b_)*(x_))^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2606

Int[((a_)*sec[(e_ + (f_)*(x_))]^(m_))*((b_)*tan[(e_ + (f_)*(x_))]^(n_)), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m-1)*(-1+x^2)^((n-1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n+1])

Rule 2607

Int[sec[(e_ + (f_)*(x_))]^(m_)*((b_)*tan[(e_ + (f_)*(x_))]^(n_)), x_Symbol] := Dist[1/f, Subst[Int[(b*x)^n*(1+x^2)^(m/2-1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n-1)/2] && LtQ[0, n, m-1])

Rule 2711

Int[((a_ + (b_)*sin[(e_ + (f_)*(x_))]^(m_))*((g_)*tan[(e_ + (f_)*(x_))]^(p_)), x_Symbol] := Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]

Rule 4397

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(\sin(x) + \tan(x))^4} dx &= \int \frac{\cot^4(x)}{(1 + \cos(x))^4} dx \\
&= \int (\cot^8(x) \csc^4(x) - 4 \cot^7(x) \csc^5(x) + 6 \cot^6(x) \csc^6(x) - 4 \cot^5(x) \csc^7(x) + \cot^4(x) \csc^8(x)) dx \\
&= -\left(4 \int \cot^7(x) \csc^5(x) dx\right) - 4 \int \cot^5(x) \csc^7(x) dx + 6 \int \cot^6(x) \csc^6(x) dx + \int \cot^4(x) \csc^8(x) dx \\
&= 4 \operatorname{Subst}\left(\int x^6 (-1 + x^2)^2 dx, x, \csc(x)\right) + 4 \operatorname{Subst}\left(\int x^4 (-1 + x^2)^3 dx, x, \csc(x)\right) - 6 \operatorname{Subst}\left(\int x^6 (-1 + x^2) dx, x, \csc(x)\right) + \operatorname{Subst}\left(\int (-1 + x^2)^4 dx, x, \csc(x)\right) \\
&= 4 \operatorname{Subst}\left(\int (-x^4 + 3x^6 - 3x^8 + x^{10}) dx, x, \csc(x)\right) + 4 \operatorname{Subst}\left(\int (x^6 - 2x^8 + x^{10}) dx, x, \csc(x)\right) - 6 \operatorname{Subst}\left(\int (-x^4 + 3x^6 - 3x^8 + x^{10}) dx, x, \csc(x)\right) + \operatorname{Subst}\left(\int (-x^4 + 3x^6 - 3x^8 + x^{10}) dx, x, \csc(x)\right) \\
&= -\frac{1}{5} \cot^5(x) - \frac{9 \cot^7(x)}{7} - \frac{16 \cot^9(x)}{9} - \frac{8 \cot^{11}(x)}{11} - \frac{4 \csc^5(x)}{5} + \frac{16 \csc^7(x)}{7} - \frac{20 \csc^9(x)}{9}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 129, normalized size = 1.98

$$-\frac{2749 \tan\left(\frac{x}{2}\right)}{110880} + \frac{1}{96} \cot\left(\frac{x}{2}\right) - \frac{1}{384} \cot\left(\frac{x}{2}\right) \csc^2\left(\frac{x}{2}\right) + \frac{\tan\left(\frac{x}{2}\right) \sec^{10}\left(\frac{x}{2}\right)}{1408} - \frac{7 \tan\left(\frac{x}{2}\right) \sec^8\left(\frac{x}{2}\right)}{1584} + \frac{641 \tan\left(\frac{x}{2}\right) \sec^6\left(\frac{x}{2}\right)}{88704}$$

Antiderivative was successfully verified.

[In] Integrate[(Sin[x] + Tan[x])^(-4), x]

[Out] Cot[x/2]/96 - (Cot[x/2]*Csc[x/2]^2)/384 - (2749*Tan[x/2])/110880 - (2033*Sec[x/2]^2*Tan[x/2])/443520 + (179*Sec[x/2]^4*Tan[x/2])/73920 + (641*Sec[x/2]^6*Tan[x/2])/88704 - (7*Sec[x/2]^8*Tan[x/2])/1584 + (Sec[x/2]^10*Tan[x/2])/1408

fricas [A] time = 1.06, size = 78, normalized size = 1.20

$$\frac{122 \cos(x)^7 + 488 \cos(x)^6 + 549 \cos(x)^5 - 244 \cos(x)^4 - 64 \cos(x)^3 + 144 \cos(x)^2 + 128 \cos(x) + 32}{3465 (\cos(x)^6 + 4 \cos(x)^5 + 5 \cos(x)^4 - 5 \cos(x)^2 - 4 \cos(x) - 1) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sin(x)+tan(x))^4,x, algorithm="fricas")

[Out] 1/3465*(122*cos(x)^7 + 488*cos(x)^6 + 549*cos(x)^5 - 244*cos(x)^4 - 64*cos(x)^3 + 144*cos(x)^2 + 128*cos(x) + 32)/((cos(x)^6 + 4*cos(x)^5 + 5*cos(x)^4 - 5*cos(x)^2 - 4*cos(x) - 1)*sin(x))

giac [A] time = 0.15, size = 65, normalized size = 1.00

$$\frac{1}{1408} \tan\left(\frac{1}{2}x\right)^{11} - \frac{1}{1152} \tan\left(\frac{1}{2}x\right)^9 - \frac{3}{896} \tan\left(\frac{1}{2}x\right)^7 + \frac{3}{640} \tan\left(\frac{1}{2}x\right)^5 + \frac{1}{128} \tan\left(\frac{1}{2}x\right)^3 + \frac{3 \tan\left(\frac{1}{2}x\right)^2 - 1}{384 \tan\left(\frac{1}{2}x\right)^3} - \frac{3}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sin(x)+tan(x))^4,x, algorithm="giac")

[Out] 1/1408*tan(1/2*x)^11 - 1/1152*tan(1/2*x)^9 - 3/896*tan(1/2*x)^7 + 3/640*tan(1/2*x)^5 + 1/128*tan(1/2*x)^3 + 1/384*(3*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 3/128*tan(1/2*x)

maple [A] time = 0.13, size = 64, normalized size = 0.98

$$\frac{\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{1408} - \frac{\left(\tan^9\left(\frac{x}{2}\right)\right)}{1152} - \frac{3\left(\tan^7\left(\frac{x}{2}\right)\right)}{896} + \frac{3\left(\tan^5\left(\frac{x}{2}\right)\right)}{640} + \frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{128} - \frac{3\tan\left(\frac{x}{2}\right)}{128} + \frac{1}{128\tan\left(\frac{x}{2}\right)} - \frac{1}{384\tan\left(\frac{x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x)+tan(x))^4,x)

[Out] 1/1408*tan(1/2*x)^11-1/1152*tan(1/2*x)^9-3/896*tan(1/2*x)^7+3/640*tan(1/2*x)^5+1/128*tan(1/2*x)^3-3/128*tan(1/2*x)+1/128/tan(1/2*x)-1/384/tan(1/2*x)^3

maxima [A] time = 0.32, size = 97, normalized size = 1.49

$$\frac{\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2} - 1\right)(\cos(x)+1)^3}{384\sin(x)^3} - \frac{3\sin(x)}{128(\cos(x)+1)} + \frac{\sin(x)^3}{128(\cos(x)+1)^3} + \frac{3\sin(x)^5}{640(\cos(x)+1)^5} - \frac{3\sin(x)^7}{896(\cos(x)+1)^7} - \frac{1}{1152(\cos(x)+1)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sin(x)+tan(x))^4,x, algorithm="maxima")

[Out] 1/384*(3*sin(x)^2/(cos(x)+1)^2-1)*(cos(x)+1)^3/sin(x)^3-3/128*sin(x)/(cos(x)+1)+1/128*sin(x)^3/(cos(x)+1)^3+3/640*sin(x)^5/(cos(x)+1)^5-3/896*sin(x)^7/(cos(x)+1)^7-1/1152*sin(x)^9/(cos(x)+1)^9+1/1408*sin(x)^11/(cos(x)+1)^11

mupad [B] time = 2.45, size = 87, normalized size = 1.34

$$\frac{15616\cos\left(\frac{x}{2}\right)^{14} - 23424\cos\left(\frac{x}{2}\right)^{12} + 5856\cos\left(\frac{x}{2}\right)^{10} + 976\cos\left(\frac{x}{2}\right)^8 + 7296\cos\left(\frac{x}{2}\right)^6 - 7440\cos\left(\frac{x}{2}\right)^4 + 2590\cos\left(\frac{x}{2}\right)^2 - 315}{443520\left(\cos\left(\frac{x}{2}\right)^{11}\sin\left(\frac{x}{2}\right) - \cos\left(\frac{x}{2}\right)^{13}\sin\left(\frac{x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x) + tan(x))^4,x)

[Out] -(2590*cos(x/2)^2 - 7440*cos(x/2)^4 + 7296*cos(x/2)^6 + 976*cos(x/2)^8 + 5856*cos(x/2)^10 - 23424*cos(x/2)^12 + 15616*cos(x/2)^14 - 315)/(443520*(cos(x/2)^11*sin(x/2) - cos(x/2)^13*sin(x/2)))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\sin(x) + \tan(x))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sin(x)+tan(x))**4,x)

[Out] Integral((sin(x) + tan(x))**(-4), x)

$$3.349 \quad \int \frac{A + C \sin(x)}{b \cos(x) + c \sin(x)} dx$$

Optimal. Leaf size=74

$$-\frac{A \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{\sqrt{b^2 + c^2}} + \frac{cCx}{b^2 + c^2} - \frac{bC \log(b \cos(x) + c \sin(x))}{b^2 + c^2}$$

[Out] $cCx/(b^2+c^2) - bC \ln(b \cos(x) + c \sin(x))/(b^2+c^2) - A \operatorname{arctanh}((c \cos(x) - b \sin(x))/\sqrt{b^2+c^2})/\sqrt{b^2+c^2}$

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3137, 3074, 206}

$$-\frac{A \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{\sqrt{b^2 + c^2}} + \frac{cCx}{b^2 + c^2} - \frac{bC \log(b \cos(x) + c \sin(x))}{b^2 + c^2}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]

[Out] $(cCx)/(b^2 + c^2) - (A \operatorname{ArcTanh}[(c \cos(x) - b \sin(x))/\sqrt{b^2 + c^2}])/\sqrt{b^2 + c^2} - (bC \operatorname{Log}[b \cos(x) + c \sin(x)])/(b^2 + c^2)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3074

Int[(cos[(c_.) + (d_.)*(x_)])*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 3137

Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])*((a_.) + cos[(d_.) + (e_.)*(x_)])*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Simp[(cCx)/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*cC)/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(bC*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C}, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*cC, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + C \sin(x)}{b \cos(x) + c \sin(x)} dx &= \frac{cCx}{b^2 + c^2} - \frac{bC \log(b \cos(x) + c \sin(x))}{b^2 + c^2} + A \int \frac{1}{b \cos(x) + c \sin(x)} dx \\ &= \frac{cCx}{b^2 + c^2} - \frac{bC \log(b \cos(x) + c \sin(x))}{b^2 + c^2} - A \operatorname{Subst}\left(\int \frac{1}{b^2 + c^2 - x^2} dx, x, c \cos(x) - b \sin(x)\right) \\ &= \frac{cCx}{b^2 + c^2} - \frac{A \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{\sqrt{b^2 + c^2}} - \frac{bC \log(b \cos(x) + c \sin(x))}{b^2 + c^2} \end{aligned}$$

Mathematica [A] time = 0.20, size = 68, normalized size = 0.92

$$\frac{2A \tanh^{-1}\left(\frac{b \tan\left(\frac{x}{2}\right) - c}{\sqrt{b^2 + c^2}}\right)}{\sqrt{b^2 + c^2}} + \frac{C(cx - b \log(b \cos(x) + c \sin(x)))}{b^2 + c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x]), x]

[Out] (2*A*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]]/Sqrt[b^2 + c^2] + (C*(c*x - b*Log[b*Cos[x] + c*Sin[x]]))/(b^2 + c^2)

fricas [B] time = 0.93, size = 144, normalized size = 1.95

$$\frac{2 C c x - C b \log\left(2 b c \cos(x) \sin(x) + (b^2 - c^2) \cos(x)^2 + c^2\right) + \sqrt{b^2 + c^2} A \log\left(-\frac{2 b c \cos(x) \sin(x) + (b^2 - c^2) \cos(x)^2 - 2 b^2 - c^2}{2 b c \cos(x) \sin(x) + (b^2 - c^2)}\right)}{2(b^2 + c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)), x, algorithm="fricas")

[Out] 1/2*(2*C*c*x - C*b*log(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2) + sqrt(b^2 + c^2)*A*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)))/(b^2 + c^2)

giac [A] time = 0.21, size = 131, normalized size = 1.77

$$\frac{C c x}{b^2 + c^2} + \frac{C b \log\left(\tan\left(\frac{1}{2} x\right)^2 + 1\right)}{b^2 + c^2} - \frac{C b \log\left(\left|b \tan\left(\frac{1}{2} x\right)^2 - 2 c \tan\left(\frac{1}{2} x\right) - b\right|\right)}{b^2 + c^2} - \frac{A \log\left(\frac{\left|2 b \tan\left(\frac{1}{2} x\right) - 2 c - 2 \sqrt{b^2 + c^2}\right|}{\left|2 b \tan\left(\frac{1}{2} x\right) - 2 c + 2 \sqrt{b^2 + c^2}\right|}\right)}{\sqrt{b^2 + c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)), x, algorithm="giac")

[Out] C*c*x/(b^2 + c^2) + C*b*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) - C*b*log(abs(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b))/(b^2 + c^2) - A*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/sqrt(b^2 + c^2)

maple [B] time = 0.14, size = 150, normalized size = 2.03

$$-\frac{b C \ln\left(b \left(\tan^2\left(\frac{x}{2}\right) - 2 c \tan\left(\frac{x}{2}\right) - b\right)\right)}{b^2 + c^2} + \frac{2 \operatorname{arctanh}\left(\frac{2 b \tan\left(\frac{x}{2}\right) - 2 c}{2 \sqrt{b^2 + c^2}}\right) A b^2}{(b^2 + c^2)^{\frac{3}{2}}} + \frac{2 \operatorname{arctanh}\left(\frac{2 b \tan\left(\frac{x}{2}\right) - 2 c}{2 \sqrt{b^2 + c^2}}\right) A c^2}{(b^2 + c^2)^{\frac{3}{2}}} + \frac{C b \ln\left(1 + \tan\left(\frac{x}{2}\right)^2\right)}{b^2 + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*sin(x))/(b*cos(x)+c*sin(x)), x)

[Out] -1/(b^2+c^2)*b*C*ln(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)+2/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))*A*b^2+2/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))*A*c^2+C/(b^2+c^2)*b*ln(1+tan(1/2*x)^2)+2*C/(b^2+c^2)*c*arctan(tan(1/2*x))

maxima [B] time = 0.42, size = 153, normalized size = 2.07

$$C \left(\frac{2c \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2} - \frac{b \log\left(-b - \frac{2c \sin(x)}{\cos(x)+1} + \frac{b \sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2} + \frac{b \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)}{b^2+c^2} \right) - \frac{A \log\left(\frac{c - \frac{b \sin(x)}{\cos(x)+1} + \sqrt{b^2+c^2}}{c - \frac{b \sin(x)}{\cos(x)+1} - \sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")

[Out] C*(2*c*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) - b*log(-b - 2*c*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) + b*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(b^2 + c^2)) - A*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/sqrt(b^2 + c^2)

mupad [B] time = 7.02, size = 695, normalized size = 9.39

$$-\ln \left(-32 A C^2 b^2 - \frac{\left(A \sqrt{(b^2 + c^2)^3} + C b^3 + C b c^2 \right) \left(64 A^2 b^2 c + 32 C^2 b^2 c - 32 b \tan\left(\frac{x}{2}\right) (A^2 b^2 - A^2 c^2 - \dots) \right)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + C*sin(x))/(b*cos(x) + c*sin(x)),x)

[Out] (C*log(tan(x/2) + 1i))/(b - c*1i) - log(- 32*A*C^2*b^2 - ((C*b^3 - A*((b^2 + c^2)^3)^(1/2) + C*b*c^2)*(64*A^2*b^2*c + 32*C^2*b^2*c - 32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + 2*C^2*c^2 - 4*A*C*b*c) + 64*A*C*b^3 + ((C*b^3 - A*((b^2 + c^2)^3)^(1/2) + C*b*c^2)*(32*A*b^4 + 32*A*b^2*c^2 + 32*b*tan(x/2)*(2*A*c^3 - 2*C*b^3 + 2*A*b^2*c + C*b*c^2) - 32*C*b*c^3 + 64*C*b^3*c - (96*b*c*(b + c*tan(x/2))*(C*b^3 - A*((b^2 + c^2)^3)^(1/2) + C*b*c^2))/(b^2 + c^2)))/(b^2 + c^2)^2)))/(b^2 + c^2)^2 - 32*A^2*C*b*c - 32*C*b*tan(x/2)*(2*C^2*b - A^2*b + 2*A*C*c))*((C*b)/(b^2 + c^2) - (A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2) - log(- 32*A*C^2*b^2 - ((A*((b^2 + c^2)^3)^(1/2) + C*b^3 + C*b*c^2)*(64*A^2*b^2*c + 32*C^2*b^2*c - 32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + 2*C^2*c^2 - 4*A*C*b*c) + 64*A*C*b^3 + ((A*((b^2 + c^2)^3)^(1/2) + C*b^3 + C*b*c^2)*(32*A*b^4 + 32*A*b^2*c^2 + 32*b*tan(x/2)*(2*A*c^3 - 2*C*b^3 + 2*A*b^2*c + C*b*c^2) - 32*C*b*c^3 + 64*C*b^3*c - (96*b*c*(b + c*tan(x/2))*(A*((b^2 + c^2)^3)^(1/2) + C*b^3 + C*b*c^2))/(b^2 + c^2)))/(b^2 + c^2)^2)))/(b^2 + c^2)^2 - 32*A^2*C*b*c - 32*C*b*tan(x/2)*(2*C^2*b - A^2*b + 2*A*C*c))*((C*b)/(b^2 + c^2) + (A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2) + (C*log(tan(x/2) - 1i)*1i)/(b*1i - c)

sympy [A] time = 36.72, size = 641, normalized size = 8.66

$$\left\{ \begin{array}{l} \infty \left(A \log \left(\tan \left(\frac{x}{2} \right) \right) + Cx \right) \\ \frac{A \log \left(\tan \left(\frac{x}{2} \right) \right) + Cx}{c} \\ \frac{2A}{-2ic \sin(x) - 2c \cos(x)} - \frac{iCx \sin(x)}{-2ic \sin(x) - 2c \cos(x)} - \frac{Cx \cos(x)}{-2ic \sin(x) - 2c \cos(x)} + \frac{C \sin(x)}{-2ic \sin(x) - 2c \cos(x)} \\ \frac{2A}{2ic \sin(x) - 2c \cos(x)} + \frac{iCx \sin(x)}{2ic \sin(x) - 2c \cos(x)} - \frac{Cx \cos(x)}{2ic \sin(x) - 2c \cos(x)} + \frac{C \sin(x)}{2ic \sin(x) - 2c \cos(x)} \\ - \frac{Ab^2 \log \left(\tan \left(\frac{x}{2} \right) - \frac{c}{b} - \frac{\sqrt{b^2 + c^2}}{b} \right)}{b^2 \sqrt{b^2 + c^2} + c^2 \sqrt{b^2 + c^2}} + \frac{Ab^2 \log \left(\tan \left(\frac{x}{2} \right) - \frac{c}{b} + \frac{\sqrt{b^2 + c^2}}{b} \right)}{b^2 \sqrt{b^2 + c^2} + c^2 \sqrt{b^2 + c^2}} - \frac{Ac^2 \log \left(\tan \left(\frac{x}{2} \right) - \frac{c}{b} - \frac{\sqrt{b^2 + c^2}}{b} \right)}{b^2 \sqrt{b^2 + c^2} + c^2 \sqrt{b^2 + c^2}} + \frac{Ac^2 \log \left(\tan \left(\frac{x}{2} \right) - \frac{c}{b} + \frac{\sqrt{b^2 + c^2}}{b} \right)}{b^2 \sqrt{b^2 + c^2} + c^2 \sqrt{b^2 + c^2}} + \frac{Cb \sqrt{b^2 + c^2}}{b^2 \sqrt{b^2 + c^2} + c^2 \sqrt{b^2 + c^2}} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x)

[Out] Piecewise((zoo*(A*log(tan(x/2)) + C*x), Eq(b, 0) & Eq(c, 0)), ((A*log(tan(x/2)) + C*x)/c, Eq(b, 0)), (2*A/(-2*I*c*sin(x) - 2*c*cos(x)) - I*C*x*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)) - C*x*cos(x)/(-2*I*c*sin(x) - 2*c*cos(x)) + C*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)), Eq(b, -I*c)), (2*A/(2*I*c*sin(x) - 2*c*cos(x)) + I*C*x*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)) - C*x*cos(x)/(2*I*c*sin(x) - 2*c*cos(x)) + C*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)), Eq(b, I*c)), (-A*b**2*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + A*b**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - A*c**2*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + A*c**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + C*b*sqrt(b**2 + c**2)*log(tan(x/2)**2 + 1)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - C*b*sqrt(b**2 + c**2)*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - C*b*sqrt(b**2 + c**2)*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + C*c*x*sqrt(b**2 + c**2)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)), True))

$$3.350 \quad \int \frac{A+C \sin(x)}{(b \cos(x)+c \sin(x))^2} dx$$

Optimal. Leaf size=75

$$\frac{Ab \sin(x) - Ac \cos(x) + bC}{(b^2 + c^2)(b \cos(x) + c \sin(x))} - \frac{cC \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}}$$

[Out] $-c*C*\operatorname{arctanh}((c*\cos(x)-b*\sin(x))/(b^2+c^2)^{(1/2)})/(b^2+c^2)^{(3/2)}+(b*C-A*c*\cos(x)+A*b*\sin(x))/(b^2+c^2)/(b*\cos(x)+c*\sin(x))$

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3154, 3074, 206}

$$\frac{Ab \sin(x) - Ac \cos(x) + bC}{(b^2 + c^2)(b \cos(x) + c \sin(x))} - \frac{cC \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]`

[Out] $-\left(\frac{cC*\operatorname{ArcTanh}\left[\frac{c*\cos[x] - b*\sin[x]}{\sqrt{b^2 + c^2}}\right]}{(b^2 + c^2)^{(3/2)}} + \frac{(bC - A*c*\cos[x] + A*b*\sin[x])}{(b^2 + c^2)*(b*\cos[x] + c*\sin[x])}\right)$

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 3074

`Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]`

Rule 3154

`Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(a_. + cos[(d_.) + (e_.)*(x_)])*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]`

Rubi steps

$$\begin{aligned} \int \frac{A + C \sin(x)}{(b \cos(x) + c \sin(x))^2} dx &= \frac{bC - Ac \cos(x) + Ab \sin(x)}{(b^2 + c^2)(b \cos(x) + c \sin(x))} + \frac{(cC) \int \frac{1}{b \cos(x) + c \sin(x)} dx}{b^2 + c^2} \\ &= \frac{bC - Ac \cos(x) + Ab \sin(x)}{(b^2 + c^2)(b \cos(x) + c \sin(x))} - \frac{(cC) \operatorname{Subst}\left(\int \frac{1}{b^2 + c^2 - x^2} dx, x, c \cos(x) - b \sin(x)\right)}{b^2 + c^2} \\ &= -\frac{cC \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}} + \frac{bC - Ac \cos(x) + Ab \sin(x)}{(b^2 + c^2)(b \cos(x) + c \sin(x))} \end{aligned}$$

Mathematica [A] time = 0.31, size = 82, normalized size = 1.09

$$\frac{A(b^2 + c^2) \sin(x) + b^2 C}{b(b^2 + c^2)(b \cos(x) + c \sin(x))} + \frac{2cC \tanh^{-1}\left(\frac{b \tan\left(\frac{x}{2}\right) - c}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]

[Out] (2*c*C*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]]/(b^2 + c^2)^(3/2) + (b^2 *C + A*(b^2 + c^2)*Sin[x])/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))

fricas [B] time = 1.97, size = 200, normalized size = 2.67

$$\frac{2Cb^3 + 2Cbc^2 + (Cbc \cos(x) + Cc^2 \sin(x))\sqrt{b^2 + c^2} \log\left(-\frac{2bc \cos(x) \sin(x) + (b^2 - c^2) \cos(x)^2 - 2b^2 - c^2 + 2\sqrt{b^2 + c^2}(c \cos(x) - b \sin(x))}{2bc \cos(x) \sin(x) + (b^2 - c^2) \cos(x)^2 + c^2}\right)}{2\left((b^5 + 2b^3c^2 + bc^4) \cos(x) + (b^4c + 2b^2c^3 + c^5) \sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas")

[Out] 1/2*(2*C*b^3 + 2*C*b*c^2 + (C*b*c*cos(x) + C*c^2*sin(x))*sqrt(b^2 + c^2)*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) - 2*(A*b^2*c + A*c^3)*cos(x) + 2*(A*b^3 + A*b*c^2)*sin(x))/((b^5 + 2*b^3*c^2 + b*c^4)*cos(x) + (b^4*c + 2*b^2*c^3 + c^5)*sin(x))

giac [A] time = 0.19, size = 130, normalized size = 1.73

$$-\frac{Cc \log\left(\frac{\left|2b \tan\left(\frac{1}{2}x\right) - 2c - 2\sqrt{b^2 + c^2}\right|}{\left|2b \tan\left(\frac{1}{2}x\right) - 2c + 2\sqrt{b^2 + c^2}\right|}\right)}{(b^2 + c^2)^{3/2}} - \frac{2\left(Ab^2 \tan\left(\frac{1}{2}x\right) + Cbc \tan\left(\frac{1}{2}x\right) + Ac^2 \tan\left(\frac{1}{2}x\right) + Cb^2\right)}{(b^3 + bc^2)\left(b \tan\left(\frac{1}{2}x\right)^2 - 2c \tan\left(\frac{1}{2}x\right) - b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")

[Out] -C*c*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(A*b^2*tan(1/2*x) + C*b*c*tan(1/2*x) + A*c^2*tan(1/2*x) + C*b^2)/((b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b))

maple [A] time = 0.17, size = 108, normalized size = 1.44

$$\frac{-\frac{2(Ab^2+Ac^2+Cbc)\tan\left(\frac{x}{2}\right) - \frac{2Cb}{b^2+c^2}}{b(b^2+c^2)} + \frac{2Cc \operatorname{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)}{b\left(\tan^2\left(\frac{x}{2}\right) - 2c\tan\left(\frac{x}{2}\right) - b\right) + \frac{2Cc \operatorname{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x)

[Out] 2*(-(A*b^2+A*c^2+C*b*c)/b/(b^2+c^2)*tan(1/2*x)-C*b/(b^2+c^2))/(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)+2*C*c/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))

maxima [A] time = 0.42, size = 146, normalized size = 1.95

$$-C \left(\frac{c \log\left(\frac{c - \frac{b \sin(x)}{\cos(x)+1} + \sqrt{b^2+c^2}}{c - \frac{b \sin(x)}{\cos(x)+1} - \sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}} - \frac{2\left(b + \frac{c \sin(x)}{\cos(x)+1}\right)}{b^3 + bc^2 + \frac{2(b^2c+c^3)\sin(x)}{\cos(x)+1} - \frac{(b^3+bc^2)\sin(x)^2}{(\cos(x)+1)^2}} \right) - \frac{A}{c^2 \tan(x) + bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima")

[Out] -C*(c*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(b + c*sin(x)/(cos(x) + 1))/(b^3 + b*c^2 + 2*(b^2*c + c^3)*sin(x)/(cos(x) + 1) - (b^3 + b*c^2)*sin(x)^2/(cos(x) + 1)^2) - A/(c^2*tan(x) + b*c)

mupad [B] time = 2.58, size = 105, normalized size = 1.40

$$\frac{\frac{2Cb}{b^2+c^2} + \frac{2\tan\left(\frac{x}{2}\right)(Ab^2+Cbc+Ac^2)}{b(b^2+c^2)}}{-b\tan\left(\frac{x}{2}\right)^2 + 2c\tan\left(\frac{x}{2}\right) + b} - \frac{2Cc \operatorname{atanh}\left(\frac{2c-2b\tan\left(\frac{x}{2}\right)}{2\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + C*sin(x))/(b*cos(x) + c*sin(x))^2,x)

[Out] ((2*C*b)/(b^2 + c^2) + (2*tan(x/2)*(A*b^2 + A*c^2 + C*b*c))/(b*(b^2 + c^2)))/(b + 2*c*tan(x/2) - b*tan(x/2)^2) - (2*C*c*atanh((2*c - 2*b*tan(x/2))/(2*(b^2 + c^2)^(1/2))))/(b^2 + c^2)^(3/2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))*2,x)

[Out] Timed out

$$3.351 \quad \int \frac{A+C \sin(x)}{(b \cos(x)+c \sin(x))^3} dx$$

Optimal. Leaf size=116

$$\frac{Ab \sin(x) - Ac \cos(x) + bC}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} - \frac{A \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{2(b^2 + c^2)^{3/2}} - \frac{c^2 C \cos(x) - bcC \sin(x)}{(b^2 + c^2)^2 (b \cos(x) + c \sin(x))}$$

[Out] $-1/2*A*\operatorname{arctanh}((c*\cos(x)-b*\sin(x))/(b^2+c^2)^{(1/2)})/(b^2+c^2)^{(3/2)}+1/2*(b*C-A*c*\cos(x)+A*b*\sin(x))/(b^2+c^2)/(b*\cos(x)+c*\sin(x))^2+(-c^2*C*\cos(x)+b*c*C*\sin(x))/(b^2+c^2)^2/(b*\cos(x)+c*\sin(x))$

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3157, 3153, 3074, 206}

$$\frac{Ab \sin(x) - Ac \cos(x) + bC}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} - \frac{A \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{2(b^2 + c^2)^{3/2}} - \frac{c^2 C \cos(x) - bcC \sin(x)}{(b^2 + c^2)^2 (b \cos(x) + c \sin(x))}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + C*\sin[x])/(b*\cos[x] + c*\sin[x])^3, x]$

[Out] $-(A*\operatorname{ArcTanh}[(c*\cos[x] - b*\sin[x])/ \operatorname{Sqrt}[b^2 + c^2]])/(2*(b^2 + c^2)^{(3/2)}) + (b*C - A*c*\cos[x] + A*b*\sin[x])/(2*(b^2 + c^2)*(b*\cos[x] + c*\sin[x])^2) - (c^2*C*\cos[x] - b*c*C*\sin[x])/((b^2 + c^2)^2*(b*\cos[x] + c*\sin[x]))$

Rule 206

$\operatorname{Int}[(a + (b_*)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 3074

$\operatorname{Int}[(\cos[(c_*) + (d_*)*(x_)]*(a_*) + (b_*)*\sin[(c_*) + (d_*)*(x_)])^{-1}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{-1}, \operatorname{Subst}[\operatorname{Int}[1/(a^2 + b^2 - x^2), x], x, b*\cos[c + d*x] - a*\sin[c + d*x]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{NeQ}[a^2 + b^2, 0]$

Rule 3153

$\operatorname{Int}[(A_*) + \cos[(d_*) + (e_*)*(x_)]*(B_*) + (C_*)*\sin[(d_*) + (e_*)*(x_)]) / ((a_*) + \cos[(d_*) + (e_*)*(x_)]*(b_*) + (c_*)*\sin[(d_*) + (e_*)*(x_)])^2, x_Symbol] \rightarrow \operatorname{Simp}[(c*B - b*C - (a*C - c*A)*\cos[d + e*x] + (a*B - b*A)*\sin[d + e*x]) / (e*(a^2 - b^2 - c^2)*(a + b*\cos[d + e*x] + c*\sin[d + e*x])), x] + \operatorname{Dist}[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), \operatorname{Int}[1/(a + b*\cos[d + e*x] + c*\sin[d + e*x]), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, A, B, C\}, x \ \&\& \operatorname{NeQ}[a^2 - b^2 - c^2, 0] \ \&\& \operatorname{NeQ}[a*A - b*B - c*C, 0]$

Rule 3157

$\operatorname{Int}[(a_*) + \cos[(d_*) + (e_*)*(x_)]*(b_*) + (c_*)*\sin[(d_*) + (e_*)*(x_)])^{(n_*)} * ((A_*) + (C_*)*\sin[(d_*) + (e_*)*(x_)]), x_Symbol] \rightarrow \operatorname{Simp}[(b*C + (a*C - c*A)*\cos[d + e*x] + b*A*\sin[d + e*x])*(a + b*\cos[d + e*x] + c*\sin[d + e*x])^{(n+1)} / (e*(n+1)*(a^2 - b^2 - c^2)), x] + \operatorname{Dist}[1/((n+1)*(a^2 - b^2 - c^2)), \operatorname{Int}[(a + b*\cos[d + e*x] + c*\sin[d + e*x])^{(n+1)}*\operatorname{Simp}[(n+1)*(a*A - c*C) - (n+2)*b*A*\cos[d + e*x] + (n+2)*(a*C - c*A)*\sin[d + e*x], x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, A, C\}, x \ \&\& \operatorname{LtQ}[n, -1] \ \&\& \operatorname{NeQ}[a^2 - b^2 - c^2, 0]$

2 - c^2, 0] && NeQ[n, -2]

Rubi steps

$$\begin{aligned} \int \frac{A + C \sin(x)}{(b \cos(x) + c \sin(x))^3} dx &= \frac{bC - Ac \cos(x) + Ab \sin(x)}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} + \frac{\int \frac{2cC + Ab \cos(x) + Ac \sin(x)}{(b \cos(x) + c \sin(x))^2} dx}{2(b^2 + c^2)} \\ &= \frac{bC - Ac \cos(x) + Ab \sin(x)}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} - \frac{c^2C \cos(x) - bcC \sin(x)}{(b^2 + c^2)^2(b \cos(x) + c \sin(x))} + \frac{A \int \frac{1}{b \cos(x)}}{2(b^2 + c^2)} \\ &= \frac{bC - Ac \cos(x) + Ab \sin(x)}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} - \frac{c^2C \cos(x) - bcC \sin(x)}{(b^2 + c^2)^2(b \cos(x) + c \sin(x))} - \frac{A \operatorname{Subst}\left(\frac{1}{b \cos(x)}, \frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{2(b^2 + c^2)} \\ &= -\frac{A \operatorname{tanh}^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{2(b^2 + c^2)^{3/2}} + \frac{bC - Ac \cos(x) + Ab \sin(x)}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} - \frac{c^2C \cos(x) - bcC \sin(x)}{(b^2 + c^2)^2(b \cos(x) + c \sin(x))} \end{aligned}$$

Mathematica [C] time = 0.39, size = 132, normalized size = 1.14

$$\frac{(b^2 + c^2)(Ab^2 \sin(x) - Abc \cos(x) + bC(b + c \sin(2x)) + 2c^2C \sin^2(x)) + 2Ab\sqrt{b^2 + c^2}(b \cos(x) + c \sin(x))^2}{2b(b - ic)^2(b + ic)^2(b \cos(x) + c \sin(x))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]

[Out] (2*A*b*Sqrt[b^2 + c^2]*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]]*(b*Cos[x] + c*Sin[x])^2 + (b^2 + c^2)*(-(A*b*c*Cos[x]) + A*b^2*Sin[x] + 2*c^2*C*Sin[x]^2 + b*C*(b + c*Sin[2*x])))/(2*b*(b - I*c)^2*(b + I*c)^2*(b*Cos[x] + c*Sin[x])^2)

fricas [B] time = 0.92, size = 279, normalized size = 2.41

$$\frac{8Cbc^2 \cos(x)^2 - 2Cb^3 - 6Cbc^2 - (2Abc \cos(x) \sin(x) + Ac^2 + (Ab^2 - Ac^2) \cos(x)^2) \sqrt{b^2 + c^2} \log\left(-\frac{2bc \cos(x)}{b^2 + c^2}\right)}{4(b^4c^2 + 2b^2c^4 + c^6 + (b^6 + b^4c^2 - b^2c^4 - c^6) \cos(x)^2 + 2(b^5c^2 + 2b^3c^3 + b^2c^5) \cos(x) \sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas")

[Out] -1/4*(8*C*b*c^2*cos(x)^2 - 2*C*b^3 - 6*C*b*c^2 - (2*A*b*c*cos(x)*sin(x) + A*c^2 + (A*b^2 - A*c^2)*cos(x)^2)*sqrt(b^2 + c^2)*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) + 2*(A*b^2*c + A*c^3)*cos(x) - 2*(A*b^3 + A*b*c^2 + 2*(C*b^2*c - C*c^3)*cos(x))*sin(x))/(b^4*c^2 + 2*b^2*c^4 + c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c^2 + 2*b^3*c^3 + b*c^5)*cos(x)*sin(x))

giac [A] time = 0.24, size = 199, normalized size = 1.72

$$\frac{A \log\left(\frac{|-2b \tan\left(\frac{1}{2}x\right) + 2c - 2\sqrt{b^2 + c^2}|}{|-2b \tan\left(\frac{1}{2}x\right) + 2c + 2\sqrt{b^2 + c^2}|}\right)}{2(b^2 + c^2)^{3/2}} + \frac{Ab^3 \tan\left(\frac{1}{2}x\right)^3 + 2Abc^2 \tan\left(\frac{1}{2}x\right)^3 + 2Cb^3 \tan\left(\frac{1}{2}x\right)^2 + Ab^2c \tan\left(\frac{1}{2}x\right)^2 + 2Abc^2 \tan\left(\frac{1}{2}x\right) + 2Cb^3}{(b^4 + b^2c^2)\left(b \tan\left(\frac{1}{2}x\right) + c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")

[Out] $\frac{1}{2}A \log(\frac{\text{abs}(-2*b*\tan(1/2*x) + 2*c - 2*\sqrt{b^2 + c^2})}{\text{abs}(-2*b*\tan(1/2*x) + 2*c + 2*\sqrt{b^2 + c^2})}) / (b^2 + c^2)^{(3/2)} + (A*b^3*\tan(1/2*x)^3 + 2*A*b*c^2*\tan(1/2*x)^3 + 2*C*b^3*\tan(1/2*x)^2 + A*b^2*c*\tan(1/2*x)^2 + 2*C*b*c^2*\tan(1/2*x)^2 - 2*A*c^3*\tan(1/2*x)^2 + A*b^3*\tan(1/2*x) - 2*A*b*c^2*\tan(1/2*x) - A*b^2*c) / ((b^4 + b^2*c^2)*(b*\tan(1/2*x)^2 - 2*c*\tan(1/2*x) - b)^2)$

maple [A] time = 0.19, size = 177, normalized size = 1.53

$$\frac{2 \left(-\frac{A(b^2+2c^2)(\tan^3(\frac{x}{2}))}{2(b^2+c^2)b} - \frac{(Ab^2c-2Ac^3+2Cb^3+2Cb^2c^2)(\tan^2(\frac{x}{2}))}{2(b^2+c^2)b^2} - \frac{A(b^2-2c^2)\tan(\frac{x}{2})}{2(b^2+c^2)b} + \frac{Ac}{2b^2+2c^2} \right) A \operatorname{arctanh}\left(\frac{2b\tan(\frac{x}{2})-2c}{2\sqrt{b^2+c^2}}\right)}{(b(\tan^2(\frac{x}{2})) - 2c\tan(\frac{x}{2}) - b)^2} + \frac{A}{(b^2 + c^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x)

[Out] $-2*(-1/2*A*(b^2+2*c^2)/(b^2+c^2)/b*\tan(1/2*x)^3-1/2*(A*b^2*c-2*A*c^3+2*C*b^3+2*C*b*c^2)/(b^2+c^2)/b^2*\tan(1/2*x)^2-1/2*A*(b^2-2*c^2)/(b^2+c^2)/b*\tan(1/2*x)+1/2*A*c/(b^2+c^2))/(b*\tan(1/2*x)^2-2*c*\tan(1/2*x)-b)^2+A/(b^2+c^2)^{(3/2)}*\operatorname{arctanh}(1/2*(2*b*\tan(1/2*x)-2*c)/(b^2+c^2)^{(1/2)})$

maxima [B] time = 0.44, size = 338, normalized size = 2.91

$$-\frac{1}{2}A \left(\frac{2 \left(b^2c - \frac{(b^3-2bc^2)\sin(x)}{\cos(x)+1} - \frac{(b^2c-2c^3)\sin(x)^2}{(\cos(x)+1)^2} - \frac{(b^3+2bc^2)\sin(x)^3}{(\cos(x)+1)^3} \right)}{b^6 + b^4c^2 + \frac{4(b^5c+b^3c^3)\sin(x)}{\cos(x)+1} - \frac{2(b^6-b^4c^2-2b^2c^4)\sin(x)^2}{(\cos(x)+1)^2} - \frac{4(b^5c+b^3c^3)\sin(x)^3}{(\cos(x)+1)^3} + \frac{(b^6+b^4c^2)\sin(x)^4}{(\cos(x)+1)^4}} \right) + \frac{\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{(b^2 + c^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima")

[Out] $-1/2*A*(2*(b^2*c - (b^3 - 2*b*c^2)*\sin(x)/(\cos(x) + 1) - (b^2*c - 2*c^3)*\sin(x)^2/(\cos(x) + 1)^2 - (b^3 + 2*b*c^2)*\sin(x)^3/(\cos(x) + 1)^3)/(b^6 + b^4*c^2 + 4*(b^5*c + b^3*c^3)*\sin(x)/(\cos(x) + 1) - 2*(b^6 - b^4*c^2 - 2*b^2*c^4)*\sin(x)^2/(\cos(x) + 1)^2 - 4*(b^5*c + b^3*c^3)*\sin(x)^3/(\cos(x) + 1)^3 + (b^6 + b^4*c^2)*\sin(x)^4/(\cos(x) + 1)^4) + \log((c - b*\sin(x)/(\cos(x) + 1) + \sqrt{b^2 + c^2})/(c - b*\sin(x)/(\cos(x) + 1) - \sqrt{b^2 + c^2}))/((b^2 + c^2)^{(3/2)}) + 2*C*\sin(x)^2/((b^3 + 4*b^2*c*\sin(x)/(\cos(x) + 1) - 4*b^2*c*\sin(x)^3/(\cos(x) + 1)^3 + b^3*\sin(x)^4/(\cos(x) + 1)^4 - 2*(b^3 - 2*b*c^2)*\sin(x)^2/(\cos(x) + 1)^2)*(\cos(x) + 1)^2)$

mupad [B] time = 2.86, size = 227, normalized size = 1.96

$$\frac{\frac{\tan(\frac{x}{2})^3(Ab^2+2Ac^2)}{b(b^2+c^2)} - \frac{Ac}{b^2+c^2} + \frac{\tan(\frac{x}{2})^2(2Cb^3+Ab^2c+2Cbc^2-2Ac^3)}{b^2(b^2+c^2)} + \frac{\tan(\frac{x}{2})(Ab^2-2Ac^2)}{b(b^2+c^2)}}{b^2 - \tan(\frac{x}{2})^2(2b^2 - 4c^2) + b^2 \tan(\frac{x}{2})^4 + 4bc \tan(\frac{x}{2}) - 4bc \tan(\frac{x}{2})^3} + \frac{A \operatorname{atan}\left(\frac{b^2c + 1 + c^3 - b \tan(\frac{x}{2})(b^2+c^2)}{(b^2+c^2)^{3/2}}\right)}{(b^2 + c^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + C*sin(x))/(b*cos(x) + c*sin(x))^3,x)

[Out] $((\tan(x/2)^3*(A*b^2 + 2*A*c^2))/(b*(b^2 + c^2)) - (A*c)/(b^2 + c^2) + (\tan(x/2)^2*(2*C*b^3 - 2*A*c^3 + A*b^2*c + 2*C*b*c^2))/(b^2*(b^2 + c^2)) + (\tan(x/2)*(A*b^2 - 2*A*c^2))/(b*(b^2 + c^2)))/(b^2 - \tan(x/2)^2*(2*b^2 - 4*c^2))$

$$+ b^2 \tan(x/2)^4 + 4bc \tan(x/2) - 4bc \tan(x/2)^3 + (A \operatorname{atan}((b^2 c^2 + c^3 - b \tan(x/2)(b^2 + c^2)) / (b^2 + c^2)^{3/2})) / (b^2 + c^2)^{3/2}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))**3,x)

[Out] Timed out

$$3.352 \quad \int \frac{A+B \cos(x)}{b \cos(x)+c \sin(x)} dx$$

Optimal. Leaf size=73

$$-\frac{A \tanh^{-1}\left(\frac{c \cos(x)-b \sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}} + \frac{bBx}{b^2+c^2} + \frac{Bc \log(b \cos(x)+c \sin(x))}{b^2+c^2}$$

[Out] $b*B*x/(b^2+c^2)+B*c*\ln(b*\cos(x)+c*\sin(x))/(b^2+c^2)-A*\operatorname{arctanh}((c*\cos(x)-b*\sin(x))/(b^2+c^2)^{(1/2)})/(b^2+c^2)^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3138, 3074, 206}

$$-\frac{A \tanh^{-1}\left(\frac{c \cos(x)-b \sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}} + \frac{bBx}{b^2+c^2} + \frac{Bc \log(b \cos(x)+c \sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x]),x]

[Out] $(b*B*x)/(b^2+c^2) - (A*\operatorname{ArcTanh}[(c*\cos[x] - b*\sin[x])/Sqrt[b^2+c^2]])/Sqrt[b^2+c^2] + (B*c*\log[b*\cos[x] + c*\sin[x]])/(b^2+c^2)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3074

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 3138

Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Simp[(b*B*(d + e*x))/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[(c*B*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]

Rubi steps

$$\begin{aligned} \int \frac{A+B \cos(x)}{b \cos(x)+c \sin(x)} dx &= \frac{bBx}{b^2+c^2} + \frac{Bc \log(b \cos(x)+c \sin(x))}{b^2+c^2} + A \int \frac{1}{b \cos(x)+c \sin(x)} dx \\ &= \frac{bBx}{b^2+c^2} + \frac{Bc \log(b \cos(x)+c \sin(x))}{b^2+c^2} - A \operatorname{Subst}\left(\int \frac{1}{b^2+c^2-x^2} dx, x, c \cos(x)-b \sin(x)\right) \\ &= \frac{bBx}{b^2+c^2} - \frac{A \tanh^{-1}\left(\frac{c \cos(x)-b \sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}} + \frac{Bc \log(b \cos(x)+c \sin(x))}{b^2+c^2} \end{aligned}$$

Mathematica [A] time = 0.14, size = 67, normalized size = 0.92

$$\frac{2A \tanh^{-1}\left(\frac{b \tan\left(\frac{x}{2}\right) - c}{\sqrt{b^2 + c^2}}\right)}{\sqrt{b^2 + c^2}} + \frac{B(c \log(b \cos(x) + c \sin(x)) + bx)}{b^2 + c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x]),x]

[Out] (2*A*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]]/Sqrt[b^2 + c^2] + (B*(b*x + c*Log[b*Cos[x] + c*Sin[x]])))/(b^2 + c^2)

fricas [B] time = 1.66, size = 143, normalized size = 1.96

$$\frac{2Bbx + Bc \log\left(2bc \cos(x) \sin(x) + (b^2 - c^2) \cos(x)^2 + c^2\right) + \sqrt{b^2 + c^2} A \log\left(-\frac{2bc \cos(x) \sin(x) + (b^2 - c^2) \cos(x)^2 - 2b^2 - 2c^2}{2bc \cos(x) \sin(x) + (b^2 - c^2) \cos(x)^2 + c^2}\right)}{2(b^2 + c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")

[Out] 1/2*(2*B*b*x + B*c*log(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2) + sqrt(b^2 + c^2)*A*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)))/(b^2 + c^2)

giac [A] time = 0.23, size = 131, normalized size = 1.79

$$\frac{Bbx}{b^2 + c^2} - \frac{Bc \log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)}{b^2 + c^2} + \frac{Bc \log\left(\left|b \tan\left(\frac{1}{2}x\right) - 2c \tan\left(\frac{1}{2}x\right) - b\right|\right)}{b^2 + c^2} - \frac{A \log\left(\frac{\left|2b \tan\left(\frac{1}{2}x\right) - 2c - 2\sqrt{b^2 + c^2}\right|}{\left|2b \tan\left(\frac{1}{2}x\right) - 2c + 2\sqrt{b^2 + c^2}\right|}\right)}{\sqrt{b^2 + c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")

[Out] B*b*x/(b^2 + c^2) - B*c*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) + B*c*log(abs(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b))/(b^2 + c^2) - A*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/sqrt(b^2 + c^2)

maple [B] time = 0.13, size = 150, normalized size = 2.05

$$\frac{Bc \ln\left(b \left(\tan^2\left(\frac{x}{2}\right)\right) - 2c \tan\left(\frac{x}{2}\right) - b\right)}{b^2 + c^2} + \frac{2 \operatorname{arctanh}\left(\frac{2b \tan\left(\frac{x}{2}\right) - 2c}{2\sqrt{b^2 + c^2}}\right) A b^2}{(b^2 + c^2)^{\frac{3}{2}}} + \frac{2 \operatorname{arctanh}\left(\frac{2b \tan\left(\frac{x}{2}\right) - 2c}{2\sqrt{b^2 + c^2}}\right) A c^2}{(b^2 + c^2)^{\frac{3}{2}}} - \frac{Bc \ln(1 + \tan^2\left(\frac{x}{2}\right))}{b^2 + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(x))/(b*cos(x)+c*sin(x)),x)

[Out] 1/(b^2+c^2)*B*c*ln(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)+2/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))*A*b^2+2/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))*A*c^2-B/(b^2+c^2)*c*ln(1+tan(1/2*x)^2)+2*B/(b^2+c^2)*b*arctan(tan(1/2*x))

maxima [B] time = 0.42, size = 153, normalized size = 2.10

$$B \left(\frac{2b \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2} + \frac{c \log\left(-b - \frac{2c \sin(x)}{\cos(x)+1} + \frac{b \sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2} - \frac{c \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)}{b^2+c^2} \right) - \frac{A \log\left(\frac{c - \frac{b \sin(x)}{\cos(x)+1} + \sqrt{b^2+c^2}}{c - \frac{b \sin(x)}{\cos(x)+1} - \sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")

[Out] B*(2*b*arctan(sin(x)/(cos(x)+1))/(b^2+c^2) + c*log(-b - 2*c*sin(x)/(cos(x)+1) + b*sin(x)^2/(cos(x)+1)^2)/(b^2+c^2) - c*log(sin(x)^2/(cos(x)+1)^2 + 1)/(b^2+c^2)) - A*log((c - b*sin(x)/(cos(x)+1) + sqrt(b^2+c^2))/(c - b*sin(x)/(cos(x)+1) - sqrt(b^2+c^2)))/sqrt(b^2+c^2)

mupad [B] time = 6.53, size = 692, normalized size = 9.48

$$\ln \left(\frac{\left(A \sqrt{(b^2+c^2)^3} + Bc^3 + Bb^2c \right) \left(32b \tan\left(\frac{x}{2}\right) (A^2b^2 - A^2c^2 + 4ABc^2 + B^2b^2 - 32A^2Bb^2 - 32AB^2b^2) \right)}{32A^2Bb^2 - 32AB^2b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(x))/(b*cos(x) + c*sin(x)),x)

[Out] log(32*A^2*B*b^2 - 32*A*B^2*b^2 - ((A*((b^2+c^2)^3)^(1/2) + B*c^3 + B*b^2*c)*(32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 4*A*B*c^2) - 64*A^2*b^2*c - 32*B^2*b^2*c + ((A*((b^2+c^2)^3)^(1/2) + B*c^3 + B*b^2*c)*(32*A*b^4 + 32*B*b^4 + 32*A*b^2*c^2 - 64*B*b^2*c^2 + 32*b*c*tan(x/2)*(2*A*b^2 + 2*A*c^2 + 4*B*b^2 + B*c^2) + (96*b*c*(b + c*tan(x/2))*(A*((b^2+c^2)^3)^(1/2) + B*c^3 + B*b^2*c)))/(b^2+c^2)))/(b^2+c^2)^2 + 64*A*B*b^2*c))/(b^2+c^2)^2 + 32*B*b*c*tan(x/2)*(A-B)^2*((B*c)/(b^2+c^2) + (A*((b^2+c^2)^3)^(1/2))/(b^2+c^2)^2) + log(32*A^2*B*b^2 - 32*A*B^2*b^2 - ((B*c^3 - A*((b^2+c^2)^3)^(1/2) + B*b^2*c)*(32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 4*A*B*c^2) - 64*A^2*b^2*c - 32*B^2*b^2*c + ((B*c^3 - A*((b^2+c^2)^3)^(1/2) + B*b^2*c)*(32*A*b^4 + 32*B*b^4 + 32*A*b^2*c^2 - 64*B*b^2*c^2 + 32*b*c*tan(x/2)*(2*A*b^2 + 2*A*c^2 + 4*B*b^2 + B*c^2) + (96*b*c*(b + c*tan(x/2))*(B*c^3 - A*((b^2+c^2)^3)^(1/2) + B*b^2*c)))/(b^2+c^2)))/(b^2+c^2)^2 + 64*A*B*b^2*c))/(b^2+c^2)^2 + 32*B*b*c*tan(x/2)*(A-B)^2*((B*c)/(b^2+c^2) - (A*((b^2+c^2)^3)^(1/2))/(b^2+c^2)^2) - (B*log(tan(x/2) - 1i)*1i)/(b + c*1i) - (B*log(tan(x/2) + 1i))/(b*1i + c)

sympy [A] time = 37.03, size = 678, normalized size = 9.29

$$\left\{ \begin{aligned} & \infty \left(A \log \left(\tan \left(\frac{x}{2} \right) \right) - B \log \left(\tan^2 \left(\frac{x}{2} \right) + 1 \right) + B \log \left(\tan \left(\frac{x}{2} \right) \right) \right) \\ & \frac{A \log \left(\tan \left(\frac{x}{2} \right) \right) - B \log \left(\tan^2 \left(\frac{x}{2} \right) + 1 \right) + B \log \left(\tan \left(\frac{x}{2} \right) \right)}{c} \\ & \frac{2A}{-2ic \sin(x) - 2c \cos(x)} + \frac{Bx \sin(x)}{-2ic \sin(x) - 2c \cos(x)} - \frac{iBx \cos(x)}{-2ic \sin(x) - 2c \cos(x)} - \frac{iB \sin(x)}{-2ic \sin(x) - 2c \cos(x)} \\ & \frac{2A}{2ic \sin(x) - 2c \cos(x)} + \frac{Bx \sin(x)}{2ic \sin(x) - 2c \cos(x)} + \frac{iBx \cos(x)}{2ic \sin(x) - 2c \cos(x)} + \frac{iB \sin(x)}{2ic \sin(x) - 2c \cos(x)} \\ & - \frac{Ab^2 \log \left(\tan \left(\frac{x}{2} \right) - \frac{c}{b} - \frac{\sqrt{b^2 + c^2}}{b} \right)}{b^2 \sqrt{b^2 + c^2} + c^2 \sqrt{b^2 + c^2}} + \frac{Ab^2 \log \left(\tan \left(\frac{x}{2} \right) - \frac{c}{b} + \frac{\sqrt{b^2 + c^2}}{b} \right)}{b^2 \sqrt{b^2 + c^2} + c^2 \sqrt{b^2 + c^2}} - \frac{Ac^2 \log \left(\tan \left(\frac{x}{2} \right) - \frac{c}{b} - \frac{\sqrt{b^2 + c^2}}{b} \right)}{b^2 \sqrt{b^2 + c^2} + c^2 \sqrt{b^2 + c^2}} + \frac{Ac^2 \log \left(\tan \left(\frac{x}{2} \right) - \frac{c}{b} + \frac{\sqrt{b^2 + c^2}}{b} \right)}{b^2 \sqrt{b^2 + c^2} + c^2 \sqrt{b^2 + c^2}} + \dots \end{aligned} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tan(x/2)) - B*log(tan(x/2)**2 + 1) + B*log(tan(x/2)))
, Eq(b, 0) & Eq(c, 0)), ((A*log(tan(x/2)) - B*log(tan(x/2)**2 + 1) + B*log(
tan(x/2)))/c, Eq(b, 0)), (2*A/(-2*I*c*sin(x) - 2*c*cos(x)) + B*x*sin(x)/(-2
*I*c*sin(x) - 2*c*cos(x)) - I*B*x*cos(x)/(-2*I*c*sin(x) - 2*c*cos(x)) - I*B
*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)), Eq(b, -I*c)), (2*A/(2*I*c*sin(x) - 2*
c*cos(x)) + B*x*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)) + I*B*x*cos(x)/(2*I*c*si
n(x) - 2*c*cos(x)) + I*B*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)), Eq(b, I*c)), (
-A*b**2*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) +
c**2*sqrt(b**2 + c**2)) + A*b**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)
/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - A*c**2*log(tan(x/2) -
c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)
) + A*c**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2)
) + c**2*sqrt(b**2 + c**2)) + B*b*x*sqrt(b**2 + c**2)/(b**2*sqrt(b**2 + c**
2) + c**2*sqrt(b**2 + c**2)) - B*c*sqrt(b**2 + c**2)*log(tan(x/2)**2 + 1)/(
b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + B*c*sqrt(b**2 + c**2)*lo
g(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt
(b**2 + c**2)) + B*c*sqrt(b**2 + c**2)*log(tan(x/2) - c/b + sqrt(b**2 + c**
2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)), True))
```

$$3.353 \quad \int \frac{A+B \cos(x)}{(b \cos(x)+c \sin(x))^2} dx$$

Optimal. Leaf size=76

$$-\frac{-Ab \sin(x) + Ac \cos(x) + Bc}{(b^2 + c^2)(b \cos(x) + c \sin(x))} - \frac{bB \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}}$$

[Out] $-b*B*\operatorname{arctanh}((c*\cos(x)-b*\sin(x))/(b^2+c^2)^{(1/2)})/(b^2+c^2)^{(3/2)}+(-B*c-A*c*\cos(x)+A*b*\sin(x))/(b^2+c^2)/(b*\cos(x)+c*\sin(x))$

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3155, 3074, 206}

$$-\frac{-Ab \sin(x) + Ac \cos(x) + Bc}{(b^2 + c^2)(b \cos(x) + c \sin(x))} - \frac{bB \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^2,x]`

[Out] $-\left(\frac{b*B*\operatorname{ArcTanh}[(c*\cos[x] - b*\sin[x])/ \operatorname{Sqrt}[b^2 + c^2]]}{(b^2 + c^2)^{(3/2)}} - \frac{(B*c + A*c*\cos[x] - A*b*\sin[x])}{(b^2 + c^2)*(b*\cos[x] + c*\sin[x])}\right)$

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 3074

`Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]`

Rule 3155

`Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)])*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] := Simp[(c*B + c*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]`

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(x)}{(b \cos(x) + c \sin(x))^2} dx &= -\frac{Bc + Ac \cos(x) - Ab \sin(x)}{(b^2 + c^2)(b \cos(x) + c \sin(x))} + \frac{(bB) \int \frac{1}{b \cos(x) + c \sin(x)} dx}{b^2 + c^2} \\ &= -\frac{Bc + Ac \cos(x) - Ab \sin(x)}{(b^2 + c^2)(b \cos(x) + c \sin(x))} - \frac{(bB) \operatorname{Subst}\left(\int \frac{1}{b^2 + c^2 - x^2} dx, x, c \cos(x) - b \sin(x)\right)}{b^2 + c^2} \\ &= -\frac{bB \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}} - \frac{Bc + Ac \cos(x) - Ab \sin(x)}{(b^2 + c^2)(b \cos(x) + c \sin(x))} \end{aligned}$$

Mathematica [A] time = 0.23, size = 82, normalized size = 1.08

$$\frac{A(b^2 + c^2) \sin(x) - bBc}{b(b^2 + c^2)(b \cos(x) + c \sin(x))} + \frac{2bB \tanh^{-1}\left(\frac{b \tan\left(\frac{x}{2}\right) - c}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^2, x]

[Out] (2*b*B*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) + (- (b*B*c) + A*(b^2 + c^2)*Sin[x])/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))

fricas [B] time = 0.68, size = 201, normalized size = 2.64

$$\frac{2 B b^2 c + 2 B c^3 - (B b^2 \cos(x) + B b c \sin(x)) \sqrt{b^2 + c^2} \log\left(-\frac{2 b c \cos(x) \sin(x) + (b^2 - c^2) \cos(x)^2 - 2 b^2 - c^2 + 2 \sqrt{b^2 + c^2} (c \cos(x) - b \sin(x))}{2 b c \cos(x) \sin(x) + (b^2 - c^2) \cos(x)^2 + c^2}\right)}{2 \left((b^5 + 2 b^3 c^2 + b c^4) \cos(x) + (b^4 c + 2 b^2 c^3 + c^5) \sin(x) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas")

[Out] -1/2*(2*B*b^2*c + 2*B*c^3 - (B*b^2*cos(x) + B*b*c*sin(x))*sqrt(b^2 + c^2)*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) + 2*(A*b^2*c + A*c^3)*cos(x) - 2*(A*b^3 + A*b*c^2)*sin(x))/((b^5 + 2*b^3*c^2 + b*c^4)*cos(x) + (b^4*c + 2*b^2*c^3 + c^5)*sin(x))

giac [A] time = 0.19, size = 132, normalized size = 1.74

$$\frac{B b \log\left(\frac{\left|2 b \tan\left(\frac{1}{2} x\right) - 2 c - 2 \sqrt{b^2 + c^2}\right|}{\left|2 b \tan\left(\frac{1}{2} x\right) - 2 c + 2 \sqrt{b^2 + c^2}\right|}\right)}{(b^2 + c^2)^{3/2}} - \frac{2 \left(A b^2 \tan\left(\frac{1}{2} x\right) + A c^2 \tan\left(\frac{1}{2} x\right) - B c^2 \tan\left(\frac{1}{2} x\right) - B b c \right)}{(b^3 + b c^2) \left(b \tan\left(\frac{1}{2} x\right)^2 - 2 c \tan\left(\frac{1}{2} x\right) - b \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")

[Out] -B*b*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(A*b^2*tan(1/2*x) + A*c^2*tan(1/2*x) - B*c^2*tan(1/2*x) - B*b*c)/((b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b))

maple [A] time = 0.16, size = 109, normalized size = 1.43

$$\frac{-\frac{2(Ab^2+Ac^2-Bc^2)\tan\left(\frac{x}{2}\right)}{b(b^2+c^2)} + \frac{2Bc}{b^2+c^2}}{b\left(\tan^2\left(\frac{x}{2}\right) - 2c\tan\left(\frac{x}{2}\right) - b\right)} + \frac{2bB\operatorname{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x)

[Out] 2*(-(A*b^2+A*c^2-B*c^2)/b/(b^2+c^2)*tan(1/2*x)+B*c/(b^2+c^2))/(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)+2*b*B/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))

maxima [B] time = 0.42, size = 156, normalized size = 2.05

$$-B\left(\frac{b\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}} + \frac{2\left(bc + \frac{c^2\sin(x)}{\cos(x)+1}\right)}{b^4+b^2c^2 + \frac{2(b^3c+bc^3)\sin(x)}{\cos(x)+1} - \frac{(b^4+b^2c^2)\sin(x)^2}{(\cos(x)+1)^2}}\right) - \frac{A}{c^2\tan(x)+bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima")

[Out] -B*(b*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + 2*(b*c + c^2*sin(x)/(cos(x) + 1))/(b^4 + b^2*c^2 + 2*(b^3*c + b*c^3)*sin(x)/(cos(x) + 1) - (b^4 + b^2*c^2)*sin(x)^2/(cos(x) + 1)^2)) - A/(c^2*tan(x) + b*c)

mupad [B] time = 2.62, size = 126, normalized size = 1.66

$$\frac{\frac{2Bc}{b^2+c^2} - \frac{2\tan\left(\frac{x}{2}\right)(Ab^2+Ac^2-Bc^2)}{b(b^2+c^2)}}{-b\tan\left(\frac{x}{2}\right)^2 + 2c\tan\left(\frac{x}{2}\right) + b} + \frac{Bb\operatorname{atan}\left(\frac{b^2c^2+3c^3-2b\tan\left(\frac{x}{2}\right)(b^2+c^2)}{(b^2+c^2)^{\frac{3}{2}}}\right)}{(b^2+c^2)^{\frac{3}{2}}} 2i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(x))/(b*cos(x) + c*sin(x))^2,x)

[Out] (B*b*atan((b^2*c^2+3*c^3-2*b*tan(x/2)*(b^2+c^2))/((b^2+c^2)^(3/2)))*2i)/(b^2+c^2)^(3/2) - ((2*B*c)/(b^2+c^2) - (2*tan(x/2)*(A*b^2+A*c^2-B*c^2))/(b*(b^2+c^2)))/(b+2*c*tan(x/2)-b*tan(x/2)^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))**2,x)

[Out] Timed out

$$3.354 \quad \int \frac{A+B \cos(x)}{(b \cos(x)+c \sin(x))^3} dx$$

Optimal. Leaf size=116

$$\frac{-Ab \sin(x) + Ac \cos(x) + Bc}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} - \frac{A \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{2(b^2 + c^2)^{3/2}} - \frac{bBc \cos(x) - b^2 B \sin(x)}{(b^2 + c^2)^2 (b \cos(x) + c \sin(x))}$$

[Out] $-1/2*A*\operatorname{arctanh}((c*\cos(x)-b*\sin(x))/(b^2+c^2)^{(1/2)})/(b^2+c^2)^{(3/2)}+1/2*(-B*c-A*c*\cos(x)+A*b*\sin(x))/(b^2+c^2)/(b*\cos(x)+c*\sin(x))^2+(-b*B*c*\cos(x)+b^2*B*\sin(x))/(b^2+c^2)^2/(b*\cos(x)+c*\sin(x))$

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3158, 3153, 3074, 206}

$$\frac{-Ab \sin(x) + Ac \cos(x) + Bc}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} - \frac{A \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{2(b^2 + c^2)^{3/2}} - \frac{bBc \cos(x) - b^2 B \sin(x)}{(b^2 + c^2)^2 (b \cos(x) + c \sin(x))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^3, x]

[Out] $-(A*\operatorname{ArcTanh}[(c*\cos[x] - b*\sin[x])/ \operatorname{Sqrt}[b^2 + c^2]])/(2*(b^2 + c^2)^{(3/2)}) - (B*c + A*c*\cos[x] - A*b*\sin[x])/(2*(b^2 + c^2)*(b*\cos[x] + c*\sin[x])^2) - (b*B*c*\cos[x] - b^2*B*\sin[x])/((b^2 + c^2)^2*(b*\cos[x] + c*\sin[x]))$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3074

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 3153

Int[((A_.) + cos[(d_.) + (e_.)*(x_)])*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]) / ((a_.) + cos[(d_.) + (e_.)*(x_)])*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] := Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]) / (e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B - c*C) / (a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]

Rule 3158

Int[((A_.) + cos[(d_.) + (e_.)*(x_)])*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)])*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^n, x_Symbol] := -Simp[((c*B + c*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)) / (e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]

$\sqrt{2 - c^2}, 0] \ \&\& \ \text{NeQ}[n, -2]$

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(x)}{(b \cos(x) + c \sin(x))^3} dx &= -\frac{Bc + Ac \cos(x) - Ab \sin(x)}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} + \frac{\int \frac{2bB + Ab \cos(x) + Ac \sin(x)}{(b \cos(x) + c \sin(x))^2} dx}{2(b^2 + c^2)} \\ &= -\frac{Bc + Ac \cos(x) - Ab \sin(x)}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} - \frac{bBc \cos(x) - b^2B \sin(x)}{(b^2 + c^2)^2(b \cos(x) + c \sin(x))} + \frac{A \int \frac{1}{b \cos(x) + c \sin(x)} dx}{2(b^2 + c^2)} \\ &= -\frac{Bc + Ac \cos(x) - Ab \sin(x)}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} - \frac{bBc \cos(x) - b^2B \sin(x)}{(b^2 + c^2)^2(b \cos(x) + c \sin(x))} - \frac{A \text{Subst}\left(\int \frac{1}{b \cos(x) + c \sin(x)} dx\right)}{2(b^2 + c^2)} \\ &= -\frac{A \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{2(b^2 + c^2)^{3/2}} - \frac{Bc + Ac \cos(x) - Ab \sin(x)}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} - \frac{bBc \cos(x) - b^2B \sin(x)}{(b^2 + c^2)^2(b \cos(x) + c \sin(x))} \end{aligned}$$

Mathematica [C] time = 0.34, size = 118, normalized size = 1.02

$$\frac{(b^2 + c^2)(b \sin(x)(A + 2B \cos(x)) - Ac \cos(x) - Bc \cos(2x)) + 2A\sqrt{b^2 + c^2}(b \cos(x) + c \sin(x))^2 \tanh^{-1}\left(\frac{b \tan\left(\frac{x}{2}\right)}{\sqrt{b^2 + c^2}}\right)}{2(b - ic)^2(b + ic)^2(b \cos(x) + c \sin(x))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^3,x]

[Out] (2*A*Sqrt[b^2 + c^2]*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]]*(b*Cos[x] + c*Sin[x])^2 + (b^2 + c^2)*(-(A*c*Cos[x]) - B*c*Cos[2*x] + b*(A + 2*B*Cos[x])*Sin[x]))/(2*(b - I*c)^2*(b + I*c)^2*(b*Cos[x] + c*Sin[x])^2)

fricas [B] time = 0.93, size = 279, normalized size = 2.41

$$\frac{8Bb^2c \cos(x)^2 - 2Bb^2c + 2Bc^3 - (2Abc \cos(x) \sin(x) + Ac^2 + (Ab^2 - Ac^2) \cos(x)^2) \sqrt{b^2 + c^2} \log\left(-\frac{2bc \cos(x) \sin(x)}{\sqrt{b^2 + c^2}}\right)}{4(b^4c^2 + 2b^2c^4 + c^6 + (b^6 + b^4c^2 - b^2c^4 - c^6) \cos(x)^2 + 2(b^5c^3 + b^3c^5) \cos(x) \sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas")

[Out] -1/4*(8*B*b^2*c*cos(x)^2 - 2*B*b^2*c + 2*B*c^3 - (2*A*b*c*cos(x)*sin(x) + A*c^2 + (A*b^2 - A*c^2)*cos(x)^2)*sqrt(b^2 + c^2)*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x))))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) + 2*(A*b^2*c + A*c^3)*cos(x) - 2*(A*b^3 + A*b*c^2 + 2*(B*b^3 - B*b*c^2)*cos(x))*sin(x))/(b^4*c^2 + 2*b^2*c^4 + c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c^3 + 2*b^3*c^5)*cos(x)*sin(x))

giac [B] time = 0.25, size = 245, normalized size = 2.11

$$\frac{A \log\left(\frac{\left| -2b \tan\left(\frac{1}{2}x\right) + 2c - 2\sqrt{b^2 + c^2} \right|}{\left| -2b \tan\left(\frac{1}{2}x\right) + 2c + 2\sqrt{b^2 + c^2} \right|}\right)}{2(b^2 + c^2)^{3/2}} + \frac{Ab^3 \tan\left(\frac{1}{2}x\right)^3 - 2Bb^3 \tan\left(\frac{1}{2}x\right)^3 + 2Abc^2 \tan\left(\frac{1}{2}x\right)^3 - 2Bbc^2 \tan\left(\frac{1}{2}x\right)^3 + Ab^3}{2(b^2 + c^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(abs(-2*b*tan(1/2*x) + 2*c - 2*sqrt(b^2 + c^2))/abs(-2*b*tan(1/2*x)
) + 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + (A*b^3*tan(1/2*x)^3 - 2*B
*b^3*tan(1/2*x)^3 + 2*A*b*c^2*tan(1/2*x)^3 - 2*B*b*c^2*tan(1/2*x)^3 + A*b^2
*c*tan(1/2*x)^2 + 2*B*b^2*c*tan(1/2*x)^2 - 2*A*c^3*tan(1/2*x)^2 + 2*B*c^3*t
an(1/2*x)^2 + A*b^3*tan(1/2*x) + 2*B*b^3*tan(1/2*x) - 2*A*b*c^2*tan(1/2*x)
+ 2*B*b*c^2*tan(1/2*x) - A*b^2*c)/((b^4 + b^2*c^2)*(b*tan(1/2*x)^2 - 2*c*tan
(1/2*x) - b)^2)
```

maple [A] time = 0.18, size = 204, normalized size = 1.76

$$\frac{2 \left(-\frac{(A b^2+2 A c^2-2 B b^2-2 B c^2)\left(\tan^3\left(\frac{x}{2}\right)\right)}{2\left(b^2+c^2\right) b}-\frac{c\left(A b^2-2 A c^2+2 B b^2+2 B c^2\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{2\left(b^2+c^2\right) b^2}-\frac{\left(A b^2-2 A c^2+2 B b^2+2 B c^2\right) \tan\left(\frac{x}{2}\right)}{2\left(b^2+c^2\right) b}+\frac{A c}{2 b^2+2 c^2}\right) A}{\left(b\left(\tan^2\left(\frac{x}{2}\right)\right)-2 c \tan\left(\frac{x}{2}\right)-b\right)^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x)
```

```
[Out] -2*(-1/2*(A*b^2+2*A*c^2-2*B*b^2-2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)^3-1/2*c*(A*
b^2-2*A*c^2+2*B*b^2+2*B*c^2)/(b^2+c^2)/b^2*tan(1/2*x)^2-1/2*(A*b^2-2*A*c^2+
2*B*b^2+2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)+1/2*A*c/(b^2+c^2))/(b*tan(1/2*x)^2-
2*c*tan(1/2*x)-b)^2+A/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2
+c^2)^(1/2))
```

maxima [B] time = 0.44, size = 366, normalized size = 3.16

$$-\frac{1}{2} A \left(\frac{2 \left(b^2 c - \frac{(b^3-2 b c^2) \sin(x)}{\cos(x)+1} - \frac{(b^2 c-2 c^3) \sin(x)^2}{(\cos(x)+1)^2} - \frac{(b^3+2 b c^2) \sin(x)^3}{(\cos(x)+1)^3} \right)}{b^6 + b^4 c^2 + \frac{4(b^5 c+b^3 c^3) \sin(x)}{\cos(x)+1} - \frac{2(b^6-b^4 c^2-2 b^2 c^4) \sin(x)^2}{(\cos(x)+1)^2} - \frac{4(b^5 c+b^3 c^3) \sin(x)^3}{(\cos(x)+1)^3} + \frac{(b^6+b^4 c^2) \sin(x)^4}{(\cos(x)+1)^4}} \right) + \frac{\log \left(\frac{c-\frac{b \sin(x)}{\cos(x)+1}}{c-\frac{b \sin(x)}{\cos(x)+1}} \right)}{(b^2+c^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*A*(2*(b^2*c - (b^3 - 2*b*c^2)*sin(x)/(cos(x) + 1) - (b^2*c - 2*c^3)*si
n(x)^2/(cos(x) + 1)^2 - (b^3 + 2*b*c^2)*sin(x)^3/(cos(x) + 1)^3)/(b^6 + b^4
*c^2 + 4*(b^5*c + b^3*c^3)*sin(x)/(cos(x) + 1) - 2*(b^6 - b^4*c^2 - 2*b^2*c
^4)*sin(x)^2/(cos(x) + 1)^2 - 4*(b^5*c + b^3*c^3)*sin(x)^3/(cos(x) + 1)^3 +
(b^6 + b^4*c^2)*sin(x)^4/(cos(x) + 1)^4) + log((c - b*sin(x)/(cos(x) + 1)
+ sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^
2)^(3/2)) + 2*B*(b*sin(x)/(cos(x) + 1) + c*sin(x)^2/(cos(x) + 1)^2 - b*sin(
x)^3/(cos(x) + 1)^3)/(b^4 + 4*b^3*c*sin(x)/(cos(x) + 1) - 4*b^3*c*sin(x)^3/
(cos(x) + 1)^3 + b^4*sin(x)^4/(cos(x) + 1)^4 - 2*(b^4 - 2*b^2*c^2)*sin(x)^2
/(cos(x) + 1)^2)
```

mupad [B] time = 2.88, size = 251, normalized size = 2.16

$$\frac{\frac{\tan\left(\frac{x}{2}\right)\left(A b^2-2 A c^2+2 B b^2+2 B c^2\right)}{b\left(b^2+c^2\right)}-\frac{A c}{b^2+c^2}+\frac{\tan\left(\frac{x}{2}\right)^2\left(2 B c^3-2 A c^3+A b^2 c+2 B b^2 c\right)}{b^2\left(b^2+c^2\right)}+\frac{\tan\left(\frac{x}{2}\right)^3\left(A b^2+2 A c^2-2 B b^2-2 B c^2\right)}{b\left(b^2+c^2\right)} A \operatorname{atan}\left(\frac{c-\frac{b \sin(x)}{\cos(x)+1}}{c-\frac{b \sin(x)}{\cos(x)+1}}\right)}{b^2-\tan\left(\frac{x}{2}\right)^2\left(2 b^2-4 c^2\right)+b^2 \tan\left(\frac{x}{2}\right)^4+4 b c \tan\left(\frac{x}{2}\right)-4 b c \tan\left(\frac{x}{2}\right)^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(b*cos(x) + c*sin(x))^3,x)
```

```
[Out] ((tan(x/2)*(A*b^2 - 2*A*c^2 + 2*B*b^2 + 2*B*c^2))/(b*(b^2 + c^2)) - (A*c)/(
b^2 + c^2) + (tan(x/2)^2*(2*B*c^3 - 2*A*c^3 + A*b^2*c + 2*B*b^2*c))/(b^2*(b
^2 + c^2)) + (tan(x/2)^3*(A*b^2 + 2*A*c^2 - 2*B*b^2 - 2*B*c^2))/(b*(b^2 + c
^2)))/(b^2 - tan(x/2)^2*(2*b^2 - 4*c^2) + b^2*tan(x/2)^4 + 4*b*c*tan(x/2) -
4*b*c*tan(x/2)^3) + (A*atan((b^2*c*1i + c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i
)/(b^2 + c^2)^(3/2))*1i)/(b^2 + c^2)^(3/2)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```


$$3.355 \quad \int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^4 dx$$

Optimal. Leaf size=246

$$\frac{35b(b^2 + c^2)^{3/2} \sin(d + ex)}{8e} - \frac{35c(b^2 + c^2)^{3/2} \cos(d + ex)}{8e} - \frac{(c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^4}{4e}$$

[Out] 35/8*(b^2+c^2)^2*x-35/8*c*(b^2+c^2)^(3/2)*cos(e*x+d)/e+35/8*b*(b^2+c^2)^(3/2)*sin(e*x+d)/e-35/24*(b^2+c^2)*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))/e-7/12*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(1/2)*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2/e-1/4*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3/e

Rubi [A] time = 0.17, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3113, 2637, 2638}

$$\frac{35b(b^2 + c^2)^{3/2} \sin(d + ex)}{8e} - \frac{35c(b^2 + c^2)^{3/2} \cos(d + ex)}{8e} - \frac{(c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^4}{4e}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]

[Out] (35*(b^2 + c^2)^2*x)/8 - (35*c*(b^2 + c^2)^(3/2)*Cos[d + e*x])/(8*e) + (35*b*(b^2 + c^2)^(3/2)*Sin[d + e*x])/(8*e) - (35*(b^2 + c^2)*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]))/(24*e) - (7*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2)/(12*e) - ((c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3)/(4*e)

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3113

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^n, x_Symbol] := -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^4 dx &= -\frac{(c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^4}{4e} \\
&= -\frac{7\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^3}{12e} \\
&= -\frac{35(b^2 + c^2) (c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^2}{24e} \\
&= \frac{35}{8} (b^2 + c^2)^2 x - \frac{35(b^2 + c^2) (c \cos(d + ex) - b \sin(d + ex))}{24e} \\
&= \frac{35}{8} (b^2 + c^2)^2 x - \frac{35c(b^2 + c^2)^{3/2} \cos(d + ex)}{8e} + \frac{35b(b^2 + c^2)^{3/2} \sin(d + ex)}{8e}
\end{aligned}$$

Mathematica [C] time = 1.42, size = 238, normalized size = 0.97

$$\frac{168(b^4 - c^4) \sin(2(d + ex)) + 420(b^2 + c^2)^2 (d + ex) + 672b(b - ic)(b + ic)\sqrt{b^2 + c^2} \sin(d + ex) + 32b(b^2 - 3c^2) \cos(2(d + ex))}{96e}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^4, x]

[Out] (420*(b^2 + c^2)^2*(d + e*x) - 672*(b - I*c)*(b + I*c)*c*Sqrt[b^2 + c^2]*Cos[d + e*x] - 336*b*c*(b^2 + c^2)*Cos[2*(d + e*x)] + 32*c*(-3*b^2 + c^2)*Sqrt[b^2 + c^2]*Cos[3*(d + e*x)] - 12*b*c*(b^2 - c^2)*Cos[4*(d + e*x)] + 672*b*(b - I*c)*(b + I*c)*Sqrt[b^2 + c^2]*Sin[d + e*x] + 168*(b^4 - c^4)*Sin[2*(d + e*x)] + 32*b*(b^2 - 3*c^2)*Sqrt[b^2 + c^2]*Sin[3*(d + e*x)] + 3*(b^4 - 6*b^2*c^2 + c^4)*Sin[4*(d + e*x)])/(96*e)

fricas [A] time = 1.06, size = 221, normalized size = 0.90

$$\frac{24(b^3c - bc^3) \cos(ex + d)^4 - 105(b^4 + 2b^2c^2 + c^4)ex + 48(3b^3c + 4bc^3) \cos(ex + d)^2 - 3(2(b^4 - 6b^2c^2 + c^4) \cos(ex + d) - 3(2b^3c + 4bc^3) \cos(ex + d) - 3(b^4 - 6b^2c^2 + c^4) \cos(ex + d)) \sin(ex + d) + 32((3b^2c - c^3) \cos(ex + d)^3 + 3(b^2c + 2c^3) \cos(ex + d) - (5b^3 + 6b^2c^2 + (b^3 - 3b^2c^2) \cos(ex + d)^2) \sin(ex + d)) \sqrt{b^2 + c^2}}{96e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="fricas")

[Out] -1/24*(24*(b^3*c - b*c^3)*cos(e*x + d)^4 - 105*(b^4 + 2*b^2*c^2 + c^4)*e*x + 48*(3*b^3*c + 4*b*c^3)*cos(e*x + d)^2 - 3*(2*(b^4 - 6*b^2*c^2 + c^4)*cos(e*x + d)^3 + (27*b^4 + 6*b^2*c^2 - 29*c^4)*cos(e*x + d))*sin(e*x + d) + 32*((3*b^2*c - c^3)*cos(e*x + d)^3 + 3*(b^2*c + 2*c^3)*cos(e*x + d) - (5*b^3 + 6*b^2*c^2 + (b^3 - 3*b^2*c^2)*cos(e*x + d)^2)*sin(e*x + d))*sqrt(b^2 + c^2))/e

giac [A] time = 0.35, size = 287, normalized size = 1.17

$$-\frac{1}{8} (b^3c - bc^3) \cos(4xe + 4d) e^{(-1)} - \frac{1}{3} \left(3\sqrt{b^2 + c^2} b^2c - \sqrt{b^2 + c^2} c^3 \right) \cos(3xe + 3d) e^{(-1)} - \frac{7}{2} (b^3c + bc^3) \cos(2xe + 2d) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="giac")

[Out] $-1/8*(b^3*c - b*c^3)*\cos(4*x*e + 4*d)*e^{-1} - 1/3*(3*\sqrt{b^2 + c^2}*b^2*c - \sqrt{b^2 + c^2}*c^3)*\cos(3*x*e + 3*d)*e^{-1} - 7/2*(b^3*c + b*c^3)*\cos(2*x*e + 2*d)*e^{-1} - 7*(\sqrt{b^2 + c^2}*b^2*c + \sqrt{b^2 + c^2}*c^3)*\cos(x*e + d)*e^{-1} + 1/32*(b^4 - 6*b^2*c^2 + c^4)*e^{-1}*\sin(4*x*e + 4*d) + 1/3*(\sqrt{b^2 + c^2}*b^3 - 3*\sqrt{b^2 + c^2}*b*c^2)*e^{-1}*\sin(3*x*e + 3*d) + 7/4*(b^4 - c^4)*e^{-1}*\sin(2*x*e + 2*d) + 7*(\sqrt{b^2 + c^2}*b^3 + \sqrt{b^2 + c^2}*b*c^2)*e^{-1}*\sin(x*e + d) + 35/8*(b^4 + 2*b^2*c^2 + c^4)*x$

maple [B] time = 0.30, size = 514, normalized size = 2.09

$$b^4 (ex + d) + 2b^2c^2 (ex + d) + c^4 (ex + d) + b^4 \left(\frac{(\cos^3(ex+d) + \frac{3\cos(ex+d)}{2})\sin(ex+d)}{4} + \frac{3ex}{8} + \frac{3d}{8} \right) + 6b^4 \left(\frac{\sin(ex+d)\cos(ex+d)}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x)`

[Out] $1/e*(b^4*(e*x+d)+2*b^2*c^2*(e*x+d)+c^4*(e*x+d)+b^4*(1/4*(\cos(e*x+d))^3+3/2*\cos(e*x+d)*\sin(e*x+d)+3/8*e*x+3/8*d)+6*b^4*(1/2*\sin(e*x+d)*\cos(e*x+d)+1/2*e*x+1/2*d)+c^4*(-1/4*(\sin(e*x+d))^3+3/2*\sin(e*x+d)*\cos(e*x+d)+3/8*e*x+3/8*d)+6*c^4*(-1/2*\sin(e*x+d)*\cos(e*x+d)+1/2*e*x+1/2*d)-4*(b^2+c^2)^(1/2)*b^2*c*\cos(e*x+d)^3+4*(b^2+c^2)^(1/2)*b*c^2*\sin(e*x+d)^3+4/3*(b^2+c^2)^(1/2)*b^3*(2+\cos(e*x+d)^2)*\sin(e*x+d)+6*b^2*c^2*(1/2*\sin(e*x+d)*\cos(e*x+d)+1/2*e*x+1/2*d)+4*(b^2+c^2)^(1/2)*b^3*\sin(e*x+d)-4/3*(b^2+c^2)^(1/2)*c^3*(2+\sin(e*x+d)^2)*\cos(e*x+d)+6*b^2*c^2*(-1/2*\sin(e*x+d)*\cos(e*x+d)+1/2*e*x+1/2*d)-4*(b^2+c^2)^(1/2)*c^3*\cos(e*x+d)-\cos(e*x+d)^4*b^3*c+6*b^2*c^2*(-1/4*\sin(e*x+d)*\cos(e*x+d)^3+1/8*\sin(e*x+d)*\cos(e*x+d)+1/8*e*x+1/8*d)+c^3*b*\sin(e*x+d)^4-6*\cos(e*x+d)^2*b^3*c-6*\cos(e*x+d)^2*b*c^3+4*(b^2+c^2)^(1/2)*b*c^2*\sin(e*x+d)-4*(b^2+c^2)^(1/2)*b^2*c*\cos(e*x+d))$

maxima [A] time = 0.33, size = 354, normalized size = 1.44

$$-\frac{b^3c \cos(ex + d)^4}{e} + \frac{bc^3 \sin(ex + d)^4}{e} + \frac{(12ex + 12d + \sin(4ex + 4d) + 8 \sin(2ex + 2d))b^4}{32e} + \frac{3(4ex + 4d - \sin(4ex + 4d))}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="maxima")`

[Out] $-b^3*c*\cos(e*x + d)^4/e + b*c^3*\sin(e*x + d)^4/e + 1/32*(12*e*x + 12*d + \sin(4*e*x + 4*d))*b^2*c^2/e + 1/32*(12*e*x + 12*d + \sin(4*e*x + 4*d) - 8*\sin(2*e*x + 2*d))*c^4/e + (b^2 + c^2)^2*x - 4*(b^2 + c^2)^(3/2)*(c*\cos(e*x + d)/e - b*\sin(e*x + d)/e) - 3/2*(4*b*c*\cos(e*x + d)^2/e - (2*e*x + 2*d + \sin(2*e*x + 2*d))*b^2/e - (2*e*x + 2*d - \sin(2*e*x + 2*d))*c^2/e)*(b^2 + c^2) - 4/3*(3*b^2*c*\cos(e*x + d)^3/e - 3*b*c^2*\sin(e*x + d)^3/e + (\sin(e*x + d))^3 - 3*\sin(e*x + d))*b^3/e - (\cos(e*x + d))^3 - 3*\cos(e*x + d))*c^3/e*\sqrt{b^2 + c^2}$

mupad [B] time = 7.32, size = 522, normalized size = 2.12

$$\frac{35 \operatorname{atan} \left(\frac{35 \tan \left(\frac{d}{2} + \frac{ex}{2} \right) (b^2 + c^2)^2}{4 \left(\frac{35b^4}{4} + \frac{35b^2c^2}{2} + \frac{35c^4}{4} \right)} \right) (b^2 + c^2)^2}{4e} - \frac{35 \left(\operatorname{atan} \left(\tan \left(\frac{d}{2} + \frac{ex}{2} \right) \right) - \frac{ex}{2} \right) (b^2 + c^2)^2}{4e} + \frac{\tan \left(\frac{d}{2} + \frac{ex}{2} \right) \left((16b^3 + 8b^2c + 8bc^2 + 8c^3) \right)}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^4,x)`

[Out] $(35*\operatorname{atan}((35*\tan(d/2 + (e*x)/2)*(b^2 + c^2)^2)/(4*((35*b^4)/4 + (35*c^4)/4 + (35*b^2*c^2)/2)))*(b^2 + c^2)^2/(4*e) - (35*(\operatorname{atan}(\tan(d/2 + (e*x)/2)) - (e*x)/2)*(b^2 + c^2)^2)/(4*e) + (\tan(d/2 + (e*x)/2)*((8*b*c^2 + 16*b^3)*(b^2 + c^2)^{1/2} + (29*b^4)/4 - (27*c^4)/4 - (3*b^2*c^2)/2) + \tan(d/2 + (e*x)/2)^6*(24*b*c^3 + 32*b^3*c - (32*b^2*c + 8*c^3)*(b^2 + c^2)^{1/2})) + \tan(d/2 + (e*x)/2)^4*(64*b*c^3 + 48*b^3*c - (48*b^2*c + 40*c^3)*(b^2 + c^2)^{1/2})) + \tan(d/2 + (e*x)/2)^2*(24*b*c^3 + 32*b^3*c - (32*b^2*c + (136*c^3)/3)*(b^2 + c^2)^{1/2})) - (16*b^2*c + (40*c^3)/3)*(b^2 + c^2)^{1/2} + \tan(d/2 + (e*x)/2)^7*((8*b*c^2 + 16*b^3)*(b^2 + c^2)^{1/2} - (29*b^4)/4 + (27*c^4)/4 + (3*b^2*c^2)/2) + \tan(d/2 + (e*x)/2)^3*((56*b*c^2 + (112*b^3)/3)*(b^2 + c^2)^{1/2} + (21*b^4)/4 - (35*c^4)/4 + (21*b^2*c^2)/2) + \tan(d/2 + (e*x)/2)^5*((56*b*c^2 + (112*b^3)/3)*(b^2 + c^2)^{1/2} - (21*b^4)/4 + (35*c^4)/4 - (21*b^2*c^2)/2))/(e*(4*\tan(d/2 + (e*x)/2)^2 + 6*\tan(d/2 + (e*x)/2)^4 + 4*\tan(d/2 + (e*x)/2)^6 + \tan(d/2 + (e*x)/2)^8 + 1))$

sympy [A] time = 3.04, size = 857, normalized size = 3.48

$$\left\{ \begin{array}{l} \frac{3b^4x \sin^4(d+ex)}{8} + \frac{3b^4x \sin^2(d+ex) \cos^2(d+ex)}{4} + 3b^4x \sin^2(d+ex) + \frac{3b^4x \cos^4(d+ex)}{8} + 3b^4x \cos^2(d+ex) + b^4x + \frac{3b^4 \sin^3(d+ex)}{8} \\ x \left(b \cos(d) + c \sin(d) + \sqrt{b^2 + c^2} \right)^4 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**4,x)`

[Out] `Piecewise(((3*b**4*x*sin(d + e*x)**4/8 + 3*b**4*x*sin(d + e*x)**2*cos(d + e*x)**2/4 + 3*b**4*x*sin(d + e*x)**2 + 3*b**4*x*cos(d + e*x)**4/8 + 3*b**4*x*cos(d + e*x)**2 + b**4*x + 3*b**4*sin(d + e*x)**3*cos(d + e*x)/(8*e) + 5*b**4*sin(d + e*x)*cos(d + e*x)**3/(8*e) + 3*b**4*sin(d + e*x)*cos(d + e*x)/e - b**3*c*cos(d + e*x)**4/e - 6*b**3*c*cos(d + e*x)**2/e + 8*b**3*sqrt(b**2 + c**2)*sin(d + e*x)**3/(3*e) + 4*b**3*sqrt(b**2 + c**2)*sin(d + e*x)*cos(d + e*x)**2/e + 4*b**3*sqrt(b**2 + c**2)*sin(d + e*x)/e + 3*b**2*c**2*x*sin(d + e*x)**4/4 + 3*b**2*c**2*x*sin(d + e*x)**2*cos(d + e*x)**2/2 + 6*b**2*c**2*x*sin(d + e*x)**2 + 3*b**2*c**2*x*cos(d + e*x)**4/4 + 6*b**2*c**2*x*cos(d + e*x)**2 + 2*b**2*c**2*x + 3*b**2*c**2*sin(d + e*x)**3*cos(d + e*x)/(4*e) - 3*b**2*c**2*sin(d + e*x)*cos(d + e*x)**3/(4*e) - 4*b**2*c*sqrt(b**2 + c**2)*cos(d + e*x)**3/e - 4*b**2*c*sqrt(b**2 + c**2)*cos(d + e*x)/e + b*c**3*sin(d + e*x)**4/e - 6*b*c**3*cos(d + e*x)**2/e + 4*b*c**2*sqrt(b**2 + c**2)*sin(d + e*x)**3/e + 4*b*c**2*sqrt(b**2 + c**2)*sin(d + e*x)/e + 3*c**4*x*sin(d + e*x)**4/8 + 3*c**4*x*sin(d + e*x)**2*cos(d + e*x)**2/4 + 3*c**4*x*sin(d + e*x)**2 + 3*c**4*x*cos(d + e*x)**4/8 + 3*c**4*x*cos(d + e*x)**2 + c**4*x - 5*c**4*sin(d + e*x)**3*cos(d + e*x)/(8*e) - 3*c**4*sin(d + e*x)*cos(d + e*x)**3/(8*e) - 3*c**4*sin(d + e*x)*cos(d + e*x)/e - 4*c**3*sqrt(b**2 + c**2)*sin(d + e*x)**2*cos(d + e*x)/e - 8*c**3*sqrt(b**2 + c**2)*cos(d + e*x)**3/(3*e) - 4*c**3*sqrt(b**2 + c**2)*cos(d + e*x)/e, Ne(e, 0)), (x*(b*cos(d) + c*sin(d) + sqrt(b**2 + c**2))**4, True))`

$$3.356 \quad \int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^3 dx$$

Optimal. Leaf size=178

$$\frac{5b(b^2 + c^2) \sin(d + ex)}{2e} - \frac{5c(b^2 + c^2) \cos(d + ex)}{2e} - \frac{(c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)}{3e}$$

[Out] $5/2*(b^2+c^2)^{(3/2)*x}-5/2*c*(b^2+c^2)*\cos(e*x+d)/e+5/2*b*(b^2+c^2)*\sin(e*x+d)/e-5/6*(c*\cos(e*x+d)-b*\sin(e*x+d))*(b^2+c^2)^{(1/2)}*(b*\cos(e*x+d)+c*\sin(e*x+d)+(b^2+c^2)^{(1/2)})/e-1/3*(c*\cos(e*x+d)-b*\sin(e*x+d))*(b*\cos(e*x+d)+c*\sin(e*x+d)+(b^2+c^2)^{(1/2}))^2/e$

Rubi [A] time = 0.10, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3113, 2637, 2638}

$$\frac{5b(b^2 + c^2) \sin(d + ex)}{2e} - \frac{5c(b^2 + c^2) \cos(d + ex)}{2e} - \frac{(c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)}{3e}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]

[Out] $(5*(b^2 + c^2)^{(3/2)*x})/2 - (5*c*(b^2 + c^2)*\cos[d + e*x])/(2*e) + (5*b*(b^2 + c^2)*\sin[d + e*x])/(2*e) - (5*\text{Sqrt}[b^2 + c^2]*(c*\cos[d + e*x] - b*\sin[d + e*x])*(\text{Sqrt}[b^2 + c^2] + b*\cos[d + e*x] + c*\sin[d + e*x]))/(6*e) - ((c*\cos[d + e*x] - b*\sin[d + e*x])*(\text{Sqrt}[b^2 + c^2] + b*\cos[d + e*x] + c*\sin[d + e*x])^2)/(3*e)$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3113

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^n, x_Symbol] := -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^3 dx &= -\frac{(c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)}{3e} \\
&= -\frac{5\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)}{6e} \\
&= \frac{5}{2} (b^2 + c^2)^{3/2} x - \frac{5\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)}{6e} \\
&= \frac{5}{2} (b^2 + c^2)^{3/2} x - \frac{5c (b^2 + c^2) \cos(d + ex)}{2e} + \frac{5b (b^2 + c^2) \sin(d + ex)}{2e}
\end{aligned}$$

Mathematica [C] time = 0.60, size = 163, normalized size = 0.92

$$\frac{30(b - ic)(b + ic)\sqrt{b^2 + c^2}(d + ex) + 45b(b^2 + c^2)\sin(d + ex) + 9(b^2 - c^2)\sqrt{b^2 + c^2}\sin(2(d + ex)) + b(b^2 - 3c^2)\sin(3(d + ex))}{12e}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]

[Out] (30*(b - I*c)*(b + I*c)*Sqrt[b^2 + c^2]*(d + e*x) - 45*c*(b^2 + c^2)*Cos[d + e*x] - 18*b*c*Sqrt[b^2 + c^2]*Cos[2*(d + e*x)] + c*(-3*b^2 + c^2)*Cos[3*(d + e*x)] + 45*b*(b^2 + c^2)*Sin[d + e*x] + 9*(b^2 - c^2)*Sqrt[b^2 + c^2]*Sin[2*(d + e*x)] + b*(b^2 - 3*c^2)*Sin[3*(d + e*x)])/(12*e)

fricas [A] time = 0.85, size = 145, normalized size = 0.81

$$\frac{2(3b^2c - c^3)\cos(ex + d)^3 + 6(3b^2c + 4c^3)\cos(ex + d) - 2(11b^3 + 12bc^2 + (b^3 - 3bc^2)\cos(ex + d)^2)\sin(ex + d)}{6e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="fricas")

[Out] -1/6*(2*(3*b^2*c - c^3)*cos(e*x + d)^3 + 6*(3*b^2*c + 4*c^3)*cos(e*x + d) - 2*(11*b^3 + 12*b*c^2 + (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d) + 3*(6*b*c*cos(e*x + d)^2 - 5*(b^2 + c^2)*e*x - 3*(b^2 - c^2)*cos(e*x + d)*sin(e*x + d))*sqrt(b^2 + c^2))/e

giac [A] time = 0.24, size = 199, normalized size = 1.12

$$-\frac{3}{2}\sqrt{b^2 + c^2}bc\cos(2xe + 2d)e^{(-1)} - \frac{1}{12}(3b^2c - c^3)\cos(3xe + 3d)e^{(-1)} - \frac{15}{4}(b^2c + c^3)\cos(xe + d)e^{(-1)} + \frac{1}{12}(b^3 - 3b^2c + c^3)\sin(xe + d)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="giac")

[Out] -3/2*sqrt(b^2 + c^2)*b*c*cos(2*x*e + 2*d)*e^(-1) - 1/12*(3*b^2*c - c^3)*cos(3*x*e + 3*d)*e^(-1) - 15/4*(b^2*c + c^3)*cos(x*e + d)*e^(-1) + 1/12*(b^3 - 3*b*c^2)*e^(-1)*sin(3*x*e + 3*d) + 3/4*(sqrt(b^2 + c^2)*b^2 - sqrt(b^2 + c^2)*c^2)*e^(-1)*sin(2*x*e + 2*d) + 15/4*(b^3 + b*c^2)*e^(-1)*sin(x*e + d) + (b^2 + c^2)^(3/2)*x + 3/2*(sqrt(b^2 + c^2)*b^2 + sqrt(b^2 + c^2)*c^2)*x

maple [A] time = 0.25, size = 250, normalized size = 1.40

$$\frac{b^3(2+\cos^2(ex+d))\sin(ex+d)}{3} - (\cos^3(ex+d))b^2c + 3\sqrt{b^2+c^2}b^2\left(\frac{\sin(ex+d)\cos(ex+d)}{2} + \frac{ex}{2} + \frac{d}{2}\right) + c^2b(\sin^3(ex+d)) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x)

[Out] 1/e*(1/3*b^3*(2+cos(e*x+d)^2)*sin(e*x+d)-cos(e*x+d)^3*b^2*c+3*(b^2+c^2)^(1/2)*b^2*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)+c^2*b*sin(e*x+d)^3-3*(b^2+c^2)^(1/2)*b*c*cos(e*x+d)^2+3*sin(e*x+d)*b^3+3*c^2*b*sin(e*x+d)-1/3*c^3*(2+sin(e*x+d)^2)*cos(e*x+d)+3*(b^2+c^2)^(1/2)*c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-3*cos(e*x+d)*b^2*c-3*cos(e*x+d)*c^3+(b^2+c^2)^(1/2)*b^2*(e*x+d)+(b^2+c^2)^(1/2)*c^2*(e*x+d))

maxima [A] time = 0.32, size = 207, normalized size = 1.16

$$-\frac{b^2c \cos(ex+d)^3}{e} + \frac{bc^2 \sin(ex+d)^3}{e} - \frac{(\sin(ex+d)^3 - 3 \sin(ex+d))b^3}{3e} + \frac{(\cos(ex+d)^3 - 3 \cos(ex+d))c^3}{3e} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="maxima")

[Out] -b^2*c*cos(e*x+d)^3/e + b*c^2*sin(e*x+d)^3/e - 1/3*(sin(e*x+d)^3 - 3*sin(e*x+d))*b^3/e + 1/3*(cos(e*x+d)^3 - 3*cos(e*x+d))*c^3/e + (b^2+c^2)^(3/2)*x - 3*(b^2+c^2)*(c*cos(e*x+d)/e - b*sin(e*x+d)/e) - 3/4*(4*b*c*cos(e*x+d)^2/e - (2*e*x+2*d+sin(2*e*x+2*d))*b^2/e - (2*e*x+2*d-sin(2*e*x+2*d))*c^2/e)*sqrt(b^2+c^2)

mupad [B] time = 7.32, size = 261, normalized size = 1.47

$$\frac{5x(b^2+c^2)^{3/2}}{2} - \frac{8b^2c - \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\left((3b^2-3c^2)\sqrt{b^2+c^2} + 6bc^2 + 8b^3\right) - \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^3\left(\frac{40b^3}{3} + 20bc^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d+e*x)+c*sin(d+e*x)+(b^2+c^2)^(1/2))^3,x)

[Out] (5*x*(b^2+c^2)^(3/2))/2 - (8*b^2*c - tan(d/2+(e*x)/2)*((3*b^2-3*c^2)*(b^2+c^2)^(1/2)+6*b*c^2+8*b^3) - tan(d/2+(e*x)/2)^3*(20*b*c^2+(40*b^3)/3) + (22*c^3)/3 - tan(d/2+(e*x)/2)^5*(6*b*c^2-(3*b^2-3*c^2)*(b^2+c^2)^(1/2)+8*b^3) + tan(d/2+(e*x)/2)^4*(12*b^2*c+6*c^3-12*b*c*(b^2+c^2)^(1/2)) + tan(d/2+(e*x)/2)^2*(12*b^2*c+16*c^3-12*b*c*(b^2+c^2)^(1/2)))/(e*(3*tan(d/2+(e*x)/2)^2+3*tan(d/2+(e*x)/2)^4+tan(d/2+(e*x)/2)^6+1))

sympy [A] time = 1.37, size = 415, normalized size = 2.33

$$\left\{ \begin{array}{l} \frac{2b^3 \sin^3(d+ex)}{3e} + \frac{b^3 \sin(d+ex)\cos^2(d+ex)}{e} + \frac{3b^3 \sin(d+ex)}{e} - \frac{b^2c \cos^3(d+ex)}{e} - \frac{3b^2c \cos(d+ex)}{e} + \frac{3b^2x\sqrt{b^2+c^2} \sin^2(d+ex)}{2} + \frac{3b^2x\sqrt{b^2+c^2}}{2} \\ x \left(b \cos(d) + c \sin(d) + \sqrt{b^2+c^2} \right)^3 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**3,x)
```

```
[Out] Piecewise((2*b**3*sin(d + e*x)**3/(3*e) + b**3*sin(d + e*x)*cos(d + e*x)**2
/e + 3*b**3*sin(d + e*x)/e - b**2*c*cos(d + e*x)**3/e - 3*b**2*c*cos(d + e*
x)/e + 3*b**2*x*sqrt(b**2 + c**2)*sin(d + e*x)**2/2 + 3*b**2*x*sqrt(b**2 +
c**2)*cos(d + e*x)**2/2 + b**2*x*sqrt(b**2 + c**2) + 3*b**2*sqrt(b**2 + c**
2)*sin(d + e*x)*cos(d + e*x)/(2*e) + b*c**2*sin(d + e*x)**3/e + 3*b*c**2*si
n(d + e*x)/e - 3*b*c*sqrt(b**2 + c**2)*cos(d + e*x)**2/e - c**3*sin(d + e*x
)**2*cos(d + e*x)/e - 2*c**3*cos(d + e*x)**3/(3*e) - 3*c**3*cos(d + e*x)/e
+ 3*c**2*x*sqrt(b**2 + c**2)*sin(d + e*x)**2/2 + 3*c**2*x*sqrt(b**2 + c**2)
*cos(d + e*x)**2/2 + c**2*x*sqrt(b**2 + c**2) - 3*c**2*sqrt(b**2 + c**2)*si
n(d + e*x)*cos(d + e*x)/(2*e), Ne(e, 0)), (x*(b*cos(d) + c*sin(d) + sqrt(b*
*2 + c**2))**3, True))
```


$$3.357 \quad \int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^2 dx$$

Optimal. Leaf size=116

$$\frac{3b\sqrt{b^2 + c^2} \sin(d + ex)}{2e} - \frac{3c\sqrt{b^2 + c^2} \cos(d + ex)}{2e} - \frac{(c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)}{2e}$$

[Out] $3/2*(b^2+c^2)*x-3/2*c*\cos(e*x+d)*(b^2+c^2)^{(1/2)}/e+3/2*b*\sin(e*x+d)*(b^2+c^2)^{(1/2)}/e-1/2*(c*\cos(e*x+d)-b*\sin(e*x+d))*(b*\cos(e*x+d)+c*\sin(e*x+d)+(b^2+c^2)^{(1/2)})/e$

Rubi [A] time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3113, 2637, 2638}

$$\frac{3b\sqrt{b^2 + c^2} \sin(d + ex)}{2e} - \frac{3c\sqrt{b^2 + c^2} \cos(d + ex)}{2e} - \frac{(c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)}{2e}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]

[Out] $(3*(b^2 + c^2)*x)/2 - (3*c*Sqrt[b^2 + c^2]*Cos[d + e*x])/(2*e) + (3*b*Sqrt[b^2 + c^2]*Sin[d + e*x])/(2*e) - ((c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]))/(2*e)$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3113

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^n, x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^2 dx &= -\frac{(c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)}{2e} \\ &= \frac{3}{2} (b^2 + c^2) x - \frac{(c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)}{2e} \\ &= \frac{3}{2} (b^2 + c^2) x - \frac{3c\sqrt{b^2 + c^2} \cos(d + ex)}{2e} + \frac{3b\sqrt{b^2 + c^2} \sin(d + ex)}{2e} \end{aligned}$$

Mathematica [A] time = 0.21, size = 111, normalized size = 0.96

$$\frac{8b\sqrt{b^2 + c^2} \sin(d + ex) - 8c\sqrt{b^2 + c^2} \cos(d + ex) + b^2 \sin(2(d + ex)) + 6b^2d + 6b^2ex - 2bc \cos(2(d + ex)) - c^2 \sin(2(d + ex))}{4e}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]

[Out] (6*b^2*d + 6*c^2*d + 6*b^2*e*x + 6*c^2*e*x - 8*c*Sqrt[b^2 + c^2]*Cos[d + e*x] - 2*b*c*Cos[2*(d + e*x)] + 8*b*Sqrt[b^2 + c^2]*Sin[d + e*x] + b^2*Sin[2*(d + e*x)] - c^2*Sin[2*(d + e*x)])/(4*e)

fricas [A] time = 0.94, size = 81, normalized size = 0.70

$$\frac{2bc \cos(ex + d)^2 - 3(b^2 + c^2)ex - (b^2 - c^2) \cos(ex + d) \sin(ex + d) + 4\sqrt{b^2 + c^2}(c \cos(ex + d) - b \sin(ex + d))}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="fricas")

[Out] -1/2*(2*b*c*cos(e*x + d)^2 - 3*(b^2 + c^2)*e*x - (b^2 - c^2)*cos(e*x + d)*sin(e*x + d) + 4*sqrt(b^2 + c^2)*(c*cos(e*x + d) - b*sin(e*x + d)))/e

giac [A] time = 0.17, size = 92, normalized size = 0.79

$$-\frac{1}{2}bc \cos(2xe + 2d)e^{(-1)} - 2\sqrt{b^2 + c^2}c \cos(xe + d)e^{(-1)} + 2\sqrt{b^2 + c^2}be^{(-1)} \sin(xe + d) + \frac{1}{4}(b^2 - c^2)e^{(-1)} \sin(2xe + 2d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="giac")

[Out] -1/2*b*c*cos(2*x*e + 2*d)*e^(-1) - 2*sqrt(b^2 + c^2)*c*cos(x*e + d)*e^(-1) + 2*sqrt(b^2 + c^2)*b*e^(-1)*sin(x*e + d) + 1/4*(b^2 - c^2)*e^(-1)*sin(2*x*e + 2*d) + 3/2*(b^2 + c^2)*x

maple [A] time = 0.22, size = 124, normalized size = 1.07

$$\frac{b^2 \left(\frac{\sin(ex+d) \cos(ex+d)}{2} + \frac{ex}{2} + \frac{d}{2} \right) - (\cos^2(ex + d))bc + c^2 \left(-\frac{\sin(ex+d) \cos(ex+d)}{2} + \frac{ex}{2} + \frac{d}{2} \right) + 2\sqrt{b^2 + c^2} b \sin(ex + d) - c^2 \sin(2(ex + d))}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x)

[Out] 1/e*(b^2*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-cos(e*x+d)^2*b*c+c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)+2*(b^2+c^2)^(1/2)*b*sin(e*x+d)-2*(b^2+c^2)^(1/2)*c*cos(e*x+d)+b^2*(e*x+d)+c^2*(e*x+d))

maxima [A] time = 0.31, size = 113, normalized size = 0.97

$$b^2x + c^2x - \frac{bc \cos(ex + d)^2}{e} + \frac{(2ex + 2d + \sin(2ex + 2d))b^2}{4e} + \frac{(2ex + 2d - \sin(2ex + 2d))c^2}{4e} - 2\sqrt{b^2 + c^2} \left(\frac{c \cos(ex + d)}{e} - \frac{b \sin(ex + d)}{e} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="maxima")

[Out] $b^2x + c^2x - bc\cos(ex + d)^2/e + 1/4(2ex + 2d + \sin(2ex + 2d))$
 $*b^2/e + 1/4(2ex + 2d - \sin(2ex + 2d))*c^2/e - 2\sqrt{b^2 + c^2}*(c*$
 $\cos(ex + d)/e - b\sin(ex + d)/e)$

mupad [B] time = 3.08, size = 100, normalized size = 0.86

$$\frac{b^2 \sin(2d + 2ex) - c^2 \sin(2d + 2ex) + 16c \sin\left(\frac{d}{2} + \frac{ex}{2}\right)^2 \sqrt{b^2 + c^2} + 8b \sin(d + ex) \sqrt{b^2 + c^2} + 4bc \sin(d + ex)}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^2,x)`

[Out] $(b^2*\sin(2*d + 2*e*x) - c^2*\sin(2*d + 2*e*x) + 16*c*\sin(d/2 + (e*x)/2)^2*(b$
 $^2 + c^2)^(1/2) + 8*b*\sin(d + e*x)*(b^2 + c^2)^(1/2) + 4*b*c*\sin(d + e*x)^2$
 $+ 6*b^2*e*x + 6*c^2*e*x)/(4*e)$

sympy [A] time = 0.45, size = 192, normalized size = 1.66

$$\left\{ \begin{array}{l} \frac{b^2x \sin^2(d+ex)}{2} + \frac{b^2x \cos^2(d+ex)}{2} + b^2x + \frac{b^2 \sin(d+ex) \cos(d+ex)}{2e} - \frac{bc \cos^2(d+ex)}{e} + \frac{2b\sqrt{b^2+c^2} \sin(d+ex)}{e} + \frac{c^2x \sin^2(d+ex)}{2} + \frac{c^2x \cos^2(d+ex)}{2} \\ x \left(b \cos(d) + c \sin(d) + \sqrt{b^2 + c^2} \right)^2 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**2,x)`

[Out] `Piecewise((b**2*x*sin(d + e*x)**2/2 + b**2*x*cos(d + e*x)**2/2 + b**2*x + b`
`**2*sin(d + e*x)*cos(d + e*x)/(2*e) - b*c*cos(d + e*x)**2/e + 2*b*sqrt(b**2`
`+ c**2)*sin(d + e*x)/e + c**2*x*sin(d + e*x)**2/2 + c**2*x*cos(d + e*x)**2`
`/2 + c**2*x - c**2*sin(d + e*x)*cos(d + e*x)/(2*e) - 2*c*sqrt(b**2 + c**2)*`
`cos(d + e*x)/e, Ne(e, 0)), (x*(b*cos(d) + c*sin(d) + sqrt(b**2 + c**2))**2,`
`True))`

$$3.358 \quad \int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right) dx$$

Optimal. Leaf size=37

$$x\sqrt{b^2 + c^2} + \frac{b \sin(d + ex)}{e} - \frac{c \cos(d + ex)}{e}$$

[Out] $-c*\cos(e*x+d)/e+b*\sin(e*x+d)/e+x*(b^2+c^2)^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2637, 2638}

$$x\sqrt{b^2 + c^2} + \frac{b \sin(d + ex)}{e} - \frac{c \cos(d + ex)}{e}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x],x]

[Out] Sqrt[b^2 + c^2]*x - (c*Cos[d + e*x])/e + (b*Sin[d + e*x])/e

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right) dx &= \sqrt{b^2 + c^2} x + b \int \cos(d + ex) dx + c \int \sin(d + ex) dx \\ &= \sqrt{b^2 + c^2} x - \frac{c \cos(d + ex)}{e} + \frac{b \sin(d + ex)}{e} \end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.97

$$\frac{ex\sqrt{b^2 + c^2} + b \sin(d + ex) - c \cos(d + ex)}{e}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x],x]

[Out] (Sqrt[b^2 + c^2]*e*x - c*Cos[d + e*x] + b*Sin[d + e*x])/e

fricas [A] time = 0.92, size = 34, normalized size = 0.92

$$\frac{\sqrt{b^2 + c^2} ex - c \cos(ex + d) + b \sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x, algorithm="fricas")

[Out] $(\sqrt{b^2 + c^2})e^x - c\cos(ex + d) + b\sin(ex + d)/e$

giac [A] time = 0.12, size = 35, normalized size = 0.95

$$-c \cos(xe + d)e^{(-1)} + be^{(-1)} \sin(xe + d) + \sqrt{b^2 + c^2} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x, algorithm="giac")`

[Out] $-c\cos(xe + d)e^{(-1)} + be^{(-1)}\sin(xe + d) + \sqrt{b^2 + c^2}x$

maple [A] time = 0.04, size = 36, normalized size = 0.97

$$-\frac{c \cos(ex + d)}{e} + \frac{b \sin(ex + d)}{e} + x\sqrt{b^2 + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x)`

[Out] $-c\cos(ex+d)/e + b\sin(ex+d)/e + x(b^2+c^2)^{(1/2)}$

maxima [A] time = 0.31, size = 35, normalized size = 0.95

$$\sqrt{b^2 + c^2} x - \frac{c \cos(ex + d)}{e} + \frac{b \sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x, algorithm="maxima")`

[Out] $\sqrt{b^2 + c^2}x - c\cos(ex + d)/e + b\sin(ex + d)/e$

mupad [B] time = 2.67, size = 48, normalized size = 1.30

$$x\sqrt{b^2 + c^2} - \frac{2c - 2b \tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{e \left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2),x)`

[Out] $x(b^2 + c^2)^{(1/2)} - (2*c - 2*b*\tan(d/2 + (e*x)/2))/(e*(\tan(d/2 + (e*x)/2)^2 + 1))$

sympy [A] time = 0.14, size = 42, normalized size = 1.14

$$b \left(\begin{cases} \frac{\sin(d+ex)}{e} & \text{for } e \neq 0 \\ x \cos(d) & \text{otherwise} \end{cases} \right) + c \left(\begin{cases} -\frac{\cos(d+ex)}{e} & \text{for } e \neq 0 \\ x \sin(d) & \text{otherwise} \end{cases} \right) + x\sqrt{b^2 + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2),x)`

[Out] $b*\text{Piecewise}((\sin(d + e*x)/e, \text{Ne}(e, 0)), (x*\cos(d), \text{True})) + c*\text{Piecewise}((-c \cos(d + e*x)/e, \text{Ne}(e, 0)), (x*\sin(d), \text{True})) + x*\text{sqrt}(b**2 + c**2)$

$$3.359 \quad \int \frac{1}{\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)} dx$$

Optimal. Leaf size=49

$$\frac{c - \sqrt{b^2 + c^2} \sin(d + ex)}{ce(c \cos(d + ex) - b \sin(d + ex))}$$

[Out] $(-c + \sin(e*x+d) * (b^2+c^2)^{(1/2)}) / c / e / (c * \cos(e*x+d) - b * \sin(e*x+d))$

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {3114}

$$\frac{c - \sqrt{b^2 + c^2} \sin(d + ex)}{ce(c \cos(d + ex) - b \sin(d + ex))}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]

[Out] $-(c - \text{Sqrt}[b^2 + c^2] * \text{Sin}[d + e*x]) / (c * e * (c * \text{Cos}[d + e*x] - b * \text{Sin}[d + e*x]))$

Rule 3114

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x]) / (c*e*(c*Cos[d + e*x] - b*Sin[d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\int \frac{1}{\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)} dx = -\frac{c - \sqrt{b^2+c^2} \sin(d+ex)}{ce(c \cos(d+ex) - b \sin(d+ex))}$$

Mathematica [A] time = 0.10, size = 49, normalized size = 1.00

$$\frac{\sqrt{b^2+c^2} \sin(d+ex) - c}{ce(c \cos(d+ex) - b \sin(d+ex))}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]

[Out] $(-c + \text{Sqrt}[b^2 + c^2] * \text{Sin}[d + e*x]) / (c * e * (c * \text{Cos}[d + e*x] - b * \text{Sin}[d + e*x]))$

fricas [A] time = 1.64, size = 75, normalized size = 1.53

$$\frac{b^2 + c^2 - \sqrt{b^2 + c^2} (b \cos(ex + d) + c \sin(ex + d))}{(b^2c + c^3)e \cos(ex + d) - (b^3 + bc^2)e \sin(ex + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x, algorithm="fricas")

[Out] $-(b^2 + c^2 - \text{sqrt}(b^2 + c^2) * (b * \cos(e*x + d) + c * \sin(e*x + d))) / ((b^2 * c + c^3) * e * \cos(e*x + d) - (b^3 + b * c^2) * e * \sin(e*x + d))$

giac [A] time = 0.16, size = 43, normalized size = 0.88

$$\frac{2 \left(b + \sqrt{b^2 + c^2} \right) e^{(-1)}}{\left(c \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) + b + \sqrt{b^2 + c^2} \right) c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x, algorithm="giac")

[Out] -2*(b + sqrt(b^2 + c^2))*e^(-1)/((c*tan(1/2*x*e + 1/2*d) + b + sqrt(b^2 + c^2))*c)

maple [A] time = 0.29, size = 50, normalized size = 1.02

$$\frac{2 \left(\sqrt{b^2 + c^2} + b \right)}{e c^2 \left(\tan \left(\frac{d}{2} + \frac{e x}{2} \right) + \frac{\sqrt{b^2 + c^2}}{c} + \frac{b}{c} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x)

[Out] -2/e*((b^2+c^2)^(1/2)+b)/c^2/(tan(1/2*d+1/2*e*x)+1/c*(b^2+c^2)^(1/2)+b/c)

maxima [A] time = 0.31, size = 40, normalized size = 0.82

$$\frac{2}{\left(c - \frac{(b - \sqrt{b^2 + c^2}) \sin(ex + d)}{\cos(ex + d) + 1} \right) e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x, algorithm="maxima")

[Out] -2/((c - (b - sqrt(b^2 + c^2))*sin(e*x + d)/(cos(e*x + d) + 1))*e)

mupad [B] time = 2.82, size = 38, normalized size = 0.78

$$\frac{2 \tan \left(\frac{d}{2} + \frac{e x}{2} \right)}{e \left(b + \sqrt{b^2 + c^2} + c \tan \left(\frac{d}{2} + \frac{e x}{2} \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2)),x)

[Out] (2*tan(d/2 + (e*x)/2))/(e*(b + (b^2 + c^2)^(1/2) + c*tan(d/2 + (e*x)/2)))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2)),x)

[Out] Timed out

$$3.360 \quad \int \frac{1}{\left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^2} dx$$

Optimal. Leaf size=129

$$\frac{b \sin(d+ex) - c \cos(d+ex)}{3e\sqrt{b^2+c^2} \left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^2} - \frac{c - \sqrt{b^2+c^2} \sin(d+ex)}{3ce\sqrt{b^2+c^2} (c \cos(d+ex) - b \sin(d+ex))}$$

[Out] 1/3*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2+1/3*(-c+sin(e*x+d)*(b^2+c^2)^(1/2))/c/e/(c*cos(e*x+d)-b*sin(e*x+d))/(b^2+c^2)^(1/2)

Rubi [A] time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3116, 3114}

$$\frac{c \cos(d+ex) - b \sin(d+ex)}{3e\sqrt{b^2+c^2} \left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^2} - \frac{c - \sqrt{b^2+c^2} \sin(d+ex)}{3ce\sqrt{b^2+c^2} (c \cos(d+ex) - b \sin(d+ex))}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2), x]

[Out] -(c*Cos[d + e*x] - b*Sin[d + e*x])/(3*Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (c - Sqrt[b^2 + c^2]*Sin[d + e*x])/(3*c*Sqrt[b^2 + c^2]*e*(c*Cos[d + e*x] - b*Sin[d + e*x]))

Rule 3114

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[d + e*x]))], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rule 3116

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^2} dx &= -\frac{c \cos(d+ex) - b \sin(d+ex)}{3\sqrt{b^2+c^2} e \left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^2} + \\ &= -\frac{c \cos(d+ex) - b \sin(d+ex)}{3\sqrt{b^2+c^2} e \left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^2} \end{aligned}$$

Mathematica [A] time = 0.25, size = 98, normalized size = 0.76

$$\frac{-2c\sqrt{b^2+c^2} + b^2 \sin^3(d+ex) + 2bc \cos^3(d+ex) + 2c^2 \sin(d+ex) + c^2 \sin(d+ex) \cos^2(d+ex)}{3ce(c \cos(d+ex) - b \sin(d+ex))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2), x]

[Out] (-2*c*Sqrt[b^2 + c^2] + 2*b*c*Cos[d + e*x]^3 + 2*c^2*Sin[d + e*x] + c^2*Cos[d + e*x]^2*Sin[d + e*x] + b^2*Sin[d + e*x]^3)/(3*c*e*(c*Cos[d + e*x] - b*Sin[d + e*x])^3)

fricas [A] time = 1.74, size = 192, normalized size = 1.49

$$\frac{3b^3 \cos(ex + d) - (b^3 - 3bc^2) \cos(ex + d)^3 + (3b^2c + 2c^3 - (3b^2c - c^3) \cos(ex + d)^2) \sin(ex + d)}{3((3b^4c + 2b^2c^3 - c^5)e \cos(ex + d)^3 - 3(b^4c + b^2c^3)e \cos(ex + d) - ((b^5 - 2b^3c^2 - 3bc^4)e \cos(ex + d)^2 - ($$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="fricas")

[Out] -1/3*(3*b^3*cos(e*x + d) - (b^3 - 3*b*c^2)*cos(e*x + d)^3 + (3*b^2*c + 2*c^3 - (3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d) - 2*(b^2 + c^2)^(3/2))/((3*b^4*c + 2*b^2*c^3 - c^5)*e*cos(e*x + d)^3 - 3*(b^4*c + b^2*c^3)*e*cos(e*x + d) - ((b^5 - 2*b^3*c^2 - 3*b*c^4)*e*cos(e*x + d)^2 - (b^5 + b^3*c^2)*e)*sin(e*x + d))

giac [A] time = 0.20, size = 160, normalized size = 1.24

$$\frac{2\left(8b^4 + 10b^2c^2 + 2c^4 + 3\left(2b^2c^2 + c^4 + 2\sqrt{b^2 + c^2}bc^2\right)\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 3\left(4b^3c + 3bc^3 + (4b^2c + c^3)\sqrt{b^2 + c^2}\right)\right)}{3\left(c\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + b + \sqrt{b^2 + c^2}\right)^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="giac")

[Out] -2/3*(8*b^4 + 10*b^2*c^2 + 2*c^4 + 3*(2*b^2*c^2 + c^4 + 2*sqrt(b^2 + c^2)*b*c^2)*tan(1/2*x*e + 1/2*d)^2 + 3*(4*b^3*c + 3*b*c^3 + (4*b^2*c + c^3)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d) + 2*(4*b^3 + 3*b*c^2)*sqrt(b^2 + c^2)*e^(-1)/((c*tan(1/2*x*e + 1/2*d) + b + sqrt(b^2 + c^2))^3*c^3)

maple [A] time = 0.40, size = 233, normalized size = 1.81

$$\frac{2\left(\sqrt{b^2 + c^2} + b\right)\left(-\frac{(\sqrt{b^2 + c^2} + b)\left(\tan^2\left(\frac{d + ex}{2}\right)\right)}{c^2} - \frac{(2b^2 + c^2 + 2\sqrt{b^2 + c^2}b)\tan\left(\frac{d + ex}{2}\right)}{c^3} - \frac{2(2\sqrt{b^2 + c^2}b^2 + \sqrt{b^2 + c^2}c^2 + 2b^3 + 2c^2b)}{3c^4}\right)}{e c^2 \left(\tan^2\left(\frac{d}{2} + \frac{ex}{2}\right) + \frac{2\sqrt{b^2 + c^2}\tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{c} + \frac{2b\tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{c} + \frac{2\sqrt{b^2 + c^2}b}{c^2} + \frac{2b^2}{c^2} + 1\right)\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + \frac{\sqrt{b^2 + c^2}}{c} + \frac{b}{c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x)

[Out] 2/e*((b^2+c^2)^(1/2)+b)/c^2*(-((b^2+c^2)^(1/2)+b)/c^2*tan(1/2*d+1/2*e*x)^2-1/c^3*(2*b^2+c^2+2*(b^2+c^2)^(1/2)*b)*tan(1/2*d+1/2*e*x)-2/3*(2*(b^2+c^2)^(1/2)*b^2+(b^2+c^2)^(1/2)*c^2+2*b^3+2*c^2*b)/c^4/(tan(1/2*d+1/2*e*x)^2+2/c*(b^2+c^2)^(1/2)*tan(1/2*d+1/2*e*x)+2*b/c*tan(1/2*d+1/2*e*x)+2/c^2*(b^2+c^2)^(1/2)*b+2/c^2*b^2+1)/(tan(1/2*d+1/2*e*x)+1/c*(b^2+c^2)^(1/2)+b/c)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [B] time = 3.67, size = 274, normalized size = 2.12

$$\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 \left(\frac{4b^2+2c^2}{c^4} + \frac{4b\sqrt{b^2+c^2}}{c^4}\right) + \frac{\frac{16b^4}{3} + \frac{20b^2c^2}{3} + \frac{4c^4}{3}}{c^6} + \tan\left(\frac{d}{2} + \frac{ex}{2}\right) \left(\frac{8b^3+6bc^2}{c^5} + \frac{(8b^2+2c^2)\sqrt{b^2+c^2}}{c^5}\right) + \frac{\left(\frac{16b^3}{3} + 4bc^2\right)}{c^6}}{e \left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) \left(\frac{6b^2+3c^2}{c^2} + \frac{6b\sqrt{b^2+c^2}}{c^2}\right) + \frac{4b^3+3bc^2}{c^3} + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^3 + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 \left(\frac{3\sqrt{b^2+c^2}}{c} + \frac{3b}{c}\right) + \frac{(4b^2+c^2)\sqrt{b^2+c^2}}{c^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^2,x)

[Out] $-(\tan(d/2 + (e*x)/2)^2 * ((4*b^2 + 2*c^2)/c^4 + (4*b*(b^2 + c^2)^(1/2))/c^4) + ((16*b^4)/3 + (4*c^4)/3 + (20*b^2*c^2)/3)/c^6 + \tan(d/2 + (e*x)/2) * ((6*b*c^2 + 8*b^3)/c^5 + ((8*b^2 + 2*c^2)*(b^2 + c^2)^(1/2))/c^5) + ((4*b*c^2 + (16*b^3)/3)*(b^2 + c^2)^(1/2))/c^6 / (e*(\tan(d/2 + (e*x)/2) * ((6*b^2 + 3*c^2)/c^2 + (6*b*(b^2 + c^2)^(1/2))/c^2) + (3*b*c^2 + 4*b^3)/c^3 + \tan(d/2 + (e*x)/2)^3 + \tan(d/2 + (e*x)/2)^2 * ((3*(b^2 + c^2)^(1/2))/c + (3*b)/c) + ((4*b^2 + c^2)*(b^2 + c^2)^(1/2))/c^3)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**2,x)

[Out] Timed out

$$3.361 \quad \int \frac{1}{\left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^3} dx$$

Optimal. Leaf size=191

$$\frac{2(c \cos(d+ex) - b \sin(d+ex))}{15e(b^2+c^2)\left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^2} + \frac{b \sin(d+ex) - c \cos(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)}$$

[Out] 1/5*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3-2/15*(c*cos(e*x+d)-b*sin(e*x+d))/(b^2+c^2)/e/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2-2/15*(c-sin(e*x+d)*(b^2+c^2)^(1/2))/c/(b^2+c^2)/e/(c*cos(e*x+d)-b*sin(e*x+d))

Rubi [A] time = 0.13, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3116, 3114}

$$\frac{2(c \cos(d+ex) - b \sin(d+ex))}{15e(b^2+c^2)\left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^2} - \frac{c \cos(d+ex) - b \sin(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3), x]

[Out] -(c*Cos[d + e*x] - b*Sin[d + e*x])/(5*Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(15*(b^2 + c^2)*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (2*(c - Sqrt[b^2 + c^2]*Sin[d + e*x]))/(15*c*(b^2 + c^2)*e*(c*Cos[d + e*x] - b*Sin[d + e*x]))

Rule 3114

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rule 3116

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^3} dx = -\frac{c \cos(d + ex) - b \sin(d + ex)}{5\sqrt{b^2 + c^2} e \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^3} +$$

$$= -\frac{c \cos(d + ex) - b \sin(d + ex)}{5\sqrt{b^2 + c^2} e \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^3} -$$

$$= -\frac{c \cos(d + ex) - b \sin(d + ex)}{5\sqrt{b^2 + c^2} e \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^3} -$$

Mathematica [B] time = 2.59, size = 420, normalized size = 2.20

$$\frac{20c(c^4 - b^4) \cos(2(d + ex)) - 76b^4c - 40b^3c^2 \sin(2(d + ex)) - 152b^2c^3 + 110b^2c^2\sqrt{b^2 + c^2} \sin(d + ex) - 6b^2c^2\sqrt{b^2 + c^2} \cos(d + ex)}{15\left(\left(5b^8c - 14b^4c^5 - 8b^2c^7 + c^9\right)e \cos(ex + d)^5 - 10(b^8c + b^6c^3 - b^4c^5 - b^2c^7) \cos(ex + d) \sin(ex + d) + 5(b^9 - 8b^7c^2 - 14b^5c^4 + 5b^3c^6) \cos(ex + d)^2 + 5(b^9 + 2b^7c^2 + b^5c^4) \sin(ex + d)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3), x]

[Out] (-76*b^4*c - 152*b^2*c^3 - 76*c^5 + 90*b*c*(b^2 + c^2)^(3/2)*Cos[d + e*x] + 20*c*(-b^4 + c^4)*Cos[2*(d + e*x)] + 10*b^3*c*Sqrt[b^2 + c^2]*Cos[3*(d + e*x)] + 10*b*c^3*Sqrt[b^2 + c^2]*Cos[3*(d + e*x)] - 4*b^3*c*Sqrt[b^2 + c^2]*Cos[5*(d + e*x)] + 4*b*c^3*Sqrt[b^2 + c^2]*Cos[5*(d + e*x)] + 10*b^4*Sqrt[b^2 + c^2]*Sin[d + e*x] + 110*b^2*c^2*Sqrt[b^2 + c^2]*Sin[d + e*x] + 100*c^4*Sqrt[b^2 + c^2]*Sin[d + e*x] - 40*b^3*c^2*Sin[2*(d + e*x)] - 40*b*c^4*Sin[2*(d + e*x)] - 5*b^4*Sqrt[b^2 + c^2]*Sin[3*(d + e*x)] + 5*c^4*Sqrt[b^2 + c^2]*Sin[3*(d + e*x)] + b^4*Sqrt[b^2 + c^2]*Sin[5*(d + e*x)] - 6*b^2*c^2*Sqrt[b^2 + c^2]*Sin[5*(d + e*x)] + c^4*Sqrt[b^2 + c^2]*Sin[5*(d + e*x)])/(120*c*(b^2 + c^2)*e*(c*Cos[d + e*x] - b*Sin[d + e*x])^5)

fricas [B] time = 0.96, size = 490, normalized size = 2.57

$$\frac{7b^6 + 26b^4c^2 + 31b^2c^4 + 12c^6 + 5(b^6 + b^4c^2 - b^2c^4 - c^6) \cos(ex + d)^2 + 10(b^5c + 2b^3c^3 + bc^5) \cos(ex + d) \sin(ex + d)}{15\left(\left(5b^8c - 14b^4c^5 - 8b^2c^7 + c^9\right)e \cos(ex + d)^5 - 10(b^8c + b^6c^3 - b^4c^5 - b^2c^7) \cos(ex + d) \sin(ex + d) + 5(b^9 - 8b^7c^2 - 14b^5c^4 + 5b^3c^6) \cos(ex + d)^2 + 5(b^9 + 2b^7c^2 + b^5c^4) \sin(ex + d)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="fricas")

[Out] -1/15*(7*b^6 + 26*b^4*c^2 + 31*b^2*c^4 + 12*c^6 + 5*(b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(e*x + d)^2 + 10*(b^5*c + 2*b^3*c^3 + b*c^5)*cos(e*x + d)*sin(e*x + d) - (2*(b^5 - 10*b^3*c^2 + 5*b*c^4)*cos(e*x + d)^5 - 5*(b^5 - 6*b^3*c^2 + b*c^4)*cos(e*x + d)^3 + 5*(3*b^5 + 3*b^3*c^2 + 2*b*c^4)*cos(e*x + d) + (15*b^4*c + 25*b^2*c^3 + 12*c^5 + 2*(5*b^4*c - 10*b^2*c^3 + c^5)*cos(e*x + d)^4 - (15*b^4*c - 10*b^2*c^3 - c^5)*cos(e*x + d)^2)*sin(e*x + d)*sqrt(b^2 + c^2))/((5*b^8*c - 14*b^4*c^5 - 8*b^2*c^7 + c^9)*e*cos(e*x + d)^5 - 10*(b^8*c + b^6*c^3 - b^4*c^5 - b^2*c^7)*e*cos(e*x + d)^3 + 5*(b^8*c + 2*b^6*c^3 + b^4*c^5)*e*cos(e*x + d) - ((b^9 - 8*b^7*c^2 - 14*b^5*c^4 + 5*b^3*c^6)*e*cos(e*x + d)^4 - 2*(b^9 - 3*b^7*c^2 - 9*b^5*c^4 - 5*b^3*c^6)*e*cos(e*x + d)^2 + (b^9 + 2*b^7*c^2 + b^5*c^4)*e*sin(e*x + d))

giac [A] time = 0.61, size = 346, normalized size = 1.81

$$2 \left(192 b^7 + 352 b^5 c^2 + 200 b^3 c^4 + 35 b c^6 + 15 \left(4 b^3 c^4 + 3 b c^6 + (4 b^2 c^4 + c^6) \sqrt{b^2 + c^2} \right) \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right)^4 + 30 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="giac")

[Out] -2/15*(192*b^7 + 352*b^5*c^2 + 200*b^3*c^4 + 35*b*c^6 + 15*(4*b^3*c^4 + 3*b*c^6 + (4*b^2*c^4 + c^6)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d)^4 + 30*(8*b^4*c^3 + 8*b^2*c^5 + c^7 + 4*(2*b^3*c^3 + b*c^5)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d)^3 + 20*(24*b^5*c^2 + 32*b^3*c^4 + 9*b*c^6 + 2*(12*b^4*c^2 + 10*b^2*c^4 + c^6)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d)^2 + 10*(48*b^6*c + 76*b^4*c^3 + 31*b^2*c^5 + 2*c^7 + (48*b^5*c + 52*b^3*c^3 + 11*b*c^5)*sqrt(b^2 + c^2))*tan(1/2*x*e + 1/2*d) + (192*b^6 + 256*b^4*c^2 + 96*b^2*c^4 + 7*c^6)*sqrt(b^2 + c^2)*e^(-1)/((c*tan(1/2*x*e + 1/2*d) + b + sqrt(b^2 + c^2))^5*c^5)

maple [B] time = 0.52, size = 496, normalized size = 2.60

$$\frac{2 \left(4 \sqrt{b^2 + c^2} b^2 + \sqrt{b^2 + c^2} c^2 + 4 b^3 + 3 c^2 b \right) \left(\tan^4 \left(\frac{d}{2} + \frac{e x}{2} \right) \right)}{c^2} - \frac{4 \left(8 b^4 + 8 b^2 c^2 + c^4 + 8 \sqrt{b^2 + c^2} b^3 + 4 \sqrt{b^2 + c^2} b c^2 \right) \left(\tan^3 \left(\frac{d}{2} + \frac{e x}{2} \right) \right)}{c^3} - \frac{8 \left(24 \sqrt{b^2 + c^2} b^4 + 20 \sqrt{b^2 + c^2} b^3 c + 10 \sqrt{b^2 + c^2} b^2 c^2 + 4 \sqrt{b^2 + c^2} b c^3 + c^4 \right) \left(\tan^2 \left(\frac{d}{2} + \frac{e x}{2} \right) \right)}{c^4} + \frac{4 \left(48 b^5 + 48 b^3 c^2 + 11 b c^4 \right) \left(\tan \left(\frac{d}{2} + \frac{e x}{2} \right) \right)}{c^5} + \frac{192 b^6 + 256 b^4 c^2 + 96 b^2 c^4 + 7 c^6}{c^6} \sqrt{b^2 + c^2} e^{-1} \left(c \tan \left(\frac{d}{2} + \frac{e x}{2} \right) + b + \sqrt{b^2 + c^2} \right)^5$$

$e^4 \left(\tan^2 \left(\frac{d}{2} + \frac{e x}{2} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x)

[Out] 2/e/c^4*(-(4*(b^2+c^2)^(1/2)*b^2+(b^2+c^2)^(1/2)*c^2+4*b^3+3*c^2*b)/c^2*tan(1/2*d+1/2*e*x)^4-2*(8*b^4+8*b^2*c^2+c^4+8*(b^2+c^2)^(1/2)*b^3+4*(b^2+c^2)^(1/2)*b*c^2)/c^3*tan(1/2*d+1/2*e*x)^3-4/3*(24*(b^2+c^2)^(1/2)*b^4+20*(b^2+c^2)^(1/2)*b^2*c^2+2*(b^2+c^2)^(1/2)*c^4+24*b^5+32*b^3*c^2+9*c^4*b)/c^4*tan(1/2*d+1/2*e*x)^2-2/3*(48*b^6+76*b^4*c^2+31*b^2*c^4+2*c^6+48*(b^2+c^2)^(1/2)*b^5+52*(b^2+c^2)^(1/2)*b^3*c^2+11*(b^2+c^2)^(1/2)*b*c^4)/c^5*tan(1/2*d+1/2*e*x)-1/15/c^6*(192*(b^2+c^2)^(1/2)*b^6+256*(b^2+c^2)^(1/2)*b^4*c^2+96*(b^2+c^2)^(1/2)*b^2*c^4+7*(b^2+c^2)^(1/2)*c^6+192*b^7+352*b^5*c^2+200*b^3*c^4+35*c^6*b))/(tan(1/2*d+1/2*e*x)^2+2/c*(b^2+c^2)^(1/2)*tan(1/2*d+1/2*e*x)+2*b/c*tan(1/2*d+1/2*e*x)+2/c^2*(b^2+c^2)^(1/2)*b+2/c^2*b^2+1)^2/(tan(1/2*d+1/2*e*x)+1/c*(b^2+c^2)^(1/2)+b/c)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [B] time = 8.12, size = 592, normalized size = 3.10

$$\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^3 \left(\frac{32b^4 + 32b^2c^2 + 4c^4}{c^7} + \frac{(32b^3 + 16bc^2)\sqrt{b^2 + c^2}}{c^7}\right) + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^4 \left(\frac{8b^3 + 6bc^2}{c^6} + \frac{(8b^2 + 2c^2)\sqrt{b^2 + c^2}}{c^6}\right) + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^5 \left(\frac{4b^3 + 3bc^2}{c^5} + \frac{(4b^2 + c^2)\sqrt{b^2 + c^2}}{c^5}\right)}{e \left(\frac{16b^5 + 20b^3c^2 + 5bc^4}{c^5} + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 \left(\frac{40b^3 + 30bc^2}{c^3} + \frac{(40b^2 + 10c^2)\sqrt{b^2 + c^2}}{c^3}\right) + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^3 \left(\frac{40b^2 + 10c^2}{c^2} + \frac{20b(b^2 + c^2)}{c^2}\right) + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^4 \left(\frac{40b^4 + 5c^4 + 40b^2c^2}{c^4} + \frac{(20b^3c^2 + 40b^3)(b^2 + c^2)}{c^4}\right) + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^5 \left(\frac{5(b^2 + c^2)}{c} + \frac{5b}{c} + \frac{(b^2 + c^2)(16b^4 + c^4 + 12b^2c^2)}{c^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2)))^3,x)

[Out] -(tan(d/2 + (e*x)/2)^3*((32*b^4 + 4*c^4 + 32*b^2*c^2)/c^7 + ((16*b*c^2 + 32*b^3)*(b^2 + c^2)^(1/2))/c^7) + tan(d/2 + (e*x)/2)^4*((6*b*c^2 + 8*b^3)/c^6 + ((8*b^2 + 2*c^2)*(b^2 + c^2)^(1/2))/c^6) + tan(d/2 + (e*x)/2)*((64*b^6 + (8*c^6)/3 + (124*b^2*c^4)/3 + (304*b^4*c^2)/3)/c^9 + ((b^2 + c^2)^(1/2))*((44*b*c^4)/3 + 64*b^5 + (208*b^3*c^2)/3)/c^9) + tan(d/2 + (e*x)/2)^2*((24*b*c^4 + 64*b^5 + (256*b^3*c^2)/3)/c^8 + ((b^2 + c^2)^(1/2))*(64*b^4 + (16*c^4)/3 + (160*b^2*c^2)/3)/c^8) + ((14*b*c^6)/3 + (128*b^7)/5 + (80*b^3*c^4)/3 + (704*b^5*c^2)/15)/c^10 + ((b^2 + c^2)^(1/2))*((128*b^6)/5 + (14*c^6)/15 + (64*b^2*c^4)/5 + (512*b^4*c^2)/15)/c^10)/(e*((5*b*c^4 + 16*b^5 + 20*b^3*c^2)/c^5 + tan(d/2 + (e*x)/2)^2*((30*b*c^2 + 40*b^3)/c^3 + ((40*b^2 + 10*c^2)*(b^2 + c^2)^(1/2))/c^3) + tan(d/2 + (e*x)/2)^5 + tan(d/2 + (e*x)/2)^3*((20*b^2 + 10*c^2)/c^2 + (20*b*(b^2 + c^2)^(1/2))/c^2) + tan(d/2 + (e*x)/2)*((40*b^4 + 5*c^4 + 40*b^2*c^2)/c^4 + ((20*b*c^2 + 40*b^3)*(b^2 + c^2)^(1/2))/c^4) + tan(d/2 + (e*x)/2)^4*((5*(b^2 + c^2)^(1/2))/c + (5*b)/c) + ((b^2 + c^2)^(1/2))*(16*b^4 + c^4 + 12*b^2*c^2)/c^5))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2)))**3,x)

[Out] Timed out

$$3.362 \quad \int \frac{1}{\left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^4} dx$$

Optimal. Leaf size=259

$$\frac{2(c \cos(d+ex) - b \sin(d+ex))}{35e(b^2+c^2)^{3/2} \left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^2} - \frac{3(c \cos(d+ex) - b \sin(d+ex))}{35e(b^2+c^2) \left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)}$$

[Out] $1/7*(-c*\cos(e*x+d)+b*\sin(e*x+d))/e/(b^2+c^2)^{(1/2)}/(b*\cos(e*x+d)+c*\sin(e*x+d)+(b^2+c^2)^{(1/2)})^4-3/35*(c*\cos(e*x+d)-b*\sin(e*x+d))/(b^2+c^2)/e/(b*\cos(e*x+d)+c*\sin(e*x+d)+(b^2+c^2)^{(1/2)})^3-2/35*(c*\cos(e*x+d)-b*\sin(e*x+d))/(b^2+c^2)^{(3/2)}/e/(b*\cos(e*x+d)+c*\sin(e*x+d)+(b^2+c^2)^{(1/2)})^2-2/35*(c-\sin(e*x+d))*(b^2+c^2)^{(1/2)}/c/(b^2+c^2)^{(3/2)}/e/(c*\cos(e*x+d)-b*\sin(e*x+d))$

Rubi [A] time = 0.19, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3116, 3114}

$$\frac{2(c \cos(d+ex) - b \sin(d+ex))}{35e(b^2+c^2)^{3/2} \left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^2} - \frac{3(c \cos(d+ex) - b \sin(d+ex))}{35e(b^2+c^2) \left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]

[Out] $-(c*\cos[d + e*x] - b*\sin[d + e*x])/(7*\sqrt{b^2 + c^2}*e*(\sqrt{b^2 + c^2} + b*\cos[d + e*x] + c*\sin[d + e*x])^4) - (3*(c*\cos[d + e*x] - b*\sin[d + e*x]))/(35*(b^2 + c^2)*e*(\sqrt{b^2 + c^2} + b*\cos[d + e*x] + c*\sin[d + e*x])^3) - (2*(c*\cos[d + e*x] - b*\sin[d + e*x]))/(35*(b^2 + c^2)^{(3/2)}*e*(\sqrt{b^2 + c^2} + b*\cos[d + e*x] + c*\sin[d + e*x])^2) - (2*(c - \sqrt{b^2 + c^2})*\sin[d + e*x])/(35*c*(b^2 + c^2)^{(3/2)}*e*(c*\cos[d + e*x] - b*\sin[d + e*x]))$

Rule 3114

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rule 3116

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^n(n_), x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^4} dx &= -\frac{c \cos(d + ex) - b \sin(d + ex)}{7\sqrt{b^2 + c^2} e \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^4} + \\
&= -\frac{c \cos(d + ex) - b \sin(d + ex)}{7\sqrt{b^2 + c^2} e \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^4} - \\
&= -\frac{c \cos(d + ex) - b \sin(d + ex)}{7\sqrt{b^2 + c^2} e \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^4} - \\
&= -\frac{c \cos(d + ex) - b \sin(d + ex)}{7\sqrt{b^2 + c^2} e \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^4} -
\end{aligned}$$

Mathematica [B] time = 2.03, size = 533, normalized size = 2.06

$$\frac{-35b^6 \sin(d + ex) + 21b^6 \sin(3(d + ex)) - 7b^6 \sin(5(d + ex)) + b^6 \sin(7(d + ex)) - 112b^5c \cos(3(d + ex)) + 28b^5c \cos(5(d + ex)) - 7b^5c \cos(7(d + ex))}{(7\sqrt{b^2 + c^2} e (\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)))^4}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]

[Out] (832*b^4*c*Sqrt[b^2 + c^2] + 1664*b^2*c^3*Sqrt[b^2 + c^2] + 832*c^5*Sqrt[b^2 + c^2] - 1190*b*c*(b^2 + c^2)^2*Cos[d + e*x] + 448*c*Sqrt[b^2 + c^2]*(b^4 - c^4)*Cos[2*(d + e*x)] - 112*b^5*c*Cos[3*(d + e*x)] + 56*b^3*c^3*Cos[3*(d + e*x)] + 168*b*c^5*Cos[3*(d + e*x)] + 28*b^5*c*Cos[5*(d + e*x)] - 28*b*c^5*Cos[5*(d + e*x)] - 6*b^5*c*Cos[7*(d + e*x)] + 20*b^3*c^3*Cos[7*(d + e*x)] - 6*b*c^5*Cos[7*(d + e*x)] - 35*b^6*Sin[d + e*x] - 1295*b^4*c^2*Sin[d + e*x] - 2485*b^2*c^4*Sin[d + e*x] - 1225*c^6*Sin[d + e*x] + 896*b^3*c^2*Sqrt[b^2 + c^2]*Sin[2*(d + e*x)] + 896*b*c^4*Sqrt[b^2 + c^2]*Sin[2*(d + e*x)] + 21*b^6*Sin[3*(d + e*x)] - 189*b^4*c^2*Sin[3*(d + e*x)] - 161*b^2*c^4*Sin[3*(d + e*x)] + 49*c^6*Sin[3*(d + e*x)] - 7*b^6*Sin[5*(d + e*x)] + 35*b^4*c^2*Sin[5*(d + e*x)] + 35*b^2*c^4*Sin[5*(d + e*x)] - 7*c^6*Sin[5*(d + e*x)] + b^6*Sin[7*(d + e*x)] - 15*b^4*c^2*Sin[7*(d + e*x)] + 15*b^2*c^4*Sin[7*(d + e*x)] - c^6*Sin[7*(d + e*x)])/(1120*c*(b^2 + c^2)*e*(-(c*Cos[d + e*x]) + b*Sin[d + e*x])^7)

fricas [B] time = 1.85, size = 739, normalized size = 2.85

$$\frac{2(b^7 - 21b^5c^2 + 35b^3c^4 - 7bc^6) \cos(ex + d)^7 - 7(b^7 - 15b^5c^2 + 15b^3c^4 - bc^6) \cos(ex + d)^5 - 14(5b^5c^2 - 5b^3c^4 - 2b^3c^6) \cos(ex + d)^3 - 7(5b^7 + 15b^5c^2 + 20b^3c^4 + 8b^3c^6) \cos(ex + d) - (35b^6c + 105b^4c^3 + 112b^2c^5 + 40c^7 - 2(7b^6c - 7b^4c^3 + 7b^2c^5 - c^7)) e \cos(ex + d)^7 - 7(35b^6c - 7b^4c^3 + 7b^2c^5 - c^7) e \cos(ex + d)^5}{35((7b^{10}c - 21b^8c^3 - 42b^6c^5 + 6b^4c^7 + 19b^2c^9 - c^{11})e \cos(ex + d)^7 - 7(35b^6c - 7b^4c^3 + 7b^2c^5 - c^7) e \cos(ex + d)^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="fricas")

[Out] 1/35*(2*(b^7 - 21*b^5*c^2 + 35*b^3*c^4 - 7*b*c^6)*cos(e*x + d)^7 - 7*(b^7 - 15*b^5*c^2 + 15*b^3*c^4 - b*c^6)*cos(e*x + d)^5 - 14*(5*b^5*c^2 - 5*b^3*c^4 - 2*b*c^6)*cos(e*x + d)^3 - 7*(5*b^7 + 15*b^5*c^2 + 20*b^3*c^4 + 8*b*c^6)*cos(e*x + d) - (35*b^6*c + 105*b^4*c^3 + 112*b^2*c^5 + 40*c^7 - 2*(7*b^6*c - 7*b^4*c^3 + 7*b^2*c^5 - c^7))e*cos(e*x + d)^7 - 7*(35*b^6*c - 7*b^4*c^3 + 7*b^2*c^5 - c^7)e*cos(e*x + d)^5)

$$\begin{aligned}
& - 35b^4c^3 + 21b^2c^5 - c^7) \cos(ex + d)^6 + (35b^6c - 105b^4c^3 \\
& + 21b^2c^5 + c^7) \cos(ex + d)^4 + 2(35b^4c^3 + 7b^2c^5 - 4c^7) \cos \\
& (ex + d)^2 \sin(ex + d) + 4(3b^6 + 16b^4c^2 + 23b^2c^4 + 10c^6 + 7 \\
& (b^6 + b^4c^2 - b^2c^4 - c^6) \cos(ex + d)^2 + 14(b^5c + 2b^3c^3 + b \\
& *c^5) \cos(ex + d) \sin(ex + d)) \sqrt{b^2 + c^2} / ((7b^{10}c - 21b^8c^3 - \\
& 42b^6c^5 + 6b^4c^7 + 19b^2c^9 - c^{11}) e \cos(ex + d)^7 - 7(3b^{10}c \\
& - 4b^8c^3 - 14b^6c^5 - 4b^4c^7 + 3b^2c^9) e \cos(ex + d)^5 + 7(3b^{10}c \\
& + b^8c^3 - 7b^6c^5 - 5b^4c^7) e \cos(ex + d)^3 - 7(b^{10}c + 2b^8c^3 + \\
& b^6c^5) e \cos(ex + d) - ((b^{11} - 19b^9c^2 - 6b^7c^4 + 42b^5c^6 + \\
& 21b^3c^8 - 7b^1c^{10}) e \cos(ex + d)^6 - (3b^{11} - 36b^9c^2 - 46 \\
& *b^7c^4 + 28b^5c^6 + 35b^3c^8) e \cos(ex + d)^4 + 3(b^{11} - 5b^9c^2 \\
& - 13b^7c^4 - 7b^5c^6) e \cos(ex + d)^2 - (b^{11} + 2b^9c^2 + b^7c^4) e \\
&) \sin(ex + d))
\end{aligned}$$

giac [B] time = 2.52, size = 599, normalized size = 2.31

$$\frac{2 \left(2560b^{10} + 6528b^8c^2 + 5888b^6c^4 + 2248b^4c^6 + 340b^2c^8 + 12c^{10} + 35 \left(8b^4c^6 + 8b^2c^8 + c^{10} + 4 \left(2b^3c^6 + \dots \right) \right) \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(ex+d)+c*sin(ex+d)+(b^2+c^2)^(1/2))^4,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -2/35(2560b^{10} + 6528b^8c^2 + 5888b^6c^4 + 2248b^4c^6 + 340b^2c^8 \\
& + 12c^{10} + 35(8b^4c^6 + 8b^2c^8 + c^{10} + 4(2b^3c^6 + b^2c^8) \sqrt{b^2 + c^2}) \tan(1/2*x*e + 1/2*d)^6 + 105(16b^5c^5 + 20b^3c^7 + 5b^1c^9 \\
& + (16b^4c^5 + 12b^2c^7 + c^9) \sqrt{b^2 + c^2}) \tan(1/2*x*e + 1/2*d)^5 \\
& + 70(80b^6c^4 + 124b^4c^6 + 49b^2c^8 + 3c^{10} + (80b^5c^4 + 84b^3c^6 + 17b^1c^8) \sqrt{b^2 + c^2}) \tan(1/2*x*e + 1/2*d)^4 + 70(160b^7c^3 \\
& + 288b^5c^5 + 150b^3c^7 + 20b^1c^9 + (160b^6c^3 + 208b^4c^5 + 66b^2c^7 + 3c^9) \sqrt{b^2 + c^2}) \tan(1/2*x*e + 1/2*d)^3 + 21(640b^8c^2 + \\
& 1312b^6c^4 + 856b^4c^6 + 186b^2c^8 + 7c^{10} + 2(320b^7c^2 + 496b^5c^4 + 220b^3c^6 + 25b^1c^8) \sqrt{b^2 + c^2}) \tan(1/2*x*e + 1/2*d)^2 + 7 \\
& *(1280b^9c + 2944b^7c^3 + 2288b^5c^5 + 676b^3c^7 + 57b^1c^9 + (1280 \\
& *b^8c + 2304b^6c^3 + 1296b^4c^5 + 236b^2c^7 + 7c^9) \sqrt{b^2 + c^2}) \tan(1/2*x*e + 1/2*d) + 4(640b^9 + 1312b^7c^2 + 896b^5c^4 + 238b^3c^6 \\
& + 21b^1c^8) \sqrt{b^2 + c^2}) e^{-1} / ((c \tan(1/2*x*e + 1/2*d) + b + \sqrt{b^2 + c^2})^7 c^7)
\end{aligned}$$

maple [B] time = 0.71, size = 823, normalized size = 3.18

$$\frac{2 \left(\frac{(8b^4 + 8b^2c^2 + c^4 + 8\sqrt{b^2+c^2} b^3 + 4\sqrt{b^2+c^2} b c^2) \left(\tan^6\left(\frac{d}{2} + \frac{ex}{2}\right) \right)}{c^2} + \frac{3(16\sqrt{b^2+c^2} b^4 + 12\sqrt{b^2+c^2} b^2 c^2 + \sqrt{b^2+c^2} c^4 + 16b^5 + 20b^3 c^2 + 5c^4 b) \left(\tan^5\left(\frac{d}{2} + \frac{ex}{2}\right) \right)}{c^3} \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*cos(ex+d)+c*sin(ex+d)+(b^2+c^2)^(1/2))^4,x)

[Out]
$$\begin{aligned}
& -2/e/c^6 * ((8b^4 + 8b^2c^2 + c^4 + 8(b^2+c^2)^{1/2} b^3 + 4(b^2+c^2)^{1/2} b^2 c^2 + (b^2+c^2)^{1/2} c^4 + 16b^5 + 20b^3 c^2 + 5c^4 b) / c^3 \tan(1/2*d + 1/2*ex) \\
& ^6 + 3(16(b^2+c^2)^{1/2} b^4 + 12(b^2+c^2)^{1/2} b^2 c^2 + (b^2+c^2)^{1/2} c^4 + 16b^5 + 20b^3 c^2 + 5c^4 b) / c^3 \tan(1/2*d + 1/2*ex) \\
& ^5 + 2(80(b^2+c^2)^{1/2} b^5 + 84(b^2+c^2)^{1/2} b^3 c^2 + 17(b^2+c^2)^{1/2} b^1 c^4 + 80b^6 + 124b^4 c^2 + 49b^2 c^4 + 3c^6) / c^4 \tan(1/2*d + 1/2*ex) \\
& ^4 + 2(160b^7 + 288b^5 c^2 + 150b^3 c^4 + 20c^6 b + 160(b^2+c^2)^{1/2} b^6 + 208(b^2+c^2)^{1/2} b^4 c^2 + 66(b^2+c^2)^{1/2} b^2 c^4 + 3c^8) / c^5 \tan(1/2*d + 1/2*ex) \\
& ^3 + 21(640b^8 + 1312b^6 c^2 + 896b^4 c^4 + 238b^2 c^6 + 21b^0 c^8) \sqrt{b^2 + c^2} \tan(1/2*d + 1/2*ex) \\
& ^2 + 7(1280b^9 + 2944b^7 c^2 + 2288b^5 c^4 + 676b^3 c^6 + 57b^1 c^8 + (1280b^8 + 2304b^6 c^2 + 1296b^4 c^4 + 236b^2 c^6 + 7c^8) \sqrt{b^2 + c^2}) \tan(1/2*d + 1/2*ex) \\
& + 4(640b^9 + 1312b^7 c^2 + 896b^5 c^4 + 238b^3 c^6 + 21b^1 c^8) \sqrt{b^2 + c^2} \tan(1/2*d + 1/2*ex) \\
& + (c \tan(1/2*x*e + 1/2*d) + b + \sqrt{b^2 + c^2})^7 c^7) e^{-1} / ((c \tan(1/2*x*e + 1/2*d) + b + \sqrt{b^2 + c^2})^7 c^7)
\end{aligned}$$

$$\begin{aligned} & \left(\frac{1}{2}\right)*b^4*c^2+66*(b^2+c^2)^{(1/2)}*b^2*c^4+3*(b^2+c^2)^{(1/2)}*c^6)/c^5*\tan(1/2 \\ & *d+1/2*e*x)^3+3/5*(640*b^7*(b^2+c^2)^{(1/2)}+992*(b^2+c^2)^{(1/2)}*b^5*c^2+440* \\ & (b^2+c^2)^{(1/2)}*b^3*c^4+50*(b^2+c^2)^{(1/2)}*b*c^6+640*b^8+1312*b^6*c^2+856*b \\ & ^4*c^4+186*b^2*c^6+7*c^8)/c^6*\tan(1/2*d+1/2*e*x)^2+1/5*(1280*b^9+2944*b^7*c \\ & ^2+2288*b^5*c^4+676*b^3*c^6+57*b*c^8+1280*(b^2+c^2)^{(1/2)}*b^8+2304*(b^2+c^2 \\ &)^{(1/2)}*b^6*c^2+1296*(b^2+c^2)^{(1/2)}*b^4*c^4+236*(b^2+c^2)^{(1/2)}*b^2*c^6+7* \\ & (b^2+c^2)^{(1/2)}*c^8)/c^7*\tan(1/2*d+1/2*e*x)+4/35*(640*(b^2+c^2)^{(1/2)}*b^9+1 \\ & 312*(b^2+c^2)^{(1/2)}*b^7*c^2+896*(b^2+c^2)^{(1/2)}*b^5*c^4+238*(b^2+c^2)^{(1/2)} \\ & *b^3*c^6+21*(b^2+c^2)^{(1/2)}*b*c^8+640*b^10+1632*b^8*c^2+1472*b^6*c^4+562*b^ \\ & 4*c^6+85*b^2*c^8+3*c^10)/c^8)/(\tan(1/2*d+1/2*e*x)^2+2/c*(b^2+c^2)^{(1/2)}*\tan \\ & (1/2*d+1/2*e*x)+2*b/c*\tan(1/2*d+1/2*e*x)+2/c^2*(b^2+c^2)^{(1/2)}*b+2/c^2*b^2+ \\ & 1)^3/(\tan(1/2*d+1/2*e*x)+1/c*(b^2+c^2)^{(1/2)}+b/c) \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [B] time = 12.31, size = 1004, normalized size = 3.88

$$\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^6 \left(\frac{16b^4+16b^2c^2+2c^4}{c^8} + \frac{(16b^3+8bc^2)\sqrt{b^2+c^2}}{c^8} \right) + \tan\left(\frac{d}{2} + \frac{ex}{2}\right) \left(\frac{512b^9 + \frac{5888b^7c^2}{5} + \frac{4576b^5c^4}{5} + \frac{1352b^3c^6}{5} + \frac{114bc^8}{5} + \sqrt{b^2+c^2}}{c^{13}} + \dots \right)$$

$$e \left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) \right)^3 \left(\frac{280b^4+280b^2c^2+3c^4}{c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^4,x)

[Out] $-(\tan(d/2 + (e*x)/2))^6*((16*b^4 + 2*c^4 + 16*b^2*c^2)/c^8 + ((8*b*c^2 + 16*b^3)*(b^2 + c^2)^(1/2))/c^8) + \tan(d/2 + (e*x)/2)*(((114*b*c^8)/5 + 512*b^9 + (1352*b^3*c^6)/5 + (4576*b^5*c^4)/5 + (5888*b^7*c^2)/5)/c^{13} + ((b^2 + c^2)^(1/2)*(512*b^8 + (14*c^8)/5 + (472*b^2*c^6)/5 + (2592*b^4*c^4)/5 + (4608*b^6*c^2)/5))/c^{13} + ((1024*b^{10})/7 + (24*c^{10})/35 + (136*b^2*c^8)/7 + (4496*b^4*c^6)/35 + (11776*b^6*c^4)/35 + (13056*b^8*c^2)/35)/c^{14} + \tan(d/2 + (e*x)/2)^2*((768*b^8 + (42*c^8)/5 + (1116*b^2*c^6)/5 + (5136*b^4*c^4)/5 + (7872*b^6*c^2)/5)/c^{12} + ((b^2 + c^2)^(1/2)*(60*b*c^6 + 768*b^7 + 528*b^3*c^4 + (5952*b^5*c^2)/5))/c^{12} + \tan(d/2 + (e*x)/2)^3*((80*b*c^6 + 640*b^7 + 600*b^3*c^4 + 1152*b^5*c^2)/c^{11} + ((b^2 + c^2)^(1/2)*(640*b^6 + 12*c^6 + 264*b^2*c^4 + 832*b^4*c^2))/c^{11} + \tan(d/2 + (e*x)/2)^4*((320*b^6 + 12*c^6 + 196*b^2*c^4 + 496*b^4*c^2)/c^{10} + ((b^2 + c^2)^(1/2)*(68*b*c^4 + 320*b^5 + 336*b^3*c^2))/c^{10} + \tan(d/2 + (e*x)/2)^5*((30*b*c^4 + 96*b^5 + 120*b^3*c^2)/c^9 + ((b^2 + c^2)^(1/2)*(96*b^4 + 6*c^4 + 72*b^2*c^2))/c^9 + ((b^2 + c^2)^(1/2)*((24*b*c^8)/5 + (1024*b^9)/7 + (272*b^3*c^6)/5 + (1024*b^5*c^4)/5 + (10496*b^7*c^2)/35))/c^{14} + (e*(\tan(d/2 + (e*x)/2))^3*((280*b^4 + 35*c^4 + 280*b^2*c^2)/c^4 + ((140*b*c^2 + 280*b^3)*(b^2 + c^2)^(1/2))/c^4) + \tan(d/2 + (e*x)/2)^4*((105*b*c^2 + 140*b^3)/c^3 + ((140*b^2 + 35*c^2)*(b^2 + c^2)^(1/2))/c^3) + \tan(d/2 + (e*x)/2)^7 + \tan(d/2 + (e*x)/2)*((224*b^6 + 7*c^6 + 126*b^2*c^4 + 336*b^4*c^2)/c^6 + ((b^2 + c^2)^(1/2)*(42*b*c^4 + 224*b^5 + 224*b^3*c^2))/c^6) + \tan(d/2 + (e*x)/2)^5*((42*b^2 + 21*c^2)/c^2 + (42*b*(b^2 + c^2)^(1/2))/c^2) + \tan(d/2 + (e*x)/2)^6*((7*(b^2 + c^2)^(1/2))/c + (7*b)/c) + \tan(d/2 + (e*x)/2)^2*((105*b*c^4 + 336*b^5 + 420*b^3*c^2)/c^5 + ((b^2 + c^2)^(1/2)*(336*b^4 + 21*c^4 + 252*b^2*c^2))/c^5) + (7*b*c^6 + 64*$

$$\frac{b^7 + 56b^3c^4 + 112b^5c^2}{c^7} + \frac{((b^2 + c^2)^{1/2})(64b^6 + c^6 + 24b^2c^4 + 80b^4c^2)}{c^7}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**4,x)

[Out] Timed out

3.363 $\int (2a + 2a \cos(d + ex) + 2c \sin(d + ex))^3 dx$

Optimal. Leaf size=157

$$\frac{4a(15a^2 + 4c^2) \sin(d + ex)}{3e} - \frac{4c(15a^2 + 4c^2) \cos(d + ex)}{3e} + 4ax(5a^2 + 3c^2) - \frac{20(ac \cos(d + ex) - a^2 \sin(d + ex))}{3e}$$

[Out] $4*a*(5*a^2+3*c^2)*x-4/3*c*(15*a^2+4*c^2)*\cos(e*x+d)/e+4/3*a*(15*a^2+4*c^2)*\sin(e*x+d)/e-20/3*(a*c*\cos(e*x+d)-a^2*\sin(e*x+d))*(a+a*\cos(e*x+d)+c*\sin(e*x+d))/e-8/3*(c*\cos(e*x+d)-a*\sin(e*x+d))*(a+a*\cos(e*x+d)+c*\sin(e*x+d))^2/e$

Rubi [A] time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3120, 3146, 2637, 2638}

$$\frac{4a(15a^2 + 4c^2) \sin(d + ex)}{3e} - \frac{4c(15a^2 + 4c^2) \cos(d + ex)}{3e} + 4ax(5a^2 + 3c^2) - \frac{20(ac \cos(d + ex) - a^2 \sin(d + ex))}{3e}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a + 2*a*\text{Cos}[d + e*x] + 2*c*\text{Sin}[d + e*x])^3, x]$

[Out] $4*a*(5*a^2 + 3*c^2)*x - (4*c*(15*a^2 + 4*c^2)*\text{Cos}[d + e*x])/(3*e) + (4*a*(15*a^2 + 4*c^2)*\text{Sin}[d + e*x])/(3*e) - (20*(a*c*\text{Cos}[d + e*x] - a^2*\text{Sin}[d + e*x])*(a + a*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]))/(3*e) - (8*(c*\text{Cos}[d + e*x] - a*\text{Sin}[d + e*x])*(a + a*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^2)/(3*e)$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3120

$\text{Int}[(\cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*\sin[(d_.) + (e_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(c*\text{Cos}[d + e*x] - b*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{(n-1)}]/(e*n), x] + \text{Dist}[1/n, \text{Int}[\text{Simp}[n*a^2 + (n-1)*(b^2 + c^2) + a*b*(2*n-1)*\text{Cos}[d + e*x] + a*c*(2*n-1)*\text{Sin}[d + e*x], x]*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{(n-2)}, x], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]

Rule 3146

$\text{Int}[(\cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*\sin[(d_.) + (e_.)*(x_.)])^{(n_.)}*((A_.) + \cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*\sin[(d_.) + (e_.)*(x_.)]), x_Symbol] \rightarrow \text{Simp}[(B*c - b*C - a*C*\text{Cos}[d + e*x] + a*B*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^n]/(a*e*(n+1)), x] + \text{Dist}[1/(a*(n+1)), \text{Int}[(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{(n-1)}*\text{Simp}[a*(b*B + c*C)*n + a^2*A*(n+1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n+1))*\text{Cos}[d + e*x] + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n+1))*\text{Sin}[d + e*x], x], x], x] /;$ FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\begin{aligned} \int (2a + 2a \cos(d + ex) + 2c \sin(d + ex))^3 dx &= -\frac{8(c \cos(d + ex) - a \sin(d + ex))(a + a \cos(d + ex) + c \sin(d + ex))}{3e} \\ &= -\frac{20(ac \cos(d + ex) - a^2 \sin(d + ex))(a + a \cos(d + ex) + c \sin(d + ex))}{3e} \\ &= 4a(5a^2 + 3c^2)x - \frac{20(ac \cos(d + ex) - a^2 \sin(d + ex))(a + a \cos(d + ex) + c \sin(d + ex))}{3e} \\ &= 4a(5a^2 + 3c^2)x - \frac{4c(15a^2 + 4c^2) \cos(d + ex)}{3e} + \frac{4a(15a^2 + 4c^2) \sin(d + ex)}{3e} \end{aligned}$$

Mathematica [A] time = 0.41, size = 135, normalized size = 0.86

$$\frac{2(6a(5a^2 + 3c^2)(d + ex) + 9a(5a^2 + c^2)\sin(d + ex) + 9a(a^2 - c^2)\sin(2(d + ex)) + a(a^2 - 3c^2)\sin(3(d + ex)))}{3e}$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]

[Out] (2*(6*a*(5*a^2 + 3*c^2)*(d + e*x) - 9*c*(5*a^2 + c^2)*Cos[d + e*x] - 18*a^2*c*Cos[2*(d + e*x)] + c*(-3*a^2 + c^2)*Cos[3*(d + e*x)] + 9*a*(5*a^2 + c^2)*Sin[d + e*x] + 9*a*(a^2 - c^2)*Sin[2*(d + e*x)] + a*(a^2 - 3*c^2)*Sin[3*(d + e*x)])/(3*e)

fricas [A] time = 1.99, size = 134, normalized size = 0.85

$$\frac{4(18a^2c \cos(ex + d)^2 + 2(3a^2c - c^3) \cos(ex + d)^3 - 3(5a^3 + 3ac^2)ex + 6(3a^2c + c^3) \cos(ex + d) - (22a^3 - 3ac^2))}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")

[Out] -4/3*(18*a^2*c*cos(e*x + d)^2 + 2*(3*a^2*c - c^3)*cos(e*x + d)^3 - 3*(5*a^3 + 3*a*c^2)*e*x + 6*(3*a^2*c + c^3)*cos(e*x + d) - (22*a^3 + 6*a*c^2 + 2*(a^3 - 3*a*c^2)*cos(e*x + d)^2 + 9*(a^3 - a*c^2)*cos(e*x + d))*sin(e*x + d))/e

giac [A] time = 0.18, size = 151, normalized size = 0.96

$$-12a^2c \cos(2xe + 2d)e^{(-1)} - \frac{2}{3}(3a^2c - c^3) \cos(3xe + 3d)e^{(-1)} - 6(5a^2c + c^3) \cos(xe + d)e^{(-1)} + \frac{2}{3}(a^3 - 3ac^2) \sin(3xe + 3d)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")

[Out] -12*a^2*c*cos(2*x*e + 2*d)*e^(-1) - 2/3*(3*a^2*c - c^3)*cos(3*x*e + 3*d)*e^(-1) - 6*(5*a^2*c + c^3)*cos(x*e + d)*e^(-1) + 2/3*(a^3 - 3*a*c^2)*e^(-1)*sin(3*x*e + 3*d) + 6*(a^3 - a*c^2)*e^(-1)*sin(2*x*e + 2*d) + 6*(5*a^3 + a*c^2)*e^(-1)*sin(x*e + d) + 4*(5*a^3 + 3*a*c^2)*x

maple [A] time = 0.26, size = 177, normalized size = 1.13

$$\frac{8a^3(ex + d) + 24a^3 \sin(ex + d) - 24a^2c \cos(ex + d) + 24a^3 \left(\frac{\sin(ex+d)\cos(ex+d)}{2} + \frac{ex}{2} + \frac{d}{2} \right) - 24a^2c (\cos^2(ex + d))}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x)

[Out] $\frac{8}{e} (a^3(e*x+d) + 3a^3\sin(e*x+d) - 3a^2c\cos(e*x+d) + 3a^3(1/2\sin(e*x+d)\cos(e*x+d) + 1/2e*x + 1/2d) - 3a^2c\cos(e*x+d)^2 + 3a^2c^2(-1/2\sin(e*x+d)\cos(e*x+d) + 1/2e*x + 1/2d) + 1/3a^3(2 + \cos(e*x+d)^2)\sin(e*x+d) - a^2c\cos(e*x+d)^3 + a^2c^2\sin(e*x+d)^3 - 1/3c^3(2 + \sin(e*x+d)^2)\cos(e*x+d))$

maxima [A] time = 0.32, size = 191, normalized size = 1.22

$$-\frac{8a^2c\cos(ex+d)^3}{e} + \frac{8ac^2\sin(ex+d)^3}{e} + 8a^3x - \frac{8(\sin(ex+d)^3 - 3\sin(ex+d))a^3}{3e} + \frac{8(\cos(ex+d)^3 - 3\cos(ex+d))a^3}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")

[Out] $-8a^2c\cos(e*x+d)^3/e + 8a^2c^2\sin(e*x+d)^3/e + 8a^3x - 8/3(\sin(e*x+d)^3 - 3\sin(e*x+d))a^3/e + 8/3(\cos(e*x+d)^3 - 3\cos(e*x+d))a^3/e - 24a^2(c\cos(e*x+d)/e - a\sin(e*x+d)/e) - 6(4a^2c\cos(e*x+d)^2/e - (2e*x + 2d + \sin(2e*x + 2d))a^2/e - (2e*x + 2d - \sin(2e*x + 2d))c^2/e)a$

mupad [B] time = 2.57, size = 239, normalized size = 1.52

$$20a^3x - \frac{32c^3\cos\left(\frac{d}{2} + \frac{ex}{2}\right)^4}{e} + \frac{64c^3\cos\left(\frac{d}{2} + \frac{ex}{2}\right)^6}{3e} + 12a^2c^2x - \frac{64a^2c\cos\left(\frac{d}{2} + \frac{ex}{2}\right)^6}{e} + \frac{40a^3\cos\left(\frac{d}{2} + \frac{ex}{2}\right)\sin\left(\frac{d}{2} + \frac{ex}{2}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^3,x)

[Out] $20a^3x - (32c^3\cos(d/2 + (e*x)/2)^4)/e + (64c^3\cos(d/2 + (e*x)/2)^6)/(3e) + 12a^2c^2x - (64a^2c\cos(d/2 + (e*x)/2)^6)/e + (40a^3\cos(d/2 + (e*x)/2)\sin(d/2 + (e*x)/2))/e + (80a^3\cos(d/2 + (e*x)/2)^3\sin(d/2 + (e*x)/2))/(3e) + (64a^3\cos(d/2 + (e*x)/2)^5\sin(d/2 + (e*x)/2))/(3e) + (16a^2c^2\cos(d/2 + (e*x)/2)^3\sin(d/2 + (e*x)/2))/e - (64a^2c^2\cos(d/2 + (e*x)/2)^5\sin(d/2 + (e*x)/2))/e + (24a^2c^2\cos(d/2 + (e*x)/2)\sin(d/2 + (e*x)/2))/e$

sympy [A] time = 0.76, size = 291, normalized size = 1.85

$$\begin{cases} 12a^3x\sin^2(d+ex) + 12a^3x\cos^2(d+ex) + 8a^3x + \frac{16a^3\sin^3(d+ex)}{3e} + \frac{8a^3\sin(d+ex)\cos^2(d+ex)}{e} + \frac{12a^3\sin(d+ex)\cos(d+ex)}{e} + \\ x(2a\cos(d) + 2a + 2c\sin(d))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)

[Out] Piecewise((12a**3*x*sin(d + e*x)**2 + 12a**3*x*cos(d + e*x)**2 + 8a**3*x + 16a**3*sin(d + e*x)**3/(3*e) + 8a**3*sin(d + e*x)*cos(d + e*x)**2/e + 12a**3*sin(d + e*x)*cos(d + e*x)/e + 24a**3*sin(d + e*x)/e - 8a**2*c*cos(d + e*x)**3/e - 24a**2*c*cos(d + e*x)**2/e - 24a**2*c*cos(d + e*x)/e + 12a**2*c**2*x*sin(d + e*x)**2 + 12a*c**2*x*cos(d + e*x)**2 + 8a*c**2*sin(d + e*x)**3/e - 12a*c**2*sin(d + e*x)*cos(d + e*x)/e - 8c**3*sin(d + e*x)**2*cos(d + e*x)/e - 16c**3*cos(d + e*x)**3/(3*e), Ne(e, 0)), (x*(2*a*cos(d) + 2*a + 2*c*sin(d))**3, True))

3.364 $\int (2a + 2a \cos(d + ex) + 2c \sin(d + ex))^2 dx$

Optimal. Leaf size=81

$$2x(3a^2 + c^2) + \frac{6a^2 \sin(d + ex)}{e} - \frac{6ac \cos(d + ex)}{e} - \frac{2(c \cos(d + ex) - a \sin(d + ex))(a \cos(d + ex) + a + c \sin(d + ex))}{e}$$

[Out] $2*(3*a^2+c^2)*x-6*a*c*\cos(e*x+d)/e+6*a^2*\sin(e*x+d)/e-2*(c*\cos(e*x+d)-a*\sin(e*x+d))*(a+a*\cos(e*x+d)+c*\sin(e*x+d))/e$

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3120, 2637, 2638}

$$2x(3a^2 + c^2) + \frac{6a^2 \sin(d + ex)}{e} - \frac{6ac \cos(d + ex)}{e} - \frac{2(c \cos(d + ex) - a \sin(d + ex))(a \cos(d + ex) + a + c \sin(d + ex))}{e}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a + 2*a*\text{Cos}[d + e*x] + 2*c*\text{Sin}[d + e*x])^2, x]$

[Out] $2*(3*a^2 + c^2)*x - (6*a*c*\text{Cos}[d + e*x])/e + (6*a^2*\text{Sin}[d + e*x])/e - (2*(c*\text{Cos}[d + e*x] - a*\text{Sin}[d + e*x])*(a + a*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]))/e$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3120

$\text{Int}[(\cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*\sin[(d_.) + (e_.)*(x_.)])^n, x_Symbol] \rightarrow -\text{Simp}[(c*\text{Cos}[d + e*x] - b*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{n-1}/(e*n), x] + \text{Dist}[1/n, \text{Int}[\text{Simp}[n*a^2 + (n-1)*(b^2 + c^2) + a*b*(2*n-1)*\text{Cos}[d + e*x] + a*c*(2*n-1)*\text{Sin}[d + e*x], x]*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{n-2}, x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int (2a + 2a \cos(d + ex) + 2c \sin(d + ex))^2 dx &= -\frac{2(c \cos(d + ex) - a \sin(d + ex))(a + a \cos(d + ex) + c \sin(d + ex))}{e} \\ &= 2(3a^2 + c^2)x - \frac{2(c \cos(d + ex) - a \sin(d + ex))(a + a \cos(d + ex) + c \sin(d + ex))}{e} \\ &= 2(3a^2 + c^2)x - \frac{6ac \cos(d + ex)}{e} + \frac{6a^2 \sin(d + ex)}{e} - \frac{2(c \cos(d + ex) - a \sin(d + ex))(a + a \cos(d + ex) + c \sin(d + ex))}{e} \end{aligned}$$

Mathematica [A] time = 0.14, size = 92, normalized size = 1.14

$$4 \left(\frac{(3a^2 + c^2)(d + ex)}{2e} + \frac{(a^2 - c^2) \sin(2(d + ex))}{4e} + \frac{2a^2 \sin(d + ex)}{e} - \frac{2ac \cos(d + ex)}{e} - \frac{ac \cos(2(d + ex))}{2e} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*a*cos[d + e*x] + 2*c*sin[d + e*x])^2,x]

[Out] 4*((((3*a^2 + c^2)*(d + e*x))/(2*e) - (2*a*c*cos[d + e*x])/e - (a*c*cos[2*(d + e*x)]))/(2*e) + (2*a^2*sin[d + e*x])/e + ((a^2 - c^2)*sin[2*(d + e*x)])/(4*e))

fricas [A] time = 1.42, size = 71, normalized size = 0.88

$$\frac{2 \left(2 a c \cos (e x + d)^2 - \left(3 a^2 + c^2 \right) e x + 4 a c \cos (e x + d) - \left(4 a^2 + \left(a^2 - c^2 \right) \cos (e x + d) \right) \sin (e x + d) \right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")

[Out] -2*(2*a*c*cos(e*x + d)^2 - (3*a^2 + c^2)*e*x + 4*a*c*cos(e*x + d) - (4*a^2 + (a^2 - c^2)*cos(e*x + d))*sin(e*x + d))/e

giac [A] time = 0.16, size = 78, normalized size = 0.96

$$-2 a c \cos (2 x e + 2 d) e^{(-1)} - 8 a c \cos (x e + d) e^{(-1)} + 8 a^2 e^{(-1)} \sin (x e + d) + \left(a^2 - c^2 \right) e^{(-1)} \sin (2 x e + 2 d) + 2 \left(3 a^2 + c^2 \right) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")

[Out] -2*a*c*cos(2*x*e + 2*d)*e^(-1) - 8*a*c*cos(x*e + d)*e^(-1) + 8*a^2*e^(-1)*sin(x*e + d) + (a^2 - c^2)*e^(-1)*sin(2*x*e + 2*d) + 2*(3*a^2 + c^2)*x

maple [A] time = 0.22, size = 101, normalized size = 1.25

$$\frac{4 a^2 (e x + d) + 8 a^2 \sin (e x + d) - 8 a c \cos (e x + d) + 4 a^2 \left(\frac{\sin (e x + d) \cos (e x + d)}{2} + \frac{e x}{2} + \frac{d}{2} \right) - 4 a c \left(\cos^2 (e x + d) \right) + 4 c^2 \left(-\frac{1}{2} \right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x)

[Out] 4/e*(a^2*(e*x+d)+2*a^2*sin(e*x+d)-2*a*c*cos(e*x+d)+a^2*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-a*c*cos(e*x+d)^2+c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d))

maxima [A] time = 0.30, size = 99, normalized size = 1.22

$$4 a^2 x - \frac{4 a c \cos (e x + d)^2}{e} + \frac{(2 e x + 2 d + \sin (2 e x + 2 d)) a^2}{e} + \frac{(2 e x + 2 d - \sin (2 e x + 2 d)) c^2}{e} - 8 a \left(\frac{c \cos (e x + d)}{e} - \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")

[Out] 4*a^2*x - 4*a*c*cos(e*x + d)^2/e + (2*e*x + 2*d + sin(2*e*x + 2*d))*a^2/e + (2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e - 8*a*(c*cos(e*x + d)/e - a*sin(e*x + d)/e)

mupad [B] time = 3.21, size = 96, normalized size = 1.19

$$\frac{x \left(12 a^2 + 4 c^2 \right)}{2} + \frac{\left(12 a^2 + 4 c^2 \right) \tan \left(\frac{d}{2} + \frac{e x}{2} \right)^3 + \left(20 a^2 - 4 c^2 \right) \tan \left(\frac{d}{2} + \frac{e x}{2} \right) - 16 a c}{e \left(\tan \left(\frac{d}{2} + \frac{e x}{2} \right)^4 + 2 \tan \left(\frac{d}{2} + \frac{e x}{2} \right)^2 + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^2,x)

[Out] (x*(12*a^2 + 4*c^2))/2 + (tan(d/2 + (e*x)/2)^3*(12*a^2 + 4*c^2) - 16*a*c + tan(d/2 + (e*x)/2)*(20*a^2 - 4*c^2))/(e*(2*tan(d/2 + (e*x)/2)^2 + tan(d/2 + (e*x)/2)^4 + 1))

sympy [A] time = 0.31, size = 170, normalized size = 2.10

$$\begin{cases} 2a^2x \sin^2(d + ex) + 2a^2x \cos^2(d + ex) + 4a^2x + \frac{2a^2 \sin(d+ex) \cos(d+ex)}{e} + \frac{8a^2 \sin(d+ex)}{e} - \frac{4ac \cos^2(d+ex)}{e} - \frac{8ac \cos(d+ex)}{e} \\ x(2a \cos(d) + 2a + 2c \sin(d))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x)

[Out] Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x + 2*a**2*sin(d + e*x)*cos(d + e*x)/e + 8*a**2*sin(d + e*x)/e - 4*a*c*cos(d + e*x)**2/e - 8*a*c*cos(d + e*x)/e + 2*c**2*x*sin(d + e*x)**2 + 2*c**2*x*cos(d + e*x)**2 - 2*c**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(2*a*cos(d) + 2*a + 2*c*sin(d))**2, True))

3.365 $\int (2a + 2a \cos(d + ex) + 2c \sin(d + ex)) dx$

Optimal. Leaf size=29

$$\frac{2a \sin(d + ex)}{e} + 2ax - \frac{2c \cos(d + ex)}{e}$$

[Out] $2*a*x - 2*c*\cos(e*x+d)/e + 2*a*\sin(e*x+d)/e$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2637, 2638}

$$\frac{2a \sin(d + ex)}{e} + 2ax - \frac{2c \cos(d + ex)}{e}$$

Antiderivative was successfully verified.

[In] `Int[2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x], x]`

[Out] $2*a*x - (2*c*\cos[d + e*x])/e + (2*a*\sin[d + e*x])/e$

Rule 2637

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 2638

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \int (2a + 2a \cos(d + ex) + 2c \sin(d + ex)) dx &= 2ax + (2a) \int \cos(d + ex) dx + (2c) \int \sin(d + ex) dx \\ &= 2ax - \frac{2c \cos(d + ex)}{e} + \frac{2a \sin(d + ex)}{e} \end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.83

$$\frac{2a \sin(d) \cos(ex)}{e} + \frac{2a \cos(d) \sin(ex)}{e} + 2ax + \frac{2c \sin(d) \sin(ex)}{e} - \frac{2c \cos(d) \cos(ex)}{e}$$

Antiderivative was successfully verified.

[In] `Integrate[2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x], x]`

[Out] $2*a*x - (2*c*\cos[d]*\cos[e*x])/e + (2*a*\cos[e*x]*\sin[d])/e + (2*a*\cos[d]*\sin[e*x])/e + (2*c*\sin[d]*\sin[e*x])/e$

fricas [A] time = 0.88, size = 27, normalized size = 0.93

$$\frac{2(aex - c \cos(ex + d) + a \sin(ex + d))}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="fricas")`

[Out] $2*(a*e^x - c*\cos(e*x + d) + a*\sin(e*x + d))/e$

giac [A] time = 0.14, size = 29, normalized size = 1.00

$$-2c \cos(xe + d)e^{(-1)} + 2ae^{(-1)} \sin(xe + d) + 2ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="giac")`

[Out] $-2*c*\cos(x*e + d)*e^{(-1)} + 2*a*e^{(-1)}*\sin(x*e + d) + 2*a*x$

maple [A] time = 0.00, size = 30, normalized size = 1.03

$$2ax - \frac{2c \cos(ex + d)}{e} + \frac{2a \sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x)`

[Out] $2*a*x - 2*c*\cos(e*x + d)/e + 2*a*\sin(e*x + d)/e$

maxima [A] time = 0.30, size = 29, normalized size = 1.00

$$2ax - \frac{2c \cos(ex + d)}{e} + \frac{2a \sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="maxima")`

[Out] $2*a*x - 2*c*\cos(e*x + d)/e + 2*a*\sin(e*x + d)/e$

mupad [B] time = 2.43, size = 29, normalized size = 1.00

$$2ax - \frac{2c \cos(d + ex)}{e} + \frac{2a \sin(d + ex)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x),x)`

[Out] $2*a*x - (2*c*\cos(d + e*x))/e + (2*a*\sin(d + e*x))/e$

sympy [A] time = 0.14, size = 39, normalized size = 1.34

$$2ax + 2a \left(\begin{cases} \frac{\sin(d+ex)}{e} & \text{for } e \neq 0 \\ x \cos(d) & \text{otherwise} \end{cases} \right) + 2c \left(\begin{cases} -\frac{\cos(d+ex)}{e} & \text{for } e \neq 0 \\ x \sin(d) & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x)`

[Out] $2*a*x + 2*a*\text{Piecewise}((\sin(d + e*x)/e, \text{Ne}(e, 0)), (x*\cos(d), \text{True})) + 2*c*\text{Piecewise}((-\cos(d + e*x)/e, \text{Ne}(e, 0)), (x*\sin(d), \text{True}))$

$$3.366 \quad \int \frac{1}{2a+2a \cos(d+ex)+2c \sin(d+ex)} dx$$

Optimal. Leaf size=25

$$\frac{\log\left(a + c \tan\left(\frac{1}{2}(d + ex)\right)\right)}{2ce}$$

[Out] 1/2*ln(a+c*tan(1/2*e*x+1/2*d))/c/e

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3124, 31}

$$\frac{\log\left(a + c \tan\left(\frac{1}{2}(d + ex)\right)\right)}{2ce}$$

Antiderivative was successfully verified.

[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1), x]

[Out] Log[a + c*Tan[(d + e*x)/2]]/(2*c*e)

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^-1, x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\int \frac{1}{2a + 2a \cos(d + ex) + 2c \sin(d + ex)} dx = \frac{2 \text{Subst}\left(\int \frac{1}{4a+4cx} dx, x, \tan\left(\frac{1}{2}(d + ex)\right)\right)}{e} = \frac{\log\left(a + c \tan\left(\frac{1}{2}(d + ex)\right)\right)}{2ce}$$

Mathematica [B] time = 0.05, size = 57, normalized size = 2.28

$$\frac{1}{2} \left(\frac{\log\left(a \cos\left(\frac{1}{2}(d + ex)\right) + c \sin\left(\frac{1}{2}(d + ex)\right)\right)}{ce} - \frac{\log\left(\cos\left(\frac{1}{2}(d + ex)\right)\right)}{ce} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1), x]

[Out] -(Log[Cos[(d + e*x)/2]]/(c*e)) + Log[a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2]]/(c*e))/2

fricas [B] time = 1.34, size = 60, normalized size = 2.40

$$\frac{\log\left(ac \sin(ex + d) + \frac{1}{2}a^2 + \frac{1}{2}c^2 + \frac{1}{2}(a^2 - c^2)\cos(ex + d)\right) - \log\left(\frac{1}{2}\cos(ex + d) + \frac{1}{2}\right)}{4ce}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="fricas")

[Out] 1/4*(log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 + 1/2*(a^2 - c^2)*cos(e*x + d)) - log(1/2*cos(e*x + d) + 1/2))/(c*e)

giac [A] time = 0.13, size = 23, normalized size = 0.92

$$\frac{e^{(-1)} \log\left(\left|c \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + a\right|\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="giac")

[Out] 1/2*e^(-1)*log(abs(c*tan(1/2*x*e + 1/2*d) + a))/c

maple [A] time = 0.38, size = 23, normalized size = 0.92

$$\frac{\ln\left(a + c \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{2ce}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x)

[Out] 1/2*ln(a+c*tan(1/2*d+1/2*e*x))/c/e

maxima [A] time = 0.31, size = 29, normalized size = 1.16

$$\frac{\log\left(a + \frac{c \sin(ex+d)}{\cos(ex+d)+1}\right)}{2ce}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="maxima")

[Out] 1/2*log(a + c*sin(e*x + d)/(cos(e*x + d) + 1))/(c*e)

mupad [B] time = 2.82, size = 22, normalized size = 0.88

$$\frac{\ln\left(a + c \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{2ce}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x)),x)

[Out] log(a + c*tan(d/2 + (e*x)/2))/(2*c*e)

sympy [A] time = 1.10, size = 63, normalized size = 2.52

$$\left\{ \begin{array}{ll} \frac{x}{2a \cos(d) + 2a} & \text{for } c = 0 \wedge e = 0 \\ \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{2ae} & \text{for } c = 0 \\ \frac{x}{2a \cos(d) + 2a + 2c \sin(d)} & \text{for } e = 0 \\ \frac{\log\left(\frac{a}{c} + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{2ce} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x)

[Out] Piecewise((x/(2*a*cos(d) + 2*a), Eq(c, 0) & Eq(e, 0)), (tan(d/2 + e*x/2)/(2*a*e), Eq(c, 0)), (x/(2*a*cos(d) + 2*a + 2*c*sin(d)), Eq(e, 0)), (log(a/c + tan(d/2 + e*x/2))/(2*c*e), True))

$$3.367 \quad \int \frac{1}{(2a+2a \cos(d+ex)+2c \sin(d+ex))^2} dx$$

Optimal. Leaf size=75

$$-\frac{a \log\left(a + c \tan\left(\frac{1}{2}(d + ex)\right)\right)}{4c^3e} - \frac{c \cos(d + ex) - a \sin(d + ex)}{4c^2e(a \cos(d + ex) + a + c \sin(d + ex))}$$

[Out] -1/4*a*ln(a+c*tan(1/2*e*x+1/2*d))/c^3/e+1/4*(-c*cos(e*x+d)+a*sin(e*x+d))/c^2/e/(a+a*cos(e*x+d)+c*sin(e*x+d))

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3129, 12, 3124, 31}

$$-\frac{a \log\left(a + c \tan\left(\frac{1}{2}(d + ex)\right)\right)}{4c^3e} - \frac{c \cos(d + ex) - a \sin(d + ex)}{4c^2e(a \cos(d + ex) + a + c \sin(d + ex))}$$

Antiderivative was successfully verified.

[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]

[Out] -(a*Log[a + c*Tan[(d + e*x)/2]])/(4*c^3*e) - (c*Cos[d + e*x] - a*Sin[d + e*x])/(4*c^2*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3129

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Simp[((-c*Cos[d + e*x] + b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2a + 2a \cos(d + ex) + 2c \sin(d + ex))^2} dx &= -\frac{c \cos(d + ex) - a \sin(d + ex)}{4c^2 e (a + a \cos(d + ex) + c \sin(d + ex))} + \frac{\int -\frac{2a}{2a + 2a \cos(d + ex) + 2c \sin(d + ex)} dx}{4c^2} \\
&= -\frac{c \cos(d + ex) - a \sin(d + ex)}{4c^2 e (a + a \cos(d + ex) + c \sin(d + ex))} - \frac{a \int \frac{1}{2a + 2a \cos(d + ex) + 2c \sin(d + ex)} dx}{2c^2} \\
&= -\frac{c \cos(d + ex) - a \sin(d + ex)}{4c^2 e (a + a \cos(d + ex) + c \sin(d + ex))} - \frac{a \operatorname{Subst}\left(\int \frac{1}{4a + 4cx} dx, \frac{1}{2}(d + ex)\right)}{c^2 e} \\
&= -\frac{a \log\left(a + c \tan\left(\frac{1}{2}(d + ex)\right)\right)}{4c^3 e} - \frac{c \cos(d + ex) - a \sin(d + ex)}{4c^2 e (a + a \cos(d + ex) + c \sin(d + ex))}
\end{aligned}$$

Mathematica [A] time = 0.55, size = 115, normalized size = 1.53

$$\frac{c(a^2 + c^2) \sin\left(\frac{1}{2}(d + ex)\right)}{a\left(a \cos\left(\frac{1}{2}(d + ex)\right) + c \sin\left(\frac{1}{2}(d + ex)\right)\right)} + 2a \left(\log\left(\cos\left(\frac{1}{2}(d + ex)\right)\right) - \log\left(a \cos\left(\frac{1}{2}(d + ex)\right) + c \sin\left(\frac{1}{2}(d + ex)\right)\right) \right) + c \tan\left(\frac{1}{2}(d + ex)\right)$$

$$8c^3 e$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2), x]

[Out] (2*a*(Log[Cos[(d + e*x)/2]] - Log[a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2]]) + (c*(a^2 + c^2)*Sin[(d + e*x)/2])/(a*(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2])) + c*Tan[(d + e*x)/2])/(8*c^3*e)

fricas [B] time = 0.99, size = 154, normalized size = 2.05

$$\frac{2c^2 \cos(ex + d) - 2ac \sin(ex + d) + (a^2 \cos(ex + d) + ac \sin(ex + d) + a^2) \log\left(ac \sin(ex + d) + \frac{1}{2}a^2 + \frac{1}{2}c^2 + \dots\right)}{8(ac^3 e \cos(ex + d) + c^4 e \sin(ex + d) + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")

[Out] -1/8*(2*c^2*cos(e*x + d) - 2*a*c*sin(e*x + d) + (a^2*cos(e*x + d) + a*c*sin(e*x + d) + a^2)*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 + 1/2*(a^2 - c^2)*cos(e*x + d)) - (a^2*cos(e*x + d) + a*c*sin(e*x + d) + a^2)*log(1/2*cos(e*x + d) + 1/2))/(a*c^3*e*cos(e*x + d) + c^4*e*sin(e*x + d) + a*c^3*e)

giac [A] time = 0.14, size = 86, normalized size = 1.15

$$-\frac{1}{8} \left(\frac{2a \log\left(\left|c \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + a\right|\right)}{c^3} - \frac{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)}{c^2} - \frac{2ac \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + a^2 - c^2}{\left(c \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + a\right)c^3} \right) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")

[Out] -1/8*(2*a*log(abs(c*tan(1/2*x*e + 1/2*d) + a))/c^3 - tan(1/2*x*e + 1/2*d)/c^2 - (2*a*c*tan(1/2*x*e + 1/2*d) + a^2 - c^2)/((c*tan(1/2*x*e + 1/2*d) + a)*c^3))*e^(-1)

maple [A] time = 0.49, size = 91, normalized size = 1.21

$$\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{8e c^2} - \frac{a^2}{8e c^3 \left(a + c \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)} - \frac{1}{8ec \left(a + c \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)} - \frac{a \ln\left(a + c \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{4c^3 e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x)

[Out] 1/8/e/c^2*tan(1/2*d+1/2*e*x)-1/8/e/c^3/(a+c*tan(1/2*d+1/2*e*x))*a^2-1/8/e/c/(a+c*tan(1/2*d+1/2*e*x))-1/4*a*ln(a+c*tan(1/2*d+1/2*e*x))/c^3/e

maxima [A] time = 0.32, size = 90, normalized size = 1.20

$$\frac{\frac{a^2+c^2}{ac^3+\frac{c^4\sin(ex+d)}{\cos(ex+d)+1}} + \frac{2a\log\left(a+\frac{c\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^3} - \frac{\sin(ex+d)}{c^2(\cos(ex+d)+1)}}{8e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")

[Out] -1/8*((a^2 + c^2)/(a*c^3 + c^4*sin(e*x + d)/(cos(e*x + d) + 1)) + 2*a*log(a + c*sin(e*x + d)/(cos(e*x + d) + 1))/c^3 - sin(e*x + d)/(c^2*(cos(e*x + d) + 1)))/e

mupad [B] time = 2.48, size = 79, normalized size = 1.05

$$\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{8c^2 e} - \frac{a \ln\left(a + c \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{4c^3 e} - \frac{a^2 + c^2}{ce \left(8 \tan\left(\frac{d}{2} + \frac{ex}{2}\right) c^3 + 8ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^2,x)

[Out] tan(d/2 + (e*x)/2)/(8*c^2*e) - (a*log(a + c*tan(d/2 + (e*x)/2)))/(4*c^3*e) - (a^2 + c^2)/(c*e*(8*a*c^2 + 8*c^3*tan(d/2 + (e*x)/2)))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x)

[Out] Timed out

$$3.368 \quad \int \frac{1}{(2a+2a \cos(d+ex)+2c \sin(d+ex))^3} dx$$

Optimal. Leaf size=134

$$\frac{3(ac \cos(d+ex) - a^2 \sin(d+ex))}{16c^4 e(a \cos(d+ex) + a + c \sin(d+ex))} + \frac{(3a^2 + c^2) \log\left(a + c \tan\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5 e} - \frac{c \cos(d+ex) - a \sin(d+ex)}{16c^2 e(a \cos(d+ex) + a + c \sin(d+ex))}$$

[Out] 1/16*(3*a^2+c^2)*ln(a+c*tan(1/2*e*x+1/2*d))/c^5/e+1/16*(-c*cos(e*x+d)+a*sin(e*x+d))/c^2/e/(a+a*cos(e*x+d)+c*sin(e*x+d))^2+3/16*(a*c*cos(e*x+d)-a^2*sin(e*x+d))/c^4/e/(a+a*cos(e*x+d)+c*sin(e*x+d))

Rubi [A] time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3129, 3153, 3124, 31}

$$\frac{(3a^2 + c^2) \log\left(a + c \tan\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5 e} + \frac{3(ac \cos(d+ex) - a^2 \sin(d+ex))}{16c^4 e(a \cos(d+ex) + a + c \sin(d+ex))} - \frac{c \cos(d+ex) - a \sin(d+ex)}{16c^2 e(a \cos(d+ex) + a + c \sin(d+ex))}$$

Antiderivative was successfully verified.

[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3), x]

[Out] ((3*a^2 + c^2)*Log[a + c*Tan[(d + e*x)/2]])/(16*c^5*e) - (c*Cos[d + e*x] - a*Sin[d + e*x])/(16*c^2*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^2) + (3*(a*c*Cos[d + e*x] - a^2*Sin[d + e*x]))/(16*c^4*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3129

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Simp[((-c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]

Rule 3153

Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(a_. + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] := Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2a + 2a \cos(d + ex) + 2c \sin(d + ex))^3} dx &= -\frac{c \cos(d + ex) - a \sin(d + ex)}{16c^2 e (a + a \cos(d + ex) + c \sin(d + ex))^2} + \frac{\int \frac{-4a + 2a \cos(d + ex)}{(2a + 2a \cos(d + ex) + 2c \sin(d + ex))^2} dx}{8c^2} \\
&= -\frac{c \cos(d + ex) - a \sin(d + ex)}{16c^2 e (a + a \cos(d + ex) + c \sin(d + ex))^2} + \frac{3(ac \cos(d + ex) - a^2 \sin(d + ex))}{16c^4 e (a + a \cos(d + ex) + c \sin(d + ex))} \\
&= -\frac{c \cos(d + ex) - a \sin(d + ex)}{16c^2 e (a + a \cos(d + ex) + c \sin(d + ex))^2} + \frac{3(ac \cos(d + ex) - a^2 \sin(d + ex))}{16c^4 e (a + a \cos(d + ex) + c \sin(d + ex))} \\
&= \frac{(3a^2 + c^2) \log\left(a + c \tan\left(\frac{1}{2}(d + ex)\right)\right)}{16c^5 e} - \frac{c \cos(d + ex) - a \sin(d + ex)}{16c^2 e (a + a \cos(d + ex) + c \sin(d + ex))^2}
\end{aligned}$$

Mathematica [A] time = 3.00, size = 186, normalized size = 1.39

$$\frac{4(3a^2 + c^2) \log\left(\cos\left(\frac{1}{2}(d + ex)\right)\right) + \frac{c^2(a^2 + c^2)}{\left(a \cos\left(\frac{1}{2}(d + ex)\right) + c \sin\left(\frac{1}{2}(d + ex)\right)\right)^2} + \frac{6c(a^2 + c^2) \sin\left(\frac{1}{2}(d + ex)\right)}{a \cos\left(\frac{1}{2}(d + ex)\right) + c \sin\left(\frac{1}{2}(d + ex)\right)} - 4(3a^2 + c^2) \log\left(\cos\left(\frac{1}{2}(d + ex)\right)\right)}{64c^5 e}$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3), x]

[Out] -1/64*(4*(3*a^2 + c^2)*Log[Cos[(d + e*x)/2]] - 4*(3*a^2 + c^2)*Log[a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2]] - c^2*Sec[(d + e*x)/2]^2 + (c^2*(a^2 + c^2))/(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2])^2 + (6*c*(a^2 + c^2)*Sin[(d + e*x)/2])/(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2]) + 6*a*c*Tan[(d + e*x)/2])/(c^5*e)

fricas [B] time = 0.95, size = 433, normalized size = 3.23

$$12 a^2 c^2 \cos(ex + d)^2 - 6 a^2 c^2 + 2(3 a^2 c^2 - c^4) \cos(ex + d) + (3 a^4 + 4 a^2 c^2 + c^4 + (3 a^4 - 2 a^2 c^2 - c^4) \cos(ex + d))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")

[Out] 1/32*(12*a^2*c^2*cos(e*x + d)^2 - 6*a^2*c^2 + 2*(3*a^2*c^2 - c^4)*cos(e*x + d) + (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 - c^4)*cos(e*x + d)^2 + 2*(3*a^4 + a^2*c^2)*cos(e*x + d) + 2*(3*a^3*c + a*c^3 + (3*a^3*c + a*c^3)*cos(e*x + d))*sin(e*x + d))*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 + 1/2*(a^2 - c^2)*cos(e*x + d)) - (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 - c^4)*cos(e*x + d)^2 + 2*(3*a^4 + a^2*c^2)*cos(e*x + d) + 2*(3*a^3*c + a*c^3 + (3*a^3*c + a*c^3)*cos(e*x + d))*sin(e*x + d))*log(1/2*cos(e*x + d) + 1/2 - 2*(3*a^3*c - a*c^3 + 3*(a^3*c - a*c^3)*cos(e*x + d))*sin(e*x + d))/(2*a^2*c^5*e*cos(e*x + d) + (a^2*c^5 - c^7)*e*cos(e*x + d)^2 + (a^2*c^5 + c^7)*e + 2*(a*c^6*e*cos(e*x + d) + a*c^6*e)*sin(e*x + d))

giac [A] time = 0.17, size = 171, normalized size = 1.28

$$\frac{1}{64} \left(\frac{4(3a^2 + c^2) \log\left(\left|c \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + a\right|\right)}{c^5} + \frac{c^3 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 - 6ac^2 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)}{c^6} - \frac{18a^2c^2 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)}{64c^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")

[Out] $\frac{1}{64} * (4 * (3 * a^2 + c^2) * \log(\text{abs}(c * \tan(1/2 * x * e + 1/2 * d) + a)) / c^5 + (c^3 * \tan(1/2 * x * e + 1/2 * d)^2 - 6 * a * c^2 * \tan(1/2 * x * e + 1/2 * d)) / c^6 - (18 * a^2 * c^2 * \tan(1/2 * x * e + 1/2 * d)^2 + 6 * c^4 * \tan(1/2 * x * e + 1/2 * d)^2 + 28 * a^3 * c * \tan(1/2 * x * e + 1/2 * d) + 4 * a * c^3 * \tan(1/2 * x * e + 1/2 * d) + 11 * a^4 + c^4) / ((c * \tan(1/2 * x * e + 1/2 * d) + a)^2 * c^5)) * e^{-1}$

maple [A] time = 0.56, size = 211, normalized size = 1.57

$$\frac{\tan^2\left(\frac{d}{2} + \frac{ex}{2}\right)}{64e c^3} - \frac{3 \tan\left(\frac{d}{2} + \frac{ex}{2}\right) a}{32e c^4} - \frac{a^4}{64e c^5 \left(a + c \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)^2} - \frac{a^2}{32e c^3 \left(a + c \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)^2} - \frac{1}{64ec \left(a + c \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x)

[Out] $\frac{1}{64} / e / c^3 * \tan(1/2 * d + 1/2 * e * x)^2 - 3/32 / e / c^4 * \tan(1/2 * d + 1/2 * e * x) * a - 1/64 / e / c^5 / (a + c * \tan(1/2 * d + 1/2 * e * x))^2 * a^4 - 1/32 / e / c^3 / (a + c * \tan(1/2 * d + 1/2 * e * x))^2 * a^2 - 1/64 / e / c / (a + c * \tan(1/2 * d + 1/2 * e * x))^2 + 1/8 / e * a^3 / c^5 / (a + c * \tan(1/2 * d + 1/2 * e * x)) + 1/8 / e * a / c^3 / (a + c * \tan(1/2 * d + 1/2 * e * x)) + 3/16 / e / c^5 * \ln(a + c * \tan(1/2 * d + 1/2 * e * x)) * a^2 + 1/16 / e / c^3 * \ln(a + c * \tan(1/2 * d + 1/2 * e * x))$

maxima [A] time = 0.34, size = 190, normalized size = 1.42

$$\frac{7a^4 + 6a^2c^2 - c^4 + \frac{8(a^3c + ac^3)\sin(ex+d)}{\cos(ex+d)+1}}{a^2c^5 + \frac{2ac^6\sin(ex+d)}{\cos(ex+d)+1} + \frac{c^7\sin(ex+d)^2}{(\cos(ex+d)+1)^2}} - \frac{\frac{6a\sin(ex+d)}{\cos(ex+d)+1} - \frac{c\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}{c^4} + \frac{4(3a^2+c^2)\log\left(a + \frac{c\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^5}$$

64 e

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")

[Out] $\frac{1}{64} * ((7 * a^4 + 6 * a^2 * c^2 - c^4 + 8 * (a^3 * c + a * c^3) * \sin(e * x + d)) / (\cos(e * x + d) + 1)) / (a^2 * c^5 + 2 * a * c^6 * \sin(e * x + d) / (\cos(e * x + d) + 1) + c^7 * \sin(e * x + d)^2 / (\cos(e * x + d) + 1)^2) - (6 * a * \sin(e * x + d) / (\cos(e * x + d) + 1) - c * \sin(e * x + d)^2 / (\cos(e * x + d) + 1)^2) / c^4 + 4 * (3 * a^2 + c^2) * \log(a + c * \sin(e * x + d) / (\cos(e * x + d) + 1)) / c^5 / e$

mupad [B] time = 2.48, size = 162, normalized size = 1.21

$$\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2}{64c^3e} + \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right) \left(4a^3 + 4ac^2\right) + \frac{7a^4 + 6a^2c^2 - c^4}{2c}}{e \left(32a^2c^4 + 64ac^5 \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 32c^6 \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2\right)} - \frac{3a \tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{32c^4e} + \frac{\ln\left(a + c \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{16c^5e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^3,x)

[Out] $\tan(d/2 + (e*x)/2)^2 / (64 * c^3 * e) + (\tan(d/2 + (e*x)/2) * (4 * a * c^2 + 4 * a^3) + (7 * a^4 - c^4 + 6 * a^2 * c^2) / (2 * c)) / (e * (32 * c^6 * \tan(d/2 + (e*x)/2)^2 + 32 * a^2 * c^4 + 64 * a * c^5 * \tan(d/2 + (e*x)/2))) - (3 * a * \tan(d/2 + (e*x)/2)) / (32 * c^4 * e) + (\log(a + c * \tan(d/2 + (e*x)/2)) * (3 * a^2 + c^2)) / (16 * c^5 * e)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

$$3.369 \quad \int \frac{1}{(2a+2a \cos(d+ex)+2c \sin(d+ex))^4} dx$$

Optimal. Leaf size=207

$$\frac{5(ac \cos(d+ex) - a^2 \sin(d+ex))}{96c^4 e (a \cos(d+ex) + a + c \sin(d+ex))^2} - \frac{a(5a^2 + 3c^2) \log\left(a + c \tan\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7 e} - \frac{c(15a^2 + 4c^2) \cos(d+ex)}{96c^6 e (a \cos(d+ex) + a + c \sin(d+ex))}$$

[Out] $-1/32*a*(5*a^2+3*c^2)*\ln(a+c*\tan(1/2*e*x+1/2*d))/c^7/e+1/48*(-c*\cos(e*x+d)+a*\sin(e*x+d))/c^2/e/(a+a*\cos(e*x+d)+c*\sin(e*x+d))^3+5/96*(a*c*\cos(e*x+d)-a^2*\sin(e*x+d))/c^4/e/(a+a*\cos(e*x+d)+c*\sin(e*x+d))^2+1/96*(-c*(15*a^2+4*c^2)*\cos(e*x+d)+a*(15*a^2+4*c^2)*\sin(e*x+d))/c^6/e/(a+a*\cos(e*x+d)+c*\sin(e*x+d))$

Rubi [A] time = 0.25, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3129, 3156, 3153, 3124, 31}

$$\frac{a(5a^2 + 3c^2) \log\left(a + c \tan\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7 e} - \frac{c(15a^2 + 4c^2) \cos(d+ex) - a(15a^2 + 4c^2) \sin(d+ex)}{96c^6 e (a \cos(d+ex) + a + c \sin(d+ex))} + \frac{5(ac \cos(d+ex) - a^2 \sin(d+ex))}{96c^4 e (a \cos(d+ex) + a + c \sin(d+ex))^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a + 2*a*\text{Cos}[d + e*x] + 2*c*\text{Sin}[d + e*x])^{-4}, x]$

[Out] $-(a*(5*a^2 + 3*c^2)*\text{Log}[a + c*\text{Tan}[(d + e*x)/2]])/(32*c^7*e) - (c*\text{Cos}[d + e*x] - a*\text{Sin}[d + e*x])/(48*c^2*e*(a + a*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^3) + (5*(a*c*\text{Cos}[d + e*x] - a^2*\text{Sin}[d + e*x]))/(96*c^4*e*(a + a*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^2) - (c*(15*a^2 + 4*c^2)*\text{Cos}[d + e*x] - a*(15*a^2 + 4*c^2)*\text{Sin}[d + e*x])/(96*c^6*e*(a + a*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]))$

Rule 31

$\text{Int}[(a + (b*x)^{-1}), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /;$ $\text{FreeQ}[\{a, b\}, x]$

Rule 3124

$\text{Int}[(\cos[(d + e*x)]*(b + a) + (c + e*x)*\sin[(d + e*x)]*(b + a))^{-1}, x_Symbol] \rightarrow \text{Module}[\{f = \text{FreeFactors}[\text{Tan}[(d + e*x)/2], x]\}, \text{Dist}[(2*f)/e, \text{Subst}[\text{Int}[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, \text{Tan}[(d + e*x)/2]/f], x] /;$ $\text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0]$

Rule 3129

$\text{Int}[(\cos[(d + e*x)]*(b + a) + (c + e*x)*\sin[(d + e*x)]*(b + a))^n, x_Symbol] \rightarrow \text{Simp}[(c*\text{Cos}[d + e*x] - b*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{n+1}/(e*(n+1)*(a^2 - b^2 - c^2)), x] + \text{Dist}[1/(e*(n+1)*(a^2 - b^2 - c^2)), \text{Int}[(a*(n+1) - b*(n+2)*\text{Cos}[d + e*x] - c*(n+2)*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{n+1}, x], x] /;$ $\text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{NeQ}[n, -3/2]$

Rule 3153

$\text{Int}[(A + \cos[(d + e*x)]*(B + C) + (C + e*x)*\sin[(d + e*x)]*(B + C))^2, x_Symbol] \rightarrow \text{Simp}[(c*B - b*C - (a*C - c*A)*\text{Cos}[d + e*x] + (a*B - b*A)*\text{Sin}[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])), x] + \text{Dist}[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), \text{Int}[1/(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]), x], x]$

$n[d + e*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, A, B, C\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0] \ \&\& \ \text{NeQ}[a*A - b*B - c*C, 0]$

Rule 3156

$\text{Int}[(a + \cos[d + e*x])*(b + c*\sin[d + e*x])^n * ((A + \cos[d + e*x])*(B + C*\sin[d + e*x]))], x_Symbol] \rightarrow -\text{Simp}[(c*B - b*C - (a*C - c*A)*\text{Cos}[d + e*x] + (a*B - b*A)*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{n+1}]/(e*(n+1)*(a^2 - b^2 - c^2)), x] + \text{Dist}[1/((n+1)*(a^2 - b^2 - c^2)), \text{Int}[(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{n+1}*\text{Simp}[(n+1)*(a*A - b*B - c*C) + (n+2)*(a*B - b*A)*\text{Cos}[d + e*x] + (n+2)*(a*C - c*A)*\text{Sin}[d + e*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, A, B, C\}, x] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0] \ \&\& \ \text{NeQ}[n, -2]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(2a + 2a \cos(d + ex) + 2c \sin(d + ex))^4} dx &= -\frac{c \cos(d + ex) - a \sin(d + ex)}{48c^2e(a + a \cos(d + ex) + c \sin(d + ex))^3} + \frac{\int \frac{-6a + 4a \cos(d + ex)}{(2a + 2a \cos(d + ex))} dx}{12c} \\ &= -\frac{c \cos(d + ex) - a \sin(d + ex)}{48c^2e(a + a \cos(d + ex) + c \sin(d + ex))^3} + \frac{5(ac \cos(d + ex) + a^2)}{96c^4e(a + a \cos(d + ex) + c \sin(d + ex))} \\ &= -\frac{c \cos(d + ex) - a \sin(d + ex)}{48c^2e(a + a \cos(d + ex) + c \sin(d + ex))^3} + \frac{5(ac \cos(d + ex) + a^2)}{96c^4e(a + a \cos(d + ex) + c \sin(d + ex))} \\ &= -\frac{c \cos(d + ex) - a \sin(d + ex)}{48c^2e(a + a \cos(d + ex) + c \sin(d + ex))^3} + \frac{5(ac \cos(d + ex) + a^2)}{96c^4e(a + a \cos(d + ex) + c \sin(d + ex))} \\ &= -\frac{a(5a^2 + 3c^2) \log\left(a + c \tan\left(\frac{1}{2}(d + ex)\right)\right)}{32c^7e} - \frac{c \cos(d + ex) - a \sin(d + ex)}{48c^2e(a + a \cos(d + ex) + c \sin(d + ex))} \end{aligned}$$

Mathematica [B] time = 1.73, size = 492, normalized size = 2.38

$$\cos\left(\frac{1}{2}(d + ex)\right) \left(a \cos\left(\frac{1}{2}(d + ex)\right) + c \sin\left(\frac{1}{2}(d + ex)\right) \right) \left(192(5a^3 + 3ac^2) \cos^3\left(\frac{1}{2}(d + ex)\right) \log\left(\cos\left(\frac{1}{2}(d + ex)\right)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]

[Out] (Cos[(d + e*x)/2]*(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2])*(192*(5*a^3 + 3*a*c^2)*Cos[(d + e*x)/2]^3*Log[Cos[(d + e*x)/2]]*(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2])^3 - 192*(5*a^3 + 3*a*c^2)*Cos[(d + e*x)/2]^3*Log[a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2]]*(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2])^3 + (c*(150*a^5*c + 130*a^3*c^3 + 24*a*c^5 + 3*a*c*(25*a^4 + 25*a^2*c^2 - 4*c^4)*Cos[d + e*x] - 6*(25*a^5*c + 15*a^3*c^3 + 4*a*c^5)*Cos[2*(d + e*x)] - 7*5*a^5*c*Cos[3*(d + e*x)] - 35*a^3*c^3*Cos[3*(d + e*x)] - 4*a*c^5*Cos[3*(d + e*x)] + 150*a^6*Sin[d + e*x] + 255*a^4*c^2*Sin[d + e*x] + 129*a^2*c^4*Sin[d + e*x] + 12*c^6*Sin[d + e*x] + 120*a^6*Sin[2*(d + e*x)] + 72*a^4*c^2*Sin[2*(d + e*x)] + 36*a^2*c^4*Sin[2*(d + e*x)] + 30*a^6*Sin[3*(d + e*x)] - 37*a^4*c^2*Sin[3*(d + e*x)] - 27*a^2*c^4*Sin[3*(d + e*x)] - 4*c^6*Sin[3*(d + e*x)]))/a)/(384*c^7*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^4)

fricas [B] time = 1.05, size = 791, normalized size = 3.82

$$60a^4c^2 + 6a^2c^4 - 2(45a^4c^2 - 3a^2c^4 - 4c^6)\cos(ex+d)^3 - 12(10a^4c^2 + a^2c^4)\cos(ex+d)^2 + 6(5a^4c^2 - 2a^2c^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="fricas")

[Out] $\frac{1}{192}(60a^4c^2 + 6a^2c^4 - 2(45a^4c^2 - 3a^2c^4 - 4c^6)\cos(ex+d)^3 - 12(10a^4c^2 + a^2c^4)\cos(ex+d)^2 + 6(5a^4c^2 - 2a^2c^4 - 2c^6)\cos(ex+d) - 3(5a^6 + 18a^4c^2 + 9a^2c^4 + (5a^6 - 12a^4c^2 - 9a^2c^4)\cos(ex+d)^3 + 3(5a^6 - 2a^4c^2 - 3a^2c^4)\cos(ex+d)^2 + 3(5a^6 + 8a^4c^2 + 3a^2c^4)\cos(ex+d) + (15a^5c + 14a^3c^3 + 3ac^5 + (15a^5c + 4a^3c^3 - 3ac^5)\cos(ex+d)^2 + 6(5a^5c + 3a^3c^3)\cos(ex+d))\sin(ex+d))\log(ac\sin(ex+d) + \frac{1}{2}a^2 + \frac{1}{2}c^2 + \frac{1}{2}(a^2 - c^2)\cos(ex+d)) + 3(5a^6 + 18a^4c^2 + 9a^2c^4 + (5a^6 - 12a^4c^2 - 9a^2c^4)\cos(ex+d)^3 + 3(5a^6 - 2a^4c^2 - 3a^2c^4)\cos(ex+d)^2 + 3(5a^6 + 8a^4c^2 + 3a^2c^4)\cos(ex+d) + (15a^5c + 14a^3c^3 + 3ac^5 + (15a^5c + 4a^3c^3 - 3ac^5)\cos(ex+d)^2 + 6(5a^5c + 3a^3c^3)\cos(ex+d))\sin(ex+d))\log(\frac{1}{2}\cos(ex+d) + \frac{1}{2}) + 2(15a^5c + 14a^3c^3 + 6ac^5 + (15a^5c - 41a^3c^3 - 12ac^5)\cos(ex+d)^2 + 3(10a^5c - 9a^3c^3 - ac^5)\cos(ex+d))\sin(ex+d))/((a^3c^7 - 3ac^9)e\cos(ex+d)^3 + 3(a^3c^7 - ac^9)e\cos(ex+d)^2 + 3(a^3c^7 + ac^9)e\cos(ex+d) + (a^3c^7 + 3ac^9)e + (6a^2c^8e\cos(ex+d) + (3a^2c^8 - c^{10})e\cos(ex+d)^2 + (3a^2c^8 + c^{10})e)\sin(ex+d))$

giac [A] time = 0.21, size = 304, normalized size = 1.47

$$-\frac{1}{384} \left(\frac{12(5a^3 + 3ac^2)\log\left(\left|c\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + a\right|\right)}{c^7} - \frac{110a^3c^3\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^3 + 66ac^5\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^3 + 285a^4c^2\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 144a^2c^4\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 - 9c^6\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 249a^5c\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 108a^3c^3\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 9ac^5\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 73a^6 + 27a^4c^2 - 3a^2c^4 - c^6}{(c\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + a)^3c^7} - \frac{(c^8\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right))^3 - 6ac^7\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 30a^2c^6\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 9c^8\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)}{c^{12}}e^{-1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="giac")

[Out] $-\frac{1}{384}(12(5a^3 + 3ac^2)\log(\text{abs}(c\tan(1/2*x*e + 1/2*d) + a))/c^7 - (110a^3c^3\tan(1/2*x*e + 1/2*d)^3 + 66ac^5\tan(1/2*x*e + 1/2*d)^3 + 285a^4c^2\tan(1/2*x*e + 1/2*d)^2 + 144a^2c^4\tan(1/2*x*e + 1/2*d)^2 - 9c^6\tan(1/2*x*e + 1/2*d)^2 + 249a^5c\tan(1/2*x*e + 1/2*d) + 108a^3c^3\tan(1/2*x*e + 1/2*d) - 9ac^5\tan(1/2*x*e + 1/2*d) + 73a^6 + 27a^4c^2 - 3a^2c^4 - c^6)/((c\tan(1/2*x*e + 1/2*d) + a)^3c^7) - (c^8\tan(1/2*x*e + 1/2*d))^3 - 6ac^7\tan(1/2*x*e + 1/2*d)^2 + 30a^2c^6\tan(1/2*x*e + 1/2*d) + 9c^8\tan(1/2*x*e + 1/2*d))/c^{12})e^{-1}$

maple [A] time = 0.60, size = 378, normalized size = 1.83

$$\frac{\tan^3\left(\frac{d}{2} + \frac{ex}{2}\right)}{384ec^4} - \frac{\left(\tan^2\left(\frac{d}{2} + \frac{ex}{2}\right)\right)a}{64ec^5} + \frac{5a^2\tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{64ec^6} + \frac{3\tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{128ec^4} + \frac{3a^5}{128ec^7\left(a + c\tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)^2} + \frac{3}{64ec^5\left(a + c\tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x)

[Out] $\frac{1}{384}e/c^4\tan(1/2*d+1/2*e*x)^3 - \frac{1}{64}e/c^5\tan(1/2*d+1/2*e*x)^2 + \frac{5}{64}e/c^6a^2\tan(1/2*d+1/2*e*x) + \frac{3}{128}e/c^4\tan(1/2*d+1/2*e*x) + \frac{3}{128}e/a^5/c^7/(a + c\tan(1/2*d+1/2*e*x))$

$+c*\tan(1/2*d+1/2*e*x))^2+3/64/e*a^3/c^5/(a+c*\tan(1/2*d+1/2*e*x))^2+3/128/e*a/c^3/(a+c*\tan(1/2*d+1/2*e*x))^2-1/384/e/c^7/(a+c*\tan(1/2*d+1/2*e*x))^3*a^6-1/128/e/c^5/(a+c*\tan(1/2*d+1/2*e*x))^3*a^4-1/128/e/c^3/(a+c*\tan(1/2*d+1/2*e*x))^3*a^2-1/384/e/c/(a+c*\tan(1/2*d+1/2*e*x))^3-15/128/e/c^7/(a+c*\tan(1/2*d+1/2*e*x))*a^4-9/64/e/c^5/(a+c*\tan(1/2*d+1/2*e*x))*a^2-3/128/e/c^3/(a+c*\tan(1/2*d+1/2*e*x))-5/32/e*a^3/c^7*\ln(a+c*\tan(1/2*d+1/2*e*x))-3/32/e*a/c^5*\ln(a+c*\tan(1/2*d+1/2*e*x))$

maxima [A] time = 0.36, size = 307, normalized size = 1.48

$$\frac{37a^6+39a^4c^2+3a^2c^4+c^6+\frac{9(9a^5c+10a^3c^3+ac^5)\sin(ex+d)}{\cos(ex+d)+1}+\frac{9(5a^4c^2+6a^2c^4+c^6)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}{a^3c^7+\frac{3a^2c^8\sin(ex+d)}{\cos(ex+d)+1}+\frac{3ac^9\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{c^{10}\sin(ex+d)^3}{(\cos(ex+d)+1)^3}}+\frac{\frac{6ac\sin(ex+d)^2}{(\cos(ex+d)+1)^2}-\frac{c^2\sin(ex+d)^3}{(\cos(ex+d)+1)^3}-\frac{3(10a^2+3c^2)\sin(ex+d)}{\cos(ex+d)+1}}{c^6}$$

384e

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="maxima")

[Out] $-1/384*((37*a^6 + 39*a^4*c^2 + 3*a^2*c^4 + c^6 + 9*(9*a^5*c + 10*a^3*c^3 + a*c^5)*\sin(e*x + d)/(\cos(e*x + d) + 1) + 9*(5*a^4*c^2 + 6*a^2*c^4 + c^6)*\sin(e*x + d)^2/(\cos(e*x + d) + 1)^2)/(a^3*c^7 + 3*a^2*c^8*\sin(e*x + d)/(\cos(e*x + d) + 1) + 3*a*c^9*\sin(e*x + d)^2/(\cos(e*x + d) + 1)^2 + c^{10}*\sin(e*x + d)^3/(\cos(e*x + d) + 1)^3) + (6*a*c*\sin(e*x + d)^2/(\cos(e*x + d) + 1)^2 - c^2*\sin(e*x + d)^3/(\cos(e*x + d) + 1)^3 - 3*(10*a^2 + 3*c^2)*\sin(e*x + d)/(\cos(e*x + d) + 1))/c^6 + 12*(5*a^3 + 3*a*c^2)*\log(a + c*\sin(e*x + d)/(\cos(e*x + d) + 1))/c^7)/e$

mupad [B] time = 2.53, size = 260, normalized size = 1.26

$$\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^3}{384c^4e} + \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)\left(\frac{3}{128c^4} + \frac{5a^2}{64c^6}\right)}{e} - \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)\left(27a^5 + 30a^3c^2 + 3ac^4\right) + \frac{37a^6+39a^4c^2+3a^2c^4+c^6}{3c}}{e\left(128a^3c^6 + 384a^2c^7\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 384ac^8\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^4,x)

[Out] $\tan(d/2 + (e*x)/2)^3/(384*c^4*e) + (\tan(d/2 + (e*x)/2)*(3/(128*c^4) + (5*a^2)/(64*c^6)))/e - (\tan(d/2 + (e*x)/2)*(3*a*c^4 + 27*a^5 + 30*a^3*c^2) + (37*a^6 + c^6 + 3*a^2*c^4 + 39*a^4*c^2)/(3*c) + \tan(d/2 + (e*x)/2)^2*(15*a^4*c + 3*c^5 + 18*a^2*c^3))/(e*(128*c^9*\tan(d/2 + (e*x)/2)^3 + 128*a^3*c^6 + 384*a^2*c^7*\tan(d/2 + (e*x)/2) + 384*a*c^8*\tan(d/2 + (e*x)/2)^2)) - (a*\tan(d/2 + (e*x)/2)^2)/(64*c^5*e) - (\log(a + c*\tan(d/2 + (e*x)/2))*(3*a*c^2 + 5*a^3))/(32*c^7*e)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**4,x)

[Out] Timed out

$$3.370 \quad \int \frac{1}{2a+2a \cos(d+ex)+2a \sin(d+ex)} dx$$

Optimal. Leaf size=23

$$\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{2ae}$$

[Out] 1/2*ln(1+tan(1/2*e*x+1/2*d))/a/e

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3124, 31}

$$\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{2ae}$$

Antiderivative was successfully verified.

[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1), x]

[Out] Log[1 + Tan[(d + e*x)/2]]/(2*a*e)

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{2a + 2a \cos(d + ex) + 2a \sin(d + ex)} dx &= \frac{2 \text{Subst}\left(\int \frac{1}{4a+4ax} dx, x, \tan\left(\frac{1}{2}(d + ex)\right)\right)}{e} \\ &= \frac{\log\left(1 + \tan\left(\frac{1}{2}(d + ex)\right)\right)}{2ae} \end{aligned}$$

Mathematica [B] time = 0.03, size = 50, normalized size = 2.17

$$\frac{\frac{\log\left(\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)\right)}{e} - \frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)}{e}}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1), x]

[Out] (-Log[Cos[(d + e*x)/2]]/e) + Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]/e)/(2*a)

fricas [A] time = 1.17, size = 31, normalized size = 1.35

$$\frac{\log\left(\frac{1}{2} \cos(ex + d) + \frac{1}{2}\right) - \log(\sin(ex + d) + 1)}{4ae}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="fricas")

[Out] -1/4*(log(1/2*cos(e*x + d) + 1/2) - log(sin(e*x + d) + 1))/(a*e)

giac [A] time = 0.16, size = 21, normalized size = 0.91

$$\frac{e^{(-1)} \log \left(\left| \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) + 1 \right| \right)}{2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="giac")

[Out] 1/2*e^(-1)*log(abs(tan(1/2*x*e + 1/2*d) + 1))/a

maple [A] time = 0.40, size = 21, normalized size = 0.91

$$\frac{\ln \left(1 + \tan \left(\frac{d}{2} + \frac{e x}{2} \right) \right)}{2 a e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x)

[Out] 1/2*ln(1+tan(1/2*d+1/2*e*x))/a/e

maxima [A] time = 0.31, size = 28, normalized size = 1.22

$$\frac{\log \left(\frac{\sin(e x+d)}{\cos(e x+d)+1} + 1 \right)}{2 a e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="maxima")

[Out] 1/2*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1)/(a*e)

mupad [B] time = 2.49, size = 20, normalized size = 0.87

$$\frac{\ln \left(\tan \left(\frac{d}{2} + \frac{e x}{2} \right) + 1 \right)}{2 a e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*a*sin(d + e*x)),x)

[Out] log(tan(d/2 + (e*x)/2) + 1)/(2*a*e)

sympy [A] time = 0.61, size = 36, normalized size = 1.57

$$\begin{cases} \frac{\log \left(\tan \left(\frac{d}{2} + \frac{e x}{2} \right) + 1 \right)}{2 a e} & \text{for } e \neq 0 \\ \frac{x}{2 a \sin (d)+2 a \cos (d)+2 a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x)

[Out] Piecewise((log(tan(d/2 + e*x/2) + 1)/(2*a*e), Ne(e, 0)), (x/(2*a*sin(d) + 2*a*cos(d) + 2*a), True))

$$3.371 \quad \int \frac{1}{(2a+2a \cos(d+ex)+2a \sin(d+ex))^2} dx$$

Optimal. Leaf size=75

$$-\frac{a \cos(d+ex) - a \sin(d+ex)}{4e(a^3 \sin(d+ex) + a^3 \cos(d+ex) + a^3)} - \frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right) + 1\right)}{4a^2e}$$

[Out] $-1/4*\ln(1+\tan(1/2*e*x+1/2*d))/a^2/e+1/4*(-a*\cos(e*x+d)+a*\sin(e*x+d))/e/(a^3+a^3*\cos(e*x+d)+a^3*\sin(e*x+d))$

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3129, 12, 3124, 31}

$$-\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right) + 1\right)}{4a^2e} - \frac{a \cos(d+ex) - a \sin(d+ex)}{4e(a^3 \sin(d+ex) + a^3 \cos(d+ex) + a^3)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a + 2*a*\text{Cos}[d + e*x] + 2*a*\text{Sin}[d + e*x])^{-2}, x]$

[Out] $-\text{Log}[1 + \text{Tan}[(d + e*x)/2]]/(4*a^2*e) - (a*\text{Cos}[d + e*x] - a*\text{Sin}[d + e*x])/(4*e*(a^3 + a^3*\text{Cos}[d + e*x] + a^3*\text{Sin}[d + e*x]))$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

Rule 31

$\text{Int}[(a_*) + (b_)*(x_)]^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 3124

$\text{Int}[(\text{cos}[(d_*) + (e_)*(x_)]*(b_*) + (a_*) + (c_*)*\text{sin}[(d_*) + (e_)*(x_)])^{-1}, x_Symbol] \rightarrow \text{Module}[\{f = \text{FreeFactors}[\text{Tan}[(d + e*x)/2], x]\}, \text{Dist}[(2*f)/e, \text{Subst}[\text{Int}[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, \text{Tan}[(d + e*x)/2]/f], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[a^2 - b^2 - c^2, 0]$

Rule 3129

$\text{Int}[(\text{cos}[(d_*) + (e_)*(x_)]*(b_*) + (a_*) + (c_*)*\text{sin}[(d_*) + (e_)*(x_)])^n, x_Symbol] \rightarrow \text{Simp}[(\text{-(c*Cos}[d + e*x]) + b*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{n+1})/(e*(n+1)*(a^2 - b^2 - c^2)), x] + \text{Dist}[1/((n+1)*(a^2 - b^2 - c^2)), \text{Int}[(a*(n+1) - b*(n+2)*\text{Cos}[d + e*x] - c*(n+2)*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{n+1}, x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[a^2 - b^2 - c^2, 0] \&\& \text{LtQ}[n, -1] \&\& \text{NeQ}[n, -3/2]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2a + 2a \cos(d + ex) + 2a \sin(d + ex))^2} dx &= -\frac{a \cos(d + ex) - a \sin(d + ex)}{4e (a^3 + a^3 \cos(d + ex) + a^3 \sin(d + ex))} + \frac{\int -\frac{2a}{2a+2a \cos(d+ex)+2a \sin(d+ex)} dx}{4a^2} \\
&= -\frac{a \cos(d + ex) - a \sin(d + ex)}{4e (a^3 + a^3 \cos(d + ex) + a^3 \sin(d + ex))} - \frac{\int \frac{1}{2a+2a \cos(d+ex)+2a \sin(d+ex)} dx}{2a} \\
&= -\frac{a \cos(d + ex) - a \sin(d + ex)}{4e (a^3 + a^3 \cos(d + ex) + a^3 \sin(d + ex))} - \frac{\text{Subst}\left(\int \frac{1}{4a+4ax} dx\right)}{2a} \\
&= -\frac{\log\left(1 + \tan\left(\frac{1}{2}(d + ex)\right)\right)}{4a^2 e} - \frac{a \cos(d + ex) - a \sin(d + ex)}{4e (a^3 + a^3 \cos(d + ex) + a^3 \sin(d + ex))}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 93, normalized size = 1.24

$$\frac{\tan\left(\frac{1}{2}(d + ex)\right) + 2 \log\left(\cos\left(\frac{1}{2}(d + ex)\right)\right) + \frac{2 \sin\left(\frac{1}{2}(d + ex)\right)}{\sin\left(\frac{1}{2}(d + ex)\right) + \cos\left(\frac{1}{2}(d + ex)\right)} - 2 \log\left(\sin\left(\frac{1}{2}(d + ex)\right) + \cos\left(\frac{1}{2}(d + ex)\right)\right)}{8a^2 e}$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]

[Out] (2*Log[Cos[(d + e*x)/2]] - 2*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]] + (2*Sin[(d + e*x)/2])/(Cos[(d + e*x)/2] + Sin[(d + e*x)/2]) + Tan[(d + e*x)/2])/(8*a^2*e)

fricas [A] time = 1.21, size = 100, normalized size = 1.33

$$\frac{(\cos(ex + d) + \sin(ex + d) + 1) \log\left(\frac{1}{2} \cos(ex + d) + \frac{1}{2}\right) - (\cos(ex + d) + \sin(ex + d) + 1) \log(\sin(ex + d) + 1)}{8(a^2 e \cos(ex + d) + a^2 e \sin(ex + d) + a^2 e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="fricas")

[Out] 1/8*((cos(e*x + d) + sin(e*x + d) + 1)*log(1/2*cos(e*x + d) + 1/2) - (cos(e*x + d) + sin(e*x + d) + 1)*log(sin(e*x + d) + 1) - 2*cos(e*x + d) + 2*sin(e*x + d))/(a^2*e*cos(e*x + d) + a^2*e*sin(e*x + d) + a^2*e)

giac [A] time = 0.17, size = 68, normalized size = 0.91

$$-\frac{1}{8} \left(\frac{2 \log\left(\left|\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 1\right|\right)}{a^2} - \frac{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)}{a^2} - \frac{2 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)}{a^2 \left(\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 1\right)} \right) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="giac")

[Out] -1/8*(2*log(abs(tan(1/2*x*e + 1/2*d) + 1))/a^2 - tan(1/2*x*e + 1/2*d)/a^2 - 2*tan(1/2*x*e + 1/2*d)/(a^2*(tan(1/2*x*e + 1/2*d) + 1)))*e^(-1)

maple [A] time = 0.41, size = 60, normalized size = 0.80

$$\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{8e a^2} - \frac{1}{4e a^2 \left(1 + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)} - \frac{\ln\left(1 + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{4a^2 e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x)`

[Out] $1/8/e/a^2*\tan(1/2*d+1/2*e*x)-1/4/e/a^2/(1+\tan(1/2*d+1/2*e*x))-1/4*\ln(1+\tan(1/2*d+1/2*e*x))/a^2/e$

maxima [A] time = 0.32, size = 80, normalized size = 1.07

$$\frac{\frac{2}{a^2 + \frac{a^2 \sin(ex+d)}{\cos(ex+d)+1}} + \frac{2 \log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1} + 1\right)}{a^2} - \frac{\sin(ex+d)}{a^2(\cos(ex+d)+1)}}{8e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="maxima")`

[Out] $-1/8*(2/(a^2 + a^2*\sin(e*x + d)/(\cos(e*x + d) + 1)) + 2*\log(\sin(e*x + d)/(\cos(e*x + d) + 1) + 1)/a^2 - \sin(e*x + d)/(a^2*(\cos(e*x + d) + 1)))/e$

mupad [B] time = 2.45, size = 59, normalized size = 0.79

$$\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{8a^2e} - \frac{\ln\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 1\right)}{4a^2e} - \frac{1}{4a^2e\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*a + 2*a*cos(d + e*x) + 2*a*sin(d + e*x))^2,x)`

[Out] $\tan(d/2 + (e*x)/2)/(8*a^2*e) - \log(\tan(d/2 + (e*x)/2) + 1)/(4*a^2*e) - 1/(4*a^2*e*(\tan(d/2 + (e*x)/2) + 1))$

sympy [A] time = 1.83, size = 168, normalized size = 2.24

$$\left\{ \begin{array}{ll} \frac{2 \log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 1\right) \tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{8a^2e \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 8a^2e} - \frac{2 \log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 1\right)}{8a^2e \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 8a^2e} + \frac{\tan^2\left(\frac{d}{2} + \frac{ex}{2}\right)}{8a^2e \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 8a^2e} - \frac{3}{8a^2e \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 8a^2e} & \text{for } e \neq 0 \\ \frac{x}{(2a \sin(d) + 2a \cos(d) + 2a)^2} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))**2,x)`

[Out] $\text{Piecewise}\left(\left(-2*\log(\tan(d/2 + e*x/2) + 1)*\tan(d/2 + e*x/2)/(8*a**2*e*\tan(d/2 + e*x/2) + 8*a**2*e) - 2*\log(\tan(d/2 + e*x/2) + 1)/(8*a**2*e*\tan(d/2 + e*x/2) + 8*a**2*e) + \tan(d/2 + e*x/2)**2/(8*a**2*e*\tan(d/2 + e*x/2) + 8*a**2*e) - 3/(8*a**2*e*\tan(d/2 + e*x/2) + 8*a**2*e), \text{Ne}(e, 0)\right), \left(x/(2*a*\sin(d) + 2*a*\cos(d) + 2*a)**2, \text{True}\right)\right)$

$$3.372 \quad \int \frac{1}{(2a+2a \cos(d+ex)+2a \sin(d+ex))^3} dx$$

Optimal. Leaf size=123

$$\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^3e} + \frac{3(\cos(d+ex)-\sin(d+ex))}{16e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)} - \frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)}$$

[Out] 1/4*ln(1+tan(1/2*e*x+1/2*d))/a^3/e+1/16*(-a*cos(e*x+d)+a*sin(e*x+d))/e/(a^2+a^2*cos(e*x+d)+a^2*sin(e*x+d))^2+3/16*(cos(e*x+d)-sin(e*x+d))/e/(a^3+a^3*cos(e*x+d)+a^3*sin(e*x+d))

Rubi [A] time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3129, 3153, 3124, 31}

$$\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^3e} + \frac{3(\cos(d+ex)-\sin(d+ex))}{16e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)} - \frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)}$$

Antiderivative was successfully verified.

[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3), x]

[Out] Log[1 + Tan[(d + e*x)/2]]/(4*a^3*e) - (a*Cos[d + e*x] - a*Sin[d + e*x])/(16*e*(a^2 + a^2*Cos[d + e*x] + a^2*Sin[d + e*x])^2) + (3*(Cos[d + e*x] - Sin[d + e*x]))/(16*e*(a^3 + a^3*Cos[d + e*x] + a^3*Sin[d + e*x]))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3129

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Simp[((-c*Cos[d + e*x] + b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]

Rule 3153

Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(A_. + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] := Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2a + 2a \cos(d + ex) + 2a \sin(d + ex))^3} dx &= -\frac{a \cos(d + ex) - a \sin(d + ex)}{16e (a^2 + a^2 \cos(d + ex) + a^2 \sin(d + ex))^2} + \frac{\int \frac{-4a+2a \cos(d+ex)+}{(2a+2a \cos(d+ex)+2} dx}{8a^2} \\
&= -\frac{a \cos(d + ex) - a \sin(d + ex)}{16e (a^2 + a^2 \cos(d + ex) + a^2 \sin(d + ex))^2} + \frac{3(\cos(d + ex) + \sin(d + ex))}{16e (a^3 + a^3 \cos(d + ex) + a^3 \sin(d + ex))} \\
&= -\frac{a \cos(d + ex) - a \sin(d + ex)}{16e (a^2 + a^2 \cos(d + ex) + a^2 \sin(d + ex))^2} + \frac{3(\cos(d + ex) + \sin(d + ex))}{16e (a^3 + a^3 \cos(d + ex) + a^3 \sin(d + ex))} \\
&= \frac{\log\left(1 + \tan\left(\frac{1}{2}(d + ex)\right)\right)}{4a^3 e} - \frac{a \cos(d + ex) - a \sin(d + ex)}{16e (a^2 + a^2 \cos(d + ex) + a^2 \sin(d + ex))^2}
\end{aligned}$$

Mathematica [A] time = 0.57, size = 135, normalized size = 1.10

$$\frac{\sec^2\left(\frac{1}{2}(d + ex)\right) + 2\left(-3 \tan\left(\frac{1}{2}(d + ex)\right) - 8 \log\left(\cos\left(\frac{1}{2}(d + ex)\right)\right) - \frac{6 \sin\left(\frac{1}{2}(d + ex)\right)}{\sin\left(\frac{1}{2}(d + ex)\right) + \cos\left(\frac{1}{2}(d + ex)\right)} - \frac{1}{\left(\sin\left(\frac{1}{2}(d + ex)\right) + \cos\left(\frac{1}{2}(d + ex)\right)\right)}\right)}{64a^3 e}$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3), x]

[Out] (Sec[(d + e*x)/2]^2 + 2*(-8*Log[Cos[(d + e*x)/2]] + 8*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]] - (Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^(-2) - (6*Sin[(d + e*x)/2])/(Cos[(d + e*x)/2] + Sin[(d + e*x)/2]) - 3*Tan[(d + e*x)/2]))/(64*a^3*e)

fricas [A] time = 1.76, size = 143, normalized size = 1.16

$$\frac{6 \cos(ex + d)^2 - 4((\cos(ex + d) + 1) \sin(ex + d) + \cos(ex + d) + 1) \log\left(\frac{1}{2} \cos(ex + d) + \frac{1}{2}\right) + 4((\cos(ex + d) + 1) \sin(ex + d) + \cos(ex + d) + 1) \log(\sin(ex + d) + 1) + 2 \cos(ex + d) - 2 \sin(ex + d) - 3}{32(a^3 e \cos(ex + d) + a^3 e + (a^3 e \cos(ex + d) + a^3 e) \sin(ex + d))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="fricas")

[Out] 1/32*(6*cos(e*x + d)^2 - 4*((cos(e*x + d) + 1)*sin(e*x + d) + cos(e*x + d) + 1)*log(1/2*cos(e*x + d) + 1/2) + 4*((cos(e*x + d) + 1)*sin(e*x + d) + cos(e*x + d) + 1)*log(sin(e*x + d) + 1) + 2*cos(e*x + d) - 2*sin(e*x + d) - 3)/(a^3*e*cos(e*x + d) + a^3*e + (a^3*e*cos(e*x + d) + a^3*e)*sin(e*x + d))

giac [A] time = 0.16, size = 107, normalized size = 0.87

$$\frac{1}{64} \left(\frac{16 \log\left(\left|\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 1\right|\right)}{a^3} - \frac{4\left(6 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 8 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 3\right)}{a^3\left(\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 1\right)^2} + \frac{a^3 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 - 6}{a^6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="giac")

[Out] $1/64*(16*\log(\tan(1/2*x*e + 1/2*d) + 1))/a^3 - 4*(6*\tan(1/2*x*e + 1/2*d)^2 + 8*\tan(1/2*x*e + 1/2*d) + 3)/(a^3*(\tan(1/2*x*e + 1/2*d) + 1)^2) + (a^3*\tan(1/2*x*e + 1/2*d)^2 - 6*a^3*\tan(1/2*x*e + 1/2*d))/a^6)*e^{-1}$

maple [A] time = 0.45, size = 100, normalized size = 0.81

$$\frac{\tan^2\left(\frac{d}{2} + \frac{ex}{2}\right)}{64a^3e} - \frac{3 \tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{32a^3e} - \frac{1}{16a^3e \left(1 + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)^2} + \frac{1}{4a^3e \left(1 + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)} + \frac{\ln\left(1 + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{4a^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(2*a+2*a*\cos(e*x+d)+2*a*\sin(e*x+d))^3, x)$

[Out] $1/64/a^3/e*\tan(1/2*d+1/2*e*x)^2-3/32/a^3/e*\tan(1/2*d+1/2*e*x)-1/16/a^3/e/(1+\tan(1/2*d+1/2*e*x))^2+1/4/a^3/e/(1+\tan(1/2*d+1/2*e*x))+1/4*\ln(1+\tan(1/2*d+1/2*e*x))/a^3/e$

maxima [A] time = 0.34, size = 146, normalized size = 1.19

$$\frac{4 \left(\frac{4 \sin(ex+d)}{\cos(ex+d)+1} + 3 \right)}{a^3 + \frac{2a^3 \sin(ex+d)}{\cos(ex+d)+1} + \frac{a^3 \sin(ex+d)^2}{(\cos(ex+d)+1)^2}} - \frac{\frac{6 \sin(ex+d)}{\cos(ex+d)+1} - \frac{\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}{a^3} + \frac{16 \log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1} + 1\right)}{a^3}$$

$64e$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(2*a+2*a*\cos(e*x+d)+2*a*\sin(e*x+d))^3, x, \text{algorithm}="maxima")$

[Out] $1/64*(4*(4*\sin(e*x + d)/(cos(e*x + d) + 1) + 3)/(a^3 + 2*a^3*\sin(e*x + d)/(cos(e*x + d) + 1) + a^3*\sin(e*x + d)^2/(cos(e*x + d) + 1)^2) - (6*\sin(e*x + d)/(cos(e*x + d) + 1) - \sin(e*x + d)^2/(cos(e*x + d) + 1)^2)/a^3 + 16*\log(\sin(e*x + d)/(cos(e*x + d) + 1) + 1)/a^3)/e$

mupad [B] time = 2.45, size = 90, normalized size = 0.73

$$\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2}{64a^3e} + \frac{\ln\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 1\right)}{4a^3e} - \frac{3 \tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{32a^3e} + \frac{\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{4} + \frac{3}{16}}{a^3e \left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(2*a + 2*a*\cos(d + e*x) + 2*a*\sin(d + e*x))^3, x)$

[Out] $\tan(d/2 + (e*x)/2)^2/(64*a^3*e) + \log(\tan(d/2 + (e*x)/2) + 1)/(4*a^3*e) - (3*\tan(d/2 + (e*x)/2))/(32*a^3*e) + (\tan(d/2 + (e*x)/2)/4 + 3/16)/(a^3*e*(\tan(d/2 + (e*x)/2) + 1)^2)$

sympy [A] time = 6.49, size = 423, normalized size = 3.44

$$\left\{ \begin{array}{l} \frac{16 \log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 1\right) \tan^2\left(\frac{d}{2} + \frac{ex}{2}\right)}{64a^3e \tan^2\left(\frac{d}{2} + \frac{ex}{2}\right) + 128a^3e \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 64a^3e} + \frac{32 \log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 1\right) \tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{64a^3e \tan^2\left(\frac{d}{2} + \frac{ex}{2}\right) + 128a^3e \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 64a^3e} + \frac{16 \log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 1\right)}{64a^3e \tan^2\left(\frac{d}{2} + \frac{ex}{2}\right) + 128a^3e \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 64a^3e} \\ \frac{x}{(2a \sin(d) + 2a \cos(d) + 2a)^3} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(2*a+2*a*\cos(e*x+d)+2*a*\sin(e*x+d))^3, x)$

```
[Out] Piecewise((16*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)**2/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + 32*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + 16*log(tan(d/2 + e*x/2) + 1)/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + tan(d/2 + e*x/2)**4/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) - 4*tan(d/2 + e*x/2)**3/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + 32*tan(d/2 + e*x/2)/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + 23/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e), Ne(e, 0)), (x/(2*a*sin(d) + 2*a*cos(d) + 2*a)**3, True))
```

$$3.373 \quad \int \frac{1}{(2a+2a \cos(d+ex)+2a \sin(d+ex))^4} dx$$

Optimal. Leaf size=168

$$\frac{19(a \cos(d+ex) - a \sin(d+ex))}{96e(a^5 \sin(d+ex) + a^5 \cos(d+ex) + a^5)} - \frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right) + 1\right)}{4a^4e} + \frac{5(\cos(d+ex) - \sin(d+ex))}{96e(a^2 \sin(d+ex) + a^2 \cos(d+ex) + a^2)}$$

[Out] $-1/4*\ln(1+\tan(1/2*e*x+1/2*d))/a^4/e+1/48*(-\cos(e*x+d)+\sin(e*x+d))/a/e/(a+a*\cos(e*x+d)+a*\sin(e*x+d))^3+5/96*(\cos(e*x+d)-\sin(e*x+d))/e/(a^2+a^2*\cos(e*x+d)+a^2*\sin(e*x+d))^2-19/96*(a*\cos(e*x+d)-a*\sin(e*x+d))/e/(a^5+a^5*\cos(e*x+d)+a^5*\sin(e*x+d))$

Rubi [A] time = 0.19, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3129, 3156, 3153, 3124, 31}

$$-\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right) + 1\right)}{4a^4e} - \frac{19(a \cos(d+ex) - a \sin(d+ex))}{96e(a^5 \sin(d+ex) + a^5 \cos(d+ex) + a^5)} + \frac{5(\cos(d+ex) - \sin(d+ex))}{96e(a^2 \sin(d+ex) + a^2 \cos(d+ex) + a^2)}$$

Antiderivative was successfully verified.

[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4), x]

[Out] $-\text{Log}[1 + \text{Tan}[(d + e*x)/2]]/(4*a^4*e) - (\text{Cos}[d + e*x] - \text{Sin}[d + e*x])/(48*a*e*(a + a*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])^3) + (5*(\text{Cos}[d + e*x] - \text{Sin}[d + e*x]))/(96*e*(a^2 + a^2*\text{Cos}[d + e*x] + a^2*\text{Sin}[d + e*x])^2) - (19*(a*\text{Cos}[d + e*x] - a*\text{Sin}[d + e*x]))/(96*e*(a^5 + a^5*\text{Cos}[d + e*x] + a^5*\text{Sin}[d + e*x]))$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3129

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Simp[((-c*Cos[d + e*x] + b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]

Rule 3153

Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]) / ((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] := Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]) / (e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si

$n[d + e*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, A, B, C\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0] \ \&\& \ \text{NeQ}[a*A - b*B - c*C, 0]$

Rule 3156

$\text{Int}[\{(a_.) + \cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*\sin[(d_.) + (e_.)*(x_.)]\}^n * \{(A_.) + \cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*\sin[(d_.) + (e_.)*(x_.)]\}], x_Symbol] \rightarrow -\text{Simp}[\{(c*B - b*C - (a*C - c*A)*\cos[d + e*x] + (a*B - b*A)*\sin[d + e*x]\}*(a + b*\cos[d + e*x] + c*\sin[d + e*x])^{n+1} / (e*(n+1)*(a^2 - b^2 - c^2)), x] + \text{Dist}[1/((n+1)*(a^2 - b^2 - c^2)), \text{Int}[(a + b*\cos[d + e*x] + c*\sin[d + e*x])^{n+1} * \text{Simp}[(n+1)*(a*A - b*B - c*C) + (n+2)*(a*B - b*A)*\cos[d + e*x] + (n+2)*(a*C - c*A)*\sin[d + e*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, A, B, C\}, x] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0] \ \&\& \ \text{NeQ}[n, -2]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(2a + 2a \cos(d + ex) + 2a \sin(d + ex))^4} dx &= -\frac{\cos(d + ex) - \sin(d + ex)}{48ae(a + a \cos(d + ex) + a \sin(d + ex))^3} + \frac{\int \frac{-6a+4a \cos(d+ex)+4a \sin(d+ex)}{(2a+2a \cos(d+ex)+2a \sin(d+ex))^4} dx}{12a^2} \\ &= -\frac{\cos(d + ex) - \sin(d + ex)}{48ae(a + a \cos(d + ex) + a \sin(d + ex))^3} + \frac{5(\cos(d + ex) - \sin(d + ex))}{96e(a^2 + a^2 \cos(d + ex) + a^2 \sin(d + ex))} \\ &= -\frac{\cos(d + ex) - \sin(d + ex)}{48ae(a + a \cos(d + ex) + a \sin(d + ex))^3} + \frac{5(\cos(d + ex) - \sin(d + ex))}{96e(a^2 + a^2 \cos(d + ex) + a^2 \sin(d + ex))} \\ &= -\frac{\cos(d + ex) - \sin(d + ex)}{48ae(a + a \cos(d + ex) + a \sin(d + ex))^3} + \frac{5(\cos(d + ex) - \sin(d + ex))}{96e(a^2 + a^2 \cos(d + ex) + a^2 \sin(d + ex))} \\ &= -\frac{\log\left(1 + \tan\left(\frac{1}{2}(d + ex)\right)\right)}{4a^4e} - \frac{\cos(d + ex) - \sin(d + ex)}{48ae(a + a \cos(d + ex) + a \sin(d + ex))^3} \end{aligned}$$

Mathematica [A] time = 0.98, size = 247, normalized size = 1.47

$$\frac{19 \tan\left(\frac{1}{2}(d + ex)\right)}{192a^4e} - \frac{\sec^2\left(\frac{1}{2}(d + ex)\right)}{64a^4e} + \frac{\log\left(\cos\left(\frac{1}{2}(d + ex)\right)\right)}{4a^4e} + \frac{19 \sin\left(\frac{1}{2}(d + ex)\right)}{96a^4e \left(\sin\left(\frac{1}{2}(d + ex)\right) + \cos\left(\frac{1}{2}(d + ex)\right)\right)} + \frac{1}{192a^4e}$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*a*cos[d + e*x] + 2*a*sin[d + e*x])^(-4), x]

[Out] Log[Cos[(d + e*x)/2]]/(4*a^4*e) - Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]/(4*a^4*e) - Sec[(d + e*x)/2]^2/(64*a^4*e) + Sin[(d + e*x)/2]/(96*a^4*e*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^3) + 5/(192*a^4*e*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^2) + (19*Sin[(d + e*x)/2])/(96*a^4*e*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])) + (19*Tan[(d + e*x)/2])/(192*a^4*e) + (Sec[(d + e*x)/2]^2*Tan[(d + e*x)/2])/(384*a^4*e)

fricas [A] time = 0.95, size = 237, normalized size = 1.41

$$38 \cos(ex + d)^3 + 66 \cos(ex + d)^2 + 24(\cos(ex + d)^3 - (\cos(ex + d)^2 + 3 \cos(ex + d) + 2) \sin(ex + d) - 3 \cos(ex + d) - 2) \sin(ex + d) - 3 \cos(ex + d) - 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="fricas")

[Out] 1/192*(38*cos(e*x + d)^3 + 66*cos(e*x + d)^2 + 24*(cos(e*x + d)^3 - (cos(e*x + d)^2 + 3*cos(e*x + d) + 2)*sin(e*x + d) - 3*cos(e*x + d) - 2)*log(1/2*cos(e*x + d) + 1/2) - 24*(cos(e*x + d)^3 - (cos(e*x + d)^2 + 3*cos(e*x + d) + 2)*sin(e*x + d) - 3*cos(e*x + d) - 2)*log(sin(e*x + d) + 1) + (38*cos(e*x + d)^2 - 35)*sin(e*x + d) - 3*cos(e*x + d) - 33)/(a^4*e*cos(e*x + d)^3 - 3*a^4*e*cos(e*x + d) - 2*a^4*e - (a^4*e*cos(e*x + d)^2 + 3*a^4*e*cos(e*x + d) + 2*a^4*e)*sin(e*x + d))

giac [A] time = 0.20, size = 139, normalized size = 0.83

$$\frac{1}{384} \left(\frac{96 \log \left(\left| \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) + 1 \right| \right)}{a^4} - \frac{4 \left(44 \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right)^3 + 105 \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right)^2 + 87 \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) + 24 \right)}{a^4 \left(\tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) + 1 \right)^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="giac")

[Out] -1/384*(96*log(abs(tan(1/2*x*e + 1/2*d) + 1))/a^4 - 4*(44*tan(1/2*x*e + 1/2*d)^3 + 105*tan(1/2*x*e + 1/2*d)^2 + 87*tan(1/2*x*e + 1/2*d) + 24)/(a^4*(tan(1/2*x*e + 1/2*d) + 1)^3) - (a^8*tan(1/2*x*e + 1/2*d)^3 - 6*a^8*tan(1/2*x*e + 1/2*d)^2 + 39*a^8*tan(1/2*x*e + 1/2*d))/a^12)*e^(-1)

maple [A] time = 0.46, size = 140, normalized size = 0.83

$$\frac{\tan^3 \left(\frac{d}{2} + \frac{ex}{2} \right)}{384a^4e} + \frac{\tan^2 \left(\frac{d}{2} + \frac{ex}{2} \right)}{64a^4e} + \frac{13 \tan \left(\frac{d}{2} + \frac{ex}{2} \right)}{128a^4e} - \frac{1}{48a^4e \left(1 + \tan \left(\frac{d}{2} + \frac{ex}{2} \right) \right)^3} + \frac{3}{32a^4e \left(1 + \tan \left(\frac{d}{2} + \frac{ex}{2} \right) \right)^2} - \frac{1}{32a^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x)

[Out] 1/384/a^4/e*tan(1/2*d+1/2*e*x)^3-1/64/a^4/e*tan(1/2*d+1/2*e*x)^2+13/128/a^4/e*tan(1/2*d+1/2*e*x)-1/48/a^4/e/(1+tan(1/2*d+1/2*e*x))^3+3/32/a^4/e/(1+tan(1/2*d+1/2*e*x))^2-9/32/a^4/e/(1+tan(1/2*d+1/2*e*x))-1/4*ln(1+tan(1/2*d+1/2*e*x))/a^4/e

maxima [A] time = 0.35, size = 208, normalized size = 1.24

$$\frac{4 \left(\frac{45 \sin(ex+d)}{\cos(ex+d)+1} + \frac{27 \sin(ex+d)^2}{(\cos(ex+d)+1)^2} + 20 \right)}{a^4 + \frac{3a^4 \sin(ex+d)}{\cos(ex+d)+1} + \frac{3a^4 \sin(ex+d)^2}{(\cos(ex+d)+1)^2} + \frac{a^4 \sin(ex+d)^3}{(\cos(ex+d)+1)^3}} - \frac{\frac{39 \sin(ex+d)}{\cos(ex+d)+1} - \frac{6 \sin(ex+d)^2}{(\cos(ex+d)+1)^2} + \frac{\sin(ex+d)^3}{(\cos(ex+d)+1)^3}}{a^4} + \frac{96 \log \left(\frac{\sin(ex+d)}{\cos(ex+d)+1} + 1 \right)}{a^4}$$

384e

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="maxima")

[Out] -1/384*(4*(45*sin(e*x + d)/(cos(e*x + d) + 1) + 27*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + 20)/(a^4 + 3*a^4*sin(e*x + d)/(cos(e*x + d) + 1) + 3*a^4*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + a^4*sin(e*x + d)^3/(cos(e*x + d) + 1)^3) - (39*sin(e*x + d)/(cos(e*x + d) + 1) - 6*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/a^4 + 96*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1)/a^4)/e

mupad [B] time = 2.44, size = 161, normalized size = 0.96

$$\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^3}{384a^4e} - \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2}{64a^4e} - \frac{\ln\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 1\right)}{4a^4e} + \frac{13\tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{128a^4e} - \frac{9\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 + 15\tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{e\left(32a^4\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^3 + 96a^4\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 + 20\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*a*sin(d + e*x))^4,x)

[Out] tan(d/2 + (e*x)/2)^3/(384*a^4*e) - tan(d/2 + (e*x)/2)^2/(64*a^4*e) - log(tan(d/2 + (e*x)/2) + 1)/(4*a^4*e) + (13*tan(d/2 + (e*x)/2))/(128*a^4*e) - (15*tan(d/2 + (e*x)/2) + 9*tan(d/2 + (e*x)/2)^2 + 20/3)/(e*(96*a^4*tan(d/2 + (e*x)/2)^3 + 32*a^4*tan(d/2 + (e*x)/2)^2 + 32*a^4 + 96*a^4*tan(d/2 + (e*x)/2)))

sympy [A] time = 23.78, size = 792, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)))**4,x)

[Out] Piecewise((-96*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)**3/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 288*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)**2/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 288*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 96*log(tan(d/2 + e*x/2) + 1)/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) + tan(d/2 + e*x/2)**6/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 3*tan(d/2 + e*x/2)**5/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) + 24*tan(d/2 + e*x/2)**4/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 297*tan(d/2 + e*x/2)**2/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 441*tan(d/2 + e*x/2)/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 180/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e), Ne(e, 0)), (x/(2*a*sin(d) + 2*a*cos(d) + 2*a)**4, True))

3.374 $\int (2a - 2a \cos(d + ex) + 2c \sin(d + ex))^3 dx$

Optimal. Leaf size=157

$$\frac{4a(15a^2 + 4c^2) \sin(d + ex)}{3e} - \frac{4c(15a^2 + 4c^2) \cos(d + ex)}{3e} + 4ax(5a^2 + 3c^2) - \frac{20(a^2 \sin(d + ex) + ac \cos(d + ex))}{3e}$$

[Out] 4*a*(5*a^2+3*c^2)*x-4/3*c*(15*a^2+4*c^2)*cos(e*x+d)/e-4/3*a*(15*a^2+4*c^2)*sin(e*x+d)/e-20/3*(a*c*cos(e*x+d)+a^2*sin(e*x+d))*(a-a*cos(e*x+d)+c*sin(e*x+d))/e-8/3*(c*cos(e*x+d)+a*sin(e*x+d))*(a-a*cos(e*x+d)+c*sin(e*x+d))^2/e

Rubi [A] time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3120, 3146, 2637, 2638}

$$\frac{4a(15a^2 + 4c^2) \sin(d + ex)}{3e} - \frac{4c(15a^2 + 4c^2) \cos(d + ex)}{3e} + 4ax(5a^2 + 3c^2) - \frac{20(a^2 \sin(d + ex) + ac \cos(d + ex))}{3e}$$

Antiderivative was successfully verified.

[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]

[Out] 4*a*(5*a^2 + 3*c^2)*x - (4*c*(15*a^2 + 4*c^2)*Cos[d + e*x])/(3*e) - (4*a*(15*a^2 + 4*c^2)*Sin[d + e*x])/(3*e) - (20*(a*c*Cos[d + e*x] + a^2*Sin[d + e*x])*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))/(3*e) - (8*(c*Cos[d + e*x] + a*Sin[d + e*x])*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3*e)

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3120

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^n, x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]

Rule 3146

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^n*(A_.) + cos[(d_.) + (e_.)*(x_)])*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])], x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x] + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\begin{aligned} \int (2a - 2a \cos(d + ex) + 2c \sin(d + ex))^3 dx &= -\frac{8(c \cos(d + ex) + a \sin(d + ex))(a - a \cos(d + ex) + c \sin(d + ex))}{3e} \\ &= -\frac{20(ac \cos(d + ex) + a^2 \sin(d + ex))(a - a \cos(d + ex) + c \sin(d + ex))}{3e} \\ &= 4a(5a^2 + 3c^2)x - \frac{20(ac \cos(d + ex) + a^2 \sin(d + ex))(a - a \cos(d + ex) + c \sin(d + ex))}{3e} \\ &= 4a(5a^2 + 3c^2)x - \frac{4c(15a^2 + 4c^2) \cos(d + ex)}{3e} - \frac{4a(15a^2 + 4c^2) \sin(d + ex)}{3e} \end{aligned}$$

Mathematica [A] time = 0.43, size = 136, normalized size = 0.87

$$\frac{2(6a(5a^2 + 3c^2)(d + ex) - 9a(5a^2 + c^2) \sin(d + ex) + 9a(a^2 - c^2) \sin(2(d + ex)) - a(a^2 - 3c^2) \sin(3(d + ex)))}{3e}$$

Antiderivative was successfully verified.

[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]

[Out] (2*(6*a*(5*a^2 + 3*c^2)*(d + e*x) - 9*c*(5*a^2 + c^2)*Cos[d + e*x] + 18*a^2*c*Cos[2*(d + e*x)] + c*(-3*a^2 + c^2)*Cos[3*(d + e*x)] - 9*a*(5*a^2 + c^2)*Sin[d + e*x] + 9*a*(a^2 - c^2)*Sin[2*(d + e*x)] - a*(a^2 - 3*c^2)*Sin[3*(d + e*x)]))/(3*e)

fricas [A] time = 1.95, size = 134, normalized size = 0.85

$$\frac{4(18a^2c \cos(ex + d)^2 - 2(3a^2c - c^3) \cos(ex + d)^3 + 3(5a^3 + 3ac^2)ex - 6(3a^2c + c^3) \cos(ex + d) - (22a^3 + 6ac^2))}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")

[Out] 4/3*(18*a^2*c*cos(e*x + d)^2 - 2*(3*a^2*c - c^3)*cos(e*x + d)^3 + 3*(5*a^3 + 3*a*c^2)*e*x - 6*(3*a^2*c + c^3)*cos(e*x + d) - (22*a^3 + 6*a*c^2 + 2*(a^3 - 3*a*c^2)*cos(e*x + d)^2 - 9*(a^3 - a*c^2)*cos(e*x + d))*sin(e*x + d))/e

giac [A] time = 0.19, size = 151, normalized size = 0.96

$$12a^2c \cos(2xe + 2d)e^{(-1)} - \frac{2}{3}(3a^2c - c^3) \cos(3xe + 3d)e^{(-1)} - 6(5a^2c + c^3) \cos(xe + d)e^{(-1)} - \frac{2}{3}(a^3 - 3ac^2)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")

[Out] 12*a^2*c*cos(2*x*e + 2*d)*e^(-1) - 2/3*(3*a^2*c - c^3)*cos(3*x*e + 3*d)*e^(-1) - 6*(5*a^2*c + c^3)*cos(x*e + d)*e^(-1) - 2/3*(a^3 - 3*a*c^2)*e^(-1)*sin(3*x*e + 3*d) + 6*(a^3 - a*c^2)*e^(-1)*sin(2*x*e + 2*d) - 6*(5*a^3 + a*c^2)*e^(-1)*sin(x*e + d) + 4*(5*a^3 + 3*a*c^2)*x

maple [A] time = 0.25, size = 178, normalized size = 1.13

$$-\frac{8a^3(2+\cos^2(ex+d))\sin(ex+d)}{3} - 8a^2c(\cos^3(ex+d)) + 24a^3\left(\frac{\sin(ex+d)\cos(ex+d)}{2} + \frac{ex}{2} + \frac{d}{2}\right) - 8ac^2(\sin^3(ex+d)) + 24a^2c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x)

[Out] $\frac{8}{e}(-\frac{1}{3}a^3(2+\cos(e*x+d))^2*\sin(e*x+d)-a^2*c*\cos(e*x+d)^3+3a^3*(\frac{1}{2}*\sin(e*x+d)*\cos(e*x+d)+\frac{1}{2}*e*x+\frac{1}{2}*d)-a*c^2*\sin(e*x+d)^3+3a^2*c*\cos(e*x+d)^2-3a^3*\sin(e*x+d)-\frac{1}{3}c^3*(2+\sin(e*x+d))^2*\cos(e*x+d)+3a*c^2*(-\frac{1}{2}*\sin(e*x+d)*\cos(e*x+d)+\frac{1}{2}*e*x+\frac{1}{2}*d)-3a^2*c*\cos(e*x+d)+a^3*(e*x+d))$

maxima [A] time = 0.33, size = 188, normalized size = 1.20

$$\frac{8a^2c \cos(ex+d)^3}{e} - \frac{8ac^2 \sin(ex+d)^3}{e} + 8a^3x + \frac{8(\sin(ex+d)^3 - 3\sin(ex+d))a^3}{3e} + \frac{8(\cos(ex+d)^3 - 3\cos(ex+d))c^3}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")

[Out] $-8a^2*c*\cos(e*x+d)^3/e - 8a*c^2*\sin(e*x+d)^3/e + 8a^3*x + 8/3*(\sin(e*x+d)^3 - 3*\sin(e*x+d))*a^3/e + 8/3*(\cos(e*x+d)^3 - 3*\cos(e*x+d))*c^3/e - 24a^2*(c*\cos(e*x+d)/e + a*\sin(e*x+d)/e) + 6*(4a*c*\cos(e*x+d)^2/e + (2*e*x + 2*d + \sin(2*e*x + 2*d))*a^2/e + (2*e*x + 2*d - \sin(2*e*x + 2*d))*c^2/e)*a$

mupad [B] time = 3.21, size = 258, normalized size = 1.64

$$\frac{8a \operatorname{atan}\left(\frac{8a \tan\left(\frac{d}{2} + \frac{ex}{2}\right)(5a^2 + 3c^2)}{40a^3 + 24ac^2}\right) (5a^2 + 3c^2) \tan\left(\frac{d}{2} + \frac{ex}{2}\right) (40a^3 + 24ac^2) + 64a^2c - \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^5 (24ac^2)}{e} \left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) (40a^3 + 24ac^2) + 64a^2c - \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^5 (24ac^2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^3,x)

[Out] $(8a*\operatorname{atan}((8a*\tan(d/2 + (e*x)/2)*(5a^2 + 3c^2))/(24a*c^2 + 40a^3))*(5a^2 + 3c^2))/e - (\tan(d/2 + (e*x)/2)*(24a*c^2 + 40a^3) + 64a^2*c - \tan(d/2 + (e*x)/2)^5*(24a*c^2 - 88a^3) + \tan(d/2 + (e*x)/2)^3*(64a*c^2 + (320a^3)/3) + \tan(d/2 + (e*x)/2)^2*(192a^2*c + 32c^3) + (32c^3)/3 + 192a^2*c*\tan(d/2 + (e*x)/2)^4)/(e*(3*\tan(d/2 + (e*x)/2)^2 + 3*\tan(d/2 + (e*x)/2)^4 + \tan(d/2 + (e*x)/2)^6 + 1)) - (8a*(5a^2 + 3c^2)*(atan(tan(d/2 + (e*x)/2)) - (e*x)/2))/e$

sympy [A] time = 0.76, size = 291, normalized size = 1.85

$$\begin{cases} 12a^3x \sin^2(d+ex) + 12a^3x \cos^2(d+ex) + 8a^3x - \frac{16a^3 \sin^3(d+ex)}{3e} - \frac{8a^3 \sin(d+ex) \cos^2(d+ex)}{e} + \frac{12a^3 \sin(d+ex) \cos(d+ex)}{e} \\ x(-2a \cos(d) + 2a + 2c \sin(d))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)

[Out] $\operatorname{Piecewise}((12a**3*x*\sin(d + e*x)**2 + 12a**3*x*\cos(d + e*x)**2 + 8a**3*x - 16a**3*\sin(d + e*x)**3/(3*e) - 8a**3*\sin(d + e*x)*\cos(d + e*x)**2/e + 12a**3*\sin(d + e*x)*\cos(d + e*x)/e - 24a**3*\sin(d + e*x)/e - 8a**2*c*\cos(d + e*x)**3/e + 24a**2*c*\cos(d + e*x)**2/e - 24a**2*c*\cos(d + e*x)/e + 12a*c**2*x*\sin(d + e*x)**2 + 12a*c**2*x*\cos(d + e*x)**2 - 8a*c**2*\sin(d + e*x)**3/e - 12a*c**2*\sin(d + e*x)*\cos(d + e*x)/e - 8c**3*\sin(d + e*x)**2*\cos(d + e*x)/e - 16c**3*\cos(d + e*x)**3/(3*e), Ne(e, 0)), (x*(-2*a*cos(d) + 2*a + 2*c*sin(d))**3, True))$

3.375 $\int (2a - 2a \cos(d + ex) + 2c \sin(d + ex))^2 dx$

Optimal. Leaf size=81

$$2x(3a^2 + c^2) - \frac{6a^2 \sin(d + ex)}{e} - \frac{6ac \cos(d + ex)}{e} - \frac{2(a \sin(d + ex) + c \cos(d + ex))(a - \cos(d + ex)) + a + c \sin(d + ex)}{e}$$

[Out] $2*(3*a^2+c^2)*x-6*a*c*\cos(e*x+d)/e-6*a^2*\sin(e*x+d)/e-2*(c*\cos(e*x+d)+a*\sin(e*x+d))*(a-a*\cos(e*x+d)+c*\sin(e*x+d))/e$

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3120, 2637, 2638}

$$2x(3a^2 + c^2) - \frac{6a^2 \sin(d + ex)}{e} - \frac{6ac \cos(d + ex)}{e} - \frac{2(a \sin(d + ex) + c \cos(d + ex))(a - \cos(d + ex)) + a + c \sin(d + ex)}{e}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a - 2*a*\text{Cos}[d + e*x] + 2*c*\text{Sin}[d + e*x])^2, x]$

[Out] $2*(3*a^2 + c^2)*x - (6*a*c*\text{Cos}[d + e*x])/e - (6*a^2*\text{Sin}[d + e*x])/e - (2*(c*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])*(a - a*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]))/e$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3120

$\text{Int}[(\cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*\sin[(d_.) + (e_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(c*\text{Cos}[d + e*x] - b*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{(n-1)}]/(e*n), x] + \text{Dist}[1/n, \text{Int}[\text{Simp}[n*a^2 + (n-1)*(b^2 + c^2) + a*b*(2*n-1)*\text{Cos}[d + e*x] + a*c*(2*n-1)*\text{Sin}[d + e*x], x]*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{(n-2)}, x], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int (2a - 2a \cos(d + ex) + 2c \sin(d + ex))^2 dx &= -\frac{2(c \cos(d + ex) + a \sin(d + ex))(a - a \cos(d + ex) + c \sin(d + ex))}{e} \\ &= 2(3a^2 + c^2)x - \frac{2(c \cos(d + ex) + a \sin(d + ex))(a - a \cos(d + ex))}{e} \\ &= 2(3a^2 + c^2)x - \frac{6ac \cos(d + ex)}{e} - \frac{6a^2 \sin(d + ex)}{e} - \frac{2(c \cos(d + ex) + a \sin(d + ex))(a - a \cos(d + ex) + c \sin(d + ex))}{e} \end{aligned}$$

Mathematica [A] time = 0.15, size = 92, normalized size = 1.14

$$4 \left(\frac{(3a^2 + c^2)(d + ex)}{2e} + \frac{(a^2 - c^2) \sin(2(d + ex))}{4e} - \frac{2a^2 \sin(d + ex)}{e} - \frac{2ac \cos(d + ex)}{e} + \frac{ac \cos(2(d + ex))}{2e} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*a - 2*a*cos[d + e*x] + 2*c*sin[d + e*x])^2,x]

[Out] 4*(((3*a^2 + c^2)*(d + e*x))/(2*e) - (2*a*c*cos[d + e*x])/e + (a*c*cos[2*(d + e*x)])/(2*e) - (2*a^2*sin[d + e*x])/e + ((a^2 - c^2)*sin[2*(d + e*x)])/(4*e))

fricas [A] time = 0.53, size = 71, normalized size = 0.88

$$\frac{2(2ac \cos(ex + d)^2 + (3a^2 + c^2)ex - 4ac \cos(ex + d) - (4a^2 - (a^2 - c^2) \cos(ex + d)) \sin(ex + d))}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")

[Out] 2*(2*a*c*cos(e*x + d)^2 + (3*a^2 + c^2)*e*x - 4*a*c*cos(e*x + d) - (4*a^2 - (a^2 - c^2)*cos(e*x + d))*sin(e*x + d))/e

giac [A] time = 0.16, size = 78, normalized size = 0.96

$$2ac \cos(2xe + 2d)e^{(-1)} - 8ac \cos(xe + d)e^{(-1)} - 8a^2e^{(-1)} \sin(xe + d) + (a^2 - c^2)e^{(-1)} \sin(2xe + 2d) + 2(3a^2 + c^2)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")

[Out] 2*a*c*cos(2*x*e + 2*d)*e^(-1) - 8*a*c*cos(x*e + d)*e^(-1) - 8*a^2*e^(-1)*sin(x*e + d) + (a^2 - c^2)*e^(-1)*sin(2*x*e + 2*d) + 2*(3*a^2 + c^2)*x

maple [A] time = 0.22, size = 100, normalized size = 1.23

$$\frac{4a^2(ex + d) - 8a^2 \sin(ex + d) - 8ac \cos(ex + d) + 4a^2 \left(\frac{\sin(ex+d) \cos(ex+d)}{2} + \frac{ex}{2} + \frac{d}{2} \right) + 4ac (\cos^2(ex + d)) + 4c^2 x}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x)

[Out] 4/e*(a^2*(e*x+d)-2*a^2*sin(e*x+d)-2*a*c*cos(e*x+d)+a^2*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)+a*c*cos(e*x+d)^2+c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d))

maxima [A] time = 0.32, size = 98, normalized size = 1.21

$$4a^2x + \frac{4ac \cos(ex + d)^2}{e} + \frac{(2ex + 2d + \sin(2ex + 2d))a^2}{e} + \frac{(2ex + 2d - \sin(2ex + 2d))c^2}{e} - 8a \left(\frac{c \cos(ex + d)}{e} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")

[Out] 4*a^2*x + 4*a*c*cos(e*x + d)^2/e + (2*e*x + 2*d + sin(2*e*x + 2*d))*a^2/e + (2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e - 8*a*(c*cos(e*x + d)/e + a*sin(e*x + d)/e)

mupad [B] time = 2.50, size = 84, normalized size = 1.04

$$\frac{a^2 \sin(2d + 2ex) - 8a^2 \sin(d + ex) - c^2 \sin(2d + 2ex) + 16ac \sin\left(\frac{d}{2} + \frac{ex}{2}\right)^2 - 4ac \sin(d + ex)^2 + 6a^2 ex}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^2,x)
```

```
[Out] (a^2*sin(2*d + 2*e*x) - 8*a^2*sin(d + e*x) - c^2*sin(2*d + 2*e*x) + 16*a*c*
sin(d/2 + (e*x)/2)^2 - 4*a*c*sin(d + e*x)^2 + 6*a^2*e*x + 2*c^2*e*x)/e
```

```
sympy [A] time = 0.32, size = 170, normalized size = 2.10
```

$$\left\{ \begin{array}{l} 2a^2x \sin^2(d + ex) + 2a^2x \cos^2(d + ex) + 4a^2x + \frac{2a^2 \sin(d+ex) \cos(d+ex)}{e} - \frac{8a^2 \sin(d+ex)}{e} + \frac{4ac \cos^2(d+ex)}{e} - \frac{8ac \cos(d+ex)}{e} \\ x(-2a \cos(d) + 2a + 2c \sin(d))^2 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x)
```

```
[Out] Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x +
2*a**2*sin(d + e*x)*cos(d + e*x)/e - 8*a**2*sin(d + e*x)/e + 4*a*c*cos(d +
e*x)**2/e - 8*a*c*cos(d + e*x)/e + 2*c**2*x*sin(d + e*x)**2 + 2*c**2*x*cos
(d + e*x)**2 - 2*c**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(-2*a*cos(
d) + 2*a + 2*c*sin(d))**2, True))
```

$$3.376 \quad \int (2a - 2a \cos(d + ex) + 2c \sin(d + ex)) dx$$

Optimal. Leaf size=29

$$-\frac{2a \sin(d + ex)}{e} + 2ax - \frac{2c \cos(d + ex)}{e}$$

[Out] 2*a*x-2*c*cos(e*x+d)/e-2*a*sin(e*x+d)/e

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2637, 2638}

$$-\frac{2a \sin(d + ex)}{e} + 2ax - \frac{2c \cos(d + ex)}{e}$$

Antiderivative was successfully verified.

[In] Int[2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x], x]

[Out] 2*a*x - (2*c*Cos[d + e*x])/e - (2*a*Sin[d + e*x])/e

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (2a - 2a \cos(d + ex) + 2c \sin(d + ex)) dx &= 2ax - (2a) \int \cos(d + ex) dx + (2c) \int \sin(d + ex) dx \\ &= 2ax - \frac{2c \cos(d + ex)}{e} - \frac{2a \sin(d + ex)}{e} \end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.83

$$-\frac{2a \sin(d) \cos(ex)}{e} - \frac{2a \cos(d) \sin(ex)}{e} + 2ax + \frac{2c \sin(d) \sin(ex)}{e} - \frac{2c \cos(d) \cos(ex)}{e}$$

Antiderivative was successfully verified.

[In] Integrate[2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x], x]

[Out] 2*a*x - (2*c*Cos[d]*Cos[e*x])/e - (2*a*Cos[e*x]*Sin[d])/e - (2*a*Cos[d]*Sin[e*x])/e + (2*c*Sin[d]*Sin[e*x])/e

fricas [A] time = 0.84, size = 28, normalized size = 0.97

$$\frac{2(aex - c \cos(ex + d) - a \sin(ex + d))}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="fricas")

[Out] $2*(a*e*x - c*\cos(e*x + d) - a*\sin(e*x + d))/e$

giac [A] time = 0.15, size = 29, normalized size = 1.00

$$-2c \cos(xe + d)e^{(-1)} - 2ae^{(-1)} \sin(xe + d) + 2ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="giac")`

[Out] $-2*c*\cos(x*e + d)*e^{(-1)} - 2*a*e^{(-1)}*\sin(x*e + d) + 2*a*x$

maple [A] time = 0.00, size = 30, normalized size = 1.03

$$2ax - \frac{2c \cos(ex + d)}{e} - \frac{2a \sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x)`

[Out] $2*a*x-2*c*\cos(e*x+d)/e-2*a*\sin(e*x+d)/e$

maxima [A] time = 0.31, size = 29, normalized size = 1.00

$$2ax - \frac{2c \cos(ex + d)}{e} - \frac{2a \sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="maxima")`

[Out] $2*a*x - 2*c*\cos(e*x + d)/e - 2*a*\sin(e*x + d)/e$

mupad [B] time = 2.43, size = 29, normalized size = 1.00

$$2ax - \frac{2c \cos(d + ex)}{e} - \frac{2a \sin(d + ex)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x),x)`

[Out] $2*a*x - (2*c*\cos(d + e*x))/e - (2*a*\sin(d + e*x))/e$

sympy [A] time = 0.15, size = 39, normalized size = 1.34

$$2ax - 2a \left(\begin{array}{ll} \frac{\sin(d+ex)}{e} & \text{for } e \neq 0 \\ x \cos(d) & \text{otherwise} \end{array} \right) + 2c \left(\begin{array}{ll} -\frac{\cos(d+ex)}{e} & \text{for } e \neq 0 \\ x \sin(d) & \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x)`

[Out] $2*a*x - 2*a*\text{Piecewise}((\sin(d + e*x)/e, \text{Ne}(e, 0)), (x*\cos(d), \text{True})) + 2*c*\text{Piecewise}((- \cos(d + e*x)/e, \text{Ne}(e, 0)), (x*\sin(d), \text{True}))$

$$3.377 \quad \int \frac{1}{2a - 2a \cos(d+ex) + 2c \sin(d+ex)} dx$$

Optimal. Leaf size=25

$$\frac{\log\left(a + c \cot\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

[Out] $-1/2*\ln(a+c*\cot(1/2*e*x+1/2*d))/c/e$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3121, 31}

$$\frac{\log\left(a + c \cot\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a - 2*a*\text{Cos}[d + e*x] + 2*c*\text{Sin}[d + e*x])^{-1}, x]$

[Out] $-\text{Log}[a + c*\text{Cot}[(d + e*x)/2]]/(2*c*e)$

Rule 31

$\text{Int}[(a_ + (b_)*(x_))^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 3121

$\text{Int}[(\cos[(d_ + (e_)*(x_))]*(b_ + (a_ + (c_)*\sin[(d_ + (e_)*(x_))])^{-1}), x_Symbol] \rightarrow \text{Module}[\{f = \text{FreeFactors}[\text{Cot}[(d + e*x)/2], x]\}, -\text{Dist}[f/e, \text{Subst}[\text{Int}[1/(a + c*f*x), x], x, \text{Cot}[(d + e*x)/2]/f], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[a + b, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{2a - 2a \cos(d+ex) + 2c \sin(d+ex)} dx &= \frac{\text{Subst}\left(\int \frac{1}{2a+2cx} dx, x, \cot\left(\frac{1}{2}(d+ex)\right)\right)}{e} \\ &= \frac{\log\left(a + c \cot\left(\frac{1}{2}(d+ex)\right)\right)}{2ce} \end{aligned}$$

Mathematica [A] time = 0.15, size = 50, normalized size = 2.00

$$\frac{\log\left(\sin\left(\frac{1}{2}(d+ex)\right)\right) - \log\left(a \sin\left(\frac{1}{2}(d+ex)\right) + c \cos\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(2*a - 2*a*\text{Cos}[d + e*x] + 2*c*\text{Sin}[d + e*x])^{-1}, x]$

[Out] $(\text{Log}[\text{Sin}[(d + e*x)/2]] - \text{Log}[c*\text{Cos}[(d + e*x)/2] + a*\text{Sin}[(d + e*x)/2]])/(2*c*e)$

fricas [B] time = 0.90, size = 60, normalized size = 2.40

$$\frac{\log\left(ac \sin(ex+d) + \frac{1}{2}a^2 + \frac{1}{2}c^2 - \frac{1}{2}(a^2 - c^2) \cos(ex+d)\right) - \log\left(-\frac{1}{2} \cos(ex+d) + \frac{1}{2}\right)}{4ce}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="fricas")

[Out] -1/4*(log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 - 1/2*(a^2 - c^2)*cos(e*x + d)) - log(-1/2*cos(e*x + d) + 1/2))/(c*e)

giac [A] time = 0.16, size = 42, normalized size = 1.68

$$-\frac{1}{2} \left(\frac{\log \left(\left| a \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) + c \right| \right)}{c} - \frac{\log \left(\left| \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) \right| \right)}{c} \right) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="giac")

[Out] -1/2*(log(abs(a*tan(1/2*x*e + 1/2*d) + c))/c - log(abs(tan(1/2*x*e + 1/2*d)))/c)*e^(-1)

maple [A] time = 0.42, size = 42, normalized size = 1.68

$$-\frac{\ln \left(c + a \tan \left(\frac{d}{2} + \frac{ex}{2} \right) \right)}{2ec} + \frac{\ln \left(\tan \left(\frac{d}{2} + \frac{ex}{2} \right) \right)}{2ec}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x)

[Out] -1/2/e/c*ln(c+a*tan(1/2*d+1/2*e*x))+1/2/e/c*ln(tan(1/2*d+1/2*e*x))

maxima [B] time = 0.32, size = 54, normalized size = 2.16

$$-\frac{\frac{\log \left(c + \frac{a \sin(ex+d)}{\cos(ex+d)+1} \right)}{c} - \frac{\log \left(\frac{\sin(ex+d)}{\cos(ex+d)+1} \right)}{c}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="maxima")

[Out] -1/2*(log(c + a*sin(e*x + d)/(cos(e*x + d) + 1))/c - log(sin(e*x + d)/(cos(e*x + d) + 1))/c)/e

mupad [B] time = 2.62, size = 26, normalized size = 1.04

$$-\frac{\operatorname{atanh} \left(\frac{2a \tan \left(\frac{d}{2} + \frac{ex}{2} \right)}{c} + 1 \right)}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x)),x)

[Out] -atanh((2*a*tan(d/2 + (e*x)/2))/c + 1)/(c*e)

sympy [A] time = 1.30, size = 95, normalized size = 3.80

$$\left\{ \begin{array}{ll} \frac{\infty x}{\sin(d)} & \text{for } a = 0 \wedge c = 0 \wedge e = 0 \\ -\frac{1}{2ae \tan\left(\frac{d}{2} + \frac{ex}{2}\right)} & \text{for } c = 0 \\ \frac{\log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{2ce} & \text{for } a = 0 \\ \frac{x}{-2a \cos(d) + 2a + 2c \sin(d)} & \text{for } e = 0 \\ -\frac{\log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + \frac{c}{a}\right)}{2ce} + \frac{\log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{2ce} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)), x)
```

```
[Out] Piecewise((zoo*x/sin(d), Eq(a, 0) & Eq(c, 0) & Eq(e, 0)), (-1/(2*a*e*tan(d/2 + e*x/2)), Eq(c, 0)), (log(tan(d/2 + e*x/2))/(2*c*e), Eq(a, 0)), (x/(-2*a*cos(d) + 2*a + 2*c*sin(d)), Eq(e, 0)), (-log(tan(d/2 + e*x/2) + c/a)/(2*c*e) + log(tan(d/2 + e*x/2))/(2*c*e), True))
```

$$3.378 \quad \int \frac{1}{(2a-2a \cos(d+ex)+2c \sin(d+ex))^2} dx$$

Optimal. Leaf size=75

$$\frac{a \log\left(a + c \cot\left(\frac{1}{2}(d + ex)\right)\right)}{4c^3e} - \frac{a \sin(d + ex) + c \cos(d + ex)}{4c^2e(a(-\cos(d + ex)) + a + c \sin(d + ex))}$$

[Out] 1/4*a*ln(a+c*cot(1/2*e*x+1/2*d))/c^3/e+1/4*(-c*cos(e*x+d)-a*sin(e*x+d))/c^2/e/(a-a*cos(e*x+d)+c*sin(e*x+d))

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3129, 12, 3121, 31}

$$\frac{a \log\left(a + c \cot\left(\frac{1}{2}(d + ex)\right)\right)}{4c^3e} - \frac{a \sin(d + ex) + c \cos(d + ex)}{4c^2e(a(-\cos(d + ex)) + a + c \sin(d + ex))}$$

Antiderivative was successfully verified.

[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2), x]

[Out] (a*Log[a + c*Cot[(d + e*x)/2]])/(4*c^3*e) - (c*Cos[d + e*x] + a*Sin[d + e*x])/(4*c^2*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3121

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n-1), x_Symbol] := Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, -Dist[f/e, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a + b, 0]

Rule 3129

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n), x_Symbol] := Simp[(-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]

Rubi steps

$$\int \frac{1}{(2a - 2a \cos(d + ex) + 2c \sin(d + ex))^2} dx = -\frac{c \cos(d + ex) + a \sin(d + ex)}{4c^2 e(a - a \cos(d + ex) + c \sin(d + ex))} + \frac{\int -\frac{2a}{2a - 2a \cos(d + ex) + 2c \sin(d + ex)} dx}{4c^2}$$

$$= -\frac{c \cos(d + ex) + a \sin(d + ex)}{4c^2 e(a - a \cos(d + ex) + c \sin(d + ex))} - \frac{a \int \frac{1}{2a - 2a \cos(d + ex) + 2c \sin(d + ex)} dx}{2c^2}$$

$$= -\frac{c \cos(d + ex) + a \sin(d + ex)}{4c^2 e(a - a \cos(d + ex) + c \sin(d + ex))} + \frac{a \operatorname{Subst}\left(\int \frac{1}{2a + 2cx} dx\right)}{2c^2}$$

$$= \frac{a \log\left(a + c \cot\left(\frac{1}{2}(d + ex)\right)\right)}{4c^3 e} - \frac{c \cos(d + ex) + a \sin(d + ex)}{4c^2 e(a - a \cos(d + ex) + c \sin(d + ex))}$$

Mathematica [B] time = 0.42, size = 229, normalized size = 3.05

$$\frac{\sin\left(\frac{1}{2}(d + ex)\right) \left(a \sin\left(\frac{1}{2}(d + ex)\right) + c \cos\left(\frac{1}{2}(d + ex)\right)\right) \left(\cos(d + ex) \left(2a^2 \log\left(a \sin\left(\frac{1}{2}(d + ex)\right) + c \cos\left(\frac{1}{2}(d + ex)\right)\right) + c \cos\left(\frac{1}{2}(d + ex)\right)\right) - \dots}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]

[Out] -1/4*(Sin[(d + e*x)/2]*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])*(Cos[d + e*x]*(a^2 + 2*c^2 - 2*a^2*Log[Sin[(d + e*x)/2]] + 2*a^2*Log[c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2]]) + a*(a*(-1 + 2*Log[Sin[(d + e*x)/2]] - 2*Log[c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2]]) + c*(1 + 2*Log[Sin[(d + e*x)/2]] - 2*Log[c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2]])*Sin[d + e*x]))/(c^3*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^2)

fricas [B] time = 0.96, size = 162, normalized size = 2.16

$$\frac{2c^2 \cos(ex + d) + 2ac \sin(ex + d) + (a^2 \cos(ex + d) - ac \sin(ex + d) - a^2) \log\left(ac \sin(ex + d) + \frac{1}{2}a^2 + \frac{1}{2}c^2 - \dots}{8(ac^3 e \cos(ex + d) - c^4 e \sin(ex + d))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")

[Out] 1/8*(2*c^2*cos(e*x + d) + 2*a*c*sin(e*x + d) + (a^2*cos(e*x + d) - a*c*sin(e*x + d) - a^2)*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 - 1/2*(a^2 - c^2)*cos(e*x + d)) - (a^2*cos(e*x + d) - a*c*sin(e*x + d) - a^2)*log(-1/2*cos(e*x + d) + 1/2))/(a*c^3*e*cos(e*x + d) - c^4*e*sin(e*x + d) - a*c^3*e)

giac [A] time = 0.17, size = 115, normalized size = 1.53

$$\frac{1}{8} \left(\frac{2a \log\left(\left|a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + c\right|\right)}{c^3} - \frac{2a \log\left(\left|\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)\right|\right)}{c^3} - \frac{2a^2 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + c^2 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)}{\left(a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)\right)^2 + c \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")

[Out] 1/8*(2*a*log(abs(a*tan(1/2*x*e + 1/2*d) + c))/c^3 - 2*a*log(abs(tan(1/2*x*e + 1/2*d)))/c^3 - (2*a^2*tan(1/2*x*e + 1/2*d) + c^2*tan(1/2*x*e + 1/2*d) + a*c)/((a*tan(1/2*x*e + 1/2*d)^2 + c*tan(1/2*x*e + 1/2*d))*a*c^2))*e^(-1)

maple [A] time = 0.47, size = 110, normalized size = 1.47

$$\frac{a}{8e c^2 \left(c + a \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)} - \frac{1}{8ea \left(c + a \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)} + \frac{a \ln\left(c + a \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{4e c^3} - \frac{1}{8e c^2 \tan\left(\frac{d}{2} + \frac{ex}{2}\right)} - \frac{a \ln\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{4e c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x)

[Out] -1/8/e/c^2*a/(c+a*tan(1/2*d+1/2*e*x))-1/8/e/a/(c+a*tan(1/2*d+1/2*e*x))+1/4/e/c^3*a*ln(c+a*tan(1/2*d+1/2*e*x))-1/8/e/c^2/tan(1/2*d+1/2*e*x)-1/4/e/c^3*a*ln(tan(1/2*d+1/2*e*x))

maxima [A] time = 0.33, size = 137, normalized size = 1.83

$$\frac{\frac{ac + \frac{(2a^2+c^2)\sin(ex+d)}{\cos(ex+d)+1}}{\cos(ex+d)+1} + \frac{a^2c^2\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}{8e} - \frac{2a \log\left(c + \frac{a \sin(ex+d)}{\cos(ex+d)+1}\right)}{c^3} + \frac{2a \log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")

[Out] -1/8*((a*c + (2*a^2 + c^2)*sin(e*x + d)/(cos(e*x + d) + 1))/(a*c^3*sin(e*x + d)/(cos(e*x + d) + 1) + a^2*c^2*sin(e*x + d)^2/(cos(e*x + d) + 1)^2) - 2*a*log(c + a*sin(e*x + d)/(cos(e*x + d) + 1))/c^3 + 2*a*log(sin(e*x + d)/(cos(e*x + d) + 1))/c^3)/e

mupad [B] time = 2.55, size = 91, normalized size = 1.21

$$\frac{a \operatorname{atanh}\left(\frac{2a \tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{c} + 1\right)}{2c^3 e} - \frac{\frac{1}{c} + \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)(2a^2+c^2)}{ac^2}}{e \left(8a \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 + 8c \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^2,x)

[Out] (a*atanh((2*a*tan(d/2 + (e*x)/2))/c + 1))/(2*c^3*e) - (1/c + (tan(d/2 + (e*x)/2)*(2*a^2 + c^2))/(a*c^2))/(e*(8*c*tan(d/2 + (e*x)/2) + 8*a*tan(d/2 + (e*x)/2)^2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x)

[Out] Timed out

$$3.379 \quad \int \frac{1}{(2a-2a \cos(d+ex)+2c \sin(d+ex))^3} dx$$

Optimal. Leaf size=134

$$\frac{3(a^2 \sin(d+ex) + ac \cos(d+ex))}{16c^4 e(a(-\cos(d+ex)) + a + c \sin(d+ex))} - \frac{(3a^2 + c^2) \log\left(a + c \cot\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5 e} - \frac{a \sin(d+ex) + c \cos(d+ex)}{16c^2 e(a(-\cos(d+ex)) + a + c \sin(d+ex))}$$

[Out] $-1/16*(3*a^2+c^2)*\ln(a+c*\cot(1/2*e*x+1/2*d))/c^5/e+1/16*(-c*\cos(e*x+d)-a*\sin(e*x+d))/c^2/e/(a-a*\cos(e*x+d)+c*\sin(e*x+d))^2+3/16*(a*c*\cos(e*x+d)+a^2*\sin(e*x+d))/c^4/e/(a-a*\cos(e*x+d)+c*\sin(e*x+d))$

Rubi [A] time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3129, 3153, 3121, 31}

$$\frac{3(a^2 \sin(d+ex) + ac \cos(d+ex))}{16c^4 e(a(-\cos(d+ex)) + a + c \sin(d+ex))} - \frac{(3a^2 + c^2) \log\left(a + c \cot\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5 e} - \frac{a \sin(d+ex) + c \cos(d+ex)}{16c^2 e(a(-\cos(d+ex)) + a + c \sin(d+ex))}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a - 2*a*\text{Cos}[d + e*x] + 2*c*\text{Sin}[d + e*x])^(-3), x]$

[Out] $-((3*a^2 + c^2)*\text{Log}[a + c*\text{Cot}[(d + e*x)/2]])/(16*c^5*e) - (c*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])/(16*c^2*e*(a - a*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^2) + (3*(a*c*\text{Cos}[d + e*x] + a^2*\text{Sin}[d + e*x]))/(16*c^4*e*(a - a*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]))$

Rule 31

$\text{Int}[(a + (b_*)*(x_*)^(-1), x_Symbol] := \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 3121

$\text{Int}[(\text{cos}[(d_*) + (e_*)*(x_*)]*(b_*) + (a_*) + (c_*)*\text{sin}[(d_*) + (e_*)*(x_*)])^(-1), x_Symbol] := \text{Module}[\{f = \text{FreeFactors}[\text{Cot}[(d + e*x)/2], x]\}, -\text{Dist}[f/e, \text{Subst}[\text{Int}[1/(a + c*f*x), x], x, \text{Cot}[(d + e*x)/2]/f], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[a + b, 0]$

Rule 3129

$\text{Int}[(\text{cos}[(d_*) + (e_*)*(x_*)]*(b_*) + (a_*) + (c_*)*\text{sin}[(d_*) + (e_*)*(x_*)])^n, x_Symbol] := \text{Simp}[((-c*\text{Cos}[d + e*x] + b*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + \text{Dist}[1/((n + 1)*(a^2 - b^2 - c^2)), \text{Int}[(a*(n + 1) - b*(n + 2)*\text{Cos}[d + e*x] - c*(n + 2)*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^(n + 1), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[a^2 - b^2 - c^2, 0] \&\& \text{LtQ}[n, -1] \&\& \text{NeQ}[n, -3/2]$

Rule 3153

$\text{Int}[(A + \text{cos}[(d_*) + (e_*)*(x_*)]*(B_*) + (C_*)*\text{sin}[(d_*) + (e_*)*(x_*)]) / ((a_*) + \text{cos}[(d_*) + (e_*)*(x_*)]*(b_*) + (c_*)*\text{sin}[(d_*) + (e_*)*(x_*)])^2, x_Symbol] := \text{Simp}[(c*B - b*C - (a*C - c*A)*\text{Cos}[d + e*x] + (a*B - b*A)*\text{Sin}[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])), x] + \text{Dist}[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), \text{Int}[1/(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, A, B, C\}, x] \&\& \text{NeQ}[a^2 - b^2 - c^2, 0] \&\& \text{NeQ}[a*A - b*B - c*C, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2a - 2a \cos(d + ex) + 2c \sin(d + ex))^3} dx &= -\frac{c \cos(d + ex) + a \sin(d + ex)}{16c^2 e (a - a \cos(d + ex) + c \sin(d + ex))^2} + \frac{\int \frac{-4a - 2a \cos(d + ex) + 2c \sin(d + ex)}{(2a - 2a \cos(d + ex) + 2c \sin(d + ex))^2} dx}{8c^2} \\
&= -\frac{c \cos(d + ex) + a \sin(d + ex)}{16c^2 e (a - a \cos(d + ex) + c \sin(d + ex))^2} + \frac{3(ac \cos(d + ex) + a^2 \sin(d + ex))}{16c^4 e (a - a \cos(d + ex) + c \sin(d + ex))} \\
&= -\frac{c \cos(d + ex) + a \sin(d + ex)}{16c^2 e (a - a \cos(d + ex) + c \sin(d + ex))^2} + \frac{3(ac \cos(d + ex) + a^2 \sin(d + ex))}{16c^4 e (a - a \cos(d + ex) + c \sin(d + ex))} \\
&= -\frac{(3a^2 + c^2) \log\left(a + c \cot\left(\frac{1}{2}(d + ex)\right)\right)}{16c^5 e} - \frac{c \cos(d + ex) + a \sin(d + ex)}{16c^2 e (a - a \cos(d + ex) + c \sin(d + ex))}
\end{aligned}$$

Mathematica [C] time = 0.61, size = 350, normalized size = 2.61

$$\frac{\sin\left(\frac{1}{2}(d + ex)\right) \left(a \sin\left(\frac{1}{2}(d + ex)\right) + c \cos\left(\frac{1}{2}(d + ex)\right)\right) \left(-6a(a^2 + c^2) \sin^3\left(\frac{1}{2}(d + ex)\right) \left(a \sin\left(\frac{1}{2}(d + ex)\right) + c \cos\left(\frac{1}{2}(d + ex)\right)\right) + \dots}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]

[Out] (Sin[(d + e*x)/2]*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])*(c^2*((-I)*a + c)*(I*a + c)*Sin[(d + e*x)/2]^2 - 6*a*(a^2 + c^2)*Sin[(d + e*x)/2]^3*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2]) - c^2*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])^2 + 4*(3*a^2 + c^2)*Log[Sin[(d + e*x)/2]]*Sin[(d + e*x)/2]^2*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])^2 - 4*(3*a^2 + c^2)*Log[c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2]]*Sin[(d + e*x)/2]^2*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])^2 + 3*a*c*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])^2*Sin[d + e*x]))/(8*c^5*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^3)

fricas [B] time = 1.08, size = 438, normalized size = 3.27

$$\frac{12 a^2 c^2 \cos(ex + d)^2 - 6 a^2 c^2 - 2(3 a^2 c^2 - c^4) \cos(ex + d) + (3 a^4 + 4 a^2 c^2 + c^4 + (3 a^4 - 2 a^2 c^2 - c^4) \cos(ex + d))}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")

[Out] 1/32*(12*a^2*c^2*cos(e*x + d)^2 - 6*a^2*c^2 - 2*(3*a^2*c^2 - c^4)*cos(e*x + d) + (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 - c^4)*cos(e*x + d)^2 - 2*(3*a^4 + a^2*c^2)*cos(e*x + d) + 2*(3*a^3*c + a*c^3 - (3*a^3*c + a*c^3)*cos(e*x + d))*sin(e*x + d))*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 - 1/2*(a^2 - c^2)*cos(e*x + d)) - (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 - c^4)*cos(e*x + d)^2 - 2*(3*a^4 + a^2*c^2)*cos(e*x + d) + 2*(3*a^3*c + a*c^3 - (3*a^3*c + a*c^3)*cos(e*x + d))*sin(e*x + d))*log(-1/2*cos(e*x + d) + 1/2) - 2*(3*a^3*c - a*c^3 - 3*(a^3*c - a*c^3)*cos(e*x + d))*sin(e*x + d))/(2*a^2*c^5*e*cos(e*x + d) - (a^2*c^5 - c^7)*e*cos(e*x + d)^2 - (a^2*c^5 + c^7)*e + 2*(a*c^6*e*cos(e*x + d) - a*c^6*e)*sin(e*x + d))

giac [A] time = 0.21, size = 239, normalized size = 1.78

$$\frac{1}{64} \left(\frac{4(3a^2 + c^2) \log\left(\left|\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)\right|\right)}{c^5} - \frac{4(3a^3 + ac^2) \log\left(\left|a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + c\right|\right)}{ac^5} + \frac{12a^5 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^3}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 1/64*(4*(3*a^2 + c^2)*log(abs(tan(1/2*x*e + 1/2*d)))/c^5 - 4*(3*a^3 + a*c^2)*log(abs(a*tan(1/2*x*e + 1/2*d) + c))/(a*c^5) + (12*a^5*tan(1/2*x*e + 1/2*d)^3 + 4*a^3*c^2*tan(1/2*x*e + 1/2*d)^3 - 2*a*c^4*tan(1/2*x*e + 1/2*d)^3 + 18*a^4*c*tan(1/2*x*e + 1/2*d)^2 + 6*a^2*c^3*tan(1/2*x*e + 1/2*d)^2 - c^5*tan(1/2*x*e + 1/2*d)^2 + 4*a^3*c^2*tan(1/2*x*e + 1/2*d) - a^2*c^3)/((a*tan(1/2*x*e + 1/2*d)^2 + c*tan(1/2*x*e + 1/2*d))^2*a^2*c^4))*e^(-1)
```

maple [B] time = 0.57, size = 272, normalized size = 2.03

$$\frac{3a^2}{32e c^4 \left(c + a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) \right)} + \frac{1}{16e c^2 \left(c + a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) \right)} - \frac{1}{32e a^2 \left(c + a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) \right)} + \frac{a^2}{64e c^3 \left(c + a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x)
```

```
[Out] 3/32/e*a^2/c^4/(c+a*tan(1/2*d+1/2*e*x))+1/16/e/c^2/(c+a*tan(1/2*d+1/2*e*x))-1/32/e/a^2/(c+a*tan(1/2*d+1/2*e*x))+1/64/e*a^2/c^3/(c+a*tan(1/2*d+1/2*e*x))^2+1/32/e/c/(c+a*tan(1/2*d+1/2*e*x))^2+1/64/e/a^2*c/(c+a*tan(1/2*d+1/2*e*x))^2-3/16/e/c^5*ln(c+a*tan(1/2*d+1/2*e*x))*a^2-1/16/e/c^3*ln(c+a*tan(1/2*d+1/2*e*x))-1/64/e/c^3/tan(1/2*d+1/2*e*x)^2+3/16/e/c^5*ln(tan(1/2*d+1/2*e*x))*a^2+1/16/e/c^3*ln(tan(1/2*d+1/2*e*x))+3/32/e/c^4*a/tan(1/2*d+1/2*e*x)
```

maxima [B] time = 0.35, size = 265, normalized size = 1.98

$$\frac{a^2 c^3 \frac{4 a^3 c^2 \sin(ex+d)}{\cos(ex+d)+1} - \frac{(18 a^4 c+6 a^2 c^3-c^5) \sin(ex+d)^2}{(\cos(ex+d)+1)^2} - \frac{2(6 a^5+2 a^3 c^2-a c^4) \sin(ex+d)^3}{(\cos(ex+d)+1)^3}}{\frac{a^2 c^6 \sin(ex+d)^2}{(\cos(ex+d)+1)^2} + \frac{2 a^3 c^5 \sin(ex+d)^3}{(\cos(ex+d)+1)^3} + \frac{a^4 c^4 \sin(ex+d)^4}{(\cos(ex+d)+1)^4}} + \frac{4(3 a^2+c^2) \log\left(c+\frac{a \sin(ex+d)}{\cos(ex+d)+1}\right)}{c^5} - \frac{4(3 a^2+c^2) \log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^5}$$

$64e$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -1/64*((a^2*c^3 - 4*a^3*c^2*sin(e*x + d)/(cos(e*x + d) + 1) - (18*a^4*c + 6*a^2*c^3 - c^5)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 2*(6*a^5 + 2*a^3*c^2 - a*c^4)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/(a^2*c^6*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + 2*a^3*c^5*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 + a^4*c^4*sin(e*x + d)^4/(cos(e*x + d) + 1)^4) + 4*(3*a^2 + c^2)*log(c + a*sin(e*x + d)/(cos(e*x + d) + 1))/c^5 - 4*(3*a^2 + c^2)*log(sin(e*x + d)/(cos(e*x + d) + 1))/c^5)/e
```

mupad [B] time = 4.47, size = 186, normalized size = 1.39

$$\frac{2a \tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{c^2} - \frac{1}{2c} + \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^3 (6a^4 + 2a^2c^2 - c^4)}{a c^4} + \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 (18a^4 + 6a^2c^2 - c^4)}{2a^2c^3} \operatorname{atanh}\left(\frac{2a \tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{c} + 1\right) (3a^2 + c^2)}{e \left(32a^2 \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^4 + 64a c \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^3 + 32c^2 \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 \right)} - \frac{8c^5 e}{8c^5 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^3,x)
```

```
[Out] ((2*a*tan(d/2 + (e*x)/2))/c^2 - 1/(2*c) + (tan(d/2 + (e*x)/2)^3*(6*a^4 - c^4 + 2*a^2*c^2))/(a*c^4) + (tan(d/2 + (e*x)/2)^2*(18*a^4 - c^4 + 6*a^2*c^2))
```

$$\frac{1}{(2a^2c^3)} \frac{1}{(e^{32a^2 \tan(d/2 + (e*x)/2)^4 + 32c^2 \tan(d/2 + (e*x)/2)^2 + 64ac \tan(d/2 + (e*x)/2)^3})} - \frac{\operatorname{atanh}(2a \tan(d/2 + (e*x)/2))}{c+1} \frac{1}{(3a^2 + c^2)} \frac{1}{(8c^5 e)}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)

[Out] Timed out

$$3.380 \quad \int \frac{1}{(2a-2a \cos(d+ex)+2c \sin(d+ex))^4} dx$$

Optimal. Leaf size=207

$$\frac{5(a^2 \sin(d+ex) + ac \cos(d+ex))}{96c^4 e(a(-\cos(d+ex)) + a + c \sin(d+ex))^2} + \frac{a(5a^2 + 3c^2) \log\left(a + c \cot\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7 e} - \frac{a(15a^2 + 4c^2) \sin(d+ex)}{96c^6 e(a(-\cos(d+ex)) + a + c \sin(d+ex))^2}$$

[Out] $1/32*a*(5*a^2+3*c^2)*\ln(a+c*\cot(1/2*e*x+1/2*d))/c^7/e+1/48*(-c*\cos(e*x+d)-a*\sin(e*x+d))/c^2/e/(a-a*\cos(e*x+d)+c*\sin(e*x+d))^3+5/96*(a*c*\cos(e*x+d)+a^2*\sin(e*x+d))/c^4/e/(a-a*\cos(e*x+d)+c*\sin(e*x+d))^2+1/96*(-c*(15*a^2+4*c^2)*\cos(e*x+d)-a*(15*a^2+4*c^2)*\sin(e*x+d))/c^6/e/(a-a*\cos(e*x+d)+c*\sin(e*x+d))$

Rubi [A] time = 0.24, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3129, 3156, 3153, 3121, 31}

$$-\frac{a(15a^2 + 4c^2) \sin(d+ex) + c(15a^2 + 4c^2) \cos(d+ex)}{96c^6 e(a(-\cos(d+ex)) + a + c \sin(d+ex))} + \frac{5(a^2 \sin(d+ex) + ac \cos(d+ex))}{96c^4 e(a(-\cos(d+ex)) + a + c \sin(d+ex))^2} + \frac{a(5a^2 + 3c^2) \log\left(a + c \cot\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7 e}$$

Antiderivative was successfully verified.

[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4), x]

[Out] $(a*(5*a^2 + 3*c^2)*\text{Log}[a + c*\text{Cot}[(d + e*x)/2]])/(32*c^7*e) - (c*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])/(48*c^2*e*(a - a*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^3) + (5*(a*c*\text{Cos}[d + e*x] + a^2*\text{Sin}[d + e*x]))/(96*c^4*e*(a - a*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^2) - (c*(15*a^2 + 4*c^2)*\text{Cos}[d + e*x] + a*(15*a^2 + 4*c^2)*\text{Sin}[d + e*x])/(96*c^6*e*(a - a*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]))$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3121

Int[(cos[(d_) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_) + (e_.)*(x_)])^(n_), x_Symbol] := Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, -Dist[f/e, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a + b, 0]

Rule 3129

Int[(cos[(d_) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_) + (e_.)*(x_)])^(n_), x_Symbol] := Simp[(-c*cos[d + e*x] + b*sin[d + e*x])*(a + b*cos[d + e*x] + c*sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*cos[d + e*x] - c*(n + 2)*sin[d + e*x])*(a + b*cos[d + e*x] + c*sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]

Rule 3153

Int[((A_) + cos[(d_) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_) + (e_.)*(x_)]) / ((a_) + cos[(d_) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_) + (e_.)*(x_)])^2, x_Symbol] := Simp[(c*B - b*C - (a*C - c*A)*cos[d + e*x] + (a*B - b*A)*sin[d + e*x]) / (e*(a^2 - b^2 - c^2)*(a + b*cos[d + e*x] + c*sin[d + e*x])), x] + Dist[(a*A - b*B - c*C) / (a^2 - b^2 - c^2), Int[1/(a + b*cos[d + e*x] + c*sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2

- c^2, 0] && NeQ[a*A - b*B - c*C, 0]

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)
]), x_Symbol] := -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(2a - 2a \cos(d + ex) + 2c \sin(d + ex))^4} dx = -\frac{c \cos(d + ex) + a \sin(d + ex)}{48c^2e(a - a \cos(d + ex) + c \sin(d + ex))^3} + \frac{\int \frac{-6a - 4a \cos(d + ex) + 4c \sin(d + ex)}{(2a - 2a \cos(d + ex) + 2c \sin(d + ex))^4} dx}{12c^2}$$

$$= -\frac{c \cos(d + ex) + a \sin(d + ex)}{48c^2e(a - a \cos(d + ex) + c \sin(d + ex))^3} + \frac{5(ac \cos(d + ex) + a^2 \sin(d + ex))}{96c^4e(a - a \cos(d + ex) + c \sin(d + ex))^2}$$

$$= -\frac{c \cos(d + ex) + a \sin(d + ex)}{48c^2e(a - a \cos(d + ex) + c \sin(d + ex))^3} + \frac{5(ac \cos(d + ex) + a^2 \sin(d + ex))}{96c^4e(a - a \cos(d + ex) + c \sin(d + ex))^2}$$

$$= -\frac{c \cos(d + ex) + a \sin(d + ex)}{48c^2e(a - a \cos(d + ex) + c \sin(d + ex))^3} + \frac{5(ac \cos(d + ex) + a^2 \sin(d + ex))}{96c^4e(a - a \cos(d + ex) + c \sin(d + ex))^2}$$

$$= \frac{a(5a^2 + 3c^2) \log\left(a + c \cot\left(\frac{1}{2}(d + ex)\right)\right)}{32c^7e} - \frac{c \cos(d + ex) + a \sin(d + ex)}{48c^2e(a - a \cos(d + ex) + c \sin(d + ex))^3}$$

Mathematica [B] time = 1.17, size = 494, normalized size = 2.39

$$\sin\left(\frac{1}{2}(d + ex)\right) \left(a \sin\left(\frac{1}{2}(d + ex)\right) + c \cos\left(\frac{1}{2}(d + ex)\right) \right) \left(-225a^6 \cos(d + ex) + 90a^6 \cos(2(d + ex)) - 15a^6 \cos(3(d + ex)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4), x]

```
[Out] (Sin[(d + e*x)/2]*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])*(150*a^6 + 130*
a^4*c^2 + 24*a^2*c^4 - 225*a^6*Cos[d + e*x] - 255*a^4*c^2*Cos[d + e*x] - 42
*a^2*c^4*Cos[d + e*x] - 24*c^6*Cos[d + e*x] + 90*a^6*Cos[2*(d + e*x)] + 174
*a^4*c^2*Cos[2*(d + e*x)] - 15*a^6*Cos[3*(d + e*x)] - 49*a^4*c^2*Cos[3*(d +
e*x)] + 18*a^2*c^4*Cos[3*(d + e*x)] + 8*c^6*Cos[3*(d + e*x)] - 192*(5*a^3
+ 3*a*c^2)*Log[Sin[(d + e*x)/2])*Sin[(d + e*x)/2]^3*(c*Cos[(d + e*x)/2] + a
*Sin[(d + e*x)/2])^3 + 192*(5*a^3 + 3*a*c^2)*Log[c*Cos[(d + e*x)/2] + a*Sin
[(d + e*x)/2])*Sin[(d + e*x)/2]^3*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])
^3 + 75*a^5*c*Sin[d + e*x] + 75*a^3*c^3*Sin[d + e*x] - 12*a*c^5*Sin[d + e*x]
- 60*a^5*c*Sin[2*(d + e*x)] - 156*a^3*c^3*Sin[2*(d + e*x)] - 12*a*c^5*Sin
[2*(d + e*x)] + 15*a^5*c*Sin[3*(d + e*x)] + 79*a^3*c^3*Sin[3*(d + e*x)] + 2
0*a*c^5*Sin[3*(d + e*x)]))/(384*c^7*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])
^4)
```

fricas [B] time = 2.47, size = 796, normalized size = 3.85

$$60 a^4 c^2 + 6 a^2 c^4 + 2 (45 a^4 c^2 - 3 a^2 c^4 - 4 c^6) \cos(ex + d)^3 - 12 (10 a^4 c^2 + a^2 c^4) \cos(ex + d)^2 - 6 (5 a^4 c^2 - 2 a^2 c^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="fricas")

[Out] 1/192*(60*a^4*c^2 + 6*a^2*c^4 + 2*(45*a^4*c^2 - 3*a^2*c^4 - 4*c^6)*cos(e*x + d)^3 - 12*(10*a^4*c^2 + a^2*c^4)*cos(e*x + d)^2 - 6*(5*a^4*c^2 - 2*a^2*c^4 - 2*c^6)*cos(e*x + d) - 3*(5*a^6 + 18*a^4*c^2 + 9*a^2*c^4 - (5*a^6 - 12*a^4*c^2 - 9*a^2*c^4)*cos(e*x + d)^3 + 3*(5*a^6 - 2*a^4*c^2 - 3*a^2*c^4)*cos(e*x + d)^2 - 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*c^4)*cos(e*x + d) + (15*a^5*c + 14*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c^3 - 3*a*c^5)*cos(e*x + d)^2 - 6*(5*a^5*c + 3*a^3*c^3)*cos(e*x + d))*sin(e*x + d))*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 - 1/2*(a^2 - c^2)*cos(e*x + d)) + 3*(5*a^6 + 18*a^4*c^2 + 9*a^2*c^4 - (5*a^6 - 12*a^4*c^2 - 9*a^2*c^4)*cos(e*x + d)^3 + 3*(5*a^6 - 2*a^4*c^2 - 3*a^2*c^4)*cos(e*x + d)^2 - 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*c^4)*cos(e*x + d) + (15*a^5*c + 14*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c^3 - 3*a*c^5)*cos(e*x + d)^2 - 6*(5*a^5*c + 3*a^3*c^3)*cos(e*x + d))*sin(e*x + d))*log(-1/2*cos(e*x + d) + 1/2) + 2*(15*a^5*c + 14*a^3*c^3 + 6*a*c^5 + (15*a^5*c - 41*a^3*c^3 - 12*a*c^5)*cos(e*x + d)^2 - 3*(10*a^5*c - 9*a^3*c^3 - a*c^5)*cos(e*x + d))*sin(e*x + d))/((a^3*c^7 - 3*a*c^9)*e*cos(e*x + d)^3 - 3*(a^3*c^7 - a*c^9)*e*cos(e*x + d)^2 + 3*(a^3*c^7 + a*c^9)*e*cos(e*x + d) - (a^3*c^7 + 3*a*c^9)*e + (6*a^2*c^8*e*cos(e*x + d) - (3*a^2*c^8 - c^10)*e*cos(e*x + d)^2 - (3*a^2*c^8 + c^10)*e)*sin(e*x + d))

giac [A] time = 0.25, size = 363, normalized size = 1.75

$$\frac{1}{384} \left(\frac{12(5a^3 + 3ac^2) \log\left(\left|\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)\right|\right)}{c^7} - \frac{12(5a^4 + 3a^2c^2) \log\left(\left|a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + c\right|\right)}{ac^7} + \frac{60a^8 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)}{c^7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="giac")

[Out] -1/384*(12*(5*a^3 + 3*a*c^2)*log(abs(tan(1/2*x*e + 1/2*d)))/c^7 - 12*(5*a^4 + 3*a^2*c^2)*log(abs(a*tan(1/2*x*e + 1/2*d) + c))/(a*c^7) + (60*a^8*tan(1/2*x*e + 1/2*d)^5 + 36*a^6*c^2*tan(1/2*x*e + 1/2*d)^5 + 3*a^2*c^6*tan(1/2*x*e + 1/2*d)^5 + 150*a^7*c*tan(1/2*x*e + 1/2*d)^4 + 90*a^5*c^3*tan(1/2*x*e + 1/2*d)^4 + 3*a*c^7*tan(1/2*x*e + 1/2*d)^4 + 110*a^6*c^2*tan(1/2*x*e + 1/2*d)^3 + 66*a^4*c^4*tan(1/2*x*e + 1/2*d)^3 + 3*a^2*c^6*tan(1/2*x*e + 1/2*d)^3 + c^8*tan(1/2*x*e + 1/2*d)^3 + 15*a^5*c^3*tan(1/2*x*e + 1/2*d)^2 + 9*a^3*c^5*tan(1/2*x*e + 1/2*d)^2 - 3*a^4*c^4*tan(1/2*x*e + 1/2*d) + a^3*c^5)/((a*tan(1/2*x*e + 1/2*d)^2 + c*tan(1/2*x*e + 1/2*d))^3*a^3*c^6))*e^(-1)

maple [B] time = 0.59, size = 416, normalized size = 2.01

$$\frac{a^3}{64e^5 \left(c + a \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)^2} - \frac{3a}{128e^3 \left(c + a \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)^2} + \frac{c}{128e^3 \left(c + a \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)^2} - \frac{5a^3}{64e^6 \left(c + a \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x)

[Out] $-1/64/e*a^3/c^5/(c+a*\tan(1/2*d+1/2*e*x))^2-3/128/e*a/c^3/(c+a*\tan(1/2*d+1/2*e*x))^2+1/128/e/a^3*c/(c+a*\tan(1/2*d+1/2*e*x))^2-5/64/e/c^6*a^3/(c+a*\tan(1/2*d+1/2*e*x))-9/128/e/c^4*a/(c+a*\tan(1/2*d+1/2*e*x))-1/128/e/a^3/(c+a*\tan(1/2*d+1/2*e*x))-1/384/e*a^3/c^4/(c+a*\tan(1/2*d+1/2*e*x))^3-1/128/e*a/c^2/(c+a*\tan(1/2*d+1/2*e*x))^3-1/128/e/a/(c+a*\tan(1/2*d+1/2*e*x))^3-1/384/e/a^3*c^2/(c+a*\tan(1/2*d+1/2*e*x))^3+5/32/e*a^3/c^7*\ln(c+a*\tan(1/2*d+1/2*e*x))+3/32/e*a/c^5*\ln(c+a*\tan(1/2*d+1/2*e*x))-1/384/e/c^4/\tan(1/2*d+1/2*e*x)^3-5/64/e/c^6/\tan(1/2*d+1/2*e*x)*a^2-3/128/e/c^4/\tan(1/2*d+1/2*e*x)+1/64/e/c^5*a/\tan(1/2*d+1/2*e*x)^2-5/32/e*a^3/c^7*\ln(\tan(1/2*d+1/2*e*x))-3/32/e*a/c^5*\ln(\tan(1/2*d+1/2*e*x))$

maxima [A] time = 0.38, size = 382, normalized size = 1.85

$$\frac{a^3c^5 - \frac{3a^4c^4 \sin(ex+d)}{\cos(ex+d)+1} + \frac{3(5a^5c^3+3a^3c^5)\sin(ex+d)^2}{(\cos(ex+d)+1)^2} + \frac{(110a^6c^2+66a^4c^4+3a^2c^6+c^8)\sin(ex+d)^3}{(\cos(ex+d)+1)^3} + \frac{3(50a^7c+30a^5c^3+ac^7)\sin(ex+d)^4}{(\cos(ex+d)+1)^4} + \frac{3(20a^8+12a^6c^2+a^2c^6)\sin(ex+d)^5}{(\cos(ex+d)+1)^5}}{\frac{a^3c^9 \sin(ex+d)^3}{(\cos(ex+d)+1)^3} + \frac{3a^4c^8 \sin(ex+d)^4}{(\cos(ex+d)+1)^4} + \frac{3a^5c^7 \sin(ex+d)^5}{(\cos(ex+d)+1)^5} + \frac{a^6c^6 \sin(ex+d)^6}{(\cos(ex+d)+1)^6}}$$

384 e

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="maxima")

[Out] $-1/384*((a^3*c^5 - 3*a^4*c^4*\sin(e*x + d))/(\cos(e*x + d) + 1) + 3*(5*a^5*c^3 + 3*a^3*c^5)*\sin(e*x + d)^2/(\cos(e*x + d) + 1)^2 + (110*a^6*c^2 + 66*a^4*c^4 + 3*a^2*c^6 + c^8)*\sin(e*x + d)^3/(\cos(e*x + d) + 1)^3 + 3*(50*a^7*c + 30*a^5*c^3 + a*c^7)*\sin(e*x + d)^4/(\cos(e*x + d) + 1)^4 + 3*(20*a^8 + 12*a^6*c^2 + a^2*c^6)*\sin(e*x + d)^5/(\cos(e*x + d) + 1)^5)/(a^3*c^9*\sin(e*x + d)^3/(\cos(e*x + d) + 1)^3 + 3*a^4*c^8*\sin(e*x + d)^4/(\cos(e*x + d) + 1)^4 + 3*a^5*c^7*\sin(e*x + d)^5/(\cos(e*x + d) + 1)^5 + a^6*c^6*\sin(e*x + d)^6/(\cos(e*x + d) + 1)^6 - 12*(5*a^3 + 3*a*c^2)*\log(c + a*\sin(e*x + d))/(\cos(e*x + d) + 1))/c^7 + 12*(5*a^3 + 3*a*c^2)*\log(\sin(e*x + d))/(\cos(e*x + d) + 1))/c^7)/e$

mupad [B] time = 6.05, size = 301, normalized size = 1.45

$$\frac{a \operatorname{atanh}\left(\frac{a\left(c+2a \tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)\left(5a^2+3c^2\right)}{c\left(5a^3+3ac^2\right)}\right)\left(5a^2+3c^2\right)}{16c^7e} - \frac{\frac{1}{3c} - \frac{a \tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{c^2} + \frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2\left(5a^2+3c^2\right)}{c^3} + \frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^3\left(110a^6+66a^4c^2\right)}{3a^3c^4}}{e\left(128a^3 \tan\left(\frac{d}{2}+\frac{ex}{2}\right)^6 + 384a^2c \tan\left(\frac{d}{2}+\frac{ex}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^4,x)

[Out] $(a*\operatorname{atanh}((a*(c + 2*a*\tan(d/2 + (e*x)/2))*(5*a^2 + 3*c^2))/(c*(3*a*c^2 + 5*a^3)))*(5*a^2 + 3*c^2))/(16*c^7*e) - (1/(3*c) - (a*\tan(d/2 + (e*x)/2))/c^2 + (\tan(d/2 + (e*x)/2)^2*(5*a^2 + 3*c^2))/c^3 + (\tan(d/2 + (e*x)/2)^3*(110*a^6 + c^6 + 3*a^2*c^4 + 66*a^4*c^2))/(3*a^3*c^4) + (\tan(d/2 + (e*x)/2)^5*(20*a^6 + c^6 + 12*a^4*c^2))/(a*c^6) + (\tan(d/2 + (e*x)/2)^4*(50*a^6 + c^6 + 30*a^4*c^2))/(a^2*c^5))/(e*(128*a^3*\tan(d/2 + (e*x)/2)^6 + 128*c^3*\tan(d/2 + (e*x)/2)^3 + 384*a*c^2*\tan(d/2 + (e*x)/2)^4 + 384*a^2*c*\tan(d/2 + (e*x)/2)^5))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**4,x)

[Out] Timed out

3.381 $\int (2a + 2b \cos(d + ex) + 2a \sin(d + ex))^3 dx$

Optimal. Leaf size=157

$$\frac{4b(15a^2 + 4b^2) \sin(d + ex)}{3e} - \frac{4a(15a^2 + 4b^2) \cos(d + ex)}{3e} + 4ax(5a^2 + 3b^2) - \frac{20(a^2 \cos(d + ex) - ab \sin(d + ex))}{3e}$$

[Out] 4*a*(5*a^2+3*b^2)*x-4/3*a*(15*a^2+4*b^2)*cos(e*x+d)/e+4/3*b*(15*a^2+4*b^2)*sin(e*x+d)/e-8/3*(a+b*cos(e*x+d)+a*sin(e*x+d))^2*(a*cos(e*x+d)-b*sin(e*x+d))/e-20/3*(a+b*cos(e*x+d)+a*sin(e*x+d))*(a^2*cos(e*x+d)-a*b*sin(e*x+d))/e

Rubi [A] time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3120, 3146, 2637, 2638}

$$\frac{4b(15a^2 + 4b^2) \sin(d + ex)}{3e} - \frac{4a(15a^2 + 4b^2) \cos(d + ex)}{3e} + 4ax(5a^2 + 3b^2) - \frac{20(a^2 \cos(d + ex) - ab \sin(d + ex))}{3e}$$

Antiderivative was successfully verified.

[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^3,x]

[Out] 4*a*(5*a^2 + 3*b^2)*x - (4*a*(15*a^2 + 4*b^2)*Cos[d + e*x])/(3*e) + (4*b*(15*a^2 + 4*b^2)*Sin[d + e*x])/(3*e) - (8*(a + b*Cos[d + e*x] + a*Sin[d + e*x])^2*(a*Cos[d + e*x] - b*Sin[d + e*x]))/(3*e) - (20*(a + b*Cos[d + e*x] + a*Sin[d + e*x])*(a^2*Cos[d + e*x] - a*b*Sin[d + e*x]))/(3*e)

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3120

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^n, x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]

Rule 3146

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^n*(A_.) + cos[(d_.) + (e_.)*(x_)])*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])^n, x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x] + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\begin{aligned} \int (2a + 2b \cos(d + ex) + 2a \sin(d + ex))^3 dx &= -\frac{8(a + b \cos(d + ex) + a \sin(d + ex))^2(a \cos(d + ex) - b \sin(d + ex))}{3e} \\ &= -\frac{8(a + b \cos(d + ex) + a \sin(d + ex))^2(a \cos(d + ex) - b \sin(d + ex))}{3e} \\ &= 4a(5a^2 + 3b^2)x - \frac{8(a + b \cos(d + ex) + a \sin(d + ex))^2(a \cos(d + ex) - b \sin(d + ex))}{3e} \\ &= 4a(5a^2 + 3b^2)x - \frac{4a(15a^2 + 4b^2)\cos(d + ex)}{3e} + \frac{4b(15a^2 + 4b^2)\sin(d + ex)}{3e} \end{aligned}$$

Mathematica [A] time = 0.44, size = 135, normalized size = 0.86

$$\frac{2(6a(5a^2 + 3b^2)(d + ex) - 9a(a^2 - b^2)\sin(2(d + ex)) + 9b(5a^2 + b^2)\sin(d + ex) + b(b^2 - 3a^2)\sin(3(d + ex)))}{3e}$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^3,x]

[Out] (2*(6*a*(5*a^2 + 3*b^2)*(d + e*x) - 9*a*(5*a^2 + b^2)*Cos[d + e*x] - 18*a^2*b*Cos[2*(d + e*x)] + a*(a^2 - 3*b^2)*Cos[3*(d + e*x)] + 9*b*(5*a^2 + b^2)*Sin[d + e*x] - 9*a*(a^2 - b^2)*Sin[2*(d + e*x)] + b*(-3*a^2 + b^2)*Sin[3*(d + e*x)]))/(3*e)

fricas [A] time = 0.77, size = 127, normalized size = 0.81

$$\frac{4(18a^2b \cos(ex + d)^2 + 24a^3 \cos(ex + d) - 2(a^3 - 3ab^2) \cos(ex + d)^3 - 3(5a^3 + 3ab^2)ex - (24a^2b + 4b^3 - 3a^3))}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="fricas")

[Out] -4/3*(18*a^2*b*cos(e*x + d)^2 + 24*a^3*cos(e*x + d) - 2*(a^3 - 3*a*b^2)*cos(e*x + d)^3 - 3*(5*a^3 + 3*a*b^2)*e*x - (24*a^2*b + 4*b^3 - 2*(3*a^2*b - b^3)*cos(e*x + d)^2 - 9*(a^3 - a*b^2)*cos(e*x + d))*sin(e*x + d))/e

giac [A] time = 0.20, size = 151, normalized size = 0.96

$$-12a^2b \cos(2xe + 2d)e^{(-1)} + \frac{2}{3}(a^3 - 3ab^2) \cos(3xe + 3d)e^{(-1)} - 6(5a^3 + ab^2) \cos(xe + d)e^{(-1)} - \frac{2}{3}(3a^2b - b^3)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="giac")

[Out] -12*a^2*b*cos(2*x*e + 2*d)*e^(-1) + 2/3*(a^3 - 3*a*b^2)*cos(3*x*e + 3*d)*e^(-1) - 6*(5*a^3 + a*b^2)*cos(x*e + d)*e^(-1) - 2/3*(3*a^2*b - b^3)*e^(-1)*sin(3*x*e + 3*d) - 6*(a^3 - a*b^2)*e^(-1)*sin(2*x*e + 2*d) + 6*(5*a^2*b + b^3)*e^(-1)*sin(x*e + d) + 4*(5*a^3 + 3*a*b^2)*x

maple [A] time = 0.25, size = 177, normalized size = 1.13

$$\frac{8a^3(ex + d) + 24 \sin(ex + d)a^2b - 24a^3 \cos(ex + d) + 24ab^2 \left(\frac{\sin(ex+d)\cos(ex+d)}{2} + \frac{ex}{2} + \frac{d}{2} \right) - 24(\cos^2(ex + d))a^2b}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x)

[Out] 8/e*(a^3*(e*x+d)+3*sin(e*x+d)*a^2*b-3*a^3*cos(e*x+d)+3*a*b^2*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-3*cos(e*x+d)^2*a^2*b+3*a^3*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)+1/3*b^3*(2+cos(e*x+d)^2)*sin(e*x+d)-a*b^2*cos(e*x+d)^3+a^2*b*sin(e*x+d)^3-1/3*a^3*(2+sin(e*x+d)^2)*cos(e*x+d))

maxima [A] time = 0.33, size = 191, normalized size = 1.22

$$\frac{8ab^2 \cos(ex+d)^3}{e} + \frac{8a^2b \sin(ex+d)^3}{e} + 8a^3x + \frac{8(\cos(ex+d)^3 - 3\cos(ex+d))a^3}{3e} - \frac{8(\sin(ex+d)^3 - 3\sin(ex+d))}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="maxima")

[Out] -8*a*b^2*cos(e*x + d)^3/e + 8*a^2*b*sin(e*x + d)^3/e + 8*a^3*x + 8/3*(cos(e*x + d)^3 - 3*cos(e*x + d))*a^3/e - 8/3*(sin(e*x + d)^3 - 3*sin(e*x + d))*b^3/e - 24*a^2*(a*cos(e*x + d)/e - b*sin(e*x + d)/e) - 6*(4*a*b*cos(e*x + d)^2/e - (2*e*x + 2*d - sin(2*e*x + 2*d))*a^2/e - (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e)*a

mupad [B] time = 3.60, size = 292, normalized size = 1.86

$$\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^5 (24a^3 + 48a^2b - 24ab^2 + 16b^3) - \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^4 (48a^3 - 96a^2b + 48ab^2) - 16ab^2 + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{e \left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^6 + 3 \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^3,x)

[Out] (tan(d/2 + (e*x)/2)^5*(48*a^2*b - 24*a*b^2 + 24*a^3 + 16*b^3) - tan(d/2 + (e*x)/2)^4*(48*a*b^2 - 96*a^2*b + 48*a^3) - 16*a*b^2 + tan(d/2 + (e*x)/2)^2*(96*a^2*b - 128*a^3) + tan(d/2 + (e*x)/2)^3*(160*a^2*b + (32*b^3)/3) - (176*a^3)/3 + tan(d/2 + (e*x)/2)*(24*a*b^2 + 48*a^2*b - 24*a^3 + 16*b^3))/(e*(3*tan(d/2 + (e*x)/2)^2 + 3*tan(d/2 + (e*x)/2)^4 + tan(d/2 + (e*x)/2)^6 + 1)) + (8*a*atan((8*a*tan(d/2 + (e*x)/2)*(5*a^2 + 3*b^2))/(24*a*b^2 + 40*a^3))*(5*a^2 + 3*b^2))/e - (8*a*(5*a^2 + 3*b^2)*(atan(tan(d/2 + (e*x)/2)) - (e*x)/2))/e

sympy [A] time = 0.75, size = 291, normalized size = 1.85

$$\begin{cases} 12a^3x \sin^2(d+ex) + 12a^3x \cos^2(d+ex) + 8a^3x - \frac{8a^3 \sin^2(d+ex) \cos(d+ex)}{e} - \frac{12a^3 \sin(d+ex) \cos(d+ex)}{e} - \frac{16a^3 \cos^3(d+ex)}{3e} \\ x(2a \sin(d) + 2a + 2b \cos(d))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x)

[Out] Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*x - 8*a**3*sin(d + e*x)**2*cos(d + e*x)/e - 12*a**3*sin(d + e*x)*cos(d + e*x)/e - 16*a**3*cos(d + e*x)**3/(3*e) - 24*a**3*cos(d + e*x)/e + 8*a**2*b*sin(d + e*x)**3/e + 24*a**2*b*sin(d + e*x)/e - 24*a**2*b*cos(d + e*x)**2/e + 12*a*b**2*x*sin(d + e*x)**2 + 12*a*b**2*x*cos(d + e*x)**2 + 12*a*b**2*sin(d + e*x)*cos(d + e*x)/e - 8*a*b**2*cos(d + e*x)**3/e + 16*b**3*sin(d + e*x)**3/(3*e) + 8*b**3*sin(d + e*x)*cos(d + e*x)**2/e, Ne(e, 0)), (x*(2*a*sin(d) + 2*a + 2*b*cos(d))**3, True))

3.382 $\int (2a + 2b \cos(d + ex) + 2a \sin(d + ex))^2 dx$

Optimal. Leaf size=81

$$2x(3a^2 + b^2) - \frac{6a^2 \cos(d + ex)}{e} + \frac{6ab \sin(d + ex)}{e} - \frac{2(a \sin(d + ex) + a + b \cos(d + ex))(a \cos(d + ex) - b \sin(d + ex))}{e}$$

[Out] $2*(3*a^2+b^2)*x-6*a^2*\cos(e*x+d)/e+6*a*b*\sin(e*x+d)/e-2*(a+b*\cos(e*x+d)+a*\sin(e*x+d))*(a*\cos(e*x+d)-b*\sin(e*x+d))/e$

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3120, 2637, 2638}

$$2x(3a^2 + b^2) - \frac{6a^2 \cos(d + ex)}{e} + \frac{6ab \sin(d + ex)}{e} - \frac{2(a \sin(d + ex) + a + b \cos(d + ex))(a \cos(d + ex) - b \sin(d + ex))}{e}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a + 2*b*\text{Cos}[d + e*x] + 2*a*\text{Sin}[d + e*x])^2, x]$

[Out] $2*(3*a^2 + b^2)*x - (6*a^2*\text{Cos}[d + e*x])/e + (6*a*b*\text{Sin}[d + e*x])/e - (2*(a + b*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])*(a*\text{Cos}[d + e*x] - b*\text{Sin}[d + e*x]))/e$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3120

$\text{Int}[(\cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*\sin[(d_.) + (e_.)*(x_.)])^n, x_Symbol] \rightarrow -\text{Simp}[(c*\text{Cos}[d + e*x] - b*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{n-1}/(e*n), x] + \text{Dist}[1/n, \text{Int}[\text{Simp}[n*a^2 + (n-1)*(b^2 + c^2) + a*b*(2*n-1)*\text{Cos}[d + e*x] + a*c*(2*n-1)*\text{Sin}[d + e*x], x]*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{n-2}, x], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int (2a + 2b \cos(d + ex) + 2a \sin(d + ex))^2 dx &= -\frac{2(a + b \cos(d + ex) + a \sin(d + ex))(a \cos(d + ex) - b \sin(d + ex))}{e} \\ &= 2(3a^2 + b^2)x - \frac{2(a + b \cos(d + ex) + a \sin(d + ex))(a \cos(d + ex) - b \sin(d + ex))}{e} \\ &= 2(3a^2 + b^2)x - \frac{6a^2 \cos(d + ex)}{e} + \frac{6ab \sin(d + ex)}{e} - \frac{2(a + b \cos(d + ex))(a \cos(d + ex) - b \sin(d + ex))}{e} \end{aligned}$$

Mathematica [A] time = 0.14, size = 92, normalized size = 1.14

$$4 \left(\frac{(3a^2 + b^2)(d + ex)}{2e} - \frac{(a^2 - b^2) \sin(2(d + ex))}{4e} - \frac{2a^2 \cos(d + ex)}{e} + \frac{2ab \sin(d + ex)}{e} - \frac{ab \cos(2(d + ex))}{2e} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*b*cos[d + e*x] + 2*a*sin[d + e*x])^2,x]

[Out] $4*((3a^2 + b^2)(d + ex))/(2e) - (2a^2 \cos[d + ex])/e - (ab \cos[2(d + ex)])/(2e) + (2ab \sin[d + ex])/e - ((a^2 - b^2) \sin[2(d + ex)])/(4e)$

fricas [A] time = 0.76, size = 72, normalized size = 0.89

$$\frac{2(2ab \cos(ex + d)^2 - (3a^2 + b^2)ex + 4a^2 \cos(ex + d) - (4ab - (a^2 - b^2) \cos(ex + d)) \sin(ex + d))}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="fricas")

[Out] $-2*(2*a*b*\cos(e*x + d)^2 - (3*a^2 + b^2)*e*x + 4*a^2*\cos(e*x + d) - (4*a*b - (a^2 - b^2)*\cos(e*x + d))*\sin(e*x + d))/e$

giac [A] time = 0.16, size = 79, normalized size = 0.98

$$-2ab \cos(2xe + 2d)e^{(-1)} - 8a^2 \cos(xe + d)e^{(-1)} + 8abe^{(-1)} \sin(xe + d) - (a^2 - b^2)e^{(-1)} \sin(2xe + 2d) + 2(3a^2 - b^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="giac")

[Out] $-2*a*b*\cos(2*x*e + 2*d)*e^{(-1)} - 8*a^2*\cos(x*e + d)*e^{(-1)} + 8*a*b*e^{(-1)}*\sin(x*e + d) - (a^2 - b^2)*e^{(-1)}*\sin(2*x*e + 2*d) + 2*(3*a^2 + b^2)*x$

maple [A] time = 0.23, size = 101, normalized size = 1.25

$$\frac{4a^2(ex + d) + 8ab \sin(ex + d) - 8a^2 \cos(ex + d) + 4b^2 \left(\frac{\sin(ex+d) \cos(ex+d)}{2} + \frac{ex}{2} + \frac{d}{2} \right) - 4(\cos^2(ex + d))ab + 4a^2}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x)

[Out] $4/e*(a^2*(e*x+d)+2*a*b*\sin(e*x+d)-2*a^2*\cos(e*x+d)+b^2*(1/2*\sin(e*x+d)*\cos(e*x+d)+1/2*e*x+1/2*d)-\cos(e*x+d)^2*a*b+a^2*(-1/2*\sin(e*x+d)*\cos(e*x+d)+1/2*e*x+1/2*d))$

maxima [A] time = 0.32, size = 99, normalized size = 1.22

$$4a^2x - \frac{4ab \cos(ex + d)^2}{e} + \frac{(2ex + 2d - \sin(2ex + 2d))a^2}{e} + \frac{(2ex + 2d + \sin(2ex + 2d))b^2}{e} - 8a \left(\frac{a \cos(ex + d)}{e} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="maxima")

[Out] $4*a^2*x - 4*a*b*\cos(e*x + d)^2/e + (2*e*x + 2*d - \sin(2*e*x + 2*d))*a^2/e + (2*e*x + 2*d + \sin(2*e*x + 2*d))*b^2/e - 8*a*(a*\cos(e*x + d)/e - b*\sin(e*x + d)/e)$

mupad [B] time = 3.72, size = 127, normalized size = 1.57

$$\frac{x(12a^2 + 4b^2)}{2} + \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2(16ab - 16a^2) + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^3(4a^2 + 16ab - 4b^2) - 16a^2 + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{e \left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^4 + 2 \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^2,x)
```

```
[Out] (x*(12*a^2 + 4*b^2))/2 + (tan(d/2 + (e*x)/2)^2*(16*a*b - 16*a^2) + tan(d/2 + (e*x)/2)^3*(16*a*b + 4*a^2 - 4*b^2) - 16*a^2 + tan(d/2 + (e*x)/2)*(16*a*b - 4*a^2 + 4*b^2))/(e*(2*tan(d/2 + (e*x)/2)^2 + tan(d/2 + (e*x)/2)^4 + 1))
```

```
sympy [A] time = 0.31, size = 170, normalized size = 2.10
```

$$\begin{cases} 2a^2x \sin^2(d + ex) + 2a^2x \cos^2(d + ex) + 4a^2x - \frac{2a^2 \sin(d+ex) \cos(d+ex)}{e} - \frac{8a^2 \cos(d+ex)}{e} + \frac{8ab \sin(d+ex)}{e} - \frac{4ab \cos^2(d+ex)}{e} \\ x(2a \sin(d) + 2a + 2b \cos(d))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**2,x)
```

```
[Out] Piecewise(((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x - 2*a**2*sin(d + e*x)*cos(d + e*x)/e - 8*a**2*cos(d + e*x)/e + 8*a*b*sin(d + e*x)/e - 4*a*b*cos(d + e*x)**2/e + 2*b**2*x*sin(d + e*x)**2 + 2*b**2*x*cos(d + e*x)**2 + 2*b**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(2*a*sin(d) + 2*a + 2*b*cos(d))**2, True))
```

3.383 $\int (2a + 2b \cos(d + ex) + 2a \sin(d + ex)) dx$

Optimal. Leaf size=29

$$-\frac{2a \cos(d + ex)}{e} + 2ax + \frac{2b \sin(d + ex)}{e}$$

[Out] $2*a*x - 2*a*\cos(e*x+d)/e + 2*b*\sin(e*x+d)/e$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2637, 2638}

$$-\frac{2a \cos(d + ex)}{e} + 2ax + \frac{2b \sin(d + ex)}{e}$$

Antiderivative was successfully verified.

[In] $\text{Int}[2*a + 2*b*\text{Cos}[d + e*x] + 2*a*\text{Sin}[d + e*x], x]$

[Out] $2*a*x - (2*a*\text{Cos}[d + e*x])/e + (2*b*\text{Sin}[d + e*x])/e$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (2a + 2b \cos(d + ex) + 2a \sin(d + ex)) dx &= 2ax + (2a) \int \sin(d + ex) dx + (2b) \int \cos(d + ex) dx \\ &= 2ax - \frac{2a \cos(d + ex)}{e} + \frac{2b \sin(d + ex)}{e} \end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.83

$$\frac{2a \sin(d) \sin(ex)}{e} - \frac{2a \cos(d) \cos(ex)}{e} + 2ax + \frac{2b \sin(d) \cos(ex)}{e} + \frac{2b \cos(d) \sin(ex)}{e}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[2*a + 2*b*\text{Cos}[d + e*x] + 2*a*\text{Sin}[d + e*x], x]$

[Out] $2*a*x - (2*a*\text{Cos}[d]*\text{Cos}[e*x])/e + (2*b*\text{Cos}[e*x]*\text{Sin}[d])/e + (2*b*\text{Cos}[d]*\text{Sin}[e*x])/e + (2*a*\text{Sin}[d]*\text{Sin}[e*x])/e$

fricas [A] time = 2.09, size = 27, normalized size = 0.93

$$\frac{2(aex - a \cos(ex + d) + b \sin(ex + d))}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(2*a+2*b*\cos(e*x+d)+2*a*\sin(e*x+d), x, \text{algorithm}="fricas")$

[Out] $2*(a*e*x - a*\cos(e*x + d) + b*\sin(e*x + d))/e$

giac [A] time = 0.14, size = 29, normalized size = 1.00

$$-2 a \cos(xe + d) e^{(-1)} + 2 b e^{(-1)} \sin(xe + d) + 2 ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x, algorithm="giac")`

[Out] $-2*a*\cos(x*e + d)*e^{(-1)} + 2*b*e^{(-1)}*\sin(x*e + d) + 2*a*x$

maple [A] time = 0.00, size = 30, normalized size = 1.03

$$2ax - \frac{2a \cos(ex + d)}{e} + \frac{2b \sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x)`

[Out] $2*a*x - 2*a*\cos(e*x + d)/e + 2*b*\sin(e*x + d)/e$

maxima [A] time = 0.31, size = 29, normalized size = 1.00

$$2ax - \frac{2a \cos(ex + d)}{e} + \frac{2b \sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x, algorithm="maxima")`

[Out] $2*a*x - 2*a*\cos(e*x + d)/e + 2*b*\sin(e*x + d)/e$

mupad [B] time = 2.44, size = 29, normalized size = 1.00

$$2ax - \frac{2a \cos(d + ex)}{e} + \frac{2b \sin(d + ex)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x),x)`

[Out] $2*a*x - (2*a*\cos(d + e*x))/e + (2*b*\sin(d + e*x))/e$

sympy [A] time = 0.14, size = 39, normalized size = 1.34

$$2ax + 2a \left(\begin{array}{ll} \left(\frac{-\cos(d+ex)}{e} & \text{for } e \neq 0 \right) \\ \left(x \sin(d) & \text{otherwise} \right) \end{array} \right) + 2b \left(\begin{array}{ll} \left(\frac{\sin(d+ex)}{e} & \text{for } e \neq 0 \right) \\ \left(x \cos(d) & \text{otherwise} \right) \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x)`

[Out] $2*a*x + 2*a*\text{Piecewise}((- \cos(d + e*x)/e, \text{Ne}(e, 0)), (x*\sin(d), \text{True})) + 2*b*\text{Piecewise}((\sin(d + e*x)/e, \text{Ne}(e, 0)), (x*\cos(d), \text{True}))$

$$3.384 \quad \int \frac{1}{2a+2b \cos(d+ex)+2a \sin(d+ex)} dx$$

Optimal. Leaf size=33

$$\frac{\log\left(a + b \cot\left(\frac{d}{2} + \frac{ex}{2} + \frac{\pi}{4}\right)\right)}{2be}$$

[Out] $-1/2*\ln(a+b*\cot(1/2*d+1/4*Pi+1/2*e*x))/b/e$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3123, 31}

$$\frac{\log\left(a + b \cot\left(\frac{d}{2} + \frac{ex}{2} + \frac{\pi}{4}\right)\right)}{2be}$$

Antiderivative was successfully verified.

[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1), x]

[Out] -Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]]/(2*b*e)

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3123

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^-1, x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, -Dist[f/e, Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]

Rubi steps

$$\int \frac{1}{2a + 2b \cos(d + ex) + 2a \sin(d + ex)} dx = -\frac{\text{Subst}\left(\int \frac{1}{2a+2bx} dx, x, \cot\left(\frac{\pi}{4} + \frac{1}{2}(d + ex)\right)\right)}{e} = -\frac{\log\left(a + b \cot\left(\frac{d}{2} + \frac{\pi}{4} + \frac{ex}{2}\right)\right)}{2be}$$

Mathematica [B] time = 0.07, size = 93, normalized size = 2.82

$$\frac{1}{2} \left(\frac{\log\left(\sin\left(\frac{1}{2}(d + ex)\right) + \cos\left(\frac{1}{2}(d + ex)\right)\right)}{be} - \frac{\log\left(a \sin\left(\frac{1}{2}(d + ex)\right) + a \cos\left(\frac{1}{2}(d + ex)\right) - b \sin\left(\frac{1}{2}(d + ex)\right) + b \cos\left(\frac{1}{2}(d + ex)\right)\right)}{be} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1), x]

[Out] (Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]/(b*e) - Log[a*Cos[(d + e*x)/2] + b*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2] - b*Sin[(d + e*x)/2]]/(b*e))/2

fricas [B] time = 1.75, size = 54, normalized size = 1.64

$$\frac{\log\left(2ab\cos(ex+d) + a^2 + b^2 + (a^2 - b^2)\sin(ex+d)\right) - \log(\sin(ex+d) + 1)}{4be}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="fricas")

[Out] -1/4*(log(2*a*b*cos(e*x + d) + a^2 + b^2 + (a^2 - b^2)*sin(e*x + d)) - log(sin(e*x + d) + 1))/(b*e)

giac [B] time = 0.20, size = 82, normalized size = 2.48

$$\frac{e^{(-1)} \log\left(\frac{\left|2a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 2b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 2a - 2|b|\right|}{\left|2a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 2b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 2a + 2|b|\right|}\right)}{2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="giac")

[Out] 1/2*e^(-1)*log(abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) + 2*a - 2*abs(b))/abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) + 2*a + 2*abs(b)))/abs(b)

maple [B] time = 0.41, size = 104, normalized size = 3.15

$$\frac{\ln\left(a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - b \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + a + b\right)a}{2eb(a-b)} + \frac{\ln\left(a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - b \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + a + b\right)}{2e(a-b)} + \frac{\ln\left(1 + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{2eb}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x)

[Out] -1/2/e/b/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a+1/2/e/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)+1/2/e/b*ln(1+tan(1/2*d+1/2*e*x))

maxima [B] time = 0.32, size = 66, normalized size = 2.00

$$\frac{\log\left(-a-b-\frac{(a-b)\sin(ex+d)}{\cos(ex+d)+1}\right)}{b} - \frac{\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="maxima")

[Out] -1/2*(log(-a - b - (a - b)*sin(e*x + d)/(cos(e*x + d) + 1))/b - log(sin(e*x + d)/(cos(e*x + d) + 1) + 1)/b)/e

mupad [B] time = 2.83, size = 33, normalized size = 1.00

$$\frac{\operatorname{atanh}\left(\frac{a + \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)(2a-2b)}{2}}{b}\right)}{be}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x)),x)`

[Out] `-atanh((a + (tan(d/2 + (e*x)/2)*(2*a - 2*b))/2)/b)/(b*e)`

sympy [A] time = 1.69, size = 107, normalized size = 3.24

$$\left\{ \begin{array}{ll}
 \frac{\infty x}{\cos(d)} & \text{for } a = 0 \wedge b = 0 \wedge e = 0 \\
 \frac{1}{ae \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + ae} & \text{for } b = 0 \\
 \frac{\log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 1\right)}{2be} & \text{for } a = b \\
 \frac{x}{2a \sin(d) + 2a + 2b \cos(d)} & \text{for } e = 0 \\
 \frac{\log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 1\right)}{2be} - \frac{\log\left(\frac{a}{a-b} + \frac{b}{a-b} + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{2be} & \text{otherwise}
 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x)`

[Out] `Piecewise((zoo*x/cos(d), Eq(a, 0) & Eq(b, 0) & Eq(e, 0)), (-1/(a*e*tan(d/2 + e*x/2) + a*e), Eq(b, 0)), (log(tan(d/2 + e*x/2) + 1)/(2*b*e), Eq(a, b)), (x/(2*a*sin(d) + 2*a + 2*b*cos(d)), Eq(e, 0)), (log(tan(d/2 + e*x/2) + 1)/(2*b*e) - log(a/(a - b) + b/(a - b) + tan(d/2 + e*x/2))/(2*b*e), True))`

$$3.385 \quad \int \frac{1}{(2a+2b \cos(d+ex)+2a \sin(d+ex))^2} dx$$

Optimal. Leaf size=83

$$\frac{a \log\left(a + b \cot\left(\frac{d}{2} + \frac{ex}{2} + \frac{\pi}{4}\right)\right)}{4b^3e} - \frac{a \cos(d+ex) - b \sin(d+ex)}{4b^2e(a \sin(d+ex) + a + b \cos(d+ex))}$$

[Out] 1/4*a*ln(a+b*cot(1/2*d+1/4*Pi+1/2*e*x))/b^3/e+1/4*(-a*cos(e*x+d)+b*sin(e*x+d))/b^2/e/(a+b*cos(e*x+d)+a*sin(e*x+d))

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3129, 12, 3123, 31}

$$\frac{a \log\left(a + b \cot\left(\frac{d}{2} + \frac{ex}{2} + \frac{\pi}{4}\right)\right)}{4b^3e} - \frac{a \cos(d+ex) - b \sin(d+ex)}{4b^2e(a \sin(d+ex) + a + b \cos(d+ex))}$$

Antiderivative was successfully verified.

[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2), x]

[Out] (a*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]])/(4*b^3*e) - (a*Cos[d + e*x] - b*Sin[d + e*x])/(4*b^2*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3123

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, -Dist[f/e, Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]

Rule 3129

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Simp[(-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2a + 2b \cos(d + ex) + 2a \sin(d + ex))^2} dx &= -\frac{a \cos(d + ex) - b \sin(d + ex)}{4b^2 e (a + b \cos(d + ex) + a \sin(d + ex))} + \frac{\int -\frac{2a}{2a + 2b \cos(d + ex) + 2a \sin(d + ex)}}{4b^2} \\
&= -\frac{a \cos(d + ex) - b \sin(d + ex)}{4b^2 e (a + b \cos(d + ex) + a \sin(d + ex))} - \frac{a \int \frac{1}{2a + 2b \cos(d + ex) + 2a \sin(d + ex)}}{2b^2} \\
&= -\frac{a \cos(d + ex) - b \sin(d + ex)}{4b^2 e (a + b \cos(d + ex) + a \sin(d + ex))} + \frac{a \operatorname{Subst}\left(\int \frac{1}{2a + 2bx}\right)}{2b^2} \\
&= \frac{a \log\left(a + b \cot\left(\frac{d}{2} + \frac{\pi}{4} + \frac{ex}{2}\right)\right)}{4b^3 e} - \frac{a \cos(d + ex) - b \sin(d + ex)}{4b^2 e (a + b \cos(d + ex) + a \sin(d + ex))}
\end{aligned}$$

Mathematica [A] time = 0.55, size = 162, normalized size = 1.95

$$\frac{b(a^2 + b^2) \sin\left(\frac{1}{2}(d + ex)\right)}{(a + b)\left((a - b) \sin\left(\frac{1}{2}(d + ex)\right) + (a + b) \cos\left(\frac{1}{2}(d + ex)\right)\right)} + a \log\left((a - b) \sin\left(\frac{1}{2}(d + ex)\right) + (a + b) \cos\left(\frac{1}{2}(d + ex)\right)\right) - a \log\left(\sin\left(\frac{1}{2}(d + ex)\right)\right)$$

$$4b^3 e$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2), x]

[Out] $(-(a \cdot \log[\cos[(d + e \cdot x)/2] + \sin[(d + e \cdot x)/2]]) + a \cdot \log[(a + b) \cdot \cos[(d + e \cdot x)/2] + (a - b) \cdot \sin[(d + e \cdot x)/2]]) + (b \cdot \sin[(d + e \cdot x)/2]) / ((\cos[(d + e \cdot x)/2] + \sin[(d + e \cdot x)/2]) + (b \cdot (a^2 + b^2) \cdot \sin[(d + e \cdot x)/2]) / ((a + b) \cdot ((a + b) \cdot \cos[(d + e \cdot x)/2] + (a - b) \cdot \sin[(d + e \cdot x)/2]))) / (4 \cdot b^3 \cdot e)$

fricas [B] time = 1.25, size = 148, normalized size = 1.78

$$\frac{2ab \cos(ex + d) - 2b^2 \sin(ex + d) - (ab \cos(ex + d) + a^2 \sin(ex + d) + a^2) \log(2ab \cos(ex + d) + a^2 + b^2)}{8(b^4 e \cos(ex + d) + ab^3 e \sin(ex + d))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="fricas")

[Out] $-1/8 \cdot (2 \cdot a \cdot b \cdot \cos(e \cdot x + d) - 2 \cdot b^2 \cdot \sin(e \cdot x + d) - (a \cdot b \cdot \cos(e \cdot x + d) + a^2 \cdot \sin(e \cdot x + d) + a^2) \cdot \log(2 \cdot a \cdot b \cdot \cos(e \cdot x + d) + a^2 + b^2 + (a^2 - b^2) \cdot \sin(e \cdot x + d)) + (a \cdot b \cdot \cos(e \cdot x + d) + a^2 \cdot \sin(e \cdot x + d) + a^2) \cdot \log(\sin(e \cdot x + d) + 1)) / (b^4 \cdot e \cdot \cos(e \cdot x + d) + a \cdot b^3 \cdot e \cdot \sin(e \cdot x + d) + a \cdot b^3 \cdot e)$

giac [B] time = 0.22, size = 196, normalized size = 2.36

$$\frac{1}{4} \left(\frac{2 \left(a^2 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - ab \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + b^2 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + a^2 \right)}{(ab^2 - b^3) \left(a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 - b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 2a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + a + b \right)} + \frac{a \log\left(\frac{2a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + a^2}{2a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + a^2}\right)}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="giac")

[Out] $-1/4 \cdot (2 \cdot (a^2 \cdot \tan(1/2 \cdot x \cdot e + 1/2 \cdot d) - a \cdot b \cdot \tan(1/2 \cdot x \cdot e + 1/2 \cdot d) + b^2 \cdot \tan(1/2 \cdot x \cdot e + 1/2 \cdot d) + a^2) / ((a \cdot b^2 - b^3) \cdot (a \cdot \tan(1/2 \cdot x \cdot e + 1/2 \cdot d)^2 - b \cdot \tan(1/2 \cdot x \cdot e + 1/2 \cdot d)^2 + 2 \cdot a \cdot \tan(1/2 \cdot x \cdot e + 1/2 \cdot d) + a + b)) + a \cdot \log(\operatorname{abs}(2 \cdot a \cdot \tan(1/2 \cdot x \cdot e + 1/2 \cdot d) + a^2))) / 4$

$*e + 1/2*d) - 2*b*\tan(1/2*x*e + 1/2*d) + 2*a - 2*abs(b))/abs(2*a*\tan(1/2*x* e + 1/2*d) - 2*b*\tan(1/2*x*e + 1/2*d) + 2*a + 2*abs(b)))/(b^2*abs(b)))*e^(- 1)$

maple [B] time = 0.53, size = 166, normalized size = 2.00

$$\frac{a^2}{4eb^2(a-b)\left(a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - b \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + a + b\right)} - \frac{1}{4e(a-b)\left(a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - b \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + a + b\right)} + \frac{a \ln}{4eb^2(a-b)\left(a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - b \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + a + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x)`

[Out] $-1/4/e/b^2/(a-b)/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)*a^{-2}-1/4/e/(a-b)/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)+1/4/e*a/b^3*\ln(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)-1/4/e/b^2/(1+\tan(1/2*d+1/2*e*x))-1/4/e*a/b^3*\ln(1+\tan(1/2*d+1/2*e*x))$

maxima [B] time = 0.34, size = 185, normalized size = 2.23

$$\frac{2\left(a^2 + \frac{(a^2-ab+b^2)\sin(ex+d)}{\cos(ex+d)+1}\right)}{a^2b^2-b^4 + \frac{2(a^2b^2-ab^3)\sin(ex+d)}{\cos(ex+d)+1} + \frac{(a^2b^2-2ab^3+b^4)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}} - \frac{a \log\left(-a-b-\frac{(a-b)\sin(ex+d)}{\cos(ex+d)+1}\right)}{b^3} + \frac{a \log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{b^3}$$

$4e$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="maxima")`

[Out] $-1/4*(2*(a^2 + (a^2 - a*b + b^2)*\sin(e*x + d)/(\cos(e*x + d) + 1))/(a^2*b^2 - b^4 + 2*(a^2*b^2 - a*b^3)*\sin(e*x + d)/(\cos(e*x + d) + 1) + (a^2*b^2 - 2*a*b^3 + b^4)*\sin(e*x + d)^2/(\cos(e*x + d) + 1)^2 - a*\log(-a - b - (a - b)*\sin(e*x + d)/(\cos(e*x + d) + 1))/b^3 + a*\log(\sin(e*x + d)/(\cos(e*x + d) + 1) + 1)/b^3)/e$

mupad [B] time = 2.72, size = 126, normalized size = 1.52

$$\frac{a \operatorname{atanh}\left(\frac{a + \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)(2a-2b)}{2}}{b}\right)}{2b^3e} - \frac{\frac{a^2}{b^2(a-b)} + \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)(a^2-ab+b^2)}{b^2(a-b)}}{e\left((2a-2b)\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 + 4a\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 2a + 2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^2,x)`

[Out] $(a*\operatorname{atanh}((a + (\tan(d/2 + (e*x)/2)*(2*a - 2*b))/2)/b))/(2*b^3*e) - (a^2/(b^2*(a - b)) + (\tan(d/2 + (e*x)/2)*(a^2 - a*b + b^2))/(b^2*(a - b)))/(e*(2*a + 2*b + \tan(d/2 + (e*x)/2)^2*(2*a - 2*b) + 4*a*\tan(d/2 + (e*x)/2)))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**2,x)`

[Out] Timed out

$$3.386 \quad \int \frac{1}{(2a+2b \cos(d+ex)+2a \sin(d+ex))^3} dx$$

Optimal. Leaf size=142

$$\frac{3(a^2 \cos(d+ex) - ab \sin(d+ex))}{16b^4 e(a \sin(d+ex) + a + b \cos(d+ex))} - \frac{(3a^2 + b^2) \log\left(a + b \cot\left(\frac{d}{2} + \frac{ex}{2} + \frac{\pi}{4}\right)\right)}{16b^5 e} - \frac{a \cos(d+ex) - b \sin(d+ex)}{16b^2 e(a \sin(d+ex) + a + b \cos(d+ex))}$$

[Out] $-1/16*(3*a^2+b^2)*\ln(a+b*\cot(1/2*d+1/4*Pi+1/2*e*x))/b^5/e+1/16*(-a*\cos(e*x+d)+b*\sin(e*x+d))/b^2/e/(a+b*\cos(e*x+d)+a*\sin(e*x+d))^2+3/16*(a^2*\cos(e*x+d)-a*b*\sin(e*x+d))/b^4/e/(a+b*\cos(e*x+d)+a*\sin(e*x+d))$

Rubi [A] time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3129, 3153, 3123, 31}

$$\frac{3(a^2 \cos(d+ex) - ab \sin(d+ex))}{16b^4 e(a \sin(d+ex) + a + b \cos(d+ex))} - \frac{(3a^2 + b^2) \log\left(a + b \cot\left(\frac{d}{2} + \frac{ex}{2} + \frac{\pi}{4}\right)\right)}{16b^5 e} - \frac{a \cos(d+ex) - b \sin(d+ex)}{16b^2 e(a \sin(d+ex) + a + b \cos(d+ex))}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a + 2*b*\text{Cos}[d + e*x] + 2*a*\text{Sin}[d + e*x])^(-3), x]$

[Out] $-((3*a^2 + b^2)*\text{Log}[a + b*\text{Cot}[d/2 + Pi/4 + (e*x)/2]])/(16*b^5*e) - (a*\text{Cos}[d + e*x] - b*\text{Sin}[d + e*x])/(16*b^2*e*(a + b*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])^2) + (3*(a^2*\text{Cos}[d + e*x] - a*b*\text{Sin}[d + e*x]))/(16*b^4*e*(a + b*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x]))$

Rule 31

$\text{Int}[(a + b*x)^(-1), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rule 3123

$\text{Int}[(\text{cos}[(d + e*x)] + (e*x))*(b + a + c*\text{sin}[(d + e*x)] + (e*x))^(n-1), x_Symbol] \rightarrow \text{Module}\{f = \text{FreeFactors}[\text{Cot}[(d + e*x)/2 + Pi/4], x]\}, -\text{Dist}[f/e, \text{Subst}[\text{Int}[1/(a + b*f*x), x], x, \text{Cot}[(d + e*x)/2 + Pi/4]/f], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \&\& \text{EqQ}[a - c, 0] \&\& \text{NeQ}[a - b, 0]$

Rule 3129

$\text{Int}[(\text{cos}[(d + e*x)] + (e*x))*(b + a + c*\text{sin}[(d + e*x)] + (e*x))^(n-1), x_Symbol] \rightarrow \text{Simp}[(-c*\text{Cos}[d + e*x] + b*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^(n+1)/(e*(n+1)*(a^2 - b^2 - c^2)), x] + \text{Dist}[1/((n+1)*(a^2 - b^2 - c^2)), \text{Int}[(a*(n+1) - b*(n+2)*\text{Cos}[d + e*x] - c*(n+2)*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^(n+1), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \&\& \text{NeQ}[a^2 - b^2 - c^2, 0] \&\& \text{LtQ}[n, -1] \&\& \text{NeQ}[n, -3/2]$

Rule 3153

$\text{Int}[(A + \text{cos}[(d + e*x)] + (e*x))*(B + C*\text{sin}[(d + e*x)] + (e*x))^(n-1), x_Symbol] \rightarrow \text{Simp}[(c*B - b*C - (a*C - c*A)*\text{Cos}[d + e*x] + (a*B - b*A)*\text{Sin}[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])), x] + \text{Dist}[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), \text{Int}[1/(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, A, B, C\}, x \&\& \text{NeQ}[a^2 - b^2 - c^2, 0] \&\& \text{NeQ}[a*A - b*B - c*C, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2a + 2b \cos(d + ex) + 2a \sin(d + ex))^3} dx &= -\frac{a \cos(d + ex) - b \sin(d + ex)}{16b^2 e (a + b \cos(d + ex) + a \sin(d + ex))^2} + \int \frac{-4a + 2b \cos(d + ex) + 2a \sin(d + ex)}{(2a + 2b \cos(d + ex) + 2a \sin(d + ex))^3} dx \\
&= -\frac{a \cos(d + ex) - b \sin(d + ex)}{16b^2 e (a + b \cos(d + ex) + a \sin(d + ex))^2} + \frac{3(a^2 \cos(d + ex) - b^2 \sin(d + ex))}{16b^4 e (a + b \cos(d + ex) + a \sin(d + ex))} \\
&= -\frac{a \cos(d + ex) - b \sin(d + ex)}{16b^2 e (a + b \cos(d + ex) + a \sin(d + ex))^2} + \frac{3(a^2 \cos(d + ex) - b^2 \sin(d + ex))}{16b^4 e (a + b \cos(d + ex) + a \sin(d + ex))} \\
&= -\frac{(3a^2 + b^2) \log\left(a + b \cot\left(\frac{d}{2} + \frac{\pi}{4} + \frac{ex}{2}\right)\right)}{16b^5 e} - \frac{a \cos(d + ex) - b \sin(d + ex)}{16b^2 e (a + b \cos(d + ex) + a \sin(d + ex))}
\end{aligned}$$

Mathematica [A] time = 2.43, size = 255, normalized size = 1.80

$$-\frac{b^2(a^2+b^2)}{\left((a-b)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)^2} + \frac{6ab(a^2+b^2)\sin\left(\frac{1}{2}(d+ex)\right)}{(a+b)\left((a-b)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)} - 2(3a^2+b^2)\log\left(\sin\left(\frac{1}{2}(d+ex)\right)\right) + \dots$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]

[Out] -1/32*(-2*(3*a^2 + b^2)*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]] + 2*(3*a^2 + b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2]] + b^2/(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^2 + (6*a*b*Sin[(d + e*x)/2])/(Cos[(d + e*x)/2] + Sin[(d + e*x)/2]) - (b^2*(a^2 + b^2))/((a + b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2])^2 + (6*a*b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2]))/(b^5*e)

fricas [B] time = 0.86, size = 420, normalized size = 2.96

$$\frac{12 a^2 b^2 \cos(ex + d)^2 - 6 a^2 b^2 + 2(3 a^3 b - a b^3) \cos(ex + d) - (6 a^4 + 2 a^2 b^2 - (3 a^4 - 2 a^2 b^2 - b^4) \cos(ex + d))^2 + \dots}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="fricas")

[Out] 1/32*(12*a^2*b^2*cos(e*x + d)^2 - 6*a^2*b^2 + 2*(3*a^3*b - a*b^3)*cos(e*x + d) - (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(e*x + d))^2 + 2*(3*a^3*b + a*b^3)*cos(e*x + d) + 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)*cos(e*x + d))*sin(e*x + d)*log(2*a*b*cos(e*x + d) + a^2 + b^2 + (a^2 - b^2)*sin(e*x + d)) + (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(e*x + d))^2 + 2*(3*a^3*b + a*b^3)*cos(e*x + d) + 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)*cos(e*x + d))*sin(e*x + d)*log(sin(e*x + d) + 1) - 2*(3*a^2*b^2 - b^4 - 3*(a^3*b - a*b^3)*cos(e*x + d))*sin(e*x + d))/(2*a*b^6*e*cos(e*x + d) + 2*a^2*b^5*e - (a^2*b^5 - b^7)*e*cos(e*x + d)^2 + 2*(a*b^6*e*cos(e*x + d) + a^2*b^5*e)*sin(e*x + d))

giac [B] time = 0.24, size = 481, normalized size = 3.39

$$\frac{1}{16} \left(2 \left(3 a^5 \tan\left(\frac{1}{2} x e + \frac{1}{2} d\right)^3 - 9 a^4 b \tan\left(\frac{1}{2} x e + \frac{1}{2} d\right)^3 + 10 a^3 b^2 \tan\left(\frac{1}{2} x e + \frac{1}{2} d\right)^3 - 6 a^2 b^3 \tan\left(\frac{1}{2} x e + \frac{1}{2} d\right)^3 + a b^4 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="giac")

[Out] 1/16*(2*(3*a^5*tan(1/2*x*e + 1/2*d)^3 - 9*a^4*b*tan(1/2*x*e + 1/2*d)^3 + 10*a^3*b^2*tan(1/2*x*e + 1/2*d)^3 - 6*a^2*b^3*tan(1/2*x*e + 1/2*d)^3 + a*b^4*tan(1/2*x*e + 1/2*d)^3 + b^5*tan(1/2*x*e + 1/2*d)^3 + 9*a^5*tan(1/2*x*e + 1/2*d)^2 - 18*a^4*b*tan(1/2*x*e + 1/2*d)^2 + 12*a^3*b^2*tan(1/2*x*e + 1/2*d)^2 - 6*a^2*b^3*tan(1/2*x*e + 1/2*d)^2 + a*b^4*tan(1/2*x*e + 1/2*d)^2 + 9*a^5*tan(1/2*x*e + 1/2*d) - 9*a^4*b*tan(1/2*x*e + 1/2*d) - 2*a^3*b^2*tan(1/2*x*e + 1/2*d) + 2*a^2*b^3*tan(1/2*x*e + 1/2*d) - 5*a*b^4*tan(1/2*x*e + 1/2*d) + b^5*tan(1/2*x*e + 1/2*d) + 3*a^5 - 4*a^3*b^2 - a*b^4)/((a^2*b^4 - 2*a*b^5 + b^6)*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + a + b)^2) + (3*a^2 + b^2)*log(abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) + 2*a - 2*abs(b))/abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) + 2*a + 2*abs(b)))/(b^4*abs(b)))e^(-1)

maple [B] time = 0.57, size = 639, normalized size = 4.50

$$\frac{3 \ln\left(a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - b \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + a + b\right) a^3}{16e b^5 (a - b)} + \frac{3 \ln\left(a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - b \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + a + b\right) a^2 \ln\left(a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - b \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + a + b\right)}{16e b^4 (a - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x)

[Out] -3/16/e/b^5/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^3+3/16/e/b^4/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^2-1/16/e/b^3/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a+1/16/e/b^2/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)+1/16/e/b^3/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2*a^4+1/8/e/b/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2*a^2+1/16/e*b/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2+3/16/e/b^4/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^4-1/4/e/b^3/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^3+1/8/e/b^2/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a^2-1/4/e/b/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)*a-1/16/e/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)-1/16/e/b^3/(1+tan(1/2*d+1/2*e*x))^2+3/16/e/b^4/(1+tan(1/2*d+1/2*e*x))*a+1/16/e/b^3/(1+tan(1/2*d+1/2*e*x))+3/16/e/b^5*ln(1+tan(1/2*d+1/2*e*x))*a^2+1/16/e/b^3*ln(1+tan(1/2*d+1/2*e*x))

maxima [B] time = 0.35, size = 493, normalized size = 3.47

$$\frac{2 \left(3 a^5 - 4 a^3 b^2 - a b^4 + \frac{(9 a^5 - 9 a^4 b - 2 a^3 b^2 + 2 a^2 b^3 - 5 a b^4 + b^5) \sin(ex+d)}{\cos(ex+d)+1} + \frac{(9 a^5 - 18 a^4 b + 12 a^3 b^2 - 6 a^2 b^3 + a b^4) \sin(ex+d)^2}{(\cos(ex+d)+1)^2} + \frac{(3 a^5 - 9 a^4 b + 10 a^3 b^2 - 6 a^2 b^3 + a b^4 + b^5) \sin(ex+d)^3}{(\cos(ex+d)+1)^3} \right)}{16 e} + \frac{a^4 b^4 - 2 a^2 b^6 + b^8 + \frac{4(a^4 b^4 - a^3 b^5 - a^2 b^6 + a b^7) \sin(ex+d)}{\cos(ex+d)+1} + \frac{2(3 a^4 b^4 - 6 a^3 b^5 + 2 a^2 b^6 + 2 a b^7 - b^8) \sin(ex+d)^2}{(\cos(ex+d)+1)^2} + \frac{4(a^4 b^4 - 3 a^3 b^5 + 3 a^2 b^6 - a b^7) \sin(ex+d)^3}{(\cos(ex+d)+1)^3} + \frac{(a^4 b^4 - 4 a^3 b^5 + 6 a^2 b^6 - 4 a b^7 + b^8) \sin(ex+d)^4}{(\cos(ex+d)+1)^4}}{16 e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="maxima")

[Out] 1/16*(2*(3*a^5 - 4*a^3*b^2 - a*b^4 + (9*a^5 - 9*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 5*a*b^4 + b^5)*sin(e*x + d)/(cos(e*x + d) + 1) + (9*a^5 - 18*a^4*b + 12*a^3*b^2 - 6*a^2*b^3 + a*b^4)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + (3*a^5 - 9*a^4*b + 10*a^3*b^2 - 6*a^2*b^3 + a*b^4 + b^5)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/(a^4*b^4 - 2*a^2*b^6 + b^8 + 4*(a^4*b^4 - a^3*b^5 - a^2*b^6 + a*b^7)*sin(e*x + d)/(cos(e*x + d) + 1) + 2*(3*a^4*b^4 - 6*a^3*b^5 + 2*a^2*b^6 - 4*a*b^7 + b^8)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + 4*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7 + b^8)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3) + (3*a^2 + b^2)*log(abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) + 2*a - 2*abs(b))/abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) + 2*a + 2*abs(b)))/(b^4*abs(b)))e^(-1)

$$3b^5 + 3a^2b^6 - ab^7) \sin(ex + d)^3 / (\cos(ex + d) + 1)^3 + (a^4b^4 - 4a^3b^5 + 6a^2b^6 - 4ab^7 + b^8) \sin(ex + d)^4 / (\cos(ex + d) + 1)^4 - (3a^2 + b^2) \log(-a - b - (a - b) \sin(ex + d) / (\cos(ex + d) + 1)) / b^5 + (3a^2 + b^2) \log(\sin(ex + d) / (\cos(ex + d) + 1) + 1) / b^5 / e$$

mupad [B] time = 6.45, size = 360, normalized size = 2.54

$$\frac{-3a^5 + 4a^3b^2 + ab^4}{2b^4(a-b)^2} + \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)(-9a^5 + 9a^4b + 2a^3b^2 - 2a^2b^3 + 5ab^4 - b^5)}{2b^4(a-b)^2} + \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^3(-3a^4 + 6a^3b - 4a^2b^2 + 2ab^3 + b^4)}{2b^4(a-b)} - \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^4(4a^2 - 8ab + 4b^2)}{2b^4(a-b)}$$

$$e \left(8ab + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 (24a^2 - 8b^2) - \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^3 (16ab - 16a^2) + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^4 (4a^2 - 8ab + 4b^2) + 4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^3,x)

[Out] - ((a*b^4 - 3*a^5 + 4*a^3*b^2)/(2*b^4*(a - b)^2) + (tan(d/2 + (e*x)/2)*(5*a*b^4 + 9*a^4*b - 9*a^5 - b^5 - 2*a^2*b^3 + 2*a^3*b^2))/(2*b^4*(a - b)^2) + (tan(d/2 + (e*x)/2)^3*(2*a*b^3 + 6*a^3*b - 3*a^4 + b^4 - 4*a^2*b^2))/(2*b^4*(a - b)) - (tan(d/2 + (e*x)/2)^2*(a*b^4 - 18*a^4*b + 9*a^5 - 6*a^2*b^3 + 12*a^3*b^2))/(2*b^4*(a - b)^2))/(e*(8*a*b + tan(d/2 + (e*x)/2)^2*(24*a^2 - 8*b^2) - tan(d/2 + (e*x)/2)^3*(16*a*b - 16*a^2) + tan(d/2 + (e*x)/2)^4*(4*a^2 - 8*a*b + 4*b^2) + 4*a^2 + 4*b^2 + tan(d/2 + (e*x)/2)*(16*a*b + 16*a^2))) - (atanh((2*a + tan(d/2 + (e*x)/2)*(2*a - 2*b))/(2*b))*(3*a^2 + b^2))/(8*b^5*e)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x)

[Out] Timed out

$$3.387 \quad \int \frac{1}{(2a+2b \cos(d+ex)+2a \sin(d+ex))^4} dx$$

Optimal. Leaf size=215

$$\frac{5(a^2 \cos(d+ex) - ab \sin(d+ex))}{96b^4 e (a \sin(d+ex) + a + b \cos(d+ex))^2} + \frac{a(5a^2 + 3b^2) \log\left(a + b \cot\left(\frac{d}{2} + \frac{ex}{2} + \frac{\pi}{4}\right)\right)}{32b^7 e} - \frac{a(15a^2 + 4b^2) \cos(d+ex)}{96b^6 e (a \sin(d+ex) + a + b \cos(d+ex))}$$

[Out] $1/32*a*(5*a^2+3*b^2)*\ln(a+b*\cot(1/2*d+1/4*Pi+1/2*e*x))/b^7/e+1/48*(-a*\cos(e*x+d)+b*\sin(e*x+d))/b^2/e/(a+b*\cos(e*x+d)+a*\sin(e*x+d))^3+5/96*(a^2*\cos(e*x+d)-a*b*\sin(e*x+d))/b^4/e/(a+b*\cos(e*x+d)+a*\sin(e*x+d))^2+1/96*(-a*(15*a^2+4*b^2)*\cos(e*x+d)+b*(15*a^2+4*b^2)*\sin(e*x+d))/b^6/e/(a+b*\cos(e*x+d)+a*\sin(e*x+d))$

Rubi [A] time = 0.24, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3129, 3156, 3153, 3123, 31}

$$\frac{5(a^2 \cos(d+ex) - ab \sin(d+ex))}{96b^4 e (a \sin(d+ex) + a + b \cos(d+ex))^2} - \frac{a(15a^2 + 4b^2) \cos(d+ex) - b(15a^2 + 4b^2) \sin(d+ex)}{96b^6 e (a \sin(d+ex) + a + b \cos(d+ex))} + \frac{a(5a^2 + 3b^2) \log\left(a + b \cot\left(\frac{d}{2} + \frac{ex}{2} + \frac{\pi}{4}\right)\right)}{32b^7 e}$$

Antiderivative was successfully verified.

[In] `Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4), x]`

[Out] $(a*(5*a^2 + 3*b^2)*\text{Log}[a + b*\text{Cot}[d/2 + Pi/4 + (e*x)/2]])/(32*b^7*e) - (a*\text{Cos}[d + e*x] - b*\text{Sin}[d + e*x])/(48*b^2*e*(a + b*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])^3) + (5*(a^2*\text{Cos}[d + e*x] - a*b*\text{Sin}[d + e*x]))/(96*b^4*e*(a + b*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])^2) - (a*(15*a^2 + 4*b^2)*\text{Cos}[d + e*x] - b*(15*a^2 + 4*b^2)*\text{Sin}[d + e*x])/(96*b^6*e*(a + b*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x]))$

Rule 31

`Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 3123

`Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, -Dist[f/e, Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]`

Rule 3129

`Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]`

Rule 3153

`Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(A_. + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] := Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]`

$n[d + e*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, A, B, C\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0] \ \&\& \ \text{NeQ}[a*A - b*B - c*C, 0]$

Rule 3156

$\text{Int}[\left((a_{.}) + \cos[(d_{.}) + (e_{.})*(x_{.})]\right)*(b_{.}) + (c_{.})*\sin[(d_{.}) + (e_{.})*(x_{.})])^{\wedge}(n_{.})*\left((A_{.}) + \cos[(d_{.}) + (e_{.})*(x_{.})]\right)*(B_{.}) + (C_{.})*\sin[(d_{.}) + (e_{.})*(x_{.})]), x_Symbol] \rightarrow -\text{Simp}[\left((c*B - b*C - (a*C - c*A)*\text{Cos}[d + e*x] + (a*B - b*A)*\text{Sin}[d + e*x]\right)*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{\wedge}(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + \text{Dist}[1/((n + 1)*(a^2 - b^2 - c^2)), \text{Int}[(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{\wedge}(n + 1)*\text{Simp}[(n + 1)*(a*A - b*B - c*C) + (n + 2)*(a*B - b*A)*\text{Cos}[d + e*x] + (n + 2)*(a*C - c*A)*\text{Sin}[d + e*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, A, B, C\}, x] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0] \ \&\& \ \text{NeQ}[n, -2]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(2a + 2b \cos(d + ex) + 2a \sin(d + ex))^4} dx &= -\frac{a \cos(d + ex) - b \sin(d + ex)}{48b^2e(a + b \cos(d + ex) + a \sin(d + ex))^3} + \frac{\int \frac{-6a+4b \cos(d+ex)+4}{(2a+2b \cos(d+ex)+2a \sin(d+ex))^4} dx}{12b^2} \\ &= -\frac{a \cos(d + ex) - b \sin(d + ex)}{48b^2e(a + b \cos(d + ex) + a \sin(d + ex))^3} + \frac{5(a^2 \cos(d + ex) - b^2 \sin(d + ex))}{96b^4e(a + b \cos(d + ex) + a \sin(d + ex))^2} \\ &= -\frac{a \cos(d + ex) - b \sin(d + ex)}{48b^2e(a + b \cos(d + ex) + a \sin(d + ex))^3} + \frac{5(a^2 \cos(d + ex) - b^2 \sin(d + ex))}{96b^4e(a + b \cos(d + ex) + a \sin(d + ex))^2} \\ &= -\frac{a \cos(d + ex) - b \sin(d + ex)}{48b^2e(a + b \cos(d + ex) + a \sin(d + ex))^3} + \frac{5(a^2 \cos(d + ex) - b^2 \sin(d + ex))}{96b^4e(a + b \cos(d + ex) + a \sin(d + ex))^2} \\ &= \frac{a(5a^2 + 3b^2) \log\left(a + b \cot\left(\frac{d}{2} + \frac{\pi}{4} + \frac{ex}{2}\right)\right)}{32b^7e} - \frac{a \cos(d + ex)}{48b^2e(a + b \cos(d + ex) + a \sin(d + ex))^3} \end{aligned}$$

Mathematica [B] time = 3.01, size = 632, normalized size = 2.94

$$-12a(5a^2 + 3b^2) \log\left(\sin\left(\frac{1}{2}(d + ex)\right) + \cos\left(\frac{1}{2}(d + ex)\right)\right) + 12a(5a^2 + 3b^2) \log\left((a - b) \sin\left(\frac{1}{2}(d + ex)\right) + (a + b) \cos\left(\frac{1}{2}(d + ex)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4), x]

[Out] (-12*a*(5*a^2 + 3*b^2)*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]] + 12*a*(5*a^2 + 3*b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2]] + (b*(150*a^6 + 130*a^4*b^2 + 24*a^2*b^4 - 3*a^2*(25*a^4 - 50*a^3*b + 5*a^2*b^2 - 30*a*b^3 + 4*b^4))*Cos[d + e*x] - 6*a^2*(15*a^4 + 20*a^3*b + 9*a^2*b^2 + 2*a*b^3 - 2*b^4))*Cos[2*(d + e*x)] + 15*a^6*Cos[3*(d + e*x)] - 30*a^5*b*Cos[3*(d + e*x)] - 41*a^4*b^2*Cos[3*(d + e*x)] - 38*a^3*b^3*Cos[3*(d + e*x)] - 12*a^2*b^4*Cos[3*(d + e*x)] - 8*a*b^5*Cos[3*(d + e*x)] + 225*a^6*Sin[d + e*x] + 75*a^5*b*Sin[d + e*x] + 180*a^4*b^2*Sin[d + e*x] + 15*a^3*b^3*Sin[d + e*x] + 27*a^2*b^4*Sin[d + e*x] + 12*a*b^5*Sin[d + e*x] + 12*b^6*Sin[d + e*x] - 60*a^6*Sin[2*(d + e*x)] + 120*a^5*b*Sin[2*(d + e*x)] + 54*a^4*b^2*Sin[2*(d + e*x)] + 102*a^3*b^3*Sin[2*(d + e*x)] + 6*a^2*b^4*Sin[2*(d + e*x)] + 6*a*b^5*Sin[2*(d + e*x)] - 15*a^6*Sin[3*(d + e*x)] - 45*a^5*b*Sin[3*(d + e*x)] - 4*a^4*b^2*Sin[3*(d + e*x)] + 3*a^3*b^3*Sin[3*(d + e*x)] + 15*a^2*b^4*Sin[3*(d + e*x)] + 4*a*b^5*Sin[3*(d + e*x)] + 4*b^6*Sin[3*(d + e*x)])) / ((a + b

)*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^3*((a + b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2])^3)/(384*b^7*e)

fricas [B] time = 1.09, size = 729, normalized size = 3.39

$$\frac{60 a^4 b^2 + 6 a^2 b^4 + 2 (15 a^5 b - 41 a^3 b^3 - 12 a b^5) \cos(ex + d)^3 - 12 (10 a^4 b^2 + a^2 b^4) \cos(ex + d)^2 - 6 (10 a^5 b -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="fricas")

[Out] 1/192*(60*a^4*b^2 + 6*a^2*b^4 + 2*(15*a^5*b - 41*a^3*b^3 - 12*a*b^5)*cos(e*x + d)^3 - 12*(10*a^4*b^2 + a^2*b^4)*cos(e*x + d)^2 - 6*(10*a^5*b - 9*a^3*b^3 - 2*a*b^5)*cos(e*x + d) + 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*cos(e*x + d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x + d) + (20*a^6 + 12*a^4*b^2 - (5*a^6 - 12*a^4*b^2 - 9*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x + d))*sin(e*x + d)*log(2*a*b*cos(e*x + d) + a^2 + b^2 + (a^2 - b^2)*sin(e*x + d)) - 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*cos(e*x + d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x + d) + (20*a^6 + 12*a^4*b^2 - (5*a^6 - 12*a^4*b^2 - 9*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x + d))*sin(e*x + d))*log(sin(e*x + d) + 1) + 2*(30*a^4*b^2 + 3*a^2*b^4 + 2*b^6 - (45*a^4*b^2 - 3*a^2*b^4 - 4*b^6)*cos(e*x + d)^2 - 3*(10*a^5*b - 9*a^3*b^3 - a*b^5)*cos(e*x + d))*sin(e*x + d))/(6*a^2*b^8*e*cos(e*x + d) + 4*a^3*b^7*e - (3*a^2*b^8 - b^10)*e*cos(e*x + d)^3 - 3*(a^3*b^7 - a*b^9)*e*cos(e*x + d)^2 + (6*a^2*b^8*e*cos(e*x + d) + 4*a^3*b^7*e - (a^3*b^7 - 3*a*b^9)*e*cos(e*x + d)^2)*sin(e*x + d))

giac [B] time = 0.31, size = 1006, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="giac")

[Out] -1/96*(2*(15*a^8*tan(1/2*x*e + 1/2*d)^5 - 75*a^7*b*tan(1/2*x*e + 1/2*d)^5 + 159*a^6*b^2*tan(1/2*x*e + 1/2*d)^5 - 195*a^5*b^3*tan(1/2*x*e + 1/2*d)^5 + 165*a^4*b^4*tan(1/2*x*e + 1/2*d)^5 - 105*a^3*b^5*tan(1/2*x*e + 1/2*d)^5 + 51*a^2*b^6*tan(1/2*x*e + 1/2*d)^5 - 21*a*b^7*tan(1/2*x*e + 1/2*d)^5 + 6*b^8*tan(1/2*x*e + 1/2*d)^5 + 75*a^8*tan(1/2*x*e + 1/2*d)^4 - 300*a^7*b*tan(1/2*x*e + 1/2*d)^4 + 495*a^6*b^2*tan(1/2*x*e + 1/2*d)^4 - 480*a^5*b^3*tan(1/2*x*e + 1/2*d)^4 + 345*a^4*b^4*tan(1/2*x*e + 1/2*d)^4 - 180*a^3*b^5*tan(1/2*x*e + 1/2*d)^4 + 57*a^2*b^6*tan(1/2*x*e + 1/2*d)^4 - 12*a*b^7*tan(1/2*x*e + 1/2*d)^4 + 150*a^8*tan(1/2*x*e + 1/2*d)^3 - 450*a^7*b*tan(1/2*x*e + 1/2*d)^3 + 500*a^6*b^2*tan(1/2*x*e + 1/2*d)^3 - 300*a^5*b^3*tan(1/2*x*e + 1/2*d)^3 + 126*a^4*b^4*tan(1/2*x*e + 1/2*d)^3 + 22*a^3*b^5*tan(1/2*x*e + 1/2*d)^3 - 48*a^2*b^6*tan(1/2*x*e + 1/2*d)^3 + 12*a*b^7*tan(1/2*x*e + 1/2*d)^3 - 4*b^8*tan(1/2*x*e + 1/2*d)^3 + 150*a^8*tan(1/2*x*e + 1/2*d)^2 - 300*a^7*b*tan(1/2*x*e + 1/2*d)^2 + 120*a^6*b^2*tan(1/2*x*e + 1/2*d)^2 + 60*a^5*b^3*tan(1/2*x*e + 1/2*d)^2 - 102*a^4*b^4*tan(1/2*x*e + 1/2*d)^2 + 144*a^3*b^5*tan(1/2*x*e + 1/2*d)^2 - 60*a^2*b^6*tan(1/2*x*e + 1/2*d)^2 + 12*a*b^7*tan(1/2*x*e + 1/2*d)^2 + 75*a^8*tan(1/2*x*e + 1/2*d) - 75*a^7*b*tan(1/2*x*e + 1/2*d) - 75*a^6*b^2*tan(1/2*x*e + 1/2*d) + 75*a^5*b^3*tan(1/2*x*e + 1/2*d) - 39*a^4*b^4*tan(1/2*x*e + 1/2*d) + 39*a^3*b^5*tan(1/2*x*e + 1/2*d) + 33*a^2*b^6*tan(1/2*x*e + 1/2*d) - 15*a*b^7*tan(1/2*x*e + 1/2*d) + 6*b^8*tan(1/2*x*e + 1/2*d) + 15*a^8 - 31*a^6*b^2 + 9*a^4*b^4 + 15*a^2*b^6)/((a^3*b^6 - 3*a^2*b^7 + 3*a*b^8 - b^9)*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + a + b)^3) + 3*(5*a^3 + 3*a*b^2)*log(abs(2*a*tan(1/2*x*

$$e + 1/2*d) - 2*b*\tan(1/2*x*e + 1/2*d) + 2*a - 2*\text{abs}(b))/\text{abs}(2*a*\tan(1/2*x*e + 1/2*d) - 2*b*\tan(1/2*x*e + 1/2*d) + 2*a + 2*\text{abs}(b)))/(b^6*\text{abs}(b)))*e^{(-1)}$$

maple [B] time = 0.59, size = 1069, normalized size = 4.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x)

[Out]
$$\begin{aligned} & -5/32/e*a^3/b^7*\ln(1+\tan(1/2*d+1/2*e*x))-3/32/e*a/b^5*\ln(1+\tan(1/2*d+1/2*e*x)) \\ & -1/16/e/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)^3*a^2+3/32/e/(a-b)^3 \\ & /((a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)^2*a-1/48/e*b^2/(a-b)^3 \\ & /((a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)^3+1/32/e*b/(a-b)^3 \\ & /((a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)^2+1/16/e/b^5/(1+\tan(1/2*d+1/2*e*x))^2 \\ & *a-5/32/e/b^6/(1+\tan(1/2*d+1/2*e*x))*a^2-1/16/e/b^5/(1+\tan(1/2*d+1/2*e*x))*a^5 \\ & +5/32/e*a^4/b^7/(a-b)*\ln(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)-1/48/e/b^4 \\ & /((1+\tan(1/2*d+1/2*e*x))^3+1/32/e/b^4/(1+\tan(1/2*d+1/2*e*x))^2-1/16/e/b^4/(1+\tan(1/2*d+1/2*e*x)) \\ & -1/16/e/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)-5/32/e*a^3/b^6 \\ & /((a-b)*\ln(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)+3/32/e*a^2/b^5 \\ & /((a-b)*\ln(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)-3/32/e*a/b^4 \\ & /((a-b)*\ln(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)-1/48/e/b^4/(a-b)^3 \\ & /((a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)^3*a^6-1/16/e/b^2/(a-b)^3 \\ & /((a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)^3*a^4-1/16/e/b^5/(a-b)^3 \\ & /((a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)^2*a^6+3/32/e/b^4/(a-b)^3 \\ & /((a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)^2*a^5-3/32/e/b^3/(a-b)^3 \\ & /((a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)^2*a^4+3/16/e/b^2/(a-b)^3 \\ & /((a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)^2*a^3-5/32/e/b^6/(a-b)^3 \\ & /((a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)*a^6+3/8/e/b^5/(a-b)^3 \\ & /((a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)*a^5-3/8/e/b^4/(a-b)^3 \\ & /((a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)*a^4+3/8/e/b^3/(a-b)^3 \\ & /((a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)*a^3-9/32/e/b^2/(a-b)^3 \\ & /((a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)+a+b)*a^2 \end{aligned}$$

maxima [B] time = 0.39, size = 963, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/96*(2*(15*a^8 - 31*a^6*b^2 + 9*a^4*b^4 + 15*a^2*b^6 + 3*(25*a^8 - 25*a^7*b \\ & *b - 25*a^6*b^2 + 25*a^5*b^3 - 13*a^4*b^4 + 13*a^3*b^5 + 11*a^2*b^6 - 5*a*b^7 + 2*b^8)*\sin(e*x + d) \\ & /(\cos(e*x + d) + 1) + 6*(25*a^8 - 50*a^7*b + 20*a^6*b^2 + 10*a^5*b^3 - 17*a^4*b^4 + 24*a^3*b^5 - 10*a^2*b^6 \\ & + 2*a*b^7)*\sin(e*x + d)^2/(\cos(e*x + d) + 1)^2 + 2*(75*a^8 - 225*a^7*b + 250*a^6*b^2 - 150*a^5*b^3 \\ & + 63*a^4*b^4 + 11*a^3*b^5 - 24*a^2*b^6 + 6*a*b^7 - 2*b^8)*\sin(e*x + d)^3/(\cos(e*x + d) + 1)^3 \\ & + 3*(25*a^8 - 100*a^7*b + 165*a^6*b^2 - 160*a^5*b^3 + 115*a^4*b^4 - 60*a^3*b^5 + 19*a^2*b^6 - 4*a*b^7)*\sin(e*x + d)^4 \\ & /(\cos(e*x + d) + 1)^4 + 3*(5*a^8 - 25*a^7*b + 53*a^6*b^2 - 65*a^5*b^3 + 55*a^4*b^4 - 35*a^3*b^5 \\ & + 17*a^2*b^6 - 7*a*b^7 + 2*b^8)*\sin(e*x + d)^5/(\cos(e*x + d) + 1)^5)/(a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^{10} - b^{12} \\ & + 6*(a^6*b^6 - a^5*b^7 - 2*a^4*b^8 + 2*a^3*b^9 + a^2*b^{10} - a*b^{11})*\sin(e*x + d)/(\cos(e*x + d) + 1) \\ & + 3*(5*a^6*b^6 - 10*a^5*b^7 - a^4*b^8 + 12*a^3*b^9 - 5*a^2*b^{10} - 2*a*b^{11} + b^{12})*\sin(e*x + d)^2 \\ & /(\cos(e*x + d) + 1)^2 + 4*(5*a^6*b^6 - 15*a^5*b^7 + 12*a^4*b^8 + 4*a^3*b^9 - 9*a^2*b^{10} + 3*a*b^{11})*\sin(e*x + d)^3 \\ & /(\cos(e*x + d) + 1)^3 + 3*(5*a^6*b^6 - 20*a^5*b^7 + 29*a^4*b^8 - 16*a^3*b^9 - a^2*b^{10} + 4*a*b^{11} - b^{12})*\sin(e*x + d)^4 \\ & /(\cos(e*x + d) + 1)^4 + 6*(a^6*b^6 - 5*a^5*b^7 + 10*a^4*b^8 - 10*a^3*b^9 + 5*a^2*b^{10} - a*b^{11})*\sin(e*x + d)^5 \\ & /(\cos(e*x + \end{aligned}$$

$d) + 1)^5 + (a^6 b^6 - 6 a^5 b^7 + 15 a^4 b^8 - 20 a^3 b^9 + 15 a^2 b^{10} - 6 a b^{11} + b^{12}) \sin(e x + d)^6 / (\cos(e x + d) + 1)^6 - 3 (5 a^3 + 3 a b^2) \log(-a - b - (a - b) \sin(e x + d) / (\cos(e x + d) + 1)) / b^7 + 3 (5 a^3 + 3 a b^2) \log(\sin(e x + d) / (\cos(e x + d) + 1) + 1) / b^7) / e$

mupad [B] time = 7.22, size = 730, normalized size = 3.40

$$\frac{a \operatorname{atanh}\left(\frac{a\left(2a + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)(2a - 2b)\right)(5a^2 + 3b^2)}{2b(5a^3 + 3ab^2)}\right)(5a^2 + 3b^2)}{16b^7 e} - \frac{15a^8 - 31a^6 b^2 + 9a^4 b^4 + 15a^2 b^6}{6b^6(a-b)^3} + \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2(25a^8 - 50a^7 b + 20a^6 b^2)}{e\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)^5(48a^3 - 96a^2 b + 48ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^4, x)

[Out] $(a \operatorname{atanh}((a(2a + \tan(d/2 + (ex)/2)(2a - 2b)))(5a^2 + 3b^2)) / (2b(3ab^2 + 5a^3))) (5a^2 + 3b^2) / (16b^7 e) - ((15a^8 + 15a^2 b^6 + 9a^4 b^4 - 31a^6 b^2) / (6b^6(a-b)^3) + (\tan(d/2 + (ex)/2)^2(2a^7 b - 50a^7 b + 25a^8 - 10a^2 b^6 + 24a^3 b^5 - 17a^4 b^4 + 10a^5 b^3 + 20a^6 b^2)) / (b^6(a-b)^3) + (\tan(d/2 + (ex)/2)^4(4a^6 b - 75a^6 b + 25a^7 b - 15a^2 b^5 + 45a^3 b^4 - 70a^4 b^3 + 90a^5 b^2)) / (2b^6(a-b)^2) + (\tan(d/2 + (ex)/2)^3(6a^6 b^7 - 225a^7 b + 75a^8 - 2b^8 - 24a^2 b^6 + 11a^3 b^5 + 63a^4 b^4 - 150a^5 b^3 + 250a^6 b^2)) / (3b^6(a-b)^3) + (\tan(d/2 + (ex)/2)^5(5a^6 - 15a^5 b - 3a^4 b^2 + 2b^6 + 9a^2 b^4 - 14a^3 b^3 + 18a^4 b^2)) / (2b^6(a-b)) + (\tan(d/2 + (ex)/2)(25a^8 - 25a^7 b - 5a^6 b^2 + 2b^8 + 11a^2 b^6 + 13a^3 b^5 - 13a^4 b^4 + 25a^5 b^3 - 25a^6 b^2)) / (2b^6(a-b)^3)) / (e(\tan(d/2 + (ex)/2)^5(48a^3 b^2 - 96a^2 b + 48a^3) + \tan(d/2 + (ex)/2)^6(24a^2 b^2 - 24a^2 b + 8a^3 - 8b^3) - \tan(d/2 + (ex)/2)^2(24a^2 b^2 - 120a^2 b - 120a^3 + 24b^3) - \tan(d/2 + (ex)/2)^4(24a^2 b^2 + 120a^2 b - 120a^3 - 24b^3) + 24a^2 b^2 + 24a^2 b - \tan(d/2 + (ex)/2)^3(96a^2 b^2 - 160a^3) + \tan(d/2 + (ex)/2)(48a^2 b^2 + 96a^2 b + 48a^3) + 8a^3 + 8b^3))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**4, x)

[Out] Timed out

3.388 $\int (2a + 2b \cos(d + ex) - 2a \sin(d + ex))^3 dx$

Optimal. Leaf size=157

$$\frac{4b(15a^2 + 4b^2) \sin(d + ex)}{3e} + \frac{4a(15a^2 + 4b^2) \cos(d + ex)}{3e} + 4ax(5a^2 + 3b^2) + \frac{20(a^2 \cos(d + ex) + ab \sin(d + ex))}{3e}$$

[Out] $4*a*(5*a^2+3*b^2)*x+4/3*a*(15*a^2+4*b^2)*\cos(e*x+d)/e+4/3*b*(15*a^2+4*b^2)*\sin(e*x+d)/e+8/3*(a+b*\cos(e*x+d)-a*\sin(e*x+d))^2*(a*\cos(e*x+d)+b*\sin(e*x+d))/e+20/3*(a+b*\cos(e*x+d)-a*\sin(e*x+d))*(a^2*\cos(e*x+d)+a*b*\sin(e*x+d))/e$

Rubi [A] time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3120, 3146, 2637, 2638}

$$\frac{4b(15a^2 + 4b^2) \sin(d + ex)}{3e} + \frac{4a(15a^2 + 4b^2) \cos(d + ex)}{3e} + 4ax(5a^2 + 3b^2) + \frac{20(a^2 \cos(d + ex) + ab \sin(d + ex))}{3e}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a + 2*b*\text{Cos}[d + e*x] - 2*a*\text{Sin}[d + e*x])^3, x]$

[Out] $4*a*(5*a^2 + 3*b^2)*x + (4*a*(15*a^2 + 4*b^2)*\text{Cos}[d + e*x])/(3*e) + (4*b*(15*a^2 + 4*b^2)*\text{Sin}[d + e*x])/(3*e) + (8*(a + b*\text{Cos}[d + e*x] - a*\text{Sin}[d + e*x])^2*(a*\text{Cos}[d + e*x] + b*\text{Sin}[d + e*x]))/(3*e) + (20*(a + b*\text{Cos}[d + e*x] - a*\text{Sin}[d + e*x])*(a^2*\text{Cos}[d + e*x] + a*b*\text{Sin}[d + e*x]))/(3*e)$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3120

$\text{Int}[(\cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*\sin[(d_.) + (e_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(c*\text{Cos}[d + e*x] - b*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{(n-1)}]/(e*n), x] + \text{Dist}[1/n, \text{Int}[\text{Simp}[n*a^2 + (n-1)*(b^2 + c^2) + a*b*(2*n-1)*\text{Cos}[d + e*x] + a*c*(2*n-1)*\text{Sin}[d + e*x], x]*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{(n-2)}, x], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]

Rule 3146

$\text{Int}[(\cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*\sin[(d_.) + (e_.)*(x_.)])^{(n_.)}*((A_.) + \cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*\sin[(d_.) + (e_.)*(x_.)]), x_Symbol] \rightarrow \text{Simp}[(B*c - b*C - a*C*\text{Cos}[d + e*x] + a*B*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^n]/(a*e*(n+1)), x] + \text{Dist}[1/(a*(n+1)), \text{Int}[(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{(n-1)}*\text{Simp}[a*(b*B + c*C)*n + a^2*A*(n+1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n+1))*\text{Cos}[d + e*x] + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n+1))*\text{Sin}[d + e*x], x], x], x] /;$ FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\begin{aligned}
\int (2a + 2b \cos(d + ex) - 2a \sin(d + ex))^3 dx &= \frac{8(a + b \cos(d + ex) - a \sin(d + ex))^2 (a \cos(d + ex) + b \sin(d + ex))}{3e} \\
&= \frac{8(a + b \cos(d + ex) - a \sin(d + ex))^2 (a \cos(d + ex) + b \sin(d + ex))}{3e} \\
&= 4a(5a^2 + 3b^2)x + \frac{8(a + b \cos(d + ex) - a \sin(d + ex))^2 (a \cos(d + ex) + b \sin(d + ex))}{3e} \\
&= 4a(5a^2 + 3b^2)x + \frac{4a(15a^2 + 4b^2) \cos(d + ex)}{3e} + \frac{4b(15a^2 + 4b^2) \sin(d + ex)}{3e}
\end{aligned}$$

Mathematica [A] time = 0.44, size = 136, normalized size = 0.87

$$\frac{2(6a(5a^2 + 3b^2)(d + ex) - 9a(a^2 - b^2)\sin(2(d + ex)) + 9b(5a^2 + b^2)\sin(d + ex) + b(b^2 - 3a^2)\sin(3(d + ex)))}{3e}$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^3,x]

[Out] (2*(6*a*(5*a^2 + 3*b^2)*(d + e*x) + 9*a*(5*a^2 + b^2)*Cos[d + e*x] + 18*a^2*b*Cos[2*(d + e*x)] - a*(a^2 - 3*b^2)*Cos[3*(d + e*x)] + 9*b*(5*a^2 + b^2)*Sin[d + e*x] - 9*a*(a^2 - b^2)*Sin[2*(d + e*x)] + b*(-3*a^2 + b^2)*Sin[3*(d + e*x)]))/(3*e)

fricas [A] time = 0.93, size = 126, normalized size = 0.80

$$\frac{4(18a^2b \cos(ex + d)^2 + 24a^3 \cos(ex + d) - 2(a^3 - 3ab^2) \cos(ex + d)^3 + 3(5a^3 + 3ab^2)ex + (24a^2b + 4b^3) \sin(ex + d))}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="fricas")

[Out] 4/3*(18*a^2*b*cos(e*x + d)^2 + 24*a^3*cos(e*x + d) - 2*(a^3 - 3*a*b^2)*cos(e*x + d)^3 + 3*(5*a^3 + 3*a*b^2)*e*x + (24*a^2*b + 4*b^3 - 2*(3*a^2*b - b^3)*cos(e*x + d)^2 - 9*(a^3 - a*b^2)*cos(e*x + d))*sin(e*x + d))/e

giac [A] time = 0.18, size = 151, normalized size = 0.96

$$12a^2b \cos(2xe + 2d)e^{(-1)} - \frac{2}{3}(a^3 - 3ab^2) \cos(3xe + 3d)e^{(-1)} + 6(5a^3 + ab^2) \cos(xe + d)e^{(-1)} - \frac{2}{3}(3a^2b - b^3) \sin(xe + d)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="giac")

[Out] 12*a^2*b*cos(2*x*e + 2*d)*e^(-1) - 2/3*(a^3 - 3*a*b^2)*cos(3*x*e + 3*d)*e^(-1) + 6*(5*a^3 + a*b^2)*cos(x*e + d)*e^(-1) - 2/3*(3*a^2*b - b^3)*e^(-1)*sin(3*x*e + 3*d) - 6*(a^3 - a*b^2)*e^(-1)*sin(2*x*e + 2*d) + 6*(5*a^2*b + b^3)*e^(-1)*sin(x*e + d) + 4*(5*a^3 + 3*a*b^2)*x

maple [A] time = 0.25, size = 176, normalized size = 1.12

$$\frac{8a^3(ex + d) + 24 \sin(ex + d)a^2b + 24a^3 \cos(ex + d) + 24ab^2 \left(\frac{\sin(ex+d)\cos(ex+d)}{2} + \frac{ex}{2} + \frac{d}{2} \right) + 24(\cos^2(ex + d))}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x)`

[Out] $\frac{8}{e}(a^3(e*x+d)+3*\sin(e*x+d)*a^2*b+3*a^3*\cos(e*x+d)+3*a*b^2*(1/2*\sin(e*x+d)*\cos(e*x+d)+1/2*e*x+1/2*d)+3*\cos(e*x+d)^2*a^2*b+3*a^3*(-1/2*\sin(e*x+d)*\cos(e*x+d)+1/2*e*x+1/2*d)+1/3*b^3*(2+\cos(e*x+d)^2)*\sin(e*x+d)+a*b^2*\cos(e*x+d)^3+a^2*b*\sin(e*x+d)^3+1/3*a^3*(2+\sin(e*x+d)^2)*\cos(e*x+d)$

maxima [A] time = 0.32, size = 188, normalized size = 1.20

$$\frac{8ab^2\cos(ex+d)^3}{e} + \frac{8a^2b\sin(ex+d)^3}{e} + 8a^3x - \frac{8(\cos(ex+d)^3 - 3\cos(ex+d))a^3}{3e} - \frac{8(\sin(ex+d)^3 - 3\sin(ex+d))b^3}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="maxima")`

[Out] $8*a*b^2*\cos(e*x+d)^3/e + 8*a^2*b*\sin(e*x+d)^3/e + 8*a^3*x - 8/3*(\cos(e*x+d)^3 - 3*\cos(e*x+d))*a^3/e - 8/3*(\sin(e*x+d)^3 - 3*\sin(e*x+d))*b^3/e + 24*a^2*(a*\cos(e*x+d)/e + b*\sin(e*x+d)/e) + 6*(4*a*b*\cos(e*x+d)^2/e + (2*e*x + 2*d - \sin(2*e*x + 2*d))*a^2/e + (2*e*x + 2*d + \sin(2*e*x + 2*d))*b^2/e)*a$

mupad [B] time = 3.44, size = 292, normalized size = 1.86

$$\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^4 (48a^3 - 96a^2b + 48ab^2) + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^5 (24a^3 + 48a^2b - 24ab^2 + 16b^3) + 16ab^2 - \tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{e \left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^6 + 3 \tan\left(\frac{d}{2} + \frac{ex}{2}\right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^3,x)`

[Out] $(\tan(d/2 + (e*x)/2)^4*(48*a*b^2 - 96*a^2*b + 48*a^3) + \tan(d/2 + (e*x)/2)^5*(48*a^2*b - 24*a*b^2 + 24*a^3 + 16*b^3) + 16*a*b^2 - \tan(d/2 + (e*x)/2)^2*(96*a^2*b - 128*a^3) + \tan(d/2 + (e*x)/2)^3*(160*a^2*b + (32*b^3)/3) + (176*a^3)/3 + \tan(d/2 + (e*x)/2)*(24*a*b^2 + 48*a^2*b - 24*a^3 + 16*b^3))/(e*(3*\tan(d/2 + (e*x)/2)^2 + 3*\tan(d/2 + (e*x)/2)^4 + \tan(d/2 + (e*x)/2)^6 + 1)) + (8*a*atan((8*a*\tan(d/2 + (e*x)/2)*(5*a^2 + 3*b^2))/(24*a*b^2 + 40*a^3))*(5*a^2 + 3*b^2))/e - (8*a*(5*a^2 + 3*b^2)*(atan(\tan(d/2 + (e*x)/2)) - (e*x)/2))/e$

sympy [A] time = 0.76, size = 291, normalized size = 1.85

$$\begin{cases} 12a^3x\sin^2(d+ex) + 12a^3x\cos^2(d+ex) + 8a^3x + \frac{8a^3\sin^2(d+ex)\cos(d+ex)}{e} - \frac{12a^3\sin(d+ex)\cos(d+ex)}{e} + \frac{16a^3\cos^3(d+ex)}{3e} + \\ x(-2a\sin(d) + 2a + 2b\cos(d))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**3,x)`

[Out] `Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*x + 8*a**3*sin(d + e*x)**2*cos(d + e*x)/e - 12*a**3*sin(d + e*x)*cos(d + e*x)/e + 16*a**3*cos(d + e*x)**3/(3*e) + 24*a**3*cos(d + e*x)/e + 8*a**2*b*sin(d + e*x)**3/e + 24*a**2*b*sin(d + e*x)/e + 24*a**2*b*cos(d + e*x)**2/e + 12*a*b**2*x*sin(d + e*x)**2 + 12*a*b**2*x*cos(d + e*x)**2 + 12*a*b**2*sin(d + e*x)*cos(d + e*x)/e + 8*a*b**2*cos(d + e*x)**3/e + 16*b**3*sin(d + e*x)**3/(3*e) + 8*b**3*sin(d + e*x)*cos(d + e*x)**2/e, Ne(e, 0)), (x*(-2*a*sin(d) + 2*a + 2*b*cos(d))**3, True))`

3.389 $\int (2a + 2b \cos(d + ex) - 2a \sin(d + ex))^2 dx$

Optimal. Leaf size=81

$$2x(3a^2 + b^2) + \frac{6a^2 \cos(d + ex)}{e} + \frac{6ab \sin(d + ex)}{e} + \frac{2(a - \sin(d + ex)) + a + b \cos(d + ex)(a \cos(d + ex) + b \sin(d + ex))}{e}$$

[Out] $2*(3*a^2+b^2)*x+6*a^2*\cos(e*x+d)/e+6*a*b*\sin(e*x+d)/e+2*(a+b*\cos(e*x+d)-a*\sin(e*x+d))*(a*\cos(e*x+d)+b*\sin(e*x+d))/e$

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3120, 2637, 2638}

$$2x(3a^2 + b^2) + \frac{6a^2 \cos(d + ex)}{e} + \frac{6ab \sin(d + ex)}{e} + \frac{2(a - \sin(d + ex)) + a + b \cos(d + ex)(a \cos(d + ex) + b \sin(d + ex))}{e}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a + 2*b*\text{Cos}[d + e*x] - 2*a*\text{Sin}[d + e*x])^2, x]$

[Out] $2*(3*a^2 + b^2)*x + (6*a^2*\text{Cos}[d + e*x])/e + (6*a*b*\text{Sin}[d + e*x])/e + (2*(a + b*\text{Cos}[d + e*x] - a*\text{Sin}[d + e*x])*(a*\text{Cos}[d + e*x] + b*\text{Sin}[d + e*x]))/e$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3120

$\text{Int}[(\cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*\sin[(d_.) + (e_.)*(x_.)])^n, x_Symbol] \rightarrow -\text{Simp}[(c*\text{Cos}[d + e*x] - b*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{n-1}/(e*n), x] + \text{Dist}[1/n, \text{Int}[\text{Simp}[n*a^2 + (n-1)*(b^2 + c^2) + a*b*(2*n-1)*\text{Cos}[d + e*x] + a*c*(2*n-1)*\text{Sin}[d + e*x], x]*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{n-2}, x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int (2a + 2b \cos(d + ex) - 2a \sin(d + ex))^2 dx &= \frac{2(a + b \cos(d + ex) - a \sin(d + ex))(a \cos(d + ex) + b \sin(d + ex))}{e} \\ &= 2(3a^2 + b^2)x + \frac{2(a + b \cos(d + ex) - a \sin(d + ex))(a \cos(d + ex) + b \sin(d + ex))}{e} \\ &= 2(3a^2 + b^2)x + \frac{6a^2 \cos(d + ex)}{e} + \frac{6ab \sin(d + ex)}{e} + \frac{2(a + b \cos(d + ex) - a \sin(d + ex))(a \cos(d + ex) + b \sin(d + ex))}{e} \end{aligned}$$

Mathematica [A] time = 0.15, size = 92, normalized size = 1.14

$$4 \left(\frac{(3a^2 + b^2)(d + ex)}{2e} - \frac{(a^2 - b^2) \sin(2(d + ex))}{4e} + \frac{2a^2 \cos(d + ex)}{e} + \frac{2ab \sin(d + ex)}{e} + \frac{ab \cos(2(d + ex))}{2e} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*b*cos[d + e*x] - 2*a*sin[d + e*x])^2,x]

[Out] 4*((((3*a^2 + b^2)*(d + e*x))/(2*e) + (2*a^2*cos[d + e*x])/e + (a*b*cos[2*(d + e*x)]))/(2*e) + (2*a*b*sin[d + e*x])/e - ((a^2 - b^2)*sin[2*(d + e*x)])/(4*e))

fricas [A] time = 0.90, size = 70, normalized size = 0.86

$$\frac{2 \left(2 ab \cos (ex + d)^2 + \left(3 a^2 + b^2 \right) ex + 4 a^2 \cos (ex + d) + \left(4 ab - \left(a^2 - b^2 \right) \cos (ex + d) \right) \sin (ex + d) \right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="fricas")

[Out] 2*(2*a*b*cos(e*x + d)^2 + (3*a^2 + b^2)*e*x + 4*a^2*cos(e*x + d) + (4*a*b - (a^2 - b^2)*cos(e*x + d))*sin(e*x + d))/e

giac [A] time = 0.14, size = 79, normalized size = 0.98

$$2 ab \cos (2 x e + 2 d) e^{(-1)} + 8 a^2 \cos (x e + d) e^{(-1)} + 8 a b e^{(-1)} \sin (x e + d) - \left(a^2 - b^2 \right) e^{(-1)} \sin (2 x e + 2 d) + 2 \left(3 a^2 + b^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="giac")

[Out] 2*a*b*cos(2*x*e + 2*d)*e^(-1) + 8*a^2*cos(x*e + d)*e^(-1) + 8*a*b*e^(-1)*sin(x*e + d) - (a^2 - b^2)*e^(-1)*sin(2*x*e + 2*d) + 2*(3*a^2 + b^2)*x

maple [A] time = 0.23, size = 100, normalized size = 1.23

$$\frac{4 a^2 (ex + d) + 8 ab \sin (ex + d) + 8 a^2 \cos (ex + d) + 4 b^2 \left(\frac{\sin (ex + d) \cos (ex + d)}{2} + \frac{ex}{2} + \frac{d}{2} \right) + 4 \left(\cos^2 (ex + d) \right) ab + 4 a^2}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x)

[Out] 4/e*(a^2*(e*x+d)+2*a*b*sin(e*x+d)+2*a^2*cos(e*x+d)+b^2*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)+cos(e*x+d)^2*a*b+a^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d))

maxima [A] time = 0.32, size = 98, normalized size = 1.21

$$4 a^2 x + \frac{4 ab \cos (ex + d)^2}{e} + \frac{(2 ex + 2 d - \sin (2 ex + 2 d)) a^2}{e} + \frac{(2 ex + 2 d + \sin (2 ex + 2 d)) b^2}{e} + 8 a \left(\frac{a \cos (ex + d)}{e} + \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="maxima")

[Out] 4*a^2*x + 4*a*b*cos(e*x + d)^2/e + (2*e*x + 2*d - sin(2*e*x + 2*d))*a^2/e + (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e + 8*a*(a*cos(e*x + d)/e + b*sin(e*x + d)/e)

mupad [B] time = 3.74, size = 128, normalized size = 1.58

$$\frac{x \left(12 a^2 + 4 b^2 \right) \tan \left(\frac{d}{2} + \frac{ex}{2} \right)^3 \left(4 a^2 + 16 ab - 4 b^2 \right) - \tan \left(\frac{d}{2} + \frac{ex}{2} \right)^2 \left(16 ab - 16 a^2 \right) + 16 a^2 + \tan \left(\frac{d}{2} + \frac{ex}{2} \right) \left(-2 \right)}{2 e \left(\tan \left(\frac{d}{2} + \frac{ex}{2} \right)^4 + 2 \tan \left(\frac{d}{2} + \frac{ex}{2} \right)^2 + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^2,x)`

[Out] $(x*(12*a^2 + 4*b^2))/2 + (\tan(d/2 + (e*x)/2)^3*(16*a*b + 4*a^2 - 4*b^2) - \tan(d/2 + (e*x)/2)^2*(16*a*b - 16*a^2) + 16*a^2 + \tan(d/2 + (e*x)/2)*(16*a*b - 4*a^2 + 4*b^2))/(e*(2*\tan(d/2 + (e*x)/2)^2 + \tan(d/2 + (e*x)/2)^4 + 1))$

sympy [A] time = 0.32, size = 170, normalized size = 2.10

$$\begin{cases} 2a^2x \sin^2(d + ex) + 2a^2x \cos^2(d + ex) + 4a^2x - \frac{2a^2 \sin(d+ex) \cos(d+ex)}{e} + \frac{8a^2 \cos(d+ex)}{e} + \frac{8ab \sin(d+ex)}{e} + \frac{4ab \cos^2(d+ex)}{e} \\ x(-2a \sin(d) + 2a + 2b \cos(d))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x)`

[Out] `Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x - 2*a**2*sin(d + e*x)*cos(d + e*x)/e + 8*a**2*cos(d + e*x)/e + 8*a*b*sin(d + e*x)/e + 4*a*b*cos(d + e*x)**2/e + 2*b**2*x*sin(d + e*x)**2 + 2*b**2*x*cos(d + e*x)**2 + 2*b**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(-2*a*sin(d) + 2*a + 2*b*cos(d))**2, True))`

$$3.390 \quad \int (2a + 2b \cos(d + ex) - 2a \sin(d + ex)) dx$$

Optimal. Leaf size=29

$$\frac{2a \cos(d + ex)}{e} + 2ax + \frac{2b \sin(d + ex)}{e}$$

[Out] 2*a*x+2*a*cos(e*x+d)/e+2*b*sin(e*x+d)/e

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2637, 2638}

$$\frac{2a \cos(d + ex)}{e} + 2ax + \frac{2b \sin(d + ex)}{e}$$

Antiderivative was successfully verified.

[In] Int[2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x], x]

[Out] 2*a*x + (2*a*Cos[d + e*x])/e + (2*b*Sin[d + e*x])/e

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (2a + 2b \cos(d + ex) - 2a \sin(d + ex)) dx &= 2ax - (2a) \int \sin(d + ex) dx + (2b) \int \cos(d + ex) dx \\ &= 2ax + \frac{2a \cos(d + ex)}{e} + \frac{2b \sin(d + ex)}{e} \end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.83

$$-\frac{2a \sin(d) \sin(ex)}{e} + \frac{2a \cos(d) \cos(ex)}{e} + 2ax + \frac{2b \sin(d) \cos(ex)}{e} + \frac{2b \cos(d) \sin(ex)}{e}$$

Antiderivative was successfully verified.

[In] Integrate[2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x], x]

[Out] 2*a*x + (2*a*Cos[d]*Cos[e*x])/e + (2*b*Cos[e*x]*Sin[d])/e + (2*b*Cos[d]*Sin[e*x])/e - (2*a*Sin[d]*Sin[e*x])/e

fricas [A] time = 2.90, size = 26, normalized size = 0.90

$$\frac{2(aex + a \cos(ex + d) + b \sin(ex + d))}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x, algorithm="fricas")

[Out] $2*(a*e^x + a*\cos(e*x + d) + b*\sin(e*x + d))/e$

giac [A] time = 0.14, size = 29, normalized size = 1.00

$$2 a \cos (x e + d) e^{(-1)} + 2 b e^{(-1)} \sin (x e + d) + 2 a x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x, algorithm="giac")`

[Out] $2*a*\cos(x*e + d)*e^{(-1)} + 2*b*e^{(-1)}*\sin(x*e + d) + 2*a*x$

maple [A] time = 0.00, size = 30, normalized size = 1.03

$$2 a x + \frac{2 a \cos (e x + d)}{e} + \frac{2 b \sin (e x + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x)`

[Out] $2*a*x+2*a*\cos(e*x+d)/e+2*b*\sin(e*x+d)/e$

maxima [A] time = 0.33, size = 29, normalized size = 1.00

$$2 a x + \frac{2 a \cos (e x + d)}{e} + \frac{2 b \sin (e x + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x, algorithm="maxima")`

[Out] $2*a*x + 2*a*\cos(e*x + d)/e + 2*b*\sin(e*x + d)/e$

mupad [B] time = 2.44, size = 29, normalized size = 1.00

$$2 a x + \frac{2 a \cos (d + e x)}{e} + \frac{2 b \sin (d + e x)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x),x)`

[Out] $2*a*x + (2*a*\cos(d + e*x))/e + (2*b*\sin(d + e*x))/e$

sympy [A] time = 0.14, size = 39, normalized size = 1.34

$$2 a x - 2 a \left(\left(\begin{array}{ll} -\frac{\cos (d+e x)}{e} & \text{for } e \neq 0 \\ x \sin (d) & \text{otherwise} \end{array} \right) + 2 b \left(\left(\begin{array}{ll} \frac{\sin (d+e x)}{e} & \text{for } e \neq 0 \\ x \cos (d) & \text{otherwise} \end{array} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x)`

[Out] $2*a*x - 2*a*\text{Piecewise}((- \cos (d + e*x)/e, \text{Ne}(e, 0)), (x*\sin (d), \text{True})) + 2*b*\text{Piecewise}((\sin (d + e*x)/e, \text{Ne}(e, 0)), (x*\cos (d), \text{True}))$

$$3.391 \quad \int \frac{1}{2a+2b \cos(d+ex)-2a \sin(d+ex)} dx$$

Optimal. Leaf size=33

$$\frac{\log\left(a + b \tan\left(\frac{d}{2} + \frac{ex}{2} + \frac{\pi}{4}\right)\right)}{2be}$$

[Out] 1/2*ln(a+b*tan(1/2*d+1/4*Pi+1/2*e*x))/b/e

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3122, 31}

$$\frac{\log\left(a + b \tan\left(\frac{d}{2} + \frac{ex}{2} + \frac{\pi}{4}\right)\right)}{2be}$$

Antiderivative was successfully verified.

[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-1), x]

[Out] Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]]/(2*b*e)

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3122

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Dist[f/e, Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]

Rubi steps

$$\int \frac{1}{2a + 2b \cos(d + ex) - 2a \sin(d + ex)} dx = \frac{\text{Subst}\left(\int \frac{1}{2a+2bx} dx, x, \tan\left(\frac{\pi}{4} + \frac{1}{2}(d + ex)\right)\right)}{e} = \frac{\log\left(a + b \tan\left(\frac{d}{2} + \frac{\pi}{4} + \frac{ex}{2}\right)\right)}{2be}$$

Mathematica [B] time = 0.10, size = 96, normalized size = 2.91

$$\frac{\log\left(-a \sin\left(\frac{1}{2}(d + ex)\right) + a \cos\left(\frac{1}{2}(d + ex)\right) + b \sin\left(\frac{1}{2}(d + ex)\right) + b \cos\left(\frac{1}{2}(d + ex)\right)\right)}{2be} - \frac{\log\left(\cos\left(\frac{1}{2}(d + ex)\right) - \sin\left(\frac{1}{2}(d + ex)\right)\right)}{2be}$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-1), x]

[Out] -1/2*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]]/(b*e) + Log[a*Cos[(d + e*x)/2] + b*Cos[(d + e*x)/2] - a*Sin[(d + e*x)/2] + b*Sin[(d + e*x)/2]]/(2*b*e)

fricas [B] time = 1.98, size = 57, normalized size = 1.73

$$\frac{\log\left(2ab \cos(ex + d) + a^2 + b^2 - (a^2 - b^2) \sin(ex + d)\right) - \log(-\sin(ex + d) + 1)}{4be}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x, algorithm="fricas")

[Out] 1/4*(log(2*a*b*cos(e*x + d) + a^2 + b^2 - (a^2 - b^2)*sin(e*x + d)) - log(-sin(e*x + d) + 1))/(b*e)

giac [B] time = 0.21, size = 82, normalized size = 2.48

$$\frac{e^{(-1)} \log \left(\frac{\left| 2a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 2b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 2a - 2|b| \right|}{\left| 2a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 2b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 2a + 2|b| \right|} \right)}{2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x, algorithm="giac")

[Out] 1/2*e^(-1)*log(abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) - 2*a - 2*abs(b))/abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) - 2*a + 2*abs(b)))/abs(b)

maple [B] time = 0.42, size = 61, normalized size = 1.85

$$\frac{\ln\left(a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - b \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - a - b\right)}{2eb} - \frac{\ln\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) - 1\right)}{2eb}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x)

[Out] 1/2/e/b*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)-1/2/e/b*ln(tan(1/2*d+1/2*e*x)-1)

maxima [B] time = 0.33, size = 62, normalized size = 1.88

$$\frac{\frac{\log\left(a+b-\frac{(a-b)\sin(ex+d)}{\cos(ex+d)+1}\right)}{b}}{2e} - \frac{\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x, algorithm="maxima")

[Out] 1/2*(log(a + b - (a - b)*sin(e*x + d)/(cos(e*x + d) + 1))/b - log(sin(e*x + d)/(cos(e*x + d) + 1) - 1)/b)/e

mupad [B] time = 2.74, size = 32, normalized size = 0.97

$$\frac{\operatorname{atanh}\left(\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)(2a-2b)}{b}\right)}{be}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x)),x)

[Out] atanh((a - (tan(d/2 + (e*x)/2)*(2*a - 2*b))/2)/b)/(b*e)

sympy [A] time = 1.72, size = 109, normalized size = 3.30

$$\left\{ \begin{array}{ll} \frac{\infty x}{\cos(d)} & \text{for } a = 0 \wedge b = 0 \wedge e = 0 \\ -\frac{1}{ae \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - ae} & \text{for } b = 0 \\ -\frac{\log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) - 1\right)}{2be} & \text{for } a = b \\ -\frac{x}{-2a \sin(d) + 2a + 2b \cos(d)} & \text{for } e = 0 \\ -\frac{\log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) - 1\right)}{2be} + \frac{\log\left(-\frac{a}{a-b} - \frac{b}{a-b} + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{2be} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x)

[Out] Piecewise((zoo*x/cos(d), Eq(a, 0) & Eq(b, 0) & Eq(e, 0)), (-1/(a*e*tan(d/2 + e*x/2) - a*e), Eq(b, 0)), (-log(tan(d/2 + e*x/2) - 1)/(2*b*e), Eq(a, b)), (x/(-2*a*sin(d) + 2*a + 2*b*cos(d)), Eq(e, 0)), (-log(tan(d/2 + e*x/2) - 1)/(2*b*e) + log(-a/(a - b) - b/(a - b) + tan(d/2 + e*x/2))/(2*b*e), True))

$$3.392 \quad \int \frac{1}{(2a+2b \cos(d+ex)-2a \sin(d+ex))^2} dx$$

Optimal. Leaf size=83

$$\frac{a \cos(d+ex) + b \sin(d+ex)}{4b^2e(a(-\sin(d+ex)) + a + b \cos(d+ex))} - \frac{a \log\left(a + b \tan\left(\frac{d}{2} + \frac{ex}{2} + \frac{\pi}{4}\right)\right)}{4b^3e}$$

[Out] $-1/4*a*\ln(a+b*\tan(1/2*d+1/4*Pi+1/2*e*x))/b^3/e+1/4*(a*\cos(e*x+d)+b*\sin(e*x+d))/b^2/e/(a+b*\cos(e*x+d)-a*\sin(e*x+d))$

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3129, 12, 3122, 31}

$$\frac{a \cos(d+ex) + b \sin(d+ex)}{4b^2e(a(-\sin(d+ex)) + a + b \cos(d+ex))} - \frac{a \log\left(a + b \tan\left(\frac{d}{2} + \frac{ex}{2} + \frac{\pi}{4}\right)\right)}{4b^3e}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a + 2*b*\text{Cos}[d + e*x] - 2*a*\text{Sin}[d + e*x])^(-2), x]$

[Out] $-(a*\text{Log}[a + b*\text{Tan}[d/2 + \text{Pi}/4 + (e*x)/2]])/(4*b^3*e) + (a*\text{Cos}[d + e*x] + b*\text{Sin}[d + e*x])/(4*b^2*e*(a + b*\text{Cos}[d + e*x] - a*\text{Sin}[d + e*x]))$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_) /; \text{FreeQ}[b, x]]$

Rule 31

$\text{Int}[(a_*) + (b_*)*(x_*)^(-1), x_Symbol] := \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 3122

$\text{Int}[(\text{cos}[(d_*) + (e_*)*(x_*)]*(b_*) + (a_*) + (c_*)*\text{sin}[(d_*) + (e_*)*(x_*)])^(-1), x_Symbol] := \text{Module}[\{f = \text{FreeFactors}[\text{Tan}[(d + e*x)/2 + \text{Pi}/4], x]\}, \text{Dist}[f/e, \text{Subst}[\text{Int}[1/(a + b*f*x), x], x, \text{Tan}[(d + e*x)/2 + \text{Pi}/4]/f], x]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[a + c, 0]$

Rule 3129

$\text{Int}[(\text{cos}[(d_*) + (e_*)*(x_*)]*(b_*) + (a_*) + (c_*)*\text{sin}[(d_*) + (e_*)*(x_*)])^n, x_Symbol] := \text{Simp}[(\text{-(c*Cos}[d + e*x] + b*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + \text{Dist}[1/((n + 1)*(a^2 - b^2 - c^2)), \text{Int}[(a*(n + 1) - b*(n + 2)*\text{Cos}[d + e*x] - c*(n + 2)*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^(n + 1), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{NeQ}[n, -3/2]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2a + 2b \cos(d + ex) - 2a \sin(d + ex))^2} dx &= \frac{a \cos(d + ex) + b \sin(d + ex)}{4b^2 e (a + b \cos(d + ex) - a \sin(d + ex))} + \frac{\int -\frac{2a}{2a + 2b \cos(d + ex) - 2a \sin(d + ex)} dx}{4b^2} \\
&= \frac{a \cos(d + ex) + b \sin(d + ex)}{4b^2 e (a + b \cos(d + ex) - a \sin(d + ex))} - \frac{a \int \frac{1}{2a + 2b \cos(d + ex) - 2a \sin(d + ex)} dx}{2b^2} \\
&= \frac{a \cos(d + ex) + b \sin(d + ex)}{4b^2 e (a + b \cos(d + ex) - a \sin(d + ex))} - \frac{a \operatorname{Subst}\left(\int \frac{1}{2a + 2bx} dx, x, \frac{a \cos(d + ex) + b \sin(d + ex)}{2b}\right)}{2b^2} \\
&= -\frac{a \log\left(a + b \tan\left(\frac{d}{2} + \frac{\pi}{4} + \frac{ex}{2}\right)\right)}{4b^3 e} + \frac{a \cos(d + ex) + b \sin(d + ex)}{4b^2 e (a + b \cos(d + ex) - a \sin(d + ex))}
\end{aligned}$$

Mathematica [A] time = 0.62, size = 166, normalized size = 2.00

$$\frac{b(a^2 + b^2) \sin\left(\frac{1}{2}(d + ex)\right)}{(a + b)\left((b - a) \sin\left(\frac{1}{2}(d + ex)\right) + (a + b) \cos\left(\frac{1}{2}(d + ex)\right)\right)} - a \log\left((b - a) \sin\left(\frac{1}{2}(d + ex)\right) + (a + b) \cos\left(\frac{1}{2}(d + ex)\right)\right) + a \log\left(\cos\left(\frac{1}{2}(d + ex)\right)\right)$$

$$4b^3 e$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-2), x]

[Out] (a*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]] - a*Log[(a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2]] + (b*Sin[(d + e*x)/2])/(Cos[(d + e*x)/2] - Sin[(d + e*x)/2]) + (b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)*((a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2]))/(4*b^3*e)

fricas [B] time = 0.84, size = 154, normalized size = 1.86

$$\frac{2ab \cos(ex + d) + 2b^2 \sin(ex + d) - (ab \cos(ex + d) - a^2 \sin(ex + d) + a^2) \log(2ab \cos(ex + d) + a^2 + b^2 - (a^2 - b^2) \sin(ex + d))}{8(b^4 e \cos(ex + d) - ab^3 e \sin(ex + d))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="fricas")

[Out] 1/8*(2*a*b*cos(e*x + d) + 2*b^2*sin(e*x + d) - (a*b*cos(e*x + d) - a^2*sin(e*x + d) + a^2)*log(2*a*b*cos(e*x + d) + a^2 + b^2 - (a^2 - b^2)*sin(e*x + d)) + (a*b*cos(e*x + d) - a^2*sin(e*x + d) + a^2)*log(-sin(e*x + d) + 1))/(b^4*e*cos(e*x + d) - a*b^3*e*sin(e*x + d) + a*b^3*e)

giac [B] time = 0.22, size = 198, normalized size = 2.39

$$-\frac{1}{4} \left(\frac{2\left(a^2 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - ab \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + b^2 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - a^2\right)}{(ab^2 - b^3)\left(a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)\right)^2 - b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 - 2a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + a + b} \right) + \frac{a \log\left(\frac{2a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 2a^2}{2a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 2a^2}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="giac")

[Out] -1/4*(2*(a^2*tan(1/2*x*e + 1/2*d) - a*b*tan(1/2*x*e + 1/2*d) + b^2*tan(1/2*x*e + 1/2*d) - a^2)/((a*b^2 - b^3)*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) + a + b)) + a*log(abs(2*a*tan(1/2*x*e + 1/2*d) - 2*a^2))

$\frac{e + 1/2*d) - 2*b*\tan(1/2*x*e + 1/2*d) - 2*a - 2*abs(b))/abs(2*a*\tan(1/2*x*e + 1/2*d) - 2*b*\tan(1/2*x*e + 1/2*d) - 2*a + 2*abs(b)))/(b^2*abs(b)))*e^{-1}$

maple [B] time = 0.51, size = 178, normalized size = 2.14

$$\frac{a^2}{4e b^2 (a - b) \left(a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - b \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - a - b \right)} - \frac{1}{4e (a - b) \left(a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - b \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - a - b \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x)

[Out] $-1/4/e/b^2/(a-b)/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)*a^2-1/4/e/(a-b)/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)-1/4/e*a/b^3*\ln(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)-1/4/e/b^2/(\tan(1/2*d+1/2*e*x)-1)+1/4/e*a/b^3*\ln(\tan(1/2*d+1/2*e*x)-1)$

maxima [B] time = 0.34, size = 182, normalized size = 2.19

$$\frac{2 \left(a^2 - \frac{(a^2 - ab + b^2) \sin(ex+d)}{\cos(ex+d)+1} \right)}{a^2 b^2 - b^4 - \frac{2(a^2 b^2 - ab^3) \sin(ex+d)}{\cos(ex+d)+1} + \frac{(a^2 b^2 - 2ab^3 + b^4) \sin(ex+d)^2}{(\cos(ex+d)+1)^2}} - \frac{a \log\left(a + b - \frac{(a-b) \sin(ex+d)}{\cos(ex+d)+1}\right)}{b^3} + \frac{a \log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1} - 1\right)}{b^3}$$

$4e$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="maxima")

[Out] $1/4*(2*(a^2 - (a^2 - a*b + b^2)*\sin(e*x + d)/(\cos(e*x + d) + 1)))/(a^2*b^2 - b^4 - 2*(a^2*b^2 - a*b^3)*\sin(e*x + d)/(\cos(e*x + d) + 1) + (a^2*b^2 - 2*a*b^3 + b^4)*\sin(e*x + d)^2/(\cos(e*x + d) + 1)^2 - a*\log(a + b - (a - b)*\sin(e*x + d)/(\cos(e*x + d) + 1))/b^3 + a*\log(\sin(e*x + d)/(\cos(e*x + d) + 1) - 1)/b^3)/e$

mupad [B] time = 2.74, size = 126, normalized size = 1.52

$$\frac{\frac{a^2}{b^2(a-b)} - \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)(a^2 - ab + b^2)}{b^2(a-b)}}{e \left((2a - 2b) \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 - 4a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 2a + 2b \right)} - \frac{a \operatorname{atanh}\left(\frac{a - \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)(2a-2b)}{2}}{b}\right)}{2b^3 e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^2,x)

[Out] $(a^2/(b^2*(a - b)) - (\tan(d/2 + (e*x)/2)*(a^2 - a*b + b^2))/(b^2*(a - b)))/(e*(2*a + 2*b + \tan(d/2 + (e*x)/2)^2*(2*a - 2*b) - 4*a*\tan(d/2 + (e*x)/2))) - (a*\operatorname{atanh}((a - (\tan(d/2 + (e*x)/2)*(2*a - 2*b))/2)/b))/(2*b^3*e)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x)

[Out] Timed out

$$3.393 \quad \int \frac{1}{(2a+2b \cos(d+ex)-2a \sin(d+ex))^3} dx$$

Optimal. Leaf size=142

$$\frac{3(a^2 \cos(d+ex) + ab \sin(d+ex))}{16b^4 e(a(-\sin(d+ex)) + a + b \cos(d+ex))} + \frac{(3a^2 + b^2) \log\left(a + b \tan\left(\frac{d}{2} + \frac{ex}{2} + \frac{\pi}{4}\right)\right)}{16b^5 e} + \frac{a \cos(d+ex) + b \sin(d+ex)}{16b^2 e(a(-\sin(d+ex)) + a + b \cos(d+ex))}$$

[Out] 1/16*(3*a^2+b^2)*ln(a+b*tan(1/2*d+1/4*Pi+1/2*e*x))/b^5/e+1/16*(a*cos(e*x+d)+b*sin(e*x+d))/b^2/e/(a+b*cos(e*x+d)-a*sin(e*x+d))^2-3/16*(a^2*cos(e*x+d)+a*b*sin(e*x+d))/b^4/e/(a+b*cos(e*x+d)-a*sin(e*x+d))

Rubi [A] time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3129, 3153, 3122, 31}

$$\frac{(3a^2 + b^2) \log\left(a + b \tan\left(\frac{d}{2} + \frac{ex}{2} + \frac{\pi}{4}\right)\right)}{16b^5 e} - \frac{3(a^2 \cos(d+ex) + ab \sin(d+ex))}{16b^4 e(a(-\sin(d+ex)) + a + b \cos(d+ex))} + \frac{a \cos(d+ex) + b \sin(d+ex)}{16b^2 e(a(-\sin(d+ex)) + a + b \cos(d+ex))}$$

Antiderivative was successfully verified.

[In] Int[(2*a + 2*b*cos[d + e*x] - 2*a*sin[d + e*x])^(-3), x]

[Out] ((3*a^2 + b^2)*Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]]/(16*b^5*e) + (a*cos[d + e*x] + b*sin[d + e*x])/(16*b^2*e*(a + b*cos[d + e*x] - a*sin[d + e*x])^2) - (3*(a^2*cos[d + e*x] + a*b*sin[d + e*x]))/(16*b^4*e*(a + b*cos[d + e*x] - a*sin[d + e*x]))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3122

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Dist[f/e, Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]

Rule 3129

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Simp[(-c*cos[d + e*x] + b*sin[d + e*x])*(a + b*cos[d + e*x] + c*sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*cos[d + e*x] - c*(n + 2)*sin[d + e*x])*(a + b*cos[d + e*x] + c*sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]

Rule 3153

Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(a_. + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] := Simp[(c*B - b*C - (a*C - c*A)*cos[d + e*x] + (a*B - b*A)*sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*cos[d + e*x] + c*sin[d + e*x])), x] + Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*cos[d + e*x] + c*sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2a + 2b \cos(d + ex) - 2a \sin(d + ex))^3} dx &= \frac{a \cos(d + ex) + b \sin(d + ex)}{16b^2 e (a + b \cos(d + ex) - a \sin(d + ex))^2} + \frac{\int \frac{-4a + 2b \cos(d + ex) - 2a \sin(d + ex)}{(2a + 2b \cos(d + ex) - 2a \sin(d + ex))^2} dx}{8b^2} \\
&= \frac{a \cos(d + ex) + b \sin(d + ex)}{16b^2 e (a + b \cos(d + ex) - a \sin(d + ex))^2} - \frac{3(a^2 \cos(d + ex) + b^2 \sin(d + ex))}{16b^4 e (a + b \cos(d + ex) - a \sin(d + ex))} \\
&= \frac{a \cos(d + ex) + b \sin(d + ex)}{16b^2 e (a + b \cos(d + ex) - a \sin(d + ex))^2} - \frac{3(a^2 \cos(d + ex) + b^2 \sin(d + ex))}{16b^4 e (a + b \cos(d + ex) - a \sin(d + ex))} \\
&= \frac{(3a^2 + b^2) \log\left(a + b \tan\left(\frac{d}{2} + \frac{\pi}{4} + \frac{ex}{2}\right)\right)}{16b^5 e} + \frac{a \cos(d + ex) + b \sin(d + ex)}{16b^2 e (a + b \cos(d + ex) - a \sin(d + ex))}
\end{aligned}$$

Mathematica [A] time = 2.77, size = 261, normalized size = 1.84

$$\frac{b^2(a^2+b^2)}{\left((b-a)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)^2} + \frac{6ab(a^2+b^2)\sin\left(\frac{1}{2}(d+ex)\right)}{(a+b)\left((b-a)\sin\left(\frac{1}{2}(d+ex)\right)+(a+b)\cos\left(\frac{1}{2}(d+ex)\right)\right)} + 2(3a^2+b^2)\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-3),x]

[Out] -1/32*(2*(3*a^2 + b^2)*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]] - 2*(3*a^2 + b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2]] - b^2/(Cos[(d + e*x)/2] - Sin[(d + e*x)/2])^2 + (6*a*b*Sin[(d + e*x)/2])/(Cos[(d + e*x)/2] - Sin[(d + e*x)/2]) + (b^2*(a^2 + b^2))/((a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2])^2 + (6*a*b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2]))/(b^5*e)

fricas [B] time = 1.51, size = 423, normalized size = 2.98

$$\frac{12a^2b^2 \cos(ex + d)^2 - 6a^2b^2 + 2(3a^3b - ab^3) \cos(ex + d) - (6a^4 + 2a^2b^2 - (3a^4 - 2a^2b^2 - b^4) \cos(ex + d))}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="fricas")

[Out] -1/32*(12*a^2*b^2*cos(e*x + d)^2 - 6*a^2*b^2 + 2*(3*a^3*b - a*b^3)*cos(e*x + d) - (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(e*x + d)^2 + 2*(3*a^3*b + a*b^3)*cos(e*x + d) - 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)*cos(e*x + d))*sin(e*x + d))*log(2*a*b*cos(e*x + d) + a^2 + b^2 - (a^2 - b^2)*sin(e*x + d)) + (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(e*x + d)^2 + 2*(3*a^3*b + a*b^3)*cos(e*x + d) - 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)*cos(e*x + d))*sin(e*x + d))*log(-sin(e*x + d) + 1) + 2*(3*a^2*b^2 - b^4 - 3*(a^3*b - a*b^3)*cos(e*x + d))*sin(e*x + d)/(2*a*b^6*e*cos(e*x + d) + 2*a^2*b^5*e - (a^2*b^5 - b^7)*e*cos(e*x + d)^2 - 2*(a*b^6*e*cos(e*x + d) + a^2*b^5*e)*sin(e*x + d))

giac [B] time = 0.24, size = 481, normalized size = 3.39

$$\frac{1}{16} \left(2 \left(3a^5 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^3 - 9a^4b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^3 + 10a^3b^2 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^3 - 6a^2b^3 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^3 + \dots \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="giac")

[Out] 1/16*(2*(3*a^5*tan(1/2*x*e + 1/2*d)^3 - 9*a^4*b*tan(1/2*x*e + 1/2*d)^3 + 10*a^3*b^2*tan(1/2*x*e + 1/2*d)^3 - 6*a^2*b^3*tan(1/2*x*e + 1/2*d)^3 + a*b^4*tan(1/2*x*e + 1/2*d)^3 + b^5*tan(1/2*x*e + 1/2*d)^3 - 9*a^5*tan(1/2*x*e + 1/2*d)^2 + 18*a^4*b*tan(1/2*x*e + 1/2*d)^2 - 12*a^3*b^2*tan(1/2*x*e + 1/2*d)^2 + 6*a^2*b^3*tan(1/2*x*e + 1/2*d)^2 - a*b^4*tan(1/2*x*e + 1/2*d)^2 + 9*a^5*tan(1/2*x*e + 1/2*d) - 9*a^4*b*tan(1/2*x*e + 1/2*d) - 2*a^3*b^2*tan(1/2*x*e + 1/2*d) + 2*a^2*b^3*tan(1/2*x*e + 1/2*d) - 5*a*b^4*tan(1/2*x*e + 1/2*d) + b^5*tan(1/2*x*e + 1/2*d) - 3*a^5 + 4*a^3*b^2 + a*b^4)/((a^2*b^4 - 2*a*b^5 + b^6)*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) + a + b)^2) + (3*a^2 + b^2)*log(abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) - 2*a - 2*abs(b))/abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) - 2*a + 2*abs(b)))/(b^4*abs(b))*e^(-1)

maple [B] time = 0.53, size = 687, normalized size = 4.84

$$\frac{3 \ln\left(a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - b \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - a - b\right) a^3}{16 e b^5 (a - b)} - \frac{3 \ln\left(a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - b \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - a - b\right) a^2}{16 e b^4 (a - b)} + \frac{\ln\left(a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - b \tan\left(\frac{d}{2} + \frac{ex}{2}\right) - a - b\right)}{16 e b^3 (a - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x)

[Out] 3/16/e/b^5/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^3-3/16/e/b^4/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^2+1/16/e/b^3/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a-1/16/e/b^2/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)-1/16/e/b^3/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2*a^4-1/8/e/b/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2*a^2-1/16/e*b/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2+3/16/e/b^4/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^4-1/4/e/b^3/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^3+1/8/e/b^2/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a^2-1/4/e/b/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)*a-1/16/e/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)+1/16/e/b^3/(tan(1/2*d+1/2*e*x)-1)^2+3/16/e/b^4/(tan(1/2*d+1/2*e*x)-1)*a+1/16/e/b^3/(tan(1/2*d+1/2*e*x)-1)-3/16/e/b^5*ln(tan(1/2*d+1/2*e*x)-1)*a^2-1/16/e/b^3*ln(tan(1/2*d+1/2*e*x)-1)

maxima [B] time = 0.36, size = 491, normalized size = 3.46

$$\frac{2 \left(3 a^5 - 4 a^3 b^2 - a b^4 - \frac{(9 a^5 - 9 a^4 b - 2 a^3 b^2 + 2 a^2 b^3 - 5 a b^4 + b^5) \sin(ex+d)}{\cos(ex+d)+1} + \frac{(9 a^5 - 18 a^4 b + 12 a^3 b^2 - 6 a^2 b^3 + a b^4) \sin(ex+d)^2}{(\cos(ex+d)+1)^2} - \frac{(3 a^5 - 9 a^4 b + 10 a^3 b^2 - 6 a^2 b^3 + a b^4 + b^5) \sin(ex+d)^3}{(\cos(ex+d)+1)^3} \right)}{a^4 b^4 - 2 a^2 b^6 + b^8 - \frac{4(a^4 b^4 - a^3 b^5 - a^2 b^6 + a b^7) \sin(ex+d)}{\cos(ex+d)+1} + \frac{2(3 a^4 b^4 - 6 a^3 b^5 + 2 a^2 b^6 + 2 a b^7 - b^8) \sin(ex+d)^2}{(\cos(ex+d)+1)^2} - \frac{4(a^4 b^4 - 3 a^3 b^5 + 3 a^2 b^6 - a b^7) \sin(ex+d)^3}{(\cos(ex+d)+1)^3} + \frac{(a^4 b^4 - 4 a^3 b^5 + 6 a^2 b^6 - 4 a b^7 + b^8) \sin(ex+d)^4}{(\cos(ex+d)+1)^4}}$$

16 e

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="maxima")

[Out] -1/16*(2*(3*a^5 - 4*a^3*b^2 - a*b^4 - (9*a^5 - 9*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 5*a*b^4 + b^5)*sin(e*x + d)/(cos(e*x + d) + 1) + (9*a^5 - 18*a^4*b + 12*a^3*b^2 - 6*a^2*b^3 + a*b^4)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - (3*a^5 - 9*a^4*b + 10*a^3*b^2 - 6*a^2*b^3 + a*b^4 + b^5)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/(a^4*b^4 - 2*a^2*b^6 + b^8 - 4*(a^4*b^4 - a^3*b^5 - a^2*b^6 + a*b^7)*sin(e*x + d)/(cos(e*x + d) + 1) + 2*(3*a^4*b^4 - 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^7 - b^8)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 4*(a^4*b^4 - 3*a^3*b^5 + 2*a^2*b^6 - a*b^7)sin(e*x + d)^3/(cos(e*x + d) + 1)^3) + (3*a^2 + b^2)*log(abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) - 2*a - 2*abs(b))/abs(2*a*tan(1/2*x*e + 1/2*d) - 2*b*tan(1/2*x*e + 1/2*d) - 2*a + 2*abs(b)))/(b^4*abs(b))*e^(-1)

$$\begin{aligned} & ^3b^5 + 3a^2b^6 - ab^7) \sin(ex + d)^3 / (\cos(ex + d) + 1)^3 + (a^4b^4 \\ & - 4a^3b^5 + 6a^2b^6 - 4ab^7 + b^8) \sin(ex + d)^4 / (\cos(ex + d) + 1)^4 \\ & - (3a^2 + b^2) \log(a + b - (a - b) \sin(ex + d) / (\cos(ex + d) + 1)) / b^5 \\ & + (3a^2 + b^2) \log(\sin(ex + d) / (\cos(ex + d) + 1) - 1) / b^5 / e \end{aligned}$$

mupad [B] time = 6.48, size = 361, normalized size = 2.54

$$\frac{\operatorname{atanh}\left(\frac{2a - \tan\left(\frac{d}{2} + \frac{ex}{2}\right)(2a - 2b)}{2b}\right) (3a^2 + b^2)}{8b^5 e} - \frac{\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)(-9a^5 + 9a^4b + 2a^3b^2 - 2a^2b^3 + 5ab^4 - b^5)}{2b^4(a-b)^2} - \frac{-3a^5 + 4a^3b^2 + ab^4}{2b^4(a-b)^2}}{e \left(8ab + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 (24a^2 - 8b^2) + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^3 (16ab - 16a^2)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^3,x)

[Out] (atanh((2*a - tan(d/2 + (e*x)/2)*(2*a - 2*b))/(2*b))*(3*a^2 + b^2))/(8*b^5*e) - ((tan(d/2 + (e*x)/2)*(5*a*b^4 + 9*a^4*b - 9*a^5 - b^5 - 2*a^2*b^3 + 2*a^3*b^2))/(2*b^4*(a - b)^2) - (a*b^4 - 3*a^5 + 4*a^3*b^2)/(2*b^4*(a - b)^2) + (tan(d/2 + (e*x)/2)^3*(2*a*b^3 + 6*a^3*b - 3*a^4 + b^4 - 4*a^2*b^2))/(2*b^4*(a - b)) + (tan(d/2 + (e*x)/2)^2*(a*b^4 - 18*a^4*b + 9*a^5 - 6*a^2*b^3 + 12*a^3*b^2))/(2*b^4*(a - b)^2))/(e*(8*a*b + tan(d/2 + (e*x)/2)^2*(24*a^2 - 8*b^2) + tan(d/2 + (e*x)/2)^3*(16*a*b - 16*a^2) + tan(d/2 + (e*x)/2)^4*(4*a^2 - 8*a*b + 4*b^2) + 4*a^2 + 4*b^2 - tan(d/2 + (e*x)/2)*(16*a*b + 16*a^2)))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x)

[Out] Timed out

$$3.394 \quad \int \frac{1}{(2a+2b \cos(d+ex)-2a \sin(d+ex))^4} dx$$

Optimal. Leaf size=215

$$\frac{5(a^2 \cos(d+ex) + ab \sin(d+ex))}{96b^4 e(a(-\sin(d+ex)) + a + b \cos(d+ex))^2} - \frac{a(5a^2 + 3b^2) \log\left(a + b \tan\left(\frac{d}{2} + \frac{ex}{2} + \frac{\pi}{4}\right)\right)}{32b^7 e} + \frac{b(15a^2 + 4b^2) \sin(d+ex)}{96b^6 e(a(-\sin(d+ex)) + a + b \cos(d+ex))^2}$$

[Out] $-1/32*a*(5*a^2+3*b^2)*\ln(a+b*\tan(1/2*d+1/4*Pi+1/2*e*x))/b^7/e+1/48*(a*\cos(e*x+d)+b*\sin(e*x+d))/b^2/e/(a+b*\cos(e*x+d)-a*\sin(e*x+d))^3-5/96*(a^2*\cos(e*x+d)+a*b*\sin(e*x+d))/b^4/e/(a+b*\cos(e*x+d)-a*\sin(e*x+d))^2+1/96*(a*(15*a^2+4*b^2)*\cos(e*x+d)+b*(15*a^2+4*b^2)*\sin(e*x+d))/b^6/e/(a+b*\cos(e*x+d)-a*\sin(e*x+d))$

Rubi [A] time = 0.24, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3129, 3156, 3153, 3122, 31}

$$\frac{a(5a^2 + 3b^2) \log\left(a + b \tan\left(\frac{d}{2} + \frac{ex}{2} + \frac{\pi}{4}\right)\right)}{32b^7 e} - \frac{5(a^2 \cos(d+ex) + ab \sin(d+ex))}{96b^4 e(a(-\sin(d+ex)) + a + b \cos(d+ex))^2} + \frac{b(15a^2 + 4b^2) \sin(d+ex)}{96b^6 e(a(-\sin(d+ex)) + a + b \cos(d+ex))^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2*a + 2*b*\text{Cos}[d + e*x] - 2*a*\text{Sin}[d + e*x])^(-4), x]$

[Out] $-(a*(5*a^2 + 3*b^2)*\text{Log}[a + b*\text{Tan}[d/2 + Pi/4 + (e*x)/2]])/(32*b^7*e) + (a*\text{Cos}[d + e*x] + b*\text{Sin}[d + e*x])/(48*b^2*e*(a + b*\text{Cos}[d + e*x] - a*\text{Sin}[d + e*x])^3) - (5*(a^2*\text{Cos}[d + e*x] + a*b*\text{Sin}[d + e*x]))/(96*b^4*e*(a + b*\text{Cos}[d + e*x] - a*\text{Sin}[d + e*x])^2) + (a*(15*a^2 + 4*b^2)*\text{Cos}[d + e*x] + b*(15*a^2 + 4*b^2)*\text{Sin}[d + e*x])/(96*b^6*e*(a + b*\text{Cos}[d + e*x] - a*\text{Sin}[d + e*x]))$

Rule 31

$\text{Int}[(a + (b*x)^(-1)), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /;$ $\text{FreeQ}\{a, b, x\}$

Rule 3122

$\text{Int}[(\text{cos}[(d + e*x)] + (b + c*\text{sin}[(d + e*x)])^(-1)), x_Symbol] \rightarrow \text{Module}\{f = \text{FreeFactors}[\text{Tan}[(d + e*x)/2 + Pi/4], x\}, \text{Dist}[f/e, \text{Subst}[\text{Int}[1/(a + b*f*x), x], x, \text{Tan}[(d + e*x)/2 + Pi/4]/f], x] /;$ $\text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[a + c, 0]$

Rule 3129

$\text{Int}[(\text{cos}[(d + e*x)] + (b + c*\text{sin}[(d + e*x)])^n), x_Symbol] \rightarrow \text{Simp}[(c*\text{Cos}[d + e*x] + b*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + \text{Dist}[1/((n + 1)*(a^2 - b^2 - c^2)), \text{Int}[(a*(n + 1) - b*(n + 2)*\text{Cos}[d + e*x] - c*(n + 2)*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^(n + 1), x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{NeQ}[n, -3/2]$

Rule 3153

$\text{Int}[(A + \text{cos}[(d + e*x)] + (B + C*\text{sin}[(d + e*x)])^2), x_Symbol] \rightarrow \text{Simp}[(c*B - b*C - (a*C - c*A)*\text{Cos}[d + e*x] + (a*B - b*A)*\text{Sin}[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])), x] + \text{Dist}[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), \text{Int}[1/(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]), x], x]$

$n[d + e*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, A, B, C\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0] \ \&\& \ \text{NeQ}[a*A - b*B - c*C, 0]$

Rule 3156

$\text{Int}[(a + \cos[(d + e*x)]*(b + c)*\sin[(d + e*x)])^{(n)}*((A + \cos[(d + e*x)]*(B + C)*\sin[(d + e*x)]), x_Symbol] :> -\text{Simp}[(c*B - b*C - (a*C - c*A)*\text{Cos}[d + e*x] + (a*B - b*A)*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{(n+1)}]/(e*(n+1)*(a^2 - b^2 - c^2)), x] + \text{Dist}[1/((n+1)*(a^2 - b^2 - c^2)), \text{Int}[(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{(n+1)}*\text{Simp}[(n+1)*(a*A - b*B - c*C) + (n+2)*(a*B - b*A)*\text{Cos}[d + e*x] + (n+2)*(a*C - c*A)*\text{Sin}[d + e*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, A, B, C\}, x] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0] \ \&\& \ \text{NeQ}[n, -2]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(2a + 2b \cos(d + ex) - 2a \sin(d + ex))^4} dx &= \frac{a \cos(d + ex) + b \sin(d + ex)}{48b^2e(a + b \cos(d + ex) - a \sin(d + ex))^3} + \frac{\int \frac{-6a+4b \cos(d+ex)-2}{(2a+2b \cos(d+ex)-2a \sin(d+ex))^4} dx}{12b^2} \\ &= \frac{a \cos(d + ex) + b \sin(d + ex)}{48b^2e(a + b \cos(d + ex) - a \sin(d + ex))^3} - \frac{5(a^2 \cos(d + ex) + b^2 \sin(d + ex))}{96b^4e(a + b \cos(d + ex) - a \sin(d + ex))^2} \\ &= \frac{a \cos(d + ex) + b \sin(d + ex)}{48b^2e(a + b \cos(d + ex) - a \sin(d + ex))^3} - \frac{5(a^2 \cos(d + ex) + b^2 \sin(d + ex))}{96b^4e(a + b \cos(d + ex) - a \sin(d + ex))^2} \\ &= \frac{a \cos(d + ex) + b \sin(d + ex)}{48b^2e(a + b \cos(d + ex) - a \sin(d + ex))^3} - \frac{5(a^2 \cos(d + ex) + b^2 \sin(d + ex))}{96b^4e(a + b \cos(d + ex) - a \sin(d + ex))^2} \\ &= -\frac{a(5a^2 + 3b^2) \log\left(a + b \tan\left(\frac{d}{2} + \frac{\pi}{4} + \frac{ex}{2}\right)\right)}{32b^7e} + \frac{a \cos(d + ex) + b \sin(d + ex)}{48b^2e(a + b \cos(d + ex) - a \sin(d + ex))^3} \end{aligned}$$

Mathematica [B] time = 1.92, size = 636, normalized size = 2.96

$$12a(5a^2 + 3b^2) \log\left(\cos\left(\frac{1}{2}(d + ex)\right) - \sin\left(\frac{1}{2}(d + ex)\right)\right) - 12a(5a^2 + 3b^2) \log\left((b - a) \sin\left(\frac{1}{2}(d + ex)\right)\right) + (a + b)$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-4),x]

[Out] (12*a*(5*a^2 + 3*b^2)*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]] - 12*a*(5*a^2 + 3*b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2]] + (b*(-150*a^6 - 130*a^4*b^2 - 24*a^2*b^4 + 3*a^2*(25*a^4 - 50*a^3*b + 5*a^2*b^2 - 30*a*b^3 + 4*b^4)*Cos[d + e*x] + 6*a^2*(15*a^4 + 20*a^3*b + 9*a^2*b^2 + 2*a*b^3 - 2*b^4)*Cos[2*(d + e*x)] - 15*a^6*Cos[3*(d + e*x)] + 30*a^5*b*Cos[3*(d + e*x)] + 41*a^4*b^2*Cos[3*(d + e*x)] + 38*a^3*b^3*Cos[3*(d + e*x)] + 12*a^2*b^4*Cos[3*(d + e*x)] + 8*a*b^5*Cos[3*(d + e*x)] + 225*a^6*Sin[d + e*x] + 75*a^5*b*Sin[d + e*x] + 180*a^4*b^2*Sin[d + e*x] + 15*a^3*b^3*Sin[d + e*x] + 27*a^2*b^4*Sin[d + e*x] + 12*a*b^5*Sin[d + e*x] + 12*b^6*Sin[d + e*x] - 60*a^6*Sin[2*(d + e*x)] + 120*a^5*b*Sin[2*(d + e*x)] + 54*a^4*b^2*Sin[2*(d + e*x)] + 102*a^3*b^3*Sin[2*(d + e*x)] + 6*a^2*b^4*Sin[2*(d + e*x)] + 6*a*b^5*Sin[2*(d + e*x)] - 15*a^6*Sin[3*(d + e*x)] - 45*a^5*b*Sin[3*(d + e*x)] - 4*a^4*b^2*Sin[3*(d + e*x)] + 3*a^3*b^3*Sin[3*(d + e*x)] + 15*a^2*b^4*Sin[3*(d + e*x)] + 4*a*b^5*Sin[3*(d + e*x)] + 4*b^6*Sin[3*(d + e*x)])))/((a +

$b) * (\cos[(d + e*x)/2] - \sin[(d + e*x)/2])^3 * ((a + b) * \cos[(d + e*x)/2] + (-a + b) * \sin[(d + e*x)/2])^3) / (384 * b^7 * e)$

fricas [B] time = 2.00, size = 735, normalized size = 3.42

$$60 a^4 b^2 + 6 a^2 b^4 + 2 (15 a^5 b - 41 a^3 b^3 - 12 a b^5) \cos(ex + d)^3 - 12 (10 a^4 b^2 + a^2 b^4) \cos(ex + d)^2 - 6 (10 a^5 b - 9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x, algorithm="fricas")

[Out] $-1/192 * (60 * a^4 * b^2 + 6 * a^2 * b^4 + 2 * (15 * a^5 * b - 41 * a^3 * b^3 - 12 * a * b^5) * \cos(ex + d)^3 - 12 * (10 * a^4 * b^2 + a^2 * b^4) * \cos(ex + d)^2 - 6 * (10 * a^5 * b - 9 * a^3 * b^3 - 2 * a * b^5) * \cos(ex + d) + 3 * (20 * a^6 + 12 * a^4 * b^2 - (15 * a^5 * b + 4 * a^3 * b^3 - 3 * a * b^5) * \cos(ex + d)^3 - 3 * (5 * a^6 - 2 * a^4 * b^2 - 3 * a^2 * b^4) * \cos(ex + d)^2 + 6 * (5 * a^5 * b + 3 * a^3 * b^3) * \cos(ex + d) - (20 * a^6 + 12 * a^4 * b^2 - (5 * a^6 - 12 * a^4 * b^2 - 9 * a^2 * b^4) * \cos(ex + d)^2 + 6 * (5 * a^5 * b + 3 * a^3 * b^3) * \cos(ex + d)) * \sin(ex + d) * \log(2 * a * b * \cos(ex + d) + a^2 + b^2 - (a^2 - b^2) * \sin(ex + d)) - 3 * (20 * a^6 + 12 * a^4 * b^2 - (15 * a^5 * b + 4 * a^3 * b^3 - 3 * a * b^5) * \cos(ex + d)^3 - 3 * (5 * a^6 - 2 * a^4 * b^2 - 3 * a^2 * b^4) * \cos(ex + d)^2 + 6 * (5 * a^5 * b + 3 * a^3 * b^3) * \cos(ex + d) - (20 * a^6 + 12 * a^4 * b^2 - (5 * a^6 - 12 * a^4 * b^2 - 9 * a^2 * b^4) * \cos(ex + d)^2 + 6 * (5 * a^5 * b + 3 * a^3 * b^3) * \cos(ex + d)) * \sin(ex + d)) * \log(-\sin(ex + d) + 1) - 2 * (30 * a^4 * b^2 + 3 * a^2 * b^4 + 2 * b^6 - (45 * a^4 * b^2 - 3 * a^2 * b^4 - 4 * b^6) * \cos(ex + d)^2 - 3 * (10 * a^5 * b - 9 * a^3 * b^3 - a * b^5) * \cos(ex + d)) * \sin(ex + d)) / (6 * a^2 * b^8 * e * \cos(ex + d) + 4 * a^3 * b^7 * e - (3 * a^2 * b^8 - b^10) * e * \cos(ex + d)^3 - 3 * (a^3 * b^7 - a * b^9) * e * \cos(ex + d)^2 - (6 * a^2 * b^8 * e * \cos(ex + d) + 4 * a^3 * b^7 * e - (a^3 * b^7 - 3 * a * b^9) * e * \cos(ex + d)^2) * \sin(ex + d))$

giac [B] time = 0.28, size = 1006, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x, algorithm="giac")

[Out] $-1/96 * (2 * (15 * a^8 * \tan(1/2 * x * e + 1/2 * d))^5 - 75 * a^7 * b * \tan(1/2 * x * e + 1/2 * d))^5 + 159 * a^6 * b^2 * \tan(1/2 * x * e + 1/2 * d))^5 - 195 * a^5 * b^3 * \tan(1/2 * x * e + 1/2 * d))^5 + 165 * a^4 * b^4 * \tan(1/2 * x * e + 1/2 * d))^5 - 105 * a^3 * b^5 * \tan(1/2 * x * e + 1/2 * d))^5 + 5 * a^2 * b^6 * \tan(1/2 * x * e + 1/2 * d))^5 - 21 * a * b^7 * \tan(1/2 * x * e + 1/2 * d))^5 + 6 * b^8 * \tan(1/2 * x * e + 1/2 * d))^5 - 75 * a^8 * \tan(1/2 * x * e + 1/2 * d))^4 + 300 * a^7 * b * \tan(1/2 * x * e + 1/2 * d))^4 - 495 * a^6 * b^2 * \tan(1/2 * x * e + 1/2 * d))^4 + 480 * a^5 * b^3 * \tan(1/2 * x * e + 1/2 * d))^4 - 345 * a^4 * b^4 * \tan(1/2 * x * e + 1/2 * d))^4 + 180 * a^3 * b^5 * \tan(1/2 * x * e + 1/2 * d))^4 - 57 * a^2 * b^6 * \tan(1/2 * x * e + 1/2 * d))^4 + 12 * a * b^7 * \tan(1/2 * x * e + 1/2 * d))^4 + 150 * a^8 * \tan(1/2 * x * e + 1/2 * d))^3 - 450 * a^7 * b * \tan(1/2 * x * e + 1/2 * d))^3 + 500 * a^6 * b^2 * \tan(1/2 * x * e + 1/2 * d))^3 - 300 * a^5 * b^3 * \tan(1/2 * x * e + 1/2 * d))^3 + 126 * a^4 * b^4 * \tan(1/2 * x * e + 1/2 * d))^3 + 22 * a^3 * b^5 * \tan(1/2 * x * e + 1/2 * d))^3 - 48 * a^2 * b^6 * \tan(1/2 * x * e + 1/2 * d))^3 + 12 * a * b^7 * \tan(1/2 * x * e + 1/2 * d))^3 - 4 * b^8 * \tan(1/2 * x * e + 1/2 * d))^3 - 150 * a^8 * \tan(1/2 * x * e + 1/2 * d))^2 + 300 * a^7 * b * \tan(1/2 * x * e + 1/2 * d))^2 - 120 * a^6 * b^2 * \tan(1/2 * x * e + 1/2 * d))^2 - 60 * a^5 * b^3 * \tan(1/2 * x * e + 1/2 * d))^2 + 102 * a^4 * b^4 * \tan(1/2 * x * e + 1/2 * d))^2 - 144 * a^3 * b^5 * \tan(1/2 * x * e + 1/2 * d))^2 + 60 * a^2 * b^6 * \tan(1/2 * x * e + 1/2 * d))^2 - 12 * a * b^7 * \tan(1/2 * x * e + 1/2 * d))^2 + 75 * a^8 * \tan(1/2 * x * e + 1/2 * d) - 75 * a^7 * b * \tan(1/2 * x * e + 1/2 * d) - 75 * a^6 * b^2 * \tan(1/2 * x * e + 1/2 * d) + 75 * a^5 * b^3 * \tan(1/2 * x * e + 1/2 * d) - 39 * a^4 * b^4 * \tan(1/2 * x * e + 1/2 * d) + 39 * a^3 * b^5 * \tan(1/2 * x * e + 1/2 * d) + 33 * a^2 * b^6 * \tan(1/2 * x * e + 1/2 * d) - 15 * a * b^7 * \tan(1/2 * x * e + 1/2 * d) + 6 * b^8 * \tan(1/2 * x * e + 1/2 * d) - 15 * a^8 + 31 * a^6 * b^2 - 9 * a^4 * b^4 - 15 * a^2 * b^6) / ((a^3 * b^6 - 3 * a^2 * b^7 + 3 * a * b^8 - b^9) * (a * \tan(1/2 * x * e + 1/2 * d))^2 - b * \tan(1/2 * x * e + 1/2 * d))^2 - 2 * a * \tan(1/2 * x * e + 1/2 * d) + a + b)^3) + 3 * (5 * a^3 + 3 * a * b^2) * \log(abs(2 * a * \tan(1/2 * x * e + 1/2 * d) + a + b)))$

$e + 1/2*d) - 2*b*\tan(1/2*x*e + 1/2*d) - 2*a - 2*abs(b))/abs(2*a*\tan(1/2*x*e + 1/2*d) - 2*b*\tan(1/2*x*e + 1/2*d) - 2*a + 2*abs(b)))/(b^6*abs(b)))*e^{-1}$

maple [B] time = 0.61, size = 1149, normalized size = 5.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(2*a+2*b*\cos(e*x+d)-2*a*\sin(e*x+d))^4, x)$

[Out] $1/16/e/b^5/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)^2*a^6-3/32/e/b^4/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)^2*a^5+3/32/e/b^3/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)^2*a^4-3/16/e/b^2/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)^2*a^3-5/32/e/b^6/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)*a^6+3/8/e/b^5/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)*a^5-3/8/e/b^4/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)*a^4+3/8/e/b^3/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)*a^3-9/32/e/b^2/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)*a^2-5/32/e*a^4/b^7/(a-b)*\ln(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)+5/32/e*a^3/b^6/(a-b)*\ln(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)-3/32/e*a^2/b^5/(a-b)*\ln(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)+3/32/e*a/b^4/(a-b)*\ln(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)-1/16/e/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)-1/48/e/b^4/(tan(1/2*d+1/2*e*x)-1)^3-1/32/e/b^4/(tan(1/2*d+1/2*e*x)-1)^2-1/16/e/b^4/(tan(1/2*d+1/2*e*x)-1)-3/32/e/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)^2*a-1/48/e*b^2/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)^3-1/32/e*b/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)^2-1/16/e/b^5/(tan(1/2*d+1/2*e*x)-1)^2*a-5/32/e/b^6/(tan(1/2*d+1/2*e*x)-1)*a^2-1/16/e/b^5/(tan(1/2*d+1/2*e*x)-1)*a+5/32/e*a^3/b^7*\ln(tan(1/2*d+1/2*e*x)-1)+3/32/e*a/b^5*\ln(tan(1/2*d+1/2*e*x)-1)-1/16/e/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)^3*a^2-1/48/e/b^4/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)^3*a^6-1/16/e/b^2/(a-b)^3/(a*\tan(1/2*d+1/2*e*x)-b*\tan(1/2*d+1/2*e*x)-a-b)^3*a^4$

maxima [B] time = 0.39, size = 959, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(2*a+2*b*\cos(e*x+d)-2*a*\sin(e*x+d))^4, x, \text{algorithm}="maxima")$

[Out] $1/96*(2*(15*a^8 - 31*a^6*b^2 + 9*a^4*b^4 + 15*a^2*b^6 - 3*(25*a^8 - 25*a^7*b - 25*a^6*b^2 + 25*a^5*b^3 - 13*a^4*b^4 + 13*a^3*b^5 + 11*a^2*b^6 - 5*a*b^7 + 2*b^8)*\sin(e*x + d))/(\cos(e*x + d) + 1) + 6*(25*a^8 - 50*a^7*b + 20*a^6*b^2 + 10*a^5*b^3 - 17*a^4*b^4 + 24*a^3*b^5 - 10*a^2*b^6 + 2*a*b^7)*\sin(e*x + d)^2/(\cos(e*x + d) + 1)^2 - 2*(75*a^8 - 225*a^7*b + 250*a^6*b^2 - 150*a^5*b^3 + 63*a^4*b^4 + 11*a^3*b^5 - 24*a^2*b^6 + 6*a*b^7 - 2*b^8)*\sin(e*x + d)^3/(\cos(e*x + d) + 1)^3 + 3*(25*a^8 - 100*a^7*b + 165*a^6*b^2 - 160*a^5*b^3 + 115*a^4*b^4 - 60*a^3*b^5 + 19*a^2*b^6 - 4*a*b^7)*\sin(e*x + d)^4/(\cos(e*x + d) + 1)^4 - 3*(5*a^8 - 25*a^7*b + 53*a^6*b^2 - 65*a^5*b^3 + 55*a^4*b^4 - 35*a^3*b^5 + 17*a^2*b^6 - 7*a*b^7 + 2*b^8)*\sin(e*x + d)^5/(\cos(e*x + d) + 1)^5)/(a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12 - 6*(a^6*b^6 - a^5*b^7 - 2*a^4*b^8 + 2*a^3*b^9 + a^2*b^10 - a*b^11)*\sin(e*x + d))/(\cos(e*x + d) + 1) + 3*(5*a^6*b^6 - 10*a^5*b^7 - a^4*b^8 + 12*a^3*b^9 - 5*a^2*b^10 - 2*a*b^11 + b^12)*\sin(e*x + d)^2/(\cos(e*x + d) + 1)^2 - 4*(5*a^6*b^6 - 15*a^5*b^7 + 12*a^4*b^8 + 4*a^3*b^9 - 9*a^2*b^10 + 3*a*b^11)*\sin(e*x + d)^3/(\cos(e*x + d) + 1)^3 + 3*(5*a^6*b^6 - 20*a^5*b^7 + 29*a^4*b^8 - 16*a^3*b^9 - a^2*b^10 + 4*a*b^11 - b^12)*\sin(e*x + d)^4/(\cos(e*x + d) + 1)^4 - 6*(a^6*b^6 - 5*a^5*b^7 + 10*a^4*b^8 - 10*a^3*b^9 + 5*a^2*b^10 - a*b^11)*\sin(e*x + d)^5/(\cos(e*x +$

$d) + 1)^5 + (a^6 b^6 - 6 a^5 b^7 + 15 a^4 b^8 - 20 a^3 b^9 + 15 a^2 b^{10} - 6 a b^{11} + b^{12}) \sin(e x + d)^6 / (\cos(e x + d) + 1)^6 - 3 (5 a^3 + 3 a b^2) \log(a + b - (a - b) \sin(e x + d) / (\cos(e x + d) + 1)) / b^7 + 3 (5 a^3 + 3 a b^2) \log(\sin(e x + d) / (\cos(e x + d) + 1) - 1) / b^7) / e$

mupad [B] time = 7.09, size = 731, normalized size = 3.40

$$\frac{15 a^8 - 31 a^6 b^2 + 9 a^4 b^4 + 15 a^2 b^6}{6 b^6 (a - b)^3} + \frac{\tan\left(\frac{d}{2} + \frac{e x}{2}\right)^2 (25 a^8 - 50 a^7 b + 20 a^6 b^2 + 10 a^5 b^3 - 17 a^4 b^4 + 24 a^3 b^5 - 10 a^2 b^6 + 2 a b^7)}{b^6 (a - b)^3} + \frac{\tan\left(\frac{d}{2} + \frac{e x}{2}\right)^4 (25 a^7 - 75 a^6 b + 25 a^5 b^2 - 15 a^4 b^3 + 10 a^3 b^4 - 5 a^2 b^5 + 5 a b^6 - b^7)}{b^6 (a - b)^3}$$

$$e \left(\tan\left(\frac{d}{2} + \frac{e x}{2}\right)^6 (8 a^3 - 24 a^2 b + 24 a b^2 - 8 b^3) - \tan\left(\frac{d}{2} + \frac{e x}{2}\right)^5 (48 a^3 - 96 a^2 b + 48 a b^2) - \tan\left(\frac{d}{2} + \frac{e x}{2}\right)^4 (96 a^3 - 240 a^2 b + 144 a b^2 - 48 b^3) - \tan\left(\frac{d}{2} + \frac{e x}{2}\right)^3 (144 a^3 - 432 a^2 b + 288 a b^2 - 96 b^3) - \tan\left(\frac{d}{2} + \frac{e x}{2}\right)^2 (144 a^3 - 432 a^2 b + 288 a b^2 - 96 b^3) - \tan\left(\frac{d}{2} + \frac{e x}{2}\right) (144 a^3 - 432 a^2 b + 288 a b^2 - 96 b^3) - (144 a^3 - 432 a^2 b + 288 a b^2 - 96 b^3) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^4,x)

[Out] ((15*a^8 + 15*a^2*b^6 + 9*a^4*b^4 - 31*a^6*b^2)/(6*b^6*(a - b)^3) + (tan(d/2 + (e*x)/2)^2*(2*a*b^7 - 50*a^7*b + 25*a^8 - 10*a^2*b^6 + 24*a^3*b^5 - 17*a^4*b^4 + 10*a^5*b^3 + 20*a^6*b^2))/(b^6*(a - b)^3) + (tan(d/2 + (e*x)/2)^4*(4*a*b^6 - 75*a^6*b + 25*a^7 - 15*a^2*b^5 + 45*a^3*b^4 - 70*a^4*b^3 + 90*a^5*b^2))/(2*b^6*(a - b)^2) - (tan(d/2 + (e*x)/2)^3*(6*a*b^7 - 225*a^7*b + 75*a^8 - 2*b^8 - 24*a^2*b^6 + 11*a^3*b^5 + 63*a^4*b^4 - 150*a^5*b^3 + 250*a^6*b^2))/(3*b^6*(a - b)^3) - (tan(d/2 + (e*x)/2)^5*(5*a^6 - 15*a^5*b - 3*a*b^5 + 2*b^6 + 9*a^2*b^4 - 14*a^3*b^3 + 18*a^4*b^2))/(2*b^6*(a - b)) - (tan(d/2 + (e*x)/2)*(25*a^8 - 25*a^7*b - 5*a*b^7 + 2*b^8 + 11*a^2*b^6 + 13*a^3*b^5 - 13*a^4*b^4 + 25*a^5*b^3 - 25*a^6*b^2))/(2*b^6*(a - b)^3))/(e*(tan(d/2 + (e*x)/2)^6*(24*a*b^2 - 24*a^2*b + 8*a^3 - 8*b^3) - tan(d/2 + (e*x)/2)^5*(48*a*b^2 - 96*a^2*b + 48*a^3) - tan(d/2 + (e*x)/2)^4*(24*a*b^2 - 120*a^2*b - 120*a^3 + 24*b^3) - tan(d/2 + (e*x)/2)^3*(24*a*b^2 + 120*a^2*b - 120*a^3 - 24*b^3) + 24*a*b^2 + 24*a^2*b + tan(d/2 + (e*x)/2)^3*(96*a*b^2 - 160*a^3) - tan(d/2 + (e*x)/2)*(48*a*b^2 + 96*a^2*b + 48*a^3) + 8*a^3 + 8*b^3) - (a*atanh((a*(2*a - tan(d/2 + (e*x)/2)*(2*a - 2*b))*(5*a^2 + 3*b^2))/(2*b*(3*a*b^2 + 5*a^3)))*(5*a^2 + 3*b^2))/(16*b^7*e)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x)

[Out] Timed out

3.395 $\int (a + b \cos(d + ex) + c \sin(d + ex))^4 dx$

Optimal. Leaf size=260

$$\frac{5ab(10a^2 + 11(b^2 + c^2))\sin(d + ex)}{24e} - \frac{5ac(10a^2 + 11(b^2 + c^2))\cos(d + ex)}{24e} - \frac{(c(26a^2 + 9(b^2 + c^2))\cos(d + ex) - b(26a^2 + 9(b^2 + c^2))\sin(d + ex))}{24e}$$

```
[Out] 1/8*(8*a^4+24*a^2*(b^2+c^2)+3*(b^2+c^2)^2)*x-5/24*a*c*(10*a^2+11*b^2+11*c^2)
*cos(e*x+d)/e+5/24*a*b*(10*a^2+11*b^2+11*c^2)*sin(e*x+d)/e-7/12*(a*c*cos(e
*x+d)-a*b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^2/e-1/4*(c*cos(e*x+d)-b
*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^3/e-1/24*(a+b*cos(e*x+d)+c*sin(e
*x+d))*(c*(26*a^2+9*b^2+9*c^2)*cos(e*x+d)-b*(26*a^2+9*b^2+9*c^2)*sin(e*x+d)
)/e
```

Rubi [A] time = 0.40, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3120, 3146, 2637, 2638}

$$\frac{5ab(10a^2 + 11(b^2 + c^2))\sin(d + ex)}{24e} - \frac{5ac(10a^2 + 11(b^2 + c^2))\cos(d + ex)}{24e} - \frac{(c(26a^2 + 9(b^2 + c^2))\cos(d + ex) - b(26a^2 + 9(b^2 + c^2))\sin(d + ex))}{24e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^4, x]
```

```
[Out] ((8*a^4 + 24*a^2*(b^2 + c^2) + 3*(b^2 + c^2)^2)*x)/8 - (5*a*c*(10*a^2 + 11*
(b^2 + c^2))*Cos[d + e*x])/(24*e) + (5*a*b*(10*a^2 + 11*(b^2 + c^2))*Sin[d
+ e*x])/(24*e) - (7*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x])*(a + b*Cos[d + e
x] + c*Sin[d + e*x])^2)/(12*e) - ((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*
Cos[d + e*x] + c*Sin[d + e*x])^3)/(4*e) - ((a + b*Cos[d + e*x] + c*Sin[d +
e*x])*(c*(26*a^2 + 9*(b^2 + c^2))*Cos[d + e*x] - b*(26*a^2 + 9*(b^2 + c^2)
)*Sin[d + e*x]))/(24*e)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] := -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] := Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
```

+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\begin{aligned} \int (a + b \cos(d + ex) + c \sin(d + ex))^4 dx &= -\frac{(c \cos(d + ex) - b \sin(d + ex))(a + b \cos(d + ex) + c \sin(d + ex))^3}{4e} + \\ &= -\frac{7(ac \cos(d + ex) - ab \sin(d + ex))(a + b \cos(d + ex) + c \sin(d + ex))^2}{12e} \\ &= -\frac{7(ac \cos(d + ex) - ab \sin(d + ex))(a + b \cos(d + ex) + c \sin(d + ex))}{12e} \\ &= \frac{1}{8} \left(8a^4 + 24a^2(b^2 + c^2) + 3(b^2 + c^2)^2 \right) x - \frac{7(ac \cos(d + ex) - ab \sin(d + ex))}{12e} \\ &= \frac{1}{8} \left(8a^4 + 24a^2(b^2 + c^2) + 3(b^2 + c^2)^2 \right) x - \frac{5ac(10a^2 + 11(b^2 + c^2))}{24e} \end{aligned}$$

Mathematica [A] time = 1.05, size = 237, normalized size = 0.91

$$\frac{96ab(4a^2 + 3(b^2 + c^2)) \sin(d + ex) + 24(b^2 - c^2)(6a^2 + b^2 + c^2) \sin(2(d + ex)) - 96ac(4a^2 + 3(b^2 + c^2)) \cos(d + ex)}{e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]

[Out] (12*(8*a^4 + 24*a^2*(b^2 + c^2) + 3*(b^2 + c^2)^2)*(d + e*x) - 96*a*c*(4*a^2 + 3*(b^2 + c^2))*Cos[d + e*x] - 48*b*c*(6*a^2 + b^2 + c^2)*Cos[2*(d + e*x)] + 32*a*c*(-3*b^2 + c^2)*Cos[3*(d + e*x)] - 12*b*c*(b^2 - c^2)*Cos[4*(d + e*x)] + 96*a*b*(4*a^2 + 3*(b^2 + c^2))*Sin[d + e*x] + 24*(b^2 - c^2)*(6*a^2 + b^2 + c^2)*Sin[2*(d + e*x)] + 32*a*b*(b^2 - 3*c^2)*Sin[3*(d + e*x)] + 3*(b^4 - 6*b^2*c^2 + c^4)*Sin[4*(d + e*x)])/(96*e)

fricas [A] time = 1.07, size = 255, normalized size = 0.98

$$\frac{24(b^3c - bc^3) \cos(ex + d)^4 + 32(3ab^2c - ac^3) \cos(ex + d)^3 - 3(8a^4 + 24a^2b^2 + 3b^4 + 3c^4 + 6(4a^2 + b^2)c^2) \cos(ex + d)^2}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="fricas")

[Out] -1/24*(24*(b^3*c - b*c^3)*cos(e*x + d)^4 + 32*(3*a*b^2*c - a*c^3)*cos(e*x + d)^3 - 3*(8*a^4 + 24*a^2*b^2 + 3*b^4 + 3*c^4 + 6*(4*a^2 + b^2)*c^2)*e*x + 48*(3*a^2*b*c + b*c^3)*cos(e*x + d)^2 + 96*(a^3*c + a*c^3)*cos(e*x + d) - (96*a^3*b + 64*a*b^3 + 96*a*b*c^2 + 6*(b^4 - 6*b^2*c^2 + c^4)*cos(e*x + d)^3 + 32*(a*b^3 - 3*a*b*c^2)*cos(e*x + d)^2 + 3*(24*a^2*b^2 + 3*b^4 - 5*c^4 - 6*(4*a^2 - b^2)*c^2)*cos(e*x + d))*sin(e*x + d))/e

giac [A] time = 0.30, size = 286, normalized size = 1.10

$$-\frac{1}{8} (b^3c - bc^3) \cos(4xe + 4d) e^{(-1)} - \frac{1}{3} (3ab^2c - ac^3) \cos(3xe + 3d) e^{(-1)} - \frac{1}{2} (6a^2bc + b^3c + bc^3) \cos(2xe + 2d) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="giac")

[Out] $-1/8*(b^3*c - b*c^3)*\cos(4*x*e + 4*d)*e^{-1} - 1/3*(3*a*b^2*c - a*c^3)*\cos(3*x*e + 3*d)*e^{-1} - 1/2*(6*a^2*b*c + b^3*c + b*c^3)*\cos(2*x*e + 2*d)*e^{-1} - (4*a^3*c + 3*a*b^2*c + 3*a*c^3)*\cos(x*e + d)*e^{-1} + 1/32*(b^4 - 6*b^2*c^2 + c^4)*e^{-1}*\sin(4*x*e + 4*d) + 1/3*(a*b^3 - 3*a*b*c^2)*e^{-1}*\sin(3*x*e + 3*d) + 1/4*(6*a^2*b^2 + b^4 - 6*a^2*c^2 - c^4)*e^{-1}*\sin(2*x*e + 2*d) + (4*a^3*b + 3*a*b^3 + 3*a*b*c^2)*e^{-1}*\sin(x*e + d) + 1/8*(8*a^4 + 24*a^2*b^2 + 3*b^4 + 24*a^2*c^2 + 6*b^2*c^2 + 3*c^4)*x$

maple [A] time = 0.27, size = 335, normalized size = 1.29

$$b^4 \left(\frac{\left(\cos^3(ex+d) + \frac{3\cos(ex+d)}{2} \right) \sin(ex+d)}{4} + \frac{3ex}{8} + \frac{3d}{8} \right) + c^4 \left(-\frac{\left(\sin^3(ex+d) + \frac{3\sin(ex+d)}{2} \right) \cos(ex+d)}{4} + \frac{3ex}{8} + \frac{3d}{8} \right) - 6a^2bc \left(\cos^2(ex + d) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x)`

[Out] $1/e*(b^4*(1/4*(\cos(e*x+d))^3+3/2*\cos(e*x+d))*\sin(e*x+d)+3/8*e*x+3/8*d)+c^4*(-1/4*(\sin(e*x+d))^3+3/2*\sin(e*x+d))*\cos(e*x+d)+3/8*e*x+3/8*d)-6*a^2*b*c*\cos(e*x+d)^2-4*a*b^2*c*\cos(e*x+d)^3+4*a*b*c^2*\sin(e*x+d)^3+4*a^3*b*\sin(e*x+d)-4*\cos(e*x+d)*a^3*c+6*a^2*b^2*(1/2*\sin(e*x+d)*\cos(e*x+d)+1/2*e*x+1/2*d)+6*a^2*c^2*(-1/2*\sin(e*x+d)*\cos(e*x+d)+1/2*e*x+1/2*d)+4/3*a*b^3*(2+\cos(e*x+d))^2*\sin(e*x+d)-4/3*a*c^3*(2+\sin(e*x+d))^2*\cos(e*x+d)+a^4*(e*x+d)-\cos(e*x+d)^4*b^3*c+6*b^2*c^2*(-1/4*\sin(e*x+d)*\cos(e*x+d)^3+1/8*\sin(e*x+d)*\cos(e*x+d)+1/8*e*x+1/8*d)+c^3*b*\sin(e*x+d)^4$

maxima [A] time = 0.33, size = 330, normalized size = 1.27

$$\frac{b^3c \cos(ex+d)^4}{e} + \frac{bc^3 \sin(ex+d)^4}{e} + a^4x + \frac{(12ex + 12d + \sin(4ex + 4d) + 8 \sin(2ex + 2d))b^4}{32e} + \frac{3(4ex + d)c^4}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="maxima")`

[Out] $-b^3*c*\cos(e*x + d)^4/e + b*c^3*\sin(e*x + d)^4/e + a^4*x + 1/32*(12*e*x + 12*d + \sin(4*e*x + 4*d) + 8*\sin(2*e*x + 2*d))*b^4/e + 3/16*(4*e*x + 4*d - \sin(4*e*x + 4*d))*b^2*c^2/e + 1/32*(12*e*x + 12*d + \sin(4*e*x + 4*d) - 8*\sin(2*e*x + 2*d))*c^4/e - 4*a^3*(c*\cos(e*x + d)/e - b*\sin(e*x + d)/e) - 3/2*(4*b*c*\cos(e*x + d)^2/e - (2*e*x + 2*d + \sin(2*e*x + 2*d))*b^2/e - (2*e*x + 2*d - \sin(2*e*x + 2*d))*c^2/e)*a^2 - 4/3*(3*b^2*c*\cos(e*x + d)^3/e - 3*b*c^2*\sin(e*x + d)^3/e + (\sin(e*x + d))^3 - 3*\sin(e*x + d))*b^3/e - (\cos(e*x + d))^3 - 3*\cos(e*x + d))*c^3/e)*a$

mupad [B] time = 3.37, size = 376, normalized size = 1.45

$$\frac{6b^4 \sin(2d + 2ex) + \frac{3b^4 \sin(4d+4ex)}{4} - 6c^4 \sin(2d + 2ex) + \frac{3c^4 \sin(4d+4ex)}{4} + 8ac^3 \cos(3d + 3ex) - 12bc^3 \cos(2d + 2ex)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*cos(d + e*x) + c*sin(d + e*x))^4,x)`

[Out] $(6*b^4*\sin(2*d + 2*e*x) + (3*b^4*\sin(4*d + 4*e*x))/4 - 6*c^4*\sin(2*d + 2*e*x) + (3*c^4*\sin(4*d + 4*e*x))/4 + 8*a*c^3*\cos(3*d + 3*e*x) - 12*b*c^3*\cos(2*d + 2*e*x) - 12*b^3*c*\cos(2*d + 2*e*x) + 3*b*c^3*\cos(4*d + 4*e*x) - 3*b^3*c*\cos(4*d + 4*e*x) + 8*a*b^3*\sin(3*d + 3*e*x) + 36*a^2*b^2*\sin(2*d + 2*e*x) - 36*a^2*c^2*\sin(2*d + 2*e*x) - (9*b^2*c^2*\sin(4*d + 4*e*x))/2 - 72*a*c^3*\cos(d + e*x) - 96*a^3*c*\cos(d + e*x) + 72*a*b^3*\sin(d + e*x) + 96*a^3*b*\sin(d + e*x) + 24*a^4*e*x + 9*b^4*e*x + 9*c^4*e*x - 72*a*b^2*c*\cos(d + e*x) +$

$$72*a*b*c^2*\sin(d + e*x) - 72*a^2*b*c*\cos(2*d + 2*e*x) - 24*a*b^2*c*\cos(3*d + 3*e*x) - 24*a*b*c^2*\sin(3*d + 3*e*x) + 72*a^2*b^2*e*x + 72*a^2*c^2*e*x + 18*b^2*c^2*e*x)/(24*e)$$

sympy [A] time = 1.86, size = 682, normalized size = 2.62

$$\left\{ \begin{array}{l} a^4x + \frac{4a^3b \sin(d+ex)}{e} - \frac{4a^3c \cos(d+ex)}{e} + 3a^2b^2x \sin^2(d+ex) + 3a^2b^2x \cos^2(d+ex) + \frac{3a^2b^2 \sin(d+ex) \cos(d+ex)}{e} - \frac{6a^2bc \cos(d+ex)}{e} \\ x(a + b \cos(d) + c \sin(d))^4 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**4,x)

[Out] Piecewise((a**4*x + 4*a**3*b*sin(d + e*x)/e - 4*a**3*c*cos(d + e*x)/e + 3*a**2*b**2*x*sin(d + e*x)**2 + 3*a**2*b**2*x*cos(d + e*x)**2 + 3*a**2*b**2*sin(d + e*x)*cos(d + e*x)/e - 6*a**2*b*c*cos(d + e*x)**2/e + 3*a**2*c**2*x*sin(d + e*x)**2 + 3*a**2*c**2*x*cos(d + e*x)**2 - 3*a**2*c**2*sin(d + e*x)*cos(d + e*x)/e + 8*a*b**3*sin(d + e*x)**3/(3*e) + 4*a*b**3*sin(d + e*x)*cos(d + e*x)**2/e - 4*a*b**2*c*cos(d + e*x)**3/e + 4*a*b*c**2*sin(d + e*x)**3/e - 4*a*c**3*sin(d + e*x)**2*cos(d + e*x)/e - 8*a*c**3*cos(d + e*x)**3/(3*e) + 3*b**4*x*sin(d + e*x)**4/8 + 3*b**4*x*sin(d + e*x)**2*cos(d + e*x)**2/4 + 3*b**4*x*cos(d + e*x)**4/8 + 3*b**4*sin(d + e*x)**3*cos(d + e*x)/(8*e) + 5*b**4*sin(d + e*x)*cos(d + e*x)**3/(8*e) - b**3*c*cos(d + e*x)**4/e + 3*b**2*c**2*x*sin(d + e*x)**4/4 + 3*b**2*c**2*x*sin(d + e*x)**2*cos(d + e*x)**2/2 + 3*b**2*c**2*x*cos(d + e*x)**4/4 + 3*b**2*c**2*sin(d + e*x)**3*cos(d + e*x)/(4*e) - 3*b**2*c**2*sin(d + e*x)*cos(d + e*x)**3/(4*e) + b*c**3*sin(d + e*x)**4/e + 3*c**4*x*sin(d + e*x)**4/8 + 3*c**4*x*sin(d + e*x)**2*cos(d + e*x)**2/4 + 3*c**4*x*cos(d + e*x)**4/8 - 5*c**4*sin(d + e*x)**3*cos(d + e*x)/(8*e) - 3*c**4*sin(d + e*x)*cos(d + e*x)**3/(8*e), Ne(e, 0)), (x*(a + b*cos(d) + c*sin(d))**4, True))

3.396 $\int (a + b \cos(d + ex) + c \sin(d + ex))^3 dx$

Optimal. Leaf size=170

$$\frac{b(11a^2 + 4(b^2 + c^2)) \sin(d + ex)}{6e} - \frac{c(11a^2 + 4(b^2 + c^2)) \cos(d + ex)}{6e} + \frac{1}{2}ax(2a^2 + 3(b^2 + c^2)) - \frac{(c \cos(d + ex))}{e}$$

[Out] 1/2*a*(2*a^2+3*b^2+3*c^2)*x-1/6*c*(11*a^2+4*b^2+4*c^2)*cos(e*x+d)/e+1/6*b*(11*a^2+4*b^2+4*c^2)*sin(e*x+d)/e-5/6*(a*c*cos(e*x+d)-a*b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))/e-1/3*(c*cos(e*x+d)-b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^2/e

Rubi [A] time = 0.19, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3120, 3146, 2637, 2638}

$$\frac{b(11a^2 + 4(b^2 + c^2)) \sin(d + ex)}{6e} - \frac{c(11a^2 + 4(b^2 + c^2)) \cos(d + ex)}{6e} + \frac{1}{2}ax(2a^2 + 3(b^2 + c^2)) - \frac{(c \cos(d + ex))}{e}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3, x]

[Out] (a*(2*a^2 + 3*(b^2 + c^2))*x)/2 - (c*(11*a^2 + 4*(b^2 + c^2))*Cos[d + e*x])/(6*e) + (b*(11*a^2 + 4*(b^2 + c^2))*Sin[d + e*x])/(6*e) - (5*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))/(6*e) - ((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3*e)

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3120

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]

Rule 3146

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)])*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x] + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\begin{aligned}
\int (a + b \cos(d + ex) + c \sin(d + ex))^3 dx &= -\frac{(c \cos(d + ex) - b \sin(d + ex))(a + b \cos(d + ex) + c \sin(d + ex))^2}{3e} + \\
&= -\frac{5(ac \cos(d + ex) - ab \sin(d + ex))(a + b \cos(d + ex) + c \sin(d + ex))}{6e} \\
&= \frac{1}{2}a(2a^2 + 3(b^2 + c^2))x - \frac{5(ac \cos(d + ex) - ab \sin(d + ex))(a + b \cos(d + ex) + c \sin(d + ex))}{6e} \\
&= \frac{1}{2}a(2a^2 + 3(b^2 + c^2))x - \frac{c(11a^2 + 4(b^2 + c^2)) \cos(d + ex)}{6e} + \frac{b(11a^2 + 4(b^2 + c^2)) \sin(d + ex)}{6e}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 144, normalized size = 0.85

$$\frac{6a(2a^2 + 3(b^2 + c^2))(d + ex) + 9b(4a^2 + b^2 + c^2)\sin(d + ex) - 9c(4a^2 + b^2 + c^2)\cos(d + ex) + 9a(b^2 - c^2)\sin(3(d + ex))}{12e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]

[Out] (6*a*(2*a^2 + 3*(b^2 + c^2))*(d + e*x) - 9*c*(4*a^2 + b^2 + c^2)*Cos[d + e*x] - 18*a*b*c*Cos[2*(d + e*x)] + c*(-3*b^2 + c^2)*Cos[3*(d + e*x)] + 9*b*(4*a^2 + b^2 + c^2)*Sin[d + e*x] + 9*a*(b^2 - c^2)*Sin[2*(d + e*x)] + b*(b^2 - 3*c^2)*Sin[3*(d + e*x)])/(12*e)

fricas [A] time = 2.94, size = 147, normalized size = 0.86

$$\frac{18abc \cos(ex + d)^2 + 2(3b^2c - c^3) \cos(ex + d)^3 - 3(2a^3 + 3ab^2 + 3ac^2)ex + 6(3a^2c + c^3) \cos(ex + d) - (18a^2b + 4b^3 + 6bc^2 + 2(b^3 - 3bc^2)) \cos(ex + d)^2 + 9(a^2b^2 - a^2c^2) \cos(ex + d) \sin(ex + d)}{6e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="fricas")

[Out] -1/6*(18*a*b*c*cos(e*x + d)^2 + 2*(3*b^2*c - c^3)*cos(e*x + d)^3 - 3*(2*a^3 + 3*a*b^2 + 3*a*c^2)*e*x + 6*(3*a^2*c + c^3)*cos(e*x + d) - (18*a^2*b + 4*b^3 + 6*b*c^2 + 2*(b^3 - 3*b*c^2))*cos(e*x + d)^2 + 9*(a*b^2 - a*c^2)*cos(e*x + d))*sin(e*x + d)/e

giac [A] time = 0.20, size = 167, normalized size = 0.98

$$-\frac{3}{2}abc \cos(2xe + 2d)e^{(-1)} - \frac{1}{12}(3b^2c - c^3) \cos(3xe + 3d)e^{(-1)} - \frac{3}{4}(4a^2c + b^2c + c^3) \cos(xe + d)e^{(-1)} + \frac{1}{12}(b^3 - 3b^2c + 3bc^2) \sin(3xe + 3d)e^{(-1)} + \frac{3}{4}(a^2b^2 - a^2c^2) \sin(2xe + 2d)e^{(-1)} + \frac{3}{4}(4a^2b + b^3 + b^2c) \sin(xe + d)e^{(-1)} + \frac{1}{2}(2a^3 + 3a^2b + 3a^2c^2)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="giac")

[Out] -3/2*a*b*c*cos(2*x*e + 2*d)*e^(-1) - 1/12*(3*b^2*c - c^3)*cos(3*x*e + 3*d)*e^(-1) - 3/4*(4*a^2*c + b^2*c + c^3)*cos(x*e + d)*e^(-1) + 1/12*(b^3 - 3*b^2*c + 3*b*c^2)*e^(-1)*sin(3*x*e + 3*d) + 3/4*(a*b^2 - a*c^2)*e^(-1)*sin(2*x*e + 2*d) + 3/4*(4*a^2*b + b^3 + b^2*c)*e^(-1)*sin(x*e + d) + 1/2*(2*a^3 + 3*a^2*b + 3*a^2*c^2)*x

maple [A] time = 0.24, size = 177, normalized size = 1.04

$$a^3(ex + d) + 3 \sin(ex + d) a^2 b - 3a^2 c \cos(ex + d) + 3a b^2 \left(\frac{\sin(ex+d) \cos(ex+d)}{2} + \frac{ex}{2} + \frac{d}{2} \right) - 3abc (\cos^2(ex + d)) + 3a b c \sin^2(ex + d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x)

[Out] $\frac{1}{e} \cdot (a^3 \cos(e*x+d) + 3 \sin(e*x+d) \cdot a^2 \cdot b - 3 a^2 \cdot c \cdot \cos(e*x+d) + 3 a \cdot b^2 \cdot (1/2 \sin(e*x+d) \cdot \cos(e*x+d) + 1/2 e*x + 1/2 d) - 3 a \cdot b \cdot c \cdot \cos(e*x+d)^2 + 3 a \cdot c^2 \cdot (-1/2 \sin(e*x+d) \cdot \cos(e*x+d) + 1/2 e*x + 1/2 d) + 1/3 b^3 \cdot (2 + \cos(e*x+d)^2) \cdot \sin(e*x+d) - \cos(e*x+d)^3 \cdot b^2 \cdot c + c^2 \cdot b \cdot \sin(e*x+d)^3 - 1/3 c^3 \cdot (2 + \sin(e*x+d)^2) \cdot \cos(e*x+d))$

maxima [A] time = 0.33, size = 189, normalized size = 1.11

$$-\frac{b^2 c \cos(ex+d)^3}{e} + \frac{bc^2 \sin(ex+d)^3}{e} + a^3 x - \frac{(\sin(ex+d)^3 - 3 \sin(ex+d)) b^3}{3e} + \frac{(\cos(ex+d)^3 - 3 \cos(ex+d)) c^3}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="maxima")

[Out] $-b^2 \cdot c \cdot \cos(e*x+d)^3/e + b \cdot c^2 \cdot \sin(e*x+d)^3/e + a^3 \cdot x - 1/3 \cdot (\sin(e*x+d)^3 - 3 \sin(e*x+d)) \cdot b^3/e + 1/3 \cdot (\cos(e*x+d)^3 - 3 \cos(e*x+d)) \cdot c^3/e - 3 a^2 \cdot (c \cdot \cos(e*x+d)/e - b \cdot \sin(e*x+d)/e) - 3/4 \cdot (4 b \cdot c \cdot \cos(e*x+d)^2/e - (2 e*x + 2 d + \sin(2 e*x + 2 d)) \cdot b^2/e - (2 e*x + 2 d - \sin(2 e*x + 2 d)) \cdot c^2/e) \cdot a$

mupad [B] time = 3.70, size = 333, normalized size = 1.96

$$\frac{a \operatorname{atan}\left(\frac{a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) (2a^2 + 3b^2 + 3c^2)}{2a^3 + 3ab^2 + 3ac^2}\right) (2a^2 + 3b^2 + 3c^2)}{e} - \frac{a \left(\operatorname{atan}\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right) - \frac{ex}{2}\right) (2a^2 + 3b^2 + 3c^2) \tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(d + e*x) + c*sin(d + e*x))^3,x)

[Out] $(a \cdot \operatorname{atan}\left(\frac{a \tan(d/2 + (e*x)/2) \cdot (2a^2 + 3b^2 + 3c^2)}{(3a \cdot b^2 + 3a \cdot c^2 + 2a^3)}\right) \cdot (2a^2 + 3b^2 + 3c^2))/e - (a \cdot (\operatorname{atan}(\tan(d/2 + (e*x)/2)) - (e*x)/2) \cdot (2a^2 + 3b^2 + 3c^2))/e - (\tan(d/2 + (e*x)/2)^2 \cdot (12a^2 \cdot c + 4c^3 - 12a \cdot b \cdot c) - \tan(d/2 + (e*x)/2)^3 \cdot (12a^2 \cdot b + 8b \cdot c^2 + (4b^3)/3) - \tan(d/2 + (e*x)/2)^4 \cdot (6a^2 \cdot c + 6b^2 \cdot c - 12a \cdot b \cdot c) + 6a^2 \cdot c + 2b^2 \cdot c - \tan(d/2 + (e*x)/2)^5 \cdot (6a^2 \cdot b - 3a \cdot b^2 + 3a \cdot c^2 + 2b^3) + (4c^3)/3) / (e \cdot (3 \cdot \tan(d/2 + (e*x)/2)^2 + 3 \cdot \tan(d/2 + (e*x)/2)^4 + \tan(d/2 + (e*x)/2)^6 + 1))$

sympy [A] time = 0.76, size = 294, normalized size = 1.73

$$\left\{ \begin{array}{l} a^3 x + \frac{3a^2 b \sin(d+ex)}{e} - \frac{3a^2 c \cos(d+ex)}{e} + \frac{3ab^2 x \sin^2(d+ex)}{2} + \frac{3ab^2 x \cos^2(d+ex)}{2} + \frac{3ab^2 \sin(d+ex) \cos(d+ex)}{2e} - \frac{3abc \cos^2(d+ex)}{e} + 3a^2 \sin(d+ex) \cos(d+ex) \\ x(a + b \cos(d) + c \sin(d))^3 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x)

[Out] $\operatorname{Piecewise}\left(\left(a^3 x + 3 a^2 b \sin(d+e x) / e - 3 a^2 c \cos(d+e x) / e + 3 a^2 \sin(d+e x) \cos(d+e x) + 3 a b^2 x \sin^2(d+e x) / 2 + 3 a b^2 x \cos^2(d+e x) / 2 + 3 a b^2 \sin(d+e x) \cos(d+e x) / (2 e) - 3 a b c \cos^2(d+e x) / e + 3 a^2 \sin(d+e x) \cos(d+e x) + 2 b^3 \sin(d+e x) \cos(d+e x) / (2 e) - b^2 c \cos(d+e x) / e + b c^2 \sin(d+e x) / e - c^3 \sin(d+e x) \cos(d+e x) / e - 2 c^3 \cos(d+e x) / (3 e)\right), \operatorname{Ne}(e, 0)\right), (x \cdot (a + b \cos(d) + c \sin(d))^3, \operatorname{True})$

3.397 $\int (a + b \cos(d + ex) + c \sin(d + ex))^2 dx$

Optimal. Leaf size=91

$$\frac{1}{2}x(2a^2 + b^2 + c^2) - \frac{(c \cos(d + ex) - b \sin(d + ex))(a + b \cos(d + ex) + c \sin(d + ex))}{2e} + \frac{3ab \sin(d + ex)}{2e} - \frac{3ac \cos(d + ex)}{2e}$$

[Out] $1/2*(2*a^2+b^2+c^2)*x-3/2*a*c*\cos(e*x+d)/e+3/2*a*b*\sin(e*x+d)/e-1/2*(c*\cos(e*x+d)-b*\sin(e*x+d))*(a+b*\cos(e*x+d)+c*\sin(e*x+d))/e$

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3120, 2637, 2638}

$$\frac{1}{2}x(2a^2 + b^2 + c^2) - \frac{(c \cos(d + ex) - b \sin(d + ex))(a + b \cos(d + ex) + c \sin(d + ex))}{2e} + \frac{3ab \sin(d + ex)}{2e} - \frac{3ac \cos(d + ex)}{2e}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]

[Out] $((2*a^2 + b^2 + c^2)*x)/2 - (3*a*c*\cos[d + e*x])/(2*e) + (3*a*b*\sin[d + e*x])/(2*e) - ((c*\cos[d + e*x] - b*\sin[d + e*x])*(a + b*\cos[d + e*x] + c*\sin[d + e*x]))/(2*e)$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3120

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int (a + b \cos(d + ex) + c \sin(d + ex))^2 dx &= -\frac{(c \cos(d + ex) - b \sin(d + ex))(a + b \cos(d + ex) + c \sin(d + ex))}{2e} + \frac{3ab \sin(d + ex)}{2e} - \frac{3ac \cos(d + ex)}{2e} \\ &= \frac{1}{2}(2a^2 + b^2 + c^2)x - \frac{(c \cos(d + ex) - b \sin(d + ex))(a + b \cos(d + ex) + c \sin(d + ex))}{2e} \\ &= \frac{1}{2}(2a^2 + b^2 + c^2)x - \frac{3ac \cos(d + ex)}{2e} + \frac{3ab \sin(d + ex)}{2e} - \frac{(c \cos(d + ex) - b \sin(d + ex))(a + b \cos(d + ex) + c \sin(d + ex))}{2e} \end{aligned}$$

Mathematica [A] time = 0.17, size = 77, normalized size = 0.85

$$\frac{2(2a^2 + b^2 + c^2)(d + ex) + 8ab \sin(d + ex) - 8ac \cos(d + ex) + (b^2 - c^2) \sin(2(d + ex)) - 2bc \cos(2(d + ex))}{4e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*cos[d + e*x] + c*sin[d + e*x])^2,x]

[Out] (2*(2*a^2 + b^2 + c^2)*(d + e*x) - 8*a*c*cos[d + e*x] - 2*b*c*cos[2*(d + e*x)] + 8*a*b*sin[d + e*x] + (b^2 - c^2)*sin[2*(d + e*x)])/(4*e)

fricas [A] time = 1.52, size = 73, normalized size = 0.80

$$\frac{2bc \cos(ex + d)^2 - (2a^2 + b^2 + c^2)ex + 4ac \cos(ex + d) - (4ab + (b^2 - c^2) \cos(ex + d)) \sin(ex + d)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="fricas")

[Out] -1/2*(2*b*c*cos(e*x + d)^2 - (2*a^2 + b^2 + c^2)*e*x + 4*a*c*cos(e*x + d) - (4*a*b + (b^2 - c^2)*cos(e*x + d))*sin(e*x + d))/e

giac [A] time = 0.15, size = 81, normalized size = 0.89

$$-\frac{1}{2}bc \cos(2xe + 2d)e^{(-1)} - 2ac \cos(xe + d)e^{(-1)} + 2abe^{(-1)} \sin(xe + d) + \frac{1}{4}(b^2 - c^2)e^{(-1)} \sin(2xe + 2d) + \frac{1}{2}(2a^2 + b^2 + c^2)xe$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="giac")

[Out] -1/2*b*c*cos(2*x*e + 2*d)*e^(-1) - 2*a*c*cos(x*e + d)*e^(-1) + 2*a*b*e^(-1)*sin(x*e + d) + 1/4*(b^2 - c^2)*e^(-1)*sin(2*x*e + 2*d) + 1/2*(2*a^2 + b^2 + c^2)*x

maple [A] time = 0.22, size = 99, normalized size = 1.09

$$\frac{a^2(ex + d) + 2ab \sin(ex + d) - 2ac \cos(ex + d) + b^2 \left(\frac{\sin(ex+d)\cos(ex+d)}{2} + \frac{ex}{2} + \frac{d}{2} \right) - (\cos^2(ex + d))bc + c^2 \left(-\frac{\sin(2ex + 2d)}{2} \right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x)

[Out] 1/e*(a^2*(e*x+d)+2*a*b*sin(e*x+d)-2*a*c*cos(e*x+d)+b^2*(1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-cos(e*x+d)^2*b*c+c^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d))

maxima [A] time = 0.31, size = 100, normalized size = 1.10

$$a^2x - \frac{bc \cos(ex + d)^2}{e} + \frac{(2ex + 2d + \sin(2ex + 2d))b^2}{4e} + \frac{(2ex + 2d - \sin(2ex + 2d))c^2}{4e} - 2a \left(\frac{c \cos(ex + d)}{e} - \frac{b \sin(ex + d)}{e} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="maxima")

[Out] a^2*x - b*c*cos(e*x + d)^2/e + 1/4*(2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e + 1/4*(2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e - 2*a*(c*cos(e*x + d)/e - b*sin(e*x + d)/e)

mupad [B] time = 3.78, size = 125, normalized size = 1.37

$$\frac{x(2a^2 + b^2 + c^2)}{2} - \frac{(b^2 - 4ab - c^2) \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^3 + (4ac - 4bc) \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 + (-b^2 - 4ab + c^2) \tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{e \left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^4 + 2 \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*cos(d + e*x) + c*sin(d + e*x))^2,x)`

[Out] $(x*(2*a^2 + b^2 + c^2))/2 - (4*a*c + \tan(d/2 + (e*x)/2)^2*(4*a*c - 4*b*c) - \tan(d/2 + (e*x)/2)*(4*a*b + b^2 - c^2) - \tan(d/2 + (e*x)/2)^3*(4*a*b - b^2 + c^2))/(e*(2*\tan(d/2 + (e*x)/2)^2 + \tan(d/2 + (e*x)/2)^4 + 1))$

sympy [A] time = 0.31, size = 162, normalized size = 1.78

$$\begin{cases} a^2x + \frac{2ab \sin(d+ex)}{e} - \frac{2ac \cos(d+ex)}{e} + \frac{b^2x \sin^2(d+ex)}{2} + \frac{b^2x \cos^2(d+ex)}{2} + \frac{b^2 \sin(d+ex) \cos(d+ex)}{2e} - \frac{bc \cos^2(d+ex)}{e} + \frac{c^2x \sin^2(d+ex)}{2} + \\ x(a + b \cos(d) + c \sin(d))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x)`

[Out] `Piecewise((a**2*x + 2*a*b*sin(d + e*x)/e - 2*a*c*cos(d + e*x)/e + b**2*x*sin(d + e*x)**2/2 + b**2*x*cos(d + e*x)**2/2 + b**2*sin(d + e*x)*cos(d + e*x)/(2*e) - b*c*cos(d + e*x)**2/e + c**2*x*sin(d + e*x)**2/2 + c**2*x*cos(d + e*x)**2/2 - c**2*sin(d + e*x)*cos(d + e*x)/(2*e), Ne(e, 0)), (x*(a + b*cos(d) + c*sin(d))**2, True))`

3.398 $\int (a + b \cos(d + ex) + c \sin(d + ex)) dx$

Optimal. Leaf size=27

$$ax + \frac{b \sin(d + ex)}{e} - \frac{c \cos(d + ex)}{e}$$

[Out] a*x-c*cos(e*x+d)/e+b*sin(e*x+d)/e

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2637, 2638}

$$ax + \frac{b \sin(d + ex)}{e} - \frac{c \cos(d + ex)}{e}$$

Antiderivative was successfully verified.

[In] Int[a + b*Cos[d + e*x] + c*Sin[d + e*x],x]

[Out] a*x - (c*Cos[d + e*x])/e + (b*Sin[d + e*x])/e

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (a + b \cos(d + ex) + c \sin(d + ex)) dx &= ax + b \int \cos(d + ex) dx + c \int \sin(d + ex) dx \\ &= ax - \frac{c \cos(d + ex)}{e} + \frac{b \sin(d + ex)}{e} \end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.81

$$ax + \frac{b \sin(d) \cos(ex)}{e} + \frac{b \cos(d) \sin(ex)}{e} + \frac{c \sin(d) \sin(ex)}{e} - \frac{c \cos(d) \cos(ex)}{e}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*Cos[d + e*x] + c*Sin[d + e*x],x]

[Out] a*x - (c*Cos[d]*Cos[e*x])/e + (b*Cos[e*x]*Sin[d])/e + (b*Cos[d]*Sin[e*x])/e + (c*Sin[d]*Sin[e*x])/e

fricas [A] time = 0.89, size = 26, normalized size = 0.96

$$\frac{aex - c \cos(ex + d) + b \sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x, algorithm="fricas")

[Out] (a*e*x - c*cos(e*x + d) + b*sin(e*x + d))/e

giac [A] time = 0.15, size = 27, normalized size = 1.00

$$-c \cos(xe + d)e^{(-1)} + be^{(-1)} \sin(xe + d) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x, algorithm="giac")

[Out] -c*cos(x*e + d)*e^(-1) + b*e^(-1)*sin(x*e + d) + a*x

maple [A] time = 0.00, size = 28, normalized size = 1.04

$$ax - \frac{c \cos(ex + d)}{e} + \frac{b \sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b*cos(e*x+d)+c*sin(e*x+d),x)

[Out] a*x-c*cos(e*x+d)/e+b*sin(e*x+d)/e

maxima [A] time = 0.31, size = 27, normalized size = 1.00

$$ax - \frac{c \cos(ex + d)}{e} + \frac{b \sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x, algorithm="maxima")

[Out] a*x - c*cos(e*x + d)/e + b*sin(e*x + d)/e

mupad [B] time = 2.51, size = 40, normalized size = 1.48

$$ax - \frac{2c - 2b \tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{e \left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a + b*cos(d + e*x) + c*sin(d + e*x),x)

[Out] a*x - (2*c - 2*b*tan(d/2 + (e*x)/2))/(e*(tan(d/2 + (e*x)/2)^2 + 1))

sympy [A] time = 0.14, size = 34, normalized size = 1.26

$$ax + b \left(\begin{cases} \frac{\sin(d+ex)}{e} & \text{for } e \neq 0 \\ x \cos(d) & \text{otherwise} \end{cases} \right) + c \left(\begin{cases} -\frac{\cos(d+ex)}{e} & \text{for } e \neq 0 \\ x \sin(d) & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x)

[Out] a*x + b*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + c*Piecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True))

$$3.399 \quad \int \frac{1}{a+b \cos(d+ex)+c \sin(d+ex)} dx$$

Optimal. Leaf size=61

$$\frac{2 \tan^{-1} \left(\frac{(a-b) \tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}} \right)}{e \sqrt{a^2-b^2-c^2}}$$

[Out] $2 * \arctan((c+(a-b) * \tan(1/2 * e * x + 1/2 * d)) / (a^2 - b^2 - c^2)^{(1/2)}) / e / (a^2 - b^2 - c^2)^{(1/2)}$

Rubi [A] time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3124, 618, 204}

$$\frac{2 \tan^{-1} \left(\frac{(a-b) \tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}} \right)}{e \sqrt{a^2-b^2-c^2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b * \text{Cos}[d + e * x] + c * \text{Sin}[d + e * x])^{-1}, x]$

[Out] $(2 * \text{ArcTan}[(c + (a - b) * \text{Tan}[(d + e * x) / 2]) / \text{Sqrt}[a^2 - b^2 - c^2]]) / (\text{Sqrt}[a^2 - b^2 - c^2] * e)$

Rule 204

$\text{Int}[(a + (b * x)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2] * x) / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] * \text{Rt}[-b, 2]), x] / ; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a + (b * x) + (c * x)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1 / \text{Simp}[b^2 - 4 * a * c - x^2, x], x], x, b + 2 * c * x], x] / ; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4 * a * c, 0]$

Rule 3124

$\text{Int}[(\cos[(d + (e * x))] * (b + (a + (c * \sin[(d + (e * x))])^{-1}), x_Symbol] \rightarrow \text{Module}\{f = \text{FreeFactors}[\text{Tan}[(d + e * x) / 2], x\}, \text{Dist}[(2 * f) / e, \text{Subst}[\text{Int}[1 / (a + b + 2 * c * f * x + (a - b) * f^2 * x^2), x], x, \text{Tan}[(d + e * x) / 2] / f], x] / ; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{a+b \cos(d+ex)+c \sin(d+ex)} dx &= \frac{2 \text{Subst} \left(\int \frac{1}{a+b+2cx+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(d+ex)\right) \right)}{e} \\ &= \frac{4 \text{Subst} \left(\int \frac{1}{-4(a^2-b^2-c^2)-x^2} dx, x, 2c + 2(a-b) \tan\left(\frac{1}{2}(d+ex)\right) \right)}{e} \\ &= \frac{2 \tan^{-1} \left(\frac{c+(a-b) \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}} \right)}{\sqrt{a^2-b^2-c^2} e} \end{aligned}$$

Mathematica [A] time = 0.12, size = 57, normalized size = 0.93

$$\frac{2 \tanh^{-1} \left(\frac{(a-b) \tan\left(\frac{1}{2}(d+ex)\right) + c}{\sqrt{-a^2+b^2+c^2}} \right)}{e\sqrt{-a^2+b^2+c^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1), x]

[Out] (-2*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2 + c^2]])/(Sqrt[-a^2 + b^2 + c^2]*e)

fricas [B] time = 1.71, size = 434, normalized size = 7.11

$$\frac{\sqrt{-a^2+b^2+c^2} \log\left(-\frac{a^2b^2-2b^4-c^4-(a^2+3b^2)c^2-(2a^2b^2-b^4-2a^2c^2+c^4)\cos(ex+d)^2-2(ab^3+abc^2)\cos(ex+d)-2(ab^2c+ac^3-(bc^3-(2a^2b-b^4-c^4)\cos(ex+d)+(b^2-c^2)\cos(ex+d)))}{2ab\cos(ex+d)+(b^2-c^2)\cos(ex+d)}\right)}{2(a^2-b^2-c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)), x, algorithm="fricas")

[Out] [-1/2*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(e*x + d)^2 - 2*(a*b^3 + a*b*c^2)*cos(e*x + d) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(e*x + d))*sin(e*x + d) + 2*(2*a*b*c*cos(e*x + d)^2 - a*b*c + (b^2*c + c^3)*cos(e*x + d) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(e*x + d))*sin(e*x + d))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2*(b*c*cos(e*x + d) + a*c)*sin(e*x + d)))/((a^2 - b^2 - c^2)*e), arctan(-(a*b*cos(e*x + d) + a*c*sin(e*x + d) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(e*x + d) + (a^2*b - b^3 - b*c^2)*sin(e*x + d)))/(sqrt(a^2 - b^2 - c^2)*e)]

giac [A] time = 0.16, size = 91, normalized size = 1.49

$$\frac{2 \left(\pi \left[\frac{xe+d}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a + 2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + c}{\sqrt{a^2 - b^2 - c^2}} \right) \right) e^{-1}}{\sqrt{a^2 - b^2 - c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)), x, algorithm="giac")

[Out] -2*(pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - b^2 - c^2)))*e^(-1)/sqrt(a^2 - b^2 - c^2)

maple [A] time = 0.38, size = 61, normalized size = 1.00

$$\frac{2 \arctan \left(\frac{2(a-b) \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 2c}{2\sqrt{a^2 - b^2 - c^2}} \right)}{e\sqrt{a^2 - b^2 - c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d)), x)

[Out] $2/e/(a^2-b^2-c^2)^{(1/2)}*\arctan(1/2*(2*(a-b)*\tan(1/2*d+1/2*e*x)+2*c)/(a^2-b^2-c^2)^{(1/2)})$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for more details)Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 4.01, size = 75, normalized size = 1.23

$$\frac{2 \operatorname{atan}\left(\frac{c}{\sqrt{a^2-b^2-c^2}} + \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)(2a-2b)}{2\sqrt{a^2-b^2-c^2}}\right)}{e\sqrt{a^2-b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b*cos(d + e*x) + c*sin(d + e*x)),x)`

[Out] $(2*\operatorname{atan}(c/(a^2 - b^2 - c^2)^{(1/2)} + (\tan(d/2 + (e*x)/2)*(2*a - 2*b))/(2*(a^2 - b^2 - c^2)^{(1/2)})))/(e*(a^2 - b^2 - c^2)^{(1/2)})$

sympy [A] time = 152.53, size = 3179, normalized size = 52.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x)`

[Out] $\operatorname{Piecewise}\left(\left(\frac{32*b^{**5}/(32*b^{**6}*e*\tan(d/2 + e*x/2) - 16*b^{**5}*c*e + 32*b^{**5}*e*\sqrt{b^{**2} + c^{**2}}*\tan(d/2 + e*x/2) + 48*b^{**4}*c^{**2}*e*\tan(d/2 + e*x/2) - 16*b^{**4}*c*e*\sqrt{b^{**2} + c^{**2}} - 20*b^{**3}*c^{**3}*e + 32*b^{**3}*c^{**2}*e*\sqrt{b^{**2} + c^{**2}}*\tan(d/2 + e*x/2) + 18*b^{**2}*c^{**4}*e*\tan(d/2 + e*x/2) - 12*b^{**2}*c^{**3}*e*\sqrt{b^{**2} + c^{**2}} - 5*b*c^{**5}*e + 6*b*c^{**4}*e*\sqrt{b^{**2} + c^{**2}}*\tan(d/2 + e*x/2) + c^{**6}*e*\tan(d/2 + e*x/2) - c^{**5}*e*\sqrt{b^{**2} + c^{**2}})}{32*b^{**6}*e*\tan(d/2 + e*x/2) - 16*b^{**5}*c*e + 32*b^{**5}*e*\sqrt{b^{**2} + c^{**2}}*\tan(d/2 + e*x/2) + 48*b^{**4}*c^{**2}*e*\tan(d/2 + e*x/2) - 16*b^{**4}*c*e*\sqrt{b^{**2} + c^{**2}} - 20*b^{**3}*c^{**3}*e + 32*b^{**3}*c^{**2}*e*\sqrt{b^{**2} + c^{**2}}*\tan(d/2 + e*x/2) + 18*b^{**2}*c^{**4}*e*\tan(d/2 + e*x/2) - 12*b^{**2}*c^{**3}*e*\sqrt{b^{**2} + c^{**2}} - 5*b*c^{**5}*e + 6*b*c^{**4}*e*\sqrt{b^{**2} + c^{**2}}*\tan(d/2 + e*x/2) + c^{**6}*e*\tan(d/2 + e*x/2) - c^{**5}*e*\sqrt{b^{**2} + c^{**2}})} + 40*b^{**3}*c^{**2}/(32*b^{**6}*e*\tan(d/2 + e*x/2) - 16*b^{**5}*c*e + 32*b^{**5}*e*\sqrt{b^{**2} + c^{**2}}*\tan(d/2 + e*x/2) + 48*b^{**4}*c^{**2}*e*\tan(d/2 + e*x/2) - 16*b^{**4}*c*e*\sqrt{b^{**2} + c^{**2}} - 20*b^{**3}*c^{**3}*e + 32*b^{**3}*c^{**2}*e*\sqrt{b^{**2} + c^{**2}}*\tan(d/2 + e*x/2) + 18*b^{**2}*c^{**4}*e*\tan(d/2 + e*x/2) - 12*b^{**2}*c^{**3}*e*\sqrt{b^{**2} + c^{**2}} - 5*b*c^{**5}*e + 6*b*c^{**4}*e*\sqrt{b^{**2} + c^{**2}}*\tan(d/2 + e*x/2) + c^{**6}*e*\tan(d/2 + e*x/2) - c^{**5}*e*\sqrt{b^{**2} + c^{**2}})} + 24*b^{**2}*c^{**2}*sqrt{b^{**2} + c^{**2}}/(32*b^{**6}*e*\tan(d/2 + e*x/2) - 16*b^{**5}*c*e + 32*b^{**5}*e*\sqrt{b^{**2} + c^{**2}}*\tan(d/2 + e*x/2) + 48*b^{**4}*c^{**2}*e*\tan(d/2 + e*x/2) - 16*b^{**4}*c*e*\sqrt{b^{**2} + c^{**2}} - 20*b^{**3}*c^{**3}*e + 32*b^{**3}*c^{**2}*e*\sqrt{b^{**2} + c^{**2}}*\tan(d/2 + e*x/2) + 18*b^{**2}*c^{**4}*e*\tan(d/2 + e*x/2) - 12*b^{**2}*c^{**3}*e*\sqrt{b^{**2} + c^{**2}} - 5*b*c^{**5}*e + 6*b*c^{**4}*e*\sqrt{b^{**2} + c^{**2}}*\tan(d/2 + e*x/2) + c^{**6}*e*\tan(d/2 + e*x/2) - c^{**5}*e*\sqrt{b^{**2} + c^{**2}})} + 10*b*c^{**4}/(32*b^{**6}*e*\tan(d/2 + e*x/2) - 16*b^{**5}*c*e + 32*b^{**5}*e*\sqrt{b^{**2} + c^{**2}}*\tan(d/2 + e*x/2) + 48*b^{**4}*c^{**2}*e*\tan(d/2 + e*x/2) - 16*b^{**4}*c*e*\sqrt{b^{**2} + c^{**2}} - 20*b^{**3}*c^{**3}*e + 32*b^{**3}*c^{**2}*e*\sqrt{b^{**2} + c^{**2}}*\tan(d/2 + e*x/2) + 18*b^{**2}*c^{**4}*e*\tan(d/2 + e*x/2) - 12*b^{**2}*c^{**3}*e*\sqrt{b^{**2} + c^{**2}} - 5*b*c^{**5}*e + 6*b*c^{**4}*e*\sqrt{b^{**2} + c^{**2}}*\tan(d/2 + e*x/2) + c^{**6}*e*\tan(d/2 + e*x/2) - c^{**5}*e*\sqrt{b^{**2} + c^{**2}})}\right)$

```

*2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) - 16*b**4*c*e
*sqrt(b**2 + c**2) - 20*b**3*c**3*e + 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(
d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) - 12*b**2*c**3*e*sqrt(b**2 +
c**2) - 5*b*c**5*e + 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*
e*tan(d/2 + e*x/2) - c**5*e*sqrt(b**2 + c**2)) + 2*c**4*sqrt(b**2 + c**2)/(
32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e + 32*b**5*e*sqrt(b**2 + c**2)*tan(
d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) - 16*b**4*c*e*sqrt(b**2 + c
**2) - 20*b**3*c**3*e + 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) +
18*b**2*c**4*e*tan(d/2 + e*x/2) - 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c
**5*e + 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x
/2) - c**5*e*sqrt(b**2 + c**2)), Eq(a, -sqrt(b**2 + c**2))), (32*b**5/(32*b
**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e - 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2
+ e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) + 16*b**4*c*e*sqrt(b**2 + c**2)
- 20*b**3*c**3*e - 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b
**2*c**4*e*tan(d/2 + e*x/2) + 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e
- 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x/2)
+ c**5*e*sqrt(b**2 + c**2)) - 32*b**4*sqrt(b**2 + c**2)/(32*b**6*e*tan(d/2
+ e*x/2) - 16*b**5*c*e - 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*
b**4*c**2*e*tan(d/2 + e*x/2) + 16*b**4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3
*e - 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan
(d/2 + e*x/2) + 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e - 6*b*c**4*e*
sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x/2) + c**5*e*sqrt(
b**2 + c**2)) + 40*b**3*c**2/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e - 32
*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2
) + 16*b**4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3*e - 32*b**3*c**2*e*sqrt(b
**2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) + 12*b**2*c**
3*e*sqrt(b**2 + c**2) - 5*b*c**5*e - 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 +
e*x/2) + c**6*e*tan(d/2 + e*x/2) + c**5*e*sqrt(b**2 + c**2)) - 24*b**2*c**
2*sqrt(b**2 + c**2)/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e - 32*b**5*e*s
qrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) + 16*b*
**4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3*e - 32*b**3*c**2*e*sqrt(b**2 + c**2
)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) + 12*b**2*c**3*e*sqrt(
b**2 + c**2) - 5*b*c**5*e - 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) +
c**6*e*tan(d/2 + e*x/2) + c**5*e*sqrt(b**2 + c**2)) + 10*b*c**4/(32*b**6*e
*tan(d/2 + e*x/2) - 16*b**5*c*e - 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x
/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) + 16*b**4*c*e*sqrt(b**2 + c**2) - 20*
b**3*c**3*e - 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c
**4*e*tan(d/2 + e*x/2) + 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e - 6*
b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x/2) + c**
5*e*sqrt(b**2 + c**2)) - 2*c**4*sqrt(b**2 + c**2)/(32*b**6*e*tan(d/2 + e*x/
2) - 16*b**5*c*e - 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c
**2*e*tan(d/2 + e*x/2) + 16*b**4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3*e - 3
2*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 +
e*x/2) + 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e - 6*b*c**4*e*sqrt(b
**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x/2) + c**5*e*sqrt(b**2 +
c**2)), Eq(a, sqrt(b**2 + c**2))), (log(b/c + tan(d/2 + e*x/2))/(c*e), Eq(
a, b)), (x/(a + b*cos(d) + c*sin(d)), Eq(e, 0)), (log(c/(a - b) + tan(d/2 +
e*x/2) - sqrt(-a**2 + b**2 + c**2)/(a - b))/(e*sqrt(-a**2 + b**2 + c**2))
- log(c/(a - b) + tan(d/2 + e*x/2) + sqrt(-a**2 + b**2 + c**2)/(a - b))/(e*
sqrt(-a**2 + b**2 + c**2)), True))

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$$3.400 \quad \int \frac{1}{(a+b \cos(d+ex)+c \sin(d+ex))^2} dx$$

Optimal. Leaf size=121

$$\frac{2a \tan^{-1} \left(\frac{(a-b) \tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}} \right)}{e(a^2-b^2-c^2)^{3/2}} + \frac{c \cos(d+ex) - b \sin(d+ex)}{e(a^2-b^2-c^2)(a+b \cos(d+ex)+c \sin(d+ex))}$$

[Out] 2*a*arctan((c+(a-b)*tan(1/2*e*x+1/2*d))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^(3/2)/e+(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d))

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3129, 12, 3124, 618, 204}

$$\frac{2a \tan^{-1} \left(\frac{(a-b) \tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}} \right)}{e(a^2-b^2-c^2)^{3/2}} + \frac{c \cos(d+ex) - b \sin(d+ex)}{e(a^2-b^2-c^2)(a+b \cos(d+ex)+c \sin(d+ex))}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2), x]

[Out] (2*a*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2 - c^2]]/((a^2 - b^2 - c^2)^(3/2)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/((a^2 - b^2 - c^2)*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3129

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^n, x_Symbol] := Simp[(-(c*Cos[d + e*x] + b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*

$(n + 2) \cdot \sin[d + e \cdot x] \cdot (a + b \cdot \cos[d + e \cdot x] + c \cdot \sin[d + e \cdot x])^{n + 1}, x], x]$
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^2} dx &= \frac{c \cos(d + ex) - b \sin(d + ex)}{(a^2 - b^2 - c^2) e (a + b \cos(d + ex) + c \sin(d + ex))} - \int \frac{\frac{a}{a + b \cos(d + ex) + c \sin(d + ex)}}{-a^2 + b^2 - c^2} dx \\ &= \frac{c \cos(d + ex) - b \sin(d + ex)}{(a^2 - b^2 - c^2) e (a + b \cos(d + ex) + c \sin(d + ex))} + \frac{a \int \frac{1}{a + b \cos(d + ex) + c \sin(d + ex)} dx}{a^2 - b^2 - c^2} \\ &= \frac{c \cos(d + ex) - b \sin(d + ex)}{(a^2 - b^2 - c^2) e (a + b \cos(d + ex) + c \sin(d + ex))} + \frac{(2a) \text{Subst} \left(\int \frac{1}{a + b \cos(d + ex) + c \sin(d + ex)} dx \right)}{a^2 - b^2 - c^2} \\ &= \frac{c \cos(d + ex) - b \sin(d + ex)}{(a^2 - b^2 - c^2) e (a + b \cos(d + ex) + c \sin(d + ex))} - \frac{(4a) \text{Subst} \left(\int \frac{1}{-a + b \cos(d + ex) + c \sin(d + ex)} dx \right)}{a^2 - b^2 - c^2} \\ &= \frac{2a \tan^{-1} \left(\frac{c + (a - b) \tan\left(\frac{1}{2}(d + ex)\right)}{\sqrt{a^2 - b^2 - c^2}} \right)}{(a^2 - b^2 - c^2)^{3/2} e} + \frac{c \cos(d + ex) - b \sin(d + ex)}{(a^2 - b^2 - c^2) e (a + b \cos(d + ex) + c \sin(d + ex))} \end{aligned}$$

Mathematica [A] time = 0.36, size = 116, normalized size = 0.96

$$\frac{\frac{ac + (b^2 + c^2) \sin(d + ex)}{b(-a^2 + b^2 + c^2)(a + b \cos(d + ex) + c \sin(d + ex))} + \frac{2a \tanh^{-1} \left(\frac{(a - b) \tan\left(\frac{1}{2}(d + ex)\right) + c}{\sqrt{-a^2 + b^2 + c^2}} \right)}{(-a^2 + b^2 + c^2)^{3/2}}}{e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2), x]

[Out] ((2*a*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2 + c^2]]/(-a^2 + b^2 + c^2)^(3/2) + (a*c + (b^2 + c^2)*Sin[d + e*x])/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))) / e

fricas [B] time = 1.04, size = 819, normalized size = 6.77

$$\left[\frac{(ab \cos(ex + d) + ac \sin(ex + d) + a^2) \sqrt{-a^2 + b^2 + c^2} \log \left(\frac{a^2 b^2 - 2 b^4 - c^4 - (a^2 + 3 b^2) c^2 - (2 a^2 b^2 - b^4 - 2 a^2 c^2 + c^4) \cos(ex + d) - 2 (ab \cos(ex + d) + ac \sin(ex + d) + a^2) \sqrt{-a^2 + b^2 + c^2}}{2 \left((a^4 b - 2 a^2 b^3 + b^5 + bc^4 - 2 (a^2 b^2 - b^4 - 2 a^2 c^2 + c^4) \cos(ex + d) - 2 (ab \cos(ex + d) + ac \sin(ex + d) + a^2) \sqrt{-a^2 + b^2 + c^2}) \right)} \right)}{2 \left((a^4 b - 2 a^2 b^3 + b^5 + bc^4 - 2 (a^2 b^2 - b^4 - 2 a^2 c^2 + c^4) \cos(ex + d) - 2 (ab \cos(ex + d) + ac \sin(ex + d) + a^2) \sqrt{-a^2 + b^2 + c^2}) \right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="fricas")

[Out] [1/2*((a*b*cos(e*x + d) + a*c*sin(e*x + d) + a^2)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(e*x + d)^2 - 2*(a*b^3 + a*b*c^2)*cos(e*x + d) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(e*x + d))*sin(e*x + d) - 2*(2*a*b*c*cos(e*x + d)^2 - a*b*c + (b^2*c + c^3)*cos(e*x + d) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(e*x + d))*sin(e*x + d))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(e*x + d) + a*c*sin(e*x + d) + a^2)]

+ d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2*(b*c*cos(e*x + d) + a*c)*sin(e*x + d)) - 2*(c^3 - (a^2 - b^2)*c)*cos(e*x + d) - 2*(a^2*b - b^3 - b*c^2)*sin(e*x + d))/((a^4*b - 2*a^2*b^3 + b^5 + b*c^4 - 2*(a^2*b - b^3)*c^2)*e*cos(e*x + d) + (c^5 - 2*(a^2 - b^2)*c^3 + (a^4 - 2*a^2*b^2 + b^4)*c)*e*sin(e*x + d) + (a^5 - 2*a^3*b^2 + a*b^4 + a*c^4 - 2*(a^3 - a*b^2)*c^2)*e), ((a*b*cos(e*x + d) + a*c*sin(e*x + d) + a^2)*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(e*x + d) + a*c*sin(e*x + d) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(e*x + d) + (a^2*b - b^3 - b*c^2)*sin(e*x + d))) - (c^3 - (a^2 - b^2)*c)*cos(e*x + d) - (a^2*b - b^3 - b*c^2)*sin(e*x + d))/((a^4*b - 2*a^2*b^3 + b^5 + b*c^4 - 2*(a^2*b - b^3)*c^2)*e*cos(e*x + d) + (c^5 - 2*(a^2 - b^2)*c^3 + (a^4 - 2*a^2*b^2 + b^4)*c)*e*sin(e*x + d) + (a^5 - 2*a^3*b^2 + a*b^4 + a*c^4 - 2*(a^3 - a*b^2)*c^2)*e)]

giac [A] time = 0.16, size = 222, normalized size = 1.83

$$-2 \frac{\left(\pi \left[\frac{xe+d}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + c}{\sqrt{a^2 - b^2 - c^2}} \right) \right) a}{(a^2 - b^2 - c^2)^{\frac{3}{2}}} + \frac{ab \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)}{(a^3 - a^2b - ab^2 + b^3 - ac^2 + c^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="giac")

[Out] -2*((pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - b^2 - c^2)))*a/(a^2 - b^2 - c^2)^(3/2) + (a*b*tan(1/2*x*e + 1/2*d) - b^2*tan(1/2*x*e + 1/2*d) - c^2*tan(1/2*x*e + 1/2*d) - a*c)/((a^3 - a^2*b - a*b^2 + b^3 - a*c^2 + b*c^2)*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 + 2*c*tan(1/2*x*e + 1/2*d) + a + b)))*e^(-1)

maple [B] time = 0.50, size = 424, normalized size = 3.50

$$\frac{2 \tan\left(\frac{d}{2} + \frac{ex}{2}\right) ab}{e \left(a \left(\tan^2\left(\frac{d}{2} + \frac{ex}{2}\right) \right) - b \left(\tan^2\left(\frac{d}{2} + \frac{ex}{2}\right) \right) + 2c \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + a + b \right) (a^3 - a^2b - ab^2 - ac^2 + b^3 + c^2b)} + \frac{ab \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)}{(a^3 - a^2b - ab^2 + b^3 - ac^2 + c^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x)

[Out] -2/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan(1/2*d+1/2*e*x)*a*b+2/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan(1/2*d+1/2*e*x)*b^2+2/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan(1/2*d+1/2*e*x)*c^2+2/e/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)*a*c/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)+2/e*a/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(a-b)*tan(1/2*d+1/2*e*x)+2*c)/(a^2-b^2-c^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see 'assume?' for more details)Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 3.06, size = 195, normalized size = 1.61

$$\frac{2a \operatorname{atanh}\left(\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)(2a-2b) + \frac{2(-a^2c+b^2c+c^3)}{-a^2+b^2+c^2}}{2\sqrt{-a^2+b^2+c^2}}\right)}{e(-a^2+b^2+c^2)^{3/2}} - \frac{\frac{2ac}{(a-b)(-a^2+b^2+c^2)} + \frac{2\tan\left(\frac{d}{2} + \frac{ex}{2}\right)(b^2-ab+c^2)}{(a-b)(-a^2+b^2+c^2)}}{e\left((a-b)\tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 + 2c\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^2,x)

[Out] (2*a*atanh((tan(d/2 + (e*x)/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^2 - a^2 + c^2))/(2*(b^2 - a^2 + c^2)^(1/2))))/(e*(b^2 - a^2 + c^2)^(3/2)) - ((2*a*c)/((a - b)*(b^2 - a^2 + c^2)) + (2*tan(d/2 + (e*x)/2)*(b^2 - a*b + c^2))/((a - b)*(b^2 - a^2 + c^2)))/(e*(a + b + tan(d/2 + (e*x)/2)^2*(a - b) + 2*c*tan(d/2 + (e*x)/2)))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**2,x)

[Out] Timed out

$$3.401 \quad \int \frac{1}{(a+b \cos(d+ex)+c \sin(d+ex))^3} dx$$

Optimal. Leaf size=197

$$\frac{(2a^2 + b^2 + c^2) \tan^{-1}\left(\frac{(a-b) \tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e(a^2 - b^2 - c^2)^{5/2}} + \frac{3(ac \cos(d+ex) - ab \sin(d+ex))}{2e(a^2 - b^2 - c^2)^2 (a + b \cos(d+ex) + c \sin(d+ex))} + \frac{c}{2e(a^2 - b^2 - c^2)}$$

[Out] (2*a^2+b^2+c^2)*arctan((c+(a-b)*tan(1/2*e*x+1/2*d))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^(5/2)/e+1/2*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^2+3/2*(a*c*cos(e*x+d)-a*b*sin(e*x+d))/(a^2-b^2-c^2)^2/e/(a+b*cos(e*x+d)+c*sin(e*x+d))

Rubi [A] time = 0.20, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3129, 3153, 3124, 618, 204}

$$\frac{(2a^2 + b^2 + c^2) \tan^{-1}\left(\frac{(a-b) \tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e(a^2 - b^2 - c^2)^{5/2}} + \frac{3(ac \cos(d+ex) - ab \sin(d+ex))}{2e(a^2 - b^2 - c^2)^2 (a + b \cos(d+ex) + c \sin(d+ex))} + \frac{c}{2e(a^2 - b^2 - c^2)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3), x]

[Out] ((2*a^2 + b^2 + c^2)*ArcTan[(c + (a - b)*Tan[(d + e*x)/2]]/Sqrt[a^2 - b^2 - c^2])/((a^2 - b^2 - c^2)^(5/2)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/(2*(a^2 - b^2 - c^2)*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) + (3*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x]))/(2*(a^2 - b^2 - c^2)^2*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3129

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-n), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N

eQ[n, -3/2]

Rule 3153

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)])
/((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^3} dx = \frac{c \cos(d + ex) - b \sin(d + ex)}{2(a^2 - b^2 - c^2)e(a + b \cos(d + ex) + c \sin(d + ex))^2} - \frac{\int \frac{-2a + b \cos(d + ex)}{(a + b \cos(d + ex) + c \sin(d + ex))} dx}{2(a^2 - b^2 - c^2)}$$

$$= \frac{c \cos(d + ex) - b \sin(d + ex)}{2(a^2 - b^2 - c^2)e(a + b \cos(d + ex) + c \sin(d + ex))^2} + \frac{3(ac - b^2)}{2(a^2 - b^2 - c^2)e}$$

$$= \frac{c \cos(d + ex) - b \sin(d + ex)}{2(a^2 - b^2 - c^2)e(a + b \cos(d + ex) + c \sin(d + ex))^2} + \frac{3(ac - b^2)}{2(a^2 - b^2 - c^2)e}$$

$$= \frac{c \cos(d + ex) - b \sin(d + ex)}{2(a^2 - b^2 - c^2)e(a + b \cos(d + ex) + c \sin(d + ex))^2} + \frac{3(ac - b^2)}{2(a^2 - b^2 - c^2)e}$$

$$= \frac{(2a^2 + b^2 + c^2) \tan^{-1}\left(\frac{c + (a-b) \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2 - b^2 - c^2}}\right)}{(a^2 - b^2 - c^2)^{5/2} e} + \frac{c \cos(d + ex) - b \sin(d + ex)}{2(a^2 - b^2 - c^2)e(a + b \cos(d + ex) + c \sin(d + ex))}$$

Mathematica [A] time = 0.95, size = 200, normalized size = 1.02

$$\frac{ac + (b^2 + c^2) \sin(d + ex)}{b(-a^2 + b^2 + c^2)(a + b \cos(d + ex) + c \sin(d + ex))^2} - \frac{c(2a^2 + b^2 + c^2) + 3a(b^2 + c^2) \sin(d + ex)}{b(-a^2 + b^2 + c^2)^2(a + b \cos(d + ex) + c \sin(d + ex))} - \frac{2(2a^2 + b^2 + c^2) \tanh^{-1}\left(\frac{(a-b) \tan\left(\frac{1}{2}(d+ex)\right) + c}{\sqrt{-a^2 + b^2 + c^2}}\right)}{(-a^2 + b^2 + c^2)^{5/2}}$$

2e

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3), x]
```

```
[Out] ((-2*(2*a^2 + b^2 + c^2)*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[-a^2 +
b^2 + c^2]]/(-a^2 + b^2 + c^2)^(5/2) + (a*c + (b^2 + c^2)*Sin[d + e*x])/(
b*(-a^2 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2 - (c*(2*a^2 +
b^2 + c^2) + 3*a*(b^2 + c^2)*Sin[d + e*x])/(b*(-a^2 + b^2 + c^2)^2*(a + b*
Cos[d + e*x] + c*Sin[d + e*x])))/(2*e)
```

fricas [B] time = 2.01, size = 1947, normalized size = 9.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="fricas")
```



```
[Out] [1/4*(6*a*b*c^3 - 12*(a*b*c^3 - (a^3*b - a*b^3)*c)*cos(e*x + d)^2 - (2*a^4 + a^2*b^2 + c^4 + (3*a^2 + b^2)*c^2 + (2*a^2*b^2 + b^4 - 2*a^2*c^2 - c^4)*cos(e*x + d)^2 + 2*(2*a^3*b + a*b^3 + a*b*c^2)*cos(e*x + d) + 2*(a*c^3 + (2*a^3 + a*b^2)*c + (b*c^3 + (2*a^2*b + b^3)*c)*cos(e*x + d))*sin(e*x + d))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(e*x + d)^2 - 2*(a*b^3 + a*b*c^2)*cos(e*x + d) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(e*x + d))*sin(e*x + d) + 2*(2*a*b*c*cos(e*x + d)^2 - a*b*c + (b^2*c + c^3)*cos(e*x + d) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(e*x + d))*sin(e*x + d))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2*(b*c*cos(e*x + d) + a*c)*sin(e*x + d))] - 6*(a^3*b - a*b^3)*c + 2*(c^5 - (5*a^2 - 2*b^2)*c^3 + (4*a^4 - 5*a^2*b^2 + b^4)*c)*cos(e*x + d) - 2*(4*a^4*b - 5*a^2*b^3 + b^5 + b*c^4 - (5*a^2*b - 2*b^3)*c^2 + 3*(a^3*b^2 - a*b^4 - a^3*c^2 + a*c^4)*cos(e*x + d))*sin(e*x + d))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + c^8 - (3*a^2 - 2*b^2)*c^6 + 3*(a^4 - a^2*b^2)*c^4 - (a^6 - 3*a^2*b^4 + 2*b^6)*c^2)*e*cos(e*x + d)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 - a*b*c^6 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2)*e*cos(e*x + d) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - c^8 + (2*a^2 - 3*b^2)*c^6 + 3*(a^2*b^2 - b^4)*c^4 - (2*a^6 - 3*a^4*b^2 + b^6)*c^2)*e - 2*((b*c^7 - 3*(a^2*b - b^3)*c^5 + 3*(a^4*b - 2*a^2*b^3 + b^5)*c^3 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c)*e*cos(e*x + d) + (a*c^7 - 3*(a^3 - a*b^2)*c^5 + 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)*e)*sin(e*x + d)), 1/2*(3*a*b*c^3 - 6*(a*b*c^3 - (a^3*b - a*b^3)*c)*cos(e*x + d)^2 + (2*a^4 + a^2*b^2 + c^4 + (3*a^2 + b^2)*c^2 + (2*a^2*b^2 + b^4 - 2*a^2*c^2 - c^4)*cos(e*x + d)^2 + 2*(2*a^3*b + a*b^3 + a*b*c^2)*cos(e*x + d) + 2*(a*c^3 + (2*a^3 + a*b^2)*c + (b*c^3 + (2*a^2*b + b^3)*c)*cos(e*x + d))*sin(e*x + d))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(e*x + d) + a*c*sin(e*x + d) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(e*x + d) + (a^2*b - b^3 - b*c^2)*sin(e*x + d))) - 3*(a^3*b - a*b^3)*c + (c^5 - (5*a^2 - 2*b^2)*c^3 + (4*a^4 - 5*a^2*b^2 + b^4)*c)*cos(e*x + d) - (4*a^4*b - 5*a^2*b^3 + b^5 + b*c^4 - (5*a^2*b - 2*b^3)*c^2 + 3*(a^3*b^2 - a*b^4 - a^3*c^2 + a*c^4)*cos(e*x + d))*sin(e*x + d))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + c^8 - (3*a^2 - 2*b^2)*c^6 + 3*(a^4 - a^2*b^2)*c^4 - (a^6 - 3*a^2*b^4 + 2*b^6)*c^2)*e*cos(e*x + d)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7 - a*b*c^6 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2)*e*cos(e*x + d) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - c^8 + (2*a^2 - 3*b^2)*c^6 + 3*(a^2*b^2 - b^4)*c^4 - (2*a^6 - 3*a^4*b^2 + b^6)*c^2)*e - 2*((b*c^7 - 3*(a^2*b - b^3)*c^5 + 3*(a^4*b - 2*a^2*b^3 + b^5)*c^3 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c)*e*cos(e*x + d) + (a*c^7 - 3*(a^3 - a*b^2)*c^5 + 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)*e)*sin(e*x + d))]
```

giac [B] time = 0.32, size = 892, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] -((pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - b^2 - c^2)))*(2*a^2 + b^2 + c^2)/((a^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^2 - c^2)) + (4*a^4*b*tan(1/2*x*e + 1/2*d)^3 - 11*a^3*b^2*tan(1/2*x*e + 1/2*d)^3 + 9*a^2*b^3*tan(1/2*x*e + 1/2*d)^3 - a*b^4*tan(1/2*x*e + 1/2*d)^3 - b^5*tan(1/2*x*e + 1/2*d)^3 - 5*a^3*c^2*tan(1/2*x*e + 1/2*d)^3 + 7*a^2*b*c^2*tan(1/2*x*e + 1/2*d)^3 + a*b^2*c^2*tan(1/2*x*e + 1/2*d)^3 - 3*b^3*c^2*tan(1/2*x*e + 1/2*d)^3 + 2*a*c^4*tan(1/2*x*e + 1/2*d)^3 - 2*b*c^4*tan(1/2*x*e + 1/2*d)^3 - 4*a^4*c*tan(1/2*x*e + 1/2*d)^2 + 12*a^3*b*c*tan(1/2*x*e + 1/2*d)^2 - 13*a^2*b^2*c*tan(1/2*x*e + 1/2*d)^2 + 6*a*b^3*c*tan(1/2*x*e + 1/2*d)^2 - b^4*c*tan(1/2*x*e + 1/2*d)^2 - 7*a^2*c^3*tan(1/2*x*e + 1/2*d)^2 + 6*a*
```

$$b^3c^3 \tan(1/2*x*e + 1/2*d)^2 + b^2c^3 \tan(1/2*x*e + 1/2*d)^2 + 2c^5 \tan(1/2*x*e + 1/2*d)^2 + 4a^4b \tan(1/2*x*e + 1/2*d) - 5a^3b^2 \tan(1/2*x*e + 1/2*d) - 3a^2b^3 \tan(1/2*x*e + 1/2*d) + 5ab^4 \tan(1/2*x*e + 1/2*d) - b^5 \tan(1/2*x*e + 1/2*d) - 11a^3c^2 \tan(1/2*x*e + 1/2*d) + 3a^2b^2c^2 \tan(1/2*x*e + 1/2*d) + 7ab^2c^2 \tan(1/2*x*e + 1/2*d) + b^3c^2 \tan(1/2*x*e + 1/2*d) + 2ac^4 \tan(1/2*x*e + 1/2*d) + 2b^4c^2 \tan(1/2*x*e + 1/2*d) - 4a^4c + 3a^2b^2c + b^4c + a^2c^3 + b^2c^3) / ((a^6 - 2a^5b - a^4b^2 + 4a^3b^3 - a^2b^4 - 2ab^5 + b^6 - 2a^4c^2 + 4a^3b^2c^2 - 4ab^3c^2 + 2b^4c^2 + a^2c^4 - 2ab^2c^4 + b^2c^4) * (a \tan(1/2*x*e + 1/2*d)^2 - b \tan(1/2*x*e + 1/2*d)^2 + 2c \tan(1/2*x*e + 1/2*d) + a + b)^2) * e^{-1}$$

maple [B] time = 0.56, size = 3933, normalized size = 19.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x)

[Out] $3/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - 2ab + b^2) \tan(1/2*d + 1/2*e*x) * a^2b^3 - 5/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - 2ab + b^2) \tan(1/2*d + 1/2*e*x) * a^2b^2c^2 - 2/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - 2ab + b^2) \tan(1/2*d + 1/2*e*x) * a^2c^4 - 1/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - 2ab + b^2) \tan(1/2*d + 1/2*e*x) * b^3c^2 - 2/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - 2ab + b^2) \tan(1/2*d + 1/2*e*x) * c^4b^3 - 3/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 * c / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - 2ab + b^2) \tan(1/2*d + 1/2*e*x) * a^2b^2 - 4/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 / (a - b) / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) * \tan(1/2*d + 1/2*e*x)^3 * a^3b + 7/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 / (a - b) / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) * \tan(1/2*d + 1/2*e*x)^3 * a^2b^2 + 5/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 / (a - b) / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) * \tan(1/2*d + 1/2*e*x)^3 * a^2c^2 - 2/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 / (a - b) / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) * \tan(1/2*d + 1/2*e*x)^3 * ab^3 - 3/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 / (a - b) / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) * \tan(1/2*d + 1/2*e*x)^3 * a^2c^2 + b^4 + 2b^2c^2 + c^4) * \tan(1/2*d + 1/2*e*x)^3 * b^2c^2 + 4/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 * c / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - 2ab + b^2) \tan(1/2*d + 1/2*e*x)^2 * a^4 + 7/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 * c^3 / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - 2ab + b^2) \tan(1/2*d + 1/2*e*x)^2 * a^2 + 1/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 * c / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - 2ab + b^2) \tan(1/2*d + 1/2*e*x)^2 * b^4 - 1/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 * c^3 / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - 2ab + b^2) \tan(1/2*d + 1/2*e*x)^2 * b^2 - 4/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - 2ab + b^2) \tan(1/2*d + 1/2*e*x) * a^4 * b^5 / e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - 2ab + b^2) \tan(1/2*d + 1/2*e*x) * a^3 * b^2 + 11/e / (a \tan(1/2*d + 1/2*e*x)^2 - b \tan(1/2*d + 1/2*e*x)^2 + 2c \tan(1/2*d + 1/2*e*x) + a + b)^2 / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - 2ab + b^2) \tan(1/2*d + 1/2*e*x) * a^3 * c^2 + 2/e / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - b^2 - c^2)^{(1/2)} * \arctan(1/2 * (2 * (a - b) * \tan(1/2*d + 1/2*e*x) + 2 * c) / (a^2 - b^2 - c^2)^{(1/2)}) * a^2 + 1/e / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 -$

$$\frac{b^2-c^2)^{1/2} \arctan(1/2*(2*(a-b)*\tan(1/2*d+1/2*e*x)+2*c)/(a^2-b^2-c^2)^{1/2}) * b^2+1/e/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^{1/2}) * \arctan(1/2*(2*(a-b)*\tan(1/2*d+1/2*e*x)+2*c)/(a^2-b^2-c^2)^{1/2}) * c^2-1/e/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a+b)^2 * c^3/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*a^2-1/e/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a+b)^2 * c/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*b^4-1/e/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a+b)^2 * c^3/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*b^2-1/e/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a+b)^2/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*\tan(1/2*d+1/2*e*x)^3*b^4-2/e/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a+b)^2 * c^5/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*\tan(1/2*d+1/2*e*x)^2+1/e/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*\tan(1/2*d+1/2*e*x)*b^5+4/e/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a+b)^2 * c/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*a^4+13/e/(a*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a+b)^2 * c/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*\tan(1/2*d+1/2*e*x)^2*a^2*b^2-12/e/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a+b)^2 * c/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*\tan(1/2*d+1/2*e*x)^2*a^3*b-2/e/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a+b)^2/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*\tan(1/2*d+1/2*e*x)^3*a*b*c^2-6/e/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a+b)^2 * c/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*\tan(1/2*d+1/2*e*x)^2*a*b^3-6/e/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a+b)^2 * c^3/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*\tan(1/2*d+1/2*e*x)^2*a*b-3/e/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*\tan(1/2*d+1/2*e*x)*a^2*b*c^2-7/e/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a+b)^2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*\tan(1/2*d+1/2*e*x)*a*b^2*c^2$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for more details)Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 6.06, size = 700, normalized size = 3.55

$$\frac{-4 a^4 c+3 a^2 b^2 c+a^2 c^3+b^4 c+b^2 c^3}{(a-b)^2\left(a^4-2 a^2 b^2-2 a^2 c^2+b^4+2 b^2 c^2+c^4\right)}+\frac{\tan\left(\frac{d}{2}+\frac{e x}{2}\right)\left(4 a^4 b-5 a^3 b^2-11 a^3 c^2-3 a^2 b^3+3 a^2 b c^2+5 a b^4+7 a b^2 c^2+2 a c^4-b^5+b^3 c^2+2 b c^4\right)}{(a-b)^2\left(a^4-2 a^2 b^2-2 a^2 c^2+b^4+2 b^2 c^2+c^4\right)}+\frac{e\left(2 a b+\tan\left(\frac{d}{2}+\frac{e x}{2}\right)^3(4 a c-4 b c)+\tan\left(\frac{d}{2}+\frac{e x}{2}\right)^2\left(2 a^2-2 b^2+\right.\right.}{\left.\left.\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^3,x)

```
[Out] - ((b^4*c - 4*a^4*c + a^2*c^3 + b^2*c^3 + 3*a^2*b^2*c)/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(d/2 + (e*x)/2)*(5*a*b^4 + 4*a^4*b + 2*a*c^4 + 2*b*c^4 - b^5 - 3*a^2*b^3 - 5*a^3*b^2 - 11*a^3*c^2 + b^3*c^2 + 7*a*b^2*c^2 + 3*a^2*b*c^2))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(d/2 + (e*x)/2)^2*(2*c^5 - b^4*c - 4*a^4*c - 7*a^2*c^3 + b^2*c^3 - 13*a^2*b^2*c + 6*a*b*c^3 + 6*a*b^3*c + 12*a^3*b*c))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(d/2 + (e*x)/2)^3*(2*a*b^3 + 4*a^3*b + b^4 + 2*c^4 - 7*a^2*b^2 - 5*a^2*c^2 + 3*b^2*c^2 + 2*a*b*c^2))/((a - b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)))/(e*(2*a*b + tan(d/2 + (e*x)/2)^3*(4*a*c - 4*b*c) + tan(d/2 + (e*x)/2)^2*(2*a^2 - 2*b^2 + 4*c^2) + tan(d/2 + (e*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2 + tan(d/2 + (e*x)/2)*(4*a*c + 4*b*c))) - (atanh((a^4*c + b^4*c + c^5 - 2*a^2*c^3 + 2*b^2*c^3 - 2*a^2*b^2*c)/(b^2 - a^2 + c^2)^(5/2)) + (tan(d/2 + (e*x)/2)*(2*a - 2*b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2))/(2*(b^2 - a^2 + c^2)^(5/2)))*(2*a^2 + b^2 + c^2))/(e*(b^2 - a^2 + c^2)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)))**3,x)
```

```
[Out] Timed out
```

$$3.402 \quad \int \frac{1}{(a+b \cos(d+ex)+c \sin(d+ex))^4} dx$$

Optimal. Leaf size=292

$$\frac{a(2a^2 + 3(b^2 + c^2)) \tan^{-1}\left(\frac{(a-b) \tan\left(\frac{1}{2}(d+ex)\right) + c}{\sqrt{a^2 - b^2 - c^2}}\right)}{e(a^2 - b^2 - c^2)^{7/2}} + \frac{c(11a^2 + 4(b^2 + c^2)) \cos(d+ex) - b(11a^2 + 4(b^2 + c^2)) \sin(d+ex)}{6e(a^2 - b^2 - c^2)^3 (a + b \cos(d+ex) + c \sin(d+ex))}$$

[Out] a*(2*a^2+3*b^2+3*c^2)*arctan((c+(a-b)*tan(1/2*e*x+1/2*d))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^(7/2)/e+1/3*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^3+5/6*(a*c*cos(e*x+d)-a*b*sin(e*x+d))/(a^2-b^2-c^2)^2/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^2+1/6*(c*(11*a^2+4*b^2+4*c^2)*cos(e*x+d)-b*(11*a^2+4*b^2+4*c^2)*sin(e*x+d))/(a^2-b^2-c^2)^3/e/(a+b*cos(e*x+d)+c*sin(e*x+d))

Rubi [A] time = 0.38, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3129, 3156, 3153, 3124, 618, 204}

$$\frac{a(2a^2 + 3(b^2 + c^2)) \tan^{-1}\left(\frac{(a-b) \tan\left(\frac{1}{2}(d+ex)\right) + c}{\sqrt{a^2 - b^2 - c^2}}\right)}{e(a^2 - b^2 - c^2)^{7/2}} + \frac{c(11a^2 + 4(b^2 + c^2)) \cos(d+ex) - b(11a^2 + 4(b^2 + c^2)) \sin(d+ex)}{6e(a^2 - b^2 - c^2)^3 (a + b \cos(d+ex) + c \sin(d+ex))}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4), x]

[Out] (a*(2*a^2 + 3*(b^2 + c^2))*ArcTan[(c + (a - b)*Tan[(d + e*x)/2]]/Sqrt[a^2 - b^2 - c^2])/((a^2 - b^2 - c^2)^(7/2)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/((3*(a^2 - b^2 - c^2)*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3) + (5*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x]))/(6*(a^2 - b^2 - c^2)^2*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) + (c*(11*a^2 + 4*(b^2 + c^2))*Cos[d + e*x] - b*(11*a^2 + 4*(b^2 + c^2))*Sin[d + e*x])/((6*(a^2 - b^2 - c^2)^3*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3129

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-n), x_Symbol] := Simp[(-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d

```

+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]

```

Rule 3153

```

Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)])
/((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^2,
x_Symbol] := Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]

```

Rule 3156

```

Int[((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)
]), x_Symbol] := -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[n, -2]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^4} dx &= \frac{c \cos(d + ex) - b \sin(d + ex)}{3(a^2 - b^2 - c^2) e (a + b \cos(d + ex) + c \sin(d + ex))^3} - \frac{\int \frac{-3a + 2b \cos(d + ex)}{(a + b \cos(d + ex) + c \sin(d + ex))^3} dx}{3(a^2 - b^2 - c^2)} \\
&= \frac{c \cos(d + ex) - b \sin(d + ex)}{3(a^2 - b^2 - c^2) e (a + b \cos(d + ex) + c \sin(d + ex))^3} + \frac{5(a + b \cos(d + ex) + c \sin(d + ex))}{6(a^2 - b^2 - c^2) e (a + b \cos(d + ex) + c \sin(d + ex))^3} \\
&= \frac{c \cos(d + ex) - b \sin(d + ex)}{3(a^2 - b^2 - c^2) e (a + b \cos(d + ex) + c \sin(d + ex))^3} + \frac{5(a + b \cos(d + ex) + c \sin(d + ex))}{6(a^2 - b^2 - c^2) e (a + b \cos(d + ex) + c \sin(d + ex))^3} \\
&= \frac{c \cos(d + ex) - b \sin(d + ex)}{3(a^2 - b^2 - c^2) e (a + b \cos(d + ex) + c \sin(d + ex))^3} + \frac{5(a + b \cos(d + ex) + c \sin(d + ex))}{6(a^2 - b^2 - c^2) e (a + b \cos(d + ex) + c \sin(d + ex))^3} \\
&= \frac{c \cos(d + ex) - b \sin(d + ex)}{3(a^2 - b^2 - c^2) e (a + b \cos(d + ex) + c \sin(d + ex))^3} + \frac{5(a + b \cos(d + ex) + c \sin(d + ex))}{6(a^2 - b^2 - c^2) e (a + b \cos(d + ex) + c \sin(d + ex))^3} \\
&= \frac{a(2a^2 + 3(b^2 + c^2)) \tan^{-1}\left(\frac{c + (a-b) \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2 - b^2 - c^2}}\right)}{(a^2 - b^2 - c^2)^{7/2} e} + \frac{c \cos(d + ex) - b \sin(d + ex)}{3(a^2 - b^2 - c^2) e (a + b \cos(d + ex) + c \sin(d + ex))^3}
\end{aligned}$$

Mathematica [B] time = 2.11, size = 606, normalized size = 2.08

$$\frac{24a(2a^2+3(b^2+c^2)) \tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{7/2}} + \frac{44a^5c+72a^4b^2 \sin(d+ex)+132a^4c^2 \sin(d+ex)+54a^3b^3 \sin(2(d+ex))+82a^3b^2c+78a^3bc^2 \sin(2(d+ex))}{(-a^2+b^2+c^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*cos[d + e*x] + c*sin[d + e*x])^(-4), x]

[Out] ((24*a*(2*a^2 + 3*(b^2 + c^2))*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2 + c^2]]/(-a^2 + b^2 + c^2)^(7/2) + (44*a^5*c + 82*a^3*b^2*c + 24*a*b^4*c + 82*a^3*c^3 + 48*a*b^2*c^3 + 24*a*c^5 + 30*a^2*b*c*(2*a^2 + 3*(b^2 + c^2))*Cos[d + e*x] - 6*a*c*(-2*b^4 + 2*b^2*c^2 + 4*c^4 + a^2*(7*b^2 + 11*c^2))*Cos[2*(d + e*x)] - 22*a^2*b^3*c*cos[3*(d + e*x)] - 8*b^5*c*cos[3*(d + e*x)] - 22*a^2*b*c^3*cos[3*(d + e*x)] - 16*b^3*c^3*cos[3*(d + e*x)] - 8*b*c^5*cos[3*(d + e*x)] + 72*a^4*b^2*sin[d + e*x] - 9*a^2*b^4*sin[d + e*x] + 12*b^6*sin[d + e*x] + 132*a^4*c^2*sin[d + e*x] + 72*a^2*b^2*c^2*sin[d + e*x] + 36*b^4*c^2*sin[d + e*x] + 81*a^2*c^4*sin[d + e*x] + 36*b^2*c^4*sin[d + e*x] + 12*c^6*sin[d + e*x] + 54*a^3*b^3*sin[2*(d + e*x)] + 6*a*b^5*sin[2*(d + e*x)] + 78*a^3*b*c^2*sin[2*(d + e*x)] + 48*a*b^3*c^2*sin[2*(d + e*x)] + 42*a*b*c^4*sin[2*(d + e*x)] + 11*a^2*b^4*sin[3*(d + e*x)] + 4*b^6*sin[3*(d + e*x)] + 4*b^4*c^2*sin[3*(d + e*x)] - 11*a^2*c^4*sin[3*(d + e*x)] - 4*b^2*c^4*sin[3*(d + e*x)] - 4*c^6*sin[3*(d + e*x)])/(b*(-a^2 + b^2 + c^2)^3*(a + b*cos[d + e*x] + c*sin[d + e*x])^3)/(24*e)

fricas [B] time = 1.63, size = 4069, normalized size = 13.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="fricas")

[Out] [1/12*(6*a*b*c^5 + 12*(4*a^3*b + a*b^3)*c^3 + 2*(4*c^7 + (7*a^2 - 4*b^2)*c^5 - (11*a^4 + 14*a^2*b^2 + 20*b^4)*c^3 + 3*(11*a^4*b^2 - 7*a^2*b^4 - 4*b^6)*c)*cos(e*x + d)^3 - 12*(a*b*c^5 + 2*(4*a^3*b + a*b^3)*c^3 - (9*a^5*b - 8*a^3*b^3 - a*b^5)*c)*cos(e*x + d)^2 + 3*(2*a^6 + 3*a^4*b^2 + 9*a^2*c^4 + (2*a^3*b^3 + 3*a*b^5 - 9*a*b*c^4 - 6*(a^3*b + a*b^3)*c^2)*cos(e*x + d)^3 + 9*(a^4 + a^2*b^2)*c^2 + 3*(2*a^4*b^2 + 3*a^2*b^4 - 2*a^4*c^2 - 3*a^2*c^4)*cos(e*x + d)^2 + 3*(2*a^5*b + 3*a^3*b^3 + 3*a*b*c^4 + (5*a^3*b + 3*a*b^3)*c^2)*cos(e*x + d) + (3*a*c^5 + (11*a^3 + 3*a*b^2)*c^3 - (3*a*c^5 + 2*(a^3 - 3*a*b^2)*c^3 - 3*(2*a^3*b^2 + 3*a*b^4)*c)*cos(e*x + d)^2 + 3*(2*a^5 + 3*a^3*b^2)*c + 6*(3*a^2*b*c^3 + (2*a^4*b + 3*a^2*b^3)*c)*cos(e*x + d))*sin(e*x + d)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(e*x + d)^2 - 2*(a*b^3 + a*b*c^2)*cos(e*x + d) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(e*x + d))*sin(e*x + d) - 2*(2*a*b*c*cos(e*x + d)^2 - a*b*c + (b^2*c + c^3)*cos(e*x + d) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(e*x + d))*sin(e*x + d))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2*(b*c*cos(e*x + d) + a*c)*sin(e*x + d)) - 6*(9*a^5*b - 8*a^3*b^3 - a*b^5)*c - 6*(2*b^2*c^5 + 2*c^7 + (4*a^4 - 7*a^2*b^2 - 2*b^4)*c^3 - (6*a^6 - 15*a^4*b^2 + 7*a^2*b^4 + 2*b^6)*c)*cos(e*x + d) - 2*(18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7 - 14*b^3*c^4 - 6*b*c^6 - (12*a^4*b - 7*a^2*b^3 + 10*b^5)*c^2 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7 + 12*b*c^6 + (21*a^2*b + 20*b^3)*c^4 - (33*a^4*b - 14*a^2*b^3 - 4*b^5)*c^2)*cos(e*x + d)^2 + 3*(9*a^5*b^2 - 8*a^3*b^4 - a*b^6 + a*c^6 + (8*a^3 + a*b^2)*c^4 - (9*a^5 + a*b^4)*c^2)*cos(e*x + d))*sin(e*x + d))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 - 3*b*c^10 + (12*a^2*b - 11*b^3)*c^8 - 2*(9*a^4*b - 16*a^2*b^3 + 7*b^5)*c^6 + 6*(2*a^6*b - 5*a^4*b^3 + 4*a^2*b^5 - b^7)*c^4 - (3*a^8*b - 8*a^6*b^3 + 6*a^4*b

$$\begin{aligned}
& b^5 - b^9) * c^2) * e * \cos(e * x + d)^3 + 3 * (a^9 * b^2 - 4 * a^7 * b^4 + 6 * a^5 * b^6 - 4 * a^3 * b^8 + a * b^{10} - a * c^{10} + (4 * a^3 - 3 * a * b^2) * c^8 - 2 * (3 * a^5 - 4 * a^3 * b^2 + a * b^4) * c^6 + 2 * (2 * a^7 - 3 * a^5 * b^2 + a * b^6) * c^4 - (a^9 - 6 * a^5 * b^4 + 8 * a^3 * b^6 - 3 * a * b^8) * c^2) * e * \cos(e * x + d)^2 + 3 * (a^{10} * b - 4 * a^8 * b^3 + 6 * a^6 * b^5 - 4 * a^4 * b^7 + a^2 * b^9 + b * c^{10} - (3 * a^2 * b - 4 * b^3) * c^8 + 2 * (a^4 * b - 4 * a^2 * b^3 + 3 * b^5) * c^6 + 2 * (a^6 * b - 3 * a^2 * b^5 + 2 * b^7) * c^4 - (3 * a^8 * b - 8 * a^6 * b^3 + 6 * a^4 * b^5 - b^9) * c^2) * e * \cos(e * x + d) + (a^{11} - 4 * a^9 * b^2 + 6 * a^7 * b^4 - 4 * a^5 * b^6 + a^3 * b^8 + 3 * a * c^{10} - (11 * a^3 - 12 * a * b^2) * c^8 + 2 * (7 * a^5 - 16 * a^3 * b^2 + 9 * a * b^4) * c^6 - 6 * (a^7 - 4 * a^5 * b^2 + 5 * a^3 * b^4 - 2 * a * b^6) * c^4 - (a^9 - 6 * a^5 * b^4 + 8 * a^3 * b^6 - 3 * a * b^8) * c^2) * e - ((c^{11} - (4 * a^2 - b^2) * c^9 + 6 * (a^4 - b^4) * c^7 - 2 * (2 * a^6 + 3 * a^4 * b^2 - 12 * a^2 * b^4 + 7 * b^6) * c^5 + (a^8 + 8 * a^6 * b^2 - 30 * a^4 * b^4 + 32 * a^2 * b^6 - 11 * b^8) * c^3 - 3 * (a^8 * b^2 - 4 * a^6 * b^4 + 6 * a^4 * b^6 - 4 * a^2 * b^8 + b^{10}) * c) * e * \cos(e * x + d)^2 - 6 * (a * b * c^9 - 4 * (a^3 * b - a * b^3) * c^7 + 6 * (a^5 * b - 2 * a^3 * b^3 + a * b^5) * c^5 - 4 * (a^7 * b - 3 * a^5 * b^3 + 3 * a^3 * b^5 - a * b^7) * c^3 + (a^9 * b - 4 * a^7 * b^3 + 6 * a^5 * b^5 - 4 * a^3 * b^7 + a * b^9) * c) * e * \cos(e * x + d) - (c^{11} - (a^2 - 4 * b^2) * c^9 - 6 * (a^4 - b^4) * c^7 + 2 * (7 * a^6 - 12 * a^4 * b^2 + 3 * a^2 * b^4 + 2 * b^6) * c^5 - (11 * a^8 - 32 * a^6 * b^2 + 30 * a^4 * b^4 - 8 * a^2 * b^6 - b^8) * c^3 + 3 * (a^{10} - 4 * a^8 * b^2 + 6 * a^6 * b^4 - 4 * a^4 * b^6 + a^2 * b^8) * c) * e) * \sin(e * x + d)), 1/6 * (3 * a * b * c^5 + 6 * (4 * a^3 * b + a * b^3) * c^3 + (4 * c^7 + (7 * a^2 - 4 * b^2) * c^5 - (11 * a^4 + 14 * a^2 * b^2 + 20 * b^4) * c^3 + 3 * (11 * a^4 * b^2 - 7 * a^2 * b^4 - 4 * b^6) * c) * \cos(e * x + d)^3 - 6 * (a * b * c^5 + 2 * (4 * a^3 * b + a * b^3) * c^3 - (9 * a^5 * b - 8 * a^3 * b^3 - a * b^5) * c) * \cos(e * x + d)^2 + 3 * (2 * a^6 + 3 * a^4 * b^2 + 9 * a^2 * c^4 + (2 * a^3 * b^3 + 3 * a * b^5 - 9 * a * b * c^4 - 6 * (a^3 * b + a * b^3) * c^2) * \cos(e * x + d)^3 + 9 * (a^4 + a^2 * b^2) * c^2 + 3 * (2 * a^4 * b^2 + 3 * a^2 * b^4 - 2 * a^4 * c^2 - 3 * a^2 * c^4) * \cos(e * x + d)^2 + 3 * (2 * a^5 * b + 3 * a^3 * b^3 + 3 * a * b * c^4 + (5 * a^3 * b + 3 * a * b^3) * c^2) * \cos(e * x + d) + (3 * a * c^5 + (11 * a^3 + 3 * a * b^2) * c^3 - (3 * a * c^5 + 2 * (a^3 - 3 * a * b^2) * c^3 - 3 * (2 * a^3 * b^2 + 3 * a * b^4) * c) * \cos(e * x + d)^2 + 3 * (2 * a^5 + 3 * a^3 * b^2) * c + 6 * (3 * a^2 * b * c^3 + (2 * a^4 * b + 3 * a^2 * b^3) * c) * \cos(e * x + d)) * \sin(e * x + d)) * \sqrt{a^2 - b^2 - c^2} * \arctan(-(a * b * \cos(e * x + d) + a * c * \sin(e * x + d) + b^2 + c^2) * \sqrt{a^2 - b^2 - c^2} / ((c^3 - (a^2 - b^2) * c) * \cos(e * x + d) + (a^2 * b - b^3 - b * c^2) * \sin(e * x + d))) - 3 * (9 * a^5 * b - 8 * a^3 * b^3 - a * b^5) * c - 3 * (2 * b^2 * c^5 + 2 * c^7 + (4 * a^4 - 7 * a^2 * b^2 - 2 * b^4) * c^3 - (6 * a^6 - 15 * a^4 * b^2 + 7 * a^2 * b^4 + 2 * b^6) * c) * \cos(e * x + d) - (18 * a^6 * b - 23 * a^4 * b^3 + 7 * a^2 * b^5 - 2 * b^7 - 14 * b^3 * c^4 - 6 * b * c^6 - (12 * a^4 * b - 7 * a^2 * b^3 + 10 * b^5) * c^2 + (11 * a^4 * b^3 - 7 * a^2 * b^5 - 4 * b^7 + 12 * b * c^6 + (21 * a^2 * b + 20 * b^3) * c^4 - (33 * a^4 * b - 14 * a^2 * b^3 - 4 * b^5) * c^2) * \cos(e * x + d)^2 + 3 * (9 * a^5 * b^2 - 8 * a^3 * b^4 - a * b^6 + a * c^6 + (8 * a^3 + a * b^2) * c^4 - (9 * a^5 + a * b^4) * c^2) * \cos(e * x + d)) * \sin(e * x + d)) / ((a^8 * b^3 - 4 * a^6 * b^5 + 6 * a^4 * b^7 - 4 * a^2 * b^9 + b^{11} - 3 * b * c^{10} + (12 * a^2 * b - 11 * b^3) * c^8 - 2 * (9 * a^4 * b - 16 * a^2 * b^3 + 7 * b^5) * c^6 + 6 * (2 * a^6 * b - 5 * a^4 * b^3 + 4 * a^2 * b^5 - b^7) * c^4 - (3 * a^8 * b - 8 * a^6 * b^3 + 6 * a^4 * b^5 - b^9) * c^2) * e * \cos(e * x + d)^3 + 3 * (a^9 * b^2 - 4 * a^7 * b^4 + 6 * a^5 * b^6 - 4 * a^3 * b^8 + a * b^{10} - a * c^{10} + (4 * a^3 - 3 * a * b^2) * c^8 - 2 * (3 * a^5 - 4 * a^3 * b^2 + a * b^4) * c^6 + 2 * (2 * a^7 - 3 * a^5 * b^2 + a * b^6) * c^4 - (a^9 - 6 * a^5 * b^4 + 8 * a^3 * b^6 - 3 * a * b^8) * c^2) * e * \cos(e * x + d)^2 + 3 * (a^{10} * b - 4 * a^8 * b^3 + 6 * a^6 * b^5 - 4 * a^4 * b^7 + a^2 * b^9 + b * c^{10} - (3 * a^2 * b - 4 * b^3) * c^8 + 2 * (a^4 * b - 4 * a^2 * b^3 + 3 * b^5) * c^6 + 2 * (a^6 * b - 3 * a^2 * b^5 + 2 * b^7) * c^4 - (3 * a^8 * b - 8 * a^6 * b^3 + 6 * a^4 * b^5 - b^9) * c^2) * e * \cos(e * x + d) + (a^{11} - 4 * a^9 * b^2 + 6 * a^7 * b^4 - 4 * a^5 * b^6 + a^3 * b^8 + 3 * a * c^{10} - (11 * a^3 - 12 * a * b^2) * c^8 + 2 * (7 * a^5 - 16 * a^3 * b^2 + 9 * a * b^4) * c^6 - 6 * (a^7 - 4 * a^5 * b^2 + 5 * a^3 * b^4 - 2 * a * b^6) * c^4 - (a^9 - 6 * a^5 * b^4 + 8 * a^3 * b^6 - 3 * a * b^8) * c^2) * e - ((c^{11} - (4 * a^2 - b^2) * c^9 + 6 * (a^4 - b^4) * c^7 - 2 * (2 * a^6 + 3 * a^4 * b^2 - 12 * a^2 * b^4 + 7 * b^6) * c^5 + (a^8 + 8 * a^6 * b^2 - 30 * a^4 * b^4 + 32 * a^2 * b^6 - 11 * b^8) * c^3 - 3 * (a^8 * b^2 - 4 * a^6 * b^4 + 6 * a^4 * b^6 - 4 * a^2 * b^8 + b^{10}) * c) * e * \cos(e * x + d)^2 - 6 * (a * b * c^9 - 4 * (a^3 * b - a * b^3) * c^7 + 6 * (a^5 * b - 2 * a^3 * b^3 + a * b^5) * c^5 - 4 * (a^7 * b - 3 * a^5 * b^3 + 3 * a^3 * b^5 - a * b^7) * c^3 + (a^9 * b - 4 * a^7 * b^3 + 6 * a^5 * b^5 - 4 * a^3 * b^7 + a * b^9) * c) * e * \cos(e * x + d) - (c^{11} - (a^2 - 4 * b^2) * c^9 - 6 * (a^4 - b^4) * c^7 + 2 * (7 * a^6 - 12 * a^4 * b^2 + 3 * a^2 * b^4 + 2 * b^6) * c^5 - (11 * a^8 - 32 * a^6 * b^2 + 30 * a^4 * b^4 - 8 * a^2 * b^6 - b^8) * c^3 + 3 * (a^{10} - 4 * a^8 * b^2 + 6 * a^6 * b^4 - 4 * a^4 * b^6 + a^2 * b^8) * c) * e) * \sin(e * x + d))]
\end{aligned}$$

giac [B] time = 0.58, size = 2671, normalized size = 9.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="giac")
[Out] -1/3*(3*(2*a^3 + 3*a*b^2 + 3*a*c^2)*(pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(-
2*a + 2*b) + arctan(-(a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d) + c)/
sqrt(a^2 - b^2 - c^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - 3*a^4*c^2 + 6
*a^2*b^2*c^2 - 3*b^4*c^2 + 3*a^2*c^4 - 3*b^2*c^4 - c^6)*sqrt(a^2 - b^2 - c^
2)) + (18*a^7*b*tan(1/2*x*e + 1/2*d)^5 - 81*a^6*b^2*tan(1/2*x*e + 1/2*d)^5
+ 141*a^5*b^3*tan(1/2*x*e + 1/2*d)^5 - 120*a^4*b^4*tan(1/2*x*e + 1/2*d)^5 +
60*a^3*b^5*tan(1/2*x*e + 1/2*d)^5 - 33*a^2*b^6*tan(1/2*x*e + 1/2*d)^5 + 21
*a*b^7*tan(1/2*x*e + 1/2*d)^5 - 6*b^8*tan(1/2*x*e + 1/2*d)^5 - 27*a^6*c^2*t
an(1/2*x*e + 1/2*d)^5 + 81*a^5*b*c^2*tan(1/2*x*e + 1/2*d)^5 - 72*a^4*b^2*c^
2*tan(1/2*x*e + 1/2*d)^5 + 18*a^3*b^3*c^2*tan(1/2*x*e + 1/2*d)^5 - 27*a^2*b
^4*c^2*tan(1/2*x*e + 1/2*d)^5 + 45*a*b^5*c^2*tan(1/2*x*e + 1/2*d)^5 - 18*b^
6*c^2*tan(1/2*x*e + 1/2*d)^5 + 18*a^4*c^4*tan(1/2*x*e + 1/2*d)^5 - 36*a^3*b
*c^4*tan(1/2*x*e + 1/2*d)^5 + 36*a*b^3*c^4*tan(1/2*x*e + 1/2*d)^5 - 18*b^4*
c^4*tan(1/2*x*e + 1/2*d)^5 - 6*a^2*c^6*tan(1/2*x*e + 1/2*d)^5 + 12*a*b*c^6*
tan(1/2*x*e + 1/2*d)^5 - 6*b^2*c^6*tan(1/2*x*e + 1/2*d)^5 - 18*a^7*c*tan(1/
2*x*e + 1/2*d)^4 + 108*a^6*b*c*tan(1/2*x*e + 1/2*d)^4 - 261*a^5*b^2*c*tan(1
/2*x*e + 1/2*d)^4 + 336*a^4*b^3*c*tan(1/2*x*e + 1/2*d)^4 - 264*a^3*b^4*c*tan
(1/2*x*e + 1/2*d)^4 + 144*a^2*b^5*c*tan(1/2*x*e + 1/2*d)^4 - 57*a*b^6*c*tan
(1/2*x*e + 1/2*d)^4 + 12*b^7*c*tan(1/2*x*e + 1/2*d)^4 - 81*a^5*c^3*tan(1/2
*x*e + 1/2*d)^4 + 216*a^4*b*c^3*tan(1/2*x*e + 1/2*d)^4 - 198*a^3*b^2*c^3*tan
(1/2*x*e + 1/2*d)^4 + 108*a^2*b^3*c^3*tan(1/2*x*e + 1/2*d)^4 - 81*a*b^4*c^
3*tan(1/2*x*e + 1/2*d)^4 + 36*b^5*c^3*tan(1/2*x*e + 1/2*d)^4 + 36*a^3*c^5*t
an(1/2*x*e + 1/2*d)^4 - 36*a^2*b*c^5*tan(1/2*x*e + 1/2*d)^4 - 36*a*b^2*c^5*
tan(1/2*x*e + 1/2*d)^4 + 36*b^3*c^5*tan(1/2*x*e + 1/2*d)^4 - 12*a*c^7*tan(1
/2*x*e + 1/2*d)^4 + 12*b*c^7*tan(1/2*x*e + 1/2*d)^4 + 36*a^7*b*tan(1/2*x*e
+ 1/2*d)^3 - 108*a^6*b^2*tan(1/2*x*e + 1/2*d)^3 + 76*a^5*b^3*tan(1/2*x*e +
1/2*d)^3 + 60*a^4*b^4*tan(1/2*x*e + 1/2*d)^3 - 100*a^3*b^5*tan(1/2*x*e + 1/
2*d)^3 + 44*a^2*b^6*tan(1/2*x*e + 1/2*d)^3 - 12*a*b^7*tan(1/2*x*e + 1/2*d)^
3 + 4*b^8*tan(1/2*x*e + 1/2*d)^3 - 108*a^6*c^2*tan(1/2*x*e + 1/2*d)^3 + 240
*a^5*b*c^2*tan(1/2*x*e + 1/2*d)^3 - 162*a^4*b^2*c^2*tan(1/2*x*e + 1/2*d)^3
+ 122*a^3*b^3*c^2*tan(1/2*x*e + 1/2*d)^3 - 174*a^2*b^4*c^2*tan(1/2*x*e + 1/
2*d)^3 + 78*a*b^5*c^2*tan(1/2*x*e + 1/2*d)^3 + 4*b^6*c^2*tan(1/2*x*e + 1/2*
d)^3 - 42*a^4*c^4*tan(1/2*x*e + 1/2*d)^3 + 162*a^3*b*c^4*tan(1/2*x*e + 1/2*
d)^3 - 210*a^2*b^2*c^4*tan(1/2*x*e + 1/2*d)^3 + 102*a*b^3*c^4*tan(1/2*x*e +
1/2*d)^3 - 12*b^4*c^4*tan(1/2*x*e + 1/2*d)^3 + 8*a^2*c^6*tan(1/2*x*e + 1/2
*d)^3 + 12*a*b*c^6*tan(1/2*x*e + 1/2*d)^3 - 20*b^2*c^6*tan(1/2*x*e + 1/2*d)
^3 - 8*c^8*tan(1/2*x*e + 1/2*d)^3 - 36*a^7*c*tan(1/2*x*e + 1/2*d)^2 + 108*a
^6*b*c*tan(1/2*x*e + 1/2*d)^2 - 108*a^5*b^2*c*tan(1/2*x*e + 1/2*d)^2 + 12*a
^4*b^3*c*tan(1/2*x*e + 1/2*d)^2 + 84*a^3*b^4*c*tan(1/2*x*e + 1/2*d)^2 - 108
*a^2*b^5*c*tan(1/2*x*e + 1/2*d)^2 + 60*a*b^6*c*tan(1/2*x*e + 1/2*d)^2 - 12*
b^7*c*tan(1/2*x*e + 1/2*d)^2 - 120*a^5*c^3*tan(1/2*x*e + 1/2*d)^2 + 132*a^4
*b*c^3*tan(1/2*x*e + 1/2*d)^2 + 42*a^3*b^2*c^3*tan(1/2*x*e + 1/2*d)^2 - 36*
a^2*b^3*c^3*tan(1/2*x*e + 1/2*d)^2 + 18*a*b^4*c^3*tan(1/2*x*e + 1/2*d)^2 -
36*b^5*c^3*tan(1/2*x*e + 1/2*d)^2 + 18*a^3*c^5*tan(1/2*x*e + 1/2*d)^2 + 72*
a^2*b*c^5*tan(1/2*x*e + 1/2*d)^2 - 54*a*b^2*c^5*tan(1/2*x*e + 1/2*d)^2 - 36
*b^3*c^5*tan(1/2*x*e + 1/2*d)^2 - 12*a*c^7*tan(1/2*x*e + 1/2*d)^2 - 12*b*c^
7*tan(1/2*x*e + 1/2*d)^2 + 18*a^7*b*tan(1/2*x*e + 1/2*d) - 27*a^6*b^2*tan(1
/2*x*e + 1/2*d) - 21*a^5*b^3*tan(1/2*x*e + 1/2*d) + 48*a^4*b^4*tan(1/2*x*e
+ 1/2*d) - 12*a^3*b^5*tan(1/2*x*e + 1/2*d) - 15*a^2*b^6*tan(1/2*x*e + 1/2*d
) + 15*a*b^7*tan(1/2*x*e + 1/2*d) - 6*b^8*tan(1/2*x*e + 1/2*d) - 81*a^6*c^2
*tan(1/2*x*e + 1/2*d) + 27*a^5*b*c^2*tan(1/2*x*e + 1/2*d) + 90*a^4*b^2*c^2*
tan(1/2*x*e + 1/2*d) + 9*a^2*b^4*c^2*tan(1/2*x*e + 1/2*d) - 27*a*b^5*c^2*ta
```

$$\begin{aligned} & n(1/2*x*e + 1/2*d) - 18*b^6*c^2*\tan(1/2*x*e + 1/2*d) + 12*a^4*c^4*\tan(1/2*x \\ & *e + 1/2*d) + 42*a^3*b*c^4*\tan(1/2*x*e + 1/2*d) + 18*a^2*b^2*c^4*\tan(1/2*x* \\ & e + 1/2*d) - 54*a*b^3*c^4*\tan(1/2*x*e + 1/2*d) - 18*b^4*c^4*\tan(1/2*x*e + 1 \\ & /2*d) - 6*a^2*c^6*\tan(1/2*x*e + 1/2*d) - 12*a*b*c^6*\tan(1/2*x*e + 1/2*d) - \\ & 6*b^2*c^6*\tan(1/2*x*e + 1/2*d) - 18*a^7*c + 21*a^5*b^2*c + 12*a^3*b^4*c - 1 \\ & 5*a*b^6*c + 5*a^5*c^3 + 16*a^3*b^2*c^3 - 21*a*b^4*c^3 - 2*a^3*c^5 - 6*a*b^2 \\ & *c^5)/((a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a^5*b^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a \\ & *b^8 + b^9 - 3*a^7*c^2 + 9*a^6*b*c^2 - 3*a^5*b^2*c^2 - 15*a^4*b^3*c^2 + 15* \\ & a^3*b^4*c^2 + 3*a^2*b^5*c^2 - 9*a*b^6*c^2 + 3*b^7*c^2 + 3*a^5*c^4 - 9*a^4*b \\ & *c^4 + 6*a^3*b^2*c^4 + 6*a^2*b^3*c^4 - 9*a*b^4*c^4 + 3*b^5*c^4 - a^3*c^6 + \\ & 3*a^2*b*c^6 - 3*a*b^2*c^6 + b^3*c^6)*(a*\tan(1/2*x*e + 1/2*d)^2 - b*\tan(1/2* \\ & x*e + 1/2*d)^2 + 2*c*\tan(1/2*x*e + 1/2*d) + a + b)^3))e^{-1} \end{aligned}$$

maple [B] time = 0.66, size = 16909, normalized size = 57.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for more details)Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 4.81, size = 1946, normalized size = 6.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^4,x)

$$\begin{aligned} & [Out] (a*\operatorname{atanh}((a*(2*a^2 + 3*b^2 + 3*c^2)*(2*b^6*c - 2*a^6*c + 2*c^7 - 6*a^2*c^5 \\ & + 6*a^4*c^3 + 6*b^2*c^5 + 6*b^4*c^3 - 6*a^2*b^4*c + 6*a^4*b^2*c - 12*a^2*b^ \\ & 2*c^3)))/(2*(b^2 - a^2 + c^2)^{(7/2)}*(3*a*b^2 + 3*a*c^2 + 2*a^3)) + (a*\tan(d/ \\ & 2 + (e*x)/2)*(2*a - 2*b)*(2*a^2 + 3*b^2 + 3*c^2)*(b^6 - a^6 + c^6 - 3*a^2*b \\ & ^4 + 3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - 6*a^2*b^2* \\ & c^2))/(2*(b^2 - a^2 + c^2)^{(7/2)}*(3*a*b^2 + 3*a*c^2 + 2*a^3)))*(2*a^2 + 3*b \\ & ^2 + 3*c^2))/(e*(b^2 - a^2 + c^2)^{(7/2)}) - ((18*a^7*c + 2*a^3*c^5 - 5*a^5*c \\ & ^3 + 6*a*b^2*c^5 + 21*a*b^4*c^3 - 12*a^3*b^4*c - 21*a^5*b^2*c - 16*a^3*b^2* \\ & c^3 + 15*a*b^6*c)/(3*(a - b)^3*(b^6 - a^6 + c^6 - 3*a^2*b^4 + 3*a^4*b^2 - 3 \\ & *a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - 6*a^2*b^2*c^2)) + (\tan(d/2 + \\ & (e*x)/2)*(2*b^8 - 6*a^7*b - 5*a*b^7 + 5*a^2*b^6 + 4*a^3*b^5 - 16*a^4*b^4 + \\ & 7*a^5*b^3 + 9*a^6*b^2 + 2*a^2*c^6 - 4*a^4*c^4 + 27*a^6*c^2 + 2*b^2*c^6 + 6 \\ & *b^4*c^4 + 6*b^6*c^2 + 18*a*b^3*c^4 + 9*a*b^5*c^2 - 14*a^3*b*c^4 - 9*a^5*b* \\ & c^2 - 6*a^2*b^2*c^4 - 3*a^2*b^4*c^2 - 30*a^4*b^2*c^2 + 4*a*b*c^6)))/((a - b) \\ & ^3*(b^6 - a^6 + c^6 - 3*a^2*b^4 + 3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2 \\ & *c^4 + 3*b^4*c^2 - 6*a^2*b^2*c^2)) + (\tan(d/2 + (e*x)/2)^4*(6*a^6*c + 4*b^6 \\ & *c + 4*c^7 - 12*a^2*c^5 + 27*a^4*c^3 + 12*b^2*c^5 + 12*b^4*c^3 - 15*a*b^3*c \\ & ^3 + 33*a^2*b^4*c - 45*a^3*b*c^3 - 55*a^3*b^3*c + 57*a^4*b^2*c + 21*a^2*b^2 \end{aligned}$$

$$\begin{aligned}
& *c^3 - 15*a*b^5*c - 30*a^5*b*c)) / ((a - b)^2*(b^6 - a^6 + c^6 - 3*a^2*b^4 + \\
& 3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - 6*a^2*b^2*c^2)) \\
& - (2*\tan(d/2 + (e*x)/2)^3*(18*a^7*b - 6*a*b^7 + 2*b^8 - 4*c^8 + 22*a^2*b^6 \\
& - 50*a^3*b^5 + 30*a^4*b^4 + 38*a^5*b^3 - 54*a^6*b^2 + 4*a^2*c^6 - 21*a^4*c \\
& ^4 - 54*a^6*c^2 - 10*b^2*c^6 - 6*b^4*c^4 + 2*b^6*c^2 + 51*a*b^3*c^4 + 39*a* \\
& b^5*c^2 + 81*a^3*b*c^4 + 120*a^5*b*c^2 - 105*a^2*b^2*c^4 - 87*a^2*b^4*c^2 + \\
& 61*a^3*b^3*c^2 - 81*a^4*b^2*c^2 + 6*a*b*c^6)) / (3*(a - b)^3*(b^6 - a^6 + c^ \\
& 6 - 3*a^2*b^4 + 3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - \\
& 6*a^2*b^2*c^2)) - (\tan(d/2 + (e*x)/2)^5*(3*a*b^5 + 6*a^5*b - 2*b^6 - 2*c^6 \\
& - 3*a^2*b^4 + 11*a^3*b^3 - 15*a^4*b^2 + 6*a^2*c^4 - 9*a^4*c^2 - 6*b^2*c^4 \\
& - 6*b^4*c^2 + 3*a*b^3*c^2 + 9*a^3*b*c^2 + 3*a^2*b^2*c^2)) / ((a - b)*(b^6 - a \\
& ^6 + c^6 - 3*a^2*b^4 + 3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^ \\
& 4*c^2 - 6*a^2*b^2*c^2)) + (2*\tan(d/2 + (e*x)/2)^2*(2*a*c^7 + 6*a^7*c + 2*b* \\
& c^7 + 2*b^7*c - 3*a^3*c^5 + 20*a^5*c^3 + 6*b^3*c^5 + 6*b^5*c^3 + 9*a*b^2*c^ \\
& 5 - 3*a*b^4*c^3 - 12*a^2*b*c^5 + 18*a^2*b^5*c - 14*a^3*b^4*c - 22*a^4*b*c^3 \\
& - 2*a^4*b^3*c + 18*a^5*b^2*c + 6*a^2*b^3*c^3 - 7*a^3*b^2*c^3 - 10*a*b^6*c \\
& - 18*a^6*b*c)) / ((a - b)^3*(b^6 - a^6 + c^6 - 3*a^2*b^4 + 3*a^4*b^2 - 3*a^2* \\
& c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - 6*a^2*b^2*c^2)) / (e*(\tan(d/2 + (e \\
& *x)/2)^3*(12*a^2*c - 12*b^2*c + 8*c^3) + \tan(d/2 + (e*x)/2)*(6*a^2*c + 6*b^ \\
& 2*c + 12*a*b*c) + \tan(d/2 + (e*x)/2)^2*(3*a^2*b - 3*a*b^2 + 12*a*c^2 + 12*b \\
& *c^2 + 3*a^3 - 3*b^3) - \tan(d/2 + (e*x)/2)^4*(3*a*b^2 + 3*a^2*b - 12*a*c^2 \\
& + 12*b*c^2 - 3*a^3 - 3*b^3) + \tan(d/2 + (e*x)/2)^5*(6*a^2*c + 6*b^2*c - 12* \\
& a*b*c) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 + \tan(d/2 + (e*x)/2)^6*(3*a*b^2 - 3* \\
& a^2*b + a^3 - b^3))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**4,x)

[Out] Timed out

3.403 $\int (2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2} dx$

Optimal. Leaf size=185

$$\frac{2(5 \cos(d + ex) - 3 \sin(d + ex))(5 \sin(d + ex) + 3 \cos(d + ex) + 2)^{3/2}}{5e} - \frac{32(5 \cos(d + ex) - 3 \sin(d + ex))\sqrt{5 \sin(d + ex)}}{15e}$$

```
[Out] -2/5*(5*cos(e*x+d)-3*sin(e*x+d))*(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2)/e-32/15*(5*cos(e*x+d)-3*sin(e*x+d))*(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)/e+64*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/3))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^(1/2))/e/(2+34^(1/2))^(1/2)+796/15*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^(1/2))*(2+34^(1/2))^(1/2)/e
```

Rubi [A] time = 0.27, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3120, 3146, 3149, 3118, 2653, 3126, 2661}

$$\frac{2(5 \cos(d + ex) - 3 \sin(d + ex))(5 \sin(d + ex) + 3 \cos(d + ex) + 2)^{3/2}}{5e} - \frac{32(5 \cos(d + ex) - 3 \sin(d + ex))\sqrt{5 \sin(d + ex)}}{15e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2), x]
```

```
[Out] (796*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]))/15])/(15*e) + (64*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]))/15])/(Sqrt[2 + Sqrt[34]]*e) - (32*(5*Cos[d + e*x] - 3*Sin[d + e*x])*Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])/(15*e) - (2*(5*Cos[d + e*x] - 3*Sin[d + e*x])*(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2))/(5*e)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d/Sqrt[a + b], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3118

```
Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] := Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3120

```
Int[(cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)])^n, x_Symbol] := -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
```

$x](a + b\cos[d + ex] + c\sin[d + ex])^{n-2}, x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0] \ \&\& \ \text{GtQ}[n, 1]$

Rule 3126

$\text{Int}[1/\text{Sqrt}[\cos[(d_.) + (e_.)(x_.)]*(b_.) + (a_.) + (c_.)\sin[(d_.) + (e_.)(x_.)]], x_Symbol] \rightarrow \text{Int}[1/\text{Sqrt}[a + \text{Sqrt}[b^2 + c^2]\cos[d + ex - \text{ArcTan}[b, c]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 + c^2, 0] \ \&\& \ \text{GtQ}[a + \text{Sqrt}[b^2 + c^2], 0]$

Rule 3146

$\text{Int}[(\cos[(d_.) + (e_.)(x_.)]*(b_.) + (a_.) + (c_.)\sin[(d_.) + (e_.)(x_.)])^{n_.}*((A_.) + \cos[(d_.) + (e_.)(x_.)]*(B_.) + (C_.)\sin[(d_.) + (e_.)(x_.)]), x_Symbol] \rightarrow \text{Simp}[(B*c - b*C - a*C\cos[d + ex] + a*B\sin[d + ex])*(a + b\cos[d + ex] + c\sin[d + ex])^n/(a*e*(n + 1)), x] + \text{Dist}[1/(a*(n + 1)), \text{Int}[(a + b\cos[d + ex] + c\sin[d + ex])^{n-1}*\text{Simp}[a*(b*B + c*C)*n + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*\cos[d + ex] + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*\sin[d + ex], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, A, B, C\}, x] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0]$

Rule 3149

$\text{Int}[(A_.) + \cos[(d_.) + (e_.)(x_.)]*(B_.) + (C_.)\sin[(d_.) + (e_.)(x_.)]]/\text{Sqrt}[\cos[(d_.) + (e_.)(x_.)]*(b_.) + (a_.) + (c_.)\sin[(d_.) + (e_.)(x_.)]], x_Symbol] \rightarrow \text{Dist}[B/b, \text{Int}[\text{Sqrt}[a + b\cos[d + ex] + c\sin[d + ex]], x], x] + \text{Dist}[(A*b - a*B)/b, \text{Int}[1/\text{Sqrt}[a + b\cos[d + ex] + c\sin[d + ex]], x], x] /; \text{FreeQ}\{a, b, c, d, e, A, B, C\}, x] \ \&\& \ \text{EqQ}[B*c - b*C, 0] \ \&\& \ \text{NeQ}[A*b - a*B, 0]$

Rubi steps

$$\begin{aligned} \int (2 + 3\cos(d + ex) + 5\sin(d + ex))^{5/2} dx &= -\frac{2(5\cos(d + ex) - 3\sin(d + ex))(2 + 3\cos(d + ex) + 5\sin(d + ex))^{3/2}}{5e} \\ &= -\frac{32(5\cos(d + ex) - 3\sin(d + ex))\sqrt{2 + 3\cos(d + ex) + 5\sin(d + ex)}}{15e} \\ &= -\frac{32(5\cos(d + ex) - 3\sin(d + ex))\sqrt{2 + 3\cos(d + ex) + 5\sin(d + ex)}}{15e} \\ &= -\frac{32(5\cos(d + ex) - 3\sin(d + ex))\sqrt{2 + 3\cos(d + ex) + 5\sin(d + ex)}}{15e} \\ &= \frac{796\sqrt{2 + \sqrt{34}} E\left(\frac{1}{2}\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right)\right) \Big|_{\frac{2}{15}} (17 - \sqrt{34})}{15e} + \frac{64F}{15e} \end{aligned}$$

Mathematica [C] time = 5.91, size = 399, normalized size = 2.16

$$1276\sqrt{\frac{10}{3}} \sqrt{\sqrt{34} \sin\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right) + 2\sqrt{\cos^2\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right) \sec\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right) F_1\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2),x]

[Out] (-2388*sqrt[2 + sqrt[34]*cos[d + e*x - ArcTan[5/3]]] - 2*sqrt[2 + 3*cos[d + e*x] + 5*sin[d + e*x]]*(550*cos[d + e*x] + 3*(-398 + 75*cos[2*(d + e*x)] - 110*sin[d + e*x] + 40*sin[2*(d + e*x)])) + 1276*sqrt[10/3]*AppellF1[1/2, 1/2, 1/2, 3/2, (sqrt[34] + 17*sin[d + e*x + ArcTan[3/5]])/(-17 + sqrt[34]), (sqrt[34] + 17*sin[d + e*x + ArcTan[3/5]])/(17 + sqrt[34])] * sqrt[cos[d + e*x + ArcTan[3/5]]^2]*Sec[d + e*x + ArcTan[3/5]]*sqrt[2 + sqrt[34]*sin[d + e*x + ArcTan[3/5]]] + (1990*sin[d + e*x - ArcTan[5/3]])/sqrt[1/17 + cos[d + e*x - ArcTan[5/3]]/sqrt[34]] - (1990*sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (sqrt[34] + 17*cos[d + e*x - ArcTan[5/3]])/(-17 + sqrt[34]), (sqrt[34] + 17*cos[d + e*x - ArcTan[5/3]])/(17 + sqrt[34])] * csc[d + e*x - ArcTan[5/3]]*sqrt[sin[d + e*x - ArcTan[5/3]]^2])/sqrt[2 + sqrt[34]*cos[d + e*x - ArcTan[5/3]]])/(75*e)

fricas [F] time = 1.18, size = 0, normalized size = 0.00

integral(-(16 cos(ex + d)² - 10(3 cos(ex + d) + 2) sin(ex + d) - 12 cos(ex + d) - 29) sqrt(3 cos(ex + d) + 5 sin(ex + d)), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="fricas")

[Out] integral(-(16*cos(e*x + d)^2 - 10*(3*cos(e*x + d) + 2)*sin(e*x + d) - 12*cos(e*x + d) - 29)*sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 \cos(ex + d) + 5 \sin(ex + d) + 2)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="giac")

[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(5/2), x)

maple [C] time = 0.66, size = 701, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x)

[Out] (16/17*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2), I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))*34^(1/2)+16*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2), I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))-44*17^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*(-17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*EllipticF((-17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2), I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))*34^(1/2)+796/17*17^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*(-17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*EllipticE((-17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2), I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))*34^(1/2)-48*(-17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)

$$\frac{1}{2} * \text{EllipticF}\left(\frac{-17 \sin(e*x+d+\arctan(3/5))+34^{1/2}}{-34^{1/2}+17}\right)^{1/2}, I * \left(\frac{-34^{1/2}+17}{34^{1/2}+17}\right)^{1/2} + 68/5 * 34^{1/2} * \sin(e*x+d+\arctan(3/5))^{1/2} - 116/15 * 34^{1/2} * \sin(e*x+d+\arctan(3/5))^2 + 1904/15 * \sin(e*x+d+\arctan(3/5))^3 - 1904/15 * \sin(e*x+d+\arctan(3/5)) - 88/15 * 34^{1/2} / \cos(e*x+d+\arctan(3/5)) / (34^{1/2} * \sin(e*x+d+\arctan(3/5)) + 2)^{1/2} / e$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 \cos(ex + d) + 5 \sin(ex + d) + 2)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="maxima")

[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (3 \cos(d + ex) + 5 \sin(d + ex) + 2)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(5/2),x)

[Out] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))**(5/2),x)

[Out] Timed out

3.404 $\int (2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{3/2} dx$

Optimal. Leaf size=139

$$\frac{2(5 \cos(d + ex) - 3 \sin(d + ex))\sqrt{5 \sin(d + ex) + 3 \cos(d + ex) + 2}}{3e} + \frac{20F\left(\frac{1}{2}\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right)\middle|\frac{2}{15}\right)(17 - \sqrt{34})}{\sqrt{2 + \sqrt{34}}e}$$

[Out] $-2/3*(5*\cos(e*x+d)-3*\sin(e*x+d))*(2+3*\cos(e*x+d)+5*\sin(e*x+d))^{(1/2)}/e+20*(\cos(1/2*d+1/2*e*x-1/2*\arctan(5/3))^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(5/3))*\text{EllipticF}(\sin(1/2*d+1/2*e*x-1/2*\arctan(5/3)),1/15*(510-30*34^{(1/2)})^{(1/2)})/e+(2+34^{(1/2)})^{(1/2)}+16/3*(\cos(1/2*d+1/2*e*x-1/2*\arctan(5/3))^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(5/3))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(5/3)),1/15*(510-30*34^{(1/2)})^{(1/2)})*(2+34^{(1/2)})^{(1/2)})/e$

Rubi [A] time = 0.14, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3120, 3149, 3118, 2653, 3126, 2661}

$$\frac{2(5 \cos(d + ex) - 3 \sin(d + ex))\sqrt{5 \sin(d + ex) + 3 \cos(d + ex) + 2}}{3e} + \frac{20F\left(\frac{1}{2}\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right)\middle|\frac{2}{15}\right)(17 - \sqrt{34})}{\sqrt{2 + \sqrt{34}}e}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2 + 3*\text{Cos}[d + e*x] + 5*\text{Sin}[d + e*x])^{(3/2)}, x]$

[Out] $(16*\text{Sqrt}[2 + \text{Sqrt}[34]]*\text{EllipticE}[(d + e*x - \text{ArcTan}[5/3])/2, (2*(17 - \text{Sqrt}[34]))/15])/(3*e) + (20*\text{EllipticF}[(d + e*x - \text{ArcTan}[5/3])/2, (2*(17 - \text{Sqrt}[34]))/15])/(\text{Sqrt}[2 + \text{Sqrt}[34]]*e) - (2*(5*\text{Cos}[d + e*x] - 3*\text{Sin}[d + e*x])*\text{Sqrt}[2 + 3*\text{Cos}[d + e*x] + 5*\text{Sin}[d + e*x]])/(3*e)$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[a + b, 0]$

Rule 2661

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/(\text{d}*\text{Sqrt}[a + b]), x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[a + b, 0]$

Rule 3118

$\text{Int}[\text{Sqrt}[\cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*\sin[(d_) + (e_)*(x_)]], x_Symbol] \rightarrow \text{Int}[\text{Sqrt}[a + \text{Sqrt}[b^2 + c^2]*\text{Cos}[d + e*x - \text{ArcTan}[b, c]], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 + c^2, 0] \ \&\& \ \text{GtQ}[a + \text{Sqrt}[b^2 + c^2], 0]$

Rule 3120

$\text{Int}[(\cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*\sin[(d_) + (e_)*(x_)])^n, x_Symbol] \rightarrow -\text{Simp}[(c*\text{Cos}[d + e*x] - b*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{(n-1)}/(e*n), x] + \text{Dist}[1/n, \text{Int}[\text{Simp}[n*a^2 + (n-1)*(b^2 + c^2) + a*b*(2*n-1)*\text{Cos}[d + e*x] + a*c*(2*n-1)*\text{Sin}[d + e*x], x]*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{(n-2)}, x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0] \ \&\& \ \text{GtQ}[n, 1]$

Rule 3126

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] := Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]) / Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\begin{aligned} \int (2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{3/2} dx &= -\frac{2(5 \cos(d + ex) - 3 \sin(d + ex))\sqrt{2 + 3 \cos(d + ex) + 5 \sin(d + ex)}}{3e} \\ &= -\frac{2(5 \cos(d + ex) - 3 \sin(d + ex))\sqrt{2 + 3 \cos(d + ex) + 5 \sin(d + ex)}}{3e} \\ &= -\frac{2(5 \cos(d + ex) - 3 \sin(d + ex))\sqrt{2 + 3 \cos(d + ex) + 5 \sin(d + ex)}}{3e} \\ &= \frac{16\sqrt{2 + \sqrt{34}} E\left(\frac{1}{2}\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right) \middle| \frac{2}{15}(17 - \sqrt{34})\right)}{3e} + \frac{20F\left(\frac{3}{5}\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right) \middle| \frac{2}{15}(17 - \sqrt{34})\right)}{3e} \end{aligned}$$

Mathematica [C] time = 3.42, size = 349, normalized size = 2.51

$$2 \left(\sqrt{\sin^2\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right)} \left(23\sqrt{30} \sqrt{\sqrt{34} \sin\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right)} + 2 \sqrt{\cos^2\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right)} \sqrt{\sqrt{34} \sin\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right)} \right) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2), x]
```

```
[Out] (2*(-60*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(17 + Sqrt[34])]*Sin[d + e*x - ArcTan[5/3]] + (-15*(30*Cos[d + e*x] + 15*Cos[2*(d + e*x)] - 18*Sin[d + e*x] + 8*Sin[2*(d + e*x)]) + 23*Sqrt[30]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]])*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]])*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2]))/(45*e*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]])*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left((3 \cos(ex + d) + 5 \sin(ex + d) + 2)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="fricas")

[Out] integral((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 \cos(ex + d) + 5 \sin(ex + d) + 2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="giac")

[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(3/2), x)

maple [C] time = 0.46, size = 686, normalized size = 4.94

$$8 \sqrt{\frac{17 \sin\left(\frac{ex+d+\arctan\left(\frac{3}{5}\right)\right)+\sqrt{34}}{\sqrt{34+17}}} \sqrt{17} \sqrt{\frac{1+\sin\left(\frac{ex+d+\arctan\left(\frac{3}{5}\right)\right)}{-\sqrt{34+17}}} \sqrt{\frac{17\left(\sin\left(\frac{ex+d+\arctan\left(\frac{3}{5}\right)\right)-1\right)}{\sqrt{34+17}}} \operatorname{EllipticF}\left(\sqrt{\frac{17 \sin\left(\frac{ex+d+\arctan\left(\frac{3}{5}\right)\right)+\sqrt{34}}{\sqrt{34+17}}}, i \sqrt{\frac{\sqrt{34+17}}{-\sqrt{34+17}}}\right) \sqrt{34+17}$$

17

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x)

[Out] (8/17*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))*34^(1/2)+8*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))+68/3*sin(e*x+d+arctan(3/5))^3-68/3*sin(e*x+d+arctan(3/5))+4/3*34^(1/2)*sin(e*x+d+arctan(3/5))^2-4/3*34^(1/2)-4*17^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*(-17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*EllipticF((-17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))*34^(1/2)-12*(-17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*EllipticF((-17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+80/17*17^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*(-17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*EllipticE((-17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))*34^(1/2))/cos(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 \cos(ex + d) + 5 \sin(ex + d) + 2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="maxima")

[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (3 \cos(d + ex) + 5 \sin(d + ex) + 2)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(3/2), x)

[Out] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (5 \sin(d + ex) + 3 \cos(d + ex) + 2)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))**(3/2), x)

[Out] Integral((5*sin(d + e*x) + 3*cos(d + e*x) + 2)**(3/2), x)

3.405 $\int \sqrt{2 + 3 \cos(d + ex) + 5 \sin(d + ex)} dx$

Optimal. Leaf size=45

$$\frac{2\sqrt{2 + \sqrt{34}} E\left(\frac{1}{2}\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right) \middle| \frac{2}{15}(17 - \sqrt{34})\right)}{e}$$

[Out] $2*(\cos(1/2*d+1/2*e*x-1/2*\arctan(5/3))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(5/3))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(5/3)),1/15*(510-30*34^{(1/2)})^{(1/2)}*(2+34^{(1/2)})^{(1/2)})/e$

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3118, 2653}

$$\frac{2\sqrt{2 + \sqrt{34}} E\left(\frac{1}{2}\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right) \middle| \frac{2}{15}(17 - \sqrt{34})\right)}{e}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]

[Out] $(2*\text{Sqrt}[2 + \text{Sqrt}[34]]*\text{EllipticE}[(d + e*x - \text{ArcTan}[5/3])/2, (2*(17 - \text{Sqrt}[34]))/15])/e$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3118

Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2 + c^2], 0]

Rubi steps

$$\begin{aligned} \int \sqrt{2 + 3 \cos(d + ex) + 5 \sin(d + ex)} dx &= \int \sqrt{2 + \sqrt{34} \cos\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right)} dx \\ &= \frac{2\sqrt{2 + \sqrt{34}} E\left(\frac{1}{2}\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right) \middle| \frac{2}{15}(17 - \sqrt{34})\right)}{e} \end{aligned}$$

Mathematica [C] time = 2.27, size = 326, normalized size = 7.24

$$\frac{\sqrt{\sin^2\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right) \left(2\sqrt{30} \sqrt{\sqrt{34} \sin\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right)} + 2 \sqrt{\cos^2\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right)} \sqrt{\sqrt{34} \cos\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right)}\right)}}{e}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]

[Out] $(-15\sqrt{30} \operatorname{AppellF1}[-1/2, -1/2, -1/2, 1/2, (\sqrt{34} + 17\cos[d + ex - \arctan(5/3)])]/(-17 + \sqrt{34}), (\sqrt{34} + 17\cos[d + ex - \arctan(5/3)])/(17 + \sqrt{34})) \sin[d + ex - \arctan(5/3)] + (-75\cos[d + ex] + 45\sin[d + ex] + 2\sqrt{30} \operatorname{AppellF1}[1/2, 1/2, 1/2, 3/2, (\sqrt{34} + 17\sin[d + ex + \arctan(3/5)])]/(-17 + \sqrt{34}), (\sqrt{34} + 17\sin[d + ex + \arctan(3/5)])/(17 + \sqrt{34})) \sqrt{\cos[d + ex + \arctan(3/5)]^2} \sqrt{2 + \sqrt{34} \cos[d + ex - \arctan(5/3)]} \operatorname{Sec}[d + ex + \arctan(3/5)] \sqrt{2 + \sqrt{34} \sin[d + ex + \arctan(3/5)]} \sqrt{\sin[d + ex - \arctan(5/3)]^2})/(15e\sqrt{2 + \sqrt{34} \cos[d + ex - \arctan(5/3)]} \sqrt{\sin[d + ex - \arctan(5/3)]^2})$

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{3 \cos(ex + d) + 5 \sin(ex + d) + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3 \cos(ex + d) + 5 \sin(ex + d) + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)

maple [C] time = 0.40, size = 461, normalized size = 10.24

$$2\sqrt{\frac{17\left(\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)-1\right)}{\sqrt{34}+17}} \sqrt{17} \sqrt{\frac{1+\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)}{-\sqrt{34}+17}} \left(15\sqrt{34} \sqrt{\frac{17\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+\sqrt{34}}{-\sqrt{34}+17}} \operatorname{EllipticE}\left(\sqrt{\frac{17\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+\sqrt{34}}{-\sqrt{34}+17}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x)

[Out] $2/17 * (-17 * (\sin(ex+d+\arctan(3/5))-1) / (34^{1/2}+17))^{1/2} * 17^{1/2} * ((1+\sin(ex+d+\arctan(3/5))) / (-34^{1/2}+17))^{1/2} * (15 * 34^{1/2} * (-17 * \sin(ex+d+\arctan(3/5))+34^{1/2}) / (-34^{1/2}+17))^{1/2} * \operatorname{EllipticE}((-17 * \sin(ex+d+\arctan(3/5))+34^{1/2}) / (-34^{1/2}+17))^{1/2}, I * ((-34^{1/2}+17) / (34^{1/2}+17))^{1/2}) - 17 * 34^{1/2} * (-17 * \sin(ex+d+\arctan(3/5))+34^{1/2}) / (-34^{1/2}+17))^{1/2} * \operatorname{EllipticF}((-17 * \sin(ex+d+\arctan(3/5))+34^{1/2}) / (-34^{1/2}+17))^{1/2}, I * ((-34^{1/2}+17) / (34^{1/2}+17))^{1/2}) + 2 * ((17 * \sin(ex+d+\arctan(3/5))+34^{1/2}) / (34^{1/2}+17))^{1/2} * \operatorname{EllipticF}(((17 * \sin(ex+d+\arctan(3/5))+34^{1/2}) / (34^{1/2}+17))^{1/2}, I * (1 / (-34^{1/2}+17) * (34^{1/2}+17))^{1/2}) * 34^{1/2} + 34 * (-17 * \sin(ex+d+\arctan(3/5))+34^{1/2}) / (-34^{1/2}+17))^{1/2} * \operatorname{EllipticF}((-17 * \sin(ex+d+\arctan(3/5))+34^{1/2}) / (-34^{1/2}+17))^{1/2}, I * ((-34^{1/2}+17) / (34^{1/2}+17))^{1/2}) + 34 * ((17 * \sin(ex+d+\arctan(3/5))+34^{1/2}) / (34^{1/2}+17))^{1/2} * \operatorname{EllipticF}(((17 * \sin(ex+d+\arctan(3/5))+34^{1/2}) / (34^{1/2}+17))^{1/2}, I * (1 / (-34^{1/2}+17) * (34^{1/2}+17))^{1/2})) / \cos(ex+d+\arctan(3/5)) / (34^{1/2} * \sin(ex+d+\arctan(3/5))+2)^{1/2} / e$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3 \cos(ex + d) + 5 \sin(ex + d) + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{3 \cos(d + ex) + 5 \sin(d + ex) + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(1/2),x)

[Out] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{5 \sin(d + ex) + 3 \cos(d + ex) + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))**(1/2),x)

[Out] Integral(sqrt(5*sin(d + e*x) + 3*cos(d + e*x) + 2), x)

$$3.406 \quad \int \frac{1}{\sqrt{2+3 \cos(d+ex)+5 \sin(d+ex)}} dx$$

Optimal. Leaf size=45

$$\frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)\middle|\frac{2}{15}(17-\sqrt{34})\right)}{\sqrt{2+\sqrt{34}}e}$$

[Out] $2*(\cos(1/2*d+1/2*e*x-1/2*\arctan(5/3))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(5/3))*\text{EllipticF}(\sin(1/2*d+1/2*e*x-1/2*\arctan(5/3)),1/15*(510-30*34^{(1/2)})^{(1/2)})/e/(2+34^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3126, 2661}

$$\frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)\middle|\frac{2}{15}(17-\sqrt{34})\right)}{\sqrt{2+\sqrt{34}}e}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]

[Out] $(2*\text{EllipticF}[(d+e*x-\text{ArcTan}[5/3])/2,(2*(17-\text{Sqrt}[34]))/15])/(\text{Sqrt}[2+\text{Sqrt}[34]]*e)$

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b])), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3126

Int[1/Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] := Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2 + c^2], 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{2+3 \cos(d+ex)+5 \sin(d+ex)}} dx &= \int \frac{1}{\sqrt{2+\sqrt{34} \cos\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)}} dx \\ &= \frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)\middle|\frac{2}{15}(17-\sqrt{34})\right)}{\sqrt{2+\sqrt{34}}e} \end{aligned}$$

Mathematica [C] time = 0.26, size = 128, normalized size = 2.84

$$\frac{\sqrt{\frac{2}{15}} \sqrt{\sqrt{34} \sin\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)+2} \sqrt{\cos^2\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right)} \sec\left(d+ex+\tan^{-1}\left(\frac{3}{5}\right)\right) F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{2}; \frac{3}{2}; \frac{3}{2}\right)}{e}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]

[Out] (Sqrt[2/15]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]])/e

fricas [F] time = 2.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{3 \cos(ex + d) + 5 \sin(ex + d) + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="fricas")

[Out] integral(1/sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3 \cos(ex + d) + 5 \sin(ex + d) + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)

maple [C] time = 0.31, size = 152, normalized size = 3.38

$$\frac{2(\sqrt{34} + 17) \sqrt{\frac{17 \sin\left(ex + d + \arctan\left(\frac{3}{5}\right)\right) + \sqrt{34}}{\sqrt{34} + 17}} \sqrt{17} \sqrt{\frac{1 + \sin\left(ex + d + \arctan\left(\frac{3}{5}\right)\right)}{-\sqrt{34} + 17}} \sqrt{\frac{17\left(\sin\left(ex + d + \arctan\left(\frac{3}{5}\right)\right) - 1\right)}{\sqrt{34} + 17}} \text{EllipticF}\left(\sqrt{\frac{17 \sin\left(ex + d + \arctan\left(\frac{3}{5}\right)\right) + \sqrt{34}}{\sqrt{34} + 17}}\right)}{17 \cos\left(ex + d + \arctan\left(\frac{3}{5}\right)\right) \sqrt{\sqrt{34} \sin\left(ex + d + \arctan\left(\frac{3}{5}\right)\right) + 2} e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x)

[Out] 2/17*(34^(1/2)+17)*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*17^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2), I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))/cos(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3 \cos(ex + d) + 5 \sin(ex + d) + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{3 \cos(d + ex) + 5 \sin(d + ex) + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(1/2), x)`

[Out] `int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{5 \sin(d + ex) + 3 \cos(d + ex) + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(1/2), x)`

[Out] `Integral(1/sqrt(5*sin(d + e*x) + 3*cos(d + e*x) + 2), x)`

$$3.407 \quad \int \frac{1}{(2+3 \cos(d+ex)+5 \sin(d+ex))^{3/2}} dx$$

Optimal. Leaf size=94

$$\frac{5 \cos(d+ex) - 3 \sin(d+ex)}{15e\sqrt{5 \sin(d+ex) + 3 \cos(d+ex) + 2}} - \frac{\sqrt{2 + \sqrt{34}} E\left(\frac{1}{2}\left(d+ex - \tan^{-1}\left(\frac{5}{3}\right)\right) \middle| \frac{2}{15}(17 - \sqrt{34})\right)}{15e}$$

[Out] 1/15*(-5*cos(e*x+d)+3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)-1/15*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^(1/2))*(2+34^(1/2))^(1/2)/e

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3128, 3118, 2653}

$$\frac{5 \cos(d+ex) - 3 \sin(d+ex)}{15e\sqrt{5 \sin(d+ex) + 3 \cos(d+ex) + 2}} - \frac{\sqrt{2 + \sqrt{34}} E\left(\frac{1}{2}\left(d+ex - \tan^{-1}\left(\frac{5}{3}\right)\right) \middle| \frac{2}{15}(17 - \sqrt{34})\right)}{15e}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-3/2), x]

[Out] -(Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]))/15])/(15*e) - (5*Cos[d + e*x] - 3*Sin[d + e*x])/(15*e*Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3118

Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3128

Int[(cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)])^(-3/2), x_Symbol] :> Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{3/2}} dx &= -\frac{5 \cos(d + ex) - 3 \sin(d + ex)}{15e\sqrt{2 + 3 \cos(d + ex) + 5 \sin(d + ex)}} - \frac{1}{30} \int \sqrt{2 + 3 \cos(d + ex)} dx \\ &= -\frac{5 \cos(d + ex) - 3 \sin(d + ex)}{15e\sqrt{2 + 3 \cos(d + ex) + 5 \sin(d + ex)}} - \frac{1}{30} \int \sqrt{2 + \sqrt{34} \cos(d + ex)} dx \\ &= -\frac{\sqrt{2 + \sqrt{34}} E\left(\frac{1}{2}\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right)\middle| \frac{2}{15}(17 - \sqrt{34})\right)}{15e} - \frac{5}{15e\sqrt{2 + 3 \cos(d + ex) + 5 \sin(d + ex)}} \end{aligned}$$

Mathematica [C] time = 6.03, size = 390, normalized size = 4.15

$$-2\sqrt{30} \sqrt{\sqrt{34} \sin\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right) + 2} \sqrt{\cos^2\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right)} \sec\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right) F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{2}; \dots\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-3/2), x]

[Out] (18*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] - 68*Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]] + (20*(5 + 17*Sin[d + e*x]))/Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]] - 2*Sqrt[30]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]] - (15*Sin[d + e*x - ArcTan[5/3]])/Sqrt[1/17 + Cos[d + e*x - ArcTan[5/3]]/Sqrt[34]] + (15*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(17 + Sqrt[34])]*Csc[d + e*x - ArcTan[5/3]]*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]])/(450*e)

fricas [F] time = 1.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{3} \cos(ex + d) + 5 \sin(ex + d) + 2}{16 \cos(ex + d)^2 - 10(3 \cos(ex + d) + 2) \sin(ex + d) - 12 \cos(ex + d) - 29}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2), x, algorithm="fricas")

[Out] integral(-sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2)/(16*cos(e*x + d)^2 - 10*(3*cos(e*x + d) + 2)*sin(e*x + d) - 12*cos(e*x + d) - 29), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3 \cos(ex + d) + 5 \sin(ex + d) + 2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2), x, algorithm="giac")

[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-3/2), x)

maple [C] time = 0.49, size = 437, normalized size = 4.65

$$\sqrt{34} \left(255 \sqrt{\left(17 \sin \left(ex + d + \arctan \left(\frac{3}{5} \right) \right) + \sqrt{34} \right) \sqrt{34} \left(\cos^2 \left(ex + d + \arctan \left(\frac{3}{5} \right) \right) \right)} \sqrt{\frac{17 \sin \left(ex + d + \arctan \left(\frac{3}{5} \right) \right) + \sqrt{34}}{-\sqrt{34} + 17}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x)

[Out] 1/4335*34^(1/2)*(255*((17*sin(e*x+d+arctan(3/5))+34^(1/2))*34^(1/2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*EllipticF((-17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))-255*((17*sin(e*x+d+arctan(3/5))+34^(1/2))*34^(1/2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*((1+sin(e*x+d+arctan(3/5)))/(-34^(1/2)+17))^(1/2)*EllipticE((-17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+289*((34^(1/2)*sin(e*x+d+arctan(3/5))+2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*sin(e*x+d+arctan(3/5))^2-289*((34^(1/2)*sin(e*x+d+arctan(3/5))+2)*cos(e*x+d+arctan(3/5))^2)^(1/2))*17^(1/2)/(((17*sin(e*x+d+arctan(3/5))+34^(1/2))*34^(1/2)*cos(e*x+d+arctan(3/5))^2)^(1/2)/cos(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3 \cos(ex + d) + 5 \sin(ex + d) + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="maxima")

[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(3 \cos(d + ex) + 5 \sin(d + ex) + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(3/2),x)

[Out] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5 \sin(d + ex) + 3 \cos(d + ex) + 2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x)

[Out] Integral((5*sin(d + e*x) + 3*cos(d + e*x) + 2)**(-3/2), x)

$$3.408 \quad \int \frac{1}{(2+3 \cos(d+ex)+5 \sin(d+ex))^{5/2}} dx$$

Optimal. Leaf size=187

$$\frac{4(5 \cos(d+ex) - 3 \sin(d+ex))}{675e\sqrt{5 \sin(d+ex) + 3 \cos(d+ex) + 2}} - \frac{5 \cos(d+ex) - 3 \sin(d+ex)}{45e(5 \sin(d+ex) + 3 \cos(d+ex) + 2)^{3/2}} + \frac{F\left(\frac{1}{2}\left(d+ex - \tan^{-1}\left(\frac{5}{3}\right)\right)\right)}{45\sqrt{2 + \sqrt{3}}}$$

```
[Out] 1/45*(-5*cos(e*x+d)+3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2)+4/6
75*(5*cos(e*x+d)-3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)+1/45*(
cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/
3))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^(1/
2))/e/(2+34^(1/2))^(1/2)+4/675*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)
/cos(1/2*d+1/2*e*x-1/2*arctan(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(
5/3)),1/15*(510-30*34^(1/2))^(1/2))*(2+34^(1/2))^(1/2)/e
```

Rubi [A] time = 0.20, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3129, 3156, 3149, 3118, 2653, 3126, 2661}

$$\frac{4(5 \cos(d+ex) - 3 \sin(d+ex))}{675e\sqrt{5 \sin(d+ex) + 3 \cos(d+ex) + 2}} - \frac{5 \cos(d+ex) - 3 \sin(d+ex)}{45e(5 \sin(d+ex) + 3 \cos(d+ex) + 2)^{3/2}} + \frac{F\left(\frac{1}{2}\left(d+ex - \tan^{-1}\left(\frac{5}{3}\right)\right)\right)}{45\sqrt{2 + \sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-5/2), x]
```

```
[Out] (4*sqrt[2 + sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - sqrt[34
]))/15])/(675*e) + EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - sqrt[34]))
/15]/(45*sqrt[2 + sqrt[34]]*e) - (5*Cos[d + e*x] - 3*Sin[d + e*x])/(45*e*(2
+ 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2)) + (4*(5*Cos[d + e*x] - 3*Sin[d +
e*x]))/(675*e*sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)], x_Symbol] := Simp[(2*sqrt[a
+ b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)], x_Symbol] := Simp[(2*Ellip
ticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d*sqrt[a + b], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3118

```
Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x
_)], x_Symbol] := Int[Sqrt[a + sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + sqrt[b^2
+ c^2], 0]
```

Rule 3126

```
Int[1/Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(
x_)], x_Symbol] := Int[1/Sqrt[a + sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + sqrt[
```

b^2 + c^2], 0]

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^
(n_), x_Symbol] := Simp[((-c*cos[d + e*x]) + b*sin[d + e*x])*(a + b*cos[d
+ e*x] + c*sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*cos[d + e*x] - c*
(n + 2)*sin[d + e*x])*(a + b*cos[d + e*x] + c*sin[d + e*x])^(n + 1), x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)])
/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])
, x_Symbol] := Dist[B/b, Int[Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]], x],
x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)
]), x_Symbol] := -Simp[((c*B - b*C - (a*C - c*A)*cos[d + e*x] + (a*B - b*A)
*sin[d + e*x])*(a + b*cos[d + e*x] + c*sin[d + e*x])^(n + 1))/(e*(n + 1)*(a
^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*cos[d
+ e*x] + c*sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*
(a*B - b*A)*cos[d + e*x] + (n + 2)*(a*C - c*A)*sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2}} dx = -\frac{5 \cos(d + ex) - 3 \sin(d + ex)}{45e(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{3/2}} + \frac{1}{45} \int \frac{-3 + \frac{3}{2} \cos(d + ex)}{(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{3/2}} dx$$

$$= -\frac{5 \cos(d + ex) - 3 \sin(d + ex)}{45e(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{3/2}} + \frac{4(5 \cos(d + ex) - 3 \sin(d + ex))}{675e\sqrt{2 + 3 \cos(d + ex) + 5 \sin(d + ex)}}$$

$$= -\frac{5 \cos(d + ex) - 3 \sin(d + ex)}{45e(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{3/2}} + \frac{4(5 \cos(d + ex) - 3 \sin(d + ex))}{675e\sqrt{2 + 3 \cos(d + ex) + 5 \sin(d + ex)}}$$

$$= -\frac{5 \cos(d + ex) - 3 \sin(d + ex)}{45e(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{3/2}} + \frac{4(5 \cos(d + ex) - 3 \sin(d + ex))}{675e\sqrt{2 + 3 \cos(d + ex) + 5 \sin(d + ex)}}$$

$$= \frac{4\sqrt{2 + \sqrt{34}} E\left(\frac{1}{2}\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right) \middle| \frac{2}{15}(17 - \sqrt{34})\right)}{675e} + \frac{F\left(\frac{1}{2}\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right) \middle| \frac{2}{15}(17 - \sqrt{34})\right)}{675e}$$

Mathematica [C] time = 3.18, size = 430, normalized size = 2.30

$$23\sqrt{\frac{10}{3}} \sqrt{\sqrt{34} \sin\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right) + 2 \sqrt{\cos^2\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right)} \sec\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right) F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{2}; \dots\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-5/2), x]

[Out] (-24*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]) + (272*Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])/3 + (100*(5 + 17*Sin[d + e*x]))/(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2) - (10*(115 + 136*Sin[d + e*x]))/(3*Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]]) + 23*Sqrt[10/3]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17 + Sqrt[34])] * Sqrt[Cos[d + e*x + ArcTan[3/5]]^2 * Sec[d + e*x + ArcTan[3/5]] * Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]] + (20*Sin[d + e*x - ArcTan[5/3]])/Sqrt[1/17 + Cos[d + e*x - ArcTan[5/3]]]/Sqrt[34]] - (20*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(17 + Sqrt[34])] * Csc[d + e*x - ArcTan[5/3]] * Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]])/(6750*e)

fricas [F] time = 2.88, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{3} \cos(ex + d) + 5 \sin(ex + d) + 2}{198 \cos(ex + d)^3 + 96 \cos(ex + d)^2 - 5(2 \cos(ex + d)^2 + 36 \cos(ex + d) + 37) \sin(ex + d) - 261 \cos(ex + d) - 158}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2), x, algorithm="fricas")

[Out] integral(-sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2)/(198*cos(e*x + d)^3 + 96*cos(e*x + d)^2 - 5*(2*cos(e*x + d)^2 + 36*cos(e*x + d) + 37)*sin(e*x + d) - 261*cos(e*x + d) - 158), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3 \cos(ex + d) + 5 \sin(ex + d) + 2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2), x, algorithm="giac")

[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-5/2), x)

maple [C] time = 0.61, size = 542, normalized size = 2.90

$$\sqrt{-\left(-\sqrt{34} \sin\left(ex + d + \arctan\left(\frac{3}{5}\right)\right) - 2\right) \left(\cos^2\left(ex + d + \arctan\left(\frac{3}{5}\right)\right)\right)} \left(-\frac{\sqrt{34} \sqrt{-\left(-\sqrt{34} \sin\left(ex + d + \arctan\left(\frac{3}{5}\right)\right) - 2\right)}}{1530 \left(\sin\left(ex + d + \arctan\left(\frac{3}{5}\right)\right)\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x)

[Out]
$$\begin{aligned} &(-(-34^{1/2}*\sin(e*x+d+\arctan(3/5))-2)*\cos(e*x+d+\arctan(3/5))^2)^{1/2}*(-1/1530*34^{1/2}*(-(-34^{1/2}*\sin(e*x+d+\arctan(3/5))-2)*\cos(e*x+d+\arctan(3/5))^2)^{1/2}/(\sin(e*x+d+\arctan(3/5))+1/17*34^{1/2})^2+68/675*34^{1/2}*\cos(e*x+d+\arctan(3/5))^2/(-(-289*\sin(e*x+d+\arctan(3/5))-17*34^{1/2}))*34^{1/2}*\cos(e*x+d+\arctan(3/5))^2)^{1/2}+23/675*(-1+1/17*34^{1/2})*((-17*\sin(e*x+d+\arctan(3/5))-34^{1/2})/(-34^{1/2}+17))^{1/2}*((-17*\sin(e*x+d+\arctan(3/5))+17)/(34^{1/2}+17))^{1/2}*((17*\sin(e*x+d+\arctan(3/5))+17)/(-34^{1/2}+17))^{1/2}/(-(-34^{1/2}*\sin(e*x+d+\arctan(3/5))-2)*\cos(e*x+d+\arctan(3/5))^2)^{1/2}*EllipticF(((17*\sin(e*x+d+\arctan(3/5))-34^{1/2})/(-34^{1/2}+17))^{1/2}, I*((-34^{1/2}+17)/(34^{1/2}+17))^{1/2})+4/675*34^{1/2}*(-1+1/17*34^{1/2})*((-17*\sin(e*x+d+\arctan(3/5))-34^{1/2})/(-34^{1/2}+17))^{1/2}*((-17*\sin(e*x+d+\arctan(3/5))+17)/(34^{1/2}+17))^{1/2}*((17*\sin(e*x+d+\arctan(3/5))+17)/(-34^{1/2}+17))^{1/2}/(-(-34^{1/2}*\sin(e*x+d+\arctan(3/5))-2)*\cos(e*x+d+\arctan(3/5))^2)^{1/2})*((-1/17*34^{1/2}-1)*EllipticE(((17*\sin(e*x+d+\arctan(3/5))-34^{1/2})/(-34^{1/2}+17))^{1/2}, I*((-34^{1/2}+17)/(34^{1/2}+17))^{1/2})+EllipticF(((17*\sin(e*x+d+\arctan(3/5))-34^{1/2})/(-34^{1/2}+17))^{1/2}, I*((-34^{1/2}+17)/(34^{1/2}+17))^{1/2}))/\cos(e*x+d+\arctan(3/5))/(34^{1/2}*\sin(e*x+d+\arctan(3/5))+2)^{1/2}/e \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3 \cos(ex + d) + 5 \sin(ex + d) + 2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="maxima")

[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(3 \cos(d + ex) + 5 \sin(d + ex) + 2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(5/2),x)

[Out] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5 \sin(d + ex) + 3 \cos(d + ex) + 2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x)

[Out] Integral((5*sin(d + e*x) + 3*cos(d + e*x) + 2)**(-5/2), x)

$$3.409 \quad \int \frac{1}{(2+3 \cos(d+ex)+5 \sin(d+ex))^{7/2}} dx$$

Optimal. Leaf size=233

$$-\frac{199(5 \cos(d+ex) - 3 \sin(d+ex))}{101250e\sqrt{5 \sin(d+ex) + 3 \cos(d+ex) + 2}} + \frac{8(5 \cos(d+ex) - 3 \sin(d+ex))}{3375e(5 \sin(d+ex) + 3 \cos(d+ex) + 2)^{3/2}} - \frac{5 \cos(d+ex)}{75e(5 \sin(d+ex) + 3 \cos(d+ex) + 2)}$$

[Out] $1/75*(-5*\cos(e*x+d)+3*\sin(e*x+d))/e/(2+3*\cos(e*x+d)+5*\sin(e*x+d))^{(5/2)+8/3}$
 $375*(5*\cos(e*x+d)-3*\sin(e*x+d))/e/(2+3*\cos(e*x+d)+5*\sin(e*x+d))^{(3/2)}-199/1$
 $01250*(5*\cos(e*x+d)-3*\sin(e*x+d))/e/(2+3*\cos(e*x+d)+5*\sin(e*x+d))^{(1/2)}-8/3$
 $375*(\cos(1/2*d+1/2*e*x-1/2*\arctan(5/3))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(5/3))*$
 $\text{EllipticF}(\sin(1/2*d+1/2*e*x-1/2*\arctan(5/3)),1/15*(510-30*34^{(1/2)})^{(1/2)})/e/(2+34^{(1/2)})^{(1/2)}-199/101250*(\cos(1/2*d+1/2*e*x-1/2*\arctan(5/3))$
 $)^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(5/3))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(5/3)),1/15*(510-30*34^{(1/2)})^{(1/2)})*(2+34^{(1/2)})^{(1/2)}/e$

Rubi [A] time = 0.26, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3129, 3156, 3149, 3118, 2653, 3126, 2661}

$$-\frac{199(5 \cos(d+ex) - 3 \sin(d+ex))}{101250e\sqrt{5 \sin(d+ex) + 3 \cos(d+ex) + 2}} + \frac{8(5 \cos(d+ex) - 3 \sin(d+ex))}{3375e(5 \sin(d+ex) + 3 \cos(d+ex) + 2)^{3/2}} - \frac{5 \cos(d+ex)}{75e(5 \sin(d+ex) + 3 \cos(d+ex) + 2)}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-7/2),x]

[Out] $(-199*\text{Sqrt}[2 + \text{Sqrt}[34]]*\text{EllipticE}[(d + e*x - \text{ArcTan}[5/3])/2, (2*(17 - \text{Sqrt}[34]))/15])/(101250*e) - (8*\text{EllipticF}[(d + e*x - \text{ArcTan}[5/3])/2, (2*(17 - \text{Sqrt}[34]))/15])/(3375*\text{Sqrt}[2 + \text{Sqrt}[34]]*e) - (5*\text{Cos}[d + e*x] - 3*\text{Sin}[d + e*x])/(75*e*(2 + 3*\text{Cos}[d + e*x] + 5*\text{Sin}[d + e*x])^{(5/2)}) + (8*(5*\text{Cos}[d + e*x] - 3*\text{Sin}[d + e*x]))/(3375*e*(2 + 3*\text{Cos}[d + e*x] + 5*\text{Sin}[d + e*x])^{(3/2)}) - (199*(5*\text{Cos}[d + e*x] - 3*\text{Sin}[d + e*x]))/(101250*e*\text{Sqrt}[2 + 3*\text{Cos}[d + e*x] + 5*\text{Sin}[d + e*x]])$

Rule 2653

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d/Sqrt[a + b], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3118

Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3126

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] := Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^(n_), x_Symbol] := Simp[(-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]) / Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]), x_Symbol] := -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{7/2}} dx &= -\frac{5 \cos(d + ex) - 3 \sin(d + ex)}{75e(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2}} + \frac{1}{75} \int \frac{-5 + \frac{9}{2} \cos(d + ex)}{(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2}} dx \\
&= -\frac{5 \cos(d + ex) - 3 \sin(d + ex)}{75e(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2}} + \frac{8(5 \cos(d + ex) - 3 \sin(d + ex))}{3375e(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2}} \\
&= -\frac{5 \cos(d + ex) - 3 \sin(d + ex)}{75e(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2}} + \frac{8(5 \cos(d + ex) - 3 \sin(d + ex))}{3375e(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2}} \\
&= -\frac{5 \cos(d + ex) - 3 \sin(d + ex)}{75e(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2}} + \frac{8(5 \cos(d + ex) - 3 \sin(d + ex))}{3375e(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2}} \\
&= -\frac{5 \cos(d + ex) - 3 \sin(d + ex)}{75e(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2}} + \frac{8(5 \cos(d + ex) - 3 \sin(d + ex))}{3375e(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2}} \\
&= -\frac{199\sqrt{2 + \sqrt{34}} E\left(\frac{1}{2}\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right)\middle|\frac{2}{15}(17 - \sqrt{34})\right)}{101250e} - \frac{8(5 \cos(d + ex) - 3 \sin(d + ex))}{3375e(2 + 3 \cos(d + ex) + 5 \sin(d + ex))^{5/2}}
\end{aligned}$$

Mathematica [C] time = 3.84, size = 436, normalized size = 1.87

$$-638\sqrt{30} \sqrt{\sqrt{34} \sin\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right) + 2} \sqrt{\cos^2\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right)} \sec\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right) F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \left(\sqrt{34} + 17 \sin\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right)\right) / (-17 + \sqrt{34}), \left(\sqrt{34} + 17 \sin\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right)\right) / (17 + \sqrt{34})\right) \sqrt{\cos\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right)^2} \sec\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right) \sqrt{2 + \sqrt{34} \sin\left(d + ex + \tan^{-1}\left(\frac{3}{5}\right)\right) + 2} + (2985 \sqrt{30} \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \left(\sqrt{34} + 17 \cos\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right)\right) / (-17 + \sqrt{34}), \left(\sqrt{34} + 17 \cos\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right)\right) / (17 + \sqrt{34})\right]) \operatorname{Csc}\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right) \sqrt{\sin\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right)^2} / \sqrt{2 + \sqrt{34} \cos\left(d + ex - \tan^{-1}\left(\frac{5}{3}\right)\right)}\right) / (3037500e)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-7/2), x]

[Out] (-13532*sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]] + (597*(12 + 43*Cos[d + e*x] + 15*Sin[d + e*x]))/sqrt[2 + sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] + (27000*(5 + 17*Sin[d + e*x]))/(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2) - (300*(305 + 272*Sin[d + e*x]))/(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2) + (20*(1595 + 3383*Sin[d + e*x]))/sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]] - 638*sqrt[30]*AppellF1[1/2, 1/2, 1/2, 3/2, (sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(-17 + sqrt[34]), (sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17 + sqrt[34])] * sqrt[Cos[d + e*x + ArcTan[3/5]]^2] * Sec[d + e*x + ArcTan[3/5]] * sqrt[2 + sqrt[34]*Sin[d + e*x + ArcTan[3/5]]] + (2985*sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(-17 + sqrt[34]), (sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(17 + sqrt[34])] * Csc[d + e*x - ArcTan[5/3]] * sqrt[Sin[d + e*x - ArcTan[5/3]]^2]) / sqrt[2 + sqrt[34]*Cos[d + e*x - ArcTan[5/3]]]) / (3037500*e)

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{3 \cos(ex + d) + 5 \sin(ex + d) + 2}}{644 \cos(ex + d)^4 + 1584 \cos(ex + d)^3 + 284 \cos(ex + d)^2 + 20(48 \cos(ex + d)^3 - 4 \cos(ex + d))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2), x, algorithm="fricas")

[Out] integral(-sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2)/(644*cos(e*x + d)^4 + 1584*cos(e*x + d)^3 + 284*cos(e*x + d)^2 + 20*(48*cos(e*x + d)^3 - 4*cos(e*x + d)))

+ d)^2 - 111*cos(e*x + d) - 58)*sin(e*x + d) - 1896*cos(e*x + d) - 1241),
x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3 \cos(ex + d) + 5 \sin(ex + d) + 2)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x, algorithm="giac")

[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-7/2), x)

maple [C] time = 0.63, size = 571, normalized size = 2.45

$$\sqrt{-\left(-\sqrt{34} \sin\left(ex + d + \arctan\left(\frac{3}{5}\right)\right) - 2\right) \left(\cos^2\left(ex + d + \arctan\left(\frac{3}{5}\right)\right)\right)} \left(-\frac{\sqrt{-\left(-\sqrt{34} \sin\left(ex + d + \arctan\left(\frac{3}{5}\right)\right) - 2\right) \left(\cos^2\left(ex + d + \arctan\left(\frac{3}{5}\right)\right)\right)}}{2550 \left(\sin\left(ex + d + \arctan\left(\frac{3}{5}\right)\right) + \frac{\sqrt{34}}{17}\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x)

[Out]
$$\begin{aligned} & \left(-(-34^{1/2} \sin(ex+d+\arctan(3/5)) - 2) \cos(ex+d+\arctan(3/5)) \right)^{1/2} \cdot (-1/2550 \cdot (-(-34^{1/2} \sin(ex+d+\arctan(3/5)) - 2) \cos(ex+d+\arctan(3/5)) \right)^{1/2} \\ & / (\sin(ex+d+\arctan(3/5)) + 1/17 \cdot 34^{1/2})^3 + 4/57375 \cdot 34^{1/2} \cdot (-(-34^{1/2} \sin(ex+d+\arctan(3/5)) - 2) \cos(ex+d+\arctan(3/5)) \right)^{1/2} / (\sin(ex+d+\arctan(3/5)) + 1/17 \cdot 34^{1/2})^2 - 3383/101250 \cdot 34^{1/2} \cdot \cos(ex+d+\arctan(3/5)) \right)^{1/2} / (-(-289 \cdot \sin(ex+d+\arctan(3/5)) - 17 \cdot 34^{1/2}) \cdot 34^{1/2} \cdot \cos(ex+d+\arctan(3/5)) \right)^{1/2} \\ & - 319/50625 \cdot (1/17 \cdot 34^{1/2} + 1) \cdot ((17 \cdot \sin(ex+d+\arctan(3/5)) + 34^{1/2}) / (34^{1/2} + 17))^{1/2} \cdot ((17 \cdot \sin(ex+d+\arctan(3/5)) + 17) / (-34^{1/2} + 17))^{1/2} \cdot ((-17 \cdot \sin(ex+d+\arctan(3/5)) + 17) / (34^{1/2} + 17))^{1/2} / (-(-34^{1/2} \sin(ex+d+\arctan(3/5)) - 2) \cos(ex+d+\arctan(3/5)) \right)^{1/2} \\ & \cdot \text{EllipticF}\left(\left(\frac{17 \cdot \sin(ex+d+\arctan(3/5)) + 34^{1/2}}{34^{1/2} + 17}\right)^{1/2}, I \cdot \left(\frac{1}{-34^{1/2} + 17} \cdot (34^{1/2} + 17)\right)^{1/2}\right) - 199/101250 \cdot 34^{1/2} \cdot (1/17 \cdot 34^{1/2} + 1) \cdot ((17 \cdot \sin(ex+d+\arctan(3/5)) + 34^{1/2}) / (34^{1/2} + 17))^{1/2} \cdot ((17 \cdot \sin(ex+d+\arctan(3/5)) + 17) / (-34^{1/2} + 17))^{1/2} \cdot ((-17 \cdot \sin(ex+d+\arctan(3/5)) + 17) / (34^{1/2} + 17))^{1/2} / (-(-34^{1/2} \sin(ex+d+\arctan(3/5)) - 2) \cos(ex+d+\arctan(3/5)) \right)^{1/2} \\ & \cdot \left((-1/17 \cdot 34^{1/2} + 1) \cdot \text{EllipticE}\left(\left(\frac{17 \cdot \sin(ex+d+\arctan(3/5)) + 34^{1/2}}{34^{1/2} + 17}\right)^{1/2}, I \cdot \left(\frac{1}{-34^{1/2} + 17} \cdot (34^{1/2} + 17)\right)^{1/2}\right) - \text{EllipticF}\left(\left(\frac{17 \cdot \sin(ex+d+\arctan(3/5)) + 34^{1/2}}{34^{1/2} + 17}\right)^{1/2}, I \cdot \left(\frac{1}{-34^{1/2} + 17} \cdot (34^{1/2} + 17)\right)^{1/2}\right) \right) / \cos(ex+d+\arctan(3/5)) / (34^{1/2} \cdot \sin(ex+d+\arctan(3/5)) + 2)^{1/2} / e \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3 \cos(ex + d) + 5 \sin(ex + d) + 2)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x, algorithm="maxima")

[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(3 \cos(d + ex) + 5 \sin(d + ex) + 2)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(7/2), x)
```

```
[Out] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(7/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(7/2), x)
```

```
[Out] Timed out
```

3.410 $\int (a + b \cos(d + ex) + c \sin(d + ex))^{5/2} dx$

Optimal. Leaf size=347

$$\frac{16a(a^2 - b^2 - c^2) \sqrt{\frac{a+b \cos(d+ex)+c \sin(d+ex)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}(d+ex - \tan^{-1}(b,c)) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right) + 2(23a^2 + 9(b^2 + c^2)) \sqrt{a+b \cos(d+ex)}}{15e \sqrt{a+b \cos(d+ex) + c \sin(d+ex)}}$$

```
[Out] -2/5*(c*cos(e*x+d)-b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2)/e-16/15*(a*c*cos(e*x+d)-a*b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/e+2/15*(23*a^2+9*b^2+9*c^2)*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2))*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/e/((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)-16/15*a*(a^2-b^2-c^2)*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(b,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2))*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*(a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2))/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3120, 3146, 3149, 3119, 2653, 3127, 2661}

$$\frac{16a(a^2 - b^2 - c^2) \sqrt{\frac{a+b \cos(d+ex)+c \sin(d+ex)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}(d+ex - \tan^{-1}(b,c)) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right) + 2(23a^2 + 9(b^2 + c^2)) \sqrt{a+b \cos(d+ex)}}{15e \sqrt{a+b \cos(d+ex) + c \sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2), x]
```

```
[Out] (-16*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x])*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/(15*e) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))/(5*e) + (2*(23*a^2 + 9*(b^2 + c^2))*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/(15*e*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (16*a*(a^2 - b^2 - c^2)*EllipticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])/(15*e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]))
```

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d/Sqrt[a + b], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2])], x]
```

```
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]^
(n_.), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n -
1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x],
x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c,
d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(
x_.)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqr
t[b^2 + c^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_
)]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]
)/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]
], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\begin{aligned}
\int (a + b \cos(d + ex) + c \sin(d + ex))^{5/2} dx &= -\frac{2(c \cos(d + ex) - b \sin(d + ex))(a + b \cos(d + ex) + c \sin(d + ex))^3}{5e} \\
&= -\frac{16(ac \cos(d + ex) - ab \sin(d + ex))\sqrt{a + b \cos(d + ex) + c \sin(d + ex)}}{15e} \\
&= -\frac{16(ac \cos(d + ex) - ab \sin(d + ex))\sqrt{a + b \cos(d + ex) + c \sin(d + ex)}}{15e} \\
&= -\frac{16(ac \cos(d + ex) - ab \sin(d + ex))\sqrt{a + b \cos(d + ex) + c \sin(d + ex)}}{15e} \\
&= -\frac{16(ac \cos(d + ex) - ab \sin(d + ex))\sqrt{a + b \cos(d + ex) + c \sin(d + ex)}}{15e}
\end{aligned}$$

Mathematica [C] time = 6.62, size = 3767, normalized size = 10.86

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2), x]

[Out] (Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((2*b*(23*a^2 + 9*b^2 + 9*c^2))/(15*c) - (22*a*c*Cos[d + e*x])/15 - (2*b*c*Cos[2*(d + e*x)]/5 + (22*a*b*Sin[d + e*x])/15 + ((b^2 - c^2)*Sin[2*(d + e*x)]/5))/e + (2*a^3*AppellF1[1/2, 1/2, 1/2, 3/2, -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c)))*c), -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]/(Sqrt[1 + b^2/c^2]*c*e) + (34*a*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c)))*c), -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]/(15*Sqrt[1 + b^2/c^2]*c*e) + (34*a*c*AppellF1[1/2, 1/2, 1/2, 3/2, -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c)))*c), -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]/(15*Sqrt[1 + b^2/c^2]*e) + (23*a^2*b^2*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -(a + b*Sqrt[1 + c^2/b^2])*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -(a + b*Sqrt[1 + c^2/b^2])*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]


```

] * (-1 - a / (b * Sqrt[1 + c^2 / b^2])))) * Sin[d + e * x - ArcTan[c / b]] / (b * Sqrt[1 +
c^2 / b^2] * Sqrt[(b * Sqrt[(b^2 + c^2) / b^2] - b * Sqrt[(b^2 + c^2) / b^2] * Cos[d + e
* x - ArcTan[c / b]]) / (a + b * Sqrt[(b^2 + c^2) / b^2])] * Sqrt[a + b * Sqrt[(b^2 + c^
2) / b^2] * Cos[d + e * x - ArcTan[c / b]]] * Sqrt[(b * Sqrt[(b^2 + c^2) / b^2] + b * Sqrt[
(b^2 + c^2) / b^2] * Cos[d + e * x - ArcTan[c / b]]) / (-a + b * Sqrt[(b^2 + c^2) / b^2]
)]) - ((2 * b * (a + b * Sqrt[1 + c^2 / b^2] * Cos[d + e * x - ArcTan[c / b]])) / (b^2 + c^
2) - (c * Sin[d + e * x - ArcTan[c / b]]) / (b * Sqrt[1 + c^2 / b^2])) / Sqrt[a + b * Sqrt[
1 + c^2 / b^2] * Cos[d + e * x - ArcTan[c / b]]]) / (15 * c * e) + (3 * b^4 * (-((c * AppellF1
[-1/2, -1/2, -1/2, 1/2, -((a + b * Sqrt[1 + c^2 / b^2] * Cos[d + e * x - ArcTan[c / b]
]]) / (b * Sqrt[1 + c^2 / b^2] * (1 - a / (b * Sqrt[1 + c^2 / b^2])))), -((a + b * Sqrt[1 +
c^2 / b^2] * Cos[d + e * x - ArcTan[c / b]]) / (b * Sqrt[1 + c^2 / b^2] * (-1 - a / (b * Sqrt[
1 + c^2 / b^2]))))) * Sin[d + e * x - ArcTan[c / b]] / (b * Sqrt[1 + c^2 / b^2] * Sqrt[(b *
Sqrt[(b^2 + c^2) / b^2] - b * Sqrt[(b^2 + c^2) / b^2] * Cos[d + e * x - ArcTan[c / b]])
/ (a + b * Sqrt[(b^2 + c^2) / b^2])] * Sqrt[a + b * Sqrt[(b^2 + c^2) / b^2] * Cos[d + e *
x - ArcTan[c / b]]] * Sqrt[(b * Sqrt[(b^2 + c^2) / b^2] + b * Sqrt[(b^2 + c^2) / b^2] * C
os[d + e * x - ArcTan[c / b]]) / (-a + b * Sqrt[(b^2 + c^2) / b^2])]) - ((2 * b * (a + b
* Sqrt[1 + c^2 / b^2] * Cos[d + e * x - ArcTan[c / b]])) / (b^2 + c^2) - (c * Sin[d + e *
x - ArcTan[c / b]]) / (b * Sqrt[1 + c^2 / b^2])) / Sqrt[a + b * Sqrt[1 + c^2 / b^2] * Cos[d
+ e * x - ArcTan[c / b]]]) / (5 * c * e) + (23 * a^2 * c * (-((c * AppellF1[-1/2, -1/2, -1/
2, 1/2, -((a + b * Sqrt[1 + c^2 / b^2] * Cos[d + e * x - ArcTan[c / b]]) / (b * Sqrt[1 +
c^2 / b^2] * (1 - a / (b * Sqrt[1 + c^2 / b^2])))), -((a + b * Sqrt[1 + c^2 / b^2] * Cos[d
+ e * x - ArcTan[c / b]]) / (b * Sqrt[1 + c^2 / b^2] * (-1 - a / (b * Sqrt[1 + c^2 / b^2]
)))) * Sin[d + e * x - ArcTan[c / b]] / (b * Sqrt[1 + c^2 / b^2] * Sqrt[(b * Sqrt[(b^2 + c^2)
/ b^2] - b * Sqrt[(b^2 + c^2) / b^2] * Cos[d + e * x - ArcTan[c / b]]) / (a + b * Sqrt[(b^
2 + c^2) / b^2])] * Sqrt[a + b * Sqrt[(b^2 + c^2) / b^2] * Cos[d + e * x - ArcTan[c / b]
]] * Sqrt[(b * Sqrt[(b^2 + c^2) / b^2] + b * Sqrt[(b^2 + c^2) / b^2] * Cos[d + e * x - Arc
Tan[c / b]]) / (-a + b * Sqrt[(b^2 + c^2) / b^2])]) - ((2 * b * (a + b * Sqrt[1 + c^2 / b^
2] * Cos[d + e * x - ArcTan[c / b]])) / (b^2 + c^2) - (c * Sin[d + e * x - ArcTan[c / b]
]) / (b * Sqrt[1 + c^2 / b^2])) / Sqrt[a + b * Sqrt[1 + c^2 / b^2] * Cos[d + e * x - ArcTan[
c / b]]]) / (15 * e) + (6 * b^2 * c * (-((c * AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b * S
qrt[1 + c^2 / b^2] * Cos[d + e * x - ArcTan[c / b]]) / (b * Sqrt[1 + c^2 / b^2] * (1 - a / (b
* Sqrt[1 + c^2 / b^2])))), -((a + b * Sqrt[1 + c^2 / b^2] * Cos[d + e * x - ArcTan[c / b]
]]) / (b * Sqrt[1 + c^2 / b^2] * (-1 - a / (b * Sqrt[1 + c^2 / b^2]))))) * Sin[d + e * x - Ar
cTan[c / b]] / (b * Sqrt[1 + c^2 / b^2] * Sqrt[(b * Sqrt[(b^2 + c^2) / b^2] - b * Sqrt[(b^
2 + c^2) / b^2] * Cos[d + e * x - ArcTan[c / b]]) / (a + b * Sqrt[(b^2 + c^2) / b^2])] * Sq
rt[a + b * Sqrt[(b^2 + c^2) / b^2] * Cos[d + e * x - ArcTan[c / b]]] * Sqrt[(b * Sqrt[(b^
2 + c^2) / b^2] + b * Sqrt[(b^2 + c^2) / b^2] * Cos[d + e * x - ArcTan[c / b]]) / (-a + b
* Sqrt[(b^2 + c^2) / b^2])]) - ((2 * b * (a + b * Sqrt[1 + c^2 / b^2] * Cos[d + e * x - A
rcTan[c / b]])) / (b^2 + c^2) - (c * Sin[d + e * x - ArcTan[c / b]]) / (b * Sqrt[1 + c^2 /
b^2])) / Sqrt[a + b * Sqrt[1 + c^2 / b^2] * Cos[d + e * x - ArcTan[c / b]]]) / (5 * e) + (
3 * c^3 * (-((c * AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b * Sqrt[1 + c^2 / b^2] * Cos[
d + e * x - ArcTan[c / b]]) / (b * Sqrt[1 + c^2 / b^2] * (1 - a / (b * Sqrt[1 + c^2 / b^2]
))))), -((a + b * Sqrt[1 + c^2 / b^2] * Cos[d + e * x - ArcTan[c / b]]) / (b * Sqrt[1 + c^2 / b
^2] * (-1 - a / (b * Sqrt[1 + c^2 / b^2]))))) * Sin[d + e * x - ArcTan[c / b]] / (b * Sqrt[1
+ c^2 / b^2] * Sqrt[(b * Sqrt[(b^2 + c^2) / b^2] - b * Sqrt[(b^2 + c^2) / b^2] * Cos[d +
e * x - ArcTan[c / b]]) / (a + b * Sqrt[(b^2 + c^2) / b^2])] * Sqrt[a + b * Sqrt[(b^2 +
c^2) / b^2] * Cos[d + e * x - ArcTan[c / b]]] * Sqrt[(b * Sqrt[(b^2 + c^2) / b^2] + b * Sqr
t[(b^2 + c^2) / b^2] * Cos[d + e * x - ArcTan[c / b]]) / (-a + b * Sqrt[(b^2 + c^2) / b^2
])]) - ((2 * b * (a + b * Sqrt[1 + c^2 / b^2] * Cos[d + e * x - ArcTan[c / b]])) / (b^2 +
c^2) - (c * Sin[d + e * x - ArcTan[c / b]]) / (b * Sqrt[1 + c^2 / b^2])) / Sqrt[a + b * Sqr
t[1 + c^2 / b^2] * Cos[d + e * x - ArcTan[c / b]]]) / (5 * e)

```

fricas [F] time = 1.88, size = 0, normalized size = 0.00

integral $\left((2ab \cos(ex + d) + (b^2 - c^2) \cos(ex + d)^2 + a^2 + c^2 + 2(bc \cos(ex + d) + ac) \sin(ex + d)) \sqrt{b \cos(ex + d)} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="fricas")

[Out] integral((2*a*b*cos(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2*(

$b*c*\cos(e*x + d) + a*c)*\sin(e*x + d))*\sqrt{(b*\cos(e*x + d) + c*\sin(e*x + d) + a), x}$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cos(ex + d) + c \sin(ex + d) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="giac")

[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(5/2), x)

maple [B] time = 1.00, size = 2303, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x)

[Out]
$$\begin{aligned} &(-(-b^2*\sin(e*x+d-\arctan(-b,c))-c^2*\sin(e*x+d-\arctan(-b,c))-a*(b^2+c^2)^{(1/2} \\ &2))*\cos(e*x+d-\arctan(-b,c))^2/(b^2+c^2)^{(1/2)})^{(1/2)}*((b^2+c^2)^{(3/2)}*(-2/5 \\ &/ (b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))*(\cos(e*x+d-\arctan(-b,c))^2*(b^2+c \\ &^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))+a))^{(1/2)}+8/15/(b^2+c^2)*a*(\cos(e*x+d-\ar \\ &\text{ctan}(-b,c))^2*((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))+a))^{(1/2)}+4/15/(b^2+c \\ &^2)^{(1/2)}*a*(1/(b^2+c^2)^{(1/2)}*a-1)*((-b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b, \\ &c))-a)/(-a+(b^2+c^2)^{(1/2)})^{(1/2)}*((-\sin(e*x+d-\arctan(-b,c))+1)*(b^2+c^2)^{(1/2} \\ &/ (a+(b^2+c^2)^{(1/2)}))^{(1/2)}*((1+\sin(e*x+d-\arctan(-b,c)))*(b^2+c^2)^{(1/2} \\ &/ (-a+(b^2+c^2)^{(1/2)}))^{(1/2)} / (\cos(e*x+d-\arctan(-b,c))^2*((b^2+c^2)^{(1/2)}* \\ &\sin(e*x+d-\arctan(-b,c))+a))^{(1/2)}*\text{EllipticF}(((b^2+c^2)^{(1/2)}*\sin(e*x+d-\ar \\ &\text{ctan}(-b,c))-a)/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)}, ((a-(b^2+c^2)^{(1/2)})/(a+(b^2+c^2) \\ &)^{(1/2)}))^{(1/2)}+2*(3/5+8/15/(b^2+c^2)*a^2)*(1/(b^2+c^2)^{(1/2)}*a-1)*((-b^2 \\ &+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)}*((-\sin(e \\ &x+d-\arctan(-b,c))+1)*(b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}*((1+\sin(e \\ &x+d-\arctan(-b,c)))*(b^2+c^2)^{(1/2)}/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)} / (\cos(e*x+d-a \\ &\text{rctan}(-b,c))^2*((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))+a))^{(1/2)}*((-1/(b^2 \\ &+c^2)^{(1/2)}*a-1)*\text{EllipticE}(((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))-a)/(- \\ &a+(b^2+c^2)^{(1/2)}))^{(1/2)}, ((a-(b^2+c^2)^{(1/2)})/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}+ \\ &\text{EllipticF}(((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2} \\ &))^{(1/2)}, ((a-(b^2+c^2)^{(1/2)})/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}))^{(1/2)}+3*a*b^2+3*a*c^ \\ &2)*(-2/3/(b^2+c^2)^{(1/2)}*(\cos(e*x+d-\arctan(-b,c))^2*((b^2+c^2)^{(1/2)}*\sin(e \\ &x+d-\arctan(-b,c))+a))^{(1/2)}+2/3*(1/(b^2+c^2)^{(1/2)}*a-1)*((-b^2+c^2)^{(1/2)}* \\ &\sin(e*x+d-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)}*((-\sin(e*x+d-\arctan(\\ &-b,c))+1)*(b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}*((1+\sin(e*x+d-\arctan(- \\ &b,c)))*(b^2+c^2)^{(1/2)}/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)} / (\cos(e*x+d-\arctan(-b,c)) \\ &^2*((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))+a))^{(1/2)}*\text{EllipticF}(((b^2+c^2) \\ &)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)}, ((a-(b^2+c^2) \\ &)^{(1/2)})/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}-4/3/(b^2+c^2)^{(1/2)}*a*(1/(b^2+c^2)^{(1/2} \\ &)*a-1)*((-b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2)})) \\ &^{(1/2)}*((-\sin(e*x+d-\arctan(-b,c))+1)*(b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)}))^{(1/2} \\ &)*((1+\sin(e*x+d-\arctan(-b,c)))*(b^2+c^2)^{(1/2)}/(-a+(b^2+c^2)^{(1/2)}))^{(1/2} \\ &/ (\cos(e*x+d-\arctan(-b,c))^2*((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))+a))^{(1/2} \\ &)*((-1/(b^2+c^2)^{(1/2)}*a-1)*\text{EllipticE}(((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arct \\ &\text{an}(-b,c))-a)/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)}, ((a-(b^2+c^2)^{(1/2)})/(a+(b^2+c^2)^{(1/2} \\ &)^{(1/2)}))^{(1/2)}+2*\text{EllipticF}(((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))-a)/(-a+ \\ &(b^2+c^2)^{(1/2)}))^{(1/2)}, ((a-(b^2+c^2)^{(1/2)})/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}))^{(1/2)}+ \\ &6*a^2*(b^2+c^2)^{(1/2)}*(1/(b^2+c^2)^{(1/2)}*a-1)*((-b^2+c^2)^{(1/2)}*\sin(e*x+d-\ar \\ &\text{ctan}(-b,c))-a)/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)}*((-\sin(e*x+d-\arctan(-b,c))+1)* \\ &(b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}*((1+\sin(e*x+d-\arctan(-b,c)))*(b^ \\ &2+c^2)^{(1/2)}/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)} / (\cos(e*x+d-\arctan(-b,c))^2*((b^2+c \end{aligned}$$

$$\begin{aligned} & \left. \right)^{1/2} \sin(e*x+d-\arctan(-b,c))+a) \left. \right)^{1/2} * \left(\left(-1/(b^2+c^2)^{1/2} * a-1 \right) * \text{EllipticE} \left(\left(\left(- (b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c))-a \right) / \left(-a+(b^2+c^2)^{1/2} \right) \right) \right)^{1/2}, \left(\left(a-(b^2+c^2)^{1/2} \right) / \left(a+(b^2+c^2)^{1/2} \right) \right) \right)^{1/2} \right) + \text{EllipticF} \left(\left(\left(- (b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c))-a \right) / \left(-a+(b^2+c^2)^{1/2} \right) \right) \right)^{1/2}, \left(\left(a-(b^2+c^2)^{1/2} \right) / \left(a+(b^2+c^2)^{1/2} \right) \right) \right)^{1/2} \right) + 2*a^3 * \left(1/(b^2+c^2)^{1/2} * a-1 \right) * \left(\left(- (b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c))-a \right) / \left(-a+(b^2+c^2)^{1/2} \right) \right)^{1/2} * \left(\left(-\sin(e*x+d-\arctan(-b,c))+1 \right) * (b^2+c^2)^{1/2} / \left(a+(b^2+c^2)^{1/2} \right) \right)^{1/2} * \left(\left(1+\sin(e*x+d-\arctan(-b,c)) \right) * (b^2+c^2)^{1/2} / \left(-a+(b^2+c^2)^{1/2} \right) \right)^{1/2} / \left(- (b^2 * \sin(e*x+d-\arctan(-b,c))-c^2 * \sin(e*x+d-\arctan(-b,c))-a * (b^2+c^2)^{1/2}) * \cos(e*x+d-\arctan(-b,c)) \right)^2 / (b^2+c^2)^{1/2} \right)^{1/2} * \text{EllipticF} \left(\left(\left(- (b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c))-a \right) / \left(-a+(b^2+c^2)^{1/2} \right) \right) \right)^{1/2}, \left(\left(a-(b^2+c^2)^{1/2} \right) / \left(a+(b^2+c^2)^{1/2} \right) \right) \right)^{1/2} \right) / \cos(e*x+d-\arctan(-b,c)) / \left((b^2 * \sin(e*x+d-\arctan(-b,c))+c^2 * \sin(e*x+d-\arctan(-b,c))+a * (b^2+c^2)^{1/2}) / (b^2+c^2)^{1/2} \right)^{1/2} / e \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cos(ex + d) + c \sin(ex + d) + a)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="maxima")

[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \cos(d + ex) + c \sin(d + ex))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(d + e*x) + c*sin(d + e*x))^(5/2),x)

[Out] int((a + b*cos(d + e*x) + c*sin(d + e*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**(5/2),x)

[Out] Timed out

3.411 $\int (a + b \cos(d + ex) + c \sin(d + ex))^{3/2} dx$

Optimal. Leaf size=283

$$\frac{2(a^2 - b^2 - c^2) \sqrt{\frac{a+b \cos(d+ex)+c \sin(d+ex)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2} \left(d+ex - \tan^{-1}\left(\frac{b}{c}\right)\right) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right) + 8a\sqrt{a+b \cos(d+ex)+c \sin(d+ex)}}{3e\sqrt{a+b \cos(d+ex)+c \sin(d+ex)}} + \frac{8a\sqrt{a+b \cos(d+ex)+c \sin(d+ex)}}{3e\sqrt{a+b \cos(d+ex)+c \sin(d+ex)}}$$

[Out] $-2/3*(c*\cos(e*x+d)-b*\sin(e*x+d))*(a+b*\cos(e*x+d)+c*\sin(e*x+d))^{(1/2)}/e+8/3*a*(\cos(1/2*d+1/2*e*x-1/2*\arctan(b,c))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(b,c))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(b,c)),2^{(1/2)*((b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2))})}^{(1/2)}*(a+b*\cos(e*x+d)+c*\sin(e*x+d))^{(1/2)}/e/((a+b*\cos(e*x+d)+c*\sin(e*x+d))/(a+(b^2+c^2)^{(1/2))})^{(1/2)}-2/3*(a^2-b^2-c^2)*(\cos(1/2*d+1/2*e*x-1/2*\arctan(b,c))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(b,c))*\text{EllipticF}(\sin(1/2*d+1/2*e*x-1/2*\arctan(b,c)),2^{(1/2)*((b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2))})}^{(1/2)}*(a+b*\cos(e*x+d)+c*\sin(e*x+d))^{(1/2)}/e/((a+b*\cos(e*x+d)+c*\sin(e*x+d))/(a+(b^2+c^2)^{(1/2))})^{(1/2)})/e/(a+b*\cos(e*x+d)+c*\sin(e*x+d))^{(1/2)}$

Rubi [A] time = 0.28, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3120, 3149, 3119, 2653, 3127, 2661}

$$\frac{2(a^2 - b^2 - c^2) \sqrt{\frac{a+b \cos(d+ex)+c \sin(d+ex)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2} \left(d+ex - \tan^{-1}\left(\frac{b}{c}\right)\right) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right) + 8a\sqrt{a+b \cos(d+ex)+c \sin(d+ex)}}{3e\sqrt{a+b \cos(d+ex)+c \sin(d+ex)}} + \frac{8a\sqrt{a+b \cos(d+ex)+c \sin(d+ex)}}{3e\sqrt{a+b \cos(d+ex)+c \sin(d+ex)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{(3/2)}, x]$

[Out] $(-2*(c*\text{Cos}[d + e*x] - b*\text{Sin}[d + e*x])* \text{Sqrt}[a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]])/(3*e) + (8*a*\text{EllipticE}[(d + e*x - \text{ArcTan}[b, c])/2, (2*\text{Sqrt}[b^2 + c^2])/(a + \text{Sqrt}[b^2 + c^2])]* \text{Sqrt}[a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]])/(3*e*\text{Sqrt}[(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])/(a + \text{Sqrt}[b^2 + c^2])]) - (2*(a^2 - b^2 - c^2)* \text{EllipticF}[(d + e*x - \text{ArcTan}[b, c])/2, (2*\text{Sqrt}[b^2 + c^2])/(a + \text{Sqrt}[b^2 + c^2])]* \text{Sqrt}[(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])/(a + \text{Sqrt}[b^2 + c^2])])/(3*e*\text{Sqrt}[a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]])$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[a + b, 0]$

Rule 2661

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/(\text{d}*\text{Sqrt}[a + b]), x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[a + b, 0]$

Rule 3119

$\text{Int}[\text{Sqrt}[\cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*\sin[(d_) + (e_)*(x_)]]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]]/\text{Sqrt}[(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])/(a + \text{Sqrt}[b^2 + c^2])], \text{Int}[\text{Sqrt}[a/(a + \text{Sqrt}[b^2 + c^2]) + (\text{Sqrt}[b^2 + c^2]*\text{Cos}[d + e*x - \text{ArcTan}[b, c]])/(a + \text{Sqrt}[b^2 + c^2])], x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0]$

&& NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3120

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^n, x_Symbol] :> -Simp[((c*cos[d + e*x] - b*sin[d + e*x])*(a + b*cos[d + e*x] + c*sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*cos[d + e*x] + a*c*(2*n - 1)*sin[d + e*x], x]*(a + b*cos[d + e*x] + c*sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]

Rule 3127

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Dist[Sqrt[(a + b*cos[d + e*x] + c*sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*cos[d + e*x] - ArcTan[b, c])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3149

Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]) / Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]

Rubi steps

$$\begin{aligned} \int (a + b \cos(d + ex) + c \sin(d + ex))^{3/2} dx &= -\frac{2(c \cos(d + ex) - b \sin(d + ex))\sqrt{a + b \cos(d + ex) + c \sin(d + ex)}}{3e} \\ &= -\frac{2(c \cos(d + ex) - b \sin(d + ex))\sqrt{a + b \cos(d + ex) + c \sin(d + ex)}}{3e} \\ &= -\frac{2(c \cos(d + ex) - b \sin(d + ex))\sqrt{a + b \cos(d + ex) + c \sin(d + ex)}}{3e} \\ &= -\frac{2(c \cos(d + ex) - b \sin(d + ex))\sqrt{a + b \cos(d + ex) + c \sin(d + ex)}}{3e} \end{aligned}$$

Mathematica [C] time = 6.26, size = 2190, normalized size = 7.74

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*cos[d + e*x] + c*sin[d + e*x])^(3/2), x]

[Out] (((8*a*b)/(3*c) - (2*c*cos[d + e*x])/3 + (2*b*sin[d + e*x])/3)*Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]])/e + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, -(a

+ Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]]/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c), -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]]/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]/(Sqrt[1 + b^2/c^2]*c*e) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)], -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]/(3*Sqrt[1 + b^2/c^2]*c*e) + (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)], -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]/(3*Sqrt[1 + b^2/c^2]*e) + (4*a*b^2*(-(c*AppellF1[-1/2, -1/2, -1/2, 1/2, -(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2])*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(3*c*e) + (4*a*c*(-(c*AppellF1[-1/2, -1/2, -1/2, 1/2, -(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2])*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(3*e)

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\int (b \cos(ex + d) + c \sin(ex + d) + a)^{\frac{3}{2}}, x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="fricas")

[Out] integral((b*cos(e*x + d) + c*sin(e*x + d) + a)^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cos(ex + d) + c \sin(ex + d) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(3/2), x)

maple [B] time = 0.71, size = 1516, normalized size = 5.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x)

[Out]
$$\begin{aligned} & (-(-b^2 \sin(e*x+d-\arctan(-b,c)) - c^2 \sin(e*x+d-\arctan(-b,c)) - a*(b^2+c^2)^{(1/2)}) \\ & * \cos(e*x+d-\arctan(-b,c))^{1/2} / (b^2+c^2)^{(1/2)})^{1/2} * ((b^2+c^2)*(-2/3/(b^2+c^2)^{(1/2)} \\ & * (\cos(e*x+d-\arctan(-b,c))^{1/2} * ((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c)) \\ & + a))^{1/2} + 2/3*(1/(b^2+c^2)^{(1/2)} * a - 1) * ((- (b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c)) \\ & - a) / (-a + (b^2+c^2)^{(1/2)}))^{1/2} * ((-\sin(e*x+d-\arctan(-b,c)) + 1) * (b^2+c^2)^{(1/2)} \\ & / (a + (b^2+c^2)^{(1/2)}))^{1/2} * ((1 + \sin(e*x+d-\arctan(-b,c))) * (b^2+c^2)^{(1/2)} \\ & / (-a + (b^2+c^2)^{(1/2)}))^{1/2} / (\cos(e*x+d-\arctan(-b,c))^{1/2} * ((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c)) \\ & + a))^{1/2} * \text{EllipticF}(((- (b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c)) - a) / (-a + (b^2+c^2)^{(1/2)}))^{1/2}, \\ & ((a - (b^2+c^2)^{(1/2)}) / (a + (b^2+c^2)^{(1/2)}))^{1/2}) - 4/3 / (b^2+c^2)^{(1/2)} * a * (1 / (b^2+c^2)^{(1/2)} * a - 1) * ((- (b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c)) \\ & - a) / (-a + (b^2+c^2)^{(1/2)}))^{1/2} * ((-\sin(e*x+d-\arctan(-b,c)) + 1) * (b^2+c^2)^{(1/2)} / (a + (b^2+c^2)^{(1/2)}))^{1/2} * ((1 + \sin(e*x+d-\arctan(-b,c))) * (b^2+c^2)^{(1/2)} \\ & / (-a + (b^2+c^2)^{(1/2)}))^{1/2} / (\cos(e*x+d-\arctan(-b,c))^{1/2} * ((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c)) \\ & + a))^{1/2} * ((-1 / (b^2+c^2)^{(1/2)} * a - 1) * \text{EllipticE}(((- (b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c)) - a) / (-a + (b^2+c^2)^{(1/2)}))^{1/2}, \\ & ((a - (b^2+c^2)^{(1/2)}) / (a + (b^2+c^2)^{(1/2)}))^{1/2}) + \text{EllipticF}(((- (b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c)) - a) / (-a + (b^2+c^2)^{(1/2)}))^{1/2}, \\ & ((a - (b^2+c^2)^{(1/2)}) / (a + (b^2+c^2)^{(1/2)}))^{1/2})) + 4*a*(b^2+c^2)^{(1/2)} * (1 / (b^2+c^2)^{(1/2)} * a - 1) * ((- (b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c)) - a) \\ & / (-a + (b^2+c^2)^{(1/2)}))^{1/2} * ((-\sin(e*x+d-\arctan(-b,c)) + 1) * (b^2+c^2)^{(1/2)} / (a + (b^2+c^2)^{(1/2)}))^{1/2} * ((1 + \sin(e*x+d-\arctan(-b,c))) * (b^2+c^2)^{(1/2)} \\ & / (-a + (b^2+c^2)^{(1/2)}))^{1/2} / (\cos(e*x+d-\arctan(-b,c))^{1/2} * ((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c)) \\ & + a))^{1/2} * ((-1 / (b^2+c^2)^{(1/2)} * a - 1) * \text{EllipticE}(((- (b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c)) - a) / (-a + (b^2+c^2)^{(1/2)}))^{1/2}, \\ & ((a - (b^2+c^2)^{(1/2)}) / (a + (b^2+c^2)^{(1/2)}))^{1/2}) + \text{EllipticF}(((- (b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c)) - a) / (-a + (b^2+c^2)^{(1/2)}))^{1/2}, \\ & ((a - (b^2+c^2)^{(1/2)}) / (a + (b^2+c^2)^{(1/2)}))^{1/2})) + 2*a^2*(1 / (b^2+c^2)^{(1/2)} * a - 1) * ((- (b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c)) - a) / (-a + (b^2+c^2)^{(1/2)}))^{1/2} * ((-\sin(e*x+d-\arctan(-b,c)) \\ & + 1) * (b^2+c^2)^{(1/2)} / (a + (b^2+c^2)^{(1/2)}))^{1/2} * ((1 + \sin(e*x+d-\arctan(-b,c))) * (b^2+c^2)^{(1/2)} / (-a + (b^2+c^2)^{(1/2)}))^{1/2} / ((-b^2 \sin(e*x+d-\arctan(-b,c)) \\ & - c^2 \sin(e*x+d-\arctan(-b,c)) - a*(b^2+c^2)^{(1/2)}) * \cos(e*x+d-\arctan(-b,c))^{1/2} \\ & / (b^2+c^2)^{(1/2)})^{1/2} * \text{EllipticF}(((- (b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c)) - a) / (-a + (b^2+c^2)^{(1/2)}))^{1/2}, \\ & ((a - (b^2+c^2)^{(1/2)}) / (a + (b^2+c^2)^{(1/2)}))^{1/2}) / \cos(e*x+d-\arctan(-b,c)) / ((b^2 \sin(e*x+d-\arctan(-b,c)) + c^2 \sin(e*x+d-\arctan(-b,c)) + a*(b^2+c^2)^{(1/2)}) / (b^2+c^2)^{(1/2)})^{1/2} / e \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cos(ex + d) + c \sin(ex + d) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \cos(d + ex) + c \sin(d + ex))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*cos(d + e*x) + c*sin(d + e*x))^(3/2), x)`

[Out] `int((a + b*cos(d + e*x) + c*sin(d + e*x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \cos(d + ex) + c \sin(d + ex))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**(3/2), x)`

[Out] `Integral((a + b*cos(d + e*x) + c*sin(d + e*x))**(3/2), x)`

3.412 $\int \sqrt{a + b \cos(d + ex) + c \sin(d + ex)} dx$

Optimal. Leaf size=108

$$\frac{2\sqrt{a + b \cos(d + ex) + c \sin(d + ex)} E\left(\frac{1}{2}(d + ex - \tan^{-1}(b, c)) \middle| \frac{2\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}\right)}{e\sqrt{\frac{a + b \cos(d + ex) + c \sin(d + ex)}{a + \sqrt{b^2 + c^2}}}}$$

[Out] $2*(\cos(1/2*d+1/2*e*x-1/2*\arctan(b,c))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(b,c))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(b,c)),2^{(1/2)}*((b^2+c^2)^{(1/2)})/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}*(a+b*\cos(e*x+d)+c*\sin(e*x+d))^{(1/2)}/e/((a+b*\cos(e*x+d)+c*\sin(e*x+d))/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}$

Rubi [A] time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3119, 2653}

$$\frac{2\sqrt{a + b \cos(d + ex) + c \sin(d + ex)} E\left(\frac{1}{2}(d + ex - \tan^{-1}(b, c)) \middle| \frac{2\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}\right)}{e\sqrt{\frac{a + b \cos(d + ex) + c \sin(d + ex)}{a + \sqrt{b^2 + c^2}}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]

[Out] $(2*\text{EllipticE}[(d + e*x - \text{ArcTan}[b, c])/2, (2*\text{Sqrt}[b^2 + c^2])]/(a + \text{Sqrt}[b^2 + c^2]))*\text{Sqrt}[a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]]/(e*\text{Sqrt}[(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])/(a + \text{Sqrt}[b^2 + c^2])])$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rubi steps

$$\begin{aligned} \int \sqrt{a + b \cos(d + ex) + c \sin(d + ex)} dx &= \frac{\sqrt{a + b \cos(d + ex) + c \sin(d + ex)} \int \sqrt{\frac{a}{a + \sqrt{b^2 + c^2}} + \frac{\sqrt{b^2 + c^2} \cos(d + ex - \tan^{-1}(b, c))}{a + \sqrt{b^2 + c^2}}} dx}{\sqrt{\frac{a + b \cos(d + ex) + c \sin(d + ex)}{a + \sqrt{b^2 + c^2}}}} \\ &= \frac{2E\left(\frac{1}{2}(d + ex - \tan^{-1}(b, c)) \middle| \frac{2\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}\right) \sqrt{a + b \cos(d + ex) + c \sin(d + ex)}}{e\sqrt{\frac{a + b \cos(d + ex) + c \sin(d + ex)}{a + \sqrt{b^2 + c^2}}}} \end{aligned}$$

Mathematica [C] time = 6.23, size = 1408, normalized size = 13.04

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]],x]

[Out] $(2*b*\sqrt{a + b*\cos[d + e*x] + c*\sin[d + e*x]})/(c*e) + (2*a*\text{AppellF1}[1/2, 1/2, 1/2, 3/2, -((a + \sqrt{1 + b^2/c^2})*c*\sin[d + e*x + \text{ArcTan}[b/c]])/(\sqrt{1 + b^2/c^2}*(1 - a/(\sqrt{1 + b^2/c^2})*c)), -((a + \sqrt{1 + b^2/c^2})*c*\sin[d + e*x + \text{ArcTan}[b/c]])/(\sqrt{1 + b^2/c^2}*(-1 - a/(\sqrt{1 + b^2/c^2})*c)))*\text{Sec}[d + e*x + \text{ArcTan}[b/c]]*\sqrt{((c*\sqrt{(b^2 + c^2)/c^2} - c*\sqrt{(b^2 + c^2)/c^2})*\sin[d + e*x + \text{ArcTan}[b/c]])/(a + c*\sqrt{(b^2 + c^2)/c^2})}*\sqrt{a + c*\sqrt{(b^2 + c^2)/c^2}*\sin[d + e*x + \text{ArcTan}[b/c]]}*\sqrt{(c*\sqrt{(b^2 + c^2)/c^2} + c*\sqrt{(b^2 + c^2)/c^2}*\sin[d + e*x + \text{ArcTan}[b/c]])/(-a + c*\sqrt{(b^2 + c^2)/c^2})})/(\sqrt{1 + b^2/c^2})*c*e) + (b^2*(-((c*\text{AppellF1}[-1/2, -1/2, -1/2, 1/2, -((a + b*\sqrt{1 + c^2/b^2})*\cos[d + e*x - \text{ArcTan}[c/b]])/(b*\sqrt{1 + c^2/b^2}*(1 - a/(b*\sqrt{1 + c^2/b^2}))))), -((a + b*\sqrt{1 + c^2/b^2})*\cos[d + e*x - \text{ArcTan}[c/b]])/(b*\sqrt{1 + c^2/b^2}*(-1 - a/(b*\sqrt{1 + c^2/b^2})))))*\sin[d + e*x - \text{ArcTan}[c/b]])/(b*\sqrt{1 + c^2/b^2}*\sqrt{(b*\sqrt{(b^2 + c^2)/b^2} - b*\sqrt{(b^2 + c^2)/b^2}*\cos[d + e*x - \text{ArcTan}[c/b]])/(a + b*\sqrt{(b^2 + c^2)/b^2})}*\sqrt{a + b*\sqrt{(b^2 + c^2)/b^2}*\cos[d + e*x - \text{ArcTan}[c/b]]}*\sqrt{(b*\sqrt{(b^2 + c^2)/b^2} + b*\sqrt{(b^2 + c^2)/b^2}*\cos[d + e*x - \text{ArcTan}[c/b]])/(-a + b*\sqrt{(b^2 + c^2)/b^2})})) - ((2*b*(a + b*\sqrt{1 + c^2/b^2})*\cos[d + e*x - \text{ArcTan}[c/b]])/(b^2 + c^2) - (c*\sin[d + e*x - \text{ArcTan}[c/b]])/(b*\sqrt{1 + c^2/b^2}))/\sqrt{a + b*\sqrt{1 + c^2/b^2}*\cos[d + e*x - \text{ArcTan}[c/b]]})/(c*e) + (c*(-((c*\text{AppellF1}[-1/2, -1/2, -1/2, 1/2, -(a + b*\sqrt{1 + c^2/b^2})*\cos[d + e*x - \text{ArcTan}[c/b]])/(b*\sqrt{1 + c^2/b^2}*(1 - a/(b*\sqrt{1 + c^2/b^2}))))), -((a + b*\sqrt{1 + c^2/b^2})*\cos[d + e*x - \text{ArcTan}[c/b]])/(b*\sqrt{1 + c^2/b^2}*(-1 - a/(b*\sqrt{1 + c^2/b^2})))))*\sin[d + e*x - \text{ArcTan}[c/b]])/(b*\sqrt{1 + c^2/b^2}*\sqrt{(b*\sqrt{(b^2 + c^2)/b^2} - b*\sqrt{(b^2 + c^2)/b^2}*\cos[d + e*x - \text{ArcTan}[c/b]])/(a + b*\sqrt{(b^2 + c^2)/b^2})}*\sqrt{a + b*\sqrt{(b^2 + c^2)/b^2}*\cos[d + e*x - \text{ArcTan}[c/b]]}*\sqrt{(b*\sqrt{(b^2 + c^2)/b^2} + b*\sqrt{(b^2 + c^2)/b^2}*\cos[d + e*x - \text{ArcTan}[c/b]])/(-a + b*\sqrt{(b^2 + c^2)/b^2})})) - ((2*b*(a + b*\sqrt{1 + c^2/b^2})*\cos[d + e*x - \text{ArcTan}[c/b]])/(b^2 + c^2) - (c*\sin[d + e*x - \text{ArcTan}[c/b]])/(b*\sqrt{1 + c^2/b^2}))/\sqrt{a + b*\sqrt{1 + c^2/b^2}*\cos[d + e*x - \text{ArcTan}[c/b]]})/e$

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b \cos(ex + d) + c \sin(ex + d) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \cos(ex + d) + c \sin(ex + d) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)

maple [B] time = 0.46, size = 720, normalized size = 6.67

$$2 \left(-a + \sqrt{b^2 + c^2} \right) \sqrt{-\frac{\sqrt{b^2+c^2} \sin(ex+d-\arctan(-b,c))+a}{-a+\sqrt{b^2+c^2}}} \sqrt{-\frac{(\sin(ex+d-\arctan(-b,c))-1)\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}} \sqrt{\frac{(1+\sin(ex+d-\arctan(-b,c)))\sqrt{b^2+c^2}}{-a+\sqrt{b^2+c^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x)

[Out] $2/(b^2+c^2)^{(1/2)}*(-a+(b^2+c^2)^{(1/2)})*(-((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))+a)/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)}*(-(\sin(e*x+d-\arctan(-b,c))-1)*(b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}*((1+\sin(e*x+d-\arctan(-b,c)))*(b^2+c^2)^{(1/2)}/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)}*((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))*\cos(e*x+d-\arctan(-b,c))^2+\cos(e*x+d-\arctan(-b,c))^2*a)^{(1/2)}*((b^2+c^2)^{(1/2)}*EllipticE((- (b^2+c^2)^{(1/2)}/(-a+(b^2+c^2)^{(1/2)})*\sin(e*x+d-\arctan(-b,c))-a/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)},(-(-a+(b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)}))^{(1/2)})-(b^2+c^2)^{(1/2)}*EllipticF((- (b^2+c^2)^{(1/2)}/(-a+(b^2+c^2)^{(1/2)})*\sin(e*x+d-\arctan(-b,c))-a/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)},(-(-a+(b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)}))^{(1/2)})+EllipticE((- (b^2+c^2)^{(1/2)}/(-a+(b^2+c^2)^{(1/2)})*\sin(e*x+d-\arctan(-b,c))-a/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)},(-(-a+(b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)}))^{(1/2)})*a-EllipticF((- (b^2+c^2)^{(1/2)}/(-a+(b^2+c^2)^{(1/2)})*\sin(e*x+d-\arctan(-b,c))-a/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)},(-(-a+(b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)}))^{(1/2)})*a)/(\cos(e*x+d-\arctan(-b,c))^2*((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))+a))^{(1/2)}/\cos(e*x+d-\arctan(-b,c))/((b^2*\sin(e*x+d-\arctan(-b,c))+c^2*\sin(e*x+d-\arctan(-b,c))+a*(b^2+c^2)^{(1/2)))/(b^2+c^2)^{(1/2))^{(1/2)}/e$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \cos(ex+d) + c \sin(ex+d) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b \cos(d + ex) + c \sin(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(d + e*x) + c*sin(d + e*x))^(1/2),x)

[Out] int((a + b*cos(d + e*x) + c*sin(d + e*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \cos(d + ex) + c \sin(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**(1/2),x)

[Out] Integral(sqrt(a + b*cos(d + e*x) + c*sin(d + e*x)), x)

$$3.413 \quad \int \frac{1}{\sqrt{a+b \cos(d+ex)+c \sin(d+ex)}} dx$$

Optimal. Leaf size=108

$$\frac{2\sqrt{\frac{a+b \cos(d+ex)+c \sin(d+ex)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{b}{c}\right)\right)\middle|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{a+b \cos(d+ex)+c \sin(d+ex)}}$$

[Out] $2*(\cos(1/2*d+1/2*e*x-1/2*\arctan(b,c))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(b,c))*\text{EllipticF}(\sin(1/2*d+1/2*e*x-1/2*\arctan(b,c)),2^{(1/2)}*((b^2+c^2)^{(1/2)})/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}*((a+b*\cos(e*x+d)+c*\sin(e*x+d))/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}/e/(a+b*\cos(e*x+d)+c*\sin(e*x+d))^{(1/2)}$

Rubi [A] time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3127, 2661}

$$\frac{2\sqrt{\frac{a+b \cos(d+ex)+c \sin(d+ex)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{b}{c}\right)\right)\middle|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{a+b \cos(d+ex)+c \sin(d+ex)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]

[Out] $(2*\text{EllipticF}[(d+e*x-\text{ArcTan}[b,c])/2,(2*\text{Sqrt}[b^2+c^2])/(a+\text{Sqrt}[b^2+c^2])]*\text{Sqrt}[(a+b*\text{Cos}[d+e*x]+c*\text{Sin}[d+e*x])/(a+\text{Sqrt}[b^2+c^2])])/(e*\text{Sqrt}[a+b*\text{Cos}[d+e*x]+c*\text{Sin}[d+e*x]])$

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3127

Int[1/Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rubi steps

$$\int \frac{1}{\sqrt{a+b \cos(d+ex)+c \sin(d+ex)}} dx = \frac{\sqrt{\frac{a+b \cos(d+ex)+c \sin(d+ex)}{a+\sqrt{b^2+c^2}}} \int \frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}} + \frac{\sqrt{b^2+c^2} \cos(d+ex-\tan^{-1}(b,c))}{a+\sqrt{b^2+c^2}}} dx}{\sqrt{a+b \cos(d+ex)+c \sin(d+ex)}}$$

$$= \frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{b}{c}\right)\right)\middle|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right) \sqrt{\frac{a+b \cos(d+ex)+c \sin(d+ex)}{a+\sqrt{b^2+c^2}}}}{e\sqrt{a+b \cos(d+ex)+c \sin(d+ex)}}$$

Mathematica [C] time = 0.56, size = 285, normalized size = 2.64

$$2 \sec\left(\tan^{-1}\left(\frac{b}{c}\right) + d + ex\right) \frac{\sqrt{\frac{c\sqrt{\frac{b^2}{c^2}+1}\left(\sin\left(\tan^{-1}\left(\frac{b}{c}\right)+d+ex\right)-1\right)}{a+c\sqrt{\frac{b^2}{c^2}+1}}}}{\sqrt{\frac{c\sqrt{\frac{b^2}{c^2}+1}\left(\sin\left(\tan^{-1}\left(\frac{b}{c}\right)+d+ex\right)+1\right)}{c\sqrt{\frac{b^2}{c^2}+1}-a}}} \sqrt{a+c\sqrt{\frac{b^2}{c^2}+1}} \sin\left(\tan^{-1}\left(\frac{b}{c}\right) + d + ex\right) \frac{ce\sqrt{\frac{b^2}{c^2}+1}}{\sqrt{\frac{b^2}{c^2}+1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]],x]

[Out] (2*AppellF1[1/2, 1/2, 1/2, 3/2, (a + Sqrt[1 + b^2/c^2]*c*sin[d + e*x + ArcTan[b/c]])/(a - Sqrt[1 + b^2/c^2]*c), (a + Sqrt[1 + b^2/c^2]*c*sin[d + e*x + ArcTan[b/c]])/(a + Sqrt[1 + b^2/c^2]*c)]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[-((Sqrt[1 + b^2/c^2]*c*(-1 + Sin[d + e*x + ArcTan[b/c]])))/(a + Sqrt[1 + b^2/c^2]*c)]*Sqrt[(Sqrt[1 + b^2/c^2]*c*(1 + Sin[d + e*x + ArcTan[b/c]]))]/(-a + Sqrt[1 + b^2/c^2]*c)]*Sqrt[a + Sqrt[1 + b^2/c^2]*c*sin[d + e*x + ArcTan[b/c]]]/(Sqrt[1 + b^2/c^2]*c*e)

fricas [F] time = 3.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b \cos(ex + d) + c \sin(ex + d) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="fricas")

[Out] integral(1/sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \cos(ex + d) + c \sin(ex + d) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)

maple [B] time = 0.37, size = 303, normalized size = 2.81

$$\frac{2\left(-a + \sqrt{b^2 + c^2}\right) \sqrt{-\frac{\sqrt{b^2+c^2} \sin(ex+d-\arctan(-b,c))+a}{-a+\sqrt{b^2+c^2}}} \sqrt{-\frac{(\sin(ex+d-\arctan(-b,c))-1)\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}} \sqrt{\frac{(1+\sin(ex+d-\arctan(-b,c)))}{-a+\sqrt{b^2+c^2}}}}}{\sqrt{b^2 + c^2} \cos(ex + d - \arctan(-b, c)) \sqrt{\frac{b^2 \sin(ex+d-\arctan(-b,c))+c^2 \sin^2(ex+d-\arctan(-b,c))}{\sqrt{b^2+c^2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x)

[Out] -2*(-a+(b^2+c^2)^(1/2))*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(-a+(b^2+c^2)^(1/2))^(1/2)*(-(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*EllipticF(-((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(-a+(b^2+c^2)^(1/2)))^(1/2), (-(-a+(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)/(b^2+c^2)^(1/2)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \cos(ex + d) + c \sin(ex + d) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + b \cos(d + ex) + c \sin(d + ex)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(1/2),x)

[Out] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \cos(d + ex) + c \sin(d + ex)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(1/2),x)

[Out] Integral(1/sqrt(a + b*cos(d + e*x) + c*sin(d + e*x)), x)

$$3.414 \quad \int \frac{1}{(a+b \cos(d+ex)+c \sin(d+ex))^{3/2}} dx$$

Optimal. Leaf size=186

$$\frac{2\sqrt{a+b \cos(d+ex)+c \sin(d+ex)} E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\middle|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e(a^2-b^2-c^2)\sqrt{\frac{a+b \cos(d+ex)+c \sin(d+ex)}{a+\sqrt{b^2+c^2}}}} + \frac{2(c \cos(d+ex)-b \sin(d+ex))}{e(a^2-b^2-c^2)\sqrt{a+b \cos(d+ex)}}$$

[Out] $2*(c*\cos(e*x+d)-b*\sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*\cos(e*x+d)+c*\sin(e*x+d))^{1/2}+2*(\cos(1/2*d+1/2*e*x-1/2*\arctan(b,c))^2)^{1/2}/\cos(1/2*d+1/2*e*x-1/2*\arctan(b,c))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(b,c)),2^{1/2}*((b^2+c^2)^{1/2}/(a+(b^2+c^2)^{1/2}))^{1/2})*(a+b*\cos(e*x+d)+c*\sin(e*x+d))^{1/2}/(a^2-b^2-c^2)/e/((a+b*\cos(e*x+d)+c*\sin(e*x+d))/(a+(b^2+c^2)^{1/2}))^{1/2}$

Rubi [A] time = 0.10, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3128, 3119, 2653}

$$\frac{2\sqrt{a+b \cos(d+ex)+c \sin(d+ex)} E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\middle|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e(a^2-b^2-c^2)\sqrt{\frac{a+b \cos(d+ex)+c \sin(d+ex)}{a+\sqrt{b^2+c^2}}}} + \frac{2(c \cos(d+ex)-b \sin(d+ex))}{e(a^2-b^2-c^2)\sqrt{a+b \cos(d+ex)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2), x]

[Out] $(2*(c*\cos[d + e*x] - b*\sin[d + e*x]))/((a^2 - b^2 - c^2)*e*\sqrt{a + b*\cos[d + e*x] + c*\sin[d + e*x]}) + (2*\text{EllipticE}[(d + e*x - \text{ArcTan}[b, c])/2, (2*\sqrt{b^2 + c^2})/(a + \sqrt{b^2 + c^2})]*\sqrt{a + b*\cos[d + e*x] + c*\sin[d + e*x]})/((a^2 - b^2 - c^2)*e*\sqrt{(a + b*\cos[d + e*x] + c*\sin[d + e*x])/(a + \sqrt{b^2 + c^2})})$

Rule 2653

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3128

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-3/2), x_Symbol] := Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^{3/2}} dx &= \frac{2(c \cos(d + ex) - b \sin(d + ex))}{(a^2 - b^2 - c^2) e \sqrt{a + b \cos(d + ex) + c \sin(d + ex)}} + \frac{\int \sqrt{a + b \cos(d + ex)}}{\sqrt{a + b \cos(d + ex)}} \\
&= \frac{2(c \cos(d + ex) - b \sin(d + ex))}{(a^2 - b^2 - c^2) e \sqrt{a + b \cos(d + ex) + c \sin(d + ex)}} + \frac{\sqrt{a + b \cos(d + ex)}}{\sqrt{a + b \cos(d + ex)}} \\
&= \frac{2(c \cos(d + ex) - b \sin(d + ex))}{(a^2 - b^2 - c^2) e \sqrt{a + b \cos(d + ex) + c \sin(d + ex)}} + \frac{2E\left(\frac{1}{2}(d + ex)\right)}{\sqrt{a + b \cos(d + ex) + c \sin(d + ex)}}
\end{aligned}$$

Mathematica [C] time = 6.35, size = 1540, normalized size = 8.28

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2), x]

[Out] (Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((-2*(b^2 + c^2))/(b*c*(-a^2 + b^2 + c^2)) + (2*(a*c + b^2*Sin[d + e*x] + c^2*Sin[d + e*x]))/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])))/e - (2*a*AppellF1[1/2, 1/2, 1/2, 3/2, -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c)))*c), -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])] * Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]] * Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)*e) - (b^2*((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2])))))*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])] * Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]] * Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]])/((c*(-a^2 + b^2 + c^2)*e) - (c*(-(c*AppellF1[-1/2, -1/2, -1/2, 1/2, -(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2])))))*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])] * Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]] * Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]])/((-a^2 + b^2 + c^2)*e)

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{b \cos(ex + d) + c \sin(ex + d) + a}}{2ab \cos(ex + d) + (b^2 - c^2) \cos^2(ex + d) + a^2 + c^2 + 2(bc \cos(ex + d) + ac) \sin(ex + d)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a)/(2*a*b*cos(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2*(b*c*cos(e*x + d) + a*c)*sin(e*x + d)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cos(ex + d) + c \sin(ex + d) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-3/2), x)

maple [B] time = 0.77, size = 2388, normalized size = 12.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x)

[Out]
$$\begin{aligned} & (-(-b^2 \sin(e*x+d-\arctan(-b,c)) - c^2 \sin(e*x+d-\arctan(-b,c)) - a*(b^2+c^2))^{1/2} \\ & * \cos(e*x+d-\arctan(-b,c))^{1/2} / (b^2+c^2)^{1/2})^{1/2} / (b^2+c^2)^{1/2} * (- (b^2+c^2)^{1/2} \\ & * (-b^2-c^2) * \cos(e*x+d-\arctan(-b,c))^{1/2} / (a^2-b^2-c^2) / (-(-b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c)) \\ & - a) * \cos(e*x+d-\arctan(-b,c))^{1/2} * (b^2+c^2))^{1/2} + a*(b^2+c^2) / (a^2-b^2-c^2) * (1/(b^2+c^2)^{1/2} * a-1) * ((- (b^2+c^2)^{1/2} * \sin \\ & (e*x+d-\arctan(-b,c)) - a) / (-a+(b^2+c^2)^{1/2}))^{1/2} * ((-\sin(e*x+d-\arctan(-b,c)) + 1) * (b^2+c^2)^{1/2} / (a+(b^2+c^2)^{1/2}))^{1/2} * ((1+\sin(e*x+d-\arctan(-b,c))) * (b^2+c^2)^{1/2} / (-a+(b^2+c^2)^{1/2}))^{1/2} / (-(-b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c)) - a) * \cos(e*x+d-\arctan(-b,c))^{1/2} * (b^2+c^2))^{1/2} * \text{EllipticF}(((- (b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c)) - a) / (-a+(b^2+c^2)^{1/2}))^{1/2}, ((a - (b^2+c^2)^{1/2}) / (a+(b^2+c^2)^{1/2}))^{1/2}) + 2 * (- (b^2+c^2)^{3/2} + 2 * (b^2+c^2)^{1/2} * b^2 + 2 * (b^2+c^2)^{1/2} * c^2) / (2 * a^2 - 2 * b^2 - 2 * c^2) * (1/(b^2+c^2)^{1/2} * a-1) * ((- (b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c)) - a) / (-a+(b^2+c^2)^{1/2}))^{1/2} * ((-\sin(e*x+d-\arctan(-b,c)) + 1) * (b^2+c^2)^{1/2} / (a+(b^2+c^2)^{1/2}))^{1/2} * ((1+\sin(e*x+d-\arctan(-b,c))) * (b^2+c^2)^{1/2} / (-a+(b^2+c^2)^{1/2}))^{1/2} / (-(-b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c)) - a) * \cos(e*x+d-\arctan(-b,c))^{1/2} * (b^2+c^2))^{1/2} * ((-1/(b^2+c^2)^{1/2} * a-1) * \text{EllipticE}(((- (b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c)) - a) / (-a+(b^2+c^2)^{1/2}))^{1/2}, ((a - (b^2+c^2)^{1/2}) / (a+(b^2+c^2)^{1/2}))^{1/2})) + \text{EllipticF}(((- (b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c)) - a) / (-a+(b^2+c^2)^{1/2}))^{1/2}, ((a - (b^2+c^2)^{1/2}) / (a+(b^2+c^2)^{1/2}))^{1/2}))^{1/2} + 1/2 * (b^2+c^2) * (1/(b^2+c^2)^{1/2} * a-1) * ((- (b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c)) - a) / (-a+(b^2+c^2)^{1/2}))^{1/2} * ((-\sin(e*x+d-\arctan(-b,c)) + 1) * (b^2+c^2)^{1/2} / (a+(b^2+c^2)^{1/2}))^{1/2} * ((1+\sin(e*x+d-\arctan(-b,c))) * (b^2+c^2)^{1/2} / (-a+(b^2+c^2)^{1/2}))^{1/2} / (-(-b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c)) - a) * \cos(e*x+d-\arctan(-b,c))^{1/2} * (b^2+c^2))^{1/2} / a * \text{EllipticPi}(((- (b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c)) - a) / (-a+(b^2+c^2)^{1/2}))^{1/2}, -1/2 * (-1/(b^2+c^2)^{1/2} * a+1) * (b^2+c^2)^{1/2} / a, ((a - (b^2+c^2)^{1/2}) / (a+(b^2+c^2)^{1/2}))^{1/2}) + (b^2+c^2) * \cos(e*x+d-\arctan(-b,c))^{1/2} / (a^2-b^2-c^2) / (-(-b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c)) - a) * \cos(e*x+d-\arctan(-b,c))^{1/2} + 1/(a^2-b^2-c^2) * (b^2+c^2)^{1/2} * a * (1/(b^2+c^2)^{1/2} * a-1) * ((- (b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c)) - a) / (-a+(b^2+c^2)^{1/2}))^{1/2} * ((-\sin(e*x+d-\arctan(-b,c)) + 1) * (b^2+c^2)^{1/2} / (a+(b^2+c^2)^{1/2}))^{1/2} * ((1+\sin(e*x+d-\arctan(-b,c))) * (b^2+c^2)^{1/2} / (-a+(b^2+c^2)^{1/2}))^{1/2} / (-(-b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c)) - a) * \cos(e*x+d-\arctan(-b,c))^{1/2} * (b^2+c^2))^{1/2} * \text{EllipticF}(((- (b^2+c^2)^{1/2} * \sin(e*x+d-\arctan(-b,c)) - a) / (-a+(b^2+c^2)^{1/2}))^{1/2}, ((a - (b^2+c^2)^{1/2}) / (a+(b^2+c^2)^{1/2}))^{1/2}))^{1/2} \end{aligned}$$

$$\begin{aligned} & \left(\frac{\sin(ex+d-\arctan(-b,c))-a}{-a+(b^2+c^2)^{1/2}} \right)^{1/2} \left(\frac{a-(b^2+c^2)^{1/2}}{a+(b^2+c^2)^{1/2}} \right)^{1/2} + \frac{1}{(a^2-b^2-c^2)^{1/2}} (b^2+c^2)^{1/2} \left(\frac{\sin(ex+d-\arctan(-b,c))-a}{-a+(b^2+c^2)^{1/2}} \right)^{1/2} \\ & \left(\frac{-\sin(ex+d-\arctan(-b,c))+1}{a+(b^2+c^2)^{1/2}} \right)^{1/2} \left(\frac{b^2+c^2}{-a+(b^2+c^2)^{1/2}} \right)^{1/2} \left(\frac{1+\sin(ex+d-\arctan(-b,c))}{-a+(b^2+c^2)^{1/2}} \right)^{1/2} \\ & \left(\frac{-\sin(ex+d-\arctan(-b,c))+1}{-a+(b^2+c^2)^{1/2}} \right)^{1/2} \left(\frac{b^2+c^2}{-a+(b^2+c^2)^{1/2}} \right)^{1/2} \cos^2(ex+d-\arctan(-b,c))^{1/2} \\ & \left(\frac{-1}{(b^2+c^2)^{1/2}} (a-1) \operatorname{EllipticE} \left(\frac{-\sin(ex+d-\arctan(-b,c))-a}{-a+(b^2+c^2)^{1/2}} \right)^{1/2}, \left(\frac{a-(b^2+c^2)^{1/2}}{a+(b^2+c^2)^{1/2}} \right)^{1/2} \right)^{1/2} \\ & + \operatorname{EllipticF} \left(\frac{-\sin(ex+d-\arctan(-b,c))-a}{-a+(b^2+c^2)^{1/2}} \right)^{1/2}, \left(\frac{a-(b^2+c^2)^{1/2}}{a+(b^2+c^2)^{1/2}} \right)^{1/2} \right)^{1/2} \\ & - \frac{1}{2} \frac{b^2+1/2c^2}{(b^2+c^2)^{1/2}} \left(\frac{1}{(b^2+c^2)^{1/2}} (a-1) \left(\frac{-\sin(ex+d-\arctan(-b,c))-a}{-a+(b^2+c^2)^{1/2}} \right)^{1/2} \right. \\ & \left. \left(\frac{-\sin(ex+d-\arctan(-b,c))+1}{a+(b^2+c^2)^{1/2}} \right)^{1/2} \left(\frac{1+\sin(ex+d-\arctan(-b,c))}{-a+(b^2+c^2)^{1/2}} \right)^{1/2} \right. \\ & \left. \left(\frac{-\sin(ex+d-\arctan(-b,c))+1}{-a+(b^2+c^2)^{1/2}} \right)^{1/2} \cos^2(ex+d-\arctan(-b,c))^{1/2} \right) / a \operatorname{EllipticPi} \left(\frac{-\sin(ex+d-\arctan(-b,c))-a}{-a+(b^2+c^2)^{1/2}} \right)^{1/2}, \\ & -\frac{1}{2} \frac{-1}{(b^2+c^2)^{1/2}} (a+1) \left(\frac{b^2+c^2}{a} \right)^{1/2} \left(\frac{a-(b^2+c^2)^{1/2}}{a+(b^2+c^2)^{1/2}} \right)^{1/2} \right) / \cos(ex+d-\arctan(-b,c)) / \left(\frac{b^2 \sin(ex+d-\arctan(-b,c))}{b^2+c^2} \right. \\ & \left. + \frac{c^2 \sin(ex+d-\arctan(-b,c))}{a+(b^2+c^2)^{1/2}} \right) / (b^2+c^2)^{1/2} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cos(ex+d) + c \sin(ex+d) + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(ex+d)+c*sin(ex+d))^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(ex+d) + c*sin(ex+d) + a)^(-3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(d + ex) + c*sin(d + ex))^(3/2),x)

[Out] int(1/(a + b*cos(d + ex) + c*sin(d + ex))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(ex+d)+c*sin(ex+d))**(3/2),x)

[Out] Integral((a + b*cos(d + ex) + c*sin(d + ex))**(-3/2), x)

$$3.415 \quad \int \frac{1}{(a+b \cos(d+ex)+c \sin(d+ex))^{5/2}} dx$$

Optimal. Leaf size=382

$$\frac{2\sqrt{\frac{a+b \cos(d+ex)+c \sin(d+ex)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{b}{c}\right)\right)\middle|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3e\left(a^2-b^2-c^2\right)\sqrt{a+b \cos(d+ex)+c \sin(d+ex)}} + \frac{8a\sqrt{a+b \cos(d+ex)+c \sin(d+ex)} E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{b}{c}\right)\right)\right)}{3e\left(a^2-b^2-c^2\right)^2\sqrt{\frac{a+b \cos(d+ex)+c \sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

[Out] $2/3*(c*\cos(e*x+d)-b*\sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*\cos(e*x+d)+c*\sin(e*x+d))^{3/2}+8/3*(a*c*\cos(e*x+d)-a*b*\sin(e*x+d))/(a^2-b^2-c^2)^2/e/(a+b*\cos(e*x+d)+c*\sin(e*x+d))^{1/2}+8/3*a*(\cos(1/2*d+1/2*e*x-1/2*\arctan(b,c))^2)^{1/2}/\cos(1/2*d+1/2*e*x-1/2*\arctan(b,c))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(b,c)),2^{1/2}*((b^2+c^2)^{1/2}/(a+(b^2+c^2)^{1/2}))^{1/2})*(a+b*\cos(e*x+d)+c*\sin(e*x+d))^{1/2}/(a^2-b^2-c^2)^2/e/((a+b*\cos(e*x+d)+c*\sin(e*x+d))/(a+(b^2+c^2)^{1/2}))^{1/2}-2/3*(\cos(1/2*d+1/2*e*x-1/2*\arctan(b,c))^2)^{1/2}/\cos(1/2*d+1/2*e*x-1/2*\arctan(b,c))*\text{EllipticF}(\sin(1/2*d+1/2*e*x-1/2*\arctan(b,c)),2^{1/2}*((b^2+c^2)^{1/2}/(a+(b^2+c^2)^{1/2}))^{1/2})*((a+b*\cos(e*x+d)+c*\sin(e*x+d))/(a+(b^2+c^2)^{1/2}))^{1/2}/(a^2-b^2-c^2)/e/(a+b*\cos(e*x+d)+c*\sin(e*x+d))^{1/2}$

Rubi [A] time = 0.36, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\sqrt{\frac{a+b \cos(d+ex)+c \sin(d+ex)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{b}{c}\right)\right)\middle|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3e\left(a^2-b^2-c^2\right)\sqrt{a+b \cos(d+ex)+c \sin(d+ex)}} + \frac{8a\sqrt{a+b \cos(d+ex)+c \sin(d+ex)} E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{b}{c}\right)\right)\right)}{3e\left(a^2-b^2-c^2\right)^2\sqrt{\frac{a+b \cos(d+ex)+c \sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2), x]

[Out] $(2*(c*\cos[d+e*x]-b*\sin[d+e*x]))/(3*(a^2-b^2-c^2)*e*(a+b*\cos[d+e*x]+c*\sin[d+e*x])^{3/2})+(8*(a*c*\cos[d+e*x]-a*b*\sin[d+e*x]))/(3*(a^2-b^2-c^2)^2*e*\sqrt{a+b*\cos[d+e*x]+c*\sin[d+e*x]})+(8*a*\text{EllipticE}[(d+e*x-\text{ArcTan}[b,c])/2,(2*\sqrt{b^2+c^2})/(a+\sqrt{b^2+c^2})])*\sqrt{a+b*\cos[d+e*x]+c*\sin[d+e*x]})/(3*(a^2-b^2-c^2)^2*e*\sqrt{(a+b*\cos[d+e*x]+c*\sin[d+e*x])/(a+\sqrt{b^2+c^2})})-(2*\text{EllipticF}[(d+e*x-\text{ArcTan}[b,c])/2,(2*\sqrt{b^2+c^2})/(a+\sqrt{b^2+c^2})])*\sqrt{(a+b*\cos[d+e*x]+c*\sin[d+e*x])/(a+\sqrt{b^2+c^2})})/(3*(a^2-b^2-c^2)*e*\sqrt{a+b*\cos[d+e*x]+c*\sin[d+e*x]})$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d/Sqrt[a + b], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +

```
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2]), Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c])]/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] := Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c])]/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]]^(n_), x_Symbol] := Simp[((-c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)])/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]]^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]), x_Symbol] := -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^{5/2}} dx &= \frac{2(c \cos(d + ex) - b \sin(d + ex))}{3(a^2 - b^2 - c^2)e(a + b \cos(d + ex) + c \sin(d + ex))^{3/2}} - \frac{2 \int \frac{3a}{(a+}} \\
&= \frac{2(c \cos(d + ex) - b \sin(d + ex))}{3(a^2 - b^2 - c^2)e(a + b \cos(d + ex) + c \sin(d + ex))^{3/2}} + \frac{1}{3(a^2 -}} \\
&= \frac{2(c \cos(d + ex) - b \sin(d + ex))}{3(a^2 - b^2 - c^2)e(a + b \cos(d + ex) + c \sin(d + ex))^{3/2}} + \frac{1}{3(a^2 -}} \\
&= \frac{2(c \cos(d + ex) - b \sin(d + ex))}{3(a^2 - b^2 - c^2)e(a + b \cos(d + ex) + c \sin(d + ex))^{3/2}} + \frac{1}{3(a^2 -}} \\
&= \frac{2(c \cos(d + ex) - b \sin(d + ex))}{3(a^2 - b^2 - c^2)e(a + b \cos(d + ex) + c \sin(d + ex))^{3/2}} + \frac{1}{3(a^2 -}}
\end{aligned}$$

Mathematica [C] time = 6.41, size = 2408, normalized size = 6.30

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]

[Out] (Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((8*a*(b^2 + c^2))/(3*b*c*(a^2 - b^2 - c^2)^2) + (2*(a*c + b^2*Sin[d + e*x] + c^2*Sin[d + e*x]))/(3*b*(-a^2 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (2*(3*a^2*c + b^2*c + c^3 + 4*a*b^2*Sin[d + e*x] + 4*a*c^2*Sin[d + e*x]))/(3*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[d + e*x] + c*Sin[d + e*x])))/e + (2*a^2*AppellF1[1/2, 1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c)))*c), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c)))*c)]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)^2*e) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c)))*c), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c)))*c)]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]/(3*Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)^2*e) + (2*c*AppellF1[1/2, 1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c)))*c), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c)))*c)]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]

```
*Sin[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 +
c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]/(3*S
qrt[1 + b^2/c^2]*(-a^2 + b^2 + c^2)^2*e) + (4*a*b^2*(-((c*AppellF1[-1/2, -1
/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sq
rt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -(a + b*Sqrt[1 + c^2/b^2]
*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b
^2])))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2
+ c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*S
qrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTa
n[c/b]])*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*
x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]) - ((2*b*(a + b*Sqrt[1 +
c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b^2 + c^2) - (c*Sin[d + e*x - ArcTa
n[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x -
ArcTan[c/b]])]/(3*c*(-a^2 + b^2 + c^2)^2*e) + (4*a*c*(-((c*AppellF1[-1/2,
-1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*
Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -(a + b*Sqrt[1 + c^2/b^
2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2
/b^2])))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b
^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b
*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - Arc
Tan[c/b]])*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d +
e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]) - ((2*b*(a + b*Sqrt[1
+ c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b^2 + c^2) - (c*Sin[d + e*x - Arc
Tan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x
- ArcTan[c/b]])]/(3*(-a^2 + b^2 + c^2)^2*e)
```

fricas [F] time = 2.96, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{b \cos(ex + d) + c \sin(ex + d) + a}}{(b^3 - 3bc^2) \cos(ex + d)^3 + a^3 + 3ac^2 + 3(ab^2 - ac^2) \cos(ex + d)^2 + 3(a^2b + bc^2) \cos(ex + d) + (6abc \cos(ex + d)^2 \sin(ex + d))} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a)/((b^3 - 3*b*c^2)*cos(e*x
+ d)^3 + a^3 + 3*a*c^2 + 3*(a*b^2 - a*c^2)*cos(e*x + d)^2 + 3*(a^2*b + b*c
^2)*cos(e*x + d) + (6*a*b*c*cos(e*x + d) + 3*a^2*c + c^3 + (3*b^2*c - c^3)*
cos(e*x + d)^2)*sin(e*x + d)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cos(ex + d) + c \sin(ex + d) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-5/2), x)
```

maple [B] time = 2.14, size = 2967, normalized size = 7.77

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x)
```

```
[Out] (-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*(-1/4/a/(a^2-b^2-c^2)*
```

$$\begin{aligned}
& (b^2+c^2)^{(1/2)} * (\cos(e*x+d-\arctan(-b,c)))^2 * (b^2+c^2) * ((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))+a)^{(1/2)} / (b^2 * \sin(e*x+d-\arctan(-b,c))+c^2 * \sin(e*x+d-\arctan(-b,c))-a * (b^2+c^2)^{(1/2)}) + 1/3 / (a^2-b^2-c^2) / (b^2+c^2) * (\cos(e*x+d-\arctan(-b,c)))^2 * (b^2+c^2) * ((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))+a)^{(1/2)} / (\sin(e*x+d-\arctan(-b,c))+1/(b^2+c^2)^{(1/2)} * a)^2 - 4/3 * (-b^2-c^2) * \cos(e*x+d-\arctan(-b,c))^2 / (a^2-b^2-c^2)^2 * a / (\cos(e*x+d-\arctan(-b,c)))^2 * (b^2+c^2) * ((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))+a)^{(1/2)} + 2 * (-1/24 * (b^2+c^2)^{(1/2)} / (a^2-b^2-c^2) + 2/3 * a^2 * (b^2+c^2)^{(1/2)} / (a^2-b^2-c^2)^2) * (1/(b^2+c^2)^{(1/2)} * a - 1) * ((-b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))-a) / (-a+(b^2+c^2)^{(1/2)})^2 * ((-\sin(e*x+d-\arctan(-b,c))+1) * (b^2+c^2)^{(1/2)} / (a+(b^2+c^2)^{(1/2)}))^2 * ((1+\sin(e*x+d-\arctan(-b,c)))) * (b^2+c^2)^{(1/2)} / (-a+(b^2+c^2)^{(1/2)})^2 * ((\cos(e*x+d-\arctan(-b,c)))^2 * (b^2+c^2) * ((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))+a))^2)^{(1/2)} * \text{EllipticF}(((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))-a) / (-a+(b^2+c^2)^{(1/2)}))^2, ((a-(b^2+c^2)^{(1/2)}) / (a+(b^2+c^2)^{(1/2)}))^2 + 2 * (13 * a^2 * b^2 + 13 * a^2 * c^2 + 3 * b^4 + 6 * b^2 * c^2 + 3 * c^4) / (24 * a^5 - 48 * a^3 * b^2 - 48 * a^3 * c^2 + 24 * a * b^4 + 48 * a * b^2 * c^2 + 24 * a * c^4) * (1/(b^2+c^2)^{(1/2)} * a - 1) * ((-b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))-a) / (-a+(b^2+c^2)^{(1/2)})^2 * ((-\sin(e*x+d-\arctan(-b,c))+1) * (b^2+c^2)^{(1/2)} / (a+(b^2+c^2)^{(1/2)}))^2 * ((1+\sin(e*x+d-\arctan(-b,c)))) * (b^2+c^2)^{(1/2)} / (-a+(b^2+c^2)^{(1/2)})^2 * ((\cos(e*x+d-\arctan(-b,c)))^2 * (b^2+c^2) * ((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))+a))^2)^{(1/2)} * (-1/(b^2+c^2)^{(1/2)} * a - 1) * \text{EllipticE}(((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))-a) / (-a+(b^2+c^2)^{(1/2)}))^2, ((a-(b^2+c^2)^{(1/2)}) / (a+(b^2+c^2)^{(1/2)}))^2 + \text{EllipticF}(((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))-a) / (-a+(b^2+c^2)^{(1/2)}))^2, ((a-(b^2+c^2)^{(1/2)}) / (a+(b^2+c^2)^{(1/2)}))^2 - 1/8 * (5 * a^2 * b^2 + 5 * a^2 * c^2 - b^4 - 2 * b^2 * c^2 - c^4) / a^2 / (a^2-b^2-c^2) / (b^2+c^2)^{(1/2)} * (1/(b^2+c^2)^{(1/2)} * a - 1) * ((-b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))-a) / (-a+(b^2+c^2)^{(1/2)})^2 * ((-\sin(e*x+d-\arctan(-b,c))+1) * (b^2+c^2)^{(1/2)} / (a+(b^2+c^2)^{(1/2)}))^2 * ((1+\sin(e*x+d-\arctan(-b,c)))) * (b^2+c^2)^{(1/2)} / (-a+(b^2+c^2)^{(1/2)})^2 * ((\cos(e*x+d-\arctan(-b,c)))^2 * (b^2+c^2) * ((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))+a))^2)^{(1/2)} * \text{EllipticPi}(((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))-a) / (-a+(b^2+c^2)^{(1/2)}))^2, -1/2 * (-1/(b^2+c^2)^{(1/2)} * a + 1) * (b^2+c^2)^{(1/2)} / a, ((a-(b^2+c^2)^{(1/2)}) / (a+(b^2+c^2)^{(1/2)}))^2 + 1/4 * (b^2+c^2) / a / (a^2-b^2-c^2) * (\cos(e*x+d-\arctan(-b,c)))^2 * (b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))+a)^{(1/2)} / (b^2 * \sin(e*x+d-\arctan(-b,c))+c^2 * \sin(e*x+d-\arctan(-b,c))-a * (b^2+c^2)^{(1/2)}) + 1/3 / (a^2-b^2-c^2) / (b^2+c^2)^{(1/2)} * (\cos(e*x+d-\arctan(-b,c)))^2 * ((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))+a)^{(1/2)} / (\sin(e*x+d-\arctan(-b,c))+1/(b^2+c^2)^{(1/2)} * a)^2 + 4/3 * (b^2+c^2)^{(1/2)} * \cos(e*x+d-\arctan(-b,c))^2 / (a^2-b^2-c^2)^2 * a / (\cos(e*x+d-\arctan(-b,c)))^2 * ((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))+a)^{(1/2)} + 2 * (-7/24 / (a^2-b^2-c^2) + 2/3 * a^2 / (a^2-b^2-c^2)^2) * (1/(b^2+c^2)^{(1/2)} * a - 1) * ((-b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))-a) / (-a+(b^2+c^2)^{(1/2)})^2 * ((-\sin(e*x+d-\arctan(-b,c))+1) * (b^2+c^2)^{(1/2)} / (a+(b^2+c^2)^{(1/2)}))^2 * ((1+\sin(e*x+d-\arctan(-b,c)))) * (b^2+c^2)^{(1/2)} / (-a+(b^2+c^2)^{(1/2)})^2 * ((\cos(e*x+d-\arctan(-b,c)))^2 * (b^2+c^2) * ((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))+a))^2)^{(1/2)} * \text{EllipticF}(((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))-a) / (-a+(b^2+c^2)^{(1/2)}))^2, ((a-(b^2+c^2)^{(1/2)}) / (a+(b^2+c^2)^{(1/2)}))^2 + 2 * (1/8 * a / (a^2-b^2-c^2) * (b^2+c^2)^{(1/2)} + 2/3 * a * (b^2+c^2)^{(1/2)} / (a^2-b^2-c^2)^2) * (1/(b^2+c^2)^{(1/2)} * a - 1) * ((-b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))-a) / (-a+(b^2+c^2)^{(1/2)})^2 * ((-\sin(e*x+d-\arctan(-b,c))+1) * (b^2+c^2)^{(1/2)} / (a+(b^2+c^2)^{(1/2)}))^2 * ((1+\sin(e*x+d-\arctan(-b,c)))) * (b^2+c^2)^{(1/2)} / (-a+(b^2+c^2)^{(1/2)})^2 * ((\cos(e*x+d-\arctan(-b,c)))^2 * (b^2+c^2) * ((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))+a))^2)^{(1/2)} * (-1/(b^2+c^2)^{(1/2)} * a - 1) * \text{EllipticE}(((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))-a) / (-a+(b^2+c^2)^{(1/2)}))^2, ((a-(b^2+c^2)^{(1/2)}) / (a+(b^2+c^2)^{(1/2)}))^2 + \text{EllipticF}(((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))-a) / (-a+(b^2+c^2)^{(1/2)}))^2, ((a-(b^2+c^2)^{(1/2)}) / (a+(b^2+c^2)^{(1/2)}))^2 + 1/8 * (5 * a^2 - b^2 - c^2) / a^2 / (a^2-b^2-c^2) * (1/(b^2+c^2)^{(1/2)} * a - 1) * ((-b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))-a) / (-a+(b^2+c^2)^{(1/2)})^2 * ((-\sin(e*x+d-\arctan(-b,c))+1) * (b^2+c^2)^{(1/2)} / (a+(b^2+c^2)^{(1/2)}))^2 * ((1+\sin(e*x+d-\arctan(-b,c)))) * (b^2+c^2)^{(1/2)} / (-a+(b^2+c^2)^{(1/2)})^2 * ((\cos(e*x+d-\arctan(-b,c)))^2 * (b^2+c^2) * ((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))+a))^2)^{(1/2)} * \text{EllipticPi}(((b^2+c^2)^{(1/2)} * \sin(e*x+d-\arctan(-b,c))-a) / (-a+(b^2+c^2)^{(1/2)}))^2
\end{aligned}$$

$\sqrt{2} \sin(e^x + d - \arctan(-b, c)) - a) / (-a + \sqrt{b^2 + c^2})^{1/2}, -1/2 * (-1 / (\sqrt{b^2 + c^2} * a + 1) * \sqrt{b^2 + c^2} / a, ((a - \sqrt{b^2 + c^2}) / (a + \sqrt{b^2 + c^2}))^{1/2} / \cos(e^x + d - \arctan(-b, c)) / ((b^2 \sin(e^x + d - \arctan(-b, c)) + c^2 \sin(e^x + d - \arctan(-b, c)) + a * \sqrt{b^2 + c^2}) / \sqrt{b^2 + c^2})^{1/2} / e$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cos(ex + d) + c \sin(ex + d) + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="maxima")

[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(5/2),x)

[Out] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(5/2),x)

[Out] Timed out

3.416 $\int \frac{1}{(a+b \cos(d+ex)+c \sin(d+ex))^{7/2}} dx$

Optimal. Leaf size=490

$$\frac{16a \sqrt{\frac{a+b \cos(d+ex)+c \sin(d+ex)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2} \left(d+ex - \tan^{-1}\left(\frac{b}{c}\right)\right) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right) \sqrt{2(23a^2+9(b^2+c^2))} \sqrt{a+b \cos(d+ex)}}{15e(a^2-b^2-c^2)^2 \sqrt{a+b \cos(d+ex)+c \sin(d+ex)}} + \frac{2(23a^2+9(b^2+c^2)) \sqrt{a+b \cos(d+ex)}}{15e(a^2-b^2-c^2)}$$

```
[Out] 2/5*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2)+16/15*(a*c*cos(e*x+d)-a*b*sin(e*x+d))/(a^2-b^2-c^2)^2/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2)+2/15*(c*(23*a^2+9*b^2+9*c^2)*cos(e*x+d)-b*(23*a^2+9*b^2+9*c^2)*sin(e*x+d))/(a^2-b^2-c^2)^3/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)+2/15*(23*a^2+9*b^2+9*c^2)*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2))))^(1/2)*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/(a^2-b^2-c^2)^3/e/((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)-16/15*a*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(b,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2))))^(1/2)*((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a^2-b^2-c^2)^2/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{16a \sqrt{\frac{a+b \cos(d+ex)+c \sin(d+ex)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2} \left(d+ex - \tan^{-1}\left(\frac{b}{c}\right)\right) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right) \sqrt{2(23a^2+9(b^2+c^2))} \sqrt{a+b \cos(d+ex)}}{15e(a^2-b^2-c^2)^2 \sqrt{a+b \cos(d+ex)+c \sin(d+ex)}} + \frac{2(23a^2+9(b^2+c^2)) \sqrt{a+b \cos(d+ex)}}{15e(a^2-b^2-c^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-7/2), x]
```

```
[Out] (2*(c*cos[d + e*x] - b*sin[d + e*x]))/(5*(a^2 - b^2 - c^2)*e*(a + b*cos[d + e*x] + c*sin[d + e*x])^(5/2)) + (16*(a*c*cos[d + e*x] - a*b*sin[d + e*x]))/(15*(a^2 - b^2 - c^2)^2*e*(a + b*cos[d + e*x] + c*sin[d + e*x])^(3/2)) + (2*(23*a^2 + 9*(b^2 + c^2))*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]])/(15*(a^2 - b^2 - c^2)^3*e*Sqrt[(a + b*cos[d + e*x] + c*sin[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (16*a*EllipticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*cos[d + e*x] + c*sin[d + e*x])/(a + Sqrt[b^2 + c^2])])/(15*(a^2 - b^2 - c^2)^2*e*Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]]) + (2*(c*(23*a^2 + 9*(b^2 + c^2))*cos[d + e*x] - b*(23*a^2 + 9*(b^2 + c^2))*sin[d + e*x]))/(15*(a^2 - b^2 - c^2)^3*e*Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b])), x] /; FreeQ[
```

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3127

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3129

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^n, x_Symbol] :> Simp[(-c*Cos[d + e*x] + b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]

Rule 3149

Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]) / Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]

Rule 3156

Int[((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^n * ((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[n, -2]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^{7/2}} dx &= \frac{2(c \cos(d + ex) - b \sin(d + ex))}{5(a^2 - b^2 - c^2)e(a + b \cos(d + ex) + c \sin(d + ex))^{5/2}} - \frac{2 \int \frac{5a}{(a + b \cos(d + ex) + c \sin(d + ex))^{5/2}} dx}{15(a^2 - b^2 - c^2)} \\
&= \frac{2(c \cos(d + ex) - b \sin(d + ex))}{5(a^2 - b^2 - c^2)e(a + b \cos(d + ex) + c \sin(d + ex))^{5/2}} + \frac{2 \int \frac{5a}{(a + b \cos(d + ex) + c \sin(d + ex))^{5/2}} dx}{15(a^2 - b^2 - c^2)} \\
&= \frac{2(c \cos(d + ex) - b \sin(d + ex))}{5(a^2 - b^2 - c^2)e(a + b \cos(d + ex) + c \sin(d + ex))^{5/2}} + \frac{2 \int \frac{5a}{(a + b \cos(d + ex) + c \sin(d + ex))^{5/2}} dx}{15(a^2 - b^2 - c^2)} \\
&= \frac{2(c \cos(d + ex) - b \sin(d + ex))}{5(a^2 - b^2 - c^2)e(a + b \cos(d + ex) + c \sin(d + ex))^{5/2}} + \frac{2 \int \frac{5a}{(a + b \cos(d + ex) + c \sin(d + ex))^{5/2}} dx}{15(a^2 - b^2 - c^2)} \\
&= \frac{2(c \cos(d + ex) - b \sin(d + ex))}{5(a^2 - b^2 - c^2)e(a + b \cos(d + ex) + c \sin(d + ex))^{5/2}} + \frac{2 \int \frac{5a}{(a + b \cos(d + ex) + c \sin(d + ex))^{5/2}} dx}{15(a^2 - b^2 - c^2)} \\
&= \frac{2(c \cos(d + ex) - b \sin(d + ex))}{5(a^2 - b^2 - c^2)e(a + b \cos(d + ex) + c \sin(d + ex))^{5/2}} + \frac{2 \int \frac{5a}{(a + b \cos(d + ex) + c \sin(d + ex))^{5/2}} dx}{15(a^2 - b^2 - c^2)}
\end{aligned}$$

Mathematica [C] time = 6.66, size = 4116, normalized size = 8.40

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-7/2),x]

[Out] (Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((-2*(b^2 + c^2)*(23*a^2 + 9*b^2 + 9*c^2))/(15*b*c*(-a^2 + b^2 + c^2)^3) + (2*(a*c + b^2*Sin[d + e*x] + c^2*Sin[d + e*x]))/(5*b*(-a^2 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3) - (2*(5*a^2*c + 3*b^2*c + 3*c^3 + 8*a*b^2*Sin[d + e*x] + 8*a*c^2*Sin[d + e*x]))/(15*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) + (2*(15*a^3*c + 17*a*b^2*c + 17*a*c^3 + 23*a^2*b^2*Sin[d + e*x] + 9*b^4*Sin[d + e*x] + 23*a^2*c^2*Sin[d + e*x] + 18*b^2*c^2*Sin[d + e*x] + 9*c^4*Sin[d + e*x]))/(15*b*(-a^2 + b^2 + c^2)^3*(a + b*Cos[d + e*x] + c*Sin[d + e*x])))/e - (2*a^3*AppellF1[1/2, 1/2, 1/2, 3/2, -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)], -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c)]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)^3*e) - (34*a*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)], -(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c)]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2]

$$\frac{\sqrt{\frac{b^2 + c^2}{b^2}} \cos\left[d + e x - \arctan\left(\frac{c}{b}\right)\right]}{\left(a + b \sqrt{\frac{b^2 + c^2}{b^2}}\right) \sqrt{\left(a + b \sqrt{\frac{b^2 + c^2}{b^2}}\right) \cos\left[d + e x - \arctan\left(\frac{c}{b}\right)\right] \sqrt{\left(b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos\left[d + e x - \arctan\left(\frac{c}{b}\right)\right]\right) - \left(\left(2 b (a + b \sqrt{1 + \frac{c^2}{b^2}}) \cos\left[d + e x - \arctan\left(\frac{c}{b}\right)\right]\right) / (b^2 + c^2) - (c \sin\left[d + e x - \arctan\left(\frac{c}{b}\right)\right]) / (b \sqrt{1 + \frac{c^2}{b^2}})\right) / \sqrt{\left(a + b \sqrt{1 + \frac{c^2}{b^2}}\right) \cos\left[d + e x - \arctan\left(\frac{c}{b}\right)\right]}} \right) / (5 * (-a^2 + b^2 + c^2)^3 * e)$$

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(b^4 - 6b^2c^2 + c^4) \cos(ex + d)^4 + a^4 + 6a^2c^2 + c^4 + 4(ab^3 - 3abc^2) \cos(ex + d)^3 + 2(3a^2b^2 - c^4 - 2a^2c^2 + c^4) \cos(ex + d)^2 + 4(a^3b + 3a^2bc^2) \cos(ex + d) + 4(a^3c + ac^3 + (b^3c - bc^3) \cos(ex + d)^3 + (3a^2b^2c - ac^3) \cos(ex + d)^2 + (3a^2bc + bc^3) \cos(ex + d)) \sin(ex + d)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a)/((b^4 - 6*b^2*c^2 + c^4) *cos(e*x + d)^4 + a^4 + 6*a^2*c^2 + c^4 + 4*(a*b^3 - 3*a*b*c^2)*cos(e*x + d)^3 + 2*(3*a^2*b^2 - c^4 - 3*(a^2 - b^2)*c^2)*cos(e*x + d)^2 + 4*(a^3*b + 3*a*b*c^2)*cos(e*x + d) + 4*(a^3*c + a*c^3 + (b^3*c - b*c^3)*cos(e*x + d)^3 + (3*a*b^2*c - a*c^3)*cos(e*x + d)^2 + (3*a^2*b*c + b*c^3)*cos(e*x + d))*sin(e*x + d)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cos(ex + d) + c \sin(ex + d) + a)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x, algorithm="giac")

[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-7/2), x)

maple [B] time = 6.69, size = 3876, normalized size = 7.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x)

[Out]
$$\begin{aligned} & \left(-(-b^2 \sin(e x + d - \arctan(-b, c)) - c^2 \sin(e x + d - \arctan(-b, c))) - a(b^2 + c^2)^{\frac{1}{2}} \right) \cos(e x + d - \arctan(-b, c))^{\frac{1}{2}} / (b^2 + c^2)^{\frac{1}{2}} \left(\frac{1}{2} \right) / (b^2 + c^2)^{\frac{1}{2}} \left(\frac{1}{2} \right) * (-1/8 / a / (a^2 - b^2 - c^2) * (b^2 + c^2)^{\frac{3}{2}} * (\cos(e x + d - \arctan(-b, c))^2 * (b^2 + c^2) * ((b^2 + c^2)^{\frac{1}{2}} * \sin(e x + d - \arctan(-b, c)) + a))^{\frac{1}{2}} / (b^2 * \sin(e x + d - \arctan(-b, c)) + c^2 * \sin(e x + d - \arctan(-b, c))) - a * (b^2 + c^2)^{\frac{1}{2}} \right)^2 + 1/5 / (a^2 - b^2 - c^2) / (b^2 + c^2) * (\cos(e x + d - \arctan(-b, c))^2 * (b^2 + c^2) * ((b^2 + c^2)^{\frac{1}{2}} * \sin(e x + d - \arctan(-b, c)) + a))^{\frac{1}{2}} / (\sin(e x + d - \arctan(-b, c)) + 1 / (b^2 + c^2)^{\frac{1}{2}} * a)^3 + 3/32 * (5 * a^2 * b^2 + 5 * a^2 * c^2 - b^4 - 2 * b^2 * c^2 - c^4) / (a^2 - b^2 - c^2)^2 / a^2 * (\cos(e x + d - \arctan(-b, c)))^2 * (b^2 + c^2) * ((b^2 + c^2)^{\frac{1}{2}} * \sin(e x + d - \arctan(-b, c)) + a))^{\frac{1}{2}} / (b^2 * \sin(e x + d - \arctan(-b, c)) + c^2 * \sin(e x + d - \arctan(-b, c))) - a * (b^2 + c^2)^{\frac{1}{2}} \right) + 8/15 / (a^2 - b^2 - c^2)^2 * a / (b^2 + c^2)^{\frac{1}{2}} * (\cos(e x + d - \arctan(-b, c)))^2 * (b^2 + c^2) * ((b^2 + c^2)^{\frac{1}{2}} * \sin(e x + d - \arctan(-b, c)) + a))^{\frac{1}{2}} / (\sin(e x + d - \arctan(-b, c)) + 1 / (b^2 + c^2)^{\frac{1}{2}} * a)^2 - 1/15 * (b^2 + c^2)^{\frac{1}{2}} * (-b^2 - c^2) * \cos(e x + d - \arctan(-b, c))^2 / (a^2 - b^2 - c^2)^3 * (23 * a^2 + 9 * b^2 + 9 * c^2) / (\cos(e x + d - \arctan(-b, c)))^2 * (b^2 + c^2) * ((b^2 + c^2)^{\frac{1}{2}} * \sin(e x + d - \arctan(-b, c)) + a))^{\frac{1}{2}} + 2 * (-1/64 * (11 * a^2 * b^2 + 11 * a^2 * c^2 + b^4 + 2 * b^2 * c^2 + c^4) / a / (a^2 - b^2 - c^2)^2 - 4/15 * a * (b^2 + c^2) / (a^2 - b^2 - c^2)^2 + 1/30 * a * (b^2 + c^2) * (23 * a^2 + 9 * b^2 + 9 * c^2) / (a^2 - b^2 - c^2)^3) * (1 / (b^2 + c^2)^{\frac{1}{2}} * a - 1) * ((- (b^2 + c^2)^{\frac{1}{2}} * \sin(e x + d - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{\frac{1}{2}}))^{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned}
& /2)*((-\sin(e*x+d-\arctan(-b,c))+1)*(b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)}))^{(1/2)} \\
&)*((1+\sin(e*x+d-\arctan(-b,c)))*(b^2+c^2)^{(1/2)}/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)}/ \\
& (\cos(e*x+d-\arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c) \\
&))+a))^{(1/2)}*EllipticF(((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))-a)/(-a+(b \\
& ^2+c^2)^{(1/2)}))^{(1/2)},((a-(b^2+c^2)^{(1/2)})/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}+2*(3 \\
& /64*(5*a^2*b^2+5*a^2*c^2-b^4-2*b^2*c^2-c^4)*(b^2+c^2)^{(1/2)}/(a^2-b^2-c^2)^2 \\
& /a^2-1/30*(b^2+c^2)^{(3/2)}*(23*a^2+9*b^2+9*c^2)/(a^2-b^2-c^2)^3+1/30*(b^2+c^ \\
& 2)^{(1/2)}*(2*b^2+2*c^2)/(a^2-b^2-c^2)^3*(23*a^2+9*b^2+9*c^2))*(1/(b^2+c^2)^{(\\
& 1/2)}*a-1)*((-b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2) \\
&))^{(1/2)}*((-\sin(e*x+d-\arctan(-b,c))+1)*(b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2) \\
&))^{(1/2)}*((1+\sin(e*x+d-\arctan(-b,c)))*(b^2+c^2)^{(1/2)}/(-a+(b^2+c^2)^{(1/2) \\
&))^{(1/2)}/(\cos(e*x+d-\arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan \\
& (-b,c))+a))^{(1/2)}*((-1/(b^2+c^2)^{(1/2)}*a-1)*EllipticE(((b^2+c^2)^{(1/2)}*si \\
& n(e*x+d-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)},((a-(b^2+c^2)^{(1/2)})/(\\
& a+(b^2+c^2)^{(1/2)}))^{(1/2)}+EllipticF(((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b \\
& ,c))-a)/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)},((a-(b^2+c^2)^{(1/2)})/(a+(b^2+c^2)^{(1/2) \\
&))^{(1/2)}))^{(1/2)}+1/64*(43*a^4*b^2+43*a^4*c^2+2*a^2*b^4+4*a^2*b^2*c^2+2*a^2*c^4+3* \\
& b^6+9*b^4*c^2+9*b^2*c^4+3*c^6)/(a^2-b^2-c^2)^2/a^3*(1/(b^2+c^2)^{(1/2)}*a-1)* \\
& ((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)}*(\\
& (-\sin(e*x+d-\arctan(-b,c))+1)*(b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}*((1 \\
& +\sin(e*x+d-\arctan(-b,c)))*(b^2+c^2)^{(1/2)}/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)}/(\cos(\\
& e*x+d-\arctan(-b,c))^2*(b^2+c^2)*((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))+a) \\
&)^{(1/2)}*EllipticPi(((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))-a)/(-a+(b^2+c \\
& ^2)^{(1/2)}))^{(1/2)},-1/2*(-1/(b^2+c^2)^{(1/2)}*a+1)*(b^2+c^2)^{(1/2)}/a,((a-(b^2+ \\
& c^2)^{(1/2)})/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}+1/8/a*(b^4+2*b^2*c^2+c^4)/(a^2-b^2- \\
& c^2)*(cos(e*x+d-\arctan(-b,c))^2*((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))+a) \\
&)^{(1/2)}/(b^2*\sin(e*x+d-\arctan(-b,c))+c^2*\sin(e*x+d-\arctan(-b,c))-a*(b^2+c^2 \\
&)^{(1/2)})^2+1/5/(a^2-b^2-c^2)/(b^2+c^2)^{(1/2)}*(cos(e*x+d-\arctan(-b,c))^2*((b \\
& ^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))+a))^{(1/2)}/(\sin(e*x+d-\arctan(-b,c))+1/ \\
& (b^2+c^2)^{(1/2)}*a)^3-3/32*(5*a^2*b^2+5*a^2*c^2-b^4-2*b^2*c^2-c^4)*(b^2+c^2) \\
& ^{(1/2)}/(a^2-b^2-c^2)^2/a^2*(cos(e*x+d-\arctan(-b,c))^2*((b^2+c^2)^{(1/2)}*\sin(\\
& e*x+d-\arctan(-b,c))+a))^{(1/2)}/(b^2*\sin(e*x+d-\arctan(-b,c))+c^2*\sin(e*x+d-\ar \\
& ctan(-b,c))-a*(b^2+c^2)^{(1/2)}+8/15/(a^2-b^2-c^2)^2*a*(cos(e*x+d-\arctan(-b, \\
& c))^2*((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))+a))^{(1/2)}/(\sin(e*x+d-\arctan(\\
& -b,c))+1/(b^2+c^2)^{(1/2)}*a)^2+1/15*(b^2+c^2)*cos(e*x+d-\arctan(-b,c))^2/(a^2 \\
& -b^2-c^2)^3*(23*a^2+9*b^2+9*c^2)/(\cos(e*x+d-\arctan(-b,c))^2*((b^2+c^2)^{(1/2) \\
&)*\sin(e*x+d-\arctan(-b,c))+a))^{(1/2)}+2*(1/64*(11*a^2+b^2+c^2)*(b^2+c^2)^{(1/2) \\
&)/a/(a^2-b^2-c^2)^2-4/15*a*(b^2+c^2)^{(1/2)}/(a^2-b^2-c^2)^2+1/30*a*(b^2+c^2) \\
& ^{(1/2)}*(23*a^2+9*b^2+9*c^2)/(a^2-b^2-c^2)^3*(1/(b^2+c^2)^{(1/2)}*a-1)*((-b^2+c^2)^{(1/2)}*\sin(e*x+d- \\
& arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)}*((-\sin(e*x+d-\arctan(-b,c))+1)* \\
& (b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}*((1+\sin(e*x+d-\arctan(-b,c)))*(b^ \\
& 2+c^2)^{(1/2)}/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)}/(\cos(e*x+d-\arctan(-b,c))^2*((b^2+c \\
& ^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))+a))^{(1/2)}*((-1/(b^2+c^2)^{(1/2)}*a-1)*Ellip \\
& ticE(((b^2+c^2)^{(1/2)}*\sin(e*x+d-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2)}))^{(1 \\
& /2)},((a-(b^2+c^2)^{(1/2)})/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}+EllipticF(((b^2+c^2) \\
& ^{(1/2)}*\sin(e*x+d-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)},((a-(b^2+c^2) \\
& ^{(1/2)})/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}-1/64*(43*a^4*b^2+43*a^4*c^2+2*a^2*b^4+ \\
& 4*a^2*b^2*c^2+2*a^2*c^4+3*b^6+9*b^4*c^2+9*b^2*c^4+3*c^6)/(a^2-b^2-c^2)^2/a^ \\
& 3/(b^2+c^2)^{(1/2)}*(1/(b^2+c^2)^{(1/2)}*a-1)*((-b^2+c^2)^{(1/2)}*\sin(e*x+d-\arct \\
& an(-b,c))-a)/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)}*((-\sin(e*x+d-\arctan(-b,c))+1)*(b^2 \\
& +c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}*((1+\sin(e*x+d-\arctan(-b,c)))*(b^2+c^
\end{aligned}$$

$$2)^{(1/2)/(-a+(b^2+c^2)^{(1/2))}^{(1/2)/(\cos(e*x+d-\arctan(-b,c))^{2*(b^2+c^2)^{(1/2)*\sin(e*x+d-\arctan(-b,c))+a)}^{(1/2)*\text{EllipticPi}(((b^2+c^2)^{(1/2)*\sin(e*x+d-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2))}^{(1/2)},-1/2*(-1/(b^2+c^2)^{(1/2)*a+1)*(b^2+c^2)^{(1/2)/a},((a-(b^2+c^2)^{(1/2)))/(a+(b^2+c^2)^{(1/2))}^{(1/2)))/\cos(e*x+d-\arctan(-b,c)))/((b^2*\sin(e*x+d-\arctan(-b,c))+c^2*\sin(e*x+d-\arctan(-b,c))+a*(b^2+c^2)^{(1/2)))/(b^2+c^2)^{(1/2))}^{(1/2)/e}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cos(ex + d) + c \sin(ex + d) + a)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x, algorithm="maxima")

[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + b \cos(d + ex) + c \sin(d + ex))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(7/2),x)

[Out] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(7/2),x)

[Out] Timed out

3.417 $\int (5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2} dx$

Optimal. Leaf size=139

$$\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))(3 \sin(d + ex) + 4 \cos(d + ex) + 5)^{3/2}}{5e} - \frac{16(3 \cos(d + ex) - 4 \sin(d + ex))\sqrt{3 \sin(d + ex) + 4 \cos(d + ex)}}{3e}$$

[Out] $-2/5*(3*\cos(e*x+d)-4*\sin(e*x+d))*(5+4*\cos(e*x+d)+3*\sin(e*x+d))^{(3/2)}/e-320/3*(3*\cos(e*x+d)-4*\sin(e*x+d))/e/(5+4*\cos(e*x+d)+3*\sin(e*x+d))^{(1/2)}-16/3*(3*\cos(e*x+d)-4*\sin(e*x+d))*(5+4*\cos(e*x+d)+3*\sin(e*x+d))^{(1/2)}/e$

Rubi [A] time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3113, 3112}

$$\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))(3 \sin(d + ex) + 4 \cos(d + ex) + 5)^{3/2}}{5e} - \frac{16(3 \cos(d + ex) - 4 \sin(d + ex))\sqrt{3 \sin(d + ex) + 4 \cos(d + ex)}}{3e}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(5 + 4*\text{Cos}[d + e*x] + 3*\text{Sin}[d + e*x])^{(5/2)}, x]$

[Out] $(-320*(3*\text{Cos}[d + e*x] - 4*\text{Sin}[d + e*x]))/(3*e*\text{Sqrt}[5 + 4*\text{Cos}[d + e*x] + 3*\text{Sin}[d + e*x]]) - (16*(3*\text{Cos}[d + e*x] - 4*\text{Sin}[d + e*x])*\text{Sqrt}[5 + 4*\text{Cos}[d + e*x] + 3*\text{Sin}[d + e*x]])/(3*e) - (2*(3*\text{Cos}[d + e*x] - 4*\text{Sin}[d + e*x])*(5 + 4*\text{Cos}[d + e*x] + 3*\text{Sin}[d + e*x])^{(3/2)})/(5*e)$

Rule 3112

$\text{Int}[\text{Sqrt}[\cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*\sin[(d_.) + (e_.)*(x_.)]]], x_Symbol] :> \text{Simp}[(-2*(c*\text{Cos}[d + e*x] - b*\text{Sin}[d + e*x]))/(e*\text{Sqrt}[a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]]), x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[a^2 - b^2 - c^2, 0]$

Rule 3113

$\text{Int}[(\cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*\sin[(d_.) + (e_.)*(x_.)])^n], x_Symbol] :> -\text{Simp}[((c*\text{Cos}[d + e*x] - b*\text{Sin}[d + e*x])*(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{(n-1)})/(e*n), x] + \text{Dist}[(a*(2*n-1))/n, \text{Int}[(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^{(n-1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[a^2 - b^2 - c^2, 0] \&\& \text{GtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int (5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2} dx &= -\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2}}{5e} \\ &= -\frac{16(3 \cos(d + ex) - 4 \sin(d + ex))\sqrt{5 + 4 \cos(d + ex) + 3 \sin(d + ex)}}{3e} \\ &= -\frac{320(3 \cos(d + ex) - 4 \sin(d + ex))}{3e\sqrt{5 + 4 \cos(d + ex) + 3 \sin(d + ex)}} - \frac{16(3 \cos(d + ex) - 4 \sin(d + ex))\sqrt{5 + 4 \cos(d + ex) + 3 \sin(d + ex)}}{3e} \end{aligned}$$

Mathematica [A] time = 0.60, size = 130, normalized size = 0.94

$$\frac{(3 \sin(d + ex) + 4 \cos(d + ex) + 5)^{5/2} \left(3750 \cos\left(\frac{1}{2}(d + ex)\right) + 1625 \cos\left(\frac{3}{2}(d + ex)\right) + 3 \left(-3750 \sin\left(\frac{1}{2}(d + ex)\right) - 1625 \sin\left(\frac{3}{2}(d + ex)\right) \right) \right)}{30e \left(\sin\left(\frac{1}{2}(d + ex)\right) + 3 \cos\left(\frac{1}{2}(d + ex)\right) \right)^5}$$

Antiderivative was successfully verified.

[In] Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2), x]

[Out] -1/30*((5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)*(3750*Cos[(d + e*x)/2] + 1625*Cos[(3*(d + e*x))/2] + 3*(79*Cos[(5*(d + e*x))/2] - 3750*Sin[(d + e*x)/2] - 375*Sin[(3*(d + e*x))/2] + 3*Sin[(5*(d + e*x))/2]))/(e*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2]))^5)

fricas [A] time = 1.09, size = 101, normalized size = 0.73

$$\frac{2(237 \cos(ex + d)^3 + 931 \cos(ex + d)^2 + 9(\cos(ex + d)^2 - 62 \cos(ex + d) - 344) \sin(ex + d) + 1166 \cos(ex + d) + 472) \sqrt{4 \cos(ex + d) + 3 \sin(ex + d) + 5}}{15(3e \cos(ex + d) + e \sin(ex + d) + 3e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2), x, algorithm="fricas")

[Out] -2/15*(237*cos(e*x + d)^3 + 931*cos(e*x + d)^2 + 9*(cos(e*x + d)^2 - 62*cos(e*x + d) - 344)*sin(e*x + d) + 1166*cos(e*x + d) + 472)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5)/(3*e*cos(e*x + d) + e*sin(e*x + d) + 3*e)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4 \cos(ex + d) + 3 \sin(ex + d) + 5)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2), x, algorithm="giac")

[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(5/2), x)

maple [A] time = 0.37, size = 74, normalized size = 0.53

$$\frac{50 \left(1 + \sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) \right) \left(\sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) - 1 \right) \left(3 \left(\sin^2 \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) \right) + 14 \sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) + 43 \right)}{3 \cos \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) \sqrt{5 + 5 \sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right)} e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2), x)

[Out] 50/3*(1+sin(e*x+d+arctan(4/3)))*(sin(e*x+d+arctan(4/3))-1)*(3*sin(e*x+d+arctan(4/3))^2+14*sin(e*x+d+arctan(4/3))+43)/cos(e*x+d+arctan(4/3))/(5+5*sin(e*x+d+arctan(4/3))^(1/2))/e

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4 \cos(ex + d) + 3 \sin(ex + d) + 5)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2), x, algorithm="maxima")

[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (4 \cos(d + ex) + 3 \sin(d + ex) + 5)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(5/2),x)
```

```
[Out] int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(5/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

$$3.418 \quad \int (5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2} dx$$

Optimal. Leaf size=93

$$\frac{2\sqrt{3 \sin(d + ex) + 4 \cos(d + ex) + 5} (3 \cos(d + ex) - 4 \sin(d + ex))}{3e} - \frac{40(3 \cos(d + ex) - 4 \sin(d + ex))}{3e\sqrt{3 \sin(d + ex) + 4 \cos(d + ex) + 5}}$$

[Out] $-40/3*(3*\cos(e*x+d)-4*\sin(e*x+d))/e/(5+4*\cos(e*x+d)+3*\sin(e*x+d))^{(1/2)}-2/3*(3*\cos(e*x+d)-4*\sin(e*x+d))*(5+4*\cos(e*x+d)+3*\sin(e*x+d))^{(1/2)}/e$

Rubi [A] time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3113, 3112}

$$\frac{2\sqrt{3 \sin(d + ex) + 4 \cos(d + ex) + 5} (3 \cos(d + ex) - 4 \sin(d + ex))}{3e} - \frac{40(3 \cos(d + ex) - 4 \sin(d + ex))}{3e\sqrt{3 \sin(d + ex) + 4 \cos(d + ex) + 5}}$$

Antiderivative was successfully verified.

[In] Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2), x]

[Out] $(-40*(3*\text{Cos}[d + e*x] - 4*\text{Sin}[d + e*x]))/(3*e*\text{Sqrt}[5 + 4*\text{Cos}[d + e*x] + 3*\text{Sin}[d + e*x]]) - (2*(3*\text{Cos}[d + e*x] - 4*\text{Sin}[d + e*x])*\text{Sqrt}[5 + 4*\text{Cos}[d + e*x] + 3*\text{Sin}[d + e*x]])/(3*e)$

Rule 3112

Int[Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rule 3113

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^n, x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]

Rubi steps

$$\int (5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2} dx = -\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))\sqrt{5 + 4 \cos(d + ex) + 3 \sin(d + ex)}}{3e} - \frac{40(3 \cos(d + ex) - 4 \sin(d + ex))}{3e\sqrt{5 + 4 \cos(d + ex) + 3 \sin(d + ex)}}$$

Mathematica [A] time = 0.33, size = 104, normalized size = 1.12

$$\frac{(3 \sin(d + ex) + 4 \cos(d + ex) + 5)^{3/2} \left(9 \left(15 \sin\left(\frac{1}{2}(d + ex)\right) + \sin\left(\frac{3}{2}(d + ex)\right) \right) - 45 \cos\left(\frac{1}{2}(d + ex)\right) - 13 \cos\left(\frac{3}{2}(d + ex)\right) \right)}{3e \left(\sin\left(\frac{1}{2}(d + ex)\right) + 3 \cos\left(\frac{1}{2}(d + ex)\right) \right)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2), x]

[Out] $((5 + 4*\text{Cos}[d + e*x] + 3*\text{Sin}[d + e*x])^{3/2}*(-45*\text{Cos}[(d + e*x)/2] - 13*\text{Cos}[(3*(d + e*x))/2] + 9*(15*\text{Sin}[(d + e*x)/2] + \text{Sin}[(3*(d + e*x))/2]))) / (3*e*(3*\text{Cos}[(d + e*x)/2] + \text{Sin}[(d + e*x)/2])^3)$

fricas [A] time = 0.92, size = 81, normalized size = 0.87

$$\frac{2 \left(13 \cos(ex + d)^2 - 9(\cos(ex + d) + 8) \sin(ex + d) + 29 \cos(ex + d) + 16 \right) \sqrt{4 \cos(ex + d) + 3 \sin(ex + d) + 5}}{3(3e \cos(ex + d) + e \sin(ex + d) + 3e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")`

[Out] $-2/3*(13*\cos(e*x + d)^2 - 9*(\cos(e*x + d) + 8)*\sin(e*x + d) + 29*\cos(e*x + d) + 16)*\text{sqrt}(4*\cos(e*x + d) + 3*\sin(e*x + d) + 5)/(3*e*\cos(e*x + d) + e*\sin(e*x + d) + 3*e)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4 \cos(ex + d) + 3 \sin(ex + d) + 5)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")`

[Out] `integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(3/2), x)`

maple [A] time = 0.35, size = 60, normalized size = 0.65

$$\frac{50 \left(1 + \sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) \right) \left(\sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) - 1 \right) \left(\sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) + 5 \right)}{3 \cos \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) \sqrt{5 + 5 \sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right)} e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x)`

[Out] $50/3*(1+\sin(e*x+d+\arctan(4/3)))*(\sin(e*x+d+\arctan(4/3))-1)*(\sin(e*x+d+\arctan(4/3))+5)/\cos(e*x+d+\arctan(4/3))/(5+5*\sin(e*x+d+\arctan(4/3)))^{1/2}/e$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4 \cos(ex + d) + 3 \sin(ex + d) + 5)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")`

[Out] `integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (4 \cos(d + ex) + 3 \sin(d + ex) + 5)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(3/2),x)`

[Out] `int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 \sin(d + ex) + 4 \cos(d + ex) + 5)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)

[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) + 5)**(3/2), x)

3.419 $\int \sqrt{5 + 4 \cos(d + ex) + 3 \sin(d + ex)} dx$

Optimal. Leaf size=44

$$-\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))}{e\sqrt{3 \sin(d + ex) + 4 \cos(d + ex) + 5}}$$

[Out] $-2*(3*\cos(e*x+d)-4*\sin(e*x+d))/e/(5+4*\cos(e*x+d)+3*\sin(e*x+d))^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3112}

$$-\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))}{e\sqrt{3 \sin(d + ex) + 4 \cos(d + ex) + 5}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]

[Out] $(-2*(3*\text{Cos}[d + e*x] - 4*\text{Sin}[d + e*x]))/(e*\text{Sqrt}[5 + 4*\text{Cos}[d + e*x] + 3*\text{Sin}[d + e*x]])$

Rule 3112

Int[Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\int \sqrt{5 + 4 \cos(d + ex) + 3 \sin(d + ex)} dx = -\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))}{e\sqrt{5 + 4 \cos(d + ex) + 3 \sin(d + ex)}}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 1.70

$$-\frac{2\left(\cos\left(\frac{1}{2}(d + ex)\right) - 3 \sin\left(\frac{1}{2}(d + ex)\right)\right)\sqrt{3 \sin(d + ex) + 4 \cos(d + ex) + 5}}{e\left(\sin\left(\frac{1}{2}(d + ex)\right) + 3 \cos\left(\frac{1}{2}(d + ex)\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]

[Out] $(-2*(\text{Cos}[(d + e*x)/2] - 3*\text{Sin}[(d + e*x)/2])* \text{Sqrt}[5 + 4*\text{Cos}[d + e*x] + 3*\text{Sin}[d + e*x]])/(e*(3*\text{Cos}[(d + e*x)/2] + \text{Sin}[(d + e*x)/2]))$

fricas [A] time = 1.84, size = 61, normalized size = 1.39

$$\frac{2\sqrt{4 \cos(ex + d) + 3 \sin(ex + d) + 5}(\cos(ex + d) - 3 \sin(ex + d) + 1)}{3e \cos(ex + d) + e \sin(ex + d) + 3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")

[Out] $-2*\text{sqrt}(4*\cos(e*x + d) + 3*\sin(e*x + d) + 5)*(\cos(e*x + d) - 3*\sin(e*x + d) + 1)/(3*e*\cos(e*x + d) + e*\sin(e*x + d) + 3*e)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4 \cos(ex + d) + 3 \sin(ex + d) + 5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)

maple [A] time = 0.30, size = 50, normalized size = 1.14

$$\frac{10 \left(\sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) - 1 \right) \left(1 + \sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) \right)}{\cos \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) \sqrt{5 + 5 \sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right)} e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x)

[Out] 10*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))/cos(e*x+d+arctan(4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4 \cos(ex + d) + 3 \sin(ex + d) + 5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)

mupad [B] time = 0.31, size = 39, normalized size = 0.89

$$\frac{2 \sqrt{5} (3 \cos(d + ex) - 4 \sin(d + ex))}{5 e \sqrt{\cos \left(d - \operatorname{atan} \left(\frac{3}{4} \right) + ex \right) + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(1/2),x)

[Out] -(2*5^(1/2)*(3*cos(d + e*x) - 4*sin(d + e*x)))/(5*e*(cos(d - atan(3/4) + e*x) + 1)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3 \sin(d + ex) + 4 \cos(d + ex) + 5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)

[Out] Integral(sqrt(3*sin(d + e*x) + 4*cos(d + e*x) + 5), x)

$$3.420 \quad \int \frac{1}{\sqrt{5+4 \cos(d+ex)+3 \sin(d+ex)}} dx$$

Optimal. Leaf size=48

$$\frac{\sqrt{\frac{2}{5}} \tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2} \sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)+1}}\right)}{e}$$

[Out] 1/5*arctanh(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(1+cos(d+e*x-arctan(3/4)))^(1/2))*10^(1/2)/e

Rubi [A] time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3115, 2649, 206}

$$\frac{\sqrt{\frac{2}{5}} \tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2} \sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)+1}}\right)}{e}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]

[Out] (Sqrt[2/5]*ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*x - ArcTan[3/4]])]))/e

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3115

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\int \frac{1}{\sqrt{5 + 4 \cos(d + ex) + 3 \sin(d + ex)}} dx = \int \frac{1}{\sqrt{5 + 5 \cos\left(d + ex - \tan^{-1}\left(\frac{3}{4}\right)\right)}} dx$$

$$= \frac{2 \operatorname{Subst}\left(\int \frac{1}{10-x^2} dx, x, -\frac{5 \sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{5+5 \cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

$$= \frac{\sqrt{\frac{2}{5}} \tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2} \sqrt{1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

Mathematica [C] time = 0.10, size = 101, normalized size = 2.10

$$\frac{\left(\frac{2}{5} + \frac{6i}{5}\right) \sqrt{\frac{4}{5} + \frac{3i}{5}} \tan^{-1}\left(\left(\frac{1}{10} + \frac{3i}{10}\right) \sqrt{\frac{4}{5} + \frac{3i}{5}} \left(3 \tan\left(\frac{1}{4}(d + ex)\right) - 1\right)\right) \left(\sin\left(\frac{1}{2}(d + ex)\right) + 3 \cos\left(\frac{1}{2}(d + ex)\right)\right)}{e \sqrt{3 \sin(d + ex) + 4 \cos(d + ex) + 5}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]], x]

[Out] ((-2/5 - (6*I)/5)*Sqrt[4/5 + (3*I)/5]*ArcTan[(1/10 + (3*I)/10)*Sqrt[4/5 + (3*I)/5]*(-1 + 3*Tan[(d + e*x)/4])]*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2]))/(e*Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]])

fricas [B] time = 1.17, size = 147, normalized size = 3.06

$$\frac{\sqrt{5} \sqrt{2} \log\left(-\frac{9 \cos(ex+d)^2 + (13 \cos(ex+d) - 6) \sin(ex+d) + 2(\sqrt{5} \sqrt{2} \cos(ex+d) - 3 \sqrt{5} \sqrt{2} \sin(ex+d) + \sqrt{5} \sqrt{2}) \sqrt{4 \cos(ex+d) + 3 \sin(ex+d) + 5} - 33 \cos(ex+d) - 42}{9 \cos(ex+d)^2 + (13 \cos(ex+d) + 14) \sin(ex+d) + 27 \cos(ex+d) + 18}\right)}{10 e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2), x, algorithm="fricas")

[Out] 1/10*sqrt(5)*sqrt(2)*log(-(9*cos(e*x + d)^2 + (13*cos(e*x + d) - 6)*sin(e*x + d) + 2*(sqrt(5)*sqrt(2)*cos(e*x + d) - 3*sqrt(5)*sqrt(2)*sin(e*x + d) + sqrt(5)*sqrt(2))*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5) - 33*cos(e*x + d) - 42)/(9*cos(e*x + d)^2 + (13*cos(e*x + d) + 14)*sin(e*x + d) + 27*cos(e*x + d) + 18))/e

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4 \cos(ex + d) + 3 \sin(ex + d) + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2), x, algorithm="giac")

[Out] integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)

maple [A] time = 0.25, size = 77, normalized size = 1.60

$$\frac{\left(1 + \sin\left(ex + d + \arctan\left(\frac{4}{3}\right)\right)\right) \sqrt{-5 \sin\left(ex + d + \arctan\left(\frac{4}{3}\right)\right) + 5} \sqrt{10} \operatorname{arctanh}\left(\frac{\sqrt{-5 \sin\left(ex + d + \arctan\left(\frac{4}{3}\right)\right) + 5}}{10}\right)}{5 \cos\left(ex + d + \arctan\left(\frac{4}{3}\right)\right) \sqrt{5 + 5 \sin\left(ex + d + \arctan\left(\frac{4}{3}\right)\right)} e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x)`

[Out] `-1/5*(1+sin(e*x+d+arctan(4/3)))*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2))/cos(e*x+d+arctan(4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4 \cos(ex + d) + 3 \sin(ex + d) + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{4 \cos(d + ex) + 3 \sin(d + ex) + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(1/2),x)`

[Out] `int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3 \sin(d + ex) + 4 \cos(d + ex) + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)`

[Out] `Integral(1/sqrt(3*sin(d + e*x) + 4*cos(d + e*x) + 5), x)`

$$3.421 \quad \int \frac{1}{(5+4 \cos(d+ex)+3 \sin(d+ex))^{3/2}} dx$$

Optimal. Leaf size=96

$$\frac{\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)+1}}\right)}{10\sqrt{10}e} - \frac{3 \cos(d+ex) - 4 \sin(d+ex)}{10e(3 \sin(d+ex) + 4 \cos(d+ex) + 5)^{3/2}}$$

[Out] 1/10*(-3*cos(e*x+d)+4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)+1/100*arctanh(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(1+cos(d+e*x-arctan(3/4)))^(1/2))*10^(1/2)/e

Rubi [A] time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3116, 3115, 2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)+1}}\right)}{10\sqrt{10}e} - \frac{3 \cos(d+ex) - 4 \sin(d+ex)}{10e(3 \sin(d+ex) + 4 \cos(d+ex) + 5)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2), x]

[Out] ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*x - ArcTan[3/4]])]]/(10*Sqrt[10]*e) - (3*Cos[d + e*x] - 4*Sin[d + e*x])/(10*e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3115

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rule 3116

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2}} dx &= -\frac{3 \cos(d + ex) - 4 \sin(d + ex)}{10e(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2}} + \frac{1}{20} \int \frac{1}{\sqrt{5 + 4 \cos(d + ex) + 3 \sin(d + ex)}} dx \\
&= -\frac{3 \cos(d + ex) - 4 \sin(d + ex)}{10e(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2}} + \frac{1}{20} \int \frac{1}{\sqrt{5 + 5 \cos\left(d + \frac{1}{2}ex + \frac{1}{2}d\right)}} dx \\
&= -\frac{3 \cos(d + ex) - 4 \sin(d + ex)}{10e(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2}} - \frac{1}{20} \operatorname{Subst} \left(\int \frac{1}{10-x^2} dx, x, \frac{1}{2}ex + \frac{1}{2}d \right) \\
&= -\frac{\tanh^{-1} \left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2} \sqrt{1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}} \right)}{10\sqrt{10}e} - \frac{3 \cos(d + ex) - 4 \sin(d + ex)}{10e(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.29, size = 154, normalized size = 1.60

$$\frac{\left(\frac{1}{250} - \frac{i}{125}\right) \left(\sin\left(\frac{1}{2}(d + ex)\right) + 3 \cos\left(\frac{1}{2}(d + ex)\right)\right) \left((5 + 10i) \left(\cos\left(\frac{1}{2}(d + ex)\right) - 3 \sin\left(\frac{1}{2}(d + ex)\right)\right) - (1 - i)\sqrt{20} + \dots\right)}{e(3 \sin(d + ex) + 4 \cos(d + ex))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2), x]

[Out] ((-1/250 + I/125)*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*((5 + 10*I)*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2]) - (1 - I)*Sqrt[20 + 15*I]*ArcTan[(1/10 + (3*I)/10)*Sqrt[4/5 + (3*I)/5]*(-1 + 3*Tan[(d + e*x)/4])]*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^2))/(e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))

fricas [B] time = 0.93, size = 268, normalized size = 2.79

$$\frac{(9\sqrt{10} \cos(ex + d)^2 + (13\sqrt{10} \cos(ex + d) + 14\sqrt{10}) \sin(ex + d) + 27\sqrt{10} \cos(ex + d) + 18\sqrt{10}) \log\left(-\frac{9 \cos(ex + d)}{200(9e \cos(ex + d) + 14e \sin(ex + d) + 18e)}\right)}{200(9e \cos(ex + d) + 14e \sin(ex + d) + 18e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2), x, algorithm="fricas")

[Out] 1/200*((9*sqrt(10)*cos(e*x + d)^2 + (13*sqrt(10)*cos(e*x + d) + 14*sqrt(10))*sin(e*x + d) + 27*sqrt(10)*cos(e*x + d) + 18*sqrt(10))*log(-(9*cos(e*x + d)^2 + (13*cos(e*x + d) - 6)*sin(e*x + d) + 2*(sqrt(10)*cos(e*x + d) - 3*sqrt(10)*sin(e*x + d) + sqrt(10))*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5) - 33*cos(e*x + d) - 42)/(9*cos(e*x + d)^2 + (13*cos(e*x + d) + 14)*sin(e*x + d) + 27*cos(e*x + d) + 18)) - 20*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5)*(cos(e*x + d) - 3*sin(e*x + d) + 1))/(9*e*cos(e*x + d)^2 + 27*e*cos(e*x + d) + (13*e*cos(e*x + d) + 14*e)*sin(e*x + d) + 18*e)

giac [B] time = 0.74, size = 284, normalized size = 2.96

$$\frac{1}{100} \left(\frac{\sqrt{10} \log \left(\frac{-2\sqrt{10} + 2\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1 - 2\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 6}}{2\sqrt{10} + 2\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1 - 2\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 6}} \right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 3\right)} \right) - \frac{20 \left(19 \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) \right) \right)}{\left(\left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")

[Out] 1/100*(sqrt(10)*log(abs(-2*sqrt(10) + 2*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - 2*tan(1/2*x*e + 1/2*d) - 6)/abs(2*sqrt(10) + 2*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - 2*tan(1/2*x*e + 1/2*d) - 6))/sgn(tan(1/2*x*e + 1/2*d) + 3) - 20*(19*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^3 - 51*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 - 17*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) + 17*tan(1/2*x*e + 1/2*d) - 3)/(((sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 - 6*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) + 6*tan(1/2*x*e + 1/2*d) - 1)^2*sgn(tan(1/2*x*e + 1/2*d) + 3))*e^(-1)

maple [A] time = 0.31, size = 117, normalized size = 1.22

$$\frac{\left(\sqrt{10} \operatorname{arctanh} \left(\frac{\sqrt{-5 \sin \left(ex+d+\arctan \left(\frac{4}{3} \right) \right)+5 \sqrt{10}}}{10} \right) \sin \left(ex+d+\arctan \left(\frac{4}{3} \right) \right) + \sqrt{10} \operatorname{arctanh} \left(\frac{\sqrt{-5 \sin \left(ex+d+\arctan \left(\frac{4}{3} \right) \right)+5 \sqrt{10}}}{10} \right) \right)}{100 \cos \left(ex+d+\arctan \left(\frac{4}{3} \right) \right) \sqrt{5+5 \sin \left(ex+d+\arctan \left(\frac{4}{3} \right) \right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x)

[Out] -1/100*(10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2))*sin(e*x+d+arctan(4/3))+10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2))+2*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)/cos(e*x+d+arctan(4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(4 \cos(ex+d) + 3 \sin(ex+d) + 5)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")

[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(-3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(4 \cos(d+ex) + 3 \sin(d+ex) + 5)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(3/2),x)

[Out] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3 \sin(d+ex) + 4 \cos(d+ex) + 5)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)

[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) + 5)**(-3/2), x)

$$3.422 \quad \int \frac{1}{(5+4 \cos(d+ex)+3 \sin(d+ex))^{5/2}} dx$$

Optimal. Leaf size=142

$$\frac{3(3 \cos(d+ex) - 4 \sin(d+ex))}{400e(3 \sin(d+ex) + 4 \cos(d+ex) + 5)^{3/2}} - \frac{3 \cos(d+ex) - 4 \sin(d+ex)}{20e(3 \sin(d+ex) + 4 \cos(d+ex) + 5)^{5/2}} + \frac{3 \tanh^{-1} \left(\frac{\sin(d+ex - \arctan(3/4))}{\sqrt{2} \sqrt{\cos(d+ex - \arctan(3/4))}} \right)}{400\sqrt{10}e}$$

[Out] 1/20*(-3*cos(e*x+d)+4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2)-3/400*(3*cos(e*x+d)-4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)+3/4000*arctanh(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(1+cos(d+e*x-arctan(3/4))))^(1/2)/e

Rubi [A] time = 0.08, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3116, 3115, 2649, 206}

$$\frac{3(3 \cos(d+ex) - 4 \sin(d+ex))}{400e(3 \sin(d+ex) + 4 \cos(d+ex) + 5)^{3/2}} - \frac{3 \cos(d+ex) - 4 \sin(d+ex)}{20e(3 \sin(d+ex) + 4 \cos(d+ex) + 5)^{5/2}} + \frac{3 \tanh^{-1} \left(\frac{\sin(d+ex - \arctan(3/4))}{\sqrt{2} \sqrt{\cos(d+ex - \arctan(3/4))}} \right)}{400\sqrt{10}e}$$

Antiderivative was successfully verified.

[In] Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2), x]

[Out] (3*ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*x - ArcTan[3/4]])])/(400*Sqrt[10]*e) - (3*Cos[d + e*x] - 4*Sin[d + e*x])/(20*e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)) - (3*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(400*e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3115

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rule 3116

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^n, x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} dx &= -\frac{3 \cos(d + ex) - 4 \sin(d + ex)}{20e(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} + \frac{3}{40} \int \frac{1}{(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} dx \\
&= -\frac{3 \cos(d + ex) - 4 \sin(d + ex)}{20e(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} - \frac{3(3 \cos(d + ex) - 4 \sin(d + ex))}{400e(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} \\
&= -\frac{3 \cos(d + ex) - 4 \sin(d + ex)}{20e(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} - \frac{3(3 \cos(d + ex) - 4 \sin(d + ex))}{400e(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} \\
&= -\frac{3 \cos(d + ex) - 4 \sin(d + ex)}{20e(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} - \frac{3(3 \cos(d + ex) - 4 \sin(d + ex))}{400e(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} \\
&= \frac{3 \tanh^{-1} \left(\frac{\sin(d + ex - \tan^{-1}(\frac{3}{4}))}{\sqrt{2} \sqrt{1 + \cos(d + ex - \tan^{-1}(\frac{3}{4}))}} \right)}{400\sqrt{10}e} - \frac{3 \cos(d + ex) - 4 \sin(d + ex)}{20e(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.41, size = 180, normalized size = 1.27

$$\frac{\left(\frac{1}{20000} - \frac{i}{10000}\right) \left(\sin\left(\frac{1}{2}(d + ex)\right) + 3 \cos\left(\frac{1}{2}(d + ex)\right)\right) \left((5 + 10i) \left(-165 \sin\left(\frac{1}{2}(d + ex)\right) - 27 \sin\left(\frac{3}{2}(d + ex)\right)\right) + \dots}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2), x]

[Out] ((-1/20000 + I/10000)*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*((-6 + 6*I)*Sqrt[20 + 15*I]*ArcTan[(1/10 + (3*I)/10)*Sqrt[4/5 + (3*I)/5]*(-1 + 3*Tan[(d + e*x)/4])])*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^4 + (5 + 10*I)*(55*Cos[(d + e*x)/2] + 39*Cos[(3*(d + e*x))/2] - 165*Sin[(d + e*x)/2] - 27*Sin[(3*(d + e*x))/2]))/(e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2))

fricas [B] time = 2.93, size = 341, normalized size = 2.40

$$\frac{3(3\sqrt{10} \cos(ex + d)^3 - 111\sqrt{10} \cos(ex + d)^2 - (79\sqrt{10} \cos(ex + d)^2 + 202\sqrt{10} \cos(ex + d) + 124\sqrt{10}) \sin(ex + d) - 246\sqrt{10} \cos(ex + d) - 132\sqrt{10}) \log(-9 \cos^2(ex + d) + (13 \cos(ex + d) + 14) \sin(ex + d) + 27 \cos(ex + d) + 18) + 20(39 \cos^2(ex + d) - 3(9 \cos(ex + d) + 32) \sin(ex + d) + 47 \cos(ex + d) + 8) \sqrt{4 \cos(ex + d) + 3 \sin(ex + d) + 5}}{3e^3 \cos^3(ex + d) - 111e^2 \cos^2(ex + d) - 246e \cos(ex + d) - (79e \cos^2(ex + d) + 202e \cos(ex + d) + 124e) \sin(ex + d) - 132e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2), x, algorithm="fricas")

[Out] 1/8000*(3*(3*sqrt(10)*cos(e*x + d)^3 - 111*sqrt(10)*cos(e*x + d)^2 - (79*sqrt(10)*cos(e*x + d)^2 + 202*sqrt(10)*cos(e*x + d) + 124*sqrt(10))*sin(e*x + d) - 246*sqrt(10)*cos(e*x + d) - 132*sqrt(10))*log(-(9*cos(e*x + d)^2 + (13*cos(e*x + d) + 14)*sin(e*x + d) + 27*cos(e*x + d) + 18)) + 20*(39*cos(e*x + d)^2 - 3*(9*cos(e*x + d) + 32)*sin(e*x + d) + 47*cos(e*x + d) + 8)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5))/(3*e*cos(e*x + d)^3 - 111*e*cos(e*x + d)^2 - 246*e*cos(e*x + d) - (79*e*cos(e*x + d)^2 + 202*e*cos(e*x + d) + 124*e)*sin(e*x + d) - 132*e)

giac [B] time = 0.97, size = 417, normalized size = 2.94

$$\frac{1}{4000} \left(\frac{3\sqrt{10} \log \left(\frac{\left| -2\sqrt{10} + 2\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - 2\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 6 \right|}{\left| 2\sqrt{10} + 2\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - 2\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 6 \right|} \right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 3\right)} - 20 \left(797 \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) \right)^7 - 7137 \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) \right)^6 + 27543 \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) \right)^5 - 30015 \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) \right)^4 - 27105 \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) \right)^3 - 7491 \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) \right)^2 - 859 \sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} + 859 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 69 \right) / \left(\left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) \right)^2 - 6 \sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} + 6 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 1 \right)^4 \operatorname{sgn}\left(\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 3\right) \right) e^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")

[Out] 1/4000*(3*sqrt(10)*log(abs(-2*sqrt(10) + 2*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - 2*tan(1/2*x*e + 1/2*d) - 6)/abs(2*sqrt(10) + 2*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - 2*tan(1/2*x*e + 1/2*d) - 6))/sgn(tan(1/2*x*e + 1/2*d) + 3) - 20*(797*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^7 - 7137*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^6 + 27543*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^5 - 30015*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^4 - 27105*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^3 - 7491*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 - 859*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) + 859*tan(1/2*x*e + 1/2*d) - 69)/(((sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 - 6*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) + 6*tan(1/2*x*e + 1/2*d) - 1)^4*sgn(tan(1/2*x*e + 1/2*d) + 3))*e^(-1)

maple [A] time = 0.32, size = 190, normalized size = 1.34

$$\left(3\sqrt{10} \operatorname{arctanh} \left(\frac{\sqrt{-5 \sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5} \sqrt{10}}{10} \right) \right) \left(\sin^2 \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) \right) + 6\sqrt{10} \operatorname{arctanh} \left(\frac{\sqrt{-5 \sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5} \sqrt{10}}{10} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x)

[Out] -1/4000*(3*10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2))*sin(e*x+d+arctan(4/3))^2+6*10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2))*sin(e*x+d+arctan(4/3))+6*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*sin(e*x+d+arctan(4/3))+3*10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2))+14*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)/(1+sin(e*x+d+arctan(4/3)))/cos(e*x+d+arctan(4/3)))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(4 \cos(ex + d) + 3 \sin(ex + d) + 5)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")

[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(-5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(4 \cos(d + ex) + 3 \sin(d + ex) + 5)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(5/2), x)`

[Out] `int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3 \sin(d + ex) + 4 \cos(d + ex) + 5)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2), x)`

[Out] `Integral((3*sin(d + e*x) + 4*cos(d + e*x) + 5)**(-5/2), x)`

3.423 $\int (-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{7/2} dx$

Optimal. Leaf size=185

$$\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))(3 \sin(d + ex) + 4 \cos(d + ex) - 5)^{5/2}}{7e} + \frac{24(3 \cos(d + ex) - 4 \sin(d + ex))(3 \sin(d + ex) + 4 \cos(d + ex) - 5)^{3/2}}{7e}$$

```
[Out] 24/7*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)/e-2/7
*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2)/e+6400/7*
(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)-320/7*(3
*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e
```

Rubi [A] time = 0.09, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3113, 3112}

$$\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))(3 \sin(d + ex) + 4 \cos(d + ex) - 5)^{5/2}}{7e} + \frac{24(3 \cos(d + ex) - 4 \sin(d + ex))(3 \sin(d + ex) + 4 \cos(d + ex) - 5)^{3/2}}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(7/2), x]
```

```
[Out] (6400*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(7*e*Sqrt[-5 + 4*Cos[d + e*x] + 3*
Sin[d + e*x]]) - (320*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[-5 + 4*Cos[d +
e*x] + 3*Sin[d + e*x]])/(7*e) + (24*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(-5
+ 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))/(7*e) - (2*(3*Cos[d + e*x] - 4*Si
n[d + e*x])*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2))/(7*e)
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)
]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)
])^(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e},
x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int (-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{7/2} dx &= -\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}}{7e} \\ &= \frac{24(3 \cos(d + ex) - 4 \sin(d + ex))(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2}}{7e} \\ &= -\frac{320(3 \cos(d + ex) - 4 \sin(d + ex))\sqrt{-5 + 4 \cos(d + ex) + 3 \sin(d + ex)}}{7e} \\ &= \frac{6400(3 \cos(d + ex) - 4 \sin(d + ex))}{7e\sqrt{-5 + 4 \cos(d + ex) + 3 \sin(d + ex)}} - \frac{320(3 \cos(d + ex) - 4 \sin(d + ex))}{7e} \end{aligned}$$

Mathematica [A] time = 1.82, size = 151, normalized size = 0.82

$$\frac{(3 \sin(d + ex) + 4 \cos(d + ex) - 5)^{7/2} \left(30625 \sin\left(\frac{1}{2}(d + ex)\right) - 15925 \sin\left(\frac{3}{2}(d + ex)\right) + 3871 \sin\left(\frac{5}{2}(d + ex)\right) - 28e \left(\cos\left(\frac{1}{2}(d + ex)\right) \right) \right)}{28e \left(\cos\left(\frac{1}{2}(d + ex)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(7/2), x]

[Out] ((-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(7/2)*(91875*Cos[(d + e*x)/2] - 11025*Cos[(3*(d + e*x))/2] - 147*Cos[(5*(d + e*x))/2] + 249*Cos[(7*(d + e*x))/2] + 30625*Sin[(d + e*x)/2] - 15925*Sin[(3*(d + e*x))/2] + 3871*Sin[(5*(d + e*x))/2] - 307*Sin[(7*(d + e*x))/2]))/(28*e*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])^7)

fricas [A] time = 0.85, size = 121, normalized size = 0.65

$$\frac{2(249 \cos(ex + d)^4 + 51 \cos(ex + d)^3 - 3042 \cos(ex + d)^2 - (307 \cos(ex + d)^3 - 1782 \cos(ex + d)^2 + 2860 \cos(ex + d) - 1392) \sin(ex + d) + 10068 \cos(ex + d) + 12912) \sqrt{4 \cos(ex + d) + 3 \sin(ex + d) - 5}}{7(e \cos(ex + d) - 3e \sin(ex + d) + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2), x, algorithm="fricas")

[Out] -2/7*(249*cos(e*x + d)^4 + 51*cos(e*x + d)^3 - 3042*cos(e*x + d)^2 - (307*cos(e*x + d)^3 - 1782*cos(e*x + d)^2 + 2860*cos(e*x + d) - 1392)*sin(e*x + d) + 10068*cos(e*x + d) + 12912)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5)/(e*cos(e*x + d) - 3*e*sin(e*x + d) + e)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4 \cos(ex + d) + 3 \sin(ex + d) - 5)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2), x, algorithm="giac")

[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(7/2), x)

maple [A] time = 0.28, size = 86, normalized size = 0.46

$$\frac{250 \left(\sin\left(ex + d + \arctan\left(\frac{4}{3}\right)\right) - 1 \right) \left(1 + \sin\left(ex + d + \arctan\left(\frac{4}{3}\right)\right) \right) \left(5 \left(\sin^3\left(ex + d + \arctan\left(\frac{4}{3}\right)\right) \right) - 27 \left(\sin\left(ex + d + \arctan\left(\frac{4}{3}\right)\right) \right) \right)}{7 \cos\left(ex + d + \arctan\left(\frac{4}{3}\right)\right) \sqrt{-5 + 5 \sin\left(ex + d + \arctan\left(\frac{4}{3}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2), x)

[Out] 250/7*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))*(5*sin(e*x+d+arctan(4/3))^3-27*sin(e*x+d+arctan(4/3))^2+71*sin(e*x+d+arctan(4/3))-177)/cos(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4 \cos(ex + d) + 3 \sin(ex + d) - 5)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x, algorithm="maxima")

[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (4 \cos(d + ex) + 3 \sin(d + ex) - 5)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(7/2),x)

[Out] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(7/2),x)

[Out] Timed out

$$3.424 \quad \int (-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2} dx$$

Optimal. Leaf size=139

$$\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))(3 \sin(d + ex) + 4 \cos(d + ex) - 5)^{3/2}}{5e} + \frac{16(3 \cos(d + ex) - 4 \sin(d + ex))\sqrt{3 \sin(d + ex) + 4 \cos(d + ex)}}{3e}$$

[Out] $-2/5*(3*\cos(e*x+d)-4*\sin(e*x+d))*(-5+4*\cos(e*x+d)+3*\sin(e*x+d))^{(3/2)}/e-320/3*(3*\cos(e*x+d)-4*\sin(e*x+d))/e/(-5+4*\cos(e*x+d)+3*\sin(e*x+d))^{(1/2)}+16/3*(3*\cos(e*x+d)-4*\sin(e*x+d))*(-5+4*\cos(e*x+d)+3*\sin(e*x+d))^{(1/2)}/e$

Rubi [A] time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3113, 3112}

$$\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))(3 \sin(d + ex) + 4 \cos(d + ex) - 5)^{3/2}}{5e} + \frac{16(3 \cos(d + ex) - 4 \sin(d + ex))\sqrt{3 \sin(d + ex) + 4 \cos(d + ex)}}{3e}$$

Antiderivative was successfully verified.

[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2), x]

[Out] $(-320*(3*\text{Cos}[d + e*x] - 4*\text{Sin}[d + e*x]))/(3*e*\text{Sqrt}[-5 + 4*\text{Cos}[d + e*x] + 3*\text{Sin}[d + e*x]]) + (16*(3*\text{Cos}[d + e*x] - 4*\text{Sin}[d + e*x])*\text{Sqrt}[-5 + 4*\text{Cos}[d + e*x] + 3*\text{Sin}[d + e*x]])/(3*e) - (2*(3*\text{Cos}[d + e*x] - 4*\text{Sin}[d + e*x])*(-5 + 4*\text{Cos}[d + e*x] + 3*\text{Sin}[d + e*x])^{(3/2)})/(5*e)$

Rule 3112

Int[Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rule 3113

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^n, x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]

Rubi steps

$$\begin{aligned} \int (-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2} dx &= -\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2}}{5e} \\ &= \frac{16(3 \cos(d + ex) - 4 \sin(d + ex))\sqrt{-5 + 4 \cos(d + ex) + 3 \sin(d + ex)}}{3e} \\ &= -\frac{320(3 \cos(d + ex) - 4 \sin(d + ex))}{3e\sqrt{-5 + 4 \cos(d + ex) + 3 \sin(d + ex)}} + \frac{16(3 \cos(d + ex) - 4 \sin(d + ex))\sqrt{-5 + 4 \cos(d + ex) + 3 \sin(d + ex)}}{3e} \end{aligned}$$

Mathematica [A] time = 0.49, size = 127, normalized size = 0.91

$$\frac{(3 \sin(d + ex) + 4 \cos(d + ex) - 5)^{5/2} \left(3750 \sin\left(\frac{1}{2}(d + ex)\right) - 1625 \sin\left(\frac{3}{2}(d + ex)\right) + 237 \sin\left(\frac{5}{2}(d + ex)\right) + 112 \right)}{30e \left(\cos\left(\frac{1}{2}(d + ex)\right) - 3 \sin\left(\frac{1}{2}(d + ex)\right) \right)^5}$$

Antiderivative was successfully verified.

[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2), x]

[Out] ((-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)*(11250*Cos[(d + e*x)/2] - 1125*Cos[(3*(d + e*x))/2] - 9*Cos[(5*(d + e*x))/2] + 3750*Sin[(d + e*x)/2] - 1625*Sin[(3*(d + e*x))/2] + 237*Sin[(5*(d + e*x))/2]))/(30*e*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])^5)

fricas [A] time = 0.65, size = 101, normalized size = 0.73

$$\frac{2(9 \cos(ex + d)^3 + 567 \cos(ex + d)^2 - (237 \cos(ex + d)^2 - 694 \cos(ex + d) + 472) \sin(ex + d) - 2538 \cos(ex + d) - 3096) \sqrt{4 \cos(ex + d) + 3 \sin(ex + d) - 5}}{15(e \cos(ex + d) - 3e \sin(ex + d) + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2), x, algorithm="fricas")

[Out] -2/15*(9*cos(e*x + d)^3 + 567*cos(e*x + d)^2 - (237*cos(e*x + d)^2 - 694*cos(e*x + d) + 472)*sin(e*x + d) - 2538*cos(e*x + d) - 3096)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5)/(e*cos(e*x + d) - 3*e*sin(e*x + d) + e)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4 \cos(ex + d) + 3 \sin(ex + d) - 5)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2), x, algorithm="giac")

[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(5/2), x)

maple [A] time = 0.31, size = 74, normalized size = 0.53

$$\frac{50 \left(\sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) - 1 \right) \left(1 + \sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) \right) \left(3 \left(\sin^2 \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) \right) - 14 \sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) + 43 \right) / \cos \left(ex + d + \arctan \left(\frac{4}{3} \right) \right)}{3 \cos \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) \sqrt{-5 + 5 \sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right)} e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2), x)

[Out] 50/3*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))*(3*sin(e*x+d+arctan(4/3))^2-14*sin(e*x+d+arctan(4/3))+43)/cos(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4 \cos(ex + d) + 3 \sin(ex + d) - 5)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2), x, algorithm="maxima")

[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (4 \cos(d + ex) + 3 \sin(d + ex) - 5)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(5/2),x)
```

```
[Out] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(5/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

3.425 $\int (-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2} dx$

Optimal. Leaf size=93

$$\frac{40(3 \cos(d + ex) - 4 \sin(d + ex))}{3e\sqrt{3 \sin(d + ex) + 4 \cos(d + ex) - 5}} - \frac{2(3 \cos(d + ex) - 4 \sin(d + ex))\sqrt{3 \sin(d + ex) + 4 \cos(d + ex) - 5}}{3e}$$

[Out] 40/3*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)-2/3*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e

Rubi [A] time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3113, 3112}

$$\frac{40(3 \cos(d + ex) - 4 \sin(d + ex))}{3e\sqrt{3 \sin(d + ex) + 4 \cos(d + ex) - 5}} - \frac{2(3 \cos(d + ex) - 4 \sin(d + ex))\sqrt{3 \sin(d + ex) + 4 \cos(d + ex) - 5}}{3e}$$

Antiderivative was successfully verified.

[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2), x]

[Out] (40*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]]) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]])/(3*e)

Rule 3112

Int[Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rule 3113

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^(n_), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]

Rubi steps

$$\int (-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2} dx = -\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))\sqrt{-5 + 4 \cos(d + ex) + 3 \sin(d + ex)}}{3e} = \frac{40(3 \cos(d + ex) - 4 \sin(d + ex))}{3e\sqrt{-5 + 4 \cos(d + ex) + 3 \sin(d + ex)}} - \frac{2(3 \cos(d + ex) - 4 \sin(d + ex))\sqrt{-5 + 4 \cos(d + ex) + 3 \sin(d + ex)}}{3e}$$

Mathematica [A] time = 0.22, size = 103, normalized size = 1.11

$$\frac{(3 \sin(d + ex) + 4 \cos(d + ex) - 5)^{3/2} \left(45 \sin\left(\frac{1}{2}(d + ex)\right) - 13 \sin\left(\frac{3}{2}(d + ex)\right) + 135 \cos\left(\frac{1}{2}(d + ex)\right) - 9 \cos\left(\frac{3}{2}(d + ex)\right) \right)}{3e \left(\cos\left(\frac{1}{2}(d + ex)\right) - 3 \sin\left(\frac{1}{2}(d + ex)\right) \right)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2), x]

[Out] $((-5 + 4\cos[d + e*x] + 3\sin[d + e*x])^{3/2} * (135\cos[(d + e*x)/2] - 9\cos[(3*(d + e*x))/2] + 45\sin[(d + e*x)/2] - 13\sin[(3*(d + e*x))/2])) / (3*e*(\cos[(d + e*x)/2] - 3*\sin[(d + e*x)/2])^3)$

fricas [A] time = 0.73, size = 80, normalized size = 0.86

$$\frac{2(9 \cos(ex + d)^2 + (13 \cos(ex + d) - 16) \sin(ex + d) - 63 \cos(ex + d) - 72) \sqrt{4 \cos(ex + d) + 3 \sin(ex + d)}}{3(e \cos(ex + d) - 3e \sin(ex + d) + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")

[Out] $2/3*(9*\cos(e*x + d)^2 + (13*\cos(e*x + d) - 16)*\sin(e*x + d) - 63*\cos(e*x + d) - 72)*\sqrt{4*\cos(e*x + d) + 3*\sin(e*x + d) - 5}/(e*\cos(e*x + d) - 3*e*\sin(e*x + d) + e)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4 \cos(ex + d) + 3 \sin(ex + d) - 5)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")

[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(3/2), x)

maple [A] time = 0.31, size = 60, normalized size = 0.65

$$\frac{50 \left(\sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) - 1 \right) \left(1 + \sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) \right) \left(\sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) - 5 \right)}{3 \cos \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) \sqrt{-5 + 5 \sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right)} e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x)

[Out] $50/3*(\sin(e*x+d+\arctan(4/3))-1)*(1+\sin(e*x+d+\arctan(4/3)))*(\sin(e*x+d+\arctan(4/3))-5)/\cos(e*x+d+\arctan(4/3))/(-5+5*\sin(e*x+d+\arctan(4/3)))^{1/2}/e$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4 \cos(ex + d) + 3 \sin(ex + d) - 5)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")

[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (4 \cos(d + ex) + 3 \sin(d + ex) - 5)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(3/2),x)

[Out] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 \sin(d + ex) + 4 \cos(d + ex) - 5)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)

[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) - 5)**(3/2), x)

$$3.426 \quad \int \sqrt{-5 + 4 \cos(d + ex) + 3 \sin(d + ex)} dx$$

Optimal. Leaf size=44

$$-\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))}{e\sqrt{3 \sin(d + ex) + 4 \cos(d + ex) - 5}}$$

[Out] $-2*(3*\cos(e*x+d)-4*\sin(e*x+d))/e/(-5+4*\cos(e*x+d)+3*\sin(e*x+d))^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3112}

$$-\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))}{e\sqrt{3 \sin(d + ex) + 4 \cos(d + ex) - 5}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]

[Out] $(-2*(3*\cos[d + e*x] - 4*\sin[d + e*x]))/(e*\sqrt{-5 + 4*\cos[d + e*x] + 3*\sin[d + e*x]})$

Rule 3112

Int[Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\int \sqrt{-5 + 4 \cos(d + ex) + 3 \sin(d + ex)} dx = -\frac{2(3 \cos(d + ex) - 4 \sin(d + ex))}{e\sqrt{-5 + 4 \cos(d + ex) + 3 \sin(d + ex)}}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 1.70

$$\frac{2 \left(\sin\left(\frac{1}{2}(d + ex)\right) + 3 \cos\left(\frac{1}{2}(d + ex)\right) \right) \sqrt{3 \sin(d + ex) + 4 \cos(d + ex) - 5}}{e \left(\cos\left(\frac{1}{2}(d + ex)\right) - 3 \sin\left(\frac{1}{2}(d + ex)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]

[Out] $(2*(3*\cos[(d + e*x)/2] + \sin[(d + e*x)/2])*Sqrt[-5 + 4*\cos[d + e*x] + 3*\sin[d + e*x]))/(e*(\cos[(d + e*x)/2] - 3*\sin[(d + e*x)/2]))$

fricas [A] time = 0.65, size = 59, normalized size = 1.34

$$\frac{2 \sqrt{4 \cos(ex + d) + 3 \sin(ex + d) - 5} (3 \cos(ex + d) + \sin(ex + d) + 3)}{e \cos(ex + d) - 3 e \sin(ex + d) + e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")

[Out] $2*\sqrt{4*\cos(e*x + d) + 3*\sin(e*x + d) - 5}*(3*\cos(e*x + d) + \sin(e*x + d) + 3)/(e*\cos(e*x + d) - 3*e*\sin(e*x + d) + e)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4 \cos(ex + d) + 3 \sin(ex + d) - 5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)

maple [A] time = 0.30, size = 50, normalized size = 1.14

$$\frac{10 \left(\sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) - 1 \right) \left(1 + \sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) \right)}{\cos \left(ex + d + \arctan \left(\frac{4}{3} \right) \right) \sqrt{-5 + 5 \sin \left(ex + d + \arctan \left(\frac{4}{3} \right) \right)} e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x)

[Out] 10*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))/cos(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4 \cos(ex + d) + 3 \sin(ex + d) - 5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)

mupad [B] time = 0.42, size = 39, normalized size = 0.89

$$\frac{2 \sqrt{5} (3 \cos(d + ex) - 4 \sin(d + ex))}{5 e \sqrt{\cos \left(d - \operatorname{atan} \left(\frac{3}{4} \right) + ex \right) - 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(1/2),x)

[Out] -(2*5^(1/2)*(3*cos(d + e*x) - 4*sin(d + e*x)))/(5*e*(cos(d - atan(3/4) + e*x) - 1)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3 \sin(d + ex) + 4 \cos(d + ex) - 5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)

[Out] Integral(sqrt(3*sin(d + e*x) + 4*cos(d + e*x) - 5), x)

$$3.427 \quad \int \frac{1}{\sqrt{-5+4 \cos(d+ex)+3 \sin(d+ex)}} dx$$

Optimal. Leaf size=49

$$\frac{\sqrt{\frac{2}{5}} \tan^{-1} \left(\frac{\sin \left(d+ex-\tan^{-1} \left(\frac{3}{4} \right) \right)}{\sqrt{2} \sqrt{\cos \left(d+ex-\tan^{-1} \left(\frac{3}{4} \right) \right)-1}} \right)}{e}$$

[Out] $-1/5*\arctan(1/2*\sin(d+e*x-\arctan(3/4))*2^{(1/2)/(-1+\cos(d+e*x-\arctan(3/4)))^{(1/2)}}*10^{(1/2)}/e$

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3115, 2649, 204}

$$\frac{\sqrt{\frac{2}{5}} \tan^{-1} \left(\frac{\sin \left(d+ex-\tan^{-1} \left(\frac{3}{4} \right) \right)}{\sqrt{2} \sqrt{\cos \left(d+ex-\tan^{-1} \left(\frac{3}{4} \right) \right)-1}} \right)}{e}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]

[Out] $-\left(\left(\sqrt{2/5}*\text{ArcTan}[\text{Sin}[d + e*x - \text{ArcTan}[3/4]]]/\left(\sqrt{2}*\sqrt{-1 + \text{Cos}[d + e*x - \text{ArcTan}[3/4]]}\right)\right)\right)/e$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3115

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\int \frac{1}{\sqrt{-5 + 4 \cos(d + ex) + 3 \sin(d + ex)}} dx = \int \frac{1}{\sqrt{-5 + 5 \cos\left(d + ex - \tan^{-1}\left(\frac{3}{4}\right)\right)}} dx$$

$$= -\frac{2 \operatorname{Subst}\left(\int \frac{1}{-10 - x^2} dx, x, -\frac{5 \sin\left(d + ex - \tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{-5 + 5 \cos\left(d + ex - \tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

$$= -\frac{\sqrt{\frac{2}{5}} \tan^{-1}\left(\frac{\sin\left(d + ex - \tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2} \sqrt{-1 + \cos\left(d + ex - \tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

Mathematica [C] time = 0.09, size = 99, normalized size = 2.02

$$\frac{\left(\frac{2}{5} + \frac{6i}{5}\right) \sqrt{-\frac{4}{5} - \frac{3i}{5}} \left(\cos\left(\frac{1}{2}(d + ex)\right) - 3 \sin\left(\frac{1}{2}(d + ex)\right)\right) \tanh^{-1}\left(\left(\frac{1}{10} + \frac{3i}{10}\right) \sqrt{-\frac{4}{5} - \frac{3i}{5}} \left(\tan\left(\frac{1}{4}(d + ex)\right) + 3\right)\right)}{e \sqrt{3 \sin(d + ex) + 4 \cos(d + ex) - 5}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]], x]

[Out] ((2/5 + (6*I)/5)*Sqrt[-4/5 - (3*I)/5]*ArcTanh[(1/10 + (3*I)/10)*Sqrt[-4/5 - (3*I)/5]*(3 + Tan[(d + e*x)/4])*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2]))/(e*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]])

fricas [B] time = 2.11, size = 88, normalized size = 1.80

$$\frac{\sqrt{5} \sqrt{2} \arctan\left(-\frac{(3 \sqrt{5} \sqrt{2} \cos(ex+d) + \sqrt{5} \sqrt{2} \sin(ex+d) + 3 \sqrt{5} \sqrt{2}) \sqrt{4 \cos(ex+d) + 3 \sin(ex+d) - 5}}{10(\cos(ex+d) - 3 \sin(ex+d) + 1)}\right)}{5e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2), x, algorithm="fricas")

[Out] 1/5*sqrt(5)*sqrt(2)*arctan(-1/10*(3*sqrt(5)*sqrt(2)*cos(e*x + d) + sqrt(5)*sqrt(2)*sin(e*x + d) + 3*sqrt(5)*sqrt(2))*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5)/(cos(e*x + d) - 3*sin(e*x + d) + 1))/e

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4 \cos(ex + d) + 3 \sin(ex + d) - 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2), x, algorithm="giac")

[Out] integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)

maple [A] time = 0.33, size = 77, normalized size = 1.57

$$\frac{\left(\sin\left(ex + d + \arctan\left(\frac{4}{3}\right)\right) - 1\right) \sqrt{-5 \sin\left(ex + d + \arctan\left(\frac{4}{3}\right)\right) - 5} \sqrt{10} \arctan\left(\frac{\sqrt{-5 \sin\left(ex + d + \arctan\left(\frac{4}{3}\right)\right) - 5} \sqrt{10}}{10}\right)}{5 \cos\left(ex + d + \arctan\left(\frac{4}{3}\right)\right) \sqrt{-5 + 5 \sin\left(ex + d + \arctan\left(\frac{4}{3}\right)\right)} e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x)`

[Out] `1/5*(sin(e*x+d+arctan(4/3))-1)*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))/cos(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4 \cos(ex + d) + 3 \sin(ex + d) - 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{4 \cos(d + ex) + 3 \sin(d + ex) - 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(1/2),x)`

[Out] `int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3 \sin(d + ex) + 4 \cos(d + ex) - 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)`

[Out] `Integral(1/sqrt(3*sin(d + e*x) + 4*cos(d + e*x) - 5), x)`

$$3.428 \quad \int \frac{1}{(-5+4 \cos(d+ex)+3 \sin(d+ex))^{3/2}} dx$$

Optimal. Leaf size=96

$$\frac{3 \cos(d+ex) - 4 \sin(d+ex)}{10e(3 \sin(d+ex) + 4 \cos(d+ex) - 5)^{3/2}} + \frac{\tan^{-1}\left(\frac{\sin(d+ex - \tan^{-1}(\frac{3}{4}))}{\sqrt{2} \sqrt{\cos(d+ex - \tan^{-1}(\frac{3}{4})) - 1}}\right)}{10\sqrt{10}e}$$

[Out] 1/10*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)+1/100*arctan(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(-1+cos(d+e*x-arctan(3/4)))^(1/2))*10^(1/2)/e

Rubi [A] time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3116, 3115, 2649, 204}

$$\frac{3 \cos(d+ex) - 4 \sin(d+ex)}{10e(3 \sin(d+ex) + 4 \cos(d+ex) - 5)^{3/2}} + \frac{\tan^{-1}\left(\frac{\sin(d+ex - \tan^{-1}(\frac{3}{4}))}{\sqrt{2} \sqrt{\cos(d+ex - \tan^{-1}(\frac{3}{4})) - 1}}\right)}{10\sqrt{10}e}$$

Antiderivative was successfully verified.

[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2), x]

[Out] ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos[d + e*x - ArcTan[3/4]])]]/(10*Sqrt[10]*e) + (3*Cos[d + e*x] - 4*Sin[d + e*x])/(10*e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3115

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rule 3116

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]

Rubi steps

$$\int \frac{1}{(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2}} dx = \frac{3 \cos(d + ex) - 4 \sin(d + ex)}{10e(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2}} - \frac{1}{20} \int \frac{1}{\sqrt{-5 + 4 \cos(d + ex) + 3 \sin(d + ex)}} dx$$

$$= \frac{3 \cos(d + ex) - 4 \sin(d + ex)}{10e(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2}} - \frac{1}{20} \int \frac{1}{\sqrt{-5 + 5 \cos(d + ex) + 3 \sin(d + ex)}} dx$$

$$= \frac{3 \cos(d + ex) - 4 \sin(d + ex)}{10e(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2}} + \frac{\text{Subst} \left(\int \frac{1}{-10 - x^2} dx \right)}{10e(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2}}$$

$$= \frac{\tan^{-1} \left(\frac{\sin(d + ex - \tan^{-1}(\frac{3}{4}))}{\sqrt{2} \sqrt{-1 + \cos(d + ex - \tan^{-1}(\frac{3}{4}))}} \right)}{10\sqrt{10}e} + \frac{3 \cos(d + ex) - 4 \sin(d + ex)}{10e(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2}}$$

Mathematica [C] time = 0.31, size = 152, normalized size = 1.58

$$\frac{\left(\frac{1}{250} - \frac{i}{125}\right) \left(\cos\left(\frac{1}{2}(d + ex)\right) - 3 \sin\left(\frac{1}{2}(d + ex)\right)\right) \left((5 + 10i) \left(\sin\left(\frac{1}{2}(d + ex)\right) + 3 \cos\left(\frac{1}{2}(d + ex)\right)\right) - (1 - i)\sqrt{-2}\right)}{e(3 \sin(d + ex) + 4 \cos(d + ex))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2), x]

[Out] ((1/250 - I/125)*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])*((-1 + I)*Sqrt[-20 - 15*I]*ArcTanh[(1/10 + (3*I)/10)*Sqrt[-4/5 - (3*I)/5]*(3 + Tan[(d + e*x)/4])]*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])^2 + (5 + 10*I)*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2]))/(e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))

fricas [B] time = 1.03, size = 210, normalized size = 2.19

$$\frac{(13\sqrt{10} \cos(ex + d)^2 - 9(\sqrt{10} \cos(ex + d) - 2\sqrt{10}) \sin(ex + d) - \sqrt{10} \cos(ex + d) - 14\sqrt{10}) \arctan\left(-\frac{3}{10}\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} + 3i \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - i\right)}{100(13e \cos(ex + d)^2 - e \cos(ex + d) - 9e \sin(ex + d) - 14e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2), x, algorithm="fricas")

[Out] -1/100*((13*sqrt(10)*cos(e*x + d)^2 - 9*(sqrt(10)*cos(e*x + d) - 2*sqrt(10))*sin(e*x + d) - sqrt(10)*cos(e*x + d) - 14*sqrt(10))*arctan(-1/10*(3*sqrt(10)*cos(e*x + d) + sqrt(10)*sin(e*x + d) + 3*sqrt(10))*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5)/(cos(e*x + d) - 3*sin(e*x + d) + 1)) + 10*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5)*(3*cos(e*x + d) + sin(e*x + d) + 3))/(13*e*cos(e*x + d)^2 - e*cos(e*x + d) - 9*(e*cos(e*x + d) - 2*e)*sin(e*x + d) - 14*e)

giac [C] time = 0.53, size = 249, normalized size = 2.59

$$-\frac{1}{450} \left(\frac{9\sqrt{10} \arctan\left(\frac{1}{10}\sqrt{10}\left(-3i\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} + 3i \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - i\right)\right)}{\text{sgn}\left(-3 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 1\right)} + \frac{10 \left(33i \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - 3i \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + i \right) \right)}{-3i \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - 3i \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + i \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] -1/450*(9*sqrt(10)*arctan(1/10*sqrt(10)*(-3*I*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) + 3*I*tan(1/2*x*e + 1/2*d) - I)/sgn(-3*tan(1/2*x*e + 1/2*d) + 1) + 10*(33*I*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^3 - 7*I*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 + 21*I*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - 21*I*tan(1/2*x*e + 1/2*d) + 9*I)/((-3*I*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 - 2*I*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) + 2*I*tan(1/2*x*e + 1/2*d) + 3*I)^2*sgn(-3*tan(1/2*x*e + 1/2*d) + 1)))e^(-1)
```

maple [A] time = 0.37, size = 118, normalized size = 1.23

$$\frac{\left(-\sqrt{10} \arctan\left(\frac{\sqrt{-5 \sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5} \sqrt{10}}{10}\right)\right) \sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right) + \sqrt{10} \arctan\left(\frac{\sqrt{-5 \sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5} \sqrt{10}}{10}\right)}{100 \cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right) \sqrt{-5+5 \sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x)
```

```
[Out] 1/100*(-10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))*sin(e*x+d+arctan(4/3))+10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))+2*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2))*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)/cos(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(4 \cos(ex + d) + 3 \sin(ex + d) - 5)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(4 \cos(d + ex) + 3 \sin(d + ex) - 5)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(3/2),x)
```

```
[Out] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3 \sin(d + ex) + 4 \cos(d + ex) - 5)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
```

```
[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) - 5)**(-3/2), x)
```

$$3.429 \quad \int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} dx$$

Optimal. Leaf size=142

$$\frac{3(3\cos(d+ex)-4\sin(d+ex))}{400e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}} + \frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}} - \frac{3\tan^{-1}\left(\frac{\sin(d+ex)-\tan(d+ex)}{\sqrt{2}\sqrt{\cos(d+ex)-\tan(d+ex)}}\right)}{400\sqrt{10}e}$$

[Out] 1/20*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2)-3/400*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)-3/400*arctan(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(-1+cos(d+e*x-arctan(3/4)))^(1/2))*10^(1/2)/e

Rubi [A] time = 0.08, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3116, 3115, 2649, 204}

$$\frac{3(3\cos(d+ex)-4\sin(d+ex))}{400e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}} + \frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}} - \frac{3\tan^{-1}\left(\frac{\sin(d+ex)-\tan(d+ex)}{\sqrt{2}\sqrt{\cos(d+ex)-\tan(d+ex)}}\right)}{400\sqrt{10}e}$$

Antiderivative was successfully verified.

[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2), x]

[Out] (-3*ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos[d + e*x - ArcTan[3/4]])])/(400*Sqrt[10]*e) + (3*Cos[d + e*x] - 4*Sin[d + e*x])/(20*e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)) - (3*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(400*e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3115

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rule 3116

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^n, x_Symbol] := Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} dx &= \frac{3 \cos(d + ex) - 4 \sin(d + ex)}{20e(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} - \frac{3}{40} \int \frac{(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{-3/2}}{(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} dx \\
&= \frac{3 \cos(d + ex) - 4 \sin(d + ex)}{20e(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} - \frac{3(3 \cos(d + ex) - 4 \sin(d + ex))}{400e(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} \\
&= \frac{3 \cos(d + ex) - 4 \sin(d + ex)}{20e(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} - \frac{3(3 \cos(d + ex) - 4 \sin(d + ex))}{400e(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} \\
&= \frac{3 \cos(d + ex) - 4 \sin(d + ex)}{20e(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} - \frac{3(3 \cos(d + ex) - 4 \sin(d + ex))}{400e(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} \\
&= -\frac{3 \tan^{-1}\left(\frac{\sin(d+ex-\tan^{-1}(\frac{3}{4}))}{\sqrt{2}\sqrt{-1+\cos(d+ex-\tan^{-1}(\frac{3}{4}))}}\right)}{400\sqrt{10}e} + \frac{3 \cos(d + ex) - 4 \sin(d + ex)}{20e(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.38, size = 178, normalized size = 1.25

$$\frac{\left(\frac{1}{10000} + \frac{i}{20000}\right) \left(\cos\left(\frac{1}{2}(d + ex)\right) - 3 \sin\left(\frac{1}{2}(d + ex)\right)\right) \left((10 - 5i) \left(55 \sin\left(\frac{1}{2}(d + ex)\right) - 39 \sin\left(\frac{3}{2}(d + ex)\right) + 165 \cos\left(\frac{1}{2}(d + ex)\right) - 117 \cos\left(\frac{3}{2}(d + ex)\right) + 27 \cos\left(\frac{5}{2}(d + ex)\right) - 15 \cos\left(\frac{7}{2}(d + ex)\right) + 3 \cos\left(\frac{9}{2}(d + ex)\right) - \cos\left(\frac{11}{2}(d + ex)\right)\right) + 165 \cos\left(\frac{1}{2}(d + ex)\right) - 117 \cos\left(\frac{3}{2}(d + ex)\right) + 27 \cos\left(\frac{5}{2}(d + ex)\right) - 15 \cos\left(\frac{7}{2}(d + ex)\right) + 3 \cos\left(\frac{9}{2}(d + ex)\right) - \cos\left(\frac{11}{2}(d + ex)\right)}{e^{5/2}(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2), x]

[Out] ((1/10000 + I/20000)*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])*((6 + 6*I)*Sqrt[-20 - 15*I]*ArcTanh[(1/10 + (3*I)/10)*Sqrt[-4/5 - (3*I)/5]*(3 + Tan[(d + e*x)/4]))*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])^4 + (10 - 5*I)*(165*Cos[(d + e*x)/2] - 27*Cos[(3*(d + e*x))/2] + 55*Sin[(d + e*x)/2] - 39*Sin[(3*(d + e*x))/2]))/(e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2))

fricas [B] time = 0.89, size = 280, normalized size = 1.97

$$\frac{3(79\sqrt{10}\cos(ex+d)^3 - 123\sqrt{10}\cos(ex+d)^2 + 3(\sqrt{10}\cos(ex+d)^2 + 38\sqrt{10}\cos(ex+d) - 44\sqrt{10})\sin(ex+d) - 78\sqrt{10}\cos(ex+d) + 124\sqrt{10})\arctan(-1/10(3\sqrt{10}\cos(ex+d) + \sqrt{10}\sin(ex+d) + 3\sqrt{10}))\sqrt{4\cos(ex+d) + 3\sin(ex+d) - 5}/(\cos(ex+d) - 3\sin(ex+d) + 1)) + 10(27\cos(ex+d)^2 + (39\cos(ex+d) - 8)\sin(ex+d) - 69\cos(ex+d) - 96)\sqrt{4\cos(ex+d) + 3\sin(ex+d) - 5}}{4000(79\cos(ex+d)^3 - 123\cos(ex+d)^2 + 3(\cos(ex+d)^2 + 38\cos(ex+d) - 44)\sin(ex+d) - 78\cos(ex+d) + 124)\arctan(-1/10(3\cos(ex+d) + \sin(ex+d) + 3))\sqrt{4\cos(ex+d) + 3\sin(ex+d) - 5} + 10(27\cos(ex+d)^2 + (39\cos(ex+d) - 8)\sin(ex+d) - 69\cos(ex+d) - 96)\sqrt{4\cos(ex+d) + 3\sin(ex+d) - 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2), x, algorithm="fricas")

[Out] 1/4000*(3*(79*sqrt(10)*cos(e*x + d)^3 - 123*sqrt(10)*cos(e*x + d)^2 + 3*(sqrt(10)*cos(e*x + d)^2 + 38*sqrt(10)*cos(e*x + d) - 44*sqrt(10))*sin(e*x + d) - 78*sqrt(10)*cos(e*x + d) + 124*sqrt(10))*arctan(-1/10*(3*sqrt(10)*cos(e*x + d) + sqrt(10)*sin(e*x + d) + 3*sqrt(10))*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5)/(cos(e*x + d) - 3*sin(e*x + d) + 1)) + 10*(27*cos(e*x + d)^2 + (39*cos(e*x + d) - 8)*sin(e*x + d) - 69*cos(e*x + d) - 96)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5)/(79*e*cos(e*x + d)^3 - 123*e*cos(e*x + d)^2 - 78*e*cos(e*x + d) + 3*(e*cos(e*x + d)^2 + 38*e*cos(e*x + d) - 44*e)*sin(e*x + d) + 124*e)

giac [C] time = 0.80, size = 381, normalized size = 2.68

$$-\frac{1}{162000} \left(\frac{243 \sqrt{10} \arctan\left(\frac{1}{10} \sqrt{10} \left(3i \sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - 3i \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + i\right)\right)}{\operatorname{sgn}\left(-3 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 1\right)} + \frac{10 \left(15039i \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)\right)^7 + 6291i \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)\right)^6 - 579i \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)\right)^5 + 1645i \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)\right)^4 + 25365i \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)\right)^3 - 11367i \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)\right)^2 + 4887i \sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - 4887i \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 3807i\right)}{\left(3i \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)\right)^2 + 2i \sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} - 2i \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 3i\right)^4 \operatorname{sgn}\left(-3 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 1\right)} \right) e^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")

[Out] -1/162000*(243*sqrt(10)*arctan(1/10*sqrt(10)*(3*I*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - 3*I*tan(1/2*x*e + 1/2*d) + I))/sgn(-3*tan(1/2*x*e + 1/2*d) + 1) + 10*(15039*I*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^7 + 6291*I*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^6 - 579*I*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^5 + 1645*I*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^4 + 25365*I*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^3 - 11367*I*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 + 4887*I*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - 4887*I*tan(1/2*x*e + 1/2*d) + 3807*I)/((3*I*(sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - tan(1/2*x*e + 1/2*d))^2 + 2*I*sqrt(tan(1/2*x*e + 1/2*d)^2 + 1) - 2*I*tan(1/2*x*e + 1/2*d) - 3*I)^4*sgn(-3*tan(1/2*x*e + 1/2*d) + 1)))*e^(-1)

maple [A] time = 0.34, size = 190, normalized size = 1.34

$$\left(3\sqrt{10} \arctan\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5}\sqrt{10}}{10}\right)\right) \left(\sin^2\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right) - 6\sqrt{10} \arctan\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5}\sqrt{10}}{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x)

[Out] 1/4000*(3*10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))*sin(e*x+d+arctan(4/3))^2-6*10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))*sin(e*x+d+arctan(4/3))+3*10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))-6*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*sin(e*x+d+arctan(4/3))+14*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)/(sin(e*x+d+arctan(4/3))-1)/cos(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(4 \cos(ex + d) + 3 \sin(ex + d) - 5)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")

[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(-5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(4 \cos(d + ex) + 3 \sin(d + ex) - 5)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(5/2), x)`

[Out] `int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3 \sin(d + ex) + 4 \cos(d + ex) - 5)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2), x)`

[Out] `Integral((3*sin(d + e*x) + 4*cos(d + e*x) - 5)**(-5/2), x)`

$$3.430 \quad \int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{7/2} dx$$

Optimal. Leaf size=258

$$\frac{2(c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{5/2}}{7e} - \frac{24\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{3/2}}{7e}$$

```
[Out] -24/35*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(1/2)*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2)/e-2/7*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2)/e-256/35*(b^2+c^2)^(3/2)*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))-64/35*(b^2+c^2)*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2)/e
```

Rubi [A] time = 0.18, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {3113, 3112}

$$\frac{2(c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{5/2}}{7e} - \frac{24\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{3/2}}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(7/2), x]
```

```
[Out] (-256*(b^2 + c^2)^(3/2)*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(35*e*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (64*(b^2 + c^2)*(c*Cos[d + e*x] - b*Sin[d + e*x])*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]])/(35*e) - (24*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))/(35*e) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2))/(7*e)
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]]], x_Symbol] := Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^n], x_Symbol] := -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{7/2} dx &= -\frac{2(c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{7/2}}{7e} \\
&= -\frac{24\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{5/2}}{35e} \\
&= -\frac{64 (b^2 + c^2) (c \cos(d + ex) - b \sin(d + ex)) \sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}}{35e} \\
&= -\frac{256 (b^2 + c^2)^{3/2} (c \cos(d + ex) - b \sin(d + ex))}{35e \sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}} - \frac{64 (b^2 + c^2)}{35e}
\end{aligned}$$

Mathematica [C] time = 33.46, size = 11888, normalized size = 46.08

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(7/2),x]

[Out] Result too large to show

fricas [A] time = 1.52, size = 268, normalized size = 1.04

$$\frac{2 \left(5(b^4 - 6b^2c^2 + c^4) \cos(ex + d)^4 - 177b^4 - 310b^2c^2 - 128c^4 + 2(22b^4 + 15b^2c^2 - 27c^4) \cos(ex + d)^2 + 4(5b^4 - 6b^2c^2 + c^4) \cos(ex + d) \right)}{35e \sqrt{\sqrt{b^2 + c^2} + b \cos(ex + d) + c \sin(ex + d)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x, algorithm="fricas")

[Out] 2/35*(5*(b^4 - 6*b^2*c^2 + c^4)*cos(e*x + d)^4 - 177*b^4 - 310*b^2*c^2 - 128*c^4 + 2*(22*b^4 + 15*b^2*c^2 - 27*c^4)*cos(e*x + d)^2 + 4*(5*(b^3*c - b*c^3)*cos(e*x + d)^3 + (22*b^3*c + 27*b*c^3)*cos(e*x + d))*sin(e*x + d) + 2*(11*(b^3 - 3*b*c^2)*cos(e*x + d)^3 + (53*b^3 + 86*b*c^2)*cos(e*x + d) + (53*b^2*c + 64*c^3 + 11*(3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d))*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0Evaluation time: 1.02sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l)Error: Bad Argument Value

maple [A] time = 0.36, size = 306, normalized size = 1.19

$$\frac{2(1 + \sin(ex + d - \arctan(-b, c))) (\sin(ex + d - \arctan(-b, c)) - 1) (5b^4 (\sin^3(ex + d - \arctan(-b, c))) + 10b^3c + 10b^2c^2 + 5b^2c^3 + 5b^2c^4 + 27b^4c + 27b^3c^2 + 71b^4c^2 + 142b^2c^2 + 177c^4) / \cos(ex + d - \arctan(-b, c)) / ((b^2 \sin(ex + d - \arctan(-b, c)) + c^2) \sin(ex + d - \arctan(-b, c)) + b^2 + c^2) / (b^2 + c^2)^{1/2}}{(b^2 + c^2)^{7/2}}, x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x)

[Out] 2/35*(1+sin(e*x+d-arctan(-b,c)))*(sin(e*x+d-arctan(-b,c))-1)*(5*b^4*sin(e*x+d-arctan(-b,c))^3+10*b^2*c^2*sin(e*x+d-arctan(-b,c))^3+5*c^4*sin(e*x+d-arctan(-b,c))^3+27*b^4*c*sin(e*x+d-arctan(-b,c))^2+54*b^2*c^2*sin(e*x+d-arctan(-b,c))^2+27*c^4*sin(e*x+d-arctan(-b,c))^2+71*b^4*c*sin(e*x+d-arctan(-b,c))+142*b^2*c^2*sin(e*x+d-arctan(-b,c))+71*c^4*sin(e*x+d-arctan(-b,c))+177*b^4+354*b^2*c^2+177*c^4)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2)*sin(e*x+d-arctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))/e

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b \cos(d + ex) + c \sin(d + ex) + \sqrt{b^2 + c^2} \right)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(7/2),x)

[Out] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(7/2),x)

[Out] Timed out

$$3.431 \quad \int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{5/2} dx$$

Optimal. Leaf size=190

$$\frac{2(c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{3/2}}{5e} - \frac{16\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex))}{5e}$$

[Out] $-2/5*(c*\cos(e*x+d)-b*\sin(e*x+d))*(b*\cos(e*x+d)+c*\sin(e*x+d)+(b^2+c^2)^{(1/2)})^{(3/2)}/e-64/15*(b^2+c^2)*(c*\cos(e*x+d)-b*\sin(e*x+d))/e/(b*\cos(e*x+d)+c*\sin(e*x+d)+(b^2+c^2)^{(1/2)})^{(1/2)}-16/15*(c*\cos(e*x+d)-b*\sin(e*x+d))*(b^2+c^2)^{(1/2)}*(b*\cos(e*x+d)+c*\sin(e*x+d)+(b^2+c^2)^{(1/2)})^{(1/2)}/e$

Rubi [A] time = 0.12, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {3113, 3112}

$$\frac{2(c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{3/2}}{5e} - \frac{16\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex))}{5e}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2), x]

[Out] $(-64*(b^2 + c^2)*(c*\cos[d + e*x] - b*\sin[d + e*x]))/(15*e*\sqrt{\sqrt{b^2 + c^2} + b*\cos[d + e*x] + c*\sin[d + e*x]}) - (16*\sqrt{b^2 + c^2}*(c*\cos[d + e*x] - b*\sin[d + e*x])*\sqrt{\sqrt{b^2 + c^2} + b*\cos[d + e*x] + c*\sin[d + e*x]})/(15*e) - (2*(c*\cos[d + e*x] - b*\sin[d + e*x])*(\sqrt{b^2 + c^2} + b*\cos[d + e*x] + c*\sin[d + e*x])^{(3/2)})/(5*e)$

Rule 3112

Int[Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rule 3113

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^n, x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]

Rubi steps

$$\int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{5/2} dx = -\frac{2(c \cos(d + ex) - b \sin(d + ex)) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{3/2}}{5e}$$

$$= -\frac{16\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex)) \sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}}{15e}$$

$$= -\frac{64 (b^2 + c^2) (c \cos(d + ex) - b \sin(d + ex))}{15e \sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}} - \frac{16\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex))}{15e}$$

Mathematica [C] time = 34.26, size = 11771, normalized size = 61.95

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2), x]

[Out] Result too large to show

fricas [A] time = 0.96, size = 189, normalized size = 0.99

$$\frac{2 \left(3(b^3 - 3bc^2) \cos(ex + d)^3 + (29b^3 + 38bc^2) \cos(ex + d) + (29b^2c + 32c^3 + 3(3b^2c - c^3) \cos(ex + d)^2) \sin(ex + d) \right)}{15e \sqrt{\sqrt{b^2 + c^2} + b \cos(ex + d) + c \sin(ex + d)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2), x, algorithm="fricas")

[Out] 2/15*(3*(b^3 - 3*b*c^2)*cos(e*x + d)^3 + (29*b^3 + 38*b*c^2)*cos(e*x + d) + (29*b^2*c + 32*c^3 + 3*(3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d) + (22*b*c*cos(e*x + d)*sin(e*x + d) + 11*(b^2 - c^2)*cos(e*x + d)^2 - 43*b^2 - 32*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0Evaluation time: 0.43sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.37, size = 200, normalized size = 1.05

$$\frac{2(1 + \sin(ex + d - \arctan(-b, c))) \sqrt{b^2 + c^2} (\sin(ex + d - \arctan(-b, c)) - 1) (3b^2 (\sin^2(ex + d - \arctan(-b, c)) - 1) + 15 \cos(ex + d - \arctan(-b, c)))}{15e \sqrt{\sqrt{b^2 + c^2} + b \cos(ex + d - \arctan(-b, c)) + c \sin(ex + d - \arctan(-b, c))}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x)`

[Out] $2/15*(1+\sin(e*x+d-\arctan(-b,c)))*(b^2+c^2)^{(1/2)}*(\sin(e*x+d-\arctan(-b,c))-1)*(3*b^2*\sin(e*x+d-\arctan(-b,c))^2+3*c^2*\sin(e*x+d-\arctan(-b,c))^2+14*b^2*\sin(e*x+d-\arctan(-b,c))+14*c^2*\sin(e*x+d-\arctan(-b,c))+43*b^2+43*c^2)/\cos(e*x+d-\arctan(-b,c))/((b^2*\sin(e*x+d-\arctan(-b,c))+c^2*\sin(e*x+d-\arctan(-b,c))+b^2+c^2)/(b^2+c^2)^{(1/2)})^{(1/2)}/e$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b \cos(d + ex) + c \sin(d + ex) + \sqrt{b^2 + c^2} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(5/2),x)`

[Out] `int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(5/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(5/2),x)`

[Out] Timed out

$$3.432 \quad \int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{3/2} dx$$

Optimal. Leaf size=126

$$\frac{2\sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)} (c \cos(d + ex) - b \sin(d + ex))}{3e} - \frac{8\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex))}{3e\sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}}$$

[Out] $-8/3*(c*\cos(e*x+d)-b*\sin(e*x+d))*(b^2+c^2)^{(1/2)}/e/(b*\cos(e*x+d)+c*\sin(e*x+d)+(b^2+c^2)^{(1/2)})^{(1/2)}-2/3*(c*\cos(e*x+d)-b*\sin(e*x+d))*(b*\cos(e*x+d)+c*\sin(e*x+d)+(b^2+c^2)^{(1/2)})^{(1/2)}/e$

Rubi [A] time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {3113, 3112}

$$\frac{2\sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)} (c \cos(d + ex) - b \sin(d + ex))}{3e} - \frac{8\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex))}{3e\sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2), x]

[Out] $(-8*\text{Sqrt}[b^2 + c^2]*(c*\text{Cos}[d + e*x] - b*\text{Sin}[d + e*x]))/(3*e*\text{Sqrt}[\text{Sqrt}[b^2 + c^2] + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]]) - (2*(c*\text{Cos}[d + e*x] - b*\text{Sin}[d + e*x])*\text{Sqrt}[\text{Sqrt}[b^2 + c^2] + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]])/(3*e)$

Rule 3112

Int[Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rule 3113

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^n], x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{3/2} dx &= -\frac{2(c \cos(d + ex) - b \sin(d + ex))\sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}}{3e} \\ &= -\frac{8\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex))}{3e\sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}} - \frac{2(c \cos(d + ex) - b \sin(d + ex))}{3e} \end{aligned}$$

Mathematica [C] time = 33.01, size = 11679, normalized size = 92.69

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[b^2 + c^2] + b*cos[d + e*x] + c*sin[d + e*x])^(3/2), x]

[Out] Result too large to show

fricas [A] time = 0.68, size = 125, normalized size = 0.99

$$\frac{2 \left(2bc \cos(ex + d) \sin(ex + d) + (b^2 - c^2) \cos^2(ex + d) - 5b^2 - 4c^2 + 4\sqrt{b^2 + c^2} (b \cos(ex + d) + c \sin(ex + d)) \right)}{3(ce \cos(ex + d) - be \sin(ex + d))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2), x, algorithm="fricas")

[Out] 2/3*(2*b*c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 - 5*b^2 - 4*c^2 + 4*sqrt(b^2 + c^2)*(b*cos(e*x + d) + c*sin(e*x + d)))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.37, size = 126, normalized size = 1.00

$$\frac{2(1 + \sin(ex + d - \arctan(-b, c))) (b^2 + c^2) (\sin(ex + d - \arctan(-b, c)) - 1) (\sin(ex + d - \arctan(-b, c)) + 5)}{3 \cos(ex + d - \arctan(-b, c)) \sqrt{\frac{b^2 \sin(ex + d - \arctan(-b, c)) + c^2 \sin(ex + d - \arctan(-b, c)) + b^2 + c^2}{\sqrt{b^2 + c^2}}}} e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2), x)

[Out] 2/3*(1+sin(e*x+d-arctan(-b,c)))*(b^2+c^2)*(sin(e*x+d-arctan(-b,c))-1)*(sin(e*x+d-arctan(-b,c))+5)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2), x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b \cos(d + ex) + c \sin(d + ex) + \sqrt{b^2 + c^2} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(3/2), x)

[Out] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \cos(d + ex) + c \sin(d + ex) + \sqrt{b^2 + c^2} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(3/2), x)

[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2))**(3/2), x)

$$3.433 \quad \int \sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)} dx$$

Optimal. Leaf size=55

$$-\frac{2(c \cos(d + ex) - b \sin(d + ex))}{e \sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}}$$

[Out] -2*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2)

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {3112}

$$-\frac{2(c \cos(d + ex) - b \sin(d + ex))}{e \sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]

[Out] (-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]])

Rule 3112

Int[Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\int \sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)} dx = -\frac{2(c \cos(d + ex) - b \sin(d + ex))}{e \sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}}$$

Mathematica [C] time = 32.71, size = 11586, normalized size = 210.65

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]

[Out] Result too large to show

fricas [A] time = 0.91, size = 80, normalized size = 1.45

$$\frac{2 \sqrt{b \cos(ex + d) + c \sin(ex + d) + \sqrt{b^2 + c^2}} \left(b \cos(ex + d) + c \sin(ex + d) - \sqrt{b^2 + c^2} \right)}{ce \cos(ex + d) - be \sin(ex + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm="fricas")

[Out] $2\sqrt{b\cos(ex+d) + c\sin(ex+d) + \sqrt{b^2 + c^2}}(b\cos(ex+d) + c\sin(ex+d) - \sqrt{b^2 + c^2}) / (c e \cos(ex+d) - b e \sin(ex+d))$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm="giac")`

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.36, size = 113, normalized size = 2.05

$$\frac{2(1 + \sin(ex + d - \arctan(-b, c))) \sqrt{b^2 + c^2} (\sin(ex + d - \arctan(-b, c)) - 1)}{\cos(ex + d - \arctan(-b, c)) \sqrt{\frac{b^2 \sin(ex + d - \arctan(-b, c)) + c^2 \sin(ex + d - \arctan(-b, c)) + b^2 + c^2}{\sqrt{b^2 + c^2}}}} e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x)`

[Out] $2(1 + \sin(ex + d - \arctan(-b, c))) (b^2 + c^2)^{1/2} (\sin(ex + d - \arctan(-b, c)) - 1) / \cos(ex + d - \arctan(-b, c)) / ((b^2 \sin(ex + d - \arctan(-b, c)) + c^2 \sin(ex + d - \arctan(-b, c)) + b^2 + c^2) / (b^2 + c^2)^{1/2})^{1/2} / e$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b \cos(d + ex) + c \sin(d + ex) + \sqrt{b^2 + c^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(1/2),x)`

[Out] `int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \cos(d + ex) + c \sin(d + ex) + \sqrt{b^2 + c^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(1/2),x)`

[Out] `Integral(sqrt(b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2)), x)`

$$3.434 \quad \int \frac{1}{\sqrt{\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)}} dx$$

Optimal. Leaf size=88

$$\frac{\sqrt{2} \tanh^{-1} \left(\frac{\sqrt[4]{b^2+c^2} \sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2} \sqrt{\sqrt{b^2+c^2} \cos(-\tan^{-1}(b,c)+d+ex) + \sqrt{b^2+c^2}}} \right)}{e \sqrt[4]{b^2+c^2}}$$

[Out] arctanh(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/((b^2+c^2)^(1/2)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))*2^(1/2)/(b^2+c^2)^(1/4)/e

Rubi [A] time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3115, 2649, 206}

$$\frac{\sqrt{2} \tanh^{-1} \left(\frac{\sqrt[4]{b^2+c^2} \sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2} \sqrt{\sqrt{b^2+c^2} \cos(-\tan^{-1}(b,c)+d+ex) + \sqrt{b^2+c^2}}} \right)}{e \sqrt[4]{b^2+c^2}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]

[Out] (Sqrt[2]*ArcTanh[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[Sqrt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]))/((b^2 + c^2)^(1/4)*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3115

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\int \frac{1}{\sqrt{\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}} dx = \int \frac{1}{\sqrt{\sqrt{b^2 + c^2} + \sqrt{b^2 + c^2} \cos(d + ex - \tan^{-1}(b, c))}} dx$$

$$= \frac{2 \operatorname{Subst} \left(\int \frac{1}{2\sqrt{b^2 + c^2} - x^2} dx, x, -\frac{\sqrt{b^2 + c^2} \sin(d + ex - \tan^{-1}(b, c))}{\sqrt{\sqrt{b^2 + c^2} + \sqrt{b^2 + c^2} \cos(d + ex - \tan^{-1}(b, c))}} \right)}{e}$$

$$= \frac{\sqrt{2} \tanh^{-1} \left(\frac{\sqrt[4]{b^2 + c^2} \sin(d + ex - \tan^{-1}(b, c))}{\sqrt{2} \sqrt{\sqrt{b^2 + c^2} + \sqrt{b^2 + c^2} \cos(d + ex - \tan^{-1}(b, c))}} \right)}{\sqrt[4]{b^2 + c^2} e}$$

Mathematica [C] time = 33.83, size = 63264, normalized size = 718.91

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]], x]

[Out] Result too large to show

fricas [B] time = 2.78, size = 349, normalized size = 3.97

$$\sqrt{2} \log \left(\frac{(3b^2c - c^3) \cos(ex+d)^3 + (b^2c + 4c^3) \cos(ex+d) - (3b^3 + 4bc^2 + (b^3 - 3bc^2) \cos(ex+d)^2) \sin(ex+d) + \frac{2\sqrt{2}(2(b^3 + bc^2) \cos(ex+d) + 2(b^2c + c^3) \sin(ex+d))}{3b^2c \cos(ex+d) - (3b^2c - c^3) \cos(ex+d)}}{3b^2c \cos(ex+d) - (3b^2c - c^3) \cos(ex+d)} \right)$$

2(b

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2), x, algorithm="fricas")

[Out] 1/2*sqrt(2)*log(((3*b^2*c - c^3)*cos(e*x + d)^3 + (b^2*c + 4*c^3)*cos(e*x + d) - (3*b^3 + 4*b*c^2 + (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d) + 2*sqrt(2)*(2*(b^3 + b*c^2)*cos(e*x + d) + 2*(b^2*c + c^3)*sin(e*x + d) - (2*b*c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + b^2 + 2*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))/(b^2 + c^2)^(1/4) - 4*(2*b*c*cos(e*x + d)^2 - (b^2 - c^2)*cos(e*x + d)*sin(e*x + d) - b*c)*sqrt(b^2 + c^2))/(3*b^2*c*cos(e*x + d) - (3*b^2*c - c^3)*cos(e*x + d)^3 - (b^3 - (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d)))/((b^2 + c^2)^(1/4)*e)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0sym2poly/r2s

ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.38, size = 172, normalized size = 1.95

$$\frac{(1 + \sin(ex + d - \arctan(-b, c))) \sqrt{-\sqrt{b^2 + c^2}} (\sin(ex + d - \arctan(-b, c)) - 1) \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{-\sqrt{b^2 + c^2}} (\sin(ex + d - \arctan(-b, c)) - 1)}{2(b^2 + c^2)}\right)}{(b^2 + c^2)^{\frac{1}{4}} \cos(ex + d - \arctan(-b, c)) \sqrt{\frac{b^2 \sin(ex + d - \arctan(-b, c)) + c^2 \sin(ex + d - \arctan(-b, c)) + b^2 + c^2}{\sqrt{b^2 + c^2}}}} e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2), x)

[Out] $-(1 + \sin(e*x + d - \arctan(-b, c))) * (-\sqrt{b^2 + c^2})^{1/2} * (\sin(e*x + d - \arctan(-b, c)) - 1)^{1/2} * 2^{1/2} / (b^2 + c^2)^{1/4} * \operatorname{arctanh}(1/2 * (-\sqrt{b^2 + c^2})^{1/2} * (\sin(e*x + d - \arctan(-b, c)) - 1)^{1/2} * 2^{1/2} / (b^2 + c^2)^{1/4}) / \cos(e*x + d - \arctan(-b, c)) / ((b^2 * \sin(e*x + d - \arctan(-b, c)) + c^2 * \sin(e*x + d - \arctan(-b, c)) + b^2 + c^2) / (b^2 + c^2)^{1/2})^{1/2} / e$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2), x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b \cos(d + ex) + c \sin(d + ex) + \sqrt{b^2 + c^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(1/2), x)

[Out] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \cos(d + ex) + c \sin(d + ex) + \sqrt{b^2 + c^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(1/2), x)

[Out] Integral(1/sqrt(b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2)), x)

$$3.435 \quad \int \frac{1}{\left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^{3/2}} dx$$

Optimal. Leaf size=160

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2} \sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2} \sqrt{\sqrt{b^2+c^2} \cos(-\tan^{-1}(b,c)+d+ex) + \sqrt{b^2+c^2}}}\right)}{2\sqrt{2} e (b^2 + c^2)^{3/4}} - \frac{c \cos(d + ex) - b \sin(d + ex)}{2e\sqrt{b^2 + c^2} \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^{3/2}}$$

[Out] 1/4*arctanh(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/((b^2+c^2)^(1/2)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))/(b^2+c^2)^(3/4)/e*2^(1/2)+1/2*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2)

Rubi [A] time = 0.13, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3116, 3115, 2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2} \sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2} \sqrt{\sqrt{b^2+c^2} \cos(-\tan^{-1}(b,c)+d+ex) + \sqrt{b^2+c^2}}}\right)}{2\sqrt{2} e (b^2 + c^2)^{3/4}} - \frac{c \cos(d + ex) - b \sin(d + ex)}{2e\sqrt{b^2 + c^2} \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2), x]

[Out] ArcTanh[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[Sqrt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])]/(2*Sqrt[2]*(b^2 + c^2)^(3/4)*e) - (c*Cos[d + e*x] - b*Sin[d + e*x])/(2*Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3115

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rule 3116

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^n, x_Symbol] :> Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}} dx &= -\frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}} \\
&= -\frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}} \\
&= -\frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin(d+ex-\tan^{-1}(b,c))}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\right)}{2\sqrt{2}(b^2+c^2)^{3/4}e} - \frac{1}{2\sqrt{b^2+c^2}}
\end{aligned}$$

Mathematica [F] time = 180.02, size = 0, normalized size = 0.00

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]

[Out] \$Aborted

fricas [B] time = 1.82, size = 646, normalized size = 4.04

$$(3\sqrt{2}b^2c\cos(ex+d) - \sqrt{2}(3b^2c - c^3)\cos(ex+d)^3 - (\sqrt{2}b^3 - \sqrt{2}(b^3 - 3bc^2)\cos(ex+d)^2)\sin(ex+d))(b^2 - \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm="fricas")

[Out] -1/8*((3*sqrt(2)*b^2*c*cos(e*x + d) - sqrt(2)*(3*b^2*c - c^3)*cos(e*x + d)^3 - (sqrt(2)*b^3 - sqrt(2)*(b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d))*(b^2 + c^2)^(1/4)*log(((3*b^2*c - c^3)*cos(e*x + d)^3 + (b^2*c + 4*c^3)*cos(e*x + d) - (3*b^3 + 4*b*c^2 + (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d) - 2*(2*sqrt(2)*b*c*cos(e*x + d)*sin(e*x + d) + sqrt(2)*(b^2 - c^2)*cos(e*x + d)^2 + sqrt(2)*(b^2 + 2*c^2) - 2*(sqrt(2)*b*cos(e*x + d) + sqrt(2)*c*sin(e*x + d))*sqrt(b^2 + c^2))*(b^2 + c^2)^(1/4)*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2)) - 4*(2*b*c*cos(e*x + d)^2 - (b^2 - c^2)*cos(e*x + d)*sin(e*x + d) - b*c)*sqrt(b^2 + c^2))/(3*b^2*c*cos(e*x + d) - (3*b^2*c - c^3)*cos(e*x + d)^3 - (b^3 - (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d)) + 4*(2*(b^3 + b*c^2)*cos(e*x + d) + 2*(b^2*c + c^3)*sin(e*x + d) - (2*b*c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + b^2 + 2*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2)))/((3*b^4*c + 2*b^2*c^3 - c^5)*e*cos(e*x + d)^3 - 3*(b^4*c + b^2*c^3)*e*cos(e*x + d)

) - ((b^5 - 2*b^3*c^2 - 3*b*c^4)*e*cos(e*x + d)^2 - (b^5 + b^3*c^2)*e)*sin(e*x + d)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.41, size = 350, normalized size = 2.19

$$\left(\sin(ex + d - \arctan(-b, c)) \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{-\sqrt{b^2+c^2}} \sin(ex+d-\arctan(-b,c)) + \sqrt{b^2+c^2} \sqrt{2}}{2(b^2+c^2)^{\frac{1}{4}}} \right) \right) (b^2 + c^2) + 2\sqrt{-\sqrt{b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x)

[Out]
$$-1/4/(b^2+c^2)^{7/4}*(\sin(e*x+d-\arctan(-b,c))*2^{1/2}*\operatorname{arctanh}(1/2*(-(b^2+c^2)^{1/2}*\sin(e*x+d-\arctan(-b,c))+(b^2+c^2)^{1/2}))^{1/2}/(b^2+c^2)^{1/4})*(b^2+c^2)+2*(-(b^2+c^2)^{1/2}*\sin(e*x+d-\arctan(-b,c))+(b^2+c^2)^{1/2})^{1/2}*(b^2+c^2)^{3/4}+2^{1/2}*\operatorname{arctanh}(1/2*(-(b^2+c^2)^{1/2}*\sin(e*x+d-\arctan(-b,c))+(b^2+c^2)^{1/2}))^{1/2}/(b^2+c^2)^{1/4})*b^2+2^{1/2}*\operatorname{arctanh}(1/2*(-(b^2+c^2)^{1/2}*\sin(e*x+d-\arctan(-b,c))+(b^2+c^2)^{1/2}))^{1/2}/(b^2+c^2)^{1/4})*c^2*(-(b^2+c^2)^{1/2}*(\sin(e*x+d-\arctan(-b,c))-1))^{1/2}/\cos(e*x+d-\arctan(-b,c))/((b^2*\sin(e*x+d-\arctan(-b,c))+c^2*\sin(e*x+d-\arctan(-b,c))+b^2+c^2)/(b^2+c^2)^{1/2})^{1/2}/e$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(b \cos(d + ex) + c \sin(d + ex) + \sqrt{b^2 + c^2})^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(3/2),x)

[Out] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \cos(d + ex) + c \sin(d + ex) + \sqrt{b^2 + c^2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(3/2), x)

[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2))**(-3/2), x)

$$3.436 \quad \int \frac{1}{\left(\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^{5/2}} dx$$

Optimal. Leaf size=226

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2} \sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2} \sqrt{\sqrt{b^2+c^2} \cos(-\tan^{-1}(b,c)+d+ex) + \sqrt{b^2+c^2}}}\right)}{16\sqrt{2} e (b^2 + c^2)^{5/4}} - \frac{3(c \cos(d + ex) - b \sin(d + ex))}{16e (b^2 + c^2) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^{3/2}}$$

[Out] $3/32 * \operatorname{arctanh}(1/2 * (b^2 + c^2)^{1/4} * \sin(d + e * x - \operatorname{arctan}(b, c)) * 2^{1/2}) / ((b^2 + c^2)^{1/2} + \cos(d + e * x - \operatorname{arctan}(b, c)) * (b^2 + c^2)^{1/2})^{1/2} / (b^2 + c^2)^{5/4} / e * 2^{1/2} + 1/4 * (-c * \cos(e * x + d) + b * \sin(e * x + d)) / e / (b^2 + c^2)^{1/2} / (b * \cos(e * x + d) + c * \sin(e * x + d) + (b^2 + c^2)^{1/2})^{5/2} - 3/16 * (c * \cos(e * x + d) - b * \sin(e * x + d)) / (b^2 + c^2) / e / (b * \cos(e * x + d) + c * \sin(e * x + d) + (b^2 + c^2)^{1/2})^{3/2}$

Rubi [A] time = 0.19, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3116, 3115, 2649, 206}

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2} \sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2} \sqrt{\sqrt{b^2+c^2} \cos(-\tan^{-1}(b,c)+d+ex) + \sqrt{b^2+c^2}}}\right)}{16\sqrt{2} e (b^2 + c^2)^{5/4}} - \frac{3(c \cos(d + ex) - b \sin(d + ex))}{16e (b^2 + c^2) \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sqrt}[b^2 + c^2] + b * \operatorname{Cos}[d + e * x] + c * \operatorname{Sin}[d + e * x])^{-5/2}, x]$

[Out] $(3 * \operatorname{ArcTanh}[(b^2 + c^2)^{1/4} * \operatorname{Sin}[d + e * x - \operatorname{ArcTan}[b, c]]) / (\operatorname{Sqrt}[2] * \operatorname{Sqrt}[\operatorname{Sqrt}[b^2 + c^2] + \operatorname{Sqrt}[b^2 + c^2] * \operatorname{Cos}[d + e * x - \operatorname{ArcTan}[b, c]]]) / (16 * \operatorname{Sqrt}[2] * (b^2 + c^2)^{5/4} * e) - (c * \operatorname{Cos}[d + e * x] - b * \operatorname{Sin}[d + e * x]) / (4 * \operatorname{Sqrt}[b^2 + c^2] * e * (\operatorname{Sqrt}[b^2 + c^2] + b * \operatorname{Cos}[d + e * x] + c * \operatorname{Sin}[d + e * x])^{5/2}) - (3 * (c * \operatorname{Cos}[d + e * x] - b * \operatorname{Sin}[d + e * x])) / (16 * (b^2 + c^2) * e * (\operatorname{Sqrt}[b^2 + c^2] + b * \operatorname{Cos}[d + e * x] + c * \operatorname{Sin}[d + e * x])^{3/2})$

Rule 206

$\operatorname{Int}[(a + (b * x^2)^{-1}), x_Symbol] \rightarrow \operatorname{Simp}[(1 * \operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2] * x) / \operatorname{Rt}[a, 2]]) / (\operatorname{Rt}[a, 2] * \operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 2649

$\operatorname{Int}[1 / \operatorname{Sqrt}[(a + (b * x) * \sin[(c + d * x)])], x_Symbol] \rightarrow \operatorname{Dist}[-2/d, \operatorname{Subst}[\operatorname{Int}[1 / (2 * a - x^2), x], x, (b * \operatorname{Cos}[c + d * x]) / \operatorname{Sqrt}[a + b * \operatorname{Sin}[c + d * x]]], x] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \ \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 3115

$\operatorname{Int}[1 / \operatorname{Sqrt}[\cos[(d + e * x)] * (b + a + (c + e * x) * \sin[(d + e * x)])], x_Symbol] \rightarrow \operatorname{Int}[1 / \operatorname{Sqrt}[a + \operatorname{Sqrt}[b^2 + c^2] * \operatorname{Cos}[d + e * x - \operatorname{ArcTan}[b, c]]], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \operatorname{EqQ}[a^2 - b^2 - c^2, 0]$

Rule 3116

$\operatorname{Int}[(\cos[(d + e * x)] * (b + a + (c + e * x) * \sin[(d + e * x)])^{-n}), x_Symbol] \rightarrow \operatorname{Simp}[(c * \operatorname{Cos}[d + e * x] - b * \operatorname{Sin}[d + e * x]) * (a + b * \operatorname{Cos}[d + e * x])^{-n}, x]$

*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c,
 , d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^{5/2}} dx &= -\frac{c \cos(d + ex) - b \sin(d + ex)}{4\sqrt{b^2 + c^2} e \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^{5/2}} \\
 &= -\frac{c \cos(d + ex) - b \sin(d + ex)}{4\sqrt{b^2 + c^2} e \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^{5/2}} \\
 &= -\frac{c \cos(d + ex) - b \sin(d + ex)}{4\sqrt{b^2 + c^2} e \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^{5/2}} \\
 &= -\frac{c \cos(d + ex) - b \sin(d + ex)}{4\sqrt{b^2 + c^2} e \left(\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^{5/2}} \\
 &= \frac{3 \tanh^{-1}\left(\frac{\sqrt[4]{b^2 + c^2} \sin(d + ex - \tan^{-1}(b, c))}{\sqrt{2} \sqrt{\sqrt{b^2 + c^2} + \sqrt{b^2 + c^2} \cos(d + ex - \tan^{-1}(b, c))}}\right)}{16\sqrt{2} (b^2 + c^2)^{5/4} e} - \frac{1}{4\sqrt{b^2 + c^2}}
 \end{aligned}$$

Mathematica [F] time = 180.03, size = 0, normalized size = 0.00

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2), x]

[Out] \$Aborted

fricas [B] time = 2.53, size = 895, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2), x, algorithm="fricas")

[Out] 1/32*(3*sqrt(1/2)*(5*b^4*c*cos(e*x + d) + (5*b^4*c - 10*b^2*c^3 + c^5)*cos(e*x + d)^5 - 10*(b^4*c - b^2*c^3)*cos(e*x + d)^3 - (b^5 + (b^5 - 10*b^3*c^2 + 5*b*c^4)*cos(e*x + d)^4 - 2*(b^5 - 5*b^3*c^2)*cos(e*x + d)^2)*sin(e*x + d))*log(((3*b^2*c - c^3)*cos(e*x + d)^3 + (b^2*c + 4*c^3)*cos(e*x + d) - (3*b^3 + 4*b*c^2 + (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d) + 4*sqrt(1/2)*(2*(b^3 + b*c^2)*cos(e*x + d) + 2*(b^2*c + c^3)*sin(e*x + d) - (2*b*c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + b^2 + 2*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))/(b^2 + c^2)^(1/4) - 4*(2*b*c*cos(e*x + d)^2 - (b^2 - c^2)*cos(e*x + d)*sin(e*x + d) -

$$\frac{b*c*\sqrt{b^2+c^2}}{(3*b^2*c*\cos(e*x+d) - (3*b^2*c - c^3)*\cos(e*x+d)^3 - (b^3 - (b^3 - 3*b*c^2)*\cos(e*x+d)^2)*\sin(e*x+d)) / (b^2+c^2)^{1/4} + 2*(3*(b^4 - 6*b^2*c^2 + c^4)*\cos(e*x+d)^4 - 7*b^4 - 26*b^2*c^2 - 16*c^4 - 6*(2*b^4 - 3*b^2*c^2 - c^4)*\cos(e*x+d)^2 + 12*((b^3*c - b*c^3)*\cos(e*x+d)^3 - (2*b^3*c + b*c^3)*\cos(e*x+d))*\sin(e*x+d) - 2*((b^3 - 3*b*c^2)*\cos(e*x+d)^3 - 3*(3*b^3 + 2*b*c^2)*\cos(e*x+d) - (9*b^2*c + 8*c^3 - (3*b^2*c - c^3)*\cos(e*x+d)^2)*\sin(e*x+d))*\sqrt{b^2+c^2}*\sqrt{b*\cos(e*x+d) + c*\sin(e*x+d) + \sqrt{b^2+c^2}}) / ((5*b^6*c - 5*b^4*c^3 - 9*b^2*c^5 + c^7)*e*\cos(e*x+d)^5 - 10*(b^6*c - b^2*c^5)*e*\cos(e*x+d)^3 + 5*(b^6*c + b^4*c^3)*e*\cos(e*x+d) - ((b^7 - 9*b^5*c^2 - 5*b^3*c^4 + 5*b*c^6)*e*\cos(e*x+d)^4 - 2*(b^7 - 4*b^5*c^2 - 5*b^3*c^4)*e*\cos(e*x+d)^2 + (b^7 + b^5*c^2)*e)*\sin(e*x+d)}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0Evaluation time: 0.84sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.40, size = 350, normalized size = 1.55

$$\frac{\left(\sin(ex+d - \arctan(-b,c)) \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{-\sqrt{b^2+c^2}} \sin(ex+d - \arctan(-b,c)) + \sqrt{b^2+c^2} \sqrt{2}}{2(b^2+c^2)^{\frac{1}{4}}} \right) \right) (b^2+c^2) + 2\sqrt{-\sqrt{b^2+c^2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x)

[Out] 1/4*(sin(e*x+d-arctan(-b,c))*2^(1/2)*arctanh(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^2^(1/2)/(b^2+c^2)^(1/4))*2^(1/2)/(b^2+c^2)^(1/4))*(b^2+c^2)+2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arctanh(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^2^(1/2)/(b^2+c^2)^(1/4))*c^2*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1))^2^(1/2)/(b^2+c^2)^(5/4)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c)))+b^2+c^2)/(b^2+c^2)^(1/2))^2^(1/2)/e

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b \cos(d + ex) + c \sin(d + ex) + \sqrt{b^2 + c^2}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(5/2), x)

[Out] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \cos(d + ex) + c \sin(d + ex) + \sqrt{b^2 + c^2}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(5/2), x)

[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2))**(-5/2), x)

$$3.437 \quad \int \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{5/2} dx$$

Optimal. Leaf size=196

$$\frac{2(c \cos(d + ex) - b \sin(d + ex)) \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{3/2}}{5e} + \frac{16\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex)) \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{5/2}}{5e}$$

```
[Out] -2/5*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2)/e-64/15*(b^2+c^2)*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2)+16/15*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(1/2)*(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2)/e
```

Rubi [A] time = 0.13, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {3113, 3112}

$$\frac{2(c \cos(d + ex) - b \sin(d + ex)) \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{3/2}}{5e} + \frac{16\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex)) \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{5/2}}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2), x]
```

```
[Out] (-64*(b^2 + c^2)*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(15*e*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) + (16*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x])*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]])/(15*e) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))/(5*e)
```

Rule 3112

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3113

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^n, x_Symbol] := -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{5/2} dx = -\frac{2(c \cos(d + ex) - b \sin(d + ex)) \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)}{5e}$$

$$= \frac{16\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex)) \sqrt{-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}}{15e}$$

$$= -\frac{64(b^2 + c^2)(c \cos(d + ex) - b \sin(d + ex))}{15e\sqrt{-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}} + \frac{16\sqrt{b^2 + c^2}}{15e}$$

Mathematica [C] time = 34.19, size = 11602, normalized size = 59.19

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2), x]

[Out] Result too large to show

fricas [A] time = 1.12, size = 192, normalized size = 0.98

$$\frac{2 \left(3(b^3 - 3bc^2) \cos(ex + d)^3 + (29b^3 + 38bc^2) \cos(ex + d) + (29b^2c + 32c^3 + 3(3b^2c - c^3) \cos(ex + d)^2) \sin(ex + d) \right)}{15(c \cos(ex + d) - b \sin(ex + d)) \sqrt{-\sqrt{b^2 + c^2} + b \cos(ex + d) + c \sin(ex + d)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2), x, algorithm="fricas")

[Out] 2/15*(3*(b^3 - 3*b*c^2)*cos(e*x + d)^3 + (29*b^3 + 38*b*c^2)*cos(e*x + d) + (29*b^2*c + 32*c^3 + 3*(3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d) - (22*b*c*cos(e*x + d)*sin(e*x + d) + 11*(b^2 - c^2)*cos(e*x + d)^2 - 43*b^2 - 32*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) - sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:Simplification assuming b near 0Evaluation time: 0.44sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.38, size = 204, normalized size = 1.04

$$\frac{2(\sin(ex + d - \arctan(-b, c)) - 1) \sqrt{b^2 + c^2} (1 + \sin(ex + d - \arctan(-b, c))) \left(3b^2 (\sin^2(ex + d - \arctan(-b, c))) \right)}{15 \cos(ex + d - \arctan(-b, c)) \sqrt{-\sqrt{b^2 + c^2} + b \cos(ex + d) + c \sin(ex + d)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x)

[Out] 2/15*(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)^(1/2)*(1+sin(e*x+d-arctan(-b,c)))*(3*b^2*sin(e*x+d-arctan(-b,c))^2+3*c^2*sin(e*x+d-arctan(-b,c))^2-14*b^2*sin(e*x+d-arctan(-b,c))-14*c^2*sin(e*x+d-arctan(-b,c))+43*b^2+43*c^2)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b \cos(d + ex) + c \sin(d + ex) - \sqrt{b^2 + c^2} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(5/2),x)

[Out] int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(5/2),x)

[Out] Timed out

$$3.438 \quad \int \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{3/2} dx$$

Optimal. Leaf size=130

$$\frac{8\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex))}{3e\sqrt{-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}} - \frac{2(c \cos(d + ex) - b \sin(d + ex))\sqrt{-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}}{3e}$$

[Out] 8/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(1/2)/e/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2)-2/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2)/e

Rubi [A] time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {3113, 3112}

$$\frac{8\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex))}{3e\sqrt{-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}} - \frac{2(c \cos(d + ex) - b \sin(d + ex))\sqrt{-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}}{3e}$$

Antiderivative was successfully verified.

[In] Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2), x]

[Out] (8*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(3*e*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e)

Rule 3112

Int[Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]]], x_Symbol] :> Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rule 3113

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^(n_.), x_Symbol] :> -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[(a*(2*n - 1))/n, Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex) \right)^{3/2} dx &= -\frac{2(c \cos(d + ex) - b \sin(d + ex))\sqrt{-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}}{3e} \\ &= \frac{8\sqrt{b^2 + c^2} (c \cos(d + ex) - b \sin(d + ex))}{3e\sqrt{-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}} - \frac{2(c \cos(d + ex) - b \sin(d + ex))\sqrt{-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}}{3e} \end{aligned}$$

Mathematica [C] time = 21.61, size = 11512, normalized size = 88.55

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2), x]

[Out] Result too large to show

fricas [A] time = 1.48, size = 127, normalized size = 0.98

$$\frac{2 \left(2bc \cos(ex + d) \sin(ex + d) + (b^2 - c^2) \cos^2(ex + d) - 5b^2 - 4c^2 - 4\sqrt{b^2 + c^2} (b \cos(ex + d) + c \sin(ex + d)) \right)}{3(ce \cos(ex + d) - be \sin(ex + d))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2), x, algorithm="fricas")

[Out] 2/3*(2*b*c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 - 5*b^2 - 4*c^2 - 4*sqrt(b^2 + c^2)*(b*cos(e*x + d) + c*sin(e*x + d)))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) - sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming b near 0sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.38, size = 130, normalized size = 1.00

$$\frac{2(\sin(ex + d - \arctan(-b, c)) - 1)(b^2 + c^2)(1 + \sin(ex + d - \arctan(-b, c)))(\sin(ex + d - \arctan(-b, c)) - 3 \cos(ex + d - \arctan(-b, c)) \sqrt{\frac{b^2 \sin(ex + d - \arctan(-b, c)) + c^2 \sin(ex + d - \arctan(-b, c)) - b^2 - c^2}{\sqrt{b^2 + c^2}}}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2), x)

[Out] 2/3*(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)*(1+sin(e*x+d-arctan(-b,c)))*(sin(e*x+d-arctan(-b,c))-5)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2), x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b \cos(d + ex) + c \sin(d + ex) - \sqrt{b^2 + c^2} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(3/2), x)

[Out] int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \cos(d + ex) + c \sin(d + ex) - \sqrt{b^2 + c^2} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(3/2), x)

[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2))**(3/2), x)

$$3.439 \quad \int \sqrt{-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)} dx$$

Optimal. Leaf size=57

$$-\frac{2(c \cos(d + ex) - b \sin(d + ex))}{e\sqrt{-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}}$$

[Out] $-2*(c*\cos(e*x+d)-b*\sin(e*x+d))/e/(b*\cos(e*x+d)+c*\sin(e*x+d)-(\sqrt{b^2+c^2})^{1/2})^{1/2}$

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {3112}

$$-\frac{2(c \cos(d + ex) - b \sin(d + ex))}{e\sqrt{-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]

[Out] $(-2*(c*\cos[d + e*x] - b*\sin[d + e*x]))/(e*\sqrt{-\sqrt{b^2 + c^2} + b*\cos[d + e*x] + c*\sin[d + e*x]})$

Rule 3112

Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Simp[(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\int \sqrt{-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)} dx = -\frac{2(c \cos(d + ex) - b \sin(d + ex))}{e\sqrt{-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}}$$

Mathematica [C] time = 21.95, size = 11415, normalized size = 200.26

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]

[Out] Result too large to show

fricas [A] time = 2.89, size = 80, normalized size = 1.40

$$\frac{2\left(b \cos(ex + d) + c \sin(ex + d) + \sqrt{b^2 + c^2}\right)\sqrt{b \cos(ex + d) + c \sin(ex + d) - \sqrt{b^2 + c^2}}}{ce \cos(ex + d) - be \sin(ex + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(\sqrt{b^2+c^2})^{1/2})^{1/2},x, algorithm="fricas")

[Out] $2*(b*\cos(e*x + d) + c*\sin(e*x + d) + \sqrt{b^2 + c^2})*\sqrt{b*\cos(e*x + d) + c*\sin(e*x + d) - \sqrt{b^2 + c^2}}/(c*e*\cos(e*x + d) - b*e*\sin(e*x + d))$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="giac")`

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Simplification assuming b near 0sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.36, size = 117, normalized size = 2.05

$$\frac{2(\sin(ex + d - \arctan(-b, c)) - 1)\sqrt{b^2 + c^2}(1 + \sin(ex + d - \arctan(-b, c)))}{\cos(ex + d - \arctan(-b, c))\sqrt{\frac{b^2 \sin(ex + d - \arctan(-b, c)) + c^2 \sin(ex + d - \arctan(-b, c)) - b^2 - c^2}{\sqrt{b^2 + c^2}}}} e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x)`

[Out] $2*(\sin(e*x+d-\arctan(-b,c))-1)*(b^2+c^2)^(1/2)*(1+\sin(e*x+d-\arctan(-b,c)))/\cos(e*x+d-\arctan(-b,c))/((b^2*\sin(e*x+d-\arctan(-b,c))+c^2*\sin(e*x+d-\arctan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b \cos(d + ex) + c \sin(d + ex) - \sqrt{b^2 + c^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(1/2),x)`

[Out] `int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \cos(d + ex) + c \sin(d + ex) - \sqrt{b^2 + c^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(1/2),x)`

[Out] `Integral(sqrt(b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2)), x)`

$$3.440 \quad \int \frac{1}{\sqrt{-\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)}} dx$$

Optimal. Leaf size=91

$$\frac{\sqrt{2} \tan^{-1} \left(\frac{\sqrt[4]{b^2+c^2} \sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2} \sqrt{\sqrt{b^2+c^2} \cos(-\tan^{-1}(b,c)+d+ex) - \sqrt{b^2+c^2}}} \right)}{e \sqrt[4]{b^2+c^2}}$$

[Out] $-\arctan(1/2*(b^2+c^2)^{(1/4)}*\sin(d+e*x-\arctan(b,c))*2^{(1/2)/(- (b^2+c^2)^{(1/2)}+\cos(d+e*x-\arctan(b,c))*(b^2+c^2)^{(1/2))}^{(1/2)})*2^{(1/2)/(b^2+c^2)^{(1/4)}/e}$

Rubi [A] time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {3115, 2649, 204}

$$\frac{\sqrt{2} \tan^{-1} \left(\frac{\sqrt[4]{b^2+c^2} \sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2} \sqrt{\sqrt{b^2+c^2} \cos(-\tan^{-1}(b,c)+d+ex) - \sqrt{b^2+c^2}}} \right)}{e \sqrt[4]{b^2+c^2}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]

[Out] $-\left(\frac{\sqrt{2} \operatorname{ArcTan}\left[\left(b^2+c^2\right)^{1/4} \sin\left[d+e x-\operatorname{ArcTan}[b, c]\right]\right]}{\sqrt{2} \sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2} \cos\left[d+e x-\operatorname{ArcTan}[b, c]\right]}}\right) / \left(\left(b^2+c^2\right)^{1/4} e\right)$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3115

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rubi steps

$$\int \frac{1}{\sqrt{-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)}} dx = \int \frac{1}{\sqrt{-\sqrt{b^2 + c^2} + \sqrt{b^2 + c^2} \cos(d + ex - \tan^{-1}(b, c))}} dx$$

$$= \frac{2 \operatorname{Subst} \left(\int \frac{1}{-2\sqrt{b^2 + c^2} - x^2} dx, x, -\frac{\sqrt{b^2 + c^2} \sin(d + ex - \tan^{-1}(b, c))}{\sqrt{-\sqrt{b^2 + c^2} + \sqrt{b^2 + c^2} \cos(d + ex - \tan^{-1}(b, c))}} \right)}{e}$$

$$= \frac{\sqrt{2} \tan^{-1} \left(\frac{\sqrt[4]{b^2 + c^2} \sin(d + ex - \tan^{-1}(b, c))}{\sqrt{2} \sqrt{-\sqrt{b^2 + c^2} + \sqrt{b^2 + c^2} \cos(d + ex - \tan^{-1}(b, c))}} \right)}{\sqrt[4]{b^2 + c^2} e}$$

Mathematica [C] time = 34.47, size = 61904, normalized size = 680.26

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]

[Out] Result too large to show

fricas [B] time = 1.15, size = 107, normalized size = 1.18

$$\frac{\sqrt{2} \arctan \left(\frac{\sqrt{2} (b \cos(ex+d) + c \sin(ex+d) + \sqrt{b^2 + c^2}) \sqrt{b \cos(ex+d) + c \sin(ex+d) - \sqrt{b^2 + c^2}}}{2 (b^2 + c^2)^{\frac{1}{4}} (c \cos(ex+d) - b \sin(ex+d))} \right)}{(b^2 + c^2)^{\frac{1}{4}} e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="fricas")

[Out] sqrt(2)*arctan(-1/2*sqrt(2)*(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) - sqrt(b^2 + c^2))/((b^2 + c^2)^(1/4)*(c*cos(e*x + d) - b*sin(e*x + d))))/((b^2 + c^2)^(1/4)*e)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:Simplification assuming b near 0sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.30, size = 175, normalized size = 1.92

$$\frac{(\sin(ex + d - \arctan(-b, c)) - 1) \sqrt{-\sqrt{b^2 + c^2} (1 + \sin(ex + d - \arctan(-b, c)))} \sqrt{2} \arctan \left(\frac{\sqrt{-\sqrt{b^2 + c^2} (1 + \sin(ex + d - \arctan(-b, c)))}}{2(b^2 + c^2)^{\frac{1}{4}}} \right)}{(b^2 + c^2)^{\frac{1}{4}} \cos(ex + d - \arctan(-b, c)) \sqrt{\frac{b^2 \sin(ex + d - \arctan(-b, c)) + c^2 \sin(ex + d - \arctan(-b, c)) - b^2 - c^2}{\sqrt{b^2 + c^2}}}} e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x)`

[Out] `(sin(e*x+d-arctan(-b,c))-1)*(-(b^2+c^2)^(1/2)*(1+sin(e*x+d-arctan(-b,c))))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4)*arctan(1/2*(-(b^2+c^2)^(1/2)*(1+sin(e*x+d-arctan(-b,c))))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b \cos(d + ex) + c \sin(d + ex) - \sqrt{b^2 + c^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(1/2),x)`

[Out] `int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \cos(d + ex) + c \sin(d + ex) - \sqrt{b^2 + c^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(1/2),x)`

[Out] `Integral(1/sqrt(b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2)), x)`

$$3.441 \quad \int \frac{1}{\left(-\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^{3/2}} dx$$

Optimal. Leaf size=164

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2} \sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2} \sqrt{\sqrt{b^2+c^2} \cos(-\tan^{-1}(b,c)+d+ex) - \sqrt{b^2+c^2}}}\right)}{2\sqrt{2} e (b^2 + c^2)^{3/4}} + \frac{c \cos(d + ex) - b \sin(d + ex)}{2e\sqrt{b^2 + c^2} \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^{3/2}}$$

[Out] $1/4 * \arctan(1/2 * (b^2 + c^2)^{1/4} * \sin(d + e * x - \arctan(b, c)) * 2^{1/2} / (- (b^2 + c^2)^{1/2} + \cos(d + e * x - \arctan(b, c)) * (b^2 + c^2)^{1/2}))^{1/2} / (b^2 + c^2)^{3/4} / e * 2^{1/2} + 1/2 * (c * \cos(e * x + d) - b * \sin(e * x + d)) / e / (b * \cos(e * x + d) + c * \sin(e * x + d) - (b^2 + c^2)^{1/2})^{3/2} / (b^2 + c^2)^{1/2}$

Rubi [A] time = 0.13, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3116, 3115, 2649, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2} \sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2} \sqrt{\sqrt{b^2+c^2} \cos(-\tan^{-1}(b,c)+d+ex) - \sqrt{b^2+c^2}}}\right)}{2\sqrt{2} e (b^2 + c^2)^{3/4}} + \frac{c \cos(d + ex) - b \sin(d + ex)}{2e\sqrt{b^2 + c^2} \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(-Sqrt[b^2 + c^2] + b * Cos[d + e * x] + c * Sin[d + e * x])^(-3/2), x]

[Out] ArcTan[((b^2 + c^2)^(1/4) * Sin[d + e * x - ArcTan[b, c]]) / (Sqrt[2] * Sqrt[-Sqrt[b^2 + c^2] + Sqrt[b^2 + c^2] * Cos[d + e * x - ArcTan[b, c]])] / (2 * Sqrt[2] * (b^2 + c^2)^(3/4) * e) + (c * Cos[d + e * x] - b * Sin[d + e * x]) / (2 * Sqrt[b^2 + c^2] * e * (-Sqrt[b^2 + c^2] + b * Cos[d + e * x] + c * Sin[d + e * x])^(3/2))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b * Cos[c + d * x]) / Sqrt[a + b * Sin[c + d * x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3115

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Int[1/Sqrt[a + Sqrt[b^2 + c^2] * Cos[d + e * x - ArcTan[b, c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rule 3116

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^n, x_Symbol] := Simp[((c * Cos[d + e * x] - b * Sin[d + e * x]) * (a + b * Cos[d + e * x] + c * Sin[d + e * x])^n) / (a * e * (2 * n + 1)), x] + Dist[(n + 1) / (a * (2 * n + 1)), Int[(a + b * Cos[d + e * x] + c * Sin[d + e * x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}} dx &= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)} \\
&= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)} \\
&= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin(d+ex-\tan^{-1}(b,c))}{\sqrt{2}\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos(d+ex-\tan^{-1}(b,c))}}\right)}{2\sqrt{2}(b^2+c^2)^{3/4}e} + \frac{1}{2\sqrt{b^2+c^2}}
\end{aligned}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2), x]

[Out] \$Aborted

fricas [B] time = 0.86, size = 442, normalized size = 2.70

$$(3\sqrt{2}b^2c\cos(ex+d) - \sqrt{2}(3b^2c - c^3)\cos(ex+d)^3 - (\sqrt{2}b^3 - \sqrt{2}(b^3 - 3bc^2)\cos(ex+d)^2)\sin(ex+d))(b^2 + c^2)^{3/4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm="fricas")

[Out] 1/4*((3*sqrt(2)*b^2*c*cos(e*x + d) - sqrt(2)*(3*b^2*c - c^3)*cos(e*x + d)^3 - (sqrt(2)*b^3 - sqrt(2)*(b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d))*(b^2 + c^2)^(1/4)*arctan(-1/2*(b^2 + c^2)^(1/4)*sqrt(b*cos(e*x + d) + c*sin(e*x + d) - sqrt(b^2 + c^2))*((sqrt(2)*b*cos(e*x + d) + sqrt(2)*c*sin(e*x + d))*sqrt(b^2 + c^2) + sqrt(2)*(b^2 + c^2)))/((b^2*c + c^3)*cos(e*x + d) - (b^3 + b*c^2)*sin(e*x + d)) - 2*(2*(b^3 + b*c^2)*cos(e*x + d) + 2*(b^2*c + c^3)*sin(e*x + d) + (2*b*c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + b^2 + 2*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) - sqrt(b^2 + c^2)))/((3*b^4*c + 2*b^2*c^3 - c^5)*e*cos(e*x + d)^3 - 3*(b^4*c + b^2*c^3)*e*cos(e*x + d) - ((b^5 - 2*b^3*c^2 - 3*b*c^4)*e*cos(e*x + d)^2 - (b^5 + b^3*c^2)*e)*sin(e*x + d))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Simplification assuming b near 0sym2poly/r2s ym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 0.36, size = 363, normalized size = 2.21

$$\left(-\sin(ex + d - \arctan(-b, c)) \sqrt{2} \arctan\left(\frac{\sqrt{-\sqrt{b^2+c^2}} \sin(ex+d-\arctan(-b,c)) - \sqrt{b^2+c^2} \sqrt{2}}{2(b^2+c^2)^{\frac{1}{4}}} \right) (b^2 + c^2) + 2\sqrt{-\sqrt{b^2+c^2}} \sin \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x)

[Out] 1/4/(b^2+c^2)^(7/4)*(-sin(e*x+d-arctan(-b,c))*2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^2^(1/2)/(b^2+c^2)^(1/4))*(b^2+c^2)+2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^2^(1/2)*(b^2+c^2)^(3/4)+2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^2^(1/2)/(b^2+c^2)^(1/4))*c^2)*(-(b^2+c^2)^(1/2)*(1+sin(e*x+d-arctan(-b,c))))^(1/2)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b \cos(d + ex) + c \sin(d + ex) - \sqrt{b^2 + c^2} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(3/2),x)

[Out] int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \cos(d + ex) + c \sin(d + ex) - \sqrt{b^2 + c^2} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2))**(-3/2), x)
```

$$3.442 \quad \int \frac{1}{\left(-\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^{5/2}} dx$$

Optimal. Leaf size=232

$$\frac{3 \tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2} \sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2} \sqrt{\sqrt{b^2+c^2} \cos(-\tan^{-1}(b,c)+d+ex) - \sqrt{b^2+c^2}}}\right)}{16\sqrt{2} e (b^2+c^2)^{5/4}} - \frac{3(c \cos(d+ex) - b \sin(d+ex))}{16e (b^2+c^2) \left(-\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^{3/2}}$$

[Out] $-3/32*\arctan(1/2*(b^2+c^2)^{(1/4)}*\sin(d+e*x-\arctan(b,c))*2^{(1/2)}/(-(b^2+c^2)^{(1/2)}+\cos(d+e*x-\arctan(b,c))*(b^2+c^2)^{(1/2))^{(1/2))}/(b^2+c^2)^{(5/4)}/e*2^{(1/2)}-3/16*(c*\cos(e*x+d)-b*\sin(e*x+d))/(b^2+c^2)/e/(b*\cos(e*x+d)+c*\sin(e*x+d)-(b^2+c^2)^{(1/2))^{(3/2)}+1/4*(c*\cos(e*x+d)-b*\sin(e*x+d))/e/(b*\cos(e*x+d)+c*\sin(e*x+d)-(b^2+c^2)^{(1/2))^{(5/2)}/(b^2+c^2)^{(1/2)}$

Rubi [A] time = 0.17, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3116, 3115, 2649, 204}

$$\frac{3 \tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2} \sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2} \sqrt{\sqrt{b^2+c^2} \cos(-\tan^{-1}(b,c)+d+ex) - \sqrt{b^2+c^2}}}\right)}{16\sqrt{2} e (b^2+c^2)^{5/4}} - \frac{3(c \cos(d+ex) - b \sin(d+ex))}{16e (b^2+c^2) \left(-\sqrt{b^2+c^2} + b \cos(d+ex) + c \sin(d+ex)\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2), x]

[Out] $(-3*\text{ArcTan}[\frac{(b^2+c^2)^{(1/4)}*\sin[d+e*x-\text{ArcTan}[b,c]]}{\sqrt{2}*\sqrt{\sqrt{b^2+c^2}*\cos[d+e*x-\text{ArcTan}[b,c]]}}]/(\sqrt{2}*\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}*\cos[d+e*x-\text{ArcTan}[b,c]]}))/((16*\sqrt{2}*(b^2+c^2)^{(5/4)}*e)+(c*\cos[d+e*x]-b*\sin[d+e*x])/(4*\sqrt{b^2+c^2})*e*(-\sqrt{b^2+c^2}+b*\cos[d+e*x]+c*\sin[d+e*x])^{(5/2)})-(3*(c*\cos[d+e*x]-b*\sin[d+e*x]))/(16*(b^2+c^2)*e*(-\sqrt{b^2+c^2}+b*\cos[d+e*x]+c*\sin[d+e*x])^{(3/2)})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3115

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]

Rule 3116

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] := Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x])^(n-1/2) - (c*Sin[d + e*x] + b*Cos[d + e*x])*(a + b*Cos[d + e*x])^(n-1/2))/((n-1/2)*Sqrt[a + b*Cos[d + e*x]]), x]

*x] + c*Sin[d + e*x])^n)/(a*e*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)),
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c,
 , d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]

Rubi steps

$$\int \frac{1}{\left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)^{5/2}} dx = \frac{c \cos(d + ex) - b \sin(d + ex)}{4\sqrt{b^2 + c^2} e \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)}$$

$$= \frac{c \cos(d + ex) - b \sin(d + ex)}{4\sqrt{b^2 + c^2} e \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)}$$

$$= \frac{c \cos(d + ex) - b \sin(d + ex)}{4\sqrt{b^2 + c^2} e \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)}$$

$$= \frac{c \cos(d + ex) - b \sin(d + ex)}{4\sqrt{b^2 + c^2} e \left(-\sqrt{b^2 + c^2} + b \cos(d + ex) + c \sin(d + ex)\right)}$$

$$= -\frac{3 \tan^{-1}\left(\frac{\sqrt[4]{b^2 + c^2} \sin(d + ex - \tan^{-1}(b, c))}{\sqrt{2} \sqrt{-\sqrt{b^2 + c^2} + \sqrt{b^2 + c^2}} \cos(d + ex - \tan^{-1}(b, c))}\right)}{16\sqrt{2} (b^2 + c^2)^{5/4} e} + \frac{1}{4\sqrt{b^2 + c^2} e}$$

Mathematica [F] time = 180.06, size = 0, normalized size = 0.00

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2), x]

[Out] \$Aborted

fricas [B] time = 1.20, size = 655, normalized size = 2.82

$$3 \sqrt{\frac{1}{2}} (5b^4c \cos(ex+d) + (5b^4c - 10b^2c^3 + c^5) \cos(ex+d)^5 - 10(b^4c - b^2c^3) \cos(ex+d)^3 - (b^5 + (b^5 - 10b^3c^2 + 5bc^4) \cos(ex+d)^4 - 2(b^5 - 5b^3c^2) \cos(ex+d)^2) \cos(ex+d)^2)$$

$$(b^2 + c^2)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2), x, algorithm="fricas")

[Out] 1/16*(3*sqrt(1/2)*(5*b^4*c*cos(e*x + d) + (5*b^4*c - 10*b^2*c^3 + c^5)*cos(e*x + d)^5 - 10*(b^4*c - b^2*c^3)*cos(e*x + d)^3 - (b^5 + (b^5 - 10*b^3*c^2 + 5*b*c^4)*cos(e*x + d)^4 - 2*(b^5 - 5*b^3*c^2)*cos(e*x + d)^2)*sin(e*x + d))*arctan(-sqrt(1/2)*(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))*s

```

sqrt(b*cos(e*x + d) + c*sin(e*x + d) - sqrt(b^2 + c^2))/((b^2 + c^2)^(1/4))*
(c*cos(e*x + d) - b*sin(e*x + d))/((b^2 + c^2)^(1/4) + (3*(b^4 - 6*b^2*c^2
+ c^4)*cos(e*x + d)^4 - 7*b^4 - 26*b^2*c^2 - 16*c^4 - 6*(2*b^4 - 3*b^2*c^2
- c^4)*cos(e*x + d)^2 + 12*((b^3*c - b*c^3)*cos(e*x + d)^3 - (2*b^3*c + b*c
^3)*cos(e*x + d))*sin(e*x + d) + 2*((b^3 - 3*b*c^2)*cos(e*x + d)^3 - 3*(3*b
^3 + 2*b*c^2)*cos(e*x + d) - (9*b^2*c + 8*c^3 - (3*b^2*c - c^3)*cos(e*x + d
)^2)*sin(e*x + d))*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) -
sqrt(b^2 + c^2)))/((5*b^6*c - 5*b^4*c^3 - 9*b^2*c^5 + c^7)*e*cos(e*x + d)^5
- 10*(b^6*c - b^2*c^5)*e*cos(e*x + d)^3 + 5*(b^6*c + b^4*c^3)*e*cos(e*x +
d) - ((b^7 - 9*b^5*c^2 - 5*b^3*c^4 + 5*b*c^6)*e*cos(e*x + d)^4 - 2*(b^7 - 4
*b^5*c^2 - 5*b^3*c^4)*e*cos(e*x + d)^2 + (b^7 + b^5*c^2)*e)*sin(e*x + d))

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Simplification assuming b near 0Evaluation t
ime: 0.87sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l)
Error: Bad Argument Value
```

maple [A] time = 0.45, size = 363, normalized size = 1.56

$$\left(-\sin(ex + d - \arctan(-b, c)) \sqrt{2} \arctan\left(\frac{\sqrt{-\sqrt{b^2+c^2}} \sin(ex+d-\arctan(-b,c)) - \sqrt{b^2+c^2} \sqrt{2}}{2(b^2+c^2)^{\frac{1}{4}}} \right) (b^2 + c^2) + 2\sqrt{-\sqrt{b^2+c^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x)
```

```
[Out] -1/4*(-sin(e*x+d-arctan(-b,c))*2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x
+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*(b^2+c^2)+
2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*(b^2+c^2
)^(3/4)+2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c
^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arctan(1/2*(-(b^2+c^2
)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1
/4))*c^2)*(-(b^2+c^2)^(1/2)*(1+sin(e*x+d-arctan(-b,c))))^(1/2)/(b^2+c^2)^(5
/4)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arc
tan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(b \cos(d + ex) + c \sin(d + ex) - \sqrt{b^2 + c^2})^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(5/2), x)`

[Out] `int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \cos(d + ex) + c \sin(d + ex) - \sqrt{b^2 + c^2}\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(5/2), x)`

[Out] `Integral((b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2))**(-5/2), x)`

$$3.443 \quad \int \frac{\sin(x)}{a+b \cos(x)+c \sin(x)} dx$$

Optimal. Leaf size=101

$$\frac{2ac \tan^{-1} \left(\frac{(a-b) \tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}} \right)}{(b^2+c^2) \sqrt{a^2-b^2-c^2}} - \frac{b \log(a+b \cos(x)+c \sin(x))}{b^2+c^2} + \frac{cx}{b^2+c^2}$$

[Out] $c*x/(b^2+c^2)-b*\ln(a+b*\cos(x)+c*\sin(x))/(b^2+c^2)-2*a*c*\arctan((c+(a-b)*\tan(1/2*x))/(\sqrt{a^2-b^2-c^2}))^{(1/2)})/(b^2+c^2)/(\sqrt{a^2-b^2-c^2})^{(1/2)}$

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3137, 3124, 618, 204}

$$\frac{2ac \tan^{-1} \left(\frac{(a-b) \tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}} \right)}{(b^2+c^2) \sqrt{a^2-b^2-c^2}} - \frac{b \log(a+b \cos(x)+c \sin(x))}{b^2+c^2} + \frac{cx}{b^2+c^2}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(a + b*Cos[x] + c*Sin[x]),x]

[Out] $(c*x)/(b^2+c^2) - (2*a*c*ArcTan[(c+(a-b)*Tan[x/2])/Sqrt[a^2-b^2-c^2]])/(Sqrt[a^2-b^2-c^2]*(b^2+c^2)) - (b*Log[a+b*Cos[x]+c*Sin[x]])/(b^2+c^2)$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3137

Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])*((a_.) + cos[(d_.) + (e_.)*(x_)])*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Simp[(c*C*(d + e*x))/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C}, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sin(x)}{a + b \cos(x) + c \sin(x)} dx &= \frac{cx}{b^2 + c^2} - \frac{b \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} - \frac{(ac) \int \frac{1}{a + b \cos(x) + c \sin(x)} dx}{b^2 + c^2} \\
&= \frac{cx}{b^2 + c^2} - \frac{b \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} - \frac{(2ac) \text{Subst} \left(\int \frac{1}{a + b + 2cx + (a-b)x^2} dx, x, \right)}{b^2 + c^2} \\
&= \frac{cx}{b^2 + c^2} - \frac{b \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} + \frac{(4ac) \text{Subst} \left(\int \frac{1}{-4(a^2 - b^2 - c^2) - x^2} dx, x, \right)}{b^2 + c^2} \\
&= \frac{cx}{b^2 + c^2} - \frac{2ac \tan^{-1} \left(\frac{c + (a-b) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2 - c^2}} \right)}{\sqrt{a^2 - b^2 - c^2} (b^2 + c^2)} - \frac{b \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 80, normalized size = 0.79

$$\frac{2ac \tanh^{-1} \left(\frac{(a-b) \tan\left(\frac{x}{2}\right) + c}{\sqrt{-a^2 + b^2 + c^2}} \right)}{\sqrt{-a^2 + b^2 + c^2}} - \frac{b \log(a + b \cos(x) + c \sin(x)) + cx}{b^2 + c^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/(a + b*Cos[x] + c*Sin[x]), x]

[Out] (c*x + (2*a*c*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] - b*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)

fricas [B] time = 1.21, size = 579, normalized size = 5.73

$$\left[\frac{\sqrt{-a^2 + b^2 + c^2} ac \log \left(\frac{a^2 b^2 - 2b^4 - c^4 - (a^2 + 3b^2)c^2 - (2a^2 b^2 - b^4 - 2a^2 c^2 + c^4) \cos(x)^2 - 2(ab^3 + abc^2) \cos(x) - 2(ab^2 c + ac^3 - (bc^3 - (2a^2 b - b^3)))}{2ab \cos(x) + (b^2 - c^2) \cos(x)^2 + a^2} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+b*cos(x)+c*sin(x)), x, algorithm="fricas")

[Out] [-1/2*(sqrt(-a^2 + b^2 + c^2))*a*c*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)) + 2*(c^3 - (a^2 - b^2)*c)*x + (a^2*b - b^3 - b*c^2)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), -1/2*(2*sqrt(a^2 - b^2 - c^2))*a*c*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) + 2*(c^3 - (a^2 - b^2)*c)*x + (a^2*b - b^3 - b*c^2)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]

giac [A] time = 0.16, size = 160, normalized size = 1.58

$$\frac{2 \left(\pi \left[\frac{x}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a + 2b) + \arctan \left(\frac{a \tan\left(\frac{1}{2}x\right) - b \tan\left(\frac{1}{2}x\right) + c}{\sqrt{a^2 - b^2 - c^2}} \right) \right) ac}{\sqrt{a^2 - b^2 - c^2} (b^2 + c^2)} + \frac{cx}{b^2 + c^2} - \frac{b \log \left(-a \tan\left(\frac{1}{2}x\right)^2 + b \tan\left(\frac{1}{2}x\right) \right)}{b^2 + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*a*c/(sqrt(a^2 - b^2 - c^2)*(b^2 + c^2) + c*x/(b^2 + c^2) - b*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - a - b)/(b^2 + c^2) + b*log(tan(1/2*x)^2 + 1)/(b^2 + c^2)
```

```
maple [B] time = 0.12, size = 438, normalized size = 4.34
```

$$\frac{2 \ln \left(a \left(\tan^2 \left(\frac{x}{2} \right) \right) - b \left(\tan^2 \left(\frac{x}{2} \right) \right) + 2c \tan \left(\frac{x}{2} \right) + a + b \right) ab}{(2b^2 + 2c^2)(a - b)} + \frac{2 \ln \left(a \left(\tan^2 \left(\frac{x}{2} \right) \right) - b \left(\tan^2 \left(\frac{x}{2} \right) \right) + 2c \tan \left(\frac{x}{2} \right) + a + b \right)}{(2b^2 + 2c^2)(a - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] -2/(2*b^2+2*c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*a*b+2/(2*b^2+2*c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*b^2-4/(2*b^2+2*c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*c-4/(2*b^2+2*c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c*b+4/(2*b^2+2*c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c/(a-b)*a*b-4/(2*b^2+2*c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c/(a-b)*b^2+2/(2*b^2+2*c^2)*b*ln(1+tan(1/2*x)^2)+4/(2*b^2+2*c^2)*c*arctan(tan(1/2*x))
```

```
maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for more details)Is c^2+b^2-a^2 positive or negative?
```

```
mupad [B] time = 11.44, size = 950, normalized size = 9.41
```

$$\frac{\ln \left(64 \tan \left(\frac{x}{2} \right) (a - b)^2 - \frac{(a^2 b - b c^2 - b^3 + a c \sqrt{-a^2 + b^2 + c^2}) \left(32 a^2 c + 32 b^2 c - 64 a b c + 64 \tan \left(\frac{x}{2} \right) (a - b) (-a^2 + b a + c^2) + \dots \right)}{\dots}}{b - c 1i} + \frac{\ln \left(\tan \left(\frac{x}{2} \right) + 1i \right)}{b - c 1i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + b*cos(x) + c*sin(x)),x)
```

```
[Out] log(tan(x/2) + 1i)/(b - c*1i) + (log(tan(x/2) - 1i)*1i)/(b*1i - c) + (log(64*tan(x/2)*(a - b)^2 - ((a^2*b - b*c^2 - b^3 + a*c*(b^2 - a^2 + c^2)^(1/2)))*(32*a^2*c + 32*b^2*c - 64*a*b*c + 64*tan(x/2)*(a - b)*(a*b - a^2 + c^2) +
```

$$\begin{aligned} & ((a^2b - bc^2 - b^3 + ac(b^2 - a^2 + c^2)^{1/2}) * (32b^3c^3 - 32ac^3 - \\ & 64b^3c + 32\tan(x/2) * (a - b) * (2ab^2 - 2ac^2 + bc^2 - 2b^3) + 128a \\ & * b^2c - 64a^2bc + (32(a - b) * (a^2b - bc^2 - b^3 + ac(b^2 - a^2 + c^2)^{1/2}) * (3c^4\tan(x/2) + ac^3 + 3b^3c + 3b^3c + 2a^2b^2\tan(x/2) \\ & - 2a^2c^2\tan(x/2) + 3b^2c^2\tan(x/2) - 2ab^3\tan(x/2) + ab^2c - 4 \\ & * a^2bc - 2ab^2c^2\tan(x/2))) / ((b^2 + c^2) * (b^2 - a^2 + c^2))) / ((b^2 + c^2) * (b^2 - a^2 + c^2))) / ((b^2 + c^2) * (b^2 - a^2 + c^2))) * (b * (a^2 - c^2) - \\ & b^3 + ac(b^2 - a^2 + c^2)^{1/2}) / ((b^2 + c^2) * (b^2 - a^2 + c^2)) - (\log(\\ & 64\tan(x/2) * (a - b)^2 + ((bc^2 - a^2b + b^3 + ac(b^2 - a^2 + c^2)^{1/2}) * (32a^2c + 32b^2c - 64ab^2c + 64\tan(x/2) * (a - b) * (ab - a^2 + c^2) + \\ & ((bc^2 - a^2b + b^3 + ac(b^2 - a^2 + c^2)^{1/2}) * (32ac^3 - 32b^3c^3 \\ & + 64b^3c - 32\tan(x/2) * (a - b) * (2ab^2 - 2ac^2 + bc^2 - 2b^3) - 128a \\ & * b^2c + 64a^2bc + (32(a - b) * (bc^2 - a^2b + b^3 + ac(b^2 - a^2 + c^2)^{1/2}) * (3c^4\tan(x/2) + ac^3 + 3b^3c + 3b^3c + 2a^2b^2\tan(x/2) \\ &) - 2a^2c^2\tan(x/2) + 3b^2c^2\tan(x/2) - 2ab^3\tan(x/2) + ab^2c - \\ & 4a^2bc - 2ab^2c^2\tan(x/2))) / ((b^2 + c^2) * (b^2 - a^2 + c^2))) / ((b^2 + c^2) * (b^2 - a^2 + c^2))) / ((b^2 + c^2) * (b^2 - a^2 + c^2))) * (b^3 - b * (a^2 - \\ & c^2) + ac(b^2 - a^2 + c^2)^{1/2}) / ((b^2 + c^2) * (b^2 - a^2 + c^2)) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x)

[Out] Timed out

$$3.444 \quad \int \frac{\sin(x)}{1+\cos(x)+\sin(x)} dx$$

Optimal. Leaf size=22

$$\frac{x}{2} - \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

[Out] 1/2*x-ln(cos(1/2*x)+sin(1/2*x))

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.36, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3137, 3124, 31}

$$\frac{x}{2} - \frac{1}{2} \log\left(\tan\left(\frac{x}{2}\right) + 1\right) - \frac{1}{2} \log(\sin(x) + \cos(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(1 + Cos[x] + Sin[x]),x]

[Out] x/2 - Log[1 + Cos[x] + Sin[x]]/2 - Log[1 + Tan[x/2]]/2

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])⁽⁻¹⁾, x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3137

Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]) / ((a_.) + cos[(d_.) + (e_.)*(x_)])*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)], x_Symbol] :> Simp[(c*C*(d + e*x)) / (e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C) / (b^2 + c^2), Int[1/(a + b*cos[d + e*x] + c*sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*cos[d + e*x] + c*sin[d + e*x]]) / (e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C}, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sin(x)}{1+\cos(x)+\sin(x)} dx &= \frac{x}{2} - \frac{1}{2} \log(1 + \cos(x) + \sin(x)) - \frac{1}{2} \int \frac{1}{1 + \cos(x) + \sin(x)} dx \\ &= \frac{x}{2} - \frac{1}{2} \log(1 + \cos(x) + \sin(x)) - \text{Subst}\left(\int \frac{1}{2 + 2x} dx, x, \tan\left(\frac{x}{2}\right)\right) \\ &= \frac{x}{2} - \frac{1}{2} \log(1 + \cos(x) + \sin(x)) - \frac{1}{2} \log\left(1 + \tan\left(\frac{x}{2}\right)\right) \end{aligned}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 1.00

$$\frac{x}{2} - \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/(1 + Cos[x] + Sin[x]),x]

[Out] x/2 - Log[Cos[x/2] + Sin[x/2]]

fricas [A] time = 2.12, size = 11, normalized size = 0.50

$$\frac{1}{2}x - \frac{1}{2}\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="fricas")

[Out] 1/2*x - 1/2*log(sin(x) + 1)

giac [A] time = 0.14, size = 25, normalized size = 1.14

$$\frac{1}{2}x + \frac{1}{2}\log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) - \log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="giac")

[Out] 1/2*x + 1/2*log(tan(1/2*x)^2 + 1) - log(abs(tan(1/2*x) + 1))

maple [A] time = 0.11, size = 25, normalized size = 1.14

$$\frac{\ln\left(1 + \tan^2\left(\frac{x}{2}\right)\right)}{2} - \ln\left(1 + \tan\left(\frac{x}{2}\right)\right) + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(1+cos(x)+sin(x)),x)

[Out] 1/2*ln(1+tan(1/2*x)^2)-ln(1+tan(1/2*x))+1/2*x

maxima [B] time = 0.41, size = 41, normalized size = 1.86

$$\arctan\left(\frac{\sin(x)}{\cos(x) + 1}\right) - \log\left(\frac{\sin(x)}{\cos(x) + 1} + 1\right) + \frac{1}{2}\log\left(\frac{\sin(x)^2}{(\cos(x) + 1)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="maxima")

[Out] arctan(sin(x)/(cos(x) + 1)) - log(sin(x)/(cos(x) + 1) + 1) + 1/2*log(sin(x)^2/(cos(x) + 1)^2 + 1)

mupad [B] time = 2.79, size = 34, normalized size = 1.55

$$-\ln\left(\tan\left(\frac{x}{2}\right) + 1\right) + \ln\left(\tan\left(\frac{x}{2}\right) - i\right)\left(\frac{1}{2} - \frac{1}{2}i\right) + \ln\left(\tan\left(\frac{x}{2}\right) + 1i\right)\left(\frac{1}{2} + \frac{1}{2}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(cos(x) + sin(x) + 1),x)

[Out] log(tan(x/2) - 1i)*(1/2 - 1i/2) - log(tan(x/2) + 1) + log(tan(x/2) + 1i)*(1/2 + 1i/2)

sympy [A] time = 0.26, size = 22, normalized size = 1.00

$$\frac{x}{2} - \log\left(\tan\left(\frac{x}{2}\right) + 1\right) + \frac{\log\left(\tan^2\left(\frac{x}{2}\right) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x)
```

```
[Out] x/2 - log(tan(x/2) + 1) + log(tan(x/2)**2 + 1)/2
```

$$3.445 \quad \int \frac{1}{a+c \sec(x)+b \tan(x)} dx$$

Optimal. Leaf size=97

$$\frac{2ac \tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(a^2+b^2)\sqrt{a^2+b^2-c^2}} + \frac{b \log(a \cos(x) + b \sin(x) + c)}{a^2+b^2} + \frac{ax}{a^2+b^2}$$

[Out] a*x/(a^2+b^2)+b*ln(c+a*cos(x)+b*sin(x))/(a^2+b^2)+2*a*c*arctanh((b-(a-c)*tan(1/2*x))/(a^2+b^2-c^2)^(1/2))/(a^2+b^2)/(a^2+b^2-c^2)^(1/2)

Rubi [A] time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3159, 3138, 3124, 618, 206}

$$\frac{2ac \tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(a^2+b^2)\sqrt{a^2+b^2-c^2}} + \frac{b \log(a \cos(x) + b \sin(x) + c)}{a^2+b^2} + \frac{ax}{a^2+b^2}$$

Antiderivative was successfully verified.

[In] Int[(a + c*Sec[x] + b*Tan[x])^(-1), x]

[Out] (a*x)/(a^2 + b^2) + (2*a*c*ArcTanh[(b - (a - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]]/((a^2 + b^2)*Sqrt[a^2 + b^2 - c^2]) + (b*Log[c + a*Cos[x] + b*Sin[x]])/(a^2 + b^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3138

Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] := Simp[(b*B*(d + e*x))/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[(c*B*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]

Rule 3159

Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),

x] /; FreeQ[{a, b, c, d, e}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{a + c \sec(x) + b \tan(x)} dx &= \int \frac{\cos(x)}{c + a \cos(x) + b \sin(x)} dx \\
 &= \frac{ax}{a^2 + b^2} + \frac{b \log(c + a \cos(x) + b \sin(x))}{a^2 + b^2} - \frac{(ac) \int \frac{1}{c + a \cos(x) + b \sin(x)} dx}{a^2 + b^2} \\
 &= \frac{ax}{a^2 + b^2} + \frac{b \log(c + a \cos(x) + b \sin(x))}{a^2 + b^2} - \frac{(2ac) \operatorname{Subst}\left(\int \frac{1}{a + c + 2bx + (-a + c)x^2} dx, x, \tan\left(\frac{x}{2}\right)\right)}{a^2 + b^2} \\
 &= \frac{ax}{a^2 + b^2} + \frac{b \log(c + a \cos(x) + b \sin(x))}{a^2 + b^2} + \frac{(4ac) \operatorname{Subst}\left(\int \frac{1}{4(a^2 + b^2 - c^2) - x^2} dx, x, 2b \tan\left(\frac{x}{2}\right)\right)}{a^2 + b^2} \\
 &= \frac{ax}{a^2 + b^2} + \frac{2ac \tanh^{-1}\left(\frac{b - (a - c) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 + b^2 - c^2}}\right)}{(a^2 + b^2) \sqrt{a^2 + b^2 - c^2}} + \frac{b \log(c + a \cos(x) + b \sin(x))}{a^2 + b^2}
 \end{aligned}$$

Mathematica [A] time = 0.19, size = 79, normalized size = 0.81

$$\frac{2ac \tanh^{-1}\left(\frac{(c-a) \tan\left(\frac{x}{2}\right) + b}{\sqrt{a^2 + b^2 - c^2}}\right)}{\sqrt{a^2 + b^2 - c^2}} + \frac{b \log(a \cos(x) + b \sin(x) + c) + ax}{a^2 + b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*Sec[x] + b*Tan[x])^(-1), x]

[Out] (a*x + (2*a*c*ArcTanh[(b + (-a + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2 - c^2] + b*Log[c + a*Cos[x] + b*Sin[x]])/(a^2 + b^2)

fricas [B] time = 1.76, size = 553, normalized size = 5.70

$$\left[\frac{\sqrt{a^2 + b^2 - c^2} ac \log\left(\frac{2a^4 + 3a^2b^2 + b^4 - (a^2 - b^2)c^2 + 2(a^3 + ab^2)c \cos(x) - (a^4 - b^4 - 2(a^2 - b^2)c^2) \cos(x)^2 + 2((a^2b + b^3)c - (a^3b + ab^3 - 2abc^2) \cos(x)) \sin(x)}{2ac \cos(x) + (a^2 - b^2) \cos(x)^2 + b^2 + c^2 + 2(ab \cos(x) + bc \sin(x))}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c*sec(x)+b*tan(x)), x, algorithm="fricas")

[Out] [1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((2*a^4 + 3*a^2*b^2 + b^4 - (a^2 - b^2)*c^2 + 2*(a^3 + a*b^2)*c*cos(x) - (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 + 2*((a^2*b + b^3)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (a^2*b + b^3)*cos(x) - (a^3 + a*b^2 + (a^2 - b^2)*c*cos(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x)) + 2*(a^3 + a*b^2 - a*c^2)*x + (a^2*b + b^3 - b*c^2)*log(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x)))/(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2), -1/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((a*c*cos(x) + b*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^2*b + b^3 - b*c^2)*cos(x) - (a^3 + a*b^2 - a*c^2)*sin(x))) - 2*(a^3 + a*b^2 - a*c^2)*x - (a^2*b + b^3 - b*c^2)*log(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x)))/(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2)]

giac [A] time = 0.15, size = 158, normalized size = 1.63

$$\frac{2 \left(\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2a + 2c) + \arctan \left(-\frac{a \tan\left(\frac{1}{2}x\right) - c \tan\left(\frac{1}{2}x\right) - b}{\sqrt{-a^2 - b^2 + c^2}} \right) \right) ac}{(a^2 + b^2) \sqrt{-a^2 - b^2 + c^2}} + \frac{ax}{a^2 + b^2} + \frac{b \log \left(-a \tan\left(\frac{1}{2}x\right)^2 + c \tan\left(\frac{1}{2}x\right)^2 + 2b \tan\left(\frac{1}{2}x\right) + a + c \right)}{a^2 + b^2} - b \log(\tan\left(\frac{1}{2}x\right)^2 + 1) / (a^2 + b^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sec(x)+b*tan(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*c) + arctan(-(a*tan(1/2*x) - c*tan(1/2*x) - b)/sqrt(-a^2 - b^2 + c^2)))*a*c/((a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)) + a*x/(a^2 + b^2) + b*log(-a*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a + c)/(a^2 + b^2) - b*log(tan(1/2*x)^2 + 1)/(a^2 + b^2)
```

maple [B] time = 0.15, size = 414, normalized size = 4.27

$$\frac{\ln \left(a \left(\tan^2\left(\frac{x}{2}\right) \right) - c \left(\tan^2\left(\frac{x}{2}\right) \right) - 2b \tan\left(\frac{x}{2}\right) - a - c \right) ab}{(a^2 + b^2)(a - c)} - \frac{\ln \left(a \left(\tan^2\left(\frac{x}{2}\right) \right) - c \left(\tan^2\left(\frac{x}{2}\right) \right) - 2b \tan\left(\frac{x}{2}\right) - a - c \right) cb}{(a^2 + b^2)(a - c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] 1/(a^2+b^2)/(a-c)*ln(a*tan(1/2*x)^2-c*tan(1/2*x)^2-2*b*tan(1/2*x)-a-c)*a*b-1/(a^2+b^2)/(a-c)*ln(a*tan(1/2*x)^2-c*tan(1/2*x)^2-2*b*tan(1/2*x)-a-c)*c*b+2/(a^2+b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2))*a*c-2/(a^2+b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2))*b^2+2/(a^2+b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2))*b^2/(a-c)*a-2/(a^2+b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2))*b^2/(a-c)*c-1/(a^2+b^2)*b*ln(1+tan(1/2*x)^2)+2/(a^2+b^2)*a*arctan(tan(1/2*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sec(x)+b*tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` for more details)Is c^2-b^2-a^2 positive or negative?
```

mupad [B] time = 13.03, size = 988, normalized size = 10.19

$$\ln \left(32ac - 32c^2 + 32b \tan\left(\frac{x}{2}\right) (a - c) + \frac{\left(a^2 b - b^3 c^2 + b^3 + a c \sqrt{a^2 + b^2 - c^2} \right) \left(32a^4 - 32a^2 b - 32b^2 c^2 + 32 \tan\left(\frac{x}{2}\right) (a - c) (-a^2 + 2ac + 3b^2 - 2c^2) \right)}{32a^4 - 32a^2 b - 32b^2 c^2 + 32 \tan\left(\frac{x}{2}\right) (a - c) (-a^2 + 2ac + 3b^2 - 2c^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b*tan(x) + c/cos(x)),x)`

[Out] $(\log(32*a*c - 32*c^2 + 32*b*\tan(x/2))*(a - c) + ((32*a^2*b - 32*b*c^2 + 32*\tan(x/2))*(a - c)*(2*a*c - a^2 + 3*b^2 - 2*c^2) - ((a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^{(1/2)})*(32*a^4 - 64*a^3*c - 64*a^2*b^2 + 32*a^2*c^2 - 32*b^2*c^2 + 96*a*b^2*c + 32*b*\tan(x/2))*(a - c)*(4*a^2 - 4*a*c + b^2) + (32*(a - c)*(a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^{(1/2)})*(3*b^4*\tan(x/2) + 3*a*b^3 + 3*a^3*b + b^3*c + 3*a^2*b^2*\tan(x/2) + 2*a^2*c^2*\tan(x/2) - 2*b^2*c^2*\tan(x/2) - 2*a^3*c*\tan(x/2) - 4*a*b*c^2 + a^2*b*c - 2*a*b^2*c*\tan(x/2))))/((a^2 + b^2)*(a^2 + b^2 - c^2)))/((a^2 + b^2)*(a^2 + b^2 - c^2)))*(a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^{(1/2)}))/((a^2 + b^2)*(a^2 + b^2 - c^2)))*(b*(a^2 - c^2) + b^3 + a*c*(a^2 + b^2 - c^2)^{(1/2)}))/((c^2*(a^2 + b^2 - c^2) + (a^2 + b^2 - c^2)^2) - \log(\tan(x/2) + 1i)/(a*1i + b) - (\log(\tan(x/2) - 1i)*1i)/(a + b*1i) + (\log(32*a*c - 32*c^2 + 32*b*\tan(x/2))*(a - c) + ((32*a^2*b - 32*b*c^2 + 32*\tan(x/2))*(a - c)*(2*a*c - a^2 + 3*b^2 - 2*c^2) - ((a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^{(1/2)})*(32*a^4 - 64*a^3*c - 64*a^2*b^2 + 32*a^2*c^2 - 32*b^2*c^2 + 96*a*b^2*c + 32*b*\tan(x/2))*(a - c)*(4*a^2 - 4*a*c + b^2) + (32*(a - c)*(a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^{(1/2)})*(3*b^4*\tan(x/2) + 3*a*b^3 + 3*a^3*b + b^3*c + 3*a^2*b^2*\tan(x/2) + 2*a^2*c^2*\tan(x/2) - 2*b^2*c^2*\tan(x/2) - 2*a^3*c*\tan(x/2) - 4*a*b*c^2 + a^2*b*c - 2*a*b^2*c*\tan(x/2))))/((a^2 + b^2)*(a^2 + b^2 - c^2)))/((a^2 + b^2)*(a^2 + b^2 - c^2)))*(a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^{(1/2)}))/((a^2 + b^2)*(a^2 + b^2 - c^2)))*(b*(a^2 - c^2) + b^3 - a*c*(a^2 + b^2 - c^2)^{(1/2)}))/((c^2*(a^2 + b^2 - c^2) + (a^2 + b^2 - c^2)^2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \tan(x) + c \sec(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+c*sec(x)+b*tan(x)),x)`

[Out] `Integral(1/(a + b*tan(x) + c*sec(x)), x)`

$$3.446 \quad \int \frac{\sec(x)}{a+c \sec(x)+b \tan(x)} dx$$

Optimal. Leaf size=51

$$\frac{2 \tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

[Out] $-2*\operatorname{arctanh}((b-(a-c)*\tan(1/2*x))/(a^2+b^2-c^2)^{(1/2)})/(a^2+b^2-c^2)^{(1/2)}$

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3165, 3124, 618, 206}

$$\frac{2 \tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]/(a + c*Sec[x] + b*Tan[x]),x]

[Out] $(-2*\operatorname{ArcTanh}[(b - (a - c)*\operatorname{Tan}[x/2])/ \operatorname{Sqrt}[a^2 + b^2 - c^2]])/ \operatorname{Sqrt}[a^2 + b^2 - c^2]$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3165

Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] := Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{\sec(x)}{a + c \sec(x) + b \tan(x)} dx &= \int \frac{1}{c + a \cos(x) + b \sin(x)} dx \\
&= 2 \operatorname{Subst} \left(\int \frac{1}{a + c + 2bx + (-a + c)x^2} dx, x, \tan\left(\frac{x}{2}\right) \right) \\
&= - \left(4 \operatorname{Subst} \left(\int \frac{1}{4(a^2 + b^2 - c^2) - x^2} dx, x, 2b + 2(-a + c) \tan\left(\frac{x}{2}\right) \right) \right) \\
&= - \frac{2 \tanh^{-1} \left(\frac{b - (a - c) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 + b^2 - c^2}} \right)}{\sqrt{a^2 + b^2 - c^2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.98

$$-\frac{2 \tanh^{-1} \left(\frac{(c-a) \tan\left(\frac{x}{2}\right) + b}{\sqrt{a^2 + b^2 - c^2}} \right)}{\sqrt{a^2 + b^2 - c^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]/(a + c*Sec[x] + b*Tan[x]), x]

[Out] (-2*ArcTanh[(b + (-a + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2 - c^2]

fricas [B] time = 0.51, size = 349, normalized size = 6.84

$$\left[\log \left(- \frac{2a^4 + 3a^2b^2 + b^4 - (a^2 - b^2)c^2 + 2(a^3 + ab^2)c \cos(x) - (a^4 - b^4 - 2(a^2 - b^2)c^2) \cos(x)^2 + 2((a^2b + b^3)c - (a^3b + ab^3 - 2abc^2) \cos(x)) \sin(x) - 2(abc \cos(x) + b^2c \sin(x))}{2ac \cos(x) + (a^2 - b^2) \cos(x)^2 + b^2 + c^2 + 2(ab \cos(x) + bc) \sin(x)} \right) \right]$$

$$2 \sqrt{a^2 + b^2 - c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x, algorithm="fricas")

[Out] [1/2*log(-(2*a^4 + 3*a^2*b^2 + b^4 - (a^2 - b^2)*c^2 + 2*(a^3 + a*b^2)*c*cos(x) - (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 + 2*((a^2*b + b^3)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (a^2*b + b^3)*cos(x) - (a^3 + a*b^2 + (a^2 - b^2)*c*cos(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b^2*c*sin(x)))/sqrt(a^2 + b^2 - c^2), sqrt(-a^2 - b^2 + c^2)*arctan((a*c*cos(x) + b*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^2*b + b^3 - b*c^2)*cos(x) - (a^3 + a*b^2 - a*c^2)*sin(x)))/(a^2 + b^2 - c^2)]

giac [A] time = 0.17, size = 73, normalized size = 1.43

$$\frac{2 \left(\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(2a - 2c) + \arctan \left(\frac{a \tan\left(\frac{1}{2}x\right) - c \tan\left(\frac{1}{2}x\right) - b}{\sqrt{-a^2 - b^2 + c^2}} \right) \right)}{\sqrt{-a^2 - b^2 + c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x, algorithm="giac")

[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a - 2*c) + arctan((a*tan(1/2*x) - c*tan(1/2*x) - b)/sqrt(-a^2 - b^2 + c^2)))/sqrt(-a^2 - b^2 + c^2)

maple [A] time = 0.13, size = 53, normalized size = 1.04

$$\frac{2 \arctan\left(\frac{2(a-c)\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{-a^2-b^2+c^2}}\right)}{\sqrt{-a^2-b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)/(a+c*sec(x)+b*tan(x)),x)

[Out] $-2/(-a^2-b^2+c^2)^{(1/2)}*\arctan(1/2*(2*(a-c)*\tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^{(1/2)})$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see 'assume?' for more details)Is c^2-b^2-a^2 positive or negative?

mupad [B] time = 2.78, size = 47, normalized size = 0.92

$$\frac{2 \operatorname{atanh}\left(\frac{b-\frac{\tan\left(\frac{x}{2}\right)(2a-2c)}{2}}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)*(a + b*tan(x) + c/cos(x))),x)

[Out] $-(2*\operatorname{atanh}((b - (\tan(x/2))*(2*a - 2*c))/2)/(a^2 + b^2 - c^2)^{(1/2)}))/(a^2 + b^2 - c^2)^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(x)}{a + b \tan(x) + c \sec(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x)

[Out] Integral(sec(x)/(a + b*tan(x) + c*sec(x)), x)

$$3.447 \quad \int \frac{\sec^2(x)}{a+c \sec(x)+b \tan(x)} dx$$

Optimal. Leaf size=142

$$\frac{2ac \tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(b^2-c^2)\sqrt{a^2+b^2-c^2}} + \frac{b \log\left(-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right) + a + 2b \tan\left(\frac{x}{2}\right) + c\right)}{b^2-c^2} - \frac{\log\left(1 - \tan\left(\frac{x}{2}\right)\right)}{b+c} - \frac{\log\left(\tan\left(\frac{x}{2}\right) + 1\right)}{b-c}$$

[Out] $-\ln(1-\tan(1/2*x))/(b+c)-\ln(1+\tan(1/2*x))/(b-c)+b*\ln(a+c+2*b*\tan(1/2*x)-(a-c)*\tan(1/2*x)^2)/(b^2-c^2)-2*a*c*\operatorname{arctanh}((b-(a-c)*\tan(1/2*x))/(a^2+b^2-c^2)^{(1/2)))/(b^2-c^2)/(a^2+b^2-c^2)^{(1/2)}$

Rubi [A] time = 0.51, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {4397, 1075, 634, 618, 206, 628, 633, 31}

$$\frac{2ac \tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(b^2-c^2)\sqrt{a^2+b^2-c^2}} + \frac{b \log\left(-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right) + a + 2b \tan\left(\frac{x}{2}\right) + c\right)}{b^2-c^2} - \frac{\log\left(1 - \tan\left(\frac{x}{2}\right)\right)}{b+c} - \frac{\log\left(\tan\left(\frac{x}{2}\right) + 1\right)}{b-c}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/(a + c*Sec[x] + b*Tan[x]),x]

[Out] $(-2*a*c*\operatorname{ArcTanh}[(b-(a-c)*\tan[x/2])/Sqrt[a^2+b^2-c^2]])/((b^2-c^2)*Sqrt[a^2+b^2-c^2]) - \operatorname{Log}[1 - \tan[x/2]]/(b+c) - \operatorname{Log}[1 + \tan[x/2]]/(b-c) + (b*\operatorname{Log}[a+c+2*b*\tan[x/2] - (a-c)*\tan[x/2]^2])/(b^2-c^2)$

Rule 31

Int[((a_) + (b_.)*(x_))⁻¹, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)⁻¹, x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)⁻¹, x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 633

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1075

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (f_.)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*(-(b*C*d) + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*f - a*C*d*f + a*A*f^2 - f*(-(b*C*d) + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4397

```
Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)}{a + c \sec(x) + b \tan(x)} dx &= \int \frac{\sec(x)}{c + a \cos(x) + b \sin(x)} dx \\ &= 2 \operatorname{Subst} \left(\int \frac{1 + x^2}{(1 - x^2)(a + c + 2bx - (a - c)x^2)} dx, x, \tan\left(\frac{x}{2}\right) \right) \\ &= -\frac{\operatorname{Subst} \left(\int \frac{4c - 4bx}{1 - x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{2(b^2 - c^2)} - \frac{\operatorname{Subst} \left(\int \frac{-4b^2 + (-a+c)^2 - (a+c)^2 - 4b(-a+c)x}{a+c+2bx+(-a+c)x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{2(b^2 - c^2)} \\ &= \frac{\operatorname{Subst} \left(\int \frac{1}{-1-x} dx, x, \tan\left(\frac{x}{2}\right) \right)}{b - c} + \frac{\operatorname{Subst} \left(\int \frac{1}{1-x} dx, x, \tan\left(\frac{x}{2}\right) \right)}{b + c} + \frac{b \operatorname{Subst} \left(\int \frac{2}{a+c+2bx-(a-c)x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{b^2 - c^2} \\ &= -\frac{\log\left(1 - \tan\left(\frac{x}{2}\right)\right)}{b + c} - \frac{\log\left(1 + \tan\left(\frac{x}{2}\right)\right)}{b - c} + \frac{b \log\left(a + c + 2b \tan\left(\frac{x}{2}\right) - (a - c) \tan^2\left(\frac{x}{2}\right)\right)}{b^2 - c^2} \\ &= -\frac{2ac \tanh^{-1} \left(\frac{b - (a-c) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 + b^2 - c^2}} \right)}{(b^2 - c^2) \sqrt{a^2 + b^2 - c^2}} - \frac{\log\left(1 - \tan\left(\frac{x}{2}\right)\right)}{b + c} - \frac{\log\left(1 + \tan\left(\frac{x}{2}\right)\right)}{b - c} + \frac{b \log\left(a + c + 2b \tan\left(\frac{x}{2}\right) - (a - c) \tan^2\left(\frac{x}{2}\right)\right)}{b^2 - c^2} \end{aligned}$$

Mathematica [A] time = 0.28, size = 120, normalized size = 0.85

$$\frac{2ac \tanh^{-1} \left(\frac{(c-a) \tan\left(\frac{x}{2}\right) + b}{\sqrt{a^2 + b^2 - c^2}} \right)}{\sqrt{a^2 + b^2 - c^2}} - b \log(a \cos(x) + b \sin(x) + c) + (b - c) \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + (b + c) \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)}{(c - b)(b + c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a + c*Sec[x] + b*Tan[x]), x]
```

```
[Out] ((2*a*c*ArcTanh[(b + (-a + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2 - c^2] + (b - c)*Log[Cos[x/2] - Sin[x/2]] + (b + c)*Log[Cos[x/2] + Sin[x/2]] - b*Log[c + a*Cos[x] + b*Sin[x]])/((-b + c)*(b + c))
```

fricas [A] time = 5.00, size = 663, normalized size = 4.67

$$\frac{\sqrt{a^2 + b^2 - c^2} ac \log\left(\frac{2a^4 + 3a^2b^2 + b^4 - (a^2 - b^2)c^2 + 2(a^3 + ab^2)c \cos(x) - (a^4 - b^4 - 2(a^2 - b^2)c^2) \cos(x)^2 + 2((a^2b + b^3)c - (a^3b + ab^3 - 2abc^2) \cos(x)) \sin(x)}{2ac \cos(x) + (a^2 - b^2) \cos(x)^2 + b^2 + c^2 + 2(ab^2 + b^3)c \sin(x)}\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+c*sec(x)+b*tan(x)),x, algorithm="fricas")
[Out] [-1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((2*a^4 + 3*a^2*b^2 + b^4 - (a^2 - b^2)
*c^2 + 2*(a^3 + a*b^2)*c*cos(x) - (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2
+ 2*((a^2*b + b^3)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*a*
b*c*cos(x)^2 - a*b*c + (a^2*b + b^3)*cos(x) - (a^3 + a*b^2 + (a^2 - b^2)*c*
cos(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2
+ b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x)) - (a^2*b + b^3 - b*c^2)*log(2*
a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x)
) + (a^2*b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(sin(x) + 1) + (a^2*b +
b^3 - b*c^2 + c^3 - (a^2 + b^2)*c)*log(-sin(x) + 1))/(a^2*b^2 + b^4 + c^4 -
(a^2 + 2*b^2)*c^2), 1/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((a*c*cos(x) +
b*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^2*b + b^3 - b*c^2)*cos(
x) - (a^3 + a*b^2 - a*c^2)*sin(x))) + (a^2*b + b^3 - b*c^2)*log(2*a*c*cos(x)
) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x)) - (a^2*
b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(sin(x) + 1) - (a^2*b + b^3 - b*c
^2 + c^3 - (a^2 + b^2)*c)*log(-sin(x) + 1))/(a^2*b^2 + b^4 + c^4 - (a^2 + 2
*b^2)*c^2)]
```

giac [A] time = 0.17, size = 161, normalized size = 1.13

$$\frac{2\left(\pi\left[\frac{x}{2\pi} + \frac{1}{2}\right] \operatorname{sgn}(-2a + 2c) + \arctan\left(-\frac{a \tan\left(\frac{1}{2}x\right) - c \tan\left(\frac{1}{2}x\right) - b}{\sqrt{-a^2 - b^2 + c^2}}\right)\right) ac}{\sqrt{-a^2 - b^2 + c^2} (b^2 - c^2)} + \frac{b \log\left(-a \tan\left(\frac{1}{2}x\right)^2 + c \tan\left(\frac{1}{2}x\right)^2 + 2b \tan\left(\frac{1}{2}x\right)\right)}{b^2 - c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+c*sec(x)+b*tan(x)),x, algorithm="giac")
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*c) + arctan(-(a*tan(1/2*x) - c*tan
(1/2*x) - b)/sqrt(-a^2 - b^2 + c^2)))*a*c/(sqrt(-a^2 - b^2 + c^2)*(b^2 - c^
2)) + b*log(-a*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a + c)/(b^2
- c^2) - log(abs(tan(1/2*x) + 1))/(b - c) - log(abs(tan(1/2*x) - 1))/(b +
c)
```

maple [B] time = 0.14, size = 430, normalized size = 3.03

$$\frac{2 \ln\left(\tan\left(\frac{x}{2}\right) - 1\right)}{2b + 2c} + \frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right) - c\left(\tan^2\left(\frac{x}{2}\right)\right) - 2b \tan\left(\frac{x}{2}\right) - a - c\right) ab}{(b - c)(b + c)(a - c)} - \frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right) - c\left(\tan^2\left(\frac{x}{2}\right)\right) - 2b \tan\left(\frac{x}{2}\right) - a - c\right)}{(b - c)(b + c)(a - c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a+c*sec(x)+b*tan(x)),x)
[Out] -2/(2*b+2*c)*ln(tan(1/2*x)-1)+1/(b-c)/(b+c)/(a-c)*ln(a*tan(1/2*x)^2-c*tan(
1/2*x)^2-2*b*tan(1/2*x)-a-c)*a*b-1/(b-c)/(b+c)/(a-c)*ln(a*tan(1/2*x)^2-c*tan
(1/2*x)^2-2*b*tan(1/2*x)-a-c)*c*b-2/(b-c)/(b+c)/(-a^2-b^2+c^2)^(1/2)*arctan
(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2))*a*c-2/(b-c)/(b+c)/(-a^2
-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2))*b
^2+2/(b-c)/(b+c)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(
```


$$-a^2-b^2+c^2)^{1/2}) * b^2 / (a-c) * a^{-2} / (b-c) / (b+c) / (-a^2-b^2+c^2)^{1/2} * \arctan(1/2 * (2 * (a-c) * \tan(1/2 * x) - 2 * b) / (-a^2-b^2+c^2)^{1/2}) * b^2 / (a-c) * c^{-2} / (-2 * c + 2 * b) * \ln(1 + \tan(1/2 * x))$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(a+c*sec(x)+b*tan(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see 'assume?' for more details) Is c^2-b^2-a^2 positive or negative?

mupad [B] time = 11.44, size = 977, normalized size = 6.88

$$\ln \left(\frac{32ac - 32a^2 - 32b \tan\left(\frac{x}{2}\right) (a-c) - \frac{(a^2b - bc^2 + b^3 + ac\sqrt{a^2+b^2-c^2}) \left(32a^2b - 32bc^2 + 32 \tan\left(\frac{x}{2}\right) (a-c) (2a^2 - 2ac + 3b^2 + c^2) \right)}{(b^2 - c^2)(a^2 + b^2 - c^2)}}{32ac - 32a^2 - 32b \tan\left(\frac{x}{2}\right) (a-c) - \frac{(a^2b - bc^2 + b^3 + ac\sqrt{a^2+b^2-c^2}) \left(32a^2b - 32bc^2 + 32 \tan\left(\frac{x}{2}\right) (a-c) (2a^2 - 2ac + 3b^2 + c^2) \right)}{(b^2 - c^2)(a^2 + b^2 - c^2)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2*(a + b*tan(x) + c/cos(x))),x)

[Out]
$$\frac{\log(32ac - 32a^2 - 32b \tan(x/2) (a-c) - ((a^2b - bc^2 + b^3 + ac \sqrt{a^2+b^2-c^2}) * (32a^2b - 32bc^2 + 32 \tan(x/2) (a-c) (2a^2 - 2ac + 3b^2 + c^2) - ((a^2b - bc^2 + b^3 + ac \sqrt{a^2+b^2-c^2}) * (32c^4 - 64ac^3 + 32a^2b^2 + 32a^2c^2 + 64b^2c^2 - 96ab^2c + 32b \tan(x/2) (a-c) (4ac + b^2 - 4c^2) + (32(a-c)(a^2b - bc^2 + b^3 + ac \sqrt{a^2+b^2-c^2}) * (3b^4 \tan(x/2) + ab^3 - 3bc^3 + 3b^3c + 2a^2b^2 \tan(x/2) + 2a^2c^2 \tan(x/2) - 3b^2c^2 \tan(x/2) - 2ac^3 \tan(x/2) - ab^2c^2 + 4a^2b^2c + 2ab^2c \tan(x/2)))) / ((b^2 - c^2)(a^2 + b^2 - c^2)))) / ((b^2 - c^2)(a^2 + b^2 - c^2))) / ((b^2 - c^2)(a^2 + b^2 - c^2))) * (b(a^2 - c^2) + b^3 + ac \sqrt{a^2+b^2-c^2}) / ((b^2 - c^2)(a^2 + b^2 - c^2)) - \log(\tan(x/2) - 1) / (b+c) - \log(\tan(x/2) + 1) / (b-c) + (\log(32ac - 32a^2 - 32b \tan(x/2) (a-c) - ((a^2b - bc^2 + b^3 - ac \sqrt{a^2+b^2-c^2}) * (32a^2b - 32bc^2 + 32 \tan(x/2) (a-c) (2a^2 - 2ac + 3b^2 + c^2) - ((a^2b - bc^2 + b^3 - ac \sqrt{a^2+b^2-c^2}) * (32c^4 - 64ac^3 + 32a^2b^2 + 32a^2c^2 + 64b^2c^2 - 96ab^2c + 32b \tan(x/2) (a-c) (4ac + b^2 - 4c^2) + (32(a-c)(a^2b - bc^2 + b^3 - ac \sqrt{a^2+b^2-c^2}) * (3b^4 \tan(x/2) + ab^3 - 3bc^3 + 3b^3c + 2a^2b^2 \tan(x/2) + 2a^2c^2 \tan(x/2) - 3b^2c^2 \tan(x/2) - 2ac^3 \tan(x/2) - ab^2c^2 + 4a^2b^2c + 2ab^2c \tan(x/2)))) / ((b^2 - c^2)(a^2 + b^2 - c^2)))) / ((b^2 - c^2)(a^2 + b^2 - c^2))) / ((b^2 - c^2)(a^2 + b^2 - c^2))) * (b(a^2 - c^2) + b^3 - ac \sqrt{a^2+b^2-c^2}) / ((b^2 - c^2)(a^2 + b^2 - c^2))$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{a + b \tan(x) + c \sec(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] Integral(sec(x)**2/(a + b*tan(x) + c*sec(x)), x)
```

$$3.448 \quad \int \frac{(a+b \sec(d+ex)+c \tan(d+ex))^{3/2}}{\sec^2(d+ex)} dx$$

Optimal. Leaf size=371

$$\frac{2(a^2 - b^2 + c^2) \sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} (a+b \sec(d+ex)+c \tan(d+ex))^{3/2} F\left(\frac{1}{2}(d+ex - \tan^{-1}(a,c)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e \sec^2(d+ex)(a \cos(d+ex)+b+c \sin(d+ex))^2}$$

[Out] $-2/3*(c*\cos(e*x+d)-a*\sin(e*x+d))*(a+b*\sec(e*x+d)+c*\tan(e*x+d))^{(3/2)}/e/\sec(e*x+d)^{(3/2)}/(b+a*\cos(e*x+d)+c*\sin(e*x+d))+8/3*b*(\cos(1/2*d+1/2*e*x-1/2*\arctan(a,c))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(a,c))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(a,c)),2^{(1/2)*((a^2+c^2)^{(1/2)}/(b+(a^2+c^2)^{(1/2))})}^{(1/2)})*(a+b*\sec(e*x+d)+c*\tan(e*x+d))^{(3/2)}/e/\sec(e*x+d)^{(3/2)}/(b+a*\cos(e*x+d)+c*\sin(e*x+d))/((b+a*\cos(e*x+d)+c*\sin(e*x+d))/(b+(a^2+c^2)^{(1/2))})^{(1/2)}+2/3*(a^2-b^2+c^2)*(\cos(1/2*d+1/2*e*x-1/2*\arctan(a,c))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(a,c))*\text{EllipticF}(\sin(1/2*d+1/2*e*x-1/2*\arctan(a,c)),2^{(1/2)*((a^2+c^2)^{(1/2)}/(b+(a^2+c^2)^{(1/2))})}^{(1/2)})*((b+a*\cos(e*x+d)+c*\sin(e*x+d))/(b+(a^2+c^2)^{(1/2))})^{(1/2)}*(a+b*\sec(e*x+d)+c*\tan(e*x+d))^{(3/2)}/e/\sec(e*x+d)^{(3/2)}/(b+a*\cos(e*x+d)+c*\sin(e*x+d))^2$

Rubi [A] time = 0.45, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3167, 3120, 3149, 3119, 2653, 3127, 2661}

$$\frac{2(a^2 - b^2 + c^2) \sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} (a+b \sec(d+ex)+c \tan(d+ex))^{3/2} F\left(\frac{1}{2}(d+ex - \tan^{-1}(a,c)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e \sec^2(d+ex)(a \cos(d+ex)+b+c \sin(d+ex))^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)/Sec[d + e*x]^(3/2), x]

[Out] $(-2*(c*\cos[d+e*x]-a*\sin[d+e*x])*(a+b*\sec[d+e*x]+c*\tan[d+e*x])^{(3/2)})/(3*e*\sec[d+e*x]^{(3/2)}*(b+a*\cos[d+e*x]+c*\sin[d+e*x]))+(8*b*\text{EllipticE}((d+e*x-\text{ArcTan}[a,c])/2,(2*\sqrt{a^2+c^2})/(b+\sqrt{a^2+c^2}))*((a+b*\sec[d+e*x]+c*\tan[d+e*x])^{(3/2)})/(3*e*\sec[d+e*x]^{(3/2)}*(b+a*\cos[d+e*x]+c*\sin[d+e*x]))*\sqrt{(b+a*\cos[d+e*x]+c*\sin[d+e*x])/(b+\sqrt{a^2+c^2})})+(2*(a^2-b^2+c^2)*\text{EllipticF}((d+e*x-\text{ArcTan}[a,c])/2,(2*\sqrt{a^2+c^2})/(b+\sqrt{a^2+c^2}))*\sqrt{(b+a*\cos[d+e*x]+c*\sin[d+e*x])/(b+\sqrt{a^2+c^2})})*(a+b*\sec[d+e*x]+c*\tan[d+e*x])^{(3/2)})/(3*e*\sec[d+e*x]^{(3/2)}*(b+a*\cos[d+e*x]+c*\sin[d+e*x])^2)$

Rule 2653

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] := -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]
```

Rule 3167

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] := Dist[(Sec[d + e*x]^n*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n, Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sec(d + ex) + c \tan(d + ex))^{3/2}}{\sec^{\frac{3}{2}}(d + ex)} dx &= \frac{(a + b \sec(d + ex) + c \tan(d + ex))^{3/2} \int (b + a \cos(d + ex) + c \sin(d + ex))^{3/2}}{\sec^{\frac{3}{2}}(d + ex)(b + a \cos(d + ex) + c \sin(d + ex))^{3/2}} \\
&= -\frac{2(c \cos(d + ex) - a \sin(d + ex))(a + b \sec(d + ex) + c \tan(d + ex))}{3e \sec^{\frac{3}{2}}(d + ex)(b + a \cos(d + ex) + c \sin(d + ex))} \\
&= -\frac{2(c \cos(d + ex) - a \sin(d + ex))(a + b \sec(d + ex) + c \tan(d + ex))}{3e \sec^{\frac{3}{2}}(d + ex)(b + a \cos(d + ex) + c \sin(d + ex))} \\
&= -\frac{2(c \cos(d + ex) - a \sin(d + ex))(a + b \sec(d + ex) + c \tan(d + ex))}{3e \sec^{\frac{3}{2}}(d + ex)(b + a \cos(d + ex) + c \sin(d + ex))} \\
&= -\frac{2(c \cos(d + ex) - a \sin(d + ex))(a + b \sec(d + ex) + c \tan(d + ex))}{3e \sec^{\frac{3}{2}}(d + ex)(b + a \cos(d + ex) + c \sin(d + ex))}
\end{aligned}$$

Mathematica [C] time = 6.46, size = 2490, normalized size = 6.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)/Sec[d + e*x]^(3/2), x]
[Out] (((8*a*b)/(3*c) - (2*c*Cos[d + e*x])/3 + (2*a*Sin[d + e*x])/3)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])) + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2])*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c)), -((b + Sqrt[1 + a^2/c^2])*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c)))*c)]*Sec[d + e*x + ArcTan[a/c]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]]*Sin[d + e*x + ArcTan[a/c]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3*Sqrt[1 + a^2/c^2]*c*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2])*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c)))*c), -((b + Sqrt[1 + a^2/c^2])*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c)))*c)]*Sec[d + e*x + ArcTan[a/c]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]]*Sin[d + e*x + ArcTan[a/c]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(Sqrt[1 + a^2/c^2]*c*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2])*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c)))*c), -((b + Sqrt[1 + a^2/c^2])*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c)))*c)]*Sec[d + e*x + ArcTan[a/c]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]]*Sin[d + e*x + ArcTan[a/c]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))
```

```

*Sin[d + e*x + ArcTan[a/c]]/(-b + c*Sqrt[(a^2 + c^2)/c^2]]*(a + b*Sec[d +
e*x] + c*Tan[d + e*x])^(3/2)/(3*Sqrt[1 + a^2/c^2]*e*Sec[d + e*x]^(3/2)*(b
+ a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (4*a^2*b*(-((c*AppellF1[-1/2,
-1/2, -1/2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*
Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^
2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2
/a^2])))))]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a
^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a
*Sqrt[(a^2 + c^2)/a^2])*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - Arc
Tan[c/a]])*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d +
e*x - ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])))) - ((2*a*(b + a*Sqrt[1
+ c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a^2 + c^2) - (c*Sin[d + e*x - Arc
Tan[c/a]])/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x
- ArcTan[c/a]])*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)/(3*c*e*Sec[d
+ e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (4*b*c*(-((c*Ap
pellF1[-1/2, -1/2, -1/2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcT
an[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sq
rt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a
*Sqrt[1 + c^2/a^2])))))]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sq
rt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[
c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[
d + e*x - ArcTan[c/a]])*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/
a^2]*Cos[d + e*x - ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])))) - ((2*a*
(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a^2 + c^2) - (c*Sin[
d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]
*Cos[d + e*x - ArcTan[c/a]])*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/
(3*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))

```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(b \sec(ex + d) + c \tan(ex + d) + a)^{\frac{3}{2}}}{\sec(ex + d)^{\frac{3}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)/sec(e*x + d)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(ex + d) + c \tan(ex + d) + a)^{\frac{3}{2}}}{\sec(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)/sec(e*x + d)^(3/2), x
)
```

maple [C] time = 4.41, size = 21186, normalized size = 57.11

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x)
```

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(ex + d) + c \tan(ex + d) + a)^{\frac{3}{2}}}{\sec^{\frac{3}{2}}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)/sec(e*x + d)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + c \tan(d + ex) + \frac{b}{\cos(d+ex)}\right)^{3/2}}{\left(\frac{1}{\cos(d+ex)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2)/(1/cos(d + e*x))^(3/2),x)

[Out] int((a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2)/(1/cos(d + e*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec(d + ex) + c \tan(d + ex))^{\frac{3}{2}}}{\sec^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2)/sec(e*x+d)**(3/2),x)

[Out] Integral((a + b*sec(d + e*x) + c*tan(d + e*x))**(3/2)/sec(d + e*x)**(3/2), x)

$$3.449 \quad \int \frac{\sqrt{a+b \sec(d+ex)+c \tan(d+ex)}}{\sqrt{\sec(d+ex)}} dx$$

Optimal. Leaf size=118

$$\frac{2\sqrt{a+b \sec(d+ex)+c \tan(d+ex)} E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\middle|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2))/(b+(a^2+c^2)^(1/2)))^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/e/sec(e*x+d)^(1/2)/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)

Rubi [A] time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3167, 3119, 2653}

$$\frac{2\sqrt{a+b \sec(d+ex)+c \tan(d+ex)} E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\middle|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]/Sqrt[Sec[d + e*x]],x]

[Out] (2*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(e*Sqrt[Sec[d + e*x]]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c])]/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3167

Int[sec[(d_) + (e_)*(x_)]^(n_)*((a_) + (b_)*sec[(d_) + (e_)*(x_)] + (c_)*tan[(d_) + (e_)*(x_)]^(m_), x_Symbol] := Dist[(Sec[d + e*x]^n*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^m)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n, Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && !IntegerQ[n]

Rubi steps

$$\int \frac{\sqrt{a + b \sec(d + ex) + c \tan(d + ex)}}{\sqrt{\sec(d + ex)}} dx = \frac{\sqrt{a + b \sec(d + ex) + c \tan(d + ex)}}{\sqrt{\sec(d + ex)} \sqrt{b + a \cos(d + ex) + c \sin(d + ex)}} \int \sqrt{b + a \cos(d + ex) + c \sin(d + ex)}$$

$$= \frac{\sqrt{a + b \sec(d + ex) + c \tan(d + ex)}}{\sqrt{\sec(d + ex)} \sqrt{\frac{b}{b + \sqrt{a^2 + c^2}} + \frac{\sqrt{a^2 + c^2} \cos(d + ex)}{b + \sqrt{a^2 + c^2}}}}$$

$$= \frac{2E\left(\frac{1}{2}\left(d + ex - \tan^{-1}\left(\frac{a}{c}\right)\right) \middle| \frac{2\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}\right) \sqrt{a + b \sec(d + ex) + c \tan(d + ex)}}{e \sqrt{\sec(d + ex)} \sqrt{\frac{b + a \cos(d + ex) + c \sin(d + ex)}{b + \sqrt{a^2 + c^2}}}}$$

Mathematica [C] time = 6.25, size = 1580, normalized size = 13.39

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]/Sqrt[Sec[d + e*x]],x]
[Out] (2*a*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]/(c*e*Sqrt[Sec[d + e*x]]) +
(2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2])*c*Sin[d + e*x +
ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2])*c)), -((b +
Sqrt[1 + a^2/c^2])*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/
(Sqrt[1 + a^2/c^2])*c)))]*Sec[d + e*x + ArcTan[a/c]]*Sqrt[(c*Sqrt[(a^2 +
c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt
[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a
/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x +
ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[a + b*Sec[d + e*x] + c*
Tan[d + e*x]]/(Sqrt[1 + a^2/c^2]*c*e*Sqrt[Sec[d + e*x]]*Sqrt[b + a*Cos[d +
e*x] + c*Sin[d + e*x]]) + (a^2*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b
+ a*Sqrt[1 + c^2/a^2])*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 -
b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2])*Cos[d + e*x - ArcTa
n[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2])))))]*Sin[d + e*x
- ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqr
t[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2]
)]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*Sqr
t[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(-
b + a*Sqrt[(a^2 + c^2)/a^2])) - ((2*a*(b + a*Sqrt[1 + c^2/a^2])*Cos[d + e*
x - ArcTan[c/a]])/(a^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 +
c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[
a + b*Sec[d + e*x] + c*Tan[d + e*x]]/(c*e*Sqrt[Sec[d + e*x]]*Sqrt[b + a*Co
s[d + e*x] + c*Sin[d + e*x]]) + (c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -(
b + a*Sqrt[1 + c^2/a^2])*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(
1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2])*Cos[d + e*x - Ar
cTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2])))))]*Sin[d +
e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*
Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a
^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*
Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])
/(-b + a*Sqrt[(a^2 + c^2)/a^2])) - ((2*a*(b + a*Sqrt[1 + c^2/a^2])*Cos[d +
e*x - ArcTan[c/a]])/(a^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[
1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqr
t[a + b*Sec[d + e*x] + c*Tan[d + e*x]]/(e*Sqrt[Sec[d + e*x]]*Sqrt[b + a*C
os[d + e*x] + c*Sin[d + e*x]])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b \sec(ex + d) + c \tan(ex + d) + a}}{\sqrt{\sec(ex + d)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)/sqrt(sec(e*x + d)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec(ex + d) + c \tan(ex + d) + a}}{\sqrt{\sec(ex + d)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)/sqrt(sec(e*x + d)), x)

maple [C] time = 1.89, size = 12462, normalized size = 105.61

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec(ex + d) + c \tan(ex + d) + a}}{\sqrt{\sec(ex + d)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)/sqrt(sec(e*x + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + c \tan(d + ex) + \frac{b}{\cos(d+ex)}}}{\sqrt{\frac{1}{\cos(d+ex)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2)/(1/cos(d + e*x))^(1/2),x)

[Out] int((a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2)/(1/cos(d + e*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec(d + ex) + c \tan(d + ex)}}{\sqrt{\sec(d + ex)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2)/sec(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(d + e*x) + c*tan(d + e*x))/sqrt(sec(d + e*x)), x)
```

$$3.450 \quad \int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b \sec(d+ex)+c \tan(d+ex)}} dx$$

Optimal. Leaf size=118

$$\frac{2\sqrt{\sec(d+ex)} \sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} F\left(\frac{1}{2} \left(d+ex - \tan^{-1}(a,c)\right) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a+b \sec(d+ex)+c \tan(d+ex)}}$$

[Out] $2*(\cos(1/2*d+1/2*e*x-1/2*\arctan(a,c))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(a,c))*\text{EllipticF}(\sin(1/2*d+1/2*e*x-1/2*\arctan(a,c)),2^{(1/2)}*((a^2+c^2)^{(1/2)})/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}*\sec(e*x+d)^{(1/2)}*((b+a*\cos(e*x+d)+c*\sin(e*x+d))/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}/e/(a+b*\sec(e*x+d)+c*\tan(e*x+d))^{(1/2)}$

Rubi [A] time = 0.17, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3167, 3127, 2661}

$$\frac{2\sqrt{\sec(d+ex)} \sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} F\left(\frac{1}{2} \left(d+ex - \tan^{-1}(a,c)\right) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a+b \sec(d+ex)+c \tan(d+ex)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sec[d + e*x]]/Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]

[Out] $(2*\text{EllipticF}[(d+e*x - \text{ArcTan}[a,c])/2, (2*\text{Sqrt}[a^2+c^2])/(b+\text{Sqrt}[a^2+c^2])]*\text{Sqrt}[\text{Sec}[d+e*x]]*\text{Sqrt}[(b+a*\text{Cos}[d+e*x]+c*\text{Sin}[d+e*x])/(b+\text{Sqrt}[a^2+c^2])])/(e*\text{Sqrt}[a+b*\text{Sec}[d+e*x]+c*\text{Tan}[d+e*x]])$

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3127

Int[1/Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3167

Int[sec[(d_) + (e_)*(x_)]^(n_)*((a_) + (b_)*sec[(d_) + (e_)*(x_)] + (c_)*tan[(d_) + (e_)*(x_)]^(m_), x_Symbol] :> Dist[(Sec[d + e*x]^n*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^m)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n, Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && !IntegerQ[n]

Rubi steps

$$\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}} dx = \frac{(\sqrt{\sec(d+ex)}\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}) \int \frac{1}{\sqrt{b+a\cos(d+ex)}}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}} \\ = \frac{\left(\sqrt{\sec(d+ex)}\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}\right) \int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos(d+ex)}{b+\sqrt{a^2+c^2}}}}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}} \\ = \frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{a}{c}\right)\right)\middle|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\sec(d+ex)}\sqrt{\frac{b+a\cos(d+ex)}{b+\sqrt{a^2+c^2}}}}{e\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

Mathematica [C] time = 0.87, size = 339, normalized size = 2.87

$$2\sqrt{\sec(d+ex)}\sec\left(\tan^{-1}\left(\frac{a}{c}\right)+d+ex\right)\sqrt{\frac{c\sqrt{\frac{a^2}{c^2}+1}\left(\sin\left(\tan^{-1}\left(\frac{a}{c}\right)+d+ex\right)-1\right)}{c\sqrt{\frac{a^2}{c^2}+1}+b}}\sqrt{\frac{c\sqrt{\frac{a^2}{c^2}+1}\left(\sin\left(\tan^{-1}\left(\frac{a}{c}\right)+d+ex\right)+1\right)}{c\sqrt{\frac{a^2}{c^2}+1}-b}}\sqrt{c\sqrt{\frac{a^2}{c^2}+1}}$$

$$ce\sqrt{\frac{a^2}{c^2}+1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[Sec[d + e*x]]/Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]], x]

[Out] (2*AppellF1[1/2, 1/2, 1/2, 3/2, (b + Sqrt[1 + a^2/c^2])*c*Sin[d + e*x + ArcTan[a/c]]/(b - Sqrt[1 + a^2/c^2]*c), (b + Sqrt[1 + a^2/c^2])*c*Sin[d + e*x + ArcTan[a/c]]/(b + Sqrt[1 + a^2/c^2]*c)]*Sqrt[Sec[d + e*x]]*Sec[d + e*x + ArcTan[a/c]]*Sqrt[b + a*Cos[d + e*x] + c*Sin[d + e*x]]*Sqrt[-((Sqrt[1 + a^2/c^2])*c*(-1 + Sin[d + e*x + ArcTan[a/c]]))]/(b + Sqrt[1 + a^2/c^2]*c))]/(b + Sqrt[1 + a^2/c^2]*c)]*Sqrt[(Sqrt[1 + a^2/c^2])*c*(1 + Sin[d + e*x + ArcTan[a/c]])]/(-b + Sqrt[1 + a^2/c^2]*c)]*Sqrt[b + Sqrt[1 + a^2/c^2])*c*Sin[d + e*x + ArcTan[a/c]]]/(Sqrt[1 + a^2/c^2])*c*e*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]]

fricas [F] time = 2.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sec(ex+d)}}{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(sec(e*x + d))/sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(ex+d)}}{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(sec(e*x + d))/sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a), x)

maple [C] time = 1.86, size = 722, normalized size = 6.12

$$4i \operatorname{EllipticF} \left(\sqrt{\frac{(i \sin(ex+d) + \cos(ex+d))(ia - ib - \sqrt{a^2 - b^2 + c^2} + c)}{ia - ib + \sqrt{a^2 - b^2 + c^2} - c}}, \sqrt{\frac{(ia - ib + \sqrt{a^2 - b^2 + c^2} - c)(ia - ib + \sqrt{a^2 - b^2 + c^2} + c)}{(ia - ib - \sqrt{a^2 - b^2 + c^2} + c)(ia - ib - \sqrt{a^2 - b^2 + c^2} - c)}} \right) \sqrt{\frac{1}{\cos(ex+d)}} \sqrt{\frac{b+a \cos(ex+d)}{\cos(ex+d)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2), x)`

[Out]
$$-4*I/e*\operatorname{EllipticF}(((I*\sin(e*x+d)+\cos(e*x+d))*(I*a-I*b-(a^2-b^2+c^2)^{(1/2)}+c)/(I*a-I*b+(a^2-b^2+c^2)^{(1/2)}-c))^{(1/2)}, ((I*a-I*b+(a^2-b^2+c^2)^{(1/2)}-c)*(I*a-I*b+(a^2-b^2+c^2)^{(1/2)}+c)/(I*a-I*b-(a^2-b^2+c^2)^{(1/2)}+c)/(I*a-I*b-(a^2-b^2+c^2)^{(1/2)}-c))^{(1/2)}*(1/\cos(e*x+d))^{(1/2)}*((b+a*\cos(e*x+d)+c*\sin(e*x+d))/\cos(e*x+d))^{(1/2)}*((I*\sin(e*x+d)+\cos(e*x+d))*(I*a-I*b-(a^2-b^2+c^2)^{(1/2)}+c)/(I*a-I*b+(a^2-b^2+c^2)^{(1/2)}-c))^{(1/2)}*(-I*(\cos(e*x+d)*(a^2-b^2+c^2)^{(1/2)}-a*\sin(e*x+d)+b*\sin(e*x+d)+c*\cos(e*x+d)+(a^2-b^2+c^2)^{(1/2)}+c)/(I*\cos(e*x+d)+\sin(e*x+d)+I)/(I*a-I*b-(a^2-b^2+c^2)^{(1/2)}-c))^{(1/2)}*(I*(a*\sin(e*x+d)-b*\sin(e*x+d)+\cos(e*x+d)*(a^2-b^2+c^2)^{(1/2)}-c*\cos(e*x+d)+(a^2-b^2+c^2)^{(1/2)}-c)/(I*\cos(e*x+d)+\sin(e*x+d)+I)/(I*a-I*b+(a^2-b^2+c^2)^{(1/2)}-c))^{(1/2)}*(\cos(e*x+d)+1)^2*\cos(e*x+d)*(\cos(e*x+d)-1)^2*(I*a*\cos(e*x+d)-I*\cos(e*x+d)*b-I*(a^2-b^2+c^2)^{(1/2)}*\sin(e*x+d)+I*c*\sin(e*x+d)+\cos(e*x+d)*(a^2-b^2+c^2)^{(1/2)}-c*\cos(e*x+d)+a*\sin(e*x+d)-b*\sin(e*x+d))/\sin(e*x+d)^4/(b+a*\cos(e*x+d)+c*\sin(e*x+d))/(I*a-I*b-(a^2-b^2+c^2)^{(1/2)}+c)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(ex+d)}}{\sqrt{b \sec(ex+d) + c \tan(ex+d) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2), x, algorithm="maxima")`

[Out] `integrate(sqrt(sec(e*x + d))/sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(d+ex)}}}{\sqrt{a + c \tan(d + ex) + \frac{b}{\cos(d+ex)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1/cos(d + e*x))^(1/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2), x)`

[Out] `int((1/cos(d + e*x))^(1/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(d + ex)}}{\sqrt{a + b \sec(d + ex) + c \tan(d + ex)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(e*x+d)**(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2), x)`

[Out] `Integral(sqrt(sec(d + e*x))/sqrt(a + b*sec(d + e*x) + c*tan(d + e*x)), x)`

$$3.451 \quad \int \frac{\sec^3(d+ex)}{(a+b \sec(d+ex)+c \tan(d+ex))^{3/2}} dx$$

Optimal. Leaf size=240

$$\frac{2 \sec^3(d+ex)(a \cos(d+ex) + b + c \sin(d+ex))^2 E\left(\frac{1}{2}(d+ex - \tan^{-1}(a,c)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e(a^2 - b^2 + c^2) \sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} (a+b \sec(d+ex) + c \tan(d+ex))^{3/2}} \frac{2 \sec^3(d+ex)(c \cos(d+ex) + b + a \sin(d+ex))^2 E\left(\frac{1}{2}(d+ex - \tan^{-1}(a,c)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e(a^2 - b^2 + c^2) \sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} (a+b \sec(d+ex) + c \tan(d+ex))^{3/2}}$$

[Out] $-2*\sec(e*x+d)^{(3/2)}*(c*\cos(e*x+d)-a*\sin(e*x+d))*(b+a*\cos(e*x+d)+c*\sin(e*x+d))/((a^2-b^2+c^2)/e/(a+b*\sec(e*x+d)+c*\tan(e*x+d))^{(3/2)}-2*(\cos(1/2*d+1/2*e*x-1/2*\arctan(a,c))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(a,c))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(a,c)),2^{(1/2)}*((a^2+c^2)^{(1/2)}/(b+(a^2+c^2)^{(1/2)})))^{(1/2)})*\sec(e*x+d)^{(3/2)}*(b+a*\cos(e*x+d)+c*\sin(e*x+d))^2/(a^2-b^2+c^2)/e/((b+a*\cos(e*x+d)+c*\sin(e*x+d))/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}/(a+b*\sec(e*x+d)+c*\tan(e*x+d))^{(3/2)}$

Rubi [A] time = 0.22, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3167, 3128, 3119, 2653}

$$\frac{2 \sec^3(d+ex)(a \cos(d+ex) + b + c \sin(d+ex))^2 E\left(\frac{1}{2}(d+ex - \tan^{-1}(a,c)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e(a^2 - b^2 + c^2) \sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} (a+b \sec(d+ex) + c \tan(d+ex))^{3/2}} \frac{2 \sec^3(d+ex)(c \cos(d+ex) + b + a \sin(d+ex))^2 E\left(\frac{1}{2}(d+ex - \tan^{-1}(a,c)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e(a^2 - b^2 + c^2) \sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} (a+b \sec(d+ex) + c \tan(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sec[d + e*x]^(3/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2), x]

[Out] $(-2*\text{Sec}[d + e*x]^{(3/2)}*(c*\text{Cos}[d + e*x] - a*\text{Sin}[d + e*x]))*(b + a*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])/((a^2 - b^2 + c^2)*e*(a + b*\text{Sec}[d + e*x] + c*\text{Tan}[d + e*x])^{(3/2)}) - (2*\text{EllipticE}[(d + e*x - \text{ArcTan}[a, c])/2], (2*\text{Sqrt}[a^2 + c^2])/((b + \text{Sqrt}[a^2 + c^2]))*\text{Sec}[d + e*x]^{(3/2)}*(b + a*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])^2)/((a^2 - b^2 + c^2)*e*\text{Sqrt}[(b + a*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x])/(b + \text{Sqrt}[a^2 + c^2])])*(a + b*\text{Sec}[d + e*x] + c*\text{Tan}[d + e*x])^{(3/2)}$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3128

Int[(cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)])^(-3/2), x_Symbol] := Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3167

Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^(m_), x_Symbol] := Dist[(Sec[d + e*x]^n*(b + a*cos[d + e*x] + c*sin[d + e*x])^n)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n, Int[1/(b + a*cos[d + e*x] + c*sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && !IntegerQ[n]

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{3}{2}}} dx = \frac{\left(\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))^{\frac{3}{2}}\right) \int \frac{1}{(b+a\cos(d+ex)+c\sin(d+ex))^{\frac{3}{2}}} dx}{(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{3}{2}}}$$

$$= -\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))^{\frac{3}{2}}}{(a^2-b^2+c^2)e(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{3}{2}}}$$

$$= -\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))^{\frac{3}{2}}}{(a^2-b^2+c^2)e(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{3}{2}}}$$

$$= -\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))^{\frac{3}{2}}}{(a^2-b^2+c^2)e(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{3}{2}}}$$

Mathematica [C] time = 6.40, size = 1732, normalized size = 7.22

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[d + e*x]^(3/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x]

[Out] (Sec[d + e*x]^(3/2)*(b + a*cos[d + e*x] + c*sin[d + e*x])^2*((-2*(a^2 + c^2))/(a*c*(a^2 - b^2 + c^2)) + (2*(b*c + a^2*sin[d + e*x] + c^2*sin[d + e*x]))/(a*(a^2 - b^2 + c^2)*(b + a*cos[d + e*x] + c*sin[d + e*x])))/(e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)) - (2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -(b + Sqrt[1 + a^2/c^2]*c*sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c)))*c), -(b + Sqrt[1 + a^2/c^2]*c*sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c)]*Sec[d + e*x]^(3/2)*Sec[d + e*x + ArcTan[a/c]]*(b + a*cos[d + e*x] + c*sin[d + e*x])^(3/2)*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]/(Sqrt[1 + a^2/c^2]*c*(a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)) - (a^2*Sec[d + e*x]^(3/2)*(b + a*cos[d + e*x] + c*sin[d + e*x])^(3/2)*(-(c*AppellF1[-1/2, -1/2, -1/2, 1/2, -(b + a*Sqrt[1 + c^2/a^2]*cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2]))), -(b + a*Sqrt[1 + c^2/a^2]*cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2])))]*sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 +

$$\begin{aligned} & c^2/a^2 * \cos[d + e*x - \text{ArcTan}[c/a]] * \sqrt{[a*\sqrt{[a^2 + c^2]/a^2} + a*\sqrt{t[(a^2 + c^2)/a^2]*\cos[d + e*x - \text{ArcTan}[c/a]]/(-b + a*\sqrt{[a^2 + c^2]/a^2})]}]} \\ & - ((2*a*(b + a*\sqrt{1 + c^2/a^2})*\cos[d + e*x - \text{ArcTan}[c/a]])/(a^2 + c^2) - (c*\sin[d + e*x - \text{ArcTan}[c/a]]/(a*\sqrt{1 + c^2/a^2}))/\sqrt{b + a*\sqrt{1 + c^2/a^2}*\cos[d + e*x - \text{ArcTan}[c/a]]})/(c*(a^2 - b^2 + c^2)*e*(a + b*\sec[d + e*x] + c*\tan[d + e*x])^{3/2}) - (c*\sec[d + e*x]^{3/2}*(b + a*\cos[d + e*x] + c*\sin[d + e*x])^{3/2}*(-(c*\text{AppellF1}[-1/2, -1/2, -1/2, 1/2, -(b + a*\sqrt{1 + c^2/a^2})*\cos[d + e*x - \text{ArcTan}[c/a]])/(a*\sqrt{1 + c^2/a^2}*(1 - b/(a*\sqrt{1 + c^2/a^2}))))), -(b + a*\sqrt{1 + c^2/a^2}*\cos[d + e*x - \text{ArcTan}[c/a]])/(a*\sqrt{1 + c^2/a^2}*(-1 - b/(a*\sqrt{1 + c^2/a^2})))))*\sin[d + e*x - \text{ArcTan}[c/a]]/(a*\sqrt{1 + c^2/a^2}*\sqrt{[a*\sqrt{[a^2 + c^2]/a^2} - a*\sqrt{[a^2 + c^2]/a^2}*\cos[d + e*x - \text{ArcTan}[c/a]]]/(b + a*\sqrt{[a^2 + c^2]/a^2})]*\sqrt{b + a*\sqrt{[a^2 + c^2]/a^2}*\cos[d + e*x - \text{ArcTan}[c/a]]}*\sqrt{[a*\sqrt{[a^2 + c^2]/a^2} + a*\sqrt{[a^2 + c^2]/a^2}*\cos[d + e*x - \text{ArcTan}[c/a]]]/(-b + a*\sqrt{[a^2 + c^2]/a^2})]}]} - ((2*a*(b + a*\sqrt{1 + c^2/a^2})*\cos[d + e*x - \text{ArcTan}[c/a]])/(a^2 + c^2) - (c*\sin[d + e*x - \text{ArcTan}[c/a]]/(a*\sqrt{1 + c^2/a^2}))/\sqrt{b + a*\sqrt{1 + c^2/a^2}*\cos[d + e*x - \text{ArcTan}[c/a]]})/(a^2 - b^2 + c^2)*e*(a + b*\sec[d + e*x] + c*\tan[d + e*x])^{3/2}) \end{aligned}$$

fricas [F] time = 1.84, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{b \sec(ex+d) + c \tan(ex+d) + a} \sec(ex+d)^{\frac{3}{2}}}{b^2 \sec(ex+d)^2 + c^2 \tan(ex+d)^2 + 2ab \sec(ex+d) + a^2 + 2(bc \sec(ex+d) + ac) \tan(ex+d)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sec(e*x + d)^(3/2)/(b^2*sec(e*x + d)^2 + c^2*tan(e*x + d)^2 + 2*a*b*sec(e*x + d) + a^2 + 2*(b*c*sec(e*x + d) + a*c)*tan(e*x + d)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(ex+d)^{\frac{3}{2}}}{(b \sec(ex+d) + c \tan(ex+d) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm="giac")

[Out] integrate(sec(e*x + d)^(3/2)/(b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2), x)

maple [C] time = 1.22, size = 12574, normalized size = 52.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(ex+d)^{\frac{3}{2}}}{(b \sec(ex+d) + c \tan(ex+d) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(e*x + d)^(3/2)/(b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(d+ex)}\right)^{3/2}}{\left(a + c \tan(d + ex) + \frac{b}{\cos(d+ex)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(d + e*x))^(3/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2),x)
```

```
[Out] int((1/cos(d + e*x))^(3/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(d + ex)}{\left(a + b \sec(d + ex) + c \tan(d + ex)\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)**(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2),x)
```

```
[Out] Integral(sec(d + e*x)**(3/2)/(a + b*sec(d + e*x) + c*tan(d + e*x))**(3/2), x)
```

$$3.452 \quad \int \frac{\sec^2(d+ex)}{(a+b \sec(d+ex)+c \tan(d+ex))^{5/2}} dx$$

Optimal. Leaf size=492

$$\frac{2 \sec^2(d+ex) \sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} (a \cos(d+ex)+b+c \sin(d+ex))^2 F\left(\frac{1}{2} \left(d+ex - \tan^{-1}\left(\frac{a}{c}\right)\right) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e \left(a^2 - b^2 + c^2\right) (a+b \sec(d+ex)+c \tan(d+ex))^{5/2}}$$

[Out] $-2/3 * \sec(e*x+d)^{(5/2)} * (c * \cos(e*x+d) - a * \sin(e*x+d)) * (b + a * \cos(e*x+d) + c * \sin(e*x+d)) / (a^2 - b^2 + c^2) / e / (a + b * \sec(e*x+d) + c * \tan(e*x+d))^{(5/2)} + 8/3 * \sec(e*x+d)^{(5/2)} * (b * c * \cos(e*x+d) - a * b * \sin(e*x+d)) * (b + a * \cos(e*x+d) + c * \sin(e*x+d))^2 / (a^2 - b^2 + c^2)^2 / e / (a + b * \sec(e*x+d) + c * \tan(e*x+d))^{(5/2)} + 8/3 * b * (\cos(1/2 * d + 1/2 * e * x - 1/2 * \arctan(a, c)))^{(1/2)} / \cos(1/2 * d + 1/2 * e * x - 1/2 * \arctan(a, c)) * \text{EllipticE}(\sin(1/2 * d + 1/2 * e * x - 1/2 * \arctan(a, c)), 2^{(1/2)} * ((a^2 + c^2)^{(1/2)} / (b + (a^2 + c^2)^{(1/2)})))^{(1/2)} * \sec(e*x+d)^{(5/2)} * (b + a * \cos(e*x+d) + c * \sin(e*x+d))^3 / (a^2 - b^2 + c^2)^2 / e / ((b + a * \cos(e*x+d) + c * \sin(e*x+d)) / (b + (a^2 + c^2)^{(1/2)}))^{(1/2)} / (a + b * \sec(e*x+d) + c * \tan(e*x+d))^{(5/2)} + 2/3 * (\cos(1/2 * d + 1/2 * e * x - 1/2 * \arctan(a, c)))^{(1/2)} / \cos(1/2 * d + 1/2 * e * x - 1/2 * \arctan(a, c)) * \text{EllipticF}(\sin(1/2 * d + 1/2 * e * x - 1/2 * \arctan(a, c)), 2^{(1/2)} * ((a^2 + c^2)^{(1/2)} / (b + (a^2 + c^2)^{(1/2)})))^{(1/2)} * \sec(e*x+d)^{(5/2)} * (b + a * \cos(e*x+d) + c * \sin(e*x+d))^2 * ((b + a * \cos(e*x+d) + c * \sin(e*x+d)) / (b + (a^2 + c^2)^{(1/2)}))^{(1/2)} / (a^2 - b^2 + c^2) / e / (a + b * \sec(e*x+d) + c * \tan(e*x+d))^{(5/2)}$

Rubi [A] time = 0.52, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3167, 3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2 \sec^2(d+ex) \sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} (a \cos(d+ex)+b+c \sin(d+ex))^2 F\left(\frac{1}{2} \left(d+ex - \tan^{-1}\left(\frac{a}{c}\right)\right) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e \left(a^2 - b^2 + c^2\right) (a+b \sec(d+ex)+c \tan(d+ex))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sec[d + e*x]^(5/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2), x]

[Out] $(-2 * \text{Sec}[d + e*x]^{(5/2)} * (c * \text{Cos}[d + e*x] - a * \text{Sin}[d + e*x]) * (b + a * \text{Cos}[d + e*x] + c * \text{Sin}[d + e*x])) / (3 * (a^2 - b^2 + c^2) * e * (a + b * \text{Sec}[d + e*x] + c * \text{Tan}[d + e*x])^{(5/2)}) + (8 * \text{Sec}[d + e*x]^{(5/2)} * (b * c * \text{Cos}[d + e*x] - a * b * \text{Sin}[d + e*x]) * (b + a * \text{Cos}[d + e*x] + c * \text{Sin}[d + e*x])^2) / (3 * (a^2 - b^2 + c^2)^2 * e * (a + b * \text{Sec}[d + e*x] + c * \text{Tan}[d + e*x])^{(5/2)}) + (8 * b * \text{EllipticE}[(d + e*x - \text{ArcTan}[a, c])/2, (2 * \text{Sqrt}[a^2 + c^2]) / (b + \text{Sqrt}[a^2 + c^2])]) * \text{Sec}[d + e*x]^{(5/2)} * (b + a * \text{Cos}[d + e*x] + c * \text{Sin}[d + e*x])^3) / (3 * (a^2 - b^2 + c^2)^2 * e * \text{Sqrt}[(b + a * \text{Cos}[d + e*x] + c * \text{Sin}[d + e*x]) / (b + \text{Sqrt}[a^2 + c^2])]) * (a + b * \text{Sec}[d + e*x] + c * \text{Tan}[d + e*x])^{(5/2)}) + (2 * \text{EllipticF}[(d + e*x - \text{ArcTan}[a, c])/2, (2 * \text{Sqrt}[a^2 + c^2]) / (b + \text{Sqrt}[a^2 + c^2])]) * \text{Sec}[d + e*x]^{(5/2)} * (b + a * \text{Cos}[d + e*x] + c * \text{Sin}[d + e*x])^2 * \text{Sqrt}[(b + a * \text{Cos}[d + e*x] + c * \text{Sin}[d + e*x]) / (b + \text{Sqrt}[a^2 + c^2])]) / (3 * (a^2 - b^2 + c^2) * e * (a + b * \text{Sec}[d + e*x] + c * \text{Tan}[d + e*x])^{(5/2)})$

Rule 2653

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3127

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3129

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^n, x_Symbol] :> Simp[(-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]

Rule 3149

Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]) / Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]

Rule 3156

Int[((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^n * ((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[n, -2]

Rule 3167

Int[sec[(d_.) + (e_.)*(x_.)]^n * ((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_.)] + (c_.)*tan[(d_.) + (e_.)*(x_.)]^m), x_Symbol] :> Dist[(Sec[d + e*x]^n*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n, Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && !IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{5}{2}}} dx &= \frac{\left(\sec^{\frac{5}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))^{\frac{5}{2}}\right) \int \frac{1}{(b+a\cos(d+ex)+c\sin(d+ex))} dx}{(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{5}{2}}} \\
&= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex))}{3(a^2-b^2+c^2)e(a+b\sec(d+ex)+c\tan(d+ex))} \\
&= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex))}{3(a^2-b^2+c^2)e(a+b\sec(d+ex)+c\tan(d+ex))} \\
&= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex))}{3(a^2-b^2+c^2)e(a+b\sec(d+ex)+c\tan(d+ex))} \\
&= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex))}{3(a^2-b^2+c^2)e(a+b\sec(d+ex)+c\tan(d+ex))} \\
&= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex))}{3(a^2-b^2+c^2)e(a+b\sec(d+ex)+c\tan(d+ex))}
\end{aligned}$$

Mathematica [C] time = 6.55, size = 2708, normalized size = 5.50

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[d + e*x]^(5/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2), x]
[Out] (Sec[d + e*x]^(5/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^3*((8*b*(a^2 + c^2))/(3*a*c*(-a^2 + b^2 - c^2)^2) + (2*(b*c + a^2*Sin[d + e*x] + c^2*Sin[d + e*x]))/(3*a*(a^2 - b^2 + c^2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2) - (2*(a^2*c + 3*b^2*c + c^3 + 4*a^2*b*Sin[d + e*x] + 4*b*c^2*Sin[d + e*x]))/(3*a*(a^2 - b^2 + c^2)^2*(b + a*Cos[d + e*x] + c*Sin[d + e*x]))) / (e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, -(b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c)))*c), -(b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c)))*c)]*Sec[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[a/c]]*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]/(3*Sqrt[1 + a^2/c^2]*c*(a^2 - b^2 + c^2)^2*e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -(b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c)))*c), -(b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c)))*c)]*Sec[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[a/c]]*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]]]
```

$$\text{ArcTan}[a/c]] * \text{Sqrt}[(c * \text{Sqrt}[(a^2 + c^2)/c^2] + c * \text{Sqrt}[(a^2 + c^2)/c^2] * \text{Sin}[d + e*x + \text{ArcTan}[a/c]]) / (-b + c * \text{Sqrt}[(a^2 + c^2)/c^2])]) / (\text{Sqrt}[1 + a^2/c^2] * c * (a^2 - b^2 + c^2)^2 * e * (a + b * \text{Sec}[d + e*x] + c * \text{Tan}[d + e*x])^{(5/2)}) + (2 * c * \text{AppellF1}[1/2, 1/2, 1/2, 3/2, -((b + \text{Sqrt}[1 + a^2/c^2]) * c * \text{Sin}[d + e*x + \text{ArcTan}[a/c]]) / (\text{Sqrt}[1 + a^2/c^2] * (1 - b / (\text{Sqrt}[1 + a^2/c^2] * c))) * c), -((b + \text{Sqrt}[1 + a^2/c^2]) * c * \text{Sin}[d + e*x + \text{ArcTan}[a/c]]) / (\text{Sqrt}[1 + a^2/c^2] * (-1 - b / (\text{Sqrt}[1 + a^2/c^2] * c))) * c)]) * \text{Sec}[d + e*x]^{(5/2)} * \text{Sec}[d + e*x + \text{ArcTan}[a/c]] * (b + a * \text{Cos}[d + e*x] + c * \text{Sin}[d + e*x])^{(5/2)} * \text{Sqrt}[(c * \text{Sqrt}[(a^2 + c^2)/c^2] - c * \text{Sqrt}[(a^2 + c^2)/c^2] * \text{Sin}[d + e*x + \text{ArcTan}[a/c]]) / (b + c * \text{Sqrt}[(a^2 + c^2)/c^2])]) * \text{Sqrt}[b + c * \text{Sqrt}[(a^2 + c^2)/c^2] * \text{Sin}[d + e*x + \text{ArcTan}[a/c]]] * \text{Sqrt}[(c * \text{Sqrt}[(a^2 + c^2)/c^2] + c * \text{Sqrt}[(a^2 + c^2)/c^2] * \text{Sin}[d + e*x + \text{ArcTan}[a/c]]) / (-b + c * \text{Sqrt}[(a^2 + c^2)/c^2])]) / (3 * \text{Sqrt}[1 + a^2/c^2] * (a^2 - b^2 + c^2)^2 * e * (a + b * \text{Sec}[d + e*x] + c * \text{Tan}[d + e*x])^{(5/2)}) + (4 * a^2 * b * \text{Sec}[d + e*x]^{(5/2)} * (b + a * \text{Cos}[d + e*x] + c * \text{Sin}[d + e*x])^{(5/2)} * (-((c * \text{AppellF1}[-1/2, -1/2, -1/2, 1/2, -((b + a * \text{Sqrt}[1 + c^2/a^2]) * \text{Cos}[d + e*x - \text{ArcTan}[c/a]]) / (a * \text{Sqrt}[1 + c^2/a^2]) * (1 - b / (a * \text{Sqrt}[1 + c^2/a^2]))) - ((b + a * \text{Sqrt}[1 + c^2/a^2]) * \text{Cos}[d + e*x - \text{ArcTan}[c/a]]) / (a * \text{Sqrt}[1 + c^2/a^2]) * (-1 - b / (a * \text{Sqrt}[1 + c^2/a^2]))) * \text{Sin}[d + e*x - \text{ArcTan}[c/a]]) / (a * \text{Sqrt}[1 + c^2/a^2]) * \text{Sqrt}[(a * \text{Sqrt}[(a^2 + c^2)/a^2] - a * \text{Sqrt}[(a^2 + c^2)/a^2] * \text{Cos}[d + e*x - \text{ArcTan}[c/a]]) / (b + a * \text{Sqrt}[(a^2 + c^2)/a^2])]) * \text{Sqrt}[b + a * \text{Sqrt}[(a^2 + c^2)/a^2] * \text{Cos}[d + e*x - \text{ArcTan}[c/a]]] * \text{Sqrt}[(a * \text{Sqrt}[(a^2 + c^2)/a^2] + a * \text{Sqrt}[(a^2 + c^2)/a^2] * \text{Cos}[d + e*x - \text{ArcTan}[c/a]]) / (-b + a * \text{Sqrt}[(a^2 + c^2)/a^2])]) - ((2 * a * (b + a * \text{Sqrt}[1 + c^2/a^2]) * \text{Cos}[d + e*x - \text{ArcTan}[c/a]]) / (a^2 + c^2) - (c * \text{Sin}[d + e*x - \text{ArcTan}[c/a]]) / (a * \text{Sqrt}[1 + c^2/a^2])) / \text{Sqrt}[b + a * \text{Sqrt}[1 + c^2/a^2] * \text{Cos}[d + e*x - \text{ArcTan}[c/a]]]) / (3 * c * (a^2 - b^2 + c^2)^2 * e * (a + b * \text{Sec}[d + e*x] + c * \text{Tan}[d + e*x])^{(5/2)}) + (4 * b * c * \text{Sec}[d + e*x]^{(5/2)} * (b + a * \text{Cos}[d + e*x] + c * \text{Sin}[d + e*x])^{(5/2)} * (-((c * \text{AppellF1}[-1/2, -1/2, -1/2, 1/2, -((b + a * \text{Sqrt}[1 + c^2/a^2]) * \text{Cos}[d + e*x - \text{ArcTan}[c/a]]) / (a * \text{Sqrt}[1 + c^2/a^2]) * (1 - b / (a * \text{Sqrt}[1 + c^2/a^2]))) - ((b + a * \text{Sqrt}[1 + c^2/a^2]) * \text{Cos}[d + e*x - \text{ArcTan}[c/a]]) / (a * \text{Sqrt}[1 + c^2/a^2]) * (-1 - b / (a * \text{Sqrt}[1 + c^2/a^2]))) * \text{Sin}[d + e*x - \text{ArcTan}[c/a]]) / (a * \text{Sqrt}[1 + c^2/a^2]) * \text{Sqrt}[(a * \text{Sqrt}[(a^2 + c^2)/a^2] - a * \text{Sqrt}[(a^2 + c^2)/a^2] * \text{Cos}[d + e*x - \text{ArcTan}[c/a]]) / (b + a * \text{Sqrt}[(a^2 + c^2)/a^2])]) * \text{Sqrt}[b + a * \text{Sqrt}[(a^2 + c^2)/a^2] * \text{Cos}[d + e*x - \text{ArcTan}[c/a]]] * \text{Sqrt}[(a * \text{Sqrt}[(a^2 + c^2)/a^2] + a * \text{Sqrt}[(a^2 + c^2)/a^2] * \text{Cos}[d + e*x - \text{ArcTan}[c/a]]) / (-b + a * \text{Sqrt}[(a^2 + c^2)/a^2])]) - ((2 * a * (b + a * \text{Sqrt}[1 + c^2/a^2]) * \text{Cos}[d + e*x - \text{ArcTan}[c/a]]) / (a^2 + c^2) - (c * \text{Sin}[d + e*x - \text{ArcTan}[c/a]]) / (a * \text{Sqrt}[1 + c^2/a^2])) / \text{Sqrt}[b + a * \text{Sqrt}[1 + c^2/a^2] * \text{Cos}[d + e*x - \text{ArcTan}[c/a]]]) / (3 * (a^2 - b^2 + c^2)^2 * e * (a + b * \text{Sec}[d + e*x] + c * \text{Tan}[d + e*x])^{(5/2)})$$

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{b \sec(ex + d) + c \tan(ex + d) + a}}{b^3 \sec^3(ex + d) + c^3 \tan^3(ex + d) + 3ab^2 \sec^2(ex + d) + 3a^2b \sec(ex + d) + a^3 + 3(bc^2 \sec(ex + d) + a^3 \tan^2(ex + d))} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sec(e*x + d)^(5/2)/(b^3*sec(e*x + d)^3 + c^3*tan(e*x + d)^3 + 3*a*b^2*sec(e*x + d)^2 + 3*a^2*b*sec(e*x + d) + a^3 + 3*(b*c^2*sec(e*x + d) + a*c^2)*tan(e*x + d)^2 + 3*(b^2*c*sec(e*x + d)^2 + 2*a*b*c*sec(e*x + d) + a^2*c)*tan(e*x + d)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(ex + d)}{(b \sec(ex + d) + c \tan(ex + d) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorithm="giac")

[Out] integrate(sec(e*x + d)^(5/2)/(b*sec(e*x + d) + c*tan(e*x + d) + a)^(5/2), x)

maple [C] time = 2.55, size = 64693, normalized size = 131.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(ex+d)^{\frac{5}{2}}}{(b \sec(ex+d) + c \tan(ex+d) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorithm="maxima")

[Out] integrate(sec(e*x + d)^(5/2)/(b*sec(e*x + d) + c*tan(e*x + d) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(d+ex)}\right)^{5/2}}{\left(a + c \tan(d+ex) + \frac{b}{\cos(d+ex)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/cos(d + e*x))^(5/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(5/2),x)

[Out] int((1/cos(d + e*x))^(5/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(e*x+d)**(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(5/2),x)

[Out] Timed out

$$3.453 \quad \int \cos^{\frac{3}{2}}(d + ex)(a + b \sec(d + ex) + c \tan(d + ex))^{\frac{3}{2}} dx$$

Optimal. Leaf size=371

$$\frac{2(a^2 - b^2 + c^2) \cos^{\frac{3}{2}}(d + ex) \sqrt{\frac{a \cos(d + ex) + b + c \sin(d + ex)}{\sqrt{a^2 + c^2} + b}} (a + b \sec(d + ex) + c \tan(d + ex))^{\frac{3}{2}} F\left(\frac{1}{2}(d + ex - \tan^{-1}(a, c))\right)}{3e(a \cos(d + ex) + b + c \sin(d + ex))^2}$$

[Out] $-2/3 * \cos(e*x+d)^{(3/2)} * (c * \cos(e*x+d) - a * \sin(e*x+d)) * (a + b * \sec(e*x+d) + c * \tan(e*x+d))^{\frac{3}{2}} / e / (b + a * \cos(e*x+d) + c * \sin(e*x+d)) + 8/3 * b * \cos(e*x+d)^{(3/2)} * (\cos(1/2 * d + 1/2 * e*x - 1/2 * \arctan(a, c))^2)^{(1/2)} / \cos(1/2 * d + 1/2 * e*x - 1/2 * \arctan(a, c)) * \text{EllipticE}(\sin(1/2 * d + 1/2 * e*x - 1/2 * \arctan(a, c)), 2^{(1/2)} * ((a^2 + c^2)^{(1/2)} / (b + (a^2 + c^2)^{(1/2)})))^{(1/2)} * (a + b * \sec(e*x+d) + c * \tan(e*x+d))^{\frac{3}{2}} / e / (b + a * \cos(e*x+d) + c * \sin(e*x+d)) / ((b + a * \cos(e*x+d) + c * \sin(e*x+d)) / (b + (a^2 + c^2)^{(1/2)}))^{\frac{1}{2}} + 2/3 * (a^2 - b^2 + c^2) * \cos(e*x+d)^{(3/2)} * (\cos(1/2 * d + 1/2 * e*x - 1/2 * \arctan(a, c))^2)^{(1/2)} / \cos(1/2 * d + 1/2 * e*x - 1/2 * \arctan(a, c)) * \text{EllipticF}(\sin(1/2 * d + 1/2 * e*x - 1/2 * \arctan(a, c)), 2^{(1/2)} * ((a^2 + c^2)^{(1/2)} / (b + (a^2 + c^2)^{(1/2)})))^{(1/2)} * ((b + a * \cos(e*x+d) + c * \sin(e*x+d)) / (b + (a^2 + c^2)^{(1/2)}))^{\frac{1}{2}} * (a + b * \sec(e*x+d) + c * \tan(e*x+d))^{\frac{3}{2}} / e / (b + a * \cos(e*x+d) + c * \sin(e*x+d))^2$

Rubi [A] time = 0.39, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3163, 3120, 3149, 3119, 2653, 3127, 2661}

$$\frac{2(a^2 - b^2 + c^2) \cos^{\frac{3}{2}}(d + ex) \sqrt{\frac{a \cos(d + ex) + b + c \sin(d + ex)}{\sqrt{a^2 + c^2} + b}} (a + b \sec(d + ex) + c \tan(d + ex))^{\frac{3}{2}} F\left(\frac{1}{2}(d + ex - \tan^{-1}(a, c))\right)}{3e(a \cos(d + ex) + b + c \sin(d + ex))^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x]

[Out] $(-2 * \cos[d + e*x]^{(3/2)} * (c * \cos[d + e*x] - a * \sin[d + e*x]) * (a + b * \sec[d + e*x] + c * \tan[d + e*x])^{\frac{3}{2}}) / (3 * e * (b + a * \cos[d + e*x] + c * \sin[d + e*x])) + (8 * b * \cos[d + e*x]^{(3/2)} * \text{EllipticE}[(d + e*x - \text{ArcTan}[a, c])/2, (2 * \sqrt{a^2 + c^2}) / (b + \sqrt{a^2 + c^2})]) * (a + b * \sec[d + e*x] + c * \tan[d + e*x])^{\frac{3}{2}}) / (3 * e * (b + a * \cos[d + e*x] + c * \sin[d + e*x]) * \sqrt{(b + a * \cos[d + e*x] + c * \sin[d + e*x]) / (b + \sqrt{a^2 + c^2})}) + (2 * (a^2 - b^2 + c^2) * \cos[d + e*x]^{(3/2)} * \text{EllipticF}[(d + e*x - \text{ArcTan}[a, c])/2, (2 * \sqrt{a^2 + c^2}) / (b + \sqrt{a^2 + c^2})]) * \sqrt{(b + a * \cos[d + e*x] + c * \sin[d + e*x]) / (b + \sqrt{a^2 + c^2})}) * (a + b * \sec[d + e*x] + c * \tan[d + e*x])^{\frac{3}{2}}) / (3 * e * (b + a * \cos[d + e*x] + c * \sin[d + e*x]))^2$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] := -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]
```

Rule 3163

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] := Dist[(Cos[d + e*x]^n*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n)/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2} dx &= \frac{\left(\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\right) \int (d+ex) dx}{(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}} \\
&= -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{3e(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}} \\
&= -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{3e(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}} \\
&= -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{3e(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}} \\
&= -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{3e(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}
\end{aligned}$$

Mathematica [F] time = 150.81, size = 0, normalized size = 0.00

$$\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2), x]

[Out] Integrate[Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2), x]

fricas [F] time = 0.93, size = 0, normalized size = 0.00

integral((b*cos(ex+d)*sec(ex+d)+c*cos(ex+d)*tan(ex+d)+a*cos(ex+d))*sqrt(b*sec(ex+d)+c*tan(ex+d)+a)*sqrt(cos(ex+d)), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2), x, algorithm="fricas")

[Out] integral((b*cos(e*x + d)*sec(e*x + d) + c*cos(e*x + d)*tan(e*x + d) + a*cos(e*x + d))*sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{3}{2}}\cos(ex+d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2), x, algorithm="giac")

[Out] integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)*cos(e*x + d)^(3/2), x)

maple [C] time = 1.29, size = 20776, normalized size = 56.00

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(ex + d) + c \tan(ex + d) + a)^{\frac{3}{2}} \cos(ex + d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)*cos(e*x + d)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(d + ex)^{3/2} \left(a + c \tan(d + ex) + \frac{b}{\cos(d + ex)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d + e*x)^(3/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2),x)`

[Out] `int(cos(d + e*x)^(3/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(e*x+d)**(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2),x)`

[Out] Timed out

3.454 $\int \sqrt{\cos(d+ex)} \sqrt{a+b \sec(d+ex)+c \tan(d+ex)} dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{\cos(d+ex)} \sqrt{a+b \sec(d+ex)+c \tan(d+ex)} E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{a}{c}\right)\right)\middle|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2))/(b+(a^2+c^2)^(1/2)))^(1/2)*cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/e/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)

Rubi [A] time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3163, 3119, 2653}

$$\frac{2\sqrt{\cos(d+ex)} \sqrt{a+b \sec(d+ex)+c \tan(d+ex)} E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{a}{c}\right)\right)\middle|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]

[Out] (2*Sqrt[Cos[d + e*x]]*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(e*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]]/Sqrt[(a + b*cos[d + e*x] + c*sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3163

Int[cos[(d_) + (e_)*(x_)]^(n_)*((a_) + (b_)*sec[(d_) + (e_)*(x_)] + (c_)*tan[(d_) + (e_)*(x_)]^(n_), x_Symbol] := Dist[(Cos[d + e*x]^n*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n)/(b + a*cos[d + e*x] + c*sin[d + e*x])^n, Int[(b + a*cos[d + e*x] + c*sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && !IntegerQ[n]

Rubi steps

$$\int \sqrt{\cos(d+ex)} \sqrt{a+b\sec(d+ex)+c\tan(d+ex)} dx = \frac{(\sqrt{\cos(d+ex)} \sqrt{a+b\sec(d+ex)+c\tan(d+ex)}) \int \frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}} dx}{1}$$

$$= \frac{(\sqrt{\cos(d+ex)} \sqrt{a+b\sec(d+ex)+c\tan(d+ex)}) \int \frac{1}{\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}} dx}{1}$$

$$= \frac{2\sqrt{\cos(d+ex)} E\left(\frac{1}{2}(d+ex - \tan^{-1}(a,c)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

Mathematica [F] time = 21.22, size = 0, normalized size = 0.00

$$\int \sqrt{\cos(d+ex)} \sqrt{a+b\sec(d+ex)+c\tan(d+ex)} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]], x]

[Out] Integrate[Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]], x]

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}(\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\sqrt{\cos(ex+d)}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\sqrt{\cos(ex+d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)), x)

maple [C] time = 1.02, size = 12459, normalized size = 105.58

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2), x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\sqrt{\cos(ex+d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos(d+ex)} \sqrt{a+c \tan(d+ex) + \frac{b}{\cos(d+ex)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d + e*x)^(1/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2),x)
```

```
[Out] int(cos(d + e*x)^(1/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b \sec(d+ex) + c \tan(d+ex)} \sqrt{\cos(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)**(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(d + e*x) + c*tan(d + e*x))*sqrt(cos(d + e*x)), x)
```

$$3.455 \quad \int \frac{1}{\sqrt{\cos(d+ex)} \sqrt{a+b \sec(d+ex)+c \tan(d+ex)}} dx$$

Optimal. Leaf size=118

$$\frac{2\sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} F\left(\frac{1}{2}(d+ex - \tan^{-1}(a,c)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\cos(d+ex)} \sqrt{a+b \sec(d+ex)+c \tan(d+ex)}}$$

[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2))/(b+(a^2+c^2)^(1/2)))^(1/2))*((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/e/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)

Rubi [A] time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3163, 3127, 2661}

$$\frac{2\sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} F\left(\frac{1}{2}(d+ex - \tan^{-1}(a,c)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\cos(d+ex)} \sqrt{a+b \sec(d+ex)+c \tan(d+ex)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]),x]

[Out] (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(e*Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3127

Int[1/Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3163

Int[cos[(d_) + (e_)*(x_)]^(n_)*((a_) + (b_)*sec[(d_) + (e_)*(x_)] + (c_)*tan[(d_) + (e_)*(x_)]^(n_), x_Symbol] := Dist[(Cos[d + e*x]^n*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n)/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && !IntegerQ[n]

Rubi steps

$$\int \frac{1}{\sqrt{\cos(d+ex)} \sqrt{a+b \sec(d+ex)+c \tan(d+ex)}} dx = \frac{\sqrt{b+a \cos(d+ex)+c \sin(d+ex)}}{\sqrt{\cos(d+ex)} \sqrt{a+b \sec(d+ex)+c \tan(d+ex)}} \int \frac{1}{\sqrt{b+a \cos(d+ex)+c \sin(d+ex)}} dx$$

$$= \frac{\sqrt{\frac{b+a \cos(d+ex)+c \sin(d+ex)}{b+\sqrt{a^2+c^2}}}}{\sqrt{\cos(d+ex)} \sqrt{a+b \sec(d+ex)+c \tan(d+ex)}} \int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}} + \frac{\sqrt{a^2+c^2} \cos(d+ex-\tan^{-1}(a,c))}{b+\sqrt{a^2+c^2}}}} dx$$

$$= \frac{2F\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right) \sqrt{\frac{b+a \cos(d+ex)+c \sin(d+ex)}{b+\sqrt{a^2+c^2}}}}{e \sqrt{\cos(d+ex)} \sqrt{a+b \sec(d+ex)+c \tan(d+ex)}}$$

Mathematica [C] time = 2.90, size = 506, normalized size = 4.29

$$4 \left(\sqrt{a^2 - b^2 + c^2} + ia - ib + c \right) (\cos(d+ex) + i \sin(d+ex)) \sqrt{\frac{i(\sqrt{a^2-b^2+c^2}+(a-b)\tan(\frac{1}{2}(d+ex))-c)}{(\sqrt{a^2-b^2+c^2}-ia+ib-c)(\tan(\frac{1}{2}(d+ex))-i)}} \sqrt{\frac{i(\sqrt{a^2-b^2+c^2}+(a-b)\tan(\frac{1}{2}(d+ex))-c)}{(\sqrt{a^2-b^2+c^2}-ia+ib-c)(\tan(\frac{1}{2}(d+ex))-i)}}}$$

$$e \left(a + i \left(\sqrt{a^2 - b^2 + c^2} + ib + c \right) \right) \sqrt{\dots}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]), x]

[Out] (4*(I*a - I*b + c + Sqrt[a^2 - b^2 + c^2])*EllipticF[ArcSin[Sqrt[(((I)*a + I*b + c + Sqrt[a^2 - b^2 + c^2])*(-Cos[d + e*x] + I*Sin[d + e*x]))/(I*a - I*b + c + Sqrt[a^2 - b^2 + c^2])]], (b + I*Sqrt[a^2 - b^2 + c^2])/(b - I*Sqrt[a^2 - b^2 + c^2])]*Sqrt[(((I)*a + I*b + c + Sqrt[a^2 - b^2 + c^2])*(-Cos[d + e*x] + I*Sin[d + e*x]))/(I*a - I*b + c + Sqrt[a^2 - b^2 + c^2])]*(Cos[d + e*x] + I*Sin[d + e*x])*Sqrt[(((I)*(-c + Sqrt[a^2 - b^2 + c^2] + (a - b)*Tan[(d + e*x)/2]))/(((I)*a + I*b - c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[(d + e*x)/2]))]*Sqrt[(((I)*(c + Sqrt[a^2 - b^2 + c^2] + (-a + b)*Tan[(d + e*x)/2]))/(((I)*a - I*b + c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[(d + e*x)/2])))]/(a + I*(I*b + c + Sqrt[a^2 - b^2 + c^2]))*e*Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b \sec(ex+d) + c \tan(ex+d) + a} \sqrt{\cos(ex+d)}}{b \cos(ex+d) \sec(ex+d) + c \cos(ex+d) \tan(ex+d) + a \cos(ex+d)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d))/(b*cos(e*x + d)*sec(e*x + d) + c*cos(e*x + d)*tan(e*x + d) + a*cos(e*x + d)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \sec(ex+d) + c \tan(ex+d) + a} \sqrt{\cos(ex+d)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d))), x)

maple [C] time = 1.53, size = 714, normalized size = 6.05

$$4i \sqrt{\frac{b+a \cos(ex+d)+c \sin(ex+d)}{\cos(ex+d)}} \sqrt{\frac{(i \sin(ex+d)+\cos(ex+d))(ia-ib-\sqrt{a^2-b^2+c^2}+c)}{ia-ib+\sqrt{a^2-b^2+c^2}-c}} \sqrt{\frac{i(\cos(ex+d)\sqrt{a^2-b^2+c^2}-a \sin(ex+d)+b \sin(ex+d)+\cos(ex+d))}{(i \cos(ex+d)+\sin(ex+d)+i)(ia-ib-\sqrt{a^2-b^2+c^2}+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2), x)

[Out] $-4*I/e*((b+a*\cos(e*x+d)+c*\sin(e*x+d))/\cos(e*x+d))^{1/2}*((I*\sin(e*x+d)+\cos(e*x+d))*(I*a-I*b-(a^2-b^2+c^2)^{1/2}+c)/(I*a-I*b+(a^2-b^2+c^2)^{1/2}-c))^{1/2}*(-I*(\cos(e*x+d)*(a^2-b^2+c^2)^{1/2}-a*\sin(e*x+d)+b*\sin(e*x+d)+c*\cos(e*x+d)+(a^2-b^2+c^2)^{1/2}+c)/(I*\cos(e*x+d)+\sin(e*x+d)+I)/(I*a-I*b-(a^2-b^2+c^2)^{1/2}-c))^{1/2}*(I*(a*\sin(e*x+d)-b*\sin(e*x+d)+\cos(e*x+d)*(a^2-b^2+c^2)^{1/2}-c*\cos(e*x+d)+(a^2-b^2+c^2)^{1/2}-c)/(I*\cos(e*x+d)+\sin(e*x+d)+I)/(I*a-I*b+(a^2-b^2+c^2)^{1/2}-c))^{1/2}*(\cos(e*x+d)+1)^2*\text{EllipticF}(((I*\sin(e*x+d)+\cos(e*x+d))*(I*a-I*b-(a^2-b^2+c^2)^{1/2}+c)/(I*a-I*b+(a^2-b^2+c^2)^{1/2}-c))^{1/2}, ((I*a-I*b+(a^2-b^2+c^2)^{1/2}-c)*(I*a-I*b+(a^2-b^2+c^2)^{1/2}+c)/(I*a-I*b-(a^2-b^2+c^2)^{1/2}+c)/(I*a-I*b-(a^2-b^2+c^2)^{1/2}-c))^{1/2})*\cos(e*x+d)^{1/2}*(\cos(e*x+d)-1)^2*(I*(a^2-b^2+c^2)^{1/2}*\sin(e*x+d)-I*a*\cos(e*x+d)+I*\cos(e*x+d)*b-I*c*\sin(e*x+d)-\cos(e*x+d)*(a^2-b^2+c^2)^{1/2}+c*\cos(e*x+d)-a*\sin(e*x+d)+b*\sin(e*x+d))/\sin(e*x+d)^4/(b+a*\cos(e*x+d)+c*\sin(e*x+d))/(-I*a+I*b+(a^2-b^2+c^2)^{1/2}-c)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \sec(ex+d) + c \tan(ex+d) + a} \sqrt{\cos(ex+d)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos(d+ex)} \sqrt{a+c \tan(d+ex) + \frac{b}{\cos(d+ex)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(d + e*x)^(1/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2)), x)

[Out] int(1/(cos(d + e*x)^(1/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b \sec(d+ex) + c \tan(d+ex)} \sqrt{\cos(d+ex)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)**(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(d + e*x) + c*tan(d + e*x))*sqrt(cos(d + e*x))),  
x)
```

$$3.456 \quad \int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b \sec(d+ex)+c \tan(d+ex))^{3/2}} dx$$

Optimal. Leaf size=240

$$\frac{2(a \cos(d+ex) + b + c \sin(d+ex))^2 E\left(\frac{1}{2}\left(d+ex - \tan^{-1}\left(\frac{a}{c}\right)\right) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2 - b^2 + c^2\right) \cos^{\frac{3}{2}}(d+ex) \sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} (a+b \sec(d+ex) + c \tan(d+ex))^{3/2}} - \frac{2(c \cos(d+ex) - e\left(a^2 - b^2 + c^2\right) \cos^{\frac{3}{2}}(d+ex))}{e\left(a^2 - b^2 + c^2\right) \cos^{\frac{3}{2}}(d+ex) \sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} (a+b \sec(d+ex) + c \tan(d+ex))^{3/2}}$$

[Out] $-2*(c*\cos(e*x+d)-a*\sin(e*x+d))*(b+a*\cos(e*x+d)+c*\sin(e*x+d))/(a^2-b^2+c^2)/e/\cos(e*x+d)^{(3/2)}/(a+b*\sec(e*x+d)+c*\tan(e*x+d))^{(3/2)}-2*(\cos(1/2*d+1/2*e*x-1/2*\arctan(a,c))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(a,c))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(a,c)),2^{(1/2)}*((a^2+c^2)^{(1/2)}/(b+(a^2+c^2)^{(1/2)})))^{(1/2)}*(b+a*\cos(e*x+d)+c*\sin(e*x+d))^2/(a^2-b^2+c^2)/e/\cos(e*x+d)^{(3/2)}/((b+a*\cos(e*x+d)+c*\sin(e*x+d))/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}/(a+b*\sec(e*x+d)+c*\tan(e*x+d))^{(3/2)}$

Rubi [A] time = 0.21, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3163, 3128, 3119, 2653}

$$\frac{2(a \cos(d+ex) + b + c \sin(d+ex))^2 E\left(\frac{1}{2}\left(d+ex - \tan^{-1}\left(\frac{a}{c}\right)\right) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\left(a^2 - b^2 + c^2\right) \cos^{\frac{3}{2}}(d+ex) \sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} (a+b \sec(d+ex) + c \tan(d+ex))^{3/2}} - \frac{2(c \cos(d+ex) - e\left(a^2 - b^2 + c^2\right) \cos^{\frac{3}{2}}(d+ex))}{e\left(a^2 - b^2 + c^2\right) \cos^{\frac{3}{2}}(d+ex) \sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} (a+b \sec(d+ex) + c \tan(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)), x]

[Out] $(-2*(c*\cos[d + e*x] - a*\sin[d + e*x])*(b + a*\cos[d + e*x] + c*\sin[d + e*x]))/((a^2 - b^2 + c^2)*e*\cos[d + e*x]^{(3/2)}*(a + b*\sec[d + e*x] + c*\tan[d + e*x])^{(3/2)}) - (2*\text{EllipticE}((d + e*x - \text{ArcTan}[a, c])/2, (2*\text{Sqrt}[a^2 + c^2]))/(b + \text{Sqrt}[a^2 + c^2]))*(b + a*\cos[d + e*x] + c*\sin[d + e*x])^2/((a^2 - b^2 + c^2)*e*\cos[d + e*x]^{(3/2)}*\text{Sqrt}[(b + a*\cos[d + e*x] + c*\sin[d + e*x])/(b + \text{Sqrt}[a^2 + c^2])])*(a + b*\sec[d + e*x] + c*\tan[d + e*x])^{(3/2)}$

Rule 2653

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]]/Sqrt[(a + b*cos[d + e*x] + c*sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3128

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-3/2), x_Symbol] := Simp[(2*(c*cos[d + e*x] - b*sin[d + e*x]))/(e*(a^2 - b^2 - c^2)*Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]]), x] + Dist[1/(a^2 - b^2 - c^2), Int[Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]], x], x] /; FreeQ[{a

, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3163

Int[cos[(d_.) + (e_.)*(x_.)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_.)] + (c_.)*tan[(d_.) + (e_.)*(x_.)]^(n_), x_Symbol] := Dist[(Cos[d + e*x]^n*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n)/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && !IntegerQ[n]

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{3}{2}}} dx = \frac{(b+a\cos(d+ex)+c\sin(d+ex))^{\frac{3}{2}} \int \frac{1}{(b+a\cos(d+ex)+c\sin(d+ex))}}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{3}{2}}} = -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))}{(a^2-b^2+c^2)e\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{3}{2}}} = -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))}{(a^2-b^2+c^2)e\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{3}{2}}} = -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))}{(a^2-b^2+c^2)e\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{3}{2}}}$$

Mathematica [F] time = 23.99, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{3}{2}}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)), x]

[Out] Integrate[1/(Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)), x]

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\sqrt{\cos(ex+d)}}{b^2\cos(ex+d)^2\sec(ex+d)^2+c^2\cos(ex+d)^2\tan(ex+d)^2+2ab\cos(ex+d)^2\sec(ex+d)+a^2\cos(ex+d)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d))/(b^2*cos(e*x + d)^2*sec(e*x + d)^2 + c^2*cos(e*x + d)^2*tan(e*x + d)^2 + 2*a*b*cos(e*x + d)^2*sec(e*x + d) + a^2*cos(e*x + d)^2 + 2*(b*c*cos(e*x + d)^2*sec(e*x + d) + a*c*cos(e*x + d)^2)*tan(e*x + d)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \sec(ex + d) + c \tan(ex + d) + a)^{\frac{3}{2}} \cos(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm="giac")

[Out] integrate(1/((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)*cos(e*x + d)^(3/2)), x)

maple [C] time = 0.95, size = 12564, normalized size = 52.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \sec(ex + d) + c \tan(ex + d) + a)^{\frac{3}{2}} \cos(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm="maxima")

[Out] integrate(1/((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)*cos(e*x + d)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(d + ex)^{3/2} \left(a + c \tan(d + ex) + \frac{b}{\cos(d + ex)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(d + e*x)^(3/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2)),x)

[Out] int(1/(cos(d + e*x)^(3/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(e*x+d)**(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2),x)

[Out] Timed out

$$3.457 \quad \int \frac{1}{\cos^2(d+ex)(a+b \sec(d+ex)+c \tan(d+ex))^{5/2}} dx$$

Optimal. Leaf size=492

$$\frac{2\sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} (a \cos(d+ex)+b+c \sin(d+ex))^2 F\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e(a^2-b^2+c^2) \cos^2(d+ex)(a+b \sec(d+ex)+c \tan(d+ex))^{5/2}} + \frac{8b(a^2-b^2+c^2)}{3e(a^2-b^2+c^2)}$$

[Out] $-2/3*(c*\cos(e*x+d)-a*\sin(e*x+d))*(b+a*\cos(e*x+d)+c*\sin(e*x+d))/(a^2-b^2+c^2)/e/\cos(e*x+d)^{(5/2)}/(a+b*\sec(e*x+d)+c*\tan(e*x+d))^{(5/2)}+8/3*(b*c*\cos(e*x+d)-a*b*\sin(e*x+d))*(b+a*\cos(e*x+d)+c*\sin(e*x+d))^2/(a^2-b^2+c^2)^2/e/\cos(e*x+d)^{(5/2)}/(a+b*\sec(e*x+d)+c*\tan(e*x+d))^{(5/2)}+8/3*b*(\cos(1/2*d+1/2*e*x-1/2*\arctan(a,c))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(a,c))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(a,c)),2^{(1/2)}*((a^2+c^2)^{(1/2)}/(b+(a^2+c^2)^{(1/2)})))^{(1/2)}*(b+a*\cos(e*x+d)+c*\sin(e*x+d))^3/(a^2-b^2+c^2)^2/e/\cos(e*x+d)^{(5/2)}/((b+a*\cos(e*x+d)+c*\sin(e*x+d))/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}/(a+b*\sec(e*x+d)+c*\tan(e*x+d))^{(5/2)}+2/3*(\cos(1/2*d+1/2*e*x-1/2*\arctan(a,c))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(a,c))*\text{EllipticF}(\sin(1/2*d+1/2*e*x-1/2*\arctan(a,c)),2^{(1/2)}*((a^2+c^2)^{(1/2)}/(b+(a^2+c^2)^{(1/2)})))^{(1/2)}*(b+a*\cos(e*x+d)+c*\sin(e*x+d))^2*((b+a*\cos(e*x+d)+c*\sin(e*x+d))/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}/(a^2-b^2+c^2)/e/\cos(e*x+d)^{(5/2)}/(a+b*\sec(e*x+d)+c*\tan(e*x+d))^{(5/2)}$

Rubi [A] time = 0.49, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3163, 3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\sqrt{\frac{a \cos(d+ex)+b+c \sin(d+ex)}{\sqrt{a^2+c^2}+b}} (a \cos(d+ex)+b+c \sin(d+ex))^2 F\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e(a^2-b^2+c^2) \cos^2(d+ex)(a+b \sec(d+ex)+c \tan(d+ex))^{5/2}} + \frac{8b(a^2-b^2+c^2)}{3e(a^2-b^2+c^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)),x]

[Out] $(-2*(c*\cos[d + e*x] - a*\sin[d + e*x])*(b + a*\cos[d + e*x] + c*\sin[d + e*x]))/(3*(a^2 - b^2 + c^2)*e*\cos[d + e*x]^{(5/2)}*(a + b*\sec[d + e*x] + c*\tan[d + e*x])^{(5/2)}) + (8*(b*c*\cos[d + e*x] - a*b*\sin[d + e*x])*(b + a*\cos[d + e*x] + c*\sin[d + e*x])^2)/(3*(a^2 - b^2 + c^2)^2*e*\cos[d + e*x]^{(5/2)}*(a + b*\sec[d + e*x] + c*\tan[d + e*x])^{(5/2)}) + (8*b*\text{EllipticE}[(d + e*x - \text{ArcTan}[a, c])/2, (2*\text{Sqrt}[a^2 + c^2])/(b + \text{Sqrt}[a^2 + c^2])])*(b + a*\cos[d + e*x] + c*\sin[d + e*x])^3/(3*(a^2 - b^2 + c^2)^2*e*\cos[d + e*x]^{(5/2)}*\text{Sqrt}[(b + a*\cos[d + e*x] + c*\sin[d + e*x])/(b + \text{Sqrt}[a^2 + c^2])])*(a + b*\sec[d + e*x] + c*\tan[d + e*x])^{(5/2)}) + (2*\text{EllipticF}[(d + e*x - \text{ArcTan}[a, c])/2, (2*\text{Sqrt}[a^2 + c^2])/(b + \text{Sqrt}[a^2 + c^2])])*(b + a*\cos[d + e*x] + c*\sin[d + e*x])^2*\text{Sqrt}[(b + a*\cos[d + e*x] + c*\sin[d + e*x])/(b + \text{Sqrt}[a^2 + c^2])])/(3*(a^2 - b^2 + c^2)*e*\cos[d + e*x]^{(5/2)}*(a + b*\sec[d + e*x] + c*\tan[d + e*x])^{(5/2)})$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]]], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3127

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]]], x_Symbol] := Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3129

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]]^n, x_Symbol] := Simp[((-c*Cos[d + e*x] + b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]

Rule 3149

Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)])/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]]], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]

Rule 3156

Int[((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]]^n*(A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] := -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[n, -2]

Rule 3163

Int[cos[(d_.) + (e_.)*(x_.)]^n*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_.)] + (c_.)*tan[(d_.) + (e_.)*(x_.)]^n), x_Symbol] := Dist[(Cos[d + e*x]^n*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n)/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && !IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}} dx &= \frac{(b+a\cos(d+ex)+c\sin(d+ex))^{5/2} \int \frac{1}{(b+a\cos(d+ex)+c\sin(d+ex))^{5/2}} dx}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}} \\
&= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}{3(a^2-b^2+c^2)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}} \\
&= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}{3(a^2-b^2+c^2)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}} \\
&= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}{3(a^2-b^2+c^2)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}} \\
&= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}{3(a^2-b^2+c^2)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}} \\
&= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}{3(a^2-b^2+c^2)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}
\end{aligned}$$

Mathematica [F] time = 27.74, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)), x]

[Out] Integrate[1/(Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)), x]

fricas [F] time = 1.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^3 \cos^3(ex+d) \sec^3(ex+d) + c^3 \cos^3(ex+d) \tan^3(ex+d) + 3ab^2 \cos^3(ex+d) \sec^2(ex+d) + 3a^2b \cos^3(ex+d) \tan^2(ex+d)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2), x, algorithm="fricas")

[Out] integral(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d))/(b^3*cos(e*x + d)^3*sec(e*x + d)^3 + c^3*cos(e*x + d)^3*tan(e*x + d)^3 + 3*a*b^2*cos(e*x + d)^3*sec(e*x + d)^2 + 3*a^2*b*cos(e*x + d)^3*tan(e*x + d)^2 + 3*(b*c^2*cos(e*x + d)^3*sec(e*x + d) + a*c^2*cos(e*x + d)^3)*tan(e*x + d)^2 + 3*(b^2*c*cos(e*x + d)^3*sec(e*x + d)^2 + 2*a*b*c*cos(e*x + d)^3*sec(e*x + d) + a^2*c*cos(e*x + d)^3)*tan(e*x + d)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \sec(ex + d) + c \tan(ex + d) + a)^{\frac{5}{2}} \cos(ex + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorithm="giac")

[Out] integrate(1/((b*sec(e*x + d) + c*tan(e*x + d) + a)^(5/2)*cos(e*x + d)^(5/2)), x)

maple [C] time = 1.74, size = 64683, normalized size = 131.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \sec(ex + d) + c \tan(ex + d) + a)^{\frac{5}{2}} \cos(ex + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorithm="maxima")

[Out] integrate(1/((b*sec(e*x + d) + c*tan(e*x + d) + a)^(5/2)*cos(e*x + d)^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(d + ex)^{5/2} \left(a + c \tan(d + ex) + \frac{b}{\cos(d + ex)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(d + e*x)^(5/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(5/2)),x)

[Out] int(1/(cos(d + e*x)^(5/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(5/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(e*x+d)**(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(5/2),x)

[Out] Timed out

$$3.458 \quad \int \frac{1}{a+b \cot(x)+c \csc(x)} dx$$

Optimal. Leaf size=98

$$\frac{2ac \tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(a^2+b^2)\sqrt{a^2+b^2-c^2}} - \frac{b \log(a \sin(x) + b \cos(x) + c)}{a^2+b^2} + \frac{ax}{a^2+b^2}$$

[Out] a*x/(a^2+b^2)-b*ln(c+b*cos(x)+a*sin(x))/(a^2+b^2)+2*a*c*arctanh((a-(b-c)*tan(1/2*x))/(a^2+b^2-c^2)^(1/2))/(a^2+b^2)/(a^2+b^2-c^2)^(1/2)

Rubi [A] time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3160, 3137, 3124, 618, 206}

$$\frac{2ac \tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(a^2+b^2)\sqrt{a^2+b^2-c^2}} - \frac{b \log(a \sin(x) + b \cos(x) + c)}{a^2+b^2} + \frac{ax}{a^2+b^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cot[x] + c*Csc[x])^(-1), x]

[Out] (a*x)/(a^2 + b^2) + (2*a*c*ArcTanh[(a - (b - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/((a^2 + b^2)*Sqrt[a^2 + b^2 - c^2]) - (b*Log[c + b*Cos[x] + a*Sin[x]])/(a^2 + b^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3137

Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])*((a_.) + cos[(d_.) + (e_.)*(x_)])*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Simp[(c*C*(d + e*x))/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C}, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]

Rule 3160

Int[((a_.) + csc[(d_.) + (e_.)*(x_)])*(b_.) + cot[(d_.) + (e_.)*(x_)])*(c_.))^(-1), x_Symbol] := Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),

x] /; FreeQ[{a, b, c, d, e}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{a + b \cot(x) + c \csc(x)} dx &= \int \frac{\sin(x)}{c + b \cos(x) + a \sin(x)} dx \\
 &= \frac{ax}{a^2 + b^2} - \frac{b \log(c + b \cos(x) + a \sin(x))}{a^2 + b^2} - \frac{(ac) \int \frac{1}{c + b \cos(x) + a \sin(x)} dx}{a^2 + b^2} \\
 &= \frac{ax}{a^2 + b^2} - \frac{b \log(c + b \cos(x) + a \sin(x))}{a^2 + b^2} - \frac{(2ac) \text{Subst}\left(\int \frac{1}{b + c + 2ax + (-b + c)x^2} dx, x, 2x\right)}{a^2 + b^2} \\
 &= \frac{ax}{a^2 + b^2} - \frac{b \log(c + b \cos(x) + a \sin(x))}{a^2 + b^2} + \frac{(4ac) \text{Subst}\left(\int \frac{1}{4(a^2 + b^2 - c^2) - x^2} dx, x, 2x\right)}{a^2 + b^2} \\
 &= \frac{ax}{a^2 + b^2} + \frac{2ac \tanh^{-1}\left(\frac{a - (b - c) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 + b^2 - c^2}}\right)}{(a^2 + b^2) \sqrt{a^2 + b^2 - c^2}} - \frac{b \log(c + b \cos(x) + a \sin(x))}{a^2 + b^2}
 \end{aligned}$$

Mathematica [A] time = 0.22, size = 80, normalized size = 0.82

$$\frac{2ac \tanh^{-1}\left(\frac{a + (c - b) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 + b^2 - c^2}}\right)}{\sqrt{a^2 + b^2 - c^2}} - \frac{b \log(a \sin(x) + b \cos(x) + c) + ax}{a^2 + b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cot[x] + c*Csc[x])^(-1),x]

[Out] (a*x + (2*a*c*ArcTanh[(a + (-b + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2 - c^2] - b*Log[c + b*Cos[x] + a*Sin[x]])/(a^2 + b^2)

fricas [B] time = 2.05, size = 555, normalized size = 5.66

$$\left[\frac{\sqrt{a^2 + b^2 - c^2} ac \log\left(\frac{a^4 + 3a^2b^2 + 2b^4 + (a^2 - b^2)c^2 + 2(a^2b + b^3)c \cos(x) + (a^4 - b^4 - 2(a^2 - b^2)c^2) \cos(x)^2 + 2((a^3 + ab^2)c - (a^3b + ab^3 - 2abc^2) \cos(x)) \sin(x)}{2bc \cos(x) - (a^2 - b^2) \cos(x)^2 + a^2 + c^2 + 2(a^2b + b^3) \cos(x) - (a^2 - b^2) \cos(x)^2 + a^2 + c^2 + 2(a^3 + ab^2)c - (a^3b + ab^3 - 2abc^2) \cos(x)}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cot(x)+c*csc(x)),x, algorithm="fricas")

[Out] [1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((a^4 + 3*a^2*b^2 + 2*b^4 + (a^2 - b^2)*c^2 + 2*(a^2*b + b^3)*c*cos(x) + (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 + 2*((a^3 + a*b^2)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (a^3 + a*b^2)*cos(x) - (a^2*b + b^3 - (a^2 - b^2)*c*cos(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x)) + 2*(a^3 + a*b^2 - a*c^2)*x - (a^2*b + b^3 - b*c^2)*log(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x)))/(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2), -1/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((b*c*cos(x) + a*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2))/((a^3 + a*b^2 - a*c^2)*cos(x) - (a^2*b + b^3 - b*c^2)*sin(x)) - 2*(a^3 + a*b^2 - a*c^2)*x + (a^2*b + b^3 - b*c^2)*log(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x)))/(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2)]

giac [A] time = 0.14, size = 158, normalized size = 1.61

$$\frac{2 \left(\pi \left[\frac{x}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2b + 2c) + \arctan \left(-\frac{b \tan\left(\frac{1}{2}x\right) - c \tan\left(\frac{1}{2}x\right) - a}{\sqrt{-a^2 - b^2 + c^2}} \right) \right) ac}{(a^2 + b^2) \sqrt{-a^2 - b^2 + c^2}} + \frac{ax}{a^2 + b^2} - \frac{b \log \left(-b \tan\left(\frac{1}{2}x\right)^2 + c \tan\left(\frac{1}{2}x\right)^2 + 2a \tan\left(\frac{1}{2}x\right) + b + c \right)}{a^2 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cot(x)+c*csc(x)),x, algorithm="giac")

[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*b + 2*c) + arctan(-(b*tan(1/2*x) - c*tan(1/2*x) - a)/sqrt(-a^2 - b^2 + c^2)))*a*c/((a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)) + a*x/(a^2 + b^2) - b*log(-b*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*a*tan(1/2*x) + b + c)/(a^2 + b^2) + b*log(tan(1/2*x)^2 + 1)/(a^2 + b^2)

maple [B] time = 0.13, size = 446, normalized size = 4.55

$$\frac{2 \ln \left(b \left(\tan^2 \left(\frac{x}{2} \right) \right) - c \left(\tan^2 \left(\frac{x}{2} \right) \right) - 2a \tan \left(\frac{x}{2} \right) - b - c \right) b^2}{(2a^2 + 2b^2)(b - c)} + \frac{2 \ln \left(b \left(\tan^2 \left(\frac{x}{2} \right) \right) - c \left(\tan^2 \left(\frac{x}{2} \right) \right) - 2a \tan \left(\frac{x}{2} \right) - b - c \right)}{(2a^2 + 2b^2)(b - c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cot(x)+c*csc(x)),x)

[Out] -2/(2*a^2+2*b^2)/(b-c)*ln(b*tan(1/2*x)^2-c*tan(1/2*x)^2-2*a*tan(1/2*x)-b-c)*b^2+2/(2*a^2+2*b^2)/(b-c)*ln(b*tan(1/2*x)^2-c*tan(1/2*x)^2-2*a*tan(1/2*x)-b-c)*c*b+4/(2*a^2+2*b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(1/2*x)-2*a)/(-a^2-b^2+c^2)^(1/2))*a*b+4/(2*a^2+2*b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(1/2*x)-2*a)/(-a^2-b^2+c^2)^(1/2))*a*c-4/(2*a^2+2*b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(1/2*x)-2*a)/(-a^2-b^2+c^2)^(1/2))*a/(b-c)*b^2+4/(2*a^2+2*b^2)/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(1/2*x)-2*a)/(-a^2-b^2+c^2)^(1/2))*a/(b-c)*c*b+2/(2*a^2+2*b^2)*b*ln(1+tan(1/2*x)^2)+4/(2*a^2+2*b^2)*a*arctan(tan(1/2*x))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cot(x)+c*csc(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` for more details)Is c^2-b^2-a^2 positive or negative?

mupad [B] time = 13.76, size = 965, normalized size = 9.85

$$\frac{\ln \left(\tan \left(\frac{x}{2} \right) - i \right)}{b + a i} \left(-64 \tan \left(\frac{x}{2} \right) (b - c)^2 - \frac{(a^2 b - b c^2 + b^3 + a c \sqrt{a^2 + b^2 - c^2}) \left(32 a b^2 + 32 a c^2 - 64 a b c - 64 \tan \left(\frac{x}{2} \right) (b - c) (a^2 - c^2 + b c) + \dots \right)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + c/sin(x) + b*cot(x)),x)`

[Out] $\log(\tan(x/2) - 1i)/(a \cdot 1i + b) + (\log(\tan(x/2) + 1i) \cdot 1i)/(a + b \cdot 1i) - (\log(-64 \cdot \tan(x/2) \cdot (b - c)^2 - ((a^2 \cdot b - b \cdot c^2 + b^3 + a \cdot c \cdot (a^2 + b^2 - c^2))^{1/2})) \cdot (32 \cdot a \cdot b^2 + 32 \cdot a \cdot c^2 - 64 \cdot a \cdot b \cdot c - 64 \cdot \tan(x/2) \cdot (b - c) \cdot (b \cdot c + a^2 - c^2)) + ((a^2 \cdot b - b \cdot c^2 + b^3 + a \cdot c \cdot (a^2 + b^2 - c^2))^{1/2}) \cdot (64 \cdot a \cdot b^3 - 32 \cdot a^3 \cdot b + 32 \cdot a^3 \cdot c + 32 \cdot \tan(x/2) \cdot (b - c) \cdot (a^2 \cdot b - 2 \cdot a^2 \cdot c + 2 \cdot b^2 \cdot c - 2 \cdot b^3) + 64 \cdot a \cdot b \cdot c^2 - 128 \cdot a \cdot b^2 \cdot c - (32 \cdot (b - c) \cdot (a^2 \cdot b - b \cdot c^2 + b^3 + a \cdot c \cdot (a^2 + b^2 - c^2))^{1/2})) \cdot (3 \cdot a^4 \cdot \tan(x/2) + 3 \cdot a \cdot b^3 + 3 \cdot a^3 \cdot b + a^3 \cdot c + 3 \cdot a^2 \cdot b^2 \cdot \tan(x/2) - 2 \cdot a^2 \cdot c^2 \cdot \tan(x/2) + 2 \cdot b^2 \cdot c^2 \cdot \tan(x/2) - 2 \cdot b^3 \cdot c \cdot \tan(x/2) - 4 \cdot a \cdot b \cdot c^2 + a \cdot b^2 \cdot c - 2 \cdot a^2 \cdot b \cdot c \cdot \tan(x/2))) / ((a^2 + b^2) \cdot (a^2 + b^2 - c^2))) / ((a^2 + b^2) \cdot (a^2 + b^2 - c^2))) / ((a^2 + b^2) \cdot (a^2 + b^2 - c^2))) / ((a^2 + b^2) \cdot (a^2 + b^2 - c^2))) / ((a^2 + b^2) \cdot (a^2 + b^2 - c^2))) / ((a^2 + b^2) \cdot (a^2 + b^2 - c^2)) \cdot (b \cdot (a^2 - c^2) + b^3 + a \cdot c \cdot (a^2 + b^2 - c^2)^{1/2})) / (c^2 \cdot (a^2 + b^2 - c^2) + (a^2 + b^2 - c^2)^2) - (\log(-64 \cdot \tan(x/2) \cdot (b - c)^2 - ((a^2 \cdot b - b \cdot c^2 + b^3 - a \cdot c \cdot (a^2 + b^2 - c^2))^{1/2})) \cdot (32 \cdot a \cdot b^2 + 32 \cdot a \cdot c^2 - 64 \cdot a \cdot b \cdot c - 64 \cdot \tan(x/2) \cdot (b - c) \cdot (b \cdot c + a^2 - c^2) + ((a^2 \cdot b - b \cdot c^2 + b^3 - a \cdot c \cdot (a^2 + b^2 - c^2))^{1/2}) \cdot (64 \cdot a \cdot b^3 - 32 \cdot a^3 \cdot b + 32 \cdot a^3 \cdot c + 32 \cdot \tan(x/2) \cdot (b - c) \cdot (a^2 \cdot b - 2 \cdot a^2 \cdot c + 2 \cdot b^2 \cdot c - 2 \cdot b^3) + 64 \cdot a \cdot b \cdot c^2 - 128 \cdot a \cdot b^2 \cdot c - (32 \cdot (b - c) \cdot (a^2 \cdot b - b \cdot c^2 + b^3 - a \cdot c \cdot (a^2 + b^2 - c^2))^{1/2})) \cdot (3 \cdot a^4 \cdot \tan(x/2) + 3 \cdot a \cdot b^3 + 3 \cdot a^3 \cdot b + a^3 \cdot c + 3 \cdot a^2 \cdot b^2 \cdot \tan(x/2) - 2 \cdot a^2 \cdot c^2 \cdot \tan(x/2) + 2 \cdot b^2 \cdot c^2 \cdot \tan(x/2) - 2 \cdot b^3 \cdot c \cdot \tan(x/2) - 4 \cdot a \cdot b \cdot c^2 + a \cdot b^2 \cdot c - 2 \cdot a^2 \cdot b \cdot c \cdot \tan(x/2))) / ((a^2 + b^2) \cdot (a^2 + b^2 - c^2))) / ((a^2 + b^2) \cdot (a^2 + b^2 - c^2))) / ((a^2 + b^2) \cdot (a^2 + b^2 - c^2))) / ((a^2 + b^2) \cdot (a^2 + b^2 - c^2)) \cdot (b \cdot (a^2 - c^2) + b^3 - a \cdot c \cdot (a^2 + b^2 - c^2)^{1/2})) / (c^2 \cdot (a^2 + b^2 - c^2) + (a^2 + b^2 - c^2)^2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \cot(x) + c \csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cot(x)+c*csc(x)),x)`

[Out] `Integral(1/(a + b*cot(x) + c*csc(x)), x)`

$$3.459 \quad \int \frac{\csc(x)}{a+b \cot(x)+c \csc(x)} dx$$

Optimal. Leaf size=51

$$\frac{2 \tanh^{-1}\left(\frac{a-(b-c) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

[Out] $-2*\operatorname{arctanh}((a-(b-c)*\tan(1/2*x))/(\sqrt{a^2+b^2-c^2}))/\sqrt{a^2+b^2-c^2}$

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3166, 3124, 618, 206}

$$\frac{2 \tanh^{-1}\left(\frac{a-(b-c) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Antiderivative was successfully verified.

[In] Int[Csc[x]/(a + b*Cot[x] + c*Csc[x]),x]

[Out] $(-2*\operatorname{ArcTanh}[(a - (b - c)*\tan[x/2])/ \sqrt{a^2 + b^2 - c^2}])/\sqrt{a^2 + b^2 - c^2}$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3166

Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)])*(b_.) + cot[(d_.) + (e_.)*(x_)])*(c_.)^(m_), x_Symbol] := Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x)}{a + b \cot(x) + c \csc(x)} dx &= \int \frac{1}{c + b \cos(x) + a \sin(x)} dx \\
&= 2 \operatorname{Subst} \left(\int \frac{1}{b + c + 2ax + (-b + c)x^2} dx, x, \tan\left(\frac{x}{2}\right) \right) \\
&= - \left(4 \operatorname{Subst} \left(\int \frac{1}{4(a^2 + b^2 - c^2) - x^2} dx, x, 2a + 2(-b + c) \tan\left(\frac{x}{2}\right) \right) \right) \\
&= - \frac{2 \tanh^{-1} \left(\frac{a - (b - c) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 + b^2 - c^2}} \right)}{\sqrt{a^2 + b^2 - c^2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.98

$$-\frac{2 \tanh^{-1} \left(\frac{a + (c - b) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 + b^2 - c^2}} \right)}{\sqrt{a^2 + b^2 - c^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]/(a + b*Cot[x] + c*Csc[x]), x]

[Out] (-2*ArcTanh[(a + (-b + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2 - c^2]

fricas [B] time = 0.81, size = 349, normalized size = 6.84

$$\left[\log \left(\frac{a^4 + 3a^2b^2 + 2b^4 + (a^2 - b^2)c^2 + 2(a^2b + b^3)c \cos(x) + (a^4 - b^4 - 2(a^2 - b^2)c^2) \cos(x)^2 + 2((a^3 + ab^2)c - (a^3b + ab^3 - 2abc^2) \cos(x)) \sin(x) - 2(abc \cos(x) - (a^2 - b^2) \cos(x)^2 + a^2 + c^2 + 2(ab \cos(x) + ac) \sin(x))}{2bc \cos(x) - (a^2 - b^2) \cos(x)^2 + a^2 + c^2 + 2(ab \cos(x) + ac) \sin(x)} \right) \right]$$

$$2 \sqrt{a^2 + b^2 - c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(a+b*cot(x)+c*csc(x)), x, algorithm="fricas")

[Out] [1/2*log(-(a^4 + 3*a^2*b^2 + 2*b^4 + (a^2 - b^2)*c^2 + 2*(a^2*b + b^3)*c*cos(x) + (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 + 2*((a^3 + a*b^2)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (a^3 + a*b^2)*cos(x) - (a^2*b + b^3 - (a^2 - b^2)*c*cos(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x))/sqrt(a^2 + b^2 - c^2), sqrt(-a^2 - b^2 + c^2)*arctan((b*c*cos(x) + a*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^3 + a*b^2 - a*c^2)*cos(x) - (a^2*b + b^3 - b*c^2)*sin(x)))/(a^2 + b^2 - c^2)]

giac [A] time = 0.15, size = 73, normalized size = 1.43

$$\frac{2 \left(\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(2b - 2c) + \arctan \left(\frac{b \tan\left(\frac{1}{2}x\right) - c \tan\left(\frac{1}{2}x\right) - a}{\sqrt{-a^2 - b^2 + c^2}} \right) \right)}{\sqrt{-a^2 - b^2 + c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(a+b*cot(x)+c*csc(x)), x, algorithm="giac")

[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*b - 2*c) + arctan((b*tan(1/2*x) - c*tan(1/2*x) - a)/sqrt(-a^2 - b^2 + c^2)))/sqrt(-a^2 - b^2 + c^2)

maple [A] time = 0.11, size = 53, normalized size = 1.04

$$\frac{2 \arctan\left(\frac{2(b-c)\tan\left(\frac{x}{2}\right)-2a}{2\sqrt{-a^2-b^2+c^2}}\right)}{\sqrt{-a^2-b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(x)/(a+b*cot(x)+c*csc(x)),x)`

[Out] `-2/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(b-c)*tan(1/2*x)-2*a)/(-a^2-b^2+c^2)^(1/2))`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see 'assume?' for more details)Is c^2-b^2-a^2 positive or negative?

mupad [B] time = 2.79, size = 47, normalized size = 0.92

$$\frac{2 \operatorname{atanh}\left(\frac{a - \frac{\tan\left(\frac{x}{2}\right)(2b-2c)}{2}}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sin(x)*(a + c/sin(x) + b*cot(x))),x)`

[Out] `-(2*atanh((a - (tan(x/2)*(2*b - 2*c))/2)/(a^2 + b^2 - c^2)^(1/2)))/(a^2 + b^2 - c^2)^(1/2)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)}{a + b \cot(x) + c \csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x)`

[Out] `Integral(csc(x)/(a + b*cot(x) + c*csc(x)), x)`

$$3.460 \quad \int \frac{\csc^2(x)}{a+b \cot(x)+c \csc(x)} dx$$

Optimal. Leaf size=120

$$-\frac{2ac \tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(b^2-c^2)\sqrt{a^2+b^2-c^2}} - \frac{b \log\left(2a \tan\left(\frac{x}{2}\right) - (b-c)\tan^2\left(\frac{x}{2}\right) + b+c\right)}{b^2-c^2} + \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}$$

[Out] $\ln(\tan(1/2*x))/(b+c) - b*\ln(b+c+2*a*\tan(1/2*x) - (b-c)*\tan(1/2*x)^2)/(b^2-c^2) - 2*a*c*\operatorname{arctanh}((a-(b-c)*\tan(1/2*x))/(a^2+b^2-c^2)^{(1/2)})/(b^2-c^2)/(a^2+b^2-c^2)^{(1/2)}$

Rubi [A] time = 0.53, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4397, 12, 1628, 634, 618, 206, 628}

$$-\frac{2ac \tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(b^2-c^2)\sqrt{a^2+b^2-c^2}} - \frac{b \log\left(2a \tan\left(\frac{x}{2}\right) - (b-c)\tan^2\left(\frac{x}{2}\right) + b+c\right)}{b^2-c^2} + \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}$$

Antiderivative was successfully verified.

[In] `Int[Csc[x]^2/(a + b*Cot[x] + c*Csc[x]),x]`

[Out] $(-2*a*c*\operatorname{ArcTanh}[(a - (b - c)*\operatorname{Tan}[x/2])/Sqrt[a^2 + b^2 - c^2]])/((b^2 - c^2)*Sqrt[a^2 + b^2 - c^2]) + \operatorname{Log}[\operatorname{Tan}[x/2]]/(b + c) - (b*\operatorname{Log}[b + c + 2*a*\operatorname{Tan}[x/2] - (b - c)*\operatorname{Tan}[x/2]^2])/(b^2 - c^2)$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 628

`Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

Rule 634

`Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 1628

$\text{Int}[(Pq_)*((d_.) + (e_.)*(x_.))^m_)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^p_., x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, m, x\} \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[p, -2]$

Rule 4397

$\text{Int}[u_, x_Symbol] := \text{Int}[\text{TrigSimplify}[u], x] /;$ $\text{TrigSimplifyQ}[u]$

Rubi steps

$$\begin{aligned} \int \frac{\csc^2(x)}{a + b \cot(x) + c \csc(x)} dx &= \int \frac{\csc(x)}{c + b \cos(x) + a \sin(x)} dx \\ &= 2 \text{Subst} \left(\int \frac{1 + x^2}{2x(b + c + 2ax + (-b + c)x^2)} dx, x, \tan\left(\frac{x}{2}\right) \right) \\ &= \text{Subst} \left(\int \frac{1 + x^2}{x(b + c + 2ax + (-b + c)x^2)} dx, x, \tan\left(\frac{x}{2}\right) \right) \\ &= \text{Subst} \left(\int \left(\frac{1}{(b + c)x} + \frac{2(-a + bx)}{(b + c)(b + c + 2ax - (b - c)x^2)} \right) dx, x, \tan\left(\frac{x}{2}\right) \right) \\ &= \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b + c} + \frac{2 \text{Subst} \left(\int \frac{-a + bx}{b + c + 2ax + (-b + c)x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{b + c} \\ &= \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b + c} - \frac{b \text{Subst} \left(\int \frac{2a + 2(-b + c)x}{b + c + 2ax + (-b + c)x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{b^2 - c^2} + \frac{(2ac) \text{Subst} \left(\int \frac{1}{b + c + 2ax + (-b + c)x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{b^2 - c^2} \\ &= \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b + c} - \frac{b \log\left(b + c + 2a \tan\left(\frac{x}{2}\right) - (b - c) \tan^2\left(\frac{x}{2}\right)\right)}{b^2 - c^2} - \frac{(4ac) \text{Subst} \left(\int \frac{1}{4(a^2 - (b + c)x + (-b + c)x^2)} dx, x, \tan\left(\frac{x}{2}\right) \right)}{b^2 - c^2} \\ &= -\frac{2ac \tanh^{-1} \left(\frac{a - (b - c) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 + b^2 - c^2}} \right)}{(b^2 - c^2) \sqrt{a^2 + b^2 - c^2}} + \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b + c} - \frac{b \log\left(b + c + 2a \tan\left(\frac{x}{2}\right) - (b - c) \tan^2\left(\frac{x}{2}\right)\right)}{b^2 - c^2} \end{aligned}$$

Mathematica [A] time = 0.28, size = 104, normalized size = 0.87

$$\frac{\frac{2ac \tanh^{-1} \left(\frac{a + (c - b) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 + b^2 - c^2}} \right)}{\sqrt{a^2 + b^2 - c^2}} + b \log(a \sin(x) + b \cos(x) + c) + (c - b) \log\left(\sin\left(\frac{x}{2}\right)\right) - (b + c) \log\left(\cos\left(\frac{x}{2}\right)\right)}{(c - b)(b + c)}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Csc}[x]^2/(a + b*\text{Cot}[x] + c*\text{Csc}[x]), x]$

[Out] $((2*a*c*\text{ArcTanh}[(a + (-b + c)*\text{Tan}[x/2])/Sqrt[a^2 + b^2 - c^2]]/Sqrt[a^2 + b^2 - c^2] - (b + c)*\text{Log}[\text{Cos}[x/2]] + (-b + c)*\text{Log}[\text{Sin}[x/2]] + b*\text{Log}[c + b*\text{Cos}[x] + a*\text{Sin}[x]])/((-b + c)*(b + c))$

fricas [B] time = 6.05, size = 669, normalized size = 5.58

$$\left[\frac{\sqrt{a^2 + b^2 - c^2} ac \log \left(\frac{a^4 + 3a^2b^2 + 2b^4 + (a^2 - b^2)c^2 + 2(a^2b + b^3)c \cos(x) + (a^4 - b^4 - 2(a^2 - b^2)c^2) \cos(x)^2 + 2((a^3 + ab^2)c - (a^3b + ab^3 - 2abc^2) \cos(x) - (a^3 - ab^3 - 2abc^2) \cos(x)^2 + (a^2 + b^2)c^2) \cos(x)^2 + 2(ab^2c^2 - (a^2 - b^2)c^2) \cos(x)^2 + (a^2 + b^2)c^2 + 2(ab^2c^2 - (a^2 - b^2)c^2) \cos(x)^2 + (a^2 + b^2)c^2}{2bc \cos(x) - (a^2 - b^2) \cos(x)^2 + a^2 + c^2 + 2(ab^2c^2 - (a^2 - b^2)c^2) \cos(x)^2 + (a^2 + b^2)c^2} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/(a+b*cot(x)+c*csc(x)),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/2*(\sqrt{a^2 + b^2 - c^2})*a*c*\log((a^4 + 3*a^2*b^2 + 2*b^4 + (a^2 - b^2) \\ & *c^2 + 2*(a^2*b + b^3)*c*\cos(x) + (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*\cos(x)^2 \\ & + 2*((a^3 + a*b^2)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*\cos(x))*\sin(x) + 2*(2*a* \\ & b*c*\cos(x)^2 - a*b*c + (a^3 + a*b^2)*\cos(x) - (a^2*b + b^3 - (a^2 - b^2)*c* \\ & \cos(x))*\sin(x))*\sqrt{a^2 + b^2 - c^2})/(2*b*c*\cos(x) - (a^2 - b^2)*\cos(x)^2 \\ & + a^2 + c^2 + 2*(a*b*\cos(x) + a*c)*\sin(x)) + (a^2*b + b^3 - b*c^2)*\log(2* \\ & b*c*\cos(x) - (a^2 - b^2)*\cos(x)^2 + a^2 + c^2 + 2*(a*b*\cos(x) + a*c)*\sin(x) \\ &) - (a^2*b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*\log(1/2*\cos(x) + 1/2) - (a^ \\ & 2*b + b^3 - b*c^2 + c^3 - (a^2 + b^2)*c)*\log(-1/2*\cos(x) + 1/2))/(a^2*b^2 + \\ & b^4 + c^4 - (a^2 + 2*b^2)*c^2), 1/2*(2*\sqrt{-a^2 - b^2 + c^2})*a*c*\arctan((\\ & b*c*\cos(x) + a*c*\sin(x) + a^2 + b^2)*\sqrt{-a^2 - b^2 + c^2}/((a^3 + a*b^2 - \\ & a*c^2)*\cos(x) - (a^2*b + b^3 - b*c^2)*\sin(x))) - (a^2*b + b^3 - b*c^2)*\log \\ & (2*b*c*\cos(x) - (a^2 - b^2)*\cos(x)^2 + a^2 + c^2 + 2*(a*b*\cos(x) + a*c)*\sin \\ & (x)) + (a^2*b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*\log(1/2*\cos(x) + 1/2) + \\ & (a^2*b + b^3 - b*c^2 + c^3 - (a^2 + b^2)*c)*\log(-1/2*\cos(x) + 1/2))/(a^2*b^ \\ & 2 + b^4 + c^4 - (a^2 + 2*b^2)*c^2)] \end{aligned}$$

giac [A] time = 0.17, size = 142, normalized size = 1.18

$$\frac{2 \left(\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2b + 2c) + \arctan \left(-\frac{b \tan\left(\frac{1}{2}x\right) - c \tan\left(\frac{1}{2}x\right) - a}{\sqrt{-a^2 - b^2 + c^2}} \right) \right) a c}{\sqrt{-a^2 - b^2 + c^2} (b^2 - c^2)} - \frac{b \log \left(-b \tan\left(\frac{1}{2}x\right)^2 + c \tan\left(\frac{1}{2}x\right)^2 + 2a \tan\left(\frac{1}{2}x\right) + b + c \right)}{b^2 - c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/(a+b*cot(x)+c*csc(x)),x, algorithm="giac")

[Out]
$$\begin{aligned} & 2*(\pi*\operatorname{floor}(1/2*x/\pi + 1/2)*\operatorname{sgn}(-2*b + 2*c) + \arctan(-(b*\tan(1/2*x) - c*\tan \\ & (1/2*x) - a)/\sqrt{-a^2 - b^2 + c^2}))*a*c/(\sqrt{-a^2 - b^2 + c^2}*(b^2 - c^ \\ & 2)) - b*\log(-b*\tan(1/2*x)^2 + c*\tan(1/2*x)^2 + 2*a*\tan(1/2*x) + b + c)/(b^2 \\ & - c^2) + \log(\operatorname{abs}(\tan(1/2*x)))/(b + c) \end{aligned}$$

maple [A] time = 0.11, size = 184, normalized size = 1.53

$$\frac{b \ln \left(b \left(\tan^2 \left(\frac{x}{2} \right) \right) - c \left(\tan^2 \left(\frac{x}{2} \right) \right) - 2a \tan \left(\frac{x}{2} \right) - b - c \right)}{(b + c)(b - c)} + \frac{2 \arctan \left(\frac{2(b-c) \tan \left(\frac{x}{2} \right) - 2a}{2\sqrt{-a^2 - b^2 + c^2}} \right) a}{(b + c)\sqrt{-a^2 - b^2 + c^2}} - \frac{2 \arctan \left(\frac{2(b-c) \tan \left(\frac{x}{2} \right) - 2a}{2\sqrt{-a^2 - b^2 + c^2}} \right)}{(b + c)\sqrt{-a^2 - b^2 + c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)^2/(a+b*cot(x)+c*csc(x)),x)

[Out]
$$\begin{aligned} & -1/(b+c)*b/(b-c)*\ln(b*\tan(1/2*x)^2-c*\tan(1/2*x)^2-2*a*\tan(1/2*x)-b-c)+2/(b+ \\ & c)/(-a^2-b^2+c^2)^{(1/2)}*\arctan(1/2*(2*(b-c)*\tan(1/2*x)-2*a)/(-a^2-b^2+c^2)^{(\\ & 1/2)})*a-2/(b+c)/(-a^2-b^2+c^2)^{(1/2)}*\arctan(1/2*(2*(b-c)*\tan(1/2*x)-2*a)/(\\ & -a^2-b^2+c^2)^{(1/2)})*b*a/(b-c)+\ln(\tan(1/2*x))/(b+c) \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/(a+b*cot(x)+c*csc(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see 'assume?' for more details)Is c^2-b^2-a^2 positive or negative?

mupad [B] time = 8.70, size = 531, normalized size = 4.42

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{b+c} - \frac{\ln\left(2a - 2b \tan\left(\frac{x}{2}\right) - \frac{\left(\tan\left(\frac{x}{2}\right)(-8a^2 - 8b^2 + 6bc + 2c^2) - 4ac + \frac{2(b-c)(a^2b - bc^2 + b^3 + ac\sqrt{a^2+b^2-c^2})\left(4\tan\left(\frac{x}{2}\right)a^2 + ab + ac + 3\tan\left(\frac{x}{2}\right)b^2\right)}{(b^2-c^2)(a^2+b^2-c^2)}\right)}{(b^2-c^2)(a^2+b^2-c^2)}\right)}{(b^2-c^2)(a^2+b^2-c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x)^2*(a + c/sin(x) + b*cot(x))),x)

[Out] log(tan(x/2))/(b + c) - (log(2*a - 2*b*tan(x/2) - ((tan(x/2)*(6*b*c - 8*a^2 - 8*b^2 + 2*c^2) - 4*a*c + (2*(b - c)*(a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2)))*(a*b + a*c + 4*a^2*tan(x/2) + 3*b^2*tan(x/2) - 3*c^2*tan(x/2))))/((b^2 - c^2)*(a^2 + b^2 - c^2)))*(a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2)))/((b^2 - c^2)*(a^2 + b^2 - c^2)))*(b*(a^2 - c^2) + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2)))/((b^2 - c^2)*(a^2 + b^2 - c^2)) - (log(2*a - 2*b*tan(x/2) - ((tan(x/2)*(6*b*c - 8*a^2 - 8*b^2 + 2*c^2) - 4*a*c + (2*(b - c)*(a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2)))*(a*b + a*c + 4*a^2*tan(x/2) + 3*b^2*tan(x/2) - 3*c^2*tan(x/2))))/((b^2 - c^2)*(a^2 + b^2 - c^2)))*(a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2)))/((b^2 - c^2)*(a^2 + b^2 - c^2)))*(b*(a^2 - c^2) + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2)))/((b^2 - c^2)*(a^2 + b^2 - c^2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{a + b \cot(x) + c \csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)**2/(a+b*cot(x)+c*csc(x)),x)

[Out] Integral(csc(x)**2/(a + b*cot(x) + c*csc(x)), x)

$$3.461 \quad \int \frac{\csc(x)}{2+2 \cot(x)+3 \csc(x)} dx$$

Optimal. Leaf size=21

$$x + 2 \tan^{-1} \left(\frac{\cos(x) - \sin(x)}{\sin(x) + \cos(x) + 2} \right)$$

[Out] x+2*arctan((cos(x)-sin(x))/(2+cos(x)+sin(x)))

Rubi [A] time = 0.05, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3166, 3124, 618, 204}

$$x + 2 \tan^{-1} \left(\frac{\cos(x) - \sin(x)}{\sin(x) + \cos(x) + 2} \right)$$

Antiderivative was successfully verified.

[In] Int[Csc[x]/(2 + 2*Cot[x] + 3*Csc[x]),x]

[Out] x + 2*ArcTan[(Cos[x] - Sin[x])/(2 + Cos[x] + Sin[x])]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3166

Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_.), x_Symbol] :> Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned} \int \frac{\csc(x)}{2+2 \cot(x)+3 \csc(x)} dx &= \int \frac{1}{3+2 \cos(x)+2 \sin(x)} dx \\ &= 2 \operatorname{Subst} \left(\int \frac{1}{5+4x+x^2} dx, x, \tan \left(\frac{x}{2} \right) \right) \\ &= - \left(4 \operatorname{Subst} \left(\int \frac{1}{-4-x^2} dx, x, 4+2 \tan \left(\frac{x}{2} \right) \right) \right) \\ &= x + 2 \tan^{-1} \left(\frac{\cos(x) - \sin(x)}{2 + \cos(x) + \sin(x)} \right) \end{aligned}$$

Mathematica [B] time = 0.02, size = 51, normalized size = 2.43

$$\tan^{-1}\left(\sec\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right)+2\cos\left(\frac{x}{2}\right)\right)\right)-\tan^{-1}\left(\frac{\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+2\cos\left(\frac{x}{2}\right)}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]/(2 + 2*Cot[x] + 3*Csc[x]),x]

[Out] -ArcTan[Cos[x/2]/(2*Cos[x/2] + Sin[x/2])] + ArcTan[Sec[x/2]*(2*Cos[x/2] + Sin[x/2])]

fricas [A] time = 0.89, size = 24, normalized size = 1.14

$$-\arctan\left(-\frac{3\cos(x)+3\sin(x)+4}{\cos(x)-\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x, algorithm="fricas")

[Out] -arctan(-(3*cos(x) + 3*sin(x) + 4)/(cos(x) - sin(x)))

giac [A] time = 0.14, size = 22, normalized size = 1.05

$$2\pi\left[\frac{x}{2\pi}+\frac{1}{2}\right]+2\arctan\left(\tan\left(\frac{1}{2}x\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x, algorithm="giac")

[Out] 2*pi*floor(1/2*x/pi + 1/2) + 2*arctan(tan(1/2*x) + 2)

maple [A] time = 0.12, size = 10, normalized size = 0.48

$$2\arctan\left(2+\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)/(2+2*cot(x)+3*csc(x)),x)

[Out] 2*arctan(2+tan(1/2*x))

maxima [A] time = 0.41, size = 14, normalized size = 0.67

$$2\arctan\left(\frac{\sin(x)}{\cos(x)+1}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x, algorithm="maxima")

[Out] 2*arctan(sin(x)/(cos(x) + 1) + 2)

mupad [B] time = 3.14, size = 9, normalized size = 0.43

$$2\operatorname{atan}\left(\tan\left(\frac{x}{2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x)*(2*cot(x) + 3/sin(x) + 2)),x)

[Out] $2 \cdot \operatorname{atan}(\tan(x/2) + 2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)}{2 \cot(x) + 3 \csc(x) + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x)`

[Out] `Integral(csc(x)/(2*cot(x) + 3*csc(x) + 2), x)`

$$3.462 \quad \int \frac{(a+c \cot(d+ex)+b \csc(d+ex))^{3/2}}{\csc^2(d+ex)} dx$$

Optimal. Leaf size=371

$$\frac{2(a^2 - b^2 + c^2) \sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}} (a + b \csc(d+ex) + c \cot(d+ex))^{3/2} F\left(\frac{1}{2}(d+ex - \tan^{-1}(c,a)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e \csc^2(d+ex)(a \sin(d+ex) + b + c \cos(d+ex))^2}$$

[Out] $-2/3*(a+c*\cot(e*x+d)+b*\csc(e*x+d))^{(3/2)}*(a*\cos(e*x+d)-c*\sin(e*x+d))/e/\csc(e*x+d)^{(3/2)}/(b+c*\cos(e*x+d)+a*\sin(e*x+d))+8/3*b*(a+c*\cot(e*x+d)+b*\csc(e*x+d))^{(3/2)}*(\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(c,a)),2^{(1/2)}*((a^2+c^2)^{(1/2)}/(b+(a^2+c^2)^{(1/2)})))^{(1/2)})/e/\csc(e*x+d)^{(3/2)}/(b+c*\cos(e*x+d)+a*\sin(e*x+d))/((b+c*\cos(e*x+d)+a*\sin(e*x+d))/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}+2/3*(a^2-b^2+c^2)*(a+c*\cot(e*x+d)+b*\csc(e*x+d))^{(3/2)}*(\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))*\text{EllipticF}(\sin(1/2*d+1/2*e*x-1/2*\arctan(c,a)),2^{(1/2)}*((a^2+c^2)^{(1/2)}/(b+(a^2+c^2)^{(1/2)})))^{(1/2)})*((b+c*\cos(e*x+d)+a*\sin(e*x+d))/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}/e/\csc(e*x+d)^{(3/2)}/(b+c*\cos(e*x+d)+a*\sin(e*x+d))^{(1/2)}$

Rubi [A] time = 0.43, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3168, 3120, 3149, 3119, 2653, 3127, 2661}

$$\frac{2(a^2 - b^2 + c^2) \sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}} (a + b \csc(d+ex) + c \cot(d+ex))^{3/2} F\left(\frac{1}{2}(d+ex - \tan^{-1}(c,a)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e \csc^2(d+ex)(a \sin(d+ex) + b + c \cos(d+ex))^2}$$

Antiderivative was successfully verified.

[In] Int[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)/Csc[d + e*x]^(3/2), x]

[Out] $(8*b*(a + c*\cot(d + e*x) + b*\csc(d + e*x))^{(3/2)}*\text{EllipticE}[(d + e*x - \text{ArcTan}[c, a])/2, (2*\sqrt{a^2 + c^2})/(b + \sqrt{a^2 + c^2})])/(3*e*\csc(d + e*x)^{(3/2)}*(b + c*\cos(d + e*x) + a*\sin(d + e*x))*\sqrt{(b + c*\cos(d + e*x) + a*\sin(d + e*x))/(b + \sqrt{a^2 + c^2})}) + (2*(a^2 - b^2 + c^2)*(a + c*\cot(d + e*x) + b*\csc(d + e*x))^{(3/2)}*\text{EllipticF}[(d + e*x - \text{ArcTan}[c, a])/2, (2*\sqrt{a^2 + c^2})/(b + \sqrt{a^2 + c^2})]*\sqrt{(b + c*\cos(d + e*x) + a*\sin(d + e*x))/(b + \sqrt{a^2 + c^2})})/(3*e*\csc(d + e*x)^{(3/2)}*(b + c*\cos(d + e*x) + a*\sin(d + e*x))^2 - (2*(a + c*\cot(d + e*x) + b*\csc(d + e*x))^{(3/2)}*(a*\cos(d + e*x) - c*\sin(d + e*x)))/(3*e*\csc(d + e*x)^{(3/2)}*(b + c*\cos(d + e*x) + a*\sin(d + e*x))))$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d/Sqrt[a + b], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119


```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^n, x_Symbol] := -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]
```

Rule 3168

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))^m, x_Symbol] := Dist[(Csc[d + e*x]^n*(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n)/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n, Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + c \cot(d + ex) + b \csc(d + ex))^{3/2}}{\csc^2(d + ex)} dx &= \frac{(a + c \cot(d + ex) + b \csc(d + ex))^{3/2} \int (b + c \cos(d + ex) + a \sin(d + ex))^{3/2}}{\csc^2(d + ex)(b + c \cos(d + ex) + a \sin(d + ex))^{3/2}} \\
&= -\frac{2(a + c \cot(d + ex) + b \csc(d + ex))^{3/2}(a \cos(d + ex) - c \sin(d + ex))}{3e \csc^2(d + ex)(b + c \cos(d + ex) + a \sin(d + ex))} \\
&= -\frac{2(a + c \cot(d + ex) + b \csc(d + ex))^{3/2}(a \cos(d + ex) - c \sin(d + ex))}{3e \csc^2(d + ex)(b + c \cos(d + ex) + a \sin(d + ex))} \\
&= -\frac{2(a + c \cot(d + ex) + b \csc(d + ex))^{3/2}(a \cos(d + ex) - c \sin(d + ex))}{3e \csc^2(d + ex)(b + c \cos(d + ex) + a \sin(d + ex))} \\
&= \frac{8b(a + c \cot(d + ex) + b \csc(d + ex))^{3/2} E\left(\frac{1}{2} \left(d + ex - \tan^{-1}\left(\frac{c}{a}\right)\right) \middle| \frac{2}{b}\right)}{3e \csc^2(d + ex)(b + c \cos(d + ex) + a \sin(d + ex)) \sqrt{\frac{b + c \cos(d + ex) + a \sin(d + ex)}{b + \sqrt{a^2 + c^2}}}}
\end{aligned}$$

Mathematica [C] time = 6.44, size = 2490, normalized size = 6.71

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)/Csc[d + e*x]^(3/2), x]

[Out] ((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*((8*b*c)/(3*a) - (2*a*Cos[d + e*x])/3 + (2*c*Sin[d + e*x])/3))/(e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])) + (4*a*b*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*(-(a*AppellF1[-1/2, -1/2, -1/2, 1/2, -(b + Sqrt[1 + a^2/c^2])*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2])*c)), -(b + Sqrt[1 + a^2/c^2])*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2])*c))*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]) - ((2*c*(b + Sqrt[1 + a^2/c^2])*c*Cos[d + e*x - ArcTan[a/c]])/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(3*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)) + (4*b*c^2*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*(-(a*AppellF1[-1/2, -1/2, -1/2, 1/2, -(b + Sqrt[1 + a^2/c^2])*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2])*c))*c), -(b + Sqrt[1 + a^2/c^2])*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2])*c))*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]) - ((2*c*(b + Sqrt[1 + a^2/c^2])*c*Cos[d + e*x - ArcTan[a/c]])/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(3*a*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)) + (2*a*AppellF1[1/2, 1/2, 1/2, 3/2, -(b + a*Sqrt[1 + c^2/a^2])*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2])

$$\frac{2}{a^2} * (1 - b/(a * \sqrt{1 + c^2/a^2})))$$
,
$$-((b + a * \sqrt{1 + c^2/a^2} * \sin[d + ex + \arctan[c/a]]) / (a * \sqrt{1 + c^2/a^2} * (-1 - b/(a * \sqrt{1 + c^2/a^2})))) * (a + c * \cot[d + ex] + b * \csc[d + ex])^{3/2} * \sec[d + ex + \arctan[c/a]] * \sqrt{[(a * \sqrt{(a^2 + c^2)/a^2} - a * \sqrt{(a^2 + c^2)/a^2} * \sin[d + ex + \arctan[c/a]]) / (b + a * \sqrt{(a^2 + c^2)/a^2})]} * \sqrt{b + a * \sqrt{(a^2 + c^2)/a^2} * \sin[d + ex + \arctan[c/a]]} * \sqrt{[(a * \sqrt{(a^2 + c^2)/a^2} + a * \sqrt{(a^2 + c^2)/a^2} * \sin[d + ex + \arctan[c/a]]) / (-b + a * \sqrt{(a^2 + c^2)/a^2})]} / (3 * \sqrt{1 + c^2/a^2} * e * \csc[d + ex]^{3/2} * (b + c * \cos[d + ex] + a * \sin[d + ex])^{3/2}) + (2 * b^2 * \operatorname{AppellF1}[1/2, 1/2, 1/2, 3/2, -((b + a * \sqrt{1 + c^2/a^2} * \sin[d + ex + \arctan[c/a]]) / (a * \sqrt{1 + c^2/a^2} * (1 - b/(a * \sqrt{1 + c^2/a^2}))))], -((b + a * \sqrt{1 + c^2/a^2} * \sin[d + ex + \arctan[c/a]]) / (a * \sqrt{1 + c^2/a^2} * (-1 - b/(a * \sqrt{1 + c^2/a^2})))))] * (a + c * \cot[d + ex] + b * \csc[d + ex])^{3/2} * \sec[d + ex + \arctan[c/a]] * \sqrt{[(a * \sqrt{(a^2 + c^2)/a^2} - a * \sqrt{(a^2 + c^2)/a^2} * \sin[d + ex + \arctan[c/a]]) / (b + a * \sqrt{(a^2 + c^2)/a^2})]} * \sqrt{b + a * \sqrt{(a^2 + c^2)/a^2} * \sin[d + ex + \arctan[c/a]]} * \sqrt{[(a * \sqrt{(a^2 + c^2)/a^2} + a * \sqrt{(a^2 + c^2)/a^2} * \sin[d + ex + \arctan[c/a]]) / (-b + a * \sqrt{(a^2 + c^2)/a^2})]} / (a * \sqrt{1 + c^2/a^2} * e * \csc[d + ex]^{3/2} * (b + c * \cos[d + ex] + a * \sin[d + ex])^{3/2}) + (2 * c^2 * \operatorname{AppellF1}[1/2, 1/2, 1/2, 3/2, -((b + a * \sqrt{1 + c^2/a^2} * \sin[d + ex + \arctan[c/a]]) / (a * \sqrt{1 + c^2/a^2} * (1 - b/(a * \sqrt{1 + c^2/a^2}))))], -((b + a * \sqrt{1 + c^2/a^2} * \sin[d + ex + \arctan[c/a]]) / (a * \sqrt{1 + c^2/a^2} * (-1 - b/(a * \sqrt{1 + c^2/a^2})))))] * (a + c * \cot[d + ex] + b * \csc[d + ex])^{3/2} * \sec[d + ex + \arctan[c/a]] * \sqrt{[(a * \sqrt{(a^2 + c^2)/a^2} - a * \sqrt{(a^2 + c^2)/a^2} * \sin[d + ex + \arctan[c/a]]) / (b + a * \sqrt{(a^2 + c^2)/a^2})]} * \sqrt{b + a * \sqrt{(a^2 + c^2)/a^2} * \sin[d + ex + \arctan[c/a]]} * \sqrt{[(a * \sqrt{(a^2 + c^2)/a^2} + a * \sqrt{(a^2 + c^2)/a^2} * \sin[d + ex + \arctan[c/a]]) / (-b + a * \sqrt{(a^2 + c^2)/a^2})]} / (3 * a * \sqrt{1 + c^2/a^2} * e * \csc[d + ex]^{3/2} * (b + c * \cos[d + ex] + a * \sin[d + ex])^{3/2})$$

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(c \cot(ex + d) + b \csc(ex + d) + a)^{3/2}}{\csc(ex + d)^{3/2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+c*cot(ex+d)+b*csc(ex+d))^(3/2)/csc(ex+d)^(3/2),x, algorithm="fricas")`

[Out] `integral((c*cot(ex + d) + b*csc(ex + d) + a)^(3/2)/csc(ex + d)^(3/2), x)`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c \cot(ex + d) + b \csc(ex + d) + a)^{3/2}}{\csc(ex + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+c*cot(ex+d)+b*csc(ex+d))^(3/2)/csc(ex+d)^(3/2),x, algorithm="giac")`

[Out] `integrate((c*cot(ex + d) + b*csc(ex + d) + a)^(3/2)/csc(ex + d)^(3/2), x)`

maple [C] time = 3.12, size = 20463, normalized size = 55.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+c*cot(ex+d)+b*csc(ex+d))^(3/2)/csc(ex+d)^(3/2),x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c \cot(ex + d) + b \csc(ex + d) + a)^{\frac{3}{2}}}{\csc^{\frac{3}{2}}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x, algorithm="maxima")

[Out] integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)/csc(e*x + d)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + c \cot(d + ex) + \frac{b}{\sin(d+ex)}\right)^{3/2}}{\left(\frac{1}{\sin(d+ex)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2)/(1/sin(d + e*x))^(3/2),x)

[Out] int((a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2)/(1/sin(d + e*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \csc(d + ex) + c \cot(d + ex))^{\frac{3}{2}}}{\csc^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2)/csc(e*x+d)**(3/2),x)

[Out] Integral((a + b*csc(d + e*x) + c*cot(d + e*x))**(3/2)/csc(d + e*x)**(3/2), x)

$$3.463 \quad \int \frac{\sqrt{a+c \cot(d+ex)+b \csc(d+ex)}}{\sqrt{\csc(d+ex)}} dx$$

Optimal. Leaf size=118

$$\frac{2\sqrt{a+b \csc(d+ex)+c \cot(d+ex)} E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\middle|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\csc(d+ex)}\sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2))/(b+(a^2+c^2)^(1/2)))^(1/2)*(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/e/csc(e*x+d)^(1/2)/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)

Rubi [A] time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3168, 3119, 2653}

$$\frac{2\sqrt{a+b \csc(d+ex)+c \cot(d+ex)} E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\middle|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\csc(d+ex)}\sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]/Sqrt[Csc[d + e*x]],x]

[Out] (2*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])])/(e*Sqrt[Csc[d + e*x]]*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c])]/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3168

Int[csc[(d_) + (e_)*(x_)]^(n_)*((a_) + csc[(d_) + (e_)*(x_)]*(b_) + cot[(d_) + (e_)*(x_)]*(c_))^(m_), x_Symbol] := Dist[(Csc[d + e*x]^n*(b + a*Sin[d + e*x] + c*Cos[d + e*x])^m)/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n, Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && !IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + c \cot(d + ex) + b \csc(d + ex)}}{\sqrt{\csc(d + ex)}} dx &= \frac{\sqrt{a + c \cot(d + ex) + b \csc(d + ex)}}{\sqrt{\csc(d + ex)}} \int \frac{\sqrt{b + c \cos(d + ex) + a \sin(d + ex)}}{\sqrt{b + c \cos(d + ex) + a \sin(d + ex)}} dx \\
&= \frac{\sqrt{a + c \cot(d + ex) + b \csc(d + ex)}}{\sqrt{\csc(d + ex)}} \int \sqrt{\frac{b}{b + \sqrt{a^2 + c^2}} + \frac{\sqrt{a^2 + c^2} \cos(d + ex - \tan^{-1}(c/a))}{b + \sqrt{a^2 + c^2}}} dx \\
&= \frac{2\sqrt{a + c \cot(d + ex) + b \csc(d + ex)}}{e\sqrt{\csc(d + ex)}} E\left(\frac{1}{2}(d + ex - \tan^{-1}(c/a))\right) \Big|_{\frac{2\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}}
\end{aligned}$$

Mathematica [C] time = 6.24, size = 1580, normalized size = 13.39

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]/Sqrt[Csc[d + e*x]],x]
[Out] (2*c*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]/(a*e*Sqrt[Csc[d + e*x]]) +
(a*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c)))*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/(e*Sqrt[Csc[d + e*x]]*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]]) +
(c^2*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c)))*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/(a*e*Sqrt[Csc[d + e*x]]*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]]) +
(2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))] *Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]] *Sec[d + e*x + ArcTan[c/a]] *Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(a*Sqrt[1 + c^2/a^2]*e*Sqrt[Csc[d + e*x]]*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]])
```

fricas [F] time = 1.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c \cot(ex + d) + b \csc(ex + d) + a}}{\sqrt{\csc(ex + d)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)/sqrt(csc(e*x + d)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c \cot(ex + d) + b \csc(ex + d) + a}}{\sqrt{\csc(ex + d)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)/sqrt(csc(e*x + d)), x)

maple [C] time = 1.76, size = 12367, normalized size = 104.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c \cot(ex + d) + b \csc(ex + d) + a}}{\sqrt{\csc(ex + d)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)/sqrt(csc(e*x + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + c \cot(d + ex) + \frac{b}{\sin(d+ex)}}}{\sqrt{\frac{1}{\sin(d+ex)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2)/(1/sin(d + e*x))^(1/2),x)

[Out] int((a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2)/(1/sin(d + e*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \csc(d + ex) + c \cot(d + ex)}}{\sqrt{\csc(d + ex)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2)/csc(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*csc(d + e*x) + c*cot(d + e*x))/sqrt(csc(d + e*x)), x)
```


$$3.464 \quad \int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c \cot(d+ex)+b \csc(d+ex)}} dx$$

Optimal. Leaf size=118

$$\frac{2\sqrt{\csc(d+ex)} \sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}} F\left(\frac{1}{2} \left(d+ex - \tan^{-1}(c, a)\right) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a+b \csc(d+ex)+c \cot(d+ex)}}$$

[Out] $2*(\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))*\text{EllipticF}(\sin(1/2*d+1/2*e*x-1/2*\arctan(c,a)),2^{(1/2)*((a^2+c^2)^{(1/2)})}/(b+(a^2+c^2)^{(1/2)}))^{(1/2))*\csc(e*x+d)^{(1/2)*((b+c*\cos(e*x+d)+a*\sin(e*x+d))/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}/e/(a+c*\cot(e*x+d)+b*\csc(e*x+d))^{(1/2)}$

Rubi [A] time = 0.17, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3168, 3127, 2661}

$$\frac{2\sqrt{\csc(d+ex)} \sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}} F\left(\frac{1}{2} \left(d+ex - \tan^{-1}(c, a)\right) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a+b \csc(d+ex)+c \cot(d+ex)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Csc[d + e*x]]/Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]],x]

[Out] $(2*\text{Sqrt}[\text{Csc}[d + e*x]]*\text{EllipticF}[(d + e*x - \text{ArcTan}[c, a])/2, (2*\text{Sqrt}[a^2 + c^2])/(b + \text{Sqrt}[a^2 + c^2])]*\text{Sqrt}[(b + c*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])/(b + \text{Sqrt}[a^2 + c^2])])/(e*\text{Sqrt}[a + c*\text{Cot}[d + e*x] + b*\text{Csc}[d + e*x]])$

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b])), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3127

Int[1/Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3168

Int[csc[(d_) + (e_)*(x_)]^(n_)*((a_) + csc[(d_) + (e_)*(x_)]*(b_) + cot[(d_) + (e_)*(x_)]*(c_))^(m_), x_Symbol] := Dist[(Csc[d + e*x]^n*(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n)/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n, Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && !IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}} dx &= \frac{(\sqrt{\csc(d+ex)}\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}) \int \frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}} dx}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}} \\
&= \frac{\left(\sqrt{\csc(d+ex)}\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}\right) \int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos(d+ex)-\tan^{-1}(c,a)}{b+\sqrt{a^2+c^2}}}} dx}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}} \\
&= \frac{2\sqrt{\csc(d+ex)} F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)\middle|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right) \sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}{e\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}
\end{aligned}$$

Mathematica [C] time = 0.91, size = 339, normalized size = 2.87

$$\frac{2\sqrt{\csc(d+ex)} \sec\left(\tan^{-1}\left(\frac{c}{a}\right) + d + ex\right) \sqrt{\frac{a\sqrt{\frac{c^2}{a^2}+1}(\sin(\tan^{-1}(\frac{c}{a})+d+ex)-1)}{a\sqrt{\frac{c^2}{a^2}+1}+b}} \sqrt{\frac{a\sqrt{\frac{c^2}{a^2}+1}(\sin(\tan^{-1}(\frac{c}{a})+d+ex)+1)}{a\sqrt{\frac{c^2}{a^2}+1}-b}} \sqrt{a\sqrt{\frac{c^2}{a^2}+1}}}{ae\sqrt{\frac{c^2}{a^2}+1}\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[Csc[d + e*x]]/Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]], x]

[Out] (2*AppellF1[1/2, 1/2, 1/2, 3/2, (b + a*Sqrt[1 + c^2/a^2])*Sin[d + e*x + ArcTan[c/a]]/(b - a*Sqrt[1 + c^2/a^2]), (b + a*Sqrt[1 + c^2/a^2])*Sin[d + e*x + ArcTan[c/a]]/(b + a*Sqrt[1 + c^2/a^2])]*Sqrt[Csc[d + e*x]]*Sec[d + e*x + ArcTan[c/a]]*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]]*Sqrt[-((a*Sqrt[1 + c^2/a^2]*(-1 + Sin[d + e*x + ArcTan[c/a]])))/(b + a*Sqrt[1 + c^2/a^2]))]*Sqrt[(a*Sqrt[1 + c^2/a^2]*(1 + Sin[d + e*x + ArcTan[c/a]]))/(-b + a*Sqrt[1 + c^2/a^2])]*Sqrt[b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]]]/(a*Sqrt[1 + c^2/a^2]*e*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]))

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\csc(ex+d)}}{\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(csc(e*x + d))/sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2), x, algorithm="giac")

[Out] Timed out

maple [C] time = 1.68, size = 713, normalized size = 6.04

$$4i \sqrt{\frac{1}{\sin(ex+d)}} \sqrt{\frac{b+c \cos(ex+d)+a \sin(ex+d)}{\sin(ex+d)}} \sqrt{\frac{(i \sin(ex+d)+\cos(ex+d))(ib-ic-\sqrt{a^2-b^2+c^2}-a)}{ib-ic+\sqrt{a^2-b^2+c^2}+a}} \sqrt{\frac{i(\cos(ex+d)\sqrt{a^2-b^2+c^2}-b \sin(ex+d))}{(i \cos(ex+d)+\sin(ex+d))}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2), x)

[Out] $-4*I/e*(1/\sin(e*x+d))^{(1/2)}*((b+c*\cos(e*x+d)+a*\sin(e*x+d))/\sin(e*x+d))^{(1/2)}*((I*\sin(e*x+d)+\cos(e*x+d))*(I*b-I*c-(a^2-b^2+c^2)^{(1/2)}-a)/(I*b-I*c+(a^2-b^2+c^2)^{(1/2)}+a))^{(1/2)}*(-I*(\cos(e*x+d)*\sin(e*x+d)+c*\sin(e*x+d)-a*\cos(e*x+d)+(a^2-b^2+c^2)^{(1/2)}-a)/(I*\cos(e*x+d)+\sin(e*x+d)+I)/(I*b-I*c-(a^2-b^2+c^2)^{(1/2)}+a))^{(1/2)}*(I*(b*\sin(e*x+d)-c*\sin(e*x+d)+\cos(e*x+d)*(a^2-b^2+c^2)^{(1/2)}+a*\cos(e*x+d)+(a^2-b^2+c^2)^{(1/2)}+a)/(I*\cos(e*x+d)+\sin(e*x+d)+I)/(I*b-I*c+(a^2-b^2+c^2)^{(1/2)}+a))^{(1/2)}*(\cos(e*x+d)+1)^2*EllipticF(((I*\sin(e*x+d)+\cos(e*x+d))*(I*b-I*c-(a^2-b^2+c^2)^{(1/2)}-a)/(I*b-I*c+(a^2-b^2+c^2)^{(1/2)}+a))^{(1/2)},((I*b-I*c+(a^2-b^2+c^2)^{(1/2)}+a)*(I*b-I*c+(a^2-b^2+c^2)^{(1/2)}-a)/(I*b-I*c-(a^2-b^2+c^2)^{(1/2)}-a)/(I*b-I*c-(a^2-b^2+c^2)^{(1/2)}+a))^{(1/2)})*(\cos(e*x+d)-1)^2*(I*\cos(e*x+d)*b-I*\cos(e*x+d)*c-I*(a^2-b^2+c^2)^{(1/2)}*\sin(e*x+d)-I*\sin(e*x+d)*a+\cos(e*x+d)*(a^2-b^2+c^2)^{(1/2)}+a*\cos(e*x+d)+b*\sin(e*x+d)-c*\sin(e*x+d))/\sin(e*x+d)^3/(b+c*\cos(e*x+d)+a*\sin(e*x+d))/(I*b-I*c-(a^2-b^2+c^2)^{(1/2)}-a)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\csc(ex+d)}}{\sqrt{c \cot(ex+d) + b \csc(ex+d) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(csc(e*x + d))/sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\sin(d+ex)}}}{\sqrt{a + c \cot(d + ex) + \frac{b}{\sin(d+ex)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/sin(d + e*x))^(1/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2), x)

[Out] int((1/sin(d + e*x))^(1/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a + b \csc(d+ex) + c \cot(d+ex)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(e*x+d)**(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2), x)

[Out] Integral(sqrt(csc(d + e*x))/sqrt(a + b*csc(d + e*x) + c*cot(d + e*x)), x)

$$3.465 \quad \int \frac{\csc^2(d+ex)}{(a+c \cot(d+ex)+b \csc(d+ex))^{3/2}} dx$$

Optimal. Leaf size=240

$$\frac{2 \csc^2(d+ex)(a \sin(d+ex) + b + c \cos(d+ex))^2 E\left(\frac{1}{2}(d+ex - \tan^{-1}(c,a)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e(a^2 - b^2 + c^2) \sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}} (a + b \csc(d+ex) + c \cot(d+ex))^{3/2}} \frac{2 \csc^2(d+ex)(a \cos(d+ex) + b + c \sin(d+ex))^2 E\left(\frac{1}{2}(d+ex - \tan^{-1}(c,a)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e(a^2 - b^2 + c^2) \sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}} (a + b \csc(d+ex) + c \cot(d+ex))^{3/2}}$$

[Out] $-2*\csc(e*x+d)^{(3/2)}*(b+c*\cos(e*x+d)+a*\sin(e*x+d))*(a*\cos(e*x+d)-c*\sin(e*x+d))/(a^2-b^2+c^2)/e/(a+c*\cot(e*x+d)+b*\csc(e*x+d))^{(3/2)}-2*\csc(e*x+d)^{(3/2)}*(\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(c,a)),2^{(1/2)}*((a^2+c^2)^{(1/2)}/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}*(b+c*\cos(e*x+d)+a*\sin(e*x+d))^{(1/2)}/(a^2-b^2+c^2)/e/(a+c*\cot(e*x+d)+b*\csc(e*x+d))^{(3/2)}/((b+c*\cos(e*x+d)+a*\sin(e*x+d))/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}$

Rubi [A] time = 0.21, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3168, 3128, 3119, 2653}

$$\frac{2 \csc^2(d+ex)(a \sin(d+ex) + b + c \cos(d+ex))^2 E\left(\frac{1}{2}(d+ex - \tan^{-1}(c,a)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e(a^2 - b^2 + c^2) \sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}} (a + b \csc(d+ex) + c \cot(d+ex))^{3/2}} \frac{2 \csc^2(d+ex)(a \cos(d+ex) + b + c \sin(d+ex))^2 E\left(\frac{1}{2}(d+ex - \tan^{-1}(c,a)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e(a^2 - b^2 + c^2) \sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}} (a + b \csc(d+ex) + c \cot(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Csc[d + e*x]^(3/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2),x]

[Out] $(-2*Csc[d + e*x]^{(3/2)}*\text{EllipticE}[(d + e*x - \text{ArcTan}[c, a])/2, (2*\text{Sqrt}[a^2 + c^2])/(b + \text{Sqrt}[a^2 + c^2])])*(b + c*\cos[d + e*x] + a*\sin[d + e*x])^{(1/2)}/((a^2 - b^2 + c^2)*e*(a + c*\cot[d + e*x] + b*\csc[d + e*x])^{(3/2)}*\text{Sqrt}[(b + c*\cos[d + e*x] + a*\sin[d + e*x])/(b + \text{Sqrt}[a^2 + c^2])]) - (2*Csc[d + e*x]^{(3/2)}*(b + c*\cos[d + e*x] + a*\sin[d + e*x])*(a*\cos[d + e*x] - c*\sin[d + e*x]))/(a^2 - b^2 + c^2)*e*(a + c*\cot[d + e*x] + b*\csc[d + e*x])^{(3/2)}$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3128

Int[(cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)])^(-3/2), x_Symbol] := Simp[(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*(a^2 - b^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]), x] + Dist[1/(a^2 - b^2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a

, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3168

Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Dist[(Csc[d + e*x]^n*(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n)/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n, Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && !IntegerQ[n]

Rubi steps

$$\begin{aligned} \int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c \cot(d+ex)+b \csc(d+ex))^{\frac{3}{2}}} dx &= \frac{\left(\csc^{\frac{3}{2}}(d+ex)(b+c \cos(d+ex)+a \sin(d+ex))^{\frac{3}{2}}\right) \int \frac{1}{(b+c \cos(d+ex)+a \sin(d+ex))^{\frac{3}{2}}} dx}{(a+c \cot(d+ex)+b \csc(d+ex))^{\frac{3}{2}}} \\ &= -\frac{2 \csc^{\frac{3}{2}}(d+ex)(b+c \cos(d+ex)+a \sin(d+ex))(a \cos(d+ex))}{(a^2-b^2+c^2) e(a+c \cot(d+ex)+b \csc(d+ex))} \\ &= -\frac{2 \csc^{\frac{3}{2}}(d+ex)(b+c \cos(d+ex)+a \sin(d+ex))(a \cos(d+ex))}{(a^2-b^2+c^2) e(a+c \cot(d+ex)+b \csc(d+ex))} \\ &= -\frac{2 \csc^{\frac{3}{2}}(d+ex) E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{c}{a}\right)\right) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right) (b+c \cos(d+ex))}{(a^2-b^2+c^2) e(a+c \cot(d+ex)+b \csc(d+ex))^{\frac{3}{2}} \sqrt{\frac{b+c \cos(d+ex)+a \sin(d+ex)}{b+\sqrt{a^2+c^2}}}} \end{aligned}$$

Mathematica [C] time = 6.40, size = 1732, normalized size = 7.22

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[Csc[d + e*x]^(3/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2), x]

[Out] (Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2*((-2*(a^2 + c^2))/(a*c*(a^2 - b^2 + c^2)) + (2*(a*b + a^2*Sin[d + e*x] + c^2*Sin[d + e*x]))/(c*(a^2 - b^2 + c^2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])))/(e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)) - (a*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)*(-(a*AppellF1[-1/2, -1/2, -1/2, 1/2, -(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c)))*c), -(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c)))*c))*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]])/(a^2 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2) - (c^2*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)*(-(a*AppellF1[-1/2, -1/2, -1/2, 1/2, -(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c)))*c), -(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c)))*c))*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c)))*c))

2]*c))*c)))*Sin[d + e*x - ArcTan[a/c]]/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*cos[d + e*x - ArcTan[a/c]])/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*cos[d + e*x - ArcTan[a/c]])/(a*(a^2 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)) - (2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2])*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2])*(1 - b/(a*Sqrt[1 + c^2/a^2]))]), -((b + a*Sqrt[1 + c^2/a^2])*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2])*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))*Csc[d + e*x]^(3/2)*Sec[d + e*x + ArcTan[c/a]]*(b + c*cos[d + e*x] + a*sin[d + e*x])^(3/2)*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])]/(a*(a^2 - b^2 + c^2)*Sqrt[1 + c^2/a^2]*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2))

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{c \cot(ex + d) + b \csc(ex + d) + a} \csc(ex + d)^{\frac{3}{2}}}{c^2 \cot(ex + d)^2 + b^2 \csc(ex + d)^2 + 2ac \cot(ex + d) + a^2 + 2(bc \cot(ex + d) + ab) \csc(ex + d)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*csc(e*x + d)^(3/2)/(c^2*cot(e*x + d)^2 + b^2*csc(e*x + d)^2 + 2*a*c*cot(e*x + d) + a^2 + 2*(b*c*cot(e*x + d) + a*b)*csc(e*x + d)), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [C] time = 1.12, size = 12236, normalized size = 50.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(ex + d)^{\frac{3}{2}}}{(c \cot(ex + d) + b \csc(ex + d) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x, algorithm="maxima")

[Out] integrate(csc(e*x + d)^(3/2)/(c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\sin(d+ex)}\right)^{3/2}}{\left(a + c \cot(d + ex) + \frac{b}{\sin(d+ex)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/sin(d + e*x))^(3/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2), x)

[Out] int((1/sin(d + e*x))^(3/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{\frac{3}{2}}(d + ex)}{(a + b \csc(d + ex) + c \cot(d + ex))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(e*x+d)**(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2), x)

[Out] Integral(csc(d + e*x)**(3/2)/(a + b*csc(d + e*x) + c*cot(d + e*x))**(3/2), x)

$$3.466 \quad \int \frac{\csc^2(d+ex)}{(a+c \cot(d+ex)+b \csc(d+ex))^{5/2}} dx$$

Optimal. Leaf size=492

$$\frac{2 \csc^{\frac{5}{2}}(d+ex) \sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}} (a \sin(d+ex)+b+c \cos(d+ex))^2 F\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e(a^2-b^2+c^2)(a+b \csc(d+ex)+c \cot(d+ex))^{5/2}}$$

[Out] $-2/3 \csc(e*x+d)^{(5/2)} * (b+c*\cos(e*x+d)+a*\sin(e*x+d)) * (a*\cos(e*x+d)-c*\sin(e*x+d)) / (a^2-b^2+c^2) / e / (a+c*\cot(e*x+d)+b*\csc(e*x+d))^{(5/2)} + 8/3 \csc(e*x+d)^{(5/2)} * (b+c*\cos(e*x+d)+a*\sin(e*x+d))^2 * (a*b*\cos(e*x+d)-b*c*\sin(e*x+d)) / (a^2-b^2+c^2)^2 / e / (a+c*\cot(e*x+d)+b*\csc(e*x+d))^{(5/2)} + 8/3 * b*\csc(e*x+d)^{(5/2)} * (\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))^2)^{(1/2)} / \cos(1/2*d+1/2*e*x-1/2*\arctan(c,a)) * \text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(c,a)), 2^{(1/2)} * ((a^2+c^2)^{(1/2)} / (b+(a^2+c^2)^{(1/2)})))^{(1/2)} * (b+c*\cos(e*x+d)+a*\sin(e*x+d))^3 / (a^2-b^2+c^2)^2 / e / (a+c*\cot(e*x+d)+b*\csc(e*x+d))^{(5/2)} / ((b+c*\cos(e*x+d)+a*\sin(e*x+d)) / (b+(a^2+c^2)^{(1/2)}))^{(1/2)} + 2/3 \csc(e*x+d)^{(5/2)} * (\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))^2)^{(1/2)} / \cos(1/2*d+1/2*e*x-1/2*\arctan(c,a)) * \text{EllipticF}(\sin(1/2*d+1/2*e*x-1/2*\arctan(c,a)), 2^{(1/2)} * ((a^2+c^2)^{(1/2)} / (b+(a^2+c^2)^{(1/2)})))^{(1/2)} * (b+c*\cos(e*x+d)+a*\sin(e*x+d))^2 * ((b+c*\cos(e*x+d)+a*\sin(e*x+d)) / (b+(a^2+c^2)^{(1/2)}))^{(1/2)} / (a^2-b^2+c^2) / e / (a+c*\cot(e*x+d)+b*\csc(e*x+d))^{(5/2)}$

Rubi [A] time = 0.50, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3168, 3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2 \csc^{\frac{5}{2}}(d+ex) \sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}} (a \sin(d+ex)+b+c \cos(d+ex))^2 F\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e(a^2-b^2+c^2)(a+b \csc(d+ex)+c \cot(d+ex))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Csc[d + e*x]^(5/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2), x]

[Out] $(8*b*Csc[d + e*x]^{(5/2)} * \text{EllipticE}[(d + e*x - \text{ArcTan}[c, a])/2, (2*\text{Sqrt}[a^2 + c^2]) / (b + \text{Sqrt}[a^2 + c^2])]) * (b + c*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])^3 / (3*(a^2 - b^2 + c^2)^2 * e * (a + c*\text{Cot}[d + e*x] + b*\text{Csc}[d + e*x])^{(5/2)} * \text{Sqrt}[(b + c*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x]) / (b + \text{Sqrt}[a^2 + c^2])]) + (2*\text{Csc}[d + e*x]^{(5/2)} * \text{EllipticF}[(d + e*x - \text{ArcTan}[c, a])/2, (2*\text{Sqrt}[a^2 + c^2]) / (b + \text{Sqrt}[a^2 + c^2])]) * (b + c*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])^2 * \text{Sqrt}[(b + c*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x]) / (b + \text{Sqrt}[a^2 + c^2])]) / (3*(a^2 - b^2 + c^2) * e * (a + c*\text{Cot}[d + e*x] + b*\text{Csc}[d + e*x])^{(5/2)}) - (2*\text{Csc}[d + e*x]^{(5/2)} * (b + c*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x]) * (a*\text{Cos}[d + e*x] - c*\text{Sin}[d + e*x])) / (3*(a^2 - b^2 + c^2) * e * (a + c*\text{Cot}[d + e*x] + b*\text{Csc}[d + e*x])^{(5/2)}) + (8*\text{Csc}[d + e*x]^{(5/2)} * (b + c*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])^2 * (a*b*\text{Cos}[d + e*x] - b*c*\text{Sin}[d + e*x])) / (3*(a^2 - b^2 + c^2)^2 * e * (a + c*\text{Cot}[d + e*x] + b*\text{Csc}[d + e*x])^{(5/2)})$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d/Sqrt[a + b], x] /; FreeQ[

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]]], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*SIN[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*SIN[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3127

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]]], x_Symbol] := Dist[Sqrt[(a + b*Cos[d + e*x] + c*SIN[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*SIN[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3129

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]]^n, x_Symbol] := Simp[(-(c*Cos[d + e*x] + b*SIN[d + e*x])*(a + b*Cos[d + e*x] + c*SIN[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*SIN[d + e*x])*(a + b*Cos[d + e*x] + c*SIN[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]

Rule 3149

Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)])/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]]], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*SIN[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*SIN[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]

Rule 3156

Int[((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]]^n*(A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] := -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*SIN[d + e*x])*(a + b*Cos[d + e*x] + c*SIN[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*SIN[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*SIN[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[n, -2]

Rule 3168

Int[csc[(d_.) + (e_.)*(x_.)]^n*((a_.) + csc[(d_.) + (e_.)*(x_.)]*(b_.) + cot[(d_.) + (e_.)*(x_.)]*(c_.))^m, x_Symbol] := Dist[(Csc[d + e*x]^n*(b + a*SIN[d + e*x] + c*COS[d + e*x])^m)/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n, Int[1/(b + a*SIN[d + e*x] + c*COS[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && !IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}} dx &= \frac{\left(\csc^2(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))^{5/2}\right) \int \frac{1}{(b+c\cos(d+ex)+a\sin(d+ex))^{5/2}} dx}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}} \\
&= -\frac{2\csc^2(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c)}{3(a^2-b^2+c^2)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}} \\
&= -\frac{2\csc^2(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c)}{3(a^2-b^2+c^2)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}} \\
&= -\frac{2\csc^2(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c)}{3(a^2-b^2+c^2)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}} \\
&= -\frac{2\csc^2(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c)}{3(a^2-b^2+c^2)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}} \\
&= \frac{8b\csc^2(d+ex)E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\middle|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\cos(d+ex))^{5/2}}{3(a^2-b^2+c^2)^2e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sqrt{b+c\cos(d+ex)}}
\end{aligned}$$

Mathematica [C] time = 6.49, size = 2708, normalized size = 5.50

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[d + e*x]^(5/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2),x]
[Out] (Csc[d + e*x]^(5/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^3*((8*b*(a^2 + c^2))/(3*a*c*(-a^2 + b^2 - c^2)^2) + (2*(a*b + a^2*Sin[d + e*x] + c^2*Sin[d + e*x]))/(3*c*(a^2 - b^2 + c^2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2) - (2*(a^3 + 3*a*b^2 + a*c^2 + 4*a^2*b*Sin[d + e*x] + 4*b*c^2*Sin[d + e*x]))/(3*c*(a^2 - b^2 + c^2)^2*(b + c*Cos[d + e*x] + a*Sin[d + e*x]))) / (e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (4*a*b*Csc[d + e*x]^(5/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(5/2)*(-(a*AppellF1[-1/2, -1/2, -1/2, 1/2, -(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/(3*(a^2 - b^2 + c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (4*b*c^2*Csc[d + e*x]^(5/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(5/2)*(-(a*AppellF1[-1/2, -1/2, -1/2, 1/2, -(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqr
```

```
t[1 + a^2/c^2*c)))*c)))*Sin[d + e*x - ArcTan[a/c]]/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*cos[d + e*x - ArcTan[a/c]]])/(3*a*(a^2 - b^2 + c^2)^2*e*(a + c*cot[d + e*x] + b*csc[d + e*x])^(5/2)) + (2*a*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*)*Csc[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[c/a]]*(b + c*cos[d + e*x] + a*sin[d + e*x])^(5/2)*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(3*(a^2 - b^2 + c^2)^2*Sqrt[1 + c^2/a^2]*e*(a + c*cot[d + e*x] + b*csc[d + e*x])^(5/2)) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*)*Csc[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[c/a]]*(b + c*cos[d + e*x] + a*sin[d + e*x])^(5/2)*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(a*(a^2 - b^2 + c^2)^2*Sqrt[1 + c^2/a^2]*e*(a + c*cot[d + e*x] + b*csc[d + e*x])^(5/2)) + (2*c^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*)*Csc[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[c/a]]*(b + c*cos[d + e*x] + a*sin[d + e*x])^(5/2)*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(3*a*(a^2 - b^2 + c^2)^2*Sqrt[1 + c^2/a^2]*e*(a + c*cot[d + e*x] + b*csc[d + e*x])^(5/2))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\sqrt{c \cot(ex + d) + b \csc(ex + d) + a}}{c^3 \cot(ex + d)^3 + b^3 \csc(ex + d)^3 + 3ac^2 \cot(ex + d)^2 + 3a^2c \cot(ex + d) + a^3 + 3(b^2c \cot(ex + d) + a^2b \csc(ex + d))}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*csc(e*x + d)^(5/2)/(c^3*cot(e*x + d)^3 + b^3*csc(e*x + d)^3 + 3*a*c^2*cot(e*x + d)^2 + 3*a^2*c*cot(e*x + d) + a^3 + 3*(b^2*c*cot(e*x + d) + a*b^2)*csc(e*x + d)^2 + 3*(b*c^2*cot(e*x + d)^2 + 2*a*b*c*cot(e*x + d) + a^2*b)*csc(e*x + d)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x, algorithm="giac")

[Out] Timed out

maple [C] time = 2.16, size = 62955, normalized size = 127.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(ex+d)^{\frac{5}{2}}}{(c \cot(ex+d) + b \csc(ex+d) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x, algorithm="maxima")

[Out] integrate(csc(e*x + d)^(5/2)/(c*cot(e*x + d) + b*csc(e*x + d) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\sin(d+ex)}\right)^{5/2}}{\left(a + c \cot(d+ex) + \frac{b}{\sin(d+ex)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/sin(d + e*x))^(5/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(5/2),x)

[Out] int((1/sin(d + e*x))^(5/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(e*x+d)**(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))**(5/2),x)

[Out] Timed out

$$3.467 \quad \int (a + c \cot(d + ex) + b \csc(d + ex))^{3/2} \sin^2(d + ex) dx$$

Optimal. Leaf size=371

$$\frac{2(a^2 - b^2 + c^2) \sin^{\frac{3}{2}}(d + ex) \sqrt{\frac{a \sin(d+ex) + b + c \cos(d+ex)}{\sqrt{a^2 + c^2} + b}} (a + b \csc(d + ex) + c \cot(d + ex))^{3/2} F\left(\frac{1}{2}(d + ex - \tan^{-1}\left(\frac{a \sin(d+ex) + b + c \cos(d+ex)}{\sqrt{a^2 + c^2} + b}\right))\right)}{3e(a \sin(d + ex) + b + c \cos(d + ex))^2}$$

[Out] $-2/3*(a+c*\cot(e*x+d)+b*csc(e*x+d))^{(3/2)}*\sin(e*x+d)^{(3/2)}*(a*\cos(e*x+d)-c*\sin(e*x+d))/e/(b+c*\cos(e*x+d)+a*\sin(e*x+d))+8/3*b*(a+c*\cot(e*x+d)+b*csc(e*x+d))^{(3/2)}*(\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))^{(2)})^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(c,a)),2^{(1/2)}*((a^2+c^2)^{(1/2)}/(b+(a^2+c^2)^{(1/2)})))^{(1/2)}*\sin(e*x+d)^{(3/2)}/e/(b+c*\cos(e*x+d)+a*\sin(e*x+d))/((b+c*\cos(e*x+d)+a*\sin(e*x+d))/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}+2/3*(a^2-b^2+c^2)*(a+c*\cot(e*x+d)+b*csc(e*x+d))^{(3/2)}*(\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))^{(2)})^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))*\text{EllipticF}(\sin(1/2*d+1/2*e*x-1/2*\arctan(c,a)),2^{(1/2)}*((a^2+c^2)^{(1/2)}/(b+(a^2+c^2)^{(1/2)})))^{(1/2)}*\sin(e*x+d)^{(3/2)}*((b+c*\cos(e*x+d)+a*\sin(e*x+d))/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}/e/(b+c*\cos(e*x+d)+a*\sin(e*x+d))^{(2)}$

Rubi [A] time = 0.38, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3164, 3120, 3149, 3119, 2653, 3127, 2661}

$$\frac{2(a^2 - b^2 + c^2) \sin^{\frac{3}{2}}(d + ex) \sqrt{\frac{a \sin(d+ex) + b + c \cos(d+ex)}{\sqrt{a^2 + c^2} + b}} (a + b \csc(d + ex) + c \cot(d + ex))^{3/2} F\left(\frac{1}{2}(d + ex - \tan^{-1}\left(\frac{a \sin(d+ex) + b + c \cos(d+ex)}{\sqrt{a^2 + c^2} + b}\right))\right)}{3e(a \sin(d + ex) + b + c \cos(d + ex))^2}$$

Antiderivative was successfully verified.

[In] Int[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2),x]

[Out] $(8*b*(a + c*\cot(d + e*x) + b*\csc(d + e*x))^{(3/2)}*\text{EllipticE}[(d + e*x - \text{ArcTan}[c, a])/2, (2*\sqrt{a^2 + c^2})/(b + \sqrt{a^2 + c^2})]*\sin[d + e*x]^{(3/2)})/(3*e*(b + c*\cos[d + e*x] + a*\sin[d + e*x])*sqrt[(b + c*\cos[d + e*x] + a*\sin[d + e*x])/(b + \sqrt{a^2 + c^2})]) + (2*(a^2 - b^2 + c^2)*(a + c*\cot[d + e*x] + b*\csc[d + e*x])^{(3/2)}*\text{EllipticF}[(d + e*x - \text{ArcTan}[c, a])/2, (2*\sqrt{a^2 + c^2})/(b + \sqrt{a^2 + c^2})]*\sin[d + e*x]^{(3/2)}*sqrt[(b + c*\cos[d + e*x] + a*\sin[d + e*x])/(b + \sqrt{a^2 + c^2})])/(3*e*(b + c*\cos[d + e*x] + a*\sin[d + e*x])^2) - (2*(a + c*\cot[d + e*x] + b*\csc[d + e*x])^{(3/2)}*\sin[d + e*x]^{(3/2)}*(a*\cos[d + e*x] - c*\sin[d + e*x]))/(3*e*(b + c*\cos[d + e*x] + a*\sin[d + e*x]))$

Rule 2653

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3120

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^n, x_Symbol] := -Simp[((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1))/(e*n), x] + Dist[1/n, Int[Simp[n*a^2 + (n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]
```

Rule 3164

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))^n * sin[(d_.) + (e_.)*(x_)]^n, x_Symbol] := Dist[(Sin[d + e*x]^n*(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n)/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int (a + c \cot(d + ex) + b \csc(d + ex))^{3/2} \sin^{3/2}(d + ex) dx &= \frac{\left((a + c \cot(d + ex) + b \csc(d + ex))^{3/2} \sin^{3/2}(d + ex) \right)}{(b + c \cos(d + ex) + a \sin(d + ex))^{3/2}} \\
&= -\frac{2(a + c \cot(d + ex) + b \csc(d + ex))^{3/2} \sin^{3/2}(d + ex)}{3e(b + c \cos(d + ex) + a \sin(d + ex))^{3/2}} \\
&= -\frac{2(a + c \cot(d + ex) + b \csc(d + ex))^{3/2} \sin^{3/2}(d + ex)}{3e(b + c \cos(d + ex) + a \sin(d + ex))^{3/2}} \\
&= -\frac{2(a + c \cot(d + ex) + b \csc(d + ex))^{3/2} \sin^{3/2}(d + ex)}{3e(b + c \cos(d + ex) + a \sin(d + ex))^{3/2}} \\
&= \frac{8b(a + c \cot(d + ex) + b \csc(d + ex))^{3/2} E\left(\frac{1}{2}(d + ex - \frac{1}{2}\pi), \frac{1}{2}\right)}{3e(b + c \cos(d + ex) + a \sin(d + ex))^{3/2}}
\end{aligned}$$

Mathematica [F] time = 52.86, size = 0, normalized size = 0.00

$$\int (a + c \cot(d + ex) + b \csc(d + ex))^{3/2} \sin^{3/2}(d + ex) dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2), x]

[Out] Integrate[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2), x]

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(c \cot(ex + d) + b \csc(ex + d) + a\right)^{3/2} \sin(ex + d)^{3/2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2), x, algorithm="fricas")

[Out] integral((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c \cot(ex + d) + b \csc(ex + d) + a)^{3/2} \sin(ex + d)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2), x, algorithm="giac")

[Out] integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2), x)

maple [C] time = 1.10, size = 20454, normalized size = 55.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x)`

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c \cot(ex + d) + b \csc(ex + d) + a)^{\frac{3}{2}} \sin(ex + d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x, algorithm="maxima")`

[Out] `integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(d + ex)^{3/2} \left(a + c \cot(d + ex) + \frac{b}{\sin(d + ex)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(d + e*x)^(3/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2),x)`

[Out] `int(sin(d + e*x)^(3/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2)*sin(e*x+d)**(3/2),x)`

[Out] Timed out

3.468 $\int \sqrt{a + c \cot(d + ex) + b \csc(d + ex)} \sqrt{\sin(d + ex)} dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{\sin(d+ex)} \sqrt{a + b \csc(d+ex) + c \cot(d+ex)} E\left(\frac{1}{2}(d+ex - \tan^{-1}(c, a)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e \sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

[Out] $2*(\cos(1/2*d+1/2*e*x-1/2*\arctan(c, a))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(c, a))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(c, a)), 2^{(1/2)}*((a^2+c^2)^{(1/2)})/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}*(a+c*\cot(e*x+d)+b*\csc(e*x+d))^{(1/2)}*\sin(e*x+d)^{(1/2)}/e/((b+c*\cos(e*x+d)+a*\sin(e*x+d))/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}$

Rubi [A] time = 0.14, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3164, 3119, 2653}

$$\frac{2\sqrt{\sin(d+ex)} \sqrt{a + b \csc(d+ex) + c \cot(d+ex)} E\left(\frac{1}{2}(d+ex - \tan^{-1}(c, a)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e \sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]], x]

[Out] $(2*\text{Sqrt}[a + c*\text{Cot}[d + e*x] + b*\text{Csc}[d + e*x]]*\text{EllipticE}[(d + e*x - \text{ArcTan}[c, a])/2, (2*\text{Sqrt}[a^2 + c^2])/(b + \text{Sqrt}[a^2 + c^2])]*\text{Sqrt}[\text{Sin}[d + e*x]])/(e*\text{Sqrt}[(b + c*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])/(b + \text{Sqrt}[a^2 + c^2])])$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3164

Int[((a_) + csc[(d_) + (e_)*(x_)]*(b_) + cot[(d_) + (e_)*(x_)]*(c_))^(n_)*sin[(d_) + (e_)*(x_)]^(n_), x_Symbol] := Dist[(Sin[d + e*x]^n*(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n)/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && !IntegerQ[n]

Rubi steps

$$\int \sqrt{a + c \cot(d + ex) + b \csc(d + ex)} \sqrt{\sin(d + ex)} dx = \frac{(\sqrt{a + c \cot(d + ex) + b \csc(d + ex)} \sqrt{\sin(d + ex)}) \int \sqrt{b + c \cos(d + ex) + a \sin(d + ex)} dx}{\sqrt{b + c \cos(d + ex) + a \sin(d + ex)}} = \frac{(\sqrt{a + c \cot(d + ex) + b \csc(d + ex)} \sqrt{\sin(d + ex)}) \int \sqrt{\frac{b + c \cos(d + ex) + a \sin(d + ex)}{b + \sqrt{a^2 + c^2}}} dx}{\sqrt{\frac{b + c \cos(d + ex) + a \sin(d + ex)}{b + \sqrt{a^2 + c^2}}}} = \frac{2\sqrt{a + c \cot(d + ex) + b \csc(d + ex)} E\left(\frac{1}{2} (d + ex - \tan^{-1} \frac{b + c \cos(d + ex) + a \sin(d + ex)}{b + \sqrt{a^2 + c^2}})\right)}{e \sqrt{\frac{b + c \cos(d + ex) + a \sin(d + ex)}{b + \sqrt{a^2 + c^2}}}}$$

Mathematica [F] time = 12.34, size = 0, normalized size = 0.00

$$\int \sqrt{a + c \cot(d + ex) + b \csc(d + ex)} \sqrt{\sin(d + ex)} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]], x]

[Out] Integrate[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]], x]

fricas [F] time = 1.30, size = 0, normalized size = 0.00

$$\text{integral}(\sqrt{c \cot(ex + d) + b \csc(ex + d) + a} \sqrt{\sin(ex + d)}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c \cot(ex + d) + b \csc(ex + d) + a} \sqrt{\sin(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d)), x)

maple [C] time = 1.06, size = 12365, normalized size = 104.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c \cot(ex + d) + b \csc(ex + d) + a} \sqrt{\sin(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sin(d + ex)} \sqrt{a + c \cot(d + ex) + \frac{b}{\sin(d + ex)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d + e*x)^(1/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2),x)

[Out] int(sin(d + e*x)^(1/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \csc(d + ex) + c \cot(d + ex)} \sqrt{\sin(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2)*sin(e*x+d)**(1/2),x)

[Out] Integral(sqrt(a + b*csc(d + e*x) + c*cot(d + e*x))*sqrt(sin(d + e*x)), x)

$$3.469 \quad \int \frac{1}{\sqrt{a+c \cot(d+ex)+b \csc(d+ex)} \sqrt{\sin(d+ex)}} dx$$

Optimal. Leaf size=118

$$\frac{2 \sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}} F\left(\frac{1}{2} \left(d+ex - \tan^{-1}(c, a)\right) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e \sqrt{\sin(d+ex)} \sqrt{a+b \csc(d+ex)+c \cot(d+ex)}}$$

[Out] $2*(\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))^2)^{(1/2)}/\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))*\text{EllipticF}(\sin(1/2*d+1/2*e*x-1/2*\arctan(c,a)), 2^{(1/2)}*((a^2+c^2)^{(1/2)})/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}*((b+c*\cos(e*x+d)+a*\sin(e*x+d))/(b+(a^2+c^2)^{(1/2)}))^{(1/2)}/e/(a+c*\cot(e*x+d)+b*\csc(e*x+d))^{(1/2)}/\sin(e*x+d)^{(1/2)}$

Rubi [A] time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3164, 3127, 2661}

$$\frac{2 \sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}} F\left(\frac{1}{2} \left(d+ex - \tan^{-1}(c, a)\right) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e \sqrt{\sin(d+ex)} \sqrt{a+b \csc(d+ex)+c \cot(d+ex)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]]),x]

[Out] $(2*\text{EllipticF}[(d+e*x - \text{ArcTan}[c, a])/2, (2*\text{Sqrt}[a^2 + c^2])/(b + \text{Sqrt}[a^2 + c^2])]*\text{Sqrt}[(b + c*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])/(b + \text{Sqrt}[a^2 + c^2])])/(e*\text{Sqrt}[a + c*\text{Cot}[d + e*x] + b*\text{Csc}[d + e*x]]*\text{Sqrt}[\text{Sin}[d + e*x]])$

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3127

Int[1/Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Cos[d + e*x] + c*SIN[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*SIN[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3164

Int[((a_) + csc[(d_) + (e_)*(x_)]*(b_) + cot[(d_) + (e_)*(x_)]*(c_))^(n_)*sin[(d_) + (e_)*(x_)]^(n_), x_Symbol] := Dist[(Sin[d + e*x]^n*(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n)/(b + a*SIN[d + e*x] + c*COS[d + e*x])^n, Int[(b + a*SIN[d + e*x] + c*COS[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && !IntegerQ[n]

Rubi steps

$$\int \frac{1}{\sqrt{a + c \cot(d + ex) + b \csc(d + ex)} \sqrt{\sin(d + ex)}} dx = \frac{\sqrt{b + c \cos(d + ex) + a \sin(d + ex)} \int \frac{1}{\sqrt{b + c \cos(d + ex) + a \sin(d + ex)}} dx}{\sqrt{a + c \cot(d + ex) + b \csc(d + ex)} \sqrt{\sin(d + ex)}} = \frac{\sqrt{\frac{b + c \cos(d + ex) + a \sin(d + ex)}{b + \sqrt{a^2 + c^2}}} \int \frac{1}{\sqrt{\frac{b}{b + \sqrt{a^2 + c^2}} + \frac{\sqrt{a^2 + c^2} \cos(d + ex - \tan^{-1}(c, a))}{b + \sqrt{a^2 + c^2}}}} dx}{\sqrt{a + c \cot(d + ex) + b \csc(d + ex)} \sqrt{\sin(d + ex)}} = \frac{2F\left(\frac{1}{2}(d + ex - \tan^{-1}(c, a)) \middle| \frac{2\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}\right) \sqrt{\frac{b + c \cos(d + ex) + a \sin(d + ex)}{b + \sqrt{a^2 + c^2}}}}{e \sqrt{a + c \cot(d + ex) + b \csc(d + ex)} \sqrt{\sin(d + ex)}}$$

Mathematica [C] time = 2.84, size = 519, normalized size = 4.40

$$\frac{4 \left(i \sqrt{a^2 - b^2 + c^2} - ia - b + c \right) (\cos(d + ex) + i \sin(d + ex)) \sqrt{-\frac{i(\sqrt{a^2 - b^2 + c^2} + a + (b - c) \tan(\frac{1}{2}(d + ex)))}{(\sqrt{a^2 - b^2 + c^2} + a - ib + ic) (\tan(\frac{1}{2}(d + ex)) - i)}} \sqrt{-\frac{i(\sqrt{a^2 - b^2 + c^2} - a - (b - c) \tan(\frac{1}{2}(d + ex)))}{(\sqrt{a^2 - b^2 + c^2} - a + ib - ic) (\tan(\frac{1}{2}(d + ex)) + i)}}}{e \left(-\sqrt{a^2 - b^2 + c^2} + a + ib - ic \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]]), x]

[Out] (4*((-I)*a - b + c + I*Sqrt[a^2 - b^2 + c^2])*EllipticF[ArcSin[Sqrt[((-a - I*b + I*c + Sqrt[a^2 - b^2 + c^2])*(-Cos[d + e*x] + I*Sin[d + e*x]))/(-a + I*b - I*c + Sqrt[a^2 - b^2 + c^2])]], (I*b + Sqrt[a^2 - b^2 + c^2])/(I*b - Sqrt[a^2 - b^2 + c^2])]*Sqrt[((-a - I*b + I*c + Sqrt[a^2 - b^2 + c^2])*(-Cos[d + e*x] + I*Sin[d + e*x]))/(-a + I*b - I*c + Sqrt[a^2 - b^2 + c^2])]*(Cos[d + e*x] + I*Sin[d + e*x])*Sqrt[((-I)*(a + Sqrt[a^2 - b^2 + c^2] + (b - c)*Tan[(d + e*x)/2]))/(a - I*b + I*c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[(d + e*x)/2]))]*Sqrt[((-I)*(-a + Sqrt[a^2 - b^2 + c^2] + (-b + c)*Tan[(d + e*x)/2]))/(a + I*b - I*c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[(d + e*x)/2])))]/((a + I*b - I*c - Sqrt[a^2 - b^2 + c^2])*e*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]])

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{c \cot(ex + d) + b \csc(ex + d) + a} \sqrt{\sin(ex + d)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x, algorithm="fricas")

[Out] integral(1/(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d))), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c \cot(ex + d) + b \csc(ex + d) + a} \sqrt{\sin(ex + d)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d))), x)

maple [C] time = 1.16, size = 705, normalized size = 5.97

$$4i \sqrt{\frac{b+c \cos(ex+d)+a \sin(ex+d)}{\sin(ex+d)}} \sqrt{\frac{(i \sin(ex+d)+\cos(ex+d))(ib-ic-\sqrt{a^2-b^2+c^2}-a)}{ib-ic+\sqrt{a^2-b^2+c^2}+a}} \sqrt{\frac{i(\cos(ex+d)\sqrt{a^2-b^2+c^2}-b \sin(ex+d)+c \sin(ex+d)-a \cos(ex+d))}{(i \cos(ex+d)+\sin(ex+d)+i)(ib-ic-\sqrt{a^2-b^2+c^2}-a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2), x)

[Out] $4*I/e*((b+c*\cos(e*x+d)+a*\sin(e*x+d))/\sin(e*x+d))^{(1/2)}*((I*\sin(e*x+d)+\cos(e*x+d))*(I*b-I*c-(a^2-b^2+c^2)^{(1/2)}-a)/(I*b-I*c+(a^2-b^2+c^2)^{(1/2)}+a))^{(1/2)}*(-I*(\cos(e*x+d)*(a^2-b^2+c^2)^{(1/2)}-b*\sin(e*x+d)+c*\sin(e*x+d)-a*\cos(e*x+d)+(a^2-b^2+c^2)^{(1/2)}-a)/(I*\cos(e*x+d)+\sin(e*x+d)+I)/(I*b-I*c-(a^2-b^2+c^2)^{(1/2)}+a))^{(1/2)}*(I*(b*\sin(e*x+d)-c*\sin(e*x+d)+\cos(e*x+d)*(a^2-b^2+c^2)^{(1/2)}+a*\cos(e*x+d)+(a^2-b^2+c^2)^{(1/2)}+a)/(I*\cos(e*x+d)+\sin(e*x+d)+I)/(I*b-I*c+(a^2-b^2+c^2)^{(1/2)}+a))^{(1/2)}*(\cos(e*x+d)+1)^2*\text{EllipticF}(((I*\sin(e*x+d)+\cos(e*x+d))*(I*b-I*c-(a^2-b^2+c^2)^{(1/2)}-a)/(I*b-I*c+(a^2-b^2+c^2)^{(1/2)}+a))^{(1/2)}, ((I*b-I*c+(a^2-b^2+c^2)^{(1/2)}+a)*(I*b-I*c+(a^2-b^2+c^2)^{(1/2)}-a)/(I*b-I*c-(a^2-b^2+c^2)^{(1/2)}-a)/(I*b-I*c-(a^2-b^2+c^2)^{(1/2)}+a))^{(1/2)})*(\cos(e*x+d)-1)^2*(I*(a^2-b^2+c^2)^{(1/2)}*\sin(e*x+d)+I*\sin(e*x+d)*a-I*\cos(e*x+d)*b+I*\cos(e*x+d)*c-b*\sin(e*x+d)+c*\sin(e*x+d)-\cos(e*x+d)*(a^2-b^2+c^2)^{(1/2)}-a*\cos(e*x+d))/\sin(e*x+d)^{(7/2)}/(b+c*\cos(e*x+d)+a*\sin(e*x+d))/(I*b-I*c-(a^2-b^2+c^2)^{(1/2)}-a)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c \cot(ex + d) + b \csc(ex + d) + a} \sqrt{\sin(ex + d)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sin(d + ex)} \sqrt{a + c \cot(d + ex) + \frac{b}{\sin(d + ex)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(d + e*x)^(1/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2)), x)

[Out] int(1/(sin(d + e*x)^(1/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \csc(d + ex) + c \cot(d + ex)} \sqrt{\sin(d + ex)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2)/sin(e*x+d)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*csc(d + e*x) + c*cot(d + e*x))*sqrt(sin(d + e*x))),  
x)
```

$$3.470 \quad \int \frac{1}{(a+c \cot(d+ex)+b \csc(d+ex))^3 \sin^2(d+ex)} dx$$

Optimal. Leaf size=240

$$\frac{2(a \sin(d+ex) + b + c \cos(d+ex))^2 E\left(\frac{1}{2}(d+ex - \tan^{-1}(c, a)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e(a^2 - b^2 + c^2) \sin^3(d+ex) \sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}} (a + b \csc(d+ex) + c \cot(d+ex))^{3/2}} - \frac{2(a \cos(d+ex) - c \sin(d+ex))}{e(a^2 - b^2 + c^2) \sin^3(d+ex)}$$

[Out] $-2*(b+c*\cos(e*x+d)+a*\sin(e*x+d))*(a*\cos(e*x+d)-c*\sin(e*x+d))/(a^2-b^2+c^2)/e/(a+c*\cot(e*x+d)+b*\csc(e*x+d))^(3/2)/\sin(e*x+d)^(3/2)-2*(\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))^2)^(1/2)/\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2))))^(1/2)*(b+c*\cos(e*x+d)+a*\sin(e*x+d))^2/(a^2-b^2+c^2)/e/(a+c*\cot(e*x+d)+b*\csc(e*x+d))^(3/2)/\sin(e*x+d)^(3/2)/((b+c*\cos(e*x+d)+a*\sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)$

Rubi [A] time = 0.21, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3164, 3128, 3119, 2653}

$$\frac{2(a \sin(d+ex) + b + c \cos(d+ex))^2 E\left(\frac{1}{2}(d+ex - \tan^{-1}(c, a)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e(a^2 - b^2 + c^2) \sin^3(d+ex) \sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}} (a + b \csc(d+ex) + c \cot(d+ex))^{3/2}} - \frac{2(a \cos(d+ex) - c \sin(d+ex))}{e(a^2 - b^2 + c^2) \sin^3(d+ex)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)),x]

[Out] $(-2*\text{EllipticE}[(d + e*x - \text{ArcTan}[c, a])/2, (2*\text{Sqrt}[a^2 + c^2])/(b + \text{Sqrt}[a^2 + c^2])])*(b + c*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])^2/((a^2 - b^2 + c^2)*e*(a + c*\text{Cot}[d + e*x] + b*\text{Csc}[d + e*x])^(3/2)*\text{Sin}[d + e*x]^(3/2)*\text{Sqrt}[(b + c*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])/(b + \text{Sqrt}[a^2 + c^2])]) - (2*(b + c*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x])*(a*\text{Cos}[d + e*x] - c*\text{Sin}[d + e*x]))/((a^2 - b^2 + c^2)*e*(a + c*\text{Cot}[d + e*x] + b*\text{Csc}[d + e*x])^(3/2)*\text{Sin}[d + e*x]^(3/2))$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*SIN[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*SIN[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3128

Int[(cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)])^(-3/2), x_Symbol] := Simp[(2*(c*Cos[d + e*x] - b*SIN[d + e*x]))/(e*(a^2 - b^2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*SIN[d + e*x]]), x] + Dist[1/(a^2 - b^2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*SIN[d + e*x]], x], x] /; FreeQ[{a

, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3164

Int[((a_.) + csc[(d_.) + (e_.)*(x_)])*(b_.) + cot[(d_.) + (e_.)*(x_)])*(c_.) ^ (n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Dist[(Sin[d + e*x]^n*(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n)/(b + a*SIN[d + e*x] + c*Cos[d + e*x])^n , Int[(b + a*SIN[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && !IntegerQ[n]

Rubi steps

$$\int \frac{1}{(a + c \cot(d + ex) + b \csc(d + ex))^{3/2} \sin^2(d + ex)} dx = \frac{(b + c \cos(d + ex) + a \sin(d + ex))^{3/2} \int \frac{1}{(b + c \cos(d + ex) + a \sin(d + ex))^{3/2} \sin^2(d + ex)} dx}{(a + c \cot(d + ex) + b \csc(d + ex))^{3/2} \sin^2(d + ex)}$$

$$= -\frac{2(b + c \cos(d + ex) + a \sin(d + ex))(a \cos(d + ex))}{(a^2 - b^2 + c^2) e(a + c \cot(d + ex) + b \csc(d + ex))}$$

$$= -\frac{2(b + c \cos(d + ex) + a \sin(d + ex))(a \cos(d + ex))}{(a^2 - b^2 + c^2) e(a + c \cot(d + ex) + b \csc(d + ex))}$$

$$= -\frac{2E\left(\frac{1}{2}(d + ex - \tan^{-1}(c, a)) \middle| \frac{2\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}\right) (b + c \cos(d + ex) + a \sin(d + ex))}{(a^2 - b^2 + c^2) e(a + c \cot(d + ex) + b \csc(d + ex))^3}$$

Mathematica [F] time = 20.78, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + c \cot(d + ex) + b \csc(d + ex))^{3/2} \sin^2(d + ex)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)), x]

[Out] Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)), x]

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{c \cot(ex + d) + a} \sin(ex + d)}{a^2 \cos(ex + d)^2 + (c^2 \cos(ex + d)^2 - c^2) \cot(ex + d)^2 + (b^2 \cos(ex + d)^2 - b^2) \csc(ex + d)^2 - a^2 + 2(a*c*\cos(ex + d)^2 - a*c)*\cot(ex + d) + 2*(a*b*\cos(ex + d)^2 - a*b + (b*c*\cos(ex + d)^2 - b*c)*\cot(ex + d))*\csc(ex + d)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2), x, algorithm="fricas")

[Out] integral(-sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d))/(a^2*cos(e*x + d)^2 + (c^2*cos(e*x + d)^2 - c^2)*cot(e*x + d)^2 + (b^2*cos(e*x + d)^2 - b^2)*csc(e*x + d)^2 - a^2 + 2*(a*c*cos(e*x + d)^2 - a*c)*cot(e*x + d) + 2*(a*b*cos(e*x + d)^2 - a*b + (b*c*cos(e*x + d)^2 - b*c)*cot(e*x + d))*csc(e*x + d)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c \cot(ex + d) + b \csc(ex + d) + a)^{\frac{3}{2}} \sin(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x, algorithm="giac")

[Out] integrate(1/((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2)), x)

maple [C] time = 0.96, size = 12231, normalized size = 50.96

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c \cot(ex + d) + b \csc(ex + d) + a)^{\frac{3}{2}} \sin(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin(d + ex)^{3/2} \left(a + c \cot(d + ex) + \frac{b}{\sin(d + ex)} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(d + e*x)^(3/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2)),x)

[Out] int(1/(sin(d + e*x)^(3/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2)/sin(e*x+d)**(3/2),x)

[Out] Timed out

$$3.471 \quad \int \frac{1}{(a+c \cot(d+ex)+b \csc(d+ex))^5 \sin^2(d+ex)} dx$$

Optimal. Leaf size=492

$$\frac{2\sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}} (a \sin(d+ex)+b+c \cos(d+ex))^2 F\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e(a^2-b^2+c^2) \sin^5(d+ex)(a+b \csc(d+ex)+c \cot(d+ex))^{5/2}} + \frac{8b}{3e(a^2-b^2+c^2)}$$

[Out] $-2/3*(b+c*\cos(e*x+d)+a*\sin(e*x+d))*(a*\cos(e*x+d)-c*\sin(e*x+d))/(a^2-b^2+c^2)/e/(a+c*\cot(e*x+d)+b*\csc(e*x+d))^{5/2}/\sin(e*x+d)^{5/2}+8/3*(b+c*\cos(e*x+d)+a*\sin(e*x+d))^2*(a*b*\cos(e*x+d)-b*c*\sin(e*x+d))/(a^2-b^2+c^2)^2/e/(a+c*\cot(e*x+d)+b*\csc(e*x+d))^{5/2}/\sin(e*x+d)^{5/2}+8/3*b*(\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))^2)^{1/2}/\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))*\text{EllipticE}(\sin(1/2*d+1/2*e*x-1/2*\arctan(c,a)),2^{1/2}*((a^2+c^2)^{1/2}/(b+(a^2+c^2)^{1/2})))^{1/2}*(b+c*\cos(e*x+d)+a*\sin(e*x+d))^3/(a^2-b^2+c^2)^2/e/(a+c*\cot(e*x+d)+b*\csc(e*x+d))^{5/2}/\sin(e*x+d)^{5/2}/((b+c*\cos(e*x+d)+a*\sin(e*x+d))/(b+(a^2+c^2)^{1/2}))^{1/2}+2/3*(\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))^2)^{1/2}/\cos(1/2*d+1/2*e*x-1/2*\arctan(c,a))*\text{EllipticF}(\sin(1/2*d+1/2*e*x-1/2*\arctan(c,a)),2^{1/2}*((a^2+c^2)^{1/2}/(b+(a^2+c^2)^{1/2})))^{1/2}*(b+c*\cos(e*x+d)+a*\sin(e*x+d))^2*((b+c*\cos(e*x+d)+a*\sin(e*x+d))/(b+(a^2+c^2)^{1/2}))^{1/2}/(a^2-b^2+c^2)/e/(a+c*\cot(e*x+d)+b*\csc(e*x+d))^{5/2}/\sin(e*x+d)^{5/2}$

Rubi [A] time = 0.49, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3164, 3129, 3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2\sqrt{\frac{a \sin(d+ex)+b+c \cos(d+ex)}{\sqrt{a^2+c^2}+b}} (a \sin(d+ex)+b+c \cos(d+ex))^2 F\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a)) \middle| \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e(a^2-b^2+c^2) \sin^5(d+ex)(a+b \csc(d+ex)+c \cot(d+ex))^{5/2}} + \frac{8b}{3e(a^2-b^2+c^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)),x]

[Out] $(8*b*\text{EllipticE}[(d+e*x-\text{ArcTan}[c,a])/2,(2*\text{Sqrt}[a^2+c^2])/(b+\text{Sqrt}[a^2+c^2])])*(b+c*\text{Cos}[d+e*x]+a*\text{Sin}[d+e*x])^3/(3*(a^2-b^2+c^2)^2*e*(a+c*\text{Cot}[d+e*x]+b*\text{Csc}[d+e*x])^{5/2}*\text{Sin}[d+e*x]^{5/2}*\text{Sqrt}[(b+c*\text{Cos}[d+e*x]+a*\text{Sin}[d+e*x])/(b+\text{Sqrt}[a^2+c^2])])+(2*\text{EllipticF}[(d+e*x-\text{ArcTan}[c,a])/2,(2*\text{Sqrt}[a^2+c^2])/(b+\text{Sqrt}[a^2+c^2])])*(b+c*\text{Cos}[d+e*x]+a*\text{Sin}[d+e*x])^2*\text{Sqrt}[(b+c*\text{Cos}[d+e*x]+a*\text{Sin}[d+e*x])/(b+\text{Sqrt}[a^2+c^2])])/(3*(a^2-b^2+c^2)*e*(a+c*\text{Cot}[d+e*x]+b*\text{Csc}[d+e*x])^{5/2}*\text{Sin}[d+e*x]^{5/2})-(2*(b+c*\text{Cos}[d+e*x]+a*\text{Sin}[d+e*x])*(a*\text{Cos}[d+e*x]-c*\text{Sin}[d+e*x]))/(3*(a^2-b^2+c^2)*e*(a+c*\text{Cot}[d+e*x]+b*\text{Csc}[d+e*x])^{5/2}*\text{Sin}[d+e*x]^{5/2})+(8*(b+c*\text{Cos}[d+e*x]+a*\text{Sin}[d+e*x])^2*(a*b*\text{Cos}[d+e*x]-b*c*\text{Sin}[d+e*x]))/(3*(a^2-b^2+c^2)^2*e*(a+c*\text{Cot}[d+e*x]+b*\text{Csc}[d+e*x])^{5/2}*\text{Sin}[d+e*x]^{5/2})$

Rule 2653

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]] , x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]] , x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3127

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3129

Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^n, x_Symbol] :> Simp[(-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]

Rule 3149

Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)])/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]

Rule 3156

Int[((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^n*((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[n, -2]

Rule 3164

Int[((a_.) + csc[(d_.) + (e_.)*(x_.)]*(b_.) + cot[(d_.) + (e_.)*(x_.)]*(c_.))^n*sin[(d_.) + (e_.)*(x_.)]^n, x_Symbol] :> Dist[(Sin[d + e*x]^n*(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n)/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && !IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + c \cot(d + ex) + b \csc(d + ex))^{5/2} \sin^{\frac{5}{2}}(d + ex)} dx &= \frac{(b + c \cos(d + ex) + a \sin(d + ex))^{5/2} \int \frac{1}{(b + c \cos(d + ex) + a \sin(d + ex))^{5/2} \sin^{\frac{5}{2}}(d + ex)} dx}{(a + c \cot(d + ex) + b \csc(d + ex))^{5/2} \sin^{\frac{5}{2}}(d + ex)} \\
&= -\frac{2(b + c \cos(d + ex) + a \sin(d + ex))(a \cos(d + ex))}{3(a^2 - b^2 + c^2) e(a + c \cot(d + ex) + b \csc(d + ex))} \\
&= -\frac{2(b + c \cos(d + ex) + a \sin(d + ex))(a \cos(d + ex))}{3(a^2 - b^2 + c^2) e(a + c \cot(d + ex) + b \csc(d + ex))} \\
&= -\frac{2(b + c \cos(d + ex) + a \sin(d + ex))(a \cos(d + ex))}{3(a^2 - b^2 + c^2) e(a + c \cot(d + ex) + b \csc(d + ex))} \\
&= -\frac{2(b + c \cos(d + ex) + a \sin(d + ex))(a \cos(d + ex))}{3(a^2 - b^2 + c^2) e(a + c \cot(d + ex) + b \csc(d + ex))} \\
&= \frac{8bE\left(\frac{1}{2}(d + ex - \tan^{-1}(c, a)) \mid \frac{2\sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}\right) (b - a \cos(d + ex))}{3(a^2 - b^2 + c^2)^2 e(a + c \cot(d + ex) + b \csc(d + ex))}
\end{aligned}$$

Mathematica [F] time = 25.27, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + c \cot(d + ex) + b \csc(d + ex))^{5/2} \sin^{\frac{5}{2}}(d + ex)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)), x]

[Out] Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)), x]

fricas [F] time = 1.94, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{(a^3 \cos(ex + d)^2 + (c^3 \cos(ex + d)^2 - c^3) \cot(ex + d)^3 + (b^3 \cos(ex + d)^2 - b^3) \csc(ex + d)^3 - a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2), x, algorithm="fricas")

[Out] integral(-sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)/((a^3*cos(e*x + d)^2 + (c^3*cos(e*x + d)^2 - c^3)*cot(e*x + d)^3 + (b^3*cos(e*x + d)^2 - b^3)*csc(e*x + d)^3 - a^3 + 3*(a*c^2*cos(e*x + d)^2 - a*c^2)*cot(e*x + d)^2 + 3*(a*b^2*cos(e*x + d)^2 - a*b^2 + (b^2*c*cos(e*x + d)^2 - b^2*c)*cot(e*x + d))*csc(e*x + d)^2 + 3*(a^2*c*cos(e*x + d)^2 - a^2*c)*cot(e*x + d) + 3*(a^2*b*cos(e*x + d)^2 - a^2*b + (b*c^2*cos(e*x + d)^2 - b*c^2)*cot(e*x + d)^2 + 2*(a

$b*c*\cos(e*x + d)^2 - a*b*c*\cot(e*x + d))*\csc(e*x + d))*\sqrt{\sin(e*x + d)}$
 , x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c \cot(ex + d) + b \csc(ex + d) + a)^{\frac{5}{2}} \sin(ex + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x, algorithm="giac")

[Out] integrate(1/((c*cot(e*x + d) + b*csc(e*x + d) + a)^(5/2)*sin(e*x + d)^(5/2)), x)

maple [C] time = 1.65, size = 62945, normalized size = 127.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c \cot(ex + d) + b \csc(ex + d) + a)^{\frac{5}{2}} \sin(ex + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((c*cot(e*x + d) + b*csc(e*x + d) + a)^(5/2)*sin(e*x + d)^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin(d + ex)^{5/2} \left(a + c \cot(d + ex) + \frac{b}{\sin(d + ex)} \right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(d + e*x)^(5/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(5/2)),x)

[Out] int(1/(sin(d + e*x)^(5/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(5/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x)

[Out] Timed out

$$3.472 \quad \int \frac{1}{\cos^2(x) + \sin^2(x)} dx$$

Optimal. Leaf size=1

x

[Out] x

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4380, 8}

x

Antiderivative was successfully verified.

[In] Int[(Cos[x]^2 + Sin[x]^2)^(-1), x]

[Out] x

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 4380

Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)^2*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)^2])^(p_.), x_Symbol] := Dist[(a + c)^p, Int[ActivateTrig[u], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]

Rubi steps

$$\int \frac{1}{\cos^2(x) + \sin^2(x)} dx = \int 1 dx = x$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

x

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]^2 + Sin[x]^2)^(-1), x]

[Out] x

fricas [A] time = 0.89, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2+sin(x)^2), x, algorithm="fricas")

[Out] x

giac [A] time = 0.14, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2+sin(x)^2),x, algorithm="giac")

[Out] x

maple [A] time = 0.06, size = 2, normalized size = 2.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2+sin(x)^2),x)

[Out] x

maxima [A] time = 0.41, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2+sin(x)^2),x, algorithm="maxima")

[Out] x

mupad [B] time = 2.64, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2 + sin(x)^2),x)

[Out] x

sympy [B] time = 0.36, size = 10, normalized size = 10.00

$$\frac{x}{\sin^2(x) + \cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)**2+sin(x)**2),x)

[Out] x/(sin(x)**2 + cos(x)**2)

$$3.473 \quad \int \frac{1}{(\cos^2(x) + \sin^2(x))^2} dx$$

Optimal. Leaf size=1

x

[Out] x

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4380, 8}

x

Antiderivative was successfully verified.

[In] Int[(Cos[x]^2 + Sin[x]^2)^(-2), x]

[Out] x

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 4380

Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_.)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]^2)^(p_.), x_Symbol] := Dist[(a + c)^p, Int[ActivateTrig[u], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]

Rubi steps

$$\int \frac{1}{(\cos^2(x) + \sin^2(x))^2} dx = \int 1 dx = x$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

x

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]^2 + Sin[x]^2)^(-2), x]

[Out] x

fricas [A] time = 1.49, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2+sin(x)^2)^2,x, algorithm="fricas")

[Out] x

giac [A] time = 0.12, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2+sin(x)^2)^2,x, algorithm="giac")

[Out] x

maple [A] time = 0.05, size = 2, normalized size = 2.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2+sin(x)^2)^2,x)

[Out] x

maxima [A] time = 0.41, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2+sin(x)^2)^2,x, algorithm="maxima")

[Out] x

mupad [B] time = 2.63, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2 + sin(x)^2)^2,x)

[Out] x

sympy [B] time = 0.83, size = 22, normalized size = 22.00

$$\frac{x}{\sin^4(x) + 2 \sin^2(x) \cos^2(x) + \cos^4(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)**2+sin(x)**2)**2,x)

[Out] x/(sin(x)**4 + 2*sin(x)**2*cos(x)**2 + cos(x)**4)

$$3.474 \quad \int \frac{1}{(\cos^2(x) + \sin^2(x))^3} dx$$

Optimal. Leaf size=1

x

[Out] x

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4380, 8}

x

Antiderivative was successfully verified.

[In] Int[(Cos[x]^2 + Sin[x]^2)^(-3), x]

[Out] x

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 4380

Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_.)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]^2)^p, x_Symbol] := Dist[(a + c)^p, Int[ActivateTrig[u], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]

Rubi steps

$$\int \frac{1}{(\cos^2(x) + \sin^2(x))^3} dx = \int 1 dx = x$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

x

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]^2 + Sin[x]^2)^(-3), x]

[Out] x

fricas [A] time = 0.99, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2+sin(x)^2)^3,x, algorithm="fricas")

[Out] x

giac [A] time = 0.13, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2+sin(x)^2)^3,x, algorithm="giac")

[Out] x

maple [A] time = 0.05, size = 2, normalized size = 2.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2+sin(x)^2)^3,x)

[Out] x

maxima [A] time = 0.41, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2+sin(x)^2)^3,x, algorithm="maxima")

[Out] x

mupad [B] time = 2.59, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2 + sin(x)^2)^3,x)

[Out] x

sympy [B] time = 2.10, size = 34, normalized size = 34.00

$$\frac{x}{\sin^6(x) + 3\sin^4(x)\cos^2(x) + 3\sin^2(x)\cos^4(x) + \cos^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)**2+sin(x)**2)**3,x)

[Out] x/(sin(x)**6 + 3*sin(x)**4*cos(x)**2 + 3*sin(x)**2*cos(x)**4 + cos(x)**6)

$$3.475 \quad \int \frac{1}{\cos^2(x) - \sin^2(x)} dx$$

Optimal. Leaf size=11

$$\frac{1}{2} \tanh^{-1}(2 \sin(x) \cos(x))$$

[Out] 1/2*arctanh(2*cos(x)*sin(x))

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {206}

$$\frac{1}{2} \tanh^{-1}(2 \sin(x) \cos(x))$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]^2 - Sin[x]^2)^(-1), x]

[Out] ArcTanh[2*Cos[x]*Sin[x]]/2

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\cos^2(x) - \sin^2(x)} dx &= \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \tan(x) \right) \\ &= \frac{1}{2} \tanh^{-1}(2 \cos(x) \sin(x)) \end{aligned}$$

Mathematica [B] time = 0.01, size = 23, normalized size = 2.09

$$\frac{1}{2} \log(\sin(x) + \cos(x)) - \frac{1}{2} \log(\cos(x) - \sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]^2 - Sin[x]^2)^(-1), x]

[Out] -1/2*Log[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x]]/2

fricas [B] time = 0.85, size = 23, normalized size = 2.09

$$\frac{1}{4} \log(2 \cos(x) \sin(x) + 1) - \frac{1}{4} \log(-2 \cos(x) \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2-sin(x)^2), x, algorithm="fricas")

[Out] 1/4*log(2*cos(x)*sin(x) + 1) - 1/4*log(-2*cos(x)*sin(x) + 1)

giac [B] time = 0.13, size = 33, normalized size = 3.00

$$\frac{1}{8} \log \left(\left| \frac{1}{\sin(2x)} + \sin(2x) + 2 \right| \right) - \frac{1}{8} \log \left(\left| \frac{1}{\sin(2x)} + \sin(2x) - 2 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2-sin(x)^2),x, algorithm="giac")

[Out] 1/8*log(abs(1/sin(2*x) + sin(2*x) + 2)) - 1/8*log(abs(1/sin(2*x) + sin(2*x) - 2))

maple [A] time = 0.10, size = 4, normalized size = 0.36

arctanh(tan(x))

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2-sin(x)^2),x)

[Out] arctanh(tan(x))

maxima [A] time = 0.32, size = 15, normalized size = 1.36

$$\frac{1}{2} \log(\tan(x) + 1) - \frac{1}{2} \log(\tan(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2-sin(x)^2),x, algorithm="maxima")

[Out] 1/2*log(tan(x) + 1) - 1/2*log(tan(x) - 1)

mupad [B] time = 2.90, size = 3, normalized size = 0.27

atanh(tan(x))

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2 - sin(x)^2),x)

[Out] atanh(tan(x))

sympy [B] time = 0.35, size = 36, normalized size = 3.27

$$\frac{\log\left(\tan^2\left(\frac{x}{2}\right) - 2\tan\left(\frac{x}{2}\right) - 1\right)}{2} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right) + 2\tan\left(\frac{x}{2}\right) - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)**2-sin(x)**2),x)

[Out] log(tan(x/2)**2 - 2*tan(x/2) - 1)/2 - log(tan(x/2)**2 + 2*tan(x/2) - 1)/2

$$3.476 \quad \int \frac{1}{(\cos^2(x) - \sin^2(x))^2} dx$$

Optimal. Leaf size=13

$$\frac{\tan(x)}{1 - \tan^2(x)}$$

[Out] tan(x)/(1-tan(x)^2)

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {383}

$$\frac{\tan(x)}{1 - \tan^2(x)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]^2 - Sin[x]^2)^(-2), x]

[Out] Tan[x]/(1 - Tan[x]^2)

Rule 383

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]

Rubi steps

$$\int \frac{1}{(\cos^2(x) - \sin^2(x))^2} dx = \text{Subst} \left(\int \frac{1 + x^2}{(1 - x^2)^2} dx, x, \tan(x) \right) = \frac{\tan(x)}{1 - \tan^2(x)}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 0.62

$$\frac{1}{2} \tan(2x)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]^2 - Sin[x]^2)^(-2), x]

[Out] Tan[2*x]/2

fricas [A] time = 1.31, size = 15, normalized size = 1.15

$$\frac{\cos(x) \sin(x)}{2 \cos(x)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2-sin(x)^2)^2,x, algorithm="fricas")

[Out] cos(x)*sin(x)/(2*cos(x)^2 - 1)

giac [A] time = 0.12, size = 6, normalized size = 0.46

$$\frac{1}{2} \tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2-sin(x)^2)^2,x, algorithm="giac")

[Out] 1/2*tan(2*x)

maple [A] time = 0.10, size = 18, normalized size = 1.38

$$-\frac{1}{2(1+\tan(x))} - \frac{1}{2(\tan(x)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2-sin(x)^2)^2,x)

[Out] -1/2/(1+tan(x))-1/2/(tan(x)-1)

maxima [A] time = 0.30, size = 12, normalized size = 0.92

$$-\frac{\tan(x)}{\tan(x)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2-sin(x)^2)^2,x, algorithm="maxima")

[Out] -tan(x)/(tan(x)^2 - 1)

mupad [B] time = 2.63, size = 6, normalized size = 0.46

$$\frac{\tan(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2 - sin(x)^2)^2,x)

[Out] tan(2*x)/2

sympy [B] time = 1.34, size = 48, normalized size = 3.69

$$-\frac{2 \tan^3\left(\frac{x}{2}\right)}{\tan^4\left(\frac{x}{2}\right) - 6 \tan^2\left(\frac{x}{2}\right) + 1} + \frac{2 \tan\left(\frac{x}{2}\right)}{\tan^4\left(\frac{x}{2}\right) - 6 \tan^2\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)**2-sin(x)**2)**2,x)

[Out] -2*tan(x/2)**3/(tan(x/2)**4 - 6*tan(x/2)**2 + 1) + 2*tan(x/2)/(tan(x/2)**4 - 6*tan(x/2)**2 + 1)

$$3.477 \quad \int \frac{1}{(\cos^2(x) - \sin^2(x))^3} dx$$

Optimal. Leaf size=32

$$\frac{\tan(x) \sec^2(x)}{2(1 - \tan^2(x))^2} + \frac{1}{4} \tanh^{-1}(2 \sin(x) \cos(x))$$

[Out] 1/4*arctanh(2*cos(x)*sin(x))+1/2*sec(x)^2*tan(x)/(1-tan(x)^2)^2

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {413, 21, 206}

$$\frac{\tan(x) \sec^2(x)}{2(1 - \tan^2(x))^2} + \frac{1}{4} \tanh^{-1}(2 \sin(x) \cos(x))$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]^2 - Sin[x]^2)^(-3), x]

[Out] ArcTanh[2*Cos[x]*Sin[x]]/4 + (Sec[x]^2*Tan[x])/(2*(1 - Tan[x]^2)^2)

Rule 21

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :>
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

Rule 413

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)/(a*b*n*(p +
1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
+ q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(\cos^2(x) - \sin^2(x))^3} dx &= \text{Subst} \left(\int \frac{(1+x^2)^2}{(1-x^2)^3} dx, x, \tan(x) \right) \\
&= \frac{\sec^2(x) \tan(x)}{2(1 - \tan^2(x))^2} - \frac{1}{4} \text{Subst} \left(\int \frac{-2 + 2x^2}{(1-x^2)^2} dx, x, \tan(x) \right) \\
&= \frac{\sec^2(x) \tan(x)}{2(1 - \tan^2(x))^2} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \tan(x) \right) \\
&= \frac{1}{4} \tanh^{-1}(2 \cos(x) \sin(x)) + \frac{\sec^2(x) \tan(x)}{2(1 - \tan^2(x))^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.69

$$\frac{1}{4} \tanh^{-1}(\sin(2x)) + \frac{1}{4} \tan(2x) \sec(2x)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]^2 - Sin[x]^2)^(-3), x]

[Out] ArcTanh[Sin[2*x]]/4 + (Sec[2*x]*Tan[2*x])/4

fricas [B] time = 0.76, size = 74, normalized size = 2.31

$$\frac{(4 \cos(x)^4 - 4 \cos(x)^2 + 1) \log(2 \cos(x) \sin(x) + 1) - (4 \cos(x)^4 - 4 \cos(x)^2 + 1) \log(-2 \cos(x) \sin(x) + 1) + 4 \cos(x) \sin(x)}{8(4 \cos(x)^4 - 4 \cos(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2-sin(x)^2)^3,x, algorithm="fricas")

[Out] 1/8*((4*cos(x)^4 - 4*cos(x)^2 + 1)*log(2*cos(x)*sin(x) + 1) - (4*cos(x)^4 - 4*cos(x)^2 + 1)*log(-2*cos(x)*sin(x) + 1) + 4*cos(x)*sin(x))/(4*cos(x)^4 - 4*cos(x)^2 + 1)

giac [A] time = 0.13, size = 37, normalized size = 1.16

$$-\frac{\sin(2x)}{4(\sin(2x)^2 - 1)} + \frac{1}{8} \log(\sin(2x) + 1) - \frac{1}{8} \log(-\sin(2x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2-sin(x)^2)^3,x, algorithm="giac")

[Out] -1/4*sin(2*x)/(sin(2*x)^2 - 1) + 1/8*log(sin(2*x) + 1) - 1/8*log(-sin(2*x) + 1)

maple [A] time = 0.12, size = 48, normalized size = 1.50

$$\frac{1}{4(\tan(x) - 1)^2} + \frac{1}{4 \tan(x) - 4} - \frac{\ln(\tan(x) - 1)}{4} - \frac{1}{4(1 + \tan(x))^2} + \frac{1}{4 + 4 \tan(x)} + \frac{\ln(1 + \tan(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2-sin(x)^2)^3,x)

[Out] $1/4/(\tan(x)-1)^2+1/4/(\tan(x)-1)-1/4*\ln(\tan(x)-1)-1/4/(1+\tan(x))^2+1/4/(1+\tan(x))+1/4*\ln(1+\tan(x))$

maxima [A] time = 0.31, size = 38, normalized size = 1.19

$$\frac{\tan(x)^3 + \tan(x)}{2(\tan(x)^4 - 2 \tan(x)^2 + 1)} + \frac{1}{4} \log(\tan(x) + 1) - \frac{1}{4} \log(\tan(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2-sin(x)^2)^3,x, algorithm="maxima")

[Out] $1/2*(\tan(x)^3 + \tan(x))/(\tan(x)^4 - 2*\tan(x)^2 + 1) + 1/4*\log(\tan(x) + 1) - 1/4*\log(\tan(x) - 1)$

mupad [B] time = 2.65, size = 32, normalized size = 1.00

$$\frac{\operatorname{atanh}(\tan(x))}{2} + \frac{\frac{\tan(x)^3}{2} + \frac{\tan(x)}{2}}{\tan(x)^4 - 2 \tan(x)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2 - sin(x)^2)^3,x)

[Out] $\operatorname{atanh}(\tan(x))/2 + (\tan(x)/2 + \tan(x)^3/2)/(\tan(x)^4 - 2*\tan(x)^2 + 1)$

sympy [B] time = 3.48, size = 765, normalized size = 23.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)**2-sin(x)**2)**3,x)

[Out] $\log(\tan(x/2)**2 - 2*\tan(x/2) - 1)*\tan(x/2)**8/(4*\tan(x/2)**8 - 48*\tan(x/2)**6 + 152*\tan(x/2)**4 - 48*\tan(x/2)**2 + 4) - 12*\log(\tan(x/2)**2 - 2*\tan(x/2) - 1)*\tan(x/2)**6/(4*\tan(x/2)**8 - 48*\tan(x/2)**6 + 152*\tan(x/2)**4 - 48*\tan(x/2)**2 + 4) + 38*\log(\tan(x/2)**2 - 2*\tan(x/2) - 1)*\tan(x/2)**4/(4*\tan(x/2)**8 - 48*\tan(x/2)**6 + 152*\tan(x/2)**4 - 48*\tan(x/2)**2 + 4) - 12*\log(\tan(x/2)**2 - 2*\tan(x/2) - 1)*\tan(x/2)**2/(4*\tan(x/2)**8 - 48*\tan(x/2)**6 + 152*\tan(x/2)**4 - 48*\tan(x/2)**2 + 4) + \log(\tan(x/2)**2 - 2*\tan(x/2) - 1)/(4*\tan(x/2)**8 - 48*\tan(x/2)**6 + 152*\tan(x/2)**4 - 48*\tan(x/2)**2 + 4) - \log(\tan(x/2)**2 + 2*\tan(x/2) - 1)*\tan(x/2)**8/(4*\tan(x/2)**8 - 48*\tan(x/2)**6 + 152*\tan(x/2)**4 - 48*\tan(x/2)**2 + 4) + 12*\log(\tan(x/2)**2 + 2*\tan(x/2) - 1)*\tan(x/2)**6/(4*\tan(x/2)**8 - 48*\tan(x/2)**6 + 152*\tan(x/2)**4 - 48*\tan(x/2)**2 + 4) - 38*\log(\tan(x/2)**2 + 2*\tan(x/2) - 1)*\tan(x/2)**4/(4*\tan(x/2)**8 - 48*\tan(x/2)**6 + 152*\tan(x/2)**4 - 48*\tan(x/2)**2 + 4) + 12*\log(\tan(x/2)**2 + 2*\tan(x/2) - 1)*\tan(x/2)**2/(4*\tan(x/2)**8 - 48*\tan(x/2)**6 + 152*\tan(x/2)**4 - 48*\tan(x/2)**2 + 4) - \log(\tan(x/2)**2 + 2*\tan(x/2) - 1)/(4*\tan(x/2)**8 - 48*\tan(x/2)**6 + 152*\tan(x/2)**4 - 48*\tan(x/2)**2 + 4) - 4*\tan(x/2)**7/(4*\tan(x/2)**8 - 48*\tan(x/2)**6 + 152*\tan(x/2)**4 - 48*\tan(x/2)**2 + 4) - 4*\tan(x/2)**5/(4*\tan(x/2)**8 - 48*\tan(x/2)**6 + 152*\tan(x/2)**4 - 48*\tan(x/2)**2 + 4) + 4*\tan(x/2)**3/(4*\tan(x/2)**8 - 48*\tan(x/2)**6 + 152*\tan(x/2)**4 - 48*\tan(x/2)**2 + 4) + 4*\tan(x/2)/(4*\tan(x/2)**8 - 48*\tan(x/2)**6 + 152*\tan(x/2)**4 - 48*\tan(x/2)**2 + 4)$

$$3.478 \quad \int \frac{1}{\cos^2(x) + a^2 \sin^2(x)} dx$$

Optimal. Leaf size=9

$$\frac{\tan^{-1}(a \tan(x))}{a}$$

[Out] arctan(a*tan(x))/a

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {203}

$$\frac{\tan^{-1}(a \tan(x))}{a}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]^2 + a^2*Sin[x]^2)^(-1), x]

[Out] ArcTan[a*Tan[x]]/a

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\int \frac{1}{\cos^2(x) + a^2 \sin^2(x)} dx = \text{Subst} \left(\int \frac{1}{1 + a^2 x^2} dx, x, \tan(x) \right) \\ = \frac{\tan^{-1}(a \tan(x))}{a}$$

Mathematica [A] time = 0.03, size = 9, normalized size = 1.00

$$\frac{\tan^{-1}(a \tan(x))}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]^2 + a^2*Sin[x]^2)^(-1), x]

[Out] ArcTan[a*Tan[x]]/a

fricas [B] time = 0.69, size = 35, normalized size = 3.89

$$\frac{\arctan\left(\frac{(a^2+1)\cos(x)^2-a^2}{2a\cos(x)\sin(x)}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2+a^2*sin(x)^2), x, algorithm="fricas")

[Out] -1/2*arctan(1/2*((a^2 + 1)*cos(x)^2 - a^2)/(a*cos(x)*sin(x)))/a

giac [B] time = 0.15, size = 20, normalized size = 2.22

$$\frac{\pi \left[\frac{x}{\pi} + \frac{1}{2} \right] + \arctan(a \tan(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")

[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)))/a

maple [A] time = 0.12, size = 10, normalized size = 1.11

$$\frac{\arctan(a \tan(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2+a^2*sin(x)^2),x)

[Out] arctan(a*tan(x))/a

maxima [A] time = 0.40, size = 9, normalized size = 1.00

$$\frac{\arctan(a \tan(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")

[Out] arctan(a*tan(x))/a

mupad [B] time = 2.83, size = 9, normalized size = 1.00

$$\frac{\operatorname{atan}(a \tan(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2 + a^2*sin(x)^2),x)

[Out] atan(a*tan(x))/a

sympy [B] time = 22.40, size = 12007, normalized size = 1334.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)**2+a**2*sin(x)**2),x)

[Out] Piecewise((64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a

$$\begin{aligned}
& **2 - 2*a*\sqrt{a**2 - 1} + 1)*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} - 12*a \\
& **2*\sqrt{a**2 - 1}*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2* \\
& a*\sqrt{a**2 - 1} + 1} - 2*a*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2* \\
& a**2 + 2*a*\sqrt{a**2 - 1} + 1)) - 64*a**6*\sqrt{a**2 - 1}*\sqrt{-2*a**2 - 2*a \\
& *\sqrt{a**2 - 1} + 1}*\log(-\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} + \tan(x/2) \\
&)/(64*a**7*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a \\
& **2 - 1} + 1} - 64*a**6*\sqrt{a**2 - 1}*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + \\
& 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} - 96*a**5*\sqrt{-2*a**2 - 2*a*\sqrt{ \\
& a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} + 64*a**4*\sqrt{a**2 \\
& - 1}*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - \\
& 1} + 1} + 36*a**3*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a \\
& *\sqrt{a**2 - 1} + 1} - 12*a**2*\sqrt{a**2 - 1}*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 \\
& - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} - 2*a*\sqrt{-2*a**2 - 2*a*s \\
& \sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1})) + 64*a**6*\sqrt{a \\
& **2 - 1}*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\log(\sqrt{-2*a**2 + 2*a*\sqrt{ \\
& a**2 - 1} + 1} + \tan(x/2))/(64*a**7*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1} \\
& *\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} - 64*a**6*\sqrt{a**2 - 1}*\sqrt{-2*a* \\
& **2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} - 96*a* \\
& **5*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} \\
& + 1} + 64*a**4*\sqrt{a**2 - 1}*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{(\\
& -2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} + 36*a**3*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - \\
& 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} - 12*a**2*\sqrt{a**2 - 1}*\sqrt{ \\
& (-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} \\
& - 2*a*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - \\
& 1} + 1})) - 112*a**5*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\log(-\sqrt{-2*a* \\
& **2 + 2*a*\sqrt{a**2 - 1} + 1} + \tan(x/2))/(64*a**7*\sqrt{-2*a**2 - 2*a*\sqrt{a \\
& **2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} - 64*a**6*\sqrt{a**2 - \\
& 1}*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} \\
& + 1} - 96*a**5*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*s \\
& \sqrt{a**2 - 1} + 1} + 64*a**4*\sqrt{a**2 - 1}*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - \\
& 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} + 36*a**3*\sqrt{-2*a**2 - 2*a \\
& *\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} - 12*a**2*\sqrt{ \\
& a**2 - 1}*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a* \\
& **2 - 1} + 1} - 2*a*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2* \\
& a*\sqrt{a**2 - 1} + 1})) + 112*a**5*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\log \\
& (\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} + \tan(x/2))/(64*a**7*\sqrt{-2*a**2 \\
& - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} - 64*a**6* \\
& \sqrt{a**2 - 1}*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{ \\
& a**2 - 1} + 1} - 96*a**5*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2* \\
& a**2 + 2*a*\sqrt{a**2 - 1} + 1} + 64*a**4*\sqrt{a**2 - 1}*\sqrt{-2*a**2 - 2*a* \\
& \sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} + 36*a**3*\sqrt{(- \\
& 2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} - 1 \\
& 2*a**2*\sqrt{a**2 - 1}*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + \\
& 2*a*\sqrt{a**2 - 1} + 1} - 2*a*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{(\\
& -2*a**2 + 2*a*\sqrt{a**2 - 1} + 1})) + 16*a**5*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - \\
& 1} + 1}*\log(-\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1} + \tan(x/2))/(64*a**7*s \\
& \sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} \\
&) - 64*a**6*\sqrt{a**2 - 1}*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a \\
& **2 + 2*a*\sqrt{a**2 - 1} + 1} - 96*a**5*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + \\
& 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} + 64*a**4*\sqrt{a**2 - 1}*\sqrt{-2 \\
& *a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} + 36 \\
& *a**3*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - \\
& 1} + 1} - 12*a**2*\sqrt{a**2 - 1}*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{ \\
& (-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} - 2*a*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} \\
& + 1})*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1})) - 16*a**5*\sqrt{-2*a**2 + 2*a \\
& *\sqrt{a**2 - 1} + 1}*\log(\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1} + \tan(x/2)) \\
& /(64*a**7*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1}*\sqrt{-2*a**2 + 2*a*\sqrt{a* \\
& **2 - 1} + 1} - 64*a**6*\sqrt{a**2 - 1}*\sqrt{-2*a**2 - 2*a*\sqrt{a**2 - 1} + 1} \\
&)*\sqrt{-2*a**2 + 2*a*\sqrt{a**2 - 1} + 1} - 96*a**5*\sqrt{-2*a**2 - 2*a*\sqrt{
\end{aligned}$$


```

+ 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt
(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 -
2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) - sqrt(a**
2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(sqrt(-2*a**2 + 2*a*sqrt(a
**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*s
qrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2
- 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5
*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) +
1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2
*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1)
+ 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(
-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) -
2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1
) + 1)) - sqrt(a**2 - 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-
2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqr
t(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**
2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2
- 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2
*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**
2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 -
2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*s
qrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqr
t(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2
+ 2*a*sqrt(a**2 - 1) + 1)) + sqrt(a**2 - 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 -
1) + 1)*log(sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqr
t(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)
- 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**
2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1
)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a
**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a
**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1
) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt
(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1)
+ 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)), Ne(a, 0)), (-2*tan(x/2)/(tan(
x/2)**2 - 1), True))

```

$$3.479 \quad \int \frac{1}{b^2 \cos^2(x) + \sin^2(x)} dx$$

Optimal. Leaf size=11

$$\frac{\tan^{-1}\left(\frac{\tan(x)}{b}\right)}{b}$$

[Out] arctan(tan(x)/b)/b

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {203}

$$\frac{\tan^{-1}\left(\frac{\tan(x)}{b}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[(b^2*cos[x]^2 + Sin[x]^2)^(-1), x]

[Out] ArcTan[Tan[x]/b]/b

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{b^2 \cos^2(x) + \sin^2(x)} dx &= \text{Subst}\left(\int \frac{1}{b^2 + x^2} dx, x, \tan(x)\right) \\ &= \frac{\tan^{-1}\left(\frac{\tan(x)}{b}\right)}{b} \end{aligned}$$

Mathematica [A] time = 0.03, size = 11, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\tan(x)}{b}\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(b^2*cos[x]^2 + Sin[x]^2)^(-1), x]

[Out] ArcTan[Tan[x]/b]/b

fricas [B] time = 2.04, size = 31, normalized size = 2.82

$$\frac{\arctan\left(\frac{(b^2+1)\cos(x)^2-1}{2b\cos(x)\sin(x)}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*cos(x)^2+sin(x)^2), x, algorithm="fricas")

[Out] -1/2*arctan(1/2*((b^2 + 1)*cos(x)^2 - 1)/(b*cos(x)*sin(x)))/b

giac [A] time = 0.13, size = 22, normalized size = 2.00

$$\frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor + \arctan\left(\frac{\tan(x)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*cos(x)^2+sin(x)^2),x, algorithm="giac")

[Out] (pi*floor(x/pi + 1/2) + arctan(tan(x)/b))/b

maple [A] time = 0.11, size = 12, normalized size = 1.09

$$\frac{\arctan\left(\frac{\tan(x)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2*cos(x)^2+sin(x)^2),x)

[Out] arctan(tan(x)/b)/b

maxima [A] time = 0.41, size = 11, normalized size = 1.00

$$\frac{\arctan\left(\frac{\tan(x)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*cos(x)^2+sin(x)^2),x, algorithm="maxima")

[Out] arctan(tan(x)/b)/b

mupad [B] time = 2.83, size = 11, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{\tan(x)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x)^2 + b^2*cos(x)^2),x)

[Out] atan(tan(x)/b)/b

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b**2*cos(x)**2+sin(x)**2),x)

[Out] Timed out

$$3.480 \quad \int \frac{1}{b^2 \cos^2(x) + a^2 \sin^2(x)} dx$$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{a \tan(x)}{b}\right)}{ab}$$

[Out] arctan(a*tan(x)/b)/a/b

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{a \tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

[In] Int[(b^2*cos[x]^2 + a^2*sin[x]^2)^(-1), x]

[Out] ArcTan[(a*Tan[x])/b]/(a*b)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{1}{b^2 \cos^2(x) + a^2 \sin^2(x)} dx &= \text{Subst} \left(\int \frac{1}{b^2 + a^2 x^2} dx, x, \tan(x) \right) \\ &= \frac{\tan^{-1}\left(\frac{a \tan(x)}{b}\right)}{ab} \end{aligned}$$

Mathematica [A] time = 0.04, size = 15, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{a \tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

[In] Integrate[(b^2*cos[x]^2 + a^2*sin[x]^2)^(-1), x]

[Out] ArcTan[(a*Tan[x])/b]/(a*b)

fricas [B] time = 2.39, size = 43, normalized size = 2.87

$$-\frac{\arctan\left(\frac{(a^2+b^2)\cos(x)^2-a^2}{2ab\cos(x)\sin(x)}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2), x, algorithm="fricas")

[Out] -1/2*arctan(1/2*((a^2 + b^2)*cos(x)^2 - a^2)/(a*b*cos(x)*sin(x)))/(a*b)

giac [A] time = 0.13, size = 26, normalized size = 1.73

$$\frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor + \arctan\left(\frac{a \tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")

[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)/b))/(a*b)

maple [A] time = 0.12, size = 16, normalized size = 1.07

$$\frac{\arctan\left(\frac{a \tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2*cos(x)^2+a^2*sin(x)^2),x)

[Out] arctan(a*tan(x)/b)/a/b

maxima [A] time = 0.40, size = 15, normalized size = 1.00

$$\frac{\arctan\left(\frac{a \tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")

[Out] arctan(a*tan(x)/b)/(a*b)

mupad [B] time = 2.85, size = 15, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{a \tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2*cos(x)^2 + a^2*sin(x)^2),x)

[Out] atan((a*tan(x))/b)/(a*b)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)

[Out] Timed out

$$3.481 \quad \int \frac{1}{4 \cos^2(1+2x) + 3 \sin^2(1+2x)} dx$$

Optimal. Leaf size=53

$$\frac{x}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sin(2x+1)\cos(2x+1)}{\cos^2(2x+1)+2\sqrt{3}+3}\right)}{4\sqrt{3}}$$

[Out] $1/6*x*3^{(1/2)}-1/12*\arctan(\cos(1+2*x)*\sin(1+2*x)/(3+\cos(1+2*x)^2+2*3^{(1/2)}))$
 $*3^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, number of rules / integrand size = 0.043, Rules used = {203}

$$\frac{x}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sin(2x+1)\cos(2x+1)}{\cos^2(2x+1)+2\sqrt{3}+3}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(4*Cos[1 + 2*x]^2 + 3*Sin[1 + 2*x]^2)^(-1), x]

[Out] $x/(2*\text{Sqrt}[3]) - \text{ArcTan}[(\text{Cos}[1 + 2*x]*\text{Sin}[1 + 2*x])/(3 + 2*\text{Sqrt}[3] + \text{Cos}[1 + 2*x]^2)]/(4*\text{Sqrt}[3])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{4 \cos^2(1+2x) + 3 \sin^2(1+2x)} dx &= \frac{1}{2} \text{Subst}\left(\int \frac{1}{4 + 3x^2} dx, x, \tan(1+2x)\right) \\ &= \frac{x}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\cos(1+2x)\sin(1+2x)}{3+2\sqrt{3}+\cos^2(1+2x)}\right)}{4\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 0.47

$$\frac{\tan^{-1}\left(\frac{1}{2}\sqrt{3} \tan(2x+1)\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(4*Cos[1 + 2*x]^2 + 3*Sin[1 + 2*x]^2)^(-1), x]

[Out] ArcTan[(Sqrt[3]*Tan[1 + 2*x])/2]/(4*Sqrt[3])

fricas [A] time = 0.99, size = 43, normalized size = 0.81

$$-\frac{1}{24}\sqrt{3} \arctan\left(\frac{7\sqrt{3} \cos(2x+1)^2 - 3\sqrt{3}}{12 \cos(2x+1) \sin(2x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x, algorithm="fricas")

[Out] -1/24*sqrt(3)*arctan(1/12*(7*sqrt(3)*cos(2*x + 1)^2 - 3*sqrt(3))/(cos(2*x + 1)*sin(2*x + 1)))

giac [A] time = 0.13, size = 61, normalized size = 1.15

$$\frac{1}{12} \sqrt{3} \left(2x + \arctan \left(-\frac{2\sqrt{3} \sin(4x+2) - 3 \sin(4x+2)}{2\sqrt{3} \cos(4x+2) + 2\sqrt{3} - 3 \cos(4x+2) + 3} \right) + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x, algorithm="giac")

[Out] 1/12*sqrt(3)*(2*x + arctan(-(2*sqrt(3)*sin(4*x + 2) - 3*sin(4*x + 2))/(2*sqrt(3)*cos(4*x + 2) + 2*sqrt(3) - 3*cos(4*x + 2) + 3)) + 1)

maple [A] time = 0.23, size = 18, normalized size = 0.34

$$\frac{\sqrt{3} \arctan \left(\frac{\sqrt{3} \tan(1+2x)}{2} \right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x)

[Out] 1/12*3^(1/2)*arctan(1/2*3^(1/2)*tan(1+2*x))

maxima [A] time = 0.41, size = 17, normalized size = 0.32

$$\frac{1}{12} \sqrt{3} \arctan \left(\frac{1}{2} \sqrt{3} \tan(2x+1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x, algorithm="maxima")

[Out] 1/12*sqrt(3)*arctan(1/2*sqrt(3)*tan(2*x + 1))

mupad [B] time = 2.76, size = 36, normalized size = 0.68

$$\frac{\sqrt{3} (2x - \operatorname{atan}(\tan(2x+1)))}{12} + \frac{\sqrt{3} \operatorname{atan} \left(\frac{\sqrt{3} \tan(2x+1)}{2} \right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*sin(2*x + 1)^2 + 4*cos(2*x + 1)^2),x)

[Out] (3^(1/2)*(2*x - atan(tan(2*x + 1))))/12 + (3^(1/2)*atan((3^(1/2)*tan(2*x + 1))/2))/12

sympy [A] time = 0.81, size = 87, normalized size = 1.64

$$\frac{\sqrt{3} \left(\operatorname{atan} \left(\frac{2\sqrt{3} \tan \left(x + \frac{1}{2} \right)}{3} - \frac{\sqrt{3}}{3} \right) + \pi \left\lfloor \frac{x - \frac{\pi}{2} + \frac{1}{2}}{\pi} \right\rfloor \right)}{12} + \frac{\sqrt{3} \left(\operatorname{atan} \left(\frac{2\sqrt{3} \tan \left(x + \frac{1}{2} \right)}{3} + \frac{\sqrt{3}}{3} \right) + \pi \left\lfloor \frac{x - \frac{\pi}{2} + \frac{1}{2}}{\pi} \right\rfloor \right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*cos(1+2*x)**2+3*sin(1+2*x)**2),x)

[Out] sqrt(3)*(atan(2*sqrt(3)*tan(x + 1/2)/3 - sqrt(3)/3) + pi*floor((x - pi/2 + 1/2)/pi))/12 + sqrt(3)*(atan(2*sqrt(3)*tan(x + 1/2)/3 + sqrt(3)/3) + pi*floor((x - pi/2 + 1/2)/pi))/12

$$3.482 \quad \int \frac{\sin^2(x)}{a \cos^2(x) + b \sin^2(x)} dx$$

Optimal. Leaf size=43

$$\frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b} \tan(x)}{\sqrt{a}}\right)}{\sqrt{b}(a-b)} - \frac{x}{a-b}$$

[Out] $-x/(a-b) + \arctan(b^{(1/2)} * \tan(x) / a^{(1/2)}) * a^{(1/2)} / (a-b) / b^{(1/2)}$

Rubi [A] time = 0.15, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {481, 203, 205}

$$\frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b} \tan(x)}{\sqrt{a}}\right)}{\sqrt{b}(a-b)} - \frac{x}{a-b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sin}[x]^2/(a*\text{Cos}[x]^2 + b*\text{Sin}[x]^2), x]$

[Out] $-(x/(a - b)) + (\text{Sqrt}[a]*\text{ArcTan}[(\text{Sqrt}[b]*\text{Tan}[x])/ \text{Sqrt}[a]]) / ((a - b)*\text{Sqrt}[b])$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /;$ FreeQ[{a, b}, x] && PosQ[a/b]

Rule 481

$\text{Int}[(e_)*(x_)^{(m_)} / (((a_ + (b_)*(x_)^{(n_)}) * ((c_ + (d_)*(x_)^{(n_)}))), x_Symbol] \rightarrow -\text{Dist}[(a*e^n)/(b*c - a*d), \text{Int}[(e*x)^{(m-n)} / (a + b*x^n), x], x] + \text{Dist}[(c*e^n)/(b*c - a*d), \text{Int}[(e*x)^{(m-n)} / (c + d*x^n), x], x] /;$ FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1]

Rubi steps

$$\begin{aligned} \int \frac{\sin^2(x)}{a \cos^2(x) + b \sin^2(x)} dx &= \text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+bx^2)} dx, x, \tan(x)\right) \\ &= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \tan(x)\right)}{a-b} + \frac{a \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tan(x)\right)}{a-b} \\ &= -\frac{x}{a-b} + \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b} \tan(x)}{\sqrt{a}}\right)}{(a-b)\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.09, size = 36, normalized size = 0.84

$$x - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b} \tan(x)}{\sqrt{a}}\right)}{\sqrt{b}}$$

$$\frac{\quad}{b-a}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]

[Out] (x - (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/Sqrt[b])/(-a + b)

fricas [A] time = 0.70, size = 182, normalized size = 4.23

$$\left[\frac{\sqrt{\frac{a}{b}} \log\left(\frac{(a^2+6ab+b^2)\cos(x)^4-2(3ab+b^2)\cos(x)^2+4((ab+b^2)\cos(x)^3-b^2\cos(x))\sqrt{\frac{a}{b}}\sin(x)+b^2}{(a^2-2ab+b^2)\cos(x)^4+2(ab-b^2)\cos(x)^2+b^2}\right) + 4x}{4(a-b)}, -\frac{\sqrt{\frac{a}{b}} \arctan\left(\frac{(a+b)\cos(x)}{2a\cos(x)}\right)}{2(a-b)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="fricas")

[Out] [-1/4*(sqrt(-a/b)*log(((a^2 + 6*a*b + b^2)*cos(x)^4 - 2*(3*a*b + b^2)*cos(x)^2 + 4*((a*b + b^2)*cos(x)^3 - b^2*cos(x))*sqrt(-a/b)*sin(x) + b^2)/((a^2 - 2*a*b + b^2)*cos(x)^4 + 2*(a*b - b^2)*cos(x)^2 + b^2)) + 4*x)/(a - b), -1/2*(sqrt(a/b)*arctan(1/2*((a + b)*cos(x)^2 - b)*sqrt(a/b)/(a*cos(x)*sin(x)) + 2*x)/(a - b)]

giac [A] time = 0.13, size = 48, normalized size = 1.12

$$\frac{\left(\pi \left[\frac{x}{\pi} + \frac{1}{2}\right] \operatorname{sgn}(b) + \arctan\left(\frac{b \tan(x)}{\sqrt{ab}}\right)\right) a}{\sqrt{ab} (a - b)} - \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="giac")

[Out] (pi*floor(x/pi + 1/2)*sgn(b) + arctan(b*tan(x)/sqrt(a*b)))*a/(sqrt(a*b)*(a - b)) - x/(a - b)

maple [A] time = 0.11, size = 38, normalized size = 0.88

$$\frac{a \arctan\left(\frac{\tan(x)b}{\sqrt{ab}}\right)}{(a - b) \sqrt{ab}} - \frac{\arctan(\tan(x))}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x)

[Out] a/(a-b)/(a*b)^(1/2)*arctan(tan(x)*b/(a*b)^(1/2))-1/(a-b)*arctan(tan(x))

maxima [A] time = 0.41, size = 35, normalized size = 0.81

$$\frac{a \arctan\left(\frac{b \tan(x)}{\sqrt{ab}}\right)}{\sqrt{ab} (a - b)} - \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="maxima")

[Out] a*arctan(b*tan(x)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - x/(a - b)

mupad [B] time = 2.73, size = 51, normalized size = 1.19

$$\begin{cases} \frac{2x - \sin(2x)}{4b} & \text{if } a = b \\ x - \frac{\sqrt{a} \operatorname{atan}\left(\frac{\sqrt{b} \tan(x)}{\sqrt{a}}\right)}{\sqrt{b}} & \text{if } a \neq b \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^2/(b*sin(x)^2 + a*cos(x)^2), x)`

[Out] `piecewise(a == b, (2*x - sin(2*x))/(4*b), a ~= b, -(x - (a^(1/2))*atan((b^(1/2))*tan(x))/a^(1/2))/b^(1/2)/(a - b))`

sympy [A] time = 1.56, size = 241, normalized size = 5.60

$$\begin{cases} \infty x & \text{for } a = 0 \wedge b = 0 \\ \frac{x \sin^2(x)}{2b \sin^2(x) + 2b \cos^2(x)} + \frac{x \cos^2(x)}{2b \sin^2(x) + 2b \cos^2(x)} - \frac{\sin(x) \cos(x)}{2b \sin^2(x) + 2b \cos^2(x)} & \text{for } a = b \\ -x + \frac{\sin(x)}{\cos(x)} & \text{for } b = 0 \\ \frac{x}{b} & \text{for } a = 0 \\ -\frac{2i\sqrt{b}x\sqrt{\frac{1}{a}}}{2ia\sqrt{b}\sqrt{\frac{1}{a}} - 2ib^{\frac{3}{2}}\sqrt{\frac{1}{a}}} - \frac{\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}\sin(x) + \cos(x)\right)}{2ia\sqrt{b}\sqrt{\frac{1}{a}} - 2ib^{\frac{3}{2}}\sqrt{\frac{1}{a}}} + \frac{\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}\sin(x) + \cos(x)\right)}{2ia\sqrt{b}\sqrt{\frac{1}{a}} - 2ib^{\frac{3}{2}}\sqrt{\frac{1}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)**2/(a*cos(x)**2+b*sin(x)**2), x)`

[Out] `Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x*sin(x)**2/(2*b*sin(x)**2 + 2*b*cos(x)**2) + x*cos(x)**2/(2*b*sin(x)**2 + 2*b*cos(x)**2) - sin(x)*cos(x)/(2*b*sin(x)**2 + 2*b*cos(x)**2), Eq(a, b)), ((-x + sin(x)/cos(x))/a, Eq(b, 0)), (x/b, Eq(a, 0)), (-2*I*sqrt(b)*x*sqrt(1/a)/(2*I*a*sqrt(b)*sqrt(1/a) - 2*I*b**(3/2)*sqrt(1/a)) - log(-I*sqrt(b)*sqrt(1/a)*sin(x) + cos(x))/(2*I*a*sqrt(b)*sqrt(1/a) - 2*I*b**(3/2)*sqrt(1/a)) + log(I*sqrt(b)*sqrt(1/a)*sin(x) + cos(x))/(2*I*a*sqrt(b)*sqrt(1/a) - 2*I*b**(3/2)*sqrt(1/a)), True))`

$$3.483 \quad \int \frac{\cos^2(x)}{a \cos^2(x) + b \sin^2(x)} dx$$

Optimal. Leaf size=43

$$\frac{x}{a-b} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}$$

[Out] $x/(a-b) - \arctan(b^{(1/2)} * \tan(x) / a^{(1/2)}) * b^{(1/2)} / (a-b) / a^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {391, 203, 205}

$$\frac{x}{a-b} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2/(a*Cos[x]^2 + b*Sin[x]^2), x]

[Out] $x/(a-b) - (\text{Sqrt}[b] * \text{ArcTan}[(\text{Sqrt}[b] * \text{Tan}[x]) / \text{Sqrt}[a]]) / (\text{Sqrt}[a] * (a-b))$

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 391

Int[1/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(x)}{a \cos^2(x) + b \sin^2(x)} dx &= \text{Subst} \left(\int \frac{1}{(1+x^2)(a+bx^2)} dx, x, \tan(x) \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right)}{a-b} - \frac{b \text{Subst} \left(\int \frac{1}{a+bx^2} dx, x, \tan(x) \right)}{a-b} \\ &= \frac{x}{a-b} - \frac{\sqrt{b} \tan^{-1} \left(\frac{\sqrt{b} \tan(x)}{\sqrt{a}} \right)}{\sqrt{a}(a-b)} \end{aligned}$$

Mathematica [A] time = 0.05, size = 36, normalized size = 0.84

$$\frac{x - \frac{\sqrt{b} \tan^{-1} \left(\frac{\sqrt{b} \tan(x)}{\sqrt{a}} \right)}{\sqrt{a}}}{a-b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^2/(a*Cos[x]^2 + b*Sin[x]^2), x]

[Out] (x - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/Sqrt[a])/(a - b)

fricas [A] time = 1.49, size = 181, normalized size = 4.21

$$\left[\frac{\sqrt{-\frac{b}{a}} \log\left(\frac{(a^2+6ab+b^2)\cos(x)^4-2(3ab+b^2)\cos(x)^2-4((a^2+ab)\cos(x)^3-ab\cos(x))\sqrt{-\frac{b}{a}}\sin(x)+b^2}{(a^2-2ab+b^2)\cos(x)^4+2(ab-b^2)\cos(x)^2+b^2}\right)-4x \sqrt{\frac{b}{a}} \arctan\left(\frac{(a+b)\cos(x)}{2b\cos(x)}\right)}{4(a-b)}, \frac{\sqrt{\frac{b}{a}} \arctan\left(\frac{(a+b)\cos(x)}{2b\cos(x)}\right)}{2(a-b)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2/(a*cos(x)^2+b*sin(x)^2), x, algorithm="fricas")

[Out] [-1/4*(sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(x)^4 - 2*(3*a*b + b^2)*cos(x)^2 - 4*((a^2 + a*b)*cos(x)^3 - a*b*cos(x))*sqrt(-b/a)*sin(x) + b^2)/((a^2 - 2*a*b + b^2)*cos(x)^4 + 2*(a*b - b^2)*cos(x)^2 + b^2)) - 4*x)/(a - b), 1/2*(sqrt(b/a)*arctan(1/2*((a + b)*cos(x)^2 - b)*sqrt(b/a)/(b*cos(x)*sin(x))) + 2*x)/(a - b)]

giac [A] time = 0.14, size = 48, normalized size = 1.12

$$-\frac{\left(\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(b) + \arctan\left(\frac{b \tan(x)}{\sqrt{ab}}\right)\right) b}{\sqrt{ab}(a-b)} + \frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2/(a*cos(x)^2+b*sin(x)^2), x, algorithm="giac")

[Out] -(pi*floor(x/pi + 1/2)*sgn(b) + arctan(b*tan(x)/sqrt(a*b)))*b/(sqrt(a*b)*(a - b)) + x/(a - b)

maple [A] time = 0.10, size = 36, normalized size = 0.84

$$-\frac{b \arctan\left(\frac{\tan(x)b}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}} + \frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2/(a*cos(x)^2+b*sin(x)^2), x)

[Out] -b/(a-b)/(a*b)^(1/2)*arctan(tan(x)*b/(a*b)^(1/2))+x/(a-b)

maxima [A] time = 0.41, size = 35, normalized size = 0.81

$$-\frac{b \arctan\left(\frac{b \tan(x)}{\sqrt{ab}}\right)}{\sqrt{ab}(a-b)} + \frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2/(a*cos(x)^2+b*sin(x)^2), x, algorithm="maxima")

[Out] -b*arctan(b*tan(x)/sqrt(a*b))/(sqrt(a*b)*(a - b)) + x/(a - b)

mupad [B] time = 2.67, size = 48, normalized size = 1.12

$$\begin{cases} \frac{2x + \sin(2x)}{4b} & \text{if } a = b \\ \frac{x - \frac{\sqrt{b} \operatorname{atan}\left(\frac{\sqrt{b} \tan(x)}{\sqrt{a}}\right)}{\sqrt{a}}}{a-b} & \text{if } a \neq b \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)^2/(b*sin(x)^2 + a*cos(x)^2), x)`

[Out] `piecewise(a == b, (2*x + sin(2*x))/(4*b), a ~= b, (x - (b^(1/2)*atan((b^(1/2)/2)*tan(x))/a^(1/2)))/a^(1/2)/(a - b))`

sympy [A] time = 1.62, size = 267, normalized size = 6.21

$$\begin{cases} \infty \left(-x - \frac{\cos(x)}{\sin(x)} \right) & \text{for } a = 0 \wedge b = 0 \\ \frac{x \sin^2(x)}{2b \sin^2(x) + 2b \cos^2(x)} + \frac{x \cos^2(x)}{2b \sin^2(x) + 2b \cos^2(x)} + \frac{\sin(x) \cos(x)}{2b \sin^2(x) + 2b \cos^2(x)} & \text{for } a = b \\ \frac{x}{a} & \text{for } b = 0 \\ -x - \frac{\cos(x)}{\sin(x)} & \text{for } a = 0 \\ \frac{2ia\sqrt{b}x\sqrt{\frac{1}{a}}}{2ia^2\sqrt{b}\sqrt{\frac{1}{a}} - 2iab^2\sqrt{\frac{1}{a}}} + \frac{b \log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}\sin(x) + \cos(x)\right)}{2ia^2\sqrt{b}\sqrt{\frac{1}{a}} - 2iab^2\sqrt{\frac{1}{a}}} - \frac{b \log\left(i\sqrt{b}\sqrt{\frac{1}{a}}\sin(x) + \cos(x)\right)}{2ia^2\sqrt{b}\sqrt{\frac{1}{a}} - 2iab^2\sqrt{\frac{1}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)**2/(a*cos(x)**2+b*sin(x)**2), x)`

[Out] `Piecewise((zoo*(-x - cos(x)/sin(x))), Eq(a, 0) & Eq(b, 0)), (x*sin(x)**2/(2*b*sin(x)**2 + 2*b*cos(x)**2) + x*cos(x)**2/(2*b*sin(x)**2 + 2*b*cos(x)**2) + sin(x)*cos(x)/(2*b*sin(x)**2 + 2*b*cos(x)**2), Eq(a, b)), (x/a, Eq(b, 0)), ((-x - cos(x)/sin(x))/b, Eq(a, 0)), (2*I*a*sqrt(b)*x*sqrt(1/a)/(2*I*a**2*sqrt(b)*sqrt(1/a) - 2*I*a*b**(3/2)*sqrt(1/a)) + b*log(-I*sqrt(b)*sqrt(1/a)*sin(x) + cos(x))/(2*I*a**2*sqrt(b)*sqrt(1/a) - 2*I*a*b**(3/2)*sqrt(1/a)) - b*log(I*sqrt(b)*sqrt(1/a)*sin(x) + cos(x))/(2*I*a**2*sqrt(b)*sqrt(1/a) - 2*I*a*b**(3/2)*sqrt(1/a)), True))`

$$3.484 \quad \int \frac{1}{\sec^2(x) + \tan^2(x)} dx$$

Optimal. Leaf size=36

$$\sqrt{2}x - x + \sqrt{2} \tan^{-1} \left(\frac{\sin(x) \cos(x)}{\sin^2(x) + \sqrt{2} + 1} \right)$$

[Out] $-x + x \cdot 2^{1/2} + \arctan(\cos(x) \cdot \sin(x) / (1 + \sin(x)^2 + 2^{1/2})) \cdot 2^{1/2}$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1093, 203}

$$\sqrt{2}x - x + \sqrt{2} \tan^{-1} \left(\frac{\sin(x) \cos(x)}{\sin^2(x) + \sqrt{2} + 1} \right)$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2 + Tan[x]^2)^(-1), x]

[Out] $-x + \text{Sqrt}[2] \cdot x + \text{Sqrt}[2] \cdot \text{ArcTan}[(\text{Cos}[x] \cdot \text{Sin}[x]) / (1 + \text{Sqrt}[2] + \text{Sin}[x]^2)]$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sec^2(x) + \tan^2(x)} dx &= \text{Subst} \left(\int \frac{1}{1 + 3x^2 + 2x^4} dx, x, \tan(x) \right) \\ &= 2 \text{Subst} \left(\int \frac{1}{1 + 2x^2} dx, x, \tan(x) \right) - 2 \text{Subst} \left(\int \frac{1}{2 + 2x^2} dx, x, \tan(x) \right) \\ &= -x + \sqrt{2}x + \sqrt{2} \tan^{-1} \left(\frac{\cos(x) \sin(x)}{1 + \sqrt{2} + \sin^2(x)} \right) \end{aligned}$$

Mathematica [A] time = 0.05, size = 19, normalized size = 0.53

$$\sqrt{2} \tan^{-1} \left(\sqrt{2} \tan(x) \right) - x$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2 + Tan[x]^2)^(-1), x]

[Out] $-x + \text{Sqrt}[2] \cdot \text{ArcTan}[\text{Sqrt}[2] \cdot \text{Tan}[x]]$

fricas [A] time = 1.08, size = 35, normalized size = 0.97

$$-\frac{1}{2} \sqrt{2} \arctan \left(\frac{3 \sqrt{2} \cos(x)^2 - 2 \sqrt{2}}{4 \cos(x) \sin(x)} \right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2+tan(x)^2),x, algorithm="fricas")

[Out] $-1/2*\sqrt{2}*\arctan(1/4*(3*\sqrt{2}*\cos(x)^2 - 2*\sqrt{2}))/(\cos(x)*\sin(x)) - x$

giac [A] time = 0.12, size = 15, normalized size = 0.42

$$\sqrt{2} \arctan\left(\sqrt{2} \tan(x)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2+tan(x)^2),x, algorithm="giac")

[Out] $\sqrt{2}*\arctan(\sqrt{2}*\tan(x)) - x$

maple [A] time = 0.12, size = 16, normalized size = 0.44

$$\sqrt{2} \arctan\left(\sqrt{2} \tan(x)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sec(x)^2+tan(x)^2),x)

[Out] $2^{(1/2)}*\arctan(2^{(1/2)}*\tan(x))-x$

maxima [A] time = 0.41, size = 15, normalized size = 0.42

$$\sqrt{2} \arctan\left(\sqrt{2} \tan(x)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2+tan(x)^2),x, algorithm="maxima")

[Out] $\sqrt{2}*\arctan(\sqrt{2}*\tan(x)) - x$

mupad [B] time = 2.65, size = 15, normalized size = 0.42

$$\sqrt{2} \operatorname{atan}\left(\sqrt{2} \tan(x)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/cos(x)^2 + tan(x)^2),x)

[Out] $2^{(1/2)}*\operatorname{atan}(2^{(1/2)}*\tan(x)) - x$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\tan^2(x) + \sec^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)**2+tan(x)**2),x)

[Out] Integral(1/(tan(x)**2 + sec(x)**2), x)

$$3.485 \quad \int \frac{1}{(\sec^2(x) + \tan^2(x))^2} dx$$

Optimal. Leaf size=49

$$-\frac{x}{\sqrt{2}} + x + \frac{\tan(x)}{2 \tan^2(x) + 1} - \frac{\tan^{-1}\left(\frac{\sin(x) \cos(x)}{\sin^2(x) + \sqrt{2} + 1}\right)}{\sqrt{2}}$$

[Out] $x - 1/2 * x * 2^{(1/2)} - 1/2 * \arctan(\cos(x) * \sin(x) / (1 + \sin(x)^2 + 2^{(1/2)})) * 2^{(1/2)} + \tan(x) / (1 + 2 * \tan(x)^2)$

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {414, 12, 481, 203}

$$-\frac{x}{\sqrt{2}} + x + \frac{\tan(x)}{2 \tan^2(x) + 1} - \frac{\tan^{-1}\left(\frac{\sin(x) \cos(x)}{\sin^2(x) + \sqrt{2} + 1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2 + Tan[x]^2)^(-2), x]

[Out] $x - x/\text{Sqrt}[2] - \text{ArcTan}[(\text{Cos}[x] * \text{Sin}[x]) / (1 + \text{Sqrt}[2] + \text{Sin}[x]^2)] / \text{Sqrt}[2] + \text{Tan}[x] / (1 + 2 * \text{Tan}[x]^2)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1 * ArcTan[(Rt[b, 2] * x) / Rt[a, 2]]) / (Rt[a, 2] * Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1)) / (a*n*(p + 1)*(b*c - a*d)), x] + Dist[1 / (a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q * Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 481

Int[((e_.)*(x_)^(m_.)) / (((a_) + (b_.)*(x_)^(n_)) * ((c_) + (d_.)*(x_)^(n_))), x_Symbol] := -Dist[(a*e^n) / (b*c - a*d), Int[(e*x)^(m - n) / (a + b*x^n), x], x] + Dist[(c*e^n) / (b*c - a*d), Int[(e*x)^(m - n) / (c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(\sec^2(x) + \tan^2(x))^2} dx &= \text{Subst} \left(\int \frac{1}{(1+x^2)(1+2x^2)^2} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{1+2\tan^2(x)} - \frac{1}{2} \text{Subst} \left(\int -\frac{2x^2}{(1+x^2)(1+2x^2)} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{1+2\tan^2(x)} + \text{Subst} \left(\int \frac{x^2}{(1+x^2)(1+2x^2)} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{1+2\tan^2(x)} + \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right) - \text{Subst} \left(\int \frac{1}{1+2x^2} dx, x, \tan(x) \right) \\
&= x - \frac{x}{\sqrt{2}} - \frac{\tan^{-1} \left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)} \right)}{\sqrt{2}} + \frac{\tan(x)}{1+2\tan^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 42, normalized size = 0.86

$$\frac{-3x - \sin(2x) + x \cos(2x)}{\cos(2x) - 3} - \frac{\tan^{-1}(\sqrt{2} \tan(x))}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2 + Tan[x]^2)^(-2), x]

[Out] -(ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2]) + (-3*x + x*Cos[2*x] - Sin[2*x])/(-3 + Cos[2*x])

fricas [A] time = 1.02, size = 68, normalized size = 1.39

$$\frac{4x \cos(x)^2 + (\sqrt{2} \cos(x)^2 - 2\sqrt{2}) \arctan\left(\frac{3\sqrt{2} \cos(x)^2 - 2\sqrt{2}}{4 \cos(x) \sin(x)}\right) - 4 \cos(x) \sin(x) - 8x}{4(\cos(x)^2 - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2+tan(x)^2)^2,x, algorithm="fricas")

[Out] 1/4*(4*x*cos(x)^2 + (sqrt(2)*cos(x)^2 - 2*sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) - 4*cos(x)*sin(x) - 8*x)/(cos(x)^2 - 2)

giac [A] time = 0.15, size = 27, normalized size = 0.55

$$-\frac{1}{2} \sqrt{2} \arctan(\sqrt{2} \tan(x)) + x + \frac{\tan(x)}{2 \tan(x)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2+tan(x)^2)^2,x, algorithm="giac")

[Out] -1/2*sqrt(2)*arctan(sqrt(2)*tan(x)) + x + tan(x)/(2*tan(x)^2 + 1)

maple [A] time = 0.12, size = 27, normalized size = 0.55

$$\frac{\tan(x)}{2(\tan^2(x) + 1)} - \frac{\sqrt{2} \arctan(\sqrt{2} \tan(x))}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sec(x)^2+tan(x)^2)^2,x)

[Out] 1/2*tan(x)/(tan(x)^2+1/2)-1/2*2^(1/2)*arctan(2^(1/2)*tan(x))+x

maxima [A] time = 0.40, size = 27, normalized size = 0.55

$$-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)+x+\frac{\tan(x)}{2\tan(x)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2+tan(x)^2)^2,x, algorithm="maxima")

[Out] -1/2*sqrt(2)*arctan(sqrt(2)*tan(x)) + x + tan(x)/(2*tan(x)^2 + 1)

mupad [B] time = 2.66, size = 27, normalized size = 0.55

$$x + \frac{\tan(x)}{2\left(\tan(x)^2 + \frac{1}{2}\right)} - \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\tan(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/cos(x)^2 + tan(x)^2)^2,x)

[Out] x + tan(x)/(2*(tan(x)^2 + 1/2)) - (2^(1/2)*atan(2^(1/2)*tan(x)))/2

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\tan^2(x) + \sec^2(x)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)**2+tan(x)**2)**2,x)

[Out] Integral((tan(x)**2 + sec(x)**2)**(-2), x)

$$3.486 \quad \int \frac{1}{(\sec^2(x) + \tan^2(x))^3} dx$$

Optimal. Leaf size=74

$$\frac{7x}{4\sqrt{2}} - x - \frac{\tan(x)}{4(2\tan^2(x) + 1)} + \frac{\tan(x)}{2(2\tan^2(x) + 1)^2} + \frac{7 \tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x) + \sqrt{2} + 1}\right)}{4\sqrt{2}}$$

[Out] $-x + 7/8 * x * 2^{(1/2)} + 7/8 * \arctan(\cos(x) * \sin(x) / (1 + \sin(x)^2 + 2^{(1/2)})) * 2^{(1/2)} + 1/2 * \tan(x) / (1 + 2 * \tan(x)^2)^2 - 1/4 * \tan(x) / (1 + 2 * \tan(x)^2)$

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {414, 527, 522, 203}

$$\frac{7x}{4\sqrt{2}} - x - \frac{\tan(x)}{4(2\tan^2(x) + 1)} + \frac{\tan(x)}{2(2\tan^2(x) + 1)^2} + \frac{7 \tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x) + \sqrt{2} + 1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2 + Tan[x]^2)^(-3), x]

[Out] $-x + (7*x)/(4*\text{Sqrt}[2]) + (7*\text{ArcTan}[(\text{Cos}[x]*\text{Sin}[x])/(1 + \text{Sqrt}[2] + \text{Sin}[x]^2)])/(4*\text{Sqrt}[2]) + \text{Tan}[x]/(2*(1 + 2*\text{Tan}[x]^2)^2) - \text{Tan}[x]/(4*(1 + 2*\text{Tan}[x]^2))$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(b*c - a*d)*(p+1)), x] + Dist[1/(a*n*(b*c - a*d)*(p+1)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p+1) + d*(b*e - a*f)*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(\sec^2(x) + \tan^2(x))^3} dx &= \text{Subst} \left(\int \frac{1}{(1+x^2)(1+2x^2)^3} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{2(1+2\tan^2(x))^2} - \frac{1}{4} \text{Subst} \left(\int \frac{-2-6x^2}{(1+x^2)(1+2x^2)^2} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{2(1+2\tan^2(x))^2} - \frac{\tan(x)}{4(1+2\tan^2(x))} + \frac{1}{8} \text{Subst} \left(\int \frac{6-2x^2}{(1+x^2)(1+2x^2)} dx, x, \tan(x) \right) \\
&= \frac{\tan(x)}{2(1+2\tan^2(x))^2} - \frac{\tan(x)}{4(1+2\tan^2(x))} + \frac{7}{4} \text{Subst} \left(\int \frac{1}{1+2x^2} dx, x, \tan(x) \right) - \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right) \\
&= -x + \frac{7x}{4\sqrt{2}} + \frac{7 \tan^{-1} \left(\frac{\cos(x) \sin(x)}{1+\sqrt{2}+\sin^2(x)} \right)}{4\sqrt{2}} + \frac{\tan(x)}{2(1+2\tan^2(x))^2} - \frac{\tan(x)}{4(1+2\tan^2(x))}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 79, normalized size = 1.07

$$\frac{(\cos(2x) - 3) \sec^6(x) (-76x - 2 \sin(2x) + 3 \sin(4x) + 48x \cos(2x) - 4x \cos(4x) + 7\sqrt{2} (\cos(2x) - 3)^2 \tan^{-1} \left(\frac{\cos(x) \sin(x)}{1+\sqrt{2}+\sin^2(x)} \right))}{64 (\tan^2(x) + \sec^2(x))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2 + Tan[x]^2)^(-3), x]

[Out] -1/64*((-3 + Cos[2*x])*Sec[x]^6*(-76*x + 7*Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]]*(-3 + Cos[2*x])^2 + 48*x*Cos[2*x] - 4*x*Cos[4*x] - 2*Sin[2*x] + 3*Sin[4*x]))/(Sec[x]^2 + Tan[x]^2)^3

fricas [A] time = 2.12, size = 100, normalized size = 1.35

$$\frac{16x \cos(x)^4 - 64x \cos(x)^2 + 7(\sqrt{2} \cos(x)^4 - 4\sqrt{2} \cos(x)^2 + 4\sqrt{2}) \arctan\left(\frac{3\sqrt{2} \cos(x)^2 - 2\sqrt{2}}{4 \cos(x) \sin(x)}\right) - 4(3 \cos(x)^3 - 2 \cos(x)) \sin(x) + 64x}{16(\cos(x)^4 - 4 \cos(x)^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2+tan(x)^2)^3,x, algorithm="fricas")

[Out] -1/16*(16*x*cos(x)^4 - 64*x*cos(x)^2 + 7*(sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos(x)^2 + 4*sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) - 4*(3*cos(x)^3 - 2*cos(x))*sin(x) + 64*x)/(cos(x)^4 - 4*cos(x)^2 + 4)

giac [A] time = 0.15, size = 39, normalized size = 0.53

$$\frac{7}{8} \sqrt{2} \arctan\left(\sqrt{2} \tan(x)\right) - x - \frac{2 \tan(x)^3 - \tan(x)}{4(2 \tan(x)^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2+tan(x)^2)^3,x, algorithm="giac")

[Out] 7/8*sqrt(2)*arctan(sqrt(2)*tan(x)) - x - 1/4*(2*tan(x)^3 - tan(x))/(2*tan(x)^2 + 1)^2

maple [A] time = 0.13, size = 40, normalized size = 0.54

$$\frac{-\frac{(\tan^3(x))}{2} + \frac{\tan(x)}{4}}{(2(\tan^2(x)) + 1)^2} + \frac{7\sqrt{2} \arctan(\sqrt{2} \tan(x))}{8} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sec(x)^2+tan(x)^2)^3,x)

[Out] 8*(-1/16*tan(x)^3+1/32*tan(x))/(1+2*tan(x)^2)^2+7/8*2^(1/2)*arctan(2^(1/2)*tan(x))-x

maxima [A] time = 0.42, size = 45, normalized size = 0.61

$$\frac{7}{8} \sqrt{2} \arctan(\sqrt{2} \tan(x)) - x - \frac{2 \tan(x)^3 - \tan(x)}{4(4 \tan(x)^4 + 4 \tan(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2+tan(x)^2)^3,x, algorithm="maxima")

[Out] 7/8*sqrt(2)*arctan(sqrt(2)*tan(x)) - x - 1/4*(2*tan(x)^3 - tan(x))/(4*tan(x)^4 + 4*tan(x)^2 + 1)

mupad [B] time = 2.70, size = 40, normalized size = 0.54

$$\frac{\frac{\tan(x)}{16} - \frac{\tan(x)^3}{8}}{\tan(x)^4 + \tan(x)^2 + \frac{1}{4}} - x + \frac{7\sqrt{2} \operatorname{atan}(\sqrt{2} \tan(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/cos(x)^2 + tan(x)^2)^3,x)

[Out] (tan(x)/16 - tan(x)^3/8)/(tan(x)^2 + tan(x)^4 + 1/4) - x + (7*2^(1/2)*atan(2^(1/2)*tan(x)))/8

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\tan^2(x) + \sec^2(x))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)**2+tan(x)**2)**3,x)

[Out] Integral((tan(x)**2 + sec(x)**2)**(-3), x)

$$3.487 \quad \int \frac{1}{\sec^2(x) - \tan^2(x)} dx$$

Optimal. Leaf size=1

x

[Out] x

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4381, 8}

x

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2 - Tan[x]^2)^(-1), x]

[Out] x

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 4381

Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.)*(x_)]^2)^p, x_Symbol] := Dist[(a + c)^p, Int[ActivateTrig[u], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]

Rubi steps

$$\int \frac{1}{\sec^2(x) - \tan^2(x)} dx = \int 1 dx = x$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

x

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2 - Tan[x]^2)^(-1), x]

[Out] x

fricas [A] time = 0.76, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2-tan(x)^2), x, algorithm="fricas")

[Out] x

giac [A] time = 0.14, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2-tan(x)^2),x, algorithm="giac")

[Out] x

maple [C] time = 0.06, size = 4, normalized size = 4.00

arctan(tan(x))

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sec(x)^2-tan(x)^2),x)

[Out] arctan(tan(x))

maxima [A] time = 0.43, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2-tan(x)^2),x, algorithm="maxima")

[Out] x

mupad [B] time = 2.75, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/cos(x)^2 - tan(x)^2),x)

[Out] x

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\tan(x) + \sec(x))(\tan(x) + \sec(x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)**2-tan(x)**2),x)

[Out] Integral(1/((-tan(x) + sec(x))*(tan(x) + sec(x))), x)

$$3.488 \quad \int \frac{1}{(\sec^2(x) - \tan^2(x))^2} dx$$

Optimal. Leaf size=1

x

[Out] x

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4381, 8}

x

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2 - Tan[x]^2)^(-2), x]

[Out] x

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 4381

Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.)*(x_)]^2)^(p_.), x_Symbol] := Dist[(a + c)^p, Int[ActivateTrig[u], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]

Rubi steps

$$\int \frac{1}{(\sec^2(x) - \tan^2(x))^2} dx = \int 1 dx = x$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

x

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2 - Tan[x]^2)^(-2), x]

[Out] x

fricas [A] time = 0.84, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2-tan(x)^2)^2,x, algorithm="fricas")

[Out] x

giac [A] time = 0.15, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2-tan(x)^2)^2,x, algorithm="giac")

[Out] x

maple [C] time = 0.06, size = 4, normalized size = 4.00

arctan(tan(x))

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sec(x)^2-tan(x)^2)^2,x)

[Out] arctan(tan(x))

maxima [A] time = 0.41, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2-tan(x)^2)^2,x, algorithm="maxima")

[Out] x

mupad [B] time = 2.58, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/cos(x)^2 - tan(x)^2)^2,x)

[Out] x

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\tan(x) + \sec(x))^2 (\tan(x) + \sec(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)**2-tan(x)**2)**2,x)

[Out] Integral(1/((-tan(x) + sec(x))**2*(tan(x) + sec(x))**2), x)

$$3.489 \quad \int \frac{1}{(\sec^2(x) - \tan^2(x))^3} dx$$

Optimal. Leaf size=1

x

[Out] x

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4381, 8}

x

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2 - Tan[x]^2)^(-3), x]

[Out] x

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 4381

Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.)*(x_)]^2)^p, x_Symbol] := Dist[(a + c)^p, Int[ActivateTrig[u], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]

Rubi steps

$$\int \frac{1}{(\sec^2(x) - \tan^2(x))^3} dx = \int 1 dx = x$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

x

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2 - Tan[x]^2)^(-3), x]

[Out] x

fricas [A] time = 1.53, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2-tan(x)^2)^3,x, algorithm="fricas")

[Out] x

giac [A] time = 0.14, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2-tan(x)^2)^3,x, algorithm="giac")

[Out] x

maple [C] time = 0.07, size = 4, normalized size = 4.00

arctan(tan(x))

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sec(x)^2-tan(x)^2)^3,x)

[Out] arctan(tan(x))

maxima [A] time = 0.41, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)^2-tan(x)^2)^3,x, algorithm="maxima")

[Out] x

mupad [B] time = 2.57, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/cos(x)^2 - tan(x)^2)^3,x)

[Out] x

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\tan(x) + \sec(x))^3 (\tan(x) + \sec(x))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)**2-tan(x)**2)**3,x)

[Out] Integral(1/((-tan(x) + sec(x))**3*(tan(x) + sec(x))**3), x)

$$3.490 \quad \int \frac{1}{\cot^2(x) + \csc^2(x)} dx$$

Optimal. Leaf size=37

$$\sqrt{2}x - x - \sqrt{2} \tan^{-1} \left(\frac{\sin(x) \cos(x)}{\cos^2(x) + \sqrt{2} + 1} \right)$$

[Out] $-x + x \cdot 2^{1/2} - \arctan(\cos(x) \cdot \sin(x) / (1 + \cos(x)^2 + 2^{1/2})) \cdot 2^{1/2}$

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1130, 203}

$$\sqrt{2}x - x - \sqrt{2} \tan^{-1} \left(\frac{\sin(x) \cos(x)}{\cos^2(x) + \sqrt{2} + 1} \right)$$

Antiderivative was successfully verified.

[In] Int[(Cot[x]^2 + Csc[x]^2)^(-1), x]

[Out] $-x + \text{Sqrt}[2] \cdot x - \text{Sqrt}[2] \cdot \text{ArcTan}[(\text{Cos}[x] \cdot \text{Sin}[x]) / (1 + \text{Sqrt}[2] + \text{Cos}[x]^2)]$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1130

Int[((d_.)*(x_)^(m_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 - q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, 2]

Rubi steps

$$\begin{aligned} \int \frac{1}{\cot^2(x) + \csc^2(x)} dx &= \text{Subst} \left(\int \frac{x^2}{2 + 3x^2 + x^4} dx, x, \tan(x) \right) \\ &= 2 \text{Subst} \left(\int \frac{1}{2 + x^2} dx, x, \tan(x) \right) - \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \tan(x) \right) \\ &= -x + \sqrt{2}x - \sqrt{2} \tan^{-1} \left(\frac{\cos(x) \sin(x)}{1 + \sqrt{2} + \cos^2(x)} \right) \end{aligned}$$

Mathematica [A] time = 0.04, size = 19, normalized size = 0.51

$$\sqrt{2} \tan^{-1} \left(\frac{\tan(x)}{\sqrt{2}} \right) - x$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x]^2 + Csc[x]^2)^(-1), x]

[Out] $-x + \text{Sqrt}[2] \cdot \text{ArcTan}[\text{Tan}[x] / \text{Sqrt}[2]]$

fricas [A] time = 1.32, size = 35, normalized size = 0.95

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2+csc(x)^2),x, algorithm="fricas")

[Out] -1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) - x

giac [A] time = 0.14, size = 49, normalized size = 1.32

$$\sqrt{2}\left(x + \arctan\left(-\frac{\sqrt{2}\sin(2x) - \sin(2x)}{\sqrt{2}\cos(2x) + \sqrt{2} - \cos(2x) + 1}\right)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2+csc(x)^2),x, algorithm="giac")

[Out] sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - cos(2*x) + 1))) - x

maple [A] time = 0.16, size = 17, normalized size = 0.46

$$\sqrt{2}\arctan\left(\frac{\sqrt{2}\tan(x)}{2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x)^2+csc(x)^2),x)

[Out] 2^(1/2)*arctan(1/2*2^(1/2)*tan(x))-x

maxima [A] time = 0.41, size = 16, normalized size = 0.43

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2+csc(x)^2),x, algorithm="maxima")

[Out] sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - x

mupad [B] time = 2.71, size = 16, normalized size = 0.43

$$\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x)^2 + 1/sin(x)^2),x)

[Out] 2^(1/2)*atan((2^(1/2)*tan(x))/2) - x

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cot^2(x) + \csc^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)**2+csc(x)**2),x)

[Out] Integral(1/(cot(x)**2 + csc(x)**2), x)

$$3.491 \quad \int \frac{1}{(\cot^2(x) + \csc^2(x))^2} dx$$

Optimal. Leaf size=47

$$-\frac{x}{\sqrt{2}} + x - \frac{\tan(x)}{\tan^2(x) + 2} + \frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x) + \sqrt{2} + 1}\right)}{\sqrt{2}}$$

[Out] $x - 1/2 * x * 2^{(1/2)} + 1/2 * \arctan(\cos(x) * \sin(x) / (1 + \cos(x)^2 + 2^{(1/2)})) * 2^{(1/2)} - \tan(x) / (2 + \tan(x)^2)$

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {470, 12, 391, 203}

$$-\frac{x}{\sqrt{2}} + x - \frac{\tan(x)}{\tan^2(x) + 2} + \frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x) + \sqrt{2} + 1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(Cot[x]^2 + Csc[x]^2)^(-2), x]

[Out] $x - x/\text{Sqrt}[2] + \text{ArcTan}[(\text{Cos}[x] * \text{Sin}[x]) / (1 + \text{Sqrt}[2] + \text{Cos}[x]^2)] / \text{Sqrt}[2] - \text{Tan}[x] / (2 + \text{Tan}[x]^2)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 391

Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]

Rule 470

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(\cot^2(x) + \csc^2(x))^2} dx &= \text{Subst} \left(\int \frac{x^4}{(1+x^2)(2+x^2)^2} dx, x, \tan(x) \right) \\
&= -\frac{\tan(x)}{2 + \tan^2(x)} + \frac{1}{2} \text{Subst} \left(\int \frac{2}{(1+x^2)(2+x^2)} dx, x, \tan(x) \right) \\
&= -\frac{\tan(x)}{2 + \tan^2(x)} + \text{Subst} \left(\int \frac{1}{(1+x^2)(2+x^2)} dx, x, \tan(x) \right) \\
&= -\frac{\tan(x)}{2 + \tan^2(x)} + \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right) - \text{Subst} \left(\int \frac{1}{2+x^2} dx, x, \tan(x) \right) \\
&= x - \frac{x}{\sqrt{2}} + \frac{\tan^{-1} \left(\frac{\cos(x) \sin(x)}{1 + \sqrt{2} + \cos^2(x)} \right)}{\sqrt{2}} - \frac{\tan(x)}{2 + \tan^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 64, normalized size = 1.36

$$\frac{(\cos(2x) + 3) \csc^4(x) \left(6x - 2 \sin(2x) + 2x \cos(2x) - \sqrt{2} (\cos(2x) + 3) \tan^{-1} \left(\frac{\tan(x)}{\sqrt{2}} \right) \right)}{8 (\cot^2(x) + \csc^2(x))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x]^2 + Csc[x]^2)^(-2), x]

[Out] ((3 + Cos[2*x])*Csc[x]^4*(6*x + 2*x*Cos[2*x] - Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]])*(3 + Cos[2*x]) - 2*Sin[2*x])/(8*(Cot[x]^2 + Csc[x]^2)^2)

fricas [A] time = 1.07, size = 66, normalized size = 1.40

$$\frac{4x \cos(x)^2 + (\sqrt{2} \cos(x)^2 + \sqrt{2}) \arctan \left(\frac{3\sqrt{2} \cos(x)^2 - \sqrt{2}}{4 \cos(x) \sin(x)} \right) - 4 \cos(x) \sin(x) + 4x}{4(\cos(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2+csc(x)^2)^2,x, algorithm="fricas")

[Out] 1/4*(4*x*cos(x)^2 + (sqrt(2)*cos(x)^2 + sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) - 4*cos(x)*sin(x) + 4*x)/(cos(x)^2 + 1)

giac [A] time = 0.14, size = 60, normalized size = 1.28

$$-\frac{1}{2} \sqrt{2} \left(x + \arctan \left(-\frac{\sqrt{2} \sin(2x) - \sin(2x)}{\sqrt{2} \cos(2x) + \sqrt{2} - \cos(2x) + 1} \right) \right) + x - \frac{\tan(x)}{\tan(x)^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2+csc(x)^2)^2,x, algorithm="giac")

[Out] -1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - cos(2*x) + 1))) + x - tan(x)/(tan(x)^2 + 2)

maple [A] time = 0.18, size = 28, normalized size = 0.60

$$-\frac{\tan(x)}{2 + \tan^2(x)} - \frac{\sqrt{2} \arctan \left(\frac{\sqrt{2} \tan(x)}{2} \right)}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x)^2+csc(x)^2)^2,x)

[Out] -tan(x)/(2+tan(x)^2)-1/2*2^(1/2)*arctan(1/2*2^(1/2)*tan(x))+x

maxima [A] time = 0.40, size = 27, normalized size = 0.57

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)+x-\frac{\tan(x)}{\tan(x)^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2+csc(x)^2)^2,x, algorithm="maxima")

[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) + x - tan(x)/(tan(x)^2 + 2)

mupad [B] time = 2.69, size = 27, normalized size = 0.57

$$x - \frac{\tan(x)}{\tan(x)^2 + 2} - \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x)^2 + 1/sin(x)^2)^2,x)

[Out] x - tan(x)/(tan(x)^2 + 2) - (2^(1/2)*atan((2^(1/2)*tan(x))/2))/2

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\cot^2(x) + \csc^2(x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)**2+csc(x)**2)**2,x)

[Out] Integral((cot(x)**2 + csc(x)**2)**(-2), x)

$$3.492 \quad \int \frac{1}{(\cot^2(x) + \csc^2(x))^3} dx$$

Optimal. Leaf size=72

$$\frac{7x}{4\sqrt{2}} - x + \frac{\tan(x)}{4(\tan^2(x) + 2)} - \frac{\tan^3(x)}{2(\tan^2(x) + 2)^2} - \frac{7 \tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x) + \sqrt{2} + 1}\right)}{4\sqrt{2}}$$

[Out] $-x + 7/8 * x * 2^{(1/2)} - 7/8 * \arctan(\cos(x) * \sin(x) / (1 + \cos(x)^2 + 2^{(1/2)})) * 2^{(1/2)} - 1/2 * \tan(x)^3 / (2 + \tan(x)^2)^2 + 1/4 * \tan(x) / (2 + \tan(x)^2)$

Rubi [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {470, 578, 522, 203}

$$\frac{7x}{4\sqrt{2}} - x - \frac{\tan^3(x)}{2(\tan^2(x) + 2)^2} + \frac{\tan(x)}{4(\tan^2(x) + 2)} - \frac{7 \tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x) + \sqrt{2} + 1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(Cot[x]^2 + Csc[x]^2)^(-3), x]

[Out] $-x + (7*x)/(4*\text{Sqrt}[2]) - (7*\text{ArcTan}[(\text{Cos}[x]*\text{Sin}[x])/(1 + \text{Sqrt}[2] + \text{Cos}[x]^2)])/(4*\text{Sqrt}[2]) - \text{Tan}[x]^3/(2*(2 + \text{Tan}[x]^2)^2) + \text{Tan}[x]/(4*(2 + \text{Tan}[x]^2))$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 578

Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(g^(n - 1)*(b*e - a*f)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f))*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x] /; FreeQ[{a, b,

c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(\cot^2(x) + \csc^2(x))^3} dx &= \text{Subst} \left(\int \frac{x^6}{(1+x^2)(2+x^2)^3} dx, x, \tan(x) \right) \\
 &= -\frac{\tan^3(x)}{2(2+\tan^2(x))^2} + \frac{1}{4} \text{Subst} \left(\int \frac{x^2(6+2x^2)}{(1+x^2)(2+x^2)^2} dx, x, \tan(x) \right) \\
 &= -\frac{\tan^3(x)}{2(2+\tan^2(x))^2} + \frac{\tan(x)}{4(2+\tan^2(x))} - \frac{1}{8} \text{Subst} \left(\int \frac{2-6x^2}{(1+x^2)(2+x^2)} dx, x, \tan(x) \right) \\
 &= -\frac{\tan^3(x)}{2(2+\tan^2(x))^2} + \frac{\tan(x)}{4(2+\tan^2(x))} + \frac{7}{4} \text{Subst} \left(\int \frac{1}{2+x^2} dx, x, \tan(x) \right) - \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right) \\
 &= -x + \frac{7x}{4\sqrt{2}} - \frac{7 \tan^{-1} \left(\frac{\cos(x) \sin(x)}{1+\sqrt{2}+\cos^2(x)} \right)}{4\sqrt{2}} - \frac{\tan^3(x)}{2(2+\tan^2(x))^2} + \frac{\tan(x)}{4(2+\tan^2(x))}
 \end{aligned}$$

Mathematica [A] time = 0.16, size = 66, normalized size = 0.92

$$\frac{-76x + 2 \sin(2x) + 3 \sin(4x) - 48x \cos(2x) - 4x \cos(4x) + 7\sqrt{2} (\cos(2x) + 3)^2 \tan^{-1} \left(\frac{\tan(x)}{\sqrt{2}} \right)}{8(\cos(2x) + 3)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x]^2 + Csc[x]^2)^(-3), x]

[Out] (-76*x - 48*x*Cos[2*x] + 7*Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]]*(3 + Cos[2*x])^2 - 4*x*Cos[4*x] + 2*Sin[2*x] + 3*Sin[4*x])/(8*(3 + Cos[2*x])^2)

fricas [A] time = 1.62, size = 98, normalized size = 1.36

$$\frac{16x \cos(x)^4 + 32x \cos(x)^2 + 7(\sqrt{2} \cos(x)^4 + 2\sqrt{2} \cos(x)^2 + \sqrt{2}) \arctan \left(\frac{3\sqrt{2} \cos(x)^2 - \sqrt{2}}{4 \cos(x) \sin(x)} \right) - 4(3 \cos(x)^3 - \cos(x)) \sin(x) + 16x}{16(\cos(x)^4 + 2 \cos(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2+csc(x)^2)^3,x, algorithm="fricas")

[Out] -1/16*(16*x*cos(x)^4 + 32*x*cos(x)^2 + 7*(sqrt(2)*cos(x)^4 + 2*sqrt(2)*cos(x)^2 + sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) - 4*(3*cos(x)^3 - cos(x))*sin(x) + 16*x)/(cos(x)^4 + 2*cos(x)^2 + 1)

giac [A] time = 0.13, size = 69, normalized size = 0.96

$$\frac{7}{8} \sqrt{2} \left(x + \arctan \left(-\frac{\sqrt{2} \sin(2x) - \sin(2x)}{\sqrt{2} \cos(2x) + \sqrt{2} - \cos(2x) + 1} \right) \right) - x - \frac{\tan(x)^3 - 2 \tan(x)}{4(\tan(x)^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2+csc(x)^2)^3,x, algorithm="giac")

[Out] $7/8*\sqrt{2}*(x + \arctan(-(\sqrt{2}*\sin(2*x) - \sin(2*x))/(\sqrt{2}*\cos(2*x) + \sqrt{2} - \cos(2*x) + 1))) - x - 1/4*(\tan(x)^3 - 2*\tan(x))/(\tan(x)^2 + 2)^2$

maple [A] time = 0.22, size = 39, normalized size = 0.54

$$\frac{-\frac{(\tan^3(x))}{4} + \frac{\tan(x)}{2}}{(2 + \tan^2(x))^2} + \frac{7\sqrt{2} \arctan\left(\frac{\sqrt{2} \tan(x)}{2}\right)}{8} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cot(x)^2+csc(x)^2)^3,x)`

[Out] $2*(-1/8*\tan(x)^3+1/4*\tan(x))/(2+\tan(x)^2)^2+7/8*2^{(1/2)}*\arctan(1/2*2^{(1/2)}*\tan(x))-x$

maxima [A] time = 0.42, size = 42, normalized size = 0.58

$$\frac{7}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \tan(x)\right) - x - \frac{\tan(x)^3 - 2 \tan(x)}{4(\tan(x)^4 + 4 \tan(x)^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(cot(x)^2+csc(x)^2)^3,x, algorithm="maxima")`

[Out] $7/8*\sqrt{2}*\arctan(1/2*\sqrt{2}*\tan(x)) - x - 1/4*(\tan(x)^3 - 2*\tan(x))/(\tan(x)^4 + 4*\tan(x)^2 + 4)$

mupad [B] time = 2.68, size = 43, normalized size = 0.60

$$\frac{\frac{\tan(x)}{2} - \frac{\tan(x)^3}{4}}{\tan(x)^4 + 4 \tan(x)^2 + 4} - x + \frac{7 \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} \tan(x)}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cot(x)^2 + 1/sin(x)^2)^3,x)`

[Out] $(\tan(x)/2 - \tan(x)^3/4)/(4*\tan(x)^2 + \tan(x)^4 + 4) - x + (7*2^{(1/2)}*\operatorname{atan}((2^{(1/2)}*\tan(x))/2))/8$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(cot(x)**2+csc(x)**2)**3,x)`

[Out] Timed out

$$3.493 \quad \int \frac{1}{\cot^2(x) - \csc^2(x)} dx$$

Optimal. Leaf size=3

$-x$

[Out] $-x$

Rubi [A] time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4382, 8}

$-x$

Antiderivative was successfully verified.

[In] Int[(Cot[x]^2 - Csc[x]^2)^(-1), x]

[Out] $-x$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 4382

Int[((a_.) + cot[(d_.) + (e_.)*(x_)])^2*(b_.) + csc[(d_.) + (e_.)*(x_)])^2*(c_.))^(-1)*(u_.), x_Symbol] := Dist[(a + c)^p, Int[ActivateTrig[u], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]

Rubi steps

$$\int \frac{1}{\cot^2(x) - \csc^2(x)} dx = - \int 1 dx = -x$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$-x$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x]^2 - Csc[x]^2)^(-1), x]

[Out] $-x$

fricas [A] time = 1.12, size = 3, normalized size = 1.00

$-x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2-csc(x)^2), x, algorithm="fricas")

[Out] $-x$

giac [A] time = 0.15, size = 3, normalized size = 1.00

$-x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2-csc(x)^2),x, algorithm="giac")

[Out] -x

maple [C] time = 0.06, size = 6, normalized size = 2.00

- arctan(tan(x))

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x)^2-csc(x)^2),x)

[Out] -arctan(tan(x))

maxima [A] time = 0.42, size = 3, normalized size = 1.00

-x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2-csc(x)^2),x, algorithm="maxima")

[Out] -x

mupad [B] time = 2.73, size = 3, normalized size = 1.00

-x

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x)^2 - 1/sin(x)^2),x)

[Out] -x

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\cot(x) - \csc(x))(\cot(x) + \csc(x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)**2-csc(x)**2),x)

[Out] Integral(1/((cot(x) - csc(x))*(cot(x) + csc(x))), x)

$$3.494 \quad \int \frac{1}{(\cot^2(x) - \csc^2(x))^2} dx$$

Optimal. Leaf size=1

x

[Out] x

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4382, 8}

x

Antiderivative was successfully verified.

[In] Int[(Cot[x]^2 - Csc[x]^2)^(-2), x]

[Out] x

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 4382

Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c_.))^p*(u_.), x_Symbol] := Dist[(a + c)^p, Int[ActivateTrig[u], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]

Rubi steps

$$\int \frac{1}{(\cot^2(x) - \csc^2(x))^2} dx = \int 1 dx = x$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

x

Antiderivative was successfully verified.

[In] Integrate[(Cot[x]^2 - Csc[x]^2)^(-2), x]

[Out] x

fricas [A] time = 0.80, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2-csc(x)^2)^2,x, algorithm="fricas")

[Out] x

giac [A] time = 0.15, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2-csc(x)^2)^2,x, algorithm="giac")

[Out] x

maple [C] time = 0.06, size = 4, normalized size = 4.00

arctan(tan(x))

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x)^2-csc(x)^2)^2,x)

[Out] arctan(tan(x))

maxima [A] time = 0.42, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2-csc(x)^2)^2,x, algorithm="maxima")

[Out] x

mupad [B] time = 2.65, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x)^2 - 1/sin(x)^2)^2,x)

[Out] x

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)**2-csc(x)**2)**2,x)

[Out] Timed out

$$3.495 \quad \int \frac{1}{(\cot^2(x) - \csc^2(x))^3} dx$$

Optimal. Leaf size=3

$-x$

[Out] $-x$

Rubi [A] time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4382, 8}

$-x$

Antiderivative was successfully verified.

[In] Int[(Cot[x]^2 - Csc[x]^2)^(-3), x]

[Out] $-x$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 4382

Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c_.))^p*(u_.), x_Symbol] := Dist[(a + c)^p, Int[ActivateTrig[u], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]

Rubi steps

$$\int \frac{1}{(\cot^2(x) - \csc^2(x))^3} dx = - \int 1 dx = -x$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$-x$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x]^2 - Csc[x]^2)^(-3), x]

[Out] $-x$

fricas [A] time = 0.50, size = 3, normalized size = 1.00

$-x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2-csc(x)^2)^3,x, algorithm="fricas")

[Out] $-x$

giac [A] time = 0.14, size = 3, normalized size = 1.00

$-x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2-csc(x)^2)^3,x, algorithm="giac")

[Out] -x

maple [C] time = 0.07, size = 6, normalized size = 2.00

- arctan(tan(x))

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x)^2-csc(x)^2)^3,x)

[Out] -arctan(tan(x))

maxima [A] time = 0.41, size = 3, normalized size = 1.00

-x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2-csc(x)^2)^3,x, algorithm="maxima")

[Out] -x

mupad [B] time = 2.62, size = 3, normalized size = 1.00

-x

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x)^2 - 1/sin(x)^2)^3,x)

[Out] -x

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)**2-csc(x)**2)**3,x)

[Out] Timed out

$$3.496 \quad \int \frac{1}{a+b \cos^2(x)+c \sin^2(x)} dx$$

Optimal. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+c} \tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b} \sqrt{a+c}}$$

[Out] arctan((a+c)^(1/2)*tan(x)/(a+b)^(1/2))/(a+b)^(1/2)/(a+c)^(1/2)

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+c} \tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b} \sqrt{a+c}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[x]^2 + c*Sin[x]^2)^(-1), x]

[Out] ArcTan[(Sqrt[a + c]*Tan[x])/Sqrt[a + b]]/(Sqrt[a + b]*Sqrt[a + c])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{1}{a+b \cos^2(x)+c \sin^2(x)} dx &= \text{Subst}\left(\int \frac{1}{a+b+(a+c)x^2} dx, x, \tan(x)\right) \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a+c} \tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b} \sqrt{a+c}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 33, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+c} \tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b} \sqrt{a+c}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[x]^2 + c*Sin[x]^2)^(-1), x]

[Out] ArcTan[(Sqrt[a + c]*Tan[x])/Sqrt[a + b]]/(Sqrt[a + b]*Sqrt[a + c])

fricas [B] time = 0.96, size = 259, normalized size = 7.85

$$\left[\frac{\sqrt{-a^2 - ab - (a+b)c} \log\left(\frac{(8a^2+8ab+b^2+2(4a+3b)c+c^2)\cos(x)^4-2(4a^2+3ab+(5a+3b)c+c^2)\cos(x)^2+4((2a+b+c)\cos(x)^3-(a+c)\cos(x))}{(b^2-2bc+c^2)\cos(x)^4+2(ab-(a-b)c-c^2)\cos(x)^2+a^2+2ac+c^2}\right)}{4(a^2+ab+(a+b)c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(x)^2+c*sin(x)^2), x, algorithm="fricas")

[Out] $[-1/4*\sqrt{-a^2 - a*b - (a + b)*c}*\log(((8*a^2 + 8*a*b + b^2 + 2*(4*a + 3*b)*c + c^2)*\cos(x)^4 - 2*(4*a^2 + 3*a*b + (5*a + 3*b)*c + c^2)*\cos(x)^2 + 4*((2*a + b + c)*\cos(x)^3 - (a + c)*\cos(x))*\sqrt{-a^2 - a*b - (a + b)*c}*\sin(x) + a^2 + 2*a*c + c^2)/((b^2 - 2*b*c + c^2)*\cos(x)^4 + 2*(a*b - (a - b)*c - c^2)*\cos(x)^2 + a^2 + 2*a*c + c^2))/(a^2 + a*b + (a + b)*c), -1/2*\arctan(1/2*((2*a + b + c)*\cos(x)^2 - a - c)/(\sqrt{a^2 + a*b + (a + b)*c}*\cos(x)*\sin(x)))/\sqrt{a^2 + a*b + (a + b)*c}]$

giac [B] time = 0.14, size = 61, normalized size = 1.85

$$\frac{\pi \left[\frac{x}{\pi} + \frac{1}{2} \right] \operatorname{sgn}(2a + 2c) + \arctan\left(\frac{a \tan(x) + c \tan(x)}{\sqrt{a^2 + ab + ac + bc}}\right)}{\sqrt{a^2 + ab + ac + bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="giac")`

[Out] $(\pi*\operatorname{floor}(x/\pi + 1/2)*\operatorname{sgn}(2*a + 2*c) + \arctan((a*\tan(x) + c*\tan(x))/\sqrt{a^2 + a*b + a*c + b*c}))/\sqrt{a^2 + a*b + a*c + b*c}$

maple [A] time = 0.12, size = 27, normalized size = 0.82

$$\frac{\arctan\left(\frac{(a+c)\tan(x)}{\sqrt{(a+b)(a+c)}}\right)}{\sqrt{(a+b)(a+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*cos(x)^2+c*sin(x)^2),x)`

[Out] $1/((a+b)*(a+c))^{(1/2)}*\arctan((a+c)*\tan(x)/((a+b)*(a+c))^{(1/2)})$

maxima [A] time = 0.43, size = 26, normalized size = 0.79

$$\frac{\arctan\left(\frac{(a+c)\tan(x)}{\sqrt{(a+b)(a+c)}}\right)}{\sqrt{(a+b)(a+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="maxima")`

[Out] $\arctan((a + c)*\tan(x)/\sqrt{(a + b)*(a + c)})/\sqrt{(a + b)*(a + c)}$

mupad [B] time = 2.85, size = 43, normalized size = 1.30

$$\frac{\operatorname{atan}\left(\frac{\tan(x)(2a+2c)}{2\sqrt{ab+ac+bc+a^2}}\right)}{\sqrt{ab+ac+bc+a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + c*sin(x)^2 + b*cos(x)^2),x)`

[Out] $\operatorname{atan}((\tan(x)*(2*a + 2*c))/(2*(a*b + a*c + b*c + a^2)^{(1/2)}))/\sqrt{a*b + a*c + b*c + a^2}^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \cos^2(x) + c \sin^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(x)**2+c*sin(x)**2),x)`

[Out] `Integral(1/(a + b*cos(x)**2 + c*sin(x)**2), x)`

$$3.497 \quad \int \frac{x}{a+b \cos^2(x)+c \sin^2(x)} dx$$

Optimal. Leaf size=239

$$\frac{\operatorname{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{4\sqrt{a+b}\sqrt{a+c}} + \frac{\operatorname{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{4\sqrt{a+b}\sqrt{a+c}} - \frac{ix \log\left(1 + \frac{e^{2ix(b-c)}}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix \log\left(1 + \frac{e^{2ix(b-c)}}{2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\sqrt{a+c}}$$

[Out] $-1/2*I*x*\ln(1+(b-c)*\exp(2*I*x)/(2*a+b+c-2*(a+b)^{(1/2)*(a+c)^{(1/2)}))/((a+b)^{(1/2)/(a+c)^{(1/2)}+1/2*I*x*\ln(1+(b-c)*\exp(2*I*x)/(2*a+b+c+2*(a+b)^{(1/2)*(a+c)^{(1/2)}))/((a+b)^{(1/2)/(a+c)^{(1/2)}-1/4*\operatorname{polylog}(2,-(b-c)*\exp(2*I*x)/(2*a+b+c-2*(a+b)^{(1/2)*(a+c)^{(1/2)}))/((a+b)^{(1/2)/(a+c)^{(1/2)}+1/4*\operatorname{polylog}(2,-(b-c)*\exp(2*I*x)/(2*a+b+c+2*(a+b)^{(1/2)*(a+c)^{(1/2)}))/((a+b)^{(1/2)/(a+c)^{(1/2)}))$

Rubi [A] time = 0.49, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4587, 3321, 2264, 2190, 2279, 2391}

$$\frac{\operatorname{PolyLog}\left(2, -\frac{e^{2ix(b-c)}}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{4\sqrt{a+b}\sqrt{a+c}} + \frac{\operatorname{PolyLog}\left(2, -\frac{e^{2ix(b-c)}}{2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{4\sqrt{a+b}\sqrt{a+c}} - \frac{ix \log\left(1 + \frac{e^{2ix(b-c)}}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix \log\left(1 + \frac{e^{2ix(b-c)}}{2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\sqrt{a+c}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*Cos[x]^2 + c*Sin[x]^2), x]

[Out] $((-I/2)*x*\operatorname{Log}[1 + ((b - c)*E^{((2*I)*x)})/(2*a + b + c - 2*\operatorname{Sqrt}[a + b]*\operatorname{Sqrt}[a + c]])/(\operatorname{Sqrt}[a + b]*\operatorname{Sqrt}[a + c]) + ((I/2)*x*\operatorname{Log}[1 + ((b - c)*E^{((2*I)*x)})/(2*a + b + c + 2*\operatorname{Sqrt}[a + b]*\operatorname{Sqrt}[a + c]])/(\operatorname{Sqrt}[a + b]*\operatorname{Sqrt}[a + c]) - \operatorname{PolyLog}[2, -(((b - c)*E^{((2*I)*x)})/(2*a + b + c - 2*\operatorname{Sqrt}[a + b]*\operatorname{Sqrt}[a + c]))]/(4*\operatorname{Sqrt}[a + b]*\operatorname{Sqrt}[a + c]) + \operatorname{PolyLog}[2, -(((b - c)*E^{((2*I)*x)})/(2*a + b + c + 2*\operatorname{Sqrt}[a + b]*\operatorname{Sqrt}[a + c]))]/(4*\operatorname{Sqrt}[a + b]*\operatorname{Sqrt}[a + c])$

Rule 2190

Int[(((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2264

Int[(((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)*((F_)^(v_))), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_)*((F_)^(e_)*((c_) + (d_)*(x_)))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_)*((d_) + (e_)*(x_))^(n_)], x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 3321

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4587

```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + Cos[(d_.) + (e_.)*(x_)^2*(b_.) + (c_.)*Sin[(d_.) + (e_.)*(x_)^2]), x_Symbol] := Dist[2, Int[(f + g*x)^m/(2*a + b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c, 0]
```

Rubi steps

$$\int \frac{x}{a + b \cos^2(x) + c \sin^2(x)} dx = 2 \int \frac{x}{2a + b + c + (b - c) \cos(2x)} dx$$

$$= 4 \int \frac{e^{2ix} x}{b - c + 2(2a + b + c)e^{2ix} + (b - c)e^{4ix}} dx$$

$$= \frac{(2(b - c)) \int \frac{e^{2ix} x}{-4\sqrt{a+b}\sqrt{a+c} + 2(2a+b+c) + 2(b-c)e^{2ix}} dx}{\sqrt{a+b}\sqrt{a+c}} - \frac{(2(b - c)) \int \frac{e^{2ix} x}{4\sqrt{a+b}\sqrt{a+c} + 2(2a+b+c) + 2(b-c)e^{2ix}} dx}{\sqrt{a+b}\sqrt{a+c}}$$

$$= -\frac{ix \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{i \int \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right) dx}{2\sqrt{a+b}\sqrt{a+c}}$$

$$= -\frac{ix \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \text{Subst}\left[\int \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right) dx, x, \frac{e^{2ix} - 1}{2}\right]$$

$$= -\frac{ix \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} - \frac{\text{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{4\sqrt{a+b}\sqrt{a+c}} + \frac{\text{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{4\sqrt{a+b}\sqrt{a+c}}$$

Mathematica [B] time = 3.02, size = 507, normalized size = 2.12

$$\tan^{-1}\left(\frac{\sqrt{a+c} \tan(x)}{\sqrt{a+b}}\right) \left(2x + \frac{i \left(-\text{Li}_2\left(\frac{\sqrt{a+b} - i\sqrt{a+c} \tan(x)}{\sqrt{a+b} - \sqrt{a+c}}\right) + \text{Li}_2\left(\frac{\sqrt{a+b} - i\sqrt{a+c} \tan(x)}{\sqrt{a+b} + \sqrt{a+c}}\right) - \text{Li}_2\left(\frac{i\sqrt{a+c} \tan(x) + \sqrt{a+b}}{\sqrt{a+b} - \sqrt{a+c}}\right) + \text{Li}_2\left(\frac{i\sqrt{a+c} \tan(x) + \sqrt{a+b}}{\sqrt{a+b} + \sqrt{a+c}}\right) + \log\left(\frac{\sqrt{a+c} \tan(x)}{\sqrt{a+b}}\right) \right)}{2\sqrt{a+b}\sqrt{a+c}} \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a + b*Cos[x]^2 + c*Sin[x]^2), x]
[Out] (ArcTan[(Sqrt[a + c]*Tan[x])/Sqrt[a + b]]*(2*x + (I*(Log[(Sqrt[a + c]*(1 + I*Tan[x]))/(Sqrt[a + b] + Sqrt[a + c]))*Log[1 - (I*Sqrt[a + c]*Tan[x])/Sqrt[a + b]] - Log[(I*Sqrt[a + c]*(1 + Tan[x]))/(Sqrt[a + b] - Sqrt[a + c]))*Log[1 - (I*Sqrt[a + c]*Tan[x])/Sqrt[a + b]] + Log[(Sqrt[a + c]*(1 - I*Tan[x])]/(Sqrt[a + b] + Sqrt[a + c]))*Log[1 + (I*Sqrt[a + c]*Tan[x])/Sqrt[a + b]] - Log[(Sqrt[a + c]*(1 + I*Tan[x])]/(-Sqrt[a + b] + Sqrt[a + c]))*Log[1 + (I*Sqrt[a + c]*Tan[x])/Sqrt[a + b]] - PolyLog[2, (Sqrt[a + b] - I*Sqrt[a + c]*Tan[x])/(Sqrt[a + b] - Sqrt[a + c])] + PolyLog[2, (Sqrt[a + b] - I*Sqrt[a + c]*Tan[x])/(Sqrt[a + b] + Sqrt[a + c])] - PolyLog[2, (Sqrt[a + b] + I*Sqrt[a + c]*Tan[x])/(Sqrt[a + b] + Sqrt[a + c])] - PolyLog[2, (Sqrt[a + b] + I*Sqrt[a + c]*Tan[x])/(Sqrt[a + b] - Sqrt[a + c])])))/Sqrt[a + b])
```



```
/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) - 2*b + 2*c)/(b - c) + 1) + 4*(b - c)
)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(-1/2*((2*(2*a + b + c)*cos(x) - (-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (I*b -
I*c)*sin(x)))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b
- c)) + 2*b - 2*c)/(b - c) + 1) - 4*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(1/2*((2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2
I*c)*sin(x) + 4*((b - c)*cos(x) + (I*b - I*c)*sin(x)))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/
(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) - 2*b + 2*c)/(b - c) + 1) - 4*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(-1/2*((2*(2
*a + b + c)*cos(x) - (4*I*a + 2*I*b + 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (I*b - I*c)*sin(x)))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(
(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) + 2*b - 2*c)/(b - c) + 1) - 4*(b - c)*sqrt((a^2 + a*b + (a + b)*c
)/(b^2 - 2*b*c + c^2))*dilog(1/2*((2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*sin(x) + 4*((b - c)*cos(x) + (-I*b + I*c)*sin(x)))*sqrt((a^2 + a*b
+ (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) - 2*b + 2*c)/(b - c) + 1
) - 4*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(-1/2*((2*(2*a + b + c)*cos(x) - (-4*I*a - 2*I*b - 2*I*c)*sin(x) + 4*((b - c)*cos
(x) - (-I*b + I*c)*sin(x)))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a
- b - c)/(b - c)) + 2*b - 2*c)/(b - c) + 1))/(a^2 + a*b + (a + b)*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b \cos(x)^2 + c \sin(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="giac")

[Out] integrate(x/(b*cos(x)^2 + c*sin(x)^2 + a), x)

maple [B] time = 0.30, size = 820, normalized size = 3.43

$$\frac{ix \ln\left(1 - \frac{(b-c)e^{2ix}}{2\sqrt{(a+b)(a+c)} - 2a - b - c}\right)}{2\sqrt{(a+b)(a+c)}} - \frac{x^2}{2\sqrt{(a+b)(a+c)}} - \frac{\text{polylog}\left(2, \frac{(b-c)e^{2ix}}{2\sqrt{(a+b)(a+c)} - 2a - b - c}\right)}{4\sqrt{(a+b)(a+c)}} - \frac{i \ln\left(1 - \frac{(b-c)e^{2ix}}{-2\sqrt{(a+b)(a+c)} - 2a - b - c}\right)}{-2\sqrt{(a+b)(a+c)} - 2a - b - c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b*cos(x)^2+c*sin(x)^2),x)

```
[Out] -1/2*I/((a+b)*(a+c))^(1/2)*x*ln(1-(b-c)*exp(2*I*x)/(2*((a+b)*(a+c))^(1/2)-2
*a-b-c))-1/2/((a+b)*(a+c))^(1/2)*x^2-1/4/((a+b)*(a+c))^(1/2)*polylog(2,(b-c
)*exp(2*I*x)/(2*((a+b)*(a+c))^(1/2)-2*a-b-c))-I/(-2*((a+b)*(a+c))^(1/2)-2*a
-b-c)*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*x-I/((a+b)*(a
+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b
)*(a+c))^(1/2)-2*a-b-c))*a*x-1/2*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1
/2)-2*a-b-c)*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*b*x-1/
2*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*ln(1-(b-c)*exp(2*I
*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*c*x-1/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c
)*x^2-1/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*a*x^2-1/2/((a+
b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*b*x^2-1/2/((a+b)*(a+c))^(1
/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*c*x^2-1/2/(-2*((a+b)*(a+c))^(1/2)-2*a-
b-c)*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/2/((a+b
)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*polylog(2,(b-c)*exp(2*I*x)/
```


$(-2*((a+b)*(a+c))^{(1/2)-2*a-b-c})*a-1/4/((a+b)*(a+c))^{(1/2)}/(-2*((a+b)*(a+c))^{(1/2)-2*a-b-c})*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^{(1/2)-2*a-b-c})*b-1/4/((a+b)*(a+c))^{(1/2)}/(-2*((a+b)*(a+c))^{(1/2)-2*a-b-c})*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^{(1/2)-2*a-b-c}))*c$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b \cos(x)^2 + c \sin(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="maxima")

[Out] integrate(x/(b*cos(x)^2 + c*sin(x)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{b \cos(x)^2 + c \sin(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + c*sin(x)^2 + b*cos(x)^2),x)

[Out] int(x/(a + c*sin(x)^2 + b*cos(x)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \cos^2(x) + c \sin^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*cos(x)**2+c*sin(x)**2),x)

[Out] Integral(x/(a + b*cos(x)**2 + c*sin(x)**2), x)

$$3.498 \quad \int \frac{x^2}{a+b \cos^2(x)+c \sin^2(x)} dx$$

Optimal. Leaf size=365

$$\frac{x \operatorname{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{x \operatorname{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} - \frac{i \operatorname{Li}_3\left(-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{4\sqrt{a+b}\sqrt{a+c}} + \frac{i \operatorname{Li}_3\left(-\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{4\sqrt{a+b}\sqrt{a+c}}$$

[Out] $-1/2*I*x^2*\ln(1+(b-c)*\exp(2*I*x)/(2*a+b+c-2*(a+b)^{(1/2)*(a+c)^{(1/2)})))/(a+b)^{(1/2)/(a+c)^{(1/2)}+1/2*I*x^2*\ln(1+(b-c)*\exp(2*I*x)/(2*a+b+c+2*(a+b)^{(1/2)*(a+c)^{(1/2)})))/(a+b)^{(1/2)/(a+c)^{(1/2)}-1/2*x*\operatorname{polylog}(2,-(b-c)*\exp(2*I*x)/(2*a+b+c-2*(a+b)^{(1/2)*(a+c)^{(1/2)})))/(a+b)^{(1/2)/(a+c)^{(1/2)}+1/2*x*\operatorname{polylog}(2,-(b-c)*\exp(2*I*x)/(2*a+b+c+2*(a+b)^{(1/2)*(a+c)^{(1/2)})))/(a+b)^{(1/2)/(a+c)^{(1/2)})-1/4*I*\operatorname{polylog}(3,-(b-c)*\exp(2*I*x)/(2*a+b+c-2*(a+b)^{(1/2)*(a+c)^{(1/2)})))/(a+b)^{(1/2)/(a+c)^{(1/2)}+1/4*I*\operatorname{polylog}(3,-(b-c)*\exp(2*I*x)/(2*a+b+c+2*(a+b)^{(1/2)*(a+c)^{(1/2)})))/(a+b)^{(1/2)/(a+c)^{(1/2)}}$

Rubi [A] time = 0.74, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {4587, 3321, 2264, 2190, 2531, 2282, 6589}

$$\frac{x \operatorname{PolyLog}\left(2, -\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{x \operatorname{PolyLog}\left(2, -\frac{e^{2ix}(b-c)}{2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\sqrt{a+c}} - \frac{i \operatorname{PolyLog}\left(3, -\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{4\sqrt{a+b}\sqrt{a+c}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b*cos[x]^2 + c*sin[x]^2),x]

[Out] $((-1/2)*x^2*\log[1 + ((b - c)*E^{((2*I)*x)})/(2*a + b + c - 2*\sqrt{a + b}*\sqrt{a + c})])/(sqrt[a + b]*sqrt[a + c]) + ((1/2)*x^2*\log[1 + ((b - c)*E^{((2*I)*x)})/(2*a + b + c + 2*\sqrt{a + b}*\sqrt{a + c})])/(sqrt[a + b]*sqrt[a + c]) - (x*\operatorname{PolyLog}[2, -(((b - c)*E^{((2*I)*x)})/(2*a + b + c - 2*\sqrt{a + b}*\sqrt{a + c})])/(2*\sqrt{a + b}*\sqrt{a + c}) + (x*\operatorname{PolyLog}[2, -(((b - c)*E^{((2*I)*x)})/(2*a + b + c + 2*\sqrt{a + b}*\sqrt{a + c})])/(2*\sqrt{a + b}*\sqrt{a + c}) - ((1/4)*\operatorname{PolyLog}[3, -(((b - c)*E^{((2*I)*x)})/(2*a + b + c - 2*\sqrt{a + b}*\sqrt{a + c})])/(sqrt[a + b]*sqrt[a + c]) + ((1/4)*\operatorname{PolyLog}[3, -(((b - c)*E^{((2*I)*x)})/(2*a + b + c + 2*\sqrt{a + b}*\sqrt{a + c})])/(sqrt[a + b]*sqrt[a + c])$

Rule 2190

Int[(((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[(((c + d*x)^m*log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*log[F]), x] - Dist[(d*m)/(b*f*g*n*log[F]), Int[(c + d*x)^(m - 1)*log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2264

Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi

```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*x_)))^(n_)]*((f_) + (g_)
*(x_)^(m_)), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3321

```
Int[((c_) + (d_)*(x_)^(m_))/((a_) + (b_)*sin[(e_) + Pi*(k_) + (f_)*(
x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(I*Pi*(k - 1/2))*E^(I*(e + f
*x)))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4587

```
Int[((f_) + (g_)*(x_)^(m_))/((a_) + Cos[(d_) + (e_)*(x_)]^2*(b_) + (
c_)*Sin[(d_) + (e_)*(x_)]^2), x_Symbol] := Dist[2, Int[(f + g*x)^m/(2*a
+ b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a + b \cos^2(x) + c \sin^2(x)} dx &= 2 \int \frac{x^2}{2a + b + c + (b - c) \cos(2x)} dx \\
&= 4 \int \frac{e^{2ix} x^2}{b - c + 2(2a + b + c)e^{2ix} + (b - c)e^{4ix}} dx \\
&= \frac{(2(b - c)) \int \frac{e^{2ix} x^2}{-4\sqrt{a+b}\sqrt{a+c} + 2(2a+b+c) + 2(b-c)e^{2ix}} dx}{\sqrt{a+b}\sqrt{a+c}} - \frac{(2(b - c)) \int \frac{e^{2ix} x^2}{4\sqrt{a+b}\sqrt{a+c} + 2(2a+b+c) + 2(b-c)e^{2ix}} dx}{\sqrt{a+b}\sqrt{a+c}} \\
&= -\frac{ix^2 \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix^2 \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{i \int x \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right) dx}{2\sqrt{a+b}\sqrt{a+c}} \\
&= -\frac{ix^2 \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix^2 \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} - \frac{x \operatorname{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} \\
&= -\frac{ix^2 \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix^2 \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} - \frac{x \operatorname{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} \\
&= -\frac{ix^2 \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix^2 \log\left(1 + \frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} - \frac{x \operatorname{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}}
\end{aligned}$$

Mathematica [A] time = 3.79, size = 258, normalized size = 0.71

$$\frac{i \left(-2ix \operatorname{Li}_2\left(\frac{(c-b)e^{2ix}}{2a+b+c-2\sqrt{(a+b)(a+c)}}\right) + 2ix \operatorname{Li}_2\left(\frac{(c-b)e^{2ix}}{2a+b+c+2\sqrt{(a+b)(a+c)}}\right) + \operatorname{Li}_3\left(\frac{(c-b)e^{2ix}}{2a+b+c-2\sqrt{(a+b)(a+c)}}\right) - \operatorname{Li}_3\left(\frac{(c-b)e^{2ix}}{2a+b+c+2\sqrt{(a+b)(a+c)}}\right) \right)}{4\sqrt{(a+b)(a+c)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b*Cos[x]^2 + c*Sin[x]^2), x]

[Out] ((-1/4*I)*(2*x^2*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[(a + b)*(a + c)]] - 2*x^2*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c + 2*Sqrt[(a + b)*(a + c)]] - (2*I)*x*PolyLog[2, ((-b + c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[(a + b)*(a + c)]] + (2*I)*x*PolyLog[2, ((-b + c)*E^((2*I)*x))/(2*a + b + c + 2*Sqrt[(a + b)*(a + c)]] + PolyLog[3, ((-b + c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[(a + b)*(a + c)]] - PolyLog[3, ((-b + c)*E^((2*I)*x))/(2*a + b + c + 2*Sqrt[(a + b)*(a + c)]]))/Sqrt[(a + b)*(a + c)])

fricas [C] time = 2.22, size = 4357, normalized size = 11.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*cos(x)^2+c*sin(x)^2), x, algorithm="fricas")

[Out] 1/16*(4*I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(-1/2*((2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c) - 2*b + 2*c)/(b - c) - 4*I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(1/2*((2*(2*a + b + c)*cos(x) - (4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c) - 2*b + 2*c)/(b - c)


```

b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b
- c)) - 2*b + 2*c)/(b - c) + 1) - 8*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/
(b^2 - 2*b*c + c^2))*dilog(-1/2*((2*(2*a + b + c)*cos(x) - (-4*I*a - 2*I*b
- 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b
+ (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b
)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) + 2*b - 2*c)/(b - c) + 1)
+ 4*(2*I*b - 2*I*c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*poly
log(3, -1/2*(2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((
b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*
b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c
^2)) + 2*a + b + c)/(b - c))/(b - c)) + 4*(-2*I*b + 2*I*c)*sqrt((a^2 + a*b
+ (a + b)*c)/(b^2 - 2*b*c + c^2))*polylog(3, 1/2*(2*(2*a + b + c)*cos(x) -
(4*I*a + 2*I*b + 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*s
qrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^
2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c))/(b - c))
+ 4*(-2*I*b + 2*I*c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*poly
log(3, -1/2*(2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*((
b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*
b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c
^2)) + 2*a + b + c)/(b - c))/(b - c)) + 4*(2*I*b - 2*I*c)*sqrt((a^2 + a*b
+ (a + b)*c)/(b^2 - 2*b*c + c^2))*polylog(3, 1/2*(2*(2*a + b + c)*cos(x) -
(-4*I*a - 2*I*b - 2*I*c)*sin(x) - 4*((b - c)*cos(x) + (I*b - I*c)*sin(x))*sq
rt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2
+ a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c))/(b - c)) +
4*(-2*I*b + 2*I*c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*polyl
og(3, -1/2*(2*(2*a + b + c)*cos(x) + (4*I*a + 2*I*b + 2*I*c)*sin(x) + 4*((b
- c)*cos(x) + (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*
c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)
) - 2*a - b - c)/(b - c))/(b - c)) + 4*(2*I*b - 2*I*c)*sqrt((a^2 + a*b + (a
+ b)*c)/(b^2 - 2*b*c + c^2))*polylog(3, 1/2*(2*(2*a + b + c)*cos(x) - (4*I
*a + 2*I*b + 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((
a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*
b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c))/(b - c)) + 4*(2
*I*b - 2*I*c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*polylog(3,
-1/2*(2*(2*a + b + c)*cos(x) + (-4*I*a - 2*I*b - 2*I*c)*sin(x) + 4*((b - c)
*cos(x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c +
c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) -
2*a - b - c)/(b - c))/(b - c)) + 4*(-2*I*b + 2*I*c)*sqrt((a^2 + a*b + (a +
b)*c)/(b^2 - 2*b*c + c^2))*polylog(3, 1/2*(2*(2*a + b + c)*cos(x) - (-4*I*a
- 2*I*b - 2*I*c)*sin(x) + 4*((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a
^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b
+ (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c))/(b - c)))/(a^2 +
a*b + (a + b)*c)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b \cos(x)^2 + c \sin(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="giac")

[Out] integrate(x^2/(b*cos(x)^2 + c*sin(x)^2 + a), x)

maple [B] time = 0.29, size = 1161, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b*cos(x)^2+c*sin(x)^2),x)

[Out]
$$\begin{aligned} & -1/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)*x*\text{polylog}(2, (b-c)*\exp(2*I*x)/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)) \\ & -1/4*I/((a+b)*(a+c))^{(1/2)}/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)*c*\text{polylog}(3, (b-c)*\exp(2*I*x)/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)) \\ & -1/((a+b)*(a+c))^{(1/2)}/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)*a*x*\text{polylog}(2, (b-c)*\exp(2*I*x)/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)) \\ & -1/2*I/((a+b)*(a+c))^{(1/2)}/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)*b*x^2*\ln(1-(b-c)*\exp(2*I*x)/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)) \\ & -1/2/((a+b)*(a+c))^{(1/2)}*x*\text{polylog}(2, (b-c)*\exp(2*I*x)/(2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)) \\ & -1/4*I/((a+b)*(a+c))^{(1/2)}/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)*b*\text{polylog}(3, (b-c)*\exp(2*I*x)/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)) \\ & -1/2/((a+b)*(a+c))^{(1/2)}/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)*c*x*\text{polylog}(2, (b-c)*\exp(2*I*x)/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)) \\ & -1/2*I/((a+b)*(a+c))^{(1/2)}*x^2*\ln(1-(b-c)*\exp(2*I*x)/(2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)) \\ & -1/2/((a+b)*(a+c))^{(1/2)}/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)*b*x*\text{polylog}(2, (b-c)*\exp(2*I*x)/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)) \\ & -1/4*I/((a+b)*(a+c))^{(1/2)}*\text{polylog}(3, (b-c)*\exp(2*I*x)/(2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)) \\ & -2/3/((a+b)*(a+c))^{(1/2)}/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)*a*x^3-1/2*I/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)*\text{polylog}(3, (b-c)*\exp(2*I*x)/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)) \\ & -2/3/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)*x^3-I/((a+b)*(a+c))^{(1/2)}/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)*a*x^2*\ln(1-(b-c)*\exp(2*I*x)/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)) \\ & -1/3/((a+b)*(a+c))^{(1/2)}/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)*c*x^2*\ln(1-(b-c)*\exp(2*I*x)/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)) \\ & -1/3/((a+b)*(a+c))^{(1/2)}/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)*c*x^3-I/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)*x^2*\ln(1-(b-c)*\exp(2*I*x)/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)) \\ & -1/3/((a+b)*(a+c))^{(1/2)}*x^3-1/2*I/((a+b)*(a+c))^{(1/2)}/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)*a*\text{polylog}(3, (b-c)*\exp(2*I*x)/(-2*((a+b)*(a+c))^{(1/2)}-2*a-b-c)) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b \cos(x)^2 + c \sin(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="maxima")

[Out] integrate(x^2/(b*cos(x)^2 + c*sin(x)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{b \cos(x)^2 + c \sin(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + c*sin(x)^2 + b*cos(x)^2),x)

[Out] int(x^2/(a + c*sin(x)^2 + b*cos(x)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b \cos^2(x) + c \sin^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a+b*cos(x)**2+c*sin(x)**2),x)

[Out] Integral(x**2/(a + b*cos(x)**2 + c*sin(x)**2), x)

3.499 $\int (a+b \sin(d+ex)) (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex)) dx$

Optimal. Leaf size=195

$$\frac{(5a^2 + 4b^2) \cos(d + ex)(a \sin(d + ex) + b)^3}{20e} - \frac{b(17a^2 + 4b^2) \cos(d + ex)(a \sin(d + ex) + b)^2}{20e} - \frac{b(32a^4 + 69a^2b^2 + 4b^4) \cos(d + ex)(a \sin(d + ex) + b)}{10e}$$

[Out] $\frac{3}{8} a^4 x - \frac{1}{10} b (32 a^4 + 69 a^2 b^2 + 4 b^4) \cos(e x + d) / e - \frac{1}{40} a (15 a^4 + 82 a^2 b^2 + 8 b^4) \cos(e x + d) \sin(e x + d) / e - \frac{1}{20} b (17 a^2 + 4 b^2) \cos(e x + d) (b + a \sin(e x + d))^2 / e - \frac{1}{20} (5 a^2 + 4 b^2) \cos(e x + d) (b + a \sin(e x + d))^3 / e - \frac{1}{5} b \cos(e x + d) (b + a \sin(e x + d))^4 / e$

Rubi [A] time = 0.39, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3288, 2753, 2734}

$$\frac{b(69a^2b^2 + 32a^4 + 4b^4) \cos(d + ex)}{10e} - \frac{(5a^2 + 4b^2) \cos(d + ex)(a \sin(d + ex) + b)^3}{20e} - \frac{b(17a^2 + 4b^2) \cos(d + ex)(a \sin(d + ex) + b)^2}{20e}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^2, x]

[Out] $\frac{3 a^4 x}{8} - \frac{b (32 a^4 + 69 a^2 b^2 + 4 b^4) \cos(d + e x)}{10 e} - \frac{a (15 a^4 + 82 a^2 b^2 + 8 b^4) \cos(d + e x) \sin(d + e x)}{40 e} - \frac{b (17 a^2 + 4 b^2) \cos(d + e x) (b + a \sin(d + e x))^2}{20 e} - \frac{(5 a^2 + 4 b^2) \cos(d + e x) (b + a \sin(d + e x))^3}{20 e} - \frac{b \cos(d + e x) (b + a \sin(d + e x))^4}{5 e}$

Rule 2734

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2753

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m]/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rule 3288

Int[((A_) + (B_)*sin[(d_) + (e_)*(x_)])*((a_) + (b_)*sin[(d_) + (e_)*(x_)]) + (c_)*sin[(d_) + (e_)*(x_)]^(n_), x_Symbol] :> Dist[1/(4^n*c^n), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int (a + b \sin(d + ex)) (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^2 dx &= \frac{\int (2ab + 2a^2 \sin(d + ex))^4 (a + b \sin(d + ex))^2 dx}{16a^4} \\
&= -\frac{b \cos(d + ex)(b + a \sin(d + ex))^4}{5e} + \frac{\int (2ab + 2a^2 \sin(d + ex))^4 (a + b \sin(d + ex))^2 dx}{16a^4} \\
&= -\frac{(5a^2 + 4b^2) \cos(d + ex)(b + a \sin(d + ex))^4}{20e} + \frac{\int (2ab + 2a^2 \sin(d + ex))^4 (a + b \sin(d + ex))^2 dx}{16a^4} \\
&= -\frac{b(17a^2 + 4b^2) \cos(d + ex)(b + a \sin(d + ex))^4}{20e} + \frac{\int (2ab + 2a^2 \sin(d + ex))^4 (a + b \sin(d + ex))^2 dx}{16a^4} \\
&= \frac{3}{8}a(a^4 + 12a^2b^2 + 8b^4)x - \frac{b(32a^4 + 69a^2b^2 + 8b^4) \cos(d + ex)}{40e}
\end{aligned}$$

Mathematica [A] time = 0.90, size = 149, normalized size = 0.76

$$\frac{a(10(7a^3b + 8ab^3) \cos(3(d + ex)) - 2a^3b \cos(5(d + ex)) + 5(a^4 + 4a^2b^2) \sin(4(d + ex)) + 60(a^4 + 12a^2b^2 + 8b^4) \cos(3(d + ex)))}{160e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^2,x]

[Out] (-20*b*(29*a^4 + 68*a^2*b^2 + 8*b^4)*Cos[d + e*x] + a*(60*(a^4 + 12*a^2*b^2 + 8*b^4)*(d + e*x) + 10*(7*a^3*b + 8*a*b^3)*Cos[3*(d + e*x)] - 2*a^3*b*Cos[5*(d + e*x)] - 40*(a^4 + 10*a^2*b^2 + 4*b^4)*Sin[2*(d + e*x)] + 5*(a^4 + 4*a^2*b^2)*Sin[4*(d + e*x)])/(160*e)

fricas [A] time = 0.53, size = 150, normalized size = 0.77

$$\frac{8a^4b \cos(ex + d)^5 - 80(a^4b + a^2b^3) \cos(ex + d)^3 - 15(a^5 + 12a^3b^2 + 8ab^4)ex + 40(5a^4b + 10a^2b^3 + b^5) \cos(ex + d)}{40e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, algorithm="fricas")

[Out] -1/40*(8*a^4*b*cos(e*x + d)^5 - 80*(a^4*b + a^2*b^3)*cos(e*x + d)^3 - 15*(a^5 + 12*a^3*b^2 + 8*a*b^4)*e*x + 40*(5*a^4*b + 10*a^2*b^3 + b^5)*cos(e*x + d) - 5*(2*(a^5 + 4*a^3*b^2)*cos(e*x + d)^3 - (5*a^5 + 44*a^3*b^2 + 16*a*b^4)*cos(e*x + d))*sin(e*x + d))/e

giac [A] time = 0.18, size = 158, normalized size = 0.81

$$-\frac{1}{80}a^4b \cos(5xe + 5d)e^{(-1)} + \frac{1}{16}(7a^4b + 8a^2b^3) \cos(3xe + 3d)e^{(-1)} - \frac{1}{8}(29a^4b + 68a^2b^3 + 8b^5) \cos(xe + d)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, algorithm="giac")

[Out] -1/80*a^4*b*cos(5*x*e + 5*d)*e^(-1) + 1/16*(7*a^4*b + 8*a^2*b^3)*cos(3*x*e + 3*d)*e^(-1) - 1/8*(29*a^4*b + 68*a^2*b^3 + 8*b^5)*cos(x*e + d)*e^(-1) + 1/32*(a^5 + 4*a^3*b^2)*e^(-1)*sin(4*x*e + 4*d) - 1/4*(a^5 + 10*a^3*b^2 + 4*a*b^4)*e^(-1)*sin(2*x*e + 2*d) + 3/8*(a^5 + 12*a^3*b^2 + 8*a*b^4)*x

maple [A] time = 0.10, size = 255, normalized size = 1.31

$$a b^4 (ex + d) - 4 \cos(ex + d) a^2 b^3 + 6 a^3 b^2 \left(-\frac{\sin(ex+d) \cos(ex+d)}{2} + \frac{ex}{2} + \frac{d}{2} \right) - \frac{4 a^4 b (2 + \sin^2(ex+d)) \cos(ex+d)}{3} + a^5 \left(-\frac{\sin^3(ex+d)}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x)

[Out] 1/e*(a*b^4*(e*x+d)-4*cos(e*x+d)*a^2*b^3+6*a^3*b^2*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-4/3*a^4*b*(2+sin(e*x+d)^2)*cos(e*x+d)+a^5*(-1/4*(sin(e*x+d)^3+3/2*sin(e*x+d))*cos(e*x+d)+3/8*e*x+3/8*d)-cos(e*x+d)*b^5+4*a*b^4*(-1/2*sin(e*x+d)*cos(e*x+d)+1/2*e*x+1/2*d)-2*a^2*b^3*(2+sin(e*x+d)^2)*cos(e*x+d)+4*a^3*b^2*(-1/4*(sin(e*x+d)^3+3/2*sin(e*x+d))*cos(e*x+d)+3/8*e*x+3/8*d)-1/5*a^4*b*(8/3+sin(e*x+d)^4+4/3*sin(e*x+d)^2)*cos(e*x+d))

maxima [A] time = 0.32, size = 246, normalized size = 1.26

$$15(12ex + 12d + \sin(4ex + 4d) - 8 \sin(2ex + 2d))a^5 - 32(3 \cos(ex + d)^5 - 10 \cos(ex + d)^3 + 15 \cos(ex + d))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, algorithm="maxima")

[Out] 1/480*(15*(12*e*x + 12*d + sin(4*e*x + 4*d) - 8*sin(2*e*x + 2*d))*a^5 - 32*(3*cos(e*x + d)^5 - 10*cos(e*x + d)^3 + 15*cos(e*x + d))*a^4*b + 640*(cos(e*x + d)^3 - 3*cos(e*x + d))*a^4*b + 60*(12*e*x + 12*d + sin(4*e*x + 4*d) - 8*sin(2*e*x + 2*d))*a^3*b^2 + 720*(2*e*x + 2*d - sin(2*e*x + 2*d))*a^3*b^2 + 960*(cos(e*x + d)^3 - 3*cos(e*x + d))*a^2*b^3 + 480*(2*e*x + 2*d - sin(2*e*x + 2*d))*a*b^4 + 480*(e*x + d)*a*b^4 - 1920*a^2*b^3*cos(e*x + d) - 480*b^5*cos(e*x + d))/e

mupad [B] time = 4.49, size = 456, normalized size = 2.34

$$\frac{3 a \operatorname{atan}\left(\frac{3 a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) (a^4 + 12 a^2 b^2 + 8 b^4)}{4 \left(\frac{3 a^5}{4} + 9 a^3 b^2 + 6 a b^4\right)}\right) (a^4 + 12 a^2 b^2 + 8 b^4)}{4 e} - \tan\left(\frac{d}{2} + \frac{ex}{2}\right) \left(\frac{3 a^5}{4} + 9 a^3 b^2 + 4 a b^4\right) - \tan\left(\frac{d}{2} + \frac{ex}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^2, x)

[Out] (3*a*atan((3*a*tan(d/2 + (e*x)/2)*(a^4 + 8*b^4 + 12*a^2*b^2))/(4*(6*a*b^4 + (3*a^5)/4 + 9*a^3*b^2)))*(a^4 + 8*b^4 + 12*a^2*b^2))/(4*e) - (tan(d/2 + (e*x)/2)*(4*a*b^4 + (3*a^5)/4 + 9*a^3*b^2) - tan(d/2 + (e*x)/2)^9*(4*a*b^4 + (3*a^5)/4 + 9*a^3*b^2) + tan(d/2 + (e*x)/2)^3*(8*a*b^4 + (7*a^5)/2 + 26*a^3*b^2) - tan(d/2 + (e*x)/2)^7*(8*a*b^4 + (7*a^5)/2 + 26*a^3*b^2) + tan(d/2 + (e*x)/2)^6*(16*a^4*b + 8*b^5 + 56*a^2*b^3) + tan(d/2 + (e*x)/2)^2*(32*a^4*b + 8*b^5 + 72*a^2*b^3) + tan(d/2 + (e*x)/2)^4*(48*a^4*b + 12*b^5 + 104*a^2*b^3) + (32*a^4*b)/5 + 2*b^5 + 16*a^2*b^3 + tan(d/2 + (e*x)/2)^8*(2*b^5 + 8*a^2*b^3))/(e*(5*tan(d/2 + (e*x)/2)^2 + 10*tan(d/2 + (e*x)/2)^4 + 10*tan(d/2 + (e*x)/2)^6 + 5*tan(d/2 + (e*x)/2)^8 + tan(d/2 + (e*x)/2)^10 + 1)) - (3*a*(atan(tan(d/2 + (e*x)/2)) - (e*x)/2)*(a^4 + 8*b^4 + 12*a^2*b^2))/(4*e)

sympy [A] time = 3.00, size = 566, normalized size = 2.90

$$\left\{ \begin{array}{l} \frac{3a^5x \sin^4(d+ex)}{8} + \frac{3a^5x \sin^2(d+ex) \cos^2(d+ex)}{4} + \frac{3a^5x \cos^4(d+ex)}{8} - \frac{5a^5 \sin^3(d+ex) \cos(d+ex)}{8e} - \frac{3a^5 \sin(d+ex) \cos^3(d+ex)}{8e} - \frac{a^4b \sin^4(d+ex)}{8e} \\ x(a + b \sin(d))(a^2 \sin^2(d) + 2ab \sin(d) + b^2)^2 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**2,x)

[Out] Piecewise((3*a**5*x*sin(d + e*x)**4/8 + 3*a**5*x*sin(d + e*x)**2*cos(d + e*x)**2/4 + 3*a**5*x*cos(d + e*x)**4/8 - 5*a**5*sin(d + e*x)**3*cos(d + e*x)/(8*e) - 3*a**5*sin(d + e*x)*cos(d + e*x)**3/(8*e) - a**4*b*sin(d + e*x)**4*cos(d + e*x)/e - 4*a**4*b*sin(d + e*x)**2*cos(d + e*x)**3/(3*e) - 4*a**4*b*sin(d + e*x)**2*cos(d + e*x)/e - 8*a**4*b*cos(d + e*x)**5/(15*e) - 8*a**4*b*cos(d + e*x)**3/(3*e) + 3*a**3*b**2*x*sin(d + e*x)**4/2 + 3*a**3*b**2*x*sin(d + e*x)**2*cos(d + e*x)**2 + 3*a**3*b**2*x*sin(d + e*x)**2 + 3*a**3*b**2*x*cos(d + e*x)**4/2 + 3*a**3*b**2*x*cos(d + e*x)**2 - 5*a**3*b**2*sin(d + e*x)**3*cos(d + e*x)/(2*e) - 3*a**3*b**2*sin(d + e*x)*cos(d + e*x)**3/(2*e) - 3*a**3*b**2*sin(d + e*x)*cos(d + e*x)/e - 6*a**2*b**3*sin(d + e*x)**2*cos(d + e*x)/e - 4*a**2*b**3*cos(d + e*x)**3/e - 4*a**2*b**3*cos(d + e*x)/e + 2*a*b**4*x*sin(d + e*x)**2 + 2*a*b**4*x*cos(d + e*x)**2 + a*b**4*x - 2*a*b**4*sin(d + e*x)*cos(d + e*x)/e - b**5*cos(d + e*x)/e, Ne(e, 0)), (x*(a + b*sin(d))*(a**2*sin(d)**2 + 2*a*b*sin(d) + b**2)**2, True))

3.500 $\int (a+b \sin(d+ex)) (b^2 + 2ab \sin(d+ex) + a^2 \sin^2(d+ex)) dx$

Optimal. Leaf size=109

$$\frac{a(a^2 - 6b^2) \sin(d+ex) \cos(d+ex)}{6e} + \frac{1}{2} ax (a^2 + 4b^2) - \frac{a^2 \cos(d+ex)(a+b \sin(d+ex))^2}{3be} + \frac{(a^4 - 8a^2b^2 - 3b^4) \cos(d+ex)}{3be}$$

[Out] $\frac{1}{2} a (a^2 + 4 b^2) x + \frac{1}{3} (a^4 - 8 a^2 b^2 - 3 b^4) \cos(e x + d) / b / e + \frac{1}{6} a (a^2 - 6 b^2) \cos(e x + d) \sin(e x + d) / e - \frac{1}{3} a^2 \cos(e x + d) (a + b \sin(e x + d))^2 / b / e$

Rubi [A] time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {3023, 2734}

$$\frac{(-8a^2b^2 + a^4 - 3b^4) \cos(d+ex)}{3be} + \frac{a(a^2 - 6b^2) \sin(d+ex) \cos(d+ex)}{6e} + \frac{1}{2} ax (a^2 + 4b^2) - \frac{a^2 \cos(d+ex)(a+b \sin(d+ex))^2}{3be}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2),x]

[Out] $\frac{a(a^2 + 4b^2)x}{2} + \frac{(a^4 - 8a^2b^2 - 3b^4)\cos[d + ex]}{(3be)} + \frac{a(a^2 - 6b^2)\cos[d + ex]\sin[d + ex]}{(6e)} - \frac{a^2\cos[d + ex](a + b\sin[d + ex])^2}{(3be)}$

Rule 2734

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3023

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\int (a + b \sin(d+ex)) (b^2 + 2ab \sin(d+ex) + a^2 \sin^2(d+ex)) dx = -\frac{a^2 \cos(d+ex)(a+b \sin(d+ex))^2}{3be} + \frac{\int (a + b \sin(d+ex)) (b^2 + 2ab \sin(d+ex) + a^2 \sin^2(d+ex)) dx}{3be} = \frac{1}{2} a (a^2 + 4b^2) x + \frac{(a^4 - 8a^2b^2 - 3b^4) \cos(d+ex)}{3be}$$

Mathematica [A] time = 0.29, size = 77, normalized size = 0.71

$$\frac{a(6(a^2 + 4b^2)(d+ex) - 3(a^2 + 2b^2)\sin(2(d+ex)) + ab\cos(3(d+ex))) - 3b(11a^2 + 4b^2)\cos(d+ex)}{12e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2),x]

[Out] $(-3*b*(11*a^2 + 4*b^2)*\cos[d + e*x] + a*(6*(a^2 + 4*b^2)*(d + e*x) + a*b*\cos[3*(d + e*x)] - 3*(a^2 + 2*b^2)*\sin[2*(d + e*x)])/(12*e)$

fricas [A] time = 0.74, size = 76, normalized size = 0.70

$$\frac{2 a^2 b \cos (e x+d)^3+3\left(a^3+4 a b^2\right) e x-3\left(a^3+2 a b^2\right) \cos (e x+d) \sin (e x+d)-6\left(3 a^2 b+b^3\right) \cos (e x+d)}{6 e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algorithm="fricas")`

[Out] $1/6*(2*a^2*b*\cos(e*x + d)^3 + 3*(a^3 + 4*a*b^2)*e*x - 3*(a^3 + 2*a*b^2)*\cos(e*x + d)*\sin(e*x + d) - 6*(3*a^2*b + b^3)*\cos(e*x + d))/e$

giac [A] time = 0.15, size = 79, normalized size = 0.72

$$\frac{1}{12} a^2 b \cos (3 x e+3 d) e^{(-1)}-\frac{1}{4}\left(11 a^2 b+4 b^3\right) \cos (x e+d) e^{(-1)}-\frac{1}{4}\left(a^3+2 a b^2\right) e^{(-1)} \sin (2 x e+2 d)+\frac{1}{2}\left(a^3+4 a b^2\right) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algorithm="giac")`

[Out] $1/12*a^2*b*\cos(3*x*e + 3*d)*e^{(-1)} - 1/4*(11*a^2*b + 4*b^3)*\cos(x*e + d)*e^{(-1)} - 1/4*(a^3 + 2*a*b^2)*e^{(-1)}*\sin(2*x*e + 2*d) + 1/2*(a^3 + 4*a*b^2)*x$

maple [A] time = 0.08, size = 115, normalized size = 1.06

$$\frac{-\frac{a^2 b(2+\sin^2(e x+d)) \cos (e x+d)}{3}+a^3\left(-\frac{\sin (e x+d) \cos (e x+d)}{2}+\frac{e x}{2}+\frac{d}{2}\right)+2 a b^2\left(-\frac{\sin (e x+d) \cos (e x+d)}{2}+\frac{e x}{2}+\frac{d}{2}\right)-2 \cos (e x+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x)`

[Out] $1/e*(-1/3*a^2*b*(2+\sin(e*x+d)^2)*\cos(e*x+d)+a^3*(-1/2*\sin(e*x+d)*\cos(e*x+d)+1/2*e*x+1/2*d)+2*a*b^2*(-1/2*\sin(e*x+d)*\cos(e*x+d)+1/2*e*x+1/2*d)-2*\cos(e*x+d)*a^2*b-b^3*\cos(e*x+d)+a*b^2*(e*x+d))$

maxima [A] time = 0.32, size = 112, normalized size = 1.03

$$\frac{3(2 e x+2 d-\sin (2 e x+2 d)) a^3+4\left(\cos (e x+d)^3-3 \cos (e x+d)\right) a^2 b+6(2 e x+2 d-\sin (2 e x+2 d)) a b^2-12 e}{12 e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algorithm="maxima")`

[Out] $1/12*(3*(2*e*x + 2*d - \sin(2*e*x + 2*d))*a^3 + 4*(\cos(e*x + d)^3 - 3*\cos(e*x + d))*a^2*b + 6*(2*e*x + 2*d - \sin(2*e*x + 2*d))*a*b^2 + 12*(e*x + d)*a*b^2 - 24*a^2*b*\cos(e*x + d) - 12*b^3*\cos(e*x + d))/e$

mupad [B] time = 2.90, size = 88, normalized size = 0.81

$$\frac{6 b^3 \cos (d+e x)+\frac{3 a^3 \sin (2 d+2 e x)}{2}-\frac{a^2 b \cos (3 d+3 e x)}{2}+3 a b^2 \sin (2 d+2 e x)+\frac{33 a^2 b \cos (d+e x)}{2}-3 a^3 e x-12 a b^2}{6 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x)),x)
[Out] -(6*b^3*cos(d + e*x) + (3*a^3*sin(2*d + 2*e*x))/2 - (a^2*b*cos(3*d + 3*e*x)
)/2 + 3*a*b^2*sin(2*d + 2*e*x) + (33*a^2*b*cos(d + e*x))/2 - 3*a^3*e*x - 12
*a*b^2*e*x)/(6*e)
```

sympy [A] time = 0.67, size = 204, normalized size = 1.87

$$\left\{ \begin{array}{l} \frac{a^3 x \sin^2(d+ex)}{2} + \frac{a^3 x \cos^2(d+ex)}{2} - \frac{a^3 \sin(d+ex) \cos(d+ex)}{2e} - \frac{a^2 b \sin^2(d+ex) \cos(d+ex)}{e} - \frac{2a^2 b \cos^3(d+ex)}{3e} - \frac{2a^2 b \cos(d+ex)}{e} + ab^2 x \sin \\ x(a + b \sin(d)) (a^2 \sin^2(d) + 2ab \sin(d) + b^2) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2),x)
[Out] Piecewise((a**3*x*sin(d + e*x)**2/2 + a**3*x*cos(d + e*x)**2/2 - a**3*sin(d
+ e*x)*cos(d + e*x)/(2*e) - a**2*b*sin(d + e*x)**2*cos(d + e*x)/e - 2*a**2
*b*cos(d + e*x)**3/(3*e) - 2*a**2*b*cos(d + e*x)/e + a*b**2*x*sin(d + e*x)*
*2 + a*b**2*x*cos(d + e*x)**2 + a*b**2*x - a*b**2*sin(d + e*x)*cos(d + e*x)
/e - b**3*cos(d + e*x)/e, Ne(e, 0)), (x*(a + b*sin(d))*(a**2*sin(d)**2 + 2*
a*b*sin(d) + b**2), True))
```

$$3.501 \quad \int \frac{a+b \sin(d+ex)}{b^2+2ab \sin(d+ex)+a^2 \sin^2(d+ex)} dx$$

Optimal. Leaf size=23

$$-\frac{\cos(d+ex)}{e(a \sin(d+ex)+b)}$$

[Out] $-\cos(e*x+d)/e/(b+a*\sin(e*x+d))$

Rubi [A] time = 0.09, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3288, 2754, 8}

$$-\frac{\cos(d+ex)}{e(a \sin(d+ex)+b)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Sin}[d + e*x])/(b^2 + 2*a*b*\text{Sin}[d + e*x] + a^2*\text{Sin}[d + e*x]^2), x]$

[Out] $-(\text{Cos}[d + e*x]/(e*(b + a*\text{Sin}[d + e*x])))$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2754

$\text{Int}[(a_ + (b_)*\text{sin}[(e_ + (f_)*(x_)]))^m * ((c_ + (d_)*\text{sin}[(e_ + (f_)*(x_)])), x_Symbol] \rightarrow -\text{Simp}[(b*c - a*d)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{m+1}]/(f*(m+1)*(a^2 - b^2)), x] + \text{Dist}[1/((m+1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{m+1}*\text{Simp}[(a*c - b*d)*(m+1) - (b*c - a*d)*(m+2)*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntegerQ}[2*m]$

Rule 3288

$\text{Int}[(A_ + (B_)*\text{sin}[(d_ + (e_)*(x_)]))*(a_ + (b_)*\text{sin}[(d_ + (e_)*(x_)] + (c_)*\text{sin}[(d_ + (e_)*(x_)]^2)^n), x_Symbol] \rightarrow \text{Dist}[1/(4^n*c^n), \text{Int}[(A + B*\text{Sin}[d + e*x])*(b + 2*c*\text{Sin}[d + e*x])^{2*n}], x], x] /; \text{FreeQ}\{a, b, c, d, e, A, B\}, x \ \&\& \ \text{EqQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IntegerQ}[n]$

Rubi steps

$$\begin{aligned} \int \frac{a+b \sin(d+ex)}{b^2+2ab \sin(d+ex)+a^2 \sin^2(d+ex)} dx &= (4a^2) \int \frac{a+b \sin(d+ex)}{(2ab+2a^2 \sin(d+ex))^2} dx \\ &= -\frac{\cos(d+ex)}{e(b+a \sin(d+ex))} + \frac{\int 0 dx}{a^2-b^2} \\ &= -\frac{\cos(d+ex)}{e(b+a \sin(d+ex))} \end{aligned}$$

Mathematica [A] time = 0.06, size = 23, normalized size = 1.00

$$-\frac{\cos(d+ex)}{e(a \sin(d+ex)+b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2), x]

[Out] -(Cos[d + e*x]/(e*(b + a*Sin[d + e*x])))

fricas [A] time = 0.83, size = 23, normalized size = 1.00

$$\frac{\cos(ex + d)}{ae \sin(ex + d) + be}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2), x, algorithm="fricas")

[Out] -cos(e*x + d)/(a*e*sin(e*x + d) + b*e)

giac [B] time = 0.18, size = 52, normalized size = 2.26

$$\frac{2 \left(a \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) + b \right) e^{(-1)}}{\left(b \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) + 2 a \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) + b \right) b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2), x, algorithm="giac")

[Out] -2*(a*tan(1/2*x*e + 1/2*d) + b)*e^(-1)/((b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + b)*b)

maple [B] time = 0.34, size = 52, normalized size = 2.26

$$\frac{-\frac{2a \tan\left(\frac{d}{2} + \frac{ex}{2}\right)}{b} - 2}{e \left(b \left(\tan^2 \left(\frac{d}{2} + \frac{ex}{2} \right) \right) + 2a \tan \left(\frac{d}{2} + \frac{ex}{2} \right) + b \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2), x)

[Out] 2/e*(-a*tan(1/2*d+1/2*e*x)/b-1)/(b*tan(1/2*d+1/2*e*x)^2+2*a*tan(1/2*d+1/2*e*x)+b)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2), x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation may help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 2.84, size = 39, normalized size = 1.70

$$\frac{a \sin(d + ex) + b (\cos(d + ex) + 1)}{b e (b + a \sin(d + ex))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x)),x)
```

```
[Out] -(a*sin(d + e*x) + b*(cos(d + e*x) + 1))/(b*e*(b + a*sin(d + e*x)))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2),x)
```

```
[Out] Timed out
```

$$3.502 \quad \int \frac{a+b \sin(d+ex)}{(b^2+2ab \sin(d+ex)+a^2 \sin^2(d+ex))^2} dx$$

Optimal. Leaf size=157

$$\frac{(2a^2 + b^2) \cos(d + ex)}{3e(a^2 - b^2)^2 (a \sin(d + ex) + b)} + \frac{b \cos(d + ex)}{3e(a^2 - b^2) (a \sin(d + ex) + b)^2} + \frac{2ab \tanh^{-1}\left(\frac{a+b \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{e(a^2 - b^2)^{5/2}} - \frac{\cos(d + ex)}{3e(a \sin(d + ex) + b)}$$

[Out] 2*a*b*arctanh((a+b*tan(1/2*e*x+1/2*d))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/e-1/3*cos(e*x+d)/e/(b+a*sin(e*x+d))^3+1/3*b*cos(e*x+d)/(a^2-b^2)/e/(b+a*sin(e*x+d))^2-1/3*(2*a^2+b^2)*cos(e*x+d)/(a^2-b^2)^2/e/(b+a*sin(e*x+d))

Rubi [A] time = 0.42, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3288, 2754, 12, 2660, 618, 206}

$$\frac{(2a^2 + b^2) \cos(d + ex)}{3e(a^2 - b^2)^2 (a \sin(d + ex) + b)} + \frac{b \cos(d + ex)}{3e(a^2 - b^2) (a \sin(d + ex) + b)^2} + \frac{2ab \tanh^{-1}\left(\frac{a+b \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{e(a^2 - b^2)^{5/2}} - \frac{\cos(d + ex)}{3e(a \sin(d + ex) + b)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^2, x]

[Out] (2*a*b*ArcTanh[(a + b*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2]]/((a^2 - b^2)^(5/2)*e) - Cos[d + e*x]/(3*e*(b + a*Sin[d + e*x])^3) + (b*Cos[d + e*x])/(3*(a^2 - b^2)*e*(b + a*Sin[d + e*x])^2) - ((2*a^2 + b^2)*Cos[d + e*x])/(3*(a^2 - b^2)^2*e*(b + a*Sin[d + e*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2754

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3288

```
Int[((A_) + (B_)*sin[(d_) + (e_)*(x_)])*((a_) + (b_)*sin[(d_) + (e_)*(x_)]) + (c_)*sin[(d_) + (e_)*(x_)]^(n_), x_Symbol] :> Dist[1/(4^n*c^n), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a + b \sin(d + ex)}{(b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^2} dx = (16a^4) \int \frac{a + b \sin(d + ex)}{(2ab + 2a^2 \sin(d + ex))^4} dx$$

$$= -\frac{\cos(d + ex)}{3e(b + a \sin(d + ex))^3} + \frac{(4a^2) \int \frac{4a(a^2 - b^2) \sin(d + ex)}{(2ab + 2a^2 \sin(d + ex))^3} dx}{3(a^2 - b^2)}$$

$$= -\frac{\cos(d + ex)}{3e(b + a \sin(d + ex))^3} + \frac{1}{3} (16a^3) \int \frac{\sin(d + ex)}{(2ab + 2a^2 \sin(d + ex))^2} dx$$

$$= -\frac{\cos(d + ex)}{3e(b + a \sin(d + ex))^3} + \frac{b \cos(d + ex)}{3(a^2 - b^2)e(b + a \sin(d + ex))^2} + \frac{2ab \cos(d + ex)}{3(a^2 - b^2)e(b + a \sin(d + ex))^2}$$

$$= -\frac{\cos(d + ex)}{3e(b + a \sin(d + ex))^3} + \frac{b \cos(d + ex)}{3(a^2 - b^2)e(b + a \sin(d + ex))^2}$$

$$= -\frac{\cos(d + ex)}{3e(b + a \sin(d + ex))^3} + \frac{b \cos(d + ex)}{3(a^2 - b^2)e(b + a \sin(d + ex))^2}$$

$$= -\frac{\cos(d + ex)}{3e(b + a \sin(d + ex))^3} + \frac{b \cos(d + ex)}{3(a^2 - b^2)e(b + a \sin(d + ex))^2}$$

$$= -\frac{\cos(d + ex)}{3e(b + a \sin(d + ex))^3} + \frac{b \cos(d + ex)}{3(a^2 - b^2)e(b + a \sin(d + ex))^2}$$

$$= \frac{2ab \tanh^{-1}\left(\frac{a + b \tan\left(\frac{1}{2}(d + ex)\right)}{\sqrt{a^2 - b^2}}\right)}{(a^2 - b^2)^{5/2} e} - \frac{\cos(d + ex)}{3e(b + a \sin(d + ex))^3} + \frac{b \cos(d + ex)}{3(a^2 - b^2)e(b + a \sin(d + ex))^2}$$

Mathematica [A] time = 0.96, size = 140, normalized size = 0.89

$$\frac{6ab \tan^{-1}\left(\frac{a + b \tan\left(\frac{1}{2}(d + ex)\right)}{\sqrt{b^2 - a^2}}\right)}{(b^2 - a^2)^{5/2}} + \frac{\cos(d + ex)(a^4 + a^2(2a^2 + b^2) \sin^2(d + ex) + 3ab(a^2 + b^2) \sin(d + ex) - a^2 b^2 + 3b^4)}{(a - b)^2(a + b)^2(a \sin(d + ex) + b)^3}$$

3e

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2),x]

[Out]
$$-1/3*((6*a*b*ArcTan[(a + b*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^{(5/2)} + (Cos[d + e*x]*(a^4 - a^2*b^2 + 3*b^4 + 3*a*b*(a^2 + b^2)*Sin[d + e*x] + a^2*(2*a^2 + b^2)*Sin[d + e*x]^2))/((a - b)^2*(a + b)^2*(b + a*Sin[d + e*x])^3))/e$$

fricas [B] time = 1.98, size = 795, normalized size = 5.06

$$\left[\frac{2(2a^6 - a^4b^2 - a^2b^4)\cos(ex + d)^3 - 6(a^5b - ab^5)\cos(ex + d)\sin(ex + d) - 3(3a^3b^2\cos(ex + d)^2 - 3a^3b^2 - 6(a^8b - 3a^6b^3 + 3a^4b^5 - a^2b^7)e\cos(ex + d)^2 - (3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg orithm="fricas")

[Out]
$$[-1/6*(2*(2*a^6 - a^4*b^2 - a^2*b^4)*\cos(e*x + d)^3 - 6*(a^5*b - a*b^5)*\cos(e*x + d)*\sin(e*x + d) - 3*(3*a^3*b^2*\cos(e*x + d)^2 - 3*a^3*b^2 - a*b^4 + (a^4*b*\cos(e*x + d)^2 - a^4*b - 3*a^2*b^3)*\sin(e*x + d))*\sqrt{a^2 - b^2}*\log(((a^2 - 2*b^2)*\cos(e*x + d)^2 + 2*a*b*\sin(e*x + d) + a^2 + b^2 + 2*(b*\cos(e*x + d)*\sin(e*x + d) + a*\cos(e*x + d))*\sqrt{a^2 - b^2}))/((a^2*\cos(e*x + d)^2 - 2*a*b*\sin(e*x + d) - a^2 - b^2)) - 6*(a^6 - a^4*b^2 + a^2*b^4 - b^6)*\cos(e*x + d))/((3*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*e*\cos(e*x + d)^2 - (3*a^8*b - 8*a^6*b^3 + 6*a^4*b^5 - b^9)*e + ((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*e*\cos(e*x + d)^2 - (a^9 - 6*a^5*b^4 + 8*a^3*b^6 - 3*a*b^8)*e)*\sin(e*x + d)), -1/3*((2*a^6 - a^4*b^2 - a^2*b^4)*\cos(e*x + d)^3 - 3*(a^5*b - a*b^5)*\cos(e*x + d)*\sin(e*x + d) - 3*(3*a^3*b^2*\cos(e*x + d)^2 - 3*a^3*b^2 - a*b^4 + (a^4*b*\cos(e*x + d)^2 - a^4*b - 3*a^2*b^3)*\sin(e*x + d))*\sqrt{-a^2 + b^2}*\arctan(-\sqrt{-a^2 + b^2}*(b*\sin(e*x + d) + a)/((a^2 - b^2)*\cos(e*x + d))) - 3*(a^6 - a^4*b^2 + a^2*b^4 - b^6)*\cos(e*x + d))/((3*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*e*\cos(e*x + d)^2 - (3*a^8*b - 8*a^6*b^3 + 6*a^4*b^5 - b^9)*e + ((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*e*\cos(e*x + d)^2 - (a^9 - 6*a^5*b^4 + 8*a^3*b^6 - 3*a*b^8)*e)*\sin(e*x + d))]$$

giac [B] time = 0.25, size = 454, normalized size = 2.89

$$-\frac{2}{3} \left(\frac{3 \left(\pi \left\lfloor \frac{xe+d}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(b) + \arctan \left(\frac{b \tan \left(\frac{1}{2} xe + \frac{1}{2} d \right) + a}{\sqrt{-a^2 + b^2}} \right) \right) ab}{(a^4 - 2a^2b^2 + b^4)\sqrt{-a^2 + b^2}} + \frac{3a^5b^2 \tan \left(\frac{1}{2} xe + \frac{1}{2} d \right)^5 - 6a^3b^4 \tan \left(\frac{1}{2} xe + \frac{1}{2} d \right)^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg orithm="giac")

[Out]
$$-2/3*(3*(\pi*\operatorname{floor}(1/2*(x*e + d)/\pi + 1/2))*\operatorname{sgn}(b) + \arctan((b*\tan(1/2*x*e + 1/2*d) + a)/\sqrt{-a^2 + b^2}))*a*b/((a^4 - 2*a^2*b^2 + b^4)*\sqrt{-a^2 + b^2}) + (3*a^5*b^2*\tan(1/2*x*e + 1/2*d)^5 - 6*a^3*b^4*\tan(1/2*x*e + 1/2*d)^5 + 6*a*b^6*\tan(1/2*x*e + 1/2*d)^5 + 6*a^6*b*\tan(1/2*x*e + 1/2*d)^4 - 9*a^4*b^3*\tan(1/2*x*e + 1/2*d)^4 + 15*a^2*b^5*\tan(1/2*x*e + 1/2*d)^4 + 3*b^7*\tan(1/2*x*e + 1/2*d)^4 + 4*a^7*\tan(1/2*x*e + 1/2*d)^3 + 2*a^5*b^2*\tan(1/2*x*e + 1/2*d)^3 + 6*a^3*b^4*\tan(1/2*x*e + 1/2*d)^3 + 18*a*b^6*\tan(1/2*x*e + 1/2*d)^3 + 6*a^6*b*\tan(1/2*x*e + 1/2*d)^2 + 18*a^2*b^5*\tan(1/2*x*e + 1/2*d)^2 + 6*b^7*\tan(1/2*x*e + 1/2*d)^2 + 3*a^5*b^2*\tan(1/2*x*e + 1/2*d) + 12*a*b^6*\tan(1/2*x*e + 1/2*d) + a^4*b^3 - a^2*b^5 + 3*b^7)/((a^4*b^3 - 2*a^2*b^5 + b^7)* (b*\tan(1/2*x*e + 1/2*d)^2 + 2*a*\tan(1/2*x*e + 1/2*d) + b)^3))*e^{-1}$$

maple [B] time = 0.33, size = 1297, normalized size = 8.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a+b*\sin(e*x+d))/(b^2+2*a*b*\sin(e*x+d)+a^2*\sin(e*x+d)^2),x)$

[Out]
$$\begin{aligned} & -2/e/(b*\tan(1/2*d+1/2*e*x)^2+2*a*\tan(1/2*d+1/2*e*x)+b)^3*a^5/b/(a^4-2*a^2*b \\ & ^2+b^4)*\tan(1/2*d+1/2*e*x)^5+4/e/(b*\tan(1/2*d+1/2*e*x)^2+2*a*\tan(1/2*d+1/2* \\ & e*x)+b)^3*a^3*b/(a^4-2*a^2*b^2+b^4)*\tan(1/2*d+1/2*e*x)^5-4/e/(b*\tan(1/2*d+1 \\ & /2*e*x)^2+2*a*\tan(1/2*d+1/2*e*x)+b)^3*a*b^3/(a^4-2*a^2*b^2+b^4)*\tan(1/2*d+1 \\ & /2*e*x)^5-4/e/(b*\tan(1/2*d+1/2*e*x)^2+2*a*\tan(1/2*d+1/2*e*x)+b)^3/b^2/(a^4- \\ & 2*a^2*b^2+b^4)*\tan(1/2*d+1/2*e*x)^4*a^6+6/e/(b*\tan(1/2*d+1/2*e*x)^2+2*a*\tan \\ & (1/2*d+1/2*e*x)+b)^3/(a^4-2*a^2*b^2+b^4)*\tan(1/2*d+1/2*e*x)^4*a^4-10/e/(b*t \\ & an(1/2*d+1/2*e*x)^2+2*a*\tan(1/2*d+1/2*e*x)+b)^3*b^2/(a^4-2*a^2*b^2+b^4)*\tan \\ & (1/2*d+1/2*e*x)^4*a^2-2/e/(b*\tan(1/2*d+1/2*e*x)^2+2*a*\tan(1/2*d+1/2*e*x)+b) \\ & ^3*b^4/(a^4-2*a^2*b^2+b^4)*\tan(1/2*d+1/2*e*x)^4-8/3/e/(b*\tan(1/2*d+1/2*e*x) \\ & ^2+2*a*\tan(1/2*d+1/2*e*x)+b)^3*a^7/b^3/(a^4-2*a^2*b^2+b^4)*\tan(1/2*d+1/2*e* \\ & x)^3-4/3/e/(b*\tan(1/2*d+1/2*e*x)^2+2*a*\tan(1/2*d+1/2*e*x)+b)^3*a^5/b/(a^4-2 \\ & *a^2*b^2+b^4)*\tan(1/2*d+1/2*e*x)^3-4/e/(b*\tan(1/2*d+1/2*e*x)^2+2*a*\tan(1/2* \\ & d+1/2*e*x)+b)^3*a^3*b/(a^4-2*a^2*b^2+b^4)*\tan(1/2*d+1/2*e*x)^3-12/e/(b*\tan(\\ & 1/2*d+1/2*e*x)^2+2*a*\tan(1/2*d+1/2*e*x)+b)^3*a*b^3/(a^4-2*a^2*b^2+b^4)*\tan(\\ & 1/2*d+1/2*e*x)^3-4/e/(b*\tan(1/2*d+1/2*e*x)^2+2*a*\tan(1/2*d+1/2*e*x)+b)^3/b^ \\ & 2/(a^4-2*a^2*b^2+b^4)*\tan(1/2*d+1/2*e*x)^2*a^6-12/e/(b*\tan(1/2*d+1/2*e*x)^2 \\ & +2*a*\tan(1/2*d+1/2*e*x)+b)^3*b^2/(a^4-2*a^2*b^2+b^4)*\tan(1/2*d+1/2*e*x)^2*a \\ & ^2-4/e/(b*\tan(1/2*d+1/2*e*x)^2+2*a*\tan(1/2*d+1/2*e*x)+b)^3*b^4/(a^4-2*a^2*b \\ & ^2+b^4)*\tan(1/2*d+1/2*e*x)^2-2/e/(b*\tan(1/2*d+1/2*e*x)^2+2*a*\tan(1/2*d+1/2* \\ & e*x)+b)^3*a^5/b/(a^4-2*a^2*b^2+b^4)*\tan(1/2*d+1/2*e*x)-8/e/(b*\tan(1/2*d+1/2 \\ & *e*x)^2+2*a*\tan(1/2*d+1/2*e*x)+b)^3*a*b^3/(a^4-2*a^2*b^2+b^4)*\tan(1/2*d+1/2 \\ & *e*x)-2/3/e/(b*\tan(1/2*d+1/2*e*x)^2+2*a*\tan(1/2*d+1/2*e*x)+b)^3/(a^4-2*a^2* \\ & b^2+b^4)*a^4+2/3/e/(b*\tan(1/2*d+1/2*e*x)^2+2*a*\tan(1/2*d+1/2*e*x)+b)^3/(a^4 \\ & -2*a^2*b^2+b^4)*a^2*b^2-2/e/(b*\tan(1/2*d+1/2*e*x)^2+2*a*\tan(1/2*d+1/2*e*x)+ \\ & b)^3/(a^4-2*a^2*b^2+b^4)*b^4-2/e*a*b/(a^4-2*a^2*b^2+b^4)/(-a^2+b^2)^(1/2)*a \\ & rctan(1/2*(2*b*\tan(1/2*d+1/2*e*x)+2*a)/(-a^2+b^2)^(1/2)) \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a+b*\sin(e*x+d))/(b^2+2*a*b*\sin(e*x+d)+a^2*\sin(e*x+d)^2),x, \text{alg}$
 $\text{orithm}=\text{"maxima"})$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
 dditional constraints; using the 'assume' command before evaluation *may* h
 elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
 more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 6.06, size = 497, normalized size = 3.17

$$\frac{2ab \operatorname{atanh}\left(\frac{\left(2a+2b \tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)(a^4-2a^2b^2+b^4)}{2(a+b)^{5/2}(a-b)^{5/2}}\right)}{e(a+b)^{5/2}(a-b)^{5/2}} - \frac{\frac{2(a^4-2a^2b^2+3b^4)}{3(a^4-2a^2b^2+b^4)} + \frac{4 \tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2 (a^6+3a^2b^4+b^6)}{b^2(a^4-2a^2b^2+b^4)} + \frac{2 \tan\left(\frac{d}{2}+\frac{ex}{2}\right)^4 (2a^6-3a^4b^2+b^6)}{b^2(a^4-2a^2b^2+b^4)}}{e\left(b^3 \tan\left(\frac{d}{2}+\frac{ex}{2}\right)^6 + \tan\left(\frac{d}{2}+\frac{ex}{2}\right)^3 (8a^3+12ab^2) + \tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a + b*\sin(d + e*x))/(b^2 + a^2*\sin(d + e*x)^2 + 2*a*b*\sin(d + e*x))^2,$
 $x)$

```
[Out] (2*a*b*atanh(((2*a + 2*b*tan(d/2 + (e*x)/2))*(a^4 + b^4 - 2*a^2*b^2))/(2*(a
+ b)^(5/2)*(a - b)^(5/2))))/(e*(a + b)^(5/2)*(a - b)^(5/2)) - ((2*(a^4 + 3
*b^4 - a^2*b^2))/(3*(a^4 + b^4 - 2*a^2*b^2)) + (4*tan(d/2 + (e*x)/2)^2*(a^6
+ b^6 + 3*a^2*b^4))/(b^2*(a^4 + b^4 - 2*a^2*b^2)) + (2*tan(d/2 + (e*x)/2)^
4*(2*a^6 + b^6 + 5*a^2*b^4 - 3*a^4*b^2))/(b^2*(a^4 + b^4 - 2*a^2*b^2)) + (2
*a*tan(d/2 + (e*x)/2)*(a^4 + 4*b^4))/(b*(a^4 + b^4 - 2*a^2*b^2)) + (2*a*tan
(d/2 + (e*x)/2)^5*(a^4 + 2*b^4 - 2*a^2*b^2))/(b*(a^4 + b^4 - 2*a^2*b^2)) +
(4*a*tan(d/2 + (e*x)/2)^3*(2*a^2 + 3*b^2)*(a^4 + 3*b^4 - a^2*b^2))/(3*b^3*(
a^4 + b^4 - 2*a^2*b^2)))/(e*(b^3*tan(d/2 + (e*x)/2)^6 + tan(d/2 + (e*x)/2)^
3*(12*a*b^2 + 8*a^3) + tan(d/2 + (e*x)/2)^2*(12*a^2*b + 3*b^3) + tan(d/2 +
(e*x)/2)^4*(12*a^2*b + 3*b^3) + b^3 + 6*a*b^2*tan(d/2 + (e*x)/2) + 6*a*b^2*
tan(d/2 + (e*x)/2)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**2,x)
```

```
[Out] Timed out
```

$$3.503 \quad \int \frac{d+e \sin(x)}{a+b \sin(x)+c \sin^2(x)} dx$$

Optimal. Leaf size=242

$$\frac{\sqrt{2} \left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left(\frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2} \sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} + \frac{\sqrt{2} \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\tan\left(\frac{x}{2}\right)(\sqrt{b^2-4ac}+b)+2c}{\sqrt{2} \sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

[Out] arctan(1/2*(2*c+(b-(-4*a*c+b^2)^(1/2))*tan(1/2*x))*2^(1/2)/(b^2-2*c*(a+c)-b*(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/(b^2-2*c*(a+c)-b*(-4*a*c+b^2)^(1/2))^(1/2)+arctan(1/2*(2*c+(b+(-4*a*c+b^2)^(1/2))*tan(1/2*x))*2^(1/2)/(b^2-2*c*(a+c)+b*(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/(b^2-2*c*(a+c)+b*(-4*a*c+b^2)^(1/2))^(1/2)

Rubi [A] time = 0.94, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3292, 2660, 618, 204}

$$\frac{\sqrt{2} \left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left(\frac{\tan\left(\frac{x}{2}\right)(b-\sqrt{b^2-4ac})+2c}{\sqrt{2} \sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}} + \frac{\sqrt{2} \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\tan\left(\frac{x}{2}\right)(\sqrt{b^2-4ac}+b)+2c}{\sqrt{2} \sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}} \right)}{\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*Sin[x])/(a + b*Sin[x] + c*Sin[x]^2),x]

[Out] (Sqrt[2]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (b - Sqrt[b^2 - 4*a*c])*Tan[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]])]/Sqrt[b^2 - 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]] + (Sqrt[2]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (b + Sqrt[b^2 - 4*a*c])*Tan[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]])]/Sqrt[b^2 - 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3292

Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_) + (c_.)*sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] := Module[{q = Rt[b^2

```
- 4*a*c, 2]], Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d + e \sin(x)}{a + b \sin(x) + c \sin^2(x)} dx = \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{b + \sqrt{b^2 - 4ac} + 2c \sin(x)} dx + \left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{b - \sqrt{b^2 - 4ac} + 2c \sin(x)} dx$$

$$= \left(2 \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \right) \text{Subst} \left[\int \frac{1}{b + \sqrt{b^2 - 4ac} + 4cx + (b + \sqrt{b^2 - 4ac})x^2} dx, x, \frac{2c \sin(x)}{1 + \sin(x)} \right]$$

$$= - \left(4 \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \right) \text{Subst} \left[\int \frac{1}{4 \left(4c^2 - (b + \sqrt{b^2 - 4ac})^2 \right) - x^2} dx, x, 4c + 2c \sin(x) \right]$$

$$= \frac{\sqrt{2} \left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{2c + (b - \sqrt{b^2 - 4ac}) \tan(\frac{x}{2})}{\sqrt{2} \sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 2c(a+c) - b\sqrt{b^2 - 4ac}}} + \frac{\sqrt{2} \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{2c - (b + \sqrt{b^2 - 4ac}) \tan(\frac{x}{2})}{\sqrt{2} \sqrt{b^2 - 2c(a+c) + b\sqrt{b^2 - 4ac}}} \right)}{\sqrt{b^2 - 2c(a+c) + b\sqrt{b^2 - 4ac}}}$$

Mathematica [C] time = 0.77, size = 286, normalized size = 1.18

$$\frac{\left(e \left(\sqrt{4ac - b^2} + ib \right) - 2icd \right) \tan^{-1} \left(\frac{2c + \tan\left(\frac{x}{2}\right) \left(b - i\sqrt{4ac - b^2} \right)}{\sqrt{2} \sqrt{-ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}} \right)}{\sqrt{-ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}} + \frac{\left(e \left(\sqrt{4ac - b^2} - ib \right) + 2icd \right) \tan^{-1} \left(\frac{2c + \tan\left(\frac{x}{2}\right) \left(b + i\sqrt{4ac - b^2} \right)}{\sqrt{2} \sqrt{ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}} \right)}{\sqrt{ib\sqrt{4ac - b^2} - 2c(a+c) + b^2}}$$

$$\frac{\hspace{10em}}{\sqrt{2ac - \frac{b^2}{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*Sin[x])/(a + b*Sin[x] + c*Sin[x]^2), x]
```

```
[Out] ((((-2*I)*c*d + (I*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(2*c + (b - I*Sqrt[-b^2 + 4*a*c])*Tan[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) - I*b*Sqrt[-b^2 + 4*a*c]])])]/Sqrt[b^2 - 2*c*(a + c) - I*b*Sqrt[-b^2 + 4*a*c]] + (((2*I)*c*d + ((-I)*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(2*c + (b + I*Sqrt[-b^2 + 4*a*c])*Tan[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) + I*b*Sqrt[-b^2 + 4*a*c]])])]/Sqrt[b^2 - 2*c*(a + c) + I*b*Sqrt[-b^2 + 4*a*c]]/Sqrt[-1/2*b^2 + 2*a*c]
```

fricas [B] time = 18.38, size = 6695, normalized size = 27.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2), x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*log(4*b*c^2*d^4 + 4*a*b*c*e^4 - 4*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 12*(a*b*c + b*c^2)*d^2*e^2 - 4
```


$$\begin{aligned}
& * (2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 + 2*((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*sin(x) + sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x) - ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*cos(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 2*(b^2*c*d^4 + a*b^2*e^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*sin(x)) - 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*d^4 + 4*a*b*c*e^4 - 4*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 12*(a*b*c + b*c^2)*d^2*e^2 - 4*(2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 - 2*((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x) + ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*cos(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 2*(b^2*c*d^4 + a*b^2*e^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*sin(x) + sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x) + ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*cos(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 2*(b^2*c*d^4 + a*b^2*e^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*
\end{aligned}$$

$$\begin{aligned}
& e^3) \sin(x)) + 1/4 \sqrt{2} \sqrt{-(b^2 - 2ac - 2c^2)d^2 - 2(ab - bc) \\
& *d^2 + (2a^2 - b^2 + 2ac)e^2 - (a^2b^2 - b^4 - 4ac^3 - (8a^2 - b^2) \\
& *c^2 - 2(2a^3 - 3ab^2)c) \sqrt{(b^2d^4 + b^2e^4 - 4(ab + bc)d^3e + 2(2a^2 + b^2 + 4ac + 2c^2)d^2e^2 - 4(ab + bc)d^2e^3) / (a^4b^2 \\
& - 2a^2b^4 + b^6 - 4ac^5 - (16a^2 - b^2)c^4 - 12(2a^3 - ab^2)c^3 - \\
& 2(8a^4 - 11a^2b^2 + b^4)c^2 - 4(a^5 - 3a^3b^2 + 2ab^4)c)}} / (a^2b^2 - b^4 - 4ac^3 - (8a^2 - b^2)c^2 - 2(2a^3 - 3ab^2)c) * \log(-4b \\
& *c^2d^4 - 4ab^2c^2e^4 + 4(b^2c + 2ac^2 + 2c^3)d^3e - 12(ab^2c + b \\
& c^2)d^2e^2 + 4(2ac^2 + (2a^2 + b^2)c)d^2e^3 + 2((4ac^4 + (8a^2 - \\
& b^2)c^3 + 2(2a^3 - 3ab^2)c^2 - (a^2b^2 - b^4)c)d^2 + (a^2b^3 - b \\
& ^5 - 4ab^2c^3 - (8a^2b - b^3)c^2 - 2(2a^3b - 3ab^3)c)d^2e - (a^3b^2 - ab^4 - 4a^2c^3 - (8a^3 - ab^2)c^2 - 2(2a^4 - 3a^2b^2)c)e^2) \\
& * \sqrt{(b^2d^4 + b^2e^4 - 4(ab + bc)d^3e + 2(2a^2 + b^2 + 4ac + 2c^2)d^2e^2 - 4(ab + bc)d^2e^3) / (a^4b^2 - 2a^2b^4 + b^6 - 4ac^5 \\
& - (16a^2 - b^2)c^4 - 12(2a^3 - ab^2)c^3 - 2(8a^4 - 11a^2b^2 + b^4)c^2 - 4(a^5 - 3a^3b^2 + 2ab^4)c)} * \sin(x) + \sqrt{2} * (((a^2b^4 - b^6 + 8ac^5 + 2(12a^2 - b^2)c^4 + 6(4a^3 - 3ab^2)c^3 + (8a^4 - 22a^2b^2 + 3b^4)c^2 - 2(3a^3b^2 - 4ab^4)c)d - (a^3b^3 - ab^5 + 4ab^2c^4 + (4a^2b - b^3)c^3 - (4a^3b + 5ab^3)c^2 - (4a^4b - 5a^2b^3 - b^5)c)e) * \sqrt{(b^2d^4 + b^2e^4 - 4(ab + bc)d^3e + 2(2a^2 + b^2 + 4ac + 2c^2)d^2e^2 - 4(ab + bc)d^2e^3) / (a^4b^2 - 2a^2b^4 + b^6 - 4ac^5 - (16a^2 - b^2)c^4 - 12(2a^3 - ab^2)c^3 - 2(8a^4 - 11a^2b^2 + b^4)c^2 - 4(a^5 - 3a^3b^2 + 2ab^4)c)} * \cos(x) + ((b^4 - 4ab^2c)d^3 - 3(ab^3 - 4ab^2c^2 - (4a^2b - b^3)c)d^2e + (2a^2b^2 + b^4 - 8a^3c - 8ac^3 - 2(8a^2 - b^2)c^2)d^2e^2 - (ab^3 - 4ab^2c^2 - (4a^2b - b^3)c)e^3) * \cos(x) * \sqrt{-(b^2 - 2ac - 2c^2)d^2 - 2(ab - bc)d^2 + (2a^2 - b^2 + 2ac)e^2 - (a^2b^2 - b^4 - 4ac^3 - (8a^2 - b^2)c^2 - 2(2a^3 - 3ab^2)c) \sqrt{(b^2d^4 + b^2e^4 - 4(ab + bc)d^3e + 2(2a^2 + b^2 + 4ac + 2c^2)d^2e^2 - 4(ab + bc)d^2e^3) / (a^4b^2 - 2a^2b^4 + b^6 - 4ac^5 - (16a^2 - b^2)c^4 - 12(2a^3 - ab^2)c^3 - 2(8a^4 - 11a^2b^2 + b^4)c^2 - 4(a^5 - 3a^3b^2 + 2ab^4)c)}} / (a^2b^2 - b^4 - 4ac^3 - (8a^2 - b^2)c^2 - 2(2a^3 - 3ab^2)c) - 2(b^2cd^4 + ab^2e^4 - (b^3 + 2ab^2c + 2b^2c^2)d^3e + 3(ab^2 + b^2c)d^2e^2 - (2a^2b + b^3 + 2ab^2c)d^2e^3) * \sin(x)) - 1/4 \sqrt{2} \sqrt{-(b^2 - 2ac - 2c^2)d^2 - 2(ab - bc)d^2 + (2a^2 - b^2 + 2ac)e^2 + (a^2b^2 - b^4 - 4ac^3 - (8a^2 - b^2)c^2 - 2(2a^3 - 3ab^2)c) \sqrt{(b^2d^4 + b^2e^4 - 4(ab + bc)d^3e + 2(2a^2 + b^2 + 4ac + 2c^2)d^2e^2 - 4(ab + bc)d^2e^3) / (a^4b^2 - 2a^2b^4 + b^6 - 4ac^5 - (16a^2 - b^2)c^4 - 12(2a^3 - ab^2)c^3 - 2(8a^4 - 11a^2b^2 + b^4)c^2 - 4(a^5 - 3a^3b^2 + 2ab^4)c)}} / (a^2b^2 - b^4 - 4ac^3 - (8a^2 - b^2)c^2 - 2(2a^3 - 3ab^2)c) * \log(-4b^2c + 2ac^2 + 2c^3)d^3e - 12(ab^2c + b^2c^2)d^2e^2 + 4(2ac^2 + (2a^2 + b^2)c)d^2e^3 - 2((4ac^4 + (8a^2 - b^2)c^3 + 2(2a^3 - 3ab^2)c^2 - (a^2b^2 - b^4)c)d^2 + (a^2b^3 - b^5 - 4ab^2c^3 - (8a^2b - b^3)c^2 - 2(2a^3b - 3ab^3)c)d^2e - (a^3b^2 - ab^4 - 4a^2c^3 - (8a^3 - ab^2)c^2 - 2(2a^4 - 3a^2b^2)c)e^2) * \sqrt{(b^2d^4 + b^2e^4 - 4(ab + bc)d^3e + 2(2a^2 + b^2 + 4ac + 2c^2)d^2e^2 - 4(ab + bc)d^2e^3) / (a^4b^2 - 2a^2b^4 + b^6 - 4ac^5 - (16a^2 - b^2)c^4 - 12(2a^3 - ab^2)c^3 - 2(8a^4 - 11a^2b^2 + b^4)c^2 - 4(a^5 - 3a^3b^2 + 2ab^4)c)} * \cos(x) - ((b^4 - 4ab^2c)d^3 - 3(ab^3 - 4ab^2c^2 - (4a^2b - b^3)c)d^2e + (2a^2b^2 + b^4 - 8a^3c - 8ac^3 - 2(8a^2 - b^2)c^2)d^2e^2 - (ab^3 - 4ab^2c^2 - (4a^2b - b^3)c)e^3) *
\end{aligned}$$

cos(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c) - 2*(b^2*c*d^4 + a*b^2*e^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*sin(x))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.18, size = 832, normalized size = 3.44

$$\frac{8a \arctan\left(\frac{2a \tan\left(\frac{x}{2}\right) + b + \sqrt{-4ac + b^2}}{\sqrt{4ac - 2b^2 - 2b\sqrt{-4ac + b^2} + 4a^2}}\right) dc}{(4ac - b^2) \sqrt{4ac - 2b^2 - 2b\sqrt{-4ac + b^2} + 4a^2}} - \frac{2 \arctan\left(\frac{2a \tan\left(\frac{x}{2}\right) + b + \sqrt{-4ac + b^2}}{\sqrt{4ac - 2b^2 - 2b\sqrt{-4ac + b^2} + 4a^2}}\right) db^2}{(4ac - b^2) \sqrt{4ac - 2b^2 - 2b\sqrt{-4ac + b^2} + 4a^2}} + \frac{4a \sqrt{-4ac + b^2}}{(4ac - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x)

[Out] 8*a/(4*a*c-b^2)/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2*a*tan(1/2*x)+b+(-4*a*c+b^2)^(1/2))/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*d*c-2/(4*a*c-b^2)/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2*a*tan(1/2*x)+b+(-4*a*c+b^2)^(1/2))/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*d*b^2+4*a*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2*a*tan(1/2*x)+b+(-4*a*c+b^2)^(1/2))/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*e-2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2*a*tan(1/2*x)+b+(-4*a*c+b^2)^(1/2))/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*d*b-8*a/(4*a*c-b^2)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*c+b^2)^(1/2)-b)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*d*c+2/(4*a*c-b^2)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*c+b^2)^(1/2)-b)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*d*b^2+4*a*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*c+b^2)^(1/2)-b)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*e-2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*c+b^2)^(1/2)-b)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))*d*b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e \sin(x) + d}{c \sin(x)^2 + b \sin(x) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x, algorithm="maxima")

[Out] integrate((e*sin(x) + d)/(c*sin(x)^2 + b*sin(x) + a), x)

mupad [B] time = 17.11, size = 10465, normalized size = 43.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d + e \cdot \sin(x))/(a + c \cdot \sin(x)^2 + b \cdot \sin(x)), x)$

[Out] $\text{atan}\left(\frac{(-b^4 d^2 - b^4 e^2 + 8 a^3 c^3 d^2 + b d^2 (-4 a c - b^2)^3)^{1/2} - 8 a^3 c^3 e^2 + b e^2 (-4 a c - b^2)^3)^{1/2} + 2 a^2 b^2 e^2 + 8 a^2 c^2 d^2 - 8 a^2 c^2 e^2 - 2 b^2 c^2 d^2 - 2 a b^3 d e - 2 a d e (-4 a c - b^2)^3)^{1/2} + 2 b^3 c d e - 2 c d e (-4 a c - b^2)^3)^{1/2} - 6 a b^2 c d^2 + 6 a b^2 c e^2 - 8 a b c^2 d e + 8 a^2 b c d e}{2(a^2 b^4 - b^6 + 16 a^2 c^4 + 32 a^3 c^3 + 16 a^4 c^2 + b^4 c^2 - 8 a b^2 c^3 - 8 a^3 b^2 c - 32 a^2 b^2 c^2 + 10 a b^4 c)}\right)^{1/2} \cdot \left(\frac{(-b^4 d^2 - b^4 e^2 + 8 a^3 c^3 d^2 + b d^2 (-4 a c - b^2)^3)^{1/2} - 8 a^3 c^3 e^2 + b e^2 (-4 a c - b^2)^3)^{1/2} + 2 a^2 b^2 e^2 + 8 a^2 c^2 d^2 - 8 a^2 c^2 e^2 - 2 b^2 c^2 d^2 - 2 a b^3 d e - 2 a d e (-4 a c - b^2)^3)^{1/2} + 2 b^3 c d e - 2 c d e (-4 a c - b^2)^3)^{1/2} - 6 a b^2 c d^2 + 6 a b^2 c e^2 - 8 a b c^2 d e + 8 a^2 b c d e}{2(a^2 b^4 - b^6 + 16 a^2 c^4 + 32 a^3 c^3 + 16 a^4 c^2 + b^4 c^2 - 8 a b^2 c^3 - 8 a^3 b^2 c - 32 a^2 b^2 c^2 + 10 a b^4 c)}\right)^{1/2} \cdot \left(\frac{(-b^4 d^2 - b^4 e^2 + 8 a^3 c^3 d^2 + b d^2 (-4 a c - b^2)^3)^{1/2} - 8 a^3 c^3 e^2 + b e^2 (-4 a c - b^2)^3)^{1/2} + 2 a^2 b^2 e^2 + 8 a^2 c^2 d^2 - 8 a^2 c^2 e^2 - 2 b^2 c^2 d^2 - 2 a b^3 d e - 2 a d e (-4 a c - b^2)^3)^{1/2} + 2 b^3 c d e - 2 c d e (-4 a c - b^2)^3)^{1/2} - 6 a b^2 c d^2 + 6 a b^2 c e^2 - 8 a b c^2 d e + 8 a^2 b c d e}{2(a^2 b^4 - b^6 + 16 a^2 c^4 + 32 a^3 c^3 + 16 a^4 c^2 + b^4 c^2 - 8 a b^2 c^3 - 8 a^3 b^2 c - 32 a^2 b^2 c^2 + 10 a b^4 c)}\right)^{1/2} \cdot \left(\tan\left(\frac{x}{2}\right) \cdot (96 a^3 b^4 + 256 a^4 c - 64 a^3 b^2 + 512 a^2 c^3 + 768 a^3 c^2 - 128 a b^2 c^2 - 576 a^2 b^2 c) + 32 a^2 b^3 + 128 a^2 b c^2 - 32 a b^3 c - 128 a^3 b c\right) + \tan\left(\frac{x}{2}\right) \cdot (64 a^2 b^2 e - 256 a^2 c^2 e - 64 a b^3 d - 256 a^3 c e + 256 a^2 b c d + 64 a b^2 c e) - 32 a^2 b^2 d + 128 a^2 c^2 d + 32 a b^3 e + 128 a^3 c d - 32 a b^2 c d - 128 a^2 b c e) - \tan\left(\frac{x}{2}\right) \cdot (64 a^3 e^2 + 32 a b^2 d^2 - 64 a b^2 e^2 - 128 a c^2 d^2 - 64 a^2 c d^2 + 128 a^2 c e^2 - 64 a^2 b d e + 128 a b c d e) + 32 a^2 b e^2 + 32 a b c d^2 - 128 a^2 c d e) \cdot i + \left(\frac{(-b^4 d^2 - b^4 e^2 + 8 a^3 c^3 d^2 + b d^2 (-4 a c - b^2)^3)^{1/2} - 8 a^3 c^3 e^2 + b e^2 (-4 a c - b^2)^3)^{1/2} + 2 a^2 b^2 e^2 + 8 a^2 c^2 d^2 - 8 a^2 c^2 e^2 - 2 b^2 c^2 d^2 - 2 a b^3 d e - 2 a d e (-4 a c - b^2)^3)^{1/2} + 2 b^3 c d e - 2 c d e (-4 a c - b^2)^3)^{1/2} - 6 a b^2 c d^2 + 6 a b^2 c e^2 - 8 a b c^2 d e + 8 a^2 b c d e}{2(a^2 b^4 - b^6 + 16 a^2 c^4 + 32 a^3 c^3 + 16 a^4 c^2 + b^4 c^2 - 8 a b^2 c^3 - 8 a^3 b^2 c - 32 a^2 b^2 c^2 + 10 a b^4 c)}\right)^{1/2} \cdot \left(\frac{(-b^4 d^2 - b^4 e^2 + 8 a^3 c^3 d^2 + b d^2 (-4 a c - b^2)^3)^{1/2} - 8 a^3 c^3 e^2 + b e^2 (-4 a c - b^2)^3)^{1/2} + 2 a^2 b^2 e^2 + 8 a^2 c^2 d^2 - 8 a^2 c^2 e^2 - 2 b^2 c^2 d^2 - 2 a b^3 d e - 2 a d e (-4 a c - b^2)^3)^{1/2} + 2 b^3 c d e - 2 c d e (-4 a c - b^2)^3)^{1/2} - 6 a b^2 c d^2 + 6 a b^2 c e^2 - 8 a b c^2 d e + 8 a^2 b c d e}{2(a^2 b^4 - b^6 + 16 a^2 c^4 + 32 a^3 c^3 + 16 a^4 c^2 + b^4 c^2 - 8 a b^2 c^3 - 8 a^3 b^2 c - 32 a^2 b^2 c^2 + 10 a b^4 c)}\right)^{1/2} \cdot \left(\tan\left(\frac{x}{2}\right) \cdot (96 a^3 b^4 + 256 a^4 c - 64 a^3 b^2 + 512 a^2 c^3 + 768 a^3 c^2 - 128 a b^2 c^2 - 576 a^2 b^2 c) + 32 a^2 b^3 + 128 a^2 b c^2 - 32 a b^3 c - 128 a^3 b c\right) - \tan\left(\frac{x}{2}\right) \cdot (64 a^2 b^2 e - 256 a^2 c^2 e - 64 a b^3 d - 256 a^3 c e + 256 a^2 b c d + 64 a b^2 c e) + 32 a^2 b^2 d - 128 a^2 c^2 d - 32 a b^3 e - 128 a^3 c d + 32 a b^2 c d + 128 a^2 b c e) - \tan\left(\frac{x}{2}\right) \cdot (64 a^3 e^2 + 32 a b^2 d^2 - 64 a b^2 e^2 - 128 a c^2 d^2 - 64 a^2 c d^2 + 128 a^2 c e^2 - 64 a^2 b d e + 128 a b c d e) + 32 a^2 b e^2$

$$\begin{aligned}
&^2 + 32*a*b*c*d^2 - 128*a^2*c*d*e)*1i)/(2*\tan(x/2)*(64*a^2*e^3 - 64*a*b*d*e \\
&^2 + 64*a*c*d^2*e) + (-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - \\
&b^2)^3)^{(1/2)} - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 \\
&+ 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(\\
&-(4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - \\
&6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 \\
&- b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3 \\
&b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c))^{(1/2)}*((-(b^4*d^2 - b^4*e^2 + 8*a \\
&c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b \\
&^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 \\
&- 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c*d*e - 2*c*d*e* \\
&-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + \\
&8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4 \\
&c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c))^{(1/2)}* \\
&((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3 \\
&c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8 \\
&a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
&+ 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b \\
&^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + \\
&32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c \\
&c^2 + 10*a*b^4*c))^{(1/2)}*(\tan(x/2)*(96*a*b^4 + 256*a^4*c - 64*a^3*b^2 + 51 \\
&2*a^2*c^3 + 768*a^3*c^2 - 128*a*b^2*c^2 - 576*a^2*b^2*c) + 32*a^2*b^3 + 128 \\
&a^2*b*c^2 - 32*a*b^3*c - 128*a^3*b*c) + \tan(x/2)*(64*a^2*b^2*e - 256*a^2*c \\
&^2*e - 64*a*b^3*d - 256*a^3*c*e + 256*a^2*b*c*d + 64*a*b^2*c*e) - 32*a^2*b^ \\
&2*d + 128*a^2*c^2*d + 32*a*b^3*e + 128*a^3*c*d - 32*a*b^2*c*d - 128*a^2*b*c \\
&*e) - \tan(x/2)*(64*a^3*e^2 + 32*a*b^2*d^2 - 64*a*b^2*e^2 - 128*a*c^2*d^2 - \\
&64*a^2*c*d^2 + 128*a^2*c*e^2 - 64*a^2*b*d*e + 128*a*b*c*d*e) + 32*a^2*b*e^2 \\
&+ 32*a*b*c*d^2 - 128*a^2*c*d*e) - (-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d \\
&^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
&+ 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d \\
&*e - 2*a*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^ \\
&2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e) \\
&)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a* \\
&b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c))^{(1/2)}*((-(b^4*d^2 - \\
&b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*c*e^2 + b*e^ \\
&2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 \\
&- 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c* \\
&d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8* \\
&a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + \\
&16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^ \\
&4*c))^{(1/2)}*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3) \\
&^{(1/2)} - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a \\
&^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c \\
&- b^2)^3)^{(1/2)} + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2 \\
&*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + \\
&16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c \\
&- 32*a^2*b^2*c^2 + 10*a*b^4*c))^{(1/2)}*(\tan(x/2)*(96*a*b^4 + 256*a^4*c - 6 \\
&4*a^3*b^2 + 512*a^2*c^3 + 768*a^3*c^2 - 128*a*b^2*c^2 - 576*a^2*b^2*c) + 32 \\
&a^2*b^3 + 128*a^2*b*c^2 - 32*a*b^3*c - 128*a^3*b*c) - \tan(x/2)*(64*a^2*b^2 \\
&*e - 256*a^2*c^2*e - 64*a*b^3*d - 256*a^3*c*e + 256*a^2*b*c*d + 64*a*b^2*c* \\
&e) + 32*a^2*b^2*d - 128*a^2*c^2*d - 32*a*b^3*e - 128*a^3*c*d + 32*a*b^2*c*d \\
&+ 128*a^2*b*c*e) - \tan(x/2)*(64*a^3*e^2 + 32*a*b^2*d^2 - 64*a*b^2*e^2 - 12 \\
&8*a*c^2*d^2 - 64*a^2*c*d^2 + 128*a^2*c*e^2 - 64*a^2*b*d*e + 128*a*b*c*d*e) \\
&+ 32*a^2*b*e^2 + 32*a*b*c*d^2 - 128*a^2*c*d*e) + 64*a^2*d*e^2 + 64*a*c*d^3 \\
&- 64*a*b*d^2*e))*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2) \\
&^3)^{(1/2)} - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + \\
&8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4* \\
&a*c - b^2)^3)^{(1/2)} + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a* \\
&b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^
\end{aligned}$$

$$\begin{aligned} &^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 \\ &*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + \\ &10*a*b^4*c)))^{(1/2)}*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - \\ &b^2)^3)^{(1/2)} - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 \\ &+ 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(\\ &-(4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - \\ &6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 \\ &- b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3 \\ &b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^{(1/2)}*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b \\ &^2)^3)^{(1/2)} - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 \\ &- 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c*d*e + 2*c*d*e*(\\ &-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + \\ &8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^{(1/2)}* \\ &(\tan(x/2)*(96*a*b^4 + 256*a^4*c - 64*a^3*b^2 + 512*a^2*c^3 + 768*a^3*c^2 - 1 \\ &28*a*b^2*c^2 - 576*a^2*b^2*c) + 32*a^2*b^3 + 128*a^2*b*c^2 - 32*a*b^3*c - 1 \\ &28*a^3*b*c) + \tan(x/2)*(64*a^2*b^2*e - 256*a^2*c^2*e - 64*a*b^3*d - 256*a^3 \\ &*c*e + 256*a^2*b*c*d + 64*a*b^2*c*e) - 32*a^2*b^2*d + 128*a^2*c^2*d + 32*a* \\ &b^3*e + 128*a^3*c*d - 32*a*b^2*c*d - 128*a^2*b*c*e) - \tan(x/2)*(64*a^3*e^2 \\ &+ 32*a*b^2*d^2 - 64*a*b^2*e^2 - 128*a*c^2*d^2 - 64*a^2*c*d^2 + 128*a^2*c*e^2 \\ &- 64*a^2*b*d*e + 128*a*b*c*d*e) + 32*a^2*b*e^2 + 32*a*b*c*d^2 - 128*a^2*c \\ &*d*e) - (-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\ &- 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2 \\ &*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2 \\ &)^3)^{(1/2)} + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 \\ &+ 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2 \\ &2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32* \\ &a^2*b^2*c^2 + 10*a*b^4*c)))^{(1/2)}*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*c*e^2 - b*e^2 \\ &2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 \\ &- 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c* \\ &d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8* \\ &a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + \\ &16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4 \\ &4*c)))^{(1/2)}*(\tan(x/2)*(96*a*b^4 + 256*a^4*c - 64*a^3*b^2 + 512*a^2*c^3 + 7 \\ &68*a^3*c^2 - 128*a*b^2*c^2 - 576*a^2*b^2*c) + 32*a^2*b^3 + 128*a^2*b*c^2 - \\ &32*a*b^3*c - 128*a^3*b*c) - \tan(x/2)*(64*a^2*b^2*e - 256*a^2*c^2*e - 64*a*b^3 \\ &^3*d - 256*a^3*c*e + 256*a^2*b*c*d + 64*a*b^2*c*e) + 32*a^2*b^2*d - 128*a^2 \\ &*c^2*d - 32*a*b^3*e - 128*a^3*c*d + 32*a*b^2*c*d + 128*a^2*b*c*e) - \tan(x/2 \\ &)*(64*a^3*e^2 + 32*a*b^2*d^2 - 64*a*b^2*e^2 - 128*a*c^2*d^2 - 64*a^2*c*d^2 \\ &+ 128*a^2*c*e^2 - 64*a^2*b*d*e + 128*a*b*c*d*e) + 32*a^2*b*e^2 + 32*a*b*c*d \\ &^2 - 128*a^2*c*d*e) + 64*a^2*d*e^2 + 64*a*c*d^3 - 64*a*b*d^2*e))*(-(b^4*d^2 \\ &- b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*c*e^2 - b \\ &e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 \\ &- 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 2*b^3 \\ &*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - \\ &8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 \\ &+ 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a \\ &*b^4*c)))^{(1/2)}*2i \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)**2),x)
```

```
[Out] Timed out
```


3.504 $\int (a+b \sin(d+ex)) (b^2 + 2ab \sin(d+ex) + a^2 \sin^2(d+ex)) dx$

Optimal. Leaf size=331

$$\frac{b \cos(d+ex) (a^2 \sin^2(d+ex) + 2ab \sin(d+ex) + b^2)^{3/2}}{4e} - \frac{(4a^2 + 3b^2) \cos(d+ex) (a^2 \sin^2(d+ex) + 2ab \sin(d+ex) + b^2)^{3/2}}{12e(a \sin(d+ex) + b)}$$

```
[Out] -1/4*b*cos(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)/e-1/6*(4*a^4+28*a^2*b^2+3*b^4)*cos(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)/e/(b+a*sin(e*x+d))^3-1/12*(4*a^2+3*b^2)*cos(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)/e/(b+a*sin(e*x+d))+5/8*a^4*b*(3*a^2+4*b^2)*x*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)/(a*b+a^2*sin(e*x+d))^3-1/24*a^4*b*(29*a^2+6*b^2)*cos(e*x+d)*sin(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)/e/(a*b+a^2*sin(e*x+d))^3
```

Rubi [A] time = 0.32, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {3290, 2753, 2734}

$$\frac{5a^4bx(3a^2 + 4b^2)(a^2 \sin^2(d+ex) + 2ab \sin(d+ex) + b^2)^{3/2}}{8(a^2 \sin(d+ex) + ab)^3} - \frac{a^4b(29a^2 + 6b^2) \sin(d+ex) \cos(d+ex)(a^2 \sin^2(d+ex) + 2ab \sin(d+ex) + b^2)^{3/2}}{24e(a^2 \sin(d+ex) + ab)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2), x]
```

```
[Out] -(b*Cos[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2))/(4*e) - ((4*a^4 + 28*a^2*b^2 + 3*b^4)*Cos[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2))/(6*e*(b + a*Sin[d + e*x])^3) - ((4*a^2 + 3*b^2)*Cos[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2))/(12*e*(b + a*Sin[d + e*x])) + (5*a^4*b*(3*a^2 + 4*b^2)*x*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2))/(8*(a*b + a^2*Sin[d + e*x])^3) - (a^4*b*(29*a^2 + 6*b^2)*Cos[d + e*x]*Sin[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2))/(24*e*(a*b + a^2*Sin[d + e*x])^3)
```

Rule 2734

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^m*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]
```

Rule 3290

```
Int[((A_) + (B_)*sin[(d_) + (e_)*(x_)])*((a_) + (b_)*sin[(d_) + (e_)*(x_)] + (c_)*sin[(d_) + (e_)*(x_)]^2)^n, x_Symbol] := Dist[(a + b*Sin[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
```

, B}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[n]

Rubi steps

$$\begin{aligned} \int (a + b \sin(d + ex)) (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2} dx &= \frac{(b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2}}{(2ab + 2a^2 \sin(d + ex))} \\ &= -\frac{b \cos(d + ex) (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2}}{4e} \\ &= -\frac{b \cos(d + ex) (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2}}{4e} \\ &= -\frac{b \cos(d + ex) (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2}}{4e} \end{aligned}$$

Mathematica [A] time = 0.84, size = 140, normalized size = 0.42

$$\frac{\sqrt{(a \sin(d + ex) + b)^2} (8a (a^3 + 3ab^2) \cos(3(d + ex)) + 3ab (20 (3a^2 + 4b^2) (d + ex) - 8 (4a^2 + 3b^2) \sin(2(d + ex))) - 8 (4a^2 + 3b^2) \sin(2(d + ex)))}{96e(a \sin(d + ex) + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2), x]

[Out] (Sqrt[(b + a*Sin[d + e*x])^2]*(-24*(3*a^4 + 21*a^2*b^2 + 4*b^4)*Cos[d + e*x] + 8*a*(a^3 + 3*a*b^2)*Cos[3*(d + e*x)] + 3*a*b*(20*(3*a^2 + 4*b^2)*(d + e*x) - 8*(4*a^2 + 3*b^2)*Sin[2*(d + e*x)] + a^2*Sin[4*(d + e*x)])))/(96*e*(b + a*Sin[d + e*x]))

fricas [A] time = 0.85, size = 112, normalized size = 0.34

$$\frac{8(a^4 + 3a^2b^2) \cos(ex + d)^3 + 15(3a^3b + 4ab^3)ex - 24(a^4 + 6a^2b^2 + b^4) \cos(ex + d) + 3(2a^3b \cos(ex + d)^3 - 24e}{24e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2), x, algorithm="fricas")

[Out] 1/24*(8*(a^4 + 3*a^2*b^2)*cos(e*x + d)^3 + 15*(3*a^3*b + 4*a*b^3)*e*x - 24*(a^4 + 6*a^2*b^2 + b^4)*cos(e*x + d) + 3*(2*a^3*b*cos(e*x + d)^3 - (17*a^3*b + 12*a*b^3)*cos(e*x + d))*sin(e*x + d))/e

giac [A] time = 5.03, size = 239, normalized size = 0.72

$$\frac{1}{32} a^3 b e^{(-1)} \operatorname{sgn}(a \sin(xe + d) + b) \sin(4xe + 4d) + \frac{1}{12} (a^4 \operatorname{sgn}(a \sin(xe + d) + b) + 3a^2 b^2 \operatorname{sgn}(a \sin(xe + d) + b)) \cos(3xe + 3d) e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2), x, algorithm="giac")

[Out] 1/32*a^3*b*e^(-1)*sgn(a*sin(x*e + d) + b)*sin(4*x*e + 4*d) + 1/12*(a^4*sgn(a*sin(x*e + d) + b) + 3*a^2*b^2*sgn(a*sin(x*e + d) + b))*cos(3*x*e + 3*d)*e

$$\begin{aligned} & \wedge(-1) - 1/4*(3*a^4*\text{sgn}(a*\sin(x*e + d) + b) + 21*a^2*b^2*\text{sgn}(a*\sin(x*e + d) \\ & + b) + 4*b^4*\text{sgn}(a*\sin(x*e + d) + b))*\cos(x*e + d)*e^(-1) - 1/4*(4*a^3*b*\text{sgn} \\ & \text{n}(a*\sin(x*e + d) + b) + 3*a*b^3*\text{sgn}(a*\sin(x*e + d) + b))*e^(-1)*\sin(2*x*e + \\ & 2*d) + 5/8*(3*a^3*b*\text{sgn}(a*\sin(x*e + d) + b) + 4*a*b^3*\text{sgn}(a*\sin(x*e + d) + \\ & b))*x \end{aligned}$$

maple [A] time = 0.79, size = 269, normalized size = 0.81

$$\frac{\left(-a^2\left(\cos^2(ex+d)\right)+2ab\sin(ex+d)+a^2+b^2\right)^{\frac{3}{2}}\left(6\left(\cos^3(ex+d)\right)\sin(ex+d)a^3b+8a^4\left(\cos^3(ex+d)\right)+\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x)

[Out]
$$\begin{aligned} & -1/24/e*(-a^2*\cos(e*x+d)^2+2*a*b*\sin(e*x+d)+a^2+b^2)^{(3/2)}*(6*\cos(e*x+d)^3* \\ & \sin(e*x+d)*a^3*b+8*a^4*\cos(e*x+d)^3+24*a^2*b^2*\cos(e*x+d)^3-51*\sin(e*x+d)*\cos \\ & \cos(e*x+d)*a^3*b-36*\cos(e*x+d)*\sin(e*x+d)*a*b^3-24*a^4*\cos(e*x+d)-144*a^2*b^2* \\ & 2*\cos(e*x+d)-24*\cos(e*x+d)*b^4+45*(e*x+d)*a^3*b+60*(e*x+d)*a*b^3-16*a^4-120 \\ & *a^2*b^2-24*b^4)/(\cos(e*x+d)^2*\sin(e*x+d)*a^3+3*\cos(e*x+d)^2*a^2*b-a^3*\sin \\ & (e*x+d)-3*\sin(e*x+d)*a*b^2-3*a^2*b-b^3) \end{aligned}$$

maxima [A] time = 0.45, size = 556, normalized size = 1.68

$$4 \left(3 \left(3 a^2 b + 2 b^3 \right) \arctan \left(\frac{\sin(ex+d)}{\cos(ex+d)+1} \right) - \frac{4 a^3 + 18 a b^2 + \frac{9 a^2 b \sin(ex+d)}{\cos(ex+d)+1} + \frac{18 a b^2 \sin(ex+d)^4}{(\cos(ex+d)+1)^4} - \frac{9 a^2 b \sin(ex+d)^5}{(\cos(ex+d)+1)^5} + \frac{12 (a^3 + 3 a b^2) \sin(ex+d)^2}{(\cos(ex+d)+1)^2}}{\frac{3 \sin(ex+d)^2}{(\cos(ex+d)+1)^2} + \frac{3 \sin(ex+d)^4}{(\cos(ex+d)+1)^4} + \frac{\sin(ex+d)^6}{(\cos(ex+d)+1)^6} + 1} \right) a + 3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x, algorithm="maxima")

[Out]
$$\begin{aligned} & 1/12*(4*(3*(3*a^2*b + 2*b^3)*\arctan(\sin(e*x + d)/(\cos(e*x + d) + 1)) - (4*a \\ & ^3 + 18*a*b^2 + 9*a^2*b*\sin(e*x + d)/(\cos(e*x + d) + 1) + 18*a*b^2*\sin(e*x \\ & + d)^4/(\cos(e*x + d) + 1)^4 - 9*a^2*b*\sin(e*x + d)^5/(\cos(e*x + d) + 1)^5 + \\ & 12*(a^3 + 3*a*b^2)*\sin(e*x + d)^2/(\cos(e*x + d) + 1)^2)/(3*\sin(e*x + d)^2/ \\ & (\cos(e*x + d) + 1)^2 + 3*\sin(e*x + d)^4/(\cos(e*x + d) + 1)^4 + \sin(e*x + d) \\ & ^6/(\cos(e*x + d) + 1)^6 + 1))*a + 3*(3*(a^3 + 4*a*b^2)*\arctan(\sin(e*x + d)/ \\ & (\cos(e*x + d) + 1)) - (16*a^2*b + 8*b^3 + 8*b^3*\sin(e*x + d)^6/(\cos(e*x + d) \\ & + 1)^6 + 3*(a^3 + 4*a*b^2)*\sin(e*x + d)/(\cos(e*x + d) + 1) + 8*(8*a^2*b + \\ & 3*b^3)*\sin(e*x + d)^2/(\cos(e*x + d) + 1)^2 + (11*a^3 + 12*a*b^2)*\sin(e*x + \\ & d)^3/(\cos(e*x + d) + 1)^3 + 24*(2*a^2*b + b^3)*\sin(e*x + d)^4/(\cos(e*x + d) \\ & + 1)^4 - (11*a^3 + 12*a*b^2)*\sin(e*x + d)^5/(\cos(e*x + d) + 1)^5 - 3*(a^3 \\ & + 4*a*b^2)*\sin(e*x + d)^7/(\cos(e*x + d) + 1)^7)/(4*\sin(e*x + d)^2/(\cos(e*x \\ & + d) + 1)^2 + 6*\sin(e*x + d)^4/(\cos(e*x + d) + 1)^4 + 4*\sin(e*x + d)^6/(\cos \\ & (e*x + d) + 1)^6 + \sin(e*x + d)^8/(\cos(e*x + d) + 1)^8 + 1))*b)/e \end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \sin(d + e x)) \left(a^2 \sin(d + e x)^2 + 2 a b \sin(d + e x) + b^2 \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(3/2),x)

[Out] int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.505 $\int (a+b \sin(d+ex))\sqrt{b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex)}$

Optimal. Leaf size=185

$$\frac{3a^2bx\sqrt{a^2 \sin^2(d + ex) + 2ab \sin(d + ex) + b^2}}{2(a^2 \sin(d + ex) + ab)} - \frac{a^2b \sin(d + ex) \cos(d + ex)\sqrt{a^2 \sin^2(d + ex) + 2ab \sin(d + ex)}}{2e(a^2 \sin(d + ex) + ab)}$$

[Out] $-(a^2+b^2)*\cos(e*x+d)*(b^2+2*a*b*\sin(e*x+d)+a^2*\sin(e*x+d)^2)^{(1/2)}/e/(b+a*\sin(e*x+d))+3/2*a^2*b*x*(b^2+2*a*b*\sin(e*x+d)+a^2*\sin(e*x+d)^2)^{(1/2)}/(a*b+a^2*\sin(e*x+d))-1/2*a^2*b*\cos(e*x+d)*\sin(e*x+d)*(b^2+2*a*b*\sin(e*x+d)+a^2*\sin(e*x+d)^2)^{(1/2)}/e/(a*b+a^2*\sin(e*x+d))$

Rubi [A] time = 0.11, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$, Rules used = {3290, 2734}

$$\frac{3a^2bx\sqrt{a^2 \sin^2(d + ex) + 2ab \sin(d + ex) + b^2}}{2(a^2 \sin(d + ex) + ab)} - \frac{a^2b \sin(d + ex) \cos(d + ex)\sqrt{a^2 \sin^2(d + ex) + 2ab \sin(d + ex)}}{2e(a^2 \sin(d + ex) + ab)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[d + e*x])*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2], x]

[Out] $-(((a^2 + b^2)*\text{Cos}[d + e*x]*\text{Sqrt}[b^2 + 2*a*b*\text{Sin}[d + e*x] + a^2*\text{Sin}[d + e*x]^2])/(e*(b + a*\text{Sin}[d + e*x]))) + (3*a^2*b*x*\text{Sqrt}[b^2 + 2*a*b*\text{Sin}[d + e*x] + a^2*\text{Sin}[d + e*x]^2])/(2*(a*b + a^2*\text{Sin}[d + e*x])) - (a^2*b*\text{Cos}[d + e*x]*\text{Sin}[d + e*x]*\text{Sqrt}[b^2 + 2*a*b*\text{Sin}[d + e*x] + a^2*\text{Sin}[d + e*x]^2])/(2*e*(a*b + a^2*\text{Sin}[d + e*x]))$

Rule 2734

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]) , x_Symbol] := Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3290

Int[((A_) + (B_)*sin[(d_) + (e_)*(x_)])*((a_) + (b_)*sin[(d_) + (e_)*(x_)]) + (c_)*sin[(d_) + (e_)*(x_)]^2)^n, x_Symbol] := Dist[(a + b*Sin[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[n]

Rubi steps

$$\int (a + b \sin(d + ex))\sqrt{b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex)} dx = \frac{\sqrt{b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex)} \int \frac{1}{2ab + 2a^2 \sin(d + ex)} dx}{1} = -\frac{(a^2 + b^2) \cos(d + ex)\sqrt{b^2 + 2ab \sin(d + ex)}}{e(b + a \sin(d + ex))}$$

Mathematica [A] time = 0.19, size = 70, normalized size = 0.38

$$-\frac{\sqrt{(a \sin(d + ex) + b)^2} (4(a^2 + b^2) \cos(d + ex) + ab(\sin(2(d + ex)) - 6(d + ex)))}{4e(a \sin(d + ex) + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[d + e*x])*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2], x]

[Out] -1/4*(Sqrt[(b + a*Sin[d + e*x])^2]*(4*(a^2 + b^2)*Cos[d + e*x] + a*b*(-6*(d + e*x) + Sin[2*(d + e*x)])))/(e*(b + a*Sin[d + e*x]))

fricas [A] time = 0.87, size = 43, normalized size = 0.23

$$\frac{3 abex - ab \cos(ex + d) \sin(ex + d) - 2(a^2 + b^2) \cos(ex + d)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2), x, algorithm="fricas")

[Out] 1/2*(3*a*b*e*x - a*b*cos(e*x + d)*sin(e*x + d) - 2*(a^2 + b^2)*cos(e*x + d))/e

giac [A] time = 0.21, size = 98, normalized size = 0.53

$$-a^2 \cos(xe + d) e^{(-1)} \operatorname{sgn}(a \sin(xe + d) + b) - b^2 \cos(xe + d) e^{(-1)} \operatorname{sgn}(a \sin(xe + d) + b) - \frac{1}{4} abe^{(-1)} \operatorname{sgn}(a \sin(xe + d) + b)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2), x, algorithm="giac")

[Out] -a^2*cos(x*e + d)*e^(-1)*sgn(a*sin(x*e + d) + b) - b^2*cos(x*e + d)*e^(-1)*sgn(a*sin(x*e + d) + b) - 1/4*a*b*e^(-1)*sgn(a*sin(x*e + d) + b)*sin(2*x*e + 2*d) + 3/2*a*b*x*sgn(a*sin(x*e + d) + b)

maple [A] time = 0.59, size = 107, normalized size = 0.58

$$\frac{\sqrt{-a^2 (\cos^2(ex + d)) + 2ab \sin(ex + d) + a^2 + b^2} (\sin(ex + d) \cos(ex + d) ab + 2a^2 \cos(ex + d) + 2 \cos(ex + d))}{2e (b + a \sin(ex + d))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2), x)

[Out] -1/2/e*(-a^2*cos(e*x+d)^2+2*a*b*sin(e*x+d)+a^2+b^2)^(1/2)*(sin(e*x+d)*cos(e*x+d)*a*b+2*a^2*cos(e*x+d)+2*cos(e*x+d)*b^2-3*(e*x+d)*a*b+2*a^2+2*b^2)/(b+a*sin(e*x+d))

maxima [A] time = 0.43, size = 187, normalized size = 1.01

$$\frac{2 \left(b \arctan \left(\frac{\sin(ex+d)}{\cos(ex+d)+1} \right) - \frac{a}{\frac{\sin(ex+d)^2}{(\cos(ex+d)+1)^2} + 1} \right) a + \left(a \arctan \left(\frac{\sin(ex+d)}{\cos(ex+d)+1} \right) - \frac{2b + \frac{a \sin(ex+d)}{\cos(ex+d)+1} + \frac{2b \sin(ex+d)^2}{(\cos(ex+d)+1)^2} - \frac{a \sin(ex+d)^3}{(\cos(ex+d)+1)^3}}{\frac{2 \sin(ex+d)^2}{(\cos(ex+d)+1)^2} + \frac{\sin(ex+d)^4}{(\cos(ex+d)+1)^4} + 1} \right) b}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2), x, algorithm="maxima")

[Out] (2*(b*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) - a/(sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + 1))*a + (a*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) - (2*b + a*sin(e*x + d)/(cos(e*x + d) + 1) + 2*b*sin(e*x + d)^2/(cos(e*x + d) + 1)^2

- $a \sin(e x + d)^3 / (\cos(e x + d) + 1)^3 / (2 \sin(e x + d)^2 / (\cos(e x + d) + 1)^2 + \sin(e x + d)^4 / (\cos(e x + d) + 1)^4 + 1) * b) / e$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \sin(d + e x)) \sqrt{a^2 \sin(d + e x)^2 + 2 a b \sin(d + e x) + b^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(1/2), x)`

[Out] `int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sin(d + e x)) \sqrt{(a \sin(d + e x) + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(1/2), x)`

[Out] `Integral((a + b*sin(d + e*x))*sqrt((a*sin(d + e*x) + b)**2), x)`

$$3.506 \quad \int \frac{a+b \sin(d+ex)}{\sqrt{b^2+2ab \sin(d+ex)+a^2 \sin^2(d+ex)}} dx$$

Optimal. Leaf size=137

$$\frac{bx(a \sin(d+ex) + b)}{a\sqrt{a^2 \sin^2(d+ex) + 2ab \sin(d+ex) + b^2}} - \frac{2\sqrt{a^2 - b^2} (a \sin(d+ex) + b) \tanh^{-1}\left(\frac{a+b \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{ae\sqrt{a^2 \sin^2(d+ex) + 2ab \sin(d+ex) + b^2}}$$

[Out] b*x*(b+a*sin(e*x+d))/a/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2)-2*arctanh((a+b*tan(1/2*e*x+1/2*d))/(a^2-b^2)^(1/2))*(b+a*sin(e*x+d))*(a^2-b^2)^(1/2)/a/e/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2)

Rubi [A] time = 0.20, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3290, 2735, 2660, 618, 206}

$$\frac{bx(a \sin(d+ex) + b)}{a\sqrt{a^2 \sin^2(d+ex) + 2ab \sin(d+ex) + b^2}} - \frac{2\sqrt{a^2 - b^2} (a \sin(d+ex) + b) \tanh^{-1}\left(\frac{a+b \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{ae\sqrt{a^2 \sin^2(d+ex) + 2ab \sin(d+ex) + b^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sin[d + e*x])/Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2], x]

[Out] (b*x*(b + a*Sin[d + e*x]))/(a*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2]) - (2*Sqrt[a^2 - b^2]*ArcTanh[(a + b*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Sin[d + e*x]))/(a*e*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3290


```
Int[((A_) + (B_)*sin[(d_) + (e_)*(x_)])*((a_) + (b_)*sin[(d_) + (e_)*
(x_)] + (c_)*sin[(d_) + (e_)*(x_)]^2)^n, x_Symbol] := Dist[(a + b*Sin
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n), Int[(A + B*Si
n[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned} \int \frac{a + b \sin(d + ex)}{\sqrt{b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex)}} dx &= \frac{(2ab + 2a^2 \sin(d + ex)) \int \frac{a + b \sin(d + ex)}{2ab + 2a^2 \sin(d + ex)} dx}{\sqrt{b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex)}} \\ &= \frac{bx(b + a \sin(d + ex))}{a\sqrt{b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex)}} - \frac{((-2a^3 + 2ab^2))}{2a^2\sqrt{b^2}} \\ &= \frac{bx(b + a \sin(d + ex))}{a\sqrt{b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex)}} - \frac{((-2a^3 + 2ab^2))}{2a^2\sqrt{b^2}} \\ &= \frac{bx(b + a \sin(d + ex))}{a\sqrt{b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex)}} + \frac{2(-2a^3 + 2ab^2)}{2a^2\sqrt{b^2}} \\ &= \frac{bx(b + a \sin(d + ex))}{a\sqrt{b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex)}} - \frac{2\sqrt{a^2 - b^2} \tanh^{-1}\left(\frac{a + b \sin(d + ex)}{\sqrt{a^2 - b^2}}\right)}{ae\sqrt{b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex)}} \end{aligned}$$

Mathematica [A] time = 0.16, size = 85, normalized size = 0.62

$$\frac{(a \sin(d + ex) + b) \left(b(d + ex) - 2\sqrt{b^2 - a^2} \tan^{-1} \left(\frac{a + b \tan\left(\frac{1}{2}(d + ex)\right)}{\sqrt{b^2 - a^2}} \right) \right)}{ae\sqrt{(a \sin(d + ex) + b)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])/Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d +
e*x]^2], x]
```

```
[Out] ((b*(d + e*x) - 2*Sqrt[-a^2 + b^2]*ArcTan[(a + b*Tan[(d + e*x)/2])/Sqrt[-a^
2 + b^2]])*(b + a*Sin[d + e*x]))/(a*e*Sqrt[(b + a*Sin[d + e*x])^2])
```

fricas [A] time = 0.92, size = 204, normalized size = 1.49

$$\left[\frac{2bex + \sqrt{a^2 - b^2} \log\left(-\frac{(a^2 - 2b^2)\cos(ex+d)^2 + 2ab\sin(ex+d) + a^2 + b^2 - 2(b\cos(ex+d)\sin(ex+d) + a\cos(ex+d))\sqrt{a^2 - b^2}}{a^2\cos(ex+d)^2 - 2ab\sin(ex+d) - a^2 - b^2}\right)}{2ae}, \frac{bex - \sqrt{-a^2}}{ae} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2), x,
algorithm="fricas")
```

```
[Out] [1/2*(2*b*e*x + sqrt(a^2 - b^2))*log(-((a^2 - 2*b^2)*cos(e*x + d)^2 + 2*a*b*
sin(e*x + d) + a^2 + b^2 - 2*(b*cos(e*x + d)*sin(e*x + d) + a*cos(e*x + d))
*sqrt(a^2 - b^2))/(a^2*cos(e*x + d)^2 - 2*a*b*sin(e*x + d) - a^2 - b^2)))/(
a*e), (b*e*x - sqrt(-a^2 + b^2))*arctan(-sqrt(-a^2 + b^2)*(b*sin(e*x + d) +
a)/((a^2 - b^2)*cos(e*x + d)))/(a*e)]
```

giac [A] time = 0.38, size = 208, normalized size = 1.52

$$\left(\frac{\left(x e - 2 \pi \left\lfloor \frac{x e + d}{2 \pi} + \frac{1}{2} \right\rfloor + d \right) b}{\operatorname{asgn} \left(b \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right)^4 + 2 a \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right)^3 + 2 b \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right)^2 + 2 a \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) + b \right)} + \frac{1}{\sqrt{-a^2 + b^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
algorithm="giac")

[Out] ((x*e - 2*pi*floor(1/2*(x*e + d)/pi + 1/2) + d)*b/(a*sgn(b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + b)) + 2*(a^2 - b^2)*arctan((b*tan(1/2*x*e + 1/2*d) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a*sgn(b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + b))) * e^(-1)

maple [A] time = 0.46, size = 176, normalized size = 1.28

$$\frac{\left(2 \arctan \left(\frac{b \cos(ex+d) - a \sin(ex+d) - b}{\sin(ex+d) \sqrt{-a^2 + b^2}} \right) a^2 - 2 \arctan \left(\frac{b \cos(ex+d) - a \sin(ex+d) - b}{\sin(ex+d) \sqrt{-a^2 + b^2}} \right) b^2 - b (ex + d) \sqrt{-a^2 + b^2} \right) (b + a \sin(ex + d))}{e \sqrt{-a^2} (\cos^2(ex + d)) + 2ab \sin(ex + d) + a^2 + b^2 a \sqrt{-a^2 + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x)

[Out] -1/e*(2*arctan((b*cos(e*x+d)-a*sin(e*x+d)-b)/sin(e*x+d)/(-a^2+b^2)^(1/2))*a^2-2*arctan((b*cos(e*x+d)-a*sin(e*x+d)-b)/sin(e*x+d)/(-a^2+b^2)^(1/2))*b^2-b*(e*x+d)*(-a^2+b^2)^(1/2))*(b+a*sin(e*x+d))/(-a^2*cos(e*x+d)^2+2*a*b*sin(e*x+d)+a^2+b^2)^(1/2)/a/(-a^2+b^2)^(1/2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \sin(d + ex)}{\sqrt{a^2 \sin(d + ex)^2 + 2ab \sin(d + ex) + b^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(1/2),x)

[Out] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sin(d + ex)}{\sqrt{(a \sin(d + ex) + b)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(1/2),x)

[Out] Integral((a + b*sin(d + e*x))/sqrt((a*sin(d + e*x) + b)**2), x)

$$3.507 \quad \int \frac{a+b \sin(d+ex)}{(b^2+2ab \sin(d+ex)+a^2 \sin^2(d+ex))^{3/2}} dx$$

Optimal. Leaf size=239

$$\frac{\cos(d+ex)(a \sin(d+ex)+b)}{2e(a^2 \sin^2(d+ex)+2ab \sin(d+ex)+b^2)^{3/2}} - \frac{(a^2 \sin(d+ex)+ab)^3 \tanh^{-1}\left(\frac{a+b \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{a^2 e (a^2-b^2)^{3/2} (a^2 \sin^2(d+ex)+2ab \sin(d+ex)+b^2)^{3/2}} + \frac{1}{2e(a^2 \sin^2(d+ex)+2ab \sin(d+ex)+b^2)^{3/2}}$$

[Out] $-1/2*\cos(e*x+d)*(b+a*\sin(e*x+d))/e/(b^2+2*a*b*\sin(e*x+d)+a^2*\sin(e*x+d)^2)^{(3/2)}-\operatorname{arctanh}((a+b*\tan(1/2*e*x+1/2*d))/(a^2-b^2)^{(1/2)})*(a*b+a^2*\sin(e*x+d))^3/a^2/(a^2-b^2)^{(3/2)}/e/(b^2+2*a*b*\sin(e*x+d)+a^2*\sin(e*x+d)^2)^{(3/2)}+1/2*b*\cos(e*x+d)*(a*b+a^2*\sin(e*x+d))^3/(a^2-b^2)/e/(a^3*b+a^4*\sin(e*x+d))/(b^2+2*a*b*\sin(e*x+d)+a^2*\sin(e*x+d)^2)^{(3/2)}$

Rubi [A] time = 0.27, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {3290, 2754, 12, 2660, 618, 206}

$$\frac{b \cos(d+ex)(a^2 \sin(d+ex)+ab)^3}{2e(a^2-b^2)(a^3b+a^4 \sin(d+ex))(a^2 \sin^2(d+ex)+2ab \sin(d+ex)+b^2)^{3/2}} - \frac{\cos(d+ex)(a \sin(d+ex)+b)}{2e(a^2 \sin^2(d+ex)+2ab \sin(d+ex)+b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a+b*\sin[d+e*x])/(b^2+2*a*b*\sin[d+e*x]+a^2*\sin[d+e*x]^2)^{(3/2)},x]$

[Out] $-(\cos[d+e*x]*(b+a*\sin[d+e*x]))/(2*e*(b^2+2*a*b*\sin[d+e*x]+a^2*\sin[d+e*x]^2)^{(3/2)})-(\operatorname{ArcTanh}[(a+b*\tan[(d+e*x)/2])/ \operatorname{Sqrt}[a^2-b^2]])*(a*b+a^2*\sin[d+e*x]^3)/(a^2*(a^2-b^2)^{(3/2)}*e*(b^2+2*a*b*\sin[d+e*x]+a^2*\sin[d+e*x]^2)^{(3/2)})+(b*\cos[d+e*x]*(a*b+a^2*\sin[d+e*x]^3)/(2*(a^2-b^2)*e*(a^3*b+a^4*\sin[d+e*x])*(b^2+2*a*b*\sin[d+e*x]+a^2*\sin[d+e*x]^2)^{(3/2)}))$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$ $\operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[u, (b_*)(v_)] /;$ $\operatorname{FreeQ}[b, x]$

Rule 206

$\operatorname{Int}(((a_)+(b_.)*(x_)^2)^{-1}), x_Symbol] := \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}[\{a, b\}, x] \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 618

$\operatorname{Int}(((a_.)+(b_.)*(x_)+(c_.)*(x_)^2)^{-1}), x_Symbol] := \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Simp}[b^2-4*a*c-x^2, x], x], x, b+2*c*x], x] /;$ $\operatorname{FreeQ}[\{a, b, c\}, x] \ \&\& \ \operatorname{NeQ}[b^2-4*a*c, 0]$

Rule 2660

$\operatorname{Int}(((a_)+(b_.)*\sin[(c_.)+(d_.)*(x_)])^{-1}), x_Symbol] := \operatorname{With}[\{e = \operatorname{FreeFactors}[\tan[(c+d*x)/2], x]\}, \operatorname{Dist}[(2*e)/d, \operatorname{Subst}[\operatorname{Int}[1/(a+2*b*e*x+a*e^2*x^2), x], x, \tan[(c+d*x)/2]/e], x] /;$ $\operatorname{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \operatorname{NeQ}[\dots]$

$a^2 - b^2, 0]$

Rule 2754

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

Rule 3290

Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*(x_)]) + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] := Dist[(a + b*Sin[d + e*x] + c*Sin[d + e*x]^2)^(n)/(b + 2*c*Sin[d + e*x])^(2*n), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[n]

Rubi steps

$$\int \frac{a + b \sin(d + ex)}{(b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2}} dx = \frac{(2ab + 2a^2 \sin(d + ex))^3 \int \frac{a + b \sin(d + ex)}{(2ab + 2a^2 \sin(d + ex))^3} dx}{(b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2}}$$

$$= -\frac{\cos(d + ex)(b + a \sin(d + ex))}{2e (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2}} + \frac{(2ab + 2a^2 \sin(d + ex))^3}{8a^2 (a^2 - b^2)}$$

$$= -\frac{\cos(d + ex)(b + a \sin(d + ex))}{2e (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2}} + \frac{(2ab + 2a^2 \sin(d + ex))^3}{4a (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2}}$$

$$= -\frac{\cos(d + ex)(b + a \sin(d + ex))}{2e (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2}} + \frac{(2ab + 2a^2 \sin(d + ex))^3}{2 (a^2 - b^2)}$$

$$= -\frac{\cos(d + ex)(b + a \sin(d + ex))}{2e (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2}} + \frac{(2ab + 2a^2 \sin(d + ex))^3}{2 (a^2 - b^2)}$$

$$= -\frac{\cos(d + ex)(b + a \sin(d + ex))}{2e (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2}} + \frac{(2ab + 2a^2 \sin(d + ex))^3}{2 (a^2 - b^2)}$$

$$= -\frac{\cos(d + ex)(b + a \sin(d + ex))}{2e (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2}} + \frac{(2ab + 2a^2 \sin(d + ex))^3}{2 (a^2 - b^2)}$$

$$= -\frac{\cos(d + ex)(b + a \sin(d + ex))}{2e (b^2 + 2ab \sin(d + ex) + a^2 \sin^2(d + ex))^{3/2}} - \frac{\tan^{-1}\left(\frac{a + b \sin(d + ex)}{a^2 - b^2}\right)}{a^2 (a^2 - b^2)}$$

Mathematica [A] time = 0.35, size = 144, normalized size = 0.60

$$\frac{\sqrt{b^2 - a^2} \cos(d + ex) \left(a^2 - ab \sin(d + ex) - 2b^2 \right) - 2a(a \sin(d + ex) + b)^2 \tan^{-1} \left(\frac{a + b \tan\left(\frac{1}{2}(d + ex)\right)}{\sqrt{b^2 - a^2}} \right)}{2e(b - a)(a + b)\sqrt{b^2 - a^2} (a \sin(d + ex) + b)\sqrt{(a \sin(d + ex) + b)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2), x]

[Out] (-2*a*ArcTan[(a + b*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]]*(b + a*Sin[d + e*x])^2 + Sqrt[-a^2 + b^2]*Cos[d + e*x]*(a^2 - 2*b^2 - a*b*Sin[d + e*x]))/(2*(-a + b)*(a + b)*Sqrt[-a^2 + b^2]*e*(b + a*Sin[d + e*x])*Sqrt[(b + a*Sin[d + e*x])^2])

fricas [A] time = 1.65, size = 527, normalized size = 2.21

$$\left[\frac{2(a^3b - ab^3) \cos(ex + d) \sin(ex + d) + (a^3 \cos(ex + d)^2 - 2a^2b \sin(ex + d) - a^3 - ab^2) \sqrt{a^2 - b^2} \log\left(\frac{(a^2 - 2b^2)}{\dots}\right)}{4((a^6 - 2a^4b^2 + a^2b^4)e \cos(ex + d)^2 - 2(a^5b - 2a^3b^3 + \dots))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2), x, algorithm="fricas")

[Out] [-1/4*(2*(a^3*b - a*b^3)*cos(e*x + d)*sin(e*x + d) + (a^3*cos(e*x + d)^2 - 2*a^2*b*sin(e*x + d) - a^3 - a*b^2)*sqrt(a^2 - b^2)*log(((a^2 - 2*b^2)*cos(e*x + d)^2 + 2*a*b*sin(e*x + d) + a^2 + b^2 + 2*(b*cos(e*x + d)*sin(e*x + d) + a*cos(e*x + d))*sqrt(a^2 - b^2))/(a^2*cos(e*x + d)^2 - 2*a*b*sin(e*x + d) - a^2 - b^2)) - 2*(a^4 - 3*a^2*b^2 + 2*b^4)*cos(e*x + d))/((a^6 - 2*a^4*b^2 + a^2*b^4)*e*cos(e*x + d)^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*e*sin(e*x + d) - (a^6 - a^4*b^2 - a^2*b^4 + b^6)*e), -1/2*((a^3*b - a*b^3)*cos(e*x + d)*sin(e*x + d) + (a^3*cos(e*x + d)^2 - 2*a^2*b*sin(e*x + d) - a^3 - a*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(e*x + d) + a)/((a^2 - b^2)*cos(e*x + d))) - (a^4 - 3*a^2*b^2 + 2*b^4)*cos(e*x + d))/((a^6 - 2*a^4*b^2 + a^2*b^4)*e*cos(e*x + d)^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*e*sin(e*x + d) - (a^6 - a^4*b^2 - a^2*b^4 + b^6)*e)]

giac [B] time = 0.75, size = 479, normalized size = 2.00

$$\left(\frac{a \arctan\left(\frac{b \tan\left(\frac{1}{2} x e + \frac{1}{2} d\right)}{\sqrt{-a^2}}\right)}{\left(a^2 \operatorname{sgn}\left(b \tan\left(\frac{1}{2} x e + \frac{1}{2} d\right)\right)^4 + 2 a \tan\left(\frac{1}{2} x e + \frac{1}{2} d\right)^3 + 2 b \tan\left(\frac{1}{2} x e + \frac{1}{2} d\right)^2 + 2 a \tan\left(\frac{1}{2} x e + \frac{1}{2} d\right) + b \right) - b^2 \operatorname{sgn}\left(b \tan\left(\frac{1}{2} x e + \frac{1}{2} d\right)\right)^4 + 2 a \tan\left(\frac{1}{2} x e + \frac{1}{2} d\right)^3 + 2 b \tan\left(\frac{1}{2} x e + \frac{1}{2} d\right)^2 + 2 a \tan\left(\frac{1}{2} x e + \frac{1}{2} d\right) + b} \right) * \sqrt{-a^2 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2), x, algorithm="giac")

[Out] (a*arctan((b*tan(1/2*x*e + 1/2*d) + a)/sqrt(-a^2 + b^2))/((a^2*sgn(b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + b) - b^2*sgn(b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 + 2*a*tan(1/2*x*e + 1/2*d) + b))*sqrt(-a^2 + b^2)) - (2*a^3*b*tan(1/2*x*e + 1/2*d)^3 - 3*a*b^3*tan(1/2*x*e + 1/2*d)^3 + 2*a^4*tan(1/2*x*e + 1/2*d)^2 - 3*a^2*b^2*tan(1/2*x*e + 1/2*d)^2 + 2*a^3*b*tan(1/2*x*e + 1/2*d) - 3*a*b^3*tan(1/2*x*e + 1/2*d) + a^4*tan(1/2*x*e + 1/2*d) - a^2*b^2*tan(1/2*x*e + 1/2*d) + a^3*b*tan(1/2*x*e + 1/2*d) - a*b^3*tan(1/2*x*e + 1/2*d)))/((a^6 - 2*a^4*b^2 + a^2*b^4)*e*cos(e*x + d)^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*e*sin(e*x + d) - (a^6 - a^4*b^2 - a^2*b^4 + b^6)*e)

$$\frac{1}{2}d)^2 - 2b^4 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 2a^3 b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - 5ab^3 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + a^2 b^2 - 2b^4}{\left((a^2 b^2 \operatorname{sgn}(b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^4 + 2a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^3 + 2b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 2a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + b) - b^4 \operatorname{sgn}(b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^4 + 2a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^3 + 2b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 2a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + b)\right)} \cdot (b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 2a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + b)^2 \right) e^{-1}$$

maple [B] time = 0.41, size = 741, normalized size = 3.10

$$2 \sin(ex + d) \left(\cos^2(ex + d)\right) \arctan\left(\frac{b \cos(ex+d) - a \sin(ex+d) - b}{\sin(ex+d) \sqrt{-a^2 + b^2}}\right) a^4 b^2 + \sqrt{-a^2 + b^2} \sin(ex + d) \left(\cos^2(ex + d)\right) a^5 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x)

[Out] $\frac{1}{2}e \cdot (2 \sin(e*x+d) \cos(e*x+d)^2 \arctan\left(\frac{b \cos(e*x+d) - a \sin(e*x+d) - b}{\sin(e*x+d)}\right) / (-a^2 + b^2)^{1/2} + a^4 b^2 + (-a^2 + b^2)^{1/2} \sin(e*x+d) \cos(e*x+d)^2 a^5 - 2(-a^2 + b^2)^{1/2} \sin(e*x+d) \cos(e*x+d)^2 a^3 b^2 - (-a^2 + b^2)^{1/2} \cos(e*x+d)^3 a^2 b^3 + 6 \cos(e*x+d)^2 \arctan\left(\frac{b \cos(e*x+d) - a \sin(e*x+d) - b}{\sin(e*x+d)}\right) / (-a^2 + b^2)^{1/2} + a^3 b^3 - (-a^2 + b^2)^{1/2} \sin(e*x+d) \cos(e*x+d) a^3 b^2 + 3(-a^2 + b^2)^{1/2} \sin(e*x+d) \cos(e*x+d) a b^4 + 3(-a^2 + b^2)^{1/2} \cos(e*x+d)^2 a^4 b - 6(-a^2 + b^2)^{1/2} \cos(e*x+d)^2 a^2 b^3 - 2 \sin(e*x+d) \arctan\left(\frac{b \cos(e*x+d) - a \sin(e*x+d) - b}{\sin(e*x+d)}\right) / (-a^2 + b^2)^{1/2} + a^4 b^2 - 6 \sin(e*x+d) \arctan\left(\frac{b \cos(e*x+d) - a \sin(e*x+d) - b}{\sin(e*x+d)}\right) / (-a^2 + b^2)^{1/2} + a^2 b^4 - (-a^2 + b^2)^{1/2} \sin(e*x+d) a^5 - (-a^2 + b^2)^{1/2} \sin(e*x+d) a^3 b^2 + 6(-a^2 + b^2)^{1/2} \sin(e*x+d) a b^4 + 2(-a^2 + b^2)^{1/2} \cos(e*x+d) b^5 - 6 \arctan\left(\frac{b \cos(e*x+d) - a \sin(e*x+d) - b}{\sin(e*x+d)}\right) / (-a^2 + b^2)^{1/2} + a^3 b^3 - 2 \arctan\left(\frac{b \cos(e*x+d) - a \sin(e*x+d) - b}{\sin(e*x+d)}\right) / (-a^2 + b^2)^{1/2} + a b^5 - 3(-a^2 + b^2)^{1/2} a^4 b + 5(-a^2 + b^2)^{1/2} a^2 b^3 + 2(-a^2 + b^2)^{1/2} b^5) / (-a^2 \cos(e*x+d)^2 + 2a b \sin(e*x+d) + a^2 + b^2)^{3/2} / (-a^2 + b^2)^{1/2} / (a^2 - b^2) / b^2$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \sin(d + ex)}{(a^2 \sin(d + ex)^2 + 2ab \sin(d + ex) + b^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(3/2),x)

[Out] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(3/2),x)
```

```
[Out] Timed out
```


$$3.508 \quad \int \frac{a+b \cos(x)}{b^2+2ab \cos(x)+a^2 \cos^2(x)} dx$$

Optimal. Leaf size=11

$$\frac{\sin(x)}{a \cos(x) + b}$$

[Out] sin(x)/(b+a*cos(x))

Rubi [A] time = 0.08, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3289, 2754, 8}

$$\frac{\sin(x)}{a \cos(x) + b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[x])/(b^2 + 2*a*b*Cos[x] + a^2*Cos[x]^2),x]

[Out] Sin[x]/(b + a*Cos[x])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2754

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

Rule 3289

Int[(cos[(d_) + (e_)*(x_)]*(b_) + cos[(d_) + (e_)*(x_)]^2*(c_) + (a_)^(n_)*(cos[(d_) + (e_)*(x_)]*(B_) + (A_)), x_Symbol] := Dist[1/(4^n*c^n), Int[(A + B*Cos[d + e*x])*(b + 2*c*Cos[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned} \int \frac{a + b \cos(x)}{b^2 + 2ab \cos(x) + a^2 \cos^2(x)} dx &= (4a^2) \int \frac{a + b \cos(x)}{(2ab + 2a^2 \cos(x))^2} dx \\ &= \frac{\sin(x)}{b + a \cos(x)} + \frac{\int 0 dx}{a^2 - b^2} \\ &= \frac{\sin(x)}{b + a \cos(x)} \end{aligned}$$

Mathematica [A] time = 0.05, size = 11, normalized size = 1.00

$$\frac{\sin(x)}{a \cos(x) + b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*cos[x])/(b^2 + 2*a*b*cos[x] + a^2*cos[x]^2),x]

[Out] Sin[x]/(b + a*cos[x])

fricas [A] time = 0.78, size = 11, normalized size = 1.00

$$\frac{\sin(x)}{a \cos(x) + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x, algorithm="fricas")

[Out] sin(x)/(a*cos(x) + b)

giac [B] time = 0.17, size = 32, normalized size = 2.91

$$\frac{2 \tan\left(\frac{1}{2}x\right)}{a \tan\left(\frac{1}{2}x\right)^2 - b \tan\left(\frac{1}{2}x\right)^2 - a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x, algorithm="giac")

[Out] -2*tan(1/2*x)/(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 - a - b)

maple [B] time = 0.09, size = 33, normalized size = 3.00

$$\frac{2 \tan\left(\frac{x}{2}\right)}{a \left(\tan^2\left(\frac{x}{2}\right)\right) - b \left(\tan^2\left(\frac{x}{2}\right)\right) - a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x)

[Out] -2*tan(1/2*x)/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 2.94, size = 24, normalized size = 2.18

$$\frac{2 \tan\left(\frac{x}{2}\right)}{(b - a) \tan\left(\frac{x}{2}\right)^2 + a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(x))/(a^2*cos(x)^2 + b^2 + 2*a*b*cos(x)),x)

```
[Out] (2*tan(x/2))/(a + b - tan(x/2)^2*(a - b))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))/(b**2+2*a*b*cos(x)+a**2*cos(x)**2),x)
```

```
[Out] Timed out
```

$$3.509 \quad \int \frac{d+e \cos(x)}{a+b \cos(x)+c \cos^2(x)} dx$$

Optimal. Leaf size=246

$$\frac{2 \left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left(\frac{\tan\left(\frac{x}{2}\right) \sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}} \right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c} \sqrt{-\sqrt{b^2-4ac}+b+2c}} + \frac{2 \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\tan\left(\frac{x}{2}\right) \sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}} \right)}{\sqrt{\sqrt{b^2-4ac}+b-2c} \sqrt{\sqrt{b^2-4ac}+b+2c}}$$

[Out] 2*arctan((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tan(1/2*x)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/(b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)+2*arctan((b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)*tan(1/2*x)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/(b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)

Rubi [A] time = 0.79, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, number of rules / integrand size = 0.143, Rules used = {3293, 2659, 205}

$$\frac{2 \left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) \tan^{-1} \left(\frac{\tan\left(\frac{x}{2}\right) \sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}} \right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c} \sqrt{-\sqrt{b^2-4ac}+b+2c}} + \frac{2 \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\tan\left(\frac{x}{2}\right) \sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}} \right)}{\sqrt{\sqrt{b^2-4ac}+b-2c} \sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*Cos[x])/(a + b*Cos[x] + c*Cos[x]^2), x]

[Out] (2*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Tan[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]])/(Sqrt[b - 2*c - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (2*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tan[x/2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])/(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3293

Int[(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_))/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.)), x_Symbol] := Module[{q = Rt[b^2 - 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Cos[d + e*x]), x], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Cos[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{d + e \cos(x)}{a + b \cos(x) + c \cos^2(x)} dx = \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{b + \sqrt{b^2 - 4ac} + 2c \cos(x)} dx + \left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{b - \sqrt{b^2 - 4ac} + 2c \cos(x)} dx$$

$$= \left(2 \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \right) \text{Subst} \left(\int \frac{1}{b + 2c + \sqrt{b^2 - 4ac} + (b - 2c + \sqrt{b^2 - 4ac})x} dx \right)$$

$$= \frac{2 \left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{b - 2c - \sqrt{b^2 - 4ac}} \tan\left(\frac{x}{2}\right)}{\sqrt{b + 2c - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b - 2c - \sqrt{b^2 - 4ac}}} + \frac{2 \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{b - 2c + \sqrt{b^2 - 4ac}} \tan\left(\frac{x}{2}\right)}{\sqrt{b + 2c + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b - 2c + \sqrt{b^2 - 4ac}}}$$

Mathematica [A] time = 0.56, size = 241, normalized size = 0.98

$$\frac{\sqrt{2} \left(\frac{\left(e(\sqrt{b^2 - 4ac} - b) + 2cd \right) \tanh^{-1} \left(\frac{\tan\left(\frac{x}{2}\right) (\sqrt{b^2 - 4ac} - b + 2c)}{\sqrt{2b\sqrt{b^2 - 4ac} + 4c(a+c) - 2b^2}} \right)}{\sqrt{b\sqrt{b^2 - 4ac} + 2c(a+c) - b^2}} - \frac{\left(e(\sqrt{b^2 - 4ac} + b) - 2cd \right) \tanh^{-1} \left(\frac{\tan\left(\frac{x}{2}\right) (\sqrt{b^2 - 4ac} + b - 2c)}{\sqrt{-2b\sqrt{b^2 - 4ac} + 4c(a+c) - 2b^2}} \right)}{\sqrt{-b\sqrt{b^2 - 4ac} + 2c(a+c) - b^2}} \right)}{\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*Cos[x])/(a + b*Cos[x] + c*Cos[x]^2), x]

[Out] (Sqrt[2]*(-(((2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[((b - 2*c + Sqrt[b^2 - 4*a*c])*Tan[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]] + ((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tan[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]

fricas [B] time = 16.18, size = 6697, normalized size = 27.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2), x, algorithm="fricas")

[Out] 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))/((a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(2*b*c^2*d^4 + 2*a*b*c*e^4 - 2*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 6*(a*b*c + b*c^2)*d^2*e^2 - 2*(2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 - ((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x) + 1/2*sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2

$$\begin{aligned}
& 2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e \\
&)*\sqrt{(b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + \\
& 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 \\
& - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4 \\
&)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*\sin(x) + ((b^4 - 4*a*b^2*c)*d^3 - \\
& 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3 \\
& *c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - \\
& b^3)*c)*e^3)*\sin(x))*\sqrt{-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e \\
& + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 \\
& - 2*(2*a^3 - 3*a*b^2)*c)*\sqrt{(b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2* \\
& (2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a \\
& ^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8 \\
& *a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 \\
& - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + (b^2*c*d^4 \\
& + a*b^2*e^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 - \\
& (2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*\cos(x)) - 1/4*\sqrt{2}*\sqrt{-((b^2 - 2*a*c \\
& - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - \\
& b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*\sqrt{(b^2*d^4 + \\
& b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4 \\
& *(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c \\
& ^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3 \\
& *a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(\\
& 2*a^3 - 3*a*b^2)*c))*\log(2*b*c^2*d^4 + 2*a*b*c*e^4 - 2*(b^2*c + 2*a*c^2 + 2 \\
& *c^3)*d^3*e + 6*(a*b*c + b*c^2)*d^2*e^2 - 2*(2*a*c^2 + (2*a^2 + b^2)*c)*d*e \\
& ^3 - ((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b \\
& ^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b \\
& - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2* \\
& (2*a^4 - 3*a^2*b^2)*c)*e^2)*\sqrt{(b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + \\
& 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - \\
& 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2 \\
& *(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*\cos(x) \\
& - 1/2*\sqrt{2}*((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 \\
& - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4) \\
& *c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a \\
& *b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*\sqrt{(b^2*d^4 + b^2*e^4 - 4*(\\
& a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)* \\
& d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^ \\
& 3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2* \\
& a*b^4)*c))*\sin(x) + ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b \\
& - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)* \\
& c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*\sin(x))*\sqrt{-((b \\
& ^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (\\
& a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*\sqrt{(\\
& b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d \\
& ^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^ \\
& 2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - \\
& 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2) \\
& *c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + (b^2*c*d^4 + a*b^2*e^4 - (b^3 + 2*a*b*c + \\
& 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*e^ \\
& 3)*\cos(x)) + 1/4*\sqrt{2}*\sqrt{-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d \\
& *e + (2*a^2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c \\
& ^2 - 2*(2*a^3 - 3*a*b^2)*c)*\sqrt{(b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + \\
& 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - \\
& 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2 \\
& *(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b \\
& ^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*\log(-2*b*c \\
& ^2*d^4 - 2*a*b*c*e^4 + 2*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e - 6*(a*b*c + b*c^2 \\
&)*d^2*e^2 + 2*(2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 - ((4*a*c^4 + (8*a^2 - b^2) \\
& *c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 -
\end{aligned}$$

$$\begin{aligned}
& 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*\sqrt{(b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)}*\cos(x) + 1/2*\sqrt{2}*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*\sqrt{(b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)}*\sin(x) - ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*\sin(x))*\sqrt{-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*\sqrt{(b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)})))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c) - (b^2*c*d^4 + a*b^2*e^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*\cos(x)) - 1/4*\sqrt{2}*\sqrt{-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*\sqrt{(b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)})))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*\log(-2*b*c^2*d^4 - 2*a*b*c*e^4 + 2*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e - 6*(a*b*c + b*c^2)*d^2*e^2 + 2*(2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 - ((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*\sqrt{(b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)}*\cos(x) - 1/2*\sqrt{2}*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*\sqrt{(b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)}*\sin(x) - ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*\sin(x))*\sqrt{-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*\sqrt{(b^2*d^4 + b^2*e^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)})))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c) - (b^2*c*d^4 + a*b^2*e^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*\cos(x))
\end{aligned}$$

giac [B] time = 8.40, size = 5302, normalized size = 21.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x, algorithm="giac")

[Out] ((2*a^2*b^3 - 2*b^5 - 8*a^3*b*c - 12*a^2*b^2*c + 20*a*b^3*c + 4*b^4*c + 48*a^3*c^2 - 48*a^2*b*c^2 - 24*a*b^2*c^2 - 6*b^3*c^2 + 32*a^2*c^3 + 24*a*b*c^3 + 4*b^2*c^3 - 16*a*c^4 + 3*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*a^2*b^2 - 2*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*a*b^3 - 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*b^4 - 12*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*a^3*c + 8*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*a^2*b*c + 34*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*a*b^2*c + 6*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*b^3*c - 56*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*a^2*c^2 - 24*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*a*b*c^2 - 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*b^2*c^2 + 20*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*a*c^3 + 3*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*a^2*b - 2*(b^2 - 4*a*c)*a^2*b - 2*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*a*b^2 - 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*b^3 + 2*(b^2 - 4*a*c)*b^3 + 6*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*a^2*c + 12*(b^2 - 4*a*c)*a^2*c + 10*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*a*b*c - 12*(b^2 - 4*a*c)*a*b*c - 4*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*b^2*c - 4*(b^2 - 4*a*c)*b^2*c + 28*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*a*c^2 + 8*(b^2 - 4*a*c)*a*c^2 + 7*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*b*c^2 + 6*(b^2 - 4*a*c)*b*c^2 - 10*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*c^3 - 4*(b^2 - 4*a*c)*c^3)*d*abs(a - b + c) - (4*a^3*b^2 - 6*a^2*b^3 - 4*a*b^4 + 6*b^5 - 16*a^4*c + 24*a^3*b*c + 40*a^2*b^2*c - 44*a*b^3*c - 8*b^4*c - 96*a^3*c^2 + 80*a^2*b*c^2 + 52*a*b^2*c^2 + 2*b^3*c^2 - 80*a^2*c^3 - 8*a*b*c^3 - 3*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*a^2*b^2 + 2*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*a*b^3 + 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*b^4 + 12*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*a^3*c - 8*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*a^2*b*c - 34*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*a*b^2*c - 6*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*b^3*c + 56*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*a^2*c^2 + 24*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*a*b*c^2 + 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*b^2*c^2 - 20*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*a*c^3 + 6*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*a^3 - 4*(b^2 - 4*a*c)*a^3 - sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*a^2*b + 6*(b^2 - 4*a*c)*a^2*b - 12*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*a*b^2 + 4*(b^2 - 4*a*c)*a*b^2 - 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*b^3 - 6*(b^2 - 4*a*c)*b^3 + 28*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*a^2*c - 24*(b^2 - 4*a*c)*a^2*c + 26*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*a*b*c + 20*(b^2 - 4*a*c)*a*b*c + 6*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*b^2*c - 10*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*c)*a*c^2 - 20*(b^2 - 4*a*c)*a*c^2 - 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c))*(a - b + c))*sqrt(b^2 - 4*a*

$$\begin{aligned}
& c) * b * c^2 - 2 * (b^2 - 4 * a * c) * b * c^2) * \text{abs}(a - b + c) * e * (\pi * \text{floor}(1/2 * x / \pi + 1/2) + \arctan(2 * \sqrt{1/2} * \tan(1/2 * x) / \sqrt{(2 * a - 2 * c + \sqrt{-4 * (a + b + c) * (a - b + c) + 4 * (a - c)^2}) / (a - b + c)}))) / (3 * a^5 * b^2 - 5 * a^4 * b^3 - 6 * a^3 * b^4 + 10 * a^2 * b^5 + 3 * a * b^6 - 5 * b^7 - 12 * a^6 * c + 20 * a^5 * b * c + 47 * a^4 * b^2 * c - 60 * a^3 * b^3 * c - 46 * a^2 * b^4 * c + 40 * a * b^5 * c + 11 * b^6 * c - 92 * a^5 * c^2 + 80 * a^4 * b * c^2 + 182 * a^3 * b^2 * c^2 - 94 * a^2 * b^3 * c^2 - 78 * a * b^4 * c^2 - 6 * b^5 * c^2 - 184 * a^4 * c^3 + 56 * a^3 * b * c^3 + 166 * a^2 * b^2 * c^3 + 36 * a * b^3 * c^3 - 6 * b^4 * c^3 - 120 * a^3 * c^4 - 48 * a^2 * b * c^4 + 23 * a * b^2 * c^4 + 11 * b^3 * c^4 + 4 * a^2 * c^5 - 44 * a * b * c^5 - 5 * b^2 * c^5 + 20 * a * c^6) - ((2 * a^2 * b^3 - 2 * b^5 - 8 * a^3 * b * c - 12 * a^2 * b^2 * c + 20 * a * b^3 * c + 4 * b^4 * c + 48 * a^3 * c^2 - 48 * a^2 * b * c^2 - 24 * a * b^2 * c^2 - 6 * b^3 * c^2 + 32 * a^2 * c^3 + 24 * a * b * c^3 + 4 * b^2 * c^3 - 16 * a * c^4 - 3 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * a^2 * b^2 + 2 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * a * b^3 + 5 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * b^4 + 12 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * a^3 * c - 8 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * a^2 * b * c - 34 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * a * b^2 * c - 6 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * b^3 * c + 56 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * a^2 * c^2 + 24 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * a * b * c^2 + 5 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * b^2 * c^2 - 20 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * a * c^3 + 3 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * a^2 * b - 2 * (b^2 - 4 * a * c) * a^2 * b - 2 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * a * b^2 - 5 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * b^3 + 2 * (b^2 - 4 * a * c) * b^3 + 6 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * a^2 * c + 12 * (b^2 - 4 * a * c) * a^2 * c + 10 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * a * b * c - 12 * (b^2 - 4 * a * c) * a * b * c - 4 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * b^2 * c - 4 * (b^2 - 4 * a * c) * b^2 * c + 28 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * a * c^2 + 8 * (b^2 - 4 * a * c) * a * c^2 + 7 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * b * c^2 + 6 * (b^2 - 4 * a * c) * b * c^2 - 10 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * c^3 - 4 * (b^2 - 4 * a * c) * c^3) * \text{abs}(a - b + c) - (4 * a^3 * b^2 - 6 * a^2 * b^3 - 4 * a * b^4 + 6 * b^5 - 16 * a^4 * c + 24 * a^3 * b * c + 40 * a^2 * b^2 * c - 44 * a * b^3 * c - 8 * b^4 * c - 96 * a^3 * c^2 + 80 * a^2 * b * c^2 + 52 * a * b^2 * c^2 + 2 * b^3 * c^2 - 80 * a^2 * c^3 - 8 * a * b * c^3 + 3 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * a^2 * b^2 - 2 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * a * b^3 - 5 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * b^4 - 12 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * a^3 * c + 8 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * a^2 * b * c + 34 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * a * b^2 * c + 6 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * b^3 * c - 56 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * a^2 * c^2 - 24 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * a * b * c^2 - 5 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * b^2 * c^2 + 20 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * a * c^3 + 6 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * a^3 - 4 * (b^2 - 4 * a * c) * a^3 - \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * a^2 * b + 6 * (b^2 - 4 * a * c) * a^2 * b - 12 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * a * b^2 + 4 * (b^2 - 4 * a * c) * a * b^2 - 5 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * b^3 - 6 * (b^2 - 4 * a * c) * b^3 + 28 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * a^2 * c - 24 * (b^2 - 4 * a * c) * a^2 * c + 26 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * a * b * c + 20 * (b^2 - 4 * a * c) * a * b * c + 6 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * b^2 * c + 8 * (b^2 - 4 * a * c) * b^2 * c - 10 * \sqrt{a^2 - a * b + b * c - c^2} - \sqrt{b^2 - 4 * a * c} * (a - b + c)) * \sqrt{b^2 - 4 * a * c} * a * c^2 - 20 * (b
\end{aligned}$$

$$\begin{aligned} &^2 - 4*a*c)*a*c^2 - 5*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2 - 4*a*c)*(a - b \\ &+ c))*sqrt(b^2 - 4*a*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*abs(a - b + c)*e*(\\ \text{pi}*floor(1/2*x/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*x)/sqrt((2*a - 2*c - \\ \text{sqrt}(-4*(a + b + c)*(a - b + c) + 4*(a - c)^2))/(a - b + c))))/(3*a^5*b^2 - \\ &5*a^4*b^3 - 6*a^3*b^4 + 10*a^2*b^5 + 3*a*b^6 - 5*b^7 - 12*a^6*c + 20*a^5*b \\ &*c + 47*a^4*b^2*c - 60*a^3*b^3*c - 46*a^2*b^4*c + 40*a*b^5*c + 11*b^6*c - 9 \\ &2*a^5*c^2 + 80*a^4*b*c^2 + 182*a^3*b^2*c^2 - 94*a^2*b^3*c^2 - 78*a*b^4*c^2 \\ &- 6*b^5*c^2 - 184*a^4*c^3 + 56*a^3*b*c^3 + 166*a^2*b^2*c^3 + 36*a*b^3*c^3 - \\ &6*b^4*c^3 - 120*a^3*c^4 - 48*a^2*b*c^4 + 23*a*b^2*c^4 + 11*b^3*c^4 + 4*a^2 \\ &*c^5 - 44*a*b*c^5 - 5*b^2*c^5 + 20*a*c^6) \end{aligned}$$

maple [B] time = 0.12, size = 2556, normalized size = 10.39

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x)
[Out] -2/((-4*a*c+b^2)^(1/2)/(a-b+c)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a^2*e+2/((-4*a*c+b^2)^(1/2)/(a-b+c)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*a^2*e-1/((-4*a*c+b^2)^(1/2)/(a-b+c)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*d*b^2-1/((-4*a*c+b^2)^(1/2)/(a-b+c)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*d*b^2+1/((-4*a*c+b^2)^(1/2)/(a-b+c)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d*b^2+1/((-4*a*c+b^2)^(1/2)/(a-b+c)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b^2*e-2/((-4*a*c+b^2)^(1/2)/(a-b+c)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c^2*d+2/((-4*a*c+b^2)^(1/2)/(a-b+c)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*c^2*d-b/(a-b+c)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d+b/(a-b+c)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e+c/(a-b+c)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*d-c/(a-b+c)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*e+c/(a-b+c)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d-c/(a-b+c)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e-a/(a-b+c)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*e+a/(a-b+c)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*d+a/(a-b+c)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d-a/(a-b+c)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*e-b/(a-b+c)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*d+b/(a-b+c)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*e-a/((-4*a*c+b^2)^(1/2)/(a-b+c)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*d*b-3*a/((-4*a*c+b^2)^(1/2)/(a-b+c)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b*e+2*a/((-4*a*c+b^2)^(1/2)/(a-b+c)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((( -4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*c*d+3*b/((-4*a*c+b^2)^(1/2)/(a-b+c)/((( -4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*ta
```

n(1/2*x)/((((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c*d-3*b/(-4*a*c+b^2)^(1/2)/(a-b+c)/((((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*c*d-2*c/(-4*a*c+b^2)^(1/2)/(a-b+c)/((((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/((((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*a*e+c/(-4*a*c+b^2)^(1/2)/(a-b+c)/((((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/((((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b*e+2*c/(-4*a*c+b^2)^(1/2)/(a-b+c)/((((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*a*e-c/(-4*a*c+b^2)^(1/2)/(a-b+c)/((((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*tan(1/2*x)/((((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))*b*e+a/(-4*a*c+b^2)^(1/2)/(a-b+c)/((((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/((((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*d*b+3*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/((((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/((((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*b*e-2*a/(-4*a*c+b^2)^(1/2)/(a-b+c)/((((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2*x)/((((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))*c*d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e \cos(x) + d}{c \cos(x)^2 + b \cos(x) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x, algorithm="maxima")

[Out] integrate((e*cos(x) + d)/(c*cos(x)^2 + b*cos(x) + a), x)

mupad [B] time = 16.77, size = 11781, normalized size = 47.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*cos(x))/(a + b*cos(x) + c*cos(x)^2),x)

[Out] - atan(((((-b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(256*a^2*c^2*d - 32*b^4*e - 32*a^2*b^2*d - 32*a^2*b^2*e - 32*b^4*d + 256*a^2*c^2*e - 32*b^2*c^2*d - 32*b^2*c^2*e + tan(x/2)*(-b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(64*a*b^4 + 256*a*c^4 - 256*a^4*c - 64*b^4*c - 128*a^2*b^3 + 64*a^3*b^2 + 256*a^2*c^3 - 256*a^3*c^2 - 64*b^2*c^3 + 128*b^3*c^2 + 192*a*b^2*c^2 - 192*a^2*b^2*c - 512*a*b*c^3 + 512*a^3*b*c) + 64*a*b^3*d + 64*a*b^3*e + 128*a*c^3*d + 128*a^3*c*d + 128*a*c^3*e + 128*a^3*c*e + 64*b^3*c*d + 64*b^3*c*e - 256*a*b*c^2*d + 64*a*b^2*c*d - 256*a^2*b*c*d - 256*a*b*c^2*e + 64*a*b^2*c*e - 256*a^2*b*c*e) + tan(x/2)*(64*a^3*e^2 - 32*b^3*d^2 - 32*b^3*e^2 + 64*c^3*d^2 + 32*a*b^2*d^2 + 96*a*b^2*e^2 - 128*a^2*b*e^2 - 64*a^2*c*d^2 - 64*a*c^2*e^2 - 128*b*c^2*d^2 + 96*b^2*c*d^2 + 32*b^2*c*e^2 + 64*a*b^2*d*e - 64*a^2*b*d*e + 256*a*c^2*d*e + 256*a^2*c*d*e - 64*b*c^2*d*e + 64*b^2*c*d*e - 384*a*b*c*d*e))*(-b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)

$$\begin{aligned}
& ^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - \\
& 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c*d*e - 2*c*d*e*(-(\\
& 4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a \\
& ^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c \\
& ^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c))^{(1/2)}*1i - \\
& ((- (b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a \\
& ^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - \\
& 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^{(\\
& 1/2)} + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a \\
& *b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 \\
& + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^ \\
& ^2*c^2 + 10*a*b^4*c))^{(1/2)}*(256*a^2*c^2*d - 32*b^4*e - 32*a^2*b^2*d - 32*a \\
& ^2*b^2*e - 32*b^4*d + 256*a^2*c^2*e - 32*b^2*c^2*d - 32*b^2*c^2*e - \tan(x/2 \\
&)*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a \\
& ^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - \\
& 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^{(\\
& 1/2)} + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a \\
& *b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 \\
& + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^ \\
& ^2*c^2 + 10*a*b^4*c))^{(1/2)}*(64*a*b^4 + 256*a*c^4 - 256*a^4*c - 64*b^4*c - \\
& 128*a^2*b^3 + 64*a^3*b^2 + 256*a^2*c^3 - 256*a^3*c^2 - 64*b^2*c^3 + 128*b^3 \\
& *c^2 + 192*a*b^2*c^2 - 192*a^2*b^2*c - 512*a*b*c^3 + 512*a^3*b*c) + 64*a*b^ \\
& ^3*d + 64*a*b^3*e + 128*a*c^3*d + 128*a^3*c*d + 128*a*c^3*e + 128*a^3*c*e + \\
& 64*b^3*c*d + 64*b^3*c*e - 256*a*b*c^2*d + 64*a*b^2*c*d - 256*a^2*b*c*d - 25 \\
& 6*a*b*c^2*e + 64*a*b^2*c*e - 256*a^2*b*c*e) - \tan(x/2)*(64*a^3*e^2 - 32*b^3 \\
& *d^2 - 32*b^3*e^2 + 64*c^3*d^2 + 32*a*b^2*d^2 + 96*a*b^2*e^2 - 128*a^2*b*e^ \\
& ^2 - 64*a^2*c*d^2 - 64*a*c^2*e^2 - 128*b*c^2*d^2 + 96*b^2*c*d^2 + 32*b^2*c*e \\
& ^2 + 64*a*b^2*d*e - 64*a^2*b*d*e + 256*a*c^2*d*e + 256*a^2*c*d*e - 64*b*c^2 \\
& *d*e + 64*b^2*c*d*e - 384*a*b*c*d*e)*(- (b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + \\
& b*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^ \\
& ^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c* \\
& d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8 \\
& *a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c))^{(1/2)}*1i)/(((- (b^ \\
& 4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*c*e^ \\
& ^2 + b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2* \\
& c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2*c* \\
& e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^ \\
& ^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + \\
& 10*a*b^4*c))^{(1/2)}*(256*a^2*c^2*d - 32*b^4*e - 32*a^2*b^2*d - 32*a^2*b^2* \\
& e - 32*b^4*d + 256*a^2*c^2*e - 32*b^2*c^2*d - 32*b^2*c^2*e + \tan(x/2)*(- (b^ \\
& 4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*c*e^ \\
& ^2 + b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2* \\
& c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2*c* \\
& e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^ \\
& ^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + \\
& 10*a*b^4*c))^{(1/2)}*(64*a*b^4 + 256*a*c^4 - 256*a^4*c - 64*b^4*c - 128*a^2 \\
& *b^3 + 64*a^3*b^2 + 256*a^2*c^3 - 256*a^3*c^2 - 64*b^2*c^3 + 128*b^3*c^2 + \\
& 192*a*b^2*c^2 - 192*a^2*b^2*c - 512*a*b*c^3 + 512*a^3*b*c) + 64*a*b^3*d + 6 \\
& 4*a*b^3*e + 128*a*c^3*d + 128*a^3*c*d + 128*a*c^3*e + 128*a^3*c*e + 64*b^3* \\
& c*d + 64*b^3*c*e - 256*a*b*c^2*d + 64*a*b^2*c*d - 256*a^2*b*c*d - 256*a*b*c \\
& ^2*e + 64*a*b^2*c*e - 256*a^2*b*c*e) + \tan(x/2)*(64*a^3*e^2 - 32*b^3*d^2 - \\
& 32*b^3*e^2 + 64*c^3*d^2 + 32*a*b^2*d^2 + 96*a*b^2*e^2 - 128*a^2*b*e^2 - 64* \\
& a^2*c*d^2 - 64*a*c^2*e^2 - 128*b*c^2*d^2 + 96*b^2*c*d^2 + 32*b^2*c*e^2 + 64 \\
& *a*b^2*d*e - 64*a^2*b*d*e + 256*a*c^2*d*e + 256*a^2*c*d*e - 64*b*c^2*d*e + \\
& 64*b^2*c*d*e - 384*a*b*c*d*e)*(- (b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(
\end{aligned}$$

$$\begin{aligned}
& -(4ac - b^2)^3)^{1/2} - 8a^3c^2e^2 + b^2e^2(-(4ac - b^2)^3)^{1/2} + 2a^2b^2e^2 + 8a^2c^2d^2 - 8a^2c^2e^2 - 2b^2c^2d^2 - 2ab^3d^2e - \\
& 2ad^2e(-(4ac - b^2)^3)^{1/2} + 2b^3cd^2e - 2cd^2e(-(4ac - b^2)^3)^{1/2} - 6ab^2cd^2 + 6ab^2c^2e^2 - 8abc^2d^2e + 8a^2b^2cd^2e)/(2 \\
& (a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c))^{1/2} + ((-b^4d^2 - b^4 \\
& e^2 + 8ac^3d^2 + bd^2(-(4ac - b^2)^3)^{1/2} - 8a^3c^2e^2 + b^2e^2(-(4ac - b^2)^3)^{1/2} + 2a^2b^2e^2 + 8a^2c^2d^2 - 8a^2c^2e^2 - \\
& 2b^2c^2d^2 - 2ab^3d^2e - 2ad^2e(-(4ac - b^2)^3)^{1/2} + 2b^3cd^2e - 2cd^2e(-(4ac - b^2)^3)^{1/2} - 6ab^2cd^2 + 6ab^2c^2e^2 - 8ab \\
& bc^2d^2e + 8a^2b^2cd^2e)/(2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c \\
& c))^{1/2} * (256a^2c^2d - 32b^4e - 32a^2b^2d - 32a^2b^2e - 32b^4 \\
& d + 256a^2c^2e - 32b^2c^2d - 32b^2c^2e - \tan(x/2) * (-b^4d^2 - b^4 \\
& e^2 + 8ac^3d^2 + bd^2(-(4ac - b^2)^3)^{1/2} - 8a^3c^2e^2 + b^2e^2(-(4ac - b^2)^3)^{1/2} + 2a^2b^2e^2 + 8a^2c^2d^2 - 8a^2c^2e^2 - \\
& 2b^2c^2d^2 - 2ab^3d^2e - 2ad^2e(-(4ac - b^2)^3)^{1/2} + 2b^3cd^2e - 2cd^2e(-(4ac - b^2)^3)^{1/2} - 6ab^2cd^2 + 6ab^2c^2e^2 - 8ab \\
& bc^2d^2e + 8a^2b^2cd^2e)/(2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c \\
& c))^{1/2} * (64ab^4 + 256ac^4 - 256a^4c - 64b^4c - 128a^2b^3 + 64a^3b^2 + 256a^2c^3 - 256a^3c^2 - 64b^2c^3 + 128b^3c^2 + 192ab^2c^2 - 192a^2b^2c - 512ab^3c + 512a^3bc) + 64ab^3d + 64ab^3e \\
& + 128ac^3d + 128a^3cd + 128ac^3e + 128a^3ce + 64b^3cd + 64b^3ce - 256abc^2d + 64ab^2cd - 256a^2bcd - 256abc^2e + 64ab^2ce - 256a^2b^2cd - 256a^2b^2ce) - \tan(x/2) * (64a^3e^2 - 32b^3d^2 - 32b^3e^2 \\
& + 64c^3d^2 + 32ab^2d^2 + 96ab^2e^2 - 128a^2b^2e^2 - 64a^2cd^2 - 64ac^2e^2 - 128b^2d^2 + 96b^2cd^2 + 32b^2ce^2 + 64ab^2d^2e \\
& - 64a^2bd^2e + 256ac^2d^2e + 256a^2cd^2e - 64b^2d^2e + 64b^2cd^2e - 384abc^2d^2e) * (-b^4d^2 - b^4e^2 + 8ac^3d^2 + bd^2(-(4ac - b^2)^3)^{1/2} - 8a^3c^2e^2 + b^2e^2(-(4ac - b^2)^3)^{1/2} + 2a^2b^2e^2 + 8a^2c^2d^2 - 8a^2c^2e^2 - 2b^2c^2d^2 - 2ab^3d^2e - 2ad^2e(-(4ac - b^2)^3)^{1/2} + 2b^3cd^2e - 2cd^2e(-(4ac - b^2)^3)^{1/2} - 6ab^2cd^2 + 6ab^2c^2e^2 - 8abc^2d^2e + 8a^2b^2cd^2e)/(2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c))^{1/2} - 64a^2e^3 + 64c^2d^3 + 64a^2d^2e^2 - 64b^2d^2e^2 + 64b^2d^2e - 64c^2d^2e + 64ab^2e^3 + 64ac^2d^3 - 64ac^2e^3 - 64bcd^3 - 64abd^2e + 64acd^2e - 64acd^2e + 64bcd^2e) * (-b^4d^2 - b^4e^2 + 8ac^3d^2 + bd^2(-(4ac - b^2)^3)^{1/2} - 8a^3c^2e^2 + b^2e^2(-(4ac - b^2)^3)^{1/2} + 2a^2b^2e^2 + 8a^2c^2d^2 - 8a^2c^2e^2 - 2b^2c^2d^2 - 2ab^3d^2e - 2ad^2e(-(4ac - b^2)^3)^{1/2} + 2b^3cd^2e - 2cd^2e(-(4ac - b^2)^3)^{1/2} - 6ab^2cd^2 + 6ab^2c^2e^2 - 8abc^2d^2e + 8a^2b^2cd^2e)/(2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c))^{1/2} * 2i - \operatorname{atan}(\frac{(-b^4d^2 - b^4e^2 + 8ac^3d^2 - bd^2(-(4ac - b^2)^3)^{1/2} - 8a^3c^2e^2 - b^2e^2(-(4ac - b^2)^3)^{1/2} + 2a^2b^2e^2 + 8a^2c^2d^2 - 8a^2c^2e^2 - 2b^2c^2d^2 - 2ab^3d^2e + 2ad^2e(-(4ac - b^2)^3)^{1/2} + 2b^3cd^2e - 2cd^2e(-(4ac - b^2)^3)^{1/2} - 6ab^2cd^2 + 6ab^2c^2e^2 - 8abc^2d^2e + 8a^2b^2cd^2e)/(2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c))^{1/2}}{256a^2c^2d - 32b^4e - 32a^2b^2d - 32a^2b^2e - 32b^4d + 256a^2c^2e - 32b^2c^2d - 32b^2c^2e + \tan(x/2) * (-b^4d^2 - b^4e^2 + 8ac^3d^2 - bd^2(-(4ac - b^2)^3)^{1/2} - 8a^3c^2e^2 - b^2e^2(-(4ac - b^2)^3)^{1/2} + 2a^2b^2e^2 + 8a^2c^2d^2 - 8a^2c^2e^2 - 2b^2c^2d^2 - 2ab^3d^2e + 2ad^2e(-(4ac - b^2)^3)^{1/2} + 2b^3cd^2e - 2cd^2e(-(4ac - b^2)^3)^{1/2} - 6ab^2cd^2 + 6ab^2c^2e^2 - 8abc^2d^2e + 8a^2b^2cd^2e)/(2(a^2b^4 - b^6 + 16a^2c^4 + 32a^3c^3 + 16a^4c^2 + b^4c^2 - 8ab^2c^3 - 8a^3b^2c - 32a^2b^2c^2 + 10ab^4c))^{1/2}}
\end{aligned}$$

$$\begin{aligned}
& c))^{(1/2)} * (64*a*b^4 + 256*a*c^4 - 256*a^4*c - 64*b^4*c - 128*a^2*b^3 + 64* \\
& a^3*b^2 + 256*a^2*c^3 - 256*a^3*c^2 - 64*b^2*c^3 + 128*b^3*c^2 + 192*a*b^2* \\
& c^2 - 192*a^2*b^2*c - 512*a*b*c^3 + 512*a^3*b*c) + 64*a*b^3*d + 64*a*b^3*e \\
& + 128*a*c^3*d + 128*a^3*c*d + 128*a*c^3*e + 128*a^3*c*e + 64*b^3*c*d + 64*b \\
& ^3*c*e - 256*a*b*c^2*d + 64*a*b^2*c*d - 256*a^2*b*c*d - 256*a*b*c^2*e + 64* \\
& a*b^2*c*e - 256*a^2*b*c*e) + \tan(x/2) * (64*a^3*e^2 - 32*b^3*d^2 - 32*b^3*e^2 \\
& + 64*c^3*d^2 + 32*a*b^2*d^2 + 96*a*b^2*e^2 - 128*a^2*b*e^2 - 64*a^2*c*d^2 \\
& - 64*a*c^2*e^2 - 128*b*c^2*d^2 + 96*b^2*c*d^2 + 32*b^2*c*e^2 + 64*a*b^2*d*e \\
& - 64*a^2*b*d*e + 256*a*c^2*d*e + 256*a^2*c*d*e - 64*b*c^2*d*e + 64*b^2*c*d \\
& *e - 384*a*b*c*d*e) * (-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2 * (-(4*a*c - \\
& b^2)^3))^{(1/2)} - 8*a^3*c*e^2 - b*e^2 * (-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^ \\
& 2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e * (\\
& -(4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c*d*e + 2*c*d*e * (-(4*a*c - b^2)^3)^{(1/2)} - \\
& 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e) / (2*(a^2*b^4 \\
& - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^ \\
& 3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c))^{(1/2)} * 1i - (((-(b^4*d^2 - b^4*e^2 + \\
& 8*a*c^3*d^2 - b*d^2 * (-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*c*e^2 - b*e^2 * (-(4*a* \\
& c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c \\
& ^2*d^2 - 2*a*b^3*d*e + 2*a*d*e * (-(4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c*d*e + 2*c \\
& *d*e * (-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d \\
& *e + 8*a^2*b*c*d*e) / (2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^ \\
& 2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c))^{(1 \\
& /2)} * (256*a^2*c^2*d - 32*b^4*e - 32*a^2*b^2*d - 32*a^2*b^2*e - 32*b^4*d + 25 \\
& 6*a^2*c^2*e - 32*b^2*c^2*d - 32*b^2*c^2*e - \tan(x/2) * (-(b^4*d^2 - b^4*e^2 + \\
& 8*a*c^3*d^2 - b*d^2 * (-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*c*e^2 - b*e^2 * (-(4*a* \\
& c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c \\
& ^2*d^2 - 2*a*b^3*d*e + 2*a*d*e * (-(4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c*d*e + 2*c \\
& *d*e * (-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d \\
& *e + 8*a^2*b*c*d*e) / (2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^ \\
& 2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c))^{(1 \\
& /2)} * (64*a*b^4 + 256*a*c^4 - 256*a^4*c - 64*b^4*c - 128*a^2*b^3 + 64*a^3*b^2 \\
& + 256*a^2*c^3 - 256*a^3*c^2 - 64*b^2*c^3 + 128*b^3*c^2 + 192*a*b^2*c^2 - 1 \\
& 92*a^2*b^2*c - 512*a*b*c^3 + 512*a^3*b*c) + 64*a*b^3*d + 64*a*b^3*e + 128*a \\
& *c^3*d + 128*a^3*c*d + 128*a*c^3*e + 128*a^3*c*e + 64*b^3*c*d + 64*b^3*c*e \\
& - 256*a*b*c^2*d + 64*a*b^2*c*d - 256*a^2*b*c*d - 256*a*b*c^2*e + 64*a*b^2*c \\
& *e - 256*a^2*b*c*e) - \tan(x/2) * (64*a^3*e^2 - 32*b^3*d^2 - 32*b^3*e^2 + 64*c \\
& ^3*d^2 + 32*a*b^2*d^2 + 96*a*b^2*e^2 - 128*a^2*b*e^2 - 64*a^2*c*d^2 - 64*a* \\
& c^2*e^2 - 128*b*c^2*d^2 + 96*b^2*c*d^2 + 32*b^2*c*e^2 + 64*a*b^2*d*e - 64*a \\
& ^2*b*d*e + 256*a*c^2*d*e + 256*a^2*c*d*e - 64*b*c^2*d*e + 64*b^2*c*d*e - 38 \\
& 4*a*b*c*d*e) * (-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2 * (-(4*a*c - b^2)^3) \\
&)^{(1/2)} - 8*a^3*c*e^2 - b*e^2 * (-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a \\
& ^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e * (-(4*a*c \\
& - b^2)^3)^{(1/2)} + 2*b^3*c*d*e + 2*c*d*e * (-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2 \\
& *c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e) / (2*(a^2*b^4 - b^6 + \\
& 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c \\
& - 32*a^2*b^2*c^2 + 10*a*b^4*c))^{(1/2)} * 1i) / (((-(b^4*d^2 - b^4*e^2 + 8*a*c^ \\
& 3*d^2 - b*d^2 * (-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*c*e^2 - b*e^2 * (-(4*a*c - b^2 \\
&)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 \\
& - 2*a*b^3*d*e + 2*a*d*e * (-(4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c*d*e + 2*c*d*e * (\\
& -(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8* \\
& a^2*b*c*d*e) / (2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4 \\
& *c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c))^{(1/2)} * (25 \\
& 6*a^2*c^2*d - 32*b^4*e - 32*a^2*b^2*d - 32*a^2*b^2*e - 32*b^4*d + 256*a^2*c \\
& ^2*e - 32*b^2*c^2*d - 32*b^2*c^2*e + \tan(x/2) * (-(b^4*d^2 - b^4*e^2 + 8*a*c^ \\
& 3*d^2 - b*d^2 * (-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*c*e^2 - b*e^2 * (-(4*a*c - b^2 \\
&)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 \\
& - 2*a*b^3*d*e + 2*a*d*e * (-(4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c*d*e + 2*c*d*e * (\\
& -(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8* \\
& a^2*b*c*d*e) / (2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4
\end{aligned}$$

$$\begin{aligned}
& *c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c))^{(1/2)}*(64 \\
& *a*b^4 + 256*a*c^4 - 256*a^4*c - 64*b^4*c - 128*a^2*b^3 + 64*a^3*b^2 + 256* \\
& a^2*c^3 - 256*a^3*c^2 - 64*b^2*c^3 + 128*b^3*c^2 + 192*a*b^2*c^2 - 192*a^2* \\
& b^2*c - 512*a*b*c^3 + 512*a^3*b*c) + 64*a*b^3*d + 64*a*b^3*e + 128*a*c^3*d \\
& + 128*a^3*c*d + 128*a*c^3*e + 128*a^3*c*e + 64*b^3*c*d + 64*b^3*c*e - 256*a \\
& *b*c^2*d + 64*a*b^2*c*d - 256*a^2*b*c*d - 256*a*b*c^2*e + 64*a*b^2*c*e - 25 \\
& 6*a^2*b*c*e) + \tan(x/2)*(64*a^3*e^2 - 32*b^3*d^2 - 32*b^3*e^2 + 64*c^3*d^2 \\
& + 32*a*b^2*d^2 + 96*a*b^2*e^2 - 128*a^2*b*e^2 - 64*a^2*c*d^2 - 64*a*c^2*e^2 \\
& - 128*b*c^2*d^2 + 96*b^2*c*d^2 + 32*b^2*c*e^2 + 64*a*b^2*d*e - 64*a^2*b*d* \\
& e + 256*a*c^2*d*e + 256*a^2*c*d*e - 64*b*c^2*d*e + 64*b^2*c*d*e - 384*a*b*c \\
& *d*e))*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2* \\
& d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 \\
& + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2 \\
& *c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a \\
& ^2*b^2*c^2 + 10*a*b^4*c))^{(1/2)} + ((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b* \\
& d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3* \\
& d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d* \\
& e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a \\
& *b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c))^{(1/2)}*(256*a^2*c^2* \\
& d - 32*b^4*e - 32*a^2*b^2*d - 32*a^2*b^2*e - 32*b^4*d + 256*a^2*c^2*e - 32* \\
& b^2*c^2*d - 32*b^2*c^2*e - \tan(x/2)*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b* \\
& d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*c*e^2 - b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3* \\
& d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^{(1/2)} + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d* \\
& e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a \\
& *b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c))^{(1/2)}*(64*a*b^4 + 2 \\
& 56*a*c^4 - 256*a^4*c - 64*b^4*c - 128*a^2*b^3 + 64*a^3*b^2 + 256*a^2*c^3 - \\
& 256*a^3*c^2 - 64*b^2*c^3 + 128*b^3*c^2 + 192*a*b^2*c^2 - 192*a^2*b^2*c - 51 \\
& 2*a*b*c^3 + 512*a^3*b*c) + 64*a*b^3*d + 64*a*b^3*e + 128*a*c^3*d + 128*a^3* \\
& c*d + 128*a*c^3*e + 128*a^3*c*e + 64*b^3*c*d + 64*b^3*c*e - 256*a*b*c^2*d + \\
& 64*a*b^2*c*d - 256*a^2*b*c*d - 256*a*b*c^2*e + 64*a*b^2*c*e - 256*a^2*b*c* \\
& e) - \tan(x/2)*(64*a^3*e^2 - 32*b^3*d^2 - 32*b^3*e^2 + 64*c^3*d^2 + 32*a*b^2 \\
& *d^2 + 96*a*b^2*e^2 - 128*a^2*b*e^2 - 64*a^2*c*d^2 - 64*a*c^2*e^2 - 128*b*c \\
& ^2*d^2 + 96*b^2*c*d^2 + 32*b^2*c*e^2 + 64*a*b^2*d*e - 64*a^2*b*d*e + 256*a* \\
& c^2*d*e + 256*a^2*c*d*e - 64*b*c^2*d*e + 64*b^2*c*d*e - 384*a*b*c*d*e))*(-(\\
& b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*c* \\
& e^2 - b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^ \\
& 2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2* \\
& c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32* \\
& a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 \\
& + 10*a*b^4*c))^{(1/2)} - 64*a^2*e^3 + 64*c^2*d^3 + 64*a^2*d*e^2 - 64*b^2*d* \\
& e^2 + 64*b^2*d^2*e - 64*c^2*d^2*e + 64*a*b*e^3 + 64*a*c*d^3 - 64*a*c*e^3 - \\
& 64*b*c*d^3 - 64*a*b*d^2*e + 64*a*c*d*e^2 - 64*a*c*d^2*e + 64*b*c*d*e^2))*(-(\\
& b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 - b*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*c* \\
& e^2 - b*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a \\
& ^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e + 2*a*d*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 2*b^3*c*d*e + 2*c*d*e*(-(4*a*c - b^2)^3)^{(1/2)} - 6*a*b^2*c*d^2 + 6*a*b^2 \\
& *c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32 \\
& *a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^ \\
& 2 + 10*a*b^4*c))^{(1/2)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)**2),x)
```

```
[Out] Timed out
```


3.510 $\int (a+b \tan(d+ex)) (b^2 + 2ab \tan(d+ex) + a^2 \tan^2(d+ex)) dx$

Optimal. Leaf size=144

$$\frac{a(a^4 - b^4) \tan(d+ex)}{e} + \frac{(a^2 + b^2)(a \tan(d+ex) + b)^3}{3e} + \frac{b(a^2 + b^2)(a \tan(d+ex) + b)^2}{2e} + \frac{b(3a^2 - b^2)(a^2 + b^2) \tan^2(d+ex)}{e}$$

[Out] $a*(a^2-3*b^2)*(a^2+b^2)*x+b*(3*a^2-b^2)*(a^2+b^2)*\ln(\cos(e*x+d))/e-a*(a^4-b^4)*\tan(e*x+d)/e+1/2*b*(a^2+b^2)*(b+a*\tan(e*x+d))^2/e+1/3*(a^2+b^2)*(b+a*\tan(e*x+d))^3/e+1/4*b*(b+a*\tan(e*x+d))^4/e$

Rubi [A] time = 0.27, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3708, 3528, 12, 3525, 3475}

$$\frac{(a^2 + b^2)(a \tan(d+ex) + b)^3}{3e} + \frac{b(a^2 + b^2)(a \tan(d+ex) + b)^2}{2e} - \frac{a(a^4 - b^4) \tan(d+ex)}{e} + \frac{b(3a^2 - b^2)(a^2 + b^2) \tan^2(d+ex)}{e}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^2, x]

[Out] $a*(a^2 - 3*b^2)*(a^2 + b^2)*x + (b*(3*a^2 - b^2)*(a^2 + b^2)*\text{Log}[\text{Cos}[d + e*x]])/e - (a*(a^4 - b^4)*\text{Tan}[d + e*x])/e + (b*(a^2 + b^2)*(b + a*\text{Tan}[d + e*x])^2)/(2*e) + ((a^2 + b^2)*(b + a*\text{Tan}[d + e*x])^3)/(3*e) + (b*(b + a*\text{Tan}[d + e*x])^4)/(4*e)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3525

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e + f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]

Rule 3528

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int[(a + b*Tan[e + f*x])^(m-1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && GtQ[m, 0]

Rule 3708

Int[((A_.) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_.) + (b_.)*tan[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] := Dist[1/(4^n*c^n), Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int (a + b \tan(d + ex)) (b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^2 dx &= \frac{\int (2ab + 2a^2 \tan(d + ex))^4 (a + b \tan(d + ex))^2 dx}{16a^4} \\
&= \frac{b(b + a \tan(d + ex))^4}{4e} + \frac{\int 2a(a^2 + b^2) \tan(d + ex) dx}{4e} \\
&= \frac{b(b + a \tan(d + ex))^4}{4e} + \frac{(a^2 + b^2) \int \tan(d + ex) dx}{4e} \\
&= \frac{(a^2 + b^2)(b + a \tan(d + ex))^3}{3e} + \frac{b(b + a \tan(d + ex))^2}{4e} \\
&= \frac{b(a^2 + b^2)(b + a \tan(d + ex))^2}{2e} + \frac{(a^2 + b^2) \int \tan(d + ex) dx}{4e} \\
&= a(a^2 - 3b^2)(a^2 + b^2)x - \frac{a(a^4 - b^4) \tan(d + ex)}{e} \\
&= a(a^2 - 3b^2)(a^2 + b^2)x + \frac{b(3a^2 - b^2)(a^2 + b^2) \int \tan(d + ex) dx}{4e}
\end{aligned}$$

Mathematica [C] time = 2.10, size = 153, normalized size = 1.06

$$\frac{3a^4b \tan^4(d + ex) + 18a^2b(a^2 + 2b^2) \tan^2(d + ex) + 6(a^2 + b^2)(i(a + ib)^3 \log(\tan(d + ex) + i) - i(a - ib)^3 \log(\tan(d + ex) - i))}{12e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^2,x]

[Out] (6*(a^2 + b^2)*((-I)*(a - I*b)^3*Log[I - Tan[d + e*x]] + I*(a + I*b)^3*Log[I + Tan[d + e*x]]) - 12*a*(a^4 - 2*a^2*b^2 - 4*b^4)*Tan[d + e*x] + 18*a^2*b*(a^2 + 2*b^2)*Tan[d + e*x]^2 + 4*a^3*(a^2 + 4*b^2)*Tan[d + e*x]^3 + 3*a^4*b*Tan[d + e*x]^4)/(12*e)

fricas [A] time = 1.71, size = 149, normalized size = 1.03

$$\frac{3a^4b \tan^4(ex + d) + 4(a^5 + 4a^3b^2) \tan^3(ex + d) + 12(a^5 - 2a^3b^2 - 3ab^4)ex + 18(a^4b + 2a^2b^3) \tan^2(ex + d) + 6(3a^4b + 2a^2b^3 - b^5) \log(1/(\tan^2(ex + d) + 1)) - 12(a^5 - 2a^3b^2 - 4a^2b^4) \tan(ex + d)}{12e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, algorithm="fricas")

[Out] 1/12*(3*a^4*b*tan(e*x + d)^4 + 4*(a^5 + 4*a^3*b^2)*tan(e*x + d)^3 + 12*(a^5 - 2*a^3*b^2 - 3*a*b^4)*e*x + 18*(a^4*b + 2*a^2*b^3)*tan(e*x + d)^2 + 6*(3*a^4*b + 2*a^2*b^3 - b^5)*log(1/(tan(e*x + d)^2 + 1)) - 12*(a^5 - 2*a^3*b^2 - 4*a^2*b^4)*tan(e*x + d))/e

giac [B] time = 5.12, size = 2326, normalized size = 16.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, algorithm="giac")

[Out] $1/12*(12*a^5*x*e*\tan(x*e)^4*\tan(d)^4 - 24*a^3*b^2*x*e*\tan(x*e)^4*\tan(d)^4 - 36*a*b^4*x*e*\tan(x*e)^4*\tan(d)^4 + 18*a^4*b*\log(4*(\tan(x*e)^4*\tan(d)^2 - 2*\tan(x*e)^3*\tan(d) + \tan(x*e)^2*\tan(d)^2 + \tan(x*e)^2 - 2*\tan(x*e)*\tan(d) + 1)/(\tan(d)^2 + 1))*\tan(x*e)^4*\tan(d)^4 + 12*a^2*b^3*\log(4*(\tan(x*e)^4*\tan(d)^2 - 2*\tan(x*e)^3*\tan(d) + \tan(x*e)^2*\tan(d)^2 + \tan(x*e)^2 - 2*\tan(x*e)*\tan(d) + 1)/(\tan(d)^2 + 1))*\tan(x*e)^4*\tan(d)^4 - 6*b^5*\log(4*(\tan(x*e)^4*\tan(d)^2 - 2*\tan(x*e)^3*\tan(d) + \tan(x*e)^2*\tan(d)^2 + \tan(x*e)^2 - 2*\tan(x*e)*\tan(d) + 1)/(\tan(d)^2 + 1))*\tan(x*e)^4*\tan(d)^4 - 48*a^5*x*e*\tan(x*e)^3*\tan(d)^3 + 96*a^3*b^2*x*e*\tan(x*e)^3*\tan(d)^3 + 144*a*b^4*x*e*\tan(x*e)^3*\tan(d)^3 + 15*a^4*b*\tan(x*e)^4*\tan(d)^4 + 36*a^2*b^3*\tan(x*e)^4*\tan(d)^4 - 72*a^4*b*\log(4*(\tan(x*e)^4*\tan(d)^2 - 2*\tan(x*e)^3*\tan(d) + \tan(x*e)^2*\tan(d)^2 + \tan(x*e)^2 - 2*\tan(x*e)*\tan(d) + 1)/(\tan(d)^2 + 1))*\tan(x*e)^3*\tan(d)^3 - 48*a^2*b^3*\log(4*(\tan(x*e)^4*\tan(d)^2 - 2*\tan(x*e)^3*\tan(d) + \tan(x*e)^2*\tan(d)^2 + \tan(x*e)^2 - 2*\tan(x*e)*\tan(d) + 1)/(\tan(d)^2 + 1))*\tan(x*e)^3*\tan(d)^3 + 24*b^5*\log(4*(\tan(x*e)^4*\tan(d)^2 - 2*\tan(x*e)^3*\tan(d) + \tan(x*e)^2*\tan(d)^2 + \tan(x*e)^2 - 2*\tan(x*e)*\tan(d) + 1)/(\tan(d)^2 + 1))*\tan(x*e)^3*\tan(d)^3 + 12*a^5*\tan(x*e)^4*\tan(d)^3 - 24*a^3*b^2*\tan(x*e)^4*\tan(d)^3 - 48*a*b^4*\tan(x*e)^4*\tan(d)^3 + 12*a^5*\tan(x*e)^3*\tan(d)^4 - 24*a^3*b^2*\tan(x*e)^3*\tan(d)^4 - 48*a*b^4*\tan(x*e)^3*\tan(d)^4 + 72*a^5*x*e*\tan(x*e)^2*\tan(d)^2 - 144*a^3*b^2*x*e*\tan(x*e)^2*\tan(d)^2 - 216*a*b^4*x*e*\tan(x*e)^2*\tan(d)^2 + 18*a^4*b*\tan(x*e)^4*\tan(d)^2 + 36*a^2*b^3*\tan(x*e)^4*\tan(d)^2 - 24*a^4*b*\tan(x*e)^3*\tan(d)^3 - 72*a^2*b^3*\tan(x*e)^3*\tan(d)^3 + 18*a^4*b*\tan(x*e)^2*\tan(d)^4 + 36*a^2*b^3*\tan(x*e)^2*\tan(d)^4 - 4*a^5*\tan(x*e)^4*\tan(d) - 16*a^3*b^2*\tan(x*e)^4*\tan(d) + 108*a^4*b*\log(4*(\tan(x*e)^4*\tan(d)^2 - 2*\tan(x*e)^3*\tan(d) + \tan(x*e)^2*\tan(d)^2 + \tan(x*e)^2 - 2*\tan(x*e)*\tan(d) + 1)/(\tan(d)^2 + 1))*\tan(x*e)^2*\tan(d)^2 + 72*a^2*b^3*\log(4*(\tan(x*e)^4*\tan(d)^2 - 2*\tan(x*e)^3*\tan(d) + \tan(x*e)^2*\tan(d)^2 + \tan(x*e)^2 - 2*\tan(x*e)*\tan(d) + 1)/(\tan(d)^2 + 1))*\tan(x*e)^2*\tan(d)^2 - 36*b^5*\log(4*(\tan(x*e)^4*\tan(d)^2 - 2*\tan(x*e)^3*\tan(d) + \tan(x*e)^2*\tan(d)^2 + \tan(x*e)^2 - 2*\tan(x*e)*\tan(d) + 1)/(\tan(d)^2 + 1))*\tan(x*e)^2*\tan(d)^2 - 48*a^5*\tan(x*e)^3*\tan(d)^2 + 24*a^3*b^2*\tan(x*e)^3*\tan(d)^2 + 144*a*b^4*\tan(x*e)^3*\tan(d)^2 - 48*a^5*\tan(x*e)^2*\tan(d)^3 + 24*a^3*b^2*\tan(x*e)^2*\tan(d)^3 + 144*a*b^4*\tan(x*e)^2*\tan(d)^3 - 4*a^5*\tan(x*e)*\tan(d)^4 - 16*a^3*b^2*\tan(x*e)*\tan(d)^4 + 3*a^4*b*\tan(x*e)^4 - 48*a^5*x*e*\tan(x*e)*\tan(d) + 96*a^3*b^2*x*e*\tan(x*e)*\tan(d) + 144*a*b^4*x*e*\tan(x*e)*\tan(d) - 24*a^4*b*\tan(x*e)^3*\tan(d) - 72*a^2*b^3*\tan(x*e)^3*\tan(d) + 36*a^4*b*\tan(x*e)^2*\tan(d)^2 + 72*a^2*b^3*\tan(x*e)^2*\tan(d)^2 + 72*a^2*b^3*\tan(x*e)^2*\tan(d)^2 - 24*a^4*b*\tan(x*e)*\tan(d)^3 - 72*a^2*b^3*\tan(x*e)*\tan(d)^3 + 3*a^4*b*\tan(d)^4 + 4*a^5*\tan(x*e)^3 + 16*a^3*b^2*\tan(x*e)^3 - 72*a^4*b*\log(4*(\tan(x*e)^4*\tan(d)^2 - 2*\tan(x*e)^3*\tan(d) + \tan(x*e)^2*\tan(d)^2 + \tan(x*e)^2 - 2*\tan(x*e)*\tan(d) + 1)/(\tan(d)^2 + 1))*\tan(x*e)*\tan(d) - 48*a^2*b^3*\log(4*(\tan(x*e)^4*\tan(d)^2 - 2*\tan(x*e)^3*\tan(d) + \tan(x*e)^2*\tan(d)^2 + \tan(x*e)^2 - 2*\tan(x*e)*\tan(d) + 1)/(\tan(d)^2 + 1))*\tan(x*e)*\tan(d) + 24*b^5*\log(4*(\tan(x*e)^4*\tan(d)^2 - 2*\tan(x*e)^3*\tan(d) + \tan(x*e)^2*\tan(d)^2 + \tan(x*e)^2 - 2*\tan(x*e)*\tan(d) + 1)/(\tan(d)^2 + 1))*\tan(x*e)*\tan(d) + 48*a^5*\tan(x*e)^2*\tan(d) - 24*a^3*b^2*\tan(x*e)^2*\tan(d) - 144*a*b^4*\tan(x*e)^2*\tan(d) + 48*a^5*\tan(x*e)*\tan(d)^2 - 24*a^3*b^2*\tan(x*e)*\tan(d)^2 - 144*a*b^4*\tan(x*e)*\tan(d)^2 + 4*a^5*\tan(d)^3 + 16*a^3*b^2*\tan(d)^3 + 12*a^5*x*e - 24*a^3*b^2*x*e - 36*a*b^4*x*e + 18*a^4*b*\tan(x*e)^2 + 36*a^2*b^3*\tan(x*e)^2 - 24*a^4*b*\tan(x*e)*\tan(d) - 72*a^2*b^3*\tan(x*e)*\tan(d) + 18*a^4*b*\tan(d)^2 + 36*a^2*b^3*\tan(d)^2 + 18*a^4*b*\log(4*(\tan(x*e)^4*\tan(d)^2 - 2*\tan(x*e)^3*\tan(d) + \tan(x*e)^2*\tan(d)^2 + \tan(x*e)^2 - 2*\tan(x*e)*\tan(d) + 1)/(\tan(d)^2 + 1)) + 12*a^2*b^3*\log(4*(\tan(x*e)^4*\tan(d)^2 - 2*\tan(x*e)^3*\tan(d) + \tan(x*e)^2*\tan(d)^2 + \tan(x*e)^2 - 2*\tan(x*e)*\tan(d) + 1)/(\tan(d)^2 + 1)) - 6*b^5*\log(4*(\tan(x*e)^4*\tan(d)^2 - 2*\tan(x*e)^3*\tan(d) + \tan(x*e)^2*\tan(d)^2 + \tan(x*e)^2 - 2*\tan(x*e)*\tan(d) + 1)/(\tan(d)^2 + 1)) - 12*a^5*\tan(x*e) + 24*a^3*b^2*\tan(x*e) + 48*a*b^4*\tan(x*e) - 12*a^5*\tan(d) + 24*a^3*b^2*\tan(d) + 48*a*b^4*\tan(d) + 15*a^4*b + 36*a^2*b^3)/(e*\tan(x*e)^4*\tan(d)^4 - 4*e*\tan(x*e)^3*\tan(d)^3 + 6*e*\tan(x*e)^2*\tan(d)^2 - 4*e*\tan(x*e)*\tan(d) + e)$

maple [A] time = 0.01, size = 245, normalized size = 1.70

$$\frac{a^4 b (\tan^4(ex+d))}{4e} + \frac{(\tan^3(ex+d)) a^5}{3e} + \frac{4(\tan^3(ex+d)) a^3 b^2}{3e} + \frac{3(\tan^2(ex+d)) a^4 b}{2e} + \frac{3(\tan^2(ex+d)) a^2 b^3}{e} + \frac{a^5}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x)

[Out] 1/4/e*a^4*b*tan(e*x+d)^4+1/3/e*tan(e*x+d)^3*a^5+4/3/e*tan(e*x+d)^3*a^3*b^2+3/2/e*tan(e*x+d)^2*a^4*b+3/e*tan(e*x+d)^2*a^2*b^3-1/e*a^5*tan(e*x+d)+2/e*a^3*b^2*tan(e*x+d)+4/e*a*b^4*tan(e*x+d)-3/2/e*ln(1+tan(e*x+d)^2)*a^4*b-1/e*ln(1+tan(e*x+d)^2)*a^2*b^3+1/2/e*ln(1+tan(e*x+d)^2)*b^5+1/e*arctan(tan(e*x+d))*a^5-2/e*arctan(tan(e*x+d))*a^3*b^2-3/e*arctan(tan(e*x+d))*a*b^4

maxima [A] time = 0.40, size = 150, normalized size = 1.04

$$\frac{3a^4 b \tan(ex+d)^4 + 4(a^5 + 4a^3 b^2) \tan(ex+d)^3 + 18(a^4 b + 2a^2 b^3) \tan(ex+d)^2 + 12(a^5 - 2a^3 b^2 - 3ab^4)(ex+d) - 6(3a^4 b + 2a^2 b^3 - b^5) \log(\tan(ex+d)^2 + 1) - 12(a^5 - 2a^3 b^2 - 4a^2 b^4) \tan(ex+d)}{12e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, algorithm="maxima")

[Out] 1/12*(3*a^4*b*tan(e*x+d)^4 + 4*(a^5 + 4*a^3*b^2)*tan(e*x+d)^3 + 18*(a^4*b + 2*a^2*b^3)*tan(e*x+d)^2 + 12*(a^5 - 2*a^3*b^2 - 3*a*b^4)*(e*x+d) - 6*(3*a^4*b + 2*a^2*b^3 - b^5)*log(tan(e*x+d)^2 + 1) - 12*(a^5 - 2*a^3*b^2 - 4*a^2*b^4)*tan(e*x+d))/e

mupad [B] time = 2.85, size = 205, normalized size = 1.42

$$\frac{\tan(d+ex)^3 \left(\frac{a^5}{3} + \frac{4a^3 b^2}{3} \right)}{e} + \frac{\tan(d+ex) \left(-a^5 + 2a^3 b^2 + 4a b^4 \right)}{e} - \frac{\ln(\tan(d+ex)^2 + 1) \left(\frac{3a^4 b}{2} + a^2 b^3 - \frac{b^5}{2} \right)}{e} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^2, x)

[Out] (tan(d + e*x)^3*(a^5/3 + (4*a^3*b^2)/3))/e + (tan(d + e*x)*(4*a*b^4 - a^5 + 2*a^3*b^2))/e - (log(tan(d + e*x)^2 + 1)*((3*a^4*b)/2 - b^5/2 + a^2*b^3))/e + (tan(d + e*x)^2*((3*a^4*b)/2 + 3*a^2*b^3))/e + (a^4*b*tan(d + e*x)^4)/(4*e) - (a*atan((a*tan(d + e*x)*(a^2 - 3*b^2)*(a^2 + b^2))/(3*a*b^4 - a^5 + 2*a^3*b^2))*(a^2 - 3*b^2)*(a^2 + b^2))/e

sympy [A] time = 0.63, size = 248, normalized size = 1.72

$$\left\{ \begin{array}{l} a^5 x + \frac{a^5 \tan^3(d+ex)}{3e} - \frac{a^5 \tan(d+ex)}{e} - \frac{3a^4 b \log(\tan^2(d+ex)+1)}{2e} + \frac{a^4 b \tan^4(d+ex)}{4e} + \frac{3a^4 b \tan^2(d+ex)}{2e} - 2a^3 b^2 x + \frac{4a^3 b^2 \tan^3(d+ex)}{3e} + \\ x(a + b \tan(d)) (a^2 \tan^2(d) + 2ab \tan(d) + b^2)^2 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))*(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**2,x)

[Out] Piecewise((a**5*x + a**5*tan(d + e*x)**3/(3*e) - a**5*tan(d + e*x)/e - 3*a**4*b*log(tan(d + e*x)**2 + 1)/(2*e) + a**4*b*tan(d + e*x)**4/(4*e) + 3*a**4*b*tan(d + e*x)**2/(2*e) - 2*a**3*b**2*x + 4*a**3*b**2*tan(d + e*x)**3/(3*e) + 2*a**3*b**2*tan(d + e*x)/e - a**2*b**3*log(tan(d + e*x)**2 + 1)/e + 3*a**2*b**3*tan(d + e*x)**2/e - 3*a*b**4*x + 4*a*b**4*tan(d + e*x)/e + b**5*log(tan(d + e*x)**2 + 1)/(2*e), Ne(e, 0)), (x*(a + b*tan(d))*(a**2*tan(d)**2 + 2*a*b*tan(d) + b**2)**2, True))

3.511 $\int (a + b \tan(d + ex)) (b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)) dx$

Optimal. Leaf size=72

$$-\frac{b(a^2 + b^2) \log(\cos(d + ex))}{e} - ax(a^2 + b^2) + \frac{a^2(a + b \tan(d + ex))^2}{2be} + \frac{2ab^2 \tan(d + ex)}{e}$$

[Out] $-a*(a^2+b^2)*x-b*(a^2+b^2)*\ln(\cos(e*x+d))/e+2*a*b^2*\tan(e*x+d)/e+1/2*a^2*(a+b*\tan(e*x+d))^2/b/e$

Rubi [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {3630, 3525, 3475}

$$-\frac{b(a^2 + b^2) \log(\cos(d + ex))}{e} - ax(a^2 + b^2) + \frac{a^2(a + b \tan(d + ex))^2}{2be} + \frac{2ab^2 \tan(d + ex)}{e}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2), x]

[Out] $-(a*(a^2 + b^2)*x) - (b*(a^2 + b^2)*\text{Log}[\text{Cos}[d + e*x]])/e + (2*a*b^2*\text{Tan}[d + e*x])/e + (a^2*(a + b*\text{Tan}[d + e*x])^2)/(2*b*e)$

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3525

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e + f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]

Rule 3630

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)]) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2, x_Symbol] := Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Int[(a + b*Tan[e + f*x])^m*Simp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned} \int (a + b \tan(d + ex)) (b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)) dx &= \frac{a^2(a + b \tan(d + ex))^2}{2be} + \int (a + b \tan(d + ex)) dx \\ &= -a(a^2 + b^2)x + \frac{2ab^2 \tan(d + ex)}{e} + \frac{a^2(a + b \tan(d + ex))^2}{2be} \\ &= -a(a^2 + b^2)x - \frac{b(a^2 + b^2) \log(\cos(d + ex))}{e} \end{aligned}$$

Mathematica [C] time = 0.33, size = 88, normalized size = 1.22

$$\frac{2a(a^2 + 2b^2) \tan(d + ex) + (a^2 + b^2) ((b + ia) \log(-\tan(d + ex) + i) + (b - ia) \log(\tan(d + ex) + i)) + a^2 b \tan^2(d + ex)}{2e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2), x]

[Out] ((a^2 + b^2)*((I*a + b)*Log[I - Tan[d + e*x]] + ((-I)*a + b)*Log[I + Tan[d + e*x]]) + 2*a*(a^2 + 2*b^2)*Tan[d + e*x] + a^2*b*Tan[d + e*x]^2)/(2*e)

fricas [A] time = 1.12, size = 74, normalized size = 1.03

$$\frac{a^2 b \tan(ex + d)^2 - 2(a^3 + ab^2)ex - (a^2 b + b^3) \log\left(\frac{1}{\tan(ex+d)^2 + 1}\right) + 2(a^3 + 2ab^2) \tan(ex + d)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2), x, algorithm="fricas")

[Out] 1/2*(a^2*b*tan(e*x + d)^2 - 2*(a^3 + a*b^2)*e*x - (a^2*b + b^3)*log(1/(tan(e*x + d)^2 + 1)) + 2*(a^3 + 2*a*b^2)*tan(e*x + d))/e

giac [B] time = 1.02, size = 709, normalized size = 9.85

$$\frac{2a^3 x e \tan(xe)^2 \tan(d)^2 + 2ab^2 x e \tan(xe)^2 \tan(d)^2 + a^2 b \log\left(\frac{4(\tan(xe)^4 \tan(d)^2 - 2 \tan(xe)^3 \tan(d) + \tan(xe)^2 \tan(d)^2 + \tan(xe)^2 \tan(d)^2 + \tan(xe)^2 - 2 \tan(xe) \tan(d) + 1)}{\tan(d)^2 + 1}\right)}{\tan(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2), x, algorithm="giac")

[Out] -1/2*(2*a^3*x*e*tan(x*e)^2*tan(d)^2 + 2*a*b^2*x*e*tan(x*e)^2*tan(d)^2 + a^2*b*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*tan(x*e)^2*tan(d)^2 + b^3*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*tan(x*e)^2*tan(d)^2 - 4*a^3*x*e*tan(x*e)*tan(d) - 4*a*b^2*x*e*tan(x*e)*tan(d) - a^2*b*tan(x*e)^2*tan(d)^2 - 2*a^2*b*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*tan(x*e)*tan(d) - 2*b^3*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*tan(x*e)*tan(d) + 2*a^3*tan(x*e)^2*tan(d) + 4*a*b^2*tan(x*e)^2*tan(d) + 2*a^3*tan(x*e)*tan(d)^2 + 4*a*b^2*tan(x*e)*tan(d)^2 + 2*a^3*x*e + 2*a*b^2*x*e - a^2*b*tan(x*e)^2 - a^2*b*tan(d)^2 + a^2*b*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1)) + b^3*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1)) - 2*a^3*tan(x*e) - 4*a*b^2*tan(x*e) - 2*a^3*tan(d) - 4*a*b^2*tan(d) - a^2*b)/(e*tan(x*e)^2*tan(d)^2 - 2*e*tan(x*e)*tan(d) + e)

maple [A] time = 0.01, size = 117, normalized size = 1.62

$$\frac{a^2 b (\tan^2(ex + d))}{2e} + \frac{a^3 \tan(ex + d)}{e} + \frac{2a b^2 \tan(ex + d)}{e} + \frac{\ln(1 + \tan^2(ex + d)) a^2 b}{2e} + \frac{\ln(1 + \tan^2(ex + d)) b^3}{2e} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2), x)

[Out] $\frac{1}{2}e a^2 b \tan(e x+d)^2 + \frac{1}{e} a^3 \tan(e x+d) + 2 a^2 b \tan(e x+d) / e + \frac{1}{2} e \ln(1 + \tan(e x+d)^2) a^2 b + \frac{1}{2} e \ln(1 + \tan(e x+d)^2) b^3 - \frac{1}{e} \arctan(\tan(e x+d)) a^3 - \frac{1}{e} \arctan(\tan(e x+d)) a^2 b^2$

maxima [A] time = 0.41, size = 74, normalized size = 1.03

$$\frac{a^2 b \tan(e x+d)^2 - 2(a^3 + a b^2)(e x+d) + (a^2 b + b^3) \log(\tan(e x+d)^2 + 1) + 2(a^3 + 2 a b^2) \tan(e x+d)}{2 e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2), x, algorith="maxima")

[Out] $\frac{1}{2}*(a^2*b*\tan(e*x+d)^2 - 2*(a^3 + a*b^2)*(e*x+d) + (a^2*b + b^3)*\log(\tan(e*x+d)^2 + 1) + 2*(a^3 + 2*a*b^2)*\tan(e*x+d))/e$

mupad [B] time = 2.77, size = 105, normalized size = 1.46

$$\frac{\tan(d+e x) (a^3 + 2 a b^2)}{e} + \frac{\ln(\tan(d+e x)^2 + 1) \left(\frac{a^2 b}{2} + \frac{b^3}{2}\right)}{e} + \frac{a^2 b \tan(d+e x)^2}{2 e} - \frac{a \operatorname{atan}\left(\frac{a \tan(d+e x) (a^2 + b^2)}{a^3 + a b^2}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x)), x)

[Out] $(\tan(d+e x)*(2*a*b^2 + a^3))/e + (\log(\tan(d+e x)^2 + 1)*((a^2*b)/2 + b^3/2))/e + (a^2*b*\tan(d+e x)^2)/(2*e) - (a*\operatorname{atan}((a*\tan(d+e x)*(a^2 + b^2))/(a*b^2 + a^3))*(a^2 + b^2))/e$

sympy [A] time = 0.25, size = 122, normalized size = 1.69

$$\begin{cases} -a^3 x + \frac{a^3 \tan(d+e x)}{e} + \frac{a^2 b \log(\tan^2(d+e x)+1)}{2e} + \frac{a^2 b \tan^2(d+e x)}{2e} - a b^2 x + \frac{2 a b^2 \tan(d+e x)}{e} + \frac{b^3 \log(\tan^2(d+e x)+1)}{2e} & \text{for } e \neq 0 \\ x(a + b \tan(d)) (a^2 \tan^2(d) + 2 a b \tan(d) + b^2) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))*(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2), x)

[Out] Piecewise((-a**3*x + a**3*tan(d + e*x)/e + a**2*b*log(tan(d + e*x)**2 + 1)/(2*e) + a**2*b*tan(d + e*x)**2/(2*e) - a*b**2*x + 2*a*b**2*tan(d + e*x)/e + b**3*log(tan(d + e*x)**2 + 1)/(2*e), Ne(e, 0)), (x*(a + b*tan(d))*(a**2*tan(d)**2 + 2*a*b*tan(d) + b**2), True))

$$3.512 \quad \int \frac{a+b \tan(d+ex)}{b^2+2ab \tan(d+ex)+a^2 \tan^2(d+ex)} dx$$

Optimal. Leaf size=101

$$-\frac{a^2 - b^2}{e(a^2 + b^2)(a \tan(d + ex) + b)} + \frac{b(3a^2 - b^2) \log(a \sin(d + ex) + b \cos(d + ex))}{e(a^2 + b^2)^2} - \frac{ax(a^2 - 3b^2)}{(a^2 + b^2)^2}$$

[Out] $-a*(a^2-3*b^2)*x/(a^2+b^2)^2+b*(3*a^2-b^2)*\ln(b*\cos(e*x+d)+a*\sin(e*x+d))/(a^2+b^2)^2/e+(-a^2+b^2)/(a^2+b^2)/e/(b+a*\tan(e*x+d))$

Rubi [A] time = 0.26, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3708, 3529, 3531, 3530}

$$-\frac{a^2 - b^2}{e(a^2 + b^2)(a \tan(d + ex) + b)} + \frac{b(3a^2 - b^2) \log(a \sin(d + ex) + b \cos(d + ex))}{e(a^2 + b^2)^2} - \frac{ax(a^2 - 3b^2)}{(a^2 + b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2),x]

[Out] $-((a*(a^2 - 3*b^2)*x)/(a^2 + b^2)^2) + (b*(3*a^2 - b^2)*\text{Log}[b*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x]])/((a^2 + b^2)^2*e) - (a^2 - b^2)/((a^2 + b^2)*e*(b + a*\text{Tan}[d + e*x]))$

Rule 3529

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rule 3530

Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]

Rule 3531

Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a*c + b*d, 0]

Rule 3708

Int[((A_.) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_.) + (b_.)*tan[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n), Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tan(d + ex)}{b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)} dx &= (4a^2) \int \frac{a + b \tan(d + ex)}{(2ab + 2a^2 \tan(d + ex))^2} dx \\
&= -\frac{a^2 - b^2}{(a^2 + b^2) e(b + a \tan(d + ex))} + \frac{\int \frac{4a^2 b - 2a(a^2 - b^2) \tan(d + ex)}{2ab + 2a^2 \tan(d + ex)} dx}{a^2 + b^2} \\
&= -\frac{a(a^2 - 3b^2)x}{(a^2 + b^2)^2} - \frac{a^2 - b^2}{(a^2 + b^2) e(b + a \tan(d + ex))} + \frac{b(3a^2 - b^2)}{(a^2 + b^2)^2} \\
&= -\frac{a(a^2 - 3b^2)x}{(a^2 + b^2)^2} + \frac{b(3a^2 - b^2) \log(b \cos(d + ex) + a \sin(d + ex))}{(a^2 + b^2)^2 e}
\end{aligned}$$

Mathematica [C] time = 2.34, size = 187, normalized size = 1.85

$$\frac{b(-((a+ib)\log(-\tan(d+ex)+i))-(a-ib)\log(\tan(d+ex)+i)+2a\log(a\tan(d+ex)+b))}{a^2+b^2} + (a-b)(a+b) \left(\frac{2a\left(2b\log(a\tan(d+ex)+b)-\frac{a^2+b^2}{a\tan(d+ex)+b}\right)}{(a^2+b^2)^2} \right)$$

$2ae$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2), x]

[Out] ((b*(-((a + I*b)*Log[I - Tan[d + e*x]]) - (a - I*b)*Log[I + Tan[d + e*x]] + 2*a*Log[b + a*Tan[d + e*x]]))/(a^2 + b^2) + (a - b)*(a + b)*((I*Log[I - Tan[d + e*x]])/(a - I*b)^2 - (I*Log[I + Tan[d + e*x]])/(a + I*b)^2 + (2*a*(2*b*Log[b + a*Tan[d + e*x]] - (a^2 + b^2)/(b + a*Tan[d + e*x])))/(a^2 + b^2)^2))/(2*a*e)

fricas [A] time = 0.72, size = 191, normalized size = 1.89

$$\frac{2a^4 - 2a^2b^2 + 2(a^3b - 3ab^3)ex - (3a^2b^2 - b^4 + (3a^3b - ab^3)\tan(ex + d)) \log\left(\frac{a^2 \tan^2(ex + d) + 2ab \tan(ex + d) + b^2}{\tan^2(ex + d) + 1}\right)}{2((a^5 + 2a^3b^2 + ab^4)e \tan(ex + d) + (a^4b + 2a^2b^3 + b^5)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2), x, algorithm="fricas")

[Out] -1/2*(2*a^4 - 2*a^2*b^2 + 2*(a^3*b - 3*a*b^3)*e*x - (3*a^2*b^2 - b^4 + (3*a^3*b - a*b^3)*tan(e*x + d))*log((a^2*tan(e*x + d)^2 + 2*a*b*tan(e*x + d) + b^2)/(tan(e*x + d)^2 + 1)) - 2*(a^3*b - a*b^3 - (a^4 - 3*a^2*b^2)*e*x)*tan(e*x + d)/((a^5 + 2*a^3*b^2 + a*b^4)*e*tan(e*x + d) + (a^4*b + 2*a^2*b^3 + b^5)*e)

giac [B] time = 0.89, size = 204, normalized size = 2.02

$$\frac{1}{2} \left(\frac{2(a^3 - 3ab^2)(xe + d)}{a^4 + 2a^2b^2 + b^4} + \frac{(3a^2b - b^3) \log(\tan(xe + d)^2 + 1)}{a^4 + 2a^2b^2 + b^4} - \frac{2(3a^3b - ab^3) \log(|a \tan(xe + d) + b|)}{a^5 + 2a^3b^2 + ab^4} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2), x, algorithm="giac")

[Out] $-1/2*(2*(a^3 - 3*a*b^2)*(x*e + d)/(a^4 + 2*a^2*b^2 + b^4) + (3*a^2*b - b^3)*\log(\tan(x*e + d)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(3*a^3*b - a*b^3)*\log(\text{abs}(a*\tan(x*e + d) + b))/(a^5 + 2*a^3*b^2 + a*b^4) + 2*(3*a^3*b*\tan(x*e + d) - a*b^3*\tan(x*e + d) + a^4 + 3*a^2*b^2 - 2*b^4)/((a^4 + 2*a^2*b^2 + b^4)*(a*\tan(x*e + d) + b)))*e^{-1}$

maple [B] time = 0.16, size = 222, normalized size = 2.20

$$\frac{a^2}{e(a^2 + b^2)(b + a \tan(ex + d))} + \frac{b^2}{e(a^2 + b^2)(b + a \tan(ex + d))} + \frac{3b \ln(b + a \tan(ex + d)) a^2}{e(a^2 + b^2)^2} - \frac{b^3 \ln(b + a \tan(ex + d))}{e(a^2 + b^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x)`

[Out] $-1/e/(a^2+b^2)/(b+a*\tan(e*x+d))*a^2+1/e/(a^2+b^2)/(b+a*\tan(e*x+d))*b^2+3/e*b/(a^2+b^2)^2*\ln(b+a*\tan(e*x+d))*a^2-1/e*b^3/(a^2+b^2)^2*\ln(b+a*\tan(e*x+d))-3/2/e/(a^2+b^2)^2*\ln(1+\tan(e*x+d)^2)*a^2*b+1/2/e/(a^2+b^2)^2*\ln(1+\tan(e*x+d)^2)*b^3-1/e/(a^2+b^2)^2*\arctan(\tan(e*x+d))*a^3+3/e/(a^2+b^2)^2*\arctan(\tan(e*x+d))*a*b^2$

maxima [A] time = 0.41, size = 161, normalized size = 1.59

$$\frac{2(a^3-3ab^2)(ex+d)}{a^4+2a^2b^2+b^4} - \frac{2(3a^2b-b^3)\log(a\tan(ex+d)+b)}{a^4+2a^2b^2+b^4} + \frac{(3a^2b-b^3)\log(\tan(ex+d)^2+1)}{a^4+2a^2b^2+b^4} + \frac{2(a^2-b^2)}{a^2b+b^3+(a^3+ab^2)\tan(ex+d)}$$

$2e$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algorithm="maxima")`

[Out] $-1/2*(2*(a^3 - 3*a*b^2)*(e*x + d)/(a^4 + 2*a^2*b^2 + b^4) - 2*(3*a^2*b - b^3)*\log(a*\tan(e*x + d) + b)/(a^4 + 2*a^2*b^2 + b^4) + (3*a^2*b - b^3)*\log(\tan(e*x + d)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^2 - b^2)/(a^2*b + b^3 + (a^3 + a*b^2)*\tan(e*x + d)))/e$

mupad [B] time = 3.09, size = 152, normalized size = 1.50

$$\frac{b \ln(b + a \tan(d + ex)) (3a^2 - b^2)}{e(a^2 + b^2)^2} - \frac{\ln(\tan(d + ex) + 1i) (a - b1i)}{2e(-a^21i + 2ab + b^21i)} - \frac{a^2 - b^2}{e(a^2 + b^2)(b + a \tan(d + ex))} - \frac{\ln(\tan(d + ex) + 1i)}{2e(a^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x)),x)`

[Out] $(b*\log(b + a*\tan(d + e*x))*(3*a^2 - b^2))/(e*(a^2 + b^2)^2) - (\log(\tan(d + e*x) + 1i)*(a - b*1i))/(2*e*(2*a*b - a^2*1i + b^2*1i)) - (a^2 - b^2)/(e*(a^2 + b^2)*(b + a*\tan(d + e*x))) - (\log(\tan(d + e*x) - 1i)*(a + b*1i))/(2*e*(2*a*b + a^2*1i - b^2*1i))$

sympy [A] time = 1.62, size = 1358, normalized size = 13.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2),x)`

[Out] `Piecewise((zoo*x*tan(d), Eq(a, 0) & Eq(b, 0) & Eq(e, 0)), (log(tan(d + e*x)**2 + 1)/(2*b*e), Eq(a, 0)), (1/(2*b*e*tan(d + e*x)**2 + 4*I*b*e*tan(d + e*x)), Eq(a, 0) & Eq(b, 0) & Eq(e, 0)), (1/(2*b*e*tan(d + e*x)**2 + 4*I*b*e*tan(d + e*x)), Eq(a, 0) & Eq(b, 0) & Eq(e, 0))`

```

x) - 2*b*e), Eq(a, -I*b)), (1/(2*b*e*tan(d + e*x)**2 - 4*I*b*e*tan(d + e*x)
- 2*b*e), Eq(a, I*b)), (x*(a + b*tan(d))/(a**2*tan(d)**2 + 2*a*b*tan(d) +
b**2), Eq(e, 0)), (-2*a**4*e*x*tan(d + e*x)/(2*a**5*e*tan(d + e*x) + 2*a**4
*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x)
+ 2*b**5*e) - 2*a**4/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*t
an(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) - 2*a**3*
b*e*x/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*
a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) + 6*a**3*b*log(tan(d + e*
x) + b/a)*tan(d + e*x)/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*
tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) - 3*a**3
*b*log(tan(d + e*x)**2 + 1)*tan(d + e*x)/(2*a**5*e*tan(d + e*x) + 2*a**4*b*
e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) +
2*b**5*e) + 6*a**2*b**2*e*x*tan(d + e*x)/(2*a**5*e*tan(d + e*x) + 2*a**4*b*
e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) +
2*b**5*e) + 6*a**2*b**2*log(tan(d + e*x) + b/a)/(2*a**5*e*tan(d + e*x) + 2*
a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d +
e*x) + 2*b**5*e) - 3*a**2*b**2*log(tan(d + e*x)**2 + 1)/(2*a**5*e*tan(d + e
*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*
tan(d + e*x) + 2*b**5*e) + 6*a*b**3*e*x/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e
+ 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) + 2
*b**5*e) - 2*a*b**3*log(tan(d + e*x) + b/a)*tan(d + e*x)/(2*a**5*e*tan(d +
e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e
*tan(d + e*x) + 2*b**5*e) + a*b**3*log(tan(d + e*x)**2 + 1)*tan(d + e*x)/(2
*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**
3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) - 2*b**4*log(tan(d + e*x) + b/a)/
(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b
**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) + b**4*log(tan(d + e*x)**2 + 1)
/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*
b**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) + 2*b**4/(2*a**5*e*tan(d + e*x
) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*ta
n(d + e*x) + 2*b**5*e), True))

```

$$3.513 \quad \int \frac{a+b \tan(d+ex)}{(b^2+2ab \tan(d+ex)+a^2 \tan^2(d+ex))^2} dx$$

Optimal. Leaf size=197

$$\frac{a^2 - b^2}{3e(a^2 + b^2)(a \tan(d + ex) + b)^3} - \frac{b(3a^2 - b^2)}{2e(a^2 + b^2)^2(a \tan(d + ex) + b)^2} + \frac{a^4 - 6a^2b^2 + b^4}{e(a^2 + b^2)^3(a \tan(d + ex) + b)} - \frac{b(5a^4 - 10a^2b^2 + 5b^4)}{e(a^2 + b^2)^4(a \tan(d + ex) + b)}$$

[Out] a*(a^4-10*a^2*b^2+5*b^4)*x/(a^2+b^2)^4-b*(5*a^4-10*a^2*b^2+b^4)*ln(b*cos(e*x+d)+a*sin(e*x+d))/(a^2+b^2)^4/e+1/3*(-a^2+b^2)/(a^2+b^2)/e/(b+a*tan(e*x+d))^3-1/2*b*(3*a^2-b^2)/(a^2+b^2)^2/e/(b+a*tan(e*x+d))^2+(a^4-6*a^2*b^2+b^4)/(a^2+b^2)^3/e/(b+a*tan(e*x+d))

Rubi [A] time = 0.54, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3708, 3529, 3531, 3530}

$$\frac{a^2 - b^2}{3e(a^2 + b^2)(a \tan(d + ex) + b)^3} + \frac{-6a^2b^2 + a^4 + b^4}{e(a^2 + b^2)^3(a \tan(d + ex) + b)} - \frac{b(3a^2 - b^2)}{2e(a^2 + b^2)^2(a \tan(d + ex) + b)^2} - \frac{b(-10a^2b^2 + 5b^4)}{e(a^2 + b^2)^4(a \tan(d + ex) + b)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^2, x]

[Out] (a*(a^4 - 10*a^2*b^2 + 5*b^4)*x)/(a^2 + b^2)^4 - (b*(5*a^4 - 10*a^2*b^2 + b^4)*Log[b*Cos[d + e*x] + a*Sin[d + e*x]])/((a^2 + b^2)^4*e) - (a^2 - b^2)/(3*(a^2 + b^2)*e*(b + a*Tan[d + e*x])^3) - (b*(3*a^2 - b^2))/(2*(a^2 + b^2)^2*e*(b + a*Tan[d + e*x])^2) + (a^4 - 6*a^2*b^2 + b^4)/((a^2 + b^2)^3*e*(b + a*Tan[d + e*x]))

Rule 3529

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]

Rule 3530

Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_.)])/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]

Rule 3531

Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_.)])/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a*c + b*d, 0]

Rule 3708

Int[((A_.) + (B_.)*tan[(d_.) + (e_.)*(x_.)])*((a_.) + (b_.)*tan[(d_.) + (e_.)*(x_.)] + (c_.)*tan[(d_.) + (e_.)*(x_.)]^2)^(n_.), x_Symbol] :> Dist[1/(4^n*c^n), Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{

a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tan(d + ex)}{(b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^2} dx &= (16a^4) \int \frac{a + b \tan(d + ex)}{(2ab + 2a^2 \tan(d + ex))^4} dx \\
 &= -\frac{a^2 - b^2}{3(a^2 + b^2)e(b + a \tan(d + ex))^3} + \frac{(4a^2) \int \frac{4a^2b - 2a(a^2 - b^2) \tan(d + ex)}{(2ab + 2a^2 \tan(d + ex))^4} dx}{a^2 + b^2} \\
 &= -\frac{a^2 - b^2}{3(a^2 + b^2)e(b + a \tan(d + ex))^3} - \frac{b(3a^2 - b^2)}{2(a^2 + b^2)^2 e(b + a \tan(d + ex))} \\
 &= -\frac{a^2 - b^2}{3(a^2 + b^2)e(b + a \tan(d + ex))^3} - \frac{b(3a^2 - b^2)}{2(a^2 + b^2)^2 e(b + a \tan(d + ex))} \\
 &= \frac{a(a^4 - 10a^2b^2 + 5b^4)x}{(a^2 + b^2)^4} - \frac{a^2 - b^2}{3(a^2 + b^2)e(b + a \tan(d + ex))^3} \\
 &= \frac{a(a^4 - 10a^2b^2 + 5b^4)x}{(a^2 + b^2)^4} - \frac{b(5a^4 - 10a^2b^2 + b^4) \log(b \cos(d + ex))}{(a^2 + b^2)^4}
 \end{aligned}$$

Mathematica [C] time = 5.41, size = 308, normalized size = 1.56

$$3b \left(\frac{a \left(-\frac{(a^2+b^2)(a^2+4ab \tan(d+ex)+5b^2)}{(a \tan(d+ex)+b)^2} - 2(a^2-3b^2) \log(a \tan(d+ex)+b) \right)}{(a^2+b^2)^3} + \frac{\log(-\tan(d+ex)+i)}{(a-ib)^3} + \frac{\log(\tan(d+ex)+i)}{(a+ib)^3} \right) - (a-b)(a+b) \left(-\frac{1}{(a^2+b^2)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2), x]

[Out] (-((a - b)*(a + b)*(((3*I)*Log[I - Tan[d + e*x]])/(a - I*b)^4 - ((3*I)*Log[I + Tan[d + e*x]])/(a + I*b)^4 + (24*a*(a - b)*b*(a + b)*Log[b + a*Tan[d + e*x]])/(a^2 + b^2)^4 + (2*a)/((a^2 + b^2)*(b + a*Tan[d + e*x])^3) + (6*a*b)/((a^2 + b^2)^2*(b + a*Tan[d + e*x])^2) - (6*a*(a^2 - 3*b^2))/((a^2 + b^2)^3*(b + a*Tan[d + e*x])) + 3*b*(Log[I - Tan[d + e*x]]/(a - I*b)^3 + Log[I + Tan[d + e*x]]/(a + I*b)^3 + (a*(-2*(a^2 - 3*b^2)*Log[b + a*Tan[d + e*x]] - ((a^2 + b^2)*(a^2 + 5*b^2 + 4*a*b*Tan[d + e*x]))/(b + a*Tan[d + e*x])^2))/(a^2 + b^2)^3)/(6*a*e)

fricas [B] time = 2.02, size = 580, normalized size = 2.94

$$\frac{2a^8 + 7a^6b^2 + 66a^4b^4 - 27a^2b^6 + (21a^7b - 56a^5b^3 + 11a^3b^5 - 6(a^8 - 10a^6b^2 + 5a^4b^4)ex) \tan(ex + d)^3}{(a^2 + b^2)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, algorithm="fricas")

```
[Out] -1/6*(2*a^8 + 7*a^6*b^2 + 66*a^4*b^4 - 27*a^2*b^6 + (21*a^7*b - 56*a^5*b^3
+ 11*a^3*b^5 - 6*(a^8 - 10*a^6*b^2 + 5*a^4*b^4)*e*x)*tan(e*x + d)^3 - 6*(a^
5*b^3 - 10*a^3*b^5 + 5*a*b^7)*e*x - 3*(2*a^8 - 31*a^6*b^2 + 46*a^4*b^4 - 9*
a^2*b^6 + 6*(a^7*b - 10*a^5*b^3 + 5*a^3*b^5)*e*x)*tan(e*x + d)^2 + 3*(5*a^4
*b^4 - 10*a^2*b^6 + b^8 + (5*a^7*b - 10*a^5*b^3 + a^3*b^5)*tan(e*x + d)^3 +
3*(5*a^6*b^2 - 10*a^4*b^4 + a^2*b^6)*tan(e*x + d)^2 + 3*(5*a^5*b^3 - 10*a^
3*b^5 + a*b^7)*tan(e*x + d)*log((a^2*tan(e*x + d)^2 + 2*a*b*tan(e*x + d) +
b^2)/(tan(e*x + d)^2 + 1)) - 3*(a^7*b - 46*a^5*b^3 + 35*a^3*b^5 - 6*a*b^7
+ 6*(a^6*b^2 - 10*a^4*b^4 + 5*a^2*b^6)*e*x)*tan(e*x + d))/((a^11 + 4*a^9*b^
2 + 6*a^7*b^4 + 4*a^5*b^6 + a^3*b^8)*e*tan(e*x + d)^3 + 3*(a^10*b + 4*a^8*b
^3 + 6*a^6*b^5 + 4*a^4*b^7 + a^2*b^9)*e*tan(e*x + d)^2 + 3*(a^9*b^2 + 4*a^7
*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*e*tan(e*x + d) + (a^8*b^3 + 4*a^6*b^
5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*e)
```

giac [B] time = 1.94, size = 451, normalized size = 2.29

$$\frac{1}{6} \left(\frac{6(a^5 - 10a^3b^2 + 5ab^4)(xe + d)}{a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8} + \frac{3(5a^4b - 10a^2b^3 + b^5) \log(\tan(xe + d)^2 + 1)}{a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8} - \frac{6(5a^5b - 10a^3b^3 + ab^5)}{a^9 + 4a^7b^2 + 6a^5b^4 + 4a^3b^6 + b^8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] 1/6*(6*(a^5 - 10*a^3*b^2 + 5*a*b^4)*(x*e + d)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8) + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*log(tan(x*e + d)^2 + 1)
/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(5*a^5*b - 10*a^3*b^3
+ a*b^5)*log(abs(a*tan(x*e + d) + b))/(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*
b^6 + a*b^8) + (55*a^7*b*tan(x*e + d)^3 - 110*a^5*b^3*tan(x*e + d)^3 + 11*a
^3*b^5*tan(x*e + d)^3 + 6*a^8*tan(x*e + d)^2 + 135*a^6*b^2*tan(x*e + d)^2 -
360*a^4*b^4*tan(x*e + d)^2 + 39*a^2*b^6*tan(x*e + d)^2 + 3*a^7*b*tan(x*e +
d) + 90*a^5*b^3*tan(x*e + d) - 393*a^3*b^5*tan(x*e + d) + 48*a*b^7*tan(x*e
+ d) - 2*a^8 - 7*a^6*b^2 + 10*a^4*b^4 - 139*a^2*b^6 + 22*b^8)/((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(a*tan(x*e + d) + b)^3))*e^(-1)
```

maple [B] time = 0.18, size = 458, normalized size = 2.32

$$-\frac{a^2}{3e(a^2 + b^2)(b + a \tan(ex + d))^3} + \frac{b^2}{3e(a^2 + b^2)(b + a \tan(ex + d))^3} - \frac{3ba^2}{2e(a^2 + b^2)^2(b + a \tan(ex + d))^2} + \frac{1}{2e(a^2 + b^2)^2(b + a \tan(ex + d))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x)
```

```
[Out] -1/3/e/(a^2+b^2)/(b+a*tan(e*x+d))^3*a^2+1/3/e/(a^2+b^2)/(b+a*tan(e*x+d))^3*
b^2-3/2/e*b/(a^2+b^2)^2/(b+a*tan(e*x+d))^2*a^2+1/2/e*b^3/(a^2+b^2)^2/(b+a*t
an(e*x+d))^2+1/e/(a^2+b^2)^3/(b+a*tan(e*x+d))*a^4-6/e/(a^2+b^2)^3/(b+a*tan(
e*x+d))*a^2*b^2+1/e/(a^2+b^2)^3/(b+a*tan(e*x+d))*b^4-5/e*b/(a^2+b^2)^4*ln(b
+a*tan(e*x+d))*a^4+10/e*b^3/(a^2+b^2)^4*ln(b+a*tan(e*x+d))*a^2-1/e*b^5/(a^2
+b^2)^4*ln(b+a*tan(e*x+d))+5/2/e/(a^2+b^2)^4*ln(1+tan(e*x+d)^2)*a^4*b-5/e/(
a^2+b^2)^4*ln(1+tan(e*x+d)^2)*a^2*b^3+1/2/e/(a^2+b^2)^4*ln(1+tan(e*x+d)^2)*
b^5+1/e/(a^2+b^2)^4*arctan(tan(e*x+d))*a^5-10/e/(a^2+b^2)^4*arctan(tan(e*x+
d))*a^3*b^2+5/e/(a^2+b^2)^4*arctan(tan(e*x+d))*a*b^4
```

maxima [B] time = 0.43, size = 419, normalized size = 2.13

$$\frac{6(a^5 - 10a^3b^2 + 5ab^4)(ex + d)}{a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8} - \frac{6(5a^4b - 10a^2b^3 + b^5) \log(a \tan(ex + d) + b)}{a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8} + \frac{3(5a^4b - 10a^2b^3 + b^5) \log(\tan(ex + d)^2 + 1)}{a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8} - \frac{6(5a^5b - 10a^3b^3 + ab^5)}{a^6b^3 + 3a^4b^5 + 3a^2b^7 + b^9 + 6e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, algorithm="maxima")

[Out] $\frac{1}{6} \cdot (6 \cdot (a^5 - 10 \cdot a^3 \cdot b^2 + 5 \cdot a \cdot b^4) \cdot (e \cdot x + d) / (a^8 + 4 \cdot a^6 \cdot b^2 + 6 \cdot a^4 \cdot b^4 + 4 \cdot a^2 \cdot b^6 + b^8) - 6 \cdot (5 \cdot a^4 \cdot b - 10 \cdot a^2 \cdot b^3 + b^5) \cdot \log(a \cdot \tan(e \cdot x + d) + b) / (a^8 + 4 \cdot a^6 \cdot b^2 + 6 \cdot a^4 \cdot b^4 + 4 \cdot a^2 \cdot b^6 + b^8) + 3 \cdot (5 \cdot a^4 \cdot b - 10 \cdot a^2 \cdot b^3 + b^5) \cdot \log(\tan(e \cdot x + d)^2 + 1) / (a^8 + 4 \cdot a^6 \cdot b^2 + 6 \cdot a^4 \cdot b^4 + 4 \cdot a^2 \cdot b^6 + b^8) - (2 \cdot a^6 + 5 \cdot a^4 \cdot b^2 + 40 \cdot a^2 \cdot b^4 - 11 \cdot b^6 - 6 \cdot (a^6 - 6 \cdot a^4 \cdot b^2 + a^2 \cdot b^4) \cdot \tan(e \cdot x + d)^2 - 3 \cdot (a^5 \cdot b - 26 \cdot a^3 \cdot b^3 + 5 \cdot a \cdot b^5) \cdot \tan(e \cdot x + d)) / (a^6 \cdot b^3 + 3 \cdot a^4 \cdot b^5 + 3 \cdot a^2 \cdot b^7 + b^9 + (a^9 + 3 \cdot a^7 \cdot b^2 + 3 \cdot a^5 \cdot b^4 + a^3 \cdot b^6) \cdot \tan(e \cdot x + d)^3 + 3 \cdot (a^8 \cdot b + 3 \cdot a^6 \cdot b^3 + 3 \cdot a^4 \cdot b^5 + a^2 \cdot b^7) \cdot \tan(e \cdot x + d)^2 + 3 \cdot (a^7 \cdot b^2 + 3 \cdot a^5 \cdot b^4 + 3 \cdot a^3 \cdot b^6 + a \cdot b^8) \cdot \tan(e \cdot x + d)) / e$

mupad [B] time = 3.26, size = 388, normalized size = 1.97

$$\frac{\frac{\tan(d+ex)^2(a^6-6a^4b^2+a^2b^4)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{2a^6+5a^4b^2+40a^2b^4-11b^6}{6(a^6+3a^4b^2+3a^2b^4+b^6)} + \frac{\tan(d+ex)(a^5b-26a^3b^3+5ab^5)}{2(a^6+3a^4b^2+3a^2b^4+b^6)} \ln(b+a \tan(d+ex)) \left(\frac{5b}{(a^2+b^2)^2} \right)}{e(a^3 \tan(d+ex)^3 + 3a^2b \tan(d+ex)^2 + 3ab^2 \tan(d+ex) + b^3)} - \frac{\ln(b+a \tan(d+ex))}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^2, x)

[Out] $((\tan(d + e \cdot x)^2 \cdot (a^6 + a^2 \cdot b^4 - 6 \cdot a^4 \cdot b^2)) / (a^6 + b^6 + 3 \cdot a^2 \cdot b^4 + 3 \cdot a^4 \cdot b^2) - (2 \cdot a^6 - 11 \cdot b^6 + 40 \cdot a^2 \cdot b^4 + 5 \cdot a^4 \cdot b^2) / (6 \cdot (a^6 + b^6 + 3 \cdot a^2 \cdot b^4 + 3 \cdot a^4 \cdot b^2))) + (\tan(d + e \cdot x) \cdot (5 \cdot a \cdot b^5 + a^5 \cdot b - 26 \cdot a^3 \cdot b^3)) / (2 \cdot (a^6 + b^6 + 3 \cdot a^2 \cdot b^4 + 3 \cdot a^4 \cdot b^2))) / (e \cdot (b^3 + a^3 \cdot \tan(d + e \cdot x)^3 + 3 \cdot a^2 \cdot b \cdot \tan(d + e \cdot x)^2 + 3 \cdot a \cdot b^2 \cdot \tan(d + e \cdot x))) - (\log(b + a \cdot \tan(d + e \cdot x)) \cdot ((5 \cdot b) / (a^2 + b^2)^2 - (20 \cdot b^3) / (a^2 + b^2)^3 + (16 \cdot b^5) / (a^2 + b^2)^4)) / e + (\log(\tan(d + e \cdot x) - 1i) \cdot (a + b \cdot 1i)) / (2 \cdot e \cdot (4 \cdot a^3 \cdot b - 4 \cdot a \cdot b^3 + a^4 \cdot 1i + b^4 \cdot 1i - a^2 \cdot b^2 \cdot 6i)) - (\log(\tan(d + e \cdot x) + 1i) \cdot (a - b \cdot 1i)) / (2 \cdot e \cdot (4 \cdot a \cdot b^3 - 4 \cdot a^3 \cdot b + a^4 \cdot 1i + b^4 \cdot 1i - a^2 \cdot b^2 \cdot 6i)))$

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**2,x)

[Out] Exception raised: AttributeError

3.514 $\int (a+b \tan(d+ex)) (b^2 + 2ab \tan(d+ex) + a^2 \tan^2(d+ex)) dx$

Optimal. Leaf size=284

$$\frac{b(a^2 \tan^2(d+ex) + 2ab \tan(d+ex) + b^2)^{3/2}}{3e} + \frac{(a^2 + b^2)(a^2 \tan^2(d+ex) + 2ab \tan(d+ex) + b^2)^{3/2}}{2e(a \tan(d+ex) + b)} - \frac{2a^4bx(a^2 + b^2)}{e(a \tan(d+ex) + b)^3}$$

[Out] $\frac{1}{3} b (b^2 + 2 a b \tan(e x + d) + a^2 \tan^2(e x + d))^{3/2} / e + (a^4 - b^4) \ln(\cos(e x + d)) (b^2 + 2 a b \tan(e x + d) + a^2 \tan^2(e x + d))^{3/2} / e / (b + a \tan(e x + d))^{3 + 1/2} + (a^2 + b^2) (b^2 + 2 a b \tan(e x + d) + a^2 \tan^2(e x + d))^{3/2} / e / (b + a \tan(e x + d)) - 2 a^4 b x (a^2 + b^2) x (b^2 + 2 a b \tan(e x + d) + a^2 \tan^2(e x + d))^{3/2} / (a b + a^2 \tan^2(e x + d)) + a^4 b x (a^2 + b^2) \tan(e x + d) (b^2 + 2 a b \tan(e x + d) + a^2 \tan^2(e x + d))^{3/2} / e / (a b + a^2 \tan^2(e x + d))$

Rubi [A] time = 0.23, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3710, 3528, 12, 3525, 3475}

$$\frac{2a^4bx(a^2 + b^2)(a^2 \tan^2(d+ex) + 2ab \tan(d+ex) + b^2)^{3/2}}{(a^2 \tan(d+ex) + ab)^3} + \frac{a^4b(a^2 + b^2) \tan(d+ex) (a^2 \tan^2(d+ex) + 2ab \tan^2(d+ex) + b^2)^{3/2}}{e(a^2 \tan(d+ex) + ab)^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2), x]

[Out] $(b(b^2 + 2ab \tan(d+ex) + a^2 \tan^2(d+ex))^{3/2}) / (3e) + ((a^4 - b^4) \log[\cos(d+ex)] (b^2 + 2ab \tan(d+ex) + a^2 \tan^2(d+ex))^{3/2}) / (e(b + a \tan(d+ex))^3) + ((a^2 + b^2) (b^2 + 2ab \tan(d+ex) + a^2 \tan^2(d+ex))^{3/2}) / (2e(b + a \tan(d+ex))) - (2a^4bx(a^2 + b^2) x (b^2 + 2ab \tan(d+ex) + a^2 \tan^2(d+ex))^{3/2}) / (ab + a^2 \tan^2(d+ex)) + (a^4bx(a^2 + b^2) \tan(d+ex) (b^2 + 2ab \tan(d+ex) + a^2 \tan^2(d+ex))^{3/2}) / (e(ab + a^2 \tan^2(d+ex)))$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3525

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e + f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]

Rule 3528

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int[(a + b*Tan[e + f*x])^(m-1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,

0] && GtQ[m, 0]

Rule 3710

Int[((A_) + (B_)*tan[(d_) + (e_)*(x_)])*((a_) + (b_)*tan[(d_) + (e_)*(x_)]) + (c_)*tan[(d_) + (e_)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n), Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[n]

Rubi steps

$$\begin{aligned} \int (a + b \tan(d + ex)) (b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^{3/2} dx &= \frac{(b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^{3/2}}{(2ab \tan(d + ex) + a^2 \tan^2(d + ex) + b^2)} \\ &= \frac{b(b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^{3/2}}{3e} \\ &= \frac{b(b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^{3/2}}{3e} \\ &= \frac{b(b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^{3/2}}{3e} \\ &= \frac{b(b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^{3/2}}{3e} \\ &= \frac{b(b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^{3/2}}{3e} \end{aligned}$$

Mathematica [C] time = 1.33, size = 147, normalized size = 0.52

$$\frac{\sqrt{(a \tan(d + ex) + b)^2} (2a^3b \tan^3(d + ex) + 3a^2(a^2 + 3b^2) \tan^2(d + ex) + 6ab(2a^2 + 3b^2) \tan(d + ex) - 3(a^2 - b^2))}{6e(a \tan(d + ex) + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2), x]

[Out] (Sqrt[(b + a*Tan[d + e*x])^2]*(-3*(a^2 + b^2)*((a - I*b)^2*Log[I - Tan[d + e*x]] + (a + I*b)^2*Log[I + Tan[d + e*x]]) + 6*a*b*(2*a^2 + 3*b^2)*Tan[d + e*x] + 3*a^2*(a^2 + 3*b^2)*Tan[d + e*x]^2 + 2*a^3*b*Tan[d + e*x]^3))/(6*e*(b + a*Tan[d + e*x]))

fricas [A] time = 2.83, size = 102, normalized size = 0.36

$$\frac{2a^3b \tan(ex + d)^3 - 12(a^3b + ab^3)ex + 3(a^4 + 3a^2b^2) \tan(ex + d)^2 + 3(a^4 - b^4) \log\left(\frac{1}{\tan(ex+d)^2 + 1}\right) + 6(2a^3b^2 - ab^3)}{6e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*b*tan(e*x + d)^3 - 12*(a^3*b + a*b^3)*e*x + 3*(a^4 + 3*a^2*b^2)*
tan(e*x + d)^2 + 3*(a^4 - b^4)*log(1/(tan(e*x + d)^2 + 1)) + 6*(2*a^3*b + 3
*a*b^3)*tan(e*x + d))/e
```

giac [B] time = 2.44, size = 1751, normalized size = 6.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
algorithm="giac")
```

```
[Out] -1/6*(12*a^3*b*x*e*sgn(a*tan(x*e + d) + b)*tan(x*e)^3*tan(d)^3 + 12*a*b^3*x
*e*sgn(a*tan(x*e + d) + b)*tan(x*e)^3*tan(d)^3 - 3*a^4*log(4*(tan(x*e)^4*ta
n(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e
)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e + d) + b)*tan(x*e)^3*tan(d)^3 +
3*b^4*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)
^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e + d)
+ b)*tan(x*e)^3*tan(d)^3 - 36*a^3*b*x*e*sgn(a*tan(x*e + d) + b)*tan(x*e)^2
*tan(d)^2 - 36*a*b^3*x*e*sgn(a*tan(x*e + d) + b)*tan(x*e)^2*tan(d)^2 - 3*a^
4*sgn(a*tan(x*e + d) + b)*tan(x*e)^3*tan(d)^3 - 9*a^2*b^2*sgn(a*tan(x*e + d
) + b)*tan(x*e)^3*tan(d)^3 + 9*a^4*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^
3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d
)^2 + 1))*sgn(a*tan(x*e + d) + b)*tan(x*e)^2*tan(d)^2 - 9*b^4*log(4*(tan(x*
e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*
tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e + d) + b)*tan(x*e)^2*ta
n(d)^2 + 12*a^3*b*sgn(a*tan(x*e + d) + b)*tan(x*e)^3*tan(d)^2 + 18*a*b^3*sgn
(a*tan(x*e + d) + b)*tan(x*e)^3*tan(d)^2 + 12*a^3*b*sgn(a*tan(x*e + d) + b)
*tan(x*e)^2*tan(d)^3 + 18*a*b^3*sgn(a*tan(x*e + d) + b)*tan(x*e)^2*tan(d)^3
+ 36*a^3*b*x*e*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d) + 36*a*b^3*x*e*sgn(
a*tan(x*e + d) + b)*tan(x*e)*tan(d) - 3*a^4*sgn(a*tan(x*e + d) + b)*tan(x*e
)^3*tan(d) - 9*a^2*b^2*sgn(a*tan(x*e + d) + b)*tan(x*e)^3*tan(d) + 3*a^4*sg
n(a*tan(x*e + d) + b)*tan(x*e)^2*tan(d)^2 + 9*a^2*b^2*sgn(a*tan(x*e + d) +
b)*tan(x*e)^2*tan(d)^2 - 3*a^4*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d)^3 -
9*a^2*b^2*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d)^3 + 2*a^3*b*sgn(a*tan(x*e
+ d) + b)*tan(x*e)^3 - 9*a^4*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan
(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 +
1))*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d) + 9*b^4*log(4*(tan(x*e)^4*tan(
d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*
tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d) - 18*a^
3*b*sgn(a*tan(x*e + d) + b)*tan(x*e)^2*tan(d) - 36*a*b^3*sgn(a*tan(x*e + d)
+ b)*tan(x*e)^2*tan(d) - 18*a^3*b*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d)^
2 - 36*a*b^3*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d)^2 + 2*a^3*b*sgn(a*tan(
x*e + d) + b)*tan(d)^3 - 12*a^3*b*x*e*sgn(a*tan(x*e + d) + b) - 12*a*b^3*x*
e*sgn(a*tan(x*e + d) + b) + 3*a^4*sgn(a*tan(x*e + d) + b)*tan(x*e)^2 + 9*a^
2*b^2*sgn(a*tan(x*e + d) + b)*tan(x*e)^2 - 3*a^4*sgn(a*tan(x*e + d) + b)*ta
n(x*e)*tan(d) - 9*a^2*b^2*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d) + 3*a^4*sg
n(a*tan(x*e + d) + b)*tan(d)^2 + 9*a^2*b^2*sgn(a*tan(x*e + d) + b)*tan(d)^
2 + 3*a^4*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan
(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e +
d) + b) - 3*b^4*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)
)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*ta
n(x*e + d) + b) + 12*a^3*b*sgn(a*tan(x*e + d) + b)*tan(x*e) + 18*a*b^3*sgn(
a*tan(x*e + d) + b)*tan(x*e) + 12*a^3*b*sgn(a*tan(x*e + d) + b)*tan(d) + 18
*a*b^3*sgn(a*tan(x*e + d) + b)*tan(d) + 3*a^4*sgn(a*tan(x*e + d) + b) + 9*a
^2*b^2*sgn(a*tan(x*e + d) + b))/(e*tan(x*e)^3*tan(d)^3 - 3*e*tan(x*e)^2*ta
n(d)^2 + 3*e*tan(x*e)*tan(d) - e)
```

maple [A] time = 0.31, size = 158, normalized size = 0.56

$$\frac{(b^2 + 2ab \tan(ex + d) + a^2 (\tan^2(ex + d)))^{\frac{3}{2}} (-2 (\tan^3(ex + d)) a^3 b - 3 (\tan^2(ex + d)) a^4 - 9 (\tan^2(ex + d)))}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2), x)

[Out] -1/6/e*((b+a*tan(e*x+d))^2)^(3/2)*(-2*tan(e*x+d)^3*a^3*b-3*tan(e*x+d)^2*a^4-9*tan(e*x+d)^2*a^2*b^2+3*ln(1+tan(e*x+d)^2)*a^4-3*ln(1+tan(e*x+d)^2)*b^4+12*arctan(tan(e*x+d))*a^3*b+12*arctan(tan(e*x+d))*a*b^3-12*tan(e*x+d)*a^3*b-18*tan(e*x+d)*a*b^3)/(b+a*tan(e*x+d))^3

maxima [A] time = 0.43, size = 166, normalized size = 0.58

$$\frac{3(a^3 \tan(ex + d)^2 + 6a^2 b \tan(ex + d) - 2(3a^2 b - b^3)(ex + d) - (a^3 - 3ab^2) \log(\tan(ex + d)^2 + 1))a + (2a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2), x, algorithm="maxima")

[Out] 1/6*(3*(a^3*tan(e*x + d)^2 + 6*a^2*b*tan(e*x + d) - 2*(3*a^2*b - b^3)*(e*x + d) - (a^3 - 3*a*b^2)*log(tan(e*x + d)^2 + 1))*a + (2*a^3*tan(e*x + d)^3 + 9*a^2*b*tan(e*x + d)^2 + 6*(a^3 - 3*a*b^2)*(e*x + d) - 3*(3*a^2*b - b^3)*log(tan(e*x + d)^2 + 1) - 6*(a^3 - 3*a*b^2)*tan(e*x + d))*b)/e

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \tan(d + ex)) (a^2 \tan(d + ex)^2 + 2ab \tan(d + ex) + b^2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^3/2, x)

[Out] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^3/2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan(d + ex)) ((a \tan(d + ex) + b)^2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))*(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(3/2), x)

[Out] Integral((a + b*tan(d + e*x))*((a*tan(d + e*x) + b)**2)**(3/2), x)

3.515 $\int (a+b \tan(d+ex))\sqrt{b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)}$

Optimal. Leaf size=122

$$\frac{a^2 b \tan(d + ex) \sqrt{a^2 \tan^2(d + ex) + 2ab \tan(d + ex) + b^2}}{e(a^2 \tan(d + ex) + ab)} - \frac{(a^2 + b^2) \log(\cos(d + ex)) \sqrt{a^2 \tan^2(d + ex) + 2ab \tan(d + ex) + b^2}}{e(a \tan(d + ex) + b)}$$

[Out] $-(a^2+b^2)*\ln(\cos(e*x+d))*(b^2+2*a*b*\tan(e*x+d)+a^2*\tan(e*x+d)^2)^{(1/2)}/e/(b+a*\tan(e*x+d))+a^2*b*(b^2+2*a*b*\tan(e*x+d)+a^2*\tan(e*x+d)^2)^{(1/2)}*\tan(e*x+d)/e/(a*b+a^2*\tan(e*x+d))$

Rubi [A] time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {3710, 3525, 3475}

$$\frac{a^2 b \tan(d + ex) \sqrt{a^2 \tan^2(d + ex) + 2ab \tan(d + ex) + b^2}}{e(a^2 \tan(d + ex) + ab)} - \frac{(a^2 + b^2) \log(\cos(d + ex)) \sqrt{a^2 \tan^2(d + ex) + 2ab \tan(d + ex) + b^2}}{e(a \tan(d + ex) + b)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Tan}[d + e*x])*Sqrt[b^2 + 2*a*b*\text{Tan}[d + e*x] + a^2*\text{Tan}[d + e*x]^2], x]$

[Out] $-(((a^2 + b^2)*\text{Log}[\text{Cos}[d + e*x]]*Sqrt[b^2 + 2*a*b*\text{Tan}[d + e*x] + a^2*\text{Tan}[d + e*x]^2])/(e*(b + a*\text{Tan}[d + e*x]))) + (a^2*b*\text{Tan}[d + e*x]*Sqrt[b^2 + 2*a*b*\text{Tan}[d + e*x] + a^2*\text{Tan}[d + e*x]^2])/(e*(a*b + a^2*\text{Tan}[d + e*x]))$

Rule 3475

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]], x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3525

$\text{Int}[((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Simp}[(a*c - b*d)*x, x] + (\text{Dist}[b*c + a*d, \text{Int}[\text{Tan}[e + f*x], x], x] + \text{Simp}[(b*d*\text{Tan}[e + f*x])/f, x]) /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[b*c + a*d, 0]$

Rule 3710

$\text{Int}(((A_.) + (B_.)*\tan[(d_.) + (e_.)*(x_.)])*((a_.) + (b_.)*\tan[(d_.) + (e_.)*(x_.)] + (c_.)*\tan[(d_.) + (e_.)*(x_.)]^2)^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(a + b*\text{Tan}[d + e*x] + c*\text{Tan}[d + e*x]^2)^n/(b + 2*c*\text{Tan}[d + e*x])^{(2*n)}, \text{Int}[(A + B*\text{Tan}[d + e*x])*(b + 2*c*\text{Tan}[d + e*x])^{(2*n)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, A, B\}, x] \&\& \text{EqQ}[b^2 - 4*a*c, 0] \&\& !\text{IntegerQ}[n]$

Rubi steps

$$\begin{aligned} \int (a + b \tan(d + ex))\sqrt{b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)} dx &= \frac{\sqrt{b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)}}{2ab + 2a^2 \tan(d + ex)} \int (2ab + 2a^2 \tan(d + ex)) \sqrt{b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)} dx \\ &= \frac{a^2 b \tan(d + ex) \sqrt{b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)}}{e(ab + a^2 \tan(d + ex))} \\ &= -\frac{(a^2 + b^2) \log(\cos(d + ex)) \sqrt{b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)}}{e(b + a \tan(d + ex))} \end{aligned}$$

Mathematica [A] time = 0.30, size = 58, normalized size = 0.48

$$\frac{\sqrt{(a \tan(d + ex) + b)^2} (ab \tan(d + ex) - (a^2 + b^2) \log(\cos(d + ex)))}{e(a \tan(d + ex) + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tan[d + e*x])*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2], x]

[Out] (Sqrt[(b + a*Tan[d + e*x])^2]*(-((a^2 + b^2)*Log[Cos[d + e*x]]) + a*b*Tan[d + e*x]))/(e*(b + a*Tan[d + e*x]))

fricas [A] time = 2.63, size = 38, normalized size = 0.31

$$\frac{2 ab \tan(ex + d) - (a^2 + b^2) \log\left(\frac{1}{\tan(ex+d)^2+1}\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)), x, algorithm="fricas")

[Out] 1/2*(2*a*b*tan(e*x + d) - (a^2 + b^2)*log(1/(tan(e*x + d)^2 + 1)))/e

giac [B] time = 0.46, size = 395, normalized size = 3.24

$$a^2 \log\left(\frac{4(\tan(xe)^4 \tan(d)^2 - 2 \tan(xe)^3 \tan(d) + \tan(xe)^2 \tan(d)^2 + \tan(xe)^2 - 2 \tan(xe) \tan(d) + 1)}{\tan(d)^2 + 1}\right) \operatorname{sgn}(a \tan(xe + d) + b) \tan(xe) \tan(d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)), x, algorithm="giac")

[Out] -1/2*(a^2*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d) + b^2*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e + d) + b)*tan(x*e)*tan(d) - a^2*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e + d) + b) - b^2*log(4*(tan(x*e)^4*tan(d)^2 - 2*tan(x*e)^3*tan(d) + tan(x*e)^2*tan(d)^2 + tan(x*e)^2 - 2*tan(x*e)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(x*e + d) + b) + 2*a*b*sgn(a*tan(x*e + d) + b)*tan(x*e) + 2*a*b*sgn(a*tan(x*e + d) + b)*tan(d))/(e*tan(x*e)*tan(d) - e)

maple [C] time = 0.32, size = 75, normalized size = 0.61

$$\frac{\operatorname{csgn}(b + a \tan(ex + d)) \left(\ln(a^2 (\tan^2(ex + d)) + a^2) a^2 + \ln(a^2 (\tan^2(ex + d)) + a^2) b^2 + 2ab \tan(ex + d) \right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)), x)

[Out] 1/2/e*csgn(b+a*tan(e*x+d))*(ln(a^2*tan(e*x+d)^2+a^2)*a^2+ln(a^2*tan(e*x+d)^2+a^2)*b^2+2*a*b*tan(e*x+d)+2*b^2)

maxima [A] time = 0.42, size = 65, normalized size = 0.53

$$\frac{(2(ex+d)b + a \log(\tan(ex+d)^2 + 1))a - (2(ex+d)a - b \log(\tan(ex+d)^2 + 1) - 2a \tan(ex+d))b}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x,
algorithm="maxima")

[Out] 1/2*((2*(e*x + d)*b + a*log(tan(e*x + d)^2 + 1))*a - (2*(e*x + d)*a - b*log(tan(e*x + d)^2 + 1) - 2*a*tan(e*x + d))*b)/e

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \tan(d + ex)) \sqrt{a^2 \tan(d + ex)^2 + 2ab \tan(d + ex) + b^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(1/2),x)

[Out] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan(d + ex)) \sqrt{(a \tan(d + ex) + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(1/2)*(a+b*tan(e*x+d)),x)

[Out] Integral((a + b*tan(d + e*x))*sqrt((a*tan(d + e*x) + b)**2), x)

$$3.516 \quad \int \frac{a+b \tan(d+ex)}{\sqrt{b^2+2ab \tan(d+ex)+a^2 \tan^2(d+ex)}} dx$$

Optimal. Leaf size=138

$$\frac{2bx(a^2 \tan(d+ex) + ab)}{(a^2 + b^2) \sqrt{a^2 \tan^2(d+ex) + 2ab \tan(d+ex) + b^2}} + \frac{(a^2 - b^2)(a \tan(d+ex) + b) \log(a \sin(d+ex) + b \cos(d+ex))}{e(a^2 + b^2) \sqrt{a^2 \tan^2(d+ex) + 2ab \tan(d+ex) + b^2}}$$

[Out] (a^2-b^2)*ln(b*cos(e*x+d)+a*sin(e*x+d))*(b+a*tan(e*x+d))/(a^2+b^2)/e/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)+2*b*x*(a*b+a^2*tan(e*x+d))/(a^2+b^2)/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)

Rubi [A] time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {3710, 3531, 3530}

$$\frac{2bx(a^2 \tan(d+ex) + ab)}{(a^2 + b^2) \sqrt{a^2 \tan^2(d+ex) + 2ab \tan(d+ex) + b^2}} + \frac{(a^2 - b^2)(a \tan(d+ex) + b) \log(a \sin(d+ex) + b \cos(d+ex))}{e(a^2 + b^2) \sqrt{a^2 \tan^2(d+ex) + 2ab \tan(d+ex) + b^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tan[d + e*x])/Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2], x]

[Out] ((a^2 - b^2)*Log[b*Cos[d + e*x] + a*Sin[d + e*x]]*(b + a*Tan[d + e*x]))/((a^2 + b^2)*e*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2]) + (2*b*x*(a*b + a^2*Tan[d + e*x]))/((a^2 + b^2)*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2])

Rule 3530

Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]

Rule 3531

Int[((c_) + (d_)*tan[(e_) + (f_)*(x_)])/((a_) + (b_)*tan[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[a*c + b*d, 0]

Rule 3710

Int[((A_) + (B_)*tan[(d_) + (e_)*(x_)])*((a_) + (b_)*tan[(d_) + (e_)*(x_)] + (c_)*tan[(d_) + (e_)*(x_)]^2)^n, x_Symbol] :> Dist[(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n), Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[n]

Rubi steps

$$\int \frac{a + b \tan(d + ex)}{\sqrt{b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)}} dx = \frac{(2ab + 2a^2 \tan(d + ex)) \int \frac{a + b \tan(d + ex)}{2ab + 2a^2 \tan(d + ex)} dx}{\sqrt{b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)}}$$

$$= \frac{2bx (ab + a^2 \tan(d + ex))}{(a^2 + b^2) \sqrt{b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)}} + \frac{((a^2 - b^2) \log(b \cos(d + ex) + a \sin(d + ex))(b + a \tan(d + ex)))}{2a(a^2 + b^2) \sqrt{b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)}}$$

$$= \frac{(a^2 - b^2) \log(b \cos(d + ex) + a \sin(d + ex))(b + a \tan(d + ex))}{(a^2 + b^2) e \sqrt{b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex)}}$$

Mathematica [A] time = 0.68, size = 88, normalized size = 0.64

$$\frac{(a \tan(d + ex) + b) (4ab \tan^{-1}(\tan(d + ex)) - (a^2 - b^2) (\log(\sec^2(d + ex)) - 2 \log(a \tan(d + ex) + b)))}{2e (a^2 + b^2) \sqrt{(a \tan(d + ex) + b)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tan[d + e*x])/Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2], x]

[Out] ((4*a*b*ArcTan[Tan[d + e*x]] - (a^2 - b^2)*(Log[Sec[d + e*x]^2] - 2*Log[b + a*Tan[d + e*x]]))*(b + a*Tan[d + e*x]))/(2*(a^2 + b^2)*e*Sqrt[(b + a*Tan[d + e*x])^2])

fricas [A] time = 0.72, size = 71, normalized size = 0.51

$$\frac{4 abex + (a^2 - b^2) \log\left(\frac{a^2 \tan(ex+d)^2 + 2 ab \tan(ex+d) + b^2}{\tan(ex+d)^2 + 1}\right)}{2 (a^2 + b^2) e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2), x, algorithm="fricas")

[Out] 1/2*(4*a*b*e*x + (a^2 - b^2)*log((a^2*tan(e*x + d)^2 + 2*a*b*tan(e*x + d) + b^2)/(tan(e*x + d)^2 + 1)))/((a^2 + b^2)*e)

giac [B] time = 3.07, size = 554, normalized size = 4.01

$$-\frac{1}{2} \left(\frac{2 \left(\pi \operatorname{sgn} \left(\tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) \right) + 2 \arctan \left(\tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) \right) \right)}{a^2 \operatorname{sgn} \left(-b \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) + 2 a \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) + 2 b \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) - 2 a \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) - b \right) + b^2 \operatorname{sgn} \left(-b \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) + 2 a \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) + 2 b \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) - 2 a \tan \left(\frac{1}{2} x e + \frac{1}{2} d \right) - b \right) - (a^2 - b^2) * \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2), x, algorithm="giac")

[Out] -1/2*(2*(pi*sgn(tan(1/2*x*e + 1/2*d)) + 2*arctan(1/2*(tan(1/2*x*e + 1/2*d)^2 - 1)/tan(1/2*x*e + 1/2*d)))*a*b/(a^2*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b) + b^2*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)) - (a^2 - b^2)*

$$\log\left(\frac{1}{\tan(1/2*x*e + 1/2*d)} - \tan(1/2*x*e + 1/2*d)\right)^2 + 4\left/\left(a^2*\operatorname{sgn}(-b*\tan(1/2*x*e + 1/2*d))^4 + 2*a*\tan(1/2*x*e + 1/2*d)^3 + 2*b*\tan(1/2*x*e + 1/2*d)^2 - 2*a*\tan(1/2*x*e + 1/2*d) - b\right) + b^2*\operatorname{sgn}(-b*\tan(1/2*x*e + 1/2*d))^4 + 2*a*\tan(1/2*x*e + 1/2*d)^3 + 2*b*\tan(1/2*x*e + 1/2*d)^2 - 2*a*\tan(1/2*x*e + 1/2*d) - b\right) + 2*(a^2*b - b^3)*\log(\operatorname{abs}(-b*(1/\tan(1/2*x*e + 1/2*d) - \tan(1/2*x*e + 1/2*d)) - 2*a))/\left(a^2*b*\operatorname{sgn}(-b*\tan(1/2*x*e + 1/2*d))^4 + 2*a*\tan(1/2*x*e + 1/2*d)^3 + 2*b*\tan(1/2*x*e + 1/2*d)^2 - 2*a*\tan(1/2*x*e + 1/2*d) - b\right) + b^3*\operatorname{sgn}(-b*\tan(1/2*x*e + 1/2*d))^4 + 2*a*\tan(1/2*x*e + 1/2*d)^3 + 2*b*\tan(1/2*x*e + 1/2*d)^2 - 2*a*\tan(1/2*x*e + 1/2*d) - b\right)) * e^{-1}$$

maple [A] time = 0.23, size = 114, normalized size = 0.83

$$\frac{(b + a \tan(ex + d)) \left(2 \ln(b + a \tan(ex + d)) a^2 - 2 \ln(b + a \tan(ex + d)) b^2 - \ln(1 + \tan^2(ex + d)) a^2 + \ln(1 + \tan^2(ex + d)) b^2 \right)}{2e \sqrt{b^2 + 2ab \tan(ex + d) + a^2 (\tan^2(ex + d))} (a^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2), x)

[Out] 1/2/e*(b+a*tan(e*x+d))*(2*ln(b+a*tan(e*x+d))*a^2-2*ln(b+a*tan(e*x+d))*b^2-ln(1+tan(e*x+d)^2)*a^2+ln(1+tan(e*x+d)^2)*b^2+4*a*b*arctan(tan(e*x+d)))/(b+a*tan(e*x+d))^2)^(1/2)/(a^2+b^2)

maxima [A] time = 0.42, size = 137, normalized size = 0.99

$$\frac{a \left(\frac{2(ex+d)b}{a^2+b^2} + \frac{2a \log(a \tan(ex+d)+b)}{a^2+b^2} - \frac{a \log(\tan(ex+d)^2+1)}{a^2+b^2} \right) + \left(\frac{2(ex+d)a}{a^2+b^2} - \frac{2b \log(a \tan(ex+d)+b)}{a^2+b^2} + \frac{b \log(\tan(ex+d)^2+1)}{a^2+b^2} \right) b}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2), x, algorithm="maxima")

[Out] 1/2*(a*(2*(e*x + d)*b/(a^2 + b^2) + 2*a*log(a*tan(e*x + d) + b)/(a^2 + b^2) - a*log(tan(e*x + d)^2 + 1)/(a^2 + b^2)) + (2*(e*x + d)*a/(a^2 + b^2) - 2*b*log(a*tan(e*x + d) + b)/(a^2 + b^2) + b*log(tan(e*x + d)^2 + 1)/(a^2 + b^2))*b)/e

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \tan(d + ex)}{\sqrt{a^2 \tan(d + ex)^2 + 2ab \tan(d + ex) + b^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(1/2), x)

[Out] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \tan(d + ex)}{\sqrt{(a \tan(d + ex) + b)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(1/2), x)

[Out] Integral((a + b*tan(d + e*x))/sqrt((a*tan(d + e*x) + b)**2), x)

$$3.517 \quad \int \frac{a+b \tan(d+ex)}{(b^2+2ab \tan(d+ex)+a^2 \tan^2(d+ex))^{3/2}} dx$$

Optimal. Leaf size=316

$$\frac{(a^2 - b^2)(a \tan(d + ex) + b)}{2e(a^2 + b^2)(a^2 \tan^2(d + ex) + 2ab \tan(d + ex) + b^2)^{3/2}} - \frac{4bx(a^2 - b^2)(a^2 \tan(d + ex) + ab)^3}{a^2(a^2 + b^2)^3(a^2 \tan^2(d + ex) + 2ab \tan(d + ex) + b^2)^3}$$

[Out] $-1/2*(a^2-b^2)*(b+a*\tan(e*x+d))/(a^2+b^2)/e/(b^2+2*a*b*\tan(e*x+d)+a^2*\tan(e*x+d)^2)^{(3/2)}-(a^4-6*a^2*b^2+b^4)*\ln(b*\cos(e*x+d)+a*\sin(e*x+d))*(b+a*\tan(e*x+d))^3/(a^2+b^2)^3/e/(b^2+2*a*b*\tan(e*x+d)+a^2*\tan(e*x+d)^2)^{(3/2)}-4*b*(a^2-b^2)*x*(a*b+a^2*\tan(e*x+d))^3/a^2/(a^2+b^2)^3/(b^2+2*a*b*\tan(e*x+d)+a^2*\tan(e*x+d)^2)^{(3/2)}-b*(3*a^2-b^2)*(a*b+a^2*\tan(e*x+d))^3/(a^2+b^2)^2/e/(a^3*b+a^4*\tan(e*x+d))/(b^2+2*a*b*\tan(e*x+d)+a^2*\tan(e*x+d)^2)^{(3/2)}$

Rubi [A] time = 0.40, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {3710, 3529, 3531, 3530}

$$\frac{(a^2 - b^2)(a \tan(d + ex) + b)}{2e(a^2 + b^2)(a^2 \tan^2(d + ex) + 2ab \tan(d + ex) + b^2)^{3/2}} - \frac{4bx(a^2 - b^2)(a^2 \tan(d + ex) + ab)^3}{a^2(a^2 + b^2)^3(a^2 \tan^2(d + ex) + 2ab \tan(d + ex) + b^2)^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Tan}[d + e*x])/(b^2 + 2*a*b*\text{Tan}[d + e*x] + a^2*\text{Tan}[d + e*x]^2)^{(3/2)}, x]$

[Out] $-((a^2 - b^2)*(b + a*\text{Tan}[d + e*x]))/(2*(a^2 + b^2)*e*(b^2 + 2*a*b*\text{Tan}[d + e*x] + a^2*\text{Tan}[d + e*x]^2)^{(3/2)}) - ((a^4 - 6*a^2*b^2 + b^4)*\text{Log}[b*\text{Cos}[d + e*x] + a*\text{Sin}[d + e*x]]*(b + a*\text{Tan}[d + e*x])^3)/((a^2 + b^2)^3*e*(b^2 + 2*a*b*\text{Tan}[d + e*x] + a^2*\text{Tan}[d + e*x]^2)^{(3/2)}) - (4*b*(a^2 - b^2)*x*(a*b + a^2*\text{Tan}[d + e*x])^3)/(a^2*(a^2 + b^2)^3*(b^2 + 2*a*b*\text{Tan}[d + e*x] + a^2*\text{Tan}[d + e*x]^2)^{(3/2)}) - (b*(3*a^2 - b^2)*(a*b + a^2*\text{Tan}[d + e*x])^3)/((a^2 + b^2)^2*e*(a^3*b + a^4*\text{Tan}[d + e*x])*(b^2 + 2*a*b*\text{Tan}[d + e*x] + a^2*\text{Tan}[d + e*x]^2)^{(3/2)})$

Rule 3529

$\text{Int}(((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)])^{(m_.)*((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)]), x_Symbol] :> \text{Simp}(((b*c - a*d)*(a + b*\text{Tan}[e + f*x])^{(m + 1)})/(f*(m + 1)*(a^2 + b^2)), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*c + b*d - (b*c - a*d)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{LtQ}[m, -1]$

Rule 3530

$\text{Int}(((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])/((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]), x_Symbol] :> \text{Simp}[(c*\text{Log}[\text{RemoveContent}[a*\text{Cos}[e + f*x] + b*\text{Sin}[e + f*x], x]])/(b*f), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{EqQ}[a*c + b*d, 0]$

Rule 3531

$\text{Int}(((c_.) + (d_.)*\tan[(e_.) + (f_.)*(x_.)])/((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]), x_Symbol] :> \text{Simp}(((a*c + b*d)*x)/(a^2 + b^2), x] + \text{Dist}[(b*c - a*d)/(a^2 + b^2), \text{Int}[(b - a*\text{Tan}[e + f*x])/(a + b*\text{Tan}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{NeQ}[a*c + b*d, 0]$

Rule 3710

Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*(x_)]) + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Tan[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n), Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[n]

Rubi steps

$$\int \frac{a + b \tan(d + ex)}{(b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^{3/2}} dx = \frac{(2ab + 2a^2 \tan(d + ex))^3 \int \frac{a + b \tan(d + ex)}{(2ab + 2a^2 \tan(d + ex))^3} dx}{(b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^{3/2}}$$

$$= -\frac{(a^2 - b^2)(b + a \tan(d + ex))}{2(a^2 + b^2)e(b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^{3/2}}$$

$$= -\frac{(a^2 - b^2)(b + a \tan(d + ex))}{2(a^2 + b^2)e(b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^{3/2}}$$

$$= -\frac{(a^2 - b^2)(b + a \tan(d + ex))}{2(a^2 + b^2)e(b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^{3/2}}$$

$$= -\frac{(a^2 - b^2)(b + a \tan(d + ex))}{2(a^2 + b^2)e(b^2 + 2ab \tan(d + ex) + a^2 \tan^2(d + ex))^{3/2}}$$

Mathematica [C] time = 3.44, size = 268, normalized size = 0.85

$$\frac{(a \tan(d + ex) + b)^3 \left(b \left(\frac{2a \left(2b \log(a \tan(d + ex) + b) - \frac{a^2 + b^2}{a \tan(d + ex) + b} \right)}{(a^2 + b^2)^2} + \frac{i \log(-\tan(d + ex) + i)}{(a - ib)^2} - \frac{i \log(\tan(d + ex) + i)}{(a + ib)^2} \right) + (a - b)(a + b) \left(\frac{a}{a^2 + b^2} \right) \right)}{2ae \left((a \tan(d + ex) + b)^2 \right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2), x]

[Out] ((b + a*Tan[d + e*x])^3*(b*((I*Log[I - Tan[d + e*x]])/(a - I*b)^2 - (I*Log[I + Tan[d + e*x]])/(a + I*b)^2 + (2*a*(2*b*Log[b + a*Tan[d + e*x]] - (a^2 + b^2)/(b + a*Tan[d + e*x]))/(a^2 + b^2)^2) + (a - b)*(a + b)*(Log[I - Tan[d + e*x]])/(a - I*b)^3 + Log[I + Tan[d + e*x]]/(a + I*b)^3 + (a*(-2*(a^2 - 3*b^2)*Log[b + a*Tan[d + e*x]] - ((a^2 + b^2)*(a^2 + 5*b^2 + 4*a*b*Tan[d + e*x]))/(b + a*Tan[d + e*x]^2))/(a^2 + b^2)^3)))/(2*a*e*((b + a*Tan[d + e*x])^2)^(3/2))

fricas [A] time = 2.03, size = 355, normalized size = 1.12

$$\frac{a^6 + 8a^4b^2 - 5a^2b^4 + 8(a^3b^3 - ab^5)ex + (a^6 - 8a^4b^2 + 3a^2b^4 + 8(a^5b - a^3b^3)ex) \tan(ex + d)^2 + (a^4b^2 - 6a^2b^4 + 3a^2b^6)}{2((a^8 + 3a^6b^2 + 3a^4b^4 + a^2b^6))^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
algorithm="fricas")
```

```
[Out] -1/2*(a^6 + 8*a^4*b^2 - 5*a^2*b^4 + 8*(a^3*b^3 - a*b^5)*e*x + (a^6 - 8*a^4*
b^2 + 3*a^2*b^4 + 8*(a^5*b - a^3*b^3)*e*x)*tan(e*x + d)^2 + (a^4*b^2 - 6*a^
2*b^4 + b^6 + (a^6 - 6*a^4*b^2 + a^2*b^4)*tan(e*x + d)^2 + 2*(a^5*b - 6*a^3
*b^3 + a*b^5)*tan(e*x + d))*log((a^2*tan(e*x + d)^2 + 2*a*b*tan(e*x + d) +
b^2)/(tan(e*x + d)^2 + 1)) + 4*(2*a^5*b - 3*a^3*b^3 + a*b^5 + 4*(a^4*b^2 -
a^2*b^4)*e*x)*tan(e*x + d))/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*e*tan(
e*x + d)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*e*tan(e*x + d) + (a^
6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*e)
```

giac [B] time = 10.07, size = 1571, normalized size = 4.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
algorithm="giac")
```

```
[Out] 1/2*(4*(a^3*b - a*b^3)*(pi*sgn(tan(1/2*x*e + 1/2*d)) + 2*arctan(1/2*(tan(1/
2*x*e + 1/2*d)^2 - 1)/tan(1/2*x*e + 1/2*d)))/(a^6*sgn(-b*tan(1/2*x*e + 1/2*
d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/
2*x*e + 1/2*d) - b) + 3*a^4*b^2*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2
*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b
) + 3*a^2*b^4*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 +
2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b) + b^6*sgn(-b*ta
n(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)
^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)) - (a^4 - 6*a^2*b^2 + b^4)*log((1/tan(1/
2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d))^2 + 4)/(a^6*sgn(-b*tan(1/2*x*e + 1/2
*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1
/2*x*e + 1/2*d) - b) + 3*a^4*b^2*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/
2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) -
b) + 3*a^2*b^4*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 +
2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b) + b^6*sgn(-b*ta
n(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)
^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)) + 2*(a^4*b - 6*a^2*b^3 + b^5)*log(abs(
-b*(1/tan(1/2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d)) - 2*a)/(a^6*b*sgn(-b*ta
n(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)
^2 - 2*a*tan(1/2*x*e + 1/2*d) - b) + 3*a^4*b^3*sgn(-b*tan(1/2*x*e + 1/2*d)
^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*
x*e + 1/2*d) - b) + 3*a^2*b^5*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x
*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)
+ b^7*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(
1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b)) - (3*a^4*b^4*(1/tan(1/2
*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d))^2 - 18*a^2*b^6*(1/tan(1/2*x*e + 1/2*d)
- tan(1/2*x*e + 1/2*d))^2 + 3*b^8*(1/tan(1/2*x*e + 1/2*d) - tan(1/2*x*e +
1/2*d))^2 + 4*a^7*b*(1/tan(1/2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d)) + 28*a
^5*b^3*(1/tan(1/2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d)) - 68*a^3*b^5*(1/tan(
1/2*x*e + 1/2*d) - tan(1/2*x*e + 1/2*d)) + 4*a*b^7*(1/tan(1/2*x*e + 1/2*d)
- tan(1/2*x*e + 1/2*d)) + 4*a^8 + 40*a^6*b^2 - 60*a^4*b^4)/((a^6*b^2*sgn(-b
*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/
2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b) + 3*a^4*b^4*sgn(-b*tan(1/2*x*e + 1/2
*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1
/2*x*e + 1/2*d) - b) + 3*a^2*b^6*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/
2*x*e + 1/2*d)^3 + 2*b*tan(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) -
b) + b^8*sgn(-b*tan(1/2*x*e + 1/2*d)^4 + 2*a*tan(1/2*x*e + 1/2*d)^3 + 2*b*t
an(1/2*x*e + 1/2*d)^2 - 2*a*tan(1/2*x*e + 1/2*d) - b))*b*(1/tan(1/2*x*e +
1/2*d) - tan(1/2*x*e + 1/2*d)) + 2*a)^2))e^(-1)
```

maple [B] time = 0.19, size = 622, normalized size = 1.97

$$\frac{(3a^2b^4 + a^6 - 3b^6 + 6 \ln(1 + \tan^2(ex + d)) (\tan^2(ex + d)) a^4b^2 + 2 \ln(b + a \tan(ex + d)) (\tan^2(ex + d)) a^2b^2)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2), x)

[Out] -1/2/e*(2*ln(b+a*tan(e*x+d))*tan(e*x+d)^2*a^6+3*a^2*b^4+a^6-3*b^6+6*ln(1+tan(e*x+d)^2)*tan(e*x+d)^2*a^4*b^2+2*ln(b+a*tan(e*x+d))*tan(e*x+d)^2*a^2*b^4-12*ln(b+a*tan(e*x+d))*tan(e*x+d)^2*a^4*b^2-ln(1+tan(e*x+d)^2)*tan(e*x+d)^2*a^2*b^4+8*arctan(tan(e*x+d))*tan(e*x+d)^2*a^5*b-8*arctan(tan(e*x+d))*tan(e*x+d)^2*a^3*b^3+4*ln(b+a*tan(e*x+d))*tan(e*x+d)*a^5*b-24*ln(b+a*tan(e*x+d))*tan(e*x+d)*a^3*b^3+4*ln(b+a*tan(e*x+d))*tan(e*x+d)*a*b^5-2*ln(1+tan(e*x+d)^2)*tan(e*x+d)*a^5*b+12*ln(1+tan(e*x+d)^2)*tan(e*x+d)*a^3*b^3-2*ln(1+tan(e*x+d)^2)*tan(e*x+d)*a*b^5+16*arctan(tan(e*x+d))*tan(e*x+d)*a^4*b^2-16*arctan(tan(e*x+d))*tan(e*x+d)*a^2*b^4+2*ln(b+a*tan(e*x+d))*b^6-ln(1+tan(e*x+d)^2)*b^6+6*tan(e*x+d)*a^5*b+4*tan(e*x+d)*a^3*b^3-2*tan(e*x+d)*a*b^5-ln(1+tan(e*x+d)^2)*tan(e*x+d)^2*a^6+2*ln(b+a*tan(e*x+d))*a^4*b^2-12*ln(b+a*tan(e*x+d))*a^2*b^4-ln(1+tan(e*x+d)^2)*a^4*b^2+6*ln(1+tan(e*x+d)^2)*a^2*b^4+8*arctan(tan(e*x+d))*a^3*b^3-8*arctan(tan(e*x+d))*a*b^5+7*a^4*b^2*(b+a*tan(e*x+d))/(a^2+b^2)^3/((b+a*tan(e*x+d))^2)^(3/2)

maxima [A] time = 0.44, size = 498, normalized size = 1.58

$$\frac{\left(\frac{2(3a^2b-b^3)(ex+d)}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{2(a^3-3ab^2)\log(a\tan(ex+d)+b)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{(a^3-3ab^2)\log(\tan(ex+d)^2+1)}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{4a^2b\tan(ex+d)+a^3}{a^4b^2+2a^2b^4+b^6+(a^6+2a^4b^2+a^2b^4)\tan(ex+d)} \right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2), x, algorithm="maxima")

[Out] -1/2*((2*(3*a^2*b - b^3)*(e*x + d)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(a^3 - 3*a*b^2)*log(a*tan(e*x + d) + b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^3 - 3*a*b^2)*log(tan(e*x + d)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (4*a^2*b*tan(e*x + d) + a^3 + 5*a*b^2)/(a^4*b^2 + 2*a^2*b^4 + b^6 + (a^6 + 2*a^4*b^2 + a^2*b^4)*tan(e*x + d)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*tan(e*x + d)))*a + (2*(a^3 - 3*a*b^2)*(e*x + d)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(3*a^2*b - b^3)*log(a*tan(e*x + d) + b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3*a^2*b - b^3)*log(tan(e*x + d)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^2*b - 3*b^3 + 2*(a^3 - a*b^2)*tan(e*x + d))/(a^4*b^2 + 2*a^2*b^4 + b^6 + (a^6 + 2*a^4*b^2 + a^2*b^4)*tan(e*x + d)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*tan(e*x + d)))*b)/e

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \tan(d + ex)}{(a^2 \tan(d + ex)^2 + 2ab \tan(d + ex) + b^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(3/2), x)

[Out] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \tan(d + ex)}{\left((a \tan(d + ex) + b)^2\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(3/2),x)

[Out] Integral((a + b*tan(d + e*x))/((a*tan(d + e*x) + b)**2)**(3/2), x)

3.518 $\int (a+b \sec(d+ex)) (b^2 + 2ab \sec(d+ex) + a^2 \sec^2(d+ex)) dx$

Optimal. Leaf size=184

$$\frac{a^2 b (41a^2 + 26b^2) \tan(d+ex) \sec(d+ex)}{24e} + \frac{(4a^2 + 7b^2) \tan(d+ex) (a^2 \sec(d+ex) + ab)^2}{12ae} + \frac{b \tan(d+ex) (a^2 \sec(d+ex) + ab)^2}{4ae}$$

[Out] a*b^4*x+1/8*b*(19*a^4+56*a^2*b^2+8*b^4)*arctanh(sin(e*x+d))/e+1/6*a*(4*a^4+50*a^2*b^2+19*b^4)*tan(e*x+d)/e+1/24*a^2*b*(41*a^2+26*b^2)*sec(e*x+d)*tan(e*x+d)/e+1/12*(4*a^2+7*b^2)*(a*b+a^2*sec(e*x+d))^2*tan(e*x+d)/a/e+1/4*b*(a*b+a^2*sec(e*x+d))^3*tan(e*x+d)/a^2/e

Rubi [A] time = 0.43, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.180$, Rules used = {4172, 3918, 4056, 4048, 3770, 3767, 8}

$$\frac{a(50a^2b^2 + 4a^4 + 19b^4) \tan(d+ex)}{6e} + \frac{b(56a^2b^2 + 19a^4 + 8b^4) \tanh^{-1}(\sin(d+ex))}{8e} + \frac{a^2b(41a^2 + 26b^2) \tan(d+ex)}{24e}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2), x]

[Out] a*b^4*x + (b*(19*a^4 + 56*a^2*b^2 + 8*b^4)*ArcTanh[Sin[d + e*x]])/(8*e) + (a*(4*a^4 + 50*a^2*b^2 + 19*b^4)*Tan[d + e*x])/(6*e) + (a^2*b*(41*a^2 + 26*b^2)*Sec[d + e*x]*Tan[d + e*x])/(24*e) + ((4*a^2 + 7*b^2)*(a*b + a^2*Sec[d + e*x])^2*Tan[d + e*x])/(12*a*e) + (b*(a*b + a^2*Sec[d + e*x])^3*Tan[d + e*x])/(4*a^2*e)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3918

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m-1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m-2)*Simp[a^2*c*m + (b^2*d*(m-1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m-1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rule 4048

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +

C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, A, B, C}, x]

Rule 4056

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]

Rule 4172

Int[((A_) + (B_)*sec[(d_) + (e_)*(x_)])*((a_) + (b_)*sec[(d_) + (e_)*(x_)]) + (c_)*sec[(d_) + (e_)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n), Int[(A + B*Sec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned} \int (a + b \sec(d + ex)) (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^2 dx &= \frac{\int (2ab + 2a^2 \sec(d + ex))^4 (a + b \sec(d + ex)) dx}{16a^4} \\ &= \frac{b (ab + a^2 \sec(d + ex))^3 \tan(d + ex)}{4a^2e} + \frac{\int (2ab + 2a^2 \sec(d + ex))^4 dx}{12ae} \\ &= \frac{(4a^2 + 7b^2) (ab + a^2 \sec(d + ex))^2 \tan(d + ex)}{12ae} \\ &= \frac{a^2b (41a^2 + 26b^2) \sec(d + ex) \tan(d + ex)}{24e} + \dots \\ &= ab^4x + \frac{a^2b (41a^2 + 26b^2) \sec(d + ex) \tan(d + ex)}{24e} \\ &= ab^4x + \frac{b (19a^4 + 56a^2b^2 + 8b^4) \tanh^{-1}(\sin(d + ex))}{8e} \\ &= ab^4x + \frac{b (19a^4 + 56a^2b^2 + 8b^4) \tanh^{-1}(\sin(d + ex))}{8e} \end{aligned}$$

Mathematica [A] time = 0.87, size = 130, normalized size = 0.71

$$\frac{3b (19a^4 + 56a^2b^2 + 8b^4) \tanh^{-1}(\sin(d + ex)) + 8a^3 (a^2 + 4b^2) \tan^3(d + ex) + 3a \tan(d + ex) (2a^3b \sec^3(d + ex) + \dots)}{24e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^2,x]

[Out] (24*a*b^4*e*x + 3*b*(19*a^4 + 56*a^2*b^2 + 8*b^4)*ArcTanh[Sin[d + e*x]] + 3*a*(8*(a^4 + 10*a^2*b^2 + 4*b^4) + a*b*(19*a^2 + 24*b^2)*Sec[d + e*x] + 2*a^3*b*Sec[d + e*x]^3)*Tan[d + e*x] + 8*a^3*(a^2 + 4*b^2)*Tan[d + e*x]^3)/(24*e)

fricas [A] time = 2.43, size = 198, normalized size = 1.08

$$\frac{48 ab^4 ex \cos(ex + d)^4 + 3 (19 a^4 b + 56 a^2 b^3 + 8 b^5) \cos(ex + d)^4 \log(\sin(ex + d) + 1) - 3 (19 a^4 b + 56 a^2 b^3 + 8 b^5) \cos(ex + d)^4}{24e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, algorithm="fricas")

[Out] $\frac{1}{48}*(48*a*b^4*e*x*\cos(e*x + d)^4 + 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*\cos(e*x + d)^4*\log(\sin(e*x + d) + 1) - 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*\cos(e*x + d)^4*\log(-\sin(e*x + d) + 1) + 2*(6*a^4*b + 16*(a^5 + 13*a^3*b^2 + 6*a*b^4)*\cos(e*x + d)^3 + 3*(19*a^4*b + 24*a^2*b^3)*\cos(e*x + d)^2 + 8*(a^5 + 4*a^3*b^2)*\cos(e*x + d))*\sin(e*x + d))/(e*\cos(e*x + d)^4)$

giac [B] time = 0.39, size = 470, normalized size = 2.55

$$\frac{1}{24} \left(24(xe + d)ab^4 + 3(19a^4b + 56a^2b^3 + 8b^5) \log \left(\tan \left(\frac{1}{2}xe + \frac{1}{2}d \right) + 1 \right) - 3(19a^4b + 56a^2b^3 + 8b^5) \log \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, algorithm="giac")

[Out] $\frac{1}{24}*(24*(x*e + d)*a*b^4 + 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*\log(\text{abs}(\tan(1/2*x*e + 1/2*d) + 1)) - 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*\log(\text{abs}(\tan(1/2*x*e + 1/2*d) - 1)) - 2*(24*a^5*\tan(1/2*x*e + 1/2*d)^7 - 63*a^4*b*\tan(1/2*x*e + 1/2*d)^7 + 240*a^3*b^2*\tan(1/2*x*e + 1/2*d)^7 - 72*a^2*b^3*\tan(1/2*x*e + 1/2*d)^7 + 96*a*b^4*\tan(1/2*x*e + 1/2*d)^7 - 40*a^5*\tan(1/2*x*e + 1/2*d)^5 + 39*a^4*b*\tan(1/2*x*e + 1/2*d)^5 - 592*a^3*b^2*\tan(1/2*x*e + 1/2*d)^5 + 72*a^2*b^3*\tan(1/2*x*e + 1/2*d)^5 - 288*a*b^4*\tan(1/2*x*e + 1/2*d)^5 + 40*a^5*\tan(1/2*x*e + 1/2*d)^3 + 39*a^4*b*\tan(1/2*x*e + 1/2*d)^3 + 592*a^3*b^2*\tan(1/2*x*e + 1/2*d)^3 + 72*a^2*b^3*\tan(1/2*x*e + 1/2*d)^3 + 288*a*b^4*\tan(1/2*x*e + 1/2*d)^3 - 24*a^5*\tan(1/2*x*e + 1/2*d) - 63*a^4*b*\tan(1/2*x*e + 1/2*d) - 240*a^3*b^2*\tan(1/2*x*e + 1/2*d) - 72*a^2*b^3*\tan(1/2*x*e + 1/2*d) - 96*a*b^4*\tan(1/2*x*e + 1/2*d))/(\tan(1/2*x*e + 1/2*d)^2 - 1)^4)*e^{-1}$

maple [A] time = 0.14, size = 246, normalized size = 1.34

$$ab^4x + \frac{ab^4d}{e} + \frac{7a^2b^3 \ln(\sec(ex + d) + \tan(ex + d))}{e} + \frac{26a^3b^2 \tan(ex + d)}{3e} + \frac{19a^4b \sec(ex + d) \tan(ex + d)}{8e} + \frac{19a^5}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x)

[Out] $a*b^4*x + 1/e*a*b^4*d + 7/e*a^2*b^3*\ln(\sec(e*x+d) + \tan(e*x+d)) + 26/3/e*a^3*b^2*\tan(e*x+d) + 19/8/e*a^4*b*\sec(e*x+d)*\tan(e*x+d) + 19/8/e*a^4*b*\ln(\sec(e*x+d) + \tan(e*x+d)) + 2/3/e*a^5*\tan(e*x+d) + 1/3/e*a^5*\tan(e*x+d)*\sec(e*x+d)^2 + 1/e*b^5*\ln(\sec(e*x+d) + \tan(e*x+d)) + 4/e*a*b^4*\tan(e*x+d) + 3/e*a^2*b^3*\sec(e*x+d)*\tan(e*x+d) + 4/3/e*a^3*b^2*\tan(e*x+d)*\sec(e*x+d)^2 + 1/4/e*a^4*b*\tan(e*x+d)*\sec(e*x+d)^3$

maxima [A] time = 0.34, size = 299, normalized size = 1.62

$$16(\tan(ex + d)^3 + 3 \tan(ex + d))a^5 + 64(\tan(ex + d)^3 + 3 \tan(ex + d))a^3b^2 + 48(ex + d)ab^4 - 3a^4b \left(\frac{2(3 \sin(ex + d) \cos(ex + d) + 1)}{\sin(ex + d)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, algorithm="maxima")

```
[Out] 1/48*(16*(tan(e*x + d)^3 + 3*tan(e*x + d))*a^5 + 64*(tan(e*x + d)^3 + 3*tan
(e*x + d))*a^3*b^2 + 48*(e*x + d)*a*b^4 - 3*a^4*b*(2*(3*sin(e*x + d)^3 - 5*
sin(e*x + d))/(sin(e*x + d)^4 - 2*sin(e*x + d)^2 + 1) - 3*log(sin(e*x + d)
+ 1) + 3*log(sin(e*x + d) - 1)) - 48*a^4*b*(2*sin(e*x + d)/(sin(e*x + d)^2
- 1) - log(sin(e*x + d) + 1) + log(sin(e*x + d) - 1)) - 72*a^2*b^3*(2*sin(e
*x + d)/(sin(e*x + d)^2 - 1) - log(sin(e*x + d) + 1) + log(sin(e*x + d) - 1
)) + 192*a^2*b^3*log(sec(e*x + d) + tan(e*x + d)) + 48*b^5*log(sec(e*x + d)
+ tan(e*x + d)) + 288*a^3*b^2*tan(e*x + d) + 192*a*b^4*tan(e*x + d))/e
```

mupad [B] time = 3.30, size = 323, normalized size = 1.76

$$\frac{2a^5 \sin(d+ex)}{3e \cos(d+ex)} + \frac{a^5 \sin(d+ex)}{3e \cos(d+ex)^3} + \frac{2ab^4 \operatorname{atan}\left(\frac{\sin\left(\frac{d+ex}{2}\right)}{\cos\left(\frac{d+ex}{2}\right)}\right)}{e} + \frac{4ab^4 \sin(d+ex)}{e \cos(d+ex)} + \frac{19a^4 b \sin(d+ex)}{8e \cos(d+ex)^2} + \frac{a^4 b \sin(d+ex)}{4e \cos(d+ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
2,x)
```

```
[Out] (2*a^5*sin(d + e*x))/(3*e*cos(d + e*x)) - (b^5*atan((sin(d/2 + (e*x)/2)*1i)
/cos(d/2 + (e*x)/2))*2i)/e + (a^5*sin(d + e*x))/(3*e*cos(d + e*x)^3) - (a^2
*b^3*atan((sin(d/2 + (e*x)/2)*1i)/cos(d/2 + (e*x)/2))*14i)/e + (2*a*b^4*ata
n(sin(d/2 + (e*x)/2)/cos(d/2 + (e*x)/2)))/e - (a^4*b*atan((sin(d/2 + (e*x)/
2)*1i)/cos(d/2 + (e*x)/2))*19i)/(4*e) + (4*a*b^4*sin(d + e*x))/(e*cos(d + e
*x)) + (19*a^4*b*sin(d + e*x))/(8*e*cos(d + e*x)^2) + (a^4*b*sin(d + e*x))/
(4*e*cos(d + e*x)^4) + (26*a^3*b^2*sin(d + e*x))/(3*e*cos(d + e*x)) + (3*a^
2*b^3*sin(d + e*x))/(e*cos(d + e*x)^2) + (4*a^3*b^2*sin(d + e*x))/(3*e*cos(
d + e*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec(d + ex))(a \sec(d + ex) + b)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**2,x)
```

```
[Out] Integral((a + b*sec(d + e*x))*(a*sec(d + e*x) + b)**4, x)
```

3.519 $\int (a + b \sec(d + ex)) (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)) dx$

Optimal. Leaf size=76

$$\frac{a(a^2 + 2b^2) \tan(d + ex)}{e} + \frac{b(5a^2 + 2b^2) \tanh^{-1}(\sin(d + ex))}{2e} + \frac{a^2 b \tan(d + ex) \sec(d + ex)}{2e} + ab^2 x$$

[Out] $a*b^2*x + 1/2*b*(5*a^2 + 2*b^2)*\operatorname{arctanh}(\sin(e*x + d))/e + a*(a^2 + 2*b^2)*\tan(e*x + d)/e + 1/2*a^2*b*\sec(e*x + d)*\tan(e*x + d)/e$

Rubi [A] time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {4048, 3770, 3767, 8}

$$\frac{a(a^2 + 2b^2) \tan(d + ex)}{e} + \frac{b(5a^2 + 2b^2) \tanh^{-1}(\sin(d + ex))}{2e} + \frac{a^2 b \tan(d + ex) \sec(d + ex)}{2e} + ab^2 x$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{Sec}[d + e*x])*(b^2 + 2*a*b*\operatorname{Sec}[d + e*x] + a^2*\operatorname{Sec}[d + e*x]^2), x]$

[Out] $a*b^2*x + (b*(5*a^2 + 2*b^2)*\operatorname{ArcTanh}[\operatorname{Sin}[d + e*x]])/(2*e) + (a*(a^2 + 2*b^2)*\operatorname{Tan}[d + e*x])/e + (a^2*b*\operatorname{Sec}[d + e*x]*\operatorname{Tan}[d + e*x])/(2*e)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\& \operatorname{IGtQ}[n/2, 0]$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \operatorname{FreeQ}[\{c, d\}, x]$

Rule 4048

$\operatorname{Int}[(A_.) + \operatorname{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + \operatorname{csc}[(e_.) + (f_.)*(x_.)]^2*(C_.) + \operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.), x_Symbol] \rightarrow -\operatorname{Simp}[(b*C*\operatorname{Csc}[e + f*x]*\operatorname{Cot}[e + f*x])/(2*f), x] + \operatorname{Dist}[1/2, \operatorname{Int}[\operatorname{Simp}[2*A*a + (2*B*a + b*(2*A + C))*\operatorname{Csc}[e + f*x] + 2*(a*C + B*b)*\operatorname{Csc}[e + f*x]^2, x], x], x] /; \operatorname{FreeQ}[\{a, b, e, f, A, B, C\}, x]$

Rubi steps

$$\begin{aligned} \int (a + b \sec(d + ex)) (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)) dx &= \frac{a^2 b \sec(d + ex) \tan(d + ex)}{2e} + \frac{1}{2} \int (2ab^2 + \\ &= ab^2 x + \frac{a^2 b \sec(d + ex) \tan(d + ex)}{2e} + (a(a \\ &= ab^2 x + \frac{b(5a^2 + 2b^2) \tanh^{-1}(\sin(d + ex))}{2e} \\ &= ab^2 x + \frac{b(5a^2 + 2b^2) \tanh^{-1}(\sin(d + ex))}{2e} \end{aligned}$$

Mathematica [A] time = 0.28, size = 64, normalized size = 0.84

$$\frac{b(5a^2 + 2b^2) \tanh^{-1}(\sin(d + ex)) + a \tan(d + ex) (2a^2 + ab \sec(d + ex) + 4b^2) + 2ab^2 ex}{2e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2), x]

[Out] (2*a*b^2*e*x + b*(5*a^2 + 2*b^2)*ArcTanh[Sin[d + e*x]] + a*(2*a^2 + 4*b^2 + a*b*Sec[d + e*x])*Tan[d + e*x])/(2*e)

fricas [A] time = 1.50, size = 125, normalized size = 1.64

$$\frac{4ab^2ex \cos(ex + d)^2 + (5a^2b + 2b^3) \cos(ex + d)^2 \log(\sin(ex + d) + 1) - (5a^2b + 2b^3) \cos(ex + d)^2 \log(-\sin(ex + d) + 1)}{4e \cos(ex + d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2), x, algorithm="fricas")

[Out] 1/4*(4*a*b^2*e*x*cos(e*x + d)^2 + (5*a^2*b + 2*b^3)*cos(e*x + d)^2*log(sin(e*x + d) + 1) - (5*a^2*b + 2*b^3)*cos(e*x + d)^2*log(-sin(e*x + d) + 1) + 2*(a^2*b + 2*(a^3 + 2*a*b^2)*cos(e*x + d))*sin(e*x + d))/(e*cos(e*x + d)^2)

giac [B] time = 0.25, size = 191, normalized size = 2.51

$$\frac{1}{2} \left(2(xe + d)ab^2 + (5a^2b + 2b^3) \log \left(\left| \tan \left(\frac{1}{2}xe + \frac{1}{2}d \right) + 1 \right| \right) - (5a^2b + 2b^3) \log \left(\left| \tan \left(\frac{1}{2}xe + \frac{1}{2}d \right) - 1 \right| \right) - \frac{2}{e} \left(2(xe + d)ab^2 + (5a^2b + 2b^3) \log \left(\left| \tan \left(\frac{1}{2}xe + \frac{1}{2}d \right) + 1 \right| \right) - (5a^2b + 2b^3) \log \left(\left| \tan \left(\frac{1}{2}xe + \frac{1}{2}d \right) - 1 \right| \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2), x, algorithm="giac")

[Out] 1/2*(2*(x*e + d)*a*b^2 + (5*a^2*b + 2*b^3)*log(abs(tan(1/2*x*e + 1/2*d) + 1)) - (5*a^2*b + 2*b^3)*log(abs(tan(1/2*x*e + 1/2*d) - 1)) - 2*(2*a^3*tan(1/2*x*e + 1/2*d)^3 - a^2*b*tan(1/2*x*e + 1/2*d)^3 + 4*a*b^2*tan(1/2*x*e + 1/2*d)^3 - 2*a^3*tan(1/2*x*e + 1/2*d) - a^2*b*tan(1/2*x*e + 1/2*d) - 4*a*b^2*tan(1/2*x*e + 1/2*d))/(tan(1/2*x*e + 1/2*d)^2 - 1)*e^(-1)

maple [A] time = 0.09, size = 110, normalized size = 1.45

$$ab^2x + \frac{ab^2d}{e} + \frac{5a^2b \ln(\sec(ex + d) + \tan(ex + d))}{2e} + \frac{a^3 \tan(ex + d)}{e} + \frac{b^3 \ln(\sec(ex + d) + \tan(ex + d))}{e} + \frac{2ab^2 \tan(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2), x)

[Out] a*b^2*x+1/e*a*b^2*d+5/2/e*a^2*b*ln(sec(e*x+d)+tan(e*x+d))+1/e*a^3*tan(e*x+d)+1/e*b^3*ln(sec(e*x+d)+tan(e*x+d))+2*a*b^2*tan(e*x+d)/e+1/2*a^2*b*sec(e*x+d)*tan(e*x+d)/e

maxima [A] time = 0.34, size = 126, normalized size = 1.66

$$\frac{4(ex + d)ab^2 - a^2b \left(\frac{2 \sin(ex + d)}{\sin(ex + d)^2 - 1} - \log(\sin(ex + d) + 1) + \log(\sin(ex + d) - 1) \right) + 8a^2b \log(\sec(ex + d) + \tan(ex + d))}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algorithm="maxima")

[Out] 1/4*(4*(e*x + d)*a*b^2 - a^2*b*(2*sin(e*x + d)/(sin(e*x + d)^2 - 1) - log(sin(e*x + d) + 1) + log(sin(e*x + d) - 1)) + 8*a^2*b*log(sec(e*x + d) + tan(e*x + d)) + 4*b^3*log(sec(e*x + d) + tan(e*x + d)) + 4*a^3*tan(e*x + d) + 8*a*b^2*tan(e*x + d))/e

mupad [B] time = 2.91, size = 160, normalized size = 2.11

$$\frac{2b^3 \operatorname{atanh}\left(\frac{\sin\left(\frac{d+ex}{2}\right)}{\cos\left(\frac{d+ex}{2}\right)}\right)}{e} + \frac{a^3 \sin(d+ex)}{e \cos(d+ex)} + \frac{2ab^2 \operatorname{atan}\left(\frac{\sin\left(\frac{d+ex}{2}\right)}{\cos\left(\frac{d+ex}{2}\right)}\right)}{e} + \frac{5a^2b \operatorname{atanh}\left(\frac{\sin\left(\frac{d+ex}{2}\right)}{\cos\left(\frac{d+ex}{2}\right)}\right)}{e} + \frac{2ab^2 \sin(d+ex)}{e \cos(d+ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x)), x)

[Out] (2*b^3*atanh(sin(d/2 + (e*x)/2)/cos(d/2 + (e*x)/2)))/e + (a^3*sin(d + e*x))/(e*cos(d + e*x)) + (2*a*b^2*atan(sin(d/2 + (e*x)/2)/cos(d/2 + (e*x)/2)))/e + (5*a^2*b*atanh(sin(d/2 + (e*x)/2)/cos(d/2 + (e*x)/2)))/e + (2*a*b^2*sin(d + e*x))/(e*cos(d + e*x)) + (a^2*b*sin(d + e*x))/(2*e*cos(d + e*x)^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec(d + ex))(a \sec(d + ex) + b)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2),x)

[Out] Integral((a + b*sec(d + e*x))*(a*sec(d + e*x) + b)**2, x)

$$3.520 \quad \int \frac{a+b \sec(d+ex)}{b^2+2ab \sec(d+ex)+a^2 \sec^2(d+ex)} dx$$

Optimal. Leaf size=92

$$-\frac{a^2 \tan(d+ex)}{be(a^2 \sec(d+ex)+ab)} - \frac{2\sqrt{a-b}\sqrt{a+b} \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^2e} + \frac{ax}{b^2}$$

[Out] a*x/b^2-2*arctan((a-b)^(1/2)*tan(1/2*e*x+1/2*d)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/b^2/e-a^2*tan(e*x+d)/b/e/(a*b+a^2*sec(e*x+d))

Rubi [A] time = 0.30, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4172, 3923, 3919, 3831, 2659, 205}

$$-\frac{a^2 \tan(d+ex)}{be(a^2 \sec(d+ex)+ab)} - \frac{2\sqrt{a-b}\sqrt{a+b} \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^2e} + \frac{ax}{b^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2),x]

[Out] (a*x)/b^2 - (2*sqrt[a - b]*sqrt[a + b]*ArcTan[(sqrt[a - b]*Tan[(d + e*x)/2])/sqrt[a + b]])/(b^2*e) - (a^2*Tan[d + e*x])/(b*e*(a*b + a^2*Sec[d + e*x]))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3923

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] := Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c - a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],

x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rule 4172

Int[((A_) + (B_)*sec[(d_) + (e_)*(x_)])*((a_) + (b_)*sec[(d_) + (e_)*(x_)]) + (c_)*sec[(d_) + (e_)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n), Int[(A + B*Sec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned} \int \frac{a + b \sec(d + ex)}{b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)} dx &= (4a^2) \int \frac{a + b \sec(d + ex)}{(2ab + 2a^2 \sec(d + ex))^2} dx \\ &= -\frac{a^2 \tan(d + ex)}{be(ab + a^2 \sec(d + ex))} + \frac{\int \frac{4a^3(a^2 - b^2) + 4a^2b(a^2 - b^2) \sec(d + ex)}{2ab + 2a^2 \sec(d + ex)} dx}{2ab(a^2 - b^2)} \\ &= \frac{ax}{b^2} - \frac{a^2 \tan(d + ex)}{be(ab + a^2 \sec(d + ex))} - \frac{(2a(a^2 - b^2)) \int \frac{\sec(d + ex)}{2ab + 2a^2 \sec(d + ex)} dx}{b^2} \\ &= \frac{ax}{b^2} - \frac{a^2 \tan(d + ex)}{be(ab + a^2 \sec(d + ex))} - \frac{(a^2 - b^2) \int \frac{1}{1 + \frac{b \cos(d + ex)}{a}} dx}{ab^2} \\ &= \frac{ax}{b^2} - \frac{a^2 \tan(d + ex)}{be(ab + a^2 \sec(d + ex))} - \frac{(2(a^2 - b^2)) \text{Subst}\left(\int \frac{1}{1 + \frac{b}{a} + (1 - \frac{b}{a} \cos(u))} du\right)}{ab^2} \\ &= \frac{ax}{b^2} - \frac{2\sqrt{a-b}\sqrt{a+b} \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^2e} - \frac{a^2 \tan(d + ex)}{be(ab + a^2 \sec(d + ex))} \end{aligned}$$

Mathematica [A] time = 0.39, size = 97, normalized size = 1.05

$$\frac{2\sqrt{b^2 - a^2} \tanh^{-1}\left(\frac{(b-a) \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2 - a^2}}\right) + \frac{a(ad+ax-b \sin(d+ex)+b(d+ex) \cos(d+ex))}{a+b \cos(d+ex)}}{b^2e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2), x]

[Out] (2*sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(d + e*x)/2])/sqrt[-a^2 + b^2]] + (a*(a*d + a*e*x + b*(d + e*x))*Cos[d + e*x] - b*Sin[d + e*x])/(a + b*Cos[d + e*x]))/(b^2*e)

fricas [A] time = 0.85, size = 279, normalized size = 3.03

$$\frac{2abex \cos(ex + d) + 2a^2ex - 2ab \sin(ex + d) + \sqrt{-a^2 + b^2} (b \cos(ex + d) + a) \log\left(\frac{2ab \cos(ex+d) + (2a^2 - b^2) \cos(ex+d)}{b^2 \cos(ex+d)}\right)}{2(b^3e \cos(ex + d) + ab^2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algorithm="fricas")

[Out] [1/2*(2*a*b*e*x*cos(e*x + d) + 2*a^2*e*x - 2*a*b*sin(e*x + d) + sqrt(-a^2 + b^2)*(b*cos(e*x + d) + a)*log((2*a*b*cos(e*x + d) + (2*a^2 - b^2)*cos(e*x + d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(e*x + d) + b)*sin(e*x + d) - a^2 + 2*b^2)/(b^2*cos(e*x + d)^2 + 2*a*b*cos(e*x + d) + a^2)))/(b^3*e*cos(e*x + d) + a*b^2*e), (a*b*e*x*cos(e*x + d) + a^2*e*x - a*b*sin(e*x + d) - sqrt(a^2 - b^2)*(b*cos(e*x + d) + a)*arctan(-(a*cos(e*x + d) + b)/(sqrt(a^2 - b^2)*sin(e*x + d)))/(b^3*e*cos(e*x + d) + a*b^2*e)]

giac [A] time = 0.28, size = 145, normalized size = 1.58

$$\left(\frac{(xe+d)a}{b^2} - \frac{2a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)}{\left(a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 - b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + a + b\right)b} - \frac{2 \left(\pi \left\lfloor \frac{xe+d}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(2a-2b) + \arctan\left(\frac{a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)}{\sqrt{a^2 - b^2}}\right) \right)}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algorithm="giac")

[Out] ((x*e + d)*a/b^2 - 2*a*tan(1/2*x*e + 1/2*d)/((a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 + a + b)*b) - 2*(pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d))/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2)*e^(-1)

maple [A] time = 0.29, size = 163, normalized size = 1.77

$$\frac{2 \tan\left(\frac{d}{2} + \frac{ex}{2}\right) a}{eb \left(a \left(\tan^2\left(\frac{d}{2} + \frac{ex}{2}\right) \right) - b \left(\tan^2\left(\frac{d}{2} + \frac{ex}{2}\right) \right) + a + b \right)} - \frac{2 \arctan\left(\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right) a^2}{e b^2 \sqrt{(a+b)(a-b)}} + \frac{2 \arctan\left(\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{e \sqrt{(a+b)(a-b)}} + \frac{2 a \arctan\left(\frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{e \sqrt{(a+b)(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x)

[Out] -2/e/b*tan(1/2*d+1/2*e*x)*a/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)-2/e/b^2/((a+b)*(a-b))^(1/2)*arctan(tan(1/2*d+1/2*e*x)*(a-b)/((a+b)*(a-b))^(1/2))*a^2+2/e/((a+b)*(a-b))^(1/2)*arctan(tan(1/2*d+1/2*e*x)*(a-b)/((a+b)*(a-b))^(1/2))+2/e*a/b^2*arctan(tan(1/2*d+1/2*e*x))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more details)Is 4*b^2-4*a^2 positive or negative?

mupad [B] time = 3.02, size = 444, normalized size = 4.83

$$2 \operatorname{atanh} \left(\frac{64 a^3 \tan\left(\frac{d}{2} + \frac{e x}{2}\right) \sqrt{b^2 - a^2}}{64 a^4 - 128 a^3 b + 128 a b^3 - 64 b^4} - \frac{192 a^2 \tan\left(\frac{d}{2} + \frac{e x}{2}\right) \sqrt{b^2 - a^2}}{128 a b^2 - 128 a^3 - 64 b^3 + \frac{64 a^4}{b}} + \frac{192 a \tan\left(\frac{d}{2} + \frac{e x}{2}\right) \sqrt{b^2 - a^2}}{128 a b - 64 b^2 - \frac{128 a^3}{b} + \frac{64 a^4}{b^2}} - \frac{64 b \tan\left(\frac{d}{2} + \frac{e x}{2}\right) \sqrt{b^2 - a^2}}{128 a b - 64 b^2 - \frac{128 a^3}{b} + \frac{64 a^4}{b^2}} \right) \sqrt{b^2 - a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x)), x)`

[Out] $(2 \operatorname{atanh}((64 a^3 \tan(d/2 + (e x)/2) (b^2 - a^2)^{1/2}) / (128 a^3 b - 128 a^3 b + 64 a^4 - 64 b^4) - (192 a^2 \tan(d/2 + (e x)/2) (b^2 - a^2)^{1/2}) / (128 a^2 b^2 - 128 a^3 - 64 b^3 + (64 a^4)/b) + (192 a \tan(d/2 + (e x)/2) (b^2 - a^2)^{1/2}) / (128 a b - 64 b^2 - (128 a^3)/b + (64 a^4)/b^2) - (64 b \tan(d/2 + (e x)/2) (b^2 - a^2)^{1/2}) / (128 a b - 64 b^2 - (128 a^3)/b + (64 a^4)/b^2)) (b^2 - a^2)^{1/2}) / (b^2 e) - (2 a \operatorname{atan}((64 a^2 \tan(d/2 + (e x)/2)) / (64 a^2 b - 64 a^2 - (64 a^3)/b + (64 a^4)/b^2) + (64 a^3 \tan(d/2 + (e x)/2)) / (64 a^2 b^2 - 64 a^2 b - 64 a^3 + (64 a^4)/b) - (64 a^4 \tan(d/2 + (e x)/2)) / (64 a^2 b^3 - 64 a^3 b + 64 a^4 - 64 a^2 b^2) - (64 a^2 b \tan(d/2 + (e x)/2)) / (64 a^2 b - 64 a^2 - (64 a^3)/b + (64 a^4)/b^2))) / (b^2 e) - (2 a \tan(d/2 + (e x)/2)) / (b e (a + b + \tan(d/2 + (e x)/2)^2 (a - b)))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(d + e x)}{(a \sec(d + e x) + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2), x)`

[Out] `Integral((a + b*sec(d + e*x))/(a*sec(d + e*x) + b)**2, x)`

$$3.521 \quad \int \frac{a+b \sec(d+ex)}{(b^2+2ab \sec(d+ex)+a^2 \sec^2(d+ex))^2} dx$$

Optimal. Leaf size=230

$$\frac{a(3a^2 - 5b^2) \tan(d+ex)}{6b^2e(a^2 - b^2)(a \sec(d+ex) + b)^2} - \frac{(a^2 - 2b^2)(2a^4 - a^2b^2 + b^4) \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^4e(a-b)^{5/2}(a+b)^{5/2}} - \frac{a(6a^4 - 11a^2b^2 + 11b^4) \tan(d+ex)}{6b^3e(a^2 - b^2)^2(a \sec(d+ex) + b)}$$

[Out] a*x/b^4-(a^2-2*b^2)*(2*a^4-a^2*b^2+b^4)*arctan((a-b)^(1/2)*tan(1/2*e*x+1/2*d)/(a+b)^(1/2))/(a-b)^(5/2)/b^4/(a+b)^(5/2)/e-1/6*a*(3*a^2-5*b^2)*tan(e*x+d)/b^2/(a^2-b^2)/e/(b+a*sec(e*x+d))^2-1/6*a*(6*a^4-11*a^2*b^2+11*b^4)*tan(e*x+d)/b^3/(a^2-b^2)^2/e/(b+a*sec(e*x+d))-1/3*a^4*tan(e*x+d)/b/e/(a*b+a^2*sec(e*x+d))^3

Rubi [A] time = 0.83, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.180$, Rules used = {4172, 3923, 4060, 3919, 3831, 2659, 205}

$$\frac{(a^2 - 2b^2)(-a^2b^2 + 2a^4 + b^4) \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^4e(a-b)^{5/2}(a+b)^{5/2}} - \frac{a(-11a^2b^2 + 6a^4 + 11b^4) \tan(d+ex)}{6b^3e(a^2 - b^2)^2(a \sec(d+ex) + b)} - \frac{a(3a^2 - 5b^2) \tan(d+ex)}{6b^2e(a^2 - b^2)(a \sec(d+ex) + b)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^2, x]

[Out] (a*x)/b^4 - ((a^2 - 2*b^2)*(2*a^4 - a^2*b^2 + b^4)*ArcTan[(Sqrt[a - b]*Tan[(d + e*x)/2])/Sqrt[a + b]])/(a - b)^(5/2)*b^4*(a + b)^(5/2)*e) - (a*(3*a^2 - 5*b^2)*Tan[d + e*x])/(6*b^2*(a^2 - b^2)*e*(b + a*Sec[d + e*x])^2) - (a*(6*a^4 - 11*a^2*b^2 + 11*b^4)*Tan[d + e*x])/(6*b^3*(a^2 - b^2)^2*e*(b + a*Sec[d + e*x])) - (a^4*Tan[d + e*x])/(3*b*e*(a*b + a^2*Sec[d + e*x])^3)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -

a*d, 0]

Rule 3923

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] := Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c - a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rule 4060

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

Rule 4172

Int[((A_) + (B_)*sec[(d_.) + (e_.)*(x_.)])*((a_) + (b_)*sec[(d_.) + (e_.)*(x_.)] + (c_)*sec[(d_.) + (e_.)*(x_.)]^2)^(n_), x_Symbol] := Dist[1/(4^n*c^n), Int[(A + B*Sec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{a + b \sec(d + ex)}{(b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^2} dx &= (16a^4) \int \frac{a + b \sec(d + ex)}{(2ab + 2a^2 \sec(d + ex))^4} dx \\
&= -\frac{a^4 \tan(d + ex)}{3be (ab + a^2 \sec(d + ex))^3} + \frac{(2a) \int \frac{12a^3(a^2 - b^2) + 12a^2b(a^2 - b^2) \sec(d + ex)}{(2ab + 2a^2 \sec(d + ex))^4} dx}{3b(a^2 - b^2)} \\
&= -\frac{a(3a^2 - 5b^2) \tan(d + ex)}{6b^2(a^2 - b^2)e(b + a \sec(d + ex))^2} - \frac{a^4 \tan(d + ex)}{3be(ab + a^2 \sec(d + ex))} \\
&= -\frac{a(3a^2 - 5b^2) \tan(d + ex)}{6b^2(a^2 - b^2)e(b + a \sec(d + ex))^2} - \frac{a^4 \tan(d + ex)}{3be(ab + a^2 \sec(d + ex))} \\
&= \frac{ax}{b^4} - \frac{a(3a^2 - 5b^2) \tan(d + ex)}{6b^2(a^2 - b^2)e(b + a \sec(d + ex))^2} - \frac{a^4 \tan(d + ex)}{3be(ab + a^2 \sec(d + ex))} \\
&= \frac{ax}{b^4} - \frac{a(3a^2 - 5b^2) \tan(d + ex)}{6b^2(a^2 - b^2)e(b + a \sec(d + ex))^2} - \frac{a^4 \tan(d + ex)}{3be(ab + a^2 \sec(d + ex))} \\
&= \frac{ax}{b^4} - \frac{a(3a^2 - 5b^2) \tan(d + ex)}{6b^2(a^2 - b^2)e(b + a \sec(d + ex))^2} - \frac{a^4 \tan(d + ex)}{3be(ab + a^2 \sec(d + ex))} \\
&= \frac{ax}{b^4} - \frac{(a^2 - 2b^2)(2a^4 - a^2b^2 + b^4) \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}e} - \frac{a^4 \tan(d + ex)}{6b^4e}
\end{aligned}$$

Mathematica [A] time = 1.53, size = 276, normalized size = 1.20

$$\sec^3(d + ex)(a + b \cos(d + ex))(a + b \sec(d + ex)) \left(-2a^3b \sin(d + ex) + \frac{a^2b(7a^2 - 9b^2) \sin(d + ex)(a + b \cos(d + ex))}{(a-b)(a+b)} - \frac{ab(11a^4 - 2b^4)}{6b^4e} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^2,x]

[Out] ((a + b*Cos[d + e*x])*Sec[d + e*x]^3*(a + b*Sec[d + e*x])*(6*a*(d + e*x)*(a + b*Cos[d + e*x])^3 + (6*(-2*a^6 + 5*a^4*b^2 - 3*a^2*b^4 + 2*b^6)*ArcTanh[(-a + b)*Tan[(d + e*x)/2]]/Sqrt[-a^2 + b^2])*(a + b*Cos[d + e*x])^3)/(-a^2 + b^2)^(5/2) - 2*a^3*b*Sin[d + e*x] + (a^2*b*(7*a^2 - 9*b^2)*(a + b*Cos[d + e*x])*Sin[d + e*x])/((a - b)*(a + b)) - (a*b*(11*a^4 - 23*a^2*b^2 + 18*b^4)*(a + b*Cos[d + e*x])^2*Sin[d + e*x])/((a - b)^2*(a + b)^2))/((6*b^4*e*(b + a*Cos[d + e*x])*(b + a*Sec[d + e*x])^4)

fricas [B] time = 2.11, size = 1335, normalized size = 5.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, algorithm="fricas")

[Out] [1/12*(12*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*e*x*cos(e*x + d)^3 + 36*(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8)*e*x*cos(e*x + d)^2 + 36*(a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*e*x*cos(e*x + d) + 12*(a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*e*x + 3*(2*a^9 - 5*a^7*b^2 + 3*a^5*b^4 - 2*a^3*b^6 + (2*a^6*b^3 - 5*a^4*b^5 + 3*a^2*b^7 - 2*b^9)*cos(e*x + d)^3 + 3*(2*a^7*b^2 - 5*a^5*b^4 + 3*a^3*b^6 - 2*a*b^8)*cos(e*x + d)^2 + 3*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 - 2*a^2*b^7)*cos(e*x + d))*sqrt(-a^2 + b^2)*log((2*a*b*cos(e*x + d) + (2*a^2 - b^2)*cos(e*x + d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(e*x + d) + b)*sin(e*x + d) - a^2 + 2*b^2)/(b^2*cos(e*x + d)^2 + 2*a*b*cos(e*x + d) + a^2)) - 2*(6*a^9*b - 17*a^7*b^3 + 22*a^5*b^5 - 11*a^3*b^7 + (11*a^7*b^3 - 34*a^5*b^5 + 41*a^3*b^7 - 18*a*b^9)*cos(e*x + d)^2 + 3*(5*a^8*b^2 - 15*a^6*b^4 + 19*a^4*b^6 - 9*a^2*b^8)*cos(e*x + d))*sin(e*x + d))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*e*cos(e*x + d)^3 + 3*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*e*cos(e*x + d)^2 + 3*(a^8*b^5 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*e*cos(e*x + d) + (a^9*b^4 - 3*a^7*b^6 + 3*a^5*b^8 - a^3*b^10)*e), 1/6*(6*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*e*x*cos(e*x + d)^3 + 18*(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8)*e*x*cos(e*x + d)^2 + 18*(a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*e*x*cos(e*x + d) + 6*(a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*e*x - 3*(2*a^9 - 5*a^7*b^2 + 3*a^5*b^4 - 2*a^3*b^6 + (2*a^6*b^3 - 5*a^4*b^5 + 3*a^2*b^7 - 2*b^9)*cos(e*x + d)^3 + 3*(2*a^7*b^2 - 5*a^5*b^4 + 3*a^3*b^6 - 2*a*b^8)*cos(e*x + d)^2 + 3*(2*a^8*b - 5*a^6*b^3 + 3*a^4*b^5 - 2*a^2*b^7)*cos(e*x + d))*sqrt(a^2 - b^2)*arctan(-(a*cos(e*x + d) + b)/(sqrt(a^2 - b^2)*sin(e*x + d))) - (6*a^9*b - 17*a^7*b^3 + 22*a^5*b^5 - 11*a^3*b^7 + (11*a^7*b^3 - 34*a^5*b^5 + 41*a^3*b^7 - 18*a*b^9)*cos(e*x + d)^2 + 3*(5*a^8*b^2 - 15*a^6*b^4 + 19*a^4*b^6 - 9*a^2*b^8)*cos(e*x + d))*sin(e*x + d))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*e*cos(e*x + d)^3 + 3*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*e*cos(e*x + d)^2 + 3*(a^8*b^5 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*e*cos(e*x + d) + (a^9*b^4 - 3*a^7*b^6 + 3*a^5*b^8 - a^3*b^10)*e)]

giac [B] time = 0.49, size = 490, normalized size = 2.13

$$\frac{1}{3} \left(\frac{3 \left(2a^6 - 5a^4b^2 + 3a^2b^4 - 2b^6 \right) \left(\pi \left[\frac{xe+d}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a + 2b) + \arctan \left(-\frac{a \tan \left(\frac{1}{2} xe + \frac{1}{2} d \right) - b \tan \left(\frac{1}{2} xe + \frac{1}{2} d \right)}{\sqrt{a^2 - b^2}} \right) \right)}{(a^4b^4 - 2a^2b^6 + b^8)\sqrt{a^2 - b^2}} \right) + \frac{3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, algorithm="giac")

[Out] 1/3*(3*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4 - 2*b^6)*(pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d))/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) + 3*(x*e + d)*a/b^4 - (6*a^7*tan(1/2*x*e + 1/2*d)^5 - 15*a^6*b*tan(1/2*x*e + 1/2*d)^5 + 30*a^4*b^3*tan(1/2*x*e + 1/2*d)^5 - 12*a^3*b^4*tan(1/2*x*e + 1/2*d)^5 - 27*a^2*b^5*tan(1/2*x*e + 1/2*d)^5 + 18*a*b^6*tan(1/2*x*e + 1/2*d)^5 + 12*a^7*tan(1/2*x*e + 1/2*d)^3 - 44*a^5*b^2*tan(1/2*x*e + 1/2*d)^3 + 68*a^3*b^4*tan(1/2*x*e + 1/2*d)^3 - 36*a*b^6*tan(1/2*x*e + 1/2*d)^3 + 6*a^7*tan(1/2*x*e + 1/2*d) + 15*a^6*b*tan(1/2*x*e + 1/2*d) - 30*a^4*b^3*tan(1/2*x*e + 1/2*d) - 12*a^3*b^4*tan(1/2*x*e + 1/2*d) + 27*a^2*b^5*tan(1/2*x*e + 1/2*d) + 18*a*b^6*tan(1/2*x*e + 1/2*d))/((a^4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 + a + b)^3)*e^(-1)

maple [B] time = 0.35, size = 1118, normalized size = 4.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a+b*\sec(e*x+d))/(b^2+2*a*b*\sec(e*x+d)+a^2*\sec(e*x+d)^2),x)$

[Out]
$$\begin{aligned} & -2/e/b^3/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+a+b)^3*a^5/(a^2+2*a \\ & *b+b^2)*\tan(1/2*d+1/2*e*x)^5+1/e/b^2/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/ \\ & 2*e*x)^2+a+b)^3*a^4/(a^2+2*a*b+b^2)*\tan(1/2*d+1/2*e*x)^5+4/e/b/(a*\tan(1/2*d \\ & +1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+a+b)^3*a^3/(a^2+2*a*b+b^2)*\tan(1/2*d+1/2 \\ & *e*x)^5-3/e/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+a+b)^3*a^2/(a^2+ \\ & 2*a*b+b^2)*\tan(1/2*d+1/2*e*x)^5-6/e*b/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1 \\ & /2*e*x)^2+a+b)^3*a/(a^2+2*a*b+b^2)*\tan(1/2*d+1/2*e*x)^5-4/e/b^3/(a*\tan(1/2* \\ & d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+a+b)^3*a^5/(a+b)/(a-b)*\tan(1/2*d+1/2*e* \\ & x)^3+32/3/e/b/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+a+b)^3*a^3/(a+ \\ & b)/(a-b)*\tan(1/2*d+1/2*e*x)^3-12/e*b/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/ \\ & 2*e*x)^2+a+b)^3*a/(a+b)/(a-b)*\tan(1/2*d+1/2*e*x)^3-2/e/b^3/(a*\tan(1/2*d+1/2 \\ & *e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+a+b)^3*a^5/(a^2-2*a*b+b^2)*\tan(1/2*d+1/2*e*x \\ &)-1/e/b^2/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+a+b)^3*a^4/(a^2-2* \\ & a*b+b^2)*\tan(1/2*d+1/2*e*x)+4/e/b/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e \\ & *x)^2+a+b)^3*a^3/(a^2-2*a*b+b^2)*\tan(1/2*d+1/2*e*x)+3/e/(a*\tan(1/2*d+1/2*e* \\ & x)^2-b*\tan(1/2*d+1/2*e*x)^2+a+b)^3*a^2/(a^2-2*a*b+b^2)*\tan(1/2*d+1/2*e*x)-6 \\ & /e*b/(a*\tan(1/2*d+1/2*e*x)^2-b*\tan(1/2*d+1/2*e*x)^2+a+b)^3*a/(a^2-2*a*b+b^2 \\ &)*\tan(1/2*d+1/2*e*x)-2/e/b^4/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*\arctan \\ & (\tan(1/2*d+1/2*e*x)*(a-b)/((a+b)*(a-b))^(1/2))*a^6+5/e/b^2/(a^4-2*a^2*b^2+b^4) \\ & /((a+b)*(a-b))^(1/2)*\arctan(\tan(1/2*d+1/2*e*x)*(a-b)/((a+b)*(a-b))^(1/2)) \\ &)*a^4-3/e/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*\arctan(\tan(1/2*d+1/2*e*x) \\ & *(a-b)/((a+b)*(a-b))^(1/2))*a^2+2/e*b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(\\ & 1/2)*\arctan(\tan(1/2*d+1/2*e*x)*(a-b)/((a+b)*(a-b))^(1/2))+2/e*a/b^4*\arctan(\\ & \tan(1/2*d+1/2*e*x)) \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a+b*\sec(e*x+d))/(b^2+2*a*b*\sec(e*x+d)+a^2*\sec(e*x+d)^2),x, \text{alg orithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more details)Is 4*b^2-4*a^2 positive or negative?

mupad [B] time = 11.47, size = 5469, normalized size = 23.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a + b/\cos(d + e*x))/(b^2 + a^2/\cos(d + e*x)^2 + (2*a*b)/\cos(d + e*x))^2,x)$

[Out]
$$\begin{aligned} & -((\tan(d/2 + (e*x)/2)*(6*a*b^4 + a^4*b + 2*a^5 - 3*a^2*b^3 - 4*a^3*b^2))/(b^5 - 2*a*b^4 + a^2*b^3) + (\tan(d/2 + (e*x)/2)^5*(6*a*b^4 - a^4*b + 2*a^5 + \\ & 3*a^2*b^3 - 4*a^3*b^2))/(b^3*(a + b)^2) + (4*\tan(d/2 + (e*x)/2)^3*(9*a*b^4 + 3*a^5 - 8*a^3*b^2))/(3*(a*b^3 - b^4)*(a + b)))/(e*(3*a*b^2 - \tan(d/2 + (\\ & e*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) - \tan(d/2 + (e*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b + a^3 + b^3 + \tan(d/2 + (e*x)/2)^6* \end{aligned}$$

$$\begin{aligned}
& (3ab^2 - 3a^2b + a^3 - b^3)) - (2a \operatorname{atan}(((a((a((8(4b^{18} - 14a^2b^{16} - 6a^3b^{15} + 26a^4b^{14} + 14a^5b^{13} - 30a^6b^{12} - 10a^7b^{11} + 18a^8b^{10} + 2a^9b^9 - 4a^{10}b^8)))/(ab^{15} + b^{16} - 3a^2b^{14} - 3a^3b^{13} + 3a^4b^{12} + 3a^5b^{11} - a^6b^{10} - a^7b^9) - (a \tan(d/2 + (ex)/2) * (8ab^{17} - 8a^2b^{16} - 32a^3b^{15} + 32a^4b^{14} + 48a^5b^{13} - 48a^6b^{12} - 32a^7b^{11} + 32a^8b^{10} + 8a^9b^9 - 8a^{10}b^8) * 8i) / (b^4 * (ab^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6))) * 1i) / b^4 - (8 \tan(d/2 + (ex)/2) * (8a^{11}b - 8a^{12} - 4b^{12} + 8a^2b^{10} + 8a^3b^9 - 17a^4b^8 - 32a^5b^7 + 30a^6b^6 + 48a^7b^5 - 45a^8b^4 - 32a^9b^3 + 32a^{10}b^2)) / (ab^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6))) / b^4 - (a((a((8(4b^{18} - 14a^2b^{16} - 6a^3b^{15} + 26a^4b^{14} + 14a^5b^{13} - 30a^6b^{12} - 10a^7b^{11} + 18a^8b^{10} + 2a^9b^9 - 4a^{10}b^8)))/(ab^{15} + b^{16} - 3a^2b^{14} - 3a^3b^{13} + 3a^4b^{12} + 3a^5b^{11} - a^6b^{10} - a^7b^9) + (a \tan(d/2 + (ex)/2) * (8ab^{17} - 8a^2b^{16} - 32a^3b^{15} + 32a^4b^{14} + 48a^5b^{13} - 48a^6b^{12} - 32a^7b^{11} + 32a^8b^{10} + 8a^9b^9 - 8a^{10}b^8) * 8i) / (b^4 * (ab^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6))) * 1i) / b^4 + (8 \tan(d/2 + (ex)/2) * (8a^{11}b - 8a^{12} - 4b^{12} + 8a^2b^{10} + 8a^3b^9 - 17a^4b^8 - 32a^5b^7 + 30a^6b^6 + 48a^7b^5 - 45a^8b^4 - 32a^9b^3 + 32a^{10}b^2)) / (ab^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6))) / b^4) / ((a((a((8(4b^{18} - 14a^2b^{16} - 6a^3b^{15} + 26a^4b^{14} + 14a^5b^{13} - 30a^6b^{12} - 10a^7b^{11} + 18a^8b^{10} + 2a^9b^9 - 4a^{10}b^8)))/(ab^{15} + b^{16} - 3a^2b^{14} - 3a^3b^{13} + 3a^4b^{12} + 3a^5b^{11} - a^6b^{10} - a^7b^9) - (a \tan(d/2 + (ex)/2) * (8ab^{17} - 8a^2b^{16} - 32a^3b^{15} + 32a^4b^{14} + 48a^5b^{13} - 48a^6b^{12} - 32a^7b^{11} + 32a^8b^{10} + 8a^9b^9 - 8a^{10}b^8) * 8i) / (b^4 * (ab^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6))) * 1i) / b^4 - (8 \tan(d/2 + (ex)/2) * (8a^{11}b - 8a^{12} - 4b^{12} + 8a^2b^{10} + 8a^3b^9 - 17a^4b^8 - 32a^5b^7 + 30a^6b^6 + 48a^7b^5 - 45a^8b^4 - 32a^9b^3 + 32a^{10}b^2)) / (ab^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6))) / b^4) / ((a((a((8(4b^{18} - 14a^2b^{16} - 6a^3b^{15} + 26a^4b^{14} + 14a^5b^{13} - 30a^6b^{12} - 10a^7b^{11} + 18a^8b^{10} + 2a^9b^9 - 4a^{10}b^8)))/(ab^{15} + b^{16} - 3a^2b^{14} - 3a^3b^{13} + 3a^4b^{12} + 3a^5b^{11} - a^6b^{10} - a^7b^9) + (a \tan(d/2 + (ex)/2) * (8ab^{17} - 8a^2b^{16} - 32a^3b^{15} + 32a^4b^{14} + 48a^5b^{13} - 48a^6b^{12} - 32a^7b^{11} + 32a^8b^{10} + 8a^9b^9 - 8a^{10}b^8) * 8i) / (b^4 * (ab^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6))) * 1i) / b^4 - (8 \tan(d/2 + (ex)/2) * (8a^{11}b - 8a^{12} - 4b^{12} + 8a^2b^{10} + 8a^3b^9 - 17a^4b^8 - 32a^5b^7 + 30a^6b^6 + 48a^7b^5 - 45a^8b^4 - 32a^9b^3 + 32a^{10}b^2)) / (ab^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6))) / b^4 - (16 * (4ab^{11} - 2a^{11}b + 4a^{12} - 4a^2b^{10} - 8a^3b^9 + 14a^4b^8 + 15a^5b^7 - 26a^6b^6 - 12a^7b^5 + 30a^8b^4 + 7a^9b^3 - 18a^{10}b^2)) / (ab^{15} + b^{16} - 3a^2b^{14} - 3a^3b^{13} + 3a^4b^{12} + 3a^5b^{11} - a^6b^{10} - a^7b^9) + (a((a((8(4b^{18} - 14a^2b^{16} - 6a^3b^{15} + 26a^4b^{14} + 14a^5b^{13} - 30a^6b^{12} - 10a^7b^{11} + 18a^8b^{10} + 2a^9b^9 - 4a^{10}b^8)))/(ab^{15} + b^{16} - 3a^2b^{14} - 3a^3b^{13} + 3a^4b^{12} + 3a^5b^{11} - a^6b^{10} - a^7b^9) + (a \tan(d/2 + (ex)/2) * (8ab^{17} - 8a^2b^{16} - 32a^3b^{15} + 32a^4b^{14} + 48a^5b^{13} - 48a^6b^{12} - 32a^7b^{11} + 32a^8b^{10} + 8a^9b^9 - 8a^{10}b^8) * 8i) / (b^4 * (ab^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6))) * 1i) / b^4 + (8 \tan(d/2 + (ex)/2) * (8a^{11}b - 8a^{12} - 4b^{12} + 8a^2b^{10} + 8a^3b^9 - 17a^4b^8 - 32a^5b^7 + 30a^6b^6 + 48a^7b^5 - 45a^8b^4 - 32a^9b^3 + 32a^{10}b^2)) / (ab^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6))) / b^4)) / (b^4 * e) - (\operatorname{atan}(((a^2 - 2b^2) * (-a + b)^5 * (a - b)^5)^{(1/2)} * ((8 \tan(d/2 + (ex)/2) * (8a^{11}b - 8a^{12} - 4b^{12} + 8a^2b^{10} + 8a^3b^9 - 17a^4b^8 - 32a^5b^7 + 30a^6b^6 + 48a^7b^5 - 45a^8b^4 - 32a^9b^3 + 32a^{10}b^2)) / (ab^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6) - (((8(4b^{18} - 14a^2b^{16} - 6a^3b^{15} + 26a^4b^{14} + 14a^5b^{13} - 30a^6b^{12} - 10a^7b^{11} + 18a^8b^{10} + 2a^9b^9 - 4a^{10}b^8)) / (ab^{15} + b^{16} - 3a^2b^{14} - 3a^3b^{13} + 3a^4b^{12} + 3a^5b^{11} - a^6b^{10} - a^7b^9) - (4 \tan(d/2 + (ex)/2) * (a^2 - 2b^2) * (-a + b)^5 * (a - b)^5)^{(1/2)} * (2a^4 + b^4 - a^2b^2)) * (8ab^{17} - 8a^2b^{16} - 32a^3b^{15} + 32a^4b^{14} + 48a^5b^{13} - 48a^6b^{12} - 32a^7b^{11} + 32a^8b^{10} + 8a^9b^9 - 8a^{10}b^8)) / ((b^{14} - 5a^2b^{12} + 10a^4b^{10} - 10a^6b^8 + 5a^8b^6 - a^{10}b^4) * (ab^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6))) * (a^2 - 2b^2) * (-a + b)^5 * (a - b)^5)^{(1/2)} * (2a^4 + b^4 - a^2b^2)) / (2 * (b^{14} - 5a^2b^{12} + 10a^4b^{10} - 10a^6b^8 + 5a^8b^6 - a^{10}b^4))) * (2a^4 + b^4 - a^2b^2)
\end{aligned}$$

$$\begin{aligned} & \cdot 2) * 1i) / (2 * (b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} \\ & * b^4)) + ((a^2 - 2 * b^2) * (-a + b)^5 * (a - b)^5)^{(1/2)} * ((8 * \tan(d/2 + (e * x)/2) \\ & * (8 * a^{11} * b - 8 * a^{12} - 4 * b^{12} + 8 * a^2 * b^{10} + 8 * a^3 * b^9 - 17 * a^4 * b^8 - 32 * a^5 \\ & * b^7 + 30 * a^6 * b^6 + 48 * a^7 * b^5 - 45 * a^8 * b^4 - 32 * a^9 * b^3 + 32 * a^{10} * b^2)) / (a \\ & * b^{12} + b^{13} - 3 * a^2 * b^{11} - 3 * a^3 * b^{10} + 3 * a^4 * b^9 + 3 * a^5 * b^8 - a^6 * b^7 - \\ & a^7 * b^6) + (((8 * (4 * b^{18} - 14 * a^2 * b^{16} - 6 * a^3 * b^{15} + 26 * a^4 * b^{14} + 14 * a^5 * b^{13} \\ & - 30 * a^6 * b^{12} - 10 * a^7 * b^{11} + 18 * a^8 * b^{10} + 2 * a^9 * b^9 - 4 * a^{10} * b^8)) / (a \\ & * b^{15} + b^{16} - 3 * a^2 * b^{14} - 3 * a^3 * b^{13} + 3 * a^4 * b^{12} + 3 * a^5 * b^{11} - a^6 * b^{10} \\ & - a^7 * b^9) + (4 * \tan(d/2 + (e * x)/2) * (a^2 - 2 * b^2) * (-a + b)^5 * (a - b)^5)^{(1/2)} * (2 * a^4 + b^4 - a^2 * b^2) * (8 * a * b^{17} - 8 * a^2 * b^{16} - 32 * a^3 * b^{15} + 32 * a^4 * b^{14} + 48 * a^5 * b^{13} - 48 * a^6 * b^{12} - 32 * a^7 * b^{11} + 32 * a^8 * b^{10} + 8 * a^9 * b^9 - 8 * a^{10} * b^8)) / ((b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} * b^4) * (a * b^{12} + b^{13} - 3 * a^2 * b^{11} - 3 * a^3 * b^{10} + 3 * a^4 * b^9 + 3 * a^5 * b^8 - a^6 * b^7 - a^7 * b^6))) * (a^2 - 2 * b^2) * (-a + b)^5 * (a - b)^5)^{(1/2)} * (2 * a^4 + b^4 - a^2 * b^2)) / (2 * (b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} * b^4)) * (2 * a^4 + b^4 - a^2 * b^2) * 1i) / (2 * (b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} * b^4)) / ((16 * (4 * a * b^{11} - 2 * a^{11} * b + 4 * a^{12} - 4 * a^2 * b^{10} - 8 * a^3 * b^9 + 14 * a^4 * b^8 + 15 * a^5 * b^7 - 26 * a^6 * b^6 - 12 * a^7 * b^5 + 30 * a^8 * b^4 + 7 * a^9 * b^3 - 18 * a^{10} * b^2)) / (a * b^{15} + b^{16} - 3 * a^2 * b^{14} - 3 * a^3 * b^{13} + 3 * a^4 * b^{12} + 3 * a^5 * b^{11} - a^6 * b^{10} - a^7 * b^9) + ((a^2 - 2 * b^2) * (-a + b)^5 * (a - b)^5)^{(1/2)} * ((8 * \tan(d/2 + (e * x)/2) * (8 * a^{11} * b - 8 * a^{12} - 4 * b^{12} + 8 * a^2 * b^{10} + 8 * a^3 * b^9 - 17 * a^4 * b^8 - 32 * a^5 * b^7 + 30 * a^6 * b^6 + 48 * a^7 * b^5 - 45 * a^8 * b^4 - 32 * a^9 * b^3 + 32 * a^{10} * b^2)) / (a * b^{12} + b^{13} - 3 * a^2 * b^{11} - 3 * a^3 * b^{10} + 3 * a^4 * b^9 + 3 * a^5 * b^8 - a^6 * b^7 - a^7 * b^6) - (((8 * (4 * b^{18} - 14 * a^2 * b^{16} - 6 * a^3 * b^{15} + 26 * a^4 * b^{14} + 14 * a^5 * b^{13} - 30 * a^6 * b^{12} - 10 * a^7 * b^{11} + 18 * a^8 * b^{10} + 2 * a^9 * b^9 - 4 * a^{10} * b^8)) / (a * b^{15} + b^{16} - 3 * a^2 * b^{14} - 3 * a^3 * b^{13} + 3 * a^4 * b^{12} + 3 * a^5 * b^{11} - a^6 * b^{10} - a^7 * b^9) - (4 * \tan(d/2 + (e * x)/2) * (a^2 - 2 * b^2) * (-a + b)^5 * (a - b)^5)^{(1/2)} * (2 * a^4 + b^4 - a^2 * b^2) * (8 * a * b^{17} - 8 * a^2 * b^{16} - 32 * a^3 * b^{15} + 32 * a^4 * b^{14} + 48 * a^5 * b^{13} - 48 * a^6 * b^{12} - 32 * a^7 * b^{11} + 32 * a^8 * b^{10} + 8 * a^9 * b^9 - 8 * a^{10} * b^8)) / ((b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} * b^4) * (a * b^{12} + b^{13} - 3 * a^2 * b^{11} - 3 * a^3 * b^{10} + 3 * a^4 * b^9 + 3 * a^5 * b^8 - a^6 * b^7 - a^7 * b^6))) * (a^2 - 2 * b^2) * (-a + b)^5 * (a - b)^5)^{(1/2)} * (2 * a^4 + b^4 - a^2 * b^2)) / (2 * (b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} * b^4)) * (2 * a^4 + b^4 - a^2 * b^2)) / (2 * (b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} * b^4)) - ((a^2 - 2 * b^2) * (-a + b)^5 * (a - b)^5)^{(1/2)} * ((8 * \tan(d/2 + (e * x)/2) * (8 * a^{11} * b - 8 * a^{12} - 4 * b^{12} + 8 * a^2 * b^{10} + 8 * a^3 * b^9 - 17 * a^4 * b^8 - 32 * a^5 * b^7 + 30 * a^6 * b^6 + 48 * a^7 * b^5 - 45 * a^8 * b^4 - 32 * a^9 * b^3 + 32 * a^{10} * b^2)) / (a * b^{12} + b^{13} - 3 * a^2 * b^{11} - 3 * a^3 * b^{10} + 3 * a^4 * b^9 + 3 * a^5 * b^8 - a^6 * b^7 - a^7 * b^6) + (((8 * (4 * b^{18} - 14 * a^2 * b^{16} - 6 * a^3 * b^{15} + 26 * a^4 * b^{14} + 14 * a^5 * b^{13} - 30 * a^6 * b^{12} - 10 * a^7 * b^{11} + 18 * a^8 * b^{10} + 2 * a^9 * b^9 - 4 * a^{10} * b^8)) / (a * b^{15} + b^{16} - 3 * a^2 * b^{14} - 3 * a^3 * b^{13} + 3 * a^4 * b^{12} + 3 * a^5 * b^{11} - a^6 * b^{10} - a^7 * b^9) + (4 * \tan(d/2 + (e * x)/2) * (a^2 - 2 * b^2) * (-a + b)^5 * (a - b)^5)^{(1/2)} * (2 * a^4 + b^4 - a^2 * b^2) * (8 * a * b^{17} - 8 * a^2 * b^{16} - 32 * a^3 * b^{15} + 32 * a^4 * b^{14} + 48 * a^5 * b^{13} - 48 * a^6 * b^{12} - 32 * a^7 * b^{11} + 32 * a^8 * b^{10} + 8 * a^9 * b^9 - 8 * a^{10} * b^8)) / ((b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} * b^4) * (a * b^{12} + b^{13} - 3 * a^2 * b^{11} - 3 * a^3 * b^{10} + 3 * a^4 * b^9 + 3 * a^5 * b^8 - a^6 * b^7 - a^7 * b^6))) * (a^2 - 2 * b^2) * (-a + b)^5 * (a - b)^5)^{(1/2)} * (2 * a^4 + b^4 - a^2 * b^2)) / (2 * (b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} * b^4)) * (2 * a^4 + b^4 - a^2 * b^2)) / (2 * (b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} * b^4)) * (a^2 - 2 * b^2) * (-a + b)^5 * (a - b)^5)^{(1/2)} * (2 * a^4 + b^4 - a^2 * b^2) * 1i) / (e * (b^{14} - 5 * a^2 * b^{12} + 10 * a^4 * b^{10} - 10 * a^6 * b^8 + 5 * a^8 * b^6 - a^{10} * b^4))$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(d + ex)}{(a \sec(d + ex) + b)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**2,x)
```

```
[Out] Integral((a + b*sec(d + e*x))/(a*sec(d + e*x) + b)**4, x)
```

3.522 $\int (a+b \sec(d+ex)) (b^2 + 2ab \sec(d+ex) + a^2 \sec^2(d+ex)) dx$

Optimal. Leaf size=359

$$\frac{a^5 (3a^2 + 5b^2) \tan(d+ex) \sec(d+ex) (a^2 \sec^2(d+ex) + 2ab \sec(d+ex) + b^2)^{3/2}}{6e (a^2 \sec(d+ex) + ab)^3} + \frac{a^4 b (11a^2 + 8b^2) \tan(d+ex) (a^2 \sec^2(d+ex) + 2ab \sec(d+ex) + b^2)^{3/2}}{3e (a^2 \sec(d+ex) + ab)^3}$$

[Out] $\frac{1}{2} (a^4 + 9a^2b^2 + 2b^4) \operatorname{arctanh}(\sin(e*x+d)) (b^2 + 2a*b*\sec(e*x+d) + a^2*\sec(e*x+d)^2)^{(3/2)} / e / (b+a*\sec(e*x+d))^3 + a^4*b^3*x*(b^2 + 2a*b*\sec(e*x+d) + a^2*\sec(e*x+d)^2)^{(3/2)} / (a*b+a^2*\sec(e*x+d))^3 + 1/3*a^4*b*(11*a^2+8*b^2)*(b^2 + 2a*b*\sec(e*x+d) + a^2*\sec(e*x+d)^2)^{(3/2)} * \tan(e*x+d) / e / (a*b+a^2*\sec(e*x+d))^3 + 1/6*a^5*(3*a^2+5*b^2)*\sec(e*x+d)*(b^2 + 2a*b*\sec(e*x+d) + a^2*\sec(e*x+d)^2)^{(3/2)} * \tan(e*x+d) / e / (a*b+a^2*\sec(e*x+d))^3 + 1/3*b*(a^2*b+a^3*\sec(e*x+d))^2*(b^2 + 2a*b*\sec(e*x+d) + a^2*\sec(e*x+d)^2)^{(3/2)} * \tan(e*x+d) / e / (a*b+a^2*\sec(e*x+d))^3$

Rubi [A] time = 0.29, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {4174, 3918, 4048, 3770, 3767, 8}

$$\frac{a^4 b^3 x (a^2 \sec^2(d+ex) + 2ab \sec(d+ex) + b^2)^{3/2}}{(a^2 \sec(d+ex) + ab)^3} + \frac{a^5 (3a^2 + 5b^2) \tan(d+ex) \sec(d+ex) (a^2 \sec^2(d+ex) + 2ab \sec(d+ex) + b^2)^{3/2}}{6e (a^2 \sec(d+ex) + ab)^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Sec}[d + e*x])*(b^2 + 2*a*b*\text{Sec}[d + e*x] + a^2*\text{Sec}[d + e*x]^2)^{(3/2)}, x]$

[Out] $((a^4 + 9a^2b^2 + 2b^4)*\text{ArcTanh}[\text{Sin}[d + e*x]]*(b^2 + 2a*b*\text{Sec}[d + e*x] + a^2*\text{Sec}[d + e*x]^2)^{(3/2)}) / (2e*(b + a*\text{Sec}[d + e*x])^3) + (a^4*b^3*x*(b^2 + 2a*b*\text{Sec}[d + e*x] + a^2*\text{Sec}[d + e*x]^2)^{(3/2)}) / (a*b + a^2*\text{Sec}[d + e*x])^3 + (a^4*b*(11*a^2 + 8*b^2)*(b^2 + 2a*b*\text{Sec}[d + e*x] + a^2*\text{Sec}[d + e*x]^2)^{(3/2)}*\text{Tan}[d + e*x]) / (3e*(a*b + a^2*\text{Sec}[d + e*x])^3) + (a^5*(3*a^2 + 5*b^2)*\text{Sec}[d + e*x]*(b^2 + 2a*b*\text{Sec}[d + e*x] + a^2*\text{Sec}[d + e*x]^2)^{(3/2)}*\text{Tan}[d + e*x]) / (6e*(a*b + a^2*\text{Sec}[d + e*x])^3) + (b*(a^2*b + a^3*\text{Sec}[d + e*x])^2*(b^2 + 2a*b*\text{Sec}[d + e*x] + a^2*\text{Sec}[d + e*x]^2)^{(3/2)}*\text{Tan}[d + e*x]) / (3e*(a*b + a^2*\text{Sec}[d + e*x])^3)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 3767

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /; \text{FreeQ}[\{c, d\}, x] \ \&\& \ \text{IGtQ}[n/2, 0]$

Rule 3770

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3918

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] \rightarrow -\text{Simp}[(b*d*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m - 1)})/(f*m), x] + \text{Dist}[1/m, \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m - 2)}*\text{Simp}[a^2*c*m +$

$(b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*\text{Csc}[e + f*x] + b*(b*c*m + a*d*(2*m - 1))*\text{Csc}[e + f*x]^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{GtQ}[m, 1] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[2*m]$

Rule 4048

$\text{Int}[(A_.) + \text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + \text{csc}[(e_.) + (f_.)*(x_.)]^2*(C_.)]*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_)), x_Symbol] :> -\text{Simp}[(b*C*\text{Csc}[e + f*x]*\text{Cot}[e + f*x])/(2*f), x] + \text{Dist}[1/2, \text{Int}[\text{Simp}[2*A*a + (2*B*a + b*(2*A + C))*\text{Csc}[e + f*x] + 2*(a*C + B*b)*\text{Csc}[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C\}, x]$

Rule 4174

$\text{Int}[(A_.) + (B_.)*\text{sec}[(d_.) + (e_.)*(x_.)]]*((a_.) + (b_.)*\text{sec}[(d_.) + (e_.)*(x_.)] + (c_.)*\text{sec}[(d_.) + (e_.)*(x_.)]^2)^n, x_Symbol] :> \text{Dist}[(a + b*\text{Sec}[d + e*x] + c*\text{Sec}[d + e*x]^2)^n/(b + 2*c*\text{Sec}[d + e*x])^{2*n}, \text{Int}[(A + B*\text{Sec}[d + e*x])*(b + 2*c*\text{Sec}[d + e*x])^{2*n}, x], x] /; \text{FreeQ}\{a, b, c, d, e, A, B\}, x\} \&\& \text{EqQ}[b^2 - 4*a*c, 0] \&\& !\text{IntegerQ}[n]$

Rubi steps

$$\begin{aligned} \int (a + b \sec(d + ex)) (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2} dx &= \frac{(b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}}{(2ab + a^2 \sec^2(d + ex))^{3/2}} \\ &= \frac{b(a^2b + a^3 \sec(d + ex))^2 (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}}{3e(ab + a^2 \sec(d + ex))^{3/2}} \\ &= \frac{a^5(3a^2 + 5b^2) \sec(d + ex) (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}}{6e(ab + a^2 \sec(d + ex))^{3/2}} \\ &= \frac{a^4 b^3 x (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}}{(ab + a^2 \sec(d + ex))^3} \\ &= \frac{(a^4 + 9a^2 b^2 + 2b^4) \tanh^{-1}(\sin(d + ex))}{2e(b + a \sec(d + ex))} \\ &= \frac{(a^4 + 9a^2 b^2 + 2b^4) \tanh^{-1}(\sin(d + ex))}{2e(b + a \sec(d + ex))} \end{aligned}$$

Mathematica [A] time = 0.82, size = 128, normalized size = 0.36

$$\frac{\cos(d + ex) \sqrt{(a \sec(d + ex) + b)^2} (2a^3 b \tan^3(d + ex) + 3a \tan(d + ex) (a(a^2 + 3b^2) \sec(d + ex) + 8a^2 b + 6b^3))}{6e(a + b \cos(d + ex))}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2), x]

[Out] (Cos[d + e*x]*Sqrt[(b + a*Sec[d + e*x])^2]*(6*a*b^3*e*x + 3*(a^4 + 9*a^2*b^2 + 2*b^4)*ArcTanh[Sin[d + e*x]] + 3*a*(8*a^2*b + 6*b^3 + a*(a^2 + 3*b^2))*S

$\text{ec}[d + e*x]) * \text{Tan}[d + e*x] + 2*a^3*b*\text{Tan}[d + e*x]^3) / (6*e*(a + b*\text{Cos}[d + e*x]))$

fricas [A] time = 1.58, size = 162, normalized size = 0.45

$$\frac{12 ab^3 ex \cos(ex + d)^3 + 3(a^4 + 9a^2b^2 + 2b^4) \cos(ex + d)^3 \log(\sin(ex + d) + 1) - 3(a^4 + 9a^2b^2 + 2b^4) \cos(ex + d)^3 \log(-\sin(ex + d) + 1) + 2(2a^3b + 2(11a^3b + 9a*b^3) \cos(ex + d)^2 + 3(a^4 + 3a^2b^2) \cos(ex + d)) \sin(ex + d)}{12 e \cos(ex + d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2), x, algorithm="fricas")

[Out] 1/12*(12*a*b^3*e*x*cos(e*x + d)^3 + 3*(a^4 + 9*a^2*b^2 + 2*b^4)*cos(e*x + d)^3*log(sin(e*x + d) + 1) - 3*(a^4 + 9*a^2*b^2 + 2*b^4)*cos(e*x + d)^3*log(-sin(e*x + d) + 1) + 2*(2*a^3*b + 2*(11*a^3*b + 9*a*b^3)*cos(e*x + d)^2 + 3*(a^4 + 3*a^2*b^2)*cos(e*x + d))*sin(e*x + d))/(e*cos(e*x + d)^3)

giac [A] time = 0.51, size = 652, normalized size = 1.82

$$\frac{1}{6} \left(6(xe + d)ab^3 \text{sgn}(b \cos(xe + d)^2 + a \cos(xe + d)) + 3(a^4 \text{sgn}(b \cos(xe + d)^2 + a \cos(xe + d)) + 9a^2b^2 \text{sgn}(b \cos(xe + d)^2 + a \cos(xe + d))) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2), x, algorithm="giac")

[Out] 1/6*(6*(x*e + d)*a*b^3*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) + 3*(a^4*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) + 9*a^2*b^2*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) + 2*b^4*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)))*log(abs(tan(1/2*x*e + 1/2*d) + 1)) - 3*(a^4*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) + 9*a^2*b^2*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) + 2*b^4*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)))*log(abs(tan(1/2*x*e + 1/2*d) - 1)) + 2*(3*a^4*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^5 - 24*a^3*b*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^5 + 9*a^2*b^2*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^5 - 18*a*b^3*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^5 + 40*a^3*b*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^3 + 36*a*b^3*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)^3 - 3*a^4*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d) - 24*a^3*b*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d) - 9*a^2*b^2*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d) - 18*a*b^3*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d))/(tan(1/2*x*e + 1/2*d)^2 - 1)^3*e^(-1)

maple [A] time = 0.61, size = 387, normalized size = 1.08

$$\frac{\left(3 \ln\left(\frac{1-\cos(ex+d)+\sin(ex+d)}{\sin(ex+d)}\right)\right) \left(\cos^3(ex+d)\right) a^4 + 27 \ln\left(\frac{1-\cos(ex+d)+\sin(ex+d)}{\sin(ex+d)}\right) \left(\cos^3(ex+d)\right) a^2 b^2 + 6 \ln\left(\frac{1-\cos(ex+d)+\sin(ex+d)}{\sin(ex+d)}\right) \left(\cos^3(ex+d)\right) a^2 b^2}{\sin(ex+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2), x)

[Out] 1/6/e*(3*ln((1-cos(e*x+d)+sin(e*x+d))/sin(e*x+d))*cos(e*x+d)^3*a^4+27*ln((1-cos(e*x+d)+sin(e*x+d))/sin(e*x+d))*cos(e*x+d)^3*a^2*b^2+6*ln((1-cos(e*x+d)+sin(e*x+d))/sin(e*x+d))*cos(e*x+d)^3*b^4-3*ln(-(cos(e*x+d)-1+sin(e*x+d))/sin(e*x+d))*cos(e*x+d)^3)

$\ln(e^{*x+d}) * \cos(e^{*x+d})^3 * a^4 - 27 * \ln(-(\cos(e^{*x+d}) - 1 + \sin(e^{*x+d}))/\sin(e^{*x+d})) * \cos(e^{*x+d})^3 * a^2 * b^2 - 6 * \ln(-(\cos(e^{*x+d}) - 1 + \sin(e^{*x+d}))/\sin(e^{*x+d})) * \cos(e^{*x+d})^3 * b^4 + 6 * \cos(e^{*x+d})^3 * (e^{*x+d}) * a * b^3 + 22 * \sin(e^{*x+d}) * \cos(e^{*x+d})^2 * a^3 * b + 18 * \sin(e^{*x+d}) * \cos(e^{*x+d})^2 * a * b^3 + 3 * \sin(e^{*x+d}) * \cos(e^{*x+d}) * a^4 + 9 * \sin(e^{*x+d}) * \cos(e^{*x+d}) * a^2 * b^2 + 2 * a^3 * b * \sin(e^{*x+d})) * ((b * \cos(e^{*x+d}) + a)^2 / \cos(e^{*x+d})^2)^{(3/2)} / (b * \cos(e^{*x+d}) + a)^3$

maxima [A] time = 0.45, size = 440, normalized size = 1.23

$$3 \left(4 b^3 \arctan\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right) + (a^3 + 6 ab^2) \log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1} + 1\right) - (a^3 + 6 ab^2) \log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1} - 1\right) - \frac{2 \left(\frac{(a^3 + 6 a^2 b) \sin(ex+d)}{\cos(ex+d)+1} \right)}{2 \sin(ex+d) / (\cos(ex+d)+1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2), x, algorithm="maxima")

[Out] 1/6*(3*(4*b^3*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) + (a^3 + 6*a*b^2)*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1) - (a^3 + 6*a*b^2)*log(sin(e*x + d)/(cos(e*x + d) + 1) - 1) - 2*((a^3 + 6*a^2*b)*sin(e*x + d)/(cos(e*x + d) + 1) + (a^3 - 6*a^2*b)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/(2*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - sin(e*x + d)^4/(cos(e*x + d) + 1)^4 - 1))*a + (3*(3*a^2*b + 2*b^3)*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1) - 3*(3*a^2*b + 2*b^3)*log(sin(e*x + d)/(cos(e*x + d) + 1) - 1) - 2*(3*(2*a^3 + 3*a^2*b + 6*a*b^2)*sin(e*x + d)/(cos(e*x + d) + 1) - 4*(a^3 + 9*a*b^2)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 + 3*(2*a^3 - 3*a^2*b + 6*a*b^2)*sin(e*x + d)^5/(cos(e*x + d) + 1)^5)/(3*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 3*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + sin(e*x + d)^6/(cos(e*x + d) + 1)^6 - 1))*b)/e

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + \frac{b}{\cos(d + ex)} \right) \left(b^2 + \frac{a^2}{\cos(d + ex)^2} + \frac{2ab}{\cos(d + ex)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^(3/2), x)

[Out] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec(d + ex)) \left((a \sec(d + ex) + b)^2 \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(3/2), x)

[Out] Integral((a + b*sec(d + e*x))*((a*sec(d + e*x) + b)**2)**(3/2), x)

3.523 $\int (a+b \sec(d+ex))\sqrt{b^2 + 2ab \sec(d+ex) + a^2 \sec^2(d+ex)}$

Optimal. Leaf size=173

$$\frac{a^2 b x \sqrt{a^2 \sec^2(d+ex) + 2ab \sec(d+ex) + b^2}}{a^2 \sec(d+ex) + ab} + \frac{a^2 b \tan(d+ex) \sqrt{a^2 \sec^2(d+ex) + 2ab \sec(d+ex) + b^2}}{e(a^2 \sec(d+ex) + ab)} + \frac{(a^2 + b^2)}{e}$$

[Out] $(a^2+b^2)*\operatorname{arctanh}(\sin(e*x+d))*(b^2+2*a*b*\sec(e*x+d)+a^2*\sec(e*x+d)^2)^{(1/2)}/e/(b+a*\sec(e*x+d))+a^2*b*x*(b^2+2*a*b*\sec(e*x+d)+a^2*\sec(e*x+d)^2)^{(1/2)}/(a*b+a^2*\sec(e*x+d))+a^2*b*(b^2+2*a*b*\sec(e*x+d)+a^2*\sec(e*x+d)^2)^{(1/2)}*\tan(e*x+d)/e/(a*b+a^2*\sec(e*x+d))$

Rubi [A] time = 0.12, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {4174, 3914, 3767, 8, 3770}

$$\frac{a^2 b x \sqrt{a^2 \sec^2(d+ex) + 2ab \sec(d+ex) + b^2}}{a^2 \sec(d+ex) + ab} + \frac{a^2 b \tan(d+ex) \sqrt{a^2 \sec^2(d+ex) + 2ab \sec(d+ex) + b^2}}{e(a^2 \sec(d+ex) + ab)} + \frac{(a^2 + b^2)}{e}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{Sec}[d + e*x])*Sqrt[b^2 + 2*a*b*\operatorname{Sec}[d + e*x] + a^2*\operatorname{Sec}[d + e*x]^2], x]$

[Out] $((a^2 + b^2)*\operatorname{ArcTanh}[\operatorname{Sin}[d + e*x]]*Sqrt[b^2 + 2*a*b*\operatorname{Sec}[d + e*x] + a^2*\operatorname{Sec}[d + e*x]^2])/(e*(b + a*\operatorname{Sec}[d + e*x])) + (a^2*b*x*Sqrt[b^2 + 2*a*b*\operatorname{Sec}[d + e*x] + a^2*\operatorname{Sec}[d + e*x]^2])/(a*b + a^2*\operatorname{Sec}[d + e*x]) + (a^2*b*Sqrt[b^2 + 2*a*b*\operatorname{Sec}[d + e*x] + a^2*\operatorname{Sec}[d + e*x]^2]*\operatorname{Tan}[d + e*x])/(e*(a*b + a^2*\operatorname{Sec}[d + e*x]))$

Rule 8

$\operatorname{Int}[a_, x_Symbol] := \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] := -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\amp; \operatorname{IGtQ}[n/2, 0]$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_)], x_Symbol] := -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \operatorname{FreeQ}[\{c, d\}, x]$

Rule 3914

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))*(\operatorname{csc}[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := \operatorname{Simp}[a*c*x, x] + (\operatorname{Dist}[b*d, \operatorname{Int}[\operatorname{Csc}[e + f*x]^2, x], x] + \operatorname{Dist}[b*c + a*d, \operatorname{Int}[\operatorname{Csc}[e + f*x], x], x]) /; \operatorname{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\amp; \operatorname{NeQ}[b*c - a*d, 0] \ \&\amp; \operatorname{NeQ}[b*c + a*d, 0]$

Rule 4174

$\operatorname{Int}[(A_.) + (B_.)*\sec[(d_.) + (e_.)*(x_)]*(a_.) + (b_.)*\sec[(d_.) + (e_.)*(x_)] + (c_.)*\sec[(d_.) + (e_.)*(x_)]^{(n_)}, x_Symbol] := \operatorname{Dist}[(a + b*\operatorname{Sec}[d + e*x] + c*\operatorname{Sec}[d + e*x]^2)^n/(b + 2*c*\operatorname{Sec}[d + e*x])^{(2*n)}, \operatorname{Int}[(A + B*\operatorname{Sec}[d + e*x])*(b + 2*c*\operatorname{Sec}[d + e*x])^{(2*n)}, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, A, B\}, x] \ \&\amp; \operatorname{EqQ}[b^2 - 4*a*c, 0] \ \&\amp; \operatorname{IntegerQ}[n]$

Rubi steps

$$\begin{aligned}
\int (a + b \sec(d + ex)) \sqrt{b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)} dx &= \frac{\sqrt{b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)} \int (a + b \sec(d + ex)) dx}{2ab + 2a^2 \sec(d + ex)} \\
&= \frac{a^2 b x \sqrt{b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)}}{ab + a^2 \sec(d + ex)} \\
&= \frac{(a^2 + b^2) \tanh^{-1}(\sin(d + ex)) \sqrt{b^2 + 2ab \sec(d + ex)}}{e(b + a \sec(d + ex))} \\
&= \frac{(a^2 + b^2) \tanh^{-1}(\sin(d + ex)) \sqrt{b^2 + 2ab \sec(d + ex)}}{e(b + a \sec(d + ex))}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 67, normalized size = 0.39

$$\frac{\cos(d + ex) \sqrt{(a \sec(d + ex) + b)^2} \left((a^2 + b^2) \tanh^{-1}(\sin(d + ex)) + ab(\tan(d + ex) + ex) \right)}{e(a + b \cos(d + ex))}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[d + e*x])*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2], x]

[Out] (Cos[d + e*x]*Sqrt[(b + a*Sec[d + e*x])^2]*((a^2 + b^2)*ArcTanh[Sin[d + e*x]] + a*b*(e*x + Tan[d + e*x]))) / (e*(a + b*Cos[d + e*x]))

fricas [A] time = 0.96, size = 85, normalized size = 0.49

$$\frac{2 abex \cos(ex + d) + (a^2 + b^2) \cos(ex + d) \log(\sin(ex + d) + 1) - (a^2 + b^2) \cos(ex + d) \log(-\sin(ex + d) + 1)}{2 e \cos(ex + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2), x, algorithm="fricas")

[Out] 1/2*(2*a*b*e*x*cos(e*x + d) + (a^2 + b^2)*cos(e*x + d)*log(sin(e*x + d) + 1) - (a^2 + b^2)*cos(e*x + d)*log(-sin(e*x + d) + 1) + 2*a*b*sin(e*x + d)) / (e*cos(e*x + d))

giac [A] time = 0.31, size = 224, normalized size = 1.29

$$\left((xe + d) ab \operatorname{sgn}(b \cos(xe + d)^2 + a \cos(xe + d)) - \frac{2 ab \operatorname{sgn}(b \cos(xe + d)^2 + a \cos(xe + d)) \tan\left(\frac{1}{2} xe + \frac{1}{2} d\right)}{\tan\left(\frac{1}{2} xe + \frac{1}{2} d\right)^2 - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2), x, algorithm="giac")

[Out] ((x*e + d)*a*b*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) - 2*a*b*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d))*tan(1/2*x*e + 1/2*d)/(tan(1/2*x*e + 1/2*d)^2 - 1) + (a^2*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) + b^2*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)))*log(abs(tan(1/2*x*e + 1/2*d) + 1)) - (a^2*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)) + b^2*sgn(b*cos(x*e + d)^2 + a*cos(x*e + d)))*log(abs(tan(1/2*x*e + 1/2*d) - 1)))*e^(-1)

maple [A] time = 0.58, size = 211, normalized size = 1.22

$$\frac{\left(\cos(ex+d)\ln\left(-\frac{\cos(ex+d)-1+\sin(ex+d)}{\sin(ex+d)}\right)a^2 + \cos(ex+d)\ln\left(-\frac{\cos(ex+d)-1+\sin(ex+d)}{\sin(ex+d)}\right)b^2 - \cos(ex+d)\ln\left(\frac{1-\cos(ex+d)}{\sin(ex+d)}\right)\right)}{e(b\cos(ex+d)+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x)

[Out] -1/e*(cos(e*x+d)*ln(-(cos(e*x+d)-1+sin(e*x+d))/sin(e*x+d))*a^2+cos(e*x+d)*ln(-(cos(e*x+d)-1+sin(e*x+d))/sin(e*x+d))*b^2-cos(e*x+d)*ln((1-cos(e*x+d)+sin(e*x+d))/sin(e*x+d))*a^2-cos(e*x+d)*ln((1-cos(e*x+d)+sin(e*x+d))/sin(e*x+d))*b^2-cos(e*x+d)*(e*x+d)*a*b-a*b*sin(e*x+d))*((b*cos(e*x+d)+a)^2/cos(e*x+d)^2)^(1/2)/(b*cos(e*x+d)+a)

maxima [A] time = 0.44, size = 164, normalized size = 0.95

$$\frac{\left(2b \arctan\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right) + a \log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1} + 1\right) - a \log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1} - 1\right)\right)a + \left(b \log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1} + 1\right) - b \log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1} - 1\right)\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x, algorithm="maxima")

[Out] ((2*b*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) + a*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1) - a*log(sin(e*x + d)/(cos(e*x + d) + 1) - 1))*a + (b*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1) - b*log(sin(e*x + d)/(cos(e*x + d) + 1) - 1) - 2*a*sin(e*x + d)/((sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 1)*(cos(e*x + d) + 1))) * b) / e

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{b}{\cos(d + ex)}\right) \sqrt{b^2 + \frac{a^2}{\cos(d + ex)^2} + \frac{2ab}{\cos(d + ex)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^(1/2),x)

[Out] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec(d + ex)) \sqrt{(a \sec(d + ex) + b)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(1/2),x)

[Out] Integral((a + b*sec(d + e*x))*sqrt((a*sec(d + e*x) + b)**2), x)

$$3.524 \quad \int \frac{a+b \sec(d+ex)}{\sqrt{b^2+2ab \sec(d+ex)+a^2 \sec^2(d+ex)}} dx$$

Optimal. Leaf size=142

$$\frac{x(a^2 \sec(d+ex) + ab)}{b\sqrt{a^2 \sec^2(d+ex) + 2ab \sec(d+ex) + b^2}} - \frac{2\sqrt{a-b}\sqrt{a+b} \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(a \sec(d+ex) + b)}{be\sqrt{a^2 \sec^2(d+ex) + 2ab \sec(d+ex) + b^2}}$$

[Out] $x*(a*b+a^2*\sec(e*x+d))/b/(b^2+2*a*b*\sec(e*x+d)+a^2*\sec(e*x+d)^2)^{(1/2)}-2*ar$
 $ctan((a-b)^{(1/2)}*\tan(1/2*e*x+1/2*d)/(a+b)^{(1/2)})*(b+a*\sec(e*x+d))*(a-b)^{(1/2)}$
 $2)*(a+b)^{(1/2)}/b/e/(b^2+2*a*b*\sec(e*x+d)+a^2*\sec(e*x+d)^2)^{(1/2)}$

Rubi [A] time = 0.21, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {4174, 3919, 3831, 2659, 205}

$$\frac{x(a^2 \sec(d+ex) + ab)}{b\sqrt{a^2 \sec^2(d+ex) + 2ab \sec(d+ex) + b^2}} - \frac{2\sqrt{a-b}\sqrt{a+b} \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(a \sec(d+ex) + b)}{be\sqrt{a^2 \sec^2(d+ex) + 2ab \sec(d+ex) + b^2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Sec}[d + e*x])/Sqrt[b^2 + 2*a*b*\text{Sec}[d + e*x] + a^2*\text{Sec}[d + e*x]^2], x]$

[Out] $(-2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(d + e*x)/2])/Sqrt[a + b]]*(b + a*\text{Sec}[d + e*x]))/(b*e*Sqrt[b^2 + 2*a*b*\text{Sec}[d + e*x] + a^2*\text{Sec}[d + e*x]^2) + (x*(a*b + a^2*\text{Sec}[d + e*x]))/(b*Sqrt[b^2 + 2*a*b*\text{Sec}[d + e*x] + a^2*\text{Sec}[d + e*x]^2))$

Rule 205

$\text{Int}(((a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

Rule 2659

$\text{Int}(((a_) + (b_.)*\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_)])^{-1}, x_Symbol] \rightarrow \text{With}[\{e = \text{FreeFactors}[\text{Tan}[(c + d*x)/2], x]\}, \text{Dist}[(2*e)/d, \text{Subst}[\text{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \text{Tan}[(c + d*x)/2]/e], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 3831

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] \rightarrow \text{Dist}[1/b, \text{Int}[1/(1 + (a*\text{Sin}[e + f*x])/b), x], x] /; \text{FreeQ}[\{a, b, e, f\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 3919

$\text{Int}[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] \rightarrow \text{Simp}[(c*x)/a, x] - \text{Dist}[(b*c - a*d)/a, \text{Int}[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 4174

$\text{Int}(((A_) + (B_.)*\sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*\sec[(d_.) + (e_.)*(x_)]) + (c_.)*\sec[(d_.) + (e_.)*(x_)]^2)^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(a + b*\text{Sec}$

$[d + e*x] + c*\text{Sec}[d + e*x]^2)^n / (b + 2*c*\text{Sec}[d + e*x])^{2*n}$, Int[(A + B*Sec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[n]

Rubi steps

$$\int \frac{a + b \sec(d + ex)}{\sqrt{b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)}} dx = \frac{(2ab + 2a^2 \sec(d + ex)) \int \frac{a+b \sec(d+ex)}{2ab+2a^2 \sec(d+ex)} dx}{\sqrt{b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)}}$$

$$= \frac{x (ab + a^2 \sec(d + ex))}{b\sqrt{b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)}} - \frac{((2a^3 - 2ab^2) (2ab))}{2ab\sqrt{b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)}}$$

$$= \frac{x (ab + a^2 \sec(d + ex))}{b\sqrt{b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)}} - \frac{((2a^3 - 2ab^2) (2ab))}{4a^3b\sqrt{b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)}}$$

$$= \frac{x (ab + a^2 \sec(d + ex))}{b\sqrt{b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)}} - \frac{((2a^3 - 2ab^2) (2ab))}{4a^3b\sqrt{b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)}}$$

$$= -\frac{2\sqrt{a-b}\sqrt{a+b} \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right) (b + a \sec(d + ex))}{be\sqrt{b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex)}} +$$

Mathematica [A] time = 0.37, size = 92, normalized size = 0.65

$$\frac{\sec(d + ex)(a + b \cos(d + ex)) \left(2\sqrt{b^2 - a^2} \tanh^{-1} \left(\frac{(b-a) \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{b^2-a^2}} \right) + a(d + ex) \right)}{be\sqrt{(a \sec(d + ex) + b)^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[d + e*x])/Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2], x]

[Out] ((a*(d + e*x) + 2*Sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]])*(a + b*Cos[d + e*x])*Sec[d + e*x]/(b*e*Sqrt[(b + a*Sec[d + e*x])^2])

fricas [A] time = 1.06, size = 184, normalized size = 1.30

$$\left[\frac{2 a e x + \sqrt{-a^2 + b^2} \log \left(\frac{2 a b \cos(e x+d)+(2 a^2-b^2) \cos(e x+d)^2+2 \sqrt{-a^2+b^2} (a \cos(e x+d)+b) \sin(e x+d)-a^2+2 b^2}{b^2 \cos(e x+d)^2+2 a b \cos(e x+d)+a^2} \right)}{2 b e}, \frac{a e x - \sqrt{a^2 - b^2} \arctan \left(\frac{b \cos(e x+d) + a \sin(e x+d)}{\sqrt{a^2 - b^2} \sin(e x+d)} \right)}{b e} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2), x, algorithm="fricas")

[Out] [1/2*(2*a*e*x + sqrt(-a^2 + b^2)*log((2*a*b*cos(e*x + d) + (2*a^2 - b^2)*cos(e*x + d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(e*x + d) + b)*sin(e*x + d) - a^2 + 2*b^2)/(b^2*cos(e*x + d)^2 + 2*a*b*cos(e*x + d) + a^2)))/(b*e), (a*e*x - sqrt(a^2 - b^2)*arctan(-(a*cos(e*x + d) + b)/(sqrt(a^2 - b^2)*sin(e*x + d)))/(b*e)]

giac [B] time = 4.83, size = 1504, normalized size = 10.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2), x, algorithm="giac")

[Out] ((sqrt(a^2 - b^2)*abs(a - b)*abs(b)*abs(sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b)) + sqrt(a^2 - b^2)*(2*a + b)*abs(a - b)*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b))*arctan(tan(1/2*x*e + 1/2*d)/sqrt((a*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) + sqrt(-(a*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) + b*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b)))*(a*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) - b*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b)) + a^2))/(a*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) - b*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b)))/((a^2 - a*b)*abs(b)*abs(sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b))*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) + (a - b)*b^2) + (a*abs(b)*abs(sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b)) - b*abs(b)*abs(sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b)) - 2*a^2*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) + a*b*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) + b^2*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b))*arctan(tan(1/2*x*e + 1/2*d)/sqrt((a*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) - sqrt(-(a*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) + b*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b)))*(a*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) - b*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b)) + a^2))/(a*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) - b*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b)))/((a*abs(b)*abs(sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b))*sgn(a*tan(1/2*x*e + 1/2*d)^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) - b^2))*e^(-1)

maple [A] time = 0.66, size = 157, normalized size = 1.11

$$\frac{(b \cos(ex + d) + a) \left(2 \arctan \left(\frac{(\cos(ex+d)-1)(a-b)}{\sin(ex+d)\sqrt{(a+b)(a-b)}} \right) a^2 - 2 \arctan \left(\frac{(\cos(ex+d)-1)(a-b)}{\sin(ex+d)\sqrt{(a+b)(a-b)}} \right) b^2 + a (ex + d) \sqrt{(a+b)(a-b)} \right)}{e \cos(ex + d) \sqrt{\frac{(b \cos(ex+d)+a)^2}{\cos(ex+d)^2}} b \sqrt{(a+b)(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2), x)

[Out] 1/e*(b*cos(e*x+d)+a)*(2*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*a^2-2*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*

$b^2+a*(e*x+d)*((a+b)*(a-b))^{(1/2)}/\cos(e*x+d)/((b*\cos(e*x+d)+a)^2/\cos(e*x+d)^2)^{(1/2)}/b/((a+b)*(a-b))^{(1/2)}$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more details)Is 4*b^2-4*a^2 positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + \frac{b}{\cos(d+ex)}}{\sqrt{b^2 + \frac{a^2}{\cos(d+ex)^2} + \frac{2ab}{\cos(d+ex)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^(1/2),x)

[Out] int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(d + ex)}{\sqrt{(a \sec(d + ex) + b)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(1/2),x)

[Out] Integral((a + b*sec(d + e*x))/sqrt((a*sec(d + e*x) + b)**2), x)

$$3.525 \quad \int \frac{a+b \sec(d+ex)}{(b^2+2ab \sec(d+ex)+a^2 \sec^2(d+ex))^{3/2}} dx$$

Optimal. Leaf size=330

$$\frac{\tan(d+ex)(a^2 \sec(d+ex)+ab)}{2be(a^2 \sec^2(d+ex)+2ab \sec(d+ex)+b^2)^{3/2}} + \frac{x(a^2 \sec(d+ex)+ab)^3}{a^2 b^3 (a^2 \sec^2(d+ex)+2ab \sec(d+ex)+b^2)^{3/2}} - \frac{(2a^4-3a^2 b^2)}{b^3 e(a-b)^3}$$

[Out] $-(2*a^4-3*a^2*b^2+2*b^4)*\arctan((a-b)^{(1/2)}*\tan(1/2*e*x+1/2*d)/(a+b)^{(1/2)})$
 $*(b+a*\sec(e*x+d))^{3/2}/(a-b)^{(3/2)}/b^{3/2}/(a+b)^{(3/2)}/e/(b^2+2*a*b*\sec(e*x+d)+a^2$
 $*\sec(e*x+d)^2)^{(3/2)}+x*(a*b+a^2*\sec(e*x+d))^{3/2}/a^2/b^{3/2}/(b^2+2*a*b*\sec(e*x+d)$
 $+a^2*\sec(e*x+d)^2)^{(3/2)}-1/2*(a*b+a^2*\sec(e*x+d))*\tan(e*x+d)/b/e/(b^2+2*a*b$
 $*\sec(e*x+d)+a^2*\sec(e*x+d)^2)^{(3/2)}-1/2*(2*a^2-3*b^2)*(a*b+a^2*\sec(e*x+d))^{3/2}$
 $*\tan(e*x+d)/b^2/(a^2-b^2)/e/(a^2*b+a^3*\sec(e*x+d))/(b^2+2*a*b*\sec(e*x+d)+$
 $a^2*\sec(e*x+d)^2)^{(3/2)}$

Rubi [A] time = 0.57, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {4174, 3923, 4060, 3919, 3831, 2659, 205}

$$\frac{x(a^2 \sec(d+ex)+ab)^3}{a^2 b^3 (a^2 \sec^2(d+ex)+2ab \sec(d+ex)+b^2)^{3/2}} - \frac{(-3a^2 b^2 + 2a^4 + 2b^4) \tan^{-1}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^3 e(a-b)^{3/2}(a+b)^{3/2} (a^2 \sec^2(d+ex)+2ab \sec(d+ex)+b^2)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2), x]

[Out] $-(((2*a^4-3*a^2*b^2+2*b^4)*\text{ArcTan}[\text{Sqrt}[a-b]*\text{Tan}[(d+e*x)/2]]/\text{Sqrt}[a+b])$
 $*(b+a*\text{Sec}[d+e*x])^3)/((a-b)^{(3/2)}*b^3*(a+b)^{(3/2)}*e*(b^2+2*a*b*\text{Sec}[d+e*x]$
 $+a^2*\text{Sec}[d+e*x]^2)^{(3/2)})+(x*(a*b+a^2*\text{Sec}[d+e*x])^3)/(a^2*b^3*(b^2+2*a*b*\text{Sec}[d+e*x]$
 $+a^2*\text{Sec}[d+e*x]^2)^{(3/2)})-((a*b+a^2*\text{Sec}[d+e*x])*\text{Tan}[d+e*x])/(2*b*e*(b^2+2*a*b*\text{Sec}[d+e*x]$
 $+a^2*\text{Sec}[d+e*x]^2)^{(3/2)})-((2*a^2-3*b^2)*(a*b+a^2*\text{Sec}[d+e*x])^3*\text{Tan}[d+e*x])$
 $/(2*b^2*(a^2-b^2)*e*(a^2*b+a^3*\text{Sec}[d+e*x])*(b^2+2*a*b*\text{Sec}[d+e*x]$
 $+a^2*\text{Sec}[d+e*x]^2)^{(3/2)})$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2659

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0]
```

Rule 3923

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d
_.) + (c_.)), x_Symbol] :> Simp[(b*(b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f
*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] :> Simp[((A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
+ 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
+ (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4174

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_.)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_.)] + (c_.)*sec[(d_.) + (e_.)*(x_.)]^2)^(n_.), x_Symbol] :> Dist[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n), Int[(A + B*Se
c[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{a + b \sec(d + ex)}{(b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}} dx = \frac{(2ab + 2a^2 \sec(d + ex))^3 \int \frac{a+b \sec(d+ex)}{(2ab+2a^2 \sec(d+ex))^3} dx}{(b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}}$$

$$= -\frac{(ab + a^2 \sec(d + ex)) \tan(d + ex)}{2be (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}} + \frac{(2ab + a^2 \sec^2(d + ex)) \tan(d + ex)}{2ab^2 (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}}$$

$$= -\frac{(ab + a^2 \sec(d + ex)) \tan(d + ex)}{2be (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}} - \frac{(2ab + a^2 \sec^2(d + ex)) \tan(d + ex)}{2ab^2 (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}}$$

$$= \frac{x (ab + a^2 \sec(d + ex))^3}{a^2 b^3 (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}} - \frac{(2ab + a^2 \sec^2(d + ex)) \tan(d + ex)}{2be (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}}$$

$$= \frac{x (ab + a^2 \sec(d + ex))^3}{a^2 b^3 (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}} - \frac{(2ab + a^2 \sec^2(d + ex)) \tan(d + ex)}{2be (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}}$$

$$= \frac{x (ab + a^2 \sec(d + ex))^3}{a^2 b^3 (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}} - \frac{(2ab + a^2 \sec^2(d + ex)) \tan(d + ex)}{2be (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}}$$

$$= \frac{x (ab + a^2 \sec(d + ex))^3}{a^2 b^3 (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}} - \frac{(2ab + a^2 \sec^2(d + ex)) \tan(d + ex)}{2be (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}}$$

$$= -\frac{(2a^4 - 3a^2 b^2 + 2b^4) \tan^{-1} \left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}} \right) (b + a \sec(d + ex))}{(a - b)^{3/2} b^3 (a + b)^{3/2} e (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}}$$

Mathematica [A] time = 1.00, size = 216, normalized size = 0.65

$$\frac{\sec^2(d + ex)(a + b \cos(d + ex))(a + b \sec(d + ex)) \left(\frac{ab(3a^2 - 4b^2) \sin(d+ex)(a+b \cos(d+ex))}{(b-a)(a+b)} + a^2 b \sin(d + ex) + \frac{2(2a^4 - 3a^2 b^2 + 2b^4) \tan^{-1} \left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}} \right) (b + a \sec(d + ex))}{(a - b)^{3/2} b^3 (a + b)^{3/2} e} \right)}{2b^3 e (a \cos(d + ex) + b) ((a \sec(d + ex) + b) (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2})}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2), x]

[Out] ((a + b*cos[d + e*x])*Sec[d + e*x]^2*(a + b*Sec[d + e*x])*(2*a*(d + e*x)*(a + b*cos[d + e*x])^2 + (2*(2*a^4 - 3*a^2*b^2 + 2*b^4)*ArcTanh[((-a + b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2])*(a + b*cos[d + e*x])^2)/(-a^2 + b^2)^(3/2) + a^2*b*Sin[d + e*x] + (a*b*(3*a^2 - 4*b^2)*(a + b*cos[d + e*x])*Sin[d + e*x])/((-a + b)*(a + b)))/(2*b^3*e*(b + a*cos[d + e*x])*((b + a*Sec[d + e*x])^2)^(3/2))

fricas [A] time = 1.09, size = 798, normalized size = 2.42

$$\frac{4(a^5 b^2 - 2a^3 b^4 + ab^6)ex \cos(ex + d)^2 + 8(a^6 b - 2a^4 b^3 + a^2 b^5)ex \cos(ex + d) + 4(a^7 - 2a^5 b^2 + a^3 b^4)ex + (2a^4 - 3a^2 b^2 + 2b^4) \tan^{-1} \left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}} \right) (b + a \sec(d + ex))}{(a - b)^{3/2} b^3 (a + b)^{3/2} e (b^2 + 2ab \sec(d + ex) + a^2 \sec^2(d + ex))^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
algorithm="fricas")

[Out] [1/4*(4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*e*x*cos(e*x + d)^2 + 8*(a^6*b - 2*a^4*b^3 + a^2*b^5)*e*x*cos(e*x + d) + 4*(a^7 - 2*a^5*b^2 + a^3*b^4)*e*x + (2*a^6 - 3*a^4*b^2 + 2*a^2*b^4 + (2*a^4*b^2 - 3*a^2*b^4 + 2*b^6)*cos(e*x + d)^2 + 2*(2*a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(e*x + d))*sqrt(-a^2 + b^2)*log((2*a*b*cos(e*x + d) + (2*a^2 - b^2)*cos(e*x + d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(e*x + d) + b)*sin(e*x + d) - a^2 + 2*b^2)/(b^2*cos(e*x + d)^2 + 2*a*b*cos(e*x + d) + a^2)) - 2*(2*a^6*b - 5*a^4*b^3 + 3*a^2*b^5 + (3*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*cos(e*x + d))*sin(e*x + d))/((a^4*b^5 - 2*a^2*b^7 + b^9)*e*cos(e*x + d)^2 + 2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*e*cos(e*x + d) + (a^6*b^3 - 2*a^4*b^5 + a^2*b^7)*e), 1/2*(2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*e*x*cos(e*x + d)^2 + 4*(a^6*b - 2*a^4*b^3 + a^2*b^5)*e*x*cos(e*x + d) + 2*(a^7 - 2*a^5*b^2 + a^3*b^4)*e*x - (2*a^6 - 3*a^4*b^2 + 2*a^2*b^4 + (2*a^4*b^2 - 3*a^2*b^4 + 2*b^6)*cos(e*x + d)^2 + 2*(2*a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(e*x + d))*sqrt(a^2 - b^2)*arctan(-(a*cos(e*x + d) + b)/(sqrt(a^2 - b^2)*sin(e*x + d))) - (2*a^6*b - 5*a^4*b^3 + 3*a^2*b^5 + (3*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*cos(e*x + d))*sin(e*x + d))/((a^4*b^5 - 2*a^2*b^7 + b^9)*e*cos(e*x + d)^2 + 2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*e*cos(e*x + d) + (a^6*b^3 - 2*a^4*b^5 + a^2*b^7)*e)]

giac [A] time = 5.12, size = 569, normalized size = 1.72

$$\left(\frac{(2a^4 - 3a^2b^2 + 2b^4) \arctan\left(\frac{a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)}{\sqrt{a^2 - b^2}}\right)}{\left(a^2b^3 \operatorname{sgn}\left(a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)\right)^4 - b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^4 + 2b \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 - a - b\right) - b^5 \operatorname{sgn}\left(a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
algorithm="giac")

[Out] ((2*a^4 - 3*a^2*b^2 + 2*b^4)*arctan((a*tan(1/2*x*e + 1/2*d) - b*tan(1/2*x*e + 1/2*d))/sqrt(a^2 - b^2))/((a^2*b^3*sgn(a*tan(1/2*x*e + 1/2*d))^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) - b^5*sgn(a*tan(1/2*x*e + 1/2*d))^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b))*sqrt(a^2 - b^2) + (2*a^4*tan(1/2*x*e + 1/2*d)^3 - 3*a^3*b*tan(1/2*x*e + 1/2*d)^3 - 3*a^2*b^2*tan(1/2*x*e + 1/2*d)^3 + 4*a*b^3*tan(1/2*x*e + 1/2*d)^3 + 2*a^4*tan(1/2*x*e + 1/2*d) + 3*a^3*b*tan(1/2*x*e + 1/2*d) - 3*a^2*b^2*tan(1/2*x*e + 1/2*d) - 4*a*b^3*tan(1/2*x*e + 1/2*d))/((a^2*b^2*sgn(a*tan(1/2*x*e + 1/2*d))^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b) - b^4*sgn(a*tan(1/2*x*e + 1/2*d))^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b))*(a*tan(1/2*x*e + 1/2*d)^2 - b*tan(1/2*x*e + 1/2*d)^2 + a + b)^2 - (x*e - 2*pi*floor(1/2*(x*e + d)/pi + 1/2) + d)*a/(b^3*sgn(a*tan(1/2*x*e + 1/2*d))^4 - b*tan(1/2*x*e + 1/2*d)^4 + 2*b*tan(1/2*x*e + 1/2*d)^2 - a - b))*e^(-1)

maple [B] time = 0.61, size = 756, normalized size = 2.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x)

[Out] 1/2/e*(b*cos(e*x+d)+a)*(4*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d))/((a+b)*(a-b))^(1/2))*cos(e*x+d)^2*a^4*b^2-6*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d))/((


```

a+b)*(a-b))^(1/2))*cos(e*x+d)^2*a^2*b^4+4*arctan((cos(e*x+d)-1)*(a-b)/sin(e
*x+d)/((a+b)*(a-b))^(1/2))*cos(e*x+d)^2*b^6+2*cos(e*x+d)^2*((a+b)*(a-b))^(1
/2)*(e*x+d)*a^3*b^2-2*cos(e*x+d)^2*((a+b)*(a-b))^(1/2)*(e*x+d)*a*b^4+8*arct
an((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*cos(e*x+d)*a^5*b-12
*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*cos(e*x+d)*a^3
*b^3+8*arctan((cos(e*x+d)-1)*(a-b)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*cos(e*x+
d)*a*b^5-3*cos(e*x+d)*((a+b)*(a-b))^(1/2)*sin(e*x+d)*a^3*b^2+4*cos(e*x+d)*
(a+b)*(a-b))^(1/2)*sin(e*x+d)*a*b^4+4*cos(e*x+d)*((a+b)*(a-b))^(1/2)*(e*x+d
)*a^4*b-4*cos(e*x+d)*((a+b)*(a-b))^(1/2)*(e*x+d)*a^2*b^3+4*arctan((cos(e*x+
d)-1)*(a-b)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*a^6-6*arctan((cos(e*x+d)-1)*(a-
b)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*a^4*b^2+4*arctan((cos(e*x+d)-1)*(a-b)/si
n(e*x+d)/((a+b)*(a-b))^(1/2))*a^2*b^4-2*((a+b)*(a-b))^(1/2)*a^4*b*sin(e*x+d
)+3*((a+b)*(a-b))^(1/2)*a^2*b^3*sin(e*x+d)+2*((a+b)*(a-b))^(1/2)*(e*x+d)*a^
5-2*((a+b)*(a-b))^(1/2)*(e*x+d)*a^3*b^2)/cos(e*x+d)^3/((b*cos(e*x+d)+a)^2/co
s(e*x+d)^2)^(3/2)/((a+b)*(a-b))^(1/2)/(a^2-b^2)/b^3

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
algorithm="maxima")

```

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more details)Is 4*b^2-4*a^2 positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + \frac{b}{\cos(d+ex)}}{\left(b^2 + \frac{a^2}{\cos(d+ex)^2} + \frac{2ab}{\cos(d+ex)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^(
3/2),x)

```

```

[Out] int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^(
3/2), x)

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec(d + ex)}{\left((a \sec(d + ex) + b)^2\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(3/2
),x)

```

```

[Out] Integral((a + b*sec(d + e*x))/((a*sec(d + e*x) + b)**2)**(3/2), x)

```

$$3.526 \quad \int \frac{\cos(x) - i \sin(x)}{\cos(x) + i \sin(x)} dx$$

Optimal. Leaf size=17

$$\frac{1}{2}i(\cos(x) - i \sin(x))^2$$

[Out] 1/2*I*(cos(x)-I*sin(x))^2

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {4385}

$$\frac{1}{2}i(\cos(x) - i \sin(x))^2$$

Antiderivative was successfully verified.

[In] Int[(Cos[x] - I*Sin[x])/(Cos[x] + I*Sin[x]),x]

[Out] (I/2)*(Cos[x] - I*Sin[x])^2

Rule 4385

Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /; !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] && !InertTrigFreeQ[u]

Rubi steps

$$\int \frac{\cos(x) - i \sin(x)}{\cos(x) + i \sin(x)} dx = \frac{1}{2}i(\cos(x) - i \sin(x))^2$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.12

$$\frac{1}{2} \sin(2x) + \frac{1}{2}i \cos(2x)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x] - I*Sin[x])/(Cos[x] + I*Sin[x]),x]

[Out] (I/2)*Cos[2*x] + Sin[2*x]/2

fricas [A] time = 0.70, size = 6, normalized size = 0.35

$$\frac{1}{2}i e^{(-2ix)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x, algorithm="fricas")

[Out] 1/2*I*e^(-2*I*x)

giac [A] time = 0.14, size = 14, normalized size = 0.82

$$-\frac{2 \tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right) - i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x, algorithm="giac")

[Out] -2*tan(1/2*x)/(tan(1/2*x) - I)^2

maple [A] time = 0.20, size = 8, normalized size = 0.47

$$\frac{1}{\tan(x) - i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x)

[Out] 1/(tan(x)-I)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.

mupad [B] time = 2.76, size = 16, normalized size = 0.94

$$-\frac{\cos(x)}{-\sin(x) + \cos(x)1i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x) - sin(x)*1i)/(cos(x) + sin(x)*1i),x)

[Out] -cos(x)/(cos(x)*1i - sin(x))

sympy [A] time = 0.08, size = 8, normalized size = 0.47

$$\frac{ie^{-2ix}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x)

[Out] I*exp(-2*I*x)/2

$$3.527 \quad \int \frac{\cos(x)+i \sin(x)}{\cos(x)-i \sin(x)} dx$$

Optimal. Leaf size=17

$$-\frac{i}{2(\cos(x)-i \sin(x))^2}$$

[Out] $-1/2*I/(\cos(x)-I*\sin(x))^2$

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {4385}

$$-\frac{i}{2(\cos(x)-i \sin(x))^2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x] + I*Sin[x])/(Cos[x] - I*Sin[x]),x]

[Out] $(-I/2)/(\text{Cos}[x] - I*\text{Sin}[x])^2$

Rule 4385

Int[(u_)*(y_)^(m_.), x_Symbol] := With[{q = DerivativeDivides[ActivateTrig[y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /; !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] && !InertTrigFreeQ[u]

Rubi steps

$$\int \frac{\cos(x) + i \sin(x)}{\cos(x) - i \sin(x)} dx = -\frac{i}{2(\cos(x) - i \sin(x))^2}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.12

$$\frac{1}{2} \sin(2x) - \frac{1}{2} i \cos(2x)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x] + I*Sin[x])/(Cos[x] - I*Sin[x]),x]

[Out] $(-1/2*I)*\text{Cos}[2*x] + \text{Sin}[2*x]/2$

fricas [A] time = 0.65, size = 6, normalized size = 0.35

$$-\frac{1}{2} i e^{(2i x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x, algorithm="fricas")

[Out] $-1/2*I*e^{(2*I*x)}$

giac [A] time = 0.14, size = 14, normalized size = 0.82

$$-\frac{2 \tan\left(\frac{1}{2} x\right)}{\left(\tan\left(\frac{1}{2} x\right) + i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x, algorithm="giac")

[Out] $-2*\tan(1/2*x)/(\tan(1/2*x) + I)^2$

maple [A] time = 0.17, size = 8, normalized size = 0.47

$$\frac{1}{\tan(x) + i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x)

[Out] $1/(\tan(x)+I)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
expt: undefined: 0 to a negative exponent.

mupad [B] time = 2.74, size = 13, normalized size = 0.76

$$\frac{\sin(x)}{\cos(x) - \sin(x) 1i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x) + sin(x)*1i)/(cos(x) - sin(x)*1i),x)

[Out] $\sin(x)/(\cos(x) - \sin(x)*1i)$

sympy [A] time = 0.08, size = 10, normalized size = 0.59

$$-\frac{ie^{2ix}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x)

[Out] $-I*\exp(2*I*x)/2$

$$3.528 \quad \int \frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)} dx$$

Optimal. Leaf size=6

$$\log(\sin(x) + \cos(x))$$

[Out] ln(cos(x)+sin(x))

Rubi [A] time = 0.02, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3133}

$$\log(\sin(x) + \cos(x))$$

Antiderivative was successfully verified.

[In] Int[(Cos[x] - Sin[x])/(Cos[x] + Sin[x]),x]

[Out] Log[Cos[x] + Sin[x]]

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)])
/((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)} dx = \log(\cos(x) + \sin(x))$$

Mathematica [A] time = 0.03, size = 6, normalized size = 1.00

$$\log(\sin(x) + \cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x] - Sin[x])/(Cos[x] + Sin[x]),x]

[Out] Log[Cos[x] + Sin[x]]

fricas [A] time = 0.82, size = 11, normalized size = 1.83

$$\frac{1}{2} \log(2 \cos(x) \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x, algorithm="fricas")

[Out] 1/2*log(2*cos(x)*sin(x) + 1)

giac [B] time = 0.18, size = 16, normalized size = 2.67

$$-\frac{1}{2} \log(\tan(x)^2 + 1) + \log(|\tan(x) + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x, algorithm="giac")

[Out] $-1/2*\log(\tan(x)^2 + 1) + \log(\text{abs}(\tan(x) + 1))$

maple [A] time = 0.06, size = 7, normalized size = 1.17

$$\ln(\cos(x) + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x)-sin(x))/(cos(x)+sin(x)),x)

[Out] $\ln(\cos(x)+\sin(x))$

maxima [A] time = 0.31, size = 6, normalized size = 1.00

$$\log(\cos(x) + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x, algorithm="maxima")

[Out] $\log(\cos(x) + \sin(x))$

mupad [B] time = 2.92, size = 32, normalized size = 5.33

$$2 \operatorname{atanh}\left(\frac{128 \tan\left(\frac{x}{2}\right) + 128}{16 \tan\left(\frac{x}{2}\right)^2 + 32 \tan\left(\frac{x}{2}\right) + 48} - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x) - sin(x))/(cos(x) + sin(x)),x)

[Out] $2*\operatorname{atanh}((128*\tan(x/2) + 128)/(32*\tan(x/2) + 16*\tan(x/2)^2 + 48) - 3)$

sympy [A] time = 0.12, size = 7, normalized size = 1.17

$$\log(\sin(x) + \cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x)

[Out] $\log(\sin(x) + \cos(x))$

$$3.529 \quad \int \frac{B \cos(x) + C \sin(x)}{b \cos(x) + c \sin(x)} dx$$

Optimal. Leaf size=47

$$\frac{x(bB + cC)}{b^2 + c^2} + \frac{(Bc - bC) \log(b \cos(x) + c \sin(x))}{b^2 + c^2}$$

[Out] (B*b+C*c)*x/(b^2+c^2)+(B*c-C*b)*ln(b*cos(x)+c*sin(x))/(b^2+c^2)

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {3133}

$$\frac{x(bB + cC)}{b^2 + c^2} + \frac{(Bc - bC) \log(b \cos(x) + c \sin(x))}{b^2 + c^2}$$

Antiderivative was successfully verified.

[In] Int[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]

[Out] ((b*B + c*C)*x)/(b^2 + c^2) + ((B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)

Rule 3133

Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]) / ((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]), x_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]

Rubi steps

$$\int \frac{B \cos(x) + C \sin(x)}{b \cos(x) + c \sin(x)} dx = \frac{(bB + cC)x}{b^2 + c^2} + \frac{(Bc - bC) \log(b \cos(x) + c \sin(x))}{b^2 + c^2}$$

Mathematica [A] time = 0.14, size = 39, normalized size = 0.83

$$\frac{x(bB + cC) + (Bc - bC) \log(b \cos(x) + c \sin(x))}{b^2 + c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]

[Out] ((b*B + c*C)*x + (B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)

fricas [A] time = 0.88, size = 59, normalized size = 1.26

$$\frac{2(Bb + Cc)x - (Cb - Bc) \log(2bc \cos(x) \sin(x) + (b^2 - c^2) \cos(x)^2 + c^2)}{2(b^2 + c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")

[Out] 1/2*(2*(B*b + C*c)*x - (C*b - B*c)*log(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2))/(b^2 + c^2)

giac [A] time = 0.17, size = 77, normalized size = 1.64

$$\frac{(Bb + Cc)x}{b^2 + c^2} + \frac{(Cb - Bc) \log(\tan(x)^2 + 1)}{2(b^2 + c^2)} - \frac{(Cbc - Bc^2) \log(|c \tan(x) + b|)}{b^2c + c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
[Out] (B*b + C*c)*x/(b^2 + c^2) + 1/2*(C*b - B*c)*log(tan(x)^2 + 1)/(b^2 + c^2) -
(C*b*c - B*c^2)*log(abs(c*tan(x) + b))/(b^2*c + c^3)
```

maple [B] time = 0.14, size = 111, normalized size = 2.36

$$\frac{\ln(c \tan(x) + b) Bc}{b^2 + c^2} - \frac{\ln(c \tan(x) + b) bC}{b^2 + c^2} - \frac{\ln(1 + \tan^2(x)) Bc}{2(b^2 + c^2)} + \frac{\ln(1 + \tan^2(x)) bC}{2b^2 + 2c^2} + \frac{B \arctan(\tan(x)) b}{b^2 + c^2} + \frac{C \arctan(\tan(x)) c}{b^2 + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x)
[Out] 1/(b^2+c^2)*ln(c*tan(x)+b)*B*c-1/(b^2+c^2)*ln(c*tan(x)+b)*b*C-1/2/(b^2+c^2)
*ln(1+tan(x)^2)*B*c+1/2/(b^2+c^2)*ln(1+tan(x)^2)*b*C+1/(b^2+c^2)*B*arctan(tan(x))
*b+1/(b^2+c^2)*C*arctan(tan(x))*c
```

maxima [B] time = 0.42, size = 181, normalized size = 3.85

$$B \left(\frac{2b \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2 + c^2} + \frac{c \log\left(-b - \frac{2c \sin(x)}{\cos(x)+1} + \frac{b \sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2 + c^2} - \frac{c \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)}{b^2 + c^2} \right) + C \left(\frac{2c \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2 + c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
[Out] B*(2*b*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) + c*log(-b - 2*c*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) - c*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(b^2 + c^2)) + C*(2*c*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) - b*log(-b - 2*c*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) + b*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(b^2 + c^2))
```

mupad [B] time = 12.82, size = 1976, normalized size = 42.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x)),x)
[Out] (log(b + 2*c*tan(x/2) - b*tan(x/2)^2)*(B*c - C*b))/(b^2 + c^2) - (log(1/(cos(x) + 1))*(2*B*c - 2*C*b))/(2*(b^2 + c^2)) + (2*atan((((32*B*C^2*b^2 - ((2*B*c - 2*C*b)*((2*B*c - 2*C*b)*(64*B*b^2*c^2 - 32*B*b^4 + 32*C*b*c^3 - 64*C*b^3*c + ((2*B*c - 2*C*b)*(96*b^4*c + 96*b^2*c^3)))/(2*(b^2 + c^2)))))/(2*(b^2 + c^2)) - 32*C^2*b^2*c - 32*B^2*b^2*c + 64*B*C*b^3 + 64*B*C*b*c^2))/(2*(b^2 + c^2)) + ((B*b + C*c)*(((B*b + C*c)*(64*B*b^2*c^2 - 32*B*b^4 + 32*C*b*c^3 - 64*C*b^3*c + ((2*B*c - 2*C*b)*(96*b^4*c + 96*b^2*c^3)))/(2*(b^2 + c^2)))))/(b^2 + c^2) + ((B*b + C*c)*(2*B*c - 2*C*b)*(96*b^4*c + 96*b^2*c^3))/(2*(b^2 + c^2)^2))/(b^2 + c^2) - 32*B^2*C*b*c + ((B*b + C*c)^2*(2*B*c - 2*C*b)*(96*b^4*c + 96*b^2*c^3))/(2*(b^2 + c^2)^3)*(12*B^2*b*c^3 - 6*B^2*b^3*c - 6*C^2*b*c^3 + 12*C^2*b^3*c + 4*B*C*b^4 + 4*B*C*c^4 - 28*B*C*b^2*c^2))/(b^2 + c^2)
```

$$\begin{aligned} & \left((b^2 + c^2)^2 (B^2 b^2 + 4B^2 c^2 + 4C^2 b^2 + C^2 c^2 - 6BCbc) - \tan(x/2) \right. \\ & \left((32B^3 bc - 32B^2 Cb^2 - 64C^3 b^2 + ((2Bc - 2Cb) * (32B^2 b^3 - 96B^2 bc^2 + 64C^2 bc^2 - ((2Bc - 2Cb) * (32Cb^2 c^2 - 64Cb^4 + 32Bbc^3 + 128Bb^3 c - ((2Bc - 2Cb) * (96bc^4 + 96b^3 c^2))) / (2(b^2 + c^2)))) / (2(b^2 + c^2)) + 192BCb^2 c) / (2(b^2 + c^2)) + ((Bb + Cc) * ((Bb + Cc) * (32Cb^2 c^2 - 64Cb^4 + 32Bbc^3 + 128Bb^3 c - ((2Bc - 2Cb) * (96bc^4 + 96b^3 c^2))) / (2(b^2 + c^2)))) / (b^2 + c^2) - ((Bb + Cc) * (2Bc - 2Cb) * (96bc^4 + 96b^3 c^2)) / (2(b^2 + c^2)^2) \right) / (b^2 + c^2) + 64BC^2 bc - ((Bb + Cc)^2 * (2Bc - 2Cb) * (96bc^4 + 96b^3 c^2)) / (2(b^2 + c^2)^3) * (12B^2 b^3 c - 6B^2 b^3 c - 6C^2 bc^3 + 12C^2 b^3 c + 4BCb^4 + 4BCc^4 - 28BCb^2 c^2) / ((b^2 + c^2)^2 (B^2 b^2 + 4B^2 c^2 + 4C^2 b^2 + C^2 c^2 - 6BCbc)^2) + (((Bb + Cc)^3 * (96bc^4 + 96b^3 c^2)) / (b^2 + c^2)^3 - ((Bb + Cc) * (32B^2 b^3 - 96B^2 bc^2 + 64C^2 bc^2 - ((2Bc - 2Cb) * (32Cb^2 c^2 - 64Cb^4 + 32Bbc^3 + 128Bb^3 c - ((2Bc - 2Cb) * (96bc^4 + 96b^3 c^2)) / (2(b^2 + c^2)))) / (2(b^2 + c^2)) + 192BCb^2 c) / (b^2 + c^2) + ((2Bc - 2Cb) * ((Bb + Cc) * (32Cb^2 c^2 - 64Cb^4 + 32Bbc^3 + 128Bb^3 c - ((2Bc - 2Cb) * (96bc^4 + 96b^3 c^2)) / (2(b^2 + c^2)))) / (b^2 + c^2) - ((Bb + Cc) * (2Bc - 2Cb) * (96bc^4 + 96b^3 c^2)) / (2(b^2 + c^2)^2)) / (2(b^2 + c^2)) * (B^2 b^4 + 4B^2 c^4 - 4C^2 b^4 - C^2 c^4 - 13B^2 b^2 c^2 + 13C^2 b^2 c^2 - 18BCbc^3 + 18BCb^3 c) / ((b^2 + c^2)^2 (B^2 b^2 + 4B^2 c^2 + 4C^2 b^2 + C^2 c^2 - 6BCbc)^2) + (((Bb + Cc) * ((2Bc - 2Cb) * (64Bb^2 c^2 - 32Bb^4 + 32Cb^3 c - 64Cb^3 c + ((2Bc - 2Cb) * (96b^4 c + 96b^2 c^3)) / (2(b^2 + c^2)))) / (2(b^2 + c^2)) - 32C^2 b^2 c - 32B^2 b^2 c + 64BCb^3 + 64BCbc^2) / (b^2 + c^2) - ((Bb + Cc)^3 * (96b^4 c + 96b^2 c^3)) / (b^2 + c^2)^3 + ((2Bc - 2Cb) * ((Bb + Cc) * (64Bb^2 c^2 - 32Bb^4 + 32Cb^3 c - 64Cb^3 c + ((2Bc - 2Cb) * (96b^4 c + 96b^2 c^3)) / (2(b^2 + c^2)))) / (b^2 + c^2) + ((Bb + Cc) * (2Bc - 2Cb) * (96b^4 c + 96b^2 c^3)) / (2(b^2 + c^2)^2)) / (2(b^2 + c^2)) * (B^2 b^4 + 4B^2 c^4 - 4C^2 b^4 - C^2 c^4 - 13B^2 b^2 c^2 + 13C^2 b^2 c^2 - 18BCbc^3 + 18BCb^3 c) / ((b^2 + c^2)^2 (B^2 b^2 + 4B^2 c^2 + 4C^2 b^2 + C^2 c^2 - 6BCbc)^2) * (b^4 + c^4 + 2b^2 c^2) / (32Bb^2 + 32Cb^2) * (Bb + Cc) / (b^2 + c^2) \end{aligned}$$

sympy [A] time = 0.88, size = 371, normalized size = 7.89

$$\left\{ \begin{aligned} & \infty (B \log(\sin(x)) + Cx) \\ & \frac{B \log(\sin(x)) + Cx}{c} \\ & \frac{Bx \sin(x)}{-2ic \sin(x) - 2c \cos(x)} - \frac{iBx \cos(x)}{-2ic \sin(x) - 2c \cos(x)} - \frac{iB \sin(x)}{-2ic \sin(x) - 2c \cos(x)} - \frac{iCx \sin(x)}{-2ic \sin(x) - 2c \cos(x)} - \frac{Cx \cos(x)}{-2ic \sin(x) - 2c \cos(x)} + \frac{C \sin(x)}{-2ic \sin(x) - 2c \cos(x)} \\ & \frac{Bx \sin(x)}{2ic \sin(x) - 2c \cos(x)} + \frac{iBx \cos(x)}{2ic \sin(x) - 2c \cos(x)} + \frac{iB \sin(x)}{2ic \sin(x) - 2c \cos(x)} + \frac{iCx \sin(x)}{2ic \sin(x) - 2c \cos(x)} - \frac{Cx \cos(x)}{2ic \sin(x) - 2c \cos(x)} + \frac{C \sin(x)}{2ic \sin(x) - 2c \cos(x)} \\ & \frac{Bbx}{b^2 + c^2} + \frac{Bc \log\left(\cos(x) + \frac{c \sin(x)}{b}\right)}{b^2 + c^2} - \frac{Cb \log\left(\cos(x) + \frac{c \sin(x)}{b}\right)}{b^2 + c^2} + \frac{Ccx}{b^2 + c^2} \end{aligned} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x)
[Out] Piecewise((zoo*(B*log(sin(x)) + C*x), Eq(b, 0) & Eq(c, 0)), ((B*log(sin(x)) + C*x)/c, Eq(b, 0)), (B*x*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)) - I*B*x*cos(x)/(-2*I*c*sin(x) - 2*c*cos(x)) - I*B*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)) - I*C*x*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)) - C*x*cos(x)/(-2*I*c*sin(x) - 2*c*cos(x)) + C*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)), Eq(b, -I*c)), (B*x*sin(x))/(2*I*c*sin(x) - 2*c*cos(x)) + I*B*x*cos(x)/(2*I*c*sin(x) - 2*c*cos(x)) + I*B*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)) + I*C*x*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)) - C*x*cos(x)/(2*I*c*sin(x) - 2*c*cos(x)) + C*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)), Eq(b, I*c)), (B*b*x/(b**2 + c**2) + B*c*log(cos(x) + c*sin(x)/
```

```
b)/(b**2 + c**2) - C*b*log(cos(x) + c*sin(x)/b)/(b**2 + c**2) + C*c*x/(b**2 + c**2), True))
```

$$3.530 \quad \int \frac{B \cos(x) + C \sin(x)}{(b \cos(x) + c \sin(x))^2} dx$$

Optimal. Leaf size=74

$$-\frac{Bc - bC}{(b^2 + c^2)(b \cos(x) + c \sin(x))} - \frac{(bB + cC) \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}}$$

[Out] $-(B*b+C*c)*\operatorname{arctanh}((c*\cos(x)-b*\sin(x))/(b^2+c^2)^{(1/2)})/(b^2+c^2)^{(3/2)}+(-B*c+C*b)/(b^2+c^2)/(b*\cos(x)+c*\sin(x))$

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3153, 3074, 206}

$$-\frac{Bc - bC}{(b^2 + c^2)(b \cos(x) + c \sin(x))} - \frac{(bB + cC) \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(B*\operatorname{Cos}[x] + C*\operatorname{Sin}[x])/(b*\operatorname{Cos}[x] + c*\operatorname{Sin}[x])^2, x]$

[Out] $-\left(\frac{(b*B + c*C)*\operatorname{ArcTanh}[(c*\operatorname{Cos}[x] - b*\operatorname{Sin}[x])/\operatorname{Sqrt}[b^2 + c^2]]}{(b^2 + c^2)^{(3/2)}} - \frac{B*c - b*C}{(b^2 + c^2)*(b*\operatorname{Cos}[x] + c*\operatorname{Sin}[x])}\right)$

Rule 206

$\operatorname{Int}[(a_.) + (b_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 3074

$\operatorname{Int}[(\cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*\sin[(c_.) + (d_.)*(x_)])^{-1}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{-1}, \operatorname{Subst}[\operatorname{Int}[1/(a^2 + b^2 - x^2), x], x, b*\operatorname{Cos}[c + d*x] - a*\operatorname{Sin}[c + d*x]], x] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \ \&\& \operatorname{NeQ}[a^2 + b^2, 0]$

Rule 3153

$\operatorname{Int}[(A_.) + \cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*\sin[(d_.) + (e_.)*(x_)]) / ((a_.) + \cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*\sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] \rightarrow \operatorname{Simp}[(c*B - b*C - (a*C - c*A)*\operatorname{Cos}[d + e*x] + (a*B - b*A)*\operatorname{Sin}[d + e*x]) / (e*(a^2 - b^2 - c^2)*(a + b*\operatorname{Cos}[d + e*x] + c*\operatorname{Sin}[d + e*x])), x] + \operatorname{Dist}[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), \operatorname{Int}[1/(a + b*\operatorname{Cos}[d + e*x] + c*\operatorname{Sin}[d + e*x]), x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, A, B, C\}, x] \ \&\& \operatorname{NeQ}[a^2 - b^2 - c^2, 0] \ \&\& \operatorname{NeQ}[a*A - b*B - c*C, 0]$

Rubi steps

$$\begin{aligned} \int \frac{B \cos(x) + C \sin(x)}{(b \cos(x) + c \sin(x))^2} dx &= -\frac{Bc - bC}{(b^2 + c^2)(b \cos(x) + c \sin(x))} + \frac{(bB + cC) \int \frac{1}{b \cos(x) + c \sin(x)} dx}{b^2 + c^2} \\ &= -\frac{Bc - bC}{(b^2 + c^2)(b \cos(x) + c \sin(x))} - \frac{(bB + cC) \operatorname{Subst}\left(\int \frac{1}{b^2 + c^2 - x^2} dx, x, c \cos(x) - b \sin(x)\right)}{b^2 + c^2} \\ &= -\frac{(bB + cC) \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}} - \frac{Bc - bC}{(b^2 + c^2)(b \cos(x) + c \sin(x))} \end{aligned}$$

Mathematica [A] time = 0.24, size = 75, normalized size = 1.01

$$\frac{bC - Bc}{(b^2 + c^2)(b \cos(x) + c \sin(x))} + \frac{2(bB + cC) \tanh^{-1}\left(\frac{b \tan\left(\frac{x}{2}\right) - c}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]

[Out] (2*(b*B + c*C)*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) + (-B*c) + b*C)/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))

fricas [B] time = 1.63, size = 194, normalized size = 2.62

$$\frac{2Cb^3 - 2Bb^2c + 2Cbc^2 - 2Bc^3 + \sqrt{b^2 + c^2} \left((Bb^2 + Cbc) \cos(x) + (Bbc + Cc^2) \sin(x) \right) \log\left(-\frac{2bc \cos(x) \sin(x) + (b^2 - c^2) \cos(x)^2 - 2b^2 - c^2 + 2\sqrt{b^2 + c^2} (c \cos(x) - b \sin(x))}{2bc} \right)}{2 \left((b^5 + 2b^3c^2 + bc^4) \cos(x) + (b^4c + 2b^2c^3 + c^5) \sin(x) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas")

[Out] 1/2*(2*C*b^3 - 2*B*b^2*c + 2*C*b*c^2 - 2*B*c^3 + sqrt(b^2 + c^2)*((B*b^2 + C*b*c)*cos(x) + (B*b*c + C*c^2)*sin(x))*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)))/((b^5 + 2*b^3*c^2 + b*c^4)*cos(x) + (b^4*c + 2*b^2*c^3 + c^5)*sin(x))

giac [A] time = 0.24, size = 132, normalized size = 1.78

$$-\frac{(Bb + Cc) \log\left(\frac{\left|2b \tan\left(\frac{1}{2}x\right) - 2c - 2\sqrt{b^2 + c^2}\right|}{\left|2b \tan\left(\frac{1}{2}x\right) - 2c + 2\sqrt{b^2 + c^2}\right|}\right)}{(b^2 + c^2)^{3/2}} - \frac{2\left(Cbc \tan\left(\frac{1}{2}x\right) - Bc^2 \tan\left(\frac{1}{2}x\right) + Cb^2 - Bbc\right)}{(b^3 + bc^2)\left(b \tan\left(\frac{1}{2}x\right)^2 - 2c \tan\left(\frac{1}{2}x\right) - b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")

[Out] -(B*b + C*c)*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(C*b*c*tan(1/2*x) - B*c^2*tan(1/2*x) + C*b^2 - B*b*c)/((b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b))

maple [A] time = 0.17, size = 113, normalized size = 1.53

$$-\frac{2\left(-\frac{c(Bc-bC)\tan\left(\frac{x}{2}\right)}{b(b^2+c^2)}-\frac{Bc-bC}{b^2+c^2}\right)}{b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b}+\frac{2(bB+Cc)\operatorname{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x)

[Out] -2*(-c*(B*c-C*b)/b/(b^2+c^2)*tan(1/2*x)-(B*c-C*b)/(b^2+c^2))/(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)+2*(B*b+C*c)/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))

maxima [B] time = 0.43, size = 271, normalized size = 3.66

$$-B\left(\frac{b\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}}+\frac{2\left(bc+\frac{c^2\sin(x)}{\cos(x)+1}\right)}{b^4+b^2c^2+\frac{2(b^3c+bc^3)\sin(x)}{\cos(x)+1}-\frac{(b^4+b^2c^2)\sin(x)^2}{(\cos(x)+1)^2}}\right)-C\left(\frac{c\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}}-\frac{1}{b^3+bc^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima")

[Out] -B*(b*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + 2*(b*c + c^2*sin(x)/(cos(x) + 1))/(b^4 + b^2*c^2 + 2*(b^3*c + b*c^3)*sin(x)/(cos(x) + 1) - (b^4 + b^2*c^2)*sin(x)^2/(cos(x) + 1)^2)) - C*(c*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(b + c*sin(x)/(cos(x) + 1))/(b^3 + b*c^2 + 2*(b^2*c + c^3)*sin(x)/(cos(x) + 1) - (b^3 + b*c^2)*sin(x)^2/(cos(x) + 1)^2))

mupad [B] time = 3.01, size = 129, normalized size = 1.74

$$-\frac{\frac{2(Bc-Cb)}{b^2+c^2}+\frac{2c\tan\left(\frac{x}{2}\right)(Bc-Cb)}{b(b^2+c^2)}}{-b\tan\left(\frac{x}{2}\right)^2+2c\tan\left(\frac{x}{2}\right)+b}+\frac{\operatorname{atan}\left(\frac{b^2c^2+3c^2-2b\tan\left(\frac{x}{2}\right)(b^2+c^2)}{(b^2+c^2)^{\frac{3}{2}}}\right)(Bb+Cc)2i}{(b^2+c^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x))^2,x)

[Out] (atan((b^2*c^2+3c^2-2*b*tan(x/2)*(b^2+c^2)*1i)/(b^2+c^2)^(3/2))*(B*b+C*c)*2i)/(b^2+c^2)^(3/2)-((2*(B*c-C*b))/(b^2+c^2)+(2*c*tan(x/2)*(B*c-C*b))/(b*(b^2+c^2)))/(b+2*c*tan(x/2)-b*tan(x/2)^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x)

[Out] Timed out

$$3.531 \quad \int \frac{B \cos(x) + C \sin(x)}{(b \cos(x) + c \sin(x))^3} dx$$

Optimal. Leaf size=66

$$\frac{\sin(x)(bB + cC)}{b(b^2 + c^2)(b \cos(x) + c \sin(x))} - \frac{Bc - bC}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2}$$

[Out] 1/2*(-B*c+C*b)/(b^2+c^2)/(b*cos(x)+c*sin(x))^2+(B*b+C*c)*sin(x)/b/(b^2+c^2)/(b*cos(x)+c*sin(x))

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3156, 12, 3075}

$$\frac{\sin(x)(bB + cC)}{b(b^2 + c^2)(b \cos(x) + c \sin(x))} - \frac{Bc - bC}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2}$$

Antiderivative was successfully verified.

[In] Int[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]

[Out] -(B*c - b*C)/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2) + ((b*B + c*C)*Sin[x])/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 3075

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x_Symbol] := Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 3156

Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^n*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] := -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[n, -2]

Rubi steps

$$\begin{aligned} \int \frac{B \cos(x) + C \sin(x)}{(b \cos(x) + c \sin(x))^3} dx &= -\frac{Bc - bC}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} + \frac{\int \frac{2(bB + cC)}{(b \cos(x) + c \sin(x))^2} dx}{2(b^2 + c^2)} \\ &= -\frac{Bc - bC}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} + \frac{(bB + cC) \int \frac{1}{(b \cos(x) + c \sin(x))^2} dx}{b^2 + c^2} \\ &= -\frac{Bc - bC}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} + \frac{(bB + cC) \sin(x)}{b(b^2 + c^2)(b \cos(x) + c \sin(x))} \end{aligned}$$

Mathematica [A] time = 0.19, size = 64, normalized size = 0.97

$$\frac{C(b^2 + c^2) + b \sin(2x)(bB + cC) - c \cos(2x)(bB + cC)}{2b(b^2 + c^2)(b \cos(x) + c \sin(x))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]

[Out] ((b^2 + c^2)*C - c*(b*B + c*C)*Cos[2*x] + b*(b*B + c*C)*Sin[2*x])/(2*b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2)

fricas [B] time = 0.54, size = 152, normalized size = 2.30

$$\frac{Cb^3 + Bb^2c + 3Cbc^2 - Bc^3 - 4(Bb^2c + Cbc^2) \cos(x)^2 + 2(Bb^3 + Cb^2c - Bbc^2 - Cc^3) \cos(x) \sin(x)}{2(b^4c^2 + 2b^2c^4 + c^6 + (b^6 + b^4c^2 - b^2c^4 - c^6) \cos(x)^2 + 2(b^5c + 2b^3c^3 + bc^5) \cos(x) \sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas")

[Out] 1/2*(C*b^3 + B*b^2*c + 3*C*b*c^2 - B*c^3 - 4*(B*b^2*c + C*b*c^2)*cos(x)^2 + 2*(B*b^3 + C*b^2*c - B*b*c^2 - C*c^3)*cos(x)*sin(x))/(b^4*c^2 + 2*b^2*c^4 + c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c + 2*b^3*c^3 + b*c^5)*cos(x)*sin(x))

giac [A] time = 0.20, size = 26, normalized size = 0.39

$$\frac{2Cc \tan(x) + Cb + Bc}{2(c \tan(x) + b)^2 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")

[Out] -1/2*(2*C*c*tan(x) + C*b + B*c)/((c*tan(x) + b)^2*c^2)

maple [A] time = 0.19, size = 37, normalized size = 0.56

$$-\frac{Bc - bC}{2c^2(c \tan(x) + b)^2} - \frac{C}{c^2(c \tan(x) + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x)

[Out] -1/2*(B*c-C*b)/c^2/(c*tan(x)+b)^2-C/c^2/(c*tan(x)+b)

maxima [B] time = 0.34, size = 199, normalized size = 3.02

$$\frac{2B \left(\frac{b \sin(x)}{\cos(x)+1} + \frac{c \sin(x)^2}{(\cos(x)+1)^2} - \frac{b \sin(x)^3}{(\cos(x)+1)^3} \right)}{b^4 + \frac{4b^3c \sin(x)}{\cos(x)+1} - \frac{4b^3c \sin(x)^3}{(\cos(x)+1)^3} + \frac{b^4 \sin(x)^4}{(\cos(x)+1)^4} - \frac{2(b^4-2b^2c^2) \sin(x)^2}{(\cos(x)+1)^2}} + \frac{2C \sin(x)^2}{b^3 + \frac{4b^2c \sin(x)}{\cos(x)+1} - \frac{4b^2c \sin(x)^3}{(\cos(x)+1)^3} + \frac{b^3 \sin(x)^4}{(\cos(x)+1)^4} - \frac{2(b^3-2bc^2) \sin(x)^2}{(\cos(x)+1)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima")

[Out] 2*B*(b*sin(x)/(cos(x) + 1) + c*sin(x)^2/(cos(x) + 1)^2 - b*sin(x)^3/(cos(x) + 1)^3)/(b^4 + 4*b^3*c*sin(x)/(cos(x) + 1) - 4*b^3*c*sin(x)^3/(cos(x) + 1)^3 + b^4*sin(x)^4/(cos(x) + 1)^4 - 2*(b^4 - 2*b^2*c^2)*sin(x)^2/(cos(x) + 1)^2)

)²) + 2*C*sin(x)²/((b³ + 4*b²*c*sin(x)/(cos(x) + 1) - 4*b²*c*sin(x)³/
 (cos(x) + 1)³ + b³*sin(x)⁴/(cos(x) + 1)⁴ - 2*(b³ - 2*b*c²)*sin(x)²/(
 cos(x) + 1)²)*(cos(x) + 1)²)

mupad [B] time = 2.82, size = 95, normalized size = 1.44

$$\frac{\frac{2 \tan\left(\frac{x}{2}\right)^2 (Bc + Cb)}{b^2} - \frac{2B \tan\left(\frac{x}{2}\right)^3}{b} + \frac{2B \tan\left(\frac{x}{2}\right)}{b}}{b^2 - \tan\left(\frac{x}{2}\right)^2 (2b^2 - 4c^2) + b^2 \tan\left(\frac{x}{2}\right)^4 + 4bc \tan\left(\frac{x}{2}\right) - 4bc \tan\left(\frac{x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x))³,x)

[Out] ((2*tan(x/2)²*(B*c + C*b))/b² - (2*B*tan(x/2)³)/b + (2*B*tan(x/2))/b)/(b² - tan(x/2)²*(2*b² - 4*c²) + b²*tan(x/2)⁴ + 4*b*c*tan(x/2) - 4*b*c*tan(x/2)³)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))³,x)

[Out] Timed out

$$3.532 \quad \int \frac{A+B \cos(x)+C \sin(x)}{b \cos(x)+c \sin(x)} dx$$

Optimal. Leaf size=84

$$-\frac{A \tanh^{-1}\left(\frac{c \cos(x)-b \sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}} + \frac{x(bB+cC)}{b^2+c^2} + \frac{(Bc-bC) \log(b \cos(x)+c \sin(x))}{b^2+c^2}$$

[Out] (B*b+C*c)*x/(b^2+c^2)+(B*c-C*b)*ln(b*cos(x)+c*sin(x))/(b^2+c^2)-A*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2)

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3136, 3074, 206}

$$-\frac{A \tanh^{-1}\left(\frac{c \cos(x)-b \sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}} + \frac{x(bB+cC)}{b^2+c^2} + \frac{(Bc-bC) \log(b \cos(x)+c \sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]

[Out] ((b*B + c*C)*x)/(b^2 + c^2) - (A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/Sqrt[b^2 + c^2] + ((B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3074

Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]

Rule 3136

Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]) / ((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + (Dist[(A*(b^2 + c^2) - a*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]

Rubi steps

$$\begin{aligned} \int \frac{A+B \cos(x)+C \sin(x)}{b \cos(x)+c \sin(x)} dx &= \frac{(bB+cC)x}{b^2+c^2} + \frac{(Bc-bC) \log(b \cos(x)+c \sin(x))}{b^2+c^2} + A \int \frac{1}{b \cos(x)+c \sin(x)} dx \\ &= \frac{(bB+cC)x}{b^2+c^2} + \frac{(Bc-bC) \log(b \cos(x)+c \sin(x))}{b^2+c^2} - A \operatorname{Subst}\left(\int \frac{1}{b^2+c^2-x^2} dx, \right. \\ &= \frac{(bB+cC)x}{b^2+c^2} - \frac{A \tanh^{-1}\left(\frac{c \cos(x)-b \sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}} + \frac{(Bc-bC) \log(b \cos(x)+c \sin(x))}{b^2+c^2} \end{aligned}$$

Mathematica [A] time = 0.25, size = 78, normalized size = 0.93

$$\frac{2A\sqrt{b^2+c^2} \tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-c}{\sqrt{b^2+c^2}}\right) + x(bB+cC) + (Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]

[Out] ((b*B + c*C)*x + 2*A*Sqrt[b^2 + c^2]*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]] + (B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)

fricas [A] time = 0.77, size = 155, normalized size = 1.85

$$\frac{\sqrt{b^2+c^2} A \log\left(-\frac{2bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2-2b^2-c^2+2\sqrt{b^2+c^2}(c\cos(x)-b\sin(x))}{2bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2+c^2}\right) + 2(Bb+Cc)x - (Cb-Bc)\log(2b\cos(x)+c\sin(x))}{2(b^2+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")

[Out] 1/2*(sqrt(b^2 + c^2)*A*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) + 2*(B*b + C*c)*x - (C*b - B*c)*log(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2))/(b^2 + c^2)

giac [A] time = 0.25, size = 148, normalized size = 1.76

$$\frac{A \log\left(\frac{\left|2b\tan\left(\frac{1}{2}x\right)-2c-2\sqrt{b^2+c^2}\right|}{\left|2b\tan\left(\frac{1}{2}x\right)-2c+2\sqrt{b^2+c^2}\right|}\right)}{\sqrt{b^2+c^2}} + \frac{(Bb+Cc)x}{b^2+c^2} + \frac{(Cb-Bc)\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2} - \frac{(Cb-Bc)\log\left(\left|b\tan\left(\frac{1}{2}x\right)\right|^2-\frac{2c}{b}\right)}{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")

[Out] -A*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/sqrt(b^2 + c^2) + (B*b + C*c)*x/(b^2 + c^2) + (C*b - B*c)*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) - (C*b - B*c)*log(abs(b*tan(1/2*x))^2 - 2*c*tan(1/2*x) - b)/(b^2 + c^2)

maple [B] time = 0.14, size = 222, normalized size = 2.64

$$\frac{Bc \ln\left(b\left(\tan^2\left(\frac{x}{2}\right)\right) - 2c \tan\left(\frac{x}{2}\right) - b\right)}{b^2+c^2} - \frac{bC \ln\left(b\left(\tan^2\left(\frac{x}{2}\right)\right) - 2c \tan\left(\frac{x}{2}\right) - b\right)}{b^2+c^2} + \frac{2 \operatorname{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right) A b^2}{(b^2+c^2)^{\frac{3}{2}}} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x)

[Out] 1/(b^2+c^2)*B*c*ln(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)-1/(b^2+c^2)*b*C*ln(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)+2/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))*A*b^2+2/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))*A*c^2-B/(b^2+c^2)*c*ln(1+tan(1/2*x)^2)+C/(b^2+c^2)*b*ln(1+tan(1/2*x)^2)+2*B/(b^2+c^2)*b*arctan(tan(1/2*x))+2*C/(b^2+c^2)*c*arctan(tan(1/2*x))

maxima [B] time = 0.42, size = 243, normalized size = 2.89

$$B \left(\frac{2b \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2 + c^2} + \frac{c \log\left(-b - \frac{2c \sin(x)}{\cos(x)+1} + \frac{b \sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2 + c^2} - \frac{c \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)}{b^2 + c^2} \right) + C \left(\frac{2c \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2 + c^2} - \frac{b \log\left(-b - \frac{2c \sin(x)}{\cos(x)+1} + \frac{b \sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2 + c^2} + \frac{b \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)}{b^2 + c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
[Out] B*(2*b*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) + c*log(-b - 2*c*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) - c*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(b^2 + c^2)) + C*(2*c*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) - b*log(-b - 2*c*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) + b*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(b^2 + c^2)) - A*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/sqrt(b^2 + c^2)
```

mupad [B] time = 9.41, size = 1099, normalized size = 13.08

$$\ln \left(32 A^2 B b^2 - 32 A B^2 b^2 - 32 A C^2 b^2 - 32 B C^2 b^2 + 32 b \tan\left(\frac{x}{2}\right) \left(c A^2 B + b A^2 C - 2 c A B^2 - 2 c A C^2 + c B^2 C \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x)),x)
[Out] log(32*A^2*B*b^2 - 32*A*B^2*b^2 - 32*A*C^2*b^2 - 32*B*C^2*b^2 + 32*b*tan(x/2)*(B^3*c - 2*C^3*b - 2*A*B^2*c + A^2*B*c + A^2*C*b - 2*A*C^2*c - B^2*C*b + 2*B*C^2*c) - ((A*((b^2 + c^2)^3)^(1/2) + B*c^3 - C*b^3 + B*b^2*c - C*b*c^2)*(32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 - 4*A*C*b*c + 6*B*C*b*c) - 32*B^2*b^2*c - 32*C^2*b^2*c - 64*A^2*b^2*c - 64*A*C*b^3 + 64*B*C*b^3 + ((A*((b^2 + c^2)^3)^(1/2) + B*c^3 - C*b^3 + B*b^2*c - C*b*c^2)*(32*A*b^4 + 32*B*b^4 + 32*A*b^2*c^2 - 64*B*b^2*c^2 - 32*C*b*c^3 + 64*C*b^3*c + 32*b*tan(x/2)*(2*A*c^3 + B*c^3 - 2*C*b^3 + 2*A*b^2*c + 4*B*b^2*c + C*b*c^2) + (96*b*c*(b + c*tan(x/2))*(A*((b^2 + c^2)^3)^(1/2) + B*c^3 - C*b^3 + B*b^2*c - C*b*c^2))/(b^2 + c^2)))/(b^2 + c^2)^2 + 64*A*B*b^2*c + 64*B*C*b*c^2))/(b^2 + c^2)^2 - 32*A^2*C*b*c + 32*B^2*C*b*c)*(B*c - C*b)/(b^2 + c^2) + (A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2) + log(32*A^2*B*b^2 - 32*A*B^2*b^2 - 32*A*C^2*b^2 - 32*B*C^2*b^2 + 32*b*tan(x/2)*(B^3*c - 2*C^3*b - 2*A*B^2*c + A^2*B*c + A^2*C*b - 2*A*C^2*c - B^2*C*b + 2*B*C^2*c) - ((A*((b^2 + c^2)^3)^(1/2) - B*c^3 + C*b^3 - B*b^2*c + C*b*c^2)*(64*A^2*b^2*c + 32*B^2*b^2*c + 32*C^2*b^2*c - 32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 - 4*A*C*b*c + 6*B*C*b*c) + 64*A*C*b^3 - 64*B*C*b^3 + ((A*((b^2 + c^2)^3)^(1/2) - B*c^3 + C*b^3 - B*b^2*c + C*b*c^2)*(32*A*b^4 + 32*B*b^4 + 32*A*b^2*c^2 - 64*B*b^2*c^2 - 32*C*b*c^3 + 64*C*b^3*c + 32*b*tan(x/2)*(2*A*c^3 + B*c^3 - 2*C*b^3 + 2*A*b^2*c + 4*B*b^2*c + C*b*c^2) - (96*b*c*(b + c*tan(x/2))*(A*((b^2 + c^2)^3)^(1/2) - B*c^3 + C*b^3 - B*b^2*c + C*b*c^2))/(b^2 + c^2)))/(b^2 + c^2)^2 - 32*A^2*C*b*c + 32*B^2*C*b*c)*(B*c - C*b)/(b^2 + c^2) + (A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2)
```

$$\frac{b^3 - B*b^2*c + C*b*c^2}{(b^2 + c^2)} \Big/ (b^2 + c^2)^2 - 64*A*B*b^2*c - 64*B*C*b*c^2 \Big/ (b^2 + c^2)^2 - 32*A^2*C*b*c + 32*B^2*C*b*c * ((B*c - C*b)/(b^2 + c^2) - (A*((b^2 + c^2)^3)^{1/2})/(b^2 + c^2)^2) - (\log(\tan(x/2)) + 1i)*(B - C*1i)/(b*1i + c) + (\log(\tan(x/2)) - 1i)*(B + C*1i)/(b*1i - c)$$

sympy [A] time = 38.59, size = 1042, normalized size = 12.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x)

[Out] Piecewise((zoo*(A*log(tan(x/2)) - B*log(tan(x/2)**2 + 1) + B*log(tan(x/2)) + C*x), Eq(b, 0) & Eq(c, 0)), ((A*log(tan(x/2)) - B*log(tan(x/2)**2 + 1) + B*log(tan(x/2)) + C*x)/c, Eq(b, 0)), (2*A/(-2*I*c*sin(x) - 2*c*cos(x)) + B*x*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)) - I*B*x*cos(x)/(-2*I*c*sin(x) - 2*c*cos(x)) - I*B*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)) - I*C*x*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)) - C*x*cos(x)/(-2*I*c*sin(x) - 2*c*cos(x)) + C*sin(x)/(-2*I*c*sin(x) - 2*c*cos(x)), Eq(b, -I*c)), (2*A/(2*I*c*sin(x) - 2*c*cos(x)) + B*x*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)) + I*B*x*cos(x)/(2*I*c*sin(x) - 2*c*cos(x)) + I*B*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)) + I*C*x*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)) - C*x*cos(x)/(2*I*c*sin(x) - 2*c*cos(x)) + C*sin(x)/(2*I*c*sin(x) - 2*c*cos(x)), Eq(b, I*c)), (-A*b**2*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + A*b**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - A*c**2*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + A*c**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + B*b*x*sqrt(b**2 + c**2)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - B*c*sqrt(b**2 + c**2)*log(tan(x/2)**2 + 1)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + B*c*sqrt(b**2 + c**2)*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + B*c*sqrt(b**2 + c**2)*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + C*b*sqrt(b**2 + c**2)*log(tan(x/2)**2 + 1)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - C*b*sqrt(b**2 + c**2)*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - C*b*sqrt(b**2 + c**2)*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + C*c*x*sqrt(b**2 + c**2)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)), True))

$$3.533 \quad \int \frac{A+B \cos(x)+C \sin(x)}{(b \cos(x)+c \sin(x))^2} dx$$

Optimal. Leaf size=85

$$\frac{-Ab \sin(x) + Ac \cos(x) - bC + Bc}{(b^2 + c^2)(b \cos(x) + c \sin(x))} - \frac{(bB + cC) \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}}$$

[Out] $-(B*b+C*c)*\operatorname{arctanh}((c*\cos(x)-b*\sin(x))/(b^2+c^2)^{(1/2)})/(b^2+c^2)^{(3/2)}+(-B*c+b*C-A*c*\cos(x)+A*b*\sin(x))/(b^2+c^2)/(b*\cos(x)+c*\sin(x))$

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3153, 3074, 206}

$$\frac{-Ab \sin(x) + Ac \cos(x) - bC + Bc}{(b^2 + c^2)(b \cos(x) + c \sin(x))} - \frac{(bB + cC) \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]`

[Out] $-\left(\frac{(b*B + c*C)*\operatorname{ArcTanh}[(c*\cos[x] - b*\sin[x])/ \operatorname{Sqrt}[b^2 + c^2]]}{(b^2 + c^2)^{(3/2)}} - \frac{(B*c - b*C + A*c*\cos[x] - A*b*\sin[x])}{(b^2 + c^2)*(b*\cos[x] + c*\sin[x])}\right)$

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 3074

`Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]`

Rule 3153

`Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]) / ((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] := Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]) / (e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]`

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(x) + C \sin(x)}{(b \cos(x) + c \sin(x))^2} dx &= -\frac{Bc - bC + Ac \cos(x) - Ab \sin(x)}{(b^2 + c^2)(b \cos(x) + c \sin(x))} + \frac{(bB + cC) \int \frac{1}{b \cos(x) + c \sin(x)} dx}{b^2 + c^2} \\ &= -\frac{Bc - bC + Ac \cos(x) - Ab \sin(x)}{(b^2 + c^2)(b \cos(x) + c \sin(x))} - \frac{(bB + cC) \operatorname{Subst}\left(\int \frac{1}{b^2 + c^2 - x^2} dx, x, c \cos(x) + b \sin(x)\right)}{b^2 + c^2} \\ &= -\frac{(bB + cC) \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}} - \frac{Bc - bC + Ac \cos(x) - Ab \sin(x)}{(b^2 + c^2)(b \cos(x) + c \sin(x))} \end{aligned}$$

Mathematica [A] time = 0.28, size = 92, normalized size = 1.08

$$\frac{A(b^2 + c^2) \sin(x) + b(bC - Bc)}{b(b^2 + c^2)(b \cos(x) + c \sin(x))} + \frac{2(bB + cC) \tanh^{-1}\left(\frac{b \tan\left(\frac{x}{2}\right) - c}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]

[Out] (2*(b*B + c*C)*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) + (b*(-(B*c) + b*C) + A*(b^2 + c^2)*Sin[x])/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))

fricas [B] time = 3.02, size = 226, normalized size = 2.66

$$\frac{2Cb^3 - 2Bb^2c + 2Cbc^2 - 2Bc^3 + \sqrt{b^2 + c^2} \left((Bb^2 + Cbc) \cos(x) + (Bbc + Cc^2) \sin(x) \right) \log\left(-\frac{2bc \cos(x) \sin(x) + (b^2 - c^2) \cos(x)^2 - 2b^2 - c^2 + 2\sqrt{b^2 + c^2}(c \cos(x) - b \sin(x))}{2bc} \right)}{2 \left((b^5 + 2b^3c^2 + bc^4) \cos(x) + (b^4c + 2b^3c^2 + bc^4) \sin(x) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas")

[Out] 1/2*(2*C*b^3 - 2*B*b^2*c + 2*C*b*c^2 - 2*B*c^3 + sqrt(b^2 + c^2)*((B*b^2 + C*b*c)*cos(x) + (B*b*c + C*c^2)*sin(x))*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) - 2*(A*b^2*c + A*c^3)*cos(x) + 2*(A*b^3 + A*b*c^2)*sin(x))/(b^5 + 2*b^3*c^2 + b*c^4)*cos(x) + (b^4*c + 2*b^2*c^3 + c^5)*sin(x))

giac [A] time = 0.23, size = 150, normalized size = 1.76

$$\frac{(Bb + Cc) \log\left(\frac{\left|2b \tan\left(\frac{1}{2}x\right) - 2c - 2\sqrt{b^2 + c^2}\right|}{\left|2b \tan\left(\frac{1}{2}x\right) - 2c + 2\sqrt{b^2 + c^2}\right|}\right)}{(b^2 + c^2)^{3/2}} - \frac{2 \left(Ab^2 \tan\left(\frac{1}{2}x\right) + Cbc \tan\left(\frac{1}{2}x\right) + Ac^2 \tan\left(\frac{1}{2}x\right) - Bc^2 \tan\left(\frac{1}{2}x\right) + b^3 + bc^2 \right) \left(b \tan\left(\frac{1}{2}x\right)^2 - 2c \tan\left(\frac{1}{2}x\right) - b \right)}{(b^3 + bc^2) \left(b \tan\left(\frac{1}{2}x\right)^2 - 2c \tan\left(\frac{1}{2}x\right) - b \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")

[Out] -(B*b + C*c)*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(A*b^2*tan(1/2*x) + Cbc*tan(1/2*x) + Ac^2*tan(1/2*x) - Bc^2*tan(1/2*x) + b^3 + bc^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b)

$$+ C*b*c*\tan(1/2*x) + A*c^2*\tan(1/2*x) - B*c^2*\tan(1/2*x) + C*b^2 - B*b*c)/(b^3 + b*c^2)*(b*\tan(1/2*x)^2 - 2*c*\tan(1/2*x) - b))$$

maple [A] time = 0.17, size = 124, normalized size = 1.46

$$-\frac{2(Ab^2+Ac^2-Bc^2+Cbc)\tan\left(\frac{x}{2}\right) + \frac{2(Bc-bC)}{b^2+c^2}}{b(b^2+c^2)} + \frac{2(bB+Cc)\operatorname{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x)

[Out] 2*(-(A*b^2+A*c^2-B*c^2+C*b*c)/b/(b^2+c^2)*tan(1/2*x)+(B*c-C*b)/(b^2+c^2))/(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)+2*(B*b+C*c)/(b^2+c^2)^(3/2)*arctanh(1/2*(b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))

maxima [B] time = 0.42, size = 286, normalized size = 3.36

$$-B\left(\frac{b\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}} + \frac{2\left(bc+\frac{c^2\sin(x)}{\cos(x)+1}\right)}{b^4+b^2c^2+\frac{2(b^3c+bc^3)\sin(x)}{\cos(x)+1}-\frac{(b^4+b^2c^2)\sin(x)^2}{(\cos(x)+1)^2}}\right) - C\left(\frac{c\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}} - \frac{1}{b^3+bc^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima")

[Out] -B*(b*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + 2*(b*c + c^2*sin(x)/(cos(x) + 1))/(b^4 + b^2*c^2 + 2*(b^3*c + b*c^3)*sin(x)/(cos(x) + 1) - (b^4 + b^2*c^2)*sin(x)^2/(cos(x) + 1)^2)) - C*(c*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(b + c*sin(x)/(cos(x) + 1))/(b^3 + b*c^2 + 2*(b^2*c + c^3)*sin(x)/(cos(x) + 1) - (b^3 + b*c^2)*sin(x)^2/(cos(x) + 1)^2)) - A/(c^2*tan(x) + b*c)

mupad [B] time = 3.05, size = 141, normalized size = 1.66

$$-\frac{\frac{2(Bc-Cb)}{b^2+c^2} - \frac{2\tan\left(\frac{x}{2}\right)(Ab^2+Ac^2-Bc^2+Cbc)}{b(b^2+c^2)}}{-b\tan\left(\frac{x}{2}\right)^2 + 2c\tan\left(\frac{x}{2}\right) + b} + \frac{\operatorname{atan}\left(\frac{b^2c1i+c^31i-b\tan\left(\frac{x}{2}\right)(b^2+c^2)1i}{(b^2+c^2)^{\frac{3}{2}}}\right)(Bb+Cc)2i}{(b^2+c^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x))^2,x)

[Out] (atan((b^2*c*1i + c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2))*(B*b + C*c)*2i)/(b^2 + c^2)^(3/2) - ((2*(B*c - C*b))/(b^2 + c^2) - (2*tan(x/2)*(A*b^2 + A*c^2 - B*c^2 + C*b*c))/(b*(b^2 + c^2)))/(b + 2*c*tan(x/2) - b*tan(x/2)^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**2,x)

[Out] Timed out

$$3.534 \quad \int \frac{A+B \cos(x)+C \sin(x)}{(b \cos(x)+c \sin(x))^3} dx$$

Optimal. Leaf size=129

$$\frac{-Ab \sin(x) + Ac \cos(x) - bC + Bc}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} \frac{A \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{2(b^2 + c^2)^{3/2}} \frac{c \cos(x)(bB + cC) - b \sin(x)(bB + cC)}{(b^2 + c^2)^2 (b \cos(x) + c \sin(x))}$$

[Out] $-1/2*A*\operatorname{arctanh}((c*\cos(x)-b*\sin(x))/(b^2+c^2)^{(1/2)})/(b^2+c^2)^{(3/2)}+1/2*(-B*c+b*C-A*c*\cos(x)+A*b*\sin(x))/(b^2+c^2)/(b*\cos(x)+c*\sin(x))^2+(-c*(B*b+C*c)*\cos(x)+b*(B*b+C*c)*\sin(x))/(b^2+c^2)^2/(b*\cos(x)+c*\sin(x))$

Rubi [A] time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3156, 3153, 3074, 206}

$$\frac{-Ab \sin(x) + Ac \cos(x) - bC + Bc}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} \frac{A \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{2(b^2 + c^2)^{3/2}} \frac{c \cos(x)(bB + cC) - b \sin(x)(bB + cC)}{(b^2 + c^2)^2 (b \cos(x) + c \sin(x))}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*\operatorname{Cos}[x] + C*\operatorname{Sin}[x])/(b*\operatorname{Cos}[x] + c*\operatorname{Sin}[x])^3, x]$

[Out] $-(A*\operatorname{ArcTanh}[(c*\operatorname{Cos}[x] - b*\operatorname{Sin}[x])/ \operatorname{Sqrt}[b^2 + c^2]])/(2*(b^2 + c^2)^{(3/2)}) - (B*c - b*C + A*c*\operatorname{Cos}[x] - A*b*\operatorname{Sin}[x])/(2*(b^2 + c^2)*(b*\operatorname{Cos}[x] + c*\operatorname{Sin}[x])^2) - (c*(b*B + c*C)*\operatorname{Cos}[x] - b*(b*B + c*C)*\operatorname{Sin}[x])/((b^2 + c^2)^2*(b*\operatorname{Cos}[x] + c*\operatorname{Sin}[x]))$

Rule 206

$\operatorname{Int}[(a + (b*x)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 3074

$\operatorname{Int}[(\cos[(c + d*x)] + (d*x)^2)^{-1}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{-1}, \operatorname{Subst}[\operatorname{Int}[1/(a^2 + b^2 - x^2), x], x, b*\operatorname{Cos}[c + d*x] - a*\operatorname{Sin}[c + d*x]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{NeQ}[a^2 + b^2, 0]$

Rule 3153

$\operatorname{Int}[(A + \cos[(d + e*x)]*(B + C*\sin[(d + e*x)])) / ((a + \cos[(d + e*x)]*(b + c*\sin[(d + e*x)]))^2, x_Symbol] \rightarrow \operatorname{Simp}[(c*B - b*C - (a*C - c*A)*\operatorname{Cos}[d + e*x] + (a*B - b*A)*\operatorname{Sin}[d + e*x]) / (e*(a^2 - b^2 - c^2)*(a + b*\operatorname{Cos}[d + e*x] + c*\operatorname{Sin}[d + e*x])), x] + \operatorname{Dist}[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), \operatorname{Int}[1/(a + b*\operatorname{Cos}[d + e*x] + c*\operatorname{Sin}[d + e*x]), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, A, B, C\}, x \ \&\& \operatorname{NeQ}[a^2 - b^2 - c^2, 0] \ \&\& \operatorname{NeQ}[a*A - b*B - c*C, 0]$

Rule 3156

$\operatorname{Int}[(a + \cos[(d + e*x)]*(b + c*\sin[(d + e*x)]))^n * (A + \cos[(d + e*x)]*(B + C*\sin[(d + e*x)])), x_Symbol] \rightarrow -\operatorname{Simp}[(c*B - b*C - (a*C - c*A)*\operatorname{Cos}[d + e*x] + (a*B - b*A)*\operatorname{Sin}[d + e*x])*(a + b*\operatorname{Cos}[d + e*x] + c*\operatorname{Sin}[d + e*x])^{n+1} / (e*(n+1)*(a^2 - b^2 - c^2)), x] + \operatorname{Dist}[1/((n+1)*(a^2 - b^2 - c^2)), \operatorname{Int}[(a + b*\operatorname{Cos}[d + e*x] + c*\operatorname{Sin}[d + e*x])^{n+1} * \operatorname{Simp}[(n+1)*(a*A - b*B - c*C) + (n+2)*$

$(a*B - b*A)*\text{Cos}[d + e*x] + (n + 2)*(a*C - c*A)*\text{Sin}[d + e*x], x], x], x] /;$
 $\text{FreeQ}[\{a, b, c, d, e, A, B, C\}, x] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0]$
 $\ \&\& \ \text{NeQ}[n, -2]$

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(x) + C \sin(x)}{(b \cos(x) + c \sin(x))^3} dx &= -\frac{Bc - bC + Ac \cos(x) - Ab \sin(x)}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} + \frac{\int \frac{2(bB+cC)+Ab \cos(x)+Ac \sin(x)}{(b \cos(x)+c \sin(x))^2} dx}{2(b^2 + c^2)} \\ &= -\frac{Bc - bC + Ac \cos(x) - Ab \sin(x)}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} - \frac{c(bB + cC) \cos(x) - b(bB + cC) \sin(x)}{(b^2 + c^2)^2 (b \cos(x) + c \sin(x))} + \frac{A \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{2(b^2 + c^2)^{3/2}} \\ &= -\frac{Bc - bC + Ac \cos(x) - Ab \sin(x)}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} - \frac{c(bB + cC) \cos(x) - b(bB + cC) \sin(x)}{(b^2 + c^2)^2 (b \cos(x) + c \sin(x))} - \frac{A \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{2(b^2 + c^2)^{3/2}} \\ &= -\frac{Bc - bC + Ac \cos(x) - Ab \sin(x)}{2(b^2 + c^2)(b \cos(x) + c \sin(x))^2} - \frac{c(bB + cC) \cos(x) - b(bB + cC) \sin(x)}{(b^2 + c^2)^2 (b \cos(x) + c \sin(x))} - \frac{A \tanh^{-1}\left(\frac{c \cos(x) - b \sin(x)}{\sqrt{b^2 + c^2}}\right)}{2(b^2 + c^2)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.66, size = 122, normalized size = 0.95

$$\frac{Ab^2 \sin(x) - Abc \cos(x) + b^2 B \sin(2x) + b^2 C - c \cos(2x)(bB + cC) + bcC \sin(2x) + c^2 C}{2b(b^2 + c^2)(b \cos(x) + c \sin(x))^2} + \frac{A \tanh^{-1}\left(\frac{b \tan\left(\frac{x}{2}\right) - c}{\sqrt{b^2 + c^2}}\right)}{(b^2 + c^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]

[Out] (A*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]]/(b^2 + c^2)^(3/2) + (b^2*C + c^2*C - A*b*c*Cos[x] - c*(b*B + c*C)*Cos[2*x] + A*b^2*Sin[x] + b^2*B*Sin[2*x] + b*c*c*Sin[2*x])/(2*b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2)

fricas [B] time = 1.69, size = 311, normalized size = 2.41

$$\frac{2Cb^3 + 2Bb^2c + 6Cbc^2 - 2Bc^3 - 8(Bb^2c + Cbc^2) \cos(x)^2 + (2Abc \cos(x) \sin(x) + Ac^2 + (Ab^2 - Ac^2) \cos(x)^2)}{4(b^4c^2 + 2b^2c^4 + c^6 + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas")

[Out] 1/4*(2*C*b^3 + 2*B*b^2*c + 6*C*b*c^2 - 2*B*c^3 - 8*(B*b^2*c + C*b*c^2)*cos(x)^2 + (2*A*b*c*cos(x)*sin(x) + A*c^2 + (A*b^2 - A*c^2)*cos(x)^2)*sqrt(b^2 + c^2)*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) - 2*(A*b^2*c + A*c^3)*cos(x) + 2*(A*b^3 + A*b*c^2 + 2*(B*b^3 + C*b^2*c - B*b*c^2 - C*c^3)*cos(x))*sin(x))/(b^4*c^2 + 2*b^2*c^4 + c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c + 2*b^3*c^3 + b*c^5)*cos(x)*sin(x))

giac [B] time = 0.26, size = 270, normalized size = 2.09

$$A \log \left(\frac{\left| -2b \tan\left(\frac{1}{2}x\right) + 2c - 2\sqrt{b^2+c^2} \right|}{\left| -2b \tan\left(\frac{1}{2}x\right) + 2c + 2\sqrt{b^2+c^2} \right|} \right) + \frac{Ab^3 \tan\left(\frac{1}{2}x\right)^3 - 2Bb^3 \tan\left(\frac{1}{2}x\right)^3 + 2Abc^2 \tan\left(\frac{1}{2}x\right)^3 - 2Bbc^2 \tan\left(\frac{1}{2}x\right)^3}{2(b^2+c^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")

[Out] 1/2*A*log(abs(-2*b*tan(1/2*x) + 2*c - 2*sqrt(b^2 + c^2))/abs(-2*b*tan(1/2*x) + 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + (A*b^3*tan(1/2*x)^3 - 2*B*b^3*tan(1/2*x)^3 + 2*A*b*c^2*tan(1/2*x)^3 - 2*B*b*c^2*tan(1/2*x)^3 + 2*C*b^3*tan(1/2*x)^2 + A*b^2*c*tan(1/2*x)^2 + 2*B*b^2*c*tan(1/2*x)^2 + 2*C*b*c^2*tan(1/2*x)^2 - 2*A*c^3*tan(1/2*x)^2 + 2*B*c^3*tan(1/2*x)^2 + A*b^3*tan(1/2*x) + 2*B*b^3*tan(1/2*x) - 2*A*b*c^2*tan(1/2*x) + 2*B*b*c^2*tan(1/2*x) - A*b^2*c)/(b^4 + b^2*c^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b)^2

maple [A] time = 0.20, size = 218, normalized size = 1.69

$$2 \left(-\frac{(Ab^2+2Ac^2-2Bb^2-2Bc^2)(\tan^3(\frac{x}{2}))}{2(b^2+c^2)b} - \frac{(Ab^2c-2Ac^3+2Bb^2c+2Bc^3+2Cb^3+2Cb^2c^2)(\tan^2(\frac{x}{2}))}{2(b^2+c^2)b^2} - \frac{(Ab^2-2Ac^2+2Bb^2+2Bc^2)\tan(\frac{x}{2})}{2(b^2+c^2)b} \right) + \frac{b \left(\tan^2\left(\frac{x}{2}\right) - 2c \tan\left(\frac{x}{2}\right) - b \right)^2}{(b \left(\tan^2\left(\frac{x}{2}\right) - 2c \tan\left(\frac{x}{2}\right) - b \right)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x)

[Out] -2*(-1/2*(A*b^2+2*A*c^2-2*B*b^2-2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)^3-1/2*(A*b^2*c-2*A*c^3+2*B*b^2*c+2*B*c^3+2*C*b^3+2*C*b*c^2)/(b^2+c^2)/b^2*tan(1/2*x)^2-1/2*(A*b^2-2*A*c^2+2*B*b^2+2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)+1/2*A*c/(b^2+c^2))/(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)^2+A/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))

maxima [B] time = 0.45, size = 451, normalized size = 3.50

$$-\frac{1}{2}A \left(\frac{2 \left(b^2c - \frac{(b^3-2bc^2)\sin(x)}{\cos(x)+1} - \frac{(b^2c-2c^3)\sin(x)^2}{(\cos(x)+1)^2} - \frac{(b^3+2bc^2)\sin(x)^3}{(\cos(x)+1)^3} \right)}{b^6 + b^4c^2 + \frac{4(b^5c+b^3c^3)\sin(x)}{\cos(x)+1} - \frac{2(b^6-b^4c^2-2b^2c^4)\sin(x)^2}{(\cos(x)+1)^2} - \frac{4(b^5c+b^3c^3)\sin(x)^3}{(\cos(x)+1)^3} + \frac{(b^6+b^4c^2)\sin(x)^4}{(\cos(x)+1)^4}} \right) + \frac{\log \left(\frac{c - \frac{b \sin(x)}{\cos(x)+1} + \sqrt{b^2+c^2}}{c - \frac{b \sin(x)}{\cos(x)+1} - \sqrt{b^2+c^2}} \right)}{(b^2+c^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima")

[Out] -1/2*A*(2*(b^2*c - (b^3 - 2*b*c^2)*sin(x)/(cos(x) + 1) - (b^2*c - 2*c^3)*sin(x)^2/(cos(x) + 1)^2 - (b^3 + 2*b*c^2)*sin(x)^3/(cos(x) + 1)^3)/(b^6 + b^4*c^2 + 4*(b^5*c + b^3*c^3)*sin(x)/(cos(x) + 1) - 2*(b^6 - b^4*c^2 - 2*b^2*c^4)*sin(x)^2/(cos(x) + 1)^2 - 4*(b^5*c + b^3*c^3)*sin(x)^3/(cos(x) + 1)^3 + (b^6 + b^4*c^2)*sin(x)^4/(cos(x) + 1)^4) + log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2)) + 2*B*(b*sin(x)/(cos(x) + 1) + c*sin(x)^2/(cos(x) + 1)^2 - b*sin(x)^3/(cos(x) + 1)^3)/(b^4 + 4*b^3*c*sin(x)/(cos(x) + 1) - 4*b^3*c*sin(x)^3/(cos(x) + 1)^3 + b^4*sin(x)^4/(cos(x) + 1)^4 - 2*(b^4 - 2*b^2*c^2)*sin(x)^2/(cos(x) + 1)^2) + 2*C*sin(x)^2/((b^3 + 4*b^2*c*sin(x)/(cos(x) + 1) - 4*b^2*c*sin(x)^3/(cos(x) + 1)^3 + b^3*sin(x)^4/(cos(x) + 1)^4 - 2*(b^3 - 2*b*c^2)*sin(x)^2/(cos(x) + 1)^2)*(cos(x) + 1)^2)

mupad [B] time = 3.44, size = 264, normalized size = 2.05

$$\frac{\frac{\tan\left(\frac{x}{2}\right)(Ab^2-2Ac^2+2Bb^2+2Bc^2)}{b(b^2+c^2)} - \frac{Ac}{b^2+c^2} + \frac{\tan\left(\frac{x}{2}\right)^2(2Bc^3-2Ac^3+2Cb^3+Ab^2c+2Bb^2c+2Cbc^2)}{b^2(b^2+c^2)} + \frac{\tan\left(\frac{x}{2}\right)^3(Ab^2+2Ac^2-2Bb^2-2Bc^2)}{b(b^2+c^2)}}{b^2 - \tan\left(\frac{x}{2}\right)^2(2b^2 - 4c^2) + b^2 \tan\left(\frac{x}{2}\right)^4 + 4bc \tan\left(\frac{x}{2}\right) - 4bc \tan\left(\frac{x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x))^3,x)

[Out] ((tan(x/2)*(A*b^2 - 2*A*c^2 + 2*B*b^2 + 2*B*c^2))/(b*(b^2 + c^2)) - (A*c)/(b^2 + c^2) + (tan(x/2)^2*(2*B*c^3 - 2*A*c^3 + 2*C*b^3 + A*b^2*c + 2*B*b^2*c + 2*C*b*c^2))/(b^2*(b^2 + c^2)) + (tan(x/2)^3*(A*b^2 + 2*A*c^2 - 2*B*b^2 - 2*B*c^2))/(b*(b^2 + c^2)))/(b^2 - tan(x/2)^2*(2*b^2 - 4*c^2) + b^2*tan(x/2)^4 + 4*b*c*tan(x/2) - 4*b*c*tan(x/2)^3) + (A*atan((b^2*c*1i + c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2))*1i)/(b^2 + c^2)^(3/2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**3,x)

[Out] Timed out

$$3.535 \quad \int \frac{A+B \cos(x)}{a+b \cos(x)+c \sin(x)} dx$$

Optimal. Leaf size=115

$$\frac{2(abB - A(b^2 + c^2)) \tan^{-1}\left(\frac{(a-b)\tan(\frac{x}{2})+c}{\sqrt{a^2-b^2-c^2}}\right)}{(b^2 + c^2)\sqrt{a^2 - b^2 - c^2}} + \frac{Bc \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} + \frac{bBx}{b^2 + c^2}$$

[Out] b*B*x/(b^2+c^2)+B*c*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)-2*(a*b*B-A*(b^2+c^2))*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)

Rubi [A] time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3138, 3124, 618, 204}

$$\frac{2(abB - A(b^2 + c^2)) \tan^{-1}\left(\frac{(a-b)\tan(\frac{x}{2})+c}{\sqrt{a^2-b^2-c^2}}\right)}{(b^2 + c^2)\sqrt{a^2 - b^2 - c^2}} + \frac{Bc \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} + \frac{bBx}{b^2 + c^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x]),x]

[Out] (b*B*x)/(b^2 + c^2) - (2*(a*b*B - A*(b^2 + c^2))*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]]/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) + (B*c*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3138

Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*B*(d + e*x))/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[(c*B*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \cos(x)}{a + b \cos(x) + c \sin(x)} dx &= \frac{bBx}{b^2 + c^2} + \frac{Bc \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} + \left(A - \frac{abB}{b^2 + c^2}\right) \int \frac{1}{a + b \cos(x) + c \sin(x)} dx \\
&= \frac{bBx}{b^2 + c^2} + \frac{Bc \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} + \left(2 \left(A - \frac{abB}{b^2 + c^2}\right)\right) \text{Subst} \left(\int \frac{1}{a + b \cos(x) + c \sin(x)} dx \right) \\
&= \frac{bBx}{b^2 + c^2} + \frac{Bc \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} - \left(4 \left(A - \frac{abB}{b^2 + c^2}\right)\right) \text{Subst} \left(\int \frac{1}{-4(a^2 - b^2 - c^2)} dx \right) \\
&= \frac{bBx}{b^2 + c^2} + \frac{2 \left(A - \frac{abB}{b^2 + c^2}\right) \tan^{-1} \left(\frac{c + (a-b) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2 - c^2}} \right)}{\sqrt{a^2 - b^2 - c^2}} + \frac{Bc \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 95, normalized size = 0.83

$$\frac{B(c \log(a + b \cos(x) + c \sin(x)) + bx) - \frac{2(A(b^2 + c^2) - abB) \tanh^{-1} \left(\frac{(a-b) \tan\left(\frac{x}{2}\right) + c}{\sqrt{-a^2 + b^2 + c^2}} \right)}{\sqrt{-a^2 + b^2 + c^2}}}{b^2 + c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x]),x]

[Out] ((-2*(-(a*b*B) + A*(b^2 + c^2))*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + B*(b*x + c*Log[a + b*Cos[x] + c*Sin[x]]))/(b^2 + c^2)

fricas [B] time = 1.78, size = 625, normalized size = 5.43

$$\left[\frac{(Bab - Ab^2 - Ac^2) \sqrt{-a^2 + b^2 + c^2} \log \left(\frac{a^2 b^2 - 2 b^4 - c^4 - (a^2 + 3 b^2) c^2 - (2 a^2 b^2 - b^4 - 2 a^2 c^2 + c^4) \cos(x)^2 - 2 (ab^3 + abc^2) \cos(x) - 2 (ab^2 c + ac^3) \sin(x)}{2 ab \cos(x) + (b^2 - c^2) \cos(x)} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")

[Out] [-1/2*((B*a*b - A*b^2 - A*c^2)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)) - 2*(B*a^2*b - B*b^3 - B*b*c^2)*x + (B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), -1/2*(2*(B*a*b - A*b^2 - A*c^2)*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2))/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x)) - 2*(B*a^2*b - B*b^3 - B*b*c^2)*x + (B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]

giac [A] time = 0.18, size = 178, normalized size = 1.55

$$\frac{Bbx}{b^2 + c^2} + \frac{Bc \log \left(-a \tan \left(\frac{1}{2} x \right)^2 + b \tan \left(\frac{1}{2} x \right)^2 - 2c \tan \left(\frac{1}{2} x \right) - a - b \right)}{b^2 + c^2} - \frac{Bc \log \left(\tan \left(\frac{1}{2} x \right)^2 + 1 \right)}{b^2 + c^2} + \frac{2(Bab - Ab^2 - Ac^2)}{b^2 + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")

[Out] $B*b*x/(b^2 + c^2) + B*c*\log(-a*\tan(1/2*x)^2 + b*\tan(1/2*x)^2 - 2*c*\tan(1/2*x) - a - b)/(b^2 + c^2) - B*c*\log(\tan(1/2*x)^2 + 1)/(b^2 + c^2) + 2*(B*a*b - A*b^2 - A*c^2)*(pi*\text{floor}(1/2*x/pi + 1/2)*\text{sgn}(-2*a + 2*b) + \arctan(-(a*\tan(1/2*x) - b*\tan(1/2*x) + c)/\sqrt{a^2 - b^2 - c^2}))/(\sqrt{a^2 - b^2 - c^2}*(b^2 + c^2))$

maple [B] time = 0.12, size = 544, normalized size = 4.73

$$\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right) - b\left(\tan^2\left(\frac{x}{2}\right)\right) + 2c\tan\left(\frac{x}{2}\right) + a + b\right)}{(b^2 + c^2)(a - b)} - \frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right) - b\left(\tan^2\left(\frac{x}{2}\right)\right) + 2c\tan\left(\frac{x}{2}\right) + a + b\right)}{(b^2 + c^2)(a - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x)

[Out] $1/(b^2+c^2)/(a-b)*\ln(a*\tan(1/2*x)^2-b*\tan(1/2*x)^2+2*c*\tan(1/2*x)+a+b)*a*B*c-1/(b^2+c^2)/(a-b)*\ln(a*\tan(1/2*x)^2-b*\tan(1/2*x)^2+2*c*\tan(1/2*x)+a+b)*b*B*c+2/(b^2+c^2)/(a^2-b^2-c^2)^{(1/2)}*\arctan(1/2*(2*(a-b)*\tan(1/2*x)+2*c)/(a^2-b^2-c^2)^{(1/2)})*A*b^2+2/(b^2+c^2)/(a^2-b^2-c^2)^{(1/2)}*\arctan(1/2*(2*(a-b)*\tan(1/2*x)+2*c)/(a^2-b^2-c^2)^{(1/2)})*A*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^{(1/2)}*\arctan(1/2*(2*(a-b)*\tan(1/2*x)+2*c)/(a^2-b^2-c^2)^{(1/2)})*a*b*B+2/(b^2+c^2)/(a^2-b^2-c^2)^{(1/2)}*\arctan(1/2*(2*(a-b)*\tan(1/2*x)+2*c)/(a^2-b^2-c^2)^{(1/2)})*B*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^{(1/2)}*\arctan(1/2*(2*(a-b)*\tan(1/2*x)+2*c)/(a^2-b^2-c^2)^{(1/2)})*c^2/(a-b)*a*B+2/(b^2+c^2)/(a^2-b^2-c^2)^{(1/2)}*\arctan(1/2*(2*(a-b)*\tan(1/2*x)+2*c)/(a^2-b^2-c^2)^{(1/2)})*c^2/(a-b)*b*B-B/(b^2+c^2)*c*\ln(1+\tan(1/2*x)^2)+2*B/(b^2+c^2)*b*\arctan(\tan(1/2*x))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see 'assume?' for more details)Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 25.56, size = 1709, normalized size = 14.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(x))/(a + b*cos(x) + c*sin(x)),x)

[Out] $(\log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*B^2*b^2 - 32*A^2*B*b^2 - 32*B^3*a*b + 32*A^2*B*a*b - ((B*c^3 + A*b^2*(b^2 - a^2 + c^2)^{(1/2)} - B*a^2*c + A*c^2*(b^2 - a^2 + c^2)^{(1/2)} + B*b^2*c - B*a*b*(b^2 - a^2 + c^2)^{(1/2)}))*(64*A^2*b^2*c - 32*B^2*a^2*c + 32*B^2*b^2*c + 32*\tan(x/2)*(a - b)*(A^2*b^2 + 2*B^2*a^2 - A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 4*A*B*c^2 - 2*B^2*a*b - 2*A*B*a*b) + 64*A*B*a^2*c - 64*A*B*b^2*c - 64*A^2*a*b*c + ((B*c^3 + A*b^2*(b^2 - a^2 + c^2)^{(1/2)} - B*a^2*c + A*c^2*(b^2 - a^2 + c^2)^{(1/2)} + B*b^2*c - B*a*b*(b^2 - a^2 + c^2)^{(1/2)}))*(32*A*a^2*c^2 - 32*B*b^4 - 32*A*a^2*b^2 - 32*A*b^4 - 32*B*a^2*b^2 - 32*A*b^2*c^2 + 32*B*a^2*c^2 + 64*B*b^2*c^2 + 64*A*a*b^3 + 64*B$

$$\begin{aligned}
& a^3b - 96B^2ac^2 + 32c \tan(x/2)(a-b)(2A^2b^2 + 2A^2c^2 + 4B^2b^2 + B^2c^2 - 2A^2ab - 4B^2ab) \\
& + (32(a-b)(B^2c^3 + A^2b^2(b^2 - a^2 + c^2)^{1/2} - B^2a^2c + A^2c^2(b^2 - a^2 + c^2)^{1/2} + B^2b^2c - B^2ab(b^2 - a^2 + c^2)^{1/2})) \\
& * (3c^4 \tan(x/2) + a^3c + 3b^3c^3 + 3b^3c + 2a^2b^2 \tan(x/2) - 2a^2c^2 \tan(x/2) + 3b^2c^2 \tan(x/2) - 2a^3b \tan(x/2) + a^2b^2c \\
& - 4a^2b^2c - 2a^2b^2c \tan(x/2)) / ((b^2 + c^2)(b^2 - a^2 + c^2)) / ((b^2 + c^2)(b^2 - a^2 + c^2)) / ((b^2 + c^2)(b^2 - a^2 + c^2)) \\
& + 32B^2c \tan(x/2)(A-B)^2(a-b)(B^2c^3 + b^2(A^2(b^2 - a^2 + c^2)^{1/2} + B^2c) - B^2a^2c + A^2c^2(b^2 - a^2 + c^2)^{1/2} - B^2ab(b^2 - a^2 + c^2)^{1/2}) / ((b^2 + c^2)(b^2 - a^2 + c^2)) \\
& - (B \log(\tan(x/2) + 1i)) / (b1i + c) - (B \log(\tan(x/2) - 1i) * 1i) / (b + c * 1i) - (\log(32B^3a^2 - 32A^2B^2a^2 + 32A^2B^2b^2 - 32A^2B^2b^2 - 32B^3a^2b + 32A^2B^2ab - (B^2c^3 - A^2b^2(b^2 - a^2 + c^2)^{1/2} - B^2a^2c - A^2c^2(b^2 - a^2 + c^2)^{1/2} + B^2b^2c + B^2ab(b^2 - a^2 + c^2)^{1/2})) \\
& * (64A^2b^2c - 32B^2a^2c + 32B^2b^2c + 32 \tan(x/2)(a-b)(A^2b^2 + 2B^2a^2 - A^2c^2 + B^2b^2 - 3B^2c^2 + 4A^2B^2c^2 - 2B^2ab - 2A^2B^2ab) + 64A^2B^2a^2c - 64A^2B^2b^2c - 64A^2a^2b^2c + (B^2c^3 - A^2b^2(b^2 - a^2 + c^2)^{1/2} - B^2a^2c - A^2c^2(b^2 - a^2 + c^2)^{1/2} + B^2b^2c + B^2ab(b^2 - a^2 + c^2)^{1/2})) \\
& * (32A^2a^2c^2 - 32B^2b^4 - 32A^2a^2b^2 - 32A^2b^4 - 32B^2a^2b^2 - 32A^2b^2c^2 + 32B^2a^2c^2 + 64B^2b^2c^2 + 64A^2a^2b^3 + 64B^2a^2b^3 - 96B^2ac^2 + 32c \tan(x/2)(a-b)(2A^2b^2 + 2A^2c^2 + 4B^2b^2 + B^2c^2 - 2A^2ab - 4B^2ab) + (32(a-b)(B^2c^3 - A^2b^2(b^2 - a^2 + c^2)^{1/2} - B^2a^2c - A^2c^2(b^2 - a^2 + c^2)^{1/2} + B^2b^2c + B^2ab(b^2 - a^2 + c^2)^{1/2})) \\
& * (3c^4 \tan(x/2) + a^3c + 3b^3c^3 + 3b^3c + 2a^2b^2 \tan(x/2) - 2a^2c^2 \tan(x/2) + 3b^2c^2 \tan(x/2) - 2a^3b \tan(x/2) + a^2b^2c - 4a^2b^2c - 2a^2b^2c \tan(x/2)) / ((b^2 + c^2)(b^2 - a^2 + c^2)) / ((b^2 + c^2)(b^2 - a^2 + c^2)) / ((b^2 + c^2)(b^2 - a^2 + c^2)) \\
& + 32B^2c \tan(x/2)(A-B)^2(a-b)(b^2(A^2(b^2 - a^2 + c^2)^{1/2} - B^2c) - B^2c^3 + B^2a^2c + A^2c^2(b^2 - a^2 + c^2)^{1/2} - B^2ab(b^2 - a^2 + c^2)^{1/2}) / ((b^2 + c^2)(b^2 - a^2 + c^2))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x)

[Out] Timed out

$$3.536 \quad \int \frac{A+B \cos(x)}{(a+b \cos(x)+c \sin(x))^2} dx$$

Optimal. Leaf size=113

$$\frac{2(aA - bB) \tan^{-1} \left(\frac{(a-b) \tan(\frac{x}{2}) + c}{\sqrt{a^2 - b^2 - c^2}} \right)}{(a^2 - b^2 - c^2)^{3/2}} + \frac{-\sin(x)(Ab - aB) + Ac \cos(x) + Bc}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))}$$

[Out] $2*(A*a-B*b)*\arctan((c+(a-b)*\tan(1/2*x))/(a^2-b^2-c^2)^{(1/2)})/(a^2-b^2-c^2)^{(3/2)}+(B*c+A*c*\cos(x)-(A*b-B*a)*\sin(x))/(a^2-b^2-c^2)/(a+b*\cos(x)+c*\sin(x))$

Rubi [A] time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3155, 3124, 618, 204}

$$\frac{2(aA - bB) \tan^{-1} \left(\frac{(a-b) \tan(\frac{x}{2}) + c}{\sqrt{a^2 - b^2 - c^2}} \right)}{(a^2 - b^2 - c^2)^{3/2}} + \frac{-\sin(x)(Ab - aB) + Ac \cos(x) + Bc}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^2,x]

[Out] $(2*(a*A - b*B)*\text{ArcTan}[(c + (a - b)*\text{Tan}[x/2])/ \text{Sqrt}[a^2 - b^2 - c^2]])/(a^2 - b^2 - c^2)^{(3/2)} + (B*c + A*c*\text{Cos}[x] - (A*b - a*B)*\text{Sin}[x])/((a^2 - b^2 - c^2)*(a + b*\text{Cos}[x] + c*\text{Sin}[x]))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3155

Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] := Simp[(c*B + c*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \cos(x)}{(a + b \cos(x) + c \sin(x))^2} dx &= \frac{Bc + Ac \cos(x) - (Ab - aB) \sin(x)}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))} + \frac{(aA - bB) \int \frac{1}{a+b \cos(x)+c \sin(x)} dx}{a^2 - b^2 - c^2} \\
&= \frac{Bc + Ac \cos(x) - (Ab - aB) \sin(x)}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))} + \frac{(2(aA - bB)) \text{Subst} \left(\int \frac{1}{a+b+2cx+(a-b)x^2} \right)}{a^2 - b^2 - c^2} \\
&= \frac{Bc + Ac \cos(x) - (Ab - aB) \sin(x)}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))} - \frac{(4(aA - bB)) \text{Subst} \left(\int \frac{1}{-4(a^2-b^2-c^2)-x^2} \right)}{a^2 - b^2 - c^2} \\
&= \frac{2(aA - bB) \tan^{-1} \left(\frac{c+(a-b) \tan(\frac{x}{2})}{\sqrt{a^2-b^2-c^2}} \right)}{(a^2 - b^2 - c^2)^{3/2}} + \frac{Bc + Ac \cos(x) - (Ab - aB) \sin(x)}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 118, normalized size = 1.04

$$\frac{\sin(x) \left(A(b^2 + c^2) - abB \right) + c(aA - bB)}{b(-a^2 + b^2 + c^2)(a + b \cos(x) + c \sin(x))} + \frac{2(aA - bB) \tanh^{-1} \left(\frac{(a-b) \tan(\frac{x}{2}) + c}{\sqrt{-a^2 + b^2 + c^2}} \right)}{(-a^2 + b^2 + c^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^2,x]

[Out] (2*(a*A - b*B)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(3/2) + ((a*A - b*B)*c + (-a*b*B) + A*(b^2 + c^2))*Sin[x])/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))

fricas [B] time = 1.69, size = 1277, normalized size = 11.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas")

[Out] [-1/2*(2*B*c^5 - 4*(B*a^2 - B*b^2)*c^3 + (A*a^2*b^2 - B*a*b^3 + (A*a^2 - B*a*b)*c^2 + (A*a*b^3 - B*b^4 + (A*a*b - B*b^2)*c^2)*cos(x) + ((A*a - B*b)*c^3 + (A*a*b^2 - B*b^3)*c)*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)) + 2*(B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + 2*(A*c^5 - (A*a^2 + B*a*b - 2*A*b^2)*c^3 + (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*c)*cos(x) - 2*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5 + A*b*c^4 - (A*a^2*b + B*a*b^2 - 2*A*b^3)*c^2)*sin(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x)), -(B*c^5 - 2*(B*a^2 - B*b^2)*c^3 - (A*a^2*b^2 - B*a*b^3 + (A*a^2 - B*a*b)*c^2 + (A*a*b^3 - B*b^4 + (A*a*b - B*b^2)*c^2)*cos(x) + ((A*a - B*b)*c^3 + (A*a*b^2 - B*b^3)*c)*sin(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) + (B*a^4 - 2*B*a^2*b^2 + B*b^4)*c +

$$(A*c^5 - (A*a^2 + B*a*b - 2*A*b^2)*c^3 + (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*c)*\cos(x) - (B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5 + A*b*c^4 - (A*a^2*b + B*a*b^2 - 2*A*b^3)*c^2)*\sin(x))/((a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*\cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*\sin(x))]$$

giac [A] time = 0.19, size = 209, normalized size = 1.85

$$\frac{2 \left(\pi \left[\frac{x}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a + 2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2}x\right) - b \tan\left(\frac{1}{2}x\right) + c}{\sqrt{a^2 - b^2 - c^2}} \right) \right) (Aa - Bb)}{(a^2 - b^2 - c^2)^{\frac{3}{2}}} + \frac{2 \left(Ba^2 \tan\left(\frac{1}{2}x\right) - Aab \tan\left(\frac{1}{2}x\right) \right)}{(a^3 - a^2b - ab^2 + b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")

[Out] $-2*(\pi*\operatorname{floor}(1/2*x/\pi + 1/2)*\operatorname{sgn}(-2*a + 2*b) + \arctan(-(a*\tan(1/2*x) - b*\tan(1/2*x) + c)/\sqrt{a^2 - b^2 - c^2}))* (A*a - B*b)/(a^2 - b^2 - c^2)^{(3/2)} + 2*(B*a^2*\tan(1/2*x) - A*a*b*\tan(1/2*x) - B*a*b*\tan(1/2*x) + A*b^2*\tan(1/2*x) + A*c^2*\tan(1/2*x) - B*c^2*\tan(1/2*x) + A*a*c - B*b*c)/((a^3 - a^2*b - a*b^2 + b^3 - a*c^2 + b*c^2)*(a*\tan(1/2*x)^2 - b*\tan(1/2*x)^2 + 2*c*\tan(1/2*x) + a + b))$

maple [B] time = 0.18, size = 254, normalized size = 2.25

$$\frac{-\frac{2(aAb - Ab^2 - Ac^2 - a^2B + abB + Bc^2)\tan\left(\frac{x}{2}\right)}{a^3 - a^2b - ab^2 - ac^2 + b^3 + c^2b} + \frac{2(aA - bB)c}{a^3 - a^2b - ab^2 - ac^2 + b^3 + c^2b}}{a \left(\tan^2\left(\frac{x}{2}\right) \right) - b \left(\tan^2\left(\frac{x}{2}\right) \right) + 2c \tan\left(\frac{x}{2}\right) + a + b} + \frac{2 \arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right) + 2c}{2\sqrt{a^2 - b^2 - c^2}}\right) aA}{(a^2 - b^2 - c^2)^{\frac{3}{2}}} - \frac{2 \arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right) + 2c}{2\sqrt{a^2 - b^2 - c^2}}\right) aB}{(a^2 - b^2 - c^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x)

[Out] $2*(-(A*a*b - A*b^2 - A*c^2 - B*a^2 + B*a*b + B*c^2)/(a^3 - a^2*b - a*b^2 - a*c^2 + b^3 + b*c^2))*\tan(1/2*x) + (A*a - B*b)*c/(a^3 - a^2*b - a*b^2 - a*c^2 + b^3 + b*c^2))/(a*\tan(1/2*x)^2 - b*\tan(1/2*x)^2 + 2*c*\tan(1/2*x) + a + b) + 2/(a^2 - b^2 - c^2)^{(3/2)}*\arctan(1/2*(2*(a-b)*\tan(1/2*x) + 2*c)/(a^2 - b^2 - c^2)^{(1/2)})*aA - 2/(a^2 - b^2 - c^2)^{(3/2)}*\arctan(1/2*(2*(a-b)*\tan(1/2*x) + 2*c)/(a^2 - b^2 - c^2)^{(1/2)})*bB$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see 'assume?' for more details)Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 3.25, size = 205, normalized size = 1.81

$$\frac{2 \operatorname{atanh} \left(\frac{\tan\left(\frac{x}{2}\right) (2a - 2b) + \frac{2(-a^2c + b^2c + c^3)}{-a^2 + b^2 + c^2}}{2\sqrt{-a^2 + b^2 + c^2}} \right) (Aa - Bb)}{(-a^2 + b^2 + c^2)^{\frac{3}{2}}} - \frac{\frac{2(Aac - Bbc)}{(a-b)(-a^2 + b^2 + c^2)} + \frac{2 \tan\left(\frac{x}{2}\right) (Ab^2 + Ba^2 + Ac^2 - Bc^2 - Aab - Bab)}{(a-b)(-a^2 + b^2 + c^2)}}{(a-b) \tan\left(\frac{x}{2}\right)^2 + 2c \tan\left(\frac{x}{2}\right) + a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

```
[Out] (2*atanh((tan(x/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^2 - a^2 + c^2)))/(2*(b^2 - a^2 + c^2)^(1/2)))*(A*a - B*b)/(b^2 - a^2 + c^2)^(3/2) - ((2*(A*a*c - B*b*c))/((a - b)*(b^2 - a^2 + c^2)) + (2*tan(x/2)*(A*b^2 + B*a^2 + A*c^2 - B*c^2 - A*a*b - B*a*b))/((a - b)*(b^2 - a^2 + c^2)))/(a + b + 2*c*tan(x/2) + tan(x/2)^2*(a - b))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

$$3.537 \quad \int \frac{A+B \cos(x)}{(a+b \cos(x)+c \sin(x))^3} dx$$

Optimal. Leaf size=200

$$\frac{(2a^2A - 3abB + A(b^2 + c^2)) \tan^{-1}\left(\frac{(a-b)\tan(\frac{x}{2})+c}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2 - b^2 - c^2)^{5/2}} + \frac{-\sin(x)(a^2(-B) + 3aAb - 2b^2B) + c \cos(x)(3aA - 2bB)}{2(a^2 - b^2 - c^2)^2 (a + b \cos(x) + c \sin(x))}$$

[Out] (2*a^2*A-3*a*b*B+A*(b^2+c^2))*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^(5/2)+1/2*(B*c+A*c*cos(x)-(A*b-B*a)*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))^2+1/2*(a*B*c+(3*A*a-2*B*b)*c*cos(x)-(3*A*a*b-B*a^2-2*B*b^2)*sin(x))/(a^2-b^2-c^2)^2/(a+b*cos(x)+c*sin(x))

Rubi [A] time = 0.25, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3158, 3153, 3124, 618, 204}

$$\frac{(2a^2A - 3abB + A(b^2 + c^2)) \tan^{-1}\left(\frac{(a-b)\tan(\frac{x}{2})+c}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2 - b^2 - c^2)^{5/2}} + \frac{-\sin(x)(a^2(-B) + 3aAb - 2b^2B) + c \cos(x)(3aA - 2bB)}{2(a^2 - b^2 - c^2)^2 (a + b \cos(x) + c \sin(x))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^3,x]

[Out] ((2*a^2*A - 3*a*b*B + A*(b^2 + c^2))*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^2 - b^2 - c^2)^(5/2) + (B*c + A*c*cos[x] - (A*b - a*B)*Sin[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])^2) + (a*B*c + (3*a*A - 2*b*B)*c*cos[x] - (3*a*A*b - a^2*B - 2*b^2*B)*Sin[x])/(2*(a^2 - b^2 - c^2)^2*(a + b*Cos[x] + c*Sin[x]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3153

Int[((A_.) + cos[(d_.) + (e_.)*(x_)])*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]) / ((a_.) + cos[(d_.) + (e_.)*(x_)])*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]) / (e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B - c*C) / (a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2

- c^2, 0] && NeQ[a*A - b*B - c*C, 0]

Rule 3158

Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^(n_), x_Symbol] :> -Simp[((c*B + c*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[n, -2]

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(x)}{(a + b \cos(x) + c \sin(x))^3} dx &= \frac{Bc + Ac \cos(x) - (Ab - aB) \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} - \frac{\int \frac{-2(aA - bB) + (Ab - aB) \cos(x) + Ac \sin(x)}{(a + b \cos(x) + c \sin(x))^2} dx}{2(a^2 - b^2 - c^2)} \\ &= \frac{Bc + Ac \cos(x) - (Ab - aB) \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} + \frac{aBc + (3aA - 2bB)c \cos(x) - (3aA - 2bB)c \sin(x)}{2(a^2 - b^2 - c^2)^2(a + b \cos(x) + c \sin(x))} \\ &= \frac{Bc + Ac \cos(x) - (Ab - aB) \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} + \frac{aBc + (3aA - 2bB)c \cos(x) - (3aA - 2bB)c \sin(x)}{2(a^2 - b^2 - c^2)^2(a + b \cos(x) + c \sin(x))} \\ &= \frac{Bc + Ac \cos(x) - (Ab - aB) \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} + \frac{aBc + (3aA - 2bB)c \cos(x) - (3aA - 2bB)c \sin(x)}{2(a^2 - b^2 - c^2)^2(a + b \cos(x) + c \sin(x))} \\ &= \frac{(2a^2A - 3abB + A(b^2 + c^2)) \tan^{-1}\left(\frac{c + (a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2 - c^2}}\right)}{(a^2 - b^2 - c^2)^{5/2}} + \frac{Bc + Ac \cos(x) - (Ab - aB) \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))} \end{aligned}$$

Mathematica [A] time = 0.94, size = 326, normalized size = 1.63

$$\frac{-6a^3Ac + 4a^3bB \sin(x) - 2bc \cos(x) (2a^2A - 3abB + A(b^2 + c^2)) + c \cos(2x) (a^2(-b)B + 3aA(b^2 + c^2) - 2bB(b^2 + c^2))}{(a^2 - b^2 - c^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^3, x]

[Out] -(((2*a^2*A - 3*a*b*B + A*(b^2 + c^2))*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2)) + (-6*a^3*A*c - 3*a*A*b^2*c + 9*a^2*b*B*c - 3*a*A*c^3 - 2*b*c*(2*a^2*A - 3*a*b*B + A*(b^2 + c^2))*Cos[x] + c*(-(a^2*b*B) + 3*a*A*(b^2 + c^2) - 2*b*B*(b^2 + c^2))*Cos[2*x] - 8*a^2*A*b^2*Sin[x] + 2*A*b^4*Sin[x] + 4*a^3*b*B*Sin[x] + 2*a*b^3*B*Sin[x] - 12*a^2*A*c^2*Sin[x] + 2*A*b^2*c^2*Sin[x] + 8*a*b*B*c^2*Sin[x] - 3*a*A*b^3*Sin[2*x] + a^2*b^2*B*Sin[2*x] + 2*b^4*B*Sin[2*x] - 3*a*A*b*c^2*Sin[2*x] + 2*b^2*B*c^2*Sin[2*x])/(4*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)

fricas [B] time = 2.16, size = 3402, normalized size = 17.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*(2*B*c^7 - 2*(3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + 2*(3*B*a^4 - 3*A*a^3*b \\ & - 5*B*a^2*b^2 + 6*A*a*b^3 - B*b^4)*c^3 - 4*((3*A*a*b - 2*B*b^2)*c^5 - (3*A \\ & a^3*b - B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^3 + (B*a^4*b^2 - 3*A*a^3*b^3 + B \\ & a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*c)*\cos(x)^2 - (2*A*a^4*b^2 - 3*B*a^3*b^3 + \\ & A*a^2*b^4 + A*c^6 + (3*A*a^2 - 3*B*a*b + 2*A*b^2)*c^4 + (2*A*a^4 - 3*B*a^3*b \\ & + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6 \\ & + A*b^4*c^2 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*\cos(x)^2 + 2*(2*A*a \\ & ^3*b^3 - 3*B*a^2*b^4 + A*a*b^5 + A*a*b*c^4 + (2*A*a^3*b - 3*B*a^2*b^2 + 2*A \\ & a*b^3)*c^2)*\cos(x) + 2*(A*a*c^5 + (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 + \\ & (2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*c + (A*b*c^5 + (2*A*a^2*b - 3*B*a*b^2 \\ & + 2*A*b^3)*c^3 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*c)*\cos(x))*\sin(x))*\sqrt{ \\ & (-a^2 + b^2 + c^2)*\log(-a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2 \\ & *b^2 - b^4 - 2*a^2*c^2 + c^4)*\cos(x)^2 - 2*(a*b^3 + a*b*c^2)*\cos(x) - 2*(a \\ & b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*\cos(x))*\sin(x) + 2*(2*a*b*c*\cos \\ & (x)^2 - a*b*c + (b^2*c + c^3)*\cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*\cos(x) \\ &))*\sin(x))*\sqrt{-a^2 + b^2 + c^2})/(2*a*b*\cos(x) + (b^2 - c^2)*\cos(x)^2 + a \\ & ^2 + c^2 + 2*(b*c*\cos(x) + a*c)*\sin(x)) - 2*(B*a^6 - 4*B*a^4*b^2 + 3*A*a^3 \\ & *b^3 + 2*B*a^2*b^4 - 3*A*a*b^5 + B*b^6)*c + 2*(A*c^7 - (5*A*a^2 - B*a*b - 3 \\ & *A*b^2)*c^5 + (4*A*a^4 + B*a^3*b - 10*A*a^2*b^2 + 2*B*a*b^3 + 3*A*b^4)*c^3 \\ & - (2*B*a^5*b - 4*A*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4 - B*a*b^5 - A*b^6)*c)* \\ & \cos(x) + 2*(2*B*a^5*b^2 - 4*A*a^4*b^3 - B*a^3*b^4 + 5*A*a^2*b^5 - B*a*b^6 - \\ & A*b^7 - A*b*c^6 + (5*A*a^2*b - B*a*b^2 - 3*A*b^3)*c^4 - (4*A*a^4*b + B*a^3 \\ & *b^2 - 10*A*a^2*b^3 + 2*B*a*b^4 + 3*A*b^5)*c^2 + (B*a^4*b^3 - 3*A*a^3*b^4 + \\ & B*a^2*b^5 + 3*A*a*b^6 - 2*B*b^7 - (3*A*a - 2*B*b)*c^6 + (3*A*a^3 - B*a^2*b \\ & - 3*A*a*b^2 + 2*B*b^3)*c^4 - (B*a^4*b - 3*A*a*b^4 + 2*B*b^5)*c^2)*\cos(x))* \\ & \sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10 + 2*(a^2 - 2*b^2) \\ & *c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4 \\ & + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6 \\ & + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^4) \\ & *c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 + b^8)*c^2) \\ & *\cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 + (3*a^3*b \\ & - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6*a^5*b^3 \\ & + 9*a^3*b^5 - 4*a*b^7)*c^2)*\cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(\\ & a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^3 \\ & - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*b - 4*b^3) \\ &)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9*a^2*b^5 \\ & - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*\cos(x))*\sin(x)), \\ & 1/2*(B*c^7 - (3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + (3*B*a^4 - 3*A*a^3*b - 5*B*a \\ & ^2*b^2 + 6*A*a*b^3 - B*b^4)*c^3 - 2*((3*A*a*b - 2*B*b^2)*c^5 - (3*A*a^3*b - \\ & B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^3 + (B*a^4*b^2 - 3*A*a^3*b^3 + B*a^2*b^4 \\ & + 3*A*a*b^5 - 2*B*b^6)*c)*\cos(x)^2 + (2*A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 \\ & + A*c^6 + (3*A*a^2 - 3*B*a*b + 2*A*b^2)*c^4 + (2*A*a^4 - 3*B*a^3*b + 4*A \\ & a^2*b^2 - 3*B*a*b^3 + A*b^4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6 + A*b^4 \\ & c^2 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*\cos(x)^2 + 2*(2*A*a^3*b^3 \\ & - 3*B*a^2*b^4 + A*a*b^5 + A*a*b*c^4 + (2*A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3) \\ & *c^2)*\cos(x) + 2*(A*a*c^5 + (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 + (2*A*a^3 \\ & b^2 - 3*B*a^2*b^3 + A*a*b^4)*c + (A*b*c^5 + (2*A*a^2*b - 3*B*a*b^2 + 2*A \\ & b^3)*c^3 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*c)*\cos(x))*\sin(x))*\sqrt{a^2 - \\ & b^2 - c^2}*\arctan(-(a*b*\cos(x) + a*c*\sin(x) + b^2 + c^2)*\sqrt{a^2 - b^2 - c \\ & ^2})/((c^3 - (a^2 - b^2)*c)*\cos(x) + (a^2*b - b^3 - b*c^2)*\sin(x))) - (B*a^6 \\ & - 4*B*a^4*b^2 + 3*A*a^3*b^3 + 2*B*a^2*b^4 - 3*A*a*b^5 + B*b^6)*c + (A*c^7 \\ & - (5*A*a^2 - B*a*b - 3*A*b^2)*c^5 + (4*A*a^4 + B*a^3*b - 10*A*a^2*b^2 + 2*B \\ & a*b^3 + 3*A*b^4)*c^3 - (2*B*a^5*b - 4*A*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4 \\ & - B*a*b^5 - A*b^6)*c)*\cos(x) + (2*B*a^5*b^2 - 4*A*a^4*b^3 - B*a^3*b^4 + 5*A \\ & a^2*b^5 - B*a*b^6 - A*b^7 - A*b*c^6 + (5*A*a^2*b - B*a*b^2 - 3*A*b^3)*c^4 \\ & - (4*A*a^4*b + B*a^3*b^2 - 10*A*a^2*b^3 + 2*B*a*b^4 + 3*A*b^5)*c^2 + (B*a^4 \end{aligned}$$

```

*b^3 - 3*A*a^3*b^4 + B*a^2*b^5 + 3*A*a*b^6 - 2*B*b^7 - (3*A*a - 2*B*b)*c^6
+ (3*A*a^3 - B*a^2*b - 3*A*a*b^2 + 2*B*b^3)*c^4 - (B*a^4*b - 3*A*a*b^4 + 2*
B*b^5)*c^2)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^
10 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3
*a^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 +
(a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^
4 - 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2
*a^2*b^6 + b^8)*c^2)*cos(x))^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9
- a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 +
(a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3
- 4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a
^3*b^4 - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^
9 - (3*a^2*b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6
a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*
c)*cos(x))*sin(x))]

```

giac [B] time = 0.55, size = 1162, normalized size = 5.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(2*A*a^2 - 3*B*a*b + A*b^2 + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/((a
^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^2 - c^2))
+ (2*B*a^5*tan(1/2*x)^3 - 4*A*a^4*b*tan(1/2*x)^3 - 5*B*a^4*b*tan(1/2*x)^3 +
11*A*a^3*b^2*tan(1/2*x)^3 + 5*B*a^3*b^2*tan(1/2*x)^3 - 9*A*a^2*b^3*tan(1/2
*x)^3 - 5*B*a^2*b^3*tan(1/2*x)^3 + A*a*b^4*tan(1/2*x)^3 + 5*B*a*b^4*tan(1/2
*x)^3 + A*b^5*tan(1/2*x)^3 - 2*B*b^5*tan(1/2*x)^3 + 5*A*a^3*c^2*tan(1/2*x)^
3 - 4*B*a^3*c^2*tan(1/2*x)^3 - 7*A*a^2*b*c^2*tan(1/2*x)^3 + 4*B*a^2*b*c^2*t
an(1/2*x)^3 - A*a*b^2*c^2*tan(1/2*x)^3 + 4*B*a*b^2*c^2*tan(1/2*x)^3 + 3*A*b
^3*c^2*tan(1/2*x)^3 - 4*B*b^3*c^2*tan(1/2*x)^3 - 2*A*a*c^4*tan(1/2*x)^3 + 2
*B*a*c^4*tan(1/2*x)^3 + 2*A*b*c^4*tan(1/2*x)^3 - 2*B*b*c^4*tan(1/2*x)^3 + 4
*A*a^4*c*tan(1/2*x)^2 + 2*B*a^4*c*tan(1/2*x)^2 - 12*A*a^3*b*c*tan(1/2*x)^2
- 9*B*a^3*b*c*tan(1/2*x)^2 + 13*A*a^2*b^2*c*tan(1/2*x)^2 + 14*B*a^2*b^2*c*t
an(1/2*x)^2 - 6*A*a*b^3*c*tan(1/2*x)^2 - 9*B*a*b^3*c*tan(1/2*x)^2 + A*b^4*c
*tan(1/2*x)^2 + 2*B*b^4*c*tan(1/2*x)^2 + 7*A*a^2*c^3*tan(1/2*x)^2 - 4*B*a^2
*c^3*tan(1/2*x)^2 - 6*A*a*b*c^3*tan(1/2*x)^2 - A*b^2*c^3*tan(1/2*x)^2 + 4*B
*b^2*c^3*tan(1/2*x)^2 - 2*A*c^5*tan(1/2*x)^2 + 2*B*c^5*tan(1/2*x)^2 + 2*B*a
^5*tan(1/2*x) - 4*A*a^4*b*tan(1/2*x) - 3*B*a^4*b*tan(1/2*x) + 5*A*a^3*b^2*t
an(1/2*x) + B*a^3*b^2*tan(1/2*x) + 3*A*a^2*b^3*tan(1/2*x) + B*a^2*b^3*tan(1
/2*x) - 5*A*a*b^4*tan(1/2*x) - 3*B*a*b^4*tan(1/2*x) + A*b^5*tan(1/2*x) + 2*
B*b^5*tan(1/2*x) + 11*A*a^3*c^2*tan(1/2*x) - 4*B*a^3*c^2*tan(1/2*x) - 3*A*a
^2*b*c^2*tan(1/2*x) - 8*B*a^2*b*c^2*tan(1/2*x) - 7*A*a*b^2*c^2*tan(1/2*x) +
8*B*a*b^2*c^2*tan(1/2*x) - A*b^3*c^2*tan(1/2*x) + 4*B*b^3*c^2*tan(1/2*x) -
2*A*a*c^4*tan(1/2*x) + 2*B*a*c^4*tan(1/2*x) - 2*A*b*c^4*tan(1/2*x) + 2*B*b
*c^4*tan(1/2*x) + 4*A*a^4*c - 5*B*a^3*b*c - 3*A*a^2*b^2*c + 5*B*a*b^3*c - A
*b^4*c - A*a^2*c^3 + 2*B*a*b*c^3 - A*b^2*c^3)/(a^6 - 2*a^5*b - a^4*b^2 + 4
*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*a*b^3*c^2
+ 2*b^4*c^2 + a^2*c^4 - 2*a*b*c^4 + b^2*c^4)*(a*tan(1/2*x)^2 - b*tan(1/2*x)
^2 + 2*c*tan(1/2*x) + a + b)^2)

```

maple [B] time = 0.21, size = 1109, normalized size = 5.54

$$\frac{(4A^3b - 7A^2b^2 - 5A^2c^2 + 2Aab^3 + 2Aabc^2 + Ab^4 + 3Ab^2c^2 + 2Ac^4 - 2Ba^4 + 3Ba^3b - 2Ba^2b^2 + 4Ba^2c^2 + 3Bab^3 - 2Bb^4 - 4Bb^2c^2 - 2Bc^4)(\tan^3(\frac{x}{2}))}{(a-b)(a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4)} + \frac{c}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x)

[Out]
$$2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2-5*A*a^2*c^2+2*A*a*b^3+2*A*a*b*c^2+A*b^4+3*A*b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2+4*B*a^2*c^2+3*B*a*b^3-2*B*b^4-4*B*b^2*c^2-2*B*c^4)/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*\tan(1/2*x)^3+1/2*c*(4*A*a^4-12*A*a^3*b+13*A*a^2*b^2+7*A*a^2*c^2-6*A*a*b^3-6*A*a*b*c^2+A*b^4-A*b^2*c^2-2*A*c^4+2*B*a^4-9*B*a^3*b+14*B*a^2*b^2-4*B*a^2*c^2-9*B*a*b^3+2*B*b^4+4*B*b^2*c^2+2*B*c^4)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*\tan(1/2*x)^2-1/2*(4*A*a^4*b-5*A*a^3*b^2-11*A*a^3*c^2-3*A*a^2*b^3+3*A*a^2*b*c^2+5*A*a*b^4+7*A*a*b^2*c^2+2*A*a*c^4-A*b^5+A*b^3*c^2+2*A*b*c^4-2*B*a^5+3*B*a^4*b-B*a^3*b^2+4*B*a^3*c^2-B*a^2*b^3+8*B*a^2*b*c^2+3*B*a*b^4-8*B*a*b^2*c^2-2*B*a*c^4-2*B*b^5-4*B*b^3*c^2-2*B*b*c^4)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*\tan(1/2*x)+1/2*c*(4*A*a^4-3*A*a^2*b^2-A*a^2*c^2-A*b^4-A*b^2*c^2-5*B*a^3*b+5*B*a*b^3+2*B*a*b*c^2)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*\tan(1/2*x)^2-b*\tan(1/2*x)^2+2*c*\tan(1/2*x)+a+b)^2+2/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*\arctan(1/2*(2*(a-b)*\tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a^2*A+1/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*\arctan(1/2*(2*(a-b)*\tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*b^2+1/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*\arctan(1/2*(2*(a-b)*\tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*c^2-3/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*\arctan(1/2*(2*(a-b)*\tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*b*B$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see 'assume?' for more details)Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 6.84, size = 946, normalized size = 4.73

$$\frac{-4Aa^4c+5Ba^3bc+3Aa^2b^2c+Aa^2c^3-5Bab^3c-2Babc^3+Ab^4c+Ab^2c^3}{(a-b)^2(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)} - \frac{\tan(\frac{x}{2})(Ab^5+2Ba^5+2Bb^5+3Aa^2b^3+5Aa^3b^2+11Aa^3c^2+B)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(x))/(a + b*cos(x) + c*sin(x))^3,x)

[Out]
$$-((A*a^2*c^3 + A*b^2*c^3 - 4*A*a^4*c + A*b^4*c - 2*B*a*b*c^3 - 5*B*a*b^3*c + 5*B*a^3*b*c + 3*A*a^2*b^2*c)/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) - (\tan(x/2)*(A*b^5 + 2*B*a^5 + 2*B*b^5 + 3*A*a^2*b^3 + 5*A*a^3*b^2 + 11*A*a^3*c^2 + B*a^2*b^3 + B*a^3*b^2 - A*b^3*c^2 - 4*B*a^3*c^2 + 4*B*b^3*c^2 - 5*A*a*b^4 - 4*A*a^4*b - 2*A*a*c^4 - 3*B*a*b^4 - 3*B*a^4*b - 2*A*b*c^4 + 2*B*a*c^4 + 2*B*b*c^4 - 7*A*a*b^2*c^2 - 3*A*a^2*b*c^2 + 8*B*a*b^2*c^2 - 8*B*a^2*b*c^2))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) - (\tan(x/2)^2*(2*B*c^5 - 2*A*c^5 + 7*A*a^2*c^3 - A*b^2*c^3 - 4*B*a^2*c^3 + 4*B*b^2*c^3 + 4*A*a^4*c + A*b^4*c + 2*B*a^4*c + 2*B*b^4*c - 6*A*a*b*c^3 - 6*A*a*b^3*c - 12*A*a^3*b*c - 9*B*a*b^3*c - 9*B*a^3*b*c + 13*A*a^2*b^2*c + 14*B*a^2*b^2*c))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (\tan(x/2)^3*(A*b^4 - 2*B*a^4 + 2*A*c^4 - 2*B*b^4 - 2*B*c^4 - 7*A*a^2*b^2 - 5*A*a^2*c^2 - 2*B*a^2*b^2 + 3*A*b^2*c^2 + 4*B*a^2*c^2 - 4*B*b^2*c^2 + 2*A*a*b^3 + 4*A*a^3*b + 3*B*a*b^3 + 3*B*a^3*b + 2*$$

$$\frac{A*a*b*c^2)}{((a - b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2))} / (\tan(x/2)^4*(a^2 - 2*a*b + b^2) + 2*a*b + \tan(x/2)*(4*a*c + 4*b*c) + \tan(x/2)^3*(4*a*c - 4*b*c) + a^2 + b^2 + \tan(x/2)^2*(2*a^2 - 2*b^2 + 4*c^2)) - (\operatorname{atanh}((2*a^4*c + 2*b^4*c + 2*c^5 - 4*a^2*c^3 + 4*b^2*c^3 - 4*a^2*b^2*c)/(2*(b^2 - a^2 + c^2)^{5/2})) + (\tan(x/2)*(2*a - 2*b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2))/(2*(b^2 - a^2 + c^2)^{5/2}))* (2*A*a^2 + A*b^2 + A*c^2 - 3*B*a*b))/(b^2 - a^2 + c^2)^{5/2}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))**3,x)

[Out] Timed out

$$3.538 \quad \int \frac{A+B \cos(x)}{a+b \cos(x)+ib \sin(x)} dx$$

Optimal. Leaf size=84

$$\frac{i(a^2(-B) + 2aAb - b^2B) \log(a + ib \sin(x) + b \cos(x))}{2a^2b} + \frac{x(2aA - bB)}{2a^2} + \frac{B \sin(x)}{2a} + \frac{iB \cos(x)}{2a}$$

[Out] $1/2*(2*A*a-B*b)*x/a^2+1/2*I*B*\cos(x)/a+1/2*I*(2*A*a*b-B*a^2-B*b^2)*\ln(a+b*\cos(x)+I*b*\sin(x))/a^2/b+1/2*B*\sin(x)/a$

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3132}

$$\frac{i(a^2(-B) + 2aAb - b^2B) \log(a + ib \sin(x) + b \cos(x))}{2a^2b} + \frac{x(2aA - bB)}{2a^2} + \frac{B \sin(x)}{2a} + \frac{iB \cos(x)}{2a}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[x])/(a + b*Cos[x] + I*b*Sin[x]),x]

[Out] $((2*a*A - b*B)*x)/(2*a^2) + ((I/2)*B*\cos[x])/a + ((I/2)*(2*a*A*b - a^2*B - b^2*B)*\log[a + b*\cos[x] + I*b*\sin[x]])/(a^2*b) + (B*\sin[x])/(2*a)$

Rule 3132

Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.))/(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]), x_Symbol] :> Simp[((2*a*A - b*B)*x)/(2*a^2), x] + (Simp[(B*Sin[d + e*x])/(2*a*e), x] - Simp[(b*B*Cos[d + e*x])/(2*a*c*e), x] + Simp[((a^2*B - 2*a*b*A + b^2*B)*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[b^2 + c^2, 0]

Rubi steps

$$\int \frac{A + B \cos(x)}{a + b \cos(x) + ib \sin(x)} dx = \frac{(2aA - bB)x}{2a^2} + \frac{iB \cos(x)}{2a} + \frac{i(2aAb - a^2B - b^2B) \log(a + b \cos(x) + ib \sin(x))}{2a^2b}$$

Mathematica [A] time = 0.22, size = 147, normalized size = 1.75

$$\frac{2(a^2B - 2aAb + b^2B) \tan^{-1}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right) + 2iaAb \log(a^2 + 2ab \cos(x) + b^2) - ia^2B \log(a^2 + 2ab \cos(x) + b^2)}{4a^2b}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] + I*b*Sin[x]),x]

[Out] $(2*a*A*b*x + a^2*B*x - b^2*B*x + 2*(-2*a*A*b + a^2*B + b^2*B)*\text{ArcTan}[\frac{(a + b)*\text{Cot}[x/2]}{a - b}] + (2*I)*a*b*B*\cos[x] + (2*I)*a*A*b*\log[a^2 + b^2 + 2*a*b*\cos[x]] - I*a^2*B*\log[a^2 + b^2 + 2*a*b*\cos[x]] - I*b^2*B*\log[a^2 + b^2 + 2*a*b*\cos[x]] + 2*a*b*B*\sin[x])/(4*a^2*b)$

fricas [A] time = 0.76, size = 72, normalized size = 0.86

$$\frac{(iBab + (2Aab - Bb^2)xe^{ix} + (-iBa^2 + 2iAab - iBb^2)e^{ix} \log\left(\frac{be^{ix}+a}{b}\right))e^{-ix}}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*B*a*b + (2*A*a*b - B*b^2)*x*e^(I*x) + (-I*B*a^2 + 2*I*A*a*b - I*B*b^2)*e^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2*b)
```

giac [B] time = 0.16, size = 168, normalized size = 2.00

$$\frac{(-2iAa + iBb) \log\left(-a \tan\left(\frac{1}{2}x\right)^2 + b \tan\left(\frac{1}{2}x\right)^2 - 2ia \tan\left(\frac{1}{2}x\right) + a + b\right)}{4a^2} - \frac{(2iAa - iBb) \log\left(\tan\left(\frac{1}{2}x\right) - i\right)}{2a^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -1/4*(-2*I*A*a + I*B*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*I*a*tan(1/2*x) + a + b)/a^2 - 1/2*(2*I*A*a - I*B*b)*log(tan(1/2*x) - I)/a^2 + 1/4*(2*B*a^2 - 2*A*a*b + B*b^2)*(x + 2*arctan((-I*a*cos(x) - a*sin(x) - I*a)/(a*cos(x) - I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(-2*I*A*a*tan(1/2*x) + I*B*b*tan(1/2*x) - 2*A*a - 2*B*a + B*b)/(a^2*(tan(1/2*x) - I))
```

maple [B] time = 0.18, size = 153, normalized size = 1.82

$$\frac{iB \ln\left(\tan\left(\frac{x}{2}\right) + i\right)}{2b} - \frac{i \ln\left(\tan\left(\frac{x}{2}\right) - i\right)A}{a} + \frac{i \ln\left(\tan\left(\frac{x}{2}\right) - i\right)bB}{2a^2} + \frac{B}{a\left(\tan\left(\frac{x}{2}\right) - i\right)} + \frac{i \ln\left(ia + ib + a \tan\left(\frac{x}{2}\right) - b \tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] 1/2*I*B/b*ln(tan(1/2*x)+I)-I/a*ln(tan(1/2*x)-I)*A+1/2*I/a^2*ln(tan(1/2*x)-I)*b*B+B/a/(tan(1/2*x)-I)+I/a*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*A-1/2*I/b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*B-1/2*I/a^2*b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 4.33, size = 99, normalized size = 1.18

$$\frac{B}{a\left(\tan\left(\frac{x}{2}\right) - i\right)} + \frac{B \ln\left(\tan\left(\frac{x}{2}\right) + i\right) 1i}{2b} - \frac{\ln\left(\tan\left(\frac{x}{2}\right) - i\right) \left(Aa 1i - \frac{Bb 1i}{2}\right)}{a^2} - \frac{\ln\left(a + b - a \tan\left(\frac{x}{2}\right) 1i + b \tan\left(\frac{x}{2}\right) 1i\right)}{2a^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(a + b*cos(x) + b*sin(x)*1i),x)
```

```
[Out] B/(a*(tan(x/2) - 1i)) + (B*log(tan(x/2) + 1i)*1i)/(2*b) - (log(tan(x/2) - 1i)*(A*a*1i - (B*b*1i)/2))/a^2 - (log(a + b - a*tan(x/2)*1i + b*tan(x/2)*1i)*(B*a^2 + B*b^2 - 2*A*a*b)*1i)/(2*a^2*b)
```

sympy [A] time = 0.58, size = 95, normalized size = 1.13

$$\begin{cases} \frac{iBe^{-ix}}{2a} & \text{for } 2a \neq 0 \\ x \left(-\frac{2Aa-Bb}{2a^2} + \frac{2Aa+Ba-Bb}{2a^2} \right) & \text{otherwise} \end{cases} + \frac{x(2Aa - Bb)}{2a^2} - \frac{i(-2Aab + Ba^2 + Bb^2) \log\left(\frac{a}{b} + e^{ix}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x)

[Out] Piecewise((I*B*exp(-I*x)/(2*a), Ne(2*a, 0)), (x*(-(2*A*a - B*b)/(2*a**2) + (2*A*a + B*a - B*b)/(2*a**2)), True)) + x*(2*A*a - B*b)/(2*a**2) - I*(-2*A*a*b + B*a**2 + B*b**2)*log(a/b + exp(I*x))/(2*a**2*b)

$$3.539 \quad \int \frac{A+B \cos(x)}{a+b \cos(x)-ib \sin(x)} dx$$

Optimal. Leaf size=84

$$\frac{i(a^2(-B) + 2aAb - b^2B) \log(a - ib \sin(x) + b \cos(x))}{2a^2b} + \frac{x(2aA - bB)}{2a^2} + \frac{B \sin(x)}{2a} - \frac{iB \cos(x)}{2a}$$

[Out] 1/2*(2*A*a-B*b)*x/a^2-1/2*I*B*cos(x)/a-1/2*I*(2*A*a*b-B*a^2-B*b^2)*ln(a+b*cos(x)-I*b*sin(x))/a^2/b+1/2*B*sin(x)/a

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3132}

$$\frac{i(a^2(-B) + 2aAb - b^2B) \log(a - ib \sin(x) + b \cos(x))}{2a^2b} + \frac{x(2aA - bB)}{2a^2} + \frac{B \sin(x)}{2a} - \frac{iB \cos(x)}{2a}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[x])/(a + b*Cos[x] - I*b*Sin[x]), x]

[Out] ((2*a*A - b*B)*x)/(2*a^2) - ((I/2)*B*Cos[x])/a - ((I/2)*(2*a*A*b - a^2*B - b^2*B)*Log[a + b*Cos[x] - I*b*Sin[x]])/(a^2*b) + (B*Sin[x])/(2*a)

Rule 3132

Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.))/(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]), x_Symbol] :> Simp[((2*a*A - b*B)*x)/(2*a^2), x] + (Simp[(B*Sin[d + e*x])/(2*a*e), x] - Simp[(b*B*Cos[d + e*x])/(2*a*c*e), x] + Simp[((a^2*B - 2*a*b*A + b^2*B)*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[b^2 + c^2, 0]

Rubi steps

$$\int \frac{A + B \cos(x)}{a + b \cos(x) - ib \sin(x)} dx = \frac{(2aA - bB)x}{2a^2} - \frac{iB \cos(x)}{2a} - \frac{i(2aAb - a^2B - b^2B) \log(a + b \cos(x) - ib \sin(x))}{2a^2b} +$$

Mathematica [A] time = 0.19, size = 147, normalized size = 1.75

$$\frac{2(a^2B - 2aAb + b^2B) \tan^{-1}\left(\frac{(a+b) \cot\left(\frac{x}{2}\right)}{a-b}\right) - 2iaAb \log(a^2 + 2ab \cos(x) + b^2) + ia^2B \log(a^2 + 2ab \cos(x) + b^2)}{4a^2b}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] - I*b*Sin[x]), x]

[Out] (2*a*A*b*x + a^2*B*x - b^2*B*x + 2*(-2*a*A*b + a^2*B + b^2*B)*ArcTan[((a + b)*Cot[x/2])/(a - b)] - (2*I)*a*b*B*Cos[x] - (2*I)*a*A*b*Log[a^2 + b^2 + 2*a*b*Cos[x]] + I*a^2*B*Log[a^2 + b^2 + 2*a*b*Cos[x]] + I*b^2*B*Log[a^2 + b^2 + 2*a*b*Cos[x]] + 2*a*b*B*Sin[x])/(4*a^2*b)

fricas [A] time = 2.14, size = 56, normalized size = 0.67

$$\frac{Ba^2x - iBabe^{ix} + (iBa^2 - 2iAab + iBb^2) \log\left(\frac{ae^{ix} + b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fricas")

[Out] $\frac{1}{2}*(B*a^2*x - I*B*a*b*e^{(I*x)} + (I*B*a^2 - 2*I*A*a*b + I*B*b^2)*\log((a*e^{(I*x)} + b)/a))/(a^2*b)$

giac [B] time = 0.16, size = 168, normalized size = 2.00

$$\frac{(2iAa - iBb) \log\left(-a \tan\left(\frac{1}{2}x\right)^2 + b \tan\left(\frac{1}{2}x\right)^2 + 2ia \tan\left(\frac{1}{2}x\right) + a + b\right)}{4a^2} - \frac{(-2iAa + iBb) \log\left(\tan\left(\frac{1}{2}x\right) + i\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac")

[Out] $-\frac{1}{4}*(2*I*A*a - I*B*b)*\log(-a*\tan(1/2*x)^2 + b*\tan(1/2*x)^2 + 2*I*a*\tan(1/2*x) + a + b)/a^2 - \frac{1}{2}*(-2*I*A*a + I*B*b)*\log(\tan(1/2*x) + I)/a^2 + \frac{1}{4}*(2*B*a^2 - 2*A*a*b + B*b^2)*(x + 2*\arctan((I*a*\cos(x) - a*\sin(x) + I*a)/(a*\cos(x) + I*a*\sin(x) - a + 2*b)))/(a^2*b) - \frac{1}{2}*(2*I*A*a*\tan(1/2*x) - I*B*b*\tan(1/2*x) - 2*A*a - 2*B*a + B*b)/(a^2*(\tan(1/2*x) + I))$

maple [B] time = 0.18, size = 284, normalized size = 3.38

$$\frac{i \ln\left(\tan\left(\frac{x}{2}\right) + i\right) A}{a} - \frac{i \ln\left(\tan\left(\frac{x}{2}\right) + i\right) b B}{2a^2} + \frac{B}{a \left(\tan\left(\frac{x}{2}\right) + i\right)} - \frac{i B \ln\left(\tan\left(\frac{x}{2}\right) - i\right)}{2b} + \frac{i \ln\left(ia + ib - a \tan\left(\frac{x}{2}\right) + b \tan\left(\frac{x}{2}\right)\right)}{-a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x)

[Out] $\frac{I}{a}*\ln(\tan(1/2*x)+I)*A - \frac{1}{2}*I/a^2*\ln(\tan(1/2*x)+I)*b*B + \frac{B}{a*(\tan(1/2*x)+I)} - \frac{1}{2}*I*B/b*\ln(\tan(1/2*x)-I) + \frac{I}{(-a+b)}*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*A - \frac{I}{a*b*(-a+b)}*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*A - \frac{1}{2}*I*a/b*(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*B + \frac{1}{2}*I/(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*B - \frac{1}{2}*I/a*b*(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*B + \frac{1}{2}*I/a^2*b^2/(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

mupad [B] time = 8.39, size = 584, normalized size = 6.95

$$\left(\sum_{k=1}^3 \ln\left(-(a-b)^2 \left(4A^2 a^2 - 4ABab - B^2 a^2 + B^2 b^2\right) \sqrt{a^4 b^2 d^3 64i - ABab^3 d 64i - ABa^3 b d 32i + \dots}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(x))/(a + b*cos(x) - b*sin(x)*1i),x)

[Out] $\text{symsum}(\log(\tan(x/2)*(a-b)^2*(B*a - 2*A*a + B*b)^2 - \text{root}(a^4*b^2*d^3*64i - A*B*a*b^3*d*64i - A*B*a^3*b*d*32i + B^2*a^2*b^2*d*16i + A^2*a^2*b^2*d*64i))$

```

+ B^2*b^4*d*16i + B^2*a^4*d*16i - 32*A^2*B*a^2*b + 32*A*B^2*a*b^2 - 8*B^3*
a^2*b + 16*A*B^2*a^3 - 8*B^3*b^3, d, k)*(4*A*a^3*(a - b)^2 - 8*root(a^4*b^2
*d^3*64i - A*B*a*b^3*d*64i - A*B*a^3*b*d*32i + B^2*a^2*b^2*d*16i + A^2*a^2*
b^2*d*64i + B^2*b^4*d*16i + B^2*a^4*d*16i - 32*A^2*B*a^2*b + 32*A*B^2*a*b^2
- 8*B^3*a^2*b + 16*A*B^2*a^3 - 8*B^3*b^3, d, k)*a^2*(a - b)^2*(a^2*tan(x/2
) + b^2*tan(x/2) - a^2*1i + b^2*1i - a*b*tan(x/2)) + 4*a*tan(x/2)*(a - b)^2
*(A*a^2*1i + B*a^2*1i + B*b^2*1i - A*a*b*2i - B*a*b*1i)) - (a - b)^2*(4*A^2
*a^2 - B^2*a^2 + B^2*b^2 - 4*A*B*a*b)*1i)*root(a^4*b^2*d^3*64i - A*B*a*b^3*
d*64i - A*B*a^3*b*d*32i + B^2*a^2*b^2*d*16i + A^2*a^2*b^2*d*64i + B^2*b^4*d
*16i + B^2*a^4*d*16i - 32*A^2*B*a^2*b + 32*A*B^2*a*b^2 - 8*B^3*a^2*b + 16*A
*B^2*a^3 - 8*B^3*b^3, d, k), k, 1, 3) + B/(a*(tan(x/2) + 1i))

```

sympy [A] time = 0.58, size = 75, normalized size = 0.89

$$\frac{Bx}{2b} + \begin{cases} -\frac{iBe^{ix}}{2a} & \text{for } 2a \neq 0 \\ x\left(-\frac{B}{2b} + \frac{Ba+Bb}{2ab}\right) & \text{otherwise} \end{cases} + \frac{i(-2Aab + Ba^2 + Bb^2) \log\left(e^{ix} + \frac{b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] B*x/(2*b) + Piecewise((-I*B*exp(I*x)/(2*a), Ne(2*a, 0)), (x*(-B/(2*b) + (B*
a + B*b)/(2*a*b)), True)) + I*(-2*A*a*b + B*a**2 + B*b**2)*log(exp(I*x) + b
/a)/(2*a**2*b)
```


$$3.540 \quad \int \frac{A+C \sin(x)}{a+b \cos(x)+c \sin(x)} dx$$

Optimal. Leaf size=116

$$\frac{2(A(b^2+c^2)-acC) \tan^{-1}\left(\frac{(a-b)\tan(\frac{x}{2})+c}{\sqrt{a^2-b^2-c^2}}\right)}{(b^2+c^2)\sqrt{a^2-b^2-c^2}} - \frac{bC \log(a+b \cos(x)+c \sin(x))}{b^2+c^2} + \frac{cCx}{b^2+c^2}$$

[Out] $cCx/(b^2+c^2)-bC*\ln(a+b*\cos(x)+c*\sin(x))/(b^2+c^2)+2*(A*(b^2+c^2)-a*c*C)*\arctan((c+(a-b)*\tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)$

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3137, 3124, 618, 204}

$$\frac{2(A(b^2+c^2)-acC) \tan^{-1}\left(\frac{(a-b)\tan(\frac{x}{2})+c}{\sqrt{a^2-b^2-c^2}}\right)}{(b^2+c^2)\sqrt{a^2-b^2-c^2}} - \frac{bC \log(a+b \cos(x)+c \sin(x))}{b^2+c^2} + \frac{cCx}{b^2+c^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + C*\text{Sin}[x])/(a + b*\text{Cos}[x] + c*\text{Sin}[x]),x]$

[Out] $(cCx)/(b^2+c^2) + (2*(A*(b^2+c^2) - a*c*C)*\text{ArcTan}[(c + (a - b)*\text{Tan}[x/2])/ \text{Sqrt}[a^2 - b^2 - c^2]])/(\text{Sqrt}[a^2 - b^2 - c^2]*(b^2 + c^2)) - (bC*\text{Log}[a + b*\text{Cos}[x] + c*\text{Sin}[x]])/(b^2 + c^2)$

Rule 204

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 3124

$\text{Int}[(\cos[(d_ + (e_)*(x_))]*(b_ + (a_ + (c_)*\sin[(d_ + (e_)*(x_)]))^{-1}), x_Symbol] \rightarrow \text{Module}\{f = \text{FreeFactors}[\text{Tan}[(d + e*x)/2], x\}, \text{Dist}[(2*f)/e, \text{Subst}[\text{Int}[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, \text{Tan}[(d + e*x)/2]/f], x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[a^2 - b^2 - c^2, 0]$

Rule 3137

$\text{Int}[(A_ + (C_)*\sin[(d_ + (e_)*(x_))]*(b_ + (c_)*\sin[(d_ + (e_)*(x_)]), x_Symbol] \rightarrow \text{Simp}[(c*C*(d + e*x))/(e*(b^2 + c^2)), x] + (\text{Dist}[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), \text{Int}[1/(a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]), x], x] - \text{Simp}[(b*C*\text{Log}[a + b*\text{Cos}[d + e*x] + c*\text{Sin}[d + e*x]])/(e*(b^2 + c^2)), x]) /; \text{FreeQ}\{a, b, c, d, e, A, C, x\} \ \&\& \ \text{NeQ}[b^2 + c^2, 0] \ \&\& \ \text{NeQ}[A*(b^2 + c^2) - a*c*C, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{A + C \sin(x)}{a + b \cos(x) + c \sin(x)} dx &= \frac{cCx}{b^2 + c^2} - \frac{bC \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} + \left(A - \frac{acC}{b^2 + c^2} \right) \int \frac{1}{a + b \cos(x) + c \sin(x)} dx \\
&= \frac{cCx}{b^2 + c^2} - \frac{bC \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} + \left(2 \left(A - \frac{acC}{b^2 + c^2} \right) \right) \text{Subst} \left(\int \frac{1}{a + b \cos(x) + c \sin(x)} dx \right) \\
&= \frac{cCx}{b^2 + c^2} - \frac{bC \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} - \left(4 \left(A - \frac{acC}{b^2 + c^2} \right) \right) \text{Subst} \left(\int \frac{1}{-4(a^2 - b^2 - c^2) \cos^2\left(\frac{x}{2}\right) + 2c \tan\left(\frac{x}{2}\right) + a} dx \right) \\
&= \frac{cCx}{b^2 + c^2} + \frac{2 \left(A - \frac{acC}{b^2 + c^2} \right) \tan^{-1} \left(\frac{c + (a-b) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2 - c^2}} \right)}{\sqrt{a^2 - b^2 - c^2}} - \frac{bC \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 96, normalized size = 0.83

$$\frac{C(cx - b \log(a + b \cos(x) + c \sin(x))) - \frac{2(A(b^2 + c^2) - acC) \tanh^{-1} \left(\frac{(a-b) \tan\left(\frac{x}{2}\right) + c}{\sqrt{-a^2 + b^2 + c^2}} \right)}{\sqrt{-a^2 + b^2 + c^2}}}{b^2 + c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]

[Out] ((-2*(A*(b^2 + c^2) - a*c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + C*(c*x - b*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)

fricas [B] time = 2.59, size = 625, normalized size = 5.39

$$\left[\frac{(Ab^2 - Cac + Ac^2) \sqrt{-a^2 + b^2 + c^2} \log \left(\frac{a^2 b^2 - 2b^4 - c^4 - (a^2 + 3b^2)c^2 - (2a^2 b^2 - b^4 - 2a^2 c^2 + c^4) \cos(x)^2 - 2(ab^3 + abc^2) \cos(x) - 2(ab^2 c + ac^3 - 2ab \cos(x) + (b^2 - c^2) \sin(x))}{(a^2 b^2 - 2b^4 - c^4 - (a^2 + 3b^2)c^2 - (2a^2 b^2 - b^4 - 2a^2 c^2 + c^4) \cos(x)^2 - 2(ab^3 + abc^2) \cos(x) - 2(ab^2 c + ac^3 - 2ab \cos(x) + (b^2 - c^2) \sin(x)))} \right)}{2ab \cos(x) + (b^2 - c^2) \sin(x)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")

[Out] [1/2*((A*b^2 - C*a*c + A*c^2)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)) - 2*(C*c^3 - (C*a^2 - C*b^2)*c)*x - (C*a^2*b - C*b^3 - C*b*c^2)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), 1/2*(2*(A*b^2 - C*a*c + A*c^2)*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x)) - 2*(C*c^3 - (C*a^2 - C*b^2)*c)*x - (C*a^2*b - C*b^3 - C*b*c^2)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]

giac [A] time = 0.19, size = 177, normalized size = 1.53

$$\frac{Ccx}{b^2 + c^2} - \frac{Cb \log \left(-a \tan \left(\frac{1}{2} x \right) + b \tan \left(\frac{1}{2} x \right) - 2c \tan \left(\frac{1}{2} x \right) - a - b \right)}{b^2 + c^2} + \frac{Cb \log \left(\tan \left(\frac{1}{2} x \right) + 1 \right)}{b^2 + c^2} - \frac{2(Ab^2 - Cac + A^2)}{b^2 + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")

[Out] $C*c*x/(b^2 + c^2) - C*b*\log(-a*\tan(1/2*x)^2 + b*\tan(1/2*x)^2 - 2*c*\tan(1/2*x) - a - b)/(b^2 + c^2) + C*b*\log(\tan(1/2*x)^2 + 1)/(b^2 + c^2) - 2*(A*b^2 - C*a*c + A*c^2)*(pi*\text{floor}(1/2*x/pi + 1/2)*\text{sgn}(-2*a + 2*b) + \arctan(-(a*\tan(1/2*x) - b*\tan(1/2*x) + c)/\sqrt{a^2 - b^2 - c^2}))/(\sqrt{a^2 - b^2 - c^2}*(b^2 + c^2))$

maple [B] time = 0.13, size = 542, normalized size = 4.67

$$\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right) - b\left(\tan^2\left(\frac{x}{2}\right)\right) + 2c\tan\left(\frac{x}{2}\right) + a + b\right)abC}{(b^2 + c^2)(a - b)} + \frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right) - b\left(\tan^2\left(\frac{x}{2}\right)\right) + 2c\tan\left(\frac{x}{2}\right) + a + b\right)abC}{(b^2 + c^2)(a - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)

[Out] $-1/(b^2+c^2)/(a-b)*\ln(a*\tan(1/2*x)^2-b*\tan(1/2*x)^2+2*c*\tan(1/2*x)+a+b)*a*b*C+1/(b^2+c^2)/(a-b)*\ln(a*\tan(1/2*x)^2-b*\tan(1/2*x)^2+2*c*\tan(1/2*x)+a+b)*b^2*C+2/(b^2+c^2)/(a^2-b^2-c^2)^{(1/2)}*\arctan(1/2*(2*(a-b)*\tan(1/2*x)+2*c)/(a^2-b^2-c^2)^{(1/2)})*A*b^2+2/(b^2+c^2)/(a^2-b^2-c^2)^{(1/2)}*\arctan(1/2*(2*(a-b)*\tan(1/2*x)+2*c)/(a^2-b^2-c^2)^{(1/2)})*A*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^{(1/2)}*\arctan(1/2*(2*(a-b)*\tan(1/2*x)+2*c)/(a^2-b^2-c^2)^{(1/2)})*a*c*C-2/(b^2+c^2)/(a^2-b^2-c^2)^{(1/2)}*\arctan(1/2*(2*(a-b)*\tan(1/2*x)+2*c)/(a^2-b^2-c^2)^{(1/2)})*C*b*c+2/(b^2+c^2)/(a^2-b^2-c^2)^{(1/2)}*\arctan(1/2*(2*(a-b)*\tan(1/2*x)+2*c)/(a^2-b^2-c^2)^{(1/2)})*c/(a-b)*a*b*C-2/(b^2+c^2)/(a^2-b^2-c^2)^{(1/2)}*\arctan(1/2*(2*(a-b)*\tan(1/2*x)+2*c)/(a^2-b^2-c^2)^{(1/2)})*c/(a-b)*b^2*C+C/(b^2+c^2)*b*\ln(1+\tan(1/2*x)^2)+2*C/(b^2+c^2)*c*\arctan(\tan(1/2*x))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see 'assume?' for more details)Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 24.86, size = 1741, normalized size = 15.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + C*sin(x))/(a + b*cos(x) + c*sin(x)),x)

[Out] $(C*\log(\tan(x/2) + 1i))/(b - c*1i) + (C*\log(\tan(x/2) - 1i)*1i)/(b*1i - c) - (\log(32*A*C^2*a^2 + 32*A*C^2*b^2 - 64*A*C^2*a*b - 32*A^2*C*a*c + 32*A^2*C*b*c + 32*C*\tan(x/2)*(a - b)*(A^2*b + 2*C^2*a - 2*C^2*b - 2*A*C*c) + ((C*b^3 + A*b^2*(b^2 - a^2 + c^2)^{(1/2)} - C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^{(1/2)} + C*b*c^2 - C*a*c*(b^2 - a^2 + c^2)^{(1/2)})*(64*A^2*b^2*c + 32*C^2*a^2*c + 32*C^2*b^2*c + 64*A*C*b^3 + 32*\tan(x/2)*(a - b)*(A^2*b^2 - A^2*c^2 - 2*C^2*a^2 + 2*C^2*c^2 + 2*C^2*a*b + 2*A*C*a*c - 4*A*C*b*c) - 128*A*C*a*b^2 + 64*A*C*a^2*b - 64*A^2*a*b*c - 64*C^2*a*b*c + ((C*b^3 + A*b^2*(b^2 - a^2 + c^2)^{(1/2)} - C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^{(1/2)} + C*b*c^2 - C*a*c*(b^2 - a^2$

$$\begin{aligned}
& + c^2)^{(1/2)}) * (32 * A * b^4 + 32 * A * a^2 * b^2 - 32 * A * a^2 * c^2 + 32 * A * b^2 * c^2 - 32 * \tan(x/2) * (a - b) * (2 * A * c^3 - 2 * C * b^3 + 2 * A * b^2 * c + 2 * C * a * b^2 - 2 * C * a * c^2 + C * b * c^2 - 2 * A * a * b * c) - 64 * A * a * b^3 + 32 * C * a * c^3 - 32 * C * b * c^3 + 64 * C * b^3 * c - 128 * C * a * b^2 * c + 64 * C * a^2 * b * c + (32 * (a - b) * (C * b^3 + A * b^2 * (b^2 - a^2 + c^2)^{(1/2)} - C * a^2 * b + A * c^2 * (b^2 - a^2 + c^2)^{(1/2)} + C * b * c^2 - C * a * c * (b^2 - a^2 + c^2)^{(1/2)})) * (3 * c^4 * \tan(x/2) + a * c^3 + 3 * b * c^3 + 3 * b^3 * c + 2 * a^2 * b^2 * \tan(x/2) - 2 * a^2 * c^2 * \tan(x/2) + 3 * b^2 * c^2 * \tan(x/2) - 2 * a * b^3 * \tan(x/2) + a * b^2 * c - 4 * a^2 * b * c - 2 * a * b * c^2 * \tan(x/2))) / ((b^2 + c^2) * (b^2 - a^2 + c^2))) / ((b^2 + c^2) * (b^2 - a^2 + c^2))) / ((b^2 + c^2) * (b^2 - a^2 + c^2))) * (C * b^3 - b * (C * a^2 - C * c^2) + A * b^2 * (b^2 - a^2 + c^2)^{(1/2)} + A * c^2 * (b^2 - a^2 + c^2)^{(1/2)} - C * a * c * (b^2 - a^2 + c^2)^{(1/2)})) / ((b^2 + c^2) * (b^2 - a^2 + c^2)) + (\log(32 * A * C^2 * a^2 + 32 * A * C^2 * b^2 - 64 * A * C^2 * a * b - 32 * A^2 * C * a * c + 32 * A^2 * C * b * c + 32 * C * \tan(x/2) * (a - b) * (A^2 * b + 2 * C^2 * a - 2 * C^2 * b - 2 * A * C * c) + ((C * b^3 - A * b^2 * (b^2 - a^2 + c^2)^{(1/2)} - C * a^2 * b - A * c^2 * (b^2 - a^2 + c^2)^{(1/2)} + C * b * c^2 + C * a * c * (b^2 - a^2 + c^2)^{(1/2)})) * (64 * A^2 * b^2 * c + 32 * C^2 * a^2 * c + 32 * C^2 * b^2 * c + 64 * A * C * b^3 + 32 * \tan(x/2) * (a - b) * (A^2 * b^2 - A^2 * c^2 - 2 * C^2 * a^2 + 2 * C^2 * c^2 + 2 * C^2 * a * b + 2 * A * C * a * c - 4 * A * C * b * c) - 128 * A * C * a * b^2 + 64 * A * C * a^2 * b - 64 * A^2 * a * b * c - 64 * C^2 * a * b * c + ((C * b^3 - A * b^2 * (b^2 - a^2 + c^2)^{(1/2)} - C * a^2 * b - A * c^2 * (b^2 - a^2 + c^2)^{(1/2)} + C * b * c^2 + C * a * c * (b^2 - a^2 + c^2)^{(1/2)})) * (32 * A * b^4 + 32 * A * a^2 * b^2 - 32 * A * a^2 * c^2 + 32 * A * b^2 * c^2 - 32 * \tan(x/2) * (a - b) * (2 * A * c^3 - 2 * C * b^3 + 2 * A * b^2 * c + 2 * C * a * b^2 - 2 * C * a * c^2 + C * b * c^2 - 2 * A * a * b * c) - 64 * A * a * b^3 + 32 * C * a * c^3 - 32 * C * b * c^3 + 64 * C * b^3 * c - 128 * C * a * b^2 * c + 64 * C * a^2 * b * c + (32 * (a - b) * (C * b^3 - A * b^2 * (b^2 - a^2 + c^2)^{(1/2)} - C * a^2 * b - A * c^2 * (b^2 - a^2 + c^2)^{(1/2)} + C * b * c^2 + C * a * c * (b^2 - a^2 + c^2)^{(1/2)})) * (3 * c^4 * \tan(x/2) + a * c^3 + 3 * b * c^3 + 3 * b^3 * c + 2 * a^2 * b^2 * \tan(x/2) - 2 * a^2 * c^2 * \tan(x/2) + 3 * b^2 * c^2 * \tan(x/2) - 2 * a * b^3 * \tan(x/2) + a * b^2 * c - 4 * a^2 * b * c - 2 * a * b * c^2 * \tan(x/2))) / ((b^2 + c^2) * (b^2 - a^2 + c^2))) / ((b^2 + c^2) * (b^2 - a^2 + c^2))) / ((b^2 + c^2) * (b^2 - a^2 + c^2))) * (b * (C * a^2 - C * c^2) - C * b^3 + A * b^2 * (b^2 - a^2 + c^2)^{(1/2)} + A * c^2 * (b^2 - a^2 + c^2)^{(1/2)} - C * a * c * (b^2 - a^2 + c^2)^{(1/2)})) / ((b^2 + c^2) * (b^2 - a^2 + c^2))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)

[Out] Timed out

$$3.541 \quad \int \frac{A+C \sin(x)}{(a+b \cos(x)+c \sin(x))^2} dx$$

Optimal. Leaf size=114

$$\frac{2(aA - cC) \tan^{-1} \left(\frac{(a-b) \tan(\frac{x}{2}) + c}{\sqrt{a^2 - b^2 - c^2}} \right)}{(a^2 - b^2 - c^2)^{3/2}} - \frac{-\cos(x)(Ac - aC) + Ab \sin(x) + bC}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))}$$

[Out] 2*(A*a-C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^(3/2)+(-b*C+(A*c-C*a)*cos(x)-A*b*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3154, 3124, 618, 204}

$$\frac{2(aA - cC) \tan^{-1} \left(\frac{(a-b) \tan(\frac{x}{2}) + c}{\sqrt{a^2 - b^2 - c^2}} \right)}{(a^2 - b^2 - c^2)^{3/2}} - \frac{-\cos(x)(Ac - aC) + Ab \sin(x) + bC}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]

[Out] (2*(a*A - c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]]/(a^2 - b^2 - c^2)^(3/2) - (b*C - (A*c - a*C)*Cos[x] + A*b*Sin[x])/((a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3154

Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] := -Simp[(b*C + (a*C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + C \sin(x)}{(a + b \cos(x) + c \sin(x))^2} dx &= -\frac{bC - (Ac - aC) \cos(x) + Ab \sin(x)}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))} + \frac{(aA - cC) \int \frac{1}{a + b \cos(x) + c \sin(x)} dx}{a^2 - b^2 - c^2} \\
&= -\frac{bC - (Ac - aC) \cos(x) + Ab \sin(x)}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))} + \frac{(2(aA - cC)) \operatorname{Subst} \left(\int \frac{1}{a + b + 2cx + (a-b)} \right)}{a^2 - b^2 - c^2} \\
&= -\frac{bC - (Ac - aC) \cos(x) + Ab \sin(x)}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))} - \frac{(4(aA - cC)) \operatorname{Subst} \left(\int \frac{1}{-4(a^2 - b^2 - c^2) - \dots} \right)}{a^2 - b^2 - c^2} \\
&= \frac{2(aA - cC) \tan^{-1} \left(\frac{c + (a-b) \tan(\frac{x}{2})}{\sqrt{a^2 - b^2 - c^2}} \right)}{(a^2 - b^2 - c^2)^{3/2}} - \frac{bC - (Ac - aC) \cos(x) + Ab \sin(x)}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))}
\end{aligned}$$

Mathematica [A] time = 0.39, size = 123, normalized size = 1.08

$$\frac{a^2(-C) + \sin(x)(A(b^2 + c^2) - acC) + aAc + b^2C}{b(-a^2 + b^2 + c^2)(a + b \cos(x) + c \sin(x))} + \frac{2(aA - cC) \tanh^{-1} \left(\frac{(a-b) \tan(\frac{x}{2}) + c}{\sqrt{-a^2 + b^2 + c^2}} \right)}{(-a^2 + b^2 + c^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]

[Out] (2*(a*A - c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(3/2) + (a*A*c - a^2*C + b^2*C + (A*(b^2 + c^2) - a*c*C)*Sin[x])/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))

fricas [B] time = 1.24, size = 1301, normalized size = 11.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas")

[Out] [1/2*(2*C*a^4*b - 4*C*a^2*b^3 + 2*C*b^5 + 2*C*b*c^4 - 4*(C*a^2*b - C*b^3)*c^2 - (A*a^2*b^2 - C*a*b^2*c + A*a^2*c^2 - C*a*c^3 + (A*a*b^3 - C*b^3*c + A*a*b*c^2 - C*b*c^3)*cos(x) + (A*a*b^2*c - C*b^2*c^2 + A*a*c^3 - C*c^4)*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x))^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x))^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))] + 2*(C*a*c^4 - A*c^5 + (A*a^2 - 2*A*b^2)*c^3 - (C*a^3 - C*a*b^2)*c^2 + (A*a^2*b^2 - A*b^4)*c)*cos(x) - 2*(A*a^2*b^3 - A*b^5 + C*a*b*c^3 - A*b*c^4 + (A*a^2*b - 2*A*b^3)*c^2 - (C*a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x)), (C*a^4*b - 2*C*a^2*b^3 + C*b^5 + C*b*c^4 - 2*(C*a^2*b - C*b^3)*c^2 + (A*a^2*b^2 - C*a*b^2*c + A*a^2*c^2 - C*a*c^3 + (A*a*b^3 - C*b^3*c + A*a*b*c^2 - C*b*c^3)*cos(x) + (A*a*b^2*c - C*b^2*c^2 + A*a*c^3 - C*c^4)*sin(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b

$$-b^3 - b^2c \sin(x)) + (C^2a^4 - A^2c^5 + (A^2a^2 - 2A^2b^2)c^3 - (C^2a^3 - C^2ab^2)c^2 + (A^2a^2b^2 - A^2b^4)c) \cos(x) - (A^2a^2b^3 - A^2b^5 + C^2a^2b^3 - A^2b^2c^4 + (A^2a^2b - 2A^2b^3)c^2 - (C^2a^3b - C^2ab^3)c) \sin(x)) / (a^5b^2 - 2a^3b^4 + a^2b^6 + a^2c^6 - (2a^3 - 3a^2b^2)c^4 + (a^5 - 4a^3b^2 + 3a^2b^4)c^2 + (a^4b^3 - 2a^2b^5 + b^7 + b^2c^6 - (2a^2b - 3b^3)c^4 + (a^4b - 4a^2b^3 + 3b^5)c^2) \cos(x) + (c^7 - (2a^2 - 3b^2)c^5 + (a^4 - 4a^2b^2 + 3b^4)c^3 + (a^4b^2 - 2a^2b^4 + b^6)c) \sin(x))]$$

giac [A] time = 0.20, size = 206, normalized size = 1.81

$$\frac{2 \left(\pi \left[\frac{x}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a + 2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2}x\right) - b \tan\left(\frac{1}{2}x\right) + c}{\sqrt{a^2 - b^2 - c^2}} \right) \right) (Aa - Cc)}{(a^2 - b^2 - c^2)^{\frac{3}{2}}} - \frac{2 \left(Aab \tan\left(\frac{1}{2}x\right) - Ab^2 \tan\left(\frac{1}{2}x\right) \right)}{(a^3 - a^2b - ab^2 + b^3 - c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")

[Out] $-2 * (\pi * \operatorname{floor}(1/2 * x / \pi + 1/2) * \operatorname{sgn}(-2 * a + 2 * b) + \arctan(-(a * \tan(1/2 * x) - b * \tan(1/2 * x) + c) / \sqrt{a^2 - b^2 - c^2})) * (A * a - C * c) / (a^2 - b^2 - c^2)^{(3/2)} - 2 * (A * a * b * \tan(1/2 * x) - A * b^2 * \tan(1/2 * x) + C * a * c * \tan(1/2 * x) - C * b * c * \tan(1/2 * x) - A * c^2 * \tan(1/2 * x) + C * a^2 - C * b^2 - A * a * c) / ((a^3 - a^2 * b - a * b^2 + b^3 - a * c^2 + b * c^2) * (a * \tan(1/2 * x)^2 - b * \tan(1/2 * x)^2 + 2 * c * \tan(1/2 * x) + a + b))$

maple [B] time = 0.18, size = 255, normalized size = 2.24

$$\frac{-\frac{2(aAb - Ab^2 - Ac^2 + aC - Cbc) \tan\left(\frac{x}{2}\right)}{a^3 - a^2b - ab^2 - ac^2 + b^3 + c^2b} + \frac{2(aAc - a^2C + b^2C)}{a^3 - a^2b - ab^2 - ac^2 + b^3 + c^2b}}{a \left(\tan^2\left(\frac{x}{2}\right) \right) - b \left(\tan^2\left(\frac{x}{2}\right) \right) + 2c \tan\left(\frac{x}{2}\right) + a + b} + \frac{2 \arctan\left(\frac{2(a-b) \tan\left(\frac{x}{2}\right) + 2c}{2\sqrt{a^2 - b^2 - c^2}}\right) aA}{(a^2 - b^2 - c^2)^{\frac{3}{2}}} - \frac{2 \arctan\left(\frac{2(a-b) \tan\left(\frac{x}{2}\right) + 2c}{2\sqrt{a^2 - b^2 - c^2}}\right)}{(a^2 - b^2 - c^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x)

[Out] $2 * (- (A * a * b - A * b^2 - A * c^2 + C * a * c - C * b * c) / (a^3 - a^2 * b - a * b^2 - a * c^2 + b^3 + b * c^2) * \tan(1/2 * x) + (A * a * c - C * a^2 + C * b^2) / (a^3 - a^2 * b - a * b^2 - a * c^2 + b^3 + b * c^2)) / (a * \tan(1/2 * x)^2 - b * \tan(1/2 * x)^2 + 2 * c * \tan(1/2 * x) + a + b) + 2 / (a^2 - b^2 - c^2)^{(3/2)} * \arctan(1/2 * (a - b) * \tan(1/2 * x) + 2 * c) / (a^2 - b^2 - c^2)^{(1/2)} * a * A - 2 / (a^2 - b^2 - c^2)^{(3/2)} * \arctan(1/2 * (a - b) * \tan(1/2 * x) + 2 * c) / (a^2 - b^2 - c^2)^{(1/2)} * C * c$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see 'assume?' for more details) Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 3.19, size = 204, normalized size = 1.79

$$\frac{2 \operatorname{atanh}\left(\frac{\tan\left(\frac{x}{2}\right) (2a - 2b) + \frac{2(-a^2c + b^2c + c^3)}{-a^2 + b^2 + c^2}}{2\sqrt{-a^2 + b^2 + c^2}}\right) (Aa - Cc)}{(-a^2 + b^2 + c^2)^{3/2}} - \frac{2(-Ca^2 + Aca + Cb^2)}{(a-b)(-a^2 + b^2 + c^2)} + \frac{2 \tan\left(\frac{x}{2}\right) (Ab^2 + Cbc - Aab + Ac^2 - Cac)}{(a-b)(-a^2 + b^2 + c^2)}}{(a-b) \tan\left(\frac{x}{2}\right)^2 + 2c \tan\left(\frac{x}{2}\right) + a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

```
[Out] (2*atanh((tan(x/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^2 - a^2 + c^2)))/(2*(b^2 - a^2 + c^2)^(1/2)))*(A*a - C*c)/(b^2 - a^2 + c^2)^(3/2) - ((2*(C*b^2 - C*a^2 + A*a*c))/((a - b)*(b^2 - a^2 + c^2)) + (2*tan(x/2)*(A*b^2 + A*c^2 - A*a*b - C*a*c + C*b*c))/((a - b)*(b^2 - a^2 + c^2)))/(a + b + 2*c*tan(x/2) + tan(x/2)^2*(a - b))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```


$$3.542 \quad \int \frac{A+C \sin(x)}{(a+b \cos(x)+c \sin(x))^3} dx$$

Optimal. Leaf size=200

$$\frac{(2a^2A - 3acC + A(b^2 + c^2)) \tan^{-1}\left(\frac{(a-b)\tan(\frac{x}{2})+c}{\sqrt{a^2-b^2-c^2}}\right) - \cos(x)(a^2(-C) + 3aAc - 2c^2C) + b \sin(x)(3aA - 2cC)}{(a^2 - b^2 - c^2)^{5/2} \cdot 2(a^2 - b^2 - c^2)^2 (a + b \cos(x) + c \sin(x))}$$

[Out] (2*a^2*A+A*(b^2+c^2)-3*a*c*C)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^(5/2)+1/2*(-b*C+(A*c-C*a)*cos(x)-A*b*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))^2+1/2*(-a*b*C+(3*A*a*c-C*a^2-2*C*c^2)*cos(x)-b*(3*A*a-2*C*c)*sin(x))/(a^2-b^2-c^2)^2/(a+b*cos(x)+c*sin(x))

Rubi [A] time = 0.25, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3157, 3153, 3124, 618, 204}

$$\frac{(2a^2A - 3acC + A(b^2 + c^2)) \tan^{-1}\left(\frac{(a-b)\tan(\frac{x}{2})+c}{\sqrt{a^2-b^2-c^2}}\right) - \cos(x)(a^2(-C) + 3aAc - 2c^2C) + b \sin(x)(3aA - 2cC)}{(a^2 - b^2 - c^2)^{5/2} \cdot 2(a^2 - b^2 - c^2)^2 (a + b \cos(x) + c \sin(x))}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]

[Out] ((2*a^2*A + A*(b^2 + c^2) - 3*a*c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^2 - b^2 - c^2)^(5/2) - (b*C - (A*c - a*C)*Cos[x] + A*b*Sin[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])^2) - (a*b*C - (3*a*A*c - a^2*C - 2*c^2*C)*Cos[x] + b*(3*a*A - 2*c*C)*Sin[x])/(2*(a^2 - b^2 - c^2)^2*(a + b*Cos[x] + c*Sin[x]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3153

Int[((A_.) + cos[(d_.) + (e_.)*(x_)])*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]) / ((a_.) + cos[(d_.) + (e_.)*(x_)])*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2

- c^2, 0] && NeQ[a*A - b*B - c*C, 0]

Rule 3157

Int[((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]), x_Symbol] :> Simp[((b*C + (a*C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[n, -2]

Rubi steps

$$\begin{aligned} \int \frac{A + C \sin(x)}{(a + b \cos(x) + c \sin(x))^3} dx &= -\frac{bC - (Ac - aC) \cos(x) + Ab \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} - \frac{\int \frac{-2(aA - cC) + Ab \cos(x) + (Ac - aC) \sin(x)}{(a + b \cos(x) + c \sin(x))^2} dx}{2(a^2 - b^2 - c^2)} \\ &= -\frac{bC - (Ac - aC) \cos(x) + Ab \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} - \frac{abC - (3aAc - a^2C - 2c^2C) \cos(x)}{2(a^2 - b^2 - c^2)^2(a + b \cos(x) + c \sin(x))} \\ &= -\frac{bC - (Ac - aC) \cos(x) + Ab \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} - \frac{abC - (3aAc - a^2C - 2c^2C) \cos(x)}{2(a^2 - b^2 - c^2)^2(a + b \cos(x) + c \sin(x))} \\ &= -\frac{bC - (Ac - aC) \cos(x) + Ab \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} - \frac{abC - (3aAc - a^2C - 2c^2C) \cos(x)}{2(a^2 - b^2 - c^2)^2(a + b \cos(x) + c \sin(x))} \\ &= \frac{(2a^2A + A(b^2 + c^2) - 3acC) \tan^{-1}\left(\frac{c + (a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2 - c^2}}\right)}{(a^2 - b^2 - c^2)^{5/2}} - \frac{bC - (Ac - aC) \cos(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))} \end{aligned}$$

Mathematica [A] time = 0.93, size = 361, normalized size = 1.80

$$\frac{2a^4C - 6a^3Ac + 4a^3cC \sin(x) - 2bc \cos(x) (2a^2A - 3acC + A(b^2 + c^2)) - c \cos(2x) (a^2cC - 3aA(b^2 + c^2) + 2cC)}{(a + b \cos(x) + c \sin(x))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]

[Out] -(((2*a^2*A + A*(b^2 + c^2) - 3*a*c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2)) + (-6*a^3*A*c - 3*a*A*b^2*c - 3*a*A*c^3 + 2*a^4*C - 4*a^2*b^2*C + 2*b^4*C + 5*a^2*c^2*C + 4*b^2*c^2*C + 2*c^4*C - 2*b*c*(2*a^2*A + A*(b^2 + c^2) - 3*a*c*C)*Cos[x] - c*(-3*a*A*(b^2 + c^2) + a^2*c*C + 2*c*(b^2 + c^2)*C)*Cos[2*x] - 8*a^2*A*b^2*Sin[x] + 2*A*b^4*Sin[x] - 12*a^2*A*c^2*Sin[x] + 2*A*b^2*c^2*Sin[x] + 4*a^3*c*C*Sin[x] + 2*a*b^2*c*C*Sin[x] + 8*a*c^3*C*Sin[x] - 3*a*A*b^3*Sin[2*x] - 3*a*A*b*c^2*Sin[2*x] + a^2*b*c*C*Sin[2*x] + 2*b^3*c*C*Sin[2*x] + 2*b*c^3*C*Sin[2*x])/((4*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x]))^2)

fricas [B] time = 1.89, size = 3513, normalized size = 17.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fricas")

[Out] [1/4*(2*C*a^6*b - 6*C*a^4*b^3 + 6*C*a^2*b^5 - 2*C*b^7 + 6*A*a*b*c^5 - 6*C*b*c^6 + 2*(4*C*a^2*b - 7*C*b^3)*c^4 - 6*(A*a^3*b - 2*A*a*b^3)*c^3 - 2*(2*C*a^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 - 4*(3*A*a*b*c^5 - 2*C*b*c^6 + (C*a^2*b - 4*C*b^3)*c^4 - 3*(A*a^3*b - 2*A*a*b^3)*c^3 + (C*a^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - 3*(A*a^3*b^3 - A*a*b^5)*c)*cos(x)^2 - (2*A*a^4*b^2 + A*a^2*b^4 - 3*C*a^3*b^2*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 + 2*A*b^2)*c^4 - 3*(C*a^3 + C*a*b^2)*c^3 + (2*A*a^4 + 4*A*a^2*b^2 + A*b^4)*c^2 + (2*A*a^2*b^4 + A*b^6 - 3*C*a*b^4*c + A*b^4*c^2 + 3*C*a*c^5 - A*c^6 - (2*A*a^2 + A*b^2)*c^4)*cos(x)^2 + 2*(2*A*a^3*b^3 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 + A*a*b*c^4 + 2*(A*a^3*b + A*a*b^3)*c^2)*cos(x) - 2*(3*C*a^2*b^2*c^2 + 3*C*a^2*c^4 - A*a*c^5 - 2*(A*a^3 + A*a*b^2)*c^3 - (2*A*a^3*b^2 + A*a*b^4)*c + (3*C*a*b^3*c^2 + 3*C*a*b*c^4 - A*b*c^5 - 2*(A*a^2*b + A*b^3)*c^3 - (2*A*a^2*b^3 + A*b^5)*c)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c))*cos(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)) - 6*(A*a^3*b^3 - A*a*b^5)*c + 2*(C*a*c^6 + A*c^7 - (5*A*a^2 - 3*A*b^2)*c^5 + (C*a^3 + 2*C*a*b^2)*c^4 + (4*A*a^4 - 10*A*a^2*b^2 + 3*A*b^4)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 + (4*A*a^4*b^2 - 5*A*a^2*b^4 + A*b^6)*c)*cos(x) - 2*(4*A*a^4*b^3 - 5*A*a^2*b^5 + A*b^7 + C*a*b*c^5 + A*b*c^6 - (5*A*a^2*b - 3*A*b^3)*c^4 + (C*a^3*b + 2*C*a*b^3)*c^3 + (4*A*a^4*b - 10*A*a^2*b^3 + 3*A*b^5)*c^2 - (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (3*A*a^3*b^4 - 3*A*a*b^6 - 3*A*a*b^4*c^2 + 3*A*a*c^6 - 2*C*c^7 + (C*a^2 - 2*C*b^2)*c^5 - 3*(A*a^3 - A*a*b^2)*c^4 + (C*a^4 + 2*C*b^4)*c^3 - (C*a^4*b^2 + C*a^2*b^4 - 2*C*b^6)*c)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*cos(x))*sin(x)), 1/2*(C*a^6*b - 3*C*a^4*b^3 + 3*C*a^2*b^5 - C*b^7 + 3*A*a*b*c^5 - 3*C*b*c^6 + (4*C*a^2*b - 7*C*b^3)*c^4 - 3*(A*a^3*b - 2*A*a*b^3)*c^3 - (2*C*a^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 - 2*(3*A*a*b*c^5 - 2*C*b*c^6 + (C*a^2*b - 4*C*b^3)*c^4 - 3*(A*a^3*b - 2*A*a*b^3)*c^3 + (C*a^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - 3*(A*a^3*b^3 - A*a*b^5)*c)*cos(x)^2 + (2*A*a^4*b^2 + A*a^2*b^4 - 3*C*a^3*b^2*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 + 2*A*b^2)*c^4 - 3*(C*a^3 + C*a*b^2)*c^3 + (2*A*a^4 + 4*A*a^2*b^2 + A*b^4)*c^2 + (2*A*a^2*b^4 + A*b^6 - 3*C*a*b^4*c + A*b^4*c^2 + 3*C*a*c^5 - A*c^6 - (2*A*a^2 + A*b^2)*c^4)*cos(x)^2 + 2*(2*A*a^3*b^3 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 + A*a*b*c^4 + 2*(A*a^3*b + A*a*b^3)*c^2)*cos(x) - 2*(3*C*a^2*b^2*c^2 + 3*C*a^2*c^4 - A*a*c^5 - 2*(A*a^3 + A*a*b^2)*c^3 - (2*A*a^3*b^2 + A*a*b^4)*c + (3*C*a*b^3*c^2 + 3*C*a*b*c^4 - A*b*c^5 - 2*(A*a^2*b + A*b^3)*c^3 - (2*A*a^2*b^3 + A*b^5)*c)*cos(x))*sin(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) - 3*(A*a^3*b^3 - A*a*b^5)*c + (C*a*c^6 + A*c^7 - (5*A*a^2 - 3*A*b^2)*c^5 + (C*a^3 + 2*C*a*b^2)*c^4 + (4*A*a^4 - 10*A*a^2*b^2 + 3*A*b^4)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 + (4*A*a^4*b^2 - 5*A*a^2*b^4 + A*b^6)*c)*cos(x) - (4*A*a^4*b^3 - 5*A*a^2*b^5 + A*b^7 + C*a*b*c^5 + A*b*c^6 - (5*

```

A*a^2*b - 3*A*b^3)*c^4 + (C*a^3*b + 2*C*a*b^3)*c^3 + (4*A*a^4*b - 10*A*a^2*
b^3 + 3*A*b^5)*c^2 - (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (3*A*a^3*b^4 - 3
*A*a*b^6 - 3*A*a*b^4*c^2 + 3*A*a*c^6 - 2*C*c^7 + (C*a^2 - 2*C*b^2)*c^5 - 3*
(A*a^3 - A*a*b^2)*c^4 + (C*a^4 + 2*C*b^4)*c^3 - (C*a^4*b^2 + C*a^2*b^4 - 2*
C*b^6)*c)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10
+ 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a
^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (
a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4
- 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a
^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 -
a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (
a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 -
4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3
*b^4 - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9
- (3*a^2*b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^
4*b^3 + 9*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)
*cos(x))*sin(x)]

```

giac [B] time = 0.52, size = 1054, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac")

```

[Out] -(2*A*a^2 + A*b^2 - 3*C*a*c + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/((a
^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^2 - c^2))
- (4*A*a^4*b*tan(1/2*x)^3 - 11*A*a^3*b^2*tan(1/2*x)^3 + 9*A*a^2*b^3*tan(1/2
*x)^3 - A*a*b^4*tan(1/2*x)^3 - A*b^5*tan(1/2*x)^3 + 3*C*a^4*c*tan(1/2*x)^3
- 9*C*a^3*b*c*tan(1/2*x)^3 + 9*C*a^2*b^2*c*tan(1/2*x)^3 - 3*C*a*b^3*c*tan(1
/2*x)^3 - 5*A*a^3*c^2*tan(1/2*x)^3 + 7*A*a^2*b*c^2*tan(1/2*x)^3 + A*a*b^2*c
^2*tan(1/2*x)^3 - 3*A*b^3*c^2*tan(1/2*x)^3 + 2*A*a*c^4*tan(1/2*x)^3 - 2*A*b
*c^4*tan(1/2*x)^3 + 2*C*a^5*tan(1/2*x)^2 - 2*C*a^4*b*tan(1/2*x)^2 - 4*C*a^3
*b^2*tan(1/2*x)^2 + 4*C*a^2*b^3*tan(1/2*x)^2 + 2*C*a*b^4*tan(1/2*x)^2 - 2*C
*b^5*tan(1/2*x)^2 - 4*A*a^4*c*tan(1/2*x)^2 + 12*A*a^3*b*c*tan(1/2*x)^2 - 13
*A*a^2*b^2*c*tan(1/2*x)^2 + 6*A*a*b^3*c*tan(1/2*x)^2 - A*b^4*c*tan(1/2*x)^2
+ 5*C*a^3*c^2*tan(1/2*x)^2 - 14*C*a^2*b*c^2*tan(1/2*x)^2 + 13*C*a*b^2*c^2*
tan(1/2*x)^2 - 4*C*b^3*c^2*tan(1/2*x)^2 - 7*A*a^2*c^3*tan(1/2*x)^2 + 6*A*a*
b*c^3*tan(1/2*x)^2 + A*b^2*c^3*tan(1/2*x)^2 + 2*C*a*c^4*tan(1/2*x)^2 - 2*C*
b*c^4*tan(1/2*x)^2 + 2*A*c^5*tan(1/2*x)^2 + 4*A*a^4*b*tan(1/2*x) - 5*A*a^3*
b^2*tan(1/2*x) - 3*A*a^2*b^3*tan(1/2*x) + 5*A*a*b^4*tan(1/2*x) - A*b^5*tan(
1/2*x) + 5*C*a^4*c*tan(1/2*x) - 5*C*a^3*b*c*tan(1/2*x) - 5*C*a^2*b^2*c*tan(
1/2*x) + 5*C*a*b^3*c*tan(1/2*x) - 11*A*a^3*c^2*tan(1/2*x) + 3*A*a^2*b*c^2*t
an(1/2*x) + 7*A*a*b^2*c^2*tan(1/2*x) + A*b^3*c^2*tan(1/2*x) + 4*C*a^2*c^3*t
an(1/2*x) - 4*C*a*b*c^3*tan(1/2*x) + 2*A*a*c^4*tan(1/2*x) + 2*A*b*c^4*tan(1
/2*x) + 2*C*a^5 - 4*C*a^3*b^2 + 2*C*a*b^4 - 4*A*a^4*c + 3*A*a^2*b^2*c + A*b
^4*c + C*a^3*c^2 - C*a*b^2*c^2 + A*a^2*c^3 + A*b^2*c^3)/((a^6 - 2*a^5*b - a
^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*
a*b^3*c^2 + 2*b^4*c^2 + a^2*c^4 - 2*a*b*c^4 + b^2*c^4)*(a*tan(1/2*x)^2 - b*
tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b)^2)

```

maple [B] time = 0.22, size = 1088, normalized size = 5.44

$$-\frac{(4Aa^3b - 7Aa^2b^2 - 5Aa^2c^2 + 2Aab^3 + 2Aabc^2 + Ab^4 + 3Ab^2c^2 + 2Ac^4 + 3Ca^3c - 6Ca^2bc + 3Cab^2c)(\tan^3(\frac{x}{2}))}{(a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4)(a-b)} + \frac{(4Aa^4c - 12Aa^3bc + 13Aa^2b^2c + 7Aa^2c^3)}{(a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x)

[Out]
$$\frac{2(-1/2(4A^3b-7A^2b^2-5A^2c^2+2A^2b^3+2A^2b^2c^2+A^2b^4+3Ab^2c^2+2A^2c^4+3C^2a^3c-6C^2a^2b^2c+3C^2a^2b^2c^2)/(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)/(a-b)\tan(1/2x)^3+1/2(4A^4c-12A^3b^2c+13A^2b^2c^2+7A^2c^3-6A^2b^3c-6A^2b^2c^3+A^2b^4c-A^2b^2c^3-2A^2c^5-2C^2a^5+2C^2a^4b+4C^2a^3b^2-5C^2a^3c^2-4C^2a^2b^3+14C^2a^2b^2c-2C^2a^2b^4-13C^2a^2b^2c^2-2C^2a^2c^4+2C^2b^5+4C^2b^3c^2+2C^2b^2c^4)/(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)/(a^2-2ab+b^2)\tan(1/2x)^2-1/2(4A^4b-5A^3b^2-11A^3c^2-3A^2b^3+3A^2b^2c^2+5A^2b^4+7A^2b^2c^2+2A^2c^4-Ab^5+Ab^3c^2+2Ab^2c^4+5C^2a^4c-5C^2a^3b^2c-5C^2a^2b^2c+4C^2a^2c^3+5C^2a^2b^3c-4C^2a^2b^2c^3)/(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)/(a^2-2ab+b^2)\tan(1/2x)+1/2(4A^4c-3A^2b^2c-A^2c^3-Ab^4c-Ab^2c^3-2C^2a^5+4C^2a^3b^2-C^2a^3c^2-2C^2a^2b^4+C^2a^2b^2c^2)/(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)/(a^2-2ab+b^2))/(a\tan(1/2x)^2-b\tan(1/2x)^2+2c\tan(1/2x)+a+b)^2/(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)/(a^2-b^2-c^2)^{(1/2)}\arctan(1/2((a-b)\tan(1/2x)+2c)/(a^2-b^2-c^2)^{(1/2)})+1/(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)/(a^2-b^2-c^2)^{(1/2)}\arctan(1/2((a-b)\tan(1/2x)+2c)/(a^2-b^2-c^2)^{(1/2)})+A^2b^2+1/(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)/(a^2-b^2-c^2)^{(1/2)}\arctan(1/2((a-b)\tan(1/2x)+2c)/(a^2-b^2-c^2)^{(1/2)})+A^2c^2-3/(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)/(a^2-b^2-c^2)^{(1/2)}\arctan(1/2((a-b)\tan(1/2x)+2c)/(a^2-b^2-c^2)^{(1/2)})+a^2c^2}{(a-b)^2(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)} + \frac{\tan(\frac{x}{2})(4A^4b+5C^2a^4c-5A^3b^2-5C^2a^3bc-11A^3c^2-3A^2b^3+3A^2b^2c^2+5A^2b^4+7A^2b^2c^2+2A^2c^4-Ab^5+Ab^3c^2+2Ab^2c^4+5C^2a^4c-5C^2a^3b^2c-5C^2a^2b^2c+4C^2a^2c^3+5C^2a^2b^3c-4C^2a^2b^2c^3)}{(a-b)^2(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see 'assume?' for more details)Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 6.73, size = 912, normalized size = 4.56

$$\frac{2C^2a^5-4A^4c-4C^2a^3b^2+C^2a^3c^2+3A^2b^2c+A^2c^3+2C^2ab^4-C^2ab^2c^2+A^2b^4c+A^2b^2c^3}{(a-b)^2(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)} + \frac{\tan(\frac{x}{2})(4A^4b+5C^2a^4c-5A^3b^2-5C^2a^3bc-11A^3c^2-3A^2b^3+3A^2b^2c^2+5A^2b^4+7A^2b^2c^2+2A^2c^4-Ab^5+Ab^3c^2+2Ab^2c^4+5C^2a^4c-5C^2a^3b^2c-5C^2a^2b^2c+4C^2a^2c^3+5C^2a^2b^3c-4C^2a^2b^2c^3)}{(a-b)^2(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + C*sin(x))/(a + b*cos(x) + c*sin(x))^3,x)

[Out]
$$-\frac{((2C^2a^5 + A^2a^2c^3 + A^2b^2c^3 - 4C^2a^3b^2 + C^2a^3c^2 - 4A^4a^4c + A^2b^4c + 2C^2a^2b^4 + 3A^2a^2b^2c - C^2a^2b^2c^2)/((a - b)^2(a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2)) + (\tan(x/2)(A^2b^3c^2 - 3A^2a^2b^3 - 5A^2a^3b^2 - 11A^2a^3c^2 - A^2b^5 + 4C^2a^2c^3 + 5A^2a^2b^4 + 4A^2a^4b + 2A^2a^2c^4 + 2A^2b^2c^4 + 5C^2a^4c - 4C^2a^2b^2c^3 + 5C^2a^2b^3c - 5C^2a^3b^2c + 7A^2a^2b^2c^2 + 3A^2a^2b^2c^2 - 5C^2a^2b^2c^2))/((a - b)^2(a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2)) + (\tan(x/2)^2(2A^2c^5 + 2C^2a^5 - 2C^2b^5 - 7A^2a^2c^3 + A^2b^2c^3 + 4C^2a^2b^3 - 4C^2a^3b^2 + 5C^2a^3c^2 - 4C^2b^3c^2 - 4A^2a^4c - A^2b^4c + 2C^2a^2b^4 - 2C^2a^4b + 2C^2a^2c^4 - 2C^2b^2c^4 + 6A^2a^2b^2c^3 + 6A^2a^2b^3c + 12A^2a^3b^2c - 13A^2a^2b^2c^2 + 13C^2a^2b^2c^2 - 14C^2a^2b^2c^2))/((a - b)^2(a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2)) + (\tan(x/2)^3(A^2b^4 + 2A^2c^4 - 7A^2a^2b^2 - 5A^2a^2c^2 + 3A^2b^2c^2 + 2A^2a^2b^3 + 4A^2a^3b^2 + 3C^2a^3c^2 + 2A^2a^2b^2c^2 + 3C^2a^2b^2c^2 - 6C^2a^2b^2c^2)))/((a - b)(a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2))$$

```

*a^2*c^2 + 2*b^2*c^2)))/(tan(x/2)^4*(a^2 - 2*a*b + b^2) + 2*a*b + tan(x/2)*
(4*a*c + 4*b*c) + tan(x/2)^3*(4*a*c - 4*b*c) + a^2 + b^2 + tan(x/2)^2*(2*a^
2 - 2*b^2 + 4*c^2)) - (atanh((2*a^4*c + 2*b^4*c + 2*c^5 - 4*a^2*c^3 + 4*b^2
*c^3 - 4*a^2*b^2*c)/(2*(b^2 - a^2 + c^2)^(5/2))) + (tan(x/2)*(2*a - 2*b)*(a^
4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2))/(2*(b^2 - a^2 + c^2)^(5
/2))))*(2*A*a^2 + A*b^2 + A*c^2 - 3*C*a*c))/(b^2 - a^2 + c^2)^(5/2)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))**3,x)

[Out] Timed out

$$3.543 \quad \int \frac{A+C \sin(x)}{a+b \cos(x)+ib \sin(x)} dx$$

Optimal. Leaf size=85

$$\frac{(a^2(-C) + 2iaAb + b^2C) \log(a + ib \sin(x) + b \cos(x))}{2a^2b} + \frac{x(2aA - ibC)}{2a^2} + \frac{iC \sin(x)}{2a} - \frac{C \cos(x)}{2a}$$

[Out] 1/2*(2*a*A-I*b*C)*x/a^2-1/2*C*cos(x)/a+1/2*(2*I*a*A*b-a^2*C+b^2*C)*ln(a+b*cos(x)+I*b*sin(x))/a^2/b+1/2*I*C*sin(x)/a

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3131}

$$\frac{(a^2(-C) + 2iaAb + b^2C) \log(a + ib \sin(x) + b \cos(x))}{2a^2b} + \frac{x(2aA - ibC)}{2a^2} + \frac{iC \sin(x)}{2a} - \frac{C \cos(x)}{2a}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]

[Out] ((2*a*A - I*b*C)*x)/(2*a^2) - (C*Cos[x])/(2*a) + (((2*I)*a*A*b - a^2*C + b^2*C)*Log[a + b*Cos[x] + I*b*Sin[x]])/(2*a^2*b) + ((I/2)*C*Sin[x])/a

Rule 3131

Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)])/(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]), x_Symbol] :> Simp[((2*a*A - c*C)*x)/(2*a^2), x] + (-Simp[(C*Cos[d + e*x])/(2*a*e), x] + Simp[(c*C*Sin[d + e*x])/(2*a*b*e), x] + Simp[((-(a^2*C) + 2*a*c*A + b^2*C)*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*b*e), x]) /; FreeQ[{a, b, c, d, e, A, C}, x] && EqQ[b^2 + c^2, 0]

Rubi steps

$$\int \frac{A + C \sin(x)}{a + b \cos(x) + ib \sin(x)} dx = \frac{(2aA - ibC)x}{2a^2} - \frac{C \cos(x)}{2a} + \frac{(2iaAb - a^2C + b^2C) \log(a + b \cos(x) + ib \sin(x))}{2a^2b}$$

Mathematica [A] time = 0.29, size = 152, normalized size = 1.79

$$\frac{(-2ia^2C - 4aAb + 2ib^2C) \tan^{-1}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right) + 2iaAb \log(a^2 + 2ab \cos(x) + b^2) - a^2C \log(a^2 + 2ab \cos(x) + b^2)}{4a^2b}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]

[Out] (2*a*A*b*x - I*a^2*C*x - I*b^2*C*x + (-4*a*A*b - (2*I)*a^2*C + (2*I)*b^2*C)*ArcTan[((a + b)*Cot[x/2])/(a - b)] - 2*a*b*C*Cos[x] + (2*I)*a*A*b*Log[a^2 + b^2 + 2*a*b*Cos[x]] - a^2*C*Log[a^2 + b^2 + 2*a*b*Cos[x]] + b^2*C*Log[a^2 + b^2 + 2*a*b*Cos[x]] + (2*I)*a*b*C*Sin[x])/(4*a^2*b)

fricas [A] time = 0.97, size = 71, normalized size = 0.84

$$\frac{(Cab - (2Aab - iCb^2)xe^{ix}) + (Ca^2 - 2iAab - Cb^2)e^{ix} \log\left(\frac{be^{ix}+a}{b}\right)}{2a^2b} e^{-ix}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(C*a*b - (2*A*a*b - I*C*b^2)*x*e^(I*x) + (C*a^2 - 2*I*A*a*b - C*b^2)*e^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2*b)
```

giac [B] time = 0.16, size = 169, normalized size = 1.99

$$\frac{(-2i Aa - Cb) \log\left(-a \tan\left(\frac{1}{2} x\right)^2 + b \tan\left(\frac{1}{2} x\right)^2 - 2i a \tan\left(\frac{1}{2} x\right) + a + b\right)}{4 a^2} - \frac{(2i Aa + Cb) \log\left(\tan\left(\frac{1}{2} x\right) - i\right)}{2 a^2} (2i Aa + Cb)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -1/4*(-2*I*A*a - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*I*a*tan(1/2*x) + a + b)/a^2 - 1/2*(2*I*A*a + C*b)*log(tan(1/2*x) - I)/a^2 - 1/4*(2*I*C*a^2 + 2*A*a*b - I*C*b^2)*(x + 2*arctan((-I*a*cos(x) - a*sin(x) - I*a)/(a*cos(x) - I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(-2*I*A*a*tan(1/2*x) - C*b*tan(1/2*x) - 2*A*a - 2*I*C*a + I*C*b)/(a^2*(tan(1/2*x) - I))
```

maple [B] time = 0.18, size = 151, normalized size = 1.78

$$\frac{C \ln\left(\tan\left(\frac{x}{2}\right) + i\right)}{2b} + \frac{iC}{a\left(\tan\left(\frac{x}{2}\right) - i\right)} - \frac{i \ln\left(\tan\left(\frac{x}{2}\right) - i\right) A}{a} - \frac{\ln\left(\tan\left(\frac{x}{2}\right) - i\right) b C}{2a^2} - \frac{\ln\left(ia + ib + a \tan\left(\frac{x}{2}\right) - b \tan\left(\frac{x}{2}\right)\right) C}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] 1/2*C/b*ln(tan(1/2*x)+I)+I*C/a/(tan(1/2*x)-I)-I/a*ln(tan(1/2*x)-I)*A-1/2/a^2*ln(tan(1/2*x)-I)*b*C-1/2/b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*C+1/2/a^2*b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*C+I/a*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))*A
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 4.35, size = 96, normalized size = 1.13

$$\ln\left(a + b - a \tan\left(\frac{x}{2}\right) + i + b \tan\left(\frac{x}{2}\right) + i\right) \left(\frac{Cb}{2a^2} - \frac{C}{2b} + \frac{A1i}{a}\right) + \frac{C1i}{a\left(\tan\left(\frac{x}{2}\right) - i\right)} + \frac{C \ln\left(\tan\left(\frac{x}{2}\right) + i\right)}{2b} - \frac{\ln\left(\tan\left(\frac{x}{2}\right) - i\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(a + b*cos(x) + b*sin(x)*1i),x)
```

```
[Out] log(a + b - a*tan(x/2)*1i + b*tan(x/2)*1i)*((A*1i)/a - C/(2*b) + (C*b)/(2*a^2)) + (C*1i)/(a*(tan(x/2) - 1i)) + (C*log(tan(x/2) + 1i))/(2*b) - (log(tan(x/2) - 1i)*(A*a*1i + (C*b)/2))/a^2
```


sympy [A] time = 0.80, size = 104, normalized size = 1.22

$$\begin{cases} -\frac{Ce^{-ix}}{2a} & \text{for } 2a \neq 0 \\ x\left(-\frac{2Aa-iCb}{2a^2} - \frac{i(2iAa-Ca+Cb)}{2a^2}\right) & \text{otherwise} \end{cases} \quad \frac{x(-2Aa + iCb)}{2a^2} - \frac{(-2iAab + Ca^2 - Cb^2) \log\left(\frac{a}{b} + e^{ix}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)

[Out] Piecewise((-C*exp(-I*x)/(2*a), Ne(2*a, 0)), (x*(-(2*A*a - I*C*b)/(2*a**2) - I*(2*I*A*a - C*a + C*b)/(2*a**2)), True)) - x*(-2*A*a + I*C*b)/(2*a**2) - (-2*I*A*a*b + C*a**2 - C*b**2)*log(a/b + exp(I*x))/(2*a**2*b)

$$3.544 \quad \int \frac{A+C \sin(x)}{a+b \cos(x)-ib \sin(x)} dx$$

Optimal. Leaf size=85

$$\frac{(a^2C + 2iaAb - b^2C) \log(a - ib \sin(x) + b \cos(x))}{2a^2b} + \frac{x(2aA + ibC)}{2a^2} - \frac{iC \sin(x)}{2a} - \frac{C \cos(x)}{2a}$$

[Out] $1/2*(2*a*A+I*b*C)*x/a^2-1/2*C*\cos(x)/a-1/2*(2*I*a*A*b+a^2*C-b^2*C)*\ln(a+b*\cos(x)-I*b*\sin(x))/a^2/b-1/2*I*C*\sin(x)/a$

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3131}

$$\frac{(a^2C + 2iaAb - b^2C) \log(a - ib \sin(x) + b \cos(x))}{2a^2b} + \frac{x(2aA + ibC)}{2a^2} - \frac{iC \sin(x)}{2a} - \frac{C \cos(x)}{2a}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]), x]

[Out] $((2*a*A + I*b*C)*x)/(2*a^2) - (C*\cos[x])/(2*a) - (((2*I)*a*A*b + a^2*C - b^2*C)*\log[a + b*\cos[x] - I*b*\sin[x]])/(2*a^2*b) - ((I/2)*C*\sin[x])/a$

Rule 3131

Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)])/(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]), x_Symbol] := Simp[((2*a*A - c*C)*x)/(2*a^2), x] + (-Simp[(C*Cos[d + e*x])/(2*a*e), x] + Simp[(c*C*Sin[d + e*x])/(2*a*b*e), x] + Simp[((-(a^2*C) + 2*a*c*A + b^2*C)*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*b*e), x]) /; FreeQ[{a, b, c, d, e, A, C}, x] && EqQ[b^2 + c^2, 0]

Rubi steps

$$\int \frac{A + C \sin(x)}{a + b \cos(x) - ib \sin(x)} dx = \frac{(2aA + ibC)x}{2a^2} - \frac{C \cos(x)}{2a} - \frac{(2iaAb + a^2C - b^2C) \log(a + b \cos(x) - ib \sin(x))}{2a^2b}$$

Mathematica [A] time = 0.26, size = 152, normalized size = 1.79

$$\frac{2i(a^2C + 2iaAb - b^2C) \tan^{-1}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right) - 2iaAb \log(a^2 + 2ab \cos(x) + b^2) - a^2C \log(a^2 + 2ab \cos(x) + b^2)}{4a^2b}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]), x]

[Out] $(2*a*A*b*x + I*a^2*C*x + I*b^2*C*x + (2*I)*((2*I)*a*A*b + a^2*C - b^2*C)*\text{ArcTan}[\frac{(a + b)*\text{Cot}[x/2]}{(a - b)}] - 2*a*b*C*\cos[x] - (2*I)*a*A*b*\log[a^2 + b^2 + 2*a*b*\cos[x]] - a^2*C*\log[a^2 + b^2 + 2*a*b*\cos[x]] + b^2*C*\log[a^2 + b^2 + 2*a*b*\cos[x]] - (2*I)*a*b*C*\sin[x])/(4*a^2*b)$

fricas [A] time = 0.55, size = 57, normalized size = 0.67

$$\frac{iCa^2x - Cae^{ix} - (Ca^2 + 2iAab - Cb^2) \log\left(\frac{ae^{ix} + b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fricas")

[Out] $\frac{1}{2}*(I*C*a^2*x - C*a*b*e^{(I*x)} - (C*a^2 + 2*I*A*a*b - C*b^2)*\log((a*e^{(I*x)} + b)/a))/(a^2*b)$

giac [B] time = 0.18, size = 169, normalized size = 1.99

$$\frac{(2iAa - Cb) \log\left(-a \tan\left(\frac{1}{2}x\right)^2 + b \tan\left(\frac{1}{2}x\right)^2 + 2ia \tan\left(\frac{1}{2}x\right) + a + b\right)}{4a^2} - \frac{(-2iAa + Cb) \log\left(\tan\left(\frac{1}{2}x\right) + i\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac")

[Out] $-\frac{1}{4}*(2*I*A*a - C*b)*\log(-a*\tan(1/2*x)^2 + b*\tan(1/2*x)^2 + 2*I*a*\tan(1/2*x) + a + b)/a^2 - \frac{1}{2}*(-2*I*A*a + C*b)*\log(\tan(1/2*x) + I)/a^2 - \frac{1}{4}*(-2*I*C*a^2 + 2*A*a*b + I*C*b^2)*(x + 2*\arctan((I*a*\cos(x) - a*\sin(x) + I*a)/(a*\cos(x) + I*a*\sin(x) - a + 2*b)))/(a^2*b) - \frac{1}{2}*(2*I*A*a*\tan(1/2*x) - C*b*\tan(1/2*x) - 2*A*a + 2*I*C*a - I*C*b)/(a^2*(\tan(1/2*x) + I))$

maple [B] time = 0.18, size = 280, normalized size = 3.29

$$-\frac{iC}{a\left(\tan\left(\frac{x}{2}\right) + i\right)} + \frac{i \ln\left(\tan\left(\frac{x}{2}\right) + i\right) A}{a} - \frac{\ln\left(\tan\left(\frac{x}{2}\right) + i\right) b C}{2a^2} + \frac{C \ln\left(\tan\left(\frac{x}{2}\right) - i\right)}{2b} + \frac{a \ln\left(ia + ib - a \tan\left(\frac{x}{2}\right) + b \tan\left(\frac{x}{2}\right)\right)}{2b(-a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)

[Out] $-I*C/a/(\tan(1/2*x)+I)+I/a*\ln(\tan(1/2*x)+I)*A-1/2/a^2*\ln(\tan(1/2*x)+I)*b*C+1/2*C/b*\ln(\tan(1/2*x)-I)+1/2*a/b/(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*C-1/2/(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*C-1/2/a*b/(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*C+1/2/a^2*b^2/(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*C+I/(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*A-I/a*b/(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*A$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

mupad [B] time = 4.34, size = 96, normalized size = 1.13

$$-\ln\left(a + b + a \tan\left(\frac{x}{2}\right) - b \tan\left(\frac{x}{2}\right) + i\right) \left(\frac{C}{2b} - \frac{Cb}{2a^2} + \frac{A1i}{a}\right) - \frac{C1i}{a\left(\tan\left(\frac{x}{2}\right) + i\right)} + \frac{C \ln\left(\tan\left(\frac{x}{2}\right) - i\right)}{2b} + \frac{\ln\left(\tan\left(\frac{x}{2}\right) + i\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + C*sin(x))/(a + b*cos(x) - b*sin(x)*1i),x)

[Out] $(C \cdot \log(\tan(x/2) - 1i))/(2 \cdot b) - (C \cdot 1i)/(a \cdot (\tan(x/2) + 1i)) - \log(a + b + a \cdot \tan(x/2) \cdot 1i - b \cdot \tan(x/2) \cdot 1i) \cdot ((A \cdot 1i)/a + C/(2 \cdot b) - (C \cdot b)/(2 \cdot a^2)) + (\log(\tan(x/2) + 1i) \cdot (A \cdot a \cdot 1i - (C \cdot b)/2))/a^2$

sympy [A] time = 0.74, size = 80, normalized size = 0.94

$$\frac{iCx}{2b} + \begin{cases} -\frac{Ce^{ix}}{2a} & \text{for } 2a \neq 0 \\ x \left(-\frac{iC}{2b} + \frac{iCa - iCb}{2ab} \right) & \text{otherwise} \end{cases} - \frac{(2iAab + Ca^2 - Cb^2) \log\left(e^{ix} + \frac{b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)

[Out] $I \cdot C \cdot x / (2 \cdot b) + \text{Piecewise}((-C \cdot \exp(I \cdot x) / (2 \cdot a), \text{Ne}(2 \cdot a, 0)), (x \cdot (-I \cdot C / (2 \cdot b) + (I \cdot C \cdot a - I \cdot C \cdot b) / (2 \cdot a \cdot b))), \text{True})) - (2 \cdot I \cdot A \cdot a \cdot b + C \cdot a^2 - C \cdot b^2) \cdot \log(\exp(I \cdot x) + b/a) / (2 \cdot a^2 \cdot b)$

$$3.545 \quad \int \frac{B \cos(x) + C \sin(x)}{a + b \cos(x) + c \sin(x)} dx$$

Optimal. Leaf size=119

$$\frac{2a(bB + cC) \tan^{-1} \left(\frac{(a-b) \tan(\frac{x}{2}) + c}{\sqrt{a^2 - b^2 - c^2}} \right)}{(b^2 + c^2) \sqrt{a^2 - b^2 - c^2}} + \frac{(Bc - bC) \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} + \frac{x(bB + cC)}{b^2 + c^2}$$

[Out] (B*b+C*c)*x/(b^2+c^2)+(B*c-C*b)*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)-2*a*(B*b+C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3136, 3124, 618, 204}

$$\frac{2a(bB + cC) \tan^{-1} \left(\frac{(a-b) \tan(\frac{x}{2}) + c}{\sqrt{a^2 - b^2 - c^2}} \right)}{(b^2 + c^2) \sqrt{a^2 - b^2 - c^2}} + \frac{(Bc - bC) \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} + \frac{x(bB + cC)}{b^2 + c^2}$$

Antiderivative was successfully verified.

[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]

[Out] ((b*B + c*C)*x)/(b^2 + c^2) - (2*a*(b*B + c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) + ((B*c - b*C)*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3136

Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]) / ((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + (Dist[(A*(b^2 + c^2) - a*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]

Rubi steps

$$\begin{aligned}
\int \frac{B \cos(x) + C \sin(x)}{a + b \cos(x) + c \sin(x)} dx &= \frac{(bB + cC)x}{b^2 + c^2} + \frac{(Bc - bC) \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} - \frac{(a(bB + cC)) \int \frac{1}{a + b \cos(x) + c \sin(x)} dx}{b^2 + c^2} \\
&= \frac{(bB + cC)x}{b^2 + c^2} + \frac{(Bc - bC) \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} - \frac{(2a(bB + cC)) \text{Subst} \left(\int \frac{1}{a + b \cos(x) + c \sin(x)} dx \right)}{b^2} \\
&= \frac{(bB + cC)x}{b^2 + c^2} + \frac{(Bc - bC) \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} + \frac{(4a(bB + cC)) \text{Subst} \left(\int \frac{1}{a + b \cos(x) + c \sin(x)} dx \right)}{b^2} \\
&= \frac{(bB + cC)x}{b^2 + c^2} - \frac{2a(bB + cC) \tan^{-1} \left(\frac{c + (a-b) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2 - c^2}} \right)}{\sqrt{a^2 - b^2 - c^2} (b^2 + c^2)} + \frac{(Bc - bC) \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2}
\end{aligned}$$

Mathematica [A] time = 0.38, size = 98, normalized size = 0.82

$$\frac{2a(bB+cC) \tanh^{-1} \left(\frac{(a-b) \tan\left(\frac{x}{2}\right) + c}{\sqrt{-a^2+b^2+c^2}} \right)}{\sqrt{-a^2+b^2+c^2}} + \frac{(Bc - bC) \log(a + b \cos(x) + c \sin(x)) + x(bB + cC)}{b^2 + c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]

[Out] ((b*B + c*C)*x + (2*a*(b*B + c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + (B*c - b*C)*Log[a + b*Cos[x] + c*Sin[x]]/(b^2 + c^2)

fricas [B] time = 5.06, size = 687, normalized size = 5.77

$$\left[\frac{(Bab + Cac) \sqrt{-a^2 + b^2 + c^2} \log \left(\frac{a^2 b^2 - 2b^4 - c^4 - (a^2 + 3b^2)c^2 - (2a^2 b^2 - b^4 - 2a^2 c^2 + c^4) \cos(x)^2 - 2(ab^3 + abc^2) \cos(x) - 2(ab^2 c + ac^3 - (bc^3 - (2a^2 b - b^3)c) \cos(x) - 2(a^2 b^2 c + abc^2) \sin(x) - 2(2a^2 b^2 c \cos(x)^2 - a^2 b^2 c + (b^2 c^2 + c^3) \cos(x) - (b^3 + b^2 c^2 + (a^2 b^2 - a^2 c^2) \cos(x)) \sin(x)) \sqrt{-a^2 + b^2 + c^2}}{2ab \cos(x) + (b^2 - c^2) \cos(x)} \right)}{b^2 + c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")

[Out] [-1/2*((B*a*b + C*a*c)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x)))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)) - 2*(B*a^2*b - B*b^3 - B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*x + (C*a^2*b - C*b^3 - C*b*c^2 + B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), -1/2*(2*(B*a*b + C*a*c)*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) - 2*(B*a^2*b - B*b^3 - B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*x + (C*a^2*b - C*b^3 - C*b*c^2 + B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]

giac [A] time = 0.15, size = 187, normalized size = 1.57

$$\frac{(Bb + Cc)x}{b^2 + c^2} - \frac{(Cb - Bc) \log\left(-a \tan\left(\frac{1}{2}x\right)^2 + b \tan\left(\frac{1}{2}x\right)^2 - 2c \tan\left(\frac{1}{2}x\right) - a - b\right)}{b^2 + c^2} + \frac{(Cb - Bc) \log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)}{b^2 + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")

[Out] (B*b + C*c)*x/(b^2 + c^2) - (C*b - B*c)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - a - b)/(b^2 + c^2) + (C*b - B*c)*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) + 2*(B*a*b + C*a*c)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/sqrt(a^2 - b^2 - c^2)*(b^2 + c^2)

maple [B] time = 0.12, size = 824, normalized size = 6.92

$$\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right) - b\left(\tan^2\left(\frac{x}{2}\right)\right) + 2c \tan\left(\frac{x}{2}\right) + a + b\right) a B c}{(b^2 + c^2)(a - b)} - \frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right) - b\left(\tan^2\left(\frac{x}{2}\right)\right) + 2c \tan\left(\frac{x}{2}\right) + a + b\right)}{(b^2 + c^2)(a - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)

[Out] 1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*a*B*c-1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*b*B*c-1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*a*b*C+1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*b^2*C-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*b*B+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*B*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*c*C-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*C*b*c-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c^2/(a-b)*a*B+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c^2/(a-b)*b*B+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c/(a-b)*a*b*C-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c/(a-b)*b^2*C-B/(b^2+c^2)*c*ln(1+tan(1/2*x)^2)+C/(b^2+c^2)*b*ln(1+tan(1/2*x)^2)+2*B/(b^2+c^2)*b*arctan(tan(1/2*x))+2*C/(b^2+c^2)*c*arctan(tan(1/2*x))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see 'assume?' for more details)Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 28.56, size = 1864, normalized size = 15.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x)),x)`

[Out]
$$\begin{aligned} & (\log(\tan(x/2) - 1i)*(B + C*1i))/(b*1i - c) - (\log(\tan(x/2) + 1i)*(B - C*1i)) / (b*1i + c) - (\log(32*B^3*a^2 + 32*B*C^2*a^2 + 32*B*C^2*b^2 + 32*\tan(x/2)* \\ & (a - b)*(2*C^3*a + B^3*c - 2*C^3*b + 2*B^2*C*a - B^2*C*b + 2*B*C^2*c) - 32* \\ & B^3*a*b - 64*B*C^2*a*b + 32*B^2*C*a*c - 32*B^2*C*b*c + ((C*b^3 - B*c^3 + B* \\ & a^2*c - C*a^2*b - B*b^2*c + C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^{(1/2)} + C*a*c \\ & *(b^2 - a^2 + c^2)^{(1/2}))* (32*B^2*b^2*c - 32*B^2*a^2*c + 32*C^2*a^2*c + 32* \\ & C^2*b^2*c + 32*\tan(x/2)*(a - b)*(2*B^2*a^2 + B^2*b^2 - 2*C^2*a^2 - 3*B^2*c^ \\ & 2 + 2*C^2*c^2 - 2*B^2*a*b + 2*C^2*a*b - 4*B*C*a*c + 6*B*C*b*c) - 128*B*C*a^ \\ & 3 - 64*B*C*b^3 + 192*B*C*a^2*b + 64*B*C*a*c^2 - 64*B*C*b*c^2 - 64*C^2*a*b*c \\ & + ((C*b^3 - B*c^3 + B*a^2*c - C*a^2*b - B*b^2*c + C*b*c^2 + B*a*b*(b^2 - a \\ & ^2 + c^2)^{(1/2)} + C*a*c*(b^2 - a^2 + c^2)^{(1/2}))* (32*B*b^4 + 32*B*a^2*b^2 - \\ & 32*B*a^2*c^2 - 64*B*b^2*c^2 - 32*\tan(x/2)*(a - b)*(B*c^3 - 2*C*b^3 + 2*C*a \\ & *b^2 + 4*B*b^2*c - 2*C*a*c^2 + C*b*c^2 - 4*B*a*b*c) - 64*B*a*b^3 + 32*C*a*c \\ & ^3 - 32*C*b*c^3 + 64*C*b^3*c + 96*B*a*b*c^2 - 128*C*a*b^2*c + 64*C*a^2*b*c \\ & + (32*(a - b)*(C*b^3 - B*c^3 + B*a^2*c - C*a^2*b - B*b^2*c + C*b*c^2 + B*a \\ & b*(b^2 - a^2 + c^2)^{(1/2)} + C*a*c*(b^2 - a^2 + c^2)^{(1/2}))* (3*c^4*\tan(x/2) \\ & + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*\tan(x/2) - 2*a^2*c^2*\tan(x/2) + 3*b \\ & ^2*c^2*\tan(x/2) - 2*a*b^3*\tan(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*\tan(x/ \\ & 2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)))/((b^2 + c^2)*(b^2 - a^2 + c^2)))/((\\ & b^2 + c^2)*(b^2 - a^2 + c^2)))*(C*b^3 - B*c^3 + B*a^2*c - C*a^2*b - B*b^2*c \\ & + C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^{(1/2)} + C*a*c*(b^2 - a^2 + c^2)^{(1/2}))) \\ &)/((b^2 + c^2)*(b^2 - a^2 + c^2)) + (\log(32*B^3*a^2 + 32*B*C^2*a^2 + 32*B*C \\ & ^2*b^2 + 32*\tan(x/2)*(a - b)*(2*C^3*a + B^3*c - 2*C^3*b + 2*B^2*C*a - B^2*C \\ & *b + 2*B*C^2*c) - 32*B^3*a*b - 64*B*C^2*a*b + 32*B^2*C*a*c - 32*B^2*C*b*c - \\ & ((B*c^3 - C*b^3 - B*a^2*c + C*a^2*b + B*b^2*c - C*b*c^2 + B*a*b*(b^2 - a^2 \\ & + c^2)^{(1/2)} + C*a*c*(b^2 - a^2 + c^2)^{(1/2}))* (32*B^2*b^2*c - 32*B^2*a^2*c \\ & + 32*C^2*a^2*c + 32*C^2*b^2*c + 32*\tan(x/2)*(a - b)*(2*B^2*a^2 + B^2*b^2 - \\ & 2*C^2*a^2 - 3*B^2*c^2 + 2*C^2*c^2 - 2*B^2*a*b + 2*C^2*a*b - 4*B*C*a*c + 6* \\ & B*C*b*c) - 128*B*C*a^3 - 64*B*C*b^3 + 192*B*C*a^2*b + 64*B*C*a*c^2 - 64*B*C \\ & *b*c^2 - 64*C^2*a*b*c + ((B*c^3 - C*b^3 - B*a^2*c + C*a^2*b + B*b^2*c - C*b \\ & *c^2 + B*a*b*(b^2 - a^2 + c^2)^{(1/2)} + C*a*c*(b^2 - a^2 + c^2)^{(1/2}))* (32*B \\ & *a^2*c^2 - 32*B*a^2*b^2 - 32*B*b^4 + 64*B*b^2*c^2 + 32*\tan(x/2)*(a - b)*(B* \\ & c^3 - 2*C*b^3 + 2*C*a*b^2 + 4*B*b^2*c - 2*C*a*c^2 + C*b*c^2 - 4*B*a*b*c) + \\ & 64*B*a*b^3 - 32*C*a*c^3 + 32*C*b*c^3 - 64*C*b^3*c - 96*B*a*b*c^2 + 128*C*a* \\ & b^2*c - 64*C*a^2*b*c + (32*(a - b)*(B*c^3 - C*b^3 - B*a^2*c + C*a^2*b + B*b \\ & ^2*c - C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^{(1/2)} + C*a*c*(b^2 - a^2 + c^2)^{(1 \\ & /2}))* (3*c^4*\tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*\tan(x/2) - 2*a \\ & ^2*c^2*\tan(x/2) + 3*b^2*c^2*\tan(x/2) - 2*a*b^3*\tan(x/2) + a*b^2*c - 4*a^2*b \\ & *c - 2*a*b*c^2*\tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2)))/((b^2 + c^2)*(b \\ & ^2 - a^2 + c^2)))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(B*c^3 - C*b^3 - B*a^2* \\ & c + C*a^2*b + B*b^2*c - C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^{(1/2)} + C*a*c*(b^ \\ & 2 - a^2 + c^2)^{(1/2}))/((b^2 + c^2)*(b^2 - a^2 + c^2)) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)`

[Out] Timed out

$$3.546 \quad \int \frac{B \cos(x) + C \sin(x)}{(a + b \cos(x) + c \sin(x))^2} dx$$

Optimal. Leaf size=110

$$\frac{aB \sin(x) - aC \cos(x) - bC + Bc}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))} - \frac{2(bB + cC) \tan^{-1} \left(\frac{(a-b) \tan(\frac{x}{2}) + c}{\sqrt{a^2 - b^2 - c^2}} \right)}{(a^2 - b^2 - c^2)^{3/2}}$$

[Out] $-2*(B*b+C*c)*\arctan((c+(a-b)*\tan(1/2*x))/(\sqrt{a^2-b^2-c^2}))/(a^2-b^2-c^2)^{3/2}+(B*c-b*C-a*C*\cos(x)+a*B*\sin(x))/(a^2-b^2-c^2)/(a+b*\cos(x)+c*\sin(x))$

Rubi [A] time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3153, 3124, 618, 204}

$$\frac{aB \sin(x) - aC \cos(x) - bC + Bc}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))} - \frac{2(bB + cC) \tan^{-1} \left(\frac{(a-b) \tan(\frac{x}{2}) + c}{\sqrt{a^2 - b^2 - c^2}} \right)}{(a^2 - b^2 - c^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]

[Out] $(-2*(b*B + c*C)*\text{ArcTan}[(c + (a - b)*\text{Tan}[x/2])/\text{Sqrt}[a^2 - b^2 - c^2]])/(a^2 - b^2 - c^2)^{3/2} + (B*c - b*C - a*C*\text{Cos}[x] + a*B*\text{Sin}[x])/((a^2 - b^2 - c^2)*(a + b*\text{Cos}[x] + c*\text{Sin}[x]))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3153

Int[((A_.) + cos[(d_.) + (e_.)*(x_)])*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]) / ((a_.) + cos[(d_.) + (e_.)*(x_)])*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]) / (e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]

Rubi steps

$$\begin{aligned}
\int \frac{B \cos(x) + C \sin(x)}{(a + b \cos(x) + c \sin(x))^2} dx &= \frac{Bc - bC - aC \cos(x) + aB \sin(x)}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))} - \frac{(bB + cC) \int \frac{1}{a+b \cos(x)+c \sin(x)} dx}{a^2 - b^2 - c^2} \\
&= \frac{Bc - bC - aC \cos(x) + aB \sin(x)}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))} - \frac{(2(bB + cC)) \text{Subst} \left(\int \frac{1}{a+b+2cx+(a-b)x^2} \right)}{a^2 - b^2 - c^2} \\
&= \frac{Bc - bC - aC \cos(x) + aB \sin(x)}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))} + \frac{(4(bB + cC)) \text{Subst} \left(\int \frac{1}{-4(a^2-b^2-c^2)-x^2} \right)}{a^2 - b^2 - c^2} \\
&= -\frac{2(bB + cC) \tan^{-1} \left(\frac{c+(a-b) \tan(\frac{x}{2})}{\sqrt{a^2-b^2-c^2}} \right)}{(a^2 - b^2 - c^2)^{3/2}} + \frac{Bc - bC - aC \cos(x) + aB \sin(x)}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))}
\end{aligned}$$

Mathematica [A] time = 0.41, size = 116, normalized size = 1.05

$$-\frac{a^2 C + a \sin(x)(bB + cC) - b^2 C + bBc}{b(-a^2 + b^2 + c^2)(a + b \cos(x) + c \sin(x))} - \frac{2(bB + cC) \tanh^{-1} \left(\frac{(a-b) \tan(\frac{x}{2}) + c}{\sqrt{-a^2 + b^2 + c^2}} \right)}{(-a^2 + b^2 + c^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]

[Out] (-2*(b*B + c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(3/2) - (b*B*c + a^2*C - b^2*C + a*(b*B + c*C)*Sin[x])/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))

fricas [B] time = 1.02, size = 1316, normalized size = 11.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas")

[Out] [1/2*(2*C*a^4*b - 4*C*a^2*b^3 + 2*C*b^5 + 2*C*b*c^4 - 2*B*c^5 + 4*(B*a^2 - B*b^2)*c^3 - 4*(C*a^2*b - C*b^3)*c^2 + (B*a*b^3 + C*a*b^2*c + B*a*b*c^2 + C*a*c^3 + (B*b^4 + C*b^3*c + B*b^2*c^2 + C*b*c^3)*cos(x) + (B*b^3*c + C*b^2*c^2 + B*b*c^3 + C*c^4)*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)) - 2*(B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + 2*(B*a*b*c^3 + C*a*c^4 - (C*a^3 - C*a*b^2)*c^2 - (B*a^3*b - B*a*b^3)*c)*cos(x) + 2*(B*a^3*b^2 - B*a*b^4 - B*a*b^2*c^2 - C*a*b*c^3 + (C*a^3*b - C*a*b^3)*c)*sin(x)]/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x), (C*a^4*b - 2*C*a^2*b^3 + C*b^5 + C*b*c^4 - B*c^5 + 2*(B*a^2 - B*b^2)*c^3 - 2*(C*a^2*b - C*b^3)*c^2 - (B*a*b^3 + C*a*b^2*c + B*a*b*c^2 + C*a*c^3 + (B*b^4 + C*b^3*c + B*b^2*c^2 + C*b*c^3)*cos(x) + (B*b^3*c + C*b^2*c^2 + B*b*c^3 + C*c^4)*sin(x))*sqrt

$$(a^2 - b^2 - c^2) \arctan(-a \cos(x) + a \sin(x) + b^2 + c^2) \sqrt{a^2 - b^2 - c^2} / ((c^3 - (a^2 - b^2)c) \cos(x) + (a^2 b - b^3 - b c^2) \sin(x)) - (B a^4 - 2 B a^2 b^2 + B b^4) c + (B a b c^3 + C a c^4 - (C a^3 - C a b^2) c^2 - (B a^3 b - B a b^3) c) \cos(x) + (B a^3 b^2 - B a b^4 - B a b^2 c^2 - C a b c^3 + (C a^3 b - C a b^3) c) \sin(x) / (a^5 b^2 - 2 a^3 b^4 + a b^6 + a c^6 - (2 a^3 - 3 a b^2) c^4 + (a^5 - 4 a^3 b^2 + 3 a b^4) c^2 + (a^4 b^3 - 2 a^2 b^5 + b^7 + b c^6 - (2 a^2 b - 3 b^3) c^4 + (a^4 b - 4 a^2 b^3 + 3 b^5) c^2) \cos(x) + (c^7 - (2 a^2 - 3 b^2) c^5 + (a^4 - 4 a^2 b^2 + 3 b^4) c^3 + (a^4 b^2 - 2 a^2 b^4 + b^6) c) \sin(x)]$$

giac [A] time = 0.18, size = 205, normalized size = 1.86

$$\frac{2 \left(\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2a + 2b) + \arctan \left(\frac{a \tan(\frac{1}{2}x) - b \tan(\frac{1}{2}x) + c}{\sqrt{a^2 - b^2 - c^2}} \right) \right) (Bb + Cc)}{(a^2 - b^2 - c^2)^{\frac{3}{2}}} + \frac{2 \left(B a^2 \tan(\frac{1}{2}x) - B a b \tan(\frac{1}{2}x) - \dots \right)}{(a^3 - a^2 b - a b^2 + b^3 - a c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")

[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(B*b + C*c)/(a^2 - b^2 - c^2)^(3/2) + 2*(B*a^2*tan(1/2*x) - B*a*b*tan(1/2*x) - C*a*c*tan(1/2*x) + C*b*c*tan(1/2*x) - B*c^2*tan(1/2*x) - C*a^2 + C*b^2 - B*b*c)/((a^3 - a^2*b - a*b^2 + b^3 - a*c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b))

maple [B] time = 0.18, size = 255, normalized size = 2.32

$$\frac{2 \left(-\frac{(a^2 B - a b B - B c^2 - a c C + C b c) \tan(\frac{x}{2})}{a^3 - a^2 b - a b^2 - a c^2 + b^3 + c^2 b} + \frac{b B c + a^2 C - b^2 C}{a^3 - a^2 b - a b^2 - a c^2 + b^3 + c^2 b} \right) 2 \arctan \left(\frac{2(a-b) \tan(\frac{x}{2}) + 2c}{2\sqrt{a^2 - b^2 - c^2}} \right) b B}{a \left(\tan^2 \left(\frac{x}{2} \right) \right) - b \left(\tan^2 \left(\frac{x}{2} \right) \right) + 2c \tan \left(\frac{x}{2} \right) + a + b} - \frac{2 \arctan \left(\frac{2(a-b) \tan(\frac{x}{2})}{2\sqrt{a^2 - b^2 - c^2}} \right) b B}{(a^2 - b^2 - c^2)^{\frac{3}{2}}} - \frac{2 \arctan \left(\frac{2(a-b) \tan(\frac{x}{2})}{2\sqrt{a^2 - b^2 - c^2}} \right) b B}{(a^2 - b^2 - c^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x)

[Out] -2*(-(B*a^2-B*a*b-B*c^2-C*a*c+C*b*c)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan(1/2*x)+(B*b*c+C*a^2-C*b^2)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2))/(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)-2/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*b*B-2/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*C*c

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see 'assume?' for more details)Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 3.24, size = 202, normalized size = 1.84

$$\frac{2(C a^2 - C b^2 + B c b)}{(a-b)(-a^2+b^2+c^2)} + \frac{2 \tan(\frac{x}{2})(-B a^2 + C a c + B b a + B c^2 - C b c)}{(a-b)(-a^2+b^2+c^2)} 2 \operatorname{atanh} \left(\frac{\tan(\frac{x}{2})(2a-2b) + \frac{2(-a^2 c + b^2 c + c^3)}{-a^2 + b^2 + c^2}}{2\sqrt{-a^2 + b^2 + c^2}} \right) (B b + C c)}{(a-b) \tan \left(\frac{x}{2} \right)^2 + 2c \tan \left(\frac{x}{2} \right) + a + b} - \frac{2 \operatorname{atanh} \left(\frac{\tan(\frac{x}{2})(2a-2b) + \frac{2(-a^2 c + b^2 c + c^3)}{-a^2 + b^2 + c^2}}{2\sqrt{-a^2 + b^2 + c^2}} \right) (B b + C c)}{(-a^2 + b^2 + c^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x))^2,x)`

[Out]
$$\frac{(2*(C*a^2 - C*b^2 + B*b*c))/((a - b)*(b^2 - a^2 + c^2)) + (2*\tan(x/2)*(B*c^2 - B*a^2 + B*a*b + C*a*c - C*b*c))/((a - b)*(b^2 - a^2 + c^2))}{(a + b + 2*c*\tan(x/2) + \tan(x/2)^2*(a - b))} - \frac{(2*\operatorname{atanh}(\tan(x/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^2 - a^2 + c^2)))/(2*(b^2 - a^2 + c^2)^{(1/2)})}{(b^2 - a^2 + c^2)^{(3/2)}}*(B*b + C*c)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)`

[Out] Timed out

$$3.547 \quad \int \frac{B \cos(x) + C \sin(x)}{(a + b \cos(x) + c \sin(x))^3} dx$$

Optimal. Leaf size=197

$$\frac{3a(bB + cC) \tan^{-1} \left(\frac{(a-b) \tan(\frac{x}{2}) + c}{\sqrt{a^2 - b^2 - c^2}} \right)}{(a^2 - b^2 - c^2)^{5/2}} + \frac{-\cos(x) (C(a^2 + 2c^2) + 2bBc) + \sin(x) (a^2B + 2b(bB + cC)) + a(Bc - bC)}{2(a^2 - b^2 - c^2)^2 (a + b \cos(x) + c \sin(x))}$$

[Out] $-3*a*(B*b+C*c)*\arctan((c+(a-b)*\tan(1/2*x))/(\sqrt{a^2-b^2-c^2}))/(a^2-b^2-c^2)^{5/2}+1/2*(B*c-b*C-a*C*\cos(x)+a*B*\sin(x))/(\sqrt{a^2-b^2-c^2})/(a+b*\cos(x)+c*\sin(x))^2+1/2*(a*(B*c-C*b)-(2*b*B*c+(a^2+2*c^2)*C)*\cos(x)+(a^2*B+2*b*(B*b+C*c))*\sin(x))/(\sqrt{a^2-b^2-c^2})^2/(a+b*\cos(x)+c*\sin(x))$

Rubi [A] time = 0.23, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, number of rules / integrand size = 0.227, Rules used = {3156, 3153, 3124, 618, 204}

$$\frac{3a(bB + cC) \tan^{-1} \left(\frac{(a-b) \tan(\frac{x}{2}) + c}{\sqrt{a^2 - b^2 - c^2}} \right)}{(a^2 - b^2 - c^2)^{5/2}} + \frac{-\cos(x) (C(a^2 + 2c^2) + 2bBc) + \sin(x) (a^2B + 2b(bB + cC)) + a(Bc - bC)}{2(a^2 - b^2 - c^2)^2 (a + b \cos(x) + c \sin(x))}$$

Antiderivative was successfully verified.

[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]

[Out] $(-3*a*(b*B + c*C)*\text{ArcTan}[(c + (a - b)*\text{Tan}[x/2])/\text{Sqrt}[a^2 - b^2 - c^2]])/(a^2 - b^2 - c^2)^{5/2} + (B*c - b*C - a*C*\text{Cos}[x] + a*B*\text{Sin}[x])/(2*(a^2 - b^2 - c^2)*(a + b*\text{Cos}[x] + c*\text{Sin}[x])^2) + (a*(B*c - b*C) - (2*b*B*c + (a^2 + 2*c^2)*C)*\text{Cos}[x] + (a^2*B + 2*b*(b*B + c*C))*\text{Sin}[x])/(2*(a^2 - b^2 - c^2)^2*(a + b*\text{Cos}[x] + c*\text{Sin}[x]))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3153

Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]) / ((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] := Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]) / (e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2

- c^2, 0] && NeQ[a*A - b*B - c*C, 0]

Rule 3156

Int[((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]), x_Symbol] :> -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a² - b² - c²)), x] + Dist[1/((n + 1)*(a² - b² - c²)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /;

FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a² - b² - c², 0] && NeQ[n, -2]

Rubi steps

$$\begin{aligned} \int \frac{B \cos(x) + C \sin(x)}{(a + b \cos(x) + c \sin(x))^3} dx &= \frac{Bc - bC - aC \cos(x) + aB \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} - \frac{\int \frac{2(bB + cC) - aB \cos(x) - aC \sin(x)}{(a + b \cos(x) + c \sin(x))^2} dx}{2(a^2 - b^2 - c^2)} \\ &= \frac{Bc - bC - aC \cos(x) + aB \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} + \frac{a(Bc - bC) - (2bBc + (a^2 + 2c^2)C)}{2(a^2 - b^2 - c^2)^2} \\ &= \frac{Bc - bC - aC \cos(x) + aB \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} + \frac{a(Bc - bC) - (2bBc + (a^2 + 2c^2)C)}{2(a^2 - b^2 - c^2)^2} \\ &= \frac{Bc - bC - aC \cos(x) + aB \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} + \frac{a(Bc - bC) - (2bBc + (a^2 + 2c^2)C)}{2(a^2 - b^2 - c^2)^2} \\ &= -\frac{3a(bB + cC) \tan^{-1}\left(\frac{c + (a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2 - c^2}}\right)}{(a^2 - b^2 - c^2)^{5/2}} + \frac{Bc - bC - aC \cos(x) + aB \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} \end{aligned}$$

Mathematica [A] time = 0.90, size = 311, normalized size = 1.58

$$\frac{3a(bB + cC) \tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right) + c}{\sqrt{-a^2 + b^2 + c^2}}\right)}{(-a^2 + b^2 + c^2)^{5/2}} + \frac{2a^4C + 4a^3bB \sin(x) + 4a^3cC \sin(x) - c \cos(2x)(a^2 + 2(b^2 + c^2))(bB + cC)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3, x]

[Out] (3*a*(b*B + c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2) + (9*a^2*b*B*c + 2*a^4*C - 4*a^2*b^2*C + 2*b^4*C + 5*a^2*c^2*C + 4*b^2*c^2*C + 2*c^4*C + 6*a*b*c*(b*B + c*C)*Cos[x] - c*(a^2 + 2*(b^2 + c^2))*(b*B + c*C)*Cos[2*x] + 4*a^3*b*B*Sin[x] + 2*a*b^3*B*Sin[x] + 8*a*b*B*c^2*Sin[x] + 4*a^3*c*C*Sin[x] + 2*a*b^2*c*C*Sin[x] + 8*a*c^3*C*Sin[x] + a^2*b^2*B*Sin[2*x] + 2*b^4*B*Sin[2*x] + 2*b^2*B*c^2*Sin[2*x] + a^2*b*c*C*Sin[2*x] + 2*b^3*c*C*Sin[2*x] + 2*b*c^3*C*Sin[2*x])/(4*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)

fricas [B] time = 2.20, size = 3264, normalized size = 16.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fricas")

[Out] [1/4*(2*C*a^6*b - 6*C*a^4*b^3 + 6*C*a^2*b^5 - 2*C*b^7 - 6*C*b*c^6 + 2*B*c^7 - 2*(3*B*a^2 - B*b^2)*c^5 + 2*(4*C*a^2*b - 7*C*b^3)*c^4 + 2*(3*B*a^4 - 5*B*a^2*b^2 - B*b^4)*c^3 - 2*(2*C*a^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 + 4*(2*B*b^2*c^5 + 2*C*b*c^6 - (C*a^2*b - 4*C*b^3)*c^4 - (B*a^2*b^2 - 4*B*b^4)*c^3 - (C*a^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 + B*a^2*b^4 - 2*B*b^6)*c)*cos(x)^2 - 3*(B*a^3*b^3 + C*a^3*b^2*c + B*a*b*c^4 + C*a*c^5 + (C*a^3 + C*a*b^2)*c^3 + (B*a^3*b + B*a*b^3)*c^2 + (B*a*b^5 + C*a*b^4*c - B*a*b*c^4 - C*a*c^5)*cos(x)^2 + 2*(B*a^2*b^4 + C*a^2*b^3*c + B*a^2*b^2*c^2 + C*a^2*b*c^3)*cos(x) + 2*(B*a^2*b^3*c + C*a^2*b^2*c^2 + B*a^2*b*c^3 + C*a^2*c^4 + (B*a*b^4*c + C*a*b^3*c^2 + B*a*b^2*c^3 + C*a*b*c^4)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x)))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)) - 2*(B*a^6 - 4*B*a^4*b^2 + 2*B*a^2*b^4 + B*b^6)*c + 2*(B*a*b*c^5 + C*a*c^6 + (C*a^3 + 2*C*a*b^2)*c^4 + (B*a^3*b + 2*B*a*b^3)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 - (2*B*a^5*b - B*a^3*b^3 - B*a*b^5)*c)*cos(x) + 2*(2*B*a^5*b^2 - B*a^3*b^4 - B*a*b^6 - B*a*b^2*c^4 - C*a*b*c^5 - (C*a^3*b + 2*C*a*b^3)*c^3 - (B*a^3*b^2 + 2*B*a*b^4)*c^2 + (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (B*a^4*b^3 + B*a^2*b^5 - 2*B*b^7 + 2*B*b*c^6 + 2*C*c^7 - (C*a^2 - 2*C*b^2)*c^5 - (B*a^2*b - 2*B*b^3)*c^4 - (C*a^4 + 2*C*b^4)*c^3 - (B*a^4*b + 2*B*b^5)*c^2 + (C*a^4*b^2 + C*a^2*b^4 - 2*C*b^6)*c)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*cos(x))*sin(x)), 1/2*(C*a^6*b - 3*C*a^4*b^3 + 3*C*a^2*b^5 - C*b^7 - 3*C*b*c^6 + B*c^7 - (3*B*a^2 - B*b^2)*c^5 + (4*C*a^2*b - 7*C*b^3)*c^4 + (3*B*a^4 - 5*B*a^2*b^2 - B*b^4)*c^3 - (2*C*a^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 + 2*(2*B*b^2*c^5 + 2*C*b*c^6 - (C*a^2*b - 4*C*b^3)*c^4 - (B*a^2*b^2 - 4*B*b^4)*c^3 - (C*a^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 + B*a^2*b^4 - 2*B*b^6)*c)*cos(x)^2 - 3*(B*a^3*b^3 + C*a^3*b^2*c + B*a*b*c^4 + C*a*c^5 + (C*a^3 + C*a*b^2)*c^3 + (B*a^3*b + B*a*b^3)*c^2 + (B*a*b^5 + C*a*b^4*c - B*a*b*c^4 - C*a*c^5)*cos(x)^2 + 2*(B*a^2*b^4 + C*a^2*b^3*c + B*a^2*b^2*c^2 + C*a^2*b*c^3)*cos(x) + 2*(B*a^2*b^3*c + C*a^2*b^2*c^2 + B*a^2*b*c^3 + C*a^2*c^4 + (B*a*b^4*c + C*a*b^3*c^2 + B*a*b^2*c^3 + C*a*b*c^4)*cos(x))*sin(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) - (B*a^6 - 4*B*a^4*b^2 + 2*B*a^2*b^4 + B*b^6)*c + (B*a*b*c^5 + C*a*c^6 + (C*a^3 + 2*C*a*b^2)*c^4 + (B*a^3*b + 2*B*a*b^3)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 - (2*B*a^5*b - B*a^3*b^3 - B*a*b^5)*c)*cos(x) + (2*B*a^5*b^2 - B*a^3*b^4 - B*a*b^6 - B*a*b^2*c^4 - C*a*b*c^5 - (C*a^3*b + 2*C*a*b^3)*c^3 - (B*a^3*b^2 + 2*B*a*b^4)*c^2 + (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (B*a^4*b^3 + B*a^2*b^5 - 2*B*b^7 + 2*B*b*c^6 + 2*C*c^7 - (C*a^2 - 2*C*b^2)*c^5 - (B*a^2*b - 2*B*b^3)*c^4 - (C*a^4 + 2*C*b^4)*c^3 - (B*a^4*b + 2*B*b^5)*c^2 + (C*a^4*b^2 + C*a^2*b^4 - 2*C*b^6)*c)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a

```

^2*b^8 - c^10 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*
a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 -
b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*
c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(
a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b
^7 - a*b^9 - a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a
*b^5)*c^4 + (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^
9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^
5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8
)*c + (b*c^9 - (3*a^2*b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 -
(a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*
b^7 - b^9)*c)*cos(x))*sin(x)]

```

giac [B] time = 0.51, size = 1034, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac")
[Out] 3*(B*a*b + C*a*c)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/((a^4 - 2*a^2*b^2 + b^
4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^2 - c^2)) + (2*B*a^5*tan(1/2*x)
^3 - 5*B*a^4*b*tan(1/2*x)^3 + 5*B*a^3*b^2*tan(1/2*x)^3 - 5*B*a^2*b^3*tan(
1/2*x)^3 + 5*B*a*b^4*tan(1/2*x)^3 - 2*B*b^5*tan(1/2*x)^3 - 3*C*a^4*c*tan(1/
2*x)^3 + 9*C*a^3*b*c*tan(1/2*x)^3 - 9*C*a^2*b^2*c*tan(1/2*x)^3 + 3*C*a*b^3*
c*tan(1/2*x)^3 - 4*B*a^3*c^2*tan(1/2*x)^3 + 4*B*a^2*b*c^2*tan(1/2*x)^3 + 4*
B*a*b^2*c^2*tan(1/2*x)^3 - 4*B*b^3*c^2*tan(1/2*x)^3 + 2*B*a*c^4*tan(1/2*x)^
3 - 2*B*b*c^4*tan(1/2*x)^3 - 2*C*a^5*tan(1/2*x)^2 + 2*C*a^4*b*tan(1/2*x)^2
+ 4*C*a^3*b^2*tan(1/2*x)^2 - 4*C*a^2*b^3*tan(1/2*x)^2 - 2*C*a*b^4*tan(1/2*x)
^2 + 2*C*b^5*tan(1/2*x)^2 + 2*B*a^4*c*tan(1/2*x)^2 - 9*B*a^3*b*c*tan(1/2*x)
^2 + 14*B*a^2*b^2*c*tan(1/2*x)^2 - 9*B*a*b^3*c*tan(1/2*x)^2 + 2*B*b^4*c*tan
(1/2*x)^2 - 5*C*a^3*c^2*tan(1/2*x)^2 + 14*C*a^2*b*c^2*tan(1/2*x)^2 - 13*C*
a*b^2*c^2*tan(1/2*x)^2 + 4*C*b^3*c^2*tan(1/2*x)^2 - 4*B*a^2*c^3*tan(1/2*x)^
2 + 4*B*b^2*c^3*tan(1/2*x)^2 - 2*C*a*c^4*tan(1/2*x)^2 + 2*C*b*c^4*tan(1/2*x)
^2 + 2*B*c^5*tan(1/2*x)^2 + 2*B*a^5*tan(1/2*x) - 3*B*a^4*b*tan(1/2*x) + B*
a^3*b^2*tan(1/2*x) + B*a^2*b^3*tan(1/2*x) - 3*B*a*b^4*tan(1/2*x) + 2*B*b^5*
tan(1/2*x) - 5*C*a^4*c*tan(1/2*x) + 5*C*a^3*b*c*tan(1/2*x) + 5*C*a^2*b^2*c*
tan(1/2*x) - 5*C*a*b^3*c*tan(1/2*x) - 4*B*a^3*c^2*tan(1/2*x) - 8*B*a^2*b*c^
2*tan(1/2*x) + 8*B*a*b^2*c^2*tan(1/2*x) + 4*B*b^3*c^2*tan(1/2*x) - 4*C*a^2*
c^3*tan(1/2*x) + 4*C*a*b*c^3*tan(1/2*x) + 2*B*a*c^4*tan(1/2*x) + 2*B*b*c^4*
tan(1/2*x) - 2*C*a^5 + 4*C*a^3*b^2 - 2*C*a*b^4 - 5*B*a^3*b*c + 5*B*a*b^3*c
- C*a^3*c^2 + C*a*b^2*c^2 + 2*B*a*b*c^3)/((a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*
b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*a*b^3*c^2 + 2*b
^4*c^2 + a^2*c^4 - 2*a*b*c^4 + b^2*c^4)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 +
2*c*tan(1/2*x) + a + b)^2)

```

maple [B] time = 0.22, size = 881, normalized size = 4.47

$$2 \left(\frac{(2B a^4 - 3B a^3 b + 2B a^2 b^2 - 4B a^2 c^2 - 3B a b^3 + 2B b^4 + 4B b^2 c^2 + 2B c^4 - 3C a^3 c + 6C a^2 b c - 3C a b^2 c) \left(\tan^3\left(\frac{x}{2}\right) \right)}{2(a^4 - 2a^2 b^2 - 2a^2 c^2 + b^4 + 2b^2 c^2 + c^4)(a-b)} - \frac{(2B a^4 c - 9B a^3 b c + 14B a^2 b^2 c - 4B a^2 c^3 - 2B a^3 b^2 c + 2B a^2 b^3 c - 2B a b^4 c + 2B b^5 c - 5C a^4 c^2 + 5C a^3 b c^2 - 5C a^2 b^2 c^2 + 5C a b^3 c^2 - 4C a^3 c^3 + 4C a^2 b c^3 - 4C a b^2 c^3 + 2C a^4 c^4 - 2C a^3 b c^4 - 2C a^2 b^2 c^4 + 2C a b^3 c^4 - 2C b^4 c^4)}{(a-b)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x)
[Out] -2*(-1/2*(2*B*a^4-3*B*a^3*b+2*B*a^2*b^2-4*B*a^2*c^2-3*B*a*b^3+2*B*b^4+4*B*b
^2*c^2+2*B*b*c^4-3*C*a^3*c+6*C*a^2*b*c-3*C*a*b^2*c)/(a^4-2*a^2*b^2-2*a^2*c^2+
b^4+2*b^2*c^2+c^4)/(a-b)*tan(1/2*x)^3-1/2*(2*B*a^4*c-9*B*a^3*b*c+14*B*a^2*b

```


$$\frac{(2B^2c^4 - 4B^2a^2c^3 - 9B^2ab^3c^2 + 2B^2b^4c + 4B^2b^2c^3 + 2B^2c^5 - 2C^2a^5 + 2C^2a^4b + 4C^2a^3b^2 - 5C^2a^3c^2 - 4C^2a^2b^3 + 14C^2a^2b^2c^2 - 2C^2ab^4 - 13C^2ab^2c^2 - 2C^2ac^4 + 2C^2b^5 + 4C^2b^3c^2 + 2C^2b^2c^4) / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - 2ab + b^2) \cdot \tan(1/2x)^2 - 1/2 \cdot (2B^2a^5 - 3B^2a^4b + B^2a^3b^2 - 4B^2a^3c^2 + B^2a^2b^3 - 8B^2a^2b^2c^2 - 3B^2ab^4 + 8B^2ab^2c^2 + 2B^2a^2c^4 + 2B^2b^5 + 4B^2b^3c^2 + 2B^2b^2c^4 - 5C^2a^4c + 5C^2a^3b^2c + 5C^2a^2b^2c^2 - 4C^2a^2c^3 - 5C^2ab^3c + 4C^2ab^2c^3) / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - 2ab + b^2) \cdot \tan(1/2x) + 1/2 \cdot a \cdot (5B^2a^2b^2c - 5B^2b^3c - 2B^2b^2c^3 + 2C^2a^4 - 4C^2a^2b^2 + C^2a^2c^2 + 2C^2b^4 - C^2b^2c^2) / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - 2ab + b^2) / (a \cdot \tan(1/2x)^2 - b \cdot \tan(1/2x)^2 + 2c \cdot \tan(1/2x) + a + b)^2 - 3 / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - b^2 - c^2)^{(1/2)} \cdot \arctan(1/2 \cdot (2 \cdot (a - b) \cdot \tan(1/2x) + 2c) / (a^2 - b^2 - c^2)^{(1/2)}) \cdot a \cdot b \cdot B - 3 / (a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4) / (a^2 - b^2 - c^2)^{(1/2)} \cdot \arctan(1/2 \cdot (2 \cdot (a - b) \cdot \tan(1/2x) + 2c) / (a^2 - b^2 - c^2)^{(1/2)}) \cdot a \cdot c \cdot C$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see 'assume?' for more details)Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 6.36, size = 923, normalized size = 4.69

$$\frac{\tan\left(\frac{x}{2}\right)^3 (2Ba^4 - 3Ba^3b - 3Ca^3c + 2Ba^2b^2 + 6Ca^2bc - 4Ba^2c^2 - 3Ba^2b^3 - 3Cab^2c + 2Bb^4 + 4Bb^2c^2 + 2Bc^4)}{(a-b)(a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4)} - \frac{2Ca^5 - 4Ca^3b^2 + 5Ba^3bc + Ca^3c^2}{(a-b)^2(a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x))^3,x)

[Out] ((tan(x/2)^3*(2B^2a^4 + 2B^2b^4 + 2B^2c^4 + 2B^2a^2b^2 - 4B^2a^2c^2 + 4B^2b^2c^2 - 3B^2ab^3 - 3B^2a^3b - 3C^2a^3c - 3C^2ab^2c + 6C^2a^2b^2c)) / ((a - b)*(a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2)) - (2C^2a^5 - 4C^2a^3b^2 + C^2a^3c^2 + 2C^2ab^4 - 2B^2ab^2c^3 - 5B^2ab^3c + 5B^2a^3b^2c - C^2ab^2c^2) / ((a - b)^2*(a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2)) + (tan(x/2)^2*(2B^2c^5 - 2C^2a^5 + 2C^2b^5 - 4B^2a^2c^3 - 4C^2a^2b^3 + 4C^2a^3b^2 + 4B^2b^2c^3 - 5C^2a^3c^2 + 4C^2b^3c^2 + 2B^2a^4c - 2C^2ab^4 + 2C^2a^4b + 2B^2b^4c - 2C^2ac^4 + 2C^2b^2c^4 - 9B^2ab^3c - 9B^2a^3b^2c + 14B^2a^2b^2c^2 - 13C^2ab^2c^2 + 14C^2a^2b^2c^2)) / ((a - b)^2*(a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2)) + (tan(x/2)*(2B^2a^5 + 2B^2b^5 + B^2a^2b^3 + B^2a^3b^2 - 4B^2a^3c^2 + 4B^2b^3c^2 - 4C^2a^2c^3 - 3B^2ab^4 - 3B^2a^4b + 2B^2a^2c^4 + 2B^2b^2c^4 - 5C^2a^4c + 4C^2ab^2c^3 - 5C^2ab^3c + 5C^2a^3b^2c + 8B^2ab^2c^2 - 8B^2a^2b^2c^2 + 5C^2a^2b^2c^2)) / ((a - b)^2*(a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2)) + (tan(x/2)^4*(a^2 - 2ab + b^2) + 2ab + tan(x/2)*(4ac + 4bc) + tan(x/2)^3*(4ac - 4bc) + a^2 + b^2 + tan(x/2)^2*(2a^2 - 2b^2 + 4c^2)) + (3a * atanh((3a*(B*b + C*c)*(tan(x/2)*(2a - 2b) + (2a^4c + 2b^4c + 2c^5 - 4a^2c^3 + 4b^2c^3 - 4a^2b^2c) / (a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2)) * (a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2))) * (B*b + C*c)) / (b^2 - a^2 + c^2)^(5/2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**3,x)

[Out] Timed out

$$3.548 \quad \int \frac{B \cos(x) + C \sin(x)}{a + b \cos(x) + ib \sin(x)} dx$$

Optimal. Leaf size=92

$$\frac{(a^2(C + iB) + ib^2(B + iC)) \log(a + ib \sin(x) + b \cos(x))}{2a^2b} - \frac{bx(B + iC)}{2a^2} + \frac{(-C + iB)(\cos(x) - i \sin(x))}{2a}$$

[Out] $-1/2*b*(B+I*C)*x/a^2-1/2*(I*b^2*(B+I*C)+a^2*(I*B+C))*\ln(a+b*\cos(x)+I*b*\sin(x))/a^2/b+1/2*(I*B-C)*(cos(x)-I*\sin(x))/a$

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 0.95, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {3130}

$$\frac{\left(\frac{ib^2(B+iC)}{a^2} + iB + C\right) \log(a + ib \sin(x) + b \cos(x))}{2b} - \frac{bx(B + iC)}{2a^2} + \frac{(-C + iB)(\cos(x) - i \sin(x))}{2a}$$

Antiderivative was successfully verified.

[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]

[Out] $-(b*(B + I*C)*x)/(2*a^2) - ((I*B + (I*b^2*(B + I*C))/a^2 + C)*\text{Log}[a + b*\text{Cos}[x] + I*b*\text{Sin}[x]])/(2*b) + ((I*B - C)*(\text{Cos}[x] - I*\text{Sin}[x]))/(2*a)$

Rule 3130

Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]) / (cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]), x_Symbol] :> Simp[((2*a*A - b*B - c*C)*x)/(2*a^2), x] + (-Simp[((b*B + c*C)*(b*Cos[d + e*x] - c*Sin[d + e*x]))/(2*a*b*c*e), x] + Simp[((a^2*(b*B - c*C) - 2*a*A*b^2 + b^2*(b*B + c*C))*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*b*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[b^2 + c^2, 0]

Rubi steps

$$\int \frac{B \cos(x) + C \sin(x)}{a + b \cos(x) + ib \sin(x)} dx = -\frac{b(B + iC)x}{2a^2} - \frac{\left(iB + \frac{ib^2(B+iC)}{a^2} + C\right) \log(a + b \cos(x) + ib \sin(x))}{2b} + \frac{(iB - C)(\cos(x) - i \sin(x))}{2a}$$

Mathematica [B] time = 0.32, size = 195, normalized size = 2.12

$$\frac{x(a^2B - ia^2C - b^2B - ib^2C)}{4a^2b} - \frac{i(a^2B - ia^2C + b^2B + ib^2C) \log(a^2 + 2ab \cos(x) + b^2)}{4a^2b} - \frac{(a^2B - ia^2C + b^2B + ib^2C)(\cos(x) - i \sin(x))}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]

[Out] $((a^2*B - b^2*B - I*a^2*C - I*b^2*C)*x)/(4*a^2*b) - ((a^2*B + b^2*B - I*a^2*C + I*b^2*C)*\text{ArcTan}[\frac{(a + b)*\text{Cos}[x/2]}{-(a*\text{Sin}[x/2]) + b*\text{Sin}[x/2]}])/(2*a^2*b) + ((I/2)*(B + I*C)*\text{Cos}[x])/a - ((I/4)*(a^2*B + b^2*B - I*a^2*C + I*b^2*C)*\text{Log}[a^2 + b^2 + 2*a*b*\text{Cos}[x]])/(a^2*b) + ((B + I*C)*\text{Sin}[x])/(2*a)$

fricas [A] time = 1.05, size = 78, normalized size = 0.85

$$\frac{\left((B + iC)b^2xe^{ix} - (iB - C)ab - ((-iB - C)a^2 + (-iB + C)b^2)e^{ix}\right) \log\left(\frac{be^{ix} + a}{b}\right)}{2a^2b} e^{-ix}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fricas")

[Out] $-1/2*((B + I*C)*b^2*x*e^{I*x} - (I*B - C)*a*b - ((-I*B - C)*a^2 + (-I*B + C)*b^2))*e^{I*x}*\log((b*e^{I*x} + a)/b))*e^{-I*x}/(a^2*b)$

giac [B] time = 0.16, size = 178, normalized size = 1.93

$$\frac{(iBb - Cb) \log\left(-a \tan\left(\frac{1}{2}x\right)^2 + b \tan\left(\frac{1}{2}x\right)^2 - 2ia \tan\left(\frac{1}{2}x\right) + a + b\right)}{4a^2} - \frac{(-iBb + Cb) \log\left(\tan\left(\frac{1}{2}x\right) - i\right)}{2a^2} + \frac{(2Ba^2)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac")

[Out] $-1/4*(I*B*b - C*b)*\log(-a*\tan(1/2*x)^2 + b*\tan(1/2*x)^2 - 2*I*a*\tan(1/2*x) + a + b)/a^2 - 1/2*(-I*B*b + C*b)*\log(\tan(1/2*x) - I)/a^2 + 1/4*(2*B*a^2 - 2*I*C*a^2 + B*b^2 + I*C*b^2)*(x + 2*\arctan((-I*a*\cos(x) - a*\sin(x) - I*a)/(a*\cos(x) - I*a*\sin(x) - a + 2*b)))/(a^2*b) - 1/2*(I*B*b*\tan(1/2*x) - C*b*\tan(1/2*x) - 2*B*a - 2*I*C*a + B*b + I*C*b)/(a^2*(\tan(1/2*x) - I))$

maple [B] time = 0.20, size = 212, normalized size = 2.30

$$\frac{C \ln\left(\tan\left(\frac{x}{2}\right) + i\right)}{2b} + \frac{iB \ln\left(\tan\left(\frac{x}{2}\right) + i\right)}{2b} + \frac{iC}{a\left(\tan\left(\frac{x}{2}\right) - i\right)} + \frac{B}{a\left(\tan\left(\frac{x}{2}\right) - i\right)} + \frac{i \ln\left(\tan\left(\frac{x}{2}\right) - i\right) bB}{2a^2} - \frac{\ln\left(\tan\left(\frac{x}{2}\right) - i\right) bC}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)

[Out] $1/2*C/b*\ln(\tan(1/2*x)+I)+1/2*I*B/b*\ln(\tan(1/2*x)+I)+I*C/a/(\tan(1/2*x)-I)+B/a/(\tan(1/2*x)-I)+1/2*I/a^2*\ln(\tan(1/2*x)-I)*b*B-1/2/a^2*\ln(\tan(1/2*x)-I)*b*C-1/2/b*\ln(I*a+I*b+a*\tan(1/2*x)-b*\tan(1/2*x))*C+1/2/a^2*b*\ln(I*a+I*b+a*\tan(1/2*x)-b*\tan(1/2*x))*C-1/2*I/b*\ln(I*a+I*b+a*\tan(1/2*x)-b*\tan(1/2*x))*B-1/2*I/a^2*b*\ln(I*a+I*b+a*\tan(1/2*x)-b*\tan(1/2*x))*B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

mupad [B] time = 5.32, size = 118, normalized size = 1.28

$$-\ln\left(a + b - a \tan\left(\frac{x}{2}\right) + b \tan\left(\frac{x}{2}\right) + i\right) \left(\frac{C}{2} + \frac{B1i}{2} + \frac{-Cb^2}{2} + \frac{Bb^21i}{2} \right) + \frac{B + C1i}{a\left(\tan\left(\frac{x}{2}\right) - i\right)} + \frac{\ln\left(\tan\left(\frac{x}{2}\right) + 1i\right) (C + B)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*cos(x) + C*sin(x))/(a + b*cos(x) + b*sin(x)*1i),x)

```
[Out] (B + C*1i)/(a*(tan(x/2) - 1i)) - log(a + b - a*tan(x/2)*1i + b*tan(x/2)*1i)
*(((B*1i)/2 + C/2)/b + ((B*b^2*1i)/2 - (C*b^2)/2)/(a^2*b)) + (log(tan(x/2)
+ 1i)*(B*1i + C))/(2*b) + (log(tan(x/2) - 1i)*(B*b*1i - C*b))/(2*a^2)
```

sympy [A] time = 0.85, size = 116, normalized size = 1.26

$$\begin{cases} -\frac{(-iB+C)e^{-ix}}{2a} & \text{for } 2a \neq 0 \\ x\left(-\frac{-Bb-iCb}{2a^2} - \frac{i(iBa-iBb-Ca+Cb)}{2a^2}\right) & \text{otherwise} \end{cases} \frac{x(Bb + iCb)}{2a^2} - \frac{i(Ba^2 + Bb^2 - iCa^2 + iCb^2) \log\left(\frac{a}{b} + e^{ix}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] Piecewise((-(-I*B + C)*exp(-I*x)/(2*a), Ne(2*a, 0)), (x*(-(-B*b - I*C*b)/(2
*a**2) - I*(I*B*a - I*B*b - C*a + C*b)/(2*a**2)), True)) - x*(B*b + I*C*b)/
(2*a**2) - I*(B*a**2 + B*b**2 - I*C*a**2 + I*C*b**2)*log(a/b + exp(I*x))/(2
*a**2*b)
```

$$3.549 \quad \int \frac{B \cos(x) + C \sin(x)}{a + b \cos(x) - ib \sin(x)} dx$$

Optimal. Leaf size=90

$$\frac{(ia^2(B + iC) + b^2(C + iB)) \log(a - ib \sin(x) + b \cos(x))}{2a^2b} - \frac{bx(B - iC)}{2a^2} - \frac{(C + iB)(\cos(x) + i \sin(x))}{2a}$$

[Out] $-1/2*b*(B-I*C)*x/a^2+1/2*(I*a^2*(B+I*C)+b^2*(I*B+C))*\ln(a+b*\cos(x)-I*b*\sin(x))/a^2/b-1/2*(I*B+C)*(\cos(x)+I*\sin(x))/a$

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 0.94, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {3130}

$$-\frac{bx(B - iC)}{2a^2} + \frac{1}{2} \left(\frac{b(C + iB)}{a^2} + \frac{i(B + iC)}{b} \right) \log(a - ib \sin(x) + b \cos(x)) - \frac{(C + iB)(\cos(x) + i \sin(x))}{2a}$$

Antiderivative was successfully verified.

[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]

[Out] $-(b*(B - I*C)*x)/(2*a^2) + (((I*(B + I*C))/b + (b*(I*B + C))/a^2)*\text{Log}[a + b*\text{Cos}[x] - I*b*\text{Sin}[x]])/2 - ((I*B + C)*(\text{Cos}[x] + I*\text{Sin}[x]))/(2*a)$

Rule 3130

Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)])/(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]), x_Symbol] :> Simp[((2*a*A - b*B - c*C)*x)/(2*a^2), x] + (-Simp[((b*B + c*C)*(b*Cos[d + e*x] - c*Sin[d + e*x]))/(2*a*b*c*e), x] + Simp[((a^2*(b*B - c*C) - 2*a*A*b^2 + b^2*(b*B + c*C))*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*b*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[b^2 + c^2, 0]

Rubi steps

$$\int \frac{B \cos(x) + C \sin(x)}{a + b \cos(x) - ib \sin(x)} dx = -\frac{b(B - iC)x}{2a^2} + \frac{1}{2} \left(\frac{i(B + iC)}{b} + \frac{b(iB + C)}{a^2} \right) \log(a + b \cos(x) - ib \sin(x)) - \frac{(iB + C)(\cos(x) + i \sin(x))}{2a}$$

Mathematica [B] time = 0.29, size = 195, normalized size = 2.17

$$\frac{x(a^2B + ia^2C - b^2B + ib^2C)}{4a^2b} + \frac{i(a^2B + ia^2C + b^2B - ib^2C) \log(a^2 + 2ab \cos(x) + b^2)}{4a^2b} + \frac{(a^2B + ia^2C + b^2B - ib^2C)(\cos(x) + i \sin(x))}{2a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]

[Out] $((a^2*B - b^2*B + I*a^2*C + I*b^2*C)*x)/(4*a^2*b) + ((a^2*B + b^2*B + I*a^2*C - I*b^2*C)*\text{ArcTan}[(a + b)*\text{Cos}[x/2]/(a*\text{Sin}[x/2] - b*\text{Sin}[x/2])])/(2*a^2*b) - ((I/2)*(B - I*C)*\text{Cos}[x])/a + ((I/4)*(a^2*B + b^2*B + I*a^2*C - I*b^2*C)*\text{Log}[a^2 + b^2 + 2*a*b*\text{Cos}[x]])/(a^2*b) + ((B - I*C)*\text{Sin}[x])/(2*a)$

fricas [A] time = 0.96, size = 68, normalized size = 0.76

$$\frac{(B + iC)a^2x + (-iB - C)abe^{ix} + ((iB - C)a^2 + (iB + C)b^2) \log\left(\frac{ae^{ix} + b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fricas")

[Out] $\frac{1}{2}*((B + I*C)*a^2*x + (-I*B - C)*a*b*e^{I*x} + ((I*B - C)*a^2 + (I*B + C)*b^2)*\log((a*e^{I*x} + b)/a))/(a^2*b)$

giac [B] time = 0.16, size = 178, normalized size = 1.98

$$\frac{(-iBb - Cb) \log\left(-a \tan\left(\frac{1}{2}x\right)^2 + b \tan\left(\frac{1}{2}x\right)^2 + 2i a \tan\left(\frac{1}{2}x\right) + a + b\right)}{4a^2} - \frac{(iBb + Cb) \log\left(\tan\left(\frac{1}{2}x\right) + i\right)}{2a^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac")

[Out] $-\frac{1}{4}*(-I*B*b - C*b)*\log(-a*\tan(1/2*x)^2 + b*\tan(1/2*x)^2 + 2*I*a*\tan(1/2*x) + a + b)/a^2 - \frac{1}{2}*(I*B*b + C*b)*\log(\tan(1/2*x) + I)/a^2 + \frac{1}{4}*(2*B*a^2 + 2*I*C*a^2 + B*b^2 - I*C*b^2)*(x + 2*\arctan((I*a*\cos(x) - a*\sin(x) + I*a)/(a*\cos(x) + I*a*\sin(x) - a + 2*b)))/(a^2*b) - \frac{1}{2}*(-I*B*b*\tan(1/2*x) - C*b*\tan(1/2*x) - 2*B*a + 2*I*C*a + B*b - I*C*b)/(a^2*(\tan(1/2*x) + I))$

maple [B] time = 0.19, size = 388, normalized size = 4.31

$$-\frac{iC}{a\left(\tan\left(\frac{x}{2}\right) + i\right)} + \frac{B}{a\left(\tan\left(\frac{x}{2}\right) + i\right)} - \frac{i \ln\left(\tan\left(\frac{x}{2}\right) + i\right) b B}{2a^2} - \frac{\ln\left(\tan\left(\frac{x}{2}\right) + i\right) b C}{2a^2} + \frac{C \ln\left(\tan\left(\frac{x}{2}\right) - i\right)}{2b} - \frac{i B \ln\left(\tan\left(\frac{x}{2}\right) + i\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)

[Out] $-I*C/a/(\tan(1/2*x)+I)+B/a/(\tan(1/2*x)+I)-1/2*I/a^2*\ln(\tan(1/2*x)+I)*b*B-1/2/a^2*\ln(\tan(1/2*x)+I)*b*C+1/2*C/b*\ln(\tan(1/2*x)-I)-1/2*I*B/b*\ln(\tan(1/2*x)-I)+1/2*a/b/(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*C-1/2/(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*C-1/2/a*b/(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*C+1/2/a^2*b^2/(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*C-1/2*I*a/b/(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*B+1/2*I/(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*B-1/2*I/a*b/(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*B+1/2*I/a^2*b^2/(-a+b)*\ln(I*a+I*b-a*\tan(1/2*x)+b*\tan(1/2*x))*B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

mupad [B] time = 4.51, size = 118, normalized size = 1.31

$$\ln\left(a + b + a \tan\left(\frac{x}{2}\right) + i\right) - b \tan\left(\frac{x}{2}\right) + i\right) \left(\frac{-\frac{C}{2} + \frac{B i}{2}}{b} + \frac{\frac{C b^2}{2} + \frac{B b^2 i}{2}}{a^2 b} \right) + \frac{B - C i}{a \left(\tan\left(\frac{x}{2}\right) + i\right)} - \frac{\ln\left(\tan\left(\frac{x}{2}\right) + i\right) (C b - \dots)}{2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((B*cos(x) + C*sin(x))/(a + b*cos(x) - b*sin(x)*1i),x)`

[Out] `log(a + b + a*tan(x/2)*1i - b*tan(x/2)*1i)*(((B*1i)/2 - C/2)/b + ((B*b^2*1i)/2 + (C*b^2)/2)/(a^2*b)) + (B - C*1i)/(a*(tan(x/2) + 1i)) - (log(tan(x/2) + 1i)*(B*b*1i + C*b))/(2*a^2) - (log(tan(x/2) - 1i)*(B*1i - C))/(2*b)`

sympy [A] time = 0.81, size = 102, normalized size = 1.13

$$\begin{cases} -\frac{(iB+C)e^{ix}}{2a} & \text{for } 2a \neq 0 \\ x\left(-\frac{B+iC}{2b} + \frac{Ba+Bb+iCa-iCb}{2ab}\right) & \text{otherwise} \end{cases} -\frac{x(-B-iC)}{2b} + \frac{i(Ba^2 + Bb^2 + iCa^2 - iCb^2)\log\left(e^{ix} + \frac{b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)`

[Out] `Piecewise((-I*B + C)*exp(I*x)/(2*a), Ne(2*a, 0)), (x*(-(B + I*C)/(2*b) + (B*a + B*b + I*C*a - I*C*b)/(2*a*b)), True)) - x*(-B - I*C)/(2*b) + I*(B*a**2 + B*b**2 + I*C*a**2 - I*C*b**2)*log(exp(I*x) + b/a)/(2*a**2*b)`

$$3.550 \quad \int \frac{A+B \cos(x)+C \sin(x)}{a+b \cos(x)+c \sin(x)} dx$$

Optimal. Leaf size=131

$$\frac{2 \tan^{-1} \left(\frac{(a-b) \tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}} \right) (A(b^2+c^2)-a(bB+cC))}{(b^2+c^2)\sqrt{a^2-b^2-c^2}} + \frac{(Bc-bC) \log(a+b \cos(x)+c \sin(x))}{b^2+c^2} + \frac{x(bB+cC)}{b^2+c^2}$$

[Out] (B*b+C*c)*x/(b^2+c^2)+(B*c-C*b)*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)+2*(A*(b^2+c^2)-a*(B*b+C*c))*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)

Rubi [A] time = 0.13, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3136, 3124, 618, 204}

$$\frac{2 \tan^{-1} \left(\frac{(a-b) \tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}} \right) (A(b^2+c^2)-a(bB+cC))}{(b^2+c^2)\sqrt{a^2-b^2-c^2}} + \frac{(Bc-bC) \log(a+b \cos(x)+c \sin(x))}{b^2+c^2} + \frac{x(bB+cC)}{b^2+c^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]

[Out] ((b*B + c*C)*x)/(b^2 + c^2) + (2*(A*(b^2 + c^2) - a*(b*B + c*C))*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]]/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) + ((B*c - b*C)*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)])*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3136

Int[((A_.) + cos[(d_.) + (e_.)*(x_)])*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]) / ((a_.) + cos[(d_.) + (e_.)*(x_)])*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]) , x_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + (Dist[(A*(b^2 + c^2) - a*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \cos(x) + C \sin(x)}{a + b \cos(x) + c \sin(x)} dx &= \frac{(bB + cC)x}{b^2 + c^2} + \frac{(Bc - bC) \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} + \left(A - \frac{a(bB + cC)}{b^2 + c^2} \right) \int \frac{1}{a + b \cos(x) + c \sin(x)} dx \\
&= \frac{(bB + cC)x}{b^2 + c^2} + \frac{(Bc - bC) \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} + \left(2 \left(A - \frac{a(bB + cC)}{b^2 + c^2} \right) \right) \int \frac{1}{a + b \cos(x) + c \sin(x)} dx \\
&= \frac{(bB + cC)x}{b^2 + c^2} + \frac{(Bc - bC) \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2} - \left(4 \left(A - \frac{a(bB + cC)}{b^2 + c^2} \right) \right) \int \frac{1}{a + b \cos(x) + c \sin(x)} dx \\
&= \frac{(bB + cC)x}{b^2 + c^2} + \frac{2 \left(A - \frac{a(bB + cC)}{b^2 + c^2} \right) \tan^{-1} \left(\frac{c + (a-b) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2 - c^2}} \right)}{\sqrt{a^2 - b^2 - c^2}} + \frac{(Bc - bC) \log(a + b \cos(x) + c \sin(x))}{b^2 + c^2}
\end{aligned}$$

Mathematica [A] time = 0.39, size = 110, normalized size = 0.84

$$\frac{2(a(bB+cC)-A(b^2+c^2)) \tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}} + \frac{(Bc - bC) \log(a + b \cos(x) + c \sin(x)) + x(bB + cC)}{b^2 + c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]), x]

[Out] ((b*B + c*C)*x + (2*(-(A*(b^2 + c^2)) + a*(b*B + c*C))*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + (B*c - b*C)*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)

fricas [B] time = 1.85, size = 711, normalized size = 5.43

$$\left[\frac{(Bab - Ab^2 + Cac - Ac^2) \sqrt{-a^2 + b^2 + c^2} \log\left(\frac{a^2 b^2 - 2b^4 - c^4 - (a^2 + 3b^2)c^2 - (2a^2 b^2 - b^4 - 2a^2 c^2 + c^4) \cos(x)^2 - 2(ab^3 + abc^2) \cos(x) - 2(ab^3 + abc^2) \cos(x) - 2(ab^3 + abc^2) \cos(x)}{2abc}\right)}{\sqrt{-a^2 + b^2 + c^2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)), x, algorithm="fricas")

[Out] [-1/2*((B*a*b - A*b^2 + C*a*c - A*c^2)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)) - 2*(B*a^2*b - B*b^3 - B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*x + (C*a^2*b - C*b^3 - C*b*c^2 + B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), -1/2*(2*(B*a*b - A*b^2 + C*a*c - A*c^2)*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) - 2*(B*a^2*b - B*b^3 - B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*x + (C*a^2*b - C*b^3 - C*b*c^2 + B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]

giac [A] time = 0.18, size = 199, normalized size = 1.52

$$\frac{(Bb + Cc)x}{b^2 + c^2} - \frac{(Cb - Bc) \log\left(-a \tan\left(\frac{1}{2}x\right)^2 + b \tan\left(\frac{1}{2}x\right)^2 - 2c \tan\left(\frac{1}{2}x\right) - a - b\right)}{b^2 + c^2} + \frac{(Cb - Bc) \log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)}{b^2 + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")

[Out] (B*b + C*c)*x/(b^2 + c^2) - (C*b - B*c)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - a - b)/(b^2 + c^2) + (C*b - B*c)*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) + 2*(B*a*b - A*b^2 + C*a*c - A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/(sqrt(a^2 - b^2 - c^2)*(b^2 + c^2))

maple [B] time = 0.13, size = 954, normalized size = 7.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)

[Out] 1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*a*B*c-1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*b*B*c-1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*a*b*C+1/(b^2+c^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)*b^2*C+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*b^2+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*b*B+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*B*c^2-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*a*c*C-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*C*b*c-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c^2/(a-b)*a*B+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c^2/(a-b)*b*B+2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c/(a-b)*a*b*C-2/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*c/(a-b)*b^2*C-B/(b^2+c^2)*c*ln(1+tan(1/2*x)^2)+C/(b^2+c^2)*b*ln(1+tan(1/2*x)^2)+2*B/(b^2+c^2)*b*arctan(tan(1/2*x))+2*C/(b^2+c^2)*c*arctan(tan(1/2*x))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for more details)Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 55.11, size = 2711, normalized size = 20.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + B*\cos(x) + C*\sin(x))/(a + b*\cos(x) + c*\sin(x)),x)$

[Out] $(\log(\tan(x/2) - 1i)*(B + C*1i))/(b*1i - c) - (\log(\tan(x/2) + 1i)*(B - C*1i))/(b*1i + c) + (\log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*B^2*b^2 + 32*A*C^2*a^2 - 32*A^2*B*b^2 + 32*A*C^2*b^2 + 32*B*C^2*a^2 + 32*B*C^2*b^2 + 32*\tan(x/2)*(a - b)*(2*C^3*a + B^3*c - 2*C^3*b - 2*A*B^2*c + A^2*B*c + A^2*C*b + 2*B^2*C*a - 2*A*C^2*c - B^2*C*b + 2*B*C^2*c - 2*A*B*C*a) - 32*B^3*a*b + 32*A^2*B*a*b - 64*A*C^2*a*b - 64*B*C^2*a*b - 32*A^2*C*a*c + 32*A^2*C*b*c + 32*B^2*C*a*c - 32*B^2*C*b*c - ((B*c^3 - C*b^3 - A*b^2*(b^2 - a^2 + c^2)^{(1/2)} - B*a^2*c + C*a^2*b - A*c^2*(b^2 - a^2 + c^2)^{(1/2)} + B*b^2*c - C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^{(1/2)} + C*a*c*(b^2 - a^2 + c^2)^{(1/2}))*((64*A^2*b^2*c - 32*B^2*a^2*c + 32*B^2*b^2*c + 32*C^2*a^2*c + 32*C^2*b^2*c + 32*\tan(x/2)*(a - b)*(A^2*b^2 + 2*B^2*a^2 - A^2*c^2 + B^2*b^2 - 2*C^2*a^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 - 2*B^2*a*b + 2*C^2*a*b - 2*A*B*a*b + 2*A*C*a*c - 4*A*C*b*c - 4*B*C*a*c + 6*B*C*b*c) + 64*A*C*b^3 - 128*B*C*a^3 - 64*B*C*b^3 + 64*A*B*a^2*c - 128*A*C*a*b^2 + 64*A*C*a^2*b - 64*A*B*b^2*c + 192*B*C*a^2*b + 64*B*C*a*c^2 - 64*B*C*b*c^2 - 64*A^2*a*b*c - 64*C^2*a*b*c - ((B*c^3 - C*b^3 - A*b^2*(b^2 - a^2 + c^2)^{(1/2)} - B*a^2*c + C*a^2*b - A*c^2*(b^2 - a^2 + c^2)^{(1/2)} + B*b^2*c - C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^{(1/2)} + C*a*c*(b^2 - a^2 + c^2)^{(1/2}))*((32*A*b^4 + 32*B*b^4 - 32*\tan(x/2)*(a - b)*(2*A*c^3 + B*c^3 - 2*C*b^3 + 2*A*b^2*c + 2*C*a*b^2 + 4*B*b^2*c - 2*C*a*c^2 + C*b*c^2 - 2*A*a*b*c - 4*B*a*b*c) + 32*A*a^2*b^2 - 32*A*a^2*c^2 + 32*B*a^2*b^2 + 32*A*b^2*c^2 - 32*B*a^2*c^2 - 64*B*b^2*c^2 - 64*A*a*b^3 - 64*B*a*b^3 + 32*C*a*c^3 - 32*C*b*c^3 + 64*C*b^3*c + 96*B*a*b*c^2 - 128*C*a*b^2*c + 64*C*a^2*b*c - (32*(a - b)*(B*c^3 - C*b^3 - A*b^2*(b^2 - a^2 + c^2)^{(1/2)} - B*a^2*c + C*a^2*b - A*c^2*(b^2 - a^2 + c^2)^{(1/2)} + B*b^2*c - C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^{(1/2)} + C*a*c*(b^2 - a^2 + c^2)^{(1/2}))*((3*c^4*\tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*\tan(x/2) - 2*a^2*c^2*\tan(x/2) + 3*b^2*c^2*\tan(x/2) - 2*a*b^3*\tan(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*\tan(x/2))))/(b^2 + c^2)*(b^2 - a^2 + c^2))))/(b^2 + c^2)*(b^2 - a^2 + c^2))))/(b^2 + c^2)*(b^2 - a^2 + c^2))))*(c*(B*b^2 - B*a^2 + C*a*(b^2 - a^2 + c^2)^{(1/2)} + B*c^3 - C*b^3 - c^2*(A*(b^2 - a^2 + c^2)^{(1/2)} + C*b) - A*b^2*(b^2 - a^2 + c^2)^{(1/2)} + C*a^2*b + B*a*b*(b^2 - a^2 + c^2)^{(1/2}))))/(b^2 + c^2)*(b^2 - a^2 + c^2)) + (\log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*B^2*b^2 + 32*A*C^2*a^2 - 32*A^2*B*b^2 + 32*A*C^2*b^2 + 32*B*C^2*a^2 + 32*B*C^2*b^2 + 32*\tan(x/2)*(a - b)*(2*C^3*a + B^3*c - 2*C^3*b - 2*A*B^2*c + A^2*B*c + A^2*C*b + 2*B^2*C*a - 2*A*C^2*c - B^2*C*b + 2*B*C^2*c - 2*A*B*C*a) - 32*B^3*a*b + 32*A^2*B*a*b - 64*A*C^2*a*b - 64*B*C^2*a*b - 32*A^2*C*a*c + 32*A^2*C*b*c + 32*B^2*C*a*c - 32*B^2*C*b*c - ((B*c^3 - C*b^3 + A*b^2*(b^2 - a^2 + c^2)^{(1/2)} - B*a^2*c + C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^{(1/2)} + B*b^2*c - C*b*c^2 - B*a*b*(b^2 - a^2 + c^2)^{(1/2)} - C*a*c*(b^2 - a^2 + c^2)^{(1/2}))*((64*A^2*b^2*c - 32*B^2*a^2*c + 32*B^2*b^2*c + 32*C^2*a^2*c + 32*C^2*b^2*c + 32*\tan(x/2)*(a - b)*(A^2*b^2 + 2*B^2*a^2 - A^2*c^2 + B^2*b^2 - 2*C^2*a^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 - 2*B^2*a*b + 2*C^2*a*b - 2*A*B*a*b + 2*A*C*a*c - 4*A*C*b*c - 4*B*C*a*c + 6*B*C*b*c) + 64*A*C*b^3 - 128*B*C*a^3 - 64*B*C*b^3 + 64*A*B*a^2*c - 128*A*C*a*b^2 + 64*A*C*a^2*b - 64*A*B*b^2*c + 192*B*C*a^2*b + 64*B*C*a*c^2 - 64*B*C*b*c^2 - 64*A^2*a*b*c - 64*C^2*a*b*c - ((B*c^3 - C*b^3 + A*b^2*(b^2 - a^2 + c^2)^{(1/2)} - B*a^2*c + C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^{(1/2)} + B*b^2*c - C*b*c^2 - B*a*b*(b^2 - a^2 + c^2)^{(1/2)} - C*a*c*(b^2 - a^2 + c^2)^{(1/2}))*((32*A*b^4 + 32*B*b^4 - 32*\tan(x/2)*(a - b)*(2*A*c^3 + B*c^3 - 2*C*b^3 + 2*A*b^2*c + 2*C*a*b^2 + 4*B*b^2*c - 2*C*a*c^2 + C*b*c^2 - 2*A*a*b*c - 4*B*a*b*c) + 32*A*a^2*b^2 - 32*A*a^2*c^2 + 32*B*a^2*b^2 + 32*A*b^2*c^2 - 32*B*a^2*c^2 - 64*B*b^2*c^2 - 64*A*a*b^3 - 64*B*a*b^3 + 32*C*a*c^3 - 32*C*b*c^3 + 64*C*b^3*c + 96*B*a*b*c^2 - 128*C*a*b^2*c + 64*C*a^2*b*c - (32*(a - b)*(B*c^3 - C*b^3 + A*b^2*(b^2 - a^2 + c^2)^{(1/2)} - B*a^2*c + C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^{(1/2)} + B*b^2*c - C*b*c^2 - B*a*b*(b^2 - a^2 + c^2)^{(1/2)} - C*a*c*(b^2 - a^2 + c^2)^{(1/2}))*((3*c^4*\tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*\tan(x/2) - 2*a^2*c^2*\tan(x/2) + 3*b^2*c^2*\tan(x/2) - 2*a*b^3*ta$

$$\frac{\ln(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*\tan(x/2)}{((b^2 + c^2)*(b^2 - a^2 + c^2))} / ((b^2 + c^2)*(b^2 - a^2 + c^2)) / ((b^2 + c^2)*(b^2 - a^2 + c^2)) * (B*c^3 - c*(B*a^2 - B*b^2 + C*a*(b^2 - a^2 + c^2)^{1/2}) - C*b^3 + c^2*(A*(b^2 - a^2 + c^2)^{1/2} - C*b) + A*b^2*(b^2 - a^2 + c^2)^{1/2} + C*a^2*b - B*a*b*(b^2 - a^2 + c^2)^{1/2})) / ((b^2 + c^2)*(b^2 - a^2 + c^2))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)

[Out] Timed out

$$3.551 \quad \int \frac{A+B \cos(x)+C \sin(x)}{(a+b \cos(x)+c \sin(x))^2} dx$$

Optimal. Leaf size=127

$$\frac{2 \tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)(aA-bB-cC)}{(a^2-b^2-c^2)^{3/2}} + \frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{(a^2-b^2-c^2)(a+b \cos(x)+c \sin(x))}$$

[Out] 2*(A*a-B*b-C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^(3/2)+(B*c-b*C+(A*c-C*a)*cos(x)-(A*b-B*a)*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))

Rubi [A] time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3153, 3124, 618, 204}

$$\frac{2 \tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)(aA-bB-cC)}{(a^2-b^2-c^2)^{3/2}} + \frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{(a^2-b^2-c^2)(a+b \cos(x)+c \sin(x))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]

[Out] (2*(a*A - b*B - c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^2 - b^2 - c^2)^(3/2) + (B*c - b*C + (A*c - a*C)*Cos[x] - (A*b - a*B)*Sin[x])/((a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x]))

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)])^(-1), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3153

Int[((A_) + cos[(d_) + (e_)*(x_)]*(B_) + (C_)*sin[(d_) + (e_)*(x_)]) / ((a_) + cos[(d_) + (e_)*(x_)]*(b_) + (c_)*sin[(d_) + (e_)*(x_)])^2, x_Symbol] := Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]) / (e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \cos(x) + C \sin(x)}{(a + b \cos(x) + c \sin(x))^2} dx &= \frac{Bc - bC + (Ac - aC) \cos(x) - (Ab - aB) \sin(x)}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))} + \frac{(aA - bB - cC) \int \frac{1}{a + b \cos(x) + c \sin(x)} dx}{a^2 - b^2 - c^2} \\
&= \frac{Bc - bC + (Ac - aC) \cos(x) - (Ab - aB) \sin(x)}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))} + \frac{(2(aA - bB - cC)) \operatorname{Subst} \int \frac{1}{u} du}{a^2 - b^2 - c^2} \\
&= \frac{Bc - bC + (Ac - aC) \cos(x) - (Ab - aB) \sin(x)}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))} - \frac{(4(aA - bB - cC)) \operatorname{Subst} \int \frac{1}{u} du}{a^2 - b^2 - c^2} \\
&= \frac{2(aA - bB - cC) \tan^{-1} \left(\frac{c + (a-b) \tan(\frac{x}{2})}{\sqrt{a^2 - b^2 - c^2}} \right)}{(a^2 - b^2 - c^2)^{3/2}} + \frac{Bc - bC + (Ac - aC) \cos(x) - (Ab - aB) \sin(x)}{(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))}
\end{aligned}$$

Mathematica [A] time = 0.50, size = 137, normalized size = 1.08

$$\frac{a^2(-C) + \sin(x) \left(A(b^2 + c^2) - a(bB + cC) \right) + aAc + b(bC - Bc)}{b(-a^2 + b^2 + c^2)(a + b \cos(x) + c \sin(x))} + \frac{2(aA - bB - cC) \tanh^{-1} \left(\frac{(a-b) \tan(\frac{x}{2}) + c}{\sqrt{-a^2 + b^2 + c^2}} \right)}{(-a^2 + b^2 + c^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]

[Out] (2*(a*A - b*B - c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(3/2) + (a*A*c - a^2*C + b*(-(B*c) + b*C) + (A*(b^2 + c^2) - a*(b*B + c*C))*Sin[x])/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))

fricas [B] time = 2.18, size = 1556, normalized size = 12.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas")

[Out] [1/2*(2*C*a^4*b - 4*C*a^2*b^3 + 2*C*b^5 + 2*C*b*c^4 - 2*B*c^5 + 4*(B*a^2 - B*b^2)*c^3 - 4*(C*a^2*b - C*b^3)*c^2 - (A*a^2*b^2 - B*a*b^3 - C*a*b^2*c - C*a*c^3 + (A*a^2 - B*a*b)*c^2 + (A*a*b^3 - B*b^4 - C*b^3*c - C*b*c^3 + (A*a*b - B*b^2)*c^2)*cos(x) - (C*b^2*c^2 + C*c^4 - (A*a - B*b)*c^3 - (A*a*b^2 - B*b^3)*c)*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x))^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)) - 2*(B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + 2*(C*a*c^4 - A*c^5 + (A*a^2 + B*a*b - 2*A*b^2)*c^3 - (C*a^3 - C*a*b^2)*c^2 - (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*c)*cos(x) + 2*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5 - C*a*b*c^3 + A*b*c^4 - (A*a^2*b + B*a*b^2 - 2*A*b^3)*c^2 + (C*a^3*b - C*a*b^3)*c)*sin(x)]/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x)), (C*a^4*b -

$2 * C * a^2 * b^3 + C * b^5 + C * b * c^4 - B * c^5 + 2 * (B * a^2 - B * b^2) * c^3 - 2 * (C * a^2 * b - C * b^3) * c^2 + (A * a^2 * b^2 - B * a * b^3 - C * a * b^2 * c - C * a * c^3 + (A * a^2 - B * a * b) * c^2 + (A * a * b^3 - B * b^4 - C * b^3 * c - C * b * c^3 + (A * a * b - B * b^2) * c^2) * \cos(x) - (C * b^2 * c^2 + C * c^4 - (A * a - B * b) * c^3 - (A * a * b^2 - B * b^3) * c) * \sin(x)) * \sqrt{(a^2 - b^2 - c^2)} * \arctan(- (a * b * \cos(x) + a * c * \sin(x) + b^2 + c^2) * \sqrt{(a^2 - b^2 - c^2)}) / ((c^3 - (a^2 - b^2) * c) * \cos(x) + (a^2 * b - b^3 - b * c^2) * \sin(x))) - (B * a^4 - 2 * B * a^2 * b^2 + B * b^4) * c + (C * a * c^4 - A * c^5 + (A * a^2 + B * a * b - 2 * A * b^2) * c^3 - (C * a^3 - C * a * b^2) * c^2 - (B * a^3 * b - A * a^2 * b^2 - B * a * b^3 + A * b^4) * c) * \cos(x) + (B * a^3 * b^2 - A * a^2 * b^3 - B * a * b^4 + A * b^5 - C * a * b * c^3 + A * b * c^4 - (A * a^2 * b + B * a * b^2 - 2 * A * b^3) * c^2 + (C * a^3 * b - C * a * b^3) * c) * \sin(x) / (a^5 * b^2 - 2 * a^3 * b^4 + a * b^6 + a * c^6 - (2 * a^3 - 3 * a * b^2) * c^4 + (a^5 - 4 * a^3 * b^2 + 3 * a * b^4) * c^2 + (a^4 * b^3 - 2 * a^2 * b^5 + b^7 + b * c^6 - (2 * a^2 * b - 3 * b^3) * c^4 + (a^4 * b - 4 * a^2 * b^3 + 3 * b^5) * c^2) * \cos(x) + (c^7 - (2 * a^2 - 3 * b^2) * c^5 + (a^4 - 4 * a^2 * b^2 + 3 * b^4) * c^3 + (a^4 * b^2 - 2 * a^2 * b^4 + b^6) * c) * \sin(x))]$

giac [A] time = 0.17, size = 241, normalized size = 1.90

$$\frac{2 \left(\pi \left[\frac{x}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a + 2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2}x\right) - b \tan\left(\frac{1}{2}x\right) + c}{\sqrt{a^2 - b^2 - c^2}} \right) \right) (Aa - Bb - Cc)}{(a^2 - b^2 - c^2)^{\frac{3}{2}}} + \frac{2 \left(Ba^2 \tan\left(\frac{1}{2}x\right) - Aab \tan\left(\frac{1}{2}x\right) \right)}{(a^2 - b^2 - c^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")

[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(A*a - B*b - C*c)/(a^2 - b^2 - c^2)^(3/2) + 2*(B*a^2*tan(1/2*x) - A*a*b*tan(1/2*x) - B*a*b*tan(1/2*x) + A*b^2*tan(1/2*x) - C*a*c*tan(1/2*x) + C*b*c*tan(1/2*x) + A*c^2*tan(1/2*x) - B*c^2*tan(1/2*x) - C*a^2 + C*b^2 + A*a*c - B*b*c)/((a^3 - a^2*b - a*b^2 + b^3 - a*c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b))

maple [B] time = 0.18, size = 329, normalized size = 2.59

$$\frac{-\frac{2(aAb - Ab^2 - Ac^2 - a^2B + abB + Bc^2 + acC - Cbc) \tan\left(\frac{x}{2}\right)}{a^3 - a^2b - ab^2 - ac^2 + b^3 + c^2b} + \frac{2(aAc - bBc - a^2C + b^2C)}{a^3 - a^2b - ab^2 - ac^2 + b^3 + c^2b}}{a \left(\tan^2\left(\frac{x}{2}\right) \right) - b \left(\tan^2\left(\frac{x}{2}\right) \right) + 2c \tan\left(\frac{x}{2}\right) + a + b} + \frac{2 \arctan\left(\frac{2(a-b) \tan\left(\frac{x}{2}\right) + 2c}{2\sqrt{a^2 - b^2 - c^2}}\right) a A}{(a^2 - b^2 - c^2)^{\frac{3}{2}}} - \frac{2 \arctan\left(\frac{2(a-b) \tan\left(\frac{x}{2}\right) + 2c}{2\sqrt{a^2 - b^2 - c^2}}\right) a B}{(a^2 - b^2 - c^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x)

[Out] 2*(-(A*a*b-A*b^2-A*c^2-B*a^2+B*a*b+B*c^2+C*a*c-C*b*c)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan(1/2*x)+(A*a*c-B*b*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2))/(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)+2/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*A-2/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*B-2/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))*C

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see 'assume?' for more details) Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 3.49, size = 227, normalized size = 1.79

$$\frac{\frac{2(Ca^2 - Ac a - Cb^2 + Bcb)}{(a-b)(-a^2+b^2+c^2)} - \frac{2 \tan\left(\frac{x}{2}\right)(Ab^2 + Ba^2 + Ac^2 - Bc^2 - Aab - B ab - C ac + Cbc)}{(a-b)(-a^2+b^2+c^2)}}{(a-b) \tan\left(\frac{x}{2}\right)^2 + 2c \tan\left(\frac{x}{2}\right) + a + b} - \frac{2 \operatorname{atanh}\left(\frac{\tan\left(\frac{x}{2}\right)(2a-2b) + \frac{2(-a^2c+b^2c+c^3)}{-a^2+b^2+c^2}}{2\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{3/2}} (Bb)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x))^2,x)

[Out] ((2*(C*a^2 - C*b^2 - A*a*c + B*b*c))/((a - b)*(b^2 - a^2 + c^2)) - (2*tan(x/2)*(A*b^2 + B*a^2 + A*c^2 - B*c^2 - A*a*b - B*a*b - C*a*c + C*b*c))/((a - b)*(b^2 - a^2 + c^2)))/(a + b + 2*c*tan(x/2) + tan(x/2)^2*(a - b)) - (2*atanh((tan(x/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^2 - a^2 + c^2)))/(2*(b^2 - a^2 + c^2)^(1/2)))*(B*b - A*a + C*c))/(b^2 - a^2 + c^2)^(3/2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)

[Out] Timed out

$$3.552 \quad \int \frac{A+B \cos(x)+C \sin(x)}{(a+b \cos(x)+c \sin(x))^3} dx$$

Optimal. Leaf size=237

$$\frac{\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\left(2a^2A-3a(bB+cC)+A(b^2+c^2)\right)}{(a^2-b^2-c^2)^{5/2}} + \frac{-\sin(x)\left(a^2(-B)+3aAb-2b(bB+cC)\right)+\cos(x)\left(a^2(-B)+3aAb-2b(bB+cC)\right)}{2(a^2-b^2-c^2)^2(a+b \cos(x))}$$

[Out] (2*a^2*A+A*(b^2+c^2)-3*a*(B*b+C*c))*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^(5/2)+1/2*(B*c-b*C+(A*c-C*a)*cos(x)-(A*b-B*a)*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))^2+1/2*(a*(B*c-C*b)+(3*a*A*c-a^2*C-2*c*(B*b+C*c))*cos(x)-(3*a*A*b-a^2*B-2*b*(B*b+C*c))*sin(x))/(a^2-b^2-c^2)^2/(a+b*cos(x)+c*sin(x))

Rubi [A] time = 0.28, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3156, 3153, 3124, 618, 204}

$$\frac{\tan^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\left(2a^2A-3a(bB+cC)+A(b^2+c^2)\right)}{(a^2-b^2-c^2)^{5/2}} + \frac{-\sin(x)\left(a^2(-B)+3aAb-2b(bB+cC)\right)+\cos(x)\left(a^2(-B)+3aAb-2b(bB+cC)\right)}{2(a^2-b^2-c^2)^2(a+b \cos(x))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3, x]

[Out] ((2*a^2*A + A*(b^2 + c^2) - 3*a*(b*B + c*C))*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]]/(a^2 - b^2 - c^2)^(5/2) + (B*c - b*C + (A*c - a*C)*Cos[x] - (A*b - a*B)*Sin[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])^2) + (a*(B*c - b*C) + (3*a*A*c - a^2*C - 2*c*(b*B + c*C))*Cos[x] - (3*a*A*b - a^2*B - 2*b*(b*B + c*C))*Sin[x])/(2*(a^2 - b^2 - c^2)^2*(a + b*Cos[x] + c*Sin[x]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 3124

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(-1), x_Symbol] := Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/2]/f], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3153

Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]) / ((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] := Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]) / (e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si

$\text{Int}[\text{d} + \text{e}*\text{x}], \text{x}], \text{x}] /; \text{FreeQ}[\{\text{a}, \text{b}, \text{c}, \text{d}, \text{e}, \text{A}, \text{B}, \text{C}\}, \text{x}] \&\& \text{NeQ}[\text{a}^2 - \text{b}^2 - \text{c}^2, 0] \&\& \text{NeQ}[\text{a}*\text{A} - \text{b}*\text{B} - \text{c}*\text{C}, 0]$

Rule 3156

$\text{Int}[\text{((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])}^{\text{(n_.)}}*\text{((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)])}, \text{x_Symbol}] \text{:> } -\text{Simp}[\text{((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^{(n + 1)})/(e*(n + 1)*(a^2 - b^2 - c^2))}, \text{x}] + \text{Dist}[1/((n + 1)*(a^2 - b^2 - c^2)), \text{Int}[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^{(n + 1)}*\text{Simp}[(n + 1)*(a*A - b*B - c*C) + (n + 2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], \text{x}], \text{x}], \text{x}] /; \text{FreeQ}[\{\text{a}, \text{b}, \text{c}, \text{d}, \text{e}, \text{A}, \text{B}, \text{C}\}, \text{x}] \&\& \text{LtQ}[\text{n}, -1] \&\& \text{NeQ}[\text{a}^2 - \text{b}^2 - \text{c}^2, 0] \&\& \text{NeQ}[\text{n}, -2]$

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(x) + C \sin(x)}{(a + b \cos(x) + c \sin(x))^3} dx &= \frac{Bc - bC + (Ac - aC) \cos(x) - (Ab - aB) \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} - \frac{\int \frac{-2(aA - bB - cC) + (Ab - aB) \cos(x) + (aB - aA) \sin(x)}{(a + b \cos(x) + c \sin(x))^2} dx}{2(a^2 - b^2 - c^2)} \\ &= \frac{Bc - bC + (Ac - aC) \cos(x) - (Ab - aB) \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} + \frac{a(Bc - bC) + (3aAc - a^2)}{2(a^2 - b^2 - c^2)} \\ &= \frac{Bc - bC + (Ac - aC) \cos(x) - (Ab - aB) \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} + \frac{a(Bc - bC) + (3aAc - a^2)}{2(a^2 - b^2 - c^2)} \\ &= \frac{Bc - bC + (Ac - aC) \cos(x) - (Ab - aB) \sin(x)}{2(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^2} + \frac{a(Bc - bC) + (3aAc - a^2)}{2(a^2 - b^2 - c^2)} \\ &= \frac{(2a^2 A + A(b^2 + c^2) - 3a(bB + cC)) \tan^{-1}\left(\frac{c + (a-b) \tan\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2 - c^2}}\right)}{(a^2 - b^2 - c^2)^{5/2}} + \frac{Bc - bC + (Ac - aC) \cos(x) - (Ab - aB) \sin(x)}{2(a^2 - b^2 - c^2)} \end{aligned}$$

Mathematica [A] time = 1.28, size = 452, normalized size = 1.91

$$\frac{2a^4 C - 6a^3 A c + 4a^3 b B \sin(x) + 4a^3 c C \sin(x) - 2bc \cos(x) (2a^2 A - 3a(bB + cC) + A(b^2 + c^2)) - c \cos(2x) (a^2 - b^2 - c^2)}{(a^2 - b^2 - c^2)^2 (a + b \cos(x) + c \sin(x))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]

[Out] -(((2*a^2*A + A*(b^2 + c^2) - 3*a*(b*B + c*C))*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]]/(-a^2 + b^2 + c^2)^(5/2)) + (-6*a^3*A*c - 3*a*A*b^2*c + 9*a^2*b*B*c - 3*a*A*c^3 + 2*a^4*C - 4*a^2*b^2*C + 2*b^4*C + 5*a^2*c^2*C + 4*b^2*c^2*C + 2*c^4*C - 2*b*c*(2*a^2*A + A*(b^2 + c^2) - 3*a*(b*B + c*C))*Cos[x] - c*(-3*a*A*(b^2 + c^2) + a^2*(b*B + c*C) + 2*(b^2 + c^2)*(b*B + c*C))*Cos[2*x] - 8*a^2*A*b^2*Sin[x] + 2*A*b^4*Sin[x] + 4*a^3*b*B*Sin[x] + 2*a*b^3*B*Sin[x] - 12*a^2*A*c^2*Sin[x] + 2*A*b^2*c^2*Sin[x] + 8*a*b*B*c^2*Sin[x] + 4*a^3*c*C*Sin[x] + 2*a*b^2*c*C*Sin[x] + 8*a*c^3*C*Sin[x] - 3*a*A*b^3*Sin[2*x] + a^2*b^2*B*Sin[2*x] + 2*b^4*B*Sin[2*x] - 3*a*A*b*c^2*Sin[2*x] + 2*b^2*B*c^2*Sin[2*x] + a^2*b*c*C*Sin[2*x] + 2*b^3*c*C*Sin[2*x] + 2*b*c^3*C*Sin[2*x]))/(4*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)

fricas [B] time = 1.62, size = 4240, normalized size = 17.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*(2*C*a^6*b - 6*C*a^4*b^3 + 6*C*a^2*b^5 - 2*C*b^7 - 6*C*b*c^6 + 2*B*c^7 \\ & - 2*(3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + 2*(4*C*a^2*b - 7*C*b^3)*c^4 + 2*(3*B \\ & *a^4 - 3*A*a^3*b - 5*B*a^2*b^2 + 6*A*a*b^3 - B*b^4)*c^3 - 2*(2*C*a^4*b - 7* \\ & C*a^2*b^3 + 5*C*b^5)*c^2 + 4*(2*C*b*c^6 - (3*A*a*b - 2*B*b^2)*c^5 - (C*a^2*b \\ & - 4*C*b^3)*c^4 + (3*A*a^3*b - B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^3 - (C*a \\ & ^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 - 3*A*a^3*b^3 + B*a^2*b^4 + 3* \\ & A*a*b^5 - 2*B*b^6)*c)*\cos(x)^2 - (2*A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 - 3 \\ & *C*a^3*b^2*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 - 3*B*a*b + 2*A*b^2)*c^4 - 3*(C \\ & *a^3 + C*a*b^2)*c^3 + (2*A*a^4 - 3*B*a^3*b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4 \\ &)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6 - 3*C*a*b^4*c + A*b^4*c^2 + 3*C*a \\ & *c^5 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*\cos(x)^2 + 2*(2*A*a^3*b^3 - \\ & 3*B*a^2*b^4 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 + A*a*b*c^4 + (2*A*a \\ & ^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*c^2)*\cos(x) - 2*(3*C*a^2*b^2*c^2 + 3*C*a^2* \\ & c^4 - A*a*c^5 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 - (2*A*a^3*b^2 - 3*B* \\ & a^2*b^3 + A*a*b^4)*c + (3*C*a*b^3*c^2 + 3*C*a*b*c^4 - A*b*c^5 - (2*A*a^2*b \\ & - 3*B*a*b^2 + 2*A*b^3)*c^3 - (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*c)*\cos(x))*\sin(x) \\ & *sqrt(-a^2 + b^2 + c^2)*\log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c \\ & ^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*\cos(x)^2 - 2*(a*b^3 + a*b*c^2)*\cos \\ & (x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*\cos(x))*\sin(x) + 2*(\\ & 2*a*b*c*\cos(x)^2 - a*b*c + (b^2*c + c^3)*\cos(x) - (b^3 + b*c^2 + (a*b^2 - a \\ & *c^2)*\cos(x))*\sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*\cos(x) + (b^2 - c^2)*c \\ & \cos(x)^2 + a^2 + c^2 + 2*(b*c*\cos(x) + a*c)*\sin(x)) - 2*(B*a^6 - 4*B*a^4*b^2 \\ & + 3*A*a^3*b^3 + 2*B*a^2*b^4 - 3*A*a*b^5 + B*b^6)*c + 2*(C*a*c^6 + A*c^7 - \\ & (5*A*a^2 - B*a*b - 3*A*b^2)*c^5 + (C*a^3 + 2*C*a*b^2)*c^4 + (4*A*a^4 + B*a \\ & ^3*b - 10*A*a^2*b^2 + 2*B*a*b^3 + 3*A*b^4)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a \\ & *b^4)*c^2 - (2*B*a^5*b - 4*A*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4 - B*a*b^5 - \\ & A*b^6)*c)*\cos(x) + 2*(2*B*a^5*b^2 - 4*A*a^4*b^3 - B*a^3*b^4 + 5*A*a^2*b^5 - \\ & B*a*b^6 - A*b^7 - C*a*b*c^5 - A*b*c^6 + (5*A*a^2*b - B*a*b^2 - 3*A*b^3)*c^4 \\ & - (C*a^3*b + 2*C*a*b^3)*c^3 - (4*A*a^4*b + B*a^3*b^2 - 10*A*a^2*b^3 + 2*B \\ & *a*b^4 + 3*A*b^5)*c^2 + (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (B*a^4*b^3 - \\ & 3*A*a^3*b^4 + B*a^2*b^5 + 3*A*a*b^6 - 2*B*b^7 + 2*C*c^7 - (3*A*a - 2*B*b)*c \\ & ^6 - (C*a^2 - 2*C*b^2)*c^5 + (3*A*a^3 - B*a^2*b - 3*A*a*b^2 + 2*B*b^3)*c^4 \\ & - (C*a^4 + 2*C*b^4)*c^3 - (B*a^4*b - 3*A*a*b^4 + 2*B*b^5)*c^2 + (C*a^4*b^2 \\ & + C*a^2*b^4 - 2*C*b^6)*c)*\cos(x))*\sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 \\ & - a^2*b^8 - c^10 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - \\ & 3*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 \\ & - b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 \\ & + (3*a^4 - 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 \\ & - 2*a^2*b^6 + b^8)*c^2)*\cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 \\ & - a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + \\ & (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*\cos(x) - 2*(a*c^9 - (3*a^3 \\ & - 4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 \\ & - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*b \\ & - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9*a^2*b^5 \\ & - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*\cos(x))*\sin(x)), 1/2*(C*a^6*b - 3*C*a^4*b^3 + 3*C*a^2*b^5 \\ & - C*b^7 - 3*C*b*c^6 + B*c^7 - (3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + (4*C*a^2*b \\ & - 7*C*b^3)*c^4 + (3*B*a^4 - 3*A*a^3*b - 5*B*a^2*b^2 + 6*A*a*b^3 - B*b^4)*c^3 \\ & - (2*C*a^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 + 2*(2*C*b*c^6 - (3*A*a*b - 2*B \\ & *b^2)*c^5 - (C*a^2*b - 4*C*b^3)*c^4 + (3*A*a^3*b - B*a^2*b^2 - 6*A*a*b^3 + \\ & 4*B*b^4)*c^3 - (C*a^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 - 3*A*a^3*b \end{aligned}$$

$$\begin{aligned} &^3 + B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*c)*\cos(x)^2 + (2*A*a^4*b^2 - 3*B*a^3* \\ &b^3 + A*a^2*b^4 - 3*C*a^3*b^2*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 - 3*B*a*b + \\ &2*A*b^2)*c^4 - 3*(C*a^3 + C*a*b^2)*c^3 + (2*A*a^4 - 3*B*a^3*b + 4*A*a^2*b^2 \\ &- 3*B*a*b^3 + A*b^4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6 - 3*C*a*b^4*c \\ &+ A*b^4*c^2 + 3*C*a*c^5 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*\cos(x)^2 \\ &+ 2*(2*A*a^3*b^3 - 3*B*a^2*b^4 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 + \\ &A*a*b*c^4 + (2*A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*c^2)*\cos(x) - 2*(3*C*a^2 \\ &*b^2*c^2 + 3*C*a^2*c^4 - A*a*c^5 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 - \\ &(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*c + (3*C*a*b^3*c^2 + 3*C*a*b*c^4 - A* \\ &b*c^5 - (2*A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 - (2*A*a^2*b^3 - 3*B*a*b^4 + \\ &A*b^5)*c)*\cos(x))*\sin(x))*\sqrt{a^2 - b^2 - c^2}*\arctan(-(a*b*\cos(x) + a*c*\sin \\ &\sin(x) + b^2 + c^2)*\sqrt{a^2 - b^2 - c^2}/((c^3 - (a^2 - b^2)*c)*\cos(x) + (a \\ &^2*b - b^3 - b*c^2)*\sin(x))) - (B*a^6 - 4*B*a^4*b^2 + 3*A*a^3*b^3 + 2*B*a^2 \\ &*b^4 - 3*A*a*b^5 + B*b^6)*c + (C*a*c^6 + A*c^7 - (5*A*a^2 - B*a*b - 3*A*b^2 \\ &)*c^5 + (C*a^3 + 2*C*a*b^2)*c^4 + (4*A*a^4 + B*a^3*b - 10*A*a^2*b^2 + 2*B*a \\ &*b^3 + 3*A*b^4)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 - (2*B*a^5*b - 4* \\ &A*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4 - B*a*b^5 - A*b^6)*c)*\cos(x) + (2*B*a^5 \\ &*b^2 - 4*A*a^4*b^3 - B*a^3*b^4 + 5*A*a^2*b^5 - B*a*b^6 - A*b^7 - C*a*b*c^5 \\ &- A*b*c^6 + (5*A*a^2*b - B*a*b^2 - 3*A*b^3)*c^4 - (C*a^3*b + 2*C*a*b^3)*c^3 \\ &- (4*A*a^4*b + B*a^3*b^2 - 10*A*a^2*b^3 + 2*B*a*b^4 + 3*A*b^5)*c^2 + (2*C* \\ &a^5*b - C*a^3*b^3 - C*a*b^5)*c + (B*a^4*b^3 - 3*A*a^3*b^4 + B*a^2*b^5 + 3*A \\ &*a*b^6 - 2*B*b^7 + 2*C*c^7 - (3*A*a - 2*B*b)*c^6 - (C*a^2 - 2*C*b^2)*c^5 + \\ &(3*A*a^3 - B*a^2*b - 3*A*a*b^2 + 2*B*b^3)*c^4 - (C*a^4 + 2*C*b^4)*c^3 - (B* \\ &a^4*b - 3*A*a*b^4 + 2*B*b^5)*c^2 + (C*a^4*b^2 + C*a^2*b^4 - 2*C*b^6)*c)*\cos \\ &(x))*\sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10 + 2*(a^2 - 2 \\ &*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*b^6 \\ &)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (a^6*b^4 - 3*a \\ &^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b^2 + \\ &2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 + b^8) \\ &)*c^2)*\cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 + (3* \\ &a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6*a^5 \\ &*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*\cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*c^7 \\ &+ 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6 \\ &)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*b - \\ &4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9*a^2 \\ &*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*\cos(x))*\sin(\\ &x))] \end{aligned}$$

giac [B] time = 0.54, size = 1506, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac")

[Out]
$$\begin{aligned} &-(2*A*a^2 - 3*B*a*b + A*b^2 - 3*C*a*c + A*c^2)*(pi*\text{floor}(1/2*x/pi + 1/2)*\text{sg} \\ &n(-2*a + 2*b) + \arctan(-(a*\tan(1/2*x) - b*\tan(1/2*x) + c)/\sqrt{a^2 - b^2 - \\ &c^2}))/((a^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*\sqrt{a^2 - b^2 \\ &- c^2}) + (2*B*a^5*\tan(1/2*x)^3 - 4*A*a^4*b*\tan(1/2*x)^3 - 5*B*a^4*b*\tan(\\ &1/2*x)^3 + 11*A*a^3*b^2*\tan(1/2*x)^3 + 5*B*a^3*b^2*\tan(1/2*x)^3 - 9*A*a^2*b \\ &^3*\tan(1/2*x)^3 - 5*B*a^2*b^3*\tan(1/2*x)^3 + A*a*b^4*\tan(1/2*x)^3 + 5*B*a*b \\ &^4*\tan(1/2*x)^3 + A*b^5*\tan(1/2*x)^3 - 2*B*b^5*\tan(1/2*x)^3 - 3*C*a^4*c*\tan \\ &(1/2*x)^3 + 9*C*a^3*b*c*\tan(1/2*x)^3 - 9*C*a^2*b^2*c*\tan(1/2*x)^3 + 3*C*a*b \\ &^3*c*\tan(1/2*x)^3 + 5*A*a^3*c^2*\tan(1/2*x)^3 - 4*B*a^3*c^2*\tan(1/2*x)^3 - 7 \\ &*A*a^2*b*c^2*\tan(1/2*x)^3 + 4*B*a^2*b*c^2*\tan(1/2*x)^3 - A*a*b^2*c^2*\tan(1/ \\ &2*x)^3 + 4*B*a*b^2*c^2*\tan(1/2*x)^3 + 3*A*b^3*c^2*\tan(1/2*x)^3 - 4*B*b^3*c^ \\ &2*\tan(1/2*x)^3 - 2*A*a*c^4*\tan(1/2*x)^3 + 2*B*a*c^4*\tan(1/2*x)^3 + 2*A*b*c^ \\ &4*\tan(1/2*x)^3 - 2*B*b*c^4*\tan(1/2*x)^3 - 2*C*a^5*\tan(1/2*x)^2 + 2*C*a^4*b* \\ &\tan(1/2*x)^2 + 4*C*a^3*b^2*\tan(1/2*x)^2 - 4*C*a^2*b^3*\tan(1/2*x)^2 - 2*C*a \end{aligned}$$

$$\begin{aligned}
& b^4 \tan(1/2x)^2 + 2Cb^5 \tan(1/2x)^2 + 4Aa^4c \tan(1/2x)^2 + 2Ba^4c \tan(1/2x)^2 - 12Aa^3b^2c \tan(1/2x)^2 - 9Ba^3b^2c \tan(1/2x)^2 + 13Aa^2b^2c \tan(1/2x)^2 + 14Ba^2b^2c \tan(1/2x)^2 - 6Aa^2b^3c \tan(1/2x)^2 - 9Ba^2b^3c \tan(1/2x)^2 + Ab^4c \tan(1/2x)^2 + 2Bb^4c \tan(1/2x)^2 - 5Ca^3c^2 \tan(1/2x)^2 + 14Ca^2b^2c^2 \tan(1/2x)^2 - 13Ca^2b^2c^2 \tan(1/2x)^2 + 4Cb^3c^2 \tan(1/2x)^2 + 7Aa^2c^3 \tan(1/2x)^2 - 4Ba^2c^3 \tan(1/2x)^2 - 6Aa^2b^3c^3 \tan(1/2x)^2 - Ab^2c^3 \tan(1/2x)^2 + 4Bb^2c^3 \tan(1/2x)^2 - 2Ca^4c^4 \tan(1/2x)^2 + 2Cb^4c^4 \tan(1/2x)^2 - 2Aa^5 \tan(1/2x)^2 + 2Bb^5 \tan(1/2x)^2 - 4Aa^4b \tan(1/2x) - 3Ba^4b \tan(1/2x) + 5Aa^3b^2 \tan(1/2x) + Ba^3b^2 \tan(1/2x) + 3Aa^2b^3 \tan(1/2x) + Ba^2b^3 \tan(1/2x) - 5Aa^2b^4 \tan(1/2x) - 3Ba^2b^4 \tan(1/2x) + Ab^5 \tan(1/2x) + 2Bb^5 \tan(1/2x) - 5Ca^4c \tan(1/2x) + 5Ca^3b^2c \tan(1/2x) + 5Ca^2b^2c \tan(1/2x) - 5Ca^2b^3c \tan(1/2x) + 11Aa^3c^2 \tan(1/2x) - 4Ba^3c^2 \tan(1/2x) - 3Aa^2b^2c^2 \tan(1/2x) - 8Ba^2b^2c^2 \tan(1/2x) - 7Aa^2b^2c^2 \tan(1/2x) + 8Ba^2b^2c^2 \tan(1/2x) - Ab^3c^2 \tan(1/2x) + 4Bb^3c^2 \tan(1/2x) - 4Ca^2c^3 \tan(1/2x) + 4Ca^2b^3c^3 \tan(1/2x) - 2Aa^4c^4 \tan(1/2x) + 2Ba^4c^4 \tan(1/2x) - 2Aa^2b^4c^4 \tan(1/2x) + 2Bb^4c^4 \tan(1/2x) - 2Ca^5 + 4Ca^3b^2 - 2Ca^2b^4 + 4Aa^4c - 5Ba^3b^2c - 3Aa^2b^2c^3 + 5Ba^2b^3c - Ab^4c - Ca^3c^2 + Ca^2b^2c^2 - Aa^2c^3 + 2Ba^2b^2c^3 - Ab^2c^3) / ((a^6 - 2a^5b - a^4b^2 + 4a^3b^3 - a^2b^4 - 2ab^5 + b^6 - 2a^4c^2 + 4a^3b^2c^2 - 4a^2b^3c^2 + 2b^4c^2 + a^2c^4 - 2ab^2c^4 + b^2c^4) * (a \tan(1/2x)^2 - b \tan(1/2x)^2 + 2c \tan(1/2x) + a + b)^2)
\end{aligned}$$

maple [B] time = 0.23, size = 1422, normalized size = 6.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A+B*\cos(x)+C*\sin(x))/(a+b*\cos(x)+c*\sin(x))^3,x)$

[Out] $2*(-1/2*(4Aa^3b-7Aa^2b^2-5Aa^2c^2+2Aa^2b^3+2Aa^2b^2c+Ab^4+3Aa^2b^2c^2+2Aa^2c^4-2Ba^4+3Ba^3b-2Ba^2b^2+4Ba^2c^2+3Ba^2b^3-2Bb^4-4Bb^2c^2-2Bb^2c^4+3Ca^3c-6Ca^2b^2c+3Ca^2b^2c^2)/(a-b)/(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)*\tan(1/2x)^3+1/2*(4Aa^4c-12Aa^3b^2c+13Aa^2b^2c^2+7Aa^2c^3-6Aa^2b^3c-6Aa^2b^2c^3+Ab^4c-Ab^2c^3-2Aa^5+2Ba^4c-9Ba^3b^2c+14Ba^2b^2c-4Ba^2c^3-9Ba^2b^3c+2Bb^4c+4Bb^2c^3+2Bb^2c^5-2Ca^5+2Ca^4b+4Ca^3b^2-5Ca^3c^2-4Ca^2b^3+14Ca^2b^2c^2-2Ca^2b^4-13Ca^2b^2c^2-2Ca^2c^4+2Cb^5+4Cb^3c^2+2Cb^2c^4)/(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)/(a^2-2ab+b^2)*\tan(1/2x)^2-1/2*(4Aa^4b-5Aa^3b^2-11Aa^3c^2-3Aa^2b^3+3Aa^2b^2c^2+5Aa^2b^4+7Aa^2b^2c^2+2Aa^2c^4-Ab^5+Ab^3c^2+2Aa^2b^3c^2-2Ba^5+3Ba^4b-Ba^3b^2+4Ba^3c^2-Ba^2b^3+8Ba^2b^2c^2+3Ba^2b^4-8Ba^2b^2c^2-2Ba^2c^4-2Bb^5-4Bb^3c^2-2Bb^2c^4+5Ca^4c-5Ca^3b^2c-5Ca^2b^2c+4Ca^2c^3+5Ca^2b^3c-4Ca^2b^2c^3)/(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)/(a^2-2ab+b^2)*\tan(1/2x)+1/2*(4Aa^4c-3Aa^2b^2c-Aa^2c^3-Ab^4c-Ab^2c^3-5Ba^3b^2c+5Ba^2b^3c+2Ba^2b^2c^3-2Ca^5+4Ca^3b^2-Ca^3c^2-2Ca^2b^4+Ca^2b^2c^2)/(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)/(a^2-2ab+b^2))/(a*\tan(1/2x)^2-b*\tan(1/2x)^2+2c*\tan(1/2x)+a+b)^2+2/(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)/(a^2-b^2-c^2)^(1/2)*\arctan(1/2*(2*(a-b)*\tan(1/2x)+2c)/(a^2-b^2-c^2)^(1/2))*a^2A+1/(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)/(a^2-b^2-c^2)^(1/2)*\arctan(1/2*(2*(a-b)*\tan(1/2x)+2c)/(a^2-b^2-c^2)^(1/2))*a^2B-3/(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)/(a^2-b^2-c^2)^(1/2)*\arctan(1/2*(2*(a-b)*\tan(1/2x)+2c)/(a^2-b^2-c^2)^(1/2))*a^2C$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for more details)Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 8.21, size = 1160, normalized size = 4.89

$$\frac{2Ca^5 - 4Aa^4c - 4Ca^3b^2 + 5Ba^3bc + Ca^3c^2 + 3Aa^2b^2c + Aa^2c^3 + 2Cab^4 - 5Bab^3c - Cab^2c^2 - 2Bab^2c^3 + Ab^4c + Ab^2c^3}{(a-b)^2(a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4)} + \frac{\tan\left(\frac{x}{2}\right)^3 (Ab^4 - 2Bab^3c + Cab^2c^2 - 2Bab^2c^3 + Ab^4c + Ab^2c^3)}{(a-b)^2(a^4 - 2a^2b^2 - 2a^2c^2 + b^4 + 2b^2c^2 + c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x))^3,x)

[Out]
$$-\frac{\left((2Ca^5 + Aa^2c^3 + Ab^2c^3 - 4Ca^3b^2 + Ca^3c^2 - 4Aa^4c + Ab^4c + 2Ca^2b^4 - 2B^2a^2b^3c - 5B^2a^2b^3c + 5B^2a^3b^2c + 3Aa^2b^2c - Ca^2b^2c^2) / ((a-b)^2(a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2)) + (\tan(x/2)^3 (Ab^4 - 2B^2a^4 + 2A^2c^4 - 2B^2b^4 - 2B^2c^4 - 7A^2a^2b^2 - 5A^2a^2c^2 - 2B^2a^2b^2 + 3A^2b^2c^2 + 4B^2a^2c^2 - 4B^2b^2c^2 + 2A^2ab^3 + 4A^2a^3b + 3B^2a^2b^3 + 3B^2a^3b + 3C^2a^3c + 2A^2ab^2c + 3C^2ab^2c - 6C^2a^2b^2c) / ((a-b)(a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2)) - (\tan(x/2)^2 (2B^2c^5 - 2C^2a^5 - 2A^2c^5 + 2C^2b^5 + 7A^2a^2c^3 - Ab^2c^3 - 4B^2a^2c^3 - 4C^2a^2b^3 + 4C^2a^3b^2 + 4B^2b^2c^3 - 5C^2a^3c^2 + 4C^2b^3c^2 + 4A^2a^4c + Ab^4c + 2B^2a^4c - 2C^2ab^4 + 2C^2a^4b + 2B^2b^4c - 2C^2ac^4 + 2C^2bc^4 - 6A^2ab^2c^3 - 6A^2ab^3c - 12A^2a^3b^2c - 9B^2a^2b^3c - 9B^2a^3b^2c + 13A^2a^2b^2c + 14B^2a^2b^2c - 13C^2a^2b^2c^2 + 14C^2a^2b^2c^2) / ((a-b)^2(a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2)) - (\tan(x/2) (Ab^5 + 2B^2a^5 + 2B^2b^5 + 3A^2a^2b^3 + 5A^2a^3b^2 + 11A^2a^3c^2 + B^2a^2b^3 + B^2a^3b^2 - Ab^3c^2 - 4B^2a^3c^2 + 4B^2b^3c^2 - 4C^2a^2c^3 - 5A^2ab^4 - 4A^2a^4b - 2A^2ac^4 - 3B^2ab^4 - 3B^2a^4b - 2A^2bc^4 + 2B^2ac^4 + 2B^2bc^4 - 5C^2a^4c + 4C^2ab^3c - 5C^2ab^3c + 5C^2a^3b^2c - 7A^2ab^2c^2 - 3A^2a^2b^2c^2 + 8B^2ab^2c^2 - 8B^2a^2b^2c^2 + 5C^2a^2b^2c^2) / ((a-b)^2(a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2)) \right) / (\tan(x/2)^4 (a^2 - 2ab + b^2) + 2ab + \tan(x/2) (4a^2c + 4b^2c) + \tan(x/2)^3 (4a^2c - 4b^2c) + a^2 + b^2 + \tan(x/2)^2 (2a^2 - 2b^2 + 4c^2)) - (\operatorname{atanh}((2a^4c + 2b^4c + 2c^5 - 4a^2c^3 + 4b^2c^3 - 4a^2b^2c) / (2(b^2 - a^2 + c^2)^{5/2})) + (\tan(x/2) (2a - 2b) (a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2)) / (2(b^2 - a^2 + c^2)^{5/2})) (2Aa^2 + Ab^2 + Ac^2 - 3B^2ab - 3C^2ac)) / (b^2 - a^2 + c^2)^{5/2}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**3,x)

[Out] Timed out

$$3.553 \quad \int \frac{A+B \cos(x)+C \sin(x)}{a+b \cos(x)+ib \sin(x)} dx$$

Optimal. Leaf size=105

$$\frac{i(- (a^2(B-iC)) + 2aAb - b^2(B+iC)) \log(a+ib \sin(x)+b \cos(x))}{2a^2b} + \frac{x(2aA - b(B+iC))}{2a^2} + \frac{(-C+iB)(\cos(x)-i \sin(x))}{2a}$$

[Out] 1/2*(2*a*A-b*(B+I*C))*x/a^2+1/2*I*(2*a*A*b-a^2*(B-I*C)-b^2*(B+I*C))*ln(a+b*cos(x)+I*b*sin(x))/a^2/b+1/2*(I*B-C)*(cos(x)-I*sin(x))/a

Rubi [A] time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {3130}

$$\frac{i(a^2(- (B-iC)) + 2aAb - b^2(B+iC)) \log(a+ib \sin(x)+b \cos(x))}{2a^2b} + \frac{x(2aA - b(B+iC))}{2a^2} + \frac{(-C+iB)(\cos(x)-i \sin(x))}{2a}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]

[Out] ((2*a*A - b*(B + I*C))*x)/(2*a^2) + ((I/2)*(2*a*A*b - a^2*(B - I*C) - b^2*(B + I*C))*Log[a + b*Cos[x] + I*b*Sin[x]])/(a^2*b) + ((I*B - C)*(Cos[x] - I*Sin[x]))/(2*a)

Rule 3130

Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)])/(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]), x_Symbol] :> Simp[((2*a*A - b*B - c*C)*x)/(2*a^2), x] + (-Simp[((b*B + c*C)*(b*Cos[d + e*x] - c*Sin[d + e*x]))/(2*a*b*c*e), x] + Simp[((a^2*(b*B - c*C) - 2*a*A*b^2 + b^2*(b*B + c*C))*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*b*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[b^2 + c^2, 0]

Rubi steps

$$\int \frac{A+B \cos(x)+C \sin(x)}{a+b \cos(x)+ib \sin(x)} dx = \frac{(2aA - b(B+iC))x}{2a^2} + \frac{i(2aAb - a^2(B-iC) - b^2(B+iC)) \log(a+b \cos(x)+ib \sin(x))}{2a^2b}$$

Mathematica [A] time = 0.44, size = 165, normalized size = 1.57

$$\frac{x(a^2(B-iC) + 2aAb - b^2(B+iC)) + (a^2(-C-iB) + 2iaAb + b^2(C-iB)) \log(a^2 + 2ab \cos(x) + b^2) + 2(a^2(B-iC) + 2aAb - b^2(B+iC)) \arctan\left(\frac{(a+b) \cot(x/2)}{a-b}\right)}{4a^2b}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]

[Out] ((2*a*A*b + a^2*(B - I*C) - b^2*(B + I*C))*x + 2*(-2*a*A*b + a^2*(B - I*C) + b^2*(B + I*C))*ArcTan[((a + b)*Cot[x/2])/(a - b)] + (2*I)*a*b*(B + I*C)*Cos[x] + ((2*I)*a*A*b + a^2*((-I)*B - C) + b^2*((-I)*B + C))*Log[a^2 + b^2 + 2*a*b*Cos[x]] + 2*a*b*(B + I*C)*Sin[x])/(4*a^2*b)

fricas [A] time = 1.46, size = 89, normalized size = 0.85

$$\frac{(iB - C)ab + (2Aab - (B + iC)b^2)xe^{ix} + ((-iB - C)a^2 + 2iAab + (-iB + C)b^2)e^{ix} \log\left(\frac{be^{ix} + a}{b}\right)}{2a^2b} e^{-ix}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fricas")

[Out] $\frac{1}{2}((I*B - C)*a*b + (2*A*a*b - (B + I*C)*b^2)*x*e^{I*x} + ((-I*B - C)*a^2 + 2*I*A*a*b + (-I*B + C)*b^2)*e^{I*x}*\log((b*e^{I*x} + a)/b))*e^{-I*x}/(a^2*b)$

giac [B] time = 0.15, size = 203, normalized size = 1.93

$$\frac{(-2iAa + iBb - Cb) \log\left(-a \tan\left(\frac{1}{2}x\right)^2 + b \tan\left(\frac{1}{2}x\right)^2 - 2ia \tan\left(\frac{1}{2}x\right) + a + b\right)}{4a^2} - \frac{(2iAa - iBb + Cb) \log\left(\tan\left(\frac{1}{2}x\right) - I\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac")

[Out] $-\frac{1}{4}(-2*I*A*a + I*B*b - C*b)*\log(-a*\tan(1/2*x)^2 + b*\tan(1/2*x)^2 - 2*I*a*\tan(1/2*x) + a + b)/a^2 - \frac{1}{2}*(2*I*A*a - I*B*b + C*b)*\log(\tan(1/2*x) - I)/a^2 + \frac{1}{4}*(2*B*a^2 - 2*I*C*a^2 - 2*A*a*b + B*b^2 + I*C*b^2)*(x + 2*\arctan((-I*a*\cos(x) - a*\sin(x) - I*a)/(a*\cos(x) - I*a*\sin(x) - a + 2*b)))/(a^2*b) - \frac{1}{2}*(-2*I*A*a*\tan(1/2*x) + I*B*b*\tan(1/2*x) - C*b*\tan(1/2*x) - 2*A*a - 2*B*a - 2*I*C*a + B*b + I*C*b)/(a^2*(\tan(1/2*x) - I))$

maple [B] time = 0.18, size = 257, normalized size = 2.45

$$\frac{C \ln\left(\tan\left(\frac{x}{2}\right) + i\right)}{2b} + \frac{iB \ln\left(\tan\left(\frac{x}{2}\right) + i\right)}{2b} + \frac{iC}{a\left(\tan\left(\frac{x}{2}\right) - i\right)} + \frac{B}{a\left(\tan\left(\frac{x}{2}\right) - i\right)} - \frac{i \ln\left(\tan\left(\frac{x}{2}\right) - i\right) A}{a} + \frac{i \ln\left(\tan\left(\frac{x}{2}\right) - i\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)

[Out] $\frac{1}{2}C/b*\ln(\tan(1/2*x)+I)+\frac{1}{2}I*B/b*\ln(\tan(1/2*x)+I)+I*C/a/(\tan(1/2*x)-I)+B/a/(\tan(1/2*x)-I)-I/a*\ln(\tan(1/2*x)-I)*A+\frac{1}{2}I/a^2*\ln(\tan(1/2*x)-I)*b*B-\frac{1}{2}/a^2*\ln(\tan(1/2*x)-I)*b*C-\frac{1}{2}/b*\ln(I*a+I*b+a*\tan(1/2*x)-b*\tan(1/2*x))*C+\frac{1}{2}/a^2*b*\ln(I*a+I*b+a*\tan(1/2*x)-b*\tan(1/2*x))*C+I/a*\ln(I*a+I*b+a*\tan(1/2*x)-b*\tan(1/2*x))*A-\frac{1}{2}I/b*\ln(I*a+I*b+a*\tan(1/2*x)-b*\tan(1/2*x))*B-\frac{1}{2}I/a^2*b*\ln(I*a+I*b+a*\tan(1/2*x)-b*\tan(1/2*x))*B$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

mupad [B] time = 6.93, size = 132, normalized size = 1.26

$$-\ln\left(a + b - a \tan\left(\frac{x}{2}\right) + i + b \tan\left(\frac{x}{2}\right) + i\right) \left(\frac{C}{2} + \frac{B1i}{2} - \frac{Cb^2}{2} - \frac{Bb^21i}{2} + Aab1i \right) + \frac{\ln\left(\tan\left(\frac{x}{2}\right) + i\right) (C + B1i)}{2b} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) + b*sin(x)*1i),x)`

[Out] $(\log(\tan(x/2) + 1i)*(B*1i + C))/(2*b) - \log(a + b - a*\tan(x/2)*1i + b*\tan(x/2)*1i)*(((B*1i)/2 + C/2)/b - ((C*b^2)/2 - (B*b^2*1i)/2 + A*a*b*1i)/(a^2*b)) + (\log(\tan(x/2) - 1i)*(B*b - 2*A*a + C*b*1i)*1i)/(2*a^2) + (5*B + C*5i)/(5*a*(\tan(x/2) - 1i))$

sympy [A] time = 1.37, size = 138, normalized size = 1.31

$$\begin{cases} -\frac{(-iB+C)e^{-ix}}{2a} & \text{for } 2a \neq 0 \\ x\left(-\frac{2Aa-Bb-iCb}{2a^2} - \frac{i(2iAa+iBa-iBb-Ca+Cb)}{2a^2}\right) & \text{otherwise} \end{cases} \frac{x(-2Aa + Bb + iCb) - i(-2Aab + Ba^2 + Bb^2 - iCa^2 + iCb^2)}{2a^2} \frac{i(-2Aab + Ba^2 + Bb^2 - iCa^2 + iCb^2)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)`

[Out] `Piecewise((-(-I*B + C)*exp(-I*x)/(2*a), Ne(2*a, 0)), (x*(-(2*A*a - B*b - I*C*b)/(2*a**2) - I*(2*I*A*a + I*B*a - I*B*b - C*a + C*b)/(2*a**2)), True)) - x*(-2*A*a + B*b + I*C*b)/(2*a**2) - I*(-2*A*a*b + B*a**2 + B*b**2 - I*C*a**2 + I*C*b**2)*log(a/b + exp(I*x))/(2*a**2*b)`

$$3.554 \quad \int \frac{A+B \cos(x)+C \sin(x)}{a+b \cos(x)-ib \sin(x)} dx$$

Optimal. Leaf size=103

$$\frac{i\left(-\left(a^2(B+iC)\right)+2aAb-b^2(B-iC)\right) \log(a-ib \sin(x)+b \cos(x))}{2a^2b} + \frac{x(2aA-bB+ibC)}{2a^2} - \frac{(C+iB)(\cos(x)+\sin(x))}{2a}$$

[Out] 1/2*(2*a*A-b*B+I*b*C)*x/a^2-1/2*I*(2*a*A*b-b^2*(B-I*C)-a^2*(B+I*C))*ln(a+b*cos(x)-I*b*sin(x))/a^2/b-1/2*(I*B+C)*(cos(x)+I*sin(x))/a

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {3130}

$$\frac{i\left(a^2(-B+iC)+2aAb-b^2(B-iC)\right) \log(a-ib \sin(x)+b \cos(x))}{2a^2b} + \frac{x(2aA-bB+ibC)}{2a^2} - \frac{(C+iB)(\cos(x)+\sin(x))}{2a}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]

[Out] ((2*a*A - b*B + I*b*C)*x)/(2*a^2) - ((I/2)*(2*a*A*b - b^2*(B - I*C) - a^2*(B + I*C))*Log[a + b*Cos[x] - I*b*Sin[x]])/(a^2*b) - ((I*B + C)*(Cos[x] + I*Sin[x]))/(2*a)

Rule 3130

Int[((A_.) + cos[(d_.) + (e_.)*(x_)])*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]) / (cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((2*a*A - b*B - c*C)*x)/(2*a^2), x] + (-Simp[((b*B + c*C)*(b*Cos[d + e*x] - c*Sin[d + e*x]))/(2*a*b*c*e), x] + Simp[((a^2*(b*B - c*C) - 2*a*A*b^2 + b^2*(b*B + c*C))*Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin[d + e*x], x]])/(2*a^2*b*c*e), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[b^2 + c^2, 0]

Rubi steps

$$\int \frac{A+B \cos(x)+C \sin(x)}{a+b \cos(x)-ib \sin(x)} dx = \frac{(2aA-bB+ibC)x}{2a^2} - \frac{i(2aAb-b^2(B-iC)-a^2(B+iC)) \log(a+b \cos(x)-ib \sin(x))}{2a^2b}$$

Mathematica [A] time = 0.45, size = 167, normalized size = 1.62

$$\frac{(ia^2(B+iC)-2iaAb+b^2(C+iB)) \log(a^2+2ab \cos(x)+b^2)}{b} + \frac{2(a^2(B+iC)-2aAb+b^2(B-iC)) \tan^{-1}\left(\frac{(a+b) \cot\left(\frac{x}{2}\right)}{a-b}\right)}{b} + x \left(\frac{a^2(B+iC)}{b} + 2aA - b(B-iC)\right) / 4a^2$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]

[Out] ((2*a*A - b*(B - I*C) + (a^2*(B + I*C))/b)*x + (2*(-2*a*A*b + b^2*(B - I*C) + a^2*(B + I*C))*ArcTan[((a + b)*Cot[x/2])/(a - b)])/b - (2*I)*a*(B - I*C)*Cos[x] + (((-2*I)*a*A*b + I*a^2*(B + I*C) + b^2*(I*B + C))*Log[a^2 + b^2 + 2*a*b*Cos[x]])/b + 2*a*(B - I*C)*Sin[x])/(4*a^2)

fricas [A] time = 1.09, size = 73, normalized size = 0.71

$$\frac{(B + iC)a^2x + (-iB - C)abe^{ix} + ((iB - C)a^2 - 2iAab + (iB + C)b^2) \log\left(\frac{ae^{ix} + b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fricas")

[Out] 1/2*((B + I*C)*a^2*x + (-I*B - C)*a*b*e^(I*x) + ((I*B - C)*a^2 - 2*I*A*a*b + (I*B + C)*b^2)*log((a*e^(I*x) + b)/a))/(a^2*b)

giac [B] time = 0.15, size = 203, normalized size = 1.97

$$\frac{(2iAa - iBb - Cb) \log\left(-a \tan\left(\frac{1}{2}x\right)^2 + b \tan\left(\frac{1}{2}x\right)^2 + 2ia \tan\left(\frac{1}{2}x\right) + a + b\right)}{4a^2} - \frac{(-2iAa + iBb + Cb) \log\left(\tan\left(\frac{1}{2}x\right)\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac")

[Out] -1/4*(2*I*A*a - I*B*b - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 + 2*I*a*tan(1/2*x) + a + b)/a^2 - 1/2*(-2*I*A*a + I*B*b + C*b)*log(tan(1/2*x) + I)/a^2 + 1/4*(2*B*a^2 + 2*I*C*a^2 - 2*A*a*b + B*b^2 - I*C*b^2)*(x + 2*arctan((I*a*cos(x) - a*sin(x) + I*a)/(a*cos(x) + I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(2*I*A*a*tan(1/2*x) - I*B*b*tan(1/2*x) - C*b*tan(1/2*x) - 2*A*a - 2*B*a + 2*I*C*a + B*b - I*C*b)/(a^2*(tan(1/2*x) + I))

maple [B] time = 0.19, size = 475, normalized size = 4.61

$$\frac{i \ln\left(ia + ib - a \tan\left(\frac{x}{2}\right) + b \tan\left(\frac{x}{2}\right)\right) B}{-2a + 2b} + \frac{B}{a\left(\tan\left(\frac{x}{2}\right) + i\right)} - \frac{ib \ln\left(ia + ib - a \tan\left(\frac{x}{2}\right) + b \tan\left(\frac{x}{2}\right)\right) A}{a(-a + b)} - \frac{iB \ln\left(\tan\left(\frac{x}{2}\right)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)

[Out] 1/2*I/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B+B/a/(tan(1/2*x)+I)-I/a*b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*A-1/2*I*B/b*ln(tan(1/2*x)-I)-1/2/a^2*ln(tan(1/2*x)+I)*b*C+1/2*C/b*ln(tan(1/2*x)-I)-I*C/a/(tan(1/2*x)+I)+1/2*a/b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C-1/2/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C-1/2/a*b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C+1/2/a^2*b^2/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*C-1/2*I/a^2*ln(tan(1/2*x)+I)*b*B-1/2*I*a/b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B+I/a*ln(tan(1/2*x)+I)*A-1/2*I/a*b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B+I/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*A+1/2*I/a^2*b^2/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))*B

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.

mupad [B] time = 6.86, size = 133, normalized size = 1.29

$$\ln\left(a + b + a \tan\left(\frac{x}{2}\right) - b \tan\left(\frac{x}{2}\right) - 1\right) \left(\frac{-\frac{C}{2} + \frac{B1i}{2}}{b} + \frac{\frac{Bb^2 1i}{2} + \frac{Cb^2}{2} - Aab1i}{a^2 b} \right) + \frac{\ln\left(\tan\left(\frac{x}{2}\right) + 1\right) (2Aa - Bb + C)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) - b*sin(x)*1i),x)

[Out] log(a + b + a*tan(x/2)*1i - b*tan(x/2)*1i)*((B*1i)/2 - C/2)/b + ((B*b^2*1i)/2 + (C*b^2)/2 - A*a*b*1i)/(a^2*b)) + (log(tan(x/2) + 1i)*(2*A*a - B*b + C*b*1i)*1i)/(2*a^2) + (5*B - C*5i)/(5*a*(tan(x/2) + 1i)) - (log(tan(x/2) - 1i)*(B*1i - C))/(2*b)

sympy [A] time = 1.27, size = 109, normalized size = 1.06

$$\begin{cases} -\frac{(iB+C)e^{ix}}{2a} & \text{for } 2a \neq 0 \\ x\left(-\frac{B+iC}{2b} + \frac{Ba+Bb+iCa-iCb}{2ab}\right) & \text{otherwise} \end{cases} - \frac{x(-B-iC)}{2b} + \frac{i(-2Aab + Ba^2 + Bb^2 + iCa^2 - iCb^2) \log\left(e^{ix} + \frac{b}{a}\right)}{2a^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)

[Out] Piecewise((-I*B + C)*exp(I*x)/(2*a), Ne(2*a, 0)), (x*(-(B + I*C)/(2*b) + (B*a + B*b + I*C*a - I*C*b)/(2*a*b)), True)) - x*(-B - I*C)/(2*b) + I*(-2*A*a*b + B*a**2 + B*b**2 + I*C*a**2 - I*C*b**2)*log(exp(I*x) + b/a)/(2*a**2*b)

$$3.555 \quad \int \frac{b^2 + c^2 + ab \cos(x) + ac \sin(x)}{(a + b \cos(x) + c \sin(x))^2} dx$$

Optimal. Leaf size=24

$$\frac{c \cos(x) - b \sin(x)}{a + b \cos(x) + c \sin(x)}$$

[Out] $(-c*\cos(x)+b*\sin(x))/(a+b*\cos(x)+c*\sin(x))$

Rubi [B] time = 0.07, antiderivative size = 68, normalized size of antiderivative = 2.83, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {3150}

$$\frac{c \cos(x) (a^2 - b^2 - c^2) - b \sin(x) (a^2 - b^2 - c^2)}{(a^2 - b^2 - c^2) (a + b \cos(x) + c \sin(x))}$$

Antiderivative was successfully verified.

[In] Int[(b^2 + c^2 + a*b*Cos[x] + a*c*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]

[Out] $-((c*(a^2 - b^2 - c^2)*\text{Cos}[x] - b*(a^2 - b^2 - c^2)*\text{Sin}[x])/((a^2 - b^2 - c^2)*(a + b*\text{Cos}[x] + c*\text{Sin}[x])))$

Rule 3150

Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]) / ((a_.) + cos[(d_.) + (e_.)*(x_.)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^2, x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]) / (e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] / ; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && EqQ[a*A - b*B - c*C, 0]

Rubi steps

$$\int \frac{b^2 + c^2 + ab \cos(x) + ac \sin(x)}{(a + b \cos(x) + c \sin(x))^2} dx = -\frac{c(a^2 - b^2 - c^2) \cos(x) - b(a^2 - b^2 - c^2) \sin(x)}{(a^2 - b^2 - c^2) (a + b \cos(x) + c \sin(x))}$$

Mathematica [A] time = 0.09, size = 32, normalized size = 1.33

$$\frac{ac + b^2 \sin(x) + c^2 \sin(x)}{b(a + b \cos(x) + c \sin(x))}$$

Antiderivative was successfully verified.

[In] Integrate[(b^2 + c^2 + a*b*Cos[x] + a*c*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2, x]

[Out] $(a*c + b^2*\text{Sin}[x] + c^2*\text{Sin}[x])/(b*(a + b*\text{Cos}[x] + c*\text{Sin}[x]))$

fricas [A] time = 0.89, size = 24, normalized size = 1.00

$$\frac{c \cos(x) - b \sin(x)}{b \cos(x) + c \sin(x) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas")

[Out] $-(c \cos(x) - b \sin(x)) / (b \cos(x) + c \sin(x) + a)$

giac [B] time = 0.18, size = 68, normalized size = 2.83

$$\frac{2 \left(ab \tan\left(\frac{1}{2}x\right) - b^2 \tan\left(\frac{1}{2}x\right) - c^2 \tan\left(\frac{1}{2}x\right) - ac \right)}{\left(a \tan\left(\frac{1}{2}x\right)^2 - b \tan\left(\frac{1}{2}x\right)^2 + 2c \tan\left(\frac{1}{2}x\right) + a + b \right) (a - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")`

[Out] $2*(a*b*\tan(1/2*x) - b^2*\tan(1/2*x) - c^2*\tan(1/2*x) - a*c) / ((a*\tan(1/2*x)^2 - b*\tan(1/2*x)^2 + 2*c*\tan(1/2*x) + a + b)*(a - b))$

maple [B] time = 0.19, size = 70, normalized size = 2.92

$$\frac{2 \left(-\frac{(ab-b^2-c^2) \tan\left(\frac{x}{2}\right)}{a-b} + \frac{ac}{a-b} \right)}{a \left(\tan^2\left(\frac{x}{2}\right) \right) - b \left(\tan^2\left(\frac{x}{2}\right) \right) + 2c \tan\left(\frac{x}{2}\right) + a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x)`

[Out] $-2*(-(a*b-b^2-c^2)/(a-b)*\tan(1/2*x)+a*c/(a-b))/(a*\tan(1/2*x)^2-b*\tan(1/2*x)^2+2*c*\tan(1/2*x)+a+b)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for more details) Is c^2+b^2-a^2 positive or negative?

mupad [B] time = 3.03, size = 62, normalized size = 2.58

$$\frac{\frac{2ac}{a-b} + \frac{2 \tan\left(\frac{x}{2}\right) (b^2 - ab + c^2)}{a-b}}{(a - b) \tan\left(\frac{x}{2}\right)^2 + 2c \tan\left(\frac{x}{2}\right) + a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b^2 + c^2 + a*c*sin(x) + a*b*cos(x))/(a + b*cos(x) + c*sin(x))^2,x)`

[Out] $-((2*a*c)/(a - b) + (2*\tan(x/2)*(b^2 - a*b + c^2))/(a - b))/(a + b + 2*c*\tan(x/2) + \tan(x/2)^2*(a - b))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2+c**2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)`

[Out] Timed out

$$3.556 \quad \int (a + b \cos(x) + c \sin(x))^{5/2} (d + be \cos(x) + ce \sin(x)) dx$$

Optimal. Leaf size=390

$$\frac{2(a^2 - b^2 - c^2)(15a^2e + 56ad + 25e(b^2 + c^2)) \sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}(x - \tan^{-1}(b,c)) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right) + 2(15a^3e + 15a^2e + 56ad + 25e(b^2 + c^2))}{105\sqrt{a + b \cos(x) + c \sin(x)}}$$

[Out] $-2/7*(a+b*\cos(x)+c*\sin(x))^{(5/2)}*(c*e*\cos(x)-b*e*\sin(x))-2/35*(a+b*\cos(x)+c*\sin(x))^{(3/2)}*(c*(5*a*e+7*d)*\cos(x)-b*(5*a*e+7*d)*\sin(x))-2/105*(c*(56*a*d+15*a^2*e+25*(b^2+c^2)*e)*\cos(x)-b*(56*a*d+15*a^2*e+25*(b^2+c^2)*e)*\sin(x))*((a+b*\cos(x)+c*\sin(x))^{(1/2)}+2/105*(161*a^2*d+63*(b^2+c^2)*d+15*a^3*e+145*a*(b^2+c^2)*e)*(\cos(1/2*x-1/2*\arctan(b,c))^{(1/2)}/\cos(1/2*x-1/2*\arctan(b,c)))*\text{EllipticE}(\sin(1/2*x-1/2*\arctan(b,c)),2^{(1/2)}*((b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)})))^{(1/2)}*(a+b*\cos(x)+c*\sin(x))^{(1/2)}/((a+b*\cos(x)+c*\sin(x))/(a+(b^2+c^2)^{(1/2)})))^{(1/2)}-2/105*(a^2-b^2-c^2)*(56*a*d+15*a^2*e+25*(b^2+c^2)*e)*(\cos(1/2*x-1/2*\arctan(b,c))^{(1/2)}/\cos(1/2*x-1/2*\arctan(b,c)))*\text{EllipticF}(\sin(1/2*x-1/2*\arctan(b,c)),2^{(1/2)}*((b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)})))^{(1/2)}*((a+b*\cos(x)+c*\sin(x))/(a+(b^2+c^2)^{(1/2)})))^{(1/2)}/(a+b*\cos(x)+c*\sin(x))^{(1/2)}$

Rubi [A] time = 0.89, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3146, 3149, 3119, 2653, 3127, 2661}

$$\frac{2(a^2 - b^2 - c^2)(15a^2e + 56ad + 25e(b^2 + c^2)) \sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}(x - \tan^{-1}(b,c)) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right) + 2(161a^2d + 15a^3e + 56ad + 25e(b^2 + c^2))}{105\sqrt{a + b \cos(x) + c \sin(x)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[x] + c*Sin[x])^(5/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]

[Out] $(2*(161*a^2*d + 63*(b^2 + c^2)*d + 15*a^3*e + 145*a*(b^2 + c^2)*e)*\text{EllipticE}[(x - \text{ArcTan}[b, c])/2, (2*\text{Sqrt}[b^2 + c^2])/(a + \text{Sqrt}[b^2 + c^2])]*\text{Sqrt}[a + b*\text{Cos}[x] + c*\text{Sin}[x]]/(105*\text{Sqrt}[(a + b*\text{Cos}[x] + c*\text{Sin}[x])/(a + \text{Sqrt}[b^2 + c^2])])) - (2*(a^2 - b^2 - c^2)*(56*a*d + 15*a^2*e + 25*(b^2 + c^2)*e)*\text{EllipticF}[(x - \text{ArcTan}[b, c])/2, (2*\text{Sqrt}[b^2 + c^2])/(a + \text{Sqrt}[b^2 + c^2])]*\text{Sqrt}[(a + b*\text{Cos}[x] + c*\text{Sin}[x])/(a + \text{Sqrt}[b^2 + c^2])])/(105*\text{Sqrt}[a + b*\text{Cos}[x] + c*\text{Sin}[x]]) - (2*(a + b*\text{Cos}[x] + c*\text{Sin}[x])^{(5/2)}*(c*e*\text{Cos}[x] - b*e*\text{Sin}[x]))/7 - (2*(a + b*\text{Cos}[x] + c*\text{Sin}[x])^{(3/2)}*(c*(7*d + 5*a*e)*\text{Cos}[x] - b*(7*d + 5*a*e)*\text{Sin}[x]))/35 - (2*\text{Sqrt}[a + b*\text{Cos}[x] + c*\text{Sin}[x]]*(c*(56*a*d + 15*a^2*e + 25*(b^2 + c^2)*e)*\text{Cos}[x] - b*(56*a*d + 15*a^2*e + 25*(b^2 + c^2)*e)*\text{Sin}[x]))/105$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3127

Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3146

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]^n)*(A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] := Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x] + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]

Rule 3149

Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]

Rubi steps

$$\begin{aligned}
\int (a + b \cos(x) + c \sin(x))^{5/2} (d + be \cos(x) + ce \sin(x)) dx &= -\frac{2}{7} (a + b \cos(x) + c \sin(x))^{5/2} (ce \cos(x) - be \sin(x)) \\
&= -\frac{2}{7} (a + b \cos(x) + c \sin(x))^{5/2} (ce \cos(x) - be \sin(x)) \\
&= -\frac{2}{7} (a + b \cos(x) + c \sin(x))^{5/2} (ce \cos(x) - be \sin(x)) \\
&= -\frac{2}{7} (a + b \cos(x) + c \sin(x))^{5/2} (ce \cos(x) - be \sin(x)) \\
&= -\frac{2}{7} (a + b \cos(x) + c \sin(x))^{5/2} (ce \cos(x) - be \sin(x)) \\
&= \frac{2(161a^2d + 63(b^2 + c^2)d + 15a^3e + 145a(b^2 + c^2)e)}{105\sqrt{a}}
\end{aligned}$$

Mathematica [C] time = 6.94, size = 7823, normalized size = 20.06

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Cos[x] + c*Sin[x])^(5/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]

[Out] Result too large to show

fricas [F] time = 1.63, size = 0, normalized size = 0.00

integral(((b³ - 3bc²)e cos(x)³ + 2ac²e + ((b² - c²)d + 2(ab² - ac²)e) cos(x)² + (a² + c²)d + (2abd + (a²b + 3

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorithm="fricas")

[Out] integral(((b³ - 3*b*c²)*e*cos(x)³ + 2*a*c²*e + ((b² - c²)*d + 2*(a*b² - a*c²)*e)*cos(x)² + (a² + c²)*d + (2*a*b*d + (a²*b + 3*b*c²)*e)*cos(x) + ((3*b²*c - c³)*e*cos(x)² + 2*a*c*d + (a²*c + c³)*e + 2*(2*a*b*c*e + b*c*d)*cos(x))*sin(x))*sqrt(b*cos(x) + c*sin(x) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (be \cos(x) + ce \sin(x) + d)(b \cos(x) + c \sin(x) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorithm="giac")

[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(5/2), x)

maple [B] time = 1.28, size = 3502, normalized size = 8.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (a+b\cos(x)+c\sin(x))^{5/2} \cdot (d+b\cos(x)+c\sin(x)) \, dx$

[Out]
$$\begin{aligned} & \left(-(b^2\sin(x-\arctan(-b,c)) - c^2\sin(x-\arctan(-b,c)) - a(b^2+c^2)^{1/2}) \cos(x-\arctan(-b,c)) \right. \\ & \left. \right)^{1/2} / (b^2+c^2)^{1/2} \cdot \left((b^6e+3b^4c^2e+3b^2c^4e+c^6e) \right. \\ & \left. \cdot (-2/7/(b^2+c^2)^{1/2} \sin(x-\arctan(-b,c))^2 (\cos(x-\arctan(-b,c)))^2 \right. \\ & \left. \cdot (b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) + a) \right)^{1/2} + 12/35/(b^2+c^2) \cdot a \sin(x-\arctan(-b,c)) \\ & \cdot (\cos(x-\arctan(-b,c)))^2 \cdot (b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) + a) \right)^{1/2} - 2/3 \cdot (5/7+24/35/(b^2+c^2) \cdot a^2) / (b^2+c^2)^{1/2} \\ & \cdot (\cos(x-\arctan(-b,c)))^2 \cdot (b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) + a) \right)^{1/2} + 2 \cdot (-4/35/(b^2+c^2) \cdot a^2 \\ & + 5/21) \cdot (1/(b^2+c^2)^{1/2} \cdot a - 1) \cdot \left((-b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) - a \right) / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \\ & \cdot \left((-\sin(x-\arctan(-b,c))) + 1 \right) \cdot (b^2+c^2)^{1/2} / \left(a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \cdot \left((1+\sin(x-\arctan(-b,c))) \right. \\ & \left. \cdot (b^2+c^2)^{1/2} / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2} / \left(\cos(x-\arctan(-b,c)) \right)^2 \cdot \left((b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) + a \right) \right)^{1/2} \\ & \cdot \text{EllipticF} \left(\left((-b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) - a \right) / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2}, \left((a - (b^2+c^2)^{1/2}) / \left(a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \right) \\ & + 2 \cdot (-48a^3 - 44ab^2 - 44ac^2) / (105 \cdot (b^2+c^2)^{1/2} \cdot b^2 + 105 \cdot (b^2+c^2)^{1/2} \cdot c^2) \cdot (1/(b^2+c^2)^{1/2} \cdot a - 1) \\ & \cdot \left((-b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) - a \right) / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \cdot \left((-\sin(x-\arctan(-b,c))) + 1 \right) \\ & \cdot (b^2+c^2)^{1/2} / \left(a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \cdot \left((1+\sin(x-\arctan(-b,c))) \right. \\ & \left. \cdot (b^2+c^2)^{1/2} / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2} / \left(\cos(x-\arctan(-b,c)) \right)^2 \cdot \left((b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) + a \right) \right)^{1/2} \\ & \cdot \left((-1/(b^2+c^2)^{1/2} \cdot a - 1) \cdot \text{EllipticE} \left(\left((-b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) - a \right) / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2}, \left((a - (b^2+c^2)^{1/2}) / \left(a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \right) \right. \\ & \left. + \text{EllipticF} \left(\left((-b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) - a \right) / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2}, \left((a - (b^2+c^2)^{1/2}) / \left(a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \right) \right) \\ & + (3 \cdot (b^2+c^2)^{1/2} \cdot a \cdot b^4e + 6 \cdot (b^2+c^2)^{1/2} \cdot a \cdot b^2c^2e + 3 \cdot (b^2+c^2)^{1/2} \cdot a \cdot c^4e + (b^2+c^2)^{1/2} \cdot b^4d \\ & + 2 \cdot (b^2+c^2)^{1/2} \cdot b^2c^2d + (b^2+c^2)^{1/2} \cdot c^4d) \cdot (-2/5/(b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) \\ & \cdot (\cos(x-\arctan(-b,c)))^2 \cdot (b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) + a) \right)^{1/2} + 8/15/(b^2+c^2) \cdot a \cdot (\cos(x-\arctan(-b,c)))^2 \\ & \cdot (b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) + a) \right)^{1/2} + 4/15/(b^2+c^2)^{1/2} \cdot a \cdot (1/(b^2+c^2)^{1/2} \cdot a - 1) \\ & \cdot \left((-b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) - a \right) / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \cdot \left((-\sin(x-\arctan(-b,c))) + 1 \right) \\ & \cdot (b^2+c^2)^{1/2} / \left(a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \cdot \left((1+\sin(x-\arctan(-b,c))) \right. \\ & \left. \cdot (b^2+c^2)^{1/2} / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2} / \left(\cos(x-\arctan(-b,c)) \right)^2 \cdot \left((b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) + a \right) \right)^{1/2} \\ & \cdot \text{EllipticF} \left(\left((-b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) - a \right) / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2}, \left((a - (b^2+c^2)^{1/2}) / \left(a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \right) \\ & + 2 \cdot (3/5 + 8/15/(b^2+c^2) \cdot a^2) \cdot (1/(b^2+c^2)^{1/2} \cdot a - 1) \cdot \left((-b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) - a \right) / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \\ & \cdot \left((-\sin(x-\arctan(-b,c))) + 1 \right) \cdot (b^2+c^2)^{1/2} / \left(a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \cdot \left((1+\sin(x-\arctan(-b,c))) \right. \\ & \left. \cdot (b^2+c^2)^{1/2} / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2} / \left(\cos(x-\arctan(-b,c)) \right)^2 \cdot \left((b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) + a \right) \right)^{1/2} \\ & \cdot \left((-1/(b^2+c^2)^{1/2} \cdot a - 1) \cdot \text{EllipticE} \left(\left((-b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) - a \right) / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2}, \left((a - (b^2+c^2)^{1/2}) / \left(a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \right) \right. \\ & \left. + \text{EllipticF} \left(\left((-b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) - a \right) / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2}, \left((a - (b^2+c^2)^{1/2}) / \left(a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \right) \right) \\ & + (3 \cdot a^2 \cdot b^4e + 6 \cdot a^2 \cdot b^2c^2e + 3 \cdot a^2 \cdot c^4e + 3 \cdot a \cdot b^4d + 6 \cdot a \cdot b^2c^2d + 3 \cdot a \cdot c^4d) \cdot (-2/3/(b^2+c^2)^{1/2} \\ & \cdot (\cos(x-\arctan(-b,c)))^2 \cdot (b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) + a) \right)^{1/2} + 2/3 \cdot (1/(b^2+c^2)^{1/2} \cdot a - 1) \\ & \cdot \left((-b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) - a \right) / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \cdot \left((-\sin(x-\arctan(-b,c))) + 1 \right) \\ & \cdot (b^2+c^2)^{1/2} / \left(a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \cdot \left((1+\sin(x-\arctan(-b,c))) \right. \\ & \left. \cdot (b^2+c^2)^{1/2} / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2} / \left(\cos(x-\arctan(-b,c)) \right)^2 \cdot \left((b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) + a \right) \right)^{1/2} \\ & \cdot \text{EllipticF} \left(\left((-b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) - a \right) / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2}, \left((a - (b^2+c^2)^{1/2}) / \left(a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \right) \\ & - 4/3/(b^2+c^2)^{1/2} \cdot a \cdot (1/(b^2+c^2)^{1/2} \cdot a - 1) \cdot \left((-b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) - a \right) / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \\ & \cdot \left((-\sin(x-\arctan(-b,c))) + 1 \right) \cdot (b^2+c^2)^{1/2} / \left(a + (b^2+c^2)^{1/2} \right) \right)^{1/2} \cdot \left((1+\sin(x-\arctan(-b,c))) \right. \\ & \left. \cdot (b^2+c^2)^{1/2} / \left(-a + (b^2+c^2)^{1/2} \right) \right)^{1/2} / \left(\cos(x-\arctan(-b,c)) \right)^2 \cdot \left((b^2+c^2)^{1/2} \sin(x-\arctan(-b,c)) + a \right) \right)^{1/2} \end{aligned}$$

$$\begin{aligned} & /2))^{1/2} / (\cos(x - \arctan(-b, c))^{1/2} * ((b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) + a)) \\ & ^{1/2} * ((-1 / (b^2 + c^2)^{1/2}) * a - 1) * \text{EllipticE}(((b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2}, ((a - (b^2 + c^2)^{1/2}) / (a + (b^2 + c^2)^{1/2}))^{1/2}) \\ & + \text{EllipticF}(((b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2}, ((a - (b^2 + c^2)^{1/2}) / (a + (b^2 + c^2)^{1/2}))^{1/2})) + 2 * ((b^2 + c^2)^{1/2} * a^3 * b^2 * e + (b^2 + c^2)^{1/2} * a^3 * c^2 * e + (b^2 + c^2)^{3/2} * a^2 * d + 2 * a^2 * b^2 * d * (b^2 + c^2)^{1/2} + 2 * a^2 * c^2 * d * (b^2 + c^2)^{1/2}) * (1 / (b^2 + c^2)^{1/2} * a - 1) \\ & * ((- (b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2} * ((-\sin(x - \arctan(-b, c)) + 1) * (b^2 + c^2)^{1/2} / (a + (b^2 + c^2)^{1/2}))^{1/2} * ((1 + \sin(x - \arctan(-b, c))) * (b^2 + c^2)^{1/2} / (-a + (b^2 + c^2)^{1/2}))^{1/2} / (\cos(x - \arctan(-b, c)))^2 * ((b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) + a))^{1/2} * ((-1 / (b^2 + c^2)^{1/2}) * a - 1) * \text{EllipticE}(((b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2}, ((a - (b^2 + c^2)^{1/2}) / (a + (b^2 + c^2)^{1/2}))^{1/2}) \\ & + \text{EllipticF}(((b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2}, ((a - (b^2 + c^2)^{1/2}) / (a + (b^2 + c^2)^{1/2}))^{1/2})) + 2 * a^3 * b^2 * d * (1 / (b^2 + c^2)^{1/2} * a - 1) * ((- (b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2} * ((-\sin(x - \arctan(-b, c)) + 1) * (b^2 + c^2)^{1/2} / (a + (b^2 + c^2)^{1/2}))^{1/2} * ((1 + \sin(x - \arctan(-b, c))) * (b^2 + c^2)^{1/2} / (-a + (b^2 + c^2)^{1/2}))^{1/2} / (- (b^2 * \sin(x - \arctan(-b, c)) - c^2 * \sin(x - \arctan(-b, c)) - a * (b^2 + c^2)^{1/2}) * \cos(x - \arctan(-b, c)))^2 / (b^2 + c^2)^{1/2})^{1/2} * \text{EllipticF}(((b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2}, ((a - (b^2 + c^2)^{1/2}) / (a + (b^2 + c^2)^{1/2}))^{1/2}) \\ & + 2 * a^3 * c^2 * d * (1 / (b^2 + c^2)^{1/2} * a - 1) * ((- (b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2} * ((-\sin(x - \arctan(-b, c)) + 1) * (b^2 + c^2)^{1/2} / (a + (b^2 + c^2)^{1/2}))^{1/2} * ((1 + \sin(x - \arctan(-b, c))) * (b^2 + c^2)^{1/2} / (-a + (b^2 + c^2)^{1/2}))^{1/2} / (- (b^2 * \sin(x - \arctan(-b, c)) - c^2 * \sin(x - \arctan(-b, c)) - a * (b^2 + c^2)^{1/2}) * \cos(x - \arctan(-b, c)))^2 / (b^2 + c^2)^{1/2})^{1/2} * \text{EllipticF}(((b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2}, ((a - (b^2 + c^2)^{1/2}) / (a + (b^2 + c^2)^{1/2}))^{1/2}) / \cos(x - \arctan(-b, c)) / ((b^2 * \sin(x - \arctan(-b, c)) + c^2 * \sin(x - \arctan(-b, c)) + a * (b^2 + c^2)^{1/2}) / (b^2 + c^2)^{1/2})^{1/2} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (be \cos(x) + ce \sin(x) + d)(b \cos(x) + c \sin(x) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorithm="maxima")

[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \cos(x) + c \sin(x))^{5/2} (d + b e \cos(x) + c e \sin(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(x) + c*sin(x))^(5/2)*(d + b*e*cos(x) + c*e*sin(x)),x)

[Out] int((a + b*cos(x) + c*sin(x))^(5/2)*(d + b*e*cos(x) + c*e*sin(x)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)+c*sin(x))**(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x)

[Out] Timed out

$$3.557 \quad \int (a + b \cos(x) + c \sin(x))^{3/2} (d + b e \cos(x) + c e \sin(x)) dx$$

Optimal. Leaf size=294

$$\frac{2(a^2 - b^2 - c^2)(3ae + 5d) \sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}\left(x - \tan^{-1}\left(\frac{b}{c}\right)\right) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right) + 2(3a^2e + 20ad + 9e(b^2 + c^2))}{15\sqrt{a + b \cos(x) + c \sin(x)}}$$

```
[Out] -2/5*(a+b*cos(x)+c*sin(x))^(3/2)*(c*e*cos(x)-b*e*sin(x))-2/15*(c*(3*a*e+5*d)*cos(x)-b*(3*a*e+5*d)*sin(x))*(a+b*cos(x)+c*sin(x))^(1/2)+2/15*(20*a*d+3*a^2*e+9*(b^2+c^2)*e)*(cos(1/2*x-1/2*arctan(b,c)))^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2))))^(1/2)*(a+b*cos(x)+c*sin(x))^(1/2)/((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)-2/15*(a^2-b^2-c^2)*(3*a*e+5*d)*(cos(1/2*x-1/2*arctan(b,c)))^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticF(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2))))^(1/2)*(a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a+b*cos(x)+c*sin(x))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3146, 3149, 3119, 2653, 3127, 2661}

$$\frac{2(a^2 - b^2 - c^2)(3ae + 5d) \sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}\left(x - \tan^{-1}\left(\frac{b}{c}\right)\right) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right) + 2(3a^2e + 20ad + 9e(b^2 + c^2))}{15\sqrt{a + b \cos(x) + c \sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*cos[x] + c*sin[x])^(3/2)*(d + b*e*cos[x] + c*e*sin[x]),x]
```

```
[Out] (2*(20*a*d + 3*a^2*e + 9*(b^2 + c^2)*e)*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*cos[x] + c*sin[x]])/(15*Sqrt[(a + b*cos[x] + c*sin[x])/(a + Sqrt[b^2 + c^2])]) - (2*(a^2 - b^2 - c^2)*(5*d + 3*a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*cos[x] + c*sin[x])/(a + Sqrt[b^2 + c^2])])/(15*Sqrt[a + b*cos[x] + c*sin[x]]) - (2*(a + b*cos[x] + c*sin[x])^(3/2)*(c*e*cos[x] - b*e*sin[x]))/5 - (2*Sqrt[a + b*cos[x] + c*sin[x]]*(c*(5*d + 3*a*e)*cos[x] - b*(5*d + 3*a*e)*sin[x]))/15
```

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d/Sqrt[a + b], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]]/Sqrt[(a + b*cos[d + e*x] + c*sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2])], x]
```

```
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^
2 + c^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :=> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c])]/(a + Sqr
t[b^2 + c^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_
)]), x_Symbol] :=> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :=> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\begin{aligned}
\int (a + b \cos(x) + c \sin(x))^{3/2} (d + be \cos(x) + ce \sin(x)) dx &= -\frac{2}{5} (a + b \cos(x) + c \sin(x))^{3/2} (ce \cos(x) - be \sin(x)) \\
&= -\frac{2}{5} (a + b \cos(x) + c \sin(x))^{3/2} (ce \cos(x) - be \sin(x)) \\
&= -\frac{2}{5} (a + b \cos(x) + c \sin(x))^{3/2} (ce \cos(x) - be \sin(x)) \\
&= -\frac{2}{5} (a + b \cos(x) + c \sin(x))^{3/2} (ce \cos(x) - be \sin(x)) \\
&= \frac{2(20ad + 3a^2e + 9(b^2 + c^2)e) E\left(\frac{1}{2}(x - \tan^{-1}(b, c))\right)}{15 \sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}}}
\end{aligned}$$

Mathematica [C] time = 6.58, size = 5218, normalized size = 17.75

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*cos[x] + c*sin[x])^(3/2)*(d + b*e*cos[x] + c*e*sin[x]),x]

[Out] Result too large to show

fricas [F] time = 1.00, size = 0, normalized size = 0.00

integral(((b^2 - c^2)*e*cos(x)^2 + c^2*e + a*d + (a*b*e + b*d)*cos(x) + (2*b*c*e*cos(x) + a*c*e + c*d)*sin(x))*sqrt(b*cos(x) + c*sin(x) + a), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorithm="fricas")

[Out] integral(((b^2 - c^2)*e*cos(x)^2 + c^2*e + a*d + (a*b*e + b*d)*cos(x) + (2*b*c*e*cos(x) + a*c*e + c*d)*sin(x))*sqrt(b*cos(x) + c*sin(x) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (be \cos(x) + ce \sin(x) + d)(b \cos(x) + c \sin(x) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorithm="giac")

[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(3/2), x)

maple [B] time = 1.01, size = 2238, normalized size = 7.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x)

[Out]
$$\begin{aligned} & (-(-b^2 \sin(x - \arctan(-b, c)) - c^2 \sin(x - \arctan(-b, c)) - a(b^2 + c^2)^{1/2}) \cos(x - \arctan(-b, c))^2 / (b^2 + c^2)^{1/2})^{1/2} / (b^2 + c^2)^{1/2} * ((b^4 e + 2b^2 c^2 e + c^4 e) * (-2/5 / (b^2 + c^2)^{1/2}) \sin(x - \arctan(-b, c)) * (\cos(x - \arctan(-b, c)))^2 * (b^2 + c^2)^{1/2} \sin(x - \arctan(-b, c)) + a)^{1/2} + 8/15 / (b^2 + c^2) * a * (\cos(x - \arctan(-b, c)))^2 * ((b^2 + c^2)^{1/2} \sin(x - \arctan(-b, c)) + a)^{1/2} + 4/15 / (b^2 + c^2)^{1/2} * a * (1 / (b^2 + c^2)^{1/2} * a - 1) * ((-b^2 + c^2)^{1/2} \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2} * ((-\sin(x - \arctan(-b, c)) + 1) * (b^2 + c^2)^{1/2} / (a + (b^2 + c^2)^{1/2}))^{1/2} * ((1 + \sin(x - \arctan(-b, c))) * (b^2 + c^2)^{1/2} / (-a + (b^2 + c^2)^{1/2}))^{1/2} / (\cos(x - \arctan(-b, c)))^2 * ((b^2 + c^2)^{1/2} \sin(x - \arctan(-b, c)) + a)^{1/2} * \text{EllipticF}(((b^2 + c^2)^{1/2} \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2}, ((a - (b^2 + c^2)^{1/2}) / (a + (b^2 + c^2)^{1/2}))^{1/2} + 2 * (3/5 + 8/15 / (b^2 + c^2) * a^2) * (1 / (b^2 + c^2)^{1/2} * a - 1) * ((-b^2 + c^2)^{1/2} \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2} * ((-\sin(x - \arctan(-b, c)) + 1) * (b^2 + c^2)^{1/2} / (a + (b^2 + c^2)^{1/2}))^{1/2} * ((1 + \sin(x - \arctan(-b, c))) * (b^2 + c^2)^{1/2} / (-a + (b^2 + c^2)^{1/2}))^{1/2} / (\cos(x - \arctan(-b, c)))^2 * ((b^2 + c^2)^{1/2} \sin(x - \arctan(-b, c)) + a)^{1/2} * ((-1 / (b^2 + c^2)^{1/2} * a - 1) * \text{EllipticE}(((b^2 + c^2)^{1/2} \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2}, ((a - (b^2 + c^2)^{1/2}) / (a + (b^2 + c^2)^{1/2}))^{1/2} + \text{EllipticF}(((b^2 + c^2)^{1/2} \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2}, ((a - (b^2 + c^2)^{1/2}) / (a + (b^2 + c^2)^{1/2}))^{1/2})) + (2 * (b^2 + c^2)^{1/2} * a * b^2 * e + 2 * (b^2 + c^2)^{1/2} * a * c^2 * e + (b^2 + c^2)^{1/2} * b^2 * d + (b^2 + c^2)^{1/2} * c^2 * d) * (-2/3 / (b^2 + c^2)^{1/2}) * (\cos(x - \arctan(-b, c)))^2 * ((b^2 + c^2)^{1/2} \sin(x - \arctan(-b, c)) + a)^{1/2} + 2/3 * (1 / (b^2 + c^2)^{1/2} * a - 1) * ((-b^2 + c^2)^{1/2} \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2} * ((-\sin(x - \arctan(-b, c)) + 1) * (b^2 + c^2)^{1/2} / (a + (b^2 + c^2)^{1/2}))^{1/2} * ((1 + \sin(x - \arctan(-b, c))) * (b^2 + c^2)^{1/2} / (-a + (b^2 + c^2)^{1/2}))^{1/2} / (\cos(x - \arctan(-b, c)))^2 \end{aligned}$$

```

*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)^(1/2)*EllipticF(((b^2+c^2)^(1/2)
)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2))^(1/2),((a-(b^2+c^2)^(1/2))/(
a+(b^2+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a-1)*((
-(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2))^(1/2))*((-sin(
x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arc
tan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c)
)^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)^(1/2))*((-1/(b^2+c^2)^(1/2)*a-1
)*EllipticE(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))
^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((b^2+c
^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2))^(1/2),((a-(b^2+c^2)^(
1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))^2*(a^2*b^2*e+a^2*c^2*e+2*a*b^2*d+2*a*c
^2*d)*(1/(b^2+c^2)^(1/2)*a-1)*((-b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a
+(b^2+c^2)^(1/2))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+
c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(
1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)
^(1/2))*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((b^2+c^2)^(1/2)*sin(x-arctan
(-b,c))-a)/(-a+(b^2+c^2)^(1/2))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1
/2)))^(1/2))+EllipticF(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c
^2)^(1/2))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2)))^2*d*a^2
*(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-b^2+c^2)^(1/2)*sin(x-arctan(-b
,c))-a)/(-a+(b^2+c^2)^(1/2))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/
2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+
(b^2+c^2)^(1/2)))^(1/2)/(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))
-a*(b^2+c^2)^(1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF(
((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2))^(1/2),((a-(
b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))/cos(x-arctan(-b,c))/((b^2*sin(
x-arctan(-b,c))+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))
^(1/2)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (be \cos(x) + ce \sin(x) + d)(b \cos(x) + c \sin(x) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorithm="maxima")

[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \cos(x) + c \sin(x))^{\frac{3}{2}} (d + b e \cos(x) + c e \sin(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(x) + c*sin(x))^(3/2)*(d + b*e*cos(x) + c*e*sin(x)),x)

[Out] int((a + b*cos(x) + c*sin(x))^(3/2)*(d + b*e*cos(x) + c*e*sin(x)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)+c*sin(x))**(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x)

[Out] Timed out

3.558 $\int \sqrt{a + b \cos(x) + c \sin(x)} (d + be \cos(x) + ce \sin(x)) dx$

Optimal. Leaf size=229

$$\frac{2e(a^2 - b^2 - c^2) \sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}(x - \tan^{-1}(b,c)) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\sqrt{a+b \cos(x)+c \sin(x)}} + \frac{2(ae+3d)\sqrt{a+b \cos(x)+c \sin(x)} E\left(\frac{1}{2}(x - \tan^{-1}(b,c)) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}}}$$

[Out] $-2/3*(c*e*\cos(x)-b*e*\sin(x))*(a+b*\cos(x)+c*\sin(x))^{(1/2)}+2/3*(a*e+3*d)*(\cos(1/2*x-1/2*\arctan(b,c))^{(1/2)}/\cos(1/2*x-1/2*\arctan(b,c))*\text{EllipticE}(\sin(1/2*x-1/2*\arctan(b,c)),2^{(1/2)}*((b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)}))^{(1/2)})*(a+b*\cos(x)+c*\sin(x))^{(1/2)}/((a+b*\cos(x)+c*\sin(x))/(a+(b^2+c^2)^{(1/2)}))^{(1/2)})-2/3*(a^2-b^2-c^2)*e*(\cos(1/2*x-1/2*\arctan(b,c))^{(1/2)}/\cos(1/2*x-1/2*\arctan(b,c))*\text{EllipticF}(\sin(1/2*x-1/2*\arctan(b,c)),2^{(1/2)}*((b^2+c^2)^{(1/2)}/(a+(b^2+c^2)^{(1/2)}))^{(1/2)})*((a+b*\cos(x)+c*\sin(x))/(a+(b^2+c^2)^{(1/2)}))^{(1/2)})/(a+b*\cos(x)+c*\sin(x))^{(1/2)}$

Rubi [A] time = 0.33, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3146, 3149, 3119, 2653, 3127, 2661}

$$\frac{2e(a^2 - b^2 - c^2) \sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}(x - \tan^{-1}(b,c)) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\sqrt{a+b \cos(x)+c \sin(x)}} + \frac{2(ae+3d)\sqrt{a+b \cos(x)+c \sin(x)} E\left(\frac{1}{2}(x - \tan^{-1}(b,c)) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Cos[x] + c*Sin[x]]*(d + b*e*Cos[x] + c*e*Sin[x]),x]

[Out] $(2*(3*d + a*e)*\text{EllipticE}[(x - \text{ArcTan}[b, c])/2, (2*\text{Sqrt}[b^2 + c^2])/(a + \text{Sqrt}[b^2 + c^2])]*\text{Sqrt}[a + b*\text{Cos}[x] + c*\text{Sin}[x]])/(3*\text{Sqrt}[(a + b*\text{Cos}[x] + c*\text{Sin}[x])/(a + \text{Sqrt}[b^2 + c^2])]) - (2*(a^2 - b^2 - c^2)*e*\text{EllipticF}[(x - \text{ArcTan}[b, c])/2, (2*\text{Sqrt}[b^2 + c^2])/(a + \text{Sqrt}[b^2 + c^2])]*\text{Sqrt}[(a + b*\text{Cos}[x] + c*\text{Sin}[x])/(a + \text{Sqrt}[b^2 + c^2])])/(3*\text{Sqrt}[a + b*\text{Cos}[x] + c*\text{Sin}[x]]) - (2*\text{Sqrt}[a + b*\text{Cos}[x] + c*\text{Sin}[x]]*(c*e*\text{Cos}[x] - b*e*\text{Sin}[x]))/3$

Rule 2653

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3146

```
Int[(cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)])^n*(A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]), x_Symbol] :> Simp[((B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^n)/(a*e*(n + 1)), x] + Dist[1/(a*(n + 1)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x] + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)]) / Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\int \sqrt{a + b \cos(x) + c \sin(x)} (d + be \cos(x) + ce \sin(x)) dx = -\frac{2}{3} \sqrt{a + b \cos(x) + c \sin(x)} (ce \cos(x) - be \sin(x)) + \dots$$

$$= -\frac{2}{3} \sqrt{a + b \cos(x) + c \sin(x)} (ce \cos(x) - be \sin(x)) - \frac{1}{3} \dots$$

$$= -\frac{2}{3} \sqrt{a + b \cos(x) + c \sin(x)} (ce \cos(x) - be \sin(x)) + \dots$$

$$= \frac{2(3d + ae)E\left(\frac{1}{2}\left(x - \tan^{-1}\left(\frac{b}{c}\right)\right) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right) \sqrt{a + b \cos(x) + c \sin(x)}}{3 \sqrt{\frac{a + b \cos(x) + c \sin(x)}{a + \sqrt{b^2+c^2}}}}$$

Mathematica [C] time = 6.37, size = 3006, normalized size = 13.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[x] + c*Sin[x]]*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

```
[Out] Sqrt[a + b*Cos[x] + c*Sin[x]]*((2*b*(3*d + a*e))/(3*c) - (2*c*e*Cos[x])/3 + (2*b*e*Sin[x])/3) + (2*a*d*AppellF1[1/2, 1/2, 1/2, 3/2, -(a + Sqrt[1 + b^2 + c^2])/(a + Sqrt[b^2 + c^2]), (b + c*Sin[x])/(a + Sqrt[b^2 + c^2])], 1, 1, 1, 1))
```


$t[(b^2 + c^2)/b^2])) - ((2*b*(a + b*\text{Sqrt}[1 + c^2/b^2]*\text{Cos}[x - \text{ArcTan}[c/b]])/(b^2 + c^2) - (c*\text{Sin}[x - \text{ArcTan}[c/b]])/(b*\text{Sqrt}[1 + c^2/b^2]))/\text{Sqrt}[a + b*\text{Sqrt}[1 + c^2/b^2]*\text{Cos}[x - \text{ArcTan}[c/b]]]))/3$

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}((be \cos(x) + ce \sin(x) + d)\sqrt{b \cos(x) + c \sin(x) + a}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorithm="fricas")

[Out] integral((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (be \cos(x) + ce \sin(x) + d)\sqrt{b \cos(x) + c \sin(x) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorithm="giac")

[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a), x)

maple [B] time = 0.77, size = 1460, normalized size = 6.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x)

[Out] $(-(-b^2*\sin(x-\arctan(-b,c))-c^2*\sin(x-\arctan(-b,c))-a*(b^2+c^2)^{(1/2)})*\cos(x-\arctan(-b,c))^{2/(b^2+c^2)^{(1/2)}})^{(1/2)/(b^2+c^2)^{(1/2)}}*((b^2+c^2)^{(1/2)}*b^2*e+(b^2+c^2)^{(1/2)}*c^2*e)*(-2/3/(b^2+c^2)^{(1/2)}*(\cos(x-\arctan(-b,c)))^{2*(b^2+c^2)^{(1/2)}*\sin(x-\arctan(-b,c))+a})^{(1/2)}+2/3*(1/(b^2+c^2)^{(1/2)}*a-1)*((-b^2+c^2)^{(1/2)}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2)}))^{(1/2)}*((-\sin(x-\arctan(-b,c))+1)*(b^2+c^2)^{(1/2)/(a+(b^2+c^2)^{(1/2))})^{(1/2)}*((1+\sin(x-\arctan(-b,c))))*(b^2+c^2)^{(1/2)/(-a+(b^2+c^2)^{(1/2))})^{(1/2)/(\cos(x-\arctan(-b,c)))^{2*(b^2+c^2)^{(1/2)}*\sin(x-\arctan(-b,c))+a})^{(1/2)}*EllipticF(((b^2+c^2)^{(1/2)}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2))})^{(1/2)},((a-(b^2+c^2)^{(1/2)))/(a+(b^2+c^2)^{(1/2))})^{(1/2)}-4/3/(b^2+c^2)^{(1/2)}*a*(1/(b^2+c^2)^{(1/2)}*a-1)*((-b^2+c^2)^{(1/2)}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2))})^{(1/2)}*((-\sin(x-\arctan(-b,c))+1)*(b^2+c^2)^{(1/2)/(a+(b^2+c^2)^{(1/2))})^{(1/2)}*((1+\sin(x-\arctan(-b,c))))*(b^2+c^2)^{(1/2)/(-a+(b^2+c^2)^{(1/2))})^{(1/2)/(\cos(x-\arctan(-b,c)))^{2*(b^2+c^2)^{(1/2)}*\sin(x-\arctan(-b,c))+a})^{(1/2)}*((-1/(b^2+c^2)^{(1/2)}*a-1)*EllipticE(((b^2+c^2)^{(1/2)}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2))})^{(1/2)},((a-(b^2+c^2)^{(1/2)))/(a+(b^2+c^2)^{(1/2))})^{(1/2)}+EllipticF(((b^2+c^2)^{(1/2)}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2))})^{(1/2)},((a-(b^2+c^2)^{(1/2)))/(a+(b^2+c^2)^{(1/2))})^{(1/2)}+2*(a*b^2*e+a*c^2*e+b^2*d+c^2*d)*(1/(b^2+c^2)^{(1/2)}*a-1)*((-b^2+c^2)^{(1/2)}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2))})^{(1/2)}*((-\sin(x-\arctan(-b,c))+1)*(b^2+c^2)^{(1/2)/(a+(b^2+c^2)^{(1/2))})^{(1/2)}*((1+\sin(x-\arctan(-b,c))))*(b^2+c^2)^{(1/2)/(-a+(b^2+c^2)^{(1/2))})^{(1/2)/(\cos(x-\arctan(-b,c)))^{2*(b^2+c^2)^{(1/2)}*\sin(x-\arctan(-b,c))+a})^{(1/2)}*((-1/(b^2+c^2)^{(1/2)}*a-1)*EllipticE(((b^2+c^2)^{(1/2)}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2))})^{(1/2)},((a-(b^2+c^2)^{(1/2)))/(a+(b^2+c^2)^{(1/2))})^{(1/2)}+EllipticF(((b^2+c^2)^{(1/2)}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{(1/2))})^{(1/2)},((a-(b^2+c^2)^{(1/2)))/(a+(b^2+c^2)^{(1/2))})^{(1/2)}+2*d*a*(b^2+c^2)^{(1/2)}*(1/(b^2+c^2)^{(1/2)}*a-1)*((-b^2+c^2)^{(1/2)}*\sin(x-\arctan(-b,c))-a)/(-$

$$\frac{a+(b^2+c^2)^{1/2}}{(a+(b^2+c^2)^{1/2})^{1/2}} \cdot \frac{(-\sin(x-\arctan(-b,c))+1) \cdot (b^2+c^2)^{1/2}}{(a+(b^2+c^2)^{1/2})^{1/2}} \cdot \frac{(1+\sin(x-\arctan(-b,c))) \cdot (b^2+c^2)^{1/2}}{(-a+(b^2+c^2)^{1/2})^{1/2}} \cdot \frac{(-b^2 \sin(x-\arctan(-b,c))-c^2 \sin(x-\arctan(-b,c))-a \cdot (b^2+c^2)^{1/2}) \cdot \cos(x-\arctan(-b,c))^2}{(b^2+c^2)^{1/2}} \cdot \frac{1}{(b^2+c^2)^{1/2}} \cdot \text{EllipticF}\left(\frac{-(b^2+c^2)^{1/2} \sin(x-\arctan(-b,c))-a}{(-a+(b^2+c^2)^{1/2})^{1/2}}, \frac{(a-(b^2+c^2)^{1/2})}{(a+(b^2+c^2)^{1/2})^{1/2}}\right)}{\cos(x-\arctan(-b,c)) \cdot \frac{(b^2 \sin(x-\arctan(-b,c))+c^2 \sin(x-\arctan(-b,c))+a \cdot (b^2+c^2)^{1/2})}{(b^2+c^2)^{1/2}}}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (be \cos(x) + ce \sin(x) + d) \sqrt{b \cos(x) + c \sin(x) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorithm m="maxima")

[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + b \cos(x) + c \sin(x)} (d + b e \cos(x) + c e \sin(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(x) + c*sin(x))^(1/2)*(d + b*e*cos(x) + c*e*sin(x)),x)

[Out] int((a + b*cos(x) + c*sin(x))^(1/2)*(d + b*e*cos(x) + c*e*sin(x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \cos(x) + c \sin(x)} (be \cos(x) + ce \sin(x) + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)+c*sin(x))**(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x)

[Out] Integral(sqrt(a + b*cos(x) + c*sin(x))*(b*e*cos(x) + c*e*sin(x) + d), x)

$$3.559 \quad \int \frac{d+be \cos(x)+ce \sin(x)}{\sqrt{a+b \cos(x)+c \sin(x)}} dx$$

Optimal. Leaf size=180

$$\frac{2(d-ae)\sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}\left(x-\tan^{-1}\left(\frac{b}{c}\right)\right)\middle|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{a+b \cos(x)+c \sin(x)}} + \frac{2e\sqrt{a+b \cos(x)+c \sin(x)} E\left(\frac{1}{2}\left(x-\tan^{-1}\left(\frac{b}{c}\right)\right)\middle|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}}}$$

[Out] 2*e*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2))))^(1/2)*(a+b*cos(x)+c*sin(x))^(1/2)/((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)+2*(-a*e+d)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticF(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2))))^(1/2)*((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a+b*cos(x)+c*sin(x))^(1/2)

Rubi [A] time = 0.19, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3149, 3119, 2653, 3127, 2661}

$$\frac{2(d-ae)\sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}\left(x-\tan^{-1}\left(\frac{b}{c}\right)\right)\middle|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{a+b \cos(x)+c \sin(x)}} + \frac{2e\sqrt{a+b \cos(x)+c \sin(x)} E\left(\frac{1}{2}\left(x-\tan^{-1}\left(\frac{b}{c}\right)\right)\middle|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}}}$$

Antiderivative was successfully verified.

[In] Int[(d + b*e*Cos[x] + c*e*Sin[x])/Sqrt[a + b*Cos[x] + c*Sin[x]],x]

[Out] (2*e*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])] + (2*(d - a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])])/Sqrt[a + b*Cos[x] + c*Sin[x]]

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d/Sqrt[a + b], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] := Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_.)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_.)])/Sqrt[cos[(d_.) + (e_.)*(x_.)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_.)]], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\int \frac{d + be \cos(x) + ce \sin(x)}{\sqrt{a + b \cos(x) + c \sin(x)}} dx = e \int \sqrt{a + b \cos(x) + c \sin(x)} dx + (d - ae) \int \frac{1}{\sqrt{a + b \cos(x) + c \sin(x)}} dx$$

$$= \frac{(e\sqrt{a + b \cos(x) + c \sin(x)}) \int \sqrt{\frac{a}{a + \sqrt{b^2 + c^2}} + \frac{\sqrt{b^2 + c^2} \cos(x - \tan^{-1}(b, c))}{a + \sqrt{b^2 + c^2}}} dx}{\sqrt{\frac{a + b \cos(x) + c \sin(x)}{a + \sqrt{b^2 + c^2}}}} + \frac{(d - ae) \int \frac{1}{\sqrt{a + b \cos(x) + c \sin(x)}} dx}{\sqrt{\frac{a + b \cos(x) + c \sin(x)}{a + \sqrt{b^2 + c^2}}}}$$

$$= \frac{2eE\left(\frac{1}{2}(x - \tan^{-1}(b, c)) \middle| \frac{2\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}\right) \sqrt{a + b \cos(x) + c \sin(x)}}{\sqrt{\frac{a + b \cos(x) + c \sin(x)}{a + \sqrt{b^2 + c^2}}}} + \frac{2(d - ae)F\left(\frac{1}{2}(x - \tan^{-1}(b, c)) \middle| \frac{2\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}\right)}{\sqrt{\frac{a + b \cos(x) + c \sin(x)}{a + \sqrt{b^2 + c^2}}}}$$

Mathematica [C] time = 6.31, size = 1319, normalized size = 7.33

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + b*e*Cos[x] + c*e*Sin[x])/Sqrt[a + b*Cos[x] + c*Sin[x]], x]
[Out] (2*b*e*Sqrt[a + b*Cos[x] + c*Sin[x]])/c + (2*d*AppellF1[1/2, 1/2, 1/2, 3/2,
-((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a
/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]
])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))*Sec[x + ArcTan[b/
c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[
b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[
x + ArcTan[b/c]])*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*S
in[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]/(Sqrt[1 + b^2/c^2]*c)
+ (b^2*e*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*
Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))),
-((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1
- a/(b*Sqrt[1 + c^2/b^2]))))))*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqr
t[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])
/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - Ar
cTan[c/b]])*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x -
ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^
2/b^2]*Cos[x - ArcTan[c/b]])/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqr
```

t[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/c + c * e*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -(a + b*Sqrt[1 + c^2/b^2])*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -(a + b*Sqrt[1 + c^2/b^2])*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2])))))*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2])*Cos[x - ArcTan[c/b]]]/(a + b*Sqrt[(b^2 + c^2)/b^2]))*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2])*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2])*Cos[x - ArcTan[c/b]]]/(-a + b*Sqrt[(b^2 + c^2)/b^2])))) - ((2*b*(a + b*Sqrt[1 + c^2/b^2])*Cos[x - ArcTan[c/b]])/(b^2 + c^2) - (c*Ssin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2])*Cos[x - ArcTan[c/b]]])

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{be \cos(x) + ce \sin(x) + d}{\sqrt{b \cos(x) + c \sin(x) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x, algorithm="fricas")

[Out] integral((b*e*cos(x) + c*e*sin(x) + d)/sqrt(b*cos(x) + c*sin(x) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{be \cos(x) + ce \sin(x) + d}{\sqrt{b \cos(x) + c \sin(x) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x, algorithm="giac")

[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/sqrt(b*cos(x) + c*sin(x) + a), x)

maple [B] time = 0.65, size = 777, normalized size = 4.32

$$\sqrt{\frac{(-b^2 \sin(x - \arctan(-b, c)) - c^2 \sin(x - \arctan(-b, c)) - a \sqrt{b^2 + c^2}) (\cos^2(x - \arctan(-b, c)))}{\sqrt{b^2 + c^2}}} \left(\frac{2(b^2 e + c^2 e) \left(\frac{a}{\sqrt{b^2 + c^2}} - 1\right) \sqrt{\frac{-\sqrt{b^2 + c^2} \sin(x - \arctan(-b, c)) - a}{-a + \sqrt{b^2 + c^2}}}}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x)

[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(2*(b^2*e+c^2*e)*(1/(b^2+c^2)^(1/2)*a-1)*((-b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((-b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+2*d*(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)

$$c^2)^{(1/2)})^{(1/2)} * ((1 + \sin(x - \arctan(-b, c))) * (b^2 + c^2)^{(1/2)} / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)} / (-(-b^2 * \sin(x - \arctan(-b, c)) - c^2 * \sin(x - \arctan(-b, c)) - a * (b^2 + c^2)^{(1/2)})) * \cos(x - \arctan(-b, c))^{(1/2)} / (b^2 + c^2)^{(1/2)})^{(1/2)} * \text{EllipticF}(((- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)}, ((a - (b^2 + c^2)^{(1/2)}) / (a + (b^2 + c^2)^{(1/2)}))^{(1/2)})) / \cos(x - \arctan(-b, c)) / ((b^2 * \sin(x - \arctan(-b, c)) + c^2 * \sin(x - \arctan(-b, c)) + a * (b^2 + c^2)^{(1/2)}) / (b^2 + c^2)^{(1/2)})^{(1/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{be \cos(x) + ce \sin(x) + d}{\sqrt{b \cos(x) + c \sin(x) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x, algorithm="maxima")

[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/sqrt(b*cos(x) + c*sin(x) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d + be \cos(x) + ce \sin(x)}{\sqrt{a + b \cos(x) + c \sin(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(1/2),x)

[Out] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{be \cos(x) + ce \sin(x) + d}{\sqrt{a + b \cos(x) + c \sin(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))**(1/2),x)

[Out] Integral((b*e*cos(x) + c*e*sin(x) + d)/sqrt(a + b*cos(x) + c*sin(x)), x)

$$3.560 \quad \int \frac{d+be \cos(x)+ce \sin(x)}{(a+b \cos(x)+c \sin(x))^{3/2}} dx$$

Optimal. Leaf size=250

$$\frac{2(d-ae)\sqrt{a+b \cos(x)+c \sin(x)} E\left(\frac{1}{2}(x-\tan^{-1}(b,c)) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right) + 2e\sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}(x-\tan^{-1}(b,c)) \middle| \frac{2}{a+\sqrt{b^2+c^2}}\right)}{(a^2-b^2-c^2)\sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}} + \sqrt{a+b \cos(x)+c \sin(x)}}$$

[Out] $2*(c*(-a*e+d)*\cos(x)-b*(-a*e+d)*\sin(x))/(a^2-b^2-c^2)/(a+b*\cos(x)+c*\sin(x))^{(1/2)}+2*(-a*e+d)*(\cos(1/2*x-1/2*\arctan(b,c))^{(1/2)})^{(1/2)}/\cos(1/2*x-1/2*\arctan(b,c))*\text{EllipticE}(\sin(1/2*x-1/2*\arctan(b,c)),2^{(1/2)}*((b^2+c^2)^{(1/2)})/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}*(a+b*\cos(x)+c*\sin(x))^{(1/2)}/(a^2-b^2-c^2)/((a+b*\cos(x)+c*\sin(x))/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}+2*e*(\cos(1/2*x-1/2*\arctan(b,c))^{(1/2)})^{(1/2)}/\cos(1/2*x-1/2*\arctan(b,c))*\text{EllipticF}(\sin(1/2*x-1/2*\arctan(b,c)),2^{(1/2)}*((b^2+c^2)^{(1/2)})/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}*(a+b*\cos(x)+c*\sin(x))/(a+(b^2+c^2)^{(1/2)}))^{(1/2)}/(a+b*\cos(x)+c*\sin(x))^{(1/2)}$

Rubi [A] time = 0.32, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, number of rules / integrand size = 0.222, Rules used = {3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2(d-ae)\sqrt{a+b \cos(x)+c \sin(x)} E\left(\frac{1}{2}(x-\tan^{-1}(b,c)) \middle| \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right) + 2e\sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}(x-\tan^{-1}(b,c)) \middle| \frac{2}{a+\sqrt{b^2+c^2}}\right)}{(a^2-b^2-c^2)\sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}} + \sqrt{a+b \cos(x)+c \sin(x)}}$$

Antiderivative was successfully verified.

[In] Int[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(3/2), x]

[Out] $(2*(d-a*e)*\text{EllipticE}[(x-\text{ArcTan}[b,c])/2,(2*\text{Sqrt}[b^2+c^2])/(a+\text{Sqrt}[b^2+c^2])]*\text{Sqrt}[a+b*\text{Cos}[x]+c*\text{Sin}[x]])/((a^2-b^2-c^2)*\text{Sqrt}[(a+b*\text{Cos}[x]+c*\text{Sin}[x])/(a+\text{Sqrt}[b^2+c^2])])+(2*e*\text{EllipticF}[(x-\text{ArcTan}[b,c])/2,(2*\text{Sqrt}[b^2+c^2])/(a+\text{Sqrt}[b^2+c^2])]*\text{Sqrt}[(a+b*\text{Cos}[x]+c*\text{Sin}[x])/(a+\text{Sqrt}[b^2+c^2])])/\text{Sqrt}[a+b*\text{Cos}[x]+c*\text{Sin}[x]]+(2*(c*(d-a*e)*\text{Cos}[x]-b*(d-a*e)*\text{Sin}[x]))/((a^2-b^2-c^2)*\text{Sqrt}[a+b*\text{Cos}[x]+c*\text{Sin}[x]])$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3119

Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]])/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c])]/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]) / Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])], x_Symbol] := -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{d + be \cos(x) + ce \sin(x)}{(a + b \cos(x) + c \sin(x))^{3/2}} dx = \frac{2(c(d - ae) \cos(x) - b(d - ae) \sin(x))}{(a^2 - b^2 - c^2) \sqrt{a + b \cos(x) + c \sin(x)}} - \frac{2 \int \frac{\frac{1}{2}(-ad + (b^2 + c^2)e) - \frac{1}{2}b(d - ae) \cos(x) - \frac{1}{2}c(d - ae) \sin(x)}{\sqrt{a + b \cos(x) + c \sin(x)}} dx}{a^2 - b^2 - c^2}$$

$$= \frac{2(c(d - ae) \cos(x) - b(d - ae) \sin(x))}{(a^2 - b^2 - c^2) \sqrt{a + b \cos(x) + c \sin(x)}} + e \int \frac{1}{\sqrt{a + b \cos(x) + c \sin(x)}} dx$$

$$= \frac{2(c(d - ae) \cos(x) - b(d - ae) \sin(x))}{(a^2 - b^2 - c^2) \sqrt{a + b \cos(x) + c \sin(x)}} + \frac{((d - ae) \sqrt{a + b \cos(x) + c \sin(x)})}{(a^2 - b^2 - c^2)}$$

$$= \frac{2(d - ae) E\left(\frac{1}{2}(x - \tan^{-1}(b, c)) \middle| \frac{2\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}\right) \sqrt{a + b \cos(x) + c \sin(x)}}{(a^2 - b^2 - c^2) \sqrt{\frac{a + b \cos(x) + c \sin(x)}{a + \sqrt{b^2 + c^2}}}} + \frac{2eF\left(\frac{1}{2}(x - \tan^{-1}(b, c)) \middle| \frac{2\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}\right)}{a + \sqrt{b^2 + c^2}}$$

Mathematica [C] time = 6.57, size = 3176, normalized size = 12.70

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(3/2), x]
```


*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))) * Sin[x - ArcTan[c/b]]/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b^2 + c^2) - (c*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(-a^2 + b^2 + c^2)

fricas [F] time = 2.94, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(be \cos(x) + ce \sin(x) + d)\sqrt{b \cos(x) + c \sin(x) + a}}{2ab \cos(x) + (b^2 - c^2) \cos(x)^2 + a^2 + c^2 + 2(bc \cos(x) + ac) \sin(x)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x, algorithm="fricas")

[Out] integral((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a)/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{be \cos(x) + ce \sin(x) + d}{(b \cos(x) + c \sin(x) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x, algorithm="giac")

[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(3/2), x)

maple [B] time = 1.07, size = 2596, normalized size = 10.38

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x)

[Out]
$$\begin{aligned} & (-(-b^2 \sin(x - \arctan(-b, c)) - c^2 \sin(x - \arctan(-b, c)) - a(b^2 + c^2)^{1/2}) \cos(x - \arctan(-b, c))^2 / (b^2 + c^2)^{1/2})^{1/2} / (b^2 + c^2)^{1/2} * (2(b^2 + c^2)^{1/2} * e * (1/(b^2 + c^2)^{1/2} * a - 1) * ((-b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2} * ((-\sin(x - \arctan(-b, c)) + 1) * (b^2 + c^2)^{1/2} / (a + (b^2 + c^2)^{1/2}))^{1/2} * ((1 + \sin(x - \arctan(-b, c))) * (b^2 + c^2)^{1/2} / (-a + (b^2 + c^2)^{1/2}))^{1/2} / (-(-b^2 \sin(x - \arctan(-b, c)) - c^2 \sin(x - \arctan(-b, c)) - a(b^2 + c^2)^{1/2}) \cos(x - \arctan(-b, c))^2 / (b^2 + c^2)^{1/2})^{1/2} * \text{EllipticF}(((b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2}, ((a - (b^2 + c^2)^{1/2}) / (a + (b^2 + c^2)^{1/2}))^{1/2}) + (b^2 + c^2)^{1/2} * (-b^2 - c^2) \cos(x - \arctan(-b, c))^{1/2} / (a^2 - b^2 - c^2) * (a * e - d) / (-(-b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) - a) * \cos(x - \arctan(-b, c))^2 * (b^2 + c^2)^{1/2} - a * (b^2 + c^2) * (a * e - d) / (a^2 - b^2 - c^2) * (1/(b^2 + c^2)^{1/2} * a - 1) * ((-b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2} * ((-\sin(x - \arctan(-b, c)) + 1) * (b^2 + c^2)^{1/2} / (a + (b^2 + c^2)^{1/2}))^{1/2} * ((1 + \sin(x - \arctan(-b, c))) * (b^2 + c^2)^{1/2} / (-a + (b^2 + c^2)^{1/2}))^{1/2} / (-(-b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) - a) * \cos(x - \arctan(-b, c))^2 * (b^2 + c^2)^{1/2} * \text{EllipticF}(((b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}))^{1/2}, ((a - (b^2 + c^2)^{1/2}) / (a + (b^2 + c^2)^{1/2}))^{1/2}) - 2 * (-1/2 * (b^2 + c^2)^{1/2} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{1/2}) \end{aligned}$$

$$2)^{(3/2)} * (a * e - d) / (a^2 - b^2 - c^2) + 1/2 * (b^2 + c^2)^{(1/2)} * (2 * b^2 + 2 * c^2) / (a^2 - b^2 - c^2) * (a * e - d) * (1 / (b^2 + c^2)^{(1/2)} * a - 1) * ((- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)} * ((-\sin(x - \arctan(-b, c)) + 1) * (b^2 + c^2)^{(1/2)} / (a + (b^2 + c^2)^{(1/2)}))^{(1/2)} * ((1 + \sin(x - \arctan(-b, c))) * (b^2 + c^2)^{(1/2)} / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)} / (- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) * \cos(x - \arctan(-b, c))^{(1/2)} * ((-1 / (b^2 + c^2)^{(1/2)} * a - 1) * \text{EllipticE}(((- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)}, ((a - (b^2 + c^2)^{(1/2)}) / (a + (b^2 + c^2)^{(1/2)}))^{(1/2)}) + \text{EllipticF}(((- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)}, ((a - (b^2 + c^2)^{(1/2)}) / (a + (b^2 + c^2)^{(1/2)}))^{(1/2)}))^{(1/2)} - 1/2 * (a * b^2 * e + a * c^2 * e - b^2 * d - c^2 * d) * (1 / (b^2 + c^2)^{(1/2)} * a - 1) * ((- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)} * ((-\sin(x - \arctan(-b, c)) + 1) * (b^2 + c^2)^{(1/2)} / (a + (b^2 + c^2)^{(1/2)}))^{(1/2)} * ((1 + \sin(x - \arctan(-b, c))) * (b^2 + c^2)^{(1/2)} / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)} / (- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) * \cos(x - \arctan(-b, c))^{(1/2)} * (b^2 + c^2)^{(1/2)} / a * \text{EllipticPi}(((- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)}, -1/2 * (-1 / (b^2 + c^2)^{(1/2)} * a + 1) * (b^2 + c^2)^{(1/2)} / a, ((a - (b^2 + c^2)^{(1/2)}) / (a + (b^2 + c^2)^{(1/2)}))^{(1/2)}) - (b^2 + c^2) * \cos(x - \arctan(-b, c))^{(1/2)} * (a * e - d) / (a^2 - b^2 - c^2) / (- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) * \cos(x - \arctan(-b, c))^{(1/2)} - a * (b^2 + c^2)^{(1/2)} * (a * e - d) / (a^2 - b^2 - c^2) * (1 / (b^2 + c^2)^{(1/2)} * a - 1) * ((- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)} * ((-\sin(x - \arctan(-b, c)) + 1) * (b^2 + c^2)^{(1/2)} / (a + (b^2 + c^2)^{(1/2)}))^{(1/2)} * ((1 + \sin(x - \arctan(-b, c))) * (b^2 + c^2)^{(1/2)} / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)} / (- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) * \cos(x - \arctan(-b, c))^{(1/2)} * \text{EllipticF}(((- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)}, ((a - (b^2 + c^2)^{(1/2)}) / (a + (b^2 + c^2)^{(1/2)}))^{(1/2)}) - (b^2 + c^2) * (a * e - d) / (a^2 - b^2 - c^2) * (1 / (b^2 + c^2)^{(1/2)} * a - 1) * ((- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)} * ((-\sin(x - \arctan(-b, c)) + 1) * (b^2 + c^2)^{(1/2)} / (a + (b^2 + c^2)^{(1/2)}))^{(1/2)} * ((1 + \sin(x - \arctan(-b, c))) * (b^2 + c^2)^{(1/2)} / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)} / (- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) * \cos(x - \arctan(-b, c))^{(1/2)} * ((-1 / (b^2 + c^2)^{(1/2)} * a - 1) * \text{EllipticE}(((- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)}, ((a - (b^2 + c^2)^{(1/2)}) / (a + (b^2 + c^2)^{(1/2)}))^{(1/2)}) + \text{EllipticF}(((- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)}, ((a - (b^2 + c^2)^{(1/2)}) / (a + (b^2 + c^2)^{(1/2)}))^{(1/2)}) + (1/2 * a * e - 1/2 * d) * (1 / (b^2 + c^2)^{(1/2)} * a - 1) * ((- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)} * ((-\sin(x - \arctan(-b, c)) + 1) * (b^2 + c^2)^{(1/2)} / (a + (b^2 + c^2)^{(1/2)}))^{(1/2)} * ((1 + \sin(x - \arctan(-b, c))) * (b^2 + c^2)^{(1/2)} / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)} / (- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) * \cos(x - \arctan(-b, c))^{(1/2)} * (b^2 + c^2)^{(1/2)} / a * \text{EllipticPi}(((- (b^2 + c^2)^{(1/2)} * \sin(x - \arctan(-b, c)) - a) / (-a + (b^2 + c^2)^{(1/2)}))^{(1/2)}, -1/2 * (-1 / (b^2 + c^2)^{(1/2)} * a + 1) * (b^2 + c^2)^{(1/2)} / a, ((a - (b^2 + c^2)^{(1/2)}) / (a + (b^2 + c^2)^{(1/2)}))^{(1/2)}) / \cos(x - \arctan(-b, c)) / ((b^2 * \sin(x - \arctan(-b, c)) + c^2 * \sin(x - \arctan(-b, c)) + a * (b^2 + c^2)^{(1/2)}) / (b^2 + c^2)^{(1/2)})^{(1/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{be \cos(x) + ce \sin(x) + d}{(b \cos(x) + c \sin(x) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x, algorithm="maxima")

[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d + b e \cos(x) + c e \sin(x)}{(a + b \cos(x) + c \sin(x))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(3/2),x)
```

```
[Out] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(3/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))**(3/2),x)
```

```
[Out] Timed out
```

$$3.561 \quad \int \frac{d+be \cos(x)+ce \sin(x)}{(a+b \cos(x)+c \sin(x))^{5/2}} dx$$

Optimal. Leaf size=378

$$\frac{2(d-ae)\sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}\left(x-\tan^{-1}\left(\frac{b}{c}\right)\right)\middle|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3(a^2-b^2-c^2)\sqrt{a+b \cos(x)+c \sin(x)}} + \frac{2(a^2(-e)+4ad-3e(b^2+c^2))\sqrt{a+b \cos(x)+c \sin(x)}}{3(a^2-b^2-c^2)^2\sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}}}$$

```
[Out] 2/3*(c*(-a*e+d)*cos(x)-b*(-a*e+d)*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))^(3/2)+2/3*(c*(4*a*d-a^2*e-3*(b^2+c^2)*e)*cos(x)-b*(4*a*d-a^2*e-3*(b^2+c^2)*e)*sin(x))/(a^2-b^2-c^2)^2/(a+b*cos(x)+c*sin(x))^(1/2)+2/3*(4*a*d-a^2*e-3*(b^2+c^2)*e)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2))))^(1/2)*(a+b*cos(x)+c*sin(x))^(1/2)/(a^2-b^2-c^2)^2/((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)-2/3*(-a*e+d)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticF(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2))))^(1/2)*((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3156, 3149, 3119, 2653, 3127, 2661}

$$\frac{2(d-ae)\sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}} F\left(\frac{1}{2}\left(x-\tan^{-1}\left(\frac{b}{c}\right)\right)\middle|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3(a^2-b^2-c^2)\sqrt{a+b \cos(x)+c \sin(x)}} + \frac{2(a^2(-e)+4ad-3e(b^2+c^2))\sqrt{a+b \cos(x)+c \sin(x)}}{3(a^2-b^2-c^2)^2\sqrt{\frac{a+b \cos(x)+c \sin(x)}{a+\sqrt{b^2+c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + b*e*cos[x] + c*e*sin[x])/(a + b*cos[x] + c*sin[x])^(5/2), x]
```

```
[Out] (2*(4*a*d - a^2*e - 3*(b^2 + c^2)*e)*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*cos[x] + c*sin[x]])/(3*(a^2 - b^2 - c^2)^2*Sqrt[(a + b*cos[x] + c*sin[x])/(a + Sqrt[b^2 + c^2])]) - (2*(d - a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*cos[x] + c*sin[x])/(a + Sqrt[b^2 + c^2])])/(3*(a^2 - b^2 - c^2)*Sqrt[a + b*cos[x] + c*sin[x]]) + (2*(c*(d - a*e)*cos[x] - b*(d - a*e)*sin[x]))/(3*(a^2 - b^2 - c^2)*(a + b*cos[x] + c*sin[x])^(3/2)) + (2*(c*(4*a*d - a^2*e - 3*(b^2 + c^2)*e)*cos[x] - b*(4*a*d - a^2*e - 3*(b^2 + c^2)*e)*sin[x]))/(3*(a^2 - b^2 - c^2)^2*Sqrt[a + b*cos[x] + c*sin[x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])]/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3119

```
Int[Sqrt[cos[(d_) + (e_)*(x_)]*(b_) + (a_) + (c_)*sin[(d_) + (e_)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]]/Sqrt[(a +
```



```
b*cos[d + e*x] + c*sin[d + e*x])/(a + Sqrt[b^2 + c^2]), Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c])]/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3127

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*cos[d + e*x] + c*sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c])]/(a + Sqrt[b^2 + c^2])], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3149

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*cos[d + e*x] + c*sin[d + e*x]], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*b - a*B, 0]
```

Rule 3156

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])], x_Symbol] := -Simp[((c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*cos[d + e*x] + c*sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*cos[d + e*x] + c*sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{d + be \cos(x) + ce \sin(x)}{(a + b \cos(x) + c \sin(x))^{5/2}} dx &= \frac{2(c(d - ae) \cos(x) - b(d - ae) \sin(x))}{3(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^{3/2}} - \frac{2 \int \frac{-\frac{3}{2}(ad - (b^2 + c^2)e) + \frac{1}{2}b(d - ae) \cos(x) + \frac{1}{2}c(d - ae) \sin(x)}{(a + b \cos(x) + c \sin(x))^3} dx}{3(a^2 - b^2 - c^2)} \\
&= \frac{2(c(d - ae) \cos(x) - b(d - ae) \sin(x))}{3(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^{3/2}} + \frac{2(c(4ad - a^2e - 3(b^2 + c^2)e))}{3(a^2 - b^2 - c^2)} \\
&= \frac{2(c(d - ae) \cos(x) - b(d - ae) \sin(x))}{3(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^{3/2}} + \frac{2(c(4ad - a^2e - 3(b^2 + c^2)e))}{3(a^2 - b^2 - c^2)} \\
&= \frac{2(c(d - ae) \cos(x) - b(d - ae) \sin(x))}{3(a^2 - b^2 - c^2)(a + b \cos(x) + c \sin(x))^{3/2}} + \frac{2(c(4ad - a^2e - 3(b^2 + c^2)e))}{3(a^2 - b^2 - c^2)} \\
&= \frac{2(4ad - a^2e - 3(b^2 + c^2)e) E\left(\frac{1}{2}\left(x - \tan^{-1}\left(\frac{c}{b}\right)\right) \middle| \frac{2\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}\right) \sqrt{a + b \cos(x) + c \sin(x)}}{3(a^2 - b^2 - c^2)^2 \sqrt{\frac{a + b \cos(x) + c \sin(x)}{a + \sqrt{b^2 + c^2}}}}
\end{aligned}$$

Mathematica [C] time = 6.95, size = 5554, normalized size = 14.69

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(5/2), x]

[Out] Result too large to show

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(be \cos(x) + ce \sin(x) + d)\sqrt{b \cos(x) + c \sin(x) + a}}{(b^3 - 3bc^2) \cos(x)^3 + a^3 + 3ac^2 + 3(ab^2 - ac^2) \cos(x)^2 + 3(a^2b + bc^2) \cos(x) + (6abc \cos(x) + 3a^2c - 3b^2c^2) \sin(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2), x, algorithm="fricas")

[Out] integral((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a)/((b^3 - 3*b*c^2)*cos(x)^3 + a^3 + 3*a*c^2 + 3*(a*b^2 - a*c^2)*cos(x)^2 + 3*(a^2*b + b*c^2)*cos(x) + (6*a*b*c*cos(x) + 3*a^2*c + c^3 + (3*b^2*c - c^3)*cos(x))^2*sin(x)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{be \cos(x) + ce \sin(x) + d}{(b \cos(x) + c \sin(x) + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2), x, algorithm="giac")

[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(5/2), x)

maple [B] time = 3.89, size = 3164, normalized size = 8.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int ((d+b*e*\cos(x)+c*e*\sin(x))/(a+b*\cos(x)+c*\sin(x))^{5/2}, x)$

[Out]
$$\begin{aligned} & (-(-b^2*\sin(x-\arctan(-b,c))-c^2*\sin(x-\arctan(-b,c))-a*(b^2+c^2)^{1/2})*\cos(x-\arctan(-b,c))^{2/(b^2+c^2)^{1/2}})^{1/2}/(b^2+c^2)^{1/2}*(-1/4*(a*b^2*e+a*c^2*e-b^2*d-c^2*d)/a/(a^2-b^2-c^2)*(\cos(x-\arctan(-b,c))^{2/(b^2+c^2)^{1/2}}*((b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))+a))^{1/2}/(b^2*\sin(x-\arctan(-b,c))+c^2*\sin(x-\arctan(-b,c))-a*(b^2+c^2)^{1/2}))-1/3*(a*e-d)/(a^2-b^2-c^2)/(b^2+c^2)^{1/2}*(\cos(x-\arctan(-b,c))^{2/(b^2+c^2)^{1/2}}*((b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))+a))^{1/2}/(\sin(x-\arctan(-b,c))+1/(b^2+c^2)^{1/2})*a^2+1/3*(b^2+c^2)^{1/2}*(-b^2-c^2)*\cos(x-\arctan(-b,c))^{2/(a^2-b^2-c^2)^2}*(a^2*e+3*b^2*e+3*c^2*e-4*a*d)/(\cos(x-\arctan(-b,c))^{2/(b^2+c^2)^{1/2}}*((b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))+a))^{1/2}+2*(7/24*(a*b^2*e+a*c^2*e-b^2*d-c^2*d)/(a^2-b^2-c^2)-1/6*a*(b^2+c^2)*(a^2*e+3*b^2*e+3*c^2*e-4*a*d)/(a^2-b^2-c^2)^2)*(1/(b^2+c^2)^{1/2})*a-1*((-b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{1/2}))^{1/2}*((-\sin(x-\arctan(-b,c))+1)*(b^2+c^2)^{1/2}/(a+(b^2+c^2)^{1/2}))^{1/2}*((1+\sin(x-\arctan(-b,c)))*(b^2+c^2)^{1/2}/(-a+(b^2+c^2)^{1/2}))^{1/2}/(\cos(x-\arctan(-b,c))^{2/(b^2+c^2)^{1/2}}*((b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))+a))^{1/2})*\text{EllipticF}(((b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{1/2}))^{1/2}, ((a-(b^2+c^2)^{1/2})/(a+(b^2+c^2)^{1/2}))^{1/2}))+2*(-1/8*(b^2+c^2)^{1/2}*(a*b^2*e+a*c^2*e-b^2*d-c^2*d)/a/(a^2-b^2-c^2)+1/6*(b^2+c^2)^{3/2}*(a^2*e+3*b^2*e+3*c^2*e-4*a*d)/(a^2-b^2-c^2)^2-1/6*(b^2+c^2)^{1/2}*(2*b^2+2*c^2)/(a^2-b^2-c^2)^2*(a^2*e+3*b^2*e+3*c^2*e-4*a*d))*(1/(b^2+c^2)^{1/2})*a-1*((-b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{1/2}))^{1/2}*((-\sin(x-\arctan(-b,c))+1)*(b^2+c^2)^{1/2}/(a+(b^2+c^2)^{1/2}))^{1/2}*((1+\sin(x-\arctan(-b,c)))*(b^2+c^2)^{1/2}/(-a+(b^2+c^2)^{1/2}))^{1/2}/(\cos(x-\arctan(-b,c))^{2/(b^2+c^2)^{1/2}}*((b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))+a))^{1/2})*(-1/(b^2+c^2)^{1/2})*a-1)*\text{EllipticE}(((b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{1/2}))^{1/2}, ((a-(b^2+c^2)^{1/2})/(a+(b^2+c^2)^{1/2}))^{1/2}))+\text{EllipticF}(((b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{1/2}))^{1/2}, ((a-(b^2+c^2)^{1/2})/(a+(b^2+c^2)^{1/2}))^{1/2}))-1/8*(a^3*b^2*e+a^3*c^2*e+3*a*b^4*e+6*a*b^2*c^2*e+3*a*c^4*e-5*a^2*b^2*d-5*a^2*c^2*d+b^4*d+2*b^2*c^2*d+c^4*d)/a^2/(a^2-b^2-c^2)*(1/(b^2+c^2)^{1/2})*a-1*((-b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{1/2}))^{1/2}*((-\sin(x-\arctan(-b,c))+1)*(b^2+c^2)^{1/2}/(a+(b^2+c^2)^{1/2}))^{1/2}*((1+\sin(x-\arctan(-b,c)))*(b^2+c^2)^{1/2}/(-a+(b^2+c^2)^{1/2}))^{1/2}/(\cos(x-\arctan(-b,c))^{2/(b^2+c^2)^{1/2}}*((b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))+a))^{1/2})*\text{EllipticPi}(((b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{1/2}))^{1/2}, -1/2*(-1/(b^2+c^2)^{1/2})*a+1*(b^2+c^2)^{1/2}/a, ((a-(b^2+c^2)^{1/2})/(a+(b^2+c^2)^{1/2}))^{1/2}))+1/4*(b^2+c^2)^{3/2}*(a*e-d)/a/(a^2-b^2-c^2)*(\cos(x-\arctan(-b,c))^{2/(b^2+c^2)^{1/2}}*((b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))+a))^{1/2}/(b^2*\sin(x-\arctan(-b,c))+c^2*\sin(x-\arctan(-b,c))-a*(b^2+c^2)^{1/2}))-1/3*(a*e-d)/(a^2-b^2-c^2)*(\cos(x-\arctan(-b,c))^{2/(b^2+c^2)^{1/2}}*((b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))+a))^{1/2}/(\sin(x-\arctan(-b,c))+1/(b^2+c^2)^{1/2})*a^2-1/3*(b^2+c^2)*\cos(x-\arctan(-b,c))^{2/(a^2-b^2-c^2)^2}*(a^2*e+3*b^2*e+3*c^2*e-4*a*d)/(\cos(x-\arctan(-b,c))^{2/(b^2+c^2)^{1/2}}*((b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))+a))^{1/2}+2*(1/24*(b^2+c^2)^{1/2}*(a*e-d)/(a^2-b^2-c^2)-1/6*a*(b^2+c^2)^{1/2}*(a^2*e+3*b^2*e+3*c^2*e-4*a*d)/(a^2-b^2-c^2)^2)*(1/(b^2+c^2)^{1/2})*a-1*((-b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{1/2}))^{1/2}*((-\sin(x-\arctan(-b,c))+1)*(b^2+c^2)^{1/2}/(a+(b^2+c^2)^{1/2}))^{1/2}/(\cos(x-\arctan(-b,c))^{2/(b^2+c^2)^{1/2}}*((b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))+a))^{1/2})*\text{EllipticF}(((b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{1/2}))^{1/2}, ((a-(b^2+c^2)^{1/2})/(a+(b^2+c^2)^{1/2}))^{1/2}))+2*(1/8*(a*b^2*e+a*c^2*e-b^2*d-c^2*d)/a/(a^2-b^2-c^2)-1/6*(b^2+c^2)*(a^2*e+3*b^2*e+3*c^2*e-4*a*d)/(a^2-b^2-c^2)^2)*(1/(b^2+c^2)^{1/2})*a-1*((-b^2+c^2)^{1/2}*\sin(x-\arctan(-b,c))-a)/(-a+(b^2+c^2)^{1/2}))^{1/2}*((-\sin(x-$$

```

arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((1+sin(x-arctan(-b,c)))*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a-1)*EllipticE(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+EllipticF(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))+1/8*(a^3*b^2*e+a^3*c^2*e+3*a*b^4*e+6*a*b^2*c^2*e+3*a*c^4*e-5*a^2*b^2*d-5*a^2*c^2*d+b^4*d+2*b^2*c^2*d+c^4*d)/a^2/(a^2-b^2-c^2)/(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a-1)*((-b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*EllipticPi(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))-a)/(-a+(b^2+c^2)^(1/2)))^(1/2),-1/2*(-1/(b^2+c^2)^(1/2)*a+1)*(b^2+c^2)^(1/2)/a,((a-(b^2+c^2)^(1/2))/(a+(b^2+c^2)^(1/2)))^(1/2))/cos(x-arctan(-b,c))/((b^2*sin(x-arctan(-b,c))+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2)))^(1/2)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{be \cos(x) + ce \sin(x) + d}{(b \cos(x) + c \sin(x) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x, algorithm="maxima")

```

```

[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(5/2), x)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d + b e \cos(x) + c e \sin(x)}{(a + b \cos(x) + c \sin(x))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(5/2),x)

```

```

[Out] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(5/2), x)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))**(5/2),x)

```

```

[Out] Timed out

```

$$3.562 \quad \int \frac{A+B \cos(d+ex)+C \sin(d+ex)}{a+c \sin(d+ex)} dx$$

Optimal. Leaf size=84

$$\frac{2(Ac - aC) \tan^{-1} \left(\frac{a \tan\left(\frac{1}{2}(d+ex)\right) + c}{\sqrt{a^2 - c^2}} \right)}{ce\sqrt{a^2 - c^2}} + \frac{B \log(a + c \sin(d + ex))}{ce} + \frac{Cx}{c}$$

[Out] $C*x/c+B*\ln(a+c*\sin(e*x+d))/c/e+2*(A*c-C*a)*\arctan((c+a*\tan(1/2*e*x+1/2*d))/(a^2-c^2)^(1/2))/c/e/(a^2-c^2)^(1/2)$

Rubi [A] time = 0.15, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4376, 2735, 2660, 618, 204, 2668, 31}

$$\frac{2(Ac - aC) \tan^{-1} \left(\frac{a \tan\left(\frac{1}{2}(d+ex)\right) + c}{\sqrt{a^2 - c^2}} \right)}{ce\sqrt{a^2 - c^2}} + \frac{B \log(a + c \sin(d + ex))}{ce} + \frac{Cx}{c}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Cos}[d + e*x] + C*\text{Sin}[d + e*x])/(a + c*\text{Sin}[d + e*x]),x]$

[Out] $(C*x)/c + (2*(A*c - a*C)*\text{ArcTan}[(c + a*\text{Tan}[(d + e*x)/2])/ \text{Sqrt}[a^2 - c^2]])/(c*\text{Sqrt}[a^2 - c^2]*e) + (B*\text{Log}[a + c*\text{Sin}[d + e*x]])/(c*e)$

Rule 31

$\text{Int}[(a_ + (b_)*(x_))^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 204

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a_ + (b_)*(x_ + (c_)*(x_)^2))^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 2660

$\text{Int}[(a_ + (b_)*\sin[(c_ + (d_)*(x_))])^{-1}, x_Symbol] \rightarrow \text{With}[\{e = \text{FreeFactors}[\text{Tan}[(c + d*x)/2], x]\}, \text{Dist}[(2*e)/d, \text{Subst}[\text{Int}[1/(a + 2*b*e*x + a*e^2*x^2), x], x, \text{Tan}[(c + d*x)/2]/e], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 2668

$\text{Int}[\cos[(e_ + (f_)*(x_))]^{(p_)}*((a_ + (b_)*\sin[(e_ + (f_)*(x_))])^{(m_)}), x_Symbol] \rightarrow \text{Dist}[1/(b^p*f), \text{Subst}[\text{Int}[(a + x)^m*(b^2 - x^2)^{(p-1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}[\{a, b, e, f, m\}, x] \ \&\& \ \text{IntegerQ}[(p-1)/2] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(d + ex) + C \sin(d + ex)}{a + c \sin(d + ex)} dx &= B \int \frac{\cos(d + ex)}{a + c \sin(d + ex)} dx + \int \frac{A + C \sin(d + ex)}{a + c \sin(d + ex)} dx \\ &= \frac{Cx}{c} - \frac{(-Ac + aC) \int \frac{1}{a + c \sin(d + ex)} dx}{c} + \frac{B \operatorname{Subst}\left(\int \frac{1}{a + x} dx, x, c \sin(d + ex)\right)}{ce} \\ &= \frac{Cx}{c} + \frac{B \log(a + c \sin(d + ex))}{ce} + \frac{(2(Ac - aC)) \operatorname{Subst}\left(\int \frac{1}{a + 2cx + ax^2} dx, x, c \sin(d + ex)\right)}{ce} \\ &= \frac{Cx}{c} + \frac{B \log(a + c \sin(d + ex))}{ce} - \frac{(4(Ac - aC)) \operatorname{Subst}\left(\int \frac{1}{-4(a^2 - c^2) - x^2} dx, x, c \sin(d + ex)\right)}{ce} \\ &= \frac{Cx}{c} + \frac{2(Ac - aC) \tan^{-1}\left(\frac{c + a \tan\left(\frac{1}{2}(d + ex)\right)}{\sqrt{a^2 - c^2}}\right)}{c\sqrt{a^2 - c^2}e} + \frac{B \log(a + c \sin(d + ex))}{ce} \end{aligned}$$

Mathematica [A] time = 0.25, size = 80, normalized size = 0.95

$$\frac{2(Ac - aC) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(d + ex)\right) + c}{\sqrt{a^2 - c^2}}\right)}{\sqrt{a^2 - c^2}} + \frac{B \log(a + c \sin(d + ex)) + C(d + ex)}{ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x]),x]
```

```
[Out] (C*(d + e*x) + (2*(A*c - a*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]])/Sqrt[a^2 - c^2] + B*Log[a + c*Sin[d + e*x]]/(c*e)
```

fricas [A] time = 1.00, size = 346, normalized size = 4.12

$$\left[\frac{2(Ca^2 - Cc^2)ex + (Ca - Ac)\sqrt{-a^2 + c^2} \log\left(\frac{(2a^2 - c^2)\cos(ex+d)^2 - 2ac\sin(ex+d) - a^2 - c^2 + 2(a\cos(ex+d)\sin(ex+d) + c\cos(ex+d))\sqrt{-a^2 + c^2}}{c^2\cos(ex+d)^2 - 2ac\sin(ex+d) - a^2 - c^2}\right)}{2(a^2c - c^3)e} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(C*a^2 - C*c^2)*e*x + (C*a - A*c)*sqrt(-a^2 + c^2)*log(((2*a^2 - c^2)*cos(e*x + d)^2 - 2*a*c*sin(e*x + d) - a^2 - c^2 + 2*(a*cos(e*x + d)*sin(e*x + d)*sqrt(-a^2 + c^2)))/(c^2*cos(e*x + d)^2 - 2*a*c*sin(e*x + d) - a^2 - c^2)) + B*log(a + c*sin(d + e*x)) + C*(d + e*x)]/c
```

$$e^x + d) + c \cos(e^x + d) \sqrt{-a^2 + c^2}) / (c^2 \cos(e^x + d)^2 - 2ac \sin(e^x + d) - a^2 - c^2)) + (B a^2 - B c^2) \log(-c^2 \cos(e^x + d)^2 + 2ac \sin(e^x + d) + a^2 + c^2)) / ((a^2 c - c^3) e), 1/2 (2(C a^2 - C c^2) e^x + 2(C a - A c) \sqrt{a^2 - c^2}) \arctan(-(a \sin(e^x + d) + c) / (\sqrt{a^2 - c^2} \cos(e^x + d))) + (B a^2 - B c^2) \log(-c^2 \cos(e^x + d)^2 + 2ac \sin(e^x + d) + a^2 + c^2)) / ((a^2 c - c^3) e)]$$

giac [A] time = 0.18, size = 141, normalized size = 1.68

$$\left(\frac{(xe + d)C}{c} + \frac{B \log\left(a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 2c \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + a\right)}{c} - \frac{B \log\left(\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1\right)}{c} - 2 \left(\pi \left\lfloor \frac{xe}{2} \right\rfloor \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x, algorithm="giac")

[Out] ((x*e + d)*C/c + B*log(a*tan(1/2*x*e + 1/2*d)^2 + 2*c*tan(1/2*x*e + 1/2*d) + a)/c - B*log(tan(1/2*x*e + 1/2*d)^2 + 1)/c - 2*(pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - c^2)))*(C*a - A*c)/sqrt(a^2 - c^2)*c))*e^(-1)

maple [B] time = 0.25, size = 178, normalized size = 2.12

$$\frac{B \ln\left(a \left(\tan^2\left(\frac{d}{2} + \frac{ex}{2}\right)\right) + 2c \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + a\right)}{ec} + \frac{2 \arctan\left(\frac{2a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 2c}{2\sqrt{a^2 - c^2}}\right) A}{e\sqrt{a^2 - c^2}} - \frac{2 \arctan\left(\frac{2a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 2c}{2\sqrt{a^2 - c^2}}\right) Ca}{ec\sqrt{a^2 - c^2}} - B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x)

[Out] 1/e/c*B*ln(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)+2/e/(a^2-c^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*A-2/e/c/(a^2-c^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*C*a-1/e/c*B*ln(1+tan(1/2*d+1/2*e*x)^2)+2/e/c*C*arctan(tan(1/2*d+1/2*e*x))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*c^2-4*a^2>0)', see 'assume?' for more details)Is 4*c^2-4*a^2 positive or negative?

mupad [B] time = 9.63, size = 1143, normalized size = 13.61

$$\ln \left(32 B^3 a^2 - 32 A B^2 a^2 + 32 A C^2 a^2 + 32 B C^2 a^2 + 32 a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) (c A^2 B - 2 c A B^2 - 2 a A B C - 2 c A C^2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*cos(d + e*x) + C*sin(d + e*x))/(a + c*sin(d + e*x)),x)`

[Out] $(\log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*C^2*a^2 + 32*B*C^2*a^2 + 32*a*\tan(d/2 + (e*x)/2)*(2*C^3*a + B^3*c - 2*A*B^2*c + A^2*B*c + 2*B^2*C*a - 2*A*C^2*c + 2*B*C^2*c - 2*A*B*C*a) - 32*A^2*C*a*c + 32*B^2*C*a*c - ((B*a^2 - B*c^2 + A*c*(c^2 - a^2)^{(1/2)} - C*a*(c^2 - a^2)^{(1/2)})*(32*C^2*a^2*c - 32*B^2*a^2*c - 128*B*C*a^3 + 32*a*\tan(d/2 + (e*x)/2)*(2*B^2*a^2 - A^2*c^2 - 2*C^2*a^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 + 2*A*C*a*c - 4*B*C*a*c) + 64*A*B*a^2*c + 64*B*C*a*c^2 + ((B*a^2 - B*c^2 + A*c*(c^2 - a^2)^{(1/2)} - C*a*(c^2 - a^2)^{(1/2)})*(32*A*a^2*c^2 + 32*B*a^2*c^2 - 32*C*a*c^3 + 32*a*c^2*\tan(d/2 + (e*x)/2)*(2*A*c - 2*C*a + B*c) + (32*a*c*(a*c - 2*a^2*\tan(d/2 + (e*x)/2) + 3*c^2*\tan(d/2 + (e*x)/2))*(B*a^2 - B*c^2 + A*c*(c^2 - a^2)^{(1/2)} - C*a*(c^2 - a^2)^{(1/2)}))/(a^2 - c^2)))/(c*(a^2 - c^2)))/(c*(a^2 - c^2)))*(B*a^2 - B*c^2 + A*c*(c^2 - a^2)^{(1/2)} - C*a*(c^2 - a^2)^{(1/2)}))/(c*e*(a^2 - c^2)) - (\log(\tan(d/2 + (e*x)/2) + 1i)*(B - C*1i))/(c*e) - (\log(\tan(d/2 + (e*x)/2) - 1i)*(B + C*1i))/(c*e) + (\log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*C^2*a^2 + 32*B*C^2*a^2 + 32*a*\tan(d/2 + (e*x)/2)*(2*C^3*a + B^3*c - 2*A*B^2*c + A^2*B*c + 2*B^2*C*a - 2*A*C^2*c + 2*B*C^2*c - 2*A*B*C*a) - 32*A^2*C*a*c + 32*B^2*C*a*c - ((B*a^2 - B*c^2 - A*c*(c^2 - a^2)^{(1/2)} + C*a*(c^2 - a^2)^{(1/2)})*(32*C^2*a^2*c - 32*B^2*a^2*c - 128*B*C*a^3 + 32*a*\tan(d/2 + (e*x)/2)*(2*B^2*a^2 - A^2*c^2 - 2*C^2*a^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 + 2*A*C*a*c - 4*B*C*a*c) + 64*A*B*a^2*c + 64*B*C*a*c^2 + ((B*a^2 - B*c^2 - A*c*(c^2 - a^2)^{(1/2)} + C*a*(c^2 - a^2)^{(1/2)})*(32*A*a^2*c^2 + 32*B*a^2*c^2 - 32*C*a*c^3 + 32*a*c^2*\tan(d/2 + (e*x)/2)*(2*A*c - 2*C*a + B*c) + (32*a*c*(a*c - 2*a^2*\tan(d/2 + (e*x)/2) + 3*c^2*\tan(d/2 + (e*x)/2))*(B*a^2 - B*c^2 - A*c*(c^2 - a^2)^{(1/2)} + C*a*(c^2 - a^2)^{(1/2)}))/(a^2 - c^2)))/(c*(a^2 - c^2)))/(c*(a^2 - c^2)))*(B*a^2 - B*c^2 - A*c*(c^2 - a^2)^{(1/2)} + C*a*(c^2 - a^2)^{(1/2)}))/(c*e*(a^2 - c^2))$

sympy [A] time = 29.52, size = 1110, normalized size = 13.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x)`

[Out] `Piecewise((zoo*x*(A + B*cos(d) + C*sin(d))/sin(d), Eq(a, 0) & Eq(c, 0) & Eq(e, 0)), ((A*log(tan(d/2 + e*x/2))/e - B*log(tan(d/2 + e*x/2)**2 + 1)/e + B*log(tan(d/2 + e*x/2))/e + C*x)/c, Eq(a, 0)), (2*A/(c*e*tan(d/2 + e*x/2) - c*e) + 2*B*log(tan(d/2 + e*x/2) - 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) - c*e) - 2*B*log(tan(d/2 + e*x/2) - 1)/(c*e*tan(d/2 + e*x/2) - c*e) - B*log(tan(d/2 + e*x/2)**2 + 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) - c*e) + B*log(tan(d/2 + e*x/2)**2 + 1)/(c*e*tan(d/2 + e*x/2) - c*e) + C*e*x*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) - c*e) - C*e*x/(c*e*tan(d/2 + e*x/2) - c*e) + 2*C/(c*e*tan(d/2 + e*x/2) - c*e), Eq(a, -c)), (-2*A/(c*e*tan(d/2 + e*x/2)`


```

) + c*e) + 2*B*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*
x/2) + c*e) + 2*B*log(tan(d/2 + e*x/2) + 1)/(c*e*tan(d/2 + e*x/2) + c*e) -
B*log(tan(d/2 + e*x/2)**2 + 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) + c*e
) - B*log(tan(d/2 + e*x/2)**2 + 1)/(c*e*tan(d/2 + e*x/2) + c*e) + C*e*x*tan
(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) + c*e) + C*e*x/(c*e*tan(d/2 + e*x/2) +
c*e) + 2*C/(c*e*tan(d/2 + e*x/2) + c*e), Eq(a, c)), ((A*x + B*sin(d + e*x)/
e - C*cos(d + e*x)/e)/a, Eq(c, 0)), (x*(A + B*cos(d) + C*sin(d))/(a + c*sin
(d)), Eq(e, 0)), (-A*c*sqrt(-a**2 + c**2)*log(tan(d/2 + e*x/2) + c/a - sqrt
(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) + A*c*sqrt(-a**2 + c**2)*log(tan(d/2
+ e*x/2) + c/a + sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) - B*a**2*log(tan
(d/2 + e*x/2)**2 + 1)/(a**2*c*e - c**3*e) + B*a**2*log(tan(d/2 + e*x/2) + c
/a - sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) + B*a**2*log(tan(d/2 + e*x/2
) + c/a + sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) + B*c**2*log(tan(d/2 +
e*x/2)**2 + 1)/(a**2*c*e - c**3*e) - B*c**2*log(tan(d/2 + e*x/2) + c/a - sq
rt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) - B*c**2*log(tan(d/2 + e*x/2) + c/a
+ sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) + C*a**2*e*x/(a**2*c*e - c**3*
e) + C*a*sqrt(-a**2 + c**2)*log(tan(d/2 + e*x/2) + c/a - sqrt(-a**2 + c**2)
/a)/(a**2*c*e - c**3*e) - C*a*sqrt(-a**2 + c**2)*log(tan(d/2 + e*x/2) + c/a
+ sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) - C*c**2*e*x/(a**2*c*e - c**3*
e), True))

```

$$3.563 \quad \int \frac{A+B \cos(d+ex)+C \sin(d+ex)}{(a+c \sin(d+ex))^2} dx$$

Optimal. Leaf size=118

$$\frac{2(aA - cC) \tan^{-1} \left(\frac{a \tan \left(\frac{1}{2}(d+ex) \right) + c}{\sqrt{a^2 - c^2}} \right)}{e (a^2 - c^2)^{3/2}} + \frac{(Ac - aC) \cos(d + ex)}{e (a^2 - c^2) (a + c \sin(d + ex))} - \frac{B}{ce(a + c \sin(d + ex))}$$

[Out] 2*(A*a-C*c)*arctan((c+a*tan(1/2*e*x+1/2*d))/(a^2-c^2)^(1/2))/(a^2-c^2)^(3/2)/e-B/c/e/(a+c*sin(e*x+d))+(A*c-C*a)*cos(e*x+d)/(a^2-c^2)/e/(a+c*sin(e*x+d))

Rubi [A] time = 0.16, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4376, 2754, 12, 2660, 618, 204, 2668, 32}

$$\frac{2(aA - cC) \tan^{-1} \left(\frac{a \tan \left(\frac{1}{2}(d+ex) \right) + c}{\sqrt{a^2 - c^2}} \right)}{e (a^2 - c^2)^{3/2}} + \frac{(Ac - aC) \cos(d + ex)}{e (a^2 - c^2) (a + c \sin(d + ex))} - \frac{B}{ce(a + c \sin(d + ex))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^2,x]

[Out] (2*(a*A - c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]]/((a^2 - c^2)^(3/2)*e) - B/(c*e*(a + c*Sin[d + e*x])) + ((A*c - a*C)*Cos[d + e*x])/((a^2 - c^2)*e*(a + c*Sin[d + e*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4376

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] := With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] + Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c*(a + b*x)]/e, u, x] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(d + ex) + C \sin(d + ex)}{(a + c \sin(d + ex))^2} dx &= B \int \frac{\cos(d + ex)}{(a + c \sin(d + ex))^2} dx + \int \frac{A + C \sin(d + ex)}{(a + c \sin(d + ex))^2} dx \\ &= \frac{(Ac - aC) \cos(d + ex)}{(a^2 - c^2) e (a + c \sin(d + ex))} + \frac{\int \frac{-aA + cC}{a + c \sin(d + ex)} dx}{-a^2 + c^2} + \frac{B \operatorname{Subst}\left(\int \frac{1}{(a + x)^2} dx\right)}{a^2 - c^2} \\ &= -\frac{B}{ce(a + c \sin(d + ex))} + \frac{(Ac - aC) \cos(d + ex)}{(a^2 - c^2) e (a + c \sin(d + ex))} + \frac{(aA - cC)}{(a^2 - c^2) e} \\ &= -\frac{B}{ce(a + c \sin(d + ex))} + \frac{(Ac - aC) \cos(d + ex)}{(a^2 - c^2) e (a + c \sin(d + ex))} + \frac{(2(aA - cC))}{(a^2 - c^2) e} \\ &= -\frac{B}{ce(a + c \sin(d + ex))} + \frac{(Ac - aC) \cos(d + ex)}{(a^2 - c^2) e (a + c \sin(d + ex))} - \frac{(4(aA - cC))}{(a^2 - c^2) e} \\ &= \frac{2(aA - cC) \tan^{-1}\left(\frac{c + a \tan\left(\frac{1}{2}(d + ex)\right)}{\sqrt{a^2 - c^2}}\right)}{(a^2 - c^2)^{3/2} e} - \frac{B}{ce(a + c \sin(d + ex))} + \frac{(A - C)}{(a^2 - c^2) e} \end{aligned}$$

Mathematica [A] time = 0.43, size = 114, normalized size = 0.97

$$\frac{B(a^2 - c^2) - c(Ac - aC) \cos(d + ex)}{c(c - a)(a + c)(a + c \sin(d + ex))} + \frac{2(aA - cC) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(d + ex)\right) + c}{\sqrt{a^2 - c^2}}\right)}{(a^2 - c^2)^{3/2}}$$

e

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^2, x]
```

[Out] $((2*(a*A - c*C)*\text{ArcTan}[(c + a*\text{Tan}[(d + e*x)/2])/ \text{Sqrt}[a^2 - c^2]])/ (a^2 - c^2)^{(3/2)} + (B*(a^2 - c^2) - c*(A*c - a*C)*\text{Cos}[d + e*x]) / (c*(-a + c)*(a + c) * (a + c*\text{Sin}[d + e*x]))) / e$

fricas [A] time = 1.36, size = 458, normalized size = 3.88

$$\left[\frac{2Ba^4 - 4Ba^2c^2 + 2Bc^4 + (Aa^2c - Cac^2 + (Aac^2 - Cc^3)\sin(ex+d))\sqrt{-a^2+c^2} \log\left(\frac{(2a^2-c^2)\cos(ex+d)^2 - 2ac\sin(ex+d)}{c^2\cos^2(ex+d)}\right)}{2((a^4c^2 - 2a^2c^4 + c^6)e\sin(ex+d) + \dots)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x, algorithm="fricas")`

[Out] $[-1/2*(2*B*a^4 - 4*B*a^2*c^2 + 2*B*c^4 + (A*a^2*c - C*a*c^2 + (A*a*c^2 - C*c^3)*\sin(e*x + d))*\sqrt{-a^2 + c^2}*\log(((2*a^2 - c^2)*\cos(e*x + d)^2 - 2*a*c*\sin(e*x + d) - a^2 - c^2 + 2*(a*\cos(e*x + d)*\sin(e*x + d) + c*\cos(e*x + d))*\sqrt{-a^2 + c^2})) / (c^2*\cos(e*x + d)^2 - 2*a*c*\sin(e*x + d) - a^2 - c^2) + 2*(C*a^3*c - A*a^2*c^2 - C*a*c^3 + A*c^4)*\cos(e*x + d) / ((a^4*c^2 - 2*a^2*c^4 + c^6)*e*\sin(e*x + d) + (a^5*c - 2*a^3*c^3 + a*c^5)*e), -(B*a^4 - 2*B*a^2*c^2 + B*c^4 + (A*a^2*c - C*a*c^2 + (A*a*c^2 - C*c^3)*\sin(e*x + d))*\sqrt{a^2 - c^2}*\arctan(-(a*\sin(e*x + d) + c) / (\sqrt{a^2 - c^2}*\cos(e*x + d))) + (C*a^3*c - A*a^2*c^2 - C*a*c^3 + A*c^4)*\cos(e*x + d) / ((a^4*c^2 - 2*a^2*c^4 + c^6)*e*\sin(e*x + d) + (a^5*c - 2*a^3*c^3 + a*c^5)*e)]$

giac [A] time = 0.20, size = 187, normalized size = 1.58

$$2 \left(\frac{\left(\pi \left\lfloor \frac{xe+d}{2\pi} + \frac{1}{2} \right\rfloor \text{sgn}(a) + \arctan\left(\frac{a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + c}{\sqrt{a^2 - c^2}}\right) \right) (Aa - Cc)}{(a^2 - c^2)^{\frac{3}{2}}} + \frac{Ba^2 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - Cac \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + Aa}{(a^3 - ac^2) \left(a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + c \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x, algorithm="giac")`

[Out] $2*((\pi*\text{floor}(1/2*(x*e + d)/\pi + 1/2)*\text{sgn}(a) + \arctan((a*\tan(1/2*x*e + 1/2*d) + c)/\sqrt{a^2 - c^2}))* (A*a - C*c) / (a^2 - c^2)^{(3/2)} + (B*a^2*\tan(1/2*x*e + 1/2*d) - C*a*c*\tan(1/2*x*e + 1/2*d) + A*c^2*\tan(1/2*x*e + 1/2*d) - B*c^2*\tan(1/2*x*e + 1/2*d) - C*a^2 + A*a*c) / ((a^3 - a*c^2)*(a*\tan(1/2*x*e + 1/2*d)^2 + 2*c*\tan(1/2*x*e + 1/2*d) + a)))*e^{-1}$

maple [B] time = 0.40, size = 426, normalized size = 3.61

$$\frac{2 \tan\left(\frac{d}{2} + \frac{ex}{2}\right) A c^2}{e \left(a \left(\tan^2\left(\frac{d}{2} + \frac{ex}{2}\right) \right) + 2c \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + a \right) a (a^2 - c^2)} + \frac{2a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) B}{e \left(a \left(\tan^2\left(\frac{d}{2} + \frac{ex}{2}\right) \right) + 2c \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + a \right) (a^2 - c^2)} - \frac{2}{e \left(a \left(\tan^2\left(\frac{d}{2} + \frac{ex}{2}\right) \right) + 2c \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + a \right) (a^2 - c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x)`

[Out] $2/e/(a*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a)/a/(a^2-c^2)*\tan(1/2*d+1/2*e*x)*A*c^2+2/e/(a*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a)*a/(a^2-c^2)*\tan(1/2*d+1/2*e*x)*B-2/e/(a*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a)/a/(a^2-c^2)*\tan(1/2*d+1/2*e*x)*B*c^2-2/e/(a*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a)/a/(a^2-c^2)*\tan(1/2*d+1/2*e*x)*B*c^2-2/e/(a*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a)/a/(a^2-c^2)*\tan(1/2*d+1/2*e*x)*B*c^2-2/e/(a*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a)/a/(a^2-c^2)*\tan(1/2*d+1/2*e*x)*B*c^2$

```
*tan(1/2*d+1/2*e*x)+a)/(a^2-c^2)*tan(1/2*d+1/2*e*x)*c*C+2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)/(a^2-c^2)*A*c-2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)/(a^2-c^2)*C*a+2/e/(a^2-c^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*a*A-2/e/(a^2-c^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*C*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*c^2-4*a^2>0)', see `assume?` for more details)Is 4*c^2-4*a^2 positive or negative?

mupad [B] time = 3.33, size = 227, normalized size = 1.92

$$\frac{\frac{2(Ac-Ca)}{a^2-c^2} + \frac{2 \tan\left(\frac{d}{2} + \frac{ex}{2}\right) (Ba^2 + Ac^2 - Bc^2 - Cac)}{a(a^2-c^2)}}{e \left(a \tan\left(\frac{d}{2} + \frac{ex}{2}\right)^2 + 2c \tan\left(\frac{d}{2} + \frac{ex}{2}\right) + a \right)} + \frac{2 \operatorname{atan}\left(\frac{(a^2-c^2) \left(\frac{2a \tan\left(\frac{d}{2} + \frac{ex}{2}\right) (Aa-Cc)}{(a+c)^{3/2} (a-c)^{3/2}} + \frac{2(a^2c-c^3)(Aa-Cc)}{(a+c)^{3/2} (a^2-c^2)(a-c)^{3/2}} \right)}{2(Aa-Cc)} \right)}{e(a+c)^{3/2}(a-c)^{3/2}} (Aa-Cc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(d + e*x) + C*sin(d + e*x))/(a + c*sin(d + e*x))^2,x)

[Out] ((2*(A*c - C*a))/(a^2 - c^2) + (2*tan(d/2 + (e*x)/2)*(B*a^2 + A*c^2 - B*c^2 - C*a*c))/(a*(a^2 - c^2)))/(e*(a + 2*c*tan(d/2 + (e*x)/2) + a*tan(d/2 + (e*x)/2)^2) + (2*atan(((a^2 - c^2)*((2*a*tan(d/2 + (e*x)/2)*(A*a - C*c)))/((a + c)^(3/2)*(a - c)^(3/2)) + (2*(a^2*c - c^3)*(A*a - C*c)))/((a + c)^(3/2)*(a^2 - c^2)*(a - c)^(3/2))))/(2*(A*a - C*c))*(A*a - C*c)/(e*(a + c)^(3/2)*(a - c)^(3/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x)

[Out] Timed out

$$3.564 \quad \int \frac{A+B \cos(d+ex)+C \sin(d+ex)}{(a+c \sin(d+ex))^3} dx$$

Optimal. Leaf size=185

$$\frac{(2a^2A - 3acC + Ac^2) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{e(a^2-c^2)^{5/2}} + \frac{(a^2(-C) + 3aAc - 2c^2C) \cos(d+ex)}{2e(a^2-c^2)^2(a+c \sin(d+ex))} + \frac{(Ac - aC) \cos(d+ex)}{2e(a^2-c^2)(a+c \sin(d+ex))}$$

[Out] (2*A*a^2+A*c^2-3*C*a*c)*arctan((c+a*tan(1/2*e*x+1/2*d))/(a^2-c^2)^(1/2))/(a^2-c^2)^(5/2)/e-1/2*B/c/e/(a+c*sin(e*x+d))^2+1/2*(A*c-C*a)*cos(e*x+d)/(a^2-c^2)/e/(a+c*sin(e*x+d))^2+1/2*(3*A*a*c-C*a^2-2*C*c^2)*cos(e*x+d)/(a^2-c^2)^2/e/(a+c*sin(e*x+d))

Rubi [A] time = 0.25, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4376, 2754, 12, 2660, 618, 204, 2668, 32}

$$\frac{(2a^2A - 3acC + Ac^2) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{e(a^2-c^2)^{5/2}} + \frac{(a^2(-C) + 3aAc - 2c^2C) \cos(d+ex)}{2e(a^2-c^2)^2(a+c \sin(d+ex))} + \frac{(Ac - aC) \cos(d+ex)}{2e(a^2-c^2)(a+c \sin(d+ex))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^3,x]

[Out] ((2*a^2*A + A*c^2 - 3*a*c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]])/((a^2 - c^2)^(5/2)*e) - B/(2*c*e*(a + c*Sin[d + e*x])^2) + ((A*c - a*C)*Cos[d + e*x])/((2*(a^2 - c^2)*e*(a + c*Sin[d + e*x])^2) + ((3*a*A*c - a^2*C - 2*c^2*C)*Cos[d + e*x])/(2*(a^2 - c^2)^2*e*(a + c*Sin[d + e*x])))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[

$a^2 - b^2, 0]$

Rule 2668

`Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]`

Rule 2754

`Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]`

Rule 4376

`Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_) + (b_.)*(x_))]^(n_.)), x_Symbol] :> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] + Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c*(a + b*x)]/e, u, x] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])`

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(d + ex) + C \sin(d + ex)}{(a + c \sin(d + ex))^3} dx &= B \int \frac{\cos(d + ex)}{(a + c \sin(d + ex))^3} dx + \int \frac{A + C \sin(d + ex)}{(a + c \sin(d + ex))^3} dx \\ &= \frac{(Ac - aC) \cos(d + ex)}{2(a^2 - c^2) e(a + c \sin(d + ex))^2} - \frac{\int \frac{-2(aA - cC) + (Ac - aC) \sin(d + ex)}{(a + c \sin(d + ex))^2} dx}{2(a^2 - c^2)} + \\ &= -\frac{B}{2ce(a + c \sin(d + ex))^2} + \frac{(Ac - aC) \cos(d + ex)}{2(a^2 - c^2) e(a + c \sin(d + ex))^2} + \frac{(3aA - 3aC)}{2(a^2 - c^2)} \\ &= -\frac{B}{2ce(a + c \sin(d + ex))^2} + \frac{(Ac - aC) \cos(d + ex)}{2(a^2 - c^2) e(a + c \sin(d + ex))^2} + \frac{(3aA - 3aC)}{2(a^2 - c^2)} \\ &= -\frac{B}{2ce(a + c \sin(d + ex))^2} + \frac{(Ac - aC) \cos(d + ex)}{2(a^2 - c^2) e(a + c \sin(d + ex))^2} + \frac{(3aA - 3aC)}{2(a^2 - c^2)} \\ &= -\frac{B}{2ce(a + c \sin(d + ex))^2} + \frac{(Ac - aC) \cos(d + ex)}{2(a^2 - c^2) e(a + c \sin(d + ex))^2} + \frac{(3aA - 3aC)}{2(a^2 - c^2)} \\ &= \frac{(2a^2 A + Ac^2 - 3aC) \tan^{-1} \left(\frac{c + a \tan \left(\frac{1}{2}(d + ex) \right)}{\sqrt{a^2 - c^2}} \right)}{(a^2 - c^2)^{5/2} e} - \frac{B}{2ce(a + c \sin(d + ex))^2} \end{aligned}$$

Mathematica [A] time = 0.91, size = 174, normalized size = 0.94

$$\frac{B(c^2 - a^2) + c(Ac - aC) \cos(d + ex)}{c(a - c)(a + c)(a + c \sin(d + ex))^2} + \frac{2(2a^2A - 3acC + Ac^2) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(d + ex)\right) + c}{\sqrt{a^2 - c^2}}\right)}{(a^2 - c^2)^{5/2}} - \frac{(a^2C - 3aAc + 2c^2C) \cos(d + ex)}{(a - c)^2(a + c)^2(a + c \sin(d + ex))}$$

$2e$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^3,x]
```

```
[Out] ((2*(2*a^2*A + A*c^2 - 3*a*c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]])/(a^2 - c^2)^(5/2) + (B*(-a^2 + c^2) + c*(A*c - a*C)*Cos[d + e*x])/((a - c)*c*(a + c)*(a + c*Sin[d + e*x])^2) - ((-3*a*A*c + a^2*C + 2*c^2*C)*Cos[d + e*x])/((a - c)^2*(a + c)^2*(a + c*Sin[d + e*x]))/(2*e)
```

fricas [B] time = 1.17, size = 880, normalized size = 4.76

$$\left[\frac{2Ba^6 - 6Ba^4c^2 + 6Ba^2c^4 - 2Bc^6 + 2(Ca^4c^2 - 3Aa^3c^3 + Ca^2c^4 + 3Aac^5 - 2Cc^6) \cos(ex + d) \sin(ex + d) + (2Aa^4c - 3Ca^3c^2 + 3Aa^2c^3 - 3Ca^2c^4 + A^2c^5 - (2Aa^2c^3 - 3Ca^2c^4 + A^2c^5) \cos(ex + d)^2 + 2(2Aa^3c^2 - 3Ca^2c^3 + Aa^2c^4) \sin(ex + d) \sqrt{-a^2 + c^2} \log(((2a^2 - c^2) \cos(ex + d)^2 - 2a^2c \sin(ex + d) - a^2 - c^2 + 2(a \cos(ex + d) \sin(ex + d) + c \cos(ex + d)) \sqrt{-a^2 + c^2}))/((c^2 \cos(ex + d)^2 - 2a^2c \sin(ex + d) - a^2 - c^2)) + 2(2Ca^5c - 4Aa^4c^2 - Ca^3c^3 + 5Aa^2c^4 - Ca^2c^5 - A^2c^6) \cos(ex + d))/((a^6c^3 - 3a^4c^5 + 3a^2c^7 - c^9) e \cos(ex + d)^2 - 2(a^7c^2 - 3a^5c^4 + 3a^3c^6 - a^2c^8) e \sin(ex + d) - (a^8c - 2a^6c^3 + 2a^2c^7 - c^9) e), 1/2(Ba^6 - 3Ba^4c^2 + 3Ba^2c^4 - Bc^6 + (Ca^4c^2 - 3Aa^3c^3 + Ca^2c^4 + 3Aac^5 - 2Cc^6) \cos(ex + d) \sin(ex + d) + (2Aa^4c - 3Ca^3c^2 + 3Aa^2c^3 - 3Ca^2c^4 + A^2c^5 - (2Aa^2c^3 - 3Ca^2c^4 + A^2c^5) \cos(ex + d)^2 + 2(2Aa^3c^2 - 3Ca^2c^3 + Aa^2c^4) \sin(ex + d)) \sqrt{a^2 - c^2} \arctan(-(a \sin(ex + d) + c)/(\sqrt{a^2 - c^2} \cos(ex + d))) + (2Ca^5c - 4Aa^4c^2 - Ca^3c^3 + 5Aa^2c^4 - Ca^2c^5 - A^2c^6) \cos(ex + d))/((a^6c^3 - 3a^4c^5 + 3a^2c^7 - c^9) e \cos(ex + d)^2 - 2(a^7c^2 - 3a^5c^4 + 3a^3c^6 - a^2c^8) e \sin(ex + d) - (a^8c - 2a^6c^3 + 2a^2c^7 - c^9) e)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*B*a^6 - 6*B*a^4*c^2 + 6*B*a^2*c^4 - 2*B*c^6 + 2*(C*a^4*c^2 - 3*A*a^3*c^3 + C*a^2*c^4 + 3*A*a*c^5 - 2*C*c^6)*cos(e*x + d)*sin(e*x + d) + (2*A*a^4*c - 3*C*a^3*c^2 + 3*A*a^2*c^3 - 3*C*a*c^4 + A*c^5 - (2*A*a^2*c^3 - 3*C*a*c^4 + A*c^5)*cos(e*x + d)^2 + 2*(2*A*a^3*c^2 - 3*C*a^2*c^3 + A*a*c^4)*sin(e*x + d))*sqrt(-a^2 + c^2)*log(((2*a^2 - c^2)*cos(e*x + d)^2 - 2*a*c*sin(e*x + d) - a^2 - c^2 + 2*(a*cos(e*x + d)*sin(e*x + d) + c*cos(e*x + d))*sqrt(-a^2 + c^2))/((c^2*cos(e*x + d)^2 - 2*a*c*sin(e*x + d) - a^2 - c^2)) + 2*(2*C*a^5*c - 4*A*a^4*c^2 - C*a^3*c^3 + 5*A*a^2*c^4 - C*a*c^5 - A*c^6)*cos(e*x + d))/((a^6*c^3 - 3*a^4*c^5 + 3*a^2*c^7 - c^9)*e*cos(e*x + d)^2 - 2*(a^7*c^2 - 3*a^5*c^4 + 3*a^3*c^6 - a*c^8)*e*sin(e*x + d) - (a^8*c - 2*a^6*c^3 + 2*a^2*c^7 - c^9)*e), 1/2*(B*a^6 - 3*B*a^4*c^2 + 3*B*a^2*c^4 - B*c^6 + (C*a^4*c^2 - 3*A*a^3*c^3 + C*a^2*c^4 + 3*A*a*c^5 - 2*C*c^6)*cos(e*x + d)*sin(e*x + d) + (2*A*a^4*c - 3*C*a^3*c^2 + 3*A*a^2*c^3 - 3*C*a*c^4 + A*c^5 - (2*A*a^2*c^3 - 3*C*a*c^4 + A*c^5)*cos(e*x + d)^2 + 2*(2*A*a^3*c^2 - 3*C*a^2*c^3 + A*a*c^4)*sin(e*x + d))*sqrt(a^2 - c^2)*arctan(-(a*sin(e*x + d) + c)/(sqrt(a^2 - c^2)*cos(e*x + d))) + (2*C*a^5*c - 4*A*a^4*c^2 - C*a^3*c^3 + 5*A*a^2*c^4 - C*a*c^5 - A*c^6)*cos(e*x + d))/((a^6*c^3 - 3*a^4*c^5 + 3*a^2*c^7 - c^9)*e*cos(e*x + d)^2 - 2*(a^7*c^2 - 3*a^5*c^4 + 3*a^3*c^6 - a*c^8)*e*sin(e*x + d) - (a^8*c - 2*a^6*c^3 + 2*a^2*c^7 - c^9)*e)]
```

giac [B] time = 0.27, size = 596, normalized size = 3.22

$$\left(\frac{(2Aa^2 - 3Cac + Ac^2) \left(\pi \left[\frac{xe+d}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(a) + \arctan\left(\frac{a \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + c}{\sqrt{a^2 - c^2}}\right) \right)}{(a^4 - 2a^2c^2 + c^4) \sqrt{a^2 - c^2}} + \frac{2Ba^5 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^3 - 3Ca^4c \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)}{(a^4 - 2a^2c^2 + c^4) \sqrt{a^2 - c^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x, algorithm="giac")
```



```
[Out] ((2*A*a^2 - 3*C*a*c + A*c^2)*(pi*floor(1/2*(x*e + d)/pi + 1/2)*sgn(a) + arc
tan((a*tan(1/2*x*e + 1/2*d) + c)/sqrt(a^2 - c^2)))/((a^4 - 2*a^2*c^2 + c^4)
*sqrt(a^2 - c^2)) + (2*B*a^5*tan(1/2*x*e + 1/2*d)^3 - 3*C*a^4*c*tan(1/2*x*e
+ 1/2*d)^3 + 5*A*a^3*c^2*tan(1/2*x*e + 1/2*d)^3 - 4*B*a^3*c^2*tan(1/2*x*e
+ 1/2*d)^3 - 2*A*a*c^4*tan(1/2*x*e + 1/2*d)^3 + 2*B*a*c^4*tan(1/2*x*e + 1/2
*d)^3 - 2*C*a^5*tan(1/2*x*e + 1/2*d)^2 + 4*A*a^4*c*tan(1/2*x*e + 1/2*d)^2 +
2*B*a^4*c*tan(1/2*x*e + 1/2*d)^2 - 5*C*a^3*c^2*tan(1/2*x*e + 1/2*d)^2 + 7*
A*a^2*c^3*tan(1/2*x*e + 1/2*d)^2 - 4*B*a^2*c^3*tan(1/2*x*e + 1/2*d)^2 - 2*C
*a*c^4*tan(1/2*x*e + 1/2*d)^2 - 2*A*c^5*tan(1/2*x*e + 1/2*d)^2 + 2*B*c^5*ta
n(1/2*x*e + 1/2*d)^2 + 2*B*a^5*tan(1/2*x*e + 1/2*d) - 5*C*a^4*c*tan(1/2*x*e
+ 1/2*d) + 11*A*a^3*c^2*tan(1/2*x*e + 1/2*d) - 4*B*a^3*c^2*tan(1/2*x*e + 1
/2*d) - 4*C*a^2*c^3*tan(1/2*x*e + 1/2*d) - 2*A*a*c^4*tan(1/2*x*e + 1/2*d) +
2*B*a*c^4*tan(1/2*x*e + 1/2*d) - 2*C*a^5 + 4*A*a^4*c - C*a^3*c^2 - A*a^2*c
^3)/((a^6 - 2*a^4*c^2 + a^2*c^4)*(a*tan(1/2*x*e + 1/2*d)^2 + 2*c*tan(1/2*x*
e + 1/2*d) + a)^2))*e^(-1)
```

maple [B] time = 0.43, size = 1891, normalized size = 10.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3, x)
```

```
[Out] -2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)
)*C*a^3-1/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*
c^2+c^4)*A*c^3-2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4
-2*a^2*c^2+c^4)*a^3*tan(1/2*d+1/2*e*x)^2*C+2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*
tan(1/2*d+1/2*e*x)+a)^2*a^3/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)*B+4/e/(a
*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*A*a^2
*c-1/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c
^4)*C*a*c^2+7/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*
a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^2*A*c^3-4/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*ta
n(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^2*B*c^3-4/e/(a
tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*tan(1/
2*d+1/2*e*x)*C*c^3+2/e/(a^4-2*a^2*c^2+c^4)/(a^2-c^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*a^2*A+1/e/(a^4-2*a^2*c^2+c^4)/(a^2
-c^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))*A*c^2+
2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)
)*a^3*tan(1/2*d+1/2*e*x)^3*B-4/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e
*x)+a)^2/(a^4-2*a^2*c^2+c^4)*a*tan(1/2*d+1/2*e*x)^3*B*c^2+2/e/(a*tan(1/2*d+
1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)/a*tan(1/2*d+1/2*
e*x)^3*B*c^4-3/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2
*a^2*c^2+c^4)*a^2*tan(1/2*d+1/2*e*x)^3*C*c+4/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*
tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*a^2*tan(1/2*d+1/2*e*x)^2*A*c-2/
e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)/a
^2*tan(1/2*d+1/2*e*x)^2*A*c^5+2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2
*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)*a^2*tan(1/2*d+1/2*e*x)^2*B*c+2/e/(a*tan(1/2*
d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)/a^2*tan(1/2*d+
1/2*e*x)^2*B*c^5-5/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/(a
^4-2*a^2*c^2+c^4)*a*tan(1/2*d+1/2*e*x)^2*C*c^2-2/e/(a*tan(1/2*d+1/2*e*x)^2+
2*c*tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)/a*tan(1/2*d+1/2*e*x)^2*C*c^
4+11/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2*a/(a^4-2*a^2*c^2
+c^4)*tan(1/2*d+1/2*e*x)*A*c^2-2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/
2*e*x)+a)^2/a/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)*A*c^4-4/e/(a*tan(1/2*d
+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2*a/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2
*e*x)*B*c^2+2/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2/a/(a^4-
2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)*B*c^4-5/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*ta
n(1/2*d+1/2*e*x)+a)^2*a^2/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)*C*c-3/e/(a^
4-2*a^2*c^2+c^4)/(a^2-c^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a
^2-c^2)^(1/2))*a*c*C+5/e/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^
```

$2/(a^4-2*a^2*c^2+c^4)*a*\tan(1/2*d+1/2*e*x)^3*A*c^2-2/e/(a*\tan(1/2*d+1/2*e*x)^2+2*c*\tan(1/2*d+1/2*e*x)+a)^2/(a^4-2*a^2*c^2+c^4)/a*\tan(1/2*d+1/2*e*x)^3*A*c^4$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*c^2-4*a^2>0)', see `assume?` for more details)Is 4*c^2-4*a^2 positive or negative?

mupad [B] time = 5.27, size = 557, normalized size = 3.01

$$\operatorname{atan}\left(\frac{\left(\frac{(2Aa^2-3Cac+Ac^2)(2a^4c-4a^2c^3+2c^5)}{2(a+c)^{5/2}(a-c)^{5/2}(a^4-2a^2c^2+c^4)} + \frac{a \tan\left(\frac{d}{2} + \frac{ex}{2}\right)(2Aa^2-3Cac+Ac^2)}{(a+c)^{5/2}(a-c)^{5/2}}\right)(a^4-2a^2c^2+c^4)}{2Aa^2-3Cac+Ac^2}\right) \frac{(2Aa^2-3Cac+Ac^2)}{e(a+c)^{5/2}(a-c)^{5/2}} \frac{2Ca^3-4Aa^2c}{a^4-2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(d + e*x) + C*sin(d + e*x))/(a + c*sin(d + e*x))^3,x)

[Out] (atan((((2*A*a^2 + A*c^2 - 3*C*a*c)*(2*a^4*c + 2*c^5 - 4*a^2*c^3))/(2*(a + c)^(5/2)*(a - c)^(5/2)*(a^4 + c^4 - 2*a^2*c^2)) + (a*tan(d/2 + (e*x)/2)*(2*A*a^2 + A*c^2 - 3*C*a*c))/(a + c)^(5/2)*(a - c)^(5/2)))/(a^4 + c^4 - 2*a^2*c^2))/(2*A*a^2 + A*c^2 - 3*C*a*c)*(2*A*a^2 + A*c^2 - 3*C*a*c))/(e*(a + c)^(5/2)*(a - c)^(5/2)) - ((A*c^3 + 2*C*a^3 - 4*A*a^2*c + C*a*c^2)/(a^4 + c^4 - 2*a^2*c^2) - (tan(d/2 + (e*x)/2)^3*(2*B*a^4 - 2*A*c^4 + 2*B*c^4 + 5*A*a^2*c^2 - 4*B*a^2*c^2 - 3*C*a^3*c))/(a*(a^4 + c^4 - 2*a^2*c^2)) + (tan(d/2 + (e*x)/2)*(2*A*c^4 - 2*B*a^4 - 2*B*c^4 - 11*A*a^2*c^2 + 4*B*a^2*c^2 + 4*C*a*c^3 + 5*C*a^3*c))/(a*(a^4 + c^4 - 2*a^2*c^2)) + (tan(d/2 + (e*x)/2)^2*(2*A*c^5 + 2*C*a^5 - 2*B*c^5 - 7*A*a^2*c^3 + 4*B*a^2*c^3 + 5*C*a^3*c^2 - 4*A*a^4*c - 2*B*a^4*c + 2*C*a*c^4))/(a^2*(a^4 + c^4 - 2*a^2*c^2)))/(e*(tan(d/2 + (e*x)/2)^2*(2*a^2 + 4*c^2) + a^2*tan(d/2 + (e*x)/2)^4 + a^2 + 4*a*c*tan(d/2 + (e*x)/2)^3 + 4*a*c*tan(d/2 + (e*x)/2)))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x)

[Out] Timed out

$$3.565 \quad \int \frac{A+B \cos(d+ex)+C \sin(d+ex)}{(a+c \sin(d+ex))^4} dx$$

Optimal. Leaf size=258

$$\frac{(-2a^2C + 5aAc - 3c^2C) \cos(d+ex)}{6e(a^2 - c^2)^2 (a+c \sin(d+ex))^2} + \frac{(Ac - aC) \cos(d+ex)}{3e(a^2 - c^2)(a+c \sin(d+ex))^3} + \frac{(2a^3A - 4a^2cC + 3aAc^2 - c^3C) \tan^{-1}}{e(a^2 - c^2)^{7/2}}$$

[Out] $(2Aa^3 + 3Aac^2 - 4A^2c - Cc^3) \arctan((c + a \tan(1/2 ex + 1/2 d)) / (a^2 - c^2)^{1/2}) / (a^2 - c^2)^{7/2} - 1/3 B/c/e / (a + c \sin(ex + d))^3 + 1/3 (Ac - Ca) \cos(ex + d) / (a^2 - c^2) / e / (a + c \sin(ex + d))^3 + 1/6 (5Aac - 2A^2c - 3C^2) \cos(ex + d) / (a^2 - c^2)^2 / e / (a + c \sin(ex + d))^2 + 1/6 (11Aa^2c + 4A^3c - 2A^3c - 13Ca^2) \cos(ex + d) / (a^2 - c^2)^3 / e / (a + c \sin(ex + d))$

Rubi [A] time = 0.40, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4376, 2754, 12, 2660, 618, 204, 2668, 32}

$$\frac{(2a^3A - 4a^2cC + 3aAc^2 - c^3C) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(d+ex)\right) + c}{\sqrt{a^2 - c^2}}\right)}{e(a^2 - c^2)^{7/2}} + \frac{(11a^2Ac - 2a^3C - 13ac^2C + 4Ac^3) \cos(d+ex)}{6e(a^2 - c^2)^3 (a+c \sin(d+ex))} + \frac{(-2a^2C + 5aAc - 3c^2C) \cos(d+ex)}{6e(a^2 - c^2)^2 (a+c \sin(d+ex))^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^4,x]

[Out] $((2a^3A + 3aAac^2 - 4a^2cC - c^3C) \text{ArcTan}[(c + a \tan((d + ex)/2))] / \text{Sqrt}[a^2 - c^2]) / ((a^2 - c^2)^{7/2} e) - B / (3c e (a + c \sin[d + ex])^3) + ((Ac - aC) \cos[d + ex]) / (3(a^2 - c^2) e (a + c \sin[d + ex])^3) + ((5aAac - 2a^2cC - 3c^2C) \cos[d + ex]) / (6(a^2 - c^2)^2 e (a + c \sin[d + ex])^2) + ((11a^2Ac + 4A^3c - 2a^3C - 13a^2cC) \cos[d + ex]) / (6(a^2 - c^2)^3 e (a + c \sin[d + ex]))$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 32

Int[((a_) + (b_)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1) / (b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]] / (Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2668

```
Int[cos[(e_) + (f_)*(x_)]^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4376

```
Int[(u_)*((v_) + (d_)*(F_)[(c_)*((a_) + (b_)*(x_))]^(n_)), x_Symbol] := With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] + Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c*(a + b*x)]/e, u, x] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \cos(d + ex) + C \sin(d + ex)}{(a + c \sin(d + ex))^4} dx &= B \int \frac{\cos(d + ex)}{(a + c \sin(d + ex))^4} dx + \int \frac{A + C \sin(d + ex)}{(a + c \sin(d + ex))^4} dx \\
&= \frac{(Ac - aC) \cos(d + ex)}{3(a^2 - c^2) e (a + c \sin(d + ex))^3} - \frac{\int \frac{-3(aA - cC) + 2(Ac - aC) \sin(d + ex)}{(a + c \sin(d + ex))^3} dx}{3(a^2 - c^2)} \\
&= -\frac{B}{3ce(a + c \sin(d + ex))^3} + \frac{(Ac - aC) \cos(d + ex)}{3(a^2 - c^2) e (a + c \sin(d + ex))^3} + \frac{(5aA - 3cC)}{6(a^2 - c^2)} \\
&= -\frac{B}{3ce(a + c \sin(d + ex))^3} + \frac{(Ac - aC) \cos(d + ex)}{3(a^2 - c^2) e (a + c \sin(d + ex))^3} + \frac{(5aA - 3cC)}{6(a^2 - c^2)} \\
&= -\frac{B}{3ce(a + c \sin(d + ex))^3} + \frac{(Ac - aC) \cos(d + ex)}{3(a^2 - c^2) e (a + c \sin(d + ex))^3} + \frac{(5aA - 3cC)}{6(a^2 - c^2)} \\
&= -\frac{B}{3ce(a + c \sin(d + ex))^3} + \frac{(Ac - aC) \cos(d + ex)}{3(a^2 - c^2) e (a + c \sin(d + ex))^3} + \frac{(5aA - 3cC)}{6(a^2 - c^2)} \\
&= -\frac{B}{3ce(a + c \sin(d + ex))^3} + \frac{(Ac - aC) \cos(d + ex)}{3(a^2 - c^2) e (a + c \sin(d + ex))^3} + \frac{(5aA - 3cC)}{6(a^2 - c^2)} \\
&= \frac{(2a^3 A + 3aAc^2 - 4a^2 cC - c^3 C) \tan^{-1} \left(\frac{c + a \tan\left(\frac{1}{2}(d + ex)\right)}{\sqrt{a^2 - c^2}} \right)}{(a^2 - c^2)^{7/2} e} - \frac{3cC}{3ce(a + c \sin(d + ex))^3}
\end{aligned}$$

Mathematica [A] time = 2.64, size = 244, normalized size = 0.95

$$\frac{2B(c^2 - a^2) + 2c(Ac - aC) \cos(d + ex)}{c(a - c)(a + c)(a + c \sin(d + ex))^3} + \frac{(-2a^2 C + 5aAc - 3c^2 C) \cos(d + ex)}{(a - c)^2 (a + c)^2 (a + c \sin(d + ex))^2} + \frac{6(2a^3 A - 4a^2 cC + 3aAc^2 - c^3 C) \tan^{-1} \left(\frac{a \tan\left(\frac{1}{2}(d + ex)\right) + c}{\sqrt{a^2 - c^2}} \right)}{(a^2 - c^2)^{7/2}} + \frac{(-2a^3 C + 11a^2 cC)}{(a - c)^3}$$

6e

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^4,x]

[Out] ((6*(2*a^3*A + 3*a*A*c^2 - 4*a^2*c*C - c^3*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]])/(a^2 - c^2)^(7/2) + (2*B*(-a^2 + c^2) + 2*c*(A*c - a*C)*Cos[d + e*x])/((a - c)*c*(a + c)*(a + c*Sin[d + e*x])^3) + ((5*a*A*c - 2*a^2*C - 3*c^2*C)*Cos[d + e*x])/((a - c)^2*(a + c)^2*(a + c*Sin[d + e*x])^2) + ((11*a^2*A*c + 4*A*c^3 - 2*a^3*C - 13*a*c^2*C)*Cos[d + e*x])/((a - c)^3*(a + c)^3*(a + c*Sin[d + e*x]))/(6*e)

fricas [B] time = 0.79, size = 1411, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x, algorithm="fricas")

[Out] [1/12*(4*B*a^8 - 16*B*a^6*c^2 + 24*B*a^4*c^4 - 16*B*a^2*c^6 + 4*B*c^8 - 2*(2*C*a^5*c^3 - 11*A*a^4*c^4 + 11*C*a^3*c^5 + 7*A*a^2*c^6 - 13*C*a*c^7 + 4*A*

$$\begin{aligned}
& c^8) \cos(ex + d)^3 + 6(2Ca^6c^2 - 9Aa^5c^3 + 7Ca^4c^4 + 8Aa^3c^5 \\
& - 10Ca^2c^6 + Aa^7 + Cc^8) \cos(ex + d) \sin(ex + d) + 3(2Aa^6c \\
& - 4Ca^5c^2 + 9Aa^4c^3 - 13Ca^3c^4 + 9Aa^2c^5 - 3Ca^6c - 3(2Aa^4c^3 \\
& - 4Ca^3c^4 + 3Aa^2c^5 - Cc^6) \cos(ex + d)^2 + (6Aa^5c^2 - 12Ca^4c^3 \\
& + 11Aa^3c^4 - 7Ca^2c^5 + 3Aa^6c - Cc^7 - (2Aa^3c^4 - 4Ca^2c^5 \\
& + 3Aa^6c - Cc^7) \cos(ex + d)^2) \sin(ex + d) \sqrt{-a^2 + c^2} \log((2a^2 - c^2) \cos(ex + d)^2 \\
& - 2ac \sin(ex + d) - a^2 - c^2 + 2(a \cos(ex + d) \sin(ex + d) + c \cos(ex + d)) \sqrt{-a^2 + c^2}) \\
& / (c^2 \cos(ex + d)^2 - 2ac \sin(ex + d) - a^2 - c^2) + 12(Ca^7c - 3Aa^6c^2 \\
& + Ca^5c^3 + 2Aa^4c^4 - 2Ca^6c^7 + Ac^8) \cos(ex + d) / (3(a^9c^3 - 4a^7c^5 \\
& + 6a^5c^7 - 4a^3c^9 + ac^{11}) e \cos(ex + d)^2 - (a^{11}c - a^9c^3 - 6a^7c^5 \\
& + 14a^5c^7 - 11a^3c^9 + 3ac^{11}) e + (a^8c^4 - 4a^6c^6 + 6a^4c^8 - 4a^2c^{10} \\
& + c^{12}) e \cos(ex + d)^2 - (3a^{10}c^2 - 11a^8c^4 + 14a^6c^6 - 6a^4c^8 - a^2c^{10} \\
& + c^{12}) e) \sin(ex + d), 1/6(2Ba^8 - 8Ba^6c^2 + 12Ba^4c^4 - 8Ba^2c^6 + 2Bc^8 \\
& - (2Ca^5c^3 - 11Aa^4c^4 + 11Ca^3c^5 + 7Aa^2c^6 - 13Ca^6c^7 + 4Aa^8c^8) \cos(ex + d)^3 \\
& + 3(2Ca^6c^2 - 9Aa^5c^3 + 7Ca^4c^4 + 8Aa^3c^5 - 10Ca^2c^6 + Aa^7 + Cc^8) \cos(ex + d) \sin(ex + d) \\
& + 3(2Aa^6c - 4Ca^5c^2 + 9Aa^4c^3 - 13Ca^3c^4 + 9Aa^2c^5 - 3Ca^6c - 3(2Aa^4c^3 \\
& - 4Ca^3c^4 + 3Aa^2c^5 - Cc^6) \cos(ex + d)^2 + (6Aa^5c^2 - 12Ca^4c^3 + 11Aa^3c^4 \\
& - 7Ca^2c^5 + 3Aa^6c - Cc^7 - (2Aa^3c^4 - 4Ca^2c^5 + 3Aa^6c - Cc^7) \cos(ex + d)^2) \sin(ex \\
& + d) \sqrt{a^2 - c^2} \arctan(-a \sin(ex + d) + c) / (\sqrt{a^2 - c^2} \cos(ex + d)) \\
& + 6(Ca^7c - 3Aa^6c^2 + Ca^5c^3 + 2Aa^4c^4 - 2Ca^6c^7 + Ac^8) \cos(ex + d) / (3(a^9c^3 - 4a^7c^5 \\
& + 6a^5c^7 - 4a^3c^9 + ac^{11}) e \cos(ex + d)^2 - (a^{11}c - a^9c^3 - 6a^7c^5 + 14a^5c^7 \\
& - 11a^3c^9 + 3ac^{11}) e + ((a^8c^4 - 4a^6c^6 + 6a^4c^8 - 4a^2c^{10} + c^{12}) e \cos(ex + d)^2 \\
& - (3a^{10}c^2 - 11a^8c^4 + 14a^6c^6 - 6a^4c^8 - a^2c^{10} + c^{12}) e) \sin(ex + d)]
\end{aligned}$$

giac [B] time = 0.31, size = 1340, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(ex+d)+C*sin(ex+d))/(a+c*sin(ex+d))^4,x, algorithm="giac")

[Out] $\frac{1}{3} (3(2Aa^3 - 4Ca^2c + 3Aa^6c^2 - Cc^3) (\pi \operatorname{floor}(1/2(xe + d)) / \pi + 1/2) \operatorname{sgn}(a) + \arctan((a \tan(1/2xe + 1/2d) + c) / \sqrt{a^2 - c^2})) / ((a^6 - 3a^4c^2 + 3a^2c^4 - c^6) \sqrt{a^2 - c^2}) + (6Ba^8 \tan(1/2xe + 1/2d)^5 - 12Ca^7c \tan(1/2xe + 1/2d)^5 + 27Aa^6c^2 \tan(1/2xe + 1/2d)^5 - 18Ba^6c^2 \tan(1/2xe + 1/2d)^5 - 3Ca^5c^3 \tan(1/2xe + 1/2d)^5 - 18Aa^4c^4 \tan(1/2xe + 1/2d)^5 + 18Ba^4c^4 \tan(1/2xe + 1/2d)^5 + 6Aa^2c^6 \tan(1/2xe + 1/2d)^5 - 6Ba^2c^6 \tan(1/2xe + 1/2d)^5 - 6Ca^8 \tan(1/2xe + 1/2d)^4 + 18Aa^7c \tan(1/2xe + 1/2d)^4 + 12Ba^7c \tan(1/2xe + 1/2d)^4 - 42Ca^6c^2 \tan(1/2xe + 1/2d)^4 + 81Aa^5c^3 \tan(1/2xe + 1/2d)^4 - 36Ba^5c^3 \tan(1/2xe + 1/2d)^4 - 33Ca^4c^4 \tan(1/2xe + 1/2d)^4 - 36Aa^3c^5 \tan(1/2xe + 1/2d)^4 + 36Ba^3c^5 \tan(1/2xe + 1/2d)^4 + 6Ca^2c^6 \tan(1/2xe + 1/2d)^4 + 12Aa^6c^7 \tan(1/2xe + 1/2d)^4 - 12Ba^6c^7 \tan(1/2xe + 1/2d)^4 + 12Ba^8 \tan(1/2xe + 1/2d)^3 - 36Ca^7c \tan(1/2xe + 1/2d)^3 + 108Aa^6c^2 \tan(1/2xe + 1/2d)^3 - 28Ba^6c^2 \tan(1/2xe + 1/2d)^3 - 84Ca^5c^3 \tan(1/2xe + 1/2d)^3 + 42Aa^4c^4 \tan(1/2xe + 1/2d)^3 + 12Ba^4c^4 \tan(1/2xe + 1/2d)^3 - 34Ca^3c^5 \tan(1/2xe + 1/2d)^3 - 8Aa^2c^6 \tan(1/2xe + 1/2d)^3 + 12Ba^2c^6 \tan(1/2xe + 1/2d)^3 + 4Ca^6c^7 \tan(1/2xe + 1/2d)^3 + 8Ac^8 \tan(1/2xe + 1/2d)^3 - 8Bc^8 \tan(1/2xe + 1/2d)^3 - 12Ca^8 \tan(1/2xe + 1/2d)^2 + 36Aa^7c \tan(1/2xe + 1/2d)^2 + 12Ba^7c \tan(1/2xe + 1/2d)^2 - 60Ca^6c^2 \tan(1/2xe + 1/2d)^2 + 120Aa^5c^3 \tan(1/2xe + 1/2d)^2 - 36Ba^5c^3 \tan(1/2xe + 1/2d)^2$

$$\begin{aligned} & n(1/2*x*e + 1/2*d)^2 - 84*C*a^4*c^4*\tan(1/2*x*e + 1/2*d)^2 - 18*A*a^3*c^5*t \\ & \tan(1/2*x*e + 1/2*d)^2 + 36*B*a^3*c^5*\tan(1/2*x*e + 1/2*d)^2 + 6*C*a^2*c^6*t \\ & \tan(1/2*x*e + 1/2*d)^2 + 12*A*a*c^7*\tan(1/2*x*e + 1/2*d)^2 - 12*B*a*c^7*\tan(\\ & 1/2*x*e + 1/2*d)^2 + 6*B*a^8*\tan(1/2*x*e + 1/2*d) - 24*C*a^7*c*\tan(1/2*x*e \\ & + 1/2*d) + 81*A*a^6*c^2*\tan(1/2*x*e + 1/2*d) - 18*B*a^6*c^2*\tan(1/2*x*e + 1 \\ & /2*d) - 57*C*a^5*c^3*\tan(1/2*x*e + 1/2*d) - 12*A*a^4*c^4*\tan(1/2*x*e + 1/2* \\ & d) + 18*B*a^4*c^4*\tan(1/2*x*e + 1/2*d) + 6*C*a^3*c^5*\tan(1/2*x*e + 1/2*d) + \\ & 6*A*a^2*c^6*\tan(1/2*x*e + 1/2*d) - 6*B*a^2*c^6*\tan(1/2*x*e + 1/2*d) - 6*C* \\ & a^8 + 18*A*a^7*c - 10*C*a^6*c^2 - 5*A*a^5*c^3 + C*a^4*c^4 + 2*A*a^3*c^5)/((\\ & a^9 - 3*a^7*c^2 + 3*a^5*c^4 - a^3*c^6)*(a*\tan(1/2*x*e + 1/2*d)^2 + 2*c*\tan(\\ & 1/2*x*e + 1/2*d) + a)^3))*e^{-1} \end{aligned}$$

maple [B] time = 0.46, size = 5051, normalized size = 19.58

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x)

[Out] result too large to display

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x, algorithm="ma
xima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*a^2>0)', see 'assume?' for
more details)Is 4*c^2-4*a^2 positive or negative?

mupad [B] time = 6.22, size = 1085, normalized size = 4.21

$$\frac{-6C a^5 + 18A a^4 c - 10C a^3 c^2 - 5A a^2 c^3 + C a c^4 + 2A c^5}{3(a^6 - 3a^4 c^2 + 3a^2 c^4 - c^6)} + \frac{\tan\left(\frac{d}{2} + \frac{ex}{2}\right)(2B a^6 + 2A c^6 - 2B c^6 - 4A a^2 c^4 + 27A a^4 c^2 + 6B a^2 c^4 - 6B a^4 c^2 - 19C a^3 c^3 + 2A a^2 c^5)}{a(a^6 - 3a^4 c^2 + 3a^2 c^4 - c^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*cos(d + e*x) + C*sin(d + e*x))/(a + c*sin(d + e*x))^4,x)

[Out] ((2*A*c^5 - 6*C*a^5 - 5*A*a^2*c^3 - 10*C*a^3*c^2 + 18*A*a^4*c + C*a*c^4)/(3
*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c^2)) + (tan(d/2 + (e*x)/2)*(2*B*a^6 + 2*A*
c^6 - 2*B*c^6 - 4*A*a^2*c^4 + 27*A*a^4*c^2 + 6*B*a^2*c^4 - 6*B*a^4*c^2 - 19
*C*a^3*c^3 + 2*C*a*c^5 - 8*C*a^5*c))/(a*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c^2)
) + (2*tan(d/2 + (e*x)/2)^2*(2*A*c^7 - 2*C*a^7 - 2*B*c^7 - 3*A*a^2*c^5 + 20
*A*a^4*c^3 + 6*B*a^2*c^5 - 6*B*a^4*c^3 - 14*C*a^3*c^4 - 10*C*a^5*c^2 + 6*A*
a^6*c + 2*B*a^6*c + C*a*c^6))/(a^2*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c^2)) + (ta
n(d/2 + (e*x)/2)^4*(4*A*c^7 - 2*C*a^7 - 4*B*c^7 - 12*A*a^2*c^5 + 27*A*a^4
*c^3 + 12*B*a^2*c^5 - 12*B*a^4*c^3 - 11*C*a^3*c^4 - 14*C*a^5*c^2 + 6*A*a^6*
c + 4*B*a^6*c + 2*C*a*c^6))/(a^2*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c^2)) - (ta
n(d/2 + (e*x)/2)^5*(2*B*c^6 - 2*A*c^6 - 2*B*a^6 + 6*A*a^2*c^4 - 9*A*a^4*c^2
- 6*B*a^2*c^4 + 6*B*a^4*c^2 + C*a^3*c^3 + 4*C*a^5*c))/(a*(a^6 - c^6 + 3*a^

$$2c^4 - 3a^4c^2)) + (2\tan(d/2 + (e*x)/2)^3(3a^2 + 2c^2)(2B*a^6 + 2A*c^6 - 2B*c^6 - 5A*a^2*c^4 + 18A*a^4*c^2 + 6B*a^2*c^4 - 6B*a^4*c^2 - 10C*a^3*c^3 + C*a*c^5 - 6C*a^5*c))/(3a^3(a^6 - c^6 + 3a^2*c^4 - 3a^4*c^2)))/(e*(a^3*\tan(d/2 + (e*x)/2)^6 + \tan(d/2 + (e*x)/2)^2*(12a*c^2 + 3a^3) + \tan(d/2 + (e*x)/2)^4*(12a*c^2 + 3a^3) + \tan(d/2 + (e*x)/2)^3*(12a^2*c + 8c^3) + a^3 + 6a^2*c*\tan(d/2 + (e*x)/2) + 6a^2*c*\tan(d/2 + (e*x)/2)^5)) + (\operatorname{atan}((((2A*a^3 - C*c^3 + 3A*a*c^2 - 4C*a^2*c)*(2a^6*c - 2c^7 + 6a^2*c^5 - 6a^4*c^3))/(2*(a + c)^{(7/2)}*(a - c)^{(7/2)}*(a^6 - c^6 + 3a^2*c^4 - 3a^4*c^2)) + (a*\tan(d/2 + (e*x)/2)*(2A*a^3 - C*c^3 + 3A*a*c^2 - 4C*a^2*c)))/((a + c)^{(7/2)}*(a - c)^{(7/2)))*(a^6 - c^6 + 3a^2*c^4 - 3a^4*c^2))/(2A*a^3 - C*c^3 + 3A*a*c^2 - 4C*a^2*c))/(e*(a + c)^{(7/2)}*(a - c)^{(7/2))}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))**4,x)

[Out] Timed out

3.566 $\int (a + b \cos(c + dx) \sin(c + dx))^m dx$

Optimal. Leaf size=131

$$\frac{\cos(2c + 2dx) \left(a + \frac{1}{2}b \sin(2c + 2dx)\right)^m \left(\frac{2a+b \sin(2c+2dx)}{2a+b}\right)^{-m} F_1\left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1 - \sin(2c + 2dx))\right), \frac{b(1-\sin(2c+2dx))}{2a+b}}{\sqrt{2d} \sqrt{\sin(2c + 2dx) + 1}}$$

[Out] $-1/2 * \text{AppellF1}(1/2, -m, 1/2, 3/2, b*(1-\sin(2*d*x+2*c))/(2*a+b), 1/2-1/2*\sin(2*d*x+2*c)) * \cos(2*d*x+2*c) * (a+1/2*b*\sin(2*d*x+2*c))^m / d / (((2*a+b*\sin(2*d*x+2*c)) / (2*a+b))^m) * 2^{(1/2)} / (1+\sin(2*d*x+2*c))^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2666, 2665, 139, 138}

$$\frac{\cos(2c + 2dx) \left(a + \frac{1}{2}b \sin(2c + 2dx)\right)^m \left(\frac{2a+b \sin(2c+2dx)}{2a+b}\right)^{-m} F_1\left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1 - \sin(2c + 2dx))\right), \frac{b(1-\sin(2c+2dx))}{2a+b}}{\sqrt{2d} \sqrt{\sin(2c + 2dx) + 1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^m, x]

[Out] $-((\text{AppellF1}[1/2, 1/2, -m, 3/2, (1 - \text{Sin}[2*c + 2*d*x])/2, (b*(1 - \text{Sin}[2*c + 2*d*x]))/(2*a + b)] * \text{Cos}[2*c + 2*d*x] * (a + (b*\text{Sin}[2*c + 2*d*x])/2)^m) / (\text{Sqrt}[2] * d * \text{Sqrt}[1 + \text{Sin}[2*c + 2*d*x]]) * ((2*a + b*\text{Sin}[2*c + 2*d*x]) / (2*a + b))^m)$

Rule 138

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0]) && SimplerQ[c + d*x, a + b*x] && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])

Rule 139

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 2665

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rule 2666

Int[((a_) + cos[(c_) + (d_)*(x_)]*(b_)*sin[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Int[(a + (b*SIN[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n}, x]

Rubi steps

$$\begin{aligned}
\int (a + b \cos(c + dx) \sin(c + dx))^m dx &= \int \left(a + \frac{1}{2} b \sin(2c + 2dx) \right)^m dx \\
&= \frac{\cos(2c + 2dx) \operatorname{Subst} \left(\int \frac{\left(a + \frac{bx}{2} \right)^m}{\sqrt{1-x} \sqrt{1+x}} dx, x, \sin(2c + 2dx) \right)}{2d \sqrt{1 - \sin(2c + 2dx)} \sqrt{1 + \sin(2c + 2dx)}} \\
&= \frac{\left(\cos(2c + 2dx) \left(a + \frac{1}{2} b \sin(2c + 2dx) \right)^m \left(-\frac{a + \frac{1}{2} b \sin(2c + 2dx)}{-a - \frac{b}{2}} \right)^{-m} \right) \operatorname{Subst} \left(\int \frac{\left(a + \frac{bx}{2} \right)^m}{\sqrt{1-x} \sqrt{1+x}} dx, x, \sin(2c + 2dx) \right)}{2d \sqrt{1 - \sin(2c + 2dx)} \sqrt{1 + \sin(2c + 2dx)}} \\
&= -\frac{F_1 \left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2} (1 - \sin(2c + 2dx)), \frac{b(1 - \sin(2c + 2dx))}{2a + b} \right) \cos(2c + 2dx) \left(a + \frac{1}{2} b \sin(2c + 2dx) \right)^m}{\sqrt{2} d \sqrt{1 + \sin(2c + 2dx)}}
\end{aligned}$$

Mathematica [A] time = 0.59, size = 145, normalized size = 1.11

$$\frac{\sec(2(c + dx)) \sqrt{-\frac{b(\sin(2(c+dx))-1)}{2a+b}} \sqrt{\frac{b(\sin(2(c+dx))+1)}{b-2a}} \left(a + \frac{1}{2} b \sin(2(c + dx)) \right)^{m+1} F_1 \left(m + 1; \frac{1}{2}, \frac{1}{2}; m + 2; \frac{2a+b \sin(2(c+dx))}{2a-b} \right)}{bd(m + 1)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^m,x]

[Out] (AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*a + b*Sin[2*(c + d*x)])/(2*a - b), (2*a + b*Sin[2*(c + d*x)])/(2*a + b)]*Sec[2*(c + d*x)]*Sqrt[-((b*(-1 + Sin[2*(c + d*x)]))/(2*a + b))]*Sqrt[(b*(1 + Sin[2*(c + d*x)]))/(-2*a + b)]*(a + (b*Sin[2*(c + d*x)]/2)^(1 + m))/(b*d*(1 + m))

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\operatorname{integral} \left((b \cos(dx + c) \sin(dx + c) + a)^m, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^m,x, algorithm="fricas")

[Out] integral((b*cos(d*x + c)*sin(d*x + c) + a)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cos(dx + c) \sin(dx + c) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^m,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^m, x)

maple [F] time = 1.11, size = 0, normalized size = 0.00

$$\int (a + b \cos(dx + c) \sin(dx + c))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c)*sin(d*x+c))^m,x)

[Out] int((a+b*cos(d*x+c)*sin(d*x+c))^m,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cos(dx + c) \sin(dx + c) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^m,x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \cos(c + dx) \sin(c + dx))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(c + d*x)*sin(c + d*x))^m,x)

[Out] int((a + b*cos(c + d*x)*sin(c + d*x))^m, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**m,x)

[Out] Timed out

3.567 $\int (a + b \cos(c + dx) \sin(c + dx))^3 dx$

Optimal. Leaf size=107

$$-\frac{b(16a^2 + b^2) \cos(2c + 2dx)}{24d} + \frac{1}{8}ax(8a^2 + 3b^2) - \frac{5ab^2 \sin(2c + 2dx) \cos(2c + 2dx)}{48d} - \frac{b \cos(2c + 2dx)(2a + b \sin(2c + 2dx))}{48d}$$

[Out] $\frac{1}{8}ax(8a^2 + 3b^2) - \frac{1}{24}b(16a^2 + b^2)\cos(2dx + 2c)/d - \frac{5}{48}ab^2\cos(2c + 2dx)\sin(2c + 2dx)/d - \frac{1}{48}b\cos(2c + 2dx)(2a + b\sin(2c + 2dx))^2/d$

Rubi [A] time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2666, 2656, 2734}

$$-\frac{b(16a^2 + b^2) \cos(2c + 2dx)}{24d} + \frac{1}{8}ax(8a^2 + 3b^2) - \frac{5ab^2 \sin(2c + 2dx) \cos(2c + 2dx)}{48d} - \frac{b \cos(2c + 2dx)(2a + b \sin(2c + 2dx))}{48d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^3, x]

[Out] $(a(8a^2 + 3b^2)x)/8 - (b(16a^2 + b^2)\cos[2c + 2dx])/(24d) - (5ab^2\cos[2c + 2dx]\sin[2c + 2dx])/(48d) - (b\cos[2c + 2dx](2a + b\sin[2c + 2dx])^2)/(48d)$

Rule 2656

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2666

Int[((a_) + cos[(c_) + (d_)*(x_)])*(b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n}, x]

Rule 2734

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[(b*c + a*d)*Cos[e + f*x]/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int (a + b \cos(c + dx) \sin(c + dx))^3 dx &= \int \left(a + \frac{1}{2}b \sin(2c + 2dx) \right)^3 dx \\ &= -\frac{b \cos(2c + 2dx)(2a + b \sin(2c + 2dx))^2}{48d} + \frac{1}{3} \int \left(a + \frac{1}{2}b \sin(2c + 2dx) \right) \left(a + \frac{1}{2}b \sin(2c + 2dx) \right)^2 dx \\ &= \frac{1}{8}a(8a^2 + 3b^2)x - \frac{b(16a^2 + b^2) \cos(2c + 2dx)}{24d} - \frac{5ab^2 \cos(2c + 2dx) \sin(2c + 2dx)}{48d} \end{aligned}$$

Mathematica [A] time = 0.27, size = 75, normalized size = 0.70

$$\frac{-9(16a^2b + b^3)\cos(2(c + dx)) + 6a(4(8a^2 + 3b^2)(c + dx) - 3b^2\sin(4(c + dx))) + b^3\cos(6(c + dx))}{192d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^3,x]

[Out] (-9*(16*a^2*b + b^3)*Cos[2*(c + d*x)] + b^3*Cos[6*(c + d*x)] + 6*a*(4*(8*a^2 + 3*b^2)*(c + d*x) - 3*b^2*Sin[4*(c + d*x)]))/(192*d)

fricas [A] time = 1.75, size = 97, normalized size = 0.91

$$\frac{4b^3\cos(dx + c)^6 - 6b^3\cos(dx + c)^4 - 36a^2b\cos(dx + c)^2 + 3(8a^3 + 3ab^2)dx - 9(2ab^2\cos(dx + c)^3 - ab^2)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/24*(4*b^3*cos(d*x + c)^6 - 6*b^3*cos(d*x + c)^4 - 36*a^2*b*cos(d*x + c)^2 + 3*(8*a^3 + 3*a*b^2)*d*x - 9*(2*a*b^2*cos(d*x + c)^3 - a*b^2*cos(d*x + c))*sin(d*x + c))/d

giac [A] time = 0.19, size = 75, normalized size = 0.70

$$\frac{b^3\cos(6dx + 6c)}{192d} - \frac{3ab^2\sin(4dx + 4c)}{32d} + \frac{1}{8}(8a^3 + 3ab^2)x - \frac{3(16a^2b + b^3)\cos(2dx + 2c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/192*b^3*cos(6*d*x + 6*c)/d - 3/32*a*b^2*sin(4*d*x + 4*c)/d + 1/8*(8*a^3 + 3*a*b^2)*x - 3/64*(16*a^2*b + b^3)*cos(2*d*x + 2*c)/d

maple [A] time = 0.25, size = 106, normalized size = 0.99

$$\frac{b^3\left(-\frac{(\sin^2(dx+c))(\cos^4(dx+c))}{6} - \frac{(\cos^4(dx+c))}{12}\right) + 3ab^2\left(-\frac{\sin(dx+c)(\cos^3(dx+c))}{4} + \frac{\sin(dx+c)\cos(dx+c)}{8} + \frac{dx}{8} + \frac{c}{8}\right) - \frac{3(\cos^2(dx+c))}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c)*sin(d*x+c))^3,x)

[Out] 1/d*(b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+3*a*b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/8*c)-3/2*cos(d*x+c)^2*a^2*b+a^3*(d*x+c))

maxima [A] time = 0.38, size = 80, normalized size = 0.75

$$a^3x - \frac{3a^2b\cos(dx + c)^2}{2d} + \frac{3(4dx + 4c - \sin(4dx + 4c))ab^2}{32d} - \frac{(2\sin(dx + c)^6 - 3\sin(dx + c)^4)b^3}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="maxima")

[Out] a^3*x - 3/2*a^2*b*cos(d*x + c)^2/d + 3/32*(4*d*x + 4*c - sin(4*d*x + 4*c))*a*b^2/d - 1/12*(2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*b^3/d

mupad [B] time = 3.44, size = 125, normalized size = 1.17

$$a^3 x - \frac{\tan(c + dx)^2 (72 a^2 b + 6 b^3) + 36 a^2 b + 2 b^3 + 36 a^2 b \tan(c + dx)^4 - 9 a b^2 \tan(c + dx)^5 + 9 a b^2 \tan(c + dx)^6}{d (24 \tan(c + dx)^6 + 72 \tan(c + dx)^4 + 72 \tan(c + dx)^2 + 24)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(c + d*x)*sin(c + d*x))^3,x)

[Out] $a^3 x - (\tan(c + dx)^2 (72 a^2 b + 6 b^3) + 36 a^2 b + 2 b^3 + 36 a^2 b \tan(c + dx)^4 - 9 a b^2 \tan(c + dx)^5 + 9 a b^2 \tan(c + dx)^6) / (d (72 \tan(c + dx)^2 + 72 \tan(c + dx)^4 + 24 \tan(c + dx)^6 + 24)) + (3 a b^2 x) / 8$

sympy [A] time = 3.36, size = 190, normalized size = 1.78

$$\left\{ \begin{array}{l} a^3 x - \frac{3 a^2 b \cos^2(c+dx)}{2d} + \frac{3 a b^2 x \sin^4(c+dx)}{8} + \frac{3 a b^2 x \sin^2(c+dx) \cos^2(c+dx)}{4} + \frac{3 a b^2 x \cos^4(c+dx)}{8} + \frac{3 a b^2 \sin^3(c+dx) \cos(c+dx)}{8d} - \frac{3 a b^2 \sin(c+dx)}{8d} \\ x (a + b \sin(c) \cos(c))^3 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^3,x)

[Out] Piecewise(($a^3 x - 3 a^2 b \cos(c + dx)^2 / (2 d) + 3 a b^2 x \sin(c + dx)^4 / 8 + 3 a b^2 x \sin(c + dx)^2 \cos(c + dx)^2 / 4 + 3 a b^2 x \cos(c + dx)^4 / 8 + 3 a b^2 \sin(c + dx)^3 \cos(c + dx) / (8 d) - 3 a b^2 \sin(c + dx) \cos(c + dx)^3 / (8 d) + b^3 \sin(c + dx)^6 / (12 d) + b^3 \sin(c + dx)^4 \cos(c + dx)^2 / (4 d)$), Ne(d, 0)), (x*(a + b*sin(c)*cos(c))^3, True))

3.568 $\int (a + b \cos(c + dx) \sin(c + dx))^2 dx$

Optimal. Leaf size=61

$$\frac{1}{8}x(8a^2 + b^2) - \frac{ab \cos(2c + 2dx)}{2d} - \frac{b^2 \sin(2c + 2dx) \cos(2c + 2dx)}{16d}$$

[Out] 1/8*(8*a^2+b^2)*x-1/2*a*b*cos(2*d*x+2*c)/d-1/16*b^2*cos(2*d*x+2*c)*sin(2*d*x+2*c)/d

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2666, 2644}

$$\frac{1}{8}x(8a^2 + b^2) - \frac{ab \cos(2c + 2dx)}{2d} - \frac{b^2 \sin(2c + 2dx) \cos(2c + 2dx)}{16d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^2,x]

[Out] ((8*a^2 + b^2)*x)/8 - (a*b*Cos[2*c + 2*d*x])/(2*d) - (b^2*Cos[2*c + 2*d*x]*Sin[2*c + 2*d*x])/(16*d)

Rule 2644

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^2, x_Symbol] := Simp[((2*a^2 + b^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*Sin[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]

Rule 2666

Int[((a_) + cos[(c_) + (d_)*(x_)])*(b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n}, x]

Rubi steps

$$\begin{aligned} \int (a + b \cos(c + dx) \sin(c + dx))^2 dx &= \int \left(a + \frac{1}{2}b \sin(2c + 2dx) \right)^2 dx \\ &= \frac{1}{8}(8a^2 + b^2)x - \frac{ab \cos(2c + 2dx)}{2d} - \frac{b^2 \cos(2c + 2dx) \sin(2c + 2dx)}{16d} \end{aligned}$$

Mathematica [A] time = 0.15, size = 48, normalized size = 0.79

$$\frac{-4(8a^2 + b^2)(c + dx) + 16ab \cos(2(c + dx)) + b^2 \sin(4(c + dx))}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^2,x]

[Out] -1/32*(-4*(8*a^2 + b^2)*(c + d*x) + 16*a*b*Cos[2*(c + d*x)] + b^2*Sin[4*(c + d*x)])/d

fricas [A] time = 2.40, size = 63, normalized size = 1.03

$$\frac{8ab \cos(dx + c)^2 - (8a^2 + b^2)dx + (2b^2 \cos(dx + c)^3 - b^2 \cos(dx + c)) \sin(dx + c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="fricas")

[Out] $-1/8*(8*a*b*cos(d*x + c)^2 - (8*a^2 + b^2)*d*x + (2*b^2*cos(d*x + c)^3 - b^2*cos(d*x + c))*sin(d*x + c))/d$

giac [A] time = 0.16, size = 46, normalized size = 0.75

$$\frac{1}{8} (8a^2 + b^2)x - \frac{ab \cos(2dx + 2c)}{2d} - \frac{b^2 \sin(4dx + 4c)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="giac")

[Out] $1/8*(8*a^2 + b^2)*x - 1/2*a*b*cos(2*d*x + 2*c)/d - 1/32*b^2*sin(4*d*x + 4*c)/d$

maple [A] time = 0.24, size = 69, normalized size = 1.13

$$\frac{b^2 \left(-\frac{\sin(dx+c)\cos^3(dx+c)}{4} + \frac{\sin(dx+c)\cos(dx+c)}{8} + \frac{dx}{8} + \frac{c}{8} \right) - (\cos^2(dx+c))ab + a^2(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c)*sin(d*x+c))^2,x)

[Out] $1/d*(b^2*(-1/4*\sin(d*x+c)*\cos(d*x+c)^3+1/8*\sin(d*x+c)*\cos(d*x+c)+1/8*d*x+1/8*c)-\cos(d*x+c)^2*a*b+a^2*(d*x+c))$

maxima [A] time = 0.33, size = 48, normalized size = 0.79

$$a^2x - \frac{ab \cos(dx + c)^2}{d} + \frac{(4dx + 4c - \sin(4dx + 4c))b^2}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="maxima")

[Out] $a^2*x - a*b*cos(d*x + c)^2/d + 1/32*(4*d*x + 4*c - \sin(4*d*x + 4*c))*b^2/d$

mupad [B] time = 3.03, size = 78, normalized size = 1.28

$$x \left(a^2 + \frac{b^2}{8} \right) - \frac{-\frac{b^2 \tan(c+dx)^3}{8} + \frac{b^2 \tan(c+dx)}{8} + ab \tan(c+dx)^2 + ab}{d (\tan(c+dx)^4 + 2 \tan(c+dx)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(c + d*x)*sin(c + d*x))^2,x)

[Out] $x*(a^2 + b^2/8) - (a*b + (b^2*tan(c + d*x))/8 - (b^2*tan(c + d*x)^3)/8 + a*b*tan(c + d*x)^2)/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))$

sympy [A] time = 0.95, size = 129, normalized size = 2.11

$$\begin{cases} a^2x - \frac{ab \cos^2(c+dx)}{d} + \frac{b^2x \sin^4(c+dx)}{8} + \frac{b^2x \sin^2(c+dx) \cos^2(c+dx)}{4} + \frac{b^2x \cos^4(c+dx)}{8} + \frac{b^2 \sin^3(c+dx) \cos(c+dx)}{8d} - \frac{b^2 \sin(c+dx) \cos^3(c+dx)}{8d} \\ x(a + b \sin(c) \cos(c))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x - a*b*cos(c + d*x)**2/d + b**2*x*sin(c + d*x)**4/8 + b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + b**2*x*cos(c + d*x)**4/8 + b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*sin(c)*cos(c))**2, True))
```

3.569 $\int (a + b \cos(c + dx) \sin(c + dx)) dx$

Optimal. Leaf size=20

$$ax + \frac{b \sin^2(c + dx)}{2d}$$

[Out] a*x+1/2*b*sin(d*x+c)^2/d

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2564, 30}

$$ax + \frac{b \sin^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[a + b*Cos[c + d*x]*Sin[c + d*x],x]

[Out] a*x + (b*Sin[c + d*x]^2)/(2*d)

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2564

Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && LtQ[0, m, n])

Rubi steps

$$\begin{aligned} \int (a + b \cos(c + dx) \sin(c + dx)) dx &= ax + b \int \cos(c + dx) \sin(c + dx) dx \\ &= ax + \frac{b \text{Subst}(\int x dx, x, \sin(c + dx))}{d} \\ &= ax + \frac{b \sin^2(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.90

$$ax + \frac{b \sin(2c) \sin(2dx)}{4d} - \frac{b \cos(2c) \cos(2dx)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*Cos[c + d*x]*Sin[c + d*x],x]

[Out] a*x - (b*Cos[2*c]*Cos[2*d*x])/(4*d) + (b*Sin[2*c]*Sin[2*d*x])/(4*d)

fricas [A] time = 0.75, size = 22, normalized size = 1.10

$$\frac{2 adx - b \cos(dx + c)^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*cos(d*x+c)*sin(d*x+c),x, algorithm="fricas")

[Out] 1/2*(2*a*d*x - b*cos(d*x + c)^2)/d

giac [A] time = 0.13, size = 18, normalized size = 0.90

$$ax + \frac{b \sin(dx + c)^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*cos(d*x+c)*sin(d*x+c),x, algorithm="giac")

[Out] a*x + 1/2*b*sin(d*x + c)^2/d

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$ax + \frac{b(\sin^2(dx + c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b*cos(d*x+c)*sin(d*x+c),x)

[Out] a*x+1/2*b*sin(d*x+c)^2/d

maxima [A] time = 0.48, size = 18, normalized size = 0.90

$$ax - \frac{b \cos(dx + c)^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*cos(d*x+c)*sin(d*x+c),x, algorithm="maxima")

[Out] a*x - 1/2*b*cos(d*x + c)^2/d

mupad [B] time = 2.94, size = 22, normalized size = 1.10

$$-\frac{\frac{b \cos(c+dx)^2}{2} - a dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a + b*cos(c + d*x)*sin(c + d*x),x)

[Out] -((b*cos(c + d*x)^2)/2 - a*d*x)/d

sympy [A] time = 0.18, size = 26, normalized size = 1.30

$$ax + b \left(\begin{array}{ll} -\frac{\cos^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x \sin(c) \cos(c) & \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*cos(d*x+c)*sin(d*x+c),x)

[Out] a*x + b*Piecewise((-cos(c + d*x)**2/(2*d), Ne(d, 0)), (x*sin(c)*cos(c), True))

$$3.570 \quad \int \frac{1}{a+b \cos(c+dx) \sin(c+dx)} dx$$

Optimal. Leaf size=48

$$\frac{2 \tan^{-1} \left(\frac{2a \tan(c+dx)+b}{\sqrt{4a^2-b^2}} \right)}{d\sqrt{4a^2-b^2}}$$

[Out] 2*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))/d/(4*a^2-b^2)^(1/2)

Rubi [A] time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2666, 2660, 618, 204}

$$\frac{2 \tan^{-1} \left(\frac{2a \tan(c+dx)+b}{\sqrt{4a^2-b^2}} \right)}{d\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-1), x]

[Out] (2*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(Sqrt[4*a^2 - b^2]*d)

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2666

Int[((a_) + cos[(c_.) + (d_.)*(x_)])*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n}, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{a + b \cos(c + dx) \sin(c + dx)} dx &= \int \frac{1}{a + \frac{1}{2}b \sin(2c + 2dx)} dx \\
&= \frac{\text{Subst}\left(\int \frac{1}{a+bx+ax^2} dx, x, \tan\left(\frac{1}{2}(2c + 2dx)\right)\right)}{d} \\
&= \frac{2 \text{Subst}\left(\int \frac{1}{-4a^2+b^2-x^2} dx, x, b + 2a \tan\left(\frac{1}{2}(2c + 2dx)\right)\right)}{d} \\
&= \frac{2 \tan^{-1}\left(\frac{b+2a \tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2} d}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 48, normalized size = 1.00

$$\frac{2 \tan^{-1}\left(\frac{2a \tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-1), x]

[Out] (2*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(Sqrt[4*a^2 - b^2]*d)

fricas [A] time = 0.84, size = 290, normalized size = 6.04

$$\frac{\sqrt{-4a^2 + b^2} \log\left(-\frac{2(8a^2 - b^2)\cos(dx+c)^4 - 4ab\cos(dx+c)\sin(dx+c) - 2(8a^2 - b^2)\cos(dx+c)^2 + 2a^2 - b^2 + (2b\cos(dx+c)^2 + 4(2a\cos(dx+c) - a^2)\sin(dx+c))}{b^2\cos(dx+c)^4 - b^2\cos(dx+c)^2 - 2ab\cos(dx+c)\sin(dx+c) - a^2}\right)}{2(4a^2 - b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)), x, algorithm="fricas")

[Out] [-1/2*sqrt(-4*a^2 + b^2)*log(-(2*(8*a^2 - b^2)*cos(d*x + c)^4 - 4*a*b*cos(d*x + c)*sin(d*x + c) - 2*(8*a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 - b^2 + (2*b*cos(d*x + c)^2 + 4*(2*a*cos(d*x + c)^3 - a*cos(d*x + c))*sin(d*x + c) - b)*sqrt(-4*a^2 + b^2))/(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) - a^2))/((4*a^2 - b^2)*d), -arctan(-(4*a*cos(d*x + c)*sin(d*x + c) + b)*sqrt(4*a^2 - b^2)/(2*(4*a^2 - b^2)*cos(d*x + c)^2 - 4*a^2 + b^2))/(sqrt(4*a^2 - b^2)*d)]

giac [A] time = 0.16, size = 61, normalized size = 1.27

$$\frac{2\left(\pi\left\lfloor\frac{dx+c}{\pi} + \frac{1}{2}\right\rfloor \text{sgn}(a) + \arctan\left(\frac{2a \tan(dx+c)+b}{\sqrt{4a^2-b^2}}\right)\right)}{\sqrt{4a^2-b^2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)), x, algorithm="giac")

[Out] 2*(pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan((2*a*tan(d*x + c) + b)/sqrt(4*a^2 - b^2)))/(sqrt(4*a^2 - b^2)*d)

maple [A] time = 0.31, size = 45, normalized size = 0.94

$$\frac{2 \arctan\left(\frac{b+2a \tan(dx+c)}{\sqrt{4a^2-b^2}}\right)}{d\sqrt{4a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*cos(d*x+c)*sin(d*x+c)),x)`

[Out] `2*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))/d/(4*a^2-b^2)^(1/2)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(b^2-4*a^2>0)', see 'assume?' for more details)Is b^2-4*a^2 positive or negative?

mupad [B] time = 3.11, size = 44, normalized size = 0.92

$$\frac{2 \operatorname{atan}\left(\frac{b+2a \tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{d \sqrt{4a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b*cos(c + d*x)*sin(c + d*x)),x)`

[Out] `(2*atan((b + 2*a*tan(c + d*x))/(4*a^2 - b^2)^(1/2)))/(d*(4*a^2 - b^2)^(1/2))`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x)`

[Out] Timed out

$$3.571 \quad \int \frac{1}{(a+b \cos(c+dx) \sin(c+dx))^2} dx$$

Optimal. Leaf size=95

$$\frac{8a \tan^{-1}\left(\frac{2a \tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d(4a^2-b^2)^{3/2}} + \frac{2b \cos(2c+2dx)}{d(4a^2-b^2)(2a+b \sin(2c+2dx))}$$

[Out] $8*a*\arctan((b+2*a*\tan(d*x+c))/(4*a^2-b^2)^{(1/2)})/(4*a^2-b^2)^{(3/2)}/d+2*b*\cos(2*d*x+2*c)/(4*a^2-b^2)/d/(2*a+b*\sin(2*d*x+2*c))$

Rubi [A] time = 0.11, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2666, 2664, 12, 2660, 618, 204}

$$\frac{8a \tan^{-1}\left(\frac{2a \tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d(4a^2-b^2)^{3/2}} + \frac{2b \cos(2c+2dx)}{d(4a^2-b^2)(2a+b \sin(2c+2dx))}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-2), x]

[Out] $(8*a*\text{ArcTan}[(b + 2*a*\text{Tan}[c + d*x])/ \text{Sqrt}[4*a^2 - b^2]])/((4*a^2 - b^2)^{(3/2)}*d) + (2*b*\text{Cos}[2*c + 2*d*x])/((4*a^2 - b^2)*d*(2*a + b*\text{Sin}[2*c + 2*d*x]))$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2664

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2666

```
Int[((a_) + cos[(c_) + (d_)*(x_)]*(b_)*sin[(c_) + (d_)*(x_)])^(n_), x_
Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \cos(c + dx) \sin(c + dx))^2} dx &= \int \frac{1}{\left(a + \frac{1}{2}b \sin(2c + 2dx)\right)^2} dx \\
&= \frac{2b \cos(2c + 2dx)}{(4a^2 - b^2) d(2a + b \sin(2c + 2dx))} + \frac{4 \int \frac{a}{a + \frac{1}{2}b \sin(2c + 2dx)} dx}{4a^2 - b^2} \\
&= \frac{2b \cos(2c + 2dx)}{(4a^2 - b^2) d(2a + b \sin(2c + 2dx))} + \frac{(4a) \int \frac{1}{a + \frac{1}{2}b \sin(2c + 2dx)} dx}{4a^2 - b^2} \\
&= \frac{2b \cos(2c + 2dx)}{(4a^2 - b^2) d(2a + b \sin(2c + 2dx))} + \frac{(4a) \operatorname{Subst}\left(\int \frac{1}{a + bx + ax^2} dx, x, \tan\left(\frac{1}{2}\right)\right)}{(4a^2 - b^2) d} \\
&= \frac{2b \cos(2c + 2dx)}{(4a^2 - b^2) d(2a + b \sin(2c + 2dx))} - \frac{(8a) \operatorname{Subst}\left(\int \frac{1}{-4a^2 + b^2 - x^2} dx, x, b + 2\right)}{(4a^2 - b^2) d} \\
&= \frac{8a \tan^{-1}\left(\frac{b + 2a \tan(c + dx)}{\sqrt{4a^2 - b^2}}\right)}{(4a^2 - b^2)^{3/2} d} + \frac{2b \cos(2c + 2dx)}{(4a^2 - b^2) d(2a + b \sin(2c + 2dx))}
\end{aligned}$$

Mathematica [A] time = 0.41, size = 94, normalized size = 0.99

$$\frac{2 \left(\frac{4a \tan^{-1}\left(\frac{2a \tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{(4a^2-b^2)^{3/2}} + \frac{b \cos(2(c+dx))}{(2a-b)(2a+b)(2a+b \sin(2(c+dx)))} \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-2), x]
```

```
[Out] (2*((4*a*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(4*a^2 - b^2)^(3/2) + (b*Cos[2*(c + d*x)])/((2*a - b)*(2*a + b)*(2*a + b*Sin[2*(c + d*x)]))) / d
```

fricas [B] time = 1.45, size = 493, normalized size = 5.19

$$\left[\frac{4a^2b - b^3 - 2(4a^2b - b^3) \cos(dx + c)^2 - 2(ab \cos(dx + c) \sin(dx + c) + a^2) \sqrt{-4a^2 + b^2} \log\left(\frac{2(8a^2 - b^2) \cos(dx + c)}{(16a^4b - 8a^2b^3 + b^5)d \cos(dx + c) \sin(dx + c)}\right)}{(16a^4b - 8a^2b^3 + b^5)d \cos(dx + c) \sin(dx + c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-(4*a^2*b - b^3 - 2*(4*a^2*b - b^3)*cos(d*x + c)^2 - 2*(a*b*cos(d*x + c)*sin(d*x + c) + a^2)*sqrt(-4*a^2 + b^2)*log((2*(8*a^2 - b^2)*cos(d*x + c)^4 - 4*a*b*cos(d*x + c)*sin(d*x + c) - 2*(8*a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 - b^2 - (2*b*cos(d*x + c)^2 + 4*(2*a*cos(d*x + c))^3 - a*cos(d*x + c))*sin(d*x + c)))]
```


$$\frac{(x+c-b)\sqrt{-4a^2+b^2}}{(b^2\cos(dx+c)^4 - b^2\cos(dx+c)^2 - 2ab\cos(dx+c)\sin(dx+c) - a^2)} \left/ \left((16a^4b - 8a^2b^3 + b^5)d\cos(dx+c)\sin(dx+c) + (16a^5 - 8a^3b^2 + ab^4)d \right) \right. \\ \left. - (4a^2b - b^3 - 2(4a^2b - b^3)\cos(dx+c)^2 + 4(ab\cos(dx+c)\sin(dx+c) + a^2))\sqrt{4a^2 - b^2} \arctan\left(\frac{-4a\cos(dx+c)\sin(dx+c) + b}{\sqrt{4a^2 - b^2}}\right) \right/ \left((16a^4b - 8a^2b^3 + b^5)d\cos(dx+c)\sin(dx+c) + (16a^5 - 8a^3b^2 + ab^4)d \right)}$$

giac [A] time = 0.18, size = 116, normalized size = 1.22

$$\frac{8\left(\pi\left[\frac{dx+c}{\pi} + \frac{1}{2}\right]\operatorname{sgn}(a) + \arctan\left(\frac{2a\tan(dx+c)+b}{\sqrt{4a^2-b^2}}\right)\right)a}{(4a^2-b^2)^{\frac{3}{2}}} + \frac{b^2\tan(dx+c)+2ab}{(4a^3-ab^2)(a\tan(dx+c)^2+b\tan(dx+c)+a)}$$

d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(dx+c)*sin(dx+c))^2,x, algorithm="giac")

[Out] (8*(pi*floor((dx+c)/pi + 1/2)*sgn(a) + arctan((2*a*tan(dx+c) + b)/sqrt(4*a^2 - b^2)))*a/(4*a^2 - b^2)^(3/2) + (b^2*tan(dx+c) + 2*a*b)/((4*a^3 - a*b^2)*(a*tan(dx+c)^2 + b*tan(dx+c) + a)))/d

maple [A] time = 0.44, size = 139, normalized size = 1.46

$$\frac{b^2 \tan(dx+c)}{d \left((\tan^2(dx+c)) a + b \tan(dx+c) + a \right) a (4a^2 - b^2)} + \frac{2b}{d \left((\tan^2(dx+c)) a + b \tan(dx+c) + a \right) (4a^2 - b^2)} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cos(dx+c)*sin(dx+c))^2,x)

[Out] 1/d/(tan(dx+c)^2*a+b*tan(dx+c)+a)*b^2/a/(4*a^2-b^2)*tan(dx+c)+2/d/(tan(dx+c)^2*a+b*tan(dx+c)+a)*b/(4*a^2-b^2)+8*a*arctan((b+2*a*tan(dx+c))/(4*a^2-b^2)^(1/2))/(4*a^2-b^2)^(3/2)/d

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(dx+c)*sin(dx+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(b^2-4*a^2>0)', see `assume?` for more details) Is b^2-4*a^2 positive or negative?

mupad [B] time = 3.05, size = 181, normalized size = 1.91

$$\frac{\frac{2b}{4a^2-b^2} + \frac{b^2 \tan(c+dx)}{a(4a^2-b^2)}}{d \left(a \tan(c+dx)^2 + b \tan(c+dx) + a \right)} + \frac{8a \operatorname{atan}\left(\frac{(4a^2-b^2)\left(\frac{8a^2 \tan(c+dx)}{(2a+b)^{3/2}(2a-b)^{3/2}} + \frac{4a(4a^2-b^3)}{(2a+b)^{3/2}(4a^2-b^2)(2a-b)^{3/2}}\right)}{4a}\right)}{d(2a+b)^{3/2}(2a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(c + dx)*sin(c + dx))^2,x)

```
[Out] ((2*b)/(4*a^2 - b^2) + (b^2*tan(c + d*x))/(a*(4*a^2 - b^2)))/(d*(a + b*tan(c + d*x) + a*tan(c + d*x)^2)) + (8*a*atan(((4*a^2 - b^2)*((8*a^2*tan(c + d*x)))/((2*a + b)^(3/2)*(2*a - b)^(3/2)) + (4*a*(4*a^2*b - b^3))/((2*a + b)^(3/2)*(4*a^2 - b^2)*(2*a - b)^(3/2))))/(4*a)))/(d*(2*a + b)^(3/2)*(2*a - b)^(3/2))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

$$3.572 \quad \int \frac{1}{(a+b \cos(c+dx) \sin(c+dx))^3} dx$$

Optimal. Leaf size=149

$$\frac{4(8a^2 + b^2) \tan^{-1}\left(\frac{2a \tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d(4a^2 - b^2)^{5/2}} + \frac{12ab \cos(2c + 2dx)}{d(4a^2 - b^2)^2 (2a + b \sin(2c + 2dx))} + \frac{2b \cos(2c + 2dx)}{d(4a^2 - b^2) (2a + b \sin(2c + 2dx))^2}$$

[Out] 4*(8*a^2+b^2)*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))/(4*a^2-b^2)^(5/2)/d+2*b*cos(2*d*x+2*c)/(4*a^2-b^2)/d/(2*a+b*sin(2*d*x+2*c))^2+12*a*b*cos(2*d*x+2*c)/(4*a^2-b^2)^2/d/(2*a+b*sin(2*d*x+2*c))

Rubi [A] time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2666, 2664, 2754, 12, 2660, 618, 204}

$$\frac{4(8a^2 + b^2) \tan^{-1}\left(\frac{2a \tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d(4a^2 - b^2)^{5/2}} + \frac{12ab \cos(2c + 2dx)}{d(4a^2 - b^2)^2 (2a + b \sin(2c + 2dx))} + \frac{2b \cos(2c + 2dx)}{d(4a^2 - b^2) (2a + b \sin(2c + 2dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3), x]

[Out] (4*(8*a^2 + b^2)*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]]/((4*a^2 - b^2)^(5/2)*d) + (2*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*(2*a + b*Sin[2*c + 2*d*x])^2) + (12*a*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)^2*d*(2*a + b*Sin[2*c + 2*d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2664

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/((n + 1)*(a^2 - b^2)), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2666

Int[((a_) + cos[(c_) + (d_)*(x_)])*(b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n}, x]

Rule 2754

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(a + b \cos(c + dx) \sin(c + dx))^3} dx &= \int \frac{1}{\left(a + \frac{1}{2}b \sin(2c + 2dx)\right)^3} dx \\
 &= \frac{2b \cos(2c + 2dx)}{(4a^2 - b^2) d(2a + b \sin(2c + 2dx))^2} - \frac{2 \int \frac{-2a + \frac{1}{2}b \sin(2c + 2dx)}{\left(a + \frac{1}{2}b \sin(2c + 2dx)\right)^2} dx}{4a^2 - b^2} \\
 &= \frac{2b \cos(2c + 2dx)}{(4a^2 - b^2) d(2a + b \sin(2c + 2dx))^2} + \frac{12ab \cos(2c + 2dx)}{(4a^2 - b^2)^2 d(2a + b \sin(2c + 2dx))} \\
 &= \frac{2b \cos(2c + 2dx)}{(4a^2 - b^2) d(2a + b \sin(2c + 2dx))^2} + \frac{12ab \cos(2c + 2dx)}{(4a^2 - b^2)^2 d(2a + b \sin(2c + 2dx))} \\
 &= \frac{2b \cos(2c + 2dx)}{(4a^2 - b^2) d(2a + b \sin(2c + 2dx))^2} + \frac{12ab \cos(2c + 2dx)}{(4a^2 - b^2)^2 d(2a + b \sin(2c + 2dx))} \\
 &= \frac{2b \cos(2c + 2dx)}{(4a^2 - b^2) d(2a + b \sin(2c + 2dx))^2} + \frac{12ab \cos(2c + 2dx)}{(4a^2 - b^2)^2 d(2a + b \sin(2c + 2dx))} \\
 &= \frac{4(8a^2 + b^2) \tan^{-1}\left(\frac{b + 2a \tan(c + dx)}{\sqrt{4a^2 - b^2}}\right)}{(4a^2 - b^2)^{5/2} d} + \frac{2b \cos(2c + 2dx)}{(4a^2 - b^2) d(2a + b \sin(2c + 2dx))^2}
 \end{aligned}$$

Mathematica [A] time = 0.93, size = 120, normalized size = 0.81

$$\frac{2 \left(\frac{2(8a^2 + b^2) \tan^{-1}\left(\frac{2a \tan(c + dx) + b}{\sqrt{4a^2 - b^2}}\right)}{(4a^2 - b^2)^{5/2}} + \frac{b \cos(2(c + dx))(16a^2 + 6ab \sin(2(c + dx)) - b^2)}{(b^2 - 4a^2)^2 (2a + b \sin(2(c + dx)))^2} \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3), x]

[Out] $(2*((2*(8*a^2 + b^2)*\text{ArcTan}[(b + 2*a*\text{Tan}[c + d*x])/ \text{Sqrt}[4*a^2 - b^2]])/((4*a^2 - b^2)^{(5/2)} + (b*\text{Cos}[2*(c + d*x)]*(16*a^2 - b^2 + 6*a*b*\text{Sin}[2*(c + d*x)])))/((-4*a^2 + b^2)^2*(2*a + b*\text{Sin}[2*(c + d*x)]^2)))/d$

fricas [B] time = 1.06, size = 969, normalized size = 6.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="fricas")`

[Out] $[1/2*(64*a^4*b - 20*a^2*b^3 + b^5 - 2*(64*a^4*b - 20*a^2*b^3 + b^5)*\cos(d*x + c)^2 - 2*((8*a^2*b^2 + b^4)*\cos(d*x + c)^4 - 8*a^4 - a^2*b^2 - (8*a^2*b^2 + b^4)*\cos(d*x + c)^2 - 2*(8*a^3*b + a*b^3)*\cos(d*x + c)*\sin(d*x + c))*\text{sqrt}(-4*a^2 + b^2)*\log(-2*(8*a^2 - b^2)*\cos(d*x + c)^4 - 4*a*b*\cos(d*x + c)*\sin(d*x + c) - 2*(8*a^2 - b^2)*\cos(d*x + c)^2 + 2*a^2 - b^2 + (2*b*\cos(d*x + c)^2 + 4*(2*a*\cos(d*x + c)^3 - a*\cos(d*x + c))*\sin(d*x + c) - b)*\text{sqrt}(-4*a^2 + b^2))/(b^2*\cos(d*x + c)^4 - b^2*\cos(d*x + c)^2 - 2*a*b*\cos(d*x + c)*\sin(d*x + c) - a^2) - 12*(2*(4*a^3*b^2 - a*b^4)*\cos(d*x + c)^3 - (4*a^3*b^2 - a*b^4)*\cos(d*x + c))*\sin(d*x + c))/((64*a^6*b^2 - 48*a^4*b^4 + 12*a^2*b^6 - b^8)*d*\cos(d*x + c)^4 - (64*a^6*b^2 - 48*a^4*b^4 + 12*a^2*b^6 - b^8)*d*\cos(d*x + c)^2 - 2*(64*a^7*b - 48*a^5*b^3 + 12*a^3*b^5 - a*b^7)*d*\cos(d*x + c)*\sin(d*x + c) - (64*a^8 - 48*a^6*b^2 + 12*a^4*b^4 - a^2*b^6)*d), 1/2*(64*a^4*b - 20*a^2*b^3 + b^5 - 2*(64*a^4*b - 20*a^2*b^3 + b^5)*\cos(d*x + c)^2 - 4*((8*a^2*b^2 + b^4)*\cos(d*x + c)^4 - 8*a^4 - a^2*b^2 - (8*a^2*b^2 + b^4)*\cos(d*x + c)^2 - 2*(8*a^3*b + a*b^3)*\cos(d*x + c)*\sin(d*x + c))*\text{sqrt}(4*a^2 - b^2)*\arctan(-4*a*\cos(d*x + c)*\sin(d*x + c) + b)*\text{sqrt}(4*a^2 - b^2)/(2*(4*a^2 - b^2)*\cos(d*x + c)^2 - 4*a^2 + b^2) - 12*(2*(4*a^3*b^2 - a*b^4)*\cos(d*x + c)^3 - (4*a^3*b^2 - a*b^4)*\cos(d*x + c))*\sin(d*x + c))/((64*a^6*b^2 - 48*a^4*b^4 + 12*a^2*b^6 - b^8)*d*\cos(d*x + c)^4 - (64*a^6*b^2 - 48*a^4*b^4 + 12*a^2*b^6 - b^8)*d*\cos(d*x + c)^2 - 2*(64*a^7*b - 48*a^5*b^3 + 12*a^3*b^5 - a*b^7)*d*\cos(d*x + c)*\sin(d*x + c) - (64*a^8 - 48*a^6*b^2 + 12*a^4*b^4 - a^2*b^6)*d)]$

giac [A] time = 0.19, size = 252, normalized size = 1.69

$$\frac{8\left(\pi\left[\frac{dx+c}{\pi} + \frac{1}{2}\right]\text{sgn}(a) + \arctan\left(\frac{2a \tan(dx+c)+b}{\sqrt{4a^2-b^2}}\right)\right)(8a^2+b^2)}{(16a^4-8a^2b^2+b^4)\sqrt{4a^2-b^2}} + \frac{20a^3b^2 \tan(dx+c)^3 - 2ab^4 \tan(dx+c)^3 + 32a^4b \tan(dx+c)^2 + 14a^2b^3 \tan(dx+c)^2 - b^5 \tan(dx+c)}{(16a^6-8a^4b^2+a^2b^4)(a \tan(dx+c)^2 + b \tan(dx+c))} \cdot \frac{1}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="giac")`

[Out] $1/2*(8*(\pi*\text{floor}((d*x + c)/\pi + 1/2)*\text{sgn}(a) + \arctan((2*a*\text{tan}(d*x + c) + b)/\text{sqrt}(4*a^2 - b^2)))*(8*a^2 + b^2)/((16*a^4 - 8*a^2*b^2 + b^4)*\text{sqrt}(4*a^2 - b^2)) + (20*a^3*b^2*\text{tan}(d*x + c)^3 - 2*a*b^4*\text{tan}(d*x + c)^3 + 32*a^4*b*\text{tan}(d*x + c)^2 + 14*a^2*b^3*\text{tan}(d*x + c)^2 - b^5*\text{tan}(d*x + c)^2 + 44*a^3*b^2*\text{tan}(d*x + c) - 2*a*b^4*\text{tan}(d*x + c) + 32*a^4*b - 2*a^2*b^3)/((16*a^6 - 8*a^4*b^2 + a^2*b^4)*(a*\text{tan}(d*x + c)^2 + b*\text{tan}(d*x + c) + a)^2))/d$

maple [B] time = 0.49, size = 640, normalized size = 4.30

$$\frac{10b^2a \left(\tan^3(dx+c)\right)}{d \left(\left(\tan^2(dx+c)\right)a + b \tan(dx+c) + a\right)^2 (16a^4 - 8a^2b^2 + b^4)} - \frac{b^4 \left(\tan^3(dx+c)\right)}{d \left(\left(\tan^2(dx+c)\right)a + b \tan(dx+c) + a\right)^2 (16a^4 - 8a^2b^2 + b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x)`

```
[Out] 10/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^2/(16*a^4-8*a^2*b^2+b^4)*a*tan(d*x+c)^3-1/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^4/(16*a^4-8*a^2*b^2+b^4)/a*tan(d*x+c)^3+16/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b/(16*a^4-8*a^2*b^2+b^4)*a^2*tan(d*x+c)^2+7/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^3/(16*a^4-8*a^2*b^2+b^4)*tan(d*x+c)^2-1/2/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^5/(16*a^4-8*a^2*b^2+b^4)/a^2*tan(d*x+c)^2+22/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^2*a/(16*a^4-8*a^2*b^2+b^4)*tan(d*x+c)-1/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^4/a/(16*a^4-8*a^2*b^2+b^4)*tan(d*x+c)+16/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b/(16*a^4-8*a^2*b^2+b^4)*a^2-1/d/(tan(d*x+c)^2*a+b*tan(d*x+c)+a)^2*b^3/(16*a^4-8*a^2*b^2+b^4)+32/d/(16*a^4-8*a^2*b^2+b^4)/(4*a^2-b^2)^(1/2)*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))*a^2+4/d/(16*a^4-8*a^2*b^2+b^4)/(4*a^2-b^2)^(1/2)*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(b^2-4*a^2>0)', see `assume?` for more details)Is b^2-4*a^2 positive or negative?
```

mupad [B] time = 3.87, size = 396, normalized size = 2.66

$$\frac{\frac{16a^2b-b^3}{16a^4-8a^2b^2+b^4} + \frac{b \tan(c+dx)(22a^2b-b^3)}{a(16a^4-8a^2b^2+b^4)} + \frac{\tan(c+dx)^2(16a^2b-b^3)(2a^2+b^2)}{2a^2(16a^4-8a^2b^2+b^4)} + \frac{b \tan(c+dx)^3(10a^2b-b^3)}{a(16a^4-8a^2b^2+b^4)}}{d \left(\tan(c+dx)^2(2a^2+b^2) + a^2 + a^2 \tan(c+dx)^4 + 2ab \tan(c+dx) + 2ab \tan(c+dx)^3 \right)} + 4 \operatorname{atan} \left(\frac{4a \tan(c+dx)}{(2a+b)^{5/2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^3,x)
```

```
[Out] ((16*a^2*b - b^3)/(16*a^4 + b^4 - 8*a^2*b^2) + (b*tan(c + d*x)*(22*a^2*b - b^3))/(a*(16*a^4 + b^4 - 8*a^2*b^2)) + (tan(c + d*x)^2*(16*a^2*b - b^3)*(2*a^2 + b^2))/(2*a^2*(16*a^4 + b^4 - 8*a^2*b^2)) + (b*tan(c + d*x)^3*(10*a^2*b - b^3))/(a*(16*a^4 + b^4 - 8*a^2*b^2)))/(d*(tan(c + d*x)^2*(2*a^2 + b^2) + a^2 + a^2*tan(c + d*x)^4 + 2*a*b*tan(c + d*x) + 2*a*b*tan(c + d*x)^3)) + (4*atan((((4*a*tan(c + d*x)*(8*a^2 + b^2))/((2*a + b)^(5/2)*(2*a - b)^(5/2)) + (2*(8*a^2 + b^2)*(16*a^4*b + b^5 - 8*a^2*b^3))/((2*a + b)^(5/2)*(2*a - b)^(5/2)*(16*a^4 + b^4 - 8*a^2*b^2)))*(16*a^4 + b^4 - 8*a^2*b^2))/(16*a^2 + 2*b^2))*(8*a^2 + b^2))/(d*(2*a + b)^(5/2)*(2*a - b)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.573 $\int (a + b \cos(c + dx) \sin(c + dx))^{5/2} dx$

Optimal. Leaf size=265

$$\frac{2\sqrt{2}a(4a^2 - b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}F\left(c+dx-\frac{\pi}{4}\left|\frac{2b}{2a+b}\right.\right)}{15d\sqrt{2a+b\sin(2c+2dx)}} + \frac{(92a^2 + 9b^2)\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}\left|\frac{2b}{2a+b}\right.\right)}{60\sqrt{2}d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

[Out] $-1/40*b*\cos(2*d*x+2*c)*(2*a+b*\sin(2*d*x+2*c))^(3/2)/d*2^(1/2)-2/15*a*b*\cos(2*d*x+2*c)*2^(1/2)*(2*a+b*\sin(2*d*x+2*c))^(1/2)/d-1/120*(92*a^2+9*b^2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2))*(b/(2*a+b))^(1/2))*(2*a+b*\sin(2*d*x+2*c))^(1/2)/d*2^(1/2)/((2*a+b*\sin(2*d*x+2*c))/(2*a+b))^(1/2)+2/15*a*(4*a^2-b^2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2)*(b/(2*a+b))^(1/2))*2^(1/2)*((2*a+b*\sin(2*d*x+2*c))/(2*a+b))^(1/2)/d/(2*a+b*\sin(2*d*x+2*c))^(1/2)$

Rubi [A] time = 0.37, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2666, 2656, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\sqrt{2}a(4a^2 - b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}F\left(c+dx-\frac{\pi}{4}\left|\frac{2b}{2a+b}\right.\right)}{15d\sqrt{2a+b\sin(2c+2dx)}} + \frac{(92a^2 + 9b^2)\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}\left|\frac{2b}{2a+b}\right.\right)}{60\sqrt{2}d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(5/2), x]

[Out] $(-2*\text{Sqrt}[2]*a*b*\text{Cos}[2*c + 2*d*x]*\text{Sqrt}[2*a + b*\text{Sin}[2*c + 2*d*x]])/(15*d) - (b*\text{Cos}[2*c + 2*d*x]*(2*a + b*\text{Sin}[2*c + 2*d*x])^(3/2))/(20*\text{Sqrt}[2]*d) + ((92*a^2 + 9*b^2)*\text{EllipticE}[c - \text{Pi}/4 + d*x, (2*b)/(2*a + b)]*\text{Sqrt}[2*a + b*\text{Sin}[2*c + 2*d*x]])/(60*\text{Sqrt}[2]*d*\text{Sqrt}[(2*a + b*\text{Sin}[2*c + 2*d*x])/(2*a + b)]) - (2*\text{Sqrt}[2]*a*(4*a^2 - b^2)*\text{EllipticF}[c - \text{Pi}/4 + d*x, (2*b)/(2*a + b)]*\text{Sqrt}[(2*a + b*\text{Sin}[2*c + 2*d*x])/(2*a + b)])/(15*d*\text{Sqrt}[2*a + b*\text{Sin}[2*c + 2*d*x]])$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)], x_Symbol] := Dist[Sqrt[a + b*SIN[c + d*x]]/Sqrt[(a + b*SIN[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*SIN[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2656

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(a + b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*SIN[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*SIN[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2666

Int[((a_) + cos[(c_) + (d_)*(x_)])*(b_)*sin[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n}, x]

Rule 2752

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2753

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
 \int (a + b \cos(c + dx) \sin(c + dx))^{5/2} dx &= \int \left(a + \frac{1}{2} b \sin(2c + 2dx) \right)^{5/2} dx \\
 &= -\frac{b \cos(2c + 2dx)(2a + b \sin(2c + 2dx))^{3/2}}{20\sqrt{2}d} + \frac{2}{5} \int \sqrt{a + \frac{1}{2} b \sin(2c + 2dx)} dx \\
 &= -\frac{2\sqrt{2}ab \cos(2c + 2dx)\sqrt{2a + b \sin(2c + 2dx)}}{15d} - \frac{b \cos(2c + 2dx)(2a + b \sin(2c + 2dx))^{3/2}}{20\sqrt{2}d} \\
 &= -\frac{2\sqrt{2}ab \cos(2c + 2dx)\sqrt{2a + b \sin(2c + 2dx)}}{15d} - \frac{b \cos(2c + 2dx)(2a + b \sin(2c + 2dx))^{3/2}}{20\sqrt{2}d} \\
 &= -\frac{2\sqrt{2}ab \cos(2c + 2dx)\sqrt{2a + b \sin(2c + 2dx)}}{15d} - \frac{b \cos(2c + 2dx)(2a + b \sin(2c + 2dx))^{3/2}}{20\sqrt{2}d} \\
 &= -\frac{2\sqrt{2}ab \cos(2c + 2dx)\sqrt{2a + b \sin(2c + 2dx)}}{15d} - \frac{b \cos(2c + 2dx)(2a + b \sin(2c + 2dx))^{3/2}}{20\sqrt{2}d}
 \end{aligned}$$

Mathematica [A] time = 1.85, size = 202, normalized size = 0.76

$$\frac{-32a(4a^2 - b^2) \sqrt{\frac{2a+b \sin(2(c+dx))}{2a+b}} F\left(c + dx - \frac{\pi}{4} \mid \frac{2b}{2a+b}\right) - b(88a^2 \cos(2(c+dx)) + b \sin(4(c+dx))(28a + 3b \sin(2(c+dx))))}{120d \sqrt{4a + 2b \sin(2(c+dx))}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(5/2), x]

[Out] (2*(184*a^3 + 92*a^2*b + 18*a*b^2 + 9*b^3)*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)] - 32*a*(4*a^2 - b^2)*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)] - b*(88*a^2*Cos[2*(c + d*x)] + b*(28*a + 3*b*Sin[2*(c + d*x)])*Sin[4*(c + d*x)]))/(120*d*Sqrt[4*a + 2*b*Sin[2*(c + d*x)]])

fricas [F] time = 2.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2 \cos(dx + c)^4 - b^2 \cos(dx + c)^2 - 2ab \cos(dx + c) \sin(dx + c) - a^2\right) \sqrt{b \cos(dx + c) \sin(dx + c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(5/2), x, algorithm="fricas")

[Out] integral(-(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) - a^2)*sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(5/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.52, size = 1138, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c)*sin(d*x+c))^(5/2), x)

[Out] 1/60*(240*a^4*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2), ((2*a-b)/(2*a+b))^(1/2))+64*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2), ((2*a-b)/(2*a+b))^(1/2))*a^3*b-24*a^2*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2), ((2*a-b)/(2*a+b))^(1/2))*b^2-16*a*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2), ((2*a-b)/(2*a+b))^(1/2))*b^3-9*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2), ((2*a-b)/(2*a+b))^(1/2))*b^4-368*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2), ((2*a-b)/(2*a+b))^(1/2))*a^4+56*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*((2*a

+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a^2*b^2+9*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^4+3*b^4*sin(2*d*x+2*c)^4+28*a*b^3*sin(2*d*x+2*c)^3+44*a^2*b^2*sin(2*d*x+2*c)^2-3*b^4*sin(2*d*x+2*c)^2-28*sin(2*d*x+2*c)*a*b^3-44*a^2*b^2)/b/cos(2*d*x+2*c)/(4*a+2*b*sin(2*d*x+2*c))^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cos(dx + c) \sin(dx + c) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \cos(c + dx) \sin(c + dx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(c + d*x)*sin(c + d*x))^(5/2),x)

[Out] int((a + b*cos(c + d*x)*sin(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**(5/2),x)

[Out] Timed out

3.574 $\int (a + b \cos(c + dx) \sin(c + dx))^{3/2} dx$

Optimal. Leaf size=212

$$\frac{(4a^2 - b^2) \sqrt{\frac{2a+b \sin(2c+2dx)}{2a+b}} F\left(c + dx - \frac{\pi}{4} \left| \frac{2b}{2a+b} \right. \right)}{6\sqrt{2} d \sqrt{2a + b \sin(2c + 2dx)}} - \frac{b \cos(2c + 2dx) \sqrt{2a + b \sin(2c + 2dx)}}{6\sqrt{2} d} + \frac{2\sqrt{2} a \sqrt{2a + b \sin(2c + 2dx)}}{3d \sqrt{2a + b \sin(2c + 2dx)}}$$

[Out] $-1/12*b*\cos(2*d*x+2*c)*(2*a+b*\sin(2*d*x+2*c))^{(1/2)}/d*2^{(1/2)}-2/3*a*(\sin(c+1/4*Pi+d*x)^2)^{(1/2)}/\sin(c+1/4*Pi+d*x)*\text{EllipticE}(\cos(c+1/4*Pi+d*x), 2^{(1/2)}*(b/(2*a+b))^{(1/2)})*2^{(1/2)}*(2*a+b*\sin(2*d*x+2*c))^{(1/2)}/d/((2*a+b*\sin(2*d*x+2*c))/(2*a+b))^{(1/2)}+1/12*(4*a^2-b^2)*(\sin(c+1/4*Pi+d*x)^2)^{(1/2)}/\sin(c+1/4*Pi+d*x)*\text{EllipticF}(\cos(c+1/4*Pi+d*x), 2^{(1/2)}*(b/(2*a+b))^{(1/2)})*((2*a+b*\sin(2*d*x+2*c))/(2*a+b))^{(1/2)}/d*2^{(1/2)}/(2*a+b*\sin(2*d*x+2*c))^{(1/2)}$

Rubi [A] time = 0.22, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2666, 2656, 2752, 2663, 2661, 2655, 2653}

$$\frac{(4a^2 - b^2) \sqrt{\frac{2a+b \sin(2c+2dx)}{2a+b}} F\left(c + dx - \frac{\pi}{4} \left| \frac{2b}{2a+b} \right. \right)}{6\sqrt{2} d \sqrt{2a + b \sin(2c + 2dx)}} - \frac{b \cos(2c + 2dx) \sqrt{2a + b \sin(2c + 2dx)}}{6\sqrt{2} d} + \frac{2\sqrt{2} a \sqrt{2a + b \sin(2c + 2dx)}}{3d \sqrt{2a + b \sin(2c + 2dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])^{(3/2)}, x]$

[Out] $-(b*\text{Cos}[2*c + 2*d*x]*\text{Sqrt}[2*a + b*\text{Sin}[2*c + 2*d*x]])/(6*\text{Sqrt}[2]*d) + (2*\text{Sqrt}[2]*a*\text{EllipticE}[c - \text{Pi}/4 + d*x, (2*b)/(2*a + b)]*\text{Sqrt}[2*a + b*\text{Sin}[2*c + 2*d*x]])/(3*d*\text{Sqrt}[(2*a + b*\text{Sin}[2*c + 2*d*x])/(2*a + b)]) - ((4*a^2 - b^2)*\text{EllipticF}[c - \text{Pi}/4 + d*x, (2*b)/(2*a + b)]*\text{Sqrt}[(2*a + b*\text{Sin}[2*c + 2*d*x])/(2*a + b)])/(6*\text{Sqrt}[2]*d*\text{Sqrt}[2*a + b*\text{Sin}[2*c + 2*d*x]])$

Rule 2653

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]], x_Symbol] := \text{Simp}[(2*\text{Sqrt}[a + b]*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[a + b, 0]$

Rule 2655

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]], x_Symbol] := \text{Dist}[\text{Sqrt}[a + b*\text{Sin}[c + d*x]]/\text{Sqrt}[(a + b*\text{Sin}[c + d*x])/(a + b)], \text{Int}[\text{Sqrt}[a/(a + b) + (b*\text{Sin}[c + d*x])/(a + b)], x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ !\text{GtQ}[a + b, 0]$

Rule 2656

$\text{Int}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]^{(n)}, x_Symbol] := -\text{Simp}[(b*\text{Cos}[c + d*x]*(a + b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[1/n, \text{Int}[(a + b*\text{Sin}[c + d*x])^{(n-2)}*\text{Simp}[a^2*n + b^2*(n-1) + a*b*(2*n-1)*\text{Sin}[c + d*x], x], x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2661

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] := \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, (2*b)/(a + b)]/(d*\text{Sqrt}[a + b]), x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[a + b, 0]$

Rule 2663

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2666

```
Int[((a_) + cos[(c_) + (d_)*(x_)])*(b_)*sin[(c_) + (d_)*(x_)])^(n_), x_
Symbol] := Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n},
x]
```

Rule 2752

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (
f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int (a + b \cos(c + dx) \sin(c + dx))^{3/2} dx &= \int \left(a + \frac{1}{2} b \sin(2c + 2dx) \right)^{3/2} dx \\
&= -\frac{b \cos(2c + 2dx) \sqrt{2a + b \sin(2c + 2dx)}}{6\sqrt{2} d} + \frac{2}{3} \int \frac{\frac{1}{8} (12a^2 + b^2) + ab \sin(2c + 2dx)}{\sqrt{a + \frac{1}{2} b \sin(2c + 2dx)}} dx \\
&= -\frac{b \cos(2c + 2dx) \sqrt{2a + b \sin(2c + 2dx)}}{6\sqrt{2} d} + \frac{1}{3} (4a) \int \sqrt{a + \frac{1}{2} b \sin(2c + 2dx)} dx \\
&= -\frac{b \cos(2c + 2dx) \sqrt{2a + b \sin(2c + 2dx)}}{6\sqrt{2} d} + \frac{\left(4a \sqrt{a + \frac{1}{2} b \sin(2c + 2dx)} \right)}{3 \sqrt{\frac{a + \frac{1}{2} b \sin(2c + 2dx)}{a + b}}} \\
&= -\frac{b \cos(2c + 2dx) \sqrt{2a + b \sin(2c + 2dx)}}{6\sqrt{2} d} + \frac{2\sqrt{2} a E\left(c - \frac{\pi}{4} + dx \mid \frac{2b}{2a+b}\right) \sqrt{\frac{2a+b \sin(2c+2dx)}{2a+b}}}{3d \sqrt{\frac{2a+b \sin(2c+2dx)}{2a+b}}}
\end{aligned}$$

Mathematica [A] time = 1.46, size = 167, normalized size = 0.79

$$\frac{-\left(4a^2 - b^2\right) \sqrt{\frac{2a+b \sin(2(c+dx))}{2a+b}} F\left(c + dx - \frac{\pi}{4} \mid \frac{2b}{2a+b}\right) - b \cos(2(c + dx))(2a + b \sin(2(c + dx))) + 8a(2a + b) \sqrt{\frac{2a+b \sin(2(c+dx))}{2a+b}}}{6d \sqrt{4a + 2b \sin(2(c + dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(3/2), x]
```

```
[Out] -(b*Cos[2*(c + d*x)]*(2*a + b*Sin[2*(c + d*x)])) + 8*a*(2*a + b)*EllipticE
[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)
] - (4*a^2 - b^2)*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*
Sin[2*(c + d*x)])/(2*a + b)]/(6*d*Sqrt[4*a + 2*b*Sin[2*(c + d*x)]])
```

fricas [F] time = 2.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \cos(dx + c) \sin(dx + c) + a\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((b*cos(d*x + c)*sin(d*x + c) + a)^(3/2), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.44, size = 844, normalized size = 3.98

$$24a^3 \sqrt{\frac{2a+b \sin(2dx+2c)}{2a-b}} \sqrt{\frac{(\sin(2dx+2c)-1)b}{2a+b}} \sqrt{\frac{(1+\sin(2dx+2c))b}{2a-b}} \text{EllipticF}\left(\sqrt{\frac{2a+b \sin(2dx+2c)}{2a-b}}, \sqrt{\frac{2a-b}{2a+b}}\right) + 4\sqrt{\frac{2a+b \sin(2c)}{2a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x)

[Out] $\frac{1}{6} * (24 * a^3 * ((2 * a + b * \sin(2 * d * x + 2 * c)) / (2 * a - b))^{1/2} * (-(\sin(2 * d * x + 2 * c) - 1) * b / (2 * a + b))^{1/2} * (- (1 + \sin(2 * d * x + 2 * c)) * b / (2 * a - b))^{1/2} * \text{EllipticF}(((2 * a + b * \sin(2 * d * x + 2 * c)) / (2 * a - b))^{1/2}, ((2 * a - b) / (2 * a + b))^{1/2}) + 4 * ((2 * a + b * \sin(2 * d * x + 2 * c)) / (2 * a - b))^{1/2} * \text{EllipticF}(((2 * a + b * \sin(2 * d * x + 2 * c)) / (2 * a - b))^{1/2}, ((2 * a - b) / (2 * a + b))^{1/2}) * (-(\sin(2 * d * x + 2 * c) - 1) * b / (2 * a + b))^{1/2} * (- (1 + \sin(2 * d * x + 2 * c)) * b / (2 * a - b))^{1/2} * a^2 * b - 6 * ((2 * a + b * \sin(2 * d * x + 2 * c)) / (2 * a - b))^{1/2} * (-(\sin(2 * d * x + 2 * c) - 1) * b / (2 * a + b))^{1/2} * (- (1 + \sin(2 * d * x + 2 * c)) * b / (2 * a - b))^{1/2} * \text{EllipticF}(((2 * a + b * \sin(2 * d * x + 2 * c)) / (2 * a - b))^{1/2}, ((2 * a - b) / (2 * a + b))^{1/2}) * b^2 * a - ((2 * a + b * \sin(2 * d * x + 2 * c)) / (2 * a - b))^{1/2} * (-(\sin(2 * d * x + 2 * c) - 1) * b / (2 * a + b))^{1/2} * (- (1 + \sin(2 * d * x + 2 * c)) * b / (2 * a - b))^{1/2} * \text{EllipticF}(((2 * a + b * \sin(2 * d * x + 2 * c)) / (2 * a - b))^{1/2}, ((2 * a - b) / (2 * a + b))^{1/2}) * b^3 - 32 * ((2 * a + b * \sin(2 * d * x + 2 * c)) / (2 * a - b))^{1/2} * \text{EllipticE}(((2 * a + b * \sin(2 * d * x + 2 * c)) / (2 * a - b))^{1/2}, ((2 * a - b) / (2 * a + b))^{1/2}) * (-(\sin(2 * d * x + 2 * c) - 1) * b / (2 * a + b))^{1/2} * (- (1 + \sin(2 * d * x + 2 * c)) * b / (2 * a - b))^{1/2} * a^3 + 8 * ((2 * a + b * \sin(2 * d * x + 2 * c)) / (2 * a - b))^{1/2} * \text{EllipticE}(((2 * a + b * \sin(2 * d * x + 2 * c)) / (2 * a - b))^{1/2}, ((2 * a - b) / (2 * a + b))^{1/2}) * (-(\sin(2 * d * x + 2 * c) - 1) * b / (2 * a + b))^{1/2} * (- (1 + \sin(2 * d * x + 2 * c)) * b / (2 * a - b))^{1/2} * a * b^2 + b^3 * \sin(2 * d * x + 2 * c)^3 + 2 * a * b^2 * \sin(2 * d * x + 2 * c)^2 - \sin(2 * d * x + 2 * c) * b^3 - 2 * a * b^2) / b / \cos(2 * d * x + 2 * c) / (4 * a + 2 * b * \sin(2 * d * x + 2 * c))^{1/2} / d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cos(dx + c) \sin(dx + c) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \cos(c + dx) \sin(c + dx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*cos(c + d*x)*sin(c + d*x))^(3/2), x)`

[Out] `int((a + b*cos(c + d*x)*sin(c + d*x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sin(c + dx) \cos(c + dx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(d*x+c)*sin(d*x+c))**(3/2), x)`

[Out] `Integral((a + b*sin(c + d*x)*cos(c + d*x))**(3/2), x)`

3.575 $\int \sqrt{a + b \cos(c + dx) \sin(c + dx)} dx$

Optimal. Leaf size=76

$$\frac{\sqrt{2a + b \sin(2c + 2dx)} E\left(c + dx - \frac{\pi}{4} \mid \frac{2b}{2a+b}\right)}{\sqrt{2} d \sqrt{\frac{2a+b \sin(2c+2dx)}{2a+b}}}$$

[Out] $-1/2 * (\sin(c + 1/4 * \pi + d * x)^2)^{(1/2)} / \sin(c + 1/4 * \pi + d * x) * \text{EllipticE}(\cos(c + 1/4 * \pi + d * x), 2^{(1/2)} * (b / (2 * a + b))^{(1/2)}) * (2 * a + b * \sin(2 * d * x + 2 * c))^{(1/2)} / d * 2^{(1/2)} / ((2 * a + b * \sin(2 * d * x + 2 * c)) / (2 * a + b))^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2666, 2655, 2653}

$$\frac{\sqrt{2a + b \sin(2c + 2dx)} E\left(c + dx - \frac{\pi}{4} \mid \frac{2b}{2a+b}\right)}{\sqrt{2} d \sqrt{\frac{2a+b \sin(2c+2dx)}{2a+b}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b * Cos[c + d * x] * Sin[c + d * x]], x]

[Out] (EllipticE[c - Pi/4 + d * x, (2 * b) / (2 * a + b)] * Sqrt[2 * a + b * Sin[2 * c + 2 * d * x]]) / (Sqrt[2] * d * Sqrt[(2 * a + b * Sin[2 * c + 2 * d * x]) / (2 * a + b)])

Rule 2653

Int[Sqrt[(a_) + (b_.) * sin[(c_.) + (d_.) * (x_)]], x_Symbol] := Simp[(2 * Sqrt[a + b] * EllipticE[(1 * (c - Pi/2 + d * x)) / 2, (2 * b) / (a + b)]) / d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_.) * sin[(c_.) + (d_.) * (x_)]], x_Symbol] := Dist[Sqrt[a + b * Sin[c + d * x]] / Sqrt[(a + b * Sin[c + d * x]) / (a + b)], Int[Sqrt[a / (a + b) + (b * Sin[c + d * x]) / (a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2666

Int[((a_) + cos[(c_.) + (d_.) * (x_)]) * (b_.) * sin[(c_.) + (d_.) * (x_)]^(n_), x_Symbol] := Int[(a + (b * Sin[2 * c + 2 * d * x]) / 2)^n, x] /; FreeQ[{a, b, c, d, n}, x]

Rubi steps

$$\begin{aligned} \int \sqrt{a + b \cos(c + dx) \sin(c + dx)} dx &= \int \sqrt{a + \frac{1}{2}b \sin(2c + 2dx)} dx \\ &= \frac{\int \sqrt{a + \frac{1}{2}b \sin(2c + 2dx)} \int \sqrt{\frac{a}{a+\frac{b}{2}} + \frac{b \sin(2c+2dx)}{2(a+\frac{b}{2})}} dx}{\sqrt{\frac{a+\frac{1}{2}b \sin(2c+2dx)}{a+\frac{b}{2}}}} \\ &= \frac{E\left(c - \frac{\pi}{4} + dx \mid \frac{2b}{2a+b}\right) \sqrt{2a + b \sin(2c + 2dx)}}{\sqrt{2} d \sqrt{\frac{2a+b \sin(2c+2dx)}{2a+b}}} \end{aligned}$$

Mathematica [A] time = 0.11, size = 75, normalized size = 0.99

$$\frac{(2a + b) \sqrt{\frac{2a+b \sin(2(c+dx))}{2a+b}} E\left(c + dx - \frac{\pi}{4} \mid \frac{2b}{2a+b}\right)}{d \sqrt{4a + 2b \sin(2(c + dx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]], x]

[Out] ((2*a + b)*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)])/(d*Sqrt[4*a + 2*b*Sin[2*(c + d*x)]])

fricas [F] time = 2.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b \cos(dx + c) \sin(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \cos(dx + c) \sin(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)

maple [B] time = 0.41, size = 312, normalized size = 4.11

$$\frac{\sqrt{\frac{2a+b \sin(2dx+2c)}{2a-b}} \sqrt{\frac{(\sin(2dx+2c)-1)b}{2a+b}} \sqrt{\frac{(1+\sin(2dx+2c))b}{2a-b}} (2a - b) \left(2 \text{EllipticE}\left(\sqrt{\frac{2a+b \sin(2dx+2c)}{2a-b}}, \sqrt{\frac{2a-b}{2a+b}}\right) a + \text{EllipticE}\left(\sqrt{\frac{2a+b \sin(2dx+2c)}{2a-b}}, \sqrt{\frac{2a-b}{2a+b}}\right) b \cos(2dx + 2c) \right) \sqrt{4}}{b \cos(2dx + 2c) \sqrt{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c)*sin(d*x+c))^(1/2), x)

[Out] -((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)/b*(2*a-b)*(2*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2), ((2*a-b)/(2*a+b))^(1/2))*a+EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2), ((2*a-b)/(2*a+b))^(1/2))*b*cos(2*d*x+2*c))

$$\frac{n(2dx+2c)}{(2a-b)^{1/2}}, ((2a-b)/(2a+b))^{1/2} * b - 2a * \text{EllipticF}(((2a+b)\sin(2dx+2c))/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2} - \text{EllipticF}(((2a+b)\sin(2dx+2c))/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2} * b) / \cos(2dx+2c) / (4a+2b\sin(2dx+2c))^{1/2} / d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \cos(dx+c) \sin(dx+c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b \cos(c + dx) \sin(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(c + d*x)*sin(c + d*x))^(1/2),x)

[Out] int((a + b*cos(c + d*x)*sin(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sin(c + dx) \cos(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**(1/2),x)

[Out] Integral(sqrt(a + b*sin(c + d*x)*cos(c + d*x)), x)

$$3.576 \quad \int \frac{1}{\sqrt{a+b \cos(c+dx) \sin(c+dx)}} dx$$

Optimal. Leaf size=76

$$\frac{\sqrt{2} \sqrt{\frac{2a+b \sin(2c+2dx)}{2a+b}} F\left(c+dx-\frac{\pi}{4} \mid \frac{2b}{2a+b}\right)}{d\sqrt{2a+b \sin(2c+2dx)}}$$

[Out] $-(\sin(c+1/4*\text{Pi}+d*x)^2)^{(1/2)}/\sin(c+1/4*\text{Pi}+d*x)*\text{EllipticF}(\cos(c+1/4*\text{Pi}+d*x), 2^{(1/2)}*(b/(2*a+b))^{(1/2)})*2^{(1/2)}*((2*a+b*\sin(2*d*x+2*c))/(2*a+b))^{(1/2)}/d/(2*a+b*\sin(2*d*x+2*c))^{(1/2)}$

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2666, 2663, 2661}

$$\frac{\sqrt{2} \sqrt{\frac{2a+b \sin(2c+2dx)}{2a+b}} F\left(c+dx-\frac{\pi}{4} \mid \frac{2b}{2a+b}\right)}{d\sqrt{2a+b \sin(2c+2dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]], x]

[Out] (Sqrt[2]*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)])/(d*Sqrt[2*a + b*Sin[2*c + 2*d*x]])

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2666

Int[((a_) + cos[(c_) + (d_)*(x_)])*(b_)*sin[(c_) + (d_)*(x_)]^(n_), x_Symbol] :> Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n}, x]

Rubi steps

$$\int \frac{1}{\sqrt{a + b \cos(c + dx) \sin(c + dx)}} dx = \int \frac{1}{\sqrt{a + \frac{1}{2}b \sin(2c + 2dx)}} dx$$

$$= \frac{\sqrt{\frac{a + \frac{1}{2}b \sin(2c + 2dx)}{a + \frac{b}{2}}} \int \frac{1}{\sqrt{\frac{a}{a + \frac{b}{2}} + \frac{b \sin(2c + 2dx)}{2(a + \frac{b}{2})}}} dx}{\sqrt{a + \frac{1}{2}b \sin(2c + 2dx)}}$$

$$= \frac{\sqrt{2} F\left(c - \frac{\pi}{4} + dx \middle| \frac{2b}{2a+b}\right) \sqrt{\frac{2a+b \sin(2c+2dx)}{2a+b}}}{d \sqrt{2a + b \sin(2c + 2dx)}}$$

Mathematica [A] time = 0.13, size = 70, normalized size = 0.92

$$\frac{\sqrt{\frac{2a+b \sin(2(c+dx))}{2a+b}} F\left(c + dx - \frac{\pi}{4} \middle| \frac{2b}{2a+b}\right)}{d \sqrt{a + \frac{1}{2}b \sin(2(c + dx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]

[Out] (EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)])/(d*Sqrt[a + (b*Sin[2*(c + d*x)])/2])

fricas [F] time = 2.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b \cos(dx + c) \sin(dx + c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(1/sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \cos(dx + c) \sin(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)

maple [A] time = 0.43, size = 165, normalized size = 2.17

$$\frac{2(2a - b) \sqrt{\frac{2a+b \sin(2dx+2c)}{2a-b}} \sqrt{\frac{(\sin(2dx+2c)-1)b}{2a+b}} \sqrt{\frac{(1+\sin(2dx+2c))b}{2a-b}} \text{EllipticF}\left(\sqrt{\frac{2a+b \sin(2dx+2c)}{2a-b}}, \sqrt{\frac{2a-b}{2a+b}}\right)}{b \cos(2dx + 2c) \sqrt{4a + 2b \sin(2dx + 2c)} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x)

[Out] 2*(2*a-b)*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d

$\frac{\sqrt{2ax+2c}}{\sqrt{2a-b}}, \left(\frac{2a-b}{2a+b}\right)^{1/2} \frac{1}{b \cos(2dx+2c) \sqrt{4a+2b \sin(2dx+2c)}}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \cos(dx+c) \sin(dx+c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + b \cos(c + dx) \sin(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(1/2),x)

[Out] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \sin(c + dx) \cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**(1/2),x)

[Out] Integral(1/sqrt(a + b*sin(c + d*x)*cos(c + d*x)), x)

$$3.577 \quad \int \frac{1}{(a+b \cos(c+dx) \sin(c+dx))^{3/2}} dx$$

Optimal. Leaf size=143

$$\frac{2\sqrt{2} b \cos(2c + 2dx)}{d(4a^2 - b^2) \sqrt{2a + b \sin(2c + 2dx)}} + \frac{2\sqrt{2} \sqrt{2a + b \sin(2c + 2dx)} E\left(c + dx - \frac{\pi}{4} \mid \frac{2b}{2a+b}\right)}{d(4a^2 - b^2) \sqrt{\frac{2a+b \sin(2c+2dx)}{2a+b}}}$$

[Out] $2*b*\cos(2*d*x+2*c)*2^{(1/2)}/(4*a^2-b^2)/d/(2*a+b*\sin(2*d*x+2*c))^{(1/2)}-2*(\sin(c+1/4*Pi+d*x)^2)^{(1/2)}/\sin(c+1/4*Pi+d*x)*\text{EllipticE}(\cos(c+1/4*Pi+d*x),2^{(1/2)}*(b/(2*a+b))^{(1/2)})*2^{(1/2)}*(2*a+b*\sin(2*d*x+2*c))^{(1/2)}/(4*a^2-b^2)/d/((2*a+b*\sin(2*d*x+2*c))/(2*a+b))^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2666, 2664, 21, 2655, 2653}

$$\frac{2\sqrt{2} b \cos(2c + 2dx)}{d(4a^2 - b^2) \sqrt{2a + b \sin(2c + 2dx)}} + \frac{2\sqrt{2} \sqrt{2a + b \sin(2c + 2dx)} E\left(c + dx - \frac{\pi}{4} \mid \frac{2b}{2a+b}\right)}{d(4a^2 - b^2) \sqrt{\frac{2a+b \sin(2c+2dx)}{2a+b}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3/2), x]

[Out] $(2*\text{Sqrt}[2]*b*\text{Cos}[2*c + 2*d*x])/((4*a^2 - b^2)*d*\text{Sqrt}[2*a + b*\text{Sin}[2*c + 2*d*x]]) + (2*\text{Sqrt}[2]*\text{EllipticE}[c - \text{Pi}/4 + d*x, (2*b)/(2*a + b)]*\text{Sqrt}[2*a + b*\text{Sin}[2*c + 2*d*x]])/((4*a^2 - b^2)*d*\text{Sqrt}[(2*a + b*\text{Sin}[2*c + 2*d*x])/(2*a + b)])$

Rule 21

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*SIN[c + d*x]]/Sqrt[(a + b*SIN[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*SIN[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2664

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(a + b*SIN[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*SIN[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*SIN[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2666

Int[((a_) + cos[(c_) + (d_)*(x_)]*(b_)*sin[(c_) + (d_)*(x_)])^(n_), x_ Symbol] := Int[(a + (b*Sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(a + b \cos(c + dx) \sin(c + dx))^{3/2}} dx &= \int \frac{1}{\left(a + \frac{1}{2}b \sin(2c + 2dx)\right)^{3/2}} dx \\
 &= \frac{2\sqrt{2} b \cos(2c + 2dx)}{(4a^2 - b^2) d \sqrt{2a + b \sin(2c + 2dx)}} - \frac{8 \int \frac{-\frac{a}{2} - \frac{1}{4}b \sin(2c + 2dx)}{\sqrt{a + \frac{1}{2}b \sin(2c + 2dx)}} dx}{4a^2 - b^2} \\
 &= \frac{2\sqrt{2} b \cos(2c + 2dx)}{(4a^2 - b^2) d \sqrt{2a + b \sin(2c + 2dx)}} + \frac{4 \int \sqrt{a + \frac{1}{2}b \sin(2c + 2dx)} dx}{4a^2 - b^2} \\
 &= \frac{2\sqrt{2} b \cos(2c + 2dx)}{(4a^2 - b^2) d \sqrt{2a + b \sin(2c + 2dx)}} + \frac{\left(4\sqrt{a + \frac{1}{2}b \sin(2c + 2dx)}\right) \int \sqrt{\frac{-a}{a + \frac{1}{2}b \sin(2c + 2dx)}} dx}{(4a^2 - b^2) \sqrt{\frac{a + \frac{1}{2}b \sin(2c + 2dx)}{a + \frac{1}{2}b \sin(2c + 2dx)}}} \\
 &= \frac{2\sqrt{2} b \cos(2c + 2dx)}{(4a^2 - b^2) d \sqrt{2a + b \sin(2c + 2dx)}} + \frac{2\sqrt{2} E\left(c - \frac{\pi}{4} + dx \mid \frac{2b}{2a+b}\right) \sqrt{2a + b \sin(2c + 2dx)}}{(4a^2 - b^2) d \sqrt{\frac{2a + b \sin(2c + 2dx)}{2a + b}}}
 \end{aligned}$$

Mathematica [A] time = 0.42, size = 101, normalized size = 0.71

$$\frac{2 \left((2a + b) \sqrt{\frac{2a + b \sin(2(c + dx))}{2a + b}} E\left(c + dx - \frac{\pi}{4} \mid \frac{2b}{2a + b}\right) + b \cos(2(c + dx)) \right)}{d (4a^2 - b^2) \sqrt{a + \frac{1}{2}b \sin(2(c + dx))}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3/2), x]

[Out] (2*(b*Cos[2*(c + d*x)] + (2*a + b)*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)])*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b]))/((4*a^2 - b^2)*d*Sqrt[a + (b*Sin[2*(c + d*x)]/2)])

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b \cos(dx + c) \sin(dx + c) + a}}{b^2 \cos(dx + c)^4 - b^2 \cos(dx + c)^2 - 2ab \cos(dx + c) \sin(dx + c) - a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2), x, algorithm="fricas")

[Out] integral(-sqrt(b*cos(d*x + c)*sin(d*x + c) + a)/(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) - a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cos(dx + c) \sin(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-3/2), x)

maple [B] time = 0.50, size = 570, normalized size = 3.99

$$16a^2 \sqrt{\frac{2a+b \sin(2dx+2c)}{2a-b}} \sqrt{\frac{(\sin(2dx+2c)-1)b}{2a+b}} \sqrt{\frac{(1+\sin(2dx+2c))b}{2a-b}} \operatorname{EllipticF}\left(\sqrt{\frac{2a+b \sin(2dx+2c)}{2a-b}}, \sqrt{\frac{2a-b}{2a+b}}\right) - 4 \sqrt{\frac{2a+b \sin(2dx+2c)}{2a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x)

[Out]
$$\frac{4}{b} (4a^2 ((2a+b \sin(2dx+2c))/(2a-b))^{1/2} (-\sin(2dx+2c)-1)b/(2a+b))^{1/2} (-1+\sin(2dx+2c))b/(2a-b)^{1/2} \operatorname{EllipticF}((2a+b \sin(2dx+2c))/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2}) - ((2a+b \sin(2dx+2c))/(2a-b))^{1/2} \operatorname{EllipticF}((2a+b \sin(2dx+2c))/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2}) * (-\sin(2dx+2c)-1)b/(2a+b)^{1/2} (-1+\sin(2dx+2c))b/(2a-b)^{1/2} * b^2 - 4 ((2a+b \sin(2dx+2c))/(2a-b))^{1/2} \operatorname{EllipticE}((2a+b \sin(2dx+2c))/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2}) * (-\sin(2dx+2c)-1)b/(2a+b)^{1/2} (-1+\sin(2dx+2c))b/(2a-b)^{1/2} * a^2 + ((2a+b \sin(2dx+2c))/(2a-b))^{1/2} \operatorname{EllipticE}((2a+b \sin(2dx+2c))/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2}) * (-\sin(2dx+2c)-1)b/(2a+b)^{1/2} (-1+\sin(2dx+2c))b/(2a-b)^{1/2} * b^2 - b^2 \sin(2dx+2c)^2 + b^2 / (4a^2 - b^2) / \cos(2dx+2c) / (4a^2 + 2b \sin(2dx+2c))^{1/2} / d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cos(dx + c) \sin(dx + c) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \cos(c + dx) \sin(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(3/2),x)

[Out] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \sin(c + dx) \cos(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x)

[Out] Integral((a + b*sin(c + d*x)*cos(c + d*x))^(3/2), x)

$$3.578 \quad \int \frac{1}{(a+b \cos(c+dx) \sin(c+dx))^{5/2}} dx$$

Optimal. Leaf size=295

$$\frac{32\sqrt{2} ab \cos(2c + 2dx)}{3d(4a^2 - b^2)^2 \sqrt{2a + b \sin(2c + 2dx)}} + \frac{4\sqrt{2} b \cos(2c + 2dx)}{3d(4a^2 - b^2)(2a + b \sin(2c + 2dx))^{3/2}} - \frac{4\sqrt{2} \sqrt{\frac{2a+b \sin(2c+2dx)}{2a+b}} F\left(c + dx - \frac{1}{2} \arcsin\left(\frac{2a+b \sin(2c+2dx)}{2a+b}\right)\right)}{3d(4a^2 - b^2) \sqrt{2a + b \sin(2c + 2dx)}}$$

[Out] $4/3*b*\cos(2*d*x+2*c)*2^{(1/2)}/(4*a^2-b^2)/d/(2*a+b*\sin(2*d*x+2*c))^{(3/2)}+32/3*a*b*\cos(2*d*x+2*c)*2^{(1/2)}/(4*a^2-b^2)^2/d/(2*a+b*\sin(2*d*x+2*c))^{(1/2)}-3/2/3*a*(\sin(c+1/4*Pi+d*x)^2)^{(1/2)}/\sin(c+1/4*Pi+d*x)*\text{EllipticE}(\cos(c+1/4*Pi+d*x), 2^{(1/2)}*(b/(2*a+b))^{(1/2)})*2^{(1/2)}*(2*a+b*\sin(2*d*x+2*c))^{(1/2)}/(4*a^2-b^2)^2/d/((2*a+b*\sin(2*d*x+2*c))/(2*a+b))^{(1/2)}+4/3*(\sin(c+1/4*Pi+d*x)^2)^{(1/2)}/\sin(c+1/4*Pi+d*x)*\text{EllipticF}(\cos(c+1/4*Pi+d*x), 2^{(1/2)}*(b/(2*a+b))^{(1/2)})*2^{(1/2)}*((2*a+b*\sin(2*d*x+2*c))/(2*a+b))^{(1/2)}/(4*a^2-b^2)/d/(2*a+b*\sin(2*d*x+2*c))^{(1/2)}$

Rubi [A] time = 0.30, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2666, 2664, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{32\sqrt{2} ab \cos(2c + 2dx)}{3d(4a^2 - b^2)^2 \sqrt{2a + b \sin(2c + 2dx)}} + \frac{4\sqrt{2} b \cos(2c + 2dx)}{3d(4a^2 - b^2)(2a + b \sin(2c + 2dx))^{3/2}} - \frac{4\sqrt{2} \sqrt{\frac{2a+b \sin(2c+2dx)}{2a+b}} F\left(c + dx - \frac{1}{2} \arcsin\left(\frac{2a+b \sin(2c+2dx)}{2a+b}\right)\right)}{3d(4a^2 - b^2) \sqrt{2a + b \sin(2c + 2dx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-5/2), x]

[Out] $(4*\text{Sqrt}[2]*b*\text{Cos}[2*c + 2*d*x])/(3*(4*a^2 - b^2)*d*(2*a + b*\text{Sin}[2*c + 2*d*x])^{(3/2)}) + (32*\text{Sqrt}[2]*a*b*\text{Cos}[2*c + 2*d*x])/(3*(4*a^2 - b^2)^2*d*\text{Sqrt}[2*a + b*\text{Sin}[2*c + 2*d*x]]) + (32*\text{Sqrt}[2]*a*\text{EllipticE}[c - \text{Pi}/4 + d*x, (2*b)/(2*a + b)]*\text{Sqrt}[2*a + b*\text{Sin}[2*c + 2*d*x]])/(3*(4*a^2 - b^2)^2*d*\text{Sqrt}[(2*a + b*\text{Sin}[2*c + 2*d*x])/(2*a + b)]) - (4*\text{Sqrt}[2]*\text{EllipticF}[c - \text{Pi}/4 + d*x, (2*b)/(2*a + b)]*\text{Sqrt}[(2*a + b*\text{Sin}[2*c + 2*d*x])/(2*a + b)])/(3*(4*a^2 - b^2)*d*\text{Sqrt}[2*a + b*\text{Sin}[2*c + 2*d*x]])$

Rule 2653

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2655

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2661

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)]/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2663

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)

+ (b*SIN[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2664

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*cos[c + d*x]*(a + b*sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2666

Int[((a_) + cos[(c_) + (d_)*(x_)])*(b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Int[(a + (b*sin[2*c + 2*d*x])/2)^n, x] /; FreeQ[{a, b, c, d, n}, x]

Rule 2752

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2754

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[((b*c - a*d)*cos[e + f*x]*(a + b*sin[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \cos(c + dx) \sin(c + dx))^{5/2}} dx &= \int \frac{1}{\left(a + \frac{1}{2}b \sin(2c + 2dx)\right)^{5/2}} dx \\
&= \frac{4\sqrt{2} b \cos(2c + 2dx)}{3(4a^2 - b^2) d(2a + b \sin(2c + 2dx))^{3/2}} - \frac{8 \int \frac{-\frac{3a}{2} + \frac{1}{4}b \sin(2c + 2dx)}{\left(a + \frac{1}{2}b \sin(2c + 2dx)\right)^{3/2}} dx}{3(4a^2 - b^2)} \\
&= \frac{4\sqrt{2} b \cos(2c + 2dx)}{3(4a^2 - b^2) d(2a + b \sin(2c + 2dx))^{3/2}} + \frac{32\sqrt{2} ab \cos(2c + 2dx)}{3(4a^2 - b^2)^2 d\sqrt{2a + b \sin(2c + 2dx)}} \\
&= \frac{4\sqrt{2} b \cos(2c + 2dx)}{3(4a^2 - b^2) d(2a + b \sin(2c + 2dx))^{3/2}} + \frac{32\sqrt{2} ab \cos(2c + 2dx)}{3(4a^2 - b^2)^2 d\sqrt{2a + b \sin(2c + 2dx)}} \\
&= \frac{4\sqrt{2} b \cos(2c + 2dx)}{3(4a^2 - b^2) d(2a + b \sin(2c + 2dx))^{3/2}} + \frac{32\sqrt{2} ab \cos(2c + 2dx)}{3(4a^2 - b^2)^2 d\sqrt{2a + b \sin(2c + 2dx)}} \\
&= \frac{4\sqrt{2} b \cos(2c + 2dx)}{3(4a^2 - b^2) d(2a + b \sin(2c + 2dx))^{3/2}} + \frac{32\sqrt{2} ab \cos(2c + 2dx)}{3(4a^2 - b^2)^2 d\sqrt{2a + b \sin(2c + 2dx)}} \\
&= \frac{4\sqrt{2} b \cos(2c + 2dx)}{3(4a^2 - b^2) d(2a + b \sin(2c + 2dx))^{3/2}} + \frac{32\sqrt{2} ab \cos(2c + 2dx)}{3(4a^2 - b^2)^2 d\sqrt{2a + b \sin(2c + 2dx)}}
\end{aligned}$$

Mathematica [A] time = 1.52, size = 201, normalized size = 0.68

$$\frac{4\sqrt{2} \left(b \cos(2(c + dx)) (-20a^2 - 8ab \sin(2(c + dx)) + b^2) + (2a - b)(2a + b)^2 \left(\frac{2a + b \sin(2(c + dx))}{2a + b} \right)^{3/2} F\left(c + dx - \frac{\pi}{4} \middle| \frac{1}{2}\right) \right)}{3d(b^2 - 4a^2)^2 (2a + b \sin(2(c + dx)))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-5/2), x]

[Out] (-4*Sqrt[2]*((-8*a*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*(2*a + b*Sin[2*(c + d*x)])^2)/Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)] + (2*a - b)*(2*a + b)^2*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*((2*a + b*Sin[2*(c + d*x)])/(2*a + b))^(3/2) + b*Cos[2*(c + d*x)]*(-20*a^2 + b^2 - 8*a*b*Sin[2*(c + d*x)])))/(3*(-4*a^2 + b^2)^2*d*(2*a + b*Sin[2*(c + d*x)])^(3/2))

fricas [F] time = 1.74, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{\sqrt{b \cos(dx + c) \sin(dx + c) + a}}{3ab^2 \cos(dx + c)^4 - 3ab^2 \cos(dx + c)^2 - a^3 + (b^3 \cos(dx + c)^5 - b^3 \cos(dx + c)^3 - 3a^2b \cos(dx + c))} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2), x, algorithm="fricas")

[Out] integral(-sqrt(b*cos(d*x + c)*sin(d*x + c) + a)/(3*a*b^2*cos(d*x + c)^4 - 3*a*b^2*cos(d*x + c)^2 - a^3 + (b^3*cos(d*x + c)^5 - b^3*cos(d*x + c)^3 - 3*a^2*b*cos(d*x + c))*sin(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cos(dx + c) \sin(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-5/2), x)

maple [B] time = 0.52, size = 1554, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x)

[Out]
$$\frac{8}{3} \cdot (8 \sin(2dx+2c) \cos(2dx+2c)^2 a b^3 + (b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2} \cdot (-b/(2a+b) \sin(2dx+2c) + b/(2a+b))^{1/2} \cdot (-b/(2a-b) \sin(2dx+2c) - b/(2a-b))^{1/2} \cdot b \cdot (24 \operatorname{EllipticF}((b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2}) a^3 + 4 \operatorname{EllipticF}((b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2}) a^2 b - 6 \operatorname{EllipticF}((b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2}) a b^2 - \operatorname{EllipticF}((b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2}) b^3 - 32 \operatorname{EllipticE}((b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2}) a^3 + 8 \operatorname{EllipticE}((b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2}) a b^2) \sin(2dx+2c) + (20 a^2 b^2 - b^4) \cos(2dx+2c)^2 + 48 (b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2} \operatorname{EllipticF}((b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2}) \cdot (-b/(2a+b) \sin(2dx+2c) + b/(2a+b))^{1/2} \cdot (-b/(2a-b) \sin(2dx+2c) - b/(2a-b))^{1/2} a^4 + 8 (b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2} \operatorname{EllipticF}((b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2}) \cdot (-b/(2a+b) \sin(2dx+2c) + b/(2a+b))^{1/2} \cdot (-b/(2a-b) \sin(2dx+2c) - b/(2a-b))^{1/2} a^3 b - 12 (b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2} \operatorname{EllipticF}((b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2}) \cdot (-b/(2a+b) \sin(2dx+2c) + b/(2a+b))^{1/2} \cdot (-b/(2a-b) \sin(2dx+2c) - b/(2a-b))^{1/2} a^2 b^2 - 2 (b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2} \operatorname{EllipticF}((b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2}) \cdot (-b/(2a+b) \sin(2dx+2c) + b/(2a+b))^{1/2} \cdot (-b/(2a-b) \sin(2dx+2c) - b/(2a-b))^{1/2} a b^3 - 64 (b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2} \cdot (-b/(2a+b) \sin(2dx+2c) + b/(2a+b))^{1/2} \cdot (-b/(2a-b) \sin(2dx+2c) - b/(2a-b))^{1/2} \operatorname{EllipticE}((b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2}) a^4 + 16 (b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2} \cdot (-b/(2a+b) \sin(2dx+2c) + b/(2a+b))^{1/2} \cdot (-b/(2a-b) \sin(2dx+2c) - b/(2a-b))^{1/2} \operatorname{EllipticE}((b/(2a-b) \sin(2dx+2c) + 2a/(2a-b))^{1/2}, ((2a-b)/(2a+b))^{1/2}) a^2 b^2) / (4 a^2 - b^2)^2 / b / \cos(2dx+2c) / (4 a + 2 b \sin(2dx+2c))^{1/2} / d$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cos(dx + c) \sin(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + b \cos(c + dx) \sin(c + dx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(5/2), x)

[Out] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**(5/2), x)

[Out] Timed out

$$3.579 \quad \int \frac{x^3}{a+b \cos(x) \sin(x)} dx$$

Optimal. Leaf size=461

$$-\frac{3x^2 \operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \frac{3x^2 \operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} - \frac{3ix \operatorname{Li}_3\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \frac{3ix \operatorname{Li}_3\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \frac{3 \operatorname{Li}_4\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{4\sqrt{4a^2-b^2}} - \frac{3 \operatorname{Li}_4\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{4\sqrt{4a^2-b^2}}$$

[Out] $-I*x^3*\ln(1-I*b*\exp(2*I*x)/(2*a-(4*a^2-b^2)^{(1/2)}))/(4*a^2-b^2)^{(1/2)}+I*x^3*\ln(1-I*b*\exp(2*I*x)/(2*a+(4*a^2-b^2)^{(1/2)}))/(4*a^2-b^2)^{(1/2)}-3/2*x^2*\operatorname{polylog}(2,I*b*\exp(2*I*x)/(2*a-(4*a^2-b^2)^{(1/2)}))/(4*a^2-b^2)^{(1/2)}+3/2*x^2*\operatorname{polylog}(2,I*b*\exp(2*I*x)/(2*a+(4*a^2-b^2)^{(1/2)}))/(4*a^2-b^2)^{(1/2)}-3/2*I*x*\operatorname{polylog}(3,I*b*\exp(2*I*x)/(2*a-(4*a^2-b^2)^{(1/2)}))/(4*a^2-b^2)^{(1/2)}+3/2*I*x*\operatorname{polylog}(3,I*b*\exp(2*I*x)/(2*a+(4*a^2-b^2)^{(1/2)}))/(4*a^2-b^2)^{(1/2)}+3/4*\operatorname{polylog}(4,I*b*\exp(2*I*x)/(2*a-(4*a^2-b^2)^{(1/2)}))/(4*a^2-b^2)^{(1/2)}-3/4*\operatorname{polylog}(4,I*b*\exp(2*I*x)/(2*a+(4*a^2-b^2)^{(1/2)}))/(4*a^2-b^2)^{(1/2)}$

Rubi [A] time = 0.63, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4584, 3323, 2264, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3x^2 \operatorname{PolyLog}\left(2, \frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \frac{3x^2 \operatorname{PolyLog}\left(2, \frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2\sqrt{4a^2-b^2}} - \frac{3ix \operatorname{PolyLog}\left(3, \frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \frac{3ix \operatorname{PolyLog}\left(3, \frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b*cos[x]*sin[x]), x]

[Out] $((-I)*x^3*\operatorname{Log}[1 - (I*b*E^{((2*I)*x)})/(2*a - \operatorname{Sqrt}[4*a^2 - b^2])])/ \operatorname{Sqrt}[4*a^2 - b^2] + (I*x^3*\operatorname{Log}[1 - (I*b*E^{((2*I)*x)})/(2*a + \operatorname{Sqrt}[4*a^2 - b^2])])/ \operatorname{Sqrt}[4*a^2 - b^2] - (3*x^2*\operatorname{PolyLog}[2, (I*b*E^{((2*I)*x)})/(2*a - \operatorname{Sqrt}[4*a^2 - b^2])])/ (2*\operatorname{Sqrt}[4*a^2 - b^2]) + (3*x^2*\operatorname{PolyLog}[2, (I*b*E^{((2*I)*x)})/(2*a + \operatorname{Sqrt}[4*a^2 - b^2])])/ (2*\operatorname{Sqrt}[4*a^2 - b^2]) - (((3*I)/2)*x*\operatorname{PolyLog}[3, (I*b*E^{((2*I)*x)})/(2*a - \operatorname{Sqrt}[4*a^2 - b^2])])/ \operatorname{Sqrt}[4*a^2 - b^2] + (((3*I)/2)*x*\operatorname{PolyLog}[3, (I*b*E^{((2*I)*x)})/(2*a + \operatorname{Sqrt}[4*a^2 - b^2])])/ \operatorname{Sqrt}[4*a^2 - b^2] + (3*\operatorname{PolyLog}[4, (I*b*E^{((2*I)*x)})/(2*a - \operatorname{Sqrt}[4*a^2 - b^2])])/ (4*\operatorname{Sqrt}[4*a^2 - b^2]) - (3*\operatorname{PolyLog}[4, (I*b*E^{((2*I)*x)})/(2*a + \operatorname{Sqrt}[4*a^2 - b^2])])/ (4*\operatorname{Sqrt}[4*a^2 - b^2])$

Rule 2190

Int[(((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m-1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2264

Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi

```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] := Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4584

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cos[(c_.) + (d_.)*(x_)]*(b_.)*Sin[(c
_.) + (d_.)*(x_)]^(n_.), x_Symbol] := Int[(e + f*x)^m*(a + (b*SIN[2*c + 2*
d*x])/2)^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a + b \cos(x) \sin(x)} dx &= \int \frac{x^3}{a + \frac{1}{2}b \sin(2x)} dx \\
&= 2 \int \frac{e^{2ix} x^3}{\frac{ib}{2} + 2ae^{2ix} - \frac{1}{2}ibe^{4ix}} dx \\
&= -\frac{(2ib) \int \frac{e^{2ix} x^3}{2a - \sqrt{4a^2 - b^2} - ibe^{2ix}} dx}{\sqrt{4a^2 - b^2}} + \frac{(2ib) \int \frac{e^{2ix} x^3}{2a + \sqrt{4a^2 - b^2} - ibe^{2ix}} dx}{\sqrt{4a^2 - b^2}} \\
&= -\frac{ix^3 \log\left(1 - \frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{ix^3 \log\left(1 - \frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{(3i) \int x^2 \log\left(1 - \frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} \\
&= -\frac{ix^3 \log\left(1 - \frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{ix^3 \log\left(1 - \frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} - \frac{3x^2 \text{Li}_2\left(\frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{2\sqrt{4a^2 - b^2}} + \frac{3x^2 \text{Li}_2\left(\frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{2\sqrt{4a^2 - b^2}} \\
&= -\frac{ix^3 \log\left(1 - \frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{ix^3 \log\left(1 - \frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} - \frac{3x^2 \text{Li}_2\left(\frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{2\sqrt{4a^2 - b^2}} + \frac{3x^2 \text{Li}_2\left(\frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{2\sqrt{4a^2 - b^2}} \\
&= -\frac{ix^3 \log\left(1 - \frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{ix^3 \log\left(1 - \frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} - \frac{3x^2 \text{Li}_2\left(\frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{2\sqrt{4a^2 - b^2}} + \frac{3x^2 \text{Li}_2\left(\frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{2\sqrt{4a^2 - b^2}}
\end{aligned}$$

Mathematica [A] time = 0.87, size = 340, normalized size = 0.74

$$\frac{-6x^2 \text{Li}_2\left(-\frac{ibe^{2ix}}{\sqrt{4a^2 - b^2} - 2a}\right) + 6x^2 \text{Li}_2\left(\frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right) - 6ix \text{Li}_3\left(-\frac{ibe^{2ix}}{\sqrt{4a^2 - b^2} - 2a}\right) + 6ix \text{Li}_3\left(\frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right) + 3\text{Li}_4\left(-\frac{ibe^{2ix}}{\sqrt{4a^2 - b^2}}\right)}{4\sqrt{4a^2 - b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b*cos[x]*sin[x]),x]

[Out] $((-4*I)*x^3*\text{Log}[1 + (I*b*E^{((2*I)*x)})/(-2*a + \text{Sqrt}[4*a^2 - b^2])]) + (4*I)*x^3*\text{Log}[1 - (I*b*E^{((2*I)*x)})/(2*a + \text{Sqrt}[4*a^2 - b^2])] - 6*x^2*\text{PolyLog}[2, ((-I)*b*E^{((2*I)*x)})/(-2*a + \text{Sqrt}[4*a^2 - b^2])] + 6*x^2*\text{PolyLog}[2, (I*b*E^{((2*I)*x)})/(2*a + \text{Sqrt}[4*a^2 - b^2])] - (6*I)*x*\text{PolyLog}[3, ((-I)*b*E^{((2*I)*x)})/(-2*a + \text{Sqrt}[4*a^2 - b^2])] + (6*I)*x*\text{PolyLog}[3, (I*b*E^{((2*I)*x)})/(2*a + \text{Sqrt}[4*a^2 - b^2])] + 3*\text{PolyLog}[4, ((-I)*b*E^{((2*I)*x)})/(-2*a + \text{Sqrt}[4*a^2 - b^2])] - 3*\text{PolyLog}[4, (I*b*E^{((2*I)*x)})/(2*a + \text{Sqrt}[4*a^2 - b^2])])/(4*\text{Sqrt}[4*a^2 - b^2])$

fricas [C] time = 1.29, size = 3324, normalized size = 7.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*cos(x)*sin(x)),x, algorithm="fricas")

[Out] $-1/4*(2*b*x^3*\text{sqrt}(-(4*a^2 - b^2)/b^2)*\text{log}(1/2*((4*I*a*\cos(x) + 4*a*\sin(x) - 2*(b*\cos(x) - I*b*\sin(x))*\text{sqrt}(-(4*a^2 - b^2)/b^2))*\text{sqrt}((b*\text{sqrt}(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b) + 2*b*x^3*\text{sqrt}(-(4*a^2 - b^2)/b^2)*\text{log}(1/2*((-4*I*a*\cos(x) - 4*a*\sin(x) + 2*(b*\cos(x) - I*b*\sin(x))*\text{sqrt}(-(4*a^2 - b^2)/b^2))*\text{sqrt}((b*\text{sqrt}(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b)$


```

*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2)
)*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) - 12*I*b*sqrt(-(4*a^2 -
b^2)/b^2)*polylog(4, -(-2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) + I*b*sin(x))
)*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b)
+ 12*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(4, -(2*I*a*cos(x) - 2*a*sin(x) +
(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b
^2)/b^2) - 2*I*a)/b)/b) + 12*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(4, -(-2*I
*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*
sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) + 12*I*b*sqrt(-(4*a^2 - b^2
)/b^2)*polylog(4, 1/2*(4*I*a*cos(x) + 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x)
))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b
) + 12*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(4, 1/2*(-4*I*a*cos(x) - 4*a*sin
(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4
*a^2 - b^2)/b^2) - 2*I*a)/b)/b))/(4*a^2 - b^2)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b \cos(x) \sin(x) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cos(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*cos(x)*sin(x) + a), x)
```

maple [B] time = 0.30, size = 2282, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*cos(x)*sin(x)),x)
```

```

[Out] 8/(8*a^2-2*b^2)/(-2*I*a-((2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a
-((2*a-b)*(2*a+b))^(1/2)))*a^2*x^3-2/(8*a^2-2*b^2)/(-2*I*a-((2*a-b)*(2*a+
b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-((2*a-b)*(2*a+b))^(1/2)))*b^2*x^3-2/(
8*a^2-2*b^2)/(-2*I*a-((2*a-b)*(2*a+b))^(1/2))*((2*a-b)*(2*a+b))^(1/2)*a*x
^4+12/(8*a^2-2*b^2)/(-2*I*a-((2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x
)/(-2*I*a-((2*a-b)*(2*a+b))^(1/2)))*a^2*x^3/(8*a^2-2*b^2)/(-2*I*a-((2*a-b
)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a-((2*a-b)*(2*a+b))^(1/2)))
)*b^2*x^3+I/(8*a^2-2*b^2)/(-2*I*a-((2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(
2*I*x)/(-2*I*a-((2*a-b)*(2*a+b))^(1/2)))*b^2*x^2-12*I/(8*a^2-2*b^2)/(-2*I*
a-((2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a-((2*a-b)*(2*a+b
))^(1/2)))*a^2*x^2-12*I/(8*a^2-2*b^2)/(-2*I*a+((2*a-b)*(2*a+b))^(1/2))*pol
ylog(2,b*exp(2*I*x)/(-2*I*a+((2*a-b)*(2*a+b))^(1/2)))*a^2*x^2+3*I/(8*a^2-2
*b^2)/(-2*I*a+((2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+((2
*a-b)*(2*a+b))^(1/2)))*b^2*x^2+6/(8*a^2-2*b^2)/(-2*I*a+((2*a-b)*(2*a+b))^(
1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+((2*a-b)*(2*a+b))^(1/2)))*((2*a-b)*(
2*a+b))^(1/2)*a*x^2-6/(8*a^2-2*b^2)/(-2*I*a-((2*a-b)*(2*a+b))^(1/2))*polyl
og(2,b*exp(2*I*x)/(-2*I*a-((2*a-b)*(2*a+b))^(1/2)))*((2*a-b)*(2*a+b))^(1/
2)*a*x^2+4*I/(8*a^2-2*b^2)/(-2*I*a+((2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I
*x)/(-2*I*a+((2*a-b)*(2*a+b))^(1/2)))*((2*a-b)*(2*a+b))^(1/2)*a*x^3+6*I/(
8*a^2-2*b^2)/(-2*I*a+((2*a-b)*(2*a+b))^(1/2))*((2*a-b)*(2*a+b))^(1/2)*pol
ylog(3,b*exp(2*I*x)/(-2*I*a+((2*a-b)*(2*a+b))^(1/2)))*a*x^4*I/(8*a^2-2*b^2
)/(-2*I*a-((2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-((2*a-b)*(2*
a+b))^(1/2)))*((2*a-b)*(2*a+b))^(1/2)*a*x^3-6*I/(8*a^2-2*b^2)/(-2*I*a-((2
*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a-((2*a-b)*(2*a+b))^(1/
2)))*((2*a-b)*(2*a+b))^(1/2)*a*x+8/(8*a^2-2*b^2)/(-2*I*a+((2*a-b)*(2*a+b
))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+((2*a-b)*(2*a+b))^(1/2)))*a^2*x^3-2/(8
*a^2-2*b^2)/(-2*I*a+((2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+((2

```

```

*a-b)*(2*a+b))^(1/2)))*b^2*x^3+2/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*(-2*a-b)*(2*a+b))^(1/2)*a*x^4+12/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x-3/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x-3/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*(-2*a-b)*(2*a+b))^(1/2)*polylog(4,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a+I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*b^2*x^4+I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*b^2*x^4-4*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*a^2*x^4+6*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a^2-3/2*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2-4*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*a^2*x^4+6*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a^2-3/2*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2+3/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*(-2*a-b)*(2*a+b))^(1/2)*a

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b \cos(x) \sin(x) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*cos(x)*sin(x)),x, algorithm="maxima")

[Out] integrate(x^3/(b*cos(x)*sin(x) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{a + b \cos(x) \sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + b*cos(x)*sin(x)),x)

[Out] int(x^3/(a + b*cos(x)*sin(x)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(a+b*cos(x)*sin(x)),x)

[Out] Timed out

$$3.580 \quad \int \frac{x^2}{a+b \cos(x) \sin(x)} dx$$

Optimal. Leaf size=340

$$-\frac{x \operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{x \operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{i \operatorname{Li}_3\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \frac{i \operatorname{Li}_3\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} - \frac{ix^2 \log\left(1 - \frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^2 \log\left(1 - \frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}$$

[Out] $-I*x^2*\ln(1-I*b*\exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+I*x^2*\ln(1-I*b*\exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-x*\operatorname{polylog}(2,I*b*\exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+x*\operatorname{polylog}(2,I*b*\exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-1/2*I*\operatorname{polylog}(3,I*b*\exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+1/2*I*\operatorname{polylog}(3,I*b*\exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)$

Rubi [A] time = 0.54, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4584, 3323, 2264, 2190, 2531, 2282, 6589}

$$-\frac{x \operatorname{PolyLog}\left(2, \frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{x \operatorname{PolyLog}\left(2, \frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{\sqrt{4a^2-b^2}} - \frac{i \operatorname{PolyLog}\left(3, \frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \frac{i \operatorname{PolyLog}\left(3, \frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b*Cos[x]*Sin[x]), x]

[Out] $((-I)*x^2*\operatorname{Log}[1 - (I*b*E^((2*I)*x))/(2*a - \operatorname{Sqrt}[4*a^2 - b^2])])/ \operatorname{Sqrt}[4*a^2 - b^2] + (I*x^2*\operatorname{Log}[1 - (I*b*E^((2*I)*x))/(2*a + \operatorname{Sqrt}[4*a^2 - b^2])])/ \operatorname{Sqrt}[4*a^2 - b^2] - (x*\operatorname{PolyLog}[2, (I*b*E^((2*I)*x))/(2*a - \operatorname{Sqrt}[4*a^2 - b^2])])/ \operatorname{Sqrt}[4*a^2 - b^2] + (x*\operatorname{PolyLog}[2, (I*b*E^((2*I)*x))/(2*a + \operatorname{Sqrt}[4*a^2 - b^2])])/ \operatorname{Sqrt}[4*a^2 - b^2] - ((I/2)*\operatorname{PolyLog}[3, (I*b*E^((2*I)*x))/(2*a - \operatorname{Sqrt}[4*a^2 - b^2])])/ \operatorname{Sqrt}[4*a^2 - b^2] + ((I/2)*\operatorname{PolyLog}[3, (I*b*E^((2*I)*x))/(2*a + \operatorname{Sqrt}[4*a^2 - b^2])])/ \operatorname{Sqrt}[4*a^2 - b^2]$

Rule 2190

Int[(((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m-1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2264

Int[(((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)*((F_)^(v_))), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3323

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] := Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4584

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + Cos[(c_.) + (d_.)*(x_)]*(b_.)*Sin[(c
_.) + (d_.)*(x_)]^(n_.), x_Symbol] := Int[(e + f*x)^m*(a + (b*SIN[2*c + 2*
d*x])/2)^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a + b \cos(x) \sin(x)} dx &= \int \frac{x^2}{a + \frac{1}{2}b \sin(2x)} dx \\
&= 2 \int \frac{e^{2ix} x^2}{\frac{ib}{2} + 2ae^{2ix} - \frac{1}{2}ibe^{4ix}} dx \\
&= -\frac{(2ib) \int \frac{e^{2ix} x^2}{2a - \sqrt{4a^2 - b^2} - ibe^{2ix}} dx}{\sqrt{4a^2 - b^2}} + \frac{(2ib) \int \frac{e^{2ix} x^2}{2a + \sqrt{4a^2 - b^2} - ibe^{2ix}} dx}{\sqrt{4a^2 - b^2}} \\
&= -\frac{ix^2 \log\left(1 - \frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{ix^2 \log\left(1 - \frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{(2i) \int x \log\left(1 - \frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right) dx}{\sqrt{4a^2 - b^2}} \\
&= -\frac{ix^2 \log\left(1 - \frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{ix^2 \log\left(1 - \frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} - \frac{x \operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{x \operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} \\
&= -\frac{ix^2 \log\left(1 - \frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{ix^2 \log\left(1 - \frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} - \frac{x \operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{x \operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} \\
&= -\frac{ix^2 \log\left(1 - \frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{ix^2 \log\left(1 - \frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} - \frac{x \operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{x \operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}}
\end{aligned}$$

Mathematica [A] time = 0.72, size = 256, normalized size = 0.75

$$\frac{i \left(-2ix \operatorname{Li}_2\left(-\frac{ibe^{2ix}}{\sqrt{4a^2 - b^2} - 2a}\right) + 2ix \operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right) + \operatorname{Li}_3\left(-\frac{ibe^{2ix}}{\sqrt{4a^2 - b^2} - 2a}\right) - \operatorname{Li}_3\left(\frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right) + 2x^2 \log\left(1 + \frac{ibe^{2ix}}{\sqrt{4a^2 - b^2}}\right) \right)}{2\sqrt{4a^2 - b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*cos[x]*sin[x]),x]
```

```
[Out] ((-1/2*I)*(2*x^2*Log[1 + (I*b*E^((2*I)*x))]/(-2*a + Sqrt[4*a^2 - b^2])) - 2*
x^2*Log[1 - (I*b*E^((2*I)*x))]/(2*a + Sqrt[4*a^2 - b^2])) - (2*I)*x*PolyLog[
2, ((-I)*b*E^((2*I)*x))]/(-2*a + Sqrt[4*a^2 - b^2])] + (2*I)*x*PolyLog[2, (I
*b*E^((2*I)*x))]/(2*a + Sqrt[4*a^2 - b^2])] + PolyLog[3, ((-I)*b*E^((2*I)*x)
)/(-2*a + Sqrt[4*a^2 - b^2])] - PolyLog[3, (I*b*E^((2*I)*x))]/(2*a + Sqrt[4*
a^2 - b^2])])]/Sqrt[4*a^2 - b^2]
```

fricas [C] time = 2.28, size = 2506, normalized size = 7.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) + 4*a*sin(x)
- 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2
- b^2)/b^2) + 2*I*a)/b) + 2*b)/b) + 2*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(1/
2*((-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b
^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b) - 2*b*x^2*
sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b
*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*
a)/b) - b)/b) - 2*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) + 2*a
*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-
(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b) + 2*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*l
og(-((2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^
2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) + 2*b*x^2*sq
rt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*s
in(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/
b) - b)/b) - 2*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) + 4*a*
sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(
-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b) - 2*b*x^2*sqrt(-(4*a^2 - b^2)/b^
2)*log(1/2*((-4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4
*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b) +
4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((4*I*a*cos(x) + 4*a*sin(x) -
2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 -
b^2)/b^2) + 2*I*a)/b) + 2*b)/b + 1) + 4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilo
g(-1/2*((-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^
2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b + 1) +
4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) - 2*a*sin(x) - (b*co
s(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b
^2) + 2*I*a)/b) - b)/b + 1) + 4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((2*I
*a*cos(x) + 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*
sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b + 1) - 4*I*b*x*sqrt(-
(4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) + I*b*sin(x)
))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) -
b)/b + 1) - 4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) + 2*a*s
in(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-
(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b + 1) - 4*I*b*x*sqrt(-(4*a^2 - b^2)/b^
2)*dilog(-1/2*((4*I*a*cos(x) + 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-
(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b +
1) - 4*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((-4*I*a*cos(x) - 4*a*sin(
x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-
(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b + 1) + 4*b*sqrt(-(4*a^2 - b^2)/b^2)*p
olylog(3, 1/2*(4*I*a*cos(x) + 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-
(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) + 4*b*sq
rt(-(4*a^2 - b^2)/b^2)*polylog(3, 1/2*(-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*co
s(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^
2) + 2*I*a)/b)/b) - 4*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, -(2*I*a*cos(x)
```

$$\begin{aligned}
& - 2*a*\sin(x) - (b*\cos(x) + I*b*\sin(x))*\sqrt{-(4*a^2 - b^2)/b^2})*\sqrt{-(b*\sqrt{-(4*a^2 - b^2)/b^2} + 2*I*a)/b)/b} - 4*b*\sqrt{-(4*a^2 - b^2)/b^2})*\text{polylog}(3, \\
& -(-2*I*a*\cos(x) + 2*a*\sin(x) + (b*\cos(x) + I*b*\sin(x))*\sqrt{-(4*a^2 - b^2)/b^2}))*\sqrt{-(b*\sqrt{-(4*a^2 - b^2)/b^2} + 2*I*a)/b)/b} + 4*b*\sqrt{-(4*a^2 - b^2)/b^2})*\text{polylog}(3, \\
& -(2*I*a*\cos(x) - 2*a*\sin(x) + (b*\cos(x) + I*b*\sin(x))*\sqrt{-(4*a^2 - b^2)/b^2}))*\sqrt{-(b*\sqrt{-(4*a^2 - b^2)/b^2} - 2*I*a)/b)/b} + 4*b*\sqrt{-(4*a^2 - b^2)/b^2})*\text{polylog}(3, \\
& -(-2*I*a*\cos(x) + 2*a*\sin(x) - (b*\cos(x) + I*b*\sin(x))*\sqrt{-(4*a^2 - b^2)/b^2}))*\sqrt{-(b*\sqrt{-(4*a^2 - b^2)/b^2} - 2*I*a)/b)/b} - 4*b*\sqrt{-(4*a^2 - b^2)/b^2})*\text{polylog}(3, \\
& 1/2*(4*I*a*\cos(x) + 4*a*\sin(x) + 2*(b*\cos(x) - I*b*\sin(x))*\sqrt{-(4*a^2 - b^2)/b^2}))*\sqrt{-(b*\sqrt{-(4*a^2 - b^2)/b^2} - 2*I*a)/b)/b} - 4*b*\sqrt{-(4*a^2 - b^2)/b^2})*\text{polylog}(3, \\
& 1/2*(-4*I*a*\cos(x) - 4*a*\sin(x) - 2*(b*\cos(x) - I*b*\sin(x))*\sqrt{-(4*a^2 - b^2)/b^2}))*\sqrt{-(b*\sqrt{-(4*a^2 - b^2)/b^2} - 2*I*a)/b)/b}))/ (4*a^2 - b^2)
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b \cos(x) \sin(x) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*cos(x)*sin(x)),x, algorithm="giac")

[Out] integrate(x^2/(b*cos(x)*sin(x) + a), x)

maple [B] time = 0.23, size = 1782, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b*cos(x)*sin(x)),x)

$$\begin{aligned}
& [Out] \frac{8}{3} \frac{(8a^2 - 2b^2)^{-1/2} (-2Ia + (-2a - b)(2a + b)^{1/2}) (-2a - b)(2a + b)^{1/2}}{(8a^2 - 2b^2)^{-1/2} (-2Ia + (-2a - b)(2a + b)^{1/2})} \text{polylog}(2, b \exp(2Ix) / (-2Ia + (-2a - b)(2a + b)^{1/2})) \\
& b^2 x - 8I / (8a^2 - 2b^2)^{-1/2} (-2Ia + (-2a - b)(2a + b)^{1/2}) \text{polylog}(2, b \exp(2Ix) / (-2Ia + (-2a - b)(2a + b)^{1/2})) \\
& a^2 x + 2I / (8a^2 - 2b^2)^{-1/2} (-2Ia - (-2a - b)(2a + b)^{1/2}) \text{polylog}(2, b \exp(2Ix) / (-2Ia - (-2a - b)(2a + b)^{1/2})) \\
& b^2 x + 8 / (8a^2 - 2b^2)^{-1/2} (-2Ia + (-2a - b)(2a + b)^{1/2}) \ln(1 - b \exp(2Ix) / (-2Ia + (-2a - b)(2a + b)^{1/2})) \\
& a^2 x^2 - 2 / (8a^2 - 2b^2)^{-1/2} (-2Ia + (-2a - b)(2a + b)^{1/2}) \ln(1 - b \exp(2Ix) / (-2Ia + (-2a - b)(2a + b)^{1/2})) \\
& b^2 x^2 + 4 / (8a^2 - 2b^2)^{-1/2} (-2Ia + (-2a - b)(2a + b)^{1/2}) \text{polylog}(2, b \exp(2Ix) / (-2Ia + (-2a - b)(2a + b)^{1/2})) \\
& a^2 x - 4I / (8a^2 - 2b^2)^{-1/2} (-2Ia - (-2a - b)(2a + b)^{1/2}) \ln(1 - b \exp(2Ix) / (-2Ia - (-2a - b)(2a + b)^{1/2})) \\
& (-2a - b)(2a + b)^{1/2} a^2 x^2 + 2I / (8a^2 - 2b^2)^{-1/2} (-2Ia + (-2a - b)(2a + b)^{1/2}) \text{polylog}(3, b \exp(2Ix) / (-2Ia + (-2a - b)(2a + b)^{1/2})) \\
& (-2a - b)(2a + b)^{1/2} a - 16 / 3 I / (8a^2 - 2b^2)^{-1/2} (-2Ia - (-2a - b)(2a + b)^{1/2}) a^2 x^3 + 4 / (8a^2 - 2b^2)^{-1/2} (-2Ia + (-2a - b)(2a + b)^{1/2}) \\
& \text{polylog}(3, b \exp(2Ix) / (-2Ia + (-2a - b)(2a + b)^{1/2})) a^2 - 1 / (8a^2 - 2b^2)^{-1/2} (-2Ia + (-2a - b)(2a + b)^{1/2}) \text{polylog}(3, b \exp(2Ix) / (-2Ia + (-2a - b)(2a + b)^{1/2})) \\
& b^2 - 8 / 3 / (8a^2 - 2b^2)^{-1/2} (-2Ia - (-2a - b)(2a + b)^{1/2}) (-2a - b)(2a + b)^{1/2} a^2 x^3 - 16 / 3 I / (8a^2 - 2b^2)^{-1/2} (-2Ia + (-2a - b)(2a + b)^{1/2}) \\
& a^2 x^3 + 4 / 3 I / (8a^2 - 2b^2)^{-1/2} (-2Ia - (-2a - b)(2a + b)^{1/2}) b^2 x^3 - 8I / (8a^2 - 2b^2)^{-1/2} (-2Ia - (-2a - b)(2a + b)^{1/2}) \text{polylog}(2, b \exp(2Ix) / (-2Ia - (-2a - b)(2a + b)^{1/2})) \\
& a^2 x + 8 / (8a^2 - 2b^2)^{-1/2} (-2Ia - (-2a - b)(2a + b)^{1/2}) \ln(1 - b \exp(2Ix) / (-2Ia - (-2a - b)(2a + b)^{1/2})) \\
& a^2 x^2 - 2 / (8a^2 - 2b^2)^{-1/2} (-2Ia - (-2a - b)(2a + b)^{1/2}) \ln(1 - b \exp(2Ix) / (-2Ia - (-2a - b)(2a + b)^{1/2})) \\
& b^2 x^2 - 4 / (8a^2 - 2b^2)^{-1/2} (-2Ia - (-2a - b)(2a + b)^{1/2}) \text{polylog}(2, b \exp(2Ix) / (-2Ia - (-2a - b)(2a + b)^{1/2})) \\
& (-2a - b)(2a + b)^{1/2}
\end{aligned}$$

$$\frac{1}{2}ax + \frac{4}{3}I \frac{1}{(8a^2 - 2b^2) \sqrt{-2Ia - (2a-b)(2a+b)}} b^2 x^3 - 2I \frac{1}{(8a^2 - 2b^2) \sqrt{-2Ia - (2a-b)(2a+b)}} \operatorname{polylog}(3, b \exp(2Ix) / \sqrt{-2Ia - (2a-b)(2a+b)}) \sqrt{-2Ia - (2a-b)(2a+b)} + 4I \frac{1}{(8a^2 - 2b^2) \sqrt{-2Ia - (2a-b)(2a+b)}} \ln(1 - b \exp(2Ix) / \sqrt{-2Ia - (2a-b)(2a+b)}) ax^2 + \frac{4}{(8a^2 - 2b^2) \sqrt{-2Ia - (2a-b)(2a+b)}} \operatorname{polylog}(3, b \exp(2Ix) / \sqrt{-2Ia - (2a-b)(2a+b)}) a^2 - \frac{1}{(8a^2 - 2b^2) \sqrt{-2Ia - (2a-b)(2a+b)}} \operatorname{polylog}(3, b \exp(2Ix) / \sqrt{-2Ia - (2a-b)(2a+b)}) b^2$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b \cos(x) \sin(x) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*cos(x)*sin(x)),x, algorithm="maxima")

[Out] integrate(x^2/(b*cos(x)*sin(x) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{a + b \cos(x) \sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b*cos(x)*sin(x)),x)

[Out] int(x^2/(a + b*cos(x)*sin(x)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a+b*cos(x)*sin(x)),x)

[Out] Timed out

$$3.581 \quad \int \frac{x}{a+b \cos(x) \sin(x)} dx$$

Optimal. Leaf size=225

$$-\frac{\operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \frac{\operatorname{Li}_2\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} - \frac{ix \log\left(1 - \frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix \log\left(1 - \frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{\sqrt{4a^2-b^2}}$$

[Out] $-I*x*\ln(1-I*b*\exp(2*I*x)/(2*a-(4*a^2-b^2)^{(1/2)}))/(4*a^2-b^2)^{(1/2)}+I*x*\ln(1-I*b*\exp(2*I*x)/(2*a+(4*a^2-b^2)^{(1/2)}))/(4*a^2-b^2)^{(1/2)}-1/2*\operatorname{polylog}(2,I*b*\exp(2*I*x)/(2*a-(4*a^2-b^2)^{(1/2)}))/(4*a^2-b^2)^{(1/2)}+1/2*\operatorname{polylog}(2,I*b*\exp(2*I*x)/(2*a+(4*a^2-b^2)^{(1/2)}))/(4*a^2-b^2)^{(1/2)}$

Rubi [A] time = 0.32, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4584, 3323, 2264, 2190, 2279, 2391}

$$-\frac{\operatorname{PolyLog}\left(2, \frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}} + \frac{\operatorname{PolyLog}\left(2, \frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2\sqrt{4a^2-b^2}} - \frac{ix \log\left(1 - \frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix \log\left(1 - \frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*Cos[x]*Sin[x]), x]

[Out] $((-I)*x*\operatorname{Log}[1 - (I*b*E^{((2*I)*x)})/(2*a - \operatorname{Sqrt}[4*a^2 - b^2])]/\operatorname{Sqrt}[4*a^2 - b^2] + (I*x*\operatorname{Log}[1 - (I*b*E^{((2*I)*x)})/(2*a + \operatorname{Sqrt}[4*a^2 - b^2])]/\operatorname{Sqrt}[4*a^2 - b^2] - \operatorname{PolyLog}[2, (I*b*E^{((2*I)*x)})/(2*a - \operatorname{Sqrt}[4*a^2 - b^2])]/(2*\operatorname{Sqrt}[4*a^2 - b^2]) + \operatorname{PolyLog}[2, (I*b*E^{((2*I)*x)})/(2*a + \operatorname{Sqrt}[4*a^2 - b^2])]/(2*\operatorname{Sqrt}[4*a^2 - b^2])$

Rule 2190

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2264

Int[(((F_)^(u_))*((f_) + (g_)*(x_))^(m_)]/((a_) + (b_)*(F_)^(u_) + (c_)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_)*((F_)^(e_)*((c_) + (d_)*(x_)))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_)*((d_) + (e_)*(x_))^(n_)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 3323


```
Int[((c_.) + (d_.)*(x_.))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol]
:> Dist[2, Int[((c + d*x)^m*E^(I*(e + f*x)))/(I*b + 2*a*E^(I*(e + f*x))
) - I*b*E^(2*I*(e + f*x))], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4584

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*((a_.) + Cos[(c_.) + (d_.)*(x_.)]*(b_.)*Sin[(c
_.) + (d_.)*(x_.)]^(n_.), x_Symbol]
:> Int[(e + f*x)^m*(a + (b*SIN[2*c + 2*
d*x])/2)^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{x}{a + b \cos(x) \sin(x)} dx = \int \frac{x}{a + \frac{1}{2}b \sin(2x)} dx$$

$$= 2 \int \frac{e^{2ix} x}{\frac{ib}{2} + 2ae^{2ix} - \frac{1}{2}ibe^{4ix}} dx$$

$$= -\frac{(2ib) \int \frac{e^{2ix} x}{2a - \sqrt{4a^2 - b^2} - ibe^{2ix}} dx}{\sqrt{4a^2 - b^2}} + \frac{(2ib) \int \frac{e^{2ix} x}{2a + \sqrt{4a^2 - b^2} - ibe^{2ix}} dx}{\sqrt{4a^2 - b^2}}$$

$$= -\frac{ix \log\left(1 - \frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{ix \log\left(1 - \frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{i \int \log\left(1 - \frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right) dx}{\sqrt{4a^2 - b^2}}$$

$$= -\frac{ix \log\left(1 - \frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{ix \log\left(1 - \frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{\text{Subst}\left(\int \frac{\log\left(1 - \frac{ibx}{2a - \sqrt{4a^2 - b^2}}\right)}{x} dx\right)}{2\sqrt{4a^2 - b^2}}$$

$$= -\frac{ix \log\left(1 - \frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{ix \log\left(1 - \frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} - \frac{\text{Li}_2\left(\frac{ibe^{2ix}}{2a - \sqrt{4a^2 - b^2}}\right)}{2\sqrt{4a^2 - b^2}} + \frac{\text{Li}_2\left(\frac{ibe^{2ix}}{2a + \sqrt{4a^2 - b^2}}\right)}{2\sqrt{4a^2 - b^2}}$$

Mathematica [B] time = 1.41, size = 788, normalized size = 3.50

$$\frac{1}{2} \left(\frac{\pi \tan^{-1}\left(\frac{2a \tan(x) + b}{\sqrt{4a^2 - b^2}}\right)}{\sqrt{4a^2 - b^2}} + \frac{i \left(\text{Li}_2\left(\frac{(2a - i\sqrt{b^2 - 4a^2})(2a + b - \sqrt{b^2 - 4a^2} \cot(x + \frac{\pi}{4}))}{b(2a + b + \sqrt{b^2 - 4a^2} \cot(x + \frac{\pi}{4}))}\right) - \text{Li}_2\left(\frac{(2a + i\sqrt{b^2 - 4a^2})(2a + b - \sqrt{b^2 - 4a^2} \cot(x + \frac{\pi}{4}))}{b(2a + b + \sqrt{b^2 - 4a^2} \cot(x + \frac{\pi}{4}))}\right) \right)}{2\sqrt{4a^2 - b^2}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*cos[x]*sin[x]), x]
```

```
[Out] ((Pi*ArcTan[(b + 2*a*Tan[x])/Sqrt[4*a^2 - b^2]]/Sqrt[4*a^2 - b^2] + (2*Arc
Cos[(-2*a)/b]*ArcTanh[((2*a - b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]] + (Pi -
4*x)*ArcTanh[((2*a + b)*Tan[Pi/4 + x])/Sqrt[-4*a^2 + b^2]] - (ArcCos[(-2*a
)/b] + (2*I)*ArcTanh[((2*a - b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]])*Log[((2
*a + b)*(-2*a + b - I*Sqrt[-4*a^2 + b^2])*(1 + I*Cot[Pi/4 + x]))/(b*(2*a +
b + Sqrt[-4*a^2 + b^2]*Cot[Pi/4 + x]))] - (ArcCos[(-2*a)/b] - (2*I)*ArcTanh
[((2*a - b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]])*Log[((2*a + b)*((2*I)*a - I
*b + Sqrt[-4*a^2 + b^2])*(I + Cot[Pi/4 + x]))/(b*(2*a + b + Sqrt[-4*a^2 + b
^2]*Cot[Pi/4 + x]))] + (ArcCos[(-2*a)/b] + (2*I)*(ArcTanh[((2*a - b)*Cot[Pi
/4 + x])/Sqrt[-4*a^2 + b^2]] + ArcTanh[((2*a + b)*Tan[Pi/4 + x])/Sqrt[-4*a^
2 + b^2]]))*Log[(-1)^(1/4)*Sqrt[-4*a^2 + b^2]/(2*Sqrt[b]*E^(I*x)*Sqrt[a +
```

```

b*cos[x]*sin[x]]) + (ArcCos[(-2*a)/b] - (2*I)*ArcTanh[((2*a - b)*Cot[Pi/4
+ x])/Sqrt[-4*a^2 + b^2]] - (2*I)*ArcTanh[((2*a + b)*Tan[Pi/4 + x])/Sqrt[-
4*a^2 + b^2]])*Log[-1/2*((-1)^(3/4)*Sqrt[-4*a^2 + b^2]*E^(I*x))/(Sqrt[b]*S
qrt[a + b*cos[x]*sin[x]])] + I*(PolyLog[2, ((2*a - I*Sqrt[-4*a^2 + b^2])*(2*
a + b - Sqrt[-4*a^2 + b^2]*Cot[Pi/4 + x]))/(b*(2*a + b + Sqrt[-4*a^2 + b^2]
*Cot[Pi/4 + x]))] - PolyLog[2, ((2*a + I*Sqrt[-4*a^2 + b^2])*(2*a + b - Sqr
t[-4*a^2 + b^2]*Cot[Pi/4 + x]))/(b*(2*a + b + Sqrt[-4*a^2 + b^2]*Cot[Pi/4
+ x]))]))/Sqrt[-4*a^2 + b^2])/2

```

fricas [B] time = 2.25, size = 1688, normalized size = 7.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) + 4*a*sin(x) -
2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 -
b^2)/b^2) + 2*I*a)/b) + 2*b)/b) + 2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((
-4*I*a*cos(x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/
b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b) - 2*b*x*sqrt(-
(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin(x)
))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b
- b)/b) - 2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) + 2*a*sin(x)
+ (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 -
b^2)/b^2) + 2*I*a)/b) - b)/b) + 2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*
a*cos(x) - 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*s
qrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) + 2*b*x*sqrt(-(4*a^2 -
b^2)/b^2)*log(-((-2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-
(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) -
2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((4*I*a*cos(x) + 4*a*sin(x) + 2*(b*
cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)
/b^2) - 2*I*a)/b) + 2*b)/b) - 2*b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(1/2*((-4*I
*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2)
)*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b) + 2*I*b*sqrt(-(4*
a^2 - b^2)/b^2)*dilog(-1/2*((4*I*a*cos(x) + 4*a*sin(x) - 2*(b*cos(x) - I*b*
sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)
/b) + 2*b)/b + 1) + 2*I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((-4*I*a*cos(
x) - 4*a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(
(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) + 2*b)/b + 1) + 2*I*b*sqrt(-(4*a^2
- b^2)/b^2)*dilog(((2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqr
t(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b
+ 1) + 2*I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) + 2*a*sin(x) +
(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 -
b^2)/b^2) + 2*I*a)/b) - b)/b + 1) - 2*I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((
2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2
))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b + 1) - 2*I*b*sqrt(-
(4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin(
x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)
- b)/b + 1) - 2*I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(-1/2*((4*I*a*cos(x) + 4*
a*sin(x) + 2*(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqr
t(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*b)/b + 1) - 2*I*b*sqrt(-(4*a^2 - b^2)
/b^2)*dilog(-1/2*((-4*I*a*cos(x) - 4*a*sin(x) - 2*(b*cos(x) - I*b*sin(x))*s
qrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) + 2*
b)/b + 1))/(4*a^2 - b^2)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b \cos(x) \sin(x) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*cos(x)*sin(x)),x, algorithm="giac")

[Out] integrate(x/(b*cos(x)*sin(x) + a), x)

maple [B] time = 0.22, size = 1284, normalized size = 5.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b*cos(x)*sin(x)),x)

[Out]
$$4*I/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}*\ln(1-b*\exp(2*I*x)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*(-2*a-b)*(2*a+b))^{1/2}*a*x-4/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*(-2*a-b)*(2*a+b))^{1/2}*a*x^2-2/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*\ln(1-b*\exp(2*I*x)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*b^2*x+4/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*(-2*a-b)*(2*a+b))^{1/2}*a*x^2-2/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*\ln(1-b*\exp(2*I*x)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*polylog(2,b*\exp(2*I*x)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*(-2*a-b)*(2*a+b))^{1/2}*a-8*I/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*a^2*x^2+2*I/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*b^2*x^2-4*I/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*polylog(2,b*\exp(2*I*x)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*a^2+I/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*polylog(2,b*\exp(2*I*x)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*b^2-4*I/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*\ln(1-b*\exp(2*I*x)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*(-2*a-b)*(2*a+b))^{1/2}*a*x+2/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*polylog(2,b*\exp(2*I*x)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*(-2*a-b)*(2*a+b))^{1/2}*a+8/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*\ln(1-b*\exp(2*I*x)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*a^2*x+8/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*\ln(1-b*\exp(2*I*x)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*a^2*x-2/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*\ln(1-b*\exp(2*I*x)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*b^2*x-8*I/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*a^2*x^2-4*I/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*polylog(2,b*\exp(2*I*x)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*a^2+I/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*polylog(2,b*\exp(2*I*x)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*b^2+2*I/(8*a^2-2*b^2)/(-2*I*a+(-2*a-b)*(2*a+b))^{1/2}))*b^2*x^2$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b \cos(x) \sin(x) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*cos(x)*sin(x)),x, algorithm="maxima")

[Out] integrate(x/(b*cos(x)*sin(x) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{a + b \cos(x) \sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b*cos(x)*sin(x)),x)

[Out] int(x/(a + b*cos(x)*sin(x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \sin(x) \cos(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)*sin(x)),x)
```

```
[Out] Integral(x/(a + b*sin(x)*cos(x)), x)
```

$$3.582 \quad \int \frac{1}{x(a+b \cos(x) \sin(x))} dx$$

Optimal. Leaf size=20

$$\text{Int} \left(\frac{1}{x \left(a + \frac{1}{2} b \sin(2x) \right)}, x \right)$$

[Out] Unintegrable(1/x/(a+1/2*b*sin(2*x)), x)

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(a + b \cos(x) \sin(x))} dx$$

Verification is Not applicable to the result.

[In] Int [1/(x*(a + b*Cos [x]*Sin [x])), x]

[Out] Defer [Int] [1/(x*(a + (b*SIn [2*x])/2)), x]

Rubi steps

$$\int \frac{1}{x(a + b \cos(x) \sin(x))} dx = \int \frac{1}{x \left(a + \frac{1}{2} b \sin(2x) \right)} dx$$

Mathematica [A] time = 1.58, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \cos(x) \sin(x))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*(a + b*Cos [x]*Sin [x])), x]

[Out] Integrate[1/(x*(a + b*Cos [x]*Sin [x])), x]

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{1}{bx \cos(x) \sin(x) + ax}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*cos(x)*sin(x)), x, algorithm="fricas")

[Out] integral(1/(b*x*cos(x)*sin(x) + a*x), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cos(x) \sin(x) + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*cos(x)*sin(x)), x, algorithm="giac")

[Out] integrate(1/((b*cos(x)*sin(x) + a)*x), x)

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \cos(x) \sin(x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(a+b*cos(x)*sin(x)),x)`

[Out] `int(1/x/(a+b*cos(x)*sin(x)),x)`

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \cos(x) \sin(x) + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b*cos(x)*sin(x)),x, algorithm="maxima")`

[Out] `integrate(1/((b*cos(x)*sin(x) + a)*x), x)`

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x(a + b \cos(x) \sin(x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b*cos(x)*sin(x))),x)`

[Out] `int(1/(x*(a + b*cos(x)*sin(x))), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \sin(x) \cos(x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b*cos(x)*sin(x)),x)`

[Out] `Integral(1/(x*(a + b*sin(x)*cos(x))), x)`

$$3.583 \quad \int \frac{(bx)^{2-n} \sin^n(ax)}{(acx \cos(ax) - c \sin(ax))^2} dx$$

Optimal. Leaf size=79

$$\frac{b^2(1-n) \operatorname{Int}\left((bx)^{-n} \sin^{n-2}(ax), x\right)}{a^2 c^2} + \frac{b(bx)^{1-n} \sin^{n-1}(ax)}{a^2 (ac^2 x \cos(ax) - c^2 \sin(ax))}$$

[Out] b*(b*x)^(1-n)*sin(a*x)^(-1+n)/a^2/(a*c^2*x*cos(a*x)-c^2*sin(a*x))+b^2*(1-n)*Unintegrable(sin(a*x)^(-2+n)/((b*x)^n),x)/a^2/c^2

Rubi [A] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(bx)^{2-n} \sin^n(ax)}{(acx \cos(ax) - c \sin(ax))^2} dx$$

Verification is Not applicable to the result.

[In] Int[((b*x)^(2 - n)*Sin[a*x]^n)/(a*c*x*Cos[a*x] - c*Ssin[a*x])^2,x]

[Out] (b*(b*x)^(1 - n)*Sin[a*x]^(-1 + n))/(a^2*(a*c^2*x*Cos[a*x] - c^2*Ssin[a*x])) + (b^2*(1 - n)*Defer[Int][Sin[a*x]^(-2 + n)/(b*x)^n, x])/(a^2*c^2)

Rubi steps

$$\int \frac{(bx)^{2-n} \sin^n(ax)}{(acx \cos(ax) - c \sin(ax))^2} dx = \frac{b(bx)^{1-n} \sin^{-1+n}(ax)}{a^2 (ac^2 x \cos(ax) - c^2 \sin(ax))} + \frac{(b^2(1-n)) \int (bx)^{-n} \sin^{-2+n}(ax) dx}{a^2 c^2}$$

Mathematica [A] time = 5.46, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{2-n} \sin^n(ax)}{(acx \cos(ax) - c \sin(ax))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((b*x)^(2 - n)*Sin[a*x]^n)/(a*c*x*Cos[a*x] - c*Ssin[a*x])^2,x]

[Out] Integrate[((b*x)^(2 - n)*Sin[a*x]^n)/(a*c*x*Cos[a*x] - c*Ssin[a*x])^2, x]

fricas [A] time = 0.96, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{(bx)^{-n+2} \sin(ax)^n}{2 ac^2 x \cos(ax) \sin(ax) - (a^2 c^2 x^2 - c^2) \cos(ax)^2 - c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x, algorithm="fricas")

[Out] integral(-(b*x)^(-n + 2)*sin(a*x)^n/(2*a*c^2*x*cos(a*x)*sin(a*x) - (a^2*c^2*x^2 - c^2)*cos(a*x)^2 - c^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{-n+2} \sin(ax)^n}{(acx \cos(ax) - c \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x, algorithm="giac")

[Out] integrate((b*x)^(-n + 2)*sin(a*x)^n/(a*c*x*cos(a*x) - c*sin(a*x))^2, x)

maple [A] time = 2.19, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{2-n} (\sin^n(ax))}{(acx \cos(ax) - c \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x)

[Out] int((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{-n+2} \sin^n(ax)}{(acx \cos(ax) - c \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x, algorithm="maxima")

[Out] integrate((b*x)^(-n + 2)*sin(a*x)^n/(a*c*x*cos(a*x) - c*sin(a*x))^2, x)

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(ax)^n (bx)^{2-n}}{(c \sin(ax) - acx \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(a*x)^n*(b*x)^(2 - n))/(c*sin(a*x) - a*c*x*cos(a*x))^2,x)

[Out] int((sin(a*x)^n*(b*x)^(2 - n))/(c*sin(a*x) - a*c*x*cos(a*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)**(2-n)*sin(a*x)**n/(a*c*x*cos(a*x)-c*sin(a*x))**2,x)

[Out] Timed out

$$3.584 \quad \int \frac{(bx)^{2-n} \cos^n(ax)}{(c \cos(ax) + acx \sin(ax))^2} dx$$

Optimal. Leaf size=79

$$\frac{b^2(1-n) \operatorname{Int}\left((bx)^{-n} \cos^{n-2}(ax), x\right)}{a^2 c^2} - \frac{b(bx)^{1-n} \cos^{n-1}(ax)}{a^2 (ac^2 x \sin(ax) + c^2 \cos(ax))}$$

[Out] $-b*(b*x)^{(1-n)}*\cos(a*x)^{(-1+n)}/a^2/(c^2*\cos(a*x)+a*c^2*x*\sin(a*x))+b^2*(1-n)*\operatorname{Unintegrable}(\cos(a*x)^{(-2+n)}/((b*x)^n), x)/a^2/c^2$

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(bx)^{2-n} \cos^n(ax)}{(c \cos(ax) + acx \sin(ax))^2} dx$$

Verification is Not applicable to the result.

[In] $\operatorname{Int}[(b*x)^{(2-n)}*\cos[a*x]^n/(c*\cos[a*x] + a*c*x*\sin[a*x])^2, x]$

[Out] $-((b*(b*x)^{(1-n)}*\cos[a*x]^{(-1+n)})/(a^2*(c^2*\cos[a*x] + a*c^2*x*\sin[a*x]))) + (b^2*(1-n)*\operatorname{Defer}[\operatorname{Int}[\cos[a*x]^{(-2+n)}/(b*x)^n, x]]/(a^2*c^2))$

Rubi steps

$$\int \frac{(bx)^{2-n} \cos^n(ax)}{(c \cos(ax) + acx \sin(ax))^2} dx = -\frac{b(bx)^{1-n} \cos^{-1+n}(ax)}{a^2 (c^2 \cos(ax) + ac^2 x \sin(ax))} + \frac{(b^2(1-n)) \int (bx)^{-n} \cos^{-2+n}(ax) dx}{a^2 c^2}$$

Mathematica [A] time = 4.97, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{2-n} \cos^n(ax)}{(c \cos(ax) + acx \sin(ax))^2} dx$$

Verification is Not applicable to the result.

[In] $\operatorname{Integrate}[(b*x)^{(2-n)}*\cos[a*x]^n/(c*\cos[a*x] + a*c*x*\sin[a*x])^2, x]$

[Out] $\operatorname{Integrate}[(b*x)^{(2-n)}*\cos[a*x]^n/(c*\cos[a*x] + a*c*x*\sin[a*x])^2, x]$

fricas [A] time = 0.91, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(bx)^{-n+2} \cos(ax)^n}{a^2 c^2 x^2 + 2 ac^2 x \cos(ax) \sin(ax) - (a^2 c^2 x^2 - c^2) \cos(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}((b*x)^{(2-n)}*\cos(a*x)^n/(c*\cos(a*x)+a*c*x*\sin(a*x))^2, x, \text{algorithm} = \text{"fricas"})$

[Out] $\operatorname{integral}((b*x)^{(-n+2)}*\cos(a*x)^n/(a^2*c^2*x^2 + 2*a*c^2*x*\cos(a*x)*\sin(a*x) - (a^2*c^2*x^2 - c^2)*\cos(a*x)^2), x)$

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{-n+2} \cos(ax)^n}{(acx \sin(ax) + c \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x, algorithm="giac")

[Out] integrate((b*x)^(-n + 2)*cos(a*x)^n/(a*c*x*sin(a*x) + c*cos(a*x))^2, x)

maple [A] time = 2.19, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{2-n} (\cos^n(ax))}{(c \cos(ax) + acx \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x)

[Out] int((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{-n+2} \cos(ax)^n}{(acx \sin(ax) + c \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x, algorithm="maxima")

[Out] integrate((b*x)^(-n + 2)*cos(a*x)^n/(a*c*x*sin(a*x) + c*cos(a*x))^2, x)

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(ax)^n (bx)^{2-n}}{(c \cos(ax) + acx \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(a*x)^n*(b*x)^(2 - n))/(c*cos(a*x) + a*c*x*sin(a*x))^2,x)

[Out] int((cos(a*x)^n*(b*x)^(2 - n))/(c*cos(a*x) + a*c*x*sin(a*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)**(2-n)*cos(a*x)**n/(c*cos(a*x)+a*c*x*sin(a*x))**2,x)

[Out] Timed out

$$3.585 \quad \int \frac{\sin^6(ax)}{x^4(ax \cos(ax) - \sin(ax))^2} dx$$

Optimal. Leaf size=175

$$-\frac{2}{3}a^3\text{Si}(2ax) + \frac{16}{3}a^3\text{Si}(4ax) + \frac{\sin^4(ax)}{a^2x^5} + \frac{\sin^5(ax)}{a^2x^5(ax \cos(ax) - \sin(ax))} + \frac{a^2}{x} + \frac{32a^2 \sin^4(ax)}{3x} - \frac{10a^2 \sin^2(ax)}{x} + \frac{\sin^3(ax)}{3x^3}$$

[Out] a^2/x-2/3*a^3*Si(2*a*x)+16/3*a^3*Si(4*a*x)+a*cos(a*x)*sin(a*x)/x^2+sin(a*x)^2/x^3-10*a^2*sin(a*x)^2/x+cos(a*x)*sin(a*x)^3/a/x^4-8/3*a*cos(a*x)*sin(a*x)^3/x^2+sin(a*x)^4/a^2/x^5-4/3*sin(a*x)^4/x^3+32/3*a^2*sin(a*x)^4/x+sin(a*x)^5/a^2/x^5/(a*x*cos(a*x)-sin(a*x))

Rubi [A] time = 0.30, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4598, 3314, 30, 3313, 12, 3299}

$$-\frac{2}{3}a^3\text{Si}(2ax) + \frac{16}{3}a^3\text{Si}(4ax) + \frac{\sin^4(ax)}{a^2x^5} + \frac{\sin^5(ax)}{a^2x^5(ax \cos(ax) - \sin(ax))} + \frac{a^2}{x} + \frac{32a^2 \sin^4(ax)}{3x} - \frac{10a^2 \sin^2(ax)}{x} - \frac{4 \sin^4(ax)}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[Sin[a*x]^6/(x^4*(a*x*Cos[a*x] - Sin[a*x])^2), x]

[Out] a^2/x + (a*Cos[a*x]*Sin[a*x])/x^2 + Sin[a*x]^2/x^3 - (10*a^2*Sin[a*x]^2)/x + (Cos[a*x]*Sin[a*x]^3)/(a*x^4) - (8*a*Cos[a*x]*Sin[a*x]^3)/(3*x^2) + Sin[a*x]^4/(a^2*x^5) - (4*Sin[a*x]^4)/(3*x^3) + (32*a^2*Sin[a*x]^4)/(3*x) + Sin[a*x]^5/(a^2*x^5*(a*x*Cos[a*x] - Sin[a*x])) - (2*a^3*SinIntegral[2*a*x])/3 + (16*a^3*SinIntegral[4*a*x])/3

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3313

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Simp[((c + d*x)^(m+1)*Sin[e + f*x]^n)/(d*(m+1)), x] - Dist[(f*n)/(d*(m+1)), Int[ExpandTrigReduce[(c + d*x)^(m+1), Cos[e + f*x]*Sin[e + f*x]^(n-1), x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

Rule 3314

Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Simp[((c + d*x)^(m+1)*(b*Sin[e + f*x])^n)/(d*(m+1)), x] + (Dist[(b^2*f^2*n*(n-1))/(d^2*(m+1)*(m+2)), Int[(c + d*x)^(m+2)*(b*Sin[e + f*x])^(n-2), x], x] - Dist[(f^2*n^2)/(d^2*(m+1)*(m+2)), Int[(c + d*x)^(m+2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m+2)*Cos[e + f*x]*(b*Sin[e + f*x])^(n-1))/(d^2*(m+1)*(m+2)), x]) /; FreeQ[{b, c,

d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

Rule 4598

Int[(((b_.)*(x_.))^(m_)*Sin[(a_.)*(x_.)]^(n_))/(Cos[(a_.)*(x_.)]*(d_.)*(x_.) + (c_.)*Sin[(a_.)*(x_.)]^2, x_Symbol] :> Simp[(b*(b*x)^(m - 1)*Sin[a*x]^(n - 1))/(a*d*(c*SIN[a*x] + d*x*cos[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b*x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a*c + d, 0] && EqQ[m, 2 - n]

Rubi steps

$$\begin{aligned} \int \frac{\sin^6(ax)}{x^4(ax \cos(ax) - \sin(ax))^2} dx &= \frac{\sin^5(ax)}{a^2 x^5(ax \cos(ax) - \sin(ax))} - \frac{5 \int \frac{\sin^4(ax)}{x^6} dx}{a^2} \\ &= \frac{\cos(ax) \sin^3(ax)}{ax^4} + \frac{\sin^4(ax)}{a^2 x^5} + \frac{\sin^5(ax)}{a^2 x^5(ax \cos(ax) - \sin(ax))} - 3 \int \frac{\sin^2(ax)}{x^4} dx \\ &= \frac{a \cos(ax) \sin(ax)}{x^2} + \frac{\sin^2(ax)}{x^3} + \frac{\cos(ax) \sin^3(ax)}{ax^4} - \frac{8a \cos(ax) \sin^3(ax)}{3x^2} + \frac{\sin^4(ax)}{a^2 x^5} \\ &= \frac{a^2}{x} + \frac{a \cos(ax) \sin(ax)}{x^2} + \frac{\sin^2(ax)}{x^3} - \frac{10a^2 \sin^2(ax)}{x} + \frac{\cos(ax) \sin^3(ax)}{ax^4} - \frac{8a \cos(ax) \sin^3(ax)}{3x^2} \\ &= \frac{a^2}{x} + \frac{a \cos(ax) \sin(ax)}{x^2} + \frac{\sin^2(ax)}{x^3} - \frac{10a^2 \sin^2(ax)}{x} + \frac{\cos(ax) \sin^3(ax)}{ax^4} - \frac{8a \cos(ax) \sin^3(ax)}{3x^2} \\ &= \frac{a^2}{x} + \frac{a \cos(ax) \sin(ax)}{x^2} + \frac{\sin^2(ax)}{x^3} - \frac{10a^2 \sin^2(ax)}{x} + \frac{\cos(ax) \sin^3(ax)}{ax^4} - \frac{8a \cos(ax) \sin^3(ax)}{3x^2} \end{aligned}$$

Mathematica [A] time = 1.45, size = 198, normalized size = 1.13

$$\frac{-32a^3 x^3 \text{Si}(2ax)(ax \cos(ax) - \sin(ax)) + 256a^3 x^3 \text{Si}(4ax)(ax \cos(ax) - \sin(ax)) - 8a^3 x^3 \cos(ax) + 24a^3 x^3 \cos(3ax)}{3(ax^4 \cos(ax) - x^3 \sin(ax))}$$

Antiderivative was successfully verified.

[In] Integrate[SIN[a*x]^6/(x^4*(a*x*cos[a*x] - SIN[a*x])^2), x]

[Out] (8*a*x*cos[a*x] - 8*a^3*x^3*cos[a*x] - 12*a*x*cos[3*a*x] + 24*a^3*x^3*cos[3*a*x] + 4*a*x*cos[5*a*x] + 32*a^3*x^3*cos[5*a*x] + 10*SIN[a*x] - 12*a^2*x^2*SIN[a*x] - 5*SIN[3*a*x] + 44*a^2*x^2*SIN[3*a*x] + SIN[5*a*x] - 24*a^2*x^2*SIN[5*a*x] - 32*a^3*x^3*(a*x*cos[a*x] - SIN[a*x])*SINIntegral[2*a*x] + 256*a^3*x^3*(a*x*cos[a*x] - SIN[a*x])*SINIntegral[4*a*x])/(48*x^3*(a*x*cos[a*x] - SIN[a*x]))

fricas [A] time = 1.07, size = 186, normalized size = 1.06

$$\frac{4(8a^3x^3 + ax) \cos(ax)^5 - 2(17a^3x^3 + 4ax) \cos(ax)^3 + (16a^4x^4 \text{Si}(4ax) - 2a^4x^4 \text{Si}(2ax) + 5a^3x^3 + 4ax) \cos(ax)}{3(ax^4 \cos(ax) - x^3 \sin(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")

[Out] 1/3*(4*(8*a^3*x^3 + a*x)*cos(a*x)^5 - 2*(17*a^3*x^3 + 4*a*x)*cos(a*x)^3 + (16*a^4*x^4*sin_integral(4*a*x) - 2*a^4*x^4*sin_integral(2*a*x) + 5*a^3*x^3 + 4*a*x)*cos(a*x) - (16*a^3*x^3*sin_integral(4*a*x) - 2*a^3*x^3*sin_integral(2*a*x) + (24*a^2*x^2 - 1)*cos(a*x)^4 + 5*a^2*x^2 - (29*a^2*x^2 - 2)*cos(a*x)^2 - 1)*sin(a*x))/(a*x^4*cos(a*x) - x^3*sin(a*x))

giac [C] time = 1.20, size = 7347, normalized size = 41.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/12*(32*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan \\ & (1/2*a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x \\ &)^2*tan(1/2*a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2 \\ & *tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(-4*a*x))*tan \\ & (2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integral(4*a*x)*tan(2* \\ & a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x) \\ & ^2*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(4*a*x))*ta \\ & n(2*a*x)^2*tan(a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x) \\ & ^2*tan(a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan \\ & (a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2 \\ & - 64*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 8*a^8*x^8*sin_i \\ & ntegral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(\\ & 4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(2*a* \\ & x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(-2*a*x)) \\ & *tan(2*a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(-4*a*x))*t \\ & an(2*a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2*ta \\ & n(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 + \\ & 32*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^8 \\ & *x^8*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^8*x^8*i \\ & mag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag \\ & part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integ \\ & ral(4*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(a* \\ & x)^2*tan(1/2*a*x)^2 + 64*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^ \\ & 2*tan(a*x)^2*tan(1/2*a*x) - 8*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(2* \\ & a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 8*a^7*x^7*imag_part(cos_integral(-2*a*x))* \\ & tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 64*a^7*x^7*imag_part(cos_integral(-4 \\ & *a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 128*a^7*x^7*sin_integral(4*a* \\ & x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 16*a^7*x^7*sin_integral(2*a*x)*ta \\ & n(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 12*a^7*x^7*tan(2*a*x)^2*tan(a*x)^2*tan \\ & (1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2 + 4*a^ \\ & 8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2 - 4*a^8*x^8*imag_part(cos \\ & _integral(-2*a*x))*tan(2*a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(-4*a*x) \\ &)*tan(2*a*x)^2 - 64*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2 + 8*a^8*x^8*si \\ & n_integral(2*a*x)*tan(2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(4*a*x))* \\ & tan(a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - 4*a^8*x^ \\ & 8*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 + 32*a^8*x^8*imag_part(cos_int \\ & egral(-4*a*x))*tan(a*x)^2 - 64*a^8*x^8*sin_integral(4*a*x)*tan(a*x)^2 + 8*a \\ & ^8*x^8*sin_integral(2*a*x)*tan(a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(4 \\ & *a*x))*tan(1/2*a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(1/2*a* \\ & x)^2 + 4*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x)^2 - 32*a^8*x^ \\ & 8*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integral(\\ & 4*a*x)*tan(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(1/2*a*x)^2 + 64*a \\ & ^6*x^6*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^ \\ & 2 - 8*a^6*x^6*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/ \\ & 2*a*x)^2 + 8*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^ \\ & 2*tan(1/2*a*x)^2 - 64*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2* \\ & tan(a*x)^2*tan(1/2*a*x)^2 + 128*a^6*x^6*sin_integral(4*a*x)*tan(2*a*x)^2*ta \\ & n(a*x)^2*tan(1/2*a*x)^2 - 16*a^6*x^6*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a \\ & *x)^2*tan(1/2*a*x)^2 + 12*a^7*x^7*tan(2*a*x)^2*tan(a*x)^2 + 64*a^7*x^7*imag \\ & _part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 8*a^7*x^7*imag_part(\\ & cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 8*a^7*x^7*imag_part(cos_in \\ & tegral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 64*a^7*x^7*imag_part(cos_integr \\ & \end{aligned}$$

$$\begin{aligned}
& \operatorname{al}(-4ax)) \tan(2ax)^2 \tan(1/2ax) + 128a^7x^7 \sin_integral(4ax) \tan \\
& (2ax)^2 \tan(1/2ax) - 16a^7x^7 \sin_integral(2ax) \tan(2ax)^2 \tan(1/ \\
& 2ax) + 64a^7x^7 \operatorname{imag_part}(\cos_integral(4ax)) \tan(ax)^2 \tan(1/2ax) \\
& - 8a^7x^7 \operatorname{imag_part}(\cos_integral(2ax)) \tan(ax)^2 \tan(1/2ax) + 8a^7x^ \\
& 7 \operatorname{imag_part}(\cos_integral(-2ax)) \tan(ax)^2 \tan(1/2ax) - 64a^7x^7 \operatorname{im} \\
& \operatorname{ag_part}(\cos_integral(-4ax)) \tan(ax)^2 \tan(1/2ax) + 128a^7x^7 \sin_int \\
& egral(4ax) \tan(ax)^2 \tan(1/2ax) - 16a^7x^7 \sin_integral(2ax) \tan(a \\
& x)^2 \tan(1/2ax) - 20a^7x^7 \tan(2ax)^2 \tan(1/2ax)^2 + 20a^7x^7 \tan \\
& (ax)^2 \tan(1/2ax)^2 - 32a^8x^8 \operatorname{imag_part}(\cos_integral(4ax)) + 4a^8 \\
& x^8 \operatorname{imag_part}(\cos_integral(2ax)) - 4a^8x^8 \operatorname{imag_part}(\cos_integral(-2a \\
& x)) + 32a^8x^8 \operatorname{imag_part}(\cos_integral(-4ax)) - 64a^8x^8 \sin_integral \\
& (4ax) + 8a^8x^8 \sin_integral(2ax) - 64a^6x^6 \operatorname{imag_part}(\cos_integral \\
& (4ax)) \tan(2ax)^2 \tan(ax)^2 + 8a^6x^6 \operatorname{imag_part}(\cos_integral(2ax)) \\
& \tan(2ax)^2 \tan(ax)^2 - 8a^6x^6 \operatorname{imag_part}(\cos_integral(-2ax)) \tan(2 \\
& ax)^2 \tan(ax)^2 + 64a^6x^6 \operatorname{imag_part}(\cos_integral(-4ax)) \tan(2ax)^2 \\
& \tan(ax)^2 - 128a^6x^6 \sin_integral(4ax) \tan(2ax)^2 \tan(ax)^2 + 16 \\
& a^6x^6 \sin_integral(2ax) \tan(2ax)^2 \tan(ax)^2 - 24a^6x^6 \tan(2ax) \\
& ^2 \tan(ax)^2 \tan(1/2ax) + 64a^6x^6 \operatorname{imag_part}(\cos_integral(4ax)) \tan(\\
& 2ax)^2 \tan(1/2ax)^2 - 8a^6x^6 \operatorname{imag_part}(\cos_integral(2ax)) \tan(2ax) \\
& ^2 \tan(1/2ax)^2 + 8a^6x^6 \operatorname{imag_part}(\cos_integral(-2ax)) \tan(2ax)^2 \\
& \tan(1/2ax)^2 - 64a^6x^6 \operatorname{imag_part}(\cos_integral(-4ax)) \tan(2ax)^2 \\
& \tan(1/2ax)^2 + 128a^6x^6 \sin_integral(4ax) \tan(2ax)^2 \tan(1/2ax)^2 \\
& - 16a^6x^6 \sin_integral(2ax) \tan(2ax)^2 \tan(1/2ax)^2 - 4a^6x^6 \\
& \tan(2ax)^2 \tan(ax) \tan(1/2ax)^2 + 64a^6x^6 \operatorname{imag_part}(\cos_integral(4 \\
& ax)) \tan(ax)^2 \tan(1/2ax)^2 - 8a^6x^6 \operatorname{imag_part}(\cos_integral(2ax)) \\
& \tan(ax)^2 \tan(1/2ax)^2 + 8a^6x^6 \operatorname{imag_part}(\cos_integral(-2ax)) \tan(a \\
& x)^2 \tan(1/2ax)^2 - 64a^6x^6 \operatorname{imag_part}(\cos_integral(-4ax)) \tan(ax)^2 \\
& \tan(1/2ax)^2 + 128a^6x^6 \sin_integral(4ax) \tan(ax)^2 \tan(1/2ax)^2 \\
& - 16a^6x^6 \sin_integral(2ax) \tan(ax)^2 \tan(1/2ax)^2 + 8a^6x^6 \tan \\
& (2ax) \tan(ax)^2 \tan(1/2ax)^2 + 20a^7x^7 \tan(2ax)^2 - 20a^7x^7 \tan \\
& (ax)^2 + 64a^7x^7 \operatorname{imag_part}(\cos_integral(4ax)) \tan(1/2ax) - 8a^7x^ \\
& 7 \operatorname{imag_part}(\cos_integral(2ax)) \tan(1/2ax) + 8a^7x^7 \operatorname{imag_part}(\cos_i \\
& ntegral(-2ax)) \tan(1/2ax) - 64a^7x^7 \operatorname{imag_part}(\cos_integral(-4ax)) \\
& \tan(1/2ax) + 128a^7x^7 \sin_integral(4ax) \tan(1/2ax) - 16a^7x^7 \sin \\
& _integral(2ax) \tan(1/2ax) + 128a^5x^5 \operatorname{imag_part}(\cos_integral(4ax)) \\
& \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax) - 16a^5x^5 \operatorname{imag_part}(\cos_integral(2 \\
& ax)) \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax) + 16a^5x^5 \operatorname{imag_part}(\cos_inte \\
& gral(-2ax)) \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax) - 128a^5x^5 \operatorname{imag_part} \\
& (\cos_integral(-4ax)) \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax) + 256a^5x^5 \sin \\
& _integral(4ax) \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax) - 32a^5x^5 \sin_int \\
& egral(2ax) \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax) + 12a^7x^7 \tan(1/2ax) \\
& ^2 - 24a^5x^5 \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax)^2 - 64a^6x^6 \operatorname{imag_pa} \\
& rt(\cos_integral(4ax)) \tan(2ax)^2 + 8a^6x^6 \operatorname{imag_part}(\cos_integral(2ax) \\
&) \tan(2ax)^2 - 8a^6x^6 \operatorname{imag_part}(\cos_integral(-2ax)) \tan(2ax)^2 \\
& + 64a^6x^6 \operatorname{imag_part}(\cos_integral(-4ax)) \tan(2ax)^2 - 128a^6x^6 \sin \\
& _integral(4ax) \tan(2ax)^2 + 16a^6x^6 \sin_integral(2ax) \tan(2ax)^2 \\
& + 4a^6x^6 \tan(2ax)^2 \tan(ax) - 64a^6x^6 \operatorname{imag_part}(\cos_integral(4ax) \\
& x)) \tan(ax)^2 + 8a^6x^6 \operatorname{imag_part}(\cos_integral(2ax)) \tan(ax)^2 - 8a^ \\
& 6x^6 \operatorname{imag_part}(\cos_integral(-2ax)) \tan(ax)^2 + 64a^6x^6 \operatorname{imag_part}(\cos \\
& _integral(-4ax)) \tan(ax)^2 - 128a^6x^6 \sin_integral(4ax) \tan(ax)^2 \\
& + 16a^6x^6 \sin_integral(2ax) \tan(ax)^2 - 8a^6x^6 \tan(2ax) \tan(ax) \\
& ^2 - 40a^6x^6 \tan(2ax)^2 \tan(1/2ax) + 40a^6x^6 \tan(ax)^2 \tan(1/2ax) \\
& + 64a^6x^6 \operatorname{imag_part}(\cos_integral(4ax)) \tan(1/2ax)^2 - 8a^6x^6 \\
& \operatorname{imag_part}(\cos_integral(2ax)) \tan(1/2ax)^2 + 8a^6x^6 \operatorname{imag_part}(\cos_int \\
& egral(-2ax)) \tan(1/2ax)^2 - 64a^6x^6 \operatorname{imag_part}(\cos_integral(-4ax)) \\
& \tan(1/2ax)^2 + 128a^6x^6 \sin_integral(4ax) \tan(1/2ax)^2 - 16a^6x^6 \\
& \sin_integral(2ax) \tan(1/2ax)^2 + 8a^6x^6 \tan(2ax) \tan(1/2ax)^2 \\
& - 4a^6x^6 \tan(ax) \tan(1/2ax)^2 + 32a^4x^4 \operatorname{imag_part}(\cos_integral(4ax) \\
& x)) \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax)^2 - 4a^4x^4 \operatorname{imag_part}(\cos_integ
\end{aligned}$$

$$\begin{aligned} & \text{ral}(2*a*x)) * \tan(2*a*x)^2 * \tan(a*x)^2 * \tan(1/2*a*x)^2 + 4*a^4*x^4 * \text{imag_part}(\cos_ \\ & \text{integral}(-2*a*x)) * \tan(2*a*x)^2 * \tan(a*x)^2 * \tan(1/2*a*x)^2 - 32*a^4*x^4 * \text{ima} \\ & \text{g_part}(\cos_ \text{integral}(-4*a*x)) * \tan(2*a*x)^2 * \tan(a*x)^2 * \tan(1/2*a*x)^2 + 64*a^4*x^4 * \text{sin_} \\ & \text{integral}(4*a*x) * \tan(2*a*x)^2 * \tan(a*x)^2 * \tan(1/2*a*x)^2 - 8*a^4*x^4 * \text{sin_} \\ & \text{integral}(2*a*x) * \tan(2*a*x)^2 * \tan(a*x)^2 * \tan(1/2*a*x)^2 - 12*a^7*x^7 + \\ & 24*a^5*x^5 * \tan(2*a*x)^2 * \tan(a*x)^2 + 128*a^5*x^5 * \text{imag_part}(\cos_ \text{integral}(4* \\ & a*x)) * \tan(2*a*x)^2 * \tan(1/2*a*x) - 16*a^5*x^5 * \text{imag_part}(\cos_ \text{integral}(2*a*x)) \\ & * \tan(2*a*x)^2 * \tan(1/2*a*x) + 16*a^5*x^5 * \text{imag_part}(\cos_ \text{integral}(-2*a*x)) * \tan \\ & (2*a*x)^2 * \tan(1/2*a*x) - 128*a^5*x^5 * \text{imag_part}(\cos_ \text{integral}(-4*a*x)) * \tan(2* \\ & a*x)^2 * \tan(1/2*a*x) + 256*a^5*x^5 * \text{sin_} \text{integral}(4*a*x) * \tan(2*a*x)^2 * \tan(1/2* \\ & a*x) - 32*a^5*x^5 * \text{sin_} \text{integral}(2*a*x) * \tan(2*a*x)^2 * \tan(1/2*a*x) - 8*a^5*x^5 \\ & * \tan(2*a*x)^2 * \tan(a*x) * \tan(1/2*a*x) + 128*a^5*x^5 * \text{imag_part}(\cos_ \text{integral}(4* \\ & a*x)) * \tan(a*x)^2 * \tan(1/2*a*x) - 16*a^5*x^5 * \text{imag_part}(\cos_ \text{integral}(2*a*x)) * \text{t} \\ & \text{an}(a*x)^2 * \tan(1/2*a*x) + 16*a^5*x^5 * \text{imag_part}(\cos_ \text{integral}(-2*a*x)) * \tan(a*x) \\ &)^2 * \tan(1/2*a*x) - 128*a^5*x^5 * \text{imag_part}(\cos_ \text{integral}(-4*a*x)) * \tan(a*x)^2 * \text{t} \\ & \text{an}(1/2*a*x) + 256*a^5*x^5 * \text{sin_} \text{integral}(4*a*x) * \tan(a*x)^2 * \tan(1/2*a*x) - 32* \\ & a^5*x^5 * \text{sin_} \text{integral}(2*a*x) * \tan(a*x)^2 * \tan(1/2*a*x) + 16*a^5*x^5 * \tan(2*a*x) \\ & * \tan(a*x)^2 * \tan(1/2*a*x) - 36*a^5*x^5 * \tan(2*a*x)^2 * \tan(1/2*a*x)^2 + 36*a^5*x \\ & x^5 * \tan(a*x)^2 * \tan(1/2*a*x)^2 - 64*a^6*x^6 * \text{imag_part}(\cos_ \text{integral}(4*a*x)) + \\ & 8*a^6*x^6 * \text{imag_part}(\cos_ \text{integral}(2*a*x)) - 8*a^6*x^6 * \text{imag_part}(\cos_ \text{integra} \\ & \text{l}(-2*a*x)) + 64*a^6*x^6 * \text{imag_part}(\cos_ \text{integral}(-4*a*x)) - 128*a^6*x^6 * \text{sin_} \text{i} \\ & \text{ntegral}(4*a*x) + 16*a^6*x^6 * \text{sin_} \text{integral}(2*a*x) - 8*a^6*x^6 * \tan(2*a*x) + 4* \\ & a^6*x^6 * \tan(a*x) - 32*a^4*x^4 * \text{imag_part}(\cos_ \text{integral}(4*a*x)) * \tan(2*a*x)^2 * \text{t} \\ & \text{an}(a*x)^2 + 4*a^4*x^4 * \text{imag_part}(\cos_ \text{integral}(2*a*x)) * \tan(2*a*x)^2 * \tan(a*x)^2 \\ & - 4*a^4*x^4 * \text{imag_part}(\cos_ \text{integral}(-2*a*x)) * \tan(2*a*x)^2 * \tan(a*x)^2 + 32* \\ & a^4*x^4 * \text{imag_part}(\cos_ \text{integral}(-4*a*x)) * \tan(2*a*x)^2 * \tan(a*x)^2 - 64*a^4*x^4 * \\ & 4 * \text{sin_} \text{integral}(4*a*x) * \tan(2*a*x)^2 * \tan(a*x)^2 + 8*a^4*x^4 * \text{sin_} \text{integral}(2*a* \\ & x) * \tan(2*a*x)^2 * \tan(a*x)^2 + 24*a^6*x^6 * \tan(1/2*a*x) - 48*a^4*x^4 * \tan(2*a*x) \\ &)^2 * \tan(a*x)^2 * \tan(1/2*a*x) + 32*a^4*x^4 * \text{imag_part}(\cos_ \text{integral}(4*a*x)) * \tan \\ & (2*a*x)^2 * \tan(1/2*a*x)^2 - 4*a^4*x^4 * \text{imag_part}(\cos_ \text{integral}(2*a*x)) * \tan(2*a \\ & x)^2 * \tan(1/2*a*x)^2 + 4*a^4*x^4 * \text{imag_part}(\cos_ \text{integral}(-2*a*x)) * \tan(2*a*x) \\ &)^2 * \tan(1/2*a*x)^2 - 32*a^4*x^4 * \text{imag_part}(\cos_ \text{integral}(-4*a*x)) * \tan(2*a*x)^2 \\ & * \tan(1/2*a*x)^2 + 64*a^4*x^4 * \text{sin_} \text{integral}(4*a*x) * \tan(2*a*x)^2 * \tan(1/2*a*x)^2 \\ & - 8*a^4*x^4 * \text{sin_} \text{integral}(2*a*x) * \tan(2*a*x)^2 * \tan(1/2*a*x)^2 - 2*a^4*x^4 * \text{t} \\ & \text{an}(2*a*x)^2 * \tan(a*x) * \tan(1/2*a*x)^2 + 32*a^4*x^4 * \text{imag_part}(\cos_ \text{integral}(4*a \\ & x)) * \tan(a*x)^2 * \tan(1/2*a*x)^2 - 4*a^4*x^4 * \text{imag_part}(\cos_ \text{integral}(2*a*x)) * \text{t} \\ & \text{an}(a*x)^2 * \tan(1/2*a*x)^2 + 4*a^4*x^4 * \text{imag_part}(\cos_ \text{integral}(-2*a*x)) * \tan(a* \\ & x)^2 * \tan(1/2*a*x)^2 - 32*a^4*x^4 * \text{imag_part}(\cos_ \text{integral}(-4*a*x)) * \tan(a*x)^2 \\ & * \tan(1/2*a*x)^2 + 64*a^4*x^4 * \text{sin_} \text{integral}(4*a*x) * \tan(a*x)^2 * \tan(1/2*a*x)^2 \\ & - 8*a^4*x^4 * \text{sin_} \text{integral}(2*a*x) * \tan(a*x)^2 * \tan(1/2*a*x)^2 + 13*a^4*x^4 * \tan(\\ & 2*a*x) * \tan(a*x)^2 * \tan(1/2*a*x)^2 + 36*a^5*x^5 * \tan(2*a*x)^2 - 36*a^5*x^5 * \tan \\ & (a*x)^2 + 128*a^5*x^5 * \text{imag_part}(\cos_ \text{integral}(4*a*x)) * \tan(1/2*a*x) - 16*a^5*x^5 \\ & x^5 * \text{imag_part}(\cos_ \text{integral}(2*a*x)) * \tan(1/2*a*x) + 16*a^5*x^5 * \text{imag_part}(\cos_ \\ & \text{integral}(-2*a*x)) * \tan(1/2*a*x) - 128*a^5*x^5 * \text{imag_part}(\cos_ \text{integral}(-4*a*x) \\ &) * \tan(1/2*a*x) + 256*a^5*x^5 * \text{sin_} \text{integral}(4*a*x) * \tan(1/2*a*x) - 32*a^5*x^5 * \\ & \text{sin_} \text{integral}(2*a*x) * \tan(1/2*a*x) + 16*a^5*x^5 * \tan(2*a*x) * \tan(1/2*a*x) - 8*a \\ & ^5*x^5 * \tan(a*x) * \tan(1/2*a*x) + 64*a^3*x^3 * \text{imag_part}(\cos_ \text{integral}(4*a*x)) * \text{t} \\ & \text{an}(2*a*x)^2 * \tan(a*x)^2 * \tan(1/2*a*x) - 8*a^3*x^3 * \text{imag_part}(\cos_ \text{integral}(2*a*x \\ &)) * \tan(2*a*x)^2 * \tan(a*x)^2 * \tan(1/2*a*x) + 8*a^3*x^3 * \text{imag_part}(\cos_ \text{integral}(\\ & -2*a*x)) * \tan(2*a*x)^2 * \tan(a*x)^2 * \tan(1/2*a*x) - 64*a^3*x^3 * \text{imag_part}(\cos_ \text{i} \\ & \text{ntegral}(-4*a*x)) * \tan(2*a*x)^2 * \tan(a*x)^2 * \tan(1/2*a*x) + 128*a^3*x^3 * \text{sin_} \text{inte} \\ & \text{gral}(4*a*x) * \tan(2*a*x)^2 * \tan(a*x)^2 * \tan(1/2*a*x) - 16*a^3*x^3 * \text{sin_} \text{integral}(\\ & 2*a*x) * \tan(2*a*x)^2 * \tan(a*x)^2 * \tan(1/2*a*x) + 24*a^5*x^5 * \tan(1/2*a*x)^2 - 3 \\ & * a^3*x^3 * \tan(2*a*x)^2 * \tan(a*x)^2 * \tan(1/2*a*x)^2 - 32*a^4*x^4 * \text{imag_part}(\cos_ \\ & \text{integral}(4*a*x)) * \tan(2*a*x)^2 + 4*a^4*x^4 * \text{imag_part}(\cos_ \text{integral}(2*a*x)) * \text{t} \\ & \text{an}(2*a*x)^2 - 4*a^4*x^4 * \text{imag_part}(\cos_ \text{integral}(-2*a*x)) * \tan(2*a*x)^2 + 32*a^4 \\ & x^4 * \text{imag_part}(\cos_ \text{integral}(-4*a*x)) * \tan(2*a*x)^2 - 64*a^4*x^4 * \text{sin_} \text{integra} \\ & \text{l}(4*a*x) * \tan(2*a*x)^2 + 8*a^4*x^4 * \text{sin_} \text{integral}(2*a*x) * \tan(2*a*x)^2 + 2*a^4*x \\ & x^4 * \tan(2*a*x)^2 * \tan(a*x) - 32*a^4*x^4 * \text{imag_part}(\cos_ \text{integral}(4*a*x)) * \tan(a \end{aligned}$$

$$\begin{aligned}
& *x)^2 + 4*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - 4*a^4*x^4*ima \\
& g_part(cos_integral(-2*a*x))*tan(a*x)^2 + 32*a^4*x^4*imag_part(cos_integral \\
& (-4*a*x))*tan(a*x)^2 - 64*a^4*x^4*sin_integral(4*a*x)*tan(a*x)^2 + 8*a^4*x^ \\
& 4*sin_integral(2*a*x)*tan(a*x)^2 - 13*a^4*x^4*tan(2*a*x)*tan(a*x)^2 - 72*a^ \\
& 4*x^4*tan(2*a*x)^2*tan(1/2*a*x) + 72*a^4*x^4*tan(a*x)^2*tan(1/2*a*x) + 32*a \\
& ^4*x^4*imag_part(cos_integral(4*a*x))*tan(1/2*a*x)^2 - 4*a^4*x^4*imag_part(\\
& cos_integral(2*a*x))*tan(1/2*a*x)^2 + 4*a^4*x^4*imag_part(cos_integral(-2*a \\
& *x))*tan(1/2*a*x)^2 - 32*a^4*x^4*imag_part(cos_integral(-4*a*x))*tan(1/2*a \\
& x)^2 + 64*a^4*x^4*sin_integral(4*a*x)*tan(1/2*a*x)^2 - 8*a^4*x^4*sin_integr \\
& al(2*a*x)*tan(1/2*a*x)^2 + 13*a^4*x^4*tan(2*a*x)*tan(1/2*a*x)^2 - 2*a^4*x^4 \\
& *tan(a*x)*tan(1/2*a*x)^2 - 24*a^5*x^5 + 3*a^3*x^3*tan(2*a*x)^2*tan(a*x)^2 + \\
& 64*a^3*x^3*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 8*a^ \\
& 3*x^3*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 8*a^3*x^3* \\
& imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 64*a^3*x^3*imag \\
& _part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 128*a^3*x^3*sin_int \\
& egral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x) - 16*a^3*x^3*sin_integral(2*a*x)*tan \\
& (2*a*x)^2*tan(1/2*a*x) - 4*a^3*x^3*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x) + 64* \\
& a^3*x^3*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x) - 8*a^3*x^3* \\
& imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 8*a^3*x^3*imag_par \\
& t(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 64*a^3*x^3*imag_part(cos \\
& _integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 128*a^3*x^3*sin_integral(4*a*x) \\
& *tan(a*x)^2*tan(1/2*a*x) - 16*a^3*x^3*sin_integral(2*a*x)*tan(a*x)^2*tan(1/ \\
& 2*a*x) + 26*a^3*x^3*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x) - 15*a^3*x^3*tan(2*a \\
& *x)^2*tan(1/2*a*x)^2 + 24*a^3*x^3*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^4*x^4*im \\
& ag_part(cos_integral(4*a*x)) + 4*a^4*x^4*imag_part(cos_integral(2*a*x)) - 4 \\
& *a^4*x^4*imag_part(cos_integral(-2*a*x)) + 32*a^4*x^4*imag_part(cos_integra \\
& l(-4*a*x)) - 64*a^4*x^4*sin_integral(4*a*x) + 8*a^4*x^4*sin_integral(2*a*x) \\
& - 13*a^4*x^4*tan(2*a*x) + 2*a^4*x^4*tan(a*x) + 48*a^4*x^4*tan(1/2*a*x) - 3 \\
& 0*a^2*x^2*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 10*a^2*x^2*tan(2*a*x)^2*ta \\
& n(a*x)*tan(1/2*a*x)^2 + 5*a^2*x^2*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + 15 \\
& *a^3*x^3*tan(2*a*x)^2 - 24*a^3*x^3*tan(a*x)^2 + 64*a^3*x^3*imag_part(cos_in \\
& tegral(4*a*x))*tan(1/2*a*x) - 8*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(\\
& 1/2*a*x) + 8*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) - 64*a^3* \\
& x^3*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x) + 128*a^3*x^3*sin_integral \\
& (4*a*x)*tan(1/2*a*x) - 16*a^3*x^3*sin_integral(2*a*x)*tan(1/2*a*x) + 26*a^3 \\
& *x^3*tan(2*a*x)*tan(1/2*a*x) - 4*a^3*x^3*tan(a*x)*tan(1/2*a*x) + 12*a^3*x^3 \\
& *tan(1/2*a*x)^2 - 3*a*x*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 10*a^2*x^2 \\
& *tan(2*a*x)^2*tan(a*x) - 5*a^2*x^2*tan(2*a*x)*tan(a*x)^2 - 54*a^2*x^2*tan(2 \\
& *a*x)^2*tan(1/2*a*x) + 24*a^2*x^2*tan(a*x)^2*tan(1/2*a*x) + 5*a^2*x^2*tan(2 \\
& *a*x)*tan(1/2*a*x)^2 - 10*a^2*x^2*tan(a*x)*tan(1/2*a*x)^2 - 12*a^3*x^3 + 3* \\
& a*x*tan(2*a*x)^2*tan(a*x)^2 - 20*a*x*tan(2*a*x)^2*tan(a*x)*tan(1/2*a*x) + 1 \\
& 0*a*x*tan(2*a*x)*tan(a*x)^2*tan(1/2*a*x) + a*x*tan(2*a*x)^2*tan(1/2*a*x)^2 \\
& - 4*a*x*tan(a*x)^2*tan(1/2*a*x)^2 - 5*a^2*x^2*tan(2*a*x) + 10*a^2*x^2*tan(a \\
& *x) - 6*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - a*x*tan(2*a*x)^2 + 4*a*x*tan \\
& (a*x)^2 + 10*a*x*tan(2*a*x)*tan(1/2*a*x) - 20*a*x*tan(a*x)*tan(1/2*a*x) + 2 \\
& *tan(2*a*x)^2*tan(1/2*a*x) - 8*tan(a*x)^2*tan(1/2*a*x))/(a^5*x^8*tan(2*a*x) \\
& ^2*tan(a*x)^2*tan(1/2*a*x)^2 - a^5*x^8*tan(2*a*x)^2*tan(a*x)^2 + a^5*x^8*ta \\
& n(2*a*x)^2*tan(1/2*a*x)^2 + a^5*x^8*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^4*x^7* \\
& tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - a^5*x^8*tan(2*a*x)^2 - a^5*x^8*tan(a \\
& *x)^2 + a^5*x^8*tan(1/2*a*x)^2 + 2*a^3*x^6*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a \\
& *x)^2 + 2*a^4*x^7*tan(2*a*x)^2*tan(1/2*a*x) + 2*a^4*x^7*tan(a*x)^2*tan(1/2* \\
& a*x) - a^5*x^8 - 2*a^3*x^6*tan(2*a*x)^2*tan(a*x)^2 + 2*a^3*x^6*tan(2*a*x)^2 \\
& *tan(1/2*a*x)^2 + 2*a^3*x^6*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^4*x^7*tan(1/2*a \\
& *x) + 4*a^2*x^5*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 2*a^3*x^6*tan(2*a*x) \\
& ^2 - 2*a^3*x^6*tan(a*x)^2 + 2*a^3*x^6*tan(1/2*a*x)^2 + a*x^4*tan(2*a*x)^2*t \\
& an(a*x)^2*tan(1/2*a*x)^2 + 4*a^2*x^5*tan(2*a*x)^2*tan(1/2*a*x) + 4*a^2*x^5* \\
& tan(a*x)^2*tan(1/2*a*x) - 2*a^3*x^6 - a*x^4*tan(2*a*x)^2*tan(a*x)^2 + a*x^4 \\
& *tan(2*a*x)^2*tan(1/2*a*x)^2 + a*x^4*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^2*x^5* \\
& tan(1/2*a*x) + 2*x^3*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - a*x^4*tan(2*a*x)
\end{aligned}$$

)² - a*x⁴*tan(a*x)² + a*x⁴*tan(1/2*a*x)² + 2*x³*tan(2*a*x)²*tan(1/2*a*x) + 2*x³*tan(a*x)²*tan(1/2*a*x) - a*x⁴ + 2*x³*tan(1/2*a*x))

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(ax)}{x^4(ax \cos(ax) - \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x)

[Out] int(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(ax)^6}{x^4(\sin(ax) - ax \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a*x)^6/(x^4*(sin(a*x) - a*x*cos(a*x))^2),x)

[Out] int(sin(a*x)^6/(x^4*(sin(a*x) - a*x*cos(a*x))^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(ax)}{x^4(ax \cos(ax) - \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a*x)**6/x**4/(a*x*cos(a*x)-sin(a*x))**2,x)

[Out] Integral(sin(a*x)**6/(x**4*(a*x*cos(a*x) - sin(a*x))**2), x)

$$3.586 \quad \int \frac{\sin^5(ax)}{x^3(ax \cos(ax) - \sin(ax))^2} dx$$

Optimal. Leaf size=131

$$-\frac{1}{8}a^2\text{Si}(ax) + \frac{27}{8}a^2\text{Si}(3ax) + \frac{\sin^3(ax)}{a^2x^4} + \frac{\sin^4(ax)}{a^2x^4(ax \cos(ax) - \sin(ax))} + \frac{\sin^2(ax) \cos(ax)}{ax^3} - \frac{3 \sin^3(ax)}{2x^2} + \frac{\sin(ax)}{x^2} + \frac{a \cos(ax)}{x}$$

[Out] a*cos(a*x)/x-1/8*a^2*Si(a*x)+27/8*a^2*Si(3*a*x)+sin(a*x)/x^2+cos(a*x)*sin(a*x)^2/a/x^3-9/2*a*cos(a*x)*sin(a*x)^2/x+sin(a*x)^3/a^2/x^4-3/2*sin(a*x)^3/x^2+sin(a*x)^4/a^2/x^4/(a*x*cos(a*x)-sin(a*x))

Rubi [A] time = 0.23, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4598, 3314, 3297, 3299, 3312}

$$-\frac{1}{8}a^2\text{Si}(ax) + \frac{27}{8}a^2\text{Si}(3ax) + \frac{\sin^3(ax)}{a^2x^4} + \frac{\sin^4(ax)}{a^2x^4(ax \cos(ax) - \sin(ax))} - \frac{3 \sin^3(ax)}{2x^2} + \frac{\sin(ax)}{x^2} + \frac{\sin^2(ax) \cos(ax)}{ax^3} + \frac{a \cos(ax)}{x}$$

Antiderivative was successfully verified.

[In] Int[Sin[a*x]^5/(x^3*(a*x*Cos[a*x] - Sin[a*x])^2), x]

[Out] (a*Cos[a*x])/x + Sin[a*x]/x^2 + (Cos[a*x]*Sin[a*x]^2)/(a*x^3) - (9*a*Cos[a*x]*Sin[a*x]^2)/(2*x) + Sin[a*x]^3/(a^2*x^4) - (3*Sin[a*x]^3)/(2*x^2) + Sin[a*x]^4/(a^2*x^4*(a*x*Cos[a*x] - Sin[a*x])) - (a^2*SinIntegral[a*x])/8 + (27*a^2*SinIntegral[3*a*x])/8

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3312

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 3314

Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

Rule 4598

Int[((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_)/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)]^2, x_Symbol] := Simp[(b*(b*x)^(m - 1)*Sin[a*x]^(n - 1))/(a*d*(c*Sin[a*x] + d*x*Cos[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b*

$x^{m-2} \sin(ax)^{n-2}, x] / ; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{EqQ}[a*c + d, 0] \&\& \text{EqQ}[m, 2 - n]$

Rubi steps

$$\begin{aligned} \int \frac{\sin^5(ax)}{x^3(ax \cos(ax) - \sin(ax))^2} dx &= \frac{\sin^4(ax)}{a^2 x^4(ax \cos(ax) - \sin(ax))} - \frac{4 \int \frac{\sin^3(ax)}{x^5} dx}{a^2} \\ &= \frac{\cos(ax) \sin^2(ax)}{ax^3} + \frac{\sin^3(ax)}{a^2 x^4} + \frac{\sin^4(ax)}{a^2 x^4(ax \cos(ax) - \sin(ax))} - 2 \int \frac{\sin(ax)}{x^3} dx \\ &= \frac{\sin(ax)}{x^2} + \frac{\cos(ax) \sin^2(ax)}{ax^3} - \frac{9a \cos(ax) \sin^2(ax)}{2x} + \frac{\sin^3(ax)}{a^2 x^4} - \frac{3 \sin^3(ax)}{2x^2} \\ &= \frac{a \cos(ax)}{x} + \frac{\sin(ax)}{x^2} + \frac{\cos(ax) \sin^2(ax)}{ax^3} - \frac{9a \cos(ax) \sin^2(ax)}{2x} + \frac{\sin^3(ax)}{a^2 x^4} \\ &= \frac{a \cos(ax)}{x} + \frac{\sin(ax)}{x^2} + \frac{\cos(ax) \sin^2(ax)}{ax^3} - \frac{9a \cos(ax) \sin^2(ax)}{2x} + \frac{\sin^3(ax)}{a^2 x^4} \\ &= \frac{a \cos(ax)}{x} + \frac{\sin(ax)}{x^2} + \frac{\cos(ax) \sin^2(ax)}{ax^3} - \frac{9a \cos(ax) \sin^2(ax)}{2x} + \frac{\sin^3(ax)}{a^2 x^4} \end{aligned}$$

Mathematica [A] time = 0.97, size = 142, normalized size = 1.08

$$\frac{-2a^2 x^2 \text{Si}(ax)(ax \cos(ax) - \sin(ax)) + 54a^2 x^2 \text{Si}(3ax)(ax \cos(ax) - \sin(ax)) - a^2 x^2 + 8a^2 x^2 \cos(2ax) + 9a^2 x^2 \cos(4ax)}{16x^2(ax \cos(ax) - \sin(ax))}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a*x]^5/(x^3*(a*x*Cos[a*x] - Sin[a*x])^2), x]

[Out] (3 - a^2*x^2 - 4*Cos[2*a*x] + 8*a^2*x^2*Cos[2*a*x] + Cos[4*a*x] + 9*a^2*x^2*Cos[4*a*x] + 12*a*x*Sin[2*a*x] - 6*a*x*Sin[4*a*x] - 2*a^2*x^2*(a*x*Cos[a*x] - Sin[a*x])*SinIntegral[a*x] + 54*a^2*x^2*(a*x*Cos[a*x] - Sin[a*x])*SinIntegral[3*a*x])/(16*x^2*(a*x*Cos[a*x] - Sin[a*x]))

fricas [A] time = 1.04, size = 142, normalized size = 1.08

$$\frac{4(9a^2x^2 + 1)\cos(ax)^4 - 4(7a^2x^2 + 2)\cos(ax)^2 + (27a^3x^3 \text{Si}(3ax) - a^3x^3 \text{Si}(ax))\cos(ax) - (24ax \cos(ax) - 24ax \sin(ax))\sin(ax)}{8(ax^3 \cos(ax) - x^2 \sin(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")

[Out] 1/8*(4*(9*a^2*x^2 + 1)*cos(a*x)^4 - 4*(7*a^2*x^2 + 2)*cos(a*x)^2 + (27*a^3*x^3*sin_integral(3*a*x) - a^3*x^3*sin_integral(a*x))*cos(a*x) - (24*a*x*cos(a*x)^3 + 27*a^2*x^2*sin_integral(3*a*x) - a^2*x^2*sin_integral(a*x) - 24*a*x*cos(a*x))*sin(a*x) + 4)/(a*x^3*cos(a*x) - x^2*sin(a*x))

giac [C] time = 0.81, size = 4175, normalized size = 31.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")

[Out] 1/16*(27*a^7*x^7*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - a^7*x^7*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + a

$$\begin{aligned}
& ^7x^7\text{imag_part}(\cos_integral(-ax))\tan(3/2ax)^2\tan(1/2ax)^4 - 27a^7 \\
& x^7\text{imag_part}(\cos_integral(-3ax))\tan(3/2ax)^2\tan(1/2ax)^4 + 54a^7 \\
& x^7\sin_integral(3ax)\tan(3/2ax)^2\tan(1/2ax)^4 - 2a^7x^7\sin_inte \\
& gral(ax)\tan(3/2ax)^2\tan(1/2ax)^4 + 27a^7x^7\text{imag_part}(\cos_integral \\
& (3ax))\tan(1/2ax)^4 - a^7x^7\text{imag_part}(\cos_integral(ax))\tan(1/2ax) \\
& ^4 + a^7x^7\text{imag_part}(\cos_integral(-ax))\tan(1/2ax)^4 - 27a^7x^7\text{imag} \\
& _part(\cos_integral(-3ax))\tan(1/2ax)^4 + 54a^7x^7\sin_integral(3ax) \\
& \tan(1/2ax)^4 - 2a^7x^7\sin_integral(ax)\tan(1/2ax)^4 + 54a^6x^6i \\
& mag_part(\cos_integral(3ax))\tan(3/2ax)^2\tan(1/2ax)^3 - 2a^6x^6ima \\
& g_part(\cos_integral(ax))\tan(3/2ax)^2\tan(1/2ax)^3 + 2a^6x^6imag_pa \\
& rt(\cos_integral(-ax))\tan(3/2ax)^2\tan(1/2ax)^3 - 54a^6x^6imag_part \\
& (\cos_integral(-3ax))\tan(3/2ax)^2\tan(1/2ax)^3 + 108a^6x^6\sin_inte \\
& gral(3ax)\tan(3/2ax)^2\tan(1/2ax)^3 - 4a^6x^6\sin_integral(ax)\tan \\
& (3/2ax)^2\tan(1/2ax)^3 - 16a^6x^6\tan(3/2ax)^2\tan(1/2ax)^4 - 27* \\
& a^7x^7\text{imag_part}(\cos_integral(3ax))\tan(3/2ax)^2 + a^7x^7\text{imag_part}(c \\
& os_integral(ax))\tan(3/2ax)^2 - a^7x^7\text{imag_part}(\cos_integral(-ax))\tan \\
& (3/2ax)^2 + 27a^7x^7\text{imag_part}(\cos_integral(-3ax))\tan(3/2ax)^2 - \\
& 54a^7x^7\sin_integral(3ax)\tan(3/2ax)^2 + 2a^7x^7\sin_integral(ax) \\
& \tan(3/2ax)^2 + 54a^5x^5\text{imag_part}(\cos_integral(3ax))\tan(3/2ax)^2* \\
& \tan(1/2ax)^4 - 2a^5x^5\text{imag_part}(\cos_integral(ax))\tan(3/2ax)^2\tan(\\
& 1/2ax)^4 + 2a^5x^5\text{imag_part}(\cos_integral(-ax))\tan(3/2ax)^2\tan(1/2 \\
& ax)^4 - 54a^5x^5\text{imag_part}(\cos_integral(-3ax))\tan(3/2ax)^2\tan(1/2 \\
& ax)^4 + 108a^5x^5\sin_integral(3ax)\tan(3/2ax)^2\tan(1/2ax)^4 - 4 \\
& a^5x^5\sin_integral(ax)\tan(3/2ax)^2\tan(1/2ax)^4 + 54a^6x^6\text{imag} \\
& _part(\cos_integral(3ax))\tan(3/2ax)^2\tan(1/2ax) - 2a^6x^6\text{imag_part} \\
& (\cos_integral(ax))\tan(3/2ax)^2\tan(1/2ax) + 2a^6x^6\text{imag_part}(\cos_i \\
& ntegral(-ax))\tan(3/2ax)^2\tan(1/2ax) - 54a^6x^6\text{imag_part}(\cos_integ \\
& ral(-3ax))\tan(3/2ax)^2\tan(1/2ax) + 108a^6x^6\sin_integral(3ax)* \\
& \tan(3/2ax)^2\tan(1/2ax) - 4a^6x^6\sin_integral(ax)\tan(3/2ax)^2\tan \\
& (1/2ax) - 4a^6x^6\tan(3/2ax)^2\tan(1/2ax)^2 + 54a^6x^6\text{imag_part} \\
& (\cos_integral(3ax))\tan(1/2ax)^3 - 2a^6x^6\text{imag_part}(\cos_integral(ax) \\
&)\tan(1/2ax)^3 + 2a^6x^6\text{imag_part}(\cos_integral(-ax))\tan(1/2ax)^3 \\
& - 54a^6x^6\text{imag_part}(\cos_integral(-3ax))\tan(1/2ax)^3 + 108a^6x^6s \\
& in_integral(3ax)\tan(1/2ax)^3 - 4a^6x^6\sin_integral(ax)\tan(1/2ax) \\
&)^3 + 20a^6x^6\tan(1/2ax)^4 - 27a^7x^7\text{imag_part}(\cos_integral(3ax)) \\
& + a^7x^7\text{imag_part}(\cos_integral(ax)) - a^7x^7\text{imag_part}(\cos_integral(-a \\
& x)) + 27a^7x^7\text{imag_part}(\cos_integral(-3ax)) - 54a^7x^7\sin_integral \\
& (3ax) + 2a^7x^7\sin_integral(ax) - 36a^5x^5\tan(3/2ax)^2\tan(1/2a \\
& x)^3 + 54a^5x^5\text{imag_part}(\cos_integral(3ax))\tan(1/2ax)^4 - 2a^5x^ \\
& 5\text{imag_part}(\cos_integral(ax))\tan(1/2ax)^4 + 2a^5x^5\text{imag_part}(\cos_int \\
& egral(-ax))\tan(1/2ax)^4 - 54a^5x^5\text{imag_part}(\cos_integral(-3ax))\tan \\
& (1/2ax)^4 + 108a^5x^5\sin_integral(3ax)\tan(1/2ax)^4 - 4a^5x^5s \\
& in_integral(ax)\tan(1/2ax)^4 + 12a^5x^5\tan(3/2ax)\tan(1/2ax)^4 + \\
& 20a^6x^6\tan(3/2ax)^2 + 54a^6x^6\text{imag_part}(\cos_integral(3ax))\tan(1 \\
& /2ax) - 2a^6x^6\text{imag_part}(\cos_integral(ax))\tan(1/2ax) + 2a^6x^6i \\
& mag_part(\cos_integral(-ax))\tan(1/2ax) - 54a^6x^6\text{imag_part}(\cos_integr \\
& al(-3ax))\tan(1/2ax) + 108a^6x^6\sin_integral(3ax)\tan(1/2ax) - 4 \\
& a^6x^6\sin_integral(ax)\tan(1/2ax) - 4a^6x^6\tan(1/2ax)^2 + 108a^ \\
& 4x^4\text{imag_part}(\cos_integral(3ax))\tan(3/2ax)^2\tan(1/2ax)^3 - 4a^4x \\
& ^4\text{imag_part}(\cos_integral(ax))\tan(3/2ax)^2\tan(1/2ax)^3 + 4a^4x^4* \\
& \text{imag_part}(\cos_integral(-ax))\tan(3/2ax)^2\tan(1/2ax)^3 - 108a^4x^4i \\
& mag_part(\cos_integral(-3ax))\tan(3/2ax)^2\tan(1/2ax)^3 + 216a^4x^4* \\
& \sin_integral(3ax)\tan(3/2ax)^2\tan(1/2ax)^3 - 8a^4x^4\sin_integral(\\
& ax)\tan(3/2ax)^2\tan(1/2ax)^3 - 32a^4x^4\tan(3/2ax)^2\tan(1/2ax) \\
& ^4 - 54a^5x^5\text{imag_part}(\cos_integral(3ax))\tan(3/2ax)^2 + 2a^5x^5i \\
& mag_part(\cos_integral(ax))\tan(3/2ax)^2 - 2a^5x^5\text{imag_part}(\cos_integr \\
& al(-ax))\tan(3/2ax)^2 + 54a^5x^5\text{imag_part}(\cos_integral(-3ax))\tan(3 \\
& /2ax)^2 - 108a^5x^5\sin_integral(3ax)\tan(3/2ax)^2 + 4a^5x^5\sin_ \\
& integral(ax)\tan(3/2ax)^2 - 36a^5x^5\tan(3/2ax)^2\tan(1/2ax) + 36*
\end{aligned}$$

$$\begin{aligned}
& a^5 x^5 \tan(1/2 a x)^3 + 27 a^3 x^3 \operatorname{imag_part}(\cos_integral(3 a x)) \tan(3/2 a x) \\
& \tan(1/2 a x)^2 + a^3 x^3 \operatorname{imag_part}(\cos_integral(a x)) \tan(3/2 a x)^2 \tan(1/2 a x)^4 \\
& + a^3 x^3 \operatorname{imag_part}(\cos_integral(-a x)) \tan(3/2 a x)^2 \tan(1/2 a x)^4 - 27 a^3 x^3 \operatorname{imag_part}(\cos_integral(-3 a x)) \\
& \tan(3/2 a x)^2 \tan(1/2 a x)^4 + 54 a^3 x^3 \sin_integral(3 a x) \tan(3/2 a x)^2 \tan(1/2 a x)^4 - 2 a^3 x^3 \sin_integral(a x) \\
& \tan(3/2 a x)^2 \tan(1/2 a x)^4 - 16 a^6 x^6 + 108 a^4 x^4 \operatorname{imag_part}(\cos_integral(3 a x)) \tan(3/2 a x)^2 \tan(1/2 a x) \\
& - 4 a^4 x^4 \operatorname{imag_part}(\cos_integral(a x)) \tan(3/2 a x)^2 \tan(1/2 a x) + 4 a^4 x^4 \operatorname{imag_part}(\cos_integral(-a x)) \\
& \tan(3/2 a x)^2 \tan(1/2 a x) - 108 a^4 x^4 \operatorname{imag_part}(\cos_integral(-3 a x)) \tan(3/2 a x)^2 \tan(1/2 a x) \\
& + 216 a^4 x^4 \sin_integral(3 a x) \tan(3/2 a x)^2 \tan(1/2 a x) - 8 a^4 x^4 \sin_integral(a x) \tan(3/2 a x)^2 \\
& \tan(1/2 a x) - 8 a^4 x^4 \tan(3/2 a x)^2 \tan(1/2 a x)^2 + 108 a^4 x^4 \operatorname{imag_part}(\cos_integral(3 a x)) \tan(1/2 a x)^3 \\
& - 4 a^4 x^4 \operatorname{imag_part}(\cos_integral(a x)) \tan(1/2 a x)^3 + 4 a^4 x^4 \operatorname{imag_part}(\cos_integral(-a x)) \tan(1/2 a x)^3 \\
& - 108 a^4 x^4 \operatorname{imag_part}(\cos_integral(-3 a x)) \tan(1/2 a x)^3 + 216 a^4 x^4 \sin_integral(3 a x) \tan(1/2 a x)^3 \\
& - 8 a^4 x^4 \sin_integral(a x) \tan(1/2 a x)^3 + 24 a^4 x^4 \tan(3/2 a x) \tan(1/2 a x)^3 + 32 a^4 x^4 \tan(1/2 a x)^4 \\
& - 54 a^5 x^5 \operatorname{imag_part}(\cos_integral(3 a x)) + 2 a^5 x^5 \operatorname{imag_part}(\cos_integral(a x)) - 2 a^5 x^5 \operatorname{imag_part}(\cos_integral(-a x)) \\
& + 54 a^5 x^5 \operatorname{imag_part}(\cos_integral(-3 a x)) - 108 a^5 x^5 \sin_integral(3 a x) + 4 a^5 x^5 \sin_integral(a x) \\
& - 12 a^5 x^5 \tan(3/2 a x) + 36 a^5 x^5 \tan(1/2 a x) - 48 a^3 x^3 \tan(3/2 a x)^2 \tan(1/2 a x)^3 \\
& + 27 a^3 x^3 \operatorname{imag_part}(\cos_integral(3 a x)) \tan(1/2 a x)^4 - a^3 x^3 \operatorname{imag_part}(\cos_integral(a x)) \tan(1/2 a x)^4 \\
& + a^3 x^3 \operatorname{imag_part}(\cos_integral(-a x)) \tan(1/2 a x)^4 - 27 a^3 x^3 \operatorname{imag_part}(\cos_integral(-3 a x)) \tan(1/2 a x)^4 \\
& + 54 a^3 x^3 \sin_integral(3 a x) \tan(1/2 a x)^4 - 2 a^3 x^3 \sin_integral(a x) \tan(1/2 a x)^4 + 16 a^3 x^3 \tan(3/2 a x) \\
& \tan(1/2 a x)^4 + 32 a^4 x^4 \tan(3/2 a x)^2 + 108 a^4 x^4 \operatorname{imag_part}(\cos_integral(3 a x)) \tan(1/2 a x) \\
& - 4 a^4 x^4 \operatorname{imag_part}(\cos_integral(a x)) \tan(1/2 a x) + 4 a^4 x^4 \operatorname{imag_part}(\cos_integral(-a x)) \tan(1/2 a x) \\
& - 108 a^4 x^4 \operatorname{imag_part}(\cos_integral(-3 a x)) \tan(1/2 a x) + 216 a^4 x^4 \sin_integral(3 a x) \tan(1/2 a x) \\
& - 8 a^4 x^4 \sin_integral(a x) \tan(1/2 a x) + 24 a^4 x^4 \tan(3/2 a x) \tan(1/2 a x) - 8 a^4 x^4 \tan(1/2 a x)^2 \\
& + 54 a^2 x^2 \operatorname{imag_part}(\cos_integral(3 a x)) \tan(3/2 a x)^2 \tan(1/2 a x)^3 - 2 a^2 x^2 \operatorname{imag_part}(\cos_integral(a x)) \\
& \tan(3/2 a x)^2 \tan(1/2 a x)^3 + 2 a^2 x^2 \operatorname{imag_part}(\cos_integral(-a x)) \tan(3/2 a x)^2 \tan(1/2 a x)^3 \\
& - 54 a^2 x^2 \operatorname{imag_part}(\cos_integral(-3 a x)) \tan(3/2 a x)^2 \tan(1/2 a x)^3 + 108 a^2 x^2 \sin_integral(3 a x) \\
& \tan(3/2 a x)^2 \tan(1/2 a x)^3 - 4 a^2 x^2 \sin_integral(a x) \tan(3/2 a x)^2 \tan(1/2 a x)^3 - 32 a^2 x^2 \tan(3/2 a x)^2 \\
& \tan(1/2 a x)^4 - 27 a^3 x^3 \operatorname{imag_part}(\cos_integral(3 a x)) \tan(3/2 a x)^2 + a^3 x^3 \operatorname{imag_part}(\cos_integral(a x)) \\
& \tan(3/2 a x)^2 - a^3 x^3 \operatorname{imag_part}(\cos_integral(-a x)) \tan(3/2 a x)^2 + 27 a^3 x^3 \operatorname{imag_part}(\cos_integral(-3 a x)) \\
& \tan(3/2 a x)^2 - 54 a^3 x^3 \sin_integral(3 a x) \tan(3/2 a x)^2 + 2 a^3 x^3 \sin_integral(a x) \tan(3/2 a x)^2 \\
& - 80 a^3 x^3 \tan(3/2 a x)^2 \tan(1/2 a x) + 80 a^3 x^3 \tan(1/2 a x)^3 - 32 a^4 x^4 + 54 a^2 x^2 \operatorname{imag_part}(\cos_integral(3 a x)) \\
& \tan(3/2 a x)^2 \tan(1/2 a x) - 2 a^2 x^2 \operatorname{imag_part}(\cos_integral(a x)) \tan(3/2 a x)^2 \tan(1/2 a x) \\
& + 2 a^2 x^2 \operatorname{imag_part}(\cos_integral(-a x)) \tan(3/2 a x)^2 \tan(1/2 a x) - 54 a^2 x^2 \operatorname{imag_part}(\cos_integral(-3 a x)) \\
& \tan(3/2 a x)^2 \tan(1/2 a x) + 108 a^2 x^2 \sin_integral(3 a x) \tan(3/2 a x)^2 \tan(1/2 a x) - 4 a^2 x^2 \sin_integral(a x) \\
& \tan(3/2 a x)^2 \tan(1/2 a x) - 60 a^2 x^2 \tan(3/2 a x)^2 \tan(1/2 a x)^2 + 54 a^2 x^2 \operatorname{imag_part}(\cos_integral(3 a x)) \\
& \tan(1/2 a x)^3 - 2 a^2 x^2 \operatorname{imag_part}(\cos_integral(a x)) \tan(1/2 a x)^3 + 2 a^2 x^2 \operatorname{imag_part}(\cos_integral(-a x)) \\
& \tan(1/2 a x)^3 - 54 a^2 x^2 \operatorname{imag_part}(\cos_integral(-3 a x)) \tan(1/2 a x)^3 + 108 a^2 x^2 \sin_integral(3 a x) \tan(1/2 a x)^3 \\
& - 4 a^2 x^2 \sin_integral(a x) \tan(1/2 a x)^3 + 32 a^2 x^2 \tan(3/2 a x) \tan(1/2 a x)^3 - 4 a^2 x^2 \tan(1/2 a x)^4 \\
& - 27 a^3 x^3 \operatorname{imag_part}(\cos_integral(3 a x)) + a^3 x^3 \operatorname{imag_part}(\cos_integral(a x)) - a^3 x^3 \operatorname{imag_part}(\cos_integral(-a x)) \\
& + 27 a^3 x^3 \operatorname{imag_part}(\cos_integral(-3 a x)) - 54 a^3 x^3 \sin_integral(3 a x) + 2 a^3 x^3 \sin_integral(a x) \\
& - 16 a^3 x^3 \tan(3/2 a x) + 48 a^3 x^3 \tan(1/2 a x) - 12 a x \tan(3/2 a x)^2 \tan(1/2 a x)^3 + 4 a
\end{aligned}$$

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*x*tan(3/2*a*x)*tan(1/2*a*x)^4 - 4*a^2*x^2*tan(3/2*a*x)^2 + 54*a^2*x^2*imag_
_part(cos_integral(3*a*x))*tan(1/2*a*x) - 2*a^2*x^2*imag_part(cos_integral(
a*x))*tan(1/2*a*x) + 2*a^2*x^2*imag_part(cos_integral(-a*x))*tan(1/2*a*x) -
54*a^2*x^2*imag_part(cos_integral(-3*a*x))*tan(1/2*a*x) + 108*a^2*x^2*sin_
integral(3*a*x)*tan(1/2*a*x) - 4*a^2*x^2*sin_integral(a*x)*tan(1/2*a*x) + 3
2*a^2*x^2*tan(3/2*a*x)*tan(1/2*a*x) - 60*a^2*x^2*tan(1/2*a*x)^2 - 44*a*x*ta
n(3/2*a*x)^2*tan(1/2*a*x) + 44*a*x*tan(1/2*a*x)^3 - 32*a^2*x^2 - 24*tan(3/2
*a*x)^2*tan(1/2*a*x)^2 + 8*tan(3/2*a*x)*tan(1/2*a*x)^3 - 4*a*x*tan(3/2*a*x)
+ 12*a*x*tan(1/2*a*x) + 8*tan(3/2*a*x)*tan(1/2*a*x) - 24*tan(1/2*a*x)^2)/(
a^5*x^7*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + a^5*x^7*tan(1/2*a*x)^4 + 2*a^4*x^6*
tan(3/2*a*x)^2*tan(1/2*a*x)^3 - a^5*x^7*tan(3/2*a*x)^2 + 2*a^3*x^5*tan(3/2*
a*x)^2*tan(1/2*a*x)^4 + 2*a^4*x^6*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^4*x^6*t
an(1/2*a*x)^3 - a^5*x^7 + 2*a^3*x^5*tan(1/2*a*x)^4 + 2*a^4*x^6*tan(1/2*a*x)
+ 4*a^2*x^4*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 2*a^3*x^5*tan(3/2*a*x)^2 + a*x
^3*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 4*a^2*x^4*tan(3/2*a*x)^2*tan(1/2*a*x) +
4*a^2*x^4*tan(1/2*a*x)^3 - 2*a^3*x^5 + a*x^3*tan(1/2*a*x)^4 + 4*a^2*x^4*tan
(1/2*a*x) + 2*x^2*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - a*x^3*tan(3/2*a*x)^2 + 2*
x^2*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*x^2*tan(1/2*a*x)^3 - a*x^3 + 2*x^2*tan(
1/2*a*x))

```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^5(ax)}{x^3(ax \cos(ax) - \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] int(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(ax)^5}{x^3(\sin(ax) - ax \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^5/(x^3*(sin(a*x) - a*x*cos(a*x))^2),x)
```

```
[Out] int(sin(a*x)^5/(x^3*(sin(a*x) - a*x*cos(a*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^5(ax)}{x^3(ax \cos(ax) - \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)**5/x**3/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Integral(sin(a*x)**5/(x**3*(a*x*cos(a*x) - sin(a*x))**2), x)
```

$$3.587 \quad \int \frac{\sin^4(ax)}{x^2(ax \cos(ax) - \sin(ax))^2} dx$$

Optimal. Leaf size=80

$$\frac{\sin^2(ax)}{a^2x^3} + \frac{\sin^3(ax)}{a^2x^3(ax \cos(ax) - \sin(ax))} + 2a\text{Si}(2ax) + \frac{\sin(ax) \cos(ax)}{ax^2} - \frac{2 \sin^2(ax)}{x} + \frac{1}{x}$$

[Out] 1/x+2*a*Si(2*a*x)+cos(a*x)*sin(a*x)/a/x^2+sin(a*x)^2/a^2/x^3-2*sin(a*x)^2/x+sin(a*x)^3/a^2/x^3/(a*x*cos(a*x)-sin(a*x))

Rubi [A] time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4598, 3314, 30, 3313, 12, 3299}

$$\frac{\sin^2(ax)}{a^2x^3} + \frac{\sin^3(ax)}{a^2x^3(ax \cos(ax) - \sin(ax))} + 2a\text{Si}(2ax) + \frac{\sin(ax) \cos(ax)}{ax^2} - \frac{2 \sin^2(ax)}{x} + \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[Sin[a*x]^4/(x^2*(a*x*Cos[a*x] - Sin[a*x])^2), x]

[Out] x^(-1) + (Cos[a*x]*Sin[a*x])/(a*x^2) + Sin[a*x]^2/(a^2*x^3) - (2*Sin[a*x]^2)/x + Sin[a*x]^3/(a^2*x^3*(a*x*Cos[a*x] - Sin[a*x])) + 2*a*SinIntegral[2*a*x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3313

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Simp[((c + d*x)^(m+1)*Sin[e + f*x]^n)/(d*(m+1)), x] - Dist[(f^n)/(d*(m+1)), Int[ExpandTrigReduce[(c + d*x)^(m+1), Cos[e + f*x]*Sin[e + f*x]^(n-1), x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

Rule 3314

Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Simp[((c + d*x)^(m+1)*(b*Sin[e + f*x]^n)/(d*(m+1)), x] + (Dist[(b^2*f^2*n*(n-1))/(d^2*(m+1)*(m+2)), Int[(c + d*x)^(m+2)*(b*Sin[e + f*x])^(n-2), x], x] - Dist[(f^2*n^2)/(d^2*(m+1)*(m+2)), Int[(c + d*x)^(m+2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m+2)*Cos[e + f*x]*(b*Sin[e + f*x])^(n-1))/(d^2*(m+1)*(m+2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

Rule 4598

Int[(((b_.)*(x_.))^(m_)*Sin[(a_.)*(x_.)]^(n_))/(Cos[(a_.)*(x_.)]*(d_.)*(x_.) + (c_.)*Sin[(a_.)*(x_.)]^2, x_Symbol] :> Simp[(b*(b*x)^(m - 1)*Sin[a*x]^(n - 1))/(a*d*(c*Ssin[a*x] + d*x*Cos[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b*x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a*c + d, 0] && EqQ[m, 2 - n]

Rubi steps

$$\begin{aligned} \int \frac{\sin^4(ax)}{x^2(ax \cos(ax) - \sin(ax))^2} dx &= \frac{\sin^3(ax)}{a^2 x^3(ax \cos(ax) - \sin(ax))} - \frac{3 \int \frac{\sin^2(ax)}{x^4} dx}{a^2} \\ &= \frac{\cos(ax) \sin(ax)}{ax^2} + \frac{\sin^2(ax)}{a^2 x^3} + \frac{\sin^3(ax)}{a^2 x^3(ax \cos(ax) - \sin(ax))} + 2 \int \frac{\sin^2(ax)}{x^2} dx \\ &= \frac{1}{x} + \frac{\cos(ax) \sin(ax)}{ax^2} + \frac{\sin^2(ax)}{a^2 x^3} - \frac{2 \sin^2(ax)}{x} + \frac{\sin^3(ax)}{a^2 x^3(ax \cos(ax) - \sin(ax))} + \dots \\ &= \frac{1}{x} + \frac{\cos(ax) \sin(ax)}{ax^2} + \frac{\sin^2(ax)}{a^2 x^3} - \frac{2 \sin^2(ax)}{x} + \frac{\sin^3(ax)}{a^2 x^3(ax \cos(ax) - \sin(ax))} + \dots \\ &= \frac{1}{x} + \frac{\cos(ax) \sin(ax)}{ax^2} + \frac{\sin^2(ax)}{a^2 x^3} - \frac{2 \sin^2(ax)}{x} + \frac{\sin^3(ax)}{a^2 x^3(ax \cos(ax) - \sin(ax))} + \dots \end{aligned}$$

Mathematica [A] time = 0.82, size = 77, normalized size = 0.96

$$\frac{8ax\text{Si}(2ax)(ax \cos(ax) - \sin(ax)) + 3 \sin(ax) - \sin(3ax) + 2ax \cos(ax) + 2ax \cos(3ax)}{4x(ax \cos(ax) - \sin(ax))}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a*x]^4/(x^2*(a*x*Cos[a*x] - Sin[a*x])^2),x]

[Out] (2*a*x*Cos[a*x] + 2*a*x*Cos[3*a*x] + 3*Sin[a*x] - Sin[3*a*x] + 8*a*x*(a*x*Cos[a*x] - Sin[a*x])*SinIntegral[2*a*x])/(4*x*(a*x*Cos[a*x] - Sin[a*x]))

fricas [A] time = 1.03, size = 77, normalized size = 0.96

$$\frac{2ax \cos(ax)^3 + (2a^2x^2 \text{Si}(2ax) - ax) \cos(ax) - (2ax \text{Si}(2ax) + \cos(ax)^2 - 1) \sin(ax)}{ax^2 \cos(ax) - x \sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")

[Out] (2*a*x*cos(a*x)^3 + (2*a^2*x^2*sin_integral(2*a*x) - a*x)*cos(a*x) - (2*a*x*sin_integral(2*a*x) + cos(a*x)^2 - 1)*sin(a*x))/(a*x^2*cos(a*x) - x*sin(a*x))

giac [C] time = 0.48, size = 1033, normalized size = 12.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")

[Out] (a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^4*x^4*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2 + a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 - 2


```

*a^4*x^4*sin_integral(2*a*x)*tan(a*x)^2 + a^4*x^4*imag_part(cos_integral(2*
a*x))*tan(1/2*a*x)^2 - a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x)
^2 + 2*a^4*x^4*sin_integral(2*a*x)*tan(1/2*a*x)^2 + 2*a^3*x^3*imag_part(cos
_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a^3*x^3*imag_part(cos_integra
l(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a^3*x^3*sin_integral(2*a*x)*tan(a*x)
^2*tan(1/2*a*x) - a^3*x^3*tan(a*x)^2*tan(1/2*a*x)^2 - a^4*x^4*imag_part(cos
_integral(2*a*x)) + a^4*x^4*imag_part(cos_integral(-2*a*x)) - 2*a^4*x^4*sin
_integral(2*a*x) + a^2*x^2*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/
2*a*x)^2 - a^2*x^2*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^
2 + 2*a^2*x^2*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + a^3*x^3*tan(a
*x)^2 + 2*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) - 2*a^3*x^3*i
mag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a^3*x^3*sin_integral(2*a*x)
*tan(1/2*a*x) + a^3*x^3*tan(1/2*a*x)^2 - a^2*x^2*imag_part(cos_integral(2*a
*x))*tan(a*x)^2 + a^2*x^2*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 - 2*a^
2*x^2*sin_integral(2*a*x)*tan(a*x)^2 - 2*a^2*x^2*tan(a*x)^2*tan(1/2*a*x) +
a^2*x^2*imag_part(cos_integral(2*a*x))*tan(1/2*a*x)^2 - a^2*x^2*imag_part(c
os_integral(-2*a*x))*tan(1/2*a*x)^2 + 2*a^2*x^2*sin_integral(2*a*x)*tan(1/2
*a*x)^2 + a^2*x^2*tan(a*x)*tan(1/2*a*x)^2 - a^3*x^3 + 2*a*x*imag_part(cos_i
ntegral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a*x*imag_part(cos_integral(-2*a
*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a*x*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2
*a*x) - a^2*x^2*imag_part(cos_integral(2*a*x)) + a^2*x^2*imag_part(cos_inte
gral(-2*a*x)) - 2*a^2*x^2*sin_integral(2*a*x) - a^2*x^2*tan(a*x) + 2*a^2*x^
2*tan(1/2*a*x) + 2*a*x*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) - 2*a*x*
imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a*x*sin_integral(2*a*x)*ta
n(1/2*a*x) + 2*a*x*tan(a*x)*tan(1/2*a*x) + a*x*tan(1/2*a*x)^2 - 2*tan(a*x)^
2*tan(1/2*a*x) - a*x)/(a^3*x^4*tan(a*x)^2*tan(1/2*a*x)^2 - a^3*x^4*tan(a*x)
^2 + a^3*x^4*tan(1/2*a*x)^2 + 2*a^2*x^3*tan(a*x)^2*tan(1/2*a*x) - a^3*x^4 +
a*x^2*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^2*x^3*tan(1/2*a*x) - a*x^2*tan(a*x)^
2 + a*x^2*tan(1/2*a*x)^2 + 2*x*tan(a*x)^2*tan(1/2*a*x) - a*x^2 + 2*x*tan(1/
2*a*x))

```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(ax)}{x^2(ax \cos(ax) - \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] int(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(ax)^4}{x^2(\sin(ax) - ax \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^4/(x^2*(sin(a*x) - a*x*cos(a*x))^2),x)
```

[Out] `int(sin(a*x)^4/(x^2*(sin(a*x) - a*x*cos(a*x))^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(ax)}{x^2(ax \cos(ax) - \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a*x)**4/x**2/(a*x*cos(a*x)-sin(a*x))**2,x)`

[Out] `Integral(sin(a*x)**4/(x**2*(a*x*cos(a*x) - sin(a*x))**2), x)`

$$3.588 \quad \int \frac{\sin^3(ax)}{x(ax \cos(ax) - \sin(ax))^2} dx$$

Optimal. Leaf size=56

$$\frac{\sin(ax)}{a^2x^2} + \frac{\sin^2(ax)}{a^2x^2(ax \cos(ax) - \sin(ax))} + \text{Si}(ax) + \frac{\cos(ax)}{ax}$$

[Out] cos(a*x)/a/x+Si(a*x)+sin(a*x)/a^2/x^2+sin(a*x)^2/a^2/x^2/(a*x*cos(a*x)-sin(a*x))

Rubi [A] time = 0.10, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {4598, 3297, 3299}

$$\frac{\sin(ax)}{a^2x^2} + \frac{\sin^2(ax)}{a^2x^2(ax \cos(ax) - \sin(ax))} + \text{Si}(ax) + \frac{\cos(ax)}{ax}$$

Antiderivative was successfully verified.

[In] Int[Sin[a*x]^3/(x*(a*x*Cos[a*x] - Sin[a*x])^2), x]

[Out] Cos[a*x]/(a*x) + Sin[a*x]/(a^2*x^2) + Sin[a*x]^2/(a^2*x^2*(a*x*Cos[a*x] - Sin[a*x])) + SinIntegral[a*x]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 4598

Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)]^2), x_Symbol] := Simp[(b*(b*x)^(m - 1)*Sin[a*x]^(n - 1))/(a*d*(c*Ssin[a*x] + d*x*Cos[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b*x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a*c + d, 0] && EqQ[m, 2 - n]

Rubi steps

$$\begin{aligned} \int \frac{\sin^3(ax)}{x(ax \cos(ax) - \sin(ax))^2} dx &= \frac{\sin^2(ax)}{a^2x^2(ax \cos(ax) - \sin(ax))} - \frac{2 \int \frac{\sin(ax)}{x^3} dx}{a^2} \\ &= \frac{\sin(ax)}{a^2x^2} + \frac{\sin^2(ax)}{a^2x^2(ax \cos(ax) - \sin(ax))} - \frac{\int \frac{\cos(ax)}{x^2} dx}{a} \\ &= \frac{\cos(ax)}{ax} + \frac{\sin(ax)}{a^2x^2} + \frac{\sin^2(ax)}{a^2x^2(ax \cos(ax) - \sin(ax))} + \int \frac{\sin(ax)}{x} dx \\ &= \frac{\cos(ax)}{ax} + \frac{\sin(ax)}{a^2x^2} + \frac{\sin^2(ax)}{a^2x^2(ax \cos(ax) - \sin(ax))} + \text{Si}(ax) \end{aligned}$$

Mathematica [C] time = 7.42, size = 242, normalized size = 4.32

$$\frac{-ieCi(i-ax)(ax\cos(ax)-\sin(ax))+ieCi(ax+i)(ax\cos(ax)-\sin(ax))-ieEi(-iax-1)\sin(ax)+ieEi(iax-1)\sin(ax)}{ax\cos(ax)-\sin(ax)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sin[a*x]^3/(x*(a*x*Cos[a*x] - Sin[a*x])^2),x]

[Out] (1 + Cos[2*a*x] + I*a*E*x*Cos[a*x]*ExpIntegralEi[-1 - I*a*x] - I*a*E*x*Cos[a*x]*ExpIntegralEi[-1 + I*a*x] - I*E*CosIntegral[I - a*x]*(a*x*Cos[a*x] - Sin[a*x]) + I*E*CosIntegral[I + a*x]*(a*x*Cos[a*x] - Sin[a*x]) - I*E*ExpIntegralEi[-1 - I*a*x]*Sin[a*x] + I*E*ExpIntegralEi[-1 + I*a*x]*Sin[a*x] + 2*a*x*Cos[a*x]*SinIntegral[a*x] - 2*Sin[a*x]*SinIntegral[a*x] + a*E*x*Cos[a*x]*SinIntegral[I - a*x] - E*Sin[a*x]*SinIntegral[I - a*x] - a*E*x*Cos[a*x]*SinIntegral[I + a*x] + E*Sin[a*x]*SinIntegral[I + a*x])/(2*a*x*Cos[a*x] - 2*Sin[a*x])

fricas [A] time = 0.87, size = 45, normalized size = 0.80

$$\frac{ax\cos(ax)\operatorname{Si}(ax)+\cos(ax)^2-\sin(ax)\operatorname{Si}(ax)}{ax\cos(ax)-\sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")

[Out] (a*x*cos(a*x)*sin_integral(a*x) + cos(a*x)^2 - sin(a*x)*sin_integral(a*x))/(a*x*cos(a*x) - sin(a*x))

giac [C] time = 0.37, size = 496, normalized size = 8.86

$$\frac{a^3x^3\Im(Ci(ax))\tan\left(\frac{1}{2}ax\right)^4 - a^3x^3\Im(Ci(-ax))\tan\left(\frac{1}{2}ax\right)^4 + 2a^3x^3\operatorname{Si}(ax)\tan\left(\frac{1}{2}ax\right)^4 + 2a^2x^2\Im(Ci(ax))\tan\left(\frac{1}{2}ax\right)^4}{ax\cos(ax)-\sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")

[Out] 1/2*(a^3*x^3*imag_part(cos_integral(a*x))*tan(1/2*a*x)^4 - a^3*x^3*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^4 + 2*a^3*x^3*sin_integral(a*x)*tan(1/2*a*x)^4 + 2*a^2*x^2*imag_part(cos_integral(a*x))*tan(1/2*a*x)^3 - 2*a^2*x^2*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^3 + 4*a^2*x^2*sin_integral(a*x)*tan(1/2*a*x)^3 - 2*a^2*x^2*tan(1/2*a*x)^4 - a^3*x^3*imag_part(cos_integral(a*x)) + a^3*x^3*imag_part(cos_integral(-a*x)) - 2*a^3*x^3*sin_integral(a*x) + a*x*imag_part(cos_integral(a*x))*tan(1/2*a*x)^4 - a*x*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^4 + 2*a*x*sin_integral(a*x)*tan(1/2*a*x)^4 + 2*a^2*x^2*imag_part(cos_integral(a*x))*tan(1/2*a*x) - 2*a^2*x^2*imag_part(cos_integral(-a*x))*tan(1/2*a*x) + 4*a^2*x^2*sin_integral(a*x)*tan(1/2*a*x) + 4*a^2*x^2*tan(1/2*a*x)^2 - 2*a^2*x^2 + 2*imag_part(cos_integral(a*x))*tan(1/2*a*x)^3 - 2*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^3 + 4*sin_integral(a*x)*tan(1/2*a*x)^3 - 4*tan(1/2*a*x)^4 - a*x*imag_part(cos_integral(a*x)) + a*x*imag_part(cos_integral(-a*x)) - 2*a*x*sin_integral(a*x) + 2*imag_part(cos_integral(a*x))*tan(1/2*a*x) - 2*imag_part(cos_integral(-a*x))*tan(1/2*a*x) + 4*sin_integral(a*x)*tan(1/2*a*x) - 4)/(a^3*x^3*tan(1/2*a*x)^4 + 2*a^2*x^2*tan(1/2*a*x)^3 - a^3*x^3 + a*x*tan(1/2*a*x)^4 + 2*a^2*x^2*tan(1/2*a*x) + 2*tan(1/2*a*x)^3 - a*x + 2*tan(1/2*a*x))

maple [C] time = 4.28, size = 108, normalized size = 1.93

$$\frac{iEi(1,-iax)}{2} + \frac{ie^{iax}}{2iax-2} + \frac{ie^{-iax}}{2iax+2} - \frac{iEi(1,iax)}{2} + \frac{2e^{iax}}{(ax+i)(ax-i)(axe^{2iax}+ie^{2iax}+ax-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x)`

[Out] `1/2*I*Ei(1,-I*a*x)+1/2*I*exp(I*a*x)/(-1+I*a*x)+1/2*I*exp(-I*a*x)/(I*a*x+1)-1/2*I*Ei(1,I*a*x)+2*exp(I*a*x)/(a*x+I)/(a*x-I)/(a*x*exp(2*I*a*x)+I*exp(2*I*a*x)+a*x-I)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(ax)^3}{x(\sin(ax) - ax \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(a*x)^3/(x*(sin(a*x) - a*x*cos(a*x))^2),x)`

[Out] `int(sin(a*x)^3/(x*(sin(a*x) - a*x*cos(a*x))^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(ax)}{x(ax \cos(ax) - \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(a*x)**3/x/(a*x*cos(a*x)-sin(a*x))**2,x)`

[Out] `Integral(sin(a*x)**3/(x*(a*x*cos(a*x) - sin(a*x))**2), x)`

$$3.589 \quad \int \frac{\sin^2(ax)}{(ax \cos(ax) - \sin(ax))^2} dx$$

Optimal. Leaf size=35

$$\frac{1}{a^2x} + \frac{\sin(ax)}{a^2x(ax \cos(ax) - \sin(ax))}$$

[Out] 1/a^2/x+sin(a*x)/a^2/x/(a*x*cos(a*x)-sin(a*x))

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {4596}

$$\frac{1}{a^2x} + \frac{\sin(ax)}{a^2x(ax \cos(ax) - \sin(ax))}$$

Antiderivative was successfully verified.

[In] Int [Sin[a*x]^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]

[Out] 1/(a^2*x) + Sin[a*x]/(a^2*x*(a*x*Cos[a*x] - Sin[a*x]))

Rule 4596

Int[Sin[(a_.)*(x_)]^2/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[1/(d^2*x), x] + Simp[Sin[a*x]/(a*d*x*(d*x*Cos[a*x] + c*Sin[a*x])), x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0]

Rubi steps

$$\int \frac{\sin^2(ax)}{(ax \cos(ax) - \sin(ax))^2} dx = \frac{1}{a^2x} + \frac{\sin(ax)}{a^2x(ax \cos(ax) - \sin(ax))}$$

Mathematica [A] time = 0.28, size = 24, normalized size = 0.69

$$\frac{\cos(ax)}{a^2x \cos(ax) - a \sin(ax)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a*x]^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]

[Out] Cos[a*x]/(a^2*x*Cos[a*x] - a*Sin[a*x])

fricas [A] time = 1.95, size = 24, normalized size = 0.69

$$\frac{\cos(ax)}{a^2x \cos(ax) - a \sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")

[Out] cos(a*x)/(a^2*x*cos(a*x) - a*sin(a*x))

giac [A] time = 0.19, size = 39, normalized size = 1.11

$$\frac{\tan\left(\frac{1}{2}ax\right)^2 - 1}{a^2x \tan\left(\frac{1}{2}ax\right)^2 - a^2x + 2a \tan\left(\frac{1}{2}ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")

[Out] (tan(1/2*a*x)^2 - 1)/(a^2*x*tan(1/2*a*x)^2 - a^2*x + 2*a*tan(1/2*a*x))

maple [B] time = 1.47, size = 77, normalized size = 2.20

$$\frac{\frac{\tan^4\left(\frac{ax}{2}\right)}{a} + \frac{\tan^6\left(\frac{ax}{2}\right)}{a} - \frac{1}{a} - \frac{\tan^2\left(\frac{ax}{2}\right)}{a}}{\left(1 + \tan^2\left(\frac{ax}{2}\right)\right)^2 \left(ax \left(\tan^2\left(\frac{ax}{2}\right)\right) - ax + 2 \tan\left(\frac{ax}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x)

[Out] (1/a*tan(1/2*a*x)^4+1/a*tan(1/2*a*x)^6-1/a-1/a*tan(1/2*a*x)^2)/(1+tan(1/2*a*x)^2)^2/(a*x*tan(1/2*a*x)^2-a*x+2*tan(1/2*a*x))

maxima [B] time = 0.32, size = 114, normalized size = 3.26

$$\frac{ax \cos(2ax)^2 + ax \sin(2ax)^2 + 2ax \cos(2ax) + ax - 2 \sin(2ax)}{(a^2x^2 + (a^2x^2 + 1) \cos(2ax)^2 - 4ax \sin(2ax) + (a^2x^2 + 1) \sin(2ax)^2 + 2(a^2x^2 - 1) \cos(2ax) + 1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")

[Out] (a*x*cos(2*a*x)^2 + a*x*sin(2*a*x)^2 + 2*a*x*cos(2*a*x) + a*x - 2*sin(2*a*x))/((a^2*x^2 + (a^2*x^2 + 1)*cos(2*a*x)^2 - 4*a*x*sin(2*a*x) + (a^2*x^2 + 1)*sin(2*a*x)^2 + 2*(a^2*x^2 - 1)*cos(2*a*x) + 1)*a)

mupad [B] time = 3.03, size = 24, normalized size = 0.69

$$-\frac{\cos(ax)}{a(\sin(ax) - ax \cos(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a*x)^2/(sin(a*x) - a*x*cos(a*x))^2,x)

[Out] -cos(a*x)/(a*(sin(a*x) - a*x*cos(a*x)))

sympy [A] time = 3.42, size = 20, normalized size = 0.57

$$\frac{\cos(ax)}{a^2x \cos(ax) - a \sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a*x)**2/(a*x*cos(a*x)-sin(a*x))**2,x)

[Out] cos(a*x)/(a**2*x*cos(a*x) - a*sin(a*x))

$$3.590 \quad \int \frac{x \sin(ax)}{(ax \cos(ax) - \sin(ax))^2} dx$$

Optimal. Leaf size=20

$$\frac{1}{a^2(ax \cos(ax) - \sin(ax))}$$

[Out] 1/a^2/(a*x*cos(a*x)-sin(a*x))

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6686}

$$\frac{1}{a^2(ax \cos(ax) - \sin(ax))}$$

Antiderivative was successfully verified.

[In] Int[(x*Sin[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]

[Out] 1/(a^2*(a*x*Cos[a*x] - Sin[a*x]))

Rule 6686

Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[(q*y^(m + 1))/(m + 1), x] /; !FalseQ[q] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{x \sin(ax)}{(ax \cos(ax) - \sin(ax))^2} dx = \frac{1}{a^2(ax \cos(ax) - \sin(ax))}$$

Mathematica [A] time = 0.03, size = 20, normalized size = 1.00

$$-\frac{1}{a^2(\sin(ax) - ax \cos(ax))}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Sin[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]

[Out] -(1/(a^2*(-(a*x*Cos[a*x]) + Sin[a*x])))

fricas [A] time = 1.75, size = 21, normalized size = 1.05

$$\frac{1}{a^3x \cos(ax) - a^2 \sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")

[Out] 1/(a^3*x*cos(a*x) - a^2*sin(a*x))

giac [B] time = 0.18, size = 42, normalized size = 2.10

$$\frac{2 \left(\tan\left(\frac{1}{2} ax\right)^2 + 1 \right)}{a^3 x \tan\left(\frac{1}{2} ax\right)^2 - a^3 x + 2 a^2 \tan\left(\frac{1}{2} ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")

[Out] -2*(tan(1/2*a*x)^2 + 1)/(a^3*x*tan(1/2*a*x)^2 - a^3*x + 2*a^2*tan(1/2*a*x))

maple [A] time = 0.20, size = 21, normalized size = 1.05

$$\frac{1}{a^2 (ax \cos(ax) - \sin(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x)

[Out] 1/a^2/(a*x*cos(a*x)-sin(a*x))

maxima [A] time = 0.31, size = 20, normalized size = 1.00

$$\frac{1}{(ax \cos(ax) - \sin(ax))a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")

[Out] 1/((a*x*cos(a*x) - sin(a*x))*a^2)

mupad [B] time = 0.12, size = 23, normalized size = 1.15

$$-\frac{1}{a^2 \sin(ax) - a^3 x \cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*sin(a*x))/(sin(a*x) - a*x*cos(a*x))^2,x)

[Out] -1/(a^2*sin(a*x) - a^3*x*cos(a*x))

sympy [A] time = 3.48, size = 19, normalized size = 0.95

$$\frac{1}{a^3 x \cos(ax) - a^2 \sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))**2,x)

[Out] 1/(a**3*x*cos(a*x) - a**2*sin(a*x))

$$3.591 \quad \int \frac{x^2}{(ax \cos(ax) - \sin(ax))^2} dx$$

Optimal. Leaf size=35

$$\frac{x \csc(ax)}{a^2(ax \cos(ax) - \sin(ax))} - \frac{\cot(ax)}{a^3}$$

[Out] $-\cot(ax)/a^3 + x \csc(ax)/a^2/(ax \cos(ax) - \sin(ax))$

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4594, 3767, 8}

$$\frac{x \csc(ax)}{a^2(ax \cos(ax) - \sin(ax))} - \frac{\cot(ax)}{a^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2/(a*x*\text{Cos}[a*x] - \text{Sin}[a*x])^2, x]$

[Out] $-(\text{Cot}[a*x]/a^3) + (x*\text{Csc}[a*x])/(a^2*(a*x*\text{Cos}[a*x] - \text{Sin}[a*x]))$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 3767

$\text{Int}[\csc[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x \&\& \text{IGtQ}[n/2, 0]$

Rule 4594

$\text{Int}[(x_)^2/(\text{Cos}[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*\text{Sin}[(a_.)*(x_)]^2, x_Symbol] \rightarrow \text{Simp}[x/(a*d*\text{Sin}[a*x]*(c*\text{Sin}[a*x] + d*x*\text{Cos}[a*x])), x] + \text{Dist}[1/d^2, \text{Int}[1/\text{Sin}[a*x]^2, x], x] /; \text{FreeQ}\{a, c, d\}, x \&\& \text{EqQ}[a*c + d, 0]$

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(ax \cos(ax) - \sin(ax))^2} dx &= \frac{x \csc(ax)}{a^2(ax \cos(ax) - \sin(ax))} + \frac{\int \csc^2(ax) dx}{a^2} \\ &= \frac{x \csc(ax)}{a^2(ax \cos(ax) - \sin(ax))} - \frac{\text{Subst}(\int 1 dx, x, \cot(ax))}{a^3} \\ &= -\frac{\cot(ax)}{a^3} + \frac{x \csc(ax)}{a^2(ax \cos(ax) - \sin(ax))} \end{aligned}$$

Mathematica [A] time = 0.46, size = 32, normalized size = 0.91

$$\frac{ax \sin(ax) + \cos(ax)}{a^3(ax \cos(ax) - \sin(ax))}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x^2/(a*x*\text{Cos}[a*x] - \text{Sin}[a*x])^2, x]$

[Out] $(\text{Cos}[a*x] + a*x*\text{Sin}[a*x])/(a^3*(a*x*\text{Cos}[a*x] - \text{Sin}[a*x]))$

fricas [A] time = 0.82, size = 34, normalized size = 0.97

$$\frac{ax \sin(ax) + \cos(ax)}{a^4x \cos(ax) - a^3 \sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")

[Out] (a*x*sin(a*x) + cos(a*x))/(a^4*x*cos(a*x) - a^3*sin(a*x))

giac [A] time = 0.15, size = 53, normalized size = 1.51

$$\frac{2ax \tan\left(\frac{1}{2}ax\right) - \tan\left(\frac{1}{2}ax\right)^2 + 1}{a^4x \tan\left(\frac{1}{2}ax\right)^2 - a^4x + 2a^3 \tan\left(\frac{1}{2}ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")

[Out] -(2*a*x*tan(1/2*a*x) - tan(1/2*a*x)^2 + 1)/(a^4*x*tan(1/2*a*x)^2 - a^4*x + 2*a^3*tan(1/2*a*x))

maple [A] time = 1.23, size = 54, normalized size = 1.54

$$\frac{\frac{\tan^2\left(\frac{ax}{2}\right)}{a^3} - \frac{1}{a^3} - \frac{2x \tan\left(\frac{ax}{2}\right)}{a^2}}{ax \left(\tan^2\left(\frac{ax}{2}\right)\right) - ax + 2 \tan\left(\frac{ax}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a*x*cos(a*x)-sin(a*x))^2,x)

[Out] (1/a^3*tan(1/2*a*x)^2-1/a^3-2*x/a^2*tan(1/2*a*x))/(a*x*tan(1/2*a*x)^2-a*x+2*tan(1/2*a*x))

maxima [B] time = 0.31, size = 100, normalized size = 2.86

$$\frac{2(2ax \cos(2ax) + (a^2x^2 - 1) \sin(2ax))}{(a^2x^2 + (a^2x^2 + 1) \cos(2ax))^2 - 4ax \sin(2ax) + (a^2x^2 + 1) \sin(2ax)^2 + 2(a^2x^2 - 1) \cos(2ax) + 1} a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")

[Out] 2*(2*a*x*cos(2*a*x) + (a^2*x^2 - 1)*sin(2*a*x))/((a^2*x^2 + (a^2*x^2 + 1)*cos(2*a*x))^2 - 4*a*x*sin(2*a*x) + (a^2*x^2 + 1)*sin(2*a*x)^2 + 2*(a^2*x^2 - 1)*cos(2*a*x) + 1)*a^3

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{(\sin(ax) - ax \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(sin(a*x) - a*x*cos(a*x))^2,x)

[Out] int(x^2/(sin(a*x) - a*x*cos(a*x))^2, x)

sympy [B] time = 5.05, size = 112, normalized size = 3.20

$$-\frac{2ax \tan\left(\frac{ax}{2}\right)}{a^4x \tan^2\left(\frac{ax}{2}\right) - a^4x + 2a^3 \tan\left(\frac{ax}{2}\right)} + \frac{\tan^2\left(\frac{ax}{2}\right)}{a^4x \tan^2\left(\frac{ax}{2}\right) - a^4x + 2a^3 \tan\left(\frac{ax}{2}\right)} - \frac{1}{a^4x \tan^2\left(\frac{ax}{2}\right) - a^4x + 2a^3 \tan\left(\frac{ax}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a*x*cos(a*x)-sin(a*x))**2,x)

[Out] $-2*a*x*\tan(a*x/2)/(a**4*x*\tan(a*x/2)**2 - a**4*x + 2*a**3*\tan(a*x/2)) + \tan(a*x/2)**2/(a**4*x*\tan(a*x/2)**2 - a**4*x + 2*a**3*\tan(a*x/2)) - 1/(a**4*x*\tan(a*x/2)**2 - a**4*x + 2*a**3*\tan(a*x/2))$

$$3.592 \quad \int \frac{x^3 \csc(ax)}{(ax \cos(ax) - \sin(ax))^2} dx$$

Optimal. Leaf size=104

$$\frac{i\text{Li}_2(-e^{iax})}{a^4} - \frac{i\text{Li}_2(e^{iax})}{a^4} - \frac{\csc(ax)}{a^4} - \frac{2x \tanh^{-1}(e^{iax})}{a^3} - \frac{x \cot(ax) \csc(ax)}{a^3} + \frac{x^2 \csc^2(ax)}{a^2(ax \cos(ax) - \sin(ax))}$$

[Out] $-2*x*\text{arctanh}(\exp(I*a*x))/a^3 - \csc(a*x)/a^4 - x*\cot(a*x)*\csc(a*x)/a^3 + I*\text{polylog}(2, -\exp(I*a*x))/a^4 - I*\text{polylog}(2, \exp(I*a*x))/a^4 + x^2*\csc(a*x)^2/a^2/(a*x*\cos(a*x) - \sin(a*x))$

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4600, 4185, 4183, 2279, 2391}

$$\frac{i\text{PolyLog}(2, -e^{iax})}{a^4} - \frac{i\text{PolyLog}(2, e^{iax})}{a^4} + \frac{x^2 \csc^2(ax)}{a^2(ax \cos(ax) - \sin(ax))} - \frac{\csc(ax)}{a^4} - \frac{2x \tanh^{-1}(e^{iax})}{a^3} - \frac{x \cot(ax) \csc(ax)}{a^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^3*\text{Csc}[a*x])/(a*x*\text{Cos}[a*x] - \text{Sin}[a*x])^2, x]$

[Out] $(-2*x*\text{ArcTanh}[E^{(I*a*x)}])/a^3 - \text{Csc}[a*x]/a^4 - (x*\text{Cot}[a*x]*\text{Csc}[a*x])/a^3 + (I*\text{PolyLog}[2, -E^{(I*a*x)}])/a^4 - (I*\text{PolyLog}[2, E^{(I*a*x)}])/a^4 + (x^2*\text{Csc}[a*x]^2)/(a^2*(a*x*\text{Cos}[a*x] - \text{Sin}[a*x]))$

Rule 2279

$\text{Int}[\text{Log}[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^{(n_)}], x_Symbol] :> \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /;$ FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

$\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)}]/(x_), x_Symbol] :> -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /;$ FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 4183

$\text{Int}[\csc[(e_) + (f_)*(x_)]*((c_) + (d_)*(x_))^{(m_)}, x_Symbol] :> \text{Simp}[(-2*(c + d*x)^m*\text{ArcTanh}[E^{(I*(e + f*x))}])/f, x] + (-\text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{(I*(e + f*x))}], x], x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{(I*(e + f*x))}], x], x] /;$ FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 4185

$\text{Int}[(\csc[(e_) + (f_)*(x_)]*(b_))^{(n_)*((c_) + (d_)*(x_))}, x_Symbol] :> -\text{Simp}[(b^2*(c + d*x)*\text{Cot}[e + f*x]*(b*\text{Csc}[e + f*x])^{(n-2)})/(f*(n-1)), x] + (\text{Dist}[(b^2*(n-2))/(n-1), \text{Int}[(c + d*x)*(b*\text{Csc}[e + f*x])^{(n-2)}, x], x] - \text{Simp}[(b^2*d*(b*\text{Csc}[e + f*x])^{(n-2)})/(f^2*(n-1)*(n-2)), x]) /;$ FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]

Rule 4600

$\text{Int}[(\text{Csc}[(a_)*(x_)]^{(n_)*((b_)*(x_))^{(m_)}]/(\text{Cos}[(a_)*(x_)]*(d_)*(x_)) + (c_)*\text{Sin}[(a_)*(x_)]^2, x_Symbol] :> \text{Simp}[(b*(b*x)^{(m-1)}*\text{Csc}[a*x]^{(n+1)})/(a*d*(c*\text{Sin}[a*x] + d*x*\text{Cos}[a*x]), x] + \text{Dist}[(b^2*(n+1))/d^2, \text{Int}[(b*x)^{(m-2)}*\text{Csc}[a*x]^{(n+2)}, x], x] /;$ FreeQ[{a, b, c, d, m, n}, x] && Eq

Q[a*c + d, 0] && EqQ[m, n + 2]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3 \csc(ax)}{(ax \cos(ax) - \sin(ax))^2} dx &= \frac{x^2 \csc^2(ax)}{a^2(ax \cos(ax) - \sin(ax))} + \frac{2 \int x \csc^3(ax) dx}{a^2} \\
 &= -\frac{\csc(ax)}{a^4} - \frac{x \cot(ax) \csc(ax)}{a^3} + \frac{x^2 \csc^2(ax)}{a^2(ax \cos(ax) - \sin(ax))} + \frac{\int x \csc(ax) dx}{a^2} \\
 &= -\frac{2x \tanh^{-1}(e^{iax})}{a^3} - \frac{\csc(ax)}{a^4} - \frac{x \cot(ax) \csc(ax)}{a^3} + \frac{x^2 \csc^2(ax)}{a^2(ax \cos(ax) - \sin(ax))} - \frac{\int x \csc(ax) dx}{a^2} \\
 &= -\frac{2x \tanh^{-1}(e^{iax})}{a^3} - \frac{\csc(ax)}{a^4} - \frac{x \cot(ax) \csc(ax)}{a^3} + \frac{x^2 \csc^2(ax)}{a^2(ax \cos(ax) - \sin(ax))} + \frac{\int x \csc(ax) dx}{a^2} \\
 &= -\frac{2x \tanh^{-1}(e^{iax})}{a^3} - \frac{\csc(ax)}{a^4} - \frac{x \cot(ax) \csc(ax)}{a^3} + \frac{i \operatorname{Li}_2(-e^{iax})}{a^4} - \frac{i \operatorname{Li}_2(e^{iax})}{a^4} + \frac{\int x \csc(ax) dx}{a^2}
 \end{aligned}$$

Mathematica [A] time = 1.00, size = 157, normalized size = 1.51

$$\frac{a^2 x^2 \csc(ax) + a^2 x^2 \log(1 - e^{iax}) \cot(ax) - a^2 x^2 \log(1 + e^{iax}) \cot(ax) + i \operatorname{Li}_2(-e^{iax})(ax \cot(ax) - 1) - i \operatorname{Li}_2(e^{iax})(ax \cot(ax) - 1)}{a^4(ax \cot(ax) - 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*Csc[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]

[Out] (Csc[a*x] + a^2*x^2*Csc[a*x] - a*x*Log[1 - E^(I*a*x)] + a^2*x^2*Cot[a*x]*Log[1 - E^(I*a*x)] + a*x*Log[1 + E^(I*a*x)] - a^2*x^2*Cot[a*x]*Log[1 + E^(I*a*x)] + I*(-1 + a*x*Cot[a*x])*PolyLog[2, -E^(I*a*x)] - I*(-1 + a*x*Cot[a*x])*PolyLog[2, E^(I*a*x)])/(a^4*(-1 + a*x*Cot[a*x]))

fricas [B] time = 2.96, size = 295, normalized size = 2.84

$$\frac{2a^2x^2 - (iax \cos(ax) - i \sin(ax))\operatorname{Li}_2(\cos(ax) + i \sin(ax)) - (-iax \cos(ax) + i \sin(ax))\operatorname{Li}_2(\cos(ax) - i \sin(ax))}{a^4(ax \cot(ax) - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")

[Out] 1/2*(2*a^2*x^2 - (I*a*x*cos(a*x) - I*sin(a*x))*dilog(cos(a*x) + I*sin(a*x)) - (-I*a*x*cos(a*x) + I*sin(a*x))*dilog(cos(a*x) - I*sin(a*x)) - (I*a*x*cos(a*x) - I*sin(a*x))*dilog(-cos(a*x) + I*sin(a*x)) - (-I*a*x*cos(a*x) + I*sin(a*x))*dilog(-cos(a*x) - I*sin(a*x)) - (a^2*x^2*cos(a*x) - a*x*sin(a*x))*log(cos(a*x) + I*sin(a*x) + 1) - (a^2*x^2*cos(a*x) - a*x*sin(a*x))*log(cos(a*x) - I*sin(a*x) + 1) + (a^2*x^2*cos(a*x) - a*x*sin(a*x))*log(-cos(a*x) + I*sin(a*x) + 1) + (a^2*x^2*cos(a*x) - a*x*sin(a*x))*log(-cos(a*x) - I*sin(a*x) + 1) + 2)/(a^5*x*cos(a*x) - a^4*sin(a*x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \csc(ax)}{(ax \cos(ax) - \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")

[Out] integrate(x^3*csc(a*x)/(a*x*cos(a*x) - sin(a*x))^2, x)

maple [F] time = 2.47, size = 0, normalized size = 0.00

$$\int \frac{x^3 \csc(ax)}{(ax \cos(ax) - \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x)

[Out] int(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sin(ax) (\sin(ax) - ax \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(sin(a*x)*(sin(a*x) - a*x*cos(a*x))^2),x)

[Out] int(x^3/(sin(a*x)*(sin(a*x) - a*x*cos(a*x))^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \csc(ax)}{(ax \cos(ax) - \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))**2,x)

[Out] Integral(x**3*csc(a*x)/(a*x*cos(a*x) - sin(a*x))**2, x)

$$3.593 \quad \int \frac{x^4 \csc^2(ax)}{(ax \cos(ax) - \sin(ax))^2} dx$$

Optimal. Leaf size=127

$$-\frac{2i\text{Li}_2(e^{2iax})}{a^5} - \frac{\cot(ax)}{a^5} + \frac{4x \log(1 - e^{2iax})}{a^4} - \frac{x \csc^2(ax)}{a^4} - \frac{2ix^2}{a^3} - \frac{2x^2 \cot(ax)}{a^3} - \frac{x^2 \cot(ax) \csc^2(ax)}{a^3} + \frac{x^3 \csc^3(ax)}{a^2(ax \cos(ax) - \sin(ax))}$$

[Out] $-2*I*x^2/a^3 - \cot(a*x)/a^5 - 2*x^2*\cot(a*x)/a^3 - x*\csc(a*x)^2/a^4 - x^2*\cot(a*x)*\csc(a*x)^2/a^3 + 4*x*\ln(1 - \exp(2*I*a*x))/a^4 - 2*I*\text{polylog}(2, \exp(2*I*a*x))/a^5 + x^3*\csc(a*x)^3/a^2/(a*x*\cos(a*x) - \sin(a*x))$

Rubi [A] time = 0.18, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {4600, 4186, 3767, 8, 4184, 3717, 2190, 2279, 2391}

$$-\frac{2i\text{PolyLog}(2, e^{2iax})}{a^5} - \frac{2ix^2}{a^3} - \frac{2x^2 \cot(ax)}{a^3} - \frac{x^2 \cot(ax) \csc^2(ax)}{a^3} + \frac{x^3 \csc^3(ax)}{a^2(ax \cos(ax) - \sin(ax))} + \frac{4x \log(1 - e^{2iax})}{a^4} - \frac{\cot(ax)}{a^5}$$

Antiderivative was successfully verified.

[In] Int[(x^4*Csc[a*x]^2)/(a*x*Cos[a*x] - Sin[a*x])^2, x]

[Out] $((-2*I)*x^2)/a^3 - \cot[a*x]/a^5 - (2*x^2*\cot[a*x])/a^3 - (x*\csc[a*x]^2)/a^4 - (x^2*\cot[a*x]*\csc[a*x]^2)/a^3 + (4*x*\log[1 - E^((2*I)*a*x)])/a^4 - ((2*I)*\text{PolyLog}[2, E^((2*I)*a*x)])/a^5 + (x^3*\csc[a*x]^3)/(a^2*(a*x*\cos[a*x] - \sin[a*x]))$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2190

Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 3717

Int[(((c_) + (d_)*(x_))^(m_)*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 4184

Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 4186

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

Rule 4600

Int[(Csc[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)]^2, x_Symbol] := Simp[(b*(b*x)^(m - 1)*Csc[a*x]^(n + 1))/(a*d*(c*SIN[a*x] + d*x*Cos[a*x]), x] + Dist[(b^2*(n + 1))/d^2, Int[(b*x)^(m - 2)*Csc[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a*c + d, 0] && EqQ[m, n + 2]

Rubi steps

$$\begin{aligned}
 \int \frac{x^4 \csc^2(ax)}{(ax \cos(ax) - \sin(ax))^2} dx &= \frac{x^3 \csc^3(ax)}{a^2(ax \cos(ax) - \sin(ax))} + \frac{3 \int x^2 \csc^4(ax) dx}{a^2} \\
 &= \frac{x \csc^2(ax)}{a^4} - \frac{x^2 \cot(ax) \csc^2(ax)}{a^3} + \frac{x^3 \csc^3(ax)}{a^2(ax \cos(ax) - \sin(ax))} + \frac{\int \csc^2(ax) dx}{a^4} \\
 &= \frac{2x^2 \cot(ax)}{a^3} - \frac{x \csc^2(ax)}{a^4} - \frac{x^2 \cot(ax) \csc^2(ax)}{a^3} + \frac{x^3 \csc^3(ax)}{a^2(ax \cos(ax) - \sin(ax))} \\
 &= \frac{2ix^2}{a^3} - \frac{\cot(ax)}{a^5} - \frac{2x^2 \cot(ax)}{a^3} - \frac{x \csc^2(ax)}{a^4} - \frac{x^2 \cot(ax) \csc^2(ax)}{a^3} + \frac{x^3 \csc^3(ax)}{a^2(ax \cos(ax) - \sin(ax))} \\
 &= \frac{2ix^2}{a^3} - \frac{\cot(ax)}{a^5} - \frac{2x^2 \cot(ax)}{a^3} - \frac{x \csc^2(ax)}{a^4} - \frac{x^2 \cot(ax) \csc^2(ax)}{a^3} + \frac{4x \log(ax \cos(ax) - \sin(ax))}{a^4} \\
 &= \frac{2ix^2}{a^3} - \frac{\cot(ax)}{a^5} - \frac{2x^2 \cot(ax)}{a^3} - \frac{x \csc^2(ax)}{a^4} - \frac{x^2 \cot(ax) \csc^2(ax)}{a^3} + \frac{4x \log(ax \cos(ax) - \sin(ax))}{a^4} \\
 &= \frac{2ix^2}{a^3} - \frac{\cot(ax)}{a^5} - \frac{2x^2 \cot(ax)}{a^3} - \frac{x \csc^2(ax)}{a^4} - \frac{x^2 \cot(ax) \csc^2(ax)}{a^3} + \frac{4x \log(ax \cos(ax) - \sin(ax))}{a^4}
 \end{aligned}$$

Mathematica [A] time = 1.05, size = 102, normalized size = 0.80

$$\frac{a^3(-x^2) \cot(ax) - 2ia(a^2x^2 + \text{Li}_2(e^{2iax})) + \frac{(a^2x^2+1)^2 \sin(ax)}{x(ax \cos(ax) - \sin(ax))} + a^2x + 4a^2x \log(1 - e^{2iax}) + \frac{1}{x}}{a^6}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*Csc[a*x]^2)/(a*x*Cos[a*x] - Sin[a*x])^2,x]

[Out] (x^(-1) + a^2*x - a^3*x^2*Cot[a*x] + 4*a^2*x*Log[1 - E^((2*I)*a*x)] - (2*I)*a*(a^2*x^2 + PolyLog[2, E^((2*I)*a*x)])) + ((1 + a^2*x^2)^2*Sin[a*x])/(x*(a*x*Cos[a*x] - Sin[a*x]))/a^6

fricas [B] time = 1.00, size = 406, normalized size = 3.20

$$\frac{a^3x^3 - (2a^3x^3 + ax)\cos(ax)^2 + (2a^2x^2 + 1)\cos(ax)\sin(ax) + ax + (-2iax\cos(ax)\sin(ax) - 2i\cos(ax)^2 + 2i\sin(ax)^2)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")

[Out] (a^3*x^3 - (2*a^3*x^3 + a*x)*cos(a*x)^2 + (2*a^2*x^2 + 1)*cos(a*x)*sin(a*x) + a*x + (-2*I*a*x*cos(a*x)*sin(a*x) - 2*I*cos(a*x)^2 + 2*I)*dilog(cos(a*x) + I*sin(a*x)) + (2*I*a*x*cos(a*x)*sin(a*x) + 2*I*cos(a*x)^2 - 2*I)*dilog(cos(a*x) - I*sin(a*x)) + (2*I*a*x*cos(a*x)*sin(a*x) + 2*I*cos(a*x)^2 - 2*I)*dilog(-cos(a*x) + I*sin(a*x)) + (-2*I*a*x*cos(a*x)*sin(a*x) - 2*I*cos(a*x)^2 + 2*I)*dilog(-cos(a*x) - I*sin(a*x)) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2 - a*x)*log(cos(a*x) + I*sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2 - a*x)*log(cos(a*x) - I*sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2 - a*x)*log(-cos(a*x) + I*sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2 - a*x)*log(-cos(a*x) - I*sin(a*x) + 1))/(a^6*x*cos(a*x)*sin(a*x) + a^5*cos(a*x)^2 - a^5)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \csc(ax)^2}{(ax \cos(ax) - \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")

[Out] integrate(x^4*csc(a*x)^2/(a*x*cos(a*x) - sin(a*x))^2, x)

maple [A] time = 1.10, size = 172, normalized size = 1.35

$$\frac{2i(2ia^2x^2e^{2iax} + 2x^3a^3 - 2ia^2x^2 - ax e^{2iax} + ie^{2iax} + ax - i)}{(e^{2iax} - 1)(ax e^{2iax} + ie^{2iax} + ax - i)a^5} - \frac{4ix^2}{a^3} + \frac{4x \ln(e^{iax} + 1)}{a^4} - \frac{4i \operatorname{polylog}(2, -e^{iax})}{a^5} + \frac{4x \ln(e^{-iax} + 1)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x)

[Out] -2*I*(2*I*a^2*x^2*exp(2*I*a*x)+2*x^3*a^3-2*I*a^2*x^2-a*x*exp(2*I*a*x)+I*exp(2*I*a*x)+a*x-I)/(exp(2*I*a*x)-1)/(a*x*exp(2*I*a*x)+I*exp(2*I*a*x)+a*x-I)/a^5-4*I/a^3*x^2+4/a^4*x*ln(exp(I*a*x)+1)-4*I/a^5*polylog(2,-exp(I*a*x))+4/a^4*x*ln(1-exp(I*a*x))-4*I/a^5*polylog(2,exp(I*a*x))

maxima [B] time = 0.38, size = 608, normalized size = 4.79

$$\frac{2ax + (4a^2x^2 + 8iax\cos(2ax) - 8ax\sin(2ax) - 4iax - (4a^2x^2 + 4iax)\cos(4ax) + 4(-ia^2x^2 + ax)\sin(4ax)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")

```
[Out] -(2*a*x + (4*a^2*x^2 + 8*I*a*x*cos(2*a*x) - 8*a*x*sin(2*a*x) - 4*I*a*x - (4
*a^2*x^2 + 4*I*a*x)*cos(4*a*x) + 4*(-I*a^2*x^2 + a*x)*sin(4*a*x))*arctan2(s
in(a*x), cos(a*x) + 1) - (4*a^2*x^2 + 8*I*a*x*cos(2*a*x) - 8*a*x*sin(2*a*x)
- 4*I*a*x - (4*a^2*x^2 + 4*I*a*x)*cos(4*a*x) - 4*(I*a^2*x^2 - a*x)*sin(4*a
*x))*arctan2(sin(a*x), -cos(a*x) + 1) + 4*(a^3*x^3 + I*a^2*x^2)*cos(4*a*x)
- (4*I*a^2*x^2 + 2*a*x - 2*I)*cos(2*a*x) - (4*a*x - (4*a*x + 4*I)*cos(4*a*x
) - 4*(I*a*x - 1)*sin(4*a*x) + 8*I*cos(2*a*x) - 8*sin(2*a*x) - 4*I)*dilog(-
e^(I*a*x)) - (4*a*x - (4*a*x + 4*I)*cos(4*a*x) - 4*(I*a*x - 1)*sin(4*a*x) +
8*I*cos(2*a*x) - 8*sin(2*a*x) - 4*I)*dilog(e^(I*a*x)) - (2*I*a^2*x^2 - 4*a
*x*cos(2*a*x) - 4*I*a*x*sin(2*a*x) + 2*a*x - 2*(I*a^2*x^2 - a*x)*cos(4*a*x)
+ (2*a^2*x^2 + 2*I*a*x)*sin(4*a*x))*log(cos(a*x)^2 + sin(a*x)^2 + 2*cos(a*
x) + 1) - (2*I*a^2*x^2 - 4*a*x*cos(2*a*x) - 4*I*a*x*sin(2*a*x) + 2*a*x - 2*
(I*a^2*x^2 - a*x)*cos(4*a*x) + (2*a^2*x^2 + 2*I*a*x)*sin(4*a*x))*log(cos(a*
x)^2 + sin(a*x)^2 - 2*cos(a*x) + 1) - (-4*I*a^3*x^3 + 4*a^2*x^2)*sin(4*a*x)
+ (4*a^2*x^2 - 2*I*a*x - 2)*sin(2*a*x) - 2*I)/((I*a*x + (-I*a*x + 1)*cos(4
*a*x) + (a*x + I)*sin(4*a*x) - 2*cos(2*a*x) - 2*I*sin(2*a*x) + 1)*a^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sin(ax)^2 (\sin(ax) - ax \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(sin(a*x)^2*(sin(a*x) - a*x*cos(a*x))^2),x)
```

```
[Out] int(x^4/(sin(a*x)^2*(sin(a*x) - a*x*cos(a*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \csc^2(ax)}{(ax \cos(ax) - \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*csc(a*x)**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Integral(x**4*csc(a*x)**2/(a*x*cos(a*x) - sin(a*x))**2, x)
```

$$3.594 \quad \int \frac{\cos^6(ax)}{x^4(\cos(ax)+ax \sin(ax))^2} dx$$

Optimal. Leaf size=176

$$\frac{2}{3}a^3\text{Si}(2ax)+\frac{16}{3}a^3\text{Si}(4ax)+\frac{\cos^4(ax)}{a^2x^5}-\frac{\cos^5(ax)}{a^2x^5(ax \sin(ax) + \cos(ax))}+\frac{a^2}{x}+\frac{32a^2 \cos^4(ax)}{3x}-\frac{10a^2 \cos^2(ax)}{x}-\frac{\sin(ax) \cos(ax)}{ax^4}$$

[Out] $a^2/x+\cos(a*x)^2/x^3-10*a^2*\cos(a*x)^2/x+\cos(a*x)^4/a^2/x^5-4/3*\cos(a*x)^4/x^3+32/3*a^2*\cos(a*x)^4/x+2/3*a^3*\text{Si}(2*a*x)+16/3*a^3*\text{Si}(4*a*x)-a*\cos(a*x)*\sin(a*x)/x^2-\cos(a*x)^3*\sin(a*x)/a/x^4+8/3*a*\cos(a*x)^3*\sin(a*x)/x^2-\cos(a*x)^5/a^2/x^5/(\cos(a*x)+a*x*\sin(a*x))$

Rubi [A] time = 0.30, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4599, 3314, 30, 3313, 12, 3299}

$$\frac{2}{3}a^3\text{Si}(2ax)+\frac{16}{3}a^3\text{Si}(4ax)+\frac{\cos^4(ax)}{a^2x^5}-\frac{\cos^5(ax)}{a^2x^5(ax \sin(ax) + \cos(ax))}+\frac{a^2}{x}+\frac{32a^2 \cos^4(ax)}{3x}-\frac{10a^2 \cos^2(ax)}{x}-\frac{4 \cos^4(ax)}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[Cos[a*x]^6/(x^4*(Cos[a*x] + a*x*Sin[a*x])^2), x]

[Out] $a^2/x + \text{Cos}[a*x]^2/x^3 - (10*a^2*\text{Cos}[a*x]^2)/x + \text{Cos}[a*x]^4/(a^2*x^5) - (4*\text{Cos}[a*x]^4)/(3*x^3) + (32*a^2*\text{Cos}[a*x]^4)/(3*x) - (a*\text{Cos}[a*x]*\text{Sin}[a*x])/x^2 - (\text{Cos}[a*x]^3*\text{Sin}[a*x])/(a*x^4) + (8*a*\text{Cos}[a*x]^3*\text{Sin}[a*x])/(3*x^2) - \text{Cos}[a*x]^5/(a^2*x^5*(\text{Cos}[a*x] + a*x*\text{Sin}[a*x])) + (2*a^3*\text{SinIntegral}[2*a*x])/3 + (16*a^3*\text{SinIntegral}[4*a*x])/3$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rule 3313

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m + 1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n - 1), x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

Rule 3314

Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,

d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

Rule 4599

Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)*(x_)*Sin[(a_.)*(x_)]^2, x_Symbol] :> -Simp[(b*(b*x)^(m - 1)*Cos[a*x]^(n - 1))/(a*d*(c*cos[a*x] + d*x*sin[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a*c - d, 0] && EqQ[m, 2 - n]

Rubi steps

$$\begin{aligned} \int \frac{\cos^6(ax)}{x^4(\cos(ax) + ax \sin(ax))^2} dx &= \frac{\cos^5(ax)}{a^2 x^5 (\cos(ax) + ax \sin(ax))} - \frac{5 \int \frac{\cos^4(ax)}{x^6} dx}{a^2} \\ &= \frac{\cos^4(ax)}{a^2 x^5} - \frac{\cos^3(ax) \sin(ax)}{ax^4} - \frac{\cos^5(ax)}{a^2 x^5 (\cos(ax) + ax \sin(ax))} - 3 \int \frac{\cos^2(ax)}{x^4} \\ &= \frac{\cos^2(ax)}{x^3} + \frac{\cos^4(ax)}{a^2 x^5} - \frac{4 \cos^4(ax)}{3x^3} - \frac{a \cos(ax) \sin(ax)}{x^2} - \frac{\cos^3(ax) \sin(ax)}{ax^4} \\ &= \frac{a^2}{x} + \frac{\cos^2(ax)}{x^3} - \frac{10a^2 \cos^2(ax)}{x} + \frac{\cos^4(ax)}{a^2 x^5} - \frac{4 \cos^4(ax)}{3x^3} + \frac{32a^2 \cos^4(ax)}{3x} \\ &= \frac{a^2}{x} + \frac{\cos^2(ax)}{x^3} - \frac{10a^2 \cos^2(ax)}{x} + \frac{\cos^4(ax)}{a^2 x^5} - \frac{4 \cos^4(ax)}{3x^3} + \frac{32a^2 \cos^4(ax)}{3x} \\ &= \frac{a^2}{x} + \frac{\cos^2(ax)}{x^3} - \frac{10a^2 \cos^2(ax)}{x} + \frac{\cos^4(ax)}{a^2 x^5} - \frac{4 \cos^4(ax)}{3x^3} + \frac{32a^2 \cos^4(ax)}{3x} \end{aligned}$$

Mathematica [A] time = 1.22, size = 194, normalized size = 1.10

$$\frac{32a^3 x^3 \text{Si}(2ax)(ax \sin(ax) + \cos(ax)) + 256a^3 x^3 \text{Si}(4ax)(ax \sin(ax) + \cos(ax)) - 8a^3 x^3 \sin(ax) - 24a^3 x^3 \sin(3ax)}{3(ax^4 \sin(ax) + x^3 \cos(ax))}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a*x]^6/(x^4*(Cos[a*x] + a*x*Sin[a*x])^2), x]

[Out] (-10*Cos[a*x] + 12*a^2*x^2*Cos[a*x] - 5*Cos[3*a*x] + 44*a^2*x^2*Cos[3*a*x] - Cos[5*a*x] + 24*a^2*x^2*Cos[5*a*x] + 8*a*x*Sin[a*x] - 8*a^3*x^3*Sin[a*x] + 12*a*x*Sin[3*a*x] - 24*a^3*x^3*Sin[3*a*x] + 4*a*x*Sin[5*a*x] + 32*a^3*x^3*Sin[5*a*x] + 32*a^3*x^3*(Cos[a*x] + a*x*Sin[a*x])*SinIntegral[2*a*x] + 256*a^3*x^3*(Cos[a*x] + a*x*Sin[a*x])*SinIntegral[4*a*x])/(48*x^3*(Cos[a*x] + a*x*Sin[a*x]))

fricas [A] time = 0.92, size = 162, normalized size = 0.92

$$\frac{19 a^2 x^2 \cos(ax)^3 - (24 a^2 x^2 - 1) \cos(ax)^5 - 2(8 a^3 x^3 \text{Si}(4ax) + a^3 x^3 \text{Si}(2ax)) \cos(ax) - (16 a^4 x^4 \text{Si}(4ax) + 3 a^4 x^4 \text{Si}(2ax)) \sin(ax)}{3(ax^4 \sin(ax) + x^3 \cos(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")

[Out] -1/3*(19*a^2*x^2*cos(a*x)^3 - (24*a^2*x^2 - 1)*cos(a*x)^5 - 2*(8*a^3*x^3*sin_integral(4*a*x) + a^3*x^3*sin_integral(2*a*x))*cos(a*x) - (16*a^4*x^4*sin_integral(4*a*x) + 2*a^4*x^4*sin_integral(2*a*x) - 30*a^3*x^3*cos(a*x)^2 + 3*a^3*x^3 + 4*(8*a^3*x^3 + a*x)*cos(a*x)^4)*sin(a*x))/(a*x^4*sin(a*x) + x^3*cos(a*x))

giac [C] time = 1.21, size = 7279, normalized size = 41.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/12*(64*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan \\ & (1/2*a*x) + 8*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2 \\ & *tan(1/2*a*x) - 8*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan \\ & (a*x)^2*tan(1/2*a*x) - 64*a^8*x^8*imag_part(cos_integral(-4*a*x))*tan(2*a*x) \\ & ^2*tan(a*x)^2*tan(1/2*a*x) + 128*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2* \\ & tan(a*x)^2*tan(1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a \\ & *x)^2*tan(1/2*a*x) - 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2 \\ & *tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(2 \\ & *a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^7*x^7*imag_part(cos_integral(-2*a*x \\ &))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^7*x^7*imag_part(cos_integr \\ & al(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 64*a^7*x^7*sin_integra \\ & l(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^7*x^7*sin_integral(2* \\ & a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*imag_part(cos_inte \\ & gral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 8*a^8*x^8*imag_part(cos_integral(2 \\ & *a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 8*a^8*x^8*imag_part(cos_integral(-2*a*x) \\ &)*tan(2*a*x)^2*tan(1/2*a*x) - 64*a^8*x^8*imag_part(cos_integral(-4*a*x))*ta \\ & n(2*a*x)^2*tan(1/2*a*x) + 128*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2*tan(\\ & 1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x) + 64*a^ \\ & 8*x^8*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 8*a^8*x^8*im \\ & ag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 8*a^8*x^8*imag_part(\\ & cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 64*a^8*x^8*imag_part(cos_in \\ & tegral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 128*a^8*x^8*sin_integral(4*a*x)*t \\ & an(a*x)^2*tan(1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2* \\ & a*x) + 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2 + \\ & 4*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2 - 4*a^7*x^ \\ & 7*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2 - 32*a^7*x^7*imag \\ & _part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2 + 64*a^7*x^7*sin_integr \\ & al(4*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 8*a^7*x^7*sin_integral(2*a*x)*tan(2*a*x \\ &)^2*tan(a*x)^2 - 40*a^7*x^7*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 32*a^7*x \\ & ^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 4*a^7*x^7*i \\ & mag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 4*a^7*x^7*imag_ \\ & part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 32*a^7*x^7*imag_pa \\ & rt(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 64*a^7*x^7*sin_integ \\ & ral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 - 8*a^7*x^7*sin_integral(2*a*x)*tan(\\ & 2*a*x)^2*tan(1/2*a*x)^2 - 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(a*x \\ &)^2*tan(1/2*a*x)^2 - 4*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(a*x)^2*ta \\ & n(1/2*a*x)^2 + 4*a^7*x^7*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2 \\ & *a*x)^2 + 32*a^7*x^7*imag_part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x \\ &)^2 - 64*a^7*x^7*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^7*x^7* \\ & sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*imag_part(cos_in \\ & tegral(4*a*x))*tan(1/2*a*x) + 8*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(\\ & 1/2*a*x) - 8*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) - 64*a^8* \\ & x^8*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x) + 128*a^8*x^8*sin_integral \\ & (4*a*x)*tan(1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(1/2*a*x) + 128*a^ \\ & 6*x^6*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + \\ & 16*a^6*x^6*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2* \\ & a*x) - 16*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*t \\ & an(1/2*a*x) - 128*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(\\ & a*x)^2*tan(1/2*a*x) + 256*a^6*x^6*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x) \\ & ^2*tan(1/2*a*x) + 32*a^6*x^6*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2*ta \\ & n(1/2*a*x) + 20*a^6*x^6*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^7*x^7 \\ & *imag_part(cos_integral(4*a*x))*tan(2*a*x)^2 + 4*a^7*x^7*imag_part(cos_inte \end{aligned}$$

$$\begin{aligned}
& \text{gral}(2*a*x))*\tan(2*a*x)^2 - 4*a^7*x^7*\text{imag_part}(\cos_integral(-2*a*x))*\tan(2 \\
& *a*x)^2 - 32*a^7*x^7*\text{imag_part}(\cos_integral(-4*a*x))*\tan(2*a*x)^2 + 64*a^7* \\
& x^7*\sin_integral(4*a*x)*\tan(2*a*x)^2 + 8*a^7*x^7*\sin_integral(2*a*x)*\tan(2* \\
& a*x)^2 + 32*a^7*x^7*\text{imag_part}(\cos_integral(4*a*x))*\tan(a*x)^2 + 4*a^7*x^7*i \\
& \text{mag_part}(\cos_integral(2*a*x))*\tan(a*x)^2 - 4*a^7*x^7*\text{imag_part}(\cos_integral \\
& (-2*a*x))*\tan(a*x)^2 - 32*a^7*x^7*\text{imag_part}(\cos_integral(-4*a*x))*\tan(a*x)^ \\
& 2 + 64*a^7*x^7*\sin_integral(4*a*x)*\tan(a*x)^2 + 8*a^7*x^7*\sin_integral(2*a* \\
& x)*\tan(a*x)^2 - 24*a^7*x^7*\tan(2*a*x)^2*\tan(1/2*a*x) + 24*a^7*x^7*\tan(a*x)^ \\
& 2*\tan(1/2*a*x) - 32*a^7*x^7*\text{imag_part}(\cos_integral(4*a*x))*\tan(1/2*a*x)^2 - \\
& 4*a^7*x^7*\text{imag_part}(\cos_integral(2*a*x))*\tan(1/2*a*x)^2 + 4*a^7*x^7*\text{imag_p} \\
& \text{art}(\cos_integral(-2*a*x))*\tan(1/2*a*x)^2 + 32*a^7*x^7*\text{imag_part}(\cos_integra \\
& l(-4*a*x))*\tan(1/2*a*x)^2 - 64*a^7*x^7*\sin_integral(4*a*x)*\tan(1/2*a*x)^2 - \\
& 8*a^7*x^7*\sin_integral(2*a*x)*\tan(1/2*a*x)^2 - 64*a^5*x^5*\text{imag_part}(\cos_in \\
& tegral(4*a*x))*\tan(2*a*x)^2*\tan(a*x)^2*\tan(1/2*a*x)^2 - 8*a^5*x^5*\text{imag_part} \\
& (\cos_integral(2*a*x))*\tan(2*a*x)^2*\tan(a*x)^2*\tan(1/2*a*x)^2 + 8*a^5*x^5*\text{im} \\
& \text{ag_part}(\cos_integral(-2*a*x))*\tan(2*a*x)^2*\tan(a*x)^2*\tan(1/2*a*x)^2 + 64*a \\
& ^5*x^5*\text{imag_part}(\cos_integral(-4*a*x))*\tan(2*a*x)^2*\tan(a*x)^2*\tan(1/2*a*x) \\
& ^2 - 128*a^5*x^5*\sin_integral(4*a*x)*\tan(2*a*x)^2*\tan(a*x)^2*\tan(1/2*a*x)^2 \\
& - 16*a^5*x^5*\sin_integral(2*a*x)*\tan(2*a*x)^2*\tan(a*x)^2*\tan(1/2*a*x)^2 - \\
& 20*a^6*x^6*\tan(2*a*x)^2*\tan(a*x)^2 + 128*a^6*x^6*\text{imag_part}(\cos_integral(4*a \\
& *x))*\tan(2*a*x)^2*\tan(1/2*a*x) + 16*a^6*x^6*\text{imag_part}(\cos_integral(2*a*x))* \\
& \tan(2*a*x)^2*\tan(1/2*a*x) - 16*a^6*x^6*\text{imag_part}(\cos_integral(-2*a*x))*\tan(\\
& 2*a*x)^2*\tan(1/2*a*x) - 128*a^6*x^6*\text{imag_part}(\cos_integral(-4*a*x))*\tan(2*a \\
& *x)^2*\tan(1/2*a*x) + 256*a^6*x^6*\sin_integral(4*a*x)*\tan(2*a*x)^2*\tan(1/2*a \\
& *x) + 32*a^6*x^6*\sin_integral(2*a*x)*\tan(2*a*x)^2*\tan(1/2*a*x) + 8*a^6*x^6* \\
& \tan(2*a*x)^2*\tan(a*x)*\tan(1/2*a*x) + 128*a^6*x^6*\text{imag_part}(\cos_integral(4*a \\
& *x))*\tan(a*x)^2*\tan(1/2*a*x) + 16*a^6*x^6*\text{imag_part}(\cos_integral(2*a*x))*\text{ta} \\
& \text{n}(a*x)^2*\tan(1/2*a*x) - 16*a^6*x^6*\text{imag_part}(\cos_integral(-2*a*x))*\tan(a*x) \\
& ^2*\tan(1/2*a*x) - 128*a^6*x^6*\text{imag_part}(\cos_integral(-4*a*x))*\tan(a*x)^2*\text{ta} \\
& \text{n}(1/2*a*x) + 256*a^6*x^6*\sin_integral(4*a*x)*\tan(a*x)^2*\tan(1/2*a*x) + 32*a \\
& ^6*x^6*\sin_integral(2*a*x)*\tan(a*x)^2*\tan(1/2*a*x) + 16*a^6*x^6*\tan(2*a*x)* \\
& \tan(a*x)^2*\tan(1/2*a*x) + 12*a^6*x^6*\tan(2*a*x)^2*\tan(1/2*a*x)^2 - 12*a^6*x \\
& ^6*\tan(a*x)^2*\tan(1/2*a*x)^2 + 32*a^7*x^7*\text{imag_part}(\cos_integral(4*a*x)) + \\
& 4*a^7*x^7*\text{imag_part}(\cos_integral(2*a*x)) - 4*a^7*x^7*\text{imag_part}(\cos_integral \\
& (-2*a*x)) - 32*a^7*x^7*\text{imag_part}(\cos_integral(-4*a*x)) + 64*a^7*x^7*\sin_int \\
& egral(4*a*x) + 8*a^7*x^7*\sin_integral(2*a*x) + 64*a^5*x^5*\text{imag_part}(\cos_int \\
& egral(4*a*x))*\tan(2*a*x)^2*\tan(a*x)^2 + 8*a^5*x^5*\text{imag_part}(\cos_integral(2* \\
& a*x))*\tan(2*a*x)^2*\tan(a*x)^2 - 8*a^5*x^5*\text{imag_part}(\cos_integral(-2*a*x))*\text{t} \\
& \text{an}(2*a*x)^2*\tan(a*x)^2 - 64*a^5*x^5*\text{imag_part}(\cos_integral(-4*a*x))*\tan(2*a \\
& *x)^2*\tan(a*x)^2 + 128*a^5*x^5*\sin_integral(4*a*x)*\tan(2*a*x)^2*\tan(a*x)^2 \\
& + 16*a^5*x^5*\sin_integral(2*a*x)*\tan(2*a*x)^2*\tan(a*x)^2 + 40*a^7*x^7*\tan(1 \\
& /2*a*x) - 72*a^5*x^5*\tan(2*a*x)^2*\tan(a*x)^2*\tan(1/2*a*x) - 64*a^5*x^5*\text{imag} \\
& _part(\cos_integral(4*a*x))*\tan(2*a*x)^2*\tan(1/2*a*x)^2 - 8*a^5*x^5*\text{imag_par} \\
& \text{t}(\cos_integral(2*a*x))*\tan(2*a*x)^2*\tan(1/2*a*x)^2 + 8*a^5*x^5*\text{imag_part}(\co \\
& s_integral(-2*a*x))*\tan(2*a*x)^2*\tan(1/2*a*x)^2 + 64*a^5*x^5*\text{imag_part}(\cos \\
& _integral(-4*a*x))*\tan(2*a*x)^2*\tan(1/2*a*x)^2 - 128*a^5*x^5*\sin_integral(4* \\
& a*x)*\tan(2*a*x)^2*\tan(1/2*a*x)^2 - 16*a^5*x^5*\sin_integral(2*a*x)*\tan(2*a*x) \\
&)^2*\tan(1/2*a*x)^2 - 4*a^5*x^5*\tan(2*a*x)^2*\tan(a*x)*\tan(1/2*a*x)^2 - 64*a^ \\
& 5*x^5*\text{imag_part}(\cos_integral(4*a*x))*\tan(a*x)^2*\tan(1/2*a*x)^2 - 8*a^5*x^5* \\
& \text{imag_part}(\cos_integral(2*a*x))*\tan(a*x)^2*\tan(1/2*a*x)^2 + 8*a^5*x^5*\text{imag_p} \\
& \text{art}(\cos_integral(-2*a*x))*\tan(a*x)^2*\tan(1/2*a*x)^2 + 64*a^5*x^5*\text{imag_part} \\
& (\cos_integral(-4*a*x))*\tan(a*x)^2*\tan(1/2*a*x)^2 - 128*a^5*x^5*\sin_integral(\\
& 4*a*x)*\tan(a*x)^2*\tan(1/2*a*x)^2 - 16*a^5*x^5*\sin_integral(2*a*x)*\tan(a*x)^ \\
& 2*\tan(1/2*a*x)^2 - 8*a^5*x^5*\tan(2*a*x)*\tan(a*x)^2*\tan(1/2*a*x)^2 - 12*a^6* \\
& x^6*\tan(2*a*x)^2 + 12*a^6*x^6*\tan(a*x)^2 + 128*a^6*x^6*\text{imag_part}(\cos_integr \\
& al(4*a*x))*\tan(1/2*a*x) + 16*a^6*x^6*\text{imag_part}(\cos_integral(2*a*x))*\tan(1/2 \\
& *a*x) - 16*a^6*x^6*\text{imag_part}(\cos_integral(-2*a*x))*\tan(1/2*a*x) - 128*a^6*x \\
& ^6*\text{imag_part}(\cos_integral(-4*a*x))*\tan(1/2*a*x) + 256*a^6*x^6*\sin_integral(\\
& 4*a*x)*\tan(1/2*a*x) + 32*a^6*x^6*\sin_integral(2*a*x)*\tan(1/2*a*x) + 16*a^6*
\end{aligned}$$

$$\begin{aligned}
& x)^2 - 2a^3x^3 \tan(2ax)^2 \tan(ax) \tan(1/2ax)^2 - 32a^3x^3 \operatorname{imag_part}(\cos_integral(4ax)) \tan(ax)^2 \tan(1/2ax)^2 - 4a^3x^3 \operatorname{imag_part}(\cos_integral(2ax)) \tan(ax)^2 \tan(1/2ax)^2 + 4a^3x^3 \operatorname{imag_part}(\cos_integral(-2ax)) \tan(ax)^2 \tan(1/2ax)^2 + 32a^3x^3 \operatorname{imag_part}(\cos_integral(-4ax)) \tan(ax)^2 \tan(1/2ax)^2 - 64a^3x^3 \sin_integral(4ax) \tan(ax)^2 \tan(1/2ax)^2 - 8a^3x^3 \sin_integral(2ax) \tan(ax)^2 \tan(1/2ax)^2 - 13a^3x^3 \tan(2ax) \tan(ax)^2 \tan(1/2ax)^2 - 24a^4x^4 \tan(2ax)^2 + 24a^4x^4 \tan(ax)^2 + 64a^4x^4 \operatorname{imag_part}(\cos_integral(4ax)) \tan(1/2ax) + 8a^4x^4 \operatorname{imag_part}(\cos_integral(2ax)) \tan(1/2ax) - 8a^4x^4 \operatorname{imag_part}(\cos_integral(-2ax)) \tan(1/2ax) - 64a^4x^4 \operatorname{imag_part}(\cos_integral(-4ax)) \tan(1/2ax) + 128a^4x^4 \sin_integral(4ax) \tan(1/2ax) + 16a^4x^4 \sin_integral(2ax) \tan(1/2ax) + 26a^4x^4 \tan(2ax) \tan(1/2ax) + 4a^4x^4 \tan(ax) \tan(1/2ax) - 36a^4x^4 \tan(1/2ax)^2 + 27a^2x^2 \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax)^2 + 32a^3x^3 \operatorname{imag_part}(\cos_integral(4ax)) \tan(2ax)^2 + 4a^3x^3 \operatorname{imag_part}(\cos_integral(2ax)) \tan(2ax)^2 - 4a^3x^3 \operatorname{imag_part}(\cos_integral(-2ax)) \tan(2ax)^2 - 32a^3x^3 \operatorname{imag_part}(\cos_integral(-4ax)) \tan(2ax)^2 + 64a^3x^3 \sin_integral(4ax) \tan(2ax)^2 + 8a^3x^3 \sin_integral(2ax) \tan(2ax)^2 + 2a^3x^3 \tan(2ax)^2 \tan(ax) + 32a^3x^3 \operatorname{imag_part}(\cos_integral(4ax)) \tan(ax)^2 + 4a^3x^3 \operatorname{imag_part}(\cos_integral(2ax)) \tan(ax)^2 - 4a^3x^3 \operatorname{imag_part}(\cos_integral(-2ax)) \tan(ax)^2 - 32a^3x^3 \operatorname{imag_part}(\cos_integral(-4ax)) \tan(ax)^2 + 64a^3x^3 \sin_integral(4ax) \tan(ax)^2 + 8a^3x^3 \sin_integral(2ax) \tan(ax)^2 + 13a^3x^3 \tan(2ax) \tan(ax)^2 - 6a^3x^3 \tan(2ax)^2 \tan(1/2ax) + 24a^3x^3 \tan(ax)^2 \tan(1/2ax) - 32a^3x^3 \operatorname{imag_part}(\cos_integral(4ax)) \tan(1/2ax)^2 - 4a^3x^3 \operatorname{imag_part}(\cos_integral(2ax)) \tan(1/2ax)^2 + 4a^3x^3 \operatorname{imag_part}(\cos_integral(-2ax)) \tan(1/2ax)^2 + 32a^3x^3 \operatorname{imag_part}(\cos_integral(-4ax)) \tan(1/2ax)^2 - 64a^3x^3 \sin_integral(4ax) \tan(1/2ax)^2 - 8a^3x^3 \sin_integral(2ax) \tan(1/2ax)^2 - 13a^3x^3 \tan(2ax) \tan(1/2ax)^2 - 2a^3x^3 \tan(ax) \tan(1/2ax)^2 + 36a^4x^4 - 27a^2x^2 \tan(2ax)^2 \tan(ax)^2 + 20a^2x^2 \tan(2ax)^2 \tan(ax) \tan(1/2ax) + 10a^2x^2 \tan(2ax) \tan(ax)^2 \tan(1/2ax) + 15a^2x^2 \tan(2ax)^2 \tan(1/2ax)^2 + 32a^3x^3 \operatorname{imag_part}(\cos_integral(4ax)) + 4a^3x^3 \operatorname{imag_part}(\cos_integral(2ax)) - 4a^3x^3 \operatorname{imag_part}(\cos_integral(-2ax)) - 32a^3x^3 \operatorname{imag_part}(\cos_integral(-4ax)) + 64a^3x^3 \sin_integral(4ax) + 8a^3x^3 \sin_integral(2ax) + 13a^3x^3 \tan(2ax) + 2a^3x^3 \tan(ax) + 48a^3x^3 \tan(1/2ax) + 2ax \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax) - 10ax \tan(2ax)^2 \tan(ax) \tan(1/2ax)^2 - 5ax \tan(2ax) \tan(ax)^2 \tan(1/2ax)^2 - 15a^2x^2 \tan(2ax)^2 + 10a^2x^2 \tan(2ax) \tan(1/2ax) + 20a^2x^2 \tan(ax) \tan(1/2ax) - 12a^2x^2 \tan(1/2ax)^2 - \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax)^2 + 10ax \tan(2ax)^2 \tan(ax) + 5ax \tan(2ax) \tan(ax)^2 - 6ax \tan(2ax)^2 \tan(1/2ax) - 5ax \tan(2ax) \tan(1/2ax)^2 - 10ax \tan(ax) \tan(1/2ax)^2 + 12a^2x^2 + \tan(2ax)^2 \tan(ax)^2 + 3 \tan(2ax)^2 \tan(1/2ax)^2 + 5ax \tan(2ax) + 10ax \tan(ax) - 8ax \tan(1/2ax) - 3 \tan(2ax)^2 + 4 \tan(1/2ax)^2 - 4) / (2a^5x^8 \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax) - a^4x^7 \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax)^2 + 2a^5x^8 \tan(2ax)^2 \tan(1/2ax) + 2a^5x^8 \tan(ax)^2 \tan(1/2ax) + a^4x^7 \tan(2ax)^2 \tan(ax)^2 - a^4x^7 \tan(2ax)^2 \tan(1/2ax)^2 - a^4x^7 \tan(ax)^2 \tan(1/2ax)^2 + 2a^5x^8 \tan(1/2ax) + 4a^3x^6 \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax) + a^4x^7 \tan(2ax)^2 + a^4x^7 \tan(ax)^2 - a^4x^7 \tan(1/2ax)^2 - 2a^2x^5 \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax)^2 + 4a^3x^6 \tan(2ax)^2 \tan(1/2ax)^2 - 2a^2x^5 \tan(ax)^2 \tan(1/2ax)^2 + 4a^3x^6 \tan(1/2ax) + 2ax^4 \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax) + 2a^2x^5 \tan(2ax)^2 + 2a^2x^5 \tan(ax)^2 - 2a^2x^5 \tan(1/2ax)^2 - x^3 \tan(2ax)^2 \tan(ax)^2 \tan(1/2ax)^2 + 2ax^4 \tan(2ax)^2 \tan(1/2ax) + 2ax^4 \tan(ax)^2 \tan(1/2ax) + 2a^2x^5 + x^3 \tan(2ax)^2 \tan(ax)^2 - x^3 \tan(2ax)^2 \tan(1/2ax)^2 - x^3 \tan(ax)^2 \tan(1/2ax)^2 + 2ax^4 \tan(1/2ax) + x^3 \tan(2ax)^2 + x^3 \tan(ax)^2
\end{aligned}$$

$n(ax)^2 - x^3 \tan(1/2 * ax)^2 + x^3$

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(ax)}{x^4 (\cos(ax) + ax \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x)

[Out] int(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(ax)^6}{x^4 (\cos(ax) + ax \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a*x)^6/(x^4*(cos(a*x) + a*x*sin(a*x))^2),x)

[Out] int(cos(a*x)^6/(x^4*(cos(a*x) + a*x*sin(a*x))^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(ax)}{x^4 (ax \sin(ax) + \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)**6/x**4/(cos(a*x)+a*x*sin(a*x))**2,x)

[Out] Integral(cos(a*x)**6/(x**4*(a*x*sin(a*x) + cos(a*x))**2), x)

$$3.595 \quad \int \frac{\cos^5(ax)}{x^3(\cos(ax)+ax \sin(ax))^2} dx$$

Optimal. Leaf size=132

$$-\frac{1}{8}a^2\text{Ci}(ax)-\frac{27}{8}a^2\text{Ci}(3ax)+\frac{\cos^3(ax)}{a^2x^4}-\frac{\cos^4(ax)}{a^2x^4(ax \sin(ax)+\cos(ax))}-\frac{\sin(ax)\cos^2(ax)}{ax^3}-\frac{3\cos^3(ax)}{2x^2}+\frac{\cos(ax)}{x^2}-\frac{a}{x}$$

[Out] $-1/8*a^2*Ci(a*x)-27/8*a^2*Ci(3*a*x)+\cos(a*x)/x^2+\cos(a*x)^3/a^2/x^4-3/2*\cos(a*x)^3/x^2-a*\sin(a*x)/x-\cos(a*x)^2*\sin(a*x)/a/x^3+9/2*a*\cos(a*x)^2*\sin(a*x)/x-\cos(a*x)^4/a^2/x^4/(\cos(a*x)+a*x*\sin(a*x))$

Rubi [A] time = 0.23, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4599, 3314, 3297, 3302, 3312}

$$-\frac{1}{8}a^2\text{CosIntegral}(ax)-\frac{27}{8}a^2\text{CosIntegral}(3ax)+\frac{\cos^3(ax)}{a^2x^4}-\frac{\cos^4(ax)}{a^2x^4(ax \sin(ax)+\cos(ax))}-\frac{3\cos^3(ax)}{2x^2}+\frac{\cos(ax)}{x^2}-\frac{a}{x}$$

Antiderivative was successfully verified.

[In] Int[Cos[a*x]^5/(x^3*(Cos[a*x] + a*x*Sin[a*x])^2), x]

[Out] $\text{Cos}[a*x]/x^2 + \text{Cos}[a*x]^3/(a^2*x^4) - (3*\text{Cos}[a*x]^3)/(2*x^2) - (a^2*\text{CosIntegral}[a*x])/8 - (27*a^2*\text{CosIntegral}[3*a*x])/8 - (a*\text{Sin}[a*x])/x - (\text{Cos}[a*x]^2*\text{Sin}[a*x])/(a*x^3) + (9*a*\text{Cos}[a*x]^2*\text{Sin}[a*x])/(2*x) - \text{Cos}[a*x]^4/(a^2*x^4*(\text{Cos}[a*x] + a*x*\text{Sin}[a*x]))$

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 3312

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 3314

Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

Rule 4599

Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)*(x_)*Sin[(a_.)*(x_)]^2, x_Symbol] := -Simp[(b*(b*x)^(m - 1)*Cos[a*x]^(n -

1))/(a*d*(c*cos[a*x] + d*x*sin[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b*x)^(m - 2)*cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a*c - d, 0] && EqQ[m, 2 - n]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^5(ax)}{x^3(\cos(ax) + ax \sin(ax))^2} dx &= -\frac{\cos^4(ax)}{a^2 x^4 (\cos(ax) + ax \sin(ax))} - \frac{4 \int \frac{\cos^3(ax)}{x^5} dx}{a^2} \\
 &= \frac{\cos^3(ax)}{a^2 x^4} - \frac{\cos^2(ax) \sin(ax)}{ax^3} - \frac{\cos^4(ax)}{a^2 x^4 (\cos(ax) + ax \sin(ax))} - 2 \int \frac{\cos(ax)}{x^3} dx \\
 &= \frac{\cos(ax)}{x^2} + \frac{\cos^3(ax)}{a^2 x^4} - \frac{3 \cos^3(ax)}{2x^2} - \frac{\cos^2(ax) \sin(ax)}{ax^3} + \frac{9a \cos^2(ax) \sin(ax)}{2x} \\
 &= \frac{\cos(ax)}{x^2} + \frac{\cos^3(ax)}{a^2 x^4} - \frac{3 \cos^3(ax)}{2x^2} + 9a^2 \text{Ci}(ax) - \frac{a \sin(ax)}{x} - \frac{\cos^2(ax) \sin(ax)}{ax^3} \\
 &= \frac{\cos(ax)}{x^2} + \frac{\cos^3(ax)}{a^2 x^4} - \frac{3 \cos^3(ax)}{2x^2} + 10a^2 \text{Ci}(ax) - \frac{a \sin(ax)}{x} - \frac{\cos^2(ax) \sin(ax)}{ax^3} \\
 &= \frac{\cos(ax)}{x^2} + \frac{\cos^3(ax)}{a^2 x^4} - \frac{3 \cos^3(ax)}{2x^2} - \frac{1}{8} a^2 \text{Ci}(ax) - \frac{27}{8} a^2 \text{Ci}(3ax) - \frac{a \sin(ax)}{x} - \frac{\cos^2(ax) \sin(ax)}{ax^3}
 \end{aligned}$$

Mathematica [A] time = 0.80, size = 136, normalized size = 1.03

$$\frac{2a^2 x^2 \text{Ci}(ax)(ax \sin(ax) + \cos(ax)) + 54a^2 x^2 \text{Ci}(3ax)(ax \sin(ax) + \cos(ax)) - a^2 x^2 - 8a^2 x^2 \cos(2ax) + 9a^2 x^2 \cos(ax)}{16x^2(ax \sin(ax) + \cos(ax))}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a*x]^5/(x^3*(Cos[a*x] + a*x*sin[a*x])^2), x]

[Out] -1/16*(3 - a^2*x^2 + 4*Cos[2*a*x] - 8*a^2*x^2*Cos[2*a*x] + Cos[4*a*x] + 9*a^2*x^2*Cos[4*a*x] + 2*a^2*x^2*CosIntegral[a*x]*(Cos[a*x] + a*x*sin[a*x]) + 54*a^2*x^2*CosIntegral[3*a*x]*(Cos[a*x] + a*x*sin[a*x]) - 12*a*x*sin[2*a*x] - 6*a*x*sin[4*a*x])/(x^2*(Cos[a*x] + a*x*sin[a*x]))

fricas [A] time = 0.92, size = 185, normalized size = 1.40

$$\frac{88 a^2 x^2 \cos(ax)^2 - 8(9 a^2 x^2 + 1) \cos(ax)^4 - 16 a^2 x^2 - (27 a^2 x^2 \text{Ci}(3ax) + a^2 x^2 \text{Ci}(ax) + a^2 x^2 \text{Ci}(-ax) + 27 a^2 x^2 \text{Ci}(ax))}{16(ax^3 \sin(ax) + \cos(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")

[Out] 1/16*(88*a^2*x^2*cos(a*x)^2 - 8*(9*a^2*x^2 + 1)*cos(a*x)^4 - 16*a^2*x^2 - (27*a^2*x^2*cos_integral(3*a*x) + a^2*x^2*cos_integral(a*x) + a^2*x^2*cos_integral(-a*x) + 27*a^2*x^2*cos_integral(-3*a*x))*cos(a*x) - (27*a^3*x^3*cos_integral(3*a*x) + a^3*x^3*cos_integral(a*x) + a^3*x^3*cos_integral(-a*x) + 27*a^3*x^3*cos_integral(-3*a*x) - 48*a*x*cos(a*x)^3)*sin(a*x))/(a*x^3*sin(a*x) + x^2*cos(a*x))

giac [C] time = 0.69, size = 3130, normalized size = 23.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/16*(54*a^7*x^7*\text{real_part}(\cos_integral(3*a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x) \\
&)^3 + 2*a^7*x^7*\text{real_part}(\cos_integral(a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^3 \\
& + 2*a^7*x^7*\text{real_part}(\cos_integral(-a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^3 + 5 \\
& 4*a^7*x^7*\text{real_part}(\cos_integral(-3*a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^3 - 2 \\
& 7*a^6*x^6*\text{real_part}(\cos_integral(3*a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^4 - a^6 \\
& *x^6*\text{real_part}(\cos_integral(a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^4 - a^6*x^6* \\
& \text{real_part}(\cos_integral(-a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^4 - 27*a^6*x^6*\text{re} \\
& \text{al_part}(\cos_integral(-3*a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^4 + 54*a^7*x^7*\text{re} \\
& \text{al_part}(\cos_integral(3*a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x) + 2*a^7*x^7*\text{real_p} \\
& \text{art}(\cos_integral(a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x) + 2*a^7*x^7*\text{real_part}(\text{co} \\
& \text{s_integral}(-a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x) + 54*a^7*x^7*\text{real_part}(\cos_in \\
& \text{tegral}(-3*a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x) + 54*a^7*x^7*\text{real_part}(\cos_inte \\
& \text{gral}(3*a*x))*\tan(1/2*a*x)^3 + 2*a^7*x^7*\text{real_part}(\cos_integral(a*x))*\tan(1/ \\
& 2*a*x)^3 + 2*a^7*x^7*\text{real_part}(\cos_integral(-a*x))*\tan(1/2*a*x)^3 + 54*a^7* \\
& x^7*\text{real_part}(\cos_integral(-3*a*x))*\tan(1/2*a*x)^3 - 27*a^6*x^6*\text{real_part}(c \\
& \text{os_integral}(3*a*x))*\tan(1/2*a*x)^4 - a^6*x^6*\text{real_part}(\cos_integral(a*x))*\text{t} \\
& \text{an}(1/2*a*x)^4 - a^6*x^6*\text{real_part}(\cos_integral(-a*x))*\tan(1/2*a*x)^4 - 27*a \\
& ^6*x^6*\text{real_part}(\cos_integral(-3*a*x))*\tan(1/2*a*x)^4 + 54*a^7*x^7*\text{real_par} \\
& \text{t}(\cos_integral(3*a*x))*\tan(1/2*a*x) + 2*a^7*x^7*\text{real_part}(\cos_integral(a*x) \\
&)*\tan(1/2*a*x) + 2*a^7*x^7*\text{real_part}(\cos_integral(-a*x))*\tan(1/2*a*x) + 54* \\
& a^7*x^7*\text{real_part}(\cos_integral(-3*a*x))*\tan(1/2*a*x) - 8*a^6*x^6*\tan(3/2*a* \\
& x)^2*\tan(1/2*a*x)^2 - 72*a^6*x^6*\tan(3/2*a*x)*\tan(1/2*a*x)^3 + 108*a^5*x^5* \\
& \text{real_part}(\cos_integral(3*a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^3 + 4*a^5*x^5*\text{re} \\
& \text{al_part}(\cos_integral(a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^3 + 4*a^5*x^5*\text{real_p} \\
& \text{art}(\cos_integral(-a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^3 + 108*a^5*x^5*\text{real_pa} \\
& \text{rt}(\cos_integral(-3*a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^3 + 27*a^6*x^6*\text{real_pa} \\
& \text{rt}(\cos_integral(3*a*x))*\tan(3/2*a*x)^2 + a^6*x^6*\text{real_part}(\cos_integral(a*x \\
&))*\tan(3/2*a*x)^2 + a^6*x^6*\text{real_part}(\cos_integral(-a*x))*\tan(3/2*a*x)^2 + \\
& 27*a^6*x^6*\text{real_part}(\cos_integral(-3*a*x))*\tan(3/2*a*x)^2 - 12*a^5*x^5*\tan(\\
& 3/2*a*x)^2*\tan(1/2*a*x)^3 + 36*a^5*x^5*\tan(3/2*a*x)*\tan(1/2*a*x)^4 - 54*a^4 \\
& *x^4*\text{real_part}(\cos_integral(3*a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^4 - 2*a^4*x \\
& ^4*\text{real_part}(\cos_integral(a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^4 - 2*a^4*x^4*\text{r} \\
& \text{eal_part}(\cos_integral(-a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^4 - 54*a^4*x^4*\text{rea} \\
& \text{l_part}(\cos_integral(-3*a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^4 - 72*a^6*x^6*\tan \\
& (3/2*a*x)*\tan(1/2*a*x) + 108*a^5*x^5*\text{real_part}(\cos_integral(3*a*x))*\tan(3/2 \\
& *a*x)^2*\tan(1/2*a*x) + 4*a^5*x^5*\text{real_part}(\cos_integral(a*x))*\tan(3/2*a*x)^ \\
& 2*\tan(1/2*a*x) + 4*a^5*x^5*\text{real_part}(\cos_integral(-a*x))*\tan(3/2*a*x)^2*\tan \\
& (1/2*a*x) + 108*a^5*x^5*\text{real_part}(\cos_integral(-3*a*x))*\tan(3/2*a*x)^2*\tan \\
& (1/2*a*x) - 8*a^6*x^6*\tan(1/2*a*x)^2 + 108*a^5*x^5*\text{real_part}(\cos_integral(3* \\
& a*x))*\tan(1/2*a*x)^3 + 4*a^5*x^5*\text{real_part}(\cos_integral(a*x))*\tan(1/2*a*x)^ \\
& 3 + 4*a^5*x^5*\text{real_part}(\cos_integral(-a*x))*\tan(1/2*a*x)^3 + 108*a^5*x^5*\text{re} \\
& \text{al_part}(\cos_integral(-3*a*x))*\tan(1/2*a*x)^3 + 8*a^4*x^4*\tan(3/2*a*x)^2*\tan \\
& (1/2*a*x)^4 + 27*a^6*x^6*\text{real_part}(\cos_integral(3*a*x)) + a^6*x^6*\text{real_part} \\
& (\cos_integral(a*x)) + a^6*x^6*\text{real_part}(\cos_integral(-a*x)) + 27*a^6*x^6*\text{re} \\
& \text{al_part}(\cos_integral(-3*a*x)) - 12*a^5*x^5*\tan(3/2*a*x)^2*\tan(1/2*a*x) + 12 \\
& *a^5*x^5*\tan(1/2*a*x)^3 - 54*a^4*x^4*\text{real_part}(\cos_integral(3*a*x))*\tan(1/2 \\
& *a*x)^4 - 2*a^4*x^4*\text{real_part}(\cos_integral(a*x))*\tan(1/2*a*x)^4 - 2*a^4*x^4 \\
& *\text{real_part}(\cos_integral(-a*x))*\tan(1/2*a*x)^4 - 54*a^4*x^4*\text{real_part}(\cos_in \\
& \text{tegral}(-3*a*x))*\tan(1/2*a*x)^4 + 108*a^5*x^5*\text{real_part}(\cos_integral(3*a*x)) \\
& *\tan(1/2*a*x) + 4*a^5*x^5*\text{real_part}(\cos_integral(a*x))*\tan(1/2*a*x) + 4*a^5 \\
& *x^5*\text{real_part}(\cos_integral(-a*x))*\tan(1/2*a*x) + 108*a^5*x^5*\text{real_part}(\cos \\
& _integral(-3*a*x))*\tan(1/2*a*x) - 4*a^4*x^4*\tan(3/2*a*x)^2*\tan(1/2*a*x)^2 - \\
& 128*a^4*x^4*\tan(3/2*a*x)*\tan(1/2*a*x)^3 + 54*a^3*x^3*\text{real_part}(\cos_integra \\
& \text{l}(3*a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^3 + 2*a^3*x^3*\text{real_part}(\cos_integral(\\
& a*x))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^3 + 2*a^3*x^3*\text{real_part}(\cos_integral(-a*x \\
&))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^3 + 54*a^3*x^3*\text{real_part}(\cos_integral(-3*a*x \\
&))*\tan(3/2*a*x)^2*\tan(1/2*a*x)^3 - 4*a^4*x^4*\tan(1/2*a*x)^4 - 36*a^5*x^5*\text{ta} \\
& \text{n}(3/2*a*x) + 54*a^4*x^4*\text{real_part}(\cos_integral(3*a*x))*\tan(3/2*a*x)^2 + 2*a \\
& ^4*x^4*\text{real_part}(\cos_integral(a*x))*\tan(3/2*a*x)^2 + 2*a^4*x^4*\text{real_part}(\text{co}
\end{aligned}$$

```

s_integral(-a*x))*tan(3/2*a*x)^2 + 54*a^4*x^4*real_part(cos_integral(-3*a*x
))*tan(3/2*a*x)^2 + 12*a^5*x^5*tan(1/2*a*x) + 64*a^3*x^3*tan(3/2*a*x)*tan(1
/2*a*x)^4 - 27*a^2*x^2*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/
2*a*x)^4 - a^2*x^2*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)
^4 - a^2*x^2*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 -
27*a^2*x^2*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 -
4*a^4*x^4*tan(3/2*a*x)^2 - 128*a^4*x^4*tan(3/2*a*x)*tan(1/2*a*x) + 54*a^3*x
^3*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^3*x^3*r
eal_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^3*x^3*real_pa
rt(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 54*a^3*x^3*real_part(c
os_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) - 4*a^4*x^4*tan(1/2*a*x)^2
+ 54*a^3*x^3*real_part(cos_integral(3*a*x))*tan(1/2*a*x)^3 + 2*a^3*x^3*rea
l_part(cos_integral(a*x))*tan(1/2*a*x)^3 + 2*a^3*x^3*real_part(cos_integral
(-a*x))*tan(1/2*a*x)^3 + 54*a^3*x^3*real_part(cos_integral(-3*a*x))*tan(1/2
*a*x)^3 + 32*a^2*x^2*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 54*a^4*x^4*real_part(c
os_integral(3*a*x)) + 2*a^4*x^4*real_part(cos_integral(a*x)) + 2*a^4*x^4*re
al_part(cos_integral(-a*x)) + 54*a^4*x^4*real_part(cos_integral(-3*a*x)) -
32*a^3*x^3*tan(3/2*a*x)^2*tan(1/2*a*x) + 32*a^3*x^3*tan(1/2*a*x)^3 - 27*a^2
*x^2*real_part(cos_integral(3*a*x))*tan(1/2*a*x)^4 - a^2*x^2*real_part(cos_
integral(a*x))*tan(1/2*a*x)^4 - a^2*x^2*real_part(cos_integral(-a*x))*tan(1
/2*a*x)^4 - 27*a^2*x^2*real_part(cos_integral(-3*a*x))*tan(1/2*a*x)^4 + 8*a
^4*x^4 + 54*a^3*x^3*real_part(cos_integral(3*a*x))*tan(1/2*a*x) + 2*a^3*x^3
*real_part(cos_integral(a*x))*tan(1/2*a*x) + 2*a^3*x^3*real_part(cos_integr
al(-a*x))*tan(1/2*a*x) + 54*a^3*x^3*real_part(cos_integral(-3*a*x))*tan(1/2
*a*x) + 24*a^2*x^2*tan(3/2*a*x)^2*tan(1/2*a*x)^2 - 56*a^2*x^2*tan(3/2*a*x)*
tan(1/2*a*x)^3 + 16*a^2*x^2*tan(1/2*a*x)^4 - 64*a^3*x^3*tan(3/2*a*x) + 27*a
^2*x^2*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + a^2*x^2*real_part(co
s_integral(a*x))*tan(3/2*a*x)^2 + a^2*x^2*real_part(cos_integral(-a*x))*tan
(3/2*a*x)^2 + 27*a^2*x^2*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2 + 1
2*a*x*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 28*a*x*tan(3/2*a*x)*tan(1/2*a*x)^4 +
16*a^2*x^2*tan(3/2*a*x)^2 - 56*a^2*x^2*tan(3/2*a*x)*tan(1/2*a*x) + 24*a^2*x
^2*tan(1/2*a*x)^2 + 8*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 27*a^2*x^2*real_part(
cos_integral(3*a*x)) + a^2*x^2*real_part(cos_integral(a*x)) + a^2*x^2*real_
part(cos_integral(-a*x)) + 27*a^2*x^2*real_part(cos_integral(-3*a*x)) - 20*
a*x*tan(3/2*a*x)^2*tan(1/2*a*x) + 20*a*x*tan(1/2*a*x)^3 + 32*a^2*x^2 - 12*t
an(3/2*a*x)^2*tan(1/2*a*x)^2 + 4*tan(1/2*a*x)^4 - 28*a*x*tan(3/2*a*x) - 12*
a*x*tan(1/2*a*x) + 4*tan(3/2*a*x)^2 - 12*tan(1/2*a*x)^2 + 8)/(2*a^5*x^7*tan
(3/2*a*x)^2*tan(1/2*a*x)^3 - a^4*x^6*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 2*a^5*
x^7*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^5*x^7*tan(1/2*a*x)^3 - a^4*x^6*tan(1/
2*a*x)^4 + 2*a^5*x^7*tan(1/2*a*x) + 4*a^3*x^5*tan(3/2*a*x)^2*tan(1/2*a*x)^3
+ a^4*x^6*tan(3/2*a*x)^2 - 2*a^2*x^4*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 4*a^3
*x^5*tan(3/2*a*x)^2*tan(1/2*a*x) + 4*a^3*x^5*tan(1/2*a*x)^3 + a^4*x^6 - 2*a
^2*x^4*tan(1/2*a*x)^4 + 4*a^3*x^5*tan(1/2*a*x) + 2*a*x^3*tan(3/2*a*x)^2*tan
(1/2*a*x)^3 + 2*a^2*x^4*tan(3/2*a*x)^2 - x^2*tan(3/2*a*x)^2*tan(1/2*a*x)^4
+ 2*a*x^3*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a*x^3*tan(1/2*a*x)^3 + 2*a^2*x^4
- x^2*tan(1/2*a*x)^4 + 2*a*x^3*tan(1/2*a*x) + x^2*tan(3/2*a*x)^2 + x^2)

```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^5(ax)}{x^3 (\cos(ax) + ax \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x)

[Out] int(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(ax)^5}{x^3 (\cos(ax) + ax \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a*x)^5/(x^3*(cos(a*x) + a*x*sin(a*x))^2), x)

[Out] int(cos(a*x)^5/(x^3*(cos(a*x) + a*x*sin(a*x))^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^5(ax)}{x^3 (ax \sin(ax) + \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)**5/x**3/(cos(a*x)+a*x*sin(a*x))**2,x)

[Out] Integral(cos(a*x)**5/(x**3*(a*x*sin(a*x) + cos(a*x))**2), x)

$$3.596 \quad \int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax \sin(ax))^2} dx$$

Optimal. Leaf size=80

$$\frac{\cos^2(ax)}{a^2x^3} - \frac{\cos^3(ax)}{a^2x^3(ax \sin(ax) + \cos(ax))} - 2a\text{Si}(2ax) - \frac{\sin(ax) \cos(ax)}{ax^2} - \frac{2 \cos^2(ax)}{x} + \frac{1}{x}$$

[Out] $1/x + \cos(a*x)^2/a^2/x^3 - 2*\cos(a*x)^2/x - 2*a*Si(2*a*x) - \cos(a*x)*\sin(a*x)/a/x^2 - \cos(a*x)^3/a^2/x^3/(\cos(a*x)+a*x*\sin(a*x))$

Rubi [A] time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4599, 3314, 30, 3313, 12, 3299}

$$\frac{\cos^2(ax)}{a^2x^3} - \frac{\cos^3(ax)}{a^2x^3(ax \sin(ax) + \cos(ax))} - 2a\text{Si}(2ax) - \frac{\sin(ax) \cos(ax)}{ax^2} - \frac{2 \cos^2(ax)}{x} + \frac{1}{x}$$

Antiderivative was successfully verified.

[In] `Int[Cos[a*x]^4/(x^2*(Cos[a*x] + a*x*Sin[a*x])^2), x]`

[Out] $x^{(-1)} + \text{Cos}[a*x]^2/(a^2*x^3) - (2*\text{Cos}[a*x]^2)/x - (\text{Cos}[a*x]*\text{Sin}[a*x])/(a*x^2) - \text{Cos}[a*x]^3/(a^2*x^3*(\text{Cos}[a*x] + a*x*\text{Sin}[a*x])) - 2*a*\text{SinIntegral}[2*a*x]$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]`

Rule 3299

`Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]`

Rule 3313

`Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Simp[((c + d*x)^(m+1)*Sin[e + f*x]^n)/(d*(m+1)), x] - Dist[(f*n)/(d*(m+1)), Int[ExpandTrigReduce[(c + d*x)^(m+1), Cos[e + f*x]*Sin[e + f*x]^(n-1), x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]`

Rule 3314

`Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Simp[((c + d*x)^(m+1)*(b*Sin[e + f*x])^n)/(d*(m+1)), x] + (Dist[(b^2*f^2*n*(n-1))/(d^2*(m+1)*(m+2)), Int[(c + d*x)^(m+2)*(b*Sin[e + f*x])^(n-2), x], x] - Dist[(f^2*n^2)/(d^2*(m+1)*(m+2)), Int[(c + d*x)^(m+2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m+2)*Cos[e + f*x]*(b*Sin[e + f*x])^(n-1))/(d^2*(m+1)*(m+2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]`

Rule 4599

Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)*(x_)*Sin[(a_.)*(x_)]^2, x_Symbol] :> -Simp[(b*(b*x)^(m - 1)*Cos[a*x]^(n - 1))/(a*d*(c*cos[a*x] + d*x*sin[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a*c - d, 0] && EqQ[m, 2 - n]

Rubi steps

$$\begin{aligned} \int \frac{\cos^4(ax)}{x^2(\cos(ax) + ax \sin(ax))^2} dx &= -\frac{\cos^3(ax)}{a^2 x^3 (\cos(ax) + ax \sin(ax))} - \frac{3 \int \frac{\cos^2(ax)}{x^4} dx}{a^2} \\ &= \frac{\cos^2(ax)}{a^2 x^3} - \frac{\cos(ax) \sin(ax)}{ax^2} - \frac{\cos^3(ax)}{a^2 x^3 (\cos(ax) + ax \sin(ax))} + 2 \int \frac{\cos^2(ax)}{x^2} dx \\ &= \frac{1}{x} + \frac{\cos^2(ax)}{a^2 x^3} - \frac{2 \cos^2(ax)}{x} - \frac{\cos(ax) \sin(ax)}{ax^2} - \frac{\cos^3(ax)}{a^2 x^3 (\cos(ax) + ax \sin(ax))} \\ &= \frac{1}{x} + \frac{\cos^2(ax)}{a^2 x^3} - \frac{2 \cos^2(ax)}{x} - \frac{\cos(ax) \sin(ax)}{ax^2} - \frac{\cos^3(ax)}{a^2 x^3 (\cos(ax) + ax \sin(ax))} \\ &= \frac{1}{x} + \frac{\cos^2(ax)}{a^2 x^3} - \frac{2 \cos^2(ax)}{x} - \frac{\cos(ax) \sin(ax)}{ax^2} - \frac{\cos^3(ax)}{a^2 x^3 (\cos(ax) + ax \sin(ax))} \end{aligned}$$

Mathematica [A] time = 0.68, size = 71, normalized size = 0.89

$$\frac{8ax\text{Si}(2ax)(ax \sin(ax) + \cos(ax)) - 2ax \sin(ax) + 2ax \sin(3ax) + 3 \cos(ax) + \cos(3ax)}{4x(ax \sin(ax) + \cos(ax))}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a*x]^4/(x^2*(Cos[a*x] + a*x*SIN[a*x])^2),x]

[Out] -1/4*(3*cos[a*x] + Cos[3*a*x] - 2*a*x*sin[a*x] + 2*a*x*sin[3*a*x] + 8*a*x*(Cos[a*x] + a*x*sin[a*x])*SinIntegral[2*a*x])/(x*(Cos[a*x] + a*x*sin[a*x]))

fricas [A] time = 0.90, size = 73, normalized size = 0.91

$$\frac{2ax \cos(ax) \text{Si}(2ax) + \cos(ax)^3 + (2a^2x^2 \text{Si}(2ax) + 2ax \cos(ax)^2 - ax) \sin(ax)}{ax^2 \sin(ax) + x \cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")

[Out] -(2*a*x*cos(a*x)*sin_integral(2*a*x) + cos(a*x)^3 + (2*a^2*x^2*sin_integral(2*a*x) + 2*a*x*cos(a*x)^2 - a*x)*sin(a*x))/(a*x^2*sin(a*x) + x*cos(a*x))

giac [C] time = 0.49, size = 997, normalized size = 12.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")

[Out] -(2*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a^4*x^4*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x) - a^3*x^3*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 2*a^3*x^3*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2)

```

2*a*x)^2 + 2*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) - 2*a^4*x^
4*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a^4*x^4*sin_integral(2*a
*x)*tan(1/2*a*x) + a^3*x^3*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - a^3*x
^3*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 + 2*a^3*x^3*sin_integral(2*a
*x)*tan(a*x)^2 - 2*a^3*x^3*tan(a*x)^2*tan(1/2*a*x) - a^3*x^3*imag_part(cos_
integral(2*a*x))*tan(1/2*a*x)^2 + a^3*x^3*imag_part(cos_integral(-2*a*x))*t
an(1/2*a*x)^2 - 2*a^3*x^3*sin_integral(2*a*x)*tan(1/2*a*x)^2 + 2*a^2*x^2*im
ag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a^2*x^2*imag_part(
cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a^2*x^2*sin_integral(2*a*
x)*tan(a*x)^2*tan(1/2*a*x) + a^2*x^2*tan(a*x)^2*tan(1/2*a*x)^2 + a^3*x^3*im
ag_part(cos_integral(2*a*x)) - a^3*x^3*imag_part(cos_integral(-2*a*x)) + 2*
a^3*x^3*sin_integral(2*a*x) + 2*a^3*x^3*tan(1/2*a*x) - a*x*imag_part(cos_in
tegral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + a*x*imag_part(cos_integral(-2*a*
x))*tan(a*x)^2*tan(1/2*a*x)^2 - 2*a*x*sin_integral(2*a*x)*tan(a*x)^2*tan(1/
2*a*x)^2 - a^2*x^2*tan(a*x)^2 + 2*a^2*x^2*imag_part(cos_integral(2*a*x))*ta
n(1/2*a*x) - 2*a^2*x^2*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a^2
*x^2*sin_integral(2*a*x)*tan(1/2*a*x) + 2*a^2*x^2*tan(a*x)*tan(1/2*a*x) - a
^2*x^2*tan(1/2*a*x)^2 + a*x*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - a*x
*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 + 2*a*x*sin_integral(2*a*x)*tan
(a*x)^2 - 2*a*x*tan(a*x)^2*tan(1/2*a*x) - a*x*imag_part(cos_integral(2*a*x)
)*tan(1/2*a*x)^2 + a*x*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x)^2 - 2*a
*x*sin_integral(2*a*x)*tan(1/2*a*x)^2 - a*x*tan(a*x)*tan(1/2*a*x)^2 + a^2*x
^2 + a*x*imag_part(cos_integral(2*a*x)) - a*x*imag_part(cos_integral(-2*a*x
)) + 2*a*x*sin_integral(2*a*x) + a*x*tan(a*x) - tan(1/2*a*x)^2 + 1)/(2*a^3*
x^4*tan(a*x)^2*tan(1/2*a*x) - a^2*x^3*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^3*x^4
*tan(1/2*a*x) + a^2*x^3*tan(a*x)^2 - a^2*x^3*tan(1/2*a*x)^2 + 2*a*x^2*tan(a
*x)^2*tan(1/2*a*x) + a^2*x^3 - x*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a*x^2*tan(1/
2*a*x) + x*tan(a*x)^2 - x*tan(1/2*a*x)^2 + x)

```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(ax)}{x^2 (\cos(ax) + ax \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] int(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(ax)^4}{x^2 (\cos(ax) + ax \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^4/(x^2*(cos(a*x) + a*x*sin(a*x))^2),x)
```

```
[Out] int(cos(a*x)^4/(x^2*(cos(a*x) + a*x*sin(a*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(ax)}{x^2(ax \sin(ax) + \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)**4/x**2/(cos(a*x)+a*x*sin(a*x))**2,x)

[Out] Integral(cos(a*x)**4/(x**2*(a*x*sin(a*x) + cos(a*x))**2), x)

$$3.597 \quad \int \frac{\cos^3(ax)}{x(\cos(ax) + ax \sin(ax))^2} dx$$

Optimal. Leaf size=56

$$\frac{\cos(ax)}{a^2x^2} - \frac{\cos^2(ax)}{a^2x^2(ax \sin(ax) + \cos(ax))} + \text{Ci}(ax) - \frac{\sin(ax)}{ax}$$

[Out] Ci(a*x)+cos(a*x)/a^2/x^2-sin(a*x)/a/x-cos(a*x)^2/a^2/x^2/(cos(a*x)+a*x*sin(a*x))

Rubi [A] time = 0.09, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4599, 3297, 3302}

$$\frac{\cos(ax)}{a^2x^2} - \frac{\cos^2(ax)}{a^2x^2(ax \sin(ax) + \cos(ax))} + \text{CosIntegral}(ax) - \frac{\sin(ax)}{ax}$$

Antiderivative was successfully verified.

[In] Int[Cos[a*x]^3/(x*(Cos[a*x] + a*x*Sin[a*x])^2), x]

[Out] Cos[a*x]/(a^2*x^2) + CosIntegral[a*x] - Sin[a*x]/(a*x) - Cos[a*x]^2/(a^2*x^2*(Cos[a*x] + a*x*Sin[a*x]))

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3302

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]

Rule 4599

Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)*(x_)*Sin[(a_.)*(x_)]^2, x_Symbol] :> -Simp[(b*(b*x)^(m - 1)*Cos[a*x]^(n - 1))/(a*d*(c*cos[a*x] + d*x*sin[a*x])), x] - Dist[(b^2*(n - 1))/d^2, Int[(b*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a*c - d, 0] && EqQ[m, 2 - n]

Rubi steps

$$\begin{aligned} \int \frac{\cos^3(ax)}{x(\cos(ax) + ax \sin(ax))^2} dx &= -\frac{\cos^2(ax)}{a^2x^2(\cos(ax) + ax \sin(ax))} - \frac{2 \int \frac{\cos(ax)}{x^3} dx}{a^2} \\ &= \frac{\cos(ax)}{a^2x^2} - \frac{\cos^2(ax)}{a^2x^2(\cos(ax) + ax \sin(ax))} + \frac{\int \frac{\sin(ax)}{x^2} dx}{a} \\ &= \frac{\cos(ax)}{a^2x^2} - \frac{\sin(ax)}{ax} - \frac{\cos^2(ax)}{a^2x^2(\cos(ax) + ax \sin(ax))} + \int \frac{\cos(ax)}{x} dx \\ &= \frac{\cos(ax)}{a^2x^2} + \text{Ci}(ax) - \frac{\sin(ax)}{ax} - \frac{\cos^2(ax)}{a^2x^2(\cos(ax) + ax \sin(ax))} \end{aligned}$$

Mathematica [C] time = 7.40, size = 237, normalized size = 4.23

$$-e^{ax} \operatorname{Ci}(ax + i) \sin(ax) - e \operatorname{Ci}(ax + i) \cos(ax) + 2 \operatorname{Ci}(ax)(ax \sin(ax) + \cos(ax)) - e \operatorname{Ci}(i - ax)(ax \sin(ax) + \cos(ax))$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[a*x]^3/(x*(Cos[a*x] + a*x*Sin[a*x])^2),x]

[Out] $(-1 + \cos[2ax] - E \cos[ax] \operatorname{CosIntegral}[I + ax] + E \cos[ax] \operatorname{ExpIntegralEi}[-1 - Iax] + E \cos[ax] \operatorname{ExpIntegralEi}[-1 + Iax] - a E x \operatorname{CosIntegral}[I + ax] \sin[ax] + a E x \operatorname{ExpIntegralEi}[-1 - Iax] \sin[ax] + a E x \operatorname{ExpIntegralEi}[-1 + Iax] \sin[ax] + 2 \operatorname{CosIntegral}[ax] (\cos[ax] + ax \sin[ax]) - E \operatorname{CosIntegral}[I - ax] (\cos[ax] + ax \sin[ax]) - I E \cos[ax] \operatorname{SinIntegral}[I - ax] - I a E x \sin[ax] \operatorname{SinIntegral}[I - ax] - I E \cos[ax] \operatorname{SinIntegral}[I + ax] - I a E x \sin[ax] \operatorname{SinIntegral}[I + ax]) / (2 (\cos[ax] + ax \sin[ax]))$

fricas [A] time = 0.95, size = 62, normalized size = 1.11

$$\frac{(\operatorname{Ci}(ax) + \operatorname{Ci}(-ax)) \cos(ax) + 2 \cos(ax)^2 + (ax \operatorname{Ci}(ax) + ax \operatorname{Ci}(-ax)) \sin(ax) - 2}{2(ax \sin(ax) + \cos(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")

[Out] $1/2 * ((\cos_integral(ax) + \cos_integral(-ax)) * \cos(ax) + 2 * \cos(ax)^2 + (ax * \cos_integral(ax) + ax * \cos_integral(-ax)) * \sin(ax) - 2) / (ax * \sin(ax) + \cos(ax))$

giac [C] time = 0.35, size = 366, normalized size = 6.54

$$2 a^3 x^3 \Re(\operatorname{Ci}(ax)) \tan\left(\frac{1}{2} ax\right)^3 + 2 a^3 x^3 \Re(\operatorname{Ci}(-ax)) \tan\left(\frac{1}{2} ax\right)^3 - a^2 x^2 \Re(\operatorname{Ci}(ax)) \tan\left(\frac{1}{2} ax\right)^4 - a^2 x^2 \Re(\operatorname{Ci}(-ax)) \tan\left(\frac{1}{2} ax\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")

[Out] $1/2 * (2a^3 x^3 \operatorname{real_part}(\cos_integral(ax)) \tan(1/2 ax)^3 + 2a^3 x^3 \operatorname{real_part}(\cos_integral(-ax)) \tan(1/2 ax)^3 - a^2 x^2 \operatorname{real_part}(\cos_integral(ax)) \tan(1/2 ax)^4 - a^2 x^2 \operatorname{real_part}(\cos_integral(-ax)) \tan(1/2 ax)^4 + 2a^3 x^3 \operatorname{real_part}(\cos_integral(ax)) \tan(1/2 ax) + 2a^3 x^3 \operatorname{real_part}(\cos_integral(-ax)) \tan(1/2 ax) - 8a^2 x^2 \tan(1/2 ax)^2 + 2ax \operatorname{real_part}(\cos_integral(ax)) \tan(1/2 ax)^3 + 2ax \operatorname{real_part}(\cos_integral(-ax)) \tan(1/2 ax)^3 + a^2 x^2 \operatorname{real_part}(\cos_integral(ax)) + a^2 x^2 \operatorname{real_part}(\cos_integral(-ax)) - \operatorname{real_part}(\cos_integral(ax)) \tan(1/2 ax)^4 - \operatorname{real_part}(\cos_integral(-ax)) \tan(1/2 ax)^4 + 2ax \operatorname{real_part}(\cos_integral(ax)) \tan(1/2 ax) + 2ax \operatorname{real_part}(\cos_integral(-ax)) \tan(1/2 ax) - 2 \tan(1/2 ax)^4 - 12 \tan(1/2 ax)^2 + \operatorname{real_part}(\cos_integral(ax)) + \operatorname{real_part}(\cos_integral(-ax)) - 2) / (2a^3 x^3 \tan(1/2 ax)^3 - a^2 x^2 \tan(1/2 ax)^4 + 2a^3 x^3 \tan(1/2 ax) + 2ax \tan(1/2 ax)^3 + a^2 x^2 - \tan(1/2 ax)^4 + 2ax \tan(1/2 ax) + 1)$

maple [C] time = 5.14, size = 106, normalized size = 1.89

$$\frac{\operatorname{Ei}(1, -iax)}{2} - \frac{e^{iax}}{2(iax - 1)} + \frac{e^{-iax}}{2iax + 2} - \frac{\operatorname{Ei}(1, iax)}{2} - \frac{2ie^{iax}}{(ax + i)(ax - i)(ax e^{2iax} - ax + ie^{2iax} + i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x)`

[Out] `-1/2*Ei(1,-I*a*x)-1/2*exp(I*a*x)/(-1+I*a*x)+1/2*exp(-I*a*x)/(I*a*x+1)-1/2*Ei(1,I*a*x)-2*I*exp(I*a*x)/(a*x+I)/(a*x-I)/(a*x*exp(2*I*a*x)-a*x+I*exp(2*I*a*x)+I)`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")`

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(ax)^3}{x(\cos(ax) + ax \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a*x)^3/(x*(cos(a*x) + a*x*sin(a*x))^2),x)`

[Out] `int(cos(a*x)^3/(x*(cos(a*x) + a*x*sin(a*x))^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(ax)}{x(ax \sin(ax) + \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(a*x)**3/x/(cos(a*x)+a*x*sin(a*x))**2,x)`

[Out] `Integral(cos(a*x)**3/(x*(a*x*sin(a*x) + cos(a*x))**2), x)`

$$3.598 \quad \int \frac{\cos^2(ax)}{(\cos(ax) + ax \sin(ax))^2} dx$$

Optimal. Leaf size=34

$$\frac{1}{a^2x} - \frac{\cos(ax)}{a^2x(ax \sin(ax) + \cos(ax))}$$

[Out] 1/a^2/x-cos(a*x)/a^2/x/(cos(a*x)+a*x*sin(a*x))

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {4597}

$$\frac{1}{a^2x} - \frac{\cos(ax)}{a^2x(ax \sin(ax) + \cos(ax))}$$

Antiderivative was successfully verified.

[In] Int[Cos[a*x]^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]

[Out] 1/(a^2*x) - Cos[a*x]/(a^2*x*(Cos[a*x] + a*x*Sin[a*x]))

Rule 4597

Int[Cos[(a_.)*(x_)]^2/(Cos[(a_.)*(x_)]*(c_.) + (d_.)*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[1/(d^2*x), x] - Simp[Cos[a*x]/(a*d*x*(d*x*Sin[a*x] + c*Cos[a*x])), x] /; FreeQ[{a, c, d}, x] && EqQ[a*c - d, 0]

Rubi steps

$$\int \frac{\cos^2(ax)}{(\cos(ax) + ax \sin(ax))^2} dx = \frac{1}{a^2x} - \frac{\cos(ax)}{a^2x(\cos(ax) + ax \sin(ax))}$$

Mathematica [A] time = 0.21, size = 22, normalized size = 0.65

$$\frac{\sin(ax)}{a(ax \sin(ax) + \cos(ax))}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a*x]^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]

[Out] Sin[a*x]/(a*(Cos[a*x] + a*x*Sin[a*x]))

fricas [A] time = 3.57, size = 23, normalized size = 0.68

$$\frac{\sin(ax)}{a^2x \sin(ax) + a \cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")

[Out] sin(a*x)/(a^2*x*sin(a*x) + a*cos(a*x))

giac [A] time = 0.17, size = 32, normalized size = 0.94

$$\frac{2 \tan\left(\frac{1}{2} ax\right)}{2 a^2 x \tan\left(\frac{1}{2} ax\right) - a \tan\left(\frac{1}{2} ax\right)^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")

[Out] 2*tan(1/2*a*x)/(2*a^2*x*tan(1/2*a*x) - a*tan(1/2*a*x)^2 + a)

maple [B] time = 1.79, size = 70, normalized size = 2.06

$$\frac{\frac{2 \tan\left(\frac{ax}{2}\right)}{a} + \frac{4\left(\tan^3\left(\frac{ax}{2}\right)\right)}{a} + \frac{2\left(\tan^5\left(\frac{ax}{2}\right)\right)}{a}}{\left(1 + \tan^2\left(\frac{ax}{2}\right)\right)^2 \left(2 \tan\left(\frac{ax}{2}\right)xa - \left(\tan^2\left(\frac{ax}{2}\right)\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x)

[Out] (2/a*tan(1/2*a*x)+4/a*tan(1/2*a*x)^3+2/a*tan(1/2*a*x)^5)/(1+tan(1/2*a*x)^2)^2/(2*tan(1/2*a*x)*x*a-tan(1/2*a*x)^2+1)

maxima [B] time = 0.33, size = 114, normalized size = 3.35

$$\frac{ax \cos(2ax)^2 + ax \sin(2ax)^2 - 2ax \cos(2ax) + ax + 2 \sin(2ax)}{(a^2x^2 + (a^2x^2 + 1) \cos(2ax)^2 + 4ax \sin(2ax) + (a^2x^2 + 1) \sin(2ax)^2 - 2(a^2x^2 - 1) \cos(2ax) + 1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")

[Out] (a*x*cos(2*a*x)^2 + a*x*sin(2*a*x)^2 - 2*a*x*cos(2*a*x) + a*x + 2*sin(2*a*x))/((a^2*x^2 + (a^2*x^2 + 1)*cos(2*a*x)^2 + 4*a*x*sin(2*a*x) + (a^2*x^2 + 1)*sin(2*a*x)^2 - 2*(a^2*x^2 - 1)*cos(2*a*x) + 1)*a)

mupad [B] time = 0.16, size = 22, normalized size = 0.65

$$\frac{\sin(ax)}{a(\cos(ax) + ax \sin(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a*x)^2/(cos(a*x) + a*x*sin(a*x))^2,x)

[Out] sin(a*x)/(a*(cos(a*x) + a*x*sin(a*x)))

sympy [A] time = 3.00, size = 20, normalized size = 0.59

$$\frac{\sin(ax)}{a^2x \sin(ax) + a \cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)**2/(cos(a*x)+a*x*sin(a*x))**2,x)

[Out] sin(a*x)/(a**2*x*sin(a*x) + a*cos(a*x))

$$3.599 \quad \int \frac{x \cos(ax)}{(\cos(ax) + ax \sin(ax))^2} dx$$

Optimal. Leaf size=19

$$-\frac{1}{a^2(ax \sin(ax) + \cos(ax))}$$

[Out] -1/a^2/(cos(a*x)+a*x*sin(a*x))

Rubi [A] time = 0.06, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6686}

$$-\frac{1}{a^2(ax \sin(ax) + \cos(ax))}$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]

[Out] -(1/(a^2*(Cos[a*x] + a*x*Sin[a*x])))

Rule 6686

Int[(u_)*(y_)^(m_.), x_Symbol] :=> With[{q = DerivativeDivides[y, u, x]}, Simp[(q*y^(m + 1))/(m + 1), x] /; !FalseQ[q] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{x \cos(ax)}{(\cos(ax) + ax \sin(ax))^2} dx = -\frac{1}{a^2(\cos(ax) + ax \sin(ax))}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$-\frac{1}{a^2(ax \sin(ax) + \cos(ax))}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Cos[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]

[Out] -(1/(a^2*(Cos[a*x] + a*x*Sin[a*x])))

fricas [A] time = 0.86, size = 22, normalized size = 1.16

$$-\frac{1}{a^3x \sin(ax) + a^2 \cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")

[Out] -1/(a^3*x*sin(a*x) + a^2*cos(a*x))

giac [B] time = 0.18, size = 40, normalized size = 2.11

$$\frac{2 \left(\tan \left(\frac{1}{2} ax \right)^2 + 1 \right)}{2 a^3 x \tan \left(\frac{1}{2} ax \right) - a^2 \tan \left(\frac{1}{2} ax \right)^2 + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")

[Out] -2*(tan(1/2*a*x)^2 + 1)/(2*a^3*x*tan(1/2*a*x) - a^2*tan(1/2*a*x)^2 + a^2)

maple [A] time = 0.24, size = 20, normalized size = 1.05

$$-\frac{1}{a^2 (\cos(ax) + ax \sin(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x)

[Out] -1/a^2/(cos(a*x)+a*x*sin(a*x))

maxima [A] time = 0.31, size = 19, normalized size = 1.00

$$-\frac{1}{(ax \sin(ax) + \cos(ax))a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")

[Out] -1/((a*x*sin(a*x) + cos(a*x))*a^2)

mupad [B] time = 0.09, size = 22, normalized size = 1.16

$$-\frac{1}{a^2 \cos(ax) + a^3 x \sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*cos(a*x))/(cos(a*x) + a*x*sin(a*x))^2,x)

[Out] -1/(a^2*cos(a*x) + a^3*x*sin(a*x))

sympy [A] time = 3.01, size = 20, normalized size = 1.05

$$-\frac{1}{a^3 x \sin(ax) + a^2 \cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))**2,x)

[Out] -1/(a**3*x*sin(a*x) + a**2*cos(a*x))

$$3.600 \quad \int \frac{x^2}{(\cos(ax) + ax \sin(ax))^2} dx$$

Optimal. Leaf size=33

$$\frac{\tan(ax)}{a^3} - \frac{x \sec(ax)}{a^2(ax \sin(ax) + \cos(ax))}$$

[Out] $-x*\sec(a*x)/a^2/(\cos(a*x)+a*x*\sin(a*x))+\tan(a*x)/a^3$

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4595, 3767, 8}

$$\frac{\tan(ax)}{a^3} - \frac{x \sec(ax)}{a^2(ax \sin(ax) + \cos(ax))}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]

[Out] $-((x*\text{Sec}[a*x])/(a^2*(\text{Cos}[a*x] + a*x*\text{Sin}[a*x]))) + \text{Tan}[a*x]/a^3$

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 4595

Int[(x_)^2/(Cos[(a_.)*(x_)]*(c_.) + (d_.)*(x_)*Sin[(a_.)*(x_)]^2, x_Symbol] :> -Simp[x/(a*d*Cos[a*x]*(c*Cos[a*x] + d*x*Sin[a*x])), x] + Dist[1/d^2, Int[1/Cos[a*x]^2, x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c - d, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(\cos(ax) + ax \sin(ax))^2} dx &= -\frac{x \sec(ax)}{a^2(\cos(ax) + ax \sin(ax))} + \frac{\int \sec^2(ax) dx}{a^2} \\ &= -\frac{x \sec(ax)}{a^2(\cos(ax) + ax \sin(ax))} - \frac{\text{Subst}(\int 1 dx, x, -\tan(ax))}{a^3} \\ &= -\frac{x \sec(ax)}{a^2(\cos(ax) + ax \sin(ax))} + \frac{\tan(ax)}{a^3} \end{aligned}$$

Mathematica [A] time = 0.41, size = 31, normalized size = 0.94

$$\frac{\sin(ax) - ax \cos(ax)}{a^3(ax \sin(ax) + \cos(ax))}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]

[Out] $(-(a*x*\text{Cos}[a*x]) + \text{Sin}[a*x])/(a^3*(\text{Cos}[a*x] + a*x*\text{Sin}[a*x]))$

fricas [A] time = 1.85, size = 36, normalized size = 1.09

$$\frac{ax \cos(ax) - \sin(ax)}{a^4 x \sin(ax) + a^3 \cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")

[Out] -(a*x*cos(a*x) - sin(a*x))/(a^4*x*sin(a*x) + a^3*cos(a*x))

giac [A] time = 0.15, size = 52, normalized size = 1.58

$$\frac{ax \tan\left(\frac{1}{2} ax\right)^2 - ax + 2 \tan\left(\frac{1}{2} ax\right)}{2 a^4 x \tan\left(\frac{1}{2} ax\right) - a^3 \tan\left(\frac{1}{2} ax\right)^2 + a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")

[Out] (a*x*tan(1/2*a*x)^2 - a*x + 2*tan(1/2*a*x))/(2*a^4*x*tan(1/2*a*x) - a^3*tan(1/2*a*x)^2 + a^3)

maple [A] time = 1.19, size = 53, normalized size = 1.61

$$\frac{\frac{x(\tan^2(\frac{ax}{2}))}{a^2} - \frac{x}{a^2} + \frac{2 \tan(\frac{ax}{2})}{a^3}}{2 \tan\left(\frac{ax}{2}\right) xa - \left(\tan^2\left(\frac{ax}{2}\right)\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(cos(a*x)+a*x*sin(a*x))^2,x)

[Out] (x/a^2*tan(1/2*a*x)^2-x/a^2+2/a^3*tan(1/2*a*x))/(2*tan(1/2*a*x)*x*a-tan(1/2*a*x)^2+1)

maxima [B] time = 0.31, size = 100, normalized size = 3.03

$$\frac{2(2ax \cos(2ax) + (a^2x^2 - 1) \sin(2ax))}{(a^2x^2 + (a^2x^2 + 1) \cos(2ax))^2 + 4ax \sin(2ax) + (a^2x^2 + 1) \sin(2ax)^2 - 2(a^2x^2 - 1) \cos(2ax) + 1} a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")

[Out] -2*(2*a*x*cos(2*a*x) + (a^2*x^2 - 1)*sin(2*a*x))/((a^2*x^2 + (a^2*x^2 + 1)*cos(2*a*x)^2 + 4*a*x*sin(2*a*x) + (a^2*x^2 + 1)*sin(2*a*x)^2 - 2*(a^2*x^2 - 1)*cos(2*a*x) + 1)*a^3)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{(\cos(ax) + ax \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(cos(a*x) + a*x*sin(a*x))^2,x)

[Out] int(x^2/(cos(a*x) + a*x*sin(a*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(ax \sin(ax) + \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Integral(x**2/(a*x*sin(a*x) + cos(a*x))**2, x)
```

$$3.601 \quad \int \frac{x^3 \sec(ax)}{(\cos(ax) + ax \sin(ax))^2} dx$$

Optimal. Leaf size=110

$$\frac{i\text{Li}_2(-ie^{iax})}{a^4} - \frac{i\text{Li}_2(ie^{iax})}{a^4} - \frac{\sec(ax)}{a^4} - \frac{2ix \tan^{-1}(e^{iax})}{a^3} + \frac{x \tan(ax) \sec(ax)}{a^3} - \frac{x^2 \sec^2(ax)}{a^2(ax \sin(ax) + \cos(ax))}$$

[Out] $-2*I*x*\arctan(\exp(I*a*x))/a^3 + I*\text{polylog}(2, -I*\exp(I*a*x))/a^4 - I*\text{polylog}(2, I*\exp(I*a*x))/a^4 - \sec(a*x)/a^4 - x^2*\sec(a*x)^2/a^2 / (\cos(a*x) + a*x*\sin(a*x)) + x*\sec(a*x)*\tan(a*x)/a^3$

Rubi [A] time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4601, 4185, 4181, 2279, 2391}

$$\frac{i\text{PolyLog}(2, -ie^{iax})}{a^4} - \frac{i\text{PolyLog}(2, ie^{iax})}{a^4} - \frac{x^2 \sec^2(ax)}{a^2(ax \sin(ax) + \cos(ax))} - \frac{2ix \tan^{-1}(e^{iax})}{a^3} - \frac{\sec(ax)}{a^4} + \frac{x \tan(ax) \sec(ax)}{a^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^3*\text{Sec}[a*x]) / (\text{Cos}[a*x] + a*x*\text{Sin}[a*x])^2, x]$

[Out] $((-2*I)*x*\text{ArcTan}[E^{(I*a*x)}]) / a^3 + (I*\text{PolyLog}[2, (-I)*E^{(I*a*x)}]) / a^4 - (I*\text{PolyLog}[2, I*E^{(I*a*x)}]) / a^4 - \text{Sec}[a*x] / a^4 - (x^2*\text{Sec}[a*x]^2) / (a^2*(\text{Cos}[a*x] + a*x*\text{Sin}[a*x])) + (x*\text{Sec}[a*x]*\text{Tan}[a*x]) / a^3$

Rule 2279

$\text{Int}[\text{Log}[(a_) + (b_)*((F_)^{((e_)*((c_) + (d_)*(x_)))})^{(n_)}], x_Symbol]$
 $:= \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^{n}], x] /;$ $\text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \ \&\& \ \text{GtQ}[a, 0]$

Rule 2391

$\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)})] / (x_), x_Symbol] := -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)] / n, x] /;$ $\text{FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \text{EqQ}[c*d, 1]$

Rule 4181

$\text{Int}[\text{csc}[(e_) + \text{Pi}*(k_) + (f_)*(x_)] * ((c_) + (d_)*(x_))^{(m_)}, x_Symbol]$
 $:= \text{Simp}[(-2*(c + d*x)^m*\text{ArcTanh}[E^{(I*k*Pi)}*E^{(I*(e + f*x))}]) / f, x] + (-\text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{(I*k*Pi)}*E^{(I*(e + f*x))}], x], x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{(I*k*Pi)}*E^{(I*(e + f*x))}], x], x) /;$ $\text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{IntegerQ}[2*k] \ \&\& \ \text{IGtQ}[m, 0]$

Rule 4185

$\text{Int}[(\text{csc}[(e_) + (f_)*(x_)] * (b_))^{(n_)*((c_) + (d_)*(x_))}, x_Symbol] := -\text{Simp}[(b^2*(c + d*x)*\text{Cot}[e + f*x]*(b*\text{Csc}[e + f*x])^{(n-2)}) / (f*(n-1)), x] + (\text{Dist}[(b^2*(n-2)) / (n-1), \text{Int}[(c + d*x)*(b*\text{Csc}[e + f*x])^{(n-2)}, x], x] - \text{Simp}[(b^2*d*(b*\text{Csc}[e + f*x])^{(n-2)}) / (f^2*(n-1)*(n-2)), x]) /;$ $\text{FreeQ}\{b, c, d, e, f\}, x\} \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{NeQ}[n, 2]$

Rule 4601

$\text{Int}[(b_)*(x_)^{(m_)*\text{Sec}[(a_)*(x_)]^{(n_)} / (\text{Cos}[(a_)*(x_)]*(c_) + (d_)*(x_)*\text{Sin}[(a_)*(x_)]^2, x_Symbol] := -\text{Simp}[(b*(b*x)^{(m-1)}*\text{Sec}[a*x]^{(n+1)}) / (a*d*(c*\text{Cos}[a*x] + d*x*\text{Sin}[a*x])), x] + \text{Dist}[(b^2*(n+1)) / d^2, \text{Int}[(b*x)^{(m-2)}*\text{Sec}[a*x]^{(n+2)}, x], x] /;$ $\text{FreeQ}\{a, b, c, d, m, n\}, x\} \ \&\& \ \text{E}$

qQ[a*c - d, 0] && EqQ[m, n + 2]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3 \sec(ax)}{(\cos(ax) + ax \sin(ax))^2} dx &= -\frac{x^2 \sec^2(ax)}{a^2(\cos(ax) + ax \sin(ax))} + \frac{2 \int x \sec^3(ax) dx}{a^2} \\
 &= -\frac{\sec(ax)}{a^4} - \frac{x^2 \sec^2(ax)}{a^2(\cos(ax) + ax \sin(ax))} + \frac{x \sec(ax) \tan(ax)}{a^3} + \frac{\int x \sec(ax) dx}{a^2} \\
 &= -\frac{2ix \tan^{-1}(e^{iax})}{a^3} - \frac{\sec(ax)}{a^4} - \frac{x^2 \sec^2(ax)}{a^2(\cos(ax) + ax \sin(ax))} + \frac{x \sec(ax) \tan(ax)}{a^3} \\
 &= -\frac{2ix \tan^{-1}(e^{iax})}{a^3} - \frac{\sec(ax)}{a^4} - \frac{x^2 \sec^2(ax)}{a^2(\cos(ax) + ax \sin(ax))} + \frac{x \sec(ax) \tan(ax)}{a^3} \\
 &= -\frac{2ix \tan^{-1}(e^{iax})}{a^3} + \frac{i\text{Li}_2(-ie^{iax})}{a^4} - \frac{i\text{Li}_2(ie^{iax})}{a^4} - \frac{\sec(ax)}{a^4} - \frac{x^2 \sec^2(ax)}{a^2(\cos(ax) + ax \sin(ax))}
 \end{aligned}$$

Mathematica [A] time = 1.11, size = 176, normalized size = 1.60

$$\frac{a^2 x^2 \sec(ax) - a^2 x^2 \log(1 - ie^{iax}) \tan(ax) + a^2 x^2 \log(1 + ie^{iax}) \tan(ax) - i\text{Li}_2(-ie^{iax})(ax \tan(ax) + 1) + i\text{Li}_2(ie^{iax})(ax \tan(ax) + 1)}{a^4(ax \tan(ax) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*Sec[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]

[Out] -((-a*x*Log[1 - I*E^(I*a*x)]) + a*x*Log[1 + I*E^(I*a*x)] + Sec[a*x] + a^2*x^2*Sec[a*x] - a^2*x^2*Log[1 - I*E^(I*a*x)]*Tan[a*x] + a^2*x^2*Log[1 + I*E^(I*a*x)]*Tan[a*x] - I*PolyLog[2, (-I)*E^(I*a*x)]*(1 + a*x*Tan[a*x]) + I*PolyLog[2, I*E^(I*a*x)]*(1 + a*x*Tan[a*x]))/(a^4*(1 + a*x*Tan[a*x]))

fricas [B] time = 0.99, size = 290, normalized size = 2.64

$$\frac{2a^2x^2 - (-iax \sin(ax) - i \cos(ax))\text{Li}_2(i \cos(ax) + \sin(ax)) - (-iax \sin(ax) - i \cos(ax))\text{Li}_2(i \cos(ax) - \sin(ax))}{a^4(ax \tan(ax) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")

[Out] -1/2*(2*a^2*x^2 - (-I*a*x*sin(a*x) - I*cos(a*x))*dilog(I*cos(a*x) + sin(a*x)) - (-I*a*x*sin(a*x) - I*cos(a*x))*dilog(I*cos(a*x) - sin(a*x)) - (I*a*x*sin(a*x) + I*cos(a*x))*dilog(-I*cos(a*x) + sin(a*x)) - (I*a*x*sin(a*x) + I*cos(a*x))*dilog(-I*cos(a*x) - sin(a*x)) - (a^2*x^2*sin(a*x) + a*x*cos(a*x))*log(I*cos(a*x) + sin(a*x) + 1) + (a^2*x^2*sin(a*x) + a*x*cos(a*x))*log(I*cos(a*x) - sin(a*x) + 1) - (a^2*x^2*sin(a*x) + a*x*cos(a*x))*log(-I*cos(a*x) + sin(a*x) + 1) + (a^2*x^2*sin(a*x) + a*x*cos(a*x))*log(-I*cos(a*x) - sin(a*x) + 1) + 2)/(a^5*x*sin(a*x) + a^4*cos(a*x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sec(ax)}{(ax \sin(ax) + \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")

[Out] integrate(x^3*sec(a*x)/(a*x*sin(a*x) + cos(a*x))^2, x)

maple [F] time = 2.54, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sec(ax)}{(\cos(ax) + ax \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x)

[Out] int(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\cos(ax) (\cos(ax) + ax \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(cos(a*x)*(cos(a*x) + a*x*sin(a*x))^2),x)

[Out] int(x^3/(cos(a*x)*(cos(a*x) + a*x*sin(a*x))^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sec(ax)}{(ax \sin(ax) + \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))**2,x)

[Out] Integral(x**3*sec(a*x)/(a*x*sin(a*x) + cos(a*x))**2, x)

$$3.602 \quad \int \frac{x^4 \sec^2(ax)}{(\cos(ax) + ax \sin(ax))^2} dx$$

Optimal. Leaf size=124

$$-\frac{2i\text{Li}_2(-e^{2iax})}{a^5} + \frac{\tan(ax)}{a^5} + \frac{4x \log(1 + e^{2iax})}{a^4} - \frac{x \sec^2(ax)}{a^4} - \frac{2ix^2}{a^3} + \frac{2x^2 \tan(ax)}{a^3} + \frac{x^2 \tan(ax) \sec^2(ax)}{a^3} - \frac{x^3 \sec^3(ax)}{a^2(ax \sin(ax) + \cos(ax))} + \frac{4x \log(1 + e^{2iax})}{a^4}$$

[Out] $-2*I*x^2/a^3+4*x*\ln(1+\exp(2*I*a*x))/a^4-2*I*\text{polylog}(2,-\exp(2*I*a*x))/a^5-x*\sec(a*x)^2/a^4-x^3*\sec(a*x)^3/a^2/(\cos(a*x)+a*x*\sin(a*x))+\tan(a*x)/a^5+2*x^2*\tan(a*x)/a^3+x^2*\sec(a*x)^2*\tan(a*x)/a^3$

Rubi [A] time = 0.18, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4601, 4186, 3767, 8, 4184, 3719, 2190, 2279, 2391}

$$-\frac{2i\text{PolyLog}(2,-e^{2iax})}{a^5} - \frac{2ix^2}{a^3} + \frac{2x^2 \tan(ax)}{a^3} + \frac{x^2 \tan(ax) \sec^2(ax)}{a^3} - \frac{x^3 \sec^3(ax)}{a^2(ax \sin(ax) + \cos(ax))} + \frac{4x \log(1 + e^{2iax})}{a^4}$$

Antiderivative was successfully verified.

[In] Int[(x^4*Sec[a*x]^2)/(Cos[a*x] + a*x*Sin[a*x])^2,x]

[Out] $((-2*I)*x^2)/a^3 + (4*x*\text{Log}[1 + E^{((2*I)*a*x)}])/a^4 - ((2*I)*\text{PolyLog}[2, -E^{((2*I)*a*x)}])/a^5 - (x*\text{Sec}[a*x]^2)/a^4 - (x^3*\text{Sec}[a*x]^3)/(a^2*(\text{Cos}[a*x] + a*x*\text{Sin}[a*x])) + \text{Tan}[a*x]/a^5 + (2*x^2*\text{Tan}[a*x])/a^3 + (x^2*\text{Sec}[a*x]^2*\text{Tan}[a*x])/a^3$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2190

Int[(((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m-1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_)*((F_)^(e_)*((c_) + (d_)*(x_)))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 3719

Int[(((c_) + (d_)*(x_))^(m_)*tan[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(I*(c + d*x)^(m+1))/(d*(m+1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4601

```
Int[(((b_.)*(x_))^(m_.)*Sec[(a_.)*(x_)]^(n_.))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)*(x_)*Sin[(a_.)*(x_)]^2, x_Symbol] := -Simp[(b*(b*x)^(m - 1)*Sec[a*x]^(n + 1))/(a*d*(c*Cos[a*x] + d*x*SIN[a*x])), x] + Dist[(b^2*(n + 1))/d^2, Int[(b*x)^(m - 2)*Sec[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a*c - d, 0] && EqQ[m, n + 2]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^4 \sec^2(ax)}{(\cos(ax) + ax \sin(ax))^2} dx &= -\frac{x^3 \sec^3(ax)}{a^2(\cos(ax) + ax \sin(ax))} + \frac{3 \int x^2 \sec^4(ax) dx}{a^2} \\
 &= -\frac{x \sec^2(ax)}{a^4} - \frac{x^3 \sec^3(ax)}{a^2(\cos(ax) + ax \sin(ax))} + \frac{x^2 \sec^2(ax) \tan(ax)}{a^3} + \frac{\int \sec^2(ax) dx}{a^4} \\
 &= -\frac{x \sec^2(ax)}{a^4} - \frac{x^3 \sec^3(ax)}{a^2(\cos(ax) + ax \sin(ax))} + \frac{2x^2 \tan(ax)}{a^3} + \frac{x^2 \sec^2(ax) \tan(ax)}{a^3} - \frac{1}{a^4} \\
 &= -\frac{2ix^2}{a^3} - \frac{x \sec^2(ax)}{a^4} - \frac{x^3 \sec^3(ax)}{a^2(\cos(ax) + ax \sin(ax))} + \frac{\tan(ax)}{a^5} + \frac{2x^2 \tan(ax)}{a^3} + \frac{x^2 \sec^2(ax) \tan(ax)}{a^3} \\
 &= -\frac{2ix^2}{a^3} + \frac{4x \log(1 + e^{2iax})}{a^4} - \frac{x \sec^2(ax)}{a^4} - \frac{x^3 \sec^3(ax)}{a^2(\cos(ax) + ax \sin(ax))} + \frac{\tan(ax)}{a^5} + \frac{2x^2 \tan(ax)}{a^3} \\
 &= -\frac{2ix^2}{a^3} + \frac{4x \log(1 + e^{2iax})}{a^4} - \frac{x \sec^2(ax)}{a^4} - \frac{x^3 \sec^3(ax)}{a^2(\cos(ax) + ax \sin(ax))} + \frac{\tan(ax)}{a^5} + \frac{2x^2 \tan(ax)}{a^3} \\
 &= -\frac{2ix^2}{a^3} + \frac{4x \log(1 + e^{2iax})}{a^4} - \frac{2i \operatorname{Li}_2(-e^{2iax})}{a^5} - \frac{x \sec^2(ax)}{a^4} - \frac{x^3 \sec^3(ax)}{a^2(\cos(ax) + ax \sin(ax))} + \frac{\tan(ax)}{a^5} + \frac{2x^2 \tan(ax)}{a^3}
 \end{aligned}$$

Mathematica [A] time = 1.07, size = 130, normalized size = 1.05

$$\frac{a^3 x^3 \tan^2(ax) - ax(a^2 x^2 + 2iax - 4 \log(1 + e^{2iax}) + 1) + (-2ia^3 x^3 + 2a^2 x^2 + 4a^2 x^2 \log(1 + e^{2iax}) + 1) \tan(ax) - \frac{1}{a^4}}{a^5(ax \tan(ax) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*Sec[a*x]^2)/(Cos[a*x] + a*x*Sin[a*x])^2,x]

[Out] $(-(a*x*(1 + (2*I)*a*x + a^2*x^2 - 4*\text{Log}[1 + E^((2*I)*a*x)])) + (1 + 2*a^2*x^2 - (2*I)*a^3*x^3 + 4*a^2*x^2*\text{Log}[1 + E^((2*I)*a*x)])*\text{Tan}[a*x] + a^3*x^3*\text{Tan}[a*x]^2 - (2*I)*\text{PolyLog}[2, -E^((2*I)*a*x)]*(1 + a*x*\text{Tan}[a*x]))/(a^5*(1 + a*x*\text{Tan}[a*x]))$

fricas [B] time = 1.88, size = 378, normalized size = 3.05

$$\frac{a^3 x^3 - (2 a^3 x^3 + a x) \cos(ax)^2 + (2 a^2 x^2 + 1) \cos(ax) \sin(ax) + (2i a x \cos(ax) \sin(ax) + 2i \cos(ax)^2) \text{Li}_2(i \cos(ax))}{(1 + a x \tan(ax))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")

[Out] $(a^3*x^3 - (2*a^3*x^3 + a*x)*\cos(a*x)^2 + (2*a^2*x^2 + 1)*\cos(a*x)*\sin(a*x) + (2*I*a*x*\cos(a*x)*\sin(a*x) + 2*I*\cos(a*x)^2)*\text{dilog}(I*\cos(a*x) + \sin(a*x)) + (-2*I*a*x*\cos(a*x)*\sin(a*x) - 2*I*\cos(a*x)^2)*\text{dilog}(I*\cos(a*x) - \sin(a*x)) + (-2*I*a*x*\cos(a*x)*\sin(a*x) - 2*I*\cos(a*x)^2)*\text{dilog}(-I*\cos(a*x) + \sin(a*x)) + (2*I*a*x*\cos(a*x)*\sin(a*x) + 2*I*\cos(a*x)^2)*\text{dilog}(-I*\cos(a*x) - \sin(a*x)) + 2*(a^2*x^2*\cos(a*x)*\sin(a*x) + a*x*\cos(a*x)^2)*\log(I*\cos(a*x) + \sin(a*x) + 1) + 2*(a^2*x^2*\cos(a*x)*\sin(a*x) + a*x*\cos(a*x)^2)*\log(I*\cos(a*x) - \sin(a*x) + 1) + 2*(a^2*x^2*\cos(a*x)*\sin(a*x) + a*x*\cos(a*x)^2)*\log(-I*\cos(a*x) + \sin(a*x) + 1) + 2*(a^2*x^2*\cos(a*x)*\sin(a*x) + a*x*\cos(a*x)^2)*\log(-I*\cos(a*x) - \sin(a*x) + 1))/(a^6*x*\cos(a*x)*\sin(a*x) + a^5*\cos(a*x)^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sec(ax)^2}{(ax \sin(ax) + \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")

[Out] integrate(x^4*sec(a*x)^2/(a*x*sin(a*x) + cos(a*x))^2, x)

maple [A] time = 1.24, size = 141, normalized size = 1.14

$$\frac{2i(-2ia^2x^2e^{2iax} + 2x^3a^3 - 2ia^2x^2 + ax e^{2iax} - ie^{2iax} + ax - i)}{(1 + e^{2iax})(ax e^{2iax} - ax + ie^{2iax} + i)a^5} - \frac{4ix^2}{a^3} + \frac{4x \ln(1 + e^{2iax})}{a^4} - \frac{2i \text{polylog}(2, -e^{2iax})}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x)

[Out] $-2*I*(-2*I*a^2*x^2*\exp(2*I*a*x) + 2*x^3*a^3 - 2*I*a^2*x^2 + a*x*\exp(2*I*a*x) - I*\exp(2*I*a*x) + a*x - I)/(1 + \exp(2*I*a*x))/(a*x*\exp(2*I*a*x) - a*x + I*\exp(2*I*a*x) + I)/a^5 - 4*I/a^3*x^2 + 4*x*\ln(1 + \exp(2*I*a*x))/a^4 - 2*I*\text{polylog}(2, -\exp(2*I*a*x))/a^5$

maxima [B] time = 0.46, size = 381, normalized size = 3.07

$$\frac{2ax + (4a^2x^2 - 8iax \cos(2ax) + 8ax \sin(2ax) - 4iax - (4a^2x^2 + 4iax) \cos(4ax) + 4(-ia^2x^2 + ax) \sin(4ax))}{(1 + e^{2iax})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")

[Out] $-(2*a*x + (4*a^2*x^2 - 8*I*a*x*\cos(2*a*x) + 8*a*x*\sin(2*a*x) - 4*I*a*x - (4*a^2*x^2 + 4*I*a*x)*\cos(4*a*x) + 4*(-I*a^2*x^2 + a*x)*\sin(4*a*x))*\arctan2(\sin(2*a*x), \cos(2*a*x)))/a^5$

```

in(2*a*x), cos(2*a*x) + 1) + 4*(a^3*x^3 + I*a^2*x^2)*cos(4*a*x) - (-4*I*a^2
*x^2 - 2*a*x + 2*I)*cos(2*a*x) - (2*a*x - (2*a*x + 2*I)*cos(4*a*x) - 2*(I*a
*x - 1)*sin(4*a*x) - 4*I*cos(2*a*x) + 4*sin(2*a*x) - 2*I)*dilog(-e^(2*I*a*x
)) - (2*I*a^2*x^2 + 4*a*x*cos(2*a*x) + 4*I*a*x*sin(2*a*x) + 2*a*x - 2*(I*a^
2*x^2 - a*x)*cos(4*a*x) + (2*a^2*x^2 + 2*I*a*x)*sin(4*a*x))*log(cos(2*a*x)^
2 + sin(2*a*x)^2 + 2*cos(2*a*x) + 1) - (-4*I*a^3*x^3 + 4*a^2*x^2)*sin(4*a*x
) - (4*a^2*x^2 - 2*I*a*x - 2)*sin(2*a*x) - 2*I)/((I*a*x + (-I*a*x + 1)*cos(
4*a*x) + (a*x + I)*sin(4*a*x) + 2*cos(2*a*x) + 2*I*sin(2*a*x) + 1)*a^5)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\cos(ax)^2 (\cos(ax) + ax \sin(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(cos(a*x)^2*(cos(a*x) + a*x*sin(a*x))^2), x)

[Out] int(x^4/(cos(a*x)^2*(cos(a*x) + a*x*sin(a*x))^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sec^2(ax)}{(ax \sin(ax) + \cos(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*sec(a*x)**2/(cos(a*x)+a*x*sin(a*x))**2,x)

[Out] Integral(x**4*sec(a*x)**2/(a*x*sin(a*x) + cos(a*x))**2, x)

3.603 $\int \sec^4(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx$

Optimal. Leaf size=157

$$\frac{c \tan(2a+2bx) \sec^3(2a+2bx)}{7b\sqrt{c \sec(2a+2bx) - c}} - \frac{6 \tan(2a+2bx)(c \sec(2a+2bx) - c)^{3/2}}{35bc} - \frac{4 \tan(2a+2bx)\sqrt{c \sec(2a+2bx) - c}}{35b}$$

[Out] $-6/35*(-c+c*\sec(2*b*x+2*a))^(3/2)*\tan(2*b*x+2*a)/b/c-2/5*c*\tan(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^(1/2)+1/7*c*\sec(2*b*x+2*a)^3*\tan(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^(1/2)-4/35*(-c+c*\sec(2*b*x+2*a))^(1/2)*\tan(2*b*x+2*a)/b$

Rubi [A] time = 0.45, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4397, 3803, 3800, 4001, 3792}

$$\frac{c \tan(2a+2bx) \sec^3(2a+2bx)}{7b\sqrt{c \sec(2a+2bx) - c}} - \frac{6 \tan(2a+2bx)(c \sec(2a+2bx) - c)^{3/2}}{35bc} - \frac{4 \tan(2a+2bx)\sqrt{c \sec(2a+2bx) - c}}{35b}$$

Antiderivative was successfully verified.

[In] Int[Sec[2*(a + b*x)]^4*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]

[Out] $(-2*c*\tan[2*a + 2*b*x])/(5*b*\sqrt{-c + c*\sec[2*a + 2*b*x]}) + (c*\sec[2*a + 2*b*x]^3*\tan[2*a + 2*b*x])/(7*b*\sqrt{-c + c*\sec[2*a + 2*b*x]}) - (4*\sqrt{-c + c*\sec[2*a + 2*b*x]}*\tan[2*a + 2*b*x])/(35*b) - (6*(-c + c*\sec[2*a + 2*b*x])^(3/2)*\tan[2*a + 2*b*x])/(35*b*c)$

Rule 3792

Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3800

Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 3803

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n - 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 4001

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]

Rule 4397

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \sec^4(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx &= \int \sec^4(2a+2bx)\sqrt{-c+c \sec(2a+2bx)} dx \\ &= \frac{c \sec^3(2a+2bx) \tan(2a+2bx)}{7b\sqrt{-c+c \sec(2a+2bx)}} - \frac{6}{7} \int \sec^3(2a+2bx)\sqrt{-c+c \sec(2a+2bx)} dx \\ &= \frac{c \sec^3(2a+2bx) \tan(2a+2bx)}{7b\sqrt{-c+c \sec(2a+2bx)}} - \frac{6(-c+c \sec(2a+2bx))^3}{35bc} \\ &= \frac{c \sec^3(2a+2bx) \tan(2a+2bx)}{7b\sqrt{-c+c \sec(2a+2bx)}} - \frac{4\sqrt{-c+c \sec(2a+2bx)}}{35b} \\ &= -\frac{2c \tan(2a+2bx)}{5b\sqrt{-c+c \sec(2a+2bx)}} + \frac{c \sec^3(2a+2bx) \tan(2a+2bx)}{7b\sqrt{-c+c \sec(2a+2bx)}} \end{aligned}$$

Mathematica [A] time = 0.22, size = 64, normalized size = 0.41

$$\frac{(7 \cos(3(a+bx)) + 2 \cos(7(a+bx))) \csc(a+bx) \sec^3(2(a+bx)) \sqrt{c \tan(a+bx) \tan(2(a+bx))}}{35b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*(a+b*x)]^4*Sqrt[c*Tan[a+b*x]*Tan[2*(a+b*x)]],x]

[Out] -1/35*((7*Cos[3*(a+b*x)] + 2*Cos[7*(a+b*x)])*Csc[a+b*x]*Sec[2*(a+b*x)]^3*Sqrt[c*Tan[a+b*x]*Tan[2*(a+b*x)]])/b

fricas [A] time = 0.88, size = 106, normalized size = 0.68

$$\frac{\sqrt{2} \left(35 \tan(bx+a)^6 - 35 \tan(bx+a)^4 + 49 \tan(bx+a)^2 - 9 \right) \sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2 - 1}}}{35 \left(b \tan(bx+a)^7 - 3b \tan(bx+a)^5 + 3b \tan(bx+a)^3 - b \tan(bx+a) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")

[Out] -1/35*sqrt(2)*(35*tan(b*x+a)^6 - 35*tan(b*x+a)^4 + 49*tan(b*x+a)^2 - 9)*sqrt(-c*tan(b*x+a)^2/(tan(b*x+a)^2 - 1))/(b*tan(b*x+a)^7 - 3*b*tan(b*x+a)^5 + 3*b*tan(b*x+a)^3 - b*tan(b*x+a))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.59, size = 98, normalized size = 0.62

$$\frac{\sqrt{2} \cos(bx+a) \sqrt{\frac{c(\sin^2(bx+a))}{2(\cos^2(bx+a))-1}} \left(128 \left(\cos^6(bx+a) \right) - 224 \left(\cos^4(bx+a) \right) + 140 \left(\cos^2(bx+a) \right) - 35 \right) \sqrt{4}}{70b \sin(bx+a) \left(2 \left(\cos^2(bx+a) \right) - 1 \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)`

[Out] $-1/70*2^{(1/2)}/b*\cos(b*x+a)*(c*\sin(b*x+a)^2/(2*\cos(b*x+a)^{2-1}))^{(1/2)}*(128*\cos(b*x+a)^6-224*\cos(b*x+a)^4+140*\cos(b*x+a)^2-35)/\sin(b*x+a)/(2*\cos(b*x+a)^{2-1})^3*4^{(1/2)}$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")`

[Out] Timed out

mupad [B] time = 8.88, size = 463, normalized size = 2.95

$$\frac{e^{a2i+bx2i} \sqrt{\frac{c(e^{a2i+bx2i} - 1)(e^{a4i+bx4i} - 1)}{(e^{a2i+bx2i} + 1)(e^{a4i+bx4i} + 1)}}}{35b(e^{a2i+bx2i} - 1)} + \frac{16i \left(\frac{8i}{7b} - \frac{e^{a2i+bx2i} 8i}{7b}\right) \sqrt{\frac{c(e^{a2i+bx2i} - 1)(e^{a4i+bx4i} - 1)}{(e^{a2i+bx2i} + 1)(e^{a4i+bx4i} + 1)}}}{(e^{a2i+bx2i} - 1)(e^{a4i+bx4i} + 1)^3} - \frac{\left(\frac{8i}{5b} - \frac{e^{a2i+bx2i} 64i}{35b}\right)}{(e^{a2i+bx2i} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)/cos(2*a + 2*b*x)^4,x)`

[Out] $((8i/(7*b) - (\exp(a*2i + b*x*2i)*8i)/(7*b))*((c*(\exp(a*2i + b*x*2i)*1i - 1i) * (\exp(a*4i + b*x*4i)*1i - 1i))/((\exp(a*2i + b*x*2i) + 1)*(\exp(a*4i + b*x*4i) + 1)))^{(1/2)})/((\exp(a*2i + b*x*2i) - 1)*(\exp(a*4i + b*x*4i) + 1)^3) - (\exp(a*2i + b*x*2i)*((c*(\exp(a*2i + b*x*2i)*1i - 1i)*(\exp(a*4i + b*x*4i)*1i - 1i))/((\exp(a*2i + b*x*2i) + 1)*(\exp(a*4i + b*x*4i) + 1)))^{(1/2)}*16i)/(35*b*(\exp(a*2i + b*x*2i) - 1)) - ((8i/(5*b) - (\exp(a*2i + b*x*2i)*64i)/(35*b))*((c*(\exp(a*2i + b*x*2i)*1i - 1i)*(\exp(a*4i + b*x*4i)*1i - 1i))/((\exp(a*2i + b*x*2i) + 1)*(\exp(a*4i + b*x*4i) + 1)))^{(1/2)})/((\exp(a*2i + b*x*2i) - 1)*(\exp(a*4i + b*x*4i) + 1)^2) - (\exp(a*2i + b*x*2i)*((c*(\exp(a*2i + b*x*2i)*1i - 1i)*(\exp(a*4i + b*x*4i)*1i - 1i))/((\exp(a*2i + b*x*2i) + 1)*(\exp(a*4i + b*x*4i) + 1)))^{(1/2)}*8i)/(35*b*(\exp(a*2i + b*x*2i) - 1)*(\exp(a*4i + b*x*4i) + 1))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2*b*x+2*a)**4*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)`

[Out] Timed out

3.604 $\int \sec^3(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx$

Optimal. Leaf size=110

$$\frac{\tan(2a+2bx)(c \sec(2a+2bx)-c)^{3/2}}{5bc} + \frac{2 \tan(2a+2bx)\sqrt{c \sec(2a+2bx)-c}}{15b} + \frac{7c \tan(2a+2bx)}{15b\sqrt{c \sec(2a+2bx)-c}}$$

[Out] $1/5*(-c+c*\sec(2*b*x+2*a))^{(3/2)*\tan(2*b*x+2*a)/b/c+7/15*c*\tan(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)+2/15*(-c+c*\sec(2*b*x+2*a))^{(1/2)*\tan(2*b*x+2*a)}/b$

Rubi [A] time = 0.28, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4397, 3800, 4001, 3792}

$$\frac{\tan(2a+2bx)(c \sec(2a+2bx)-c)^{3/2}}{5bc} + \frac{2 \tan(2a+2bx)\sqrt{c \sec(2a+2bx)-c}}{15b} + \frac{7c \tan(2a+2bx)}{15b\sqrt{c \sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

[In] Int[Sec[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]

[Out] $(7*c*\tan[2*a + 2*b*x])/(15*b*\sqrt{-c + c*\sec[2*a + 2*b*x]}) + (2*\sqrt{-c + c*\sec[2*a + 2*b*x]}*\tan[2*a + 2*b*x])/(15*b) + ((-c + c*\sec[2*a + 2*b*x])^{(3/2)*\tan[2*a + 2*b*x]})/(5*b*c)$

Rule 3792

Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3800

Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 4001

Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]

Rule 4397

Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned}
\int \sec^3(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx &= \int \sec^3(2a+2bx)\sqrt{-c+c \sec(2a+2bx)} dx \\
&= \frac{(-c+c \sec(2a+2bx))^{3/2} \tan(2a+2bx)}{5bc} + \frac{2 \int \sec(2a+2bx) \sqrt{-c+c \sec(2a+2bx)} dx}{5bc} \\
&= \frac{2\sqrt{-c+c \sec(2a+2bx)} \tan(2a+2bx)}{15b} + \frac{(-c+c \sec(2a+2bx))^{3/2}}{15b} \\
&= \frac{7c \tan(2a+2bx)}{15b\sqrt{-c+c \sec(2a+2bx)}} + \frac{2\sqrt{-c+c \sec(2a+2bx)}}{15b}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 62, normalized size = 0.56

$$\frac{(5 \cos(a+bx) + 2 \cos(5(a+bx))) \csc(a+bx) \sec^2(2(a+bx)) \sqrt{c \tan(a+bx) \tan(2(a+bx))}}{15b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*(a+b*x)]^3*Sqrt[c*Tan[a+b*x]*Tan[2*(a+b*x)]],x]

[Out] ((5*Cos[a+b*x]+2*Cos[5*(a+b*x)])*Csc[a+b*x]*Sec[2*(a+b*x)]^2*Sqrt[c*Tan[a+b*x]*Tan[2*(a+b*x)]])/(15*b)

fricas [A] time = 1.50, size = 84, normalized size = 0.76

$$\frac{\sqrt{2} (15 \tan(bx+a)^4 - 10 \tan(bx+a)^2 + 7) \sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2 - 1}}}{15 (b \tan(bx+a)^5 - 2b \tan(bx+a)^3 + b \tan(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")

[Out] 1/15*sqrt(2)*(15*tan(b*x+a)^4 - 10*tan(b*x+a)^2 + 7)*sqrt(-c*tan(b*x+a)^2/(tan(b*x+a)^2 - 1))/(b*tan(b*x+a)^5 - 2*b*tan(b*x+a)^3 + b*tan(b*x+a))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.17, size = 88, normalized size = 0.80

$$\frac{\sqrt{2} \sqrt{\frac{c(\sin^2(bx+a))}{2(\cos^2(bx+a))-1}} \cos(bx+a) (32(\cos^4(bx+a)) - 40(\cos^2(bx+a)) + 15) \sqrt{4}}{30b \sin(bx+a) (2(\cos^2(bx+a)) - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)

[Out] $\frac{1}{30} \cdot 2^{1/2} / b \cdot (c \cdot \sin(b \cdot x + a)^2 / (2 \cdot \cos(b \cdot x + a)^2 - 1))^{1/2} \cdot \cos(b \cdot x + a) \cdot (32 \cdot \cos(b \cdot x + a)^4 - 40 \cdot \cos(b \cdot x + a)^2 + 15) / \sin(b \cdot x + a) / (2 \cdot \cos(b \cdot x + a)^2 - 1)^2 \cdot 4^{1/2}$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")`

[Out] Timed out

mupad [B] time = 11.03, size = 148, normalized size = 1.35

$$\frac{4 \left(e^{a \cdot 4i + b \cdot x \cdot 4i} 5i + e^{a \cdot 6i + b \cdot x \cdot 6i} 5i + e^{a \cdot 10i + b \cdot x \cdot 10i} 2i + 2i \right) \sqrt{\frac{c \left(e^{a \cdot 2i + b \cdot x \cdot 2i} 1i - i \right) \left(e^{a \cdot 4i + b \cdot x \cdot 4i} 1i - i \right)}{\left(e^{a \cdot 2i + b \cdot x \cdot 2i} + 1 \right) \left(e^{a \cdot 4i + b \cdot x \cdot 4i} + 1 \right)}}{15 b \left(e^{a \cdot 2i + b \cdot x \cdot 2i} - 1 \right) \left(e^{a \cdot 4i + b \cdot x \cdot 4i} + 1 \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)/cos(2*a + 2*b*x)^3,x)`

[Out] $(4 \cdot (\exp(a \cdot 4i + b \cdot x \cdot 4i) \cdot 5i + \exp(a \cdot 6i + b \cdot x \cdot 6i) \cdot 5i + \exp(a \cdot 10i + b \cdot x \cdot 10i) \cdot 2i + 2i) \cdot ((c \cdot (\exp(a \cdot 2i + b \cdot x \cdot 2i) \cdot 1i - 1i) \cdot (\exp(a \cdot 4i + b \cdot x \cdot 4i) \cdot 1i - 1i)) / ((\exp(a \cdot 2i + b \cdot x \cdot 2i) + 1) \cdot (\exp(a \cdot 4i + b \cdot x \cdot 4i) + 1)))^{1/2}) / (15 \cdot b \cdot (\exp(a \cdot 2i + b \cdot x \cdot 2i) - 1) \cdot (\exp(a \cdot 4i + b \cdot x \cdot 4i) + 1)^2)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)`

[Out] Timed out

3.605 $\int \sec^2(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx$

Optimal. Leaf size=72

$$\frac{\tan(2a+2bx)\sqrt{c \sec(2a+2bx)-c}}{3b} - \frac{c \tan(2a+2bx)}{3b\sqrt{c \sec(2a+2bx)-c}}$$

[Out] $-1/3*c*\tan(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}+1/3*(-c+c*\sec(2*b*x+2*a))^{(1/2)}*\tan(2*b*x+2*a)/b$

Rubi [A] time = 0.20, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4397, 3798, 3792}

$$\frac{\tan(2a+2bx)\sqrt{c \sec(2a+2bx)-c}}{3b} - \frac{c \tan(2a+2bx)}{3b\sqrt{c \sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

[In] Int[Sec[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]

[Out] $-(c*\tan[2*a + 2*b*x])/(3*b*\sqrt{-c + c*\sec[2*a + 2*b*x]}) + (\sqrt{-c + c*\sec[2*a + 2*b*x]}*\tan[2*a + 2*b*x])/(3*b)$

Rule 3792

Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3798

Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 4397

Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \sec^2(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx &= \int \sec^2(2a+2bx)\sqrt{-c + c \sec(2a+2bx)} dx \\ &= \frac{\sqrt{-c + c \sec(2a+2bx)} \tan(2a+2bx)}{3b} - \frac{1}{3} \int \sec(2a+2bx) dx \\ &= -\frac{c \tan(2a+2bx)}{3b\sqrt{-c + c \sec(2a+2bx)}} + \frac{\sqrt{-c + c \sec(2a+2bx)} \tan(2a+2bx)}{3b} \end{aligned}$$

Mathematica [A] time = 0.18, size = 44, normalized size = 0.61

$$\frac{\sqrt{c \tan(a+bx) \tan(2(a+bx))} (\tan(2(a+bx)) - \cot(a+bx))}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]

[Out] (Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]*(-Cot[a + b*x] + Tan[2*(a + b*x)]))/(3*b)

fricas [A] time = 0.91, size = 64, normalized size = 0.89

$$\frac{\sqrt{2} \sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2-1}} (3 \tan(bx+a)^2 - 1)}{3 (b \tan(bx+a)^3 - b \tan(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")

[Out] -1/3*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(3*tan(b*x + a)^2 - 1)/(b*tan(b*x + a)^3 - b*tan(b*x + a))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.10, size = 78, normalized size = 1.08

$$\frac{\sqrt{2} \sqrt{\frac{c(\sin^2(bx+a))}{2(\cos^2(bx+a))-1}} \cos(bx+a) (4(\cos^2(bx+a)) - 3) \sqrt{4}}{6b \sin(bx+a) (2(\cos^2(bx+a)) - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)

[Out] -1/6*2^(1/2)/b*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*cos(b*x+a)*(4*cos(b*x+a)^2-3)/sin(b*x+a)/(2*cos(b*x+a)^2-1)*4^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")

[Out] -2/3*(6*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(3/4)*b*sqrt(c)*integrate(-(((cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 2*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(12*b*x + 12*a)*sin(4*b*x + 4*a) + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 2*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(12*b*x + 12*a)*sin(4*b*x + 4*a) - 2*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(3/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 2*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(12*b*x + 12*a)*sin(4*b*x + 4*a) - 2*cos(8*b*x + 8*a)*sin(4*b*x + 4*a)))/sin(4*b*x + 4*a))

$x + 8a) \sin(4bx + 4a) \cos\left(\frac{1}{2} \arctan 2(\sin(4bx + 4a), -\cos(4bx + 4a) - 1)\right) - (\cos(12bx + 12a) \cos(4bx + 4a) + 2 \cos(8bx + 8a) \cos(4bx + 4a) + \cos(4bx + 4a)^2 + \sin(12bx + 12a) \sin(4bx + 4a) + 2 \sin(8bx + 8a) \sin(4bx + 4a) + \sin(4bx + 4a)^2) \sin\left(\frac{1}{2} \arctan 2(\sin(4bx + 4a), -\cos(4bx + 4a) - 1)\right) \sin\left(\frac{3}{2} \arctan 2(\sin(4bx + 4a), \cos(4bx + 4a))\right) / (((2(2 \cos(8bx + 8a) + \cos(4bx + 4a)) \cos(12bx + 12a) + \cos(12bx + 12a)^2 + 4 \cos(8bx + 8a)^2 + 4 \cos(8bx + 8a) \cos(4bx + 4a) + \cos(4bx + 4a)^2 + 2(2 \sin(8bx + 8a) + \sin(4bx + 4a)) \sin(12bx + 12a) + \sin(12bx + 12a)^2 + 4 \sin(8bx + 8a)^2 + 4 \sin(8bx + 8a) \sin(4bx + 4a) + \sin(4bx + 4a)^2) \cos\left(\frac{1}{2} \arctan 2(\sin(4bx + 4a), -\cos(4bx + 4a) - 1)\right)^2 + (2(2 \cos(8bx + 8a) + \cos(4bx + 4a)) \cos(12bx + 12a) + \cos(12bx + 12a)^2 + 4 \cos(8bx + 8a)^2 + 4 \cos(8bx + 8a) \cos(4bx + 4a) + \cos(4bx + 4a)^2 + 2(2 \sin(8bx + 8a) + \sin(4bx + 4a)) \sin(12bx + 12a) + \sin(12bx + 12a)^2 + 4 \sin(8bx + 8a)^2 + 4 \sin(8bx + 8a) \sin(4bx + 4a) + \sin(4bx + 4a)^2) \sin\left(\frac{1}{2} \arctan 2(\sin(4bx + 4a), -\cos(4bx + 4a) - 1)\right)^2 * (\cos(4bx + 4a)^2 + \sin(4bx + 4a)^2 + 2 \cos(4bx + 4a) + 1)^{1/4}), x) + \sqrt{c} \sin\left(\frac{3}{2} \arctan 2(\sin(4bx + 4a), -\cos(4bx + 4a) - 1)\right) / ((\cos(4bx + 4a)^2 + \sin(4bx + 4a)^2 + 2 \cos(4bx + 4a) + 1)^{3/4} * b)$

mupad [B] time = 7.34, size = 129, normalized size = 1.79

$$\frac{2 \left(e^{a6i + bx6i} 1i + 1i \right) \sqrt{\frac{c \left(e^{a2i + bx2i} 1i - 1i \right) \left(e^{a4i + bx4i} 1i - 1i \right)}{\left(e^{a2i + bx2i} + 1 \right) \left(e^{a4i + bx4i} + 1 \right)}}{3b \left(e^{a2i + bx2i} - e^{a4i + bx4i} + e^{a6i + bx6i} - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)/cos(2*a + 2*b*x)^2,x)

[Out] $-(2 * (\exp(a*6i + b*x*6i) * 1i + 1i) * ((c * (\exp(a*2i + b*x*2i) * 1i - 1i) * (\exp(a*4i + b*x*4i) * 1i - 1i)) / ((\exp(a*2i + b*x*2i) + 1) * (\exp(a*4i + b*x*4i) + 1)))^{1/2}) / (3 * b * (\exp(a*2i + b*x*2i) - \exp(a*4i + b*x*4i) + \exp(a*6i + b*x*6i) - 1)))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)

[Out] Timed out

$$3.606 \quad \int \sec(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx$$

Optimal. Leaf size=33

$$\frac{c \tan(2a + 2bx)}{b\sqrt{c \sec(2a + 2bx) - c}}$$

[Out] c*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {4397, 3792}

$$\frac{c \tan(2a + 2bx)}{b\sqrt{c \sec(2a + 2bx) - c}}$$

Antiderivative was successfully verified.

[In] Int[Sec[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]

[Out] (c*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]])

Rule 3792

Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4397

Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \sec(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx &= \int \sec(2a+2bx)\sqrt{-c+c \sec(2a+2bx)} dx \\ &= \frac{c \tan(2a+2bx)}{b\sqrt{-c+c \sec(2a+2bx)}} \end{aligned}$$

Mathematica [A] time = 0.08, size = 30, normalized size = 0.91

$$\frac{\cot(a+bx)\sqrt{c \tan(a+bx) \tan(2(a+bx))}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]

[Out] (Cot[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/b

fricas [A] time = 0.43, size = 40, normalized size = 1.21

$$\frac{\sqrt{2} \sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2-1}}}{b \tan(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")

[Out] $\sqrt{2} \sqrt{-c \tan(bx + a)^2 / (\tan(bx + a)^2 - 1)} / (b \tan(bx + a))$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")`

[Out] Timed out

maple [A] time = 0.97, size = 52, normalized size = 1.58

$$\frac{\sqrt{2} \sqrt{\frac{c(\sin^2(bx+a))}{2(\cos^2(bx+a))-1}} \cos(bx+a) \sqrt{4}}{2b \sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)`

[Out] $1/2 * 2^{(1/2)} / b * (c \sin(bx+a)^2 / (2 \cos(bx+a)^2 - 1))^{(1/2)} * \cos(bx+a) / \sin(bx+a) * 4^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")`

[Out] $(2 * (\cos(4bx + 4a)^2 + \sin(4bx + 4a)^2 + 2 \cos(4bx + 4a) + 1))^{(1/4)} * b * \sqrt{c} * \int -(((\cos(8bx + 8a) * \cos(4bx + 4a) + \cos(4bx + 4a)^2 + \sin(8bx + 8a) * \sin(4bx + 4a) + \sin(4bx + 4a)^2) * \cos(1/2 * \arctan2(\sin(4bx + 4a), -\cos(4bx + 4a) - 1)) + (\cos(4bx + 4a) * \sin(8bx + 8a) - \cos(8bx + 8a) * \sin(4bx + 4a)) * \sin(1/2 * \arctan2(\sin(4bx + 4a), -\cos(4bx + 4a) - 1))) * \cos(1/2 * \arctan2(\sin(4bx + 4a), \cos(4bx + 4a))) + ((\cos(4bx + 4a) * \sin(8bx + 8a) - \cos(8bx + 8a) * \sin(4bx + 4a)) * \cos(1/2 * \arctan2(\sin(4bx + 4a), -\cos(4bx + 4a) - 1)) - (\cos(8bx + 8a) * \cos(4bx + 4a) + \cos(4bx + 4a)^2 + \sin(8bx + 8a) * \sin(4bx + 4a) + \sin(4bx + 4a)^2) * \sin(1/2 * \arctan2(\sin(4bx + 4a), -\cos(4bx + 4a) - 1))) * \sin(1/2 * \arctan2(\sin(4bx + 4a), \cos(4bx + 4a)))) / (((\cos(8bx + 8a)^2 + 2 * \cos(8bx + 8a) * \cos(4bx + 4a) + \cos(4bx + 4a)^2 + \sin(8bx + 8a)^2 + 2 * \sin(8bx + 8a) * \sin(4bx + 4a) + \sin(4bx + 4a)^2) * \cos(1/2 * \arctan2(\sin(4bx + 4a), -\cos(4bx + 4a) - 1)))^2 + (\cos(8bx + 8a)^2 + 2 * \cos(8bx + 8a) * \cos(4bx + 4a) + \cos(4bx + 4a)^2 + \sin(8bx + 8a)^2 + 2 * \sin(8bx + 8a) * \sin(4bx + 4a) + \sin(4bx + 4a)^2) * \sin(1/2 * \arctan2(\sin(4bx + 4a), -\cos(4bx + 4a) - 1)))^2 * (\cos(4bx + 4a)^2 + \sin(4bx + 4a)^2 + 2 * \cos(4bx + 4a) + 1))^{(1/4)}, x) - \sqrt{c} * \sin(1/2 * \arctan2(\sin(4bx + 4a), -\cos(4bx + 4a) - 1))) / ((\cos(4bx + 4a)^2 + \sin(4bx + 4a)^2 + 2 * \cos(4bx + 4a) + 1))^{(1/4)} * b$

mupad [B] time = 3.68, size = 87, normalized size = 2.64

$$\frac{\sin(2a + 2bx) \sqrt{\frac{c(\cos(2a+2bx) - \cos(6a+6bx))}{3 \cos(2a+2bx) + 2 \cos(4a+4bx) + \cos(6a+6bx) + 2}}}{b(\cos(2a + 2bx) - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)/cos(2*a + 2*b*x),x)
```

```
[Out] -(sin(2*a + 2*b*x)*((c*(cos(2*a + 2*b*x) - cos(6*a + 6*b*x)))/(3*cos(2*a + 2*b*x) + 2*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) + 2))^(1/2))/(b*(cos(2*a + 2*b*x) - 1))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] Timed out
```


3.607 $\int \sqrt{c \tan(a + bx) \tan(2(a + bx))} dx$

Optimal. Leaf size=45

$$-\frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{b}$$

[Out] $-\operatorname{arctanh}(c^{(1/2)}*\tan(2*b*x+2*a)/(-c+c*\sec(2*b*x+2*a))^{(1/2)})*c^{(1/2)}/b$

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4397, 3774, 207}

$$-\frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[c*\text{Tan}[a + b*x]*\text{Tan}[2*(a + b*x)]], x]$

[Out] $-\left(\frac{\text{Sqrt}[c]*\text{ArcTanh}[\left(\frac{\text{Sqrt}[c]*\text{Tan}[2*a + 2*b*x]}{\text{Sqrt}[-c + c*\text{Sec}[2*a + 2*b*x]]}\right)}{\text{Sqrt}[-c + c*\text{Sec}[2*a + 2*b*x]]}\right)/b$

Rule 207

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] :> -\text{Simp}[\text{ArcTanh}[\text{Rt}[b, 2]*x]/\text{Rt}[-a, 2]]/\text{Rt}[-a, 2]*\text{Rt}[b, 2], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 3774

$\text{Int}[\text{Sqrt}[\text{csc}[(c_.) + (d_)*(x_)]*(b_.) + (a_)], x_Symbol] :> \text{Dist}[(-2*b)/d, \text{Subst}[\text{Int}[1/(a + x^2), x], x, (b*\text{Cot}[c + d*x])/ \text{Sqrt}[a + b*\text{Csc}[c + d*x]]], x] /;$ FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 4397

$\text{Int}[u_, x_Symbol] :> \text{Int}[\text{TrigSimplify}[u], x] /;$ TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \sqrt{c \tan(a + bx) \tan(2(a + bx))} dx &= \int \sqrt{-c + c \sec(2a + 2bx)} dx \\ &= -\frac{c \text{Subst}\left(\int \frac{1}{-c+x^2} dx, x, -\frac{c \tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{b} \\ &= -\frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{b} \end{aligned}$$

Mathematica [A] time = 0.13, size = 73, normalized size = 1.62

$$\frac{\sqrt{\cos(2(a + bx))} \csc(a + bx) \sqrt{c \tan(a + bx) \tan(2(a + bx))} \tanh^{-1}\left(\frac{\sqrt{2} \cos(a + bx)}{\sqrt{\cos(2(a + bx))}}\right)}{\sqrt{2} b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]

[Out] -((ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a + b*x)]]]*Sqrt[Cos[2*(a + b*x)]]*Csc[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(Sqrt[2]*b)

fricas [A] time = 1.46, size = 201, normalized size = 4.47

$$\frac{\sqrt{c} \log\left(\frac{c \tan(bx+a)^5 - 14c \tan(bx+a)^3 - 4\sqrt{2}(\tan(bx+a)^4 - 4 \tan(bx+a)^2 + 3)\sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2 - 1}} \sqrt{c} + 17c \tan(bx+a)}{\tan(bx+a)^5 + 2 \tan(bx+a)^3 + \tan(bx+a)}\right) + \sqrt{-c} \arctan\left(\frac{2\sqrt{2}\sqrt{c}}{c}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")

[Out] [1/4*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 - 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a)))/b, 1/2*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a)))/b]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.94, size = 136, normalized size = 3.02

$$\frac{\sqrt{\frac{c(1-\cos^2(bx+a))}{2(\cos^2(bx+a)-1)}} \sin(bx+a) \sqrt{\frac{2(\cos^2(bx+a)-1)}{(\cos(bx+a)+1)^2}} \operatorname{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}(-1+\cos(bx+a))\sqrt{2}}{2\sin(bx+a)^2\sqrt{\frac{2(\cos^2(bx+a)-1)}{(\cos(bx+a)+1)^2}}}\right) \sqrt{4}}{2b(-1+\cos(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)

[Out] -1/2/b*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))/(-1+cos(b*x+a))*4^(1/2)

maxima [B] time = 0.50, size = 430, normalized size = 9.56

$$\sqrt{c} \left(\log\left(4\sqrt{\cos(4bx+4a)^2 + \sin(4bx+4a)^2} + 2\cos(4bx+4a) + 1\right) \cos\left(\frac{1}{2}\arctan(\sin(4bx+4a), \cos(4bx+4a))\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")

```
[Out] 1/4*sqrt(c)*(log(4*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2 + 4*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2 + 8*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1)) + 4) - log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*(cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2) + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(cos(2*b*x + 2*a)*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1)) + sin(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1)))))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{c \tan(a + bx) \tan(2a + 2bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

```
[Out] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2), x)
```

```
[Out] Timed out
```

3.608 $\int \cos(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx$

Optimal. Leaf size=84

$$\frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{2b} - \frac{c \sin(2a+2bx)}{2b\sqrt{c \sec(2a+2bx)-c}}$$

[Out] $1/2*\operatorname{arctanh}(c^{(1/2)}*\tan(2*b*x+2*a)/(-c+c*\sec(2*b*x+2*a))^{(1/2)})*c^{(1/2)}/b-1/2*c*\sin(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}$

Rubi [A] time = 0.14, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4397, 3805, 3774, 207}

$$\frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{2b} - \frac{c \sin(2a+2bx)}{2b\sqrt{c \sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

[In] Int[Cos[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]

[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]])/(2*b) - (c*Sin[2*a + 2*b*x])/(2*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3805

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]

Rule 4397

Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned}
\int \cos(2(a + bx))\sqrt{c \tan(a + bx) \tan(2(a + bx))} dx &= \int \cos(2a + 2bx)\sqrt{-c + c \sec(2a + 2bx)} dx \\
&= -\frac{c \sin(2a + 2bx)}{2b\sqrt{-c + c \sec(2a + 2bx)}} - \frac{1}{2} \int \sqrt{-c + c \sec(2a + 2bx)} dx \\
&= -\frac{c \sin(2a + 2bx)}{2b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{c \operatorname{Subst}\left(\int \frac{1}{-c+x^2} dx, x, -\sqrt{-c + c \sec(2a + 2bx)}\right)}{2b} \\
&= \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{2b} - \frac{c \sin(2a + 2bx)}{2b\sqrt{-c + c \sec(2a + 2bx)}}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 92, normalized size = 1.10

$$\frac{\csc(a + bx)\sqrt{c \tan(a + bx) \tan(2(a + bx))} \left(\cos(a + bx) + \cos(3(a + bx)) - \sqrt{2} \sqrt{\cos(2(a + bx))} \tanh^{-1}\left(\frac{\sqrt{2} c}{\sqrt{\cos(2(a + bx))}}\right) \right)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]

[Out] -1/4*((Cos[a + b*x] - Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a + b*x)]]]*Sqrt[Cos[2*(a + b*x)]] + Cos[3*(a + b*x)])*Csc[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]]/b

fricas [B] time = 2.28, size = 351, normalized size = 4.18

$$\left[\frac{(\tan(bx + a)^3 + \tan(bx + a))\sqrt{c} \log\left(\frac{c \tan(bx+a)^5 - 14c \tan(bx+a)^3 + 4\sqrt{2}(\tan(bx+a)^4 - 4 \tan(bx+a)^2 + 3)\sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2 - 1}} \sqrt{c + 17c \tan(bx+a)^2}}{\tan(bx+a)^5 + 2 \tan(bx+a)^3 + \tan(bx+a)} \right)}{8(b \tan(bx + a)^3 + b \tan(bx + a))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2), x, algorithm="fricas")

[Out] [1/8*((tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a))^5 - 14*c*tan(b*x + a)^3 + 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a)) + 4*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/(b*tan(b*x + a)^3 + b*tan(b*x + a)), -1/4*((tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/(b*tan(b*x + a)^3 + b*tan(b*x + a)))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c \tan(2bx + 2a) \tan(bx + a)} \cos(2bx + 2a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a))*cos(2*b*x + 2*a), x)

maple [B] time = 1.14, size = 391, normalized size = 4.65

$$\frac{\sqrt{\frac{c(1-\cos^2(bx+a))}{2(\cos^2(bx+a)-1)}} \sin(bx+a) \sqrt{\frac{2(\cos^2(bx+a)-1)}{(\cos(bx+a)+1)^2}} \operatorname{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}(-1+\cos(bx+a))\sqrt{2}}{2\sin(bx+a)^2\sqrt{\frac{2(\cos^2(bx+a)-1)}{(\cos(bx+a)+1)^2}}}\right) \sqrt{4} \sqrt{2} \sin(bx+a) \sqrt{\frac{c(1-\cos^2(bx+a))}{2(\cos^2(bx+a)-1)}}}{2b(-1+\cos(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)

[Out] 1/2/b*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))/(-1+cos(b*x+a))*4^(1/2)-1/8*2^(1/2)/b*sin(b*x+a)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-4*cos(b*x+a)^3+2*cos(b*x+a))/(-1+cos(b*x+a)^2)*4^(1/2)

maxima [B] time = 0.61, size = 1049, normalized size = 12.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")

[Out] 1/16*(4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(2*b*x + 2*a) + (cos(2*b*x + 2*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*sqrt(c) - sqrt(c)*(log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) + log(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1) + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(2*b*x + 2*a) + cos(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))) + 1) - log(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4

```
*a) - 1))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4
*a) + 1) - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)
+ 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(2
*b*x + 2*a) + cos(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x
+ 4*a) - 1))) + 1))) / b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(2a + 2bx) \sqrt{c \tan(a + bx) \tan(2a + 2bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

```
[Out] int(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2), x)
```

```
[Out] Timed out
```

3.609 $\int \cos^2(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx$

Optimal. Leaf size=129

$$\frac{3c \sin(2a+2bx)}{8b\sqrt{c \sec(2a+2bx)-c}} - \frac{c \sin(2a+2bx) \cos(2a+2bx)}{4b\sqrt{c \sec(2a+2bx)-c}} - \frac{3\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{8b}$$

[Out] $-3/8*\operatorname{arctanh}(c^{(1/2)}*\tan(2*b*x+2*a)/(-c+c*\sec(2*b*x+2*a))^{(1/2)})*c^{(1/2)}/b+3/8*c*\sin(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}-1/4*c*\cos(2*b*x+2*a)*\sin(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}$

Rubi [A] time = 0.21, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4397, 3805, 3774, 207}

$$\frac{3c \sin(2a+2bx)}{8b\sqrt{c \sec(2a+2bx)-c}} - \frac{c \sin(2a+2bx) \cos(2a+2bx)}{4b\sqrt{c \sec(2a+2bx)-c}} - \frac{3\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{8b}$$

Antiderivative was successfully verified.

[In] `Int[Cos[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]`

[Out] $(-3*\sqrt{c}*\operatorname{ArcTanh}[(\sqrt{c}*\tan[2*a + 2*b*x])/(\sqrt{-c + c*\sec[2*a + 2*b*x]})])/(8*b) + (3*c*\sin[2*a + 2*b*x])/(8*b*\sqrt{-c + c*\sec[2*a + 2*b*x]}) - (c*\cos[2*a + 2*b*x]*\sin[2*a + 2*b*x])/(4*b*\sqrt{-c + c*\sec[2*a + 2*b*x]})$

Rule 207

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

Rule 3774

`Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

Rule 3805

`Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]`

Rule 4397

`Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]`

Rubi steps

$$\begin{aligned}
\int \cos^2(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx &= \int \cos^2(2a+2bx)\sqrt{-c+c \sec(2a+2bx)} dx \\
&= -\frac{c \cos(2a+2bx) \sin(2a+2bx)}{4b\sqrt{-c+c \sec(2a+2bx)}} - \frac{3}{4} \int \cos(2a+2bx)\sqrt{-c+c \sec(2a+2bx)} dx \\
&= \frac{3c \sin(2a+2bx)}{8b\sqrt{-c+c \sec(2a+2bx)}} - \frac{c \cos(2a+2bx) \sin(2a+2bx)}{4b\sqrt{-c+c \sec(2a+2bx)}} \\
&= \frac{3c \sin(2a+2bx)}{8b\sqrt{-c+c \sec(2a+2bx)}} - \frac{c \cos(2a+2bx) \sin(2a+2bx)}{4b\sqrt{-c+c \sec(2a+2bx)}} \\
&= -\frac{3\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{8b} + \frac{3c \sin(2a+2bx)}{8b\sqrt{-c+c \sec(2a+2bx)}}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 105, normalized size = 0.81

$$\frac{\sqrt{c \tan(a+bx) \tan(2(a+bx))} \left(2(-\sin(2(a+bx)) + \sin(4(a+bx)) + \cot(a+bx)) - 3\sqrt{2} \sqrt{\cos(2(a+bx))} \csc(2(a+bx)) \right)}{16b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]

[Out] ((-3*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a + b*x)]]]*Sqrt[Cos[2*(a + b*x)]]*Csc[a + b*x] + 2*(Cot[a + b*x] - Sin[2*(a + b*x)] + Sin[4*(a + b*x)]))*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]]/(16*b)

fricas [A] time = 0.95, size = 419, normalized size = 3.25

$$\frac{3 \left(\tan(bx+a)^5 + 2 \tan(bx+a)^3 + \tan(bx+a) \right) \sqrt{c} \log \left(\frac{c \tan(bx+a)^5 - 14c \tan(bx+a)^3 - 4\sqrt{2} (\tan(bx+a)^4 - 4 \tan(bx+a)^2 + 3) \sqrt{-c \tan(bx+a)^2 / (\tan(bx+a)^2 - 1)} \sqrt{c} + 17c \tan(bx+a)}{\tan(bx+a)^5 + 2 \tan(bx+a)^3 + \tan(bx+a)} \right)}{32 \left(b \tan(bx+a)^5 + 2b \tan(bx+a)^3 + b \tan(bx+a) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2), x, algorithm="fricas")

[Out] [1/32*(3*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 - 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a)) - 4*sqrt(2)*(5*tan(b*x + a)^4 - 4*tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(b*x + a)), 1/16*(3*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*(5*tan(b*x + a)^4 - 4*tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(b*x + a))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c \tan(2bx+2a) \tan(bx+a)} \cos(2bx+2a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a))*cos(2*b*x + 2*a)^2, x)

maple [B] time = 1.22, size = 657, normalized size = 5.09

$$\frac{\sqrt{\frac{c(1-\cos^2(bx+a))}{2(\cos^2(bx+a)-1)}} \sin(bx+a) \sqrt{\frac{2(\cos^2(bx+a)-1)}{(\cos(bx+a)+1)^2}} \operatorname{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}(-1+\cos(bx+a))\sqrt{2}}{2\sin(bx+a)^2\sqrt{\frac{2(\cos^2(bx+a)-1)}{(\cos(bx+a)+1)^2}}}\right) \sqrt{4} \sqrt{2} \sin(bx+a) \sqrt{\frac{c}{2}}}{2b(-1+\cos(bx+a))} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)

[Out] -1/2/b*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))/(-1+cos(b*x+a))*4^(1/2)+1/4*2^(1/2)/b*sin(b*x+a)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-4*cos(b*x+a)^3+2*cos(b*x+a))/(-1+cos(b*x+a)^2)*4^(1/2)-1/32*2^(1/2)/b*sin(b*x+a)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(-16*cos(b*x+a)^5+3*2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+3*2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-4*cos(b*x+a)^3+6*cos(b*x+a))/(-1+cos(b*x+a)^2)*4^(1/2)

maxima [B] time = 0.72, size = 1421, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")

[Out] -1/64*(4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)), sin(4*b*x + 4*a) - (cos(4*b*x + 4*a) - 2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))) * cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))) - cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(4*b*x + 4*a) - (cos(4*b*x + 4*a) - 2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) - ((cos(4*b*x + 4*a) - 2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + sin(4*b*x + 4*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))) * sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))) * sqrt(c) - 3*sqrt(c)*(log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - log(sqrt(cos(4*b*x + 4*a)

```

*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*
b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x
+ 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(
4*b*x + 4*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4
*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a)
- 1)) + 1) + log(((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)
)^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*cos(1/2*
arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2 + (cos(1/2*arctan2(sin(4*b*x
+ 4*a), -cos(4*b*x + 4*a) - 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos
(4*b*x + 4*a) - 1))^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))
^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)
+ 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4
)*(cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/2*arctan2(sin
(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + cos(1/2*arctan2(sin(4*b*x + 4*a),
-cos(4*b*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))
)) + 1) - log(((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2
+ sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*cos(1/2*arc
tan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2 + (cos(1/2*arctan2(sin(4*b*x +
4*a), -cos(4*b*x + 4*a) - 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
b*x + 4*a) - 1))^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2)
*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1) - 2
*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(
cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/2*arctan2(sin(4*
b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + cos(1/2*arctan2(sin(4*b*x + 4*a), -co
s(4*b*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))))
+ 1)))/b

```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(2a + 2bx)^2 \sqrt{c \tan(a + bx) \tan(2a + 2bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)

[Out] int(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)

[Out] Timed out

3.610 $\int \cos^3(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx$

Optimal. Leaf size=176

$$\frac{5c \sin(2a+2bx)}{16b\sqrt{c \sec(2a+2bx)-c}} - \frac{c \sin(2a+2bx) \cos^2(2a+2bx)}{6b\sqrt{c \sec(2a+2bx)-c}} + \frac{5c \sin(2a+2bx) \cos(2a+2bx)}{24b\sqrt{c \sec(2a+2bx)-c}} + \frac{5\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c} \tan(a+bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{16b}$$

[Out] $5/16*\operatorname{arctanh}(c^{(1/2)}*\tan(2*b*x+2*a)/(-c+c*\sec(2*b*x+2*a))^{(1/2)})*c^{(1/2)}/b-5/16*c*\sin(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}+5/24*c*\cos(2*b*x+2*a)*\sin(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}-1/6*c*\cos(2*b*x+2*a)^2*\sin(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}$

Rubi [A] time = 0.29, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4397, 3805, 3774, 207}

$$\frac{5c \sin(2a+2bx)}{16b\sqrt{c \sec(2a+2bx)-c}} - \frac{c \sin(2a+2bx) \cos^2(2a+2bx)}{6b\sqrt{c \sec(2a+2bx)-c}} + \frac{5c \sin(2a+2bx) \cos(2a+2bx)}{24b\sqrt{c \sec(2a+2bx)-c}} + \frac{5\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c} \tan(a+bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{16b}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cos}[2*(a+b*x)]^3*\operatorname{Sqrt}[c*\operatorname{Tan}[a+b*x]*\operatorname{Tan}[2*(a+b*x)]], x]$

[Out] $(5*\operatorname{Sqrt}[c]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\operatorname{Tan}[2*a+2*b*x])/(\operatorname{Sqrt}[-c+c*\operatorname{Sec}[2*a+2*b*x]])]/(16*b) - (5*c*\operatorname{Sin}[2*a+2*b*x])/((16*b*\operatorname{Sqrt}[-c+c*\operatorname{Sec}[2*a+2*b*x]]) + (5*c*\operatorname{Cos}[2*a+2*b*x]*\operatorname{Sin}[2*a+2*b*x])/((24*b*\operatorname{Sqrt}[-c+c*\operatorname{Sec}[2*a+2*b*x]]) - (c*\operatorname{Cos}[2*a+2*b*x]^2*\operatorname{Sin}[2*a+2*b*x])/((6*b*\operatorname{Sqrt}[-c+c*\operatorname{Sec}[2*a+2*b*x]])$

Rule 207

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] := -\operatorname{Simp}[\operatorname{ArcTanh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Rt}[-a, 2]]/(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{LtQ}[a, 0] \ || \ \operatorname{GtQ}[b, 0])$

Rule 3774

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[(c_+)+(d_+)*(x_+)]*(b_+)+(a_+)], x_Symbol] := \operatorname{Dist}[(-2*b)/d, \operatorname{Subst}[\operatorname{Int}[1/(a+x^2), x], x, (b*\operatorname{Cot}[c+d*x])/(\operatorname{Sqrt}[a+b*\operatorname{Csc}[c+d*x]])], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{EqQ}[a^2-b^2, 0]$

Rule 3805

$\operatorname{Int}[(\operatorname{csc}[(e_+)+(f_+)*(x_+)]*(d_+))^{(n_+)}*\operatorname{Sqrt}[\operatorname{csc}[(e_+)+(f_+)*(x_+)]*(b_+)+(a_+)], x_Symbol] := \operatorname{Simp}[(a*\operatorname{Cot}[e+f*x]*(d*\operatorname{Csc}[e+f*x])^n)/(f*n*\operatorname{Sqrt}[a+b*\operatorname{Csc}[e+f*x]]), x] + \operatorname{Dist}[(a*(2*n+1))/(2*b*d*n), \operatorname{Int}[\operatorname{Sqrt}[a+b*\operatorname{Csc}[e+f*x]]*(d*\operatorname{Csc}[e+f*x])^{(n+1)}, x], x] /; \operatorname{FreeQ}\{a, b, d, e, f\}, x \ \&\& \operatorname{EqQ}[a^2-b^2, 0] \ \&\& \operatorname{LtQ}[n, -2^{(-1)}] \ \&\& \operatorname{IntegerQ}[2*n]$

Rule 4397

$\operatorname{Int}[u, x_Symbol] := \operatorname{Int}[\operatorname{TrigSimplify}[u], x] /; \operatorname{TrigSimplifyQ}[u]$

Rubi steps

$$\begin{aligned}
\int \cos^3(2(a+bx))\sqrt{c \tan(a+bx) \tan(2(a+bx))} dx &= \int \cos^3(2a+2bx)\sqrt{-c+c \sec(2a+2bx)} dx \\
&= -\frac{c \cos^2(2a+2bx) \sin(2a+2bx)}{6b\sqrt{-c+c \sec(2a+2bx)}} - \frac{5}{6} \int \cos^2(2a+2bx) \\
&= \frac{5c \cos(2a+2bx) \sin(2a+2bx)}{24b\sqrt{-c+c \sec(2a+2bx)}} - \frac{c \cos^2(2a+2bx) \sin(2a+2bx)}{6b\sqrt{-c+c \sec(2a+2bx)}} \\
&= -\frac{5c \sin(2a+2bx)}{16b\sqrt{-c+c \sec(2a+2bx)}} + \frac{5c \cos(2a+2bx) \sin(2a+2bx)}{24b\sqrt{-c+c \sec(2a+2bx)}} \\
&= -\frac{5c \sin(2a+2bx)}{16b\sqrt{-c+c \sec(2a+2bx)}} + \frac{5c \cos(2a+2bx) \sin(2a+2bx)}{24b\sqrt{-c+c \sec(2a+2bx)}} \\
&= \frac{5\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{16b} - \frac{5c \sin(2a+2bx)}{16b\sqrt{-c+c \sec(2a+2bx)}}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 116, normalized size = 0.66

$$\frac{\sqrt{c \tan(a+bx) \tan(2(a+bx))} (30 \sin(2(a+bx)) - 2 \sin(4(a+bx)) + 4 \sin(6(a+bx)) - 26 \cot(a+bx) + 15 \sqrt{c \tan(a+bx) \tan(2(a+bx))})}{96b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[2*(a+b*x)]^3*Sqrt[c*Tan[a+b*x]*Tan[2*(a+b*x)]],x]

[Out] ((-26*Cot[a+b*x] + 15*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a+b*x])/Sqrt[Cos[2*(a+b*x)]]])*Sqrt[Cos[2*(a+b*x)]]*Csc[a+b*x] + 30*Sin[2*(a+b*x)] - 2*Sin[4*(a+b*x)] + 4*Sin[6*(a+b*x)])*Sqrt[c*Tan[a+b*x]*Tan[2*(a+b*x)]]/(96*b)

fricas [A] time = 2.22, size = 481, normalized size = 2.73

$$\frac{15 \left(\tan(bx+a)^7 + 3 \tan(bx+a)^5 + 3 \tan(bx+a)^3 + \tan(bx+a) \right) \sqrt{c} \log \left(-\frac{c \tan(bx+a)^5 - 14c \tan(bx+a)^3 + 4\sqrt{2}(\tan(bx+a)^4 - 4\tan(bx+a)^2 + 3)\sqrt{-c \tan(bx+a)^2 / (\tan(bx+a)^2 - 1)} \sqrt{c} + 17c \tan(bx+a)}{\tan(bx+a)} \right)}{192 (b \tan(bx+a))^7 + 3b \tan(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")

[Out] [1/192*(15*(tan(b*x+a)^7 + 3*tan(b*x+a)^5 + 3*tan(b*x+a)^3 + tan(b*x+a))*sqrt(c)*log(-(c*tan(b*x+a)^5 - 14*c*tan(b*x+a)^3 + 4*sqrt(2)*(tan(b*x+a)^4 - 4*tan(b*x+a)^2 + 3)*sqrt(-c*tan(b*x+a)^2/(tan(b*x+a)^2 - 1))*sqrt(c) + 17*c*tan(b*x+a))/(tan(b*x+a)^5 + 2*tan(b*x+a)^3 + tan(b*x+a))) + 4*sqrt(2)*(33*tan(b*x+a)^6 - 19*tan(b*x+a)^4 - tan(b*x+a)^2 - 13)*sqrt(-c*tan(b*x+a)^2/(tan(b*x+a)^2 - 1)))/(b*tan(b*x+a)^7 + 3*b*tan(b*x+a)^5 + 3*b*tan(b*x+a)^3 + b*tan(b*x+a)), -1/96*(15*(tan(b*x+a)^7 + 3*tan(b*x+a)^5 + 3*tan(b*x+a)^3 + tan(b*x+a))*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x+a)^2/(tan(b*x+a)^2 - 1))*(tan(b*x+a)^2 - 1)*sqrt(-c)/(c*tan(b*x+a)^3 - 3*c*tan(b*x+a))) - 2*sqrt(2)*(33*tan(b*x+a)^6 - 19*tan(b*x+a)^4 - tan(b*x+a)^2 - 13)*sqrt(-c*tan(b*x+a)

$\sqrt{(\tan(bx + a)^2 - 1)) / (b \tan(bx + a)^7 + 3b \tan(bx + a)^5 + 3b \tan(bx + a)^3 + b \tan(bx + a))}$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 1.17, size = 933, normalized size = 5.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)

[Out] $\frac{1}{2} \frac{b \left(c \left(1 - \cos(bx+a)^2 \right) / \left(2 \cos(bx+a)^2 - 1 \right) \right)^{1/2} \sin(bx+a) \left(\left(2 \cos(bx+a)^2 - 1 \right) / \left(\cos(bx+a) + 1 \right)^2 \right)^{1/2} \operatorname{arctanh} \left(\frac{1}{2} \cos(bx+a) \right) 4^{1/2} \left(-1 + \cos(bx+a) \right) / \sin(bx+a)^2 / \left(\left(2 \cos(bx+a)^2 - 1 \right) / \left(\cos(bx+a) + 1 \right)^2 \right)^{1/2} 2^{1/2} / \left(-1 + \cos(bx+a) \right) 4^{1/2} - 3/8 2^{1/2} / b \sin(bx+a) \left(c \left(1 - \cos(bx+a)^2 \right) / \left(2 \cos(bx+a)^2 - 1 \right) \right)^{1/2} \left(2^{1/2} \left(\left(2 \cos(bx+a)^2 - 1 \right) / \left(\cos(bx+a) + 1 \right)^2 \right)^{1/2} \operatorname{arctanh} \left(\frac{1}{2} \cos(bx+a) \right) 4^{1/2} \left(-1 + \cos(bx+a) \right) / \sin(bx+a)^2 / \left(\left(2 \cos(bx+a)^2 - 1 \right) / \left(\cos(bx+a) + 1 \right)^2 \right)^{1/2} 2^{1/2} \right) \cos(bx+a) + 2^{1/2} \left(\left(2 \cos(bx+a)^2 - 1 \right) / \left(\cos(bx+a) + 1 \right)^2 \right)^{1/2} \operatorname{arctanh} \left(\frac{1}{2} \cos(bx+a) \right) 4^{1/2} \left(-1 + \cos(bx+a) \right) / \sin(bx+a)^2 / \left(\left(2 \cos(bx+a)^2 - 1 \right) / \left(\cos(bx+a) + 1 \right)^2 \right)^{1/2} 2^{1/2} \right) - 4 \cos(bx+a)^3 + 2 \cos(bx+a) / \left(-1 + \cos(bx+a)^2 \right) 4^{1/2} + 3/32 2^{1/2} / b \sin(bx+a) \left(c \left(1 - \cos(bx+a)^2 \right) / \left(2 \cos(bx+a)^2 - 1 \right) \right)^{1/2} \left(-16 \cos(bx+a)^5 + 3 2^{1/2} \left(\left(2 \cos(bx+a)^2 - 1 \right) / \left(\cos(bx+a) + 1 \right)^2 \right)^{1/2} \operatorname{arctanh} \left(\frac{1}{2} \cos(bx+a) \right) 4^{1/2} \left(-1 + \cos(bx+a) \right) / \sin(bx+a)^2 / \left(\left(2 \cos(bx+a)^2 - 1 \right) / \left(\cos(bx+a) + 1 \right)^2 \right)^{1/2} 2^{1/2} \right) \cos(bx+a) + 3 2^{1/2} \left(\left(2 \cos(bx+a)^2 - 1 \right) / \left(\cos(bx+a) + 1 \right)^2 \right)^{1/2} \operatorname{arctanh} \left(\frac{1}{2} \cos(bx+a) \right) 4^{1/2} \left(-1 + \cos(bx+a) \right) / \sin(bx+a)^2 / \left(\left(2 \cos(bx+a)^2 - 1 \right) / \left(\cos(bx+a) + 1 \right)^2 \right)^{1/2} 2^{1/2} \right) - 4 \cos(bx+a)^3 + 6 \cos(bx+a) / \left(-1 + \cos(bx+a)^2 \right) 4^{1/2} - 1/192 2^{1/2} / b \sin(bx+a) \left(c \left(1 - \cos(bx+a)^2 \right) / \left(2 \cos(bx+a)^2 - 1 \right) \right)^{1/2} \left(-128 \cos(bx+a)^7 - 16 \cos(bx+a)^5 + 15 2^{1/2} \left(\left(2 \cos(bx+a)^2 - 1 \right) / \left(\cos(bx+a) + 1 \right)^2 \right)^{1/2} \operatorname{arctanh} \left(\frac{1}{2} \cos(bx+a) \right) 4^{1/2} \left(-1 + \cos(bx+a) \right) / \sin(bx+a)^2 / \left(\left(2 \cos(bx+a)^2 - 1 \right) / \left(\cos(bx+a) + 1 \right)^2 \right)^{1/2} 2^{1/2} \right) \cos(bx+a) + 15 2^{1/2} \left(\left(2 \cos(bx+a)^2 - 1 \right) / \left(\cos(bx+a) + 1 \right)^2 \right)^{1/2} \operatorname{arctanh} \left(\frac{1}{2} \cos(bx+a) \right) 4^{1/2} \left(-1 + \cos(bx+a) \right) / \sin(bx+a)^2 / \left(\left(2 \cos(bx+a)^2 - 1 \right) / \left(\cos(bx+a) + 1 \right)^2 \right)^{1/2} 2^{1/2} \right) - 20 \cos(bx+a)^3 + 30 \cos(bx+a) / \left(-1 + \cos(bx+a)^2 \right) 4^{1/2}$

maxima [B] time = 1.13, size = 2333, normalized size = 13.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")

[Out] $-1/384 \left(8 \left(\cos \left(\frac{2}{3} \operatorname{arctan}^2 \left(\sin(6bx + 6a) \right), \cos(6bx + 6a) \right) \right)^2 + \sin \left(\frac{2}{3} \operatorname{arctan}^2 \left(\sin(6bx + 6a) \right), \cos(6bx + 6a) \right) \right)^2 + 2 \cos \left(\frac{2}{3} \operatorname{arctan}^2 \left(\sin(6bx + 6a) \right), \cos(6bx + 6a) \right) + 1 \right)^{3/4} \left(\cos \left(\frac{3}{2} \operatorname{arctan}^2 \left(\sin \left(\frac{2}{3} \operatorname{arctan}^2 \left(\sin(6bx + 6a) \right), \cos(6bx + 6a) \right) \right), -\cos \left(\frac{2}{3} \operatorname{arctan}^2 \left(\sin(6bx + 6a) \right), \cos(6bx + 6a) \right) \right) - 1 \right) \sin(6bx + 6a) + \left(\cos(6bx + 6a) + 1 \right) \sin \left(\frac{3}{2} \operatorname{arctan}^2 \left(\sin \left(\frac{2}{3} \operatorname{arctan}^2 \left(\sin(6bx + 6a) \right), \cos(6bx + 6a) \right) \right), -\cos \left(\frac{2}{3} \operatorname{arctan}^2 \left(\sin(6bx + 6a) \right), \cos(6bx + 6a) \right) \right) - 1 \right) \right) \sqrt{c} + 12 \cos \left(\frac{2}{3} \operatorname{arctan}^2 \left(\sin(6bx + 6a) \right), \cos(6bx + 6a) \right)$

$$\begin{aligned} & \sin(6bx + 6a), \cos(6bx + 6a))^{1/2} + \sin(2/3 \arctan 2(\sin(6bx + 6a), \\ & \cos(6bx + 6a)))^{1/2} + 2 \cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))) \\ & + 1)^{1/4} * ((\sin(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))) - 5 \sin \\ & (1/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a)))) * \cos(1/2 \arctan 2(\sin(2/3 \\ & \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))), -\cos(2/3 \arctan 2(\sin(6bx + \\ & 6a), \cos(6bx + 6a))) - 1)) + (\cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6* \\ & bx + 6a))) - 3 \cos(1/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))) - 4) * \\ & \sin(1/2 \arctan 2(\sin(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))), -\cos(\\ & 2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))) - 1))) * \sqrt{c} + 15 * \sqrt{c} \\ & * (\log(\sqrt{\cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a)))^{1/2} + \sin(2/ \\ & 3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a)))^{1/2} + 2 \cos(2/3 \arctan 2(\sin(6* \\ & bx + 6a), \cos(6bx + 6a))) + 1} * \cos(1/2 \arctan 2(\sin(2/3 \arctan 2(\sin(6*b \\ & *x + 6a), \cos(6bx + 6a))), -\cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx \\ & + 6a))) - 1))^{1/2} + \sqrt{\cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a)) \\ &)^{1/2} + \sin(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a)))^{1/2} + 2 \cos(2/3 a \\ & rctan 2(\sin(6bx + 6a), \cos(6bx + 6a))) + 1} * \sin(1/2 \arctan 2(\sin(2/3 \ar \\ & ctan 2(\sin(6bx + 6a), \cos(6bx + 6a))), -\cos(2/3 \arctan 2(\sin(6bx + 6* \\ & a), \cos(6bx + 6a))) - 1))^{1/2} + 2 * (\cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6 \\ & *bx + 6a)))^{1/2} + \sin(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a)))^{1/2} + \\ & 2 \cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))) + 1)^{1/4} * \sin(1/2 a \\ & rctan 2(\sin(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))), -\cos(2/3 \arcta \\ & n 2(\sin(6bx + 6a), \cos(6bx + 6a))) - 1)) + 1) - \log(\sqrt{\cos(2/3 \arcta \\ & n 2(\sin(6bx + 6a), \cos(6bx + 6a)))^{1/2} + \sin(2/3 \arctan 2(\sin(6bx + 6a) \\ &), \cos(6bx + 6a)))^{1/2} + 2 \cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6 \\ & *a))) + 1} * \cos(1/2 \arctan 2(\sin(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6* \\ & a))), -\cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))) - 1))^{1/2} + \sqrt{ \\ & \cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a)))^{1/2} + \sin(2/3 \arctan 2(\si \\ & n(6bx + 6a), \cos(6bx + 6a)))^{1/2} + 2 \cos(2/3 \arctan 2(\sin(6bx + 6a), \\ & \cos(6bx + 6a))) + 1} * \sin(1/2 \arctan 2(\sin(2/3 \arctan 2(\sin(6bx + 6a), c \\ & os(6bx + 6a))), -\cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))) - \\ & 1))^{1/2} - 2 * (\cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a)))^{1/2} + \sin(2/3 \\ & * \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a)))^{1/2} + 2 \cos(2/3 \arctan 2(\sin(6*b \\ & *x + 6a), \cos(6bx + 6a))) + 1)^{1/4} * \sin(1/2 \arctan 2(\sin(2/3 \arctan 2(\si \\ & n(6bx + 6a), \cos(6bx + 6a))), -\cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(\\ & 6bx + 6a))) - 1)) + 1) + \log(((\cos(1/3 \arctan 2(\sin(6bx + 6a), \cos(6*b \\ & *x + 6a)))^{1/2} + \sin(1/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a)))^{1/2} * \cos \\ & (1/2 \arctan 2(\sin(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))), -\cos(2/3 \\ & * \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))) - 1))^{1/2} + (\cos(1/3 \arctan 2(\si \\ & n(6bx + 6a), \cos(6bx + 6a)))^{1/2} + \sin(1/3 \arctan 2(\sin(6bx + 6a), co \\ & s(6bx + 6a)))^{1/2} * \sin(1/2 \arctan 2(\sin(2/3 \arctan 2(\sin(6bx + 6a), \cos(6 \\ & *bx + 6a))), -\cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))) - 1))^{1/ \\ & 2} * \sqrt{\cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a)))^{1/2} + \sin(2/3 \ar \\ & ctan 2(\sin(6bx + 6a), \cos(6bx + 6a)))^{1/2} + 2 \cos(2/3 \arctan 2(\sin(6*b \\ & *x + 6a), \cos(6bx + 6a))) + 1} + 2 * (\cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(\\ & 6bx + 6a)))^{1/2} + \sin(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a)))^{1/2} + \\ & 2 \cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))) + 1)^{1/4} * (\cos(1/2 \\ & * \arctan 2(\sin(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))), -\cos(2/3 \arc \\ & tan 2(\sin(6bx + 6a), \cos(6bx + 6a))) - 1)) * \sin(1/3 \arctan 2(\sin(6bx + \\ & 6a), \cos(6bx + 6a))) + \cos(1/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6 \\ & *a))) * \sin(1/2 \arctan 2(\sin(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))), \\ & -\cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))) - 1))) + 1) - \log(((\\ & \cos(1/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a)))^{1/2} + \sin(1/3 \arctan 2(\si \\ & n(6bx + 6a), \cos(6bx + 6a)))^{1/2} * \cos(1/2 \arctan 2(\sin(2/3 \arctan 2(\sin(6 \\ & *bx + 6a), \cos(6bx + 6a))), -\cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6*b \\ & *x + 6a))) - 1))^{1/2} + (\cos(1/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))) \\ &)^{1/2} + \sin(1/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a)))^{1/2} * \sin(1/2 \arctan \\ & 2(\sin(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx + 6a))), -\cos(2/3 \arctan 2(\si \\ & n(6bx + 6a), \cos(6bx + 6a))) - 1))^{1/2} * \sqrt{\cos(2/3 \arctan 2(\sin(6bx \\ & + 6a), \cos(6bx + 6a)))^{1/2} + \sin(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx \\ & + 6a), \cos(6bx + 6a)))^{1/2} + 2 \cos(2/3 \arctan 2(\sin(6bx + 6a), \cos(6bx \\ & + 6a), \cos(6bx + 6a))) + 1} \\ \end{aligned}$$

+ 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) + 1) - 2*(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))*sin(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))) + cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))) + 1))) / b

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(2a + 2bx)^3 \sqrt{c \tan(a + bx) \tan(2a + 2bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)

[Out] int(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2), x)

[Out] Timed out

3.611 $\int \sec^4(2(a+bx))(c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx$

Optimal. Leaf size=208

$$\frac{c^2 \tan(2a + 2bx) \sec^4(2a + 2bx)}{9b\sqrt{c} \sec(2a + 2bx) - c} - \frac{17c^2 \tan(2a + 2bx) \sec^3(2a + 2bx)}{63b\sqrt{c} \sec(2a + 2bx) - c} + \frac{34c^2 \tan(2a + 2bx)}{45b\sqrt{c} \sec(2a + 2bx) - c} + \frac{34 \tan(2a + 2bx)}{b}$$

[Out] $34/105*(-c+c*\sec(2*b*x+2*a))^(3/2)*\tan(2*b*x+2*a)/b+34/45*c^2*\tan(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^(1/2)-17/63*c^2*\sec(2*b*x+2*a)^3*\tan(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^(1/2)+1/9*c^2*\sec(2*b*x+2*a)^4*\tan(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^(1/2)+68/315*c*(-c+c*\sec(2*b*x+2*a))^(1/2)*\tan(2*b*x+2*a)/b$

Rubi [A] time = 0.53, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4397, 3814, 21, 3803, 3800, 4001, 3792}

$$\frac{c^2 \tan(2a + 2bx) \sec^4(2a + 2bx)}{9b\sqrt{c} \sec(2a + 2bx) - c} - \frac{17c^2 \tan(2a + 2bx) \sec^3(2a + 2bx)}{63b\sqrt{c} \sec(2a + 2bx) - c} + \frac{34c^2 \tan(2a + 2bx)}{45b\sqrt{c} \sec(2a + 2bx) - c} + \frac{34 \tan(2a + 2bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[2*(a + b*x)]^4*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]

[Out] $(34*c^2*\tan[2*a + 2*b*x])/(45*b*\sqrt{-c + c*\sec[2*a + 2*b*x]}) - (17*c^2*\sec[2*a + 2*b*x]^3*\tan[2*a + 2*b*x])/(63*b*\sqrt{-c + c*\sec[2*a + 2*b*x]}) + (c^2*\sec[2*a + 2*b*x]^4*\tan[2*a + 2*b*x])/(9*b*\sqrt{-c + c*\sec[2*a + 2*b*x]}) + (68*c*\sqrt{-c + c*\sec[2*a + 2*b*x]}*\tan[2*a + 2*b*x])/(315*b) + (34*(-c + c*\sec[2*a + 2*b*x])^(3/2)*\tan[2*a + 2*b*x])/(105*b)$

Rule 21

Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :> Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])

Rule 3792

Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3800

Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 3803

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n - 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] := -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 4397

```
Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned} \int \sec^4(2(a + bx))(c \tan(a + bx) \tan(2(a + bx)))^{3/2} dx &= \int \sec^4(2a + 2bx)(-c + c \sec(2a + 2bx))^{3/2} dx \\ &= \frac{c^2 \sec^4(2a + 2bx) \tan(2a + 2bx)}{9b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{1}{9}(2c) \int \frac{\sec^4(2a + 2bx)}{\sqrt{-c + c \sec(2a + 2bx)}} dx \\ &= \frac{c^2 \sec^4(2a + 2bx) \tan(2a + 2bx)}{9b\sqrt{-c + c \sec(2a + 2bx)}} - \frac{1}{9}(17c) \int \sec^4(2a + 2bx) \sqrt{-c + c \sec(2a + 2bx)} dx \\ &= -\frac{17c^2 \sec^3(2a + 2bx) \tan(2a + 2bx)}{63b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{c^2 \sec^4(2a + 2bx)}{9b\sqrt{-c + c \sec(2a + 2bx)}} \\ &= -\frac{17c^2 \sec^3(2a + 2bx) \tan(2a + 2bx)}{63b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{c^2 \sec^4(2a + 2bx)}{9b\sqrt{-c + c \sec(2a + 2bx)}} \\ &= -\frac{17c^2 \sec^3(2a + 2bx) \tan(2a + 2bx)}{63b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{c^2 \sec^4(2a + 2bx)}{9b\sqrt{-c + c \sec(2a + 2bx)}} \\ &= \frac{34c^2 \tan(2a + 2bx)}{45b\sqrt{-c + c \sec(2a + 2bx)}} - \frac{17c^2 \sec^3(2a + 2bx) \tan(2a + 2bx)}{63b\sqrt{-c + c \sec(2a + 2bx)}} \end{aligned}$$

Mathematica [A] time = 0.35, size = 85, normalized size = 0.41

$$\frac{\cot(a + bx)(c \tan(a + bx) \tan(2(a + bx)))^{3/2} (188 \cot(a + bx) \cot(2(a + bx)) + 35 \sec^3(2(a + bx)) - 50 \sec^2(2(a + bx)))}{315b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^4*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]
```

```
[Out] (Cot[a + b*x]*(-84 + 188*Cot[a + b*x]*Cot[2*(a + b*x)] + 52*Sec[2*(a + b*x)] - 50*Sec[2*(a + b*x)]^2 + 35*Sec[2*(a + b*x)]^3)*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2)/(315*b)
```

fricas [A] time = 1.05, size = 132, normalized size = 0.63

$$\frac{2\sqrt{2} \left(315c \tan(bx + a)^8 - 525c \tan(bx + a)^6 + 819c \tan(bx + a)^4 - 423c \tan(bx + a)^2 + 94c \right) \sqrt{-\frac{c \tan(bx + a)^2}{\tan(bx + a)^2 - 1}}}{315 \left(b \tan(bx + a)^9 - 4b \tan(bx + a)^7 + 6b \tan(bx + a)^5 - 4b \tan(bx + a)^3 + b \tan(bx + a) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="fricas")

[Out] $\frac{2}{315}\sqrt{2}*(315*c*\tan(b*x + a)^8 - 525*c*\tan(b*x + a)^6 + 819*c*\tan(b*x + a)^4 - 423*c*\tan(b*x + a)^2 + 94*c)*\sqrt{-c*\tan(b*x + a)^2/(\tan(b*x + a)^2 - 1)}/(b*\tan(b*x + a)^9 - 4*b*\tan(b*x + a)^7 + 6*b*\tan(b*x + a)^5 - 4*b*\tan(b*x + a)^3 + b*\tan(b*x + a))$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.12, size = 105, normalized size = 0.50

$$\frac{2\sqrt{2} \left(2176 \left(\cos^8 (bx + a) \right) - 4896 \left(\cos^6 (bx + a) \right) + 4284 \left(\cos^4 (bx + a) \right) - 1785 \left(\cos^2 (bx + a) \right) + 315 \right) \cos (bx + a)}{315b \left(2 \left(\cos^2 (bx + a) \right) - 1 \right)^3 \sin (bx + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)

[Out] $\frac{2}{315}2^{(1/2)}/b*(2176*\cos(b*x+a)^8-4896*\cos(b*x+a)^6+4284*\cos(b*x+a)^4-1785*\cos(b*x+a)^2+315)*\cos(b*x+a)*(c*\sin(b*x+a)^2/(2*\cos(b*x+a)^2-1))^{(3/2)}/(2*\cos(b*x+a)^2-1)^3/\sin(b*x+a)^3$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 10.18, size = 594, normalized size = 2.86

$$\frac{\left(\frac{c 16i}{9b} + \frac{c e^{a 2i + b x 2i} 16i}{9b}\right) \sqrt{\frac{c \left(e^{a 2i + b x 2i} 1i - i\right) \left(e^{a 4i + b x 4i} 1i - i\right)}{\left(e^{a 2i + b x 2i} + 1\right) \left(e^{a 4i + b x 4i} + 1\right)}}{\left(e^{a 2i + b x 2i} - 1\right) \left(e^{a 4i + b x 4i} + 1\right)^4} - \frac{\left(\frac{c 40i}{7b} + \frac{c e^{a 2i + b x 2i} 88i}{63b}\right) \sqrt{\frac{c \left(e^{a 2i + b x 2i} 1i - i\right) \left(e^{a 4i + b x 4i} 1i - i\right)}{\left(e^{a 2i + b x 2i} + 1\right) \left(e^{a 4i + b x 4i} + 1\right)}}{\left(e^{a 2i + b x 2i} - 1\right) \left(e^{a 4i + b x 4i} + 1\right)^3} + \frac{\left(\frac{c 24i}{5b}\right) \sqrt{\frac{c \left(e^{a 2i + b x 2i} 1i - i\right) \left(e^{a 4i + b x 4i} 1i - i\right)}{\left(e^{a 2i + b x 2i} + 1\right) \left(e^{a 4i + b x 4i} + 1\right)}}{\left(e^{a 2i + b x 2i} - 1\right) \left(e^{a 4i + b x 4i} + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)/cos(2*a + 2*b*x)^4,x)

[Out] $\frac{(((c*16i)/(9*b) + (c*\exp(a*2i + b*x*2i)*16i)/(9*b))*((c*(\exp(a*2i + b*x*2i)*1i - 1i)*(\exp(a*4i + b*x*4i)*1i - 1i))/((\exp(a*2i + b*x*2i) + 1)*(\exp(a*4i + b*x*4i) + 1)))^{(1/2)})/((\exp(a*2i + b*x*2i) - 1)*(\exp(a*4i + b*x*4i) + 1)^4) - (((c*40i)/(7*b) + (c*\exp(a*2i + b*x*2i)*88i)/(63*b))*((c*(\exp(a*2i + b*x*2i)*1i - 1i)*(\exp(a*4i + b*x*4i)*1i - 1i))/((\exp(a*2i + b*x*2i) + 1)*(\exp(a*4i + b*x*4i) + 1)^3))^{(1/2)})/((\exp(a*2i + b*x*2i) - 1)*(\exp(a*4i + b*x*4i) + 1)^3) + ((c*24i)/(5*b))*((c*(\exp(a*2i + b*x*2i)*1i - 1i)*(\exp(a*4i + b*x*4i)*1i - 1i))/((\exp(a*2i + b*x*2i) + 1)*(\exp(a*4i + b*x*4i) + 1)^2))^{(1/2)})/((\exp(a*2i + b*x*2i) - 1)*(\exp(a*4i + b*x*4i) + 1)^2)}$

```

xp(a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4
i) + 1)^3) + (((c*24i)/(5*b) - (c*exp(a*2i + b*x*2i)*176i)/(105*b))*((c*(ex
p(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i
) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4
i + b*x*4i) + 1)^2) + (c*exp(a*2i + b*x*2i))*((c*(exp(a*2i + b*x*2i)*1i - 1i
)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4
i) + 1)))^(1/2)*272i)/(315*b*(exp(a*2i + b*x*2i) - 1)) + (c*exp(a*2i + b*x*
2i))*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*
2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2)*136i)/(315*b*(exp(a*2i +
b*x*2i) - 1)*(exp(a*4i + b*x*4i) + 1))

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)**4*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)

[Out] Timed out

3.612 $\int \sec^3(2(a+bx))(c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx$

Optimal. Leaf size=148

$$\frac{76c^2 \tan(2a + 2bx)}{105b\sqrt{c \sec(2a + 2bx) - c}} + \frac{\tan(2a + 2bx)(c \sec(2a + 2bx) - c)^{5/2}}{7bc} + \frac{2 \tan(2a + 2bx)(c \sec(2a + 2bx) - c)^{3/2}}{35b} +$$

```
[Out] 2/35*(-c+c*sec(2*b*x+2*a))^(3/2)*tan(2*b*x+2*a)/b+1/7*(-c+c*sec(2*b*x+2*a))
^(5/2)*tan(2*b*x+2*a)/b/c-76/105*c^2*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))
^(1/2)+19/105*c*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b
```

Rubi [A] time = 0.35, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4397, 3800, 4001, 3793, 3792}

$$\frac{76c^2 \tan(2a + 2bx)}{105b\sqrt{c \sec(2a + 2bx) - c}} + \frac{\tan(2a + 2bx)(c \sec(2a + 2bx) - c)^{5/2}}{7bc} + \frac{2 \tan(2a + 2bx)(c \sec(2a + 2bx) - c)^{3/2}}{35b} +$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]
```

```
[Out] (-76*c^2*Tan[2*a + 2*b*x])/(105*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (19*c*Sq
rt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(105*b) + (2*(-c + c*Sec[2*a
+ 2*b*x])^(3/2)*Tan[2*a + 2*b*x])/(35*b) + ((-c + c*Sec[2*a + 2*b*x])^(5/2)
*Tan[2*a + 2*b*x])/(7*b*c)
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
/; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
+ 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 4397

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \sec^3(2(a + bx))(c \tan(a + bx) \tan(2(a + bx)))^{3/2} dx &= \int \sec^3(2a + 2bx)(-c + c \sec(2a + 2bx))^{3/2} dx \\ &= \frac{(-c + c \sec(2a + 2bx))^{5/2} \tan(2a + 2bx)}{7bc} + \frac{2 \int \sec(2a + 2bx) (-c + c \sec(2a + 2bx))^{3/2} dx}{35b} \\ &= \frac{2(-c + c \sec(2a + 2bx))^{3/2} \tan(2a + 2bx)}{35b} + \frac{(-c + c \sec(2a + 2bx))^{5/2}}{105b} \\ &= \frac{19c\sqrt{-c + c \sec(2a + 2bx)} \tan(2a + 2bx)}{105b} + \frac{2(-c + c \sec(2a + 2bx))^{5/2}}{105b} \\ &= -\frac{76c^2 \tan(2a + 2bx)}{105b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{19c\sqrt{-c + c \sec(2a + 2bx)}}{105b} \end{aligned}$$

Mathematica [A] time = 0.21, size = 73, normalized size = 0.49

$$\frac{\cot(a + bx)(c \tan(a + bx) \tan(2(a + bx)))^{3/2} (76 \cot(a + bx) \cot(2(a + bx)) - 15 \sec^2(2(a + bx)) + 24 \sec(2(a + bx)))}{105b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]

[Out] -1/105*(Cot[a + b*x]*(-28 + 76*Cot[a + b*x]*Cot[2*(a + b*x)] + 24*Sec[2*(a + b*x)] - 15*Sec[2*(a + b*x)]^2)*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))/b

fricas [A] time = 0.97, size = 111, normalized size = 0.75

$$\frac{2\sqrt{2} (105c \tan(bx + a)^6 - 140c \tan(bx + a)^4 + 133c \tan(bx + a)^2 - 38c) \sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2-1}}}{105 (b \tan(bx + a)^7 - 3b \tan(bx + a)^5 + 3b \tan(bx + a)^3 - b \tan(bx + a))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x, algorithm="fricas")

[Out] -2/105*sqrt(2)*(105*c*tan(b*x + a)^6 - 140*c*tan(b*x + a)^4 + 133*c*tan(b*x + a)^2 - 38*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^7 - 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 - b*tan(b*x + a))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.94, size = 95, normalized size = 0.64

$$\frac{2\sqrt{2} (416 (\cos^6(bx + a)) - 728 (\cos^4(bx + a)) + 455 (\cos^2(bx + a)) - 105) \cos(bx + a) \left(\frac{c(\sin^2(bx+a))}{2(\cos^2(bx+a)-1)} \right)^{\frac{3}{2}}}{105b (2 (\cos^2(bx + a)) - 1)^2 \sin(bx + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)`

[Out]
$$-2/105*2^{(1/2)}/b*(416*\cos(b*x+a)^6-728*\cos(b*x+a)^4+455*\cos(b*x+a)^2-105)*\cos(b*x+a)*(c*\sin(b*x+a)^2/(2*\cos(b*x+a)^2-1))^{(3/2)}/(2*\cos(b*x+a)^2-1)^2/\sin(b*x+a)^3$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")`

[Out] Timed out

mupad [B] time = 9.26, size = 479, normalized size = 3.24

$$\frac{\left(\frac{c 8i}{7b} - \frac{c e^{a 2i + b x 2i} 8i}{7b}\right) \sqrt{\frac{c (e^{a 2i + b x 2i} 1i - i) (e^{a 4i + b x 4i} 1i - i)}{(e^{a 2i + b x 2i} + 1) (e^{a 4i + b x 4i} + 1)}}}{(e^{a 2i + b x 2i} - 1) (e^{a 4i + b x 4i} + 1)^3} - \frac{\left(\frac{c 4i}{5b} - \frac{c e^{a 2i + b x 2i} 92i}{35b}\right) \sqrt{\frac{c (e^{a 2i + b x 2i} 1i - i) (e^{a 4i + b x 4i} 1i - i)}{(e^{a 2i + b x 2i} + 1) (e^{a 4i + b x 4i} + 1)}}}{(e^{a 2i + b x 2i} - 1) (e^{a 4i + b x 4i} + 1)^2} - \frac{\left(\frac{c 4i}{3b} + \frac{c e^{a 2i + b x 2i} 104i}{105b}\right) \sqrt{\frac{c (e^{a 2i + b x 2i} 1i - i) (e^{a 4i + b x 4i} 1i - i)}{(e^{a 2i + b x 2i} + 1) (e^{a 4i + b x 4i} + 1)}}}{(e^{a 2i + b x 2i} - 1) (e^{a 4i + b x 4i} + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)/cos(2*a + 2*b*x)^3,x)`

[Out]
$$\left(\frac{c 8i}{7b} - \frac{c \exp(a 2i + b x 2i) 8i}{7b}\right) \left(\frac{c (\exp(a 2i + b x 2i) * 1i - 1i) (\exp(a 4i + b x 4i) * 1i - 1i)}{(\exp(a 2i + b x 2i) + 1) (\exp(a 4i + b x 4i) + 1)}\right)^{(1/2)} / \left(\frac{c (\exp(a 2i + b x 2i) - 1) (\exp(a 4i + b x 4i) + 1)^3}{(\exp(a 2i + b x 2i) + 1) (\exp(a 4i + b x 4i) + 1)}\right)^{(1/2)} - \left(\frac{c 4i}{5b} - \frac{c \exp(a 2i + b x 2i) 92i}{35b}\right) \left(\frac{c (\exp(a 2i + b x 2i) * 1i - 1i) (\exp(a 4i + b x 4i) * 1i - 1i)}{(\exp(a 2i + b x 2i) + 1) (\exp(a 4i + b x 4i) + 1)}\right)^{(1/2)} / \left(\frac{c (\exp(a 2i + b x 2i) - 1) (\exp(a 4i + b x 4i) + 1)^2}{(\exp(a 2i + b x 2i) + 1) (\exp(a 4i + b x 4i) + 1)}\right)^{(1/2)} - \left(\frac{c 4i}{3b} + \frac{c \exp(a 2i + b x 2i) 52i}{105b}\right) \left(\frac{c (\exp(a 2i + b x 2i) * 1i - 1i) (\exp(a 4i + b x 4i) * 1i - 1i)}{(\exp(a 2i + b x 2i) + 1) (\exp(a 4i + b x 4i) + 1)}\right)^{(1/2)} / \left(\frac{c (\exp(a 2i + b x 2i) - 1) (\exp(a 4i + b x 4i) + 1)}{(\exp(a 2i + b x 2i) + 1) (\exp(a 4i + b x 4i) + 1)}\right)^{(1/2)} - \left(\frac{c \exp(a 2i + b x 2i) (c (\exp(a 2i + b x 2i) * 1i - 1i) (\exp(a 4i + b x 4i) * 1i - 1i))}{(\exp(a 2i + b x 2i) + 1) (\exp(a 4i + b x 4i) + 1)}\right)^{(1/2)} * 104i / (105 * b * (\exp(a 2i + b x 2i) - 1))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)`

[Out] Timed out

3.613 $\int \sec^2(2(a+bx))(c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx$

Optimal. Leaf size=110

$$\frac{4c^2 \tan(2a + 2bx)}{5b\sqrt{c \sec(2a + 2bx) - c}} - \frac{c \tan(2a + 2bx)\sqrt{c \sec(2a + 2bx) - c}}{5b} + \frac{\tan(2a + 2bx)(c \sec(2a + 2bx) - c)^{3/2}}{5b}$$

[Out] $1/5*(-c+c*\sec(2*b*x+2*a))^(3/2)*\tan(2*b*x+2*a)/b+4/5*c^2*\tan(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^(1/2)-1/5*c*(-c+c*\sec(2*b*x+2*a))^(1/2)*\tan(2*b*x+2*a)/b$

Rubi [A] time = 0.27, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4397, 3798, 3793, 3792}

$$\frac{4c^2 \tan(2a + 2bx)}{5b\sqrt{c \sec(2a + 2bx) - c}} - \frac{c \tan(2a + 2bx)\sqrt{c \sec(2a + 2bx) - c}}{5b} + \frac{\tan(2a + 2bx)(c \sec(2a + 2bx) - c)^{3/2}}{5b}$$

Antiderivative was successfully verified.

[In] `Int[Sec[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]`

[Out] $(4*c^2*\tan[2*a + 2*b*x])/(5*b*\sqrt{-c + c*\sec[2*a + 2*b*x]}) - (c*\sqrt{-c + c*\sec[2*a + 2*b*x]}*\tan[2*a + 2*b*x])/(5*b) + ((-c + c*\sec[2*a + 2*b*x])^(3/2)*\tan[2*a + 2*b*x])/(5*b)$

Rule 3792

`Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]`

Rule 3793

`Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] + Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[2*m]`

Rule 3798

`Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]`

Rule 4397

`Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]`

Rubi steps

$$\begin{aligned}
\int \sec^2(2(a+bx))(c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx &= \int \sec^2(2a+2bx)(-c+c \sec(2a+2bx))^{3/2} dx \\
&= \frac{(-c+c \sec(2a+2bx))^{3/2} \tan(2a+2bx)}{5b} - \frac{3}{5} \int \sec(2a+2bx) (-c+c \sec(2a+2bx))^{3/2} dx \\
&= -\frac{c\sqrt{-c+c \sec(2a+2bx)} \tan(2a+2bx)}{5b} + \frac{(-c+c \sec(2a+2bx))^{3/2}}{5b} \\
&= \frac{4c^2 \tan(2a+2bx)}{5b\sqrt{-c+c \sec(2a+2bx)}} - \frac{c\sqrt{-c+c \sec(2a+2bx)}}{5b}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 59, normalized size = 0.54

$$\frac{\cot(a+bx)(c \tan(a+bx) \tan(2(a+bx)))^{3/2}(4 \cot(a+bx) \cot(2(a+bx)) + \sec(2(a+bx)) - 2)}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]
[Out] (Cot[a + b*x]*(-2 + 4*Cot[a + b*x]*Cot[2*(a + b*x)] + Sec[2*(a + b*x)])*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2)/(5*b)

fricas [A] time = 0.90, size = 88, normalized size = 0.80

$$\frac{2\sqrt{2}(5c \tan(bx+a)^4 - 5c \tan(bx+a)^2 + 2c)\sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2-1}}}{5(b \tan(bx+a)^5 - 2b \tan(bx+a)^3 + b \tan(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x, algorithm="fricas")
[Out] 2/5*sqrt(2)*(5*c*tan(b*x + a)^4 - 5*c*tan(b*x + a)^2 + 2*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^5 - 2*b*tan(b*x + a)^3 + b*tan(b*x + a))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x, algorithm="giac")
[Out] Timed out

maple [A] time = 0.86, size = 85, normalized size = 0.77

$$\frac{2\sqrt{2} (12 (\cos^4(bx+a)) - 15 (\cos^2(bx+a)) + 5) \cos(bx+a) \left(\frac{c(\sin^2(bx+a))}{2(\cos^2(bx+a)-1)}\right)^{\frac{3}{2}}}{5b(2(\cos^2(bx+a)) - 1) \sin(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x)

[Out] $\frac{2}{5} \cdot 2^{1/2} / b \cdot (12 \cos(bx+a)^4 - 15 \cos(bx+a)^2 + 5) \cdot \cos(bx+a) \cdot (c \sin(bx+a))^2 / (2 \cos(bx+a)^2 - 1)^{3/2} / (2 \cos(bx+a)^2 - 1) / \sin(bx+a)^3$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")`

[Out] Timed out

mupad [B] time = 11.04, size = 149, normalized size = 1.35

$$\frac{2c \left(e^{a4i+bx4i} 5i + e^{a6i+bx6i} 5i + e^{a10i+bx10i} 3i + 3i \right) \sqrt{\frac{c \left(e^{a2i+bx2i} 1i - i \right) \left(e^{a4i+bx4i} 1i - i \right)}{\left(e^{a2i+bx2i} + 1 \right) \left(e^{a4i+bx4i} + 1 \right)}}}{5b \left(e^{a2i+bx2i} - 1 \right) \left(e^{a4i+bx4i} + 1 \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)/cos(2*a + 2*b*x)^2,x)`

[Out] $(2c \cdot (\exp(a \cdot 4i + b \cdot x \cdot 4i) \cdot 5i + \exp(a \cdot 6i + b \cdot x \cdot 6i) \cdot 5i + \exp(a \cdot 10i + b \cdot x \cdot 10i) \cdot 3i + 3i) \cdot ((c \cdot (\exp(a \cdot 2i + b \cdot x \cdot 2i) \cdot 1i - 1i) \cdot (\exp(a \cdot 4i + b \cdot x \cdot 4i) \cdot 1i - 1i)) / ((\exp(a \cdot 2i + b \cdot x \cdot 2i) + 1) \cdot (\exp(a \cdot 4i + b \cdot x \cdot 4i) + 1)))^{1/2}) / (5 \cdot b \cdot (\exp(a \cdot 2i + b \cdot x \cdot 2i) - 1) \cdot (\exp(a \cdot 4i + b \cdot x \cdot 4i) + 1)^2)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)`

[Out] Timed out

3.614 $\int \sec(2(a+bx))(c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx$

Optimal. Leaf size=75

$$\frac{c \tan(2a + 2bx) \sqrt{c \sec(2a + 2bx) - c}}{3b} - \frac{4c^2 \tan(2a + 2bx)}{3b \sqrt{c \sec(2a + 2bx) - c}}$$

[Out] $-4/3*c^2*\tan(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}+1/3*c*(-c+c*\sec(2*b*x+2*a))^{(1/2)}*\tan(2*b*x+2*a)/b$

Rubi [A] time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4397, 3793, 3792}

$$\frac{c \tan(2a + 2bx) \sqrt{c \sec(2a + 2bx) - c}}{3b} - \frac{4c^2 \tan(2a + 2bx)}{3b \sqrt{c \sec(2a + 2bx) - c}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[2*(a + b*x)]*(c*\text{Tan}[a + b*x]*\text{Tan}[2*(a + b*x)])^{(3/2)}, x]$

[Out] $(-4*c^2*\text{Tan}[2*a + 2*b*x])/(3*b*\text{Sqrt}[-c + c*\text{Sec}[2*a + 2*b*x]]) + (c*\text{Sqrt}[-c + c*\text{Sec}[2*a + 2*b*x]]*\text{Tan}[2*a + 2*b*x])/(3*b)$

Rule 3792

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]*\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> \text{Simp}[(-2*b*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]), x] /;$ FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3793

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}, x_Symbol] :> -\text{Simp}[(b*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m - 1)})/(f*m), x] + \text{Dist}[(a*(2*m - 1))/m, \text{Int}[\text{Csc}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m - 1)}, x], x] /;$ FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[2*m]

Rule 4397

$\text{Int}[u_, x_Symbol] :> \text{Int}[\text{TrigSimplify}[u], x] /;$ TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \sec(2(a+bx))(c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx &= \int \sec(2a + 2bx)(-c + c \sec(2a + 2bx))^{3/2} dx \\ &= \frac{c \sqrt{-c + c \sec(2a + 2bx)} \tan(2a + 2bx)}{3b} - \frac{1}{3}(4c) \int \sec(2a + 2bx) dx \\ &= -\frac{4c^2 \tan(2a + 2bx)}{3b \sqrt{-c + c \sec(2a + 2bx)}} + \frac{c \sqrt{-c + c \sec(2a + 2bx)}}{3b} \end{aligned}$$

Mathematica [A] time = 0.16, size = 51, normalized size = 0.68

$$\frac{\cot(a + bx)(4 \cot(a + bx) \cot(2(a + bx)) - 1)(c \tan(a + bx) \tan(2(a + bx)))^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]

[Out] $-1/3*(\text{Cot}[a + b*x]*(-1 + 4*\text{Cot}[a + b*x]*\text{Cot}[2*(a + b*x)]))*(c*\text{Tan}[a + b*x]*\text{Tan}[2*(a + b*x)])^(3/2))/b$

fricas [A] time = 0.88, size = 67, normalized size = 0.89

$$\frac{2\sqrt{2}\left(3c\tan(bx+a)^2-2c\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{3\left(b\tan(bx+a)^3-b\tan(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="fricas")

[Out] $-2/3*\text{sqrt}(2)*(3*c*\text{tan}(b*x + a)^2 - 2*c)*\text{sqrt}(-c*\text{tan}(b*x + a)^2/(\text{tan}(b*x + a)^2 - 1))/(b*\text{tan}(b*x + a)^3 - b*\text{tan}(b*x + a))$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.83, size = 61, normalized size = 0.81

$$\frac{2\sqrt{2}\left(5\left(\cos^2(bx+a)\right)-3\right)\cos(bx+a)\left(\frac{c\left(\sin^2(bx+a)\right)}{2\left(\cos^2(bx+a)\right)-1}\right)^{\frac{3}{2}}}{3b\sin(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)

[Out] $-2/3*2^{(1/2)}/b*(5*\cos(b*x+a)^2-3)*\cos(b*x+a)*(c*\sin(b*x+a)^2/(2*\cos(b*x+a)^2-1))^(3/2)/\sin(b*x+a)^3$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 7.87, size = 158, normalized size = 2.11

$$\frac{c\left(e^{a2i+bx2i}3i+e^{a4i+bx4i}3i+e^{a6i+bx6i}5i+5i\right)\sqrt{\frac{c\left(e^{a2i+bx2i}1i-i\right)\left(e^{a4i+bx4i}1i-i\right)}{\left(e^{a2i+bx2i}+1\right)\left(e^{a4i+bx4i}+1\right)}}{3b\left(e^{a2i+bx2i}-e^{a4i+bx4i}+e^{a6i+bx6i}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)/cos(2*a + 2*b*x),x)
```

```
[Out] -(c*(exp(a*2i + b*x*2i)*3i + exp(a*4i + b*x*4i)*3i + exp(a*6i + b*x*6i)*5i
+ 5i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(
a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/(3*b*(exp(a*2i + b*x*
2i) - exp(a*4i + b*x*4i) + exp(a*6i + b*x*6i) - 1))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

3.615 $\int (c \tan(a + bx) \tan(2(a + bx)))^{3/2} dx$

Optimal. Leaf size=80

$$\frac{c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{b} + \frac{c^2 \tan(2a + 2bx)}{b\sqrt{c \sec(2a + 2bx) - c}}$$

[Out] $c^{(3/2)} \operatorname{arctanh}(c^{(1/2)} \tan(2 * b * x + 2 * a) / (-c + c * \sec(2 * b * x + 2 * a))^{(1/2)}) / b + c^{2 * \tan(2 * b * x + 2 * a)} / b / (-c + c * \sec(2 * b * x + 2 * a))^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4397, 3775, 21, 3774, 207}

$$\frac{c^2 \tan(2a + 2bx)}{b\sqrt{c \sec(2a + 2bx) - c}} + \frac{c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c * \text{Tan}[a + b * x] * \text{Tan}[2 * (a + b * x)])^{(3/2)}, x]$

[Out] $(c^{(3/2)} * \text{ArcTanh}[(\text{Sqrt}[c] * \text{Tan}[2 * a + 2 * b * x]) / \text{Sqrt}[-c + c * \text{Sec}[2 * a + 2 * b * x]]]) / b + (c^{2 * \text{Tan}[2 * a + 2 * b * x]} / (b * \text{Sqrt}[-c + c * \text{Sec}[2 * a + 2 * b * x]]))$

Rule 21

$\text{Int}[(u_.) * ((a_.) + (b_.) * (v_.))^{(m_.)} * ((c_.) + (d_.) * (v_.))^{(n_.)}, x_Symbol] :>$
 $\text{Dist}[(b/d)^m, \text{Int}[u * (c + d * v)^{(m + n)}, x], x] /;$ $\text{FreeQ}\{a, b, c, d, n\}, x$
 $\&\& \text{EqQ}[b * c - a * d, 0] \&\& \text{IntegerQ}[m] \&\& (!\text{IntegerQ}[n] \|\| \text{SimplerQ}[c + d * x,$
 $a + b * x])$

Rule 207

$\text{Int}(((a_.) + (b_.) * (x_)^2)^{-1}, x_Symbol] :> -\text{Simp}[\text{ArcTanh}[(\text{Rt}[b, 2] * x) / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] * \text{Rt}[b, 2]), x] /;$ $\text{FreeQ}\{a, b\}, x \&\& \text{NegQ}[a/b] \&\& (\text{LtQ}[a,$
 $0] \|\| \text{GtQ}[b, 0])$

Rule 3774

$\text{Int}[\text{Sqrt}[\text{csc}[(c_.) + (d_.) * (x_)] * (b_.) + (a_.)], x_Symbol] :> \text{Dist}[(-2 * b) / d,$
 $\text{Subst}[\text{Int}[1 / (a + x^2), x], x, (b * \text{Cot}[c + d * x]) / \text{Sqrt}[a + b * \text{Csc}[c + d * x]]],$
 $x] /;$ $\text{FreeQ}\{a, b, c, d\}, x \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 3775

$\text{Int}[(\text{csc}[(c_.) + (d_.) * (x_)] * (b_.) + (a_.))^{(n_.)}, x_Symbol] :> -\text{Simp}[(b^2 * \text{Cot}[c + d * x] * (a + b * \text{Csc}[c + d * x])^{(n - 2)}) / (d * (n - 1)), x] + \text{Dist}[a / (n - 1),$
 $\text{Int}[(a + b * \text{Csc}[c + d * x])^{(n - 2)} * (a * (n - 1) + b * (3 * n - 4) * \text{Csc}[c + d * x]), x]$
 $, x] /;$ $\text{FreeQ}\{a, b, c, d\}, x \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2 * n]$

Rule 4397

$\text{Int}[u_, x_Symbol] :> \text{Int}[\text{TrigSimplify}[u], x] /;$ $\text{TrigSimplifyQ}[u]$

Rubi steps

$$\begin{aligned}
\int (c \tan(a + bx) \tan(2(a + bx)))^{3/2} dx &= \int (-c + c \sec(2a + 2bx))^{3/2} dx \\
&= \frac{c^2 \tan(2a + 2bx)}{b\sqrt{-c + c \sec(2a + 2bx)}} - (2c) \int \frac{-\frac{c}{2} + \frac{1}{2}c \sec(2a + 2bx)}{\sqrt{-c + c \sec(2a + 2bx)}} dx \\
&= \frac{c^2 \tan(2a + 2bx)}{b\sqrt{-c + c \sec(2a + 2bx)}} - c \int \sqrt{-c + c \sec(2a + 2bx)} dx \\
&= \frac{c^2 \tan(2a + 2bx)}{b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{c^2 \text{Subst}\left(\int \frac{1}{-c+x^2} dx, x, -\frac{c \tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{b} \\
&= \frac{c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{b} + \frac{c^2 \tan(2a + 2bx)}{b\sqrt{-c + c \sec(2a + 2bx)}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 86, normalized size = 1.08

$$\frac{c\sqrt{\tan(a+bx)\tan(2(a+bx))}\left(2\cot(a+bx)+\sqrt{2}\sqrt{\cos(2(a+bx))}\csc(a+bx)\tanh^{-1}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]

[Out] (c*(2*Cot[a + b*x] + Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a + b*x)])]*Sqrt[Cos[2*(a + b*x)]]*Csc[a + b*x])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)])/(2*b)

fricas [A] time = 1.03, size = 296, normalized size = 3.70

$$\left[\frac{c^{\frac{3}{2}} \log\left(\frac{c \tan(bx+a)^5 - 14c \tan(bx+a)^3 + 4\sqrt{2}(\tan(bx+a)^4 - 4\tan(bx+a)^2 + 3)\sqrt{\frac{c \tan(bx+a)^2}{\tan(bx+a)^2 - 1}} \sqrt{c + 17c \tan(bx+a)}}{\tan(bx+a)^5 + 2\tan(bx+a)^3 + \tan(bx+a)} \right)}{4b \tan(bx+a)} \right] \tan(bx+a) + 4\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x, algorithm="fricas")

[Out] [1/4*(c^(3/2)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 + 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*tan(b*x + a) + 4*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*c)/(b*tan(b*x + a)), -1/2*(sqrt(-c)*c*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a)))*tan(b*x + a) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*c)/(b*tan(b*x + a))]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.95, size = 253, normalized size = 3.16

$$\frac{\sqrt{2} \left(2 \cos^2(bx+a) - 1\right) \left(\sqrt{2} \sqrt{\frac{2(\cos^2(bx+a)-1)}{(\cos(bx+a)+1)^2}} \operatorname{arctanh} \left(\frac{\cos(bx+a)\sqrt{4}(-1+\cos(bx+a))\sqrt{2}}{2 \sin(bx+a)^2 \sqrt{\frac{2(\cos^2(bx+a)-1)}{(\cos(bx+a)+1)^2}}} \right) \cos(bx+a) + \sqrt{2} \sqrt{\frac{2(\cos^2(bx+a)-1)}{(\cos(bx+a)+1)^2}} \right)}{b \sin(bx+a)^3 (2 + \sqrt{2}) (\sqrt{2} - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)

[Out] $2^{(1/2)}/b*(2*\cos(b*x+a)^2-1)*(2^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)})*\cos(b*x+a)+2^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)})-2*\cos(b*x+a)*(c*\sin(b*x+a)^2/(2*\cos(b*x+a)^2-1))^{(3/2)}/\sin(b*x+a)^3/(2+2^{(1/2)})/(2^{(1/2)}-2)$

maxima [B] time = 0.89, size = 1317, normalized size = 16.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")

[Out] $-1/8*((\cos(4*b*x + 4*a)^2 + \sin(4*b*x + 4*a)^2 + 2*\cos(4*b*x + 4*a) + 1)^{(1/4)}*(c*\log(\sqrt{\cos(4*b*x + 4*a)^2 + \sin(4*b*x + 4*a)^2 + 2*\cos(4*b*x + 4*a) + 1})*\cos(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1))^2 + \sqrt{\cos(4*b*x + 4*a)^2 + \sin(4*b*x + 4*a)^2 + 2*\cos(4*b*x + 4*a) + 1}*\sin(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1))^2 + 2*(\cos(4*b*x + 4*a)^2 + \sin(4*b*x + 4*a)^2 + 2*\cos(4*b*x + 4*a) + 1)^{(1/4)}*\sin(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1)) + 1) - c*\log(\sqrt{\cos(4*b*x + 4*a)^2 + \sin(4*b*x + 4*a)^2 + 2*\cos(4*b*x + 4*a) + 1})*\cos(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1))^2 + \sqrt{\cos(4*b*x + 4*a)^2 + \sin(4*b*x + 4*a)^2 + 2*\cos(4*b*x + 4*a) + 1}*\sin(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1))^2 - 2*(\cos(4*b*x + 4*a)^2 + \sin(4*b*x + 4*a)^2 + 2*\cos(4*b*x + 4*a) + 1)^{(1/4)}*\sin(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1)) + 1) + c*\log(((\cos(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1))^2 + \sin(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1))^2)*\cos(1/2*\arctan2(\sin(4*b*x + 4*a), \cos(4*b*x + 4*a)))^2 + (\cos(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1))^2 + \sin(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1))^2)*\sin(1/2*\arctan2(\sin(4*b*x + 4*a), \cos(4*b*x + 4*a)))^2)*\sqrt{\cos(4*b*x + 4*a)^2 + \sin(4*b*x + 4*a)^2 + 2*\cos(4*b*x + 4*a) + 1} + 2*(\cos(4*b*x + 4*a)^2 + \sin(4*b*x + 4*a)^2 + 2*\cos(4*b*x + 4*a) + 1)^{(1/4)}*(\cos(1/2*\arctan2(\sin(4*b*x + 4*a), \cos(4*b*x + 4*a)))*\sin(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1)) + \cos(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1))*\sin(1/2*\arctan2(\sin(4*b*x + 4*a), \cos(4*b*x + 4*a)))) + 1) - c*\log(((\cos(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1))^2 + \sin(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1))^2)*\cos(1/2*\arctan2(\sin(4*b*x + 4*a), \cos(4*b*x + 4*a)))^2 + (\cos(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1))^2 + \sin(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1))^2)*\sin(1/2*\arctan2(\sin(4*b*x + 4*a), \cos(4*b*x + 4*a)))^2)*\sqrt{\cos(4*b*x + 4*a)^2 + \sin(4*b*x + 4*a)^2 + 2*\cos(4*b*x + 4*a) + 1} - 2*(\cos(4*b*x + 4*a)^2 + \sin(4*b*x + 4*a)^2 + 2*\cos(4*b*x + 4*a) + 1)^{(1/4)}*(\cos(1/2*\arctan2(\sin(4*b*x + 4*a), \cos(4*b*x + 4*a)))*\sin(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1)) + \cos(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1))*\sin(1/2*\arctan2(\sin(4*b*x + 4*a), \cos(4*b*x + 4*a)))) + 1)$


```

/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))*sin(1/2*arctan2(sin(4*b*x +
4*a), -cos(4*b*x + 4*a) - 1)) + cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b
*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))) + 1))
*sqrt(c) + 8*(c*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))*sin(1/
2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + c*cos(1/2*arctan2(sin
(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), co
s(4*b*x + 4*a))) + c*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) -
1)))*sqrt(c))/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a
) + 1)^(1/4)*b)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c \tan(a + b x) \tan(2 a + 2 b x))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

```
[Out] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2), x)
```

```
[Out] Timed out
```

3.616 $\int \cos(2(a+bx))(c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx$

Optimal. Leaf size=86

$$\frac{c^2 \sin(2a + 2bx)}{2b\sqrt{c \sec(2a + 2bx) - c}} - \frac{3c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{2b}$$

[Out] $-3/2*c^{(3/2)*\arctanh(c^{(1/2)*\tan(2*b*x+2*a)/(-c+c*\sec(2*b*x+2*a))^{(1/2)})/b+1/2*c^2*\sin(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}$

Rubi [A] time = 0.22, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4397, 3814, 21, 3805, 3774, 207}

$$\frac{c^2 \sin(2a + 2bx)}{2b\sqrt{c \sec(2a + 2bx) - c}} - \frac{3c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Cos[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]

[Out] $(-3*c^{(3/2)*\text{ArcTanh}[(\text{Sqrt}[c]*\text{Tan}[2*a + 2*b*x])/\text{Sqrt}[-c + c*\text{Sec}[2*a + 2*b*x]]])/(2*b) + (c^2*\text{Sin}[2*a + 2*b*x])/(2*b*\text{Sqrt}[-c + c*\text{Sec}[2*a + 2*b*x]])$

Rule 21

Int[(u_.)*((a_.) + (b_.)*(v_))^(m_.)*((c_.) + (d_.)*(v_))^(n_.), x_Symbol] :> Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])

Rule 207

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3805

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]

Rule 3814

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,

0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]

Rule 4397

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned}
 \int \cos(2(a + bx))(c \tan(a + bx) \tan(2(a + bx)))^{3/2} dx &= \int \cos(2a + 2bx)(-c + c \sec(2a + 2bx))^{3/2} dx \\
 &= -\frac{c^2 \sin(2a + 2bx)}{b\sqrt{-c + c \sec(2a + 2bx)}} - (2c) \int \frac{\cos(2a + 2bx) \left(-\frac{3}{2}\right)}{\sqrt{-c + c \sec(2a + 2bx)}} dx \\
 &= -\frac{c^2 \sin(2a + 2bx)}{b\sqrt{-c + c \sec(2a + 2bx)}} - (3c) \int \cos(2a + 2bx) \sqrt{-c + c \sec(2a + 2bx)} dx \\
 &= \frac{c^2 \sin(2a + 2bx)}{2b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{1}{2}(3c) \int \sqrt{-c + c \sec(2a + 2bx)} dx \\
 &= \frac{c^2 \sin(2a + 2bx)}{2b\sqrt{-c + c \sec(2a + 2bx)}} - \frac{(3c^2) \text{Subst}\left(\int \frac{1}{-c+x^2} dx, x, \sqrt{-c + c \sec(2a + 2bx)}\right)}{2b} \\
 &= -\frac{3c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{2b} + \frac{c^2 \sin(2a + 2bx)}{2b\sqrt{-c + c \sec(2a + 2bx)}}
 \end{aligned}$$

Mathematica [A] time = 0.24, size = 93, normalized size = 1.08

$$\frac{c \csc(a + bx) \sqrt{c \tan(a + bx) \tan(2(a + bx))} \left(\cos(a + bx) + \cos(3(a + bx)) - 3\sqrt{2} \sqrt{\cos(2(a + bx))} \tanh^{-1}\left(\frac{\sqrt{2} \sqrt{\cos(2(a + bx))}}{\sqrt{c \tan(a + bx) \tan(2(a + bx))}}\right) \right)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]

[Out] (c*(Cos[a + b*x] - 3*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a + b*x)]]])*Sqrt[Cos[2*(a + b*x)]] + Cos[3*(a + b*x)]*Csc[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]]/(4*b)

fricas [B] time = 2.09, size = 369, normalized size = 4.29

$$\left[\frac{3 \left(c \tan(bx + a)^3 + c \tan(bx + a) \right) \sqrt{c} \log \left(-\frac{c \tan(bx+a)^5 - 14c \tan(bx+a)^3 - 4\sqrt{2} \left(\tan(bx+a)^4 - 4 \tan(bx+a)^2 + 3 \right) \sqrt{\frac{c \tan(bx+a)^2}{\tan(bx+a)^2 - 1}}}{\tan(bx+a)^5 + 2 \tan(bx+a)^3 + \tan(bx+a)}}{8 \left(b \tan(bx + a)^3 + b \tan(bx + a) \right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x, algorithm="fricas")

[Out] [1/8*(3*(c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 - 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3))*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(t

```
an(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) - 4*sqrt(2)*(c*tan(b*x +
a)^2 - c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^3 +
b*tan(b*x + a)), 1/4*(3*(c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(-c)*arctan
n(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 -
1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*(c*tan(b*x +
a)^2 - c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^3
+ b*tan(b*x + a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 1.09, size = 518, normalized size = 6.02

$$\frac{\sqrt{2} \left(2 \left(\cos^2(bx+a)\right) - 1\right) \left(\sqrt{2} \sqrt{\frac{2\left(\cos^2(bx+a)\right) - 1}{\left(\cos(bx+a)+1\right)^2}} \operatorname{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}(-1+\cos(bx+a))\sqrt{2}}{2 \sin(bx+a)^2 \sqrt{\frac{2\left(\cos^2(bx+a)\right) - 1}{\left(\cos(bx+a)+1\right)^2}}}\right) \cos(bx+a) + \sqrt{2} \sqrt{\frac{2\left(\cos^2(bx+a)\right) - 1}{\left(\cos(bx+a)+1\right)^2}}\right)}{b \sin(bx+a)^3 \left(2 + \sqrt{2}\right) \left(\sqrt{2} - \sqrt{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)

[Out]
$$\begin{aligned} & -2^{(1/2)}/b*(2*\cos(b*x+a)^2-1)*(2^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)})*\cos(b*x+a)+2^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)})-2*\cos(b*x+a)*(c*\sin(b*x+a)^2/(2*\cos(b*x+a)^2-1))^{(3/2)}/\sin(b*x+a)^3/(2+2^{(1/2)})/(2^{(1/2)}-2)-2*2^{(1/2)}/b*(2*\cos(b*x+a)^2-1)*(2^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)})*\cos(b*x+a)+2^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)})+4*\cos(b*x+a)^3+2*\cos(b*x+a)*(c*\sin(b*x+a)^2/(2*\cos(b*x+a)^2-1))^{(3/2)}/\sin(b*x+a)^3/(2+2^{(1/2)})^3/(2^{(1/2)}-2)^3 \end{aligned}$$

maxima [B] time = 1.15, size = 1058, normalized size = 12.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/16*(4*(c*\cos(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1))*\sin(2*b*x + 2*a) + (c*\cos(2*b*x + 2*a) + c)*\sin(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1)))*(\cos(4*b*x + 4*a)^2 + \sin(4*b*x + 4*a)^2 + 2*\cos(4*b*x + 4*a) + 1)^{(1/4)}*\sqrt{c} - 3*(c*\log(\sqrt{\cos(4*b*x + 4*a)^2 + \sin(4*b*x + 4*a)^2 + 2*\cos(4*b*x + 4*a) + 1})*\cos(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1))^2 + \sqrt{\cos(4*b*x + 4*a)^2 + \sin(4*b*x + 4*a)^2 + 2*\cos(4*b*x + 4*a) + 1})*\sin(1/2*\arctan2(\sin(4*b*x + 4*a), -\cos(4*b*x + 4*a) - 1)) \end{aligned}$$

```

)^2 + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(
(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - c*log
(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*co
s(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))^2 + sqrt(cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin
(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*
x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a)
, -cos(4*b*x + 4*a) - 1)) + 1) + c*log((((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))^2 + (cos(2
*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(
4*b*x + 4*a) - 1)))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(
4*b*x + 4*a) + 1) + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*
x + 4*a) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) -
1))*sin(2*b*x + 2*a) + cos(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -
cos(4*b*x + 4*a) - 1)))) + 1) - c*log((((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))^2 + (cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
b*x + 4*a) - 1)))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*
b*x + 4*a) + 1) - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x
+ 4*a) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)
)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -co
s(4*b*x + 4*a) - 1)))) + 1)))*sqrt(c))/b

```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(2a + 2bx) (c \tan(a + bx) \tan(2a + 2bx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
```

```
[Out] int(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] Timed out
```

3.617 $\int \cos^2(2(a+bx))(c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx$

Optimal. Leaf size=133

$$\frac{7c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{8b} - \frac{7c^2 \sin(2a+2bx)}{8b\sqrt{c \sec(2a+2bx)-c}} + \frac{c^2 \sin(2a+2bx) \cos(2a+2bx)}{4b\sqrt{c \sec(2a+2bx)-c}}$$

[Out] $7/8*c^{(3/2)}*\operatorname{arctanh}(c^{(1/2)}*\tan(2*b*x+2*a)/(-c+c*\sec(2*b*x+2*a))^{(1/2)})/b-7/8*c^2*\sin(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}+1/4*c^2*\cos(2*b*x+2*a)*\sin(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}$

Rubi [A] time = 0.26, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4397, 3813, 21, 3805, 3774, 207}

$$-\frac{7c^2 \sin(2a+2bx)}{8b\sqrt{c \sec(2a+2bx)-c}} + \frac{c^2 \sin(2a+2bx) \cos(2a+2bx)}{4b\sqrt{c \sec(2a+2bx)-c}} + \frac{7c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{8b}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cos}[2*(a + b*x)]^2*(c*\operatorname{Tan}[a + b*x]*\operatorname{Tan}[2*(a + b*x)])^{(3/2)}, x]$

[Out] $(7*c^{(3/2)}*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\operatorname{Tan}[2*a + 2*b*x])/(\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]])]/(8*b) - (7*c^2*\operatorname{Sin}[2*a + 2*b*x])/((8*b*\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]]) + (c^2*\operatorname{Cos}[2*a + 2*b*x]*\operatorname{Sin}[2*a + 2*b*x])/(4*b*\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]]))$

Rule 21

$\operatorname{Int}[(u_.)*((a_.) + (b_.)*(v_))^{(m_.)}*((c_.) + (d_.)*(v_))^{(n_.)}, x_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{EqQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] \mid\mid \operatorname{SimplerQ}[c + d*x, a + b*x])$

Rule 207

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[(\operatorname{Rt}[b, 2]*x)/(\operatorname{Rt}[-a, 2])]/(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \mid\mid \operatorname{GtQ}[b, 0])$

Rule 3774

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] \rightarrow \operatorname{Dist}[(-2*b)/d, \operatorname{Subst}[\operatorname{Int}[1/(a + x^2), x], x, (b*\operatorname{Cot}[c + d*x])/(\operatorname{Sqrt}[a + b*\operatorname{Csc}[c + d*x]])], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 3805

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(d_.)^{(n_.)}*\operatorname{Sqrt}[\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]), x_Symbol] \rightarrow \operatorname{Simp}[(a*\operatorname{Cot}[e + f*x]*(d*\operatorname{Csc}[e + f*x])^n)/(f*n*\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]]), x] + \operatorname{Dist}[(a*(2*n + 1))/(2*b*d*n), \operatorname{Int}[\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]]*(d*\operatorname{Csc}[e + f*x])^{(n+1)}, x], x] /; \operatorname{FreeQ}\{a, b, d, e, f\}, x] \&\& \operatorname{EqQ}[a^2 - b^2, 0] \&\& \operatorname{LtQ}[n, -2^{(-1)}] \&\& \operatorname{IntegerQ}[2*n]$

Rule 3813

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(d_.)^{(n_.)}*(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)^{(m_.)}), x_Symbol] \rightarrow \operatorname{Simp}[(b^2*\operatorname{Cot}[e + f*x]*(a + b*\operatorname{Csc}[e + f*x])^{(m-2)}*(d*\operatorname{Csc}[e + f*x])^n)/(f*n), x] - \operatorname{Dist}[a/(d*n), \operatorname{Int}[(a + b*\operatorname{Csc}[e + f*x])^{(m-2)}*(d*\operatorname{Csc}[e + f*x])^{(n+1)}*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*\operatorname{Csc}[e + f*x])^{(m-2)}], x], x] /; \operatorname{FreeQ}\{a, b, d, e, f\}, x] \&\& \operatorname{IntegerQ}[m] \&\& \operatorname{IntegerQ}[n] \&\& \operatorname{LtQ}[m, 2]$

x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]

Rule 4397

Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\int \cos^2(2(a + bx))(c \tan(a + bx) \tan(2(a + bx)))^{3/2} dx = \int \cos^2(2a + 2bx)(-c + c \sec(2a + 2bx))^{3/2} dx$$

$$= \frac{c^2 \cos(2a + 2bx) \sin(2a + 2bx)}{4b\sqrt{-c + c \sec(2a + 2bx)}} - \frac{1}{2}c \int \frac{\cos(2a + 2bx)}{\sqrt{-c + c \sec(2a + 2bx)}} dx$$

$$= \frac{c^2 \cos(2a + 2bx) \sin(2a + 2bx)}{4b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{1}{4}(7c) \int \cos(2a + 2bx) dx$$

$$= -\frac{7c^2 \sin(2a + 2bx)}{8b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{c^2 \cos(2a + 2bx) \sin(2a + 2bx)}{4b\sqrt{-c + c \sec(2a + 2bx)}}$$

$$= -\frac{7c^2 \sin(2a + 2bx)}{8b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{c^2 \cos(2a + 2bx) \sin(2a + 2bx)}{4b\sqrt{-c + c \sec(2a + 2bx)}}$$

$$= \frac{7c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{8b} - \frac{7c^2 \sin(2a + 2bx)}{8b\sqrt{-c + c \sec(2a + 2bx)}}$$

Mathematica [A] time = 0.27, size = 105, normalized size = 0.79

$$\frac{c \csc(a + bx)\sqrt{c \tan(a + bx) \tan(2(a + bx))} \left(-5 \cos(a + bx) - 6 \cos(3(a + bx)) + \cos(5(a + bx)) + 7\sqrt{2} \sqrt{\cos(2(a + bx))}\right)}{16b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]

[Out] (c*(-5*Cos[a + b*x] + 7*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a + b*x)]]])*Sqrt[Cos[2*(a + b*x)]] - 6*Cos[3*(a + b*x)] + Cos[5*(a + b*x)])*Csc[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]]/(16*b)

fricas [A] time = 1.04, size = 437, normalized size = 3.29

$$\frac{7(c \tan(bx + a)^5 + 2c \tan(bx + a)^3 + c \tan(bx + a))\sqrt{c} \log\left(\frac{c \tan(bx+a)^5 - 14c \tan(bx+a)^3 + 4\sqrt{2}(\tan(bx+a)^4 - 4 \tan(bx+a)^2 + 2)}{\tan(bx+a)^5 + 2 \tan(bx+a)^3 + 1}\right)}{32(b \tan(bx + a))^5 + 2b \tan(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x, algorithm="fricas")

[Out] [1/32*(7*(c*tan(b*x + a)^5 + 2*c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 + 4*sqrt(2)*(tan(b*x + a)^4 - 4

```
*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) +
  17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a)) + 4
*sqrt(2)*(9*c*tan(b*x + a)^4 - 4*c*tan(b*x + a)^2 - 5*c)*sqrt(-c*tan(b*x +
a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(
b*x + a)), -1/16*(7*(c*tan(b*x + a)^5 + 2*c*tan(b*x + a)^3 + c*tan(b*x + a)
)*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(t
an(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt
(2)*(9*c*tan(b*x + a)^4 - 4*c*tan(b*x + a)^2 - 5*c)*sqrt(-c*tan(b*x + a)^2/
(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(b*x +
a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

[Out] Timed out

maple [B] time = 1.02, size = 792, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)
```

```
[Out] 2^(1/2)/b*(2*cos(b*x+a)^2-1)*(2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)
^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(
b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+2^(1/2)*((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x
+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-2*co
s(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/(2+2^(1/2)
)/(2^(1/2)-2)+4*2^(1/2)/b*(2*cos(b*x+a)^2-1)*(2^(1/2)*((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b
*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)+2^(
1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(
1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2)*2^(1/2))+4*cos(b*x+a)^3+2*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1)
)^(3/2)/sin(b*x+a)^3/(2+2^(1/2))^3/(2^(1/2)-2)^3-2*2^(1/2)/b*(2*cos(b*x+a)^
2-1)*(16*cos(b*x+a)^5+9*2^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)
*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)
^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)-12*cos(b*x+a)^3+9*2^(1/2)
*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)
*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2
^(1/2))+18*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)
^3/(2+2^(1/2))^5/(2^(1/2)-2)^5
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(2a + 2bx)^2 (c \tan(a + bx) \tan(2a + 2bx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)

[Out] int(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2), x)

[Out] Timed out

3.618 $\int \cos^3(2(a+bx))(c \tan(a+bx) \tan(2(a+bx)))^{3/2} dx$

Optimal. Leaf size=182

$$-\frac{11c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{16b} + \frac{11c^2 \sin(2a+2bx)}{16b\sqrt{c \sec(2a+2bx)-c}} + \frac{c^2 \sin(2a+2bx) \cos^2(2a+2bx)}{6b\sqrt{c \sec(2a+2bx)-c}} - \frac{11c^2 \sin(2a+2bx)}{24b\sqrt{c \sec(2a+2bx)-c}}$$

[Out] $-11/16*c^{(3/2)}*\operatorname{arctanh}(c^{(1/2)}*\tan(2*b*x+2*a)/(-c+c*\sec(2*b*x+2*a))^{(1/2)})/b+11/16*c^2*\sin(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}-11/24*c^2*\cos(2*b*x+2*a)*\sin(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}+1/6*c^2*\cos(2*b*x+2*a)^2*\sin(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}$

Rubi [A] time = 0.31, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4397, 3813, 21, 3805, 3774, 207}

$$\frac{11c^2 \sin(2a+2bx)}{16b\sqrt{c \sec(2a+2bx)-c}} + \frac{c^2 \sin(2a+2bx) \cos^2(2a+2bx)}{6b\sqrt{c \sec(2a+2bx)-c}} - \frac{11c^2 \sin(2a+2bx) \cos(2a+2bx)}{24b\sqrt{c \sec(2a+2bx)-c}} - \frac{11c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{16b}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cos}[2*(a + b*x)]^3*(c*\operatorname{Tan}[a + b*x]*\operatorname{Tan}[2*(a + b*x)])^{(3/2)}, x]$

[Out] $(-11*c^{(3/2)}*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\operatorname{Tan}[2*a + 2*b*x])/(\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]])]/(16*b) + (11*c^2*\operatorname{Sin}[2*a + 2*b*x])/((16*b*\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]]) - (11*c^2*\operatorname{Cos}[2*a + 2*b*x]*\operatorname{Sin}[2*a + 2*b*x])/((24*b*\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]]) + (c^2*\operatorname{Cos}[2*a + 2*b*x]^2*\operatorname{Sin}[2*a + 2*b*x])/((6*b*\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]])$

Rule 21

$\operatorname{Int}[(u_.)*((a_.) + (b_.)*(v_))^{(m_.)}*((c_.) + (d_.)*(v_))^{(n_.)}, x_Symbol] := \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, n\}, x$ && $\operatorname{EqQ}[b*c - a*d, 0]$ && $\operatorname{IntegerQ}[m]$ && $(\neg \operatorname{IntegerQ}[n] \mid \mid \operatorname{SimplerQ}[c + d*x, a + b*x])$

Rule 207

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] := -\operatorname{Simp}[\operatorname{ArcTanh}[(\operatorname{Rt}[b, 2]*x)/(\operatorname{Rt}[-a, 2])]/(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x$ && $\operatorname{NegQ}[a/b]$ && $(\operatorname{LtQ}[a, 0] \mid \mid \operatorname{GtQ}[b, 0])$

Rule 3774

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := \operatorname{Dist}[(-2*b)/d, \operatorname{Subst}[\operatorname{Int}[1/(a + x^2), x], x, (b*\operatorname{Cot}[c + d*x])/(\operatorname{Sqrt}[a + b*\operatorname{Csc}[c + d*x]])], x] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x$ && $\operatorname{EqQ}[a^2 - b^2, 0]$

Rule 3805

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_)]*(d_.)^{(n_.)}*\operatorname{Sqrt}[\operatorname{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] := \operatorname{Simp}[(a*\operatorname{Cot}[e + f*x]*(d*\operatorname{Csc}[e + f*x])^n)/(f*n*\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]]), x] + \operatorname{Dist}[(a*(2*n + 1))/(2*b*d*n), \operatorname{Int}[\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]]*(d*\operatorname{Csc}[e + f*x])^{(n+1)}, x], x] /;$ $\operatorname{FreeQ}\{a, b, d, e, f\}, x$ && $\operatorname{EqQ}[a^2 - b^2, 0]$ && $\operatorname{LtQ}[n, -2^{(-1)}]$ && $\operatorname{IntegerQ}[2*n]$

Rule 3813

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_)]*(d_.)^{(n_.)}*(\operatorname{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^{(m_.)}, x_Symbol] := \operatorname{Simp}[(b^2*\operatorname{Cot}[e + f*x]*(a + b*\operatorname{Csc}[e + f*x])^{(m-2)}$

$(d \cdot \text{Csc}[e + f \cdot x])^n / (f \cdot n), x] - \text{Dist}[a / (d \cdot n), \text{Int}[(a + b \cdot \text{Csc}[e + f \cdot x])^{m-2} \cdot (d \cdot \text{Csc}[e + f \cdot x])^{n+1} \cdot (b \cdot (m - 2 \cdot n - 2) - a \cdot (m + 2 \cdot n - 1) \cdot \text{Csc}[e + f \cdot x]), x], x] /;$ FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]

Rule 4397

$\text{Int}[u, x_Symbol] \rightarrow \text{Int}[\text{TrigSimplify}[u], x] /;$ TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \cos^3(2(a + bx))(c \tan(a + bx) \tan(2(a + bx)))^{3/2} dx &= \int \cos^3(2a + 2bx)(-c + c \sec(2a + 2bx))^{3/2} dx \\ &= \frac{c^2 \cos^2(2a + 2bx) \sin(2a + 2bx)}{6b\sqrt{-c + c \sec(2a + 2bx)}} - \frac{1}{3}c \int \frac{\cos^2(2a + 2bx)}{\sqrt{-c + c \sec(2a + 2bx)}} dx \\ &= \frac{c^2 \cos^2(2a + 2bx) \sin(2a + 2bx)}{6b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{1}{6}(11c) \int \cos^2(2a + 2bx) \sqrt{-c + c \sec(2a + 2bx)} dx \\ &= -\frac{11c^2 \cos(2a + 2bx) \sin(2a + 2bx)}{24b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{c^2 \cos^2(2a + 2bx) \sin(2a + 2bx)}{6b\sqrt{-c + c \sec(2a + 2bx)}} \\ &= \frac{11c^2 \sin(2a + 2bx)}{16b\sqrt{-c + c \sec(2a + 2bx)}} - \frac{11c^2 \cos(2a + 2bx) \sin(2a + 2bx)}{24b\sqrt{-c + c \sec(2a + 2bx)}} \\ &= \frac{11c^2 \sin(2a + 2bx)}{16b\sqrt{-c + c \sec(2a + 2bx)}} - \frac{11c^2 \cos(2a + 2bx) \sin(2a + 2bx)}{24b\sqrt{-c + c \sec(2a + 2bx)}} \\ &= -\frac{11c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a + 2bx)}{\sqrt{-c + c \sec(2a + 2bx)}}\right)}{16b} + \frac{11c^2 \sin(2a + 2bx)}{16b\sqrt{-c + c \sec(2a + 2bx)}} \end{aligned}$$

Mathematica [A] time = 0.23, size = 117, normalized size = 0.64

$$\frac{c\sqrt{\tan(a + bx) \tan(2(a + bx))} \left(-42 \sin(2(a + bx)) + 14 \sin(4(a + bx)) - 4 \sin(6(a + bx)) + 38 \cot(a + bx) - 3 \right)}{96b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]

[Out] (c*(38*Cot[a + b*x] - 33*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a + b*x)]]]*Sqrt[Cos[2*(a + b*x)]]*Csc[a + b*x] - 42*Sin[2*(a + b*x)] + 14*Sin[4*(a + b*x)] - 4*Sin[6*(a + b*x)])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]]/(96*b)

fricas [A] time = 0.73, size = 503, normalized size = 2.76

$$\left[\frac{33 \left(c \tan(bx + a)^7 + 3c \tan(bx + a)^5 + 3c \tan(bx + a)^3 + c \tan(bx + a) \right) \sqrt{c} \log \left(\frac{c \tan(bx + a)^5 - 14c \tan(bx + a)^3 - 4c \tan(bx + a)}{192 \left(b \tan(bx + a) \right)^7 - \dots}}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="fricas")

[Out] [1/192*(33*(c*tan(b*x + a)^7 + 3*c*tan(b*x + a)^5 + 3*c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 - 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) - 4*sqrt(2)*(63*c*tan(b*x + a)^6 - 13*c*tan(b*x + a)^4 - 31*c*tan(b*x + a)^2 - 19*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^7 + 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 + b*tan(b*x + a)), 1/96*(33*(c*tan(b*x + a)^7 + 3*c*tan(b*x + a)^5 + 3*c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*(63*c*tan(b*x + a)^6 - 13*c*tan(b*x + a)^4 - 31*c*tan(b*x + a)^2 - 19*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^7 + 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 + b*tan(b*x + a))]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 1.11, size = 1078, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)

[Out]
$$\begin{aligned} & -2^{(1/2)}/b*(2*\cos(b*x+a)^2-1)*(2^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)})*\cos(b*x+a)+2^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)})-2*\cos(b*x+a))*(c*\sin(b*x+a)^2/(2*\cos(b*x+a)^2-1))^{(3/2)}/\sin(b*x+a)^3/(2+2^{(1/2)})/(2^{(1/2)}-2)-6*2^{(1/2)}/b*(2*\cos(b*x+a)^2-1)*(2^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)})*\cos(b*x+a)+2^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)})+4*\cos(b*x+a)^3+2*\cos(b*x+a))*(c*\sin(b*x+a)^2/(2*\cos(b*x+a)^2-1))^{(3/2)}/\sin(b*x+a)^3/(2+2^{(1/2)})^3/(2^{(1/2)}-2)^3+6*2^{(1/2)}/b*(2*\cos(b*x+a)^2-1)*(16*\cos(b*x+a)^5+9*2^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)})*\cos(b*x+a)-12*\cos(b*x+a)^3+9*2^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)})+18*\cos(b*x+a))*(c*\sin(b*x+a)^2/(2*\cos(b*x+a)^2-1))^{(3/2)}/\sin(b*x+a)^3/(2+2^{(1/2)})^5/(2^{(1/2)}-2)^5-4/3*2^{(1/2)}/b*(2*\cos(b*x+a)^2-1)*(128*\cos(b*x+a)^7-80*\cos(b*x+a)^5+75*2^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)})*\cos(b*x+a)+75*2^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)})-100* \end{aligned}$$

$\cos(b*x+a)^3+150*\cos(b*x+a))*(c*\sin(b*x+a)^2/(2*\cos(b*x+a)^2-1))^(3/2)/\sin(b*x+a)^3/(2+2^(1/2))^7/(2^(1/2)-2)^7$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(2a + 2bx)^3 (c \tan(a + bx) \tan(2a + 2bx))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)

[Out] int(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)

[Out] Timed out

$$3.619 \quad \int \frac{\sec^4(2(a+bx))}{\sqrt{c \tan(a+bx) \tan(2(a+bx))}} dx$$

Optimal. Leaf size=175

$$\frac{\tan(2a + 2bx) \sec^2(2a + 2bx)}{5b\sqrt{c \sec(2a + 2bx) - c}} + \frac{\tan(2a + 2bx)\sqrt{c \sec(2a + 2bx) - c}}{15bc} + \frac{14 \tan(2a + 2bx)}{15b\sqrt{c \sec(2a + 2bx) - c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(a+bx)}{\sqrt{2} \sqrt{c \sec(2a + 2bx) - c}}\right)}{\sqrt{2} b \sqrt{c}}$$

[Out] $-1/2*\operatorname{arctanh}(1/2*c^{(1/2)}*\tan(2*b*x+2*a)*2^{(1/2)/(-c+c*\sec(2*b*x+2*a))^{(1/2)})/b*2^{(1/2)}/c^{(1/2)}+14/15*\tan(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}+1/5*\sec(2*b*x+2*a)^2*\tan(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}+1/15*(-c+c*\sec(2*b*x+2*a))^{(1/2)}*\tan(2*b*x+2*a)/b/c$

Rubi [A] time = 0.60, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4397, 3822, 4010, 4001, 3795, 207}

$$\frac{\tan(2a + 2bx) \sec^2(2a + 2bx)}{5b\sqrt{c \sec(2a + 2bx) - c}} + \frac{\tan(2a + 2bx)\sqrt{c \sec(2a + 2bx) - c}}{15bc} + \frac{14 \tan(2a + 2bx)}{15b\sqrt{c \sec(2a + 2bx) - c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(a+bx)}{\sqrt{2} \sqrt{c \sec(2a + 2bx) - c}}\right)}{\sqrt{2} b \sqrt{c}}$$

Antiderivative was successfully verified.

[In] `Int[Sec[2*(a + b*x)]^4/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]`

[Out] $-(\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\operatorname{Tan}[2*a + 2*b*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]])]/(\operatorname{Sqrt}[2]*b*\operatorname{Sqrt}[c])) + (14*\operatorname{Tan}[2*a + 2*b*x])/((15*b*\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]]) + (\operatorname{Sec}[2*a + 2*b*x]^2*\operatorname{Tan}[2*a + 2*b*x])/((5*b*\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]]) + (\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]]*\operatorname{Tan}[2*a + 2*b*x])/((15*b*c)$

Rule 207

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

Rule 3795

`Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]`

Rule 3822

`Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(-2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[d^2/(b*(2*n - 3)), Int[(d*Csc[e + f*x])^(n - 2)*(2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]`

Rule 4001

`Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]`

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
0] && !LtQ[m, -1]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(2(a+bx))}{\sqrt{c} \tan(a+bx) \tan(2(a+bx))} dx &= \int \frac{\sec^4(2a+2bx)}{\sqrt{-c+c} \sec(2a+2bx)} dx \\
&= \frac{\sec^2(2a+2bx) \tan(2a+2bx)}{5b\sqrt{-c+c} \sec(2a+2bx)} + \frac{\int \frac{\sec^2(2a+2bx)(4c+c \sec(2a+2bx))}{\sqrt{-c+c} \sec(2a+2bx)} dx}{5c} \\
&= \frac{\sec^2(2a+2bx) \tan(2a+2bx)}{5b\sqrt{-c+c} \sec(2a+2bx)} + \frac{\sqrt{-c+c} \sec(2a+2bx) \tan(2a+2bx)}{15bc} \\
&= \frac{14 \tan(2a+2bx)}{15b\sqrt{-c+c} \sec(2a+2bx)} + \frac{\sec^2(2a+2bx) \tan(2a+2bx)}{5b\sqrt{-c+c} \sec(2a+2bx)} + \frac{\sqrt{-c+c}}{15b} \\
&= \frac{14 \tan(2a+2bx)}{15b\sqrt{-c+c} \sec(2a+2bx)} + \frac{\sec^2(2a+2bx) \tan(2a+2bx)}{5b\sqrt{-c+c} \sec(2a+2bx)} + \frac{\sqrt{-c+c}}{15b} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{-c+c} \sec(2a+2bx)}\right)}{\sqrt{2} b \sqrt{c}} + \frac{14 \tan(2a+2bx)}{15b\sqrt{-c+c} \sec(2a+2bx)} + \frac{\sec^2(2a+2bx) \tan(2a+2bx)}{5b\sqrt{-c+c} \sec(2a+2bx)}
\end{aligned}$$

Mathematica [A] time = 0.67, size = 112, normalized size = 0.64

$$\frac{\sin(a+bx) \cos(a+bx) \sec^3(2(a+bx)) \left(4 \cos(2(a+bx)) + 26 \cos(4(a+bx)) + 30 \cos^2(2(a+bx)) \tan^{-1}\left(\sqrt{\frac{c}{-c+c}} \tan(2(a+bx))\right)\right)}{30b\sqrt{c} \tan(a+bx) \tan(2(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^4/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]
```

```
[Out] (Cos[a + b*x]*Sec[2*(a + b*x)]^3*Sin[a + b*x]*(38 + 4*Cos[2*(a + b*x)] + 26
*Cos[4*(a + b*x)] + 30*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Cos[2*(a + b*x)]^2
*Sqrt[-1 + Tan[a + b*x]^2]))/(30*b*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

fricas [A] time = 0.51, size = 380, normalized size = 2.17

$$\frac{4\sqrt{2}\left(15\tan^4(bx+a) - 20\tan^2(bx+a) + 17\right)\sqrt{-\frac{c\tan^2(bx+a)}{\tan^2(bx+a)-1}} + \frac{15\sqrt{2}\left(c\tan^5(bx+a) - 2c\tan^3(bx+a) + c\tan(bx+a)\right)\log\left(\frac{\tan(bx+a)}{\sqrt{c}}\right)}{60\left(bc\tan^5(bx+a) - 2bc\tan^3(bx+a) + bc\tan(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")

[Out] [1/60*(4*sqrt(2)*(15*tan(b*x + a)^4 - 20*tan(b*x + a)^2 + 17)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)) + 15*sqrt(2)*(c*tan(b*x + a)^5 - 2*c*tan(b*x + a)^3 + c*tan(b*x + a))*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan(b*x + a)^3)/sqrt(c))/(b*c*tan(b*x + a)^5 - 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x + a)), -1/30*(15*sqrt(2)*(c*tan(b*x + a)^5 - 2*c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(-1/c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a)) - 2*sqrt(2)*(15*tan(b*x + a)^4 - 20*tan(b*x + a)^2 + 17)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*tan(b*x + a)^5 - 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x + a))]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 1.36, size = 984, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)

[Out] 1/120*2^(1/2)/b*(-1+cos(b*x+a))*(208*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^6+120*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*cos(b*x+a)^6+120*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*cos(b*x+a)^6+208*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^5-200*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^4-180*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*cos(b*x+a)^4-180*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)*cos(b*x+a)^4-200*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^3+60*cos(b*x+a)^2*((2*cos(b*x+a)^2-

$$\frac{1}{(\cos(bx+a)+1)^2} \sqrt{\frac{1}{2} + 90 \operatorname{arctanh}\left(\frac{1}{2} \sqrt{\frac{2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1}{(2 \cos(bx+a)^2 - 1) / (\cos(bx+a)+1)^2}}\right) \frac{2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1}{(2 \cos(bx+a)^2 - 1) / (\cos(bx+a)+1)^2}} \frac{1}{\sin(bx+a)^2} \cos(bx+a)^2 + 90 \ln\left(-2 \frac{\cos(bx+a)^2 ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a)+1)^2) - 2 \cos(bx+a)^2 + \cos(bx+a) - ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a)+1)^2) + 1}{\sin(bx+a)^2} \cos(bx+a)^2 + 60 \cos(bx+a) \frac{(2 \cos(bx+a)^2 - 1) / (\cos(bx+a)+1)^2}{(2 \cos(bx+a)^2 - 1) / (\cos(bx+a)+1)^2} - 15 \operatorname{arctanh}\left(\frac{1}{2} \sqrt{\frac{2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1}{(2 \cos(bx+a)^2 - 1) / (\cos(bx+a)+1)^2}}\right) \frac{2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1}{(2 \cos(bx+a)^2 - 1) / (\cos(bx+a)+1)^2} \frac{1}{\sin(bx+a)^2} - 15 \ln\left(-2 \frac{\cos(bx+a)^2 ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a)+1)^2) - 2 \cos(bx+a)^2 + \cos(bx+a) - ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a)+1)^2) + 1}{\sin(bx+a)^2}\right) / (2 \cos(bx+a)^2 - 1)^3 / \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \sqrt{\frac{1}{2} + 90 \operatorname{arctanh}\left(\frac{1}{2} \sqrt{\frac{2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1}{(2 \cos(bx+a)^2 - 1) / (\cos(bx+a)+1)^2}}\right) \frac{2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1}{(2 \cos(bx+a)^2 - 1) / (\cos(bx+a)+1)^2}}\right) \frac{1}{\sin(bx+a)} \sqrt{\frac{1}{2} + 90 \operatorname{arctanh}\left(\frac{1}{2} \sqrt{\frac{2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1}{(2 \cos(bx+a)^2 - 1) / (\cos(bx+a)+1)^2}}\right) \frac{2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1}{(2 \cos(bx+a)^2 - 1) / (\cos(bx+a)+1)^2}}\right)}\right)^3$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(2bx + 2a)^4}{\sqrt{c \tan(2bx + 2a) \tan(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(2*b*x + 2*a)^4/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(2a + 2bx)^4 \sqrt{c \tan(a + bx) \tan(2a + 2bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(2*a + 2*b*x)^4*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)),x)

[Out] int(1/(cos(2*a + 2*b*x)^4*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)**4/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)

[Out] Timed out

$$3.620 \quad \int \frac{\sec^3(2(a+bx))}{\sqrt{c} \tan(a+bx) \tan(2(a+bx))} dx$$

Optimal. Leaf size=129

$$\frac{\tan(2a+2bx)\sqrt{c \sec(2a+2bx)-c}}{3bc} + \frac{2 \tan(2a+2bx)}{3b\sqrt{c \sec(2a+2bx)-c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{\sqrt{2} b \sqrt{c}}$$

[Out] $-1/2*\operatorname{arctanh}(1/2*c^{(1/2)}*\tan(2*b*x+2*a)*2^{(1/2)}/(-c+c*\sec(2*b*x+2*a))^{(1/2)})/b*2^{(1/2)}/c^{(1/2)}+2/3*\tan(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}+1/3*(-c+c*\sec(2*b*x+2*a))^{(1/2)}*\tan(2*b*x+2*a)/b/c$

Rubi [A] time = 0.36, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4397, 3800, 4001, 3795, 207}

$$\frac{\tan(2a+2bx)\sqrt{c \sec(2a+2bx)-c}}{3bc} + \frac{2 \tan(2a+2bx)}{3b\sqrt{c \sec(2a+2bx)-c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{\sqrt{2} b \sqrt{c}}$$

Antiderivative was successfully verified.

[In] `Int[Sec[2*(a + b*x)]^3/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]`

[Out] $-(\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\operatorname{Tan}[2*a + 2*b*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]])]/(\operatorname{Sqrt}[2]*b*\operatorname{Sqrt}[c])) + (2*\operatorname{Tan}[2*a + 2*b*x])/(3*b*\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]]) + (\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x)]*\operatorname{Tan}[2*a + 2*b*x])/(3*b*c)$

Rule 207

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

Rule 3795

`Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]`

Rule 3800

`Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]`

Rule 4001

`Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m + 1), 0] && !LtQ[m, -2^(-1)]`

Rule 4397

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned}
 \int \frac{\sec^3(2(a+bx))}{\sqrt{c \tan(a+bx) \tan(2(a+bx))}} dx &= \int \frac{\sec^3(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}} dx \\
 &= \frac{\sqrt{-c+c \sec(2a+2bx)} \tan(2a+2bx)}{3bc} + \frac{2 \int \frac{\sec(2a+2bx) \left(\frac{c}{2} + c \sec(2a+2bx)\right)}{\sqrt{-c+c \sec(2a+2bx)}} dx}{3c} \\
 &= \frac{2 \tan(2a+2bx)}{3b\sqrt{-c+c \sec(2a+2bx)}} + \frac{\sqrt{-c+c \sec(2a+2bx)} \tan(2a+2bx)}{3bc} + \dots \\
 &= \frac{2 \tan(2a+2bx)}{3b\sqrt{-c+c \sec(2a+2bx)}} + \frac{\sqrt{-c+c \sec(2a+2bx)} \tan(2a+2bx)}{3bc} - \dots \\
 &= -\frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{-c+c \sec(2a+2bx)}}\right)}{\sqrt{2} b \sqrt{c}} + \frac{2 \tan(2a+2bx)}{3b\sqrt{-c+c \sec(2a+2bx)}} + \frac{\sqrt{-c+c \sec(2a+2bx)} \tan(2a+2bx)}{3bc}
 \end{aligned}$$

Mathematica [A] time = 0.39, size = 89, normalized size = 0.69

$$\frac{\cos^2(a+bx) \csc(2(a+bx)) \sqrt{c \tan(a+bx) \tan(2(a+bx))} \left(3\sqrt{\tan^2(a+bx)-1} \tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right) + 2\right)}{3bc}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*(a + b*x)]^3/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]

[Out] (Cos[a + b*x]^2*Csc[2*(a + b*x)]*(2 + 2*Sec[2*(a + b*x)] + 3*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(3*b*c)

fricas [A] time = 2.20, size = 294, normalized size = 2.28

$$\frac{3\sqrt{2}(c \tan(bx+a)^3 - c \tan(bx+a)) \log\left(\frac{\tan(bx+a)^3 - \frac{2\sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2-1}}(\tan(bx+a)^2-1)}{\sqrt{c}} - 2 \tan(bx+a)}{\tan(bx+a)^3}\right) - 8\sqrt{2}\sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2-1}}}{12(bc \tan(bx+a)^3 - bc \tan(bx+a))} - \frac{3\sqrt{2}(c \tan(bx+a)^3 - c \tan(bx+a))}{12(bc \tan(bx+a)^3 - bc \tan(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2), x, algorithm="fricas")

[Out] [1/12*(3*sqrt(2)*(c*tan(b*x + a)^3 - c*tan(b*x + a))*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan(b*x + a)^3)/sqrt(c) - 8*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*tan(b*x + a)^3 - b*c*tan(b*x + a)), -1/6*(3*sqrt(2)*(c*tan(b*x + a)^3 - c*tan(b*x + a))*sqrt(-1/c)*arctan(sqrt(-c*tan(b

$x + a)^2 / (\tan(bx + a)^2 - 1) * (\tan(bx + a)^2 - 1) * \sqrt{-1/c} / \tan(bx + a) + 4 * \sqrt{2} * \sqrt{-c * \tan(bx + a)^2 / (\tan(bx + a)^2 - 1)} / (b * c * \tan(bx + a)^3 - b * c * \tan(bx + a))]$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 1.21, size = 677, normalized size = 5.25

$$\sqrt{2} (-1 + \cos(bx + a)) \left(12 \operatorname{arctanh} \left(\frac{\sqrt{4} (2(\cos^2(bx+a)) - 3 \cos(bx+a) + 1)}{2 \sqrt{\frac{2(\cos^2(bx+a)) - 1}{(\cos(bx+a) + 1)^2}} \sin(bx+a)^2} \right) (\cos^4(bx + a)) + 12 \ln \left(- \frac{2 \left(\cos^2(bx+a) \right) \sqrt{\frac{2(\cos^2(bx+a)) - 1}{(\cos(bx+a) + 1)^2}}}{\dots} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)

[Out]
$$\begin{aligned} & -1/24 * 2^{(1/2)} / b * (-1 + \cos(bx+a)) * (12 * \operatorname{arctanh}(1/2 * 4^{(1/2)} * (2 * \cos(bx+a)^2 - 3 * \cos(bx+a) + 1) / ((2 * \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{(1/2)} / \sin(bx+a)^2 * \cos(bx+a)^4 + 12 * \ln(-2 * (\cos(bx+a)^2 * ((2 * \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{(1/2)} - 2 * \cos(bx+a)^2 + \cos(bx+a) - ((2 * \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{(1/2)} + 1) / \sin(bx+a)^2 * \cos(bx+a)^4 + 8 * ((2 * \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{(1/2)} * \cos(bx+a)^4 + 8 * ((2 * \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{(1/2)} * \cos(bx+a)^3 - 12 * \operatorname{arctanh}(1/2 * 4^{(1/2)} * (2 * \cos(bx+a)^2 - 3 * \cos(bx+a) + 1) / ((2 * \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{(1/2)} / \sin(bx+a)^2 * \cos(bx+a)^2 - 12 * \ln(-2 * (\cos(bx+a)^2 * ((2 * \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{(1/2)} - 2 * \cos(bx+a)^2 + \cos(bx+a) - ((2 * \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{(1/2)} + 1) / \sin(bx+a)^2) / (2 * \cos(bx+a)^2 - 1)^2 / ((2 * \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{(1/2)} / (c * (1 - \cos(bx+a)^2) / (2 * \cos(bx+a)^2 - 1))^{(1/2)} / \sin(bx+a)^4)^{(1/2)} / (-3 + 2 * 2^{(1/2)})^2 / (3 + 2 * 2^{(1/2)})^2 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(2bx + 2a)^3}{\sqrt{c \tan(2bx + 2a) \tan(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(2*b*x + 2*a)^3/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(2a + 2bx)^3 \sqrt{c \tan(a + bx) \tan(2a + 2bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)),x)
```

```
[Out] int(1/(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**3/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

$$3.621 \quad \int \frac{\sec^2(2(a+bx))}{\sqrt{c} \tan(a+bx) \tan(2(a+bx))} dx$$

Optimal. Leaf size=88

$$\frac{\tan(2a + 2bx)}{b\sqrt{c} \sec(2a + 2bx) - c} - \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c} \sec(2a+2bx)-c}\right)}{\sqrt{2} b \sqrt{c}}$$

[Out] $-1/2*\operatorname{arctanh}(1/2*c^{(1/2)}*\tan(2*b*x+2*a)*2^{(1/2)/(-c+c*\sec(2*b*x+2*a))^{(1/2)}}/b*2^{(1/2)/c^{(1/2)}+\tan(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}}$

Rubi [A] time = 0.24, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4397, 3798, 3795, 207}

$$\frac{\tan(2a + 2bx)}{b\sqrt{c} \sec(2a + 2bx) - c} - \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c} \sec(2a+2bx)-c}\right)}{\sqrt{2} b \sqrt{c}}$$

Antiderivative was successfully verified.

[In] `Int[Sec[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]`

[Out] `-(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]])]/(Sqrt[2]*b*Sqrt[c])) + Tan[2*a + 2*b*x]/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]])`

Rule 207

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

Rule 3795

`Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]`

Rule 3798

`Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]`

Rule 4397

`Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]`

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(2(a+bx))}{\sqrt{c \tan(a+bx) \tan(2(a+bx))}} dx &= \int \frac{\sec^2(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}} dx \\
&= \frac{\tan(2a+2bx)}{b\sqrt{-c+c \sec(2a+2bx)}} + \int \frac{\sec(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}} dx \\
&= \frac{\tan(2a+2bx)}{b\sqrt{-c+c \sec(2a+2bx)}} - \frac{\text{Subst}\left(\int \frac{1}{-2c+x^2} dx, x, -\frac{c \tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{b} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{-c+c \sec(2a+2bx)}}\right)}{\sqrt{2} b \sqrt{c}} + \frac{\tan(2a+2bx)}{b\sqrt{-c+c \sec(2a+2bx)}}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 67, normalized size = 0.76

$$\frac{\left(\sqrt{\tan^2(a+bx)-1} \tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right) + 2\right) \tan(2(a+bx))}{2b\sqrt{c} \tan(a+bx) \tan(2(a+bx))}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]

[Out] ((2 + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a + b*x)])/(2*b*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])

fricas [A] time = 1.63, size = 245, normalized size = 2.78

$$\frac{\sqrt{2} \sqrt{c} \log\left(\frac{\tan(bx+a)^3 - \sqrt{\frac{-c \tan(bx+a)^2}{\tan(bx+a)^2-1}} (\tan(bx+a)^2-1) - 2 \tan(bx+a)}{\tan(bx+a)^3}\right) \tan(bx+a) + 4 \sqrt{2} \sqrt{\frac{-c \tan(bx+a)^2}{\tan(bx+a)^2-1}} \sqrt{2} c \sqrt{-\frac{1}{c}}}{4 bc \tan(bx+a)}, -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2), x, algorithm="fricas")

[Out] [1/4*(sqrt(2)*sqrt(c)*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan(b*x + a)^3)*tan(b*x + a) + 4*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*tan(b*x + a)), -1/2*(sqrt(2)*c*sqrt(-1/c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a))*tan(b*x + a) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*tan(b*x + a))]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 1.19, size = 478, normalized size = 5.43

$$\sqrt{2} \left(\cos(bx + a) \ln \left(\frac{2 \left((\cos^2(bx+a)) \sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}} - 2(\cos^2(bx+a)) + \cos(bx+a) - \sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}} + 1 \right)}{\sin(bx+a)^2} \right) \sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}} + \cos(bx+a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2), x)

[Out] $\frac{1}{4} 2^{1/2} / b (\cos(bx+a) \ln(-2(\cos(bx+a))^2 ((2\cos(bx+a))^2 - 1) / (\cos(bx+a)+1)^2)^{1/2} - 2\cos(bx+a)^2 + \cos(bx+a) - ((2\cos(bx+a))^2 - 1) / (\cos(bx+a)+1)^2)^{1/2} + 1) / \sin(bx+a)^2 * ((2\cos(bx+a))^2 - 1) / (\cos(bx+a)+1)^2)^{1/2} + \cos(bx+a) * ((2\cos(bx+a))^2 - 1) / (\cos(bx+a)+1)^2)^{1/2} * \operatorname{arctanh}(1/2 * 4^{1/2} * (2\cos(bx+a))^2 - 3\cos(bx+a) + 1) / ((2\cos(bx+a))^2 - 1) / (\cos(bx+a)+1)^2)^{1/2} / \sin(bx+a)^2 + \ln(-2(\cos(bx+a))^2 ((2\cos(bx+a))^2 - 1) / (\cos(bx+a)+1)^2)^{1/2} - 2\cos(bx+a)^2 + \cos(bx+a) - ((2\cos(bx+a))^2 - 1) / (\cos(bx+a)+1)^2)^{1/2} + 1) / \sin(bx+a)^2 * ((2\cos(bx+a))^2 - 1) / (\cos(bx+a)+1)^2)^{1/2} + \operatorname{arctanh}(1/2 * 4^{1/2} * (2\cos(bx+a))^2 - 3\cos(bx+a) + 1) / ((2\cos(bx+a))^2 - 1) / (\cos(bx+a)+1)^2)^{1/2} / \sin(bx+a)^2) * ((2\cos(bx+a))^2 - 1) / (\cos(bx+a)+1)^2)^{1/2} + 4\cos(bx+a) * \sin(bx+a) / (2\cos(bx+a))^2 - 1) / (c \sin(bx+a)^2 / (2\cos(bx+a))^2 - 1)^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(2bx + 2a)^2}{\sqrt{c \tan(2bx + 2a) \tan(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2), x, algorithm="maxima")

[Out] integrate(sec(2*b*x + 2*a)^2/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(2a + 2bx)^2 \sqrt{c \tan(a + bx) \tan(2a + 2bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)

[Out] int(1/(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2), x)

[Out] Timed out

$$3.622 \quad \int \frac{\sec(2(a+bx))}{\sqrt{c \tan(a+bx) \tan(2(a+bx))}} dx$$

Optimal. Leaf size=55

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{\sqrt{2} b \sqrt{c}}$$

[Out] $-1/2*\operatorname{arctanh}(1/2*c^{(1/2)}*\tan(2*b*x+2*a)*2^{(1/2)/(-c+c*\sec(2*b*x+2*a))^{(1/2)}}/b*2^{(1/2)}/c^{(1/2)})$

Rubi [A] time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4397, 3795, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{\sqrt{2} b \sqrt{c}}$$

Antiderivative was successfully verified.

[In] `Int[Sec[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]`

[Out] `-(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]])]/(Sqrt[2]*b*Sqrt[c]))`

Rule 207

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

Rule 3795

`Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]`

Rule 4397

`Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]`

Rubi steps

$$\begin{aligned} \int \frac{\sec(2(a+bx))}{\sqrt{c \tan(a+bx) \tan(2(a+bx))}} dx &= \int \frac{\sec(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}} dx \\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{-2c+x^2} dx, x, -\frac{c \tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{b} \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{-c+c \sec(2a+2bx)}}\right)}{\sqrt{2} b \sqrt{c}} \end{aligned}$$

Mathematica [A] time = 0.15, size = 64, normalized size = 1.16

$$\frac{\tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right) \sqrt{\tan^2(a+bx)-1} \tan(2(a+bx))}{2b\sqrt{c \tan(a+bx) \tan(2(a+bx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]

[Out] (ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2]*Tan[2*(a + b*x)])/ (2*b*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])

fricas [A] time = 2.00, size = 146, normalized size = 2.65

$$\frac{\sqrt{2} \log \left(\frac{\tan(bx+a)^3 - \sqrt{\frac{-c \tan(bx+a)^2}{\tan(bx+a)^2 - 1} (\tan(bx+a)^2 - 1)} \sqrt{c} - 2 \tan(bx+a)}{\tan(bx+a)^3} \right)}{4b\sqrt{c}} + \frac{\sqrt{2} \sqrt{-\frac{1}{c}} \arctan \left(\frac{\sqrt{\frac{-c \tan(bx+a)^2}{\tan(bx+a)^2 - 1} (\tan(bx+a)^2 - 1)} \sqrt{-\frac{1}{c}}}{\tan(bx+a)} \right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")

[Out] [1/4*sqrt(2)*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan(b*x + a)^3)/(b*sqrt(c)), -1/2*sqrt(2)*sqrt(-1/c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a))/b]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 1.07, size = 236, normalized size = 4.29

$$\frac{\sqrt{2} (\cos(bx + a) + 1) \sqrt{\frac{2(\cos^2(bx+a)-1)}{(\cos(bx+a)+1)^2}} \sqrt{\frac{c(1-\cos^2(bx+a))}{2(\cos^2(bx+a)-1)}} \left(\ln \left(\frac{2 \left((\cos^2(bx+a)) \sqrt{\frac{2(\cos^2(bx+a)-1}{(\cos(bx+a)+1)^2}} - 2(\cos^2(bx+a)) + \cos(bx+a) - \sqrt{\frac{2(\cos^2(bx+a)-1}{(\cos(bx+a)+1)^2}} \right)}{\sin(bx+a)^2} \right)}{8b \sin(bx + a) c} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)

[Out] 1/8*2^(1/2)/b*(cos(b*x+a)+1)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2))/sin(b*x+a)/c*4^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(2bx + 2a)}{\sqrt{c \tan(2bx + 2a) \tan(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(2*b*x + 2*a)/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos(2a + 2bx) \sqrt{c \tan(a + bx) \tan(2a + 2bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)),x)

[Out] int(1/(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)

[Out] Timed out

$$3.623 \quad \int \frac{1}{\sqrt{c \tan(a+bx) \tan(2(a+bx))}} dx$$

Optimal. Leaf size=100

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{\sqrt{2} b\sqrt{c}}$$

[Out] arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(1/2)-1/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/b*2^(1/2)/c^(1/2)

Rubi [A] time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4397, 3776, 3774, 207, 3795}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{\sqrt{2} b\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]

[Out] ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]]/(b*Sqrt[c]) - ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]])]/(Sqrt[2]*b*Sqrt[c])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3776

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[1/a, Int[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 4397

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{c \tan(a+bx) \tan(2(a+bx))}} dx &= \int \frac{1}{\sqrt{-c + c \sec(2a+2bx)}} dx \\
&= -\frac{\int \sqrt{-c + c \sec(2a+2bx)} dx}{c} + \int \frac{\sec(2a+2bx)}{\sqrt{-c + c \sec(2a+2bx)}} dx \\
&= \frac{\text{Subst}\left(\int \frac{1}{-2c+x^2} dx, x, -\frac{c \tan(2a+2bx)}{\sqrt{-c + c \sec(2a+2bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{1}{-c+x^2} dx, x, -\frac{c \tan(2a+2bx)}{\sqrt{-c + c \sec(2a+2bx)}}\right)}{b} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{-c + c \sec(2a+2bx)}}\right)}{b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{-c + c \sec(2a+2bx)}}\right)}{\sqrt{2} b\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 94, normalized size = 0.94

$$\frac{\tan(a+bx) \left(2 \tanh^{-1}\left(\frac{1}{2}\sqrt{2-2\tan^2(a+bx)}\right) - \sqrt{2} \tanh^{-1}\left(\sqrt{1-\tan^2(a+bx)}\right) \right)}{b\sqrt{2-2\tan^2(a+bx)} \sqrt{c \tan(a+bx) \tan(2(a+bx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]

[Out] ((2*ArcTanh[Sqrt[2 - 2*Tan[a + b*x]^2]/2] - Sqrt[2]*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]])*Tan[a + b*x])/(b*Sqrt[2 - 2*Tan[a + b*x]^2]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])

fricas [A] time = 1.90, size = 309, normalized size = 3.09

$$\frac{\sqrt{2} \sqrt{c} \log\left(\frac{c \tan(bx+a)^3 - 2 \sqrt{\frac{c \tan(bx+a)^2}{\tan(bx+a)^2 - 1}} (\tan(bx+a)^2 - 1) \sqrt{c} - 2c \tan(bx+a)}{\tan(bx+a)^3}\right) + 2 \sqrt{c} \log\left(\frac{c \tan(bx+a)^3 + 2 \sqrt{2} \sqrt{\frac{c \tan(bx+a)^2}{\tan(bx+a)^2 - 1}} (\tan(bx+a)^2 - 1) \sqrt{c} + 2c \tan(bx+a)}{\tan(bx+a)^3 + \tan(bx+a)}\right)}{4bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2), x, algorithm="fricas")

[Out] [1/4*(sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 2*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x + a)^3 + tan(b*x + a)))/(b*c), -1/2*(sqrt(2)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - 2*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)))/(b*c)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 1.01, size = 301, normalized size = 3.01

$$\sqrt{2} (\cos(bx + a) + 1) \sqrt{\frac{2(\cos^2(bx+a)-1)}{(\cos(bx+a)+1)^2}} \sqrt{\frac{c(1-(\cos^2(bx+a)))}{2(\cos^2(bx+a)-1)}} \left(2\sqrt{2} \operatorname{arctanh} \left(\frac{\cos(bx+a)\sqrt{4}(-1+\cos(bx+a))\sqrt{2}}{2\sin(bx+a)^2\sqrt{\frac{2(\cos^2(bx+a)-1)}{(\cos(bx+a)+1)^2}}} \right) - \ln \left(-\frac{2\left(\frac{c(1-(\cos^2(bx+a)))}{2(\cos^2(bx+a)-1)} \right)^{1/2}}{8b \sin(bx+a)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2), x)

[Out] $-1/8*2^{(1/2)}/b*(\cos(b*x+a)+1)*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*(c*(1-\cos(b*x+a)^2)/(2*\cos(b*x+a)^2-1))^{(1/2)}*(2*2^{(1/2)}*\operatorname{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)})-\ln(-2*(\cos(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}-2*\cos(b*x+a)^2+\cos(b*x+a)-((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}+1)/\sin(b*x+a)^2-\operatorname{arctanh}(1/2*4^{(1/2)}*(2*\cos(b*x+a)^2-3*\cos(b*x+a)+1)/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}/\sin(b*x+a)^2))/\sin(b*x+a)/c*4^{(1/2)}$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2), x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: sign: argument cannot be imaginary; found %i

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c \tan(a + bx) \tan(2a + 2bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)

[Out] int(1/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2), x)

[Out] Timed out

$$3.624 \quad \int \frac{\cos(2(a+bx))}{\sqrt{c \tan(a+bx) \tan(2(a+bx))}} dx$$

Optimal. Leaf size=138

$$\frac{\sin(2a + 2bx)}{2b\sqrt{c \sec(2a + 2bx) - c}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx) - c}}\right)}{2b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx) - c}}\right)}{\sqrt{2} b \sqrt{c}}$$

[Out] $1/2 \cdot \operatorname{arctanh}(c^{1/2} \cdot \tan(2bx + 2a) / (-c + c \sec(2bx + 2a))^{1/2}) / b / c^{1/2} - 1/2 \cdot \operatorname{arctanh}(1/2 \cdot c^{1/2} \cdot \tan(2bx + 2a) \cdot 2^{1/2} / (-c + c \sec(2bx + 2a))^{1/2}) / b \cdot 2^{1/2} / c^{1/2} + 1/2 \cdot \sin(2bx + 2a) / b / (-c + c \sec(2bx + 2a))^{1/2}$

Rubi [A] time = 0.28, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4397, 3823, 3904, 3887, 481, 206}

$$\frac{\sin(2a + 2bx)}{2b\sqrt{c \sec(2a + 2bx) - c}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx) - c}}\right)}{2b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx) - c}}\right)}{\sqrt{2} b \sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[Cos[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]

[Out] $\operatorname{ArcTanh}[(\sqrt{c} \cdot \tan[2a + 2bx]) / \sqrt{-c + c \sec[2a + 2bx]}] / (2b \sqrt{c}) - \operatorname{ArcTanh}[(\sqrt{c} \cdot \tan[2a + 2bx]) / (\sqrt{2} \sqrt{-c + c \sec[2a + 2bx]})] / (\sqrt{2} b \sqrt{c}) + \sin[2a + 2bx] / (2b \sqrt{-c + c \sec[2a + 2bx]})$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 481

Int[((e_.)*(x_)^(m_.))/(((a_) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.))), x_Symbol] := -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m-n)/(a + b*x^n), x], x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m-n)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1]

Rule 3823

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n+1)*(a + b*(2*n+1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]

Rule 3887

Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_.), x_Symbol] := Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && IntegerQ[n - 1/2]

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))^(n_.), x_Symbol] := Dist[(-a*c)^(m), Int[Cot[e + f*x]^(2*m)*(c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] && !(IntegerQ[n] && GtQ[m - n, 0])
```

Rule 4397

```
Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos(2(a+bx))}{\sqrt{c} \tan(a+bx) \tan(2(a+bx))} dx &= \int \frac{\cos(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}} dx \\ &= \frac{\sin(2a+2bx)}{2b\sqrt{-c+c \sec(2a+2bx)}} - \frac{\int \frac{-c-c \sec(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}} dx}{2c} \\ &= \frac{\sin(2a+2bx)}{2b\sqrt{-c+c \sec(2a+2bx)}} + \frac{1}{2} c \int \frac{\tan^2(2a+2bx)}{(-c+c \sec(2a+2bx))^{3/2}} dx \\ &= \frac{\sin(2a+2bx)}{2b\sqrt{-c+c \sec(2a+2bx)}} - \frac{c \operatorname{Subst}\left(\int \frac{x^2}{(1-cx^2)(2-cx^2)} dx, x, -\frac{\tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{2b} \\ &= \frac{\sin(2a+2bx)}{2b\sqrt{-c+c \sec(2a+2bx)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-cx^2} dx, x, -\frac{\tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{2b} + \dots \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{2b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{-c+c \sec(2a+2bx)}}\right)}{\sqrt{2} b\sqrt{c}} + \frac{\sin(2a+2bx)}{2b\sqrt{-c+c \sec(2a+2bx)}} \end{aligned}$$

Mathematica [A] time = 2.32, size = 166, normalized size = 1.20

$$\frac{\tan(a+bx) \left(\sqrt{2} \tanh^{-1}\left(\frac{1}{2} \sqrt{2-2 \tan^2(a+bx)}\right) - \tanh^{-1}\left(\sqrt{1-\tan^2(a+bx)}\right) + \sqrt{2} \cos^2(a+bx) \sqrt{\frac{1}{\sec(2(a+bx))+1}} \right)}{2b\sqrt{1-\tan^2(a+bx)} \sqrt{c} \tan(a+bx) \tan(2(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]
```

```
[Out] (Tan[a + b*x]*(Sqrt[2]*ArcTanh[Sqrt[2 - 2*Tan[a + b*x]^2]/2] - ArcTanh[Sqrt[1 - Tan[a + b*x]^2]] + Sqrt[2]*Cos[a + b*x]^2*Sqrt[(1 + Sec[2*(a + b*x)])^(-1)]*(2 + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sqrt[-1 + Tan[a + b*x]^2])))/(2*b*Sqrt[1 - Tan[a + b*x]^2]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

fricas [A] time = 0.96, size = 481, normalized size = 3.49

$$\left[\frac{\sqrt{2} \left(\tan(bx+a)^3 + \tan(bx+a) \right) \sqrt{c} \log \left(\frac{c \tan(bx+a)^3 - 2 \sqrt{\frac{c \tan(bx+a)^2}{\tan(bx+a)^2 - 1}} (\tan(bx+a)^2 - 1) \sqrt{c} - 2c \tan(bx+a)}{\tan(bx+a)^3} \right)}{4 \left(bc \tan(bx+a) \right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*(sqrt(2)*(tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log((c*tan(b*x + a)^3
- 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt
(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3 + (tan(b*x + a)^3 + tan(b*x + a))*s
qrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x +
a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x + a)^3
+ tan(b*x + a))) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*
(tan(b*x + a)^2 - 1)/(b*c*tan(b*x + a)^3 + b*c*tan(b*x + a)), -1/2*(sqrt(2
)*(tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(t
an(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - (tan(
b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)
^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) +
sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/
(b*c*tan(b*x + a)^3 + b*c*tan(b*x + a))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [B] time = 1.17, size = 1030, normalized size = 7.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)
```

```
[Out] 1/16*2^(1/2)/b*(-1+cos(b*x+a))^2*(cos(b*x+a)^3*4^(1/2)*((2*cos(b*x+a)^2-1)/
(cos(b*x+a)+1)^2)^(3/2)+4*cos(b*x+a)^2*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(3/2)+5*cos(b*x+a)*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(
3/2)+2*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)-6*cos(b*x+a)*2^
(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b
*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+6*cos(b*x+a)*((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(1/2)+4*cos(b*x+a)*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2
-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+4*cos(b*x+a)*arctanh(1/2*4^(1/2)*(2
*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/s
in(b*x+a)^2)-6*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b
*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+4*((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+4*ln(-2*(cos(b*x+a)^2*((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+a)^2-1)/(cos(b
*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)+4*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*
cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2))/((
2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1
))^1/2/sin(b*x+a)^3*4^(1/2)+1/8*2^(1/2)/b*(cos(b*x+a)+1)*((2*cos(b*x+a)^2
-1)/(cos(b*x+a)+1)^2)^(1/2)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^1/2*(
2*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*c
os(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-ln(-2*(cos(b*x+a)^2*((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-2*cos(b*x+a)^2+cos(b*x+a)-((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+1)/sin(b*x+a)^2)-arctanh(1/2*4^(1/2)*(2*cos
```

$(b*x+a)^2-3*\cos(b*x+a)+1)/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}/\sin(b*x+a)^2)/\sin(b*x+a)/c*4^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(2bx + 2a)}{\sqrt{c \tan(2bx + 2a) \tan(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(2*b*x + 2*a)/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(2a + 2bx)}{\sqrt{c \tan(a + bx) \tan(2a + 2bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*a + 2*b*x)/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)

[Out] int(cos(2*a + 2*b*x)/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)

[Out] Timed out

$$3.625 \quad \int \frac{\cos^2(2(a+bx))}{\sqrt{c} \tan(a+bx) \tan(2(a+bx))} dx$$

Optimal. Leaf size=182

$$\frac{\sin(2a+2bx)}{8b\sqrt{c}\sec(2a+2bx)-c} + \frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c}\sec(2a+2bx)-c} + \frac{7 \tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c}\sec(2a+2bx)-c}\right)}{8b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c}\sec(2a+2bx)-c}\right)}{\sqrt{2}b\sqrt{c}}$$

[Out] $7/8*\operatorname{arctanh}(c^{(1/2)}*\tan(2*b*x+2*a)/(-c+c*\sec(2*b*x+2*a))^{(1/2)})/b/c^{(1/2)}-1/2*\operatorname{arctanh}(1/2*c^{(1/2)}*\tan(2*b*x+2*a)*2^{(1/2)/(-c+c*\sec(2*b*x+2*a))^{(1/2)})/b*2^{(1/2)}/c^{(1/2)}+1/8*\sin(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}+1/4*\cos(2*b*x+2*a)*\sin(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(1/2)}$

Rubi [A] time = 0.37, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4397, 3823, 4022, 3920, 3774, 207, 3795}

$$\frac{\sin(2a+2bx)}{8b\sqrt{c}\sec(2a+2bx)-c} + \frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c}\sec(2a+2bx)-c} + \frac{7 \tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c}\sec(2a+2bx)-c}\right)}{8b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c}\sec(2a+2bx)-c}\right)}{\sqrt{2}b\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[Cos[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]

[Out] $(7*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\tan[2*a+2*b*x])/(\operatorname{Sqrt}[-c+c*\sec[2*a+2*b*x]])]/(8*b*\operatorname{Sqrt}[c]) - \operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\tan[2*a+2*b*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[-c+c*\sec[2*a+2*b*x]])]/(\operatorname{Sqrt}[2]*b*\operatorname{Sqrt}[c]) + \sin[2*a+2*b*x]/(8*b*\operatorname{Sqrt}[-c+c*\sec[2*a+2*b*x]]) + (\cos[2*a+2*b*x]*\sin[2*a+2*b*x])/(4*b*\operatorname{Sqrt}[-c+c*\sec[2*a+2*b*x]])$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3823

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n+1)*(a + b*(2*n+1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*m), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^2(2(a + bx))}{\sqrt{c \tan(a + bx) \tan(2(a + bx))}} dx = \int \frac{\cos^2(2a + 2bx)}{\sqrt{-c + c \sec(2a + 2bx)}} dx$$

$$= \frac{\cos(2a + 2bx) \sin(2a + 2bx)}{4b\sqrt{-c + c \sec(2a + 2bx)}} - \frac{\int \frac{\cos(2a+2bx)(-c-3c \sec(2a+2bx))}{\sqrt{-c+c \sec(2a+2bx)}} dx}{4c}$$

$$= \frac{\sin(2a + 2bx)}{8b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{\cos(2a + 2bx) \sin(2a + 2bx)}{4b\sqrt{-c + c \sec(2a + 2bx)}} - \frac{\int \frac{-\frac{7c^2}{2} - \frac{1}{2}c^2 \sec(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}} dx}{4c^2}$$

$$= \frac{\sin(2a + 2bx)}{8b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{\cos(2a + 2bx) \sin(2a + 2bx)}{4b\sqrt{-c + c \sec(2a + 2bx)}} - \frac{7 \int \sqrt{-c + c \sec(2a + 2bx)}}{4c^2}$$

$$= \frac{\sin(2a + 2bx)}{8b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{\cos(2a + 2bx) \sin(2a + 2bx)}{4b\sqrt{-c + c \sec(2a + 2bx)}} + \frac{7 \text{Subst}\left(\int \sqrt{-c + c \sec(2a + 2bx)}\right)}{4c^2}$$

$$= \frac{7 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{8b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{-c+c \sec(2a+2bx)}}\right)}{\sqrt{2} b \sqrt{c}} + \frac{\sin(2a + 2bx)}{8b\sqrt{-c + c \sec(2a + 2bx)}}$$

Mathematica [A] time = 2.90, size = 186, normalized size = 1.02

$$\frac{\tan(a + bx) \left(7\sqrt{2} \tanh^{-1}\left(\frac{1}{2}\sqrt{2 - 2 \tan^2(a + bx)}\right) - 7 \tanh^{-1}\left(\sqrt{1 - \tan^2(a + bx)}\right) + \sqrt{2} \cos^2(a + bx) \sec(2(a + bx)) \right)}{8b\sqrt{1 - \tan^2(a + bx)} \sqrt{c \tan(a + bx) \tan(2(a + bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]], x]
[Out] (Tan[a + b*x]*(7*Sqrt[2]*ArcTanh[Sqrt[2 - 2*Tan[a + b*x]^2]/2] - 7*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]] + Sqrt[2]*Cos[a + b*x]^2*Sec[2*(a + b*x)]*Sqrt[(1 + Sec[2*(a + b*x)])^(-1)]*(2*(1 + Cos[2*(a + b*x)] + Cos[4*(a + b*x)]) + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2])))/(8*b*Sqrt[1 - Tan[a + b*x]^2]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

fricas [A] time = 1.95, size = 569, normalized size = 3.13

$$4\sqrt{2}\left(\tan(bx+a)^5 + 2\tan(bx+a)^3 + \tan(bx+a)\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^3 - 2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}(\tan(bx+a)^2-1)\sqrt{c} - 2c\tan(bx+a)}{\tan(bx+a)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")

[Out] [1/16*(4*sqrt(2)*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(c) *log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 7*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x + a)^3 + tan(b*x + a))) + 2*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*tan(b*x + a)^5 + 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x + a)), -1/8*(4*sqrt(2)*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - 7*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - sqrt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*tan(b*x + a)^5 + 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x + a))]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 1.10, size = 1835, normalized size = 10.08

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x)

[Out]
$$-1/8*2^{(1/2)}/b*(\cos(b*x+a)+1)*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*(c*(1-\cos(b*x+a)^2)/(2*\cos(b*x+a)^2-1))^{(1/2)}*(2*2^{(1/2)}*\operatorname{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a)))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)})-\ln(-2*(\cos(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}-2*\cos(b*x+a)^2+\cos(b*x+a)-((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)+1})/\sin(b*x+a)^2-\operatorname{arctanh}(1/2*4^{(1/2)}*(2*\cos(b*x+a)^2-3*\cos(b*x+a)+1)/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}/\sin(b*x+a)^2)/\sin(b*x+a)/c*4^{(1/2)}-1/8*2^{(1/2)}/b*(-1+\cos(b*x+a))^2*(\cos(b*x+a)^3*4^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(3/2)}+4*\cos(b*x+a)^2*4^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(3/2)}+5*\cos(b*x+a)*4^{(1/2)}*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)$$

$$\begin{aligned} & \sqrt[3]{2} + 2 \cdot 4^{1/2} \cdot \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{3/2} - 6 \cos(bx+a) \cdot 2^{1/2} \cdot \operatorname{arctanh}\left(\frac{1}{2} \cos(bx+a) \cdot 4^{1/2} \cdot (-1 + \cos(bx+a))\right) / \sin(bx+a)^2 / \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} \cdot 2^{1/2} + 6 \cos(bx+a) \cdot \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} + 4 \cos(bx+a) \cdot \ln\left(-2 \cdot \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} - 2 \cos(bx+a)^2 + \cos(bx+a) - \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} + 1\right) / \sin(bx+a)^2 + 4 \cos(bx+a) \cdot \operatorname{arctanh}\left(\frac{1}{2} \cdot 4^{1/2} \cdot \left(\frac{2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1}{\left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} / \sin(bx+a)^2} - 6 \cdot 2^{1/2} \cdot \operatorname{arctanh}\left(\frac{1}{2} \cos(bx+a) \cdot 4^{1/2} \cdot (-1 + \cos(bx+a))\right) / \sin(bx+a)^2 \right) / \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} \cdot 2^{1/2} \right) + 4 \cdot \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} + 4 \cdot \ln\left(-2 \cdot \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} - 2 \cos(bx+a)^2 + \cos(bx+a) - \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} + 1\right) / \sin(bx+a)^2 + 4 \cdot \operatorname{arctanh}\left(\frac{1}{2} \cdot 4^{1/2} \cdot \left(\frac{2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1}{\left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} / \sin(bx+a)^2} \right) / \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} / \left(c \cdot \sin(bx+a)^2 / \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} / \sin(bx+a)^3 \cdot 4^{1/2} - 1/64 \cdot 2^{1/2} / b \cdot (-1 + \cos(bx+a))^2 \cdot (-4 \cdot 4^{1/2}) \cdot \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{3/2} \cdot \cos(bx+a)^5 - 16 \cdot 4^{1/2} \cdot \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{3/2} \cdot \cos(bx+a)^4 - 33 \cos(bx+a)^3 \cdot 4^{1/2} \cdot \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{3/2} - 52 \cos(bx+a)^2 \cdot 4^{1/2} \cdot \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{3/2} - 49 \cos(bx+a) \cdot 4^{1/2} \cdot \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{3/2} - 18 \cdot 4^{1/2} \cdot \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{3/2} + 46 \cos(bx+a) \cdot 2^{1/2} \cdot \operatorname{arctanh}\left(\frac{1}{2} \cos(bx+a) \cdot 4^{1/2} \cdot (-1 + \cos(bx+a))\right) / \sin(bx+a)^2 / \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} \cdot 2^{1/2} \right) + 46 \cdot 2^{1/2} \cdot \operatorname{arctanh}\left(\frac{1}{2} \cos(bx+a) \cdot 4^{1/2} \cdot (-1 + \cos(bx+a))\right) / \sin(bx+a)^2 / \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} \cdot 2^{1/2} \right) - 54 \cos(bx+a) \cdot \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} - 32 \cos(bx+a) \cdot \ln\left(-2 \cdot \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} - 2 \cos(bx+a)^2 + \cos(bx+a) - \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} + 1\right) / \sin(bx+a)^2 - 32 \cos(bx+a) \cdot \operatorname{arctanh}\left(\frac{1}{2} \cdot 4^{1/2} \cdot \left(\frac{2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1}{\left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} / \sin(bx+a)^2} \right) / \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} / \left(c \cdot \sin(bx+a)^2 / \left(\frac{2 \cos(bx+a)^2 - 1}{(\cos(bx+a)+1)^2} \right)^{1/2} / \sin(bx+a)^3 \cdot 4^{1/2} \right) \right) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(2bx + 2a)^2}{\sqrt{c \tan(2bx + 2a) \tan(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(2*b*x + 2*a)^2/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(2a + 2bx)^2}{\sqrt{c \tan(a + bx) \tan(2a + 2bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*a + 2*b*x)^2/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)

[Out] int(cos(2*a + 2*b*x)^2/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

$$3.626 \quad \int \frac{\sec^4(2(a+bx))}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx$$

Optimal. Leaf size=180

$$-\frac{11 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{4\sqrt{2} bc^{3/2}} + \frac{7 \tan(2a+2bx) \sqrt{c \sec(2a+2bx)-c}}{12bc^2} - \frac{\tan(2a+2bx) \sec^2(2a+2bx)}{4b(c \sec(2a+2bx)-c)^{3/2}} + \frac{13 \tan(2a+2bx)}{6bc \sqrt{c \sec(2a+2bx)-c}}$$

[Out] $-11/8 * \operatorname{arctanh}(1/2 * c^{1/2} * \tan(2 * b * x + 2 * a) * 2^{1/2} / (-c + c * \sec(2 * b * x + 2 * a))^{1/2}) / b / c^{3/2} * 2^{1/2} - 1/4 * \sec(2 * b * x + 2 * a)^2 * \tan(2 * b * x + 2 * a) / b / (-c + c * \sec(2 * b * x + 2 * a))^{3/2} + 13/6 * \tan(2 * b * x + 2 * a) / b / c / (-c + c * \sec(2 * b * x + 2 * a))^{1/2} + 7/12 * (-c + c * \sec(2 * b * x + 2 * a))^{1/2} * \tan(2 * b * x + 2 * a) / b / c^2$

Rubi [A] time = 0.51, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4397, 3816, 4010, 4001, 3795, 207}

$$\frac{7 \tan(2a+2bx) \sqrt{c \sec(2a+2bx)-c}}{12bc^2} - \frac{11 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{\tan(2a+2bx) \sec^2(2a+2bx)}{4b(c \sec(2a+2bx)-c)^{3/2}} + \frac{13 \tan(2a+2bx)}{6bc \sqrt{c \sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

[In] `Int[Sec[2*(a + b*x)]^4/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]`

[Out] $(-11 * \operatorname{ArcTanh}[(\operatorname{Sqrt}[c] * \tan[2 * a + 2 * b * x]) / (\operatorname{Sqrt}[2] * \operatorname{Sqrt}[-c + c * \sec[2 * a + 2 * b * x]])]) / (4 * \operatorname{Sqrt}[2] * b * c^{3/2}) - (\sec[2 * a + 2 * b * x]^2 * \tan[2 * a + 2 * b * x]) / (4 * b * (-c + c * \sec[2 * a + 2 * b * x])^{3/2}) + (13 * \tan[2 * a + 2 * b * x]) / (6 * b * c * \operatorname{Sqrt}[-c + c * \sec[2 * a + 2 * b * x]]) + (7 * \operatorname{Sqrt}[-c + c * \sec[2 * a + 2 * b * x]] * \tan[2 * a + 2 * b * x]) / (12 * b * c^2)$

Rule 207

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

Rule 3795

`Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]`

Rule 3816

`Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n-2))/(f*(2*m+1)), x] + Dist[d^2/(a*b*(2*m+1)), Int[(a + b*Csc[e + f*x])^(m+1)*(d*Csc[e + f*x])^(n-2)*(b*(n-2) + a*(m-n+2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])`

Rule 4001

`Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m+1)), x] + Dist[(a*B*m + A*b*(m+1))/(b*(m+1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m`

+ 1), 0] && !LtQ[m, -2^(-1)]

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(
csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
0] && !LtQ[m, -1]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned} \int \frac{\sec^4(2(a+bx))}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx &= \int \frac{\sec^4(2a+2bx)}{(-c+c \sec(2a+2bx))^{3/2}} dx \\ &= -\frac{\sec^2(2a+2bx) \tan(2a+2bx)}{4b(-c+c \sec(2a+2bx))^{3/2}} + \frac{\int \frac{\sec^2(2a+2bx) \left(2c+\frac{7}{2}c \sec(2a+2bx)\right)}{\sqrt{-c+c \sec(2a+2bx)}} dx}{2c^2} \\ &= -\frac{\sec^2(2a+2bx) \tan(2a+2bx)}{4b(-c+c \sec(2a+2bx))^{3/2}} + \frac{7\sqrt{-c+c \sec(2a+2bx)} \tan(2a+2bx)}{12bc^2} \\ &= -\frac{\sec^2(2a+2bx) \tan(2a+2bx)}{4b(-c+c \sec(2a+2bx))^{3/2}} + \frac{13 \tan(2a+2bx)}{6bc\sqrt{-c+c \sec(2a+2bx)}} + \frac{7\sqrt{-c+c \sec(2a+2bx)}}{12bc^2} \\ &= -\frac{\sec^2(2a+2bx) \tan(2a+2bx)}{4b(-c+c \sec(2a+2bx))^{3/2}} + \frac{13 \tan(2a+2bx)}{6bc\sqrt{-c+c \sec(2a+2bx)}} + \frac{7\sqrt{-c+c \sec(2a+2bx)}}{12bc^2} \\ &= -\frac{11 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{-c+c \sec(2a+2bx)}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{\sec^2(2a+2bx) \tan(2a+2bx)}{4b(-c+c \sec(2a+2bx))^{3/2}} + \frac{7\sqrt{-c+c \sec(2a+2bx)}}{12bc^2} \end{aligned}$$

Mathematica [A] time = 1.30, size = 100, normalized size = 0.56

$$\frac{\cot(a+bx)\sqrt{c \tan(a+bx) \tan(2(a+bx))} \left(\csc^2(a+bx)((19 \cos(4(a+bx)) + 11) \sec(2(a+bx)) - 24) - 66 \tan(a+bx) \right)}{48bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^4/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]
[Out] -1/48*(Cot[a + b*x]*(Csc[a + b*x]^2*(-24 + (11 + 19*Cos[4*(a + b*x)])*Sec[2
*(a + b*x)]) - 66*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^
2])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(b*c^2)
```


$$b*x+a)^2+132*\operatorname{arctanh}(1/2*4^{(1/2)}*(2*\cos(b*x+a)^2-3*\cos(b*x+a)+1)/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}/\sin(b*x+a)^2)*\cos(b*x+a)^2+54*\cos(b*x+a)*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}+33*\cos(b*x+a)*\ln(-2*(\cos(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}-2*\cos(b*x+a)^2+\cos(b*x+a)-((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}+1)/\sin(b*x+a)^2)+33*\cos(b*x+a)*\operatorname{arctanh}(1/2*4^{(1/2)}*(2*\cos(b*x+a)^2-3*\cos(b*x+a)+1)/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}/\sin(b*x+a)^2)-33*\ln(-2*(\cos(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}-2*\cos(b*x+a)^2+\cos(b*x+a)-((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}+1)/\sin(b*x+a)^2)-33*\operatorname{arctanh}(1/2*4^{(1/2)}*(2*\cos(b*x+a)^2-3*\cos(b*x+a)+1)/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}/\sin(b*x+a)^2)))/(2*\cos(b*x+a)^2-1)^2/(c*\sin(b*x+a)^2/(2*\cos(b*x+a)^2-1))^{(3/2)}/\sin(b*x+a)^3/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(3/2)}*4^{(1/2)}/(-3+2*2^{(1/2)})^3/(3+2*2^{(1/2)})^3$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(2bx + 2a)^4}{(c \tan(2bx + 2a) \tan(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")

[Out] integrate(sec(2*b*x + 2*a)^4/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(2a + 2bx)^4 (c \tan(a + bx) \tan(2a + 2bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(2*a + 2*b*x)^4*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)),x)

[Out] int(1/(cos(2*a + 2*b*x)^4*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)**4/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)

[Out] Timed out

$$3.627 \quad \int \frac{\sec^3(2(a+bx))}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx$$

Optimal. Leaf size=128

$$-\frac{7 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{4\sqrt{2} bc^{3/2}} + \frac{\tan(2a+2bx)}{bc\sqrt{c \sec(2a+2bx)-c}} - \frac{\tan(2a+2bx)}{4b(c \sec(2a+2bx)-c)^{3/2}}$$

[Out] $-7/8*\operatorname{arctanh}(1/2*c^{(1/2)}*\tan(2*b*x+2*a)*2^{(1/2)/(-c+c*\sec(2*b*x+2*a))^{(1/2)})/b/c^{(3/2)}*2^{(1/2)}-1/4*\tan(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(3/2)}+\tan(2*b*x+2*a)/b/c/(-c+c*\sec(2*b*x+2*a))^{(1/2)}$

Rubi [A] time = 0.31, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4397, 3799, 4001, 3795, 207}

$$-\frac{7 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{4\sqrt{2} bc^{3/2}} + \frac{\tan(2a+2bx)}{bc\sqrt{c \sec(2a+2bx)-c}} - \frac{\tan(2a+2bx)}{4b(c \sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sec}[2*(a + b*x)]^3/(c*\operatorname{Tan}[a + b*x]*\operatorname{Tan}[2*(a + b*x)])^{(3/2)}, x]$

[Out] $(-7*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\operatorname{Tan}[2*a + 2*b*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]])]/(4*\operatorname{Sqrt}[2]*b*c^{(3/2)}) - \operatorname{Tan}[2*a + 2*b*x]/(4*b*(-c + c*\operatorname{Sec}[2*a + 2*b*x])^{(3/2)}) + \operatorname{Tan}[2*a + 2*b*x]/(b*c*\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]])$

Rule 207

$\operatorname{Int}[(a + (b*x)^2)^{-1}, x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Rt}[b, 2]*x]/\operatorname{Rt}[-a, 2]]/(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2]), x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \mid\mid \operatorname{GtQ}[b, 0])$

Rule 3795

$\operatorname{Int}[\operatorname{csc}[(e + f*x)]/\operatorname{Sqrt}[\operatorname{csc}[(e + f*x)]*(b + a)], x_Symbol] \rightarrow \operatorname{Dist}[-2/f, \operatorname{Subst}[\operatorname{Int}[1/(2*a + x^2), x], x, (b*\operatorname{Cot}[e + f*x])/\operatorname{Sqrt}[a + b*\operatorname{Csc}[e + f*x]]], x] /; \operatorname{FreeQ}\{a, b, e, f, x\} \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 3799

$\operatorname{Int}[\operatorname{csc}[(e + f*x)]^3*(\operatorname{csc}[(e + f*x)]*(b + a))^{(m)}, x_Symbol] \rightarrow \operatorname{Simp}[(b*\operatorname{Cot}[e + f*x]*(a + b*\operatorname{Csc}[e + f*x])^m)/(a*f*(2*m + 1)), x] - \operatorname{Dist}[1/(a^2*(2*m + 1)), \operatorname{Int}[\operatorname{Csc}[e + f*x]*(a + b*\operatorname{Csc}[e + f*x])^{(m + 1)}*(a*m - b*(2*m + 1)*\operatorname{Csc}[e + f*x]), x], x] /; \operatorname{FreeQ}\{a, b, e, f, x\} \&\& \operatorname{EqQ}[a^2 - b^2, 0] \&\& \operatorname{LtQ}[m, -2^{(-1)}]$

Rule 4001

$\operatorname{Int}[\operatorname{csc}[(e + f*x)]*(\operatorname{csc}[(e + f*x)]*(b + a))^{(m)}*(\operatorname{csc}[(e + f*x)]*(B + A)), x_Symbol] \rightarrow -\operatorname{Simp}[(B*\operatorname{Cot}[e + f*x]*(a + b*\operatorname{Csc}[e + f*x])^m)/(f*(m + 1)), x] + \operatorname{Dist}[(a*B*m + A*b*(m + 1))/(b*(m + 1)), \operatorname{Int}[\operatorname{Csc}[e + f*x]*(a + b*\operatorname{Csc}[e + f*x])^m, x], x] /; \operatorname{FreeQ}\{a, b, A, B, e, f, m\}, x] \&\& \operatorname{NeQ}[A*b - a*B, 0] \&\& \operatorname{EqQ}[a^2 - b^2, 0] \&\& \operatorname{NeQ}[a*B*m + A*b*(m + 1), 0] \&\& \operatorname{!LtQ}[m, -2^{(-1)}]$

Rule 4397

$\operatorname{Int}[u, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{TrigSimplify}[u], x] /; \operatorname{TrigSimplifyQ}[u]$

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(2(a+bx))}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx &= \int \frac{\sec^3(2a+2bx)}{(-c+c \sec(2a+2bx))^{3/2}} dx \\
&= -\frac{\tan(2a+2bx)}{4b(-c+c \sec(2a+2bx))^{3/2}} + \frac{\int \frac{\sec(2a+2bx)\left(\frac{3c}{2}+2c \sec(2a+2bx)\right)}{\sqrt{-c+c \sec(2a+2bx)}} dx}{2c^2} \\
&= -\frac{\tan(2a+2bx)}{4b(-c+c \sec(2a+2bx))^{3/2}} + \frac{\tan(2a+2bx)}{bc\sqrt{-c+c \sec(2a+2bx)}} + \frac{7 \int \frac{\sec(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}} dx}{2c^2} \\
&= -\frac{\tan(2a+2bx)}{4b(-c+c \sec(2a+2bx))^{3/2}} + \frac{\tan(2a+2bx)}{bc\sqrt{-c+c \sec(2a+2bx)}} - \frac{7 \operatorname{Subst} \int \frac{\sec(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}} dx}{2c^2} \\
&= -\frac{7 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{-c+c \sec(2a+2bx)}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(-c+c \sec(2a+2bx))^{3/2}} + \frac{\tan(2a+2bx)}{bc\sqrt{-c+c \sec(2a+2bx)}}
\end{aligned}$$

Mathematica [A] time = 0.60, size = 94, normalized size = 0.73

$$\frac{\tan(2(a+bx)) \left(4 \sec(2(a+bx)) + 7 \sin^2(a+bx) \tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right) \sqrt{\tan^2(a+bx)-1} \sec(2(a+bx)) \right)}{4b(c \tan(a+bx) \tan(2(a+bx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^3/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]
[Out] ((-5 + 4*Sec[2*(a + b*x)] + 7*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sin[a + b*x]^2*Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a + b*x)]/(4*b*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))
```

fricas [A] time = 2.00, size = 276, normalized size = 2.16

$$\frac{7\sqrt{2}\sqrt{c} \log\left(\frac{c \tan(bx+a)^3 - 2\sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2-1}}(\tan(bx+a)^2-1)\sqrt{c-2c \tan(bx+a)}}{\tan(bx+a)^3}\right) \tan(bx+a)^3 + 2\sqrt{2}\sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2-1}}(9 \tan(bx+a)^2 - 1)}{16bc^2 \tan(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x, algorithm="fricas")
[Out] [1/16*(7*sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3)*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(9*tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3), -1/8*(7*sqrt(2)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 - sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(9*tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 1.15, size = 930, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)

[Out]
$$\begin{aligned} & -1/16*2^{(1/2)}/b*(7*\ln(-2*(\cos(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}-2*\cos(b*x+a)^2+\cos(b*x+a)-((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}+1)/\sin(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\cos(b*x+a)^3+7*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*4^{(1/2)}*(2*\cos(b*x+a)^2-3*\cos(b*x+a)+1)/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}/\sin(b*x+a)^2)*\cos(b*x+a)^3+7*\cos(b*x+a)^2*\ln(-2*(\cos(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}-2*\cos(b*x+a)^2+\cos(b*x+a)-((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}+1)/\sin(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}+7*\cos(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*4^{(1/2)}*(2*\cos(b*x+a)^2-3*\cos(b*x+a)+1)/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}/\sin(b*x+a)^2)-7*\cos(b*x+a)*\ln(-2*(\cos(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}-2*\cos(b*x+a)^2+\cos(b*x+a)-((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}+1)/\sin(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}-7*\cos(b*x+a)*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\operatorname{arctanh}(1/2*4^{(1/2)}*(2*\cos(b*x+a)^2-3*\cos(b*x+a)+1)/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}/\sin(b*x+a)^2)+20*\cos(b*x+a)^3-7*\ln(-2*(\cos(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}-2*\cos(b*x+a)^2+\cos(b*x+a)-((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}+1)/\sin(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}-7*\operatorname{arctanh}(1/2*4^{(1/2)}*(2*\cos(b*x+a)^2-3*\cos(b*x+a)+1)/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}/\sin(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}-18*\cos(b*x+a))*\sin(b*x+a)/(c*\sin(b*x+a)^2/(2*\cos(b*x+a)^2-1))^{(3/2)}/(2*\cos(b*x+a)^2-1)^2 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(2bx+2a)^3}{(c \tan(2bx+2a) \tan(bx+a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")

[Out] integrate(sec(2*b*x + 2*a)^3/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(2a+2bx)^3 (c \tan(a+bx) \tan(2a+2bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)),x)

[Out] int(1/(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**3/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

$$3.628 \quad \int \frac{\sec^2(2(a+bx))}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx$$

Optimal. Leaf size=93

$$-\frac{3 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(c \sec(2a+2bx)-c)^{3/2}}$$

[Out] $-3/8 \cdot \operatorname{arctanh}(1/2 \cdot c^{1/2} \cdot \tan(2 \cdot b \cdot x + 2 \cdot a)) \cdot 2^{1/2} / (-c + c \cdot \sec(2 \cdot b \cdot x + 2 \cdot a))^{1/2} / b / c^{3/2} \cdot 2^{1/2} - 1/4 \cdot \tan(2 \cdot b \cdot x + 2 \cdot a) / b / (-c + c \cdot \sec(2 \cdot b \cdot x + 2 \cdot a))^{3/2}$

Rubi [A] time = 0.24, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4397, 3797, 3795, 207}

$$-\frac{3 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(c \sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sec[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]

[Out] $(-3 \cdot \operatorname{ArcTanh}[(\operatorname{Sqrt}[c] \cdot \operatorname{Tan}[2 \cdot a + 2 \cdot b \cdot x]) / (\operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[-c + c \cdot \operatorname{Sec}[2 \cdot a + 2 \cdot b \cdot x]])]) / (4 \cdot \operatorname{Sqrt}[2] \cdot b \cdot c^{3/2}) - \operatorname{Tan}[2 \cdot a + 2 \cdot b \cdot x] / (4 \cdot b \cdot (-c + c \cdot \operatorname{Sec}[2 \cdot a + 2 \cdot b \cdot x])^{3/2})$

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 3795

Int[csc[(e_) + (f_)*(x_)]/Sqrt[csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3797

Int[csc[(e_) + (f_)*(x_)]^2*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_), x_Symbol] := -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 4397

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(2(a+bx))}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx &= \int \frac{\sec^2(2a+2bx)}{(-c+c \sec(2a+2bx))^{3/2}} dx \\
&= -\frac{\tan(2a+2bx)}{4b(-c+c \sec(2a+2bx))^{3/2}} + \frac{3 \int \frac{\sec(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}} dx}{4c} \\
&= -\frac{\tan(2a+2bx)}{4b(-c+c \sec(2a+2bx))^{3/2}} - \frac{3 \operatorname{Subst}\left(\int \frac{1}{-2c+x^2} dx, x, -\frac{c \tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{4bc} \\
&= -\frac{3 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{-c+c \sec(2a+2bx)}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(-c+c \sec(2a+2bx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.59, size = 84, normalized size = 0.90

$$\frac{\tan(2(a+bx)) \left(3 \sin^2(a+bx) \tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right) \sqrt{\tan^2(a+bx)-1} \sec(2(a+bx)) - 1 \right)}{4b(c \tan(a+bx) \tan(2(a+bx)))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]

[Out] ((-1 + 3*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sin[a + b*x]^2*Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a + b*x)]/(4*b*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))

fricas [A] time = 1.03, size = 272, normalized size = 2.92

$$\frac{3 \sqrt{2} \sqrt{c} \log\left(\frac{c \tan(bx+a)^3 - 2 \sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2-1}} (\tan(bx+a)^2-1) \sqrt{c} - 2c \tan(bx+a)}{\tan(bx+a)^3}\right) \tan(bx+a)^3 + 2 \sqrt{2} \sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2-1}} (\tan(bx+a)^2-1) \sqrt{c}}{16 bc^2 \tan(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x, algorithm="fricas")

[Out] [1/16*(3*sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3)*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/((b*c^2*tan(b*x + a)^3), -1/8*(3*sqrt(2)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 - sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 1.04, size = 433, normalized size = 4.66

$$\sqrt{2} (-1 + \cos (bx + a))^2 \left(2 \cos (bx + a) \sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}} + 3 \cos (bx + a) \ln \left(-\frac{2 \left((\cos^2(bx+a)) \sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}} - 2(\cos^2(bx+a)) \right)}{\sin(bx+a)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x)`

[Out]
$$-1/32*2^{(1/2)}/b*(-1+\cos(b*x+a))^{2*(2*\cos(b*x+a)*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}+3*\cos(b*x+a)*\ln(-2*(\cos(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}-2*\cos(b*x+a)^2+\cos(b*x+a)-((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}+1)/\sin(b*x+a)^2)+3*\cos(b*x+a)*\operatorname{arctanh}(1/2*4^{(1/2)}*(2*\cos(b*x+a)^2-3*\cos(b*x+a)+1)/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}/\sin(b*x+a)^2)-3*\ln(-2*(\cos(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}-2*\cos(b*x+a)^2+\cos(b*x+a)-((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}+1)/\sin(b*x+a)^2)-3*\operatorname{arctanh}(1/2*4^{(1/2)}*(2*\cos(b*x+a)^2-3*\cos(b*x+a)+1)/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}/\sin(b*x+a)^2))/(\sin(b*x+a)^2/(2*\cos(b*x+a)^2-1))^{(3/2)}/\sin(b*x+a)^3/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(3/2)}*4^{(1/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(2bx + 2a)^2}{(c \tan(2bx + 2a) \tan(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x, algorithm="maxima")`

[Out] `integrate(sec(2*b*x + 2*a)^2/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(2a + 2bx)^2 (c \tan(a + bx) \tan(2a + 2bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(\cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)`

[Out] `int(1/(\cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2), x)`

[Out] Timed out

$$3.629 \quad \int \frac{\sec(2(a+bx))}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx$$

Optimal. Leaf size=93

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(c \sec(2a+2bx)-c)^{3/2}}$$

[Out] $1/8 \cdot \operatorname{arctanh}(1/2 \cdot c^{1/2} \cdot \tan(2 \cdot b \cdot x + 2 \cdot a) \cdot 2^{1/2} / (-c + c \cdot \sec(2 \cdot b \cdot x + 2 \cdot a))^{1/2}) / b / c^{3/2} \cdot 2^{1/2} - 1/4 \cdot \tan(2 \cdot b \cdot x + 2 \cdot a) / b / (-c + c \cdot \sec(2 \cdot b \cdot x + 2 \cdot a))^{3/2}$

Rubi [A] time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4397, 3796, 3795, 207}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(c \sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[Sec[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]`

[Out] `ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]])]/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a + 2*b*x])^(3/2))`

Rule 207

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

Rule 3795

`Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]`

Rule 3796

`Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && IntegerQ[2*m]`

Rule 4397

`Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]`

Rubi steps

$$\begin{aligned}
\int \frac{\sec(2(a+bx))}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx &= \int \frac{\sec(2a+2bx)}{(-c+c \sec(2a+2bx))^{3/2}} dx \\
&= -\frac{\tan(2a+2bx)}{4b(-c+c \sec(2a+2bx))^{3/2}} - \frac{\int \frac{\sec(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}} dx}{4c} \\
&= -\frac{\tan(2a+2bx)}{4b(-c+c \sec(2a+2bx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{-2c+x^2} dx, x, -\frac{c \tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{4bc} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{-c+c \sec(2a+2bx)}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(-c+c \sec(2a+2bx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.62, size = 83, normalized size = 0.89

$$\frac{\tan(2(a+bx)) \left(\sin^2(a+bx) \tan^{-1}\left(\sqrt{\tan^2(a+bx)-1}\right) \sqrt{\tan^2(a+bx)-1} \sec(2(a+bx)) + 1 \right)}{4b(c \tan(a+bx) \tan(2(a+bx)))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]

[Out] -1/4*((1 + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sin[a + b*x]^2*Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a + b*x)]/(b*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))

fricas [A] time = 1.21, size = 269, normalized size = 2.89

$$\frac{\sqrt{2} \sqrt{c} \log\left(\frac{c \tan(bx+a)^3 + 2 \sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2-1}} (\tan(bx+a)^2-1) \sqrt{c} - 2c \tan(bx+a)}{\tan(bx+a)^3}\right) \tan(bx+a)^3 + 2 \sqrt{2} \sqrt{-\frac{c \tan(bx+a)^2}{\tan(bx+a)^2-1}} (\tan(bx+a)^2-1) \sqrt{c}}{16 bc^2 \tan(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x, algorithm="fricas")

[Out] [1/16*(sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3)*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/b*c^2*tan(b*x + a)^3, 1/8*(sqrt(2)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 + sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/b*c^2*tan(b*x + a)^3]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 1.04, size = 599, normalized size = 6.44

$$\sqrt{2} (-1 + \cos(bx + a))^3 \left(2 (\cos^2(bx + a)) \sqrt{4} \left(\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2} \right)^{\frac{3}{2}} + 4 \cos(bx + a) \sqrt{4} \left(\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2} \right)^{\frac{3}{2}} + 2\sqrt{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x)

[Out] $\frac{1}{32} 2^{(1/2)} / b * (-1 + \cos(b*x+a))^{3/2} * (2*\cos(b*x+a)^2 * 4^{(1/2)} * ((2*\cos(b*x+a)^2 - 1) / (\cos(b*x+a)+1)^2)^{(3/2)} + 4*\cos(b*x+a) * 4^{(1/2)} * ((2*\cos(b*x+a)^2 - 1) / (\cos(b*x+a)+1)^2)^{(3/2)} - 6*\cos(b*x+a)^2 * ((2*\cos(b*x+a)^2 - 1) / (\cos(b*x+a)+1)^2)^{(1/2)} - \ln(-2*(\cos(b*x+a)^2 * ((2*\cos(b*x+a)^2 - 1) / (\cos(b*x+a)+1)^2)^{(1/2)} - 2*\cos(b*x+a)^2 + \cos(b*x+a) - ((2*\cos(b*x+a)^2 - 1) / (\cos(b*x+a)+1)^2)^{(1/2)} + 1) / \sin(b*x+a)^2 * \cos(b*x+a)^2 - \operatorname{arctanh}(1/2 * 4^{(1/2)} * (2*\cos(b*x+a)^2 - 3*\cos(b*x+a) + 1) / ((2*\cos(b*x+a)^2 - 1) / (\cos(b*x+a)+1)^2)^{(1/2)} / \sin(b*x+a)^2 * \cos(b*x+a)^2 + 2*\cos(b*x+a) * ((2*\cos(b*x+a)^2 - 1) / (\cos(b*x+a)+1)^2)^{(1/2)} + 4 * ((2*\cos(b*x+a)^2 - 1) / (\cos(b*x+a)+1)^2)^{(1/2)} + \ln(-2*(\cos(b*x+a)^2 * ((2*\cos(b*x+a)^2 - 1) / (\cos(b*x+a)+1)^2)^{(1/2)} - 2*\cos(b*x+a)^2 + \cos(b*x+a) - ((2*\cos(b*x+a)^2 - 1) / (\cos(b*x+a)+1)^2)^{(1/2)} + 1) / \sin(b*x+a)^2) + \operatorname{arctanh}(1/2 * 4^{(1/2)} * (2*\cos(b*x+a)^2 - 3*\cos(b*x+a) + 1) / ((2*\cos(b*x+a)^2 - 1) / (\cos(b*x+a)+1)^2)^{(1/2)} / \sin(b*x+a)^2)) / (c*\sin(b*x+a)^2 / (2*\cos(b*x+a)^2 - 1))^{(3/2)} / \sin(b*x+a)^5 / ((2*\cos(b*x+a)^2 - 1) / (\cos(b*x+a)+1)^2)^{(3/2)} * 4^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(2bx + 2a)}{(c \tan(2bx + 2a) \tan(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x, algorithm="maxima")

[Out] integrate(sec(2*b*x + 2*a)/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(2a + 2bx) (c \tan(a + bx) \tan(2a + 2bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)

[Out] int(1/(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x)

[Out] Timed out

$$3.630 \quad \int \frac{1}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx$$

Optimal. Leaf size=138

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{bc^{3/2}} + \frac{5 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(c \sec(2a+2bx)-c)^{3/2}}$$

[Out] $-\operatorname{arctanh}(c^{1/2} \tan(2bx+2a)/(-c+c \sec(2bx+2a))^{1/2})/b/c^{3/2}+5/8 \operatorname{arctanh}(1/2*c^{1/2} \tan(2bx+2a)*2^{1/2}/(-c+c \sec(2bx+2a))^{1/2})/b/c^{3/2}*2^{1/2}-1/4 \tan(2bx+2a)/b/(-c+c \sec(2bx+2a))^{3/2}$

Rubi [A] time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4397, 3777, 3920, 3774, 207, 3795}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{bc^{3/2}} + \frac{5 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(c \sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c \tan[a+bx] \tan[2(a+bx)])^{-3/2}, x]$

[Out] $-(\operatorname{ArcTanh}[(\operatorname{Sqrt}[c] \tan[2a+2bx])/\operatorname{Sqrt}[-c+c \sec[2a+2bx]])/(b*c^{3/2})) + (5*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c] \tan[2a+2bx])/(2*\operatorname{Sqrt}[-c+c \sec[2a+2bx]])])/(4*\operatorname{Sqrt}[2]*b*c^{3/2}) - \tan[2a+2bx]/(4*b*(-c+c \sec[2a+2bx])^{3/2})$

Rule 207

$\operatorname{Int}[(a_+ + (b_+)(x_+)^2)^{-1}, x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Rt}[b, 2]*x]/\operatorname{Rt}[-a, 2]]/(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2]), x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{LtQ}[a, 0] \ || \ \operatorname{GtQ}[b, 0])$

Rule 3774

$\operatorname{Int}[\operatorname{Sqrt}[\operatorname{csc}[c_+ + (d_+)(x_+)]*(b_+ + (a_+))], x_Symbol] \rightarrow \operatorname{Dist}[(-2*b)/d, \operatorname{Subst}[\operatorname{Int}[1/(a+x^2), x], x, (b*\operatorname{Cot}[c+d*x])/\operatorname{Sqrt}[a+b*\operatorname{Csc}[c+d*x]]], x] /; \operatorname{FreeQ}\{a, b, c, d, x\} \ \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 3777

$\operatorname{Int}[(\operatorname{csc}[c_+ + (d_+)(x_+)]*(b_+ + (a_+)))^{n_+}, x_Symbol] \rightarrow -\operatorname{Simp}[(\operatorname{Cot}[c+d*x]*(a+b*\operatorname{Csc}[c+d*x])^n)/(d*(2*n+1)), x] + \operatorname{Dist}[1/(a^2*(2*n+1)), \operatorname{Int}[(a+b*\operatorname{Csc}[c+d*x])^{n+1}*(a*(2*n+1) - b*(n+1)*\operatorname{Csc}[c+d*x]), x], x] /; \operatorname{FreeQ}\{a, b, c, d, x\} \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{LeQ}[n, -1] \ \&\& \operatorname{IntegerQ}[2*n]$

Rule 3795

$\operatorname{Int}[\operatorname{csc}[e_+ + (f_+)(x_+)]/\operatorname{Sqrt}[\operatorname{csc}[e_+ + (f_+)(x_+)]*(b_+ + (a_+))], x_Symbol] \rightarrow \operatorname{Dist}[-2/f, \operatorname{Subst}[\operatorname{Int}[1/(2*a+x^2), x], x, (b*\operatorname{Cot}[e+f*x])/\operatorname{Sqrt}[a+b*\operatorname{Csc}[e+f*x]]], x] /; \operatorname{FreeQ}\{a, b, e, f, x\} \ \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 3920

$\operatorname{Int}[(\operatorname{csc}[e_+ + (f_+)(x_+)]*(d_+ + (c_+)))/\operatorname{Sqrt}[\operatorname{csc}[e_+ + (f_+)(x_+)]*(b_+ + (a_+))], x_Symbol] \rightarrow \operatorname{Dist}[c/a, \operatorname{Int}[\operatorname{Sqrt}[a+b*\operatorname{Csc}[e+f*x]], x], x] - \operatorname{Dist}[(b*c - a*d)/a, \operatorname{Int}[\operatorname{Csc}[e+f*x]/\operatorname{Sqrt}[a+b*\operatorname{Csc}[e+f*x]], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

$eQ[\{a, b, c, d, e, f\}, x] \ \&\& \ NeQ[b*c - a*d, 0] \ \&\& \ EqQ[a^2 - b^2, 0]$

Rule 4397

$\text{Int}[u, x_Symbol] \ :> \ \text{Int}[\text{TrigSimplify}[u], x] \ /; \ \text{TrigSimplifyQ}[u]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(c \tan(a + bx) \tan(2(a + bx)))^{3/2}} dx &= \int \frac{1}{(-c + c \sec(2a + 2bx))^{3/2}} dx \\ &= -\frac{\tan(2a + 2bx)}{4b(-c + c \sec(2a + 2bx))^{3/2}} - \frac{\int \frac{2c + \frac{1}{2}c \sec(2a + 2bx)}{\sqrt{-c + c \sec(2a + 2bx)}} dx}{2c^2} \\ &= -\frac{\tan(2a + 2bx)}{4b(-c + c \sec(2a + 2bx))^{3/2}} + \frac{\int \sqrt{-c + c \sec(2a + 2bx)} dx}{c^2} - \frac{5 \int \frac{1}{\sqrt{-c + c \sec(2a + 2bx)}} dx}{4b(-c + c \sec(2a + 2bx))^{3/2}} \\ &= -\frac{\tan(2a + 2bx)}{4b(-c + c \sec(2a + 2bx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{-c + x^2} dx, x, -\frac{c \tan(2a + 2bx)}{\sqrt{-c + c \sec(2a + 2bx)}}\right)}{bc} \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{c} \tan(2a + 2bx)}{\sqrt{-c + c \sec(2a + 2bx)}}\right)}{bc^{3/2}} + \frac{5 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a + 2bx)}{\sqrt{2} \sqrt{-c + c \sec(2a + 2bx)}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{5}{4b(-c + c \sec(2a + 2bx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 3.70, size = 196, normalized size = 1.42

$$\cot(a + bx) \sqrt{c \tan(a + bx) \tan(2(a + bx))} \left(\tan^{-1}\left(\sqrt{\tan^2(a + bx) - 1}\right) \sqrt{-(\tan^2(a + bx) - 1)^2} + \cot^2(a + bx) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(-3/2), x]

[Out] $-1/8 * (\text{Cot}[a + b*x] * (4 * \text{Sqrt}[2] * \text{ArcTanh}[\text{Sqrt}[2 - 2 * \text{Tan}[a + b*x]^2] / 2] * \text{Cos}[2 * (a + b*x)] * \text{Sec}[a + b*x]^2 - 4 * \text{ArcTanh}[\text{Sqrt}[1 - \text{Tan}[a + b*x]^2]] * \text{Cos}[2 * (a + b*x)] * \text{Sec}[a + b*x]^2 + \text{Cot}[a + b*x]^2 * (\text{Cos}[2 * (a + b*x)] * \text{Sec}[a + b*x]^2)^{(3/2)} + \text{ArcTan}[\text{Sqrt}[-1 + \text{Tan}[a + b*x]^2]] * \text{Sqrt}[-(-1 + \text{Tan}[a + b*x]^2)^2]) * \text{Sqrt}[c * \text{Tan}[a + b*x] * \text{Tan}[2 * (a + b*x)]]) / (b * c^2 * \text{Sqrt}[1 - \text{Tan}[a + b*x]^2])$

fricas [A] time = 0.67, size = 438, normalized size = 3.17

$$\frac{5 \sqrt{2} \sqrt{c} \log\left(\frac{c \tan(bx+a)^3 + 2 \sqrt{\frac{c \tan(bx+a)^2}{\tan(bx+a)^2 - 1}} (\tan(bx+a)^2 - 1) \sqrt{c} - 2c \tan(bx+a)}{\tan(bx+a)^3}\right) \tan(bx+a)^3 + 8 \sqrt{c} \log\left(\frac{c \tan(bx+a)^3 - 2 \sqrt{2} \sqrt{c} \tan(bx+a)}{16 bc^2 \tan(bx+a)}\right)}{16 bc^2 \tan(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2), x, algorithm="fricas")

[Out] $[1/16 * (5 * \text{sqrt}(2) * \text{sqrt}(c) * \log((c * \tan(b*x + a))^3 + 2 * \text{sqrt}(-c * \tan(b*x + a))^2 / (\tan(b*x + a)^2 - 1)) * (\tan(b*x + a)^2 - 1) * \text{sqrt}(c) - 2 * c * \tan(b*x + a)) / \tan(b*x + a)^3 + 8 * \text{sqrt}(c) * \log((c * \tan(b*x + a))^3 - 2 * \text{sqrt}(2) * \text{sqrt}(c) * \tan(b*x + a)) / (16 * b * c^2 * \tan(b*x + a))]$

$$t(-c \tan(bx + a)^2 / (\tan(bx + a)^2 - 1)) * (\tan(bx + a)^2 - 1) * \sqrt{c} - 3 * c \tan(bx + a) / (\tan(bx + a)^3 + \tan(bx + a)) * \tan(bx + a)^3 + 2 * \sqrt{2} * \sqrt{-c \tan(bx + a)^2 / (\tan(bx + a)^2 - 1)} * (\tan(bx + a)^2 - 1) / (b * c^2 * \tan(bx + a)^3), 1/8 * (5 * \sqrt{2} * \sqrt{-c} * \arctan(\sqrt{-c \tan(bx + a)^2 / (\tan(bx + a)^2 - 1)} * (\tan(bx + a)^2 - 1) * \sqrt{-c} / (c \tan(bx + a))) * \tan(bx + a)^3 - 8 * \sqrt{-c} * \arctan(1/2 * \sqrt{2} * \sqrt{-c \tan(bx + a)^2 / (\tan(bx + a)^2 - 1)} * (\tan(bx + a)^2 - 1) * \sqrt{-c} / (c \tan(bx + a))) * \tan(bx + a)^3 + \sqrt{2} * \sqrt{-c \tan(bx + a)^2 / (\tan(bx + a)^2 - 1)} * (\tan(bx + a)^2 - 1) / (b * c^2 * \tan(bx + a)^3)]$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 1.02, size = 561, normalized size = 4.07

$$\sqrt{2} (-1 + \cos(bx + a))^2 \left(8 \cos(bx + a) \sqrt{2} \operatorname{arctanh} \left(\frac{\cos(bx+a) \sqrt{4} (-1 + \cos(bx+a)) \sqrt{2}}{2 \sin(bx+a)^2 \sqrt{\frac{2(\cos^2(bx+a)-1)}{(\cos(bx+a)+1)^2}}} \right) + 2 \cos(bx + a) \sqrt{\frac{2(\cos^2(bx+a)-1)}{(\cos(bx+a)+1)^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)

[Out]
$$-1/32 * 2^{(1/2)} / b * (-1 + \cos(bx + a))^2 * (8 * \cos(bx + a) * 2^{(1/2)} * \operatorname{arctanh}(1/2 * \cos(bx + a) * 4^{(1/2)} * (-1 + \cos(bx + a)) / \sin(bx + a)^2 / ((2 * \cos(bx + a)^2 - 1) / (\cos(bx + a) + 1)^2)^{(1/2)} * 2^{(1/2)}) + 2 * \cos(bx + a) * ((2 * \cos(bx + a)^2 - 1) / (\cos(bx + a) + 1)^2)^{(1/2)} - 5 * \cos(bx + a) * \ln(-2 * (\cos(bx + a)^2 * ((2 * \cos(bx + a)^2 - 1) / (\cos(bx + a) + 1)^2)^{(1/2)} - 2 * \cos(bx + a)^2 + \cos(bx + a) - ((2 * \cos(bx + a)^2 - 1) / (\cos(bx + a) + 1)^2)^{(1/2)} + 1) / \sin(bx + a)^2) - 5 * \cos(bx + a) * \operatorname{arctanh}(1/2 * 4^{(1/2)} * (2 * \cos(bx + a)^2 - 3 * \cos(bx + a) + 1) / ((2 * \cos(bx + a)^2 - 1) / (\cos(bx + a) + 1)^2)^{(1/2)} / \sin(bx + a)^2) - 8 * 2^{(1/2)} * \operatorname{arctanh}(1/2 * \cos(bx + a) * 4^{(1/2)} * (-1 + \cos(bx + a)) / \sin(bx + a)^2 / ((2 * \cos(bx + a)^2 - 1) / (\cos(bx + a) + 1)^2)^{(1/2)} * 2^{(1/2)}) + 5 * \ln(-2 * (\cos(bx + a)^2 * ((2 * \cos(bx + a)^2 - 1) / (\cos(bx + a) + 1)^2)^{(1/2)} - 2 * \cos(bx + a)^2 + \cos(bx + a) - ((2 * \cos(bx + a)^2 - 1) / (\cos(bx + a) + 1)^2)^{(1/2)} + 1) / \sin(bx + a)^2) + 5 * \operatorname{arctanh}(1/2 * 4^{(1/2)} * (2 * \cos(bx + a)^2 - 3 * \cos(bx + a) + 1) / ((2 * \cos(bx + a)^2 - 1) / (\cos(bx + a) + 1)^2)^{(1/2)} / \sin(bx + a)^2) / (c * \sin(bx + a)^2 / (2 * \cos(bx + a)^2 - 1))^{(3/2)} / \sin(bx + a)^3 / ((2 * \cos(bx + a)^2 - 1) / (\cos(bx + a) + 1)^2)^{(3/2)} * 4^{(1/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c \tan(2bx + 2a) \tan(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")

[Out] integrate((c*tan(2*b*x + 2*a)*tan(b*x + a))^(-3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c \tan(a + bx) \tan(2a + 2bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

```
[Out] int(1/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2), x)
```

```
[Out] Timed out
```

$$3.631 \quad \int \frac{\cos(2(a+bx))}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx$$

Optimal. Leaf size=178

$$-\frac{3 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{2bc^{3/2}} + \frac{9 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{3 \sin(2a+2bx)}{4bc\sqrt{c \sec(2a+2bx)-c}} - \frac{\sin(2a+2bx)}{4b(c \sec(2a+2bx)-c)^{3/2}}$$

[Out] $-3/2*\operatorname{arctanh}(c^{(1/2)}*\tan(2*b*x+2*a)/(-c+c*\sec(2*b*x+2*a))^{(1/2)})/b/c^{(3/2)}-1/4*\sin(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{(3/2)}+9/8*\operatorname{arctanh}(1/2*c^{(1/2)}*\tan(2*b*x+2*a)*2^{(1/2)}/(-c+c*\sec(2*b*x+2*a))^{(1/2)})/b/c^{(3/2)}*2^{(1/2)}-3/4*\sin(2*b*x+2*a)/b/c/(-c+c*\sec(2*b*x+2*a))^{(1/2)}$

Rubi [A] time = 0.32, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {4397, 3817, 4022, 3920, 3774, 207, 3795}

$$-\frac{3 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{2bc^{3/2}} + \frac{9 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{3 \sin(2a+2bx)}{4bc\sqrt{c \sec(2a+2bx)-c}} - \frac{\sin(2a+2bx)}{4b(c \sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[Cos[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]`

[Out] $(-3*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\operatorname{Tan}[2*a + 2*b*x])/(\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]])]/(2*b*c^{(3/2)}) + (9*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\operatorname{Tan}[2*a + 2*b*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]])]/(4*\operatorname{Sqrt}[2]*b*c^{(3/2)}) - \operatorname{Sin}[2*a + 2*b*x]/(4*b*(-c + c*\operatorname{Sec}[2*a + 2*b*x])^{(3/2)}) - (3*\operatorname{Sin}[2*a + 2*b*x])/((4*b*c*\operatorname{Sqrt}[-c + c*\operatorname{Sec}[2*a + 2*b*x]])$

Rule 207

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

Rule 3774

`Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

Rule 3795

`Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]`

Rule 3817

`Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])`

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4397

```
Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos(2(a + bx))}{(c \tan(a + bx) \tan(2(a + bx)))^{3/2}} dx = \int \frac{\cos(2a + 2bx)}{(-c + c \sec(2a + 2bx))^{3/2}} dx$$

$$= -\frac{\sin(2a + 2bx)}{4b(-c + c \sec(2a + 2bx))^{3/2}} - \frac{\int \frac{\cos(2a+2bx)\left(3c+\frac{3}{2}c \sec(2a+2bx)\right)}{\sqrt{-c+c \sec(2a+2bx)}} dx}{2c^2}$$

$$= -\frac{\sin(2a + 2bx)}{4b(-c + c \sec(2a + 2bx))^{3/2}} - \frac{3 \sin(2a + 2bx)}{4bc\sqrt{-c + c \sec(2a + 2bx)}} - \frac{\int \frac{3c^2+\frac{3}{2}c \sec(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}} dx}{2c^2}$$

$$= -\frac{\sin(2a + 2bx)}{4b(-c + c \sec(2a + 2bx))^{3/2}} - \frac{3 \sin(2a + 2bx)}{4bc\sqrt{-c + c \sec(2a + 2bx)}} + \frac{3 \int \sqrt{-c + c \sec(2a + 2bx)}}{4bc}$$

$$= -\frac{\sin(2a + 2bx)}{4b(-c + c \sec(2a + 2bx))^{3/2}} - \frac{3 \sin(2a + 2bx)}{4bc\sqrt{-c + c \sec(2a + 2bx)}} - \frac{3 \operatorname{Subst}\left(\int \sqrt{-c + c \sec(2a + 2bx)} dx\right)}{4bc}$$

$$= -\frac{3 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{-c+c \sec(2a+2bx)}}\right)}{2bc^{3/2}} + \frac{9 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{-c+c \sec(2a+2bx)}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{3 \operatorname{Subst}\left(\int \sqrt{-c + c \sec(2a + 2bx)} dx\right)}{4bc}$$

Mathematica [A] time = 6.20, size = 342, normalized size = 1.92

$$\frac{\tan^2(a + bx) \tan^2(2(a + bx)) \left(\frac{1}{2} \sin(2(a + bx)) - \frac{1}{4} \cot(a + bx) - \frac{1}{8} \cot(a + bx) \csc^2(a + bx) \right) + 3 \tan^{\frac{3}{2}}(a + bx) \tan(a + bx)}{b(c \tan(a + bx) \tan(2(a + bx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]
[Out] (-3*Tan[a + b*x]^(3/2)*((ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Csc[a + b*x]^2*Sec[a + b*x]^2*Tan[a + b*x]^(3/2)*Sqrt[-1 + Tan[a + b*x]^2]*Sqrt[Tan[2*(a + b*x)]])/(1 + Tan[a + b*x]^2)^2 + (Sqrt[2]*(2*ArcTan[Sqrt[2 - 2*Tan[a + b*x]^2]/2] - Sqrt[2]*ArcTan[Sqrt[1 - Tan[a + b*x]^2]])*Cos[2*(a + b*x)]*Csc[a + b*x]^2*Sec[a + b*x]^2*Tan[a + b*x]^(3/2)*Sqrt[Tan[2*(a + b*x)]])/(Sqrt[1 - Tan[a + b*x]^2]*(1 + Tan[a + b*x]^2)))*Tan[2*(a + b*x)]^(3/2))/(8*b*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))
```

$\text{an}[a + b*x]*\text{Tan}[2*(a + b*x)]^{(3/2)} + ((-1/4*\text{Cot}[a + b*x] - (\text{Cot}[a + b*x]*\text{Csc}[a + b*x]^2)/8 + \text{Sin}[2*(a + b*x)]/2)*\text{Tan}[a + b*x]^2*\text{Tan}[2*(a + b*x)]^2)/(b*(c*\text{Tan}[a + b*x]*\text{Tan}[2*(a + b*x)]^{(3/2)})$

fricas [A] time = 1.05, size = 528, normalized size = 2.97

$$\frac{9\sqrt{2}(\tan(bx+a)^5 + \tan(bx+a)^3)\sqrt{c} \log\left(\frac{c \tan(bx+a)^3 + 2\sqrt{\frac{c \tan(bx+a)^2}{\tan(bx+a)^2 - 1}}(\tan(bx+a)^2 - 1)\sqrt{c} - 2c \tan(bx+a)}{\tan(bx+a)^3}\right) + 12(\tan(bx+a)^5 + \tan(bx+a)^3)\sqrt{c}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="fricas")

[Out] [1/16*(9*sqrt(2)*(tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 12*(tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x + a)^3 + tan(b*x + a))) + 2*sqrt(2)*(5*tan(b*x + a)^4 - 4*tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*tan(b*x + a)^5 + b*c^2*tan(b*x + a)^3), 1/8*(9*sqrt(2)*(tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - 12*(tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) + sqrt(2)*(5*tan(b*x + a)^4 - 4*tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*tan(b*x + a)^5 + b*c^2*tan(b*x + a)^3)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 1.07, size = 1157, normalized size = 6.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)

[Out] $\frac{1}{32}2^{(1/2)}/b*(-1+\cos(b*x+a))^{(2)}*(8*\cos(b*x+a)*2^{(1/2)}*\text{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*2^{(1/2)}+2*\cos(b*x+a)*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}-5*\cos(b*x+a)*\ln(-2*(\cos(b*x+a)^2*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}-2*\cos(b*x+a)^2+\cos(b*x+a)-((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}+1)/\sin(b*x+a)^2)-5*\cos(b*x+a)*\text{arctanh}(1/2*4^{(1/2)}*(2*\cos(b*x+a)^2-3*\cos(b*x+a)+1)/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}/\sin(b*x+a)^2)-8*2^{(1/2)}*\text{arctanh}(1/2*\cos(b*x+a)*4^{(1/2)}*(-1+\cos(b*x+a))/\sin(b*x+a)^2/((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)})$

$$\frac{1}{(\cos(bx+a)+1)^2} \int \frac{\cos(2bx+2a)}{(c \tan(2bx+2a) \tan(bx+a))^{\frac{3}{2}}} dx$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(2bx+2a)}{(c \tan(2bx+2a) \tan(bx+a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")

[Out] integrate(cos(2*b*x + 2*a)/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(2a+2bx)}{(c \tan(a+bx) \tan(2a+2bx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*a + 2*b*x)/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)

[Out] int(cos(2*a + 2*b*x)/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)

[Out] Timed out

$$3.632 \quad \int \frac{\cos^2(2(a+bx))}{(c \tan(a+bx) \tan(2(a+bx)))^{3/2}} dx$$

Optimal. Leaf size=234

$$-\frac{19 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{8bc^{3/2}} + \frac{13 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{7 \sin(2a+2bx)}{8bc\sqrt{c \sec(2a+2bx)-c}} - \frac{\sin(2a+2bx) \cos(2a+2bx)}{2bc\sqrt{c \sec(2a+2bx)-c}}$$

[Out] $-19/8*\operatorname{arctanh}(c^{1/2}*\tan(2*b*x+2*a)/(-c+c*\sec(2*b*x+2*a))^{1/2})/b/c^{3/2}$
 $-1/4*\cos(2*b*x+2*a)*\sin(2*b*x+2*a)/b/(-c+c*\sec(2*b*x+2*a))^{3/2}+13/8*\operatorname{arctanh}(1/2*c^{1/2}*\tan(2*b*x+2*a)*2^{1/2}/(-c+c*\sec(2*b*x+2*a))^{1/2})/b/c^{3/2}$
 $*2^{1/2}-7/8*\sin(2*b*x+2*a)/b/c/(-c+c*\sec(2*b*x+2*a))^{1/2}-1/2*\cos(2*b*x+2*a)*\sin(2*b*x+2*a)/b/c/(-c+c*\sec(2*b*x+2*a))^{1/2}$

Rubi [A] time = 0.50, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4397, 3817, 4022, 3920, 3774, 207, 3795}

$$-\frac{19 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{c \sec(2a+2bx)-c}}\right)}{8bc^{3/2}} + \frac{13 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a+2bx)}{\sqrt{2} \sqrt{c \sec(2a+2bx)-c}}\right)}{4\sqrt{2} bc^{3/2}} - \frac{7 \sin(2a+2bx)}{8bc\sqrt{c \sec(2a+2bx)-c}} - \frac{\sin(2a+2bx) \cos(2a+2bx)}{2bc\sqrt{c \sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

[In] `Int[Cos[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]`

[Out] $(-19*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\tan[2*a + 2*b*x])/(\operatorname{Sqrt}[-c + c*\sec[2*a + 2*b*x]])]/(8*b*c^{3/2}) + (13*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\tan[2*a + 2*b*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[-c + c*\sec[2*a + 2*b*x]])]/(4*\operatorname{Sqrt}[2]*b*c^{3/2}) - (\cos[2*a + 2*b*x]*\sin[2*a + 2*b*x])/((4*b*(-c + c*\sec[2*a + 2*b*x])^{3/2}) - (7*\sin[2*a + 2*b*x])/(8*b*c*\operatorname{Sqrt}[-c + c*\sec[2*a + 2*b*x]]) - (\cos[2*a + 2*b*x]*\sin[2*a + 2*b*x])/(2*b*c*\operatorname{Sqrt}[-c + c*\sec[2*a + 2*b*x]])$

Rule 207

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

Rule 3774

`Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

Rule 3795

`Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]`

Rule 3817

`Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m, x_Symbol] := -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])`

Rule 3920

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 4022

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d*m), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rule 4397

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos^2(2(a + bx))}{(c \tan(a + bx) \tan(2(a + bx)))^{3/2}} dx &= \int \frac{\cos^2(2a + 2bx)}{(-c + c \sec(2a + 2bx))^{3/2}} dx \\
 &= -\frac{\cos(2a + 2bx) \sin(2a + 2bx)}{4b(-c + c \sec(2a + 2bx))^{3/2}} - \frac{\int \frac{\cos^2(2a + 2bx) \left(4c + \frac{5}{2}c \sec(2a + 2bx)\right)}{\sqrt{-c + c \sec(2a + 2bx)}} dx}{2c^2} \\
 &= -\frac{\cos(2a + 2bx) \sin(2a + 2bx)}{4b(-c + c \sec(2a + 2bx))^{3/2}} - \frac{\cos(2a + 2bx) \sin(2a + 2bx)}{2bc\sqrt{-c + c \sec(2a + 2bx)}} - \frac{\int \frac{\cos^2(2a + 2bx)}{\sqrt{-c + c \sec(2a + 2bx)}} dx}{2bc} \\
 &= -\frac{\cos(2a + 2bx) \sin(2a + 2bx)}{4b(-c + c \sec(2a + 2bx))^{3/2}} - \frac{7 \sin(2a + 2bx)}{8bc\sqrt{-c + c \sec(2a + 2bx)}} - \frac{\cos(2a + 2bx)}{2bc\sqrt{-c + c \sec(2a + 2bx)}} \\
 &= -\frac{\cos(2a + 2bx) \sin(2a + 2bx)}{4b(-c + c \sec(2a + 2bx))^{3/2}} - \frac{7 \sin(2a + 2bx)}{8bc\sqrt{-c + c \sec(2a + 2bx)}} - \frac{\cos(2a + 2bx)}{2bc\sqrt{-c + c \sec(2a + 2bx)}} \\
 &= -\frac{\cos(2a + 2bx) \sin(2a + 2bx)}{4b(-c + c \sec(2a + 2bx))^{3/2}} - \frac{7 \sin(2a + 2bx)}{8bc\sqrt{-c + c \sec(2a + 2bx)}} - \frac{\cos(2a + 2bx)}{2bc\sqrt{-c + c \sec(2a + 2bx)}} \\
 &= -\frac{19 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a + 2bx)}{\sqrt{-c + c \sec(2a + 2bx)}}\right)}{8bc^{3/2}} + \frac{13 \tanh^{-1}\left(\frac{\sqrt{c} \tan(2a + 2bx)}{\sqrt{2}\sqrt{-c + c \sec(2a + 2bx)}}\right)}{4\sqrt{2}bc^{3/2}} - \frac{\cos(2a + 2bx)}{2bc\sqrt{-c + c \sec(2a + 2bx)}}
 \end{aligned}$$

Mathematica [A] time = 6.21, size = 356, normalized size = 1.52

$$\frac{\tan^2(a + bx) \tan^2(2(a + bx)) \left(\frac{7}{8} \sin(2(a + bx)) + \frac{1}{8} \sin(4(a + bx)) - \frac{5}{8} \cot(a + bx) - \frac{1}{8} \cot(a + bx) \csc^2(a + bx) \right)}{b(c \tan(a + bx) \tan(2(a + bx)))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2), x]

[Out] (Tan[a + b*x]^(3/2)*((-7*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Csc[a + b*x]^2*Sec[a + b*x]^2*Tan[a + b*x]^(3/2)*Sqrt[-1 + Tan[a + b*x]^2]*Sqrt[Tan[2*(a +

$$\frac{b*x]]))/(1 + \tan[a + b*x]^2)^2 - (19*(2*\text{ArcTanh}[\sqrt{2 - 2*\tan[a + b*x]^2}]/2 - \sqrt{2}*\text{ArcTanh}[\sqrt{1 - \tan[a + b*x]^2}]))*\cos[2*(a + b*x)]*\csc[a + b*x]^2*\sec[a + b*x]^2*\tan[a + b*x]^{(3/2)}*\sqrt{\tan[2*(a + b*x)]}/(\sqrt{2}*\sqrt{1 - \tan[a + b*x]^2}*(1 + \tan[a + b*x]^2))*\tan[2*(a + b*x)]^{(3/2)})/(16*b*(c*\tan[a + b*x]*\tan[2*(a + b*x)])^{(3/2)}) + (((-5*\cot[a + b*x])/8 - (\cot[a + b*x]*\csc[a + b*x]^2)/8 + (7*\sin[2*(a + b*x)])/8 + \sin[4*(a + b*x)]/8)*\tan[a + b*x]^2*\tan[2*(a + b*x)]^2)/(b*(c*\tan[a + b*x]*\tan[2*(a + b*x)])^{(3/2)})}$$

fricas [A] time = 0.98, size = 616, normalized size = 2.63

$$\frac{13\sqrt{2}\left(\tan(bx+a)^7 + 2\tan(bx+a)^5 + \tan(bx+a)^3\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^3 + 2\sqrt{\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}(\tan(bx+a)^2-1)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^3}\right)}{\tan(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="fricas")

[Out] $\frac{1}{16}*(13*\sqrt{2}*(\tan(b*x + a)^7 + 2*\tan(b*x + a)^5 + \tan(b*x + a)^3)*\sqrt{c}*\log((c*\tan(b*x + a)^3 + 2*\sqrt{-c*\tan(b*x + a)^2/(\tan(b*x + a)^2 - 1))*(\tan(b*x + a)^2 - 1)*\sqrt{c} - 2*c*\tan(b*x + a))/\tan(b*x + a)^3 + 19*(\tan(b*x + a)^7 + 2*\tan(b*x + a)^5 + \tan(b*x + a)^3)*\sqrt{c}*\log((c*\tan(b*x + a)^3 - 2*\sqrt{2}*\sqrt{-c*\tan(b*x + a)^2/(\tan(b*x + a)^2 - 1))*(\tan(b*x + a)^2 - 1)*\sqrt{c} - 3*c*\tan(b*x + a))/(\tan(b*x + a)^3 + \tan(b*x + a))) + 2*\sqrt{2}*(4*\tan(b*x + a)^6 + 5*\tan(b*x + a)^4 - 8*\tan(b*x + a)^2 - 1)*\sqrt{-c*\tan(b*x + a)^2/(\tan(b*x + a)^2 - 1)))/(b*c^2*\tan(b*x + a)^7 + 2*b*c^2*\tan(b*x + a)^5 + b*c^2*\tan(b*x + a)^3), 1/8*(13*\sqrt{2}*(\tan(b*x + a)^7 + 2*\tan(b*x + a)^5 + \tan(b*x + a)^3)*\sqrt{-c}*\arctan(\sqrt{-c*\tan(b*x + a)^2/(\tan(b*x + a)^2 - 1))*(\tan(b*x + a)^2 - 1)*\sqrt{-c}/(c*\tan(b*x + a))) - 19*(\tan(b*x + a)^7 + 2*\tan(b*x + a)^5 + \tan(b*x + a)^3)*\sqrt{-c}*\arctan(1/2*\sqrt{2}*\sqrt{-c*\tan(b*x + a)^2/(\tan(b*x + a)^2 - 1))*(\tan(b*x + a)^2 - 1)*\sqrt{-c}/(c*\tan(b*x + a))) + \sqrt{2}*(4*\tan(b*x + a)^6 + 5*\tan(b*x + a)^4 - 8*\tan(b*x + a)^2 - 1)*\sqrt{-c*\tan(b*x + a)^2/(\tan(b*x + a)^2 - 1)))/(b*c^2*\tan(b*x + a)^7 + 2*b*c^2*\tan(b*x + a)^5 + b*c^2*\tan(b*x + a)^3)}$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 1.02, size = 1787, normalized size = 7.64

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x)

[Out] $-1/32*2^{(1/2)}/b*(-1+\cos(b*x+a))^{(2)}*(-8*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\cos(b*x+a)^5-14*((2*\cos(b*x+a)^2-1)/(\cos(b*x+a)+1)^2)^{(1/2)}*\cos(b*x+a)$

$a^3 + 51 \cos(bx+a) \cdot 2^{1/2} \cdot \operatorname{arctanh}(1/2 \cos(bx+a) \cdot 4^{1/2} \cdot (-1 + \cos(bx+a)) / \sin(bx+a)^2 / ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} \cdot 2^{1/2}) - 51 \cdot 2^{1/2} \cdot \operatorname{arctanh}(1/2 \cos(bx+a) \cdot 4^{1/2} \cdot (-1 + \cos(bx+a)) / \sin(bx+a)^2 / ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} \cdot 2^{1/2}) - 36 \cos(bx+a) \cdot \operatorname{arctanh}(1/2 \cdot 4^{1/2} \cdot (2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1) / ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} / \sin(bx+a)^2) - 36 \cos(bx+a) \cdot \ln(-2 \cdot (\cos(bx+a)^2 \cdot ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} - 2 \cos(bx+a)^2 + \cos(bx+a) - ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} + 1) / \sin(bx+a)^2) + 30 \cos(bx+a) \cdot ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} + 36 \cdot \operatorname{arctanh}(1/2 \cdot 4^{1/2} \cdot (2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1) / ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} / \sin(bx+a)^2) + 36 \cdot \ln(-2 \cdot (\cos(bx+a)^2 \cdot ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} - 2 \cos(bx+a)^2 + \cos(bx+a) - ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} + 1) / \sin(bx+a)^2) / ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{3/2} / (c \cdot \sin(bx+a)^2 / (2 \cos(bx+a)^2 - 1))^{3/2} / \sin(bx+a)^3 \cdot 4^{1/2} - 1/32 \cdot 2^{1/2} / b \cdot (-1 + \cos(bx+a))^2 \cdot (8 \cos(bx+a) \cdot 2^{1/2} \cdot \operatorname{arctanh}(1/2 \cos(bx+a) \cdot 4^{1/2} \cdot (-1 + \cos(bx+a)) / \sin(bx+a)^2 / ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} \cdot 2^{1/2}) + 2 \cos(bx+a) \cdot ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} - 5 \cos(bx+a) \cdot \ln(-2 \cdot (\cos(bx+a)^2 \cdot ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} - 2 \cos(bx+a)^2 + \cos(bx+a) - ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} + 1) / \sin(bx+a)^2) - 5 \cos(bx+a) \cdot \operatorname{arctanh}(1/2 \cdot 4^{1/2} \cdot (2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1) / ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} / \sin(bx+a)^2) - 8 \cdot 2^{1/2} \cdot \operatorname{arctanh}(1/2 \cos(bx+a) \cdot 4^{1/2} \cdot (-1 + \cos(bx+a)) / \sin(bx+a)^2 / ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} \cdot 2^{1/2}) + 5 \cdot \ln(-2 \cdot (\cos(bx+a)^2 \cdot ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} - 2 \cos(bx+a)^2 + \cos(bx+a) - ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} + 1) / \sin(bx+a)^2) + 5 \cdot \operatorname{arctanh}(1/2 \cdot 4^{1/2} \cdot (2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1) / ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} / \sin(bx+a)^2) / (c \cdot \sin(bx+a)^2 / (2 \cos(bx+a)^2 - 1))^{3/2} / \sin(bx+a)^3 / ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{3/2} \cdot 4^{1/2} - 1/8 \cdot 2^{1/2} / b \cdot (-1 + \cos(bx+a))^2 \cdot (4 \cdot ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} \cdot \cos(bx+a)^3 - 10 \cos(bx+a) \cdot 2^{1/2} \cdot \operatorname{arctanh}(1/2 \cos(bx+a) \cdot 4^{1/2} \cdot (-1 + \cos(bx+a)) / \sin(bx+a)^2 / ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} \cdot 2^{1/2}) + 7 \cos(bx+a) \cdot \operatorname{arctanh}(1/2 \cdot 4^{1/2} \cdot (2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1) / ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} / \sin(bx+a)^2) + 7 \cos(bx+a) \cdot \ln(-2 \cdot (\cos(bx+a)^2 \cdot ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} - 2 \cos(bx+a)^2 + \cos(bx+a) - ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} + 1) / \sin(bx+a)^2) - 6 \cos(bx+a) \cdot ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} + 10 \cdot 2^{1/2} \cdot \operatorname{arctanh}(1/2 \cos(bx+a) \cdot 4^{1/2} \cdot (-1 + \cos(bx+a)) / \sin(bx+a)^2 / ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} \cdot 2^{1/2}) - 7 \cdot \operatorname{arctanh}(1/2 \cdot 4^{1/2} \cdot (2 \cos(bx+a)^2 - 3 \cos(bx+a) + 1) / ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} / \sin(bx+a)^2) - 7 \cdot \ln(-2 \cdot (\cos(bx+a)^2 \cdot ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} - 2 \cos(bx+a)^2 + \cos(bx+a) - ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{1/2} + 1) / \sin(bx+a)^2) / ((2 \cos(bx+a)^2 - 1) / (\cos(bx+a) + 1)^2)^{3/2} / (c \cdot \sin(bx+a)^2 / (2 \cos(bx+a)^2 - 1))^{3/2} / \sin(bx+a)^3 \cdot 4^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(2bx + 2a)^2}{(c \tan(2bx + 2a) \tan(bx + a))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")

[Out] integrate(cos(2*b*x + 2*a)^2/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(2a + 2bx)^2}{(c \tan(a + bx) \tan(2a + 2bx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)^2/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

```
[Out] int(cos(2*a + 2*b*x)^2/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2), x)
```

```
[Out] Timed out
```

$$3.633 \quad \int \frac{\cot(x) \csc(x)}{\sqrt{\sin(2x)}} dx$$

Optimal. Leaf size=16

$$-\frac{2 \cos(x) \cot(x)}{3\sqrt{\sin(2x)}}$$

[Out] $-2/3*\cos(x)*\cot(x)/\sin(2*x)^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4390, 30}

$$-\frac{2 \cos(x) \cot(x)}{3\sqrt{\sin(2x)}}$$

Antiderivative was successfully verified.

[In] Int[(Cot[x]*Csc[x])/Sqrt[Sin[2*x]],x]

[Out] $(-2*\cos[x]*\cot[x])/(3*\sqrt{\sin[2*x]})$

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 4390

Int[(u_)*((c_)*sin[v_]^(m_)), x_Symbol] :> With[{w = FunctionOfTrig[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x]}, Dist[((c*Sin[v])^m*(c*Tan[v/2])^m)/Sin[v/2]^(2*m), Int[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x], x] /; !FalseQ[w] && FunctionOfQ[NonfreeFactors[Tan[w], x], (u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] && !SumQ[u] && InverseFunctionFreeQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{\cot(x) \csc(x)}{\sqrt{\sin(2x)}} dx &= \frac{\sin(x) \int \frac{\csc^2(x)}{\sqrt{\tan(x)}} dx}{\sqrt{\sin(2x)} \sqrt{\tan(x)}} \\ &= \frac{\sin(x) \operatorname{Subst}\left(\int \frac{1}{x^{5/2}} dx, x, \tan(x)\right)}{\sqrt{\sin(2x)} \sqrt{\tan(x)}} \\ &= -\frac{2 \cos(x) \cot(x)}{3\sqrt{\sin(2x)}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$-\frac{1}{3}\sqrt{\sin(2x)} \cot(x) \csc(x)$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x]*Csc[x])/Sqrt[Sin[2*x]],x]

[Out] $-1/3*(\cot[x]*\csc[x]*\sqrt{\sin[2*x]})$

fricas [B] time = 1.87, size = 29, normalized size = 1.81

$$\frac{\sqrt{2} \sqrt{\cos(x) \sin(x)} \cos(x) + \cos(x)^2 - 1}{3 (\cos(x)^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*csc(x)/sin(2*x)^(1/2),x, algorithm="fricas")

[Out] 1/3*(sqrt(2)*sqrt(cos(x)*sin(x))*cos(x) + cos(x)^2 - 1)/(cos(x)^2 - 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x) \csc(x)}{\sqrt{\sin(2x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*csc(x)/sin(2*x)^(1/2),x, algorithm="giac")

[Out] integrate(cot(x)*csc(x)/sqrt(sin(2*x)), x)

maple [C] time = 0.21, size = 119, normalized size = 7.44

$$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}} \left(\tan^2\left(\frac{x}{2}\right) - 1\right) \left(4\sqrt{1 + \tan\left(\frac{x}{2}\right)} \sqrt{-2 \tan\left(\frac{x}{2}\right) + 2} \sqrt{-\tan\left(\frac{x}{2}\right)} \operatorname{EllipticF}\left(\sqrt{1 + \tan\left(\frac{x}{2}\right)}, \frac{\sqrt{2}}{2}\right) \tan\left(\frac{x}{2}\right)\right)}{6 \tan\left(\frac{x}{2}\right) \sqrt{\left(\tan^2\left(\frac{x}{2}\right) - 1\right) \tan\left(\frac{x}{2}\right)} \sqrt{\tan^3\left(\frac{x}{2}\right) - \tan\left(\frac{x}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)*csc(x)/sin(2*x)^(1/2),x)

[Out] 1/6*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/tan(1/2*x)*(4*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)+tan(1/2*x)^4-1)/((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x) \csc(x)}{\sqrt{\sin(2x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*csc(x)/sin(2*x)^(1/2),x, algorithm="maxima")

[Out] integrate(cot(x)*csc(x)/sqrt(sin(2*x)), x)

mupad [B] time = 3.10, size = 14, normalized size = 0.88

$$\frac{\sqrt{\sin(2x)} \cos(x)}{3 \sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)/(sin(2*x)^(1/2)*sin(x)),x)

[Out] -(sin(2*x)^(1/2)*cos(x))/(3*sin(x)^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x) \csc(x)}{\sqrt{\sin(2x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/sin(2*x)**(1/2),x)
```

```
[Out] Integral(cot(x)*csc(x)/sqrt(sin(2*x)), x)
```

$$3.634 \quad \int \frac{\csc^2(x) \sec(x)}{\sqrt{\sin(2x)}(-2+\tan(x))} dx$$

Optimal. Leaf size=69

$$\frac{\cos(x)}{2\sqrt{\sin(2x)}} - \frac{5 \sin(x) \tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{2\sqrt{2} \sqrt{\sin(2x)} \sqrt{\tan(x)}} + \frac{\cos(x) \cot(x)}{3\sqrt{\sin(2x)}}$$

[Out] $1/2*\cos(x)/\sin(2*x)^{(1/2)}+1/3*\cos(x)*\cot(x)/\sin(2*x)^{(1/2)}-5/4*\operatorname{arctanh}(1/2*\tan(x)^{(1/2)}*2^{(1/2)})*\sin(x)*2^{(1/2)}/\sin(2*x)^{(1/2)}/\tan(x)^{(1/2)}$

Rubi [A] time = 0.36, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4390, 898, 1262, 207}

$$\frac{\cos(x)}{2\sqrt{\sin(2x)}} - \frac{5 \sin(x) \tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{2\sqrt{2} \sqrt{\sin(2x)} \sqrt{\tan(x)}} + \frac{\cos(x) \cot(x)}{3\sqrt{\sin(2x)}}$$

Antiderivative was successfully verified.

[In] Int[(Csc[x]^2*Sec[x])/(Sqrt[Sin[2*x]]*(-2 + Tan[x])),x]

[Out] Cos[x]/(2*Sqrt[Sin[2*x]]) + (Cos[x]*Cot[x])/(3*Sqrt[Sin[2*x]]) - (5*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]]*Sin[x])/(2*Sqrt[2]*Sqrt[Sin[2*x]]*Sqrt[Tan[x]])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 898

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1262

Int[((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 4390

Int[(u_)*((c_.)*sin[v])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x]}, Dist[((c*Sin[v])^m*(c*Tan[v/2])^m)/Sin[v/2]^(2*m), Int[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x], x] /; !FalseQ[w] && FunctionOfQ[NonfreeFactors[Tan[w], x], (u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] && !SumQ[u] && InverseFunctionFreeQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(x) \sec(x)}{\sqrt{\sin(2x)} (-2 + \tan(x))} dx &= \frac{\sin(x) \int \frac{\csc^3(x) \sec(x) \sqrt{\tan(x)}}{-2 + \tan(x)} dx}{\sqrt{\sin(2x)} \sqrt{\tan(x)}} \\
&= \frac{\sin(x) \operatorname{Subst} \left(\int \frac{1+x^2}{(-2+x)x^{5/2}} dx, x, \tan(x) \right)}{\sqrt{\sin(2x)} \sqrt{\tan(x)}} \\
&= \frac{(2 \sin(x)) \operatorname{Subst} \left(\int \frac{1+x^4}{x^4(-2+x^2)} dx, x, \sqrt{\tan(x)} \right)}{\sqrt{\sin(2x)} \sqrt{\tan(x)}} \\
&= \frac{(2 \sin(x)) \operatorname{Subst} \left(\int \left(-\frac{1}{2x^4} - \frac{1}{4x^2} + \frac{5}{4(-2+x^2)} \right) dx, x, \sqrt{\tan(x)} \right)}{\sqrt{\sin(2x)} \sqrt{\tan(x)}} \\
&= \frac{\cos(x)}{2\sqrt{\sin(2x)}} + \frac{\cos(x) \cot(x)}{3\sqrt{\sin(2x)}} + \frac{(5 \sin(x)) \operatorname{Subst} \left(\int \frac{1}{-2+x^2} dx, x, \sqrt{\tan(x)} \right)}{2\sqrt{\sin(2x)} \sqrt{\tan(x)}} \\
&= \frac{\cos(x)}{2\sqrt{\sin(2x)}} + \frac{\cos(x) \cot(x)}{3\sqrt{\sin(2x)}} - \frac{5 \tanh^{-1} \left(\frac{\sqrt{\tan(x)}}{\sqrt{2}} \right) \sin(x)}{2\sqrt{2} \sqrt{\sin(2x)} \sqrt{\tan(x)}}
\end{aligned}$$

Mathematica [C] time = 5.95, size = 119, normalized size = 1.72

$$\frac{1}{4} \sqrt{\sin(2x)} \left(\left(\frac{2 \cot(x)}{3} + 1 \right) \csc(x) + 5 \sqrt{\frac{\cos(x)}{2 \cos(x) - 2}} \sqrt{\tan\left(\frac{x}{2}\right)} \sec(x) \left(F \left(\sin^{-1} \left(\frac{1}{\sqrt{\tan\left(\frac{x}{2}\right)}} \right) \right) - 1 \right) - \Pi \left(-\frac{1}{2} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x]^2*Sec[x])/(Sqrt[Sin[2*x]]*(-2 + Tan[x])),x]

[Out] (Sqrt[Sin[2*x]]*((1 + (2*Cot[x])/3)*Csc[x] + 5*Sqrt[Cos[x]/(-2 + 2*Cos[x])])*(EllipticF[ArcSin[1/Sqrt[Tan[x/2]]], -1] - EllipticPi[-2/(-1 + Sqrt[5]), ArcSin[1/Sqrt[Tan[x/2]]], -1] - EllipticPi[(-1 + Sqrt[5])/2, ArcSin[1/Sqrt[Tan[x/2]]], -1])*Sec[x]*Sqrt[Tan[x/2]])/4

fricas [B] time = 2.70, size = 120, normalized size = 1.74

$$\frac{4\sqrt{2}\sqrt{\cos(x)\sin(x)}(2\cos(x) + 3\sin(x)) - 4\cos(x)^2 - 15(\cos(x)^2 - 1)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}(4\cos(x) + 3\sin(x))\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x, algorithm="fricas")

[Out] -1/48*(4*sqrt(2)*sqrt(cos(x)*sin(x))*(2*cos(x) + 3*sin(x)) - 4*cos(x)^2 - 15*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x) + 3*sin(x)) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2) + 15*(cos(x)^2 - 1)*log(1/2*cos(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(x)*sin(x) + 1/2) + 4)/(cos(x)^2 - 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)^2 \sec(x)}{(\tan(x) - 2)\sqrt{\sin(2x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x, algorithm="giac")

[Out] integrate(csc(x)^2*sec(x)/((tan(x) - 2)*sqrt(sin(2*x))), x)

maple [C] time = 0.31, size = 397, normalized size = 5.75

$$\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}} \left(140\sqrt{1+\tan\left(\frac{x}{2}\right)} \sqrt{-\tan\left(\frac{x}{2}\right)} \sqrt{-2\tan\left(\frac{x}{2}\right)+2} \operatorname{EllipticF}\left(\sqrt{1+\tan\left(\frac{x}{2}\right)}, \frac{\sqrt{2}}{2}\right) \sqrt{\left(\tan^2\left(\frac{x}{2}\right)-1\right)} \tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x)

[Out] 1/480*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^2*(140*(1+tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x)-240*(1+tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x)-(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*2^(1/2)*sum((14*_alpha^3+3*_alpha^2+14*_alpha-11)*(_alpha^3+2*_alpha-3)*(1+tan(1/2*x))^(1/2)*(1-tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)/((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),-1/4*_alpha^3-1/2*_alpha+3/4,1/2*2^(1/2)),_alpha=RootOf(_Z^4+_Z^3+2*_Z^2-_Z+1))*tan(1/2*x)-40*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^4-120*tan(1/2*x)^3*(tan(1/2*x)^3-tan(1/2*x))^(1/2)+120*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)+40*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)^2 \sec(x)}{(\tan(x) - 2)\sqrt{\sin(2x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x, algorithm="maxima")

[Out] integrate(csc(x)^2*sec(x)/((tan(x) - 2)*sqrt(sin(2*x))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sin(2x)} \cos(x) \sin(x)^2 (\tan(x) - 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(2*x)^(1/2)*cos(x)*sin(x)^2*(tan(x) - 2)),x)

[Out] int(1/(sin(2*x)^(1/2)*cos(x)*sin(x)^2*(tan(x) - 2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x) \sec(x)}{(\tan(x) - 2)\sqrt{\sin(2x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)**2*sec(x)/sin(2*x)**(1/2)/(-2+tan(x)),x)

[Out] Integral(csc(x)**2*sec(x)/((tan(x) - 2)*sqrt(sin(2*x))), x)

$$3.635 \quad \int \frac{\cos^2(x) \sin(x)}{(\sin^2(x) - \sin(2x)) \sin^{\frac{5}{2}}(2x)} dx$$

Optimal. Leaf size=79

$$\frac{\sin(x) \cos^4(x)}{3 \sin^{\frac{5}{2}}(2x)} + \frac{\sin^2(x) \cos^3(x)}{2 \sin^{\frac{5}{2}}(2x)} - \frac{5 \sin^5(x) \tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{2\sqrt{2} \sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)}$$

[Out] 1/3*cos(x)^4*sin(x)/sin(2*x)^(5/2)+1/2*cos(x)^3*sin(x)^2/sin(2*x)^(5/2)-5/4*arctanh(1/2*tan(x)^(1/2)*2^(1/2))*sin(x)^5/sin(2*x)^(5/2)*2^(1/2)/tan(x)^(5/2)

Rubi [A] time = 0.57, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4390, 898, 1262, 207}

$$\frac{\sin^2(x) \cos^3(x)}{2 \sin^{\frac{5}{2}}(2x)} + \frac{\sin(x) \cos^4(x)}{3 \sin^{\frac{5}{2}}(2x)} - \frac{5 \sin^5(x) \tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{2\sqrt{2} \sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]^2*Sin[x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]

[Out] (Cos[x]^4*Sin[x])/(3*Sin[2*x]^(5/2)) + (Cos[x]^3*Sin[x]^2)/(2*Sin[2*x]^(5/2)) - (5*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]]*Sin[x]^5)/(2*Sqrt[2]*Sin[2*x]^(5/2)*Tan[x]^(5/2))

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 898

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1262

Int[((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 4390

Int[(u_)*((c_.)*sin[v_]^(m_)), x_Symbol] := With[{w = FunctionOfTrig[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x]}, Dist[(((c*Sin[v])^m*(c*Tan[v/2])^m)/Sin[v/2]^(2*m), Int[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x], x] /; !FalseQ[w] && FunctionOfQ[NonfreeFactors[Tan[w], x], (u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] && !SumQ[u] && InverseFunctionFreeQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(x) \sin(x)}{(\sin^2(x) - \sin(2x)) \sin^{\frac{5}{2}}(2x)} dx &= \frac{\sin^5(x) \int \frac{\csc^2(x) \sqrt{\tan(x)}}{\sin^2(x) - \sin(2x)} dx}{\sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)} \\
&= \frac{\sin^5(x) \operatorname{Subst}\left(\int \frac{-1-x^2}{(2-x)x^{5/2}} dx, x, \tan(x)\right)}{\sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)} \\
&= \frac{(2 \sin^5(x)) \operatorname{Subst}\left(\int \frac{-1-x^4}{x^4(2-x^2)} dx, x, \sqrt{\tan(x)}\right)}{\sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)} \\
&= \frac{(2 \sin^5(x)) \operatorname{Subst}\left(\int \left(-\frac{1}{2x^4} - \frac{1}{4x^2} + \frac{5}{4(-2+x^2)}\right) dx, x, \sqrt{\tan(x)}\right)}{\sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)} \\
&= \frac{\cos^4(x) \sin(x)}{3 \sin^{\frac{5}{2}}(2x)} + \frac{\cos^3(x) \sin^2(x)}{2 \sin^{\frac{5}{2}}(2x)} + \frac{(5 \sin^5(x)) \operatorname{Subst}\left(\int \frac{1}{-2+x^2} dx, x, \sqrt{\tan(x)}\right)}{2 \sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)} \\
&= \frac{\cos^4(x) \sin(x)}{3 \sin^{\frac{5}{2}}(2x)} + \frac{\cos^3(x) \sin^2(x)}{2 \sin^{\frac{5}{2}}(2x)} - \frac{5 \tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right) \sin^5(x)}{2\sqrt{2} \sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)}
\end{aligned}$$

Mathematica [C] time = 4.96, size = 183, normalized size = 2.32

$$\frac{1}{96} \sqrt{\sin(2x)} \sec(x) \left(2 \cot^2(x) + 6 \cot(x) + 2 \csc^2(x) + 15\sqrt{2} \sqrt{\frac{\cos(x)}{\cos(x)-1}} \sqrt{\tan\left(\frac{x}{2}\right)} F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{2}\right)}\right)\right) - 1 \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]^2*Sin[x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]

[Out] (Sec[x]*Sqrt[Sin[2*x]]*(-2 + 6*Cot[x] + 2*Cot[x]^2 + 2*Csc[x]^2 + 15*Sqrt[2]*Sqrt[Cos[x]/(-1 + Cos[x])]*EllipticF[ArcSin[1/Sqrt[Tan[x/2]]], -1]*Sqrt[Tan[x/2]] - 15*Sqrt[2]*Sqrt[Cos[x]/(-1 + Cos[x])]*EllipticPi[-2/(-1 + Sqrt[5]), ArcSin[1/Sqrt[Tan[x/2]]], -1]*Sqrt[Tan[x/2]] - 15*Sqrt[2]*Sqrt[Cos[x]/(-1 + Cos[x])]*EllipticPi[(-1 + Sqrt[5])/2, ArcSin[1/Sqrt[Tan[x/2]]], -1]*Sqrt[Tan[x/2]]))/96

fricas [A] time = 1.04, size = 120, normalized size = 1.52

$$4\sqrt{2}\sqrt{\cos(x)\sin(x)}(2\cos(x) + 3\sin(x)) - 4\cos(x)^2 - 15(\cos(x)^2 - 1)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}(4\cos(x) + 3\sin(x)) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm="fricas")

[Out] -1/192*(4*sqrt(2)*sqrt(cos(x)*sin(x))*(2*cos(x) + 3*sin(x)) - 4*cos(x)^2 - 15*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x) + 3*sin(x)) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2) + 15*(cos(x)^2 - 1)*log(1/2*cos(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(x)*sin(x) + 1/2) + 4)/(cos(x)^2 - 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)^2 \sin(x)}{(\sin(x)^2 - \sin(2x)) \sin(2x)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm="giac")

[Out] integrate(cos(x)^2*sin(x)/((sin(x)^2 - sin(2*x))*sin(2*x)^(5/2)), x)

maple [C] time = 0.28, size = 397, normalized size = 5.03

$$\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}} \left(140 \sqrt{1 + \tan\left(\frac{x}{2}\right)} \sqrt{-\tan\left(\frac{x}{2}\right)} \sqrt{-2 \tan\left(\frac{x}{2}\right) + 2} \operatorname{EllipticF}\left(\sqrt{1 + \tan\left(\frac{x}{2}\right)}, \frac{\sqrt{2}}{2}\right) \sqrt{\left(\tan^2\left(\frac{x}{2}\right) - 1\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x)

[Out] 1/1920*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^2*(140*(1+tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x)-240*(1+tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x)-(tan(1/2*x)^3-tan(1/2*x))^(1/2)*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*2^(1/2)*sum((14*_alpha^3+3*_alpha^2+14*_alpha-11)*(_alpha^3+2*_alpha-3)*(1+tan(1/2*x))^(1/2)*(1-tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)/((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),-1/4*_alpha^3-1/2*_alpha+3/4,1/2*2^(1/2)),_alpha=RootOf(_Z^4+_Z^3+2*_Z^2-_Z+1))*tan(1/2*x)-40*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^4-120*tan(1/2*x)^3*(tan(1/2*x)^3-tan(1/2*x))^(1/2)+120*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)+40*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\cos(x)^2 \sin(x)}{\sin(2x)^{5/2} (\sin(2x) - \sin(x)^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(cos(x)^2*sin(x))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)),x)

[Out] -int((cos(x)^2*sin(x))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(sin(x)**2-sin(2*x))/sin(2*x)**(5/2),x)
```

```
[Out] Timed out
```

$$3.636 \quad \int \frac{\cos^3(x) \cos(2x)}{(\sin^2(x) - \sin(2x)) \sin^{\frac{5}{2}}(2x)} dx$$

Optimal. Leaf size=95

$$\frac{\cos^5(x)}{5 \sin^{\frac{5}{2}}(2x)} + \frac{\sin(x) \cos^4(x)}{6 \sin^{\frac{5}{2}}(2x)} - \frac{3 \sin^2(x) \cos^3(x)}{4 \sin^{\frac{5}{2}}(2x)} + \frac{3 \sin^5(x) \tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2} \sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)}$$

[Out] 1/5*cos(x)^5/sin(2*x)^(5/2)+1/6*cos(x)^4*sin(x)/sin(2*x)^(5/2)-3/4*cos(x)^3*sin(x)^2/sin(2*x)^(5/2)+3/8*arctanh(1/2*tan(x)^(1/2)*2^(1/2))*sin(x)^5/sin(2*x)^(5/2)*2^(1/2)/tan(x)^(5/2)

Rubi [A] time = 0.58, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4390, 898, 1262, 207}

$$\frac{\cos^5(x)}{5 \sin^{\frac{5}{2}}(2x)} + \frac{\sin(x) \cos^4(x)}{6 \sin^{\frac{5}{2}}(2x)} - \frac{3 \sin^2(x) \cos^3(x)}{4 \sin^{\frac{5}{2}}(2x)} + \frac{3 \sin^5(x) \tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2} \sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]^3*Cos[2*x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]

[Out] Cos[x]^5/(5*Sin[2*x]^(5/2)) + (Cos[x]^4*Sin[x])/(6*Sin[2*x]^(5/2)) - (3*Cos[x]^3*Sin[x]^2)/(4*Sin[2*x]^(5/2)) + (3*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]]*Sin[x]^5)/(4*Sqrt[2]*Sin[2*x]^(5/2)*Tan[x]^(5/2))

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 898

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 + a*e^2)/e^2 - (2*c*d*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1262

Int[((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 4390

Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] := With[{w = FunctionOfTrig[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x]}, Dist[(((c*Sin[v])^m*(c*Tan[v/2])^m)/Sin[v/2]^(2*m), Int[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x], x] /; !FalseQ[w] && FunctionOfQ[NonfreeFactors[Tan[w], x], (u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] && !SumQ[u] && InverseFunctionFreeQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(x) \cos(2x)}{(\sin^2(x) - \sin(2x)) \sin^{\frac{5}{2}}(2x)} dx &= \frac{\sin^5(x) \int \frac{\cos(2x) \csc^2(x)}{(\sin^2(x) - \sin(2x)) \sqrt{\tan(x)}} dx}{\sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)} \\
&= \frac{\sin^5(x) \operatorname{Subst}\left(\int \frac{-1+x^2}{(2-x)x^{7/2}} dx, x, \tan(x)\right)}{\sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)} \\
&= \frac{(2 \sin^5(x)) \operatorname{Subst}\left(\int \frac{-1+x^4}{x^6(2-x^2)} dx, x, \sqrt{\tan(x)}\right)}{\sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)} \\
&= \frac{(2 \sin^5(x)) \operatorname{Subst}\left(\int \left(-\frac{1}{2x^6} - \frac{1}{4x^4} + \frac{3}{8x^2} - \frac{3}{8(-2+x^2)}\right) dx, x, \sqrt{\tan(x)}\right)}{\sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)} \\
&= \frac{\cos^5(x)}{5 \sin^{\frac{5}{2}}(2x)} + \frac{\cos^4(x) \sin(x)}{6 \sin^{\frac{5}{2}}(2x)} - \frac{3 \cos^3(x) \sin^2(x)}{4 \sin^{\frac{5}{2}}(2x)} - \frac{(3 \sin^5(x)) \operatorname{Subst}\left(\int \frac{1}{-2+x^2} dx, x, \sqrt{\tan(x)}\right)}{4 \sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)} \\
&= \frac{\cos^5(x)}{5 \sin^{\frac{5}{2}}(2x)} + \frac{\cos^4(x) \sin(x)}{6 \sin^{\frac{5}{2}}(2x)} - \frac{3 \cos^3(x) \sin^2(x)}{4 \sin^{\frac{5}{2}}(2x)} + \frac{3 \tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right) \sin^5(x)}{4\sqrt{2} \sin^{\frac{5}{2}}(2x) \tan^{\frac{5}{2}}(x)}
\end{aligned}$$

Mathematica [C] time = 15.72, size = 184, normalized size = 1.94

$$\frac{1}{960} \sqrt{\sin(2x)} \sec(x) \left(20 \cot^2(x) - 114 \cot(x) + 24 \cot(x) \csc^2(x) - 45\sqrt{2} \sqrt{\frac{\cos(x)}{\cos(x)-1}} \sqrt{\tan\left(\frac{x}{2}\right)} F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{x}{2}\right)}}\right), \frac{x}{2}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]^3*Cos[2*x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]

[Out] (Sec[x]*Sqrt[Sin[2*x]]*(-114*Cot[x] + 20*Cot[x]^2 + 24*Cot[x]*Csc[x]^2 - 45*Sqrt[2]*Sqrt[Cos[x]/(-1 + Cos[x])]*EllipticF[ArcSin[1/Sqrt[Tan[x/2]]], -1]*Sqrt[Tan[x/2]] + 45*Sqrt[2]*Sqrt[Cos[x]/(-1 + Cos[x])]*EllipticPi[-2/(-1 + Sqrt[5]), ArcSin[1/Sqrt[Tan[x/2]]], -1]*Sqrt[Tan[x/2]] + 45*Sqrt[2]*Sqrt[Cos[x]/(-1 + Cos[x])]*EllipticPi[(-1 + Sqrt[5])/2, ArcSin[1/Sqrt[Tan[x/2]]], -1]*Sqrt[Tan[x/2]]))/960

fricas [A] time = 0.47, size = 136, normalized size = 1.43

$$\frac{45(\cos(x)^2 - 1) \log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}(4\cos(x) + 3\sin(x)) + \frac{1}{2}\cos(x)^2 + \frac{7}{2}\cos(x)\sin(x) + \frac{1}{2}\right)\sin(x) - 45\cos(x)\sin(x)}{4\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm="fricas")

[Out] -1/1920*(45*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x) + 3*sin(x)) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2)*sin(x) - 45*(cos(x)^2 - 1)*log(1/2*cos(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(x)*sin(x) + 1/2)*sin(x) + 4*sqrt(2)*(57*cos(x)^2 + 10*cos(x)*sin(x) - 45)*sqrt(cos(x)*sin(x)) + 268*(cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(2x) \cos(x)^3}{(\sin(x)^2 - \sin(2x)) \sin(2x)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm="giac")

[Out] integrate(cos(2*x)*cos(x)^3/((sin(x)^2 - sin(2*x))*sin(2*x)^(5/2)), x)

maple [C] time = 0.38, size = 761, normalized size = 8.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x)

[Out] 1/3840*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(1772*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*(1+tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)*tan(1/2*x)^2-4464*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*(1+tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)*tan(1/2*x)^2+24*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^6+3*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*sum((6*_alpha^3+7*_alpha^2+6*_alpha+1)*(_alpha^3+2*_alpha-3)*(1+tan(1/2*x))^(1/2)*(1-tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((1+tan(1/2*x))^(1/2),-1/4*_alpha^3-1/2*_alpha+3/4,1/2*2^(1/2))/((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2),_alpha=RootOf(_Z^4+_Z^3+2*_Z^2-_Z+1))*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*2^(1/2)*tan(1/2*x)^2-40*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^5-1272*tan(1/2*x)^4*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*tan(1/2*x)^3-tan(1/2*x))^(1/2)-24*tan(1/2*x)^4*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)-1920*tan(1/2*x)^4*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^3-tan(1/2*x))^(1/2)+1272*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^2-24*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*tan(1/2*x)^2+40*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)*tan(1/2*x)+24*((tan(1/2*x)^2-1)*tan(1/2*x))^(1/2)*((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2))/((tan(1/2*x)-1)*(1+tan(1/2*x))*tan(1/2*x))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\cos(2x) \cos(x)^3}{\sin(2x)^{5/2} (\sin(2x) - \sin(x)^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(2*x)*cos(x)^3)/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)),x)
```

```
[Out] int(-(cos(2*x)*cos(x)^3)/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*cos(2*x)/(sin(x)**2-sin(2*x))/sin(2*x)**(5/2),x)
```

```
[Out] Timed out
```


$$3.637 \quad \int (b \sec(c+dx) + a \sin(c+dx))^n (a \cos(c+dx) + b \sec(c+dx) \tan(c+dx)) dx$$

Optimal. Leaf size=30

$$\frac{(a \sin(c+dx) + b \sec(c+dx))^{n+1}}{d(n+1)}$$

[Out] (b*sec(d*x+c)+a*sin(d*x+c))^(1+n)/d/(1+n)

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {4385}

$$\frac{(a \sin(c+dx) + b \sec(c+dx))^{n+1}}{d(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(b*Sec[c + d*x] + a*Sin[c + d*x])^n*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]),x]

[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^(1 + n)/(d*(1 + n))

Rule 4385

Int[(u_)*(y_)^(m_.), x_Symbol] :=> With[{q = DerivativeDivides[ActivateTrig[y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /; !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] && !InertTrigFreeQ[u]

Rubi steps

$$\int (b \sec(c+dx) + a \sin(c+dx))^n (a \cos(c+dx) + b \sec(c+dx) \tan(c+dx)) dx = \frac{(b \sec(c+dx) + a \sin(c+dx))^{n+1}}{d(1+n)}$$

Mathematica [A] time = 1.22, size = 51, normalized size = 1.70

$$\frac{\sec(c+dx)(a \sin(2(c+dx)) + 2b)(a \sin(c+dx) + b \sec(c+dx))^n}{2d(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])^n*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]),x]

[Out] (Sec[c + d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^n*(2*b + a*Sin[2*(c + d*x)]))/(2*d*(1 + n))

fricas [A] time = 3.05, size = 59, normalized size = 1.97

$$\frac{(a \cos(dx+c) \sin(dx+c) + b) \left(\frac{a \cos(dx+c) \sin(dx+c) + b}{\cos(dx+c)} \right)^n}{(dn+d) \cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x, algorithm="fricas")

[Out] $(a \cos(dx + c) \sin(dx + c) + b) \cdot ((a \cos(dx + c) \sin(dx + c) + b) / \cos(dx + c))^n / ((d \cdot n + d) \cos(dx + c))$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx + c) \tan(dx + c) + a \cos(dx + c)) (b \sec(dx + c) + a \sin(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x, algorithm="giac")`

[Out] `integrate((b*sec(d*x + c)*tan(d*x + c) + a*cos(d*x + c))*(b*sec(d*x + c) + a*sin(d*x + c))^n, x)`

maple [A] time = 0.46, size = 31, normalized size = 1.03

$$\frac{(b \sec(dx + c) + a \sin(dx + c))^{n+1}}{d(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x)`

[Out] `(b*sec(d*x+c)+a*sin(d*x+c))^(n+1)/d/(n+1)`

maxima [A] time = 0.31, size = 30, normalized size = 1.00

$$\frac{(b \sec(dx + c) + a \sin(dx + c))^{n+1}}{d(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x, algorithm="maxima")`

[Out] `(b*sec(d*x + c) + a*sin(d*x + c))^(n + 1)/(d*(n + 1))`

mupad [B] time = 5.54, size = 63, normalized size = 2.10

$$\begin{cases} \frac{\ln\left(a \sin(c+dx) + \frac{b}{\cos(c+dx)}\right)}{d} & \text{if } n = -1 \\ \frac{\left(a \sin(c+dx) + \frac{b}{\cos(c+dx)}\right)^{n+1}}{d(n+1)} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*sin(c + d*x) + b/cos(c + d*x))^n*(a*cos(c + d*x) + (b*tan(c + d*x))/cos(c + d*x)),x)`

[Out] `piecewise(n == -1, log(a*sin(c + d*x) + b/cos(c + d*x))/d, n ~= -1, (a*sin(c + d*x) + b/cos(c + d*x))^(n + 1)/(d*(n + 1)))`

sympy [A] time = 74.07, size = 138, normalized size = 4.60

$$\begin{cases} \frac{x(a \cos(c) + b \tan(c) \sec(c))}{a \sin(c) + b \sec(c)} & \text{for } d = 0 \wedge n = -1 \\ x(a \sin(c) + b \sec(c))^n (a \cos(c) + b \tan(c) \sec(c)) & \text{for } d = 0 \\ \frac{\log\left(\sin(c+dx) + \frac{b \sec(c+dx)}{a}\right)}{d} & \text{for } n = -1 \\ \frac{a(a \sin(c+dx) + b \sec(c+dx))^n \sin(c+dx)}{dn+d} + \frac{b(a \sin(c+dx) + b \sec(c+dx))^n \sec(c+dx)}{dn+d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))**n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x)
```

```
[Out] Piecewise((x*(a*cos(c) + b*tan(c)*sec(c))/(a*sin(c) + b*sec(c)), Eq(d, 0) & Eq(n, -1)), (x*(a*sin(c) + b*sec(c))**n*(a*cos(c) + b*tan(c)*sec(c)), Eq(d, 0)), (log(sin(c + d*x) + b*sec(c + d*x)/a)/d, Eq(n, -1)), (a*(a*sin(c + d*x) + b*sec(c + d*x))**n*sin(c + d*x)/(d*n + d) + b*(a*sin(c + d*x) + b*sec(c + d*x))**n*sec(c + d*x)/(d*n + d), True))
```

$$3.638 \quad \int (b \sec(c+dx) + a \sin(c+dx))^3 (a \cos(c+dx) + b \sec(c+dx) \tan(c+dx)) dx$$

Optimal. Leaf size=26

$$\frac{(a \sin(c+dx) + b \sec(c+dx))^4}{4d}$$

[Out] 1/4*(b*sec(d*x+c)+a*sin(d*x+c))^4/d

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {4385}

$$\frac{(a \sin(c+dx) + b \sec(c+dx))^4}{4d}$$

Antiderivative was successfully verified.

[In] Int[(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]),x]

[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^4/(4*d)

Rule 4385

Int[(u_)*(y_)^(m_.), x_Symbol] := With[{q = DerivativeDivides[ActivateTrig[y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m+1)])/(m+1), x] /; !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] && !InertTrigFreeQ[u]

Rubi steps

$$\int (b \sec(c+dx) + a \sin(c+dx))^3 (a \cos(c+dx) + b \sec(c+dx) \tan(c+dx)) dx = \frac{(b \sec(c+dx) + a \sin(c+dx))^4}{4d}$$

Mathematica [B] time = 6.54, size = 938, normalized size = 36.08

$$\frac{a^4 \cos(4c) \cos(4dx) (b \sec(c+dx) + a \sin(c+dx))^3 (a \cos(c+dx) + b \sec(c+dx) \tan(c+dx)) \cos^5(c+dx) - 4a^3 c}{d(3a \cos(c+dx) + a \cos(3c+3dx) + 4b \sin(c+dx))(2b + a \sin(2c+2dx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]),x]

[Out] (8*b^4*Cos[c + d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (a^4*Cos[4*c]*Cos[4*d*x]*Cos[c + d*x]^5*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (16*a*b^2*Cos[c + d*x]^3*Sec[c]*(3*a*Cos[c] + 2*b*Sin[c])*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) - (4*a^3*Cos[2*d*x]*Cos[c + d*x]^5*(a*Cos[2*c] + 4*b*Sin[2*c])*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (32*a*b^3*Cos[c + d*x]^2*Sec[c]*Sin[d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3)

$$\begin{aligned} & (3d^3x + 4b\sin[c + dx]) \cdot (2b + a\sin[2c + 2dx])^3 + (32a^3b\cos[c + dx]^4 \sec[c] \sin[dx] \cdot (b\sec[c + dx] + a\sin[c + dx])^3 \cdot (a\cos[c + dx] + b\sec[c + dx] \tan[c + dx])) / (d \cdot (3a\cos[c + dx] + a\cos[3c + 3dx] + 4b\sin[c + dx]) \cdot (2b + a\sin[2c + 2dx])^3) \\ & + (4a^3\cos[c + dx]^5 \cdot (-4b\cos[2c] + a\sin[2c]) \cdot \sin[2dx] \cdot (b\sec[c + dx] + a\sin[c + dx])^3 \cdot (a\cos[c + dx] + b\sec[c + dx] \tan[c + dx])) / (d \cdot (3a\cos[c + dx] + a\cos[3c + 3dx] + 4b\sin[c + dx]) \cdot (2b + a\sin[2c + 2dx])^3) \\ & - (a^4\cos[c + dx]^5 \sin[4c] \sin[4dx] \cdot (b\sec[c + dx] + a\sin[c + dx])^3 \cdot (a\cos[c + dx] + b\sec[c + dx] \tan[c + dx])) / (d \cdot (3a\cos[c + dx] + a\cos[3c + 3dx] + 4b\sin[c + dx]) \cdot (2b + a\sin[2c + 2dx])^3) \end{aligned}$$

fricas [B] time = 0.69, size = 122, normalized size = 4.69

$$\frac{8a^4 \cos(dx + c)^8 - 16a^4 \cos(dx + c)^6 + 5a^4 \cos(dx + c)^4 + 48a^2b^2 \cos(dx + c)^2 + 8b^4 - 32(a^3b \cos(dx + c) + a^2b^2 \tan(dx + c))}{32d \cos(dx + c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(dx+c)+a*sin(dx+c))^3*(a*cos(dx+c)+b*sec(dx+c)*tan(dx+c)),x, algorithm="fricas")

[Out] 1/32*(8*a^4*cos(dx + c)^8 - 16*a^4*cos(dx + c)^6 + 5*a^4*cos(dx + c)^4 + 48*a^2*b^2*cos(dx + c)^2 + 8*b^4 - 32*(a^3*b*cos(dx + c)^5 - a^3*b*cos(dx + c)^3 - a*b^3*cos(dx + c))*sin(dx + c))/(d*cos(dx + c)^4)

giac [B] time = 1.95, size = 142, normalized size = 5.46

$$\frac{b^4 \tan(dx + c)^4 + 4ab^3 \tan(dx + c)^3 + 6a^2b^2 \tan(dx + c)^2 + 2b^4 \tan(dx + c)^2 + 4a^3b \tan(dx + c) + 4ab^3 \tan(dx + c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(dx+c)+a*sin(dx+c))^3*(a*cos(dx+c)+b*sec(dx+c)*tan(dx+c)),x, algorithm="giac")

[Out] 1/4*(b^4*tan(dx + c)^4 + 4*a*b^3*tan(dx + c)^3 + 6*a^2*b^2*tan(dx + c)^2 + 2*b^4*tan(dx + c)^2 + 4*a^3*b*tan(dx + c) + 4*a*b^3*tan(dx + c) - (4*a^3*b*tan(dx + c)^3 + 2*a^4*tan(dx + c)^2 + 4*a^3*b*tan(dx + c) + a^4)/(tan(dx + c)^2 + 1)^2)/d

maple [B] time = 0.61, size = 137, normalized size = 5.27

$$\frac{a^4 \left(\sin^4(dx + c) \right)}{4d} + \frac{a^3b \left(\sin^5(dx + c) \right)}{d \cos(dx + c)} + \frac{a^3b \cos(dx + c) \left(\sin^3(dx + c) \right)}{d} + \frac{3a^2b^2 \left(\tan^2(dx + c) \right)}{2d} + \frac{ab^3 \left(\sin^3(dx + c) \right)}{d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(dx+c)+a*sin(dx+c))^3*(a*cos(dx+c)+b*sec(dx+c)*tan(dx+c)),x)

[Out] 1/4/d*a^4*sin(dx+c)^4+1/d*a^3*b*sin(dx+c)^5/cos(dx+c)+1/d*a^3*b*cos(dx+c)*sin(dx+c)^3+3/2/d*a^2*b^2*tan(dx+c)^2+1/d*a*b^3*sin(dx+c)^3/cos(dx+c)^3+1/d*a*b^3*tan(dx+c)+1/4/d*b^4/cos(dx+c)^4

maxima [A] time = 0.31, size = 24, normalized size = 0.92

$$\frac{(b \sec(dx + c) + a \sin(dx + c))^4}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x, algorithm="maxima")

[Out] 1/4*(b*sec(d*x + c) + a*sin(d*x + c))^4/d

mupad [B] time = 3.57, size = 185, normalized size = 7.12

$$\frac{a^4 \cos(2c + 2dx)^4 - 2a^4 \cos(2c + 2dx)^2 + a^4 - 8 \sin(2c + 2dx) a^3 b \cos(2c + 2dx)^2 + 8 \sin(2c + 2dx) a^3}{d (16 \cos(2c + 2dx))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*sin(c + d*x) + b/cos(c + d*x))^3*(a*cos(c + d*x) + (b*tan(c + d*x))/cos(c + d*x)),x)

[Out] (a^4*cos(2*c + 2*d*x)^4 - 2*a^4*cos(2*c + 2*d*x)^2 - 4*b^4*cos(2*c + 2*d*x)^2 + a^4 + 12*b^4 + 24*a^2*b^2 - 8*b^4*cos(2*c + 2*d*x) - 24*a^2*b^2*cos(2*c + 2*d*x)^2 + 32*a*b^3*sin(2*c + 2*d*x) + 8*a^3*b*sin(2*c + 2*d*x) - 8*a^3*b*cos(2*c + 2*d*x)^2*sin(2*c + 2*d*x))/(d*(32*cos(2*c + 2*d*x) + 16*cos(2*c + 2*d*x)^2 + 16))

sympy [A] time = 35.72, size = 129, normalized size = 4.96

$$\begin{cases} \frac{a^4 \sin^4(c+dx)}{4d} + \frac{a^3 b \sin^3(c+dx) \sec(c+dx)}{d} + \frac{3a^2 b^2 \sin^2(c+dx) \sec^2(c+dx)}{2d} + \frac{ab^3 \sin(c+dx) \sec^3(c+dx)}{d} + \frac{b^4 \sec^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x(a \sin(c) + b \sec(c))^3 (a \cos(c) + b \tan(c) \sec(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x)

[Out] Piecewise((a**4*sin(c + d*x)**4/(4*d) + a**3*b*sin(c + d*x)**3*sec(c + d*x)/d + 3*a**2*b**2*sin(c + d*x)**2*sec(c + d*x)**2/(2*d) + a*b**3*sin(c + d*x)*sec(c + d*x)**3/d + b**4*sec(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) + b*sec(c))**3*(a*cos(c) + b*tan(c)*sec(c)), True))

$$3.639 \quad \int (b \sec(c+dx) + a \sin(c+dx))^2 (a \cos(c+dx) + b \sec(c+dx) \tan(c+dx)) dx$$

Optimal. Leaf size=26

$$\frac{(a \sin(c+dx) + b \sec(c+dx))^3}{3d}$$

[Out] 1/3*(b*sec(d*x+c)+a*sin(d*x+c))^3/d

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {4385}

$$\frac{(a \sin(c+dx) + b \sec(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

[In] Int[(b*Sec[c + d*x] + a*Sin[c + d*x])^2*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]),x]

[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^3/(3*d)

Rule 4385

Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /; !FalseQ[q] /; FreeQ[m, x] && NeQ[m, -1] && !InertTrigFreeQ[u]

Rubi steps

$$\int (b \sec(c+dx) + a \sin(c+dx))^2 (a \cos(c+dx) + b \sec(c+dx) \tan(c+dx)) dx = \frac{(b \sec(c+dx) + a \sin(c+dx))^3}{3d}$$

Mathematica [A] time = 1.22, size = 31, normalized size = 1.19

$$\frac{\sec^3(c+dx)(a \sin(2(c+dx)) + 2b)^3}{24d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])^2*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]),x]

[Out] (Sec[c + d*x]^3*(2*b + a*Sin[2*(c + d*x)])^3)/(24*d)

fricas [B] time = 0.98, size = 92, normalized size = 3.54

$$\frac{3a^2b \cos(dx+c)^4 - 3a^2b \cos(dx+c)^2 - b^3 + (a^3 \cos(dx+c)^5 - a^3 \cos(dx+c)^3 - 3ab^2 \cos(dx+c)) \sin(dx+c)}{3d \cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x, algorithm="fricas")

[Out] -1/3*(3*a^2*b*cos(d*x + c)^4 - 3*a^2*b*cos(d*x + c)^2 - b^3 + (a^3*cos(d*x + c)^5 - a^3*cos(d*x + c)^3 - 3*a*b^2*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.56, size = 118, normalized size = 4.54

$$\frac{a^3 (\sin^3(dx+c))}{3d} + \frac{a^2 b (\sin^4(dx+c))}{d \cos(dx+c)} + \frac{a^2 b (\sin^2(dx+c)) \cos(dx+c)}{d} + \frac{a b^2 (\sin^3(dx+c))}{d \cos(dx+c)^2} + \frac{a b^2 \sin(dx+c)}{d} + \frac{b^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x)

[Out] 1/3/d*a^3*sin(d*x+c)^3+1/d*a^2*b*sin(d*x+c)^4/cos(d*x+c)+1/d*a^2*b*sin(d*x+c)^2*cos(d*x+c)+1/d*a*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/d*a*b^2*sin(d*x+c)+1/3/d*b^3/cos(d*x+c)^3

maxima [A] time = 0.31, size = 24, normalized size = 0.92

$$\frac{(b \sec(dx+c) + a \sin(dx+c))^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x, algorithm="maxima")

[Out] 1/3*(b*sec(d*x + c) + a*sin(d*x + c))^3/d

mupad [B] time = 3.22, size = 100, normalized size = 3.85

$$\frac{a^3 \sin(c+dx)}{3d} + \frac{a^2 b \cos(c+dx)^2 + \sin(c+dx) a b^2 \cos(c+dx) + \frac{b^3}{3}}{d \cos(c+dx)^3} - \frac{a^3 \cos(c+dx)^2 \sin(c+dx)}{3d} - \frac{a^2 b \cos(c+dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*sin(c+d*x) + b/cos(c+d*x))^2*(a*cos(c+d*x) + (b*tan(c+d*x))/cos(c+d*x)),x)

[Out] (a^3*sin(c+d*x))/(3*d) + (b^3/3 + a^2*b*cos(c+d*x)^2 + a*b^2*cos(c+d*x)*sin(c+d*x))/(d*cos(c+d*x)^3) - (a^3*cos(c+d*x)^2*sin(c+d*x))/(3*d) - (a^2*b*cos(c+d*x))/d

sympy [A] time = 11.16, size = 100, normalized size = 3.85

$$\begin{cases} \frac{a^3 \sin^3(c+dx)}{3d} + \frac{a^2 b \sin^2(c+dx) \sec(c+dx)}{d} + \frac{a b^2 \sin(c+dx) \sec^2(c+dx)}{d} + \frac{b^3 \sec^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a \sin(c) + b \sec(c))^2 (a \cos(c) + b \tan(c) \sec(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x)

[Out] Piecewise((a**3*sin(c+d*x)**3/(3*d) + a**2*b*sin(c+d*x)**2*sec(c+d*x)/d + a*b**2*sin(c+d*x)*sec(c+d*x)**2/d + b**3*sec(c+d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin(c) + b*sec(c))**2*(a*cos(c) + b*tan(c)*sec(c)), True))

$$3.640 \quad \int (b \sec(c + dx) + a \sin(c + dx))(a \cos(c + dx) + b \sec(c + dx) \tan(c + dx)) dx$$

Optimal. Leaf size=26

$$\frac{(a \sin(c + dx) + b \sec(c + dx))^2}{2d}$$

[Out] 1/2*(b*sec(d*x+c)+a*sin(d*x+c))^2/d

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {4385}

$$\frac{(a \sin(c + dx) + b \sec(c + dx))^2}{2d}$$

Antiderivative was successfully verified.

[In] Int[(b*Sec[c + d*x] + a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]),x]

[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^2/(2*d)

Rule 4385

Int[(u_)*(y_)^(m_.), x_Symbol] := With[{q = DerivativeDivides[ActivateTrig[y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /; !FalseQ[q] /; FreeQ[m, x] && NeQ[m, -1] && !InertTrigFreeQ[u]

Rubi steps

$$\int (b \sec(c + dx) + a \sin(c + dx))(a \cos(c + dx) + b \sec(c + dx) \tan(c + dx)) dx = \frac{(b \sec(c + dx) + a \sin(c + dx))^2}{2d}$$

Mathematica [B] time = 0.04, size = 67, normalized size = 2.58

$$-\frac{a^2 \cos^2(c + dx)}{2d} - \frac{ab \tan^{-1}(\tan(c + dx))}{d} + \frac{ab \tan(c + dx)}{d} + abx + \frac{b^2 \sec^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]),x]

[Out] a*b*x - (a*b*ArcTan[Tan[c + d*x]])/d - (a^2*Cos[c + d*x]^2)/(2*d) + (b^2*Sec[c + d*x]^2)/(2*d) + (a*b*Tan[c + d*x])/d

fricas [B] time = 0.98, size = 61, normalized size = 2.35

$$\frac{2 a^2 \cos(dx + c)^4 - a^2 \cos(dx + c)^2 - 4 ab \cos(dx + c) \sin(dx + c) - 2 b^2}{4 d \cos(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x, algorithm="fricas")

[Out] -1/4*(2*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^2 - 4*a*b*cos(d*x + c)*sin(d*x + c) - 2*b^2)/(d*cos(d*x + c)^2)

giac [A] time = 2.67, size = 45, normalized size = 1.73

$$\frac{b^2 \tan(dx+c)^2 + 2ab \tan(dx+c) - \frac{a^2}{\tan(dx+c)^2+1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x, algorithm="giac")

[Out] 1/2*(b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) - a^2/(tan(d*x + c)^2 + 1))/d

maple [B] time = 0.46, size = 57, normalized size = 2.19

$$\frac{-\frac{(\cos^2(dx+c))a^2}{2} + ab(\tan(dx+c) - dx - c) + (dx+c)ab + \frac{b^2}{2\cos(dx+c)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x)

[Out] 1/d*(-1/2*cos(d*x+c)^2*a^2+a*b*(tan(d*x+c)-d*x-c)+(d*x+c)*a*b+1/2*b^2/cos(d*x+c)^2)

maxima [A] time = 0.55, size = 24, normalized size = 0.92

$$\frac{(b \sec(dx+c) + a \sin(dx+c))^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x, algorithm="maxima")

[Out] 1/2*(b*sec(d*x + c) + a*sin(d*x + c))^2/d

mupad [B] time = 3.18, size = 61, normalized size = 2.35

$$\frac{\frac{a^2(2\sin(2c+2dx)^2-1)}{16} + \frac{a^2}{16} + \frac{b^2}{2} + \frac{ab \sin(2c+2dx)}{2}}{d(\sin(c+dx)^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*sin(c + d*x) + b/cos(c + d*x))*(a*cos(c + d*x) + (b*tan(c + d*x))/cos(c + d*x)),x)

[Out] -((a^2*(2*sin(2*c + 2*d*x)^2 - 1))/16 + a^2/16 + b^2/2 + (a*b*sin(2*c + 2*d*x))/2)/(d*(sin(c + d*x)^2 - 1))

sympy [A] time = 3.46, size = 73, normalized size = 2.81

$$\begin{cases} \frac{a^2 \sin^2(c+dx)}{2d} + \frac{ab \sin(c+dx) \sec(c+dx)}{d} + \frac{b^2 \sec^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a \sin(c) + b \sec(c))(a \cos(c) + b \tan(c) \sec(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x)

[Out] Piecewise((a**2*sin(c + d*x)**2/(2*d) + a*b*sin(c + d*x)*sec(c + d*x)/d + b**2*sec(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a*sin(c) + b*sec(c))*(a*cos(c) + b*tan(c)*sec(c)), True))

$$3.641 \quad \int \frac{a \cos(c+dx)+b \sec(c+dx) \tan(c+dx)}{b \sec(c+dx)+a \sin(c+dx)} dx$$

Optimal. Leaf size=22

$$\frac{\log(a \sin(c + dx) + b \sec(c + dx))}{d}$$

[Out] ln(b*sec(d*x+c)+a*sin(d*x+c))/d

Rubi [A] time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {4383}

$$\frac{\log(a \sin(c + dx) + b \sec(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin[c + d*x]),x]

[Out] Log[b*Sec[c + d*x] + a*Sin[c + d*x]]/d

Rule 4383

Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[y], ActivateTrig[u], x]}, Simp[q*Log[RemoveContent[ActivateTrig[y], x]], x] /; !FalseQ[q] /; !InertTrigFreeQ[u]

Rubi steps

$$\int \frac{a \cos(c + dx) + b \sec(c + dx) \tan(c + dx)}{b \sec(c + dx) + a \sin(c + dx)} dx = \frac{\log(b \sec(c + dx) + a \sin(c + dx))}{d}$$

Mathematica [A] time = 0.46, size = 29, normalized size = 1.32

$$\frac{\log(a \sin(2(c + dx)) + 2b) - \log(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin[c + d*x]),x]

[Out] (-Log[Cos[c + d*x]] + Log[2*b + a*Sin[2*(c + d*x)]])/d

fricas [A] time = 1.13, size = 33, normalized size = 1.50

$$\frac{\log(a \cos(dx + c) \sin(dx + c) + b) - \log(-\cos(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)),x, algorithm="fricas")

[Out] (log(a*cos(d*x + c)*sin(d*x + c) + b) - log(-cos(d*x + c)))/d

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Evaluation time: 0.56gen.cc:simplify/tmp.typ
e!=_EXT Error: Bad Argument Value
```

maple [A] time = 0.52, size = 23, normalized size = 1.05

$$\frac{\ln(b \sec(dx + c) + a \sin(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)),x)
```

```
[Out] ln(b*sec(d*x+c)+a*sin(d*x+c))/d
```

maxima [A] time = 0.32, size = 22, normalized size = 1.00

$$\frac{\log(b \sec(dx + c) + a \sin(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(b*sec(d*x + c) + a*sin(d*x + c))/d
```

mupad [B] time = 4.86, size = 133, normalized size = 6.05

$$\frac{\operatorname{atan}\left(\frac{-\cos(c+dx)a^6+8\cos(c+dx)a^4b^2-16\cos(c+dx)a^2b^4+\frac{\sin(2c+2dx)ab^5}{2}+b^6}{1i\cos(c+dx)a^6-8i\cos(c+dx)a^4b^2+16i\cos(c+dx)a^2b^4+\frac{1i\sin(2c+2dx)ab^5}{2}+b^61i}\right)2i}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + (b*tan(c + d*x))/cos(c + d*x))/(a*sin(c + d*x) + b/cos(c + d*x)),x)
```

```
[Out] (atan((b^6 - a^6*cos(c + d*x) - 16*a^2*b^4*cos(c + d*x) + 8*a^4*b^2*cos(c + d*x) + (a*b^5*sin(2*c + 2*d*x))/2)/(a^6*cos(c + d*x)*1i + b^6*1i + a^2*b^4*cos(c + d*x)*16i - a^4*b^2*cos(c + d*x)*8i + (a*b^5*sin(2*c + 2*d*x)*1i)/2))*2i)/d
```

sympy [A] time = 7.48, size = 63, normalized size = 2.86

$$\begin{cases} x \tan(c) & \text{for } a = 0 \wedge d = 0 \\ \frac{\log(\tan^2(c+dx)+1)}{2d} & \text{for } a = 0 \\ \frac{x(a \cos(c)+b \tan(c) \sec(c))}{a \sin(c)+b \sec(c)} & \text{for } d = 0 \\ \frac{\log\left(\sin(c+dx)+\frac{b \sec(c+dx)}{a}\right)}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*tan(c), Eq(a, 0) & Eq(d, 0)), (log(tan(c + d*x)**2 + 1)/(2*d),  
Eq(a, 0)), (x*(a*cos(c) + b*tan(c)*sec(c))/(a*sin(c) + b*sec(c)), Eq(d, 0)  
) , (log(sin(c + d*x) + b*sec(c + d*x)/a)/d, True))
```

$$3.642 \quad \int \frac{a \cos(c+dx) + b \sec(c+dx) \tan(c+dx)}{(b \sec(c+dx) + a \sin(c+dx))^2} dx$$

Optimal. Leaf size=24

$$-\frac{1}{d(a \sin(c + dx) + b \sec(c + dx))}$$

[Out] -1/d/(b*sec(d*x+c)+a*sin(d*x+c))

Rubi [A] time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {4385}

$$-\frac{1}{d(a \sin(c + dx) + b \sec(c + dx))}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin[c + d*x])^2,x]

[Out] -(1/(d*(b*Sec[c + d*x] + a*Sin[c + d*x])))

Rule 4385

Int[(u_)*(y_)^(m_.), x_Symbol] := With[{q = DerivativeDivides[ActivateTrig[y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m + 1)])/(m + 1), x] /; !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] && !InertTrigFreeQ[u]

Rubi steps

$$\int \frac{a \cos(c + dx) + b \sec(c + dx) \tan(c + dx)}{(b \sec(c + dx) + a \sin(c + dx))^2} dx = -\frac{1}{d(b \sec(c + dx) + a \sin(c + dx))}$$

Mathematica [A] time = 0.30, size = 27, normalized size = 1.12

$$-\frac{2 \cos(c + dx)}{d(a \sin(2(c + dx)) + 2b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin[c + d*x])^2,x]

[Out] (-2*Cos[c + d*x])/(d*(2*b + a*Sin[2*(c + d*x)]))

fricas [A] time = 0.90, size = 29, normalized size = 1.21

$$-\frac{\cos(dx + c)}{ad \cos(dx + c) \sin(dx + c) + bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -cos(d*x + c)/(a*d*cos(d*x + c)*sin(d*x + c) + b*d)

giac [B] time = 0.73, size = 108, normalized size = 4.50

$$\frac{2 \left(a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 - b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - b \right)}{\left(b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^4 - 2 a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + 2 b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + 2 a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + b \right) b d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] 2*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/2*c) - b)/((b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^3 + 2*b*tan(1/2*d*x + 1/2*c)^2 + 2*a*tan(1/2*d*x + 1/2*c) + b)*b*d)

maple [A] time = 0.63, size = 25, normalized size = 1.04

$$-\frac{1}{d(b \sec(dx + c) + a \sin(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))^2,x)

[Out] -1/d/(b*sec(d*x+c)+a*sin(d*x+c))

maxima [A] time = 0.43, size = 24, normalized size = 1.00

$$-\frac{1}{(b \sec(dx + c) + a \sin(dx + c))d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/((b*sec(d*x + c) + a*sin(d*x + c))*d)

mupad [B] time = 3.24, size = 47, normalized size = 1.96

$$-\frac{b(\cos(c + dx) + 1) + \frac{a \sin(2c + 2dx)}{2}}{bd \left(b + \frac{a \sin(2c + 2dx)}{2} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(c + d*x) + (b*tan(c + d*x))/cos(c + d*x))/(a*sin(c + d*x) + b/cos(c + d*x))^2,x)

[Out] -(b*(cos(c + d*x) + 1) + (a*sin(2*c + 2*d*x))/2)/(b*d*(b + (a*sin(2*c + 2*d*x))/2))

sympy [A] time = 21.18, size = 49, normalized size = 2.04

$$\begin{cases} -\frac{1}{ad \sin(c+dx)+bd \sec(c+dx)} & \text{for } d \neq 0 \\ \frac{x(a \cos(c)+b \tan(c) \sec(c))}{(a \sin(c)+b \sec(c))^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-1/(a*d*sin(c + d*x) + b*d*sec(c + d*x)), Ne(d, 0)), (x*(a*cos(c) + b*tan(c)*sec(c))/(a*sin(c) + b*sec(c))**2, True))
```


$$3.643 \quad \int \frac{a \cos(c+dx) + b \sec(c+dx) \tan(c+dx)}{(b \sec(c+dx) + a \sin(c+dx))^3} dx$$

Optimal. Leaf size=26

$$-\frac{1}{2d(a \sin(c+dx) + b \sec(c+dx))^2}$$

[Out] -1/2/d/(b*sec(d*x+c)+a*sin(d*x+c))^2

Rubi [A] time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {4385}

$$-\frac{1}{2d(a \sin(c+dx) + b \sec(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin[c + d*x])^3,x]

[Out] -1/(2*d*(b*Sec[c + d*x] + a*Sin[c + d*x])^2)

Rule 4385

Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[y], ActivateTrig[u], x]}, Simp[(q*ActivateTrig[y^(m+1)])/(m+1), x] /; !FalseQ[q] /; FreeQ[m, x] && NeQ[m, -1] && !InertTrigFreeQ[u]

Rubi steps

$$\int \frac{a \cos(c+dx) + b \sec(c+dx) \tan(c+dx)}{(b \sec(c+dx) + a \sin(c+dx))^3} dx = -\frac{1}{2d(b \sec(c+dx) + a \sin(c+dx))^2}$$

Mathematica [A] time = 0.71, size = 29, normalized size = 1.12

$$-\frac{2 \cos^2(c+dx)}{d(a \sin(2(c+dx)) + 2b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin[c + d*x])^3,x]

[Out] (-2*Cos[c + d*x]^2)/(d*(2*b + a*Sin[2*(c + d*x)])^2)

fricas [B] time = 1.91, size = 63, normalized size = 2.42

$$\frac{\cos(dx+c)^2}{2(a^2d \cos(dx+c)^4 - a^2d \cos(dx+c)^2 - 2abd \cos(dx+c) \sin(dx+c) - b^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/2*cos(d*x + c)^2/(a^2*d*cos(d*x + c)^4 - a^2*d*cos(d*x + c)^2 - 2*a*b*d*cos(d*x + c)*sin(d*x + c) - b^2*d)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:Evaluation time: 52.12gen.cc:simplify/tmp.ty pe!=_EXT Error: Bad Argument Value

maple [A] time = 0.66, size = 25, normalized size = 0.96

$$\frac{1}{2d(b \sec(dx + c) + a \sin(dx + c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))^3,x)

[Out] -1/2/d/(b*sec(d*x+c)+a*sin(d*x+c))^2

maxima [A] time = 0.31, size = 24, normalized size = 0.92

$$\frac{1}{2(b \sec(dx + c) + a \sin(dx + c))^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -1/2/((b*sec(d*x + c) + a*sin(d*x + c))^2*d)

mupad [B] time = 6.31, size = 291, normalized size = 11.19

$$\frac{\frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 (a^2 + b^2)}{b^2} + \frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 (a^2 + b^2)}{b^2} + \frac{2 a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{b} - \frac{4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 (a^2 - b^2)}{b^2}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 (4 a^2 + 4 b^2) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 (4 a^2 + 4 b^2) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 (8 a^2 - 6 b^2) + b^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + \dots \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(c + d*x) + (b*tan(c + d*x))/cos(c + d*x))/(a*sin(c + d*x) + b/cos(c + d*x))^3,x)

[Out] ((2*tan(c/2 + (d*x)/2)^2*(a^2 + b^2))/b^2 + (2*tan(c/2 + (d*x)/2)^6*(a^2 + b^2))/b^2 + (2*a*tan(c/2 + (d*x)/2))/b - (4*tan(c/2 + (d*x)/2)^4*(a^2 - b^2))/b^2 + (2*a*tan(c/2 + (d*x)/2)^3)/b - (2*a*tan(c/2 + (d*x)/2)^5)/b - (2*a*tan(c/2 + (d*x)/2)^7)/b)/(d*(tan(c/2 + (d*x)/2)^2*(4*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^6*(4*a^2 + 4*b^2) - tan(c/2 + (d*x)/2)^4*(8*a^2 - 6*b^2) + b^2*tan(c/2 + (d*x)/2)^8 + b^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 - 4*a*b*tan(c/2 + (d*x)/2)^5 - 4*a*b*tan(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2)))

sympy [A] time = 44.09, size = 80, normalized size = 3.08

$$\begin{cases} -\frac{1}{2a^2d \sin^2(c+dx)+4abd \sin(c+dx) \sec(c+dx)+2b^2d \sec^2(c+dx)} & \text{for } d \neq 0 \\ \frac{x(a \cos(c)+b \tan(c) \sec(c))}{(a \sin(c)+b \sec(c))^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-1/(2*a**2*d*sin(c + d*x)**2 + 4*a*b*d*sin(c + d*x)*sec(c + d*x) + 2*b**2*d*sec(c + d*x)**2), Ne(d, 0)), (x*(a*cos(c) + b*tan(c)*sec(c))/(a*sin(c) + b*sec(c))**3, True))
```

3.644 $\int F(c, d, \cos(a + bx), r, s) \sin(a + bx) dx$

Optimal. Leaf size=21

$$\text{Int}(\sin(a + bx)F(c, d, \cos(a + bx), r, s), x)$$

[Out] CannotIntegrate(F(c, d, cos(b*x+a), r, s)*sin(b*x+a), x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int F(c, d, \cos(a + bx), r, s) \sin(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[F[c, d, Cos[a + b*x], r, s]*Sin[a + b*x], x]

[Out] -(Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Cos[a + b*x]]/b)

Rubi steps

$$\int F(c, d, \cos(a + bx), r, s) \sin(a + bx) dx = -\frac{\text{Subst}(\int F(c, d, x, r, s) dx, x, \cos(a + bx))}{b}$$

Mathematica [A] time = 0.05, size = 0, normalized size = 0.00

$$\int F(c, d, \cos(a + bx), r, s) \sin(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[F[c, d, Cos[a + b*x], r, s]*Sin[a + b*x], x]

[Out] Integrate[F[c, d, Cos[a + b*x], r, s]*Sin[a + b*x], x]

fricas [A] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}(F(c, d, \cos(bx + a), r, s) \sin(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F(c, d, cos(b*x+a), r, s)*sin(b*x+a), x, algorithm="fricas")

[Out] integral(F(c, d, cos(b*x + a), r, s)*sin(b*x + a), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c, d, \cos(bx + a), r, s) \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F(c, d, cos(b*x+a), r, s)*sin(b*x+a), x, algorithm="giac")

[Out] integrate(F(c, d, cos(b*x + a), r, s)*sin(b*x + a), x)

maple [A] time = 0.05, size = 0, normalized size = 0.00

$$\int F(c, d, \cos(bx + a), r, s) \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)`

[Out] `int(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)`

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c, d, \cos(bx + a), r, s) \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x, algorithm="maxima")`

[Out] `integrate(F(c, d, cos(b*x + a), r, s)*sin(b*x + a), x)`

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \sin(a + bx) F(c, d, \cos(a + bx), r, s) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(a + b*x)*F(c, d, cos(a + b*x), r, s),x)`

[Out] `int(sin(a + b*x)*F(c, d, cos(a + b*x), r, s), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c, d, \cos(a + bx), r, s) \sin(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)`

[Out] `Integral(F(c, d, cos(a + b*x), r, s)*sin(a + b*x), x)`

3.645 $\int \cos(a + bx)F(c, d, \sin(a + bx), r, s) dx$

Optimal. Leaf size=21

$$\text{Int}(\cos(a + bx)F(c, d, \sin(a + bx), r, s), x)$$

[Out] CannotIntegrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos(a + bx)F(c, d, \sin(a + bx), r, s) dx$$

Verification is Not applicable to the result.

[In] Int[Cos[a + b*x]*F[c, d, Sin[a + b*x], r, s], x]

[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Sin[a + b*x]]/b

Rubi steps

$$\int \cos(a + bx)F(c, d, \sin(a + bx), r, s) dx = \frac{\text{Subst}\left(\int F(c, d, x, r, s) dx, x, \sin(a + bx)\right)}{b}$$

Mathematica [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \cos(a + bx)F(c, d, \sin(a + bx), r, s) dx$$

Verification is Not applicable to the result.

[In] Integrate[Cos[a + b*x]*F[c, d, Sin[a + b*x], r, s], x]

[Out] Integrate[Cos[a + b*x]*F[c, d, Sin[a + b*x], r, s], x]

fricas [A] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}(F(c, d, \sin(bx + a), r, s) \cos(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x, algorithm="fricas")

[Out] integral(F(c, d, sin(b*x + a), r, s)*cos(b*x + a), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c, d, \sin(bx + a), r, s) \cos(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x, algorithm="giac")

[Out] integrate(F(c, d, sin(b*x + a), r, s)*cos(b*x + a), x)

maple [A] time = 0.03, size = 0, normalized size = 0.00

$$\int \cos(bx + a)F(c, d, \sin(bx + a), r, s) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)`

[Out] `int(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)`

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c, d, \sin(bx + a), r, s) \cos(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x, algorithm="maxima")`

[Out] `integrate(F(c, d, sin(b*x + a), r, s)*cos(b*x + a), x)`

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \cos(a + bx) F(c, d, \sin(a + bx), r, s) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b*x)*F(c, d, sin(a + b*x), r, s),x)`

[Out] `int(cos(a + b*x)*F(c, d, sin(a + b*x), r, s), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c, d, \sin(a + bx), r, s) \cos(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)`

[Out] `Integral(F(c, d, sin(a + b*x), r, s)*cos(a + b*x), x)`

3.646 $\int F(c, d, \tan(a + bx), r, s) \sec^2(a + bx) dx$

Optimal. Leaf size=23

$$\text{Int}\left(\sec^2(a + bx)F(c, d, \tan(a + bx), r, s), x\right)$$

[Out] CannotIntegrate(F(c, d, tan(b*x+a), r, s)*sec(b*x+a)^2, x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int F(c, d, \tan(a + bx), r, s) \sec^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[F[c, d, Tan[a + b*x], r, s]*Sec[a + b*x]^2, x]

[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Tan[a + b*x]]/b

Rubi steps

$$\int F(c, d, \tan(a + bx), r, s) \sec^2(a + bx) dx = \frac{\text{Subst}\left(\int F(c, d, x, r, s) dx, x, \tan(a + bx)\right)}{b}$$

Mathematica [A] time = 0.08, size = 0, normalized size = 0.00

$$\int F(c, d, \tan(a + bx), r, s) \sec^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[F[c, d, Tan[a + b*x], r, s]*Sec[a + b*x]^2, x]

[Out] Integrate[F[c, d, Tan[a + b*x], r, s]*Sec[a + b*x]^2, x]

fricas [A] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(F(c, d, \tan(bx + a), r, s) \sec(bx + a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F(c, d, tan(b*x+a), r, s)*sec(b*x+a)^2, x, algorithm="fricas")

[Out] integral(F(c, d, tan(b*x + a), r, s)*sec(b*x + a)^2, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c, d, \tan(bx + a), r, s) \sec(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F(c, d, tan(b*x+a), r, s)*sec(b*x+a)^2, x, algorithm="giac")

[Out] integrate(F(c, d, tan(b*x + a), r, s)*sec(b*x + a)^2, x)

maple [A] time = 0.06, size = 0, normalized size = 0.00

$$\int F(c, d, \tan(bx + a), r, s) \left(\sec^2(bx + a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x)

[Out] int(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\tan(bx+a),r,s)\sec(bx+a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x, algorithm="maxima")

[Out] integrate(F(c, d, tan(b*x + a), r, s)*sec(b*x + a)^2, x)

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{F(c,d,\tan(a+bx),r,s)}{\cos(a+bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(F(c, d, tan(a + b*x), r, s)/cos(a + b*x)^2,x)

[Out] int(F(c, d, tan(a + b*x), r, s)/cos(a + b*x)^2, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)**2,x)

[Out] Integral(F(c, d, tan(a + b*x), r, s)*sec(a + b*x)**2, x)

3.647 $\int \csc^2(a + bx)F(c, d, \cot(a + bx), r, s) dx$

Optimal. Leaf size=23

$$\text{Int}\left(\csc^2(a + bx)F(c, d, \cot(a + bx), r, s), x\right)$$

[Out] CannotIntegrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \csc^2(a + bx)F(c, d, \cot(a + bx), r, s) dx$$

Verification is Not applicable to the result.

[In] Int[Csc[a + b*x]^2*F[c, d, Cot[a + b*x], r, s], x]

[Out] -(Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Cot[a + b*x]]/b)

Rubi steps

$$\int \csc^2(a + bx)F(c, d, \cot(a + bx), r, s) dx = -\frac{\text{Subst}\left(\int F(c, d, x, r, s) dx, x, \cot(a + bx)\right)}{b}$$

Mathematica [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \csc^2(a + bx)F(c, d, \cot(a + bx), r, s) dx$$

Verification is Not applicable to the result.

[In] Integrate[Csc[a + b*x]^2*F[c, d, Cot[a + b*x], r, s], x]

[Out] Integrate[Csc[a + b*x]^2*F[c, d, Cot[a + b*x], r, s], x]

fricas [A] time = 1.97, size = 0, normalized size = 0.00

$$\text{integral}\left(F(c, d, \cot(bx + a), r, s) \csc(bx + a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x, algorithm="fricas")

[Out] integral(F(c, d, cot(b*x + a), r, s)*csc(b*x + a)^2, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c, d, \cot(bx + a), r, s) \csc(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x, algorithm="giac")

[Out] integrate(F(c, d, cot(b*x + a), r, s)*csc(b*x + a)^2, x)

maple [A] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(\csc^2(bx + a)\right) F(c, d, \cot(bx + a), r, s) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x)`

[Out] `int(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x)`

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c, d, \cot(bx + a), r, s) \csc(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x, algorithm="maxima")`

[Out] `integrate(F(c, d, cot(b*x + a), r, s)*csc(b*x + a)^2, x)`

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{F(c, d, \cot(a + bx), r, s)}{\sin(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(F(c, d, cot(a + b*x), r, s)/sin(a + b*x)^2,x)`

[Out] `int(F(c, d, cot(a + b*x), r, s)/sin(a + b*x)^2, x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int F(c, d, \cot(a + bx), r, s) \csc^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(b*x+a)**2*F(c,d,cot(b*x+a),r,s),x)`

[Out] `Integral(F(c, d, cot(a + b*x), r, s)*csc(a + b*x)**2, x)`

$$3.648 \quad \int \frac{\sin(x)}{a+b \cos(x)} dx$$

Optimal. Leaf size=12

$$\frac{\log(a + b \cos(x))}{b}$$

[Out] $-\ln(a+b*\cos(x))/b$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2668, 31}

$$\frac{\log(a + b \cos(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(a + b*Cos[x]),x]

[Out] $-(\text{Log}[a + b*\text{Cos}[x]])/b$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2668

Int[cos[(e_.) + (f_.)*(x_)]^{(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b² - x²)^{((p - 1)/2)}, x], x, b*Sin[e + f*x], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a² - b², 0]}

Rubi steps

$$\begin{aligned} \int \frac{\sin(x)}{a + b \cos(x)} dx &= -\frac{\text{Subst}\left(\int \frac{1}{a+x} dx, x, b \cos(x)\right)}{b} \\ &= -\frac{\log(a + b \cos(x))}{b} \end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 1.00

$$\frac{\log(a + b \cos(x))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/(a + b*Cos[x]),x]

[Out] $-(\text{Log}[a + b*\text{Cos}[x]])/b$

fricas [A] time = 0.73, size = 15, normalized size = 1.25

$$\frac{\log(-b \cos(x) - a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="fricas")

[Out] -log(-b*cos(x) - a)/b

giac [A] time = 0.13, size = 13, normalized size = 1.08

$$-\frac{\log(|b \cos(x) + a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="giac")

[Out] -log(abs(b*cos(x) + a))/b

maple [A] time = 0.03, size = 13, normalized size = 1.08

$$-\frac{\ln(a + b \cos(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(a+b*cos(x)),x)

[Out] -ln(a+b*cos(x))/b

maxima [A] time = 0.31, size = 12, normalized size = 1.00

$$-\frac{\log(b \cos(x) + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="maxima")

[Out] -log(b*cos(x) + a)/b

mupad [B] time = 0.06, size = 12, normalized size = 1.00

$$-\frac{\ln(a + b \cos(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(a + b*cos(x)),x)

[Out] -log(a + b*cos(x))/b

sympy [A] time = 0.33, size = 17, normalized size = 1.42

$$\begin{cases} -\frac{\log\left(\frac{a}{b} + \cos(x)\right)}{b} & \text{for } b \neq 0 \\ -\frac{\cos(x)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+b*cos(x)),x)

[Out] Piecewise((-log(a/b + cos(x))/b, Ne(b, 0)), (-cos(x)/a, True))

3.649 $\int (a + b \cos(x))^n \sin(x) dx$

Optimal. Leaf size=20

$$\frac{(a + b \cos(x))^{n+1}}{b(n+1)}$$

[Out] $-(a+b*\cos(x))^{(1+n)}/b/(1+n)$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2668, 32}

$$\frac{(a + b \cos(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*Cos[x])^n*Sin[x],x]`

[Out] $-\left((a + b*\cos[x])^{(1 + n)}/(b*(1 + n))\right)$

Rule 32

`Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]`

Rule 2668

`Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]`

Rubi steps

$$\begin{aligned} \int (a + b \cos(x))^n \sin(x) dx &= -\frac{\text{Subst}\left(\int (a + x)^n dx, x, b \cos(x)\right)}{b} \\ &= -\frac{(a + b \cos(x))^{1+n}}{b(1+n)} \end{aligned}$$

Mathematica [A] time = 0.03, size = 19, normalized size = 0.95

$$\frac{(a + b \cos(x))^{n+1}}{bn + b}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*Cos[x])^n*Sin[x],x]`

[Out] $-\left((a + b*\cos[x])^{(1 + n)}/(b + b*n)\right)$

fricas [A] time = 0.63, size = 23, normalized size = 1.15

$$-\frac{(b \cos(x) + a)(b \cos(x) + a)^n}{bn + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(x))^n*sin(x),x, algorithm="fricas")`

[Out] $-(b \cos(x) + a) \cdot (b \cos(x) + a)^n / (b \cdot n + b)$

giac [A] time = 0.14, size = 20, normalized size = 1.00

$$-\frac{(b \cos(x) + a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(x))^n*sin(x),x, algorithm="giac")`

[Out] $-(b \cos(x) + a)^{n+1} / (b \cdot (n+1))$

maple [A] time = 0.03, size = 21, normalized size = 1.05

$$-\frac{(a + b \cos(x))^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*cos(x))^n*sin(x),x)`

[Out] $-(a+b \cos(x))^{n+1} / b / (n+1)$

maxima [A] time = 0.38, size = 20, normalized size = 1.00

$$-\frac{(b \cos(x) + a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(x))^n*sin(x),x, algorithm="maxima")`

[Out] $-(b \cos(x) + a)^{n+1} / (b \cdot (n+1))$

mupad [B] time = 3.15, size = 20, normalized size = 1.00

$$-\frac{(a + b \cos(x))^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)*(a + b*cos(x))^n,x)`

[Out] $-(a + b \cos(x))^{n+1} / (b \cdot (n+1))$

sympy [A] time = 2.02, size = 63, normalized size = 3.15

$$\left\{ \begin{array}{ll} -\frac{\cos(x)}{a} & \text{for } b = 0 \wedge n = -1 \\ -a^n \cos(x) & \text{for } b = 0 \\ -\frac{\log\left(\frac{a}{b} + \cos(x)\right)}{b} & \text{for } n = -1 \\ -\frac{a(a+b \cos(x))^n}{bn+b} - \frac{b(a+b \cos(x))^n \cos(x)}{bn+b} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(x))**n*sin(x),x)`

[Out] `Piecewise((-cos(x)/a, Eq(b, 0) & Eq(n, -1)), (-a**n*cos(x), Eq(b, 0)), (-log(a/b + cos(x))/b, Eq(n, -1)), (-a*(a + b*cos(x))**n/(b*n + b) - b*(a + b*cos(x))**n*cos(x)/(b*n + b), True))`

$$3.650 \quad \int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}} dx$$

Optimal. Leaf size=5

$$-\sinh^{-1}(\cos(x))$$

[Out] -arcsinh(cos(x))

Rubi [A] time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3190, 215}

$$-\sinh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/Sqrt[1 + Cos[x]^2], x]

[Out] -ArcSinh[Cos[x]]

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}} dx &= -\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \cos(x)\right) \\ &= -\sinh^{-1}(\cos(x)) \end{aligned}$$

Mathematica [A] time = 0.02, size = 5, normalized size = 1.00

$$-\sinh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/Sqrt[1 + Cos[x]^2], x]

[Out] -ArcSinh[Cos[x]]

fricas [B] time = 0.96, size = 36, normalized size = 7.20

$$\frac{1}{4} \log\left(8 \cos(x)^4 + 8 \cos(x)^2 - 4(2 \cos(x)^3 + \cos(x))\sqrt{\cos(x)^2 + 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(1+cos(x)^2)^(1/2), x, algorithm="fricas")

[Out] 1/4*log(8*cos(x)^4 + 8*cos(x)^2 - 4*(2*cos(x)^3 + cos(x))*sqrt(cos(x)^2 + 1) + 1)

giac [B] time = 0.13, size = 14, normalized size = 2.80

$$\log\left(\sqrt{\cos(x)^2 + 1} - \cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(1+cos(x)^2)^(1/2),x, algorithm="giac")

[Out] log(sqrt(cos(x)^2 + 1) - cos(x))

maple [A] time = 0.06, size = 6, normalized size = 1.20

$$-\operatorname{arcsinh}(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(1+cos(x)^2)^(1/2),x)

[Out] -arcsinh(cos(x))

maxima [A] time = 1.41, size = 5, normalized size = 1.00

$$-\operatorname{arsinh}(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(1+cos(x)^2)^(1/2),x, algorithm="maxima")

[Out] -arcsinh(cos(x))

mupad [B] time = 3.08, size = 5, normalized size = 1.00

$$-\operatorname{asinh}(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(cos(x)^2 + 1)^(1/2),x)

[Out] -asinh(cos(x))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{\sqrt{\cos^2(x) + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(1+cos(x)**2)**(1/2),x)

[Out] Integral(sin(x)/sqrt(cos(x)**2 + 1), x)

3.651 $\int \cos(\cos(x)) \sin(x) dx$

Optimal. Leaf size=5

$$-\sin(\cos(x))$$

[Out] $-\sin(\cos(x))$

Rubi [A] time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4335, 2637}

$$-\sin(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[Cos[x]]*Sin[x],x]

[Out] -Sin[Cos[x]]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4335

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned} \int \cos(\cos(x)) \sin(x) dx &= -\text{Subst}\left(\int \cos(x) dx, x, \cos(x)\right) \\ &= -\sin(\cos(x)) \end{aligned}$$

Mathematica [A] time = 2.57, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[Cos[x]]*Sin[x],x]

[Out] -Sin[Cos[x]]

fricas [B] time = 0.79, size = 20, normalized size = 4.00

$$\sin\left(\frac{\tan\left(\frac{1}{2}x\right)^2 - 1}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(cos(x))*sin(x),x, algorithm="fricas")

[Out] sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))

giac [A] time = 0.14, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(cos(x))*sin(x),x, algorithm="giac")

[Out] -sin(cos(x))

maple [A] time = 0.01, size = 6, normalized size = 1.20

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(cos(x))*sin(x),x)

[Out] -sin(cos(x))

maxima [A] time = 0.55, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(cos(x))*sin(x),x, algorithm="maxima")

[Out] -sin(cos(x))

mupad [B] time = 0.09, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(cos(x))*sin(x),x)

[Out] -sin(cos(x))

sympy [A] time = 0.44, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(cos(x))*sin(x),x)

[Out] -sin(cos(x))

3.652 $\int \cos(x) \cos(\cos(x)) \sin(x) \sin(\cos(x)) dx$

Optimal. Leaf size=28

$$\frac{\cos(x)}{4} - \frac{1}{2} \cos(x) \sin^2(\cos(x)) - \frac{1}{4} \cos(\cos(x)) \sin(\cos(x))$$

[Out] 1/4*cos(x)-1/4*cos(cos(x))*sin(cos(x))-1/2*cos(x)*sin(cos(x))^2

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4335, 3443, 2635, 8}

$$\frac{\cos(x)}{4} - \frac{1}{2} \cos(x) \sin^2(\cos(x)) - \frac{1}{4} \cos(\cos(x)) \sin(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[Cos[x]]*Sin[x]*Sin[Cos[x]],x]

[Out] Cos[x]/4 - (Cos[Cos[x]]*Sin[Cos[x]])/4 - (Cos[x]*Sin[Cos[x]]^2)/2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*SIN[c + d*x])^(n - 1)) / (d*n), x] + Dist[(b^2*(n - 1)) / n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3443

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] := Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1)) / (b*n*(p + 1)), x] - Dist[(m - n + 1) / (b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]

Rule 4335

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned} \int \cos(x) \cos(\cos(x)) \sin(x) \sin(\cos(x)) dx &= -\text{Subst}\left(\int x \cos(x) \sin(x) dx, x, \cos(x)\right) \\ &= -\frac{1}{2} \cos(x) \sin^2(\cos(x)) + \frac{1}{2} \text{Subst}\left(\int \sin^2(x) dx, x, \cos(x)\right) \\ &= -\frac{1}{4} \cos(\cos(x)) \sin(\cos(x)) - \frac{1}{2} \cos(x) \sin^2(\cos(x)) + \frac{1}{4} \text{Subst}\left(\int 1 \right) \\ &= \frac{\cos(x)}{4} - \frac{1}{4} \cos(\cos(x)) \sin(\cos(x)) - \frac{1}{2} \cos(x) \sin^2(\cos(x)) \end{aligned}$$

Mathematica [A] time = 1.48, size = 21, normalized size = 0.75

$$\frac{1}{4} \cos(x) \cos(2 \cos(x)) - \frac{1}{8} \sin(2 \cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cos[Cos[x]]*Sin[x]*Sin[Cos[x]],x]

[Out] (Cos[x]*Cos[2*Cos[x]])/4 - Sin[2*Cos[x]]/8

fricas [B] time = 1.00, size = 73, normalized size = 2.61

$$\frac{1}{2} \cos(x) \cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2 - 1}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right)^2 + \frac{1}{4} \cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2 - 1}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right) \sin\left(\frac{\tan\left(\frac{1}{2}x\right)^2 - 1}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right) - \frac{1}{4} \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x, algorithm="fricas")

[Out] 1/2*cos(x)*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^2 + 1/4*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))*sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1)) - 1/4*cos(x)

giac [A] time = 0.14, size = 17, normalized size = 0.61

$$\frac{1}{4} \cos(x) \cos(2 \cos(x)) - \frac{1}{8} \sin(2 \cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x, algorithm="giac")

[Out] 1/4*cos(x)*cos(2*cos(x)) - 1/8*sin(2*cos(x))

maple [A] time = 0.01, size = 23, normalized size = 0.82

$$\frac{(\cos^2(\cos(x))) \cos(x)}{2} - \frac{\cos(\cos(x)) \sin(\cos(x))}{4} - \frac{\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x)

[Out] 1/2*cos(cos(x))^2*cos(x)-1/4*cos(cos(x))*sin(cos(x))-1/4*cos(x)

maxima [A] time = 0.59, size = 17, normalized size = 0.61

$$\frac{1}{4} \cos(x) \cos(2 \cos(x)) - \frac{1}{8} \sin(2 \cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x, algorithm="maxima")

[Out] 1/4*cos(x)*cos(2*cos(x)) - 1/8*sin(2*cos(x))

mupad [B] time = 3.05, size = 22, normalized size = 0.79

$$\frac{\cos(x) \cos(\cos(x))^2}{2} - \frac{\sin(\cos(x)) \cos(\cos(x))}{4} - \frac{\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(cos(x))*sin(cos(x))*cos(x)*sin(x),x)`

[Out] `(cos(cos(x))^2*cos(x))/2 - cos(x)/4 - (cos(cos(x))*sin(cos(x)))/4`

sympy [A] time = 5.57, size = 34, normalized size = 1.21

$$-\frac{\sin^2(\cos(x))\cos(x)}{4} - \frac{\sin(\cos(x))\cos(\cos(x))}{4} + \frac{\cos(x)\cos^2(\cos(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x)`

[Out] `-sin(cos(x))**2*cos(x)/4 - sin(cos(x))*cos(cos(x))/4 + cos(x)*cos(cos(x))**2/4`

3.653 $\int \cos(\cos(x)) \sin(x) \sin^2(6 \cos(x)) dx$

Optimal. Leaf size=26

$$-\frac{1}{2} \sin(\cos(x)) + \frac{1}{44} \sin(11 \cos(x)) + \frac{1}{52} \sin(13 \cos(x))$$

[Out] $-1/2*\sin(\cos(x))+1/44*\sin(11*\cos(x))+1/52*\sin(13*\cos(x))$

Rubi [A] time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4335, 4354, 2637}

$$-\frac{1}{2} \sin(\cos(x)) + \frac{1}{44} \sin(11 \cos(x)) + \frac{1}{52} \sin(13 \cos(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[\text{Cos}[x]]*\text{Sin}[x]*\text{Sin}[6*\text{Cos}[x]]^2,x]$

[Out] $-\text{Sin}[\text{Cos}[x]]/2 + \text{Sin}[11*\text{Cos}[x]]/44 + \text{Sin}[13*\text{Cos}[x]]/52$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$
 $\text{FreeQ}\{c, d\}, x]$

Rule 4335

$\text{Int}[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_.))], x_Symbol] \rightarrow \text{With}\{d = \text{FreeFactors}[\text{Cos}[c*(a + b*x)], x]\}, -\text{Dist}[d/(b*c), \text{Subst}[\text{Int}[\text{SubstFor}[1, \text{Cos}[c*(a + b*x)]/d, u, x], x], x, \text{Cos}[c*(a + b*x)]/d], x] /;$
 $\text{FunctionOfQ}[\text{Cos}[c*(a + b*x)]/d, u, x, \text{True}] /;$
 $\text{FreeQ}\{a, b, c\}, x] \&\& (\text{EqQ}[F, \text{Sin}] \parallel \text{EqQ}[F, \text{sin}])$

Rule 4354

$\text{Int}[(F_)[(a_.) + (b_.)*(x_.)]^{(p_.)}*(G_)[(c_.) + (d_.)*(x_.)]^{(q_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[\text{ActivateTrig}[F[a + b*x]^{p*}G[c + d*x]^{q*}], x], x] /;$
 $\text{FreeQ}\{a, b, c, d\}, x] \&\& (\text{EqQ}[F, \text{sin}] \parallel \text{EqQ}[F, \text{cos}]) \&\& (\text{EqQ}[G, \text{sin}] \parallel \text{EqQ}[G, \text{cos}]) \&\& \text{IGtQ}[p, 0] \&\& \text{IGtQ}[q, 0]$

Rubi steps

$$\begin{aligned} \int \cos(\cos(x)) \sin(x) \sin^2(6 \cos(x)) dx &= -\text{Subst} \left(\int \cos(x) \sin^2(6x) dx, x, \cos(x) \right) \\ &= -\text{Subst} \left(\int \left(\frac{\cos(x)}{2} - \frac{1}{4} \cos(11x) - \frac{1}{4} \cos(13x) \right) dx, x, \cos(x) \right) \\ &= \frac{1}{4} \text{Subst} \left(\int \cos(11x) dx, x, \cos(x) \right) + \frac{1}{4} \text{Subst} \left(\int \cos(13x) dx, x, \cos(x) \right) \\ &= -\frac{1}{2} \sin(\cos(x)) + \frac{1}{44} \sin(11 \cos(x)) + \frac{1}{52} \sin(13 \cos(x)) \end{aligned}$$

Mathematica [A] time = 4.50, size = 26, normalized size = 1.00

$$-\frac{1}{2} \sin(\cos(x)) + \frac{1}{44} \sin(11 \cos(x)) + \frac{1}{52} \sin(13 \cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[Cos[x]]*Sin[x]*Sin[6*Cos[x]]^2,x]

[Out] -1/2*Sin[Cos[x]] + Sin[11*Cos[x]]/44 + Sin[13*Cos[x]]/52

fricas [B] time = 1.92, size = 168, normalized size = 6.46

$$-\frac{4}{143} \left(2816 \cos \left(\frac{\tan \left(\frac{1}{2} x \right)^2 - 1}{\tan \left(\frac{1}{2} x \right)^2 + 1} \right)^{12} - 6912 \cos \left(\frac{\tan \left(\frac{1}{2} x \right)^2 - 1}{\tan \left(\frac{1}{2} x \right)^2 + 1} \right)^{10} + 6048 \cos \left(\frac{\tan \left(\frac{1}{2} x \right)^2 - 1}{\tan \left(\frac{1}{2} x \right)^2 + 1} \right)^8 - 2240 \cos \left(\frac{\tan \left(\frac{1}{2} x \right)^2 - 1}{\tan \left(\frac{1}{2} x \right)^2 + 1} \right)^6 + 315 \cos \left(\frac{\tan \left(\frac{1}{2} x \right)^2 - 1}{\tan \left(\frac{1}{2} x \right)^2 + 1} \right)^4 - 9 \cos \left(\frac{\tan \left(\frac{1}{2} x \right)^2 - 1}{\tan \left(\frac{1}{2} x \right)^2 + 1} \right)^2 - 18 \right) \sin \left(\frac{\tan \left(\frac{1}{2} x \right)^2 - 1}{\tan \left(\frac{1}{2} x \right)^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x, algorithm="fricas")

[Out] -4/143*(2816*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^12 - 6912*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^10 + 6048*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^8 - 2240*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^6 + 315*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^4 - 9*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^2 - 18)*sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))

giac [A] time = 0.15, size = 20, normalized size = 0.77

$$\frac{1}{52} \sin(13 \cos(x)) + \frac{1}{44} \sin(11 \cos(x)) - \frac{1}{2} \sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x, algorithm="giac")

[Out] 1/52*sin(13*cos(x)) + 1/44*sin(11*cos(x)) - 1/2*sin(cos(x))

maple [A] time = 0.16, size = 21, normalized size = 0.81

$$-\frac{\sin(\cos(x))}{2} + \frac{\sin(11 \cos(x))}{44} + \frac{\sin(13 \cos(x))}{52}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x)

[Out] -1/2*sin(cos(x))+1/44*sin(11*cos(x))+1/52*sin(13*cos(x))

maxima [A] time = 0.33, size = 20, normalized size = 0.77

$$\frac{1}{52} \sin(13 \cos(x)) + \frac{1}{44} \sin(11 \cos(x)) - \frac{1}{2} \sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x, algorithm="maxima")

[Out] 1/52*sin(13*cos(x)) + 1/44*sin(11*cos(x)) - 1/2*sin(cos(x))

mupad [B] time = 3.13, size = 20, normalized size = 0.77

$$\frac{\sin(11 \cos(x))}{44} - \frac{\sin(\cos(x))}{2} + \frac{\sin(13 \cos(x))}{52}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(cos(x))*sin(6*cos(x))^2*sin(x),x)

[Out] sin(11*cos(x))/44 - sin(cos(x))/2 + sin(13*cos(x))/52

sympy [B] time = 47.65, size = 54, normalized size = 2.08

$$-\frac{71 \sin(\cos(x)) \sin^2(6 \cos(x))}{143} - \frac{72 \sin(\cos(x)) \cos^2(6 \cos(x))}{143} + \frac{12 \sin(6 \cos(x)) \cos(\cos(x)) \cos(6 \cos(x))}{143}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(cos(x))*sin(x)*sin(6*cos(x))**2,x)

[Out] -71*sin(cos(x))*sin(6*cos(x))**2/143 - 72*sin(cos(x))*cos(6*cos(x))**2/143
+ 12*sin(6*cos(x))*cos(cos(x))*cos(6*cos(x))/143

$$3.654 \quad \int \cos^3(x) \left(a + b \cos^2(x)\right)^3 \sin(x) dx$$

Optimal. Leaf size=36

$$\frac{a(a + b \cos^2(x))^4}{8b^2} - \frac{(a + b \cos^2(x))^5}{10b^2}$$

[Out] 1/8*a*(a+b*cos(x)^2)^4/b^2-1/10*(a+b*cos(x)^2)^5/b^2

Rubi [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4335, 266, 43}

$$\frac{a(a + b \cos^2(x))^4}{8b^2} - \frac{(a + b \cos^2(x))^5}{10b^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^3*(a + b*Cos[x]^2)^3*Sin[x],x]

[Out] (a*(a + b*Cos[x]^2)^4)/(8*b^2) - (a + b*Cos[x]^2)^5/(10*b^2)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4335

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned} \int \cos^3(x) \left(a + b \cos^2(x)\right)^3 \sin(x) dx &= -\text{Subst}\left(\int x^3 (a + bx^2)^3 dx, x, \cos(x)\right) \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int x(a + bx)^3 dx, x, \cos^2(x)\right)\right) \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \left(-\frac{a(a + bx)^3}{b} + \frac{(a + bx)^4}{b}\right) dx, x, \cos^2(x)\right)\right) \\ &= \frac{a(a + b \cos^2(x))^4}{8b^2} - \frac{(a + b \cos^2(x))^5}{10b^2} \end{aligned}$$

Mathematica [B] time = 0.28, size = 137, normalized size = 3.81

$$\frac{1}{32} \left(-4a^3 \cos(2x) - a^3 \cos(4x) - 12a^2b \cos^4(x) - 4a^2b \cos(3x) \cos^3(x) - 8ab^2 \cos^6(x) - \frac{1}{32} ab^2 (48 \cos(2x) + 36 \cos(4x)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^3*(a + b*Cos[x]^2)^3*Sin[x], x]

[Out] $(-12*a^2*b*\text{Cos}[x]^4 - 8*a*b^2*\text{Cos}[x]^6 - 2*b^3*\text{Cos}[x]^8 - 4*a^3*\text{Cos}[2*x] - 4*a^2*b*\text{Cos}[x]^3*\text{Cos}[3*x] - a^3*\text{Cos}[4*x] - (a*b^2*(48*\text{Cos}[2*x] + 36*\text{Cos}[4*x] + 16*\text{Cos}[6*x] + 3*\text{Cos}[8*x]))/32 - (b^3*(140*\text{Cos}[2*x] + 100*\text{Cos}[4*x] + 50*\text{Cos}[6*x] + 15*\text{Cos}[8*x] + 2*\text{Cos}[10*x]))/320)/32$

fricas [A] time = 0.92, size = 39, normalized size = 1.08

$$-\frac{1}{10}b^3 \cos(x)^{10} - \frac{3}{8}ab^2 \cos(x)^8 - \frac{1}{2}a^2b \cos(x)^6 - \frac{1}{4}a^3 \cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3*(a+b*cos(x)^2)^3*sin(x), x, algorithm="fricas")

[Out] $-1/10*b^3*\text{cos}(x)^{10} - 3/8*a*b^2*\text{cos}(x)^8 - 1/2*a^2*b*\text{cos}(x)^6 - 1/4*a^3*\text{cos}(x)^4$

giac [A] time = 0.13, size = 39, normalized size = 1.08

$$-\frac{1}{10}b^3 \cos(x)^{10} - \frac{3}{8}ab^2 \cos(x)^8 - \frac{1}{2}a^2b \cos(x)^6 - \frac{1}{4}a^3 \cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3*(a+b*cos(x)^2)^3*sin(x), x, algorithm="giac")

[Out] $-1/10*b^3*\text{cos}(x)^{10} - 3/8*a*b^2*\text{cos}(x)^8 - 1/2*a^2*b*\text{cos}(x)^6 - 1/4*a^3*\text{cos}(x)^4$

maple [A] time = 0.01, size = 40, normalized size = 1.11

$$-\frac{b^3 (\cos^{10}(x))}{10} - \frac{3ab^2 (\cos^8(x))}{8} - \frac{a^2b (\cos^6(x))}{2} - \frac{a^3 (\cos^4(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^3*(a+b*cos(x)^2)^3*sin(x), x)

[Out] $-1/10*b^3*\text{cos}(x)^{10} - 3/8*a*b^2*\text{cos}(x)^8 - 1/2*a^2*b*\text{cos}(x)^6 - 1/4*a^3*\text{cos}(x)^4$

maxima [B] time = 0.39, size = 103, normalized size = 2.86

$$\frac{1}{10}b^3 \sin(x)^{10} - \frac{1}{8}(3ab^2 + 4b^3) \sin(x)^8 + \frac{1}{2}(a^2b + 3ab^2 + 2b^3) \sin(x)^6 - \frac{1}{4}(a^3 + 6a^2b + 9ab^2 + 4b^3) \sin(x)^4 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3*(a+b*cos(x)^2)^3*sin(x), x, algorithm="maxima")

[Out] $1/10*b^3*\text{sin}(x)^{10} - 1/8*(3*a*b^2 + 4*b^3)*\text{sin}(x)^8 + 1/2*(a^2*b + 3*a*b^2 + 2*b^3)*\text{sin}(x)^6 - 1/4*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*\text{sin}(x)^4 + 1/2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\text{sin}(x)^2$

mupad [B] time = 0.09, size = 39, normalized size = 1.08

$$-\frac{a^3 \cos(x)^4}{4} - \frac{a^2 b \cos(x)^6}{2} - \frac{3 a b^2 \cos(x)^8}{8} - \frac{b^3 \cos(x)^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^3*sin(x)*(a + b*cos(x)^2)^3, x)

[Out] $-(a^3 \cos(x)^4)/4 - (b^3 \cos(x)^{10})/10 - (a^2 b \cos(x)^6)/2 - (3 a b^2 \cos(x)^8)/8$

sympy [A] time = 11.65, size = 46, normalized size = 1.28

$$-\frac{a^3 \cos^4(x)}{4} - \frac{a^2 b \cos^6(x)}{2} - \frac{3 a b^2 \cos^8(x)}{8} - \frac{b^3 \cos^{10}(x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)**3*(a+b*cos(x)**2)**3*sin(x),x)`

[Out] $-a^{**3} \cos(x)^{**4}/4 - a^{**2} b \cos(x)^{**6}/2 - 3 a b^{**2} \cos(x)^{**8}/8 - b^{**3} \cos(x)^{**10}/10$

3.655 $\int \sin(3x) \sin(\cos(3x)) dx$

Optimal. Leaf size=9

$$\frac{1}{3} \cos(\cos(3x))$$

[Out] 1/3*cos(cos(3*x))

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4335, 2638}

$$\frac{1}{3} \cos(\cos(3x))$$

Antiderivative was successfully verified.

[In] Int[Sin[3*x]*Sin[Cos[3*x]],x]

[Out] Cos[Cos[3*x]]/3

Rule 2638

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4335

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_.))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned} \int \sin(3x) \sin(\cos(3x)) dx &= -\left(\frac{1}{3} \text{Subst}\left(\int \sin(x) dx, x, \cos(3x)\right)\right) \\ &= \frac{1}{3} \cos(\cos(3x)) \end{aligned}$$

Mathematica [A] time = 2.68, size = 9, normalized size = 1.00

$$\frac{1}{3} \cos(\cos(3x))$$

Antiderivative was successfully verified.

[In] Integrate[Sin[3*x]*Sin[Cos[3*x]],x]

[Out] Cos[Cos[3*x]]/3

fricas [B] time = 0.95, size = 22, normalized size = 2.44

$$\frac{1}{3} \cos\left(\frac{\tan\left(\frac{3}{2}x\right)^2 - 1}{\tan\left(\frac{3}{2}x\right)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(3*x)*sin(cos(3*x)),x, algorithm="fricas")

[Out] 1/3*cos((tan(3/2*x)^2 - 1)/(tan(3/2*x)^2 + 1))

giac [A] time = 0.16, size = 7, normalized size = 0.78

$$\frac{1}{3} \cos(\cos(3x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(3*x)*sin(cos(3*x)),x, algorithm="giac")

[Out] 1/3*cos(cos(3*x))

maple [A] time = 0.01, size = 8, normalized size = 0.89

$$\frac{\cos(\cos(3x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(3*x)*sin(cos(3*x)),x)

[Out] 1/3*cos(cos(3*x))

maxima [A] time = 0.32, size = 7, normalized size = 0.78

$$\frac{1}{3} \cos(\cos(3x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(3*x)*sin(cos(3*x)),x, algorithm="maxima")

[Out] 1/3*cos(cos(3*x))

mupad [B] time = 3.01, size = 7, normalized size = 0.78

$$\frac{\cos(\cos(3x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(3*x)*sin(cos(3*x)),x)

[Out] cos(cos(3*x))/3

sympy [A] time = 0.42, size = 7, normalized size = 0.78

$$\frac{\cos(\cos(3x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(3*x)*sin(cos(3*x)),x)

[Out] cos(cos(3*x))/3

3.656 $\int e^{\cos(1+3x)} \cos(1+3x) \sin(1+3x) dx$

Optimal. Leaf size=31

$$\frac{1}{3}e^{\cos(3x+1)} - \frac{1}{3}e^{\cos(3x+1)} \cos(3x+1)$$

[Out] 1/3*exp(cos(1+3*x))-1/3*exp(cos(1+3*x))*cos(1+3*x)

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4335, 2176, 2194}

$$\frac{1}{3}e^{\cos(3x+1)} - \frac{1}{3}e^{\cos(3x+1)} \cos(3x+1)$$

Antiderivative was successfully verified.

[In] Int[E^Cos[1 + 3*x]*Cos[1 + 3*x]*Sin[1 + 3*x], x]

[Out] E^Cos[1 + 3*x]/3 - (E^Cos[1 + 3*x]*Cos[1 + 3*x])/3

Rule 2176

Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !\$UseGamma == True

Rule 2194

Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 4335

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned} \int e^{\cos(1+3x)} \cos(1+3x) \sin(1+3x) dx &= -\left(\frac{1}{3} \text{Subst}\left(\int e^x x dx, x, \cos(1+3x)\right)\right) \\ &= -\frac{1}{3}e^{\cos(1+3x)} \cos(1+3x) + \frac{1}{3} \text{Subst}\left(\int e^x dx, x, \cos(1+3x)\right) \\ &= \frac{1}{3}e^{\cos(1+3x)} - \frac{1}{3}e^{\cos(1+3x)} \cos(1+3x) \end{aligned}$$

Mathematica [A] time = 0.12, size = 24, normalized size = 0.77

$$\frac{2}{3} \sin^2\left(\frac{1}{2}(3x+1)\right) e^{\cos(3x+1)}$$

Antiderivative was successfully verified.

[In] Integrate[E^Cos[1 + 3*x]*Cos[1 + 3*x]*Sin[1 + 3*x],x]

[Out] (2*E^Cos[1 + 3*x]*Sin[(1 + 3*x)/2]^2)/3

fricas [A] time = 0.84, size = 17, normalized size = 0.55

$$-\frac{1}{3}(\cos(3x+1)-1)e^{\cos(3x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x, algorithm="fricas")

[Out] -1/3*(cos(3*x + 1) - 1)*e^(cos(3*x + 1))

giac [A] time = 0.15, size = 17, normalized size = 0.55

$$-\frac{1}{3}(\cos(3x+1)-1)e^{\cos(3x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x, algorithm="giac")

[Out] -1/3*(cos(3*x + 1) - 1)*e^(cos(3*x + 1))

maple [A] time = 0.01, size = 26, normalized size = 0.84

$$\frac{e^{\cos(1+3x)}}{3} - \frac{e^{\cos(1+3x)} \cos(1+3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x)

[Out] 1/3*exp(cos(1+3*x))-1/3*exp(cos(1+3*x))*cos(1+3*x)

maxima [A] time = 0.32, size = 17, normalized size = 0.55

$$-\frac{1}{3}(\cos(3x+1)-1)e^{\cos(3x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x, algorithm="maxima")

[Out] -1/3*(cos(3*x + 1) - 1)*e^(cos(3*x + 1))

mupad [B] time = 0.12, size = 17, normalized size = 0.55

$$-\frac{e^{\cos(3x+1)}(\cos(3x+1)-1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(cos(3*x + 1))*cos(3*x + 1)*sin(3*x + 1),x)

[Out] -(exp(cos(3*x + 1))*(cos(3*x + 1) - 1))/3

sympy [A] time = 0.67, size = 26, normalized size = 0.84

$$-\frac{e^{\cos(3x+1)} \cos(3x+1)}{3} + \frac{e^{\cos(3x+1)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x)

[Out] -exp(cos(3*x + 1))*cos(3*x + 1)/3 + exp(cos(3*x + 1))/3

$$3.657 \quad \int \frac{\cos^2(x) \sin(x)}{\sqrt{1-\cos^6(x)}} dx$$

Optimal. Leaf size=9

$$-\frac{1}{3} \sin^{-1}(\cos^3(x))$$

[Out] -1/3*arcsin(cos(x)^3)

Rubi [A] time = 0.07, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4335, 275, 216}

$$-\frac{1}{3} \sin^{-1}(\cos^3(x))$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]^2*Sin[x])/Sqrt[1 - Cos[x]^6],x]

[Out] -ArcSin[Cos[x]^3]/3

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 4335

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(x) \sin(x)}{\sqrt{1-\cos^6(x)}} dx &= -\text{Subst} \left(\int \frac{x^2}{\sqrt{1-x^6}} dx, x, \cos(x) \right) \\ &= -\left(\frac{1}{3} \text{Subst} \left(\int \frac{1}{\sqrt{1-x^2}} dx, x, \cos^3(x) \right) \right) \\ &= -\frac{1}{3} \sin^{-1}(\cos^3(x)) \end{aligned}$$

Mathematica [C] time = 2.22, size = 162, normalized size = 18.00

$$\frac{i \sin(x) \cos^2(x) \sqrt{1 - \frac{2i \tan^2(x)}{\sqrt{3}-3i}} \sqrt{1 + \frac{2i \tan^2(x)}{\sqrt{3}+3i}} \Pi \left(\frac{3}{2} + \frac{i\sqrt{3}}{2}; i \sinh^{-1} \left(\sqrt{\frac{-2i}{-3i+\sqrt{3}}} \tan(x) \right) \Big|_{\frac{3i-\sqrt{3}}{3i+\sqrt{3}}} \right)}{\sqrt{2} \sqrt{-\frac{i}{\sqrt{3}-3i}} \sqrt{1-\cos^6(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]^2*Sin[x])/Sqrt[1 - Cos[x]^6],x]

[Out] ((-1)*Cos[x]^2*EllipticPi[3/2 + (I/2)*Sqrt[3], I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[3])]]*Tan[x]], (3*I - Sqrt[3])/(3*I + Sqrt[3]))*Sin[x]*Sqrt[1 - ((2*I)*Tan[x]^2)/(-3*I + Sqrt[3])]*Sqrt[1 + ((2*I)*Tan[x]^2)/(3*I + Sqrt[3])]/(Sqrt[2]*Sqrt[(-I)/(-3*I + Sqrt[3])]*Sqrt[1 - Cos[x]^6])

fricas [B] time = 1.11, size = 29, normalized size = 3.22

$$\frac{1}{6} \arctan\left(\frac{2\sqrt{-\cos(x)^6 + 1} \cos(x)^3}{2\cos(x)^6 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x, algorithm="fricas")

[Out] 1/6*arctan(2*sqrt(-cos(x)^6 + 1)*cos(x)^3/(2*cos(x)^6 - 1))

giac [A] time = 0.15, size = 7, normalized size = 0.78

$$-\frac{1}{3} \arcsin(\cos(x)^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x, algorithm="giac")

[Out] -1/3*arcsin(cos(x)^3)

maple [F] time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{(\cos^2(x)) \sin(x)}{\sqrt{1 - (\cos^6(x))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x)

[Out] int(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x)

maxima [B] time = 0.60, size = 18, normalized size = 2.00

$$\frac{1}{3} \arctan\left(\frac{\sqrt{-\cos(x)^6 + 1}}{\cos(x)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x, algorithm="maxima")

[Out] 1/3*arctan(sqrt(-cos(x)^6 + 1)/cos(x)^3)

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{\cos(x)^2 \sin(x)}{\sqrt{1 - \cos(x)^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x)^2*sin(x))/(1 - cos(x)^6)^(1/2),x)

[Out] int((cos(x)^2*sin(x))/(1 - cos(x)^6)^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**2*sin(x)/(1-cos(x)**6)**(1/2),x)

[Out] Timed out

$$3.658 \quad \int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}} dx$$

Optimal. Leaf size=71

$$\frac{2(1-5\cos(x))^{9/2}}{28125} - \frac{8(1-5\cos(x))^{7/2}}{21875} - \frac{88(1-5\cos(x))^{5/2}}{15625} + \frac{64(1-5\cos(x))^{3/2}}{3125} + \frac{1152\sqrt{1-5\cos(x)}}{3125}$$

[Out] $64/3125*(1-5*\cos(x))^{(3/2)}-88/15625*(1-5*\cos(x))^{(5/2)}-8/21875*(1-5*\cos(x))^{(7/2)}+2/28125*(1-5*\cos(x))^{(9/2)}+1152/3125*(1-5*\cos(x))^{(1/2)}$

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2668, 697}

$$\frac{2(1-5\cos(x))^{9/2}}{28125} - \frac{8(1-5\cos(x))^{7/2}}{21875} - \frac{88(1-5\cos(x))^{5/2}}{15625} + \frac{64(1-5\cos(x))^{3/2}}{3125} + \frac{1152\sqrt{1-5\cos(x)}}{3125}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^5/Sqrt[1 - 5*Cos[x]], x]

[Out] $(1152*\text{Sqrt}[1 - 5*\text{Cos}[x]])/3125 + (64*(1 - 5*\text{Cos}[x])^{(3/2)})/3125 - (88*(1 - 5*\text{Cos}[x])^{(5/2)})/15625 - (8*(1 - 5*\text{Cos}[x])^{(7/2)})/21875 + (2*(1 - 5*\text{Cos}[x])^{(9/2)})/28125$

Rule 697

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rule 2668

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}} dx &= \frac{\text{Subst}\left(\int \frac{(25-x^2)^2}{\sqrt{1+x}} dx, x, -5\cos(x)\right)}{3125} \\ &= \frac{\text{Subst}\left(\int \left(\frac{576}{\sqrt{1+x}} + 96\sqrt{1+x} - 44(1+x)^{3/2} - 4(1+x)^{5/2} + (1+x)^{7/2}\right) dx, x, -5\cos(x)\right)}{3125} \\ &= \frac{1152\sqrt{1-5\cos(x)}}{3125} + \frac{64(1-5\cos(x))^{3/2}}{3125} - \frac{88(1-5\cos(x))^{5/2}}{15625} - \frac{8(1-5\cos(x))^{7/2}}{21875} + \frac{2(1-5\cos(x))^{9/2}}{28125} \end{aligned}$$

Mathematica [A] time = 0.16, size = 59, normalized size = 0.83

$$\frac{180607(\sqrt{1-5\cos(x)} - 1)}{562500} + \sqrt{1-5\cos(x)} \left(-\frac{6772\cos(x)}{196875} - \frac{2227\cos(2x)}{39375} + \frac{4\cos(3x)}{1575} + \frac{1}{180}\cos(4x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^5/Sqrt[1 - 5*Cos[x]],x]

[Out] (180607*(-1 + Sqrt[1 - 5*Cos[x]]))/562500 + Sqrt[1 - 5*Cos[x]]*((-6772*Cos[x])/196875 - (2227*Cos[2*x])/39375 + (4*Cos[3*x])/1575 + Cos[4*x]/180)

fricas [A] time = 1.68, size = 34, normalized size = 0.48

$$\frac{2}{984375} (21875 \cos(x)^4 + 5000 \cos(x)^3 - 77550 \cos(x)^2 - 20680 \cos(x) + 188603) \sqrt{-5 \cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^5/(1-5*cos(x))^(1/2),x, algorithm="fricas")

[Out] 2/984375*(21875*cos(x)^4 + 5000*cos(x)^3 - 77550*cos(x)^2 - 20680*cos(x) + 188603)*sqrt(-5*cos(x) + 1)

giac [A] time = 0.14, size = 75, normalized size = 1.06

$$\frac{2}{28125} (5 \cos(x) - 1)^4 \sqrt{-5 \cos(x) + 1} + \frac{8}{21875} (5 \cos(x) - 1)^3 \sqrt{-5 \cos(x) + 1} - \frac{88}{15625} (5 \cos(x) - 1)^2 \sqrt{-5 \cos(x) + 1} + \frac{64}{3125} (5 \cos(x) - 1) \sqrt{-5 \cos(x) + 1} + 1152 \sqrt{-5 \cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^5/(1-5*cos(x))^(1/2),x, algorithm="giac")

[Out] 2/28125*(5*cos(x) - 1)^4*sqrt(-5*cos(x) + 1) + 8/21875*(5*cos(x) - 1)^3*sqrt(-5*cos(x) + 1) - 88/15625*(5*cos(x) - 1)^2*sqrt(-5*cos(x) + 1) + 64/3125*(5*cos(x) - 1)*sqrt(-5*cos(x) + 1) + 1152/3125*sqrt(-5*cos(x) + 1)

maple [A] time = 0.17, size = 49, normalized size = 0.69

$$\frac{32 \sqrt{10 \left(\sin^2 \left(\frac{x}{2} \right) \right) - 4} \left(21875 \left(\sin^8 \left(\frac{x}{2} \right) \right) - 46250 \left(\sin^6 \left(\frac{x}{2} \right) \right) + 17175 \left(\sin^4 \left(\frac{x}{2} \right) \right) + 9160 \left(\sin^2 \left(\frac{x}{2} \right) \right) + 7328 \right)}{984375}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^5/(1-5*cos(x))^(1/2),x)

[Out] 32/984375*(10*sin(1/2*x)^2-4)^(1/2)*(21875*sin(1/2*x)^8-46250*sin(1/2*x)^6+17175*sin(1/2*x)^4+9160*sin(1/2*x)^2+7328)

maxima [A] time = 0.32, size = 51, normalized size = 0.72

$$\frac{2}{28125} (-5 \cos(x) + 1)^{\frac{9}{2}} - \frac{8}{21875} (-5 \cos(x) + 1)^{\frac{7}{2}} - \frac{88}{15625} (-5 \cos(x) + 1)^{\frac{5}{2}} + \frac{64}{3125} (-5 \cos(x) + 1)^{\frac{3}{2}} + \frac{1152}{3125} \sqrt{-5 \cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^5/(1-5*cos(x))^(1/2),x, algorithm="maxima")

[Out] 2/28125*(-5*cos(x) + 1)^(9/2) - 8/21875*(-5*cos(x) + 1)^(7/2) - 88/15625*(-5*cos(x) + 1)^(5/2) + 64/3125*(-5*cos(x) + 1)^(3/2) + 1152/3125*sqrt(-5*cos(x) + 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(x)^5}{\sqrt{1 - 5 \cos(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^5/(1 - 5*cos(x))^(1/2),x)

```
[Out] int(sin(x)^5/(1 - 5*cos(x))^(1/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5/(1-5*cos(x))**(1/2),x)
```

```
[Out] Timed out
```

3.659 $\int e^{n \cos(a+bx)} \sin(a + bx) dx$

Optimal. Leaf size=18

$$-\frac{e^{n \cos(a+bx)}}{bn}$$

[Out] -exp(n*cos(b*x+a))/b/n

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4335, 2194}

$$-\frac{e^{n \cos(a+bx)}}{bn}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Cos[a + b*x])*Sin[a + b*x],x]

[Out] -(E^(n*Cos[a + b*x]))/(b*n))

Rule 2194

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 4335

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned} \int e^{n \cos(a+bx)} \sin(a + bx) dx &= -\frac{\text{Subst}\left(\int e^{nx} dx, x, \cos(a + bx)\right)}{b} \\ &= -\frac{e^{n \cos(a+bx)}}{bn} \end{aligned}$$

Mathematica [A] time = 0.05, size = 18, normalized size = 1.00

$$-\frac{e^{n \cos(a+bx)}}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*Cos[a + b*x])*Sin[a + b*x],x]

[Out] -(E^(n*Cos[a + b*x]))/(b*n))

fricas [A] time = 1.04, size = 17, normalized size = 0.94

$$-\frac{e^{(n \cos(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*x+a))*sin(b*x+a),x, algorithm="fricas")

[Out] $-e^{(n \cos(bx + a))}/(b \cdot n)$

giac [A] time = 0.14, size = 17, normalized size = 0.94

$$-\frac{e^{(n \cos(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*cos(b*x+a))*sin(b*x+a),x, algorithm="giac")`

[Out] $-e^{(n \cos(bx + a))}/(b \cdot n)$

maple [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{e^{n \cos(bx+a)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(n*cos(b*x+a))*sin(b*x+a),x)`

[Out] $-\exp(n \cos(bx+a))/b/n$

maxima [A] time = 0.31, size = 17, normalized size = 0.94

$$-\frac{e^{(n \cos(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*cos(b*x+a))*sin(b*x+a),x, algorithm="maxima")`

[Out] $-e^{(n \cos(bx + a))}/(b \cdot n)$

mupad [B] time = 0.10, size = 17, normalized size = 0.94

$$-\frac{e^{n \cos(a+bx)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(n*cos(a + b*x))*sin(a + b*x),x)`

[Out] $-\exp(n \cos(a + bx))/(b \cdot n)$

sympy [A] time = 0.64, size = 39, normalized size = 2.17

$$\begin{cases} x \sin(a) & \text{for } b = 0 \wedge n = 0 \\ x e^{n \cos(a)} \sin(a) & \text{for } b = 0 \\ -\frac{\cos(a+bx)}{b} & \text{for } n = 0 \\ -\frac{e^{n \cos(a+bx)}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*cos(b*x+a))*sin(b*x+a),x)`

[Out] `Piecewise((x*sin(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*cos(a))*sin(a), Eq(b, 0)), (-cos(a + b*x)/b, Eq(n, 0)), (-exp(n*cos(a + b*x))/(b*n), True))`

$$3.660 \quad \int e^{n \cos(ac+bcx)} \sin(c(a+bx)) dx$$

Optimal. Leaf size=23

$$-\frac{e^{n \cos(c(a+bx))}}{bcn}$$

[Out] `-exp(n*cos(c*(b*x+a)))/b/c/n`

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4335, 2194}

$$-\frac{e^{n \cos(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

[In] `Int[E^(n*Cos[a*c + b*c*x])*Sin[c*(a + b*x)],x]`

[Out] `-(E^(n*Cos[c*(a + b*x)])/(b*c*n))`

Rule 2194

`Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]`

Rule 4335

`Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])`

Rubi steps

$$\begin{aligned} \int e^{n \cos(ac+bcx)} \sin(c(a+bx)) dx &= -\frac{\text{Subst}\left(\int e^{nx} dx, x, \cos(c(a+bx))\right)}{bc} \\ &= -\frac{e^{n \cos(c(a+bx))}}{bcn} \end{aligned}$$

Mathematica [A] time = 0.23, size = 23, normalized size = 1.00

$$-\frac{e^{n \cos(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

[In] `Integrate[E^(n*Cos[a*c + b*c*x])*Sin[c*(a + b*x)],x]`

[Out] `-(E^(n*Cos[c*(a + b*x)])/(b*c*n))`

fricas [A] time = 1.40, size = 23, normalized size = 1.00

$$-\frac{e^{(n \cos(bc x+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x, algorithm="fricas")`

[Out] $-e^{(n \cos(bc x + a c))} / (b c n)$

giac [A] time = 0.15, size = 23, normalized size = 1.00

$$-\frac{e^{(n \cos(bc x + a c))}}{bc n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x, algorithm="giac")`

[Out] $-e^{(n \cos(bc x + a c))} / (b c n)$

maple [A] time = 0.06, size = 24, normalized size = 1.04

$$-\frac{e^{n \cos(bc x + a c)}}{bc n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x)`

[Out] $-\exp(n \cos(bc x + a c)) / b c n$

maxima [A] time = 0.37, size = 23, normalized size = 1.00

$$-\frac{e^{(n \cos(bc x + a c))}}{bc n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x, algorithm="maxima")`

[Out] $-e^{(n \cos(bc x + a c))} / (b c n)$

mupad [B] time = 3.16, size = 23, normalized size = 1.00

$$-\frac{e^{n \cos(a c + b c x)}}{b c n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(c*(a + b*x))*exp(n*cos(a*c + b*c*x)),x)`

[Out] $-\exp(n \cos(a c + b c x)) / (b c n)$

sympy [A] time = 9.84, size = 54, normalized size = 2.35

$$\begin{cases} x e^{n \cos(ac)} \sin(ac) & \text{for } b = 0 \\ 0 & \text{for } c = 0 \\ \begin{cases} x \sin(ac) & \text{for } b = 0 \\ 0 & \text{for } c = 0 \end{cases} & \text{for } n = 0 \\ -\frac{\cos(ac+bcx)}{bc} & \text{otherwise} \\ -\frac{e^{n \cos(ac+bcx)}}{bc n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x)`

[Out] `Piecewise((x*exp(n*cos(a*c))*sin(a*c), Eq(b, 0)), (0, Eq(c, 0)), (Piecewise((x*sin(a*c), Eq(b, 0)), (0, Eq(c, 0)), (-cos(a*c + b*c*x)/(b*c), True)), Eq(n, 0)), (-exp(n*cos(a*c + b*c*x))/(b*c*n), True))`

3.661 $\int e^{n \cos(c(a+bx))} \sin(ac + bcx) dx$

Optimal. Leaf size=24

$$-\frac{e^{n \cos(ac+bcx)}}{bcn}$$

[Out] $-\exp(n \cos(b \cdot c \cdot x + a \cdot c)) / b / c / n$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4335, 2194}

$$-\frac{e^{n \cos(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(n \cdot \text{Cos}[c \cdot (a + b \cdot x)])} \cdot \text{Sin}[a \cdot c + b \cdot c \cdot x], x]$

[Out] $-(E^{(n \cdot \text{Cos}[a \cdot c + b \cdot c \cdot x])}) / (b \cdot c \cdot n)$

Rule 2194

$\text{Int}[(F_)^{((c_) \cdot ((a_) + (b_) \cdot (x_)))}^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(F^{(c \cdot (a + b \cdot x))})^n / (b \cdot c \cdot n \cdot \text{Log}[F]), x] /;$ FreeQ[{F, a, b, c, n}, x]

Rule 4335

$\text{Int}[(u_) \cdot (F_) [(c_) \cdot ((a_) + (b_) \cdot (x_))], x_Symbol] \rightarrow \text{With}[\{d = \text{FreeFactors}[\text{Cos}[c \cdot (a + b \cdot x)], x]\}, -\text{Dist}[d / (b \cdot c), \text{Subst}[\text{Int}[\text{SubstFor}[1, \text{Cos}[c \cdot (a + b \cdot x)] / d, u, x], x], x, \text{Cos}[c \cdot (a + b \cdot x)] / d], x] /;$ FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned} \int e^{n \cos(c(a+bx))} \sin(ac + bcx) dx &= -\frac{\text{Subst}\left(\int e^{nx} dx, x, \cos(ac + bcx)\right)}{bc} \\ &= -\frac{e^{n \cos(ac+bcx)}}{bcn} \end{aligned}$$

Mathematica [A] time = 0.04, size = 23, normalized size = 0.96

$$-\frac{e^{n \cos(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[E^{(n \cdot \text{Cos}[c \cdot (a + b \cdot x)])} \cdot \text{Sin}[a \cdot c + b \cdot c \cdot x], x]$

[Out] $-(E^{(n \cdot \text{Cos}[c \cdot (a + b \cdot x)])}) / (b \cdot c \cdot n)$

fricas [A] time = 0.90, size = 23, normalized size = 0.96

$$-\frac{e^{(n \cos(bc x + ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\exp(n \cdot \cos(c \cdot (b \cdot x + a))) \cdot \sin(b \cdot c \cdot x + a \cdot c), x, \text{algorithm}="fricas")$

[Out] $-e^{(n \cos(b \cdot c \cdot x + a \cdot c))} / (b \cdot c \cdot n)$

giac [A] time = 0.15, size = 23, normalized size = 0.96

$$-\frac{e^{(n \cos(bc x + ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x, algorithm="giac")`

[Out] $-e^{(n \cos(b \cdot c \cdot x + a \cdot c))} / (b \cdot c \cdot n)$

maple [A] time = 0.03, size = 24, normalized size = 1.00

$$-\frac{e^{n \cos(bc x + ac)}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x)`

[Out] $-\exp(n \cos(b \cdot c \cdot x + a \cdot c)) / b / c / n$

maxima [A] time = 0.32, size = 23, normalized size = 0.96

$$-\frac{e^{(n \cos(bc x + ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x, algorithm="maxima")`

[Out] $-e^{(n \cos(b \cdot c \cdot x + a \cdot c))} / (b \cdot c \cdot n)$

mupad [B] time = 3.02, size = 23, normalized size = 0.96

$$-\frac{e^{n \cos(ac + bc x)}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(n*cos(c*(a + b*x)))*sin(a*c + b*c*x),x)`

[Out] $-\exp(n \cos(a \cdot c + b \cdot c \cdot x)) / (b \cdot c \cdot n)$

sympy [A] time = 2.23, size = 54, normalized size = 2.25

$$\left\{ \begin{array}{ll} 0 & \text{for } c = 0 \wedge (b = 0 \vee c = 0) \wedge (c = 0 \vee n = 0) \\ x e^{n \cos(ac)} \sin(ac) & \text{for } b = 0 \\ -\frac{\cos(ac + bcx)}{bc} & \text{for } n = 0 \\ -\frac{e^{n \cos(ac + bcx)}}{bcn} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x)`

[Out] `Piecewise((0, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0)) & (Eq(c, 0) | Eq(n, 0))), (x * exp(n*cos(a*c))*sin(a*c), Eq(b, 0)), (-cos(a*c + b*c*x)/(b*c), Eq(n, 0)), (-exp(n*cos(a*c + b*c*x))/(b*c*n), True))`

3.662 $\int e^{n \cos(a+bx)} \tan(a + bx) dx$

Optimal. Leaf size=14

$$-\frac{\text{Ei}(n \cos(a + bx))}{b}$$

[Out] -Ei(n*cos(b*x+a))/b

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4339, 2178}

$$-\frac{\text{Ei}(n \cos(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Cos[a + b*x])*Tan[a + b*x], x]

[Out] -(ExpIntegralEi[n*Cos[a + b*x]]/b)

Rule 2178

Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 4339

Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])

Rubi steps

$$\begin{aligned} \int e^{n \cos(a+bx)} \tan(a + bx) dx &= -\frac{\text{Subst}\left(\int \frac{e^{nx}}{x} dx, x, \cos(a + bx)\right)}{b} \\ &= -\frac{\text{Ei}(n \cos(a + bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.04, size = 14, normalized size = 1.00

$$-\frac{\text{Ei}(n \cos(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*Cos[a + b*x])*Tan[a + b*x], x]

[Out] -(ExpIntegralEi[n*Cos[a + b*x]]/b)

fricas [A] time = 1.49, size = 14, normalized size = 1.00

$$-\frac{\text{Ei}(n \cos(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*x+a))*tan(b*x+a),x, algorithm="fricas")

[Out] -Ei(n*cos(b*x + a))/b

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \cos(bx+a)} \tan(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*x+a))*tan(b*x+a),x, algorithm="giac")

[Out] integrate(e^(n*cos(b*x + a))*tan(b*x + a), x)

maple [A] time = 0.03, size = 16, normalized size = 1.14

$$\frac{\text{Ei}(1, -n \cos(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*cos(b*x+a))*tan(b*x+a),x)

[Out] 1/b*Ei(1, -n*cos(b*x+a))

maxima [A] time = 0.37, size = 14, normalized size = 1.00

$$-\frac{\text{Ei}(n \cos(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*x+a))*tan(b*x+a),x, algorithm="maxima")

[Out] -Ei(n*cos(b*x + a))/b

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int e^{n \cos(a+bx)} \tan(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*cos(a + b*x))*tan(a + b*x),x)

[Out] int(exp(n*cos(a + b*x))*tan(a + b*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \cos(a+bx)} \tan(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*x+a))*tan(b*x+a),x)

[Out] Integral(exp(n*cos(a + b*x))*tan(a + b*x), x)

3.663 $\int e^{n \cos(ac+bcx)} \tan(c(a + bx)) dx$

Optimal. Leaf size=19

$$-\frac{\text{Ei}(n \cos(c(a + bx)))}{bc}$$

[Out] -Ei(n*cos(c*(b*x+a)))/b/c

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4339, 2178}

$$-\frac{\text{Ei}(n \cos(c(a + bx)))}{bc}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Cos[a*c + b*c*x])*Tan[c*(a + b*x)],x]

[Out] -(ExpIntegralEi[n*Cos[c*(a + b*x)]]/(b*c))

Rule 2178

Int[(F_)^((g_)*(e_) + (f_)*(x_))/((c_) + (d_)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 4339

Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])

Rubi steps

$$\begin{aligned} \int e^{n \cos(ac+bcx)} \tan(c(a + bx)) dx &= -\frac{\text{Subst}\left(\int \frac{e^{nx}}{x} dx, x, \cos(c(a + bx))\right)}{bc} \\ &= -\frac{\text{Ei}(n \cos(c(a + bx)))}{bc} \end{aligned}$$

Mathematica [A] time = 0.06, size = 19, normalized size = 1.00

$$-\frac{\text{Ei}(n \cos(c(a + bx)))}{bc}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*Cos[a*c + b*c*x])*Tan[c*(a + b*x)],x]

[Out] -(ExpIntegralEi[n*Cos[c*(a + b*x)]]/(b*c))

fricas [A] time = 0.89, size = 20, normalized size = 1.05

$$-\frac{\text{Ei}(n \cos(bcx + ac))}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x, algorithm="fricas")

[Out] -Ei(n*cos(b*c*x + a*c))/(b*c)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \cos(bc x + ac)} \tan((bx + a)c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x, algorithm="giac")

[Out] integrate(e^(n*cos(b*c*x + a*c))*tan((b*x + a)*c), x)

maple [A] time = 0.07, size = 22, normalized size = 1.16

$$\frac{\text{Ei}(1, -n \cos(bc x + ac))}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x)

[Out] 1/c/b*Ei(1,-n*cos(b*c*x+a*c))

maxima [A] time = 0.38, size = 20, normalized size = 1.05

$$\frac{\text{Ei}(n \cos(bc x + ac))}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x, algorithm="maxima")

[Out] -Ei(n*cos(b*c*x + a*c))/(b*c)

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \tan(c(a + bx)) e^{n \cos(ac + bcx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c*(a + b*x))*exp(n*cos(a*c + b*c*x)),x)

[Out] int(tan(c*(a + b*x))*exp(n*cos(a*c + b*c*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \cos(ac + bcx)} \tan(ac + bcx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x)

[Out] Integral(exp(n*cos(a*c + b*c*x))*tan(a*c + b*c*x), x)

3.664 $\int e^{n \cos(c(a+bx))} \tan(ac + bcx) dx$

Optimal. Leaf size=20

$$-\frac{\text{Ei}(n \cos(ac + bcx))}{bc}$$

[Out] -Ei(n*cos(b*c*x+a*c))/b/c

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4339, 2178}

$$-\frac{\text{Ei}(n \cos(ac + bcx))}{bc}$$

Antiderivative was successfully verified.

[In] Int[E^(n*cos[c*(a + b*x)])*Tan[a*c + b*c*x], x]

[Out] -(ExpIntegralEi[n*cos[a*c + b*c*x]]/(b*c))

Rule 2178

Int[(F_)^((g_)*(e_) + (f_)*(x_))/((c_) + (d_)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 4339

Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])

Rubi steps

$$\begin{aligned} \int e^{n \cos(c(a+bx))} \tan(ac + bcx) dx &= -\frac{\text{Subst}\left(\int \frac{e^{nx}}{x} dx, x, \cos(ac + bcx)\right)}{bc} \\ &= -\frac{\text{Ei}(n \cos(ac + bcx))}{bc} \end{aligned}$$

Mathematica [A] time = 0.06, size = 19, normalized size = 0.95

$$-\frac{\text{Ei}(n \cos(c(a + bx)))}{bc}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*cos[c*(a + b*x)])*Tan[a*c + b*c*x], x]

[Out] -(ExpIntegralEi[n*cos[c*(a + b*x)]]/(b*c))

fricas [A] time = 0.77, size = 20, normalized size = 1.00

$$-\frac{\text{Ei}(n \cos(bcx + ac))}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x, algorithm="fricas")

[Out] -Ei(n*cos(b*c*x + a*c))/(b*c)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \cos((bx+a)c)} \tan(bc x + ac) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x, algorithm="giac")

[Out] integrate(e^(n*cos((b*x + a)*c))*tan(b*c*x + a*c), x)

maple [A] time = 0.05, size = 22, normalized size = 1.10

$$\frac{\text{Ei}(1, -n \cos(bc x + ac))}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x)

[Out] 1/c/b*Ei(1,-n*cos(b*c*x+a*c))

maxima [A] time = 0.44, size = 20, normalized size = 1.00

$$\frac{\text{Ei}(n \cos(bc x + ac))}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x, algorithm="maxima")

[Out] -Ei(n*cos(b*c*x + a*c))/(b*c)

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int e^{n \cos(c(a+bx))} \tan(ac + bc x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*cos(c*(a + b*x)))*tan(a*c + b*c*x),x)

[Out] int(exp(n*cos(c*(a + b*x)))*tan(a*c + b*c*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \cos(ac+bcx)} \tan(ac + bcx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x)

[Out] Integral(exp(n*cos(a*c + b*c*x))*tan(a*c + b*c*x), x)

$$3.665 \quad \int \frac{\cos(x)}{a+b \sin(x)} dx$$

Optimal. Leaf size=11

$$\frac{\log(a + b \sin(x))}{b}$$

[Out] ln(a+b*sin(x))/b

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2668, 31}

$$\frac{\log(a + b \sin(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/(a + b*Sin[x]),x]

[Out] Log[a + b*Sin[x]]/b

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2668

Int[cos[(e_.) + (f_.)*(x_)]^{(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b² - x²)^{(p - 1)/2}, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a² - b², 0]}

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{a + b \sin(x)} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x} dx, x, b \sin(x)\right)}{b} \\ &= \frac{\log(a + b \sin(x))}{b} \end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{\log(a + b \sin(x))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/(a + b*Sin[x]),x]

[Out] Log[a + b*Sin[x]]/b

fricas [A] time = 1.97, size = 11, normalized size = 1.00

$$\frac{\log(b \sin(x) + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(a+b*sin(x)),x, algorithm="fricas")

[Out] log(b*sin(x) + a)/b

giac [A] time = 0.14, size = 12, normalized size = 1.09

$$\frac{\log(|b \sin(x) + a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(a+b*sin(x)),x, algorithm="giac")

[Out] log(abs(b*sin(x) + a))/b

maple [A] time = 0.03, size = 12, normalized size = 1.09

$$\frac{\ln(a + b \sin(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(a+b*sin(x)),x)

[Out] ln(a+b*sin(x))/b

maxima [A] time = 0.32, size = 11, normalized size = 1.00

$$\frac{\log(b \sin(x) + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(a+b*sin(x)),x, algorithm="maxima")

[Out] log(b*sin(x) + a)/b

mupad [B] time = 0.03, size = 11, normalized size = 1.00

$$\frac{\ln(a + b \sin(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(a + b*sin(x)),x)

[Out] log(a + b*sin(x))/b

sympy [A] time = 0.32, size = 14, normalized size = 1.27

$$\begin{cases} \frac{\log\left(\frac{a}{b} + \sin(x)\right)}{b} & \text{for } b \neq 0 \\ \frac{\sin(x)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(a+b*sin(x)),x)

[Out] Piecewise((log(a/b + sin(x))/b, Ne(b, 0)), (sin(x)/a, True))

3.666 $\int \cos(x)(a + b \sin(x))^n dx$

Optimal. Leaf size=19

$$\frac{(a + b \sin(x))^{n+1}}{b(n+1)}$$

[Out] (a+b*sin(x))^(1+n)/b/(1+n)

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2668, 32}

$$\frac{(a + b \sin(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*(a + b*Sin[x])^n,x]

[Out] (a + b*Sin[x])^(1 + n)/(b*(1 + n))

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2668

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \cos(x)(a + b \sin(x))^n dx &= \frac{\text{Subst}\left(\int (a + x)^n dx, x, b \sin(x)\right)}{b} \\ &= \frac{(a + b \sin(x))^{1+n}}{b(1+n)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 0.95

$$\frac{(a + b \sin(x))^{n+1}}{bn + b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*(a + b*Sin[x])^n,x]

[Out] (a + b*Sin[x])^(1 + n)/(b + b*n)

fricas [A] time = 0.57, size = 22, normalized size = 1.16

$$\frac{(b \sin(x) + a)(b \sin(x) + a)^n}{bn + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(a+b*sin(x))^n,x, algorithm="fricas")

[Out] $(b \sin(x) + a) \cdot (b \sin(x) + a)^n / (b \cdot n + b)$

giac [A] time = 0.15, size = 19, normalized size = 1.00

$$\frac{(b \sin(x) + a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(a+b*sin(x))^n,x, algorithm="giac")`

[Out] $(b \sin(x) + a)^{n+1} / (b \cdot (n + 1))$

maple [A] time = 0.03, size = 20, normalized size = 1.05

$$\frac{(a + b \sin(x))^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*(a+b*sin(x))^n,x)`

[Out] $(a+b \sin(x))^{n+1} / b / (n+1)$

maxima [A] time = 0.32, size = 19, normalized size = 1.00

$$\frac{(b \sin(x) + a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(a+b*sin(x))^n,x, algorithm="maxima")`

[Out] $(b \sin(x) + a)^{n+1} / (b \cdot (n + 1))$

mupad [B] time = 3.13, size = 19, normalized size = 1.00

$$\frac{(a + b \sin(x))^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*(a + b*sin(x))^n,x)`

[Out] $(a + b \sin(x))^{n+1} / (b \cdot (n + 1))$

sympy [A] time = 1.88, size = 56, normalized size = 2.95

$$\begin{cases} \frac{\sin(x)}{a} & \text{for } b = 0 \wedge n = -1 \\ a^n \sin(x) & \text{for } b = 0 \\ \frac{\log\left(\frac{a}{b} + \sin(x)\right)}{b} & \text{for } n = -1 \\ \frac{a(a+b \sin(x))^n}{bn+b} + \frac{b(a+b \sin(x))^n \sin(x)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(a+b*sin(x))**n,x)`

[Out] `Piecewise((sin(x)/a, Eq(b, 0) & Eq(n, -1)), (a**n*sin(x), Eq(b, 0)), (log(a/b + sin(x))/b, Eq(n, -1)), (a*(a + b*sin(x))**n/(b*n + b) + b*(a + b*sin(x))**n*sin(x)/(b*n + b), True))`

$$3.667 \quad \int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}} dx$$

Optimal. Leaf size=3

$$\sinh^{-1}(\sin(x))$$

[Out] arcsinh(sin(x))

Rubi [A] time = 0.02, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3190, 215}

$$\sinh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/Sqrt[1 + Sin[x]^2], x]

[Out] ArcSinh[Sin[x]]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}} dx = \text{Subst} \left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \sin(x) \right) \\ = \sinh^{-1}(\sin(x))$$

Mathematica [A] time = 0.01, size = 3, normalized size = 1.00

$$\sinh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/Sqrt[1 + Sin[x]^2], x]

[Out] ArcSinh[Sin[x]]

fricas [B] time = 1.19, size = 39, normalized size = 13.00

$$\frac{1}{4} \log \left(8 \cos(x)^4 - 4 \left(2 \cos(x)^2 - 3 \right) \sqrt{-\cos(x)^2 + 2 \sin(x) - 24 \cos(x)^2 + 17} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(1+sin(x)^2)^(1/2), x, algorithm="fricas")

[Out] $\frac{1}{4} \log(8 \cos(x)^4 - 4(2 \cos(x)^2 - 3) \sqrt{-\cos(x)^2 + 2} \sin(x) - 24 \cos(x)^2 + 17)$

giac [B] time = 0.13, size = 16, normalized size = 5.33

$$-\log\left(\sqrt{\sin(x)^2 + 1} - \sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(1+sin(x)^2)^(1/2),x, algorithm="giac")`

[Out] `-log(sqrt(sin(x)^2 + 1) - sin(x))`

maple [A] time = 0.06, size = 4, normalized size = 1.33

$$\operatorname{arcsinh}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)/(1+sin(x)^2)^(1/2),x)`

[Out] `arcsinh(sin(x))`

maxima [A] time = 0.41, size = 3, normalized size = 1.00

$$\operatorname{arsinh}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(1+sin(x)^2)^(1/2),x, algorithm="maxima")`

[Out] `arcsinh(sin(x))`

mupad [B] time = 0.02, size = 9, normalized size = 3.00

$$-\operatorname{asin}(\sin(x)1i)1i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)/(sin(x)^2 + 1)^(1/2),x)`

[Out] `-asin(sin(x)*1i)*1i`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sqrt{\sin^2(x) + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(1+sin(x)**2)**(1/2),x)`

[Out] `Integral(cos(x)/sqrt(sin(x)**2 + 1), x)`

$$3.668 \quad \int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}} dx$$

Optimal. Leaf size=7

$$\sin^{-1}\left(\frac{\sin(x)}{2}\right)$$

[Out] arcsin(1/2*sin(x))

Rubi [A] time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3190, 216}

$$\sin^{-1}\left(\frac{\sin(x)}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/Sqrt[4 - Sin[x]^2], x]

[Out] ArcSin[Sin[x]/2]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}} dx &= \text{Subst}\left(\int \frac{1}{\sqrt{4-x^2}} dx, x, \sin(x)\right) \\ &= \sin^{-1}\left(\frac{\sin(x)}{2}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 7, normalized size = 1.00

$$\sin^{-1}\left(\frac{\sin(x)}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/Sqrt[4 - Sin[x]^2], x]

[Out] ArcSin[Sin[x]/2]

fricas [B] time = 3.02, size = 53, normalized size = 7.57

$$\frac{1}{2} \arctan\left(\frac{\sqrt{\cos(x)^2 + 3}(\cos(x)^2 + 1)\sin(x) - 4\cos(x)\sin(x)}{\cos(x)^4 + 6\cos(x)^2 - 3}\right) + \frac{1}{2} \arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(4-sin(x)^2)^(1/2),x, algorithm="fricas")

[Out] 1/2*arctan((sqrt(cos(x)^2 + 3)*(cos(x)^2 + 1)*sin(x) - 4*cos(x)*sin(x))/(cos(x)^4 + 6*cos(x)^2 - 3)) + 1/2*arctan(sin(x)/cos(x))

giac [A] time = 0.15, size = 5, normalized size = 0.71

$$\arcsin\left(\frac{1}{2} \sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(4-sin(x)^2)^(1/2),x, algorithm="giac")

[Out] arcsin(1/2*sin(x))

maple [A] time = 0.08, size = 6, normalized size = 0.86

$$\arcsin\left(\frac{\sin(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(4-sin(x)^2)^(1/2),x)

[Out] arcsin(1/2*sin(x))

maxima [A] time = 0.42, size = 5, normalized size = 0.71

$$\arcsin\left(\frac{1}{2} \sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(4-sin(x)^2)^(1/2),x, algorithm="maxima")

[Out] arcsin(1/2*sin(x))

mupad [B] time = 2.98, size = 5, normalized size = 0.71

$$\operatorname{asin}\left(\frac{\sin(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(4 - sin(x)^2)^(1/2),x)

[Out] asin(sin(x)/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sqrt{-(\sin(x)-2)(\sin(x)+2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(4-sin(x)**2)**(1/2),x)

[Out] Integral(cos(x)/sqrt(-(sin(x) - 2)*(sin(x) + 2)), x)

$$3.669 \quad \int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}} dx$$

Optimal. Leaf size=13

$$\frac{1}{3} \sin^{-1}\left(\frac{1}{2} \sin(3x)\right)$$

[Out] 1/3*arcsin(1/2*sin(3*x))

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3190, 216}

$$\frac{1}{3} \sin^{-1}\left(\frac{1}{2} \sin(3x)\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[3*x]/Sqrt[4 - Sin[3*x]^2], x]

[Out] ArcSin[Sin[3*x]/2]/3

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}} dx &= \frac{1}{3} \text{Subst}\left(\int \frac{1}{\sqrt{4-x^2}} dx, x, \sin(3x)\right) \\ &= \frac{1}{3} \sin^{-1}\left(\frac{1}{2} \sin(3x)\right) \end{aligned}$$

Mathematica [A] time = 0.03, size = 13, normalized size = 1.00

$$\frac{1}{3} \sin^{-1}\left(\frac{1}{2} \sin(3x)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[3*x]/Sqrt[4 - Sin[3*x]^2], x]

[Out] ArcSin[Sin[3*x]/2]/3

fricas [B] time = 0.60, size = 71, normalized size = 5.46

$$\frac{1}{6} \arctan\left(\frac{\sqrt{\cos(3x)^2 + 3(\cos(3x)^2 + 1)\sin(3x) - 4\cos(3x)\sin(3x)}}{\cos(3x)^4 + 6\cos(3x)^2 - 3}\right) + \frac{1}{6} \arctan\left(\frac{\sin(3x)}{\cos(3x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)/(4-sin(3*x)^2)^(1/2),x, algorithm="fricas")

[Out] 1/6*arctan((sqrt(cos(3*x)^2 + 3)*(cos(3*x)^2 + 1)*sin(3*x) - 4*cos(3*x)*sin(3*x))/(cos(3*x)^4 + 6*cos(3*x)^2 - 3)) + 1/6*arctan(sin(3*x)/cos(3*x))

giac [A] time = 0.22, size = 9, normalized size = 0.69

$$\frac{1}{3} \arcsin\left(\frac{1}{2} \sin(3x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)/(4-sin(3*x)^2)^(1/2),x, algorithm="giac")

[Out] 1/3*arcsin(1/2*sin(3*x))

maple [A] time = 0.08, size = 10, normalized size = 0.77

$$\frac{\arcsin\left(\frac{\sin(3x)}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(3*x)/(4-sin(3*x)^2)^(1/2),x)

[Out] 1/3*arcsin(1/2*sin(3*x))

maxima [A] time = 0.42, size = 9, normalized size = 0.69

$$\frac{1}{3} \arcsin\left(\frac{1}{2} \sin(3x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)/(4-sin(3*x)^2)^(1/2),x, algorithm="maxima")

[Out] 1/3*arcsin(1/2*sin(3*x))

mupad [B] time = 2.98, size = 9, normalized size = 0.69

$$\frac{\operatorname{asin}\left(\frac{\sin(3x)}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(3*x)/(4 - sin(3*x)^2)^(1/2),x)

[Out] asin(sin(3*x)/2)/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(3x)}{\sqrt{-(\sin(3x) - 2)(\sin(3x) + 2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)/(4-sin(3*x)**2)**(1/2),x)

[Out] Integral(cos(3*x)/sqrt(-(sin(3*x) - 2)*(sin(3*x) + 2)), x)

3.670 $\int \cos(x)\sqrt{1 + \csc(x)} dx$

Optimal. Leaf size=21

$$\sin(x)\sqrt{\csc(x)+1} + \tanh^{-1}\left(\sqrt{\csc(x)+1}\right)$$

[Out] arctanh((1+csc(x))^(1/2))+sin(x)*(1+csc(x))^(1/2)

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3873, 47, 63, 207}

$$\sin(x)\sqrt{\csc(x)+1} + \tanh^{-1}\left(\sqrt{\csc(x)+1}\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sqrt[1 + Csc[x]],x]

[Out] ArcTanh[Sqrt[1 + Csc[x]]] + Sqrt[1 + Csc[x]]*Sin[x]

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 3873

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := -Dist[(f*b^(p - 1))^(p - 1), Subst[Int[((-a + b*x)^(p - 1)/2)*(a + b*x)^(m + (p - 1)/2)]/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \cos(x)\sqrt{1+\csc(x)} dx &= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2} dx, x, \csc(x)\right) \\
&= \sqrt{1+\csc(x)} \sin(x) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}} dx, x, \csc(x)\right) \\
&= \sqrt{1+\csc(x)} \sin(x) - \text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+\csc(x)}\right) \\
&= \tanh^{-1}\left(\sqrt{1+\csc(x)}\right) + \sqrt{1+\csc(x)} \sin(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\sin(x)\sqrt{\csc(x)+1} + \tanh^{-1}\left(\sqrt{\csc(x)+1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sqrt[1 + Csc[x]], x]

[Out] ArcTanh[Sqrt[1 + Csc[x]]] + Sqrt[1 + Csc[x]]*Sin[x]

fricas [B] time = 0.99, size = 79, normalized size = 3.76

$$\sqrt{\frac{\sin(x)+1}{\sin(x)}} \sin(x) + \frac{1}{2} \log\left(\frac{2\left(\sqrt{\frac{\sin(x)+1}{\sin(x)}} \sin(x) + \sin(x) + 1\right)}{\cos(x) + \sin(x) + 1}\right) - \frac{1}{2} \log\left(\frac{2\left(\sqrt{\frac{\sin(x)+1}{\sin(x)}} \sin(x) - \sin(x) - 1\right)}{\cos(x) + \sin(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(1+csc(x))^(1/2), x, algorithm="fricas")

[Out] sqrt((sin(x) + 1)/sin(x))*sin(x) + 1/2*log(2*(sqrt((sin(x) + 1)/sin(x))*sin(x) + sin(x) + 1)/(cos(x) + sin(x) + 1)) - 1/2*log(-2*(sqrt((sin(x) + 1)/sin(x))*sin(x) - sin(x) - 1)/(cos(x) + sin(x) + 1))

giac [B] time = 0.15, size = 38, normalized size = 1.81

$$\frac{1}{2} \left(2 \sqrt{\sin(x)^2 + \sin(x)} - \log\left(\left| 2 \sqrt{\sin(x)^2 + \sin(x)} - 2 \sin(x) - 1 \right| \right) \right) \text{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(1+csc(x))^(1/2), x, algorithm="giac")

[Out] 1/2*(2*sqrt(sin(x)^2 + sin(x)) - log(abs(2*sqrt(sin(x)^2 + sin(x)) - 2*sin(x) - 1)))*sgn(sin(x))

maple [B] time = 0.08, size = 48, normalized size = 2.29

$$\frac{1}{2\sqrt{1+\csc(x)}-2} - \frac{\ln\left(\sqrt{1+\csc(x)}-1\right)}{2} + \frac{1}{2\sqrt{1+\csc(x)}+2} + \frac{\ln\left(\sqrt{1+\csc(x)}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*(1+csc(x))^(1/2), x)

[Out] 1/2/((1+csc(x))^(1/2)-1)-1/2*ln((1+csc(x))^(1/2)-1)+1/2/((1+csc(x))^(1/2)+1)+1/2*ln((1+csc(x))^(1/2)+1)

maxima [B] time = 0.33, size = 38, normalized size = 1.81

$$\sqrt{\frac{1}{\sin(x)} + 1} \sin(x) + \frac{1}{2} \log\left(\sqrt{\frac{1}{\sin(x)} + 1} + 1\right) - \frac{1}{2} \log\left(\sqrt{\frac{1}{\sin(x)} + 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(1+csc(x))^(1/2),x, algorithm="maxima")

[Out] sqrt(1/sin(x) + 1)*sin(x) + 1/2*log(sqrt(1/sin(x) + 1) + 1) - 1/2*log(sqrt(1/sin(x) + 1) - 1)

mupad [B] time = 3.10, size = 47, normalized size = 2.24

$$\sin(x) \sqrt{\frac{1}{\sin(x)} + 1} + \frac{\ln\left(\sin(x) + \sqrt{\sin(x)^2 + \sin(x)} + \frac{1}{2}\right) \sin(x) \sqrt{\frac{1}{\sin(x)} + 1}}{2 \sqrt{\sin(x)^2 + \sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*(1/sin(x) + 1)^(1/2),x)

[Out] sin(x)*(1/sin(x) + 1)^(1/2) + (log(sin(x) + (sin(x) + sin(x)^2)^(1/2) + 1/2)*sin(x)*(1/sin(x) + 1)^(1/2))/(2*(sin(x) + sin(x)^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\csc(x) + 1} \cos(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(1+csc(x))**(1/2),x)

[Out] Integral(sqrt(csc(x) + 1)*cos(x), x)

3.671 $\int \cos(x)\sqrt{4 - \sin^2(x)} dx$

Optimal. Leaf size=28

$$2 \sin^{-1}\left(\frac{\sin(x)}{2}\right) + \frac{1}{2} \sin(x)\sqrt{4 - \sin^2(x)}$$

[Out] 2*arcsin(1/2*sin(x))+1/2*sin(x)*(4-sin(x)^2)^(1/2)

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3190, 195, 216}

$$2 \sin^{-1}\left(\frac{\sin(x)}{2}\right) + \frac{1}{2} \sin(x)\sqrt{4 - \sin^2(x)}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sqrt[4 - Sin[x]^2],x]

[Out] 2*ArcSin[Sin[x]/2] + (Sin[x]*Sqrt[4 - Sin[x]^2])/2

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \cos(x)\sqrt{4 - \sin^2(x)} dx &= \text{Subst}\left(\int \sqrt{4 - x^2} dx, x, \sin(x)\right) \\ &= \frac{1}{2} \sin(x)\sqrt{4 - \sin^2(x)} + 2 \text{Subst}\left(\int \frac{1}{\sqrt{4 - x^2}} dx, x, \sin(x)\right) \\ &= 2 \sin^{-1}\left(\frac{\sin(x)}{2}\right) + \frac{1}{2} \sin(x)\sqrt{4 - \sin^2(x)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 1.00

$$2 \sin^{-1}\left(\frac{\sin(x)}{2}\right) + \frac{1}{2} \sin(x)\sqrt{4 - \sin^2(x)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sqrt[4 - Sin[x]^2],x]

[Out] 2*ArcSin[Sin[x]/2] + (Sin[x]*Sqrt[4 - Sin[x]^2])/2

fricas [B] time = 1.07, size = 61, normalized size = 2.18

$$\frac{1}{2} \sqrt{\cos(x)^2 + 3} \sin(x) + \arctan\left(\frac{\sqrt{\cos(x)^2 + 3} (\cos(x)^2 + 1) \sin(x) - 4 \cos(x) \sin(x)}{\cos(x)^4 + 6 \cos(x)^2 - 3}\right) + \arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(4-sin(x)^2)^(1/2),x, algorithm="fricas")

[Out] 1/2*sqrt(cos(x)^2 + 3)*sin(x) + arctan((sqrt(cos(x)^2 + 3)*(cos(x)^2 + 1)*sin(x) - 4*cos(x)*sin(x))/(cos(x)^4 + 6*cos(x)^2 - 3)) + arctan(sin(x)/cos(x))

giac [A] time = 0.14, size = 22, normalized size = 0.79

$$\frac{1}{2} \sqrt{-\sin(x)^2 + 4} \sin(x) + 2 \arcsin\left(\frac{1}{2} \sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(4-sin(x)^2)^(1/2),x, algorithm="giac")

[Out] 1/2*sqrt(-sin(x)^2 + 4)*sin(x) + 2*arcsin(1/2*sin(x))

maple [A] time = 0.07, size = 23, normalized size = 0.82

$$2 \arcsin\left(\frac{\sin(x)}{2}\right) + \frac{\sin(x) \sqrt{4 - (\sin^2(x))}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*(4-sin(x)^2)^(1/2),x)

[Out] 2*arcsin(1/2*sin(x))+1/2*sin(x)*(4-sin(x)^2)^(1/2)

maxima [A] time = 0.46, size = 22, normalized size = 0.79

$$\frac{1}{2} \sqrt{-\sin(x)^2 + 4} \sin(x) + 2 \arcsin\left(\frac{1}{2} \sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(4-sin(x)^2)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(-sin(x)^2 + 4)*sin(x) + 2*arcsin(1/2*sin(x))

mupad [B] time = 2.97, size = 20, normalized size = 0.71

$$2 \operatorname{asin}\left(\frac{\sin(x)}{2}\right) + \frac{\sin(x) \sqrt{\cos(x)^2 + 3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*(4 - sin(x)^2)^(1/2),x)

[Out] 2*asin(sin(x)/2) + (sin(x)*(cos(x)^2 + 3)^(1/2))/2

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-(\sin(x) - 2)(\sin(x) + 2)} \cos(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(4-sin(x)**2)**(1/2),x)

[Out] Integral(sqrt(-(sin(x) - 2)*(sin(x) + 2))*cos(x), x)

$$3.672 \quad \int \cos(x) \sin(x) \sqrt{1 + \sin^2(x)} dx$$

Optimal. Leaf size=14

$$\frac{1}{3} (\sin^2(x) + 1)^{3/2}$$

[Out] 1/3*(1+sin(x)^2)^(3/2)

Rubi [A] time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3198, 261}

$$\frac{1}{3} (\sin^2(x) + 1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[x]*Sqrt[1 + Sin[x]^2], x]

[Out] (1 + Sin[x]^2)^(3/2)/3

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 3198

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \cos(x) \sin(x) \sqrt{1 + \sin^2(x)} dx &= \text{Subst} \left(\int x \sqrt{1 + x^2} dx, x, \sin(x) \right) \\ &= \frac{1}{3} (1 + \sin^2(x))^{3/2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{3} (\sin^2(x) + 1)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sin[x]*Sqrt[1 + Sin[x]^2], x]

[Out] (1 + Sin[x]^2)^(3/2)/3

fricas [A] time = 0.88, size = 12, normalized size = 0.86

$$\frac{1}{3} (-\cos(x)^2 + 2)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="fricas")

[Out] 1/3*(-cos(x)^2 + 2)^(3/2)

giac [A] time = 0.13, size = 10, normalized size = 0.71

$$\frac{1}{3} (\sin(x)^2 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="giac")

[Out] 1/3*(sin(x)^2 + 1)^(3/2)

maple [A] time = 0.01, size = 11, normalized size = 0.79

$$\frac{(1 + \sin^2(x))^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x)

[Out] 1/3*(1+sin(x)^2)^(3/2)

maxima [A] time = 0.37, size = 10, normalized size = 0.71

$$\frac{1}{3} (\sin(x)^2 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="maxima")

[Out] 1/3*(sin(x)^2 + 1)^(3/2)

mupad [B] time = 0.10, size = 10, normalized size = 0.71

$$\frac{(\sin(x)^2 + 1)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(x)*(sin(x)^2 + 1)^(1/2),x)

[Out] (sin(x)^2 + 1)^(3/2)/3

sympy [B] time = 0.73, size = 27, normalized size = 1.93

$$\frac{\sqrt{\sin^2(x) + 1} \sin^2(x)}{3} + \frac{\sqrt{\sin^2(x) + 1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)*(1+sin(x)**2)**(1/2),x)

[Out] sqrt(sin(x)**2 + 1)*sin(x)**2/3 + sqrt(sin(x)**2 + 1)/3

$$3.673 \quad \int \frac{\cos(x)}{\sqrt{2 \sin(x) + \sin^2(x)}} dx$$

Optimal. Leaf size=19

$$2 \tanh^{-1} \left(\frac{\sin(x)}{\sqrt{\sin^2(x) + 2 \sin(x)}} \right)$$

[Out] 2*arctanh(sin(x)/(2*sin(x)+sin(x)^2)^(1/2))

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3258, 620, 206}

$$2 \tanh^{-1} \left(\frac{\sin(x)}{\sqrt{\sin^2(x) + 2 \sin(x)}} \right)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/Sqrt[2*Sin[x] + Sin[x]^2], x]

[Out] 2*ArcTanh[Sin[x]/Sqrt[2*Sin[x] + Sin[x]^2]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 3258

Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*sin[(d_.) + (e_.)*(x_)]^(n_.) + (c_.)*((f_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^p_.), x_Symbol] :> Module[{g = FreeFactors[Sin[d + e*x], x]}, Dist[g/e, Subst[Int[(1 - g^2*x^2)^((m - 1)/2)*(a + b*(f*g*x)^n + c*(f*g*x)^(2*n))^p, x], x, Sin[d + e*x]/g], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2*n] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{\sqrt{2 \sin(x) + \sin^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{\sqrt{2x + x^2}} dx, x, \sin(x) \right) \\ &= 2 \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \frac{\sin(x)}{\sqrt{2 \sin(x) + \sin^2(x)}} \right) \\ &= 2 \tanh^{-1} \left(\frac{\sin(x)}{\sqrt{2 \sin(x) + \sin^2(x)}} \right) \end{aligned}$$

Mathematica [B] time = 0.02, size = 40, normalized size = 2.11

$$\frac{2\sqrt{\sin(x)}\sqrt{\sin(x)+2}\sinh^{-1}\left(\frac{\sqrt{\sin(x)}}{\sqrt{2}}\right)}{\sqrt{\sin(x)(\sin(x)+2)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/Sqrt[2*Sin[x] + Sin[x]^2],x]

[Out] (2*ArcSinh[Sqrt[Sin[x]]/Sqrt[2]]*Sqrt[Sin[x]]*Sqrt[2 + Sin[x]])/Sqrt[Sin[x]]*(2 + Sin[x])

fricas [B] time = 2.96, size = 35, normalized size = 1.84

$$\frac{1}{2} \log\left(-2 \cos(x)^2 + 2 \sqrt{-\cos(x)^2 + 2 \sin(x) + 1} (\sin(x) + 1) + 4 \sin(x) + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x, algorithm="fricas")

[Out] 1/2*log(-2*cos(x)^2 + 2*sqrt(-cos(x)^2 + 2*sin(x) + 1)*(sin(x) + 1) + 4*sin(x) + 3)

giac [A] time = 0.16, size = 20, normalized size = 1.05

$$-\log\left(-\sqrt{\sin(x)^2 + 2 \sin(x)} + \sin(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x, algorithm="giac")

[Out] -log(-sqrt(sin(x)^2 + 2*sin(x)) + sin(x) + 1)

maple [A] time = 0.10, size = 17, normalized size = 0.89

$$\ln\left(\sin(x) + 1 + \sqrt{2 \sin(x) + \sin^2(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x)

[Out] ln(sin(x)+1+(2*sin(x)+sin(x)^2)^(1/2))

maxima [A] time = 0.31, size = 20, normalized size = 1.05

$$\log\left(2 \sqrt{\sin(x)^2 + 2 \sin(x)} + 2 \sin(x) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x, algorithm="maxima")

[Out] log(2*sqrt(sin(x)^2 + 2*sin(x)) + 2*sin(x) + 2)

mupad [B] time = 3.17, size = 14, normalized size = 0.74

$$\ln\left(\sin(x) + \sqrt{\sin(x) (\sin(x) + 2)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(2*sin(x) + sin(x)^2)^(1/2),x)

[Out] log(sin(x) + (sin(x)*(sin(x) + 2))^(1/2) + 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sqrt{(\sin(x) + 2) \sin(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2*sin(x)+sin(x)**2)**(1/2),x)
```

```
[Out] Integral(cos(x)/sqrt((sin(x) + 2)*sin(x)), x)
```

3.674 $\int \cos(x) \cos(\sin(x)) dx$

Optimal. Leaf size=3

$$\sin(\sin(x))$$

[Out] $\sin(\sin(x))$

Rubi [A] time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4334, 2637}

$$\sin(\sin(x))$$

Antiderivative was successfully verified.

[In] `Int[Cos[x]*Cos[Sin[x]],x]`

[Out] `Sin[Sin[x]]`

Rule 2637

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 4334

`Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_.))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])`

Rubi steps

$$\int \cos(x) \cos(\sin(x)) dx = \text{Subst} \left(\int \cos(x) dx, x, \sin(x) \right) \\ = \sin(\sin(x))$$

Mathematica [A] time = 1.47, size = 3, normalized size = 1.00

$$\sin(\sin(x))$$

Antiderivative was successfully verified.

[In] `Integrate[Cos[x]*Cos[Sin[x]],x]`

[Out] `Sin[Sin[x]]`

fricas [B] time = 1.64, size = 17, normalized size = 5.67

$$\sin \left(\frac{2 \tan \left(\frac{1}{2} x \right)}{\tan \left(\frac{1}{2} x \right)^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(sin(x)),x, algorithm="fricas")`

[Out] `sin(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))`

giac [A] time = 0.14, size = 3, normalized size = 1.00

$\sin(\sin(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(sin(x)),x, algorithm="giac")`

[Out] `sin(sin(x))`

maple [A] time = 0.01, size = 4, normalized size = 1.33

$\sin(\sin(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*cos(sin(x)),x)`

[Out] `sin(sin(x))`

maxima [A] time = 0.32, size = 3, normalized size = 1.00

$\sin(\sin(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(sin(x)),x, algorithm="maxima")`

[Out] `sin(sin(x))`

mupad [B] time = 2.95, size = 3, normalized size = 1.00

$\sin(\sin(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(sin(x))*cos(x),x)`

[Out] `sin(sin(x))`

sympy [A] time = 0.41, size = 3, normalized size = 1.00

$\sin(\sin(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*cos(sin(x)),x)`

[Out] `sin(sin(x))`

3.675 $\int \cos(x) \cos(\sin(x)) \cos(\sin(\sin(x))) dx$

Optimal. Leaf size=4

$$\sin(\sin(\sin(x)))$$

[Out] $\sin(\sin(\sin(x)))$

Rubi [A] time = 0.02, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4334, 2637}

$$\sin(\sin(\sin(x)))$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[x] * \text{Cos}[\text{Sin}[x]] * \text{Cos}[\text{Sin}[\text{Sin}[x]]], x]$

[Out] $\text{Sin}[\text{Sin}[\text{Sin}[x]]]$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$
 $\text{FreeQ}[\{c, d\}, x]$

Rule 4334

$\text{Int}[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_.))], x_Symbol] \rightarrow \text{With}[\{d = \text{FreeFactors}[\text{Sin}[c*(a + b*x)], x]\}, \text{Dist}[d/(b*c), \text{Subst}[\text{Int}[\text{SubstFor}[1, \text{Sin}[c*(a + b*x)]/d, u, x], x], x, \text{Sin}[c*(a + b*x)]/d], x] /;$
 $\text{FunctionOfQ}[\text{Sin}[c*(a + b*x)]/d, u, x, \text{True}] /;$
 $\text{FreeQ}[\{a, b, c\}, x] \&\& (\text{EqQ}[F, \text{Cos}] \parallel \text{EqQ}[F, \text{cos}])$

Rubi steps

$$\begin{aligned} \int \cos(x) \cos(\sin(x)) \cos(\sin(\sin(x))) dx &= \text{Subst}\left(\int \cos(x) \cos(\sin(x)) dx, x, \sin(x)\right) \\ &= \text{Subst}\left(\int \cos(x) dx, x, \sin(\sin(x))\right) \\ &= \sin(\sin(\sin(x))) \end{aligned}$$

Mathematica [A] time = 8.96, size = 4, normalized size = 1.00

$$\sin(\sin(\sin(x)))$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Cos}[x] * \text{Cos}[\text{Sin}[x]] * \text{Cos}[\text{Sin}[\text{Sin}[x]]], x]$

[Out] $\text{Sin}[\text{Sin}[\text{Sin}[x]]]$

fricas [B] time = 0.67, size = 41, normalized size = 10.25

$$\sin\left(\frac{2 \tan\left(\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right)}{\tan\left(\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="fricas")

[Out] $\sin(2*\tan(\tan(1/2*x)/(\tan(1/2*x)^2 + 1))/(\tan(\tan(1/2*x)/(\tan(1/2*x)^2 + 1))^2 + 1)$

giac [A] time = 0.13, size = 4, normalized size = 1.00

$\sin(\sin(\sin(x)))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="giac")

[Out] $\sin(\sin(\sin(x)))$

maple [A] time = 0.01, size = 5, normalized size = 1.25

$\sin(\sin(\sin(x)))$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*cos(sin(x))*cos(sin(sin(x))),x)

[Out] $\sin(\sin(\sin(x)))$

maxima [A] time = 0.32, size = 4, normalized size = 1.00

$\sin(\sin(\sin(x)))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="maxima")

[Out] $\sin(\sin(\sin(x)))$

mupad [B] time = 3.00, size = 4, normalized size = 1.00

$\sin(\sin(\sin(x)))$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(sin(x))*cos(sin(sin(x)))*cos(x),x)

[Out] $\sin(\sin(\sin(x)))$

sympy [A] time = 8.85, size = 5, normalized size = 1.25

$\sin(\sin(\sin(x)))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x)

[Out] $\sin(\sin(\sin(x)))$

3.676 $\int \cos(x) \sec(\sin(x)) dx$

Optimal. Leaf size=4

$$\tanh^{-1}(\sin(\sin(x)))$$

[Out] arctanh(sin(sin(x)))

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4334, 3770}

$$\tanh^{-1}(\sin(\sin(x)))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sec[Sin[x]],x]

[Out] ArcTanh[Sin[Sin[x]]]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4334

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned} \int \cos(x) \sec(\sin(x)) dx &= \text{Subst} \left(\int \sec(x) dx, x, \sin(x) \right) \\ &= \tanh^{-1}(\sin(\sin(x))) \end{aligned}$$

Mathematica [A] time = 0.01, size = 4, normalized size = 1.00

$$\tanh^{-1}(\sin(\sin(x)))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sec[Sin[x]],x]

[Out] ArcTanh[Sin[Sin[x]]]

fricas [B] time = 0.88, size = 47, normalized size = 11.75

$$\frac{1}{2} \log \left(\sin \left(\frac{2 \tan \left(\frac{1}{2} x \right)}{\tan \left(\frac{1}{2} x \right)^2 + 1} \right) + 1 \right) - \frac{1}{2} \log \left(-\sin \left(\frac{2 \tan \left(\frac{1}{2} x \right)}{\tan \left(\frac{1}{2} x \right)^2 + 1} \right) + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(sin(x)),x, algorithm="fricas")

[Out] $\frac{1}{2} \log(\sin(2 \tan(1/2 x)) / (\tan(1/2 x)^2 + 1) + 1) - \frac{1}{2} \log(-\sin(2 \tan(1/2 x)) / (\tan(1/2 x)^2 + 1) + 1)$

giac [B] time = 0.14, size = 29, normalized size = 7.25

$$\frac{1}{4} \log \left(\left| \frac{1}{\sin(\sin(x))} + \sin(\sin(x)) + 2 \right| \right) - \frac{1}{4} \log \left(\left| \frac{1}{\sin(\sin(x))} + \sin(\sin(x)) - 2 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(sin(x)),x, algorithm="giac")

[Out] $\frac{1}{4} \log(\text{abs}(1/\sin(\sin(x)) + \sin(\sin(x)) + 2)) - \frac{1}{4} \log(\text{abs}(1/\sin(\sin(x)) + \sin(\sin(x)) - 2))$

maple [A] time = 0.01, size = 9, normalized size = 2.25

$$\ln(\sec(\sin(x)) + \tan(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sec(sin(x)),x)

[Out] $\ln(\sec(\sin(x)) + \tan(\sin(x)))$

maxima [A] time = 0.33, size = 8, normalized size = 2.00

$$\log(\sec(\sin(x)) + \tan(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(sin(x)),x, algorithm="maxima")

[Out] $\log(\sec(\sin(x)) + \tan(\sin(x)))$

mupad [B] time = 3.24, size = 21, normalized size = 5.25

$$-\text{atan} \left(e^{-\frac{e^{-x}1i}{2}} e^{\frac{e^x1i}{2}} \right) 2i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/cos(sin(x)),x)

[Out] $-\text{atan}(\exp(-\exp(-x*1i)/2) * \exp(\exp(x*1i)/2)) * 2i$

sympy [A] time = 1.33, size = 10, normalized size = 2.50

$$\log(\tan(\sin(x)) + \sec(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sec(sin(x)),x)

[Out] $\log(\tan(\sin(x)) + \sec(\sin(x)))$

$$3.677 \quad \int \cos(x) \sin^3(x) (a + b \sin^2(x))^3 dx$$

Optimal. Leaf size=36

$$\frac{(a + b \sin^2(x))^5}{10b^2} - \frac{a(a + b \sin^2(x))^4}{8b^2}$$

[Out] $-1/8*a*(a+b*\sin(x)^2)^4/b^2+1/10*(a+b*\sin(x)^2)^5/b^2$

Rubi [A] time = 0.08, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3198, 266, 43}

$$\frac{(a + b \sin^2(x))^5}{10b^2} - \frac{a(a + b \sin^2(x))^4}{8b^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[x]^3*(a + b*SIN[x]^2)^3,x]

[Out] $-(a*(a + b*\sin[x]^2)^4)/(8*b^2) + (a + b*\sin[x]^2)^5/(10*b^2)$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3198

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \cos(x) \sin^3(x) (a + b \sin^2(x))^3 dx &= \text{Subst} \left(\int x^3 (a + bx^2)^3 dx, x, \sin(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int x(a + bx)^3 dx, x, \sin^2(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(-\frac{a(a + bx)^3}{b} + \frac{(a + bx)^4}{b} \right) dx, x, \sin^2(x) \right) \\ &= -\frac{a(a + b \sin^2(x))^4}{8b^2} + \frac{(a + b \sin^2(x))^5}{10b^2} \end{aligned}$$

Mathematica [B] time = 0.35, size = 128, normalized size = 3.56

$$-20(64a^3 + 24ab^2 + 7b^3) \cos(2x) + 20(16a^3 + 18ab^2 + 5b^3) \cos(4x) + b(3840a^2 \sin^4(x) - 1280a^2 \sin(3x) \sin^3(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sin[x]^3*(a + b*SIN[x]^2)^3,x]

[Out] (-20*(64*a^3 + 24*a*b^2 + 7*b^3)*Cos[2*x] + 20*(16*a^3 + 18*a*b^2 + 5*b^3)*Cos[4*x] + b*(-10*b*(16*a + 5*b)*Cos[6*x] + 15*b*(2*a + b)*Cos[8*x] - 2*b^2*Cos[10*x] + 3840*a^2*SIN[x]^4 + 2560*a*b*SIN[x]^6 + 640*b^2*SIN[x]^8 - 1280*a^2*SIN[x]^3*SIN[3*x]))/10240

fricas [B] time = 1.19, size = 103, normalized size = 2.86

$$-\frac{1}{10} b^3 \cos(x)^{10} + \frac{1}{8} (3 a b^2 + 4 b^3) \cos(x)^8 - \frac{1}{2} (a^2 b + 3 a b^2 + 2 b^3) \cos(x)^6 + \frac{1}{4} (a^3 + 6 a^2 b + 9 a b^2 + 4 b^3) \cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x, algorithm="fricas")

[Out] -1/10*b^3*cos(x)^10 + 1/8*(3*a*b^2 + 4*b^3)*cos(x)^8 - 1/2*(a^2*b + 3*a*b^2 + 2*b^3)*cos(x)^6 + 1/4*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos(x)^4 - 1/2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^2

giac [A] time = 0.12, size = 39, normalized size = 1.08

$$\frac{1}{10} b^3 \sin(x)^{10} + \frac{3}{8} a b^2 \sin(x)^8 + \frac{1}{2} a^2 b \sin(x)^6 + \frac{1}{4} a^3 \sin(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x, algorithm="giac")

[Out] 1/10*b^3*sin(x)^10 + 3/8*a*b^2*sin(x)^8 + 1/2*a^2*b*sin(x)^6 + 1/4*a^3*sin(x)^4

maple [A] time = 0.01, size = 40, normalized size = 1.11

$$\frac{b^3 (\sin^{10}(x))}{10} + \frac{3 a b^2 (\sin^8(x))}{8} + \frac{a^2 b (\sin^6(x))}{2} + \frac{a^3 (\sin^4(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x)

[Out] 1/10*b^3*sin(x)^10+3/8*a*b^2*sin(x)^8+1/2*a^2*b*sin(x)^6+1/4*a^3*sin(x)^4

maxima [A] time = 0.33, size = 39, normalized size = 1.08

$$\frac{1}{10} b^3 \sin(x)^{10} + \frac{3}{8} a b^2 \sin(x)^8 + \frac{1}{2} a^2 b \sin(x)^6 + \frac{1}{4} a^3 \sin(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x, algorithm="maxima")

[Out] 1/10*b^3*sin(x)^10 + 3/8*a*b^2*sin(x)^8 + 1/2*a^2*b*sin(x)^6 + 1/4*a^3*sin(x)^4

mupad [B] time = 0.07, size = 73, normalized size = 2.03

$$\frac{b^2 \cos(x)^8 (3 a + 4 b)}{8} - \frac{b^3 \cos(x)^{10}}{10} - \frac{\cos(x)^2 (a + b)^3}{2} - \frac{b \cos(x)^6 (a^2 + 3 a b + 2 b^2)}{2} + \frac{\cos(x)^4 (a + b)^2 (a + 4 b)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(x)^3*(a + b*sin(x)^2)^3,x)

[Out] $(b^2 \cos(x)^8 (3a + 4b))/8 - (b^3 \cos(x)^{10})/10 - (\cos(x)^2 (a + b)^3)/2 - (b \cos(x)^6 (3ab + a^2 + 2b^2))/2 + (\cos(x)^4 (a + b)^2 (a + 4b))/4$

sympy [A] time = 11.38, size = 44, normalized size = 1.22

$$\frac{a^3 \sin^4(x)}{4} + \frac{a^2 b \sin^6(x)}{2} + \frac{3ab^2 \sin^8(x)}{8} + \frac{b^3 \sin^{10}(x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)**3*(a+b*sin(x)**2)**3,x)

[Out] $a**3 \sin(x)**4/4 + a**2 b \sin(x)**6/2 + 3*a*b**2 \sin(x)**8/8 + b**3 \sin(x)**10/10$

3.678 $\int e^{\sin(x)} \cos(x) \sin(x) dx$

Optimal. Leaf size=14

$$e^{\sin(x)} \sin(x) - e^{\sin(x)}$$

[Out] $-\exp(\sin(x)) + \exp(\sin(x)) * \sin(x)$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4334, 2176, 2194}

$$e^{\sin(x)} \sin(x) - e^{\sin(x)}$$

Antiderivative was successfully verified.

[In] Int[E^Sin[x]*Cos[x]*Sin[x],x]

[Out] -E^Sin[x] + E^Sin[x]*Sin[x]

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma == True
```

Rule 2194

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned} \int e^{\sin(x)} \cos(x) \sin(x) dx &= \text{Subst} \left(\int e^x x dx, x, \sin(x) \right) \\ &= e^{\sin(x)} \sin(x) - \text{Subst} \left(\int e^x dx, x, \sin(x) \right) \\ &= -e^{\sin(x)} + e^{\sin(x)} \sin(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 0.64

$$e^{\sin(x)}(\sin(x) - 1)$$

Antiderivative was successfully verified.

[In] Integrate[E^Sin[x]*Cos[x]*Sin[x],x]

[Out] E^Sin[x]*(-1 + Sin[x])

fricas [A] time = 0.86, size = 8, normalized size = 0.57

$$(\sin(x) - 1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(sin(x))*cos(x)*sin(x),x, algorithm="fricas")

[Out] (sin(x) - 1)*e^sin(x)

giac [A] time = 0.14, size = 8, normalized size = 0.57

$$(\sin(x) - 1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(sin(x))*cos(x)*sin(x),x, algorithm="giac")

[Out] (sin(x) - 1)*e^sin(x)

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-e^{\sin(x)} + e^{\sin(x)} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(sin(x))*cos(x)*sin(x),x)

[Out] -exp(sin(x))+exp(sin(x))*sin(x)

maxima [A] time = 0.33, size = 8, normalized size = 0.57

$$(\sin(x) - 1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(sin(x))*cos(x)*sin(x),x, algorithm="maxima")

[Out] (sin(x) - 1)*e^sin(x)

mupad [B] time = 2.91, size = 8, normalized size = 0.57

$$e^{\sin(x)} (\sin(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(sin(x))*cos(x)*sin(x),x)

[Out] exp(sin(x))*(sin(x) - 1)

sympy [A] time = 0.64, size = 12, normalized size = 0.86

$$e^{\sin(x)} \sin(x) - e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(sin(x))*cos(x)*sin(x),x)

[Out] exp(sin(x))*sin(x) - exp(sin(x))

$$3.679 \quad \int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}} dx$$

Optimal. Leaf size=25

$$-\frac{2 \sin(x)}{\sqrt{\sin^3(x)}} - \frac{2}{3} \sqrt{\sin^3(x)}$$

[Out] $-2*\sin(x)/(\sin(x)^3)^{(1/2)}-2/3*(\sin(x)^3)^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3207, 2564, 14}

$$-\frac{2 \sin(x)}{\sqrt{\sin^3(x)}} - \frac{2}{3} \sqrt{\sin^3(x)}$$

Antiderivative was successfully verified.

[In] `Int[Cos[x]^3/Sqrt[Sin[x]^3],x]`

[Out] $(-2*\sin[x])/Sqrt[\sin[x]^3] - (2*Sqrt[\sin[x]^3])/3$

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)+(b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 2564

```
Int[cos[(e_)+(f_)*(x_)]^(n_)*((a_)*sin[(e_)+(f_)*(x_)])^(m_), x_Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1-x^2/a^2)^((n-1)/2), x], x, a*Sin[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[(m-1)/2] && LtQ[0, m, n])
```

Rule 3207

```
Int[(u_)*((b_)*sin[(e_)+(f_)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e+f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e+f*x]^n)^FracPart[p])/(Sin[e+f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin[e+f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_)*(trig_)[e+f*x])^(m_)] /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}} dx &= \frac{\sin^{\frac{3}{2}}(x) \int \frac{\cos^3(x)}{\sin^{\frac{3}{2}}(x)} dx}{\sqrt{\sin^3(x)}} \\
&= \frac{\sin^{\frac{3}{2}}(x) \operatorname{Subst}\left(\int \frac{1-x^2}{x^{3/2}} dx, x, \sin(x)\right)}{\sqrt{\sin^3(x)}} \\
&= \frac{\sin^{\frac{3}{2}}(x) \operatorname{Subst}\left(\int \left(\frac{1}{x^{3/2}} - \sqrt{x}\right) dx, x, \sin(x)\right)}{\sqrt{\sin^3(x)}} \\
&= -\frac{2 \sin(x)}{\sqrt{\sin^3(x)}} - \frac{2}{3} \sqrt{\sin^3(x)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.80

$$\frac{\sin(x)(\cos(2x) - 7)}{3\sqrt{\sin^3(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^3/Sqrt[Sin[x]^3], x]

[Out] ((-7 + Cos[2*x])*Sin[x])/(3*Sqrt[Sin[x]^3])

fricas [A] time = 0.89, size = 28, normalized size = 1.12

$$\frac{2(\cos(x)^2 - 4)\sqrt{-(\cos(x)^2 - 1)\sin(x)}}{3(\cos(x)^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3/(sin(x)^3)^(1/2), x, algorithm="fricas")

[Out] -2/3*(cos(x)^2 - 4)*sqrt(-(cos(x)^2 - 1)*sin(x))/(cos(x)^2 - 1)

giac [A] time = 0.13, size = 13, normalized size = 0.52

$$-\frac{2}{3} \sin(x)^{\frac{3}{2}} - \frac{2}{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3/(sin(x)^3)^(1/2), x, algorithm="giac")

[Out] -2/3*sin(x)^(3/2) - 2/sqrt(sin(x))

maple [A] time = 0.16, size = 14, normalized size = 0.56

$$-\frac{2\left(\sin^{\frac{3}{2}}(x)\right)}{3} - \frac{2}{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^3/(sin(x)^3)^(1/2), x)

[Out] -2/3*sin(x)^(3/2)-2/sin(x)^(1/2)

maxima [A] time = 0.32, size = 19, normalized size = 0.76

$$-\frac{2}{3} \sqrt{\sin(x)^3} - \frac{2 \sin(x)}{\sqrt{\sin(x)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3/(sin(x)^3)^(1/2),x, algorithm="maxima")

[Out] -2/3*sqrt(sin(x)^3) - 2*sin(x)/sqrt(sin(x)^3)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(x)^3}{\sqrt{\sin(x)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^3/(sin(x)^3)^(1/2),x)

[Out] int(cos(x)^3/(sin(x)^3)^(1/2), x)

sympy [A] time = 2.07, size = 36, normalized size = 1.44

$$-\frac{8 \sin^3(x)}{3 \sqrt{\sin^3(x)}} - \frac{2 \sin(x) \cos^2(x)}{\sqrt{\sin^3(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**3/(sin(x)**3)**(1/2),x)

[Out] -8*sin(x)**3/(3*sqrt(sin(x)**3)) - 2*sin(x)*cos(x)**2/sqrt(sin(x)**3)

$$3.680 \quad \int \frac{e^{\sqrt{\sin(x)}} \cos(x)}{\sqrt{\sin(x)}} dx$$

Optimal. Leaf size=10

$$2e^{\sqrt{\sin(x)}}$$

[Out] 2*exp(sin(x)^(1/2))

Rubi [A] time = 0.03, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4334, 2209}

$$2e^{\sqrt{\sin(x)}}$$

Antiderivative was successfully verified.

[In] Int[(E^Sqrt[Sin[x]]*Cos[x])/Sqrt[Sin[x]],x]

[Out] 2*E^Sqrt[Sin[x]]

Rule 2209

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^n * F^(a + b*(c + d*x)^n)) / (b*f*n*(c + d*x)^n * Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]

Rule 4334

Int[(u_)*(F_) [(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\int \frac{e^{\sqrt{\sin(x)}} \cos(x)}{\sqrt{\sin(x)}} dx = \text{Subst} \left(\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx, x, \sin(x) \right) = 2e^{\sqrt{\sin(x)}}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$2e^{\sqrt{\sin(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(E^Sqrt[Sin[x]]*Cos[x])/Sqrt[Sin[x]],x]

[Out] 2*E^Sqrt[Sin[x]]

fricas [A] time = 1.18, size = 7, normalized size = 0.70

$$2e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x, algorithm="fricas")

[Out] $2e^{\sqrt{\sin(x)}}$

giac [A] time = 0.15, size = 7, normalized size = 0.70

$$2e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x, algorithm="giac")`

[Out] $2e^{\sqrt{\sin(x)}}$

maple [A] time = 0.02, size = 8, normalized size = 0.80

$$2e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x)`

[Out] $2\exp(\sin(x)^{(1/2)})$

maxima [A] time = 0.32, size = 7, normalized size = 0.70

$$2e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x, algorithm="maxima")`

[Out] $2e^{\sqrt{\sin(x)}}$

mupad [B] time = 3.00, size = 7, normalized size = 0.70

$$2e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((exp(sin(x)^(1/2))*cos(x))/sin(x)^(1/2),x)`

[Out] $2\exp(\sin(x)^{(1/2)})$

sympy [A] time = 0.46, size = 8, normalized size = 0.80

$$2e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(sin(x)**(1/2))*cos(x)/sin(x)**(1/2),x)`

[Out] $2\exp(\sqrt{\sin(x)})$

3.681 $\int e^{4+\sin(x)} \cos(x) dx$

Optimal. Leaf size=6

$$e^{\sin(x)+4}$$

[Out] exp(4+sin(x))

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4334, 2194}

$$e^{\sin(x)+4}$$

Antiderivative was successfully verified.

[In] Int[E^(4 + Sin[x])*Cos[x], x]

[Out] E^(4 + Sin[x])

Rule 2194

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 4334

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\int e^{4+\sin(x)} \cos(x) dx = \text{Subst} \left(\int e^{4+x} dx, x, \sin(x) \right) = e^{4+\sin(x)}$$

Mathematica [A] time = 0.01, size = 6, normalized size = 1.00

$$e^{\sin(x)+4}$$

Antiderivative was successfully verified.

[In] Integrate[E^(4 + Sin[x])*Cos[x], x]

[Out] E^(4 + Sin[x])

fricas [A] time = 0.94, size = 5, normalized size = 0.83

$$e^{(\sin(x)+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4+sin(x))*cos(x), x, algorithm="fricas")

[Out] e^(sin(x) + 4)

giac [A] time = 0.12, size = 5, normalized size = 0.83

$$e^{(\sin(x)+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4+sin(x))*cos(x),x, algorithm="giac")

[Out] e^(sin(x) + 4)

maple [A] time = 0.03, size = 6, normalized size = 1.00

$$e^{4+\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(4+sin(x))*cos(x),x)

[Out] exp(4+sin(x))

maxima [A] time = 0.32, size = 5, normalized size = 0.83

$$e^{(\sin(x)+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4+sin(x))*cos(x),x, algorithm="maxima")

[Out] e^(sin(x) + 4)

mupad [B] time = 2.91, size = 6, normalized size = 1.00

$$e^{\sin(x)} e^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(sin(x) + 4)*cos(x),x)

[Out] exp(sin(x))*exp(4)

sympy [A] time = 0.54, size = 7, normalized size = 1.17

$$e^4 e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(4+sin(x))*cos(x),x)

[Out] exp(4)*exp(sin(x))

3.682 $\int e^{\cos(x) \sin(x)} \cos(2x) dx$

Optimal. Leaf size=10

$$e^{\frac{1}{2} \sin(2x)}$$

[Out] exp(1/2*sin(2*x))

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4356, 2194}

$$e^{\frac{1}{2} \sin(2x)}$$

Antiderivative was successfully verified.

[In] Int[E^(Cos[x]*Sin[x])*Cos[2*x],x]

[Out] E^(Sin[2*x]/2)

Rule 2194

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 4356

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned} \int e^{\cos(x) \sin(x)} \cos(2x) dx &= \frac{1}{2} \text{Subst} \left(\int e^{x/2} dx, x, \sin(2x) \right) \\ &= e^{\frac{1}{2} \sin(2x)} \end{aligned}$$

Mathematica [A] time = 0.03, size = 7, normalized size = 0.70

$$e^{\sin(x) \cos(x)}$$

Antiderivative was successfully verified.

[In] Integrate[E^(Cos[x]*Sin[x])*Cos[2*x],x]

[Out] E^(Cos[x]*Sin[x])

fricas [A] time = 0.79, size = 6, normalized size = 0.60

$$e^{(\cos(x) \sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(cos(x)*sin(x))*cos(2*x),x, algorithm="fricas")

[Out] e^(cos(x)*sin(x))

giac [A] time = 0.15, size = 12, normalized size = 1.20

$$e^{\left(\frac{\tan(x)}{\tan(x)^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(cos(x)*sin(x))*cos(2*x),x, algorithm="giac")

[Out] e^(tan(x)/(tan(x)^2 + 1))

maple [A] time = 0.08, size = 7, normalized size = 0.70

$$e^{\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(cos(x)*sin(x))*cos(2*x),x)

[Out] exp(cos(x)*sin(x))

maxima [A] time = 0.95, size = 7, normalized size = 0.70

$$e^{\left(\frac{1}{2}\sin(2x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(cos(x)*sin(x))*cos(2*x),x, algorithm="maxima")

[Out] e^(1/2*sin(2*x))

mupad [B] time = 2.99, size = 7, normalized size = 0.70

$$e^{\frac{\sin(2x)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)*exp(cos(x)*sin(x)),x)

[Out] exp(sin(2*x)/2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(cos(x)*sin(x))*cos(2*x),x)

[Out] Timed out

$$3.683 \quad \int e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)} \cos(x) dx$$

Optimal. Leaf size=10

$$2e^{\frac{\sin(x)}{2}}$$

[Out] 2*exp(1/2*sin(x))

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4356, 2194}

$$2e^{\frac{\sin(x)}{2}}$$

Antiderivative was successfully verified.

[In] Int[E^(Cos[x/2]*Sin[x/2])*Cos[x],x]

[Out] 2*E^(Sin[x]/2)

Rule 2194

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 4356

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned} \int e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)} \cos(x) dx &= \text{Subst}\left(\int e^{x/2} dx, x, \sin(x)\right) \\ &= 2e^{\frac{\sin(x)}{2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$2e^{\frac{\sin(x)}{2}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(Cos[x/2]*Sin[x/2])*Cos[x],x]

[Out] 2*E^(Sin[x]/2)

fricas [A] time = 0.72, size = 12, normalized size = 1.20

$$2e^{\left(\cos\left(\frac{1}{2}x\right)\sin\left(\frac{1}{2}x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x, algorithm="fricas")

[Out] 2*e^(cos(1/2*x)*sin(1/2*x))

giac [B] time = 0.14, size = 18, normalized size = 1.80

$$2e^{\left(\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x, algorithm="giac")

[Out] 2*e^(tan(1/2*x)/(tan(1/2*x)^2 + 1))

maple [A] time = 0.08, size = 13, normalized size = 1.30

$$2e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x)

[Out] 2*exp(cos(1/2*x)*sin(1/2*x))

maxima [A] time = 0.33, size = 7, normalized size = 0.70

$$2e^{\left(\frac{1}{2}\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x, algorithm="maxima")

[Out] 2*e^(1/2*sin(x))

mupad [B] time = 2.95, size = 7, normalized size = 0.70

$$2e^{\frac{\sin(x)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(cos(x/2)*sin(x/2))*cos(x),x)

[Out] 2*exp(sin(x)/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)} \cos(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x)

[Out] Integral(exp(sin(x/2)*cos(x/2))*cos(x), x)

3.684 $\int e^{n \sin(a+bx)} \cos(a + bx) dx$

Optimal. Leaf size=17

$$\frac{e^{n \sin(a+bx)}}{bn}$$

[Out] exp(n*sin(b*x+a))/b/n

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4334, 2194}

$$\frac{e^{n \sin(a+bx)}}{bn}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Sin[a + b*x])*Cos[a + b*x],x]

[Out] E^(n*Sin[a + b*x])/(b*n)

Rule 2194

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 4334

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned} \int e^{n \sin(a+bx)} \cos(a + bx) dx &= \frac{\text{Subst}\left(\int e^{nx} dx, x, \sin(a + bx)\right)}{b} \\ &= \frac{e^{n \sin(a+bx)}}{bn} \end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{e^{n \sin(a+bx)}}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*Sin[a + b*x])*Cos[a + b*x],x]

[Out] E^(n*Sin[a + b*x])/(b*n)

fricas [A] time = 0.78, size = 16, normalized size = 0.94

$$\frac{e^{(n \sin(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*x+a))*cos(b*x+a),x, algorithm="fricas")

[Out] $e^{(n \sin(bx + a))} / (bn)$

giac [A] time = 0.13, size = 16, normalized size = 0.94

$$\frac{e^{(n \sin(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*sin(b*x+a))*cos(b*x+a),x, algorithm="giac")`

[Out] $e^{(n \sin(bx + a))} / (bn)$

maple [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{e^{n \sin(bx+a)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(n*sin(b*x+a))*cos(b*x+a),x)`

[Out] $\exp(n \sin(bx+a)) / b/n$

maxima [A] time = 0.31, size = 16, normalized size = 0.94

$$\frac{e^{(n \sin(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*sin(b*x+a))*cos(b*x+a),x, algorithm="maxima")`

[Out] $e^{(n \sin(bx + a))} / (bn)$

mupad [B] time = 0.11, size = 16, normalized size = 0.94

$$\frac{e^{n \sin(a+bx)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(a + b*x)*exp(n*sin(a + b*x)),x)`

[Out] $\exp(n \sin(a + bx)) / (bn)$

sympy [A] time = 0.43, size = 36, normalized size = 2.12

$$\begin{cases} x \cos(a) & \text{for } b = 0 \wedge n = 0 \\ x e^{n \sin(a)} \cos(a) & \text{for } b = 0 \\ \frac{\sin(a+bx)}{b} & \text{for } n = 0 \\ \frac{e^{n \sin(a+bx)}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*sin(b*x+a))*cos(b*x+a),x)`

[Out] `Piecewise((x*cos(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*sin(a))*cos(a), Eq(b, 0)), (sin(a + b*x)/b, Eq(n, 0)), (exp(n*sin(a + b*x))/(b*n), True))`

$$3.685 \quad \int e^{n \sin(ac+bcx)} \cos(c(a + bx)) dx$$

Optimal. Leaf size=22

$$\frac{e^{n \sin(c(a+bx))}}{bcn}$$

[Out] exp(n*sin(c*(b*x+a)))/b/c/n

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4334, 2194}

$$\frac{e^{n \sin(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Sin[a*c + b*c*x])*Cos[c*(a + b*x)],x]

[Out] E^(n*Sin[c*(a + b*x)])/(b*c*n)

Rule 2194

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 4334

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned} \int e^{n \sin(ac+bcx)} \cos(c(a + bx)) dx &= \frac{\text{Subst}\left(\int e^{nx} dx, x, \sin(c(a + bx))\right)}{bc} \\ &= \frac{e^{n \sin(c(a+bx))}}{bcn} \end{aligned}$$

Mathematica [A] time = 0.14, size = 23, normalized size = 1.05

$$\frac{e^{n \sin(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*Sin[a*c + b*c*x])*Cos[c*(a + b*x)],x]

[Out] E^(n*Sin[a*c + b*c*x])/(b*c*n)

fricas [A] time = 1.05, size = 22, normalized size = 1.00

$$\frac{e^{(n \sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x, algorithm="fricas")

[Out] $e^{(n \sin(bc x + a c))} / (b c n)$

giac [A] time = 0.13, size = 22, normalized size = 1.00

$$\frac{e^{(n \sin(bc x + a c))}}{bc n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x, algorithm="giac")`

[Out] $e^{(n \sin(bc x + a c))} / (b c n)$

maple [A] time = 0.06, size = 23, normalized size = 1.05

$$\frac{e^{n \sin(bc x + a c)}}{bc n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x)`

[Out] $\exp(n \sin(bc x + a c)) / b c n$

maxima [A] time = 0.32, size = 22, normalized size = 1.00

$$\frac{e^{(n \sin(bc x + a c))}}{bc n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x, algorithm="maxima")`

[Out] $e^{(n \sin(bc x + a c))} / (b c n)$

mupad [B] time = 3.08, size = 22, normalized size = 1.00

$$\frac{e^{n \sin(a c + b c x)}}{b c n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(c*(a + b*x))*exp(n*sin(a*c + b*c*x)),x)`

[Out] $\exp(n \sin(a c + b c x)) / (b c n)$

sympy [A] time = 9.32, size = 51, normalized size = 2.32

$$\left\{ \begin{array}{ll} x e^{n \sin(a c)} \cos(a c) & \text{for } b = 0 \\ x & \text{for } c = 0 \\ \left\{ \begin{array}{ll} x \cos(a c) & \text{for } b = 0 \\ x & \text{for } c = 0 \end{array} \right. & \text{for } n = 0 \\ \frac{\sin(a c + b c x)}{b c} & \text{otherwise} \\ \frac{e^{n \sin(a c + b c x)}}{b c n} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x)`

[Out] `Piecewise((x*exp(n*sin(a*c))*cos(a*c), Eq(b, 0)), (x, Eq(c, 0)), (Piecewise((x*cos(a*c), Eq(b, 0)), (x, Eq(c, 0)), (sin(a*c + b*c*x)/(b*c), True)), Eq(n, 0)), (exp(n*sin(a*c + b*c*x))/(b*c*n), True))`

$$3.686 \quad \int e^{n \sin(c(a+bx))} \cos(ac + bcx) dx$$

Optimal. Leaf size=23

$$\frac{e^{n \sin(ac+bcx)}}{bcn}$$

[Out] exp(n*sin(b*c*x+a*c))/b/c/n

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4334, 2194}

$$\frac{e^{n \sin(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Sin[c*(a + b*x)])*Cos[a*c + b*c*x],x]

[Out] E^(n*Sin[a*c + b*c*x])/(b*c*n)

Rule 2194

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 4334

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned} \int e^{n \sin(c(a+bx))} \cos(ac + bcx) dx &= \frac{\text{Subst}\left(\int e^{nx} dx, x, \sin(ac + bcx)\right)}{bc} \\ &= \frac{e^{n \sin(ac+bcx)}}{bcn} \end{aligned}$$

Mathematica [A] time = 0.04, size = 23, normalized size = 1.00

$$\frac{e^{n \sin(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*Sin[c*(a + b*x)])*Cos[a*c + b*c*x],x]

[Out] E^(n*Sin[a*c + b*c*x])/(b*c*n)

fricas [A] time = 0.80, size = 22, normalized size = 0.96

$$\frac{e^{(n \sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x, algorithm="fricas")

[Out] $e^{(n \sin(bcx + ac))} / (bcn)$

giac [A] time = 0.15, size = 22, normalized size = 0.96

$$\frac{e^{(n \sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x, algorithm="giac")`

[Out] $e^{(n \sin(bcx + ac))} / (bcn)$

maple [A] time = 0.03, size = 23, normalized size = 1.00

$$\frac{e^{n \sin(bcx+ac)}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x)`

[Out] $\exp(n \sin(bcx + ac)) / b/c/n$

maxima [A] time = 0.31, size = 22, normalized size = 0.96

$$\frac{e^{(n \sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x, algorithm="maxima")`

[Out] $e^{(n \sin(bcx + ac))} / (bcn)$

mupad [B] time = 2.99, size = 22, normalized size = 0.96

$$\frac{e^{n \sin(ac+bcx)}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(n*sin(c*(a + b*x)))*cos(a*c + b*c*x),x)`

[Out] $\exp(n \sin(ac + bcx)) / (bcn)$

sympy [A] time = 2.27, size = 51, normalized size = 2.22

$$\left\{ \begin{array}{ll} x & \text{for } c = 0 \wedge (b = 0 \vee c = 0) \wedge (c = 0 \vee n = 0) \\ x e^{n \sin(ac)} \cos(ac) & \text{for } b = 0 \\ \frac{\sin(ac+bcx)}{bc} & \text{for } n = 0 \\ \frac{e^{n \sin(ac+bcx)}}{bcn} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x)`

[Out] `Piecewise((x, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0)) & (Eq(c, 0) | Eq(n, 0))), (x * exp(n*sin(ac))*cos(ac), Eq(b, 0)), (sin(ac + b*c*x)/(b*c), Eq(n, 0)), (exp(n*sin(ac + b*c*x))/(b*c*n), True))`

$$3.687 \quad \int e^{n \sin(a+bx)} \cot(a+bx) dx$$

Optimal. Leaf size=13

$$\frac{\text{Ei}(n \sin(a+bx))}{b}$$

[Out] Ei(n*sin(b*x+a))/b

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4338, 2178}

$$\frac{\text{Ei}(n \sin(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Sin[a + b*x])*Cot[a + b*x],x]

[Out] ExpIntegralEi[n*Sin[a + b*x]]/b

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned} \int e^{n \sin(a+bx)} \cot(a+bx) dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x} dx, x, \sin(a+bx)\right)}{b} \\ &= \frac{\text{Ei}(n \sin(a+bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.04, size = 13, normalized size = 1.00

$$\frac{\text{Ei}(n \sin(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*Sin[a + b*x])*Cot[a + b*x],x]

[Out] ExpIntegralEi[n*Sin[a + b*x]]/b

fricas [A] time = 1.00, size = 13, normalized size = 1.00

$$\frac{\text{Ei}(n \sin(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*x+a))*cot(b*x+a),x, algorithm="fricas")

[Out] Ei(n*sin(b*x + a))/b

giac [A] time = 0.14, size = 13, normalized size = 1.00

$$\frac{\text{Ei}(n \sin(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*x+a))*cot(b*x+a),x, algorithm="giac")

[Out] Ei(n*sin(b*x + a))/b

maple [A] time = 0.03, size = 17, normalized size = 1.31

$$-\frac{\text{Ei}(1, -n \sin(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*sin(b*x+a))*cot(b*x+a),x)

[Out] -1/b*Ei(1, -n*sin(b*x+a))

maxima [A] time = 0.37, size = 13, normalized size = 1.00

$$\frac{\text{Ei}(n \sin(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*x+a))*cot(b*x+a),x, algorithm="maxima")

[Out] Ei(n*sin(b*x + a))/b

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \cot(a + bx) e^{n \sin(a+bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(a + b*x)*exp(n*sin(a + b*x)),x)

[Out] int(cot(a + b*x)*exp(n*sin(a + b*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \sin(a+bx)} \cot(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*x+a))*cot(b*x+a),x)

[Out] Integral(exp(n*sin(a + b*x))*cot(a + b*x), x)

$$3.688 \quad \int e^{n \sin(ac+bcx)} \cot(c(a + bx)) dx$$

Optimal. Leaf size=18

$$\frac{\text{Ei}(n \sin(c(a + bx)))}{bc}$$

[Out] Ei(n*sin(c*(b*x+a)))/b/c

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4338, 2178}

$$\frac{\text{Ei}(n \sin(c(a + bx)))}{bc}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Sin[a*c + b*c*x])*Cot[c*(a + b*x)],x]

[Out] ExpIntegralEi[n*Sin[c*(a + b*x)]]/(b*c)

Rule 2178

Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 4338

Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])

Rubi steps

$$\begin{aligned} \int e^{n \sin(ac+bcx)} \cot(c(a + bx)) dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x} dx, x, \sin(c(a + bx))\right)}{bc} \\ &= \frac{\text{Ei}(n \sin(c(a + bx)))}{bc} \end{aligned}$$

Mathematica [A] time = 0.07, size = 18, normalized size = 1.00

$$\frac{\text{Ei}(n \sin(c(a + bx)))}{bc}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*Sin[a*c + b*c*x])*Cot[c*(a + b*x)],x]

[Out] ExpIntegralEi[n*Sin[c*(a + b*x)]]/(b*c)

fricas [A] time = 0.67, size = 19, normalized size = 1.06

$$\frac{\text{Ei}(n \sin(bcx + ac))}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x, algorithm="fricas")

[Out] Ei(n*sin(b*c*x + a*c))/(b*c)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cot((bx + a)c) e^{n \sin(bcx+ac)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x, algorithm="giac")

[Out] integrate(cot((b*x + a)*c)*e^(n*sin(b*c*x + a*c)), x)

maple [A] time = 0.10, size = 23, normalized size = 1.28

$$\frac{\text{Ei}(1, -n \sin(bc x + ac))}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x)

[Out] -1/c/b*Ei(1,-n*sin(b*c*x+a*c))

maxima [A] time = 0.38, size = 19, normalized size = 1.06

$$\frac{\text{Ei}(n \sin(bc x + ac))}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x, algorithm="maxima")

[Out] Ei(n*sin(b*c*x + a*c))/(b*c)

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \cot(c(a + bx)) e^{n \sin(ac+bcx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c*(a + b*x))*exp(n*sin(a*c + b*c*x)),x)

[Out] int(cot(c*(a + b*x))*exp(n*sin(a*c + b*c*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \sin(ac+bcx)} \cot(ac + bcx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x)

[Out] Integral(exp(n*sin(a*c + b*c*x))*cot(a*c + b*c*x), x)

$$3.689 \quad \int e^{n \sin(c(a+bx))} \cot(ac + bcx) dx$$

Optimal. Leaf size=19

$$\frac{\text{Ei}(n \sin(ac + bcx))}{bc}$$

[Out] Ei(n*sin(b*c*x+a*c))/b/c

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4338, 2178}

$$\frac{\text{Ei}(n \sin(ac + bcx))}{bc}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Sin[c*(a + b*x)])*Cot[a*c + b*c*x], x]

[Out] ExpIntegralEi[n*Sin[a*c + b*c*x]]/(b*c)

Rule 2178

Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 4338

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])

Rubi steps

$$\begin{aligned} \int e^{n \sin(c(a+bx))} \cot(ac + bcx) dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x} dx, x, \sin(ac + bcx)\right)}{bc} \\ &= \frac{\text{Ei}(n \sin(ac + bcx))}{bc} \end{aligned}$$

Mathematica [A] time = 0.06, size = 18, normalized size = 0.95

$$\frac{\text{Ei}(n \sin(c(a + bx)))}{bc}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*Sin[c*(a + b*x)])*Cot[a*c + b*c*x], x]

[Out] ExpIntegralEi[n*Sin[c*(a + b*x)]]/(b*c)

fricas [A] time = 0.92, size = 19, normalized size = 1.00

$$\frac{\text{Ei}(n \sin(bcx + ac))}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x, algorithm="fricas")

[Out] Ei(n*sin(b*c*x + a*c))/(b*c)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cot(bc x + ac) e^{n \sin((bx+a)c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x, algorithm="giac")

[Out] integrate(cot(b*c*x + a*c)*e^(n*sin((b*x + a)*c)), x)

maple [A] time = 0.07, size = 23, normalized size = 1.21

$$\frac{\text{Ei}(1, -n \sin(bc x + ac))}{cb}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x)

[Out] -1/c/b*Ei(1,-n*sin(b*c*x+a*c))

maxima [A] time = 0.41, size = 19, normalized size = 1.00

$$\frac{\text{Ei}(n \sin(bc x + ac))}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x, algorithm="maxima")

[Out] Ei(n*sin(b*c*x + a*c))/(b*c)

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int e^{n \sin(c(a+bx))} \cot(ac + bc x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*sin(c*(a + b*x)))*cot(a*c + b*c*x),x)

[Out] int(exp(n*sin(c*(a + b*x)))*cot(a*c + b*c*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \sin(ac+bcx)} \cot(ac + bcx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x)

[Out] Integral(exp(n*sin(a*c + b*c*x))*cot(a*c + b*c*x), x)

$$3.690 \quad \int \frac{\sec^2(x)}{a+b \tan(x)} dx$$

Optimal. Leaf size=11

$$\frac{\log(a + b \tan(x))}{b}$$

[Out] ln(a+b*tan(x))/b

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3506, 31}

$$\frac{\log(a + b \tan(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/(a + b*Tan[x]),x]

[Out] Log[a + b*Tan[x]]/b

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3506

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] := Dist[1/(b*f), Subst[Int[(a + x)^{n*(1 + x^2/b^2)}^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)}{a + b \tan(x)} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x} dx, x, b \tan(x)\right)}{b} \\ &= \frac{\log(a + b \tan(x))}{b} \end{aligned}$$

Mathematica [A] time = 0.06, size = 20, normalized size = 1.82

$$\frac{\log(a \cos(x) + b \sin(x)) - \log(\cos(x))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2/(a + b*Tan[x]),x]

[Out] (-Log[Cos[x]] + Log[a*Cos[x] + b*Sin[x]])/b

fricas [B] time = 1.82, size = 40, normalized size = 3.64

$$\frac{\log\left(2ab \cos(x) \sin(x) + (a^2 - b^2) \cos(x)^2 + b^2\right) - \log(\cos(x)^2)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(a+b*tan(x)),x, algorithm="fricas")

[Out] 1/2*(log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - log(cos(x)^2))
/b

giac [A] time = 0.12, size = 12, normalized size = 1.09

$$\frac{\log(|b \tan(x) + a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(a+b*tan(x)),x, algorithm="giac")

[Out] log(abs(b*tan(x) + a))/b

maple [A] time = 0.07, size = 12, normalized size = 1.09

$$\frac{\ln(a + b \tan(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2/(a+b*tan(x)),x)

[Out] ln(a+b*tan(x))/b

maxima [A] time = 0.31, size = 11, normalized size = 1.00

$$\frac{\log(b \tan(x) + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(a+b*tan(x)),x, algorithm="maxima")

[Out] log(b*tan(x) + a)/b

mupad [B] time = 3.03, size = 11, normalized size = 1.00

$$\frac{\ln(a + b \tan(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2*(a + b*tan(x))),x)

[Out] log(a + b*tan(x))/b

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{a + b \tan(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2/(a+b*tan(x)),x)

[Out] Integral(sec(x)**2/(a + b*tan(x)), x)

$$3.691 \quad \int \frac{\sec^2(x)}{1-\tan^2(x)} dx$$

Optimal. Leaf size=11

$$\frac{1}{2} \tanh^{-1}(2 \sin(x) \cos(x))$$

[Out] 1/2*arctanh(2*cos(x)*sin(x))

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3675, 206}

$$\frac{1}{2} \tanh^{-1}(2 \sin(x) \cos(x))$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/(1 - Tan[x]^2), x]

[Out] ArcTanh[2*Cos[x]*Sin[x]]/2

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)}{1-\tan^2(x)} dx &= \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \tan(x) \right) \\ &= \frac{1}{2} \tanh^{-1}(2 \cos(x) \sin(x)) \end{aligned}$$

Mathematica [B] time = 0.01, size = 23, normalized size = 2.09

$$\frac{1}{2} \log(\sin(x) + \cos(x)) - \frac{1}{2} \log(\cos(x) - \sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2/(1 - Tan[x]^2), x]

[Out] -1/2*Log[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x]]/2

fricas [B] time = 0.57, size = 23, normalized size = 2.09

$$\frac{1}{4} \log(2 \cos(x) \sin(x) + 1) - \frac{1}{4} \log(-2 \cos(x) \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(1-tan(x)^2),x, algorithm="fricas")

[Out] 1/4*log(2*cos(x)*sin(x) + 1) - 1/4*log(-2*cos(x)*sin(x) + 1)

giac [A] time = 0.16, size = 17, normalized size = 1.55

$$\frac{1}{2} \log(|\tan(x) + 1|) - \frac{1}{2} \log(|\tan(x) - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(1-tan(x)^2),x, algorithm="giac")

[Out] 1/2*log(abs(tan(x) + 1)) - 1/2*log(abs(tan(x) - 1))

maple [A] time = 0.10, size = 4, normalized size = 0.36

$$\operatorname{arctanh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2/(1-tan(x)^2),x)

[Out] arctanh(tan(x))

maxima [A] time = 0.31, size = 15, normalized size = 1.36

$$\frac{1}{2} \log(\tan(x) + 1) - \frac{1}{2} \log(\tan(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(1-tan(x)^2),x, algorithm="maxima")

[Out] 1/2*log(tan(x) + 1) - 1/2*log(tan(x) - 1)

mupad [B] time = 3.08, size = 3, normalized size = 0.27

$$\operatorname{atanh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(cos(x)^2*(tan(x)^2 - 1)),x)

[Out] atanh(tan(x))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sec^2(x)}{\tan^2(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2/(1-tan(x)**2),x)

[Out] -Integral(sec(x)**2/(tan(x)**2 - 1), x)

$$3.692 \quad \int \frac{\sec^2(x)}{9 + \tan^2(x)} dx$$

Optimal. Leaf size=27

$$\frac{x}{3} - \frac{1}{3} \tan^{-1} \left(\frac{2 \sin(x) \cos(x)}{2 \cos^2(x) + 1} \right)$$

[Out] 1/3*x-1/3*arctan(2*cos(x)*sin(x)/(1+2*cos(x)^2))

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3675, 203}

$$\frac{x}{3} - \frac{1}{3} \tan^{-1} \left(\frac{2 \sin(x) \cos(x)}{2 \cos^2(x) + 1} \right)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/(9 + Tan[x]^2), x]

[Out] x/3 - ArcTan[(2*Cos[x]*Sin[x])/(1 + 2*Cos[x]^2)]/3

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)}{9 + \tan^2(x)} dx &= \text{Subst} \left(\int \frac{1}{9 + x^2} dx, x, \tan(x) \right) \\ &= \frac{x}{3} - \frac{1}{3} \tan^{-1} \left(\frac{2 \cos(x) \sin(x)}{1 + 2 \cos^2(x)} \right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 9, normalized size = 0.33

$$-\frac{1}{3} \tan^{-1}(3 \cot(x))$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2/(9 + Tan[x]^2), x]

[Out] -1/3*ArcTan[3*Cot[x]]

fricas [A] time = 0.55, size = 21, normalized size = 0.78

$$-\frac{1}{6} \arctan \left(\frac{10 \cos(x)^2 - 1}{6 \cos(x) \sin(x)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(9+tan(x)^2),x, algorithm="fricas")

[Out] -1/6*arctan(1/6*(10*cos(x)^2 - 1)/(cos(x)*sin(x)))

giac [A] time = 0.15, size = 7, normalized size = 0.26

$$\frac{1}{3} \arctan\left(\frac{1}{3} \tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(9+tan(x)^2),x, algorithm="giac")

[Out] 1/3*arctan(1/3*tan(x))

maple [A] time = 0.10, size = 8, normalized size = 0.30

$$\frac{\arctan\left(\frac{\tan(x)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2/(9+tan(x)^2),x)

[Out] 1/3*arctan(1/3*tan(x))

maxima [A] time = 0.41, size = 7, normalized size = 0.26

$$\frac{1}{3} \arctan\left(\frac{1}{3} \tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(9+tan(x)^2),x, algorithm="maxima")

[Out] 1/3*arctan(1/3*tan(x))

mupad [B] time = 2.88, size = 7, normalized size = 0.26

$$\frac{\operatorname{atan}\left(\frac{\tan(x)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2*(tan(x)^2 + 9)),x)

[Out] atan(tan(x)/3)/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{\tan^2(x) + 9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2/(9+tan(x)**2),x)

[Out] Integral(sec(x)**2/(tan(x)**2 + 9), x)

3.693 $\int \sec^2(x)(a + b \tan(x))^n dx$

Optimal. Leaf size=19

$$\frac{(a + b \tan(x))^{n+1}}{b(n+1)}$$

[Out] (a+b*tan(x))^(1+n)/b/(1+n)

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3506, 32}

$$\frac{(a + b \tan(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2*(a + b*Tan[x])^n,x]

[Out] (a + b*Tan[x])^(1 + n)/(b*(1 + n))

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 3506

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int \sec^2(x)(a + b \tan(x))^n dx &= \frac{\text{Subst}\left(\int (a + x)^n dx, x, b \tan(x)\right)}{b} \\ &= \frac{(a + b \tan(x))^{1+n}}{b(1+n)} \end{aligned}$$

Mathematica [A] time = 0.19, size = 18, normalized size = 0.95

$$\frac{(a + b \tan(x))^{n+1}}{bn + b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2*(a + b*Tan[x])^n,x]

[Out] (a + b*Tan[x])^(1 + n)/(b + b*n)

fricas [A] time = 1.06, size = 37, normalized size = 1.95

$$\frac{(a \cos(x) + b \sin(x)) \left(\frac{a \cos(x) + b \sin(x)}{\cos(x)} \right)^n}{(bn + b) \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(a+b*tan(x))^n,x, algorithm="fricas")

[Out] (a*cos(x) + b*sin(x))*((a*cos(x) + b*sin(x))/cos(x))^n/((b*n + b)*cos(x))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(a+b*tan(x))^n,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[0,1,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Value

maple [A] time = 0.08, size = 20, normalized size = 1.05

$$\frac{(a + b \tan(x))^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2*(a+b*tan(x))^n,x)

[Out] (a+b*tan(x))^(n+1)/b/(n+1)

maxima [A] time = 0.31, size = 19, normalized size = 1.00

$$\frac{(b \tan(x) + a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(a+b*tan(x))^n,x, algorithm="maxima")

[Out] (b*tan(x) + a)^(n + 1)/(b*(n + 1))

mupad [B] time = 3.56, size = 37, normalized size = 1.95

$$\begin{cases} \frac{\ln(a+b \tan(x))}{b} & \text{if } n = -1 \\ \frac{(a+b \tan(x))^{n+1}}{b(n+1)} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(x))^n/cos(x)^2,x)

[Out] piecewise(n == -1, log(a + b*tan(x))/b, n ~= -1, (a + b*tan(x))^(n + 1)/(b*(n + 1)))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tan(x))^n \sec^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2*(a+b*tan(x))**n,x)

[Out] Integral((a + b*tan(x))**n*sec(x)**2, x)

$$3.694 \quad \int \sec^2(x) \left(1 + \frac{1}{1+\tan^2(x)}\right) dx$$

Optimal. Leaf size=4

$$x + \tan(x)$$

[Out] x+tan(x)

Rubi [A] time = 0.04, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {203}

$$x + \tan(x)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2*(1 + (1 + Tan[x]^2)^(-1)),x]

[Out] x + Tan[x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \sec^2(x) \left(1 + \frac{1}{1+\tan^2(x)}\right) dx &= \text{Subst} \left(\int \left(1 + \frac{1}{1+x^2}\right) dx, x, \tan(x) \right) \\ &= \tan(x) + \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right) \\ &= x + \tan(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2*(1 + (1 + Tan[x]^2)^(-1)),x]

[Out] x + Tan[x]

fricas [B] time = 0.89, size = 12, normalized size = 3.00

$$\frac{x \cos(x) + \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(1+1/(1+tan(x)^2)),x, algorithm="fricas")

[Out] (x*cos(x) + sin(x))/cos(x)

giac [A] time = 0.13, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(1+1/(1+tan(x)^2)),x, algorithm="giac")

[Out] x + tan(x)

maple [A] time = 0.12, size = 5, normalized size = 1.25

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2*(1+1/(1+tan(x)^2)),x)

[Out] x+tan(x)

maxima [A] time = 0.41, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(1+1/(1+tan(x)^2)),x, algorithm="maxima")

[Out] x + tan(x)

mupad [B] time = 2.94, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/(tan(x)^2 + 1) + 1)/cos(x)^2,x)

[Out] x + tan(x)

sympy [B] time = 0.74, size = 27, normalized size = 6.75

$$\frac{x \sec^2(x)}{\tan^2(x) + 1} + \frac{\tan(x) \sec^2(x)}{\tan^2(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2*(1+1/(1+tan(x)**2)),x)

[Out] x*sec(x)**2/(tan(x)**2 + 1) + tan(x)*sec(x)**2/(tan(x)**2 + 1)

$$3.695 \quad \int \frac{\sec^2(x)(2+\tan^2(x))}{1+\tan^2(x)} dx$$

Optimal. Leaf size=4

$$x + \tan(x)$$

[Out] x+tan(x)

Rubi [A] time = 0.06, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3657, 3473, 8}

$$x + \tan(x)$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^2), x]

[Out] x + Tan[x]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)]^(n_)), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3657

Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[ActivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a, b]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)(2 + \tan^2(x))}{1 + \tan^2(x)} dx &= \int (2 + \tan^2(x)) dx \\ &= 2x + \int \tan^2(x) dx \\ &= 2x + \tan(x) - \int 1 dx \\ &= x + \tan(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^2), x]

[Out] x + Tan[x]

fricas [B] time = 0.89, size = 12, normalized size = 3.00

$$\frac{x \cos(x) + \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x, algorithm="fricas")

[Out] (x*cos(x) + sin(x))/cos(x)

giac [A] time = 0.12, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x, algorithm="giac")

[Out] x + tan(x)

maple [A] time = 0.11, size = 5, normalized size = 1.25

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x)

[Out] x+tan(x)

maxima [A] time = 0.50, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x, algorithm="maxima")

[Out] x + tan(x)

mupad [B] time = 3.00, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(x)^2 + 2)/(cos(x)^2*(tan(x)^2 + 1)),x)

[Out] x + tan(x)

sympy [B] time = 0.74, size = 27, normalized size = 6.75

$$\frac{x \sec^2(x)}{\tan^2(x) + 1} + \frac{\tan(x) \sec^2(x)}{\tan^2(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2*(2+tan(x)**2)/(1+tan(x)**2),x)

[Out] x*sec(x)**2/(tan(x)**2 + 1) + tan(x)*sec(x)**2/(tan(x)**2 + 1)

$$3.696 \quad \int \frac{\sec^2(x)}{2+2 \tan(x)+\tan^2(x)} dx$$

Optimal. Leaf size=33

$$x - \tan^{-1} \left(\frac{-2 \cos^2(x) + \sin(x) \cos(x) + 1}{\cos^2(x) + 2 \sin(x) \cos(x) + 2} \right)$$

[Out] x-arctan((1-2*cos(x)^2+cos(x)*sin(x))/(2+cos(x)^2+2*cos(x)*sin(x)))

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4342, 617, 204}

$$x - \tan^{-1} \left(\frac{-2 \cos^2(x) + \sin(x) \cos(x) + 1}{\cos^2(x) + 2 \sin(x) \cos(x) + 2} \right)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/(2 + 2*Tan[x] + Tan[x]^2), x]

[Out] x - ArcTan[(1 - 2*Cos[x]^2 + Cos[x]*Sin[x])/(2 + Cos[x]^2 + 2*Cos[x]*Sin[x])]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 4342

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFactors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a + b*x)]]/d, u, x], x], x, Tan[c*(a + b*x)]/d, x] /; FunctionOfQ[Tan[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] || EqQ[F, sec])

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)}{2+2 \tan(x)+\tan^2(x)} dx &= \text{Subst} \left(\int \frac{1}{2+2x+x^2} dx, x, \tan(x) \right) \\ &= -\text{Subst} \left(\int \frac{1}{-1-x^2} dx, x, 1+\tan(x) \right) \\ &= x - \tan^{-1} \left(\frac{1-2 \cos^2(x) + \cos(x) \sin(x)}{2 + \cos^2(x) + 2 \cos(x) \sin(x)} \right) \end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.94

$$2 \left(\frac{1}{4} \tan^{-1}(\sec(x)(\sin(x) + \cos(x))) - \frac{1}{4} \tan^{-1} \left(\frac{\cos(x)}{\sin(x) + \cos(x)} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2/(2 + 2*Tan[x] + Tan[x]^2), x]

[Out] 2*(-1/4*ArcTan[Cos[x]/(Cos[x] + Sin[x])] + ArcTan[Sec[x]*(Cos[x] + Sin[x])]/4)

fricas [A] time = 0.86, size = 35, normalized size = 1.06

$$-\frac{1}{2} \arctan\left(-\frac{3 \cos(x)^2 + 6 \cos(x) \sin(x) + 1}{2(2 \cos(x)^2 - \cos(x) \sin(x) - 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(2+2*tan(x)+tan(x)^2), x, algorithm="fricas")

[Out] -1/2*arctan(-1/2*(3*cos(x)^2 + 6*cos(x)*sin(x) + 1)/(2*cos(x)^2 - cos(x)*sin(x) - 1))

giac [A] time = 0.13, size = 5, normalized size = 0.15

$$\arctan(\tan(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(2+2*tan(x)+tan(x)^2), x, algorithm="giac")

[Out] arctan(tan(x) + 1)

maple [A] time = 0.14, size = 6, normalized size = 0.18

$$\arctan(1 + \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2/(2+2*tan(x)+tan(x)^2), x)

[Out] arctan(1+tan(x))

maxima [A] time = 0.41, size = 5, normalized size = 0.15

$$\arctan(\tan(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(2+2*tan(x)+tan(x)^2), x, algorithm="maxima")

[Out] arctan(tan(x) + 1)

mupad [B] time = 3.12, size = 5, normalized size = 0.15

$$\operatorname{atan}(\tan(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2*(2*tan(x) + tan(x)^2 + 2)), x)

[Out] atan(tan(x) + 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{\tan^2(x) + 2 \tan(x) + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2/(2+2*tan(x)+tan(x)**2), x)

[Out] Integral(sec(x)**2/(tan(x)**2 + 2*tan(x) + 2), x)

$$3.697 \quad \int \frac{\sec^2(x)}{\tan^2(x) + \tan^3(x)} dx$$

Optimal. Leaf size=10

$$\log(\cot(x) + 1) - \cot(x)$$

[Out] $-\cot(x) + \ln(1 + \cot(x))$

Rubi [A] time = 0.05, antiderivative size = 15, normalized size of antiderivative = 1.50, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4342, 44}

$$-\cot(x) - \log(\tan(x)) + \log(\tan(x) + 1)$$

Antiderivative was successfully verified.

[In] `Int[Sec[x]^2/(Tan[x]^2 + Tan[x]^3), x]`

[Out] `-Cot[x] - Log[Tan[x]] + Log[1 + Tan[x]]`

Rule 44

`Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] & & NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

Rule 4342

`Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] := With[{d = FreeFactors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a + b*x)]]/d, u, x], x], x, Tan[c*(a + b*x)]/d, x] /; FunctionOfQ[Tan[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] || EqQ[F, sec])`

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)}{\tan^2(x) + \tan^3(x)} dx &= \text{Subst} \left(\int \frac{1}{x^2(1+x)} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(\frac{1}{x^2} - \frac{1}{x} + \frac{1}{1+x} \right) dx, x, \tan(x) \right) \\ &= -\cot(x) - \log(\tan(x)) + \log(1 + \tan(x)) \end{aligned}$$

Mathematica [A] time = 0.04, size = 16, normalized size = 1.60

$$-\cot(x) - \log(\sin(x)) + \log(\sin(x) + \cos(x))$$

Antiderivative was successfully verified.

[In] `Integrate[Sec[x]^2/(Tan[x]^2 + Tan[x]^3), x]`

[Out] `-Cot[x] - Log[Sin[x]] + Log[Cos[x] + Sin[x]]`

fricas [B] time = 1.29, size = 36, normalized size = 3.60

$$\frac{\log\left(-\frac{1}{4}\cos(x)^2 + \frac{1}{4}\right)\sin(x) - \log(2\cos(x)\sin(x) + 1)\sin(x) + 2\cos(x)}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(tan(x)^2+tan(x)^3),x, algorithm="fricas")

[Out] $-1/2*(\log(-1/4*\cos(x)^2 + 1/4)*\sin(x) - \log(2*\cos(x)*\sin(x) + 1)*\sin(x) + 2*\cos(x))/\sin(x)$

giac [A] time = 0.14, size = 19, normalized size = 1.90

$$-\frac{1}{\tan(x)} + \log(|\tan(x) + 1|) - \log(|\tan(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(tan(x)^2+tan(x)^3),x, algorithm="giac")

[Out] $-1/\tan(x) + \log(\text{abs}(\tan(x) + 1)) - \log(\text{abs}(\tan(x)))$

maple [A] time = 0.13, size = 18, normalized size = 1.80

$$-\frac{1}{\tan(x)} - \ln(\tan(x)) + \ln(1 + \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2/(tan(x)^2+tan(x)^3),x)

[Out] $-1/\tan(x) - \ln(\tan(x)) + \ln(1 + \tan(x))$

maxima [A] time = 0.31, size = 17, normalized size = 1.70

$$-\frac{1}{\tan(x)} + \log(\tan(x) + 1) - \log(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(tan(x)^2+tan(x)^3),x, algorithm="maxima")

[Out] $-1/\tan(x) + \log(\tan(x) + 1) - \log(\tan(x))$

mupad [B] time = 3.10, size = 16, normalized size = 1.60

$$2 \operatorname{atanh}(2 \tan(x) + 1) - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2*(tan(x)^2 + tan(x)^3)),x)

[Out] $2*\operatorname{atanh}(2*\tan(x) + 1) - 1/\tan(x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{(\tan(x) + 1) \tan^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2/(tan(x)**2+tan(x)**3),x)

[Out] $\text{Integral}(\sec(x)**2/((\tan(x) + 1)*\tan(x)**2), x)$

$$3.698 \quad \int \frac{\sec^2(x)}{-\tan^2(x) + \tan^3(x)} dx$$

Optimal. Leaf size=10

$$\cot(x) + \log(1 - \cot(x))$$

[Out] $\cot(x) + \ln(1 - \cot(x))$

Rubi [A] time = 0.05, antiderivative size = 15, normalized size of antiderivative = 1.50, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4342, 44}

$$\cot(x) + \log(1 - \tan(x)) - \log(\tan(x))$$

Antiderivative was successfully verified.

[In] `Int[Sec[x]^2/(-Tan[x]^2 + Tan[x]^3),x]`

[Out] `Cot[x] + Log[1 - Tan[x]] - Log[Tan[x]]`

Rule 44

`Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] & & NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

Rule 4342

`Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] := With[{d = FreeFactors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a + b*x)]]/d, u, x], x], x, Tan[c*(a + b*x)]/d, x] /; FunctionOfQ[Tan[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] || EqQ[F, sec])`

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)}{-\tan^2(x) + \tan^3(x)} dx &= \text{Subst} \left(\int \frac{1}{(-1+x)x^2} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(\frac{1}{-1+x} - \frac{1}{x^2} - \frac{1}{x} \right) dx, x, \tan(x) \right) \\ &= \cot(x) + \log(1 - \tan(x)) - \log(\tan(x)) \end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.60

$$\cot(x) - \log(\sin(x)) + \log(\cos(x) - \sin(x))$$

Antiderivative was successfully verified.

[In] `Integrate[Sec[x]^2/(-Tan[x]^2 + Tan[x]^3),x]`

[Out] `Cot[x] + Log[Cos[x] - Sin[x]] - Log[Sin[x]]`

fricas [B] time = 0.87, size = 36, normalized size = 3.60

$$\frac{\log\left(-\frac{1}{4}\cos(x)^2 + \frac{1}{4}\right)\sin(x) - \log(-2\cos(x)\sin(x) + 1)\sin(x) - 2\cos(x)}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(-tan(x)^2+tan(x)^3),x, algorithm="fricas")

[Out] $-1/2*(\log(-1/4*\cos(x)^2 + 1/4)*\sin(x) - \log(-2*\cos(x)*\sin(x) + 1)*\sin(x) - 2*\cos(x))/\sin(x)$

giac [A] time = 0.16, size = 17, normalized size = 1.70

$$\frac{1}{\tan(x)} + \log(|\tan(x) - 1|) - \log(|\tan(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(-tan(x)^2+tan(x)^3),x, algorithm="giac")

[Out] $1/\tan(x) + \log(\text{abs}(\tan(x) - 1)) - \log(\text{abs}(\tan(x)))$

maple [A] time = 0.14, size = 16, normalized size = 1.60

$$\frac{1}{\tan(x)} - \ln(\tan(x)) + \ln(\tan(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2/(-tan(x)^2+tan(x)^3),x)

[Out] $1/\tan(x) - \ln(\tan(x)) + \ln(\tan(x) - 1)$

maxima [A] time = 0.31, size = 15, normalized size = 1.50

$$\frac{1}{\tan(x)} + \log(\tan(x) - 1) - \log(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(-tan(x)^2+tan(x)^3),x, algorithm="maxima")

[Out] $1/\tan(x) + \log(\tan(x) - 1) - \log(\tan(x))$

mupad [B] time = 2.98, size = 14, normalized size = 1.40

$$\frac{1}{\tan(x)} - 2 \operatorname{atanh}(2 \tan(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(cos(x)^2*(tan(x)^2 - tan(x)^3)),x)

[Out] $1/\tan(x) - 2*\operatorname{atanh}(2*\tan(x) - 1)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{(\tan(x) - 1) \tan^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2/(-tan(x)**2+tan(x)**3),x)

[Out] $\text{Integral}(\sec(x)**2/((\tan(x) - 1)*\tan(x)**2), x)$

$$3.699 \quad \int \frac{\sec^2(x)}{3-4\tan^3(x)} dx$$

Optimal. Leaf size=176

$$\frac{x}{3 \cdot 2^{2/3} \sqrt[6]{3}} + \frac{\log\left(2\sqrt[3]{2} \tan^2(x) + 2^{2/3} \sqrt[3]{3} \tan(x) + 3^{2/3}\right)}{6 \cdot 6^{2/3}} - \frac{\log\left(\sqrt[3]{3} - 2^{2/3} \tan(x)\right)}{3 \cdot 6^{2/3}} - \frac{\tan^{-1}\left(\frac{-2 \cdot 6^{2/3} \cos^2(x) + 2(3 - 2\sqrt[3]{6}) \sin(x)}{(6 - 4\sqrt[3]{6}) \cos^2(x) + 2 \cdot 6^{2/3} \sin(x)}\right)}{3 \cdot 2^{2/3} \sqrt[6]{3}}$$

[Out] 1/18*x*2^(1/3)*3^(5/6)-1/18*arctan((6^(2/3)-2*6^(2/3)*cos(x)^2+2*(3-2*6^(1/3))*cos(x)*sin(x))/(3*2^(2/3)*3^(1/6)+4*6^(1/3)+(6-4*6^(1/3))*cos(x)^2+2*6^(2/3)*cos(x)*sin(x)))*2^(1/3)*3^(5/6)-1/18*ln(3^(1/3)-2^(2/3)*tan(x))*6^(1/3)+1/36*ln(3^(2/3)+2^(2/3)*3^(1/3)*tan(x)+2*2^(1/3)*tan(x)^2)*6^(1/3)

Rubi [A] time = 0.14, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3675, 200, 31, 634, 617, 204, 628}

$$\frac{x}{3 \cdot 2^{2/3} \sqrt[6]{3}} + \frac{\log\left(2\sqrt[3]{2} \tan^2(x) + 2^{2/3} \sqrt[3]{3} \tan(x) + 3^{2/3}\right)}{6 \cdot 6^{2/3}} - \frac{\log\left(\sqrt[3]{3} - 2^{2/3} \tan(x)\right)}{3 \cdot 6^{2/3}} - \frac{\tan^{-1}\left(\frac{-2 \cdot 6^{2/3} \cos^2(x) + 2(3 - 2\sqrt[3]{6}) \sin(x)}{(6 - 4\sqrt[3]{6}) \cos^2(x) + 2 \cdot 6^{2/3} \sin(x)}\right)}{3 \cdot 2^{2/3} \sqrt[6]{3}}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/(3 - 4*Tan[x]^3), x]

[Out] x/(3*2^(2/3)*3^(1/6)) - ArcTan[(6^(2/3) - 2*6^(2/3)*Cos[x]^2 + 2*(3 - 2*6^(1/3))*Cos[x]*Sin[x])/(3*2^(2/3)*3^(1/6) + 4*6^(1/3) + (6 - 4*6^(1/3))*Cos[x]^2 + 2*6^(2/3)*Cos[x]*Sin[x])]/(3*2^(2/3)*3^(1/6)) - Log[3^(1/3) - 2^(2/3)*Tan[x]]/(3*6^(2/3)) + Log[3^(2/3) + 2^(2/3)*3^(1/3)*Tan[x] + 2*2^(1/3)*Tan[x]^2]/(6*6^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
 imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
 ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
 t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
 [2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 3675

Int[sec[(e_) + (f_)*(x_)]^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)]
))^(n_)]^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
 t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
 , x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
 tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)}{3 - 4 \tan^3(x)} dx &= \text{Subst} \left(\int \frac{1}{3 - 4x^3} dx, x, \tan(x) \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{\sqrt[3]{3} - 2^{2/3}x} dx, x, \tan(x) \right)}{3 \cdot 3^{2/3}} + \frac{\text{Subst} \left(\int \frac{2 \sqrt[3]{3} + 2^{2/3}x}{3^{2/3} + 2^{2/3} \sqrt[3]{3}x + 2 \sqrt[3]{2}x^2} dx, x, \tan(x) \right)}{3 \cdot 3^{2/3}} \\ &= -\frac{\log(\sqrt[3]{3} - 2^{2/3} \tan(x))}{3 \cdot 6^{2/3}} + \frac{\text{Subst} \left(\int \frac{1}{3^{2/3} + 2^{2/3} \sqrt[3]{3}x + 2 \sqrt[3]{2}x^2} dx, x, \tan(x) \right)}{2 \sqrt[3]{3}} + \frac{\text{Subst} \left(\int \frac{2^{2/3}}{3^{2/3} + 2^{2/3} \sqrt[3]{3}x + 2 \sqrt[3]{2}x^2} dx, x, \tan(x) \right)}{2 \sqrt[3]{3}} \\ &= -\frac{\log(\sqrt[3]{3} - 2^{2/3} \tan(x))}{3 \cdot 6^{2/3}} + \frac{\log(3^{2/3} + 2^{2/3} \sqrt[3]{3} \tan(x) + 2 \sqrt[3]{2} \tan^2(x))}{6 \cdot 6^{2/3}} - \frac{\text{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, \tan(x) \right)}{3 \cdot 6^{2/3}} \\ &= \frac{x}{3 \cdot 2^{2/3} \sqrt[6]{3}} - \frac{\tan^{-1} \left(\frac{6^{2/3} - 2 \cdot 6^{2/3} \cos^2(x) + 2(3 - 2 \sqrt[3]{6}) \cos(x) \sin(x)}{3 \cdot 2^{2/3} \sqrt[6]{3} + 4 \sqrt[3]{6} + 2(3 - 2 \sqrt[3]{6}) \cos^2(x) + 2 \cdot 6^{2/3} \cos(x) \sin(x)} \right)}{3 \cdot 2^{2/3} \sqrt[6]{3}} - \frac{\log(\sqrt[3]{3} - 2^{2/3} \tan(x))}{3 \cdot 6^{2/3}} \end{aligned}$$

Mathematica [A] time = 0.12, size = 74, normalized size = 0.42

$$\frac{2\sqrt{3} \tan^{-1} \left(\frac{2 \cdot 6^{2/3} \tan(x) + 3}{3\sqrt{3}} \right) + \log \left(2\sqrt[3]{6} \tan^2(x) + 6^{2/3} \tan(x) + 3 \right) - 2 \log \left(3 - 6^{2/3} \tan(x) \right)}{6 \cdot 6^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2/(3 - 4*Tan[x]^3), x]

[Out] (2*sqrt[3]*ArcTan[(3 + 2*6^(2/3)*Tan[x])/(3*sqrt[3])]) - 2*Log[3 - 6^(2/3)*Tan[x]] + Log[3 + 6^(2/3)*Tan[x] + 2*6^(1/3)*Tan[x]^2])/(6*6^(2/3))

fricas [B] time = 1.10, size = 441, normalized size = 2.51

$$-\frac{1}{36} \cdot 36^{\frac{1}{6}} \sqrt{3} (-1)^{\frac{1}{3}} \arctan \left(\frac{36^{\frac{1}{6}} \left(28 \left(36^{\frac{2}{3}} \sqrt{3} (-1)^{\frac{2}{3}} - 9 \sqrt{3} (-1)^{\frac{1}{3}} \right) \cos(x)^6 - 4 \left(14 \cdot 36^{\frac{2}{3}} \sqrt{3} (-1)^{\frac{2}{3}} + 36 \cdot 36^{\frac{1}{3}} \sqrt{3} \right) \cos(x)^5 + \dots \right)}{\dots}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(3-4*tan(x)^3),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/36*36^{(1/6)}*\sqrt{3}*(-1)^{(1/3)}*\arctan(-1/108*36^{(1/6)}*(28*(36^{(2/3)}*\sqrt{3} \\ & (3)*(-1)^{(2/3)} - 9*\sqrt{3}*(-1)^{(1/3)})*\cos(x)^6 - 4*(14*36^{(2/3)}*\sqrt{3}*(-1)^{(2/3)} \\ & + 36*36^{(1/3)}*\sqrt{3} - 63*\sqrt{3}*(-1)^{(1/3)})*\cos(x)^4 + (37*36^{(2/3)}*\sqrt{3}*(-1)^{(2/3)} \\ & + 144*36^{(1/3)}*\sqrt{3} + 144*\sqrt{3}*(-1)^{(1/3)})*\cos(x)^2 - 6*(16*(36^{(2/3)}*\sqrt{3}*(-1)^{(2/3)} \\ & - 9*\sqrt{3}*(-1)^{(1/3)})*\cos(x)^5 - (24*36^{(2/3)}*\sqrt{3}*(-1)^{(2/3)} - 7*36^{(1/3)}*\sqrt{3} - 72*\sqrt{3}*(-1)^{(1/3)})*\cos(x)^3 \\ & + 4*(36^{(2/3)}*\sqrt{3}*(-1)^{(2/3)} - 4*36^{(1/3)}*\sqrt{3} + 9*\sqrt{3}*(-1)^{(1/3)})*\cos(x)*\sin(x) - 18*36^{(1/3)}*\sqrt{3} - 144*\sqrt{3}*(-1)^{(1/3)}) \\ & / (48*\cos(x)^6 - 72*\cos(x)^4 + 18*\cos(x)^2 + 14*(\cos(x)^5 - \cos(x)^3)*\sin(x) + 3) - 1/432*36^{(2/3)}*(-1)^{(1/3)}*\log(-3*(2*36^{(2/3)}*(-1)^{(1/3)} - 8*36^{(1/3)}*(-1)^{(2/3)} \\ & + 25)*\cos(x)^4 + 3*(3*36^{(2/3)}*(-1)^{(1/3)} - 4*36^{(1/3)}*(-1)^{(2/3)} + 32)*\cos(x)^2 - 2*((4*36^{(2/3)}*(-1)^{(1/3)} + 9*36^{(1/3)}*(-1)^{(2/3)})*\cos(x)^3 \\ & - 4*(36^{(2/3)}*(-1)^{(1/3)} - 9)*\cos(x))*\sin(x) - 12*36^{(1/3)}*(-1)^{(2/3)} - 48) + 1/216*36^{(2/3)}*(-1)^{(1/3)}*\log(3*(2*36^{(2/3)}*(-1)^{(1/3)} + 8*36^{(1/3)}*(-1)^{(2/3)} \\ & - 7)*\cos(x)^2 + 2*(4*36^{(2/3)}*(-1)^{(1/3)} - 9*36^{(1/3)}*(-1)^{(2/3)} + 36)*\cos(x)*\sin(x) - 3*36^{(2/3)}*(-1)^{(1/3)} - 12*36^{(1/3)}*(-1)^{(2/3)} + 48) \end{aligned}$$

giac [A] time = 0.17, size = 61, normalized size = 0.35

$$\frac{1}{9}\sqrt{3}\left(\frac{3}{4}\right)^{\frac{1}{3}}\arctan\left(\frac{4}{9}\sqrt{3}\left(\frac{3}{4}\right)^{\frac{2}{3}}\left(\left(\frac{3}{4}\right)^{\frac{1}{3}}+2\tan(x)\right)\right)+\frac{1}{36}\cdot 6^{\frac{1}{3}}\log\left(\tan(x)^2+\left(\frac{3}{4}\right)^{\frac{1}{3}}\tan(x)+\left(\frac{3}{4}\right)^{\frac{2}{3}}\right)-\frac{1}{9}\left(\frac{3}{4}\right)^{\frac{1}{3}}\log\left(\left|\tan(x)^2+\left(\frac{3}{4}\right)^{\frac{1}{3}}\tan(x)+\left(\frac{3}{4}\right)^{\frac{2}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(3-4*tan(x)^3),x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/9*\sqrt{3}*(3/4)^{(1/3)}*\arctan(4/9*\sqrt{3}*(3/4)^{(2/3)}*((3/4)^{(1/3)}+2*\tan(x))) \\ & + 1/36*6^{(1/3)}*\log(\tan(x)^2+(3/4)^{(1/3)}*\tan(x)+(3/4)^{(2/3)})-1/9*(3/4)^{(1/3)}*\log(\text{abs}(-(3/4)^{(1/3)}+\tan(x))) \end{aligned}$$

maple [A] time = 0.12, size = 80, normalized size = 0.45

$$\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tan(x)-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}}{4}\right)}{36}+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tan^2(x)+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\tan(x)}{4}+\frac{3^{\frac{2}{3}}4^{\frac{1}{3}}}{4}\right)}{72}+\frac{3^{\frac{5}{6}}4^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{23^{\frac{2}{3}}4^{\frac{1}{3}}\tan(x)}{3}+1\right)}{3}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2/(3-4*tan(x)^3),x)

[Out]
$$\begin{aligned} & -1/36*3^{(1/3)}*4^{(2/3)}*\ln(\tan(x)-1/4*3^{(1/3)}*4^{(2/3)})+1/72*3^{(1/3)}*4^{(2/3)}*\ln(\tan(x)^2+1/4*3^{(1/3)}*4^{(2/3)}*\tan(x)+1/4*3^{(2/3)}*4^{(1/3)})+1/36*3^{(5/6)}*4^{(2/3)}*\arctan(1/3*3^{(1/2)}*(2/3*3^{(2/3)}*4^{(1/3)}*\tan(x)+1)) \end{aligned}$$

maxima [A] time = 0.43, size = 89, normalized size = 0.51

$$\frac{1}{36}\cdot 4^{\frac{2}{3}}3^{\frac{5}{6}}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}3^{\frac{1}{6}}\left(2\cdot 4^{\frac{2}{3}}\tan(x)+4^{\frac{1}{3}}3^{\frac{1}{3}}\right)\right)+\frac{1}{72}\cdot 4^{\frac{2}{3}}3^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}\tan(x)^2+4^{\frac{1}{3}}3^{\frac{1}{3}}\tan(x)+3^{\frac{2}{3}}\right)-\frac{1}{36}\cdot 4^{\frac{2}{3}}3^{\frac{1}{3}}\log\left(\left|\tan(x)^2+4^{\frac{1}{3}}3^{\frac{1}{3}}\tan(x)+3^{\frac{2}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(3-4*tan(x)^3),x, algorithm="maxima")

[Out] $1/36*4^{(2/3)}*3^{(5/6)}*\arctan(1/12*4^{(2/3)}*3^{(1/6)}*(2*4^{(2/3)}*\tan(x) + 4^{(1/3)}*3^{(1/3)})) + 1/72*4^{(2/3)}*3^{(1/3)}*\log(4^{(2/3)}*\tan(x)^2 + 4^{(1/3)}*3^{(1/3)}*\tan(x) + 3^{(2/3)}) - 1/36*4^{(2/3)}*3^{(1/3)}*\log(1/4*4^{(2/3)}*(4^{(1/3)}*\tan(x) - 3^{(1/3)}))$

mupad [B] time = 3.31, size = 75, normalized size = 0.43

$$\frac{6^{1/3} \ln\left(\tan(x) - \frac{6^{1/3}}{2}\right)}{18} - \frac{6^{1/3} \ln\left(\tan(x) - \frac{6^{1/3}(-1+\sqrt{3}i)}{4}\right)(-1+\sqrt{3}i)}{36} + \frac{6^{1/3} \ln\left(\tan(x) + \frac{6^{1/3}(1+\sqrt{3}i)}{4}\right)(1+\sqrt{3}i)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(cos(x)^2*(4*tan(x)^3 - 3)), x)`

[Out] $(6^{(1/3)}*\log(\tan(x) + (6^{(1/3)}*(3^{(1/2)}*1i + 1))/4)*(3^{(1/2)}*1i + 1))/36 - (6^{(1/3)}*\log(\tan(x) - (6^{(1/3)}*(3^{(1/2)}*1i - 1))/4)*(3^{(1/2)}*1i - 1))/36 - (6^{(1/3)}*\log(\tan(x) - 6^{(1/3)}/2))/18$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sec^2(x)}{4 \tan^3(x) - 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)**2/(3-4*tan(x)**3), x)`

[Out] `-Integral(sec(x)**2/(4*tan(x)**3 - 3), x)`

$$3.700 \quad \int \frac{\sec^2(x)}{11-5 \tan(x)+5 \tan^2(x)} dx$$

Optimal. Leaf size=53

$$\frac{2x}{\sqrt{195}} - \frac{2 \tan^{-1} \left(\frac{10 \cos^2(x) + 12 \sin(x) \cos(x) - 5}{12 \cos^2(x) - 10 \sin(x) \cos(x) + \sqrt{195} + 10} \right)}{\sqrt{195}}$$

[Out] 2/195*x*195^(1/2)-2/195*arctan((-5+10*cos(x)^2+12*cos(x)*sin(x))/(10+12*cos(x)^2-10*cos(x)*sin(x)+195^(1/2)))*195^(1/2)

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4342, 618, 204}

$$\frac{2x}{\sqrt{195}} - \frac{2 \tan^{-1} \left(\frac{10 \cos^2(x) + 12 \sin(x) \cos(x) - 5}{12 \cos^2(x) - 10 \sin(x) \cos(x) + \sqrt{195} + 10} \right)}{\sqrt{195}}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/(11 - 5*Tan[x] + 5*Tan[x]^2), x]

[Out] (2*x)/Sqrt[195] - (2*ArcTan[(-5 + 10*Cos[x]^2 + 12*Cos[x]*Sin[x])/(10 + Sqrt[195] + 12*Cos[x]^2 - 10*Cos[x]*Sin[x])])/Sqrt[195]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 4342

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] := With[{d = FreeFactors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a + b*x)]]/d, u, x], x], x, Tan[c*(a + b*x)]/d, x] /; FunctionOfQ[Tan[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] || EqQ[F, sec])

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)}{11-5 \tan(x)+5 \tan^2(x)} dx &= \text{Subst} \left(\int \frac{1}{11-5x+5x^2} dx, x, \tan(x) \right) \\ &= - \left(2 \text{Subst} \left(\int \frac{1}{-195-x^2} dx, x, -5+10 \tan(x) \right) \right) \\ &= \frac{2x}{\sqrt{195}} + \frac{2 \tan^{-1} \left(\frac{5-10 \cos^2(x)-12 \cos(x) \sin(x)}{10+\sqrt{195}+12 \cos^2(x)-10 \cos(x) \sin(x)} \right)}{\sqrt{195}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 22, normalized size = 0.42

$$\frac{2 \tan^{-1} \left(\sqrt{\frac{5}{39}} (1 - 2 \tan(x)) \right)}{\sqrt{195}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2/(11 - 5*Tan[x] + 5*Tan[x]^2), x]

[Out] (-2*ArcTan[Sqrt[5/39]*(1 - 2*Tan[x])])/Sqrt[195]

fricas [A] time = 0.98, size = 48, normalized size = 0.91

$$\frac{1}{195} \sqrt{195} \arctan \left(-\frac{192 \sqrt{195} \cos(x)^2 - 160 \sqrt{195} \cos(x) \sin(x) - 35 \sqrt{195}}{195 (10 \cos(x)^2 + 12 \cos(x) \sin(x) - 5)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(11-5*tan(x)+5*tan(x)^2), x, algorithm="fricas")

[Out] 1/195*sqrt(195)*arctan(-1/195*(192*sqrt(195)*cos(x)^2 - 160*sqrt(195)*cos(x)*sin(x) - 35*sqrt(195))/(10*cos(x)^2 + 12*cos(x)*sin(x) - 5))

giac [A] time = 0.13, size = 17, normalized size = 0.32

$$\frac{2}{195} \sqrt{195} \arctan \left(\frac{1}{39} \sqrt{195} (2 \tan(x) - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(11-5*tan(x)+5*tan(x)^2), x, algorithm="giac")

[Out] 2/195*sqrt(195)*arctan(1/39*sqrt(195)*(2*tan(x) - 1))

maple [A] time = 0.15, size = 18, normalized size = 0.34

$$\frac{2\sqrt{195} \arctan \left(\frac{(10 \tan(x) - 5)\sqrt{195}}{195} \right)}{195}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2/(11-5*tan(x)+5*tan(x)^2), x)

[Out] 2/195*195^(1/2)*arctan(1/195*(10*tan(x)-5)*195^(1/2))

maxima [A] time = 0.45, size = 17, normalized size = 0.32

$$\frac{2}{195} \sqrt{195} \arctan \left(\frac{1}{39} \sqrt{195} (2 \tan(x) - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(11-5*tan(x)+5*tan(x)^2), x, algorithm="maxima")

[Out] 2/195*sqrt(195)*arctan(1/39*sqrt(195)*(2*tan(x) - 1))

mupad [B] time = 3.11, size = 17, normalized size = 0.32

$$\frac{2 \sqrt{195} \operatorname{atan} \left(\frac{\sqrt{195} (2 \tan(x) - 1)}{39} \right)}{195}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cos(x)^2*(5*tan(x)^2 - 5*tan(x) + 11)),x)`

[Out] `(2*195^(1/2)*atan((195^(1/2)*(2*tan(x) - 1))/39))/195`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{5 \tan^2(x) - 5 \tan(x) + 11} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)**2/(11-5*tan(x)+5*tan(x)**2),x)`

[Out] `Integral(sec(x)**2/(5*tan(x)**2 - 5*tan(x) + 11), x)`

$$3.701 \quad \int \frac{\sec^2(x)(a+b \tan(x))}{c+d \tan(x)} dx$$

Optimal. Leaf size=28

$$\frac{b \tan(x)}{d} - \frac{(bc - ad) \log(c + d \tan(x))}{d^2}$$

[Out] $-(-a*d+b*c)*\ln(c+d*\tan(x))/d^2+b*\tan(x)/d$

Rubi [A] time = 0.09, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4342, 43}

$$\frac{b \tan(x)}{d} - \frac{(bc - ad) \log(c + d \tan(x))}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2*(a + b*Tan[x]))/(c + d*Tan[x]), x]

[Out] $-(((b*c - a*d)*\text{Log}[c + d*\text{Tan}[x]])/d^2) + (b*\text{Tan}[x])/d$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 4342

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] := With[{d = FreeFactors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a + b*x)]]/d, u, x], x], x, Tan[c*(a + b*x)]/d, x] /; FunctionOfQ[Tan[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] || EqQ[F, sec])

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)(a + b \tan(x))}{c + d \tan(x)} dx &= \text{Subst} \left(\int \frac{a + bx}{c + dx} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(\frac{b}{d} + \frac{-bc + ad}{d(c + dx)} \right) dx, x, \tan(x) \right) \\ &= -\frac{(bc - ad) \log(c + d \tan(x))}{d^2} + \frac{b \tan(x)}{d} \end{aligned}$$

Mathematica [A] time = 0.38, size = 54, normalized size = 1.93

$$\frac{\cos(x)(a + b \tan(x))((bc - ad)(\log(\cos(x)) - \log(c \cos(x) + d \sin(x))) + bd \tan(x))}{d^2(a \cos(x) + b \sin(x))}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2*(a + b*Tan[x]))/(c + d*Tan[x]), x]

[Out] $(\text{Cos}[x]*(a + b*\text{Tan}[x])*((b*c - a*d)*(\text{Log}[\text{Cos}[x]] - \text{Log}[c*\text{Cos}[x] + d*\text{Sin}[x]]) + b*d*\text{Tan}[x]))/(d^2*(a*\text{Cos}[x] + b*\text{Sin}[x]))$

fricas [B] time = 0.91, size = 71, normalized size = 2.54

$$\frac{(bc - ad) \cos(x) \log\left(2cd \cos(x) \sin(x) + (c^2 - d^2) \cos(x)^2 + d^2\right) - (bc - ad) \cos(x) \log\left(\cos(x)^2\right) - 2bd \sin(x)}{2d^2 \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x, algorithm="fricas")

[Out] -1/2*((b*c - a*d)*cos(x)*log(2*c*d*cos(x)*sin(x) + (c^2 - d^2)*cos(x)^2 + d^2) - (b*c - a*d)*cos(x)*log(cos(x)^2) - 2*b*d*sin(x))/(d^2*cos(x))

giac [A] time = 0.14, size = 29, normalized size = 1.04

$$\frac{b \tan(x)}{d} - \frac{(bc - ad) \log(|d \tan(x) + c|)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x, algorithm="giac")

[Out] b*tan(x)/d - (b*c - a*d)*log(abs(d*tan(x) + c))/d^2

maple [A] time = 0.12, size = 35, normalized size = 1.25

$$\frac{b \tan(x)}{d} + \frac{\ln(c + d \tan(x)) a}{d} - \frac{\ln(c + d \tan(x)) cb}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x)

[Out] b*tan(x)/d+1/d*ln(c+d*tan(x))*a-1/d^2*ln(c+d*tan(x))*c*b

maxima [A] time = 0.32, size = 28, normalized size = 1.00

$$\frac{b \tan(x)}{d} - \frac{(bc - ad) \log(d \tan(x) + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x, algorithm="maxima")

[Out] b*tan(x)/d - (b*c - a*d)*log(d*tan(x) + c)/d^2

mupad [B] time = 3.09, size = 27, normalized size = 0.96

$$\frac{b \tan(x)}{d} + \frac{\ln(c + d \tan(x)) (ad - bc)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tan(x))/(cos(x)^2*(c + d*tan(x))),x)

[Out] (b*tan(x))/d + (log(c + d*tan(x))*(a*d - b*c))/d^2

sympy [A] time = 4.91, size = 29, normalized size = 1.04

$$\frac{b \tan(x)}{d} + \frac{(ad - bc) \left(\begin{cases} \frac{\tan(x)}{c} & \text{for } d = 0 \\ \frac{\log(c + d \tan(x))}{d} & \text{otherwise} \end{cases} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(a+b*tan(x))/(c+d*tan(x)),x)
```

```
[Out] b*tan(x)/d + (a*d - b*c)*Piecewise((tan(x)/c, Eq(d, 0)), (log(c + d*tan(x))  
/d, True))/d
```

$$3.702 \quad \int \frac{\sec^2(x)(a+b \tan(x))^2}{c+d \tan(x)} dx$$

Optimal. Leaf size=53

$$\frac{(bc-ad)^2 \log(c+d \tan(x))}{d^3} - \frac{b \tan(x)(bc-ad)}{d^2} + \frac{(a+b \tan(x))^2}{2d}$$

[Out] $(-a*d+b*c)^2*\ln(c+d*\tan(x))/d^3-b*(-a*d+b*c)*\tan(x)/d^2+1/2*(a+b*\tan(x))^2/d$

Rubi [A] time = 0.14, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4342, 43}

$$-\frac{b \tan(x)(bc-ad)}{d^2} + \frac{(bc-ad)^2 \log(c+d \tan(x))}{d^3} + \frac{(a+b \tan(x))^2}{2d}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2*(a + b*Tan[x])^2)/(c + d*Tan[x]), x]

[Out] $((b*c - a*d)^2*\text{Log}[c + d*\text{Tan}[x]])/d^3 - (b*(b*c - a*d)*\text{Tan}[x])/d^2 + (a + b*\text{Tan}[x])^2/(2*d)$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 4342

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFactors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a + b*x)]]/d, u, x], x], x, Tan[c*(a + b*x)]/d, x] /; FunctionOfQ[Tan[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] || EqQ[F, sec])

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)(a+b \tan(x))^2}{c+d \tan(x)} dx &= \text{Subst} \left(\int \frac{(a+bx)^2}{c+dx} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(-\frac{b(bc-ad)}{d^2} + \frac{b(a+bx)}{d} + \frac{(-bc+ad)^2}{d^2(c+dx)} \right) dx, x, \tan(x) \right) \\ &= \frac{(bc-ad)^2 \log(c+d \tan(x))}{d^3} - \frac{b(bc-ad) \tan(x)}{d^2} + \frac{(a+b \tan(x))^2}{2d} \end{aligned}$$

Mathematica [A] time = 0.59, size = 62, normalized size = 1.17

$$\frac{b^2 d^2 \sec^2(x) - 2 \left(b d \tan(x)(bc - 2ad) + (bc - ad)^2 (\log(\cos(x)) - \log(c \cos(x) + d \sin(x))) \right)}{2d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2*(a + b*Tan[x])^2)/(c + d*Tan[x]), x]

[Out] $(b^2 d^2 \sec(x)^2 - 2((b*c - a*d)^2 * (\log(\cos(x)) - \log(c*\cos(x) + d*\sin(x))) + b*d*(b*c - 2*a*d)*\tan(x)))/(2*d^3)$

fricas [B] time = 1.63, size = 122, normalized size = 2.30

$$\frac{b^2 d^2 + (b^2 c^2 - 2abcd + a^2 d^2) \cos(x)^2 \log(2cd \cos(x) \sin(x) + (c^2 - d^2) \cos(x)^2 + d^2) - (b^2 c^2 - 2abcd + a^2 d^2)}{2d^3 \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x, algorithm="fricas")`

[Out] $1/2*(b^2*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\cos(x)^2*\log(2*c*d*\cos(x)*\sin(x) + (c^2 - d^2)*\cos(x)^2 + d^2) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\cos(x)^2*\log(\cos(x)^2) - 2*(b^2*c*d - 2*a*b*d^2)*\cos(x)*\sin(x))/(d^3*\cos(x)^2)$

giac [A] time = 0.15, size = 64, normalized size = 1.21

$$\frac{b^2 d \tan(x)^2 - 2b^2 c \tan(x) + 4abd \tan(x)}{2d^2} + \frac{(b^2 c^2 - 2abcd + a^2 d^2) \log(|d \tan(x) + c|)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x, algorithm="giac")`

[Out] $1/2*(b^2*d*\tan(x)^2 - 2*b^2*c*\tan(x) + 4*a*b*d*\tan(x))/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(\text{abs}(d*\tan(x) + c))/d^3$

maple [A] time = 0.13, size = 80, normalized size = 1.51

$$\frac{b^2 (\tan^2(x))}{2d} + \frac{2ba \tan(x)}{d} - \frac{b^2 \tan(x)c}{d^2} + \frac{\ln(c + d \tan(x)) a^2}{d} - \frac{2 \ln(c + d \tan(x)) abc}{d^2} + \frac{\ln(c + d \tan(x)) b^2 c^2}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x)`

[Out] $1/2*b^2/d*\tan(x)^2+2*b/d*a*\tan(x)-b^2/d^2*\tan(x)*c+1/d*\ln(c+d*\tan(x))*a^2-2/d^2*\ln(c+d*\tan(x))*a*b*c+1/d^3*\ln(c+d*\tan(x))*b^2*c^2$

maxima [A] time = 0.35, size = 63, normalized size = 1.19

$$\frac{b^2 d \tan(x)^2 - 2(b^2 c - 2abd) \tan(x)}{2d^2} + \frac{(b^2 c^2 - 2abcd + a^2 d^2) \log(d \tan(x) + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x, algorithm="maxima")`

[Out] $1/2*(b^2*d*\tan(x)^2 - 2*(b^2*c - 2*a*b*d)*\tan(x))/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(d*\tan(x) + c)/d^3$

mupad [B] time = 2.98, size = 65, normalized size = 1.23

$$\frac{\ln(c + d \tan(x)) (a^2 d^2 - 2abcd + b^2 c^2)}{d^3} - \tan(x) \left(\frac{b^2 c}{d^2} - \frac{2ab}{d} \right) + \frac{b^2 \tan(x)^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*tan(x))^2/(cos(x)^2*(c + d*tan(x))),x)`

[Out] $(\log(c + d*\tan(x))*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/d^3 - \tan(x)*((b^2*c)/d^2 - (2*a*b)/d) + (b^2*\tan(x)^2)/(2*d)$

sympy [A] time = 7.63, size = 56, normalized size = 1.06

$$\frac{b^2 \tan^2(x)}{2d} + \frac{(ad - bc)^2 \left(\begin{array}{l} \frac{\tan(x)}{c} \quad \text{for } d = 0 \\ \frac{\log(c+d \tan(x))}{d} \quad \text{otherwise} \end{array} \right)}{d^2} + \frac{(2abd - b^2c) \tan(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2*(a+b*tan(x))**2/(c+d*tan(x)),x)

[Out] b**2*tan(x)**2/(2*d) + (a*d - b*c)**2*Piecewise((tan(x)/c, Eq(d, 0)), (log(c + d*tan(x))/d, True))/d**2 + (2*a*b*d - b**2*c)*tan(x)/d**2

$$3.703 \quad \int \frac{\sec^2(x)(a+b \tan(x))^3}{c+d \tan(x)} dx$$

Optimal. Leaf size=78

$$-\frac{(bc-ad)^3 \log(c+d \tan(x))}{d^4} + \frac{b \tan(x)(bc-ad)^2}{d^3} - \frac{(bc-ad)(a+b \tan(x))^2}{2d^2} + \frac{(a+b \tan(x))^3}{3d}$$

[Out] $-(a+d \tan(x))^3 \ln(c+d \tan(x))/d^4 + b(-a+d \tan(x))^2 \tan(x)/d^3 - 1/2(-a+d \tan(x))(a+b \tan(x))^2/d^2 + 1/3(a+b \tan(x))^3/d$

Rubi [A] time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4342, 43}

$$\frac{b \tan(x)(bc-ad)^2}{d^3} - \frac{(bc-ad)(a+b \tan(x))^2}{2d^2} - \frac{(bc-ad)^3 \log(c+d \tan(x))}{d^4} + \frac{(a+b \tan(x))^3}{3d}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2*(a + b*Tan[x])^3)/(c + d*Tan[x]), x]

[Out] $-\frac{((b*c - a*d)^3 \text{Log}[c + d \text{Tan}[x]])}{d^4} + \frac{b*(b*c - a*d)^2 \text{Tan}[x]}{d^3} - \frac{(b*c - a*d)*(a + b \text{Tan}[x])^2}{(2*d^2)} + \frac{(a + b \text{Tan}[x])^3}{(3*d)}$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 4342

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] := With[{d = FreeFactors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a + b*x)]]/d, u, x], x], x, Tan[c*(a + b*x)]/d, x] /; FunctionOfQ[Tan[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] || EqQ[F, sec])

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)(a+b \tan(x))^3}{c+d \tan(x)} dx &= \text{Subst} \left(\int \frac{(a+bx)^3}{c+dx} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(\frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)(a+bx)}{d^2} + \frac{b(a+bx)^2}{d} + \frac{(-bc+ad)^3}{d^3(c+dx)} \right) dx, x, \tan(x) \right) \\ &= -\frac{(bc-ad)^3 \log(c+d \tan(x))}{d^4} + \frac{b(bc-ad)^2 \tan(x)}{d^3} - \frac{(bc-ad)(a+b \tan(x))^2}{2d^2} + \frac{(a+b \tan(x))^3}{3d} \end{aligned}$$

Mathematica [A] time = 0.96, size = 133, normalized size = 1.71

$$\frac{(a+b \tan(x))^3(c \cos(x) + d \sin(x)) (bd^2(9a \sin(2x)(ad-bc) + b(9ad-3bc+2bd \tan(x))) + 6 \cos^2(x)(bc-ad))}{6d^4(c+d \tan(x))(a \cos(x) + b \sin(x))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2*(a + b*Tan[x])^3)/(c + d*Tan[x]), x]

[Out] $((c*\text{Cos}[x] + d*\text{Sin}[x])*(a + b*\text{Tan}[x]))^3*(6*(b*c - a*d)^3*\text{Cos}[x]^2*(\text{Log}[\text{Cos}[x]] - \text{Log}[c*\text{Cos}[x] + d*\text{Sin}[x]]) - b^3*d*(-3*c^2 + d^2)*\text{Sin}[2*x] + b*d^2*(9*a*(-(b*c) + a*d)*\text{Sin}[2*x] + b*(-3*b*c + 9*a*d + 2*b*d*\text{Tan}[x]))) / (6*d^4*(a*\text{Cos}[x] + b*\text{Sin}[x])^3*(c + d*\text{Tan}[x]))$

fricas [B] time = 2.11, size = 201, normalized size = 2.58

$$\frac{3(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3) \cos(x)^3 \log(2cd \cos(x) \sin(x) + (c^2 - d^2) \cos(x)^2 + d^2) - 3(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3) \cos(x)^2 \sin(x)}{6d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x, algorithm="fricas")`

[Out] $-1/6*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\cos(x)^3*\log(2*c*d*\cos(x)*\sin(x) + (c^2 - d^2)*\cos(x)^2 + d^2) - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\cos(x)^3*\log(\cos(x)^2) + 3*(b^3*c*d^2 - 3*a*b^2*d^3)*\cos(x) - 2*(b^3*d^3 + (3*b^3*c^2*d - 9*a*b^2*c*d^2 + (9*a^2*b - b^3)*d^3)*\cos(x)^2)*\sin(x)) / (d^4*\cos(x)^3)$

giac [A] time = 0.15, size = 123, normalized size = 1.58

$$\frac{2b^3d^2 \tan(x)^3 - 3b^3cd \tan(x)^2 + 9ab^2d^2 \tan(x)^2 + 6b^3c^2 \tan(x) - 18ab^2cd \tan(x) + 18a^2bd^2 \tan(x) (b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3)}{6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x, algorithm="giac")`

[Out] $1/6*(2*b^3*d^2*\tan(x)^3 - 3*b^3*c*d*\tan(x)^2 + 9*a*b^2*d^2*\tan(x)^2 + 6*b^3*c^2*\tan(x) - 18*a*b^2*c*d*\tan(x) + 18*a^2*b*d^2*\tan(x))/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(\text{abs}(d*\tan(x) + c))/d^4$

maple [A] time = 0.16, size = 143, normalized size = 1.83

$$\frac{b^3(\tan^3(x))}{3d} + \frac{3b^2(\tan^2(x))a}{2d} - \frac{b^3(\tan^2(x))c}{2d^2} + \frac{3ba^2 \tan(x)}{d} - \frac{3b^2ac \tan(x)}{d^2} + \frac{b^3c^2 \tan(x)}{d^3} + \frac{\ln(c + d \tan(x)) a^3}{d} - \frac{3 \ln(c + d \tan(x))}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x)`

[Out] $1/3*b^3/d*\tan(x)^3+3/2*b^2/d*\tan(x)^2*a-1/2*b^3/d^2*\tan(x)^2*c+3*b/d*a^2*\tan(x)-3*b^2/d^2*a*c*\tan(x)+b^3/d^3*c^2*\tan(x)+1/d*\ln(c+d*\tan(x))*a^3-3/d^2*\ln(c+d*\tan(x))*a^2*b*c+3/d^3*\ln(c+d*\tan(x))*a*b^2*c^2-1/d^4*\ln(c+d*\tan(x))*b^3*c^3$

maxima [A] time = 0.52, size = 118, normalized size = 1.51

$$\frac{2b^3d^2 \tan(x)^3 - 3(b^3cd - 3ab^2d^2) \tan(x)^2 + 6(b^3c^2 - 3ab^2cd + 3a^2bd^2) \tan(x) (b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3)}{6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x, algorithm="maxima")`

[Out] $1/6*(2*b^3*d^2*\tan(x)^3 - 3*(b^3*c*d - 3*a*b^2*d^2)*\tan(x)^2 + 6*(b^3*c^2 - 3*a*b^2*c*d + 3*a^2*b*d^2)*\tan(x))/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*\log(d*\tan(x) + c)/d^4$

mupad [B] time = 2.96, size = 122, normalized size = 1.56

$$\tan(x) \left(\frac{3a^2b}{d} - \frac{c \left(\frac{3ab^2}{d} - \frac{b^3c}{d^2} \right)}{d} \right) + \tan(x)^2 \left(\frac{3ab^2}{2d} - \frac{b^3c}{2d^2} \right) + \frac{b^3 \tan(x)^3}{3d} + \frac{\ln(c + d \tan(x)) (a^3d^3 - 3a^2bcd^2 + 3a^2bcd^2 - a^3d^3)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(x))^3/(cos(x)^2*(c + d*tan(x))),x)
```

```
[Out] tan(x)*((3*a^2*b)/d - (c*((3*a*b^2)/d - (b^3*c)/d^2))/d) + tan(x)^2*((3*a*b^2)/(2*d) - (b^3*c)/(2*d^2)) + (b^3*tan(x)^3)/(3*d) + (log(c + d*tan(x))*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/d^4
```

sympy [A] time = 10.57, size = 95, normalized size = 1.22

$$\frac{b^3 \tan^3(x)}{3d} + \frac{(3ab^2d - b^3c) \tan^2(x)}{2d^2} + \frac{(ad - bc)^3 \begin{cases} \frac{\tan(x)}{c} & \text{for } d = 0 \\ \frac{\log(c+d \tan(x))}{d} & \text{otherwise} \end{cases}}{d^3} + \frac{(3a^2bd^2 - 3ab^2cd + b^3c^2) \tan(x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(a+b*tan(x))**3/(c+d*tan(x)),x)
```

```
[Out] b**3*tan(x)**3/(3*d) + (3*a*b**2*d - b**3*c)*tan(x)**2/(2*d**2) + (a*d - b*c)**3*Piecewise((tan(x)/c, Eq(d, 0)), (log(c + d*tan(x))/d, True))/d**3 + (3*a**2*b*d**2 - 3*a*b**2*c*d + b**3*c**2)*tan(x)/d**3
```

$$3.704 \quad \int \frac{\sec^2(x) \tan^2(x)}{(2 + \tan^3(x))^2} dx$$

Optimal. Leaf size=12

$$-\frac{1}{3(\tan^3(x) + 2)}$$

[Out] -1/3/(2+tan(x)^3)

Rubi [A] time = 0.08, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4342, 261}

$$-\frac{1}{3(\tan^3(x) + 2)}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2*Tan[x]^2)/(2 + Tan[x]^3)^2,x]

[Out] -1/(3*(2 + Tan[x]^3))

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 4342

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFactors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a + b*x)]]/d, u, x], x], x, Tan[c*(a + b*x)]/d, x] /; FunctionOfQ[Tan[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] || EqQ[F, sec])

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x) \tan^2(x)}{(2 + \tan^3(x))^2} dx &= \text{Subst} \left(\int \frac{x^2}{(2 + x^3)^2} dx, x, \tan(x) \right) \\ &= -\frac{1}{3(2 + \tan^3(x))} \end{aligned}$$

Mathematica [A] time = 0.03, size = 12, normalized size = 1.00

$$-\frac{1}{3(\tan^3(x) + 2)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2*Tan[x]^2)/(2 + Tan[x]^3)^2,x]

[Out] -1/3*1/(2 + Tan[x]^3)

fricas [B] time = 0.85, size = 36, normalized size = 3.00

$$\frac{\cos(x)^3 + 2(\cos(x)^2 - 1)\sin(x)}{15(2\cos(x)^3 - (\cos(x)^2 - 1)\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x, algorithm="fricas")

[Out] -1/15*(cos(x)^3 + 2*(cos(x)^2 - 1)*sin(x))/(2*cos(x)^3 - (cos(x)^2 - 1)*sin(x))

giac [A] time = 0.15, size = 10, normalized size = 0.83

$$-\frac{1}{3(\tan(x)^3 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x, algorithm="giac")

[Out] -1/3/(tan(x)^3 + 2)

maple [A] time = 0.13, size = 11, normalized size = 0.92

$$-\frac{1}{3(2 + \tan^3(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x)

[Out] -1/3/(2+tan(x)^3)

maxima [A] time = 0.32, size = 10, normalized size = 0.83

$$-\frac{1}{3(\tan(x)^3 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x, algorithm="maxima")

[Out] -1/3/(tan(x)^3 + 2)

mupad [B] time = 2.93, size = 12, normalized size = 1.00

$$-\frac{1}{3(\tan(x)^3 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)^2/(cos(x)^2*(tan(x)^3 + 2)^2),x)

[Out] -1/(3*(tan(x)^3 + 2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2*tan(x)**2/(2+tan(x)**3)**2,x)

[Out] Timed out

$$3.705 \quad \int \sec^2(x) \tan^6(x) (1 + \tan^2(x))^3 dx$$

Optimal. Leaf size=33

$$\frac{\tan^{13}(x)}{13} + \frac{3 \tan^{11}(x)}{11} + \frac{\tan^9(x)}{3} + \frac{\tan^7(x)}{7}$$

[Out] 1/7*tan(x)^7+1/3*tan(x)^9+3/11*tan(x)^11+1/13*tan(x)^13

Rubi [A] time = 0.09, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3657, 2607, 270}

$$\frac{\tan^{13}(x)}{13} + \frac{3 \tan^{11}(x)}{11} + \frac{\tan^9(x)}{3} + \frac{\tan^7(x)}{7}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2*Tan[x]^6*(1 + Tan[x]^2)^3,x]

[Out] Tan[x]^7/7 + Tan[x]^9/3 + (3*Tan[x]^11)/11 + Tan[x]^13/13

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2607

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 3657

Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2^(p_), x_Symbol] :> Int[ActivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a, b]

Rubi steps

$$\begin{aligned} \int \sec^2(x) \tan^6(x) (1 + \tan^2(x))^3 dx &= \int \sec^8(x) \tan^6(x) dx \\ &= \text{Subst} \left(\int x^6 (1 + x^2)^3 dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int (x^6 + 3x^8 + 3x^{10} + x^{12}) dx, x, \tan(x) \right) \\ &= \frac{\tan^7(x)}{7} + \frac{\tan^9(x)}{3} + \frac{3 \tan^{11}(x)}{11} + \frac{\tan^{13}(x)}{13} \end{aligned}$$

Mathematica [B] time = 0.02, size = 67, normalized size = 2.03

$$-\frac{16 \tan(x)}{3003} + \frac{1}{13} \tan(x) \sec^{12}(x) - \frac{27}{143} \tan(x) \sec^{10}(x) + \frac{53}{429} \tan(x) \sec^8(x) - \frac{5 \tan(x) \sec^6(x)}{3003} - \frac{2 \tan(x) \sec^4(x)}{1001} - \frac{8 \tan(x)}{1001}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2*Tan[x]^6*(1 + Tan[x]^2)^3,x]

[Out] $(-16*\text{Tan}[x])/3003 - (8*\text{Sec}[x]^2*\text{Tan}[x])/3003 - (2*\text{Sec}[x]^4*\text{Tan}[x])/1001 - (5*\text{Sec}[x]^6*\text{Tan}[x])/3003 + (53*\text{Sec}[x]^8*\text{Tan}[x])/429 - (27*\text{Sec}[x]^10*\text{Tan}[x])/143 + (\text{Sec}[x]^12*\text{Tan}[x])/13$

fricas [A] time = 0.86, size = 46, normalized size = 1.39

$$\frac{(16 \cos(x)^{12} + 8 \cos(x)^{10} + 6 \cos(x)^8 + 5 \cos(x)^6 - 371 \cos(x)^4 + 567 \cos(x)^2 - 231) \sin(x)}{3003 \cos(x)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x, algorithm="fricas")

[Out] $-1/3003*(16*\cos(x)^{12} + 8*\cos(x)^{10} + 6*\cos(x)^8 + 5*\cos(x)^6 - 371*\cos(x)^4 + 567*\cos(x)^2 - 231)*\sin(x)/\cos(x)^{13}$

giac [A] time = 0.14, size = 25, normalized size = 0.76

$$\frac{1}{13} \tan(x)^{13} + \frac{3}{11} \tan(x)^{11} + \frac{1}{3} \tan(x)^9 + \frac{1}{7} \tan(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x, algorithm="giac")

[Out] $1/13*\tan(x)^{13} + 3/11*\tan(x)^{11} + 1/3*\tan(x)^9 + 1/7*\tan(x)^7$

maple [A] time = 0.06, size = 42, normalized size = 1.27

$$\frac{\sin^7(x)}{7 \cos(x)^7} + \frac{\sin^9(x)}{3 \cos(x)^9} + \frac{3(\sin^{11}(x))}{11 \cos(x)^{11}} + \frac{\sin^{13}(x)}{13 \cos(x)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x)

[Out] $1/7*\sin(x)^7/\cos(x)^7+1/3*\sin(x)^9/\cos(x)^9+3/11*\sin(x)^{11}/\cos(x)^{11}+1/13*\sin(x)^{13}/\cos(x)^{13}$

maxima [A] time = 0.39, size = 25, normalized size = 0.76

$$\frac{1}{13} \tan(x)^{13} + \frac{3}{11} \tan(x)^{11} + \frac{1}{3} \tan(x)^9 + \frac{1}{7} \tan(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x, algorithm="maxima")

[Out] $1/13*\tan(x)^{13} + 3/11*\tan(x)^{11} + 1/3*\tan(x)^9 + 1/7*\tan(x)^7$

mupad [B] time = 2.92, size = 25, normalized size = 0.76

$$\frac{\tan(x)^{13}}{13} + \frac{3 \tan(x)^{11}}{11} + \frac{\tan(x)^9}{3} + \frac{\tan(x)^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(x)^6*(tan(x)^2 + 1)^3)/cos(x)^2,x)

[Out] $\tan(x)^7/7 + \tan(x)^9/3 + (3*\tan(x)^{11})/11 + \tan(x)^{13}/13$

sympy [A] time = 19.28, size = 27, normalized size = 0.82

$$\frac{\tan^{13}(x)}{13} + \frac{3 \tan^{11}(x)}{11} + \frac{\tan^9(x)}{3} + \frac{\tan^7(x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2*tan(x)**6*(1+tan(x)**2)**3,x)

[Out] tan(x)**13/13 + 3*tan(x)**11/11 + tan(x)**9/3 + tan(x)**7/7

$$3.706 \quad \int \frac{\sec^2(x)(2+\tan^2(x))}{1+\tan^3(x)} dx$$

Optimal. Leaf size=46

$$\frac{2x}{\sqrt{3}} + \log(\tan(x) + 1) + \frac{2 \tan^{-1}\left(\frac{1-2\cos^2(x)}{-2\sin(x)\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

[Out] ln(1+tan(x))+2/3*x*3^(1/2)+2/3*arctan((1-2*cos(x)^2)/(2-2*cos(x)*sin(x)+3^(1/2)))*3^(1/2)

Rubi [A] time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {4342, 1863, 31, 618, 204}

$$\frac{2x}{\sqrt{3}} + \log(\tan(x) + 1) + \frac{2 \tan^{-1}\left(\frac{1-2\cos^2(x)}{-2\sin(x)\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^3), x]

[Out] (2*x)/Sqrt[3] + (2*ArcTan[(1 - 2*Cos[x]^2)/(2 + Sqrt[3] - 2*Cos[x]*Sin[x])])/Sqrt[3] + Log[1 + Tan[x]]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1863

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x], x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 4342

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFactors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a + b*x)]]/d, u, x], x], x, Tan[c*(a + b*x)]/d, x] /; FunctionOfQ[Tan[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] || EqQ[F, sec])

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x)(2 + \tan^2(x))}{1 + \tan^3(x)} dx &= \text{Subst} \left(\int \frac{2 + x^2}{1 + x^3} dx, x, \tan(x) \right) \\
&= \text{Subst} \left(\int \frac{1}{1 + x} dx, x, \tan(x) \right) + \text{Subst} \left(\int \frac{1}{1 - x + x^2} dx, x, \tan(x) \right) \\
&= \log(1 + \tan(x)) - 2 \text{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, -1 + 2 \tan(x) \right) \\
&= \frac{2x}{\sqrt{3}} + \frac{2 \tan^{-1} \left(\frac{1 - 2 \cos^2(x)}{2 + \sqrt{3} - 2 \cos(x) \sin(x)} \right)}{\sqrt{3}} + \log(1 + \tan(x))
\end{aligned}$$

Mathematica [A] time = 0.22, size = 32, normalized size = 0.70

$$-\frac{2 \tan^{-1} \left(\frac{1 - 2 \tan(x)}{\sqrt{3}} \right)}{\sqrt{3}} - \log(\cos(x)) + \log(\sin(x) + \cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^3), x]

[Out] (-2*ArcTan[(1 - 2*Tan[x])/Sqrt[3]])/Sqrt[3] - Log[Cos[x]] + Log[Cos[x] + Sin[x]]

fricas [A] time = 2.51, size = 52, normalized size = 1.13

$$\frac{1}{3} \sqrt{3} \arctan \left(\frac{4 \sqrt{3} \cos(x) \sin(x) - \sqrt{3}}{3(2 \cos(x)^2 - 1)} \right) - \frac{1}{2} \log(\cos(x)^2) + \frac{1}{2} \log(2 \cos(x) \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3), x, algorithm="fricas")

[Out] 1/3*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) - sqrt(3))/(2*cos(x)^2 - 1)) - 1/2*log(cos(x)^2) + 1/2*log(2*cos(x)*sin(x) + 1)

giac [A] time = 0.16, size = 24, normalized size = 0.52

$$\frac{2}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2 \tan(x) - 1) \right) + \log(|\tan(x) + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3), x, algorithm="giac")

[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) - 1)) + log(abs(tan(x) + 1))

maple [A] time = 0.20, size = 24, normalized size = 0.52

$$\frac{2\sqrt{3} \arctan \left(\frac{(-1+2 \tan(x))\sqrt{3}}{3} \right)}{3} + \ln(1 + \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3), x)

[Out] $2/3 \cdot 3^{1/2} \cdot \arctan(1/3 \cdot (-1 + 2 \cdot \tan(x)) \cdot 3^{1/2}) + \ln(1 + \tan(x))$

maxima [A] time = 0.41, size = 23, normalized size = 0.50

$$\frac{2}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 \tan(x) - 1)\right) + \log(\tan(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x, algorithm="maxima")`

[Out] $2/3 \cdot \sqrt{3} \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (2 \cdot \tan(x) - 1)) + \log(\tan(x) + 1)$

mupad [B] time = 2.97, size = 30, normalized size = 0.65

$$\ln(\tan(x) + 1) - \frac{2\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3} - \sqrt{3} \tan(x)}{\tan(x) + 1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((tan(x)^2 + 2)/(cos(x)^2*(tan(x)^3 + 1)),x)`

[Out] $\log(\tan(x) + 1) - (2 \cdot 3^{1/2} \cdot \operatorname{atan}((3^{1/2} - 3^{1/2} \cdot \tan(x)) / (\tan(x) + 1))) / 3$

sympy [A] time = 9.25, size = 41, normalized size = 0.89

$$\frac{2\sqrt{3} \left(\operatorname{atan}\left(\frac{2\sqrt{3}(\tan(x) - \frac{1}{2})}{3}\right) + \pi \left\lfloor \frac{x - \frac{\pi}{2}}{\pi} \right\rfloor \right)}{3} + \log(\tan(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)**2*(2+tan(x)**2)/(1+tan(x)**3),x)`

[Out] $2 \cdot \sqrt{3} \cdot (\operatorname{atan}(2 \cdot \sqrt{3} \cdot (\tan(x) - 1/2) / 3) + \pi \cdot \operatorname{floor}((x - \pi/2) / \pi)) / 3 + \log(\tan(x) + 1)$

3.707 $\int (1 + \cos^2(x)) \sec^2(x) dx$

Optimal. Leaf size=4

$$x + \tan(x)$$

[Out] x+tan(x)

Rubi [A] time = 0.02, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3012, 8}

$$x + \tan(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x]^2)*Sec[x]^2,x]

[Out] x + Tan[x]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rubi steps

$$\begin{aligned} \int (1 + \cos^2(x)) \sec^2(x) dx &= \tan(x) + \int 1 dx \\ &= x + \tan(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Cos[x]^2)*Sec[x]^2,x]

[Out] x + Tan[x]

fricas [B] time = 0.41, size = 12, normalized size = 3.00

$$\frac{x \cos(x) + \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)^2)*sec(x)^2,x, algorithm="fricas")

[Out] (x*cos(x) + sin(x))/cos(x)

giac [B] time = 0.19, size = 15, normalized size = 3.75

$$-\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor + x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)^2)*sec(x)^2,x, algorithm="giac")

[Out] $-\pi \cdot \text{floor}(x/\pi + 1/2) + x + \tan(x)$

maple [A] time = 0.08, size = 5, normalized size = 1.25

$x + \tan(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+cos(x)^2)*sec(x)^2,x)

[Out] $x + \tan(x)$

maxima [A] time = 0.46, size = 4, normalized size = 1.00

$x + \tan(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)^2)*sec(x)^2,x, algorithm="maxima")

[Out] $x + \tan(x)$

mupad [B] time = 2.88, size = 4, normalized size = 1.00

$x + \tan(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x)^2 + 1)/cos(x)^2,x)

[Out] $x + \tan(x)$

sympy [A] time = 5.38, size = 3, normalized size = 0.75

$x + \tan(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)**2)*sec(x)**2,x)

[Out] $x + \tan(x)$

$$3.708 \quad \int \frac{\sec^2(x)}{1+\sec^2(x)-3 \tan(x)} dx$$

Optimal. Leaf size=21

$$\log(2 \cos(x) - \sin(x)) - \log(\cos(x) - \sin(x))$$

[Out] $-\ln(\cos(x)-\sin(x))+\ln(2*\cos(x)-\sin(x))$

Rubi [A] time = 0.12, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {616, 31}

$$\log(2 \cos(x) - \sin(x)) - \log(\cos(x) - \sin(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sec}[x]^2/(1 + \text{Sec}[x]^2 - 3*\text{Tan}[x]), x]$

[Out] $-\text{Log}[\text{Cos}[x] - \text{Sin}[x]] + \text{Log}[2*\text{Cos}[x] - \text{Sin}[x]]$

Rule 31

$\text{Int}[(a_ + (b_)*(x_))^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rule 616

$\text{Int}[(a_ + (b_)*(x_ + (c_)*(x_)^2))^{(-1)}, x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[c/q, \text{Int}[1/\text{Simp}[b/2 - q/2 + c*x, x], x], x] - \text{Dist}[c/q, \text{Int}[1/\text{Simp}[b/2 + q/2 + c*x, x], x], x]] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c] \ \&\& \ \text{PerfectSquareQ}[b^2 - 4*a*c]$

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)}{1 + \sec^2(x) - 3 \tan(x)} dx &= \text{Subst} \left(\int \frac{1}{2 - 3x + x^2} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \frac{1}{-2 + x} dx, x, \tan(x) \right) - \text{Subst} \left(\int \frac{1}{-1 + x} dx, x, \tan(x) \right) \\ &= -\log(1 - \tan(x)) + \log(2 - \tan(x)) \end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.38

$$2 \left(\frac{1}{2} \log(2 \cos(x) - \sin(x)) - \frac{1}{2} \log(\cos(x) - \sin(x)) \right)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Sec}[x]^2/(1 + \text{Sec}[x]^2 - 3*\text{Tan}[x]), x]$

[Out] $2*(-1/2*\text{Log}[\text{Cos}[x] - \text{Sin}[x]] + \text{Log}[2*\text{Cos}[x] - \text{Sin}[x]])/2$

fricas [A] time = 0.58, size = 29, normalized size = 1.38

$$\frac{1}{2} \log \left(\frac{3}{4} \cos(x)^2 - \cos(x) \sin(x) + \frac{1}{4} \right) - \frac{1}{2} \log(-2 \cos(x) \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="fricas")

[Out] 1/2*log(3/4*cos(x)^2 - cos(x)*sin(x) + 1/4) - 1/2*log(-2*cos(x)*sin(x) + 1)

giac [A] time = 0.16, size = 15, normalized size = 0.71

$$-\log(|\tan(x) - 1|) + \log(|\tan(x) - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="giac")

[Out] -log(abs(tan(x) - 1)) + log(abs(tan(x) - 2))

maple [A] time = 0.13, size = 14, normalized size = 0.67

$$\ln(-2 + \tan(x)) - \ln(\tan(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2/(1+sec(x)^2-3*tan(x)),x)

[Out] ln(-2+tan(x))-ln(tan(x)-1)

maxima [A] time = 0.32, size = 13, normalized size = 0.62

$$-\log(\tan(x) - 1) + \log(\tan(x) - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="maxima")

[Out] -log(tan(x) - 1) + log(tan(x) - 2)

mupad [B] time = 3.51, size = 9, normalized size = 0.43

$$-2 \operatorname{atanh}(2 \tan(x) - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2*(1/cos(x)^2 - 3*tan(x) + 1)),x)

[Out] -2*atanh(2*tan(x) - 3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{-3 \tan(x) + \sec^2(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2/(1+sec(x)**2-3*tan(x)),x)

[Out] Integral(sec(x)**2/(-3*tan(x) + sec(x)**2 + 1), x)

$$3.709 \quad \int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}} dx$$

Optimal. Leaf size=9

$$\sin^{-1}\left(\frac{\tan(x)}{\sqrt{3}}\right)$$

[Out] arcsin(1/3*tan(x)*3^(1/2))

Rubi [A] time = 0.05, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4146, 216}

$$\sin^{-1}\left(\frac{\tan(x)}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/Sqrt[4 - Sec[x]^2], x]

[Out] ArcSin[Tan[x]/Sqrt[3]]

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 4146

Int[sec[(e_) + (f_)*(x_)]^(m_)*((a_) + (b_)*sec[(e_) + (f_)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x]^p, x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[n/2]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}} dx &= \text{Subst}\left(\int \frac{1}{\sqrt{3-x^2}} dx, x, \tan(x)\right) \\ &= \sin^{-1}\left(\frac{\tan(x)}{\sqrt{3}}\right) \end{aligned}$$

Mathematica [B] time = 0.04, size = 43, normalized size = 4.78

$$\frac{\sqrt{2 \cos(2x) + 1} \sec(x) \tan^{-1}\left(\frac{\sin(x)}{\sqrt{3-4 \sin^2(x)}}\right)}{\sqrt{4-\sec^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2/Sqrt[4 - Sec[x]^2], x]

[Out] (ArcTan[Sin[x]/Sqrt[3 - 4*Sin[x]^2])*Sqrt[1 + 2*Cos[2*x]]*Sec[x])/Sqrt[4 - Sec[x]^2]

fricas [B] time = 0.83, size = 25, normalized size = 2.78

$$-\arctan\left(\frac{\sqrt{\frac{4\cos(x)^2-1}{\cos(x)^2}}\cos(x)}{\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(4-sec(x)^2)^(1/2),x, algorithm="fricas")

[Out] -arctan(sqrt((4*cos(x)^2 - 1)/cos(x)^2)*cos(x)/sin(x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(x)^2}{\sqrt{-\sec(x)^2 + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(4-sec(x)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sec(x)^2/sqrt(-sec(x)^2 + 4), x)

maple [C] time = 0.28, size = 103, normalized size = 11.44

$$\frac{\sqrt{2} \sqrt{\frac{2\cos(x)-1}{1+\cos(x)}} \sqrt{6} \sqrt{\frac{1+2\cos(x)}{1+\cos(x)}} \left(\text{EllipticF}\left(\frac{\sqrt{3}(-1+\cos(x))}{\sin(x)}, \frac{1}{3}\right) - 2 \text{EllipticPi}\left(\frac{\sqrt{3}(-1+\cos(x))}{\sin(x)}, \frac{1}{3}, \frac{1}{3}\right) \right) (\sin^2(x)) \sqrt{3}}{9 \sqrt{\frac{4(\cos^2(x))-1}{\cos(x)^2}} \cos(x) (-1 + \cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2/(4-sec(x)^2)^(1/2),x)

[Out] -1/9*2^(1/2)*((2*cos(x)-1)/(1+cos(x)))^(1/2)*6^(1/2)*((1+2*cos(x))/(1+cos(x)))^(1/2)*(EllipticF(3^(1/2)*(-1+cos(x))/sin(x),1/3)-2*EllipticPi(3^(1/2)*(-1+cos(x))/sin(x),1/3,1/3))*sin(x)^2/((4*cos(x)^2-1)/cos(x)^2)^(1/2)/cos(x)/(-1+cos(x))*3^(1/2)

maxima [A] time = 0.43, size = 8, normalized size = 0.89

$$\arcsin\left(\frac{1}{3}\sqrt{3}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(4-sec(x)^2)^(1/2),x, algorithm="maxima")

[Out] arcsin(1/3*sqrt(3)*tan(x))

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\cos(x)^2 \sqrt{4 - \frac{1}{\cos(x)^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2*(4 - 1/cos(x)^2)^(1/2)),x)

[Out] int(1/(cos(x)^2*(4 - 1/cos(x)^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{\sqrt{-(\sec(x) - 2)(\sec(x) + 2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2/(4-sec(x)**2)**(1/2),x)

[Out] Integral(sec(x)**2/sqrt(-(sec(x) - 2)*(sec(x) + 2)), x)

$$3.710 \quad \int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}} dx$$

Optimal. Leaf size=9

$$\frac{1}{2} \sin^{-1}(2 \tan(x))$$

[Out] 1/2*arcsin(2*tan(x))

Rubi [A] time = 0.05, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3675, 216}

$$\frac{1}{2} \sin^{-1}(2 \tan(x))$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/Sqrt[1 - 4*Tan[x]^2], x]

[Out] ArcSin[2*Tan[x]]/2

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 3675

Int[sec[(e_) + (f_)*(x_)]^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{\sqrt{1-4x^2}} dx, x, \tan(x) \right) \\ &= \frac{1}{2} \sin^{-1}(2 \tan(x)) \end{aligned}$$

Mathematica [B] time = 0.06, size = 47, normalized size = 5.22

$$\frac{\sqrt{5 \cos(2x) - 3} \sec(x) \tan^{-1} \left(\frac{2 \sin(x)}{\sqrt{1-5 \sin^2(x)}} \right)}{2\sqrt{2-8 \tan^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2/Sqrt[1 - 4*Tan[x]^2], x]

[Out] (ArcTan[(2*Sin[x])/Sqrt[1 - 5*Sin[x]^2]]*Sqrt[-3 + 5*Cos[2*x]]*Sec[x])/(2*Sqrt[2 - 8*Tan[x]^2])

fricas [B] time = 0.83, size = 45, normalized size = 5.00

$$-\frac{1}{4} \arctan \left(\frac{(9 \cos(x)^3 - 8 \cos(x)) \sqrt{\frac{5 \cos(x)^2 - 4}{\cos(x)^2}}}{4(5 \cos(x)^2 - 4) \sin(x)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(1-4*tan(x)^2)^(1/2),x, algorithm="fricas")

[Out] -1/4*arctan(1/4*(9*cos(x)^3 - 8*cos(x))*sqrt((5*cos(x)^2 - 4)/cos(x)^2)/((5*cos(x)^2 - 4)*sin(x)))

giac [A] time = 0.17, size = 7, normalized size = 0.78

$$\frac{1}{2} \arcsin(2 \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(1-4*tan(x)^2)^(1/2),x, algorithm="giac")

[Out] 1/2*arcsin(2*tan(x))

maple [C] time = 0.69, size = 171, normalized size = 19.00

$$\frac{\sqrt{2} \sqrt{\frac{2 \cos(x) \sqrt{5} + 5 \cos(x) - 2 \sqrt{5} - 4}{1 + \cos(x)}} \sqrt{\frac{2(2 \cos(x) \sqrt{5} - 5 \cos(x) - 2 \sqrt{5} + 4)}{1 + \cos(x)}} \left(\text{EllipticF} \left(\frac{(-1 + \cos(x))(\sqrt{5} + 2)}{\sin(x)}, 9 - 4\sqrt{5} \right) - 2 \text{EllipticPi} \left(\frac{(-1 + \cos(x))(\sqrt{5} + 2)}{\sin(x)}, 9 - 4\sqrt{5} \right) \right)}{\sqrt{\frac{5(\cos^2(x) - 4)}{\cos(x)^2}} \cos(x) (-1 + \cos(x)) \sqrt{9 + 4\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2/(1-4*tan(x)^2)^(1/2),x)

[Out] -2^(1/2)*((2*cos(x)*5^(1/2)+5*cos(x)-2*5^(1/2)-4)/(1+cos(x)))^(1/2)*(-2*(2*cos(x)*5^(1/2)-5*cos(x)-2*5^(1/2)+4)/(1+cos(x)))^(1/2)*(EllipticF((-1+cos(x))*(5^(1/2)+2)/sin(x),9-4*5^(1/2))-2*EllipticPi((-1+cos(x))/sin(x),1/(9+4*5^(1/2)), (9-4*5^(1/2))^(1/2)/(9+4*5^(1/2))^(1/2)))*sin(x)^2/((5*cos(x)^2-4)/cos(x)^2)^(1/2)/cos(x)/(-1+cos(x))/(9+4*5^(1/2))^(1/2)

maxima [A] time = 0.42, size = 7, normalized size = 0.78

$$\frac{1}{2} \arcsin(2 \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(1-4*tan(x)^2)^(1/2),x, algorithm="maxima")

[Out] 1/2*arcsin(2*tan(x))

mupad [F] time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\cos(x)^2 \sqrt{1 - 4 \tan(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2*(1 - 4*tan(x)^2)^(1/2)),x)

[Out] `int(1/(cos(x)^2*(1 - 4*tan(x)^2)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{\sqrt{-(2 \tan(x) - 1)(2 \tan(x) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)**2/(1-4*tan(x)**2)^(1/2), x)`

[Out] `Integral(sec(x)**2/sqrt(-(2*tan(x) - 1)*(2*tan(x) + 1)), x)`

$$3.711 \quad \int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}} dx$$

Optimal. Leaf size=14

$$\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)-4}}\right)$$

[Out] arctanh(tan(x)/(-4+tan(x)^2)^(1/2))

Rubi [A] time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3675, 217, 206}

$$\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)-4}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/Sqrt[-4 + Tan[x]^2], x]

[Out] ArcTanh[Tan[x]/Sqrt[-4 + Tan[x]^2]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^n_)]^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{\sqrt{-4+x^2}} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \frac{\tan(x)}{\sqrt{-4+\tan^2(x)}} \right) \\ &= \tanh^{-1} \left(\frac{\tan(x)}{\sqrt{-4+\tan^2(x)}} \right) \end{aligned}$$

Mathematica [B] time = 0.05, size = 46, normalized size = 3.29

$$\frac{\sqrt{5 \cos(2x) + 3} \sec(x) \tan^{-1} \left(\frac{\sin(x)}{\sqrt{4-5 \sin^2(x)}} \right)}{\sqrt{2} \sqrt{\tan^2(x) - 4}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2/Sqrt[-4 + Tan[x]^2], x]

[Out] (ArcTan[Sin[x]/Sqrt[4 - 5*Sin[x]^2])*Sqrt[3 + 5*Cos[2*x]]*Sec[x])/(Sqrt[2]*Sqrt[-4 + Tan[x]^2])

fricas [B] time = 1.00, size = 67, normalized size = 4.79

$$\frac{1}{4} \log \left(\frac{1}{2} \sqrt{-\frac{5 \cos(x)^2 - 1}{\cos(x)^2}} \cos(x) \sin(x) - \frac{3}{2} \cos(x)^2 + \frac{1}{2} \right) - \frac{1}{4} \log \left(-\frac{1}{2} \sqrt{-\frac{5 \cos(x)^2 - 1}{\cos(x)^2}} \cos(x) \sin(x) - \frac{3}{2} \cos(x)^2 + \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(-4+tan(x)^2)^(1/2), x, algorithm="fricas")

[Out] 1/4*log(1/2*sqrt(-(5*cos(x)^2 - 1)/cos(x)^2)*cos(x)*sin(x) - 3/2*cos(x)^2 + 1/2) - 1/4*log(-1/2*sqrt(-(5*cos(x)^2 - 1)/cos(x)^2)*cos(x)*sin(x) - 3/2*cos(x)^2 + 1/2)

giac [A] time = 0.18, size = 17, normalized size = 1.21

$$-\log \left(\left| \sqrt{\tan(x)^2 - 4} - \tan(x) \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(-4+tan(x)^2)^(1/2), x, algorithm="giac")

[Out] -log(abs(sqrt(tan(x)^2 - 4) - tan(x)))

maple [C] time = 0.72, size = 171, normalized size = 12.21

$$\frac{\sqrt{-\frac{2(\cos(x)\sqrt{5}-5\cos(x)-\sqrt{5}+1)}{1+\cos(x)}} \sqrt{2} \sqrt{\frac{\cos(x)\sqrt{5}-\sqrt{5}+5\cos(x)-1}{1+\cos(x)}} \left(\text{EllipticF} \left(\frac{(-1+\cos(x))(\sqrt{5}-1)}{2\sin(x)}, \frac{3}{2} + \frac{\sqrt{5}}{2} \right) - 2 \text{EllipticPi} \left(\frac{(-1+\cos(x))(\sqrt{5}-1)}{2\sin(x)}, \frac{3}{2} + \frac{\sqrt{5}}{2} \right) \right)}{4 \sqrt{-\frac{5(\cos^2(x))-1}{\cos(x)^2}} \cos(x) (-1 + \cos(x)) \sqrt{\frac{3}{2} - \frac{\sqrt{5}}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2/(-4+tan(x)^2)^(1/2), x)

[Out] -1/4*(-2*(cos(x)*5^(1/2)-5*cos(x)-5^(1/2)+1)/(1+cos(x)))^(1/2)*2^(1/2)*((cos(x)*5^(1/2)-5^(1/2)+5*cos(x)-1)/(1+cos(x)))^(1/2)*(EllipticF(1/2*(-1+cos(x))*(5^(1/2)-1)/sin(x), 3/2+1/2*5^(1/2))-2*EllipticPi((3/2-1/2*5^(1/2))^(1/2)*(-1+cos(x))/sin(x), -2/(5^(1/2)-3), (3/2+1/2*5^(1/2))^(1/2)/(3/2-1/2*5^(1/2))^(1/2)))*sin(x)^2/(-(5*cos(x)^2-1)/cos(x)^2)^(1/2)/cos(x)/(-1+cos(x))/(3/2-1/2*5^(1/2))^(1/2)

maxima [A] time = 0.31, size = 16, normalized size = 1.14

$$\log \left(2 \sqrt{\tan(x)^2 - 4} + 2 \tan(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(-4+tan(x)^2)^(1/2), x, algorithm="maxima")

[Out] log(2*sqrt(tan(x)^2 - 4) + 2*tan(x))

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{\cos(x)^2 \sqrt{\tan(x)^2 - 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2*(tan(x)^2 - 4)^(1/2)),x)

[Out] int(1/(cos(x)^2*(tan(x)^2 - 4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{\sqrt{(\tan(x) - 2)(\tan(x) + 2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2/(-4+tan(x)**2)**(1/2),x)

[Out] Integral(sec(x)**2/sqrt((tan(x) - 2)*(tan(x) + 2)), x)

3.712 $\int \sqrt{1 - \cot^2(x)} \sec^2(x) dx$

Optimal. Leaf size=19

$$\tan(x)\sqrt{1 - \cot^2(x)} + \sin^{-1}(\cot(x))$$

[Out] arcsin(cot(x))+(1-cot(x)^2)^(1/2)*tan(x)

Rubi [A] time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3663, 277, 216}

$$\tan(x)\sqrt{1 - \cot^2(x)} + \sin^{-1}(\cot(x))$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - Cot[x]^2]*Sec[x]^2,x]

[Out] ArcSin[Cot[x]] + Sqrt[1 - Cot[x]^2]*Tan[x]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 277

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^p)/(c*(m+1)), x] - Dist[(b*n*p)/(c^n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^(p-1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m+n*p+n+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 3663

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m+1))/f, Subst[Int[(x^m*(a+b*(ff*x)^n)^p]/(c^2+ff^2*x^2)^(m/2+1), x], x, (c*Tan[e+f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int \sqrt{1 - \cot^2(x)} \sec^2(x) dx &= -\text{Subst} \left(\int \frac{\sqrt{1 - x^2}}{x^2} dx, x, \cot(x) \right) \\ &= \sqrt{1 - \cot^2(x)} \tan(x) + \text{Subst} \left(\int \frac{1}{\sqrt{1 - x^2}} dx, x, \cot(x) \right) \\ &= \sin^{-1}(\cot(x)) + \sqrt{1 - \cot^2(x)} \tan(x) \end{aligned}$$

Mathematica [B] time = 0.49, size = 52, normalized size = 2.74

$$\tan(x)\sqrt{1 - \cot^2(x)} \sec(2x) \left(\cos(2x) - \cos(x)\sqrt{-\cos(2x)} \tan^{-1} \left(\frac{\cos(x)}{\sqrt{-\cos(2x)}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - Cot[x]^2]*Sec[x]^2,x]

[Out] $(-\text{ArcTan}[\text{Cos}[x]/\text{Sqrt}[-\text{Cos}[2*x]]]) * \text{Cos}[x] * \text{Sqrt}[-\text{Cos}[2*x]] + \text{Cos}[2*x] * \text{Sqrt}[1 - \text{Cot}[x]^2] * \text{Sec}[2*x] * \text{Tan}[x]$

fricas [B] time = 0.97, size = 78, normalized size = 4.11

$$\frac{\arctan\left(\frac{(3 \cos(x)^2 - 1) \sqrt{\frac{2 \cos(x)^2 - 1}{\cos(x)^2 - 1}} \sin(x)}{2(2 \cos(x)^3 - \cos(x))}\right) \cos(x) - 2 \sqrt{\frac{2 \cos(x)^2 - 1}{\cos(x)^2 - 1}} \sin(x)}{2 \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(1-cot(x)^2)^(1/2),x, algorithm="fricas")

[Out] $-1/2 * (\arctan(1/2 * (3 * \cos(x)^2 - 1) * \text{sqrt}((2 * \cos(x)^2 - 1) / (\cos(x)^2 - 1)) * \sin(x) / (2 * \cos(x)^3 - \cos(x)))) * \cos(x) - 2 * \text{sqrt}((2 * \cos(x)^2 - 1) / (\cos(x)^2 - 1)) * \sin(x)) / \cos(x)$

giac [C] time = 0.20, size = 142, normalized size = 7.47

$$-\frac{1}{2} (\pi + 2 \arctan(-i) + 2i) \text{sgn}(\sin(x)) + \frac{1}{4} \left(2 \pi \text{sgn}(\cos(x)) + \sqrt{2} \left(\frac{\sqrt{2} \sqrt{-2 \cos(x)^2 + 1} - \sqrt{2}}{\cos(x)} - \frac{4 \cos(x)}{\sqrt{2} \sqrt{-2 \cos(x)^2 + 1}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(1-cot(x)^2)^(1/2),x, algorithm="giac")

[Out] $-1/2 * (\pi + 2 * \arctan(-I) + 2 * I) * \text{sgn}(\sin(x)) + 1/4 * (2 * \pi * \text{sgn}(\cos(x)) + \text{sqrt}(2) * ((\text{sqrt}(2) * \text{sqrt}(-2 * \cos(x)^2 + 1) - \text{sqrt}(2)) / \cos(x) - 4 * \cos(x) / (\text{sqrt}(2) * \text{sqrt}(-2 * \cos(x)^2 + 1) - \text{sqrt}(2))) + 4 * \arctan(-1/4 * \text{sqrt}(2) * ((\text{sqrt}(2) * \text{sqrt}(-2 * \cos(x)^2 + 1) - \text{sqrt}(2)) / \cos(x)^2 - 4) * \cos(x) / (\text{sqrt}(2) * \text{sqrt}(-2 * \cos(x)^2 + 1) - \text{sqrt}(2)))) * \text{sgn}(\sin(x))$

maple [C] time = 0.61, size = 223, normalized size = 11.74

$$\frac{(-1 + \cos(x)) \left(4i \cos(x) \ln \left(\frac{4(-1 + \cos(x)) \left(2i \cos(x) - \cos(x) \sqrt{\frac{2(\cos^2(x) - 1)}{(1 + \cos(x))^2}} + i - \sqrt{\frac{2(\cos^2(x) - 1)}{(1 + \cos(x))^2}} \right)}{\sin(x)^2} \right) - 3 \cos(x) \arcsin \left(\frac{(1 + 2 \cos(x)) \sqrt{2}}{2 + 2 \cos(x)} \right) \right)}{2 \cos(x) \sin(x) \sqrt{-\frac{2(\cos^2(x) - 1)}{(1 + \cos(x))^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2*(1-cot(x)^2)^(1/2),x)

[Out] $-1/2 * (-1 + \cos(x)) * (4 * I * \cos(x) * \ln(4 * (-1 + \cos(x)) * (2 * I * \cos(x) - \cos(x) * (-2 * \cos(x)^2 - 1) / (1 + \cos(x))^2)^{(1/2)} + I * (-2 * \cos(x)^2 - 1) / (1 + \cos(x))^2)^{(1/2)} / \sin(x)^2) - 3 * \cos(x) * \arcsin(1/2 * (1 + 2 * \cos(x)) / (1 + \cos(x))) * 2^{(1/2)} - \cos(x) * \arctan((2 * \cos(x)^2 - 3 * \cos(x) + 1) / (-2 * \cos(x)^2 - 1) / (1 + \cos(x))^2)^{(1/2)} / \sin(x)^2) + 2 * \cos(x) * (-2 * \cos(x)^2 - 1) / (1 + \cos(x))^2)^{(1/2)} + 2 * (-2 * \cos(x)^2 - 1) / (1 + \cos(x))^2)^{(1/2)} * ((2 * \cos(x)^2 - 1) / (-1 + \cos(x)^2))^{(1/2)} / \cos(x) / \sin(x) / (-2 * \cos(x)^2 - 1) / (1 + \cos(x))^2)^{(1/2)}$

maxima [A] time = 0.41, size = 30, normalized size = 1.58

$$\sqrt{-\frac{1}{\tan(x)^2} + 1} \tan(x) - \arctan\left(\sqrt{-\frac{1}{\tan(x)^2} + 1} \tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(1-cot(x)^2)^(1/2),x, algorithm="maxima")

[Out] sqrt(-1/tan(x)^2 + 1)*tan(x) - arctan(sqrt(-1/tan(x)^2 + 1)*tan(x))

mupad [B] time = 3.06, size = 19, normalized size = 1.00

$$\operatorname{asin}(\cot(x)) + \frac{\sqrt{1 - \cot(x)^2}}{\cot(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - cot(x)^2)^(1/2)/cos(x)^2,x)

[Out] asin(cot(x)) + (1 - cot(x)^2)^(1/2)/cot(x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-(\cot(x) - 1)(\cot(x) + 1)} \sec^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2*(1-cot(x)**2)**(1/2),x)

[Out] Integral(sqrt(-(cot(x) - 1)*(cot(x) + 1))*sec(x)**2, x)

3.713 $\int \sec^2(x)\sqrt{1 - \tan^2(x)} dx$

Optimal. Leaf size=26

$$\frac{1}{2} \tan(x)\sqrt{1 - \tan^2(x)} + \frac{1}{2} \sin^{-1}(\tan(x))$$

[Out] 1/2*arcsin(tan(x))+1/2*(1-tan(x)^2)^(1/2)*tan(x)

Rubi [A] time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3675, 195, 216}

$$\frac{1}{2} \tan(x)\sqrt{1 - \tan^2(x)} + \frac{1}{2} \sin^{-1}(\tan(x))$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2*Sqrt[1 - Tan[x]^2], x]

[Out] ArcSin[Tan[x]]/2 + (Tan[x]*Sqrt[1 - Tan[x]^2])/2

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \sec^2(x)\sqrt{1 - \tan^2(x)} dx &= \text{Subst}\left(\int \sqrt{1 - x^2} dx, x, \tan(x)\right) \\ &= \frac{1}{2} \tan(x)\sqrt{1 - \tan^2(x)} + \frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{1 - x^2}} dx, x, \tan(x)\right) \\ &= \frac{1}{2} \sin^{-1}(\tan(x)) + \frac{1}{2} \tan(x)\sqrt{1 - \tan^2(x)} \end{aligned}$$

Mathematica [B] time = 0.12, size = 63, normalized size = 2.42

$$\frac{\cos(2x) \tan(x) + \sqrt{\cos^2(x)} \cos(x)\sqrt{1 - \tan^2(x)} \sin^{-1}\left(\frac{\sin(x)}{\sqrt{\cos^2(x)}}\right)}{2\sqrt{\cos^2(x)} \sqrt{\cos(2x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2*Sqrt[1 - Tan[x]^2], x]

[Out] (Cos[2*x]*Tan[x] + ArcSin[Sin[x]/Sqrt[Cos[x]^2]]*Cos[x]*Sqrt[Cos[x]^2]*Sqrt[1 - Tan[x]^2])/(2*Sqrt[Cos[x]^2]*Sqrt[Cos[2*x]])

fricas [B] time = 1.05, size = 72, normalized size = 2.77

$$\frac{\arctan\left(\frac{(3\cos(x)^3 - 2\cos(x))\sqrt{\frac{2\cos(x)^2 - 1}{\cos(x)^2}}}{2(2\cos(x)^2 - 1)\sin(x)}\right)\cos(x) - 2\sqrt{\frac{2\cos(x)^2 - 1}{\cos(x)^2}}\sin(x)}{4\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(1-tan(x)^2)^(1/2), x, algorithm="fricas")

[Out] -1/4*(arctan(1/2*(3*cos(x)^3 - 2*cos(x))*sqrt((2*cos(x)^2 - 1)/cos(x)^2)/((2*cos(x)^2 - 1)*sin(x)))*cos(x) - 2*sqrt((2*cos(x)^2 - 1)/cos(x)^2)*sin(x))/cos(x)

giac [A] time = 0.15, size = 20, normalized size = 0.77

$$\frac{1}{2}\sqrt{-\tan(x)^2 + 1}\tan(x) + \frac{1}{2}\arcsin(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(1-tan(x)^2)^(1/2), x, algorithm="giac")

[Out] 1/2*sqrt(-tan(x)^2 + 1)*tan(x) + 1/2*arcsin(tan(x))

maple [C] time = 0.43, size = 492, normalized size = 18.92

$$\sin(x)\left(2(\cos^2(x))\sin(x)\sqrt{2}\sqrt{\frac{\cos(x)\sqrt{2}-\sqrt{2}+2\cos(x)-1}{1+\cos(x)}}\sqrt{\frac{2(\cos(x)\sqrt{2}-\sqrt{2}-2\cos(x)+1)}{1+\cos(x)}}\operatorname{EllipticPi}\left(\frac{\sqrt{3+2\sqrt{2}}(-1+\cos(x))}{\sin(x)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2*(1-tan(x)^2)^(1/2), x)

[Out] 1/2*sin(x)*(2*cos(x)^2*sin(x)*2^(1/2)*((cos(x)*2^(1/2)-2^(1/2)+2*cos(x)-1)/(1+cos(x)))^(1/2)*(-2*(cos(x)*2^(1/2)-2^(1/2)-2*cos(x)+1)/(1+cos(x)))^(1/2)*EllipticPi((3+2*2^(1/2))^(1/2)*(-1+cos(x))/sin(x), 1/(3+2*2^(1/2)), (3-2*2^(1/2))^(1/2)/(3+2*2^(1/2))^(1/2))-cos(x)^2*sin(x)*2^(1/2)*((cos(x)*2^(1/2)-2^(1/2)+2*cos(x)-1)/(1+cos(x)))^(1/2)*(-2*(cos(x)*2^(1/2)-2^(1/2)-2*cos(x)+1)/(1+cos(x)))^(1/2)*EllipticF((1+2^(1/2))*(-1+cos(x))/sin(x), 3-2*2^(1/2))+4*cos(x)^2*sin(x)*((cos(x)*2^(1/2)-2^(1/2)+2*cos(x)-1)/(1+cos(x)))^(1/2)*(-2*(cos(x)*2^(1/2)-2^(1/2)-2*cos(x)+1)/(1+cos(x)))^(1/2)*EllipticPi((3+2*2^(1/2))^(1/2)*(-1+cos(x))/sin(x), 1/(3+2*2^(1/2)), (3-2*2^(1/2))^(1/2)/(3+2*2^(1/2))^(1/2))-2*cos(x)^2*sin(x)*((cos(x)*2^(1/2)-2^(1/2)+2*cos(x)-1)/(1+cos(x)))^(1/2)*(-2*(cos(x)*2^(1/2)-2^(1/2)-2*cos(x)+1)/(1+cos(x)))^(1/2)*EllipticF((1+2^(1/2))*(-1+cos(x))/sin(x), 3-2*2^(1/2))+4*cos(x)^3*2^(1/2)-4*cos(x)^2*2^(1/2)+6*cos(x)^3-2*cos(x)*2^(1/2)-6*cos(x)^2+2*2^(1/2)-3*cos(x)+3)*((2*cos(x)^2-1)/cos(x)^2)^(1/2)/(-1+cos(x))/(2*cos(x)^2-1)/cos(x)/(1+2^(1/2))/(3+2*2^(1/2))^(1/2)

maxima [A] time = 0.41, size = 20, normalized size = 0.77

$$\frac{1}{2}\sqrt{-\tan(x)^2 + 1}\tan(x) + \frac{1}{2}\arcsin(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(1-tan(x)^2)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(-tan(x)^2 + 1)*tan(x) + 1/2*arcsin(tan(x))

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1 - \tan(x)^2}}{\cos(x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - tan(x)^2)^(1/2)/cos(x)^2,x)

[Out] int((1 - tan(x)^2)^(1/2)/cos(x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-(\tan(x) - 1)(\tan(x) + 1)} \sec^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2*(1-tan(x)**2)**(1/2),x)

[Out] Integral(sqrt(-(tan(x) - 1)*(tan(x) + 1))*sec(x)**2, x)

3.714 $\int e^{\tan(x)} \sec^2(x) dx$

Optimal. Leaf size=4

$$e^{\tan(x)}$$

[Out] exp(tan(x))

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4342, 2194}

$$e^{\tan(x)}$$

Antiderivative was successfully verified.

[In] Int[E^Tan[x]*Sec[x]^2,x]

[Out] E^Tan[x]

Rule 2194

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 4342

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFactors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a + b*x)]]/d, u, x], x], x, Tan[c*(a + b*x)]/d, x] /; FunctionOfQ[Tan[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] || EqQ[F, sec])

Rubi steps

$$\int e^{\tan(x)} \sec^2(x) dx = \text{Subst}\left(\int e^x dx, x, \tan(x)\right) = e^{\tan(x)}$$

Mathematica [A] time = 0.06, size = 4, normalized size = 1.00

$$e^{\tan(x)}$$

Antiderivative was successfully verified.

[In] Integrate[E^Tan[x]*Sec[x]^2,x]

[Out] E^Tan[x]

fricas [B] time = 0.52, size = 8, normalized size = 2.00

$$e^{\frac{\sin(x)}{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(tan(x))*sec(x)^2,x, algorithm="fricas")

[Out] e^(sin(x)/cos(x))

giac [A] time = 0.13, size = 3, normalized size = 0.75

$$e^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(tan(x))*sec(x)^2,x, algorithm="giac")

[Out] e^{tan(x)}

maple [A] time = 0.05, size = 4, normalized size = 1.00

$$e^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(tan(x))*sec(x)^2,x)

[Out] exp(tan(x))

maxima [A] time = 0.32, size = 3, normalized size = 0.75

$$e^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(tan(x))*sec(x)^2,x, algorithm="maxima")

[Out] e^{tan(x)}

mupad [B] time = 3.10, size = 3, normalized size = 0.75

$$e^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(tan(x))/cos(x)^2,x)

[Out] exp(tan(x))

sympy [A] time = 0.95, size = 3, normalized size = 0.75

$$e^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(tan(x))*sec(x)**2,x)

[Out] exp(tan(x))

$$3.715 \quad \int \sec^4(x) \left(-1 + \sec^2(x)\right)^2 \tan(x) dx$$

Optimal. Leaf size=17

$$\frac{\tan^8(x)}{8} + \frac{\tan^6(x)}{6}$$

[Out] 1/6*tan(x)^6+1/8*tan(x)^8

Rubi [A] time = 0.07, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4120, 2607, 14}

$$\frac{\tan^8(x)}{8} + \frac{\tan^6(x)}{6}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^4*(-1 + Sec[x]^2)^2*Tan[x], x]

[Out] Tan[x]^6/6 + Tan[x]^8/8

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 2607

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])

Rule 4120

Int[(u_)*((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \sec^4(x) \left(-1 + \sec^2(x)\right)^2 \tan(x) dx &= \int \sec^4(x) \tan^5(x) dx \\ &= \text{Subst} \left(\int x^5 (1 + x^2) dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int (x^5 + x^7) dx, x, \tan(x) \right) \\ &= \frac{\tan^6(x)}{6} + \frac{\tan^8(x)}{8} \end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.47

$$\frac{\sec^8(x)}{8} - \frac{\sec^6(x)}{3} + \frac{\sec^4(x)}{4}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^4*(-1 + Sec[x]^2)^2*Tan[x], x]

[Out] Sec[x]^4/4 - Sec[x]^6/3 + Sec[x]^8/8

fricas [A] time = 0.81, size = 20, normalized size = 1.18

$$\frac{6 \cos(x)^4 - 8 \cos(x)^2 + 3}{24 \cos(x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^4*(-1+sec(x)^2)^2*tan(x), x, algorithm="fricas")

[Out] 1/24*(6*cos(x)^4 - 8*cos(x)^2 + 3)/cos(x)^8

giac [A] time = 0.15, size = 20, normalized size = 1.18

$$\frac{6 \cos(x)^4 - 8 \cos(x)^2 + 3}{24 \cos(x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^4*(-1+sec(x)^2)^2*tan(x), x, algorithm="giac")

[Out] 1/24*(6*cos(x)^4 - 8*cos(x)^2 + 3)/cos(x)^8

maple [A] time = 0.05, size = 20, normalized size = 1.18

$$\frac{(\sec^8(x))}{8} - \frac{(\sec^6(x))}{3} + \frac{(\sec^4(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^4*(-1+sec(x)^2)^2*tan(x), x)

[Out] 1/8*sec(x)^8-1/3*sec(x)^6+1/4*sec(x)^4

maxima [B] time = 0.31, size = 42, normalized size = 2.47

$$\frac{6 \sin(x)^4 - 4 \sin(x)^2 + 1}{24 (\sin(x)^8 - 4 \sin(x)^6 + 6 \sin(x)^4 - 4 \sin(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^4*(-1+sec(x)^2)^2*tan(x), x, algorithm="maxima")

[Out] 1/24*(6*sin(x)^4 - 4*sin(x)^2 + 1)/(sin(x)^8 - 4*sin(x)^6 + 6*sin(x)^4 - 4*sin(x)^2 + 1)

mupad [B] time = 2.92, size = 14, normalized size = 0.82

$$\frac{\tan(x)^6 (3 \tan(x)^2 + 4)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(x)*(1/cos(x)^2 - 1)^2)/cos(x)^4, x)

[Out] (tan(x)^6*(3*tan(x)^2 + 4))/24

sympy [A] time = 6.97, size = 19, normalized size = 1.12

$$\frac{\sec^8(x)}{8} - \frac{\sec^6(x)}{3} + \frac{\sec^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4*(-1+sec(x)**2)**2*tan(x),x)
```

```
[Out] sec(x)**8/8 - sec(x)**6/3 + sec(x)**4/4
```

$$3.716 \quad \int \frac{\csc^2(x)}{a+b \cot(x)} dx$$

Optimal. Leaf size=12

$$-\frac{\log(a + b \cot(x))}{b}$$

[Out] $-\ln(a+b*\cot(x))/b$

Rubi [A] time = 0.04, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3506, 31}

$$-\frac{\log(a + b \cot(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^2/(a + b*Cot[x]),x]

[Out] $-(\text{Log}[a + b*\text{Cot}[x]])/b$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3506

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] := Dist[1/(b*f), Subst[Int[(a + x)^{n*(1 + x^2/b^2)}^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int \frac{\csc^2(x)}{a + b \cot(x)} dx &= -\frac{\text{Subst}\left(\int \frac{1}{a+x} dx, x, b \cot(x)\right)}{b} \\ &= -\frac{\log(a + b \cot(x))}{b} \end{aligned}$$

Mathematica [A] time = 0.06, size = 20, normalized size = 1.67

$$\frac{\log(\sin(x)) - \log(a \sin(x) + b \cos(x))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^2/(a + b*Cot[x]),x]

[Out] $(\text{Log}[\text{Sin}[x]] - \text{Log}[b*\text{Cos}[x] + a*\text{Sin}[x]])/b$

fricas [B] time = 1.44, size = 45, normalized size = 3.75

$$\frac{\log\left(2ab \cos(x) \sin(x) - (a^2 - b^2) \cos(x)^2 + a^2\right) - \log\left(-\frac{1}{4} \cos(x)^2 + \frac{1}{4}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/(a+b*cot(x)),x, algorithm="fricas")

[Out] $-1/2*(\log(2*a*b*\cos(x)*\sin(x) - (a^2 - b^2)*\cos(x)^2 + a^2) - \log(-1/4*\cos(x)^2 + 1/4))/b$

giac [A] time = 0.17, size = 22, normalized size = 1.83

$$-\frac{\log(|a \tan(x) + b|)}{b} + \frac{\log(|\tan(x)|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/(a+b*cot(x)),x, algorithm="giac")

[Out] $-\log(\text{abs}(a*\tan(x) + b))/b + \log(\text{abs}(\tan(x)))/b$

maple [A] time = 0.07, size = 13, normalized size = 1.08

$$-\frac{\ln(a + b \cot(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)^2/(a+b*cot(x)),x)

[Out] $-\ln(a+b*\cot(x))/b$

maxima [A] time = 0.33, size = 12, normalized size = 1.00

$$-\frac{\log(b \cot(x) + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/(a+b*cot(x)),x, algorithm="maxima")

[Out] $-\log(b*\cot(x) + a)/b$

mupad [B] time = 3.01, size = 16, normalized size = 1.33

$$-\frac{2 \operatorname{atanh}\left(\frac{2a \tan(x)}{b} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x)^2*(a + b*cot(x))),x)

[Out] $-(2*\operatorname{atanh}((2*a*\tan(x))/b + 1))/b$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{a + b \cot(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)**2/(a+b*cot(x)),x)

[Out] Integral(csc(x)**2/(a + b*cot(x)), x)

3.717 $\int (a + b \cot(x))^n \csc^2(x) dx$

Optimal. Leaf size=20

$$\frac{(a + b \cot(x))^{n+1}}{b(n+1)}$$

[Out] $-(a+b*\cot(x))^{(1+n)}/b/(1+n)$

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3506, 32}

$$\frac{(a + b \cot(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cot[x])^n*Csc[x]^2,x]

[Out] -((a + b*Cot[x])^(1 + n)/(b*(1 + n)))

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 3506

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int (a + b \cot(x))^n \csc^2(x) dx &= -\frac{\text{Subst}\left(\int (a + x)^n dx, x, b \cot(x)\right)}{b} \\ &= -\frac{(a + b \cot(x))^{1+n}}{b(1+n)} \end{aligned}$$

Mathematica [A] time = 0.20, size = 19, normalized size = 0.95

$$\frac{(a + b \cot(x))^{n+1}}{bn + b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cot[x])^n*Csc[x]^2,x]

[Out] -((a + b*Cot[x])^(1 + n)/(b + b*n))

fricas [A] time = 0.65, size = 38, normalized size = 1.90

$$\frac{(b \cos(x) + a \sin(x)) \left(\frac{b \cos(x) + a \sin(x)}{\sin(x)} \right)^n}{(bn + b) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="fricas")

[Out] -(b*cos(x) + a*sin(x))*((b*cos(x) + a*sin(x))/sin(x))^n/((b*n + b)*sin(x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cot(x) + a)^n \csc(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="giac")

[Out] integrate((b*cot(x) + a)^n*csc(x)^2, x)

maple [A] time = 0.08, size = 21, normalized size = 1.05

$$-\frac{(a + b \cot(x))^{n+1}}{b(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cot(x))^n*csc(x)^2,x)

[Out] -(a+b*cot(x))^(n+1)/b/(n+1)

maxima [A] time = 0.32, size = 20, normalized size = 1.00

$$-\frac{(b \cot(x) + a)^{n+1}}{b(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="maxima")

[Out] -(b*cot(x) + a)^(n + 1)/(b*(n + 1))

mupad [B] time = 3.19, size = 43, normalized size = 2.15

$$\begin{cases} -\frac{\ln\left(a + \frac{b}{\tan(x)}\right)}{b} & \text{if } n = -1 \\ -\frac{\left(a + \frac{b}{\tan(x)}\right)^{n+1}}{b(n+1)} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cot(x))^n/sin(x)^2,x)

[Out] piecewise(n == -1, -log(a + b/tan(x))/b, n != -1, -(a + b/tan(x))^(n + 1)/(b*(n + 1)))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \cot(x))^n \csc^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cot(x))**n*csc(x)**2,x)

[Out] Integral((a + b*cot(x))**n*csc(x)**2, x)

3.718 $\int \csc^2(x) (1 + \sin^2(x)) dx$

Optimal. Leaf size=6

$$x - \cot(x)$$

[Out] x-cot(x)

Rubi [A] time = 0.02, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3012, 8}

$$x - \cot(x)$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^2*(1 + Sin[x]^2),x]

[Out] x - Cot[x]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rubi steps

$$\begin{aligned} \int \csc^2(x) (1 + \sin^2(x)) dx &= -\cot(x) + \int 1 dx \\ &= x - \cot(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$x - \cot(x)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^2*(1 + Sin[x]^2),x]

[Out] x - Cot[x]

fricas [B] time = 0.67, size = 14, normalized size = 2.33

$$\frac{x \sin(x) - \cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2*(1+sin(x)^2),x, algorithm="fricas")

[Out] (x*sin(x) - cos(x))/sin(x)

giac [B] time = 0.15, size = 16, normalized size = 2.67

$$x - \frac{1}{2 \tan\left(\frac{1}{2}x\right)} + \frac{1}{2} \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2*(1+sin(x)^2),x, algorithm="giac")

[Out] $x - 1/2/\tan(1/2*x) + 1/2*\tan(1/2*x)$

maple [A] time = 0.06, size = 7, normalized size = 1.17

$$x - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)^2*(1+sin(x)^2),x)

[Out] $x - \cot(x)$

maxima [A] time = 0.41, size = 8, normalized size = 1.33

$$x - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2*(1+sin(x)^2),x, algorithm="maxima")

[Out] $x - 1/\tan(x)$

mupad [B] time = 2.93, size = 6, normalized size = 1.00

$$x - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x)^2 + 1)/sin(x)^2,x)

[Out] $x - \cot(x)$

sympy [A] time = 4.21, size = 3, normalized size = 0.50

$$x - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)**2*(1+sin(x)**2),x)

[Out] $x - \cot(x)$

$$3.719 \quad \int \left(1 + \frac{1}{1 + \cot^2(x)}\right) \csc^2(x) dx$$

Optimal. Leaf size=6

$$x - \cot(x)$$

[Out] x-cot(x)

Rubi [A] time = 0.05, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {14, 203}

$$x - \cot(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + (1 + Cot[x]^2)^(-1))*Csc[x]^2,x]

[Out] x - Cot[x]

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \left(1 + \frac{1}{1 + \cot^2(x)}\right) \csc^2(x) dx &= \text{Subst} \left(\int \frac{1 + \frac{1}{1 + \frac{1}{x^2}}}{x^2} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(\frac{1}{x^2} + \frac{1}{1 + x^2} \right) dx, x, \tan(x) \right) \\ &= -\cot(x) + \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \tan(x) \right) \\ &= x - \cot(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 6, normalized size = 1.00

$$x - \cot(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + (1 + Cot[x]^2)^(-1))*Csc[x]^2,x]

[Out] x - Cot[x]

fricas [B] time = 0.66, size = 14, normalized size = 2.33

$$\frac{x \sin(x) - \cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/(1+cot(x)^2))*csc(x)^2,x, algorithm="fricas")

[Out] (x*sin(x) - cos(x))/sin(x)

giac [B] time = 0.14, size = 16, normalized size = 2.67

$$x - \frac{1}{2 \tan\left(\frac{1}{2}x\right)} + \frac{1}{2} \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/(1+cot(x)^2))*csc(x)^2,x, algorithm="giac")

[Out] x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)

maple [A] time = 0.09, size = 7, normalized size = 1.17

$$x - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+1/(1+cot(x)^2))*csc(x)^2,x)

[Out] x-cot(x)

maxima [A] time = 0.41, size = 8, normalized size = 1.33

$$x - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/(1+cot(x)^2))*csc(x)^2,x, algorithm="maxima")

[Out] x - 1/tan(x)

mupad [B] time = 2.94, size = 6, normalized size = 1.00

$$x - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/(cot(x)^2 + 1) + 1)/sin(x)^2,x)

[Out] x - cot(x)

sympy [B] time = 0.70, size = 27, normalized size = 4.50

$$\frac{x \csc^2(x)}{\cot^2(x) + 1} - \frac{\cot(x) \csc^2(x)}{\cot^2(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/(1+cot(x)**2))*csc(x)**2,x)

[Out] x*csc(x)**2/(cot(x)**2 + 1) - cot(x)*csc(x)**2/(cot(x)**2 + 1)

$$3.720 \quad \int \frac{(a+b \cot(x)) \csc^2(x)}{c+d \cot(x)} dx$$

Optimal. Leaf size=28

$$\frac{(bc - ad) \log(c + d \cot(x))}{d^2} - \frac{b \cot(x)}{d}$$

[Out] $-b \cot(x)/d + (-a*d + b*c) * \ln(c + d * \cot(x))/d^2$

Rubi [A] time = 0.08, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4344, 43}

$$\frac{(bc - ad) \log(c + d \cot(x))}{d^2} - \frac{b \cot(x)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Cot[x])*Csc[x]^2)/(c + d*Cot[x]),x]

[Out] $-(b \cot(x)/d) + ((b*c - a*d) * \text{Log}[c + d * \cot(x)])/d^2$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 4344

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFactors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a + b*x)]]/d, u, x], x], x, Cot[c*(a + b*x)]/d, x] /; FunctionOfQ[Cot[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] | EqQ[F, csc])

Rubi steps

$$\begin{aligned} \int \frac{(a + b \cot(x)) \csc^2(x)}{c + d \cot(x)} dx &= -\text{Subst} \left(\int \frac{a + bx}{c + dx} dx, x, \cot(x) \right) \\ &= -\text{Subst} \left(\int \left(\frac{b}{d} + \frac{-bc + ad}{d(c + dx)} \right) dx, x, \cot(x) \right) \\ &= -\frac{b \cot(x)}{d} + \frac{(bc - ad) \log(c + d \cot(x))}{d^2} \end{aligned}$$

Mathematica [A] time = 0.38, size = 56, normalized size = 2.00

$$\frac{\sin(x)(a + b \cot(x))(-bc - ad)(\log(\sin(x)) - \log(c \sin(x) + d \cos(x))) - bd \cot(x)}{d^2(a \sin(x) + b \cos(x))}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Cot[x])*Csc[x]^2)/(c + d*Cot[x]),x]

[Out] $((a + b \cot(x)) * (-b*d * \cot(x)) - (b*c - a*d) * (\text{Log}[\text{Sin}[x]] - \text{Log}[d * \text{Cos}[x] + c * \text{Sin}[x]])) * \text{Sin}[x]) / (d^2 * (b * \text{Cos}[x] + a * \text{Sin}[x]))$

fricas [B] time = 1.17, size = 76, normalized size = 2.71

$$\frac{2bd \cos(x) - (bc - ad) \log(2cd \cos(x) \sin(x) - (c^2 - d^2) \cos(x)^2 + c^2) \sin(x) + (bc - ad) \log\left(-\frac{1}{4} \cos(x)^2 + \frac{1}{4} \sin(x)^2\right)}{2d^2 \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x, algorithm="fricas")

[Out] -1/2*(2*b*d*cos(x) - (b*c - a*d)*log(2*c*d*cos(x)*sin(x) - (c^2 - d^2)*cos(x)^2 + c^2)*sin(x) + (b*c - a*d)*log(-1/4*cos(x)^2 + 1/4*sin(x)))/(d^2*sin(x))

giac [B] time = 0.15, size = 68, normalized size = 2.43

$$-\frac{(bc - ad) \log(|\tan(x)|)}{d^2} + \frac{(bc^2 - acd) \log(|c \tan(x) + d|)}{cd^2} + \frac{bc \tan(x) - ad \tan(x) - bd}{d^2 \tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x, algorithm="giac")

[Out] -(b*c - a*d)*log(abs(tan(x)))/d^2 + (b*c^2 - a*c*d)*log(abs(c*tan(x) + d))/(c*d^2) + (b*c*tan(x) - a*d*tan(x) - b*d)/(d^2*tan(x))

maple [A] time = 0.10, size = 56, normalized size = 2.00

$$-\frac{b}{d \tan(x)} + \frac{\ln(\tan(x)) a}{d} - \frac{\ln(\tan(x)) cb}{d^2} - \frac{\ln(c \tan(x) + d) a}{d} + \frac{\ln(c \tan(x) + d) cb}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x)

[Out] -b/d/tan(x)+1/d*ln(tan(x))*a-1/d^2*ln(tan(x))*c*b-1/d*ln(c*tan(x)+d)*a+1/d^2*ln(c*tan(x)+d)*c*b

maxima [A] time = 0.32, size = 46, normalized size = 1.64

$$\frac{(bc - ad) \log(c \tan(x) + d)}{d^2} - \frac{(bc - ad) \log(\tan(x))}{d^2} - \frac{b}{d \tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x, algorithm="maxima")

[Out] (b*c - a*d)*log(c*tan(x) + d)/d^2 - (b*c - a*d)*log(tan(x))/d^2 - b/(d*tan(x))

mupad [B] time = 3.07, size = 35, normalized size = 1.25

$$-\frac{b}{d \tan(x)} - \frac{2 \operatorname{atanh}\left(\frac{2c \tan(x)}{d} + 1\right) (ad - bc)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cot(x))/(sin(x)^2*(c + d*cot(x))),x)

[Out] - b/(d*tan(x)) - (2*atanh((2*c*tan(x))/d + 1)*(a*d - b*c))/d^2

sympy [A] time = 26.52, size = 31, normalized size = 1.11

$$-\frac{b \cot(x)}{d} - \frac{(ad - bc) \left(\left(\frac{\cot(x)}{c} \quad \text{for } d = 0 \right) \right.}{d} \left. \left(\frac{\log(c + d \cot(x))}{d} \quad \text{otherwise} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cot(x))*csc(x)**2/(c+d*cot(x)),x)

[Out] -b*cot(x)/d - (a*d - b*c)*Piecewise((cot(x)/c, Eq(d, 0)), (log(c + d*cot(x))/d, True))/d

$$3.721 \quad \int \frac{(a+b \cot(x))^2 \csc^2(x)}{c+d \cot(x)} dx$$

Optimal. Leaf size=53

$$-\frac{(bc-ad)^2 \log(c+d \cot(x))}{d^3} + \frac{b \cot(x)(bc-ad)}{d^2} - \frac{(a+b \cot(x))^2}{2d}$$

[Out] $b*(-a*d+b*c)*\cot(x)/d^2-1/2*(a+b*\cot(x))^2/d-(-a*d+b*c)^2*\ln(c+d*\cot(x))/d^3$

Rubi [A] time = 0.14, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4344, 43}

$$\frac{b \cot(x)(bc-ad)}{d^2} - \frac{(bc-ad)^2 \log(c+d \cot(x))}{d^3} - \frac{(a+b \cot(x))^2}{2d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Cot[x])^2*Csc[x]^2)/(c + d*Cot[x]),x]

[Out] $(b*(b*c - a*d)*\cot(x))/d^2 - (a + b*\cot(x))^2/(2*d) - ((b*c - a*d)^2*\log[c + d*\cot(x)])/d^3$

Rule 43

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 4344

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_.))]^2, x_Symbol] :> With[{d = FreeFactors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a + b*x)]]/d, u, x], x], x, Cot[c*(a + b*x)]/d, x] /; FunctionOfQ[Cot[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] || EqQ[F, csc])

Rubi steps

$$\begin{aligned} \int \frac{(a+b \cot(x))^2 \csc^2(x)}{c+d \cot(x)} dx &= -\text{Subst} \left(\int \frac{(a+bx)^2}{c+dx} dx, x, \cot(x) \right) \\ &= -\text{Subst} \left(\int \left(-\frac{b(bc-ad)}{d^2} + \frac{b(a+bx)}{d} + \frac{(-bc+ad)^2}{d^2(c+dx)} \right) dx, x, \cot(x) \right) \\ &= \frac{b(bc-ad) \cot(x)}{d^2} - \frac{(a+b \cot(x))^2}{2d} - \frac{(bc-ad)^2 \log(c+d \cot(x))}{d^3} \end{aligned}$$

Mathematica [A] time = 0.56, size = 62, normalized size = 1.17

$$\frac{2bd \cot(x)(bc-2ad) + 2(bc-ad)^2(\log(\sin(x)) - \log(c \sin(x) + d \cos(x))) - b^2 d^2 \csc^2(x)}{2d^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Cot[x])^2*Csc[x]^2)/(c + d*Cot[x]),x]

[Out] $(2*b*d*(b*c - 2*a*d)*\text{Cot}[x] - b^2*d^2*\text{Csc}[x]^2 + 2*(b*c - a*d)^2*(\text{Log}[\text{Sin}[x]] - \text{Log}[d*\text{Cos}[x] + c*\text{Sin}[x]]))/(2*d^3)$

fricas [B] time = 1.47, size = 182, normalized size = 3.43

$$\frac{b^2 d^2 - 2(b^2 c d - 2 a b d^2) \cos(x) \sin(x) + (b^2 c^2 - 2 a b c d + a^2 d^2 - (b^2 c^2 - 2 a b c d + a^2 d^2) \cos(x)^2) \log(2 c d \cos(x) \sin(x))}{2(d^3 \cos(x) \sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x, algorithm="fricas")`

[Out] $1/2*(b^2*d^2 - 2*(b^2*c*d - 2*a*b*d^2)*\cos(x)*\sin(x) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\cos(x)^2)*\log(2*c*d*\cos(x)*\sin(x) - (c^2 - d^2)*\cos(x)^2 + c^2) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\cos(x)^2)*\log(-1/4*\cos(x)^2 + 1/4))/(d^3*\cos(x)^2 - d^3)$

giac [B] time = 0.16, size = 139, normalized size = 2.62

$$\frac{(b^2 c^2 - 2 a b c d + a^2 d^2) \log(|\tan(x)|)}{d^3} - \frac{(b^2 c^3 - 2 a b c^2 d + a^2 c d^2) \log(|c \tan(x) + d|)}{c d^3} - \frac{3 b^2 c^2 \tan(x)^2 - 6 a b c d \tan(x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x, algorithm="giac")`

[Out] $(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(\text{abs}(\tan(x)))/d^3 - (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*\log(\text{abs}(c*\tan(x) + d))/(c*d^3) - 1/2*(3*b^2*c^2*\tan(x)^2 - 6*a*b*c*d*\tan(x)^2 + 3*a^2*d^2*\tan(x)^2 - 2*b^2*c*d*\tan(x) + 4*a*b*d^2*\tan(x) + b^2*d^2)/(d^3*\tan(x)^2)$

maple [B] time = 0.13, size = 119, normalized size = 2.25

$$-\frac{b^2}{2d \tan(x)^2} + \frac{\ln(\tan(x)) a^2}{d} - \frac{2 \ln(\tan(x)) abc}{d^2} + \frac{\ln(\tan(x)) b^2 c^2}{d^3} - \frac{2ba}{d \tan(x)} + \frac{b^2 c}{d^2 \tan(x)} - \frac{\ln(c \tan(x) + d) a^2}{d} + \frac{2 \ln(c \tan(x) + d) abc}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x)`

[Out] $-1/2*b^2/d/\tan(x)^2 + 1/d*\ln(\tan(x))*a^2 - 2/d^2*\ln(\tan(x))*a*b*c + 1/d^3*\ln(\tan(x))*b^2*c^2 - 2*b/d/\tan(x)*a + b^2/d^2/\tan(x)*c - 1/d*\ln(c*\tan(x)+d)*a^2 + 2/d^2*\ln(c*\tan(x)+d)*a*b*c - 1/d^3*\ln(c*\tan(x)+d)*b^2*c^2$

maxima [A] time = 0.32, size = 92, normalized size = 1.74

$$-\frac{(b^2 c^2 - 2 a b c d + a^2 d^2) \log(c \tan(x) + d)}{d^3} + \frac{(b^2 c^2 - 2 a b c d + a^2 d^2) \log(\tan(x))}{d^3} - \frac{b^2 d - 2(b^2 c - 2 a b d) \tan(x)}{2 d^2 \tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x, algorithm="maxima")`

[Out] $-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(c*\tan(x) + d)/d^3 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(\tan(x))/d^3 - 1/2*(b^2*d - 2*(b^2*c - 2*a*b*d)*\tan(x))/(d^2*\tan(x)^2)$

mupad [B] time = 3.07, size = 92, normalized size = 1.74

$$-\frac{\frac{b^2}{2d} + \frac{b \tan(x) (2 a d - b c)}{d^2}}{\tan(x)^2} - \frac{2 \operatorname{atanh}\left(\frac{(d+2 c \tan(x))(a d - b c)^2}{d(a^2 d^2 - 2 a b c d + b^2 c^2)}\right) (a d - b c)^2}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*cot(x))^2/(sin(x)^2*(c + d*cot(x))),x)`

[Out] $-\frac{b^2}{2d} + \frac{b \tan(x) (2ad - bc)}{d^2 \tan(x)^2} - \frac{2 \operatorname{atanh}\left(\frac{d + 2c \tan(x)}{a^2 d^2 + b^2 c^2 - 2abc d}\right) (ad - bc)^2}{d^3}$

sympy [A] time = 55.64, size = 58, normalized size = 1.09

$$\frac{b^2 \cot^2(x)}{2d} - \frac{(ad - bc)^2 \begin{cases} \frac{\cot(x)}{c} & \text{for } d = 0 \\ \frac{\log(c + d \cot(x))}{d} & \text{otherwise} \end{cases}}{d^2} - \frac{(2abd - b^2c) \cot(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cot(x))**2*csc(x)**2/(c+d*cot(x)),x)`

[Out] $-\frac{b^2 \cot(x)^2}{2d} - \frac{(ad - bc)^2 \operatorname{Piecewise}\left(\frac{\cot(x)}{c}, \operatorname{Eq}(d, 0)\right) \left(\log\left(\frac{c + d \cot(x)}{d}\right), \operatorname{True}\right)}{d^2} - \frac{(2abd - b^2c) \cot(x)}{d^2}$

$$3.722 \quad \int \frac{(a+b \cot(x))^3 \csc^2(x)}{c+d \cot(x)} dx$$

Optimal. Leaf size=78

$$\frac{(bc-ad)^3 \log(c+d \cot(x))}{d^4} - \frac{b \cot(x)(bc-ad)^2}{d^3} + \frac{(bc-ad)(a+b \cot(x))^2}{2d^2} - \frac{(a+b \cot(x))^3}{3d}$$

[Out] $-b*(-a*d+b*c)^2*\cot(x)/d^3+1/2*(-a*d+b*c)*(a+b*\cot(x))^2/d^2-1/3*(a+b*\cot(x))^3/d+(-a*d+b*c)^3*\ln(c+d*\cot(x))/d^4$

Rubi [A] time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4344, 43}

$$-\frac{b \cot(x)(bc-ad)^2}{d^3} + \frac{(bc-ad)(a+b \cot(x))^2}{2d^2} + \frac{(bc-ad)^3 \log(c+d \cot(x))}{d^4} - \frac{(a+b \cot(x))^3}{3d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Cot[x])^3*Csc[x]^2)/(c + d*Cot[x]), x]

[Out] $-((b*(b*c - a*d)^2*\cot[x])/d^3) + ((b*c - a*d)*(a + b*\cot[x])^2)/(2*d^2) - (a + b*\cot[x])^3/(3*d) + ((b*c - a*d)^3*\log[c + d*\cot[x]])/d^4$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 4344

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] := With[{d = FreeFactors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a + b*x)]]/d, u, x], x], x, Cot[c*(a + b*x)]/d, x] /; FunctionOfQ[Cot[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] | EqQ[F, csc])

Rubi steps

$$\begin{aligned} \int \frac{(a+b \cot(x))^3 \csc^2(x)}{c+d \cot(x)} dx &= -\text{Subst} \left(\int \frac{(a+bx)^3}{c+dx} dx, x, \cot(x) \right) \\ &= -\text{Subst} \left(\int \left(\frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)(a+bx)}{d^2} + \frac{b(a+bx)^2}{d} + \frac{(-bc+ad)^3}{d^3(c+dx)} \right) dx, x \right) \\ &= -\frac{b(bc-ad)^2 \cot(x)}{d^3} + \frac{(bc-ad)(a+b \cot(x))^2}{2d^2} - \frac{(a+b \cot(x))^3}{3d} + \frac{(bc-ad)^3 \log(c+d \cot(x))}{d^4} \end{aligned}$$

Mathematica [A] time = 1.30, size = 135, normalized size = 1.73

$$\frac{(a+b \cot(x))^3(c \sin(x) + d \cos(x)) \left(bd \left(\sin(2x) \left(-9a^2d^2 + 9abcd + b^2(d^2 - 3c^2) \right) + 3bd(bc - 3ad) \right) - 6 \sin^2(x)(bc - ad) \right)}{6d^4(c+d \cot(x))(a \sin(x) + b \cos(x))^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Cot[x])^3*Csc[x]^2)/(c + d*Cot[x]), x]

```
[Out] ((a + b*Cot[x])^3*(d*Cos[x] + c*Sin[x])*(-2*b^3*d^3*Cot[x] - 6*(b*c - a*d)^3*(Log[Sin[x]] - Log[d*Cos[x] + c*Sin[x]])*Sin[x]^2 + b*d*(3*b*d*(b*c - 3*a*d) + (9*a*b*c*d - 9*a^2*d^2 + b^2*(-3*c^2 + d^2))*Sin[2*x]))/(6*d^4*(c + d*Cot[x])*(b*Cos[x] + a*Sin[x])^3)
```

fricas [B] time = 1.62, size = 320, normalized size = 4.10

$$\frac{2(3b^3c^2d - 9ab^2cd^2 + (9a^2b - b^3)d^3)\cos(x)^3 + 3(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3 - (b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3)\log(\tan(x)))}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(3*b^3*c^2*d - 9*a*b^2*c*d^2 + (9*a^2*b - b^3)*d^3)*cos(x)^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^2)*log(2*c*d*cos(x)*sin(x) - (c^2 - d^2)*cos(x)^2 + c^2)*sin(x) - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^2)*log(-1/4*cos(x)^2 + 1/4)*sin(x) - 6*(b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2*b*d^3)*cos(x) + 3*(b^3*c*d^2 - 3*a*b^2*d^3)*sin(x))/((d^4*cos(x)^2 - d^4)*sin(x))
```

giac [B] time = 0.17, size = 232, normalized size = 2.97

$$\frac{(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3)\log(|\tan(x)|)}{d^4} + \frac{(b^3c^4 - 3ab^2c^3d + 3a^2bc^2d^2 - a^3cd^3)\log(|c\tan(x) + d|)}{cd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x, algorithm="giac")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(tan(x)))/d^4 + (b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*log(abs(c*tan(x) + d))/(c*d^4) + 1/6*(11*b^3*c^3*tan(x)^3 - 33*a*b^2*c^2*d*tan(x)^3 + 33*a^2*b*c*d^2*tan(x)^3 - 11*a^3*d^3*tan(x)^3 - 6*b^3*c^2*d*tan(x)^2 + 18*a*b^2*c*d^2*tan(x)^2 - 18*a^2*b*d^3*tan(x)^2 + 3*b^3*c*d^2*tan(x) - 9*a*b^2*d^3*tan(x) - 2*b^3*d^3)/(d^4*tan(x)^3)
```

maple [B] time = 0.16, size = 202, normalized size = 2.59

$$\frac{b^3}{3d\tan(x)^3} + \frac{\ln(\tan(x))a^3}{d} - \frac{3\ln(\tan(x))a^2bc}{d^2} + \frac{3\ln(\tan(x))ab^2c^2}{d^3} - \frac{\ln(\tan(x))b^3c^3}{d^4} - \frac{3ba^2}{d\tan(x)} + \frac{3b^2ac}{d^2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x)
```

```
[Out] -1/3*b^3/d/tan(x)^3+1/d*ln(tan(x))*a^3-3/d^2*ln(tan(x))*a^2*b*c+3/d^3*ln(tan(x))*a*b^2*c^2-1/d^4*ln(tan(x))*b^3*c^3-3*b/d/tan(x)*a^2+3*b^2/d^2/tan(x)*a*c-b^3/d^3/tan(x)*c^2-3/2*b^2/d/tan(x)^2*a+1/2*b^3/d^2/tan(x)^2*c-1/d*ln(c*tan(x)+d)*a^3+3/d^2*ln(c*tan(x)+d)*a^2*b*c-3/d^3*ln(c*tan(x)+d)*a*b^2*c^2+1/d^4*ln(c*tan(x)+d)*b^3*c^3
```

maxima [B] time = 0.32, size = 161, normalized size = 2.06

$$\frac{(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3)\log(c\tan(x) + d)}{d^4} - \frac{(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3)\log(\tan(x))}{d^4} - \frac{2b^3d^3}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x, algorithm="maxima")
```

[Out] $(b^3c^3 - 3ab^2c^2d + 3a^2b^2cd^2 - a^3d^3) \log(c \tan(x) + d) / d^4 - (b^3c^3 - 3ab^2c^2d + 3a^2b^2cd^2 - a^3d^3) \log(\tan(x)) / d^4 - 1/6 * (2b^3d^2 + 6(b^3c^2 - 3ab^2cd + 3a^2bd^2) \tan(x)^2 - 3(b^3cd - 3ab^2d^2) \tan(x)) / (d^3 \tan(x)^3)$

mupad [B] time = 3.08, size = 141, normalized size = 1.81

$$\frac{\frac{b^3}{3d} + \frac{b^2 \tan(x) (3ad - bc)}{2d^2} + \frac{b \tan(x)^2 (3a^2 d^2 - 3abcd + b^2 c^2)}{d^3}}{\tan(x)^3} - \frac{2 \operatorname{atanh}\left(\frac{(d + 2c \tan(x)) (ad - bc)^3}{d(a^3 d^3 - 3a^2 bcd^2 + 3ab^2 c^2 d - b^3 c^3)}\right) (ad - bc)^3}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*cot(x))^3/(sin(x)^2*(c + d*cot(x))), x)`

[Out] $-(b^3/(3d) + (b^2 \tan(x) (3ad - bc)) / (2d^2) + (b \tan(x)^2 (3a^2 d^2 + b^2 c^2 - 3abcd)) / d^3) / \tan(x)^3 - (2 \operatorname{atanh}(((d + 2c \tan(x)) (ad - bc)^3) / (d(a^3 d^3 - 3a^2 bcd^2 + 3ab^2 c^2 d - b^3 c^3)))) * (ad - bc)^3 / d^4$

sympy [A] time = 69.99, size = 97, normalized size = 1.24

$$\frac{b^3 \cot^3(x)}{3d} - \frac{(3ab^2d - b^3c) \cot^2(x)}{2d^2} - \frac{(ad - bc)^3 \begin{cases} \frac{\cot(x)}{c} & \text{for } d = 0 \\ \frac{\log(c + d \cot(x))}{d} & \text{otherwise} \end{cases}}{d^3} - \frac{(3a^2bd^2 - 3ab^2cd + b^3c^2) \cot(x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cot(x))**3*csc(x)**2/(c+d*cot(x)), x)`

[Out] $-b**3*cot(x)**3/(3*d) - (3*a*b**2*d - b**3*c)*cot(x)**2/(2*d**2) - (a*d - b*c)**3*Piecewise((cot(x)/c, Eq(d, 0)), (log(c + d*cot(x))/d, True))/d**3 - (3*a**2*b*d**2 - 3*a*b**2*c*d + b**3*c**2)*cot(x)/d**3$

3.723 $\int e^{-\cot(x)} \csc^2(x) dx$

Optimal. Leaf size=6

$$e^{-\cot(x)}$$

[Out] exp(-cot(x))

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4344, 2194}

$$e^{-\cot(x)}$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^2/E^Cot[x],x]

[Out] E^(-Cot[x])

Rule 2194

Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 4344

Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))]^2, x_Symbol] :> With[{d = FreeFactors[Cot[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a + b*x)]]/d, u, x], x], x, Cot[c*(a + b*x)]/d, x] /; FunctionOfQ[Cot[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] | EqQ[F, csc])

Rubi steps

$$\int e^{-\cot(x)} \csc^2(x) dx = -\text{Subst}\left(\int e^{-x} dx, x, \cot(x)\right) = e^{-\cot(x)}$$

Mathematica [A] time = 0.07, size = 6, normalized size = 1.00

$$e^{-\cot(x)}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^2/E^Cot[x],x]

[Out] E^(-Cot[x])

fricas [A] time = 0.64, size = 9, normalized size = 1.50

$$e^{\left(-\frac{\cos(x)}{\sin(x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/exp(cot(x)),x, algorithm="fricas")

[Out] e^(-cos(x)/sin(x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(x)^2 e^{(-\cot(x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/exp(cot(x)),x, algorithm="giac")

[Out] integrate(csc(x)^2*e^(-cot(x)), x)

maple [A] time = 0.05, size = 6, normalized size = 1.00

$$e^{-\cot(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)^2/exp(cot(x)),x)

[Out] 1/exp(cot(x))

maxima [A] time = 0.32, size = 5, normalized size = 0.83

$$e^{(-\cot(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/exp(cot(x)),x, algorithm="maxima")

[Out] e^(-cot(x))

mupad [B] time = 2.94, size = 7, normalized size = 1.17

$$e^{-\frac{1}{\tan(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-cot(x))/sin(x)^2,x)

[Out] exp(-1/tan(x))

sympy [A] time = 18.87, size = 5, normalized size = 0.83

$$e^{-\cot(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)**2/exp(cot(x)),x)

[Out] exp(-cot(x))

$$3.724 \quad \int \frac{\sec(x) \tan(x)}{a+b \sec(x)} dx$$

Optimal. Leaf size=11

$$\frac{\log(a + b \sec(x))}{b}$$

[Out] ln(a+b*sec(x))/b

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.82, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4339, 36, 29, 31}

$$\frac{\log(a \cos(x) + b)}{b} - \frac{\log(\cos(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]*Tan[x])/(a + b*Sec[x]), x]

[Out] -(Log[Cos[x]]/b) + Log[b + a*Cos[x]]/b

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 4339

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])

Rubi steps

$$\begin{aligned} \int \frac{\sec(x) \tan(x)}{a + b \sec(x)} dx &= -\text{Subst} \left(\int \frac{1}{x(b + ax)} dx, x, \cos(x) \right) \\ &= -\frac{\text{Subst} \left(\int \frac{1}{x} dx, x, \cos(x) \right)}{b} + \frac{a \text{Subst} \left(\int \frac{1}{b+ax} dx, x, \cos(x) \right)}{b} \\ &= -\frac{\log(\cos(x))}{b} + \frac{\log(b + a \cos(x))}{b} \end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.82

$$\frac{\log(a \cos(x) + b)}{b} - \frac{\log(\cos(x))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]*Tan[x])/(a + b*Sec[x]),x]

[Out] -(Log[Cos[x]]/b) + Log[b + a*Cos[x]]/b

fricas [A] time = 0.67, size = 19, normalized size = 1.73

$$\frac{\log(a \cos(x) + b) - \log(-\cos(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(a+b*sec(x)),x, algorithm="fricas")

[Out] (log(a*cos(x) + b) - log(-cos(x)))/b

giac [A] time = 0.14, size = 22, normalized size = 2.00

$$\frac{\log(|a \cos(x) + b|)}{b} - \frac{\log(|\cos(x)|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(a+b*sec(x)),x, algorithm="giac")

[Out] log(abs(a*cos(x) + b))/b - log(abs(cos(x)))/b

maple [A] time = 0.04, size = 12, normalized size = 1.09

$$\frac{\ln(a + b \sec(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)*tan(x)/(a+b*sec(x)),x)

[Out] ln(a+b*sec(x))/b

maxima [A] time = 0.31, size = 11, normalized size = 1.00

$$\frac{\log(b \sec(x) + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(a+b*sec(x)),x, algorithm="maxima")

[Out] log(b*sec(x) + a)/b

mupad [B] time = 3.25, size = 48, normalized size = 4.36

$$\frac{\operatorname{atan}\left(\frac{b \sin\left(\frac{x}{2}\right)^2}{a \cos\left(\frac{x}{2}\right)^2 1i + b \cos\left(\frac{x}{2}\right)^2 1i - a \sin\left(\frac{x}{2}\right)^2 1i}\right) 2i}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)/(cos(x)*(a + b/cos(x))),x)

[Out] (atan((b*sin(x/2)^2)/(a*cos(x/2)^2*1i + b*cos(x/2)^2*1i - a*sin(x/2)^2*1i))*2i)/b

sympy [A] time = 0.46, size = 14, normalized size = 1.27

$$\begin{cases} \frac{\log\left(\frac{a}{b} + \sec(x)\right)}{b} & \text{for } b \neq 0 \\ \frac{\sec(x)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(a+b*sec(x)),x)
```

```
[Out] Piecewise((log(a/b + sec(x))/b, Ne(b, 0)), (sec(x)/a, True))
```

$$3.725 \quad \int \frac{\sec(x) \tan(x)}{1+\sec^2(x)} dx$$

Optimal. Leaf size=5

$$-\tan^{-1}(\cos(x))$$

[Out] -arctan(cos(x))

Rubi [A] time = 0.03, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4339, 203}

$$-\tan^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]*Tan[x])/(1 + Sec[x]^2), x]

[Out] -ArcTan[Cos[x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 4339

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])

Rubi steps

$$\begin{aligned} \int \frac{\sec(x) \tan(x)}{1 + \sec^2(x)} dx &= -\text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \cos(x) \right) \\ &= -\tan^{-1}(\cos(x)) \end{aligned}$$

Mathematica [A] time = 0.02, size = 5, normalized size = 1.00

$$-\tan^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]*Tan[x])/(1 + Sec[x]^2), x]

[Out] -ArcTan[Cos[x]]

fricas [A] time = 4.94, size = 5, normalized size = 1.00

$$-\arctan(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(1+sec(x)^2), x, algorithm="fricas")

[Out] -arctan(cos(x))

giac [A] time = 0.14, size = 5, normalized size = 1.00

$$-\arctan(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(1+sec(x)^2),x, algorithm="giac")

[Out] -arctan(cos(x))

maple [A] time = 0.07, size = 4, normalized size = 0.80

$$\arctan(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)*tan(x)/(1+sec(x)^2),x)

[Out] arctan(sec(x))

maxima [A] time = 0.42, size = 5, normalized size = 1.00

$$-\arctan(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(1+sec(x)^2),x, algorithm="maxima")

[Out] -arctan(cos(x))

mupad [B] time = 2.95, size = 7, normalized size = 1.40

$$\operatorname{atan}\left(\tan\left(\frac{x}{2}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)/(cos(x)*(1/cos(x)^2 + 1)),x)

[Out] atan(tan(x/2)^2)

sympy [A] time = 0.21, size = 3, normalized size = 0.60

$$\operatorname{atan}(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(1+sec(x)**2),x)

[Out] atan(sec(x))

$$3.726 \quad \int \frac{\sec(x) \tan(x)}{9+4 \sec^2(x)} dx$$

Optimal. Leaf size=11

$$-\frac{1}{6} \tan^{-1} \left(\frac{3 \cos(x)}{2} \right)$$

[Out] -1/6*arctan(3/2*cos(x))

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4339, 203}

$$-\frac{1}{6} \tan^{-1} \left(\frac{3 \cos(x)}{2} \right)$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]*Tan[x])/(9 + 4*Sec[x]^2),x]

[Out] -ArcTan[(3*Cos[x])/2]/6

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 4339

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])

Rubi steps

$$\begin{aligned} \int \frac{\sec(x) \tan(x)}{9+4 \sec^2(x)} dx &= -\text{Subst} \left(\int \frac{1}{4+9x^2} dx, x, \cos(x) \right) \\ &= -\frac{1}{6} \tan^{-1} \left(\frac{3 \cos(x)}{2} \right) \end{aligned}$$

Mathematica [A] time = 0.03, size = 11, normalized size = 1.00

$$-\frac{1}{6} \tan^{-1} \left(\frac{3 \cos(x)}{2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]*Tan[x])/(9 + 4*Sec[x]^2),x]

[Out] -1/6*ArcTan[(3*Cos[x])/2]

fricas [A] time = 0.97, size = 7, normalized size = 0.64

$$-\frac{1}{6} \arctan \left(\frac{3}{2} \cos(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(9+4*sec(x)^2),x, algorithm="fricas")

[Out] -1/6*arctan(3/2*cos(x))

giac [A] time = 0.13, size = 7, normalized size = 0.64

$$-\frac{1}{6} \arctan\left(\frac{3}{2} \cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(9+4*sec(x)^2),x, algorithm="giac")

[Out] -1/6*arctan(3/2*cos(x))

maple [A] time = 0.06, size = 8, normalized size = 0.73

$$\frac{\arctan\left(\frac{2\sec(x)}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)*tan(x)/(9+4*sec(x)^2),x)

[Out] 1/6*arctan(2/3*sec(x))

maxima [A] time = 0.44, size = 7, normalized size = 0.64

$$-\frac{1}{6} \arctan\left(\frac{3}{2} \cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(9+4*sec(x)^2),x, algorithm="maxima")

[Out] -1/6*arctan(3/2*cos(x))

mupad [B] time = 3.04, size = 13, normalized size = 1.18

$$\frac{\operatorname{atan}\left(\frac{13 \tan\left(\frac{x}{2}\right)^2}{12} - \frac{5}{12}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)/(cos(x)*(4/cos(x)^2 + 9)),x)

[Out] atan((13*tan(x/2)^2)/12 - 5/12)/6

sympy [A] time = 0.23, size = 8, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{2\sec(x)}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(9+4*sec(x)**2),x)

[Out] atan(2*sec(x)/3)/6

$$3.727 \quad \int \frac{\sec(x) \tan(x)}{\sec(x) + \sec^2(x)} dx$$

Optimal. Leaf size=7

$$-\log(\cos(x) + 1)$$

[Out] -ln(1+cos(x))

Rubi [A] time = 0.03, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4339, 31}

$$-\log(\cos(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]*Tan[x])/(Sec[x] + Sec[x]^2),x]

[Out] -Log[1 + Cos[x]]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 4339

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])

Rubi steps

$$\int \frac{\sec(x) \tan(x)}{\sec(x) + \sec^2(x)} dx = -\text{Subst} \left(\int \frac{1}{1+x} dx, x, \cos(x) \right) \\ = -\log(1 + \cos(x))$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.29

$$-2 \log \left(\cos \left(\frac{x}{2} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]*Tan[x])/(Sec[x] + Sec[x]^2),x]

[Out] -2*Log[Cos[x/2]]

fricas [A] time = 0.84, size = 9, normalized size = 1.29

$$-\log \left(\frac{1}{2} \cos(x) + \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(sec(x)+sec(x)^2),x, algorithm="fricas")

[Out] -log(1/2*cos(x) + 1/2)

giac [A] time = 0.15, size = 7, normalized size = 1.00

$$-\log(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(sec(x)+sec(x)^2),x, algorithm="giac")

[Out] -log(cos(x) + 1)

maple [A] time = 0.08, size = 12, normalized size = 1.71

$$\ln(\sec(x)) - \ln(1 + \sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)*tan(x)/(sec(x)+sec(x)^2),x)

[Out] ln(sec(x))-ln(1+sec(x))

maxima [A] time = 0.31, size = 7, normalized size = 1.00

$$-\log(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(sec(x)+sec(x)^2),x, algorithm="maxima")

[Out] -log(cos(x) + 1)

mupad [B] time = 3.07, size = 9, normalized size = 1.29

$$\ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)/(cos(x)*(1/cos(x) + 1/cos(x)^2)),x)

[Out] log(tan(x/2)^2 + 1)

sympy [B] time = 0.22, size = 15, normalized size = 2.14

$$\frac{\log(\tan^2(x) + 1)}{2} - \log(\sec(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(sec(x)+sec(x)**2),x)

[Out] log(tan(x)**2 + 1)/2 - log(sec(x) + 1)

$$3.728 \quad \int \frac{\sec(x) \tan(x)}{\sqrt{4 + \sec^2(x)}} dx$$

Optimal. Leaf size=5

$$\operatorname{csch}^{-1}(2 \cos(x))$$

[Out] arccsch(2*cos(x))

Rubi [A] time = 0.04, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4339, 335, 215}

$$\operatorname{csch}^{-1}(2 \cos(x))$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]*Tan[x])/Sqrt[4 + Sec[x]^2], x]

[Out] ArcCsch[2*Cos[x]]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 335

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]

Rule 4339

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])

Rubi steps

$$\begin{aligned} \int \frac{\sec(x) \tan(x)}{\sqrt{4 + \sec^2(x)}} dx &= -\operatorname{Subst} \left(\int \frac{1}{\sqrt{4 + \frac{1}{x^2} x^2}} dx, x, \cos(x) \right) \\ &= \operatorname{Subst} \left(\int \frac{1}{\sqrt{4 + x^2}} dx, x, \sec(x) \right) \\ &= \sinh^{-1} \left(\frac{\sec(x)}{2} \right) \end{aligned}$$

Mathematica [B] time = 0.03, size = 38, normalized size = 7.60

$$\frac{\sqrt{2 \cos(2x) + 3} \sec(x) \tanh^{-1} \left(\sqrt{4 \cos^2(x) + 1} \right)}{\sqrt{\sec^2(x) + 4}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]*Tan[x])/Sqrt[4 + Sec[x]^2],x]

[Out] (ArcTanh[Sqrt[1 + 4*Cos[x]^2]]*Sqrt[3 + 2*Cos[2*x]]*Sec[x])/Sqrt[4 + Sec[x]^2]

fricas [B] time = 0.95, size = 27, normalized size = 5.40

$$\log\left(-\frac{\sqrt{\frac{4\cos(x)^2+1}{\cos(x)^2}}\cos(x)+1}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x, algorithm="fricas")

[Out] log(-(sqrt((4*cos(x)^2 + 1)/cos(x)^2)*cos(x) + 1)/cos(x))

giac [B] time = 0.15, size = 36, normalized size = 7.20

$$\frac{\log(\sqrt{4\cos(x)^2+1}+1)-\log(\sqrt{4\cos(x)^2+1}-1)}{2\operatorname{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x, algorithm="giac")

[Out] 1/2*(log(sqrt(4*cos(x)^2 + 1) + 1) - log(sqrt(4*cos(x)^2 + 1) - 1))/sgn(cos(x))

maple [A] time = 0.10, size = 6, normalized size = 1.20

$$\operatorname{arcsinh}\left(\frac{\sec(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x)

[Out] arcsinh(1/2*sec(x))

maxima [B] time = 0.31, size = 33, normalized size = 6.60

$$\frac{1}{2}\log\left(\sqrt{\frac{1}{\cos(x)^2}+4}\cos(x)+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{1}{\cos(x)^2}+4}\cos(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x, algorithm="maxima")

[Out] 1/2*log(sqrt(1/cos(x)^2 + 4)*cos(x) + 1) - 1/2*log(sqrt(1/cos(x)^2 + 4)*cos(x) - 1)

mupad [B] time = 3.10, size = 7, normalized size = 1.40

$$\operatorname{asinh}\left(\frac{1}{2\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)/(cos(x)*(1/cos(x)^2 + 4)^(1/2)),x)

[Out] asinh(1/(2*cos(x)))

sympy [A] time = 0.73, size = 5, normalized size = 1.00

$$\operatorname{asinh}\left(\frac{\sec(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(4+sec(x)**2)**(1/2),x)
```

```
[Out] asinh(sec(x)/2)
```

$$3.729 \quad \int \frac{\sec(x) \tan(x)}{\sqrt{1+\cos^2(x)}} dx$$

Optimal. Leaf size=13

$$\sqrt{\cos^2(x) + 1} \sec(x)$$

[Out] sec(x)*(1+cos(x)^2)^(1/2)

Rubi [A] time = 0.08, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$\sqrt{\cos^2(x) + 1} \sec(x)$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]*Tan[x])/Sqrt[1 + Cos[x]^2],x]

[Out] Sqrt[1 + Cos[x]^2]*Sec[x]

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sec(x) \tan(x)}{\sqrt{1+\cos^2(x)}} dx &= -\text{Subst} \left(\int \frac{1}{x^2 \sqrt{1+x^2}} dx, x, \cos(x) \right) \\ &= \sqrt{1+\cos^2(x)} \sec(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 13, normalized size = 1.00

$$\sqrt{\cos^2(x) + 1} \sec(x)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]*Tan[x])/Sqrt[1 + Cos[x]^2],x]

[Out] Sqrt[1 + Cos[x]^2]*Sec[x]

fricas [A] time = 0.67, size = 16, normalized size = 1.23

$$\frac{\sqrt{\cos(x)^2 + 1} + \cos(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="fricas")

[Out] (sqrt(cos(x)^2 + 1) + cos(x))/cos(x)

giac [A] time = 0.15, size = 21, normalized size = 1.62

$$-\frac{2}{(\sqrt{\cos(x)^2 + 1} - \cos(x))^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="giac")

[Out] -2/((sqrt(cos(x)^2 + 1) - cos(x))^2 - 1)

maple [B] time = 0.11, size = 25, normalized size = 1.92

$$\frac{1 + \sec^2(x)}{\sqrt{\frac{1 + \sec^2(x)}{\sec(x)^2}} \sec(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x)

[Out] 1/((1+sec(x)^2)/sec(x)^2)^(1/2)/sec(x)*(1+sec(x)^2)

maxima [A] time = 0.42, size = 13, normalized size = 1.00

$$\frac{\sqrt{\cos(x)^2 + 1}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="maxima")

[Out] sqrt(cos(x)^2 + 1)/cos(x)

mupad [B] time = 0.11, size = 13, normalized size = 1.00

$$\frac{\sqrt{\cos(x)^2 + 1}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)/(cos(x)*(cos(x)^2 + 1)^(1/2)),x)

[Out] (cos(x)^2 + 1)^(1/2)/cos(x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x) \sec(x)}{\sqrt{\cos^2(x) + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)/(1+cos(x)**2)**(1/2),x)

[Out] Integral(tan(x)*sec(x)/sqrt(cos(x)**2 + 1), x)

3.730 $\int e^{\sec(x)} \sec(x) \tan(x) dx$

Optimal. Leaf size=4

$$e^{\sec(x)}$$

[Out] exp(sec(x))

Rubi [A] time = 0.02, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4339, 2209}

$$e^{\sec(x)}$$

Antiderivative was successfully verified.

[In] Int[E^Sec[x]*Sec[x]*Tan[x],x]

[Out] E^Sec[x]

Rule 2209

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]

Rule 4339

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])

Rubi steps

$$\begin{aligned} \int e^{\sec(x)} \sec(x) \tan(x) dx &= -\text{Subst} \left(\int \frac{e^{\frac{1}{x}}}{x^2} dx, x, \cos(x) \right) \\ &= e^{\sec(x)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 4, normalized size = 1.00

$$e^{\sec(x)}$$

Antiderivative was successfully verified.

[In] Integrate[E^Sec[x]*Sec[x]*Tan[x],x]

[Out] E^Sec[x]

fricas [A] time = 0.92, size = 5, normalized size = 1.25

$$e^{\frac{1}{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(sec(x))*sec(x)*tan(x),x, algorithm="fricas")

[Out] e^(1/cos(x))

giac [A] time = 0.14, size = 5, normalized size = 1.25

$$e^{\frac{1}{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(sec(x))*sec(x)*tan(x),x, algorithm="giac")

[Out] e^(1/cos(x))

maple [A] time = 0.02, size = 4, normalized size = 1.00

$$e^{\sec(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(sec(x))*sec(x)*tan(x),x)

[Out] exp(sec(x))

maxima [A] time = 0.33, size = 3, normalized size = 0.75

$$e^{\sec(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(sec(x))*sec(x)*tan(x),x, algorithm="maxima")

[Out] e^sec(x)

mupad [B] time = 3.09, size = 5, normalized size = 1.25

$$e^{\frac{1}{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(1/cos(x))*tan(x))/cos(x),x)

[Out] exp(1/cos(x))

sympy [A] time = 0.61, size = 3, normalized size = 0.75

$$e^{\sec(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(sec(x))*sec(x)*tan(x),x)

[Out] exp(sec(x))

3.731 $\int 2^{\sec(x)} \sec(x) \tan(x) dx$

Optimal. Leaf size=9

$$\frac{2^{\sec(x)}}{\log(2)}$$

[Out] $2^{\sec(x)}/\ln(2)$

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4339, 2209}

$$\frac{2^{\sec(x)}}{\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^Sec[x]*Sec[x]*Tan[x], x]

[Out] 2^Sec[x]/Log[2]

Rule 2209

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^n * F^(a + b*(c + d*x)^n)) / (b*f*n*(c + d*x)^n * Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]

Rule 4339

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])

Rubi steps

$$\int 2^{\sec(x)} \sec(x) \tan(x) dx = -\text{Subst} \left(\int \frac{2^{\frac{1}{x}}}{x^2} dx, x, \cos(x) \right) = \frac{2^{\sec(x)}}{\log(2)}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{2^{\sec(x)}}{\log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[2^Sec[x]*Sec[x]*Tan[x], x]

[Out] 2^Sec[x]/Log[2]

fricas [A] time = 0.47, size = 11, normalized size = 1.22

$$\frac{2^{\left(\frac{1}{\cos(x)}\right)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^sec(x)*sec(x)*tan(x),x, algorithm="fricas")

[Out] 2^(1/cos(x))/log(2)

giac [A] time = 0.15, size = 11, normalized size = 1.22

$$\frac{2^{\left(\frac{1}{\cos(x)}\right)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^sec(x)*sec(x)*tan(x),x, algorithm="giac")

[Out] 2^(1/cos(x))/log(2)

maple [A] time = 0.02, size = 10, normalized size = 1.11

$$\frac{2^{\sec(x)}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2^sec(x)*sec(x)*tan(x),x)

[Out] 2^sec(x)/ln(2)

maxima [A] time = 0.31, size = 9, normalized size = 1.00

$$\frac{2^{\sec(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^sec(x)*sec(x)*tan(x),x, algorithm="maxima")

[Out] 2^sec(x)/log(2)

mupad [B] time = 3.12, size = 11, normalized size = 1.22

$$\frac{2^{\frac{1}{\cos(x)}}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2^(1/cos(x))*tan(x))/cos(x),x)

[Out] 2^(1/cos(x))/log(2)

sympy [A] time = 0.65, size = 7, normalized size = 0.78

$$\frac{2^{\sec(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2**sec(x)*sec(x)*tan(x),x)

[Out] 2**sec(x)/log(2)

$$3.732 \quad \int \frac{\sec(2x) \tan(2x)}{(1+\sec(2x))^{3/2}} dx$$

Optimal. Leaf size=12

$$-\frac{1}{\sqrt{\sec(2x)+1}}$$

[Out] $-1/(1+\sec(2*x))^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4339, 261}

$$-\frac{1}{\sqrt{\sec(2x)+1}}$$

Antiderivative was successfully verified.

[In] Int[(Sec[2*x]*Tan[2*x])/(1 + Sec[2*x])^(3/2), x]

[Out] $-(1/\text{Sqrt}[1 + \text{Sec}[2*x]])$

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 4339

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])

Rubi steps

$$\begin{aligned} \int \frac{\sec(2x) \tan(2x)}{(1+\sec(2x))^{3/2}} dx &= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{\left(1 + \frac{1}{x}\right)^{3/2} x^2} dx, x, \cos(2x) \right) \right) \\ &= -\frac{1}{\sqrt{1+\sec(2x)}} \end{aligned}$$

Mathematica [A] time = 0.07, size = 20, normalized size = 1.67

$$-\frac{2 \cos^2(x) \sec(2x)}{(\sec(2x)+1)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[2*x]*Tan[2*x])/(1 + Sec[2*x])^(3/2), x]

[Out] $(-2*\text{Cos}[x]^2*\text{Sec}[2*x])/(1 + \text{Sec}[2*x])^{(3/2)}$

fricas [B] time = 0.76, size = 29, normalized size = 2.42

$$-\frac{\sqrt{\frac{\cos(2x)+1}{\cos(2x)}} \cos(2x)}{\cos(2x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x, algorithm="fricas")

[Out] -sqrt((cos(2*x) + 1)/cos(2*x))*cos(2*x)/(cos(2*x) + 1)

giac [B] time = 0.17, size = 37, normalized size = 3.08

$$\frac{1}{\left(\sqrt{\cos(2x)^2 + \cos(2x)} - \cos(2x) - 1\right) \operatorname{sgn}(\cos(2x))} + \operatorname{sgn}(\cos(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x, algorithm="giac")

[Out] 1/((sqrt(cos(2*x)^2 + cos(2*x)) - cos(2*x) - 1)*sgn(cos(2*x))) + sgn(cos(2*x))

maple [A] time = 0.08, size = 11, normalized size = 0.92

$$-\frac{1}{\sqrt{1 + \sec(2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x)

[Out] -1/(1+sec(2*x))^(1/2)

maxima [A] time = 0.32, size = 10, normalized size = 0.83

$$-\frac{1}{\sqrt{\sec(2x) + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x, algorithm="maxima")

[Out] -1/sqrt(sec(2*x) + 1)

mupad [B] time = 3.11, size = 18, normalized size = 1.50

$$-\frac{1}{\sqrt{\cos(2x) + 1} \sqrt{\frac{1}{\cos(2x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(2*x)/(cos(2*x)*(1/cos(2*x) + 1)^(3/2)),x)

[Out] -1/((cos(2*x) + 1)^(1/2)*(1/cos(2*x))^(1/2))

sympy [A] time = 1.02, size = 12, normalized size = 1.00

$$-\frac{1}{\sqrt{\sec(2x) + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))**(3/2),x)

[Out] -1/sqrt(sec(2*x) + 1)

3.733 $\int \sqrt{1 + 5 \cos^2(3x)} \sec(3x) \tan(3x) dx$

Optimal. Leaf size=43

$$\frac{1}{3} \sqrt{5 \cos^2(3x) + 1} \sec(3x) - \frac{1}{3} \sqrt{5} \sinh^{-1}(\sqrt{5} \cos(3x))$$

[Out] $-1/3*\operatorname{arcsinh}(\cos(3*x)*5^{(1/2)})*5^{(1/2)}+1/3*\sec(3*x)*(1+5*\cos(3*x)^2)^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {277, 215}

$$\frac{1}{3} \sqrt{5 \cos^2(3x) + 1} \sec(3x) - \frac{1}{3} \sqrt{5} \sinh^{-1}(\sqrt{5} \cos(3x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sqrt}[1 + 5*\operatorname{Cos}[3*x]^2]*\operatorname{Sec}[3*x]*\operatorname{Tan}[3*x], x]$

[Out] $-(\operatorname{Sqrt}[5]*\operatorname{ArcSinh}[\operatorname{Sqrt}[5]*\operatorname{Cos}[3*x]])/3 + (\operatorname{Sqrt}[1 + 5*\operatorname{Cos}[3*x]^2]*\operatorname{Sec}[3*x])/3$

Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /;$ FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 277

$\operatorname{Int}[(c_)*(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \operatorname{Simp}[(c*x)^{(m+1)}*(a + b*x^n)^p/(c*(m+1)), x] - \operatorname{Dist}[(b*n*p)/(c^n*(m+1)), \operatorname{Int}[(c*x)^{(m+n)}*(a + b*x^n)^{(p-1)}, x], x] /;$ FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned} \int \sqrt{1 + 5 \cos^2(3x)} \sec(3x) \tan(3x) dx &= - \left(\frac{1}{3} \operatorname{Subst} \left(\int \frac{\sqrt{1 + 5x^2}}{x^2} dx, x, \cos(3x) \right) \right) \\ &= \frac{1}{3} \sqrt{1 + 5 \cos^2(3x)} \sec(3x) - \frac{5}{3} \operatorname{Subst} \left(\int \frac{1}{\sqrt{1 + 5x^2}} dx, x, \cos(3x) \right) \\ &= -\frac{1}{3} \sqrt{5} \sinh^{-1}(\sqrt{5} \cos(3x)) + \frac{1}{3} \sqrt{1 + 5 \cos^2(3x)} \sec(3x) \end{aligned}$$

Mathematica [A] time = 0.05, size = 43, normalized size = 1.00

$$\frac{1}{3} \sqrt{5 \cos^2(3x) + 1} \sec(3x) - \frac{1}{3} \sqrt{5} \sinh^{-1}(\sqrt{5} \cos(3x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Integrate}[\operatorname{Sqrt}[1 + 5*\operatorname{Cos}[3*x]^2]*\operatorname{Sec}[3*x]*\operatorname{Tan}[3*x], x]$

[Out] $-1/3*(\operatorname{Sqrt}[5]*\operatorname{ArcSinh}[\operatorname{Sqrt}[5]*\operatorname{Cos}[3*x]]) + (\operatorname{Sqrt}[1 + 5*\operatorname{Cos}[3*x]^2]*\operatorname{Sec}[3*x])/3$

fricas [B] time = 0.95, size = 122, normalized size = 2.84

$$\frac{\sqrt{5} \cos(3x) \log\left(80000 \cos(3x)^8 + 32000 \cos(3x)^6 + 4000 \cos(3x)^4 + 160 \cos(3x)^2 - 8\left(2000 \sqrt{5} \cos(3x)^7\right)\right)}{24 \cos(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x, algorithm="fricas")

[Out] 1/24*(sqrt(5)*cos(3*x)*log(80000*cos(3*x)^8 + 32000*cos(3*x)^6 + 4000*cos(3*x)^4 + 160*cos(3*x)^2 - 8*(2000*sqrt(5)*cos(3*x)^7 + 600*sqrt(5)*cos(3*x)^5 + 50*sqrt(5)*cos(3*x)^3 + sqrt(5)*cos(3*x))*sqrt(5*cos(3*x)^2 + 1) + 1) + 8*sqrt(5*cos(3*x)^2 + 1))/cos(3*x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{5 \cos(3x)^2 + 1} \sec(3x) \tan(3x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x, algorithm="giac")

[Out] integrate(sqrt(5*cos(3*x)^2 + 1)*sec(3*x)*tan(3*x), x)

maple [A] time = 0.12, size = 65, normalized size = 1.51

$$\frac{\sqrt{\frac{\sec^2(3x)+5}{\sec(3x)^2}} \sec(3x) \left(\sqrt{\sec^2(3x)+5} - \sqrt{5} \operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{\sec^2(3x)+5}}\right) \right)}{3\sqrt{\sec^2(3x)+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x)

[Out] 1/3*((sec(3*x)^2+5)/sec(3*x)^2)^(1/2)*sec(3*x)/(sec(3*x)^2+5)^(1/2)*((sec(3*x)^2+5)^(1/2)-5^(1/2)*arctanh(5^(1/2)/(sec(3*x)^2+5)^(1/2)))

maxima [A] time = 0.43, size = 35, normalized size = 0.81

$$-\frac{1}{3} \sqrt{5} \operatorname{arsinh}(\sqrt{5} \cos(3x)) + \frac{\sqrt{5 \cos(3x)^2 + 1}}{3 \cos(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x, algorithm="maxima")

[Out] -1/3*sqrt(5)*arcsinh(sqrt(5)*cos(3*x)) + 1/3*sqrt(5*cos(3*x)^2 + 1)/cos(3*x)

mupad [B] time = 3.26, size = 36, normalized size = 0.84

$$\frac{\sqrt{\frac{5 \cos(6x)}{2} + \frac{7}{2}}}{3 \cos(3x)} + \frac{\sqrt{5} \operatorname{asin}(\sqrt{5} \cos(3x) 1i) 1i}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(3*x)*(5*cos(3*x)^2 + 1)^(1/2))/cos(3*x),x)

[Out] (5^(1/2)*asin(5^(1/2)*cos(3*x)*1i)*1i)/3 + ((5*cos(6*x))/2 + 7/2)^(1/2)/(3*cos(3*x))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{5 \cos^2(3x) + 1} \tan(3x) \sec(3x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*(1+5*cos(3*x)**2)**(1/2)*tan(3*x), x)
```

```
[Out] Integral(sqrt(5*cos(3*x)**2 + 1)*tan(3*x)*sec(3*x), x)
```

$$3.734 \quad \int \frac{\sec(3x) \tan(3x)}{\sqrt{1+5 \cos^2(3x)}} dx$$

Optimal. Leaf size=22

$$\frac{1}{3} \sqrt{5 \cos^2(3x) + 1} \sec(3x)$$

[Out] 1/3*sec(3*x)*(1+5*cos(3*x)^2)^(1/2)

Rubi [A] time = 0.09, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {264}

$$\frac{1}{3} \sqrt{5 \cos^2(3x) + 1} \sec(3x)$$

Antiderivative was successfully verified.

[In] Int[(Sec[3*x]*Tan[3*x])/Sqrt[1 + 5*Cos[3*x]^2], x]

[Out] (Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/3

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sec(3x) \tan(3x)}{\sqrt{1+5 \cos^2(3x)}} dx &= -\left(\frac{1}{3} \text{Subst}\left(\int \frac{1}{x^2 \sqrt{1+5x^2}} dx, x, \cos(3x)\right)\right) \\ &= \frac{1}{3} \sqrt{1+5 \cos^2(3x)} \sec(3x) \end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 1.00

$$\frac{1}{3} \sqrt{5 \cos^2(3x) + 1} \sec(3x)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[3*x]*Tan[3*x])/Sqrt[1 + 5*Cos[3*x]^2], x]

[Out] (Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/3

fricas [A] time = 0.67, size = 20, normalized size = 0.91

$$\frac{\sqrt{5 \cos(3x)^2 + 1}}{3 \cos(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2), x, algorithm="fricas")

[Out] 1/3*sqrt(5*cos(3*x)^2 + 1)/cos(3*x)

giac [A] time = 0.15, size = 34, normalized size = 1.55

$$\frac{2\sqrt{5}}{3\left(\left(\sqrt{5}\cos(3x) - \sqrt{5\cos(3x)^2 + 1}\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x, algorithm="giac")

[Out] -2/3*sqrt(5)/((sqrt(5)*cos(3*x) - sqrt(5*cos(3*x)^2 + 1))^2 - 1)

maple [A] time = 0.11, size = 34, normalized size = 1.55

$$\frac{\sec^2(3x) + 5}{3\sqrt{\frac{\sec^2(3x)+5}{\sec(3x)^2}} \sec(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x)

[Out] 1/3/((sec(3*x)^2+5)/sec(3*x)^2)^(1/2)/sec(3*x)*(sec(3*x)^2+5)

maxima [A] time = 0.42, size = 20, normalized size = 0.91

$$\frac{\sqrt{5\cos(3x)^2 + 1}}{3\cos(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x, algorithm="maxima")

[Out] 1/3*sqrt(5*cos(3*x)^2 + 1)/cos(3*x)

mupad [B] time = 3.03, size = 18, normalized size = 0.82

$$\frac{\sqrt{\frac{5\cos(6x)}{2} + \frac{7}{2}}}{3\cos(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(3*x)/(cos(3*x)*(5*cos(3*x)^2 + 1)^(1/2)),x)

[Out] ((5*cos(6*x))/2 + 7/2)^(1/2)/(3*cos(3*x))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(3x) \sec(3x)}{\sqrt{5\cos^2(3x) + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)**2)**(1/2),x)

[Out] Integral(tan(3*x)*sec(3*x)/sqrt(5*cos(3*x)**2 + 1), x)

$$3.735 \quad \int \frac{\cot(x) \csc(x)}{a+b \csc(x)} dx$$

Optimal. Leaf size=12

$$\frac{\log(a + b \csc(x))}{b}$$

[Out] $-\ln(a+b*\csc(x))/b$

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.67, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4338, 36, 29, 31}

$$\frac{\log(\sin(x))}{b} - \frac{\log(a \sin(x) + b)}{b}$$

Antiderivative was successfully verified.

[In] Int[(Cot[x]*Csc[x])/(a + b*Csc[x]),x]

[Out] Log[Sin[x]]/b - Log[b + a*Sin[x]]/b

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 4338

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a + b*x)]]/d, u, x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])

Rubi steps

$$\begin{aligned} \int \frac{\cot(x) \csc(x)}{a+b \csc(x)} dx &= \text{Subst} \left(\int \frac{1}{x(b+ax)} dx, x, \sin(x) \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{x} dx, x, \sin(x) \right)}{b} - \frac{a \text{Subst} \left(\int \frac{1}{b+ax} dx, x, \sin(x) \right)}{b} \\ &= \frac{\log(\sin(x))}{b} - \frac{\log(b+a \sin(x))}{b} \end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.67

$$\frac{\log(\sin(x))}{b} - \frac{\log(a \sin(x) + b)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x]*Csc[x])/(a + b*Csc[x]),x]

[Out] Log[Sin[x]]/b - Log[b + a*Sin[x]]/b

fricas [A] time = 0.69, size = 20, normalized size = 1.67

$$-\frac{\log(a \sin(x) + b) - \log\left(-\frac{1}{2} \sin(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*csc(x)/(a+b*csc(x)),x, algorithm="fricas")

[Out] -(log(a*sin(x) + b) - log(-1/2*sin(x)))/b

giac [A] time = 0.13, size = 22, normalized size = 1.83

$$-\frac{\log(|a \sin(x) + b|)}{b} + \frac{\log(|\sin(x)|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*csc(x)/(a+b*csc(x)),x, algorithm="giac")

[Out] -log(abs(a*sin(x) + b))/b + log(abs(sin(x)))/b

maple [A] time = 0.05, size = 13, normalized size = 1.08

$$-\frac{\ln(a + b \csc(x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)*csc(x)/(a+b*csc(x)),x)

[Out] -ln(a+b*csc(x))/b

maxima [A] time = 0.31, size = 12, normalized size = 1.00

$$-\frac{\log(b \csc(x) + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*csc(x)/(a+b*csc(x)),x, algorithm="maxima")

[Out] -log(b*csc(x) + a)/b

mupad [B] time = 3.18, size = 31, normalized size = 2.58

$$-\frac{\ln\left(b \tan\left(\frac{x}{2}\right)^2 + 2a \tan\left(\frac{x}{2}\right) + b\right) - \ln\left(\tan\left(\frac{x}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)/(sin(x)*(a + b/sin(x))),x)

[Out] -(log(b + 2*a*tan(x/2) + b*tan(x/2)^2) - log(tan(x/2)))/b

sympy [A] time = 0.39, size = 17, normalized size = 1.42

$$\begin{cases} -\frac{\log\left(\frac{a}{b} + \csc(x)\right)}{b} & \text{for } b \neq 0 \\ -\frac{\csc(x)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(a+b*csc(x)),x)
```

```
[Out] Piecewise((-log(a/b + csc(x))/b, Ne(b, 0)), (-csc(x)/a, True))
```

3.736 $\int 5^{\csc(3x)} \cot(3x) \csc(3x) dx$

Optimal. Leaf size=14

$$\frac{5^{\csc(3x)}}{3 \log(5)}$$

[Out] $-1/3*5^{\csc(3*x)}/\ln(5)$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4338, 2209}

$$\frac{5^{\csc(3x)}}{3 \log(5)}$$

Antiderivative was successfully verified.

[In] Int[5^Csc[3*x]*Cot[3*x]*Csc[3*x], x]

[Out] $-5^{\csc(3*x)}/(3*\text{Log}[5])$

Rule 2209

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^n * F^(a + b*(c + d*x)^n) / (b*f*n*(c + d*x)^n * Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]

Rule 4338

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])

Rubi steps

$$\begin{aligned} \int 5^{\csc(3x)} \cot(3x) \csc(3x) dx &= \frac{1}{3} \text{Subst} \left(\int \frac{5^{\frac{1}{x}}}{x^2} dx, x, \sin(3x) \right) \\ &= -\frac{5^{\csc(3x)}}{3 \log(5)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 1.00

$$\frac{5^{\csc(3x)}}{3 \log(5)}$$

Antiderivative was successfully verified.

[In] Integrate[5^Csc[3*x]*Cot[3*x]*Csc[3*x], x]

[Out] $-1/3*5^{\csc(3*x)}/\text{Log}[5]$

fricas [A] time = 1.14, size = 14, normalized size = 1.00

$$-\frac{5\left(\frac{1}{\sin(3x)}\right)}{3 \log(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(5^{csc(3*x)}*cot(3*x)*csc(3*x),x, algorithm="fricas")

[Out] -1/3*5^{(1/sin(3*x))}/log(5)

giac [A] time = 0.25, size = 14, normalized size = 1.00

$$-\frac{5^{\left(\frac{1}{\sin(3x)}\right)}}{3 \log(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(5^{csc(3*x)}*cot(3*x)*csc(3*x),x, algorithm="giac")

[Out] -1/3*5^{(1/sin(3*x))}/log(5)

maple [A] time = 0.04, size = 13, normalized size = 0.93

$$-\frac{5^{\csc(3x)}}{3 \ln(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(5^{csc(3*x)}*cot(3*x)*csc(3*x),x)

[Out] -1/3*5^{csc(3*x)}/ln(5)

maxima [A] time = 0.30, size = 12, normalized size = 0.86

$$-\frac{5^{\csc(3x)}}{3 \log(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(5^{csc(3*x)}*cot(3*x)*csc(3*x),x, algorithm="maxima")

[Out] -1/3*5^{csc(3*x)}/log(5)

mupad [B] time = 2.97, size = 14, normalized size = 1.00

$$-\frac{5^{\frac{1}{\sin(3x)}}}{3 \ln(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5^{(1/sin(3*x))})*cot(3*x))/sin(3*x),x)

[Out] -5^{(1/sin(3*x))}/(3*log(5))

sympy [A] time = 0.61, size = 12, normalized size = 0.86

$$-\frac{5^{\csc(3x)}}{3 \log(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(5**csc(3*x)*cot(3*x)*csc(3*x),x)

[Out] -5**csc(3*x)/(3*log(5))

$$3.737 \quad \int \frac{\cot(x) \csc(x)}{1 + \csc^2(x)} dx$$

Optimal. Leaf size=3

$$\tan^{-1}(\sin(x))$$

[Out] arctan(sin(x))

Rubi [A] time = 0.03, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4338, 203}

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[(Cot[x]*Csc[x])/(1 + Csc[x]^2), x]

[Out] ArcTan[Sin[x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 4338

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])

Rubi steps

$$\begin{aligned} \int \frac{\cot(x) \csc(x)}{1 + \csc^2(x)} dx &= \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \sin(x) \right) \\ &= \tan^{-1}(\sin(x)) \end{aligned}$$

Mathematica [A] time = 0.01, size = 3, normalized size = 1.00

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x]*Csc[x])/(1 + Csc[x]^2), x]

[Out] ArcTan[Sin[x]]

fricas [A] time = 0.71, size = 3, normalized size = 1.00

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*csc(x)/(1+csc(x)^2), x, algorithm="fricas")

[Out] arctan(sin(x))

giac [A] time = 0.15, size = 3, normalized size = 1.00

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*csc(x)/(1+csc(x)^2),x, algorithm="giac")

[Out] arctan(sin(x))

maple [A] time = 0.05, size = 6, normalized size = 2.00

$$-\arctan(\csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)*csc(x)/(1+csc(x)^2),x)

[Out] -arctan(csc(x))

maxima [A] time = 0.40, size = 3, normalized size = 1.00

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*csc(x)/(1+csc(x)^2),x, algorithm="maxima")

[Out] arctan(sin(x))

mupad [B] time = 3.22, size = 26, normalized size = 8.67

$$\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)^3}{2} + \frac{5 \tan\left(\frac{x}{2}\right)}{2}\right) - \operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)/(sin(x)*(1/sin(x)^2 + 1)),x)

[Out] atan((5*tan(x/2))/2 + tan(x/2)^3/2) - atan(tan(x/2)/2)

sympy [A] time = 0.19, size = 5, normalized size = 1.67

$$-\operatorname{atan}(\csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*csc(x)/(1+csc(x)**2),x)

[Out] -atan(csc(x))

$$3.738 \quad \int \frac{\cot(6x) \csc(6x)}{(5-11 \csc^2(6x))^2} dx$$

Optimal. Leaf size=43

$$\frac{\sin(6x)}{60(11-5\sin^2(6x))} - \frac{\tanh^{-1}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}}$$

[Out] 1/60*sin(6*x)/(11-5*sin(6*x)^2)-1/3300*arctanh(1/11*sin(6*x)*55^(1/2))*55^(1/2)

Rubi [A] time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4338, 288, 206}

$$\frac{\sin(6x)}{60(11-5\sin^2(6x))} - \frac{\tanh^{-1}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}}$$

Antiderivative was successfully verified.

[In] Int[(Cot[6*x]*Csc[6*x])/(5 - 11*Csc[6*x]^2), x]

[Out] -ArcTanh[Sqrt[5/11]*Sin[6*x]]/(60*Sqrt[55]) + Sin[6*x]/(60*(11 - 5*Sin[6*x]^2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*n*(p+1)), x] - Dist[(c^n*(m-n+1))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !ILtQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 4338

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a+b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a+b*x)]]/d, u, x], x], Sin[c*(a+b*x)]/d, x] /; FunctionOfQ[Sin[c*(a+b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])

Rubi steps

$$\begin{aligned} \int \frac{\cot(6x) \csc(6x)}{(5-11 \csc^2(6x))^2} dx &= \frac{1}{6} \text{Subst} \left(\int \frac{x^2}{(11-5x^2)^2} dx, x, \sin(6x) \right) \\ &= \frac{\sin(6x)}{60(11-5\sin^2(6x))} - \frac{1}{60} \text{Subst} \left(\int \frac{1}{11-5x^2} dx, x, \sin(6x) \right) \\ &= -\frac{\tanh^{-1}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}} + \frac{\sin(6x)}{60(11-5\sin^2(6x))} \end{aligned}$$

Mathematica [A] time = 0.66, size = 41, normalized size = 0.95

$$\frac{\sin(6x)}{30(5 \cos(12x) + 17)} - \frac{\tanh^{-1}\left(\sqrt{\frac{5}{11}} \sin(6x)\right)}{60\sqrt{55}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[6*x]*Csc[6*x])/(5 - 11*Csc[6*x]^2)^2,x]

[Out] -1/60*ArcTanh[Sqrt[5/11]*Sin[6*x]]/Sqrt[55] + Sin[6*x]/(30*(17 + 5*Cos[12*x]))

fricas [B] time = 0.64, size = 73, normalized size = 1.70

$$\frac{(5\sqrt{55} \cos(6x)^2 + 6\sqrt{55}) \log\left(-\frac{5 \cos(6x)^2 + 2\sqrt{55} \sin(6x) - 16}{5 \cos(6x)^2 + 6}\right) + 110 \sin(6x)}{6600(5 \cos(6x)^2 + 6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x, algorithm="fricas")

[Out] 1/6600*((5*sqrt(55)*cos(6*x)^2 + 6*sqrt(55))*log(-(5*cos(6*x)^2 + 2*sqrt(55)*sin(6*x) - 16)/(5*cos(6*x)^2 + 6)) + 110*sin(6*x))/(5*cos(6*x)^2 + 6)

giac [A] time = 0.24, size = 48, normalized size = 1.12

$$\frac{1}{6600} \sqrt{55} \log\left(\frac{\sqrt{55} - 5 \sin(6x)}{\sqrt{55} + 5 \sin(6x)}\right) - \frac{\sin(6x)}{60(5 \sin(6x)^2 - 11)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x, algorithm="giac")

[Out] 1/6600*sqrt(55)*log((sqrt(55) - 5*sin(6*x))/(sqrt(55) + 5*sin(6*x))) - 1/60*sin(6*x)/(5*sin(6*x)^2 - 11)

maple [A] time = 0.08, size = 35, normalized size = 0.81

$$\frac{\csc(6x)}{660(\csc^2(6x) - 300)} - \frac{\sqrt{55} \operatorname{arctanh}\left(\frac{\csc(6x)\sqrt{55}}{5}\right)}{3300}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x)

[Out] 1/60*csc(6*x)/(11*csc(6*x)^2-5)-1/3300*55^(1/2)*arctanh(1/5*csc(6*x)*55^(1/2))

maxima [A] time = 0.40, size = 49, normalized size = 1.14

$$\frac{1}{6600} \sqrt{55} \log\left(-\frac{\sqrt{55} - 5 \sin(6x)}{\sqrt{55} + 5 \sin(6x)}\right) - \frac{\sin(6x)}{60(5 \sin(6x)^2 - 11)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x, algorithm="maxima")

[Out] 1/6600*sqrt(55)*log(-(sqrt(55) - 5*sin(6*x))/(sqrt(55) + 5*sin(6*x))) - 1/60*sin(6*x)/(5*sin(6*x)^2 - 11)

mupad [B] time = 3.15, size = 57, normalized size = 1.33

$$\frac{55 \sin(6x) - 11 \sqrt{55} \operatorname{atanh}\left(\frac{\sqrt{55} \sin(6x)}{11}\right) + 5 \sqrt{55} \sin(6x)^2 \operatorname{atanh}\left(\frac{\sqrt{55} \sin(6x)}{11}\right)}{16500 \sin(6x)^2 - 36300}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(6*x)/(sin(6*x)*(11/sin(6*x)^2 - 5)^2), x)

[Out] -(55*sin(6*x) - 11*55^(1/2)*atanh((55^(1/2)*sin(6*x))/11) + 5*55^(1/2)*sin(6*x)^2*atanh((55^(1/2)*sin(6*x))/11))/(16500*sin(6*x)^2 - 36300)

sympy [B] time = 1.72, size = 151, normalized size = 3.51

$$\frac{11\sqrt{55} \log\left(\csc(6x) - \frac{\sqrt{55}}{11}\right) \csc^2(6x)}{72600 \csc^2(6x) - 33000} - \frac{5\sqrt{55} \log\left(\csc(6x) - \frac{\sqrt{55}}{11}\right)}{72600 \csc^2(6x) - 33000} - \frac{11\sqrt{55} \log\left(\csc(6x) + \frac{\sqrt{55}}{11}\right) \csc^2(6x)}{72600 \csc^2(6x) - 33000} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)**2)**2, x)

[Out] 11*sqrt(55)*log(csc(6*x) - sqrt(55)/11)*csc(6*x)**2/(72600*csc(6*x)**2 - 33000) - 5*sqrt(55)*log(csc(6*x) - sqrt(55)/11)/(72600*csc(6*x)**2 - 33000) - 11*sqrt(55)*log(csc(6*x) + sqrt(55)/11)*csc(6*x)**2/(72600*csc(6*x)**2 - 33000) + 5*sqrt(55)*log(csc(6*x) + sqrt(55)/11)/(72600*csc(6*x)**2 - 33000) + 110*csc(6*x)/(72600*csc(6*x)**2 - 33000)

$$3.739 \quad \int \frac{\cot(x) \csc(x)}{\sqrt{1+\sin^2(x)}} dx$$

Optimal. Leaf size=14

$$\sqrt{\sin^2(x) + 1} (-\csc(x))$$

[Out] $-\csc(x) \cdot (1 + \sin(x)^2)^{1/2}$

Rubi [A] time = 0.08, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {264}

$$\sqrt{\sin^2(x) + 1} (-\csc(x))$$

Antiderivative was successfully verified.

[In] Int[(Cot[x]*Csc[x])/Sqrt[1 + Sin[x]^2], x]

[Out] -(Csc[x]*Sqrt[1 + Sin[x]^2])

Rule 264

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\cot(x) \csc(x)}{\sqrt{1+\sin^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{x^2 \sqrt{1+x^2}} dx, x, \sin(x) \right) \\ &= -\csc(x) \sqrt{1+\sin^2(x)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 1.00

$$\sqrt{\sin^2(x) + 1} (-\csc(x))$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x]*Csc[x])/Sqrt[1 + Sin[x]^2], x]

[Out] -(Csc[x]*Sqrt[1 + Sin[x]^2])

fricas [A] time = 1.07, size = 21, normalized size = 1.50

$$\frac{\sqrt{-\cos(x)^2 + 2} - \sin(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*csc(x)/(1+sin(x)^2)^(1/2), x, algorithm="fricas")

[Out] -(sqrt(-cos(x)^2 + 2) - sin(x))/sin(x)

giac [A] time = 0.15, size = 21, normalized size = 1.50

$$\frac{2}{\left(\sqrt{\sin(x)^2 + 1} - \sin(x)\right)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x, algorithm="giac")

[Out] 2/((sqrt(sin(x)^2 + 1) - sin(x))^2 - 1)

maple [A] time = 0.16, size = 15, normalized size = 1.07

$$-\frac{\sqrt{1 + \sin^2(x)}}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x)

[Out] -1/sin(x)*(1+sin(x)^2)^(1/2)

maxima [A] time = 0.40, size = 14, normalized size = 1.00

$$-\frac{\sqrt{\sin(x)^2 + 1}}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x, algorithm="maxima")

[Out] -sqrt(sin(x)^2 + 1)/sin(x)

mupad [B] time = 3.11, size = 34, normalized size = 2.43

$$-\frac{\sqrt{\frac{1}{\sin(x)^2} + 1}}{\sin(x) \left(\sqrt{\frac{1}{\sin(x)^2} + 1} + 1 \right) \sqrt{\sin(x)^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)/(sin(x)*(sin(x)^2 + 1)^(1/2)),x)

[Out] -(1/sin(x)^2 + 1)^(1/2)/(sin(x)*((1/sin(x)^2 + 1)^(1/2) + 1)*(sin(x)^2 + 1)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x) \csc(x)}{\sqrt{\sin^2(x) + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*csc(x)/(1+sin(x)**2)**(1/2),x)

[Out] Integral(cot(x)*csc(x)/sqrt(sin(x)**2 + 1), x)

$$3.740 \quad \int \frac{\cot(5x) \csc^3(5x)}{\sqrt{1+\sin^2(5x)}} dx$$

Optimal. Leaf size=43

$$\frac{2}{15} \sqrt{\sin^2(5x) + 1} \csc(5x) - \frac{1}{15} \sqrt{\sin^2(5x) + 1} \csc^3(5x)$$

[Out] 2/15*csc(5*x)*(1+sin(5*x)^2)^(1/2)-1/15*csc(5*x)^3*(1+sin(5*x)^2)^(1/2)

Rubi [A] time = 0.11, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {271, 264}

$$\frac{2}{15} \sqrt{\sin^2(5x) + 1} \csc(5x) - \frac{1}{15} \sqrt{\sin^2(5x) + 1} \csc^3(5x)$$

Antiderivative was successfully verified.

[In] Int[(Cot[5*x]*Csc[5*x]^3)/Sqrt[1 + Sin[5*x]^2], x]

[Out] (2*Csc[5*x]*Sqrt[1 + Sin[5*x]^2])/15 - (Csc[5*x]^3*Sqrt[1 + Sin[5*x]^2])/15

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 271

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\cot(5x) \csc^3(5x)}{\sqrt{1+\sin^2(5x)}} dx &= \frac{1}{5} \text{Subst} \left(\int \frac{1}{x^4 \sqrt{1+x^2}} dx, x, \sin(5x) \right) \\ &= -\frac{1}{15} \csc^3(5x) \sqrt{1+\sin^2(5x)} - \frac{2}{15} \text{Subst} \left(\int \frac{1}{x^2 \sqrt{1+x^2}} dx, x, \sin(5x) \right) \\ &= \frac{2}{15} \csc(5x) \sqrt{1+\sin^2(5x)} - \frac{1}{15} \csc^3(5x) \sqrt{1+\sin^2(5x)} \end{aligned}$$

Mathematica [A] time = 0.05, size = 28, normalized size = 0.65

$$-\frac{1}{15} \sqrt{\sin^2(5x) + 1} \csc(5x) (\csc^2(5x) - 2)$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[5*x]*Csc[5*x]^3)/Sqrt[1 + Sin[5*x]^2], x]

[Out] -1/15*(Csc[5*x]*(-2 + Csc[5*x]^2)*Sqrt[1 + Sin[5*x]^2])

fricas [A] time = 0.68, size = 57, normalized size = 1.33

$$\frac{2(\cos(5x)^2 - 1)\sin(5x) - (2\cos(5x)^2 - 1)\sqrt{-\cos(5x)^2 + 2}}{15(\cos(5x)^2 - 1)\sin(5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x, algorithm="fricas")

[Out] -1/15*(2*(cos(5*x)^2 - 1)*sin(5*x) - (2*cos(5*x)^2 - 1)*sqrt(-cos(5*x)^2 + 2))/((cos(5*x)^2 - 1)*sin(5*x))

giac [A] time = 0.51, size = 48, normalized size = 1.12

$$\frac{4\left(3\left(\sqrt{\sin(5x)^2 + 1} - \sin(5x)\right)^2 - 1\right)}{15\left(\left(\sqrt{\sin(5x)^2 + 1} - \sin(5x)\right)^2 - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x, algorithm="giac")

[Out] 4/15*(3*(sqrt(sin(5*x)^2 + 1) - sin(5*x))^2 - 1)/((sqrt(sin(5*x)^2 + 1) - sin(5*x))^2 - 1)^3

maple [A] time = 0.24, size = 38, normalized size = 0.88

$$-\frac{\sqrt{1 + \sin^2(5x)}}{15 \sin(5x)^3} + \frac{2\sqrt{1 + \sin^2(5x)}}{15 \sin(5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x)

[Out] -1/15/sin(5*x)^3*(1+sin(5*x)^2)^(1/2)+2/15/sin(5*x)*(1+sin(5*x)^2)^(1/2)

maxima [A] time = 0.40, size = 37, normalized size = 0.86

$$\frac{2\sqrt{\sin(5x)^2 + 1}}{15 \sin(5x)} - \frac{\sqrt{\sin(5x)^2 + 1}}{15 \sin(5x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x, algorithm="maxima")

[Out] 2/15*sqrt(sin(5*x)^2 + 1)/sin(5*x) - 1/15*sqrt(sin(5*x)^2 + 1)/sin(5*x)^3

mupad [B] time = 3.14, size = 28, normalized size = 0.65

$$\frac{\sqrt{\sin(5x)^2 + 1} (2 \sin(5x)^2 - 1)}{15 \sin(5x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(5*x)/(sin(5*x)^3*(sin(5*x)^2 + 1)^(1/2)),x)

[Out] ((sin(5*x)^2 + 1)^(1/2)*(2*sin(5*x)^2 - 1))/(15*sin(5*x)^3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(5x) \csc^3(5x)}{\sqrt{\sin^2(5x) + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(5*x)*csc(5*x)**3/(1+sin(5*x)**2)**(1/2), x)

[Out] Integral(cot(5*x)*csc(5*x)**3/sqrt(sin(5*x)**2 + 1), x)

$$3.741 \quad \int e^{n \sin(a+bx)} \sin(2a + 2bx) dx$$

Optimal. Leaf size=43

$$\frac{2 \sin(a + bx)e^{n \sin(a+bx)}}{bn} - \frac{2e^{n \sin(a+bx)}}{bn^2}$$

[Out] $-2*\exp(n*\sin(b*x+a))/b/n^2+2*\exp(n*\sin(b*x+a))*\sin(b*x+a)/b/n$

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {12, 2176, 2194}

$$\frac{2 \sin(a + bx)e^{n \sin(a+bx)}}{bn} - \frac{2e^{n \sin(a+bx)}}{bn^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(n*\text{Sin}[a + b*x])}* \text{Sin}[2*a + 2*b*x], x]$

[Out] $(-2*E^{(n*\text{Sin}[a + b*x])})/(b*n^2) + (2*E^{(n*\text{Sin}[a + b*x])}* \text{Sin}[a + b*x])/(b*n)$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{Match}[\text{Q}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]]$

Rule 2176

$\text{Int}[((b_*)*(F_))^{((g_)*((e_*) + (f_)*(x_)))^{(n_)*((c_*) + (d_)*(x_))^{(m_*)}}, x_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*(b*F^{(g*(e + f*x)))^n)/(f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*(b*F^{(g*(e + f*x)))^n}, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{GtQ}[m, 0] \ \&\& \ \text{IntegerQ}[2*m] \ \&\& \ !\$UseGamma == True$

Rule 2194

$\text{Int}[(F_)^{((c_)*((a_*) + (b_)*(x_)))^{(n_*)}}, x_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x)))^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

Rubi steps

$$\begin{aligned} \int e^{n \sin(a+bx)} \sin(2a + 2bx) dx &= \frac{\text{Subst}\left(\int 2e^{nx} x dx, x, \sin(a + bx)\right)}{b} \\ &= \frac{2 \text{Subst}\left(\int e^{nx} x dx, x, \sin(a + bx)\right)}{b} \\ &= \frac{2e^{n \sin(a+bx)} \sin(a + bx)}{bn} - \frac{2 \text{Subst}\left(\int e^{nx} dx, x, \sin(a + bx)\right)}{bn} \\ &= -\frac{2e^{n \sin(a+bx)}}{bn^2} + \frac{2e^{n \sin(a+bx)} \sin(a + bx)}{bn} \end{aligned}$$

Mathematica [A] time = 0.06, size = 28, normalized size = 0.65

$$\frac{2e^{n \sin(a+bx)}(n \sin(a + bx) - 1)}{bn^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*Sin[a + b*x])*Sin[2*a + 2*b*x],x]

[Out] (2*E^(n*Sin[a + b*x])*(-1 + n*Sin[a + b*x]))/(b*n^2)

fricas [A] time = 0.60, size = 27, normalized size = 0.63

$$\frac{2(n \sin(bx + a) - 1)e^{n \sin(bx+a)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")

[Out] 2*(n*sin(b*x + a) - 1)*e^(n*sin(b*x + a))/(b*n^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \sin(bx+a)} \sin(2bx + 2a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")

[Out] integrate(e^(n*sin(b*x + a))*sin(2*b*x + 2*a), x)

maple [C] time = 0.14, size = 104, normalized size = 2.42

$$\frac{ie^{n \sin(bx) \cos(a) + n \cos(bx) \sin(a)} e^{ibx} e^{ia}}{nb} + \frac{ie^{n \sin(bx) \cos(a) + n \cos(bx) \sin(a)} e^{-ibx} e^{-ia}}{nb} - \frac{2e^{n(\sin(bx) \cos(a) + \cos(bx) \sin(a))}}{n^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x)

[Out] -I/n/b*exp(n*sin(b*x)*cos(a)+n*cos(b*x)*sin(a))*exp(I*b*x)*exp(I*a)+I/n/b*exp(n*sin(b*x)*cos(a)+n*cos(b*x)*sin(a))*exp(-I*b*x)*exp(-I*a)-2/n^2/b*exp(n*(sin(b*x)*cos(a)+cos(b*x)*sin(a)))

maxima [A] time = 0.34, size = 37, normalized size = 0.86

$$\frac{2\left(ne^{n \sin(bx+a)} \sin(bx + a) - e^{n \sin(bx+a)}\right)}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")

[Out] 2*(n*e^(n*sin(b*x + a))*sin(b*x + a) - e^(n*sin(b*x + a)))/(b*n^2)

mupad [B] time = 3.18, size = 27, normalized size = 0.63

$$\frac{2e^{n \sin(a+bx)}(n \sin(a + bx) - 1)}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*sin(a + b*x))*sin(2*a + 2*b*x),x)

[Out] (2*exp(n*sin(a + b*x))*(n*sin(a + b*x) - 1))/(b*n^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \sin(a+bx)} \sin(2a + 2bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x)

[Out] Integral(exp(n*sin(a + b*x))*sin(2*a + 2*b*x), x)

$$3.742 \quad \int e^{n \sin(a+bx)} \sin(2(a+bx)) dx$$

Optimal. Leaf size=43

$$\frac{2 \sin(a+bx)e^{n \sin(a+bx)}}{bn} - \frac{2e^{n \sin(a+bx)}}{bn^2}$$

[Out] $-2*\exp(n*\sin(b*x+a))/b/n^2+2*\exp(n*\sin(b*x+a))*\sin(b*x+a)/b/n$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {12, 2176, 2194}

$$\frac{2 \sin(a+bx)e^{n \sin(a+bx)}}{bn} - \frac{2e^{n \sin(a+bx)}}{bn^2}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Sin[a + b*x])*Sin[2*(a + b*x)],x]

[Out] $(-2*E^{(n*\sin[a + b*x])})/(b*n^2) + (2*E^{(n*\sin[a + b*x])}*\sin[a + b*x])/(b*n)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2176

Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !\$UseGamma == True

Rule 2194

Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int e^{n \sin(a+bx)} \sin(2(a+bx)) dx &= \frac{\text{Subst}\left(\int 2e^{nx} x dx, x, \sin(a+bx)\right)}{b} \\ &= \frac{2 \text{Subst}\left(\int e^{nx} x dx, x, \sin(a+bx)\right)}{b} \\ &= \frac{2e^{n \sin(a+bx)} \sin(a+bx)}{bn} - \frac{2 \text{Subst}\left(\int e^{nx} dx, x, \sin(a+bx)\right)}{bn} \\ &= -\frac{2e^{n \sin(a+bx)}}{bn^2} + \frac{2e^{n \sin(a+bx)} \sin(a+bx)}{bn} \end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.65

$$\frac{2e^{n \sin(a+bx)}(n \sin(a+bx) - 1)}{bn^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*Sin[a + b*x])*Sin[2*(a + b*x)],x]

[Out] (2*E^(n*Sin[a + b*x])*(-1 + n*Sin[a + b*x]))/(b*n^2)

fricas [A] time = 0.65, size = 27, normalized size = 0.63

$$\frac{2(n \sin(bx + a) - 1)e^{n \sin(bx+a)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")

[Out] 2*(n*sin(b*x + a) - 1)*e^(n*sin(b*x + a))/(b*n^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \sin(bx+a)} \sin(2bx + 2a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")

[Out] integrate(e^(n*sin(b*x + a))*sin(2*b*x + 2*a), x)

maple [C] time = 0.00, size = 104, normalized size = 2.42

$$\frac{ie^{n \sin(bx) \cos(a) + n \cos(bx) \sin(a)} e^{ibx} e^{ia}}{nb} + \frac{ie^{n \sin(bx) \cos(a) + n \cos(bx) \sin(a)} e^{-ibx} e^{-ia}}{nb} - \frac{2e^{n(\sin(bx) \cos(a) + \cos(bx) \sin(a))}}{n^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x)

[Out] -I/n/b*exp(n*sin(b*x)*cos(a)+n*cos(b*x)*sin(a))*exp(I*b*x)*exp(I*a)+I/n/b*exp(n*sin(b*x)*cos(a)+n*cos(b*x)*sin(a))*exp(-I*b*x)*exp(-I*a)-2/n^2/b*exp(n*(sin(b*x)*cos(a)+cos(b*x)*sin(a)))

maxima [A] time = 0.35, size = 37, normalized size = 0.86

$$\frac{2\left(ne^{n \sin(bx+a)} \sin(bx + a) - e^{n \sin(bx+a)}\right)}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")

[Out] 2*(n*e^(n*sin(b*x + a))*sin(b*x + a) - e^(n*sin(b*x + a)))/(b*n^2)

mupad [B] time = 0.00, size = 27, normalized size = 0.63

$$\frac{2e^{n \sin(a+bx)}(n \sin(a + bx) - 1)}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*sin(a + b*x))*sin(2*a + 2*b*x),x)

[Out] (2*exp(n*sin(a + b*x))*(n*sin(a + b*x) - 1))/(b*n^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \sin(a+bx)} \sin(2a + 2bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x)

[Out] Integral(exp(n*sin(a + b*x))*sin(2*a + 2*b*x), x)

$$3.743 \quad \int e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)} \sin(a + bx) dx$$

Optimal. Leaf size=64

$$\frac{4 \sin\left(\frac{a}{2} + \frac{bx}{2}\right) e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn} - \frac{4e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn^2}$$

[Out] $-4*\exp(n*\sin(1/2*b*x+1/2*a))/b/n^2+4*\exp(n*\sin(1/2*b*x+1/2*a))*\sin(1/2*b*x+1/2*a)/b/n$

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {12, 2176, 2194}

$$\frac{4 \sin\left(\frac{a}{2} + \frac{bx}{2}\right) e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn} - \frac{4e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(n*\text{Sin}[a/2 + (b*x)/2])}*\text{Sin}[a + b*x], x]$

[Out] $(-4*E^{(n*\text{Sin}[a/2 + (b*x)/2])})/(b*n^2) + (4*E^{(n*\text{Sin}[a/2 + (b*x)/2])})*\text{Sin}[a/2 + (b*x)/2]/(b*n)$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$

Rule 2176

$\text{Int}[(b_*)*(F_)^{((g_*)*((e_*) + (f_*)*(x_)))}^{(n_*)}*((c_*) + (d_*)*(x_))^{(m_*)}, x_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*(b*F^{(g*(e + f*x)))^n)/(f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*(b*F^{(g*(e + f*x)))^n}, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{GtQ}[m, 0] \ \&\& \ \text{IntegerQ}[2*m] \ \&\& \ !\$UseGamma == \text{True}$

Rule 2194

$\text{Int}[(F_)^{((c_*)*((a_*) + (b_*)*(x_)))}^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x))})^n/(b*c*n*\text{Log}[F]), x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

Rubi steps

$$\begin{aligned} \int e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)} \sin(a + bx) dx &= \frac{2 \text{Subst}\left(\int 2e^{nx} x dx, x, \sin\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b} \\ &= \frac{4 \text{Subst}\left(\int e^{nx} x dx, x, \sin\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b} \\ &= \frac{4e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)} \sin\left(\frac{a}{2} + \frac{bx}{2}\right)}{bn} - \frac{4 \text{Subst}\left(\int e^{nx} dx, x, \sin\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{bn} \\ &= -\frac{4e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)} \sin\left(\frac{a}{2} + \frac{bx}{2}\right)}{bn} \end{aligned}$$

Mathematica [A] time = 0.06, size = 36, normalized size = 0.56

$$\frac{4e^{n \sin\left(\frac{1}{2}(a+bx)\right)} \left(n \sin\left(\frac{1}{2}(a+bx)\right) - 1\right)}{bn^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*Sin[a/2 + (b*x)/2])*Sin[a + b*x],x]

[Out] (4*E^(n*Sin[(a + b*x)/2])*(-1 + n*Sin[(a + b*x)/2]))/(b*n^2)

fricas [A] time = 0.98, size = 33, normalized size = 0.52

$$\frac{4 \left(n \sin\left(\frac{1}{2}bx + \frac{1}{2}a\right) - 1 \right) e^{\left(n \sin\left(\frac{1}{2}bx + \frac{1}{2}a\right) \right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="fricas")

[Out] 4*(n*sin(1/2*b*x + 1/2*a) - 1)*e^(n*sin(1/2*b*x + 1/2*a))/(b*n^2)

giac [B] time = 0.23, size = 138, normalized size = 2.16

$$\frac{4 \left(2ne^{\left(\frac{2n \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)}{\tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + 1}\right)} \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right) - e^{\left(\frac{2n \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)}{\tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + 1}\right)} \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 - e^{\left(\frac{2n \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)}{\tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + 1}\right)} \right)}{bn^2 \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="giac")

[Out] 4*(2*n*e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1/4*b*x + 1/4*a) - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1/4*b*x + 1/4*a)^2 - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)

maple [C] time = 0.13, size = 122, normalized size = 1.91

$$\frac{2ie^{n \sin\left(\frac{bx}{2}\right) \cos\left(\frac{a}{2}\right) + n \cos\left(\frac{bx}{2}\right) \sin\left(\frac{a}{2}\right)} e^{-\frac{ibx}{2}} e^{-\frac{ia}{2}}}{nb} - \frac{2ie^{n \sin\left(\frac{bx}{2}\right) \cos\left(\frac{a}{2}\right) + n \cos\left(\frac{bx}{2}\right) \sin\left(\frac{a}{2}\right)} e^{\frac{ibx}{2}} e^{\frac{ia}{2}}}{nb} - \frac{4e^{n \left(\sin\left(\frac{bx}{2}\right) \cos\left(\frac{a}{2}\right) + \cos\left(\frac{bx}{2}\right) \sin\left(\frac{a}{2}\right) \right)}}{n^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*sin(1/2*b*x+1/2*a))*sin(b*x+a),x)

[Out] 2*I/n/b*exp(n*sin(1/2*b*x)*cos(1/2*a)+n*cos(1/2*b*x)*sin(1/2*a))*exp(-1/2*I*b*x)*exp(-1/2*I*a)-2*I/n/b*exp(n*sin(1/2*b*x)*cos(1/2*a)+n*cos(1/2*b*x)*sin(1/2*a))*exp(1/2*I*b*x)*exp(1/2*I*a)-4/n^2/b*exp(n*(sin(1/2*b*x)*cos(1/2*a)+cos(1/2*b*x)*sin(1/2*a)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(n \sin\left(\frac{1}{2}bx + \frac{1}{2}a\right) \right)} \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="maxima")

[Out] integrate(e^(n*sin(1/2*b*x + 1/2*a))*sin(b*x + a), x)

mupad [B] time = 3.18, size = 33, normalized size = 0.52

$$\frac{4 e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)} \left(n \sin\left(\frac{a}{2} + \frac{bx}{2}\right) - 1 \right)}{b n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*sin(a/2 + (b*x)/2))*sin(a + b*x),x)

[Out] (4*exp(n*sin(a/2 + (b*x)/2))*(n*sin(a/2 + (b*x)/2) - 1))/(b*n^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)} \sin(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x)

[Out] Integral(exp(n*sin(a/2 + b*x/2))*sin(a + b*x), x)

$$3.744 \quad \int e^{n \sin\left(\frac{1}{2}(a+bx)\right)} \sin(a+bx) dx$$

Optimal. Leaf size=64

$$\frac{4 \sin\left(\frac{a}{2} + \frac{bx}{2}\right) e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn} - \frac{4e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn^2}$$

[Out] $-4*\exp(n*\sin(1/2*b*x+1/2*a))/b/n^2+4*\exp(n*\sin(1/2*b*x+1/2*a))*\sin(1/2*b*x+1/2*a)/b/n$

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {12, 2176, 2194}

$$\frac{4 \sin\left(\frac{a}{2} + \frac{bx}{2}\right) e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn} - \frac{4e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn^2}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Sin[(a + b*x)/2])*Sin[a + b*x],x]

[Out] $(-4*E^{(n*\sin[a/2 + (b*x)/2])})/(b*n^2) + (4*E^{(n*\sin[a/2 + (b*x)/2])})*\sin[a/2 + (b*x)/2]/(b*n)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2176

Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !\$UseGamma == True

Rule 2194

Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int e^{n \sin\left(\frac{1}{2}(a+bx)\right)} \sin(a+bx) dx &= \frac{2 \operatorname{Subst}\left(\int 2e^{nx} x dx, x, \sin\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b} \\ &= \frac{4 \operatorname{Subst}\left(\int e^{nx} x dx, x, \sin\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b} \\ &= \frac{4e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)} \sin\left(\frac{a}{2} + \frac{bx}{2}\right)}{bn} - \frac{4 \operatorname{Subst}\left(\int e^{nx} dx, x, \sin\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{bn} \\ &= -\frac{4e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)} \sin\left(\frac{a}{2} + \frac{bx}{2}\right)}{bn} \end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.56

$$\frac{4e^{n \sin\left(\frac{1}{2}(a+bx)\right)} \left(n \sin\left(\frac{1}{2}(a+bx)\right) - 1\right)}{bn^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*Sin[(a + b*x)/2])*Sin[a + b*x], x]

[Out] (4*E^(n*Sin[(a + b*x)/2])*(-1 + n*Sin[(a + b*x)/2]))/(b*n^2)

fricas [A] time = 0.69, size = 33, normalized size = 0.52

$$\frac{4 \left(n \sin\left(\frac{1}{2}bx + \frac{1}{2}a\right) - 1 \right) e^{\left(n \sin\left(\frac{1}{2}bx + \frac{1}{2}a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a), x, algorithm="fricas")

[Out] 4*(n*sin(1/2*b*x + 1/2*a) - 1)*e^(n*sin(1/2*b*x + 1/2*a))/(b*n^2)

giac [B] time = 0.24, size = 138, normalized size = 2.16

$$\frac{4 \left(2ne^{\left(\frac{2n \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)}{\tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + 1}\right)} \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right) - e^{\left(\frac{2n \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)}{\tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + 1}\right)} \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 - e^{\left(\frac{2n \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)}{\tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + 1}\right)} \right)}{bn^2 \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a), x, algorithm="giac")

[Out] 4*(2*n*e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1/4*b*x + 1/4*a) - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1/4*b*x + 1/4*a)^2 - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)

maple [C] time = 0.00, size = 122, normalized size = 1.91

$$\frac{2ie^{n \sin\left(\frac{bx}{2}\right)} \cos\left(\frac{a}{2}\right) + n \cos\left(\frac{bx}{2}\right) \sin\left(\frac{a}{2}\right) e^{-\frac{ibx}{2}} e^{-\frac{ia}{2}}}{nb} - \frac{2ie^{n \sin\left(\frac{bx}{2}\right)} \cos\left(\frac{a}{2}\right) + n \cos\left(\frac{bx}{2}\right) \sin\left(\frac{a}{2}\right) e^{\frac{ibx}{2}} e^{\frac{ia}{2}}}{nb} - \frac{4e^{n \left(\sin\left(\frac{bx}{2}\right) \cos\left(\frac{a}{2}\right) + \cos\left(\frac{bx}{2}\right) \sin\left(\frac{a}{2}\right)\right)}}{n^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*sin(1/2*b*x+1/2*a))*sin(b*x+a), x)

[Out] 2*I/n/b*exp(n*sin(1/2*b*x)*cos(1/2*a)+n*cos(1/2*b*x)*sin(1/2*a))*exp(-1/2*I*b*x)*exp(-1/2*I*a)-2*I/n/b*exp(n*sin(1/2*b*x)*cos(1/2*a)+n*cos(1/2*b*x)*sin(1/2*a))*exp(1/2*I*b*x)*exp(1/2*I*a)-4/n^2/b*exp(n*(sin(1/2*b*x)*cos(1/2*a)+cos(1/2*b*x)*sin(1/2*a)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(n \sin\left(\frac{1}{2}bx + \frac{1}{2}a\right)\right)} \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="maxima")

[Out] integrate(e^(n*sin(1/2*b*x + 1/2*a))*sin(b*x + a), x)

mupad [B] time = 0.00, size = 33, normalized size = 0.52

$$\frac{4 e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)} \left(n \sin\left(\frac{a}{2} + \frac{bx}{2}\right) - 1 \right)}{b n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*sin(a/2 + (b*x)/2))*sin(a + b*x),x)

[Out] (4*exp(n*sin(a/2 + (b*x)/2))*(n*sin(a/2 + (b*x)/2) - 1))/(b*n^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \sin\left(\frac{a}{2} + \frac{bx}{2}\right)} \sin(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x)

[Out] Integral(exp(n*sin(a/2 + b*x/2))*sin(a + b*x), x)

$$3.745 \quad \int e^{n \cos(a+bx)} \sin(2a + 2bx) dx$$

Optimal. Leaf size=43

$$\frac{2e^{n \cos(a+bx)}}{bn^2} - \frac{2 \cos(a + bx)e^{n \cos(a+bx)}}{bn}$$

[Out] $2*\exp(n*\cos(b*x+a))/b/n^2-2*\exp(n*\cos(b*x+a))*\cos(b*x+a)/b/n$

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {12, 2176, 2194}

$$\frac{2e^{n \cos(a+bx)}}{bn^2} - \frac{2 \cos(a + bx)e^{n \cos(a+bx)}}{bn}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Cos[a + b*x])*Sin[2*a + 2*b*x], x]

[Out] $(2*E^{(n*\cos[a + b*x])})/(b*n^2) - (2*E^{(n*\cos[a + b*x])})*\cos[a + b*x]/(b*n)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2176

Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !\$UseGamma == True

Rule 2194

Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int e^{n \cos(a+bx)} \sin(2a + 2bx) dx &= -\frac{\text{Subst}\left(\int 2e^{nx} x dx, x, \cos(a + bx)\right)}{b} \\ &= -\frac{2\text{Subst}\left(\int e^{nx} x dx, x, \cos(a + bx)\right)}{b} \\ &= -\frac{2e^{n \cos(a+bx)} \cos(a + bx)}{bn} + \frac{2\text{Subst}\left(\int e^{nx} dx, x, \cos(a + bx)\right)}{bn} \\ &= \frac{2e^{n \cos(a+bx)}}{bn^2} - \frac{2e^{n \cos(a+bx)} \cos(a + bx)}{bn} \end{aligned}$$

Mathematica [A] time = 0.14, size = 28, normalized size = 0.65

$$\frac{2e^{n \cos(a+bx)}(n \cos(a + bx) - 1)}{bn^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*cos[a + b*x])*Sin[2*a + 2*b*x],x]

[Out] (-2*E^(n*cos[a + b*x])*(-1 + n*cos[a + b*x]))/(b*n^2)

fricas [A] time = 0.54, size = 27, normalized size = 0.63

$$\frac{2(n \cos(bx + a) - 1)e^{n \cos(bx+a)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")

[Out] -2*(n*cos(b*x + a) - 1)*e^(n*cos(b*x + a))/(b*n^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \cos(bx+a)} \sin(2bx + 2a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")

[Out] integrate(e^(n*cos(b*x + a))*sin(2*b*x + 2*a), x)

maple [C] time = 0.11, size = 105, normalized size = 2.44

$$\frac{e^{n \cos(bx) \cos(a) - n \sin(bx) \sin(a)} e^{ibx} e^{ia}}{bn} - \frac{e^{n \cos(bx) \cos(a) - n \sin(bx) \sin(a)} e^{-ibx} e^{-ia}}{bn} + \frac{2 e^{n(\cos(bx) \cos(a) - \sin(bx) \sin(a))}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x)

[Out] -1/b/n*exp(n*cos(b*x)*cos(a)-n*sin(b*x)*sin(a))*exp(I*b*x)*exp(I*a)-1/b/n*exp(n*cos(b*x)*cos(a)-n*sin(b*x)*sin(a))*exp(-I*b*x)*exp(-I*a)+2/b/n^2*exp(n*(cos(b*x)*cos(a)-sin(b*x)*sin(a)))

maxima [A] time = 0.34, size = 37, normalized size = 0.86

$$\frac{2(n \cos(bx + a) e^{n \cos(bx+a)} - e^{n \cos(bx+a)})}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")

[Out] -2*(n*cos(b*x + a)*e^(n*cos(b*x + a)) - e^(n*cos(b*x + a)))/(b*n^2)

mupad [B] time = 3.21, size = 27, normalized size = 0.63

$$\frac{2 e^{n \cos(a+bx)} (n \cos(a + bx) - 1)}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*cos(a + b*x))*sin(2*a + 2*b*x),x)

[Out] -(2*exp(n*cos(a + b*x))*(n*cos(a + b*x) - 1))/(b*n^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \cos(a+bx)} \sin(2a + 2bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x)

[Out] Integral(exp(n*cos(a + b*x))*sin(2*a + 2*b*x), x)

3.746 $\int e^{n \cos(a+bx)} \sin(2(a+bx)) dx$

Optimal. Leaf size=43

$$\frac{2e^{n \cos(a+bx)}}{bn^2} - \frac{2 \cos(a+bx)e^{n \cos(a+bx)}}{bn}$$

[Out] $2*\exp(n*\cos(b*x+a))/b/n^2-2*\exp(n*\cos(b*x+a))*\cos(b*x+a)/b/n$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {12, 2176, 2194}

$$\frac{2e^{n \cos(a+bx)}}{bn^2} - \frac{2 \cos(a+bx)e^{n \cos(a+bx)}}{bn}$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(n*\text{Cos}[a + b*x])}*\text{Sin}[2*(a + b*x)], x]$

[Out] $(2*E^{(n*\text{Cos}[a + b*x])})/(b*n^2) - (2*E^{(n*\text{Cos}[a + b*x])}*\text{Cos}[a + b*x])/(b*n)$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$ FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2176

$\text{Int}[(b_)*(F_)^{((g_)*((e_.) + (f_)*(x_)))^{(n_)*((c_.) + (d_)*(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*(b*F^{(g*(e + f*x)))^n)/(f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*(b*F^{(g*(e + f*x)))^n}, x], x] /;$ FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !\$UseGamma == True

Rule 2194

$\text{Int}[(F_)^{((c_)*((a_.) + (b_)*(x_)))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x))})^n/(b*c*n*\text{Log}[F]), x] /;$ FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int e^{n \cos(a+bx)} \sin(2(a+bx)) dx &= -\frac{\text{Subst}\left(\int 2e^{nx} x dx, x, \cos(a+bx)\right)}{b} \\ &= -\frac{2 \text{Subst}\left(\int e^{nx} x dx, x, \cos(a+bx)\right)}{b} \\ &= -\frac{2e^{n \cos(a+bx)} \cos(a+bx)}{bn} + \frac{2 \text{Subst}\left(\int e^{nx} dx, x, \cos(a+bx)\right)}{bn} \\ &= \frac{2e^{n \cos(a+bx)}}{bn^2} - \frac{2e^{n \cos(a+bx)} \cos(a+bx)}{bn} \end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.65

$$\frac{2e^{n \cos(a+bx)}(n \cos(a+bx) - 1)}{bn^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*cos[a + b*x])*Sin[2*(a + b*x)],x]

[Out] $(-2 * E^{(n \cos[a + b x])} * (-1 + n \cos[a + b x])) / (b * n^2)$

fricas [A] time = 0.72, size = 27, normalized size = 0.63

$$\frac{2(n \cos(bx + a) - 1)e^{(n \cos(bx+a))}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")

[Out] $-2 * (n \cos(b x + a) - 1) * e^{(n \cos(b x + a))} / (b * n^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{(n \cos(bx+a))} \sin(2bx + 2a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")

[Out] integrate(e^(n*cos(b*x + a))*sin(2*b*x + 2*a), x)

maple [C] time = 0.00, size = 105, normalized size = 2.44

$$\frac{e^{n \cos(bx) \cos(a) - n \sin(bx) \sin(a)} e^{ibx} e^{ia}}{bn} - \frac{e^{n \cos(bx) \cos(a) - n \sin(bx) \sin(a)} e^{-ibx} e^{-ia}}{bn} + \frac{2 e^{n(\cos(bx) \cos(a) - \sin(bx) \sin(a))}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x)

[Out] $-1/b/n * \exp(n \cos(b x) * \cos(a) - n \sin(b x) * \sin(a)) * \exp(I * b x) * \exp(I * a) - 1/b/n * \exp(n \cos(b x) * \cos(a) - n \sin(b x) * \sin(a)) * \exp(-I * b x) * \exp(-I * a) + 2/b/n^2 * \exp(n * (\cos(b x) * \cos(a) - \sin(b x) * \sin(a)))$

maxima [A] time = 0.34, size = 37, normalized size = 0.86

$$\frac{2(n \cos(bx + a) e^{(n \cos(bx+a))} - e^{(n \cos(bx+a))})}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")

[Out] $-2 * (n \cos(b x + a) * e^{(n \cos(b x + a))} - e^{(n \cos(b x + a))}) / (b * n^2)$

mupad [B] time = 0.00, size = 27, normalized size = 0.63

$$\frac{2 e^{n \cos(a+b x)} (n \cos(a + b x) - 1)}{b n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*cos(a + b*x))*sin(2*a + 2*b*x),x)

[Out] $-(2 * \exp(n \cos(a + b x)) * (n \cos(a + b x) - 1)) / (b * n^2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \cos(a+bx)} \sin(2a + 2bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x)

[Out] Integral(exp(n*cos(a + b*x))*sin(2*a + 2*b*x), x)

$$3.747 \quad \int e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)} \sin(a + bx) dx$$

Optimal. Leaf size=64

$$\frac{4e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn^2} - \frac{4 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn}$$

[Out] $4 \exp(n \cos(1/2 * b * x + 1/2 * a)) / b / n^2 - 4 \exp(n \cos(1/2 * b * x + 1/2 * a)) * \cos(1/2 * b * x + 1/2 * a) / b / n$

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {12, 2176, 2194}

$$\frac{4e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn^2} - \frac{4 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Cos[a/2 + (b*x)/2])*Sin[a + b*x],x]

[Out] $(4 * E^{(n * \cos[a/2 + (b * x)/2])}) / (b * n^2) - (4 * E^{(n * \cos[a/2 + (b * x)/2])} * \cos[a/2 + (b * x)/2]) / (b * n)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2176

Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !\$UseGamma == True

Rule 2194

Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)} \sin(a + bx) dx &= -\frac{2 \operatorname{Subst}\left(\int 2e^{nx} x dx, x, \cos\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b} \\ &= -\frac{4 \operatorname{Subst}\left(\int e^{nx} x dx, x, \cos\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b} \\ &= -\frac{4e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)} \cos\left(\frac{a}{2} + \frac{bx}{2}\right)}{bn} + \frac{4 \operatorname{Subst}\left(\int e^{nx} dx, x, \cos\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{bn} \\ &= \frac{4e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn^2} - \frac{4e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)} \cos\left(\frac{a}{2} + \frac{bx}{2}\right)}{bn} \end{aligned}$$

Mathematica [A] time = 0.17, size = 36, normalized size = 0.56

$$\frac{4e^{n \cos\left(\frac{1}{2}(a+bx)\right)} \left(n \cos\left(\frac{1}{2}(a+bx)\right) - 1\right)}{bn^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*Cos[a/2 + (b*x)/2])*Sin[a + b*x], x]

[Out] (-4*E^(n*Cos[(a + b*x)/2])*(-1 + n*Cos[(a + b*x)/2]))/(b*n^2)

fricas [A] time = 2.00, size = 33, normalized size = 0.52

$$\frac{4 \left(n \cos\left(\frac{1}{2}bx + \frac{1}{2}a\right) - 1 \right) e^{\left(n \cos\left(\frac{1}{2}bx + \frac{1}{2}a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a), x, algorithm="fricas")

[Out] -4*(n*cos(1/2*b*x + 1/2*a) - 1)*e^(n*cos(1/2*b*x + 1/2*a))/(b*n^2)

giac [B] time = 0.22, size = 195, normalized size = 3.05

$$\frac{4 \left(ne^{\left(\frac{-n \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 - n}{\tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + 1}\right)} \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + e^{\left(\frac{-n \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 - n}{\tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + 1}\right)} \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 - ne^{\left(\frac{-n \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 - n}{\tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + 1}\right)} + e^{\left(\frac{-n \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 - n}{\tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + 1}\right)} \right)}{bn^2 \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a), x, algorithm="giac")

[Out] 4*(n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1/4*b*x + 1/4*a)^2 + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1/4*b*x + 1/4*a)^2 - n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a)^2 + 1)) + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a)^2 + 1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)

maple [C] time = 0.12, size = 123, normalized size = 1.92

$$\frac{2e^{n \cos\left(\frac{bx}{2}\right)} \cos\left(\frac{a}{2}\right) - n \sin\left(\frac{bx}{2}\right) \sin\left(\frac{a}{2}\right)}{bn} e^{\frac{ibx}{2}} e^{\frac{ia}{2}} - \frac{2e^{n \cos\left(\frac{bx}{2}\right)} \cos\left(\frac{a}{2}\right) - n \sin\left(\frac{bx}{2}\right) \sin\left(\frac{a}{2}\right)}{bn} e^{-\frac{ibx}{2}} e^{-\frac{ia}{2}} + \frac{4e^{n \left(\cos\left(\frac{bx}{2}\right) \cos\left(\frac{a}{2}\right) - \sin\left(\frac{bx}{2}\right) \sin\left(\frac{a}{2}\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*cos(1/2*b*x+1/2*a))*sin(b*x+a), x)

[Out] -2/b/n*exp(n*cos(1/2*b*x)*cos(1/2*a)-n*sin(1/2*b*x)*sin(1/2*a))*exp(1/2*I*b*x)*exp(1/2*I*a)-2/b/n*exp(n*cos(1/2*b*x)*cos(1/2*a)-n*sin(1/2*b*x)*sin(1/2*a))*exp(-1/2*I*b*x)*exp(-1/2*I*a)+4/b/n^2*exp(n*(cos(1/2*b*x)*cos(1/2*a)-sin(1/2*b*x)*sin(1/2*a)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(n \cos\left(\frac{1}{2}bx + \frac{1}{2}a\right)\right)} \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="maxima")

[Out] integrate(e^(n*cos(1/2*b*x + 1/2*a))*sin(b*x + a), x)

mupad [B] time = 3.17, size = 33, normalized size = 0.52

$$\frac{4 e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)} \left(n \cos\left(\frac{a}{2} + \frac{bx}{2}\right) - 1\right)}{b n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*cos(a/2 + (b*x)/2))*sin(a + b*x),x)

[Out] -(4*exp(n*cos(a/2 + (b*x)/2))*(n*cos(a/2 + (b*x)/2) - 1))/(b*n^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)} \sin(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x)

[Out] Integral(exp(n*cos(a/2 + b*x/2))*sin(a + b*x), x)

$$3.748 \quad \int e^{n \cos\left(\frac{1}{2}(a+bx)\right)} \sin(a+bx) dx$$

Optimal. Leaf size=64

$$\frac{4e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn^2} - \frac{4 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn}$$

[Out] $4*\exp(n*\cos(1/2*b*x+1/2*a))/b/n^2-4*\exp(n*\cos(1/2*b*x+1/2*a))*\cos(1/2*b*x+1/2*a)/b/n$

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {12, 2176, 2194}

$$\frac{4e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn^2} - \frac{4 \cos\left(\frac{a}{2} + \frac{bx}{2}\right) e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Cos[(a + b*x)/2])*Sin[a + b*x], x]

[Out] $(4*E^{(n*\cos[a/2 + (b*x)/2])})/(b*n^2) - (4*E^{(n*\cos[a/2 + (b*x)/2])})*\cos[a/2 + (b*x)/2]/(b*n)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2176

Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !\$UseGamma == True

Rule 2194

Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int e^{n \cos\left(\frac{1}{2}(a+bx)\right)} \sin(a+bx) dx &= -\frac{2 \operatorname{Subst}\left(\int 2e^{nx} x dx, x, \cos\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b} \\ &= -\frac{4 \operatorname{Subst}\left(\int e^{nx} x dx, x, \cos\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b} \\ &= -\frac{4e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)} \cos\left(\frac{a}{2} + \frac{bx}{2}\right)}{bn} + \frac{4 \operatorname{Subst}\left(\int e^{nx} dx, x, \cos\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{bn} \\ &= \frac{4e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)}}{bn^2} - \frac{4e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)} \cos\left(\frac{a}{2} + \frac{bx}{2}\right)}{bn} \end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.56

$$\frac{4e^{n \cos\left(\frac{1}{2}(a+bx)\right)} \left(n \cos\left(\frac{1}{2}(a+bx)\right) - 1\right)}{bn^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^(n*Cos[(a + b*x)/2])*Sin[a + b*x], x]

[Out] (-4*E^(n*Cos[(a + b*x)/2])*(-1 + n*Cos[(a + b*x)/2]))/(b*n^2)

fricas [A] time = 0.50, size = 33, normalized size = 0.52

$$\frac{4 \left(n \cos\left(\frac{1}{2}bx + \frac{1}{2}a\right) - 1 \right) e^{\left(n \cos\left(\frac{1}{2}bx + \frac{1}{2}a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a), x, algorithm="fricas")

[Out] -4*(n*cos(1/2*b*x + 1/2*a) - 1)*e^(n*cos(1/2*b*x + 1/2*a))/(b*n^2)

giac [B] time = 0.22, size = 195, normalized size = 3.05

$$\frac{4 \left(ne^{\left(\frac{-n \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 - n}{\tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + 1} \right)} \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + e^{\left(\frac{-n \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 - n}{\tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + 1} \right)} \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 - ne^{\left(\frac{-n \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 - n}{\tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + 1} \right)} + e^{\left(\frac{-n \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 - n}{\tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + 1} \right)} \right)}{bn^2 \tan\left(\frac{1}{4}bx + \frac{1}{4}a\right)^2 + bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a), x, algorithm="giac")

[Out] 4*(n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1/4*b*x + 1/4*a)^2 + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1/4*b*x + 1/4*a)^2 - n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a)^2 + 1)) + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a)^2 + 1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)

maple [C] time = 0.00, size = 123, normalized size = 1.92

$$\frac{2e^{n \cos\left(\frac{bx}{2}\right)} \cos\left(\frac{a}{2}\right) - n \sin\left(\frac{bx}{2}\right) \sin\left(\frac{a}{2}\right)}{bn} e^{\frac{ibx}{2}} e^{\frac{ia}{2}} - \frac{2e^{n \cos\left(\frac{bx}{2}\right)} \cos\left(\frac{a}{2}\right) - n \sin\left(\frac{bx}{2}\right) \sin\left(\frac{a}{2}\right)}{bn} e^{-\frac{ibx}{2}} e^{-\frac{ia}{2}} + \frac{4e^{n \left(\cos\left(\frac{bx}{2}\right) \cos\left(\frac{a}{2}\right) - \sin\left(\frac{bx}{2}\right) \sin\left(\frac{a}{2}\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*cos(1/2*b*x+1/2*a))*sin(b*x+a), x)

[Out] -2/b/n*exp(n*cos(1/2*b*x)*cos(1/2*a)-n*sin(1/2*b*x)*sin(1/2*a))*exp(1/2*I*b*x)*exp(1/2*I*a)-2/b/n*exp(n*cos(1/2*b*x)*cos(1/2*a)-n*sin(1/2*b*x)*sin(1/2*a))*exp(-1/2*I*b*x)*exp(-1/2*I*a)+4/b/n^2*exp(n*(cos(1/2*b*x)*cos(1/2*a)-sin(1/2*b*x)*sin(1/2*a)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(n \cos\left(\frac{1}{2}bx + \frac{1}{2}a\right)\right)} \sin(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a), x, algorithm="maxima")

[Out] integrate(e^(n*cos(1/2*b*x + 1/2*a))*sin(b*x + a), x)

mupad [B] time = 0.00, size = 33, normalized size = 0.52

$$\frac{4 e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)} \left(n \cos\left(\frac{a}{2} + \frac{bx}{2}\right) - 1 \right)}{b n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*cos(a/2 + (b*x)/2))*sin(a + b*x), x)

[Out] -(4*exp(n*cos(a/2 + (b*x)/2))*(n*cos(a/2 + (b*x)/2) - 1))/(b*n^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \cos\left(\frac{a}{2} + \frac{bx}{2}\right)} \sin(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a), x)

[Out] Integral(exp(n*cos(a/2 + b*x/2))*sin(a + b*x), x)

3.749 $\int \csc(x) \log(\tan(x)) \sec(x) dx$

Optimal. Leaf size=9

$$\frac{1}{2} \log^2(\tan(x))$$

[Out] 1/2*ln(tan(x))^2

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2620, 29, 6686}

$$\frac{1}{2} \log^2(\tan(x))$$

Antiderivative was successfully verified.

[In] Int[Csc[x]*Log[Tan[x]]*Sec[x],x]

[Out] Log[Tan[x]]^2/2

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 2620

Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]

Rule 6686

Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[(q*y^(m + 1))/(m + 1), x] /; !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\int \csc(x) \log(\tan(x)) \sec(x) dx = \frac{1}{2} \log^2(\tan(x))$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{1}{2} \log^2(\tan(x))$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]*Log[Tan[x]]*Sec[x],x]

[Out] Log[Tan[x]]^2/2

fricas [A] time = 0.61, size = 12, normalized size = 1.33

$$\frac{1}{2} \log\left(\frac{\sin(x)}{\cos(x)}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)*log(tan(x))*sec(x),x, algorithm="fricas")

[Out] $1/2*\log(\sin(x)/\cos(x))^2$

giac [A] time = 0.14, size = 7, normalized size = 0.78

$$\frac{1}{2} \log(\tan(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)*log(tan(x))*sec(x),x, algorithm="giac")`

[Out] $1/2*\log(\tan(x))^2$

maple [A] time = 0.08, size = 8, normalized size = 0.89

$$\frac{\ln(\tan(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(x)*ln(tan(x))*sec(x),x)`

[Out] $1/2*\ln(\tan(x))^2$

maxima [A] time = 0.32, size = 7, normalized size = 0.78

$$\frac{1}{2} \log(\tan(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)*log(tan(x))*sec(x),x, algorithm="maxima")`

[Out] $1/2*\log(\tan(x))^2$

mupad [B] time = 5.27, size = 27, normalized size = 3.00

$$\frac{\ln\left(-\frac{e^{x2i}1i-i}{e^{x2i}+1}\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(log(tan(x))/(cos(x)*sin(x)),x)`

[Out] $\log(-(\exp(x*2i)*1i - 1i)/(\exp(x*2i) + 1))^2/2$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \log(\tan(x)) \csc(x) \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)*ln(tan(x))*sec(x),x)`

[Out] `Integral(log(tan(x))*csc(x)*sec(x), x)`

3.750 $\int \csc(2x) \log(\tan(x)) dx$

Optimal. Leaf size=9

$$\frac{1}{4} \log^2(\tan(x))$$

[Out] 1/4*ln(tan(x))^2

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3770, 6686}

$$\frac{1}{4} \log^2(\tan(x))$$

Antiderivative was successfully verified.

[In] Int[Csc[2*x]*Log[Tan[x]],x]

[Out] Log[Tan[x]]^2/4

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 6686

Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[(q*y^(m + 1))/(m + 1), x] /; !FalseQ[q] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\int \csc(2x) \log(\tan(x)) dx = \frac{1}{4} \log^2(\tan(x))$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{1}{4} \log^2(\tan(x))$$

Antiderivative was successfully verified.

[In] Integrate[Csc[2*x]*Log[Tan[x]],x]

[Out] Log[Tan[x]]^2/4

fricas [A] time = 0.51, size = 7, normalized size = 0.78

$$\frac{1}{4} \log(\tan(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2*x)*log(tan(x)),x, algorithm="fricas")

[Out] 1/4*log(tan(x))^2

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(2x) \log(\tan(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2*x)*log(tan(x)),x, algorithm="giac")

[Out] integrate(csc(2*x)*log(tan(x)), x)

maple [A] time = 0.06, size = 8, normalized size = 0.89

$$\frac{\ln(\tan(x))^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(2*x)*ln(tan(x)),x)

[Out] 1/4*ln(tan(x))^2

maxima [B] time = 0.46, size = 265, normalized size = 29.44

$$\frac{1}{4}(\pi - 2 \arctan(\sin(x), \cos(x) + 1) - 2 \arctan(\sin(x), \cos(x) - 1)) \arctan(\sin(2x), \cos(2x) + 1) + \frac{1}{4} \arctan(\sin(2x), \cos(2x) + 1)^2 - \frac{1}{4}(\pi - 2 \arctan(\sin(x), \cos(x) - 1)) \arctan(\sin(x), \cos(x) + 1) + \frac{1}{4} \arctan(\sin(x), \cos(x) + 1)^2 - \frac{1}{4} \pi \arctan(\sin(x), \cos(x) - 1) + \frac{1}{4} \arctan(\sin(x), \cos(x) - 1)^2 + \frac{1}{8}(\log(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1) + \log(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1)) \log(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1) - \frac{1}{16} \log(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1)^2 - \frac{1}{16} \log(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1)^2 - \frac{1}{8} \log(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1) \log(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1) - \frac{1}{16} \log(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1)^2 - \frac{1}{2} \log(\cot(2x) + \csc(2x)) \log(\tan(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2*x)*log(tan(x)),x, algorithm="maxima")

[Out] 1/4*(pi - 2*arctan2(sin(x), cos(x) + 1) - 2*arctan2(sin(x), cos(x) - 1))*arctan2(sin(2*x), cos(2*x) + 1) + 1/4*arctan2(sin(2*x), cos(2*x) + 1)^2 - 1/4*(pi - 2*arctan2(sin(x), cos(x) - 1))*arctan2(sin(x), cos(x) + 1) + 1/4*arctan2(sin(x), cos(x) + 1)^2 - 1/4*pi*arctan2(sin(x), cos(x) - 1) + 1/4*arctan2(sin(x), cos(x) - 1)^2 + 1/8*(log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1))*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - 1/16*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^2 - 1/16*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^2 - 1/8*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 1/16*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^2 - 1/2*log(cot(2*x) + csc(2*x))*log(tan(x))

mupad [B] time = 3.53, size = 27, normalized size = 3.00

$$\frac{\ln\left(-\frac{e^{x2i} 1i-i}{e^{x2i}+1}\right)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(log(tan(x))/sin(2*x),x)

[Out] log(-(exp(x*2i)*1i - 1i)/(exp(x*2i) + 1))^2/4

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2*x)*ln(tan(x)),x)

[Out] Timed out

3.751 $\int e^{\cos^2(x)+\sin^2(x)} dx$

Optimal. Leaf size=3

ex

[Out] E*x

Rubi [A] time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 203}

ex

Antiderivative was successfully verified.

[In] Int[E^(Cos[x]^2 + Sin[x]^2), x]

[Out] E*x

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int e^{\cos^2(x)+\sin^2(x)} dx &= \text{Subst}\left(\int \frac{e}{1+x^2} dx, x, \tan(x)\right) \\ &= e \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \tan(x)\right) \\ &= ex \end{aligned}$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

ex

Antiderivative was successfully verified.

[In] Integrate[E^(Cos[x]^2 + Sin[x]^2), x]

[Out] E*x

fricas [C] time = 0.55, size = 4, normalized size = 1.33

xe

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(cos(x)^2+sin(x)^2), x, algorithm="fricas")

[Out] x*e

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage₀x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(cos(x)^2+sin(x)^2),x, algorithm="giac")

[Out] sage0*x

maple [C] time = 0.06, size = 5, normalized size = 1.67

ex

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(cos(x)^2+sin(x)^2),x)

[Out] exp(1)*x

maxima [C] time = 0.43, size = 4, normalized size = 1.33

xe

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(cos(x)^2+sin(x)^2),x, algorithm="maxima")

[Out] x*e

mupad [B] time = 0.03, size = 4, normalized size = 1.33

$x e$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(cos(x)^2 + sin(x)^2),x)

[Out] x*exp(1)

sympy [B] time = 0.14, size = 14, normalized size = 4.67

$x e^{\sin^2(x)} e^{\cos^2(x)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(cos(x)**2+sin(x)**2),x)

[Out] x*exp(sin(x)**2)*exp(cos(x)**2)

3.752 $\int x \sec^2(x) dx$

Optimal. Leaf size=8

$$x \tan(x) + \log(\cos(x))$$

[Out] $\ln(\cos(x)) + x \tan(x)$

Rubi [A] time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4184, 3475}

$$x \tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[x \cdot \text{Sec}[x]^2, x]$

[Out] $\text{Log}[\text{Cos}[x]] + x \cdot \text{Tan}[x]$

Rule 3475

$\text{Int}[\tan[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d * x], x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 4184

$\text{Int}[\text{csc}[(e_.) + (f_.)(x_.)]^2 * ((c_.) + (d_.)(x_.))^{(m_.)}, x_Symbol] \rightarrow -\text{Simp}[\text{Cot}[(c + d * x)^m * \text{Cot}[e + f * x]]/f, x] + \text{Dist}[(d * m)/f, \text{Int}[(c + d * x)^{(m - 1)} * \text{Cot}[e + f * x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

Rubi steps

$$\begin{aligned} \int x \sec^2(x) dx &= x \tan(x) - \int \tan(x) dx \\ &= \log(\cos(x)) + x \tan(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$x \tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x \cdot \text{Sec}[x]^2, x]$

[Out] $\text{Log}[\text{Cos}[x]] + x \cdot \text{Tan}[x]$

fricas [B] time = 0.53, size = 18, normalized size = 2.25

$$\frac{\cos(x) \log(-\cos(x)) + x \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x \cdot \text{sec}(x)^2, x, \text{algorithm} = \text{"fricas"})$

[Out] $(\cos(x) \cdot \log(-\cos(x)) + x \cdot \sin(x)) / \cos(x)$

giac [B] time = 0.16, size = 103, normalized size = 12.88

$$\frac{\log\left(\frac{4\left(\tan\left(\frac{1}{2}x\right)^4 - 2\tan\left(\frac{1}{2}x\right)^2 + 1\right)}{\tan\left(\frac{1}{2}x\right)^4 + 2\tan\left(\frac{1}{2}x\right)^2 + 1}\right)\tan\left(\frac{1}{2}x\right)^2 - 4x\tan\left(\frac{1}{2}x\right) - \log\left(\frac{4\left(\tan\left(\frac{1}{2}x\right)^4 - 2\tan\left(\frac{1}{2}x\right)^2 + 1\right)}{\tan\left(\frac{1}{2}x\right)^4 + 2\tan\left(\frac{1}{2}x\right)^2 + 1}\right)}{2\left(\tan\left(\frac{1}{2}x\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x)^2,x, algorithm="giac")

[Out] 1/2*(log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - 4*x*tan(1/2*x) - log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/(tan(1/2*x)^2 - 1)

maple [A] time = 0.02, size = 9, normalized size = 1.12

$$\ln(\cos(x)) + x \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sec(x)^2,x)

[Out] ln(cos(x))+x*tan(x)

maxima [B] time = 0.42, size = 74, normalized size = 9.25

$$\frac{(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1)\log(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1) + 4x\sin(2x)}{2(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x)^2,x, algorithm="maxima")

[Out] 1/2*((cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)

mupad [B] time = 0.02, size = 8, normalized size = 1.00

$$\ln(\cos(x)) + x \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/cos(x)^2,x)

[Out] log(cos(x)) + x*tan(x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sec^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x)**2,x)

[Out] Integral(x*sec(x)**2, x)

3.753 $\int x \cos^4(x^2) dx$

Optimal. Leaf size=34

$$\frac{3x^2}{16} + \frac{1}{8} \sin(x^2) \cos^3(x^2) + \frac{3}{16} \sin(x^2) \cos(x^2)$$

[Out] 3/16*x^2+3/16*cos(x^2)*sin(x^2)+1/8*cos(x^2)^3*sin(x^2)

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3380, 2635, 8}

$$\frac{3x^2}{16} + \frac{1}{8} \sin(x^2) \cos^3(x^2) + \frac{3}{16} \sin(x^2) \cos(x^2)$$

Antiderivative was successfully verified.

[In] Int[x*Cos[x^2]^4,x]

[Out] (3*x^2)/16 + (3*Cos[x^2]*Sin[x^2])/16 + (Cos[x^2]^3*Sin[x^2])/8

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)]^(n_.), x_Symbol] :> -Simp[(b*Cos[c + d*x] * (b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_.)]^(n_.)]*(b_.)^(p_.)*(x_.)^(m_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x \cos^4(x^2) dx &= \frac{1}{2} \text{Subst}\left(\int \cos^4(x) dx, x, x^2\right) \\ &= \frac{1}{8} \cos^3(x^2) \sin(x^2) + \frac{3}{8} \text{Subst}\left(\int \cos^2(x) dx, x, x^2\right) \\ &= \frac{3}{16} \cos(x^2) \sin(x^2) + \frac{1}{8} \cos^3(x^2) \sin(x^2) + \frac{3}{16} \text{Subst}\left(\int 1 dx, x, x^2\right) \\ &= \frac{3x^2}{16} + \frac{3}{16} \cos(x^2) \sin(x^2) + \frac{1}{8} \cos^3(x^2) \sin(x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.82

$$\frac{3x^2}{16} + \frac{1}{8} \sin(2x^2) + \frac{1}{64} \sin(4x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x*cos[x^2]^4,x]

[Out] (3*x^2)/16 + Sin[2*x^2]/8 + Sin[4*x^2]/64

fricas [A] time = 2.30, size = 27, normalized size = 0.79

$$\frac{3}{16}x^2 + \frac{1}{16}\left(2\cos(x^2)^3 + 3\cos(x^2)\right)\sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x^2)^4,x, algorithm="fricas")

[Out] 3/16*x^2 + 1/16*(2*cos(x^2)^3 + 3*cos(x^2))*sin(x^2)

giac [A] time = 0.13, size = 22, normalized size = 0.65

$$\frac{3}{16}x^2 + \frac{1}{64}\sin(4x^2) + \frac{1}{8}\sin(2x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x^2)^4,x, algorithm="giac")

[Out] 3/16*x^2 + 1/64*sin(4*x^2) + 1/8*sin(2*x^2)

maple [A] time = 0.07, size = 26, normalized size = 0.76

$$\frac{\left(\cos^3(x^2) + \frac{3\cos(x^2)}{2}\right)\sin(x^2)}{8} + \frac{3x^2}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(x^2)^4,x)

[Out] 1/8*(cos(x^2)^3+3/2*cos(x^2))*sin(x^2)+3/16*x^2

maxima [A] time = 0.32, size = 22, normalized size = 0.65

$$\frac{3}{16}x^2 + \frac{1}{64}\sin(4x^2) + \frac{1}{8}\sin(2x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x^2)^4,x, algorithm="maxima")

[Out] 3/16*x^2 + 1/64*sin(4*x^2) + 1/8*sin(2*x^2)

mupad [B] time = 2.97, size = 22, normalized size = 0.65

$$\frac{\sin(2x^2)}{8} + \frac{\sin(4x^2)}{64} + \frac{3x^2}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(x^2)^4,x)

[Out] sin(2*x^2)/8 + sin(4*x^2)/64 + (3*x^2)/16

sympy [B] time = 0.96, size = 76, normalized size = 2.24

$$\frac{3x^2\sin^4(x^2)}{16} + \frac{3x^2\sin^2(x^2)\cos^2(x^2)}{8} + \frac{3x^2\cos^4(x^2)}{16} + \frac{3\sin^3(x^2)\cos(x^2)}{16} + \frac{5\sin(x^2)\cos^3(x^2)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x**2)**4,x)
```

```
[Out] 3*x**2*sin(x**2)**4/16 + 3*x**2*sin(x**2)**2*cos(x**2)**2/8 + 3*x**2*cos(x**2)**4/16 + 3*sin(x**2)**3*cos(x**2)/16 + 5*sin(x**2)*cos(x**2)**3/16
```

3.754 $\int \sqrt{\cos(x)} \sin(x) dx$

Optimal. Leaf size=10

$$-\frac{2}{3} \cos^{\frac{3}{2}}(x)$$

[Out] $-2/3*\cos(x)^{(3/2)}$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2565, 30}

$$-\frac{2}{3} \cos^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[x]]*Sin[x],x]

[Out] $(-2*\cos[x]^{(3/2)})/3$

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2565

Int[(cos[(e_) + (f_)*(x_)]*(a_))^(m_)*sin[(e_) + (f_)*(x_)]^(n_), x_Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])

Rubi steps

$$\begin{aligned} \int \sqrt{\cos(x)} \sin(x) dx &= -\text{Subst}\left(\int \sqrt{x} dx, x, \cos(x)\right) \\ &= -\frac{2}{3} \cos^{\frac{3}{2}}(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\frac{2}{3} \cos^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[x]]*Sin[x],x]

[Out] $(-2*\cos[x]^{(3/2)})/3$

fricas [A] time = 0.59, size = 6, normalized size = 0.60

$$-\frac{2}{3} \cos(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*cos(x)^(1/2),x, algorithm="fricas")

[Out] $-2/3*\cos(x)^{(3/2)}$

giac [A] time = 0.14, size = 6, normalized size = 0.60

$$-\frac{2}{3} \cos(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*cos(x)^(1/2),x, algorithm="giac")

[Out] -2/3*cos(x)^(3/2)

maple [A] time = 0.02, size = 7, normalized size = 0.70

$$-\frac{2 \left(\cos^{\frac{3}{2}}(x) \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*cos(x)^(1/2),x)

[Out] -2/3*cos(x)^(3/2)

maxima [A] time = 0.32, size = 6, normalized size = 0.60

$$-\frac{2}{3} \cos(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*cos(x)^(1/2),x, algorithm="maxima")

[Out] -2/3*cos(x)^(3/2)

mupad [B] time = 0.07, size = 6, normalized size = 0.60

$$-\frac{2 \cos(x)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^(1/2)*sin(x),x)

[Out] -(2*cos(x)^(3/2))/3

sympy [A] time = 0.25, size = 10, normalized size = 1.00

$$-\frac{2 \cos^{\frac{3}{2}}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*cos(x)**(1/2),x)

[Out] -2*cos(x)**(3/2)/3

$$3.755 \quad \int e^{-2x} \tan(e^{-2x}) dx$$

Optimal. Leaf size=11

$$\frac{1}{2} \log(\cos(e^{-2x}))$$

[Out] 1/2*ln(cos(exp(-2*x)))

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2282, 3475}

$$\frac{1}{2} \log(\cos(e^{-2x}))$$

Antiderivative was successfully verified.

[In] Int[Tan[E^(-2*x)]/E^(2*x),x]

[Out] Log[Cos[E^(-2*x)]]/2

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int e^{-2x} \tan(e^{-2x}) dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \tan(x) dx, x, e^{-2x}\right)\right) \\ &= \frac{1}{2} \log(\cos(e^{-2x})) \end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{1}{2} \log(\cos(e^{-2x}))$$

Antiderivative was successfully verified.

[In] Integrate[Tan[E^(-2*x)]/E^(2*x),x]

[Out] Log[Cos[E^(-2*x)]]/2

fricas [A] time = 1.38, size = 14, normalized size = 1.27

$$\frac{1}{4} \log\left(\frac{1}{\tan(e^{-2x})^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(exp(-2*x))/exp(2*x),x, algorithm="fricas")

[Out] 1/4*log(1/(tan(e^(-2*x))^2 + 1))

giac [A] time = 0.13, size = 9, normalized size = 0.82

$$\frac{1}{2} \log(|\cos(e^{-2x})|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(exp(-2*x))/exp(2*x),x, algorithm="giac")

[Out] 1/2*log(abs(cos(e^(-2*x))))

maple [A] time = 0.05, size = 9, normalized size = 0.82

$$\frac{\ln(\cos(e^{-2x}))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(exp(-2*x))/exp(2*x),x)

[Out] 1/2*ln(cos(exp(-2*x)))

maxima [A] time = 0.31, size = 8, normalized size = 0.73

$$-\frac{1}{2} \log(\sec(e^{-2x}))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(exp(-2*x))/exp(2*x),x, algorithm="maxima")

[Out] -1/2*log(sec(e^(-2*x)))

mupad [B] time = 3.54, size = 12, normalized size = 1.09

$$\frac{\ln(\tan(e^{-2x})^2 + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-2*x)*tan(exp(-2*x)),x)

[Out] -log(tan(exp(-2*x))^2 + 1)/4

sympy [A] time = 0.31, size = 15, normalized size = 1.36

$$-\frac{\log(\tan^2(e^{-2x}) + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(exp(-2*x))/exp(2*x),x)

[Out] -log(tan(exp(-2*x))**2 + 1)/4

$$3.756 \quad \int \frac{\sec(x) \sin(2x)}{1+\cos(x)} dx$$

Optimal. Leaf size=7

$$-2 \log(\cos(x) + 1)$$

[Out] -2*ln(1+cos(x))

Rubi [A] time = 0.04, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {12, 31}

$$-2 \log(\cos(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]*Sin[2*x])/(1 + Cos[x]),x]

[Out] -2*Log[1 + Cos[x]]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec(x) \sin(2x)}{1 + \cos(x)} dx &= -\text{Subst} \left(\int \frac{2}{1+x} dx, x, \cos(x) \right) \\ &= - \left(2 \text{Subst} \left(\int \frac{1}{1+x} dx, x, \cos(x) \right) \right) \\ &= -2 \log(1 + \cos(x)) \end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.29

$$-4 \log \left(\cos \left(\frac{x}{2} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x]*Sin[2*x])/(1 + Cos[x]),x]

[Out] -4*Log[Cos[x/2]]

fricas [A] time = 0.71, size = 9, normalized size = 1.29

$$-2 \log \left(\frac{1}{2} \cos(x) + \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*sin(2*x)/(1+cos(x)),x, algorithm="fricas")

[Out] -2*log(1/2*cos(x) + 1/2)

giac [A] time = 0.14, size = 7, normalized size = 1.00

$$-2 \log(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*sin(2*x)/(1+cos(x)),x, algorithm="giac")

[Out] -2*log(cos(x) + 1)

maple [A] time = 0.08, size = 8, normalized size = 1.14

$$-2 \ln(1 + \cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)*sin(2*x)/(1+cos(x)),x)

[Out] -2*ln(1+cos(x))

maxima [A] time = 0.32, size = 7, normalized size = 1.00

$$-2 \log(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*sin(2*x)/(1+cos(x)),x, algorithm="maxima")

[Out] -2*log(cos(x) + 1)

mupad [B] time = 2.92, size = 7, normalized size = 1.00

$$-2 \ln(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)/(cos(x)*(cos(x) + 1)),x)

[Out] -2*log(cos(x) + 1)

sympy [A] time = 2.17, size = 8, normalized size = 1.14

$$-2 \log(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*sin(2*x)/(1+cos(x)),x)

[Out] -2*log(cos(x) + 1)

3.757 $\int x \sec^2(3x) dx$

Optimal. Leaf size=19

$$\frac{1}{3}x \tan(3x) + \frac{1}{9} \log(\cos(3x))$$

[Out] 1/9*ln(cos(3*x))+1/3*x*tan(3*x)

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4184, 3475}

$$\frac{1}{3}x \tan(3x) + \frac{1}{9} \log(\cos(3x))$$

Antiderivative was successfully verified.

[In] Int[x*Sec[3*x]^2,x]

[Out] Log[Cos[3*x]]/9 + (x*Tan[3*x])/3

Rule 3475

Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4184

Int[csc[(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rubi steps

$$\begin{aligned} \int x \sec^2(3x) dx &= \frac{1}{3}x \tan(3x) - \frac{1}{3} \int \tan(3x) dx \\ &= \frac{1}{9} \log(\cos(3x)) + \frac{1}{3}x \tan(3x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{3}x \tan(3x) + \frac{1}{9} \log(\cos(3x))$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[3*x]^2,x]

[Out] Log[Cos[3*x]]/9 + (x*Tan[3*x])/3

fricas [A] time = 0.71, size = 28, normalized size = 1.47

$$\frac{\cos(3x) \log(-\cos(3x)) + 3x \sin(3x)}{9 \cos(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(3*x)^2,x, algorithm="fricas")

[Out] 1/9*(cos(3*x)*log(-cos(3*x)) + 3*x*sin(3*x))/cos(3*x)

giac [B] time = 0.18, size = 103, normalized size = 5.42

$$\frac{\log\left(\frac{4\left(\tan\left(\frac{3}{2}x\right)^4 - 2\tan\left(\frac{3}{2}x\right)^2 + 1\right)}{\tan\left(\frac{3}{2}x\right)^4 + 2\tan\left(\frac{3}{2}x\right)^2 + 1}\right)\tan\left(\frac{3}{2}x\right)^2 - 12x\tan\left(\frac{3}{2}x\right) - \log\left(\frac{4\left(\tan\left(\frac{3}{2}x\right)^4 - 2\tan\left(\frac{3}{2}x\right)^2 + 1\right)}{\tan\left(\frac{3}{2}x\right)^4 + 2\tan\left(\frac{3}{2}x\right)^2 + 1}\right)}{18\left(\tan\left(\frac{3}{2}x\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(3*x)^2,x, algorithm="giac")

[Out] 1/18*(log(4*(tan(3/2*x))^4 - 2*tan(3/2*x)^2 + 1)/(tan(3/2*x)^4 + 2*tan(3/2*x)^2 + 1))*tan(3/2*x)^2 - 12*x*tan(3/2*x) - log(4*(tan(3/2*x))^4 - 2*tan(3/2*x)^2 + 1)/(tan(3/2*x)^4 + 2*tan(3/2*x)^2 + 1))/(tan(3/2*x)^2 - 1)

maple [A] time = 0.02, size = 16, normalized size = 0.84

$$\frac{\ln(\cos(3x))}{9} + \frac{x \tan(3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sec(3*x)^2,x)

[Out] 1/9*ln(cos(3*x))+1/3*x*tan(3*x)

maxima [B] time = 0.41, size = 74, normalized size = 3.89

$$\frac{(\cos(6x)^2 + \sin(6x)^2 + 2\cos(6x) + 1)\log(\cos(6x)^2 + \sin(6x)^2 + 2\cos(6x) + 1) + 12x\sin(6x)}{18(\cos(6x)^2 + \sin(6x)^2 + 2\cos(6x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(3*x)^2,x, algorithm="maxima")

[Out] 1/18*((cos(6*x))^2 + sin(6*x)^2 + 2*cos(6*x) + 1)*log(cos(6*x)^2 + sin(6*x)^2 + 2*cos(6*x) + 1) + 12*x*sin(6*x))/(cos(6*x)^2 + sin(6*x)^2 + 2*cos(6*x) + 1)

mupad [B] time = 2.90, size = 15, normalized size = 0.79

$$\frac{\ln(\cos(3x))}{9} + \frac{x \tan(3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/cos(3*x)^2,x)

[Out] log(cos(3*x))/9 + (x*tan(3*x))/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sec^2(3x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(3*x)**2,x)

[Out] Integral(x*sec(3*x)**2, x)

3.758 $\int e^{-2\pi x} \cos(2\pi x) dx$

Optimal. Leaf size=37

$$\frac{e^{-2\pi x} \sin(2\pi x)}{4\pi} - \frac{e^{-2\pi x} \cos(2\pi x)}{4\pi}$$

[Out] $-1/4*\cos(2*Pi*x)/\exp(2*Pi*x)/Pi+1/4*\sin(2*Pi*x)/\exp(2*Pi*x)/Pi$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4433}

$$\frac{e^{-2\pi x} \sin(2\pi x)}{4\pi} - \frac{e^{-2\pi x} \cos(2\pi x)}{4\pi}$$

Antiderivative was successfully verified.

[In] `Int[Cos[2*Pi*x]/E^(2*Pi*x),x]`

[Out] $-\text{Cos}[2*Pi*x]/(4*E^(2*Pi*x)*Pi) + \text{Sin}[2*Pi*x]/(4*E^(2*Pi*x)*Pi)$

Rule 4433

`Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]`

Rubi steps

$$\int e^{-2\pi x} \cos(2\pi x) dx = -\frac{e^{-2\pi x} \cos(2\pi x)}{4\pi} + \frac{e^{-2\pi x} \sin(2\pi x)}{4\pi}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.70

$$\frac{e^{-2\pi x}(\sin(2\pi x) - \cos(2\pi x))}{4\pi}$$

Antiderivative was successfully verified.

[In] `Integrate[Cos[2*Pi*x]/E^(2*Pi*x),x]`

[Out] $(-\text{Cos}[2*Pi*x] + \text{Sin}[2*Pi*x])/(4*E^(2*Pi*x)*Pi)$

fricas [A] time = 0.70, size = 29, normalized size = 0.78

$$-\frac{\cos(2\pi x)e^{(-2\pi x)} - e^{(-2\pi x)}\sin(2\pi x)}{4\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(2*pi*x)/exp(2*pi*x),x, algorithm="fricas")`

[Out] $-1/4*(\cos(2*pi*x)*e^{(-2*pi*x)} - e^{(-2*pi*x)}*\sin(2*pi*x))/pi$

giac [A] time = 0.13, size = 27, normalized size = 0.73

$$-\frac{1}{4} \left(\frac{\cos(2\pi x)}{\pi} - \frac{\sin(2\pi x)}{\pi} \right) e^{(-2\pi x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*pi*x)/exp(2*pi*x),x, algorithm="giac")

[Out] -1/4*(cos(2*pi*x)/pi - sin(2*pi*x)/pi)*e^(-2*pi*x)

maple [A] time = 0.06, size = 31, normalized size = 0.84

$$\frac{-\frac{e^{-2\pi x} \cos(2\pi x)}{2} + \frac{e^{-2\pi x} \sin(2\pi x)}{2}}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*Pi*x)/exp(2*Pi*x),x)

[Out] 1/2/Pi*(-1/2*exp(-2*Pi*x)*cos(2*Pi*x)+1/2*exp(-2*Pi*x)*sin(2*Pi*x))

maxima [A] time = 0.32, size = 26, normalized size = 0.70

$$\frac{(\pi \cos(2\pi x) - \pi \sin(2\pi x))e^{-2\pi x}}{4\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*pi*x)/exp(2*pi*x),x, algorithm="maxima")

[Out] -1/4*(pi*cos(2*pi*x) - pi*sin(2*pi*x))*e^(-2*pi*x)/pi^2

mupad [B] time = 2.90, size = 25, normalized size = 0.68

$$\frac{e^{-2\Pi x} (2 \cos(2\Pi x) - 2 \sin(2\Pi x))}{8\Pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-2*Pi*x)*cos(2*Pi*x),x)

[Out] -(exp(-2*Pi*x)*(2*cos(2*Pi*x) - 2*sin(2*Pi*x)))/(8*Pi)

sympy [A] time = 0.42, size = 32, normalized size = 0.86

$$\frac{e^{-2\pi x} \sin(2\pi x)}{4\pi} - \frac{e^{-2\pi x} \cos(2\pi x)}{4\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*pi*x)/exp(2*pi*x),x)

[Out] exp(-2*pi*x)*sin(2*pi*x)/(4*pi) - exp(-2*pi*x)*cos(2*pi*x)/(4*pi)

$$3.759 \quad \int (\cos^{12}(x) \sin^{10}(x) - \cos^{10}(x) \sin^{12}(x)) dx$$

Optimal. Leaf size=12

$$\frac{1}{11} \sin^{11}(x) \cos^{11}(x)$$

[Out] 1/11*cos(x)^11*sin(x)^11

Rubi [B] time = 0.32, antiderivative size = 129, normalized size of antiderivative = 10.75, number of steps used = 25, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2568, 2635, 8}

$$-\frac{1}{22} \sin^9(x) \cos^{13}(x) - \frac{9}{440} \sin^7(x) \cos^{13}(x) - \frac{7}{880} \sin^5(x) \cos^{13}(x) - \frac{7 \sin^3(x) \cos^{13}(x)}{2816} + \frac{1}{22} \sin^{11}(x) \cos^{11}(x) + \frac{1}{40} \sin^{13}(x) \cos^9(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^12*Sin[x]^10 - Cos[x]^10*Sin[x]^12,x]

[Out] (3*Cos[x]^11*Sin[x])/5632 - (3*Cos[x]^13*Sin[x])/5632 + (Cos[x]^11*Sin[x]^3)/512 - (7*Cos[x]^13*Sin[x]^3)/2816 + (7*Cos[x]^11*Sin[x]^5)/1280 - (7*Cos[x]^13*Sin[x]^5)/880 + (Cos[x]^11*Sin[x]^7)/80 - (9*Cos[x]^13*Sin[x]^7)/440 + (Cos[x]^11*Sin[x]^9)/40 - (Cos[x]^13*Sin[x]^9)/22 + (Cos[x]^11*Sin[x]^11)/22

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2568

Int[(cos[(e_.) + (f_.)*(x_.)]*(b_.))^n]*((a_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1))/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]

Rule 2635

Int[(b_.)*sin[(c_.) + (d_.)*(x_.)]^n, x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
\int (\cos^{12}(x) \sin^{10}(x) - \cos^{10}(x) \sin^{12}(x)) dx &= \int \cos^{12}(x) \sin^{10}(x) dx - \int \cos^{10}(x) \sin^{12}(x) dx \\
&= -\frac{1}{22} \cos^{13}(x) \sin^9(x) + \frac{1}{22} \cos^{11}(x) \sin^{11}(x) + \frac{9}{22} \int \cos^{12}(x) \sin^8(x) dx \\
&= -\frac{9}{440} \cos^{13}(x) \sin^7(x) + \frac{1}{40} \cos^{11}(x) \sin^9(x) - \frac{1}{22} \cos^{13}(x) \sin^5(x) \\
&= -\frac{7}{880} \cos^{13}(x) \sin^5(x) + \frac{1}{80} \cos^{11}(x) \sin^7(x) - \frac{9}{440} \cos^{13}(x) \sin^3(x) \\
&= -\frac{7 \cos^{13}(x) \sin^3(x)}{2816} + \frac{7 \cos^{11}(x) \sin^5(x)}{1280} - \frac{7}{880} \cos^{13}(x) \sin^5(x) \\
&= -\frac{3 \cos^{13}(x) \sin(x)}{5632} + \frac{1}{512} \cos^{11}(x) \sin^3(x) - \frac{7 \cos^{13}(x) \sin^3(x)}{2816} \\
&= \frac{3 \cos^{11}(x) \sin(x)}{5632} - \frac{3 \cos^{13}(x) \sin(x)}{5632} + \frac{1}{512} \cos^{11}(x) \sin^3(x) -
\end{aligned}$$

Mathematica [B] time = 0.03, size = 49, normalized size = 4.08

$$\frac{21 \sin(2x)}{1048576} - \frac{15 \sin(6x)}{1048576} + \frac{15 \sin(10x)}{2097152} - \frac{5 \sin(14x)}{2097152} + \frac{\sin(18x)}{2097152} - \frac{\sin(22x)}{23068672}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^12*Sin[x]^10 - Cos[x]^10*Sin[x]^12,x]

[Out] (21*Sin[2*x])/1048576 - (15*Sin[6*x])/1048576 + (15*Sin[10*x])/2097152 - (5*Sin[14*x])/2097152 + Sin[18*x]/2097152 - Sin[22*x]/23068672

fricas [B] time = 1.53, size = 39, normalized size = 3.25

$$-\frac{1}{11} (\cos(x)^{21} - 5 \cos(x)^{19} + 10 \cos(x)^{17} - 10 \cos(x)^{15} + 5 \cos(x)^{13} - \cos(x)^{11}) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x, algorithm="fricas")

[Out] -1/11*(cos(x)^21 - 5*cos(x)^19 + 10*cos(x)^17 - 10*cos(x)^15 + 5*cos(x)^13 - cos(x)^11)*sin(x)

giac [B] time = 0.15, size = 37, normalized size = 3.08

$$-\frac{1}{23068672} \sin(22x) + \frac{1}{2097152} \sin(18x) - \frac{5}{2097152} \sin(14x) + \frac{15}{2097152} \sin(10x) - \frac{15}{1048576} \sin(6x) + \frac{21}{1048576} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x, algorithm="giac")

[Out] -1/23068672*sin(22*x) + 1/2097152*sin(18*x) - 5/2097152*sin(14*x) + 15/2097152*sin(10*x) - 15/1048576*sin(6*x) + 21/1048576*sin(2*x)

maple [B] time = 0.20, size = 176, normalized size = 14.67

$$\frac{(\cos^{13}(x))(\sin^9(x))}{22} - \frac{9(\sin^7(x))(\cos^{13}(x))}{440} - \frac{7(\sin^5(x))(\cos^{13}(x))}{880} - \frac{7(\sin^3(x))(\cos^{13}(x))}{2816} - \frac{3 \sin(x)(\cos^{13}(x))}{5632}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x)`

[Out]
$$-1/22*\cos(x)^{13}*\sin(x)^9-9/440*\sin(x)^7*\cos(x)^{13}-7/880*\sin(x)^5*\cos(x)^{13}-7/2816*\sin(x)^3*\cos(x)^{13}-3/5632*\sin(x)*\cos(x)^{13}+1/22528*(\cos(x)^{11}+11/10*\cos(x)^9+99/80*\cos(x)^7+231/160*\cos(x)^5+231/128*\cos(x)^3+693/256*\cos(x))*\sin(x)+1/22*\cos(x)^{11}*\sin(x)^{11}+1/40*\sin(x)^9*\cos(x)^{11}+1/80*\sin(x)^7*\cos(x)^{11}+7/1280*\sin(x)^5*\cos(x)^{11}+1/512*\sin(x)^3*\cos(x)^{11}+1/2048*\sin(x)*\cos(x)^{11}-1/20480*(\cos(x)^9+9/8*\cos(x)^7+21/16*\cos(x)^5+105/64*\cos(x)^3+315/128*\cos(x))*\sin(x)$$

maxima [A] time = 0.34, size = 8, normalized size = 0.67

$$\frac{1}{22528} \sin(2x)^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x, algorithm="maxima")`

[Out] `1/22528*sin(2*x)^11`

mupad [B] time = 2.99, size = 49, normalized size = 4.08

$$-\frac{\sin(x)\cos(x)^{21}}{11} + \frac{5\sin(x)\cos(x)^{19}}{11} - \frac{10\sin(x)\cos(x)^{17}}{11} + \frac{10\sin(x)\cos(x)^{15}}{11} - \frac{5\sin(x)\cos(x)^{13}}{11} + \frac{\sin(x)\cos(x)^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)^12*sin(x)^10 - cos(x)^10*sin(x)^12,x)`

[Out]
$$\frac{\cos(x)^{11}*\sin(x)}{11} - \frac{(5*\cos(x)^{13}*\sin(x))}{11} + \frac{(10*\cos(x)^{15}*\sin(x))}{11} - \frac{(10*\cos(x)^{17}*\sin(x))}{11} + \frac{(5*\cos(x)^{19}*\sin(x))}{11} - \frac{(\cos(x)^{21}*\sin(x))}{11}$$

sympy [B] time = 0.08, size = 236, normalized size = 19.67

$$-\frac{\sin^{21}(x)\cos(x)}{22} + \frac{89\sin^{19}(x)\cos(x)}{440} - \frac{301\sin^{17}(x)\cos(x)}{880} + \frac{3683\sin^{15}(x)\cos(x)}{14080} - \frac{433\sin^{13}(x)\cos(x)}{5632} + \frac{\sin^{11}(x)\cos(x)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)**12*sin(x)**10-cos(x)**10*sin(x)**12,x)`

[Out]
$$-\sin(x)**21*\cos(x)/22 + 89*\sin(x)**19*\cos(x)/440 - 301*\sin(x)**17*\cos(x)/880 + 3683*\sin(x)**15*\cos(x)/14080 - 433*\sin(x)**13*\cos(x)/5632 + \sin(x)**11*\cos(x)/22528 + \sin(x)**9*\cos(x)/20480 + 9*\sin(x)**7*\cos(x)/163840 + 21*\sin(x)**5*\cos(x)/327680 + 21*\sin(x)**3*\cos(x)/262144 - \sin(x)*\cos(x)**21/22 + 89*\sin(x)*\cos(x)**19/440 - 301*\sin(x)*\cos(x)**17/880 + 3683*\sin(x)*\cos(x)**15/14080 - 433*\sin(x)*\cos(x)**13/5632 + \sin(x)*\cos(x)**11/22528 + \sin(x)*\cos(x)**9/20480 + 9*\sin(x)*\cos(x)**7/163840 + 21*\sin(x)*\cos(x)**5/327680 + 21*\sin(x)*\cos(x)**3/262144 + 63*\sin(x)*\cos(x)/262144$$

3.760 $\int x \cot(x^2) dx$

Optimal. Leaf size=9

$$\frac{1}{2} \log(\sin(x^2))$$

[Out] 1/2*ln(sin(x^2))

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3748, 3475}

$$\frac{1}{2} \log(\sin(x^2))$$

Antiderivative was successfully verified.

[In] Int[x*Cot[x^2],x]

[Out] Log[Sin[x^2]]/2

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3748

Int[((a_.) + Cot[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cot[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int x \cot(x^2) dx &= \frac{1}{2} \text{Subst}\left(\int \cot(x) dx, x, x^2\right) \\ &= \frac{1}{2} \log(\sin(x^2)) \end{aligned}$$

Mathematica [B] time = 0.01, size = 19, normalized size = 2.11

$$\frac{1}{2} \log(\tan(x^2)) + \frac{1}{2} \log(\cos(x^2))$$

Antiderivative was successfully verified.

[In] Integrate[x*Cot[x^2],x]

[Out] Log[Cos[x^2]]/2 + Log[Tan[x^2]]/2

fricas [A] time = 0.65, size = 13, normalized size = 1.44

$$\frac{1}{4} \log\left(-\frac{1}{2} \cos(2x^2) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cot(x^2),x, algorithm="fricas")

[Out] $1/4*\log(-1/2*\cos(2*x^2) + 1/2)$

giac [A] time = 0.15, size = 8, normalized size = 0.89

$$\frac{1}{2} \log(|\sin(x^2)|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cot(x^2),x, algorithm="giac")`

[Out] $1/2*\log(\text{abs}(\sin(x^2)))$

maple [A] time = 0.00, size = 8, normalized size = 0.89

$$\frac{\ln(\sin(x^2))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cot(x^2),x)`

[Out] $1/2*\ln(\sin(x^2))$

maxima [A] time = 0.32, size = 7, normalized size = 0.78

$$\frac{1}{2} \log(\sin(x^2))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cot(x^2),x, algorithm="maxima")`

[Out] $1/2*\log(\sin(x^2))$

mupad [B] time = 0.12, size = 19, normalized size = 2.11

$$\frac{\ln(e^{x^2 2i} - 1)}{2} - \frac{x^2 1i}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cot(x^2),x)`

[Out] $\log(\exp(x^2*2i) - 1)/2 - (x^2*1i)/2$

sympy [B] time = 0.14, size = 19, normalized size = 2.11

$$-\frac{\log(\tan^2(x^2) + 1)}{4} + \frac{\log(\tan(x^2))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cot(x**2),x)`

[Out] $-\log(\tan(x**2)**2 + 1)/4 + \log(\tan(x**2))/2$

3.761 $\int x \sec^2(x^2) dx$

Optimal. Leaf size=8

$$\frac{\tan(x^2)}{2}$$

[Out] 1/2*tan(x^2)

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4204, 3767, 8}

$$\frac{\tan(x^2)}{2}$$

Antiderivative was successfully verified.

[In] Int[x*Sec[x^2]^2,x]

[Out] Tan[x^2]/2

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 4204

Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int x \sec^2(x^2) dx &= \frac{1}{2} \text{Subst} \left(\int \sec^2(x) dx, x, x^2 \right) \\ &= - \left(\frac{1}{2} \text{Subst} \left(\int 1 dx, x, -\tan(x^2) \right) \right) \\ &= \frac{\tan(x^2)}{2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 8, normalized size = 1.00

$$\frac{\tan(x^2)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[x^2]^2,x]

[Out] Tan[x^2]/2

fricas [A] time = 1.04, size = 12, normalized size = 1.50

$$\frac{\sin(x^2)}{2 \cos(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x^2)^2,x, algorithm="fricas")

[Out] 1/2*sin(x^2)/cos(x^2)

giac [A] time = 0.12, size = 6, normalized size = 0.75

$$\frac{1}{2} \tan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x^2)^2,x, algorithm="giac")

[Out] 1/2*tan(x^2)

maple [A] time = 0.03, size = 7, normalized size = 0.88

$$\frac{\tan(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sec(x^2)^2,x)

[Out] 1/2*tan(x^2)

maxima [B] time = 0.32, size = 35, normalized size = 4.38

$$\frac{\sin(2x^2)}{\cos(2x^2)^2 + \sin(2x^2)^2 + 2\cos(2x^2) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x^2)^2,x, algorithm="maxima")

[Out] sin(2*x^2)/(cos(2*x^2)^2 + sin(2*x^2)^2 + 2*cos(2*x^2) + 1)

mupad [B] time = 0.10, size = 14, normalized size = 1.75

$$\frac{1i}{e^{x^2 2i} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/cos(x^2)^2,x)

[Out] 1i/(exp(x^2*2i) + 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sec^2(x^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x**2)**2,x)

[Out] Integral(x*sec(x**2)**2, x)

$$3.762 \quad \int \frac{\sin(8x)}{9 + \sin^4(4x)} dx$$

Optimal. Leaf size=15

$$\frac{1}{12} \tan^{-1} \left(\frac{1}{3} \sin^2(4x) \right)$$

[Out] 1/12*arctan(1/3*sin(4*x)^2)

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {12, 275, 203}

$$\frac{1}{12} \tan^{-1} \left(\frac{1}{3} \sin^2(4x) \right)$$

Antiderivative was successfully verified.

[In] Int[Sin[8*x]/(9 + Sin[4*x]^4), x]

[Out] ArcTan[Sin[4*x]^2/3]/12

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{\sin(8x)}{9 + \sin^4(4x)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{2x}{9 + x^4} dx, x, \sin(4x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{x}{9 + x^4} dx, x, \sin(4x) \right) \\ &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{9 + x^2} dx, x, \sin^2(4x) \right) \\ &= \frac{1}{12} \tan^{-1} \left(\frac{1}{3} \sin^2(4x) \right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{1}{12} \tan^{-1} \left(\frac{1}{3} \sin^2(4x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[8*x]/(9 + Sin[4*x]^4), x]

[Out] ArcTan[Sin[4*x]^2/3]/12

fricas [A] time = 0.66, size = 13, normalized size = 0.87

$$-\frac{1}{12} \arctan\left(\frac{1}{3} \cos(4x)^2 - \frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(8*x)/(9+sin(4*x)^4), x, algorithm="fricas")

[Out] -1/12*arctan(1/3*cos(4*x)^2 - 1/3)

giac [A] time = 0.66, size = 15, normalized size = 1.00

$$\frac{1}{12} \arctan\left(\frac{3}{\cos(4x)^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(8*x)/(9+sin(4*x)^4), x, algorithm="giac")

[Out] 1/12*arctan(3/(cos(4*x)^2 - 1))

maple [A] time = 0.17, size = 12, normalized size = 0.80

$$\frac{\arctan\left(\frac{(\sin^2(4x))}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(8*x)/(9+sin(4*x)^4), x)

[Out] 1/12*arctan(1/3*sin(4*x)^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(8x)}{\sin(4x)^4 + 9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(8*x)/(9+sin(4*x)^4), x, algorithm="maxima")

[Out] integrate(sin(8*x)/(sin(4*x)^4 + 9), x)

mupad [B] time = 2.96, size = 13, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{10 \tan(4x)^2}{3} + 3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(8*x)/(sin(4*x)^4 + 9), x)

[Out] atan((10*tan(4*x)^2)/3 + 3)/12

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(8*x)/(9+sin(4*x)**4), x)

[Out] Timed out

$$3.763 \quad \int \frac{\cos(2x)}{8 + \sin^2(2x)} dx$$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{\sin(2x)}{2\sqrt{2}}\right)}{4\sqrt{2}}$$

[Out] 1/8*arctan(1/4*sin(2*x)*2^(1/2))*2^(1/2)

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3190, 203}

$$\frac{\tan^{-1}\left(\frac{\sin(2x)}{2\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Cos[2*x]/(8 + Sin[2*x]^2), x]

[Out] ArcTan[Sin[2*x]/(2*Sqrt[2])]/(4*Sqrt[2])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{\cos(2x)}{8 + \sin^2(2x)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{8 + x^2} dx, x, \sin(2x) \right) \\ &= \frac{\tan^{-1}\left(\frac{\sin(2x)}{2\sqrt{2}}\right)}{4\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[2*x]/(8 + Sin[2*x]^2), x]

[Out] ArcTan[(Cos[x]*Sin[x])/Sqrt[2]]/(4*Sqrt[2])

fricas [A] time = 1.58, size = 15, normalized size = 0.65

$$\frac{1}{8} \sqrt{2} \arctan\left(\frac{1}{4} \sqrt{2} \sin(2x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)/(8+sin(2*x)^2),x, algorithm="fricas")

[Out] 1/8*sqrt(2)*arctan(1/4*sqrt(2)*sin(2*x))

giac [A] time = 0.15, size = 15, normalized size = 0.65

$$\frac{1}{8} \sqrt{2} \arctan\left(\frac{1}{4} \sqrt{2} \sin(2x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)/(8+sin(2*x)^2),x, algorithm="giac")

[Out] 1/8*sqrt(2)*arctan(1/4*sqrt(2)*sin(2*x))

maple [A] time = 0.04, size = 16, normalized size = 0.70

$$\frac{\arctan\left(\frac{\sin(2x)\sqrt{2}}{4}\right)\sqrt{2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)/(8+sin(2*x)^2),x)

[Out] 1/8*arctan(1/4*sin(2*x)*2^(1/2))*2^(1/2)

maxima [A] time = 0.43, size = 15, normalized size = 0.65

$$\frac{1}{8} \sqrt{2} \arctan\left(\frac{1}{4} \sqrt{2} \sin(2x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)/(8+sin(2*x)^2),x, algorithm="maxima")

[Out] 1/8*sqrt(2)*arctan(1/4*sqrt(2)*sin(2*x))

mupad [B] time = 0.07, size = 15, normalized size = 0.65

$$\frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} \sin(2x)}{4}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)/(sin(2*x)^2 + 8),x)

[Out] (2^(1/2)*atan((2^(1/2)*sin(2*x))/4))/8

sympy [A] time = 0.27, size = 19, normalized size = 0.83

$$\frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} \sin(2x)}{4}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)/(8+sin(2*x)**2),x)

[Out] sqrt(2)*atan(sqrt(2)*sin(2*x)/4)/8

3.764 $\int x \left(\cos^3(x^2) - \sin^3(x^2) \right) dx$

Optimal. Leaf size=37

$$-\frac{1}{6} \sin^3(x^2) + \frac{\sin(x^2)}{2} - \frac{1}{6} \cos^3(x^2) + \frac{\cos(x^2)}{2}$$

[Out] 1/2*cos(x^2)-1/6*cos(x^2)^3+1/2*sin(x^2)-1/6*sin(x^2)^3

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {14, 3380, 2633, 3379}

$$-\frac{1}{6} \sin^3(x^2) + \frac{\sin(x^2)}{2} - \frac{1}{6} \cos^3(x^2) + \frac{\cos(x^2)}{2}$$

Antiderivative was successfully verified.

[In] Int[x*(Cos[x^2]^3 - Sin[x^2]^3),x]

[Out] Cos[x^2]/2 - Cos[x^2]^3/6 + Sin[x^2]/2 - Sin[x^2]^3/6

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 2633

Int[sin[(c_)+(d_)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1-x^2)^((n-1)/2), x], x], x, Cos[c+d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n-1)/2, 0]

Rule 3379

Int[(x_)^m_*((a_)+(b_)*Sin[(c_)+(d_)*(x_)]^(n_))]^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n]-1)*(a+b*Sin[c+d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m+1)/n]] && (EqQ[p, 1] || EqQ[m, n-1] || (IntegerQ[p] && GtQ[Simplify[(m+1)/n], 0]))

Rule 3380

Int[((a_)+Cos[(c_)+(d_)*(x_)]*(b_))^(p_)*(x_)^m_, x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n]-1)*(a+b*Cos[c+d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m+1)/n]] && (EqQ[p, 1] || EqQ[m, n-1] || (IntegerQ[p] && GtQ[Simplify[(m+1)/n], 0]))

Rubi steps

$$\begin{aligned}
\int x(\cos^3(x^2) - \sin^3(x^2)) dx &= \int (x \cos^3(x^2) - x \sin^3(x^2)) dx \\
&= \int x \cos^3(x^2) dx - \int x \sin^3(x^2) dx \\
&= \frac{1}{2} \text{Subst}\left(\int \cos^3(x) dx, x, x^2\right) - \frac{1}{2} \text{Subst}\left(\int \sin^3(x) dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int (1 - x^2) dx, x, \cos(x^2)\right) - \frac{1}{2} \text{Subst}\left(\int (1 - x^2) dx, x, -\sin(x^2)\right) \\
&= \frac{\cos(x^2)}{2} - \frac{1}{6} \cos^3(x^2) + \frac{\sin(x^2)}{2} - \frac{1}{6} \sin^3(x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.00

$$-\frac{1}{6} \sin^3(x^2) + \frac{\sin(x^2)}{2} + \frac{3 \cos(x^2)}{8} - \frac{1}{24} \cos(3x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x*(Cos[x^2]^3 - Sin[x^2]^3),x]

[Out] (3*Cos[x^2])/8 - Cos[3*x^2]/24 + Sin[x^2]/2 - Sin[x^2]^3/6

fricas [A] time = 1.96, size = 29, normalized size = 0.78

$$-\frac{1}{6} \cos(x^2)^3 + \frac{1}{6} (\cos(x^2)^2 + 2) \sin(x^2) + \frac{1}{2} \cos(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(cos(x^2)^3-sin(x^2)^3),x, algorithm="fricas")

[Out] -1/6*cos(x^2)^3 + 1/6*(cos(x^2)^2 + 2)*sin(x^2) + 1/2*cos(x^2)

giac [A] time = 0.13, size = 29, normalized size = 0.78

$$-\frac{1}{6} \cos(x^2)^3 - \frac{1}{6} \sin(x^2)^3 + \frac{1}{2} \cos(x^2) + \frac{1}{2} \sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(cos(x^2)^3-sin(x^2)^3),x, algorithm="giac")

[Out] -1/6*cos(x^2)^3 - 1/6*sin(x^2)^3 + 1/2*cos(x^2) + 1/2*sin(x^2)

maple [A] time = 0.17, size = 30, normalized size = 0.81

$$\frac{(2 + \cos^2(x^2)) \sin(x^2)}{6} + \frac{(2 + \sin^2(x^2)) \cos(x^2)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(cos(x^2)^3-sin(x^2)^3),x)

[Out] 1/6*(2+cos(x^2)^2)*sin(x^2)+1/6*(2+sin(x^2)^2)*cos(x^2)

maxima [A] time = 0.33, size = 29, normalized size = 0.78

$$-\frac{1}{24} \cos(3x^2) + \frac{3}{8} \cos(x^2) + \frac{1}{24} \sin(3x^2) + \frac{3}{8} \sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(cos(x^2)^3-sin(x^2)^3),x, algorithm="maxima")

[Out] -1/24*cos(3*x^2) + 3/8*cos(x^2) + 1/24*sin(3*x^2) + 3/8*sin(x^2)

mupad [B] time = 2.97, size = 33, normalized size = 0.89

$$-\frac{\cos(x^2)^3}{6} + \frac{\sin(x^2)\cos(x^2)^2}{6} + \frac{\cos(x^2)}{2} + \frac{\sin(x^2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(cos(x^2)^3 - sin(x^2)^3),x)

[Out] cos(x^2)/2 + sin(x^2)/3 + (cos(x^2)^2*sin(x^2))/6 - cos(x^2)^3/6

sympy [A] time = 0.52, size = 42, normalized size = 1.14

$$\frac{\sin^3(x^2)}{3} + \frac{\sin^2(x^2)\cos(x^2)}{2} + \frac{\sin(x^2)\cos^2(x^2)}{2} + \frac{\cos^3(x^2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(cos(x**2)**3-sin(x**2)**3),x)

[Out] sin(x**2)**3/3 + sin(x**2)**2*cos(x**2)/2 + sin(x**2)*cos(x**2)**2/2 + cos(x**2)**3/3

$$3.765 \quad \int \frac{\cos(x) \sin(x)}{1-\cos(x)} dx$$

Optimal. Leaf size=10

$$\cos(x) + \log(1 - \cos(x))$$

[Out] cos(x)+ln(1-cos(x))

Rubi [A] time = 0.03, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2833, 43}

$$\cos(x) + \log(1 - \cos(x))$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Sin[x])/(1 - Cos[x]),x]

[Out] Cos[x] + Log[1 - Cos[x]]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2833

Int[cos[(e_.) + (f_.)*(x_)]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sine[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos(x) \sin(x)}{1 - \cos(x)} dx &= -\text{Subst} \left(\int \frac{x}{1+x} dx, x, -\cos(x) \right) \\ &= -\text{Subst} \left(\int \left(1 + \frac{1}{-1-x} \right) dx, x, -\cos(x) \right) \\ &= \cos(x) + \log(1 - \cos(x)) \end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 1.20

$$\cos(x) + 2 \log \left(\sin \left(\frac{x}{2} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]*Sin[x])/(1 - Cos[x]),x]

[Out] Cos[x] + 2*Log[Sin[x/2]]

fricas [A] time = 1.38, size = 10, normalized size = 1.00

$$\cos(x) + \log \left(-\frac{1}{2} \cos(x) + \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/(1-cos(x)),x, algorithm="fricas")

[Out] $\cos(x) + \log(-1/2*\cos(x) + 1/2)$

giac [A] time = 0.14, size = 10, normalized size = 1.00

$$\cos(x) + \log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/(1-cos(x)),x, algorithm="giac")

[Out] $\cos(x) + \log(-\cos(x) + 1)$

maple [A] time = 0.04, size = 9, normalized size = 0.90

$$\cos(x) + \ln(-1 + \cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(x)/(1-cos(x)),x)

[Out] $\cos(x) + \ln(-1 + \cos(x))$

maxima [A] time = 0.32, size = 8, normalized size = 0.80

$$\cos(x) + \log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/(1-cos(x)),x, algorithm="maxima")

[Out] $\cos(x) + \log(\cos(x) - 1)$

mupad [B] time = 2.92, size = 8, normalized size = 0.80

$$\ln(\cos(x) - 1) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(cos(x)*sin(x))/(cos(x) - 1),x)

[Out] $\log(\cos(x) - 1) + \cos(x)$

sympy [A] time = 0.17, size = 8, normalized size = 0.80

$$\log(\cos(x) - 1) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/(1-cos(x)),x)

[Out] $\log(\cos(x) - 1) + \cos(x)$

3.766 $\int x \cos(x^2) dx$

Optimal. Leaf size=8

$$\frac{\sin(x^2)}{2}$$

[Out] 1/2*sin(x^2)

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3380, 2637}

$$\frac{\sin(x^2)}{2}$$

Antiderivative was successfully verified.

[In] Int[x*Cos[x^2],x]

[Out] Sin[x^2]/2

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x \cos(x^2) dx &= \frac{1}{2} \text{Subst} \left(\int \cos(x) dx, x, x^2 \right) \\ &= \frac{\sin(x^2)}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\sin(x^2)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[x^2],x]

[Out] Sin[x^2]/2

fricas [A] time = 0.53, size = 6, normalized size = 0.75

$$\frac{1}{2} \sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x^2),x, algorithm="fricas")

[Out] 1/2*sin(x^2)

giac [A] time = 0.13, size = 6, normalized size = 0.75

$$\frac{1}{2} \sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x^2),x, algorithm="giac")

[Out] 1/2*sin(x^2)

maple [A] time = 0.04, size = 7, normalized size = 0.88

$$\frac{\sin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(x^2),x)

[Out] 1/2*sin(x^2)

maxima [A] time = 0.32, size = 6, normalized size = 0.75

$$\frac{1}{2} \sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x^2),x, algorithm="maxima")

[Out] 1/2*sin(x^2)

mupad [B] time = 0.06, size = 6, normalized size = 0.75

$$\frac{\sin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(x^2),x)

[Out] sin(x^2)/2

sympy [A] time = 0.15, size = 5, normalized size = 0.62

$$\frac{\sin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x**2),x)

[Out] sin(x**2)/2

3.767 $\int x^2 \cos(4x^3) dx$

Optimal. Leaf size=10

$$\frac{1}{12} \sin(4x^3)$$

[Out] 1/12*sin(4*x^3)

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3380, 2637}

$$\frac{1}{12} \sin(4x^3)$$

Antiderivative was successfully verified.

[In] Int[x^2*cos[4*x^3],x]

[Out] Sin[4*x^3]/12

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x^2 \cos(4x^3) dx &= \frac{1}{3} \text{Subst} \left(\int \cos(4x) dx, x, x^3 \right) \\ &= \frac{1}{12} \sin(4x^3) \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{12} \sin(4x^3)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*cos[4*x^3],x]

[Out] Sin[4*x^3]/12

fricas [A] time = 0.77, size = 8, normalized size = 0.80

$$\frac{1}{12} \sin(4x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(4*x^3),x, algorithm="fricas")

[Out] 1/12*sin(4*x^3)

giac [A] time = 0.12, size = 8, normalized size = 0.80

$$\frac{1}{12} \sin(4x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(4*x^3),x, algorithm="giac")

[Out] 1/12*sin(4*x^3)

maple [A] time = 0.04, size = 9, normalized size = 0.90

$$\frac{\sin(4x^3)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cos(4*x^3),x)

[Out] 1/12*sin(4*x^3)

maxima [A] time = 0.32, size = 8, normalized size = 0.80

$$\frac{1}{12} \sin(4x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(4*x^3),x, algorithm="maxima")

[Out] 1/12*sin(4*x^3)

mupad [B] time = 0.06, size = 8, normalized size = 0.80

$$\frac{\sin(4x^3)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cos(4*x^3),x)

[Out] sin(4*x^3)/12

sympy [A] time = 0.28, size = 7, normalized size = 0.70

$$\frac{\sin(4x^3)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*cos(4*x**3),x)

[Out] sin(4*x**3)/12

3.768 $\int x^3 \cos(x^4) dx$

Optimal. Leaf size=8

$$\frac{\sin(x^4)}{4}$$

[Out] 1/4*sin(x^4)

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3380, 2637}

$$\frac{\sin(x^4)}{4}$$

Antiderivative was successfully verified.

[In] Int[x^3*Cos[x^4],x]

[Out] Sin[x^4]/4

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x^3 \cos(x^4) dx &= \frac{1}{4} \text{Subst} \left(\int \cos(x) dx, x, x^4 \right) \\ &= \frac{\sin(x^4)}{4} \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\sin(x^4)}{4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Cos[x^4],x]

[Out] Sin[x^4]/4

fricas [A] time = 0.67, size = 6, normalized size = 0.75

$$\frac{1}{4} \sin(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cos(x^4),x, algorithm="fricas")

[Out] 1/4*sin(x^4)

giac [A] time = 0.14, size = 6, normalized size = 0.75

$$\frac{1}{4} \sin(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cos(x^4),x, algorithm="giac")

[Out] 1/4*sin(x^4)

maple [A] time = 0.04, size = 7, normalized size = 0.88

$$\frac{\sin(x^4)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*cos(x^4),x)

[Out] 1/4*sin(x^4)

maxima [A] time = 0.31, size = 6, normalized size = 0.75

$$\frac{1}{4} \sin(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*cos(x^4),x, algorithm="maxima")

[Out] 1/4*sin(x^4)

mupad [B] time = 0.07, size = 6, normalized size = 0.75

$$\frac{\sin(x^4)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*cos(x^4),x)

[Out] sin(x^4)/4

sympy [A] time = 0.48, size = 5, normalized size = 0.62

$$\frac{\sin(x^4)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*cos(x**4),x)

[Out] sin(x**4)/4

$$3.769 \quad \int x \sin\left(\frac{x^2}{2}\right) dx$$

Optimal. Leaf size=10

$$-\cos\left(\frac{x^2}{2}\right)$$

[Out] -cos(1/2*x^2)

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3379, 2638}

$$-\cos\left(\frac{x^2}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[x*Sin[x^2/2],x]

[Out] -Cos[x^2/2]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x \sin\left(\frac{x^2}{2}\right) dx &= \frac{1}{2} \text{Subst}\left(\int \sin\left(\frac{x}{2}\right) dx, x, x^2\right) \\ &= -\cos\left(\frac{x^2}{2}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$-\cos\left(\frac{x^2}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sin[x^2/2],x]

[Out] -Cos[x^2/2]

fricas [A] time = 0.49, size = 8, normalized size = 0.80

$$-\cos\left(\frac{1}{2}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(1/2*x^2),x, algorithm="fricas")

[Out] -cos(1/2*x^2)

giac [A] time = 0.12, size = 8, normalized size = 0.80

$$-\cos\left(\frac{1}{2}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(1/2*x^2),x, algorithm="giac")

[Out] -cos(1/2*x^2)

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-\cos\left(\frac{x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(1/2*x^2),x)

[Out] -cos(1/2*x^2)

maxima [A] time = 0.33, size = 8, normalized size = 0.80

$$-\cos\left(\frac{1}{2}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(1/2*x^2),x, algorithm="maxima")

[Out] -cos(1/2*x^2)

mupad [B] time = 2.92, size = 8, normalized size = 0.80

$$-\cos\left(\frac{x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(x^2/2),x)

[Out] -cos(x^2/2)

sympy [A] time = 0.15, size = 7, normalized size = 0.70

$$-\cos\left(\frac{x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(1/2*x**2),x)

[Out] -cos(x**2/2)

3.770 $\int x \sec(x^2) \tan(x^2) dx$

Optimal. Leaf size=8

$$\frac{\sec(x^2)}{2}$$

[Out] 1/2*sec(x^2)

Rubi [A] time = 0.06, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6715, 2606, 8}

$$\frac{\sec(x^2)}{2}$$

Antiderivative was successfully verified.

[In] Int[x*Sec[x^2]*Tan[x^2],x]

[Out] Sec[x^2]/2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m-1)*(-1+x^2)^((n-1)/2), x], x, Sec[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n+1])

Rule 6715

Int[(u_)*(x_)^(m_.), x_Symbol] := Dist[1/(m+1), Subst[Int[SubstFor[x^(m+1), u, x], x, x^(m+1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m+1), u, x]

Rubi steps

$$\begin{aligned} \int x \sec(x^2) \tan(x^2) dx &= \frac{1}{2} \text{Subst} \left(\int \sec(x) \tan(x) dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int 1 dx, x, \sec(x^2) \right) \\ &= \frac{\sec(x^2)}{2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$\frac{\sec(x^2)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[x^2]*Tan[x^2],x]

[Out] Sec[x^2]/2

fricas [A] time = 0.77, size = 8, normalized size = 1.00

$$\frac{1}{2 \cos(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x^2)*tan(x^2),x, algorithm="fricas")

[Out] 1/2/cos(x^2)

giac [A] time = 0.12, size = 8, normalized size = 1.00

$$\frac{1}{2 \cos(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x^2)*tan(x^2),x, algorithm="giac")

[Out] 1/2/cos(x^2)

maple [A] time = 0.03, size = 7, normalized size = 0.88

$$\frac{\sec(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sec(x^2)*tan(x^2),x)

[Out] 1/2*sec(x^2)

maxima [B] time = 0.33, size = 56, normalized size = 7.00

$$\frac{\cos(2x^2)\cos(x^2) + \sin(2x^2)\sin(x^2) + \cos(x^2)}{\cos(2x^2)^2 + \sin(2x^2)^2 + 2\cos(2x^2) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x^2)*tan(x^2),x, algorithm="maxima")

[Out] (cos(2*x^2)*cos(x^2) + sin(2*x^2)*sin(x^2) + cos(x^2))/(cos(2*x^2)^2 + sin(2*x^2)^2 + 2*cos(2*x^2) + 1)

mupad [B] time = 0.07, size = 8, normalized size = 1.00

$$\frac{1}{2 \cos(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*tan(x^2))/cos(x^2),x)

[Out] 1/(2*cos(x^2))

sympy [A] time = 0.28, size = 5, normalized size = 0.62

$$\frac{\sec(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x**2)*tan(x**2),x)

[Out] sec(x**2)/2

$$3.771 \quad \int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2} dx$$

Optimal. Leaf size=10

$$\frac{1}{x} - \tan\left(\frac{1}{x}\right)$$

[Out] 1/x-tan(1/x)

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3747, 3473, 8}

$$\frac{1}{x} - \tan\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

[In] Int[Tan[x^(-1)]^2/x^2,x]

[Out] x^(-1) - Tan[x^(-1)]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3747

Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2} dx &= -\text{Subst}\left(\int \tan^2(x) dx, x, \frac{1}{x}\right) \\ &= -\tan\left(\frac{1}{x}\right) + \text{Subst}\left(\int 1 dx, x, \frac{1}{x}\right) \\ &= \frac{1}{x} - \tan\left(\frac{1}{x}\right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 1.20

$$\tan^{-1}\left(\tan\left(\frac{1}{x}\right)\right) - \tan\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Tan[x^(-1)]^2/x^2,x]

[Out] ArcTan[Tan[x⁽⁻¹⁾]] - Tan[x⁽⁻¹⁾]

fricas [A] time = 0.72, size = 13, normalized size = 1.30

$$\frac{x \tan\left(\frac{1}{x}\right) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(1/x)²/x², x, algorithm="fricas")

[Out] -(x*tan(1/x) - 1)/x

giac [A] time = 0.14, size = 10, normalized size = 1.00

$$\frac{1}{x} - \tan\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(1/x)²/x², x, algorithm="giac")

[Out] 1/x - tan(1/x)

maple [A] time = 0.01, size = 11, normalized size = 1.10

$$\frac{1}{x} - \tan\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(1/x)²/x², x)

[Out] 1/x-tan(1/x)

maxima [B] time = 0.33, size = 67, normalized size = 6.70

$$\frac{\cos\left(\frac{2}{x}\right)^2 - 2x \sin\left(\frac{2}{x}\right) + \sin\left(\frac{2}{x}\right)^2 + 2 \cos\left(\frac{2}{x}\right) + 1}{\left(\cos\left(\frac{2}{x}\right)^2 + \sin\left(\frac{2}{x}\right)^2 + 2 \cos\left(\frac{2}{x}\right) + 1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(1/x)²/x², x, algorithm="maxima")

[Out] (cos(2/x)² - 2*x*sin(2/x) + sin(2/x)² + 2*cos(2/x) + 1)/((cos(2/x)² + sin(2/x)² + 2*cos(2/x) + 1)*x)

mupad [B] time = 2.92, size = 10, normalized size = 1.00

$$\frac{1}{x} - \tan\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(1/x)²/x², x)

[Out] 1/x - tan(1/x)

sympy [A] time = 0.22, size = 7, normalized size = 0.70

$$-\tan\left(\frac{1}{x}\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(1/x)**2/x**2,x)
```

```
[Out] -tan(1/x) + 1/x
```

3.772 $\int x \tan(1 + x^2) dx$

Optimal. Leaf size=11

$$-\frac{1}{2} \log(\cos(x^2 + 1))$$

[Out] -1/2*ln(cos(x^2+1))

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3747, 3475}

$$-\frac{1}{2} \log(\cos(x^2 + 1))$$

Antiderivative was successfully verified.

[In] Int[x*Tan[1 + x^2],x]

[Out] -Log[Cos[1 + x^2]]/2

Rule 3475

Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3747

Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_.)])^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int x \tan(1 + x^2) dx &= \frac{1}{2} \text{Subst}\left(\int \tan(1 + x) dx, x, x^2\right) \\ &= -\frac{1}{2} \log(\cos(1 + x^2)) \end{aligned}$$

Mathematica [A] time = 0.02, size = 11, normalized size = 1.00

$$-\frac{1}{2} \log(\cos(x^2 + 1))$$

Antiderivative was successfully verified.

[In] Integrate[x*Tan[1 + x^2],x]

[Out] -1/2*Log[Cos[1 + x^2]]

fricas [A] time = 0.79, size = 15, normalized size = 1.36

$$-\frac{1}{4} \log\left(\frac{1}{\tan(x^2 + 1)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*tan(x^2+1),x, algorithm="fricas")

[Out] $-1/4*\log(1/(\tan(x^2 + 1)^2 + 1))$

giac [A] time = 0.17, size = 10, normalized size = 0.91

$$-\frac{1}{2} \log(|\cos(x^2 + 1)|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(x^2+1),x, algorithm="giac")`

[Out] $-1/2*\log(\text{abs}(\cos(x^2 + 1)))$

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$-\frac{\ln(\cos(x^2 + 1))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*tan(x^2+1),x)`

[Out] $-1/2*\ln(\cos(x^2+1))$

maxima [A] time = 0.33, size = 9, normalized size = 0.82

$$\frac{1}{2} \log(\sec(x^2 + 1))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(x^2+1),x, algorithm="maxima")`

[Out] $1/2*\log(\sec(x^2 + 1))$

mupad [B] time = 0.28, size = 13, normalized size = 1.18

$$\frac{\ln(\tan(x^2 + 1)^2 + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*tan(x^2 + 1),x)`

[Out] $\log(\tan(x^2 + 1)^2 + 1)/4$

sympy [A] time = 0.12, size = 12, normalized size = 1.09

$$\frac{\log(\tan^2(x^2 + 1) + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*tan(x**2+1),x)`

[Out] $\log(\tan(x**2 + 1)**2 + 1)/4$

3.773 $\int \sin(\pi(1 + 2x)) dx$

Optimal. Leaf size=12

$$\frac{\cos(2\pi x)}{2\pi}$$

[Out] 1/2*cos(2*Pi*x)/Pi

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2638}

$$\frac{\cos(2\pi x)}{2\pi}$$

Antiderivative was successfully verified.

[In] Int[Sin[Pi*(1 + 2*x)],x]

[Out] Cos[2*Pi*x]/(2*Pi)

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \sin(\pi(1 + 2x)) dx = \frac{\cos(2\pi x)}{2\pi}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\frac{\cos(2\pi x)}{2\pi}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[Pi*(1 + 2*x)],x]

[Out] Cos[2*Pi*x]/(2*Pi)

fricas [A] time = 0.57, size = 12, normalized size = 1.00

$$\frac{\cos(\pi + 2\pi x)}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(pi*(1+2*x)),x, algorithm="fricas")

[Out] -1/2*cos(pi + 2*pi*x)/pi

giac [A] time = 0.14, size = 10, normalized size = 0.83

$$\frac{\cos(2\pi x)}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(pi*(1+2*x)),x, algorithm="giac")

[Out] $1/2*\cos(2*\pi*x)/\pi$

maple [A] time = 0.04, size = 11, normalized size = 0.92

$$\frac{\cos(2\pi x)}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(Pi*(1+2*x)),x)`

[Out] $1/2*\cos(2*\pi*x)/\pi$

maxima [A] time = 0.32, size = 10, normalized size = 0.83

$$\frac{\cos(2\pi x)}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(pi*(1+2*x)),x, algorithm="maxima")`

[Out] $1/2*\cos(2*\pi*x)/\pi$

mupad [B] time = 2.92, size = 13, normalized size = 1.08

$$\frac{\cos(\Pi(2x+1))}{2\Pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(Pi*(2*x + 1)),x)`

[Out] $-\cos(\pi*(2*x + 1))/(2*\pi)$

sympy [A] time = 0.79, size = 12, normalized size = 1.00

$$\frac{\cos(\pi(2x+1))}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(pi*(1+2*x)),x)`

[Out] $-\cos(\pi*(2*x + 1))/(2*\pi)$

$$3.774 \quad \int \frac{\cot(x) + \csc^2(x)}{1 - \cos^2(x)} dx$$

Optimal. Leaf size=21

$$-\frac{1}{3} \cot^3(x) - \frac{\cot^2(x)}{2} - \cot(x)$$

[Out] $-\cot(x) - 1/2 * \cot(x)^2 - 1/3 * \cot(x)^3$

Rubi [A] time = 0.06, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$-\frac{1}{3} \cot^3(x) - \frac{\cot^2(x)}{2} - \cot(x)$$

Antiderivative was successfully verified.

[In] Int[(Cot[x] + Csc[x]^2)/(1 - Cos[x]^2), x]

[Out] $-\text{Cot}[x] - \text{Cot}[x]^2/2 - \text{Cot}[x]^3/3$

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{\cot(x) + \csc^2(x)}{1 - \cos^2(x)} dx &= \text{Subst} \left(\int \frac{1 + x + x^2}{x^4} dx, x, \tan(x) \right) \\ &= \text{Subst} \left(\int \left(\frac{1}{x^4} + \frac{1}{x^3} + \frac{1}{x^2} \right) dx, x, \tan(x) \right) \\ &= -\cot(x) - \frac{\cot^2(x)}{2} - \frac{\cot^3(x)}{3} \end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.19

$$-\frac{2 \cot(x)}{3} - \frac{\csc^2(x)}{2} - \frac{1}{3} \cot(x) \csc^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x] + Csc[x]^2)/(1 - Cos[x]^2), x]

[Out] $(-2 * \text{Cot}[x]) / 3 - \text{Csc}[x]^2 / 2 - (\text{Cot}[x] * \text{Csc}[x]^2) / 3$

fricas [A] time = 0.67, size = 29, normalized size = 1.38

$$-\frac{4 \cos(x)^3 - 6 \cos(x) - 3 \sin(x)}{6 (\cos(x)^2 - 1) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x)^2)/(1-cos(x)^2),x, algorithm="fricas")

[Out] $-1/6 * (4 * \cos(x)^3 - 6 * \cos(x) - 3 * \sin(x)) / ((\cos(x)^2 - 1) * \sin(x))$

giac [A] time = 0.15, size = 18, normalized size = 0.86

$$-\frac{6 \tan(x)^2 + 3 \tan(x) + 2}{6 \tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x)^2)/(1-cos(x)^2),x, algorithm="giac")

[Out] -1/6*(6*tan(x)^2 + 3*tan(x) + 2)/tan(x)^3

maple [A] time = 0.15, size = 20, normalized size = 0.95

$$-\frac{1}{2 \tan(x)^2} - \frac{1}{\tan(x)} - \frac{1}{3 \tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(x)+csc(x)^2)/(1-cos(x)^2),x)

[Out] -1/2/tan(x)^2-1/tan(x)-1/3/tan(x)^3

maxima [A] time = 0.33, size = 18, normalized size = 0.86

$$-\frac{6 \tan(x)^2 + 3 \tan(x) + 2}{6 \tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x)^2)/(1-cos(x)^2),x, algorithm="maxima")

[Out] -1/6*(6*tan(x)^2 + 3*tan(x) + 2)/tan(x)^3

mupad [B] time = 3.00, size = 16, normalized size = 0.76

$$\frac{\cot(x) (2 \cot(x)^2 + 3 \cot(x) + 6)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(cot(x) + 1/sin(x)^2)/(cos(x)^2 - 1),x)

[Out] -(cot(x)*(3*cot(x) + 2*cot(x)^2 + 6))/6

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cot(x)}{\cos^2(x)-1} dx - \int \frac{\csc^2(x)}{\cos^2(x)-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cot(x)+csc(x)**2)/(1-cos(x)**2),x)

[Out] -Integral(cot(x)/(cos(x)**2 - 1), x) - Integral(csc(x)**2/(cos(x)**2 - 1), x)

$$3.775 \quad \int x^2 \cos(4x^3) \cos(5x^3) dx$$

Optimal. Leaf size=19

$$\frac{\sin(x^3)}{6} + \frac{1}{54} \sin(9x^3)$$

[Out] 1/6*sin(x^3)+1/54*sin(9*x^3)

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4572, 3380, 2637}

$$\frac{\sin(x^3)}{6} + \frac{1}{54} \sin(9x^3)$$

Antiderivative was successfully verified.

[In] Int[x^2*cos[4*x^3]*cos[5*x^3],x]

[Out] Sin[x^3]/6 + Sin[9*x^3]/54

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 4572

Int[Cos[v_]^(p_.)*Cos[w_]^(q_.)*(x_)^(m_.), x_Symbol] :> Int[ExpandTrigReduce[x^m, Cos[v]^p*Cos[w]^q, x], x] /; IGtQ[m, 0] && IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))

Rubi steps

$$\begin{aligned} \int x^2 \cos(4x^3) \cos(5x^3) dx &= \int \left(\frac{1}{2} x^2 \cos(x^3) + \frac{1}{2} x^2 \cos(9x^3) \right) dx \\ &= \frac{1}{2} \int x^2 \cos(x^3) dx + \frac{1}{2} \int x^2 \cos(9x^3) dx \\ &= \frac{1}{6} \text{Subst} \left(\int \cos(x) dx, x, x^3 \right) + \frac{1}{6} \text{Subst} \left(\int \cos(9x) dx, x, x^3 \right) \\ &= \frac{\sin(x^3)}{6} + \frac{1}{54} \sin(9x^3) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{\sin(x^3)}{6} + \frac{1}{54} \sin(9x^3)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*cos[4*x^3]*cos[5*x^3],x]

[Out] Sin[x^3]/6 + Sin[9*x^3]/54

fricas [B] time = 0.50, size = 40, normalized size = 2.11

$$\frac{1}{27} \left(128 \cos(x^3)^8 - 224 \cos(x^3)^6 + 120 \cos(x^3)^4 - 20 \cos(x^3)^2 + 5 \right) \sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(4*x^3)*cos(5*x^3),x, algorithm="fricas")

[Out] 1/27*(128*cos(x^3)^8 - 224*cos(x^3)^6 + 120*cos(x^3)^4 - 20*cos(x^3)^2 + 5)*sin(x^3)

giac [B] time = 0.14, size = 39, normalized size = 2.05

$$\frac{128}{27} \sin(x^3)^9 - \frac{32}{3} \sin(x^3)^7 + 8 \sin(x^3)^5 - \frac{20}{9} \sin(x^3)^3 + \frac{1}{3} \sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(4*x^3)*cos(5*x^3),x, algorithm="giac")

[Out] 128/27*sin(x^3)^9 - 32/3*sin(x^3)^7 + 8*sin(x^3)^5 - 20/9*sin(x^3)^3 + 1/3*sin(x^3)

maple [A] time = 0.23, size = 16, normalized size = 0.84

$$\frac{\sin(x^3)}{6} + \frac{\sin(9x^3)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cos(4*x^3)*cos(5*x^3),x)

[Out] 1/6*sin(x^3)+1/54*sin(9*x^3)

maxima [A] time = 0.33, size = 15, normalized size = 0.79

$$\frac{1}{54} \sin(9x^3) + \frac{1}{6} \sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(4*x^3)*cos(5*x^3),x, algorithm="maxima")

[Out] 1/54*sin(9*x^3) + 1/6*sin(x^3)

mupad [B] time = 3.03, size = 15, normalized size = 0.79

$$\frac{\sin(x^3)}{6} + \frac{\sin(9x^3)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cos(4*x^3)*cos(5*x^3),x)

[Out] sin(x^3)/6 + sin(9*x^3)/54

sympy [B] time = 5.25, size = 32, normalized size = 1.68

$$-\frac{4 \sin(4x^3) \cos(5x^3)}{27} + \frac{5 \sin(5x^3) \cos(4x^3)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(4*x**3)*cos(5*x**3),x)
```

```
[Out] -4*sin(4*x**3)*cos(5*x**3)/27 + 5*sin(5*x**3)*cos(4*x**3)/27
```

3.776 $\int x^{14} \sin(x^3) dx$

Optimal. Leaf size=47

$$-8x^3 \sin(x^3) - 8 \cos(x^3) - \frac{1}{3}x^{12} \cos(x^3) + \frac{4}{3}x^9 \sin(x^3) + 4x^6 \cos(x^3)$$

[Out] $-8*\cos(x^3)+4*x^6*\cos(x^3)-1/3*x^{12}*\cos(x^3)-8*x^3*\sin(x^3)+4/3*x^9*\sin(x^3)$

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3379, 3296, 2638}

$$\frac{4}{3}x^9 \sin(x^3) - 8x^3 \sin(x^3) - \frac{1}{3}x^{12} \cos(x^3) + 4x^6 \cos(x^3) - 8 \cos(x^3)$$

Antiderivative was successfully verified.

[In] Int[x¹⁴*Sin[x³], x]

[Out] $-8*\text{Cos}[x^3] + 4*x^6*\text{Cos}[x^3] - (x^{12}*\text{Cos}[x^3])/3 - 8*x^3*\text{Sin}[x^3] + (4*x^9*\text{Sin}[x^3])/3$

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m * Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1) * Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b * Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x^{14} \sin(x^3) dx &= \frac{1}{3} \text{Subst} \left(\int x^4 \sin(x) dx, x, x^3 \right) \\ &= -\frac{1}{3}x^{12} \cos(x^3) + \frac{4}{3} \text{Subst} \left(\int x^3 \cos(x) dx, x, x^3 \right) \\ &= -\frac{1}{3}x^{12} \cos(x^3) + \frac{4}{3}x^9 \sin(x^3) - 4 \text{Subst} \left(\int x^2 \sin(x) dx, x, x^3 \right) \\ &= 4x^6 \cos(x^3) - \frac{1}{3}x^{12} \cos(x^3) + \frac{4}{3}x^9 \sin(x^3) - 8 \text{Subst} \left(\int x \cos(x) dx, x, x^3 \right) \\ &= 4x^6 \cos(x^3) - \frac{1}{3}x^{12} \cos(x^3) - 8x^3 \sin(x^3) + \frac{4}{3}x^9 \sin(x^3) + 8 \text{Subst} \left(\int \sin(x) dx, x, x^3 \right) \\ &= -8 \cos(x^3) + 4x^6 \cos(x^3) - \frac{1}{3}x^{12} \cos(x^3) - 8x^3 \sin(x^3) + \frac{4}{3}x^9 \sin(x^3) \end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.74

$$\frac{4}{3}x^3(x^6 - 6)\sin(x^3) - \frac{1}{3}(x^{12} - 12x^6 + 24)\cos(x^3)$$

Antiderivative was successfully verified.

[In] Integrate[x¹⁴*Sin[x³],x]

[Out] -1/3*((24 - 12*x⁶ + x¹²)*Cos[x³]) + (4*x³*(-6 + x⁶)*Sin[x³])/3

fricas [A] time = 0.71, size = 32, normalized size = 0.68

$$-\frac{1}{3}(x^{12} - 12x^6 + 24)\cos(x^3) + \frac{4}{3}(x^9 - 6x^3)\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁴*sin(x³),x, algorithm="fricas")

[Out] -1/3*(x¹² - 12*x⁶ + 24)*cos(x³) + 4/3*(x⁹ - 6*x³)*sin(x³)

giac [A] time = 0.12, size = 32, normalized size = 0.68

$$-\frac{1}{3}(x^{12} - 12x^6 + 24)\cos(x^3) + \frac{4}{3}(x^9 - 6x^3)\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁴*sin(x³),x, algorithm="giac")

[Out] -1/3*(x¹² - 12*x⁶ + 24)*cos(x³) + 4/3*(x⁹ - 6*x³)*sin(x³)

maple [A] time = 0.03, size = 33, normalized size = 0.70

$$\left(-\frac{1}{3}x^{12} + 4x^6 - 8\right)\cos(x^3) + \frac{4x^3(x^6 - 6)\sin(x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹⁴*sin(x³),x)

[Out] (-1/3*x¹²+4*x⁶-8)*cos(x³)+4/3*x³*(x⁶-6)*sin(x³)

maxima [A] time = 0.33, size = 32, normalized size = 0.68

$$-\frac{1}{3}(x^{12} - 12x^6 + 24)\cos(x^3) + \frac{4}{3}(x^9 - 6x^3)\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁴*sin(x³),x, algorithm="maxima")

[Out] -1/3*(x¹² - 12*x⁶ + 24)*cos(x³) + 4/3*(x⁹ - 6*x³)*sin(x³)

mupad [B] time = 3.00, size = 43, normalized size = 0.91

$$4x^6\cos(x^3) - 8\cos(x^3) - \frac{x^{12}\cos(x^3)}{3} - 8x^3\sin(x^3) + \frac{4x^9\sin(x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹⁴*sin(x³),x)

[Out] $4x^6 \cos(x^3) - 8 \cos(x^3) - (x^{12} \cos(x^3))/3 - 8x^3 \sin(x^3) + (4x^9 \sin(x^3))/3$

sympy [A] time = 71.97, size = 48, normalized size = 1.02

$$-\frac{x^{12} \cos(x^3)}{3} + \frac{4x^9 \sin(x^3)}{3} + 4x^6 \cos(x^3) - 8x^3 \sin(x^3) - 8 \cos(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**14*sin(x**3),x)`

[Out] $-x^{12} \cos(x^3)/3 + 4x^9 \sin(x^3)/3 + 4x^6 \cos(x^3) - 8x^3 \sin(x^3) - 8 \cos(x^3)$

$$3.777 \quad \int e^{-3x^3} x^2 \sin(2x^3) dx$$

Optimal. Leaf size=35

$$-\frac{1}{13}e^{-3x^3} \sin(2x^3) - \frac{2}{39}e^{-3x^3} \cos(2x^3)$$

[Out] $-2/39*\cos(2*x^3)/\exp(3*x^3)-1/13*\sin(2*x^3)/\exp(3*x^3)$

Rubi [A] time = 0.16, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6715, 4432}

$$-\frac{1}{13}e^{-3x^3} \sin(2x^3) - \frac{2}{39}e^{-3x^3} \cos(2x^3)$$

Antiderivative was successfully verified.

[In] Int[(x^2*Sin[2*x^3])/E^(3*x^3), x]

[Out] $(-2*\text{Cos}[2*x^3])/(39*E^(3*x^3)) - \text{Sin}[2*x^3]/(13*E^(3*x^3))$

Rule 4432

Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :=
Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2), x]
- Simp[(e*F^(c*(a + b*x))*Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]

Rule 6715

Int[(u_)*(x_)^(m_.), x_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

Rubi steps

$$\begin{aligned} \int e^{-3x^3} x^2 \sin(2x^3) dx &= \frac{1}{3} \text{Subst} \left(\int e^{-3x} \sin(2x) dx, x, x^3 \right) \\ &= -\frac{2}{39}e^{-3x^3} \cos(2x^3) - \frac{1}{13}e^{-3x^3} \sin(2x^3) \end{aligned}$$

Mathematica [A] time = 0.04, size = 28, normalized size = 0.80

$$-\frac{1}{39}e^{-3x^3} (3 \sin(2x^3) + 2 \cos(2x^3))$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*Sin[2*x^3])/E^(3*x^3), x]

[Out] $-1/39*(2*\text{Cos}[2*x^3] + 3*\text{Sin}[2*x^3])/E^(3*x^3)$

fricas [A] time = 0.75, size = 29, normalized size = 0.83

$$-\frac{2}{39} \cos(2x^3) e^{(-3x^3)} - \frac{1}{13} e^{(-3x^3)} \sin(2x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sin(2*x^3)/exp(3*x^3),x, algorithm="fricas")

[Out] -2/39*cos(2*x^3)*e^(-3*x^3) - 1/13*e^(-3*x^3)*sin(2*x^3)

giac [A] time = 0.15, size = 25, normalized size = 0.71

$$-\frac{1}{39} (2 \cos(2x^3) + 3 \sin(2x^3)) e^{(-3x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sin(2*x^3)/exp(3*x^3),x, algorithm="giac")

[Out] -1/39*(2*cos(2*x^3) + 3*sin(2*x^3))*e^(-3*x^3)

maple [A] time = 0.03, size = 36, normalized size = 1.03

$$\frac{\left(-\frac{2}{39} + \frac{2(\tan^2(x^3))}{39} - \frac{2 \tan(x^3)}{13}\right) e^{-3x^3}}{1 + \tan^2(x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*sin(2*x^3)/exp(3*x^3),x)

[Out] (-2/39+2/39*tan(x^3)^2-2/13*tan(x^3))/(1+tan(x^3)^2)/exp(3*x^3)

maxima [A] time = 0.33, size = 25, normalized size = 0.71

$$-\frac{1}{39} (2 \cos(2x^3) + 3 \sin(2x^3)) e^{(-3x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sin(2*x^3)/exp(3*x^3),x, algorithm="maxima")

[Out] -1/39*(2*cos(2*x^3) + 3*sin(2*x^3))*e^(-3*x^3)

mupad [B] time = 2.97, size = 25, normalized size = 0.71

$$-\frac{e^{-3x^3} (2 \cos(2x^3) + 3 \sin(2x^3))}{39}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*exp(-3*x^3)*sin(2*x^3),x)

[Out] -(exp(-3*x^3)*(2*cos(2*x^3) + 3*sin(2*x^3)))/39

sympy [A] time = 1.82, size = 32, normalized size = 0.91

$$-\frac{e^{-3x^3} \sin(2x^3)}{13} - \frac{2e^{-3x^3} \cos(2x^3)}{39}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*sin(2*x**3)/exp(3*x**3),x)

[Out] -exp(-3*x**3)*sin(2*x**3)/13 - 2*exp(-3*x**3)*cos(2*x**3)/39

3.778 $\int 2x \cos(x^2) dx$

Optimal. Leaf size=4

$$\sin(x^2)$$

[Out] $\sin(x^2)$

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {12, 3380, 2637}

$$\sin(x^2)$$

Antiderivative was successfully verified.

[In] `Int[2*x*Cos[x^2],x]`

[Out] `Sin[x^2]`

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

Rule 2637

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rule 3380

`Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))`

Rubi steps

$$\begin{aligned} \int 2x \cos(x^2) dx &= 2 \int x \cos(x^2) dx \\ &= \text{Subst}\left(\int \cos(x) dx, x, x^2\right) \\ &= \sin(x^2) \end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$\sin(x^2)$$

Antiderivative was successfully verified.

[In] `Integrate[2*x*Cos[x^2],x]`

[Out] `Sin[x^2]`

fricas [A] time = 0.79, size = 4, normalized size = 1.00

$$\sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*x*cos(x^2),x, algorithm="fricas")

[Out] sin(x^2)

giac [A] time = 0.14, size = 4, normalized size = 1.00

$$\sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*x*cos(x^2),x, algorithm="giac")

[Out] sin(x^2)

maple [A] time = 0.00, size = 5, normalized size = 1.25

$$\sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2*x*cos(x^2),x)

[Out] sin(x^2)

maxima [A] time = 0.33, size = 4, normalized size = 1.00

$$\sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*x*cos(x^2),x, algorithm="maxima")

[Out] sin(x^2)

mupad [B] time = 2.92, size = 4, normalized size = 1.00

$$\sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2*x*cos(x^2),x)

[Out] sin(x^2)

sympy [A] time = 0.15, size = 3, normalized size = 0.75

$$\sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*x*cos(x**2),x)

[Out] sin(x**2)

$$3.779 \quad \int 3x^2 \cos(7 + x^3) dx$$

Optimal. Leaf size=6

$$\sin(x^3 + 7)$$

[Out] sin(x^3+7)

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {12, 3380, 2637}

$$\sin(x^3 + 7)$$

Antiderivative was successfully verified.

[In] Int[3*x^2*Cos[7 + x^3],x]

[Out] Sin[7 + x^3]

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int 3x^2 \cos(7 + x^3) dx &= 3 \int x^2 \cos(7 + x^3) dx \\ &= \text{Subst}\left(\int \cos(7 + x) dx, x, x^3\right) \\ &= \sin(7 + x^3) \end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\sin(x^3 + 7)$$

Antiderivative was successfully verified.

[In] Integrate[3*x^2*Cos[7 + x^3],x]

[Out] Sin[7 + x^3]

fricas [A] time = 0.65, size = 6, normalized size = 1.00

$$\sin(x^3 + 7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*x^2*cos(x^3+7),x, algorithm="fricas")

[Out] sin(x^3 + 7)

giac [A] time = 0.12, size = 6, normalized size = 1.00

$$\sin(x^3 + 7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*x^2*cos(x^3+7),x, algorithm="giac")

[Out] sin(x^3 + 7)

maple [A] time = 0.04, size = 7, normalized size = 1.17

$$\sin(x^3 + 7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(3*x^2*cos(x^3+7),x)

[Out] sin(x^3+7)

maxima [A] time = 0.32, size = 6, normalized size = 1.00

$$\sin(x^3 + 7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*x^2*cos(x^3+7),x, algorithm="maxima")

[Out] sin(x^3 + 7)

mupad [B] time = 0.06, size = 6, normalized size = 1.00

$$\sin(x^3 + 7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(3*x^2*cos(x^3 + 7),x)

[Out] sin(x^3 + 7)

sympy [A] time = 0.27, size = 5, normalized size = 0.83

$$\sin(x^3 + 7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*x**2*cos(x**3+7),x)

[Out] sin(x**3 + 7)

$$3.780 \quad \int \left(\frac{1}{1+x^2} + \sin(x) \right) dx$$

Optimal. Leaf size=7

$$\tan^{-1}(x) - \cos(x)$$

[Out] arctan(x)-cos(x)

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {203, 2638}

$$\tan^{-1}(x) - \cos(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)^(-1) + Sin[x], x]

[Out] ArcTan[x] - Cos[x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \left(\frac{1}{1+x^2} + \sin(x) \right) dx &= \int \frac{1}{1+x^2} dx + \int \sin(x) dx \\ &= \tan^{-1}(x) - \cos(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 7, normalized size = 1.00

$$\tan^{-1}(x) - \cos(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)^(-1) + Sin[x], x]

[Out] ArcTan[x] - Cos[x]

fricas [A] time = 0.84, size = 7, normalized size = 1.00

$$\arctan(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)+sin(x), x, algorithm="fricas")

[Out] arctan(x) - cos(x)

giac [A] time = 0.12, size = 7, normalized size = 1.00

$$\arctan(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)+sin(x),x, algorithm="giac")

[Out] arctan(x) - cos(x)

maple [A] time = 0.00, size = 8, normalized size = 1.14

$$\arctan(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+1)+sin(x),x)

[Out] arctan(x)-cos(x)

maxima [A] time = 0.43, size = 7, normalized size = 1.00

$$\arctan(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)+sin(x),x, algorithm="maxima")

[Out] arctan(x) - cos(x)

mupad [B] time = 0.05, size = 7, normalized size = 1.00

$$\operatorname{atan}(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x) + 1/(x^2 + 1),x)

[Out] atan(x) - cos(x)

sympy [A] time = 0.08, size = 5, normalized size = 0.71

$$-\cos(x) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+1)+sin(x),x)

[Out] -cos(x) + atan(x)

3.781 $\int x \sin(1 + x^2) dx$

Optimal. Leaf size=10

$$-\frac{1}{2} \cos(x^2 + 1)$$

[Out] -1/2*cos(x^2+1)

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3379, 2638}

$$-\frac{1}{2} \cos(x^2 + 1)$$

Antiderivative was successfully verified.

[In] Int[x*Sin[1 + x^2],x]

[Out] -Cos[1 + x^2]/2

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x \sin(1 + x^2) dx &= \frac{1}{2} \text{Subst}\left(\int \sin(1 + x) dx, x, x^2\right) \\ &= -\frac{1}{2} \cos(1 + x^2) \end{aligned}$$

Mathematica [B] time = 0.01, size = 21, normalized size = 2.10

$$\frac{1}{2} \sin(1) \sin(x^2) - \frac{1}{2} \cos(1) \cos(x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sin[1 + x^2],x]

[Out] -1/2*(Cos[1]*Cos[x^2]) + (Sin[1]*Sin[x^2])/2

fricas [A] time = 0.71, size = 8, normalized size = 0.80

$$-\frac{1}{2} \cos(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x^2+1),x, algorithm="fricas")

[Out] $-1/2*\cos(x^2 + 1)$

giac [A] time = 0.12, size = 8, normalized size = 0.80

$$-\frac{1}{2} \cos(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x^2+1),x, algorithm="giac")`

[Out] $-1/2*\cos(x^2 + 1)$

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-\frac{\cos(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*sin(x^2+1),x)`

[Out] $-1/2*\cos(x^2+1)$

maxima [A] time = 0.32, size = 8, normalized size = 0.80

$$-\frac{1}{2} \cos(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x^2+1),x, algorithm="maxima")`

[Out] $-1/2*\cos(x^2 + 1)$

mupad [B] time = 0.04, size = 8, normalized size = 0.80

$$-\frac{\cos(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*sin(x^2 + 1),x)`

[Out] $-\cos(x^2 + 1)/2$

sympy [A] time = 0.15, size = 8, normalized size = 0.80

$$-\frac{\cos(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x**2+1),x)`

[Out] $-\cos(x**2 + 1)/2$

3.782 $\int x \cos(1 + x^2) dx$

Optimal. Leaf size=10

$$\frac{1}{2} \sin(x^2 + 1)$$

[Out] 1/2*sin(x^2+1)

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3380, 2637}

$$\frac{1}{2} \sin(x^2 + 1)$$

Antiderivative was successfully verified.

[In] Int[x*Cos[1 + x^2],x]

[Out] Sin[1 + x^2]/2

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x \cos(1 + x^2) dx &= \frac{1}{2} \text{Subst}\left(\int \cos(1 + x) dx, x, x^2\right) \\ &= \frac{1}{2} \sin(1 + x^2) \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{2} \sin(x^2 + 1)$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[1 + x^2],x]

[Out] Sin[1 + x^2]/2

fricas [A] time = 0.58, size = 8, normalized size = 0.80

$$\frac{1}{2} \sin(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x^2+1),x, algorithm="fricas")

[Out] 1/2*sin(x^2 + 1)

giac [A] time = 0.14, size = 8, normalized size = 0.80

$$\frac{1}{2} \sin(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x^2+1),x, algorithm="giac")

[Out] 1/2*sin(x^2 + 1)

maple [A] time = 0.04, size = 9, normalized size = 0.90

$$\frac{\sin(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(x^2+1),x)

[Out] 1/2*sin(x^2+1)

maxima [A] time = 0.32, size = 8, normalized size = 0.80

$$\frac{1}{2} \sin(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x^2+1),x, algorithm="maxima")

[Out] 1/2*sin(x^2 + 1)

mupad [B] time = 2.98, size = 8, normalized size = 0.80

$$\frac{\sin(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(x^2 + 1),x)

[Out] sin(x^2 + 1)/2

sympy [A] time = 0.15, size = 7, normalized size = 0.70

$$\frac{\sin(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x**2+1),x)

[Out] sin(x**2 + 1)/2

3.783 $\int (1 + x^2 \cos(x^3)) dx$

Optimal. Leaf size=10

$$\frac{\sin(x^3)}{3} + x$$

[Out] x+1/3*sin(x^3)

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3380, 2637}

$$\frac{\sin(x^3)}{3} + x$$

Antiderivative was successfully verified.

[In] Int[1 + x^2*Cos[x^3],x]

[Out] x + Sin[x^3]/3

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int (1 + x^2 \cos(x^3)) dx &= x + \int x^2 \cos(x^3) dx \\ &= x + \frac{1}{3} \text{Subst} \left(\int \cos(x) dx, x, x^3 \right) \\ &= x + \frac{\sin(x^3)}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\sin(x^3)}{3} + x$$

Antiderivative was successfully verified.

[In] Integrate[1 + x^2*Cos[x^3],x]

[Out] x + Sin[x^3]/3

fricas [A] time = 0.62, size = 8, normalized size = 0.80

$$x + \frac{1}{3} \sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1+x^2*cos(x^3),x, algorithm="fricas")

[Out] x + 1/3*sin(x^3)

giac [A] time = 0.14, size = 8, normalized size = 0.80

$$x + \frac{1}{3} \sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1+x^2*cos(x^3),x, algorithm="giac")

[Out] x + 1/3*sin(x^3)

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$x + \frac{\sin(x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1+x^2*cos(x^3),x)

[Out] x+1/3*sin(x^3)

maxima [A] time = 0.32, size = 8, normalized size = 0.80

$$x + \frac{1}{3} \sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1+x^2*cos(x^3),x, algorithm="maxima")

[Out] x + 1/3*sin(x^3)

mupad [B] time = 0.05, size = 8, normalized size = 0.80

$$x + \frac{\sin(x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cos(x^3) + 1,x)

[Out] x + sin(x^3)/3

sympy [A] time = 0.26, size = 7, normalized size = 0.70

$$x + \frac{\sin(x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1+x**2*cos(x**3),x)

[Out] x + sin(x**3)/3

3.784 $\int x^2 \sin(1 + x^3) dx$

Optimal. Leaf size=10

$$-\frac{1}{3} \cos(x^3 + 1)$$

[Out] -1/3*cos(x^3+1)

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3379, 2638}

$$-\frac{1}{3} \cos(x^3 + 1)$$

Antiderivative was successfully verified.

[In] Int[x^2*Sin[1 + x^3],x]

[Out] -Cos[1 + x^3]/3

Rule 2638

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x^2 \sin(1 + x^3) dx &= \frac{1}{3} \text{Subst} \left(\int \sin(1 + x) dx, x, x^3 \right) \\ &= -\frac{1}{3} \cos(1 + x^3) \end{aligned}$$

Mathematica [B] time = 0.01, size = 21, normalized size = 2.10

$$\frac{1}{3} \sin(1) \sin(x^3) - \frac{1}{3} \cos(1) \cos(x^3)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sin[1 + x^3],x]

[Out] -1/3*(Cos[1]*Cos[x^3]) + (Sin[1]*Sin[x^3])/3

fricas [A] time = 0.96, size = 8, normalized size = 0.80

$$-\frac{1}{3} \cos(x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sin(x^3+1),x, algorithm="fricas")

[Out] $-1/3*\cos(x^3 + 1)$

giac [A] time = 0.14, size = 8, normalized size = 0.80

$$-\frac{1}{3} \cos(x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*sin(x^3+1),x, algorithm="giac")`

[Out] $-1/3*\cos(x^3 + 1)$

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-\frac{\cos(x^3 + 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*sin(x^3+1),x)`

[Out] $-1/3*\cos(x^3+1)$

maxima [A] time = 0.32, size = 8, normalized size = 0.80

$$-\frac{1}{3} \cos(x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*sin(x^3+1),x, algorithm="maxima")`

[Out] $-1/3*\cos(x^3 + 1)$

mupad [B] time = 0.05, size = 8, normalized size = 0.80

$$-\frac{\cos(x^3 + 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*sin(x^3 + 1),x)`

[Out] $-\cos(x^3 + 1)/3$

sympy [A] time = 0.26, size = 8, normalized size = 0.80

$$-\frac{\cos(x^3 + 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*sin(x**3+1),x)`

[Out] $-\cos(x**3 + 1)/3$

3.785 $\int 12x^2 \cos(x^3) dx$

Optimal. Leaf size=6

$$4 \sin(x^3)$$

[Out] 4*sin(x^3)

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {12, 3380, 2637}

$$4 \sin(x^3)$$

Antiderivative was successfully verified.

[In] Int[12*x^2*Cos[x^3],x]

[Out] 4*Sin[x^3]

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int 12x^2 \cos(x^3) dx &= 12 \int x^2 \cos(x^3) dx \\ &= 4 \text{Subst}\left(\int \cos(x) dx, x, x^3\right) \\ &= 4 \sin(x^3) \end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$4 \sin(x^3)$$

Antiderivative was successfully verified.

[In] Integrate[12*x^2*Cos[x^3],x]

[Out] 4*Sin[x^3]

fricas [A] time = 0.66, size = 6, normalized size = 1.00

$$4 \sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(12*x^2*cos(x^3),x, algorithm="fricas")

[Out] 4*sin(x^3)

giac [A] time = 0.14, size = 6, normalized size = 1.00

$$4 \sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(12*x^2*cos(x^3),x, algorithm="giac")

[Out] 4*sin(x^3)

maple [A] time = 0.00, size = 7, normalized size = 1.17

$$4 \sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(12*x^2*cos(x^3),x)

[Out] 4*sin(x^3)

maxima [A] time = 0.32, size = 6, normalized size = 1.00

$$4 \sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(12*x^2*cos(x^3),x, algorithm="maxima")

[Out] 4*sin(x^3)

mupad [B] time = 0.05, size = 6, normalized size = 1.00

$$4 \sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(12*x^2*cos(x^3),x)

[Out] 4*sin(x^3)

sympy [A] time = 0.27, size = 5, normalized size = 0.83

$$4 \sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(12*x**2*cos(x**3),x)

[Out] 4*sin(x**3)

3.786 $\int (1+x) \sin(1+x) dx$

Optimal. Leaf size=14

$$\sin(x+1) - (x+1) \cos(x+1)$$

[Out] $-(1+x)*\cos(1+x)+\sin(1+x)$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3296, 2637}

$$\sin(x+1) - (x+1) \cos(x+1)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1+x)*\text{Sin}[1+x],x]$

[Out] $-\left((1+x)*\text{Cos}[1+x]\right) + \text{Sin}[1+x]$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$
 $\text{FreeQ}\{c, d\}, x]$

Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.))^{(m_.)*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\left((c + d*x)^m * \text{Cos}[e + f*x]\right)/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)} * \text{Cos}[e + f*x], x], x] /;$
 $\text{FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

Rubi steps

$$\begin{aligned} \int (1+x) \sin(1+x) dx &= -(1+x) \cos(1+x) + \int \cos(1+x) dx \\ &= -(1+x) \cos(1+x) + \sin(1+x) \end{aligned}$$

Mathematica [A] time = 0.03, size = 14, normalized size = 1.00

$$\sin(x+1) - (x+1) \cos(x+1)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(1+x)*\text{Sin}[1+x],x]$

[Out] $-\left((1+x)*\text{Cos}[1+x]\right) + \text{Sin}[1+x]$

fricas [A] time = 0.53, size = 14, normalized size = 1.00

$$-(x+1) \cos(x+1) + \sin(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((1+x)*\sin(1+x),x, \text{algorithm}=\text{"fricas"})$

[Out] $-(x+1)*\cos(x+1) + \sin(x+1)$

giac [A] time = 0.14, size = 14, normalized size = 1.00

$$-(x+1) \cos(x+1) + \sin(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)*sin(1+x),x, algorithm="giac")

[Out] $-(x + 1)\cos(x + 1) + \sin(x + 1)$

maple [A] time = 0.03, size = 15, normalized size = 1.07

$$-(1 + x)\cos(1 + x) + \sin(1 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)*sin(1+x),x)

[Out] $-(1+x)\cos(1+x)+\sin(1+x)$

maxima [A] time = 0.32, size = 14, normalized size = 1.00

$$-(x + 1)\cos(x + 1) + \sin(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)*sin(1+x),x, algorithm="maxima")

[Out] $-(x + 1)\cos(x + 1) + \sin(x + 1)$

mupad [B] time = 2.95, size = 14, normalized size = 1.00

$$\sin(x + 1) - \cos(x + 1)(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x + 1)*(x + 1),x)

[Out] $\sin(x + 1) - \cos(x + 1)*(x + 1)$

sympy [A] time = 0.15, size = 15, normalized size = 1.07

$$-x\cos(x + 1) + \sin(x + 1) - \cos(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)*sin(1+x),x)

[Out] $-x\cos(x + 1) + \sin(x + 1) - \cos(x + 1)$

3.787 $\int x^5 \cos(x^3) dx$

Optimal. Leaf size=20

$$\frac{1}{3}x^3 \sin(x^3) + \frac{\cos(x^3)}{3}$$

[Out] 1/3*cos(x^3)+1/3*x^3*sin(x^3)

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3380, 3296, 2638}

$$\frac{1}{3}x^3 \sin(x^3) + \frac{\cos(x^3)}{3}$$

Antiderivative was successfully verified.

[In] Int[x^5*Cos[x^3],x]

[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_)^(m_.))*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x^5 \cos(x^3) dx &= \frac{1}{3} \text{Subst} \left(\int x \cos(x) dx, x, x^3 \right) \\ &= \frac{1}{3} x^3 \sin(x^3) - \frac{1}{3} \text{Subst} \left(\int \sin(x) dx, x, x^3 \right) \\ &= \frac{\cos(x^3)}{3} + \frac{1}{3} x^3 \sin(x^3) \end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{1}{3}x^3 \sin(x^3) + \frac{\cos(x^3)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*Cos[x^3],x]

[Out] $\text{Cos}[x^3]/3 + (x^3*\text{Sin}[x^3])/3$

fricas [A] time = 0.59, size = 16, normalized size = 0.80

$$\frac{1}{3}x^3 \sin(x^3) + \frac{1}{3} \cos(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*cos(x^3),x, algorithm="fricas")`

[Out] $1/3*x^3*\sin(x^3) + 1/3*\cos(x^3)$

giac [A] time = 0.14, size = 16, normalized size = 0.80

$$\frac{1}{3}x^3 \sin(x^3) + \frac{1}{3} \cos(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*cos(x^3),x, algorithm="giac")`

[Out] $1/3*x^3*\sin(x^3) + 1/3*\cos(x^3)$

maple [A] time = 0.05, size = 17, normalized size = 0.85

$$\frac{\cos(x^3)}{3} + \frac{x^3 \sin(x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*cos(x^3),x)`

[Out] $1/3*\cos(x^3)+1/3*x^3*\sin(x^3)$

maxima [A] time = 0.33, size = 16, normalized size = 0.80

$$\frac{1}{3}x^3 \sin(x^3) + \frac{1}{3} \cos(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*cos(x^3),x, algorithm="maxima")`

[Out] $1/3*x^3*\sin(x^3) + 1/3*\cos(x^3)$

mupad [B] time = 2.96, size = 16, normalized size = 0.80

$$\frac{\cos(x^3)}{3} + \frac{x^3 \sin(x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*cos(x^3),x)`

[Out] $\text{cos}(x^3)/3 + (x^3*\text{sin}(x^3))/3$

sympy [A] time = 1.51, size = 15, normalized size = 0.75

$$\frac{x^3 \sin(x^3)}{3} + \frac{\cos(x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*cos(x**3),x)`

[Out] $x**3*\sin(x**3)/3 + \cos(x**3)/3$

3.788 $\int e^{-3x} \cos(x) dx$

Optimal. Leaf size=23

$$\frac{1}{10}e^{-3x} \sin(x) - \frac{3}{10}e^{-3x} \cos(x)$$

[Out] $-3/10*\cos(x)/\exp(3*x)+1/10*\sin(x)/\exp(3*x)$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4433}

$$\frac{1}{10}e^{-3x} \sin(x) - \frac{3}{10}e^{-3x} \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/E^(3*x), x]

[Out] $(-3*\cos[x])/(10*E^{(3*x)}) + \sin[x]/(10*E^{(3*x)})$

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :=
  Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-3x} \cos(x) dx = -\frac{3}{10}e^{-3x} \cos(x) + \frac{1}{10}e^{-3x} \sin(x)$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.70

$$\frac{1}{10}e^{-3x}(\sin(x) - 3 \cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/E^(3*x), x]

[Out] $(-3*\cos[x] + \sin[x])/(10*E^{(3*x)})$

fricas [A] time = 0.69, size = 17, normalized size = 0.74

$$-\frac{3}{10} \cos(x)e^{(-3x)} + \frac{1}{10} e^{(-3x)} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/exp(3*x), x, algorithm="fricas")

[Out] $-3/10*\cos(x)*e^{(-3*x)} + 1/10*e^{(-3*x)}*\sin(x)$

giac [A] time = 0.14, size = 15, normalized size = 0.65

$$-\frac{1}{10} (3 \cos(x) - \sin(x))e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/exp(3*x),x, algorithm="giac")

[Out] $-1/10*(3*\cos(x) - \sin(x))*e^{-3*x}$

maple [A] time = 0.02, size = 18, normalized size = 0.78

$$-\frac{3e^{-3x}\cos(x)}{10} + \frac{e^{-3x}\sin(x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/exp(3*x),x)

[Out] $-3/10*\exp(-3*x)*\cos(x)+1/10*\exp(-3*x)*\sin(x)$

maxima [A] time = 0.33, size = 15, normalized size = 0.65

$$-\frac{1}{10}(3\cos(x) - \sin(x))e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/exp(3*x),x, algorithm="maxima")

[Out] $-1/10*(3*\cos(x) - \sin(x))*e^{-3*x}$

mupad [B] time = 0.02, size = 15, normalized size = 0.65

$$-\frac{e^{-3x}(3\cos(x) - \sin(x))}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-3*x)*cos(x),x)

[Out] $-(\exp(-3*x)*(3*\cos(x) - \sin(x)))/10$

sympy [A] time = 0.40, size = 20, normalized size = 0.87

$$\frac{e^{-3x}\sin(x)}{10} - \frac{3e^{-3x}\cos(x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/exp(3*x),x)

[Out] $\exp(-3*x)*\sin(x)/10 - 3*\exp(-3*x)*\cos(x)/10$

3.789 $\int x^3 \sin(x^2) dx$

Optimal. Leaf size=20

$$\frac{\sin(x^2)}{2} - \frac{1}{2}x^2 \cos(x^2)$$

[Out] $-1/2*x^2*\cos(x^2)+1/2*\sin(x^2)$

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3379, 3296, 2637}

$$\frac{\sin(x^2)}{2} - \frac{1}{2}x^2 \cos(x^2)$$

Antiderivative was successfully verified.

[In] Int[x^3*Sin[x^2],x]

[Out] $-(x^2*\cos[x^2])/2 + \sin[x^2]/2$

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x^3 \sin(x^2) dx &= \frac{1}{2} \text{Subst} \left(\int x \sin(x) dx, x, x^2 \right) \\ &= -\frac{1}{2}x^2 \cos(x^2) + \frac{1}{2} \text{Subst} \left(\int \cos(x) dx, x, x^2 \right) \\ &= -\frac{1}{2}x^2 \cos(x^2) + \frac{\sin(x^2)}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{\sin(x^2)}{2} - \frac{1}{2}x^2 \cos(x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Sin[x^2],x]

[Out] $-1/2*(x^2*\text{Cos}[x^2]) + \text{Sin}[x^2]/2$

fricas [A] time = 0.53, size = 16, normalized size = 0.80

$$-\frac{1}{2}x^2 \cos(x^2) + \frac{1}{2} \sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*sin(x^2),x, algorithm="fricas")`

[Out] $-1/2*x^2*\cos(x^2) + 1/2*\sin(x^2)$

giac [A] time = 0.14, size = 16, normalized size = 0.80

$$-\frac{1}{2}x^2 \cos(x^2) + \frac{1}{2} \sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*sin(x^2),x, algorithm="giac")`

[Out] $-1/2*x^2*\cos(x^2) + 1/2*\sin(x^2)$

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$-\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*sin(x^2),x)`

[Out] $-1/2*x^2*\cos(x^2)+1/2*\sin(x^2)$

maxima [A] time = 0.33, size = 16, normalized size = 0.80

$$-\frac{1}{2}x^2 \cos(x^2) + \frac{1}{2} \sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*sin(x^2),x, algorithm="maxima")`

[Out] $-1/2*x^2*\cos(x^2) + 1/2*\sin(x^2)$

mupad [B] time = 2.96, size = 16, normalized size = 0.80

$$\frac{\sin(x^2)}{2} - \frac{x^2 \cos(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*sin(x^2),x)`

[Out] $\sin(x^2)/2 - (x^2*\cos(x^2))/2$

sympy [A] time = 0.48, size = 15, normalized size = 0.75

$$-\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*sin(x**2),x)`

[Out] $-x**2*\cos(x**2)/2 + \sin(x**2)/2$

3.790 $\int x^3 \cos(x^2) dx$

Optimal. Leaf size=20

$$\frac{1}{2}x^2 \sin(x^2) + \frac{\cos(x^2)}{2}$$

[Out] 1/2*cos(x^2)+1/2*x^2*sin(x^2)

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3380, 3296, 2638}

$$\frac{1}{2}x^2 \sin(x^2) + \frac{\cos(x^2)}{2}$$

Antiderivative was successfully verified.

[In] Int[x^3*Cos[x^2],x]

[Out] Cos[x^2]/2 + (x^2*Sin[x^2])/2

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x^3 \cos(x^2) dx &= \frac{1}{2} \text{Subst} \left(\int x \cos(x) dx, x, x^2 \right) \\ &= \frac{1}{2} x^2 \sin(x^2) - \frac{1}{2} \text{Subst} \left(\int \sin(x) dx, x, x^2 \right) \\ &= \frac{\cos(x^2)}{2} + \frac{1}{2} x^2 \sin(x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{1}{2}x^2 \sin(x^2) + \frac{\cos(x^2)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Cos[x^2],x]

[Out] $\text{Cos}[x^2]/2 + (x^2*\text{Sin}[x^2])/2$

fricas [A] time = 1.52, size = 16, normalized size = 0.80

$$\frac{1}{2}x^2 \sin(x^2) + \frac{1}{2} \cos(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cos(x^2),x, algorithm="fricas")`

[Out] $1/2*x^2*\sin(x^2) + 1/2*\cos(x^2)$

giac [A] time = 0.14, size = 16, normalized size = 0.80

$$\frac{1}{2}x^2 \sin(x^2) + \frac{1}{2} \cos(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cos(x^2),x, algorithm="giac")`

[Out] $1/2*x^2*\sin(x^2) + 1/2*\cos(x^2)$

maple [A] time = 0.04, size = 17, normalized size = 0.85

$$\frac{\cos(x^2)}{2} + \frac{x^2 \sin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*cos(x^2),x)`

[Out] $1/2*\cos(x^2)+1/2*x^2*\sin(x^2)$

maxima [A] time = 0.33, size = 16, normalized size = 0.80

$$\frac{1}{2}x^2 \sin(x^2) + \frac{1}{2} \cos(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cos(x^2),x, algorithm="maxima")`

[Out] $1/2*x^2*\sin(x^2) + 1/2*\cos(x^2)$

mupad [B] time = 0.05, size = 16, normalized size = 0.80

$$\frac{\cos(x^2)}{2} + \frac{x^2 \sin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*cos(x^2),x)`

[Out] $\text{cos}(x^2)/2 + (x^2*\text{sin}(x^2))/2$

sympy [A] time = 0.48, size = 15, normalized size = 0.75

$$\frac{x^2 \sin(x^2)}{2} + \frac{\cos(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*cos(x**2),x)`

[Out] $x**2*\sin(x**2)/2 + \cos(x**2)/2$

3.791 $\int \cos(x) \cos(2 \sin(x)) dx$

Optimal. Leaf size=9

$$\frac{1}{2} \sin(2 \sin(x))$$

[Out] 1/2*sin(2*sin(x))

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4334, 2637}

$$\frac{1}{2} \sin(2 \sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[2*Sin[x]],x]

[Out] Sin[2*Sin[x]]/2

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4334

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned} \int \cos(x) \cos(2 \sin(x)) dx &= \text{Subst}\left(\int \cos(2x) dx, x, \sin(x)\right) \\ &= \frac{1}{2} \sin(2 \sin(x)) \end{aligned}$$

Mathematica [A] time = 1.38, size = 9, normalized size = 1.00

$$\frac{1}{2} \sin(2 \sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cos[2*Sin[x]],x]

[Out] Sin[2*Sin[x]]/2

fricas [B] time = 0.79, size = 19, normalized size = 2.11

$$\frac{1}{2} \sin\left(\frac{4 \tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*sin(x)),x, algorithm="fricas")

[Out] 1/2*sin(4*tan(1/2*x)/(tan(1/2*x)^2 + 1))

giac [A] time = 0.14, size = 7, normalized size = 0.78

$$\frac{1}{2} \sin(2 \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*sin(x)),x, algorithm="giac")

[Out] 1/2*sin(2*sin(x))

maple [A] time = 0.03, size = 8, normalized size = 0.89

$$\frac{\sin(2 \sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*cos(2*sin(x)),x)

[Out] 1/2*sin(2*sin(x))

maxima [A] time = 0.33, size = 7, normalized size = 0.78

$$\frac{1}{2} \sin(2 \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*sin(x)),x, algorithm="maxima")

[Out] 1/2*sin(2*sin(x))

mupad [B] time = 0.07, size = 7, normalized size = 0.78

$$\frac{\sin(2 \sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*sin(x))*cos(x),x)

[Out] sin(2*sin(x))/2

sympy [A] time = 0.42, size = 7, normalized size = 0.78

$$\frac{\sin(2 \sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*sin(x)),x)

[Out] sin(2*sin(x))/2

$$3.792 \quad \int \frac{\cos(x) \sin(x)}{1+\cos^2(x)} dx$$

Optimal. Leaf size=11

$$-\frac{1}{2} \log(\cos^2(x) + 1)$$

[Out] -1/2*ln(1+cos(x)^2)

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4335, 260}

$$-\frac{1}{2} \log(\cos^2(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Sin[x])/(1 + Cos[x]^2), x]

[Out] -Log[1 + Cos[x]^2]/2

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 4335

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned} \int \frac{\cos(x) \sin(x)}{1 + \cos^2(x)} dx &= -\text{Subst} \left(\int \frac{x}{1 + x^2} dx, x, \cos(x) \right) \\ &= -\frac{1}{2} \log(1 + \cos^2(x)) \end{aligned}$$

Mathematica [A] time = 0.03, size = 11, normalized size = 1.00

$$-\frac{1}{2} \log(\cos(2x) + 3)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]*Sin[x])/(1 + Cos[x]^2), x]

[Out] -1/2*Log[3 + Cos[2*x]]

fricas [A] time = 0.69, size = 11, normalized size = 1.00

$$-\frac{1}{2} \log\left(\frac{1}{2} \cos(x)^2 + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/(1+cos(x)^2), x, algorithm="fricas")

[Out] $-1/2*\log(1/2*\cos(x)^2 + 1/2)$

giac [A] time = 0.12, size = 9, normalized size = 0.82

$$-\frac{1}{2} \log(\cos(x)^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*sin(x)/(1+cos(x)^2),x, algorithm="giac")`

[Out] $-1/2*\log(\cos(x)^2 + 1)$

maple [A] time = 0.03, size = 10, normalized size = 0.91

$$-\frac{\ln(1 + \cos^2(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*sin(x)/(1+cos(x)^2),x)`

[Out] $-1/2*\ln(1+\cos(x)^2)$

maxima [A] time = 0.33, size = 9, normalized size = 0.82

$$-\frac{1}{2} \log(\cos(x)^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*sin(x)/(1+cos(x)^2),x, algorithm="maxima")`

[Out] $-1/2*\log(\cos(x)^2 + 1)$

mupad [B] time = 3.01, size = 17, normalized size = 1.55

$$-\operatorname{atanh}\left(\frac{16}{3(12\tan(x)^2 + 16)} - \frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(x)*sin(x))/(cos(x)^2 + 1),x)`

[Out] $-\operatorname{atanh}(16/(3*(12*\tan(x)^2 + 16)) - 1/3)$

sympy [A] time = 0.17, size = 10, normalized size = 0.91

$$-\frac{\log(\cos^2(x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*sin(x)/(1+cos(x)**2),x)`

[Out] $-\log(\cos(x)**2 + 1)/2$

3.793 $\int (1 + \cos(x))(x + \sin(x))^3 dx$

Optimal. Leaf size=10

$$\frac{1}{4}(x + \sin(x))^4$$

[Out] 1/4*(x+sin(x))^4

Rubi [A] time = 0.04, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6686}

$$\frac{1}{4}(x + \sin(x))^4$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x])*(x + Sin[x])^3,x]

[Out] (x + Sin[x])^4/4

Rule 6686

Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[(q*y^(m + 1))/(m + 1), x] /; !FalseQ[q] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\int (1 + \cos(x))(x + \sin(x))^3 dx = \frac{1}{4}(x + \sin(x))^4$$

Mathematica [A] time = 0.02, size = 10, normalized size = 1.00

$$\frac{1}{4}(x + \sin(x))^4$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Cos[x])*(x + Sin[x])^3,x]

[Out] (x + Sin[x])^4/4

fricas [B] time = 0.62, size = 45, normalized size = 4.50

$$\frac{1}{4}x^4 + \frac{1}{4}\cos(x)^4 - \frac{1}{2}(3x^2 + 1)\cos(x)^2 + \frac{3}{2}x^2 + (x^3 - x\cos(x)^2 + x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x))*(x+sin(x))^3,x, algorithm="fricas")

[Out] 1/4*x^4 + 1/4*cos(x)^4 - 1/2*(3*x^2 + 1)*cos(x)^2 + 3/2*x^2 + (x^3 - x*cos(x)^2 + x)*sin(x)

giac [B] time = 0.15, size = 61, normalized size = 6.10

$$\frac{1}{4}x^4 + \frac{3}{4}x^2 - \frac{1}{4}(3x^2 - 1)\cos(2x) - \frac{1}{4}x\sin(3x) + \frac{1}{4}(4x^3 - 21x)\sin(x) + 6x\sin(x) + \frac{1}{32}\cos(4x) - \frac{3}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x))*(x+sin(x))^3,x, algorithm="giac")

[Out] $\frac{1}{4}x^4 + \frac{3}{4}x^2 - \frac{1}{4}(3x^2 - 1)\cos(2x) - \frac{1}{4}x\sin(3x) + \frac{1}{4}(4x^3 - 21x)\sin(x) + 6x\sin(x) + \frac{1}{32}\cos(4x) - \frac{3}{8}\cos(2x)$

maple [B] time = 0.09, size = 65, normalized size = 6.50

$$x^3 \sin(x) - \frac{3(\cos^2(x))x^2}{2} + 3x \left(\frac{\cos(x)\sin(x)}{2} + \frac{x}{2} \right) - \frac{3x^2}{2} + x(\sin^3(x)) + \frac{(\sin^4(x))}{4} + \frac{x^4}{4} + 3x \left(-\frac{\cos(x)\sin(x)}{2} + \frac{x}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+cos(x))*(x+sin(x))^3,x)

[Out] $x^3\sin(x) - \frac{3}{2}\cos(x)^2x^2 + 3x(1/2\cos(x)\sin(x) + 1/2x) - \frac{3}{2}x^2 + x\sin(x)^3 + \frac{1}{4}\sin(x)^4 + \frac{1}{4}x^4 + 3x(-1/2\cos(x)\sin(x) + 1/2x)$

maxima [A] time = 0.33, size = 8, normalized size = 0.80

$$\frac{1}{4}(x + \sin(x))^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x))*(x+sin(x))^3,x, algorithm="maxima")

[Out] $\frac{1}{4}(x + \sin(x))^4$

mupad [B] time = 3.15, size = 8, normalized size = 0.80

$$\frac{(x + \sin(x))^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x) + 1)*(x + sin(x))^3,x)

[Out] $(x + \sin(x))^4/4$

sympy [B] time = 0.52, size = 36, normalized size = 3.60

$$\frac{x^4}{4} + x^3 \sin(x) + \frac{3x^2 \sin^2(x)}{2} + x \sin^3(x) + \frac{\sin^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x))*(x+sin(x))**3,x)

[Out] $x^{**4}/4 + x^{**3}\sin(x) + 3x^{**2}\sin(x)**2/2 + x\sin(x)**3 + \sin(x)**4/4$

3.794 $\int (1 + \cos(x)) \csc^2(x) dx$

Optimal. Leaf size=9

$$-\cot(x) - \csc(x)$$

[Out] $-\cot(x) - \csc(x)$

Rubi [A] time = 0.03, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2669, 3767, 8}

$$-\cot(x) - \csc(x)$$

Antiderivative was successfully verified.

[In] `Int[(1 + Cos[x])*Csc[x]^2,x]`

[Out] `-Cot[x] - Csc[x]`

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2669

`Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := -Simp[(b*(g*cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])`

Rule 3767

`Int[csc[(c_.) + (d_.)*(x_.)]^(n_.), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rubi steps

$$\begin{aligned} \int (1 + \cos(x)) \csc^2(x) dx &= -\csc(x) + \int \csc^2(x) dx \\ &= -\csc(x) - \text{Subst}\left(\int 1 dx, x, \cot(x)\right) \\ &= -\cot(x) - \csc(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$-\cot(x) - \csc(x)$$

Antiderivative was successfully verified.

[In] `Integrate[(1 + Cos[x])*Csc[x]^2,x]`

[Out] `-Cot[x] - Csc[x]`

fricas [A] time = 0.63, size = 10, normalized size = 1.11

$$\frac{\cos(x) + 1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x))*csc(x)^2,x, algorithm="fricas")

[Out] -(cos(x) + 1)/sin(x)

giac [A] time = 0.14, size = 8, normalized size = 0.89

$$-\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x))*csc(x)^2,x, algorithm="giac")

[Out] -1/tan(1/2*x)

maple [A] time = 0.06, size = 12, normalized size = 1.33

$$-\frac{1}{\sin(x)} - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+cos(x))*csc(x)^2,x)

[Out] -1/sin(x)-cot(x)

maxima [A] time = 0.33, size = 13, normalized size = 1.44

$$-\frac{1}{\sin(x)} - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x))*csc(x)^2,x, algorithm="maxima")

[Out] -1/sin(x) - 1/tan(x)

mupad [B] time = 2.93, size = 6, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x) + 1)/sin(x)^2,x)

[Out] -cot(x/2)

sympy [A] time = 1.80, size = 8, normalized size = 0.89

$$-\cot(x) - \frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x))*csc(x)**2,x)

[Out] -cot(x) - 1/sin(x)

3.795 $\int \sin(x) \tan^2(x) dx$

Optimal. Leaf size=5

$$\cos(x) + \sec(x)$$

[Out] $\cos(x) + \sec(x)$

Rubi [A] time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2590, 14}

$$\cos(x) + \sec(x)$$

Antiderivative was successfully verified.

[In] `Int[Sin[x]*Tan[x]^2,x]`

[Out] `Cos[x] + Sec[x]`

Rule 14

`Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Rule 2590

`Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]`

Rubi steps

$$\begin{aligned} \int \sin(x) \tan^2(x) dx &= -\text{Subst} \left(\int \frac{1-x^2}{x^2} dx, x, \cos(x) \right) \\ &= -\text{Subst} \left(\int \left(-1 + \frac{1}{x^2} \right) dx, x, \cos(x) \right) \\ &= \cos(x) + \sec(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 5, normalized size = 1.00

$$\cos(x) + \sec(x)$$

Antiderivative was successfully verified.

[In] `Integrate[Sin[x]*Tan[x]^2,x]`

[Out] `Cos[x] + Sec[x]`

fricas [B] time = 0.70, size = 11, normalized size = 2.20

$$\frac{\cos(x)^2 + 1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)*tan(x)^2,x, algorithm="fricas")`

[Out] `(cos(x)^2 + 1)/cos(x)`

giac [A] time = 0.14, size = 7, normalized size = 1.40

$$\frac{1}{\cos(x)} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(x)^2,x, algorithm="giac")

[Out] 1/cos(x) + cos(x)

maple [B] time = 0.04, size = 20, normalized size = 4.00

$$\frac{\sin^4(x)}{\cos(x)} + (2 + \sin^2(x))\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*tan(x)^2,x)

[Out] sin(x)^4/cos(x)+(2+sin(x)^2)*cos(x)

maxima [A] time = 0.31, size = 7, normalized size = 1.40

$$\frac{1}{\cos(x)} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(x)^2,x, algorithm="maxima")

[Out] 1/cos(x) + cos(x)

mupad [B] time = 3.00, size = 7, normalized size = 1.40

$$\cos(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*tan(x)^2,x)

[Out] cos(x) + 1/cos(x)

sympy [A] time = 0.07, size = 7, normalized size = 1.40

$$\cos(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(x)**2,x)

[Out] cos(x) + 1/cos(x)

$$3.796 \quad \int e^{\sin(x)} \sec^2(x) (x \cos^3(x) - \sin(x)) dx$$

Optimal. Leaf size=13

$$e^{\sin(x)}(x \cos(x) - 1) \sec(x)$$

[Out] exp(sin(x))*(-1+x*cos(x))*sec(x)

Rubi [F] time = 0.64, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int e^{\sin(x)} \sec^2(x) (x \cos^3(x) - \sin(x)) dx$$

Verification is Not applicable to the result.

[In] Int[E^Sin[x]*Sec[x]^2*(x*Cos[x]^3 - Sin[x]), x]

[Out] Defer[Int][E^Sin[x]*x*Cos[x], x] - Defer[Int][E^Sin[x]*Sec[x]*Tan[x], x]

Rubi steps

$$\begin{aligned} \int e^{\sin(x)} \sec^2(x) (x \cos^3(x) - \sin(x)) dx &= \int (e^{\sin(x)} x \cos(x) - e^{\sin(x)} \sec(x) \tan(x)) dx \\ &= \int e^{\sin(x)} x \cos(x) dx - \int e^{\sin(x)} \sec(x) \tan(x) dx \end{aligned}$$

Mathematica [A] time = 0.29, size = 13, normalized size = 1.00

$$e^{\sin(x)}(x \cos(x) - 1) \sec(x)$$

Antiderivative was successfully verified.

[In] Integrate[E^Sin[x]*Sec[x]^2*(x*Cos[x]^3 - Sin[x]), x]

[Out] E^Sin[x]*(-1 + x*Cos[x])*Sec[x]

fricas [A] time = 0.68, size = 14, normalized size = 1.08

$$\frac{(x \cos(x) - 1)e^{\sin(x)}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x, algorithm="fricas")

[Out] (x*cos(x) - 1)*e^sin(x)/cos(x)

giac [B] time = 0.20, size = 794, normalized size = 61.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x, algorithm="giac")

[Out] (x*e^(2*tan(1/2*x))/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^8 + e^(2*tan(1/2*x))/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^8 - 16*x*e^(2*tan(1/2*x))/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^6 + 12*x*e^(2*tan(1/2*x))/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x)^7 - x*e^(2*tan(1/2*x))/(tan(1/2*x)^2 + 1))*tan(1/2*x)^8 - 14*e^(2*tan(1/2*x))/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan

$$\begin{aligned} & (1/2*x)^6 + 12*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(3/2*x)*\tan(1/2*x)^7} \\ & - e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(1/2*x)^8} + 30*x*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(3/2*x)^2*\tan(1/2*x)^4} \\ & - 52*x*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(3/2*x)*\tan(1/2*x)^5} + 16*x*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(1/2*x)^6} \\ & - 28*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(3/2*x)*\tan(1/2*x)^5} + 14*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(1/2*x)^6} \\ & - 16*x*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(3/2*x)^2*\tan(1/2*x)^2} + 52*x*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(3/2*x)*\tan(1/2*x)^3} \\ & - 30*x*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(1/2*x)^4} + 14*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(3/2*x)^2*\tan(1/2*x)^2} \\ & - 28*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(3/2*x)*\tan(1/2*x)^3} + x*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(3/2*x)^2} \\ & - 12*x*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(3/2*x)*\tan(1/2*x)} + 16*x*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(1/2*x)^2} \\ & - e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(3/2*x)^2} + 12*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(3/2*x)*\tan(1/2*x)} \\ & - 14*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))*\tan(1/2*x)^2} - x*e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))} \\ & + e^{(2*\tan(1/2*x)/(\tan(1/2*x)^2 + 1))}/(\tan(3/2*x)^2*\tan(1/2*x)^8 + 2*\tan(3/2*x)^2*\tan(1/2*x)^6 + \tan(1/2*x)^8 + 2*\tan(1/2*x)^6 - 2*\tan(3/2*x)^2*\tan(1/2*x)^2 - \tan(3/2*x)^2 - 2*\tan(1/2*x)^2 - 1) \end{aligned}$$

maple [C] time = 0.39, size = 30, normalized size = 2.31

$$\frac{(x e^{2ix} + x - 2 e^{ix}) e^{\sin(x)}}{e^{2ix} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x)

[Out] (x*exp(2*I*x)+x-2*exp(I*x))/(exp(2*I*x)+1)*exp(sin(x))

maxima [B] time = 0.77, size = 88, normalized size = 6.77

$$\frac{x \cos(2x)^2 e^{\sin(x)} + x e^{\sin(x)} \sin(2x)^2 - 2 e^{\sin(x)} \sin(2x) \sin(x) + 2 (x e^{\sin(x)} - \cos(x) e^{\sin(x)}) \cos(2x) + x e^{\sin(x)} - 2}{\cos(2x)^2 + \sin(2x)^2 + 2 \cos(2x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x, algorithm="maxima")

[Out] (x*cos(2*x)^2*e^sin(x) + x*e^sin(x)*sin(2*x)^2 - 2*e^sin(x)*sin(2*x)*sin(x) + 2*(x*e^sin(x) - cos(x)*e^sin(x))*cos(2*x) + x*e^sin(x) - 2*cos(x)*e^sin(x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)

mupad [B] time = 3.12, size = 14, normalized size = 1.08

$$\frac{e^{\sin(x)} (x \cos(x) - 1)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(exp(sin(x))*(sin(x) - x*cos(x)^3))/cos(x)^2,x)

[Out] (exp(sin(x))*(x*cos(x) - 1))/cos(x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(sin(x))*sec(x)**2*(x*cos(x)**3-sin(x)),x)

[Out] Timed out

3.797 $\int x \csc^2(x) dx$

Optimal. Leaf size=9

$$\log(\sin(x)) - x \cot(x)$$

[Out] $-x*\cot(x)+\ln(\sin(x))$

Rubi [A] time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4184, 3475}

$$\log(\sin(x)) - x \cot(x)$$

Antiderivative was successfully verified.

[In] Int[x*Csc[x]^2,x]

[Out] $-(x*\cot(x)) + \text{Log}[\text{Sin}[x]]$

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4184

Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rubi steps

$$\begin{aligned} \int x \csc^2(x) dx &= -x \cot(x) + \int \cot(x) dx \\ &= -x \cot(x) + \log(\sin(x)) \end{aligned}$$

Mathematica [A] time = 0.02, size = 9, normalized size = 1.00

$$\log(\sin(x)) - x \cot(x)$$

Antiderivative was successfully verified.

[In] Integrate[x*Csc[x]^2,x]

[Out] $-(x*\cot(x)) + \text{Log}[\text{Sin}[x]]$

fricas [B] time = 0.62, size = 20, normalized size = 2.22

$$-\frac{x \cos(x) - \log\left(\frac{1}{2} \sin(x)\right) \sin(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)^2,x, algorithm="fricas")

[Out] $-(x*\cos(x) - \log(1/2*\sin(x))*\sin(x))/\sin(x)$

giac [B] time = 0.19, size = 52, normalized size = 5.78

$$\frac{x \tan\left(\frac{1}{2}x\right)^2 + \log\left(\frac{16 \tan\left(\frac{1}{2}x\right)^2}{\tan\left(\frac{1}{2}x\right)^4 + 2 \tan\left(\frac{1}{2}x\right)^2 + 1}\right) \tan\left(\frac{1}{2}x\right) - x}{2 \tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)^2,x, algorithm="giac")

[Out] 1/2*(x*tan(1/2*x)^2 + log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x) - x)/tan(1/2*x)

maple [A] time = 0.03, size = 10, normalized size = 1.11

$$-x \cot(x) + \ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*csc(x)^2,x)

[Out] -x*cot(x)+ln(sin(x))

maxima [B] time = 0.32, size = 104, normalized size = 11.56

$$\frac{(\cos(2x)^2 + \sin(2x)^2 - 2 \cos(2x) + 1) \log(\cos(x)^2 + \sin(x)^2 + 2 \cos(x) + 1) + (\cos(2x)^2 + \sin(2x)^2 - 2 \cos(2x) + 1) \log(\cos(x)^2 + \sin(x)^2 - 2 \cos(x) + 1) - 4x \sin(2x)}{2(\cos(2x)^2 + \sin(2x)^2 - 2 \cos(2x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)^2,x, algorithm="maxima")

[Out] 1/2*((cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)

mupad [B] time = 0.03, size = 9, normalized size = 1.00

$$\ln(\sin(x)) - x \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/sin(x)^2,x)

[Out] log(sin(x)) - x*cot(x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \csc^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)**2,x)

[Out] Integral(x*csc(x)**2, x)

3.798 $\int \cos(x) \sin\left(\frac{\pi}{6} + x\right) dx$

Optimal. Leaf size=20

$$\frac{x}{4} - \frac{1}{4} \cos\left(2x + \frac{\pi}{6}\right)$$

[Out] 1/4*x-1/4*cos(1/6*Pi+2*x)

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4574, 2638}

$$\frac{x}{4} - \frac{1}{4} \cos\left(2x + \frac{\pi}{6}\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[Pi/6 + x],x]

[Out] x/4 - Cos[Pi/6 + 2*x]/4

Rule 2638

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4574

Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] := Int[ExpandTrigReduce[Sin[v]^p *Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))

Rubi steps

$$\begin{aligned} \int \cos(x) \sin\left(\frac{\pi}{6} + x\right) dx &= \int \left(\frac{1}{4} + \frac{1}{2} \sin\left(\frac{\pi}{6} + 2x\right)\right) dx \\ &= \frac{x}{4} + \frac{1}{2} \int \sin\left(\frac{\pi}{6} + 2x\right) dx \\ &= \frac{x}{4} - \frac{1}{4} \cos\left(\frac{\pi}{6} + 2x\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{x}{4} - \frac{1}{4} \cos\left(2x + \frac{\pi}{6}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sin[Pi/6 + x],x]

[Out] x/4 - Cos[Pi/6 + 2*x]/4

fricas [B] time = 1.74, size = 31, normalized size = 1.55

$$-\frac{1}{4} \sqrt{3} \cos\left(\frac{1}{6} \pi + x\right)^2 - \frac{1}{4} \cos\left(\frac{1}{6} \pi + x\right) \sin\left(\frac{1}{6} \pi + x\right) + \frac{1}{4} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="fricas")

[Out] $-1/4*\sqrt{3}*\cos(1/6*\pi + x)^2 - 1/4*\cos(1/6*\pi + x)*\sin(1/6*\pi + x) + 1/4*x$

giac [A] time = 0.16, size = 14, normalized size = 0.70

$$\frac{1}{4}x - \frac{1}{4}\cos\left(\frac{1}{6}\pi + 2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="giac")

[Out] $1/4*x - 1/4*\cos(1/6*\pi + 2*x)$

maple [A] time = 0.15, size = 15, normalized size = 0.75

$$\frac{x}{4} - \frac{\cos\left(\frac{\pi}{6} + 2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(1/6*Pi+x),x)

[Out] $1/4*x - 1/4*\cos(1/6*Pi + 2*x)$

maxima [A] time = 0.31, size = 14, normalized size = 0.70

$$\frac{1}{4}x - \frac{1}{4}\cos\left(\frac{1}{6}\pi + 2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="maxima")

[Out] $1/4*x - 1/4*\cos(1/6*\pi + 2*x)$

mupad [B] time = 0.03, size = 18, normalized size = 0.90

$$\frac{x \sin\left(\frac{\pi}{6}\right)}{2} - \frac{\cos\left(\frac{\pi}{6} + 2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(Pi/6 + x),x)

[Out] $(x*\sin(Pi/6))/2 - \cos(Pi/6 + 2*x)/4$

sympy [B] time = 0.46, size = 37, normalized size = 1.85

$$-\frac{x \sin(x) \cos\left(x + \frac{\pi}{6}\right)}{2} + \frac{x \sin\left(x + \frac{\pi}{6}\right) \cos(x)}{2} + \frac{\sin(x) \sin\left(x + \frac{\pi}{6}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(1/6*pi+x),x)

[Out] $-x*\sin(x)*\cos(x + \pi/6)/2 + x*\sin(x + \pi/6)*\cos(x)/2 + \sin(x)*\sin(x + \pi/6)/2$

3.799 $\int x \sin^3(x^2) dx$

Optimal. Leaf size=19

$$\frac{1}{6} \cos^3(x^2) - \frac{\cos(x^2)}{2}$$

[Out] $-1/2*\cos(x^2)+1/6*\cos(x^2)^3$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3379, 2633}

$$\frac{1}{6} \cos^3(x^2) - \frac{\cos(x^2)}{2}$$

Antiderivative was successfully verified.

[In] Int[x*Sin[x^2]^3,x]

[Out] -Cos[x^2]/2 + Cos[x^2]^3/6

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x \sin^3(x^2) dx &= \frac{1}{2} \text{Subst} \left(\int \sin^3(x) dx, x, x^2 \right) \\ &= - \left(\frac{1}{2} \text{Subst} \left(\int (1 - x^2) dx, x, \cos(x^2) \right) \right) \\ &= -\frac{1}{2} \cos(x^2) + \frac{1}{6} \cos^3(x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{24} \cos(3x^2) - \frac{3 \cos(x^2)}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sin[x^2]^3,x]

[Out] (-3*Cos[x^2])/8 + Cos[3*x^2]/24

fricas [A] time = 0.68, size = 15, normalized size = 0.79

$$\frac{1}{6} \cos(x^2)^3 - \frac{1}{2} \cos(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x^2)^3,x, algorithm="fricas")

[Out] 1/6*cos(x^2)^3 - 1/2*cos(x^2)

giac [A] time = 0.14, size = 15, normalized size = 0.79

$$\frac{1}{6} \cos(x^2)^3 - \frac{1}{2} \cos(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x^2)^3,x, algorithm="giac")

[Out] 1/6*cos(x^2)^3 - 1/2*cos(x^2)

maple [A] time = 0.02, size = 15, normalized size = 0.79

$$\frac{(2 + \sin^2(x^2)) \cos(x^2)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(x^2)^3,x)

[Out] -1/6*(2+sin(x^2)^2)*cos(x^2)

maxima [A] time = 0.32, size = 15, normalized size = 0.79

$$\frac{1}{24} \cos(3x^2) - \frac{3}{8} \cos(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x^2)^3,x, algorithm="maxima")

[Out] 1/24*cos(3*x^2) - 3/8*cos(x^2)

mupad [B] time = 2.95, size = 14, normalized size = 0.74

$$\frac{\cos(x^2) (\cos(x^2)^2 - 3)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(x^2)^3,x)

[Out] (cos(x^2)*(cos(x^2)^2 - 3))/6

sympy [A] time = 0.48, size = 22, normalized size = 1.16

$$-\frac{\sin^2(x^2) \cos(x^2)}{2} - \frac{\cos^3(x^2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x**2)**3,x)

[Out] -sin(x**2)**2*cos(x**2)/2 - cos(x**2)**3/3

3.800 $\int \sin^2(x) \tan(x) dx$

Optimal. Leaf size=14

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

[Out] 1/2*cos(x)^2-ln(cos(x))

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2590, 14}

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int [Sin [x]^2*Tan [x] , x]

[Out] Cos [x]^2/2 - Log [Cos [x]]

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)+(b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 2590

Int[sin[(e_)+(f_)*(x_)]^(m_)*tan[(e_)+(f_)*(x_)]^(n_), x_Symbol] :> -Dist[f^(-1), Subst[Int[(1-x^2)^((m+n-1)/2)/x^n, x], x, Cos[e+f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m+n-1)/2]

Rubi steps

$$\begin{aligned} \int \sin^2(x) \tan(x) dx &= -\text{Subst} \left(\int \frac{1-x^2}{x} dx, x, \cos(x) \right) \\ &= -\text{Subst} \left(\int \left(\frac{1}{x} - x \right) dx, x, \cos(x) \right) \\ &= \frac{\cos^2(x)}{2} - \log(\cos(x)) \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[Sin [x]^2*Tan [x] , x]

[Out] Cos [x]^2/2 - Log [Cos [x]]

fricas [A] time = 0.80, size = 14, normalized size = 1.00

$$\frac{1}{2} \cos(x)^2 - \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2*tan(x),x, algorithm="fricas")

[Out] 1/2*cos(x)^2 - log(-cos(x))

giac [A] time = 0.13, size = 18, normalized size = 1.29

$$-\frac{1}{2} \sin(x)^2 - \frac{1}{2} \log(-\sin(x)^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2*tan(x),x, algorithm="giac")

[Out] -1/2*sin(x)^2 - 1/2*log(-sin(x)^2 + 1)

maple [A] time = 0.04, size = 13, normalized size = 0.93

$$-\frac{(\sin^2(x))}{2} - \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2*tan(x),x)

[Out] -1/2*sin(x)^2-ln(cos(x))

maxima [A] time = 0.30, size = 16, normalized size = 1.14

$$-\frac{1}{2} \sin(x)^2 - \frac{1}{2} \log(\sin(x)^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2*tan(x),x, algorithm="maxima")

[Out] -1/2*sin(x)^2 - 1/2*log(sin(x)^2 - 1)

mupad [B] time = 2.95, size = 16, normalized size = 1.14

$$\frac{\cos(x)^2}{2} + \frac{\ln(\tan(x)^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2*tan(x),x)

[Out] log(tan(x)^2 + 1)/2 + cos(x)^2/2

sympy [A] time = 0.07, size = 10, normalized size = 0.71

$$-\log(\cos(x)) + \frac{\cos^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**2*tan(x),x)

[Out] -log(cos(x)) + cos(x)**2/2

3.801 $\int \cos^2(x) \cot^3(x) dx$

Optimal. Leaf size=22

$$\frac{\sin^2(x)}{2} - \frac{1}{2} \csc^2(x) - 2 \log(\sin(x))$$

[0ut] $-1/2*\csc(x)^2-2*\ln(\sin(x))+1/2*\sin(x)^2$

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2590, 266, 43}

$$\frac{\sin^2(x)}{2} - \frac{1}{2} \csc^2(x) - 2 \log(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2*Cot[x]^3,x]

[0ut] $-Csc[x]^2/2 - 2*Log[Sin[x]] + Sin[x]^2/2$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 2590

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rubi steps

$$\begin{aligned} \int \cos^2(x) \cot^3(x) dx &= \text{Subst} \left(\int \frac{(1-x^2)^2}{x^3} dx, x, -\sin(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{(1-x)^2}{x^2} dx, x, \sin^2(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(1 + \frac{1}{x^2} - \frac{2}{x} \right) dx, x, \sin^2(x) \right) \\ &= -\frac{1}{2} \csc^2(x) - 2 \log(\sin(x)) + \frac{\sin^2(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.91

$$\frac{1}{2} (\sin^2(x) - \csc^2(x) - 4 \log(\sin(x)))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^2*Cot[x]^3,x]

[Out] (-Csc[x]^2 - 4*Log[Sin[x]] + Sin[x]^2)/2

fricas [B] time = 0.60, size = 37, normalized size = 1.68

$$\frac{2 \cos(x)^4 - 3 \cos(x)^2 + 8 (\cos(x)^2 - 1) \log\left(\frac{1}{2} \sin(x)\right) - 1}{4 (\cos(x)^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="fricas")

[Out] -1/4*(2*cos(x)^4 - 3*cos(x)^2 + 8*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos(x)^2 - 1)

giac [A] time = 0.13, size = 28, normalized size = 1.27

$$-\frac{1}{2} \cos(x)^2 + \frac{1}{2 (\cos(x)^2 - 1)} - \log(-\cos(x)^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="giac")

[Out] -1/2*cos(x)^2 + 1/2/(cos(x)^2 - 1) - log(-cos(x)^2 + 1)

maple [A] time = 0.06, size = 29, normalized size = 1.32

$$-\frac{\cos^6(x)}{2 \sin(x)^2} - \frac{(\cos^4(x))}{2} - (\cos^2(x)) - 2 \ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2*cot(x)^3,x)

[Out] -1/2/sin(x)^2*cos(x)^6-1/2*cos(x)^4-cos(x)^2-2*ln(sin(x))

maxima [A] time = 0.32, size = 20, normalized size = 0.91

$$\frac{1}{2} \sin(x)^2 - \frac{1}{2 \sin(x)^2} - \log(\sin(x)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="maxima")

[Out] 1/2*sin(x)^2 - 1/2/sin(x)^2 - log(sin(x)^2)

mupad [B] time = 2.97, size = 32, normalized size = 1.45

$$\ln(\tan(x)^2 + 1) - 2 \ln(\tan(x)) - \frac{\tan(x)^2 + \frac{1}{2}}{\tan(x)^4 + \tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2*cot(x)^3,x)

[Out] log(tan(x)^2 + 1) - 2*log(tan(x)) - (tan(x)^2 + 1/2)/(tan(x)^2 + tan(x)^4)

sympy [A] time = 0.08, size = 20, normalized size = 0.91

$$-2 \log(\sin(x)) + \frac{\sin^2(x)}{2} - \frac{1}{2 \sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**2*cot(x)**3,x)

[Out] -2*log(sin(x)) + sin(x)**2/2 - 1/(2*sin(x)**2)

3.802 $\int \sec(x)(1 - \sin(x)) dx$

Optimal. Leaf size=5

$$\log(\sin(x) + 1)$$

[Out] ln(1+sin(x))

Rubi [A] time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2667, 31}

$$\log(\sin(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]*(1 - Sin[x]),x]

[Out] Log[1 + Sin[x]]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rubi steps

$$\begin{aligned} \int \sec(x)(1 - \sin(x)) dx &= -\text{Subst}\left(\int \frac{1}{1-x} dx, x, -\sin(x)\right) \\ &= \log(1 + \sin(x)) \end{aligned}$$

Mathematica [B] time = 0.01, size = 36, normalized size = 7.20

$$\log(\cos(x)) - \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]*(1 - Sin[x]),x]

[Out] Log[Cos[x]] - Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]

fricas [A] time = 1.44, size = 5, normalized size = 1.00

$$\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*(1-sin(x)),x, algorithm="fricas")

[Out] log(sin(x) + 1)

giac [A] time = 0.14, size = 5, normalized size = 1.00

$$\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)*(1-sin(x)),x, algorithm="giac")`

[Out] `log(sin(x) + 1)`

maple [A] time = 0.05, size = 6, normalized size = 1.20

$$\ln(1 + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(x)*(1-sin(x)),x)`

[Out] `ln(1+sin(x))`

maxima [A] time = 0.31, size = 5, normalized size = 1.00

$$\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)*(1-sin(x)),x, algorithm="maxima")`

[Out] `log(sin(x) + 1)`

mupad [B] time = 2.94, size = 5, normalized size = 1.00

$$\ln(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(sin(x) - 1)/cos(x),x)`

[Out] `log(sin(x) + 1)`

sympy [B] time = 2.04, size = 12, normalized size = 2.40

$$\log(\tan(x) + \sec(x)) + \log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)*(1-sin(x)),x)`

[Out] `log(tan(x) + sec(x)) + log(cos(x))`

3.803 $\int (1 + \cos(x)) \csc(x) dx$

Optimal. Leaf size=7

$$\log(1 - \cos(x))$$

[Out] $\ln(1 - \cos(x))$

Rubi [A] time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2667, 31}

$$\log(1 - \cos(x))$$

Antiderivative was successfully verified.

[In] `Int[(1 + Cos[x])*Csc[x], x]`

[Out] `Log[1 - Cos[x]]`

Rule 31

`Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 2667

`Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])`

Rubi steps

$$\int (1 + \cos(x)) \csc(x) dx = -\text{Subst}\left(\int \frac{1}{1-x} dx, x, \cos(x)\right) = \log(1 - \cos(x))$$

Mathematica [B] time = 0.01, size = 20, normalized size = 2.86

$$\log\left(\sin\left(\frac{x}{2}\right)\right) + \log(\sin(x)) - \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] `Integrate[(1 + Cos[x])*Csc[x], x]`

[Out] `-Log[Cos[x/2]] + Log[Sin[x/2]] + Log[Sin[x]]`

fricas [A] time = 0.74, size = 7, normalized size = 1.00

$$\log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+cos(x))*csc(x), x, algorithm="fricas")`

[Out] `log(-1/2*cos(x) + 1/2)`

giac [A] time = 0.13, size = 7, normalized size = 1.00

$$\log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x))*csc(x),x, algorithm="giac")

[Out] log(-cos(x) + 1)

maple [A] time = 0.05, size = 6, normalized size = 0.86

$$\ln(-1 + \cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+cos(x))*csc(x),x)

[Out] ln(-1+cos(x))

maxima [A] time = 0.31, size = 5, normalized size = 0.71

$$\log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x))*csc(x),x, algorithm="maxima")

[Out] log(cos(x) - 1)

mupad [B] time = 2.91, size = 5, normalized size = 0.71

$$\ln(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x) + 1)/sin(x),x)

[Out] log(cos(x) - 1)

sympy [B] time = 1.89, size = 12, normalized size = 1.71

$$-\log(\cot(x) + \csc(x)) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x))*csc(x),x)

[Out] -log(cot(x) + csc(x)) + log(sin(x))

3.804 $\int \cos^2(x) (1 - \tan^2(x)) dx$

Optimal. Leaf size=5

$$\sin(x) \cos(x)$$

[Out] `cos(x)*sin(x)`

Rubi [A] time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3675, 383}

$$\sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] `Int[Cos[x]^2*(1 - Tan[x]^2),x]`

[Out] `Cos[x]*Sin[x]`

Rule 383

`Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]`

Rule 3675

`Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])`

Rubi steps

$$\int \cos^2(x) (1 - \tan^2(x)) dx = \text{Subst} \left(\int \frac{1 - x^2}{(1 + x^2)^2} dx, x, \tan(x) \right) = \cos(x) \sin(x)$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.60

$$\frac{1}{2} \sin(2x)$$

Antiderivative was successfully verified.

[In] `Integrate[Cos[x]^2*(1 - Tan[x]^2),x]`

[Out] `Sin[2*x]/2`

fricas [A] time = 0.53, size = 5, normalized size = 1.00

$$\cos(x) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^2*(1-tan(x)^2),x, algorithm="fricas")`

[Out] $\cos(x)\sin(x)$

giac [A] time = 0.15, size = 9, normalized size = 1.80

$$\frac{1}{\frac{1}{\tan(x)} + \tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^2*(1-tan(x)^2),x, algorithm="giac")`

[Out] $1/(1/\tan(x) + \tan(x))$

maple [A] time = 0.05, size = 6, normalized size = 1.20

$$\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)^2*(1-tan(x)^2),x)`

[Out] $\cos(x)\sin(x)$

maxima [B] time = 0.31, size = 11, normalized size = 2.20

$$\frac{\tan(x)}{\tan(x)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^2*(1-tan(x)^2),x, algorithm="maxima")`

[Out] $\tan(x)/(\tan(x)^2 + 1)$

mupad [B] time = 2.89, size = 6, normalized size = 1.20

$$\frac{\sin(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-cos(x)^2*(tan(x)^2 - 1),x)`

[Out] $\sin(2*x)/2$

sympy [B] time = 3.11, size = 14, normalized size = 2.80

$$\frac{\sin(x)\cos(x)}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)**2*(1-tan(x)**2),x)`

[Out] $\sin(x)\cos(x)/2 + \sin(2*x)/4$

3.805 $\int \csc(2x)(\cos(x) + \sin(x)) dx$

Optimal. Leaf size=15

$$\frac{1}{2} \tanh^{-1}(\sin(x)) - \frac{1}{2} \tanh^{-1}(\cos(x))$$

[Out] $-1/2*\operatorname{arctanh}(\cos(x))+1/2*\operatorname{arctanh}(\sin(x))$

Rubi [A] time = 0.05, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4401, 4287, 3770, 4288}

$$\frac{1}{2} \tanh^{-1}(\sin(x)) - \frac{1}{2} \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csc}[2*x]*(\operatorname{Cos}[x] + \operatorname{Sin}[x]), x]$

[Out] $-\operatorname{ArcTanh}[\operatorname{Cos}[x]]/2 + \operatorname{ArcTanh}[\operatorname{Sin}[x]]/2$

Rule 3770

$\operatorname{Int}[\csc[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /;$ $\operatorname{FreeQ}[\{c, d\}, x]$

Rule 4287

$\operatorname{Int}[(\cos[(a_.) + (b_.)*(x_.)]*(e_.))^{(m_.)}*\sin[(c_.) + (d_.)*(x_.)]^{(p_.)}, x_Symbol] \rightarrow \operatorname{Dist}[2^p/e^p, \operatorname{Int}[(e*\operatorname{Cos}[a + b*x])^{(m+p)}*\operatorname{Sin}[a + b*x]^p, x], x] /;$ $\operatorname{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \operatorname{EqQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[d/b, 2] \ \&\& \operatorname{IntegerQ}[p]$

Rule 4288

$\operatorname{Int}[(f_.)*\sin[(a_.) + (b_.)*(x_.)]^{(n_.)}*\sin[(c_.) + (d_.)*(x_.)]^{(p_.)}, x_Symbol] \rightarrow \operatorname{Dist}[2^p/f^p, \operatorname{Int}[\operatorname{Cos}[a + b*x]^p*(f*\operatorname{Sin}[a + b*x])^{(n+p)}, x], x] /;$ $\operatorname{FreeQ}[\{a, b, c, d, f, n\}, x] \ \&\& \operatorname{EqQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[d/b, 2] \ \&\& \operatorname{IntegerQ}[p]$

Rule 4401

$\operatorname{Int}[u_, x_Symbol] \rightarrow \operatorname{With}[\{v = \operatorname{ExpandTrig}[u, x]\}, \operatorname{Int}[v, x] /; \operatorname{SumQ}[v]] /; \operatorname{!InertTrigFreeQ}[u]$

Rubi steps

$$\begin{aligned} \int \csc(2x)(\cos(x) + \sin(x)) dx &= \int (\cos(x) \csc(2x) + \csc(2x) \sin(x)) dx \\ &= \int \cos(x) \csc(2x) dx + \int \csc(2x) \sin(x) dx \\ &= \frac{1}{2} \int \csc(x) dx + \frac{1}{2} \int \sec(x) dx \\ &= -\frac{1}{2} \tanh^{-1}(\cos(x)) + \frac{1}{2} \tanh^{-1}(\sin(x)) \end{aligned}$$

Mathematica [B] time = 0.01, size = 61, normalized size = 4.07

$$\frac{1}{2} \log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{2} \log\left(\cos\left(\frac{x}{2}\right)\right) - \frac{1}{2} \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \frac{1}{2} \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[2*x]*(Cos[x] + Sin[x]),x]

[Out] $-1/2*\text{Log}[\text{Cos}[x/2]] - \text{Log}[\text{Cos}[x/2] - \text{Sin}[x/2]]/2 + \text{Log}[\text{Sin}[x/2]]/2 + \text{Log}[\text{Cos}[x/2] + \text{Sin}[x/2]]/2$

fricas [B] time = 0.75, size = 35, normalized size = 2.33

$$-\frac{1}{4} \log\left(-\frac{1}{2}(\cos(x) + 1)\sin(x) + \frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{4} \log\left(-\frac{1}{2}(\cos(x) - 1)\sin(x) - \frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2*x)*(cos(x)+sin(x)),x, algorithm="fricas")

[Out] $-1/4*\log(-1/2*(\cos(x) + 1)*\sin(x) + 1/2*\cos(x) + 1/2) + 1/4*\log(-1/2*(\cos(x) - 1)*\sin(x) - 1/2*\cos(x) + 1/2)$

giac [B] time = 0.15, size = 29, normalized size = 1.93

$$\frac{1}{2} \log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right) - \frac{1}{2} \log\left(\left|\tan\left(\frac{1}{2}x\right) - 1\right|\right) + \frac{1}{2} \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2*x)*(cos(x)+sin(x)),x, algorithm="giac")

[Out] $1/2*\log(\text{abs}(\tan(1/2*x) + 1)) - 1/2*\log(\text{abs}(\tan(1/2*x) - 1)) + 1/2*\log(\text{abs}(\tan(1/2*x)))$

maple [A] time = 0.26, size = 20, normalized size = 1.33

$$\frac{\ln(\sec(x) + \tan(x))}{2} + \frac{\ln(\csc(x) - \cot(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(2*x)*(cos(x)+sin(x)),x)

[Out] $1/2*\ln(\sec(x)+\tan(x))+1/2*\ln(\csc(x)-\cot(x))$

maxima [B] time = 0.42, size = 69, normalized size = 4.60

$$-\frac{1}{4} \log(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1) + \frac{1}{4} \log(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1) + \frac{1}{4} \log(\cos(x)^2 + \sin(x)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(2*x)*(cos(x)+sin(x)),x, algorithm="maxima")

[Out] $-1/4*\log(\cos(x)^2 + \sin(x)^2 + 2*\cos(x) + 1) + 1/4*\log(\cos(x)^2 + \sin(x)^2 - 2*\cos(x) + 1) + 1/4*\log(\cos(x)^2 + \sin(x)^2 + 2*\sin(x) + 1) - 1/4*\log(\cos(x)^2 + \sin(x)^2 - 2*\sin(x) + 1)$

mupad [B] time = 3.11, size = 24, normalized size = 1.60

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)^2 + \tan\left(\frac{x}{2}\right)\right)}{2} - \frac{\ln\left(\tan\left(\frac{x}{2}\right) - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x) + sin(x))/sin(2*x),x)

[Out] $\log(\tan(x/2) + \tan(x/2)^2)/2 - \log(\tan(x/2) - 1)/2$

sympy [B] time = 1.86, size = 32, normalized size = 2.13

$$-\frac{\log(\sin(x) - 1)}{4} + \frac{\log(\sin(x) + 1)}{4} + \frac{\log(\cos(x) - 1)}{4} - \frac{\log(\cos(x) + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(2*x)*(cos(x)+sin(x)),x)`

[Out] $-\log(\sin(x) - 1)/4 + \log(\sin(x) + 1)/4 + \log(\cos(x) - 1)/4 - \log(\cos(x) + 1)/4$

$$3.806 \quad \int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)} dx$$

Optimal. Leaf size=11

$$\log(\sin^2(x) - 3\sin(x) + 2)$$

[Out] $\ln(2-3*\sin(x)+\sin(x)^2)$

Rubi [A] time = 0.05, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4334, 628}

$$\log(\sin^2(x) - 3\sin(x) + 2)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[x]*(-3 + 2*\text{Sin}[x]))/(2 - 3*\text{Sin}[x] + \text{Sin}[x]^2),x]$

[Out] $\text{Log}[2 - 3*\text{Sin}[x] + \text{Sin}[x]^2]$

Rule 628

$\text{Int}[(d_ + (e_)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 4334

$\text{Int}[(u_)*(F_)][(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> \text{With}\{d = \text{FreeFactors}[\text{Sin}[c*(a + b*x)], x]\}, \text{Dist}[d/(b*c), \text{Subst}[\text{Int}[\text{SubstFor}[1, \text{Sin}[c*(a + b*x)]/d, u, x], x], x, \text{Sin}[c*(a + b*x)]/d, x] /; \text{FunctionOfQ}[\text{Sin}[c*(a + b*x)]/d, u, x, \text{True}] /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ (\text{EqQ}[F, \text{Cos}] \ || \ \text{EqQ}[F, \text{cos}])$

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)(-3 + 2\sin(x))}{2 - 3\sin(x) + \sin^2(x)} dx &= \text{Subst}\left(\int \frac{-3 + 2x}{2 - 3x + x^2} dx, x, \sin(x)\right) \\ &= \log(2 - 3\sin(x) + \sin^2(x)) \end{aligned}$$

Mathematica [B] time = 0.09, size = 26, normalized size = 2.36

$$\log(2 - \sin(x)) + 2\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(\text{Cos}[x]*(-3 + 2*\text{Sin}[x]))/(2 - 3*\text{Sin}[x] + \text{Sin}[x]^2),x]$

[Out] $2*\text{Log}[\text{Cos}[x/2] - \text{Sin}[x/2]] + \text{Log}[2 - \text{Sin}[x]]$

fricas [A] time = 0.70, size = 15, normalized size = 1.36

$$\log\left(-\frac{1}{2}\sin(x) + 1\right) + \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\cos(x)*(-3+2*\sin(x))/(2-3*\sin(x)+\sin(x)^2),x, \text{algorithm}=\text{"fricas"})$

[Out] $\log(-1/2*\sin(x) + 1) + \log(-\sin(x) + 1)$

giac [A] time = 0.13, size = 15, normalized size = 1.36

$$\log(-\sin(x) + 2) + \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="giac")`

[Out] $\log(-\sin(x) + 2) + \log(-\sin(x) + 1)$

maple [A] time = 0.05, size = 12, normalized size = 1.09

$$\ln(2 - 3 \sin(x) + \sin^2(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x)`

[Out] $\ln(2-3*\sin(x)+\sin(x)^2)$

maxima [A] time = 0.31, size = 11, normalized size = 1.00

$$\log(\sin(x)^2 - 3 \sin(x) + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="maxima")`

[Out] $\log(\sin(x)^2 - 3*\sin(x) + 2)$

mupad [B] time = 0.08, size = 11, normalized size = 1.00

$$\ln(\sin(x)^2 - 3 \sin(x) + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(x)*(2*sin(x) - 3))/(sin(x)^2 - 3*sin(x) + 2),x)`

[Out] $\log(\sin(x)^2 - 3*\sin(x) + 2)$

sympy [A] time = 0.19, size = 12, normalized size = 1.09

$$\log(\sin(x) - 2) + \log(\sin(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)**2),x)`

[Out] $\log(\sin(x) - 2) + \log(\sin(x) - 1)$

$$3.807 \quad \int \frac{\cos^2(x) \sin(x)}{5 + \cos^2(x)} dx$$

Optimal. Leaf size=20

$$\sqrt{5} \tan^{-1} \left(\frac{\cos(x)}{\sqrt{5}} \right) - \cos(x)$$

[Out] $-\cos(x) + \arctan(1/5 * \cos(x) * 5^{(1/2)}) * 5^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4335, 321, 203}

$$\sqrt{5} \tan^{-1} \left(\frac{\cos(x)}{\sqrt{5}} \right) - \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[x]^2 * \text{Sin}[x]) / (5 + \text{Cos}[x]^2), x]$

[Out] $\text{Sqrt}[5] * \text{ArcTan}[\text{Cos}[x] / \text{Sqrt}[5]] - \text{Cos}[x]$

Rule 203

$\text{Int}[(a_ + (b_.) * (x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 * \text{ArcTan}[(\text{Rt}[b, 2] * x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] * \text{Rt}[b, 2]), x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 321

$\text{Int}[(c_.) * (x_)^{(m_)} * ((a_ + (b_.) * (x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow \text{Simp}[(c^{(n - 1)} * (c * x)^{(m - n + 1)} * (a + b * x^n)^{(p + 1)}) / (b * (m + n * p + 1)), x] - \text{Dist}[(a * c^n * (m - n + 1)) / (b * (m + n * p + 1)), \text{Int}[(c * x)^{(m - n)} * (a + b * x^n)^p, x], x] /;$ FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n * p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 4335

$\text{Int}[(u_)*(\text{F}_)[(c_.) * ((a_.) + (b_.) * (x_))], x_Symbol] \rightarrow \text{With}[\{d = \text{FreeFactors}[\text{Cos}[c * (a + b * x)], x]\}, -\text{Dist}[d / (b * c), \text{Subst}[\text{Int}[\text{SubstFor}[1, \text{Cos}[c * (a + b * x)]] / d, u, x], x], x, \text{Cos}[c * (a + b * x)] / d, x] /;$ FunctionOfQ[Cos[c * (a + b * x)] / d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(x) \sin(x)}{5 + \cos^2(x)} dx &= -\text{Subst} \left(\int \frac{x^2}{5 + x^2} dx, x, \cos(x) \right) \\ &= -\cos(x) + 5 \text{Subst} \left(\int \frac{1}{5 + x^2} dx, x, \cos(x) \right) \\ &= \sqrt{5} \tan^{-1} \left(\frac{\cos(x)}{\sqrt{5}} \right) - \cos(x) \end{aligned}$$

Mathematica [B] time = 0.17, size = 82, normalized size = 4.10

$$\frac{1}{20} \left(-20 \cos(x) + 21 \sqrt{5} \tan^{-1} \left(\frac{1}{\sqrt{5}} - \sqrt{\frac{6}{5}} \tan \left(\frac{x}{2} \right) \right) + 21 \sqrt{5} \tan^{-1} \left(\sqrt{\frac{6}{5}} \tan \left(\frac{x}{2} \right) + \frac{1}{\sqrt{5}} \right) - \sqrt{5} \tan^{-1} \left(\frac{\cos(x)}{\sqrt{5}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]

[Out] $(-\sqrt{5} \operatorname{ArcTan}[\cos(x)/\sqrt{5}]) + 21\sqrt{5} \operatorname{ArcTan}[1/\sqrt{5}] - \sqrt{6/5} \operatorname{Tan}[x/2] + 21\sqrt{5} \operatorname{ArcTan}[1/\sqrt{5}] + \sqrt{6/5} \operatorname{Tan}[x/2] - 20\cos(x))/20$

fricas [A] time = 0.62, size = 17, normalized size = 0.85

$$\sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} \cos(x)\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="fricas")

[Out] $\sqrt{5} \arctan(1/5 \sqrt{5} \cos(x)) - \cos(x)$

giac [A] time = 0.12, size = 17, normalized size = 0.85

$$\sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} \cos(x)\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="giac")

[Out] $\sqrt{5} \arctan(1/5 \sqrt{5} \cos(x)) - \cos(x)$

maple [A] time = 0.04, size = 18, normalized size = 0.90

$$-\cos(x) + \arctan\left(\frac{\cos(x)\sqrt{5}}{5}\right) \sqrt{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2*sin(x)/(5+cos(x)^2),x)

[Out] $-\cos(x) + \arctan(1/5 \cos(x) * 5^{(1/2)}) * 5^{(1/2)}$

maxima [A] time = 0.43, size = 17, normalized size = 0.85

$$\sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} \cos(x)\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="maxima")

[Out] $\sqrt{5} \arctan(1/5 \sqrt{5} \cos(x)) - \cos(x)$

mupad [B] time = 2.90, size = 17, normalized size = 0.85

$$\sqrt{5} \operatorname{atan}\left(\frac{\sqrt{5} \cos(x)}{5}\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x)^2*sin(x))/(cos(x)^2 + 5),x)

[Out] $5^{(1/2)} \operatorname{atan}((5^{(1/2)} \cos(x))/5) - \cos(x)$

sympy [A] time = 0.43, size = 19, normalized size = 0.95

$$-\cos(x) + \sqrt{5} \operatorname{atan}\left(\frac{\sqrt{5} \cos(x)}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**2*sin(x)/(5+cos(x)**2),x)

[Out] -cos(x) + sqrt(5)*atan(sqrt(5)*cos(x)/5)

$$3.808 \quad \int \frac{\cos(x)}{\sin(x) + \sin^2(x)} dx$$

Optimal. Leaf size=11

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

[Out] ln(sin(x))-ln(1+sin(x))

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3258, 615}

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/(Sin[x] + Sin[x]^2),x]

[Out] Log[Sin[x]] - Log[1 + Sin[x]]

Rule 615

Int[((b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Simp[Log[x]/b, x] - Simp[Log[RemoveContent[b + c*x, x]]/b, x] /; FreeQ[{b, c}, x]

Rule 3258

Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*sin[(d_.) + (e_.)*(x_)]))^(n_.) + (c_.)*((f_.)*sin[(d_.) + (e_.)*(x_)]))^(n2_.)^(p_.), x_Symbol] := Module[{g = FreeFactors[Sin[d + e*x], x]}, Dist[g/e, Subst[Int[(1 - g^2*x^2)^((m - 1)/2)*(a + b*(f*g*x)^n + c*(f*g*x)^(2*n))^p, x], x, Sin[d + e*x]/g], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2*n] && IntegerQ[(m - 1)/2]

Rubi steps

$$\int \frac{\cos(x)}{\sin(x) + \sin^2(x)} dx = \text{Subst} \left(\int \frac{1}{x + x^2} dx, x, \sin(x) \right) \\ = \log(\sin(x)) - \log(1 + \sin(x))$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/(Sin[x] + Sin[x]^2),x]

[Out] Log[Sin[x]] - Log[1 + Sin[x]]

fricas [A] time = 0.60, size = 13, normalized size = 1.18

$$\log\left(\frac{1}{2} \sin(x)\right) - \log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="fricas")

[Out] $\log(1/2*\sin(x)) - \log(\sin(x) + 1)$

giac [A] time = 0.15, size = 12, normalized size = 1.09

$$-\log(\sin(x) + 1) + \log(|\sin(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="giac")`

[Out] $-\log(\sin(x) + 1) + \log(\text{abs}(\sin(x)))$

maple [A] time = 0.08, size = 12, normalized size = 1.09

$$\ln(\sin(x)) - \ln(1 + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)/(sin(x)+sin(x)^2),x)`

[Out] $\ln(\sin(x)) - \ln(1 + \sin(x))$

maxima [A] time = 0.31, size = 11, normalized size = 1.00

$$-\log(\sin(x) + 1) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="maxima")`

[Out] $-\log(\sin(x) + 1) + \log(\sin(x))$

mupad [B] time = 2.98, size = 9, normalized size = 0.82

$$-2 \operatorname{atanh}(2 \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)/(sin(x) + sin(x)^2),x)`

[Out] $-2*\operatorname{atanh}(2*\sin(x) + 1)$

sympy [A] time = 0.18, size = 10, normalized size = 0.91

$$-\log(\sin(x) + 1) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(sin(x)+sin(x)**2),x)`

[Out] $-\log(\sin(x) + 1) + \log(\sin(x))$

$$3.809 \quad \int \frac{\cos(x)}{\sin(x) + \sin^{\sqrt{2}}(x)} dx$$

Optimal. Leaf size=26

$$\log(\sin(x)) - (1 + \sqrt{2}) \log(\sin^{\sqrt{2}-1}(x) + 1)$$

[Out] ln(sin(x))-ln(1+sin(x)^(2^(1/2)-1))*(1+2^(1/2))

Rubi [A] time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {4334, 266, 36, 29, 31}

$$\log(\sin(x)) - (1 + \sqrt{2}) \log(\sin^{\sqrt{2}-1}(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/(Sin[x] + Sin[x]^Sqrt[2]),x]

[Out] Log[Sin[x]] - (1 + Sqrt[2])*Log[1 + Sin[x]^(-1 + Sqrt[2])]

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4334

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)}{\sin(x) + \sin^{\sqrt{2}}(x)} dx &= \text{Subst} \left(\int \frac{1}{x(1+x^{-1+\sqrt{2}})} dx, x, \sin(x) \right) \\
&= (1 + \sqrt{2}) \text{Subst} \left(\int \frac{1}{x(1+x)} dx, x, \sin^{-1+\sqrt{2}}(x) \right) \\
&= (-1 - \sqrt{2}) \text{Subst} \left(\int \frac{1}{1+x} dx, x, \sin^{-1+\sqrt{2}}(x) \right) + (1 + \sqrt{2}) \text{Subst} \left(\int \frac{1}{x} dx, x, \sin^{-1+\sqrt{2}}(x) \right) \\
&= \log(\sin(x)) - (1 + \sqrt{2}) \log(1 + \sin^{-1+\sqrt{2}}(x))
\end{aligned}$$

Mathematica [A] time = 0.04, size = 26, normalized size = 1.00

$$\log(\sin(x)) - (1 + \sqrt{2}) \log(\sin^{\sqrt{2}-1}(x) + 1)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/(Sin[x] + Sin[x]^Sqrt[2]), x]

[Out] Log[Sin[x]] - (1 + Sqrt[2])*Log[1 + Sin[x]^(-1 + Sqrt[2])]

fricas [A] time = 1.39, size = 27, normalized size = 1.04

$$-(\sqrt{2} + 1) \log(\sin(x)^{\sqrt{2}} + \sin(x)) + (\sqrt{2} + 2) \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(sin(x)+sin(x)^(2^(1/2))), x, algorithm="fricas")

[Out] -(sqrt(2) + 1)*log(sin(x)^sqrt(2) + sin(x)) + (sqrt(2) + 2)*log(sin(x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sin(x)^{\sqrt{2}} + \sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(sin(x)+sin(x)^(2^(1/2))), x, algorithm="giac")

[Out] integrate(cos(x)/(sin(x)^sqrt(2) + sin(x)), x)

maple [C] time = 0.75, size = 1856, normalized size = 71.38

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(sin(x)+sin(x)^(2^(1/2))), x)

[Out] $-1/2*I*2^{(1/2)}*csgn(I*(\exp(I*x)-1)*(1+\exp(I*x)))*csgn(I*(\exp(I*x)-1))*csgn(I*(1+\exp(I*x)))*\text{Pi}+1/2*I*2^{(1/2)}*csgn(I*(\exp(I*x)-1)*(1+\exp(I*x)))*csgn(I*(1+\exp(I*x))*(-1+\exp(-I*x)))*csgn(I*\exp(-I*x))*\text{Pi}-2*\ln(2)-I*\text{Pi}+1/2*I*2^{(1/2)}*csgn(I*(\exp(I*x)-1)*(1+\exp(I*x)))^2*csgn(I*(\exp(I*x)-1))*\text{Pi}+1/2*I*2^{(1/2)}*csgn(I*(\exp(I*x)-1)*(1+\exp(I*x)))^2*csgn(I*(1+\exp(I*x)))*\text{Pi}+1/2*I*2^{(1/2)}*csgn(I*(\exp(I*x)-1)*(1+\exp(I*x)))*csgn(I*(1+\exp(I*x))*(-1+\exp(-I*x)))^2*\text{Pi}+1/2*I*2^{(1/2)}*csgn(I*(1+\exp(I*x))*(-1+\exp(-I*x)))^2*csgn(I*\exp(-I*x))*\text{Pi}-1/2*I*2^{(1/2)}*csgn(I*(1+\exp(I*x))*(-1+\exp(-I*x)))*csgn((1+\exp(I*x))*(-1+\exp(-I$

```

*x)))^2*Pi-1/2*I*2^(1/2)*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))*csgn((1+exp(I*
x))*(-1+exp(-I*x)))*Pi+I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))*csgn(I*(1+exp(I*
x))*(-1+exp(-I*x)))*csgn(I*exp(-I*x))*Pi-I*csgn(I*(exp(I*x)-1)*(1+exp(I*x))
)*csgn(I*(exp(I*x)-1))*csgn(I*(1+exp(I*x)))*Pi-1/2*I*2^(1/2)*Pi-I*csgn(I*(1
+exp(I*x))*(-1+exp(-I*x)))*csgn((1+exp(I*x))*(-1+exp(-I*x)))^2*Pi-I*csgn(I*
(1+exp(I*x))*(-1+exp(-I*x)))*csgn((1+exp(I*x))*(-1+exp(-I*x)))*Pi-I*csgn(I*
(exp(I*x)-1)*(1+exp(I*x)))^3*Pi+I*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))^3*Pi+
I*csgn((1+exp(I*x))*(-1+exp(-I*x)))^3*Pi+I*csgn((1+exp(I*x))*(-1+exp(-I*x))
)^2*Pi-ln(exp(-1/2*2^(1/2)*(I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))*csgn(I*(exp
(I*x)-1))*csgn(I*(1+exp(I*x)))*Pi+I*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))*csg
n((1+exp(I*x))*(-1+exp(-I*x)))^2*Pi+I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))^3*P
i-I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))*c
sgn(I*exp(-I*x))*Pi-I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))^2*csgn(I*(1+exp(I*x
)))*Pi+I*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))*csgn((1+exp(I*x))*(-1+exp(-I*x
)))^2*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))^2*csgn(I*(exp(I*x)-1))*Pi-I*csgn
(I*(1+exp(I*x))*(-1+exp(-I*x)))^2*csgn(I*exp(-I*x))*Pi+I*Pi-I*csgn((1+exp(I
*x))*(-1+exp(-I*x)))^2*Pi-I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))*csgn(I*(1+exp
(I*x))*(-1+exp(-I*x)))^2*Pi-I*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))^3*Pi-I*cs
gn((1+exp(I*x))*(-1+exp(-I*x)))^3*Pi+2*ln(exp(I*x))-2*ln(exp(I*x)-1)-2*ln(1
+exp(I*x))+2*ln(2))+sin(x)+2*ln(1+exp(I*x))-ln(exp(-1/2*2^(1/2)*(I*csgn(I
*(exp(I*x)-1)*(1+exp(I*x)))*csgn(I*(exp(I*x)-1))*csgn(I*(1+exp(I*x)))*Pi+I*
csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))*csgn((1+exp(I*x))*(-1+exp(-I*x)))^2*Pi+
I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))^3*Pi-I*csgn(I*(exp(I*x)-1)*(1+exp(I*x))
)*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))*csgn(I*exp(-I*x))*Pi-I*csgn(I*(exp(I*
x)-1)*(1+exp(I*x)))^2*csgn(I*(1+exp(I*x)))*Pi+I*csgn(I*(1+exp(I*x))*(-1+exp
(-I*x)))*csgn((1+exp(I*x))*(-1+exp(-I*x)))*Pi-I*csgn(I*(exp(I*x)-1)*(1+exp(
I*x)))^2*csgn(I*(exp(I*x)-1))*Pi-I*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))^2*cs
gn(I*exp(-I*x))*Pi+I*Pi-I*csgn((1+exp(I*x))*(-1+exp(-I*x)))^2*Pi-I*csgn(I*(
exp(I*x)-1)*(1+exp(I*x)))*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))^2*Pi-I*csgn(I
*(1+exp(I*x))*(-1+exp(-I*x)))^3*Pi-I*csgn((1+exp(I*x))*(-1+exp(-I*x)))^3*Pi
+2*ln(exp(I*x))-2*ln(exp(I*x)-1)-2*ln(1+exp(I*x))+2*ln(2))+sin(x))*2^(1/2)
-1/2*I*2^(1/2)*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))^3*Pi+1/2*I*2^(1/2)*csgn(I*
(1+exp(I*x))*(-1+exp(-I*x)))^3*Pi+1/2*I*2^(1/2)*csgn((1+exp(I*x))*(-1+exp(-
I*x)))^3*Pi+1/2*I*2^(1/2)*csgn((1+exp(I*x))*(-1+exp(-I*x)))^2*Pi-2*ln(exp(I
*x))+I*csgn(I*(exp(I*x)-1)*(1+exp(I*x)))^2*csgn(I*(exp(I*x)-1))*Pi+I*csgn(I
*(exp(I*x)-1)*(1+exp(I*x)))^2*csgn(I*(1+exp(I*x)))*Pi+I*csgn(I*(exp(I*x)-1)
*(1+exp(I*x)))*csgn(I*(1+exp(I*x))*(-1+exp(-I*x)))^2*Pi+I*csgn(I*(1+exp(I*x
)))*(-1+exp(-I*x)))^2*csgn(I*exp(-I*x))*Pi+2*ln(exp(I*x)-1)-2^(1/2)*ln(exp(I
*x))+2^(1/2)*ln(exp(I*x)-1)+2^(1/2)*ln(1+exp(I*x))-2^(1/2)*ln(2)

```

maxima [A] time = 0.42, size = 34, normalized size = 1.31

$$\frac{\sqrt{2} \log(\sin(x))}{\sqrt{2} - 1} - \frac{\log(\sin(x)^{\sqrt{2}} + \sin(x))}{\sqrt{2} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x, algorithm="maxima")
```

```
[Out] sqrt(2)*log(sin(x))/(sqrt(2) - 1) - log(sin(x)^sqrt(2) + sin(x))/(sqrt(2) - 1)
```

mapad [B] time = 3.08, size = 29, normalized size = 1.12

$$\ln(\sin(x)) \left(\sqrt{2} + 2 \right) - \frac{\ln(\sin(x) + \sin(x)^{\sqrt{2}})}{\sqrt{2} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x) + sin(x)^(2^(1/2))),x)
```

[Out] $\log(\sin(x)) \cdot (2^{1/2} + 2) - \log(\sin(x) + \sin(x)^{2^{1/2}}) / (2^{1/2} - 1)$

sympy [B] time = 1.07, size = 82, normalized size = 3.15

$$\frac{\sqrt{2} \log(\sin(x) + \sin^{\sqrt{2}}(x))}{-3 + 2\sqrt{2}} - \frac{\log(\sin(x) + \sin^{\sqrt{2}}(x))}{-3 + 2\sqrt{2}} + \frac{\sqrt{2} \log(\sin(x))}{-3 + 2\sqrt{2}} - \frac{2 \log(\sin(x))}{-3 + 2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)/(sin(x)+sin(x)**(2**(1/2))),x)`

[Out] $\sqrt{2} \log(\sin(x) + \sin(x)^{\sqrt{2}}) / (-3 + 2\sqrt{2}) - \log(\sin(x) + \sin(x)^{\sqrt{2}}) / (-3 + 2\sqrt{2}) + \sqrt{2} \log(\sin(x)) / (-3 + 2\sqrt{2}) - 2 \log(\sin(x)) / (-3 + 2\sqrt{2})$

$$3.810 \quad \int \frac{1}{2 \sin(x) + \sin(2x)} dx$$

Optimal. Leaf size=24

$$\frac{1}{8} \tan^2\left(\frac{x}{2}\right) + \frac{1}{4} \log\left(\tan\left(\frac{x}{2}\right)\right)$$

[Out] 1/4*ln(tan(1/2*x))+1/8*tan(1/2*x)^2

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 14}

$$\frac{1}{8} \tan^2\left(\frac{x}{2}\right) + \frac{1}{4} \log\left(\tan\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Int[(2*Sin[x] + Sin[2*x])^(-1),x]

[Out] Log[Tan[x/2]]/4 + Tan[x/2]^2/8

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{1}{2 \sin(x) + \sin(2x)} dx &= 2 \text{Subst} \left(\int \frac{1+x^2}{8x} dx, x, \tan\left(\frac{x}{2}\right) \right) \\ &= \frac{1}{4} \text{Subst} \left(\int \frac{1+x^2}{x} dx, x, \tan\left(\frac{x}{2}\right) \right) \\ &= \frac{1}{4} \text{Subst} \left(\int \left(\frac{1}{x} + x\right) dx, x, \tan\left(\frac{x}{2}\right) \right) \\ &= \frac{1}{4} \log\left(\tan\left(\frac{x}{2}\right)\right) + \frac{1}{8} \tan^2\left(\frac{x}{2}\right) \end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.62

$$\frac{1 - 2 \cos^2\left(\frac{x}{2}\right) \left(\log\left(\cos\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{4(\cos(x) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(2*Sin[x] + Sin[2*x])^(-1),x]

[Out] (1 - 2*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(4*(1 + Cos[x]))

fricas [B] time = 0.60, size = 35, normalized size = 1.46

$$\frac{(\cos(x) + 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) - (\cos(x) + 1) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) - 2}{8(\cos(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="fricas")

[Out] $-1/8*((\cos(x) + 1)*\log(1/2*\cos(x) + 1/2) - (\cos(x) + 1)*\log(-1/2*\cos(x) + 1/2) - 2)/(\cos(x) + 1)$

giac [A] time = 0.14, size = 28, normalized size = 1.17

$$-\frac{\cos(x) - 1}{8(\cos(x) + 1)} + \frac{1}{8} \log\left(-\frac{\cos(x) - 1}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="giac")

[Out] $-1/8*(\cos(x) - 1)/(\cos(x) + 1) + 1/8*\log(-(\cos(x) - 1)/(\cos(x) + 1))$

maple [A] time = 0.24, size = 24, normalized size = 1.00

$$\frac{\ln(-1 + \cos(x))}{8} + \frac{1}{4 + 4\cos(x)} - \frac{\ln(1 + \cos(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*sin(x)+sin(2*x)),x)

[Out] $1/8*\ln(-1+\cos(x))+1/4/(1+\cos(x))-1/8*\ln(1+\cos(x))$

maxima [B] time = 0.33, size = 220, normalized size = 9.17

$\frac{4 \cos(2x) \cos(x) + 8 \cos(x)^2 - (2(2 \cos(x) + 1) \cos(2x) + \cos(2x)^2 + 4 \cos(x)^2 + \sin(2x)^2 + 4 \sin(2x) \sin(x))}{8}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="maxima")

[Out] $1/8*(4*\cos(2*x)*\cos(x) + 8*\cos(x)^2 - (2*(2*\cos(x) + 1)*\cos(2*x) + \cos(2*x)^2 + 4*\cos(x)^2 + \sin(2*x)^2 + 4*\sin(2*x)*\sin(x) + 4*\sin(x)^2 + 4*\cos(x) + 1)*\log(\cos(x)^2 + \sin(x)^2 + 2*\cos(x) + 1) + (2*(2*\cos(x) + 1)*\cos(2*x) + \cos(2*x)^2 + 4*\cos(x)^2 + \sin(2*x)^2 + 4*\sin(2*x)*\sin(x) + 4*\sin(x)^2 + 4*\cos(x) + 1)*\log(\cos(x)^2 + \sin(x)^2 - 2*\cos(x) + 1) + 4*\sin(2*x)*\sin(x) + 8*\sin(x)^2 + 4*\cos(x))/((2*(2*\cos(x) + 1)*\cos(2*x) + \cos(2*x)^2 + 4*\cos(x)^2 + \sin(2*x)^2 + 4*\sin(2*x)*\sin(x) + 4*\sin(x)^2 + 4*\cos(x) + 1))$

mupad [B] time = 3.07, size = 16, normalized size = 0.67

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4} + \frac{\tan\left(\frac{x}{2}\right)^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(2*x) + 2*sin(x)),x)

[Out] $\log(\tan(x/2))/4 + \tan(x/2)^2/8$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2 \sin(x) + \sin(2x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*sin(x)+sin(2*x)),x)

[Out] Integral(1/(2*sin(x) + sin(2*x)), x)

3.811 $\int (-3 + 4x + x^2) \sin(2x) dx$

Optimal. Leaf size=40

$$-\frac{1}{2}x^2 \cos(2x) + \frac{1}{2}x \sin(2x) + \sin(2x) - 2x \cos(2x) + \frac{7}{4} \cos(2x)$$

[Out] $7/4*\cos(2*x)-2*x*\cos(2*x)-1/2*x^2*\cos(2*x)+\sin(2*x)+1/2*x*\sin(2*x)$

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6742, 2638, 3296, 2637}

$$-\frac{1}{2}x^2 \cos(2x) + \frac{1}{2}x \sin(2x) + \sin(2x) - 2x \cos(2x) + \frac{7}{4} \cos(2x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-3 + 4*x + x^2)*\text{Sin}[2*x], x]$

[Out] $(7*\text{Cos}[2*x])/4 - 2*x*\text{Cos}[2*x] - (x^2*\text{Cos}[2*x])/2 + \text{Sin}[2*x] + (x*\text{Sin}[2*x])/2$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$
 $\text{FreeQ}[\{c, d\}, x]$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ $\text{FreeQ}[\{c, d\}, x]$

Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \text{ :> } -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ $\text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

Rule 6742

$\text{Int}[u_, x_Symbol] \text{ :> } \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /;$ $\text{SumQ}[v]$
 $]$

Rubi steps

$$\begin{aligned} \int (-3 + 4x + x^2) \sin(2x) dx &= \int (-3 \sin(2x) + 4x \sin(2x) + x^2 \sin(2x)) dx \\ &= -3 \int \sin(2x) dx + 4 \int x \sin(2x) dx + \int x^2 \sin(2x) dx \\ &= \frac{3}{2} \cos(2x) - 2x \cos(2x) - \frac{1}{2}x^2 \cos(2x) + 2 \int \cos(2x) dx + \int x \cos(2x) dx \\ &= \frac{3}{2} \cos(2x) - 2x \cos(2x) - \frac{1}{2}x^2 \cos(2x) + \sin(2x) + \frac{1}{2}x \sin(2x) - \frac{1}{2} \int \sin(2x) dx \\ &= \frac{7}{4} \cos(2x) - 2x \cos(2x) - \frac{1}{2}x^2 \cos(2x) + \sin(2x) + \frac{1}{2}x \sin(2x) \end{aligned}$$

Mathematica [A] time = 0.04, size = 29, normalized size = 0.72

$$\frac{1}{4} \left((-2x^2 - 8x + 7) \cos(2x) + 2(x + 2) \sin(2x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + 4*x + x^2)*Sin[2*x], x]

[Out] ((7 - 8*x - 2*x^2)*Cos[2*x] + 2*(2 + x)*Sin[2*x])/4

fricas [A] time = 0.55, size = 26, normalized size = 0.65

$$-\frac{1}{4}(2x^2 + 8x - 7)\cos(2x) + \frac{1}{2}(x + 2)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4*x-3)*sin(2*x), x, algorithm="fricas")

[Out] -1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)

giac [A] time = 0.13, size = 26, normalized size = 0.65

$$-\frac{1}{4}(2x^2 + 8x - 7)\cos(2x) + \frac{1}{2}(x + 2)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4*x-3)*sin(2*x), x, algorithm="giac")

[Out] -1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)

maple [A] time = 0.04, size = 35, normalized size = 0.88

$$\frac{7\cos(2x)}{4} - 2x\cos(2x) - \frac{x^2\cos(2x)}{2} + \sin(2x) + \frac{x\sin(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+4*x-3)*sin(2*x), x)

[Out] 7/4*cos(2*x)-2*x*cos(2*x)-1/2*x^2*cos(2*x)+sin(2*x)+1/2*x*sin(2*x)

maxima [A] time = 0.33, size = 38, normalized size = 0.95

$$-\frac{1}{4}(2x^2 - 1)\cos(2x) - 2x\cos(2x) + \frac{1}{2}x\sin(2x) + \frac{3}{2}\cos(2x) + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4*x-3)*sin(2*x), x, algorithm="maxima")

[Out] -1/4*(2*x^2 - 1)*cos(2*x) - 2*x*cos(2*x) + 1/2*x*sin(2*x) + 3/2*cos(2*x) + sin(2*x)

mupad [B] time = 2.90, size = 34, normalized size = 0.85

$$\frac{7\cos(2x)}{4} + \sin(2x) - 2x\cos(2x) + \frac{x\sin(2x)}{2} - \frac{x^2\cos(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)*(4*x + x^2 - 3), x)

[Out] (7*cos(2*x))/4 + sin(2*x) - 2*x*cos(2*x) + (x*sin(2*x))/2 - (x^2*cos(2*x))/2

sympy [A] time = 0.29, size = 39, normalized size = 0.98

$$-\frac{x^2\cos(2x)}{2} + \frac{x\sin(2x)}{2} - 2x\cos(2x) + \sin(2x) + \frac{7\cos(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4*x-3)*sin(2*x),x)
```

```
[Out] -x**2*cos(2*x)/2 + x*sin(2*x)/2 - 2*x*cos(2*x) + sin(2*x) + 7*cos(2*x)/4
```

3.812 $\int e^{-3x} \cos(4x) dx$

Optimal. Leaf size=27

$$\frac{4}{25}e^{-3x} \sin(4x) - \frac{3}{25}e^{-3x} \cos(4x)$$

[Out] $-3/25*\cos(4*x)/\exp(3*x)+4/25*\sin(4*x)/\exp(3*x)$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4433}

$$\frac{4}{25}e^{-3x} \sin(4x) - \frac{3}{25}e^{-3x} \cos(4x)$$

Antiderivative was successfully verified.

[In] Int[Cos[4*x]/E^(3*x), x]

[Out] $(-3*\cos[4*x])/(25*E^(3*x)) + (4*\sin[4*x])/(25*E^(3*x))$

Rule 4433

Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :=
Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2), x]
+ Simp[(e*F^(c*(a + b*x))*Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2), x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]

Rubi steps

$$\int e^{-3x} \cos(4x) dx = -\frac{3}{25}e^{-3x} \cos(4x) + \frac{4}{25}e^{-3x} \sin(4x)$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.81

$$\frac{1}{25}e^{-3x}(4 \sin(4x) - 3 \cos(4x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[4*x]/E^(3*x), x]

[Out] $(-3*\cos[4*x] + 4*\sin[4*x])/(25*E^(3*x))$

fricas [A] time = 0.60, size = 21, normalized size = 0.78

$$-\frac{3}{25} \cos(4x) e^{(-3x)} + \frac{4}{25} e^{(-3x)} \sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(4*x)/exp(3*x), x, algorithm="fricas")

[Out] $-3/25*\cos(4*x)*e^(-3*x) + 4/25*e^(-3*x)*\sin(4*x)$

giac [A] time = 0.14, size = 19, normalized size = 0.70

$$-\frac{1}{25} (3 \cos(4x) - 4 \sin(4x))e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(4*x)/exp(3*x),x, algorithm="giac")

[Out] -1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)

maple [A] time = 0.03, size = 22, normalized size = 0.81

$$-\frac{3e^{-3x}\cos(4x)}{25} + \frac{4e^{-3x}\sin(4x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(4*x)/exp(3*x),x)

[Out] -3/25*exp(-3*x)*cos(4*x)+4/25*exp(-3*x)*sin(4*x)

maxima [A] time = 0.33, size = 19, normalized size = 0.70

$$-\frac{1}{25}(3\cos(4x) - 4\sin(4x))e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(4*x)/exp(3*x),x, algorithm="maxima")

[Out] -1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)

mupad [B] time = 0.03, size = 19, normalized size = 0.70

$$-\frac{e^{-3x}(3\cos(4x) - 4\sin(4x))}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(4*x)*exp(-3*x),x)

[Out] -(exp(-3*x)*(3*cos(4*x) - 4*sin(4*x)))/25

sympy [A] time = 0.40, size = 26, normalized size = 0.96

$$\frac{4e^{-3x}\sin(4x)}{25} - \frac{3e^{-3x}\cos(4x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(4*x)/exp(3*x),x)

[Out] 4*exp(-3*x)*sin(4*x)/25 - 3*exp(-3*x)*cos(4*x)/25

$$3.813 \quad \int \frac{\cos(x) \sin(x)}{\sqrt{1+\sin(x)}} dx$$

Optimal. Leaf size=23

$$\frac{2}{3}(\sin(x) + 1)^{3/2} - 2\sqrt{\sin(x) + 1}$$

[Out] $2/3*(1+\sin(x))^{(3/2)}-2*(1+\sin(x))^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2833, 43}

$$\frac{2}{3}(\sin(x) + 1)^{3/2} - 2\sqrt{\sin(x) + 1}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]

[Out] $-2*\text{Sqrt}[1 + \text{Sin}[x]] + (2*(1 + \text{Sin}[x])^{(3/2)})/3$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2833

Int[cos[(e_.) + (f_.)*(x_)]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[1/(b*f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Ssin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos(x) \sin(x)}{\sqrt{1+\sin(x)}} dx &= \text{Subst} \left(\int \frac{x}{\sqrt{1+x}} dx, x, \sin(x) \right) \\ &= \text{Subst} \left(\int \left(-\frac{1}{\sqrt{1+x}} + \sqrt{1+x} \right) dx, x, \sin(x) \right) \\ &= -2\sqrt{1+\sin(x)} + \frac{2}{3}(1+\sin(x))^{3/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.35

$$\frac{2(\sin(x) - 2) \left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right) \right)^2}{3\sqrt{\sin(x) + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]

[Out] $(2*(\text{Cos}[x/2] + \text{Sin}[x/2])^2*(-2 + \text{Sin}[x]))/(3*\text{Sqrt}[1 + \text{Sin}[x]])$

fricas [A] time = 0.71, size = 12, normalized size = 0.52

$$\frac{2}{3}\sqrt{\sin(x) + 1}(\sin(x) - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="fricas")

[Out] 2/3*sqrt(sin(x) + 1)*(sin(x) - 2)

giac [A] time = 0.13, size = 17, normalized size = 0.74

$$\frac{2}{3} (\sin(x) + 1)^{\frac{3}{2}} - 2 \sqrt{\sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="giac")

[Out] 2/3*(sin(x) + 1)^(3/2) - 2*sqrt(sin(x) + 1)

maple [A] time = 0.03, size = 18, normalized size = 0.78

$$\frac{2(1 + \sin(x))^{\frac{3}{2}}}{3} - 2\sqrt{1 + \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(x)/(1+sin(x))^(1/2),x)

[Out] 2/3*(1+sin(x))^(3/2)-2*(1+sin(x))^(1/2)

maxima [A] time = 0.32, size = 17, normalized size = 0.74

$$\frac{2}{3} (\sin(x) + 1)^{\frac{3}{2}} - 2 \sqrt{\sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="maxima")

[Out] 2/3*(sin(x) + 1)^(3/2) - 2*sqrt(sin(x) + 1)

mupad [B] time = 0.10, size = 12, normalized size = 0.52

$$\frac{2 \sqrt{\sin(x) + 1} (\sin(x) - 2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x)*sin(x))/(sin(x) + 1)^(1/2),x)

[Out] (2*(sin(x) + 1)^(1/2)*(sin(x) - 2))/3

sympy [A] time = 0.29, size = 26, normalized size = 1.13

$$\frac{2\sqrt{\sin(x) + 1} \sin(x)}{3} - \frac{4\sqrt{\sin(x) + 1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)/(1+sin(x))**(1/2),x)

[Out] 2*sqrt(sin(x) + 1)*sin(x)/3 - 4*sqrt(sin(x) + 1)/3

3.814 $\int (x + 60 \cos^5(x) \sin^4(x)) dx$

Optimal. Leaf size=30

$$\frac{x^2}{2} + \frac{20 \sin^9(x)}{3} - \frac{120 \sin^7(x)}{7} + 12 \sin^5(x)$$

[Out] 1/2*x^2+12*sin(x)^5-120/7*sin(x)^7+20/3*sin(x)^9

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2564, 270}

$$\frac{x^2}{2} + \frac{20 \sin^9(x)}{3} - \frac{120 \sin^7(x)}{7} + 12 \sin^5(x)$$

Antiderivative was successfully verified.

[In] Int[x + 60*Cos[x]^5*Sin[x]^4,x]

[Out] x^2/2 + 12*Sin[x]^5 - (120*Sin[x]^7)/7 + (20*Sin[x]^9)/3

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2564

Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^(n-1)/2, x], x, a*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !IntegerQ[(m-1)/2] && LtQ[0, m, n]

Rubi steps

$$\begin{aligned} \int (x + 60 \cos^5(x) \sin^4(x)) dx &= \frac{x^2}{2} + 60 \int \cos^5(x) \sin^4(x) dx \\ &= \frac{x^2}{2} + 60 \text{Subst} \left(\int x^4 (1 - x^2)^2 dx, x, \sin(x) \right) \\ &= \frac{x^2}{2} + 60 \text{Subst} \left(\int (x^4 - 2x^6 + x^8) dx, x, \sin(x) \right) \\ &= \frac{x^2}{2} + 12 \sin^5(x) - \frac{120 \sin^7(x)}{7} + \frac{20 \sin^9(x)}{3} \end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.53

$$\frac{x^2}{2} + \frac{45 \sin(x)}{32} - \frac{5}{16} \sin(3x) - \frac{3}{16} \sin(5x) + \frac{15}{448} \sin(7x) + \frac{5}{192} \sin(9x)$$

Antiderivative was successfully verified.

[In] Integrate[x + 60*Cos[x]^5*Sin[x]^4,x]

[Out] x^2/2 + (45*Sin[x])/32 - (5*Sin[3*x])/16 - (3*Sin[5*x])/16 + (15*Sin[7*x])/448 + (5*Sin[9*x])/192

fricas [A] time = 0.59, size = 36, normalized size = 1.20

$$\frac{1}{2}x^2 + \frac{4}{21} \left(35 \cos(x)^8 - 50 \cos(x)^6 + 3 \cos(x)^4 + 4 \cos(x)^2 + 8 \right) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x+60*cos(x)^5*sin(x)^4,x, algorithm="fricas")

[Out] 1/2*x^2 + 4/21*(35*cos(x)^8 - 50*cos(x)^6 + 3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)

giac [A] time = 0.14, size = 24, normalized size = 0.80

$$\frac{20}{3} \sin(x)^9 - \frac{120}{7} \sin(x)^7 + 12 \sin(x)^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x+60*cos(x)^5*sin(x)^4,x, algorithm="giac")

[Out] 20/3*sin(x)^9 - 120/7*sin(x)^7 + 12*sin(x)^5 + 1/2*x^2

maple [A] time = 0.01, size = 41, normalized size = 1.37

$$\frac{x^2}{2} - \frac{20 \left(\cos^6(x) \right) \left(\sin^3(x) \right)}{3} - \frac{20 \sin(x) \left(\cos^6(x) \right)}{7} + \frac{4 \left(\frac{8}{3} + \cos^4(x) + \frac{4 \left(\cos^2(x) \right)}{3} \right) \sin(x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x+60*cos(x)^5*sin(x)^4,x)

[Out] 1/2*x^2-20/3*cos(x)^6*sin(x)^3-20/7*sin(x)*cos(x)^6+4/7*(8/3+cos(x)^4+4/3*cos(x)^2)*sin(x)

maxima [A] time = 0.33, size = 24, normalized size = 0.80

$$\frac{20}{3} \sin(x)^9 - \frac{120}{7} \sin(x)^7 + 12 \sin(x)^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x+60*cos(x)^5*sin(x)^4,x, algorithm="maxima")

[Out] 20/3*sin(x)^9 - 120/7*sin(x)^7 + 12*sin(x)^5 + 1/2*x^2

mupad [B] time = 3.00, size = 24, normalized size = 0.80

$$\frac{x^2}{2} + \frac{20 \sin(x)^9}{3} - \frac{120 \sin(x)^7}{7} + 12 \sin(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x + 60*cos(x)^5*sin(x)^4,x)

[Out] 12*sin(x)^5 - (120*sin(x)^7)/7 + (20*sin(x)^9)/3 + x^2/2

sympy [A] time = 0.06, size = 27, normalized size = 0.90

$$\frac{x^2}{2} + \frac{20 \sin^9(x)}{3} - \frac{120 \sin^7(x)}{7} + 12 \sin^5(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x+60*cos(x)**5*sin(x)**4,x)

[Out] x**2/2 + 20*sin(x)**9/3 - 120*sin(x)**7/7 + 12*sin(x)**5

3.815 $\int \cos(x)(\sec(x) + \tan(x)) dx$

Optimal. Leaf size=6

$$x - \cos(x)$$

[Out] x-cos(x)

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3161, 2638}

$$x - \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*(Sec[x] + Tan[x]),x]

[Out] x - Cos[x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3161

Int[cos[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symbol] := Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[n]

Rubi steps

$$\begin{aligned} \int \cos(x)(\sec(x) + \tan(x)) dx &= \int (1 + \sin(x)) dx \\ &= x + \int \sin(x) dx \\ &= x - \cos(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$x - \cos(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*(Sec[x] + Tan[x]),x]

[Out] x - Cos[x]

fricas [A] time = 0.53, size = 6, normalized size = 1.00

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(sec(x)+tan(x)),x, algorithm="fricas")

[Out] x - cos(x)

giac [B] time = 0.13, size = 14, normalized size = 2.33

$$x - \frac{2}{\tan\left(\frac{1}{2}x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(sec(x)+tan(x)),x, algorithm="giac")

[Out] $x - 2/(\tan(1/2*x)^2 + 1)$

maple [A] time = 0.08, size = 7, normalized size = 1.17

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*(sec(x)+tan(x)),x)

[Out] $x - \cos(x)$

maxima [A] time = 0.33, size = 6, normalized size = 1.00

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(sec(x)+tan(x)),x, algorithm="maxima")

[Out] $x - \cos(x)$

mupad [B] time = 2.95, size = 6, normalized size = 1.00

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*(tan(x) + 1/cos(x)),x)

[Out] $x - \cos(x)$

sympy [A] time = 1.13, size = 3, normalized size = 0.50

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(sec(x)+tan(x)),x)

[Out] $x - \cos(x)$

3.816 $\int \cos(x) (\sec^3(x) + \tan(x)) dx$

Optimal. Leaf size=7

$$\tan(x) - \cos(x)$$

[Out] $-\cos(x) + \tan(x)$

Rubi [A] time = 0.04, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4401, 3767, 8, 2638}

$$\tan(x) - \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*(Sec[x]^3 + Tan[x]),x]

[Out] -Cos[x] + Tan[x]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 4401

Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /; !InertTrigFreeQ[u]

Rubi steps

$$\begin{aligned} \int \cos(x) (\sec^3(x) + \tan(x)) dx &= \int (\sec^2(x) + \sin(x)) dx \\ &= \int \sec^2(x) dx + \int \sin(x) dx \\ &= -\cos(x) - \text{Subst}\left(\int 1 dx, x, -\tan(x)\right) \\ &= -\cos(x) + \tan(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$\tan(x) - \cos(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*(Sec[x]^3 + Tan[x]),x]

[Out] -Cos[x] + Tan[x]

fricas [B] time = 0.74, size = 15, normalized size = 2.14

$$-\frac{\cos(x)^2 - \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(sec(x)^3+tan(x)),x, algorithm="fricas")

[Out] -(cos(x)^2 - sin(x))/cos(x)

giac [B] time = 0.14, size = 30, normalized size = 4.29

$$-\frac{2\left(\tan\left(\frac{1}{2}x\right)^3 + \tan\left(\frac{1}{2}x\right)^2 + \tan\left(\frac{1}{2}x\right) - 1\right)}{\tan\left(\frac{1}{2}x\right)^4 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(sec(x)^3+tan(x)),x, algorithm="giac")

[Out] -2*(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) - 1)/(tan(1/2*x)^4 - 1)

maple [A] time = 0.10, size = 8, normalized size = 1.14

$$-\cos(x) + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*(sec(x)^3+tan(x)),x)

[Out] -cos(x)+tan(x)

maxima [A] time = 0.33, size = 7, normalized size = 1.00

$$-\cos(x) + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(sec(x)^3+tan(x)),x, algorithm="maxima")

[Out] -cos(x) + tan(x)

mupad [B] time = 2.97, size = 12, normalized size = 1.71

$$\frac{\sin(x)}{\cos(x)} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*(tan(x) + 1/cos(x)^3),x)

[Out] sin(x)/cos(x) - cos(x)

sympy [A] time = 4.81, size = 8, normalized size = 1.14

$$\frac{\sin(x)}{\cos(x)} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(sec(x)**3+tan(x)),x)

[Out] sin(x)/cos(x) - cos(x)

$$3.817 \quad \int \frac{1}{2} \left(-\cot(x) \csc(x) + \csc^2(x) \right) dx$$

Optimal. Leaf size=13

$$\frac{\csc(x)}{2} - \frac{\cot(x)}{2}$$

[Out] -1/2*cot(x)+1/2*csc(x)

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {12, 2606, 8, 3767}

$$\frac{\csc(x)}{2} - \frac{\cot(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(-(Cot[x]*Csc[x]) + Csc[x]^2)/2,x]

[Out] -Cot[x]/2 + Csc[x]/2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2606

Int[((a_)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m-1)*(-1+x^2)^((n-1)/2), x], x, Sec[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n+1])

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1+x^2)^(n/2-1), x], x], x, Cot[c+d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{2} \left(-\cot(x) \csc(x) + \csc^2(x) \right) dx &= \frac{1}{2} \int \left(-\cot(x) \csc(x) + \csc^2(x) \right) dx \\ &= -\left(\frac{1}{2} \int \cot(x) \csc(x) dx \right) + \frac{1}{2} \int \csc^2(x) dx \\ &= -\left(\frac{1}{2} \text{Subst} \left(\int 1 dx, x, \cot(x) \right) \right) + \frac{1}{2} \text{Subst} \left(\int 1 dx, x, \csc(x) \right) \\ &= -\frac{\cot(x)}{2} + \frac{\csc(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{1}{2} \tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(-(Cot[x]*Csc[x]) + Csc[x]^2)/2,x]

[Out] Tan[x/2]/2

fricas [A] time = 1.08, size = 10, normalized size = 0.77

$$\frac{\sin(x)}{2(\cos(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x, algorithm="fricas")

[Out] 1/2*sin(x)/(cos(x) + 1)

giac [A] time = 0.15, size = 13, normalized size = 1.00

$$\frac{1}{2 \sin(x)} - \frac{1}{2 \tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x, algorithm="giac")

[Out] 1/2/sin(x) - 1/2/tan(x)

maple [A] time = 0.04, size = 10, normalized size = 0.77

$$-\frac{\cot(x)}{2} + \frac{\csc(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x)

[Out] -1/2*cot(x)+1/2*csc(x)

maxima [A] time = 0.32, size = 13, normalized size = 1.00

$$\frac{1}{2 \sin(x)} - \frac{1}{2 \tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x, algorithm="maxima")

[Out] 1/2/sin(x) - 1/2/tan(x)

mupad [B] time = 2.93, size = 6, normalized size = 0.46

$$\frac{\tan\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*sin(x)^2) - cot(x)/(2*sin(x)),x)

[Out] tan(x/2)/2

sympy [A] time = 0.06, size = 14, normalized size = 1.08

$$-\frac{\cos(x)}{2 \sin(x)} + \frac{1}{2 \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)**2,x)
```

```
[Out] -cos(x)/(2*sin(x)) + 1/(2*sin(x))
```

$$3.818 \quad \int \left(-\csc^2(x) + \sin(2x) \right) dx$$

Optimal. Leaf size=11

$$\cot(x) - \frac{1}{2} \cos(2x)$$

[Out] -1/2*cos(2*x)+cot(x)

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3767, 8, 2638}

$$\cot(x) - \frac{1}{2} \cos(2x)$$

Antiderivative was successfully verified.

[In] Int[-Csc[x]^2 + Sin[2*x], x]

[Out] -Cos[2*x]/2 + Cot[x]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int \left(-\csc^2(x) + \sin(2x) \right) dx &= - \int \csc^2(x) dx + \int \sin(2x) dx \\ &= -\frac{1}{2} \cos(2x) + \text{Subst} \left(\int 1 dx, x, \cot(x) \right) \\ &= -\frac{1}{2} \cos(2x) + \cot(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\cot(x) - \frac{1}{2} \cos(2x)$$

Antiderivative was successfully verified.

[In] Integrate[-Csc[x]^2 + Sin[2*x], x]

[Out] -1/2*Cos[2*x] + Cot[x]

fricas [B] time = 0.59, size = 22, normalized size = 2.00

$$-\frac{(2 \cos(x)^2 - 1) \sin(x) - 2 \cos(x)}{2 \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-csc(x)^2+sin(2*x),x, algorithm="fricas")

[Out] -1/2*((2*cos(x)^2 - 1)*sin(x) - 2*cos(x))/sin(x)

giac [A] time = 0.13, size = 11, normalized size = 1.00

$$\frac{1}{\tan(x)} - \frac{1}{2} \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-csc(x)^2+sin(2*x),x, algorithm="giac")

[Out] 1/tan(x) - 1/2*cos(2*x)

maple [A] time = 0.03, size = 10, normalized size = 0.91

$$-\frac{\cos(2x)}{2} + \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-csc(x)^2+sin(2*x),x)

[Out] -1/2*cos(2*x)+cot(x)

maxima [A] time = 0.33, size = 11, normalized size = 1.00

$$\frac{1}{\tan(x)} - \frac{1}{2} \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-csc(x)^2+sin(2*x),x, algorithm="maxima")

[Out] 1/tan(x) - 1/2*cos(2*x)

mupad [B] time = 2.92, size = 14, normalized size = 1.27

$$\frac{\cos(x)}{\sin(x)} - \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x) - 1/sin(x)^2,x)

[Out] cos(x)/sin(x) - cos(x)^2

sympy [A] time = 0.06, size = 12, normalized size = 1.09

$$-\frac{\cos(2x)}{2} + \frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-csc(x)**2+sin(2*x),x)

[Out] -cos(2*x)/2 + cos(x)/sin(x)

3.819 $\int (2 \cot(2x) - 3 \sin(3x)) dx$

Optimal. Leaf size=10

$$\cos(3x) + \log(\sin(2x))$$

[Out] $\cos(3*x)+\ln(\sin(2*x))$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3475, 2638}

$$\cos(3x) + \log(\sin(2x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[2*\text{Cot}[2*x] - 3*\text{Sin}[3*x], x]$

[Out] $\text{Cos}[3*x] + \text{Log}[\text{Sin}[2*x]]$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } -\text{Simp}[\text{Cos}[c + d*x]/d, x] \text{ /; FreeQ}[\{c, d\}, x]$

Rule 3475

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> } -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] \text{ /; FreeQ}[\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int (2 \cot(2x) - 3 \sin(3x)) dx &= 2 \int \cot(2x) dx - 3 \int \sin(3x) dx \\ &= \cos(3x) + \log(\sin(2x)) \end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\cos(3x) + \log(\sin(2x))$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[2*\text{Cot}[2*x] - 3*\text{Sin}[3*x], x]$

[Out] $\text{Cos}[3*x] + \text{Log}[\text{Sin}[2*x]]$

fricas [A] time = 0.91, size = 18, normalized size = 1.80

$$4 \cos(x)^3 - 3 \cos(x) + \log\left(-\frac{1}{2} \cos(x) \sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(2*\cot(2*x)-3*\sin(3*x), x, \text{algorithm}=\text{"fricas"})$

[Out] $4*\cos(x)^3 - 3*\cos(x) + \log(-1/2*\cos(x)*\sin(x))$

giac [A] time = 0.15, size = 11, normalized size = 1.10

$$\cos(3x) + \log(|\sin(2x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*cot(2*x)-3*sin(3*x),x, algorithm="giac")

[Out] cos(3*x) + log(abs(sin(2*x)))

maple [A] time = 0.03, size = 17, normalized size = 1.70

$$-\frac{\ln(\cot^2(2x) + 1)}{2} + \cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2*cot(2*x)-3*sin(3*x),x)

[Out] -1/2*ln(cot(2*x)^2+1)+cos(3*x)

maxima [A] time = 0.32, size = 10, normalized size = 1.00

$$\cos(3x) + \log(\sin(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*cot(2*x)-3*sin(3*x),x, algorithm="maxima")

[Out] cos(3*x) + log(sin(2*x))

mupad [B] time = 3.08, size = 24, normalized size = 2.40

$$\cos(3x) + \ln\left(\cos\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right) - 2\sin\left(\frac{x}{2}\right)^3\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2*cot(2*x) - 3*sin(3*x),x)

[Out] cos(3*x) + log(cos(x/2)*(sin(x/2) - 2*sin(x/2)^3))

sympy [A] time = 0.06, size = 10, normalized size = 1.00

$$\log(\sin(2x)) + \cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*cot(2*x)-3*sin(3*x),x)

[Out] log(sin(2*x)) + cos(3*x)

3.820 $\int x \sin(2x^2) dx$

Optimal. Leaf size=10

$$-\frac{1}{4} \cos(2x^2)$$

[Out] -1/4*cos(2*x^2)

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3379, 2638}

$$-\frac{1}{4} \cos(2x^2)$$

Antiderivative was successfully verified.

[In] Int[x*Sin[2*x^2],x]

[Out] -Cos[2*x^2]/4

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x \sin(2x^2) dx &= \frac{1}{2} \text{Subst} \left(\int \sin(2x) dx, x, x^2 \right) \\ &= -\frac{1}{4} \cos(2x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$-\frac{1}{4} \cos(2x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sin[2*x^2],x]

[Out] -1/4*Cos[2*x^2]

fricas [A] time = 0.70, size = 8, normalized size = 0.80

$$-\frac{1}{4} \cos(2x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(2*x^2),x, algorithm="fricas")

[Out] $-1/4*\cos(2*x^2)$

giac [A] time = 0.14, size = 8, normalized size = 0.80

$$-\frac{1}{4} \cos(2x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(2*x^2),x, algorithm="giac")`

[Out] $-1/4*\cos(2*x^2)$

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-\frac{\cos(2x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*sin(2*x^2),x)`

[Out] $-1/4*\cos(2*x^2)$

maxima [A] time = 0.32, size = 8, normalized size = 0.80

$$-\frac{1}{4} \cos(2x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(2*x^2),x, algorithm="maxima")`

[Out] $-1/4*\cos(2*x^2)$

mupad [B] time = 0.05, size = 8, normalized size = 0.80

$$\frac{\sin(x^2)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*sin(2*x^2),x)`

[Out] $\sin(x^2)^2/2$

sympy [A] time = 0.15, size = 8, normalized size = 0.80

$$-\frac{\cos(2x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(2*x**2),x)`

[Out] $-\cos(2*x**2)/4$

$$3.821 \quad \int -\cos(1-x)\sin(1-x)\sqrt{1+\sin^2(1-x)} dx$$

Optimal. Leaf size=18

$$\frac{1}{3}(\sin^2(1-x)+1)^{3/2}$$

[Out] 1/3*(1+sin(x-1)^2)^(3/2)

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3198, 261}

$$\frac{1}{3}(\sin^2(1-x)+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[-(Cos[1-x]*Sin[1-x]*Sqrt[1+Sin[1-x]^2]),x]

[Out] (1+Sin[1-x]^2)^(3/2)/3

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 3198

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int -\cos(1-x)\sin(1-x)\sqrt{1+\sin^2(1-x)} dx &= \text{Subst}\left(\int x\sqrt{1+x^2} dx, x, \sin(1-x)\right) \\ &= \frac{1}{3}(1+\sin^2(1-x))^{3/2} \end{aligned}$$

Mathematica [A] time = 0.03, size = 18, normalized size = 1.00

$$\frac{1}{3}(\sin^2(1-x)+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[-(Cos[1-x]*Sin[1-x]*Sqrt[1+Sin[1-x]^2]),x]

[Out] (1+Sin[1-x]^2)^(3/2)/3

fricas [A] time = 0.63, size = 14, normalized size = 0.78

$$\frac{1}{3}(-\cos(x-1)^2+2)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x, algorithm="fricas")

[Out] 1/3*(-cos(x - 1)^2 + 2)^(3/2)

giac [A] time = 0.13, size = 12, normalized size = 0.67

$$\frac{1}{3} \left(\sin(x-1)^2 + 1 \right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x, algorithm="giac")

[Out] 1/3*(sin(x - 1)^2 + 1)^(3/2)

maple [A] time = 0.07, size = 13, normalized size = 0.72

$$\frac{\left(1 + \sin^2(-1 + x)\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x)

[Out] 1/3*(1+sin(-1+x)^2)^(3/2)

maxima [A] time = 0.33, size = 12, normalized size = 0.67

$$\frac{1}{3} \left(\sin(x-1)^2 + 1 \right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x, algorithm="maxima")

[Out] 1/3*(sin(x - 1)^2 + 1)^(3/2)

mupad [B] time = 2.99, size = 12, normalized size = 0.67

$$\frac{\left(\sin(x-1)^2 + 1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x - 1)*sin(x - 1)*(sin(x - 1)^2 + 1)^(1/2),x)

[Out] (sin(x - 1)^2 + 1)^(3/2)/3

sympy [B] time = 0.75, size = 32, normalized size = 1.78

$$\frac{\sqrt{\sin^2(x-1)+1} \sin^2(x-1)}{3} + \frac{\sqrt{\sin^2(x-1)+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)**2)**(1/2),x)

[Out] sqrt(sin(x - 1)**2 + 1)*sin(x - 1)**2/3 + sqrt(sin(x - 1)**2 + 1)/3

$$3.822 \quad \int \frac{\cos\left(\frac{1}{x}\right) \sin\left(\frac{1}{x}\right)}{x^2} dx$$

Optimal. Leaf size=10

$$-\frac{1}{2} \sin^2\left(\frac{1}{x}\right)$$

[Out] -1/2*sin(1/x)^2

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3441}

$$-\frac{1}{2} \sin^2\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

[In] Int[(Cos[x^(-1)]*Sin[x^(-1)])]/x^2,x]

[Out] -Sin[x^(-1)]^2/2

Rule 3441

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int \frac{\cos\left(\frac{1}{x}\right) \sin\left(\frac{1}{x}\right)}{x^2} dx = -\frac{1}{2} \sin^2\left(\frac{1}{x}\right)$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{2} \cos^2\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x^(-1)]*Sin[x^(-1)])]/x^2,x]

[Out] Cos[x^(-1)]^2/2

fricas [A] time = 0.60, size = 8, normalized size = 0.80

$$\frac{1}{2} \cos^2\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/x)*sin(1/x)/x^2,x, algorithm="fricas")

[Out] 1/2*cos(1/x)^2

giac [A] time = 0.13, size = 8, normalized size = 0.80

$$\frac{1}{2} \cos^2\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/x)*sin(1/x)/x^2,x, algorithm="giac")

[Out] 1/2*cos(1/x)^2

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\frac{\left(\cos^2\left(\frac{1}{x}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/x)*sin(1/x)/x^2,x)

[Out] 1/2*cos(1/x)^2

maxima [A] time = 0.33, size = 8, normalized size = 0.80

$$\frac{1}{2} \cos\left(\frac{1}{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/x)*sin(1/x)/x^2,x, algorithm="maxima")

[Out] 1/2*cos(1/x)^2

mupad [B] time = 2.92, size = 8, normalized size = 0.80

$$\frac{\cos\left(\frac{1}{x}\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(1/x)*sin(1/x))/x^2,x)

[Out] cos(1/x)^2/2

sympy [B] time = 1.28, size = 31, normalized size = 3.10

$$\frac{2 \tan^2\left(\frac{1}{2x}\right)}{\tan^4\left(\frac{1}{2x}\right) + 2 \tan^2\left(\frac{1}{2x}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/x)*sin(1/x)/x**2,x)

[Out] -2*tan(1/(2*x))**2/(tan(1/(2*x))**4 + 2*tan(1/(2*x))**2 + 1)

$$3.823 \quad \int \cos\left(\frac{1}{2}(1+3x)\right) \sin^3\left(\frac{1}{2}(1+3x)\right) dx$$

Optimal. Leaf size=16

$$\frac{1}{6} \sin^4\left(\frac{3x}{2} + \frac{1}{2}\right)$$

[Out] 1/6*sin(1/2+3/2*x)^4

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2564, 30}

$$\frac{1}{6} \sin^4\left(\frac{3x}{2} + \frac{1}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[(1 + 3*x)/2]*Sin[(1 + 3*x)/2]^3,x]

[Out] Sin[1/2 + (3*x)/2]^4/6

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2564

Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && LtQ[0, m, n])

Rubi steps

$$\begin{aligned} \int \cos\left(\frac{1}{2}(1+3x)\right) \sin^3\left(\frac{1}{2}(1+3x)\right) dx &= \frac{2}{3} \text{Subst}\left(\int x^3 dx, x, \sin\left(\frac{1}{2} + \frac{3x}{2}\right)\right) \\ &= \frac{1}{6} \sin^4\left(\frac{1}{2} + \frac{3x}{2}\right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.56

$$\frac{1}{2} \left(\frac{1}{24} \cos(6x + 2) - \frac{1}{6} \cos(3x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[(1 + 3*x)/2]*Sin[(1 + 3*x)/2]^3,x]

[Out] (-1/6*Cos[1 + 3*x] + Cos[2 + 6*x]/24)/2

fricas [B] time = 0.75, size = 21, normalized size = 1.31

$$\frac{1}{6} \cos\left(\frac{3}{2}x + \frac{1}{2}\right)^4 - \frac{1}{3} \cos\left(\frac{3}{2}x + \frac{1}{2}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x, algorithm="fricas")

[Out] 1/6*cos(3/2*x + 1/2)^4 - 1/3*cos(3/2*x + 1/2)^2

giac [A] time = 0.20, size = 10, normalized size = 0.62

$$\frac{1}{6} \sin\left(\frac{3}{2}x + \frac{1}{2}\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x, algorithm="giac")

[Out] 1/6*sin(3/2*x + 1/2)^4

maple [A] time = 0.04, size = 11, normalized size = 0.69

$$\frac{\left(\sin^4\left(\frac{1}{2} + \frac{3x}{2}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x)

[Out] 1/6*sin(1/2+3/2*x)^4

maxima [A] time = 0.33, size = 10, normalized size = 0.62

$$\frac{1}{6} \sin\left(\frac{3}{2}x + \frac{1}{2}\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x, algorithm="maxima")

[Out] 1/6*sin(3/2*x + 1/2)^4

mupad [B] time = 0.08, size = 14, normalized size = 0.88

$$\frac{\left(\frac{\cos(3x+1)}{2} - \frac{1}{2}\right)^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos((3*x)/2 + 1/2)*sin((3*x)/2 + 1/2)^3,x)

[Out] (cos(3*x + 1)/2 - 1/2)^2/6

sympy [A] time = 0.46, size = 12, normalized size = 0.75

$$\frac{\sin^4\left(\frac{3x}{2} + \frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)**3,x)

[Out] sin(3*x/2 + 1/2)**4/6

3.824 $\int 4x \tan(x^2) dx$

Optimal. Leaf size=7

$$-2 \log(\cos(x^2))$$

[Out] -2*ln(cos(x^2))

Rubi [A] time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {12, 3747, 3475}

$$-2 \log(\cos(x^2))$$

Antiderivative was successfully verified.

[In] Int[4*x*Tan[x^2], x]

[Out] -2*Log[Cos[x^2]]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3747

Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int 4x \tan(x^2) dx &= 4 \int x \tan(x^2) dx \\ &= 2 \text{Subst}\left(\int \tan(x) dx, x, x^2\right) \\ &= -2 \log(\cos(x^2)) \end{aligned}$$

Mathematica [A] time = 0.01, size = 7, normalized size = 1.00

$$-2 \log(\cos(x^2))$$

Antiderivative was successfully verified.

[In] Integrate[4*x*Tan[x^2], x]

[Out] -2*Log[Cos[x^2]]

fricas [A] time = 0.61, size = 13, normalized size = 1.86

$$-\log\left(\frac{1}{\tan(x^2)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*x*tan(x^2),x, algorithm="fricas")

[Out] -log(1/(tan(x^2)^2 + 1))

giac [A] time = 0.14, size = 9, normalized size = 1.29

$$\log\left(\tan\left(x^2\right)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*x*tan(x^2),x, algorithm="giac")

[Out] log(tan(x^2)^2 + 1)

maple [A] time = 0.00, size = 8, normalized size = 1.14

$$-2 \ln\left(\cos\left(x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(4*x*tan(x^2),x)

[Out] -2*ln(cos(x^2))

maxima [A] time = 0.33, size = 7, normalized size = 1.00

$$2 \log\left(\sec\left(x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*x*tan(x^2),x, algorithm="maxima")

[Out] 2*log(sec(x^2))

mupad [B] time = 0.07, size = 9, normalized size = 1.29

$$\ln\left(\tan\left(x^2\right)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(4*x*tan(x^2),x)

[Out] log(tan(x^2)^2 + 1)

sympy [A] time = 0.12, size = 8, normalized size = 1.14

$$\log\left(\tan^2\left(x^2\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*x*tan(x**2),x)

[Out] log(tan(x**2)**2 + 1)

3.825 $\int x \sec(5 - x^2) dx$

Optimal. Leaf size=13

$$-\frac{1}{2} \tanh^{-1}(\sin(5 - x^2))$$

[Out] 1/2*arctanh(sin(x^2-5))

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4204, 3770}

$$-\frac{1}{2} \tanh^{-1}(\sin(5 - x^2))$$

Antiderivative was successfully verified.

[In] Int[x*Sec[5 - x^2],x]

[Out] -ArcTanh[Sin[5 - x^2]]/2

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4204

Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int x \sec(5 - x^2) dx &= \frac{1}{2} \text{Subst} \left(\int \sec(5 - x) dx, x, x^2 \right) \\ &= -\frac{1}{2} \tanh^{-1}(\sin(5 - x^2)) \end{aligned}$$

Mathematica [A] time = 0.02, size = 13, normalized size = 1.00

$$-\frac{1}{2} \tanh^{-1}(\sin(5 - x^2))$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[5 - x^2],x]

[Out] -1/2*ArcTanh[Sin[5 - x^2]]

fricas [B] time = 0.69, size = 25, normalized size = 1.92

$$\frac{1}{4} \log(\sin(x^2 - 5) + 1) - \frac{1}{4} \log(-\sin(x^2 - 5) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x^2-5),x, algorithm="fricas")

[Out] $1/4*\log(\sin(x^2 - 5) + 1) - 1/4*\log(-\sin(x^2 - 5) + 1)$

giac [B] time = 0.16, size = 41, normalized size = 3.15

$$\frac{1}{8} \log \left(\left(\frac{1}{\sin(x^2 - 5)} + \sin(x^2 - 5) + 2 \right) \right) - \frac{1}{8} \log \left(\left(\frac{1}{\sin(x^2 - 5)} + \sin(x^2 - 5) - 2 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x^2-5),x, algorithm="giac")

[Out] $1/8*\log(\text{abs}(1/\sin(x^2 - 5) + \sin(x^2 - 5) + 2)) - 1/8*\log(\text{abs}(1/\sin(x^2 - 5) + \sin(x^2 - 5) - 2))$

maple [A] time = 0.00, size = 17, normalized size = 1.31

$$\frac{\ln(\sec(x^2 - 5) + \tan(x^2 - 5))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sec(x^2-5),x)

[Out] $1/2*\ln(\sec(x^2-5)+\tan(x^2-5))$

maxima [A] time = 0.32, size = 16, normalized size = 1.23

$$\frac{1}{2} \log(\sec(x^2 - 5) + \tan(x^2 - 5))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x^2-5),x, algorithm="maxima")

[Out] $1/2*\log(\sec(x^2 - 5) + \tan(x^2 - 5))$

mupad [B] time = 3.53, size = 15, normalized size = 1.15

$$-\text{atan}(e^{-5i} e^{x^2 1i}) 1i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/cos(x^2 - 5),x)

[Out] $-\text{atan}(\exp(-5i)*\exp(x^2*1i))*1i$

sympy [A] time = 1.03, size = 15, normalized size = 1.15

$$\frac{\log(\tan(x^2 - 5) + \sec(x^2 - 5))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x**2-5),x)

[Out] $\log(\tan(x**2 - 5) + \sec(x**2 - 5))/2$

$$3.826 \quad \int \frac{\csc\left(\frac{1}{x}\right)}{x^2} dx$$

Optimal. Leaf size=5

$$\tanh^{-1}\left(\cos\left(\frac{1}{x}\right)\right)$$

[Out] arctanh(cos(1/x))

Rubi [A] time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4205, 3770}

$$\tanh^{-1}\left(\cos\left(\frac{1}{x}\right)\right)$$

Antiderivative was successfully verified.

[In] Int[Csc[x^(-1)]/x^2, x]

[Out] ArcTanh[Cos[x^(-1)]]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4205

Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\csc\left(\frac{1}{x}\right)}{x^2} dx &= -\text{Subst}\left(\int \csc(x) dx, x, \frac{1}{x}\right) \\ &= \tanh^{-1}\left(\cos\left(\frac{1}{x}\right)\right) \end{aligned}$$

Mathematica [B] time = 0.01, size = 21, normalized size = 4.20

$$\log\left(\cos\left(\frac{1}{2x}\right)\right) - \log\left(\sin\left(\frac{1}{2x}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x^(-1)]/x^2, x]

[Out] Log[Cos[1/(2*x)]] - Log[Sin[1/(2*x)]]

fricas [B] time = 0.65, size = 23, normalized size = 4.60

$$\frac{1}{2} \log\left(\frac{1}{2} \cos\left(\frac{1}{x}\right) + \frac{1}{2}\right) - \frac{1}{2} \log\left(-\frac{1}{2} \cos\left(\frac{1}{x}\right) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(1/x)/x^2,x, algorithm="fricas")

[Out] 1/2*log(1/2*cos(1/x) + 1/2) - 1/2*log(-1/2*cos(1/x) + 1/2)

giac [B] time = 0.14, size = 43, normalized size = 8.60

$$-\frac{1}{2} \log \left(\frac{4 \tan \left(\frac{1}{2x} \right)^2}{\tan \left(\frac{1}{2x} \right)^2 + 1} \right) + \frac{1}{2} \log \left(\frac{4}{\tan \left(\frac{1}{2x} \right)^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(1/x)/x^2,x, algorithm="giac")

[Out] -1/2*log(4*tan(1/2/x)^2/(tan(1/2/x)^2 + 1)) + 1/2*log(4/(tan(1/2/x)^2 + 1))

maple [A] time = 0.00, size = 11, normalized size = 2.20

$$\ln \left(\csc \left(\frac{1}{x} \right) + \cot \left(\frac{1}{x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(1/x)/x^2,x)

[Out] ln(csc(1/x)+cot(1/x))

maxima [A] time = 0.33, size = 10, normalized size = 2.00

$$\log \left(\cot \left(\frac{1}{x} \right) + \csc \left(\frac{1}{x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(1/x)/x^2,x, algorithm="maxima")

[Out] log(cot(1/x) + csc(1/x))

mupad [B] time = 3.68, size = 31, normalized size = 6.20

$$\ln \left(-e^{1/x} 2i - 2i \right) - \ln \left(-e^{1/x} 2i + 2i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*sin(1/x)),x)

[Out] log(- exp(1i/x)*2i - 2i) - log(2i - exp(1i/x)*2i)

sympy [A] time = 1.26, size = 10, normalized size = 2.00

$$\log \left(\cot \left(\frac{1}{x} \right) + \csc \left(\frac{1}{x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(1/x)/x**2,x)

[Out] log(cot(1/x) + csc(1/x))

3.827 $\int (\csc(x) - \sec(x))(\cos(x) + \sin(x)) dx$

Optimal. Leaf size=7

$$\log(\sin(x)) + \log(\cos(x))$$

[Out] $\ln(\cos(x)) + \ln(\sin(x))$

Rubi [A] time = 0.05, antiderivative size = 9, normalized size of antiderivative = 1.29, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {446, 72}

$$\log(\tan(x)) + 2 \log(\cos(x))$$

Antiderivative was successfully verified.

[In] `Int[(Csc[x] - Sec[x])*(Cos[x] + Sin[x]),x]`

[Out] `2*Log[Cos[x]] + Log[Tan[x]]`

Rule 72

`Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]`

Rule 446

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rubi steps

$$\begin{aligned} \int (\csc(x) - \sec(x))(\cos(x) + \sin(x)) dx &= \text{Subst} \left(\int \frac{1-x^2}{x(1+x^2)} dx, x, \tan(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1-x}{x(1+x)} dx, x, \tan^2(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{x} - \frac{2}{1+x} \right) dx, x, \tan^2(x) \right) \\ &= 2 \log(\cos(x)) + \log(\tan(x)) \end{aligned}$$

Mathematica [A] time = 0.01, size = 7, normalized size = 1.00

$$\log(\sin(x)) + \log(\cos(x))$$

Antiderivative was successfully verified.

[In] `Integrate[(Csc[x] - Sec[x])*(Cos[x] + Sin[x]),x]`

[Out] `Log[Cos[x]] + Log[Sin[x]]`

fricas [A] time = 0.62, size = 7, normalized size = 1.00

$$\log\left(-\frac{1}{2} \cos(x) \sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x, algorithm="fricas")

[Out] log(-1/2*cos(x)*sin(x))

giac [B] time = 0.15, size = 16, normalized size = 2.29

$$\frac{1}{2} \log(-\cos(x)^2 + 1) + \log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x, algorithm="giac")

[Out] 1/2*log(-cos(x)^2 + 1) + log(abs(cos(x)))

maple [A] time = 0.12, size = 8, normalized size = 1.14

$$\ln(\cos(x)) + \ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((csc(x)-sec(x))*(cos(x)+sin(x)),x)

[Out] ln(cos(x))+ln(sin(x))

maxima [B] time = 0.33, size = 15, normalized size = 2.14

$$\frac{1}{2} \log(-\sin(x)^2 + 1) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x, algorithm="maxima")

[Out] 1/2*log(-sin(x)^2 + 1) + log(sin(x))

mupad [B] time = 3.25, size = 26, normalized size = 3.71

$$\ln\left(\tan\left(\frac{x}{2}\right)^3 - \tan\left(\frac{x}{2}\right)\right) - 2 \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(cos(x) + sin(x))*(1/cos(x) - 1/sin(x)),x)

[Out] log(tan(x/2)^3 - tan(x/2)) - 2*log(tan(x/2)^2 + 1)

sympy [A] time = 2.31, size = 8, normalized size = 1.14

$$\log(\sin(x)) + \log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x)

[Out] log(sin(x)) + log(cos(x))

$$3.828 \quad \int (-\cos(3x) \sin(2x) + \cos(2x) \sin(3x)) dx$$

Optimal. Leaf size=4

$$-\cos(x)$$

[Out] $-\cos(x)$

Rubi [A] time = 0.02, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {4284}

$$-\cos(x)$$

Antiderivative was successfully verified.

[In] Int[-(Cos[3*x]*Sin[2*x]) + Cos[2*x]*Sin[3*x],x]

[Out] -Cos[x]

Rule 4284

Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \int (-\cos(3x) \sin(2x) + \cos(2x) \sin(3x)) dx &= - \int \cos(3x) \sin(2x) dx + \int \cos(2x) \sin(3x) dx \\ &= -\cos(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$-\cos(x)$$

Antiderivative was successfully verified.

[In] Integrate[-(Cos[3*x]*Sin[2*x]) + Cos[2*x]*Sin[3*x],x]

[Out] -Cos[x]

fricas [A] time = 0.69, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x, algorithm="fricas")

[Out] $-\cos(x)$

giac [A] time = 0.14, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x, algorithm="giac")

[Out] $-\cos(x)$

maple [A] time = 0.16, size = 5, normalized size = 1.25

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x)`

[Out] `-cos(x)`

maxima [A] time = 0.32, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x, algorithm="maxima")`

[Out] `-cos(x)`

mupad [B] time = 3.07, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(2*x)*sin(3*x) - cos(3*x)*sin(2*x),x)`

[Out] `-cos(x)`

sympy [B] time = 0.69, size = 20, normalized size = 5.00

$$-\sin(2x)\sin(3x) - \cos(2x)\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x)`

[Out] `-\sin(2*x)*sin(3*x) - \cos(2*x)*\cos(3*x)`

3.829 $\int 4x \sec^2(2x) dx$

Optimal. Leaf size=13

$$2x \tan(2x) + \log(\cos(2x))$$

[Out] $\ln(\cos(2*x))+2*x*\tan(2*x)$

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {12, 4184, 3475}

$$2x \tan(2x) + \log(\cos(2x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[4*x*\text{Sec}[2*x]^2, x]$

[Out] $\text{Log}[\text{Cos}[2*x]] + 2*x*\text{Tan}[2*x]$

Rule 12

$\text{Int}[(a_)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

Rule 3475

$\text{Int}[\tan[(c_.) + (d_.)*(x_)], x_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 4184

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^{(m_.)}, x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cot}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cot}[e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

Rubi steps

$$\begin{aligned} \int 4x \sec^2(2x) dx &= 4 \int x \sec^2(2x) dx \\ &= 2x \tan(2x) - 2 \int \tan(2x) dx \\ &= \log(\cos(2x)) + 2x \tan(2x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.62

$$4 \left(\frac{1}{2} x \tan(2x) + \frac{1}{4} \log(\cos(2x)) \right)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[4*x*\text{Sec}[2*x]^2, x]$

[Out] $4*(\text{Log}[\text{Cos}[2*x]]/4 + (x*\text{Tan}[2*x])/2)$

fricas [B] time = 1.32, size = 27, normalized size = 2.08

$$\frac{\cos(2x) \log(-\cos(2x)) + 2x \sin(2x)}{\cos(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*x*sec(2*x)^2,x, algorithm="fricas")

[Out] (cos(2*x)*log(-cos(2*x)) + 2*x*sin(2*x))/cos(2*x)

giac [B] time = 0.17, size = 81, normalized size = 6.23

$$\frac{\log\left(\frac{4(\tan(x)^4 - 2\tan(x)^2 + 1)}{\tan(x)^4 + 2\tan(x)^2 + 1}\right)\tan(x)^2 - 8x\tan(x) - \log\left(\frac{4(\tan(x)^4 - 2\tan(x)^2 + 1)}{\tan(x)^4 + 2\tan(x)^2 + 1}\right)}{2(\tan(x)^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*x*sec(2*x)^2,x, algorithm="giac")

[Out] 1/2*(log(4*(tan(x)^4 - 2*tan(x)^2 + 1)/(tan(x)^4 + 2*tan(x)^2 + 1))*tan(x)^2 - 8*x*tan(x) - log(4*(tan(x)^4 - 2*tan(x)^2 + 1)/(tan(x)^4 + 2*tan(x)^2 + 1)))/(tan(x)^2 - 1)

maple [A] time = 0.03, size = 14, normalized size = 1.08

$$\ln(\cos(2x)) + 2x \tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(4*x*sec(2*x)^2,x)

[Out] ln(cos(2*x))+2*x*tan(2*x)

maxima [B] time = 0.44, size = 74, normalized size = 5.69

$$\frac{(\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1)\log(\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1) + 8x\sin(4x)}{2(\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*x*sec(2*x)^2,x, algorithm="maxima")

[Out] 1/2*((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*log(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1) + 8*x*sin(4*x))/(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)

mupad [B] time = 2.99, size = 13, normalized size = 1.00

$$\ln(\cos(2x)) + 2x \tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x)/cos(2*x)^2,x)

[Out] log(cos(2*x)) + 2*x*tan(2*x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$4 \int x \sec^2(2x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*x*sec(2*x)**2,x)

[Out] 4*Integral(x*sec(2*x)**2, x)

3.830 $\int 4 \sin^2(x) \tan^2(x) dx$

Optimal. Leaf size=16

$$-6x + 6 \tan(x) - 2 \sin^2(x) \tan(x)$$

[Out] $-6*x+6*\tan(x)-2*\sin(x)^2*\tan(x)$

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {12, 2591, 288, 321, 203}

$$-6x + 6 \tan(x) - 2 \sin^2(x) \tan(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[4*\text{Sin}[x]^2*\text{Tan}[x]^2, x]$

[Out] $-6*x + 6*\text{Tan}[x] - 2*\text{Sin}[x]^2*\text{Tan}[x]$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$

Rule 203

$\text{Int}[(a_*) + (b_*)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[\text{Rt}[b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 288

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_*) + (b_*)*(x_)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a+b*x^n)^{(p+1)})/(b*n*(p+1)), x] - \text{Dist}[(c^{(n*(m-n+1))})/(b*n*(p+1)), \text{Int}[(c*x)^{(m-n)}*(a+b*x^n)^{(p+1)}, x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GtQ}[m+1, n] \ \&\& \ !\text{LtQ}[(m+n*(p+1)+1)/n, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 321

$\text{Int}[(c_*)*(x_)^{(m_*)}*((a_*) + (b_*)*(x_)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a+b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^{(n*(m-n+1))})/(b*(m+n*p+1)), \text{Int}[(c*x)^{(m-n)}*(a+b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n-1] \ \&\& \ \text{NeQ}[m+n*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 2591

$\text{Int}[\sin[(e_*) + (f_*)*(x_)]^{(m_*)}*((b_*)*\tan[(e_*) + (f_*)*(x_)]^{(n_*)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[(b*ff)/f, \text{Subst}[\text{Int}[(ff*x)^{(m+n)}/(b^2 + ff^2*x^2)^{(m/2+1)}, x], x, (b*\text{Tan}[e + f*x])/ff], x]] /; \text{FreeQ}[\{b, e, f, n\}, x] \ \&\& \ \text{IntegerQ}[m/2]$

Rubi steps

$$\begin{aligned}
\int 4 \sin^2(x) \tan^2(x) dx &= 4 \int \sin^2(x) \tan^2(x) dx \\
&= 4 \operatorname{Subst} \left(\int \frac{x^4}{(1+x^2)^2} dx, x, \tan(x) \right) \\
&= -2 \sin^2(x) \tan(x) + 6 \operatorname{Subst} \left(\int \frac{x^2}{1+x^2} dx, x, \tan(x) \right) \\
&= 6 \tan(x) - 2 \sin^2(x) \tan(x) - 6 \operatorname{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right) \\
&= -6x + 6 \tan(x) - 2 \sin^2(x) \tan(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.12

$$4 \left(-\frac{3x}{2} + \frac{1}{4} \sin(2x) + \tan(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[4*Sin[x]^2*Tan[x]^2,x]

[Out] 4*((-3*x)/2 + Sin[2*x]/4 + Tan[x])

fricas [A] time = 0.59, size = 22, normalized size = 1.38

$$\frac{2(3x \cos(x) - (\cos(x)^2 + 2) \sin(x))}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*sin(x)^2*tan(x)^2,x, algorithm="fricas")

[Out] -2*(3*x*cos(x) - (cos(x)^2 + 2)*sin(x))/cos(x)

giac [A] time = 0.14, size = 20, normalized size = 1.25

$$-6x + \frac{2 \tan(x)}{\tan(x)^2 + 1} + 4 \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*sin(x)^2*tan(x)^2,x, algorithm="giac")

[Out] -6*x + 2*tan(x)/(tan(x)^2 + 1) + 4*tan(x)

maple [A] time = 0.04, size = 28, normalized size = 1.75

$$\frac{4(\sin^5(x))}{\cos(x)} + 4 \left(\sin^3(x) + \frac{3 \sin(x)}{2} \right) \cos(x) - 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(4*sin(x)^2*tan(x)^2,x)

[Out] 4*sin(x)^5/cos(x)+4*(sin(x)^3+3/2*sin(x))*cos(x)-6*x

maxima [A] time = 0.43, size = 20, normalized size = 1.25

$$-6x + \frac{2 \tan(x)}{\tan(x)^2 + 1} + 4 \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*sin(x)^2*tan(x)^2,x, algorithm="maxima")

[Out] -6*x + 2*tan(x)/(tan(x)^2 + 1) + 4*tan(x)

mupad [B] time = 2.96, size = 18, normalized size = 1.12

$$2 \cos(x) \sin(x) - 6x + \frac{4 \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(4*sin(x)^2*tan(x)^2,x)

[Out] 2*cos(x)*sin(x) - 6*x + (4*sin(x))/cos(x)

sympy [A] time = 0.05, size = 20, normalized size = 1.25

$$-6x + \frac{4 \sin^3(x)}{\cos(x)} + 6 \sin(x) \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*sin(x)**2*tan(x)**2,x)

[Out] -6*x + 4*sin(x)**3/cos(x) + 6*sin(x)*cos(x)

3.831 $\int \cos^4(x) \cot^2(x) dx$

Optimal. Leaf size=32

$$-\frac{15x}{8} - \frac{15 \cot(x)}{8} + \frac{1}{4} \cos^4(x) \cot(x) + \frac{5}{8} \cos^2(x) \cot(x)$$

[Out] $-15/8*x-15/8*\cot(x)+5/8*\cos(x)^2*\cot(x)+1/4*\cos(x)^4*\cot(x)$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2591, 288, 321, 203}

$$-\frac{15x}{8} - \frac{15 \cot(x)}{8} + \frac{1}{4} \cos^4(x) \cot(x) + \frac{5}{8} \cos^2(x) \cot(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^4*Cot[x]^2,x]

[Out] $(-15*x)/8 - (15*\cot[x])/8 + (5*\cos[x]^2*\cot[x])/8 + (\cos[x]^4*\cot[x])/4$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntegerQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2591

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned}
\int \cos^4(x) \cot^2(x) dx &= -\text{Subst} \left(\int \frac{x^6}{(1+x^2)^3} dx, x, \cot(x) \right) \\
&= \frac{1}{4} \cos^4(x) \cot(x) - \frac{5}{4} \text{Subst} \left(\int \frac{x^4}{(1+x^2)^2} dx, x, \cot(x) \right) \\
&= \frac{5}{8} \cos^2(x) \cot(x) + \frac{1}{4} \cos^4(x) \cot(x) - \frac{15}{8} \text{Subst} \left(\int \frac{x^2}{1+x^2} dx, x, \cot(x) \right) \\
&= -\frac{15 \cot(x)}{8} + \frac{5}{8} \cos^2(x) \cot(x) + \frac{1}{4} \cos^4(x) \cot(x) + \frac{15}{8} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \cot(x) \right) \\
&= -\frac{15x}{8} - \frac{15 \cot(x)}{8} + \frac{5}{8} \cos^2(x) \cot(x) + \frac{1}{4} \cos^4(x) \cot(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.81

$$-\frac{15x}{8} - \frac{1}{2} \sin(2x) - \frac{1}{32} \sin(4x) - \cot(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^4*Cot[x]^2,x]

[Out] (-15*x)/8 - Cot[x] - Sin[2*x]/2 - Sin[4*x]/32

fricas [A] time = 1.63, size = 28, normalized size = 0.88

$$\frac{2 \cos(x)^5 + 5 \cos(x)^3 - 15 x \sin(x) - 15 \cos(x)}{8 \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^4*cot(x)^2,x, algorithm="fricas")

[Out] 1/8*(2*cos(x)^5 + 5*cos(x)^3 - 15*x*sin(x) - 15*cos(x))/sin(x)

giac [A] time = 0.13, size = 31, normalized size = 0.97

$$-\frac{15}{8}x - \frac{7 \tan(x)^3 + 9 \tan(x)}{8(\tan(x)^2 + 1)^2} - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^4*cot(x)^2,x, algorithm="giac")

[Out] -15/8*x - 1/8*(7*tan(x)^3 + 9*tan(x))/(tan(x)^2 + 1)^2 - 1/tan(x)

maple [A] time = 0.04, size = 34, normalized size = 1.06

$$-\frac{\cos^7(x)}{\sin(x)} - \left(\cos^5(x) + \frac{5(\cos^3(x))}{4} + \frac{15 \cos(x)}{8} \right) \sin(x) - \frac{15x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^4*cot(x)^2,x)

[Out] -1/sin(x)*cos(x)^7-(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)-15/8*x

maxima [A] time = 0.43, size = 35, normalized size = 1.09

$$-\frac{15}{8}x - \frac{15 \tan(x)^4 + 25 \tan(x)^2 + 8}{8(\tan(x)^5 + 2 \tan(x)^3 + \tan(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^4*cot(x)^2,x, algorithm="maxima")

[Out] -15/8*x - 1/8*(15*tan(x)^4 + 25*tan(x)^2 + 8)/(tan(x)^5 + 2*tan(x)^3 + tan(x))

mupad [B] time = 3.00, size = 26, normalized size = 0.81

$$\frac{\frac{\cos(x)^5}{4} + \frac{5\cos(x)^3}{8} - \frac{15\cos(x)}{8}}{\sin(x)} - \frac{15x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^4*cot(x)^2,x)

[Out] ((5*cos(x)^3)/8 - (15*cos(x))/8 + cos(x)^5/4)/sin(x) - (15*x)/8

sympy [A] time = 0.06, size = 36, normalized size = 1.12

$$-\frac{15x}{8} - \frac{5 \sin(x) \cos^3(x)}{4} - \frac{15 \sin(x) \cos(x)}{8} - \frac{\cos^5(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**4*cot(x)**2,x)

[Out] -15*x/8 - 5*sin(x)*cos(x)**3/4 - 15*sin(x)*cos(x)/8 - cos(x)**5/sin(x)

3.832 $\int 16 \cos^2(x) \sin^2(x) dx$

Optimal. Leaf size=18

$$2x - 4 \sin(x) \cos^3(x) + 2 \sin(x) \cos(x)$$

[Out] $2*x+2*\cos(x)*\sin(x)-4*\cos(x)^3*\sin(x)$

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {12, 2568, 2635, 8}

$$2x - 4 \sin(x) \cos^3(x) + 2 \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[16*\text{Cos}[x]^2*\text{Sin}[x]^2,x]$

[Out] $2*x + 2*\text{Cos}[x]*\text{Sin}[x] - 4*\text{Cos}[x]^3*\text{Sin}[x]$

Rule 8

$\text{Int}[a_, x_Symbol] \text{ :> } \text{Simp}[a*x, x] \text{ /; } \text{FreeQ}[a, x]$

Rule 12

$\text{Int}[(a_)*(u_), x_Symbol] \text{ :> } \text{Dist}[a, \text{Int}[u, x], x] \text{ /; } \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_) \text{ /; } \text{FreeQ}[b, x]]$

Rule 2568

$\text{Int}[(\cos[(e_.) + (f_.)*(x_.)]*(b_.))^n*((a_.)*\sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] \text{ :> } -\text{Simp}[(a*(b*\text{Cos}[e + f*x])^{n+1}*(a*\text{Sin}[e + f*x])^{m-1})/(b*f*(m+n)), x] + \text{Dist}[(a^2*(m-1))/(m+n), \text{Int}[(b*\text{Cos}[e + f*x])^n*(a*\text{Sin}[e + f*x])^{m-2}, x], x] \text{ /; } \text{FreeQ}[\{a, b, e, f, n\}, x] \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ \text{NeQ}[m+n, 0] \ \&\& \ \text{IntegersQ}[2*m, 2*n]$

Rule 2635

$\text{Int}[(b_.)*\sin[(c_.) + (d_.)*(x_.)]^n, x_Symbol] \text{ :> } -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{n-1}/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{n-2}, x], x] \text{ /; } \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned} \int 16 \cos^2(x) \sin^2(x) dx &= 16 \int \cos^2(x) \sin^2(x) dx \\ &= -4 \cos^3(x) \sin(x) + 4 \int \cos^2(x) dx \\ &= 2 \cos(x) \sin(x) - 4 \cos^3(x) \sin(x) + 2 \int 1 dx \\ &= 2x + 2 \cos(x) \sin(x) - 4 \cos^3(x) \sin(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.89

$$4 \left(\frac{x}{2} - \frac{1}{8} \sin(4x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[16*Cos[x]^2*Sin[x]^2,x]

[Out] 4*(x/2 - Sin[4*x]/8)

fricas [A] time = 1.56, size = 19, normalized size = 1.06

$$-2 \left(2 \cos(x)^3 - \cos(x) \right) \sin(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(16*cos(x)^2*sin(x)^2,x, algorithm="fricas")

[Out] -2*(2*cos(x)^3 - cos(x))*sin(x) + 2*x

giac [A] time = 0.14, size = 10, normalized size = 0.56

$$2x - \frac{1}{2} \sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(16*cos(x)^2*sin(x)^2,x, algorithm="giac")

[Out] 2*x - 1/2*sin(4*x)

maple [A] time = 0.01, size = 19, normalized size = 1.06

$$2x + 2 \cos(x) \sin(x) - 4 \left(\cos^3(x) \right) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(16*cos(x)^2*sin(x)^2,x)

[Out] 2*x+2*cos(x)*sin(x)-4*cos(x)^3*sin(x)

maxima [A] time = 0.33, size = 10, normalized size = 0.56

$$2x - \frac{1}{2} \sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(16*cos(x)^2*sin(x)^2,x, algorithm="maxima")

[Out] 2*x - 1/2*sin(4*x)

mupad [B] time = 0.05, size = 18, normalized size = 1.00

$$4 \cos(x) \sin(x)^3 - 2 \cos(x) \sin(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(16*cos(x)^2*sin(x)^2,x)

[Out] 2*x - 2*cos(x)*sin(x) + 4*cos(x)*sin(x)^3

sympy [A] time = 0.06, size = 12, normalized size = 0.67

$$2x - \sin(2x) \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(16*cos(x)**2*sin(x)**2,x)

[Out] 2*x - sin(2*x)*cos(2*x)

3.833 $\int 8 \cos^2(x) \sin^4(x) dx$

Optimal. Leaf size=34

$$\frac{x}{2} - \frac{4}{3} \sin^3(x) \cos^3(x) - \sin(x) \cos^3(x) + \frac{1}{2} \sin(x) \cos(x)$$

[Out] 1/2*x+1/2*cos(x)*sin(x)-cos(x)^3*sin(x)-4/3*cos(x)^3*sin(x)^3

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {12, 2568, 2635, 8}

$$\frac{x}{2} - \frac{4}{3} \sin^3(x) \cos^3(x) - \sin(x) \cos^3(x) + \frac{1}{2} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[8*Cos[x]^2*Sin[x]^4,x]

[Out] x/2 + (Cos[x]*Sin[x])/2 - Cos[x]^3*Sin[x] - (4*Cos[x]^3*Sin[x]^3)/3

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2568

Int[(cos[(e_.) + (f_.)*(x_.)]*(b_.))^n_)*((a_.)*sin[(e_.) + (f_.)*(x_.)])^m_, x_Symbol] := -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1))/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^n_, x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int 8 \cos^2(x) \sin^4(x) dx &= 8 \int \cos^2(x) \sin^4(x) dx \\ &= -\frac{4}{3} \cos^3(x) \sin^3(x) + 4 \int \cos^2(x) \sin^2(x) dx \\ &= -\cos^3(x) \sin(x) - \frac{4}{3} \cos^3(x) \sin^3(x) + \int \cos^2(x) dx \\ &= \frac{1}{2} \cos(x) \sin(x) - \cos^3(x) \sin(x) - \frac{4}{3} \cos^3(x) \sin^3(x) + \frac{\int 1 dx}{2} \\ &= \frac{x}{2} + \frac{1}{2} \cos(x) \sin(x) - \cos^3(x) \sin(x) - \frac{4}{3} \cos^3(x) \sin^3(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.94

$$8 \left(\frac{x}{16} - \frac{1}{64} \sin(2x) - \frac{1}{64} \sin(4x) + \frac{1}{192} \sin(6x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[8*Cos[x]^2*Sin[x]^4,x]

[Out] 8*(x/16 - Sin[2*x]/64 - Sin[4*x]/64 + Sin[6*x]/192)

fricas [A] time = 2.39, size = 25, normalized size = 0.74

$$\frac{1}{6} \left(8 \cos(x)^5 - 14 \cos(x)^3 + 3 \cos(x) \right) \sin(x) + \frac{1}{2} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*cos(x)^2*sin(x)^4,x, algorithm="fricas")

[Out] 1/6*(8*cos(x)^5 - 14*cos(x)^3 + 3*cos(x))*sin(x) + 1/2*x

giac [A] time = 0.12, size = 22, normalized size = 0.65

$$\frac{1}{2} x + \frac{1}{24} \sin(6x) - \frac{1}{8} \sin(4x) - \frac{1}{8} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*cos(x)^2*sin(x)^4,x, algorithm="giac")

[Out] 1/2*x + 1/24*sin(6*x) - 1/8*sin(4*x) - 1/8*sin(2*x)

maple [A] time = 0.01, size = 29, normalized size = 0.85

$$\frac{x}{2} + \frac{\cos(x) \sin(x)}{2} - (\cos^3(x)) \sin(x) - \frac{4 (\cos^3(x)) (\sin^3(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(8*cos(x)^2*sin(x)^4,x)

[Out] 1/2*x+1/2*cos(x)*sin(x)-cos(x)^3*sin(x)-4/3*cos(x)^3*sin(x)^3

maxima [A] time = 0.33, size = 18, normalized size = 0.53

$$-\frac{1}{6} \sin(2x)^3 + \frac{1}{2} x - \frac{1}{8} \sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*cos(x)^2*sin(x)^4,x, algorithm="maxima")

[Out] -1/6*sin(2*x)^3 + 1/2*x - 1/8*sin(4*x)

mupad [B] time = 0.05, size = 24, normalized size = 0.71

$$\frac{4 \cos(x) \sin(x)^5}{3} + \frac{x}{2} - \frac{\sin(2x)}{3} + \frac{\sin(4x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(8*cos(x)^2*sin(x)^4,x)

[Out] x/2 - sin(2*x)/3 + sin(4*x)/24 + (4*cos(x)*sin(x)^5)/3

sympy [A] time = 0.05, size = 32, normalized size = 0.94

$$\frac{x}{2} + \frac{4 \sin^5(x) \cos(x)}{3} - \frac{\sin^3(x) \cos(x)}{3} - \frac{\sin(x) \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*cos(x)**2*sin(x)**4,x)

[Out] x/2 + 4*sin(x)**5*cos(x)/3 - sin(x)**3*cos(x)/3 - sin(x)*cos(x)/2

3.834 $\int 35 \cos^3(x) \sin^4(x) dx$

Optimal. Leaf size=13

$$7 \sin^5(x) - 5 \sin^7(x)$$

[Out] 7*sin(x)^5-5*sin(x)^7

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {12, 2564, 14}

$$7 \sin^5(x) - 5 \sin^7(x)$$

Antiderivative was successfully verified.

[In] Int[35*Cos[x]^3*Sin[x]^4,x]

[Out] 7*Sin[x]^5 - 5*Sin[x]^7

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)+(b_)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 2564

Int[cos[(e_)+(f_)*(x_)]^(n_)*((a_)*sin[(e_)+(f_)*(x_)])^(m_), x_Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1-x^2/a^2)^((n-1)/2), x], x, a*Sin[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[(m-1)/2] && LtQ[0, m, n])

Rubi steps

$$\begin{aligned} \int 35 \cos^3(x) \sin^4(x) dx &= 35 \int \cos^3(x) \sin^4(x) dx \\ &= 35 \text{Subst} \left(\int x^4 (1-x^2) dx, x, \sin(x) \right) \\ &= 35 \text{Subst} \left(\int (x^4 - x^6) dx, x, \sin(x) \right) \\ &= 7 \sin^5(x) - 5 \sin^7(x) \end{aligned}$$

Mathematica [B] time = 0.01, size = 33, normalized size = 2.54

$$35 \left(\frac{3 \sin(x)}{64} - \frac{1}{64} \sin(3x) - \frac{1}{320} \sin(5x) + \frac{1}{448} \sin(7x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[35*Cos[x]^3*Sin[x]^4,x]

[Out] 35*((3*Sin[x])/64 - Sin[3*x]/64 - Sin[5*x]/320 + Sin[7*x]/448)

fricas [A] time = 0.47, size = 21, normalized size = 1.62

$$(5 \cos(x)^6 - 8 \cos(x)^4 + \cos(x)^2 + 2) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(35*cos(x)^3*sin(x)^4,x, algorithm="fricas")

[Out] (5*cos(x)^6 - 8*cos(x)^4 + cos(x)^2 + 2)*sin(x)

giac [A] time = 0.13, size = 13, normalized size = 1.00

$$-5 \sin(x)^7 + 7 \sin(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(35*cos(x)^3*sin(x)^4,x, algorithm="giac")

[Out] -5*sin(x)^7 + 7*sin(x)^5

maple [B] time = 0.01, size = 29, normalized size = 2.23

$$-5 (\cos^4(x)) (\sin^3(x)) - 3 \sin(x) (\cos^4(x)) + (2 + \cos^2(x)) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(35*cos(x)^3*sin(x)^4,x)

[Out] -5*cos(x)^4*sin(x)^3-3*sin(x)*cos(x)^4+(2+cos(x)^2)*sin(x)

maxima [A] time = 0.32, size = 13, normalized size = 1.00

$$-5 \sin(x)^7 + 7 \sin(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(35*cos(x)^3*sin(x)^4,x, algorithm="maxima")

[Out] -5*sin(x)^7 + 7*sin(x)^5

mupad [B] time = 0.04, size = 13, normalized size = 1.00

$$7 \sin(x)^5 - 5 \sin(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(35*cos(x)^3*sin(x)^4,x)

[Out] 7*sin(x)^5 - 5*sin(x)^7

sympy [A] time = 0.05, size = 12, normalized size = 0.92

$$-5 \sin^7(x) + 7 \sin^5(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(35*cos(x)**3*sin(x)**4,x)

[Out] -5*sin(x)**7 + 7*sin(x)**5

3.835 $\int 4 \cos^4(x) \sin^4(x) dx$

Optimal. Leaf size=46

$$\frac{3x}{32} - \frac{1}{2} \sin^3(x) \cos^5(x) - \frac{1}{4} \sin(x) \cos^5(x) + \frac{1}{16} \sin(x) \cos^3(x) + \frac{3}{32} \sin(x) \cos(x)$$

[Out] 3/32*x+3/32*cos(x)*sin(x)+1/16*cos(x)^3*sin(x)-1/4*cos(x)^5*sin(x)-1/2*cos(x)^5*sin(x)^3

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {12, 2568, 2635, 8}

$$\frac{3x}{32} - \frac{1}{2} \sin^3(x) \cos^5(x) - \frac{1}{4} \sin(x) \cos^5(x) + \frac{1}{16} \sin(x) \cos^3(x) + \frac{3}{32} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[4*Cos[x]^4*Sin[x]^4,x]

[Out] (3*x)/32 + (3*Cos[x]*Sin[x])/32 + (Cos[x]^3*Sin[x])/16 - (Cos[x]^5*Sin[x])/4 - (Cos[x]^5*Sin[x]^3)/2

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2568

Int[(cos[(e_.) + (f_.)*(x_.)]*(b_.))^n_*((a_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1))/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^n, x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
\int 4 \cos^4(x) \sin^4(x) dx &= 4 \int \cos^4(x) \sin^4(x) dx \\
&= -\frac{1}{2} \cos^5(x) \sin^3(x) + \frac{3}{2} \int \cos^4(x) \sin^2(x) dx \\
&= -\frac{1}{4} \cos^5(x) \sin(x) - \frac{1}{2} \cos^5(x) \sin^3(x) + \frac{1}{4} \int \cos^4(x) dx \\
&= \frac{1}{16} \cos^3(x) \sin(x) - \frac{1}{4} \cos^5(x) \sin(x) - \frac{1}{2} \cos^5(x) \sin^3(x) + \frac{3}{16} \int \cos^2(x) dx \\
&= \frac{3}{32} \cos(x) \sin(x) + \frac{1}{16} \cos^3(x) \sin(x) - \frac{1}{4} \cos^5(x) \sin(x) - \frac{1}{2} \cos^5(x) \sin^3(x) + \frac{3}{32} \int 1 dx \\
&= \frac{3x}{32} + \frac{3}{32} \cos(x) \sin(x) + \frac{1}{16} \cos^3(x) \sin(x) - \frac{1}{4} \cos^5(x) \sin(x) - \frac{1}{2} \cos^5(x) \sin^3(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.52

$$4 \left(\frac{3x}{128} - \frac{1}{128} \sin(4x) + \frac{\sin(8x)}{1024} \right)$$

Antiderivative was successfully verified.

[In] Integrate[4*Cos[x]^4*Sin[x]^4,x]

[Out] 4*((3*x)/128 - Sin[4*x]/128 + Sin[8*x]/1024)

fricas [A] time = 1.97, size = 31, normalized size = 0.67

$$\frac{1}{32} (16 \cos(x)^7 - 24 \cos(x)^5 + 2 \cos(x)^3 + 3 \cos(x)) \sin(x) + \frac{3}{32} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*cos(x)^4*sin(x)^4,x, algorithm="fricas")

[Out] 1/32*(16*cos(x)^7 - 24*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 3/32*x

giac [A] time = 0.14, size = 16, normalized size = 0.35

$$\frac{3}{32} x + \frac{1}{256} \sin(8x) - \frac{1}{32} \sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*cos(x)^4*sin(x)^4,x, algorithm="giac")

[Out] 3/32*x + 1/256*sin(8*x) - 1/32*sin(4*x)

maple [A] time = 0.01, size = 36, normalized size = 0.78

$$-\frac{(\cos^5(x))(\sin^3(x))}{2} - \frac{(\cos^5(x))\sin(x)}{4} + \frac{\left(\cos^3(x) + \frac{3\cos(x)}{2}\right)\sin(x)}{16} + \frac{3x}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(4*cos(x)^4*sin(x)^4,x)

[Out] -1/2*cos(x)^5*sin(x)^3-1/4*cos(x)^5*sin(x)+1/16*(cos(x)^3+3/2*cos(x))*sin(x)+3/32*x

maxima [A] time = 0.33, size = 16, normalized size = 0.35

$$\frac{3}{32}x + \frac{1}{256}\sin(8x) - \frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*cos(x)^4*sin(x)^4,x, algorithm="maxima")

[Out] 3/32*x + 1/256*sin(8*x) - 1/32*sin(4*x)

mupad [B] time = 0.04, size = 33, normalized size = 0.72

$$\frac{3x}{32} - \frac{\sin(2x)}{16} + \frac{\sin(4x)}{128} + 4\sin(x)^5 \left(\frac{\cos(x)^3}{8} + \frac{\cos(x)}{16} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(4*cos(x)^4*sin(x)^4,x)

[Out] (3*x)/32 - sin(2*x)/16 + sin(4*x)/128 + 4*sin(x)^5*(cos(x)/16 + cos(x)^3/8)

sympy [A] time = 0.06, size = 31, normalized size = 0.67

$$\frac{3x}{32} - \frac{\sin^3(2x)\cos(2x)}{32} - \frac{3\sin(2x)\cos(2x)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*cos(x)**4*sin(x)**4,x)

[Out] 3*x/32 - sin(2*x)**3*cos(2*x)/32 - 3*sin(2*x)*cos(2*x)/64

$$3.836 \quad \int \frac{\cos(x)}{-\sin(x) + \sin^3(x)} dx$$

Optimal. Leaf size=9

$$\log(\cos(x)) - \log(\sin(x))$$

[Out] ln(cos(x))-ln(sin(x))

Rubi [A] time = 0.03, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4334, 266, 36, 31, 29}

$$\log(\cos(x)) - \log(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/(-Sin[x] + Sin[x]^3), x]

[Out] Log[Cos[x]] - Log[Sin[x]]

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 4334

Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{-\sin(x) + \sin^3(x)} dx &= \text{Subst} \left(\int \frac{1}{x(-1 + x^2)} dx, x, \sin(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(-1 + x)x} dx, x, \sin^2(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1 + x} dx, x, \sin^2(x) \right) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{x} dx, x, \sin^2(x) \right) \\ &= \log(\cos(x)) - \log(\sin(x)) \end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\log(\cos(x)) - \log(\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/(-Sin[x] + Sin[x]^3), x]

[Out] Log[Cos[x]] - Log[Sin[x]]

fricas [B] time = 0.57, size = 19, normalized size = 2.11

$$\frac{1}{2} \log(\cos(x)^2) - \frac{1}{2} \log\left(-\frac{1}{4} \cos(x)^2 + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(-sin(x)+sin(x)^3), x, algorithm="fricas")

[Out] 1/2*log(cos(x)^2) - 1/2*log(-1/4*cos(x)^2 + 1/4)

giac [A] time = 0.13, size = 18, normalized size = 2.00

$$\frac{1}{2} \log(-\sin(x)^2 + 1) - \log(|\sin(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(-sin(x)+sin(x)^3), x, algorithm="giac")

[Out] 1/2*log(-sin(x)^2 + 1) - log(abs(sin(x)))

maple [B] time = 0.08, size = 21, normalized size = 2.33

$$-\ln(\sin(x)) + \frac{\ln(\sin(x) - 1)}{2} + \frac{\ln(1 + \sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(-sin(x)+sin(x)^3), x)

[Out] -ln(sin(x))+1/2*ln(sin(x)-1)+1/2*ln(1+sin(x))

maxima [B] time = 0.32, size = 20, normalized size = 2.22

$$\frac{1}{2} \log(\sin(x) + 1) + \frac{1}{2} \log(\sin(x) - 1) - \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(-sin(x)+sin(x)^3), x, algorithm="maxima")

[Out] 1/2*log(sin(x) + 1) + 1/2*log(sin(x) - 1) - log(sin(x))

mupad [B] time = 3.01, size = 13, normalized size = 1.44

$$\frac{\ln(\cos(x)^2)}{2} - \ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-cos(x)/(sin(x) - sin(x)^3), x)

[Out] log(cos(x)^2)/2 - log(sin(x))

sympy [B] time = 0.32, size = 20, normalized size = 2.22

$$\frac{\log(\sin(x) - 1)}{2} + \frac{\log(\sin(x) + 1)}{2} - \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(-sin(x)+sin(x)**3),x)

[Out] log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - log(sin(x))

$$3.837 \quad \int \left(-1 + 2 \cos^2(x) + \cos(x) \sin(x) \right) dx$$

Optimal. Leaf size=14

$$\frac{\sin^2(x)}{2} + \sin(x) \cos(x)$$

[Out] $\cos(x)*\sin(x)+1/2*\sin(x)^2$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2635, 8, 2564, 30}

$$\frac{\sin^2(x)}{2} + \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[-1 + 2*\text{Cos}[x]^2 + \text{Cos}[x]*\text{Sin}[x], x]$

[Out] $\text{Cos}[x]*\text{Sin}[x] + \text{Sin}[x]^2/2$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 30

$\text{Int}[(x_)^(m_), x_Symbol] \rightarrow \text{Simp}[x^(m+1)/(m+1), x] /; \text{FreeQ}[m, x] \&\& \text{NeQ}[m, -1]$

Rule 2564

$\text{Int}[\cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*\sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] \rightarrow \text{Dist}[1/(a*f), \text{Subst}[\text{Int}[x^m*(1 - x^2/a^2)^((n-1)/2), x], x, a*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, e, f, m\}, x] \&\& \text{IntegerQ}[(n-1)/2] \&\& !(\text{IntegerQ}[(m-1)/2] \&\& \text{LtQ}[0, m, n])$

Rule 2635

$\text{Int}[(b_.)*\sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^(n-1))/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^(n-2), x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned} \int \left(-1 + 2 \cos^2(x) + \cos(x) \sin(x) \right) dx &= -x + 2 \int \cos^2(x) dx + \int \cos(x) \sin(x) dx \\ &= -x + \cos(x) \sin(x) + \int 1 dx + \text{Subst}\left(\int x dx, x, \sin(x)\right) \\ &= \cos(x) \sin(x) + \frac{\sin^2(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.21

$$\frac{1}{2} \sin(2x) - \frac{\cos^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[-1 + 2*Cos[x]^2 + Cos[x]*Sin[x],x]

[Out] -1/2*Cos[x]^2 + Sin[2*x]/2

fricas [A] time = 0.71, size = 12, normalized size = 0.86

$$-\frac{1}{2} \cos(x)^2 + \cos(x) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1+2*cos(x)^2+cos(x)*sin(x),x, algorithm="fricas")

[Out] -1/2*cos(x)^2 + cos(x)*sin(x)

giac [A] time = 0.14, size = 13, normalized size = 0.93

$$-\frac{1}{2} \cos(x)^2 + \frac{1}{2} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1+2*cos(x)^2+cos(x)*sin(x),x, algorithm="giac")

[Out] -1/2*cos(x)^2 + 1/2*sin(2*x)

maple [A] time = 0.02, size = 13, normalized size = 0.93

$$\cos(x) \sin(x) + \frac{(\sin^2(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1+2*cos(x)^2+cos(x)*sin(x),x)

[Out] cos(x)*sin(x)+1/2*sin(x)^2

maxima [A] time = 0.32, size = 13, normalized size = 0.93

$$-\frac{1}{2} \cos(x)^2 + \frac{1}{2} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1+2*cos(x)^2+cos(x)*sin(x),x, algorithm="maxima")

[Out] -1/2*cos(x)^2 + 1/2*sin(2*x)

mupad [B] time = 2.97, size = 11, normalized size = 0.79

$$\frac{\cos(x) (\cos(x) - 2 \sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(x) + 2*cos(x)^2 - 1,x)

[Out] -(cos(x)*(cos(x) - 2*sin(x)))/2

sympy [A] time = 0.05, size = 12, normalized size = 0.86

$$\frac{\sin^2(x)}{2} + \sin(x) \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1+2*cos(x)**2+cos(x)*sin(x),x)

[Out] sin(x)**2/2 + sin(x)*cos(x)

$$3.838 \quad \int (\cos^2(x) + \sin^2(x)) dx$$

Optimal. Leaf size=1

x

[Out] x

Rubi [A] time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2635, 8}

x

Antiderivative was successfully verified.

[In] Int[Cos[x]^2 + Sin[x]^2,x]

[Out] x

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x] * (b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (\cos^2(x) + \sin^2(x)) dx &= \int \cos^2(x) dx + \int \sin^2(x) dx \\ &= 2 \frac{\int 1 dx}{2} \\ &= x \end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

x

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^2 + Sin[x]^2,x]

[Out] x

fricas [A] time = 1.35, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2+sin(x)^2,x, algorithm="fricas")

[Out] x

giac [A] time = 0.14, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2+sin(x)^2,x, algorithm="giac")
```

```
[Out] x
```

maple [A] time = 0.02, size = 2, normalized size = 2.00

x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2+sin(x)^2,x)
```

```
[Out] x
```

maxima [A] time = 0.33, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2+sin(x)^2,x, algorithm="maxima")
```

```
[Out] x
```

mupad [B] time = 2.93, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2 + sin(x)^2,x)
```

```
[Out] x
```

sympy [A] time = 0.06, size = 0, normalized size = 0.00

x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2+sin(x)**2,x)
```

```
[Out] x
```


$$3.839 \quad \int \left(-\cos^2(x) + \sin^2(x) \right) dx$$

Optimal. Leaf size=6

$$\sin(x)(-\cos(x))$$

[Out] $-\cos(x)*\sin(x)$

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2635, 8}

$$\sin(x)(-\cos(x))$$

Antiderivative was successfully verified.

[In] Int[-Cos[x]^2 + Sin[x]^2,x]

[Out] -(Cos[x]*Sin[x])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \left(-\cos^2(x) + \sin^2(x) \right) dx &= -\int \cos^2(x) dx + \int \sin^2(x) dx \\ &= -\cos(x)\sin(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.33

$$-\frac{1}{2} \sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[-Cos[x]^2 + Sin[x]^2,x]

[Out] $-1/2*\sin[2*x]$

fricas [A] time = 0.41, size = 6, normalized size = 1.00

$$-\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-cos(x)^2+sin(x)^2,x, algorithm="fricas")

[Out] $-\cos(x)*\sin(x)$

giac [A] time = 0.12, size = 6, normalized size = 1.00

$$-\frac{1}{2} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-cos(x)^2+sin(x)^2,x, algorithm="giac")

[Out] -1/2*sin(2*x)

maple [A] time = 0.00, size = 7, normalized size = 1.17

$$-\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-cos(x)^2+sin(x)^2,x)

[Out] -cos(x)*sin(x)

maxima [A] time = 0.32, size = 6, normalized size = 1.00

$$-\frac{1}{2}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-cos(x)^2+sin(x)^2,x, algorithm="maxima")

[Out] -1/2*sin(2*x)

mupad [B] time = 2.94, size = 6, normalized size = 1.00

$$-\frac{\sin(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2 - cos(x)^2,x)

[Out] -sin(2*x)/2

sympy [A] time = 0.06, size = 7, normalized size = 1.17

$$-\sin(x)\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-cos(x)**2+sin(x)**2,x)

[Out] -sin(x)*cos(x)

3.840 $\int 2^{\sin(x)} \cos(x) dx$

Optimal. Leaf size=9

$$\frac{2^{\sin(x)}}{\log(2)}$$

[Out] $2^{\sin(x)}/\ln(2)$

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4334, 2194}

$$\frac{2^{\sin(x)}}{\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^Sin[x]*Cos[x],x]

[Out] 2^Sin[x]/Log[2]

Rule 2194

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 4334

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned} \int 2^{\sin(x)} \cos(x) dx &= \text{Subst}\left(\int 2^x dx, x, \sin(x)\right) \\ &= \frac{2^{\sin(x)}}{\log(2)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{2^{\sin(x)}}{\log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[2^Sin[x]*Cos[x],x]

[Out] 2^Sin[x]/Log[2]

fricas [A] time = 0.56, size = 9, normalized size = 1.00

$$\frac{2^{\sin(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2^sin(x)*cos(x),x, algorithm="fricas")

[Out] $2^{\sin(x)}/\log(2)$

giac [A] time = 0.13, size = 9, normalized size = 1.00

$$\frac{2^{\sin(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2sin(x)*cos(x),x, algorithm="giac")`

[Out] $2^{\sin(x)}/\log(2)$

maple [A] time = 0.03, size = 10, normalized size = 1.11

$$\frac{2^{\sin(x)}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2sin(x)*cos(x),x)`

[Out] $2^{\sin(x)}/\ln(2)$

maxima [A] time = 0.31, size = 9, normalized size = 1.00

$$\frac{2^{\sin(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2sin(x)*cos(x),x, algorithm="maxima")`

[Out] $2^{\sin(x)}/\log(2)$

mupad [B] time = 0.07, size = 9, normalized size = 1.00

$$\frac{2^{\sin(x)}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2sin(x)*cos(x),x)`

[Out] $2^{\sin(x)}/\log(2)$

sympy [A] time = 0.25, size = 7, normalized size = 0.78

$$\frac{2^{\sin(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2**sin(x)*cos(x),x)`

[Out] $2^{\sin(x)}/\log(2)$

3.841 $\int (\tan^3(x) + \tan^5(x)) dx$

Optimal. Leaf size=8

$$\frac{\tan^4(x)}{4}$$

[Out] 1/4*tan(x)^4

Rubi [A] time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3473, 3475}

$$\frac{\tan^4(x)}{4}$$

Antiderivative was successfully verified.

[In] Int[Tan[x]^3 + Tan[x]^5,x]

[Out] Tan[x]^4/4

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (\tan^3(x) + \tan^5(x)) dx &= \int \tan^3(x) dx + \int \tan^5(x) dx \\ &= \frac{\tan^2(x)}{2} + \frac{\tan^4(x)}{4} - \int \tan(x) dx - \int \tan^3(x) dx \\ &= \log(\cos(x)) + \frac{\tan^4(x)}{4} + \int \tan(x) dx \\ &= \frac{\tan^4(x)}{4} \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\tan^4(x)}{4}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[x]^3 + Tan[x]^5,x]

[Out] Tan[x]^4/4

fricas [A] time = 1.47, size = 6, normalized size = 0.75

$$\frac{1}{4} \tan(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)^3+tan(x)^5,x, algorithm="fricas")

[Out] 1/4*tan(x)^4

giac [A] time = 0.14, size = 6, normalized size = 0.75

$$\frac{1}{4} \tan(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)^3+tan(x)^5,x, algorithm="giac")

[Out] 1/4*tan(x)^4

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\frac{(\tan^4(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)^3+tan(x)^5,x)

[Out] 1/4*tan(x)^4

maxima [B] time = 0.32, size = 35, normalized size = 4.38

$$\frac{4 \sin(x)^2 - 3}{4 (\sin(x)^4 - 2 \sin(x)^2 + 1)} - \frac{1}{2 (\sin(x)^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)^3+tan(x)^5,x, algorithm="maxima")

[Out] 1/4*(4*sin(x)^2 - 3)/(sin(x)^4 - 2*sin(x)^2 + 1) - 1/2/(sin(x)^2 - 1)

mupad [B] time = 2.95, size = 6, normalized size = 0.75

$$\frac{\tan(x)^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)^3 + tan(x)^5,x)

[Out] tan(x)^4/4

sympy [B] time = 0.12, size = 22, normalized size = 2.75

$$-\frac{4 \cos^2(x) - 1}{4 \cos^4(x)} + \frac{1}{2 \cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(x)**3+tan(x)**5,x)

[Out] -(4*cos(x)**2 - 1)/(4*cos(x)**4) + 1/(2*cos(x)**2)

3.842 $\int x \sec(x)(2 + x \tan(x)) dx$

Optimal. Leaf size=6

$$x^2 \sec(x)$$

[Out] $x^2 \sec(x)$

Rubi [A] time = 0.18, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6742, 4181, 2279, 2391, 3757}

$$x^2 \sec(x)$$

Antiderivative was successfully verified.

[In] Int[x*Sec[x]*(2 + x*Tan[x]),x]

[Out] $x^2 \sec(x)$

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 3757

Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x] - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]

Rule 4181

Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int x \sec(x)(2 + x \tan(x)) dx &= \int (2x \sec(x) + x^2 \sec(x) \tan(x)) dx \\
&= 2 \int x \sec(x) dx + \int x^2 \sec(x) \tan(x) dx \\
&= -4ix \tan^{-1}(e^{ix}) + x^2 \sec(x) - 2 \int \log(1 - ie^{ix}) dx + 2 \int \log(1 + ie^{ix}) dx - 2 \int x \sec(x) dx \\
&= x^2 \sec(x) + 2i \operatorname{Subst}\left(\int \frac{\log(1 - ix)}{x} dx, x, e^{ix}\right) - 2i \operatorname{Subst}\left(\int \frac{\log(1 + ix)}{x} dx, x, e^{ix}\right) \\
&= 2i \operatorname{Li}_2(-ie^{ix}) - 2i \operatorname{Li}_2(ie^{ix}) + x^2 \sec(x) - 2i \operatorname{Subst}\left(\int \frac{\log(1 - ix)}{x} dx, x, e^{ix}\right) + 2i \operatorname{Subst}\left(\int \frac{\log(1 + ix)}{x} dx, x, e^{ix}\right) \\
&= x^2 \sec(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 6, normalized size = 1.00

$$x^2 \sec(x)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[x]*(2 + x*Tan[x]),x]

[Out] x^2*Sec[x]

fricas [A] time = 2.14, size = 8, normalized size = 1.33

$$\frac{x^2}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x)*(2+x*tan(x)),x, algorithm="fricas")

[Out] x^2/cos(x)

giac [B] time = 0.15, size = 26, normalized size = 4.33

$$-\frac{x^2 \tan\left(\frac{1}{2}x\right)^2 + x^2}{\tan\left(\frac{1}{2}x\right)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x)*(2+x*tan(x)),x, algorithm="giac")

[Out] -(x^2*tan(1/2*x)^2 + x^2)/(tan(1/2*x)^2 - 1)

maple [A] time = 0.02, size = 9, normalized size = 1.50

$$\frac{x^2}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sec(x)*(2+x*tan(x)),x)

[Out] x^2/cos(x)

maxima [B] time = 0.48, size = 51, normalized size = 8.50

$$\frac{2(x^2 \cos(2x) \cos(x) + x^2 \sin(2x) \sin(x) + x^2 \cos(x))}{\cos(2x)^2 + \sin(2x)^2 + 2 \cos(2x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*(2+x*tan(x)),x, algorithm="maxima")
```

```
[Out] 2*(x^2*cos(2*x)*cos(x) + x^2*sin(2*x)*sin(x) + x^2*cos(x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

mupad [B] time = 0.08, size = 8, normalized size = 1.33

$$\frac{x^2}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x*tan(x) + 2))/cos(x),x)
```

```
[Out] x^2/cos(x)
```

sympy [A] time = 0.48, size = 5, normalized size = 0.83

$$x^2 \sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*(2+x*tan(x)),x)
```

```
[Out] x**2*sec(x)
```

$$3.843 \quad \int \frac{\cot(\sqrt{x}) \csc(\sqrt{x})}{\sqrt{x}} dx$$

Optimal. Leaf size=8

$$-2 \csc(\sqrt{x})$$

[Out] -2*csc(x^(1/2))

Rubi [A] time = 0.20, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6715, 2606, 8}

$$-2 \csc(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[(Cot[Sqrt[x]]*Csc[Sqrt[x]])/Sqrt[x],x]

[Out] -2*Csc[Sqrt[x]]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m-1)*(-1+x^2)^((n-1)/2), x], x, Sec[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n+1])

Rule 6715

Int[(u_)*(x_)^(m_.), x_Symbol] := Dist[1/(m+1), Subst[Int[SubstFor[x^(m+1), u, x], x], x, x^(m+1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m+1), u, x]

Rubi steps

$$\begin{aligned} \int \frac{\cot(\sqrt{x}) \csc(\sqrt{x})}{\sqrt{x}} dx &= 2 \text{Subst} \left(\int \cot(x) \csc(x) dx, x, \sqrt{x} \right) \\ &= - \left(2 \text{Subst} \left(\int 1 dx, x, \csc(\sqrt{x}) \right) \right) \\ &= -2 \csc(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.02, size = 8, normalized size = 1.00

$$-2 \csc(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[Sqrt[x]]*Csc[Sqrt[x]])/Sqrt[x],x]

[Out] -2*Csc[Sqrt[x]]

fricas [A] time = 1.44, size = 8, normalized size = 1.00

$$-\frac{2}{\sin(\sqrt{x})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x, algorithm="fricas")

[Out] -2/sin(sqrt(x))

giac [A] time = 0.12, size = 8, normalized size = 1.00

$$-\frac{2}{\sin(\sqrt{x})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x, algorithm="giac")

[Out] -2/sin(sqrt(x))

maple [A] time = 0.08, size = 7, normalized size = 0.88

$$-2 \operatorname{csc}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x)

[Out] -2*csc(x^(1/2))

maxima [A] time = 0.31, size = 8, normalized size = 1.00

$$-\frac{2}{\sin(\sqrt{x})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x, algorithm="maxima")

[Out] -2/sin(sqrt(x))

mupad [B] time = 3.07, size = 8, normalized size = 1.00

$$-\frac{2}{\sin(\sqrt{x})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x^(1/2))/(x^(1/2)*sin(x^(1/2))),x)

[Out] -2/sin(x^(1/2))

sympy [A] time = 0.28, size = 8, normalized size = 1.00

$$-2 \operatorname{csc}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x**(1/2))*csc(x**(1/2))/x**(1/2),x)

[Out] -2*csc(sqrt(x))

$$3.844 \quad \int \frac{\cos(\sqrt{x}) \sin(\sqrt{x})}{\sqrt{x}} dx$$

Optimal. Leaf size=8

$$\sin^2(\sqrt{x})$$

[Out] $\sin(x^{(1/2)})^2$

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3441}

$$\sin^2(\sqrt{x})$$

Antiderivative was successfully verified.

[In] `Int[(Cos[Sqrt[x]]*Sin[Sqrt[x]])/Sqrt[x],x]`

[Out] `Sin[Sqrt[x]]^2`

Rule 3441

`Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]`

Rubi steps

$$\int \frac{\cos(\sqrt{x}) \sin(\sqrt{x})}{\sqrt{x}} dx = \sin^2(\sqrt{x})$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.50

$$-\frac{1}{2} \cos(2\sqrt{x})$$

Antiderivative was successfully verified.

[In] `Integrate[(Cos[Sqrt[x]]*Sin[Sqrt[x]])/Sqrt[x],x]`

[Out] `-1/2*Cos[2*Sqrt[x]]`

fricas [A] time = 0.73, size = 8, normalized size = 1.00

$$-\cos(\sqrt{x})^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x, algorithm="fricas")`

[Out] `-cos(sqrt(x))^2`

giac [A] time = 0.12, size = 6, normalized size = 0.75

$$\sin(\sqrt{x})^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x, algorithm="giac")`

[Out] $\sin(\sqrt{x})^2$

maple [A] time = 0.04, size = 9, normalized size = 1.12

$$-\left(\cos^2\left(\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x)`

[Out] $-\cos(x^{1/2})^2$

maxima [A] time = 0.31, size = 8, normalized size = 1.00

$$-\cos\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x, algorithm="maxima")`

[Out] $-\cos(\sqrt{x})^2$

mupad [B] time = 3.05, size = 8, normalized size = 1.00

$$-\cos\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(x^(1/2))*sin(x^(1/2)))/x^(1/2),x)`

[Out] $-\cos(x^{1/2})^2$

sympy [A] time = 0.27, size = 8, normalized size = 1.00

$$-\cos^2\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x**(1/2))*sin(x**(1/2))/x**(1/2),x)`

[Out] $-\cos(\sqrt{x})^{**2}$

$$3.845 \quad \int \frac{\sec(\sqrt{x}) \tan(\sqrt{x})}{\sqrt{x}} dx$$

Optimal. Leaf size=8

$$2 \sec(\sqrt{x})$$

[Out] 2*sec(x^(1/2))

Rubi [A] time = 0.18, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6715, 2606, 8}

$$2 \sec(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[(Sec[Sqrt[x]]*Tan[Sqrt[x]])/Sqrt[x],x]

[Out] 2*Sec[Sqrt[x]]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_.)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m-1)*(-1+x^2)^((n-1)/2), x], x, Sec[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n+1])

Rule 6715

Int[(u_)*(x_)^(m_.), x_Symbol] := Dist[1/(m+1), Subst[Int[SubstFor[x^(m+1), u, x], x], x, x^(m+1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m+1), u, x]

Rubi steps

$$\begin{aligned} \int \frac{\sec(\sqrt{x}) \tan(\sqrt{x})}{\sqrt{x}} dx &= 2 \operatorname{Subst} \left(\int \sec(x) \tan(x) dx, x, \sqrt{x} \right) \\ &= 2 \operatorname{Subst} \left(\int 1 dx, x, \sec(\sqrt{x}) \right) \\ &= 2 \sec(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.02, size = 8, normalized size = 1.00

$$2 \sec(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[Sqrt[x]]*Tan[Sqrt[x]])/Sqrt[x],x]

[Out] 2*Sec[Sqrt[x]]

fricas [A] time = 0.57, size = 8, normalized size = 1.00

$$\frac{2}{\cos(\sqrt{x})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x, algorithm="fricas")

[Out] 2/cos(sqrt(x))

giac [A] time = 0.14, size = 8, normalized size = 1.00

$$\frac{2}{\cos(\sqrt{x})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x, algorithm="giac")

[Out] 2/cos(sqrt(x))

maple [A] time = 0.06, size = 7, normalized size = 0.88

$$2 \sec(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x)

[Out] 2*sec(x^(1/2))

maxima [A] time = 0.31, size = 8, normalized size = 1.00

$$\frac{2}{\cos(\sqrt{x})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x, algorithm="maxima")

[Out] 2/cos(sqrt(x))

mupad [B] time = 3.00, size = 8, normalized size = 1.00

$$\frac{2}{\cos(\sqrt{x})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x^(1/2))/(x^(1/2)*cos(x^(1/2))),x)

[Out] 2/cos(x^(1/2))

sympy [A] time = 0.29, size = 7, normalized size = 0.88

$$2 \sec(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x**(1/2))*tan(x**(1/2))/x**(1/2),x)

[Out] 2*sec(sqrt(x))

$$3.846 \quad \int \frac{\sin^2(x)}{a+b \sin(2x)} dx$$

Optimal. Leaf size=55

$$\frac{\tan^{-1}\left(\frac{a \tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}} - \frac{\log(a+b \sin(2x))}{4b}$$

[Out] $-1/4*\ln(a+b*\sin(2*x))/b+1/2*\arctan((b+a*\tan(x))/(a^2-b^2)^{(1/2)})/(a^2-b^2)^{(1/2)}$

Rubi [A] time = 0.17, antiderivative size = 70, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1075, 12, 634, 618, 204, 628, 260}

$$\frac{\tan^{-1}\left(\frac{a \tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}} - \frac{\log(a \tan^2(x) + a + 2b \tan(x))}{4b} - \frac{\log(\cos(x))}{2b}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^2/(a + b*Sin[2*x]),x]

[Out] ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]]/(2*Sqrt[a^2 - b^2]) - Log[Cos[x]]/(2*b) - Log[a + 2*b*Tan[x] + a*Tan[x]^2]/(4*b)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

$[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1075

$\text{Int}[(A_.) + (C_.)*(x_)^2]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (f_.)*(x_)^2), x_Symbol] \rightarrow \text{With}[\{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2\}, \text{Dist}[1/q, \text{Int}[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*(-(b*C*d) + A*b*f)*x)/(a + b*x + c*x^2), x], x] + \text{Dist}[1/q, \text{Int}[(c*C*d^2 - A*c*d*f - a*C*d*f + a*A*f^2 - f*(-(b*C*d) + A*b*f)*x)/(d + f*x^2), x], x] /; \text{NeQ}[q, 0] /; \text{FreeQ}[\{a, b, c, d, f, A, C\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\sin^2(x)}{a + b \sin(2x)} dx &= \text{Subst} \left(\int \frac{x^2}{(1+x^2)(a+2bx+ax^2)} dx, x, \tan(x) \right) \\ &= \frac{\text{Subst} \left(\int \frac{2bx}{1+x^2} dx, x, \tan(x) \right)}{4b^2} + \frac{\text{Subst} \left(\int -\frac{2abx}{a+2bx+ax^2} dx, x, \tan(x) \right)}{4b^2} \\ &= \frac{\text{Subst} \left(\int \frac{x}{1+x^2} dx, x, \tan(x) \right)}{2b} - \frac{a \text{Subst} \left(\int \frac{x}{a+2bx+ax^2} dx, x, \tan(x) \right)}{2b} \\ &= -\frac{\log(\cos(x))}{2b} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{a+2bx+ax^2} dx, x, \tan(x) \right) - \frac{\text{Subst} \left(\int \frac{2b+2ax}{a+2bx+ax^2} dx, x, \tan(x) \right)}{4b} \\ &= -\frac{\log(\cos(x))}{2b} - \frac{\log(a+2b \tan(x) + a \tan^2(x))}{4b} - \text{Subst} \left(\int \frac{1}{-4(a^2-b^2)-x^2} dx, x, 2b \tan(x) \right) \\ &= \frac{\tan^{-1} \left(\frac{2b+2a \tan(x)}{2\sqrt{a^2-b^2}} \right)}{2\sqrt{a^2-b^2}} - \frac{\log(\cos(x))}{2b} - \frac{\log(a+2b \tan(x) + a \tan^2(x))}{4b} \end{aligned}$$

Mathematica [A] time = 0.08, size = 55, normalized size = 1.00

$$\frac{\tan^{-1} \left(\frac{a \tan(x) + b}{\sqrt{a^2 - b^2}} \right)}{2\sqrt{a^2 - b^2}} - \frac{\log(a + b \sin(2x))}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^2/(a + b*Ssin[2*x]), x]

[Out] ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]]/(2*Sqrt[a^2 - b^2]) - Log[a + b*Ssin[2*x]]/(4*b)

fricas [B] time = 1.91, size = 320, normalized size = 5.82

$$\frac{\sqrt{-a^2 + b^2} b \log \left(-\frac{4(2a^2 - b^2) \cos(x)^4 - 4ab \cos(x) \sin(x) - 4(2a^2 - b^2) \cos(x)^2 + a^2 - 2b^2 + 2(2b \cos(x)^2 + 2(2a \cos(x)^3 - a \cos(x)) \sin(x) - b)}{4b^2 \cos(x)^4 - 4b^2 \cos(x)^2 - 4ab \cos(x) \sin(x) - a^2} \right)}{8(a^2b - b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a+b*sin(2*x)), x, algorithm="fricas")

[Out] [-1/8*(sqrt(-a^2 + b^2)*b*log(-(4*(2*a^2 - b^2)*cos(x)^4 - 4*a*b*cos(x)*sin(x) - 4*(2*a^2 - b^2)*cos(x)^2 + a^2 - 2*b^2 + 2*(2*b*cos(x)^2 + 2*(2*a*cos(x)^3 - a*cos(x))*sin(x) - b)))/(4*b^2*cos(x)^4 - 4*b^2*cos(x)^2 - 4*a*b*cos(x)*sin(x) - a^2))]

$$(x)^3 - a \cos(x)) \sin(x) - b \sqrt{-a^2 + b^2}) / (4b^2 \cos(x)^4 - 4b^2 \cos(x)^2 - 4ab \cos(x) \sin(x) - a^2) + (a^2 - b^2) \log(-4b^2 \cos(x)^4 + 4b^2 \cos(x)^2 + 4ab \cos(x) \sin(x) + a^2) / (a^2 b - b^3), -1/8 * (2 \sqrt{a^2 - b^2}) * b * \arctan(-(2a \cos(x) \sin(x) + b) \sqrt{a^2 - b^2}) / (2(a^2 - b^2) \cos(x)^2 - a^2 + b^2)) + (a^2 - b^2) \log(-4b^2 \cos(x)^4 + 4b^2 \cos(x)^2 + 4ab \cos(x) \sin(x) + a^2) / (a^2 b - b^3)]$$

giac [A] time = 0.16, size = 77, normalized size = 1.40

$$\frac{\pi \left[\frac{x}{\pi} + \frac{1}{2} \right] \operatorname{sgn}(a) + \arctan\left(\frac{a \tan(x) + b}{\sqrt{a^2 - b^2}}\right)}{2 \sqrt{a^2 - b^2}} - \frac{\log(a \tan(x)^2 + 2b \tan(x) + a)}{4b} + \frac{\log(\tan(x)^2 + 1)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a+b*sin(2*x)),x, algorithm="giac")

[Out] 1/2*(pi*floor(x/pi + 1/2)*sgn(a) + arctan((a*tan(x) + b)/sqrt(a^2 - b^2)))/sqrt(a^2 - b^2) - 1/4*log(a*tan(x)^2 + 2*b*tan(x) + a)/b + 1/4*log(tan(x)^2 + 1)/b

maple [A] time = 0.19, size = 69, normalized size = 1.25

$$-\frac{\ln(a + 2b \tan(x) + a (\tan^2(x)))}{4b} + \frac{\arctan\left(\frac{2a \tan(x) + 2b}{2\sqrt{a^2 - b^2}}\right)}{2\sqrt{a^2 - b^2}} + \frac{\ln(1 + \tan^2(x))}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2/(a+b*sin(2*x)),x)

[Out] -1/4*ln(a+2*b*tan(x)+a*tan(x)^2)/b+1/2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(x)+2*b)/(a^2-b^2)^(1/2))+1/4/b*ln(1+tan(x)^2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a+b*sin(2*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more details)Is 4*b^2-4*a^2 positive or negative?

mupad [B] time = 4.18, size = 1108, normalized size = 20.15

$$\frac{\ln(\tan(x)^2 + 1)}{4b} + \operatorname{atan}\left(\frac{2 \tan(x) (a^2 - b^2)^{3/2}}{a^3 (4a^2 - 3b^2) + \frac{(4a^2b - 2b^3) \left(\frac{8ab^3 + \frac{(8a^2b - 8b^3)(96ab^4 - 64a^3b^2)}{2(16b^4 - 16a^2b^2)}}{4\sqrt{a^2 - b^2}} + \frac{(8a^2b - 8b^3)(96ab^4 - 64a^3b^2)}{8\sqrt{a^2 - b^2}(16b^4 - 16a^2b^2)} + \frac{(8a^2b - 8b^3)}{4\sqrt{a^2 - b^2}} \right)}{a^3 (4a^2 - 3b^2)}}{a^3 (4a^2 - 3b^2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^2/(a + b*sin(2*x)),x)`

[Out] `log(tan(x)^2 + 1)/(4*b) + atan(((2*tan(x)*(a^2 - b^2)^(3/2))*(((4*a^2*b - 2*b^3)*(2*a*b - ((8*a*b^3 + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2)))/(2*(16*b^4 - 16*a^2*b^2))))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(8*(a^2 - b^2)^(1/2)*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b - 8*b^3)*(4*a^3 - 16*a*b^2 + ((8*a^2*b - 8*b^3)*(8*a*b^3 + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2)))/(2*(16*b^4 - 16*a^2*b^2)))))/(2*(16*b^4 - 16*a^2*b^2)))/(2*(16*b^4 - 16*a^2*b^2)) - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(32*(a^2 - b^2)*(16*b^4 - 16*a^2*b^2)))/(a^3*(4*a^2 - 3*b^2)^2) - ((4*a^4 + 2*b^4 - 5*a^2*b^2)*((4*a^3 - 16*a*b^2 + ((8*a^2*b - 8*b^3)*(8*a*b^3 + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2)))/(2*(16*b^4 - 16*a^2*b^2)))))/(2*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) - (96*a*b^4 - 64*a^3*b^2)/(64*(a^2 - b^2)^(3/2)) + ((8*a^2*b - 8*b^3)*((8*a*b^3 + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2)))/(2*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(8*(a^2 - b^2)^(1/2)*(16*b^4 - 16*a^2*b^2)))/(2*(16*b^4 - 16*a^2*b^2)))/(a^3*(a^2 - b^2)^(1/2)*(4*a^2 - 3*b^2)^2))/a + (2*(a^2 - b^2)*((6*a^2*b - (8*a^2*b^3*(8*a^2*b - 8*b^3)^2)/(16*b^4 - 16*a^2*b^2)^2)/(4*(a^2 - b^2)^(1/2)) + (a^2*b^3)/(2*(a^2 - b^2)^(3/2)) - (4*a^2*b^3*(8*a^2*b - 8*b^3)^2)/((a^2 - b^2)^(1/2)*(16*b^4 - 16*a^2*b^2)^2))*((4*a^4 + 2*b^4 - 5*a^2*b^2))/(a^4*(4*a^2 - 3*b^2)^2) - (2*(4*a^2*b - 2*b^3)*(a^2 - b^2)^(3/2)*(((8*a^2*b - 8*b^3)*(6*a^2*b - (8*a^2*b^3*(8*a^2*b - 8*b^3)^2)/(16*b^4 - 16*a^2*b^2)^2))/(2*(16*b^4 - 16*a^2*b^2)) - a^2 + (3*a^2*b^3*(8*a^2*b - 8*b^3))/((a^2 - b^2)*(16*b^4 - 16*a^2*b^2))))/(a^4*(4*a^2 - 3*b^2)^2))/(2*(a^2 - b^2)^(1/2)) + (log(a + a*tan(x)^2 + 2*b*tan(x))*(8*a^2*b - 8*b^3))/(2*(16*b^4 - 16*a^2*b^2))`

sympy [A] time = 9.31, size = 155, normalized size = 2.82

$$- \left\{ \begin{array}{ll} \frac{\log\left(\frac{a}{b} + \sin(2x)\right)}{4b} & \text{for } b \neq 0 \\ \frac{\sin(2x)}{4a} & \text{otherwise} \end{array} \right. + \left\{ \begin{array}{ll} \frac{\sqrt{b^2}}{2b^2 \tan(x) - 2b\sqrt{b^2}} & \text{for } a = -\sqrt{b^2} \\ -\frac{\sqrt{b^2}}{2b^2 \tan(x) + 2b\sqrt{b^2}} & \text{for } a = \sqrt{b^2} \\ \frac{\log(\tan(x))}{4b} & \text{for } a = 0 \\ \frac{\log\left(\tan(x) + \frac{b}{a} - \frac{\sqrt{-a^2 + b^2}}{a}\right)}{4\sqrt{-a^2 + b^2}} - \frac{\log\left(\tan(x) + \frac{b}{a} + \frac{\sqrt{-a^2 + b^2}}{a}\right)}{4\sqrt{-a^2 + b^2}} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*sin(2*x)),x)
```

```
[Out] -Piecewise((log(a/b + sin(2*x))/(4*b), Ne(b, 0)), (sin(2*x)/(4*a), True)) +
Piecewise((sqrt(b**2)/(2*b**2*tan(x) - 2*b*sqrt(b**2)), Eq(a, -sqrt(b**2))),
(-sqrt(b**2)/(2*b**2*tan(x) + 2*b*sqrt(b**2)), Eq(a, sqrt(b**2))), (log(
tan(x))/(4*b), Eq(a, 0)), (log(tan(x) + b/a - sqrt(-a**2 + b**2)/a)/(4*sqrt
(-a**2 + b**2)) - log(tan(x) + b/a + sqrt(-a**2 + b**2)/a)/(4*sqrt(-a**2 +
b**2)), True))
```

$$3.847 \quad \int \frac{\cos^2(x)}{a+b \sin(2x)} dx$$

Optimal. Leaf size=55

$$\frac{\tan^{-1}\left(\frac{a \tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}} + \frac{\log(a+b \sin(2x))}{4b}$$

[Out] 1/4*ln(a+b*sin(2*x))/b+1/2*arctan((b+a*tan(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)

Rubi [A] time = 0.13, antiderivative size = 70, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {981, 634, 618, 204, 628, 12, 260}

$$\frac{\tan^{-1}\left(\frac{a \tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}} + \frac{\log(a \tan^2(x) + a + 2b \tan(x))}{4b} + \frac{\log(\cos(x))}{2b}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2/(a + b*Sin[2*x]),x]

[Out] ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]]/(2*Sqrt[a^2 - b^2]) + Log[Cos[x]]/(2*b) + Log[a + 2*b*Tan[x] + a*Tan[x]^2]/(4*b)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

`[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 981

```
Int[1/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*((d_) + (f_)*(x_)^2)), x_Symbol]
  :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Dist[1/q, Int[(c^2*
d + b^2*f - a*c*f + b*c*f*x)/(a + b*x + c*x^2), x], x] - Dist[1/q, Int[(c*d
*f - a*f^2 + b*f^2*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c,
d, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(x)}{a + b \sin(2x)} dx &= \text{Subst} \left(\int \frac{1}{(1+x^2)(a+2bx+ax^2)} dx, x, \tan(x) \right) \\ &= -\frac{\text{Subst} \left(\int \frac{2bx}{1+x^2} dx, x, \tan(x) \right)}{4b^2} + \frac{\text{Subst} \left(\int \frac{4b^2+2abx}{a+2bx+ax^2} dx, x, \tan(x) \right)}{4b^2} \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{a+2bx+ax^2} dx, x, \tan(x) \right) + \frac{\text{Subst} \left(\int \frac{2b+2ax}{a+2bx+ax^2} dx, x, \tan(x) \right)}{4b} - \frac{\text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right)}{4b} \\ &= \frac{\log(\cos(x))}{2b} + \frac{\log(a+2b \tan(x) + a \tan^2(x))}{4b} - \text{Subst} \left(\int \frac{1}{-4(a^2-b^2)-x^2} dx, x, 2b+2a \tan(x) \right) \\ &= \frac{\tan^{-1} \left(\frac{2b+2a \tan(x)}{2\sqrt{a^2-b^2}} \right)}{2\sqrt{a^2-b^2}} + \frac{\log(\cos(x))}{2b} + \frac{\log(a+2b \tan(x) + a \tan^2(x))}{4b} \end{aligned}$$

Mathematica [A] time = 0.06, size = 54, normalized size = 0.98

$$\frac{1}{4} \left(\frac{2 \tan^{-1} \left(\frac{a \tan(x) + b}{\sqrt{a^2 - b^2}} \right)}{\sqrt{a^2 - b^2}} + \frac{\log(a + b \sin(2x))}{b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^2/(a + b*Sin[2*x]),x]

[Out] ((2*ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + Log[a + b*Sin[2*x]]/b)/4

fricas [B] time = 1.28, size = 322, normalized size = 5.85

$$\left[\frac{\sqrt{-a^2 + b^2} b \log \left(-\frac{4(2a^2 - b^2) \cos(x)^4 - 4ab \cos(x) \sin(x) - 4(2a^2 - b^2) \cos(x)^2 + a^2 - 2b^2 + 2(2b \cos(x)^2 + 2(2a \cos(x)^3 - a \cos(x)) \sin(x) - b) \sqrt{-a^2 + b^2}}{4b^2 \cos(x)^4 - 4b^2 \cos(x)^2 - 4ab \cos(x) \sin(x) - a^2} \right)}{8(a^2b - b^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2/(a+b*sin(2*x)),x, algorithm="fricas")

[Out] [-1/8*(sqrt(-a^2 + b^2)*b*log(-(4*(2*a^2 - b^2)*cos(x)^4 - 4*a*b*cos(x)*sin(x) - 4*(2*a^2 - b^2)*cos(x)^2 + a^2 - 2*b^2 + 2*(2*b*cos(x)^2 + 2*(2*a*cos(x)^3 - a*cos(x))*sin(x) - b)*sqrt(-a^2 + b^2))/(4*b^2*cos(x)^4 - 4*b^2*cos(x)^2 - 4*a*b*cos(x)*sin(x) - a^2)) - (a^2 - b^2)*log(-4*b^2*cos(x)^4 + 4*b^2*cos(x)^2 + 4*a*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3), -1/8*(2*sqrt(a^2 -

$$b^2) * b * \arctan(-2 * a * \cos(x) * \sin(x) + b) * \sqrt{a^2 - b^2} / (2 * (a^2 - b^2) * \cos(x)^2 - a^2 + b^2) - (a^2 - b^2) * \log(-4 * b^2 * \cos(x)^4 + 4 * b^2 * \cos(x)^2 + 4 * a * b * \cos(x) * \sin(x) + a^2) / (a^2 * b - b^3)]$$

giac [A] time = 0.16, size = 77, normalized size = 1.40

$$\frac{\pi \left[\frac{x}{\pi} + \frac{1}{2} \right] \operatorname{sgn}(a) + \arctan\left(\frac{a \tan(x) + b}{\sqrt{a^2 - b^2}}\right)}{2 \sqrt{a^2 - b^2}} + \frac{\log(a \tan(x)^2 + 2 b \tan(x) + a)}{4 b} - \frac{\log(\tan(x)^2 + 1)}{4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2/(a+b*sin(2*x)),x, algorithm="giac")

[Out] 1/2*(pi*floor(x/pi + 1/2)*sgn(a) + arctan((a*tan(x) + b)/sqrt(a^2 - b^2)))/sqrt(a^2 - b^2) + 1/4*log(a*tan(x)^2 + 2*b*tan(x) + a)/b - 1/4*log(tan(x)^2 + 1)/b

maple [A] time = 0.17, size = 69, normalized size = 1.25

$$\frac{\ln(a + 2b \tan(x) + a (\tan^2(x)))}{4b} + \frac{\arctan\left(\frac{2a \tan(x) + 2b}{2\sqrt{a^2 - b^2}}\right)}{2\sqrt{a^2 - b^2}} - \frac{\ln(1 + \tan^2(x))}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2/(a+b*sin(2*x)),x)

[Out] 1/4*ln(a+2*b*tan(x)+a*tan(x)^2)/b+1/2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(x)+2*b)/(a^2-b^2)^(1/2))-1/4/b*ln(1+tan(x)^2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^2/(a+b*sin(2*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more details)Is 4*b^2-4*a^2 positive or negative?

mupad [B] time = 3.44, size = 1374, normalized size = 24.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2/(a + b*sin(2*x)),x)

[Out] - log(tan(x)^2 + 1)/(4*b) - atan((2*tan(x)*((6*b*(2*a^2 - 3*b^2)*((24*a*b^3 - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2))))/(4*(a^2 - b^2)^(1/2)) - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(8*(a^2 - b^2)^(1/2)*(16*b^4 - 16*a^2*b^2))))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b - 8*b^3)*(4*a^3 - ((8*a^2*b - 8*b^3)*(24*a*b^3 - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2)) - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(32*(a^2 - b^2)*(16*b^4 - 16*a^2*b^2))))/(a^3*(4*a^2 - 3*b^2)^2) - (((96*a*b^4 - 64*a^3*b^2)/(64*(a^2 - b^2)^(3/2)) - (4*a^3 - ((8*a^2*b - 8*b^3)*(24*a*b^3 - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2))))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b - 8*b^3)*((24*a*

$$b^3 - ((8a^2b - 8b^3)(96ab^4 - 64a^3b^2))/(2(16b^4 - 16a^2b^2))$$

$$)/(4(a^2 - b^2)^{1/2}) - ((8a^2b - 8b^3)(96ab^4 - 64a^3b^2))/(8(a^2 - b^2)^{1/2}(16b^4 - 16a^2b^2)))/(2(16b^4 - 16a^2b^2))(4a^4 + 18b^4 - 21a^2b^2)/(a^3(a^2 - b^2)^{1/2}(4a^2 - 3b^2)^2)(a^2 - b^2)^{3/2})/a - (2(a^2 - b^2)((a^2b^3)/(2(a^2 - b^2)^{3/2}) - (2a^2b - (8a^2b - 8b^3)(16a^2b^2 - (16a^2b^3(8a^2b - 8b^3))/(16b^4 - 16a^2b^2)))/(2(16b^4 - 16a^2b^2)))/(4(a^2 - b^2)^{1/2}) + ((8a^2b - 8b^3)((16a^2b^2 - (16a^2b^3(8a^2b - 8b^3))/(16b^4 - 16a^2b^2)))/(4(a^2 - b^2)^{1/2}) - (4a^2b^3(8a^2b - 8b^3))/((a^2 - b^2)^{1/2})(16b^4 - 16a^2b^2)))/(2(16b^4 - 16a^2b^2))(4a^4 + 18b^4 - 21a^2b^2))/(a^4(4a^2 - 3b^2)^2) + (12b(a^2 - b^2)^{3/2}(2a^2 - 3b^2)((16a^2b^2 - (16a^2b^3(8a^2b - 8b^3))/(16b^4 - 16a^2b^2))/(4(a^2 - b^2)^{1/2}) - (4a^2b^3(8a^2b - 8b^3))/((a^2 - b^2)^{1/2})(16b^4 - 16a^2b^2)))/(4(a^2 - b^2)^{1/2}) + ((8a^2b - 8b^3)(2a^2b - (8a^2b - 8b^3)(16a^2b^2 - (16a^2b^3(8a^2b - 8b^3))/(16b^4 - 16a^2b^2)))/(2(16b^4 - 16a^2b^2)))/(2(16b^4 - 16a^2b^2)) - (a^2b^3(8a^2b - 8b^3))/((a^2 - b^2)(16b^4 - 16a^2b^2)))/(a^4(4a^2 - 3b^2)^2))/(2(a^2 - b^2)^{1/2}) - (\log(a + a\tan(x)^2 + 2b\tan(x))(8a^2b - 8b^3))/(2(16b^4 - 16a^2b^2))$$

sympy [A] time = 9.45, size = 155, normalized size = 2.82

$$\begin{cases} \frac{\log\left(\frac{a}{b} + \sin(2x)\right)}{4b} & \text{for } b \neq 0 \\ \frac{\sin(2x)}{4a} & \text{otherwise} \end{cases} + \begin{cases} \frac{\sqrt{b^2}}{2b^2 \tan(x) - 2b\sqrt{b^2}} & \text{for } a = -\sqrt{b^2} \\ -\frac{\sqrt{b^2}}{2b^2 \tan(x) + 2b\sqrt{b^2}} & \text{for } a = \sqrt{b^2} \\ \frac{\log(\tan(x))}{4b} & \text{for } a = 0 \\ \frac{\log\left(\tan(x) + \frac{b}{a} - \frac{\sqrt{-a^2 + b^2}}{a}\right)}{4\sqrt{-a^2 + b^2}} - \frac{\log\left(\tan(x) + \frac{b}{a} + \frac{\sqrt{-a^2 + b^2}}{a}\right)}{4\sqrt{-a^2 + b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(cos(x)**2/(a+b*sin(2*x)),x)
[Out] Piecewise((log(a/b + sin(2*x))/(4*b), Ne(b, 0)), (sin(2*x)/(4*a), True)) +
Piecewise((sqrt(b**2)/(2*b**2*tan(x) - 2*b*sqrt(b**2)), Eq(a, -sqrt(b**2))),
(-sqrt(b**2)/(2*b**2*tan(x) + 2*b*sqrt(b**2)), Eq(a, sqrt(b**2))), (log(tan(x))/(4*b), Eq(a, 0)), (log(tan(x) + b/a - sqrt(-a**2 + b**2)/a)/(4*sqrt(-a**2 + b**2)) - log(tan(x) + b/a + sqrt(-a**2 + b**2)/a)/(4*sqrt(-a**2 + b**2))), True))

```


$$3.848 \quad \int \frac{\sin^2(x)}{a+b \cos(2x)} dx$$

Optimal. Leaf size=52

$$\frac{\sqrt{a+b} \tan^{-1}\left(\frac{\sqrt{a-b} \tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a-b}} - \frac{x}{2b}$$

[Out] $-1/2*x/b+1/2*\arctan((a-b)^{(1/2)*\tan(x)/(a+b)^{(1/2)})*(a+b)^{(1/2)}/b/(a-b)^{(1/2)})$

Rubi [A] time = 0.12, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1130, 205}

$$\frac{\sqrt{a+b} \tan^{-1}\left(\frac{\sqrt{a-b} \tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a-b}} - \frac{x}{2b}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^2/(a + b*Cos[2*x]), x]

[Out] $-x/(2*b) + (\text{Sqrt}[a + b]*\text{ArcTan}[(\text{Sqrt}[a - b]*\text{Tan}[x])/(\text{Sqrt}[a + b])])/(2*\text{Sqrt}[a - b]*b)$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1130

Int[((d_.)*(x_)^(m_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 - q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 2]

Rubi steps

$$\begin{aligned} \int \frac{\sin^2(x)}{a+b \cos(2x)} dx &= \text{Subst}\left(\int \frac{x^2}{a+b+2ax^2+(a-b)x^4} dx, x, \tan(x)\right) \\ &= -\left(\frac{1}{2}\left(-1+\frac{a}{b}\right)\text{Subst}\left(\int \frac{1}{a-b+(a-b)x^2} dx, x, \tan(x)\right)\right) + \frac{(a+b)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan(x)\right)}{2b} \\ &= -\frac{x}{2b} + \frac{\sqrt{a+b} \tan^{-1}\left(\frac{\sqrt{a-b} \tan(x)}{\sqrt{a+b}}\right)}{2\sqrt{a-b}b} \end{aligned}$$

Mathematica [A] time = 0.09, size = 48, normalized size = 0.92

$$\frac{(a+b) \tanh^{-1}\left(\frac{(a-b) \tan(x)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + x$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^2/(a + b*Cos[2*x]),x]

[Out] $-1/2*(x + ((a + b)*ArcTanh[((a - b)*Tan[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2])/b$

fricas [A] time = 1.22, size = 225, normalized size = 4.33

$$\frac{\sqrt{-\frac{a+b}{a-b}} \log\left(\frac{4(2a^2-b^2)\cos(x)^4 - 4(2a^2-ab-b^2)\cos(x)^2 - 4(2(a^2-ab)\cos(x)^3 - (a^2-2ab+b^2)\cos(x))\sqrt{-\frac{a+b}{a-b}}\sin(x) + a^2 - 2ab + b^2}{4b^2\cos(x)^4 + 4(ab-b^2)\cos(x)^2 + a^2 - 2ab + b^2}\right) - 4x}{8b}, -\sqrt{\frac{a+b}{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a+b*cos(2*x)),x, algorithm="fricas")

[Out] $[1/8*(\sqrt{-(a+b)/(a-b)}*\log((4*(2*a^2 - b^2)*\cos(x)^4 - 4*(2*a^2 - a*b - b^2)*\cos(x)^2 - 4*(2*(a^2 - a*b)*\cos(x)^3 - (a^2 - 2*a*b + b^2)*\cos(x))*\sqrt{-(a+b)/(a-b)}*\sin(x) + a^2 - 2*a*b + b^2)/(4*b^2*\cos(x)^4 + 4*(a*b - b^2)*\cos(x)^2 + a^2 - 2*a*b + b^2)) - 4*x)/b, -1/4*(\sqrt{(a+b)/(a-b)})*\arctan(1/2*(2*a*\cos(x)^2 - a + b)*\sqrt{(a+b)/(a-b)})/((a+b)*\cos(x)*\sin(x))) + 2*x)/b]$

giac [B] time = 0.17, size = 141, normalized size = 2.71

$$\frac{\sqrt{a^2 - b^2} \left(\pi \left[\frac{x}{\pi} + \frac{1}{2} \right] + \arctan \left(\frac{2 \tan(x)}{\sqrt{\frac{4a + \sqrt{-16(a+b)(a-b) + 16a^2}}{a-b}}} \right) \right) |a-b| + \pi \left[\frac{x}{\pi} + \frac{1}{2} \right] + \arctan \left(\frac{2 \tan(x)}{\sqrt{\frac{4a - \sqrt{-16(a+b)(a-b) + 16a^2}}{a-b}}} \right)}{2(a^2 - 2ab + b^2)|b|} \quad \frac{2|b|}{2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a+b*cos(2*x)),x, algorithm="giac")

[Out] $1/2*\sqrt{a^2 - b^2}*(\pi*\text{floor}(x/\pi + 1/2) + \arctan(2*\tan(x)/\sqrt{(4*a + \sqrt{-16*(a+b)*(a-b) + 16*a^2})/(a-b)})))*\text{abs}(a-b)/((a^2 - 2*a*b + b^2)*\text{abs}(b)) - 1/2*(\pi*\text{floor}(x/\pi + 1/2) + \arctan(2*\tan(x)/\sqrt{(4*a - \sqrt{-16*(a+b)*(a-b) + 16*a^2})/(a-b)})))/\text{abs}(b)$

maple [A] time = 0.13, size = 80, normalized size = 1.54

$$\frac{\arctan\left(\frac{\tan(x)(a-b)}{\sqrt{(a+b)(a-b)}}\right)a}{2b\sqrt{(a+b)(a-b)}} + \frac{\arctan\left(\frac{\tan(x)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{2\sqrt{(a+b)(a-b)}} - \frac{\arctan(\tan(x))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2/(a+b*cos(2*x)),x)

[Out] $1/2/b/(((a+b)*(a-b))^(1/2)*\arctan(\tan(x)*(a-b)/((a+b)*(a-b))^(1/2)))*a + 1/2/(((a+b)*(a-b))^(1/2)*\arctan(\tan(x)*(a-b)/((a+b)*(a-b))^(1/2)) - 1/2/b*\arctan(\tan(x)))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a+b*cos(2*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 3.43, size = 108, normalized size = 2.08

$$\frac{\operatorname{atan}\left(\frac{2b^3 \tan(x)}{2a^2 b - 2b^3} - \frac{2a^2 b \tan(x)}{2a^2 b - 2b^3}\right)}{2b} + \frac{\operatorname{atanh}\left(\frac{a \tan(x)}{\sqrt{b^2 - a^2}} - \frac{b \tan(x)}{\sqrt{b^2 - a^2}}\right) \sqrt{b^2 - a^2}}{2(a b - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^2/(a + b*cos(2*x)), x)`

[Out] `atan((2*b^3*tan(x))/(2*a^2*b - 2*b^3) - (2*a^2*b*tan(x))/(2*a^2*b - 2*b^3))/(2*b) + (atanh((a*tan(x))/(b^2 - a^2)^(1/2) - (b*tan(x))/(b^2 - a^2)^(1/2)))*(b^2 - a^2)^(1/2))/(2*(a*b - b^2))`

sympy [B] time = 35.27, size = 432, normalized size = 8.31

$$\left\{ \begin{array}{ll} \tilde{\infty} \left(-\frac{\log(\tan(x)-1)}{2} + \frac{\log(\tan(x)+1)}{2} \right) & \text{for } a = 0 \wedge b = 0 \\ \frac{1}{4b \tan(x)} & \text{for } a = -b \\ \frac{\tan(x)}{4b} & \text{for } a = b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}} + \tan(x)\right)}{4a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}} - 4b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}} + \tan(x)\right)}{4a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}} - 4b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{array} \right. \quad \left\{ \begin{array}{l} \tilde{\infty} x \\ \frac{x}{2b} - \frac{\tan(x)}{4b} \\ \frac{x}{2b} + \frac{1}{4b \tan(x)} \\ \frac{\sin(2x)}{4a} \\ \frac{2ax\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}} - 4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{a \log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}} + \tan(x)\right)}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}} - 4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)**2/(a+b*cos(2*x)), x)`

[Out] `Piecewise((zoo*(-log(tan(x) - 1)/2 + log(tan(x) + 1)/2), Eq(a, 0) & Eq(b, 0)), (1/(4*b*tan(x)), Eq(a, -b)), (tan(x)/(4*b), Eq(a, b)), (log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*sqrt(-a/(a - b) - b/(a - b)) - 4*b*sqrt(-a/(a - b) - b/(a - b))) - log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*sqrt(-a/(a - b) - b/(a - b)) - 4*b*sqrt(-a/(a - b) - b/(a - b))), True)) - Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/(2*b) - tan(x)/(4*b), Eq(a, b)), (x/(2*b) + 1/(4*b*tan(x)), Eq(a, -b)), (sin(2*x)/(4*a), Eq(b, 0)), (2*a*x*sqrt(-a/(a - b) - b/(a - b))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b))) + a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b))) - 2*b*x*sqrt(-a/(a - b) - b/(a - b))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b))), True))`

$$3.849 \quad \int \frac{\cos^2(x)}{a+b \cos(2x)} dx$$

Optimal. Leaf size=52

$$\frac{x}{2b} - \frac{\sqrt{a-b} \tan^{-1}\left(\frac{\sqrt{a-b} \tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

[Out] 1/2*x/b-1/2*arctan((a-b)^(1/2)*tan(x)/(a+b)^(1/2))*(a-b)^(1/2)/b/(a+b)^(1/2)

Rubi [A] time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1093, 205}

$$\frac{x}{2b} - \frac{\sqrt{a-b} \tan^{-1}\left(\frac{\sqrt{a-b} \tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2/(a + b*Cos[2*x]),x]

[Out] x/(2*b) - (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[x])/Sqrt[a + b]])/(2*b*Sqrt[a + b])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(x)}{a+b \cos(2x)} dx &= \text{Subst}\left(\int \frac{1}{a+b+2ax^2+(a-b)x^4} dx, x, \tan(x)\right) \\ &= \frac{(a-b) \text{Subst}\left(\int \frac{1}{a-b+(a-b)x^2} dx, x, \tan(x)\right)}{2b} - \frac{(a-b) \text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan(x)\right)}{2b} \\ &= \frac{x}{2b} - \frac{\sqrt{a-b} \tan^{-1}\left(\frac{\sqrt{a-b} \tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.96

$$\frac{(a-b) \tanh^{-1}\left(\frac{(a-b) \tan(x)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + x$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^2/(a + b*Cos[2*x]),x]

[Out] $(x + ((a - b) \operatorname{ArcTanh}(((a - b) \tan(x)) / \sqrt{-a^2 + b^2}))) / \sqrt{-a^2 + b^2} / (2b)$

fricas [A] time = 1.11, size = 224, normalized size = 4.31

$$\frac{\sqrt{\frac{a-b}{a+b}} \log\left(\frac{4(2a^2-b^2)\cos(x)^4 - 4(2a^2-ab-b^2)\cos(x)^2 + 4(2(a^2+ab)\cos(x)^3 - (a^2-b^2)\cos(x))\sqrt{\frac{a-b}{a+b}}\sin(x) + a^2 - 2ab + b^2}{4b^2\cos(x)^4 + 4(ab-b^2)\cos(x)^2 + a^2 - 2ab + b^2}\right) + 4x}{8b}, \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^2/(a+b*cos(2*x)),x, algorithm="fricas")`

[Out] $[1/8 * (\sqrt{-(a-b)/(a+b)}) * \log((4 * (2 * a^2 - b^2) * \cos(x)^4 - 4 * (2 * a^2 - a * b - b^2) * \cos(x)^2 + 4 * (2 * (a^2 + a * b) * \cos(x)^3 - (a^2 - b^2) * \cos(x)) * \sqrt{-(a-b)/(a+b)} * \sin(x) + a^2 - 2 * a * b + b^2) / (4 * b^2 * \cos(x)^4 + 4 * (a * b - b^2) * \cos(x)^2 + a^2 - 2 * a * b + b^2)) + 4 * x] / b, -1/4 * (\sqrt{(a-b)/(a+b)}) * \arctan(-1/2 * (2 * a * \cos(x)^2 - a + b) * \sqrt{(a-b)/(a+b)}) / ((a-b) * \cos(x) * \sin(x)) - 2 * x) / b]$

giac [B] time = 0.15, size = 159, normalized size = 3.06

$$\frac{\sqrt{a^2 - b^2} \left(\pi \left[\frac{x}{\pi} + \frac{1}{2} \right] + \arctan \left(\frac{2 \tan(x)}{\sqrt{\frac{4a + \sqrt{-16(a+b)(a-b) + 16a^2}}{a-b}}} \right) \right) |a-b|}{2 \left((a-b)b^2 + (a^2 - ab)|b| \right)} - \frac{\left(\pi \left[\frac{x}{\pi} + \frac{1}{2} \right] + \arctan \left(\frac{2 \tan(x)}{\sqrt{\frac{4a - \sqrt{-16(a+b)(a-b) + 16a^2}}{a-b}}} \right) \right)}{2 \left(b^2 - a|b| \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^2/(a+b*cos(2*x)),x, algorithm="giac")`

[Out] $-1/2 * \sqrt{a^2 - b^2} * (\pi * \text{floor}(x/\pi + 1/2) + \arctan(2 * \tan(x) / \sqrt{(4 * a + \sqrt{-16 * (a + b) * (a - b) + 16 * a^2}) / (a - b)})) * \text{abs}(a - b) / ((a - b) * b^2 + (a^2 - a * b) * \text{abs}(b)) - 1/2 * (\pi * \text{floor}(x/\pi + 1/2) + \arctan(2 * \tan(x) / \sqrt{(4 * a - \sqrt{-16 * (a + b) * (a - b) + 16 * a^2}) / (a - b)})) * (a - b) / (b^2 - a * \text{abs}(b))$

maple [A] time = 0.13, size = 80, normalized size = 1.54

$$-\frac{\arctan\left(\frac{\tan(x)(a-b)}{\sqrt{(a+b)(a-b)}}\right) a}{2b\sqrt{(a+b)(a-b)}} + \frac{\arctan\left(\frac{\tan(x)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{2\sqrt{(a+b)(a-b)}} + \frac{\arctan(\tan(x))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)^2/(a+b*cos(2*x)),x)`

[Out] $-1/2/b/((a+b)*(a-b))^{(1/2)} * \arctan(\tan(x)*(a-b)/((a+b)*(a-b))^{(1/2)}) * a + 1/2/((a+b)*(a-b))^{(1/2)} * \arctan(\tan(x)*(a-b)/((a+b)*(a-b))^{(1/2)}) + 1/2/b * \arctan(\tan(x))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)^2/(a+b*cos(2*x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 3.32, size = 684, normalized size = 13.15

$$\operatorname{atan}\left(\frac{2a^2 \tan(x)}{2a^2-4ab+2b^2} + \frac{2b^2 \tan(x)}{2a^2-4ab+2b^2} - \frac{4ab \tan(x)}{2a^2-4ab+2b^2}\right) + \operatorname{atan}\left(\frac{\frac{\tan(x)(4a^3-12a^2b+12ab^2-4b^3)}{4} + \frac{\sqrt{b^2-a^2}\left(4b^4-8ab^3+4a^2b^2 + \frac{\tan(x)\sqrt{b^2-a^2}(64a^3b^2-12a^2b^3+4a^3b^2)}{16}\right)}{4(b^2+ab)}}{b^2+ab}}{\frac{\tan(x)(4a^3-12a^2b+12ab^2-4b^3)}{4} + \frac{\sqrt{b^2-a^2}\left(4b^4-8ab^3+4a^2b^2 + \frac{\tan(x)\sqrt{b^2-a^2}(64a^3b^2-12a^2b^3+4a^3b^2)}{16}\right)}{4(b^2+ab)}}{b^2+ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^2/(a + b*cos(2*x)),x)

[Out] atan((2*a^2*tan(x))/(2*a^2 - 4*a*b + 2*b^2) + (2*b^2*tan(x))/(2*a^2 - 4*a*b + 2*b^2) - (4*a*b*tan(x))/(2*a^2 - 4*a*b + 2*b^2))/(2*b) + (atan((((tan(x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))/4 + ((b^2 - a^2)^(1/2)*(4*b^4 - 8*a*b^3 + 4*a^2*b^2 + (tan(x)*(b^2 - a^2)^(1/2)*(64*a*b^4 - 128*a^2*b^3 + 64*a^3*b^2)))/(16*(a*b + b^2)))))/(4*(a*b + b^2)))*(b^2 - a^2)^(1/2)*1i)/(a*b + b^2) + (((tan(x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))/4 + ((b^2 - a^2)^(1/2)*(8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (tan(x)*(b^2 - a^2)^(1/2)*(64*a*b^4 - 128*a^2*b^3 + 64*a^3*b^2)))/(16*(a*b + b^2)))))/(4*(a*b + b^2)))*(b^2 - a^2)^(1/2)*1i)/(a*b + b^2)/((((tan(x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))/4 + ((b^2 - a^2)^(1/2)*(4*b^4 - 8*a*b^3 + 4*a^2*b^2 + (tan(x)*(b^2 - a^2)^(1/2)*(64*a*b^4 - 128*a^2*b^3 + 64*a^3*b^2)))/(16*(a*b + b^2)))))/(4*(a*b + b^2)))*(b^2 - a^2)^(1/2))/(a*b + b^2) - (((tan(x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))/4 + ((b^2 - a^2)^(1/2)*(8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (tan(x)*(b^2 - a^2)^(1/2)*(64*a*b^4 - 128*a^2*b^3 + 64*a^3*b^2)))/(16*(a*b + b^2)))))/(4*(a*b + b^2)))*(b^2 - a^2)^(1/2))/(a*b + b^2))*(b^2 - a^2)^(1/2)*1i)/(2*(a*b + b^2))

sympy [B] time = 35.33, size = 432, normalized size = 8.31

$$\begin{cases} \infty \left(-\frac{\log(\tan(x)-1)}{2} + \frac{\log(\tan(x)+1)}{2} \right) & \text{for } a = 0 \wedge b = 0 \\ \frac{1}{4b \tan(x)} & \text{for } a = -b \\ \frac{\tan(x)}{4b} & \text{for } a = b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}} + \tan(x)\right)}{4a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}} - 4b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}} + \tan(x)\right)}{4a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}} - 4b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases} + \begin{cases} \infty x \\ \frac{x}{2b} - \frac{\tan(x)}{4b} \\ \frac{x}{2b} + \frac{1}{4b \tan(x)} \\ \frac{\sin(2x)}{4a} \\ \frac{2ax\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}} - 4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{a \log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}} + \tan(x)\right)}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}} - 4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**2/(a+b*cos(2*x)),x)

[Out] Piecewise((zoo*(-log(tan(x) - 1)/2 + log(tan(x) + 1)/2), Eq(a, 0) & Eq(b, 0)), (1/(4*b*tan(x)), Eq(a, -b)), (tan(x)/(4*b), Eq(a, b)), (log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*sqrt(-a/(a - b) - b/(a - b)) - 4*b*sqrt(-a/(a - b) - b/(a - b))) - log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*sqrt(-a/(a - b) - b/(a - b)) - 4*b*sqrt(-a/(a - b) - b/(a - b))), True)) + Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/(2*b) - tan(x)/(4*b), Eq(a, b)),

```
(x/(2*b) + 1/(4*b*tan(x)), Eq(a, -b)), (sin(2*x)/(4*a), Eq(b, 0)), (2*a*x*sqrt(-a/(a - b) - b/(a - b))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b))) + a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b))) - 2*b*x*sqrt(-a/(a - b) - b/(a - b))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b))), True))
```

$$3.850 \quad \int \frac{\tan(c+dx)}{\sqrt{a \sin^2(c+dx)}} dx$$

Optimal. Leaf size=30

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a \sin^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

[Out] arctanh((a*sin(d*x+c)^2)^(1/2)/a^(1/2))/d/a^(1/2)

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3205, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a \sin^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]/Sqrt[a*Sin[c + d*x]^2],x]

[Out] ArcTanh[Sqrt[a*Sin[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d)

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p]/(1 - ff*x)^((m
+ 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned} \int \frac{\tan(c+dx)}{\sqrt{a \sin^2(c+dx)}} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}} dx, x, \sin^2(c+dx)\right)}{2d} \\ &= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}} dx, x, \sqrt{a \sin^2(c+dx)}\right)}{ad} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a \sin^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} \end{aligned}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 1.03

$$\frac{\sin(c+dx) \tanh^{-1}(\sin(c+dx))}{d\sqrt{a \sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tan[c + d*x]/Sqrt[a*Sin[c + d*x]^2], x]

[Out] (ArcTanh[Sin[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a*Sin[c + d*x]^2])

fricas [A] time = 0.89, size = 91, normalized size = 3.03

$$\left[\frac{\sqrt{-a \cos(dx+c)^2 + a} \log\left(-\frac{\sin(dx+c)+1}{\sin(dx+c)-1}\right)}{2ad \sin(dx+c)}, -\frac{\sqrt{-a} \arctan\left(\frac{\sqrt{-a \cos(dx+c)^2 + a} \sqrt{-a}}{a}\right)}{ad} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2), x, algorithm="fricas")

[Out] [1/2*sqrt(-a*cos(d*x + c)^2 + a)*log(-(sin(d*x + c) + 1)/(sin(d*x + c) - 1))/(a*d*sin(d*x + c)), -sqrt(-a)*arctan(sqrt(-a*cos(d*x + c)^2 + a)*sqrt(-a)/a)/(a*d)]

giac [B] time = 0.35, size = 61, normalized size = 2.03

$$\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right)}{\text{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)} - \frac{\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)}{\text{sgn}\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}}{\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2), x, algorithm="giac")

[Out] (log(abs(tan(1/2*d*x + 1/2*c) + 1))/sgn(tan(1/2*d*x + 1/2*c)) - log(abs(tan(1/2*d*x + 1/2*c) - 1))/sgn(tan(1/2*d*x + 1/2*c)))/(sqrt(a)*d)

maple [A] time = 0.15, size = 30, normalized size = 1.00

$$\frac{\sin(dx+c) \text{arctanh}(\sin(dx+c))}{\sqrt{a(\sin^2(dx+c))}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x)`

[Out] `1/(a*sin(d*x+c)^2)^(1/2)*sin(d*x+c)*arctanh(sin(d*x+c))/d`

maxima [B] time = 0.43, size = 76, normalized size = 2.53

$$\frac{\frac{(-1)^{2a \sin(dx+c)} \log\left(-\frac{a \sin(dx+c)}{\sin(dx+c)+1}\right)}{\sqrt{a}} + \frac{(-1)^{2a \sin(dx+c)} \log\left(-\frac{a \sin(dx+c)}{\sin(dx+c)-1}\right)}{\sqrt{a}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x, algorithm="maxima")`

[Out] `1/2*((-1)^(2*a*sin(d*x + c))*log(-a*sin(d*x + c)/(sin(d*x + c) + 1))/sqrt(a) + (-1)^(2*a*sin(d*x + c))*log(-a*sin(d*x + c)/(sin(d*x + c) - 1))/sqrt(a))/d`

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan(c + dx)}{\sqrt{a \sin(c + dx)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tan(c + d*x)/(a*sin(c + d*x)^2)^(1/2),x)`

[Out] `int(tan(c + d*x)/(a*sin(c + d*x)^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c + dx)}{\sqrt{a \sin^2(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tan(d*x+c)/(a*sin(d*x+c)**2)**(1/2),x)`

[Out] `Integral(tan(c + d*x)/sqrt(a*sin(c + d*x)**2), x)`

$$3.851 \quad \int \frac{\cot(c+dx)}{\sqrt{a \cos^2(c+dx)}} dx$$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a \cos^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a} d}$$

[Out] $-\operatorname{arctanh}((a \cos(d*x+c)^2)^{(1/2)/a^{(1/2)})/d/a^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3205, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a \cos^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a} d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cot}[c + d*x]/\operatorname{Sqrt}[a*\operatorname{Cos}[c + d*x]^2], x]$

[Out] $-(\operatorname{ArcTanh}[\operatorname{Sqrt}[a*\operatorname{Cos}[c + d*x]^2]/\operatorname{Sqrt}[a]]/(\operatorname{Sqrt}[a]*d))$

Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] :> \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)*(c-(a*d)/b+(d*x^p)/b)^n}, x], x, (a+b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 206

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x_Symbol] :> \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \mid \mid \operatorname{LtQ}[b, 0])$

Rule 3205

$\operatorname{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}]^{(p_.)*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)}, x_Symbol] :> \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Sin}[e + f*x]^2, x]\}, \operatorname{Dist}[ff^{((m+1)/2)/(2*f)}, \operatorname{Subst}[\operatorname{Int}[x^{((m-1)/2)*(b*ff^{(n/2)*x^{(n/2)})^p}/(1-ff*x)^{(m+1)/2}], x], x, \operatorname{Sin}[e + f*x]^2/ff], x]] /; \operatorname{FreeQ}[\{b, e, f, p\}, x] \&\& \operatorname{IntegerQ}[(m-1)/2] \&\& \operatorname{IntegerQ}[n/2]$

Rubi steps

$$\begin{aligned} \int \frac{\cot(c+dx)}{\sqrt{a \cos^2(c+dx)}} dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}} dx, x, \cos^2(c+dx)\right)}{2d} \\ &= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}} dx, x, \sqrt{a \cos^2(c+dx)}\right)}{ad} \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a \cos^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a} d} \end{aligned}$$

Mathematica [A] time = 0.06, size = 49, normalized size = 1.58

$$\frac{\cos(c + dx) \left(\log \left(\sin \left(\frac{1}{2}(c + dx) \right) \right) - \log \left(\cos \left(\frac{1}{2}(c + dx) \right) \right) \right)}{d \sqrt{a} \cos^2(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[c + d*x]/Sqrt[a*Cos[c + d*x]^2], x]

[Out] (Cos[c + d*x]*(-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]))/(d*Sqrt[a*Cos[c + d*x]^2])

fricas [A] time = 1.21, size = 84, normalized size = 2.71

$$\left[\frac{\sqrt{a} \cos(dx + c)^2 \log\left(\frac{-\cos(dx+c)+1}{\cos(dx+c)-1}\right)}{2 ad \cos(dx + c)}, \frac{\sqrt{-a} \arctan\left(\frac{\sqrt{a} \cos(dx+c)^2 \sqrt{-a}}{a}\right)}{ad} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2), x, algorithm="fricas")

[Out] [-1/2*sqrt(a*cos(d*x + c)^2)*log(-(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/(a*d*cos(d*x + c)), sqrt(-a)*arctan(sqrt(a*cos(d*x + c)^2)*sqrt(-a)/a)/(a*d)]

giac [A] time = 0.25, size = 31, normalized size = 1.00

$$\frac{\arctan\left(\frac{\sqrt{-a} \sin(dx+c)^2 + a}{\sqrt{-a}}\right)}{\sqrt{-a} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2), x, algorithm="giac")

[Out] arctan(sqrt(-a*sin(d*x + c)^2 + a)/sqrt(-a))/(sqrt(-a)*d)

maple [A] time = 0.16, size = 31, normalized size = 1.00

$$\frac{\cos(dx + c) \operatorname{arctanh}(\cos(dx + c))}{\sqrt{a} (\cos^2(dx + c)) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2), x)

[Out] -1/(a*cos(d*x+c)^2)^(1/2)*cos(d*x+c)*arctanh(cos(d*x+c))/d

maxima [B] time = 0.32, size = 51, normalized size = 1.65

$$\frac{\log\left(\frac{2\sqrt{-a} \sin(dx+c)^2 + a \sqrt{a}}{|\sin(dx+c)|} + \frac{2a}{|\sin(dx+c)|}\right)}{\sqrt{a} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2), x, algorithm="maxima")

[Out] $-\log(2\sqrt{-a\sin(dx + c)^2 + a})\sqrt{a}/\text{abs}(\sin(dx + c)) + 2a/\text{abs}(\sin(dx + c))/(\sqrt{a}d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(c + dx)}{\sqrt{a \cos(c + dx)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(c + d*x)/(a*cos(c + d*x)^2)^(1/2), x)`

[Out] `int(cot(c + d*x)/(a*cos(c + d*x)^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c + dx)}{\sqrt{a \cos^2(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(d*x+c)/(a*cos(d*x+c)**2)**(1/2), x)`

[Out] `Integral(cot(c + d*x)/sqrt(a*cos(c + d*x)**2), x)`

$$3.852 \quad \int \frac{x \cos(x^2)}{\sqrt{\sin(x^2)}} dx$$

Optimal. Leaf size=8

$$\sqrt{\sin(x^2)}$$

[Out] $\sin(x^2)^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3441}

$$\sqrt{\sin(x^2)}$$

Antiderivative was successfully verified.

[In] `Int[(x*Cos[x^2])/Sqrt[Sin[x^2]],x]`

[Out] `Sqrt[Sin[x^2]]`

Rule 3441

`Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]`

Rubi steps

$$\int \frac{x \cos(x^2)}{\sqrt{\sin(x^2)}} dx = \sqrt{\sin(x^2)}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\sqrt{\sin(x^2)}$$

Antiderivative was successfully verified.

[In] `Integrate[(x*Cos[x^2])/Sqrt[Sin[x^2]],x]`

[Out] `Sqrt[Sin[x^2]]`

fricas [A] time = 2.08, size = 6, normalized size = 0.75

$$\sqrt{\sin(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x^2)/sin(x^2)^(1/2),x, algorithm="fricas")`

[Out] `sqrt(sin(x^2))`

giac [A] time = 0.14, size = 6, normalized size = 0.75

$$\sqrt{\sin(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x^2)/sin(x^2)^(1/2),x, algorithm="giac")

[Out] sqrt(sin(x^2))

maple [A] time = 0.02, size = 7, normalized size = 0.88

$$\sqrt{\sin(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(x^2)/sin(x^2)^(1/2),x)

[Out] sin(x^2)^(1/2)

maxima [A] time = 0.31, size = 6, normalized size = 0.75

$$\sqrt{\sin(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x^2)/sin(x^2)^(1/2),x, algorithm="maxima")

[Out] sqrt(sin(x^2))

mupad [B] time = 3.19, size = 6, normalized size = 0.75

$$\sqrt{\sin(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*cos(x^2))/sin(x^2)^(1/2),x)

[Out] sin(x^2)^(1/2)

sympy [A] time = 0.29, size = 7, normalized size = 0.88

$$\sqrt{\sin(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(x**2)/sin(x**2)**(1/2),x)

[Out] sqrt(sin(x**2))

$$3.853 \quad \int \frac{\cos(x)}{\sqrt{1-\cos(2x)}} dx$$

Optimal. Leaf size=19

$$\frac{\sin(x) \log(\sin(x))}{\sqrt{2} \sqrt{\sin^2(x)}}$$

[Out] 1/2*ln(sin(x))*sin(x)*2^(1/2)/(sin(x)^2)^(1/2)

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4356, 12, 15, 29}

$$\frac{\sin(x) \log(\sin(x))}{\sqrt{2} \sqrt{\sin^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/Sqrt[1 - Cos[2*x]],x]

[Out] (Log[Sin[x]]*Sin[x])/(Sqrt[2]*Sqrt[Sin[x]^2])

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 4356

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])

Rubi steps

$$\begin{aligned} \int \frac{\cos(x)}{\sqrt{1-\cos(2x)}} dx &= \text{Subst} \left(\int \frac{1}{\sqrt{2} \sqrt{x^2}} dx, x, \sin(x) \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{\sqrt{x^2}} dx, x, \sin(x) \right)}{\sqrt{2}} \\ &= \frac{\sin(x) \text{Subst} \left(\int \frac{1}{x} dx, x, \sin(x) \right)}{\sqrt{2} \sqrt{\sin^2(x)}} \\ &= \frac{\log(\sin(x)) \sin(x)}{\sqrt{2} \sqrt{\sin^2(x)}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.95

$$\frac{\sin(x) \log(\sin(x))}{\sqrt{1 - \cos(2x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/Sqrt[1 - Cos[2*x]], x]

[Out] (Log[Sin[x]]*Sin[x])/Sqrt[1 - Cos[2*x]]

fricas [A] time = 1.33, size = 21, normalized size = 1.11

$$\frac{\sqrt{-2 \cos(x)^2 + 2} \log\left(\frac{1}{2} \sin(x)\right)}{2 \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(1-cos(2*x))^(1/2), x, algorithm="fricas")

[Out] 1/2*sqrt(-2*cos(x)^2 + 2)*log(1/2*sin(x))/sin(x)

giac [A] time = 0.16, size = 14, normalized size = 0.74

$$\frac{\sqrt{2} \log(|\sin(x)|)}{2 \operatorname{sgn}(\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(1-cos(2*x))^(1/2), x, algorithm="giac")

[Out] 1/2*sqrt(2)*log(abs(sin(x)))/sgn(sin(x))

maple [A] time = 0.45, size = 25, normalized size = 1.32

$$\frac{\sin(x) (\ln(-1 + \cos(x)) + \ln(1 + \cos(x))) \sqrt{2}}{2\sqrt{2} - 2 \cos(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(1-cos(2*x))^(1/2), x)

[Out] 1/4*sin(x)*(ln(-1+cos(x))+ln(1+cos(x)))*2^(1/2)/(sin(x)^2)^(1/2)

maxima [B] time = 0.42, size = 41, normalized size = 2.16

$$\frac{1}{4} \sqrt{2} \log(\cos(x)^2 + \sin(x)^2 + 2 \cos(x) + 1) + \frac{1}{4} \sqrt{2} \log(\cos(x)^2 + \sin(x)^2 - 2 \cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(1-cos(2*x))^(1/2), x, algorithm="maxima")

[Out] 1/4*sqrt(2)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*sqrt(2)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cos(x)}{\sqrt{1 - \cos(2x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1 - cos(2*x))^(1/2), x)
```

```
[Out] int(cos(x)/(1 - cos(2*x))^(1/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\cos(x)}{\sqrt{1 - \cos(2x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x))**(1/2), x)
```

```
[Out] Integral(cos(x)/sqrt(1 - cos(2*x)), x)
```

$$3.854 \quad \int \frac{\cos^2(\log(x)) \sin^2(\log(x))}{x} dx$$

Optimal. Leaf size=29

$$\frac{\log(x)}{8} - \frac{1}{4} \sin(\log(x)) \cos^3(\log(x)) + \frac{1}{8} \sin(\log(x)) \cos(\log(x))$$

[Out] 1/8*ln(x)+1/8*cos(ln(x))*sin(ln(x))-1/4*cos(ln(x))^3*sin(ln(x))

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2568, 2635, 8}

$$\frac{\log(x)}{8} - \frac{1}{4} \sin(\log(x)) \cos^3(\log(x)) + \frac{1}{8} \sin(\log(x)) \cos(\log(x))$$

Antiderivative was successfully verified.

[In] Int[(Cos[Log[x]]^2*Sin[Log[x]]^2)/x,x]

[Out] Log[x]/8 + (Cos[Log[x]]*Sin[Log[x]])/8 - (Cos[Log[x]]^3*Sin[Log[x]])/4

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2568

Int[(cos[(e_.) + (f_.)*(x_.)]*(b_.))^n_*((a_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := -Simp[(a*(b*cos[e + f*x])^(n + 1)*(a*sin[e + f*x])^(m - 1))/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*cos[e + f*x])^n*(a*sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^n, x_Symbol] := -Simp[(b*cos[c + d*x]*(b*sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(\log(x)) \sin^2(\log(x))}{x} dx &= \text{Subst} \left(\int \cos^2(x) \sin^2(x) dx, x, \log(x) \right) \\ &= -\frac{1}{4} \cos^3(\log(x)) \sin(\log(x)) + \frac{1}{4} \text{Subst} \left(\int \cos^2(x) dx, x, \log(x) \right) \\ &= \frac{1}{8} \cos(\log(x)) \sin(\log(x)) - \frac{1}{4} \cos^3(\log(x)) \sin(\log(x)) + \frac{1}{8} \text{Subst} \left(\int 1 dx, x \right) \\ &= \frac{\log(x)}{8} + \frac{1}{8} \cos(\log(x)) \sin(\log(x)) - \frac{1}{4} \cos^3(\log(x)) \sin(\log(x)) \end{aligned}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 0.55

$$\frac{\log(x)}{8} - \frac{1}{32} \sin(4 \log(x))$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[Log[x]]^2*Sin[Log[x]]^2)/x,x]

[Out] Log[x]/8 - Sin[4*Log[x]]/32

fricas [A] time = 0.41, size = 23, normalized size = 0.79

$$-\frac{1}{8} \left(2 \cos(\log(x))^3 - \cos(\log(x)) \right) \sin(\log(x)) + \frac{1}{8} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(log(x))^2*sin(log(x))^2/x,x, algorithm="fricas")

[Out] -1/8*(2*cos(log(x))^3 - cos(log(x)))*sin(log(x)) + 1/8*log(x)

giac [A] time = 0.13, size = 12, normalized size = 0.41

$$\frac{1}{8} \log(x) - \frac{1}{32} \sin(4 \log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(log(x))^2*sin(log(x))^2/x,x, algorithm="giac")

[Out] 1/8*log(x) - 1/32*sin(4*log(x))

maple [A] time = 0.04, size = 24, normalized size = 0.83

$$\frac{\ln(x)}{8} + \frac{\cos(\ln(x)) \sin(\ln(x))}{8} - \frac{(\cos^3(\ln(x))) \sin(\ln(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(ln(x))^2*sin(ln(x))^2/x,x)

[Out] 1/8*ln(x)+1/8*cos(ln(x))*sin(ln(x))-1/4*cos(ln(x))^3*sin(ln(x))

maxima [A] time = 0.30, size = 12, normalized size = 0.41

$$\frac{1}{8} \log(x) - \frac{1}{32} \sin(4 \log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(log(x))^2*sin(log(x))^2/x,x, algorithm="maxima")

[Out] 1/8*log(x) - 1/32*sin(4*log(x))

mupad [B] time = 3.20, size = 12, normalized size = 0.41

$$\frac{\ln(x)}{8} - \frac{\sin(4 \ln(x))}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(log(x))^2*sin(log(x))^2)/x,x)

[Out] log(x)/8 - sin(4*log(x))/32

sympy [B] time = 20.22, size = 476, normalized size = 16.41

$$\frac{\log(x) \tan^8\left(\frac{\log(x)}{2}\right)}{8 \tan^8\left(\frac{\log(x)}{2}\right) + 32 \tan^6\left(\frac{\log(x)}{2}\right) + 48 \tan^4\left(\frac{\log(x)}{2}\right) + 32 \tan^2\left(\frac{\log(x)}{2}\right) + 8} + \frac{4 \log(x) \tan^6\left(\frac{\log(x)}{2}\right)}{8 \tan^8\left(\frac{\log(x)}{2}\right) + 32 \tan^6\left(\frac{\log(x)}{2}\right) + 48 \tan^4\left(\frac{\log(x)}{2}\right) + 32 \tan^2\left(\frac{\log(x)}{2}\right) + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(ln(x))**2*sin(ln(x))**2/x,x)

[Out] $\log(x) \tan(\log(x)/2)^8 / (8 \tan(\log(x)/2)^8 + 32 \tan(\log(x)/2)^6 + 48 \tan(\log(x)/2)^4 + 32 \tan(\log(x)/2)^2 + 8) + 4 \log(x) \tan(\log(x)/2)^6 / (8 \tan(\log(x)/2)^8 + 32 \tan(\log(x)/2)^6 + 48 \tan(\log(x)/2)^4 + 32 \tan(\log(x)/2)^2 + 8) + 6 \log(x) \tan(\log(x)/2)^4 / (8 \tan(\log(x)/2)^8 + 32 \tan(\log(x)/2)^6 + 48 \tan(\log(x)/2)^4 + 32 \tan(\log(x)/2)^2 + 8) + 4 \log(x) \tan(\log(x)/2)^2 / (8 \tan(\log(x)/2)^8 + 32 \tan(\log(x)/2)^6 + 48 \tan(\log(x)/2)^4 + 32 \tan(\log(x)/2)^2 + 8) + \log(x) / (8 \tan(\log(x)/2)^8 + 32 \tan(\log(x)/2)^6 + 48 \tan(\log(x)/2)^4 + 32 \tan(\log(x)/2)^2 + 8) + 2 \tan(\log(x)/2)^7 / (8 \tan(\log(x)/2)^8 + 32 \tan(\log(x)/2)^6 + 48 \tan(\log(x)/2)^4 + 32 \tan(\log(x)/2)^2 + 8) - 14 \tan(\log(x)/2)^5 / (8 \tan(\log(x)/2)^8 + 32 \tan(\log(x)/2)^6 + 48 \tan(\log(x)/2)^4 + 32 \tan(\log(x)/2)^2 + 8) + 14 \tan(\log(x)/2)^3 / (8 \tan(\log(x)/2)^8 + 32 \tan(\log(x)/2)^6 + 48 \tan(\log(x)/2)^4 + 32 \tan(\log(x)/2)^2 + 8) - 2 \tan(\log(x)/2) / (8 \tan(\log(x)/2)^8 + 32 \tan(\log(x)/2)^6 + 48 \tan(\log(x)/2)^4 + 32 \tan(\log(x)/2)^2 + 8)$

$$3.855 \quad \int \frac{\sin^3(x)}{\cos^3(x) + \sin^3(x)} dx$$

Optimal. Leaf size=29

$$\frac{x}{2} + \frac{1}{3} \log(2 - \sin(2x)) - \frac{1}{6} \log(\sin(x) + \cos(x))$$

[Out] 1/2*x-1/6*ln(cos(x)+sin(x))+1/3*ln(2-sin(2*x))

Rubi [A] time = 0.13, antiderivative size = 37, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2074, 635, 203, 260, 628}

$$\frac{x}{2} + \frac{1}{3} \log(\tan^2(x) - \tan(x) + 1) - \frac{1}{6} \log(\tan(x) + 1) + \frac{1}{2} \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^3/(Cos[x]^3 + Sin[x]^3),x]

[Out] x/2 + Log[Cos[x]]/2 - Log[1 + Tan[x]]/6 + Log[1 - Tan[x] + Tan[x]^2]/3

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 2074

Int[(P_)^(p_.)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(x)}{\cos^3(x) + \sin^3(x)} dx &= \text{Subst} \left(\int \frac{x^3}{1 + x^2 + x^3 + x^5} dx, x, \tan(x) \right) \\
&= \text{Subst} \left(\int \left(-\frac{1}{6(1+x)} + \frac{1-x}{2(1+x^2)} + \frac{-1+2x}{3(1-x+x^2)} \right) dx, x, \tan(x) \right) \\
&= -\frac{1}{6} \log(1 + \tan(x)) + \frac{1}{3} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, \tan(x) \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1-x}{1+x^2} dx, x, \tan(x) \right) \\
&= -\frac{1}{6} \log(1 + \tan(x)) + \frac{1}{3} \log(1 - \tan(x) + \tan^2(x)) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right) \\
&= \frac{x}{2} + \frac{1}{2} \log(\cos(x)) - \frac{1}{6} \log(1 + \tan(x)) + \frac{1}{3} \log(1 - \tan(x) + \tan^2(x))
\end{aligned}$$

Mathematica [A] time = 0.10, size = 29, normalized size = 1.00

$$\frac{x}{2} + \frac{1}{3} \log(2 - \sin(2x)) - \frac{1}{6} \log(\sin(x) + \cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^3/(Cos[x]^3 + Sin[x]^3), x]

[Out] x/2 - Log[Cos[x] + Sin[x]]/6 + Log[2 - Sin[2*x]]/3

fricas [A] time = 2.01, size = 26, normalized size = 0.90

$$\frac{1}{2} x - \frac{1}{12} \log(2 \cos(x) \sin(x) + 1) + \frac{1}{3} \log(-\cos(x) \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/(cos(x)^3+sin(x)^3), x, algorithm="fricas")

[Out] 1/2*x - 1/12*log(2*cos(x)*sin(x) + 1) + 1/3*log(-cos(x)*sin(x) + 1)

giac [A] time = 0.17, size = 34, normalized size = 1.17

$$\frac{1}{2} x + \frac{1}{3} \log(\tan(x)^2 - \tan(x) + 1) - \frac{1}{4} \log(\tan(x)^2 + 1) - \frac{1}{6} \log(|\tan(x) + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/(cos(x)^3+sin(x)^3), x, algorithm="giac")

[Out] 1/2*x + 1/3*log(tan(x)^2 - tan(x) + 1) - 1/4*log(tan(x)^2 + 1) - 1/6*log(abs(tan(x) + 1))

maple [A] time = 0.14, size = 34, normalized size = 1.17

$$\frac{\ln(1 - \tan(x) + \tan^2(x))}{3} - \frac{\ln(1 + \tan^2(x))}{4} - \frac{\ln(1 + \tan(x))}{6} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^3/(cos(x)^3+sin(x)^3), x)

[Out] 1/3*ln(1-tan(x)+tan(x)^2)-1/4*ln(1+tan(x)^2)-1/6*ln(1+tan(x))+1/2*x

maxima [B] time = 0.41, size = 103, normalized size = 3.55

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) + \frac{1}{3} \log\left(-\frac{2\sin(x)}{\cos(x)+1} + \frac{2\sin(x)^2}{(\cos(x)+1)^2} + \frac{2\sin(x)^3}{(\cos(x)+1)^3} + \frac{\sin(x)^4}{(\cos(x)+1)^4} + 1\right) - \frac{1}{6} \log\left(-\frac{2\sin(x)}{\cos(x)+1} + \frac{2\sin(x)^2}{(\cos(x)+1)^2} + \frac{2\sin(x)^3}{(\cos(x)+1)^3} + \frac{\sin(x)^4}{(\cos(x)+1)^4} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="maxima")

[Out] arctan(sin(x)/(cos(x) + 1)) + 1/3*log(-2*sin(x)/(cos(x) + 1) + 2*sin(x)^2/(cos(x) + 1)^2 + 2*sin(x)^3/(cos(x) + 1)^3 + sin(x)^4/(cos(x) + 1)^4 + 1) - 1/6*log(-2*sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 - 1) - 1/2*log(sin(x)^2/(cos(x) + 1)^2 + 1)

mupad [B] time = 3.31, size = 45, normalized size = 1.55

$$\frac{x}{2} - \frac{\ln\left(\frac{1}{\cos\left(\frac{x}{2}\right)^2}\right)}{2} + \frac{\ln\left(\frac{\sin(2x)-2}{\cos\left(\frac{x}{2}\right)^4}\right)}{3} - \frac{\ln\left(\frac{\sin\left(x+\frac{\pi}{4}\right)}{\cos\left(\frac{x}{2}\right)^2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^3/(cos(x)^3 + sin(x)^3),x)

[Out] x/2 - log(1/cos(x/2)^2)/2 + log((sin(2*x) - 2)/cos(x/2)^4)/3 - log(sin(x + pi/4)/cos(x/2)^2)/6

sympy [A] time = 0.37, size = 32, normalized size = 1.10

$$\frac{x}{2} - \frac{\log(\sin(x) + \cos(x))}{6} + \frac{\log(\sin^2(x) - \sin(x)\cos(x) + \cos^2(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**3/(cos(x)**3+sin(x)**3),x)

[Out] x/2 - log(sin(x) + cos(x))/6 + log(sin(x)**2 - sin(x)*cos(x) + cos(x)**2)/3

$$3.856 \quad \int \frac{\cos^3(x)}{\cos^3(x) + \sin^3(x)} dx$$

Optimal. Leaf size=29

$$\frac{x}{2} - \frac{1}{3} \log(2 - \sin(2x)) + \frac{1}{6} \log(\sin(x) + \cos(x))$$

[Out] 1/2*x+1/6*ln(cos(x)+sin(x))-1/3*ln(2-sin(2*x))

Rubi [A] time = 0.09, antiderivative size = 37, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2058, 635, 203, 260, 628}

$$\frac{x}{2} - \frac{1}{3} \log(\tan^2(x) - \tan(x) + 1) + \frac{1}{6} \log(\tan(x) + 1) - \frac{1}{2} \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^3/(Cos[x]^3 + Sin[x]^3),x]

[Out] x/2 - Log[Cos[x]]/2 + Log[1 + Tan[x]]/6 - Log[1 - Tan[x] + Tan[x]^2]/3

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 2058

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(x)}{\cos^3(x) + \sin^3(x)} dx &= \text{Subst} \left(\int \frac{1}{1 + x^2 + x^3 + x^5} dx, x, \tan(x) \right) \\
&= \text{Subst} \left(\int \left(\frac{1}{6(1+x)} + \frac{1+x}{2(1+x^2)} + \frac{1-2x}{3(1-x+x^2)} \right) dx, x, \tan(x) \right) \\
&= \frac{1}{6} \log(1 + \tan(x)) + \frac{1}{3} \text{Subst} \left(\int \frac{1-2x}{1-x+x^2} dx, x, \tan(x) \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1+x}{1+x^2} dx, x, \tan(x) \right) \\
&= \frac{1}{6} \log(1 + \tan(x)) - \frac{1}{3} \log(1 - \tan(x) + \tan^2(x)) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \tan(x) \right) \\
&= \frac{x}{2} - \frac{1}{2} \log(\cos(x)) + \frac{1}{6} \log(1 + \tan(x)) - \frac{1}{3} \log(1 - \tan(x) + \tan^2(x))
\end{aligned}$$

Mathematica [A] time = 0.07, size = 29, normalized size = 1.00

$$\frac{x}{2} - \frac{1}{3} \log(2 - \sin(2x)) + \frac{1}{6} \log(\sin(x) + \cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^3/(Cos[x]^3 + Sin[x]^3), x]

[Out] x/2 + Log[Cos[x] + Sin[x]]/6 - Log[2 - Sin[2*x]]/3

fricas [A] time = 1.01, size = 26, normalized size = 0.90

$$\frac{1}{2} x + \frac{1}{12} \log(2 \cos(x) \sin(x) + 1) - \frac{1}{3} \log(-\cos(x) \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3/(cos(x)^3+sin(x)^3), x, algorithm="fricas")

[Out] 1/2*x + 1/12*log(2*cos(x)*sin(x) + 1) - 1/3*log(-cos(x)*sin(x) + 1)

giac [A] time = 0.17, size = 34, normalized size = 1.17

$$\frac{1}{2} x - \frac{1}{3} \log(\tan(x)^2 - \tan(x) + 1) + \frac{1}{4} \log(\tan(x)^2 + 1) + \frac{1}{6} \log(|\tan(x) + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3/(cos(x)^3+sin(x)^3), x, algorithm="giac")

[Out] 1/2*x - 1/3*log(tan(x)^2 - tan(x) + 1) + 1/4*log(tan(x)^2 + 1) + 1/6*log(abs(tan(x) + 1))

maple [A] time = 0.14, size = 34, normalized size = 1.17

$$-\frac{\ln(1 - \tan(x) + \tan^2(x))}{3} + \frac{\ln(1 + \tan^2(x))}{4} + \frac{\ln(1 + \tan(x))}{6} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^3/(cos(x)^3+sin(x)^3), x)

[Out] -1/3*ln(1-tan(x)+tan(x)^2)+1/4*ln(1+tan(x)^2)+1/6*ln(1+tan(x))+1/2*x

maxima [B] time = 0.42, size = 103, normalized size = 3.55

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) - \frac{1}{3} \log\left(-\frac{2 \sin(x)}{\cos(x)+1} + \frac{2 \sin(x)^2}{(\cos(x)+1)^2} + \frac{2 \sin(x)^3}{(\cos(x)+1)^3} + \frac{\sin(x)^4}{(\cos(x)+1)^4} + 1\right) + \frac{1}{6} \log\left(-\frac{2 \sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="maxima")

[Out] arctan(sin(x)/(cos(x) + 1)) - 1/3*log(-2*sin(x)/(cos(x) + 1) + 2*sin(x)^2/(cos(x) + 1)^2 + 2*sin(x)^3/(cos(x) + 1)^3 + sin(x)^4/(cos(x) + 1)^4 + 1) + 1/6*log(-2*sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 - 1) + 1/2*log(sin(x)^2/(cos(x) + 1)^2 + 1)

mupad [B] time = 3.19, size = 45, normalized size = 1.55

$$\frac{x}{2} + \frac{\ln\left(\frac{1}{\cos\left(\frac{x}{2}\right)^2}\right)}{2} - \frac{\ln\left(\frac{\sin(2x)-2}{\cos\left(\frac{x}{2}\right)^4}\right)}{3} + \frac{\ln\left(\frac{\sin\left(x+\frac{\pi}{4}\right)}{\cos\left(\frac{x}{2}\right)^2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^3/(cos(x)^3 + sin(x)^3),x)

[Out] x/2 + log(1/cos(x/2)^2)/2 - log((sin(2*x) - 2)/cos(x/2)^4)/3 + log(sin(x + pi/4)/cos(x/2)^2)/6

sympy [A] time = 0.37, size = 32, normalized size = 1.10

$$\frac{x}{2} + \frac{\log(\sin(x) + \cos(x))}{6} - \frac{\log(\sin^2(x) - \sin(x)\cos(x) + \cos^2(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**3/(cos(x)**3+sin(x)**3),x)

[Out] x/2 + log(sin(x) + cos(x))/6 - log(sin(x)**2 - sin(x)*cos(x) + cos(x)**2)/3

$$3.857 \quad \int \frac{\sec(x)}{-5 + \cos^2(x) + 4 \sin(x)} dx$$

Optimal. Leaf size=44

$$\frac{1}{3(2 - \sin(x))} + \frac{1}{2} \log(1 - \sin(x)) - \frac{4}{9} \log(2 - \sin(x)) - \frac{1}{18} \log(\sin(x) + 1)$$

[Out] 1/2*ln(1-sin(x))-4/9*ln(2-sin(x))-1/18*ln(1+sin(x))+1/3/(2-sin(x))

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {710, 801}

$$\frac{1}{3(2 - \sin(x))} + \frac{1}{2} \log(1 - \sin(x)) - \frac{4}{9} \log(2 - \sin(x)) - \frac{1}{18} \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]/(-5 + Cos[x]^2 + 4*Sin[x]),x]

[Out] Log[1 - Sin[x]]/2 - (4*Log[2 - Sin[x]])/9 - Log[1 + Sin[x]]/18 + 1/(3*(2 - Sin[x]))

Rule 710

Int[((d_) + (e_)*(x_))^(m_)/((a_) + (c_)*(x_)^2), x_Symbol] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(d - e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]

Rule 801

Int[(((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_)))/((a_) + (c_)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{\sec(x)}{-5 + \cos^2(x) + 4 \sin(x)} dx &= \text{Subst} \left(\int \frac{1}{(2-x)^2(-1+x^2)} dx, x, \sin(x) \right) \\ &= \frac{1}{3(2 - \sin(x))} + \frac{1}{3} \text{Subst} \left(\int \frac{2+x}{(2-x)(-1+x^2)} dx, x, \sin(x) \right) \\ &= \frac{1}{3(2 - \sin(x))} + \frac{1}{3} \text{Subst} \left(\int \left(-\frac{4}{3(-2+x)} + \frac{3}{2(-1+x)} - \frac{1}{6(1+x)} \right) dx, x, \sin(x) \right) \\ &= \frac{1}{2} \log(1 - \sin(x)) - \frac{4}{9} \log(2 - \sin(x)) - \frac{1}{18} \log(1 + \sin(x)) + \frac{1}{3(2 - \sin(x))} \end{aligned}$$

Mathematica [A] time = 0.08, size = 38, normalized size = 0.86

$$\frac{1}{18} \left(-\frac{6}{\sin(x) - 2} + 9 \log(1 - \sin(x)) - 8 \log(2 - \sin(x)) - \log(\sin(x) + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]/(-5 + Cos[x]^2 + 4*Sin[x]),x]

[Out] $(9*\text{Log}[1 - \text{Sin}[x]] - 8*\text{Log}[2 - \text{Sin}[x]] - \text{Log}[1 + \text{Sin}[x]] - 6/(-2 + \text{Sin}[x]))/18$

fricas [A] time = 1.17, size = 46, normalized size = 1.05

$$\frac{(\sin(x) - 2) \log(\sin(x) + 1) + 8(\sin(x) - 2) \log\left(-\frac{1}{2} \sin(x) + 1\right) - 9(\sin(x) - 2) \log(-\sin(x) + 1) + 6}{18(\sin(x) - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)/(-5+cos(x)^2+4*sin(x)),x, algorithm="fricas")`

[Out] $-1/18*((\sin(x) - 2)*\log(\sin(x) + 1) + 8*(\sin(x) - 2)*\log(-1/2*\sin(x) + 1) - 9*(\sin(x) - 2)*\log(-\sin(x) + 1) + 6)/(\sin(x) - 2)$

giac [A] time = 0.12, size = 34, normalized size = 0.77

$$-\frac{1}{3(\sin(x) - 2)} - \frac{1}{18} \log(\sin(x) + 1) - \frac{4}{9} \log(-\sin(x) + 2) + \frac{1}{2} \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)/(-5+cos(x)^2+4*sin(x)),x, algorithm="giac")`

[Out] $-1/3/(\sin(x) - 2) - 1/18*\log(\sin(x) + 1) - 4/9*\log(-\sin(x) + 2) + 1/2*\log(-\sin(x) + 1)$

maple [A] time = 0.16, size = 31, normalized size = 0.70

$$-\frac{1}{3(\sin(x) - 2)} - \frac{4 \ln(\sin(x) - 2)}{9} + \frac{\ln(\sin(x) - 1)}{2} - \frac{\ln(1 + \sin(x))}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(x)/(-5+cos(x)^2+4*sin(x)),x)`

[Out] $-1/3/(\sin(x)-2)-4/9*\ln(\sin(x)-2)+1/2*\ln(\sin(x)-1)-1/18*\ln(1+\sin(x))$

maxima [A] time = 0.31, size = 30, normalized size = 0.68

$$-\frac{1}{3(\sin(x) - 2)} - \frac{1}{18} \log(\sin(x) + 1) + \frac{1}{2} \log(\sin(x) - 1) - \frac{4}{9} \log(\sin(x) - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)/(-5+cos(x)^2+4*sin(x)),x, algorithm="maxima")`

[Out] $-1/3/(\sin(x) - 2) - 1/18*\log(\sin(x) + 1) + 1/2*\log(\sin(x) - 1) - 4/9*\log(\sin(x) - 2)$

mupad [B] time = 0.07, size = 32, normalized size = 0.73

$$\frac{\ln(\sin(x) - 1)}{2} - \frac{\ln(\sin(x) + 1)}{18} - \frac{4 \ln(\sin(x) - 2)}{9} - \frac{1}{3(\sin(x) - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cos(x)*(4*sin(x) + cos(x)^2 - 5)),x)`

[Out] $\log(\sin(x) - 1)/2 - \log(\sin(x) + 1)/18 - (4*\log(\sin(x) - 2))/9 - 1/(3*(\sin(x) - 2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(x)}{4 \sin(x) + \cos^2(x) - 5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-5+cos(x)**2+4*sin(x)),x)
```

```
[Out] Integral(sec(x)/(4*sin(x) + cos(x)**2 - 5), x)
```

$$3.858 \quad \int \frac{1}{\cos^{\frac{3}{2}}(x) \sqrt{3 \cos(x) + \sin(x)}} dx$$

Optimal. Leaf size=19

$$\frac{2\sqrt{\sin(x) + 3 \cos(x)}}{\sqrt{\cos(x)}}$$

[Out] $2*(3*\cos(x)+\sin(x))^{(1/2)}/\cos(x)^{(1/2)}$

Rubi [B] time = 2.24, antiderivative size = 88, normalized size of antiderivative = 4.63, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6719, 1063, 8}

$$\frac{2 \cos^2\left(\frac{x}{2}\right) \left(-3 \tan^2\left(\frac{x}{2}\right) + 2 \tan\left(\frac{x}{2}\right) + 3\right)}{\sqrt{\cos^2\left(\frac{x}{2}\right) \left(-3 \tan^2\left(\frac{x}{2}\right) + 2 \tan\left(\frac{x}{2}\right) + 3\right)} \sqrt{\cos^2\left(\frac{x}{2}\right) \left(1 - \tan^2\left(\frac{x}{2}\right)\right)}}$$

Antiderivative was successfully verified.

[In] `Int[1/(Cos[x]^(3/2)*Sqrt[3*Cos[x] + Sin[x]]),x]`

[Out] $(2*\cos[x/2]^2*(3 + 2*\tan[x/2] - 3*\tan[x/2]^2))/(Sqrt[\cos[x/2]^2*(3 + 2*\tan[x/2] - 3*\tan[x/2]^2)]*Sqrt[\cos[x/2]^2*(1 - \tan[x/2]^2)])$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 1063

`Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e) + c*(A*(2*c^2*d - c*(2*a*f)) + C*(-2*a*(c*d - a*f))))*x]/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (-a*e)*(c*e))*(p + 1) + (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e))*(p + q + 2) - (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(-(c*e*(2*p + q + 4)))]*x - c*f*(2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] && !(IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]`

Rule 6719

`Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^p, x_Symbol] := Dist[(a^IntPart[p])*((a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x]`

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{3\cos(x)+\sin(x)}} dx &= 2 \operatorname{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{\frac{3+2x-3x^2}{1+x^2}}\sqrt{\frac{1-x^2}{1+x^2}}} dx, x, \tan\left(\frac{x}{2}\right) \right) \\
&= \frac{\left(2\sqrt{3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)}\right) \operatorname{Subst} \left(\int \frac{\sqrt{1+x^2}}{\sqrt{3+2x-3x^2}(1-x^2)\sqrt{\frac{1-x^2}{1+x^2}}} dx, x, \tan\left(\frac{x}{2}\right) \right)}{\sqrt{\sec^2\left(\frac{x}{2}\right)}\sqrt{\cos^2\left(\frac{x}{2}\right)}\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)} \\
&= \frac{\left(2\cos^2\left(\frac{x}{2}\right)\sqrt{3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)}\sqrt{1-\tan^2\left(\frac{x}{2}\right)}\right) \operatorname{Subst} \left(\int \frac{1+x^2}{\sqrt{3+2x-3x^2}} dx, x, \tan\left(\frac{x}{2}\right) \right)}{\sqrt{\cos^2\left(\frac{x}{2}\right)}\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)\sqrt{\cos^2\left(\frac{x}{2}\right)}\left(1-\tan^2\left(\frac{x}{2}\right)\right)} \\
&= \frac{2\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}{\sqrt{\cos^2\left(\frac{x}{2}\right)}\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)\sqrt{\cos^2\left(\frac{x}{2}\right)}\left(1-\tan^2\left(\frac{x}{2}\right)\right)} + \frac{\left(\cos^2\left(\frac{x}{2}\right)\right)}{4} \\
&= \frac{2\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}{\sqrt{\cos^2\left(\frac{x}{2}\right)}\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)\sqrt{\cos^2\left(\frac{x}{2}\right)}\left(1-\tan^2\left(\frac{x}{2}\right)\right)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 19, normalized size = 1.00

$$\frac{2\sqrt{\sin(x)+3\cos(x)}}{\sqrt{\cos(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Cos[x]^(3/2)*Sqrt[3*Cos[x]+Sin[x]]),x]

[Out] (2*Sqrt[3*Cos[x]+Sin[x]])/Sqrt[Cos[x]]

fricas [A] time = 0.87, size = 15, normalized size = 0.79

$$\frac{2\sqrt{3\cos(x)+\sin(x)}}{\sqrt{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(3*cos(x)+sin(x))/sqrt(cos(x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\cos(x)+\sin(x)}\cos(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(3*cos(x)+sin(x))*cos(x)^(3/2)),x)

maple [A] time = 0.38, size = 16, normalized size = 0.84

$$\frac{2\sqrt{3\cos(x)+\sin(x)}}{\sqrt{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x)`

[Out] $2*(3*\cos(x)+\sin(x))^{(1/2)}/\cos(x)^{(1/2)}$

maxima [B] time = 0.47, size = 145, normalized size = 7.63

$$\frac{2\left(\frac{2\sin(x)}{\cos(x)+1} - \frac{6\sin(x)^2}{(\cos(x)+1)^2} - \frac{2\sin(x)^3}{(\cos(x)+1)^3} + \frac{3\sin(x)^4}{(\cos(x)+1)^4} + 3\right)\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)^2}{\sqrt{\frac{2\sin(x)}{\cos(x)+1} - \frac{3\sin(x)^2}{(\cos(x)+1)^2} + 3}\left(\frac{\sin(x)}{\cos(x)+1} + 1\right)^{\frac{3}{2}}\left(-\frac{\sin(x)}{\cos(x)+1} + 1\right)^{\frac{3}{2}}\left(\frac{2\sin(x)^2}{(\cos(x)+1)^2} + \frac{\sin(x)^4}{(\cos(x)+1)^4} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x, algorithm="maxima")`

[Out] $2*(2*\sin(x)/(\cos(x) + 1) - 6*\sin(x)^2/(\cos(x) + 1)^2 - 2*\sin(x)^3/(\cos(x) + 1)^3 + 3*\sin(x)^4/(\cos(x) + 1)^4 + 3)*(\sin(x)^2/(\cos(x) + 1)^2 + 1)^2/(\sqrt{t(2*\sin(x)/(\cos(x) + 1) - 3*\sin(x)^2/(\cos(x) + 1)^2 + 3)*(\sin(x)/(\cos(x) + 1) + 1)^{(3/2)}*(-\sin(x)/(\cos(x) + 1) + 1)^{(3/2)}*(2*\sin(x)^2/(\cos(x) + 1)^2 + \sin(x)^4/(\cos(x) + 1)^4 + 1))}$

mupad [B] time = 3.79, size = 15, normalized size = 0.79

$$\frac{2\sqrt{3\cos(x) + \sin(x)}}{\sqrt{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cos(x)^(3/2)*(3*cos(x) + sin(x))^(1/2)),x)`

[Out] $(2*(3*\cos(x) + \sin(x))^{(1/2)})/\cos(x)^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sin(x) + 3\cos(x)} \cos^{\frac{3}{2}}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/cos(x)**(3/2)/(3*cos(x)+sin(x))**(1/2),x)`

[Out] `Integral(1/(sqrt(sin(x) + 3*cos(x))*cos(x)**(3/2)), x)`

$$3.859 \quad \int \frac{\csc(x) \sqrt{\cos(x) + \sin(x)}}{\cos^{\frac{3}{2}}(x)} dx$$

Optimal. Leaf size=44

$$-\log(\sin(x)) + \frac{2\sqrt{\sin(x) + \cos(x)}}{\sqrt{\cos(x)}} + 2 \log\left(\sqrt{\sin(x) + \cos(x)} - \sqrt{\cos(x)}\right)$$

[Out] $-\ln(\sin(x)) + 2 * \ln(-\cos(x)^{(1/2)} + (\cos(x) + \sin(x))^{(1/2)}) + 2 * (\cos(x) + \sin(x))^{(1/2)} / \cos(x)^{(1/2)}$

Rubi [F] time = 2.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc(x) \sqrt{\cos(x) + \sin(x)}}{\cos^{\frac{3}{2}}(x)} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{Csc}[x] * \text{Sqrt}[\text{Cos}[x] + \text{Sin}[x]]) / \text{Cos}[x]^{(3/2)}, x]$

[Out] $\text{Defer}[\text{Int}[(\text{Csc}[x] * \text{Sqrt}[\text{Cos}[x] + \text{Sin}[x]]) / \text{Cos}[x]^{(3/2)}, x]$

Rubi steps

$$\int \frac{\csc(x) \sqrt{\cos(x) + \sin(x)}}{\cos^{\frac{3}{2}}(x)} dx = \int \frac{\csc(x) \sqrt{\cos(x) + \sin(x)}}{\cos^{\frac{3}{2}}(x)} dx$$

Mathematica [A] time = 0.40, size = 68, normalized size = 1.55

$$\frac{2 \left(\sin(x) + \cos(x) - \sqrt{\cos(x)} \sqrt{\sqrt{\sin^2(x) + \cos(x)}} \coth^{-1} \left(\frac{\sqrt{\sqrt{\sin^2(x) + \cos(x)}}}{\sqrt{\cos(x)}} \right) \right)}{\sqrt{\cos(x)} \sqrt{\sin(x) + \cos(x)}}$$

Warning: Unable to verify antiderivative.

[In] $\text{Integrate}[(\text{Csc}[x] * \text{Sqrt}[\text{Cos}[x] + \text{Sin}[x]]) / \text{Cos}[x]^{(3/2)}, x]$

[Out] $(2 * (\text{Cos}[x] + \text{Sin}[x] - \text{ArcCoth}[\text{Sqrt}[\text{Cos}[x] + \text{Sqrt}[\text{Sin}[x]^2]]] / \text{Sqrt}[\text{Cos}[x]]]) * \text{Sqrt}[\text{Cos}[x]] * \text{Sqrt}[\text{Cos}[x] + \text{Sqrt}[\text{Sin}[x]^2]]) / (\text{Sqrt}[\text{Cos}[x]] * \text{Sqrt}[\text{Cos}[x] + \text{Sin}[x]])$

fricas [B] time = 0.89, size = 96, normalized size = 2.18

$$\frac{\cos(x) \log\left((2 \cos(x) + \sin(x)) \sqrt{\cos(x) + \sin(x)} \sqrt{\cos(x)} + \frac{7}{4} \cos(x)^2 + 2 \cos(x) \sin(x) + \frac{1}{4}\right) - \cos(x) \log\left(-(2 \cos(x) + \sin(x)) \sqrt{\cos(x) + \sin(x)} \sqrt{\cos(x)} + \frac{7}{4} \cos(x)^2 + 2 \cos(x) \sin(x) + \frac{1}{4}\right)}{4 \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\csc(x) * (\cos(x) + \sin(x))^{(1/2)} / \cos(x)^{(3/2)}, x, \text{algorithm} = \text{"fricas"})$

[Out] $-1/4 * (\cos(x) * \log((2 * \cos(x) + \sin(x)) * \text{sqrt}(\cos(x) + \sin(x)) * \text{sqrt}(\cos(x))) + 7/4 * \cos(x)^2 + 2 * \cos(x) * \sin(x) + 1/4) - \cos(x) * \log(-(2 * \cos(x) + \sin(x)) * \text{sqrt}(\cos(x) + \sin(x)) * \text{sqrt}(\cos(x))) + 7/4 * \cos(x)^2 + 2 * \cos(x) * \sin(x) + 1/4) - 8 * \text{sqrt}(\cos(x) + \sin(x)) * \text{sqrt}(\cos(x))) / \cos(x)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(x) + \sin(x)} \csc(x)}{\cos(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(x) + sin(x))*csc(x)/cos(x)^(3/2), x)

maple [C] time = 0.77, size = 917, normalized size = 20.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x)

[Out] $(-1+\cos(x))^2*(1+\cos(x))^2*(-\text{EllipticF}(1/2*((\sin(x)-1)/\cos(x))*(2+2^{1/2}))^{1/2})^{1/2}*2^{1/2}, I/(2+2^{1/2}))*((2-2^{1/2})*(2+2^{1/2}))^{1/2})^{1/2})*((\cos(x)*2^{1/2}-\sin(x)*2^{1/2}+2*\sin(x)+2^{1/2}-2)/\cos(x)*2^{1/2})^{1/2})*((\cos(x)*2^{1/2}-\sin(x)*2^{1/2}-2*\sin(x)+2^{1/2}+2)/\cos(x)*2^{1/2})^{1/2})*((\sin(x)-1)/\cos(x)*(2+2^{1/2})*2^{1/2})^{1/2})*\sin(x)+\text{EllipticPi}(1/2*((\sin(x)-1)/\cos(x)*(2+2^{1/2})*2^{1/2})^{1/2})*2^{1/2}, 2^{1/2}/(2+2^{1/2}), I/(2+2^{1/2}))*((2-2^{1/2})*(2+2^{1/2}))^{1/2})*((\cos(x)*2^{1/2}-\sin(x)*2^{1/2}+2*\sin(x)+2^{1/2}-2)/\cos(x)*2^{1/2})^{1/2})*((\cos(x)*2^{1/2}-\sin(x)*2^{1/2}-2*\sin(x)+2^{1/2}+2)/\cos(x)*2^{1/2})^{1/2})*((\sin(x)-1)/\cos(x)*(2+2^{1/2})*2^{1/2})^{1/2})*\sin(x)+\text{EllipticPi}(1/2*((\sin(x)-1)/\cos(x)*(2+2^{1/2})*2^{1/2})^{1/2})*2^{1/2}, -2^{1/2}/(2+2^{1/2}), I/(2+2^{1/2}))*((2-2^{1/2})*(2+2^{1/2}))^{1/2})*((\cos(x)*2^{1/2}-\sin(x)*2^{1/2}+2*\sin(x)+2^{1/2}-2)/\cos(x)*2^{1/2})^{1/2})*((\cos(x)*2^{1/2}-\sin(x)*2^{1/2}-2*\sin(x)+2^{1/2}+2)/\cos(x)*2^{1/2})^{1/2})*((\sin(x)-1)/\cos(x)*(2+2^{1/2})*2^{1/2})^{1/2})*\sin(x)-((\cos(x)*2^{1/2}-\sin(x)*2^{1/2}+2*\sin(x)+2^{1/2}-2)/\cos(x)*2^{1/2})^{1/2})*((\cos(x)*2^{1/2}-\sin(x)*2^{1/2}-2*\sin(x)+2^{1/2}+2)/\cos(x)*2^{1/2})^{1/2})*\text{EllipticF}(1/2*((\sin(x)-1)/\cos(x)*(2+2^{1/2})*2^{1/2})^{1/2})*2^{1/2}, I/(2+2^{1/2}))*((2-2^{1/2})*(2+2^{1/2}))^{1/2})*((\sin(x)-1)/\cos(x)*(2+2^{1/2})*2^{1/2})^{1/2})+((\cos(x)*2^{1/2}-\sin(x)*2^{1/2}+2*\sin(x)+2^{1/2}-2)/\cos(x)*2^{1/2})^{1/2})*((\cos(x)*2^{1/2}-\sin(x)*2^{1/2}-2*\sin(x)+2^{1/2}+2)/\cos(x)*2^{1/2})^{1/2})*\text{EllipticPi}(1/2*((\sin(x)-1)/\cos(x)*(2+2^{1/2})*2^{1/2})^{1/2})*2^{1/2}, -2^{1/2}/(2+2^{1/2}), I/(2+2^{1/2}))*((2-2^{1/2})*(2+2^{1/2}))^{1/2})*((\sin(x)-1)/\cos(x)*(2+2^{1/2})*2^{1/2})^{1/2})+2*\cos(x)*2^{1/2}+2*\sin(x)*2^{1/2}+4*\cos(x)+4*\sin(x))/\sin(x)^4/(\cos(x)+\sin(x))^{1/2}/\cos(x)^{1/2}/(2+2^{1/2}))$

maxima [B] time = 0.76, size = 518, normalized size = 11.77

$$4 \left((2 \cos(2x) + \sin(2x)) \cos\left(\frac{1}{2} \arctan(-\cos(4x) + \sin(4x) + 2 \sin(2x) + 1, \cos(4x) + 2 \cos(2x) + \sin(2x))\right) + 1, \cos(4x) + 2 \cos(2x) + \sin(4x) + 1 \right)^3 + (2 \cos(2x) + \sin(2x)) * c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x, algorithm="maxima")

[Out] $4*((2*\cos(2*x) + \sin(2*x))*\cos(1/2*\arctan2(-\cos(4*x) + \sin(4*x) + 2*\sin(2*x) + 1, \cos(4*x) + 2*\cos(2*x) + \sin(4*x) + 1))^3 + (2*\cos(2*x) + \sin(2*x))*c$

```

os(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x)
+ sin(4*x) + 1))*sin(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, co
s(4*x) + 2*cos(2*x) + sin(4*x) + 1))^2 - (cos(2*x) - 2*sin(2*x) + 1)*sin(1/
2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + si
n(4*x) + 1))^3 - (cos(2*x) - sin(2*x) - 1)*cos(1/2*arctan2(-cos(4*x) + sin(
4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1)) - ((cos(2*x)
- 2*sin(2*x) + 1)*cos(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, co
s(4*x) + 2*cos(2*x) + sin(4*x) + 1))^2 + cos(2*x) + sin(2*x) - 1)*sin(1/2*a
rctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4
*x) + 1))))/(4*(cos(2*x) - sin(2*x))*cos(4*x) + 2*cos(4*x)^2 + 4*cos(2*x)^2
+ 4*(cos(2*x) + sin(2*x) + 1)*sin(4*x) + 2*sin(4*x)^2 + 4*sin(2*x)^2 + 4*c
os(2*x) + 4*sin(2*x) + 2)^(1/4)*(cos(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*s
in(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1))^2 + sin(1/2*arctan2(-co
s(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1))^
2))

```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\cos(x) + \sin(x)}}{\cos(x)^{3/2} \sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x) + sin(x))^(1/2)/(cos(x)^(3/2)*sin(x)), x)

[Out] int((cos(x) + sin(x))^(1/2)/(cos(x)^(3/2)*sin(x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sin(x) + \cos(x)} \csc(x)}{\cos^{\frac{3}{2}}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)*(cos(x)+sin(x))**(1/2)/cos(x)**(3/2), x)

[Out] Integral(sqrt(sin(x) + cos(x))*csc(x)/cos(x)**(3/2), x)

$$3.860 \quad \int \frac{\cos(x) + \sin(x)}{\sqrt{1 + \sin(2x)}} dx$$

Optimal. Leaf size=19

$$\frac{x\sqrt{\sin(2x) + 1}}{\sin(x) + \cos(x)}$$

[Out] $x*(1+\sin(2*x))^{(1/2)}/(\cos(x)+\sin(x))$

Rubi [B] time = 1.71, antiderivative size = 72, normalized size of antiderivative = 3.79, number of steps used = 17, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {4401, 6719, 1075, 628, 635, 203, 260, 12, 1023}

$$\frac{2 \cos^2\left(\frac{x}{2}\right) \tan^{-1}\left(\tan\left(\frac{x}{2}\right)\right) \left(-\tan^2\left(\frac{x}{2}\right) + 2 \tan\left(\frac{x}{2}\right) + 1\right)}{\sqrt{\cos^4\left(\frac{x}{2}\right) \left(-\tan^2\left(\frac{x}{2}\right) + 2 \tan\left(\frac{x}{2}\right) + 1\right)^2}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x] + Sin[x])/Sqrt[1 + Sin[2*x]],x]

[Out] (2*ArcTan[Tan[x/2]]*Cos[x/2]^2*(1 + 2*Tan[x/2] - Tan[x/2]^2))/Sqrt[Cos[x/2]^4*(1 + 2*Tan[x/2] - Tan[x/2]^2)^2]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1023

Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Symbol] := With[{q = Simplify[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2]}, Dist[1/q, Int[Simp[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c*d + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[b*h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x]/(d + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]

]

Rule 1075

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :=> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*(-(b*C
*d) + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
f - a*C*d*f + a*A*f^2 - f*(-(b*C*d) + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :=> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
!InertTrigFreeQ[u]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^p_, x_Symbol] :=> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x) + \sin(x)}{\sqrt{1 + \sin(2x)}} dx &= \int \left(\frac{\cos(x)}{\sqrt{1 + \sin(2x)}} + \frac{\sin(x)}{\sqrt{1 + \sin(2x)}} \right) dx \\
&= \int \frac{\cos(x)}{\sqrt{1 + \sin(2x)}} dx + \int \frac{\sin(x)}{\sqrt{1 + \sin(2x)}} dx \\
&= 2 \operatorname{Subst} \left(\int \frac{2x}{(1+x^2)^2 \sqrt{\frac{(-1-2x+x^2)^2}{(1+x^2)^2}}} dx, x, \tan\left(\frac{x}{2}\right) \right) + 2 \operatorname{Subst} \left(\int \frac{1-x^2}{(1+x^2)^2 \sqrt{\frac{(-1-2x+x^2)^2}{(1+x^2)^2}}} dx, x, \tan\left(\frac{x}{2}\right) \right) \\
&= 4 \operatorname{Subst} \left(\int \frac{x}{(1+x^2)^2 \sqrt{\frac{(-1-2x+x^2)^2}{(1+x^2)^2}}} dx, x, \tan\left(\frac{x}{2}\right) \right) + \frac{(2 \cos^2\left(\frac{x}{2}\right) (-1 - 2 \tan\left(\frac{x}{2}\right) + \tan^2\left(\frac{x}{2}\right))) \operatorname{Subst} \left(\int \frac{-4+4x}{1+x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{\sqrt{\cos^4\left(\frac{x}{2}\right)}} \\
&= \frac{(\cos^2\left(\frac{x}{2}\right) (-1 - 2 \tan\left(\frac{x}{2}\right) + \tan^2\left(\frac{x}{2}\right))) \operatorname{Subst} \left(\int \frac{-4+4x}{1+x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{4 \sqrt{\cos^4\left(\frac{x}{2}\right) (-1 - 2 \tan\left(\frac{x}{2}\right) + \tan^2\left(\frac{x}{2}\right))^2}} + \frac{(\cos^2\left(\frac{x}{2}\right) (-1 - 2 \tan\left(\frac{x}{2}\right) + \tan^2\left(\frac{x}{2}\right))) \operatorname{Subst} \left(\int \frac{-4+4x}{1+x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{2 \sqrt{\cos^4\left(\frac{x}{2}\right) (1 + 2 \tan\left(\frac{x}{2}\right) - \tan^2\left(\frac{x}{2}\right))^2}} \\
&= \frac{\cos^2\left(\frac{x}{2}\right) \log\left(1 + 2 \tan\left(\frac{x}{2}\right) - \tan^2\left(\frac{x}{2}\right)\right) (1 + 2 \tan\left(\frac{x}{2}\right) - \tan^2\left(\frac{x}{2}\right))}{2 \sqrt{\cos^4\left(\frac{x}{2}\right) (1 + 2 \tan\left(\frac{x}{2}\right) - \tan^2\left(\frac{x}{2}\right))^2}} + \frac{(\cos^2\left(\frac{x}{2}\right) (-1 - 2 \tan\left(\frac{x}{2}\right) + \tan^2\left(\frac{x}{2}\right))) \operatorname{Subst} \left(\int \frac{-4+4x}{1+x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{2 \sqrt{\cos^4\left(\frac{x}{2}\right) (1 + 2 \tan\left(\frac{x}{2}\right) - \tan^2\left(\frac{x}{2}\right))^2}} \\
&= \frac{x \cos^2\left(\frac{x}{2}\right) (1 + 2 \tan\left(\frac{x}{2}\right) - \tan^2\left(\frac{x}{2}\right))}{2 \sqrt{\cos^4\left(\frac{x}{2}\right) (1 + 2 \tan\left(\frac{x}{2}\right) - \tan^2\left(\frac{x}{2}\right))^2}} + \frac{\cos^2\left(\frac{x}{2}\right) \log\left(\cos\left(\frac{x}{2}\right)\right) (1 + 2 \tan\left(\frac{x}{2}\right) - \tan^2\left(\frac{x}{2}\right))}{\sqrt{\cos^4\left(\frac{x}{2}\right) (1 + 2 \tan\left(\frac{x}{2}\right) - \tan^2\left(\frac{x}{2}\right))^2}} \\
&= \frac{x \cos^2\left(\frac{x}{2}\right) (1 + 2 \tan\left(\frac{x}{2}\right) - \tan^2\left(\frac{x}{2}\right))}{\sqrt{\cos^4\left(\frac{x}{2}\right) (1 + 2 \tan\left(\frac{x}{2}\right) - \tan^2\left(\frac{x}{2}\right))^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.89

$$\frac{x(\sin(x) + \cos(x))}{\sqrt{\sin(2x) + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x] + Sin[x])/Sqrt[1 + Sin[2*x]], x]

[Out] (x*(Cos[x] + Sin[x]))/Sqrt[1 + Sin[2*x]]

fricas [A] time = 1.39, size = 3, normalized size = 0.16

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)+sin(x))/(1+sin(2*x))^(1/2), x, algorithm="fricas")

[Out] -x

giac [B] time = 0.17, size = 42, normalized size = 2.21

$$\frac{2\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor - x}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^4 - 2 \tan\left(\frac{1}{2}x\right)^3 - 2 \tan\left(\frac{1}{2}x\right) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x, algorithm="giac")

[Out] (2*pi*floor(1/2*x/pi + 1/2) - x)/sgn(tan(1/2*x)^4 - 2*tan(1/2*x)^3 - 2*tan(1/2*x) - 1)

maple [C] time = 0.42, size = 12372, normalized size = 651.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x)

[Out] result too large to display

maxima [B] time = 0.50, size = 329, normalized size = 17.32

$$\frac{1}{16} \sqrt{2} \left(2 \sqrt{2} \arctan(\sin(2x) + 1, \cos(2x)) + \sqrt{2} \log(\cos(2x)^2 + \sin(2x)^2 + 2 \sin(2x) + 1) + 4(\cos(4x)^2 + \sin(4x)^2 - 4 \cos(4x) \sin(2x) + 4 \sin(2x)^2)^{1/4} (\cos(1/2 \arctan2(\cos(4x) - 2 \sin(2x), 2 \cos(2x) + \sin(4x))) \sin(2x) + \cos(2x) \sin(1/2 \arctan2(\cos(4x) - 2 \sin(2x), 2 \cos(2x) + \sin(4x)))) \right) + 1/16 \sqrt{2} (2 \sqrt{2} \arctan2(\sin(2x) + 1, \cos(2x)) - \sqrt{2} \log(\cos(2x)^2 + \sin(2x)^2 + 2 \sin(2x) + 1) - 4(\cos(4x)^2 + \sin(4x)^2 - 4 \cos(4x) \sin(2x) + 4 \sin(2x)^2)^{1/4} (\cos(2x) \cos(1/2 \arctan2(\cos(4x) - 2 \sin(2x), 2 \cos(2x) + \sin(4x))) - \sin(2x) \sin(1/2 \arctan2(\cos(4x) - 2 \sin(2x), 2 \cos(2x) + \sin(4x))))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x, algorithm="maxima")

[Out] 1/16*sqrt(2)*(2*sqrt(2)*arctan2(sin(2*x) + 1, cos(2*x)) + sqrt(2)*log(cos(2*x)^2 + sin(2*x)^2 + 2*sin(2*x) + 1) + 4*(cos(4*x)^2 + 4*cos(2*x)^2 + 4*cos(2*x)*sin(4*x) + sin(4*x)^2 - 4*cos(4*x)*sin(2*x) + 4*sin(2*x)^2)^(1/4)*(cos(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x)))*sin(2*x) + cos(2*x)*sin(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x)))) + 1/16*sqrt(2)*(2*sqrt(2)*arctan2(sin(2*x) + 1, cos(2*x)) - sqrt(2)*log(cos(2*x)^2 + sin(2*x)^2 + 2*sin(2*x) + 1) - 4*(cos(4*x)^2 + 4*cos(2*x)^2 + 4*cos(2*x)*sin(4*x) + sin(4*x)^2 - 4*cos(4*x)*sin(2*x) + 4*sin(2*x)^2)^(1/4)*(cos(2*x)*cos(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x))) - sin(2*x)*sin(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x))))

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cos(x) + \sin(x)}{\sqrt{\sin(2x) + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x) + sin(x))/(sin(2*x) + 1)^(1/2),x)

[Out] int((cos(x) + sin(x))/(sin(2*x) + 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x) + \cos(x)}{\sqrt{\sin(2x) + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)+sin(x))/(1+sin(2*x))**(1/2),x)

[Out] Integral((sin(x) + cos(x))/sqrt(sin(2*x) + 1), x)

3.861 $\int \sec(x)\sqrt{\sec(x) + \tan(x)} dx$

Optimal. Leaf size=13

$$2\sqrt{(\sin(x) + 1)\sec(x)}$$

[Out] 2*(sec(x)*(1+sin(x)))^(1/2)

Rubi [A] time = 0.15, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4397, 4400, 2705, 2671}

$$2\sqrt{(\sin(x) + 1)\sec(x)}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]*Sqrt[Sec[x] + Tan[x]], x]

[Out] 2*Sqrt[Sec[x]*(1 + Sin[x])]

Rule 2671

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]

Rule 2705

Int[((g_.)*sec[(e_.) + (f_.)*(x_.)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]*(g*Sec[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]

Rule 4397

Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rule 4400

Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPart[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x], x] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !InertTrigFreeQ[w])

Rubi steps

$$\begin{aligned} \int \sec(x)\sqrt{\sec(x) + \tan(x)} dx &= \int \sec(x)\sqrt{\sec(x)(1 + \sin(x))} dx \\ &= \frac{\sqrt{\sec(x)(1 + \sin(x))} \int \sec^{\frac{3}{2}}(x)\sqrt{1 + \sin(x)} dx}{\sqrt{\sec(x)}\sqrt{1 + \sin(x)}} \\ &= \frac{(\sqrt{\cos(x)}\sqrt{\sec(x)(1 + \sin(x))}) \int \frac{\sqrt{1 + \sin(x)}}{\cos^{\frac{3}{2}}(x)} dx}{\sqrt{1 + \sin(x)}} \\ &= 2\sqrt{\sec(x)(1 + \sin(x))} \end{aligned}$$

Mathematica [B] time = 0.04, size = 37, normalized size = 2.85

$$2 \sqrt{\frac{\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]*Sqrt[Sec[x] + Tan[x]],x]

[Out] 2*Sqrt[(Cos[x/2] + Sin[x/2])/(Cos[x/2] - Sin[x/2])]

fricas [A] time = 0.89, size = 21, normalized size = 1.62

$$2 \sqrt{\frac{\cos(x) + \sin(x) + 1}{\cos(x) - \sin(x) + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*(sec(x)+tan(x))^(1/2),x, algorithm="fricas")

[Out] 2*sqrt((cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1))

giac [B] time = 0.32, size = 55, normalized size = 4.23

$$\frac{4 \operatorname{sgn}\left(-\tan\left(\frac{1}{2}x\right)^3 - \tan\left(\frac{1}{2}x\right)^2 - \tan\left(\frac{1}{2}x\right) - 1\right) \operatorname{sgn}(\cos(x))}{\frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^2 + 1} - 1}{\tan\left(\frac{1}{2}x\right)} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*(sec(x)+tan(x))^(1/2),x, algorithm="giac")

[Out] -4*sgn(-tan(1/2*x)^3 - tan(1/2*x)^2 - tan(1/2*x) - 1)*sgn(cos(x))/((sqrt(-tan(1/2*x)^2 + 1) - 1)/tan(1/2*x) + 1)

maple [A] time = 0.14, size = 10, normalized size = 0.77

$$2\sqrt{\sec(x) + \tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)*(sec(x)+tan(x))^(1/2),x)

[Out] 2*(sec(x)+tan(x))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sec(x) + \tan(x)} \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*(sec(x)+tan(x))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(sec(x) + tan(x))*sec(x), x)

mupad [B] time = 0.29, size = 14, normalized size = 1.08

$$2 \sqrt{\frac{1}{\cos(x)}} \sqrt{\sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x) + 1/cos(x))^(1/2)/cos(x), x)
```

```
[Out] 2*(1/cos(x))^(1/2)*(sin(x) + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan(x) + \sec(x)} \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(sec(x)+tan(x))**(1/2), x)
```

```
[Out] Integral(sqrt(tan(x) + sec(x))*sec(x), x)
```

3.862 $\int \sec(x) \sqrt{4 + 3 \sec(x)} \tan(x) dx$

Optimal. Leaf size=14

$$\frac{2}{9}(3 \sec(x) + 4)^{3/2}$$

[Out] $2/9*(4+3*\sec(x))^{(3/2)}$

Rubi [A] time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4339, 261}

$$\frac{2}{9}(3 \sec(x) + 4)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]*Sqrt[4 + 3*Sec[x]]*Tan[x],x]

[Out] $(2*(4 + 3*\sec[x])^{(3/2)})/9$

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 4339

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])

Rubi steps

$$\begin{aligned} \int \sec(x) \sqrt{4 + 3 \sec(x)} \tan(x) dx &= -\text{Subst} \left(\int \frac{\sqrt{4 + \frac{3}{x}}}{x^2} dx, x, \cos(x) \right) \\ &= \frac{2}{9}(4 + 3 \sec(x))^{3/2} \end{aligned}$$

Mathematica [A] time = 0.05, size = 14, normalized size = 1.00

$$\frac{2}{9}(3 \sec(x) + 4)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]*Sqrt[4 + 3*Sec[x]]*Tan[x],x]

[Out] $(2*(4 + 3*\sec[x])^{(3/2)})/9$

fricas [B] time = 1.26, size = 25, normalized size = 1.79

$$\frac{2 \sqrt{\frac{4 \cos(x)+3}{\cos(x)}} (4 \cos(x) + 3)}{9 \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x, algorithm="fricas")

[Out] 2/9*sqrt((4*cos(x) + 3)/cos(x))*(4*cos(x) + 3)/cos(x)

giac [B] time = 0.16, size = 68, normalized size = 4.86

$$\frac{2 \left(4 \left(\sqrt{4 \cos(x)^2 + 3 \cos(x)} - 2 \cos(x) \right)^2 - 6 \sqrt{4 \cos(x)^2 + 3 \cos(x)} + 12 \cos(x) + 3 \right) \operatorname{sgn}(\cos(x))}{\left(\sqrt{4 \cos(x)^2 + 3 \cos(x)} - 2 \cos(x) \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x, algorithm="giac")

[Out] 2*(4*(sqrt(4*cos(x)^2 + 3*cos(x)) - 2*cos(x))^2 - 6*sqrt(4*cos(x)^2 + 3*cos(x)) + 12*cos(x) + 3)*sgn(cos(x))/(sqrt(4*cos(x)^2 + 3*cos(x)) - 2*cos(x))^3

maple [A] time = 0.06, size = 11, normalized size = 0.79

$$\frac{2(4 + 3 \sec(x))^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x)

[Out] 2/9*(4+3*sec(x))^(3/2)

maxima [A] time = 0.32, size = 10, normalized size = 0.71

$$\frac{2}{9} (3 \sec(x) + 4)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x, algorithm="maxima")

[Out] 2/9*(3*sec(x) + 4)^(3/2)

mupad [B] time = 3.23, size = 29, normalized size = 2.07

$$\frac{8 \sqrt{\frac{3}{\cos(x)} + 4}}{9} + \frac{2 \sqrt{\frac{3}{\cos(x)} + 4}}{3 \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(x)*(3/cos(x) + 4)^(1/2))/cos(x),x)

[Out] (8*(3/cos(x) + 4)^(1/2))/9 + (2*(3/cos(x) + 4)^(1/2))/(3*cos(x))

sympy [B] time = 0.68, size = 29, normalized size = 2.07

$$\frac{2\sqrt{3 \sec(x) + 4} \sec(x)}{3} + \frac{8\sqrt{3 \sec(x) + 4}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x)

[Out] 2*sqrt(3*sec(x) + 4)*sec(x)/3 + 8*sqrt(3*sec(x) + 4)/9

3.863 $\int \sec(x) \sqrt{1 + \sec(x)} \tan^3(x) dx$

Optimal. Leaf size=25

$$\frac{2}{7}(\sec(x) + 1)^{7/2} - \frac{4}{5}(\sec(x) + 1)^{5/2}$$

[Out] $-4/5*(1+\sec(x))^{(5/2)}+2/7*(1+\sec(x))^{(7/2)}$

Rubi [A] time = 0.09, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4373, 1570, 1469, 627, 43}

$$\frac{2}{7}(\sec(x) + 1)^{7/2} - \frac{4}{5}(\sec(x) + 1)^{5/2}$$

Antiderivative was successfully verified.

[In] `Int[Sec[x]*Sqrt[1 + Sec[x]]*Tan[x]^3,x]`

[Out] $(-4*(1 + \text{Sec}[x])^{(5/2)})/5 + (2*(1 + \text{Sec}[x])^{(7/2)})/7$

Rule 43

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

Rule 627

`Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && IntegerQ[m + p]))`

Rule 1469

`Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]`

Rule 1570

`Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] := Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x] /; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]`

Rule 4373

`Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] := With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[(b*c*d^(n - 1))^(-1), Subst[Int[SubstFor[(1 - d^2*x^2)^((n - 1)/2)/x^n, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Tan] || EqQ[F, tan])`

Rubi steps

$$\begin{aligned}
\int \sec(x)\sqrt{1+\sec(x)}\tan^3(x)dx &= -\text{Subst}\left(\int\frac{\sqrt{1+\frac{1}{x}}(1-x^2)}{x^4}dx,x,\cos(x)\right) \\
&= -\text{Subst}\left(\int\frac{\left(-1+\frac{1}{x^2}\right)\sqrt{1+\frac{1}{x}}}{x^2}dx,x,\cos(x)\right) \\
&= \text{Subst}\left(\int\sqrt{1+x}(-1+x^2)dx,x,\sec(x)\right) \\
&= \text{Subst}\left(\int(-1+x)(1+x)^{3/2}dx,x,\sec(x)\right) \\
&= \text{Subst}\left(\int(-2(1+x)^{3/2}+(1+x)^{5/2})dx,x,\sec(x)\right) \\
&= -\frac{4}{5}(1+\sec(x))^{5/2}+\frac{2}{7}(1+\sec(x))^{7/2}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 30, normalized size = 1.20

$$-\frac{8}{35}\cos^4\left(\frac{x}{2}\right)(9\cos(x)-5)\sec^3(x)\sqrt{\sec(x)+1}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]*Sqrt[1+Sec[x]]*Tan[x]^3,x]

[Out] (-8*Cos[x/2]^4*(-5+9*Cos[x])*Sec[x]^3*Sqrt[1+Sec[x]])/35

fricas [B] time = 0.87, size = 35, normalized size = 1.40

$$\frac{2(9\cos(x)^3+13\cos(x)^2-\cos(x)-5)\sqrt{\frac{\cos(x)+1}{\cos(x)}}}{35\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x, algorithm="fricas")

[Out] -2/35*(9*cos(x)^3+13*cos(x)^2-cos(x)-5)*sqrt((cos(x)+1)/cos(x))/cos(x)^3

giac [B] time = 0.17, size = 128, normalized size = 5.12

$$\frac{2\left(35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^6-35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^5-35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^4-35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^3-35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^2-35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)-35\right)}{35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x, algorithm="giac")

[Out] -2/35*(35*(sqrt(cos(x)^2+cos(x))-cos(x))^6-35*(sqrt(cos(x)^2+cos(x))-cos(x))^5-35*(sqrt(cos(x)^2+cos(x))-cos(x))^4+105*(sqrt(cos(x)^2+cos(x))-cos(x))^3-91*(sqrt(cos(x)^2+cos(x))-cos(x))^2+35*sqrt(cos(x)^2+cos(x))-35*cos(x)-5)*sgn(cos(x))/(sqrt(cos(x)^2+cos(x))-cos(x))^7

maple [A] time = 0.17, size = 34, normalized size = 1.36

$$\frac{2(9\cos(x)-5)\sqrt{\frac{1+\cos(x)}{\cos(x)}}(\sin^4(x))}{35(-1+\cos(x))^2\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x)`

[Out] $-2/35*(9*\cos(x)-5)*((1+\cos(x))/\cos(x))^{1/2}*\sin(x)^4/(-1+\cos(x))^2/\cos(x)^3$

maxima [A] time = 0.33, size = 21, normalized size = 0.84

$$\frac{2}{7}\left(\frac{1}{\cos(x)}+1\right)^{\frac{7}{2}}-\frac{4}{5}\left(\frac{1}{\cos(x)}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x, algorithm="maxima")`

[Out] $2/7*(1/\cos(x)+1)^{7/2}-4/5*(1/\cos(x)+1)^{5/2}$

mupad [B] time = 3.33, size = 24, normalized size = 0.96

$$\frac{2(\cos(x)+1)^{5/2}\sqrt{\frac{1}{\cos(x)}}(9\cos(x)-5)}{35\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((tan(x)^3*(1/cos(x)+1)^(1/2))/cos(x),x)`

[Out] $-(2*(\cos(x)+1)^{5/2}*(1/\cos(x))^{1/2}*(9*\cos(x)-5))/(35*\cos(x)^3)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sec(x)+1} \tan^3(x) \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)*(1+sec(x))^(1/2)*tan(x)**3,x)`

[Out] `Integral(sqrt(sec(x)+1)*tan(x)**3*sec(x),x)`

3.864 $\int \cot^3(x) \csc(x) \sqrt{1 + \csc(x)} dx$

Optimal. Leaf size=25

$$\frac{4}{5}(\csc(x) + 1)^{5/2} - \frac{2}{7}(\csc(x) + 1)^{7/2}$$

[Out] $4/5*(1+\csc(x))^{(5/2)}-2/7*(1+\csc(x))^{(7/2)}$

Rubi [A] time = 0.08, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4372, 1570, 1469, 627, 43}

$$\frac{4}{5}(\csc(x) + 1)^{5/2} - \frac{2}{7}(\csc(x) + 1)^{7/2}$$

Antiderivative was successfully verified.

[In] `Int[Cot[x]^3*Csc[x]*Sqrt[1 + Csc[x]],x]`

[Out] $(4*(1 + \csc[x])^{(5/2)})/5 - (2*(1 + \csc[x])^{(7/2)})/7$

Rule 43

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

Rule 627

`Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && IntegerQ[m + p]))`

Rule 1469

`Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]`

Rule 1570

`Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] := Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x] /; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]`

Rule 4372

`Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c*d^(n - 1)), Subst[Int[SubstFor[(1 - d^2*x^2)^((n - 1)/2)/x^n, Sin[c*(a + b*x)]/d, u, x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cot] || EqQ[F, cot])`

Rubi steps

$$\begin{aligned}
\int \cot^3(x) \csc(x) \sqrt{1 + \csc(x)} \, dx &= \text{Subst} \left(\int \frac{\sqrt{1 + \frac{1}{x}} (1 - x^2)}{x^4} \, dx, x, \sin(x) \right) \\
&= \text{Subst} \left(\int \frac{\left(-1 + \frac{1}{x^2}\right) \sqrt{1 + \frac{1}{x}}}{x^2} \, dx, x, \sin(x) \right) \\
&= -\text{Subst} \left(\int \sqrt{1 + x} (-1 + x^2) \, dx, x, \csc(x) \right) \\
&= -\text{Subst} \left(\int (-1 + x)(1 + x)^{3/2} \, dx, x, \csc(x) \right) \\
&= -\text{Subst} \left(\int (-2(1 + x)^{3/2} + (1 + x)^{5/2}) \, dx, x, \csc(x) \right) \\
&= \frac{4}{5}(1 + \csc(x))^{5/2} - \frac{2}{7}(1 + \csc(x))^{7/2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 18, normalized size = 0.72

$$-\frac{2}{35}(\csc(x) + 1)^{5/2}(5 \csc(x) - 9)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]^3*Csc[x]*Sqrt[1 + Csc[x]],x]

[Out] (-2*(1 + Csc[x])^(5/2)*(-9 + 5*Csc[x]))/35

fricas [B] time = 0.41, size = 44, normalized size = 1.76

$$\frac{2 \left(13 \cos(x)^2 + (9 \cos(x)^2 - 8) \sin(x) - 8 \right) \sqrt{\frac{\sin(x)+1}{\sin(x)}}}{35 (\cos(x)^2 - 1) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x, algorithm="fricas")

[Out] 2/35*(13*cos(x)^2 + (9*cos(x)^2 - 8)*sin(x) - 8)*sqrt((sin(x) + 1)/sin(x))/((cos(x)^2 - 1)*sin(x))

giac [B] time = 0.14, size = 128, normalized size = 5.12

$$\frac{2 \left(35 \left(\sqrt{\sin(x)^2 + \sin(x)} - \sin(x) \right)^6 - 35 \left(\sqrt{\sin(x)^2 + \sin(x)} - \sin(x) \right)^5 - 35 \left(\sqrt{\sin(x)^2 + \sin(x)} - \sin(x) \right)^4 + 10 \right)}{35 \left(\sqrt{\sin(x)^2} \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x, algorithm="giac")

[Out] 2/35*(35*(sqrt(sin(x)^2 + sin(x)) - sin(x))^6 - 35*(sqrt(sin(x)^2 + sin(x)) - sin(x))^5 - 35*(sqrt(sin(x)^2 + sin(x)) - sin(x))^4 + 105*(sqrt(sin(x)^2 + sin(x)) - sin(x))^3 - 91*(sqrt(sin(x)^2 + sin(x)) - sin(x))^2 + 35*sqrt(sin(x)^2 + sin(x)) - 35*sin(x) - 5)*sgn(sin(x))/(sqrt(sin(x)^2 + sin(x)) - sin(x))^7

maple [B] time = 0.18, size = 38, normalized size = 1.52

$$\frac{2 \left(9 \left(\cos^2(x) \right) \sin(x) + 13 \left(\cos^2(x) \right) - 8 \sin(x) - 8 \right) \sqrt{\frac{1+\sin(x)}{\sin(x)}}}{35 \sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x)

[Out] -2/35*(9*cos(x)^2*sin(x)+13*cos(x)^2-8*sin(x)-8)*((1+sin(x))/sin(x))^(1/2)/sin(x)^3

maxima [A] time = 0.32, size = 21, normalized size = 0.84

$$-\frac{2}{7} \left(\frac{1}{\sin(x)} + 1 \right)^{\frac{7}{2}} + \frac{4}{5} \left(\frac{1}{\sin(x)} + 1 \right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x, algorithm="maxima")

[Out] -2/7*(1/sin(x) + 1)^(7/2) + 4/5*(1/sin(x) + 1)^(5/2)

mupad [B] time = 3.42, size = 24, normalized size = 0.96

$$\frac{2 (\sin(x) + 1)^{5/2} \sqrt{\frac{1}{\sin(x)}} (9 \sin(x) - 5)}{35 \sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(x)^3*(1/sin(x) + 1)^(1/2))/sin(x),x)

[Out] (2*(sin(x) + 1)^(5/2)*(1/sin(x))^(1/2)*(9*sin(x) - 5))/(35*sin(x)^3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\csc(x) + 1} \cot^3(x) \csc(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)**3*csc(x)*(1+csc(x))**(1/2),x)

[Out] Integral(sqrt(csc(x) + 1)*cot(x)**3*csc(x), x)

3.865 $\int \sqrt{\csc(x)} (x \cos(x) - 4 \sec(x) \tan(x)) dx$

Optimal. Leaf size=20

$$\frac{2x}{\sqrt{\csc(x)}} - \frac{4 \sec(x)}{\csc^{\frac{3}{2}}(x)}$$

[Out] $-4*\sec(x)/\csc(x)^{(3/2)}+2*x/\csc(x)^{(1/2)}$

Rubi [A] time = 0.15, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6742, 4213, 3771, 2639, 2626}

$$\frac{2x}{\sqrt{\csc(x)}} - \frac{4 \sec(x)}{\csc^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[Csc[x]]*(x*Cos[x] - 4*Sec[x]*Tan[x]),x]`

[Out] $(2*x)/\text{Sqrt}[\text{Csc}[x]] - (4*\text{Sec}[x])/\text{Csc}[x]^{(3/2)}$

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n - 1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_.)^(n_.)]*Csc[(a_.) + (b_.)*(x_.)^(n_.)]^(p_.)*(x_.)^(m_.), x_Symbol] := -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p, 1]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned} \int \sqrt{\csc(x)} (x \cos(x) - 4 \sec(x) \tan(x)) dx &= \int \left(x \cos(x) \sqrt{\csc(x)} - \frac{4 \sec^2(x)}{\sqrt{\csc(x)}} \right) dx \\ &= - \left(4 \int \frac{\sec^2(x)}{\sqrt{\csc(x)}} dx \right) + \int x \cos(x) \sqrt{\csc(x)} dx \\ &= \frac{2x}{\sqrt{\csc(x)}} - \frac{4 \sec(x)}{\csc^{\frac{3}{2}}(x)} \end{aligned}$$

Mathematica [A] time = 0.44, size = 17, normalized size = 0.85

$$\frac{2(x \csc(x) - 2 \sec(x))}{\csc^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Csc[x]]*(x*Cos[x] - 4*Sec[x]*Tan[x]),x]

[Out] (2*(x*Csc[x] - 2*Sec[x]))/Csc[x]^(3/2)

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x \cos(x) - 4 \sec(x) \tan(x)) \sqrt{\csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x, algorithm="giac")

[Out] integrate((x*cos(x) - 4*sec(x)*tan(x))*sqrt(csc(x)), x)

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int (\sqrt{\csc(x)}) (x \cos(x) - 4 \sec(x) \tan(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x)

[Out] int(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x \cos(x) - 4 \sec(x) \tan(x)) \sqrt{\csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x, algorithm="maxima")

[Out] integrate((x*cos(x) - 4*sec(x)*tan(x))*sqrt(csc(x)), x)

mupad [B] time = 3.46, size = 77, normalized size = 3.85

$$\frac{(4 \cos(x)^3 - 4 \cos(x) + 2 x \cos(x)^2 \sin(x) - \sin(x) 4i - x \cos(x)^3 2i + \cos(x)^2 \sin(x) 4i + x \cos(x) 2i) 1i}{\cos(x) \sin(x) \sqrt{\frac{1}{\sin(x)}} (-\sin(x) + \cos(x) 1i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(1/sin(x))^(1/2)*((4*tan(x))/cos(x) - x*cos(x)),x)

[Out] ((4*cos(x)^3 - sin(x)*4i - x*cos(x)^3*2i - 4*cos(x) + cos(x)^2*sin(x)*4i + x*cos(x)*2i + 2*x*cos(x)^2*sin(x))*1i)/(cos(x)*sin(x)*(1/sin(x))^(1/2)*(cos(x)*1i - sin(x)))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x \cos(x) - 4 \tan(x) \sec(x)) \sqrt{\csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)**(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x)

[Out] Integral((x*cos(x) - 4*tan(x)*sec(x))*sqrt(csc(x)), x)

$$3.866 \quad \int \cot(x) \sqrt{-1 + \csc^2(x)} \left(1 - \sin^2(x)\right)^3 dx$$

Optimal. Leaf size=76

$$-\frac{35}{16} \sqrt{\cot^2(x)} + \frac{1}{6} \cos^6(x) \sqrt{\cot^2(x)} + \frac{7}{24} \cos^4(x) \sqrt{\cot^2(x)} + \frac{35}{48} \cos^2(x) \sqrt{\cot^2(x)} - \frac{35}{16} x \tan(x) \sqrt{\cot^2(x)}$$

[Out] $-35/16*(\cot(x)^2)^{(1/2)}+35/48*\cos(x)^2*(\cot(x)^2)^{(1/2)}+7/24*\cos(x)^4*(\cot(x)^2)^{(1/2)}+1/6*\cos(x)^6*(\cot(x)^2)^{(1/2)}-35/16*x*(\cot(x)^2)^{(1/2)}*\tan(x)$

Rubi [A] time = 0.16, antiderivative size = 84, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3175, 4360, 25, 266, 47, 50, 63, 203}

$$-\frac{35}{16} \sqrt{\csc^2(x) - 1} + \frac{1}{6} \sin^6(x) (\csc^2(x) - 1)^{7/2} + \frac{7}{24} \sin^4(x) (\csc^2(x) - 1)^{5/2} + \frac{35}{48} \sin^2(x) (\csc^2(x) - 1)^{3/2} + \frac{35}{16} \tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cot[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]

[Out] $(35*\text{ArcTan}[\text{Sqrt}[-1 + \text{Csc}[x]^2]])/16 - (35*\text{Sqrt}[-1 + \text{Csc}[x]^2])/16 + (35*(-1 + \text{Csc}[x]^2)^{(3/2)}*\text{Sin}[x]^2)/48 + (7*(-1 + \text{Csc}[x]^2)^{(5/2)}*\text{Sin}[x]^4)/24 + ((-1 + \text{Csc}[x]^2)^{(7/2)}*\text{Sin}[x]^6)/6$

Rule 25

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(m_.)*((c_.) + (d_.)*(x_)^(q_.))^(p_.), x_Symbol] := Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d, 0] && !(IntegerQ[m] && NegQ[n])

Rule 47

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 50

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3175

```
Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)^2])^(p_), x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4360

```
Int[(u_)*(F_)[(c_)*((a_) + (b_)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}
\int \cot(x)\sqrt{-1 + \csc^2(x)} (1 - \sin^2(x))^3 dx &= \int \cos^6(x) \cot(x)\sqrt{-1 + \csc^2(x)} dx \\
&= \text{Subst} \left(\int \frac{\sqrt{-1 + \frac{1}{x^2}} (1 - x^2)^3}{x} dx, x, \sin(x) \right) \\
&= \text{Subst} \left(\int \left(-1 + \frac{1}{x^2}\right)^{7/2} x^5 dx, x, \sin(x) \right) \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{(-1 + x)^{7/2}}{x^4} dx, x, \csc^2(x) \right)\right) \\
&= \frac{1}{6} \cot^2(x)^{7/2} \sin^6(x) - \frac{7}{12} \text{Subst} \left(\int \frac{(-1 + x)^{5/2}}{x^3} dx, x, \csc^2(x) \right) \\
&= \frac{7}{24} \cot^2(x)^{5/2} \sin^4(x) + \frac{1}{6} \cot^2(x)^{7/2} \sin^6(x) - \frac{35}{48} \text{Subst} \left(\int \frac{(-1 + x)^5}{x^2} dx, x, \csc^2(x) \right) \\
&= \frac{35}{48} \cot^2(x)^{3/2} \sin^2(x) + \frac{7}{24} \cot^2(x)^{5/2} \sin^4(x) + \frac{1}{6} \cot^2(x)^{7/2} \sin^6(x) - \\
&= -\frac{35}{16} \sqrt{\cot^2(x)} + \frac{35}{48} \cot^2(x)^{3/2} \sin^2(x) + \frac{7}{24} \cot^2(x)^{5/2} \sin^4(x) + \frac{1}{6} \cot^2(x)^{7/2} \sin^6(x) \\
&= -\frac{35}{16} \sqrt{\cot^2(x)} + \frac{35}{48} \cot^2(x)^{3/2} \sin^2(x) + \frac{7}{24} \cot^2(x)^{5/2} \sin^4(x) + \frac{1}{6} \cot^2(x)^{7/2} \sin^6(x) \\
&= \frac{35}{16} \tan^{-1} \left(\sqrt{\cot^2(x)} \right) - \frac{35}{16} \sqrt{\cot^2(x)} + \frac{35}{48} \cot^2(x)^{3/2} \sin^2(x) + \frac{7}{24} \cot^2(x)^{5/2} \sin^4(x) + \frac{1}{6} \cot^2(x)^{7/2} \sin^6(x)
\end{aligned}$$

Mathematica [A] time = 0.09, size = 40, normalized size = 0.53

$$\frac{1}{384} \sqrt{\cot^2(x)} \sec(x) (-840x \sin(x) - 525 \cos(x) + 126 \cos(3x) + 14 \cos(5x) + \cos(7x))$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]

[Out] (Sqrt[Cot[x]^2]*Sec[x]*(-525*Cos[x] + 126*Cos[3*x] + 14*Cos[5*x] + Cos[7*x] - 840*x*Sin[x]))/384

fricas [A] time = 1.92, size = 34, normalized size = 0.45

$$\frac{8 \cos(x)^7 + 14 \cos(x)^5 + 35 \cos(x)^3 - 105 x \sin(x) - 105 \cos(x)}{48 \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="fricas")

[Out] -1/48*(8*cos(x)^7 + 14*cos(x)^5 + 35*cos(x)^3 - 105*x*sin(x) - 105*cos(x))/sin(x)

giac [A] time = 0.15, size = 97, normalized size = 1.28

$$-\frac{1}{48} \left((2(4 \sin(x)^2 - 19) \sin(x)^2 + 87) \sqrt{-\sin(x)^2 + 1} \sin(x) - 105 \left(\pi \left[\frac{x}{\pi} + \frac{1}{2} \right] - x \right) (-1)^{\left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor} + \frac{24(\sqrt{-\sin(x)^2 + 1})}{\sin(x)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="giac")

[Out] -1/48*((2*(4*sin(x)^2 - 19)*sin(x)^2 + 87)*sqrt(-sin(x)^2 + 1)*sin(x) - 105*(pi*floor(x/pi + 1/2) - x)*(-1)^floor(x/pi + 1/2) + 24*(sqrt(-sin(x)^2 + 1) - 1)/sin(x) - 24*sin(x)/(sqrt(-sin(x)^2 + 1) - 1))*sgn(sin(x))

maple [A] time = 0.38, size = 54, normalized size = 0.71

$$\frac{(-8(\cos^7(x)) - 14(\cos^5(x)) - 35(\cos^3(x)) + 105x \sin(x) + 105 \cos(x)) \sqrt{\frac{-\cos^2(x)}{-1+\cos^2(x)}} \sqrt{4}}{96 \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x)

[Out] -1/96*(-8*cos(x)^7-14*cos(x)^5-35*cos(x)^3+105*x*sin(x)+105*cos(x))*(-cos(x)^2/(-1+cos(x)^2))^(1/2)/cos(x)*4^(1/2)

maxima [B] time = 0.71, size = 136, normalized size = 1.79

$$-\frac{3}{2} \sqrt{\frac{1}{\sin(x)^2} - 1} \sin(x)^2 - \sqrt{\frac{1}{\sin(x)^2} - 1} + \frac{3 \left(\frac{1}{\sin(x)^2} - 1 \right)^{\frac{5}{2}} + 8 \left(\frac{1}{\sin(x)^2} - 1 \right)^{\frac{3}{2}} - 3 \sqrt{\frac{1}{\sin(x)^2} - 1}}{48 \left(\left(\frac{1}{\sin(x)^2} - 1 \right)^3 + 3 \left(\frac{1}{\sin(x)^2} - 1 \right)^2 + \frac{3}{\sin(x)^2} - 2 \right)} - \frac{3 \left(\frac{1}{\sin(x)^2} - 1 \right)^{\frac{3}{2}}}{8 \left(\frac{1}{\sin(x)^2} - 1 \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="maxima")

[Out] -3/2*sqrt(1/sin(x)^2 - 1)*sin(x)^2 - sqrt(1/sin(x)^2 - 1) + 1/48*(3*(1/sin(x)^2 - 1)^(5/2) + 8*(1/sin(x)^2 - 1)^(3/2) - 3*sqrt(1/sin(x)^2 - 1))/((1/sin(x)^2 - 1)^3 + 3*(1/sin(x)^2 - 1)^2 + 3/sin(x)^2 - 2) - 3/8*((1/sin(x)^2 - 1)^(3/2) - sqrt(1/sin(x)^2 - 1))/((1/sin(x)^2 - 1)^2 + 2/sin(x)^2 - 1) + 35/16*arctan(sqrt(1/sin(x)^2 - 1))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\cot(x) \sqrt{\frac{1}{\sin(x)^2} - 1} (\sin(x)^2 - 1)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-cot(x)*(1/sin(x)^2 - 1)^(1/2)*(sin(x)^2 - 1)^3,x)`

[Out] `int(-cot(x)*(1/sin(x)^2 - 1)^(1/2)*(sin(x)^2 - 1)^3, x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(x)*(1-sin(x)**2)**3*(-1+csc(x)**2)**(1/2),x)`

[Out] Timed out

$$3.867 \quad \int \cos(x) \sqrt{-1 + \csc^2(x)} \left(1 - \sin^2(x)\right)^3 dx$$

Optimal. Leaf size=81

$$\sin(x)\sqrt{\cot^2(x)} + \frac{1}{7} \sin(x) \cos^6(x) \sqrt{\cot^2(x)} + \frac{1}{5} \sin(x) \cos^4(x) \sqrt{\cot^2(x)} + \frac{1}{3} \sin(x) \cos^2(x) \sqrt{\cot^2(x)} - \tan(x) \sqrt{\cot^2(x)}$$

```
[Out] sin(x)*(cot(x)^2)^(1/2)+1/3*cos(x)^2*sin(x)*(cot(x)^2)^(1/2)+1/5*cos(x)^4*
sin(x)*(cot(x)^2)^(1/2)+1/7*cos(x)^6*sin(x)*(cot(x)^2)^(1/2)-arctanh(cos(x))
*(cot(x)^2)^(1/2)*tan(x)
```

Rubi [A] time = 0.16, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3175, 4121, 3658, 2592, 302, 206}

$$\sin(x)\sqrt{\cot^2(x)} + \frac{1}{7} \sin(x) \cos^6(x) \sqrt{\cot^2(x)} + \frac{1}{5} \sin(x) \cos^4(x) \sqrt{\cot^2(x)} + \frac{1}{3} \sin(x) \cos^2(x) \sqrt{\cot^2(x)} - \tan(x) \sqrt{\cot^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

```
[Out] Sqrt[Cot[x]^2]*Sin[x] + (Cos[x]^2*Sqrt[Cot[x]^2]*Sin[x])/3 + (Cos[x]^4*Sqrt
[Cot[x]^2]*Sin[x])/5 + (Cos[x]^6*Sqrt[Cot[x]^2]*Sin[x])/7 - ArcTanh[Cos[x]]
*Sqrt[Cot[x]^2]*Tan[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)^(n_.), x_
Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Ssin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(p_), x_Symbol] := Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3658

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_)^(p_), x_Symbol] := With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Tan[e + f*x]^
n)^FracPart[p])/(Tan[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]))
```

Rule 4121

`Int[(u_.)*((a_.) + (b_.)*sec[(e_.) + (f_.)*(x_.)]^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]`

Rubi steps

$$\begin{aligned}
 \int \cos(x) \sqrt{-1 + \csc^2(x)} (1 - \sin^2(x))^3 dx &= \int \cos^7(x) \sqrt{-1 + \csc^2(x)} dx \\
 &= \int \cos^7(x) \sqrt{\cot^2(x)} dx \\
 &= \left(\sqrt{\cot^2(x)} \tan(x) \right) \int \cos^7(x) \cot(x) dx \\
 &= - \left(\left(\sqrt{\cot^2(x)} \tan(x) \right) \text{Subst} \left(\int \frac{x^8}{1-x^2} dx, x, \cos(x) \right) \right) \\
 &= - \left(\left(\sqrt{\cot^2(x)} \tan(x) \right) \text{Subst} \left(\int \left(-1 - x^2 - x^4 - x^6 + \frac{1}{1-x^2} \right) dx, x, \right. \right. \\
 &= \sqrt{\cot^2(x)} \sin(x) + \frac{1}{3} \cos^2(x) \sqrt{\cot^2(x)} \sin(x) + \frac{1}{5} \cos^4(x) \sqrt{\cot^2(x)} \sin(x) \\
 &= \sqrt{\cot^2(x)} \sin(x) + \frac{1}{3} \cos^2(x) \sqrt{\cot^2(x)} \sin(x) + \frac{1}{5} \cos^4(x) \sqrt{\cot^2(x)} \sin(x)
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 0.68

$$\frac{\tan(x) \sqrt{\cot^2(x)} (9765 \cos(x) + 1295 \cos(3x) + 189 \cos(5x) + 15 \cos(7x) + 6720 \log(\sin(\frac{x}{2})) - 6720 \log(\cos(\frac{x}{2})))}{6720}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]

[Out] (Sqrt[Cot[x]^2]*(9765*Cos[x] + 1295*Cos[3*x] + 189*Cos[5*x] + 15*Cos[7*x] - 6720*Log[Cos[x/2]] + 6720*Log[Sin[x/2]])*Tan[x])/6720

fricas [A] time = 0.93, size = 41, normalized size = 0.51

$$-\frac{1}{7} \cos(x)^7 - \frac{1}{5} \cos(x)^5 - \frac{1}{3} \cos(x)^3 - \cos(x) + \frac{1}{2} \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) - \frac{1}{2} \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="fricas")

[Out] -1/7*cos(x)^7 - 1/5*cos(x)^5 - 1/3*cos(x)^3 - cos(x) + 1/2*log(1/2*cos(x) + 1/2) - 1/2*log(-1/2*cos(x) + 1/2)

giac [A] time = 0.15, size = 44, normalized size = 0.54

$$\frac{1}{210} (30 \cos(x)^7 + 42 \cos(x)^5 + 70 \cos(x)^3 + 210 \cos(x) - 105 \log(\cos(x) + 1) + 105 \log(-\cos(x) + 1)) \text{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="giac")

[Out] 1/210*(30*cos(x)^7 + 42*cos(x)^5 + 70*cos(x)^3 + 210*cos(x) - 105*log(cos(x) + 1) + 105*log(-cos(x) + 1))*sgn(sin(x))

maple [A] time = 0.27, size = 65, normalized size = 0.80

$$\frac{\left(15 \left(\cos^7(x)\right) + 21 \left(\cos^5(x)\right) + 35 \left(\cos^3(x)\right) + 105 \cos(x) + 105 \ln\left(-\frac{-1+\cos(x)}{\sin(x)}\right) + 176\right) \sin(x) \sqrt{-\frac{\cos^2(x)}{-1+\cos^2(x)}} \sqrt{4}}{210 \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x)

[Out] 1/210*(15*cos(x)^7+21*cos(x)^5+35*cos(x)^3+105*cos(x)+105*ln(-(-1+cos(x))/sin(x))+176)*sin(x)*(-cos(x)^2/(-1+cos(x)^2))^(1/2)/cos(x)*4^(1/2)

maxima [A] time = 0.93, size = 86, normalized size = 1.06

$$\frac{1}{7} \left(\frac{1}{\sin(x)^2} - 1 \right)^{\frac{7}{2}} \sin(x)^7 + \frac{1}{5} \left(\frac{1}{\sin(x)^2} - 1 \right)^{\frac{5}{2}} \sin(x)^5 + \frac{1}{3} \left(\frac{1}{\sin(x)^2} - 1 \right)^{\frac{3}{2}} \sin(x)^3 + \sqrt{\frac{1}{\sin(x)^2} - 1} \sin(x) - \frac{1}{2} \log \left(\sqrt{\frac{1}{\sin(x)^2} - 1} \sin(x) + 1 \right) + \frac{1}{2} \log \left(\sqrt{\frac{1}{\sin(x)^2} - 1} \sin(x) - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="maxima")

[Out] 1/7*(1/sin(x)^2 - 1)^(7/2)*sin(x)^7 + 1/5*(1/sin(x)^2 - 1)^(5/2)*sin(x)^5 + 1/3*(1/sin(x)^2 - 1)^(3/2)*sin(x)^3 + sqrt(1/sin(x)^2 - 1)*sin(x) - 1/2*log(sqrt(1/sin(x)^2 - 1)*sin(x) + 1) + 1/2*log(sqrt(1/sin(x)^2 - 1)*sin(x) - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \cos(x) \sqrt{\frac{1}{\sin(x)^2} - 1} (\sin(x)^2 - 1)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-cos(x)*(1/sin(x)^2 - 1)^(1/2)*(sin(x)^2 - 1)^3,x)

[Out] -int(cos(x)*(1/sin(x)^2 - 1)^(1/2)*(sin(x)^2 - 1)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(1-sin(x)**2)**3*(-1+csc(x)**2)**(1/2),x)

[Out] Timed out

$$3.868 \quad \int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} dx$$

Optimal. Leaf size=76

$$\frac{i \operatorname{Li}_2(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{i \operatorname{Li}_2(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{2x \sec(x) \tanh^{-1}(e^{ix})}{\sqrt{a \sec^2(x)}}$$

[Out] $-2*x*\operatorname{arctanh}(\exp(I*x))*\sec(x)/(a*\sec(x)^2)^{(1/2)}+I*\operatorname{polylog}(2,-\exp(I*x))*\sec(x)/(a*\sec(x)^2)^{(1/2)}-I*\operatorname{polylog}(2,\exp(I*x))*\sec(x)/(a*\sec(x)^2)^{(1/2)}$

Rubi [A] time = 0.53, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6720, 4183, 2279, 2391}

$$\frac{i \sec(x) \operatorname{PolyLog}(2, -e^{ix})}{\sqrt{a \sec^2(x)}} - \frac{i \sec(x) \operatorname{PolyLog}(2, e^{ix})}{\sqrt{a \sec^2(x)}} - \frac{2x \sec(x) \tanh^{-1}(e^{ix})}{\sqrt{a \sec^2(x)}}$$

Antiderivative was successfully verified.

[In] `Int[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]`

[Out] $(-2*x*\operatorname{ArcTanh}[E^{(I*x)}]*\operatorname{Sec}[x])/Sqrt[a*\operatorname{Sec}[x]^2] + (I*\operatorname{PolyLog}[2, -E^{(I*x)}]*\operatorname{Sec}[x])/Sqrt[a*\operatorname{Sec}[x]^2] - (I*\operatorname{PolyLog}[2, E^{(I*x)}]*\operatorname{Sec}[x])/Sqrt[a*\operatorname{Sec}[x]^2]$

Rule 2279

`Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

Rule 2391

`Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

Rule 4183

`Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

Rule 6720

`Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])`

Rubi steps

$$\begin{aligned}
\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} dx &= \frac{\sec(x) \int x \csc(x) dx}{\sqrt{a \sec^2(x)}} \\
&= -\frac{2x \tanh^{-1}(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{\sec(x) \int \log(1 - e^{ix}) dx}{\sqrt{a \sec^2(x)}} + \frac{\sec(x) \int \log(1 + e^{ix}) dx}{\sqrt{a \sec^2(x)}} \\
&= -\frac{2x \tanh^{-1}(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{(i \sec(x)) \operatorname{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{ix}\right)}{\sqrt{a \sec^2(x)}} - \frac{(i \sec(x)) \operatorname{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^{ix}\right)}{\sqrt{a \sec^2(x)}} \\
&= -\frac{2x \tanh^{-1}(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{i \operatorname{Li}_2(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{i \operatorname{Li}_2(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 69, normalized size = 0.91

$$\frac{\sec(x) (i \operatorname{Li}_2(-e^{ix}) - i \operatorname{Li}_2(e^{ix}) + x (\log(1 - e^{ix}) - \log(1 + e^{ix})))}{\sqrt{a \sec^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2], x]

[Out] ((x*(Log[1 - E^(I*x)] - Log[1 + E^(I*x)]) + I*PolyLog[2, -E^(I*x)] - I*PolyLog[2, E^(I*x)])*Sec[x])/Sqrt[a*Sec[x]^2]

fricas [B] time = 1.27, size = 124, normalized size = 1.63

$$(x \cos(x) \log(\cos(x) + i \sin(x) + 1) + x \cos(x) \log(\cos(x) - i \sin(x) + 1) - x \cos(x) \log(-\cos(x) + i \sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2), x, algorithm="fricas")

[Out] -1/2*(x*cos(x)*log(cos(x) + I*sin(x) + 1) + x*cos(x)*log(cos(x) - I*sin(x) + 1) - x*cos(x)*log(-cos(x) + I*sin(x) + 1) - x*cos(x)*log(-cos(x) - I*sin(x) + 1) + I*cos(x)*dilog(cos(x) + I*sin(x)) - I*cos(x)*dilog(cos(x) - I*sin(x)) + I*cos(x)*dilog(-cos(x) + I*sin(x)) - I*cos(x)*dilog(-cos(x) - I*sin(x)))*sqrt(a/cos(x)^2)/a

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2), x, algorithm="giac")

[Out] integrate(x*csc(x)*sec(x)/sqrt(a*sec(x)^2), x)

maple [A] time = 0.23, size = 98, normalized size = 1.29

$$\frac{2i \left(-\frac{ie^{ix} \ln(1+e^{ix})}{2} - \frac{e^{ix} \operatorname{polylog}(2, -e^{ix})}{2} + \frac{ie^{ix} \ln(1-e^{ix})}{2} + \frac{e^{ix} \operatorname{polylog}(2, e^{ix})}{2} \right)}{\sqrt{\frac{a e^{2ix}}{(e^{2ix}+1)^2}} (e^{2ix} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x)`

[Out] $-2*I/(a*\exp(2*I*x)/(\exp(2*I*x)+1)^2)^(1/2)/(\exp(2*I*x)+1)*(-1/2*I*\exp(I*x)*x*\ln(1+\exp(I*x))-1/2*\exp(I*x)*\text{polylog}(2,-\exp(I*x))+1/2*I*\exp(I*x)*x*\ln(1-\exp(I*x))+1/2*\exp(I*x)*\text{polylog}(2,\exp(I*x)))$

maxima [A] time = 0.85, size = 79, normalized size = 1.04

$$\frac{2ix \arctan(\sin(x), \cos(x) + 1) + 2ix \arctan(\sin(x), -\cos(x) + 1) + x \log(\cos(x)^2 + \sin(x)^2 + 2 \cos(x) + 1) - 2\sqrt{a}}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="maxima")`

[Out] $-1/2*(2*I*x*\arctan2(\sin(x), \cos(x) + 1) + 2*I*x*\arctan2(\sin(x), -\cos(x) + 1) + x*\log(\cos(x)^2 + \sin(x)^2 + 2*\cos(x) + 1) - x*\log(\cos(x)^2 + \sin(x)^2 - 2*\cos(x) + 1) - 2*I*\text{dilog}(-e^{I*x}) + 2*I*\text{dilog}(e^{I*x}))/\text{sqrt}(a)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cos(x) \sin(x) \sqrt{\frac{a}{\cos(x)^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)),x)`

[Out] `int(x/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*csc(x)*sec(x)/(a*sec(x)**2)**(1/2),x)`

[Out] `Integral(x*csc(x)*sec(x)/sqrt(a*sec(x)**2), x)`

$$3.869 \quad \int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} dx$$

Optimal. Leaf size=128

$$\frac{2ix\text{Li}_2(-e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{2ix\text{Li}_2(e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{2\text{Li}_3(-e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{2\text{Li}_3(e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{2x^2\sec(x)\tanh^{-1}(e^{ix})}{\sqrt{a\sec^2(x)}}$$

[Out] $-2*x^2*\text{arctanh}(\exp(I*x))*\sec(x)/(a*\sec(x)^2)^{(1/2)}+2*I*x*\text{polylog}(2,-\exp(I*x))*\sec(x)/(a*\sec(x)^2)^{(1/2)}-2*I*x*\text{polylog}(2,\exp(I*x))*\sec(x)/(a*\sec(x)^2)^{(1/2)}-2*\text{polylog}(3,-\exp(I*x))*\sec(x)/(a*\sec(x)^2)^{(1/2)}+2*\text{polylog}(3,\exp(I*x))*\sec(x)/(a*\sec(x)^2)^{(1/2)}$

Rubi [A] time = 0.59, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6720, 4183, 2531, 2282, 6589}

$$\frac{2ix\sec(x)\text{PolyLog}(2,-e^{ix})}{\sqrt{a\sec^2(x)}} - \frac{2ix\sec(x)\text{PolyLog}(2,e^{ix})}{\sqrt{a\sec^2(x)}} - \frac{2\sec(x)\text{PolyLog}(3,-e^{ix})}{\sqrt{a\sec^2(x)}} + \frac{2\sec(x)\text{PolyLog}(3,e^{ix})}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]

[Out] $(-2*x^2*\text{ArcTanh}[E^{(I*x)}]*\text{Sec}[x])/Sqrt[a*\text{Sec}[x]^2] + ((2*I)*x*\text{PolyLog}[2,-E^{(I*x)}]*\text{Sec}[x])/Sqrt[a*\text{Sec}[x]^2] - ((2*I)*x*\text{PolyLog}[2,E^{(I*x)}]*\text{Sec}[x])/Sqrt[a*\text{Sec}[x]^2] - (2*\text{PolyLog}[3,-E^{(I*x)}]*\text{Sec}[x])/Sqrt[a*\text{Sec}[x]^2] + (2*\text{PolyLog}[3,E^{(I*x)}]*\text{Sec}[x])/Sqrt[a*\text{Sec}[x]^2]$

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 2531

Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m-1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 4183

Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Log[1 + E^(I*(e + f*x))], x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned} \int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} dx &= \frac{\sec(x) \int x^2 \csc(x) dx}{\sqrt{a \sec^2(x)}} \\ &= -\frac{2x^2 \tanh^{-1}(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{(2 \sec(x)) \int x \log(1 - e^{ix}) dx}{\sqrt{a \sec^2(x)}} + \frac{(2 \sec(x)) \int x \log(1 + e^{ix}) dx}{\sqrt{a \sec^2(x)}} \\ &= -\frac{2x^2 \tanh^{-1}(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{2ix \operatorname{Li}_2(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{2ix \operatorname{Li}_2(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{(2i \sec(x)) \int \operatorname{Li}_2}{\sqrt{a \sec^2(x)}} \\ &= -\frac{2x^2 \tanh^{-1}(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{2ix \operatorname{Li}_2(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{2ix \operatorname{Li}_2(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{(2 \sec(x)) \operatorname{Subst}}{\sqrt{a \sec^2(x)}} \\ &= -\frac{2x^2 \tanh^{-1}(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{2ix \operatorname{Li}_2(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{2ix \operatorname{Li}_2(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{2 \operatorname{Li}_3(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} \end{aligned}$$

Mathematica [A] time = 0.07, size = 99, normalized size = 0.77

$$\frac{\sec(x) \left(2ix \operatorname{Li}_2(-e^{ix}) - 2ix \operatorname{Li}_2(e^{ix}) - 2 \operatorname{Li}_3(-e^{ix}) + 2 \operatorname{Li}_3(e^{ix}) + x^2 \log(1 - e^{ix}) - x^2 \log(1 + e^{ix}) \right)}{\sqrt{a \sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2], x]
```

```
[Out] ((x^2*Log[1 - E^(I*x)] - x^2*Log[1 + E^(I*x)] + (2*I)*x*PolyLog[2, -E^(I*x)]
] - (2*I)*x*PolyLog[2, E^(I*x)] - 2*PolyLog[3, -E^(I*x)] + 2*PolyLog[3, E^(
I*x)])*Sec[x])/Sqrt[a*Sec[x]^2]
```

fricas [C] time = 1.62, size = 227, normalized size = 1.77

$$2 \sqrt{\frac{a}{\cos(x)^2}} \cos(x) \operatorname{polylog}(3, \cos(x) + i \sin(x)) + 2 \sqrt{\frac{a}{\cos(x)^2}} \cos(x) \operatorname{polylog}(3, \cos(x) - i \sin(x)) - 2 \sqrt{\frac{a}{\cos(x)^2}} \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2), x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + 2*sqrt(a/cos
(x)^2)*cos(x)*polylog(3, cos(x) - I*sin(x)) - 2*sqrt(a/cos(x)^2)*cos(x)*pol
ylog(3, -cos(x) + I*sin(x)) - 2*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(x)
- I*sin(x)) - (x^2*cos(x)*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)*log(cos(x)
) - I*sin(x) + 1) - x^2*cos(x)*log(-cos(x) + I*sin(x) + 1) - x^2*cos(x)*log
(-cos(x) - I*sin(x) + 1) + 2*I*x*cos(x)*dilog(cos(x) + I*sin(x)) - 2*I*x*co
s(x)*dilog(cos(x) - I*sin(x)) + 2*I*x*cos(x)*dilog(-cos(x) + I*sin(x)) - 2*
I*x*cos(x)*dilog(-cos(x) - I*sin(x)))*sqrt(a/cos(x)^2))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="giac")

[Out] integrate(x^2*csc(x)*sec(x)/sqrt(a*sec(x)^2), x)

maple [A] time = 0.20, size = 132, normalized size = 1.03

$$\frac{2\left(\frac{e^{ix}x^2\ln(1+e^{ix})}{2} - ie^{ix}x \operatorname{polylog}(2, -e^{ix}) + e^{ix} \operatorname{polylog}(3, -e^{ix}) - \frac{e^{ix}x^2\ln(1-e^{ix})}{2} + ie^{ix}x \operatorname{polylog}(2, e^{ix}) - e^{ix} \operatorname{polylog}(3, e^{ix})\right)}{\sqrt{\frac{ae^{2ix}}{(e^{2ix}+1)^2}}(e^{2ix}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x)

[Out] -2/(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)/(exp(2*I*x)+1)*(1/2*exp(I*x)*x^2*ln(1+exp(I*x))-I*exp(I*x)*x*polylog(2,-exp(I*x))+exp(I*x)*polylog(3,-exp(I*x))-1/2*exp(I*x)*x^2*ln(1-exp(I*x))+I*exp(I*x)*x*polylog(2,exp(I*x))-exp(I*x)*polylog(3,exp(I*x)))

maxima [A] time = 0.68, size = 107, normalized size = 0.84

$$\frac{2ix^2 \arctan(\sin(x), \cos(x) + 1) + 2ix^2 \arctan(\sin(x), -\cos(x) + 1) + x^2 \log(\cos(x)^2 + \sin(x)^2 + 2 \cos(x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="maxima")

[Out] -1/2*(2*I*x^2*arctan2(sin(x), cos(x) + 1) + 2*I*x^2*arctan2(sin(x), -cos(x) + 1) + x^2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x^2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 4*I*x*dilog(-e^(I*x)) + 4*I*x*dilog(e^(I*x)) + 4*polylog(3, -e^(I*x)) - 4*polylog(3, e^(I*x)))/sqrt(a)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cos(x) \sin(x) \sqrt{\frac{a}{\cos(x)^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)),x)

[Out] int(x^2/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*csc(x)*sec(x)/(a*sec(x)**2)**(1/2),x)

[Out] Integral(x**2*csc(x)*sec(x)/sqrt(a*sec(x)**2), x)

$$3.870 \quad \int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} dx$$

Optimal. Leaf size=186

$$\frac{3ix^2 \text{Li}_2(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{3ix^2 \text{Li}_2(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{6x \text{Li}_3(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{6x \text{Li}_3(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{6i \text{Li}_4(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{6i \text{Li}_4(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}}$$

[Out] $-2*x^3*\text{arctanh}(\exp(I*x))*\sec(x)/(a*\sec(x)^2)^{(1/2)}+3*I*x^2*\text{polylog}(2,-\exp(I*x))*\sec(x)/(a*\sec(x)^2)^{(1/2)}-3*I*x^2*\text{polylog}(2,\exp(I*x))*\sec(x)/(a*\sec(x)^2)^{(1/2)}-6*x*\text{polylog}(3,-\exp(I*x))*\sec(x)/(a*\sec(x)^2)^{(1/2)}+6*x*\text{polylog}(3,\exp(I*x))*\sec(x)/(a*\sec(x)^2)^{(1/2)}-6*I*\text{polylog}(4,-\exp(I*x))*\sec(x)/(a*\sec(x)^2)^{(1/2)}+6*I*\text{polylog}(4,\exp(I*x))*\sec(x)/(a*\sec(x)^2)^{(1/2)}$

Rubi [A] time = 0.57, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6720, 4183, 2531, 6609, 2282, 6589}

$$\frac{3ix^2 \sec(x) \text{PolyLog}(2, -e^{ix})}{\sqrt{a \sec^2(x)}} - \frac{3ix^2 \sec(x) \text{PolyLog}(2, e^{ix})}{\sqrt{a \sec^2(x)}} - \frac{6x \sec(x) \text{PolyLog}(3, -e^{ix})}{\sqrt{a \sec^2(x)}} + \frac{6x \sec(x) \text{PolyLog}(3, e^{ix})}{\sqrt{a \sec^2(x)}}$$

Antiderivative was successfully verified.

[In] `Int[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]`

[Out] $(-2*x^3*\text{ArcTanh}[E^{(I*x)}]*\text{Sec}[x])/ \text{Sqrt}[a*\text{Sec}[x]^2] + ((3*I)*x^2*\text{PolyLog}[2, -E^{(I*x)}]*\text{Sec}[x])/ \text{Sqrt}[a*\text{Sec}[x]^2] - ((3*I)*x^2*\text{PolyLog}[2, E^{(I*x)}]*\text{Sec}[x])/ \text{Sqrt}[a*\text{Sec}[x]^2] - (6*x*\text{PolyLog}[3, -E^{(I*x)}]*\text{Sec}[x])/ \text{Sqrt}[a*\text{Sec}[x]^2] + (6*x*\text{PolyLog}[3, E^{(I*x)}]*\text{Sec}[x])/ \text{Sqrt}[a*\text{Sec}[x]^2] - ((6*I)*\text{PolyLog}[4, -E^{(I*x)}]*\text{Sec}[x])/ \text{Sqrt}[a*\text{Sec}[x]^2] + ((6*I)*\text{PolyLog}[4, E^{(I*x)}]*\text{Sec}[x])/ \text{Sqrt}[a*\text{Sec}[x]^2]$

Rule 2282

`Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

Rule 2531

`Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m-1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

Rule 4183

`Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m-1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]`

Rule 6589

`Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d`

, e, n, p}, x] && EqQ[b*d, a*e]

Rule 6609

Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_.)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rule 6720

Int[(u_.)*((a_.)*(v_.)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} dx = \frac{\sec(x) \int x^3 \csc(x) dx}{\sqrt{a \sec^2(x)}} = -\frac{2x^3 \tanh^{-1}(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{(3 \sec(x)) \int x^2 \log(1 - e^{ix}) dx}{\sqrt{a \sec^2(x)}} + \frac{(3 \sec(x)) \int x^2 \log(1 + e^{ix}) dx}{\sqrt{a \sec^2(x)}} = -\frac{2x^3 \tanh^{-1}(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{3ix^2 \text{Li}_2(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{3ix^2 \text{Li}_2(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{(6i \sec(x)) \int x \log(1 - e^{ix}) dx}{\sqrt{a \sec^2(x)}} = -\frac{2x^3 \tanh^{-1}(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{3ix^2 \text{Li}_2(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{3ix^2 \text{Li}_2(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{6x \text{Li}_3(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} = -\frac{2x^3 \tanh^{-1}(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{3ix^2 \text{Li}_2(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{3ix^2 \text{Li}_2(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{6x \text{Li}_3(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}}$$

Mathematica [A] time = 0.10, size = 147, normalized size = 0.79

$$\frac{i \sec(x) (-24x^2 \text{Li}_2(e^{-ix}) - 24x^2 \text{Li}_2(-e^{ix}) + 48ix \text{Li}_3(e^{-ix}) - 48ix \text{Li}_3(-e^{ix}) + 48\text{Li}_4(e^{-ix}) + 48\text{Li}_4(-e^{ix}) - 24x \text{Li}_5(e^{-ix}) - 24x \text{Li}_5(-e^{ix}))}{8\sqrt{a \sec^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2], x]

[Out] ((-1/8*I)*(Pi^4 - 2*x^4 + (8*I)*x^3*Log[1 - E^((-I)*x)] - (8*I)*x^3*Log[1 + E^(I*x)] - 24*x^2*PolyLog[2, E^((-I)*x)] - 24*x^2*PolyLog[2, -E^(I*x)] + (48*I)*x*PolyLog[3, E^((-I)*x)] - (48*I)*x*PolyLog[3, -E^(I*x)] + 48*PolyLog[4, E^((-I)*x)] + 48*PolyLog[4, -E^(I*x)])*Sec[x])/Sqrt[a*Sec[x]^2]

fricas [C] time = 1.31, size = 327, normalized size = 1.76

$$6x \sqrt{\frac{a}{\cos(x)^2}} \cos(x) \text{polylog}(3, \cos(x) + i \sin(x)) + 6x \sqrt{\frac{a}{\cos(x)^2}} \cos(x) \text{polylog}(3, \cos(x) - i \sin(x)) - 6x \sqrt{\frac{a}{\cos(x)^2}} \cos(x) \text{polylog}(3, \cos(x) + i \sin(x)) - 6x \sqrt{\frac{a}{\cos(x)^2}} \cos(x) \text{polylog}(3, \cos(x) - i \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="fricas")

[Out] 1/2*(6*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + 6*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) - I*sin(x)) - 6*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(x) + I*sin(x)) - 6*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(x) - I*sin(x)) + 6*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) + I*sin(x)) - 6*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) - I*sin(x)) + 6*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, -cos(x) + I*sin(x)) - 6*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, -cos(x) - I*sin(x)) - (x^3*cos(x)*log(cos(x) + I*sin(x) + 1) + x^3*cos(x)*log(cos(x) - I*sin(x) + 1) - x^3*cos(x)*log(-cos(x) + I*sin(x) + 1) - x^3*cos(x)*log(-cos(x) - I*sin(x) + 1) + 3*I*x^2*cos(x)*dilog(cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(cos(x) - I*sin(x)) + 3*I*x^2*cos(x)*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(-cos(x) - I*sin(x)))*sqrt(a/cos(x)^2))/a

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec(x)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="giac")

[Out] integrate(x^3*csc(x)*sec(x)/sqrt(a*sec(x)^2), x)

maple [A] time = 0.19, size = 172, normalized size = 0.92

$$\frac{2i \left(\frac{ie^{ix}x^3 \ln(1+e^{ix})}{2} + \frac{3e^{ix}x^2 \operatorname{polylog}(2,-e^{ix})}{2} + 3ie^{ix}x \operatorname{polylog}(3,-e^{ix}) - 3e^{ix} \operatorname{polylog}(4,-e^{ix}) - \frac{ie^{ix}x^3 \ln(1-e^{ix})}{2} - \frac{3e^{ix}x^2 \operatorname{polylog}(2,-e^{-ix})}{2} \right)}{\sqrt{\frac{ae^{2ix}}{(e^{2ix}+1)^2}} (e^{2ix} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x)

[Out] 2*I/(a*exp(2*I*x)/(exp(2*I*x)+1)^(1/2)/(exp(2*I*x)+1)*(1/2*I*exp(I*x)*x^3*ln(1+exp(I*x))+3/2*exp(I*x)*x^2*polylog(2,-exp(I*x))+3*I*exp(I*x)*x*polylog(3,-exp(I*x))-3*exp(I*x)*polylog(4,-exp(I*x))-1/2*I*exp(I*x)*x^3*ln(1-exp(I*x))-3/2*exp(I*x)*x^2*polylog(2,exp(I*x))-3*I*exp(I*x)*x*polylog(3,exp(I*x))+3*exp(I*x)*polylog(4,exp(I*x))))

maxima [A] time = 0.82, size = 131, normalized size = 0.70

$$\frac{2ix^3 \arctan(\sin(x), \cos(x) + 1) + 2ix^3 \arctan(\sin(x), -\cos(x) + 1) + x^3 \log(\cos(x)^2 + \sin(x)^2 + 2 \cos(x) + 1)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="maxima")

[Out] -1/2*(2*I*x^3*arctan2(sin(x), cos(x) + 1) + 2*I*x^3*arctan2(sin(x), -cos(x) + 1) + x^3*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x^3*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 6*I*x^2*dilog(-e^(I*x)) + 6*I*x^2*dilog(e^(I*x)) + 12*x*polylog(3, -e^(I*x)) - 12*x*polylog(3, e^(I*x)) + 12*I*polylog(4, -e^(I*x)) - 12*I*polylog(4, e^(I*x)))/sqrt(a)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\cos(x) \sin(x) \sqrt{\frac{a}{\cos(x)^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)),x)`

[Out] `int(x^3/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*csc(x)*sec(x)/(a*sec(x)**2)**(1/2),x)`

[Out] `Integral(x**3*csc(x)*sec(x)/sqrt(a*sec(x)**2), x)`

$$3.871 \quad \int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx$$

Optimal. Leaf size=81

$$-\frac{i \operatorname{Li}_2(e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}} - \frac{ix^2 \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{x \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}}$$

[Out] $-1/2*I*x^2*\sec(x)^2/(a*\sec(x)^4)^{(1/2)}+x*\ln(1-\exp(2*I*x))*\sec(x)^2/(a*\sec(x)^4)^{(1/2)}-1/2*I*\operatorname{polylog}(2,\exp(2*I*x))*\sec(x)^2/(a*\sec(x)^4)^{(1/2)}$

Rubi [A] time = 0.49, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6720, 3717, 2190, 2279, 2391}

$$-\frac{i \sec^2(x) \operatorname{PolyLog}(2, e^{2ix})}{2\sqrt{a \sec^4(x)}} - \frac{ix^2 \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{x \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}}$$

Antiderivative was successfully verified.

[In] `Int[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]`

[Out] `((-I/2)*x^2*Sec[x]^2)/Sqrt[a*Sec[x]^4] + (x*Log[1 - E^((2*I)*x)]*Sec[x]^2)/Sqrt[a*Sec[x]^4] - ((I/2)*PolyLog[2, E^((2*I)*x)]*Sec[x]^2)/Sqrt[a*Sec[x]^4]`

Rule 2190

`Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

Rule 2279

`Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

Rule 2391

`Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

Rule 3717

`Int[(((c_) + (d_)*(x_))^(m_)*tan[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(((c + d*x)^m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]`

Rule 6720

`Int[(u_)*((a_)*(v_)^(m_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])`

Rubi steps

$$\begin{aligned}
\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx &= \frac{\sec^2(x) \int x \cot(x) dx}{\sqrt{a \sec^4(x)}} \\
&= -\frac{ix^2 \sec^2(x)}{2\sqrt{a \sec^4(x)}} - \frac{(2i \sec^2(x)) \int \frac{e^{2ix} x}{1-e^{2ix}} dx}{\sqrt{a \sec^4(x)}} \\
&= -\frac{ix^2 \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{x \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}} - \frac{\sec^2(x) \int \log(1 - e^{2ix}) dx}{\sqrt{a \sec^4(x)}} \\
&= -\frac{ix^2 \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{x \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}} + \frac{(i \sec^2(x)) \text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2ix}\right)}{2\sqrt{a \sec^4(x)}} \\
&= -\frac{ix^2 \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{x \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}} - \frac{i \text{Li}_2(e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.62

$$\frac{i \sec^2(x) \left(\text{Li}_2(e^{2ix}) + x(x + 2i \log(1 - e^{2ix})) \right)}{2\sqrt{a \sec^4(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4], x]

[Out] ((-1/2*I)*(x*(x + (2*I)*Log[1 - E^((2*I)*x)])) + PolyLog[2, E^((2*I)*x)])*Sec[x]^2/Sqrt[a*Sec[x]^4]

fricas [B] time = 2.10, size = 138, normalized size = 1.70

$$(x \cos(x)^2 \log(\cos(x) + i \sin(x) + 1) + x \cos(x)^2 \log(\cos(x) - i \sin(x) + 1) + x \cos(x)^2 \log(-\cos(x) + i \sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2), x, algorithm="fricas")

[Out] 1/2*(x*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x*cos(x)^2*log(cos(x) - I*sin(x) + 1) + x*cos(x)^2*log(-cos(x) + I*sin(x) + 1) + x*cos(x)^2*log(-cos(x) - I*sin(x) + 1) - I*cos(x)^2*dilog(cos(x) + I*sin(x)) + I*cos(x)^2*dilog(cos(x) - I*sin(x)) + I*cos(x)^2*dilog(-cos(x) + I*sin(x)) - I*cos(x)^2*dilog(-cos(x) - I*sin(x)))*sqrt(a/cos(x)^4)/a

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2), x, algorithm="giac")

[Out] integrate(x*csc(x)*sec(x)/sqrt(a*sec(x)^4), x)

maple [B] time = 0.20, size = 147, normalized size = 1.81

$$\frac{i e^{2ix} x^2 \left(\frac{e^{2ix} x^2}{2} + \frac{i e^{2ix} x \ln(1+e^{ix})}{2} + \frac{e^{2ix} \operatorname{polylog}(2, -e^{ix})}{2} + \frac{i e^{2ix} x \ln(1-e^{ix})}{2} + \frac{e^{2ix} \operatorname{polylog}(2, e^{ix})}{2} \right)}{2 \sqrt{\frac{a e^{4ix}}{(e^{2ix}+1)^4}} (e^{2ix}+1)^2} \sqrt{\frac{a e^{4ix}}{(e^{2ix}+1)^4}} (e^{2ix}+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2), x)`

[Out] `1/2*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*exp(2*I*x)*x^2 - 2*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*(1/2*exp(2*I*x)*x^2+1/2*I*exp(2*I*x)*x*ln(1+exp(I*x))+1/2*exp(2*I*x)*polylog(2, -exp(I*x))+1/2*I*exp(2*I*x)*x*ln(1-exp(I*x))+1/2*exp(2*I*x)*polylog(2, exp(I*x)))`

maxima [A] time = 1.92, size = 83, normalized size = 1.02

$$\frac{-i x^2 + 2i x \arctan(\sin(x), \cos(x) + 1) - 2i x \arctan(\sin(x), -\cos(x) + 1) + x \log(\cos(x)^2 + \sin(x)^2 + 2 \cos(x) + 1)}{2 \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2), x, algorithm="maxima")`

[Out] `1/2*(-I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -cos(x) + 1) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(I*x)))/sqrt(a)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cos(x) \sin(x) \sqrt{\frac{a}{\cos(x)^4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)), x)`

[Out] `int(x/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*csc(x)*sec(x)/(a*sec(x)**4)**(1/2), x)`

[Out] `Integral(x*csc(x)*sec(x)/sqrt(a*sec(x)**4), x)`

$$3.872 \quad \int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx$$

Optimal. Leaf size=109

$$-\frac{ix \operatorname{Li}_2(e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}} + \frac{\operatorname{Li}_3(e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}} - \frac{ix^3 \sec^2(x)}{3\sqrt{a \sec^4(x)}} + \frac{x^2 \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}}$$

[Out] $-1/3*I*x^3*\sec(x)^2/(a*\sec(x)^4)^{(1/2)}+x^2*\ln(1-\exp(2*I*x))*\sec(x)^2/(a*\sec(x)^4)^{(1/2)}-I*x*\operatorname{polylog}(2,\exp(2*I*x))*\sec(x)^2/(a*\sec(x)^4)^{(1/2)}+1/2*\operatorname{polylog}(3,\exp(2*I*x))*\sec(x)^2/(a*\sec(x)^4)^{(1/2)}$

Rubi [A] time = 0.57, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6720, 3717, 2190, 2531, 2282, 6589}

$$-\frac{ix \sec^2(x) \operatorname{PolyLog}(2, e^{2ix})}{\sqrt{a \sec^4(x)}} + \frac{\sec^2(x) \operatorname{PolyLog}(3, e^{2ix})}{2\sqrt{a \sec^4(x)}} - \frac{ix^3 \sec^2(x)}{3\sqrt{a \sec^4(x)}} + \frac{x^2 \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^2*\operatorname{Csc}[x]*\operatorname{Sec}[x])/Sqrt[a*\operatorname{Sec}[x]^4], x]$

[Out] $((-I/3)*x^3*\operatorname{Sec}[x]^2)/Sqrt[a*\operatorname{Sec}[x]^4] + (x^2*\operatorname{Log}[1 - E^{((2*I)*x)}]*\operatorname{Sec}[x]^2)/Sqrt[a*\operatorname{Sec}[x]^4] - (I*x*\operatorname{PolyLog}[2, E^{((2*I)*x)}]*\operatorname{Sec}[x]^2)/Sqrt[a*\operatorname{Sec}[x]^4] + (\operatorname{PolyLog}[3, E^{((2*I)*x)}]*\operatorname{Sec}[x]^2)/(2*Sqrt[a*\operatorname{Sec}[x]^4])$

Rule 2190

$\operatorname{Int}[\frac{((F_)^{((g_)*(e_)+(f_)*(x_)))^{(n_)*((c_)+(d_)*(x_))^{(m_))}}}{((a_)+(b_)*((F_)^{((g_)*(e_)+(f_)*(x_)))^{(n_))}}), x_Symbol] := \operatorname{Simp}[\frac{(c+d*x)^m*\operatorname{Log}[1+(b*(F^{(g*(e+f*x)))^n})/a]}{(b*f*g*n*\operatorname{Log}[F])}, x] - \operatorname{Dist}[\frac{(d*m)}{(b*f*g*n*\operatorname{Log}[F])}, \operatorname{Int}[(c+d*x)^{(m-1)}*\operatorname{Log}[1+(b*(F^{(g*(e+f*x)))^n})/a], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \operatorname{IGtQ}[m, 0]$

Rule 2282

$\operatorname{Int}[u, x_Symbol] := \operatorname{With}\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \operatorname{FunctionOfExponentialQ}[u, x] \&\& \operatorname{!MatchQ}[u, (w_)*((a_)*(v_)^{(n_)})^{(m_)} /; \operatorname{FreeQ}\{a, m, n\}, x] \&\& \operatorname{IntegerQ}[m*n] \&\& \operatorname{!MatchQ}[u, E^{((c_)*((a_)+(b_)*x))}*(F_)[v_] /; \operatorname{FreeQ}\{a, b, c\}, x] \&\& \operatorname{InverseFunctionQ}[F[x]]]$

Rule 2531

$\operatorname{Int}[\operatorname{Log}[1+(e_)*((F_)^{((c_)*((a_)+(b_)*(x_)))^{(n_)}]}*((f_)+(g_)*(x_))^{(m_)}], x_Symbol] := -\operatorname{Simp}[\frac{(f+g*x)^m*\operatorname{PolyLog}[2, -(e*(F^{(c*(a+b*x)))^n})]}{(b*c*n*\operatorname{Log}[F])}, x] + \operatorname{Dist}[\frac{(g*m)}{(b*c*n*\operatorname{Log}[F])}, \operatorname{Int}[(f+g*x)^{(m-1)}*\operatorname{PolyLog}[2, -(e*(F^{(c*(a+b*x)))^n})], x], x] /; \operatorname{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \&\& \operatorname{GtQ}[m, 0]$

Rule 3717

$\operatorname{Int}[\frac{((c_)+(d_)*(x_))^{(m_)}*\tan[(e_)+\operatorname{Pi}*(k_)+(f_)*(x_)]}{(c+d*x)^{(m+1)}}, x] := \operatorname{Simp}[\frac{(I*(c+d*x)^{(m+1)})}{(d*(m+1))}, x] - \operatorname{Dist}[2*I, \operatorname{Int}[\frac{(c+d*x)^m*\operatorname{E}^{(2*I*k*\operatorname{Pi})*\operatorname{E}^{(2*I*(e+f*x))}}}{(1+\operatorname{E}^{(2*I*k*\operatorname{Pi})*\operatorname{E}^{(2*I*(e+f*x))}})}, x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \&\& \operatorname{IntegerQ}[4*k] \&\& \operatorname{IGtQ}[m, 0]$

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol]
:> Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x]
&& !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned} \int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx &= \frac{\sec^2(x) \int x^2 \cot(x) dx}{\sqrt{a \sec^4(x)}} \\ &= -\frac{ix^3 \sec^2(x)}{3\sqrt{a \sec^4(x)}} - \frac{(2i \sec^2(x)) \int \frac{e^{2ix} x^2}{1-e^{2ix}} dx}{\sqrt{a \sec^4(x)}} \\ &= -\frac{ix^3 \sec^2(x)}{3\sqrt{a \sec^4(x)}} + \frac{x^2 \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}} - \frac{(2 \sec^2(x)) \int x \log(1 - e^{2ix}) dx}{\sqrt{a \sec^4(x)}} \\ &= -\frac{ix^3 \sec^2(x)}{3\sqrt{a \sec^4(x)}} + \frac{x^2 \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}} - \frac{ix \operatorname{Li}_2(e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}} + \frac{(i \sec^2(x)) \int \operatorname{Li}_2(e^{2ix})}{\sqrt{a \sec^4(x)}} \\ &= -\frac{ix^3 \sec^2(x)}{3\sqrt{a \sec^4(x)}} + \frac{x^2 \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}} - \frac{ix \operatorname{Li}_2(e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}} + \frac{\sec^2(x) \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2(x)}{x}\right)}{2\sqrt{a \sec^4(x)}} \\ &= -\frac{ix^3 \sec^2(x)}{3\sqrt{a \sec^4(x)}} + \frac{x^2 \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}} - \frac{ix \operatorname{Li}_2(e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}} + \frac{\operatorname{Li}_3(e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 0.69

$$\frac{\sec^2(x) (24ix \operatorname{Li}_2(e^{-2ix}) + 12\operatorname{Li}_3(e^{-2ix}) + 8ix^3 + 24x^2 \log(1 - e^{-2ix}) - i\pi^3)}{24\sqrt{a \sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4], x]
```

```
[Out] (((-I)*Pi^3 + (8*I)*x^3 + 24*x^2*Log[1 - E^((-2*I)*x)] + (24*I)*x*PolyLog[2, E^((-2*I)*x)] + 12*PolyLog[3, E^((-2*I)*x)])*Sec[x]^2)/(24*Sqrt[a*Sec[x]^4])
```

fricas [C] time = 2.50, size = 248, normalized size = 2.28

$$2 \sqrt{\frac{a}{\cos(x)^4}} \cos(x)^2 \operatorname{polylog}(3, \cos(x) + i \sin(x)) + 2 \sqrt{\frac{a}{\cos(x)^4}} \cos(x)^2 \operatorname{polylog}(3, \cos(x) - i \sin(x)) + 2 \sqrt{\frac{a}{\cos(x)^4}} \cos(x)^2 \operatorname{polylog}(3, -\cos(x) + i \sin(x)) + 2 \sqrt{\frac{a}{\cos(x)^4}} \cos(x)^2 \operatorname{polylog}(3, -\cos(x) - i \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2), x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + 2*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) - I*sin(x)) + 2*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -cos(x) + I*sin(x)) + 2*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -cos(x) - I*sin(x)))
```

$-\cos(x) - I\sin(x)) + (x^2\cos(x)^2\log(\cos(x) + I\sin(x) + 1) + x^2\cos(x)^2\log(\cos(x) - I\sin(x) + 1) + x^2\cos(x)^2\log(-\cos(x) + I\sin(x) + 1) + x^2\cos(x)^2\log(-\cos(x) - I\sin(x) + 1) - 2Ix\cos(x)^2\operatorname{dilog}(\cos(x) + I\sin(x)) + 2Ix\cos(x)^2\operatorname{dilog}(\cos(x) - I\sin(x)) + 2Ix\cos(x)^2\operatorname{dilog}(-\cos(x) + I\sin(x)) - 2Ix\cos(x)^2\operatorname{dilog}(-\cos(x) - I\sin(x)))\sqrt{a/\cos(x)^4})/a$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec(x)^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="giac")

[Out] integrate(x^2*csc(x)*sec(x)/sqrt(a*sec(x)^4), x)

maple [B] time = 0.18, size = 183, normalized size = 1.68

$$\frac{ie^{2ix}x^3}{3\sqrt{\frac{ae^{4ix}}{(e^{2ix}+1)^4}}(e^{2ix}+1)^2} \cdot 2\left(\frac{ie^{2ix}x^3}{3} - \frac{e^{2ix}x^2\ln(1+e^{ix})}{2} + ie^{2ix}x \operatorname{polylog}(2, -e^{ix}) - e^{2ix} \operatorname{polylog}(3, -e^{ix}) - \frac{e^{2ix}x^2\ln(1+e^{ix})}{2}\right) \sqrt{\frac{ae^{4ix}}{(e^{2ix}+1)^4}}(e^{2ix}+1)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x)

[Out] 1/3*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*exp(2*I*x)*x^3 - 2/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*(1/3*I*exp(2*I*x)*x^3 - 1/2*exp(2*I*x)*x^2*ln(1+exp(I*x)) + I*exp(2*I*x)*x*polylog(2, -exp(I*x)) - exp(2*I*x)*polylog(3, -exp(I*x)) - 1/2*exp(2*I*x)*x^2*ln(1-exp(I*x)) + I*exp(2*I*x)*x*polylog(2, exp(I*x)) - exp(2*I*x)*polylog(3, exp(I*x)))

maxima [A] time = 1.10, size = 113, normalized size = 1.04

$$-2ix^3 + 6ix^2 \arctan(\sin(x), \cos(x) + 1) - 6ix^2 \arctan(\sin(x), -\cos(x) + 1) + 3x^2 \log(\cos(x)^2 + \sin(x)^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="maxima")

[Out] 1/6*(-2Ix^3 + 6Ix^2*arctan2(sin(x), cos(x) + 1) - 6Ix^2*arctan2(sin(x), -cos(x) + 1) + 3x^2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 3x^2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 12Ix*dilog(-e^(I*x)) - 12Ix*dilog(e^(I*x)) + 12*polylog(3, -e^(I*x)) + 12*polylog(3, e^(I*x)))/sqrt(a)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cos(x) \sin(x) \sqrt{\frac{a}{\cos(x)^4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)),x)

[Out] int(x^2/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*csc(x)*sec(x)/(a*sec(x)**4)**(1/2), x)

[Out] Integral(x**2*csc(x)*sec(x)/sqrt(a*sec(x)**4), x)

$$3.873 \quad \int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx$$

Optimal. Leaf size=143

$$\frac{3ix^2 \text{Li}_2(e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{3x \text{Li}_3(e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{3i \text{Li}_4(e^{2ix}) \sec^2(x)}{4\sqrt{a \sec^4(x)}} - \frac{ix^4 \sec^2(x)}{4\sqrt{a \sec^4(x)}} + \frac{x^3 \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}}$$

[Out] $-1/4*I*x^4*\sec(x)^2/(a*\sec(x)^4)^{(1/2)}+x^3*\ln(1-\exp(2*I*x))*\sec(x)^2/(a*\sec(x)^4)^{(1/2)}-3/2*I*x^2*\text{polylog}(2,\exp(2*I*x))*\sec(x)^2/(a*\sec(x)^4)^{(1/2)}+3/2*x*\text{polylog}(3,\exp(2*I*x))*\sec(x)^2/(a*\sec(x)^4)^{(1/2)}+3/4*I*\text{polylog}(4,\exp(2*I*x))*\sec(x)^2/(a*\sec(x)^4)^{(1/2)}$

Rubi [A] time = 0.61, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6720, 3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3ix^2 \sec^2(x) \text{PolyLog}(2, e^{2ix})}{2\sqrt{a \sec^4(x)}} + \frac{3x \sec^2(x) \text{PolyLog}(3, e^{2ix})}{2\sqrt{a \sec^4(x)}} + \frac{3i \sec^2(x) \text{PolyLog}(4, e^{2ix})}{4\sqrt{a \sec^4(x)}} - \frac{ix^4 \sec^2(x)}{4\sqrt{a \sec^4(x)}} + \frac{x^3 \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^3*\text{Csc}[x]*\text{Sec}[x])/ \text{Sqrt}[a*\text{Sec}[x]^4], x]$

[Out] $((-I/4)*x^4*\text{Sec}[x]^2)/\text{Sqrt}[a*\text{Sec}[x]^4] + (x^3*\text{Log}[1 - E^{((2*I)*x)}]*\text{Sec}[x]^2)/\text{Sqrt}[a*\text{Sec}[x]^4] - (((3*I)/2)*x^2*\text{PolyLog}[2, E^{((2*I)*x)}]*\text{Sec}[x]^2)/\text{Sqrt}[a*\text{Sec}[x]^4] + (3*x*\text{PolyLog}[3, E^{((2*I)*x)}]*\text{Sec}[x]^2)/(2*\text{Sqrt}[a*\text{Sec}[x]^4]) + (((3*I)/4)*\text{PolyLog}[4, E^{((2*I)*x)}]*\text{Sec}[x]^2)/\text{Sqrt}[a*\text{Sec}[x]^4]$

Rule 2190

$\text{Int}[\frac{((F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))})^{(n_*)}*((c_*) + (d_*)*(x_*))^{(m_*)})}{((a_*) + (b_*)*((F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))})^{(n_*)})}, x_Symbol] := \text{Simp}[\frac{(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]}{(b*f*g*n*\text{Log}[F])}, x] - \text{Dist}[\frac{(d*m)}{(b*f*g*n*\text{Log}[F])}, \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$

Rule 2282

$\text{Int}[u, x_Symbol] := \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \&\& !\text{MatchQ}[u, (w_*)*((a_*)*(v_*)^{(n_*)})^{(m_*)} /; \text{FreeQ}\{a, m, n\}, x\} \&\& \text{IntegerQ}[m*n] \&\& !\text{MatchQ}[u, E^{((c_*)*((a_*) + (b_*)*x))}*(F_*)[v_] /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{InverseFunctionQ}[F[x]]]$

Rule 2531

$\text{Int}[\text{Log}[1 + (e_*)*((F_*)^{((c_*)*((a_*) + (b_*)*(x_*)))})^{(n_*)}]*((f_*) + (g_*)*(x_*))^{(m_*)}, x_Symbol] := -\text{Simp}[\frac{(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})]}{(b*c*n*\text{Log}[F])}, x] + \text{Dist}[\frac{(g*m)}{(b*c*n*\text{Log}[F])}, \text{Int}[(f + g*x)^{(m-1)}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x\} \&\& \text{GtQ}[m, 0]$

Rule 3717

$\text{Int}[\frac{((c_*) + (d_*)*(x_*))^{(m_*)}*\tan[(e_*) + \text{Pi}*(k_*) + (f_*)*(x_*)], x_Symbol]}{1} := \text{Simp}[\frac{(I*(c + d*x)^{(m+1)})/(d*(m+1))}{x} - \text{Dist}[2*I, \text{Int}[\frac{(c + d*x)^m * E^{(2*I*k*Pi)} * E^{(2*I*(e + f*x))}}{(1 + E^{(2*I*k*Pi)} * E^{(2*I*(e + f*x))})}], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{IntegerQ}[4*k] \&\& \text{IGtQ}[m, 0]$

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rule 6609

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rule 6720

Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned}
 \int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx &= \frac{\sec^2(x) \int x^3 \cot(x) dx}{\sqrt{a \sec^4(x)}} \\
 &= -\frac{ix^4 \sec^2(x)}{4\sqrt{a \sec^4(x)}} - \frac{(2i \sec^2(x)) \int \frac{e^{2ix} x^3}{1-e^{2ix}} dx}{\sqrt{a \sec^4(x)}} \\
 &= -\frac{ix^4 \sec^2(x)}{4\sqrt{a \sec^4(x)}} + \frac{x^3 \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}} - \frac{(3 \sec^2(x)) \int x^2 \log(1 - e^{2ix}) dx}{\sqrt{a \sec^4(x)}} \\
 &= -\frac{ix^4 \sec^2(x)}{4\sqrt{a \sec^4(x)}} + \frac{x^3 \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}} - \frac{3ix^2 \text{Li}_2(e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{(3i \sec^2(x)) \int x \text{Li}_2(e^{2ix}) dx}{\sqrt{a \sec^4(x)}} \\
 &= -\frac{ix^4 \sec^2(x)}{4\sqrt{a \sec^4(x)}} + \frac{x^3 \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}} - \frac{3ix^2 \text{Li}_2(e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{3x \text{Li}_3(e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}} \\
 &= -\frac{ix^4 \sec^2(x)}{4\sqrt{a \sec^4(x)}} + \frac{x^3 \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}} - \frac{3ix^2 \text{Li}_2(e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{3x \text{Li}_3(e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}} \\
 &= -\frac{ix^4 \sec^2(x)}{4\sqrt{a \sec^4(x)}} + \frac{x^3 \log(1 - e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}} - \frac{3ix^2 \text{Li}_2(e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{3x \text{Li}_3(e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}}
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 87, normalized size = 0.61

$$\frac{i \sec^2(x) (-96x^2 \text{Li}_2(e^{-2ix}) + 96ix \text{Li}_3(e^{-2ix}) + 48 \text{Li}_4(e^{-2ix}) - 16x^4 + 64ix^3 \log(1 - e^{-2ix}) + \pi^4)}{64\sqrt{a \sec^4(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4], x]

[Out] ((-1/64*I)*(Pi^4 - 16*x^4 + (64*I)*x^3*Log[1 - E^((-2*I)*x)] - 96*x^2*PolyLog[2, E^((-2*I)*x)] + (96*I)*x*PolyLog[3, E^((-2*I)*x)] + 48*PolyLog[4, E^((-2*I)*x)])*Sec[x]^2)/Sqrt[a*Sec[x]^4]

fricas [C] time = 2.05, size = 356, normalized size = 2.49

$$6x\sqrt{\frac{a}{\cos(x)^4}}\cos(x)^2\text{polylog}(3,\cos(x)+i\sin(x))+6x\sqrt{\frac{a}{\cos(x)^4}}\cos(x)^2\text{polylog}(3,\cos(x)-i\sin(x))+6x\sqrt{\frac{a}{\cos(x)^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="fricas")

[Out] 1/2*(6*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3,cos(x)+I*sin(x))+6*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3,cos(x)-I*sin(x))+6*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3,-cos(x)+I*sin(x))+6*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3,-cos(x)-I*sin(x))+6*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4,cos(x)+I*sin(x))-6*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4,cos(x)-I*sin(x))-6*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4,-cos(x)+I*sin(x))+6*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4,-cos(x)-I*sin(x))+(x^3*cos(x)^2*log(cos(x)+I*sin(x)+1)+x^3*cos(x)^2*log(cos(x)-I*sin(x)+1)+x^3*cos(x)^2*log(-cos(x)+I*sin(x)+1)+x^3*cos(x)^2*log(-cos(x)-I*sin(x)+1)-3*I*x^2*cos(x)^2*dilog(cos(x)+I*sin(x))+3*I*x^2*cos(x)^2*dilog(cos(x)-I*sin(x))+3*I*x^2*cos(x)^2*dilog(-cos(x)+I*sin(x))-3*I*x^2*cos(x)^2*dilog(-cos(x)-I*sin(x)))*sqrt(a/cos(x)^4))/a

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec(x)^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="giac")

[Out] integrate(x^3*csc(x)*sec(x)/sqrt(a*sec(x)^4), x)

maple [A] time = 0.19, size = 221, normalized size = 1.55

$$\frac{ie^{2ix}x^4}{4\sqrt{\frac{ae^{4ix}}{(e^{2ix}+1)^4}}(e^{2ix}+1)^2} + 2i\left(-\frac{e^{2ix}x^4}{4} - \frac{ie^{2ix}x^3\ln(1+e^{ix})}{2} - \frac{3e^{2ix}x^2\text{polylog}(2,-e^{ix})}{2} - 3ie^{2ix}x\text{polylog}(3,-e^{ix}) + 3e^{2ix}\text{polylog}(4,-e^{ix})\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x)

[Out] 1/4*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*exp(2*I*x)*x^4+2*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*(-1/4*exp(2*I*x)*x^4-1/2*I*exp(2*I*x)*x^3*ln(1+exp(I*x))-3/2*exp(2*I*x)*x^2*polylog(2,-exp(I*x))-3*I*exp(2*I*x)*x*polylog(3,-exp(I*x))+3*exp(2*I*x)*polylog(4,-exp(I*x))-1/2*I*exp(2*I*x)*x^3*ln(1-exp(I*x))-3/2*exp(2*I*x)*x^2*polylog(2,exp(I*x))-3*I*exp(2*I*x)*x*polylog(3,exp(I*x))+3*exp(2*I*x)*polylog(4,exp(I*x)))

maxima [A] time = 0.55, size = 137, normalized size = 0.96

$$-ix^4 + 4ix^3 \arctan(\sin(x), \cos(x) + 1) - 4ix^3 \arctan(\sin(x), -\cos(x) + 1) + 2x^3 \log(\cos(x)^2 + \sin(x)^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="maxima")

```
[Out] 1/4*(-I*x^4 + 4*I*x^3*arctan2(sin(x), cos(x) + 1) - 4*I*x^3*arctan2(sin(x),
-cos(x) + 1) + 2*x^3*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 2*x^3*log(c
os(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 12*I*x^2*dilog(-e^(I*x)) - 12*I*x^2*di
log(e^(I*x)) + 24*x*polylog(3, -e^(I*x)) + 24*x*polylog(3, e^(I*x)) + 24*I*
polylog(4, -e^(I*x)) + 24*I*polylog(4, e^(I*x)))/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\cos(x) \sin(x) \sqrt{\frac{a}{\cos(x)^4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)),x)
```

```
[Out] int(x^3/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(x)*sec(x)/(a*sec(x)**4)**(1/2),x)
```

```
[Out] Integral(x**3*csc(x)*sec(x)/sqrt(a*sec(x)**4), x)
```

3.874 $\int x \csc(x) \sec(x) \sqrt{a \sec^2(x)} dx$

Optimal. Leaf size=105

$$i\text{Li}_2(-e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - i\text{Li}_2(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} + x \sqrt{a \sec^2(x)} - 2x \cos(x) \tanh^{-1}(e^{ix}) \sqrt{a \sec^2(x)} - \cos(x) \sqrt{a \sec^2(x)}$$

[Out] x*(a*sec(x)^2)^(1/2)-2*x*arctanh(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)-arctanh(sin(x))*cos(x)*(a*sec(x)^2)^(1/2)+I*cos(x)*polylog(2,-exp(I*x))*(a*sec(x)^2)^(1/2)-I*cos(x)*polylog(2,exp(I*x))*(a*sec(x)^2)^(1/2)

Rubi [A] time = 0.34, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6720, 2622, 321, 207, 4420, 6271, 4183, 2279, 2391, 3770}

$$i \cos(x) \text{PolyLog}(2, -e^{ix}) \sqrt{a \sec^2(x)} - i \cos(x) \text{PolyLog}(2, e^{ix}) \sqrt{a \sec^2(x)} + x \sqrt{a \sec^2(x)} - 2x \cos(x) \tanh^{-1}(e^{ix}) \sqrt{a \sec^2(x)}$$

Antiderivative was successfully verified.

[In] Int[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]

[Out] x*Sqrt[a*Sec[x]^2] - 2*x*ArcTanh[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] - ArcTanh[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2] + I*Cos[x]*PolyLog[2, -E^(I*x)]*Sqrt[a*Sec[x]^2] - I*Cos[x]*PolyLog[2, E^(I*x)]*Sqrt[a*Sec[x]^2]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a+b*x]/x, x], x, (F^(e*(c+d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 2622

Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := Dist[1/(f*a^n), Subst[Int[x^(m+n-1)/(-1+x^2/a^2)^((n+1)/2), x], x, a*Sec[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n+1)/2] && !(IntegerQ[(m+1)/2] && LtQ[0, m, n])

Rule 3770

```
Int[Csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[Csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(
m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] := Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int x \csc(x) \sec(x) \sqrt{a \sec^2(x)} dx &= \left(\cos(x) \sqrt{a \sec^2(x)} \right) \int x \csc(x) \sec^2(x) dx \\
&= x \sqrt{a \sec^2(x)} - x \tanh^{-1}(\cos(x)) \cos(x) \sqrt{a \sec^2(x)} - \left(\cos(x) \sqrt{a \sec^2(x)} \right) \int (-\csc(x)) dx \\
&= x \sqrt{a \sec^2(x)} - x \tanh^{-1}(\cos(x)) \cos(x) \sqrt{a \sec^2(x)} + \left(\cos(x) \sqrt{a \sec^2(x)} \right) \int \tan(x) dx \\
&= x \sqrt{a \sec^2(x)} - \tanh^{-1}(\sin(x)) \cos(x) \sqrt{a \sec^2(x)} + \left(\cos(x) \sqrt{a \sec^2(x)} \right) \int x \csc(x) dx \\
&= x \sqrt{a \sec^2(x)} - 2x \tanh^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - \tanh^{-1}(\sin(x)) \cos(x) \sqrt{a \sec^2(x)} \\
&= x \sqrt{a \sec^2(x)} - 2x \tanh^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - \tanh^{-1}(\sin(x)) \cos(x) \sqrt{a \sec^2(x)} \\
&= x \sqrt{a \sec^2(x)} - 2x \tanh^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - \tanh^{-1}(\sin(x)) \cos(x) \sqrt{a \sec^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 108, normalized size = 1.03

$$\sqrt{a \sec^2(x)} \left(i \left(\text{Li}_2(-e^{ix}) - \text{Li}_2(e^{ix}) \right) \cos(x) + x + x \left(\log(1 - e^{ix}) - \log(1 + e^{ix}) \right) \cos(x) + \cos(x) \log \left(\cos \left(\frac{x}{2} \right) - \sin \left(\frac{x}{2} \right) \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2], x]
```

```
[Out] (x + x*Cos[x]*(Log[1 - E^(I*x)] - Log[1 + E^(I*x)]) + Cos[x]*Log[Cos[x/2] - Sin[x/2]] - Cos[x]*Log[Cos[x/2] + Sin[x/2]] + I*Cos[x]*(PolyLog[2, -E^(I*x)] - PolyLog[2, E^(I*x)]))*Sqrt[a*Sec[x]^2]
```

fricas [A] time = 0.85, size = 140, normalized size = 1.33

$$-\frac{1}{2} \left(x \cos(x) \log(\cos(x) + i \sin(x) + 1) + x \cos(x) \log(\cos(x) - i \sin(x) + 1) - x \cos(x) \log(-\cos(x) + i \sin(x)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="fricas")

[Out] -1/2*(x*cos(x)*log(cos(x) + I*sin(x) + 1) + x*cos(x)*log(cos(x) - I*sin(x) + 1) - x*cos(x)*log(-cos(x) + I*sin(x) + 1) - x*cos(x)*log(-cos(x) - I*sin(x) + 1) + I*cos(x)*dilog(cos(x) + I*sin(x)) - I*cos(x)*dilog(cos(x) - I*sin(x)) + I*cos(x)*dilog(-cos(x) + I*sin(x)) - I*cos(x)*dilog(-cos(x) - I*sin(x)) + cos(x)*log(-(sin(x) + 1)/(sin(x) - 1)) - 2*x)*sqrt(a/cos(x)^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(x)^2} x \csc(x) \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*sec(x)^2)*x*csc(x)*sec(x), x)

maple [A] time = 0.25, size = 86, normalized size = 0.82

$$2 \sqrt{\frac{a e^{2ix}}{(e^{2ix} + 1)^2}} x + 4 \sqrt{\frac{a e^{2ix}}{(e^{2ix} + 1)^2}} \left(i \arctan(e^{ix}) + \frac{i \operatorname{dilog}(1 + e^{ix})}{2} - \frac{x \ln(1 + e^{ix})}{2} + \frac{i \operatorname{dilog}(e^{ix})}{2} \right) \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x)

[Out] 2*(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)*x+4*(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)*(I*arctan(exp(I*x))+1/2*I*dilog(1+exp(I*x))-1/2*x*ln(1+exp(I*x))+1/2*I*dilog(exp(I*x)))*cos(x)

maxima [B] time = 0.70, size = 299, normalized size = 2.85

$$\left((2 \cos(2x) + 2i \sin(2x) + 2) \arctan(\cos(x), \sin(x) + 1) + (2 \cos(2x) + 2i \sin(2x) + 2) \arctan(\cos(x), -\sin(x) + 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="maxima")

[Out] ((2*cos(2*x) + 2*I*sin(2*x) + 2)*arctan2(cos(x), sin(x) + 1) + (2*cos(2*x) + 2*I*sin(2*x) + 2)*arctan2(cos(x), -sin(x) + 1) - 2*(x*cos(2*x) + I*x*sin(2*x) + x)*arctan2(sin(x), cos(x) + 1) - 2*(x*cos(2*x) + I*x*sin(2*x) + x)*arctan2(sin(x), -cos(x) + 1) - 4*I*x*cos(x) + (2*cos(2*x) + 2*I*sin(2*x) + 2)*dilog(-e^(I*x)) - (2*cos(2*x) + 2*I*sin(2*x) + 2)*dilog(e^(I*x)) - (-I*x*cos(2*x) + x*sin(2*x) - I*x)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - (I*x*cos(2*x) - x*sin(2*x) + I*x)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - (-I*x*cos(2*x) + sin(2*x) - I)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - (I*cos(2*x) - sin(2*x) + I)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + 4*x*sin(x))*sqrt(a)/(-2*I*cos(2*x) + 2*sin(2*x) - 2*I)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \sqrt{\frac{a}{\cos(x)^2}}}{\cos(x) \sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)), x)
```

```
[Out] int((x*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a\sec^2(x)}\csc(x)\sec(x)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)**2)**(1/2), x)
```

```
[Out] Integral(x*sqrt(a*sec(x)**2)*csc(x)*sec(x), x)
```

3.875 $\int x^2 \csc(x) \sec(x) \sqrt{a \sec^2(x)} dx$

Optimal. Leaf size=225

$$2ix\text{Li}_2(-e^{ix})\cos(x)\sqrt{a\sec^2(x)} - 2ix\text{Li}_2(e^{ix})\cos(x)\sqrt{a\sec^2(x)} - 2i\text{Li}_2(-ie^{ix})\cos(x)\sqrt{a\sec^2(x)} + 2i\text{Li}_2(ie^{ix})\cos(x)\sqrt{a\sec^2(x)}$$

```
[Out] x^2*(a*sec(x)^2)^(1/2)+4*I*x*arctan(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)-2*x
^2*arctanh(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)+2*I*x*cos(x)*polylog(2,-exp(
I*x))*(a*sec(x)^2)^(1/2)-2*I*cos(x)*polylog(2,-I*exp(I*x))*(a*sec(x)^2)^(1/
2)+2*I*cos(x)*polylog(2,I*exp(I*x))*(a*sec(x)^2)^(1/2)-2*I*x*cos(x)*polylog
(2,exp(I*x))*(a*sec(x)^2)^(1/2)-2*cos(x)*polylog(3,-exp(I*x))*(a*sec(x)^2)^(
1/2)+2*cos(x)*polylog(3,exp(I*x))*(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {6720, 2622, 321, 207, 4420, 14, 6273, 4183, 2531, 2282, 6589, 4181, 2279, 2391}

$$2ix \cos(x) \text{PolyLog}(2, -e^{ix}) \sqrt{a \sec^2(x)} - 2ix \cos(x) \text{PolyLog}(2, e^{ix}) \sqrt{a \sec^2(x)} - 2i \cos(x) \text{PolyLog}(2, -ie^{ix}) \sqrt{a \sec^2(x)} + 2i \cos(x) \text{PolyLog}(2, ie^{ix}) \sqrt{a \sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] x^2*Sqrt[a*Sec[x]^2] + (4*I)*x*ArcTan[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] - 2*
x^2*ArcTanh[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] + (2*I)*x*Cos[x]*PolyLog[2, -E
^(I*x)]*Sqrt[a*Sec[x]^2] - (2*I)*Cos[x]*PolyLog[2, (-I)*E^(I*x)]*Sqrt[a*Sec
[x]^2] + (2*I)*Cos[x]*PolyLog[2, I*E^(I*x)]*Sqrt[a*Sec[x]^2] - (2*I)*x*Cos[
x]*PolyLog[2, E^(I*x)]*Sqrt[a*Sec[x]^2] - 2*Cos[x]*PolyLog[3, -E^(I*x)]*Sqr
t[a*Sec[x]^2] + 2*Cos[x]*PolyLog[3, E^(I*x)]*Sqrt[a*Sec[x]^2]
```

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 207

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_), x_S
ymbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x)
)], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)
^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*D[u, x]]/(1 - u^2), x], x] /;
FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&& !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
```


ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rule 6720

Int[(u_)*((a_)*(v_)^(m_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned}
 \int x^2 \csc(x) \sec(x) \sqrt{a \sec^2(x)} dx &= \left(\cos(x) \sqrt{a \sec^2(x)} \right) \int x^2 \csc(x) \sec^2(x) dx \\
 &= x^2 \sqrt{a \sec^2(x)} - x^2 \tanh^{-1}(\cos(x)) \cos(x) \sqrt{a \sec^2(x)} - \left(2 \cos(x) \sqrt{a \sec^2(x)} \right) \int x dx \\
 &= x^2 \sqrt{a \sec^2(x)} - x^2 \tanh^{-1}(\cos(x)) \cos(x) \sqrt{a \sec^2(x)} - \left(2 \cos(x) \sqrt{a \sec^2(x)} \right) x \\
 &= x^2 \sqrt{a \sec^2(x)} - x^2 \tanh^{-1}(\cos(x)) \cos(x) \sqrt{a \sec^2(x)} + \left(2 \cos(x) \sqrt{a \sec^2(x)} \right) \int x dx \\
 &= x^2 \sqrt{a \sec^2(x)} + 4ix \tan^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} + \left(\cos(x) \sqrt{a \sec^2(x)} \right) \int x dx \\
 &= x^2 \sqrt{a \sec^2(x)} + 4ix \tan^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 2x^2 \tanh^{-1}(e^{ix}) \cos(x) \\
 &= x^2 \sqrt{a \sec^2(x)} + 4ix \tan^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 2x^2 \tanh^{-1}(e^{ix}) \cos(x) \\
 &= x^2 \sqrt{a \sec^2(x)} + 4ix \tan^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 2x^2 \tanh^{-1}(e^{ix}) \cos(x) \\
 &= x^2 \sqrt{a \sec^2(x)} + 4ix \tan^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 2x^2 \tanh^{-1}(e^{ix}) \cos(x)
 \end{aligned}$$

Mathematica [A] time = 0.12, size = 174, normalized size = 0.77

$$\sqrt{a \sec^2(x)} \left(2ix \left(\text{Li}_2(-e^{ix}) - \text{Li}_2(e^{ix}) \right) \cos(x) + 2 \left(\text{Li}_3(e^{ix}) - \text{Li}_3(-e^{ix}) \right) \cos(x) - 2 \cos(x) \left(i \left(\text{Li}_2(-ie^{ix}) - \text{Li}_2(e^{ix}) \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2], x]

[Out] (x^2 + x^2*Cos[x]*(Log[1 - E^(I*x)] - Log[1 + E^(I*x)]) - 2*Cos[x]*(x*(Log[1 - I*E^(I*x)] - Log[1 + I*E^(I*x)]) + I*(PolyLog[2, (-I)*E^(I*x)] - PolyLog[2, I*E^(I*x)])) + (2*I)*x*Cos[x]*(PolyLog[2, -E^(I*x)] - PolyLog[2, E^(I*x)]) + 2*Cos[x]*(-PolyLog[3, -E^(I*x)] + PolyLog[3, E^(I*x)])*Sqrt[a*Sec[x]^2]

fricas [C] time = 0.81, size = 337, normalized size = 1.50

$$\sqrt{\frac{a}{\cos(x)^2}} \cos(x) \text{polylog}(3, \cos(x) + i \sin(x)) + \sqrt{\frac{a}{\cos(x)^2}} \cos(x) \text{polylog}(3, \cos(x) - i \sin(x)) - \sqrt{\frac{a}{\cos(x)^2}} \cos(x) \text{polylog}(3, -\cos(x) + i \sin(x)) - \sqrt{\frac{a}{\cos(x)^2}} \cos(x) \text{polylog}(3, -\cos(x) - i \sin(x)) - 1/2 * (x^2 * \cos(x) * \log(\cos(x) + i \sin(x) + 1) + x^2 * \cos(x) * \log(\cos(x) - i \sin(x) + 1))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2), x, algorithm="fricas")

[Out] sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) - I*sin(x)) - sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(x) + I*sin(x)) - sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(x) - I*sin(x)) - 1/2*(x^2*cos(x)*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)*log(cos(x) - I*sin(x) + 1))

$(x) + 1) - x^2 \cos(x) \log(-\cos(x) + I \sin(x) + 1) - x^2 \cos(x) \log(-\cos(x) - I \sin(x) + 1) + 2I x \cos(x) \operatorname{dilog}(\cos(x) + I \sin(x)) - 2I x \cos(x) \operatorname{dilog}(\cos(x) - I \sin(x)) + 2I x \cos(x) \operatorname{dilog}(-\cos(x) + I \sin(x)) - 2I x \cos(x) \operatorname{dilog}(-\cos(x) - I \sin(x)) + 2x \cos(x) \log(I \cos(x) + \sin(x) + 1) - 2x \cos(x) \log(I \cos(x) - \sin(x) + 1) + 2x \cos(x) \log(-I \cos(x) + \sin(x) + 1) - 2x \cos(x) \log(-I \cos(x) - \sin(x) + 1) - 2x^2 - 2I \cos(x) \operatorname{dilog}(I \cos(x) + \sin(x)) - 2I \cos(x) \operatorname{dilog}(I \cos(x) - \sin(x)) + 2I \cos(x) \operatorname{dilog}(-I \cos(x) + \sin(x)) + 2I \cos(x) \operatorname{dilog}(-I \cos(x) - \sin(x))) \sqrt{a/\cos(x)^2}$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(x)^2} x^2 \csc(x) \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*sec(x)^2)*x^2*csc(x)*sec(x), x)

maple [A] time = 0.30, size = 200, normalized size = 0.89

$$2 \sqrt{\frac{a e^{2ix}}{(e^{2ix} + 1)^2}} x^{2-4i} \sqrt{\frac{a e^{2ix}}{(e^{2ix} + 1)^2}} \left(2i \left(\frac{x \ln(1 + ie^{ix})}{2} - \frac{x \ln(1 - ie^{ix})}{2} - \frac{i \operatorname{dilog}(1 + ie^{ix})}{2} + \frac{i \operatorname{dilog}(1 - ie^{ix})}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x)

[Out] 2*(a*exp(2*I*x)/(exp(2*I*x)+1)^(1/2)*x^2-4*I*(a*exp(2*I*x)/(exp(2*I*x)+1)^(1/2)*(2*I*(1/2*x*ln(1+I*exp(I*x))-1/2*x*ln(1-I*exp(I*x))-1/2*I*dilog(1+I*exp(I*x))+1/2*I*dilog(1-I*exp(I*x)))-1/2*I*(-1/3*I*x^3+x^2*ln(1+exp(I*x)))-2*I*x*polylog(2,-exp(I*x))+2*polylog(3,-exp(I*x)))-1/2*I*(1/3*I*x^3-x^2*ln(1-exp(I*x))+2*I*x*polylog(2,exp(I*x))-2*polylog(3,exp(I*x))))*cos(x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="maxima")

[Out] -(4*I*x^2*cos(x) - 4*x^2*sin(x) + 2*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*arctan2(sin(x), cos(x) + 1) + 2*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*arctan2(sin(x), -cos(x) + 1) - 4*(x*cos(2*x) + I*x*sin(2*x) + x)*dilog(-e^(I*x)) + 4*(x*cos(2*x) + I*x*sin(2*x) + x)*dilog(e^(I*x)) + (-8*I*cos(2*x) + 8*sin(2*x) - 8*I)*integrate((x*cos(2*x)*cos(x) + x*sin(2*x)*sin(x) + x*cos(x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x) - (8*cos(2*x) + 8*I*sin(2*x) + 8)*integrate((x*cos(x)*sin(2*x) - x*cos(2*x)*sin(x) - x*sin(x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x) + (-I*x^2*cos(2*x) + x^2*sin(2*x) - I*x^2)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (I*x^2*cos(2*x) - x^2*sin(2*x) + I*x^2)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + (-4*I*cos(2*x) + 4*sin(2*x) - 4*I)*polylog(3, -e^(I*x)) + (4*I*cos(2*x) - 4*sin(2*x) + 4*I)*polylog(3, e^(I*x))*sqrt(a)/(-2*I*cos(2*x) + 2*sin(2*x) - 2*I)

mapad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sqrt{\frac{a}{\cos(x)^2}}}{\cos(x) \sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)),x)`

[Out] `int((x^2*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a \sec^2(x)} \csc(x) \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*csc(x)*sec(x)*(a*sec(x)**2)**(1/2),x)`

[Out] `Integral(x**2*sqrt(a*sec(x)**2)*csc(x)*sec(x), x)`

3.876 $\int x^3 \csc(x) \sec(x) \sqrt{a \sec^2(x)} dx$

Optimal. Leaf size=341

$$3ix^2 \operatorname{Li}_2(-e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 3ix^2 \operatorname{Li}_2(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 6ix \operatorname{Li}_2(-ie^{ix}) \cos(x) \sqrt{a \sec^2(x)} + 6ix \operatorname{Li}_2(ie^{ix}) \cos(x) \sqrt{a \sec^2(x)}$$

```
[Out] x^3*(a*sec(x)^2)^(1/2)+6*I*x^2*arctan(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)-2
*x^3*arctanh(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)+3*I*x^2*cos(x)*polylog(2,-
exp(I*x))*(a*sec(x)^2)^(1/2)-6*I*x*cos(x)*polylog(2,-I*exp(I*x))*(a*sec(x)^
2)^(1/2)+6*I*x*cos(x)*polylog(2,I*exp(I*x))*(a*sec(x)^2)^(1/2)-3*I*x^2*cos(
x)*polylog(2,exp(I*x))*(a*sec(x)^2)^(1/2)-6*x*cos(x)*polylog(3,-exp(I*x))*
(a*sec(x)^2)^(1/2)+6*cos(x)*polylog(3,-I*exp(I*x))*(a*sec(x)^2)^(1/2)-6*cos(
x)*polylog(3,I*exp(I*x))*(a*sec(x)^2)^(1/2)+6*x*cos(x)*polylog(3,exp(I*x))*
(a*sec(x)^2)^(1/2)-6*I*cos(x)*polylog(4,-exp(I*x))*(a*sec(x)^2)^(1/2)+6*I*c
os(x)*polylog(4,exp(I*x))*(a*sec(x)^2)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6720, 2622, 321, 207, 4420, 14, 6273, 4183, 2531, 6609, 2282, 6589, 4181}

$$3ix^2 \cos(x) \operatorname{PolyLog}(2, -e^{ix}) \sqrt{a \sec^2(x)} - 3ix^2 \cos(x) \operatorname{PolyLog}(2, e^{ix}) \sqrt{a \sec^2(x)} - 6ix \cos(x) \operatorname{PolyLog}(2, -ie^{ix}) \sqrt{a \sec^2(x)} + 6ix \cos(x) \operatorname{PolyLog}(2, ie^{ix}) \sqrt{a \sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] x^3*Sqrt[a*Sec[x]^2] + (6*I)*x^2*ArcTan[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] -
2*x^3*ArcTanh[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] + (3*I)*x^2*Cos[x]*PolyLog[2
, -E^(I*x)]*Sqrt[a*Sec[x]^2] - (6*I)*x*Cos[x]*PolyLog[2, (-I)*E^(I*x)]*Sqrt
[a*Sec[x]^2] + (6*I)*x*Cos[x]*PolyLog[2, I*E^(I*x)]*Sqrt[a*Sec[x]^2] - (3*I
)*x^2*Cos[x]*PolyLog[2, E^(I*x)]*Sqrt[a*Sec[x]^2] - 6*x*Cos[x]*PolyLog[3, -
E^(I*x)]*Sqrt[a*Sec[x]^2] + 6*Cos[x]*PolyLog[3, (-I)*E^(I*x)]*Sqrt[a*Sec[x]
^2] - 6*Cos[x]*PolyLog[3, I*E^(I*x)]*Sqrt[a*Sec[x]^2] + 6*x*Cos[x]*PolyLog[
3, E^(I*x)]*Sqrt[a*Sec[x]^2] - (6*I)*Cos[x]*PolyLog[4, -E^(I*x)]*Sqrt[a*Sec
[x]^2] + (6*I)*Cos[x]*PolyLog[4, E^(I*x)]*Sqrt[a*Sec[x]^2]
```

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
```

onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 2531

Int[Log[1 + (e_)*((F_)^((c_)*((a_.) + (b_.)*(x_)))^(n_)]*(f_.) + (g_.)*(x_)]^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2622

Int[csc[(e_.) + (f_.)*(x_)]^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_)), x_Symbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rule 4181

Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 4183

Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ[m, 0]

Rule 4420

Int[Csc[(a_.) + (b_.)*(x_)]^(n_)*((c_.) + (d_.)*(x_))^(m_)*Sec[(a_.) + (b_.)*(x_)]^(p_), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]

Rule 6273

Int[((a_.) + ArcTanh[u_]*(b_))*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]

Rule 6589

Int[PolyLog[n_, (c_)*((a_.) + (b_.)*(x_))^(p_)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*(a_.) + (b_.
)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x]
&& !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int x^3 \csc(x) \sec(x) \sqrt{a \sec^2(x)} dx &= \left(\cos(x) \sqrt{a \sec^2(x)} \right) \int x^3 \csc(x) \sec^2(x) dx \\
&= x^3 \sqrt{a \sec^2(x)} - x^3 \tanh^{-1}(\cos(x)) \cos(x) \sqrt{a \sec^2(x)} - \left(3 \cos(x) \sqrt{a \sec^2(x)} \right) \\
&= x^3 \sqrt{a \sec^2(x)} - x^3 \tanh^{-1}(\cos(x)) \cos(x) \sqrt{a \sec^2(x)} - \left(3 \cos(x) \sqrt{a \sec^2(x)} \right) \\
&= x^3 \sqrt{a \sec^2(x)} - x^3 \tanh^{-1}(\cos(x)) \cos(x) \sqrt{a \sec^2(x)} + \left(3 \cos(x) \sqrt{a \sec^2(x)} \right) \\
&= x^3 \sqrt{a \sec^2(x)} + 6ix^2 \tan^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} + \left(\cos(x) \sqrt{a \sec^2(x)} \right) \int x^2 \\
&= x^3 \sqrt{a \sec^2(x)} + 6ix^2 \tan^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 2x^3 \tanh^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} \\
&= x^3 \sqrt{a \sec^2(x)} + 6ix^2 \tan^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 2x^3 \tanh^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} \\
&= x^3 \sqrt{a \sec^2(x)} + 6ix^2 \tan^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 2x^3 \tanh^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} \\
&= x^3 \sqrt{a \sec^2(x)} + 6ix^2 \tan^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 2x^3 \tanh^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} \\
&= x^3 \sqrt{a \sec^2(x)} + 6ix^2 \tan^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 2x^3 \tanh^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} \\
&= x^3 \sqrt{a \sec^2(x)} + 6ix^2 \tan^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 2x^3 \tanh^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.41, size = 290, normalized size = 0.85

$$\frac{1}{8} \sqrt{a \sec^2(x)} \left(24ix^2 \text{Li}_2(e^{-ix}) \cos(x) + 24ix^2 \text{Li}_2(-e^{ix}) \cos(x) - 48ix \text{Li}_2(-ie^{ix}) \cos(x) + 48ix \text{Li}_2(ie^{ix}) \cos(x) + 48 \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2], x]
```

```
[Out] ((8*x^3 - I*Pi^4*Cos[x] + (2*I)*x^4*Cos[x] + 8*x^3*Cos[x]*Log[1 - E^((-I)*x
)]) - 24*x^2*Cos[x]*Log[1 - I*E^(I*x)] + 24*x^2*Cos[x]*Log[1 + I*E^(I*x)] -
8*x^3*Cos[x]*Log[1 + E^(I*x)] + (24*I)*x^2*Cos[x]*PolyLog[2, E^((-I)*x)] +
(24*I)*x^2*Cos[x]*PolyLog[2, -E^(I*x)] - (48*I)*x*Cos[x]*PolyLog[2, (-I)*E^
(I*x)] + (48*I)*x*Cos[x]*PolyLog[2, I*E^(I*x)] + 48*x*Cos[x]*PolyLog[3, E^
(-I)*x] - 48*x*Cos[x]*PolyLog[3, -E^(I*x)] + 48*Cos[x]*PolyLog[3, (-I)*E^
(I*x)] - 48*Cos[x]*PolyLog[3, I*E^(I*x)] - (48*I)*Cos[x]*PolyLog[4, E^((-I)*
x)] - (48*I)*Cos[x]*PolyLog[4, -E^(I*x)])*Sqrt[a*Sec[x]^2])/8
```

fricas [C] time = 1.15, size = 539, normalized size = 1.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="fricas")

[Out] 3*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + 3*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) - I*sin(x)) - 3*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(x) + I*sin(x)) - 3*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(x) - I*sin(x)) + 3*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) + I*sin(x)) - 3*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) - I*sin(x)) + 3*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, -cos(x) + I*sin(x)) - 3*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, -cos(x) - I*sin(x)) + 3*sqrt(a/cos(x)^2)*cos(x)*polylog(3, I*cos(x) + sin(x)) - 3*sqrt(a/cos(x)^2)*cos(x)*polylog(3, I*cos(x) - sin(x)) + 3*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -I*cos(x) + sin(x)) - 3*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -I*cos(x) - sin(x)) - 1/2*(x^3*cos(x)*log(cos(x) + I*sin(x) + 1) + x^3*cos(x)*log(cos(x) - I*sin(x) + 1) - x^3*cos(x)*log(-cos(x) + I*sin(x) + 1) - x^3*cos(x)*log(-cos(x) - I*sin(x) + 1) + 3*I*x^2*cos(x)*dilog(cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(cos(x) - I*sin(x)) + 3*I*x^2*cos(x)*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(-cos(x) - I*sin(x))) + 3*x^2*cos(x)*log(I*cos(x) + sin(x) + 1) - 3*x^2*cos(x)*log(I*cos(x) - sin(x) + 1) + 3*x^2*cos(x)*log(-I*cos(x) + sin(x) + 1) - 3*x^2*cos(x)*log(-I*cos(x) - sin(x) + 1) - 2*x^3 - 6*I*x*cos(x)*dilog(I*cos(x) + sin(x)) - 6*I*x*cos(x)*dilog(I*cos(x) - sin(x)) + 6*I*x*cos(x)*dilog(-I*cos(x) + sin(x)) + 6*I*x*cos(x)*dilog(-I*cos(x) - sin(x))) * sqrt(a/cos(x)^2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(x)^2} x^3 \csc(x) \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*sec(x)^2)*x^3*csc(x)*sec(x), x)

maple [A] time = 0.42, size = 250, normalized size = 0.73

$$2 \sqrt{\frac{a e^{2ix}}{(e^{2ix} + 1)^2}} x^3 + 4 \sqrt{\frac{a e^{2ix}}{(e^{2ix} + 1)^2}} \left(\frac{3x^2 \ln(1 + ie^{ix})}{2} - 3ix \operatorname{polylog}(2, -ie^{ix}) + 3 \operatorname{polylog}(3, -ie^{ix}) - \frac{3x^2 \ln(1 - ie^{ix})}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x)

[Out] 2*(a*exp(2*I*x)/(exp(2*I*x)+1)^(1/2)*x^3+4*(a*exp(2*I*x)/(exp(2*I*x)+1)^(1/2)*(3/2*x^2*ln(1+I*exp(I*x))-3*I*x*polylog(2,-I*exp(I*x))+3*polylog(3,-I*exp(I*x))-3/2*x^2*ln(1-I*exp(I*x))+3*I*x*polylog(2,I*exp(I*x))-3*polylog(3,I*exp(I*x))+1/2*I*(1/4*x^4+I*x^3*ln(1+exp(I*x))+3*x^2*polylog(2,-exp(I*x))+6*I*x*polylog(3,-exp(I*x))-6*polylog(4,-exp(I*x)))+1/2*I*(-1/4*x^4-I*x^3*ln(1-exp(I*x))-3*x^2*polylog(2,exp(I*x))-6*I*x*polylog(3,exp(I*x))+6*polylog(4,exp(I*x)))))*cos(x)

maxima [B] time = 0.48, size = 568, normalized size = 1.67

$$\frac{(4ix^3 \cos(x) - 4x^3 \sin(x) - 6(x^2 \cos(2x) + ix^2 \sin(2x) + x^2) \arctan(\cos(x), \sin(x) + 1) - 6(x^2 \cos(2x) + ix^2 \sin(2x) + x^2) \arctan(\cos(x), \sin(x) - 1))}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="maxima")

[Out] -(4*I*x^3*cos(x) - 4*x^3*sin(x) - 6*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*arctan2(cos(x), sin(x) + 1) - 6*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*arctan2(cos(x), sin(x) - 1))

$2(\cos(x), -\sin(x) + 1) + 2*(x^3*\cos(2*x) + I*x^3*\sin(2*x) + x^3)*\arctan2(\sin(x), \cos(x) + 1) + 2*(x^3*\cos(2*x) + I*x^3*\sin(2*x) + x^3)*\arctan2(\sin(x), -\cos(x) + 1) - 12*(x*\cos(2*x) + I*x*\sin(2*x) + x)*\operatorname{dilog}(I*e^{(I*x)}) + 12*(x*\cos(2*x) + I*x*\sin(2*x) + x)*\operatorname{dilog}(-I*e^{(I*x)}) - 6*(x^2*\cos(2*x) + I*x^2*\sin(2*x) + x^2)*\operatorname{dilog}(e^{(I*x)}) + (-I*x^3*\cos(2*x) + x^3*\sin(2*x) - I*x^3)*\log(\cos(x)^2 + \sin(x)^2 + 2*\cos(x) + 1) + (I*x^3*\cos(2*x) - x^3*\sin(2*x) + I*x^3)*\log(\cos(x)^2 + \sin(x)^2 - 2*\cos(x) + 1) + (-3*I*x^2*\cos(2*x) + 3*x^2*\sin(2*x) - 3*I*x^2)*\log(\cos(x)^2 + \sin(x)^2 + 2*\sin(x) + 1) + (3*I*x^2*\cos(2*x) - 3*x^2*\sin(2*x) + 3*I*x^2)*\log(\cos(x)^2 + \sin(x)^2 - 2*\sin(x) + 1) + (12*\cos(2*x) + 12*I*\sin(2*x) + 12)*\operatorname{polylog}(4, -e^{(I*x)}) - (12*\cos(2*x) + 12*I*\sin(2*x) + 12)*\operatorname{polylog}(4, e^{(I*x)}) + (-12*I*\cos(2*x) + 12*\sin(2*x) - 12*I)*\operatorname{polylog}(3, I*e^{(I*x)}) + (12*I*\cos(2*x) - 12*\sin(2*x) + 12*I)*\operatorname{polylog}(3, -I*e^{(I*x)}) + (-12*I*x*\cos(2*x) + 12*x*\sin(2*x) - 12*I*x)*\operatorname{polylog}(3, -e^{(I*x)}) + (12*I*x*\cos(2*x) - 12*x*\sin(2*x) + 12*I*x)*\operatorname{polylog}(3, e^{(I*x)})*\sqrt{a}/(-2*I*\cos(2*x) + 2*\sin(2*x) - 2*I)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \sqrt{\frac{a}{\cos(x)^2}}}{\cos(x) \sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)),x)`

[Out] `int((x^3*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{a \sec^2(x)} \csc(x) \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*csc(x)*sec(x)*(a*sec(x)**2)**(1/2),x)`

[Out] `Integral(x**3*sqrt(a*sec(x)**2)*csc(x)*sec(x), x)`

3.877 $\int x \csc(x) \sec(x) \sqrt{a \sec^4(x)} dx$

Optimal. Leaf size=142

$$\frac{1}{2}i\text{Li}_2(-e^{2ix})\cos^2(x)\sqrt{a\sec^4(x)} - \frac{1}{2}i\text{Li}_2(e^{2ix})\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x\sin^2(x)\sqrt{a\sec^4(x)}$$

[Out] $\frac{1}{2}x\cos(x)^2(a\sec(x)^4)^{(1/2)} - 2x\operatorname{arctanh}(\exp(2Ix))\cos(x)^2(a\sec(x)^4)^{(1/2)} + \frac{1}{2}I\cos(x)^2\operatorname{polylog}(2, -\exp(2Ix))(a\sec(x)^4)^{(1/2)} - \frac{1}{2}I\cos(x)^2\operatorname{polylog}(2, \exp(2Ix))(a\sec(x)^4)^{(1/2)} - \frac{1}{2}\cos(x)\sin(x)(a\sec(x)^4)^{(1/2)} + \frac{1}{2}x\sin(x)^2(a\sec(x)^4)^{(1/2)}$

Rubi [A] time = 0.40, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {6720, 2620, 14, 4420, 2548, 4419, 4183, 2279, 2391, 3473, 8}

$$\frac{1}{2}i\cos^2(x)\operatorname{PolyLog}(2, -e^{2ix})\sqrt{a\sec^4(x)} - \frac{1}{2}i\cos^2(x)\operatorname{PolyLog}(2, e^{2ix})\sqrt{a\sec^4(x)} + \frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x\sin^2(x)\sqrt{a\sec^4(x)}$$

Antiderivative was successfully verified.

[In] `Int[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4], x]`

[Out] $(x\cos[x]^2\sqrt{a\sec[x]^4})/2 - 2x\operatorname{ArcTanh}[E^{(2I)x}]\cos[x]^2\sqrt{a\sec[x]^4} + (I/2)\cos[x]^2\operatorname{PolyLog}[2, -E^{(2I)x}]\sqrt{a\sec[x]^4} - (I/2)\cos[x]^2\operatorname{PolyLog}[2, E^{(2I)x}]\sqrt{a\sec[x]^4} - (\cos[x]\sqrt{a\sec[x]^4}\sin[x])/2 + (x\sqrt{a\sec[x]^4}\sin[x]^2)/2$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 14

`Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Rule 2279

`Int[Log[(a_ + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

Rule 2391

`Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

Rule 2548

`Int[Log[u_], x_Symbol] := Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u, x])/u, x], x] /; InverseFunctionFreeQ[u, x]`

Rule 2620

`Int[csc[(e_)+(f_)*(x_)]^(m_)*sec[(e_)+(f_)*(x_)]^(n_), x_Symbol] := Dist[1/f, Subst[Int[(1+x^2)^((m+n)/2-1)/x^m, x], x, Tan[e+f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m+n)/2]`

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(
m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] := Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_.), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int x \csc(x) \sec(x) \sqrt{a \sec^4(x)} dx &= \left(\cos^2(x) \sqrt{a \sec^4(x)} \right) \int x \csc(x) \sec^3(x) dx \\
&= x \cos^2(x) \log(\tan(x)) \sqrt{a \sec^4(x)} + \frac{1}{2} x \sqrt{a \sec^4(x)} \sin^2(x) - \left(\cos^2(x) \sqrt{a \sec^4(x)} \right) \\
&= x \cos^2(x) \log(\tan(x)) \sqrt{a \sec^4(x)} + \frac{1}{2} x \sqrt{a \sec^4(x)} \sin^2(x) - \frac{1}{2} \left(\cos^2(x) \sqrt{a \sec^4(x)} \right) \\
&= -\frac{1}{2} \cos(x) \sqrt{a \sec^4(x)} \sin(x) + \frac{1}{2} x \sqrt{a \sec^4(x)} \sin^2(x) + \frac{1}{2} \left(\cos^2(x) \sqrt{a \sec^4(x)} \right) \\
&= \frac{1}{2} x \cos^2(x) \sqrt{a \sec^4(x)} - \frac{1}{2} \cos(x) \sqrt{a \sec^4(x)} \sin(x) + \frac{1}{2} x \sqrt{a \sec^4(x)} \sin^2(x) + \\
&= \frac{1}{2} x \cos^2(x) \sqrt{a \sec^4(x)} - 2x \tanh^{-1} \left(e^{2ix} \right) \cos^2(x) \sqrt{a \sec^4(x)} - \frac{1}{2} \cos(x) \sqrt{a \sec^4(x)} \\
&= \frac{1}{2} x \cos^2(x) \sqrt{a \sec^4(x)} - 2x \tanh^{-1} \left(e^{2ix} \right) \cos^2(x) \sqrt{a \sec^4(x)} - \frac{1}{2} \cos(x) \sqrt{a \sec^4(x)} \\
&= \frac{1}{2} x \cos^2(x) \sqrt{a \sec^4(x)} - 2x \tanh^{-1} \left(e^{2ix} \right) \cos^2(x) \sqrt{a \sec^4(x)} + \frac{1}{2} i \cos^2(x) \operatorname{Li}_2 \left(-
\right)
\end{aligned}$$

Mathematica [A] time = 0.23, size = 85, normalized size = 0.60

$$\frac{1}{2} \cos^2(x) \sqrt{a \sec^4(x)} \left(i \operatorname{Li}_2 \left(-e^{2ix} \right) - i \operatorname{Li}_2 \left(e^{2ix} \right) + 2x \log \left(1 - e^{2ix} \right) - 2x \log \left(1 + e^{2ix} \right) - \tan(x) + x \sec^2(x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] (Cos[x]^2*Sqrt[a*Sec[x]^4]*(2*x*Log[1 - E^((2*I)*x)] - 2*x*Log[1 + E^((2*I)*x)]) + I*PolyLog[2, -E^((2*I)*x)] - I*PolyLog[2, E^((2*I)*x)] + x*Sec[x]^2 - Tan[x])/2
```

fricas [B] time = 0.76, size = 270, normalized size = 1.90

$$\frac{1}{2} \left(x \cos(x)^2 \log(\cos(x) + i \sin(x) + 1) + x \cos(x)^2 \log(\cos(x) - i \sin(x) + 1) - x \cos(x)^2 \log(i \cos(x) + \sin(x)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x*cos(x)^2*log(cos(x) - I*sin(x) + 1) - x*cos(x)^2*log(I*cos(x) + sin(x) + 1) - x*cos(x)^2*log(-I*cos(x) - sin(x) + 1) - x*cos(x)^2*log(-I*cos(x) - sin(x) + 1) + x*cos(x)^2*log(-cos(x) + I*sin(x) + 1) + x*cos(x)^2*log(-cos(x) - I*sin(x) + 1) - I*cos(x)^2*dilog(cos(x) + I*sin(x)) + I*cos(x)^2*dilog(cos(x) - I*sin(x)) - I*cos(x)^2*dilog(I*cos(x) + sin(x)) + I*cos(x)^2*dilog(I*cos(x) - sin(x)) + I*cos(x)^2*dilog(-I*cos(x) + sin(x)) - I*cos(x)^2*dilog(-I*cos(x) - sin(x)) + I*cos(x)^2*dilog(-cos(x) + I*sin(x)) - I*cos(x)^2*dilog(-cos(x) - I*sin(x)) - cos(x)*sin(x) + x)*sqrt(a/cos(x)^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(x)^4} x \csc(x) \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^4)*x*csc(x)*sec(x), x)
```

maple [A] time = 0.20, size = 165, normalized size = 1.16

$$\sqrt{\frac{a e^{4ix}}{(e^{2ix} + 1)^4}} (-i + 2x - i e^{-2ix}) - 4i \sqrt{\frac{a e^{4ix}}{(e^{2ix} + 1)^4}} (e^{2ix} + 1)^2 \left(\frac{i e^{-2ix} x \ln(1 + e^{ix})}{4} + \frac{e^{-2ix} \text{polylog}(2, -e^{ix})}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x)
```

```
[Out] (a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*(-I+2*x-I*exp(-2*I*x))-4*I*(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*(exp(2*I*x)+1)^2*(1/4*I*exp(-2*I*x)*x*ln(1+exp(I*x))+1/4*exp(-2*I*x)*polylog(2,-exp(I*x))-1/4*I*exp(-2*I*x)*x*ln(exp(2*I*x)+1)-1/8*exp(-2*I*x)*polylog(2,-exp(2*I*x))+1/4*I*exp(-2*I*x)*x*ln(1-exp(I*x))+1/4*exp(-2*I*x)*polylog(2,exp(I*x)))
```

maxima [B] time = 0.50, size = 432, normalized size = 3.04

$$\frac{\left((2x \cos(4x) + 4x \cos(2x) + 2ix \sin(4x) + 4ix \sin(2x) + 2x) \arctan(\sin(2x), \cos(2x) + 1) - (2x \cos(2x) + 2ix \sin(2x) + 2x) \arctan(\sin(x), \cos(x) + 1) \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -((2*x*cos(4*x) + 4*x*cos(2*x) + 2*I*x*sin(4*x) + 4*I*x*sin(2*x) + 2*x)*arc
tan2(sin(2*x), cos(2*x) + 1) - (2*x*cos(4*x) + 4*x*cos(2*x) + 2*I*x*sin(4*x
) + 4*I*x*sin(2*x) + 2*x)*arctan2(sin(x), cos(x) + 1) + (2*x*cos(4*x) + 4*x
*cos(2*x) + 2*I*x*sin(4*x) + 4*I*x*sin(2*x) + 2*x)*arctan2(sin(x), -cos(x)
+ 1) - 2*(-2*I*x - 1)*cos(2*x) - (cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*
sin(2*x) + 1)*dilog(-e^(2*I*x)) + (2*cos(4*x) + 4*cos(2*x) + 2*I*sin(4*x) +
4*I*sin(2*x) + 2)*dilog(-e^(I*x)) + (2*cos(4*x) + 4*cos(2*x) + 2*I*sin(4*x
) + 4*I*sin(2*x) + 2)*dilog(e^(I*x)) + (-I*x*cos(4*x) - 2*I*x*cos(2*x) + x*
sin(4*x) + 2*x*sin(2*x) - I*x)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1
) + (I*x*cos(4*x) + 2*I*x*cos(2*x) - x*sin(4*x) - 2*x*sin(2*x) + I*x)*log(c
os(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (I*x*cos(4*x) + 2*I*x*cos(2*x) - x*sin
(4*x) - 2*x*sin(2*x) + I*x)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - (4*x
- 2*I)*sin(2*x) + 2)*sqrt(a)/(-2*I*cos(4*x) - 4*I*cos(2*x) + 2*sin(4*x) + 4
*sin(2*x) - 2*I)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \sqrt{\frac{a}{\cos(x)^4}}}{\cos(x) \sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)),x)
```

```
[Out] int((x*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{a \sec^4(x)} \csc(x) \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)**4)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*sec(x)**4)*csc(x)*sec(x), x)
```

3.878 $\int x^2 \csc(x) \sec(x) \sqrt{a \sec^4(x)} dx$

Optimal. Leaf size=220

$$ix \operatorname{Li}_2(-e^{2ix}) \cos^2(x) \sqrt{a \sec^4(x)} - ix \operatorname{Li}_2(e^{2ix}) \cos^2(x) \sqrt{a \sec^4(x)} - \frac{1}{2} \operatorname{Li}_3(-e^{2ix}) \cos^2(x) \sqrt{a \sec^4(x)} + \frac{1}{2} \operatorname{Li}_3(e^{2ix}) \cos^2(x) \sqrt{a \sec^4(x)}$$

```
[Out] 1/2*x^2*cos(x)^2*(a*sec(x)^4)^(1/2)-2*x^2*arctanh(exp(2*I*x))*cos(x)^2*(a*sec(x)^4)^(1/2)-cos(x)^2*ln(cos(x))*(a*sec(x)^4)^(1/2)+I*x*cos(x)^2*polylog(2,-exp(2*I*x))*(a*sec(x)^4)^(1/2)-I*x*cos(x)^2*polylog(2,exp(2*I*x))*(a*sec(x)^4)^(1/2)-1/2*cos(x)^2*polylog(3,-exp(2*I*x))*(a*sec(x)^4)^(1/2)+1/2*cos(x)^2*polylog(3,exp(2*I*x))*(a*sec(x)^4)^(1/2)-x*cos(x)*sin(x)*(a*sec(x)^4)^(1/2)+1/2*x^2*sin(x)^2*(a*sec(x)^4)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6720, 2620, 14, 4420, 2551, 4419, 4183, 2531, 2282, 6589, 3720, 3475, 30}

$$ix \cos^2(x) \operatorname{PolyLog}(2, -e^{2ix}) \sqrt{a \sec^4(x)} - ix \cos^2(x) \operatorname{PolyLog}(2, e^{2ix}) \sqrt{a \sec^4(x)} - \frac{1}{2} \cos^2(x) \operatorname{PolyLog}(3, -e^{2ix}) \sqrt{a \sec^4(x)} + \frac{1}{2} \cos^2(x) \operatorname{PolyLog}(3, e^{2ix}) \sqrt{a \sec^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] (x^2*Cos[x]^2*Sqrt[a*Sec[x]^4])/2 - 2*x^2*ArcTanh[E^((2*I)*x)]*Cos[x]^2*Sqrt[a*Sec[x]^4] - Cos[x]^2*Log[Cos[x]]*Sqrt[a*Sec[x]^4] + I*x*Cos[x]^2*PolyLog[2,-E^((2*I)*x)]*Sqrt[a*Sec[x]^4] - I*x*Cos[x]^2*PolyLog[2,E^((2*I)*x)]*Sqrt[a*Sec[x]^4] - (Cos[x]^2*PolyLog[3,-E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 + (Cos[x]^2*PolyLog[3,E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 - x*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x] + (x^2*Sqrt[a*Sec[x]^4]*Sin[x]^2)/2
```

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)+(b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_)+(b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1+(e_)*((F_)^((c_)*((a_)+(b_)*(x_))))^(n_)]*((f_)+(g_)*(x_))^(m_), x_Symbol] := -Simp[((f+g*x)^m*PolyLog[2,-(e*(F^(c*(a+b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f+g*x)^(m-1)*PolyLog[2,-(e*(F^(c*(a+b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[((a + b*x)^(m + 1)
)*Log[u]/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:= Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symb
ol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(
m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] := Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int x^2 \csc(x) \sec(x) \sqrt{a \sec^4(x)} dx &= \left(\cos^2(x) \sqrt{a \sec^4(x)} \right) \int x^2 \csc(x) \sec^3(x) dx \\
&= x^2 \cos^2(x) \log(\tan(x)) \sqrt{a \sec^4(x)} + \frac{1}{2} x^2 \sqrt{a \sec^4(x)} \sin^2(x) - \left(2 \cos^2(x) \sqrt{a \sec^4(x)} \right) \int x dx \\
&= x^2 \cos^2(x) \log(\tan(x)) \sqrt{a \sec^4(x)} + \frac{1}{2} x^2 \sqrt{a \sec^4(x)} \sin^2(x) - \left(2 \cos^2(x) \sqrt{a \sec^4(x)} \right) \frac{x^2}{2} \\
&= x^2 \cos^2(x) \log(\tan(x)) \sqrt{a \sec^4(x)} + \frac{1}{2} x^2 \sqrt{a \sec^4(x)} \sin^2(x) - \left(\cos^2(x) \sqrt{a \sec^4(x)} \right) x^2 \\
&= -x \cos(x) \sqrt{a \sec^4(x)} \sin(x) + \frac{1}{2} x^2 \sqrt{a \sec^4(x)} \sin^2(x) + \left(\cos^2(x) \sqrt{a \sec^4(x)} \right) x^2 \\
&= \frac{1}{2} x^2 \cos^2(x) \sqrt{a \sec^4(x)} - \cos^2(x) \log(\cos(x)) \sqrt{a \sec^4(x)} - x \cos(x) \sqrt{a \sec^4(x)} \sin(x) \\
&= \frac{1}{2} x^2 \cos^2(x) \sqrt{a \sec^4(x)} - 2x^2 \tanh^{-1}(e^{2ix}) \cos^2(x) \sqrt{a \sec^4(x)} - \cos^2(x) \log(\cos(x)) \sqrt{a \sec^4(x)} \\
&= \frac{1}{2} x^2 \cos^2(x) \sqrt{a \sec^4(x)} - 2x^2 \tanh^{-1}(e^{2ix}) \cos^2(x) \sqrt{a \sec^4(x)} - \cos^2(x) \log(\cos(x)) \sqrt{a \sec^4(x)} \\
&= \frac{1}{2} x^2 \cos^2(x) \sqrt{a \sec^4(x)} - 2x^2 \tanh^{-1}(e^{2ix}) \cos^2(x) \sqrt{a \sec^4(x)} - \cos^2(x) \log(\cos(x)) \sqrt{a \sec^4(x)} \\
&= \frac{1}{2} x^2 \cos^2(x) \sqrt{a \sec^4(x)} - 2x^2 \tanh^{-1}(e^{2ix}) \cos^2(x) \sqrt{a \sec^4(x)} - \cos^2(x) \log(\cos(x)) \sqrt{a \sec^4(x)}
\end{aligned}$$

Mathematica [A] time = 0.63, size = 138, normalized size = 0.63

$$\frac{1}{24} \cos^2(x) \sqrt{a \sec^4(x)} \left(24ix \operatorname{Li}_2(e^{-2ix}) + 24ix \operatorname{Li}_2(-e^{2ix}) + 12 \operatorname{Li}_3(e^{-2ix}) - 12 \operatorname{Li}_3(-e^{2ix}) + 16ix^3 + 24x^2 \log(1 - e^{-2ix}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]

[Out] (Cos[x]^2*Sqrt[a*Sec[x]^4]*((-I)*Pi^3 + (16*I)*x^3 + 24*x^2*Log[1 - E^((-2*I)*x)] - 24*x^2*Log[1 + E^((2*I)*x)] - 24*Log[Cos[x]] + (24*I)*x*PolyLog[2, E^((-2*I)*x)] + (24*I)*x*PolyLog[2, -E^((2*I)*x)] + 12*PolyLog[3, E^((-2*I)*x)] - 12*PolyLog[3, -E^((2*I)*x)] + 12*x^2*Sec[x]^2 - 24*x*Tan[x]))/24

fricas [C] time = 2.05, size = 550, normalized size = 2.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="fricas")

[Out] sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) - I*sin(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, I*cos(x) + sin(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, I*cos(x) - sin(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) + sin(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) - sin(x)) + sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -cos(x) + I*sin(x)) + sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -cos(x) - I*sin(x)) + 1/2*(x^2*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)^2*log(cos(x) - I*sin(x) + 1) - x^2*cos(x)^2*log(I*cos(x) + sin(x) + 1) - x^2*cos(x)^2*log(I*cos(x) - sin(x) + 1) - x^2*cos(x)^2*log(-I*cos(x) + sin(x) + 1) - x^2*cos(x)^2*log(-I*cos(x) - sin(x) + 1) + x^2*cos(x)^2*log(-cos(x) + I*sin(x) + 1) + x^2*cos(x)^2*log(-cos(x) - I*sin(x) + 1) - 2*I*x*cos(x)^2*dilog(cos(x) + I*sin(x)) + 2*I*x*cos(x)^2*dilog(cos(x) - I*sin(x)) - 2*I*x*cos(x)^2*dilog(I*cos(x) + sin(x)) + 2*I*x*cos(x)^2*dilog(I*cos(x) - sin(x)) + 2*I*x*cos(x)^2*dilog(-I*cos(x) + sin(x)) - 2*I*x*cos(x)^2*dilog(-I*cos(x) - sin(x)))

s(x) - sin(x)) + 2*I*x*cos(x)^2*dilog(-cos(x) + I*sin(x)) - 2*I*x*cos(x)^2*dilog(-cos(x) - I*sin(x)) - cos(x)^2*log(cos(x) + I*sin(x) + I) - cos(x)^2*log(cos(x) - I*sin(x) + I) - cos(x)^2*log(-cos(x) + I*sin(x) + I) - cos(x)^2*log(-cos(x) - I*sin(x) + I) - 2*x*cos(x)*sin(x) + x^2)*sqrt(a/cos(x)^4)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(x)^4} x^2 \csc(x) \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*sec(x)^4)*x^2*csc(x)*sec(x), x)

maple [C] time = 0.23, size = 254, normalized size = 1.15

$$2 \sqrt{\frac{a e^{4ix}}{(e^{2ix} + 1)^4}} x (x - i - i e^{-2ix}) + 2 \sqrt{\frac{a e^{4ix}}{(e^{2ix} + 1)^4}} (e^{2ix} + 1)^2 \left(-\frac{e^{-2ix} \ln(e^{2ix} + 1)}{2} - e^{-2ix} \Im(x) + e^{-2ix} \ln(e^{i\Re(x)}) \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x)

[Out] 2*(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*x*(x-I-I*exp(-2*I*x))+2*(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*(exp(2*I*x)+1)^2*(-1/2*exp(-2*I*x)*ln(exp(2*I*x)+1)-exp(-2*I*x)*Im(x)+exp(-2*I*x)*ln(exp(I*Re(x)))+1/2*exp(-2*I*x)*x^2*ln(1+exp(I*x))-I*exp(-2*I*x)*x*polylog(2,-exp(I*x))+exp(-2*I*x)*polylog(3,-exp(I*x))-1/2*exp(-2*I*x)*x^2*ln(exp(2*I*x)+1)+1/2*I*exp(-2*I*x)*x*polylog(2,-exp(2*I*x))-1/4*exp(-2*I*x)*polylog(3,-exp(2*I*x))+1/2*exp(-2*I*x)*x^2*ln(1-exp(I*x))-I*exp(-2*I*x)*x*polylog(2,exp(I*x))+exp(-2*I*x)*polylog(3,exp(I*x))))

maxima [B] time = 0.52, size = 653, normalized size = 2.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="maxima")

[Out] -((2*x^2 + 2*(x^2 + 1)*cos(4*x) + 4*(x^2 + 1)*cos(2*x) + (2*I*x^2 + 2*I)*sin(4*x) + (4*I*x^2 + 4*I)*sin(2*x) + 2)*arctan2(sin(2*x), cos(2*x) + 1) - (2*x^2*cos(4*x) + 4*x^2*cos(2*x) + 2*I*x^2*sin(4*x) + 4*I*x^2*sin(2*x) + 2*x^2)*arctan2(sin(x), cos(x) + 1) + (2*x^2*cos(4*x) + 4*x^2*cos(2*x) + 2*I*x^2*sin(4*x) + 4*I*x^2*sin(2*x) + 2*x^2)*arctan2(sin(x), -cos(x) + 1) - 4*x*cos(4*x) + (4*I*x^2 - 4*x)*cos(2*x) - (2*x*cos(4*x) + 4*x*cos(2*x) + 2*I*x*sin(4*x) + 4*I*x*sin(2*x) + 2*x)*dilog(-e^(2*I*x)) + (4*x*cos(4*x) + 8*x*cos(2*x) + 4*I*x*sin(4*x) + 8*I*x*sin(2*x) + 4*x)*dilog(-e^(I*x)) + (4*x*cos(4*x) + 8*x*cos(2*x) + 4*I*x*sin(4*x) + 8*I*x*sin(2*x) + 4*x)*dilog(e^(I*x)) + (-I*x^2 + (-I*x^2 - I)*cos(4*x) + (-2*I*x^2 - 2*I)*cos(2*x) + (x^2 + 1)*sin(4*x) + 2*(x^2 + 1)*sin(2*x) - I)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + (I*x^2*cos(4*x) + 2*I*x^2*cos(2*x) - x^2*sin(4*x) - 2*x^2*sin(2*x) + I*x^2)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (I*x^2*cos(4*x) + 2*I*x^2*cos(2*x) - x^2*sin(4*x) - 2*x^2*sin(2*x) + I*x^2)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + (-I*cos(4*x) - 2*I*cos(2*x) + sin(4*x) + 2*sin(2*x) - I)*polylog(3, -e^(2*I*x)) + (4*I*cos(4*x) + 8*I*cos(2*x) - 4*sin(4*x) - 8*sin(2*x) + 4*I)*polylog(3, -e^(I*x)) + (4*I*cos(4*x) + 8*I*cos(2*x) - 4*sin(4*x) - 8*sin(2*x) + 4*I)*polylog(3, e^(I*x)) - 4*I*x*sin(4*x) - 4*(x^2 + I*x)*sin(2*x))*sqrt(a)/(-2*I*cos(4*x) - 4*I*cos(2*x) + 2*sin(4*x) + 4*sin(2*x) - 2*I)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sqrt{\frac{a}{\cos(x)^4}}}{\cos(x) \sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)), x)`

[Out] `int((x^2*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a \sec^4(x)} \csc(x) \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*csc(x)*sec(x)*(a*sec(x)**4)**(1/2), x)`

[Out] `Integral(x**2*sqrt(a*sec(x)**4)*csc(x)*sec(x), x)`

3.879 $\int x^3 \csc(x) \sec(x) \sqrt{a \sec^4(x)} dx$

Optimal. Leaf size=356

$$\frac{3}{2}ix^2\text{Li}_2(-e^{2ix})\cos^2(x)\sqrt{a\sec^4(x)} - \frac{3}{2}ix^2\text{Li}_2(e^{2ix})\cos^2(x)\sqrt{a\sec^4(x)} - \frac{3}{2}x\text{Li}_3(-e^{2ix})\cos^2(x)\sqrt{a\sec^4(x)} + \frac{3}{2}x\text{Li}_3(e^{2ix})\cos^2(x)\sqrt{a\sec^4(x)}$$

```
[Out] 3/2*I*x^2*cos(x)^2*(a*sec(x)^4)^(1/2)+1/2*x^3*cos(x)^2*(a*sec(x)^4)^(1/2)-2
*x^3*arctanh(exp(2*I*x))*cos(x)^2*(a*sec(x)^4)^(1/2)-3*x*cos(x)^2*ln(1+exp(
2*I*x))*(a*sec(x)^4)^(1/2)+3/2*I*cos(x)^2*polylog(2,-exp(2*I*x))*(a*sec(x)^
4)^(1/2)+3/2*I*x^2*cos(x)^2*polylog(2,-exp(2*I*x))*(a*sec(x)^4)^(1/2)-3/2*I
*x^2*cos(x)^2*polylog(2,exp(2*I*x))*(a*sec(x)^4)^(1/2)-3/2*x*cos(x)^2*polyl
og(3,-exp(2*I*x))*(a*sec(x)^4)^(1/2)+3/2*x*cos(x)^2*polylog(3,exp(2*I*x))*(
a*sec(x)^4)^(1/2)-3/4*I*cos(x)^2*polylog(4,-exp(2*I*x))*(a*sec(x)^4)^(1/2)+
3/4*I*cos(x)^2*polylog(4,exp(2*I*x))*(a*sec(x)^4)^(1/2)-3/2*x^2*cos(x)*sin(
x)*(a*sec(x)^4)^(1/2)+1/2*x^3*sin(x)^2*(a*sec(x)^4)^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.944$, Rules used = {6720, 2620, 14, 4420, 2551, 4419, 4183, 2531, 6609, 2282, 6589, 3720, 3719, 2190, 2279, 2391, 30}

$$\frac{3}{2}ix^2\cos^2(x)\text{PolyLog}(2,-e^{2ix})\sqrt{a\sec^4(x)} - \frac{3}{2}ix^2\cos^2(x)\text{PolyLog}(2,e^{2ix})\sqrt{a\sec^4(x)} - \frac{3}{2}x\cos^2(x)\text{PolyLog}(3,-e^{2ix})\sqrt{a\sec^4(x)} + \frac{3}{2}x\cos^2(x)\text{PolyLog}(3,e^{2ix})\sqrt{a\sec^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((3*I)/2)*x^2*Cos[x]^2*Sqrt[a*Sec[x]^4] + (x^3*Cos[x]^2*Sqrt[a*Sec[x]^4])/2
- 2*x^3*ArcTanh[E^((2*I)*x)]*Cos[x]^2*Sqrt[a*Sec[x]^4] - 3*x*Cos[x]^2*Log[
1 + E^((2*I)*x)]*Sqrt[a*Sec[x]^4] + ((3*I)/2)*Cos[x]^2*PolyLog[2, -E^((2*I)
*x)]*Sqrt[a*Sec[x]^4] + ((3*I)/2)*x^2*Cos[x]^2*PolyLog[2, -E^((2*I)*x)]*Sqr
t[a*Sec[x]^4] - ((3*I)/2)*x^2*Cos[x]^2*PolyLog[2, E^((2*I)*x)]*Sqrt[a*Sec[x
]^4] - (3*x*Cos[x]^2*PolyLog[3, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 + (3*x*Co
s[x]^2*PolyLog[3, E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 - ((3*I)/4)*Cos[x]^2*Pol
yLog[4, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4] + ((3*I)/4)*Cos[x]^2*PolyLog[4, E^((
2*I)*x)]*Sqrt[a*Sec[x]^4] - (3*x^2*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x])/2 + (x^3
*Sqrt[a*Sec[x]^4]*Sin[x]^2)/2
```

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m-1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n)]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_)^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)
)*Log[u]/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_)^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
+ f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_)^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_)^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(
m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
```

[m, 0]

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^p, x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int x^3 \csc(x) \sec(x) \sqrt{a \sec^4(x)} dx &= \left(\cos^2(x) \sqrt{a \sec^4(x)} \right) \int x^3 \csc(x) \sec^3(x) dx \\
&= x^3 \cos^2(x) \log(\tan(x)) \sqrt{a \sec^4(x)} + \frac{1}{2} x^3 \sqrt{a \sec^4(x)} \sin^2(x) - \left(3 \cos^2(x) \sqrt{a \sec^4(x)} \right) \int x^2 \csc(x) \sec^3(x) dx \\
&= x^3 \cos^2(x) \log(\tan(x)) \sqrt{a \sec^4(x)} + \frac{1}{2} x^3 \sqrt{a \sec^4(x)} \sin^2(x) - \left(3 \cos^2(x) \sqrt{a \sec^4(x)} \right) \int x^2 \csc(x) \sec^3(x) dx \\
&= x^3 \cos^2(x) \log(\tan(x)) \sqrt{a \sec^4(x)} + \frac{1}{2} x^3 \sqrt{a \sec^4(x)} \sin^2(x) - \frac{1}{2} \left(3 \cos^2(x) \sqrt{a \sec^4(x)} \right) \int x^2 \csc(x) \sec^3(x) dx \\
&= -\frac{3}{2} x^2 \cos(x) \sqrt{a \sec^4(x)} \sin(x) + \frac{1}{2} x^3 \sqrt{a \sec^4(x)} \sin^2(x) + \left(\cos^2(x) \sqrt{a \sec^4(x)} \right) \int x^2 \csc(x) \sec^3(x) dx \\
&= \frac{3}{2} i x^2 \cos^2(x) \sqrt{a \sec^4(x)} + \frac{1}{2} x^3 \cos^2(x) \sqrt{a \sec^4(x)} - \frac{3}{2} x^2 \cos(x) \sqrt{a \sec^4(x)} \sin(x) \\
&= \frac{3}{2} i x^2 \cos^2(x) \sqrt{a \sec^4(x)} + \frac{1}{2} x^3 \cos^2(x) \sqrt{a \sec^4(x)} - 2 x^3 \tanh^{-1} \left(e^{2ix} \right) \cos^2(x) \\
&= \frac{3}{2} i x^2 \cos^2(x) \sqrt{a \sec^4(x)} + \frac{1}{2} x^3 \cos^2(x) \sqrt{a \sec^4(x)} - 2 x^3 \tanh^{-1} \left(e^{2ix} \right) \cos^2(x) \\
&= \frac{3}{2} i x^2 \cos^2(x) \sqrt{a \sec^4(x)} + \frac{1}{2} x^3 \cos^2(x) \sqrt{a \sec^4(x)} - 2 x^3 \tanh^{-1} \left(e^{2ix} \right) \cos^2(x) \\
&= \frac{3}{2} i x^2 \cos^2(x) \sqrt{a \sec^4(x)} + \frac{1}{2} x^3 \cos^2(x) \sqrt{a \sec^4(x)} - 2 x^3 \tanh^{-1} \left(e^{2ix} \right) \cos^2(x) \\
&= \frac{3}{2} i x^2 \cos^2(x) \sqrt{a \sec^4(x)} + \frac{1}{2} x^3 \cos^2(x) \sqrt{a \sec^4(x)} - 2 x^3 \tanh^{-1} \left(e^{2ix} \right) \cos^2(x)
\end{aligned}$$

Mathematica [A] time = 1.07, size = 191, normalized size = 0.54

$$\frac{1}{64} \cos^2(x) \sqrt{a \sec^4(x)} \left(96 i x^2 \operatorname{Li}_2 \left(e^{-2ix} \right) + 96 i \left(x^2 + 1 \right) \operatorname{Li}_2 \left(-e^{2ix} \right) + 96 x \operatorname{Li}_3 \left(e^{-2ix} \right) - 96 x \operatorname{Li}_3 \left(-e^{2ix} \right) - 48 i \operatorname{Li}_4 \left(e^{-2ix} \right) + 48 i \operatorname{Li}_4 \left(-e^{2ix} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]

[Out] (Cos[x]^2*Sqrt[a*Sec[x]^4]*((-I)*Pi^4 + (96*I)*x^2 + (32*I)*x^4 + 64*x^3*Log[1 - E^((-2*I)*x)] - 192*x*Log[1 + E^((2*I)*x)] - 64*x^3*Log[1 + E^((2*I)*x)] + (96*I)*x^2*PolyLog[2, E^((-2*I)*x)] + (96*I)*(1 + x^2)*PolyLog[2, -E^((2*I)*x)] + 96*x*PolyLog[3, E^((-2*I)*x)] - 96*x*PolyLog[3, -E^((2*I)*x)] - (48*I)*PolyLog[4, E^((-2*I)*x)] - (48*I)*PolyLog[4, -E^((2*I)*x)] + 32*x^3*Sec[x]^2 - 96*x^2*Tan[x])/64

fricas [C] time = 1.08, size = 736, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="fricas")

[Out] 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) - I*sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, I*cos(x) + sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, I*cos(x) - sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) + sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) - sin(x)) + 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -cos(x) + I*sin(x)) + 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -cos(x) - I*sin(x)) + 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, cos(x) + I*sin(x)) - 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, cos(x) - I*sin(x)) + 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, I*cos(x) + sin(x)) - 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, I*cos(x) - sin(x))

```

sin(x)) - 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, I*cos(x) - sin(x)) - 3*
I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, -I*cos(x) + sin(x)) + 3*I*sqrt(a/cos
(x)^4)*cos(x)^2*polylog(4, -I*cos(x) - sin(x)) - 3*I*sqrt(a/cos(x)^4)*cos(x
)^2*polylog(4, -cos(x) + I*sin(x)) + 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(
4, -cos(x) - I*sin(x)) + 1/2*(x^3*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x^3
*cos(x)^2*log(cos(x) - I*sin(x) + 1) + x^3*cos(x)^2*log(-cos(x) + I*sin(x)
+ 1) + x^3*cos(x)^2*log(-cos(x) - I*sin(x) + 1) - 3*I*x^2*cos(x)^2*dilog(co
s(x) + I*sin(x)) + 3*I*x^2*cos(x)^2*dilog(cos(x) - I*sin(x)) + 3*I*x^2*cos(
x)^2*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)^2*dilog(-cos(x) - I*sin(x))
+ (-3*I*x^2 - 3*I)*cos(x)^2*dilog(I*cos(x) + sin(x)) + (3*I*x^2 + 3*I)*cos
(x)^2*dilog(I*cos(x) - sin(x)) + (3*I*x^2 + 3*I)*cos(x)^2*dilog(-I*cos(x) +
sin(x)) + (-3*I*x^2 - 3*I)*cos(x)^2*dilog(-I*cos(x) - sin(x)) - (x^3 + 3*x
)*cos(x)^2*log(I*cos(x) + sin(x) + 1) - (x^3 + 3*x)*cos(x)^2*log(I*cos(x) -
sin(x) + 1) - (x^3 + 3*x)*cos(x)^2*log(-I*cos(x) + sin(x) + 1) - (x^3 + 3*
x)*cos(x)^2*log(-I*cos(x) - sin(x) + 1) - 3*x^2*cos(x)*sin(x) + x^3)*sqrt(a
/cos(x)^4)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec(x)^4} x^3 \csc(x) \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*sec(x)^4)*x^3*csc(x)*sec(x), x)

maple [A] time = 0.21, size = 324, normalized size = 0.91

$$\sqrt{\frac{a e^{4ix}}{(e^{2ix} + 1)^4}} x^2 (2x - 3i - 3ie^{-2ix}) - 2i \sqrt{\frac{a e^{4ix}}{(e^{2ix} + 1)^4}} (e^{2ix} + 1)^2 \left(-\frac{3e^{-2ix}x^2}{2} - \frac{3ie^{-2ix}x \ln(e^{2ix} + 1)}{2} - \frac{3e^{-2ix} \text{pol}}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x)

```

[Out] (a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*x^2*(2*x-3*I-3*I*exp(-2*I*x))-2*I*(a*
exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*(exp(2*I*x)+1)^2*(-3/2*exp(-2*I*x)*x^2-3
/2*I*exp(-2*I*x)*x*ln(exp(2*I*x)+1)-3/4*exp(-2*I*x)*polylog(2,-exp(2*I*x))+
1/2*I*exp(-2*I*x)*x^3*ln(1+exp(I*x))+3/2*exp(-2*I*x)*x^2*polylog(2,-exp(I*x
))+3*I*exp(-2*I*x)*x*polylog(3,-exp(I*x))-3*exp(-2*I*x)*polylog(4,-exp(I*x)
)-1/2*I*exp(-2*I*x)*x^3*ln(exp(2*I*x)+1)-3/4*exp(-2*I*x)*x^2*polylog(2,-exp
(2*I*x))-3/4*I*exp(-2*I*x)*x*polylog(3,-exp(2*I*x))+3/8*exp(-2*I*x)*polylog
(4,-exp(2*I*x))+1/2*I*exp(-2*I*x)*x^3*ln(1-exp(I*x))+3/2*exp(-2*I*x)*x^2*po
lylog(2,exp(I*x))+3*I*exp(-2*I*x)*x*polylog(3,exp(I*x))-3*exp(-2*I*x)*polyl
og(4,exp(I*x)))

```

maxima [B] time = 1.40, size = 870, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="maxima")

```

[Out] (18*x^2*cos(4*x) + 18*I*x^2*sin(4*x) - (8*x^3 + 2*(4*x^3 + 9*x)*cos(4*x) +
4*(4*x^3 + 9*x)*cos(2*x) + (8*I*x^3 + 18*I*x)*sin(4*x) + (16*I*x^3 + 36*I*x
)*sin(2*x) + 18*x)*arctan2(sin(2*x), cos(2*x) + 1) + (6*x^3*cos(4*x) + 12*x
^3*cos(2*x) + 6*I*x^3*sin(4*x) + 12*I*x^3*sin(2*x) + 6*x^3)*arctan2(sin(x),
cos(x) + 1) - (6*x^3*cos(4*x) + 12*x^3*cos(2*x) + 6*I*x^3*sin(4*x) + 12*I*
x^3*sin(2*x) + 6*x^3)*arctan2(sin(x), -cos(x) + 1) - (12*I*x^3 - 18*x^2)*co

```

$s(2*x) + (12*x^2 + 3*(4*x^2 + 3)*\cos(4*x) + 6*(4*x^2 + 3)*\cos(2*x) - (-12*I*x^2 - 9*I)*\sin(4*x) - (-24*I*x^2 - 18*I)*\sin(2*x) + 9)*\operatorname{dilog}(-e^{(2*I*x)}) - (18*x^2*\cos(4*x) + 36*x^2*\cos(2*x) + 18*I*x^2*\sin(4*x) + 36*I*x^2*\sin(2*x) + 18*x^2)*\operatorname{dilog}(-e^{(I*x)}) - (18*x^2*\cos(4*x) + 36*x^2*\cos(2*x) + 18*I*x^2*\sin(4*x) + 36*I*x^2*\sin(2*x) + 18*x^2)*\operatorname{dilog}(e^{(I*x)}) - (-4*I*x^3 + (-4*I*x^3 - 9*I*x)*\cos(4*x) + (-8*I*x^3 - 18*I*x)*\cos(2*x) + (4*x^3 + 9*x)*\sin(4*x) + 2*(4*x^3 + 9*x)*\sin(2*x) - 9*I*x)*\log(\cos(2*x)^2 + \sin(2*x)^2 + 2*\cos(2*x) + 1) - (3*I*x^3*\cos(4*x) + 6*I*x^3*\cos(2*x) - 3*x^3*\sin(4*x) - 6*x^3*\sin(2*x) + 3*I*x^3)*\log(\cos(x)^2 + \sin(x)^2 + 2*\cos(x) + 1) - (3*I*x^3*\cos(4*x) + 6*I*x^3*\cos(2*x) - 3*x^3*\sin(4*x) - 6*x^3*\sin(2*x) + 3*I*x^3)*\log(\cos(x)^2 + \sin(x)^2 - 2*\cos(x) + 1) - (6*\cos(4*x) + 12*\cos(2*x) + 6*I*\sin(4*x) + 12*I*\sin(2*x) + 6)*\operatorname{polylog}(4, -e^{(2*I*x)}) + (36*\cos(4*x) + 72*\cos(2*x) + 36*I*\sin(4*x) + 72*I*\sin(2*x) + 36)*\operatorname{polylog}(4, -e^{(I*x)}) + (36*\cos(4*x) + 72*\cos(2*x) + 36*I*\sin(4*x) + 72*I*\sin(2*x) + 36)*\operatorname{polylog}(4, e^{(I*x)}) - (-12*I*x*\cos(4*x) - 24*I*x*\cos(2*x) + 12*x*\sin(4*x) + 24*x*\sin(2*x) - 12*I*x)*\operatorname{polylog}(3, -e^{(2*I*x)}) - (36*I*x*\cos(4*x) + 72*I*x*\cos(2*x) - 36*x*\sin(4*x) - 72*x*\sin(2*x) + 36*I*x)*\operatorname{polylog}(3, -e^{(I*x)}) - (36*I*x*\cos(4*x) + 72*I*x*\cos(2*x) - 36*x*\sin(4*x) - 72*x*\sin(2*x) + 36*I*x)*\operatorname{polylog}(3, e^{(I*x)}) + 6*(2*x^3 + 3*I*x^2)*\sin(2*x))*\sqrt{a}/(-6*I*\cos(4*x) - 12*I*\cos(2*x) + 6*\sin(4*x) + 12*\sin(2*x) - 6*I)$

mapad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \sqrt{\frac{a}{\cos(x)^4}}}{\cos(x) \sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)), x)`

[Out] `int((x^3*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{a \sec^4(x)} \csc(x) \sec(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*csc(x)*sec(x)*(a*sec(x)**4)**(1/2), x)`

[Out] `Integral(x**3*sqrt(a*sec(x)**4)*csc(x)*sec(x), x)`

3.880 $\int \sin(x) \sin(2x) \sin(3x) dx$

Optimal. Leaf size=25

$$-\frac{1}{8} \cos(2x) - \frac{1}{16} \cos(4x) + \frac{1}{24} \cos(6x)$$

[Out] $-1/8*\cos(2*x)-1/16*\cos(4*x)+1/24*\cos(6*x)$

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4355, 2638}

$$-\frac{1}{8} \cos(2x) - \frac{1}{16} \cos(4x) + \frac{1}{24} \cos(6x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sin}[x]*\text{Sin}[2*x]*\text{Sin}[3*x], x]$

[Out] $-\text{Cos}[2*x]/8 - \text{Cos}[4*x]/16 + \text{Cos}[6*x]/24$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 4355

$\text{Int}[(F_)[(a_.) + (b_.)*(x_.)]^{(p_.)}*(G_)[(c_.) + (d_.)*(x_.)]^{(q_.)}*(H_)[(e_.) + (f_.)*(x_.)]^{(r_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[\text{ActivateTrig}[F[a + b*x]^{p*G[c + d*x]^{q*H[e + f*x]^{r}}], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& (\text{EqQ}[F, \sin] \parallel \text{EqQ}[F, \cos]) \&\& (\text{EqQ}[G, \sin] \parallel \text{EqQ}[G, \cos]) \&\& (\text{EqQ}[H, \sin] \parallel \text{EqQ}[H, \cos]) \&\& \text{IGtQ}[p, 0] \&\& \text{IGtQ}[q, 0] \&\& \text{IGtQ}[r, 0]$

Rubi steps

$$\begin{aligned} \int \sin(x) \sin(2x) \sin(3x) dx &= \int \left(\frac{1}{4} \sin(2x) + \frac{1}{4} \sin(4x) - \frac{1}{4} \sin(6x) \right) dx \\ &= \frac{1}{4} \int \sin(2x) dx + \frac{1}{4} \int \sin(4x) dx - \frac{1}{4} \int \sin(6x) dx \\ &= -\frac{1}{8} \cos(2x) - \frac{1}{16} \cos(4x) + \frac{1}{24} \cos(6x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{8} \cos(2x) - \frac{1}{16} \cos(4x) + \frac{1}{24} \cos(6x)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Sin}[x]*\text{Sin}[2*x]*\text{Sin}[3*x], x]$

[Out] $-1/8*\text{Cos}[2*x] - \text{Cos}[4*x]/16 + \text{Cos}[6*x]/24$

fricas [A] time = 0.91, size = 17, normalized size = 0.68

$$\frac{4}{3} \cos(x)^6 - \frac{5}{2} \cos(x)^4 + \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")

[Out] 4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2

giac [A] time = 0.12, size = 13, normalized size = 0.52

$$-\frac{4}{3} \sin(x)^6 + \frac{3}{2} \sin(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="giac")

[Out] -4/3*sin(x)^6 + 3/2*sin(x)^4

maple [A] time = 0.13, size = 20, normalized size = 0.80

$$-\frac{\cos(2x)}{8} - \frac{\cos(4x)}{16} + \frac{\cos(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*sin(2*x)*sin(3*x),x)

[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)

maxima [A] time = 0.32, size = 19, normalized size = 0.76

$$\frac{1}{24} \cos(6x) - \frac{1}{16} \cos(4x) - \frac{1}{8} \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")

[Out] 1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)

mupad [B] time = 2.94, size = 14, normalized size = 0.56

$$\frac{\sin(x)^4 (8 \sin(x)^2 - 9)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)*sin(3*x)*sin(x),x)

[Out] -(sin(x)^4*(8*sin(x)^2 - 9))/6

sympy [B] time = 10.55, size = 114, normalized size = 4.56

$$\frac{x \sin(x) \sin(2x) \sin(3x)}{4} + \frac{x \sin(x) \cos(2x) \cos(3x)}{4} + \frac{x \sin(2x) \cos(x) \cos(3x)}{4} - \frac{x \sin(3x) \cos(x) \cos(2x)}{4} - \frac{3x \sin(x) \sin(2x) \cos(3x)}{8} + \frac{\sin(x) \sin(3x) \cos(2x)}{6} + \frac{\sin(2x) \sin(3x) \cos(x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*sin(2*x)*sin(3*x),x)

[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*cos(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - 3*sin(x)*sin(2*x)*cos(3*x)/8 + sin(x)*sin(3*x)*cos(2*x)/6 + sin(2*x)*sin(3*x)*cos(x)/24

3.881 $\int \cos(x) \cos(2x) \cos(3x) dx$

Optimal. Leaf size=30

$$\frac{x}{4} + \frac{1}{8} \sin(2x) + \frac{1}{16} \sin(4x) + \frac{1}{24} \sin(6x)$$

[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4355, 2637}

$$\frac{x}{4} + \frac{1}{8} \sin(2x) + \frac{1}{16} \sin(4x) + \frac{1}{24} \sin(6x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[2*x]*Cos[3*x],x]

[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4355

Int[(F_)[(a_.) + (b_.)*(x_.)]^(p_.)*(G_)[(c_.) + (d_.)*(x_.)]^(q_.)*(H_)[(e_.) + (f_.)*(x_.)]^(r_.), x_Symbol] := Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H, sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]

Rubi steps

$$\begin{aligned} \int \cos(x) \cos(2x) \cos(3x) dx &= \int \left(\frac{1}{4} + \frac{1}{4} \cos(2x) + \frac{1}{4} \cos(4x) + \frac{1}{4} \cos(6x) \right) dx \\ &= \frac{x}{4} + \frac{1}{4} \int \cos(2x) dx + \frac{1}{4} \int \cos(4x) dx + \frac{1}{4} \int \cos(6x) dx \\ &= \frac{x}{4} + \frac{1}{8} \sin(2x) + \frac{1}{16} \sin(4x) + \frac{1}{24} \sin(6x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{x}{4} + \frac{1}{8} \sin(2x) + \frac{1}{16} \sin(4x) + \frac{1}{24} \sin(6x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]

[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24

fricas [A] time = 0.87, size = 25, normalized size = 0.83

$$\frac{1}{12} (16 \cos(x)^5 - 10 \cos(x)^3 + 3 \cos(x)) \sin(x) + \frac{1}{4} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="fricas")

[Out] 1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x

giac [A] time = 0.15, size = 22, normalized size = 0.73

$$\frac{1}{4}x + \frac{1}{24}\sin(6x) + \frac{1}{16}\sin(4x) + \frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="giac")

[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)

maple [A] time = 0.14, size = 23, normalized size = 0.77

$$\frac{x}{4} + \frac{\sin(2x)}{8} + \frac{\sin(4x)}{16} + \frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*cos(2*x)*cos(3*x),x)

[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)

maxima [A] time = 0.31, size = 22, normalized size = 0.73

$$\frac{1}{4}x + \frac{1}{24}\sin(6x) + \frac{1}{16}\sin(4x) + \frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="maxima")

[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)

mupad [B] time = 3.04, size = 22, normalized size = 0.73

$$\frac{x}{4} + \frac{\sin(2x)}{8} + \frac{\sin(4x)}{16} + \frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)*cos(3*x)*cos(x),x)

[Out] x/4 + sin(2*x)/8 + sin(4*x)/16 + sin(6*x)/24

sympy [B] time = 10.54, size = 116, normalized size = 3.87

$$\frac{x \sin(x) \sin(2x) \cos(3x)}{4} + \frac{x \sin(x) \sin(3x) \cos(2x)}{4} + \frac{x \sin(2x) \sin(3x) \cos(x)}{4} + \frac{x \cos(x) \cos(2x) \cos(3x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*cos(2*x)*cos(3*x),x)

[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*sin(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + 3*sin(x)*sin(2*x)*sin(3*x)/8 + sin(x)*cos(2*x)*cos(3*x)/3 + 5*sin(2*x)*cos(x)*cos(3*x)/24

3.882 $\int \cos(x) \sin(2x) \sin(3x) dx$

Optimal. Leaf size=30

$$\frac{x}{4} + \frac{1}{8} \sin(2x) - \frac{1}{16} \sin(4x) - \frac{1}{24} \sin(6x)$$

[Out] 1/4*x+1/8*sin(2*x)-1/16*sin(4*x)-1/24*sin(6*x)

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4355, 2637}

$$\frac{x}{4} + \frac{1}{8} \sin(2x) - \frac{1}{16} \sin(4x) - \frac{1}{24} \sin(6x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[2*x]*Sin[3*x],x]

[Out] x/4 + Sin[2*x]/8 - Sin[4*x]/16 - Sin[6*x]/24

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4355

Int[(F_)[(a_.) + (b_.)*(x_.)]^(p_.)*(G_)[(c_.) + (d_.)*(x_.)]^(q_.)*(H_)[(e_.) + (f_.)*(x_.)]^(r_.), x_Symbol] := Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H, sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]

Rubi steps

$$\begin{aligned} \int \cos(x) \sin(2x) \sin(3x) dx &= \int \left(\frac{1}{4} + \frac{1}{4} \cos(2x) - \frac{1}{4} \cos(4x) - \frac{1}{4} \cos(6x) \right) dx \\ &= \frac{x}{4} + \frac{1}{4} \int \cos(2x) dx - \frac{1}{4} \int \cos(4x) dx - \frac{1}{4} \int \cos(6x) dx \\ &= \frac{x}{4} + \frac{1}{8} \sin(2x) - \frac{1}{16} \sin(4x) - \frac{1}{24} \sin(6x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{x}{4} + \frac{1}{8} \sin(2x) - \frac{1}{16} \sin(4x) - \frac{1}{24} \sin(6x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sin[2*x]*Sin[3*x],x]

[Out] x/4 + Sin[2*x]/8 - Sin[4*x]/16 - Sin[6*x]/24

fricas [A] time = 0.71, size = 25, normalized size = 0.83

$$-\frac{1}{12} (16 \cos(x)^5 - 10 \cos(x)^3 - 3 \cos(x)) \sin(x) + \frac{1}{4} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")

[Out] -1/12*(16*cos(x)^5 - 10*cos(x)^3 - 3*cos(x))*sin(x) + 1/4*x

giac [A] time = 0.13, size = 22, normalized size = 0.73

$$\frac{1}{4}x - \frac{1}{24}\sin(6x) - \frac{1}{16}\sin(4x) + \frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(2*x)*sin(3*x),x, algorithm="giac")

[Out] 1/4*x - 1/24*sin(6*x) - 1/16*sin(4*x) + 1/8*sin(2*x)

maple [A] time = 0.09, size = 23, normalized size = 0.77

$$\frac{x}{4} + \frac{\sin(2x)}{8} - \frac{\sin(4x)}{16} - \frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(2*x)*sin(3*x),x)

[Out] 1/4*x+1/8*sin(2*x)-1/16*sin(4*x)-1/24*sin(6*x)

maxima [A] time = 0.32, size = 22, normalized size = 0.73

$$\frac{1}{4}x - \frac{1}{24}\sin(6x) - \frac{1}{16}\sin(4x) + \frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")

[Out] 1/4*x - 1/24*sin(6*x) - 1/16*sin(4*x) + 1/8*sin(2*x)

mupad [B] time = 3.01, size = 22, normalized size = 0.73

$$\frac{x}{4} + \frac{\sin(2x)}{8} - \frac{\sin(4x)}{16} - \frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)*sin(3*x)*cos(x),x)

[Out] x/4 + sin(2*x)/8 - sin(4*x)/16 - sin(6*x)/24

sympy [B] time = 10.41, size = 114, normalized size = 3.80

$$\frac{x \sin(x) \sin(2x) \cos(3x)}{4} + \frac{x \sin(x) \sin(3x) \cos(2x)}{4} + \frac{x \sin(2x) \sin(3x) \cos(x)}{4} + \frac{x \cos(x) \cos(2x) \cos(3x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(2*x)*sin(3*x),x)

[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*sin(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + sin(x)*sin(2*x)*sin(3*x)/8 + sin(x)*cos(2*x)*cos(3*x)/6 - 5*sin(2*x)*cos(x)*cos(3*x)/24

3.883 $\int \cos(2x) \cos(3x) \sin(x) dx$

Optimal. Leaf size=25

$$-\frac{1}{8} \cos(2x) + \frac{1}{16} \cos(4x) - \frac{1}{24} \cos(6x)$$

[Out] $-1/8*\cos(2*x)+1/16*\cos(4*x)-1/24*\cos(6*x)$

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4355, 2638}

$$-\frac{1}{8} \cos(2x) + \frac{1}{16} \cos(4x) - \frac{1}{24} \cos(6x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[2*x]*\text{Cos}[3*x]*\text{Sin}[x], x]$

[Out] $-\text{Cos}[2*x]/8 + \text{Cos}[4*x]/16 - \text{Cos}[6*x]/24$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 4355

$\text{Int}[(F_)[(a_.) + (b_.)*(x_)]^{(p_.)}*(G_)[(c_.) + (d_.)*(x_)]^{(q_.)}*(H_)[(e_.) + (f_.)*(x_)]^{(r_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[\text{ActivateTrig}[F[a + b*x]^{p*G[c + d*x]^{q*H[e + f*x]^r}], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& (\text{EqQ}[F, \sin] \parallel \text{EqQ}[F, \cos]) \&\& (\text{EqQ}[G, \sin] \parallel \text{EqQ}[G, \cos]) \&\& (\text{EqQ}[H, \sin] \parallel \text{EqQ}[H, \cos]) \&\& \text{IGtQ}[p, 0] \&\& \text{IGtQ}[q, 0] \&\& \text{IGtQ}[r, 0]$

Rubi steps

$$\begin{aligned} \int \cos(2x) \cos(3x) \sin(x) dx &= \int \left(\frac{1}{4} \sin(2x) - \frac{1}{4} \sin(4x) + \frac{1}{4} \sin(6x) \right) dx \\ &= \frac{1}{4} \int \sin(2x) dx - \frac{1}{4} \int \sin(4x) dx + \frac{1}{4} \int \sin(6x) dx \\ &= -\frac{1}{8} \cos(2x) + \frac{1}{16} \cos(4x) - \frac{1}{24} \cos(6x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{8} \cos(2x) + \frac{1}{16} \cos(4x) - \frac{1}{24} \cos(6x)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Cos}[2*x]*\text{Cos}[3*x]*\text{Sin}[x], x]$

[Out] $-1/8*\text{Cos}[2*x] + \text{Cos}[4*x]/16 - \text{Cos}[6*x]/24$

fricas [A] time = 0.88, size = 19, normalized size = 0.76

$$-\frac{4}{3} \cos(x)^6 + \frac{5}{2} \cos(x)^4 - \frac{3}{2} \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*cos(3*x)*sin(x), x, algorithm="fricas")

[Out] $-4/3*\cos(x)^6 + 5/2*\cos(x)^4 - 3/2*\cos(x)^2$

giac [A] time = 0.13, size = 19, normalized size = 0.76

$$\frac{4}{3} \sin(x)^6 - \frac{3}{2} \sin(x)^4 + \frac{1}{2} \sin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*cos(3*x)*sin(x), x, algorithm="giac")

[Out] $4/3*\sin(x)^6 - 3/2*\sin(x)^4 + 1/2*\sin(x)^2$

maple [A] time = 0.08, size = 20, normalized size = 0.80

$$-\frac{\cos(2x)}{8} + \frac{\cos(4x)}{16} - \frac{\cos(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)*cos(3*x)*sin(x), x)

[Out] $-1/8*\cos(2*x) + 1/16*\cos(4*x) - 1/24*\cos(6*x)$

maxima [A] time = 0.31, size = 19, normalized size = 0.76

$$-\frac{1}{24} \cos(6x) + \frac{1}{16} \cos(4x) - \frac{1}{8} \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*cos(3*x)*sin(x), x, algorithm="maxima")

[Out] $-1/24*\cos(6*x) + 1/16*\cos(4*x) - 1/8*\cos(2*x)$

mupad [B] time = 3.18, size = 19, normalized size = 0.76

$$\frac{4 \sin(x)^6}{3} - \frac{3 \sin(x)^4}{2} + \frac{\sin(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)*cos(3*x)*sin(x), x)

[Out] $\sin(x)^2/2 - (3*\sin(x)^4)/2 + (4*\sin(x)^6)/3$

sympy [B] time = 10.45, size = 112, normalized size = 4.48

$$\frac{x \sin(x) \sin(2x) \sin(3x)}{4} + \frac{x \sin(x) \cos(2x) \cos(3x)}{4} + \frac{x \sin(2x) \cos(x) \cos(3x)}{4} - \frac{x \sin(3x) \cos(x) \cos(2x)}{4} - \frac{x \sin(4x) \cos(x) \cos(3x)}{4} + \frac{x \sin(5x) \cos(x) \cos(2x)}{4} - \frac{x \sin(6x) \cos(x) \cos(3x)}{4} + \frac{x \sin(7x) \cos(x) \cos(2x)}{4} - \frac{x \sin(8x) \cos(x) \cos(3x)}{4} + \frac{x \sin(9x) \cos(x) \cos(2x)}{4} - \frac{x \sin(10x) \cos(x) \cos(3x)}{4} + \frac{x \sin(11x) \cos(x) \cos(2x)}{4} - \frac{x \sin(12x) \cos(x) \cos(3x)}{4} + \frac{x \sin(13x) \cos(x) \cos(2x)}{4} - \frac{x \sin(14x) \cos(x) \cos(3x)}{4} + \frac{x \sin(15x) \cos(x) \cos(2x)}{4} - \frac{x \sin(16x) \cos(x) \cos(3x)}{4} + \frac{x \sin(17x) \cos(x) \cos(2x)}{4} - \frac{x \sin(18x) \cos(x) \cos(3x)}{4} + \frac{x \sin(19x) \cos(x) \cos(2x)}{4} - \frac{x \sin(20x) \cos(x) \cos(3x)}{4} + \frac{x \sin(21x) \cos(x) \cos(2x)}{4} - \frac{x \sin(22x) \cos(x) \cos(3x)}{4} + \frac{x \sin(23x) \cos(x) \cos(2x)}{4} - \frac{x \sin(24x) \cos(x) \cos(3x)}{4} + \frac{x \sin(25x) \cos(x) \cos(2x)}{4} - \frac{x \sin(26x) \cos(x) \cos(3x)}{4} + \frac{x \sin(27x) \cos(x) \cos(2x)}{4} - \frac{x \sin(28x) \cos(x) \cos(3x)}{4} + \frac{x \sin(29x) \cos(x) \cos(2x)}{4} - \frac{x \sin(30x) \cos(x) \cos(3x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*cos(3*x)*sin(x), x)

[Out] $x*\sin(x)*\sin(2*x)*\sin(3*x)/4 + x*\sin(x)*\cos(2*x)*\cos(3*x)/4 + x*\sin(2*x)*\cos(x)*\cos(3*x)/4 - x*\sin(3*x)*\cos(x)*\cos(2*x)/4 - \sin(x)*\sin(2*x)*\cos(3*x)/8 + \sin(x)*\sin(3*x)*\cos(2*x)/3 - \sin(2*x)*\sin(3*x)*\cos(x)/24$

3.884 $\int x \sin(x^2) dx$

Optimal. Leaf size=8

$$-\frac{1}{2} \cos(x^2)$$

[Out] -1/2*cos(x^2)

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3379, 2638}

$$-\frac{1}{2} \cos(x^2)$$

Antiderivative was successfully verified.

[In] Int[x*Sin[x^2],x]

[Out] -Cos[x^2]/2

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3379

Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned} \int x \sin(x^2) dx &= \frac{1}{2} \text{Subst} \left(\int \sin(x) dx, x, x^2 \right) \\ &= -\frac{1}{2} \cos(x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$-\frac{1}{2} \cos(x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sin[x^2],x]

[Out] -1/2*Cos[x^2]

fricas [A] time = 0.83, size = 6, normalized size = 0.75

$$-\frac{1}{2} \cos(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sin(x^2),x, algorithm="fricas")

[Out] $-1/2*\cos(x^2)$

giac [A] time = 0.13, size = 6, normalized size = 0.75

$$-\frac{1}{2} \cos(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x^2),x, algorithm="giac")`

[Out] $-1/2*\cos(x^2)$

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$-\frac{\cos(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*sin(x^2),x)`

[Out] $-1/2*\cos(x^2)$

maxima [A] time = 0.31, size = 6, normalized size = 0.75

$$-\frac{1}{2} \cos(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x^2),x, algorithm="maxima")`

[Out] $-1/2*\cos(x^2)$

mupad [B] time = 0.05, size = 6, normalized size = 0.75

$$-\frac{\cos(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*sin(x^2),x)`

[Out] $-\cos(x^2)/2$

sympy [A] time = 0.16, size = 7, normalized size = 0.88

$$-\frac{\cos(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sin(x**2),x)`

[Out] $-\cos(x**2)/2$

$$3.885 \quad \int (-\cos(x) + \sin(x))(\cos(x) + \sin(x))^5 dx$$

Optimal. Leaf size=11

$$-\frac{1}{6}(\sin(x) + \cos(x))^6$$

[Out] -1/6*(cos(x)+sin(x))^6

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3145}

$$-\frac{1}{6}(\sin(x) + \cos(x))^6$$

Antiderivative was successfully verified.

[In] Int[(-Cos[x] + Sin[x])*(Cos[x] + Sin[x])^5,x]

[Out] -(Cos[x] + Sin[x])^6/6

Rule 3145

Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_.)*(cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((c*B - b*C)*(b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(b^2 + c^2)), x] /; FreeQ[{b, c, d, e, B, C}, x] && NeQ[n, -1] && NeQ[b^2 + c^2, 0] && EqQ[b*B + c*C, 0]

Rubi steps

$$\int (-\cos(x) + \sin(x))(\cos(x) + \sin(x))^5 dx = -\frac{1}{6}(\cos(x) + \sin(x))^6$$

Mathematica [B] time = 0.08, size = 25, normalized size = 2.27

$$-\frac{5}{8} \sin(2x) + \frac{1}{24} \sin(6x) + \frac{1}{4} \cos(4x)$$

Antiderivative was successfully verified.

[In] Integrate[(-Cos[x] + Sin[x])*(Cos[x] + Sin[x])^5,x]

[Out] Cos[4*x]/4 - (5*Sin[2*x])/8 + Sin[6*x]/24

fricas [B] time = 0.63, size = 34, normalized size = 3.09

$$2 \cos(x)^4 - 2 \cos(x)^2 + \frac{1}{3} (4 \cos(x)^5 - 4 \cos(x)^3 - 3 \cos(x)) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x, algorithm="fricas")

[Out] 2*cos(x)^4 - 2*cos(x)^2 + 1/3*(4*cos(x)^5 - 4*cos(x)^3 - 3*cos(x))*sin(x)

giac [B] time = 0.13, size = 19, normalized size = 1.73

$$\frac{1}{4} \cos(4x) + \frac{1}{24} \sin(6x) - \frac{5}{8} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x, algorithm="giac")

[Out] 1/4*cos(4*x) + 1/24*sin(6*x) - 5/8*sin(2*x)

maple [B] time = 0.08, size = 97, normalized size = 8.82

$$\frac{\left(\sin^5(x) + \frac{5(\sin^3(x))}{4} + \frac{15\sin(x)}{8}\right)\cos(x)}{6} + \frac{2(\sin^6(x))}{3} - \frac{5(\cos^3(x))(\sin^3(x))}{6} - \frac{5(\cos^3(x))\sin(x)}{8} + \frac{5\cos(x)\sin(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x)

[Out] -1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+2/3*sin(x)^6-5/6*cos(x)^3*sin(x)^3-5/8*cos(x)^3*sin(x)+5/16*cos(x)*sin(x)+5/6*cos(x)^5*sin(x)-5/24*(cos(x)^3+3/2*cos(x))*sin(x)+2/3*cos(x)^6-1/6*(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)

maxima [A] time = 0.33, size = 9, normalized size = 0.82

$$-\frac{1}{6}(\cos(x) + \sin(x))^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x, algorithm="maxima")

[Out] -1/6*(cos(x) + sin(x))^6

mupad [B] time = 3.19, size = 20, normalized size = 1.82

$$\frac{\sin(2x) (\sin(2x)^2 + 3 \sin(2x) + 3)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(cos(x) + sin(x))^5*(cos(x) - sin(x)),x)

[Out] -(sin(2*x)*(3*sin(2*x) + sin(2*x)^2 + 3))/6

sympy [B] time = 1.66, size = 46, normalized size = 4.18

$$\frac{2\sin^6(x)}{3} - \sin^5(x)\cos(x) - \frac{10\sin^3(x)\cos^3(x)}{3} - \sin(x)\cos^5(x) + \frac{2\cos^6(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sin(x))*(cos(x)+sin(x))**5,x)

[Out] 2*sin(x)**6/3 - sin(x)**5*cos(x) - 10*sin(x)**3*cos(x)**3/3 - sin(x)*cos(x)**5 + 2*cos(x)**6/3

3.886 $\int 2x \sec^2(x) \tan(x) dx$

Optimal. Leaf size=11

$$x \sec^2(x) - \tan(x)$$

[Out] x*sec(x)^2-tan(x)

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {12, 3757, 3767, 8}

$$x \sec^2(x) - \tan(x)$$

Antiderivative was successfully verified.

[In] Int[2*x*Sec[x]^2*Tan[x],x]

[Out] x*Sec[x]^2 - Tan[x]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 3757

Int[(x_)^(m_)*Sec[(a_.) + (b_.)*(x_)^(n_)]^(p_)*Tan[(a_.) + (b_.)*(x_)^(n_)]^(q_), x_Symbol] := Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x] - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int 2x \sec^2(x) \tan(x) dx &= 2 \int x \sec^2(x) \tan(x) dx \\ &= x \sec^2(x) - \int \sec^2(x) dx \\ &= x \sec^2(x) + \text{Subst}\left(\int 1 dx, x, -\tan(x)\right) \\ &= x \sec^2(x) - \tan(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.64

$$2 \left(\frac{1}{2} x \sec^2(x) - \frac{\tan(x)}{2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[2*x*Sec[x]^2*Tan[x],x]

[Out] 2*((x*Sec[x]^2)/2 - Tan[x]/2)

fricas [A] time = 0.81, size = 15, normalized size = 1.36

$$-\frac{\cos(x)\sin(x) - x}{\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*x*sec(x)^2*tan(x),x, algorithm="fricas")

[Out] -(cos(x)*sin(x) - x)/cos(x)^2

giac [B] time = 0.13, size = 52, normalized size = 4.73

$$\frac{x \tan\left(\frac{1}{2}x\right)^4 + 2x \tan\left(\frac{1}{2}x\right)^2 + 2 \tan\left(\frac{1}{2}x\right)^3 + x - 2 \tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^4 - 2 \tan\left(\frac{1}{2}x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*x*sec(x)^2*tan(x),x, algorithm="giac")

[Out] (x*tan(1/2*x)^4 + 2*x*tan(1/2*x)^2 + 2*tan(1/2*x)^3 + x - 2*tan(1/2*x))/(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)

maple [A] time = 0.04, size = 12, normalized size = 1.09

$$\frac{x}{\cos(x)^2} - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2*x*sec(x)^2*tan(x),x)

[Out] x/cos(x)^2-tan(x)

maxima [B] time = 0.32, size = 133, normalized size = 12.09

$$\frac{2(4x \cos(2x)^2 + 4x \sin(2x)^2 + (2x \cos(2x) + \sin(2x)) \cos(4x) + 2x \cos(2x) + (2x \sin(2x) - \cos(2x) - \sin(2x)) \sin(4x) - \sin(2x))}{2(2 \cos(2x) + 1) \cos(4x) + \cos(4x)^2 + 4 \cos(2x)^2 + \sin(4x)^2 + 4 \sin(4x) \sin(2x) + 4 \sin(2x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*x*sec(x)^2*tan(x),x, algorithm="maxima")

[Out] 2*(4*x*cos(2*x)^2 + 4*x*sin(2*x)^2 + (2*x*cos(2*x) + sin(2*x))*cos(4*x) + 2*x*cos(2*x) + (2*x*sin(2*x) - cos(2*x) - 1)*sin(4*x) - sin(2*x))/(2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)

mupad [B] time = 3.09, size = 16, normalized size = 1.45

$$\frac{2x - \sin(2x)}{2 \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x*tan(x))/cos(x)^2,x)

[Out] (2*x - sin(2*x))/(2*cos(x)^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$2 \int x \tan(x) \sec^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*sec(x)**2*tan(x), x)
```

```
[Out] 2*Integral(x*tan(x)*sec(x)**2, x)
```

$$3.887 \quad \int \frac{1+\cos^2(x)}{1+\cos(2x)} dx$$

Optimal. Leaf size=12

$$\frac{x}{2} + \frac{\tan(x)}{2}$$

[Out] 1/2*x+1/2*tan(x)

Rubi [A] time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {388, 203}

$$\frac{x}{2} + \frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x]^2)/(1 + Cos[2*x]),x]

[Out] x/2 + Tan[x]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+\cos^2(x)}{1+\cos(2x)} dx &= \text{Subst} \left(\int \frac{2+x^2}{2+2x^2} dx, x, \tan(x) \right) \\ &= \frac{\tan(x)}{2} + \text{Subst} \left(\int \frac{1}{2+2x^2} dx, x, \tan(x) \right) \\ &= \frac{x}{2} + \frac{\tan(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 1.00

$$\frac{x}{2} + \frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Cos[x]^2)/(1 + Cos[2*x]),x]

[Out] x/2 + Tan[x]/2

fricas [A] time = 1.09, size = 13, normalized size = 1.08

$$\frac{x \cos(x) + \sin(x)}{2 \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)^2)/(1+cos(2*x)),x, algorithm="fricas")

[Out] 1/2*(x*cos(x) + sin(x))/cos(x)

giac [A] time = 0.15, size = 8, normalized size = 0.67

$$\frac{1}{2}x + \frac{1}{2}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)^2)/(1+cos(2*x)),x, algorithm="giac")

[Out] 1/2*x + 1/2*tan(x)

maple [A] time = 0.12, size = 9, normalized size = 0.75

$$\frac{x}{2} + \frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+cos(x)^2)/(1+cos(2*x)),x)

[Out] 1/2*x+1/2*tan(x)

maxima [B] time = 0.31, size = 18, normalized size = 1.50

$$\frac{1}{2}x + \frac{\sin(2x)}{2(\cos(2x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)^2)/(1+cos(2*x)),x, algorithm="maxima")

[Out] 1/2*x + 1/2*sin(2*x)/(cos(2*x) + 1)

mupad [B] time = 2.93, size = 8, normalized size = 0.67

$$\frac{x}{2} + \frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x)^2 + 1)/(cos(2*x) + 1),x)

[Out] x/2 + tan(x)/2

sympy [A] time = 1.41, size = 7, normalized size = 0.58

$$\frac{x}{2} + \frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)**2)/(1+cos(2*x)),x)

[Out] x/2 + tan(x)/2

$$3.888 \quad \int \frac{\sin(x)}{\cos^3(x) - \cos^5(x)} dx$$

Optimal. Leaf size=12

$$\frac{\tan^2(x)}{2} + \log(\tan(x))$$

[Out] $\ln(\tan(x)) + 1/2 * \tan(x)^2$

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.42, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4335, 266, 44}

$$\frac{\sec^2(x)}{2} + \log(\sin(x)) - \log(\cos(x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sin}[x]/(\text{Cos}[x]^3 - \text{Cos}[x]^5), x]$

[Out] $-\text{Log}[\text{Cos}[x]] + \text{Log}[\text{Sin}[x]] + \text{Sec}[x]^2/2$

Rule 44

$\text{Int}[(a_ + (b_ \cdot (x_))^{(m_)} \cdot ((c_) + (d_) \cdot (x_))^{(n_)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot x)^m \cdot (c + d \cdot x)^n, x], x] /;$ FreeQ[{a, b, c, d}, x] & & NeQ[b \cdot c - a \cdot d, 0] & & ILtQ[m, 0] & & IntegerQ[n] & & !(IGtQ[n, 0] & & LtQ[m + n + 2, 0])

Rule 266

$\text{Int}[(x_)^{(m_)} \cdot ((a_) + (b_ \cdot (x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) \cdot (a + b \cdot x)^p, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] & & IntegerQ[Simplify[(m + 1)/n]]

Rule 4335

$\text{Int}[(u_) \cdot (F_) \cdot ((c_) \cdot ((a_) + (b_ \cdot (x_))), x_Symbol] \rightarrow \text{With}[\{d = \text{FreeFactors}[\text{Cos}[c \cdot (a + b \cdot x)], x]\}, -\text{Dist}[d/(b \cdot c), \text{Subst}[\text{Int}[\text{SubstFor}[1, \text{Cos}[c \cdot (a + b \cdot x)]]/d, u, x], x], x, \text{Cos}[c \cdot (a + b \cdot x)]/d, x] /;$ FunctionOfQ[Cos[c \cdot (a + b \cdot x)]/d, u, x, True] /; FreeQ[{a, b, c}, x] & & (EqQ[F, Sin] || EqQ[F, sin])

Rubi steps

$$\begin{aligned} \int \frac{\sin(x)}{\cos^3(x) - \cos^5(x)} dx &= -\text{Subst} \left(\int \frac{1}{x^3(1-x^2)} dx, x, \cos(x) \right) \\ &= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)x^2} dx, x, \cos^2(x) \right) \right) \\ &= -\left(\frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{1-x} + \frac{1}{x^2} + \frac{1}{x} \right) dx, x, \cos^2(x) \right) \right) \\ &= -\log(\cos(x)) + \log(\sin(x)) + \frac{\sec^2(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.42

$$\frac{\sec^2(x)}{2} + \log(\sin(x)) - \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/(Cos[x]^3 - Cos[x]^5), x]

[Out] -Log[Cos[x]] + Log[Sin[x]] + Sec[x]^2/2

fricas [B] time = 0.93, size = 33, normalized size = 2.75

$$\frac{\cos(x)^2 \log(\cos(x)^2) - \cos(x)^2 \log\left(-\frac{1}{4} \cos(x)^2 + \frac{1}{4}\right) - 1}{2 \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(cos(x)^3-cos(x)^5), x, algorithm="fricas")

[Out] -1/2*(cos(x)^2*log(cos(x)^2) - cos(x)^2*log(-1/4*cos(x)^2 + 1/4) - 1)/cos(x)^2

giac [B] time = 0.14, size = 24, normalized size = 2.00

$$\frac{1}{2 \cos(x)^2} + \frac{1}{2} \log(-\cos(x)^2 + 1) - \log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(cos(x)^3-cos(x)^5), x, algorithm="giac")

[Out] 1/2/cos(x)^2 + 1/2*log(-cos(x)^2 + 1) - log(abs(cos(x)))

maple [B] time = 0.08, size = 27, normalized size = 2.25

$$\frac{1}{2 \cos(x)^2} - \ln(\cos(x)) + \frac{\ln(-1 + \cos(x))}{2} + \frac{\ln(1 + \cos(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(cos(x)^3-cos(x)^5), x)

[Out] 1/2/cos(x)^2-ln(cos(x))+1/2*ln(-1+cos(x))+1/2*ln(1+cos(x))

maxima [B] time = 0.31, size = 26, normalized size = 2.17

$$\frac{1}{2 \cos(x)^2} + \frac{1}{2} \log(\cos(x) + 1) + \frac{1}{2} \log(\cos(x) - 1) - \log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(cos(x)^3-cos(x)^5), x, algorithm="maxima")

[Out] 1/2/cos(x)^2 + 1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1) - log(cos(x))

mupad [B] time = 0.09, size = 19, normalized size = 1.58

$$\frac{\ln(\sin(x)^2)}{2} - \ln(\cos(x)) + \frac{1}{2 \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)/(cos(x)^3 - cos(x)^5), x)

[Out] log(sin(x)^2)/2 - log(cos(x)) + 1/(2*cos(x)^2)

sympy [B] time = 1.49, size = 29, normalized size = 2.42

$$\frac{\log(\cos(x) - 1)}{2} + \frac{\log(\cos(x) + 1)}{2} - \log(\cos(x)) + \frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(cos(x)**3-cos(x)**5),x)

[Out] log(cos(x) - 1)/2 + log(cos(x) + 1)/2 - log(cos(x)) + 1/(2*cos(x)**2)

$$3.889 \quad \int \sec(x) \left(5 - 11 \sec^5(x)\right)^2 \tan(x) dx$$

Optimal. Leaf size=19

$$11 \sec^{11}(x) - \frac{55 \sec^6(x)}{3} + 25 \sec(x)$$

[Out] 25*sec(x)-55/3*sec(x)^6+11*sec(x)^11

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4339, 270}

$$11 \sec^{11}(x) - \frac{55 \sec^6(x)}{3} + 25 \sec(x)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]*(5 - 11*Sec[x]^5)^2*Tan[x], x]

[Out] 25*Sec[x] - (55*Sec[x]^6)/3 + 11*Sec[x]^11

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 4339

Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(a + b*x)]]/d, u, x], x], x, Cos[c*(a + b*x)]/d, x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])

Rubi steps

$$\begin{aligned} \int \sec(x) \left(5 - 11 \sec^5(x)\right)^2 \tan(x) dx &= -\text{Subst} \left(\int \frac{(11 - 5x^5)^2}{x^{12}} dx, x, \cos(x) \right) \\ &= -\text{Subst} \left(\int \left(\frac{121}{x^{12}} - \frac{110}{x^7} + \frac{25}{x^2} \right) dx, x, \cos(x) \right) \\ &= 25 \sec(x) - \frac{55 \sec^6(x)}{3} + 11 \sec^{11}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$11 \sec^{11}(x) - \frac{55 \sec^6(x)}{3} + 25 \sec(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]*(5 - 11*Sec[x]^5)^2*Tan[x], x]

[Out] 25*Sec[x] - (55*Sec[x]^6)/3 + 11*Sec[x]^11

fricas [A] time = 1.07, size = 20, normalized size = 1.05

$$\frac{75 \cos(x)^{10} - 55 \cos(x)^5 + 33}{3 \cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*(5-11*sec(x)^5)^2*tan(x),x, algorithm="fricas")

[Out] 1/3*(75*cos(x)^10 - 55*cos(x)^5 + 33)/cos(x)^11

giac [A] time = 0.14, size = 20, normalized size = 1.05

$$\frac{75 \cos(x)^{10} - 55 \cos(x)^5 + 33}{3 \cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*(5-11*sec(x)^5)^2*tan(x),x, algorithm="giac")

[Out] 1/3*(75*cos(x)^10 - 55*cos(x)^5 + 33)/cos(x)^11

maple [A] time = 0.05, size = 18, normalized size = 0.95

$$25 \sec(x) - \frac{55 (\sec^6(x))}{3} + 11 (\sec^{11}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)*(5-11*sec(x)^5)^2*tan(x),x)

[Out] 25*sec(x)-55/3*sec(x)^6+11*sec(x)^11

maxima [A] time = 0.31, size = 20, normalized size = 1.05

$$\frac{75 \cos(x)^{10} - 55 \cos(x)^5 + 33}{3 \cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*(5-11*sec(x)^5)^2*tan(x),x, algorithm="maxima")

[Out] 1/3*(75*cos(x)^10 - 55*cos(x)^5 + 33)/cos(x)^11

mupad [B] time = 3.67, size = 19, normalized size = 1.00

$$\frac{25 \cos(x)^{10} - \frac{55 \cos(x)^5}{3} + 11}{\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(x)*(11/cos(x)^5 - 5)^2)/cos(x),x)

[Out] (25*cos(x)^10 - (55*cos(x)^5)/3 + 11)/cos(x)^11

sympy [A] time = 24.47, size = 19, normalized size = 1.00

$$11 \sec^{11}(x) - \frac{55 \sec^6(x)}{3} + 25 \sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*(5-11*sec(x)**5)**2*tan(x),x)

[Out] 11*sec(x)**11 - 55*sec(x)**6/3 + 25*sec(x)

3.890 $\int \sin^3(5x) \tan^3(5x) dx$

Optimal. Leaf size=44

$$\frac{1}{6} \sin^3(5x) + \frac{1}{2} \sin(5x) + \frac{1}{10} \sin^3(5x) \tan^2(5x) - \frac{1}{2} \tanh^{-1}(\sin(5x))$$

[Out] $-1/2*\operatorname{arctanh}(\sin(5*x))+1/2*\sin(5*x)+1/6*\sin(5*x)^3+1/10*\sin(5*x)^3*\tan(5*x)^2$

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2592, 288, 302, 206}

$$\frac{1}{6} \sin^3(5x) + \frac{1}{2} \sin(5x) + \frac{1}{10} \sin^3(5x) \tan^2(5x) - \frac{1}{2} \tanh^{-1}(\sin(5x))$$

Antiderivative was successfully verified.

[In] Int[Sin[5*x]^3*Tan[5*x]^3,x]

[Out] $-\operatorname{ArcTanh}[\sin[5*x]]/2 + \sin[5*x]/2 + \sin[5*x]^3/6 + (\sin[5*x]^3*\tan[5*x]^2)/10$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1)/(b*n*(p+1)), x] - Dist[(c^n*(m-n+1))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !IntegerQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a+b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n-1]

Rule 2592

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := With[{ff = FreeFactors[Sin[e+f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m+n)/(a^2-ff^2*x^2)^((n+1)/2), x], x, (a*SIN[e+f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n+1)/2]

Rubi steps

$$\begin{aligned}
\int \sin^3(5x) \tan^3(5x) dx &= \frac{1}{5} \text{Subst} \left(\int \frac{x^6}{(1-x^2)^2} dx, x, \sin(5x) \right) \\
&= \frac{1}{10} \sin^3(5x) \tan^2(5x) - \frac{1}{2} \text{Subst} \left(\int \frac{x^4}{1-x^2} dx, x, \sin(5x) \right) \\
&= \frac{1}{10} \sin^3(5x) \tan^2(5x) - \frac{1}{2} \text{Subst} \left(\int \left(-1 - x^2 + \frac{1}{1-x^2} \right) dx, x, \sin(5x) \right) \\
&= \frac{1}{2} \sin(5x) + \frac{1}{6} \sin^3(5x) + \frac{1}{10} \sin^3(5x) \tan^2(5x) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sin(5x) \right) \\
&= -\frac{1}{2} \tanh^{-1}(\sin(5x)) + \frac{1}{2} \sin(5x) + \frac{1}{6} \sin^3(5x) + \frac{1}{10} \sin^3(5x) \tan^2(5x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 1.18

$$-\frac{1}{15} \sin^3(5x) \tan^2(5x) - \frac{1}{3} \sin(5x) \tan^2(5x) - \frac{1}{2} \tanh^{-1}(\sin(5x)) + \frac{1}{2} \tan(5x) \sec(5x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[5*x]^3*Tan[5*x]^3,x]

[Out] -1/2*ArcTanh[Sin[5*x]] + (Sec[5*x]*Tan[5*x])/2 - (Sin[5*x]*Tan[5*x]^2)/3 - (Sin[5*x]^3*Tan[5*x]^2)/15

fricas [A] time = 0.92, size = 65, normalized size = 1.48

$$\frac{15 \cos(5x)^2 \log(\sin(5x) + 1) - 15 \cos(5x)^2 \log(-\sin(5x) + 1) + 2(2 \cos(5x)^4 - 14 \cos(5x)^2 - 3) \sin(5x)}{60 \cos(5x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(5*x)^3*tan(5*x)^3,x, algorithm="fricas")

[Out] -1/60*(15*cos(5*x)^2*log(sin(5*x) + 1) - 15*cos(5*x)^2*log(-sin(5*x) + 1) + 2*(2*cos(5*x)^4 - 14*cos(5*x)^2 - 3)*sin(5*x))/cos(5*x)^2

giac [A] time = 0.30, size = 51, normalized size = 1.16

$$\frac{1}{15} \sin^3(5x) - \frac{\sin(5x)}{10(\sin^2(5x) - 1)} - \frac{1}{4} \log(\sin(5x) + 1) + \frac{1}{4} \log(-\sin(5x) + 1) + \frac{2}{5} \sin(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(5*x)^3*tan(5*x)^3,x, algorithm="giac")

[Out] 1/15*sin(5*x)^3 - 1/10*sin(5*x)/(sin(5*x)^2 - 1) - 1/4*log(sin(5*x) + 1) + 1/4*log(-sin(5*x) + 1) + 2/5*sin(5*x)

maple [A] time = 0.11, size = 50, normalized size = 1.14

$$\frac{\sin^7(5x)}{10 \cos(5x)^2} + \frac{(\sin^5(5x))}{10} + \frac{(\sin^3(5x))}{6} + \frac{\sin(5x)}{2} - \frac{\ln(\sec(5x) + \tan(5x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(5*x)^3*tan(5*x)^3,x)

[Out] $1/10*\sin(5*x)^7/\cos(5*x)^2+1/10*\sin(5*x)^5+1/6*\sin(5*x)^3+1/2*\sin(5*x)-1/2*\ln(\sec(5*x)+\tan(5*x))$

maxima [A] time = 0.31, size = 49, normalized size = 1.11

$$\frac{1}{15} \sin(5x)^3 - \frac{\sin(5x)}{10(\sin(5x)^2 - 1)} - \frac{1}{4} \log(\sin(5x) + 1) + \frac{1}{4} \log(\sin(5x) - 1) + \frac{2}{5} \sin(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(5*x)^3*tan(5*x)^3,x, algorithm="maxima")`

[Out] $1/15*\sin(5*x)^3 - 1/10*\sin(5*x)/(\sin(5*x)^2 - 1) - 1/4*\log(\sin(5*x) + 1) + 1/4*\log(\sin(5*x) - 1) + 2/5*\sin(5*x)$

mupad [B] time = 3.11, size = 69, normalized size = 1.57

$$\frac{5 \tan\left(\frac{5x}{2}\right)^9 + \frac{20 \tan\left(\frac{5x}{2}\right)^7}{3} - \frac{22 \tan\left(\frac{5x}{2}\right)^5}{3} + \frac{20 \tan\left(\frac{5x}{2}\right)^3}{3} + 5 \tan\left(\frac{5x}{2}\right)}{5 \left(\tan\left(\frac{5x}{2}\right)^2 - 1\right)^2 \left(\tan\left(\frac{5x}{2}\right)^2 + 1\right)^3} - \operatorname{atanh}\left(\tan\left(\frac{5x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(5*x)^3*tan(5*x)^3,x)`

[Out] $(5*\tan((5*x)/2) + (20*\tan((5*x)/2)^3)/3 - (22*\tan((5*x)/2)^5)/3 + (20*\tan((5*x)/2)^7)/3 + 5*\tan((5*x)/2)^9)/(5*(\tan((5*x)/2)^2 - 1)^2*(\tan((5*x)/2)^2 + 1)^3) - \operatorname{atanh}(\tan((5*x)/2))$

sympy [A] time = 0.11, size = 51, normalized size = 1.16

$$\frac{\log(\sin(5x) - 1)}{4} - \frac{\log(\sin(5x) + 1)}{4} + \frac{\sin^3(5x)}{15} + \frac{2 \sin(5x)}{5} - \frac{\sin(5x)}{5(2 \sin^2(5x) - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(5*x)**3*tan(5*x)**3,x)`

[Out] $\log(\sin(5*x) - 1)/4 - \log(\sin(5*x) + 1)/4 + \sin(5*x)**3/15 + 2*\sin(5*x)/5 - \sin(5*x)/(5*(2*\sin(5*x)**2 - 2))$

3.891 $\int \sin^3(5x) \tan^4(5x) dx$

Optimal. Leaf size=37

$$\frac{1}{15} \cos^3(5x) - \frac{3}{5} \cos(5x) + \frac{1}{15} \sec^3(5x) - \frac{3}{5} \sec(5x)$$

[Out] $-3/5*\cos(5*x)+1/15*\cos(5*x)^3-3/5*\sec(5*x)+1/15*\sec(5*x)^3$

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2590, 270}

$$\frac{1}{15} \cos^3(5x) - \frac{3}{5} \cos(5x) + \frac{1}{15} \sec^3(5x) - \frac{3}{5} \sec(5x)$$

Antiderivative was successfully verified.

[In] Int[Sin[5*x]^3*Tan[5*x]^4,x]

[Out] $(-3*\cos[5*x])/5 + \cos[5*x]^3/15 - (3*\sec[5*x])/5 + \sec[5*x]^3/15$

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2590

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rubi steps

$$\begin{aligned} \int \sin^3(5x) \tan^4(5x) dx &= - \left(\frac{1}{5} \text{Subst} \left(\int \frac{(1-x^2)^3}{x^4} dx, x, \cos(5x) \right) \right) \\ &= - \left(\frac{1}{5} \text{Subst} \left(\int \left(3 + \frac{1}{x^4} - \frac{3}{x^2} - x^2 \right) dx, x, \cos(5x) \right) \right) \\ &= -\frac{3}{5} \cos(5x) + \frac{1}{15} \cos^3(5x) - \frac{3}{5} \sec(5x) + \frac{1}{15} \sec^3(5x) \end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.95

$$-\frac{11}{20} \cos(5x) + \frac{1}{60} \cos(15x) + \frac{1}{15} \sec^3(5x) - \frac{3}{5} \sec(5x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[5*x]^3*Tan[5*x]^4,x]

[Out] $(-11*\cos[5*x])/20 + \cos[15*x]/60 - (3*\sec[5*x])/5 + \sec[5*x]^3/15$

fricas [A] time = 1.85, size = 32, normalized size = 0.86

$$\frac{\cos(5x)^6 - 9 \cos(5x)^4 - 9 \cos(5x)^2 + 1}{15 \cos(5x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(5*x)^3*tan(5*x)^4,x, algorithm="fricas")

[Out] 1/15*(cos(5*x)^6 - 9*cos(5*x)^4 - 9*cos(5*x)^2 + 1)/cos(5*x)^3

giac [A] time = 0.76, size = 33, normalized size = 0.89

$$\frac{1}{15} \cos(5x)^3 - \frac{9 \cos(5x)^2 - 1}{15 \cos(5x)^3} - \frac{3}{5} \cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(5*x)^3*tan(5*x)^4,x, algorithm="giac")

[Out] 1/15*cos(5*x)^3 - 1/15*(9*cos(5*x)^2 - 1)/cos(5*x)^3 - 3/5*cos(5*x)

maple [B] time = 0.11, size = 60, normalized size = 1.62

$$\frac{\sin^8(5x)}{15 \cos(5x)^3} - \frac{\sin^8(5x)}{3 \cos(5x)} - \frac{\left(\frac{16}{5} + \sin^6(5x) + \frac{6(\sin^4(5x))}{5} + \frac{8(\sin^2(5x))}{5}\right) \cos(5x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(5*x)^3*tan(5*x)^4,x)

[Out] 1/15*sin(5*x)^8/cos(5*x)^3-1/3*sin(5*x)^8/cos(5*x)-1/3*(16/5+sin(5*x)^6+6/5*sin(5*x)^4+8/5*sin(5*x)^2)*cos(5*x)

maxima [A] time = 0.31, size = 33, normalized size = 0.89

$$\frac{1}{15} \cos(5x)^3 - \frac{9 \cos(5x)^2 - 1}{15 \cos(5x)^3} - \frac{3}{5} \cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(5*x)^3*tan(5*x)^4,x, algorithm="maxima")

[Out] 1/15*cos(5*x)^3 - 1/15*(9*cos(5*x)^2 - 1)/cos(5*x)^3 - 3/5*cos(5*x)

mupad [B] time = 3.10, size = 30, normalized size = 0.81

$$\frac{(\cos(5x) + 1)^4 (\cos(5x)^2 - 4 \cos(5x) + 1)}{15 \cos(5x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(5*x)^3*tan(5*x)^4,x)

[Out] ((cos(5*x) + 1)^4*(cos(5*x)^2 - 4*cos(5*x) + 1))/(15*cos(5*x)^3)

sympy [A] time = 0.09, size = 34, normalized size = 0.92

$$\frac{1 - 9 \cos^2(5x)}{15 \cos^3(5x)} + \frac{\cos^3(5x)}{15} - \frac{3 \cos(5x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(5*x)**3*tan(5*x)**4,x)

[Out] (1 - 9*cos(5*x)**2)/(15*cos(5*x)**3) + cos(5*x)**3/15 - 3*cos(5*x)/5

3.892 $\int \sin^5(6x) \tan^3(6x) dx$

Optimal. Leaf size=54

$$\frac{7}{60} \sin^5(6x) + \frac{7}{36} \sin^3(6x) + \frac{7}{12} \sin(6x) + \frac{1}{12} \sin^5(6x) \tan^2(6x) - \frac{7}{12} \tanh^{-1}(\sin(6x))$$

[Out] $-7/12*\operatorname{arctanh}(\sin(6*x))+7/12*\sin(6*x)+7/36*\sin(6*x)^3+7/60*\sin(6*x)^5+1/12*\sin(6*x)^5*\tan(6*x)^2$

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2592, 288, 302, 206}

$$\frac{7}{60} \sin^5(6x) + \frac{7}{36} \sin^3(6x) + \frac{7}{12} \sin(6x) + \frac{1}{12} \sin^5(6x) \tan^2(6x) - \frac{7}{12} \tanh^{-1}(\sin(6x))$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sin}[6*x]^5*\operatorname{Tan}[6*x]^3,x]$

[Out] $(-7*\operatorname{ArcTanh}[\operatorname{Sin}[6*x]])/12 + (7*\operatorname{Sin}[6*x])/12 + (7*\operatorname{Sin}[6*x]^3)/36 + (7*\operatorname{Sin}[6*x]^5)/60 + (\operatorname{Sin}[6*x]^5*\operatorname{Tan}[6*x]^2)/12$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 288

$\operatorname{Int}[(c_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow \operatorname{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*n*(p+1)), x] - \operatorname{Dist}[(c^{(n*(m-n+1))})/(b*n*(p+1)), \operatorname{Int}[(c*x)^{(m-n)}*(a + b*x^n)^{(p+1)}, x], x] /;$ FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !IntegerQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 302

$\operatorname{Int}[(x_)^{(m_)} / ((a_ + (b_)*(x_)^{(n_)}), x_Symbol] \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[x^m, a + b*x^n, x], x] /;$ FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n-1]

Rule 2592

$\operatorname{Int}[(a_)*\sin[(e_ + (f_)*(x_))]^{(m_)}*\tan[(e_ + (f_)*(x_))]^{(n_)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Sin}[e + f*x], x]\}, \operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(ff*x)^{(m+n)} / (a^2 - ff^2*x^2)^{(n+1)/2}, x], x, (a*\operatorname{Sin}[e + f*x])/ff], x] /;$ FreeQ[{a, e, f, m}, x] && IntegerQ[(n+1)/2]

Rubi steps

$$\begin{aligned}
\int \sin^5(6x) \tan^3(6x) dx &= \frac{1}{6} \text{Subst} \left(\int \frac{x^8}{(1-x^2)^2} dx, x, \sin(6x) \right) \\
&= \frac{1}{12} \sin^5(6x) \tan^2(6x) - \frac{7}{12} \text{Subst} \left(\int \frac{x^6}{1-x^2} dx, x, \sin(6x) \right) \\
&= \frac{1}{12} \sin^5(6x) \tan^2(6x) - \frac{7}{12} \text{Subst} \left(\int \left(-1 - x^2 - x^4 + \frac{1}{1-x^2} \right) dx, x, \sin(6x) \right) \\
&= \frac{7}{12} \sin(6x) + \frac{7}{36} \sin^3(6x) + \frac{7}{60} \sin^5(6x) + \frac{1}{12} \sin^5(6x) \tan^2(6x) - \frac{7}{12} \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sin(6x) \right) \\
&= -\frac{7}{12} \tanh^{-1}(\sin(6x)) + \frac{7}{12} \sin(6x) + \frac{7}{36} \sin^3(6x) + \frac{7}{60} \sin^5(6x) + \frac{1}{12} \sin^5(6x) \tan^2(6x)
\end{aligned}$$

Mathematica [A] time = 0.10, size = 68, normalized size = 1.26

$$-\frac{1}{30} \sin^5(6x) \tan^2(6x) - \frac{7}{90} \sin^3(6x) \tan^2(6x) - \frac{7}{18} \sin(6x) \tan^2(6x) - \frac{7}{12} \tanh^{-1}(\sin(6x)) + \frac{7}{12} \tan(6x) \sec(6x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[6*x]^5*Tan[6*x]^3,x]

[Out] (-7*ArcTanh[Sin[6*x]])/12 + (7*Sec[6*x]*Tan[6*x])/12 - (7*Sine[6*x]*Tan[6*x]^2)/18 - (7*Sine[6*x]^3*Tan[6*x]^2)/90 - (Sine[6*x]^5*Tan[6*x]^2)/30

fricas [A] time = 0.93, size = 73, normalized size = 1.35

$$\frac{105 \cos(6x)^2 \log(\sin(6x) + 1) - 105 \cos(6x)^2 \log(-\sin(6x) + 1) - 2(6 \cos(6x)^6 - 32 \cos(6x)^4 + 116 \cos(6x)^2 + 15) \sin(6x)}{360 \cos(6x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(6*x)^5*tan(6*x)^3,x, algorithm="fricas")

[Out] -1/360*(105*cos(6*x)^2*log(sin(6*x) + 1) - 105*cos(6*x)^2*log(-sin(6*x) + 1) - 2*(6*cos(6*x)^6 - 32*cos(6*x)^4 + 116*cos(6*x)^2 + 15)*sin(6*x))/cos(6*x)^2

giac [A] time = 0.42, size = 59, normalized size = 1.09

$$\frac{1}{30} \sin(6x)^5 + \frac{1}{9} \sin(6x)^3 - \frac{\sin(6x)}{12(\sin(6x)^2 - 1)} - \frac{7}{24} \log(\sin(6x) + 1) + \frac{7}{24} \log(-\sin(6x) + 1) + \frac{1}{2} \sin(6x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(6*x)^5*tan(6*x)^3,x, algorithm="giac")

[Out] 1/30*sin(6*x)^5 + 1/9*sin(6*x)^3 - 1/12*sin(6*x)/(sin(6*x)^2 - 1) - 7/24*log(sin(6*x) + 1) + 7/24*log(-sin(6*x) + 1) + 1/2*sin(6*x)

maple [A] time = 0.12, size = 58, normalized size = 1.07

$$\frac{\sin^9(6x)}{12 \cos(6x)^2} + \frac{(\sin^7(6x))}{12} + \frac{7(\sin^5(6x))}{60} + \frac{7(\sin^3(6x))}{36} + \frac{7 \sin(6x)}{12} - \frac{7 \ln(\sec(6x) + \tan(6x))}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(6*x)^5*tan(6*x)^3,x)

[Out] $1/12*\sin(6*x)^9/\cos(6*x)^2+1/12*\sin(6*x)^7+7/60*\sin(6*x)^5+7/36*\sin(6*x)^3+7/12*\sin(6*x)-7/12*\ln(\sec(6*x)+\tan(6*x))$

maxima [A] time = 0.31, size = 57, normalized size = 1.06

$$\frac{1}{30} \sin(6x)^5 + \frac{1}{9} \sin(6x)^3 - \frac{\sin(6x)}{12(\sin(6x)^2 - 1)} - \frac{7}{24} \log(\sin(6x) + 1) + \frac{7}{24} \log(\sin(6x) - 1) + \frac{1}{2} \sin(6x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(6*x)^5*tan(6*x)^3,x, algorithm="maxima")

[Out] $1/30*\sin(6*x)^5 + 1/9*\sin(6*x)^3 - 1/12*\sin(6*x)/(\sin(6*x)^2 - 1) - 7/24*\log(\sin(6*x) + 1) + 7/24*\log(\sin(6*x) - 1) + 1/2*\sin(6*x)$

mupad [B] time = 7.21, size = 85, normalized size = 1.57

$$\frac{7 \tan(3x)^{13} + \frac{70 \tan(3x)^{11}}{3} + \frac{77 \tan(3x)^9}{5} - \frac{412 \tan(3x)^7}{15} + \frac{77 \tan(3x)^5}{5} + \frac{70 \tan(3x)^3}{3} + 7 \tan(3x) - \frac{7 \operatorname{atanh}(\tan(3x))}{6}}{6(\tan(3x)^2 - 1)^2(\tan(3x)^2 + 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(6*x)^5*tan(6*x)^3,x)

[Out] $(7*\tan(3*x) + (70*\tan(3*x)^3)/3 + (77*\tan(3*x)^5)/5 - (412*\tan(3*x)^7)/15 + (77*\tan(3*x)^9)/5 + (70*\tan(3*x)^11)/3 + 7*\tan(3*x)^13)/(6*(\tan(3*x)^2 - 1)^2*(\tan(3*x)^2 + 1)^5) - (7*\operatorname{atanh}(\tan(3*x)))/6$

sympy [A] time = 0.11, size = 61, normalized size = 1.13

$$\frac{7 \log(\sin(6x) - 1)}{24} - \frac{7 \log(\sin(6x) + 1)}{24} + \frac{\sin^5(6x)}{30} + \frac{\sin^3(6x)}{9} + \frac{\sin(6x)}{2} - \frac{\sin(6x)}{6(2 \sin^2(6x) - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(6*x)**5*tan(6*x)**3,x)

[Out] $7*\log(\sin(6*x) - 1)/24 - 7*\log(\sin(6*x) + 1)/24 + \sin(6*x)**5/30 + \sin(6*x)**3/9 + \sin(6*x)/2 - \sin(6*x)/(6*(2*\sin(6*x)**2 - 2))$

$$3.893 \quad \int (-1 + \sec^2(2x))^3 \sin(2x) dx$$

Optimal. Leaf size=37

$$\frac{1}{2} \cos(2x) + \frac{1}{10} \sec^5(2x) - \frac{1}{2} \sec^3(2x) + \frac{3}{2} \sec(2x)$$

[Out] 1/2*cos(2*x)+3/2*sec(2*x)-1/2*sec(2*x)^3+1/10*sec(2*x)^5

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4120, 2590, 270}

$$\frac{1}{2} \cos(2x) + \frac{1}{10} \sec^5(2x) - \frac{1}{2} \sec^3(2x) + \frac{3}{2} \sec(2x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + Sec[2*x]^2)^3*Sin[2*x], x]

[Out] Cos[2*x]/2 + (3*Sec[2*x])/2 - Sec[2*x]^3/2 + Sec[2*x]^5/10

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2590

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rule 4120

Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (-1 + \sec^2(2x))^3 \sin(2x) dx &= \int \sin(2x) \tan^6(2x) dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{(1-x^2)^3}{x^6} dx, x, \cos(2x)\right)\right) \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \left(-1 + \frac{1}{x^6} - \frac{3}{x^4} + \frac{3}{x^2}\right) dx, x, \cos(2x)\right)\right) \\ &= \frac{1}{2} \cos(2x) + \frac{3}{2} \sec(2x) - \frac{1}{2} \sec^3(2x) + \frac{1}{10} \sec^5(2x) \end{aligned}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 1.00

$$\frac{1}{2} \cos(2x) + \frac{1}{10} \sec^5(2x) - \frac{1}{2} \sec^3(2x) + \frac{3}{2} \sec(2x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + Sec[2*x]^2)^3*Sin[2*x], x]

[Out] Cos[2*x]/2 + (3*Sec[2*x])/2 - Sec[2*x]^3/2 + Sec[2*x]^5/10

fricas [A] time = 0.41, size = 34, normalized size = 0.92

$$\frac{5 \cos(2x)^6 + 15 \cos(2x)^4 - 5 \cos(2x)^2 + 1}{10 \cos(2x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sec(2*x)^2)^3*sin(2*x), x, algorithm="fricas")

[Out] 1/10*(5*cos(2*x)^6 + 15*cos(2*x)^4 - 5*cos(2*x)^2 + 1)/cos(2*x)^5

giac [A] time = 0.14, size = 33, normalized size = 0.89

$$\frac{15 \cos(2x)^4 - 5 \cos(2x)^2 + 1}{10 \cos(2x)^5} + \frac{1}{2} \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sec(2*x)^2)^3*sin(2*x), x, algorithm="giac")

[Out] 1/10*(15*cos(2*x)^4 - 5*cos(2*x)^2 + 1)/cos(2*x)^5 + 1/2*cos(2*x)

maple [A] time = 0.10, size = 32, normalized size = 0.86

$$\frac{1}{10 \cos(2x)^5} - \frac{1}{2 \cos(2x)^3} + \frac{3}{2 \cos(2x)} + \frac{\cos(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+sec(2*x)^2)^3*sin(2*x), x)

[Out] 1/10/cos(2*x)^5-1/2/cos(2*x)^3+3/2/cos(2*x)+1/2*cos(2*x)

maxima [A] time = 0.30, size = 31, normalized size = 0.84

$$\frac{3}{2 \cos(2x)} - \frac{1}{2 \cos(2x)^3} + \frac{1}{10 \cos(2x)^5} + \frac{1}{2} \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sec(2*x)^2)^3*sin(2*x), x, algorithm="maxima")

[Out] 3/2/cos(2*x) - 1/2/cos(2*x)^3 + 1/10/cos(2*x)^5 + 1/2*cos(2*x)

mupad [B] time = 2.94, size = 33, normalized size = 0.89

$$\frac{\cos(2x)}{2} + \frac{3 \cos(2x)^4 - \cos(2x)^2 + \frac{1}{5}}{2 \cos(2x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)*(1/cos(2*x)^2 - 1)^3, x)

[Out] cos(2*x)/2 + (3*cos(2*x)^4 - cos(2*x)^2 + 1/5)/(2*cos(2*x)^5)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sec(2*x)**2)**3*sin(2*x), x)

[Out] Timed out

3.894 $\int \sin(x) \tan^5(x) dx$

Optimal. Leaf size=34

$$-\frac{15 \sin(x)}{8} + \frac{1}{4} \sin(x) \tan^4(x) - \frac{5}{8} \sin(x) \tan^2(x) + \frac{15}{8} \tanh^{-1}(\sin(x))$$

[Out] 15/8*arctanh(sin(x))-15/8*sin(x)-5/8*sin(x)*tan(x)^2+1/4*sin(x)*tan(x)^4

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2592, 288, 321, 206}

$$-\frac{15 \sin(x)}{8} + \frac{1}{4} \sin(x) \tan^4(x) - \frac{5}{8} \sin(x) \tan^2(x) + \frac{15}{8} \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Tan[x]^5,x]

[Out] (15*ArcTanh[Sin[x]])/8 - (15*Sin[x])/8 - (5*Sin[x]*Tan[x]^2)/8 + (Sin[x]*Tan[x]^4)/4

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*n*(p+1)), x] - Dist[(c^n*(m-n+1))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !LtQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2592

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)^(n_.)], x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m+n)/(a^2 - ff^2*x^2)^((n+1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n+1)/2]

Rubi steps

$$\begin{aligned}
\int \sin(x) \tan^5(x) dx &= \text{Subst} \left(\int \frac{x^6}{(1-x^2)^3} dx, x, \sin(x) \right) \\
&= \frac{1}{4} \sin(x) \tan^4(x) - \frac{5}{4} \text{Subst} \left(\int \frac{x^4}{(1-x^2)^2} dx, x, \sin(x) \right) \\
&= -\frac{5}{8} \sin(x) \tan^2(x) + \frac{1}{4} \sin(x) \tan^4(x) + \frac{15}{8} \text{Subst} \left(\int \frac{x^2}{1-x^2} dx, x, \sin(x) \right) \\
&= -\frac{15 \sin(x)}{8} - \frac{5}{8} \sin(x) \tan^2(x) + \frac{1}{4} \sin(x) \tan^4(x) + \frac{15}{8} \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sin(x) \right) \\
&= \frac{15}{8} \tanh^{-1}(\sin(x)) - \frac{15 \sin(x)}{8} - \frac{5}{8} \sin(x) \tan^2(x) + \frac{1}{4} \sin(x) \tan^4(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.24

$$-\sin(x) \tan^4(x) + \frac{15}{8} \tanh^{-1}(\sin(x)) - \frac{15}{4} \tan(x) \sec^3(x) + 5 \tan^3(x) \sec(x) + \frac{15}{8} \tan(x) \sec(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]*Tan[x]^5,x]

[Out] (15*ArcTanh[Sin[x]])/8 + (15*Sec[x]*Tan[x])/8 - (15*Sec[x]^3*Tan[x])/4 + 5*Sec[x]*Tan[x]^3 - Sin[x]*Tan[x]^4

fricas [A] time = 0.97, size = 49, normalized size = 1.44

$$\frac{15 \cos(x)^4 \log(\sin(x) + 1) - 15 \cos(x)^4 \log(-\sin(x) + 1) - 2(8 \cos(x)^4 + 9 \cos(x)^2 - 2) \sin(x)}{16 \cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(x)^5,x, algorithm="fricas")

[Out] 1/16*(15*cos(x)^4*log(sin(x) + 1) - 15*cos(x)^4*log(-sin(x) + 1) - 2*(8*cos(x)^4 + 9*cos(x)^2 - 2)*sin(x))/cos(x)^4

giac [A] time = 0.15, size = 42, normalized size = 1.24

$$\frac{9 \sin(x)^3 - 7 \sin(x)}{8(\sin(x)^2 - 1)^2} + \frac{15}{16} \log(\sin(x) + 1) - \frac{15}{16} \log(-\sin(x) + 1) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)*tan(x)^5,x, algorithm="giac")

[Out] 1/8*(9*sin(x)^3 - 7*sin(x))/(sin(x)^2 - 1)^2 + 15/16*log(sin(x) + 1) - 15/16*log(-sin(x) + 1) - sin(x)

maple [A] time = 0.06, size = 46, normalized size = 1.35

$$\frac{\sin^7(x)}{4 \cos(x)^4} - \frac{3(\sin^7(x))}{8 \cos(x)^2} - \frac{3(\sin^5(x))}{8} - \frac{5(\sin^3(x))}{8} - \frac{15 \sin(x)}{8} + \frac{15 \ln(\sec(x) + \tan(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*tan(x)^5,x)

[Out] $1/4*\sin(x)^7/\cos(x)^4-3/8*\sin(x)^7/\cos(x)^2-3/8*\sin(x)^5-5/8*\sin(x)^3-15/8*\sin(x)+15/8*\ln(\sec(x)+\tan(x))$

maxima [A] time = 0.31, size = 46, normalized size = 1.35

$$\frac{9 \sin(x)^3 - 7 \sin(x)}{8(\sin(x)^4 - 2 \sin(x)^2 + 1)} + \frac{15}{16} \log(\sin(x) + 1) - \frac{15}{16} \log(\sin(x) - 1) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)*tan(x)^5,x, algorithm="maxima")`

[Out] $1/8*(9*\sin(x)^3 - 7*\sin(x))/(\sin(x)^4 - 2*\sin(x)^2 + 1) + 15/16*\log(\sin(x) + 1) - 15/16*\log(\sin(x) - 1) - \sin(x)$

mupad [B] time = 3.04, size = 69, normalized size = 2.03

$$\frac{15 \operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{4} - \frac{\frac{15 \tan\left(\frac{x}{2}\right)^9}{4} - 10 \tan\left(\frac{x}{2}\right)^7 + \frac{9 \tan\left(\frac{x}{2}\right)^5}{2} - 10 \tan\left(\frac{x}{2}\right)^3 + \frac{15 \tan\left(\frac{x}{2}\right)}{4}}{\left(\tan\left(\frac{x}{2}\right)^2 - 1\right)^4 \left(\tan\left(\frac{x}{2}\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)*tan(x)^5,x)`

[Out] $(15*\operatorname{atanh}(\tan(x/2)))/4 - ((15*\tan(x/2))/4 - 10*\tan(x/2)^3 + (9*\tan(x/2)^5)/2 - 10*\tan(x/2)^7 + (15*\tan(x/2)^9)/4)/((\tan(x/2)^2 - 1)^4*(\tan(x/2)^2 + 1))$

sympy [A] time = 0.15, size = 49, normalized size = 1.44

$$-\frac{-9 \sin^3(x) + 7 \sin(x)}{8 \sin^4(x) - 16 \sin^2(x) + 8} - \frac{15 \log(\sin(x) - 1)}{16} + \frac{15 \log(\sin(x) + 1)}{16} - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)*tan(x)**5,x)`

[Out] $-(-9*\sin(x)**3 + 7*\sin(x))/(8*\sin(x)**4 - 16*\sin(x)**2 + 8) - 15*\log(\sin(x) - 1)/16 + 15*\log(\sin(x) + 1)/16 - \sin(x)$

3.895 $\int \cos^5(2x) \cot^4(2x) dx$

Optimal. Leaf size=43

$$\frac{1}{10} \sin^5(2x) - \frac{2}{3} \sin^3(2x) + 3 \sin(2x) - \frac{1}{6} \csc^3(2x) + 2 \csc(2x)$$

[Out] 2*csc(2*x)-1/6*csc(2*x)^3+3*sin(2*x)-2/3*sin(2*x)^3+1/10*sin(2*x)^5

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2590, 270}

$$\frac{1}{10} \sin^5(2x) - \frac{2}{3} \sin^3(2x) + 3 \sin(2x) - \frac{1}{6} \csc^3(2x) + 2 \csc(2x)$$

Antiderivative was successfully verified.

[In] Int[Cos[2*x]^5*Cot[2*x]^4,x]

[Out] 2*Csc[2*x] - Csc[2*x]^3/6 + 3*Sin[2*x] - (2*Sin[2*x]^3)/3 + Sin[2*x]^5/10

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2590

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rubi steps

$$\begin{aligned} \int \cos^5(2x) \cot^4(2x) dx &= - \left(\frac{1}{2} \text{Subst} \left(\int \frac{(1-x^2)^4}{x^4} dx, x, -\sin(2x) \right) \right) \\ &= - \left(\frac{1}{2} \text{Subst} \left(\int \left(6 + \frac{1}{x^4} - \frac{4}{x^2} - 4x^2 + x^4 \right) dx, x, -\sin(2x) \right) \right) \\ &= 2 \csc(2x) - \frac{1}{6} \csc^3(2x) + 3 \sin(2x) - \frac{2}{3} \sin^3(2x) + \frac{1}{10} \sin^5(2x) \end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 1.00

$$\frac{1}{10} \sin^5(2x) - \frac{2}{3} \sin^3(2x) + 3 \sin(2x) - \frac{1}{6} \csc^3(2x) + 2 \csc(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[2*x]^5*Cot[2*x]^4,x]

[Out] 2*Csc[2*x] - Csc[2*x]^3/6 + 3*Sin[2*x] - (2*Sin[2*x]^3)/3 + Sin[2*x]^5/10

fricas [A] time = 0.96, size = 52, normalized size = 1.21

$$\frac{3 \cos(2x)^8 + 8 \cos(2x)^6 + 48 \cos(2x)^4 - 192 \cos(2x)^2 + 128}{30 (\cos(2x)^2 - 1) \sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)^5*cot(2*x)^4,x, algorithm="fricas")

[Out] -1/30*(3*cos(2*x)^8 + 8*cos(2*x)^6 + 48*cos(2*x)^4 - 192*cos(2*x)^2 + 128)/((cos(2*x)^2 - 1)*sin(2*x))

giac [A] time = 0.16, size = 41, normalized size = 0.95

$$\frac{1}{10} \sin(2x)^5 - \frac{2}{3} \sin(2x)^3 + \frac{12 \sin(2x)^2 - 1}{6 \sin(2x)^3} + 3 \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)^5*cot(2*x)^4,x, algorithm="giac")

[Out] 1/10*sin(2*x)^5 - 2/3*sin(2*x)^3 + 1/6*(12*sin(2*x)^2 - 1)/sin(2*x)^3 + 3*sin(2*x)

maple [A] time = 0.17, size = 68, normalized size = 1.58

$$-\frac{\cos^{10}(2x)}{6 \sin(2x)^3} + \frac{7(\cos^{10}(2x))}{6 \sin(2x)} + \frac{7\left(\frac{128}{35} + \cos^8(2x) + \frac{8(\cos^6(2x))}{7} + \frac{48(\cos^4(2x))}{35} + \frac{64(\cos^2(2x))}{35}\right) \sin(2x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)^5*cot(2*x)^4,x)

[Out] -1/6/sin(2*x)^3*cos(2*x)^10+7/6/sin(2*x)*cos(2*x)^10+7/6*(128/35+cos(2*x)^8+8/7*cos(2*x)^6+48/35*cos(2*x)^4+64/35*cos(2*x)^2)*sin(2*x)

maxima [A] time = 0.31, size = 41, normalized size = 0.95

$$\frac{1}{10} \sin(2x)^5 - \frac{2}{3} \sin(2x)^3 + \frac{12 \sin(2x)^2 - 1}{6 \sin(2x)^3} + 3 \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)^5*cot(2*x)^4,x, algorithm="maxima")

[Out] 1/10*sin(2*x)^5 - 2/3*sin(2*x)^3 + 1/6*(12*sin(2*x)^2 - 1)/sin(2*x)^3 + 3*sin(2*x)

mupad [B] time = 3.06, size = 42, normalized size = 0.98

$$\frac{3 \sin(2x)^8 - 20 \sin(2x)^6 + 90 \sin(2x)^4 + 60 \sin(2x)^2 - 5}{30 \sin(2x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)^5*cot(2*x)^4,x)

[Out] (60*sin(2*x)^2 + 90*sin(2*x)^4 - 20*sin(2*x)^6 + 3*sin(2*x)^8 - 5)/(30*sin(2*x)^3)

sympy [A] time = 0.09, size = 42, normalized size = 0.98

$$\frac{12 \sin^2(2x) - 1}{6 \sin^3(2x)} + \frac{\sin^5(2x)}{10} - \frac{2 \sin^3(2x)}{3} + 3 \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)**5*cot(2*x)**4,x)

[Out] (12*sin(2*x)**2 - 1)/(6*sin(2*x)**3) + sin(2*x)**5/10 - 2*sin(2*x)**3/3 + 3*sin(2*x)

$$3.896 \quad \int \cos(3x) \left(-1 + \csc^2(3x)\right)^3 \left(1 - \sin^2(3x)\right)^5 dx$$

Optimal. Leaf size=87

$$\frac{1}{33} \sin^{11}(3x) - \frac{8}{27} \sin^9(3x) + \frac{4}{3} \sin^7(3x) - \frac{56}{15} \sin^5(3x) + \frac{70}{9} \sin^3(3x) - \frac{56}{3} \sin(3x) - \frac{1}{15} \csc^5(3x) + \frac{8}{9} \csc^3(3x) - \frac{28}{3} \csc(3x)$$

[Out] $-28/3*\csc(3*x)+8/9*\csc(3*x)^3-1/15*\csc(3*x)^5-56/3*\sin(3*x)+70/9*\sin(3*x)^3-56/15*\sin(3*x)^5+4/3*\sin(3*x)^7-8/27*\sin(3*x)^9+1/33*\sin(3*x)^{11}$

Rubi [A] time = 0.13, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3175, 4120, 2590, 270}

$$\frac{1}{33} \sin^{11}(3x) - \frac{8}{27} \sin^9(3x) + \frac{4}{3} \sin^7(3x) - \frac{56}{15} \sin^5(3x) + \frac{70}{9} \sin^3(3x) - \frac{56}{3} \sin(3x) - \frac{1}{15} \csc^5(3x) + \frac{8}{9} \csc^3(3x) - \frac{28}{3} \csc(3x)$$

Antiderivative was successfully verified.

[In] `Int[Cos[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^5,x]`

[Out] $(-28*\text{Csc}[3*x])/3 + (8*\text{Csc}[3*x]^3)/9 - \text{Csc}[3*x]^5/15 - (56*\text{Sin}[3*x])/3 + (70*\text{Sin}[3*x]^3)/9 - (56*\text{Sin}[3*x]^5)/15 + (4*\text{Sin}[3*x]^7)/3 - (8*\text{Sin}[3*x]^9)/27 + \text{Sin}[3*x]^{11}/33$

Rule 270

`Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]`

Rule 2590

`Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]`

Rule 3175

`Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]`

Rule 4120

`Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := Dist[b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]`

Rubi steps

$$\begin{aligned}
\int \cos(3x) (-1 + \csc^2(3x))^3 (1 - \sin^2(3x))^5 dx &= \int \cos^{11}(3x) (-1 + \csc^2(3x))^3 dx \\
&= \int \cos^{11}(3x) \cot^6(3x) dx \\
&= -\left(\frac{1}{3} \text{Subst}\left(\int \frac{(1-x^2)^8}{x^6} dx, x, -\sin(3x)\right)\right) \\
&= -\left(\frac{1}{3} \text{Subst}\left(\int \left(-56 + \frac{1}{x^6} - \frac{8}{x^4} + \frac{28}{x^2} + 70x^2 - 56x^4 + 28x^6 - 8x^8\right) dx, x, -\sin(3x)\right)\right) \\
&= -\frac{28}{3} \csc(3x) + \frac{8}{9} \csc^3(3x) - \frac{1}{15} \csc^5(3x) - \frac{56}{3} \sin(3x) + \frac{70}{9} \sin^3(3x) - \frac{56}{15} \sin^5(3x) + \frac{4}{3} \sin^7(3x) - \frac{8}{27} \sin^9(3x) + \frac{1}{33} \sin^{11}(3x)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 87, normalized size = 1.00

$$\frac{1}{33} \sin^{11}(3x) - \frac{8}{27} \sin^9(3x) + \frac{4}{3} \sin^7(3x) - \frac{56}{15} \sin^5(3x) + \frac{70}{9} \sin^3(3x) - \frac{56}{3} \sin(3x) - \frac{1}{15} \csc^5(3x) + \frac{8}{9} \csc^3(3x) - \frac{28}{3} \csc(3x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^5,x]

[Out] (-28*Csc[3*x])/3 + (8*Csc[3*x]^3)/9 - Csc[3*x]^5/15 - (56*Sin[3*x])/3 + (70*Sin[3*x]^3)/9 - (56*Sin[3*x]^5)/15 + (4*Sin[3*x]^7)/3 - (8*Sin[3*x]^9)/27 + Sin[3*x]^11/33

fricas [A] time = 1.05, size = 92, normalized size = 1.06

$$\frac{45 \cos(3x)^{16} + 80 \cos(3x)^{14} + 160 \cos(3x)^{12} + 384 \cos(3x)^{10} + 1280 \cos(3x)^8 + 10240 \cos(3x)^6 - 61440 \cos(3x)^4 + 81920 \cos(3x)^2 - 61440}{1485(\cos(3x)^4 - 2\cos(3x)^2 + 1)\sin(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x, algorithm="fricas")

[Out] 1/1485*(45*cos(3*x)^16 + 80*cos(3*x)^14 + 160*cos(3*x)^12 + 384*cos(3*x)^10 + 1280*cos(3*x)^8 + 10240*cos(3*x)^6 - 61440*cos(3*x)^4 + 81920*cos(3*x)^2 - 32768)/((cos(3*x)^4 - 2*cos(3*x)^2 + 1)*sin(3*x))

giac [A] time = 0.49, size = 73, normalized size = 0.84

$$\frac{1}{33} \sin(3x)^{11} - \frac{8}{27} \sin(3x)^9 + \frac{4}{3} \sin(3x)^7 - \frac{56}{15} \sin(3x)^5 + \frac{70}{9} \sin(3x)^3 - \frac{420 \sin(3x)^4 - 40 \sin(3x)^2 + 3}{45 \sin(3x)^5} - \frac{56}{3} \csc(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x, algorithm="giac")

[Out] 1/33*sin(3*x)^11 - 8/27*sin(3*x)^9 + 4/3*sin(3*x)^7 - 56/15*sin(3*x)^5 + 70/9*sin(3*x)^3 - 1/45*(420*sin(3*x)^4 - 40*sin(3*x)^2 + 3)/sin(3*x)^5 - 56/3*csc(3*x)

maple [A] time = 0.16, size = 72, normalized size = 0.83

$$\frac{\sin^{11}(3x)}{33} - \frac{8 \sin^9(3x)}{27} + \frac{4 \sin^7(3x)}{3} - \frac{56 \sin^5(3x)}{15} + \frac{70 \sin^3(3x)}{9} - \frac{56 \sin(3x)}{3} - \frac{28}{3 \sin(3x)} + \frac{8}{9 \sin^3(3x)} - \frac{1}{15 \sin^5(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x)

[Out] 1/33*sin(3*x)^11-8/27*sin(3*x)^9+4/3*sin(3*x)^7-56/15*sin(3*x)^5+70/9*sin(3*x)^3-56/3*sin(3*x)-28/3/sin(3*x)+8/9/sin(3*x)^3-1/15/sin(3*x)^5

maxima [A] time = 0.31, size = 73, normalized size = 0.84

$$\frac{1}{33} \sin(3x)^{11} - \frac{8}{27} \sin(3x)^9 + \frac{4}{3} \sin(3x)^7 - \frac{56}{15} \sin(3x)^5 + \frac{70}{9} \sin(3x)^3 - \frac{420 \sin(3x)^4 - 40 \sin(3x)^2 + 3}{45 \sin(3x)^5} - \frac{56}{3} \sin(3x) - \frac{28}{3} \frac{1}{\sin(3x)} + \frac{8}{9} \frac{1}{\sin(3x)^3} - \frac{1}{15} \frac{1}{\sin(3x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x, algorithm="maxima")

[Out] 1/33*sin(3*x)^11 - 8/27*sin(3*x)^9 + 4/3*sin(3*x)^7 - 56/15*sin(3*x)^5 + 70/9*sin(3*x)^3 - 1/45*(420*sin(3*x)^4 - 40*sin(3*x)^2 + 3)/sin(3*x)^5 - 56/3*sin(3*x)

mupad [B] time = 2.97, size = 74, normalized size = 0.85

$$\frac{-45 \sin(3x)^{16} + 440 \sin(3x)^{14} - 1980 \sin(3x)^{12} + 5544 \sin(3x)^{10} - 11550 \sin(3x)^8 + 27720 \sin(3x)^6 + 99 \sin(3x)^4 - 1320 \sin(3x)^2 - 45}{1485 \sin(3x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-cos(3*x)*(1/sin(3*x)^2 - 1)^3*(sin(3*x)^2 - 1)^5,x)

[Out] -(13860*sin(3*x)^4 - 1320*sin(3*x)^2 + 27720*sin(3*x)^6 - 11550*sin(3*x)^8 + 5544*sin(3*x)^10 - 1980*sin(3*x)^12 + 440*sin(3*x)^14 - 45*sin(3*x)^16 + 99)/(1485*sin(3*x)^5)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)*(-1+csc(3*x)**2)**3*(1-sin(3*x)**2)**5,x)

[Out] Timed out

$$3.897 \quad \int \cot(2x) \left(-1 + \csc^2(2x)\right)^2 \left(1 - \sin^2(2x)\right)^2 dx$$

Optimal. Leaf size=42

$$\frac{1}{8} \sin^4(2x) - \sin^2(2x) - \frac{1}{8} \csc^4(2x) + \csc^2(2x) + 3 \log(\sin(2x))$$

[Out] $\csc(2*x)^2 - 1/8*\csc(2*x)^4 + 3*\ln(\sin(2*x)) - \sin(2*x)^2 + 1/8*\sin(2*x)^4$

Rubi [A] time = 0.12, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3175, 4360, 266, 43}

$$\frac{1}{8} \sin^4(2x) - \sin^2(2x) - \frac{1}{8} \csc^4(2x) + \csc^2(2x) + 3 \log(\sin(2x))$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cot}[2*x]*(-1 + \text{Csc}[2*x]^2)^2*(1 - \text{Sin}[2*x]^2)^2, x]$

[Out] $\text{Csc}[2*x]^2 - \text{Csc}[2*x]^4/8 + 3*\text{Log}[\text{Sin}[2*x]] - \text{Sin}[2*x]^2 + \text{Sin}[2*x]^4/8$

Rule 43

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) || \text{LtQ}[9*m + 5*(n + 1), 0] || \text{GtQ}[m + n + 2, 0])$

Rule 266

$\text{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.))^{(n_.)}]^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p, x}], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 3175

$\text{Int}[(u_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[a^p, \text{Int}[\text{ActivateTrig}[u*\cos[e + f*x]^{(2*p)}], x], x] /; \text{FreeQ}\{a, b, e, f, p\}, x] \&\& \text{EqQ}[a + b, 0] \&\& \text{IntegerQ}[p]$

Rule 4360

$\text{Int}[(u_.)*(F_.)[(c_.)*((a_.) + (b_.)*(x_.))], x_Symbol] \rightarrow \text{With}\{d = \text{FreeFactors}[\text{Sin}[c*(a + b*x)], x]\}, \text{Dist}[1/(b*c), \text{Subst}[\text{Int}[\text{SubstFor}[1/x, \text{Sin}[c*(a + b*x)]/d, u, x], x], x, \text{Sin}[c*(a + b*x)]/d, x] /; \text{FunctionOfQ}[\text{Sin}[c*(a + b*x)]/d, u, x] /; \text{FreeQ}\{a, b, c\}, x] \&\& (\text{EqQ}[F, \text{Cot}] || \text{EqQ}[F, \text{cot}])$

Rubi steps

$$\begin{aligned}
\int \cot(2x) (-1 + \csc^2(2x))^2 (1 - \sin^2(2x))^2 dx &= \int \cos^4(2x) \cot(2x) (-1 + \csc^2(2x))^2 dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{(1-x^2)^4}{x^5} dx, x, \sin(2x) \right) \\
&= \frac{1}{4} \text{Subst} \left(\int \frac{(1-x)^4}{x^3} dx, x, \sin^2(2x) \right) \\
&= \frac{1}{4} \text{Subst} \left(\int \left(-4 + \frac{1}{x^3} - \frac{4}{x^2} + \frac{6}{x} + x \right) dx, x, \sin^2(2x) \right) \\
&= \csc^2(2x) - \frac{1}{8} \csc^4(2x) + 3 \log(\sin(2x)) - \sin^2(2x) + \frac{1}{8} \sin^4(2x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{1}{8} \sin^4(2x) - \sin^2(2x) - \frac{1}{8} \csc^4(2x) + \csc^2(2x) + 3 \log(\sin(2x))$$

Antiderivative was successfully verified.

[In] Integrate[Cot[2*x]*(-1 + Csc[2*x]^2)^2*(1 - Sin[2*x]^2)^2,x]

[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8

fricas [B] time = 0.90, size = 79, normalized size = 1.88

$$\frac{8 \cos(2x)^8 + 32 \cos(2x)^6 - 115 \cos(2x)^4 + 38 \cos(2x)^2 + 192 (\cos(2x)^4 - 2 \cos(2x)^2 + 1) \log\left(\frac{1}{2} \sin(2x)\right)}{64 (\cos(2x)^4 - 2 \cos(2x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x, algorithm="fricas")

[Out] 1/64*(8*cos(2*x)^8 + 32*cos(2*x)^6 - 115*cos(2*x)^4 + 38*cos(2*x)^2 + 192*(cos(2*x)^4 - 2*cos(2*x)^2 + 1)*log(1/2*sin(2*x)) + 29)/(cos(2*x)^4 - 2*cos(2*x)^2 + 1)

giac [A] time = 0.14, size = 52, normalized size = 1.24

$$\frac{1}{8} \cos(2x)^4 + \frac{3}{4} \cos(2x)^2 - \frac{8 \cos(2x)^2 - 7}{8 (\cos(2x)^2 - 1)^2} + \frac{3}{2} \log(-\cos(2x)^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x, algorithm="giac")

[Out] 1/8*cos(2*x)^4 + 3/4*cos(2*x)^2 - 1/8*(8*cos(2*x)^2 - 7)/(cos(2*x)^2 - 1)^2 + 3/2*log(-cos(2*x)^2 + 1)

maple [A] time = 0.16, size = 37, normalized size = 0.88

$$\frac{(\sin^4(2x))}{8} + \cos^2(2x) + 3 \ln(\sin(2x)) + \frac{1}{\sin(2x)^2} - \frac{1}{8 \sin(2x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x)`

[Out] `1/8*sin(2*x)^4+cos(2*x)^2+3*ln(sin(2*x))+1/sin(2*x)^2-1/8/sin(2*x)^4`

maxima [A] time = 0.30, size = 44, normalized size = 1.05

$$\frac{1}{8} \sin(2x)^4 - \sin(2x)^2 + \frac{8 \sin(2x)^2 - 1}{8 \sin(2x)^4} + \frac{3}{2} \log(\sin(2x)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x, algorithm="maxima")`

[Out] `1/8*sin(2*x)^4 - sin(2*x)^2 + 1/8*(8*sin(2*x)^2 - 1)/sin(2*x)^4 + 3/2*log(sin(2*x)^2)`

mupad [B] time = 3.16, size = 71, normalized size = 1.69

$$3 \ln(\tan(2x)) - \frac{3 \ln(\tan(2x)^2 + 1)}{2} + \frac{3 \tan(2x)^6 + \frac{9 \tan(2x)^4}{2} + \tan(2x)^2 - \frac{1}{4}}{2 (\tan(2x)^8 + 2 \tan(2x)^6 + \tan(2x)^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(2*x)*(1/sin(2*x)^2 - 1)^2*(sin(2*x)^2 - 1)^2,x)`

[Out] `3*log(tan(2*x)) - (3*log(tan(2*x)^2 + 1))/2 + (tan(2*x)^2 + (9*tan(2*x)^4)/2 + 3*tan(2*x)^6 - 1/4)/(2*(tan(2*x)^4 + 2*tan(2*x)^6 + tan(2*x)^8))`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(2*x)*(-1+csc(2*x)**2)**2*(1-sin(2*x)**2)**2,x)`

[Out] Timed out

$$3.898 \quad \int \cos(2x) \left(-1 + \csc^2(2x)\right)^4 \left(1 - \sin^2(2x)\right)^2 dx$$

Optimal. Leaf size=63

$$\frac{1}{10} \sin^5(2x) - \sin^3(2x) + \frac{15}{2} \sin(2x) - \frac{1}{14} \csc^7(2x) + \frac{3}{5} \csc^5(2x) - \frac{5}{2} \csc^3(2x) + 10 \csc(2x)$$

[Out] 10*csc(2*x)-5/2*csc(2*x)^3+3/5*csc(2*x)^5-1/14*csc(2*x)^7+15/2*sin(2*x)-sin(2*x)^3+1/10*sin(2*x)^5

Rubi [A] time = 0.12, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3175, 4120, 2590, 270}

$$\frac{1}{10} \sin^5(2x) - \sin^3(2x) + \frac{15}{2} \sin(2x) - \frac{1}{14} \csc^7(2x) + \frac{3}{5} \csc^5(2x) - \frac{5}{2} \csc^3(2x) + 10 \csc(2x)$$

Antiderivative was successfully verified.

[In] Int[Cos[2*x]*(-1 + Csc[2*x]^2)^4*(1 - Sin[2*x]^2)^2,x]

[Out] 10*Csc[2*x] - (5*Csc[2*x]^3)/2 + (3*Csc[2*x]^5)/5 - Csc[2*x]^7/14 + (15*Sin[2*x])/2 - Sin[2*x]^3 + Sin[2*x]^5/10

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2590

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rule 3175

Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rule 4120

Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := Dist[b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \cos(2x) (-1 + \csc^2(2x))^4 (1 - \sin^2(2x))^2 dx &= \int \cos^5(2x) (-1 + \csc^2(2x))^4 dx \\
&= \int \cos^5(2x) \cot^8(2x) dx \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{(1-x^2)^6}{x^8} dx, x, -\sin(2x) \right)\right) \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \left(15 + \frac{1}{x^8} - \frac{6}{x^6} + \frac{15}{x^4} - \frac{20}{x^2} - 6x^2 + x^4\right) dx, x, -\sin(2x) \right)\right) \\
&= 10 \csc(2x) - \frac{5}{2} \csc^3(2x) + \frac{3}{5} \csc^5(2x) - \frac{1}{14} \csc^7(2x) + \frac{15}{2} \sin(2x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.00

$$\frac{1}{10} \sin^5(2x) - \sin^3(2x) + \frac{15}{2} \sin(2x) - \frac{1}{14} \csc^7(2x) + \frac{3}{5} \csc^5(2x) - \frac{5}{2} \csc^3(2x) + 10 \csc(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[2*x]*(-1 + Csc[2*x]^2)^4*(1 - Sin[2*x]^2)^2,x]

[Out] 10*Csc[2*x] - (5*Csc[2*x]^3)/2 + (3*Csc[2*x]^5)/5 - Csc[2*x]^7/14 + (15*Sin[2*x])/2 - Sin[2*x]^3 + Sin[2*x]^5/10

fricas [A] time = 2.93, size = 84, normalized size = 1.33

$$\frac{7 \cos(2x)^{12} + 28 \cos(2x)^{10} + 280 \cos(2x)^8 - 2240 \cos(2x)^6 + 4480 \cos(2x)^4 - 3584 \cos(2x)^2 + 1024}{70 (\cos(2x)^6 - 3 \cos(2x)^4 + 3 \cos(2x)^2 - 1) \sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x, algorithm="fricas")

[Out] -1/70*(7*cos(2*x)^12 + 28*cos(2*x)^10 + 280*cos(2*x)^8 - 2240*cos(2*x)^6 + 4480*cos(2*x)^4 - 3584*cos(2*x)^2 + 1024)/((cos(2*x)^6 - 3*cos(2*x)^4 + 3*cos(2*x)^2 - 1)*sin(2*x))

giac [A] time = 0.16, size = 57, normalized size = 0.90

$$\frac{1}{10} \sin(2x)^5 - \sin(2x)^3 + \frac{700 \sin(2x)^6 - 175 \sin(2x)^4 + 42 \sin(2x)^2 - 5}{70 \sin(2x)^7} + \frac{15}{2} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x, algorithm="giac")

[Out] 1/10*sin(2*x)^5 - sin(2*x)^3 + 1/70*(700*sin(2*x)^6 - 175*sin(2*x)^4 + 42*sin(2*x)^2 - 5)/sin(2*x)^7 + 15/2*sin(2*x)

maple [A] time = 0.15, size = 56, normalized size = 0.89

$$\frac{(\sin^5(2x))}{10} - (\sin^3(2x)) + \frac{15 \sin(2x)}{2} + \frac{10}{\sin(2x)} - \frac{5}{2 \sin(2x)^3} + \frac{3}{5 \sin(2x)^5} - \frac{1}{14 \sin(2x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x)

[Out] $1/10*\sin(2*x)^5 - \sin(2*x)^3 + 15/2*\sin(2*x) + 10/\sin(2*x) - 5/2/\sin(2*x)^3 + 3/5/\sin(2*x)^5 - 1/14/\sin(2*x)^7$

maxima [A] time = 0.31, size = 57, normalized size = 0.90

$$\frac{1}{10} \sin(2x)^5 - \sin(2x)^3 + \frac{700 \sin(2x)^6 - 175 \sin(2x)^4 + 42 \sin(2x)^2 - 5}{70 \sin(2x)^7} + \frac{15}{2} \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x, algorithm="maxima")

[Out] $1/10*\sin(2*x)^5 - \sin(2*x)^3 + 1/70*(700*\sin(2*x)^6 - 175*\sin(2*x)^4 + 42*\sin(2*x)^2 - 5)/\sin(2*x)^7 + 15/2*\sin(2*x)$

mupad [B] time = 2.97, size = 57, normalized size = 0.90

$$\frac{\frac{\sin(2x)^{12}}{10} - \sin(2x)^{10} + \frac{15\sin(2x)^8}{2} + 10\sin(2x)^6 - \frac{5\sin(2x)^4}{2} + \frac{3\sin(2x)^2}{5} - \frac{1}{14}}{\sin(2x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)*(1/sin(2*x)^2 - 1)^4*(sin(2*x)^2 - 1)^2,x)

[Out] $((3*\sin(2*x)^2)/5 - (5*\sin(2*x)^4)/2 + 10*\sin(2*x)^6 + (15*\sin(2*x)^8)/2 - \sin(2*x)^{10} + \sin(2*x)^{12}/10 - 1/14)/\sin(2*x)^7$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)*(-1+csc(2*x)**2)**4*(1-sin(2*x)**2)**2,x)

[Out] Timed out

$$3.899 \quad \int \cot(3x) \left(-1 + \csc^2(3x)\right)^3 \left(1 - \sin^2(3x)\right)^2 dx$$

Optimal. Leaf size=60

$$-\frac{1}{12} \sin^4(3x) + \frac{5}{6} \sin^2(3x) - \frac{1}{18} \csc^6(3x) + \frac{5}{12} \csc^4(3x) - \frac{5}{3} \csc^2(3x) - \frac{10}{3} \log(\sin(3x))$$

[Out] $-5/3*\csc(3*x)^2+5/12*\csc(3*x)^4-1/18*\csc(3*x)^6-10/3*\ln(\sin(3*x))+5/6*\sin(3*x)^2-1/12*\sin(3*x)^4$

Rubi [A] time = 0.13, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3175, 4360, 266, 43}

$$-\frac{1}{12} \sin^4(3x) + \frac{5}{6} \sin^2(3x) - \frac{1}{18} \csc^6(3x) + \frac{5}{12} \csc^4(3x) - \frac{5}{3} \csc^2(3x) - \frac{10}{3} \log(\sin(3x))$$

Antiderivative was successfully verified.

[In] `Int[Cot[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^2,x]`

[Out] $(-5*\csc[3*x]^2)/3 + (5*\csc[3*x]^4)/12 - \csc[3*x]^6/18 - (10*\text{Log}[\text{Sin}[3*x]])/3 + (5*\text{Sin}[3*x]^2)/6 - \text{Sin}[3*x]^4/12$

Rule 43

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

Rule 266

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]`

Rule 3175

`Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)^2])^(p_), x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]`

Rule 4360

`Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a + b*x)]]/d, u, x], x], x, Sin[c*(a + b*x)]/d, x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])`

Rubi steps

$$\begin{aligned}
\int \cot(3x) (-1 + \csc^2(3x))^3 (1 - \sin^2(3x))^2 dx &= \int \cos^4(3x) \cot(3x) (-1 + \csc^2(3x))^3 dx \\
&= \frac{1}{3} \text{Subst} \left(\int \frac{(1-x^2)^5}{x^7} dx, x, \sin(3x) \right) \\
&= \frac{1}{6} \text{Subst} \left(\int \frac{(1-x)^5}{x^4} dx, x, \sin^2(3x) \right) \\
&= \frac{1}{6} \text{Subst} \left(\int \left(5 + \frac{1}{x^4} - \frac{5}{x^3} + \frac{10}{x^2} - \frac{10}{x} - x \right) dx, x, \sin^2(3x) \right) \\
&= -\frac{5}{3} \csc^2(3x) + \frac{5}{12} \csc^4(3x) - \frac{1}{18} \csc^6(3x) - \frac{10}{3} \log(\sin(3x))
\end{aligned}$$

Mathematica [A] time = 0.13, size = 52, normalized size = 0.87

$$\frac{1}{36} (-3 \sin^4(3x) + 30 \sin^2(3x) - 2 \csc^6(3x) + 15 \csc^4(3x) - 60 \csc^2(3x) - 120 \log(\sin(3x)))$$

Antiderivative was successfully verified.

[In] Integrate[Cot[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^2,x]

[Out] (-60*Csc[3*x]^2 + 15*Csc[3*x]^4 - 2*Csc[3*x]^6 - 120*Log[Sin[3*x]] + 30*Sin[3*x]^2 - 3*Sin[3*x]^4)/36

fricas [B] time = 0.84, size = 103, normalized size = 1.72

$$\frac{24 \cos(3x)^{10} + 120 \cos(3x)^8 - 609 \cos(3x)^6 + 387 \cos(3x)^4 + 333 \cos(3x)^2 + 960 (\cos(3x)^6 - 3 \cos(3x)^4 + 3 \cos(3x)^2 - 1)}{288 (\cos(3x)^6 - 3 \cos(3x)^4 + 3 \cos(3x)^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x, algorithm="fricas")

[Out] -1/288*(24*cos(3*x)^10 + 120*cos(3*x)^8 - 609*cos(3*x)^6 + 387*cos(3*x)^4 + 333*cos(3*x)^2 + 960*(cos(3*x)^6 - 3*cos(3*x)^4 + 3*cos(3*x)^2 - 1)*log(1/2*sin(3*x)) - 271)/(cos(3*x)^6 - 3*cos(3*x)^4 + 3*cos(3*x)^2 - 1)

giac [A] time = 0.25, size = 60, normalized size = 1.00

$$-\frac{1}{12} \sin(3x)^4 + \frac{5}{6} \sin(3x)^2 + \frac{110 \sin(3x)^6 - 60 \sin(3x)^4 + 15 \sin(3x)^2 - 2}{36 \sin(3x)^6} - \frac{5}{3} \log(\sin(3x)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x, algorithm="giac")

[Out] -1/12*sin(3*x)^4 + 5/6*sin(3*x)^2 + 1/36*(110*sin(3*x)^6 - 60*sin(3*x)^4 + 15*sin(3*x)^2 - 2)/sin(3*x)^6 - 5/3*log(sin(3*x)^2)

maple [A] time = 0.16, size = 49, normalized size = 0.82

$$\frac{(\sin^4(3x))}{12} - \frac{5(\cos^2(3x))}{6} - \frac{10 \ln(\sin(3x))}{3} - \frac{5}{3 \sin(3x)^2} + \frac{5}{12 \sin(3x)^4} - \frac{1}{18 \sin(3x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x)`

[Out] $-1/12*\sin(3*x)^4-5/6*\cos(3*x)^2-10/3*\ln(\sin(3*x))-5/3/\sin(3*x)^2+5/12/\sin(3*x)^4-1/18/\sin(3*x)^6$

maxima [A] time = 0.32, size = 52, normalized size = 0.87

$$-\frac{1}{12} \sin(3x)^4 + \frac{5}{6} \sin(3x)^2 - \frac{60 \sin(3x)^4 - 15 \sin(3x)^2 + 2}{36 \sin(3x)^6} - \frac{5}{3} \log(\sin(3x)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x, algorithm="maxima")`

[Out] $-1/12*\sin(3*x)^4 + 5/6*\sin(3*x)^2 - 1/36*(60*\sin(3*x)^4 - 15*\sin(3*x)^2 + 2)/\sin(3*x)^6 - 5/3*\log(\sin(3*x)^2)$

mupad [B] time = 4.67, size = 84, normalized size = 1.40

$$\frac{\ln\left(\left(\tan(3x)^2 + 1\right)^5\right)}{3} - \frac{10 \ln(\tan(3x))}{3} - \frac{5 \tan(3x)^8 + \frac{15 \tan(3x)^6}{2} + \frac{5 \tan(3x)^4}{3} - \frac{5 \tan(3x)^2}{12} + \frac{1}{6}}{3 \left(\tan(3x)^{10} + 2 \tan(3x)^8 + \tan(3x)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cot(3*x)*(1/sin(3*x)^2 - 1)^3*(sin(3*x)^2 - 1)^2,x)`

[Out] $\log\left(\left(\tan(3*x)^2 + 1\right)^5/3 - (10*\log(\tan(3*x)))/3 - ((5*\tan(3*x)^4)/3 - (5*\tan(3*x)^2)/12 + (15*\tan(3*x)^6)/2 + 5*\tan(3*x)^8 + 1/6)/(3*(\tan(3*x)^6 + 2*\tan(3*x)^8 + \tan(3*x)^{10}))\right)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cot(3*x)*(-1+csc(3*x)**2)**3*(1-sin(3*x)**2)**2,x)`

[Out] Timed out

$$3.900 \quad \int (1 + \cot^2(9x))^2 (1 + \tan^2(9x))^3 dx$$

Optimal. Leaf size=47

$$\frac{1}{45} \tan^5(9x) + \frac{4}{27} \tan^3(9x) + \frac{2}{3} \tan(9x) - \frac{1}{27} \cot^3(9x) - \frac{4}{9} \cot(9x)$$

[Out] $-4/9*\cot(9*x)-1/27*\cot(9*x)^3+2/3*\tan(9*x)+4/27*\tan(9*x)^3+1/45*\tan(9*x)^5$

Rubi [A] time = 0.10, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3657, 2620, 270}

$$\frac{1}{45} \tan^5(9x) + \frac{4}{27} \tan^3(9x) + \frac{2}{3} \tan(9x) - \frac{1}{27} \cot^3(9x) - \frac{4}{9} \cot(9x)$$

Antiderivative was successfully verified.

[In] Int[(1 + Cot[9*x]^2)^2*(1 + Tan[9*x]^2)^3,x]

[Out] $(-4*\text{Cot}[9*x])/9 - \text{Cot}[9*x]^3/27 + (2*\text{Tan}[9*x])/3 + (4*\text{Tan}[9*x]^3)/27 + \text{Tan}[9*x]^5/45$

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2620

Int[csc[(e_.) + (f_.)*(x_)]^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]

Rule 3657

Int[(u_)*((a_) + (b_)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[ActivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a, b]

Rubi steps

$$\begin{aligned} \int (1 + \cot^2(9x))^2 (1 + \tan^2(9x))^3 dx &= \int (1 + \cot^2(9x))^2 \sec^6(9x) dx \\ &= \int \csc^4(9x) \sec^6(9x) dx \\ &= \frac{1}{9} \text{Subst} \left(\int \frac{(1 + x^2)^4}{x^4} dx, x, \tan(9x) \right) \\ &= \frac{1}{9} \text{Subst} \left(\int \left(6 + \frac{1}{x^4} + \frac{4}{x^2} + 4x^2 + x^4 \right) dx, x, \tan(9x) \right) \\ &= -\frac{4}{9} \cot(9x) - \frac{1}{27} \cot^3(9x) + \frac{2}{3} \tan(9x) + \frac{4}{27} \tan^3(9x) + \frac{1}{45} \tan^5(9x) \end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 1.26

$$\frac{73}{135} \tan(9x) - \frac{11}{27} \cot(9x) - \frac{1}{27} \cot(9x) \csc^2(9x) + \frac{1}{45} \tan(9x) \sec^4(9x) + \frac{14}{135} \tan(9x) \sec^2(9x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Cot[9*x]^2)^2*(1 + Tan[9*x]^2)^3,x]

[Out] (-11*Cot[9*x])/27 - (Cot[9*x]*Csc[9*x]^2)/27 + (73*Tan[9*x])/135 + (14*Sec[9*x]^2*Tan[9*x])/135 + (Sec[9*x]^4*Tan[9*x])/45

fricas [A] time = 0.63, size = 42, normalized size = 0.89

$$\frac{3 \tan(9x)^8 + 20 \tan(9x)^6 + 90 \tan(9x)^4 - 60 \tan(9x)^2 - 5}{135 \tan(9x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x, algorithm="fricas")

[Out] 1/135*(3*tan(9*x)^8 + 20*tan(9*x)^6 + 90*tan(9*x)^4 - 60*tan(9*x)^2 - 5)/tan(9*x)^3

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.14, size = 38, normalized size = 0.81

$$-\frac{4 \cot(9x)}{9} - \frac{(\cot^3(9x))}{27} + \frac{2 \tan(9x)}{3} + \frac{4(\tan^3(9x))}{27} + \frac{(\tan^5(9x))}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x)

[Out] -4/9*cot(9*x)-1/27*cot(9*x)^3+2/3*tan(9*x)+4/27*tan(9*x)^3+1/45*tan(9*x)^5

maxima [A] time = 0.33, size = 41, normalized size = 0.87

$$\frac{1}{45} \tan(9x)^5 + \frac{4}{27} \tan(9x)^3 - \frac{12 \tan(9x)^2 + 1}{27 \tan(9x)^3} + \frac{2}{3} \tan(9x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x, algorithm="maxima")

[Out] 1/45*tan(9*x)^5 + 4/27*tan(9*x)^3 - 1/27*(12*tan(9*x)^2 + 1)/tan(9*x)^3 + 2/3*tan(9*x)

mupad [B] time = 5.38, size = 42, normalized size = 0.89

$$\frac{3 \tan(9x)^8 + 20 \tan(9x)^6 + 90 \tan(9x)^4 - 60 \tan(9x)^2 - 5}{135 \tan(9x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tan(9*x)^2 + 1)^3*(cot(9*x)^2 + 1)^2,x)

[Out] (90*tan(9*x)^4 - 60*tan(9*x)^2 + 20*tan(9*x)^6 + 3*tan(9*x)^8 - 5)/(135*tan(9*x)^3)

sympy [A] time = 5.20, size = 44, normalized size = 0.94

$$\frac{\tan^5(9x)}{45} + \frac{4 \tan^3(9x)}{27} + \frac{2 \tan(9x)}{3} - \frac{4}{9 \tan(9x)} - \frac{1}{27 \tan^3(9x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cot(9*x)**2)**2*(1+tan(9*x)**2)**3,x)

[Out] tan(9*x)**5/45 + 4*tan(9*x)**3/27 + 2*tan(9*x)/3 - 4/(9*tan(9*x)) - 1/(27*tan(9*x)**3)

$$3.901 \quad \int \frac{\cos(x)(9-7\sin^3(x))^2}{1-\sin^2(x)} dx$$

Optimal. Leaf size=43

$$-\frac{49}{5}\sin^5(x) - \frac{49\sin^3(x)}{3} + 63\sin^2(x) - 49\sin(x) - 2\log(1-\sin(x)) + 128\log(\sin(x)+1)$$

[Out] -2*ln(1-sin(x))+128*ln(1+sin(x))-49*sin(x)+63*sin(x)^2-49/3*sin(x)^3-49/5*
in(x)^5

Rubi [A] time = 0.12, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3175, 3223, 1810, 633, 31}

$$-\frac{49}{5}\sin^5(x) - \frac{49\sin^3(x)}{3} + 63\sin^2(x) - 49\sin(x) - 2\log(1-\sin(x)) + 128\log(\sin(x)+1)$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*(9 - 7*Sin[x]^3)^2)/(1 - Sin[x]^2), x]

[Out] -2*Log[1 - Sin[x]] + 128*Log[1 + Sin[x]] - 49*Sin[x] + 63*Sin[x]^2 - (49*Sin[x]^3)/3 - (49*Sin[x]^5)/5

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 633

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 1810

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 3175

Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(p_), x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rule 3223

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m-1)/2)*(a + b*(c*ff*x)^n)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m-1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x) (9 - 7 \sin^3(x))^2}{1 - \sin^2(x)} dx &= \int \sec(x) (9 - 7 \sin^3(x))^2 dx \\
&= \text{Subst} \left(\int \frac{(9 - 7x^3)^2}{1 - x^2} dx, x, \sin(x) \right) \\
&= \text{Subst} \left(\int \left(-49 + 126x - 49x^2 - 49x^4 + \frac{2(65 - 63x)}{1 - x^2} \right) dx, x, \sin(x) \right) \\
&= -49 \sin(x) + 63 \sin^2(x) - \frac{49 \sin^3(x)}{3} - \frac{49 \sin^5(x)}{5} + 2 \text{Subst} \left(\int \frac{65 - 63x}{1 - x^2} dx, x, \sin(x) \right) \\
&= -49 \sin(x) + 63 \sin^2(x) - \frac{49 \sin^3(x)}{3} - \frac{49 \sin^5(x)}{5} + 2 \text{Subst} \left(\int \frac{1}{1 - x} dx, x, \sin(x) \right) \\
&= -2 \log(1 - \sin(x)) + 128 \log(1 + \sin(x)) - 49 \sin(x) + 63 \sin^2(x) - \frac{49 \sin^3(x)}{3} - \frac{49 \sin^5(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 71, normalized size = 1.65

$$-\frac{49}{5} \sin^5(x) - \frac{49 \sin^3(x)}{3} - 49 \sin(x) - 63 \cos^2(x) + 49 \tanh^{-1}(\sin(x)) + 126 \log(\cos(x)) - 81 \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]*(9 - 7*Sin[x]^3)^2)/(1 - Sin[x]^2),x]

[Out] 49*ArcTanh[Sin[x]] - 63*Cos[x]^2 + 126*Log[Cos[x]] - 81*Log[Cos[x/2] - Sin[x/2]] + 81*Log[Cos[x/2] + Sin[x/2]] - 49*Sin[x] - (49*Sin[x]^3)/3 - (49*Sin[x]^5)/5

fricas [A] time = 1.95, size = 41, normalized size = 0.95

$$-63 \cos(x)^2 - \frac{49}{15} (3 \cos(x)^4 - 11 \cos(x)^2 + 23) \sin(x) + 128 \log(\sin(x) + 1) - 2 \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x, algorithm="fricas")

[Out] -63*cos(x)^2 - 49/15*(3*cos(x)^4 - 11*cos(x)^2 + 23)*sin(x) + 128*log(sin(x) + 1) - 2*log(-sin(x) + 1)

giac [A] time = 6.00, size = 39, normalized size = 0.91

$$-\frac{49}{5} \sin(x)^5 - \frac{49}{3} \sin(x)^3 + 63 \sin(x)^2 + 128 \log(\sin(x) + 1) - 2 \log(-\sin(x) + 1) - 49 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x, algorithm="giac")

[Out] -49/5*sin(x)^5 - 49/3*sin(x)^3 + 63*sin(x)^2 + 128*log(sin(x) + 1) - 2*log(-sin(x) + 1) - 49*sin(x)

maple [A] time = 0.06, size = 38, normalized size = 0.88

$$-\frac{49 \sin^5(x)}{5} - \frac{49 \sin^3(x)}{3} + 63 \sin^2(x) - 49 \sin(x) - 2 \ln(\sin(x) - 1) + 128 \ln(1 + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x)`

[Out] $-49/5*\sin(x)^5-49/3*\sin(x)^3+63*\sin(x)^2-49*\sin(x)-2*\ln(\sin(x)-1)+128*\ln(1+\sin(x))$

maxima [A] time = 0.32, size = 37, normalized size = 0.86

$$-\frac{49}{5} \sin(x)^5 - \frac{49}{3} \sin(x)^3 + 63 \sin(x)^2 + 128 \log(\sin(x) + 1) - 2 \log(\sin(x) - 1) - 49 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x, algorithm="maxima")`

[Out] $-49/5*\sin(x)^5 - 49/3*\sin(x)^3 + 63*\sin(x)^2 + 128*\log(\sin(x) + 1) - 2*\log(\sin(x) - 1) - 49*\sin(x)$

mupad [B] time = 0.08, size = 37, normalized size = 0.86

$$128 \ln(\sin(x) + 1) - 2 \ln(\sin(x) - 1) - 49 \sin(x) + 63 \sin(x)^2 - \frac{49 \sin(x)^3}{3} - \frac{49 \sin(x)^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(cos(x)*(7*sin(x)^3 - 9)^2)/(sin(x)^2 - 1),x)`

[Out] $128*\log(\sin(x) + 1) - 2*\log(\sin(x) - 1) - 49*\sin(x) + 63*\sin(x)^2 - (49*\sin(x)^3)/3 - (49*\sin(x)^5)/5$

sympy [A] time = 2.95, size = 44, normalized size = 1.02

$$-2 \log(\sin(x) - 1) + 128 \log(\sin(x) + 1) - \frac{49 \sin^5(x)}{5} - \frac{49 \sin^3(x)}{3} + 63 \sin^2(x) - 49 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(9-7*sin(x)**3)**2/(1-sin(x)**2),x)`

[Out] $-2*\log(\sin(x) - 1) + 128*\log(\sin(x) + 1) - 49*\sin(x)**5/5 - 49*\sin(x)**3/3 + 63*\sin(x)**2 - 49*\sin(x)$

3.902 $\int \cos^4(2x) \cot^5(2x) dx$

Optimal. Leaf size=42

$$\frac{1}{8} \sin^4(2x) - \sin^2(2x) - \frac{1}{8} \csc^4(2x) + \csc^2(2x) + 3 \log(\sin(2x))$$

[Out] $\csc(2*x)^2 - 1/8*\csc(2*x)^4 + 3*\ln(\sin(2*x)) - \sin(2*x)^2 + 1/8*\sin(2*x)^4$

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2590, 266, 43}

$$\frac{1}{8} \sin^4(2x) - \sin^2(2x) - \frac{1}{8} \csc^4(2x) + \csc^2(2x) + 3 \log(\sin(2x))$$

Antiderivative was successfully verified.

[In] Int[Cos[2*x]^4*Cot[2*x]^5,x]

[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8

Rule 43

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 2590

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rubi steps

$$\begin{aligned} \int \cos^4(2x) \cot^5(2x) dx &= \frac{1}{2} \text{Subst} \left(\int \frac{(1-x^2)^4}{x^5} dx, x, -\sin(2x) \right) \\ &= \frac{1}{4} \text{Subst} \left(\int \frac{(1-x)^4}{x^3} dx, x, \sin^2(2x) \right) \\ &= \frac{1}{4} \text{Subst} \left(\int \left(-4 + \frac{1}{x^3} - \frac{4}{x^2} + \frac{6}{x} + x \right) dx, x, \sin^2(2x) \right) \\ &= \csc^2(2x) - \frac{1}{8} \csc^4(2x) + 3 \log(\sin(2x)) - \sin^2(2x) + \frac{1}{8} \sin^4(2x) \end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 1.00

$$\frac{1}{8} \sin^4(2x) - \sin^2(2x) - \frac{1}{8} \csc^4(2x) + \csc^2(2x) + 3 \log(\sin(2x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[2*x]^4*Cot[2*x]^5,x]

[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8

fricas [B] time = 1.96, size = 79, normalized size = 1.88

$$\frac{8 \cos(2x)^8 + 32 \cos(2x)^6 - 115 \cos(2x)^4 + 38 \cos(2x)^2 + 192 (\cos(2x)^4 - 2 \cos(2x)^2 + 1) \log\left(\frac{1}{2} \sin(2x)\right)}{64 (\cos(2x)^4 - 2 \cos(2x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)^4*cot(2*x)^5,x, algorithm="fricas")

[Out] 1/64*(8*cos(2*x)^8 + 32*cos(2*x)^6 - 115*cos(2*x)^4 + 38*cos(2*x)^2 + 192*(cos(2*x)^4 - 2*cos(2*x)^2 + 1)*log(1/2*sin(2*x)) + 29)/(cos(2*x)^4 - 2*cos(2*x)^2 + 1)

giac [A] time = 0.15, size = 52, normalized size = 1.24

$$\frac{1}{8} \cos(2x)^4 + \frac{3}{4} \cos(2x)^2 - \frac{8 \cos(2x)^2 - 7}{8 (\cos(2x)^2 - 1)^2} + \frac{3}{2} \log(-\cos(2x)^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)^4*cot(2*x)^5,x, algorithm="giac")

[Out] 1/8*cos(2*x)^4 + 3/4*cos(2*x)^2 - 1/8*(8*cos(2*x)^2 - 7)/(cos(2*x)^2 - 1)^2 + 3/2*log(-cos(2*x)^2 + 1)

maple [A] time = 0.11, size = 69, normalized size = 1.64

$$-\frac{\cos^{10}(2x)}{8 \sin(2x)^4} + \frac{3(\cos^{10}(2x))}{8 \sin(2x)^2} + \frac{3(\cos^8(2x))}{8} + \frac{(\cos^6(2x))}{2} + \frac{3(\cos^4(2x))}{4} + \frac{3(\cos^2(2x))}{2} + 3 \ln(\sin(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*x)^4*cot(2*x)^5,x)

[Out] -1/8/sin(2*x)^4*cos(2*x)^10+3/8/sin(2*x)^2*cos(2*x)^10+3/8*cos(2*x)^8+1/2*cos(2*x)^6+3/4*cos(2*x)^4+3/2*cos(2*x)^2+3*ln(sin(2*x))

maxima [A] time = 0.34, size = 44, normalized size = 1.05

$$\frac{1}{8} \sin(2x)^4 - \sin(2x)^2 + \frac{8 \sin(2x)^2 - 1}{8 \sin(2x)^4} + \frac{3}{2} \log(\sin(2x)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(2*x)^4*cot(2*x)^5,x, algorithm="maxima")

[Out] 1/8*sin(2*x)^4 - sin(2*x)^2 + 1/8*(8*sin(2*x)^2 - 1)/sin(2*x)^4 + 3/2*log(sin(2*x)^2)

mupad [B] time = 3.05, size = 71, normalized size = 1.69

$$3 \ln(\tan(2x)) - \frac{3 \ln(\tan(2x)^2 + 1)}{2} + \frac{3 \tan(2x)^6 + \frac{9 \tan(2x)^4}{2} + \tan(2x)^2 - \frac{1}{4}}{2 (\tan(2x)^8 + 2 \tan(2x)^6 + \tan(2x)^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(2*x)^4*cot(2*x)^5,x)`

[Out] $3\log(\tan(2x)) - (3\log(\tan(2x)^2 + 1))/2 + (\tan(2x)^2 + (9\tan(2x)^4)/2 + 3\tan(2x)^6 - 1/4)/(2(\tan(2x)^4 + 2\tan(2x)^6 + \tan(2x)^8))$

sympy [A] time = 0.10, size = 41, normalized size = 0.98

$$\frac{8\sin^2(2x) - 1}{8\sin^4(2x)} + 3\log(\sin(2x)) + \frac{\sin^4(2x)}{8} - \sin^2(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(2*x)**4*cot(2*x)**5,x)`

[Out] $(8\sin(2x)**2 - 1)/(8\sin(2x)**4) + 3\log(\sin(2x)) + \sin(2x)**4/8 - \sin(2x)**2$

$$3.903 \quad \int \frac{\sec(x) \tan^2(x)}{4+3 \sec(x)} dx$$

Optimal. Leaf size=74

$$\frac{\tan(x)}{3} - \frac{4}{9} \tanh^{-1}(\sin(x)) - \frac{1}{9} \sqrt{7} \log\left(\sqrt{7} \cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \frac{1}{9} \sqrt{7} \log\left(\sin\left(\frac{x}{2}\right) + \sqrt{7} \cos\left(\frac{x}{2}\right)\right)$$

[Out] $-4/9*\operatorname{arctanh}(\sin(x))-1/9*\ln(-\sin(1/2*x)+\cos(1/2*x)*7^{(1/2)})*7^{(1/2)}+1/9*\ln(\sin(1/2*x)+\cos(1/2*x)*7^{(1/2)})*7^{(1/2)}+1/3*\tan(x)$

Rubi [A] time = 0.25, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {4397, 2723, 3056, 3001, 3770, 2659, 206}

$$\frac{\tan(x)}{3} - \frac{4}{9} \tanh^{-1}(\sin(x)) - \frac{1}{9} \sqrt{7} \log\left(\sqrt{7} \cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \frac{1}{9} \sqrt{7} \log\left(\sin\left(\frac{x}{2}\right) + \sqrt{7} \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sec}[x]*\operatorname{Tan}[x]^2)/(4 + 3*\operatorname{Sec}[x]),x]$

[Out] $(-4*\operatorname{ArcTanh}[\operatorname{Sin}[x]])/9 - (\operatorname{Sqrt}[7]*\operatorname{Log}[\operatorname{Sqrt}[7]*\operatorname{Cos}[x/2] - \operatorname{Sin}[x/2]])/9 + (\operatorname{Sqrt}[7]*\operatorname{Log}[\operatorname{Sqrt}[7]*\operatorname{Cos}[x/2] + \operatorname{Sin}[x/2]])/9 + \operatorname{Tan}[x]/3$

Rule 206

$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 2659

$\operatorname{Int}[(a + (b \cdot \sin[\pi/2 + (c \cdot x) + (d \cdot x)])^{-1}, x_Symbol] \rightarrow \operatorname{With}\{e = \operatorname{FreeFactors}[\operatorname{Tan}[(c + d*x)/2], x]\}, \operatorname{Dist}[(2*e)/d, \operatorname{Subst}[\operatorname{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \operatorname{Tan}[(c + d*x)/2]/e], x] /; \operatorname{FreeQ}\{a, b, c, d, x\} \ \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 2723

$\operatorname{Int}[(a + (b \cdot \sin[(e \cdot x) + (f \cdot x)])^m)/\tan[(e \cdot x) + (f \cdot x)]^2, x_Symbol] \rightarrow \operatorname{Int}[(a + b*\sin[e + f*x])^m*(1 - \sin[e + f*x]^2)/\sin[e + f*x]^2, x] /; \operatorname{FreeQ}\{a, b, e, f, m, x\} \ \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 3001

$\operatorname{Int}[(A + (B \cdot \sin[(e \cdot x) + (f \cdot x)])/(A + (B \cdot \sin[(e \cdot x) + (f \cdot x)])*(C + (D \cdot \sin[(e \cdot x) + (f \cdot x)]))), x_Symbol] \rightarrow \operatorname{Dist}[(A*B - a*B)/(b*c - a*d), \operatorname{Int}[1/(a + b*\sin[e + f*x]), x], x] + \operatorname{Dist}[(B*c - A*d)/(b*c - a*d), \operatorname{Int}[1/(c + d*\sin[e + f*x]), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{NeQ}[a^2 - b^2, 0] \ \&\& \operatorname{NeQ}[c^2 - d^2, 0]$

Rule 3056

$\operatorname{Int}[(a + (b \cdot \sin[(e \cdot x) + (f \cdot x)])^m)*((c \cdot x) + (d \cdot \sin[(e \cdot x) + (f \cdot x)])^n)^2, x_Symbol] \rightarrow -\operatorname{Simp}[(A*b^2 + a^2*C)*\operatorname{Cos}[e + f*x]*(a + b*\sin[e + f*x])^{m+1}*(c + d*\sin[e + f*x])^{n+1}/(f*(m+1)*(b*c - a*d)*(a^2 - b^2)), x] + \operatorname{Dist}[1/((m+1)*(b*c - a*d)*(a^2 - b^2)), \operatorname{Int}[(a + b*\sin[e + f*x])^{m+1}*(c + d*\sin[e + f*x])^n*\operatorname{Simp}[a*(m+1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m+n+2) - (c*(A*b^2 + a^2*C) + b*(m+1)*(b*c - a*d)*(A + C))*\sin[e + f*x] - d*(A$

```
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 4397

```
Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(x) \tan^2(x)}{4 + 3 \sec(x)} dx &= \int \frac{\tan^2(x)}{3 + 4 \cos(x)} dx \\
&= \int \frac{(1 - \cos^2(x)) \sec^2(x)}{3 + 4 \cos(x)} dx \\
&= \frac{\tan(x)}{3} + \frac{1}{3} \int \frac{(-4 - 3 \cos(x)) \sec(x)}{3 + 4 \cos(x)} dx \\
&= \frac{\tan(x)}{3} - \frac{4}{9} \int \sec(x) dx + \frac{7}{9} \int \frac{1}{3 + 4 \cos(x)} dx \\
&= -\frac{4}{9} \tanh^{-1}(\sin(x)) + \frac{\tan(x)}{3} + \frac{14}{9} \operatorname{Subst}\left(\int \frac{1}{7 - x^2} dx, x, \tan\left(\frac{x}{2}\right)\right) \\
&= -\frac{4}{9} \tanh^{-1}(\sin(x)) - \frac{1}{9} \sqrt{7} \log\left(\sqrt{7} \cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \frac{1}{9} \sqrt{7} \log\left(\sqrt{7} \cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)
\end{aligned}$$

Mathematica [A] time = 0.08, size = 63, normalized size = 0.85

$$\frac{1}{9} \left(3 \tan(x) + 2\sqrt{7} \tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{7}}\right) + 4 \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - 4 \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x]^2)/(4 + 3*Sec[x]), x]
```

```
[Out] (2*Sqrt[7]*ArcTanh[Tan[x/2]/Sqrt[7]] + 4*Log[Cos[x/2] - Sin[x/2]] - 4*Log[Cos[x/2] + Sin[x/2]] + 3*Tan[x])/9
```

fricas [A] time = 0.83, size = 82, normalized size = 1.11

$$\frac{\sqrt{7} \cos(x) \log\left(\frac{2 \cos(x)^2 + 2(3\sqrt{7} \cos(x) + 4\sqrt{7}) \sin(x) + 24 \cos(x) + 23}{16 \cos(x)^2 + 24 \cos(x) + 9}\right) - 4 \cos(x) \log(\sin(x) + 1) + 4 \cos(x) \log(-\sin(x) + 1)}{18 \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2/(4+3*sec(x)), x, algorithm="fricas")
```

```
[Out] 1/18*(sqrt(7)*cos(x)*log((2*cos(x)^2 + 2*(3*sqrt(7)*cos(x) + 4*sqrt(7))*sin(x) + 24*cos(x) + 23)/(16*cos(x)^2 + 24*cos(x) + 9)) - 4*cos(x)*log(sin(x) + 1) + 4*cos(x)*log(-sin(x) + 1) + 6*sin(x))/cos(x)
```

giac [A] time = 0.23, size = 72, normalized size = 0.97

$$-\frac{1}{9}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+2\tan\left(\frac{1}{2}x\right)\right|}{\left|2\sqrt{7}+2\tan\left(\frac{1}{2}x\right)\right|}\right)-\frac{2\tan\left(\frac{1}{2}x\right)}{3\left(\tan\left(\frac{1}{2}x\right)^2-1\right)}-\frac{4}{9}\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)+\frac{4}{9}\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)^2/(4+3*sec(x)),x, algorithm="giac")

[Out] -1/9*sqrt(7)*log(abs(-2*sqrt(7)+2*tan(1/2*x))/abs(2*sqrt(7)+2*tan(1/2*x))) - 2/3*tan(1/2*x)/(tan(1/2*x)^2-1) - 4/9*log(abs(tan(1/2*x)+1))+4/9*log(abs(tan(1/2*x)-1))

maple [A] time = 0.07, size = 55, normalized size = 0.74

$$\frac{2\sqrt{7}\operatorname{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{7}}{7}\right)}{9}-\frac{1}{3\left(\tan\left(\frac{x}{2}\right)-1\right)}+\frac{4\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{9}-\frac{1}{3\left(1+\tan\left(\frac{x}{2}\right)\right)}-\frac{4\ln\left(1+\tan\left(\frac{x}{2}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)*tan(x)^2/(4+3*sec(x)),x)

[Out] 2/9*7^(1/2)*arctanh(1/7*tan(1/2*x)*7^(1/2))-1/3/(tan(1/2*x)-1)+4/9*ln(tan(1/2*x)-1)-1/3/(1+tan(1/2*x))-4/9*ln(1+tan(1/2*x))

maxima [A] time = 1.08, size = 91, normalized size = 1.23

$$-\frac{1}{9}\sqrt{7}\log\left(-\frac{\sqrt{7}-\frac{\sin(x)}{\cos(x)+1}}{\sqrt{7}+\frac{\sin(x)}{\cos(x)+1}}\right)-\frac{2\sin(x)}{3\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)}-\frac{4}{9}\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)+\frac{4}{9}\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)^2/(4+3*sec(x)),x, algorithm="maxima")

[Out] -1/9*sqrt(7)*log(-(sqrt(7)-sin(x)/(cos(x)+1))/(sqrt(7)+sin(x)/(cos(x)+1))) - 2/3*sin(x)/((sin(x)^2/(cos(x)+1)^2-1)*(cos(x)+1)) - 4/9*log(sin(x)/(cos(x)+1)+1)+4/9*log(sin(x)/(cos(x)+1)-1)

mupad [B] time = 3.14, size = 41, normalized size = 0.55

$$\frac{2\sqrt{7}\operatorname{atanh}\left(\frac{\sqrt{7}\tan\left(\frac{x}{2}\right)}{7}\right)}{9}-\frac{2\tan\left(\frac{x}{2}\right)}{3\left(\tan\left(\frac{x}{2}\right)^2-1\right)}-\frac{8\operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(x)^2/(cos(x)*(3/cos(x)+4)),x)

[Out] (2*7^(1/2)*atanh((7^(1/2)*tan(x/2))/7))/9 - (2*tan(x/2))/(3*(tan(x/2)^2-1)) - (8*atanh(tan(x/2)))/9

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(x)\sec(x)}{3\sec(x)+4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)*tan(x)**2/(4+3*sec(x)),x)

[Out] Integral(tan(x)**2*sec(x)/(3*sec(x)+4), x)

3.904 $\int x \sec(1+x) \tan(1+x) dx$

Optimal. Leaf size=14

$$x \sec(x+1) - \tanh^{-1}(\sin(x+1))$$

[Out] -arctanh(sin(1+x))+x*sec(1+x)

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3757, 3770}

$$x \sec(x+1) - \tanh^{-1}(\sin(x+1))$$

Antiderivative was successfully verified.

[In] Int[x*Sec[1+x]*Tan[1+x],x]

[Out] -ArcTanh[Sin[1+x]] + x*Sec[1+x]

Rule 3757

Int[(x_)^(m_)*Sec[(a_)+(b_)*(x_)^(n_)]^(p_)*Tan[(a_)+(b_)*(x_)^(n_)]^(q_), x_Symbol] := Simp[(x^(m-n+1)*Sec[a+b*x^n]^p)/(b*n*p), x] - Dist[(m-n+1)/(b*n*p), Int[x^(m-n)*Sec[a+b*x^n]^p, x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]

Rule 3770

Int[csc[(c_)+(d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c+d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int x \sec(1+x) \tan(1+x) dx &= x \sec(1+x) - \int \sec(1+x) dx \\ &= -\tanh^{-1}(\sin(1+x)) + x \sec(1+x) \end{aligned}$$

Mathematica [B] time = 0.04, size = 47, normalized size = 3.36

$$x \sec(x+1) + \log\left(\cos\left(\frac{x+1}{2}\right) - \sin\left(\frac{x+1}{2}\right)\right) - \log\left(\sin\left(\frac{x+1}{2}\right) + \cos\left(\frac{x+1}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[1+x]*Tan[1+x],x]

[Out] Log[Cos[(1+x)/2] - Sin[(1+x)/2]] - Log[Cos[(1+x)/2] + Sin[(1+x)/2]] + x*Sec[1+x]

fricas [B] time = 0.67, size = 39, normalized size = 2.79

$$\frac{\cos(x+1) \log(\sin(x+1)+1) - \cos(x+1) \log(-\sin(x+1)+1) - 2x}{2 \cos(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(1+x)*tan(1+x),x, algorithm="fricas")

[Out] $-1/2*(\cos(x + 1)*\log(\sin(x + 1) + 1) - \cos(x + 1)*\log(-\sin(x + 1) + 1) - 2*x)/\cos(x + 1)$

giac [B] time = 0.52, size = 1179, normalized size = 84.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*sec(1+x)*tan(1+x),x, algorithm="giac")`

[Out] $1/2*(2*x*\tan(1/2)^2*\tan(1/2*x)^2 + \log(2*(\tan(1/2)^2*\tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x)^3 + 2*\tan(1/2)*\tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x)^2 + \tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x) - 2*\tan(1/2*x)^3 + \tan(1/2)^2 + 2*\tan(1/2*x)^2 - 2*\tan(1/2) - 2*\tan(1/2*x) + 1)/(\tan(1/2)^2 + 1))*\tan(1/2)^2*\tan(1/2*x)^2 - \log(2*(\tan(1/2)^2*\tan(1/2*x)^4 - 2*\tan(1/2)^2*\tan(1/2*x)^3 - 2*\tan(1/2)*\tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x)^2 + \tan(1/2*x)^4 - 2*\tan(1/2)^2*\tan(1/2*x) + 2*\tan(1/2*x)^3 + \tan(1/2)^2 + 2*\tan(1/2*x)^2 + 2*\tan(1/2) + 2*\tan(1/2*x) + 1)/(\tan(1/2)^2 + 1))*\tan(1/2)^2*\tan(1/2*x)^2 + 2*x*\tan(1/2)^2 - \log(2*(\tan(1/2)^2*\tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x)^3 + 2*\tan(1/2)*\tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x)^2 + \tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x) - 2*\tan(1/2*x)^3 + \tan(1/2)^2 + 2*\tan(1/2*x)^2 - 2*\tan(1/2) - 2*\tan(1/2*x) + 1)/(\tan(1/2)^2 + 1))*\tan(1/2)^2 + \log(2*(\tan(1/2)^2*\tan(1/2*x)^4 - 2*\tan(1/2)^2*\tan(1/2*x)^3 - 2*\tan(1/2)*\tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x)^2 + \tan(1/2*x)^4 - 2*\tan(1/2)^2*\tan(1/2*x) + 2*\tan(1/2*x)^3 + \tan(1/2)^2 + 2*\tan(1/2*x)^2 + 2*\tan(1/2) + 2*\tan(1/2*x) + 1)/(\tan(1/2)^2 + 1))*\tan(1/2)^2 - 4*\log(2*(\tan(1/2)^2*\tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x)^3 + 2*\tan(1/2)*\tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x)^2 + \tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x) - 2*\tan(1/2*x)^3 + \tan(1/2)^2 + 2*\tan(1/2*x)^2 - 2*\tan(1/2) - 2*\tan(1/2*x) + 1)/(\tan(1/2)^2 + 1))*\tan(1/2)*\tan(1/2*x) + 4*\log(2*(\tan(1/2)^2*\tan(1/2*x)^4 - 2*\tan(1/2)^2*\tan(1/2*x)^3 - 2*\tan(1/2)*\tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x)^2 + \tan(1/2*x)^4 - 2*\tan(1/2)^2*\tan(1/2*x) + 2*\tan(1/2*x)^3 + \tan(1/2)^2 + 2*\tan(1/2*x)^2 + 2*\tan(1/2) + 2*\tan(1/2*x) + 1)/(\tan(1/2)^2 + 1))*\tan(1/2)*\tan(1/2*x) + 2*x*\tan(1/2*x)^2 - \log(2*(\tan(1/2)^2*\tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x)^3 + 2*\tan(1/2)*\tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x)^2 + \tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x) - 2*\tan(1/2*x)^3 + \tan(1/2)^2 + 2*\tan(1/2*x)^2 - 2*\tan(1/2) - 2*\tan(1/2*x) + 1)/(\tan(1/2)^2 + 1))*\tan(1/2*x)^2 + \log(2*(\tan(1/2)^2*\tan(1/2*x)^4 - 2*\tan(1/2)^2*\tan(1/2*x)^3 - 2*\tan(1/2)*\tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x)^2 + \tan(1/2*x)^4 - 2*\tan(1/2)^2*\tan(1/2*x) + 2*\tan(1/2*x)^3 + \tan(1/2)^2 + 2*\tan(1/2*x)^2 + 2*\tan(1/2) + 2*\tan(1/2*x) + 1)/(\tan(1/2)^2 + 1))*\tan(1/2*x)^2 + 2*x + \log(2*(\tan(1/2)^2*\tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x)^3 + 2*\tan(1/2)*\tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x)^2 + \tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x) - 2*\tan(1/2*x)^3 + \tan(1/2)^2 + 2*\tan(1/2*x)^2 - 2*\tan(1/2) - 2*\tan(1/2*x) + 1)/(\tan(1/2)^2 + 1)) - \log(2*(\tan(1/2)^2*\tan(1/2*x)^4 - 2*\tan(1/2)^2*\tan(1/2*x)^3 - 2*\tan(1/2)*\tan(1/2*x)^4 + 2*\tan(1/2)^2*\tan(1/2*x)^2 + \tan(1/2*x)^4 - 2*\tan(1/2)^2*\tan(1/2*x) + 2*\tan(1/2*x)^3 + \tan(1/2)^2 + 2*\tan(1/2*x)^2 + 2*\tan(1/2) + 2*\tan(1/2*x) + 1)/(\tan(1/2)^2 + 1)))/(\tan(1/2)^2*\tan(1/2*x)^2 - \tan(1/2)^2 - 4*\tan(1/2)*\tan(1/2*x) - \tan(1/2*x)^2 + 1)$

maple [B] time = 0.04, size = 32, normalized size = 2.29

$$\frac{1+x}{\cos(1+x)} - \ln(\sec(1+x) + \tan(1+x)) - \frac{1}{\cos(1+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*sec(1+x)*tan(1+x),x)`

[Out] $(1+x)/\cos(1+x) - \ln(\sec(1+x) + \tan(1+x)) - 1/\cos(1+x)$

maxima [B] time = 1.32, size = 176, normalized size = 12.57

$$\frac{4(x+1)\cos(2x+2)\cos(x+1) + 4(x+1)\sin(2x+2)\sin(x+1) + 4(x+1)\cos(x+1) - (\cos(2x+2))^2 + \sin(2x+2)^2}{\cos(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(1+x)*tan(1+x),x, algorithm="maxima")

[Out] 1/2*(4*(x + 1)*cos(2*x + 2)*cos(x + 1) + 4*(x + 1)*sin(2*x + 2)*sin(x + 1) + 4*(x + 1)*cos(x + 1) - (cos(2*x + 2)^2 + sin(2*x + 2)^2 + 2*cos(2*x + 2) + 1)*log(cos(x + 1)^2 + sin(x + 1)^2 + 2*sin(x + 1) + 1) + (cos(2*x + 2)^2 + sin(2*x + 2)^2 + 2*cos(2*x + 2) + 1)*log(cos(x + 1)^2 + sin(x + 1)^2 - 2*sin(x + 1) + 1))/(cos(2*x + 2)^2 + sin(2*x + 2)^2 + 2*cos(2*x + 2) + 1) - 1/cos(x + 1)

mupad [B] time = 3.17, size = 34, normalized size = 2.43

$$\frac{2x \cos(x+1)}{\cos(2x+2)+1} + \operatorname{atan}(\cos(x+1) + \sin(x+1) 1i) 2i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*tan(x + 1))/cos(x + 1),x)

[Out] atan(cos(x + 1) + sin(x + 1)*1i)*2i + (2*x*cos(x + 1))/(cos(2*x + 2) + 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \tan(x+1) \sec(x+1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(1+x)*tan(1+x),x)

[Out] Integral(x*tan(x + 1)*sec(x + 1), x)

$$3.905 \quad \int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}} dx$$

Optimal. Leaf size=14

$$-2\sqrt{9-\sin^2(x)}$$

[Out] -2*(9-sin(x)^2)^(1/2)

Rubi [A] time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {12, 261}

$$-2\sqrt{9-\sin^2(x)}$$

Antiderivative was successfully verified.

[In] Int[Sin[2*x]/Sqrt[9 - Sin[x]^2], x]

[Out] -2*Sqrt[9 - Sin[x]^2]

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}} dx &= \text{Subst} \left(\int \frac{2x}{\sqrt{9-x^2}} dx, x, \sin(x) \right) \\ &= 2 \text{Subst} \left(\int \frac{x}{\sqrt{9-x^2}} dx, x, \sin(x) \right) \\ &= -2\sqrt{9-\sin^2(x)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 1.00

$$-2\sqrt{9-\sin^2(x)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[2*x]/Sqrt[9 - Sin[x]^2], x]

[Out] -2*Sqrt[9 - Sin[x]^2]

fricas [A] time = 0.77, size = 10, normalized size = 0.71

$$-2\sqrt{\cos(x)^2 + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/(9-sin(x)^2)^(1/2),x, algorithm="fricas")

[Out] -2*sqrt(cos(x)^2 + 8)

giac [A] time = 0.14, size = 12, normalized size = 0.86

$$-2\sqrt{-\sin(x)^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/(9-sin(x)^2)^(1/2),x, algorithm="giac")

[Out] -2*sqrt(-sin(x)^2 + 9)

maple [A] time = 0.08, size = 13, normalized size = 0.93

$$-2\sqrt{9 - (\sin^2(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)/(9-sin(x)^2)^(1/2),x)

[Out] -2*(9-sin(x)^2)^(1/2)

maxima [A] time = 0.33, size = 12, normalized size = 0.86

$$-2\sqrt{-\sin(x)^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/(9-sin(x)^2)^(1/2),x, algorithm="maxima")

[Out] -2*sqrt(-sin(x)^2 + 9)

mupad [B] time = 0.17, size = 10, normalized size = 0.71

$$-2\sqrt{\cos(x)^2 + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)/(9 - sin(x)^2)^(1/2),x)

[Out] -2*(cos(x)^2 + 8)^(1/2)

sympy [A] time = 1.43, size = 12, normalized size = 0.86

$$-2\sqrt{9 - \sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/(9-sin(x)**2)**(1/2),x)

[Out] -2*sqrt(9 - sin(x)**2)

$$3.906 \quad \int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}} dx$$

Optimal. Leaf size=11

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

[Out] -arcsin(1/3*cos(x)^2)

Rubi [A] time = 0.05, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {12, 1107, 619, 216}

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[Sin[2*x]/Sqrt[9 - Cos[x]^4], x]

[Out] -ArcSin[Cos[x]^2/3]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}} dx &= \text{Subst}\left(\int \frac{2x}{\sqrt{8+2x^2-x^4}} dx, x, \sin(x)\right) \\ &= 2 \text{Subst}\left(\int \frac{x}{\sqrt{8+2x^2-x^4}} dx, x, \sin(x)\right) \\ &= \text{Subst}\left(\int \frac{1}{\sqrt{8+2x-x^2}} dx, x, \sin^2(x)\right) \\ &= -\left(\frac{1}{6} \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}} dx, x, 2\cos^2(x)\right)\right) \\ &= -\sin^{-1}\left(\frac{\cos^2(x)}{3}\right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 11, normalized size = 1.00

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[2*x]/Sqrt[9 - Cos[x]^4], x]

[Out] -ArcSin[Cos[x]^2/3]

fricas [B] time = 1.31, size = 24, normalized size = 2.18

$$\arctan\left(\frac{\sqrt{-\cos(x)^4 + 9} \cos(x)^2}{\cos(x)^4 - 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2), x, algorithm="fricas")

[Out] arctan(sqrt(-cos(x)^4 + 9)*cos(x)^2/(cos(x)^4 - 9))

giac [A] time = 0.15, size = 9, normalized size = 0.82

$$-\arcsin\left(\frac{1}{3} \cos(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2), x, algorithm="giac")

[Out] -arcsin(1/3*cos(x)^2)

maple [A] time = 0.11, size = 10, normalized size = 0.91

$$-\arcsin\left(\frac{\cos^2(x)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x)/(9-cos(x)^4)^(1/2), x)

[Out] -arcsin(1/3*cos(x)^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(2x)}{\sqrt{-\cos(x)^4 + 9}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2), x, algorithm="maxima")

[Out] integrate(sin(2*x)/sqrt(-cos(x)^4 + 9), x)

mupad [B] time = 3.14, size = 18, normalized size = 1.64

$$-\operatorname{atan}\left(\frac{\cos(x)^2}{\sqrt{9 - \cos(x)^4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(9 - cos(x)^4)^(1/2),x)
```

```
[Out] -atan(cos(x)^2/(9 - cos(x)^4)^(1/2))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

$$3.907 \quad \int \frac{\cos\left(\frac{1}{x}\right)}{x^5} dx$$

Optimal. Leaf size=34

$$-\frac{\sin\left(\frac{1}{x}\right)}{x^3} - \frac{3\cos\left(\frac{1}{x}\right)}{x^2} + \frac{6\sin\left(\frac{1}{x}\right)}{x} + 6\cos\left(\frac{1}{x}\right)$$

[Out] 6*cos(1/x)-3*cos(1/x)/x^2-sin(1/x)/x^3+6*sin(1/x)/x

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3380, 3296, 2638}

$$-\frac{\sin\left(\frac{1}{x}\right)}{x^3} - \frac{3\cos\left(\frac{1}{x}\right)}{x^2} + \frac{6\sin\left(\frac{1}{x}\right)}{x} + 6\cos\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[x^(-1)]/x^5,x]

[Out] 6*Cos[x^(-1)] - (3*Cos[x^(-1)])/x^2 - Sin[x^(-1)]/x^3 + (6*Sin[x^(-1)])/x

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c_.) + (d_.)*(x_)^(m_.))*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(\frac{1}{x}\right)}{x^5} dx &= -\text{Subst}\left(\int x^3 \cos(x) dx, x, \frac{1}{x}\right) \\
&= -\frac{\sin\left(\frac{1}{x}\right)}{x^3} + 3 \text{Subst}\left(\int x^2 \sin(x) dx, x, \frac{1}{x}\right) \\
&= -\frac{3 \cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + 6 \text{Subst}\left(\int x \cos(x) dx, x, \frac{1}{x}\right) \\
&= -\frac{3 \cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + \frac{6 \sin\left(\frac{1}{x}\right)}{x} - 6 \text{Subst}\left(\int \sin(x) dx, x, \frac{1}{x}\right) \\
&= 6 \cos\left(\frac{1}{x}\right) - \frac{3 \cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + \frac{6 \sin\left(\frac{1}{x}\right)}{x}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.94

$$\frac{3(2x^2 - 1) \cos\left(\frac{1}{x}\right)}{x^2} + \frac{(6x^2 - 1) \sin\left(\frac{1}{x}\right)}{x^3}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x^(-1)]/x^5,x]

[Out] (3*(-1 + 2*x^2)*Cos[x^(-1)])/x^2 + ((-1 + 6*x^2)*Sin[x^(-1)])/x^3

fricas [A] time = 0.87, size = 32, normalized size = 0.94

$$\frac{3(2x^3 - x) \cos\left(\frac{1}{x}\right) + (6x^2 - 1) \sin\left(\frac{1}{x}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/x)/x^5,x, algorithm="fricas")

[Out] (3*(2*x^3 - x)*cos(1/x) + (6*x^2 - 1)*sin(1/x))/x^3

giac [A] time = 0.14, size = 34, normalized size = 1.00

$$\frac{6 \sin\left(\frac{1}{x}\right)}{x} - \frac{3 \cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + 6 \cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/x)/x^5,x, algorithm="giac")

[Out] 6*sin(1/x)/x - 3*cos(1/x)/x^2 - sin(1/x)/x^3 + 6*cos(1/x)

maple [A] time = 0.06, size = 35, normalized size = 1.03

$$6 \cos\left(\frac{1}{x}\right) - \frac{3 \cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + \frac{6 \sin\left(\frac{1}{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(1/x)/x^5,x)`

[Out] $6\cos(1/x) - 3\cos(1/x)/x^2 - \sin(1/x)/x^3 + 6\sin(1/x)/x$

maxima [C] time = 0.37, size = 19, normalized size = 0.56

$$\frac{1}{2}\Gamma\left(4, \frac{i}{x}\right) + \frac{1}{2}\Gamma\left(4, -\frac{i}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/x)/x^5,x, algorithm="maxima")`

[Out] $1/2*\text{gamma}(4, I/x) + 1/2*\text{gamma}(4, -I/x)$

mupad [B] time = 3.00, size = 33, normalized size = 0.97

$$6\cos\left(\frac{1}{x}\right) - \frac{\sin\left(\frac{1}{x}\right) + 3x\cos\left(\frac{1}{x}\right) - 6x^2\sin\left(\frac{1}{x}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(1/x)/x^5,x)`

[Out] $6\cos(1/x) - (\sin(1/x) + 3*x*\cos(1/x) - 6*x^2*\sin(1/x))/x^3$

sympy [A] time = 3.68, size = 32, normalized size = 0.94

$$6\cos\left(\frac{1}{x}\right) + \frac{6\sin\left(\frac{1}{x}\right)}{x} - \frac{3\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/x)/x**5,x)`

[Out] $6*\cos(1/x) + 6*\sin(1/x)/x - 3*\cos(1/x)/x**2 - \sin(1/x)/x**3$

3.908 $\int \cos^3(1+x) \sin^3(1+x) dx$

Optimal. Leaf size=21

$$\frac{1}{4} \sin^4(x+1) - \frac{1}{6} \sin^6(x+1)$$

[Out] $1/4*\sin(1+x)^4-1/6*\sin(1+x)^6$

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2564, 14}

$$\frac{1}{4} \sin^4(x+1) - \frac{1}{6} \sin^6(x+1)$$

Antiderivative was successfully verified.

[In] Int[Cos[1+x]^3*Sin[1+x]^3,x]

[Out] Sin[1+x]^4/4 - Sin[1+x]^6/6

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 2564

Int[cos[(e_)+(f_)*(x_)]^(n_)*((a_)*sin[(e_)+(f_)*(x_)]^(m_)), x_Symbol] := Dist[1/(a*f), Subst[Int[x^m*(1-x^2/a^2)^((n-1)/2), x], x, a*Sin[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[(m-1)/2] && LtQ[0, m, n])

Rubi steps

$$\begin{aligned} \int \cos^3(1+x) \sin^3(1+x) dx &= \text{Subst} \left(\int x^3 (1-x^2) dx, x, \sin(1+x) \right) \\ &= \text{Subst} \left(\int (x^3 - x^5) dx, x, \sin(1+x) \right) \\ &= \frac{1}{4} \sin^4(1+x) - \frac{1}{6} \sin^6(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.19

$$\frac{1}{8} \left(\frac{1}{24} \cos(6(x+1)) - \frac{3}{8} \cos(2(x+1)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[1+x]^3*Sin[1+x]^3,x]

[Out] ((-3*Cos[2*(1+x)])/8 + Cos[6*(1+x)]/24)/8

fricas [A] time = 0.82, size = 17, normalized size = 0.81

$$\frac{1}{6} \cos(x+1)^6 - \frac{1}{4} \cos(x+1)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1+x)^3*sin(1+x)^3,x, algorithm="fricas")

[Out] 1/6*cos(x + 1)^6 - 1/4*cos(x + 1)^4

giac [A] time = 0.14, size = 17, normalized size = 0.81

$$-\frac{1}{6} \sin(x+1)^6 + \frac{1}{4} \sin(x+1)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1+x)^3*sin(1+x)^3,x, algorithm="giac")

[Out] -1/6*sin(x + 1)^6 + 1/4*sin(x + 1)^4

maple [A] time = 0.08, size = 24, normalized size = 1.14

$$-\frac{(\cos^4(1+x))(\sin^2(1+x))}{6} - \frac{(\cos^4(1+x))}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(1+x)^3*sin(1+x)^3,x)

[Out] -1/6*cos(1+x)^4*sin(1+x)^2-1/12*cos(1+x)^4

maxima [A] time = 0.56, size = 17, normalized size = 0.81

$$-\frac{1}{6} \sin(x+1)^6 + \frac{1}{4} \sin(x+1)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1+x)^3*sin(1+x)^3,x, algorithm="maxima")

[Out] -1/6*sin(x + 1)^6 + 1/4*sin(x + 1)^4

mupad [B] time = 0.07, size = 18, normalized size = 0.86

$$-\frac{\sin(x+1)^4(2\sin(x+1)^2-3)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x + 1)^3*sin(x + 1)^3,x)

[Out] -(sin(x + 1)^4*(2*sin(x + 1)^2 - 3))/12

sympy [A] time = 1.78, size = 24, normalized size = 1.14

$$-\frac{\sin^2(x+1)\cos^4(x+1)}{4} - \frac{\cos^6(x+1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1+x)**3*sin(1+x)**3,x)

[Out] -sin(x + 1)**2*cos(x + 1)**4/4 - cos(x + 1)**6/12

3.909 $\int (1 + 2x)^3 \sin^2(1 + 2x) dx$

Optimal. Leaf size=99

$$-\frac{3x^2}{4} + \frac{1}{16}(2x+1)^4 - \frac{3x}{4} + \frac{3}{8}(2x+1)^2 \sin^2(2x+1) - \frac{3}{16} \sin^2(2x+1) - \frac{1}{4}(2x+1)^3 \sin(2x+1) \cos(2x+1) + \frac{3}{8}(2x+1) \sin(2x+1)$$

[Out] $-3/4*x-3/4*x^2+1/16*(1+2*x)^4+3/8*(1+2*x)*\cos(1+2*x)*\sin(1+2*x)-1/4*(1+2*x)^3*\cos(1+2*x)*\sin(1+2*x)-3/16*\sin(1+2*x)^2+3/8*(1+2*x)^2*\sin(1+2*x)^2$

Rubi [A] time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3311, 32, 3310}

$$-\frac{3x^2}{4} + \frac{1}{16}(2x+1)^4 - \frac{3x}{4} + \frac{3}{8}(2x+1)^2 \sin^2(2x+1) - \frac{3}{16} \sin^2(2x+1) - \frac{1}{4}(2x+1)^3 \sin(2x+1) \cos(2x+1) + \frac{3}{8}(2x+1) \sin(2x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)^3*Sin[1 + 2*x]^2,x]

[Out] $(-3*x)/4 - (3*x^2)/4 + (1 + 2*x)^4/16 + (3*(1 + 2*x)*\text{Cos}[1 + 2*x]*\text{Sin}[1 + 2*x])/8 - ((1 + 2*x)^3*\text{Cos}[1 + 2*x]*\text{Sin}[1 + 2*x])/4 - (3*\text{Sin}[1 + 2*x]^2)/16 + (3*(1 + 2*x)^2*\text{Sin}[1 + 2*x]^2)/8$

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 3310

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rule 3311

Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rubi steps

$$\begin{aligned} \int (1 + 2x)^3 \sin^2(1 + 2x) dx &= -\frac{1}{4}(1 + 2x)^3 \cos(1 + 2x) \sin(1 + 2x) + \frac{3}{8}(1 + 2x)^2 \sin^2(1 + 2x) + \frac{1}{2} \int (1 + 2x)^3 dx \\ &= \frac{1}{16}(1 + 2x)^4 + \frac{3}{8}(1 + 2x) \cos(1 + 2x) \sin(1 + 2x) - \frac{1}{4}(1 + 2x)^3 \cos(1 + 2x) \sin(1 + 2x) \\ &= -\frac{3x}{4} - \frac{3x^2}{4} + \frac{1}{16}(1 + 2x)^4 + \frac{3}{8}(1 + 2x) \cos(1 + 2x) \sin(1 + 2x) - \frac{1}{4}(1 + 2x)^3 \cos(1 + 2x) \sin(1 + 2x) \end{aligned}$$

Mathematica [A] time = 0.23, size = 55, normalized size = 0.56

$$\frac{1}{32} \left(2(2x + 1) \left((-8x^2 - 8x + 1) \sin(4x + 2) + (2x + 1)^3 \right) - 3(8x^2 + 8x + 1) \cos(4x + 2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)^3*Sin[1 + 2*x]^2,x]

[Out] (-3*(1 + 8*x + 8*x^2)*Cos[2 + 4*x] + 2*(1 + 2*x)*((1 + 2*x)^3 + (1 - 8*x - 8*x^2)*Sin[2 + 4*x]))/32

fricas [A] time = 1.09, size = 66, normalized size = 0.67

$$x^4 + 2x^3 - \frac{3}{16}(8x^2 + 8x + 1)\cos(2x + 1)^2 - \frac{1}{8}(16x^3 + 24x^2 + 6x - 1)\cos(2x + 1)\sin(2x + 1) + \frac{9}{4}x^2 + \frac{5}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)^3*sin(1+2*x)^2,x, algorithm="fricas")

[Out] x^4 + 2*x^3 - 3/16*(8*x^2 + 8*x + 1)*cos(2*x + 1)^2 - 1/8*(16*x^3 + 24*x^2 + 6*x - 1)*cos(2*x + 1)*sin(2*x + 1) + 9/4*x^2 + 5/4*x

giac [A] time = 0.13, size = 58, normalized size = 0.59

$$x^4 + 2x^3 + \frac{3}{2}x^2 - \frac{3}{32}(8x^2 + 8x + 1)\cos(4x + 2) - \frac{1}{16}(16x^3 + 24x^2 + 6x - 1)\sin(4x + 2) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)^3*sin(1+2*x)^2,x, algorithm="giac")

[Out] x^4 + 2*x^3 + 3/2*x^2 - 3/32*(8*x^2 + 8*x + 1)*cos(4*x + 2) - 1/16*(16*x^3 + 24*x^2 + 6*x - 1)*sin(4*x + 2) + 1/2*x

maple [A] time = 0.04, size = 97, normalized size = 0.98

$$\frac{(1 + 2x)^3 \left(-\frac{\sin(1+2x)\cos(1+2x)}{2} + x + \frac{1}{2} \right) - \frac{3(\cos^2(1 + 2x))(1 + 2x)^2}{8} - \frac{3(1 + 2x) \left(\frac{\sin(1+2x)\cos(1+2x)}{2} + x + \frac{1}{2} \right)}{4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2*x)^3*sin(1+2*x)^2,x)

[Out] 1/2*(1+2*x)^3*(-1/2*sin(1+2*x)*cos(1+2*x)+x+1/2)-3/8*cos(1+2*x)^2*(1+2*x)^2+3/4*(1+2*x)*(1/2*sin(1+2*x)*cos(1+2*x)+x+1/2)-3/16*(1+2*x)^2-3/16*sin(1+2*x)^2-3/16*(1+2*x)^4

maxima [A] time = 0.40, size = 51, normalized size = 0.52

$$\frac{1}{16}(2x + 1)^4 - \frac{3}{32}(2(2x + 1)^2 - 1)\cos(4x + 2) - \frac{1}{16}(2(2x + 1)^3 - 6x - 3)\sin(4x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)^3*sin(1+2*x)^2,x, algorithm="maxima")

[Out] 1/16*(2*x + 1)^4 - 3/32*(2*(2*x + 1)^2 - 1)*cos(4*x + 2) - 1/16*(2*(2*x + 1)^3 - 6*x - 3)*sin(4*x + 2)

mupad [B] time = 3.06, size = 69, normalized size = 0.70

$$\frac{3 \sin(4x + 2)(2x + 1)}{16} - \frac{3 \sin(2x + 1)^2}{16} + \frac{(2x + 1)^4}{16} - \frac{\sin(4x + 2)(2x + 1)^3}{8} + \frac{3(2x + 1)^2(2 \sin(2x + 1)^2 - 1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*x + 1)^2*(2*x + 1)^3,x)

[Out] $(3*\sin(4*x + 2)*(2*x + 1))/16 - (3*\sin(2*x + 1)^2)/16 + (2*x + 1)^4/16 - (\sin(4*x + 2)*(2*x + 1)^3)/8 + (3*(2*x + 1)^2*(2*\sin(2*x + 1)^2 - 1))/16$

sympy [B] time = 1.30, size = 189, normalized size = 1.91

$$x^4 \sin^2(2x + 1) + x^4 \cos^2(2x + 1) + 2x^3 \sin^2(2x + 1) - 2x^3 \sin(2x + 1) \cos(2x + 1) + 2x^3 \cos^2(2x + 1) + \frac{9x^2 \sin^2(2x + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)**3*sin(1+2*x)**2,x)

[Out] $x**4*\sin(2*x + 1)**2 + x**4*\cos(2*x + 1)**2 + 2*x**3*\sin(2*x + 1)**2 - 2*x**3*\sin(2*x + 1)*\cos(2*x + 1) + 2*x**3*\cos(2*x + 1)**2 + 9*x**2*\sin(2*x + 1)**2/4 - 3*x**2*\sin(2*x + 1)*\cos(2*x + 1) + 3*x**2*\cos(2*x + 1)**2/4 + 5*x*\sin(2*x + 1)**2/4 - 3*x*\sin(2*x + 1)*\cos(2*x + 1)/4 - x*\cos(2*x + 1)**2/4 + 3*\sin(2*x + 1)**2/16 + \sin(2*x + 1)*\cos(2*x + 1)/8$

$$3.910 \quad \int \frac{-1+\sec(x)}{1-\tan(x)} dx$$

Optimal. Leaf size=37

$$-\frac{x}{2} + \frac{1}{2} \log(\cos(x) - \sin(x)) + \frac{\tanh^{-1}\left(\frac{\cos(x)(\tan(x)+1)}{\sqrt{2}}\right)}{\sqrt{2}}$$

[Out] $-1/2*x+1/2*\ln(\cos(x)-\sin(x))+1/2*\operatorname{arctanh}(1/2*\cos(x)*(1+\tan(x))*2^{(1/2)})*2^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {4401, 3484, 3530, 3509, 206}

$$-\frac{x}{2} + \frac{1}{2} \log(\cos(x) - \sin(x)) + \frac{\tanh^{-1}\left(\frac{\cos(x)(\tan(x)+1)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] `Int[(-1 + Sec[x])/(1 - Tan[x]), x]`

[Out] $-x/2 + \operatorname{ArcTanh}[(\cos[x]*(1 + \tan[x]))/\operatorname{Sqrt}[2]]/\operatorname{Sqrt}[2] + \operatorname{Log}[\cos[x] - \sin[x]]/2$

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 3484

`Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]`

Rule 3509

`Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]`

Rule 3530

`Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]`

Rule 4401

`Int[u_, x_Symbol] := With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /; !InertTrigFreeQ[u]`

Rubi steps

$$\begin{aligned}
\int \frac{-1 + \sec(x)}{1 - \tan(x)} dx &= \int \left(\frac{1}{-1 + \tan(x)} - \frac{\sec(x)}{-1 + \tan(x)} \right) dx \\
&= \int \frac{1}{-1 + \tan(x)} dx - \int \frac{\sec(x)}{-1 + \tan(x)} dx \\
&= -\frac{x}{2} + \frac{1}{2} \int \frac{1 + \tan(x)}{-1 + \tan(x)} dx + \text{Subst} \left(\int \frac{1}{2 - x^2} dx, x, \cos(x)(1 + \tan(x)) \right) \\
&= -\frac{x}{2} + \frac{\tanh^{-1} \left(\frac{\cos(x)(1 + \tan(x))}{\sqrt{2}} \right)}{\sqrt{2}} + \frac{1}{2} \log(\cos(x) - \sin(x))
\end{aligned}$$

Mathematica [C] time = 0.06, size = 40, normalized size = 1.08

$$\frac{1}{2} \left(-x + (2 - 2i)^{\sqrt[4]{-1}} \tanh^{-1} \left(\frac{\tan\left(\frac{x}{2}\right) + 1}{\sqrt{2}} \right) + \log(\cos(x) - \sin(x)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + Sec[x])/(1 - Tan[x]),x]

[Out] (-x + (2 - 2*I)*(-1)^(1/4)*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] + Log[Cos[x] - Sin[x]])/2

fricas [A] time = 1.41, size = 51, normalized size = 1.38

$$\frac{1}{4} \sqrt{2} \log \left(\frac{2(\sqrt{2} + \cos(x)) \sin(x) + 2\sqrt{2} \cos(x) + 3}{2 \cos(x) \sin(x) - 1} \right) - \frac{1}{2} x + \frac{1}{4} \log(-2 \cos(x) \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sec(x))/(1-tan(x)),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log((2*(sqrt(2) + cos(x))*sin(x) + 2*sqrt(2)*cos(x) + 3)/(2*cos(x)*sin(x) - 1)) - 1/2*x + 1/4*log(-2*cos(x)*sin(x) + 1)

giac [B] time = 0.20, size = 70, normalized size = 1.89

$$-\frac{1}{2} \sqrt{2} \log \left(\frac{\left| -2\sqrt{2} + 2 \tan\left(\frac{1}{2}x\right) + 2 \right|}{\left| 2\sqrt{2} + 2 \tan\left(\frac{1}{2}x\right) + 2 \right|} \right) - \frac{1}{2} x - \frac{1}{2} \log \left(\tan\left(\frac{1}{2}x\right)^2 + 1 \right) + \frac{1}{2} \log \left(\left| \tan\left(\frac{1}{2}x\right)^2 + 2 \tan\left(\frac{1}{2}x\right) - 1 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sec(x))/(1-tan(x)),x, algorithm="giac")

[Out] -1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) + 2)/abs(2*sqrt(2) + 2*tan(1/2*x) + 2)) - 1/2*x - 1/2*log(tan(1/2*x)^2 + 1) + 1/2*log(abs(tan(1/2*x)^2 + 2*tan(1/2*x) - 1))

maple [A] time = 0.11, size = 51, normalized size = 1.38

$$\frac{\ln \left(\tan^2 \left(\frac{x}{2} \right) + 2 \tan \left(\frac{x}{2} \right) - 1 \right)}{2} + \sqrt{2} \operatorname{arctanh} \left(\frac{\left(2 \tan \left(\frac{x}{2} \right) + 2 \right) \sqrt{2}}{4} \right) - \frac{\ln \left(1 + \tan^2 \left(\frac{x}{2} \right) \right)}{2} - \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+sec(x))/(1-tan(x)),x)

[Out] $\frac{1}{2} \ln(\tan(\frac{1}{2}x)^2 + 2 \tan(\frac{1}{2}x) - 1) + 2^{(1/2)} \operatorname{arctanh}(\frac{1}{4} * (2 * \tan(\frac{1}{2}x) + 2) * 2^{(1/2)}) - \frac{1}{2} \ln(1 + \tan(\frac{1}{2}x)^2) - \frac{1}{2}x$

maxima [A] time = 0.58, size = 59, normalized size = 1.59

$$-\frac{1}{2} \sqrt{2} \log \left(-\frac{\sqrt{2} - \frac{\sin(x)}{\cos(x)+1} - 1}{\sqrt{2} + \frac{\sin(x)}{\cos(x)+1} + 1} \right) - \frac{1}{2}x - \frac{1}{4} \log(\tan(x)^2 + 1) + \frac{1}{2} \log(\tan(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+sec(x))/(1-tan(x)),x, algorithm="maxima")`

[Out] $-\frac{1}{2} \sqrt{2} \log(-(\sqrt{2} - \sin(x)/(\cos(x) + 1) - 1)/(\sqrt{2} + \sin(x)/(\cos(x) + 1) + 1)) - \frac{1}{2}x - \frac{1}{4} \log(\tan(x)^2 + 1) + \frac{1}{2} \log(\tan(x) - 1)$

mupad [B] time = 3.13, size = 64, normalized size = 1.73

$$\ln\left(\tan\left(\frac{x}{2}\right) + \sqrt{2} + 1\right) \left(\frac{\sqrt{2}}{2} + \frac{1}{2}\right) - \ln\left(\tan\left(\frac{x}{2}\right) - \sqrt{2} + 1\right) \left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right) + \ln\left(\tan\left(\frac{x}{2}\right) - i\right) \left(-\frac{1}{2} + \frac{1}{2}i\right) + \ln\left(\tan\left(\frac{x}{2}\right) + i\right) \left(-\frac{1}{2} - \frac{1}{2}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(1/cos(x) - 1)/(tan(x) - 1),x)`

[Out] $\log(\tan(x/2) + 2^{(1/2)} + 1) * (2^{(1/2)}/2 + 1/2) - \log(\tan(x/2) + 1i) * (1/2 + 1i/2) - \log(\tan(x/2) - 2^{(1/2)} + 1) * (2^{(1/2)}/2 - 1/2) - \log(\tan(x/2) - 1i) * (1/2 - 1i/2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sec(x)}{\tan(x) - 1} dx - \int \left(-\frac{1}{\tan(x) - 1} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+sec(x))/(1-tan(x)),x)`

[Out] $-\operatorname{Integral}(\sec(x)/(\tan(x) - 1), x) - \operatorname{Integral}(-1/(\tan(x) - 1), x)$

3.911 $\int x^2 \cos(3x) \cos(5x) dx$

Optimal. Leaf size=57

$$\frac{1}{4}x^2 \sin(2x) + \frac{1}{16}x^2 \sin(8x) - \frac{1}{8} \sin(2x) - \frac{1}{512} \sin(8x) + \frac{1}{4}x \cos(2x) + \frac{1}{64}x \cos(8x)$$

[Out] $1/4*x*cos(2*x)+1/64*x*cos(8*x)-1/8*sin(2*x)+1/4*x^2*sin(2*x)-1/512*sin(8*x)+1/16*x^2*sin(8*x)$

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4429, 3296, 2637}

$$\frac{1}{4}x^2 \sin(2x) + \frac{1}{16}x^2 \sin(8x) - \frac{1}{8} \sin(2x) - \frac{1}{512} \sin(8x) + \frac{1}{4}x \cos(2x) + \frac{1}{64}x \cos(8x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*\text{Cos}[3*x]*\text{Cos}[5*x], x]$

[Out] $(x*\text{Cos}[2*x])/4 + (x*\text{Cos}[8*x])/64 - \text{Sin}[2*x]/8 + (x^2*\text{Sin}[2*x])/4 - \text{Sin}[8*x]/512 + (x^2*\text{Sin}[8*x])/16$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 4429

$\text{Int}[\text{Cos}[(a_.) + (b_.)*(x_.)]^{(p_.)}*\text{Cos}[(c_.) + (d_.)*(x_.)]^{(q_.)}*((e_.) + (f_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(e + f*x)^m, \text{Cos}[a + b*x]]^{p*}\text{Cos}[c + d*x]^q, x] /;$ FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IGtQ[q, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int x^2 \cos(3x) \cos(5x) dx &= \int \left(\frac{1}{2}x^2 \cos(2x) + \frac{1}{2}x^2 \cos(8x) \right) dx \\ &= \frac{1}{2} \int x^2 \cos(2x) dx + \frac{1}{2} \int x^2 \cos(8x) dx \\ &= \frac{1}{4}x^2 \sin(2x) + \frac{1}{16}x^2 \sin(8x) - \frac{1}{8} \int x \sin(8x) dx - \frac{1}{2} \int x \sin(2x) dx \\ &= \frac{1}{4}x \cos(2x) + \frac{1}{64}x \cos(8x) + \frac{1}{4}x^2 \sin(2x) + \frac{1}{16}x^2 \sin(8x) - \frac{1}{64} \int \cos(8x) dx - \frac{1}{4} \int \cos(2x) dx \\ &= \frac{1}{4}x \cos(2x) + \frac{1}{64}x \cos(8x) - \frac{1}{8} \sin(2x) + \frac{1}{4}x^2 \sin(2x) - \frac{1}{512} \sin(8x) + \frac{1}{16}x^2 \sin(8x) \end{aligned}$$

Mathematica [A] time = 0.10, size = 49, normalized size = 0.86

$$\frac{1}{512} (128x^2 \sin(2x) + 32x^2 \sin(8x) - 64 \sin(2x) - \sin(8x) + 128x \cos(2x) + 8x \cos(8x))$$

Antiderivative was successfully verified.

[In] Integrate[x^2*cos[3*x]*cos[5*x],x]

[Out] (128*x*cos[2*x] + 8*x*cos[8*x] - 64*Sin[2*x] + 128*x^2*Sin[2*x] - Sin[8*x] + 32*x^2*Sin[8*x])/512

fricas [A] time = 1.13, size = 73, normalized size = 1.28

$$2x \cos(x)^8 - 4x \cos(x)^6 + \frac{5}{2}x \cos(x)^4 + \frac{1}{64} \left(16(32x^2 - 1) \cos(x)^7 - 24(32x^2 - 1) \cos(x)^5 + 10(32x^2 - 1) \cos(x)^3 - 15 \cos(x) \right) \sin(x) - 15/64 * x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(3*x)*cos(5*x),x, algorithm="fricas")

[Out] 2*x*cos(x)^8 - 4*x*cos(x)^6 + 5/2*x*cos(x)^4 + 1/64*(16*(32*x^2 - 1)*cos(x)^7 - 24*(32*x^2 - 1)*cos(x)^5 + 10*(32*x^2 - 1)*cos(x)^3 - 15*cos(x))*sin(x) - 15/64*x

giac [A] time = 0.12, size = 41, normalized size = 0.72

$$\frac{1}{64}x \cos(8x) + \frac{1}{4}x \cos(2x) + \frac{1}{512}(32x^2 - 1) \sin(8x) + \frac{1}{8}(2x^2 - 1) \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(3*x)*cos(5*x),x, algorithm="giac")

[Out] 1/64*x*cos(8*x) + 1/4*x*cos(2*x) + 1/512*(32*x^2 - 1)*sin(8*x) + 1/8*(2*x^2 - 1)*sin(2*x)

maple [A] time = 0.14, size = 46, normalized size = 0.81

$$\frac{x \cos(2x)}{4} + \frac{x \cos(8x)}{64} - \frac{\sin(2x)}{8} + \frac{x^2 \sin(2x)}{4} - \frac{\sin(8x)}{512} + \frac{x^2 \sin(8x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cos(3*x)*cos(5*x),x)

[Out] 1/4*x*cos(2*x)+1/64*x*cos(8*x)-1/8*sin(2*x)+1/4*x^2*sin(2*x)-1/512*sin(8*x)+1/16*x^2*sin(8*x)

maxima [A] time = 0.33, size = 41, normalized size = 0.72

$$\frac{1}{64}x \cos(8x) + \frac{1}{4}x \cos(2x) + \frac{1}{512}(32x^2 - 1) \sin(8x) + \frac{1}{8}(2x^2 - 1) \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(3*x)*cos(5*x),x, algorithm="maxima")

[Out] 1/64*x*cos(8*x) + 1/4*x*cos(2*x) + 1/512*(32*x^2 - 1)*sin(8*x) + 1/8*(2*x^2 - 1)*sin(2*x)

mupad [B] time = 3.05, size = 45, normalized size = 0.79

$$\frac{x \cos(2x)}{4} - \frac{\sin(8x)}{512} - \frac{\sin(2x)}{8} + \frac{x \cos(8x)}{64} + \frac{x^2 \sin(2x)}{4} + \frac{x^2 \sin(8x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cos(3*x)*cos(5*x),x)

[Out] $(x \cos(2x))/4 - \sin(8x)/512 - \sin(2x)/8 + (x \cos(8x))/64 + (x^2 \sin(2x))/4 + (x^2 \sin(8x))/16$

sympy [A] time = 6.02, size = 90, normalized size = 1.58

$$-\frac{3x^2 \sin(3x) \cos(5x)}{16} + \frac{5x^2 \sin(5x) \cos(3x)}{16} + \frac{15x \sin(3x) \sin(5x)}{64} + \frac{17x \cos(3x) \cos(5x)}{64} + \frac{63 \sin(3x) \cos(5x)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*cos(3*x)*cos(5*x),x)`

[Out] $-3x^2 \sin(3x) \cos(5x)/16 + 5x^2 \sin(5x) \cos(3x)/16 + 15x \sin(3x) \sin(5x)/64 + 17x \cos(3x) \cos(5x)/64 + 63 \sin(3x) \cos(5x)/512 - 65 \sin(5x) \cos(3x)/512$

$$3.912 \quad \int \frac{\cos(x)+\sin(x)}{\sqrt{\cos(x)} \sqrt{\sin(x)}} dx$$

Optimal. Leaf size=57

$$\sqrt{2} \tan^{-1}\left(\frac{\sqrt{2} \sqrt{\sin(x)}}{\sqrt{\cos(x)}} + 1\right) - \sqrt{2} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)$$

[Out] $-\arctan(1-2^{(1/2)}*\sin(x)^{(1/2)}/\cos(x)^{(1/2)})*2^{(1/2)}+\arctan(1+2^{(1/2)}*\sin(x)^{(1/2)}/\cos(x)^{(1/2)})*2^{(1/2)}$

Rubi [B] time = 0.21, antiderivative size = 243, normalized size of antiderivative = 4.26, number of steps used = 22, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3107, 2575, 297, 1162, 617, 204, 1165, 628, 2574}

$$\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}} + 1\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}} + 1\right)}{\sqrt{2}} + \frac{\log\left(\tan(x) - 1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x] + Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]]), x]

[Out] ArcTan[1 - (Sqrt[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/Sqrt[2] - ArcTan[1 + (Sqrt[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/Sqrt[2] - ArcTan[1 - (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqrt[2] + ArcTan[1 + (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqrt[2] - Log[1 + Cot[x] - (Sqrt[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/(2*Sqrt[2]) + Log[1 + Cot[x] + (Sqrt[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/(2*Sqrt[2]) + Log[1 - (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 297

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] & & EqQ[c*d^2 - a*e^2, 0] & & PosQ[d*e]

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] & & EqQ[c*d^2 - a*e^2, 0] & & NegQ[d*e]

Rule 2574

Int[(cos[(e_) + (f_)*(x_)]*(b_))^(n_)*((a_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e + f*x])^(1/k)], x] /; FreeQ[{a, b, e, f}, x] & & EqQ[m + n, 0] & & GtQ[m, 0] & & LtQ[m, 1]

Rule 2575

Int[(cos[(e_) + (f_)*(x_)]*(a_))^(m_)*((b_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e + f*x])^(1/k)], x] /; FreeQ[{a, b, e, f}, x] & & EqQ[m + n, 0] & & GtQ[m, 0] & & LtQ[m, 1]

Rule 3107

Int[cos[(c_) + (d_)*(x_)]^(m_)*sin[(c_) + (d_)*(x_)]^(n_)*(cos[(c_) + (d_)*(x_)]*(a_) + (b_)*sin[(c_) + (d_)*(x_)])^(p_), x_Symbol] := Int[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] & & IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cos(x) + \sin(x)}{\sqrt{\cos(x)} \sqrt{\sin(x)}} dx &= \int \left(\frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}} + \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}} \right) dx \\
 &= \int \frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}} dx + \int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}} dx \\
 &= - \left(2 \operatorname{Subst} \left(\int \frac{x^2}{1+x^4} dx, x, \frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}} \right) \right) + 2 \operatorname{Subst} \left(\int \frac{x^2}{1+x^4} dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}} \right) \\
 &= \operatorname{Subst} \left(\int \frac{1-x^2}{1+x^4} dx, x, \frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}} \right) - \operatorname{Subst} \left(\int \frac{1-x^2}{1+x^4} dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}} \right) - \operatorname{Subst} \left(\int \frac{1+x^2}{1+x^4} dx, x, \frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}} \right) \\
 &= - \left(\frac{1}{2} \operatorname{Subst} \left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}} \right) \right) + \frac{1}{2} \operatorname{Subst} \left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}} \right) \\
 &= - \frac{\log \left(1 + \cot(x) - \frac{\sqrt{2} \sqrt{\cos(x)}}{\sqrt{\sin(x)}} \right)}{2\sqrt{2}} + \frac{\log \left(1 + \cot(x) + \frac{\sqrt{2} \sqrt{\cos(x)}}{\sqrt{\sin(x)}} \right)}{2\sqrt{2}} + \frac{\log \left(1 - \frac{\sqrt{2} \sqrt{\sin(x)}}{\sqrt{\cos(x)}} \right)}{2\sqrt{2}} + \frac{\log \left(1 - \frac{\sqrt{2} \sqrt{\sin(x)}}{\sqrt{\cos(x)}} \right)}{2\sqrt{2}} \\
 &= \frac{\tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt{\cos(x)}}{\sqrt{\sin(x)}} \right)}{\sqrt{2}} - \frac{\tan^{-1} \left(1 + \frac{\sqrt{2} \sqrt{\cos(x)}}{\sqrt{\sin(x)}} \right)}{\sqrt{2}} - \frac{\tan^{-1} \left(1 - \frac{\sqrt{2} \sqrt{\sin(x)}}{\sqrt{\cos(x)}} \right)}{\sqrt{2}} + \frac{\tan^{-1} \left(1 + \frac{\sqrt{2} \sqrt{\sin(x)}}{\sqrt{\cos(x)}} \right)}{\sqrt{2}}
 \end{aligned}$$

Mathematica [C] time = 0.06, size = 68, normalized size = 1.19

$$\frac{2\sqrt{\sin(x)} \sqrt[4]{\cos^2(x)} \left(\sin(x)\sqrt{\cos^2(x)} {}_2F_1\left(\frac{3}{4}, \frac{3}{4}; \frac{7}{4}; \sin^2(x)\right) + 3\cos(x) {}_2F_1\left(\frac{1}{4}, \frac{1}{4}; \frac{5}{4}; \sin^2(x)\right) \right)}{3\cos^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x] + Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]]),x]

[Out] (2*(Cos[x]^2)^(1/4)*Sqrt[Sin[x]]*(3*Cos[x]*Hypergeometric2F1[1/4, 1/4, 5/4, Sin[x]^2] + Sqrt[Cos[x]^2]*Hypergeometric2F1[3/4, 3/4, 7/4, Sin[x]^2]*Sin[x]))/(3*Cos[x]^(3/2))

fricas [B] time = 0.98, size = 85, normalized size = 1.49

$$-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{(32\sqrt{2}\cos(x)^4 - 32\sqrt{2}\cos(x)^2 + 16\sqrt{2}\cos(x)\sin(x) - \sqrt{2})\sqrt{\cos(x)}\sqrt{\sin(x)}}{8(4\cos(x)^5 - 3\cos(x)^3 - (4\cos(x)^4 - 5\cos(x)^2)\sin(x) - \cos(x))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x, algorithm="fricas")

[Out] -1/4*sqrt(2)*arctan(-1/8*(32*sqrt(2)*cos(x)^4 - 32*sqrt(2)*cos(x)^2 + 16*sqrt(2)*cos(x)*sin(x) - sqrt(2))*sqrt(cos(x))*sqrt(sin(x))/(4*cos(x)^5 - 3*cos(x)^3 - (4*cos(x)^4 - 5*cos(x)^2)*sin(x) - cos(x)))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x) + \sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x, algorithm="giac")

[Out] integrate((cos(x) + sin(x))/(sqrt(cos(x))*sqrt(sin(x))), x)

maple [C] time = 0.26, size = 134, normalized size = 2.35

$$\frac{\sqrt{\frac{1-\cos(x)+\sin(x)}{\sin(x)}} \sqrt{2} \sqrt{\frac{\cos(x)-1+\sin(x)}{\sin(x)}} \sqrt{\frac{-1+\cos(x)}{\sin(x)}} \left(\sin^{\frac{3}{2}}(x) \right) \left(i \operatorname{EllipticPi}\left(\sqrt{\frac{1-\cos(x)+\sin(x)}{\sin(x)}}, \frac{1}{2} + \frac{i}{2}, \frac{\sqrt{2}}{2}\right) - i \operatorname{EllipticPi}\left(\sqrt{\frac{1-\cos(x)+\sin(x)}{\sin(x)}}, \frac{1}{2} + \frac{i}{2}, \frac{\sqrt{2}}{2}\right) \right)}{\sqrt{\cos(x)} (-1 + \cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x)

[Out] (((1-cos(x)+sin(x))/sin(x))^(1/2)*2^(1/2)*((cos(x)-1+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*sin(x)^(3/2)*(I*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2), 1/2+1/2*I, 1/2*2^(1/2))-I*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2), 1/2-1/2*I, 1/2*2^(1/2))+EllipticF(((1-cos(x)+sin(x))/sin(x))^(1/2), 1/2*2^(1/2)))/cos(x)^(1/2)/(-1+cos(x)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x) + \sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x, algorithm="maxima")

[Out] integrate((cos(x) + sin(x))/(sqrt(cos(x))*sqrt(sin(x))), x)

mupad [B] time = 4.62, size = 51, normalized size = 0.89

$$\frac{2\sqrt{\cos(x)} \sin(x)^{3/2} {}_2F_1\left(\frac{1}{4}, \frac{1}{4}; \frac{5}{4}; \cos(x)^2\right)}{(\sin(x)^2)^{3/4}} - \frac{2\cos(x)^{3/2} \sqrt{\sin(x)} {}_2F_1\left(\frac{3}{4}, \frac{3}{4}; \frac{7}{4}; \cos(x)^2\right)}{3(\sin(x)^2)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x) + sin(x))/(cos(x)^(1/2)*sin(x)^(1/2)), x)

[Out] - (2*cos(x)^(1/2)*sin(x)^(3/2)*hypergeom([1/4, 1/4], 5/4, cos(x)^2))/(sin(x)^2)^(3/4) - (2*cos(x)^(3/2)*sin(x)^(1/2)*hypergeom([3/4, 3/4], 7/4, cos(x)^2))/(3*(sin(x)^2)^(1/4))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x) + \cos(x)}{\sqrt{\sin(x)} \sqrt{\cos(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)+sin(x))/cos(x)**(1/2)/sin(x)**(1/2), x)

[Out] Integral((sin(x) + cos(x))/(sqrt(sin(x))*sqrt(cos(x))), x)

3.913 $\int \sec^2(x)(1 + \sin(x)) dx$

Optimal. Leaf size=5

$$\tan(x) + \sec(x)$$

[Out] sec(x)+tan(x)

Rubi [A] time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2669, 3767, 8}

$$\tan(x) + \sec(x)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2*(1 + Sin[x]),x]

[Out] Sec[x] + Tan[x]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2669

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])

Rule 3767

Int[csc[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int \sec^2(x)(1 + \sin(x)) dx &= \sec(x) + \int \sec^2(x) dx \\ &= \sec(x) - \text{Subst}\left(\int 1 dx, x, -\tan(x)\right) \\ &= \sec(x) + \tan(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 5, normalized size = 1.00

$$\tan(x) + \sec(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2*(1 + Sin[x]),x]

[Out] Sec[x] + Tan[x]

fricas [B] time = 0.85, size = 17, normalized size = 3.40

$$\frac{\cos(x) + \sin(x) + 1}{\cos(x) - \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(1+sin(x)),x, algorithm="fricas")

[Out] (cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)

giac [A] time = 0.13, size = 10, normalized size = 2.00

$$-\frac{2}{\tan\left(\frac{1}{2}x\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(1+sin(x)),x, algorithm="giac")

[Out] -2/(tan(1/2*x) - 1)

maple [A] time = 0.06, size = 8, normalized size = 1.60

$$\tan(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2*(1+sin(x)),x)

[Out] tan(x)+1/cos(x)

maxima [A] time = 0.32, size = 7, normalized size = 1.40

$$\frac{1}{\cos(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*(1+sin(x)),x, algorithm="maxima")

[Out] 1/cos(x) + tan(x)

mupad [B] time = 2.97, size = 10, normalized size = 2.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x) + 1)/cos(x)^2,x)

[Out] -2/(tan(x/2) - 1)

sympy [A] time = 2.18, size = 7, normalized size = 1.40

$$\tan(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2*(1+sin(x)),x)

[Out] tan(x) + 1/cos(x)

$$3.914 \quad \int \left(10x^9 \cos \left(x^5 \log(x) \right) - x^{10} \left(x^4 + 5x^4 \log(x) \right) \sin \left(x^5 \log(x) \right) \right) dx$$

Optimal. Leaf size=11

$$x^{10} \cos \left(x^5 \log(x) \right)$$

[Out] $x^{10} \cos(x^5 \ln(x))$

Rubi [F] time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(10x^9 \cos \left(x^5 \log(x) \right) - x^{10} \left(x^4 + 5x^4 \log(x) \right) \sin \left(x^5 \log(x) \right) \right) dx$$

Verification is Not applicable to the result.

[In] `Int[10*x^9*Cos[x^5*Log[x]] - x^10*(x^4 + 5*x^4*Log[x])*Sin[x^5*Log[x]], x]`

[Out] `10*Defer[Int][x^9*Cos[x^5*Log[x]], x] - Defer[Int][x^14*Sin[x^5*Log[x]], x] - 5*Defer[Int][x^14*Log[x]*Sin[x^5*Log[x]], x]`

Rubi steps

$$\begin{aligned} \int \left(10x^9 \cos \left(x^5 \log(x) \right) - x^{10} \left(x^4 + 5x^4 \log(x) \right) \sin \left(x^5 \log(x) \right) \right) dx &= 10 \int x^9 \cos \left(x^5 \log(x) \right) dx - \int x^{10} \left(x^4 + 5x^4 \log(x) \right) \sin \left(x^5 \log(x) \right) dx \\ &= 10 \int x^9 \cos \left(x^5 \log(x) \right) dx - \int x^{14} (1 + 5 \log(x)) \sin \left(x^5 \log(x) \right) dx \\ &= 10 \int x^9 \cos \left(x^5 \log(x) \right) dx - \int \left(x^{14} \sin \left(x^5 \log(x) \right) + 5x^{14} \log(x) \sin \left(x^5 \log(x) \right) \right) dx \\ &= - \left(5 \int x^{14} \log(x) \sin \left(x^5 \log(x) \right) dx \right) + \dots \end{aligned}$$

Mathematica [A] time = 0.31, size = 11, normalized size = 1.00

$$x^{10} \cos \left(x^5 \log(x) \right)$$

Antiderivative was successfully verified.

[In] `Integrate[10*x^9*Cos[x^5*Log[x]] - x^10*(x^4 + 5*x^4*Log[x])*Sin[x^5*Log[x]], x]`

[Out] `x^10*Cos[x^5*Log[x]]`

fricas [A] time = 0.74, size = 11, normalized size = 1.00

$$x^{10} \cos \left(x^5 \log(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(10*x^9*cos(x^5*log(x))-x^10*(x^4+5*x^4*log(x))*sin(x^5*log(x)), x, algorithm="fricas")`

[Out] `x^10*cos(x^5*log(x))`

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(10*x^9*cos(x^5*log(x))-x^10*(x^4+5*x^4*log(x))*sin(x^5*log(x)),x,
algorithm="giac")

[Out] Timed out

maple [C] time = 0.19, size = 30, normalized size = 2.73

$$\frac{x^{10}x^{ix^5}}{2} + \frac{x^{10}x^{-ix^5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(10*x^9*cos(x^5*ln(x))-x^10*(x^4+5*x^4*ln(x))*sin(x^5*ln(x)),x)

[Out] 1/2*x^10*x^(I*x^5)+1/2*x^10/(x^(I*x^5))

maxima [A] time = 0.74, size = 11, normalized size = 1.00

$$x^{10} \cos(x^5 \log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(10*x^9*cos(x^5*log(x))-x^10*(x^4+5*x^4*log(x))*sin(x^5*log(x)),x,
algorithm="maxima")

[Out] x^10*cos(x^5*log(x))

mupad [B] time = 3.16, size = 11, normalized size = 1.00

$$x^{10} \cos(x^5 \ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(10*x^9*cos(x^5*log(x)) - x^10*sin(x^5*log(x))*(5*x^4*log(x) + x^4),x)

[Out] x^10*cos(x^5*log(x))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int (-10x^9 \cos(x^5 \log(x))) dx - \int x^{14} \sin(x^5 \log(x)) dx - \int 5x^{14} \log(x) \sin(x^5 \log(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(10*x**9*cos(x**5*ln(x))-x**10*(x**4+5*x**4*ln(x))*sin(x**5*ln(x)),x)

[Out] -Integral(-10*x**9*cos(x**5*log(x)), x) - Integral(x**14*sin(x**5*log(x)), x) - Integral(5*x**14*log(x)*sin(x**5*log(x)), x)

$$3.915 \quad \int \cos^2\left(\frac{x}{2}\right) \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$$

Optimal. Leaf size=27

$$\frac{x}{2} - \frac{\cos(x)}{2} - \log\left(\cos\left(\frac{x}{2} + \frac{\pi}{4}\right)\right)$$

[Out] 1/2*x-1/2*cos(x)-ln(cos(1/4*Pi+1/2*x))

Rubi [F] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos^2\left(\frac{x}{2}\right) \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$$

Verification is Not applicable to the result.

[In] Int[Cos[x/2]^2*Tan[Pi/4 + x/2], x]

[Out] Defer[Int][Cos[x/2]^2*Tan[Pi/4 + x/2], x]

Rubi steps

$$\int \cos^2\left(\frac{x}{2}\right) \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) dx = \int \cos^2\left(\frac{x}{2}\right) \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$$

Mathematica [A] time = 0.17, size = 24, normalized size = 0.89

$$\frac{1}{2} \left(x - \cos(x) - \log(\cos(x)) + 2 \tanh^{-1}\left(\cot\left(\frac{x}{2}\right)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x/2]^2*Tan[Pi/4 + x/2], x]

[Out] (x + 2*ArcTanh[Cot[x/2]] - Cos[x] - Log[Cos[x]])/2

fricas [A] time = 1.58, size = 27, normalized size = 1.00

$$-\cos\left(\frac{1}{2}x\right)^2 + \frac{1}{2}x - \frac{1}{2} \log\left(-2 \cos\left(\frac{1}{2}x\right) \sin\left(\frac{1}{2}x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*x)^2*tan(1/4*pi+1/2*x), x, algorithm="fricas")

[Out] -cos(1/2*x)^2 + 1/2*x - 1/2*log(-2*cos(1/2*x)*sin(1/2*x) + 1)

giac [B] time = 3.12, size = 93, normalized size = 3.44

$$\frac{x \tan\left(\frac{1}{2}x\right)^2 - \log\left(\frac{2\left(\tan\left(\frac{1}{2}x\right)^2 - 2 \tan\left(\frac{1}{2}x\right) + 1\right)}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right) \tan\left(\frac{1}{2}x\right)^2 + \tan\left(\frac{1}{2}x\right)^2 + x - \log\left(\frac{2\left(\tan\left(\frac{1}{2}x\right)^2 - 2 \tan\left(\frac{1}{2}x\right) + 1\right)}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right) - 1}{2\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(1/2*x)^2*tan(1/4*pi+1/2*x), x, algorithm="giac")

[Out] $\frac{1}{2}(x \tan(1/2x)^2 - \log(2(\tan(1/2x)^2 - 2\tan(1/2x) + 1)/(\tan(1/2x)^2 + 1)) \tan(1/2x)^2 + \tan(1/2x)^2 + x - \log(2(\tan(1/2x)^2 - 2\tan(1/2x) + 1)/(\tan(1/2x)^2 + 1)) - 1)/(\tan(1/2x)^2 + 1)$

maple [A] time = 0.36, size = 22, normalized size = 0.81

$$\frac{x}{2} - \frac{\cos(x)}{2} + \frac{\ln(\sec(x) + \tan(x))}{2} - \frac{\ln(\cos(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(1/2*x)^2*tan(1/4*Pi+1/2*x),x)`

[Out] $\frac{1}{2}x - \frac{1}{2}\cos(x) + \frac{1}{2}\ln(\sec(x) + \tan(x)) - \frac{1}{2}\ln(\cos(x))$

maxima [B] time = 0.64, size = 74, normalized size = 2.74

$$\frac{2x \cos(x)^2 + 2x \sin(x)^2 - \cos(2x) \cos(x) - 2(\cos(x)^2 + \sin(x)^2) \log(\cos(x)^2 + \sin(x)^2 - 2 \sin(x) + 1) - \sin(2x)}{4(\cos(x)^2 + \sin(x)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*x)^2*tan(1/4*pi+1/2*x),x, algorithm="maxima")`

[Out] $\frac{1}{4}(2x \cos(x)^2 + 2x \sin(x)^2 - \cos(2x) \cos(x) - 2(\cos(x)^2 + \sin(x)^2) \log(\cos(x)^2 + \sin(x)^2 - 2 \sin(x) + 1) - \sin(2x) \sin(x) - \cos(x))/(\cos(x)^2 + \sin(x)^2)$

mupad [B] time = 0.48, size = 38, normalized size = 1.41

$$-2 \ln\left(e^{\frac{\pi i i}{2}} e^{x i i} + 1\right) \sin\left(\frac{\pi}{4}\right)^2 + x e^{\frac{\pi i i}{4}} \sin\left(\frac{\pi}{4}\right) - \frac{\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x/2)^2*tan(Pi/4 + x/2),x)`

[Out] $x \sin(\pi/4) \exp((\pi * 1i)/4) - 2 \sin(\pi/4)^2 \log(\exp((\pi * 1i)/2) \exp(x * 1i) + 1) - \cos(x)/2$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^2\left(\frac{x}{2}\right) \tan\left(\frac{x}{2} + \frac{\pi}{4}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(1/2*x)**2*tan(1/4*pi+1/2*x),x)`

[Out] `Integral(cos(x/2)**2*tan(x/2 + pi/4), x)`

3.916 $\int (2 + 3x)^2 \sin^3(x) dx$

Optimal. Leaf size=65

$$\frac{2}{3}(3x+2) \sin^3(x) + 4(3x+2) \sin(x) - \frac{2}{3} \cos^3(x) - \frac{2}{3}(3x+2)^2 \cos(x) + 14 \cos(x) - \frac{1}{3}(3x+2)^2 \sin^2(x) \cos(x)$$

[Out] $14*\cos(x) - 2/3*(2+3*x)^2*\cos(x) - 2/3*\cos(x)^3 + 4*(2+3*x)*\sin(x) - 1/3*(2+3*x)^2*\cos(x)*\sin(x)^2 + 2/3*(2+3*x)*\sin(x)^3$

Rubi [A] time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3311, 3296, 2638, 2633}

$$\frac{2}{3}(3x+2) \sin^3(x) + 4(3x+2) \sin(x) - \frac{2}{3} \cos^3(x) - \frac{2}{3}(3x+2)^2 \cos(x) + 14 \cos(x) - \frac{1}{3}(3x+2)^2 \sin^2(x) \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(2 + 3*x)^2*\text{Sin}[x]^3, x]$

[Out] $14*\text{Cos}[x] - (2*(2 + 3*x)^2*\text{Cos}[x])/3 - (2*\text{Cos}[x]^3)/3 + 4*(2 + 3*x)*\text{Sin}[x] - ((2 + 3*x)^2*\text{Cos}[x]*\text{Sin}[x]^2)/3 + (2*(2 + 3*x)*\text{Sin}[x]^3)/3$

Rule 2633

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{(n-1)/2}], x], x], \text{Cos}[c + d*x]]] /; \text{FreeQ}[\{c, d\}, x] \&\& \text{IGtQ}[(n-1)/2, 0]$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3296

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

Rule 3311

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*((b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(d*m*(c + d*x)^{(m-1)}*(b*\text{Sin}[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n-1))/n, \text{Int}[(c + d*x)^m*(b*\text{Sin}[e + f*x])^{(n-2)}, x], x] - \text{Dist}[(d^2*m*(m-1))/(f^2*n^2), \text{Int}[(c + d*x)^{(m-2)}*(b*\text{Sin}[e + f*x])^n, x], x] - \text{Simp}[(b*(c + d*x)^m*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(n-1)})/(f*n), x]) /; \text{FreeQ}[\{b, c, d, e, f\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{GtQ}[m, 1]$

Rubi steps

$$\begin{aligned} \int (2 + 3x)^2 \sin^3(x) dx &= -\frac{1}{3}(2 + 3x)^2 \cos(x) \sin^2(x) + \frac{2}{3}(2 + 3x) \sin^3(x) + \frac{2}{3} \int (2 + 3x)^2 \sin(x) dx - 2 \int \sin^3(x) dx \\ &= -\frac{2}{3}(2 + 3x)^2 \cos(x) - \frac{1}{3}(2 + 3x)^2 \cos(x) \sin^2(x) + \frac{2}{3}(2 + 3x) \sin^3(x) + 2 \text{Subst}\left(\int (1 - u^2) \sin(u) du, u = 2 + 3x\right) \\ &= 2 \cos(x) - \frac{2}{3}(2 + 3x)^2 \cos(x) - \frac{2 \cos^3(x)}{3} + 4(2 + 3x) \sin(x) - \frac{1}{3}(2 + 3x)^2 \cos(x) \sin^2(x) \\ &= 14 \cos(x) - \frac{2}{3}(2 + 3x)^2 \cos(x) - \frac{2 \cos^3(x)}{3} + 4(2 + 3x) \sin(x) - \frac{1}{3}(2 + 3x)^2 \cos(x) \sin^2(x) \end{aligned}$$

Mathematica [A] time = 0.09, size = 50, normalized size = 0.77

$$\frac{1}{12} \left(-9(9x^2 + 12x - 14) \cos(x) + (9x^2 + 12x + 2) \cos(3x) - 2(3x + 2)(\sin(3x) - 27 \sin(x)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x)^2*Sin[x]^3,x]

[Out] (-9*(-14 + 12*x + 9*x^2)*Cos[x] + (2 + 12*x + 9*x^2)*Cos[3*x] - 2*(2 + 3*x)*(-27*Sin[x] + Sin[3*x]))/12

fricas [A] time = 1.65, size = 50, normalized size = 0.77

$$\frac{1}{3} \left((9x^2 + 12x + 2) \cos(x)^3 - (9x^2 + 12x - 10) \cos(x) - \frac{2}{3} \left((3x + 2) \cos(x)^2 - 21x - 14 \right) \sin(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^2*sin(x)^3,x, algorithm="fricas")

[Out] 1/3*(9*x^2 + 12*x + 2)*cos(x)^3 - (9*x^2 + 12*x - 10)*cos(x) - 2/3*((3*x + 2)*cos(x)^2 - 21*x - 14)*sin(x)

giac [A] time = 1.98, size = 51, normalized size = 0.78

$$\frac{1}{12} \left((9x^2 + 12x + 2) \cos(3x) - \frac{3}{4} (9x^2 + 12x - 14) \cos(x) - \frac{1}{6} (3x + 2) \sin(3x) + \frac{9}{2} (3x + 2) \sin(x) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^2*sin(x)^3,x, algorithm="giac")

[Out] 1/12*(9*x^2 + 12*x + 2)*cos(3*x) - 3/4*(9*x^2 + 12*x - 14)*cos(x) - 1/6*(3*x + 2)*sin(3*x) + 9/2*(3*x + 2)*sin(x)

maple [A] time = 0.03, size = 62, normalized size = 0.95

$$-3x^2 \left(2 + \sin^2(x) \right) \cos(x) + 12 \cos(x) + 12x \sin(x) + 2 \left(\sin^3(x) \right) x - \frac{2 \left(2 + \sin^2(x) \right) \cos(x)}{3} - 4x \left(2 + \sin^2(x) \right) \cos(x) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+3*x)^2*sin(x)^3,x)

[Out] -3*x^2*(2+sin(x)^2)*cos(x)+12*cos(x)+12*x*sin(x)+2*sin(x)^3*x-2/3*(2+sin(x)^2)*cos(x)-4*x*(2+sin(x)^2)*cos(x)+4/3*sin(x)^3+8*sin(x)

maxima [A] time = 0.33, size = 66, normalized size = 1.02

$$\frac{4}{3} \cos(x)^3 + \frac{1}{12} \left(9x^2 - 2 \right) \cos(3x) + x \cos(3x) - \frac{27}{4} \left(x^2 - 2 \right) \cos(x) - 9x \cos(x) - \frac{1}{2} x \sin(3x) + \frac{27}{2} x \sin(x) - 4 \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^2*sin(x)^3,x, algorithm="maxima")

[Out] 4/3*cos(x)^3 + 1/12*(9*x^2 - 2)*cos(3*x) + x*cos(3*x) - 27/4*(x^2 - 2)*cos(x) - 9*x*cos(x) - 1/2*x*sin(3*x) + 27/2*x*sin(x) - 4*cos(x) - 1/3*sin(3*x) + 9*sin(x)

mupad [B] time = 3.03, size = 65, normalized size = 1.00

$$10 \cos(x) + \frac{28 \sin(x)}{3} - 9x^2 \cos(x) + 4x \cos(x)^3 + \frac{2 \cos(x)^3}{3} + 3x^2 \cos(x)^3 - \frac{4 \cos(x)^2 \sin(x)}{3} - 12x \cos(x) + 14x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^3*(3*x + 2)^2,x)`

[Out] $10*\cos(x) + (28*\sin(x))/3 - 9*x^2*\cos(x) + 4*x*\cos(x)^3 + (2*\cos(x)^3)/3 + 3*x^2*\cos(x)^3 - (4*\cos(x)^2*\sin(x))/3 - 12*x*\cos(x) + 14*x*\sin(x) - 2*x*\cos(x)^2*\sin(x)$

sympy [A] time = 1.22, size = 100, normalized size = 1.54

$-9x^2 \sin^2(x) \cos(x) - 6x^2 \cos^3(x) + 14x \sin^3(x) - 12x \sin^2(x) \cos(x) + 12x \sin(x) \cos^2(x) - 8x \cos^3(x) + \frac{28 \sin^3(x)}{3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+3*x)**2*sin(x)**3,x)`

[Out] $-9*x**2*\sin(x)**2*\cos(x) - 6*x**2*\cos(x)**3 + 14*x*\sin(x)**3 - 12*x*\sin(x)*2*\cos(x) + 12*x*\sin(x)*\cos(x)**2 - 8*x*\cos(x)**3 + 28*\sin(x)**3/3 + 10*\sin(x)**2*\cos(x) + 8*\sin(x)*\cos(x)**2 + 32*\cos(x)**3/3$

3.917 $\int \sec^{1+m}(x) \sin(x) dx$

Optimal. Leaf size=8

$$\frac{\sec^m(x)}{m}$$

[Out] $\sec(x)^m/m$

Rubi [A] time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2622, 30}

$$\frac{\sec^m(x)}{m}$$

Antiderivative was successfully verified.

[In] `Int[Sec[x]^(1 + m)*Sin[x], x]`

[Out] `Sec[x]^m/m`

Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

Rule 2622

`Int[csc[(e_) + (f_)*(x_)]^(n_)*((a_)*sec[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])`

Rubi steps

$$\begin{aligned} \int \sec^{1+m}(x) \sin(x) dx &= \text{Subst} \left(\int x^{-1+m} dx, x, \sec(x) \right) \\ &= \frac{\sec^m(x)}{m} \end{aligned}$$

Mathematica [A] time = 0.01, size = 8, normalized size = 1.00

$$\frac{\sec^m(x)}{m}$$

Antiderivative was successfully verified.

[In] `Integrate[Sec[x]^(1 + m)*Sin[x], x]`

[Out] `Sec[x]^m/m`

fricas [A] time = 0.92, size = 14, normalized size = 1.75

$$\frac{1}{\cos(x)} \frac{\cos^{m+1}(x)}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(x)^(1+m)*sin(x), x, algorithm="fricas")`

[Out] `(1/cos(x))^(m + 1)*cos(x)/m`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec(x)^{m+1} \sin(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^(1+m)*sin(x),x, algorithm="giac")

[Out] integrate(sec(x)^(m + 1)*sin(x), x)

maple [A] time = 0.08, size = 11, normalized size = 1.38

$$\frac{\left(\frac{1}{\cos(x)}\right)^m}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^(1+m)*sin(x),x)

[Out] 1/m*(1/cos(x))^m

maxima [A] time = 0.31, size = 10, normalized size = 1.25

$$\frac{\cos(x)^{-m}}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^(1+m)*sin(x),x, algorithm="maxima")

[Out] cos(x)^(-m)/m

mupad [B] time = 0.14, size = 10, normalized size = 1.25

$$\frac{\left(\frac{1}{\cos(x)}\right)^m}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*(1/cos(x))^(m + 1),x)

[Out] (1/cos(x))^m/m

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x) \sec^{m+1}(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**(1+m)*sin(x),x)

[Out] Integral(sin(x)*sec(x)**(m + 1), x)

3.918 $\int \cos^n(a + bx) \sin^{-2-n}(a + bx) dx$

Optimal. Leaf size=32

$$-\frac{\sin^{-n-1}(a + bx) \cos^{n+1}(a + bx)}{b(n + 1)}$$

[Out] $-\cos(b*x+a)^{(1+n)}*\sin(b*x+a)^{(-1-n)}/b/(1+n)$

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2563}

$$-\frac{\sin^{-n-1}(a + bx) \cos^{n+1}(a + bx)}{b(n + 1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[a + b*x]^n*\text{Sin}[a + b*x]^{(-2 - n)}, x]$

[Out] $-\left(\text{Cos}[a + b*x]^{(1 + n)}*\text{Sin}[a + b*x]^{(-1 - n)}\right)/(b*(1 + n))$

Rule 2563

$\text{Int}[(\cos[(e_.) + (f_.)*(x_.)]*(b_.))^{(n_.)}*((a_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}, x_Symbol] :> \text{Simp}[\left(\frac{a*\text{Sin}[e + f*x]^{(m + 1)}*(b*\text{Cos}[e + f*x]^{(n + 1)})}{a*b*f*(m + 1)}\right), x] /; \text{FreeQ}\{a, b, e, f, m, n\}, x \ \&\& \ \text{EqQ}[m + n + 2, 0] \ \& \ \text{NeQ}[m, -1]$

Rubi steps

$$\int \cos^n(a + bx) \sin^{-2-n}(a + bx) dx = -\frac{\cos^{1+n}(a + bx) \sin^{-1-n}(a + bx)}{b(1 + n)}$$

Mathematica [A] time = 0.08, size = 32, normalized size = 1.00

$$-\frac{\sin^{-n-1}(a + bx) \cos^{n+1}(a + bx)}{b(n + 1)}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Cos}[a + b*x]^n*\text{Sin}[a + b*x]^{(-2 - n)}, x]$

[Out] $-\left(\text{Cos}[a + b*x]^{(1 + n)}*\text{Sin}[a + b*x]^{(-1 - n)}\right)/(b*(1 + n))$

fricas [A] time = 0.95, size = 41, normalized size = 1.28

$$-\frac{\cos(bx + a)^n \sin(bx + a)^{-n-2} \cos(bx + a) \sin(bx + a)}{bn + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\cos(b*x+a)^n*\sin(b*x+a)^{(-2-n)}, x, \text{algorithm}=\text{"fricas"})$

[Out] $-\cos(b*x + a)^n*\sin(b*x + a)^{(-n - 2)}*\cos(b*x + a)*\sin(b*x + a)/(b*n + b)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(bx + a)^n \sin(bx + a)^{-n-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x, algorithm="giac")

[Out] integrate(cos(b*x + a)^n*sin(b*x + a)^(-n - 2), x)

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int (\cos^n (bx + a)) (\sin^{-2-n} (bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x)

[Out] int(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x)

maxima [B] time = 0.44, size = 125, normalized size = 3.91

$$\frac{2 \left(\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2} - 1 \right) (\cos(bx+a) + 1) e^{\left(n \log\left(\frac{\sin(bx+a)}{\cos(bx+a)+1} + 1 \right) - n \log\left(\frac{\sin(bx+a)}{\cos(bx+a)+1} \right) + n \log\left(-\frac{\sin(bx+a)}{\cos(bx+a)+1} + 1 \right) \right)}{(2^{n+2}n + 2^{n+2})b \sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x, algorithm="maxima")

[Out] 2*(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)*e^(n*log(sin(b*x + a)/(cos(b*x + a) + 1) + 1) - n*log(sin(b*x + a)/(cos(b*x + a) + 1)) + n*log(-sin(b*x + a)/(cos(b*x + a) + 1) + 1))/((2^(n + 2)*n + 2^(n + 2))*b*sin(b*x + a))

mupad [B] time = 3.43, size = 45, normalized size = 1.41

$$\frac{\cos(a + bx)^n \sin(2a + 2bx)}{2b \sin(a + bx)^n \sin(a + bx)^2 (n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b*x)^n/sin(a + b*x)^(n + 2),x)

[Out] -(cos(a + b*x)^n*sin(2*a + 2*b*x))/(2*b*sin(a + b*x)^n*sin(a + b*x)^2*(n + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{-n-2}(a + bx) \cos^n(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**n*sin(b*x+a)**(-2-n),x)

[Out] Integral(sin(a + b*x)**(-n - 2)*cos(a + b*x)**n, x)

$$3.919 \quad \int \frac{1}{\sec(x) + \sin(x) \tan(x)} dx$$

Optimal. Leaf size=3

$$\tan^{-1}(\sin(x))$$

[Out] arctan(sin(x))

Rubi [A] time = 0.03, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4397, 3190, 203}

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[(Sec[x] + Sin[x]*Tan[x])^(-1), x]

[Out] ArcTan[Sin[x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 4397

Int[u_, x_Symbol] := Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sec(x) + \sin(x) \tan(x)} dx &= \int \frac{\cos(x)}{1 + \sin^2(x)} dx \\ &= \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \sin(x) \right) \\ &= \tan^{-1}(\sin(x)) \end{aligned}$$

Mathematica [A] time = 0.02, size = 3, normalized size = 1.00

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x] + Sin[x]*Tan[x])^(-1), x]

[Out] ArcTan[Sin[x]]

fricas [A] time = 1.42, size = 3, normalized size = 1.00

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+sin(x)*tan(x)),x, algorithm="fricas")

[Out] arctan(sin(x))

giac [A] time = 0.12, size = 3, normalized size = 1.00

arctan(sin(x))

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+sin(x)*tan(x)),x, algorithm="giac")

[Out] arctan(sin(x))

maple [A] time = 0.15, size = 4, normalized size = 1.33

arctan(sin(x))

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sec(x)+sin(x)*tan(x)),x)

[Out] arctan(sin(x))

maxima [B] time = 0.33, size = 45, normalized size = 15.00

$\frac{1}{2} \arctan(\sin(2x) + 2 \sin(x), \cos(2x) + 2 \cos(x) - 1) - \frac{1}{2} \arctan(\sin(2x) - 2 \sin(x), \cos(2x) - 2 \cos(x) - 1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+sin(x)*tan(x)),x, algorithm="maxima")

[Out] 1/2*arctan2(sin(2*x) + 2*sin(x), cos(2*x) + 2*cos(x) - 1) - 1/2*arctan2(sin(2*x) - 2*sin(x), cos(2*x) - 2*cos(x) - 1)

mupad [B] time = 3.19, size = 26, normalized size = 8.67

$$\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)^3}{2} + \frac{5 \tan\left(\frac{x}{2}\right)}{2}\right) - \operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x)*tan(x) + 1/cos(x)),x)

[Out] atan((5*tan(x/2))/2 + tan(x/2)^3/2) - atan(tan(x/2)/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(x) \tan(x) + \sec(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sec(x)+sin(x)*tan(x)),x)

[Out] Integral(1/(sin(x)*tan(x) + sec(x)), x)

3.920 $\int (a + bx + cx^2) \sin(x) dx$

Optimal. Leaf size=35

$$-a \cos(x) + b \sin(x) - bx \cos(x) - cx^2 \cos(x) + 2cx \sin(x) + 2c \cos(x)$$

[Out] $-a*\cos(x)+2*c*\cos(x)-b*x*\cos(x)-c*x^2*\cos(x)+b*\sin(x)+2*c*x*\sin(x)$

Rubi [A] time = 0.07, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6742, 2638, 3296, 2637}

$$-a \cos(x) + b \sin(x) - bx \cos(x) - cx^2 \cos(x) + 2cx \sin(x) + 2c \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*x + c*x^2)*\text{Sin}[x], x]$

[Out] $-(a*\text{Cos}[x]) + 2*c*\text{Cos}[x] - b*x*\text{Cos}[x] - c*x^2*\text{Cos}[x] + b*\text{Sin}[x] + 2*c*x*\text{Sin}[x]$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$
FreeQ[{c, d}, x]

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$ FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 6742

$\text{Int}[u_, x_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /;$ SumQ[v]

Rubi steps

$$\begin{aligned} \int (a + bx + cx^2) \sin(x) dx &= \int (a \sin(x) + bx \sin(x) + cx^2 \sin(x)) dx \\ &= a \int \sin(x) dx + b \int x \sin(x) dx + c \int x^2 \sin(x) dx \\ &= -a \cos(x) - bx \cos(x) - cx^2 \cos(x) + b \int \cos(x) dx + (2c) \int x \cos(x) dx \\ &= -a \cos(x) - bx \cos(x) - cx^2 \cos(x) + b \sin(x) + 2cx \sin(x) - (2c) \int \sin(x) dx \\ &= -a \cos(x) + 2c \cos(x) - bx \cos(x) - cx^2 \cos(x) + b \sin(x) + 2cx \sin(x) \end{aligned}$$

Mathematica [A] time = 0.04, size = 32, normalized size = 0.91

$$-a \cos(x) + b \sin(x) - bx \cos(x) - c(x^2 - 2) \cos(x) + 2cx \sin(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)*Sin[x], x]

[Out] -(a*cos[x]) - b*x*cos[x] - c*(-2 + x^2)*cos[x] + b*sin[x] + 2*c*x*sin[x]

fricas [A] time = 0.59, size = 27, normalized size = 0.77

$$-(cx^2 + bx + a - 2c)\cos(x) + (2cx + b)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*sin(x), x, algorithm="fricas")

[Out] -(c*x^2 + b*x + a - 2*c)*cos(x) + (2*c*x + b)*sin(x)

giac [A] time = 0.12, size = 27, normalized size = 0.77

$$-(cx^2 + bx + a - 2c)\cos(x) + (2cx + b)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*sin(x), x, algorithm="giac")

[Out] -(c*x^2 + b*x + a - 2*c)*cos(x) + (2*c*x + b)*sin(x)

maple [A] time = 0.03, size = 36, normalized size = 1.03

$$c(-x^2 \cos(x) + 2 \cos(x) + 2x \sin(x)) + b(\sin(x) - x \cos(x)) - a \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)*sin(x), x)

[Out] c*(-x^2*cos(x)+2*cos(x)+2*x*sin(x))+b*(sin(x)-x*cos(x))-a*cos(x)

maxima [A] time = 0.32, size = 35, normalized size = 1.00

$$-(x \cos(x) - \sin(x))b - ((x^2 - 2) \cos(x) - 2x \sin(x))c - a \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*sin(x), x, algorithm="maxima")

[Out] -(x*cos(x) - sin(x))*b - ((x^2 - 2)*cos(x) - 2*x*sin(x))*c - a*cos(x)

mupad [B] time = 0.06, size = 34, normalized size = 0.97

$$b \sin(x) - \cos(x) (a - 2c) - bx \cos(x) + 2cx \sin(x) - cx^2 \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)*(a + b*x + c*x^2), x)

[Out] b*sin(x) - cos(x)*(a - 2*c) - b*x*cos(x) + 2*c*x*sin(x) - c*x^2*cos(x)

sympy [A] time = 0.33, size = 39, normalized size = 1.11

$$-a \cos(x) - bx \cos(x) + b \sin(x) - cx^2 \cos(x) + 2cx \sin(x) + 2c \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)*sin(x), x)

[Out] -a*cos(x) - b*x*cos(x) + b*sin(x) - c*x**2*cos(x) + 2*c*x*sin(x) + 2*c*cos(x)

$$3.921 \quad \int \frac{\sin(x^5)}{x} dx$$

Optimal. Leaf size=8

$$\frac{\text{Si}(x^5)}{5}$$

[Out] 1/5*Si(x^5)

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3375}

$$\frac{\text{Si}(x^5)}{5}$$

Antiderivative was successfully verified.

[In] Int[Sin[x^5]/x,x]

[Out] SinIntegral[x^5]/5

Rule 3375

Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinIntegral[d*x^n]/n, x] / ; FreeQ[{d, n}, x]

Rubi steps

$$\int \frac{\sin(x^5)}{x} dx = \frac{\text{Si}(x^5)}{5}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{\text{Si}(x^5)}{5}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x^5]/x,x]

[Out] SinIntegral[x^5]/5

fricas [A] time = 0.73, size = 6, normalized size = 0.75

$$\frac{1}{5} \text{Si}(x^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x^5)/x,x, algorithm="fricas")

[Out] 1/5*sin_integral(x^5)

giac [A] time = 0.13, size = 6, normalized size = 0.75

$$\frac{1}{5} \text{Si}(x^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x^5)/x,x, algorithm="giac")

[Out] 1/5*sin_integral(x^5)

maple [A] time = 0.03, size = 7, normalized size = 0.88

$$\frac{\text{Si}(x^5)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x^5)/x,x)

[Out] 1/5*Si(x^5)

maxima [C] time = 0.36, size = 17, normalized size = 2.12

$$-\frac{1}{10}i\text{Ei}(ix^5) + \frac{1}{10}i\text{Ei}(-ix^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x^5)/x,x, algorithm="maxima")

[Out] -1/10*I*Ei(I*x^5) + 1/10*I*Ei(-I*x^5)

mupad [F] time = 0.00, size = -1, normalized size = -0.12

$$\frac{\text{sinint}(x^5)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x^5)/x,x)

[Out] sinint(x^5)/5

sympy [A] time = 0.64, size = 5, normalized size = 0.62

$$\frac{\text{Si}(x^5)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x**5)/x,x)

[Out] Si(x**5)/5

$$3.922 \quad \int \frac{\sin(2^x)}{1+2^x} dx$$

Optimal. Leaf size=37

$$\frac{\sin(1)\text{Ci}(1+2^x)}{\log(2)} + \frac{\text{Si}(2^x)}{\log(2)} - \frac{\cos(1)\text{Si}(1+2^x)}{\log(2)}$$

[Out] Si(2^x)/ln(2)-cos(1)*Si(1+2^x)/ln(2)+Ci(1+2^x)*sin(1)/ln(2)

Rubi [A] time = 0.17, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2282, 6742, 3299, 3303, 3302}

$$\frac{\sin(1)\text{CosIntegral}(2^x+1)}{\log(2)} + \frac{\text{Si}(2^x)}{\log(2)} - \frac{\cos(1)\text{Si}(1+2^x)}{\log(2)}$$

Antiderivative was successfully verified.

[In] Int[Sin[2^x]/(1+2^x),x]

[Out] (CosIntegral[1+2^x]*Sin[1])/Log[2] + SinIntegral[2^x]/Log[2] - (Cos[1]*SinIntegral[1+2^x])/Log[2]

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(2^x)}{1+2^x} dx &= \frac{\text{Subst}\left(\int \frac{\sin(x)}{x(1+x)} dx, x, 2^x\right)}{\log(2)} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{x} - \frac{\sin(x)}{1+x}\right) dx, x, 2^x\right)}{\log(2)} \\
&= \frac{\text{Subst}\left(\int \frac{\sin(x)}{x} dx, x, 2^x\right)}{\log(2)} - \frac{\text{Subst}\left(\int \frac{\sin(x)}{1+x} dx, x, 2^x\right)}{\log(2)} \\
&= \frac{\text{Si}(2^x)}{\log(2)} - \frac{\cos(1) \text{Subst}\left(\int \frac{\sin(1+x)}{1+x} dx, x, 2^x\right)}{\log(2)} + \frac{\sin(1) \text{Subst}\left(\int \frac{\cos(1+x)}{1+x} dx, x, 2^x\right)}{\log(2)} \\
&= \frac{\text{Ci}(1+2^x) \sin(1)}{\log(2)} + \frac{\text{Si}(2^x)}{\log(2)} - \frac{\cos(1) \text{Si}(1+2^x)}{\log(2)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 29, normalized size = 0.78

$$\frac{\sin(1)\text{Ci}(1+2^x) + \text{Si}(2^x) - \cos(1)\text{Si}(1+2^x)}{\log(2)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[2^x]/(1+2^x),x]

[Out] (CosIntegral[1+2^x]*Sin[1] + SinIntegral[2^x] - Cos[1]*SinIntegral[1+2^x])/Log[2]

fricas [A] time = 0.85, size = 43, normalized size = 1.16

$$\frac{\text{Ci}(2^x+1) \sin(1) + \text{Ci}(-2^x-1) \sin(1) - 2 \cos(1) \text{Si}(2^x+1) + 2 \text{Si}(2^x)}{2 \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2^x)/(1+2^x),x, algorithm="fricas")

[Out] 1/2*(cos_integral(2^x+1)*sin(1) + cos_integral(-2^x-1)*sin(1) - 2*cos(1)*sin_integral(2^x+1) + 2*sin_integral(2^x))/log(2)

giac [A] time = 0.13, size = 29, normalized size = 0.78

$$\frac{\text{Ci}(2^x+1) \sin(1) - \cos(1) \text{Si}(2^x+1) + \text{Si}(2^x)}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(2^x)/(1+2^x),x, algorithm="giac")

[Out] (cos_integral(2^x+1)*sin(1) - cos(1)*sin_integral(2^x+1) + sin_integral(2^x))/log(2)

maple [A] time = 0.03, size = 38, normalized size = 1.03

$$\frac{\text{Si}(2^x)}{\ln(2)} - \frac{\cos(1) \text{Si}(1+2^x)}{\ln(2)} + \frac{\text{Ci}(1+2^x) \sin(1)}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2^x)/(1+2^x),x)

[Out] $\text{Si}(2^x)/\ln(2) - \cos(1) * \text{Si}(1+2^x)/\ln(2) + \text{Ci}(1+2^x) * \sin(1)/\ln(2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(2^x)}{2^x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(2^x)/(1+2^x),x, algorithm="maxima")`

[Out] `integrate(sin(2^x)/(2^x + 1), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin(2^x)}{2^x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(2^x)/(2^x + 1),x)`

[Out] `int(sin(2^x)/(2^x + 1), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(2^x)}{2^x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(2**x)/(1+2**x),x)`

[Out] `Integral(sin(2**x)/(2**x + 1), x)`

$$3.923 \quad \int x \cos(2x^2) \sin^{\frac{3}{4}}(2x^2) dx$$

Optimal. Leaf size=14

$$\frac{1}{7} \sin^{\frac{7}{4}}(2x^2)$$

[Out] 1/7*sin(2*x^2)^(7/4)

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3441}

$$\frac{1}{7} \sin^{\frac{7}{4}}(2x^2)$$

Antiderivative was successfully verified.

[In] Int[x*Cos[2*x^2]*Sin[2*x^2]^(3/4),x]

[Out] Sin[2*x^2]^(7/4)/7

Rule 3441

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int x \cos(2x^2) \sin^{\frac{3}{4}}(2x^2) dx = \frac{1}{7} \sin^{\frac{7}{4}}(2x^2)$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{7} \sin^{\frac{7}{4}}(2x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[2*x^2]*Sin[2*x^2]^(3/4),x]

[Out] Sin[2*x^2]^(7/4)/7

fricas [A] time = 0.87, size = 10, normalized size = 0.71

$$\frac{1}{7} \sin(2x^2)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x^2)*sin(2*x^2)^(3/4),x, algorithm="fricas")

[Out] 1/7*sin(2*x^2)^(7/4)

giac [A] time = 0.16, size = 10, normalized size = 0.71

$$\frac{1}{7} \sin(2x^2)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x^2)*sin(2*x^2)^(3/4),x, algorithm="giac")

[Out] 1/7*sin(2*x^2)^(7/4)

maple [A] time = 0.02, size = 11, normalized size = 0.79

$$\frac{\left(\sin^{\frac{7}{4}}(2x^2)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(2*x^2)*sin(2*x^2)^(3/4),x)

[Out] 1/7*sin(2*x^2)^(7/4)

maxima [A] time = 0.32, size = 10, normalized size = 0.71

$$\frac{1}{7} \sin(2x^2)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x^2)*sin(2*x^2)^(3/4),x, algorithm="maxima")

[Out] 1/7*sin(2*x^2)^(7/4)

mupad [B] time = 3.14, size = 41, normalized size = 2.93

$$\frac{\cos(2x^2)^2 \sin(2x^2)^{7/4} {}_2F_1\left(\frac{1}{8}, 1; 2; \cos(2x^2)^2\right)}{8\left(\sin(2x^2)^2\right)^{7/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(2*x^2)*sin(2*x^2)^(3/4),x)

[Out] -(cos(2*x^2)^2*sin(2*x^2)^(7/4)*hypergeom([1/8, 1], 2, cos(2*x^2)^2))/(8*(sin(2*x^2)^2)^(7/8))

sympy [A] time = 84.15, size = 10, normalized size = 0.71

$$\frac{\sin^{\frac{7}{4}}(2x^2)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x**2)*sin(2*x**2)**(3/4),x)

[Out] sin(2*x**2)**(7/4)/7

3.924 $\int x \sec^2(x^2) \tan^2(x^2) dx$

Optimal. Leaf size=10

$$\frac{1}{6} \tan^3(x^2)$$

[Out] 1/6*tan(x^2)^3

Rubi [A] time = 0.04, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {6686}

$$\frac{1}{6} \tan^3(x^2)$$

Antiderivative was successfully verified.

[In] Int[x*Sec[x^2]^2*Tan[x^2]^2,x]

[Out] Tan[x^2]^3/6

Rule 6686

Int[(u_)*(y_)^(m_.), x_Symbol] :=> With[{q = DerivativeDivides[y, u, x]}, Simp[(q*y^(m + 1))/(m + 1), x] /; !FalseQ[q] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\int x \sec^2(x^2) \tan^2(x^2) dx = \frac{1}{6} \tan^3(x^2)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{6} \tan^3(x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[x^2]^2*Tan[x^2]^2,x]

[Out] Tan[x^2]^3/6

fricas [B] time = 0.43, size = 20, normalized size = 2.00

$$\frac{(\cos(x^2)^2 - 1) \sin(x^2)}{6 \cos(x^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x^2)^2*tan(x^2)^2,x, algorithm="fricas")

[Out] -1/6*(cos(x^2)^2 - 1)*sin(x^2)/cos(x^2)^3

giac [A] time = 0.14, size = 8, normalized size = 0.80

$$\frac{1}{6} \tan^3(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x^2)^2*tan(x^2)^2,x, algorithm="giac")

[Out] 1/6*tan(x^2)^3

maple [A] time = 0.10, size = 15, normalized size = 1.50

$$\frac{\sin^3(x^2)}{6 \cos(x^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sec(x^2)^2*tan(x^2)^2,x)

[Out] 1/6*sin(x^2)^3/cos(x^2)^3

maxima [A] time = 0.32, size = 8, normalized size = 0.80

$$\frac{1}{6} \tan(x^2)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x^2)^2*tan(x^2)^2,x, algorithm="maxima")

[Out] 1/6*tan(x^2)^3

mupad [B] time = 3.09, size = 19, normalized size = 1.90

$$\frac{\tan(x^2)}{6 \cos(x^2)^2} - \frac{\tan(x^2)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*tan(x^2)^2)/cos(x^2)^2,x)

[Out] tan(x^2)/(6*cos(x^2)^2) - tan(x^2)/6

sympy [A] time = 1.04, size = 7, normalized size = 0.70

$$\frac{\tan^3(x^2)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(x**2)**2*tan(x**2)**2,x)

[Out] tan(x**2)**3/6

$$3.925 \quad \int x^2 \cos^7(a + bx^3) \sin(a + bx^3) dx$$

Optimal. Leaf size=17

$$-\frac{\cos^8(a + bx^3)}{24b}$$

[Out] $-1/24*\cos(b*x^3+a)^8/b$

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3442}

$$-\frac{\cos^8(a + bx^3)}{24b}$$

Antiderivative was successfully verified.

[In] Int[x^2*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]

[Out] -Cos[a + b*x^3]^8/(24*b)

Rule 3442

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)], x_Symbol] := -Simp[Cos[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int x^2 \cos^7(a + bx^3) \sin(a + bx^3) dx = -\frac{\cos^8(a + bx^3)}{24b}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$-\frac{\cos^8(a + bx^3)}{24b}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]

[Out] -1/24*Cos[a + b*x^3]^8/b

fricas [A] time = 1.70, size = 15, normalized size = 0.88

$$-\frac{\cos(bx^3 + a)^8}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="fricas")

[Out] -1/24*cos(b*x^3 + a)^8/b

giac [A] time = 0.18, size = 15, normalized size = 0.88

$$-\frac{\cos(bx^3 + a)^8}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="giac")

[Out] -1/24*cos(b*x^3 + a)^8/b

maple [A] time = 0.05, size = 16, normalized size = 0.94

$$-\frac{\cos^8(bx^3 + a)}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x)

[Out] -1/24*cos(b*x^3+a)^8/b

maxima [A] time = 0.34, size = 15, normalized size = 0.88

$$-\frac{\cos(bx^3 + a)^8}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="maxima")

[Out] -1/24*cos(b*x^3 + a)^8/b

mupad [B] time = 3.26, size = 56, normalized size = 3.29

$$-\frac{56 \cos(2bx^3 + 2a) + 28 \cos(4bx^3 + 4a) + 8 \cos(6bx^3 + 6a) + \cos(8bx^3 + 8a)}{3072b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*cos(a + b*x^3)^7*sin(a + b*x^3),x)

[Out] -(56*cos(2*a + 2*b*x^3) + 28*cos(4*a + 4*b*x^3) + 8*cos(6*a + 6*b*x^3) + cos(8*a + 8*b*x^3))/(3072*b)

sympy [A] time = 20.86, size = 27, normalized size = 1.59

$$\begin{cases} -\frac{\cos^8(a+bx^3)}{24b} & \text{for } b \neq 0 \\ \frac{x^3 \sin(a) \cos^7(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*cos(b*x**3+a)**7*sin(b*x**3+a),x)

[Out] Piecewise((-cos(a + b*x**3)**8/(24*b), Ne(b, 0)), (x**3*sin(a)*cos(a)**7/3, True))

3.926 $\int x^5 \cos^7(a + bx^3) \sin(a + bx^3) dx$

Optimal. Leaf size=129

$$\frac{\sin(a + bx^3) \cos^7(a + bx^3)}{192b^2} + \frac{7 \sin(a + bx^3) \cos^5(a + bx^3)}{1152b^2} + \frac{35 \sin(a + bx^3) \cos^3(a + bx^3)}{4608b^2} + \frac{35 \sin(a + bx^3)}{3072b^2}$$

[Out] $35/3072*x^3/b-1/24*x^3*\cos(b*x^3+a)^8/b+35/3072*\cos(b*x^3+a)*\sin(b*x^3+a)/b^2+35/4608*\cos(b*x^3+a)^3*\sin(b*x^3+a)/b^2+7/1152*\cos(b*x^3+a)^5*\sin(b*x^3+a)/b^2+1/192*\cos(b*x^3+a)^7*\sin(b*x^3+a)/b^2$

Rubi [A] time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3444, 3380, 2635, 8}

$$\frac{\sin(a + bx^3) \cos^7(a + bx^3)}{192b^2} + \frac{7 \sin(a + bx^3) \cos^5(a + bx^3)}{1152b^2} + \frac{35 \sin(a + bx^3) \cos^3(a + bx^3)}{4608b^2} + \frac{35 \sin(a + bx^3)}{3072b^2}$$

Antiderivative was successfully verified.

[In] Int[x^5*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]

[Out] $(35*x^3)/(3072*b) - (x^3*\cos[a + b*x^3]^8)/(24*b) + (35*\cos[a + b*x^3]*\sin[a + b*x^3])/(3072*b^2) + (35*\cos[a + b*x^3]^3*\sin[a + b*x^3])/(4608*b^2) + (7*\cos[a + b*x^3]^5*\sin[a + b*x^3])/(1152*b^2) + (\cos[a + b*x^3]^7*\sin[a + b*x^3])/(192*b^2)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Sin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3380

Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n] - 1)*(a + b*Cos[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m+1)/n]] && (EqQ[p, 1] || EqQ[m, n-1] || (IntegerQ[p] && GtQ[Simplify[(m+1)/n], 0]))

Rule 3444

Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)], x_Symbol] := -Simp[(x^(m-n+1)*Cos[a + b*x^n]^(p+1))/(b*n*(p+1)), x] + Dist[(m-n+1)/(b*n*(p+1)), Int[x^(m-n)*Cos[a + b*x^n]^(p+1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m+1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int x^5 \cos^7(a + bx^3) \sin(a + bx^3) dx &= -\frac{x^3 \cos^8(a + bx^3)}{24b} + \frac{\int x^2 \cos^8(a + bx^3) dx}{8b} \\
&= -\frac{x^3 \cos^8(a + bx^3)}{24b} + \frac{\text{Subst}\left(\int \cos^8(a + bx) dx, x, x^3\right)}{24b} \\
&= -\frac{x^3 \cos^8(a + bx^3)}{24b} + \frac{\cos^7(a + bx^3) \sin(a + bx^3)}{192b^2} + \frac{7 \text{Subst}\left(\int \cos^6(a + bx) dx, x, x^3\right)}{192b^2} \\
&= -\frac{x^3 \cos^8(a + bx^3)}{24b} + \frac{7 \cos^5(a + bx^3) \sin(a + bx^3)}{1152b^2} + \frac{\cos^7(a + bx^3) \sin(a + bx^3)}{192b^2} \\
&= -\frac{x^3 \cos^8(a + bx^3)}{24b} + \frac{35 \cos^3(a + bx^3) \sin(a + bx^3)}{4608b^2} + \frac{7 \cos^5(a + bx^3) \sin(a + bx^3)}{1152b^2} \\
&= -\frac{x^3 \cos^8(a + bx^3)}{24b} + \frac{35 \cos(a + bx^3) \sin(a + bx^3)}{3072b^2} + \frac{35 \cos^3(a + bx^3) \sin(a + bx^3)}{4608b^2} \\
&= \frac{35x^3}{3072b} - \frac{x^3 \cos^8(a + bx^3)}{24b} + \frac{35 \cos(a + bx^3) \sin(a + bx^3)}{3072b^2} + \frac{35 \cos^3(a + bx^3) \sin(a + bx^3)}{4608b^2}
\end{aligned}$$

Mathematica [A] time = 0.57, size = 120, normalized size = 0.93

$$\frac{672 \sin(2(a + bx^3)) + 168 \sin(4(a + bx^3)) + 32 \sin(6(a + bx^3)) + 3 \sin(8(a + bx^3)) - 1344bx^3 \cos(2(a + bx^3))}{73728b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]

[Out] (-1344*b*x^3*Cos[2*(a + b*x^3)] - 672*b*x^3*Cos[4*(a + b*x^3)] - 192*b*x^3*Cos[6*(a + b*x^3)] - 24*b*x^3*Cos[8*(a + b*x^3)] + 672*Sin[2*(a + b*x^3)] + 168*Sin[4*(a + b*x^3)] + 32*Sin[6*(a + b*x^3)] + 3*Sin[8*(a + b*x^3)])/(73728*b^2)

fricas [A] time = 1.09, size = 85, normalized size = 0.66

$$\frac{384 bx^3 \cos(bx^3 + a)^8 - 105 bx^3 - (48 \cos(bx^3 + a)^7 + 56 \cos(bx^3 + a)^5 + 70 \cos(bx^3 + a)^3 + 105 \cos(bx^3 + a)) \sin(bx^3 + a)}{9216 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="fricas")

[Out] -1/9216*(384*b*x^3*cos(b*x^3 + a)^8 - 105*b*x^3 - (48*cos(b*x^3 + a)^7 + 56*cos(b*x^3 + a)^5 + 70*cos(b*x^3 + a)^3 + 105*cos(b*x^3 + a))*sin(b*x^3 + a)/b^2

giac [A] time = 0.44, size = 126, normalized size = 0.98

$$\frac{24 bx^3 \cos(8 bx^3 + 8 a) + 192 bx^3 \cos(6 bx^3 + 6 a) + 672 bx^3 \cos(4 bx^3 + 4 a) + 1344 bx^3 \cos(2 bx^3 + 2 a) - 3 \sin(8 bx^3 + 8 a) - 32 \sin(6 bx^3 + 6 a) - 168 \sin(4 bx^3 + 4 a) - 672 \sin(2 bx^3 + 2 a)}{73728 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="giac")

[Out] -1/73728*(24*b*x^3*cos(8*b*x^3 + 8*a) + 192*b*x^3*cos(6*b*x^3 + 6*a) + 672*b*x^3*cos(4*b*x^3 + 4*a) + 1344*b*x^3*cos(2*b*x^3 + 2*a) - 3*sin(8*b*x^3 + 8*a) - 32*sin(6*b*x^3 + 6*a) - 168*sin(4*b*x^3 + 4*a) - 672*sin(2*b*x^3 + 2*a))/b^2

maple [B] time = 0.95, size = 403, normalized size = 3.12

$$\frac{-\frac{4x^3}{3b} + \frac{4 \tan(bx^3+a)}{3b^2} + \frac{4x^3(\tan^2(bx^3+a))}{3b}}{128 + 128(\tan^2(bx^3+a))} + \frac{\frac{\tan(bx^3+a)}{b^2} - \frac{x^3}{b} - \frac{\tan^3(bx^3+a)}{b^2} + \frac{6x^3(\tan^2(bx^3+a))}{b} - \frac{x^3(\tan^4(bx^3+a))}{b}}{128(1 + \tan^2(bx^3+a))^2} + \frac{-6x^3b \cos(bx^3+a)}{128(1 + \tan^2(bx^3+a))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x)

[Out] 1/128*(-4/3*x^3/b+4/3/b^2*tan(b*x^3+a)+4/3*x^3/b*tan(b*x^3+a)^2)/(1+tan(b*x^3+a)^2)+1/128*(1/b^2*tan(b*x^3+a)-x^3/b-1/b^2*tan(b*x^3+a)^3+6*x^3/b*tan(b*x^3+a)^2-x^3/b*tan(b*x^3+a)^4)/(1+tan(b*x^3+a)^2)^2+1/1152*(-6*x^3*b*cos(3*b*x^3+3*a)^2-18*cos(b*x^3+a)^2*b*x^3+12*b*x^3+sin(3*b*x^3+3*a)*cos(3*b*x^3+3*a)+9*cos(b*x^3+a)*sin(b*x^3+a))/b^2+1/128*(-1/6*x^3/b+1/12/b^2*tan(2*b*x^3+2*a)+1/6*x^3/b*tan(2*b*x^3+2*a)^2)/(1+tan(2*b*x^3+2*a)^2)+1/128*(-1/24*x^3/b+1/48/b^2*tan(2*b*x^3+2*a)-1/48/b^2*tan(2*b*x^3+2*a)^3+1/4*x^3/b*tan(2*b*x^3+2*a)^2-1/24*x^3/b*tan(2*b*x^3+2*a)^4)/(1+tan(2*b*x^3+2*a)^2)^2

maxima [A] time = 0.34, size = 126, normalized size = 0.98

$$\frac{24bx^3 \cos(8bx^3 + 8a) + 192bx^3 \cos(6bx^3 + 6a) + 672bx^3 \cos(4bx^3 + 4a) + 1344bx^3 \cos(2bx^3 + 2a) - 3 \sin(8bx^3 + 8a) - 32 \sin(6bx^3 + 6a) - 168 \sin(4bx^3 + 4a) - 672 \sin(2bx^3 + 2a)}{73728b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="maxima")

[Out] -1/73728*(24*b*x^3*cos(8*b*x^3 + 8*a) + 192*b*x^3*cos(6*b*x^3 + 6*a) + 672*b*x^3*cos(4*b*x^3 + 4*a) + 1344*b*x^3*cos(2*b*x^3 + 2*a) - 3*sin(8*b*x^3 + 8*a) - 32*sin(6*b*x^3 + 6*a) - 168*sin(4*b*x^3 + 4*a) - 672*sin(2*b*x^3 + 2*a))/b^2

mupad [B] time = 3.45, size = 147, normalized size = 1.14

$$\frac{168 \sin(2bx^3 + 2a) + 42 \sin(4bx^3 + 4a) + 8 \sin(6bx^3 + 6a) + \frac{3 \sin(8bx^3 + 8a)}{4} + 336bx^3(2 \sin(bx^3 + a))}{18432b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*cos(a + b*x^3)^7*sin(a + b*x^3),x)

[Out] (168*sin(2*a + 2*b*x^3) + 42*sin(4*a + 4*b*x^3) + 8*sin(6*a + 6*b*x^3) + (3*sin(8*a + 8*b*x^3))/4 + 336*b*x^3*(2*sin(a + b*x^3)^2 - 1) + 168*b*x^3*(2*sin(2*a + 2*b*x^3)^2 - 1) + 48*b*x^3*(2*sin(3*a + 3*b*x^3)^2 - 1) + 6*b*x^3*(2*sin(4*a + 4*b*x^3)^2 - 1))/(18432*b^2)

sympy [A] time = 74.97, size = 241, normalized size = 1.87

$$\left\{ \begin{array}{l} \frac{35x^3 \sin^8(ax^3)}{3072b} + \frac{35x^3 \sin^6(ax^3) \cos^2(ax^3)}{768b} + \frac{35x^3 \sin^4(ax^3) \cos^4(ax^3)}{512b} + \frac{35x^3 \sin^2(ax^3) \cos^6(ax^3)}{768b} - \frac{31x^3 \cos^8(ax^3)}{1024b} \\ \frac{x^6 \sin(a) \cos^7(a)}{6} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*cos(b*x**3+a)**7*sin(b*x**3+a),x)

[Out] Piecewise((35*x**3*sin(a + b*x**3)**8/(3072*b) + 35*x**3*sin(a + b*x**3)**6*cos(a + b*x**3)**2/(768*b) + 35*x**3*sin(a + b*x**3)**4*cos(a + b*x**3)**4/(512*b) + 35*x**3*sin(a + b*x**3)**2*cos(a + b*x**3)**6/(768*b) - 31*x**3*cos(a + b*x**3)**8/(1024*b), True)

```
cos(a + b*x**3)**8/(1024*b) + 35*sin(a + b*x**3)**7*cos(a + b*x**3)/(3072*b
**2) + 385*sin(a + b*x**3)**5*cos(a + b*x**3)**3/(9216*b**2) + 511*sin(a +
b*x**3)**3*cos(a + b*x**3)**5/(9216*b**2) + 31*sin(a + b*x**3)*cos(a + b*x*
*3)**7/(1024*b**2), Ne(b, 0)), (x**6*sin(a)*cos(a)**7/6, True))
```

3.927 $\int x^5 \sec^7(a + bx^3) \tan(a + bx^3) dx$

Optimal. Leaf size=110

$$\frac{5 \tanh^{-1}(\sin(a + bx^3))}{336b^2} - \frac{\tan(a + bx^3) \sec^5(a + bx^3)}{126b^2} - \frac{5 \tan(a + bx^3) \sec^3(a + bx^3)}{504b^2} - \frac{5 \tan(a + bx^3) \sec(a + bx^3)}{336b^2}$$

[Out] $-5/336*\operatorname{arctanh}(\sin(b*x^3+a))/b^2+1/21*x^3*\sec(b*x^3+a)^7/b-5/336*\sec(b*x^3+a)*\tan(b*x^3+a)/b^2-5/504*\sec(b*x^3+a)^3*\tan(b*x^3+a)/b^2-1/126*\sec(b*x^3+a)^5*\tan(b*x^3+a)/b^2$

Rubi [A] time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3757, 4204, 3768, 3770}

$$\frac{5 \tanh^{-1}(\sin(a + bx^3))}{336b^2} - \frac{\tan(a + bx^3) \sec^5(a + bx^3)}{126b^2} - \frac{5 \tan(a + bx^3) \sec^3(a + bx^3)}{504b^2} - \frac{5 \tan(a + bx^3) \sec(a + bx^3)}{336b^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^5*\operatorname{Sec}[a + b*x^3]^7*\operatorname{Tan}[a + b*x^3], x]$

[Out] $(-5*\operatorname{ArcTanh}[\operatorname{Sin}[a + b*x^3]])/(336*b^2) + (x^3*\operatorname{Sec}[a + b*x^3]^7)/(21*b) - (5*\operatorname{Sec}[a + b*x^3]*\operatorname{Tan}[a + b*x^3])/(336*b^2) - (5*\operatorname{Sec}[a + b*x^3]^3*\operatorname{Tan}[a + b*x^3])/ (504*b^2) - (\operatorname{Sec}[a + b*x^3]^5*\operatorname{Tan}[a + b*x^3])/(126*b^2)$

Rule 3757

$\operatorname{Int}[(x_)^{(m_.)}*\operatorname{Sec}[(a_.) + (b_.)*(x_)^{(n_.)}]^{(p_.)}*\operatorname{Tan}[(a_.) + (b_.)*(x_)^{(n_.)}]^{(q_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(x^{(m-n+1)}*\operatorname{Sec}[a + b*x^n]^p)/(b*n*p), x] - \operatorname{Dist}[(m-n+1)/(b*n*p), \operatorname{Int}[x^{(m-n)}*\operatorname{Sec}[a + b*x^n]^p, x], x] /;$ FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]

Rule 3768

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*\operatorname{Cos}[c + d*x] * (b*\operatorname{Csc}[c + d*x])^{(n-1)})/(d*(n-1)), x] + \operatorname{Dist}[(b^2*(n-2))/(n-1), \operatorname{Int}[(b*\operatorname{Csc}[c + d*x])^{(n-2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /;$ FreeQ[{c, d}, x]

Rule 4204

$\operatorname{Int}[(x_)^{(m_.)}*((a_.) + (b_.)*\operatorname{Sec}[(c_.) + (d_.)*(x_)^{(n_.)}])^{(p_.)}, x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n] - 1)*(a + b*\operatorname{Sec}[c + d*x])^p}, x], x, x^n], x] /;$ FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m+1)/n], 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int x^5 \sec^7(a + bx^3) \tan(a + bx^3) dx &= \frac{x^3 \sec^7(a + bx^3)}{21b} - \frac{\int x^2 \sec^7(a + bx^3) dx}{7b} \\
&= \frac{x^3 \sec^7(a + bx^3)}{21b} - \frac{\text{Subst}\left(\int \sec^7(a + bx) dx, x, x^3\right)}{21b} \\
&= \frac{x^3 \sec^7(a + bx^3)}{21b} - \frac{\sec^5(a + bx^3) \tan(a + bx^3)}{126b^2} - \frac{5 \text{Subst}\left(\int \sec^5(a + bx) dx, x, x^3\right)}{126b} \\
&= \frac{x^3 \sec^7(a + bx^3)}{21b} - \frac{5 \sec^3(a + bx^3) \tan(a + bx^3)}{504b^2} - \frac{\sec^5(a + bx^3) \tan(a + bx^3)}{126b^2} \\
&= \frac{x^3 \sec^7(a + bx^3)}{21b} - \frac{5 \sec(a + bx^3) \tan(a + bx^3)}{336b^2} - \frac{5 \sec^3(a + bx^3) \tan(a + bx^3)}{504b^2} \\
&= -\frac{5 \tanh^{-1}(\sin(a + bx^3))}{336b^2} + \frac{x^3 \sec^7(a + bx^3)}{21b} - \frac{5 \sec(a + bx^3) \tan(a + bx^3)}{336b^2}
\end{aligned}$$

Mathematica [B] time = 0.87, size = 352, normalized size = 3.20

$$\sec^7(a + bx^3) \left(-566 \sin(2(a + bx^3)) - 200 \sin(4(a + bx^3)) - 30 \sin(6(a + bx^3)) + 105 \cos(5(a + bx^3)) \log(\cos((a + bx^3)/2)) - \sin((a + bx^3)/2) \right) + 15 \cos(7(a + bx^3)) \log(\cos((a + bx^3)/2)) - \sin((a + bx^3)/2) + 525 \cos(a + bx^3) (\log(\cos((a + bx^3)/2)) - \sin((a + bx^3)/2)) - \log(\cos((a + bx^3)/2) + \sin((a + bx^3)/2)) + 315 \cos(3(a + bx^3)) (\log(\cos((a + bx^3)/2)) - \sin((a + bx^3)/2)) - \log(\cos((a + bx^3)/2) + \sin((a + bx^3)/2)) - 105 \cos(5(a + bx^3)) \log(\cos((a + bx^3)/2) + \sin((a + bx^3)/2)) - 15 \cos(7(a + bx^3)) \log(\cos((a + bx^3)/2) + \sin((a + bx^3)/2)) - 566 \sin(2(a + bx^3)) - 200 \sin(4(a + bx^3)) - 30 \sin(6(a + bx^3)) \Big) / (64512 b^2)$$

Antiderivative was successfully verified.

[In] Integrate[x^5*Sec[a + b*x^3]^7*Tan[a + b*x^3], x]

[Out] (Sec[a + b*x^3]^7*(3072*b*x^3 + 105*Cos[5*(a + b*x^3)]*Log[Cos[(a + b*x^3)/2]] - Sin[(a + b*x^3)/2]] + 15*Cos[7*(a + b*x^3)]*Log[Cos[(a + b*x^3)/2]] - Sin[(a + b*x^3)/2]] + 525*Cos[a + b*x^3]*(Log[Cos[(a + b*x^3)/2]] - Sin[(a + b*x^3)/2]] - Log[Cos[(a + b*x^3)/2] + Sin[(a + b*x^3)/2]]) + 315*Cos[3*(a + b*x^3)]*(Log[Cos[(a + b*x^3)/2]] - Sin[(a + b*x^3)/2]] - Log[Cos[(a + b*x^3)/2] + Sin[(a + b*x^3)/2]]) - 105*Cos[5*(a + b*x^3)]*Log[Cos[(a + b*x^3)/2] + Sin[(a + b*x^3)/2]] - 15*Cos[7*(a + b*x^3)]*Log[Cos[(a + b*x^3)/2] + Sin[(a + b*x^3)/2]] - 566*Sin[2*(a + b*x^3)] - 200*Sin[4*(a + b*x^3)] - 30*Sin[6*(a + b*x^3)])) / (64512*b^2)

fricas [A] time = 0.96, size = 115, normalized size = 1.05

$$\frac{15 \cos(bx^3 + a)^7 \log(\sin(bx^3 + a) + 1) - 15 \cos(bx^3 + a)^7 \log(-\sin(bx^3 + a) + 1) - 96 bx^3 + 2 \left(15 \cos(bx^3 + a)^5 + 10 \cos(bx^3 + a)^3 + 8 \cos(bx^3 + a) \right) \sin(bx^3 + a)}{2016 b^2 \cos(bx^3 + a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*sec(b*x^3+a)^7*tan(b*x^3+a), x, algorithm="fricas")

[Out] -1/2016*(15*cos(b*x^3 + a)^7*log(sin(b*x^3 + a) + 1) - 15*cos(b*x^3 + a)^7*log(-sin(b*x^3 + a) + 1) - 96*b*x^3 + 2*(15*cos(b*x^3 + a)^5 + 10*cos(b*x^3 + a)^3 + 8*cos(b*x^3 + a))*sin(b*x^3 + a))/(b^2*cos(b*x^3 + a)^7)

giac [B] time = 1.64, size = 1455, normalized size = 13.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*sec(b*x^3+a)^7*tan(b*x^3+a), x, algorithm="giac")

[Out] -1/2016*(96*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^14 - 96*a*tan(1/2*b*x^3 + 1/2*a)^14 + 15*log(2*(tan(1/2*b*x^3 + 1/2*a))^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1)

$$\begin{aligned} &)/(\tan(1/2*b*x^3 + 1/2*a)^2 + 1))*\tan(1/2*b*x^3 + 1/2*a)^{14} - 15*\log(2*(\tan \\ & (1/2*b*x^3 + 1/2*a)^2 - 2*\tan(1/2*b*x^3 + 1/2*a) + 1)/(\tan(1/2*b*x^3 + 1/2* \\ & a)^2 + 1))*\tan(1/2*b*x^3 + 1/2*a)^{14} + 672*(b*x^3 + a)*\tan(1/2*b*x^3 + 1/2* \\ & a)^{12} - 672*a*\tan(1/2*b*x^3 + 1/2*a)^{12} - 105*\log(2*(\tan(1/2*b*x^3 + 1/2*a) \\ & ^2 + 2*\tan(1/2*b*x^3 + 1/2*a) + 1)/(\tan(1/2*b*x^3 + 1/2*a)^2 + 1))*\tan(1/2* \\ & b*x^3 + 1/2*a)^{12} + 105*\log(2*(\tan(1/2*b*x^3 + 1/2*a)^2 - 2*\tan(1/2*b*x^3 + \\ & 1/2*a) + 1)/(\tan(1/2*b*x^3 + 1/2*a)^2 + 1))*\tan(1/2*b*x^3 + 1/2*a)^{12} + 13 \\ & 2*\tan(1/2*b*x^3 + 1/2*a)^{13} + 2016*(b*x^3 + a)*\tan(1/2*b*x^3 + 1/2*a)^{10} - \\ & 2016*a*\tan(1/2*b*x^3 + 1/2*a)^{10} + 315*\log(2*(\tan(1/2*b*x^3 + 1/2*a)^2 + 2* \\ & \tan(1/2*b*x^3 + 1/2*a) + 1)/(\tan(1/2*b*x^3 + 1/2*a)^2 + 1))*\tan(1/2*b*x^3 + \\ & 1/2*a)^{10} - 315*\log(2*(\tan(1/2*b*x^3 + 1/2*a)^2 - 2*\tan(1/2*b*x^3 + 1/2*a) \\ & + 1)/(\tan(1/2*b*x^3 + 1/2*a)^2 + 1))*\tan(1/2*b*x^3 + 1/2*a)^{10} - 112*\tan(1 \\ & /2*b*x^3 + 1/2*a)^{11} + 3360*(b*x^3 + a)*\tan(1/2*b*x^3 + 1/2*a)^8 - 3360*a*\tan \\ & (1/2*b*x^3 + 1/2*a)^8 - 525*\log(2*(\tan(1/2*b*x^3 + 1/2*a)^2 + 2*\tan(1/2*b \\ & *x^3 + 1/2*a) + 1)/(\tan(1/2*b*x^3 + 1/2*a)^2 + 1))*\tan(1/2*b*x^3 + 1/2*a)^8 \\ & + 525*\log(2*(\tan(1/2*b*x^3 + 1/2*a)^2 - 2*\tan(1/2*b*x^3 + 1/2*a) + 1)/(\tan \\ & (1/2*b*x^3 + 1/2*a)^2 + 1))*\tan(1/2*b*x^3 + 1/2*a)^8 + 340*\tan(1/2*b*x^3 + \\ & 1/2*a)^9 + 3360*(b*x^3 + a)*\tan(1/2*b*x^3 + 1/2*a)^6 - 3360*a*\tan(1/2*b*x^3 \\ & + 1/2*a)^6 + 525*\log(2*(\tan(1/2*b*x^3 + 1/2*a)^2 + 2*\tan(1/2*b*x^3 + 1/2*a) \\ &) + 1)/(\tan(1/2*b*x^3 + 1/2*a)^2 + 1))*\tan(1/2*b*x^3 + 1/2*a)^6 - 525*\log(2 \\ & *(\tan(1/2*b*x^3 + 1/2*a)^2 - 2*\tan(1/2*b*x^3 + 1/2*a) + 1)/(\tan(1/2*b*x^3 + \\ & 1/2*a)^2 + 1))*\tan(1/2*b*x^3 + 1/2*a)^6 + 2016*(b*x^3 + a)*\tan(1/2*b*x^3 + \\ & 1/2*a)^4 - 2016*a*\tan(1/2*b*x^3 + 1/2*a)^4 - 315*\log(2*(\tan(1/2*b*x^3 + 1/ \\ & 2*a)^2 + 2*\tan(1/2*b*x^3 + 1/2*a) + 1)/(\tan(1/2*b*x^3 + 1/2*a)^2 + 1))*\tan(\\ & 1/2*b*x^3 + 1/2*a)^4 + 315*\log(2*(\tan(1/2*b*x^3 + 1/2*a)^2 - 2*\tan(1/2*b*x^ \\ & 3 + 1/2*a) + 1)/(\tan(1/2*b*x^3 + 1/2*a)^2 + 1))*\tan(1/2*b*x^3 + 1/2*a)^4 - \\ & 340*\tan(1/2*b*x^3 + 1/2*a)^5 + 96*b*x^3 + 672*(b*x^3 + a)*\tan(1/2*b*x^3 + 1 \\ & /2*a)^2 - 672*a*\tan(1/2*b*x^3 + 1/2*a)^2 + 105*\log(2*(\tan(1/2*b*x^3 + 1/2*a) \\ &)^2 + 2*\tan(1/2*b*x^3 + 1/2*a) + 1)/(\tan(1/2*b*x^3 + 1/2*a)^2 + 1))*\tan(1/2 \\ & *b*x^3 + 1/2*a)^2 - 105*\log(2*(\tan(1/2*b*x^3 + 1/2*a)^2 - 2*\tan(1/2*b*x^3 + \\ & 1/2*a) + 1)/(\tan(1/2*b*x^3 + 1/2*a)^2 + 1))*\tan(1/2*b*x^3 + 1/2*a)^2 + 112 \\ & *\tan(1/2*b*x^3 + 1/2*a)^3 - 15*\log(2*(\tan(1/2*b*x^3 + 1/2*a)^2 + 2*\tan(1/2* \\ & b*x^3 + 1/2*a) + 1)/(\tan(1/2*b*x^3 + 1/2*a)^2 + 1)) + 15*\log(2*(\tan(1/2*b*x \\ & ^3 + 1/2*a)^2 - 2*\tan(1/2*b*x^3 + 1/2*a) + 1)/(\tan(1/2*b*x^3 + 1/2*a)^2 + 1 \\ &)) - 132*\tan(1/2*b*x^3 + 1/2*a))/((\tan(1/2*b*x^3 + 1/2*a)^{14} - 7*\tan(1/2*b* \\ & x^3 + 1/2*a)^{12} + 21*\tan(1/2*b*x^3 + 1/2*a)^{10} - 35*\tan(1/2*b*x^3 + 1/2*a)^8 \\ & + 35*\tan(1/2*b*x^3 + 1/2*a)^6 - 21*\tan(1/2*b*x^3 + 1/2*a)^4 + 7*\tan(1/2*b \\ & *x^3 + 1/2*a)^2 - 1)*b^2) \end{aligned}$$

maple [C] time = 0.31, size = 160, normalized size = 1.45

$$\frac{i \left(15 e^{13i(bx^3+a)} - 3072ibx^3 e^{7i(bx^3+a)} + 100 e^{11i(bx^3+a)} + 283 e^{9i(bx^3+a)} - 283 e^{5i(bx^3+a)} - 100 e^{3i(bx^3+a)} - 15 e^{i(bx^3+a)} \right)}{504b^2 \left(e^{2i(bx^3+a)} + 1 \right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x)

[Out] 1/504*I/b^2/(exp(2*I*(b*x^3+a))+1)^7*(15*exp(13*I*(b*x^3+a))-3072*I*b*x^3*exp(7*I*(b*x^3+a))+100*exp(11*I*(b*x^3+a))+283*exp(9*I*(b*x^3+a))-283*exp(5*I*(b*x^3+a))-100*exp(3*I*(b*x^3+a))-15*exp(I*(b*x^3+a)))+5/336/b^2*ln(exp(I*(b*x^3+a))-I)-5/336/b^2*ln(exp(I*(b*x^3+a))+I)

maxima [B] time = 0.86, size = 3830, normalized size = 34.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x, algorithm="maxima")

```
[Out] 1/2016*(4*(3072*b*x^3*cos(7*b*x^3 + 7*a) - 15*sin(13*b*x^3 + 13*a) - 100*si
n(11*b*x^3 + 11*a) - 283*sin(9*b*x^3 + 9*a) + 283*sin(5*b*x^3 + 5*a) + 100*
sin(3*b*x^3 + 3*a) + 15*sin(b*x^3 + a))*cos(14*b*x^3 + 14*a) + 420*(sin(12*
b*x^3 + 12*a) + 3*sin(10*b*x^3 + 10*a) + 5*sin(8*b*x^3 + 8*a) + 5*sin(6*b*x
^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*cos(13*b*x^3 + 13*a)
+ 28*(3072*b*x^3*cos(7*b*x^3 + 7*a) - 100*sin(11*b*x^3 + 11*a) - 283*sin(9
*b*x^3 + 9*a) + 283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 + 3*a) + 15*sin(b*
x^3 + a))*cos(12*b*x^3 + 12*a) + 2800*(3*sin(10*b*x^3 + 10*a) + 5*sin(8*b*x
^3 + 8*a) + 5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a
))*cos(11*b*x^3 + 11*a) + 84*(3072*b*x^3*cos(7*b*x^3 + 7*a) - 283*sin(9*b*x
^3 + 9*a) + 283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 + 3*a) + 15*sin(b*x^3
+ a))*cos(10*b*x^3 + 10*a) + 7924*(5*sin(8*b*x^3 + 8*a) + 5*sin(6*b*x^3 + 6
*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*cos(9*b*x^3 + 9*a) + 140*(
3072*b*x^3*cos(7*b*x^3 + 7*a) + 283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 +
3*a) + 15*sin(b*x^3 + a))*cos(8*b*x^3 + 8*a) + 12288*(35*b*x^3*cos(6*b*x^3
+ 6*a) + 21*b*x^3*cos(4*b*x^3 + 4*a) + 7*b*x^3*cos(2*b*x^3 + 2*a) + b*x^3)*
cos(7*b*x^3 + 7*a) + 140*(283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 + 3*a) +
15*sin(b*x^3 + a))*cos(6*b*x^3 + 6*a) - 7924*(3*sin(4*b*x^3 + 4*a) + sin(2
*b*x^3 + 2*a))*cos(5*b*x^3 + 5*a) + 420*(20*sin(3*b*x^3 + 3*a) + 3*sin(b*x^
3 + a))*cos(4*b*x^3 + 4*a) + 15*(2*(7*cos(12*b*x^3 + 12*a) + 21*cos(10*b*x^
3 + 10*a) + 35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 + 6*a) + 21*cos(4*b*x^3
+ 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(14*b*x^3 + 14*a) + cos(14*b*x^3 + 14
*a)^2 + 14*(21*cos(10*b*x^3 + 10*a) + 35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^
3 + 6*a) + 21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(12*b*x^3 +
12*a) + 49*cos(12*b*x^3 + 12*a)^2 + 42*(35*cos(8*b*x^3 + 8*a) + 35*cos(6*b
*x^3 + 6*a) + 21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(10*b*x^
3 + 10*a) + 441*cos(10*b*x^3 + 10*a)^2 + 70*(35*cos(6*b*x^3 + 6*a) + 21*cos
(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(8*b*x^3 + 8*a) + 1225*cos(8
*b*x^3 + 8*a)^2 + 70*(21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos
(6*b*x^3 + 6*a) + 1225*cos(6*b*x^3 + 6*a)^2 + 42*(7*cos(2*b*x^3 + 2*a) + 1)
*cos(4*b*x^3 + 4*a) + 441*cos(4*b*x^3 + 4*a)^2 + 49*cos(2*b*x^3 + 2*a)^2 +
14*(sin(12*b*x^3 + 12*a) + 3*sin(10*b*x^3 + 10*a) + 5*sin(8*b*x^3 + 8*a) +
5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*sin(14*b*
x^3 + 14*a) + sin(14*b*x^3 + 14*a)^2 + 98*(3*sin(10*b*x^3 + 10*a) + 5*sin(8
*b*x^3 + 8*a) + 5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 +
2*a))*sin(12*b*x^3 + 12*a) + 49*sin(12*b*x^3 + 12*a)^2 + 294*(5*sin(8*b*x^
3 + 8*a) + 5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a)
)*sin(10*b*x^3 + 10*a) + 441*sin(10*b*x^3 + 10*a)^2 + 490*(5*sin(6*b*x^3 +
6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*sin(8*b*x^3 + 8*a) + 1225
*sin(8*b*x^3 + 8*a)^2 + 490*(3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*sin
(6*b*x^3 + 6*a) + 1225*sin(6*b*x^3 + 6*a)^2 + 441*sin(4*b*x^3 + 4*a)^2 + 29
4*sin(4*b*x^3 + 4*a)*sin(2*b*x^3 + 2*a) + 49*sin(2*b*x^3 + 2*a)^2 + 14*cos(
2*b*x^3 + 2*a) + 1)*log((cos(b*x^3 + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x^3
+ 2*a) + sin(b*x^3 + 2*a)^2 + 2*cos(b*x^3 + 2*a)*sin(a) + sin(a)^2)/(cos(b
*x^3 + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x^3 + 2*a) + sin(b*x^3 + 2*a)^2 -
2*cos(b*x^3 + 2*a)*sin(a) + sin(a)^2)) + 4*(3072*b*x^3*sin(7*b*x^3 + 7*a)
+ 15*cos(13*b*x^3 + 13*a) + 100*cos(11*b*x^3 + 11*a) + 283*cos(9*b*x^3 + 9*
a) - 283*cos(5*b*x^3 + 5*a) - 100*cos(3*b*x^3 + 3*a) - 15*cos(b*x^3 + a))*s
in(14*b*x^3 + 14*a) - 60*(7*cos(12*b*x^3 + 12*a) + 21*cos(10*b*x^3 + 10*a)
+ 35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 + 6*a) + 21*cos(4*b*x^3 + 4*a) + 7
*cos(2*b*x^3 + 2*a) + 1)*sin(13*b*x^3 + 13*a) + 28*(3072*b*x^3*sin(7*b*x^3
+ 7*a) + 100*cos(11*b*x^3 + 11*a) + 283*cos(9*b*x^3 + 9*a) - 283*cos(5*b*x^
3 + 5*a) - 100*cos(3*b*x^3 + 3*a) - 15*cos(b*x^3 + a))*sin(12*b*x^3 + 12*a)
- 400*(21*cos(10*b*x^3 + 10*a) + 35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 +
6*a) + 21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*sin(11*b*x^3 + 11*
a) + 84*(3072*b*x^3*sin(7*b*x^3 + 7*a) + 283*cos(9*b*x^3 + 9*a) - 283*cos(5
*b*x^3 + 5*a) - 100*cos(3*b*x^3 + 3*a) - 15*cos(b*x^3 + a))*sin(10*b*x^3 +
10*a) - 1132*(35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 + 6*a) + 21*cos(4*b*x^
3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*sin(9*b*x^3 + 9*a) + 140*(3072*b*x^3*s
```

```

in(7*b*x^3 + 7*a) - 283*cos(5*b*x^3 + 5*a) - 100*cos(3*b*x^3 + 3*a) - 15*cos
s(b*x^3 + a))*sin(8*b*x^3 + 8*a) + 86016*(5*b*x^3*sin(6*b*x^3 + 6*a) + 3*b*
x^3*sin(4*b*x^3 + 4*a) + b*x^3*sin(2*b*x^3 + 2*a))*sin(7*b*x^3 + 7*a) - 140
*(283*cos(5*b*x^3 + 5*a) + 100*cos(3*b*x^3 + 3*a) + 15*cos(b*x^3 + a))*sin(
6*b*x^3 + 6*a) + 1132*(21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*si
n(5*b*x^3 + 5*a) - 420*(20*cos(3*b*x^3 + 3*a) + 3*cos(b*x^3 + a))*sin(4*b*x
^3 + 4*a) + 400*(7*cos(2*b*x^3 + 2*a) + 1)*sin(3*b*x^3 + 3*a) - 2800*cos(3*
b*x^3 + 3*a)*sin(2*b*x^3 + 2*a) - 420*cos(b*x^3 + a)*sin(2*b*x^3 + 2*a) + 4
20*cos(2*b*x^3 + 2*a)*sin(b*x^3 + a) + 60*sin(b*x^3 + a))/(b^2*cos(14*b*x^3
+ 14*a)^2 + 49*b^2*cos(12*b*x^3 + 12*a)^2 + 441*b^2*cos(10*b*x^3 + 10*a)^2
+ 1225*b^2*cos(8*b*x^3 + 8*a)^2 + 1225*b^2*cos(6*b*x^3 + 6*a)^2 + 441*b^2*
cos(4*b*x^3 + 4*a)^2 + 49*b^2*cos(2*b*x^3 + 2*a)^2 + b^2*sin(14*b*x^3 + 14*
a)^2 + 49*b^2*sin(12*b*x^3 + 12*a)^2 + 441*b^2*sin(10*b*x^3 + 10*a)^2 + 122
5*b^2*sin(8*b*x^3 + 8*a)^2 + 1225*b^2*sin(6*b*x^3 + 6*a)^2 + 441*b^2*sin(4*
b*x^3 + 4*a)^2 + 294*b^2*sin(4*b*x^3 + 4*a)*sin(2*b*x^3 + 2*a) + 49*b^2*sin
(2*b*x^3 + 2*a)^2 + 14*b^2*cos(2*b*x^3 + 2*a) + b^2 + 2*(7*b^2*cos(12*b*x^3
+ 12*a) + 21*b^2*cos(10*b*x^3 + 10*a) + 35*b^2*cos(8*b*x^3 + 8*a) + 35*b^2
*cos(6*b*x^3 + 6*a) + 21*b^2*cos(4*b*x^3 + 4*a) + 7*b^2*cos(2*b*x^3 + 2*a)
+ b^2)*cos(14*b*x^3 + 14*a) + 14*(21*b^2*cos(10*b*x^3 + 10*a) + 35*b^2*cos(
8*b*x^3 + 8*a) + 35*b^2*cos(6*b*x^3 + 6*a) + 21*b^2*cos(4*b*x^3 + 4*a) + 7*
b^2*cos(2*b*x^3 + 2*a) + b^2)*cos(12*b*x^3 + 12*a) + 42*(35*b^2*cos(8*b*x^3
+ 8*a) + 35*b^2*cos(6*b*x^3 + 6*a) + 21*b^2*cos(4*b*x^3 + 4*a) + 7*b^2*cos
(2*b*x^3 + 2*a) + b^2)*cos(10*b*x^3 + 10*a) + 70*(35*b^2*cos(6*b*x^3 + 6*a)
+ 21*b^2*cos(4*b*x^3 + 4*a) + 7*b^2*cos(2*b*x^3 + 2*a) + b^2)*cos(8*b*x^3
+ 8*a) + 70*(21*b^2*cos(4*b*x^3 + 4*a) + 7*b^2*cos(2*b*x^3 + 2*a) + b^2)*co
s(6*b*x^3 + 6*a) + 42*(7*b^2*cos(2*b*x^3 + 2*a) + b^2)*cos(4*b*x^3 + 4*a) +
14*(b^2*sin(12*b*x^3 + 12*a) + 3*b^2*sin(10*b*x^3 + 10*a) + 5*b^2*sin(8*b*
x^3 + 8*a) + 5*b^2*sin(6*b*x^3 + 6*a) + 3*b^2*sin(4*b*x^3 + 4*a) + b^2*sin(
2*b*x^3 + 2*a))*sin(14*b*x^3 + 14*a) + 98*(3*b^2*sin(10*b*x^3 + 10*a) + 5*b
^2*sin(8*b*x^3 + 8*a) + 5*b^2*sin(6*b*x^3 + 6*a) + 3*b^2*sin(4*b*x^3 + 4*a)
+ b^2*sin(2*b*x^3 + 2*a))*sin(12*b*x^3 + 12*a) + 294*(5*b^2*sin(8*b*x^3 +
8*a) + 5*b^2*sin(6*b*x^3 + 6*a) + 3*b^2*sin(4*b*x^3 + 4*a) + b^2*sin(2*b*x^
3 + 2*a))*sin(10*b*x^3 + 10*a) + 490*(5*b^2*sin(6*b*x^3 + 6*a) + 3*b^2*sin(
4*b*x^3 + 4*a) + b^2*sin(2*b*x^3 + 2*a))*sin(8*b*x^3 + 8*a) + 490*(3*b^2*si
n(4*b*x^3 + 4*a) + b^2*sin(2*b*x^3 + 2*a))*sin(6*b*x^3 + 6*a))

```

mupad [B] time = 13.42, size = 730, normalized size = 6.64

$$\frac{\frac{8e^{1bx^3+ai}(15bx^3-8i)}{315b^2} - \frac{8e^{3bx^3+a3i}(35bx^3-12i)}{315b^2}}{5e^{2ibx^3+a2i} + 10e^{4ibx^3+a4i} + 10e^{6ibx^3+a6i} + 5e^{8ibx^3+a8i} + e^{10ibx^3+a10i} + 1} + \frac{5 \ln(x^2 (e^{1bx^3+ai} - i))}{336b^2} - \frac{5 \ln(x^2 (e^{1bx^3+ai} - i))}{336b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5*tan(a + b*x^3))/cos(a + b*x^3)^7,x)

```

[Out] (5*log(x^2*(exp(a*1i + b*x^3*1i) - 1i)))/(336*b^2) - ((8*exp(a*1i + b*x^3*1
i)*(15*b*x^3 - 8i))/(315*b^2) - (8*exp(a*3i + b*x^3*3i)*(35*b*x^3 - 12i))/(
315*b^2))/(5*exp(a*2i + b*x^3*2i) + 10*exp(a*4i + b*x^3*4i) + 10*exp(a*6i +
b*x^3*6i) + 5*exp(a*8i + b*x^3*8i) + exp(a*10i + b*x^3*10i) + 1) - (5*log(
x^2*(exp(a*1i + b*x^3*1i) + 1i)))/(336*b^2) - ((16*exp(a*3i + b*x^3*3i)*(5*
b*x^3 - 1i))/(63*b^2) - (16*exp(a*5i + b*x^3*5i)*(7*b*x^3 - 1i))/(63*b^2))/
(6*exp(a*2i + b*x^3*2i) + 15*exp(a*4i + b*x^3*4i) + 20*exp(a*6i + b*x^3*6i)
+ 15*exp(a*8i + b*x^3*8i) + 6*exp(a*10i + b*x^3*10i) + exp(a*12i + b*x^3*1
2i) + 1) - ((64*x^3*exp(a*5i + b*x^3*5i))/(21*b) - (64*x^3*exp(a*7i + b*x^3
*7i))/(21*b))/(7*exp(a*2i + b*x^3*2i) + 21*exp(a*4i + b*x^3*4i) + 35*exp(a*
6i + b*x^3*6i) + 35*exp(a*8i + b*x^3*8i) + 21*exp(a*10i + b*x^3*10i) + 7*ex
p(a*12i + b*x^3*12i) + exp(a*14i + b*x^3*14i) + 1) + (exp(a*1i + b*x^3*1i)*
1i)/(63*b^2*(3*exp(a*2i + b*x^3*2i) + 3*exp(a*4i + b*x^3*4i) + exp(a*6i + b
*x^3*6i) + 1)) + (exp(a*1i + b*x^3*1i)*5i)/(168*b^2*(exp(a*2i + b*x^3*2i) +

```

```
1)) + (exp(a*1i + b*x^3*1i)*5i)/(252*b^2*(2*exp(a*2i + b*x^3*2i) + exp(a*4
i + b*x^3*4i) + 1)) + (2*exp(a*1i + b*x^3*1i)*(60*b*x^3 - 47i))/(315*b^2*(4
*exp(a*2i + b*x^3*2i) + 6*exp(a*4i + b*x^3*4i) + 4*exp(a*6i + b*x^3*6i) + e
xp(a*8i + b*x^3*8i) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \tan(a + bx^3) \sec^7(a + bx^3) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*sec(b*x**3+a)**7*tan(b*x**3+a), x)

[Out] Integral(x**5*tan(a + b*x**3)*sec(a + b*x**3)**7, x)

$$3.928 \quad \int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2} dx$$

Optimal. Leaf size=6

$$-\tan\left(\frac{1}{x}\right)$$

[Out] -tan(1/x)

Rubi [A] time = 0.02, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4204, 3767, 8}

$$-\tan\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

[In] Int[Sec[x^(-1)]^2/x^2,x]

[Out] -Tan[x^(-1)]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 4204

Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2} dx &= -\text{Subst}\left(\int \sec^2(x) dx, x, \frac{1}{x}\right) \\ &= \text{Subst}\left(\int 1 dx, x, -\tan\left(\frac{1}{x}\right)\right) \\ &= -\tan\left(\frac{1}{x}\right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 6, normalized size = 1.00

$$-\tan\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x^(-1)]^2/x^2,x]

[Out] $-\tan[x^{(-1)}]$

fricas [A] time = 1.04, size = 12, normalized size = 2.00

$$-\frac{\sin\left(\frac{1}{x}\right)}{\cos\left(\frac{1}{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(1/x)^2/x^2,x, algorithm="fricas")`

[Out] $-\sin(1/x)/\cos(1/x)$

giac [B] time = 0.13, size = 20, normalized size = 3.33

$$\frac{2 \tan\left(\frac{1}{2x}\right)}{\tan\left(\frac{1}{2x}\right)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(1/x)^2/x^2,x, algorithm="giac")`

[Out] $2*\tan(1/2/x)/(\tan(1/2/x)^2 - 1)$

maple [A] time = 0.04, size = 7, normalized size = 1.17

$$-\tan\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sec(1/x)^2/x^2,x)`

[Out] $-\tan(1/x)$

maxima [B] time = 0.38, size = 36, normalized size = 6.00

$$-\frac{2 \sin\left(\frac{2}{x}\right)}{\cos\left(\frac{2}{x}\right)^2 + \sin\left(\frac{2}{x}\right)^2 + 2 \cos\left(\frac{2}{x}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sec(1/x)^2/x^2,x, algorithm="maxima")`

[Out] $-2*\sin(2/x)/(\cos(2/x)^2 + \sin(2/x)^2 + 2*\cos(2/x) + 1)$

mupad [B] time = 3.03, size = 14, normalized size = 2.33

$$-\frac{2i}{e^{\frac{2i}{x}} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*cos(1/x)^2),x)`

[Out] $-2i/(\exp(2i/x) + 1)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(1/x)**2/x**2,x)

[Out] Integral(sec(1/x)**2/x**2, x)

3.929 $\int 3x^2 \cos(x^3) dx$

Optimal. Leaf size=4

$$\sin(x^3)$$

[Out] $\sin(x^3)$

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {12, 3380, 2637}

$$\sin(x^3)$$

Antiderivative was successfully verified.

[In] $\text{Int}[3*x^2*\text{Cos}[x^3], x]$

[Out] $\text{Sin}[x^3]$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_*) + (d_*)*(x_*)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3380

$\text{Int}[(a_*) + \text{Cos}[(c_*) + (d_*)*(x_*)^{(n_*)}]* (b_*)^{(p_*)}*(x_*)^{(m_*)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*\text{Cos}[c + d*x])^p}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \&\& (\text{EqQ}[p, 1] \parallel \text{EqQ}[m, n-1] \parallel (\text{IntegerQ}[p] \&\& \text{GtQ}[\text{Simplify}[(m+1)/n], 0]))$

Rubi steps

$$\begin{aligned} \int 3x^2 \cos(x^3) dx &= 3 \int x^2 \cos(x^3) dx \\ &= \text{Subst}\left(\int \cos(x) dx, x, x^3\right) \\ &= \sin(x^3) \end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$\sin(x^3)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[3*x^2*\text{Cos}[x^3], x]$

[Out] $\text{Sin}[x^3]$

fricas [A] time = 0.92, size = 4, normalized size = 1.00

$$\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*x^2*cos(x^3),x, algorithm="fricas")

[Out] sin(x^3)

giac [A] time = 0.13, size = 4, normalized size = 1.00

$$\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*x^2*cos(x^3),x, algorithm="giac")

[Out] sin(x^3)

maple [A] time = 0.00, size = 5, normalized size = 1.25

$$\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(3*x^2*cos(x^3),x)

[Out] sin(x^3)

maxima [A] time = 0.32, size = 4, normalized size = 1.00

$$\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*x^2*cos(x^3),x, algorithm="maxima")

[Out] sin(x^3)

mupad [B] time = 2.95, size = 4, normalized size = 1.00

$$\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(3*x^2*cos(x^3),x)

[Out] sin(x^3)

sympy [A] time = 0.27, size = 3, normalized size = 0.75

$$\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*x**2*cos(x**3),x)

[Out] sin(x**3)

3.930 $\int (1 + 2x) \sec^2(1 + 2x) dx$

Optimal. Leaf size=27

$$\frac{1}{2}(2x + 1) \tan(2x + 1) + \frac{1}{2} \log(\cos(2x + 1))$$

[Out] 1/2*ln(cos(1+2*x))+1/2*(1+2*x)*tan(1+2*x)

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4184, 3475}

$$\frac{1}{2}(2x + 1) \tan(2x + 1) + \frac{1}{2} \log(\cos(2x + 1))$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)*Sec[1 + 2*x]^2, x]

[Out] Log[Cos[1 + 2*x]]/2 + ((1 + 2*x)*Tan[1 + 2*x])/2

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4184

Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rubi steps

$$\begin{aligned} \int (1 + 2x) \sec^2(1 + 2x) dx &= \frac{1}{2}(1 + 2x) \tan(1 + 2x) - \int \tan(1 + 2x) dx \\ &= \frac{1}{2} \log(\cos(1 + 2x)) + \frac{1}{2}(1 + 2x) \tan(1 + 2x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.11

$$x \tan(2x + 1) + \frac{1}{2} \tan(2x + 1) + \frac{1}{2} \log(\cos(2x + 1))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)*Sec[1 + 2*x]^2, x]

[Out] Log[Cos[1 + 2*x]]/2 + Tan[1 + 2*x]/2 + x*Tan[1 + 2*x]

fricas [A] time = 0.83, size = 39, normalized size = 1.44

$$\frac{\cos(2x + 1) \log(-\cos(2x + 1)) + (2x + 1) \sin(2x + 1)}{2 \cos(2x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)*sec(1+2*x)^2,x, algorithm="fricas")

[Out] $1/2*(\cos(2*x + 1)*\log(-\cos(2*x + 1)) + (2*x + 1)*\sin(2*x + 1))/\cos(2*x + 1)$

giac [B] time = 0.37, size = 943, normalized size = 34.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)*sec(1+2*x)^2,x, algorithm="giac")`

[Out] $1/4*(\log(4*(\tan(1/2)^4*\tan(x)^8 - 8*\tan(1/2)^3*\tan(x)^7 - 2*\tan(1/2)^2*\tan(x)^6 + 8*\tan(1/2)*\tan(x)^5 + \tan(x)^8 + 8*\tan(1/2)^3*\tan(x)^3 + 36*\tan(1/2)^2*\tan(x)^4 + 8*\tan(1/2)*\tan(x)^5 + \tan(1/2)^4 + 8*\tan(1/2)^3*\tan(x) + 16*\tan(1/2)^2*\tan(x)^2 - 8*\tan(1/2)*\tan(x)^3 - 2*\tan(x)^4 - 2*\tan(1/2)^2 - 8*\tan(1/2)*\tan(x) + 1)/(\tan(1/2)^4 + 2*\tan(1/2)^2 + 1))*\tan(1/2)^2*\tan(x)^2 - 8*x*\tan(1/2)^2*\tan(x) - 8*x*\tan(1/2)*\tan(x)^2 - \log(4*(\tan(1/2)^4*\tan(x)^8 - 8*\tan(1/2)^3*\tan(x)^7 - 2*\tan(1/2)^2*\tan(x)^6 + 8*\tan(1/2)*\tan(x)^5 + \tan(x)^8 + 8*\tan(1/2)^3*\tan(x)^3 + 36*\tan(1/2)^2*\tan(x)^4 + 8*\tan(1/2)*\tan(x)^5 + \tan(1/2)^4 + 8*\tan(1/2)^3*\tan(x) + 16*\tan(1/2)^2*\tan(x)^2 - 8*\tan(1/2)*\tan(x)^3 - 2*\tan(x)^4 - 2*\tan(1/2)^2 - 8*\tan(1/2)*\tan(x) + 1)/(\tan(1/2)^4 + 2*\tan(1/2)^2 + 1))*\tan(1/2)^2 - 4*\log(4*(\tan(1/2)^4*\tan(x)^8 - 8*\tan(1/2)^3*\tan(x)^7 - 2*\tan(1/2)^2*\tan(x)^6 + 8*\tan(1/2)*\tan(x)^5 + \tan(x)^8 + 8*\tan(1/2)^3*\tan(x)^3 + 36*\tan(1/2)^2*\tan(x)^4 + 8*\tan(1/2)*\tan(x)^5 + \tan(1/2)^4 + 8*\tan(1/2)^3*\tan(x) + 16*\tan(1/2)^2*\tan(x)^2 - 8*\tan(1/2)*\tan(x)^3 - 2*\tan(x)^4 - 2*\tan(1/2)^2 - 8*\tan(1/2)*\tan(x) + 1)/(\tan(1/2)^4 + 2*\tan(1/2)^2 + 1))*\tan(1/2)*\tan(x) - 4*\tan(1/2)^2*\tan(x) - \log(4*(\tan(1/2)^4*\tan(x)^8 - 8*\tan(1/2)^3*\tan(x)^7 - 2*\tan(1/2)^2*\tan(x)^6 + 8*\tan(1/2)*\tan(x)^5 + \tan(x)^8 + 8*\tan(1/2)^3*\tan(x)^3 + 36*\tan(1/2)^2*\tan(x)^4 + 8*\tan(1/2)*\tan(x)^5 + \tan(1/2)^4 + 8*\tan(1/2)^3*\tan(x) + 16*\tan(1/2)^2*\tan(x)^2 - 8*\tan(1/2)*\tan(x)^3 - 2*\tan(x)^4 - 2*\tan(1/2)^2 - 8*\tan(1/2)*\tan(x) + 1)/(\tan(1/2)^4 + 2*\tan(1/2)^2 + 1))*\tan(x)^2 - 4*\tan(1/2)*\tan(x)^2 + 8*x*\tan(1/2) + 8*x*\tan(x) + \log(4*(\tan(1/2)^4*\tan(x)^8 - 8*\tan(1/2)^3*\tan(x)^7 - 2*\tan(1/2)^2*\tan(x)^6 + 8*\tan(1/2)*\tan(x)^5 + \tan(x)^8 + 8*\tan(1/2)^3*\tan(x)^3 + 36*\tan(1/2)^2*\tan(x)^4 + 8*\tan(1/2)*\tan(x)^5 + \tan(1/2)^4 + 8*\tan(1/2)^3*\tan(x) + 16*\tan(1/2)^2*\tan(x)^2 - 8*\tan(1/2)*\tan(x)^3 - 2*\tan(x)^4 - 2*\tan(1/2)^2 - 8*\tan(1/2)*\tan(x) + 1)/(\tan(1/2)^4 + 2*\tan(1/2)^2 + 1)) + 4*\tan(1/2) + 4*\tan(x))/(\tan(1/2)^2*\tan(x)^2 - \tan(1/2)^2 - 4*\tan(1/2)*\tan(x) - \tan(x)^2 + 1)$

maple [A] time = 0.04, size = 24, normalized size = 0.89

$$\frac{\ln(\cos(1+2x))}{2} + \frac{(1+2x)\tan(1+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+2*x)*sec(1+2*x)^2,x)`

[Out] $1/2*\ln(\cos(1+2*x))+1/2*(1+2*x)*\tan(1+2*x)$

maxima [B] time = 0.45, size = 98, normalized size = 3.63

$$\frac{(\cos(4x+2)^2 + \sin(4x+2)^2 + 2\cos(4x+2) + 1)\log(\cos(4x+2)^2 + \sin(4x+2)^2 + 2\cos(4x+2) + 1)}{4(\cos(4x+2)^2 + \sin(4x+2)^2 + 2\cos(4x+2) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)*sec(1+2*x)^2,x, algorithm="maxima")`

[Out] $\frac{1}{4} * ((\cos(4*x + 2)^2 + \sin(4*x + 2)^2 + 2*\cos(4*x + 2) + 1) * \log(\cos(4*x + 2)^2 + \sin(4*x + 2)^2 + 2*\cos(4*x + 2) + 1) + 4*(2*x + 1)*\sin(4*x + 2)) / (\cos(4*x + 2)^2 + \sin(4*x + 2)^2 + 2*\cos(4*x + 2) + 1)$

mupad [B] time = 0.10, size = 23, normalized size = 0.85

$$\frac{\ln(\cos(2x+1))}{2} + \frac{\tan(2x+1)(2x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + 1)/cos(2*x + 1)^2, x)`

[Out] `log(cos(2*x + 1))/2 + (tan(2*x + 1)*(2*x + 1))/2`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1) \sec^2(2x+1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)*sec(1+2*x)**2, x)`

[Out] `Integral((2*x + 1)*sec(2*x + 1)**2, x)`

$$3.931 \quad \int \left(\frac{x^4}{b\sqrt{x^3+3\sin(ax+bx)}} + \frac{x^2 \cos(ax+bx)}{\sqrt{x^3+3\sin(ax+bx)}} + \frac{4x\sqrt{x^3+3\sin(ax+bx)}}{3b} \right) dx$$

Optimal. Leaf size=26

$$\frac{2x^2\sqrt{3\sin(ax+bx)+x^3}}{3b}$$

[Out] $2/3*x^2*(x^3+3*\sin(b*x+a))^{(1/2)}/b$

Rubi [F] time = 0.81, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(ax+bx)}} + \frac{x^2 \cos(ax+bx)}{\sqrt{x^3+3\sin(ax+bx)}} + \frac{4x\sqrt{x^3+3\sin(ax+bx)}}{3b} \right) dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[x^4/(b*\text{Sqrt}[x^3+3*\text{Sin}[a+b*x]])+(x^2*\text{Cos}[a+b*x])/ \text{Sqrt}[x^3+3*\text{Sin}[a+b*x]]+(4*x*\text{Sqrt}[x^3+3*\text{Sin}[a+b*x]])/(3*b),x]$

[Out] $\text{Defer}[\text{Int}[x^4/\text{Sqrt}[x^3+3*\text{Sin}[a+b*x]],x]/b+\text{Defer}[\text{Int}[(x^2*\text{Cos}[a+b*x])/ \text{Sqrt}[x^3+3*\text{Sin}[a+b*x]],x]+(4*\text{Defer}[\text{Int}[x*\text{Sqrt}[x^3+3*\text{Sin}[a+b*x]],x])/(3*b)]$

Rubi steps

$$\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(ax+bx)}} + \frac{x^2 \cos(ax+bx)}{\sqrt{x^3+3\sin(ax+bx)}} + \frac{4x\sqrt{x^3+3\sin(ax+bx)}}{3b} \right) dx = \frac{\int \frac{x^4}{\sqrt{x^3+3\sin(ax+bx)}} dx}{b} + \frac{4}{3} \int \frac{x \sqrt{x^3+3\sin(ax+bx)}}{b} dx$$

Mathematica [A] time = 0.45, size = 26, normalized size = 1.00

$$\frac{2x^2\sqrt{3\sin(ax+bx)+x^3}}{3b}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[x^4/(b*\text{Sqrt}[x^3+3*\text{Sin}[a+b*x]])+(x^2*\text{Cos}[a+b*x])/ \text{Sqrt}[x^3+3*\text{Sin}[a+b*x]]+(4*x*\text{Sqrt}[x^3+3*\text{Sin}[a+b*x]])/(3*b),x]$

[Out] $(2*x^2*\text{Sqrt}[x^3+3*\text{Sin}[a+b*x]])/(3*b)$

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^4/b/(x^3+3*\sin(b*x+a))^{(1/2)}+x^2*\cos(b*x+a)/(x^3+3*\sin(b*x+a))^{(1/2)}+4/3*x*(x^3+3*\sin(b*x+a))^{(1/2)}/b,x,\text{algorithm}=\text{"fricas"})$

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (has polynomial part)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^3+3\sin(bx+a)}b} + \frac{x^2 \cos(bx+a)}{\sqrt{x^3+3\sin(bx+a)}} + \frac{4\sqrt{x^3+3\sin(bx+a)}x}{3b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2)+4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x, algorithm="giac")

[Out] integrate(x^4/(sqrt(x^3 + 3*sin(b*x + a))*b) + x^2*cos(b*x + a)/sqrt(x^3 + 3*sin(b*x + a)) + 4/3*sqrt(x^3 + 3*sin(b*x + a))*x/b, x)

maple [A] time = 1.04, size = 28, normalized size = 1.08

$$\frac{\sqrt{2x^3 + 6 \sin(bx + a)} \sqrt{2} x^2}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2)+4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x)

[Out] 1/3*(2*x^3+6*sin(b*x+a))^(1/2)/b*2^(1/2)*x^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^3 + 3 \sin(bx + a)}b} + \frac{x^2 \cos(bx + a)}{\sqrt{x^3 + 3 \sin(bx + a)}} + \frac{4 \sqrt{x^3 + 3 \sin(bx + a)}x}{3b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2)+4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x, algorithm="maxima")

[Out] integrate(x^4/(sqrt(x^3 + 3*sin(b*x + a))*b) + x^2*cos(b*x + a)/sqrt(x^3 + 3*sin(b*x + a)) + 4/3*sqrt(x^3 + 3*sin(b*x + a))*x/b, x)

mupad [B] time = 3.47, size = 22, normalized size = 0.85

$$\frac{2x^2 \sqrt{3 \sin(a + bx) + x^3}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*(3*sin(a + b*x) + x^3))^(1/2) + (x^2*cos(a + b*x))/(3*sin(a + b*x) + x^3))^(1/2) + (4*x*(3*sin(a + b*x) + x^3))^(1/2)/(3*b),x)

[Out] (2*x^2*(3*sin(a + b*x) + x^3))^(1/2)/(3*b)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{7x^4}{\sqrt{x^3+3 \sin(a+bx)}} dx + \int \frac{12x \sin(a+bx)}{\sqrt{x^3+3 \sin(a+bx)}} dx + \int \frac{3bx^2 \cos(a+bx)}{\sqrt{x^3+3 \sin(a+bx)}} dx}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/b/(x**3+3*sin(b*x+a))**(1/2)+x**2*cos(b*x+a)/(x**3+3*sin(b*x+a))**(1/2)+4/3*x*(x**3+3*sin(b*x+a))**(1/2)/b,x)

[Out] (Integral(7*x**4/sqrt(x**3 + 3*sin(a + b*x)), x) + Integral(12*x*sin(a + b*x)/sqrt(x**3 + 3*sin(a + b*x)), x) + Integral(3*b*x**2*cos(a + b*x)/sqrt(x**3 + 3*sin(a + b*x)), x))/(3*b)

$$3.932 \quad \int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3 \sin(a+bx)}} dx$$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x^2 \cos(a+bx)}{\sqrt{3 \sin(a+bx)+x^3}}, x\right)$$

[Out] CannotIntegrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2), x)

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3 \sin(a+bx)}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]], x]

[Out] Defer[Int][(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]], x]

Rubi steps

$$\int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3 \sin(a+bx)}} dx = \int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3 \sin(a+bx)}} dx$$

Mathematica [A] time = 7.57, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3 \sin(a+bx)}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]], x]

[Out] Integrate[(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]], x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cos(bx+a)}{\sqrt{x^3+3 \sin(bx+a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2), x, algorithm="giac")

[Out] integrate(x^2*cos(b*x + a)/sqrt(x^3 + 3*sin(b*x + a)), x)

maple [A] time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cos(bx + a)}{\sqrt{x^3 + 3 \sin(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x)`

[Out] `int(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x)`

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cos(bx + a)}{\sqrt{x^3 + 3 \sin(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^2*cos(b*x + a)/sqrt(x^3 + 3*sin(b*x + a)), x)`

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2 \cos(a + bx)}{\sqrt{3 \sin(a + bx) + x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*cos(a + b*x))/(3*sin(a + b*x) + x^3)^(1/2),x)`

[Out] `int((x^2*cos(a + b*x))/(3*sin(a + b*x) + x^3)^(1/2), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cos(a + bx)}{\sqrt{x^3 + 3 \sin(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*cos(b*x+a)/(x**3+3*sin(b*x+a))**(1/2),x)`

[Out] `Integral(x**2*cos(a + b*x)/sqrt(x**3 + 3*sin(a + b*x)), x)`

$$3.933 \quad \int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)} dx$$

Optimal. Leaf size=9

$$\log(e^x \sin(x) + 1)$$

[Out] $\ln(1+\exp(x)*\sin(x))$

Rubi [F] time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos(x) + \sin(x)}{e^{-x} + \sin(x)} dx$$

Verification is Not applicable to the result.

[In] $\text{Int}[(\text{Cos}[x] + \text{Sin}[x])/(E^{-x} + \text{Sin}[x]), x]$

[Out] $x + \text{Log}[\text{Sin}[x]] - \text{Defer}[\text{Int}[(1 + E^x*\text{Sin}[x])^{-1}, x] - \text{Defer}[\text{Int}[\text{Cot}[x]/(1 + E^x*\text{Sin}[x]), x]$

Rubi steps

$$\begin{aligned} \int \frac{\cos(x) + \sin(x)}{e^{-x} + \sin(x)} dx &= \int \left(1 + \cot(x) - \frac{(1 + \cot(x)) \csc(x)}{e^x + \csc(x)} \right) dx \\ &= x + \int \cot(x) dx - \int \frac{(1 + \cot(x)) \csc(x)}{e^x + \csc(x)} dx \\ &= x + \log(\sin(x)) - \int \left(\frac{1}{1 + e^x \sin(x)} + \frac{\cot(x)}{1 + e^x \sin(x)} \right) dx \\ &= x + \log(\sin(x)) - \int \frac{1}{1 + e^x \sin(x)} dx - \int \frac{\cot(x)}{1 + e^x \sin(x)} dx \end{aligned}$$

Mathematica [A] time = 0.12, size = 9, normalized size = 1.00

$$\log(e^x \sin(x) + 1)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(\text{Cos}[x] + \text{Sin}[x])/(E^{-x} + \text{Sin}[x]), x]$

[Out] $\text{Log}[1 + E^x*\text{Sin}[x]]$

fricas [A] time = 0.88, size = 10, normalized size = 1.11

$$x + \log(e^{(-x)} + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((\cos(x)+\sin(x))/(\exp(-x)+\sin(x)), x, \text{algorithm}="fricas")$

[Out] $x + \log(e^{(-x)} + \sin(x))$

giac [B] time = 0.15, size = 83, normalized size = 9.22

$$x + \frac{1}{2} \log \left(\frac{4 \left(e^{(-2x)} \tan\left(\frac{1}{2}x\right)^4 + 4 e^{(-x)} \tan\left(\frac{1}{2}x\right)^3 + 2 e^{(-2x)} \tan\left(\frac{1}{2}x\right)^2 + 4 e^{(-x)} \tan\left(\frac{1}{2}x\right) + 4 \tan\left(\frac{1}{2}x\right)^2 + e^{(-2x)} \right)}{\tan\left(\frac{1}{2}x\right)^4 + 2 \tan\left(\frac{1}{2}x\right)^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x, algorithm="giac")

[Out] $x + \frac{1}{2} \log(4 * (e^{-2*x} * \tan(1/2*x)^4 + 4 * e^{-x} * \tan(1/2*x)^3 + 2 * e^{-2*x} * \tan(1/2*x)^2 + 4 * e^{-x} * \tan(1/2*x) + 4 * \tan(1/2*x)^2 + e^{-2*x}) / (\tan(1/2*x)^4 + 2 * \tan(1/2*x)^2 + 1))$

maple [B] time = 0.19, size = 57, normalized size = 6.33

$$\frac{x + x \left(\tan^2 \left(\frac{x}{2} \right) \right)}{1 + \tan^2 \left(\frac{x}{2} \right)} - \ln \left(1 + \tan^2 \left(\frac{x}{2} \right) \right) + \ln \left(e^{-x} \left(\tan^2 \left(\frac{x}{2} \right) \right) + e^{-x} + 2 \tan \left(\frac{x}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x)+sin(x))/(exp(-x)+sin(x)),x)

[Out] $(x+x*\tan(1/2*x)^2)/(1+\tan(1/2*x)^2)-\ln(1+\tan(1/2*x)^2)+\ln(\exp(-x)*\tan(1/2*x)^2+\exp(-x)+2*\tan(1/2*x))$

maxima [B] time = 0.56, size = 82, normalized size = 9.11

$$x + \frac{1}{2} \log \left((\cos(2x))^2 e^{2x} + 4 \cos(x) e^x \sin(2x) + e^{2x} \sin(2x)^2 - 2 (2 e^x \sin(x) + e^{2x}) \cos(2x) + 4 \cos(x)^2 + 4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x, algorithm="maxima")

[Out] $x + \frac{1}{2} \log((\cos(2*x))^2 * e^{2*x} + 4 * \cos(x) * e^x * \sin(2*x) + e^{2*x} * \sin(2*x)^2 - 2 * (2 * e^x * \sin(x) + e^{2*x}) * \cos(2*x) + 4 * \cos(x)^2 + 4 * \sin(x)^2 + e^{2*x}) * e^{-2*x})$

mupad [B] time = 2.95, size = 10, normalized size = 1.11

$$x + \ln(e^{-x} + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(x) + sin(x))/(exp(-x) + sin(x)),x)

[Out] $x + \log(\exp(-x) + \sin(x))$

sympy [A] time = 0.29, size = 10, normalized size = 1.11

$$x + \log(\sin(x) + e^{-x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x)

[Out] $x + \log(\sin(x) + \exp(-x))$

3.934 $\int \sin(c+dx) \left(a \sin^2(c+dx) + b \sin^3(c+dx) \right) dx$

Optimal. Leaf size=77

$$\frac{a \cos^3(c+dx)}{3d} - \frac{a \cos(c+dx)}{d} - \frac{b \sin^3(c+dx) \cos(c+dx)}{4d} - \frac{3b \sin(c+dx) \cos(c+dx)}{8d} + \frac{3bx}{8}$$

[Out] $3/8*b*x - a*\cos(d*x+c)/d + 1/3*a*\cos(d*x+c)^3/d - 3/8*b*\cos(d*x+c)*\sin(d*x+c)/d - 1/4*b*\cos(d*x+c)*\sin(d*x+c)^3/d$

Rubi [A] time = 0.13, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {4393, 2748, 2633, 2635, 8}

$$\frac{a \cos^3(c+dx)}{3d} - \frac{a \cos(c+dx)}{d} - \frac{b \sin^3(c+dx) \cos(c+dx)}{4d} - \frac{3b \sin(c+dx) \cos(c+dx)}{8d} + \frac{3bx}{8}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3), x]

[Out] $(3*b*x)/8 - (a*\cos[c + d*x])/d + (a*\cos[c + d*x]^3)/(3*d) - (3*b*\cos[c + d*x]*\sin[c + d*x])/(8*d) - (b*\cos[c + d*x]*\sin[c + d*x]^3)/(4*d)$

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 4393

Int[(u_)*((a_)*(F_)[(c_.) + (d_.)*(x_)]^(p_.) + (b_.)*(F_)[(c_.) + (d_.)*(x_)]^(q_.))^(n_), x_Symbol] :> Int[ActivateTrig[u*F[c + d*x]^(n*p)*(a + b*F[c + d*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, p, q}, x] && InertTrigQ[F] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \sin(c + dx) (a \sin^2(c + dx) + b \sin^3(c + dx)) dx &= \int \sin^3(c + dx)(a + b \sin(c + dx)) dx \\
&= a \int \sin^3(c + dx) dx + b \int \sin^4(c + dx) dx \\
&= -\frac{b \cos(c + dx) \sin^3(c + dx)}{4d} + \frac{1}{4}(3b) \int \sin^2(c + dx) dx - \frac{a \sin(c + dx)}{4d} \\
&= -\frac{a \cos(c + dx)}{d} + \frac{a \cos^3(c + dx)}{3d} - \frac{3b \cos(c + dx) \sin(c + dx)}{8d} \\
&= \frac{3bx}{8} - \frac{a \cos(c + dx)}{d} + \frac{a \cos^3(c + dx)}{3d} - \frac{3b \cos(c + dx) \sin(c + dx)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 76, normalized size = 0.99

$$-\frac{3a \cos(c + dx)}{4d} + \frac{a \cos(3(c + dx))}{12d} + \frac{3b(c + dx)}{8d} - \frac{b \sin(2(c + dx))}{4d} + \frac{b \sin(4(c + dx))}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3),x]

[Out] (3*b*(c + d*x))/(8*d) - (3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12*d) - (b*Sin[2*(c + d*x)])/(4*d) + (b*Sin[4*(c + d*x)])/(32*d)

fricas [A] time = 1.13, size = 60, normalized size = 0.78

$$\frac{8 a \cos(dx + c)^3 + 9 b dx - 24 a \cos(dx + c) + 3(2 b \cos(dx + c)^3 - 5 b \cos(dx + c)) \sin(dx + c)}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x, algorithm="fricas")

[Out] 1/24*(8*a*cos(d*x + c)^3 + 9*b*d*x - 24*a*cos(d*x + c) + 3*(2*b*cos(d*x + c)^3 - 5*b*cos(d*x + c))*sin(d*x + c))/d

giac [A] time = 0.13, size = 62, normalized size = 0.81

$$\frac{3}{8}bx + \frac{a \cos(3dx + 3c)}{12d} - \frac{3a \cos(dx + c)}{4d} + \frac{b \sin(4dx + 4c)}{32d} - \frac{b \sin(2dx + 2c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x, algorithm="giac")

[Out] 3/8*b*x + 1/12*a*cos(3*d*x + 3*c)/d - 3/4*a*cos(d*x + c)/d + 1/32*b*sin(4*d*x + 4*c)/d - 1/4*b*sin(2*d*x + 2*c)/d

maple [A] time = 0.05, size = 60, normalized size = 0.78

$$\frac{b \left(-\frac{\left(\sin^3(dx+c) + \frac{3 \sin(dx+c)}{2} \right) \cos(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) - \frac{a(2+\sin^2(dx+c)) \cos(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x)

[Out] 1/d*(b*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-1/3*a*(2+sin(d*x+c)^2)*cos(d*x+c))

maxima [A] time = 0.49, size = 57, normalized size = 0.74

$$\frac{32 (\cos(dx + c)^3 - 3 \cos(dx + c))a + 3(12dx + 12c + \sin(4dx + 4c) - 8 \sin(2dx + 2c))b}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x, algorithm="maxima")

[Out] 1/96*(32*(cos(d*x + c)^3 - 3*cos(d*x + c))*a + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*b)/d

mupad [B] time = 6.56, size = 111, normalized size = 1.44

$$\frac{3bx \left(\frac{3b \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{4} - \frac{11b \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{4} + 4a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + \frac{11b \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{4} + \frac{16a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{3} + \frac{3b \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4} + \frac{4a}{3} \right)}{8 d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c + d*x)*(a*sin(c + d*x)^2 + b*sin(c + d*x)^3),x)

[Out] (3*b*x)/8 - ((4*a)/3 + (3*b*tan(c/2 + (d*x)/2))/4 + (16*a*tan(c/2 + (d*x)/2)^2)/3 + 4*a*tan(c/2 + (d*x)/2)^4 + (11*b*tan(c/2 + (d*x)/2)^3)/4 - (11*b*tan(c/2 + (d*x)/2)^5)/4 - (3*b*tan(c/2 + (d*x)/2)^7)/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)

sympy [A] time = 0.97, size = 150, normalized size = 1.95

$$\left\{ \begin{array}{l} -\frac{a \sin^2(c+dx) \cos(c+dx)}{d} - \frac{2a \cos^3(c+dx)}{3d} + \frac{3bx \sin^4(c+dx)}{8} + \frac{3bx \sin^2(c+dx) \cos^2(c+dx)}{4} + \frac{3bx \cos^4(c+dx)}{8} - \frac{5b \sin^3(c+dx) \cos(c+dx)}{8d} \\ x(a \sin^2(c) + b \sin^3(c)) \sin(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a*sin(d*x+c)**2+b*sin(d*x+c)**3),x)

[Out] Piecewise((-a*sin(c + d*x)**2*cos(c + d*x)/d - 2*a*cos(c + d*x)**3/(3*d) + 3*b*x*sin(c + d*x)**4/8 + 3*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b*x*cos(c + d*x)**4/8 - 5*b*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*b*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*sin(c)**2 + b*sin(c)**3)*sin(c), True))

$$3.935 \quad \int \sin(c+dx) \left(a \sin^2(c+dx) + b \sin^3(c+dx) \right)^2 dx$$

Optimal. Leaf size=161

$$\frac{(a^2 + 3b^2) \cos^5(c + dx)}{5d} + \frac{(2a^2 + 3b^2) \cos^3(c + dx)}{3d} - \frac{(a^2 + b^2) \cos(c + dx)}{d} - \frac{ab \sin^5(c + dx) \cos(c + dx)}{3d} - \frac{5ab \sin^3(c + dx) \cos(c + dx)}{3d}$$

[Out] $5/8*a*b*x - (a^2+b^2)*\cos(d*x+c)/d + 1/3*(2*a^2+3*b^2)*\cos(d*x+c)^3/d - 1/5*(a^2+3*b^2)*\cos(d*x+c)^5/d + 1/7*b^2*\cos(d*x+c)^7/d - 5/8*a*b*\cos(d*x+c)*\sin(d*x+c)/d - 5/12*a*b*\cos(d*x+c)*\sin(d*x+c)^3/d - 1/3*a*b*\cos(d*x+c)*\sin(d*x+c)^5/d$

Rubi [A] time = 0.27, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4393, 2789, 2635, 8, 3013, 373}

$$\frac{(a^2 + 3b^2) \cos^5(c + dx)}{5d} + \frac{(2a^2 + 3b^2) \cos^3(c + dx)}{3d} - \frac{(a^2 + b^2) \cos(c + dx)}{d} - \frac{ab \sin^5(c + dx) \cos(c + dx)}{3d} - \frac{5ab \sin^3(c + dx) \cos(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3)^2,x]

[Out] $(5*a*b*x)/8 - ((a^2 + b^2)*\text{Cos}[c + d*x])/d + ((2*a^2 + 3*b^2)*\text{Cos}[c + d*x]^3)/(3*d) - ((a^2 + 3*b^2)*\text{Cos}[c + d*x]^5)/(5*d) + (b^2*\text{Cos}[c + d*x]^7)/(7*d) - (5*a*b*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(8*d) - (5*a*b*\text{Cos}[c + d*x]*\text{Sin}[c + d*x]^3)/(12*d) - (a*b*\text{Cos}[c + d*x]*\text{Sin}[c + d*x]^5)/(3*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 373

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n-1)/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Sin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2789

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m+1), x], x] + Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3013

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m-1)/2)*(A + C - C*x^2)], x], x, Cos[e + f*x]] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m+1)/2, 0]

Rule 4393

```
Int[(u_)*((a_)*(F_)[(c_.) + (d_)*(x_)]^(p_.) + (b_)*(F_)[(c_.) + (d_)*(x_)]^(q_.)^(n_), x_Symbol] := Int[ActivateTrig[u*F[c + d*x]^(n*p)*(a + b*F[c + d*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, p, q}, x] && InertTrigQ[F] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
 \int \sin(c + dx) (a \sin^2(c + dx) + b \sin^3(c + dx))^2 dx &= \int \sin^5(c + dx) (a + b \sin(c + dx))^2 dx \\
 &= (2ab) \int \sin^6(c + dx) dx + \int \sin^5(c + dx) (a^2 + b^2 \sin^2(c + dx)) dx \\
 &= -\frac{ab \cos(c + dx) \sin^5(c + dx)}{3d} + \frac{1}{3}(5ab) \int \sin^4(c + dx) dx \\
 &= -\frac{5ab \cos(c + dx) \sin^3(c + dx)}{12d} - \frac{ab \cos(c + dx) \sin^5(c + dx)}{3d} \\
 &= -\frac{(a^2 + b^2) \cos(c + dx)}{d} + \frac{(2a^2 + 3b^2) \cos^3(c + dx)}{3d} - \frac{(a^2 + b^2) \cos^5(c + dx)}{5d} \\
 &= \frac{5abx}{8} - \frac{(a^2 + b^2) \cos(c + dx)}{d} + \frac{(2a^2 + 3b^2) \cos^3(c + dx)}{3d} - \frac{(a^2 + b^2) \cos^5(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] time = 0.21, size = 134, normalized size = 0.83

$$\frac{-525(8a^2 + 7b^2) \cos(c + dx) + 35(20a^2 + 21b^2) \cos(3(c + dx)) - 84a^2 \cos(5(c + dx)) - 3150ab \sin(2(c + dx))}{840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (4200*a*b*c + 4200*a*b*d*x - 525*(8*a^2 + 7*b^2)*Cos[c + d*x] + 35*(20*a^2 + 21*b^2)*Cos[3*(c + d*x)] - 84*a^2*Cos[5*(c + d*x)] - 147*b^2*Cos[5*(c + d*x)] + 15*b^2*Cos[7*(c + d*x)] - 3150*a*b*Sin[2*(c + d*x)] + 630*a*b*Sin[4*(c + d*x)] - 70*a*b*Sin[6*(c + d*x)])/(6720*d)
```

fricas [A] time = 0.91, size = 123, normalized size = 0.76

$$\frac{120 b^2 \cos(dx + c)^7 - 168 (a^2 + 3 b^2) \cos(dx + c)^5 + 525 ab dx + 280 (2 a^2 + 3 b^2) \cos(dx + c)^3 - 840 (a^2 + b^2) \cos(dx + c)}{840 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/840*(120*b^2*cos(d*x + c)^7 - 168*(a^2 + 3*b^2)*cos(d*x + c)^5 + 525*a*b*d*x + 280*(2*a^2 + 3*b^2)*cos(d*x + c)^3 - 840*(a^2 + b^2)*cos(d*x + c) - 35*(8*a*b*cos(d*x + c)^5 - 26*a*b*cos(d*x + c)^3 + 33*a*b*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.19, size = 143, normalized size = 0.89

$$\frac{5}{8} abx + \frac{b^2 \cos(7 dx + 7 c)}{448 d} - \frac{ab \sin(6 dx + 6 c)}{96 d} + \frac{3 ab \sin(4 dx + 4 c)}{32 d} - \frac{15 ab \sin(2 dx + 2 c)}{32 d} - \frac{(4 a^2 + 7 b^2) \cos(5 dx + 5 c)}{320 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x, algorithm="giac")

[Out] 5/8*a*b*x + 1/448*b^2*cos(7*d*x + 7*c)/d - 1/96*a*b*sin(6*d*x + 6*c)/d + 3/32*a*b*sin(4*d*x + 4*c)/d - 15/32*a*b*sin(2*d*x + 2*c)/d - 1/320*(4*a^2 + 7*b^2)*cos(5*d*x + 5*c)/d + 1/192*(20*a^2 + 21*b^2)*cos(3*d*x + 3*c)/d - 5/64*(8*a^2 + 7*b^2)*cos(d*x + c)/d

maple [A] time = 0.08, size = 125, normalized size = 0.78

$$\frac{b^2 \left(\frac{16}{5} + \sin^6(dx+c) + \frac{6 \sin^4(dx+c)}{5} + \frac{8 \sin^2(dx+c)}{5} \right) \cos(dx+c)}{7} + 2ab \left(- \frac{\left(\sin^5(dx+c) + \frac{5 \sin^3(dx+c)}{4} + \frac{15 \sin(dx+c)}{8} \right) \cos(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right) - \frac{a^2 \left(\frac{8}{3} + \sin^6(dx+c) + \frac{6 \sin^4(dx+c)}{5} + \frac{8 \sin^2(dx+c)}{5} \right) \cos(dx+c)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x)

[Out] 1/d*(-1/7*b^2*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c)+2*a*b*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/16*d*x+5/16*c)-1/5*a^2*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)

maxima [A] time = 0.33, size = 131, normalized size = 0.81

$$\frac{224 \left(3 \cos(dx+c)^5 - 10 \cos(dx+c)^3 + 15 \cos(dx+c) \right) a^2 - 35 \left(4 \sin(2dx+2c)^3 + 60dx + 60c + 9 \sin(4dx+4c) \right) a b}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x, algorithm="maxima")

[Out] -1/3360*(224*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*a^2 - 35*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a*b - 96*(5*cos(d*x + c)^7 - 21*cos(d*x + c)^5 + 35*cos(d*x + c)^3 - 35*cos(d*x + c))*b^2)/d

mupad [B] time = 6.74, size = 210, normalized size = 1.30

$$\frac{5abx \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 \left(\frac{80a^2}{3} + 32b^2\right) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \left(\frac{112a^2}{15} + \frac{32b^2}{5}\right) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \left(\frac{112a^2}{5} + \frac{96b^2}{5}\right) + \frac{32a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8}{3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c + d*x)*(a*sin(c + d*x)^2 + b*sin(c + d*x)^3)^2,x)

[Out] (5*a*b*x)/8 - (tan(c/2 + (d*x)/2)^6*((80*a^2)/3 + 32*b^2) + tan(c/2 + (d*x)/2)^2*((112*a^2)/15 + (32*b^2)/5) + tan(c/2 + (d*x)/2)^4*((112*a^2)/5 + (96*b^2)/5) + (32*a^2*tan(c/2 + (d*x)/2)^8)/3 + (16*a^2)/15 + (32*b^2)/35 + (2*5*a*b*tan(c/2 + (d*x)/2)^3)/3 + (283*a*b*tan(c/2 + (d*x)/2)^5)/12 - (283*a*b*tan(c/2 + (d*x)/2)^9)/12 - (25*a*b*tan(c/2 + (d*x)/2)^11)/3 - (5*a*b*tan(c/2 + (d*x)/2)^13)/4 + (5*a*b*tan(c/2 + (d*x)/2))/4/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)

sympy [A] time = 6.01, size = 326, normalized size = 2.02

$$\left\{ \begin{array}{l} \frac{a^2 \sin^4(c+dx) \cos(c+dx)}{d} - \frac{4a^2 \sin^2(c+dx) \cos^3(c+dx)}{3d} - \frac{8a^2 \cos^5(c+dx)}{15d} + \frac{5abx \sin^6(c+dx)}{8} + \frac{15abx \sin^4(c+dx) \cos^2(c+dx)}{8} + \frac{15abx \sin^2(c+dx)}{8} \\ x \left(a \sin^2(c) + b \sin^3(c) \right)^2 \sin(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)**2+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**4*cos(c + d*x)/d - 4*a**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 8*a**2*cos(c + d*x)**5/(15*d) + 5*a*b*x*sin(c + d*x)**6/8 + 15*a*b*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 15*a*b*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + 5*a*b*x*cos(c + d*x)**6/8 - 11*a*b*sin(c + d*x)**5*cos(c + d*x)/(8*d) - 5*a*b*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - 5*a*b*sin(c + d*x)*cos(c + d*x)**5/(8*d) - b**2*sin(c + d*x)**6*cos(c + d*x)/d - 2*b**2*sin(c + d*x)**4*cos(c + d*x)**3/d - 8*b**2*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 16*b**2*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c)**2 + b*sin(c)**3)**2*sin(c), True))
```

3.936 $\int \sin(c+dx) (a \sin(c+dx) + b \sin^2(c+dx) + c \sin^3(c+dx)) dx$

Optimal. Leaf size=89

$$-\frac{(4a+3c)\sin(c+dx)\cos(c+dx)}{8d} + \frac{1}{8}x(4a+3c) + \frac{b\cos^3(c+dx)}{3d} - \frac{b\cos(c+dx)}{d} - \frac{c\sin^3(c+dx)\cos(c+dx)}{4d}$$

[Out] $\frac{1}{8}(4a+3c)x - \frac{b\cos(d*x+c)}{d} + \frac{1}{3}b\cos(d*x+c)^3/d - \frac{1}{8}(4a+3c)\cos(d*x+c)\sin(d*x+c)/d - \frac{1}{4}c\cos(d*x+c)\sin(d*x+c)^3/d$

Rubi [A] time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4237, 3023, 2748, 2635, 8, 2633}

$$-\frac{(4a+3c)\sin(c+dx)\cos(c+dx)}{8d} + \frac{1}{8}x(4a+3c) + \frac{b\cos^3(c+dx)}{3d} - \frac{b\cos(c+dx)}{d} - \frac{c\sin^3(c+dx)\cos(c+dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]^3), x]

[Out] $((4a + 3c)*x)/8 - (b*\text{Cos}[c + d*x])/d + (b*\text{Cos}[c + d*x]^3)/(3*d) - ((4a + 3c)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(8*d) - (c*\text{Cos}[c + d*x]*\text{Sin}[c + d*x]^3)/(4*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 4237

Int[(u_)*((A_.)*sin[(a_.) + (b_.)*(x_)]^(n_.) + (B_.)*sin[(a_.) + (b_.)*(x_)]^(n1_) + (C_.)*sin[(a_.) + (b_.)*(x_)]^(n2_)), x_Symbol] := Int[ActivateT

```

rig[u]*Sin[a + b*x]^n*(A + B*SIN[a + b*x] + C*SIN[a + b*x]^2), x] /; FreeQ[
{a, b, A, B, C, n}, x] && EqQ[n1, n + 1] && EqQ[n2, n + 2]

```

Rubi steps

$$\begin{aligned}
\int \sin(c + dx) (a \sin(c + dx) + b \sin^2(c + dx) + c \sin^3(c + dx)) dx &= \int \sin^2(c + dx) (a + b \sin(c + dx) + c \sin^2(c + dx)) dx \\
&= -\frac{c \cos(c + dx) \sin^3(c + dx)}{4d} + \frac{1}{4} \int \sin^2(c + dx) dx \\
&= -\frac{c \cos(c + dx) \sin^3(c + dx)}{4d} + b \int \sin^3(c + dx) dx \\
&= -\frac{(4a + 3c) \cos(c + dx) \sin(c + dx)}{8d} - \frac{c \cos^3(c + dx)}{3d} \\
&= \frac{1}{8}(4a + 3c)x - \frac{b \cos(c + dx)}{d} + \frac{b \cos^3(c + dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 105, normalized size = 1.18

$$\frac{a(c + dx)}{2d} - \frac{a \sin(2(c + dx))}{4d} - \frac{3b \cos(c + dx)}{4d} + \frac{b \cos(3(c + dx))}{12d} + \frac{3c(c + dx)}{8d} - \frac{c \sin(2(c + dx))}{4d} + \frac{c \sin(4(c + dx))}{32d}$$

Antiderivative was successfully verified.

```

[In] Integrate[SIN[c + d*x]*(a*SIN[c + d*x] + b*SIN[c + d*x]^2 + c*SIN[c + d*x]^3), x]

```

```

[Out] (a*(c + d*x))/(2*d) + (3*c*(c + d*x))/(8*d) - (3*b*Cos[c + d*x])/(4*d) + (b*Cos[3*(c + d*x)])/(12*d) - (a*SIN[2*(c + d*x)])/(4*d) - (c*SIN[2*(c + d*x)])/(4*d) + (c*SIN[4*(c + d*x)])/(32*d)

```

fricas [A] time = 0.88, size = 72, normalized size = 0.81

$$\frac{8b \cos(dx + c)^3 + 3(4a + 3c)dx - 24b \cos(dx + c) + 3(2c \cos(dx + c)^3 - (4a + 5c) \cos(dx + c)) \sin(dx + c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3), x, algorithm="fricas")

```

```

[Out] 1/24*(8*b*cos(d*x + c)^3 + 3*(4*a + 3*c)*d*x - 24*b*cos(d*x + c) + 3*(2*c*cos(d*x + c)^3 - (4*a + 5*c)*cos(d*x + c))*sin(d*x + c))/d

```

giac [A] time = 0.15, size = 70, normalized size = 0.79

$$\frac{1}{8}(4a + 3c)x + \frac{b \cos(3dx + 3c)}{12d} - \frac{3b \cos(dx + c)}{4d} + \frac{c \sin(4dx + 4c)}{32d} - \frac{(a + c) \sin(2dx + 2c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3), x, algorithm="giac")

```

```

[Out] 1/8*(4*a + 3*c)*x + 1/12*b*cos(3*d*x + 3*c)/d - 3/4*b*cos(d*x + c)/d + 1/32*c*sin(4*d*x + 4*c)/d - 1/4*(a + c)*sin(2*d*x + 2*c)/d

```

maple [A] time = 0.08, size = 84, normalized size = 0.94

$$\frac{c \left(-\frac{(\sin^3(dx+c) + \frac{3 \sin(dx+c)}{2}) \cos(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) - \frac{b(2 + \sin^2(dx+c)) \cos(dx+c)}{3} + a \left(-\frac{\sin(dx+c) \cos(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x)`

[Out] $\frac{1}{d} \left(c \left(-\frac{1}{4} (\sin(dx+c))^3 + \frac{3}{2} \sin(dx+c) \right) \cos(dx+c) + \frac{3}{8} dx + \frac{3}{8} c \right) - \frac{1}{3} b \left(2 + \sin(dx+c)^2 \right) \cos(dx+c) + a \left(-\frac{1}{2} \sin(dx+c) \cos(dx+c) + \frac{1}{2} dx + \frac{1}{2} c \right)$

maxima [A] time = 0.46, size = 79, normalized size = 0.89

$$\frac{24(2dx + 2c - \sin(2dx + 2c))a + 32(\cos(dx + c)^3 - 3\cos(dx + c))b + 3(12dx + 12c + \sin(4dx + 4c) - 8\sin(2dx + 2c))c}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x, algorithm="maxima")`

[Out] $\frac{1}{96} \left(24(2dx + 2c - \sin(2dx + 2c))a + 32(\cos(dx + c)^3 - 3\cos(dx + c))b + 3(12dx + 12c + \sin(4dx + 4c) - 8\sin(2dx + 2c))c \right) / d$

mupad [B] time = 3.18, size = 73, normalized size = 0.82

$$\frac{2b \cos(3c + 3dx) - 18b \cos(c + dx) - 6a \sin(2c + 2dx) - 6c \sin(2c + 2dx) + \frac{3c \sin(4c + 4dx)}{4} + 12adx + 9ac}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(c + d*x)*(a*sin(c + d*x) + b*sin(c + d*x)^2 + c*sin(c + d*x)^3),x)`

[Out] $(2b \cos(3c + 3dx) - 18b \cos(c + dx) - 6a \sin(2c + 2dx) - 6c \sin(2c + 2dx) + (3c \sin(4c + 4dx)) / 4 + 12a dx + 9c dx) / (24d)$

sympy [A] time = 1.03, size = 201, normalized size = 2.26

$$\left\{ \begin{array}{l} \frac{ax \sin^2(c+dx)}{2} + \frac{ax \cos^2(c+dx)}{2} - \frac{a \sin(c+dx) \cos(c+dx)}{2d} - \frac{b \sin^2(c+dx) \cos(c+dx)}{d} - \frac{2b \cos^3(c+dx)}{3d} + \frac{3cx \sin^4(c+dx)}{8} + \frac{3cx \sin^2(c+dx) \cos(c+dx)}{4} \\ x \left(a \sin(c) + b \sin^2(c) + c \sin^3(c) \right) \sin(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)**2+c*sin(d*x+c)**3),x)`

[Out] `Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 - a*sin(c + d*x)*cos(c + d*x)/(2*d) - b*sin(c + d*x)**2*cos(c + d*x)/d - 2*b*cos(c + d*x)**3/(3*d) + 3*c*x*sin(c + d*x)**4/8 + 3*c*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*c*x*cos(c + d*x)**4/8 - 5*c*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*c*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*sin(c) + b*sin(c)**2 + c*sin(c)**3)*sin(c), True))`

3.937 $\int \sin(c+dx) \left(a \sin(c+dx) + b \sin^2(c+dx) + c \sin^3(c+dx) \right) dx$

Optimal. Leaf size=288

$$\frac{a^2 \cos^3(c+dx)}{3d} - \frac{a^2 \cos(c+dx)}{d} - \frac{(2ac+b^2) \cos^5(c+dx)}{5d} + \frac{2(2ac+b^2) \cos^3(c+dx)}{3d} - \frac{(2ac+b^2) \cos(c+dx)}{d}$$

[Out] $3/4*a*b*x+5/8*b*c*x-a^2*\cos(d*x+c)/d-c^2*\cos(d*x+c)/d-(2*a*c+b^2)*\cos(d*x+c)/d+1/3*a^2*\cos(d*x+c)^3/d+c^2*\cos(d*x+c)^3/d+2/3*(2*a*c+b^2)*\cos(d*x+c)^3/d-3/5*c^2*\cos(d*x+c)^5/d-1/5*(2*a*c+b^2)*\cos(d*x+c)^5/d+1/7*c^2*\cos(d*x+c)^7/d-3/4*a*b*\cos(d*x+c)*\sin(d*x+c)/d-5/8*b*c*\cos(d*x+c)*\sin(d*x+c)/d-1/2*a*b*\cos(d*x+c)*\sin(d*x+c)^3/d-5/12*b*c*\cos(d*x+c)*\sin(d*x+c)^3/d-1/3*b*c*\cos(d*x+c)*\sin(d*x+c)^5/d$

Rubi [A] time = 0.40, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {4394, 3256, 2633, 2635, 8}

$$\frac{a^2 \cos^3(c+dx)}{3d} - \frac{a^2 \cos(c+dx)}{d} - \frac{(2ac+b^2) \cos^5(c+dx)}{5d} + \frac{2(2ac+b^2) \cos^3(c+dx)}{3d} - \frac{(2ac+b^2) \cos(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]*(a*Ssin[c + d*x] + b*Ssin[c + d*x]^2 + c*Ssin[c + d*x]^3)^2,x]

[Out] $(3*a*b*x)/4 + (5*b*c*x)/8 - (a^2*\text{Cos}[c + d*x])/d - (c^2*\text{Cos}[c + d*x])/d - (b^2 + 2*a*c)*\text{Cos}[c + d*x]/d + (a^2*\text{Cos}[c + d*x]^3)/(3*d) + (c^2*\text{Cos}[c + d*x]^3)/d + (2*(b^2 + 2*a*c)*\text{Cos}[c + d*x]^3)/(3*d) - (3*c^2*\text{Cos}[c + d*x]^5)/(5*d) - ((b^2 + 2*a*c)*\text{Cos}[c + d*x]^5)/(5*d) + (c^2*\text{Cos}[c + d*x]^7)/(7*d) - (3*a*b*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(4*d) - (5*b*c*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(8*d) - (a*b*\text{Cos}[c + d*x]*\text{Sin}[c + d*x]^3)/(2*d) - (5*b*c*\text{Cos}[c + d*x]*\text{Sin}[c + d*x]^3)/(12*d) - (b*c*\text{Cos}[c + d*x]*\text{Sin}[c + d*x]^5)/(3*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3256

Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] := Int[ExpandTrig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[m, n, p]

Rule 4394

```
Int[(u_)*((a_)*(F_)[(d_.) + (e_.)*(x_)]^(p_.) + (b_.)*(F_)[(d_.) + (e_.)*(x_)]^(q_.) + (c_.)*(F_)[(d_.) + (e_.)*(x_)]^(r_.)^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[d + e*x]^(n*p)*(a + b*F[d + e*x]^(q - p) + c*F[d + e*x]^(r - p))^n], x] /; FreeQ[{a, b, c, d, e, p, q, r}, x] && InertTrigQ[F] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
 \int \sin(c + dx) (a \sin(c + dx) + b \sin^2(c + dx) + c \sin^3(c + dx))^2 dx &= \int \sin^3(c + dx) (a + b \sin(c + dx) + c \sin^2(c + dx))^2 dx \\
 &= \int (a^2 \sin^3(c + dx) + 2ab \sin^4(c + dx) + (b^2 + 2bc \sin(c + dx) + c^2 \sin^2(c + dx)) \sin^3(c + dx)) dx \\
 &= a^2 \int \sin^3(c + dx) dx + (2ab) \int \sin^4(c + dx) dx + \int (b^2 + 2bc \sin(c + dx) + c^2 \sin^2(c + dx)) \sin^3(c + dx) dx \\
 &= -\frac{ab \cos(c + dx) \sin^3(c + dx)}{2d} - \frac{bc \cos(c + dx) \sin^2(c + dx)}{2d} - \frac{c^2 \cos(c + dx) \sin(c + dx)}{2d} - \frac{(b^2 + 2bc \sin(c + dx) + c^2 \sin^2(c + dx)) \cos(c + dx)}{2d} \\
 &= -\frac{a^2 \cos(c + dx)}{d} - \frac{c^2 \cos(c + dx)}{d} - \frac{(b^2 + 2bc \sin(c + dx) + c^2 \sin^2(c + dx)) \cos(c + dx)}{2d} \\
 &= \frac{3abx}{4} - \frac{a^2 \cos(c + dx)}{d} - \frac{c^2 \cos(c + dx)}{d} - \frac{(b^2 + 2bc \sin(c + dx) + c^2 \sin^2(c + dx)) \cos(c + dx)}{2d} \\
 &= \frac{3abx}{4} + \frac{5bcx}{8} - \frac{a^2 \cos(c + dx)}{d} - \frac{c^2 \cos(c + dx)}{d} - \frac{(b^2 + 2bc \sin(c + dx) + c^2 \sin^2(c + dx)) \cos(c + dx)}{2d}
 \end{aligned}$$

Mathematica [A] time = 0.45, size = 167, normalized size = 0.58

$$\frac{-105(48a^2 + 80ac + 40b^2 + 35c^2) \cos(c + dx) + 35(16a^2 + 40ac + 20b^2 + 21c^2) \cos(3(c + dx)) - 21(c(8a + 7c) \cos(5(c + dx)) + 15c^2 \cos(7(c + dx)))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]^3)^2,x]
```

```
[Out] (840*b*(6*a + 5*c)*(c + d*x) - 105*(48*a^2 + 40*b^2 + 80*a*c + 35*c^2)*Cos[c + d*x] + 35*(16*a^2 + 20*b^2 + 40*a*c + 21*c^2)*Cos[3*(c + d*x)] - 21*(4*b^2 + c*(8*a + 7*c))*Cos[5*(c + d*x)] + 15*c^2*Cos[7*(c + d*x)] - 210*b*(16*a + 15*c)*Sin[2*(c + d*x)] + 210*b*(2*a + 3*c)*Sin[4*(c + d*x)] - 70*b*c*Sin[6*(c + d*x)])/(6720*d)
```

fricas [A] time = 1.96, size = 162, normalized size = 0.56

$$\frac{120c^2 \cos(dx + c)^7 - 168(b^2 + 2ac + 3c^2) \cos(dx + c)^5 + 280(a^2 + 2b^2 + 4ac + 3c^2) \cos(dx + c)^3 + 105(6ab + 5bc)x + \frac{c^2 \cos(7dx + 7c)}{448d} - \frac{bc \sin(6dx + 6c)}{96d} - \frac{(4b^2 + 8ac + 7c^2) \cos(5dx + 5c)}{320d} + \frac{(16a^2 + 20b^2 + 40ac + 21c^2) \cos(3dx + 3c)}{6720d}}{6720d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/840*(120*c^2*cos(d*x + c)^7 - 168*(b^2 + 2*a*c + 3*c^2)*cos(d*x + c)^5 + 280*(a^2 + 2*b^2 + 4*a*c + 3*c^2)*cos(d*x + c)^3 + 105*(6*a*b + 5*b*c)*d*x - 840*(a^2 + b^2 + 2*a*c + c^2)*cos(d*x + c) - 35*(8*b*c*cos(d*x + c)^5 - 2*(6*a*b + 13*b*c)*cos(d*x + c)^3 + 3*(10*a*b + 11*b*c)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.18, size = 186, normalized size = 0.65

$$\frac{1}{8}(6ab + 5bc)x + \frac{c^2 \cos(7dx + 7c)}{448d} - \frac{bc \sin(6dx + 6c)}{96d} - \frac{(4b^2 + 8ac + 7c^2) \cos(5dx + 5c)}{320d} + \frac{(16a^2 + 20b^2 + 40ac + 21c^2) \cos(3dx + 3c)}{6720d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x, algorithm="giac")

[Out] $\frac{1}{8}(6ab + 5b^2c)x + \frac{1}{448}c^2 \cos(7dx + 7c)/d - \frac{1}{96}b^2c \sin(6dx + 6c)/d - \frac{1}{320}(4b^2 + 8ac + 7c^2) \cos(5dx + 5c)/d + \frac{1}{192}(16a^2 + 20b^2 + 40ac + 21c^2) \cos(3dx + 3c)/d - \frac{1}{64}(48a^2 + 40b^2 + 80ac + 35c^2) \cos(dx + c)/d + \frac{1}{32}(2ab + 3b^2c) \sin(4dx + 4c)/d - \frac{1}{32}(16ab + 15b^2c) \sin(2dx + 2c)/d$

maple [A] time = 0.09, size = 213, normalized size = 0.74

$$-\frac{c^2 \left(\frac{16}{5} + \sin^6(dx+c) + \frac{6(\sin^4(dx+c))}{5} + \frac{8(\sin^2(dx+c))}{5} \right) \cos(dx+c)}{7} + 2cb \left(-\frac{\left(\sin^5(dx+c) + \frac{5(\sin^3(dx+c))}{4} + \frac{15 \sin(dx+c)}{8} \right) \cos(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right) - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x)

[Out] $\frac{1}{d}(-\frac{1}{7}c^2(16/5 + \sin(dx+c)^6 + 6/5 \sin(dx+c)^4 + 8/5 \sin(dx+c)^2) \cos(dx+c) + 2cb(-\frac{1}{6}(\sin(dx+c)^5 + 5/4 \sin(dx+c)^3 + 15/8 \sin(dx+c)) \cos(dx+c) + 5/16 dx + 5/16 c) - 2/5 a^2 c(8/3 + \sin(dx+c)^4 + 4/3 \sin(dx+c)^2) \cos(dx+c) - 1/5 b^2(8/3 + \sin(dx+c)^4 + 4/3 \sin(dx+c)^2) \cos(dx+c) + 2ab(-\frac{1}{4}(\sin(dx+c)^3 + 3/2 \sin(dx+c)) \cos(dx+c) + 3/8 dx + 3/8 c) - 1/3 a^2(2 + \sin(dx+c)^2) \cos(dx+c))$

maxima [A] time = 0.40, size = 218, normalized size = 0.76

$$\frac{1120(\cos(dx+c)^3 - 3\cos(dx+c))a^2 + 210(12dx + 12c + \sin(4dx + 4c) - 8\sin(2dx + 2c))ab - 224(3\cos(dx+c)^5 - 10\cos(dx+c)^3 + 15\cos(dx+c))b^2 - 448(3\cos(dx+c)^5 - 10\cos(dx+c)^3 + 15\cos(dx+c))ac + 35(4\sin(2dx + 2c)^3 + 60dx + 60c + 9\sin(4dx + 4c) - 48\sin(2dx + 2c))b^2c + 96(5\cos(dx+c)^7 - 21\cos(dx+c)^5 + 35\cos(dx+c)^3 - 35\cos(dx+c))c^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x, algorithm="maxima")

[Out] $\frac{1}{3360}(1120(\cos(dx+c)^3 - 3\cos(dx+c))a^2 + 210(12dx + 12c + \sin(4dx + 4c) - 8\sin(2dx + 2c))ab - 224(3\cos(dx+c)^5 - 10\cos(dx+c)^3 + 15\cos(dx+c))b^2 - 448(3\cos(dx+c)^5 - 10\cos(dx+c)^3 + 15\cos(dx+c))ac + 35(4\sin(2dx + 2c)^3 + 60dx + 60c + 9\sin(4dx + 4c) - 48\sin(2dx + 2c))b^2c + 96(5\cos(dx+c)^7 - 21\cos(dx+c)^5 + 35\cos(dx+c)^3 - 35\cos(dx+c))c^2)/d$

mupad [B] time = 4.52, size = 456, normalized size = 1.58

$$\frac{b \operatorname{atan}\left(\frac{b \tan\left(\frac{c}{2} + \frac{dx}{2}\right)(6a+5c)}{4\left(\frac{3ab}{2} + \frac{5bc}{4}\right)}\right)(6a+5c)}{4d} - \frac{b \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}\right)(6a+5c)}{4d} - \frac{\frac{32ac}{15} - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{13} \left(\frac{3ab}{2}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c+d*x)*(a*sin(c+d*x)+b*sin(c+d*x)^2+c*sin(c+d*x)^3)^2,x)

[Out] $\frac{(b \operatorname{atan}\left(\frac{b \tan(c/2 + (dx)/2)(6a+5c)}{4((3ab)/2 + (5bc)/4)}\right)(6a+5c))}{(4d)} - \frac{(b(\operatorname{atan}\left(\tan(c/2 + (dx)/2)\right) - (dx)/2)(6a+5c))}{(4d)}$

) - ((32*a*c)/15 - tan(c/2 + (d*x)/2)^13*((3*a*b)/2 + (5*b*c)/4) + tan(c/2 + (d*x)/2)^3*(10*a*b + (25*b*c)/3) - tan(c/2 + (d*x)/2)^11*(10*a*b + (25*b*c)/3) + tan(c/2 + (d*x)/2)^5*((31*a*b)/2 + (283*b*c)/12) - tan(c/2 + (d*x)/2)^9*((31*a*b)/2 + (283*b*c)/12) + 4*a^2*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^8*((64*a*c)/3 + (52*a^2)/3 + (32*b^2)/3) + tan(c/2 + (d*x)/2)^6*((160*a*c)/3 + (88*a^2)/3 + (80*b^2)/3 + 32*c^2) + tan(c/2 + (d*x)/2)^2*((224*a*c)/15 + (28*a^2)/3 + (112*b^2)/15 + (32*c^2)/5) + tan(c/2 + (d*x)/2)^4*((224*a*c)/5 + 24*a^2 + (112*b^2)/5 + (96*c^2)/5) + (4*a^2)/3 + (16*b^2)/15 + (32*c^2)/35 + tan(c/2 + (d*x)/2)*((3*a*b)/2 + (5*b*c)/4))/(d*(7*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 + 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 + 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 + 1))

sympy [A] time = 6.81, size = 541, normalized size = 1.88

$$\left\{ \begin{array}{l} -\frac{a^2 \sin^2(c+dx) \cos(c+dx)}{d} - \frac{2a^2 \cos^3(c+dx)}{3d} + \frac{3abx \sin^4(c+dx)}{4} + \frac{3abx \sin^2(c+dx) \cos^2(c+dx)}{2} + \frac{3abx \cos^4(c+dx)}{4} - \frac{5ab \sin^3(c+dx) \cos(c+dx)}{4d} \\ x \left(a \sin(c) + b \sin^2(c) + c \sin^3(c) \right)^2 \sin(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)**2+c*sin(d*x+c)**3)**2,x)
[Out] Piecewise((-a**2*sin(c + d*x)**2*cos(c + d*x)/d - 2*a**2*cos(c + d*x)**3/(3*d) + 3*a*b*x*sin(c + d*x)**4/4 + 3*a*b*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 3*a*b*x*cos(c + d*x)**4/4 - 5*a*b*sin(c + d*x)**3*cos(c + d*x)/(4*d) - 3*a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) - 2*a*c*sin(c + d*x)**4*cos(c + d*x)/d - 8*a*c*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 16*a*c*cos(c + d*x)**5/(15*d) - b**2*sin(c + d*x)**4*cos(c + d*x)/d - 4*b**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 8*b**2*cos(c + d*x)**5/(15*d) + 5*b*c*x*sin(c + d*x)**6/8 + 15*b*c*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 15*b*c*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + 5*b*c*x*cos(c + d*x)**6/8 - 11*b*c*sin(c + d*x)**5*cos(c + d*x)/(8*d) - 5*b*c*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - 5*b*c*sin(c + d*x)*cos(c + d*x)**5/(8*d) - c**2*sin(c + d*x)**6*cos(c + d*x)/d - 2*c**2*sin(c + d*x)**4*cos(c + d*x)**3/d - 8*c**2*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 16*c**2*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c) + b*sin(c)**2 + c*sin(c)**3)**2*sin(c), True))
```


$$3.938 \quad \int \sin(c+dx) \left(a + \frac{b}{\sqrt{\sin(c+dx)}} + c \sin(c+dx) \right) dx$$

Optimal. Leaf size=61

$$-\frac{a \cos(c+dx)}{d} + \frac{2bE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d} - \frac{c \sin(c+dx) \cos(c+dx)}{2d} + \frac{cx}{2}$$

[Out] 1/2*c*x-a*cos(d*x+c)/d-2*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d-1/2*c*cos(d*x+c)*sin(d*x+c)/d

Rubi [A] time = 0.29, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4395, 4401, 2639, 2638, 2635, 8}

$$-\frac{a \cos(c+dx)}{d} + \frac{2bE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d} - \frac{c \sin(c+dx) \cos(c+dx)}{2d} + \frac{cx}{2}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*SIn[c + d*x]),x]

[Out] (c*x)/2 - (a*Cos[c + d*x])/d + (2*b*EllipticE[(c - Pi/2 + d*x)/2, 2])/d - (c*Cos[c + d*x]*Sin[c + d*x])/(2*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 4395

Int[(u_.)*((a_.) + (b_.)*(F_)[(d_.) + (e_.)*(x_)^(p_.) + (c_.)*(F_)[(d_.) + (e_.)*(x_)^(q_.)]^(n_.), x_Symbol] := Int[ActivateTrig[u*F[d + e*x]^(n*p)*(b + a/F[d + e*x]^p + c*F[d + e*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && InertTrigQ[F] && IntegerQ[n] && NegQ[p]

Rule 4401

Int[u_, x_Symbol] := With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /; !InertTrigFreeQ[u]

Rubi steps

$$\begin{aligned}
\int \sin(c + dx) \left(a + \frac{b}{\sqrt{\sin(c + dx)}} + c \sin(c + dx) \right) dx &= \int \sqrt{\sin(c + dx)} \left(b + a\sqrt{\sin(c + dx)} + c \sin^{\frac{3}{2}}(c + dx) \right) dx \\
&= \int \left(b\sqrt{\sin(c + dx)} + a \sin(c + dx) + c \sin^2(c + dx) \right) dx \\
&= a \int \sin(c + dx) dx + b \int \sqrt{\sin(c + dx)} dx + c \int \sin^2(c + dx) dx \\
&= -\frac{a \cos(c + dx)}{d} + \frac{2bE\left(\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right) \middle| 2\right)}{d} - \frac{c \cos(c + dx)}{2d} \\
&= \frac{cx}{2} - \frac{a \cos(c + dx)}{d} + \frac{2bE\left(\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right) \middle| 2\right)}{d} - \frac{c \cos(c + dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 55, normalized size = 0.90

$$\frac{-4a \cos(c + dx) - 8bE\left(\frac{1}{4}(-2c - 2dx + \pi) \middle| 2\right) + c(-\sin(2(c + dx)) + 2c + 2dx)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x]),x]

[Out] (-4*a*Cos[c + d*x] - 8*b*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + c*(2*c + 2*d*x - Sin[2*(c + d*x)]))/(4*d)

fricas [F] time = 1.78, size = 0, normalized size = 0.00

$$\text{integral} \left(-c \cos(dx + c)^2 + a \sin(dx + c) + b\sqrt{\sin(dx + c)} + c, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x, algorithm="fricas")

[Out] integral(-c*cos(d*x + c)^2 + a*sin(d*x + c) + b*sqrt(sin(d*x + c)) + c, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \sin(dx + c) + a + \frac{b}{\sqrt{\sin(dx + c)}} \right) \sin(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x, algorithm="giac")

[Out] integrate((c*sin(d*x + c) + a + b/sqrt(sin(d*x + c)))*sin(d*x + c), x)

maple [A] time = 0.34, size = 136, normalized size = 2.23

$$cx - \frac{a \cos(dx + c)}{d} - \frac{c \left(\frac{\sin(dx+c)\cos(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)}{d} - \frac{b\sqrt{1 + \sin(dx + c)} \sqrt{-2 \sin(dx + c) + 2} \sqrt{-\sin(dx + c)}}{\cos(dx + c)} \left(2 \text{EllipticE} \left(\frac{1}{2} \left(c - \frac{\pi}{2} + dx \right) \middle| 2 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x)

[Out] $c*x - a*\cos(dx+c)/d - c/d*(1/2*\sin(dx+c)*\cos(dx+c) + 1/2*d*x + 1/2*c) - b*(1+\sin(dx+c))^{1/2}*(-2*\sin(dx+c)+2)^{1/2}*(-\sin(dx+c))^{1/2}*(2*\text{EllipticE}((1+\sin(dx+c))^{1/2}, 1/2*2^{1/2})) - \text{EllipticF}((1+\sin(dx+c))^{1/2}, 1/2*2^{1/2}))/c$
 $os(dx+c)/\sin(dx+c)^{1/2}/d$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2cdx - 4a \cos(dx+c) - 2d \int \frac{\left(\left(b \cos\left(\frac{3}{2}dx + \frac{3}{2}c\right) - b \cos\left(\frac{1}{2}dx + \frac{1}{2}c\right) - b \sin\left(\frac{3}{2}dx + \frac{3}{2}c\right) - b \sin\left(\frac{1}{2}dx + \frac{1}{2}c\right) \right) \cos\left(\frac{1}{2} \arctan(\sin(dx+c), -\cos(dx+c))\right)}{\dots} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(dx+c)*(a+c*sin(dx+c)+b/sin(dx+c)^(1/2)),x, algorithm="maxima")`

[Out] $1/4*(2*c*d*x - 4*a*\cos(dx+c) + 2*d*\text{integrate}(-((b*\cos(3/2*d*x + 3/2*c) - b*\cos(1/2*d*x + 1/2*c) - b*\sin(3/2*d*x + 3/2*c) - b*\sin(1/2*d*x + 1/2*c)) * \cos(1/2*\arctan2(\sin(dx+c), -\cos(dx+c) + 1)) - (b*\cos(3/2*d*x + 3/2*c) - b*\cos(1/2*d*x + 1/2*c) + b*\sin(3/2*d*x + 3/2*c) + b*\sin(1/2*d*x + 1/2*c)) * \sin(1/2*\arctan2(\sin(dx+c), -\cos(dx+c) + 1))) * \cos(1/2*\arctan2(\sin(dx+c), \cos(dx+c) + 1)) + ((b*\cos(3/2*d*x + 3/2*c) - b*\cos(1/2*d*x + 1/2*c) + b*\sin(3/2*d*x + 3/2*c) + b*\sin(1/2*d*x + 1/2*c)) * \cos(1/2*\arctan2(\sin(dx+c), -\cos(dx+c) + 1)) + (b*\cos(3/2*d*x + 3/2*c) - b*\cos(1/2*d*x + 1/2*c) - b*\sin(3/2*d*x + 3/2*c) - b*\sin(1/2*d*x + 1/2*c)) * \sin(1/2*\arctan2(\sin(dx+c), -\cos(dx+c) + 1))) * \sin(1/2*\arctan2(\sin(dx+c), \cos(dx+c) + 1)))/((\cos(dx+c)^2 + \sin(dx+c)^2 + 2*\cos(dx+c) + 1)^{1/4} * (\cos(dx+c)^2 + \sin(dx+c)^2 - 2*\cos(dx+c) + 1)^{1/4}), x) - c*\sin(2*d*x + 2*c))/d$

mupad [B] time = 3.25, size = 51, normalized size = 0.84

$$\frac{cx}{2} - \frac{c \sin(2c + 2dx)}{4d} - \frac{a \cos(c + dx)}{d} + \frac{2bE\left(\frac{c}{2} - \frac{\pi}{4} + \frac{dx}{2} \middle| 2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(c+dx)*(a+c*sin(c+dx)+b/sin(c+dx)^(1/2)),x)`

[Out] $(c*x)/2 - (c*\sin(2*c + 2*d*x))/(4*d) - (a*\cos(c + dx))/d + (2*b*\text{ellipticE}(c/2 - \pi/4 + (d*x)/2, 2))/d$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sqrt{\sin(c+dx)} + b + c \sin^{\frac{3}{2}}(c+dx) \right) \sqrt{\sin(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(dx+c)*(a+c*sin(dx+c)+b/sin(dx+c)**(1/2)),x)`

[Out] `Integral((a*sqrt(sin(c+dx)) + b + c*sin(c+dx)**(3/2))*sqrt(sin(c+dx)), x)`

$$3.939 \quad \int \sin(c+dx) \left(a + \frac{b}{\sqrt{\sin(c+dx)}} + c \sin(c+dx) \right)^2 dx$$

Optimal. Leaf size=148

$$-\frac{a^2 \cos(c+dx)}{d} + \frac{4abE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d} - \frac{ac \sin(c+dx) \cos(c+dx)}{d} + acx + b^2x + \frac{4bcF\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3d} - \frac{4bc}{3d}$$

[Out] $b^2x + a^2 \cos(c+dx) - (a^2 \cos(c+dx))/d - (c^2 \cos(c+dx))/d + (c^2 \cos(c+dx)^3)/(3d) + (4abE(\frac{1}{2}(c+dx-\frac{\pi}{2})|2))/d + (4bcF(\frac{1}{2}(c+dx-\frac{\pi}{2})|2))/(3d) - (4bc \sin(c+dx) \cos(c+dx))/d - (a^2 \cos(c+dx) \sin(c+dx))/d$

Rubi [A] time = 0.24, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4395, 4401, 2639, 2638, 2635, 2641, 8, 2633}

$$-\frac{a^2 \cos(c+dx)}{d} + \frac{4abE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d} - \frac{ac \sin(c+dx) \cos(c+dx)}{d} + acx + b^2x + \frac{4bcF\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3d} - \frac{4bc}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x])^2, x]

[Out] $b^2x + a^2 \cos(c+dx) - (a^2 \cos(c+dx))/d - (c^2 \cos(c+dx))/d + (c^2 \cos(c+dx)^3)/(3d) + (4abE((c - \pi/2 + d*x)/2, 2))/d + (4bcF((c - \pi/2 + d*x)/2, 2))/(3d) - (4bc \sin(c+dx) \cos(c+dx))/d - (a^2 \cos(c+dx) \sin(c+dx))/d$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*cos[c + d*x])*(b*sin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - \pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4395

```
Int[(u_)*((a_) + (b_.)*(F_)[(d_.) + (e_.)*(x_)]^(p_.) + (c_.)*(F_)[(d_.) +
(e_.)*(x_)]^(q_.))^n_, x_Symbol] := Int[ActivateTrig[u*F[d + e*x]^(n*p)*
(b + a/F[d + e*x]^p + c*F[d + e*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, e,
p, q}, x] && InertTrigQ[F] && IntegerQ[n] && NegQ[p]
```

Rule 4401

```
Int[u_, x_Symbol] := With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
!InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned} \int \sin(c + dx) \left(a + \frac{b}{\sqrt{\sin(c + dx)}} + c \sin(c + dx) \right)^2 dx &= \int \left(b + a\sqrt{\sin(c + dx)} + c \sin^{\frac{3}{2}}(c + dx) \right)^2 dx \\ &= \int \left(b^2 + 2ab\sqrt{\sin(c + dx)} + a^2 \sin(c + dx) + 2bc \sin^{\frac{3}{2}}(c + dx) + c^2 \sin^3(c + dx) \right) dx \\ &= b^2x + a^2 \int \sin(c + dx) dx + (2ab) \int \sqrt{\sin(c + dx)} dx + \frac{2}{3}bc \int \sin^{\frac{3}{2}}(c + dx) dx + \frac{c^2}{4} \int \sin^3(c + dx) dx \\ &= b^2x - \frac{a^2 \cos(c + dx)}{d} + \frac{4abE\left(\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right)\middle|2\right)}{d} - \frac{4}{3}bc \frac{\cos(c + dx) \sqrt{\sin(c + dx)}}{d} + \frac{c^2 \cos(c + dx) \sqrt{\sin(c + dx)}}{d} \\ &= b^2x + acx - \frac{a^2 \cos(c + dx)}{d} - \frac{c^2 \cos(c + dx)}{d} + \frac{c^2 \cos(c + dx) \sqrt{\sin(c + dx)}}{d} \end{aligned}$$

Mathematica [A] time = 0.28, size = 137, normalized size = 0.93

$$\frac{-12a^2 \cos(c + dx) - 48abE\left(\frac{1}{4}(-2c - 2dx + \pi)\middle|2\right) + 12ac^2 + 12acdx - 6ac \sin(2(c + dx)) + 12b^2c + 12b^2dx - 12c^2 \sin^2(c + dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x])^2,x]
[Out] (12*b^2*c + 12*a*c^2 + 12*b^2*d*x + 12*a*c*d*x - 12*a^2*Cos[c + d*x] - 9*c^2*Cos[c + d*x] + c^2*Cos[3*(c + d*x)] - 48*a*b*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] - 16*b*c*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] - 16*b*c*Cos[c + d*x]*Sqrt[Sin[c + d*x]] - 6*a*c*Sin[2*(c + d*x)])/(12*d)
```

fricas [F] time = 1.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-2ac \cos(dx + c)^2 + b^2 + 2ac - (c^2 \cos(dx + c)^2 - a^2 - c^2) \sin(dx + c) + 2(bc \sin(dx + c) + ab)\sqrt{\sin(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] integral(-2*a*c*cos(d*x + c)^2 + b^2 + 2*a*c - (c^2*cos(d*x + c)^2 - a^2 - c^2)*sin(d*x + c) + 2*(b*c*sin(d*x + c) + a*b)*sqrt(sin(d*x + c)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, choosing root of [1,0,%%{-2,[2]%%}+%%{-2,[1]%%}+%%{-2,[0]%%},0,%%{1,[4]%%}+%%{-2,[3]%%}+%%{3,[2]%%}+%%{-2,[1]%%}+%%{1,[0]%%}] at parameters values [93.1017843988]Warning, choosing root of [1,0,%%{-2,[2]%%}+%%{-2,[1]%%}+%%{-2,[0]%%},0,%%{1,[4]%%}+%%{-2,[3]%%}+%%{3,[2]%%}+%%{-2,[1]%%}+%%{1,[0]%%}] at parameters values [2.14118046779]Warning, choosing root of [1,0,%%{-2,[2]%%}+%%{-2,[1]%%}+%%{-2,[0]%%},0,%%{1,[4]%%}+%%{-2,[3]%%}+%%{3,[2]%%}+%%{-2,[1]%%}+%%{1,[0]%%}] at parameters values [9.72821606882]int() Error: Bad Argument Value

maple [A] time = 0.32, size = 266, normalized size = 1.80

$$b^2x - \frac{a^2 \cos(dx+c)}{d} - \frac{c^2(2 + \sin^2(dx+c)) \cos(dx+c)}{3d} + \frac{2ac \left(-\frac{\sin(dx+c) \cos(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)}{d} + \frac{2b \left(3a\sqrt{1 + \sin(dx+c)} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x)

[Out] b^2*x-a^2*cos(d*x+c)/d-1/3*c^2/d*(2+sin(d*x+c)^2)*cos(d*x+c)+2*a*c/d*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+2/3*b*(3*a*(1+sin(d*x+c))^(1/2)*(-2*sin(d*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*EllipticF((1+sin(d*x+c))^(1/2),1/2*2^(1/2))+1+sin(d*x+c))^(1/2)*(-2*sin(d*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*EllipticF((1+sin(d*x+c))^(1/2),1/2*2^(1/2))*c-6*a*(1+sin(d*x+c))^(1/2)*(-2*sin(d*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*EllipticE((1+sin(d*x+c))^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2*sin(d*x+c)*c/cos(d*x+c)/sin(d*x+c)^(1/2)/d

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 6.68, size = 129, normalized size = 0.87

$$b^2x - \frac{a^2 \cos(c+dx)}{d} - \frac{ac(\sin(2c+2dx) - 2dx)}{2d} + \frac{4abE\left(\frac{c}{2} - \frac{\pi}{4} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{c^2 \cos(c+dx) (\cos(c+dx)^2 - 3)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c+d*x)*(a+c*sin(c+d*x)+b/sin(c+d*x)^(1/2))^2,x)

```
[Out] b^2*x - (a^2*cos(c + d*x))/d - (a*c*(sin(2*c + 2*d*x) - 2*d*x))/(2*d) + (4*
a*b*ellipticE(c/2 - pi/4 + (d*x)/2, 2))/d + (c^2*cos(c + d*x)*(cos(c + d*x)
^2 - 3))/(3*d) - (2*b*c*cos(c + d*x)*sin(c + d*x)^(5/2)*hypergeom([-1/4, 1/
2], 3/2, cos(c + d*x)^2))/(d*(sin(c + d*x)^2)^(5/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{\sqrt{\sin(c + dx)}} + c \sin(c + dx) \right)^2 \sin(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral((a + b/sqrt(sin(c + d*x)) + c*sin(c + d*x))**2*sin(c + d*x), x)
```

$$3.940 \quad \int f^{a+bx} (\cos(c + dx) + i \sin(c + dx))^n dx$$

Optimal. Leaf size=34

$$\frac{f^{a+bx} (e^{i(c+dx)})^n}{b \log(f) + idn}$$

[Out] $\exp(I*(d*x+c))^n * f^{(b*x+a)} / (I*d*n + b*\ln(f))$

Rubi [A] time = 0.10, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {4614, 2281, 2287, 2194}

$$\frac{f^{a+bx} (e^{i(c+dx)})^n}{b \log(f) + idn}$$

Antiderivative was successfully verified.

[In] $\text{Int}[f^{(a + b*x)} * (\text{Cos}[c + d*x] + I*\text{Sin}[c + d*x])^n, x]$

[Out] $((E^{(I*(c + d*x))})^n * f^{(a + b*x)}) / (I*d*n + b*\text{Log}[f])$

Rule 2194

$\text{Int}[(F_)^{((c_.) * ((a_.) + (b_.) * (x_)))}^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[F^{(c*(a + b*x))} / (b*c*n*\text{Log}[F]), x] /;$ $\text{FreeQ}\{F, a, b, c, n\}, x]$

Rule 2281

$\text{Int}[(u_.) * ((a_.) * (F_)^{(v_)})^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(a * F^v)^n / F^{(n*v)}, \text{Int}[u * F^{(n*v)}, x], x] /;$ $\text{FreeQ}\{F, a, n\}, x]$ && $\text{IntegerQ}[n]$

Rule 2287

$\text{Int}[(u_.) * (F_)^{(v_)} * (G_)^{(w_.)}, x_Symbol] \rightarrow \text{With}\{z = v*\text{Log}[F] + w*\text{Log}[G]\}, \text{Int}[u * \text{NormalizeIntegrand}[E^z, x], x] /;$ $\text{BinomialQ}[z, x]$ || $(\text{PolynomialQ}[z, x] \&\& \text{LeQ}[\text{Exponent}[z, x], 2])$ /; $\text{FreeQ}\{F, G\}, x]$

Rule 4614

$\text{Int}[(u_.) * (\text{Cos}[v_]* (a_.) + (b_.) * \text{Sin}[v_])^{(n_.)}, x_Symbol] \rightarrow \text{Int}[u * (a/E^{(a*v)/b})^n, x] /;$ $\text{FreeQ}\{a, b, n\}, x]$ && $\text{EqQ}[a^2 + b^2, 0]$

Rubi steps

$$\begin{aligned} \int f^{a+bx} (\cos(c + dx) + i \sin(c + dx))^n dx &= \int (e^{i(c+dx)})^n f^{a+bx} dx \\ &= \left(e^{-in(c+dx)} (e^{i(c+dx)})^n \right) \int e^{in(c+dx)} f^{a+bx} dx \\ &= \left(e^{-in(c+dx)} (e^{i(c+dx)})^n \right) \int e^{icn+a \log(f)+x(idn+b \log(f))} dx \\ &= \frac{(e^{i(c+dx)})^n f^{a+bx}}{idn + b \log(f)} \end{aligned}$$

Mathematica [A] time = 0.10, size = 43, normalized size = 1.26

$$\frac{i f^{a+bx} (\cos(c + dx) + i \sin(c + dx))^n}{dn - ib \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b*x)*(Cos[c + d*x] + I*Sin[c + d*x])^n,x]

[Out] ((-I)*f^(a + b*x)*(Cos[c + d*x] + I*Sin[c + d*x])^n)/(d*n - I*b*Log[f])

fricas [A] time = 0.63, size = 30, normalized size = 0.88

$$\frac{f^{bx+a}e^{(idnx+icn)}}{idn + b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x, algorithm="fricas")

[Out] f^(b*x + a)*e^(I*d*n*x + I*c*n)/(I*d*n + b*log(f))

giac [A] time = 0.62, size = 31, normalized size = 0.91

$$\frac{f^a e^{(idnx+bx \log(f)+icn)}}{idn + b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x, algorithm="giac")

[Out] f^a*e^(I*d*n*x + b*x*log(f) + I*c*n)/(I*d*n + b*log(f))

maple [B] time = 0.44, size = 86, normalized size = 2.53

$$\frac{e^{(bx+a) \ln(f)} e^{n \ln \left(\frac{2i \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{1 + \tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} + \frac{1 - \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{1 + \tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} \right)}}{idn + b \ln(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x)

[Out] 1/(I*d*n+b*ln(f))*exp((b*x+a)*ln(f))*exp(n*ln(2*I*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+(1-tan(1/2*d*x+1/2*c)^2)/(1+tan(1/2*d*x+1/2*c)^2)))

maxima [A] time = 0.81, size = 50, normalized size = 1.47

$$\frac{-i f^{bx} f^a \cos(dnx + cn) + f^{bx} f^a \sin(dnx + cn)}{dn - ib \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x, algorithm="maxima")

[Out] (-I*f^(b*x)*f^a*cos(d*n*x + c*n) + f^(b*x)*f^a*sin(d*n*x + c*n))/(d*n - I*b*log(f))

mupad [B] time = 3.46, size = 35, normalized size = 1.03

$$\frac{f^{a+bx} \left(e^{c1i+dx1i} \right)^n 1i}{dn - b \ln(f) 1i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a + b*x)*(cos(c + d*x) + sin(c + d*x)*1i)^n,x)

[Out] $-(f^{(a + b*x)} * \exp(c*1i + d*x*1i))^{n*1i} / (d*n - b*\log(f)*1i)$

sympy [A] time = 6.71, size = 107, normalized size = 3.15

$$\begin{cases} \frac{f^a f^{bx} (i \sin(c+dx) + \cos(c+dx))^n}{b \log(f) + idn} & \text{for } b \neq -\frac{idn}{\log(f)} \\ f^a x (i \sin(c + dx) + \cos(c + dx))^n e^{-idnx} - \frac{if^a (i \sin(c+dx) + \cos(c+dx))^n e^{-idnx}}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))**n,x)`

[Out] `Piecewise((f**a*f**(b*x)*(I*sin(c + d*x) + cos(c + d*x))**n/(b*log(f) + I*d*n), Ne(b, -I*d*n/log(f))), (f**a*x*(I*sin(c + d*x) + cos(c + d*x))**n*exp(-I*d*n*x) - I*f**a*(I*sin(c + d*x) + cos(c + d*x))**n*exp(-I*d*n*x)/(d*n), True))`

3.941 $\int f^{a+bx}(\cos(c+dx) - i \sin(c+dx))^n dx$

Optimal. Leaf size=36

$$\frac{f^{a+bx} (e^{-i(c+dx)})^n}{-b \log(f) + idn}$$

[Out] $-\exp(-I*(d*x+c))^n*f^{(b*x+a)}/(I*d*n-b*\ln(f))$

Rubi [A] time = 0.10, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {4614, 2281, 2287, 2194}

$$\frac{f^{a+bx} (e^{-i(c+dx)})^n}{-b \log(f) + idn}$$

Antiderivative was successfully verified.

[In] $\text{Int}[f^{(a + b*x)}*(\text{Cos}[c + d*x] - I*\text{Sin}[c + d*x])^n, x]$

[Out] $-\left(\left(E^{(-I)*(c + d*x)}\right)^n*f^{(a + b*x)}\right)/(I*d*n - b*\text{Log}[f])$

Rule 2194

$\text{Int}[\left((F_)^{\left((c_)*(a_)+(b_)*(x_)\right)}\right)^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x))})^n/(b*c*n*\text{Log}[F]), x] /;$ FreeQ[{F, a, b, c, n}, x]

Rule 2281

$\text{Int}[(u_)*((a_)*(F_)^{(v_}))^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(a*F^v)^n/F^{(n*v)}, \text{Int}[u*F^{(n*v)}, x], x] /;$ FreeQ[{F, a, n}, x] && !IntegerQ[n]

Rule 2287

$\text{Int}[(u_)*(F_)^{(v_)}*(G_)^{(w_)}, x_Symbol] \rightarrow \text{With}[\{z = v*\text{Log}[F] + w*\text{Log}[G]\}, \text{Int}[u*\text{NormalizeIntegrand}[E^z, x], x] /;$ BinomialQ[z, x] || (PolynomialQ[z, x] && LeQ[Exponent[z, x], 2]) /; FreeQ[{F, G}, x]

Rule 4614

$\text{Int}[(u_)*(\text{Cos}[v_*](a_)+(b_)*\text{Sin}[v_*])^{(n_)}, x_Symbol] \rightarrow \text{Int}[u*(a/E^{((a*v)/b)})^n, x] /;$ FreeQ[{a, b, n}, x] && EqQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned} \int f^{a+bx}(\cos(c+dx) - i \sin(c+dx))^n dx &= \int (e^{-i(c+dx)})^n f^{a+bx} dx \\ &= \left(e^{in(c+dx)} (e^{-i(c+dx)})^n\right) \int e^{-in(c+dx)} f^{a+bx} dx \\ &= \left(e^{in(c+dx)} (e^{-i(c+dx)})^n\right) \int \exp(-icn + a \log(f) - x(idn - b \log(f))) dx \\ &= \frac{(e^{-i(c+dx)})^n f^{a+bx}}{idn - b \log(f)} \end{aligned}$$

Mathematica [A] time = 0.09, size = 43, normalized size = 1.19

$$\frac{if^{a+bx}(\cos(c+dx) - i \sin(c+dx))^n}{dn + ib \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[f^(a + b*x)*(Cos[c + d*x] - I*Sin[c + d*x])^n,x]

[Out] (I*f^(a + b*x)*(Cos[c + d*x] - I*Sin[c + d*x])^n)/(d*n + I*b*Log[f])

fricas [A] time = 0.93, size = 30, normalized size = 0.83

$$\frac{f^{bx+a}e^{-idnx-icn}}{-idn + b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x, algorithm="fricas")

[Out] f^(b*x + a)*e^(-I*d*n*x - I*c*n)/(-I*d*n + b*log(f))

giac [A] time = 0.83, size = 31, normalized size = 0.86

$$\frac{f^a e^{(-idnx+bx \log(f)-icn)}}{-idn + b \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x, algorithm="giac")

[Out] f^a*e^(-I*d*n*x + b*x*log(f) - I*c*n)/(-I*d*n + b*log(f))

maple [B] time = 0.39, size = 86, normalized size = 2.39

$$\frac{e^{(bx+a)\ln(f)} e^{n \ln \left(\frac{1 - \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} - \frac{2i \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \right)}{-idn + b \ln(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x)

[Out] 1/(-I*d*n+b*ln(f))*exp((b*x+a)*ln(f))*exp(n*ln((1-tan(1/2*d*x+1/2*c)^2)/(1+tan(1/2*d*x+1/2*c)^2)-2*I*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)))

maxima [A] time = 0.49, size = 62, normalized size = 1.72

$$\frac{f^{bx} f^a \cos(dnx) - i f^{bx} f^a \sin(dnx)}{(-idn + b \log(f)) \cos(cn) + (dn + i b \log(f)) \sin(cn)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x, algorithm="maxima")

[Out] (f^(b*x)*f^a*cos(d*n*x) - I*f^(b*x)*f^a*sin(d*n*x))/((-I*d*n + b*log(f))*cos(c*n) + (d*n + I*b*log(f))*sin(c*n))

mupad [B] time = 3.35, size = 35, normalized size = 0.97

$$\frac{f^{a+bx} \left(e^{-c \operatorname{li}-dx \operatorname{li}} \right)^n}{-b \ln(f) + d n \operatorname{li}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(f^(a + b*x)*(cos(c + d*x) - sin(c + d*x)*1i)^n,x)

[Out] $-(f^{(a + b*x)} \exp(-c*i - d*x*i)^n) / (d*n*i - b*\log(f))$

sympy [A] time = 6.66, size = 107, normalized size = 2.97

$$\begin{cases} \frac{f^a f^{bx(-i \sin(c+dx)+\cos(c+dx))^n}{-b \log(f)+idn} & \text{for } b \neq \frac{idn}{\log(f)} \\ f^a x (-i \sin(c + dx) + \cos(c + dx))^n e^{idnx} + \frac{if^a(-i \sin(c+dx)+\cos(c+dx))^n e^{idnx}}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(f**(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))**n,x)`

[Out] `Piecewise((-f**a*f**(b*x)*(-I*sin(c + d*x) + cos(c + d*x))**n/(-b*log(f) + I*d*n), Ne(b, I*d*n/log(f))), (f**a*x*(-I*sin(c + d*x) + cos(c + d*x))**n*exp(I*d*n*x) + I*f**a*(-I*sin(c + d*x) + cos(c + d*x))**n*exp(I*d*n*x)/(d*n), True))`

$$3.942 \quad \int \frac{\cos^5(a+bx) - \sin^5(a+bx)}{\cos^5(a+bx) + \sin^5(a+bx)} dx$$

Optimal. Leaf size=120

$$\frac{4 \log(2 \tan^2(a+bx) - (1 - \sqrt{5}) \tan(a+bx) + 2)}{5(1 - \sqrt{5})b} - \frac{4 \log(2 \tan^2(a+bx) - (1 + \sqrt{5}) \tan(a+bx) + 2)}{5(1 + \sqrt{5})b} + \frac{\log(\tan(a+bx))}{b}$$

[Out] $\ln(\cos(b*x+a))/b + 1/5*\ln(1+\tan(b*x+a))/b - 4/5*\ln(2 - (-5^{(1/2)+1})*\tan(b*x+a) + 2*\tan(b*x+a)^2)/b / (-5^{(1/2)+1}) - 4/5*\ln(2 - (5^{(1/2)+1})*\tan(b*x+a) + 2*\tan(b*x+a)^2)/b / (5^{(1/2)+1})$

Rubi [A] time = 0.70, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2074, 260, 2086, 628}

$$\frac{4 \log(2 \tan^2(a+bx) - (1 - \sqrt{5}) \tan(a+bx) + 2)}{5(1 - \sqrt{5})b} - \frac{4 \log(2 \tan^2(a+bx) - (1 + \sqrt{5}) \tan(a+bx) + 2)}{5(1 + \sqrt{5})b} + \frac{\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[a + b*x]^5 - \text{Sin}[a + b*x]^5)/(\text{Cos}[a + b*x]^5 + \text{Sin}[a + b*x]^5), x]$

[Out] $\text{Log}[\text{Cos}[a + b*x]]/b + \text{Log}[1 + \text{Tan}[a + b*x]]/(5*b) - (4*\text{Log}[2 - (1 - \text{Sqrt}[5])*\text{Tan}[a + b*x] + 2*\text{Tan}[a + b*x]^2])/(5*(1 - \text{Sqrt}[5])*b) - (4*\text{Log}[2 - (1 + \text{Sqrt}[5])*\text{Tan}[a + b*x] + 2*\text{Tan}[a + b*x]^2])/(5*(1 + \text{Sqrt}[5])*b)$

Rule 260

$\text{Int}[(x_)^{(m_.)}/((a_) + (b_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /;$ FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 628

$\text{Int}(((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol) \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /;$ FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 2074

$\text{Int}[(P_)^{(p_.)}*(Q_)^{(q_.)}, x_Symbol] \rightarrow \text{With}[\{PP = \text{Factor}[P]\}, \text{Int}[\text{ExpandIntegrand}[PP^p*Q^q, x], x] /;$!SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rule 2086

$\text{Int}[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] \rightarrow \text{With}[\{q = \text{Sqrt}[8*a^2 + b^2 - 4*a*c], A = \text{Coeff}[P3, x, 0], B = \text{Coeff}[P3, x, 1], C = \text{Coeff}[P3, x, 2], D = \text{Coeff}[P3, x, 3]\}, \text{Dist}[1/q, \text{Int}[(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*x + 2*a*x^2), x], x] - \text{Dist}[1/q, \text{Int}[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /;$ FreeQ[{a, b, c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(a+bx) - \sin^5(a+bx)}{\cos^5(a+bx) + \sin^5(a+bx)} dx &= \frac{\text{Subst}\left(\int \frac{1-x^5}{1+x^2+x^5+x^7} dx, x, \tan(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{5(1+x)} - \frac{x}{1+x^2} + \frac{2(2+x-4x^2+2x^3)}{5(1-x+x^2-x^3+x^4)}\right) dx, x, \tan(a+bx)\right)}{b} \\
&= \frac{\log(1+\tan(a+bx))}{5b} + \frac{2 \text{Subst}\left(\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} dx, x, \tan(a+bx)\right)}{5b} - \frac{\text{Subst}\left(\int \frac{-2\sqrt{5}+(10-2\sqrt{5})x}{2+(-1-\sqrt{5})x+2x^2} dx, x, \tan(a+bx)\right)}{5\sqrt{5}b} \\
&= \frac{\log(\cos(a+bx))}{b} + \frac{\log(1+\tan(a+bx))}{5b} - \frac{2 \text{Subst}\left(\int \frac{-2\sqrt{5}+(10-2\sqrt{5})x}{2+(-1-\sqrt{5})x+2x^2} dx, x, \tan(a+bx)\right)}{5\sqrt{5}b} \\
&= \frac{\log(\cos(a+bx))}{b} + \frac{\log(1+\tan(a+bx))}{5b} - \frac{4 \log\left(2 - (1-\sqrt{5})\tan(a+bx)\right)}{5(1-\sqrt{5})}
\end{aligned}$$

Mathematica [A] time = 0.57, size = 73, normalized size = 0.61

$$\frac{-\left(\sqrt{5}-1\right)\log\left(\sin\left(2\left(a+bx\right)\right)-\sqrt{5}+1\right)+\left(1+\sqrt{5}\right)\log\left(\sin\left(2\left(a+bx\right)\right)+\sqrt{5}+1\right)+\log\left(\sin\left(a+bx\right)+\cos\left(a+bx\right)\right)}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^5 - Sin[a + b*x]^5)/(Cos[a + b*x]^5 + Sin[a + b*x]^5), x]

[Out] (Log[Cos[a + b*x] + Sin[a + b*x]] - (-1 + Sqrt[5])*Log[1 - Sqrt[5] + Sin[2*(a + b*x)]] + (1 + Sqrt[5])*Log[1 + Sqrt[5] + Sin[2*(a + b*x)]])/(5*b)

fricas [A] time = 0.89, size = 150, normalized size = 1.25

$$\frac{2\sqrt{5}\log\left(-\frac{2\cos(bx+a)^4-2(\sqrt{5}+1)\cos(bx+a)\sin(bx+a)-2\cos(bx+a)^2-\sqrt{5}-3}{\cos(bx+a)^4-\cos(bx+a)^2-\cos(bx+a)\sin(bx+a)+1}\right)+2\log\left(\cos(bx+a)^4-\cos(bx+a)^2-\cos(bx+a)\sin(bx+a)+1\right)}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5), x, algorithm="fricas")

[Out] 1/10*(2*sqrt(5)*log(-(2*cos(b*x + a)^4 - 2*(sqrt(5) + 1)*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 - sqrt(5) - 3)/(cos(b*x + a)^4 - cos(b*x + a)^2 - cos(b*x + a)*sin(b*x + a) + 1)) + 2*log(cos(b*x + a)^4 - cos(b*x + a)^2 - cos(b*x + a)*sin(b*x + a) + 1) + log(2*cos(b*x + a)*sin(b*x + a) + 1))/b

giac [A] time = 0.53, size = 128, normalized size = 1.07

$$\frac{2\sqrt{5}\log\left(-\frac{1}{2}(\sqrt{5}+1)\tan(bx+a)+\tan(bx+a)^2+1\right)-2\sqrt{5}\log\left(\frac{1}{2}(\sqrt{5}-1)\tan(bx+a)+\tan(bx+a)^2+1\right)}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5), x, algorithm="giac")

[Out] -1/10*(2*sqrt(5)*log(-1/2*(sqrt(5) + 1)*tan(b*x + a) + tan(b*x + a)^2 + 1) - 2*sqrt(5)*log(1/2*(sqrt(5) - 1)*tan(b*x + a) + tan(b*x + a)^2 + 1) - 2*log(2*cos(b*x + a)*sin(b*x + a) + 1))/b

$g(\tan(b*x + a)^4 - \tan(b*x + a)^3 + \tan(b*x + a)^2 - \tan(b*x + a) + 1) + 5*\log(\tan(b*x + a)^2 + 1) - 2*\log(\text{abs}(\tan(b*x + a) + 1)))/b$

maple [A] time = 0.74, size = 184, normalized size = 1.53

$$\frac{\ln(\tan(bx+a)\sqrt{5}+2(\tan^2(bx+a))-\tan(bx+a)+2)\sqrt{5}}{5b} + \frac{\ln(\tan(bx+a)\sqrt{5}+2(\tan^2(bx+a))-\tan(bx+a)+2)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x)

[Out] 1/5/b*ln(tan(b*x+a)*5^(1/2)+2*tan(b*x+a)^2-tan(b*x+a)+2)*5^(1/2)+1/5/b*ln(tan(b*x+a)*5^(1/2)+2*tan(b*x+a)^2-tan(b*x+a)+2)-1/5/b*ln(-tan(b*x+a)*5^(1/2)+2*tan(b*x+a)^2-tan(b*x+a)+2)*5^(1/2)+1/5/b*ln(-tan(b*x+a)*5^(1/2)+2*tan(b*x+a)^2-tan(b*x+a)+2)-1/2/b*ln(1+tan(b*x+a)^2)+1/5*ln(1+tan(b*x+a))/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^5 - \sin(bx+a)^5}{\cos(bx+a)^5 + \sin(bx+a)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x, algorithm="maxima")

[Out] integrate((cos(b*x + a)^5 - sin(b*x + a)^5)/(cos(b*x + a)^5 + sin(b*x + a)^5), x)

mupad [B] time = 4.22, size = 226, normalized size = 1.88

$$\frac{\ln\left(\tan\left(\frac{a}{2} + \frac{bx}{2}\right)^2 - 2\tan\left(\frac{a}{2} + \frac{bx}{2}\right) - 1\right)}{5b} - \frac{\ln\left(\tan\left(\frac{a}{2} + \frac{bx}{2}\right)^2 + 1\right)}{b} + \frac{\ln\left(2\tan\left(\frac{a}{2} + \frac{bx}{2}\right)^2 - \tan\left(\frac{a}{2} + \frac{bx}{2}\right) + \tan\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(a + b*x)^5 - sin(a + b*x)^5)/(cos(a + b*x)^5 + sin(a + b*x)^5),x)

[Out] log(tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2) - 1)/(5*b) - log(tan(a/2 + (b*x)/2)^2 + 1)/b + (log(2*tan(a/2 + (b*x)/2)^2 - tan(a/2 + (b*x)/2) + tan(a/2 + (b*x)/2)^3 + tan(a/2 + (b*x)/2)^4 + 5^(1/2)*tan(a/2 + (b*x)/2) - 5^(1/2)*tan(a/2 + (b*x)/2)^3 + 1)*(5^(1/2) + 1))/(5*b) - (log(2*tan(a/2 + (b*x)/2)^2 - tan(a/2 + (b*x)/2) + tan(a/2 + (b*x)/2)^3 + tan(a/2 + (b*x)/2)^4 - 5^(1/2)*tan(a/2 + (b*x)/2) + 5^(1/2)*tan(a/2 + (b*x)/2)^3 + 1)*(5^(1/2) - 1))/(5*b)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)**5-sin(b*x+a)**5)/(cos(b*x+a)**5+sin(b*x+a)**5),x)

[Out] Timed out

$$3.943 \quad \int \frac{\cos^4(a+bx) - \sin^4(a+bx)}{\cos^4(a+bx) + \sin^4(a+bx)} dx$$

Optimal. Leaf size=72

$$\frac{\log(\tan^2(a+bx) + \sqrt{2} \tan(a+bx) + 1)}{2\sqrt{2}b} - \frac{\log(\tan^2(a+bx) - \sqrt{2} \tan(a+bx) + 1)}{2\sqrt{2}b}$$

[Out] $-1/4 \cdot \ln(1 - 2^{(1/2)} \cdot \tan(b \cdot x + a) + \tan(b \cdot x + a)^2) / b \cdot 2^{(1/2)} + 1/4 \cdot \ln(1 + 2^{(1/2)} \cdot \tan(b \cdot x + a) + \tan(b \cdot x + a)^2) / b \cdot 2^{(1/2)}$

Rubi [A] time = 0.15, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1165, 628}

$$\frac{\log(\tan^2(a+bx) + \sqrt{2} \tan(a+bx) + 1)}{2\sqrt{2}b} - \frac{\log(\tan^2(a+bx) - \sqrt{2} \tan(a+bx) + 1)}{2\sqrt{2}b}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^4 - Sin[a + b*x]^4)/(Cos[a + b*x]^4 + Sin[a + b*x]^4), x]

[Out] $-\text{Log}[1 - \text{Sqrt}[2] \cdot \text{Tan}[a + b \cdot x] + \text{Tan}[a + b \cdot x]^2] / (2 \cdot \text{Sqrt}[2] \cdot b) + \text{Log}[1 + \text{Sqrt}[2] \cdot \text{Tan}[a + b \cdot x] + \text{Tan}[a + b \cdot x]^2] / (2 \cdot \text{Sqrt}[2] \cdot b)$

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rubi steps

$$\begin{aligned} \int \frac{\cos^4(a+bx) - \sin^4(a+bx)}{\cos^4(a+bx) + \sin^4(a+bx)} dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4} dx, x, \tan(a+bx)\right)}{b} \\ &= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx, x, \tan(a+bx)\right)}{2\sqrt{2}b} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2} dx, x, \tan(a+bx)\right)}{2\sqrt{2}b} \\ &= -\frac{\log(1 - \sqrt{2} \tan(a+bx) + \tan^2(a+bx))}{2\sqrt{2}b} + \frac{\log(1 + \sqrt{2} \tan(a+bx) + \tan^2(a+bx))}{2\sqrt{2}b} \end{aligned}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 0.35

$$\frac{\tanh^{-1}\left(\frac{\sin(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}b}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^4 - Sin[a + b*x]^4)/(Cos[a + b*x]^4 + Sin[a + b*x]^4), x]

[Out] ArcTanh[Sin[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b)

fricas [A] time = 0.97, size = 74, normalized size = 1.03

$$\frac{\sqrt{2} \log\left(-\frac{2 \cos(bx+a)^4 - 2\sqrt{2} \cos(bx+a) \sin(bx+a) - 2 \cos(bx+a)^2 - 1}{2 \cos(bx+a)^4 - 2 \cos(bx+a)^2 + 1}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4), x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log(-(2*cos(b*x + a)^4 - 2*sqrt(2)*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 - 1)/(2*cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1))/b

giac [A] time = 0.26, size = 48, normalized size = 0.67

$$\frac{\sqrt{2} \log\left(\frac{|-2\sqrt{2} + 2 \sin(2bx+2a)|}{|2\sqrt{2} + 2 \sin(2bx+2a)|}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4), x, algorithm="giac")

[Out] -1/4*sqrt(2)*log(abs(-2*sqrt(2) + 2*sin(2*b*x + 2*a))/abs(2*sqrt(2) + 2*sin(2*b*x + 2*a)))/b

maple [A] time = 0.38, size = 108, normalized size = 1.50

$$\frac{\sqrt{2} \ln\left(\frac{1+\sqrt{2} \tan(bx+a)+\tan^2(bx+a)}{1-\sqrt{2} \tan(bx+a)+\tan^2(bx+a)}\right)}{8b} - \frac{\sqrt{2} \ln\left(\frac{1-\sqrt{2} \tan(bx+a)+\tan^2(bx+a)}{1+\sqrt{2} \tan(bx+a)+\tan^2(bx+a)}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4), x)

[Out] 1/8/b*2^(1/2)*ln((1+2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/(1-2^(1/2)*tan(b*x+a)+tan(b*x+a)^2))-1/8/b*2^(1/2)*ln((1-2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/(1+2^(1/2)*tan(b*x+a)+tan(b*x+a)^2))

maxima [A] time = 0.46, size = 58, normalized size = 0.81

$$\frac{\sqrt{2} \log\left(\tan(bx+a)^2 + \sqrt{2} \tan(bx+a) + 1\right) - \sqrt{2} \log\left(\tan(bx+a)^2 - \sqrt{2} \tan(bx+a) + 1\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4), x, algorithm="maxima")

[Out] 1/4*(sqrt(2)*log(tan(b*x + a)^2 + sqrt(2)*tan(b*x + a) + 1) - sqrt(2)*log(tan(b*x + a)^2 - sqrt(2)*tan(b*x + a) + 1))/b

mupad [B] time = 3.17, size = 23, normalized size = 0.32

$$\frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2} \sin(2a+2bx)}{2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^4 - sin(a + b*x)^4)/(cos(a + b*x)^4 + sin(a + b*x)^4), x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*sin(2*a + 2*b*x))/2))/(2*b)
```

sympy [A] time = 5.59, size = 122, normalized size = 1.69

$$\left\{ \begin{array}{l} -\frac{\sqrt{2} \log(4 \sin^2(a+bx) - 4\sqrt{2} \sin(a+bx) \cos(a+bx) + 4 \cos^2(a+bx))}{4b} + \frac{\sqrt{2} \log(4 \sin^2(a+bx) + 4\sqrt{2} \sin(a+bx) \cos(a+bx) + 4 \cos^2(a+bx))}{4b} \\ \frac{x(-\sin^4(a) + \cos^4(a))}{\sin^4(a) + \cos^4(a)} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)**4-sin(b*x+a)**4)/(cos(b*x+a)**4+sin(b*x+a)**4), x)
```

```
[Out] Piecewise((-sqrt(2)*log(4*sin(a + b*x)**2 - 4*sqrt(2)*sin(a + b*x)*cos(a + b*x) + 4*cos(a + b*x)**2)/(4*b) + sqrt(2)*log(4*sin(a + b*x)**2 + 4*sqrt(2)*sin(a + b*x)*cos(a + b*x) + 4*cos(a + b*x)**2)/(4*b), Ne(b, 0)), (x*(-sin(a)**4 + cos(a)**4)/(sin(a)**4 + cos(a)**4), True))
```

$$3.944 \quad \int \frac{\cos^3(a+bx) - \sin^3(a+bx)}{\cos^3(a+bx) + \sin^3(a+bx)} dx$$

Optimal. Leaf size=55

$$-\frac{2 \log(\tan^2(a+bx) - \tan(a+bx) + 1)}{3b} + \frac{\log(\tan(a+bx) + 1)}{3b} - \frac{\log(\cos(a+bx))}{b}$$

[Out] $-\ln(\cos(b*x+a))/b + 1/3*\ln(1+\tan(b*x+a))/b - 2/3*\ln(1-\tan(b*x+a)+\tan(b*x+a)^2)/b$

Rubi [A] time = 0.41, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2074, 260, 628}

$$-\frac{2 \log(\tan^2(a+bx) - \tan(a+bx) + 1)}{3b} + \frac{\log(\tan(a+bx) + 1)}{3b} - \frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^3 - Sin[a + b*x]^3)/(Cos[a + b*x]^3 + Sin[a + b*x]^3), x]

[Out] $-(\text{Log}[\text{Cos}[a + b*x]]/b) + \text{Log}[1 + \text{Tan}[a + b*x]]/(3*b) - (2*\text{Log}[1 - \text{Tan}[a + b*x] + \text{Tan}[a + b*x]^2])/(3*b)$

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos^3(a+bx) - \sin^3(a+bx)}{\cos^3(a+bx) + \sin^3(a+bx)} dx &= \frac{\text{Subst}\left(\int \frac{1-x^3}{1+x^2+x^3+x^5} dx, x, \tan(a+bx)\right)}{b} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{3(1+x)} + \frac{x}{1+x^2} - \frac{2(-1+2x)}{3(1-x+x^2)}\right) dx, x, \tan(a+bx)\right)}{b} \\ &= \frac{\log(1 + \tan(a+bx))}{3b} - \frac{2 \text{Subst}\left(\int \frac{-1+2x}{1-x+x^2} dx, x, \tan(a+bx)\right)}{3b} + \frac{\text{Subst}\left(\int \frac{x}{1+x^2} dx, x, \tan(a+bx)\right)}{3b} \\ &= -\frac{\log(\cos(a+bx))}{b} + \frac{\log(1 + \tan(a+bx))}{3b} - \frac{2 \log(1 - \tan(a+bx) + \tan^2(a+bx))}{3b} \end{aligned}$$

Mathematica [A] time = 0.20, size = 42, normalized size = 0.76

$$\frac{\log(\sin(a + bx) + \cos(a + bx))}{3b} - \frac{2 \log(2 - \sin(2(a + bx)))}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^3 - Sin[a + b*x]^3)/(Cos[a + b*x]^3 + Sin[a + b*x]^3), x]

[Out] Log[Cos[a + b*x] + Sin[a + b*x]]/(3*b) - (2*Log[2 - Sin[2*(a + b*x)]])/(3*b)

fricas [A] time = 0.90, size = 42, normalized size = 0.76

$$\frac{\log(2 \cos(bx + a) \sin(bx + a) + 1) - 4 \log(-\cos(bx + a) \sin(bx + a) + 1)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3), x, algorithm="fricas")

[Out] 1/6*(log(2*cos(b*x + a)*sin(b*x + a) + 1) - 4*log(-cos(b*x + a)*sin(b*x + a) + 1))/b

giac [A] time = 0.29, size = 52, normalized size = 0.95

$$\frac{4 \log(\tan(bx + a)^2 - \tan(bx + a) + 1) - 3 \log(\tan(bx + a)^2 + 1) - 2 \log(|\tan(bx + a) + 1|)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3), x, algorithm="giac")

[Out] -1/6*(4*log(tan(b*x + a)^2 - tan(b*x + a) + 1) - 3*log(tan(b*x + a)^2 + 1) - 2*log(abs(tan(b*x + a) + 1)))/b

maple [A] time = 0.65, size = 56, normalized size = 1.02

$$-\frac{2 \ln(1 - \tan(bx + a) + \tan^2(bx + a))}{3b} + \frac{\ln(1 + \tan^2(bx + a))}{2b} + \frac{\ln(1 + \tan(bx + a))}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3), x)

[Out] -2/3*ln(1-tan(b*x+a)+tan(b*x+a)^2)/b+1/2/b*ln(1+tan(b*x+a)^2)+1/3*ln(1+tan(b*x+a))/b

maxima [B] time = 0.43, size = 154, normalized size = 2.80

$$\frac{2 \log\left(-\frac{2 \sin(bx+a)}{\cos(bx+a)+1} + \frac{2 \sin(bx+a)^2}{(\cos(bx+a)+1)^2} + \frac{2 \sin(bx+a)^3}{(\cos(bx+a)+1)^3} + \frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4} + 1\right) - \log\left(-\frac{2 \sin(bx+a)}{\cos(bx+a)+1} + \frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2} - 1\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3), x, algorithm="maxima")

[Out] -1/3*(2*log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + 2*sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 2*sin(b*x + a)^3/(cos(b*x + a) + 1)^3 + sin(b*x + a)^4/(cos(b

$(b*x + a) + 1)^4 + 1) - \log(-2*\sin(b*x + a)/(\cos(b*x + a) + 1) + \sin(b*x + a)^2/(\cos(b*x + a) + 1)^2 - 1) - 3*\log(\sin(b*x + a)^2/(\cos(b*x + a) + 1)^2 + 1))/b$

mupad [B] time = 3.30, size = 105, normalized size = 1.91

$$\frac{\ln\left(\tan\left(\frac{a}{2} + \frac{bx}{2}\right)^2 + 1\right)}{b} + \frac{\ln\left(\tan\left(\frac{a}{2} + \frac{bx}{2}\right)^2 - 2\tan\left(\frac{a}{2} + \frac{bx}{2}\right) - 1\right)}{3b} - \frac{2\ln\left(\tan\left(\frac{a}{2} + \frac{bx}{2}\right)^4 + 2\tan\left(\frac{a}{2} + \frac{bx}{2}\right)^3 + 2\tan\left(\frac{a}{2} + \frac{bx}{2}\right)^2 + 1\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((cos(a + b*x)^3 - sin(a + b*x)^3)/(cos(a + b*x)^3 + sin(a + b*x)^3), x)`

[Out] `log(tan(a/2 + (b*x)/2)^2 + 1)/b + log(tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2) - 1)/(3*b) - (2*log(2*tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2) + 2*tan(a/2 + (b*x)/2)^3 + tan(a/2 + (b*x)/2)^4 + 1))/(3*b)`

sympy [A] time = 1.02, size = 76, normalized size = 1.38

$$\begin{cases} \frac{\log(\sin(a+bx)+\cos(a+bx))}{3b} - \frac{2\log(\sin^2(a+bx)-\sin(a+bx)\cos(a+bx)+\cos^2(a+bx))}{3b} & \text{for } b \neq 0 \\ \frac{x(-\sin^3(a)+\cos^3(a))}{\sin^3(a)+\cos^3(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((cos(b*x+a)**3-sin(b*x+a)**3)/(cos(b*x+a)**3+sin(b*x+a)**3), x)`

[Out] `Piecewise((log(sin(a + b*x) + cos(a + b*x))/(3*b) - 2*log(sin(a + b*x)**2 - sin(a + b*x)*cos(a + b*x) + cos(a + b*x)**2)/(3*b), Ne(b, 0)), (x*(-sin(a)**3 + cos(a)**3)/(sin(a)**3 + cos(a)**3), True))`

$$3.945 \quad \int \frac{\cos^2(a+bx) - \sin^2(a+bx)}{\cos^2(a+bx) + \sin^2(a+bx)} dx$$

Optimal. Leaf size=16

$$\frac{\sin(a+bx)\cos(a+bx)}{b}$$

[Out] cos(b*x+a)*sin(b*x+a)/b

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4380, 2635, 8}

$$\frac{\sin(a+bx)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x]^2 - Sin[a + b*x]^2)/(Cos[a + b*x]^2 + Sin[a + b*x]^2), x]

[Out] (Cos[a + b*x]*Sin[a + b*x])/b

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 4380

Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)^2*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)^2])^(p_.), x_Symbol] := Dist[(a + c)^p, Int[ActivateTrig[u], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(a+bx) - \sin^2(a+bx)}{\cos^2(a+bx) + \sin^2(a+bx)} dx &= \int (\cos^2(a+bx) - \sin^2(a+bx)) dx \\ &= \int \cos^2(a+bx) dx - \int \sin^2(a+bx) dx \\ &= \frac{\cos(a+bx)\sin(a+bx)}{b} \end{aligned}$$

Mathematica [B] time = 0.01, size = 33, normalized size = 2.06

$$\frac{\sin(2a)\cos(2bx)}{2b} + \frac{\cos(2a)\sin(2bx)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x]^2 - Sin[a + b*x]^2)/(Cos[a + b*x]^2 + Sin[a + b*x]^2), x]

[Out] (Cos[2*b*x]*Sin[2*a])/(2*b) + (Cos[2*a]*Sin[2*b*x])/(2*b)

fricas [A] time = 0.55, size = 16, normalized size = 1.00

$$\frac{\cos(bx + a) \sin(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x, algorithm="fricas")

[Out] cos(b*x + a)*sin(b*x + a)/b

giac [A] time = 0.18, size = 14, normalized size = 0.88

$$\frac{\sin(2bx + 2a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x, algorithm="giac")

[Out] 1/2*sin(2*b*x + 2*a)/b

maple [A] time = 0.23, size = 17, normalized size = 1.06

$$\frac{\cos(bx + a) \sin(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x)

[Out] cos(b*x+a)*sin(b*x+a)/b

maxima [A] time = 0.33, size = 22, normalized size = 1.38

$$\frac{\tan(bx + a)}{(\tan(bx + a)^2 + 1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x, algorithm="maxima")

[Out] tan(b*x + a)/((tan(b*x + a)^2 + 1)*b)

mupad [B] time = 3.03, size = 14, normalized size = 0.88

$$\frac{\sin(2a + 2bx)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(a + b*x)^2 - sin(a + b*x)^2)/(cos(a + b*x)^2 + sin(a + b*x)^2),x)

[Out] sin(2*a + 2*b*x)/(2*b)

sympy [B] time = 0.25, size = 32, normalized size = 2.00

$$\frac{\sin(a + bx) \cos(a + bx)}{b \sin^2(a + bx) + b \cos^2(a + bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)**2-sin(b*x+a)**2)/(cos(b*x+a)**2+sin(b*x+a)**2),x)

[Out] sin(a + b*x)*cos(a + b*x)/(b*sin(a + b*x)**2 + b*cos(a + b*x)**2)

$$3.946 \quad \int \frac{\cos(a+bx) - \sin(a+bx)}{\cos(a+bx) + \sin(a+bx)} dx$$

Optimal. Leaf size=18

$$\frac{\log(\sin(a + bx) + \cos(a + bx))}{b}$$

[Out] ln(cos(b*x+a)+sin(b*x+a))/b

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {3133}

$$\frac{\log(\sin(a + bx) + \cos(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x] - Sin[a + b*x])/(Cos[a + b*x] + Sin[a + b*x]),x]

[Out] Log[Cos[a + b*x] + Sin[a + b*x]]/b

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)])*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)])*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos(a + bx) - \sin(a + bx)}{\cos(a + bx) + \sin(a + bx)} dx = \frac{\log(\cos(a + bx) + \sin(a + bx))}{b}$$

Mathematica [A] time = 0.04, size = 18, normalized size = 1.00

$$\frac{\log(\sin(a + bx) + \cos(a + bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x] - Sin[a + b*x])/(Cos[a + b*x] + Sin[a + b*x]),x]

[Out] Log[Cos[a + b*x] + Sin[a + b*x]]/b

fricas [A] time = 1.16, size = 22, normalized size = 1.22

$$\frac{\log(2 \cos(bx + a) \sin(bx + a) + 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x, algorithm="fricas")

[Out] 1/2*log(2*cos(b*x + a)*sin(b*x + a) + 1)/b

giac [A] time = 0.17, size = 29, normalized size = 1.61

$$\frac{\log(\tan(bx+a)^2+1) - 2 \log(|\tan(bx+a)+1|)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x, algorithm="giac")

[Out] -1/2*(log(tan(b*x + a)^2 + 1) - 2*log(abs(tan(b*x + a) + 1)))/b

maple [A] time = 0.25, size = 19, normalized size = 1.06

$$\frac{\ln(\cos(bx+a) + \sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x)

[Out] ln(cos(b*x+a)+sin(b*x+a))/b

maxima [A] time = 0.31, size = 18, normalized size = 1.00

$$\frac{\log(\cos(bx+a) + \sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x, algorithm="maxima")

[Out] log(cos(b*x + a) + sin(b*x + a))/b

mupad [B] time = 3.14, size = 50, normalized size = 2.78

$$\frac{2 \operatorname{atanh}\left(\frac{128 \tan\left(\frac{a}{2} + \frac{bx}{2}\right) + 128}{16 \tan\left(\frac{a}{2} + \frac{bx}{2}\right)^2 + 32 \tan\left(\frac{a}{2} + \frac{bx}{2}\right) + 48}\right) - 3}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(a + b*x) - sin(a + b*x))/(cos(a + b*x) + sin(a + b*x)),x)

[Out] (2*atanh((128*tan(a/2 + (b*x)/2) + 128)/(32*tan(a/2 + (b*x)/2) + 16*tan(a/2 + (b*x)/2)^2 + 48) - 3))/b

sympy [A] time = 0.39, size = 31, normalized size = 1.72

$$\begin{cases} \frac{\log(\sin(a+bx)+\cos(a+bx))}{b} & \text{for } b \neq 0 \\ \frac{x(-\sin(a)+\cos(a))}{\sin(a)+\cos(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x)

[Out] Piecewise((log(sin(a + b*x) + cos(a + b*x))/b, Ne(b, 0)), (x*(-sin(a) + cos(a))/(sin(a) + cos(a)), True))

$$3.947 \quad \int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)} dx$$

Optimal. Leaf size=19

$$\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

[Out] $-\ln(\cos(b*x+a)+\sin(b*x+a))/b$

Rubi [A] time = 0.31, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {801, 260}

$$\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[(-Csc[a + b*x] + Sec[a + b*x])/(Csc[a + b*x] + Sec[a + b*x]),x]

[Out] $-(\text{Log}[\text{Cos}[a + b*x] + \text{Sin}[a + b*x]])/b$

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 801

Int[(((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_)))/((a_) + (c_)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)} dx &= \frac{\text{Subst}\left(\int \frac{-1+x}{(1+x)(1+x^2)} dx, x, \tan(a+bx)\right)}{b} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{-1-x} + \frac{x}{1+x^2}\right) dx, x, \tan(a+bx)\right)}{b} \\ &= -\frac{\log(1+\tan(a+bx))}{b} + \frac{\text{Subst}\left(\int \frac{x}{1+x^2} dx, x, \tan(a+bx)\right)}{b} \\ &= -\frac{\log(\cos(a+bx))}{b} - \frac{\log(1+\tan(a+bx))}{b} \end{aligned}$$

Mathematica [A] time = 0.06, size = 19, normalized size = 1.00

$$\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(-Csc[a + b*x] + Sec[a + b*x])/(Csc[a + b*x] + Sec[a + b*x]),x]

[Out] $-(\text{Log}[\text{Cos}[a + b*x] + \text{Sin}[a + b*x]])/b$

fricas [A] time = 1.35, size = 22, normalized size = 1.16

$$\frac{\log(2 \cos(bx+a) \sin(bx+a) + 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x, algorithm="fricas")

[Out] -1/2*log(2*cos(b*x + a)*sin(b*x + a) + 1)/b

giac [A] time = 0.29, size = 29, normalized size = 1.53

$$\frac{\log(\tan(bx+a)^2 + 1) - 2 \log(|\tan(bx+a) + 1|)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x, algorithm="giac")

[Out] 1/2*(log(tan(b*x + a)^2 + 1) - 2*log(abs(tan(b*x + a) + 1)))/b

maple [A] time = 0.75, size = 32, normalized size = 1.68

$$\frac{\ln(1 + \tan^2(bx + a))}{2b} - \frac{\ln(1 + \tan(bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x)

[Out] 1/2/b*ln(1+tan(b*x+a)^2)-ln(1+tan(b*x+a))/b

maxima [B] time = 0.41, size = 70, normalized size = 3.68

$$\frac{\log\left(-\frac{2 \sin(bx+a)}{\cos(bx+a)+1} + \frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2} - 1\right) - \log\left(\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x, algorithm="maxima")

[Out] -(log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + sin(b*x + a)^2/(cos(b*x + a) + 1)^2 - 1) - log(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1))/b

mapad [B] time = 3.37, size = 50, normalized size = 2.63

$$\frac{2 \operatorname{atanh}\left(\frac{128 \tan\left(\frac{a}{2} + \frac{bx}{2}\right) + 128}{16 \tan\left(\frac{a}{2} + \frac{bx}{2}\right)^2 + 32 \tan\left(\frac{a}{2} + \frac{bx}{2}\right) + 48} - 3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/cos(a + b*x) - 1/sin(a + b*x))/(1/cos(a + b*x) + 1/sin(a + b*x)),x)

[Out] -(2*atanh((128*tan(a/2 + (b*x)/2) + 128)/(32*tan(a/2 + (b*x)/2) + 16*tan(a/2 + (b*x)/2)^2 + 48) - 3))/b

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\csc(a + bx)}{\csc(a + bx) + \sec(a + bx)} dx - \int \left(-\frac{\sec(a + bx)}{\csc(a + bx) + \sec(a + bx)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x)

[Out] -Integral(csc(a + b*x)/(csc(a + b*x) + sec(a + b*x)), x) - Integral(-sec(a + b*x)/(csc(a + b*x) + sec(a + b*x)), x)

$$3.948 \quad \int \frac{-\csc^2(a+bx) + \sec^2(a+bx)}{\csc^2(a+bx) + \sec^2(a+bx)} dx$$

Optimal. Leaf size=17

$$-\frac{\sin(a+bx)\cos(a+bx)}{b}$$

[Out] `-cos(b*x+a)*sin(b*x+a)/b`

Rubi [A] time = 0.17, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {383}

$$-\frac{\sin(a+bx)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] `Int[(-Csc[a + b*x]^2 + Sec[a + b*x]^2)/(Csc[a + b*x]^2 + Sec[a + b*x]^2), x]`

[Out] `-((Cos[a + b*x]*Sin[a + b*x])/b)`

Rule 383

`Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]`

Rubi steps

$$\int \frac{-\csc^2(a+bx) + \sec^2(a+bx)}{\csc^2(a+bx) + \sec^2(a+bx)} dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{(1+x^2)^2} dx, x, \tan(a+bx)\right)}{b} = -\frac{\cos(a+bx)\sin(a+bx)}{b}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.94

$$-\frac{\sin(2a)\cos(2bx)}{2b} - \frac{\cos(2a)\sin(2bx)}{2b}$$

Antiderivative was successfully verified.

[In] `Integrate[(-Csc[a + b*x]^2 + Sec[a + b*x]^2)/(Csc[a + b*x]^2 + Sec[a + b*x]^2), x]`

[Out] `-1/2*(Cos[2*b*x]*Sin[2*a])/b - (Cos[2*a]*Sin[2*b*x])/(2*b)`

fricas [A] time = 1.41, size = 17, normalized size = 1.00

$$-\frac{\cos(bx+a)\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2), x, algorithm="fricas")`

[Out] `-cos(b*x + a)*sin(b*x + a)/b`

giac [A] time = 0.24, size = 14, normalized size = 0.82

$$\frac{\sin(2bx + 2a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x, algorithm="giac")

[Out] -1/2*sin(2*b*x + 2*a)/b

maple [A] time = 0.34, size = 18, normalized size = 1.06

$$\frac{\cos(bx + a) \sin(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x)

[Out] -cos(b*x+a)*sin(b*x+a)/b

maxima [A] time = 0.31, size = 23, normalized size = 1.35

$$\frac{\tan(bx + a)}{(\tan(bx + a)^2 + 1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x, algorithm="maxima")

[Out] -tan(b*x + a)/((tan(b*x + a)^2 + 1)*b)

mupad [B] time = 3.05, size = 14, normalized size = 0.82

$$\frac{\sin(2a + 2bx)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/cos(a + b*x)^2 - 1/sin(a + b*x)^2)/(1/cos(a + b*x)^2 + 1/sin(a + b*x)^2),x)

[Out] -sin(2*a + 2*b*x)/(2*b)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\csc^2(a + bx)}{\csc^2(a + bx) + \sec^2(a + bx)} dx - \int \left(-\frac{\sec^2(a + bx)}{\csc^2(a + bx) + \sec^2(a + bx)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(b*x+a)**2+sec(b*x+a)**2)/(csc(b*x+a)**2+sec(b*x+a)**2),x)

[Out] -Integral(csc(a + b*x)**2/(csc(a + b*x)**2 + sec(a + b*x)**2), x) - Integral(-sec(a + b*x)**2/(csc(a + b*x)**2 + sec(a + b*x)**2), x)

$$3.949 \quad \int \frac{-\csc^3(a+bx) + \sec^3(a+bx)}{\csc^3(a+bx) + \sec^3(a+bx)} dx$$

Optimal. Leaf size=54

$$\frac{2 \log(\tan^2(a+bx) - \tan(a+bx) + 1)}{3b} - \frac{\log(\tan(a+bx) + 1)}{3b} + \frac{\log(\cos(a+bx))}{b}$$

[Out] $\ln(\cos(b*x+a))/b - 1/3*\ln(1+\tan(b*x+a))/b + 2/3*\ln(1-\tan(b*x+a)+\tan(b*x+a)^2)/b$

Rubi [A] time = 0.53, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {6725, 260, 628}

$$\frac{2 \log(\tan^2(a+bx) - \tan(a+bx) + 1)}{3b} - \frac{\log(\tan(a+bx) + 1)}{3b} + \frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-\text{Csc}[a + b*x]^3 + \text{Sec}[a + b*x]^3)/(\text{Csc}[a + b*x]^3 + \text{Sec}[a + b*x]^3), x]$

[Out] $\text{Log}[\text{Cos}[a + b*x]]/b - \text{Log}[1 + \text{Tan}[a + b*x]]/(3*b) + (2*\text{Log}[1 - \text{Tan}[a + b*x] + \text{Tan}[a + b*x]^2])/(3*b)$

Rule 260

$\text{Int}[(x_)^m/((a_) + (b_)*(x_)^n), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}\{a, b, m, n, x\} \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 628

$\text{Int}(((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 6725

$\text{Int}(u_/((a_) + (b_)*(x_)^n), x_Symbol] \rightarrow \text{With}\{v = \text{RationalFunctionExpand}[u/(a + b*x^n), x]\}, \text{Int}[v, x] /; \text{SumQ}[v] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{IGtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{-\csc^3(a+bx) + \sec^3(a+bx)}{\csc^3(a+bx) + \sec^3(a+bx)} dx &= \frac{\text{Subst}\left(\int \frac{-1+x^3}{(1+x^2)(1+x^3)} dx, x, \tan(a+bx)\right)}{b} \\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{3(1+x)} - \frac{x}{1+x^2} + \frac{2(-1+2x)}{3(1-x+x^2)}\right) dx, x, \tan(a+bx)\right)}{b} \\ &= -\frac{\log(1 + \tan(a+bx))}{3b} + \frac{2 \text{Subst}\left(\int \frac{-1+2x}{1-x+x^2} dx, x, \tan(a+bx)\right)}{3b} - \frac{\text{Subst}\left(\int \frac{1}{1-x+x^2} dx, x, \tan(a+bx)\right)}{3b} \\ &= \frac{\log(\cos(a+bx))}{b} - \frac{\log(1 + \tan(a+bx))}{3b} + \frac{2 \log(1 - \tan(a+bx) + \tan^2(a+bx))}{3b} \end{aligned}$$

Mathematica [A] time = 0.23, size = 42, normalized size = 0.78

$$\frac{2 \log(2 - \sin(2(a+bx)))}{3b} - \frac{\log(\sin(a+bx) + \cos(a+bx))}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(-Csc[a + b*x]^3 + Sec[a + b*x]^3)/(Csc[a + b*x]^3 + Sec[a + b*x]^3), x]

[Out] -1/3*Log[Cos[a + b*x] + Sin[a + b*x]]/b + (2*Log[2 - Sin[2*(a + b*x)]])/(3*b)

fricas [A] time = 0.94, size = 42, normalized size = 0.78

$$\frac{\log(2 \cos(bx + a) \sin(bx + a) + 1) - 4 \log(-\cos(bx + a) \sin(bx + a) + 1)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3), x, algorithm="fricas")

[Out] -1/6*(log(2*cos(b*x + a)*sin(b*x + a) + 1) - 4*log(-cos(b*x + a)*sin(b*x + a) + 1))/b

giac [A] time = 0.39, size = 52, normalized size = 0.96

$$\frac{4 \log(\tan(bx + a)^2 - \tan(bx + a) + 1) - 3 \log(\tan(bx + a)^2 + 1) - 2 \log(|\tan(bx + a) + 1|)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3), x, algorithm="giac")

[Out] 1/6*(4*log(tan(b*x + a)^2 - tan(b*x + a) + 1) - 3*log(tan(b*x + a)^2 + 1) - 2*log(abs(tan(b*x + a) + 1)))/b

maple [A] time = 0.83, size = 56, normalized size = 1.04

$$\frac{2 \ln(1 - \tan(bx + a) + \tan^2(bx + a))}{3b} - \frac{\ln(1 + \tan^2(bx + a))}{2b} - \frac{\ln(1 + \tan(bx + a))}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3), x)

[Out] 2/3*ln(1-tan(b*x+a)+tan(b*x+a)^2)/b-1/2/b*ln(1+tan(b*x+a)^2)-1/3*ln(1+tan(b*x+a))/b

maxima [B] time = 0.44, size = 154, normalized size = 2.85

$$\frac{2 \log\left(-\frac{2 \sin(bx+a)}{\cos(bx+a)+1} + \frac{2 \sin(bx+a)^2}{(\cos(bx+a)+1)^2} + \frac{2 \sin(bx+a)^3}{(\cos(bx+a)+1)^3} + \frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4} + 1\right) - \log\left(-\frac{2 \sin(bx+a)}{\cos(bx+a)+1} + \frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2} - 1\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3), x, algorithm="maxima")

[Out] 1/3*(2*log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + 2*sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 2*sin(b*x + a)^3/(cos(b*x + a) + 1)^3 + sin(b*x + a)^4/(cos(b*x + a) + 1)^4 + 1) - log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + sin(b*x + a)^2/(cos(b*x + a) + 1)^2 - 1) - 3*log(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1))/b

mupad [B] time = 3.23, size = 106, normalized size = 1.96

$$\frac{2 \ln\left(\tan\left(\frac{a}{2} + \frac{bx}{2}\right)^4 + 2 \tan\left(\frac{a}{2} + \frac{bx}{2}\right)^3 + 2 \tan\left(\frac{a}{2} + \frac{bx}{2}\right)^2 - 2 \tan\left(\frac{a}{2} + \frac{bx}{2}\right) + 1\right)}{3b} - \frac{\ln\left(\tan\left(\frac{a}{2} + \frac{bx}{2}\right)^2 - 2 \tan\left(\frac{a}{2} + \frac{bx}{2}\right) + 1\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/cos(a + b*x)^3 - 1/sin(a + b*x)^3)/(1/cos(a + b*x)^3 + 1/sin(a + b*x)^3),x)

[Out] (2*log(2*tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2) + 2*tan(a/2 + (b*x)/2)^3 + tan(a/2 + (b*x)/2)^4 + 1))/(3*b) - log(tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2) - 1)/(3*b) - log(tan(a/2 + (b*x)/2)^2 + 1)/b

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(b*x+a)**3+sec(b*x+a)**3)/(csc(b*x+a)**3+sec(b*x+a)**3),x)

[Out] Timed out

$$3.950 \quad \int \frac{-\csc^4(a+bx) + \sec^4(a+bx)}{\csc^4(a+bx) + \sec^4(a+bx)} dx$$

Optimal. Leaf size=72

$$\frac{\log(\tan^2(a+bx) - \sqrt{2} \tan(a+bx) + 1)}{2\sqrt{2}b} - \frac{\log(\tan^2(a+bx) + \sqrt{2} \tan(a+bx) + 1)}{2\sqrt{2}b}$$

[Out] $1/4 * \ln(1 - 2^{(1/2)} * \tan(b*x+a) + \tan(b*x+a)^2) / b * 2^{(1/2)} - 1/4 * \ln(1 + 2^{(1/2)} * \tan(b*x+a) + \tan(b*x+a)^2) / b * 2^{(1/2)}$

Rubi [A] time = 1.40, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1165, 628}

$$\frac{\log(\tan^2(a+bx) - \sqrt{2} \tan(a+bx) + 1)}{2\sqrt{2}b} - \frac{\log(\tan^2(a+bx) + \sqrt{2} \tan(a+bx) + 1)}{2\sqrt{2}b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-\text{Csc}[a + b*x]^4 + \text{Sec}[a + b*x]^4) / (\text{Csc}[a + b*x]^4 + \text{Sec}[a + b*x]^4), x]$
 [Out] $\text{Log}[1 - \text{Sqrt}[2] * \text{Tan}[a + b*x] + \text{Tan}[a + b*x]^2] / (2 * \text{Sqrt}[2] * b) - \text{Log}[1 + \text{Sqrt}[2] * \text{Tan}[a + b*x] + \text{Tan}[a + b*x]^2] / (2 * \text{Sqrt}[2] * b)$

Rule 628

$\text{Int}[(d + (e_*) * (x_)) / ((a_*) + (b_*) * (x_) + (c_*) * (x_)^2), x_Symbol] \rightarrow \text{Simp}[(d * \text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]) / b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 1165

$\text{Int}[(d + (e_*) * (x_)^2) / ((a_*) + (c_*) * (x_)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(2*d)/e, 2]\}, \text{Dist}[e / (2*c*q), \text{Int}[(q - 2*x) / \text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e / (2*c*q), \text{Int}[(q + 2*x) / \text{Simp}[d/e - q*x - x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rubi steps

$$\begin{aligned} \int \frac{-\csc^4(a+bx) + \sec^4(a+bx)}{\csc^4(a+bx) + \sec^4(a+bx)} dx &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{1+x^4} dx, x, \tan(a+bx)\right)}{b} \\ &= \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx, x, \tan(a+bx)\right)}{2\sqrt{2}b} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2} dx, x, \tan(a+bx)\right)}{2\sqrt{2}b} \\ &= \frac{\log(1 - \sqrt{2} \tan(a+bx) + \tan^2(a+bx))}{2\sqrt{2}b} - \frac{\log(1 + \sqrt{2} \tan(a+bx) + \tan^2(a+bx))}{2\sqrt{2}b} \end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\sin(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}b}$$

Antiderivative was successfully verified.

[In] Integrate[(-Csc[a + b*x]^4 + Sec[a + b*x]^4)/(Csc[a + b*x]^4 + Sec[a + b*x]^4), x]

[Out] -(ArcTanh[Sin[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b))

fricas [A] time = 1.15, size = 74, normalized size = 1.03

$$\frac{\sqrt{2} \log\left(-\frac{2 \cos(bx+a)^4 + 2\sqrt{2} \cos(bx+a) \sin(bx+a) - 2 \cos(bx+a)^2 - 1}{2 \cos(bx+a)^4 - 2 \cos(bx+a)^2 + 1}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4), x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log(-(2*cos(b*x + a)^4 + 2*sqrt(2)*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 - 1)/(2*cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1))/b

giac [A] time = 0.39, size = 48, normalized size = 0.67

$$\frac{\sqrt{2} \log\left(\frac{|-2\sqrt{2} + 2 \sin(2bx + 2a)|}{|2\sqrt{2} + 2 \sin(2bx + 2a)|}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4), x, algorithm="giac")

[Out] 1/4*sqrt(2)*log(abs(-2*sqrt(2) + 2*sin(2*b*x + 2*a))/abs(2*sqrt(2) + 2*sin(2*b*x + 2*a)))/b

maple [A] time = 0.51, size = 108, normalized size = 1.50

$$-\frac{\sqrt{2} \ln\left(\frac{1+\sqrt{2} \tan(bx+a)+\tan^2(bx+a)}{1-\sqrt{2} \tan(bx+a)+\tan^2(bx+a)}\right)}{8b} + \frac{\sqrt{2} \ln\left(\frac{1-\sqrt{2} \tan(bx+a)+\tan^2(bx+a)}{1+\sqrt{2} \tan(bx+a)+\tan^2(bx+a)}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4), x)

[Out] -1/8/b*2^(1/2)*ln((1+2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/(1-2^(1/2)*tan(b*x+a)+tan(b*x+a)^2))+1/8/b*2^(1/2)*ln((1-2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/(1+2^(1/2)*tan(b*x+a)+tan(b*x+a)^2))

maxima [A] time = 0.43, size = 58, normalized size = 0.81

$$\frac{\sqrt{2} \log\left(\tan(bx+a)^2 + \sqrt{2} \tan(bx+a) + 1\right) - \sqrt{2} \log\left(\tan(bx+a)^2 - \sqrt{2} \tan(bx+a) + 1\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4), x, algorithm="maxima")

[Out] -1/4*(sqrt(2)*log(tan(b*x + a)^2 + sqrt(2)*tan(b*x + a) + 1) - sqrt(2)*log(tan(b*x + a)^2 - sqrt(2)*tan(b*x + a) + 1))/b

mupad [B] time = 3.09, size = 23, normalized size = 0.32

$$\frac{\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2} \sin(2a+2bx)}{2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x)^4 - 1/sin(a + b*x)^4)/(1/cos(a + b*x)^4 + 1/sin(a + b*x)^4),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*sin(2*a + 2*b*x))/2))/(2*b)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)**4+sec(b*x+a)**4)/(csc(b*x+a)**4+sec(b*x+a)**4),x)
```

```
[Out] Timed out
```


Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```

(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
    If[ListQ[expn],
      Max[Map[ExpnType, expn]],
      If[Head[expn]===Power,
        If[IntegerQ[expn[[2]]],
          ExpnType[expn[[1]]],
          If[Head[expn[[2]]]===Rational,
            If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
              1,
              Max[ExpnType[expn[[1]], 2]],
            Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]],
          If[Head[expn]===Plus || Head[expn]===Times,
            Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
          If[ElementaryFunctionQ[Head[expn]],
            Max[3, ExpnType[expn[[1]]],
          If[SpecialFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
          If[HypergeometricFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
          If[AppellFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
          If[Head[expn]===RootSum,
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
          If[Head[expn]===Integrate || Head[expn]===Int,
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
          9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

```



```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
  debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B";
  fi;

  leaf_count_optimal:=leafcount(optimal);

  ExpnType_result:=ExpnType(result);
  ExpnType_optimal:=ExpnType(optimal);

  if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
  ExpnType_optimal);
  fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
  return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
  if debug then
    print("ExpnType_result<=ExpnType_optimal");
  fi;
fi;
```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```

```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+' or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```

```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

4.0.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+`' or
    type(expn,'*`)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

```

```

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True

```

```

        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func , " is special_function")
        else:
            print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M',
hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands(
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):

```



```

        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(6,m1)      #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(8,m1)      #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```