

# Computer algebra independent integration tests

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/6.2.1-c+d-x-<sup>m</sup>-a+b-cosh-<sup>n</sup>

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 183 ]. This is test number [ 165 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 ( 183 )	% 0.00 ( 0 )
Mathematica	% 98.91 ( 181 )	% 1.09 ( 2 )
Maple	% 60.11 ( 110 )	% 39.89 ( 73 )
Maxima	% 78.14 ( 143 )	% 21.86 ( 40 )
Fricas	% 81.97 ( 150 )	% 18.03 ( 33 )
Sympy	% 33.33 ( 61 )	% 66.67 ( 122 )
Giac	% 56.28 ( 103 )	% 43.72 ( 80 )
Mupad	% 38.25 ( 70 )	% 61.75 ( 113 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

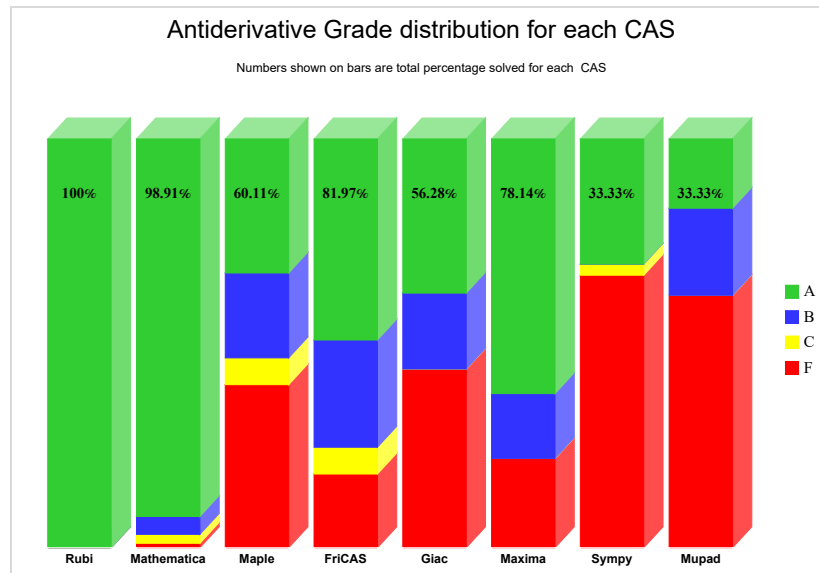
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

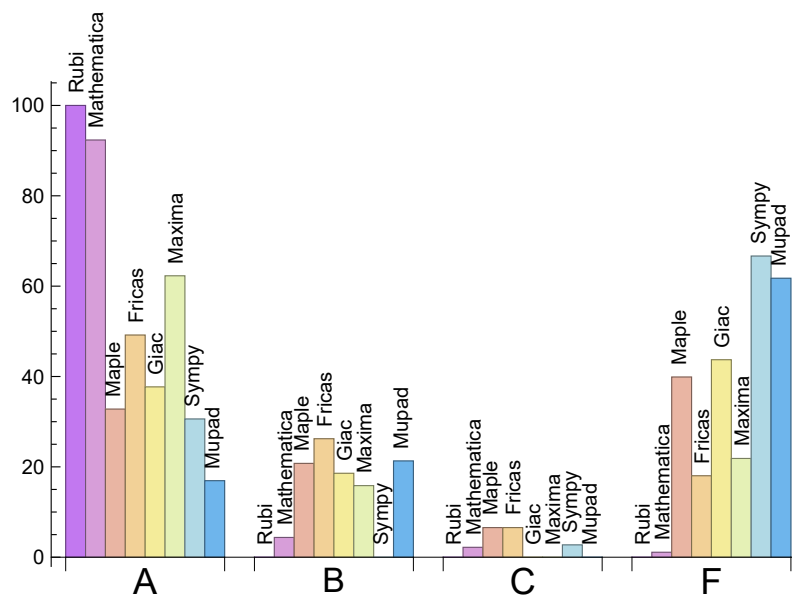
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	92.35	4.37	2.19	1.09
Maple	32.79	20.77	6.56	39.89
Maxima	62.30	15.85	0.00	21.86
Fricas	49.18	26.23	6.56	18.03
Sympy	30.60	0.00	2.73	66.67
Giac	37.70	18.58	0.00	43.72
Mupad	16.94	21.31	0.00	61.75

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	0.00 %	100.00 %	0.00 %
Maple	73	94.52 %	0.00 %	5.48 %
Maxima	40	80.00 %	0.00 %	20.00 %
Fricas	33	18.18 %	0.00 %	81.82 %
Sympy	122	74.59 %	13.93 %	11.48 %
Giac	80	100.00 %	0.00 %	0.00 %
Mupad	113	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

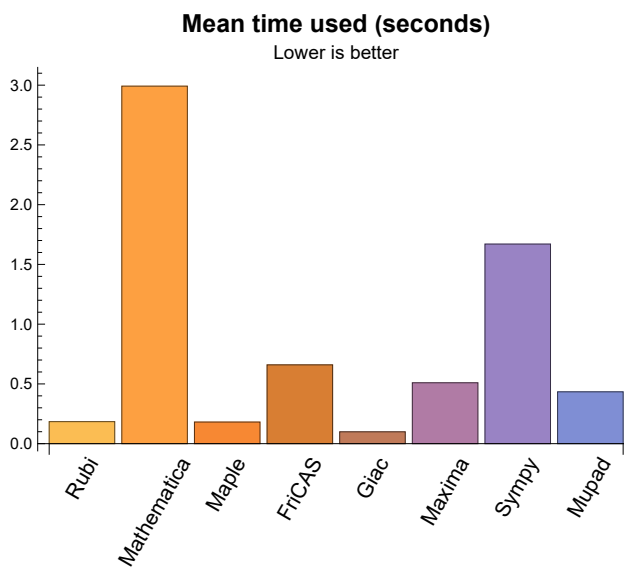
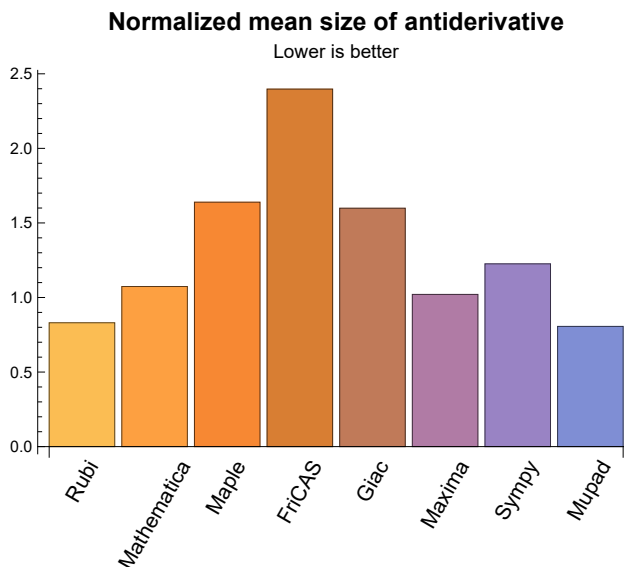
## 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	121.48	0.83	89.00	1.00
Mathematica	2.99	237.19	1.07	79.00	0.84
Maple	0.18	199.35	1.64	108.50	1.43
Maxima	0.51	133.01	1.02	100.00	0.93
Fricas	0.66	465.29	2.40	168.00	1.63
Sympy	1.67	148.85	1.23	68.00	1.13
Giac	0.10	191.42	1.60	107.00	1.15
Mupad	0.43	84.53	0.81	45.50	0.94

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.



## 1.4 list of integrals that has no closed form antiderivative

{29, 30, 34, 35, 39, 40, 68, 69, 70, 75, 79, 80, 114, 115, 119, 120, 142, 143, 147, 148, 149, 150, 154, 155, 171, 172, 176, 177, 178, 182, 183}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {32, 48, 71, 73, 112, 117, 141, 145, 173, 174, 175}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.



## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at <https://>

[ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](http://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

## 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

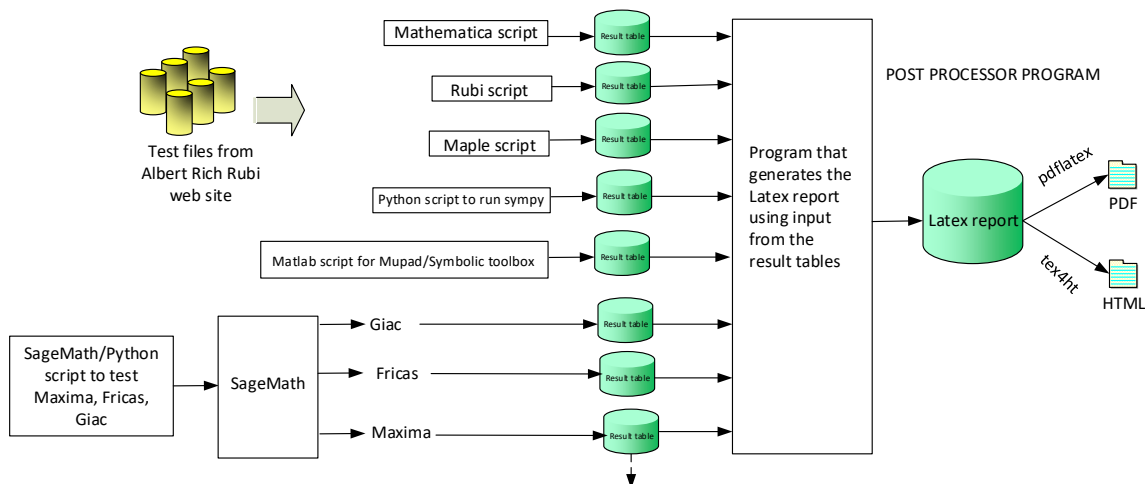
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives  $\sin(x)^2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer, the problem number.
  2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
  3. integer. Leaf size of result.
  4. integer. Leaf size of the optimal antiderivative.
  5. number. CPU time used to solve this integral. 0 if failed.
  6. string. The integral in Latex format
  7. string. The input used in CAS own syntax.
  8. string. The result (antiderivative) produced by CAS in Latex format
  9. string. The optimal antiderivative in Latex format.
  10. integer. 0 or 1. Indicates if problem has known antiderivative or not
  11. String. The result (antiderivative) in CAS own syntax.
  12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
  15. integer. Integrand leaf size.
  16. real number. Ratio of field 14 over field 15
  17. integer. 1 if result was verified or 0 if not verified.
  18. String of form "{n,n,...}" which is list of the rules used by Rubi

**High level overview of the CAS independent integration test build system**

# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 33, 34, 35, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 61, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 178, 179, 180, 181, 182, 183 }

B grade: { 28, 52, 54, 60, 62, 71, 173, 174 }

C grade: { 32, 74, 112, 117 }

F grade: { 39, 40 }

### 2.1.3 Maple

A grade: { 4, 5, 6, 12, 13, 19, 20, 21, 24, 25, 29, 30, 33, 34, 35, 39, 40, 68, 69, 75, 79, 80, 102, 103, 107, 108, 109, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 142, 143, 147, 148, 149, 150, 154, 155, 159, 160, 164, 165, 166, 171, 172, 176, 177, 178, 182, 183 }

B grade: { 1, 2, 3, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 22, 23, 28, 31, 32, 38, 99, 100, 101, 104, 105, 106, 110, 111, 112, 116, 156, 157, 158, 161, 162, 163, 167, 170, 175 }

C grade: { 63, 64, 65, 66, 67, 81, 82, 83, 84, 85, 86, 87 }

F grade: { 26, 27, 36, 37, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 70, 71, 72, 73, 74, 76, 77, 78, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 151, 152, 153, 168, 169, 173, 174, 179, 180, 181 }

### 2.1.4 Maxima

A grade: { 5, 6, 7, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 29, 30, 34, 35, 39, 40, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 66, 67, 68, 69, 70, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 101, 102, 103, 104, 107, 108, 109, 110, 113, 114, 115, 119, 120, 121, 122, 123, 127, 128, 129, 133, 134, 135, 142, 143, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 163, 164, 165, 166, 167, 171, 172, 176, 177, 178, 179, 180, 181, 182, 183 }

B grade: { 1, 2, 3, 4, 8, 9, 16, 17, 18, 19, 31, 33, 41, 42, 43, 44, 63, 64, 65, 99, 100, 105, 106, 111, 116, 118, 156, 157, 162 }

C grade: { }

F grade: { 26, 27, 28, 32, 36, 37, 38, 71, 72, 73, 74, 93, 94, 95, 96, 97, 98, 112, 117, 124, 125, 126, 130, 131, 132, 136, 137, 138, 139, 140, 141, 144, 145, 146, 168, 169, 170, 173, 174, 175 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 9, 10, 11, 12, 18, 19, 20, 23, 24, 25, 29, 30, 34, 35, 39, 40, 44, 51, 59, 65, 68, 69, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 99, 100, 101, 102, 103, 105, 106, 107, 108, 114, 115, 119, 120, 142, 143, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 171, 172, 176, 177, 178, 179, 180, 181, 182, 183 }

B grade: { 6, 7, 8, 13, 14, 15, 16, 17, 21, 22, 28, 33, 38, 41, 42, 43, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 72, 104, 109, 110, 112, 113, 117, 118, 161, 167, 170, 175 }

C grade: { 26, 27, 31, 32, 36, 37, 111, 116, 168, 169, 173, 174 }

F grade: { 70, 71, 73, 74, 95, 96, 97, 98, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 149 }

## 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 23, 24, 25, 29, 30, 34, 35, 39, 40, 68, 69, 70, 75, 79, 80, 99, 100, 101, 105, 106, 107, 113, 114, 115, 118, 119, 120, 142, 143, 147, 148, 149, 154, 155, 156, 157, 158, 162, 163, 164, 171, 172, 182, 183 }

B grade: { }

C grade: { 63, 64, 65, 66, 67 }

F grade: { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 26, 27, 28, 31, 32, 33, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 71, 72, 73, 74, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 102, 103, 104, 108, 109, 110, 111, 112, 116, 117, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 150, 151, 152, 153, 159, 160, 161, 165, 166, 167, 168, 169, 170, 173, 174, 175, 176, 177, 178, 179, 180, 181 }

## 2.1.7 Giac

A grade: { 4, 5, 10, 11, 12, 19, 20, 23, 24, 25, 29, 30, 34, 35, 39, 40, 41, 42, 43, 44, 51, 63, 64, 65, 68, 69, 70, 75, 79, 80, 101, 102, 107, 108, 113, 114, 115, 119, 120, 121, 122, 123, 124, 125, 126, 133, 134, 135, 136, 137, 142, 143, 147, 148, 149, 150, 154, 155, 158, 159, 164, 165, 171, 172, 176, 177, 178, 182, 183 }

B grade: { 1, 2, 3, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 21, 22, 33, 99, 100, 103, 104, 105, 106, 109, 110, 118, 138, 156, 157, 160, 161, 162, 163, 166, 167 }

C grade: { }

F grade: { 26, 27, 28, 31, 32, 36, 37, 38, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 66, 67, 71, 72, 73, 74, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 111, 112, 116, 117, 127, 128, 129, 130, 131, 132, 139, 140, 141, 144, 145, 146, 151, 152, 153, 168, 169, 170, 173, 174, 175, 179, 180, 181 }

## 2.1.8 Mupad

A grade: { 29, 30, 34, 35, 39, 40, 68, 69, 70, 75, 79, 80, 114, 115, 119, 120, 142, 143, 147, 148, 149, 150, 154, 155, 171, 172, 176, 177, 178, 182, 183 }

B grade: { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 23, 24, 25, 33, 71, 72, 73, 99, 100, 101, 105, 106, 107, 113, 118, 121, 122, 123, 127, 128, 129, 156, 157, 158, 162, 163, 164 }

C grade: { }

F grade: { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 26, 27, 28, 31, 32, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 74, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 102, 103, 104, 108, 109, 110, 111, 112, 116, 117, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 151, 152, 153, 159, 160, 161, 165, 166, 167, 168, 169, 170, 173, 174, 175, 179, 180, 181 }



## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	76	547	326	171	311	324	215
normalized size	1	1.00	0.84	6.01	3.58	1.88	3.42	3.56	2.36
time (sec)	N/A	0.119	0.313	0.052	0.542	0.445	2.602	0.139	0.185
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	308	222	111	202	204	143
normalized size	1	1.00	0.87	4.40	3.17	1.59	2.89	2.91	2.04
time (sec)	N/A	0.079	0.203	0.058	0.411	0.660	1.262	0.140	0.935
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	147	135	64	112	112	82
normalized size	1	1.00	0.90	3.00	2.76	1.31	2.29	2.29	1.67
time (sec)	N/A	0.047	0.152	0.050	0.577	0.656	0.559	0.134	0.905

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	53	68	30	46	46	35
normalized size	1	1.00	0.96	1.89	2.43	1.07	1.64	1.64	1.25
time (sec)	N/A	0.020	0.057	0.048	0.374	0.630	0.227	0.120	0.073

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	82	57	94	0	56	-1
normalized size	1	1.00	0.96	1.61	1.12	1.84	0.00	1.10	-0.02
time (sec)	N/A	0.101	0.078	0.075	0.545	0.481	0.000	0.116	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	133	81	150	0	615	-1
normalized size	1	1.00	0.92	1.87	1.14	2.11	0.00	8.66	-0.01
time (sec)	N/A	0.119	0.247	0.090	0.654	0.743	0.000	0.168	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	88	277	95	254	0	298	-1
normalized size	1	1.00	0.85	2.66	0.91	2.44	0.00	2.87	-0.01
time (sec)	N/A	0.163	0.555	0.099	0.477	0.533	0.000	0.141	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	132	910	382	312	660	372	332
normalized size	1	1.00	0.81	5.62	2.36	1.93	4.07	2.30	2.05
time (sec)	N/A	0.101	0.656	0.069	0.491	0.431	4.710	0.142	1.502

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	104	523	263	209	456	243	229
normalized size	1	1.00	0.78	3.90	1.96	1.56	3.40	1.81	1.71
time (sec)	N/A	0.073	0.432	0.059	0.477	0.498	2.578	0.125	1.178

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	75	262	165	123	264	136	127
normalized size	1	1.00	0.79	2.76	1.74	1.29	2.78	1.43	1.34
time (sec)	N/A	0.053	0.302	0.052	0.378	0.719	1.222	0.122	0.983

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	103	88	66	126	63	58
normalized size	1	1.00	0.93	1.87	1.60	1.20	2.29	1.15	1.05
time (sec)	N/A	0.025	0.161	0.053	0.438	0.545	0.526	0.140	0.099

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	97	72	104	0	68	-1
normalized size	1	1.00	0.82	1.24	0.92	1.33	0.00	0.87	-0.01
time (sec)	N/A	0.154	0.114	0.254	0.532	0.468	0.000	0.120	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	75	152	88	164	0	574	-1
normalized size	1	1.00	0.93	1.88	1.09	2.02	0.00	7.09	-0.01
time (sec)	N/A	0.146	0.413	0.238	0.381	0.447	0.000	0.188	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	102	299	99	278	0	330	-1
normalized size	1	1.00	0.91	2.67	0.88	2.48	0.00	2.95	-0.01
time (sec)	N/A	0.187	0.877	0.249	0.601	0.540	0.000	0.129	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	121	555	110	409	0	537	-1
normalized size	1	1.00	0.75	3.43	0.68	2.52	0.00	3.31	-0.01
time (sec)	N/A	0.181	0.856	0.250	0.472	0.838	0.000	0.132	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	385	1139	644	528	772	654	532
normalized size	1	1.00	1.71	5.06	2.86	2.35	3.43	2.91	2.36
time (sec)	N/A	0.282	0.965	0.173	0.401	0.516	7.947	0.134	1.350

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	122	634	439	343	495	414	364
normalized size	1	1.00	0.70	3.62	2.51	1.96	2.83	2.37	2.08
time (sec)	N/A	0.176	0.962	0.157	0.467	0.568	4.217	0.128	1.140

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	93	302	272	199	284	230	183
normalized size	1	1.00	0.76	2.46	2.21	1.62	2.31	1.87	1.49
time (sec)	N/A	0.103	0.555	0.157	0.415	0.536	2.248	0.156	1.218

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	52	109	143	95	126	98	77
normalized size	1	1.00	0.69	1.45	1.91	1.27	1.68	1.31	1.03
time (sec)	N/A	0.044	0.241	0.154	0.384	0.471	0.939	0.120	0.217

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	102	166	117	186	0	112	-1
normalized size	1	1.00	0.84	1.37	0.97	1.54	0.00	0.93	-0.01
time (sec)	N/A	0.240	0.232	0.300	0.411	0.797	0.000	0.135	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	196	271	145	305	0	1075	-1
normalized size	1	1.00	1.35	1.87	1.00	2.10	0.00	7.41	-0.01
time (sec)	N/A	0.237	0.533	0.303	0.461	0.530	0.000	0.211	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	218	562	145	527	0	602	-1
normalized size	1	1.00	1.18	3.05	0.79	2.86	0.00	3.27	-0.01
time (sec)	N/A	0.340	0.886	0.323	0.450	0.553	0.000	0.134	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	100	400	176	195	253	150	129
normalized size	1	1.00	0.58	2.33	1.02	1.13	1.47	0.87	0.75
time (sec)	N/A	0.146	0.415	0.170	0.363	0.562	5.700	0.120	0.368

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	90	237	132	147	209	118	94
normalized size	1	1.00	0.67	1.77	0.99	1.10	1.56	0.88	0.70
time (sec)	N/A	0.107	0.163	0.164	0.380	0.787	3.272	0.121	0.249

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	53	112	96	114	138	86	68
normalized size	1	1.00	0.66	1.40	1.20	1.42	1.72	1.08	0.85
time (sec)	N/A	0.044	0.182	0.157	0.389	1.188	1.834	0.123	0.146

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	343	0	0	493	0	0	-1
normalized size	1	1.00	1.92	0.00	0.00	2.75	0.00	0.00	-0.01
time (sec)	N/A	0.123	2.650	0.461	0.000	0.653	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	199	0	0	303	0	0	-1
normalized size	1	1.00	1.67	0.00	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.081	1.518	0.399	0.000	0.976	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	127	147	0	157	0	0	-1
normalized size	1	1.00	2.08	2.41	0.00	2.57	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.147	0.020	0.000	0.670	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.021	3.711	0.158	0.000	0.592	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.020	7.282	0.161	0.000	0.680	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	145	298	238	1332	0	0	-1
normalized size	1	1.00	1.41	2.89	2.31	12.93	0.00	0.00	-0.01
time (sec)	N/A	0.206	2.074	0.291	0.856	0.756	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	277	159	0	715	0	0	-1
normalized size	1	1.00	3.79	2.18	0.00	9.79	0.00	0.00	-0.01
time (sec)	N/A	0.133	6.303	0.261	0.000	0.614	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	51	57	72	161	0	78	50
normalized size	1	1.00	1.76	1.97	2.48	5.55	0.00	2.69	1.72
time (sec)	N/A	0.029	0.092	0.184	0.585	0.764	0.000	0.141	0.089

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.037	17.783	0.196	0.000	0.692	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.035	17.960	0.192	0.000	2.443	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	455	0	0	4735	0	0	-1
normalized size	1	1.00	1.54	0.00	0.00	16.00	0.00	0.00	-0.00
time (sec)	N/A	0.217	29.522	0.675	0.000	1.216	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	270	0	0	2613	0	0	-1
normalized size	1	1.00	1.54	0.00	0.00	14.93	0.00	0.00	-0.01
time (sec)	N/A	0.133	5.519	0.571	0.000	1.267	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	178	216	0	1243	0	0	-1
normalized size	1	1.00	1.75	2.12	0.00	12.19	0.00	0.00	-0.01
time (sec)	N/A	0.063	3.017	0.279	0.000	1.205	0.000	0.000	0.000



Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.037	180.008	0.523	0.000	0.858	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.034	180.017	0.676	0.000	2.020	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	107	0	308	523	0	232	-1
normalized size	1	1.00	0.63	0.00	1.80	3.06	0.00	1.36	-0.01
time (sec)	N/A	0.330	0.051	0.148	0.452	0.910	0.000	0.220	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	107	0	268	387	0	202	-1
normalized size	1	1.00	0.73	0.00	1.84	2.65	0.00	1.38	-0.01
time (sec)	N/A	0.243	0.103	0.155	0.356	2.983	0.000	0.181	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	105	0	230	302	0	169	-1
normalized size	1	1.00	0.85	0.00	1.87	2.46	0.00	1.37	-0.01
time (sec)	N/A	0.178	0.091	0.147	0.338	1.019	0.000	0.162	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	105	0	180	123	0	89	-1
normalized size	1	1.00	1.01	0.00	1.73	1.18	0.00	0.86	-0.01
time (sec)	N/A	0.130	0.044	0.146	0.329	0.558	0.000	0.131	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	0	104	338	0	0	-1
normalized size	1	1.00	0.99	0.00	0.87	2.84	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.365	0.139	0.329	1.047	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	150	0	115	534	0	0	-1
normalized size	1	1.00	1.01	0.00	0.77	3.58	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.730	0.139	0.540	0.908	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	191	0	115	853	0	0	-1
normalized size	1	1.00	1.10	0.00	0.66	4.90	0.00	0.00	-0.01
time (sec)	N/A	0.312	0.416	0.146	0.409	0.511	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	189	0	281	1001	0	0	-1
normalized size	1	1.00	0.79	0.00	1.18	4.19	0.00	0.00	-0.00
time (sec)	N/A	0.400	1.201	0.298	0.463	0.465	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	163	0	239	755	0	0	-1
normalized size	1	1.00	0.77	0.00	1.13	3.58	0.00	0.00	-0.00
time (sec)	N/A	0.303	0.602	0.292	0.587	0.491	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	129	0	189	590	0	0	-1
normalized size	1	1.00	0.78	0.00	1.14	3.55	0.00	0.00	-0.01
time (sec)	N/A	0.269	0.448	0.280	0.416	0.538	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	141	0	107	155	0	115	-1
normalized size	1	1.00	1.02	0.00	0.78	1.12	0.00	0.83	-0.01
time (sec)	N/A	0.214	0.127	0.278	0.496	0.511	0.000	0.172	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	570	0	116	569	0	0	-1
normalized size	1	1.00	4.01	0.00	0.82	4.01	0.00	0.00	-0.01
time (sec)	N/A	0.230	2.973	0.273	1.285	0.538	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	156	0	118	861	0	0	-1
normalized size	1	1.00	0.90	0.00	0.68	4.95	0.00	0.00	-0.01
time (sec)	N/A	0.309	1.383	0.286	0.405	0.551	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	825	0	116	1350	0	0	-1
normalized size	1	1.00	3.75	0.00	0.53	6.14	0.00	0.00	-0.00
time (sec)	N/A	0.311	3.171	0.291	0.446	0.633	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	222	0	116	1825	0	0	-1
normalized size	1	1.00	0.88	0.00	0.46	7.27	0.00	0.00	-0.00
time (sec)	N/A	0.379	0.854	0.281	1.037	0.715	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	243	0	513	2092	0	0	-1
normalized size	1	1.00	0.64	0.00	1.35	5.49	0.00	0.00	-0.00
time (sec)	N/A	0.906	4.000	0.506	1.540	0.645	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	243	0	429	1545	0	0	-1
normalized size	1	1.00	0.75	0.00	1.32	4.74	0.00	0.00	-0.00
time (sec)	N/A	0.712	2.005	0.497	0.439	0.672	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	210	0	334	1217	0	0	-1
normalized size	1	1.00	0.76	0.00	1.21	4.43	0.00	0.00	-0.00
time (sec)	N/A	0.484	0.295	0.491	0.437	0.508	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	192	0	177	253	0	0	-1
normalized size	1	1.00	0.84	0.00	0.78	1.11	0.00	0.00	-0.00
time (sec)	N/A	0.374	0.204	0.471	0.437	0.556	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	717	0	196	1344	0	0	-1
normalized size	1	1.00	2.91	0.00	0.80	5.46	0.00	0.00	-0.00
time (sec)	N/A	0.421	2.886	0.469	0.474	0.549	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	253	0	194	2058	0	0	-1
normalized size	1	1.00	0.91	0.00	0.70	7.43	0.00	0.00	-0.00
time (sec)	N/A	0.605	2.909	0.473	0.470	0.713	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	3211	0	196	3280	0	0	-1
normalized size	1	1.00	9.70	0.00	0.59	9.91	0.00	0.00	-0.00
time (sec)	N/A	0.680	6.347	0.458	0.732	0.625	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	51	133	174	191	131	145	-1
normalized size	1	1.00	0.46	1.20	1.57	1.72	1.18	1.31	-0.01
time (sec)	N/A	0.155	0.014	0.087	0.324	0.453	18.030	0.150	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	48	121	148	138	100	108	-1
normalized size	1	1.00	0.52	1.32	1.61	1.50	1.09	1.17	-0.01
time (sec)	N/A	0.106	0.013	0.082	0.553	0.628	1.404	0.150	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	48	72	117	59	66	60	-1
normalized size	1	1.00	0.62	0.94	1.52	0.77	0.86	0.78	-0.01
time (sec)	N/A	0.077	0.008	0.088	0.500	0.458	0.962	0.119	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	67	115	76	136	99	0	-1
normalized size	1	1.00	0.76	1.31	0.86	1.55	1.12	0.00	-0.01
time (sec)	N/A	0.113	0.037	0.083	1.001	0.480	2.788	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	78	126	58	179	124	0	-1
normalized size	1	1.00	0.68	1.11	0.51	1.57	1.09	0.00	-0.01
time (sec)	N/A	0.148	0.099	0.085	3.610	0.540	19.881	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.028	11.384	0.163	0.000	0.451	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.030	9.888	0.163	0.000	0.459	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.085	4.091	180.000	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F(-2)	F	F(-2)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	46	0	0	0	0	0	39
normalized size	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	1.95
time (sec)	N/A	0.048	0.388	180.000	0.000	0.000	0.000	0.000	0.968

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	16	0	0	109	0	0	42
normalized size	1	1.00	0.67	0.00	0.00	4.54	0.00	0.00	1.75
time (sec)	N/A	0.050	0.079	0.245	0.000	0.526	0.000	0.000	0.936

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	64	0	0	0	0	0	110
normalized size	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	2.34
time (sec)	N/A	0.066	0.638	180.000	0.000	0.000	0.000	0.000	1.082

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	76	0	0	0	0	0	-1
normalized size	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.092	1.027	180.000	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.046	3.191	0.141	0.000	0.525	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	205	0	161	340	0	0	-1
normalized size	1	1.00	0.86	0.00	0.68	1.43	0.00	0.00	-0.00
time (sec)	N/A	0.284	0.196	0.316	0.564	0.555	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	132	0	102	241	0	0	-1
normalized size	1	1.00	0.92	0.00	0.71	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.216	0.217	1.173	0.548	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	102	0	79	168	0	0	-1
normalized size	1	1.00	0.93	0.00	0.72	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.058	0.147	2.574	0.455	0.000	0.000	0.000



Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.020	5.931	0.140	0.000	0.627	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.036	3.490	0.143	0.000	0.500	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	55	86	0	0	-1
normalized size	1	1.00	0.92	1.24	0.93	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.074	0.039	0.102	1.378	0.456	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	55	86	0	0	-1
normalized size	1	1.00	0.92	1.24	0.93	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.021	0.129	1.226	0.449	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	55	86	0	0	-1
normalized size	1	1.00	0.92	1.24	0.93	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.071	0.033	0.120	0.745	0.438	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	55	78	0	0	-1
normalized size	1	1.00	0.92	1.24	0.93	1.32	0.00	0.00	-0.02
time (sec)	N/A	0.069	0.018	0.107	1.214	0.459	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	67	43	78	0	0	-1
normalized size	1	1.00	1.00	1.37	0.88	1.59	0.00	0.00	-0.02
time (sec)	N/A	0.069	0.021	0.119	0.768	0.439	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	52	67	55	86	0	0	-1
normalized size	1	1.00	0.95	1.22	1.00	1.56	0.00	0.00	-0.02
time (sec)	N/A	0.069	0.020	0.129	1.194	0.506	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	71	55	86	0	0	-1
normalized size	1	1.00	0.93	1.20	0.93	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.070	0.022	0.097	2.311	0.423	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	71	136	0	0	-1
normalized size	1	1.00	0.92	0.00	0.83	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.111	0.191	1.341	0.505	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	78	0	71	136	0	0	-1
normalized size	1	1.00	0.92	0.00	0.84	1.60	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.101	0.195	1.413	0.515	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	71	136	0	0	-1
normalized size	1	1.00	0.92	0.00	0.83	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.110	0.230	2.771	0.542	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	71	122	0	0	-1
normalized size	1	1.00	0.89	0.00	0.84	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.090	0.206	0.903	0.489	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	55	117	0	0	-1
normalized size	1	1.00	0.89	0.00	0.76	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.060	0.251	0.644	0.479	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	73	0	0	136	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	1.64	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.100	0.202	0.000	0.457	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	136	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.114	0.204	0.000	0.444	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.091	0.097	0.230	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.088	0.151	0.216	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.104	0.109	0.229	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.169	0.110	0.223	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	122	482	237	168	264	260	187
normalized size	1	1.00	1.37	5.42	2.66	1.89	2.97	2.92	2.10
time (sec)	N/A	0.132	0.564	0.064	2.312	0.645	1.380	0.125	1.019

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	80	240	141	102	151	148	112
normalized size	1	1.00	1.19	3.58	2.10	1.52	2.25	2.21	1.67
time (sec)	N/A	0.091	0.331	0.069	1.048	0.615	0.636	0.116	0.135

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	91	66	51	68	66	53
normalized size	1	1.00	1.16	2.02	1.47	1.13	1.51	1.47	1.18
time (sec)	N/A	0.045	0.221	0.069	0.441	0.520	0.277	0.116	0.079

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	54	94	70	111	0	69	-1
normalized size	1	1.00	0.84	1.47	1.09	1.73	0.00	1.08	-0.02
time (sec)	N/A	0.138	0.111	0.139	0.446	0.442	0.000	0.137	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	68	149	87	162	0	683	-1
normalized size	1	1.00	0.78	1.71	1.00	1.86	0.00	7.85	-0.01
time (sec)	N/A	0.172	0.317	0.141	0.420	0.589	0.000	0.162	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	90	296	98	274	0	328	-1
normalized size	1	1.00	0.73	2.41	0.80	2.23	0.00	2.67	-0.01
time (sec)	N/A	0.216	0.458	0.147	0.572	0.562	0.000	0.123	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	217	1071	527	395	779	581	452
normalized size	1	1.00	0.92	4.52	2.22	1.67	3.29	2.45	1.91
time (sec)	N/A	0.265	1.493	0.077	0.388	0.533	4.075	0.131	2.252

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	192	541	327	227	456	333	257
normalized size	1	1.00	1.14	3.22	1.95	1.35	2.71	1.98	1.53
time (sec)	N/A	0.188	0.536	0.079	0.354	0.511	1.704	0.129	1.288

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	81	211	167	113	219	155	123
normalized size	1	1.00	0.69	1.79	1.42	0.96	1.86	1.31	1.04
time (sec)	N/A	0.099	0.542	0.074	0.488	0.608	0.669	0.135	0.135

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	113	191	149	228	0	139	-1
normalized size	1	1.00	0.78	1.32	1.03	1.57	0.00	0.96	-0.01
time (sec)	N/A	0.342	0.210	0.367	0.525	0.432	0.000	0.150	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	207	308	182	359	0	1226	-1
normalized size	1	1.00	1.32	1.96	1.16	2.29	0.00	7.81	-0.01
time (sec)	N/A	0.333	0.743	0.375	0.581	0.568	0.000	0.244	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	353	618	202	596	0	706	-1
normalized size	1	1.00	1.71	2.99	0.98	2.88	0.00	3.41	-0.00
time (sec)	N/A	0.505	1.217	0.391	0.517	0.500	0.000	0.138	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	154	325	228	438	0	0	-1
normalized size	1	1.00	1.32	2.78	1.95	3.74	0.00	0.00	-0.01
time (sec)	N/A	0.273	2.299	0.244	0.695	0.642	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	472	174	0	243	0	0	-1
normalized size	1	1.00	5.36	1.98	0.00	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.201	6.379	0.213	0.000	0.406	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	70	63	71	92	76	71	53
normalized size	1	1.00	1.43	1.29	1.45	1.88	1.55	1.45	1.08
time (sec)	N/A	0.068	0.275	0.129	0.367	0.541	0.725	0.123	0.903

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.060	9.246	0.201	0.000	0.540	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	9.608	0.221	0.000	1.191	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	462	600	610	1863	0	0	-1
normalized size	1	1.00	1.81	2.35	2.39	7.31	0.00	0.00	-0.00
time (sec)	N/A	0.364	3.486	0.295	0.617	0.583	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	637	313	0	963	0	0	-1
normalized size	1	1.00	3.18	1.56	0.00	4.82	0.00	0.00	-0.00
time (sec)	N/A	0.252	6.478	0.236	0.000	0.690	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	114	108	239	385	156	207	138
normalized size	1	1.00	0.93	0.88	1.94	3.13	1.27	1.68	1.12
time (sec)	N/A	0.096	0.457	0.205	0.366	0.532	1.263	0.126	0.893



Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	30.764	0.655	0.000	0.506	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	32.277	0.808	0.000	1.130	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	53	108	120	0	0	147	117
normalized size	1	1.00	0.48	0.98	1.09	0.00	0.00	1.34	1.06
time (sec)	N/A	0.148	0.227	0.139	0.450	0.000	0.000	0.125	0.207

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	44	86	90	0	0	107	95
normalized size	1	1.00	0.50	0.98	1.02	0.00	0.00	1.22	1.08
time (sec)	N/A	0.113	0.159	0.094	0.452	0.000	0.000	0.141	0.934

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	34	64	60	0	0	67	56
normalized size	1	1.00	0.64	1.21	1.13	0.00	0.00	1.26	1.06
time (sec)	N/A	0.062	0.129	0.097	0.445	0.000	0.000	0.134	0.907

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	54	0	0	0	0	32	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.00	0.39	-0.01
time (sec)	N/A	0.132	0.075	0.126	0.000	0.000	0.000	0.117	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	75	0	0	0	0	68	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.62	-0.01
time (sec)	N/A	0.140	0.141	0.138	0.000	0.000	0.000	0.137	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	97	0	0	0	0	107	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.00	0.71	-0.01
time (sec)	N/A	0.170	0.213	0.099	0.000	0.000	0.000	0.131	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	33	62	88	0	0	0	63
normalized size	1	1.00	0.49	0.91	1.29	0.00	0.00	0.00	0.93
time (sec)	N/A	0.117	0.056	0.098	0.409	0.000	0.000	0.000	0.911

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	31	50	66	0	0	0	51
normalized size	1	1.00	0.58	0.94	1.25	0.00	0.00	0.00	0.96
time (sec)	N/A	0.097	0.044	0.085	0.424	0.000	0.000	0.000	0.071

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	22	38	44	0	0	0	39
normalized size	1	1.00	0.69	1.19	1.38	0.00	0.00	0.00	1.22
time (sec)	N/A	0.051	0.022	0.084	0.408	0.000	0.000	0.000	0.880

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.083	0.008	0.108	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	33	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.090	0.054	0.088	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	44	0	0	0	0	0	-1
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.073	0.092	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-2)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	70	0	180	0	0	192	-1
normalized size	1	1.00	0.38	0.00	0.97	0.00	0.00	1.04	-0.01
time (sec)	N/A	0.195	0.289	0.069	0.446	0.000	0.000	0.135	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	54	0	136	0	0	144	-1
normalized size	1	1.00	0.37	0.00	0.94	0.00	0.00	0.99	-0.01
time (sec)	N/A	0.149	0.219	0.069	0.414	0.000	0.000	0.130	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	56	0	92	0	0	96	-1
normalized size	1	1.00	0.63	0.00	1.03	0.00	0.00	1.08	-0.01
time (sec)	N/A	0.075	0.085	0.069	0.418	0.000	0.000	0.142	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	0	0	0	0	40	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.00	0.73	-0.02
time (sec)	N/A	0.128	0.021	0.072	0.000	0.000	0.000	0.149	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	53	0	0	0	0	112	-1
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.00	1.42	-0.01
time (sec)	N/A	0.131	0.086	0.068	0.000	0.000	0.000	0.151	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	69	0	0	0	0	170	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.00	1.56	-0.01
time (sec)	N/A	0.173	0.065	0.076	0.000	0.000	0.000	0.150	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	213	0	0	0	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	1.783	0.100	0.000	0.831	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	163	0	0	0	0	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.163	1.751	0.099	0.000	0.644	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	117	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.707	0.093	0.000	0.506	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.068	2.992	0.093	0.000	0.427	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.066	1.756	0.096	0.000	0.628	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	716	0	0	0	0	0	-1
normalized size	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.257	2.973	0.069	0.000	0.498	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	214	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.191	0.911	0.072	0.000	0.436	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	137	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.107	0.069	0.000	0.945	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.077	8.938	0.072	0.000	0.934	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.074	10.676	0.070	0.000	0.480	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.063	2.798	0.105	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	6.524	0.171	0.000	0.808	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	429	0	373	713	0	0	-1
normalized size	1	1.00	1.07	0.00	0.93	1.77	0.00	0.00	-0.00
time (sec)	N/A	0.553	2.420	0.287	0.477	0.708	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	302	0	209	493	0	0	-1
normalized size	1	1.00	1.15	0.00	0.79	1.87	0.00	0.00	-0.00
time (sec)	N/A	0.342	1.093	0.317	0.426	0.796	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	189	0	100	249	0	0	-1
normalized size	1	1.00	1.44	0.00	0.76	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.335	0.148	0.411	0.594	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	5.253	0.147	0.000	0.649	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	9.872	0.189	0.000	0.585	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	123	482	237	168	264	260	187
normalized size	1	1.00	1.38	5.42	2.66	1.89	2.97	2.92	2.10
time (sec)	N/A	0.132	0.511	0.066	0.350	0.579	1.409	0.119	0.988

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	83	240	141	102	151	148	110
normalized size	1	1.00	1.24	3.58	2.10	1.52	2.25	2.21	1.64
time (sec)	N/A	0.090	0.346	0.059	0.334	0.588	0.637	0.141	0.946

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	91	66	51	68	66	49
normalized size	1	1.00	1.02	2.02	1.47	1.13	1.51	1.47	1.09
time (sec)	N/A	0.046	0.115	0.054	0.352	0.772	0.286	0.118	0.084



Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	94	70	111	0	69	-1
normalized size	1	1.00	0.89	1.47	1.09	1.73	0.00	1.08	-0.02
time (sec)	N/A	0.121	0.147	0.101	0.389	0.556	0.000	0.120	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	71	149	87	162	0	683	-1
normalized size	1	1.00	0.82	1.71	1.00	1.86	0.00	7.85	-0.01
time (sec)	N/A	0.152	0.419	0.114	0.395	0.490	0.000	0.160	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	95	296	98	274	0	328	-1
normalized size	1	1.00	0.77	2.41	0.80	2.23	0.00	2.67	-0.01
time (sec)	N/A	0.197	0.635	0.119	0.414	0.543	0.000	0.127	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	232	1061	523	409	779	603	481
normalized size	1	1.00	0.93	4.24	2.09	1.64	3.12	2.41	1.92
time (sec)	N/A	0.285	1.525	0.081	0.386	1.020	3.607	0.146	2.636

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	252	535	324	240	456	349	281
normalized size	1	1.00	1.38	2.94	1.78	1.32	2.51	1.92	1.54
time (sec)	N/A	0.194	1.045	0.073	0.369	0.526	1.650	0.153	1.335

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	96	208	165	122	219	164	135
normalized size	1	1.00	0.83	1.79	1.42	1.05	1.89	1.41	1.16
time (sec)	N/A	0.101	0.893	0.071	0.366	0.564	0.660	0.145	0.149

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	133	202	148	230	0	148	-1
normalized size	1	1.00	0.85	1.29	0.95	1.47	0.00	0.95	-0.01
time (sec)	N/A	0.310	0.310	0.328	0.434	0.506	0.000	0.133	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	233	319	181	355	0	1227	-1
normalized size	1	1.00	1.27	1.74	0.99	1.94	0.00	6.70	-0.01
time (sec)	N/A	0.345	0.796	0.358	0.440	0.497	0.000	0.253	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	394	626	201	586	0	702	-1
normalized size	1	1.00	1.63	2.59	0.83	2.42	0.00	2.90	-0.00
time (sec)	N/A	0.428	1.320	0.380	0.453	0.519	0.000	0.149	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	384	0	0	1042	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	2.39	0.00	0.00	-0.00
time (sec)	N/A	0.821	1.617	0.482	0.000	0.617	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	247	0	0	736	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	2.30	0.00	0.00	-0.00
time (sec)	N/A	0.670	1.054	0.442	0.000	0.494	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	152	437	0	473	0	0	-1
normalized size	1	1.00	0.75	2.15	0.00	2.33	0.00	0.00	-0.00
time (sec)	N/A	0.376	0.964	0.216	0.000	0.494	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.061	0.973	0.171	0.000	1.436	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	1.023	0.167	0.000	0.539	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	823	823	11178	0	0	7116	0	0	-1
normalized size	1	1.00	13.58	0.00	0.00	8.65	0.00	0.00	-0.00
time (sec)	N/A	1.353	27.196	0.816	0.000	0.771	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	593	593	6018	0	0	4105	0	0	-1
normalized size	1	1.00	10.15	0.00	0.00	6.92	0.00	0.00	-0.00
time (sec)	N/A	1.025	22.803	0.700	0.000	0.545	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	509	585	0	1765	0	0	-1
normalized size	1	1.00	1.86	2.14	0.00	6.44	0.00	0.00	-0.00
time (sec)	N/A	0.457	5.038	0.297	0.000	0.861	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	51.534	0.587	0.000	0.616	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	54.958	0.785	0.000	0.779	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	4.531	0.148	0.000	0.591	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	447	0	375	813	0	0	-1
normalized size	1	1.00	0.82	0.00	0.69	1.50	0.00	0.00	-0.00
time (sec)	N/A	0.743	1.738	0.364	0.510	0.624	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	254	0	208	509	0	0	-1
normalized size	1	1.00	0.90	0.00	0.74	1.80	0.00	0.00	-0.00
time (sec)	N/A	0.363	0.805	0.281	0.420	0.802	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	119	0	100	249	0	0	-1
normalized size	1	1.00	0.91	0.00	0.76	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.184	0.154	0.411	0.558	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	1.203	0.146	0.000	0.663	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	5.575	0.175	0.000	0.474	0.000	0.000	0.000

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [144] had the largest ratio of [.6429]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	2	1.00	14	0.143
2	A	4	2	1.00	14	0.143
3	A	3	2	1.00	14	0.143
4	A	2	2	1.00	12	0.167
5	A	3	3	1.00	14	0.214
6	A	4	4	1.00	14	0.286
7	A	5	4	1.00	14	0.286
8	A	6	4	1.00	16	0.250
9	A	4	3	1.00	16	0.188
10	A	4	4	1.00	16	0.250
11	A	2	1	1.00	14	0.071
12	A	5	4	1.00	16	0.250
13	A	5	5	1.00	16	0.312
14	A	7	6	1.00	16	0.375
15	A	7	7	1.00	16	0.438

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	12	4	1.00	16	0.250
17	A	8	4	1.00	16	0.250
18	A	6	4	1.00	16	0.250
19	A	3	3	1.00	14	0.214
20	A	8	4	1.00	16	0.250
21	A	8	4	1.00	16	0.250
22	A	12	5	1.00	16	0.312
23	A	8	3	1.00	12	0.250
24	A	8	4	1.00	12	0.333
25	A	3	2	1.00	10	0.200
26	A	9	5	1.00	14	0.357
27	A	7	4	1.00	14	0.286
28	A	5	3	1.00	12	0.250
29	A	0	0	0.00	0	0.000
30	A	0	0	0.00	0	0.000
31	A	6	6	1.00	16	0.375
32	A	5	5	1.00	16	0.312
33	A	2	2	1.00	14	0.143
34	A	0	0	0.00	0	0.000
35	A	0	0	0.00	0	0.000
36	A	15	8	1.00	16	0.500
37	A	9	6	1.00	16	0.375
38	A	6	4	1.00	14	0.286
39	A	0	0	0.00	0	0.000
40	A	0	0	0.00	0	0.000
41	A	8	5	1.00	16	0.312

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	7	5	1.00	16	0.312
43	A	6	5	1.00	16	0.312
44	A	5	4	1.00	16	0.250
45	A	6	5	1.00	16	0.312
46	A	7	5	1.00	16	0.312
47	A	8	5	1.00	16	0.312
48	A	10	8	1.00	18	0.444
49	A	9	7	1.00	18	0.389
50	A	8	6	1.00	18	0.333
51	A	7	5	1.00	18	0.278
52	A	7	6	1.00	18	0.333
53	A	9	7	1.00	18	0.389
54	A	9	8	1.00	18	0.444
55	A	11	7	1.00	18	0.389
56	A	23	7	1.00	18	0.389
57	A	20	7	1.00	18	0.389
58	A	14	6	1.00	18	0.333
59	A	12	5	1.00	18	0.278
60	A	12	5	1.00	18	0.278
61	A	18	6	1.00	18	0.333
62	A	19	7	1.00	18	0.389
63	A	7	5	1.00	12	0.417
64	A	6	5	1.00	12	0.417
65	A	5	4	1.00	12	0.333
66	A	6	5	1.00	12	0.417
67	A	7	5	1.00	12	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
68	A	0	0	0.00	0	0.000
69	A	0	0	0.00	0	0.000
70	A	0	0	0.00	0	0.000
71	A	2	1	1.00	17	0.059
72	A	2	1	1.00	20	0.050
73	A	3	1	1.00	20	0.050
74	A	3	2	1.00	21	0.095
75	A	0	0	0.00	0	0.000
76	A	8	3	1.00	16	0.188
77	A	5	3	1.00	16	0.188
78	A	3	2	1.00	14	0.143
79	A	0	0	0.00	0	0.000
80	A	0	0	0.00	0	0.000
81	A	3	2	1.00	12	0.167
82	A	3	2	1.00	12	0.167
83	A	3	2	1.00	12	0.167
84	A	3	2	1.00	10	0.200
85	A	3	2	1.00	12	0.167
86	A	3	2	1.00	12	0.167
87	A	3	2	1.00	12	0.167
88	A	5	3	1.00	14	0.214
89	A	5	3	1.00	14	0.214
90	A	5	3	1.00	14	0.214
91	A	5	3	1.00	12	0.250
92	A	5	3	1.00	14	0.214
93	A	5	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	5	3	1.00	14	0.214
95	A	4	2	1.00	20	0.100
96	A	4	2	1.00	20	0.100
97	A	5	2	1.00	20	0.100
98	A	7	5	1.00	24	0.208
99	A	6	3	1.00	18	0.167
100	A	5	3	1.00	18	0.167
101	A	4	3	1.00	16	0.188
102	A	5	4	1.00	18	0.222
103	A	6	5	1.00	18	0.278
104	A	7	5	1.00	18	0.278
105	A	10	6	1.00	20	0.300
106	A	9	7	1.00	20	0.350
107	A	6	4	1.00	18	0.222
108	A	9	5	1.00	20	0.250
109	A	9	5	1.00	20	0.250
110	A	15	6	1.00	20	0.300
111	A	7	7	1.00	20	0.350
112	A	6	6	1.00	20	0.300
113	A	3	3	1.00	18	0.167
114	A	0	0	0.00	0	0.000
115	A	0	0	0.00	0	0.000
116	A	10	9	1.00	20	0.450
117	A	9	9	1.00	20	0.450
118	A	4	4	1.00	18	0.222
119	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	0	0	0.00	0	0.000
121	A	5	3	1.00	18	0.167
122	A	4	3	1.00	18	0.167
123	A	3	3	1.00	16	0.188
124	A	4	4	1.00	18	0.222
125	A	5	5	1.00	18	0.278
126	A	6	5	1.00	18	0.278
127	A	5	3	1.00	14	0.214
128	A	4	3	1.00	14	0.214
129	A	3	3	1.00	12	0.250
130	A	2	2	1.00	14	0.143
131	A	3	3	1.00	14	0.214
132	A	4	3	1.00	14	0.214
133	A	9	5	1.00	14	0.357
134	A	7	5	1.00	14	0.357
135	A	4	4	1.00	12	0.333
136	A	5	3	1.00	14	0.214
137	A	5	3	1.00	14	0.214
138	A	7	4	1.00	14	0.286
139	A	10	6	1.00	18	0.333
140	A	8	5	1.00	18	0.278
141	A	6	4	1.00	16	0.250
142	A	0	0	0.00	0	0.000
143	A	0	0	0.00	0	0.000
144	A	16	9	1.00	14	0.643
145	A	10	7	1.00	14	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
146	A	7	5	1.00	12	0.417
147	A	0	0	0.00	0	0.000
148	A	0	0	0.00	0	0.000
149	A	0	0	0.00	0	0.000
150	A	0	0	0.00	0	0.000
151	A	12	4	1.00	20	0.200
152	A	9	4	1.00	20	0.200
153	A	5	3	1.00	18	0.167
154	A	0	0	0.00	0	0.000
155	A	0	0	0.00	0	0.000
156	A	6	3	1.00	18	0.167
157	A	5	3	1.00	18	0.167
158	A	4	3	1.00	16	0.188
159	A	5	4	1.00	18	0.222
160	A	6	5	1.00	18	0.278
161	A	7	5	1.00	18	0.278
162	A	10	6	1.00	20	0.300
163	A	9	7	1.00	20	0.350
164	A	6	4	1.00	18	0.222
165	A	10	5	1.00	20	0.250
166	A	11	7	1.00	20	0.350
167	A	14	8	1.00	20	0.400
168	A	12	7	1.00	20	0.350
169	A	10	6	1.00	20	0.300
170	A	8	5	1.00	18	0.278
171	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
172	A	0	0	0.00	0	0.000
173	A	22	9	1.00	20	0.450
174	A	18	10	1.00	20	0.500
175	A	11	8	1.00	18	0.444
176	A	0	0	0.00	0	0.000
177	A	0	0	0.00	0	0.000
178	A	0	0	0.00	0	0.000
179	A	18	4	1.00	20	0.200
180	A	10	4	1.00	20	0.200
181	A	5	3	1.00	18	0.167
182	A	0	0	0.00	0	0.000
183	A	0	0	0.00	0	0.000



# Chapter 3

## Listing of integrals

### 3.1 $\int (c + dx)^4 \cosh(a + bx) dx$

Optimal. Leaf size=91

$$\frac{24d^4 \sinh(a + bx)}{b^5} - \frac{24d^3(c + dx) \cosh(a + bx)}{b^4} + \frac{12d^2(c + dx)^2 \sinh(a + bx)}{b^3} - \frac{4d(c + dx)^3 \cosh(a + bx)}{b^2} + \frac{(c + dx)^4 \sinh(a + bx)}{b}$$

[Out]  $-24*d^3*(d*x+c)*\cosh(b*x+a)/b^4-4*d*(d*x+c)^3*\cosh(b*x+a)/b^2+24*d^4*\sinh(b*x+a)/b^5+12*d^2*(d*x+c)^2*\sinh(b*x+a)/b^3+(d*x+c)^4*\sinh(b*x+a)/b$

**Rubi [A]** time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3296, 2637}

$$\frac{12d^2(c + dx)^2 \sinh(a + bx)}{b^3} - \frac{24d^3(c + dx) \cosh(a + bx)}{b^4} - \frac{4d(c + dx)^3 \cosh(a + bx)}{b^2} + \frac{24d^4 \sinh(a + bx)}{b^5} + \frac{(c + dx)^4 \sinh(a + bx)}{b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^4*\text{Cosh}[a + b*x], x]$

[Out]  $(-24*d^3*(c + d*x)*\text{Cosh}[a + b*x])/b^4 - (4*d*(c + d*x)^3*\text{Cosh}[a + b*x])/b^2 + (24*d^4*\text{Sinh}[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*\text{Sinh}[a + b*x])/b^3 + ((c + d*x)^4*\text{Sinh}[a + b*x])/b$

Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$   
 $\text{FreeQ}\{c, d\}, x]$

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned} \int (c + dx)^4 \cosh(a + bx) dx &= \frac{(c + dx)^4 \sinh(a + bx)}{b} - \frac{(4d) \int (c + dx)^3 \sinh(a + bx) dx}{b} \\ &= -\frac{4d(c + dx)^3 \cosh(a + bx)}{b^2} + \frac{(c + dx)^4 \sinh(a + bx)}{b} + \frac{(12d^2) \int (c + dx)^2 \cosh(a + bx) dx}{b^2} \\ &= -\frac{4d(c + dx)^3 \cosh(a + bx)}{b^2} + \frac{12d^2(c + dx)^2 \sinh(a + bx)}{b^3} + \frac{(c + dx)^4 \sinh(a + bx)}{b} \\ &= -\frac{24d^3(c + dx) \cosh(a + bx)}{b^4} - \frac{4d(c + dx)^3 \cosh(a + bx)}{b^2} + \frac{12d^2(c + dx)^2 \sinh(a + bx)}{b^3} \\ &= -\frac{24d^3(c + dx) \cosh(a + bx)}{b^4} - \frac{4d(c + dx)^3 \cosh(a + bx)}{b^2} + \frac{24d^4 \sinh(a + bx)}{b^5} + \frac{12d^2(c + dx)^2 \sinh(a + bx)}{b^3} \end{aligned}$$

**Mathematica** [A] time = 0.31, size = 76, normalized size = 0.84

$$\frac{\sinh(a + bx) \left( b^4(c + dx)^4 + 12b^2d^2(c + dx)^2 + 24d^4 \right) - 4bd(c + dx) \cosh(a + bx) \left( b^2(c + dx)^2 + 6d^2 \right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cosh[a + b*x], x]
```

```
[Out] (-4*b*d*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] + (24*d^4 + 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Sinh[a + b*x])/b^5
```

**fricas** [A] time = 0.45, size = 171, normalized size = 1.88

$$\frac{4 \left( b^3 d^4 x^3 + 3 b^3 c d^3 x^2 + b^3 c^3 d + 6 b c d^3 + 3 \left( b^3 c^2 d^2 + 2 b d^4 \right) x \right) \cosh(bx + a) - \left( b^4 d^4 x^4 + 4 b^4 c d^3 x^3 + b^4 c^4 + 12 b^2 d^2 (c + d x)^2 + 24 d^4 \right) \sinh(bx + a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a), x, algorithm="fricas")
```

```
[Out] -(4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d + 6*b*c*d^3 + 3*(b^3*c^2*d^2 + 2*b*d^4)*x)*cosh(b*x + a) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 + 12*b^2*d^2*(c + d*x)^2 + 24*d^4 + 6*(b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d + 6*b^2*c*d^3)*x)*sinh(b*x + a))/b^5
```



**giac [B]** time = 0.14, size = 324, normalized size = 3.56

$$\frac{(b^4 d^4 x^4 + 4 b^4 c d^3 x^3 + 6 b^4 c^2 d^2 x^2 - 4 b^3 d^4 x^3 + 4 b^4 c^3 d x - 12 b^3 c d^3 x^2 + b^4 c^4 - 12 b^3 c^2 d^2 x + 12 b^2 d^4 x^2 - 4 b^3 c^3 d)}{2 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^4\*cosh(b\*x+a),x, algorithm="giac")

[Out]  $\frac{1}{2}*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x - 12*b^3*c*d^3*x^2 + b^4*c^4 - 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^2 - 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*d^3 + 24*d^4)*e^{(b*x + a)}/b^5 - \frac{1}{2}*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4 + 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^2 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 + 24*b*d^4*x + 24*b*c*d^3 + 24*d^4)*e^{(-b*x - a)}/b^5$

**maple [B]** time = 0.05, size = 547, normalized size = 6.01

$$c^4 \sinh(bx + a) + \frac{d^4((bx+a)^4 \sinh(bx+a) - 4(bx+a)^3 \cosh(bx+a) + 12(bx+a)^2 \sinh(bx+a) - 24(bx+a) \cosh(bx+a) + 24 \sinh(bx+a))}{b^4} + \frac{d^4 a^4 \sinh(bx+a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^4\*cosh(b\*x+a),x)

[Out]  $\frac{1}{b}*(c^4*\sinh(b*x+a) + 1/b^4*d^4*((b*x+a)^4*\sinh(b*x+a) - 4*(b*x+a)^3*\cosh(b*x+a) + 12*(b*x+a)^2*\sinh(b*x+a) - 24*(b*x+a)*\cosh(b*x+a) + 24*\sinh(b*x+a)) + 1/b^4*d^4*a^4*\sinh(b*x+a) - 4/b^3*d^3*a^3*c*\sinh(b*x+a) + 6/b^2*d^2*a^2*c^2*\sinh(b*x+a) - 4/b*d*a*c^3*\sinh(b*x+a) - 4/b^4*d^4*a*((b*x+a)^3*\sinh(b*x+a) - 3*(b*x+a)^2*\cosh(b*x+a) + 6*(b*x+a)*\sinh(b*x+a) - 6*\cosh(b*x+a)) + 4/b^3*d^3*c*((b*x+a)^3*\sinh(b*x+a) - 3*(b*x+a)^2*\cosh(b*x+a) + 6*(b*x+a)*\sinh(b*x+a) - 6*\cosh(b*x+a)) + 6/b^4*d^4*a^2*((b*x+a)^2*\sinh(b*x+a) - 2*(b*x+a)*\cosh(b*x+a) + 2*\sinh(b*x+a)) + 6/b^2*d^2*c^2*((b*x+a)^2*\sinh(b*x+a) - 2*(b*x+a)*\cosh(b*x+a) + 2*\sinh(b*x+a)) - 4/b^4*d^4*a^3*((b*x+a)*\sinh(b*x+a) - \cosh(b*x+a)) + 4/b*d*c^3*((b*x+a)*\sinh(b*x+a) - \cosh(b*x+a)) - 12/b^2*d^2*a*c^2*((b*x+a)*\sinh(b*x+a) - \cosh(b*x+a)) - 12/b^3*d^3*a*c*((b*x+a)^2*\sinh(b*x+a) - 2*(b*x+a)*\cosh(b*x+a) + 2*\sinh(b*x+a)) + 12/b^3*d^3*a^2*c*((b*x+a)*\sinh(b*x+a) - \cosh(b*x+a))$

**maxima [B]** time = 0.54, size = 326, normalized size = 3.58

$$\frac{c^4 e^{(bx+a)}}{2b} + \frac{2(bxe^a - e^a)c^3 de^{(bx)}}{b^2} - \frac{c^4 e^{(-bx-a)}}{2b} - \frac{2(bx+1)c^3 de^{(-bx-a)}}{b^2} + \frac{3(b^2 x^2 e^a - 2bx e^a + 2e^a)c^2 d^2 e^{(bx)}}{b^3} - \frac{3(b^2 x^2 + 2bx + 2)e^a c^2 d^2 e^{(-bx-a)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^4\*cosh(b\*x+a),x, algorithm="maxima")

[Out]  $\frac{1}{2}c^4e^{(bx+a)}/b + 2(bxe^a - e^a)c^3de^{(bx)}/b^2 - \frac{1}{2}c^4e^{(-bx-a)}/b - 2(bx+1)c^3de^{(-bx-a)}/b^2 + 3(b^2x^2e^a - 2bxe^a + 2e^a)c^2d^2e^{(bx)}/b^3 - 3(b^2x^2 + 2bx + 2)c^2d^2e^{(-bx-a)}/b^3 + 2(b^3x^3e^a - 3b^2x^2e^a + 6bxe^a - 6e^a)c^3d^3e^{(bx)}/b^4 - 2(b^3x^3 + 3b^2x^2 + 6bx + 6)c^3d^3e^{(-bx-a)}/b^4 + \frac{1}{2}(b^4x^4e^a - 4b^3x^3e^a + 12b^2x^2e^a - 24bxe^a + 24e^a)d^4e^{(bx)}/b^5 - \frac{1}{2}(b^4x^4 + 4b^3x^3 + 12b^2x^2 + 24bx + 24)d^4e^{(-bx-a)}/b^5$

mupad [B] time = 0.19, size = 215, normalized size = 2.36

$$\frac{\sinh(a+bx) \left( b^4 c^4 + 12 b^2 c^2 d^2 + 24 d^4 \right)}{b^5} - \frac{4 \cosh(a+bx) \left( b^2 c^3 d + 6 c d^3 \right)}{b^4} - \frac{4 d^4 x^3 \cosh(a+bx)}{b^2} - \frac{12 x \cosh(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)\*(c + d\*x)^4,x)

[Out]  $(\sinh(a+bx)*(24d^4 + b^4c^4 + 12b^2c^2d^2))/b^5 - (4\cosh(a+bx)*(6cd^3 + b^2c^3d))/b^4 - (4d^4x^3\cosh(a+bx))/b^2 - (12x\cosh(a+bx)*(2d^4 + b^2c^2d^2))/b^4 + (d^4x^4\sinh(a+bx))/b + (4x\sinh(a+bx)*(6cd^3 + b^2c^3d))/b^3 + (6x^2\sinh(a+bx)*(2d^4 + b^2c^2d^2))/b^3 - (12cd^3x^2\cosh(a+bx))/b^2 + (4cd^3x^3\sinh(a+bx))/b$

sympy [A] time = 2.60, size = 311, normalized size = 3.42

$$\left\{ \frac{c^4 \sinh(a+bx)}{b} + \frac{4c^3 dx \sinh(a+bx)}{b} + \frac{6c^2 d^2 x^2 \sinh(a+bx)}{b} + \frac{4cd^3 x^3 \sinh(a+bx)}{b} + \frac{d^4 x^4 \sinh(a+bx)}{b} - \frac{4c^3 d \cosh(a+bx)}{b^2} - \frac{12c^2 d^2 x \cosh(a+bx)}{b^2} \right\} \left( c^4 x + 2c^3 dx^2 + 2c^2 d^2 x^3 + cd^3 x^4 + \frac{d^4 x^5}{5} \right) \cosh(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*4\*cosh(b\*x+a),x)

[Out] Piecewise((c\*\*4\*sinh(a + b\*x)/b + 4\*c\*\*3\*d\*x\*sinh(a + b\*x)/b + 6\*c\*\*2\*d\*\*2\*x\*\*2\*sinh(a + b\*x)/b + 4\*c\*d\*\*3\*x\*\*3\*sinh(a + b\*x)/b + d\*\*4\*x\*\*4\*sinh(a + b\*x)/b - 4\*c\*\*3\*d\*cosh(a + b\*x)/b\*\*2 - 12\*c\*\*2\*d\*\*2\*x\*cosh(a + b\*x)/b\*\*2 - 12\*c\*d\*\*3\*x\*\*2\*cosh(a + b\*x)/b\*\*2 - 4\*d\*\*4\*x\*\*3\*cosh(a + b\*x)/b\*\*2 + 12\*c\*\*2\*d\*\*2\*sinh(a + b\*x)/b\*\*3 + 24\*c\*d\*\*3\*x\*sinh(a + b\*x)/b\*\*3 + 12\*d\*\*4\*x\*\*2\*sinh(a + b\*x)/b\*\*3 - 24\*c\*d\*\*3\*cosh(a + b\*x)/b\*\*4 - 24\*d\*\*4\*x\*cosh(a + b\*x)/b\*\*4 + 24\*d\*\*4\*sinh(a + b\*x)/b\*\*5, Ne(b, 0)), ((c\*\*4\*x + 2\*c\*\*3\*d\*x\*\*2 + 2\*c\*\*2\*d\*\*2\*x\*\*3 + c\*d\*\*3\*x\*\*4 + d\*\*4\*x\*\*5/5)\*cosh(a), True))

## 3.2 $\int (c + dx)^3 \cosh(a + bx) dx$

**Optimal.** Leaf size=70

$$-\frac{6d^3 \cosh(a + bx)}{b^4} + \frac{6d^2(c + dx) \sinh(a + bx)}{b^3} - \frac{3d(c + dx)^2 \cosh(a + bx)}{b^2} + \frac{(c + dx)^3 \sinh(a + bx)}{b}$$

[Out]  $-6*d^3*\cosh(b*x+a)/b^4-3*d*(d*x+c)^2*\cosh(b*x+a)/b^2+6*d^2*(d*x+c)*\sinh(b*x+a)/b^3+(d*x+c)^3*\sinh(b*x+a)/b$

**Rubi [A]** time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3296, 2638}

$$\frac{6d^2(c + dx) \sinh(a + bx)}{b^3} - \frac{3d(c + dx)^2 \cosh(a + bx)}{b^2} - \frac{6d^3 \cosh(a + bx)}{b^4} + \frac{(c + dx)^3 \sinh(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^3\*Cosh[a + b\*x], x]

[Out]  $(-6*d^3*\cosh[a + b*x])/b^4 - (3*d*(c + d*x)^2*\cosh[a + b*x])/b^2 + (6*d^2*(c + d*x)*\sinh[a + b*x])/b^3 + ((c + d*x)^3*\sinh[a + b*x])/b$

### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] := -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \cosh(a + bx) dx &= \frac{(c + dx)^3 \sinh(a + bx)}{b} - \frac{(3d) \int (c + dx)^2 \sinh(a + bx) dx}{b} \\
&= -\frac{3d(c + dx)^2 \cosh(a + bx)}{b^2} + \frac{(c + dx)^3 \sinh(a + bx)}{b} + \frac{(6d^2) \int (c + dx) \cosh(a + bx) dx}{b^2} \\
&= -\frac{3d(c + dx)^2 \cosh(a + bx)}{b^2} + \frac{6d^2(c + dx) \sinh(a + bx)}{b^3} + \frac{(c + dx)^3 \sinh(a + bx)}{b} \\
&= -\frac{6d^3 \cosh(a + bx)}{b^4} - \frac{3d(c + dx)^2 \cosh(a + bx)}{b^2} + \frac{6d^2(c + dx) \sinh(a + bx)}{b^3} + \frac{(c + dx)^3 \sinh(a + bx)}{b}
\end{aligned}$$

**Mathematica [A]** time = 0.20, size = 61, normalized size = 0.87

$$\frac{b(c + dx) \sinh(a + bx) (b^2(c + dx)^2 + 6d^2) - 3d \cosh(a + bx) (b^2(c + dx)^2 + 2d^2)}{b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3\*Cosh[a + b\*x],x]

[Out] (-3\*d\*(2\*d^2 + b^2\*(c + d\*x)^2)\*Cosh[a + b\*x] + b\*(c + d\*x)\*(6\*d^2 + b^2\*(c + d\*x)^2)\*Sinh[a + b\*x])/b^4

**fricas [A]** time = 0.66, size = 111, normalized size = 1.59

$$\frac{3(b^2 d^3 x^2 + 2 b^2 c d^2 x + b^2 c^2 d + 2 d^3) \cosh(bx + a) - (b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + b^3 c^3 + 6 b c d^2 + 3(b^3 c^2 d + 2 b d^3)x)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*cosh(b\*x+a),x, algorithm="fricas")

[Out] -(3\*(b^2\*d^3\*x^2 + 2\*b^2\*c\*d^2\*x + b^2\*c^2\*d + 2\*d^3)\*cosh(b\*x + a) - (b^3\*d^3\*x^3 + 3\*b^3\*c\*d^2\*x^2 + b^3\*c^3 + 6\*b\*c\*d^2 + 3\*(b^3\*c^2\*d + 2\*b\*d^3)\*x)\*sinh(b\*x + a))/b^4

**giac [B]** time = 0.14, size = 204, normalized size = 2.91

$$\frac{(b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x - 3 b^2 d^3 x^2 + b^3 c^3 - 6 b^2 c d^2 x - 3 b^2 c^2 d + 6 b d^3 x + 6 b c d^2 - 6 d^3) e^{(bx+a)} - (b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x - 3 b^2 d^3 x^2 + b^3 c^3 - 6 b^2 c d^2 x - 3 b^2 c^2 d + 6 b d^3 x + 6 b c d^2 - 6 d^3)}{2 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*cosh(b\*x+a),x, algorithm="giac")

[Out]  $\frac{1}{2}(b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 d x - 3b^2 d^3 x^2 + b^3 c^3 - 6b^2 c d^2 x - 3b^2 c^2 d + 6b d^3 x + 6b c d^2 - 6d^3) e^{(bx+a)} / b^4 - \frac{1}{2}(b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 d x + 3b^2 d^3 x^2 + b^3 c^3 + 6b^2 c d^2 x + 3b^2 c^2 d + 6b d^3 x + 6b c d^2 + 6d^3) e^{(-bx-a)} / b^4$

**maple [B]** time = 0.06, size = 308, normalized size = 4.40

$$\frac{d^3((bx+a)^3 \sinh(bx+a) - 3(bx+a)^2 \cosh(bx+a) + 6(bx+a) \sinh(bx+a) - 6 \cosh(bx+a))}{b^3} - \frac{3d^3 a((bx+a)^2 \sinh(bx+a) - 2(bx+a) \cosh(bx+a) + 2 \sinh(bx+a))}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^3*cosh(b*x+a),x)`

[Out]  $\frac{1}{b} \left( \frac{1}{b^3 d^3} \left( (bx+a)^3 \sinh(bx+a) - 3(bx+a)^2 \cosh(bx+a) + 6(bx+a) \sinh(bx+a) - 6 \cosh(bx+a) \right) - \frac{3}{b^3 d^3} a \left( (bx+a)^2 \sinh(bx+a) - 2(bx+a) \cosh(bx+a) + 2 \sinh(bx+a) \right) + \frac{3}{b^2 d^2} c \left( (bx+a)^2 \sinh(bx+a) - 2(bx+a) \cosh(bx+a) + 2 \sinh(bx+a) \right) + \frac{3}{b^3 d^3} a^2 \left( (bx+a) \sinh(bx+a) - \cosh(bx+a) \right) - \frac{6}{b^2 d^2} a c \left( (bx+a) \sinh(bx+a) - \cosh(bx+a) \right) + \frac{3}{b d} c^2 \left( (bx+a) \sinh(bx+a) - \cosh(bx+a) \right) - \frac{1}{b^3 d^3} a^3 \sinh(bx+a) + \frac{3}{b^2 d^2} a^2 c \sinh(bx+a) - \frac{3}{b d} a c^2 \sinh(bx+a) + c^3 \sinh(bx+a) \right)$

**maxima [B]** time = 0.41, size = 222, normalized size = 3.17

$$\frac{c^3 e^{(bx+a)}}{2b} + \frac{3(bx e^a - e^a) c^2 d e^{(bx)}}{2b^2} - \frac{c^3 e^{(-bx-a)}}{2b} - \frac{3(bx+1) c^2 d e^{(-bx-a)}}{2b^2} + \frac{3(b^2 x^2 e^a - 2bx e^a + 2e^a) c d^2 e^{(bx)}}{2b^3} - \frac{3(b^2 x^2 + 2bx e^a - 2e^a) c d^2 e^{(bx)}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^3*cosh(b*x+a),x, algorithm="maxima")`

[Out]  $\frac{1}{2} c^3 e^{(bx+a)} / b + \frac{3}{2} (bx e^a - e^a) c^2 d e^{(bx)} / b^2 - \frac{1}{2} c^3 e^{(-bx-a)} / b - \frac{3}{2} (bx+1) c^2 d e^{(-bx-a)} / b^2 + \frac{3}{2} (b^2 x^2 e^a - 2bx e^a + 2e^a) c d^2 e^{(bx)} / b^3 - \frac{3}{2} (b^2 x^2 + 2bx e^a - 2e^a) c d^2 e^{(-bx-a)} / b^3 + \frac{1}{2} (b^3 x^3 e^a - 3b^2 x^2 e^a + 6bx e^a - 6e^a) d^3 e^{(bx)} / b^4 - \frac{1}{2} (b^3 x^3 + 3b^2 x^2 + 6bx + 6) d^3 e^{(-bx-a)} / b^4$

**mupad [B]** time = 0.94, size = 143, normalized size = 2.04

$$\frac{\sinh(a+bx) (b^2 c^3 + 6c d^2)}{b^3} - \frac{3 \cosh(a+bx) (b^2 c^2 d + 2d^3)}{b^4} - \frac{3 d^3 x^2 \cosh(a+bx)}{b^2} + \frac{d^3 x^3 \sinh(a+bx)}{b} + \frac{3 x^3 \sinh(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(a+b*x)*(c+d*x)^3,x)`

```
[Out] (sinh(a + b*x)*(6*c*d^2 + b^2*c^3))/b^3 - (3*cosh(a + b*x)*(2*d^3 + b^2*c^2
*d))/b^4 - (3*d^3*x^2*cosh(a + b*x))/b^2 + (d^3*x^3*sinh(a + b*x))/b + (3*x
*sinh(a + b*x)*(2*d^3 + b^2*c^2*d))/b^3 - (6*c*d^2*x*cosh(a + b*x))/b^2 + (
3*c*d^2*x^2*sinh(a + b*x))/b
```

**sympy** [A] time = 1.26, size = 202, normalized size = 2.89

$$\left\{ \begin{array}{l} \frac{c^3 \sinh(ax+bx)}{b} + \frac{3c^2 dx \sinh(ax+bx)}{b} + \frac{3cd^2 x^2 \sinh(ax+bx)}{b} + \frac{d^3 x^3 \sinh(ax+bx)}{b} - \frac{3c^2 d \cosh(ax+bx)}{b^2} - \frac{6cd^2 x \cosh(ax+bx)}{b^2} - \frac{3d^3 x^2 \cosh(ax+bx)}{b^2} \\ \left( c^3 x + \frac{3c^2 dx^2}{2} + cd^2 x^3 + \frac{d^3 x^4}{4} \right) \cosh(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cosh(b*x+a),x)
```

```
[Out] Piecewise((c**3*sinh(a + b*x)/b + 3*c**2*d*x*sinh(a + b*x)/b + 3*c*d**2*x**
2*sinh(a + b*x)/b + d**3*x**3*sinh(a + b*x)/b - 3*c**2*d*cosh(a + b*x)/b**2
- 6*c*d**2*x*cosh(a + b*x)/b**2 - 3*d**3*x**2*cosh(a + b*x)/b**2 + 6*c*d**
2*sinh(a + b*x)/b**3 + 6*d**3*x*sinh(a + b*x)/b**3 - 6*d**3*cosh(a + b*x)/b
**4, Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*cos
h(a), True))
```

### 3.3 $\int (c + dx)^2 \cosh(a + bx) dx$

Optimal. Leaf size=49

$$\frac{2d^2 \sinh(a + bx)}{b^3} - \frac{2d(c + dx) \cosh(a + bx)}{b^2} + \frac{(c + dx)^2 \sinh(a + bx)}{b}$$

[Out]  $-2*d*(d*x+c)*\cosh(b*x+a)/b^2+2*d^2*\sinh(b*x+a)/b^3+(d*x+c)^2*\sinh(b*x+a)/b$

**Rubi** [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3296, 2637}

$$-\frac{2d(c + dx) \cosh(a + bx)}{b^2} + \frac{2d^2 \sinh(a + bx)}{b^3} + \frac{(c + dx)^2 \sinh(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^2\*Cosh[a + b\*x], x]

[Out]  $(-2*d*(c + d*x)*\text{Cosh}[a + b*x])/b^2 + (2*d^2*\text{Sinh}[a + b*x])/b^3 + ((c + d*x)^2*\text{Sinh}[a + b*x])/b$

Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \cosh(a + bx) dx &= \frac{(c + dx)^2 \sinh(a + bx)}{b} - \frac{(2d) \int (c + dx) \sinh(a + bx) dx}{b} \\ &= -\frac{2d(c + dx) \cosh(a + bx)}{b^2} + \frac{(c + dx)^2 \sinh(a + bx)}{b} + \frac{(2d^2) \int \cosh(a + bx) dx}{b^2} \\ &= -\frac{2d(c + dx) \cosh(a + bx)}{b^2} + \frac{2d^2 \sinh(a + bx)}{b^3} + \frac{(c + dx)^2 \sinh(a + bx)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 44, normalized size = 0.90

$$\frac{\sinh(a + bx) (b^2(c + dx)^2 + 2d^2) - 2bd(c + dx) \cosh(a + bx)}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2\*Cosh[a + b\*x], x]

[Out] (-2\*b\*d\*(c + d\*x)\*Cosh[a + b\*x] + (2\*d^2 + b^2\*(c + d\*x)^2)\*Sinh[a + b\*x])/b^3

**fricas [A]** time = 0.66, size = 64, normalized size = 1.31

$$\frac{2(bd^2x + bcd) \cosh(bx + a) - (b^2d^2x^2 + 2b^2cdx + b^2c^2 + 2d^2) \sinh(bx + a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*cosh(b\*x+a), x, algorithm="fricas")

[Out] -(2\*(b\*d^2\*x + b\*c\*d)\*cosh(b\*x + a) - (b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2 + 2\*d^2)\*sinh(b\*x + a))/b^3

**giac [B]** time = 0.13, size = 112, normalized size = 2.29

$$\frac{(b^2d^2x^2 + 2b^2cdx + b^2c^2 - 2bd^2x - 2bcd + 2d^2)e^{(bx+a)}}{2b^3} - \frac{(b^2d^2x^2 + 2b^2cdx + b^2c^2 + 2bd^2x + 2bcd + 2d^2)e^{(-bx-a)}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*cosh(b\*x+a), x, algorithm="giac")

[Out] 1/2\*(b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2 - 2\*b\*d^2\*x - 2\*b\*c\*d + 2\*d^2)\*e^(b\*x + a)/b^3 - 1/2\*(b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2 + 2\*b\*d^2\*x + 2\*b\*c\*d + 2\*d^2)\*e^(-b\*x - a)/b^3

**maple [B]** time = 0.05, size = 147, normalized size = 3.00

$$\frac{\frac{d^2((bx+a)^2 \sinh(bx+a) - 2(bx+a) \cosh(bx+a) + 2 \sinh(bx+a))}{b^2} - \frac{2d^2a((bx+a) \sinh(bx+a) - \cosh(bx+a))}{b^2} + \frac{2dc((bx+a) \sinh(bx+a) - \cosh(bx+a))}{b}}{b} + \frac{d^2c}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*cosh(b\*x+a), x)



[Out]  $1/b*(1/b^2*d^2*((b*x+a)^2*\sinh(b*x+a)-2*(b*x+a)*\cosh(b*x+a)+2*\sinh(b*x+a))-2/b^2*d^2*a*((b*x+a)*\sinh(b*x+a)-\cosh(b*x+a))+2/b*d*c*((b*x+a)*\sinh(b*x+a)-\cosh(b*x+a))+1/b^2*d^2*a^2*\sinh(b*x+a)-2/b*d*a*c*\sinh(b*x+a)+c^2*\sinh(b*x+a))$

**maxima [B]** time = 0.58, size = 135, normalized size = 2.76

$$\frac{c^2 e^{(bx+a)}}{2b} + \frac{(bx e^a - e^a) c d e^{(bx)}}{b^2} - \frac{c^2 e^{(-bx-a)}}{2b} - \frac{(bx+1) c d e^{(-bx-a)}}{b^2} + \frac{(b^2 x^2 e^a - 2bx e^a + 2e^a) d^2 e^{(bx)}}{2b^3} - \frac{(b^2 x^2 + 2bx + 2) d^2}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*cosh(b\*x+a),x, algorithm="maxima")

[Out]  $1/2*c^2*e^{(b*x+a)}/b + (b*x*e^a - e^a)*c*d*e^{(b*x)}/b^2 - 1/2*c^2*e^{(-b*x-a)}/b - (b*x+1)*c*d*e^{(-b*x-a)}/b^2 + 1/2*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*d^2*e^{(b*x)}/b^3 - 1/2*(b^2*x^2 + 2*b*x + 2)*d^2*e^{(-b*x-a)}/b^3$

**mupad [B]** time = 0.90, size = 82, normalized size = 1.67

$$\frac{\sinh(a+bx)(b^2c^2+2d^2)}{b^3} + \frac{d^2x^2\sinh(a+bx)}{b} - \frac{2cd\cosh(a+bx)}{b^2} - \frac{2d^2x\cosh(a+bx)}{b^2} + \frac{2cdx\sinh(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a+b\*x)\*(c+d\*x)^2,x)

[Out]  $(\sinh(a+b*x)*(2*d^2+b^2*c^2))/b^3 + (d^2*x^2*\sinh(a+b*x))/b - (2*c*d*\cosh(a+b*x))/b^2 - (2*d^2*x*\cosh(a+b*x))/b^2 + (2*c*d*x*\sinh(a+b*x))/b$

**sympy [A]** time = 0.56, size = 112, normalized size = 2.29

$$\begin{cases} \frac{c^2 \sinh(a+bx)}{b} + \frac{2cdx \sinh(a+bx)}{b} + \frac{d^2x^2 \sinh(a+bx)}{b} - \frac{2cd \cosh(a+bx)}{b^2} - \frac{2d^2x \cosh(a+bx)}{b^2} + \frac{2d^2 \sinh(a+bx)}{b^3} & \text{for } b \neq 0 \\ \left( c^2x + cdx^2 + \frac{d^2x^3}{3} \right) \cosh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2\*cosh(b\*x+a),x)

[Out] Piecewise((c\*\*2\*sinh(a+b\*x)/b + 2\*c\*d\*x\*sinh(a+b\*x)/b + d\*\*2\*x\*\*2\*sinh(a+b\*x)/b - 2\*c\*d\*cosh(a+b\*x)/b\*\*2 - 2\*d\*\*2\*x\*cosh(a+b\*x)/b\*\*2 + 2\*d\*\*2\*sinh(a+b\*x)/b\*\*3, Ne(b, 0)), ((c\*\*2\*x + c\*d\*x\*\*2 + d\*\*2\*x\*\*3/3)\*cosh(a), True))

### 3.4 $\int (c + dx) \cosh(a + bx) dx$

Optimal. Leaf size=28

$$\frac{(c + dx) \sinh(a + bx)}{b} - \frac{d \cosh(a + bx)}{b^2}$$

[Out]  $-d*\cosh(b*x+a)/b^2+(d*x+c)*\sinh(b*x+a)/b$

**Rubi [A]** time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3296, 2638}

$$\frac{(c + dx) \sinh(a + bx)}{b} - \frac{d \cosh(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)*\text{Cosh}[a + b*x], x]$

[Out]  $-((d*\text{Cosh}[a + b*x])/b^2) + ((c + d*x)*\text{Sinh}[a + b*x])/b$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

Rubi steps

$$\begin{aligned} \int (c + dx) \cosh(a + bx) dx &= \frac{(c + dx) \sinh(a + bx)}{b} - \frac{d \int \sinh(a + bx) dx}{b} \\ &= -\frac{d \cosh(a + bx)}{b^2} + \frac{(c + dx) \sinh(a + bx)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 27, normalized size = 0.96

$$\frac{b(c + dx) \sinh(a + bx) - d \cosh(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)\*Cosh[a + b\*x], x]

[Out]  $(-(d*\cosh[a + b*x]) + b*(c + d*x)*\sinh[a + b*x])/b^2$

**fricas** [A] time = 0.63, size = 30, normalized size = 1.07

$$-\frac{d \cosh(bx + a) - (bdx + bc) \sinh(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*cosh(b\*x+a), x, algorithm="fricas")

[Out]  $-(d*\cosh(b*x + a) - (b*d*x + b*c)*\sinh(b*x + a))/b^2$

**giac** [A] time = 0.12, size = 46, normalized size = 1.64

$$\frac{(bdx + bc - d)e^{(bx+a)}}{2b^2} - \frac{(bdx + bc + d)e^{(-bx-a)}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*cosh(b\*x+a), x, algorithm="giac")

[Out]  $1/2*(b*d*x + b*c - d)*e^{(b*x + a)}/b^2 - 1/2*(b*d*x + b*c + d)*e^{(-b*x - a)}/b^2$

**maple** [A] time = 0.05, size = 53, normalized size = 1.89

$$\frac{\frac{d((bx+a) \sinh(bx+a) - \cosh(bx+a))}{b} - \frac{da \sinh(bx+a)}{b} + c \sinh(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)\*cosh(b\*x+a), x)

[Out]  $1/b*(1/b*d*((b*x+a)*\sinh(b*x+a) - \cosh(b*x+a)) - 1/b*d*a*\sinh(b*x+a) + c*\sinh(b*x+a))$

**maxima** [B] time = 0.37, size = 68, normalized size = 2.43

$$\frac{ce^{(bx+a)}}{2b} + \frac{(bx e^a - e^a)de^{(bx)}}{2b^2} - \frac{ce^{(-bx-a)}}{2b} - \frac{(bx + 1)de^{(-bx-a)}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*cosh(b\*x+a), x, algorithm="maxima")

[Out]  $\frac{1}{2}c e^{(bx+a)/b} + \frac{1}{2}(bx e^a - e^a) d e^{(bx)/b^2} - \frac{1}{2}c e^{(-bx-a)/b} - \frac{1}{2}(bx+1) d e^{(-bx-a)/b^2}$

**mupad** [B] time = 0.07, size = 35, normalized size = 1.25

$$\frac{c \sinh(a+bx) + dx \sinh(a+bx)}{b} - \frac{d \cosh(a+bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(a + b*x)*(c + d*x), x)`

[Out]  $(c \sinh(a+bx) + dx \sinh(a+bx))/b - (d \cosh(a+bx))/b^2$

**sympy** [A] time = 0.23, size = 46, normalized size = 1.64

$$\begin{cases} \frac{c \sinh(a+bx)}{b} + \frac{dx \sinh(a+bx)}{b} - \frac{d \cosh(a+bx)}{b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right) \cosh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*cosh(b*x+a), x)`

[Out] `Piecewise((c*sinh(a + b*x)/b + d*x*sinh(a + b*x)/b - d*cosh(a + b*x)/b**2, Ne(b, 0)), ((c*x + d*x**2/2)*cosh(a), True))`

$$3.5 \quad \int \frac{\cosh(a+bx)}{c+dx} dx$$

Optimal. Leaf size=51

$$\frac{\cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{d} + \frac{\sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{d}$$

[Out] Chi(b\*c/d+b\*x)\*cosh(a-b\*c/d)/d+Shi(b\*c/d+b\*x)\*sinh(a-b\*c/d)/d

**Rubi [A]** time = 0.10, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3303, 3298, 3301}

$$\frac{\cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{d} + \frac{\sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b\*x]/(c + d\*x), x]

[Out] (Cosh[a - (b\*c)/d]\*CoshIntegral[(b\*c)/d + b\*x])/d + (Sinh[a - (b\*c)/d]\*SinhIntegral[(b\*c)/d + b\*x])/d

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
  => Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
  && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
  => Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
  && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
  => Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x]
  + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
  && NeQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\cosh(a + bx)}{c + dx} dx = \cosh\left(a - \frac{bc}{d}\right) \int \frac{\cosh\left(\frac{bc}{d} + bx\right)}{c + dx} dx + \sinh\left(a - \frac{bc}{d}\right) \int \frac{\sinh\left(\frac{bc}{d} + bx\right)}{c + dx} dx$$

$$= \frac{\cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{d} + \frac{\sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{d}$$

**Mathematica [A]** time = 0.08, size = 49, normalized size = 0.96

$$\frac{\cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right) + \sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]/(c + d\*x),x]

[Out] (Cosh[a - (b\*c)/d]\*CoshIntegral[(b\*c)/d + b\*x] + Sinh[a - (b\*c)/d]\*SinhIntegral[(b\*c)/d + b\*x])/d

**fricas [A]** time = 0.48, size = 94, normalized size = 1.84

$$\frac{\left(\text{Ei}\left(\frac{bdx+bc}{d}\right) + \text{Ei}\left(-\frac{bdx+bc}{d}\right)\right) \cosh\left(-\frac{bc-ad}{d}\right) + \left(\text{Ei}\left(\frac{bdx+bc}{d}\right) - \text{Ei}\left(-\frac{bdx+bc}{d}\right)\right) \sinh\left(-\frac{bc-ad}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c),x, algorithm="fricas")

[Out] 1/2\*((Ei((b\*d\*x + b\*c)/d) + Ei(-(b\*d\*x + b\*c)/d))\*cosh(-(b\*c - a\*d)/d) + (Ei((b\*d\*x + b\*c)/d) - Ei(-(b\*d\*x + b\*c)/d))\*sinh(-(b\*c - a\*d)/d))/d

**giac [A]** time = 0.12, size = 56, normalized size = 1.10

$$\frac{\text{Ei}\left(\frac{bdx+bc}{d}\right) e^{\left(a - \frac{bc}{d}\right)} + \text{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(-a + \frac{bc}{d}\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c),x, algorithm="giac")

[Out] 1/2\*(Ei((b\*d\*x + b\*c)/d)\*e^(a - b\*c/d) + Ei(-(b\*d\*x + b\*c)/d)\*e^(-a + b\*c/d))/d

**maple** [A] time = 0.08, size = 82, normalized size = 1.61

$$-\frac{e^{-\frac{da-cb}{d}} \operatorname{Ei}\left(1, bx + a - \frac{da-cb}{d}\right)}{2d} - \frac{e^{\frac{da-cb}{d}} \operatorname{Ei}\left(1, -bx - a - \frac{-da+cb}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(b*x+a)/(d*x+c), x)`

[Out] `-1/2/d*exp(-(a*d-b*c)/d)*Ei(1, b*x+a-(a*d-b*c)/d)-1/2/d*exp((a*d-b*c)/d)*Ei(1, -b*x-a-(-a*d+b*c)/d)`

**maxima** [A] time = 0.55, size = 57, normalized size = 1.12

$$-\frac{e^{\left(-a+\frac{bc}{d}\right)} E_1\left(\frac{(dx+c)b}{d}\right)}{2d} - \frac{e^{\left(a-\frac{bc}{d}\right)} E_1\left(-\frac{(dx+c)b}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)/(d*x+c), x, algorithm="maxima")`

[Out] `-1/2*e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d - 1/2*e^(a - b*c/d)*exp_integral_e(1, -(d*x + c)*b/d)/d`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(a + b*x)/(c + d*x), x)`

[Out] `int(cosh(a + b*x)/(c + d*x), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)/(d*x+c), x)`

[Out] `Integral(cosh(a + b*x)/(c + d*x), x)`

$$3.6 \quad \int \frac{\cosh(a+bx)}{(c+dx)^2} dx$$

**Optimal.** Leaf size=71

$$\frac{b \sinh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{b \cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{d^2} - \frac{\cosh(a + bx)}{d(c + dx)}$$

[Out]  $-\cosh(b*x+a)/d/(d*x+c)+b*\cosh(a-b*c/d)*\text{Shi}(b*c/d+b*x)/d^2+b*\text{Chi}(b*c/d+b*x)*\sinh(a-b*c/d)/d^2$

**Rubi [A]** time = 0.12, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3297, 3303, 3298, 3301}

$$\frac{b \sinh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{b \cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{d^2} - \frac{\cosh(a + bx)}{d(c + dx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cosh}[a + b*x]/(c + d*x)^2, x]$

[Out]  $-(\text{Cosh}[a + b*x]/(d*(c + d*x))) + (b*\text{CoshIntegral}[(b*c)/d + b*x]*\text{Sinh}[a - (b*c)/d])/d^2 + (b*\text{Cosh}[a - (b*c)/d]*\text{SinhIntegral}[(b*c)/d + b*x])/d^2$

#### Rule 3297

$\text{Int}[\left((c_.) + (d_.)*(x_.)\right)^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[\left((c + d*x)^{(m + 1)}*\sin[e + f*x]/(d*(m + 1)), x\right] - \text{Dist}[f/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)}*\cos[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{LtQ}[m, -1]$

#### Rule 3298

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[(\text{I}*\text{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /; \text{FreeQ}\{c, d, e, f, fz\}, x\} \ \&\& \ \text{EqQ}[d*e - c*f*fz*I, 0]$

#### Rule 3301

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[\text{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /; \text{FreeQ}\{c, d, e, f, fz\}, x\} \ \&\& \ \text{EqQ}[d*(e - \text{Pi}/2) - c*f*fz*I, 0]$



Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cosh(a + bx)}{(c + dx)^2} dx &= -\frac{\cosh(a + bx)}{d(c + dx)} + \frac{b \int \frac{\sinh(a+bx)}{c+dx} dx}{d} \\ &= -\frac{\cosh(a + bx)}{d(c + dx)} + \frac{\left(b \cosh\left(a - \frac{bc}{d}\right)\right) \int \frac{\sinh\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{d} + \frac{\left(b \sinh\left(a - \frac{bc}{d}\right)\right) \int \frac{\cosh\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{d} \\ &= -\frac{\cosh(a + bx)}{d(c + dx)} + \frac{b \operatorname{Chi}\left(\frac{bc}{d} + bx\right) \sinh\left(a - \frac{bc}{d}\right)}{d^2} + \frac{b \cosh\left(a - \frac{bc}{d}\right) \operatorname{Shi}\left(\frac{bc}{d} + bx\right)}{d^2} \end{aligned}$$

**Mathematica [A]** time = 0.25, size = 65, normalized size = 0.92

$$\frac{b \sinh\left(a - \frac{bc}{d}\right) \operatorname{Chi}\left(b\left(\frac{c}{d} + x\right)\right) + b \cosh\left(a - \frac{bc}{d}\right) \operatorname{Shi}\left(b\left(\frac{c}{d} + x\right)\right) - \frac{d \cosh(a+bx)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]/(c + d\*x)^2,x]

[Out] (-((d\*Cosh[a + b\*x])/(c + d\*x)) + b\*CoshIntegral[b\*(c/d + x)]\*Sinh[a - (b\*c)/d] + b\*Cosh[a - (b\*c)/d]\*SinhIntegral[b\*(c/d + x)])/d^2

**fricas [B]** time = 0.74, size = 150, normalized size = 2.11

$$\frac{2 d \cosh (bx + a) - \left((bdx + bc) \operatorname{Ei}\left(\frac{bdx+bc}{d}\right) - (bdx + bc) \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right)\right) \cosh\left(-\frac{bc-ad}{d}\right) - \left((bdx + bc) \operatorname{Ei}\left(\frac{bdx+bc}{d}\right) + (bdx + bc) \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right)\right) \cosh\left(\frac{bc-ad}{d}\right)}{2\left(d^3 x + cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)^2,x, algorithm="fricas")

[Out] -1/2\*(2\*d\*cosh(b\*x + a) - ((b\*d\*x + b\*c)\*Ei((b\*d\*x + b\*c)/d) - (b\*d\*x + b\*c)\*Ei(-(b\*d\*x + b\*c)/d))\*cosh(-(b\*c - a\*d)/d) - ((b\*d\*x + b\*c)\*Ei((b\*d\*x + b\*c)/d) + (b\*d\*x + b\*c)\*Ei(-(b\*d\*x + b\*c)/d))\*cosh((b\*c - a\*d)/d)

$*c)/d) + (b*d*x + b*c)*\text{Ei}(-(b*d*x + b*c)/d))*\sinh(-(b*c - a*d)/d))/(d^3*x + c*d^2)$

**giac** [B] time = 0.17, size = 615, normalized size = 8.66

$$\frac{\left( (dx+c) \left( b - \frac{bc}{dx+c} + \frac{ad}{dx+c} \right) b^2 \text{Ei} \left( -\frac{(dx+c) \left( b - \frac{bc}{dx+c} + \frac{ad}{dx+c} \right) + bc - ad}{d} \right) e^{\left( \frac{bc-ad}{d} \right)} + b^3 c \text{Ei} \left( -\frac{(dx+c) \left( b - \frac{bc}{dx+c} + \frac{ad}{dx+c} \right) + bc - ad}{d} \right) e^{\left( \frac{bc-ad}{d} \right)} - a \right)}{2 \left( (dx+c) \left( b - \frac{bc}{dx+c} + \frac{ad}{dx+c} \right) d^4 + bcd^4 - ad^5 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)^2,x, algorithm="giac")

[Out]  $-1/2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*\text{Ei}(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d)} + b^3*c*\text{Ei}(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d)} - a*b^2*d*\text{Ei}(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{((b*c - a*d)/d)} + b^2*d*e^{-(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d)*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^4 + b*c*d^4 - a*d^5)*b) + 1/2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*\text{Ei}(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{-((b*c - a*d)/d)} + b^3*c*\text{Ei}(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{-((b*c - a*d)/d)} - a*b^2*d*\text{Ei}(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{-((b*c - a*d)/d)} - b^2*d*e^{((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d)*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^4 + b*c*d^4 - a*d^5)*b)}$

**maple** [A] time = 0.09, size = 133, normalized size = 1.87

$$-\frac{b e^{-bx-a}}{2d(bdx+cb)} + \frac{b e^{-\frac{da-cb}{d}} \text{Ei}\left(1, bx+a-\frac{da-cb}{d}\right)}{2d^2} - \frac{b e^{bx+a}}{2d^2\left(\frac{bc}{d}+bx\right)} - \frac{b e^{\frac{da-cb}{d}} \text{Ei}\left(1, -bx-a-\frac{-da+cb}{d}\right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)/(d\*x+c)^2,x)

[Out]  $-1/2*b*\exp(-b*x-a)/d/(b*d*x+b*c)+1/2*b/d^2*\exp(-(a*d-b*c)/d)*\text{Ei}(1,b*x+a-(a*d-b*c)/d)-1/2*b/d^2*\exp(b*x+a)/(b*c/d+b*x)-1/2*b/d^2*\exp((a*d-b*c)/d)*\text{Ei}(1,-b*x-a-(-a*d+b*c)/d)$

**maxima** [A] time = 0.65, size = 81, normalized size = 1.14

$$b \left( \frac{e^{\left(-a + \frac{bc}{d}\right)} E_1\left(\frac{(dx+c)b}{d}\right)}{d} - \frac{e^{\left(a - \frac{bc}{d}\right)} E_1\left(-\frac{(dx+c)b}{d}\right)}{d} \right) - \frac{\cosh(bx + a)}{(dx + c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)^2,x, algorithm="maxima")

[Out] 1/2\*b\*(e^(-a + b\*c/d)\*exp\_integral\_e(1, (d\*x + c)\*b/d)/d - e^(a - b\*c/d)\*exp\_integral\_e(1, -(d\*x + c)\*b/d)/d) - cosh(b\*x + a)/((d\*x + c)\*d)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)/(c + d\*x)^2,x)

[Out] int(cosh(a + b\*x)/(c + d\*x)^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)\*\*2,x)

[Out] Timed out

### 3.7 $\int \frac{\cosh(a+bx)}{(c+dx)^3} dx$

**Optimal.** Leaf size=104

$$\frac{b^2 \cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{2d^3} + \frac{b^2 \sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{2d^3} - \frac{b \sinh(a + bx)}{2d^2(c + dx)} - \frac{\cosh(a + bx)}{2d(c + dx)^2}$$

[Out]  $1/2*b^2*Chi(b*c/d+b*x)*cosh(a-b*c/d)/d^3-1/2*cosh(b*x+a)/d/(d*x+c)^2+1/2*b^2*Shi(b*c/d+b*x)*sinh(a-b*c/d)/d^3-1/2*b*sinh(b*x+a)/d^2/(d*x+c)$

**Rubi [A]** time = 0.16, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3297, 3303, 3298, 3301}

$$\frac{b^2 \cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{2d^3} + \frac{b^2 \sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{2d^3} - \frac{b \sinh(a + bx)}{2d^2(c + dx)} - \frac{\cosh(a + bx)}{2d(c + dx)^2}$$

Antiderivative was successfully verified.

[In] `Int[Cosh[a + b*x]/(c + d*x)^3, x]`

[Out]  $-\text{Cosh}[a + b*x]/(2*d*(c + d*x)^2) + (b^2*\text{Cosh}[a - (b*c)/d]*\text{CoshIntegral}[(b*c)/d + b*x])/(2*d^3) - (b*\text{Sinh}[a + b*x])/(2*d^2*(c + d*x)) + (b^2*\text{Sinh}[a - (b*c)/d]*\text{SinhIntegral}[(b*c)/d + b*x])/(2*d^3)$

#### Rule 3297

`Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]`

#### Rule 3298

`Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]`

#### Rule 3301

`Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]`

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh(a+bx)}{(c+dx)^3} dx &= -\frac{\cosh(a+bx)}{2d(c+dx)^2} + \frac{b \int \frac{\sinh(a+bx)}{(c+dx)^2} dx}{2d} \\ &= -\frac{\cosh(a+bx)}{2d(c+dx)^2} - \frac{b \sinh(a+bx)}{2d^2(c+dx)} + \frac{b^2 \int \frac{\cosh(a+bx)}{c+dx} dx}{2d^2} \\ &= -\frac{\cosh(a+bx)}{2d(c+dx)^2} - \frac{b \sinh(a+bx)}{2d^2(c+dx)} + \frac{\left(b^2 \cosh\left(a - \frac{bc}{d}\right)\right) \int \frac{\cosh\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{2d^2} + \frac{\left(b^2 \sinh\left(a - \frac{bc}{d}\right)\right) \text{Shi}\left(b\left(\frac{c}{d} + x\right)\right)}{2d^2} \\ &= -\frac{\cosh(a+bx)}{2d(c+dx)^2} + \frac{b^2 \cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{2d^3} - \frac{b \sinh(a+bx)}{2d^2(c+dx)} + \frac{b^2 \sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(b\left(\frac{c}{d} + x\right)\right)}{2d^3} \end{aligned}$$

**Mathematica [A]** time = 0.56, size = 88, normalized size = 0.85

$$\frac{b^2 \cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(b\left(\frac{c}{d} + x\right)\right) + b^2 \sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(b\left(\frac{c}{d} + x\right)\right) - \frac{d(b(c+dx) \sinh(a+bx) + d \cosh(a+bx))}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]/(c + d\*x)^3, x]

[Out] (b^2\*Cosh[a - (b\*c)/d]\*CoshIntegral[b\*(c/d + x)] - (d\*(d\*Cosh[a + b\*x] + b\*(c + d\*x)\*Sinh[a + b\*x]))/(c + d\*x)^2 + b^2\*Sinh[a - (b\*c)/d]\*SinhIntegral[b\*(c/d + x)]/(2\*d^3)

**fricas [B]** time = 0.53, size = 254, normalized size = 2.44

$$\frac{2d^2 \cosh(bx + a) - \left(\left(b^2 d^2 x^2 + 2b^2 cdx + b^2 c^2\right) \text{Ei}\left(\frac{bdx+bc}{d}\right) + \left(b^2 d^2 x^2 + 2b^2 cdx + b^2 c^2\right) \text{Ei}\left(-\frac{bdx+bc}{d}\right)\right) \cosh\left(-\frac{bx+a}{d}\right)}{4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)^3,x, algorithm="fricas")

[Out]  $-1/4*(2*d^2*cosh(b*x + a) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) + 2*(b*d^2*x + b*c*d)*sinh(b*x + a) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)$

**giac [B]** time = 0.14, size = 298, normalized size = 2.87

$$\frac{b^2 d^2 x^2 \operatorname{Ei}\left(\frac{bdx+bc}{d}\right) e^{\left(a-\frac{bc}{d}\right)} + b^2 d^2 x^2 \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(-a+\frac{bc}{d}\right)} + 2 b^2 c d x \operatorname{Ei}\left(\frac{bdx+bc}{d}\right) e^{\left(a-\frac{bc}{d}\right)} + 2 b^2 c d x \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(-a+\frac{bc}{d}\right)}}{4 \left(d^5 x^2 + 2 c d^4 x + c^2 d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)^3,x, algorithm="giac")

[Out]  $1/4*(b^2*d^2*x^2*Ei((b*d*x + b*c)/d)*e^{(a - b*c/d)} + b^2*d^2*x^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 2*b^2*c*d*x*Ei((b*d*x + b*c)/d)*e^{(a - b*c/d)} + 2*b^2*c*d*x*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + b^2*c^2*Ei((b*d*x + b*c)/d)*e^{(a - b*c/d)} + b^2*c^2*Ei(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - b*d^2*x*e^{(b*x + a)} + b*d^2*x*e^{(-b*x - a)} - b*c*d*e^{(b*x + a)} + b*c*d*e^{(-b*x - a)} - d^2*e^{(b*x + a)} - d^2*e^{(-b*x - a)})/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)$

**maple [B]** time = 0.10, size = 277, normalized size = 2.66

$$\frac{b^3 e^{-bx-a} x}{4d \left(b^2 d^2 x^2 + 2b^2 c d x + c^2 b^2\right)} + \frac{b^3 e^{-bx-a} c}{4d^2 \left(b^2 d^2 x^2 + 2b^2 c d x + c^2 b^2\right)} - \frac{b^2 e^{-bx-a}}{4d \left(b^2 d^2 x^2 + 2b^2 c d x + c^2 b^2\right)} - \frac{b^2 e^{-\frac{da-cb}{d}} \operatorname{Ei}\left(1, bx + a - \frac{da-cb}{d}\right)}{4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)/(d\*x+c)^3,x)

[Out]  $1/4*b^3*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*x+1/4*b^3*exp(-b*x-a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-1/4*b^2*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-1/4*b^2/d^3*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-1/4*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)^2-1/4*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)-1/4*b^2/d^3*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/d)$

**maxima** [A] time = 0.48, size = 95, normalized size = 0.91

$$b \frac{\left( \frac{e^{\left(-a + \frac{bc}{d}\right)} E_2\left(\frac{(dx+c)b}{d}\right) - e^{\left(a - \frac{bc}{d}\right)} E_2\left(-\frac{(dx+c)b}{d}\right)}{(dx+c)d} \right)}{4d} - \frac{\cosh(bx + a)}{2(dx + c)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)^3,x, algorithm="maxima")

[Out] 1/4\*b\*(e^(-a + b\*c/d)\*exp\_integral\_e(2, (d\*x + c)\*b/d)/((d\*x + c)\*d) - e^(a - b\*c/d)\*exp\_integral\_e(2, -(d\*x + c)\*b/d)/((d\*x + c)\*d))/d - 1/2\*cosh(b\*x + a)/((d\*x + c)^2\*d)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + bx)}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)/(c + d\*x)^3,x)

[Out] int(cosh(a + b\*x)/(c + d\*x)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)\*\*3,x)

[Out] Timed out

### 3.8 $\int (c + dx)^4 \cosh^2(a + bx) dx$

**Optimal.** Leaf size=162

$$\frac{3d^4 \sinh(a + bx) \cosh(a + bx)}{4b^5} - \frac{3d^3(c + dx) \cosh^2(a + bx)}{2b^4} + \frac{3d^2(c + dx)^2 \sinh(a + bx) \cosh(a + bx)}{2b^3} - \frac{d(c + dx)^3 \cosh^2(a + bx)}{b^2} + \frac{3d^4 \sinh(a + bx)}{4b^5}$$

[Out]  $\frac{3}{4}d^4x/b^4 + 1/2*d*(d*x+c)^3/b^2 + 1/10*(d*x+c)^5/d - 3/2*d^3*(d*x+c)*\cosh(b*x+a)^2/b^4 - d*(d*x+c)^3*\cosh(b*x+a)^2/b^2 + 3/4*d^4*\cosh(b*x+a)*\sinh(b*x+a)/b^5 + 3/2*d^2*(d*x+c)^2*\cosh(b*x+a)*\sinh(b*x+a)/b^3 + 1/2*(d*x+c)^4*\cosh(b*x+a)*\sinh(b*x+a)/b$

**Rubi [A]** time = 0.10, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3311, 32, 2635, 8}

$$-\frac{3d^3(c + dx) \cosh^2(a + bx)}{2b^4} + \frac{3d^2(c + dx)^2 \sinh(a + bx) \cosh(a + bx)}{2b^3} - \frac{d(c + dx)^3 \cosh^2(a + bx)}{b^2} + \frac{3d^4 \sinh(a + bx)}{4b^5}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^4\*Cosh[a + b\*x]^2,x]

[Out]  $(3*d^4*x)/(4*b^4) + (d*(c + d*x)^3)/(2*b^2) + (c + d*x)^5/(10*d) - (3*d^3*(c + d*x)*Cosh[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*Cosh[a + b*x]^2)/b^2 + (3*d^4*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^5) + (3*d^2*(c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x])/(2*b^3) + ((c + d*x)^4*Cosh[a + b*x]*Sinh[a + b*x])/(2*b)$

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x])\*(b\*Sinh[c + d\*x])^(n - 1)/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sinh[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3311



```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol
1] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1)/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

### Rubi steps

$$\begin{aligned} \int (c + dx)^4 \cosh^2(a + bx) dx &= -\frac{d(c + dx)^3 \cosh^2(a + bx)}{b^2} + \frac{(c + dx)^4 \cosh(a + bx) \sinh(a + bx)}{2b} + \frac{1}{2} \int (c + dx)^3 \cosh^2(a + bx) dx \\ &= \frac{(c + dx)^5}{10d} - \frac{3d^3(c + dx) \cosh^2(a + bx)}{2b^4} - \frac{d(c + dx)^3 \cosh^2(a + bx)}{b^2} + \frac{3d^2(c + dx)^4 \cosh^2(a + bx)}{2b^4} \\ &= \frac{d(c + dx)^3}{2b^2} + \frac{(c + dx)^5}{10d} - \frac{3d^3(c + dx) \cosh^2(a + bx)}{2b^4} - \frac{d(c + dx)^3 \cosh^2(a + bx)}{b^2} \\ &= \frac{3d^4x}{4b^4} + \frac{d(c + dx)^3}{2b^2} + \frac{(c + dx)^5}{10d} - \frac{3d^3(c + dx) \cosh^2(a + bx)}{2b^4} - \frac{d(c + dx)^3 \cosh^2(a + bx)}{b^2} \end{aligned}$$

**Mathematica [A]** time = 0.66, size = 132, normalized size = 0.81

$$\frac{-20bd(c + dx) \cosh(2(a + bx)) (2b^2(c + dx)^2 + 3d^2) + 10 \sinh(2(a + bx)) (2b^4(c + dx)^4 + 6b^2d^2(c + dx)^2 + 3d^4)}{80b^5}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^4\*Cosh[a + b\*x]^2,x]

[Out] (8\*b^5\*x\*(5\*c^4 + 10\*c^3\*d\*x + 10\*c^2\*d^2\*x^2 + 5\*c\*d^3\*x^3 + d^4\*x^4) - 20\*b\*d\*(c + d\*x)\*(3\*d^2 + 2\*b^2\*(c + d\*x)^2)\*Cosh[2\*(a + b\*x)] + 10\*(3\*d^4 + 6\*b^2\*d^2\*(c + d\*x)^2 + 2\*b^4\*(c + d\*x)^4)\*Sinh[2\*(a + b\*x)])/(80\*b^5)

**fricas [B]** time = 0.43, size = 312, normalized size = 1.93

$$\frac{2b^5d^4x^5 + 10b^5cd^3x^4 + 20b^5c^2d^2x^3 + 20b^5c^3dx^2 + 10b^5c^4x - 5(2b^3d^4x^3 + 6b^3cd^3x^2 + 2b^3c^3d + 3bcd^3 + 3(2b^2d^4x^2 + 10b^2cd^3x + 5b^2c^2d^2x + 5b^2c^3d + 5b^2c^4)) \sinh(2(a + bx))}{80b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^4\*cosh(b\*x+a)^2,x, algorithm="fricas")

[Out] 1/20\*(2\*b^5\*d^4\*x^5 + 10\*b^5\*c\*d^3\*x^4 + 20\*b^5\*c^2\*d^2\*x^3 + 20\*b^5\*c^3\*d\*x^2 + 10\*b^5\*c^4\*x - 5\*(2\*b^3\*d^4\*x^3 + 6\*b^3\*c\*d^3\*x^2 + 2\*b^3\*c^3\*d + 3\*b^2\*d^4\*x^2 + 10\*b^2\*c\*d^3\*x + 5\*b^2\*c^2\*d^2\*x + 5\*b^2\*c^3\*d + 5\*b^2\*c^4)) \sinh(2(a + bx))

$$*c*d^3 + 3*(2*b^3*c^2*d^2 + b*d^4)*x)*\cosh(b*x + a)^2 + 5*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^2*d^2 + 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 + b^2*d^4)*x^2 + 4*(2*b^4*c^3*d + 3*b^2*c*d^3)*x)*\cosh(b*x + a)*\sinh(b*x + a) - 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d + 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 + b*d^4)*x)*\sinh(b*x + a)^2)/b^5$$

**giac [B]** time = 0.14, size = 372, normalized size = 2.30

$$\frac{1}{10}d^4x^5 + \frac{1}{2}cd^3x^4 + c^2d^2x^3 + c^3dx^2 + \frac{1}{2}c^4x + \frac{(2b^4d^4x^4 + 8b^4cd^3x^3 + 12b^4c^2d^2x^2 - 4b^3d^4x^3 + 8b^4c^3dx - 12b^3cd^3x^2)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^4\*cosh(b\*x+a)^2,x, algorithm="giac")

[Out] 1/10\*d^4\*x^5 + 1/2\*c\*d^3\*x^4 + c^2\*d^2\*x^3 + c^3\*d\*x^2 + 1/2\*c^4\*x + 1/16\*(2\*b^4\*d^4\*x^4 + 8\*b^4\*c\*d^3\*x^3 + 12\*b^4\*c^2\*d^2\*x^2 - 4\*b^3\*d^4\*x^3 + 8\*b^4\*c^3\*d\*x - 12\*b^3\*c\*d^3\*x^2 + 2\*b^4\*c^4 - 12\*b^3\*c^2\*d^2\*x + 6\*b^2\*d^4\*x^2 - 4\*b^3\*c^3\*d + 12\*b^2\*c\*d^3\*x + 6\*b^2\*c^2\*d^2 - 6\*b\*d^4\*x - 6\*b\*c\*d^3 + 3\*d^4)\*e^(2\*b\*x + 2\*a)/b^5 - 1/16\*(2\*b^4\*d^4\*x^4 + 8\*b^4\*c\*d^3\*x^3 + 12\*b^4\*c^2\*d^2\*x^2 + 4\*b^3\*d^4\*x^3 + 8\*b^4\*c^3\*d\*x + 12\*b^3\*c\*d^3\*x^2 + 2\*b^4\*c^4 + 12\*b^3\*c^2\*d^2\*x + 6\*b^2\*d^4\*x^2 + 4\*b^3\*c^3\*d + 12\*b^2\*c\*d^3\*x + 6\*b^2\*c^2\*d^2 + 6\*b\*d^4\*x + 6\*b\*c\*d^3 + 3\*d^4)\*e^(-2\*b\*x - 2\*a)/b^5

**maple [B]** time = 0.07, size = 910, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^4\*cosh(b\*x+a)^2,x)

[Out] 1/b\*(c^4\*(1/2\*cosh(b\*x+a)\*sinh(b\*x+a)+1/2\*b\*x+1/2\*a)-4/b^4\*d^4\*a\*(1/2\*(b\*x+a)^3\*cosh(b\*x+a)\*sinh(b\*x+a)+1/8\*(b\*x+a)^4-3/4\*(b\*x+a)^2\*cosh(b\*x+a)^2+3/4\*(b\*x+a)\*cosh(b\*x+a)\*sinh(b\*x+a)+3/8\*(b\*x+a)^2-3/8\*cosh(b\*x+a)^2)+6/b^4\*d^4\*a^2\*(1/2\*(b\*x+a)^2\*cosh(b\*x+a)\*sinh(b\*x+a)+1/6\*(b\*x+a)^3-1/2\*(b\*x+a)\*cosh(b\*x+a)^2+1/4\*cosh(b\*x+a)\*sinh(b\*x+a)+1/4\*b\*x+1/4\*a)-4/b^4\*d^4\*a^3\*(1/2\*(b\*x+a)\*cosh(b\*x+a)\*sinh(b\*x+a)+1/4\*(b\*x+a)^2-1/4\*cosh(b\*x+a)^2)+4/b^3\*d^3\*c\*(1/2\*(b\*x+a)^3\*cosh(b\*x+a)\*sinh(b\*x+a)+1/8\*(b\*x+a)^4-3/4\*(b\*x+a)^2\*cosh(b\*x+a)^2+3/4\*(b\*x+a)\*cosh(b\*x+a)\*sinh(b\*x+a)+3/8\*(b\*x+a)^2-3/8\*cosh(b\*x+a)^2)+6/b^2\*d^2\*c^2\*(1/2\*(b\*x+a)^2\*cosh(b\*x+a)\*sinh(b\*x+a)+1/6\*(b\*x+a)^3-1/2\*(b\*x+a)\*cosh(b\*x+a)^2+1/4\*cosh(b\*x+a)\*sinh(b\*x+a)+1/4\*b\*x+1/4\*a)+4/b\*d\*c^3\*(1/2\*(b\*x+a)\*cosh(b\*x+a)\*sinh(b\*x+a)+1/4\*(b\*x+a)^2-1/4\*cosh(b\*x+a)^2)-4/b^3\*d^3\*a^3\*c\*(1/2\*cosh(b\*x+a)\*sinh(b\*x+a)+1/2\*b\*x+1/2\*a)+6/b^2\*d^2\*a^2\*c^2\*(1/2\*cosh(b\*x+a)\*sinh(b\*x+a)+1/2\*b\*x+1/2\*a)-4/b\*d\*a\*c^3\*(1/2\*cosh(b\*x+a)\*sinh(b\*x+a)+1/2\*b\*x+1/2\*a)+1/b^4\*d^4\*(1/2\*(b\*x+a)^4\*cosh(b\*x+a)\*sinh(b\*x+a)+1/10\*(b\*x+

$a)^5 - (bx+a)^3 \cosh(bx+a)^2 + 3/2 (bx+a)^2 \cosh(bx+a) \sinh(bx+a) + 1/2 (bx+a)^3 - 3/2 (bx+a) \cosh(bx+a)^2 + 3/4 \cosh(bx+a) \sinh(bx+a) + 3/4 bx + 3/4 a + 1/b^4 d^4 a^4 (1/2 \cosh(bx+a) \sinh(bx+a) + 1/2 bx + 1/2 a) - 12/b^3 d^3 c a a (1/2 (bx+a)^2 \cosh(bx+a) \sinh(bx+a) + 1/6 (bx+a)^3 - 1/2 (bx+a) \cosh(bx+a)^2 + 1/4 \cosh(bx+a) \sinh(bx+a) + 1/4 bx + 1/4 a) + 12/b^3 d^3 c a^2 (1/2 (bx+a) \cosh(bx+a) \sinh(bx+a) + 1/4 (bx+a)^2 - 1/4 \cosh(bx+a)^2) - 12/b^2 d^2 c^2 a a (1/2 (bx+a) \cosh(bx+a) \sinh(bx+a) + 1/4 (bx+a)^2 - 1/4 \cosh(bx+a)^2)$

**maxima [B]** time = 0.49, size = 382, normalized size = 2.36

$$\frac{1}{4} \left( 4x^2 + \frac{(2bx e^{2a} - e^{2a}) e^{2bx}}{b^2} - \frac{(2bx + 1) e^{(-2bx - 2a)}}{b^2} \right) c^3 d + \frac{1}{8} \left( 8x^3 + \frac{3(2b^2 x^2 e^{2a} - 2bx e^{2a} + e^{2a}) e^{2bx}}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^4\*cosh(b\*x+a)^2,x, algorithm="maxima")

[Out]  $1/4*(4*x^2 + (2*b*x*e^{(2*a)} - e^{(2*a)})*e^{(2*b*x)}/b^2 - (2*b*x + 1)*e^{(-2*b*x - 2*a)}/b^2)*c^3*d + 1/8*(8*x^3 + 3*(2*b^2*x^2*e^{(2*a)} - 2*b*x*e^{(2*a)} + e^{(2*a)})*e^{(2*b*x)}/b^3 - 3*(2*b^2*x^2 + 2*b*x + 1)*e^{(-2*b*x - 2*a)}/b^3)*c^2*d^2 + 1/8*(4*x^4 + (4*b^3*x^3*e^{(2*a)} - 6*b^2*x^2*e^{(2*a)} + 6*b*x*e^{(2*a)} - 3*e^{(2*a)})*e^{(2*b*x)}/b^4 - (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^{(-2*b*x - 2*a)}/b^4)*c*d^3 + 1/80*(8*x^5 + 5*(2*b^4*x^4*e^{(2*a)} - 4*b^3*x^3*e^{(2*a)} + 6*b^2*x^2*e^{(2*a)} - 6*b*x*e^{(2*a)} + 3*e^{(2*a)})*e^{(2*b*x)}/b^5 - 5*(2*b^4*x^4 + 4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^{(-2*b*x - 2*a)}/b^5)*d^4 + 1/8*c^4*(4*x + e^{(2*b*x + 2*a)}/b - e^{(-2*b*x - 2*a)}/b)$

**mupad [B]** time = 1.50, size = 332, normalized size = 2.05

$$\frac{c^4 x}{2} + \frac{d^4 x^5}{10} + c^3 d x^2 + \frac{c d^3 x^4}{2} + \frac{c^4 \sinh(2a + 2bx)}{4b} + \frac{3d^4 \sinh(2a + 2bx)}{8b^5} + c^2 d^2 x^3 - \frac{c^3 d \cosh(2a + 2bx)}{2b^2} - \frac{3cd}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^2\*(c + d\*x)^4,x)

[Out]  $(c^4*x)/2 + (d^4*x^5)/10 + c^3*d*x^2 + (c*d^3*x^4)/2 + (c^4*\sinh(2*a + 2*b*x))/(4*b) + (3*d^4*\sinh(2*a + 2*b*x))/(8*b^5) + c^2*d^2*x^3 - (c^3*d*cosh(2*a + 2*b*x))/(2*b^2) - (3*c*d^3*cosh(2*a + 2*b*x))/(4*b^4) - (3*d^4*x*cosh(2*a + 2*b*x))/(4*b^4) + (3*c^2*d^2*\sinh(2*a + 2*b*x))/(4*b^3) - (d^4*x^3*cosh(2*a + 2*b*x))/(2*b^2) + (d^4*x^4*\sinh(2*a + 2*b*x))/(4*b) + (3*d^4*x^2*\sinh(2*a + 2*b*x))/(4*b^3) + (3*c^2*d^2*x^2*\sinh(2*a + 2*b*x))/(2*b) + (c^3*d*x*\sinh(2*a + 2*b*x))/b + (3*c*d^3*x*\sinh(2*a + 2*b*x))/(2*b^3) - (3*c^2*d^2*x*cosh(2*a + 2*b*x))/(2*b^2) - (3*c*d^3*x^2*cosh(2*a + 2*b*x))/(2*b^2) + (c*d^3*x^3*\sinh(2*a + 2*b*x))/b$

**sympy** [A] time = 4.71, size = 660, normalized size = 4.07

$$\left\{ \begin{array}{l} -\frac{c^4 x \sinh^2(a+bx)}{2} + \frac{c^4 x \cosh^2(a+bx)}{2} - c^3 dx^2 \sinh^2(a+bx) + c^3 dx^2 \cosh^2(a+bx) - c^2 d^2 x^3 \sinh^2(a+bx) + c^2 d^2 x^3 \cosh^2(a+bx) \\ \left( c^4 x + 2c^3 dx^2 + 2c^2 d^2 x^3 + cd^3 x^4 + \frac{d^4 x^5}{5} \right) \cosh^2(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*4\*cosh(b\*x+a)\*\*2,x)

[Out] Piecewise((-c\*\*4\*x\*sinh(a + b\*x)\*\*2/2 + c\*\*4\*x\*cosh(a + b\*x)\*\*2/2 - c\*\*3\*d\*x\*\*2\*sinh(a + b\*x)\*\*2 + c\*\*3\*d\*x\*\*2\*cosh(a + b\*x)\*\*2 - c\*\*2\*d\*\*2\*x\*\*3\*sinh(a + b\*x)\*\*2 + c\*\*2\*d\*\*2\*x\*\*3\*cosh(a + b\*x)\*\*2 - c\*d\*\*3\*x\*\*4\*sinh(a + b\*x)\*\*2/2 + c\*d\*\*3\*x\*\*4\*cosh(a + b\*x)\*\*2/2 - d\*\*4\*x\*\*5\*sinh(a + b\*x)\*\*2/10 + d\*\*4\*x\*\*5\*cosh(a + b\*x)\*\*2/10 + c\*\*4\*sinh(a + b\*x)\*cosh(a + b\*x)/(2\*b) + 2\*c\*\*3\*d\*x\*sinh(a + b\*x)\*cosh(a + b\*x)/b + 3\*c\*\*2\*d\*\*2\*x\*\*2\*sinh(a + b\*x)\*cosh(a + b\*x)/b + 2\*c\*d\*\*3\*x\*\*3\*sinh(a + b\*x)\*cosh(a + b\*x)/b + d\*\*4\*x\*\*4\*sinh(a + b\*x)\*cosh(a + b\*x)/(2\*b) - c\*\*3\*d\*cosh(a + b\*x)\*\*2/b\*\*2 - 3\*c\*\*2\*d\*\*2\*x\*sinh(a + b\*x)\*\*2/(2\*b\*\*2) - 3\*c\*\*2\*d\*\*2\*x\*cosh(a + b\*x)\*\*2/(2\*b\*\*2) - 3\*c\*d\*\*3\*x\*\*2\*sinh(a + b\*x)\*\*2/(2\*b\*\*2) - 3\*c\*d\*\*3\*x\*\*2\*cosh(a + b\*x)\*\*2/(2\*b\*\*2) - d\*\*4\*x\*\*3\*sinh(a + b\*x)\*\*2/(2\*b\*\*2) - d\*\*4\*x\*\*3\*cosh(a + b\*x)\*\*2/(2\*b\*\*2) + 3\*c\*\*2\*d\*\*2\*sinh(a + b\*x)\*cosh(a + b\*x)/(2\*b\*\*3) + 3\*c\*d\*\*3\*x\*sinh(a + b\*x)\*cosh(a + b\*x)/b\*\*3 + 3\*d\*\*4\*x\*\*2\*sinh(a + b\*x)\*cosh(a + b\*x)/(2\*b\*\*3) - 3\*c\*d\*\*3\*cosh(a + b\*x)\*\*2/(2\*b\*\*4) - 3\*d\*\*4\*x\*sinh(a + b\*x)\*\*2/(4\*b\*\*4) - 3\*d\*\*4\*x\*cosh(a + b\*x)\*\*2/(4\*b\*\*4) + 3\*d\*\*4\*sinh(a + b\*x)\*cosh(a + b\*x)/(4\*b\*\*5), Ne(b, 0)), ((c\*\*4\*x + 2\*c\*\*3\*d\*x\*\*2 + 2\*c\*\*2\*d\*\*2\*x\*\*3 + c\*d\*\*3\*x\*\*4 + d\*\*4\*x\*\*5/5)\*cosh(a)\*\*2, True))

### 3.9 $\int (c + dx)^3 \cosh^2(a + bx) dx$

**Optimal.** Leaf size=134

$$\frac{3d^3 \cosh^2(a + bx)}{8b^4} + \frac{3d^2(c + dx) \sinh(a + bx) \cosh(a + bx)}{4b^3} - \frac{3d(c + dx)^2 \cosh^2(a + bx)}{4b^2} + \frac{(c + dx)^3 \sinh(a + bx)}{2b}$$

[Out]  $3/4*c*d^2*x/b^2+3/8*d^3*x^2/b^2+1/8*(d*x+c)^4/d-3/8*d^3*cosh(b*x+a)^2/b^4-3/4*d*(d*x+c)^2*cosh(b*x+a)^2/b^2+3/4*d^2*(d*x+c)*cosh(b*x+a)*sinh(b*x+a)/b^3+1/2*(d*x+c)^3*cosh(b*x+a)*sinh(b*x+a)/b$

**Rubi [A]** time = 0.07, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {3311, 32, 3310}

$$\frac{3d^2(c + dx) \sinh(a + bx) \cosh(a + bx)}{4b^3} - \frac{3d(c + dx)^2 \cosh^2(a + bx)}{4b^2} - \frac{3d^3 \cosh^2(a + bx)}{8b^4} + \frac{(c + dx)^3 \sinh(a + bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^3\*Cosh[a + b\*x]^2,x]

[Out]  $(3*c*d^2*x)/(4*b^2) + (3*d^3*x^2)/(8*b^2) + (c + d*x)^4/(8*d) - (3*d^3*Cosh[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*Cosh[a + b*x]^2)/(4*b^2) + (3*d^2*(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^3*Cosh[a + b*x]*Sinh[a + b*x])/(2*b)$

#### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(d\*(b\*Sine[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)\*(b\*Sine[e + f\*x])^(n - 2), x], x] - Simp[(b\*(c + d\*x)\*Cos[e + f\*x]\*(b\*Sine[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

#### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Sine[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*Sine[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*Sine[e + f\*x])^n, x], x])

- Simp[(b\*(c + d\*x)^m\*Cos[e + f\*x]\*(b\*Sin[e + f\*x])^(n - 1))/(f\*n), x] /;  
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

### Rubi steps

$$\begin{aligned} \int (c + dx)^3 \cosh^2(a + bx) dx &= -\frac{3d(c + dx)^2 \cosh^2(a + bx)}{4b^2} + \frac{(c + dx)^3 \cosh(a + bx) \sinh(a + bx)}{2b} + \frac{1}{2} \int (c + dx) \cosh^2(a + bx) dx \\ &= \frac{(c + dx)^4}{8d} - \frac{3d^3 \cosh^2(a + bx)}{8b^4} - \frac{3d(c + dx)^2 \cosh^2(a + bx)}{4b^2} + \frac{3d^2(c + dx) \cosh(a + bx) \sinh(a + bx)}{4b^2} \\ &= \frac{3cd^2x}{4b^2} + \frac{3d^3x^2}{8b^2} + \frac{(c + dx)^4}{8d} - \frac{3d^3 \cosh^2(a + bx)}{8b^4} - \frac{3d(c + dx)^2 \cosh^2(a + bx)}{4b^2} + \frac{3d^2(c + dx) \cosh(a + bx) \sinh(a + bx)}{4b^2} \end{aligned}$$

**Mathematica** [A] time = 0.43, size = 104, normalized size = 0.78

$$\frac{2b(c + dx) \sinh(2(a + bx)) (2b^2(c + dx)^2 + 3d^2) - 3d \cosh(2(a + bx)) (2b^2(c + dx)^2 + d^2) + 2b^4x (4c^3 + 6c^2dx + 4cd^2x^2 + d^3x^3)}{16b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3\*Cosh[a + b\*x]^2,x]

[Out] (2\*b^4\*x\*(4\*c^3 + 6\*c^2\*d\*x + 4\*c\*d^2\*x^2 + d^3\*x^3) - 3\*d\*(d^2 + 2\*b^2\*(c + d\*x)^2)\*Cosh[2\*(a + b\*x)] + 2\*b\*(c + d\*x)\*(3\*d^2 + 2\*b^2\*(c + d\*x)^2)\*Sin h[2\*(a + b\*x)])/(16\*b^4)

**fricas** [A] time = 0.50, size = 209, normalized size = 1.56

$$\frac{2b^4d^3x^4 + 8b^4cd^2x^3 + 12b^4c^2dx^2 + 8b^4c^3x - 3(2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d + d^3) \cosh(bx + a)^2 + 4(2b^3d^3x^3 + 6b^3cd^2x^2 + 2b^3c^2d + d^3) \cosh(bx + a) \sinh(bx + a) - 3(2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d + d^3) \sinh(bx + a)^2}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*cosh(b\*x+a)^2,x, algorithm="fricas")

[Out] 1/16\*(2\*b^4\*d^3\*x^4 + 8\*b^4\*c\*d^2\*x^3 + 12\*b^4\*c^2\*d\*x^2 + 8\*b^4\*c^3\*x - 3\*(2\*b^2\*d^3\*x^2 + 4\*b^2\*c\*d^2\*x + 2\*b^2\*c^2\*d + d^3)\*cosh(b\*x + a)^2 + 4\*(2\*b^3\*d^3\*x^3 + 6\*b^3\*c\*d^2\*x^2 + 2\*b^3\*c^2\*d + 3\*b\*c\*d^2 + 3\*(2\*b^3\*c^2\*d + b\*d^3)\*x)\*cosh(b\*x + a)\*sinh(b\*x + a) - 3\*(2\*b^2\*d^3\*x^2 + 4\*b^2\*c\*d^2\*x + 2\*b^2\*c^2\*d + d^3)\*sinh(b\*x + a)^2)/b^4

**giac** [B] time = 0.13, size = 243, normalized size = 1.81

$$\frac{1}{8}d^3x^4 + \frac{1}{2}cd^2x^3 + \frac{3}{4}c^2dx^2 + \frac{1}{2}c^3x + \frac{(4b^3d^3x^3 + 12b^3cd^2x^2 + 12b^3c^2dx - 6b^2d^3x^2 + 4b^3c^3 - 12b^2cd^2x - 6b^2c^2d + d^3) \cosh(bx + a)^2 + 4(2b^3d^3x^3 + 6b^3cd^2x^2 + 2b^3c^2d + d^3) \cosh(bx + a) \sinh(bx + a) - 3(2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d + d^3) \sinh(bx + a)^2}{32b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*cosh(b\*x+a)^2,x, algorithm="giac")

[Out]  $\frac{1}{8}d^3x^4 + \frac{1}{2}cd^2x^3 + \frac{3}{4}c^2d^2x^2 + \frac{1}{2}c^3x + \frac{1}{32}(4b^3d^3x^3 + 12b^3cd^2x^2 + 12b^3c^2d^2x - 6b^2d^3x^2 + 4b^3c^3 - 12b^2cd^2x - 6b^2c^2d + 6b^2d^3x + 6b^2cd^2 - 3d^3)e^{(2bx+2a)}/b^4 - \frac{1}{32}(4b^3d^3x^3 + 12b^3cd^2x^2 + 12b^3c^2d^2x + 6b^2d^3x^2 + 4b^3c^3 + 12b^2cd^2x + 6b^2c^2d + 6b^2d^3x + 6b^2cd^2 + 3d^3)e^{(-2bx-2a)}/b^4$

**maple [B]** time = 0.06, size = 523, normalized size = 3.90

$$\frac{d^3 \left( \frac{(bx+a)^3 \cosh(bx+a) \sinh(bx+a)}{2} + \frac{(bx+a)^4}{8} - \frac{3(bx+a)^2 (\cosh^2(bx+a))}{4} + \frac{3(bx+a) \cosh(bx+a) \sinh(bx+a)}{4} + \frac{3(bx+a)^2}{8} - \frac{3(\cosh^2(bx+a))}{8} \right)}{b^3} - \frac{3d^3 a \left( \frac{(bx+a)^2 \cosh(bx+a) \sinh(bx+a)}{2} \right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3\*cosh(b\*x+a)^2,x)

[Out]  $\frac{1}{b} \left( \frac{1}{b^3} d^3 \left( \frac{1}{2} (bx+a)^3 \cosh(bx+a) \sinh(bx+a) + \frac{1}{8} (bx+a)^4 - \frac{3}{4} (bx+a)^2 \cosh^2(bx+a) + \frac{3}{4} (bx+a) \cosh(bx+a) \sinh(bx+a) + \frac{3}{8} (bx+a)^2 - \frac{3}{8} \cosh^2(bx+a) \right) - \frac{3}{b^3} d^3 a \left( \frac{1}{2} (bx+a)^2 \cosh(bx+a) \sinh(bx+a) + \frac{1}{6} (bx+a)^3 - \frac{1}{2} (bx+a) \cosh(bx+a) \sinh(bx+a) + \frac{1}{4} \cosh(bx+a) \sinh(bx+a) + \frac{1}{4} bx + \frac{1}{4} a \right) + \frac{3}{b^3} d^3 a^2 \left( \frac{1}{2} (bx+a) \cosh(bx+a) \sinh(bx+a) + \frac{1}{4} (bx+a)^2 - \frac{1}{4} \cosh(bx+a) \sinh(bx+a) \right) - \frac{1}{b^3} d^3 a^3 \left( \frac{1}{2} \cosh(bx+a) \sinh(bx+a) + \frac{1}{2} bx + \frac{1}{2} a \right) + \frac{3}{b^2} c d^2 \left( \frac{1}{2} (bx+a)^2 \cosh(bx+a) \sinh(bx+a) + \frac{1}{6} (bx+a)^3 - \frac{1}{2} (bx+a) \cosh(bx+a) \sinh(bx+a) + \frac{1}{4} \cosh(bx+a) \sinh(bx+a) + \frac{1}{4} bx + \frac{1}{4} a \right) - \frac{6}{b^2} c d^2 a \left( \frac{1}{2} (bx+a) \cosh(bx+a) \sinh(bx+a) + \frac{1}{4} (bx+a)^2 - \frac{1}{4} \cosh(bx+a) \sinh(bx+a) \right) + \frac{3}{b^2} c d^2 a^2 \left( \frac{1}{2} \cosh(bx+a) \sinh(bx+a) + \frac{1}{2} bx + \frac{1}{2} a \right) + \frac{3}{b} c^2 d \left( \frac{1}{2} (bx+a) \cosh(bx+a) \sinh(bx+a) + \frac{1}{4} (bx+a)^2 - \frac{1}{4} \cosh(bx+a) \sinh(bx+a) \right) - \frac{3}{b} c^2 d a \left( \frac{1}{2} \cosh(bx+a) \sinh(bx+a) + \frac{1}{2} bx + \frac{1}{2} a \right) + c^3 \left( \frac{1}{2} \cosh(bx+a) \sinh(bx+a) + \frac{1}{2} bx + \frac{1}{2} a \right) \right)$

**maxima [B]** time = 0.48, size = 263, normalized size = 1.96

$$\frac{3}{16} \left( 4x^2 + \frac{(2bx e^{(2a)} - e^{(2a)}) e^{(2bx)}}{b^2} - \frac{(2bx+1) e^{(-2bx-2a)}}{b^2} \right) c^2 d + \frac{1}{16} \left( 8x^3 + \frac{3(2b^2 x^2 e^{(2a)} - 2bx e^{(2a)} + e^{(2a)}) e^{(2bx)}}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*cosh(b\*x+a)^2,x, algorithm="maxima")

[Out]  $\frac{3}{16} (4x^2 + (2bx e^{(2a)} - e^{(2a)}) e^{(2bx)}) / b^2 - (2bx+1) e^{(-2bx-2a)} / b^2 + \frac{1}{16} (8x^3 + 3(2b^2 x^2 e^{(2a)} - 2bx e^{(2a)} + e^{(2a)}) e^{(2bx)}) / b^3$

$$e^{(2*a)}*e^{(2*b*x)}/b^3 - 3*(2*b^2*x^2 + 2*b*x + 1)*e^{(-2*b*x - 2*a)}/b^3)*c*d^2 + 1/32*(4*x^4 + (4*b^3*x^3*e^{(2*a)} - 6*b^2*x^2*e^{(2*a)} + 6*b*x*e^{(2*a)} - 3*e^{(2*a)})*e^{(2*b*x)}/b^4 - (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^{(-2*b*x - 2*a)}/b^4)*d^3 + 1/8*c^3*(4*x + e^{(2*b*x + 2*a)}/b - e^{(-2*b*x - 2*a)}/b)$$

**mupad [B]** time = 1.18, size = 229, normalized size = 1.71

$$\frac{4b^4c^3x - \frac{3d^3\cosh(2a+2bx)}{2} + 2b^3c^3\sinh(2a+2bx) + b^4d^3x^4 - 3b^2c^2d\cosh(2a+2bx) + 6b^4c^2dx^2 + 4b^4c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(a + b*x)^2*(c + d*x)^3,x)`

[Out]  $(4*b^4*c^3*x - (3*d^3*cosh(2*a + 2*b*x))/2 + 2*b^3*c^3*sinh(2*a + 2*b*x) + b^4*d^3*x^4 - 3*b^2*c^2*d*cosh(2*a + 2*b*x) + 6*b^4*c^2*d*x^2 + 4*b^4*c*d^2*x^3 - 3*b^2*d^3*x^2*cosh(2*a + 2*b*x) + 2*b^3*d^3*x^3*sinh(2*a + 2*b*x) + 3*b*c*d^2*sinh(2*a + 2*b*x) + 3*b*d^3*x*sinh(2*a + 2*b*x) - 6*b^2*c*d^2*x*cosh(2*a + 2*b*x) + 6*b^3*c^2*d*x*sinh(2*a + 2*b*x) + 6*b^3*c*d^2*x^2*sinh(2*a + 2*b*x))/(8*b^4)$

**sympy [A]** time = 2.58, size = 456, normalized size = 3.40

$$\left\{ \begin{array}{l} -\frac{c^3x\sinh^2(a+bx)}{2} + \frac{c^3x\cosh^2(a+bx)}{2} - \frac{3c^2dx^2\sinh^2(a+bx)}{4} + \frac{3c^2dx^2\cosh^2(a+bx)}{4} - \frac{cd^2x^3\sinh^2(a+bx)}{2} + \frac{cd^2x^3\cosh^2(a+bx)}{2} - \frac{d^3x^4\sinh^2(a+bx)}{8} \\ \left( c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4} \right) \cosh^2(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)**3*cosh(b*x+a)**2,x)`

[Out] `Piecewise((-c**3*x*sinh(a + b*x)**2/2 + c**3*x*cosh(a + b*x)**2/2 - 3*c**2*d*x**2*sinh(a + b*x)**2/4 + 3*c**2*d*x**2*cosh(a + b*x)**2/4 - c*d**2*x**3*sinh(a + b*x)**2/2 + c*d**2*x**3*cosh(a + b*x)**2/2 - d**3*x**4*sinh(a + b*x)**2/8 + d**3*x**4*cosh(a + b*x)**2/8 + c**3*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 3*c**2*d*x*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 3*c*d**2*x**2*sinh(a + b*x)*cosh(a + b*x)/(2*b) + d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/(2*b) - 3*c**2*d*cosh(a + b*x)**2/(4*b**2) - 3*c*d**2*x*sinh(a + b*x)**2/(4*b**2) - 3*c*d**2*x*cosh(a + b*x)**2/(4*b**2) - 3*d**3*x**2*sinh(a + b*x)**2/(8*b**2) - 3*d**3*x**2*cosh(a + b*x)**2/(8*b**2) + 3*c*d**2*sinh(a + b*x)*cosh(a + b*x)/(4*b**3) + 3*d**3*x*sinh(a + b*x)*cosh(a + b*x)/(4*b**3) - 3*d**3*cosh(a + b*x)**2/(8*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*cosh(a)**2, True))`



### 3.10 $\int (c + dx)^2 \cosh^2(a + bx) dx$

**Optimal.** Leaf size=95

$$\frac{d^2 \sinh(a + bx) \cosh(a + bx)}{4b^3} - \frac{d(c + dx) \cosh^2(a + bx)}{2b^2} + \frac{(c + dx)^2 \sinh(a + bx) \cosh(a + bx)}{2b} + \frac{d^2 x}{4b^2} + \frac{(c + dx)^3}{6d}$$

[Out]  $1/4*d^2*x/b^2+1/6*(d*x+c)^3/d-1/2*d*(d*x+c)*\cosh(b*x+a)^2/b^2+1/4*d^2*\cosh(b*x+a)*\sinh(b*x+a)/b^3+1/2*(d*x+c)^2*\cosh(b*x+a)*\sinh(b*x+a)/b$

**Rubi [A]** time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3311, 32, 2635, 8}

$$-\frac{d(c + dx) \cosh^2(a + bx)}{2b^2} + \frac{d^2 \sinh(a + bx) \cosh(a + bx)}{4b^3} + \frac{(c + dx)^2 \sinh(a + bx) \cosh(a + bx)}{2b} + \frac{d^2 x}{4b^2} + \frac{(c + dx)^3}{6d}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^2\*Cosh[a + b\*x]^2,x]

[Out]  $(d^2*x)/(4*b^2) + (c + d*x)^3/(6*d) - (d*(c + d*x)*\text{Cosh}[a + b*x]^2)/(2*b^2) + (d^2*\text{Cosh}[a + b*x]*\text{Sinh}[a + b*x])/(4*b^3) + ((c + d*x)^2*\text{Cosh}[a + b*x]*\text{Sinh}[a + b*x])/(2*b)$

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x])\*(b\*Sin[c + d\*x])^(n - 1)/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Sin[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*Sin[e + f\*x])^(n - 2), x], x] - Dist[

$d^{2m}(m-1)/(f^{2n})$ , Int[(c + dx)^(m-2)\*(b\*Sin[e + f\*x])^n, x], x]  
 - Simp[(b\*(c + dx)^m\*Cos[e + f\*x]\*(b\*Sin[e + f\*x])^(n-1))/(f\*n), x] /;  
 FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

### Rubi steps

$$\begin{aligned} \int (c + dx)^2 \cosh^2(a + bx) dx &= -\frac{d(c + dx) \cosh^2(a + bx)}{2b^2} + \frac{(c + dx)^2 \cosh(a + bx) \sinh(a + bx)}{2b} + \frac{1}{2} \int (c + dx)^2 dx \\ &= \frac{(c + dx)^3}{6d} - \frac{d(c + dx) \cosh^2(a + bx)}{2b^2} + \frac{d^2 \cosh(a + bx) \sinh(a + bx)}{4b^3} + \frac{(c + dx)^2}{2} \\ &= \frac{d^2 x}{4b^2} + \frac{(c + dx)^3}{6d} - \frac{d(c + dx) \cosh^2(a + bx)}{2b^2} + \frac{d^2 \cosh(a + bx) \sinh(a + bx)}{4b^3} + \frac{(c + dx)^2}{2} \end{aligned}$$

**Mathematica** [A] time = 0.30, size = 75, normalized size = 0.79

$$\frac{3 \sinh(2(a + bx)) (2b^2(c + dx)^2 + d^2) - 6bd(c + dx) \cosh(2(a + bx)) + 4b^3 x (3c^2 + 3cdx + d^2 x^2)}{24b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2\*Cosh[a + b\*x]^2,x]

[Out] (4\*b^3\*x\*(3\*c^2 + 3\*c\*d\*x + d^2\*x^2) - 6\*b\*d\*(c + d\*x)\*Cosh[2\*(a + b\*x)] + 3\*(d^2 + 2\*b^2\*(c + d\*x)^2)\*Sinh[2\*(a + b\*x)])/(24\*b^3)

**fricas** [A] time = 0.72, size = 123, normalized size = 1.29

$$\frac{2b^3 d^2 x^3 + 6b^3 c d x^2 + 6b^3 c^2 x - 3(bd^2 x + bcd) \cosh(bx + a)^2 + 3(2b^2 d^2 x^2 + 4b^2 c d x + 2b^2 c^2 + d^2) \cosh(bx + a)}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*cosh(b\*x+a)^2,x, algorithm="fricas")

[Out] 1/12\*(2\*b^3\*d^2\*x^3 + 6\*b^3\*c\*d\*x^2 + 6\*b^3\*c^2\*x - 3\*(b\*d^2\*x + b\*c\*d)\*cosh(b\*x + a)^2 + 3\*(2\*b^2\*d^2\*x^2 + 4\*b^2\*c\*d\*x + 2\*b^2\*c^2 + d^2)\*cosh(b\*x + a)\*sinh(b\*x + a) - 3\*(b\*d^2\*x + b\*c\*d)\*sinh(b\*x + a)^2)/b^3

**giac** [A] time = 0.12, size = 136, normalized size = 1.43

$$\frac{1}{6} d^2 x^3 + \frac{1}{2} c d x^2 + \frac{1}{2} c^2 x + \frac{(2b^2 d^2 x^2 + 4b^2 c d x + 2b^2 c^2 - 2bd^2 x - 2bcd + d^2) e^{(2bx+2a)}}{16b^3} - \frac{(2b^2 d^2 x^2 + 4b^2 c d x + 2b^2 c^2)}{16b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*cosh(b\*x+a)^2,x, algorithm="giac")

[Out]  $\frac{1}{6}d^2x^3 + \frac{1}{2}cdx^2 + \frac{1}{2}c^2x + \frac{1}{16}(2b^2d^2x^2 + 4b^2cdx + 2b^2c^2 - 2bd^2x - 2b^2cd + d^2)e^{(2bx+2a)}/b^3 - \frac{1}{16}(2b^2d^2x^2 + 4b^2cdx + 2b^2c^2 + 2bd^2x + 2b^2cd + d^2)e^{(-2bx-2a)}/b^3$

**maple [B]** time = 0.05, size = 262, normalized size = 2.76

$$\frac{d^2 \left( \frac{(bx+a)^2 \cosh(bx+a) \sinh(bx+a)}{2} + \frac{(bx+a)^3}{6} - \frac{(bx+a) \cosh^2(bx+a)}{2} + \frac{\cosh(bx+a) \sinh(bx+a)}{4} + \frac{bx}{4} + \frac{a}{4} \right)}{b^2} - \frac{2d^2 a \left( \frac{(bx+a) \cosh(bx+a) \sinh(bx+a)}{2} + \frac{(bx+a)^2}{4} - \frac{\cosh^2(bx+a)}{4} \right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*cosh(b\*x+a)^2,x)

[Out]  $\frac{1}{b} \left( \frac{1}{b^2} d^2 \left( \frac{1}{2} (bx+a)^2 \cosh(bx+a) \sinh(bx+a) + \frac{1}{6} (bx+a)^3 - \frac{1}{2} (bx+a) \cosh(bx+a)^2 + \frac{1}{4} \cosh(bx+a) \sinh(bx+a) + \frac{1}{4} bx + \frac{1}{4} a \right) - \frac{1}{b^2} d^2 a \left( \frac{1}{2} (bx+a) \cosh(bx+a) \sinh(bx+a) + \frac{1}{4} (bx+a)^2 - \frac{1}{4} \cosh(bx+a)^2 \right) + \frac{1}{b^2} d^2 a^2 \left( \frac{1}{2} \cosh(bx+a) \sinh(bx+a) + \frac{1}{2} bx + \frac{1}{2} a \right) + \frac{2}{b} cd \left( \frac{1}{2} (bx+a) \cosh(bx+a) \sinh(bx+a) + \frac{1}{4} (bx+a)^2 - \frac{1}{4} \cosh(bx+a)^2 \right) - \frac{2}{b} cd a \left( \frac{1}{2} \cosh(bx+a) \sinh(bx+a) + \frac{1}{2} bx + \frac{1}{2} a \right) + c^2 \left( \frac{1}{2} \cosh(bx+a) \sinh(bx+a) + \frac{1}{2} bx + \frac{1}{2} a \right) \right)$

**maxima [A]** time = 0.38, size = 165, normalized size = 1.74

$$\frac{1}{8} \left( 4x^2 + \frac{(2bx e^{(2a)} - e^{(2a)}) e^{(2bx)}}{b^2} - \frac{(2bx+1) e^{(-2bx-2a)}}{b^2} \right) cd + \frac{1}{48} \left( 8x^3 + \frac{3(2b^2 x^2 e^{(2a)} - 2bx e^{(2a)} + e^{(2a)}) e^{(2bx)}}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*cosh(b\*x+a)^2,x, algorithm="maxima")

[Out]  $\frac{1}{8}(4x^2 + (2bx e^{(2a)} - e^{(2a)}) e^{(2bx)})/b^2 - (2bx+1) e^{(-2bx-2a)}/b^2 + \frac{1}{48}(8x^3 + 3(2b^2 x^2 e^{(2a)} - 2bx e^{(2a)} + e^{(2a)}) e^{(2bx)})/b^3 - 3(2b^2 x^2 + 2bx+1) e^{(-2bx-2a)}/b^3 + \frac{1}{8} c^2 (4x + e^{(2bx+2a)})/b - e^{(-2bx-2a)}/b$

**mupad [B]** time = 0.98, size = 127, normalized size = 1.34

$$\frac{c^2 x}{2} + \frac{d^2 x^3}{6} + \frac{c^2 \sinh(2a+2bx)}{4b} + \frac{d^2 \sinh(2a+2bx)}{8b^3} + \frac{cdx^2}{2} - \frac{d^2 x \cosh(2a+2bx)}{4b^2} + \frac{d^2 x^2 \sinh(2a+2bx)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(a + b*x)^2*(c + d*x)^2,x)`

[Out]  $(c^2x)/2 + (d^2x^3)/6 + (c^2\sinh(2a + 2bx))/(4b) + (d^2\sinh(2a + 2bx))/(8b^3) + (cdx^2)/2 - (d^2x\cosh(2a + 2bx))/(4b^2) + (d^2x^2\sinh(2a + 2bx))/(4b) - (cd\cosh(2a + 2bx))/(4b^2) + (cdx\sinh(2a + 2bx))/(2b)$

**sympy** [A] time = 1.22, size = 264, normalized size = 2.78

$$\left\{ \begin{array}{l} -\frac{c^2x \sinh^2(a+bx)}{2} + \frac{c^2x \cosh^2(a+bx)}{2} - \frac{cdx^2 \sinh^2(a+bx)}{2} + \frac{cdx^2 \cosh^2(a+bx)}{2} - \frac{d^2x^3 \sinh^2(a+bx)}{6} + \frac{d^2x^3 \cosh^2(a+bx)}{6} + \frac{c^2 \sinh(a+bx)}{2b} \\ \left( c^2x + cdx^2 + \frac{d^2x^3}{3} \right) \cosh^2(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)**2*cosh(b*x+a)**2,x)`

[Out] `Piecewise((-c**2*x*sinh(a + b*x)**2/2 + c**2*x*cosh(a + b*x)**2/2 - c*d*x**2*sinh(a + b*x)**2/2 + c*d*x**2*cosh(a + b*x)**2/2 - d**2*x**3*sinh(a + b*x)**2/6 + d**2*x**3*cosh(a + b*x)**2/6 + c**2*sinh(a + b*x)*cosh(a + b*x)/(2*b) + c*d*x*sinh(a + b*x)*cosh(a + b*x)/b + d**2*x**2*sinh(a + b*x)*cosh(a + b*x)/(2*b) - c*d*cosh(a + b*x)**2/(2*b**2) - d**2*x*sinh(a + b*x)**2/(4*b**2) - d**2*x*cosh(a + b*x)**2/(4*b**2) + d**2*sinh(a + b*x)*cosh(a + b*x)/(4*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*cosh(a)**2, True)`

### 3.11 $\int (c + dx) \cosh^2(a + bx) dx$

Optimal. Leaf size=55

$$-\frac{d \cosh^2(a + bx)}{4b^2} + \frac{(c + dx) \sinh(a + bx) \cosh(a + bx)}{2b} + \frac{cx}{2} + \frac{dx^2}{4}$$

[Out]  $1/2*c*x+1/4*d*x^2-1/4*d*\cosh(b*x+a)^2/b^2+1/2*(d*x+c)*\cosh(b*x+a)*\sinh(b*x+a)/b$

Rubi [A] time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {3310}

$$-\frac{d \cosh^2(a + bx)}{4b^2} + \frac{(c + dx) \sinh(a + bx) \cosh(a + bx)}{2b} + \frac{cx}{2} + \frac{dx^2}{4}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)\*Cosh[a + b\*x]^2,x]

[Out] (c\*x)/2 + (d\*x^2)/4 - (d\*Cosh[a + b\*x]^2)/(4\*b^2) + ((c + d\*x)\*Cosh[a + b\*x]\*Sinh[a + b\*x])/(2\*b)

Rule 3310

Int[((c\_.) + (d\_.)\*(x\_))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :=  
Simp[(d\*(b\*Sin[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)\*(b\*Sin[e + f\*x])^(n - 2), x], x] - Simp[(b\*(c + d\*x)\*Cos[e + f\*x]\*(b\*Sin[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int (c + dx) \cosh^2(a + bx) dx &= -\frac{d \cosh^2(a + bx)}{4b^2} + \frac{(c + dx) \cosh(a + bx) \sinh(a + bx)}{2b} + \frac{1}{2} \int (c + dx) dx \\ &= \frac{cx}{2} + \frac{dx^2}{4} - \frac{d \cosh^2(a + bx)}{4b^2} + \frac{(c + dx) \cosh(a + bx) \sinh(a + bx)}{2b} \end{aligned}$$

Mathematica [A] time = 0.16, size = 51, normalized size = 0.93

$$\frac{2b((c + dx) \sinh(2(a + bx)) + 2ac + bx(2c + dx)) - d \cosh(2(a + bx))}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)\*Cosh[a + b\*x]^2,x]

[Out]  $(-(d*\text{Cosh}[2*(a + b*x)]) + 2*b*(2*a*c + b*x*(2*c + d*x) + (c + d*x)*\text{Sinh}[2*(a + b*x)]))/ (8*b^2)$

**fricas** [A] time = 0.54, size = 66, normalized size = 1.20

$$\frac{2b^2dx^2 + 4b^2cx - d \cosh(bx + a)^2 + 4(bdx + bc) \cosh(bx + a) \sinh(bx + a) - d \sinh(bx + a)^2}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*cosh(b\*x+a)^2,x, algorithm="fricas")

[Out]  $1/8*(2*b^2*d*x^2 + 4*b^2*c*x - d*\text{cosh}(b*x + a)^2 + 4*(b*d*x + b*c)*\text{cosh}(b*x + a)*\text{sinh}(b*x + a) - d*\text{sinh}(b*x + a)^2)/b^2$

**giac** [A] time = 0.14, size = 63, normalized size = 1.15

$$\frac{1}{4}dx^2 + \frac{1}{2}cx + \frac{(2bdx + 2bc - d)e^{(2bx+2a)}}{16b^2} - \frac{(2bdx + 2bc + d)e^{(-2bx-2a)}}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*cosh(b\*x+a)^2,x, algorithm="giac")

[Out]  $1/4*d*x^2 + 1/2*c*x + 1/16*(2*b*d*x + 2*b*c - d)*e^{(2*b*x + 2*a)}/b^2 - 1/16*(2*b*d*x + 2*b*c + d)*e^{(-2*b*x - 2*a)}/b^2$

**maple** [B] time = 0.05, size = 103, normalized size = 1.87

$$\frac{d \left( \frac{(bx+a) \cosh(bx+a) \sinh(bx+a)}{2} + \frac{(bx+a)^2}{4} - \frac{(\cosh^2(bx+a))}{4} \right)}{b} - \frac{da \left( \frac{\cosh(bx+a) \sinh(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right)}{b} + c \left( \frac{\cosh(bx+a) \sinh(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)\*cosh(b\*x+a)^2,x)

[Out]  $1/b*(1/b*d*(1/2*(b*x+a)*\text{cosh}(b*x+a)*\text{sinh}(b*x+a)+1/4*(b*x+a)^2-1/4*\text{cosh}(b*x+a)^2)-1/b*d*a*(1/2*\text{cosh}(b*x+a)*\text{sinh}(b*x+a)+1/2*b*x+1/2*a)+c*(1/2*\text{cosh}(b*x+a)*\text{sinh}(b*x+a)+1/2*b*x+1/2*a))$

**maxima** [A] time = 0.44, size = 88, normalized size = 1.60

$$\frac{1}{16} \left( 4x^2 + \frac{(2bx e^{(2a)} - e^{(2a)}) e^{(2bx)}}{b^2} - \frac{(2bx + 1) e^{(-2bx-2a)}}{b^2} \right) d + \frac{1}{8} c \left( 4x + \frac{e^{(2bx+2a)}}{b} - \frac{e^{(-2bx-2a)}}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*cosh(b\*x+a)^2,x, algorithm="maxima")

[Out]  $\frac{1}{16}(4x^2 + (2bx)e^{2a} - e^{2a})e^{2bx}/b^2 - (2bx + 1)e^{-(2bx - 2a)}/b^2 * d + \frac{1}{8}c(4x + e^{2bx + 2a})/b - e^{-(2bx - 2a)}/b$

**mupad [B]** time = 0.10, size = 58, normalized size = 1.05

$$\frac{b^2 dx^2 - \frac{d \cosh(2a+2bx)}{2} + bc \sinh(2a + 2bx) + 2b^2 cx + b dx \sinh(2a + 2bx)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^2\*(c + d\*x),x)

[Out]  $(b^2 dx^2 - (d \cosh(2a + 2bx)))/2 + b^2 c \sinh(2a + 2bx) + 2b^2 cx + b dx \sinh(2a + 2bx)/(4b^2)$

**sympy [A]** time = 0.53, size = 126, normalized size = 2.29

$$\left\{ \begin{array}{l} -\frac{cx \sinh^2(a+bx)}{2} + \frac{cx \cosh^2(a+bx)}{2} - \frac{dx^2 \sinh^2(a+bx)}{4} + \frac{dx^2 \cosh^2(a+bx)}{4} + \frac{c \sinh(a+bx) \cosh(a+bx)}{2b} + \frac{dx \sinh(a+bx) \cosh(a+bx)}{2b} - \frac{d}{2b} \\ \left( cx + \frac{dx^2}{2} \right) \cosh^2(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*cosh(b\*x+a)\*\*2,x)

[Out] Piecewise((-c\*x\*sinh(a + b\*x)\*\*2/2 + c\*x\*cosh(a + b\*x)\*\*2/2 - d\*x\*\*2\*sinh(a + b\*x)\*\*2/4 + d\*x\*\*2\*cosh(a + b\*x)\*\*2/4 + c\*sinh(a + b\*x)\*cosh(a + b\*x)/(2\*b) + d\*x\*sinh(a + b\*x)\*cosh(a + b\*x)/(2\*b) - d\*cosh(a + b\*x)\*\*2/(4\*b\*\*2), Ne(b, 0)), ((c\*x + d\*x\*\*2/2)\*cosh(a)\*\*2, True))

### 3.12 $\int \frac{\cosh^2(a+bx)}{c+dx} dx$

**Optimal.** Leaf size=78

$$\frac{\cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{2d} + \frac{\sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{2d} + \frac{\log(c + dx)}{2d}$$

[Out]  $1/2*\text{Chi}(2*b*c/d+2*b*x)*\cosh(2*a-2*b*c/d)/d+1/2*\ln(d*x+c)/d+1/2*\text{Shi}(2*b*c/d+2*b*x)*\sinh(2*a-2*b*c/d)/d$

**Rubi [A]** time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3312, 3303, 3298, 3301}

$$\frac{\cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{2d} + \frac{\sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{2d} + \frac{\log(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b\*x]^2/(c + d\*x), x]

[Out]  $(\text{Cosh}[2*a - (2*b*c)/d]*\text{CoshIntegral}[(2*b*c)/d + 2*b*x])/(2*d) + \text{Log}[c + d*x]/(2*d) + (\text{Sinh}[2*a - (2*b*c)/d]*\text{SinhIntegral}[(2*b*c)/d + 2*b*x])/(2*d)$

#### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

#### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

#### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 3312



```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

### Rubi steps

$$\begin{aligned} \int \frac{\cosh^2(a + bx)}{c + dx} dx &= \int \left( \frac{1}{2(c + dx)} + \frac{\cosh(2a + 2bx)}{2(c + dx)} \right) dx \\ &= \frac{\log(c + dx)}{2d} + \frac{1}{2} \int \frac{\cosh(2a + 2bx)}{c + dx} dx \\ &= \frac{\log(c + dx)}{2d} + \frac{1}{2} \cosh\left(2a - \frac{2bc}{d}\right) \int \frac{\cosh\left(\frac{2bc}{d} + 2bx\right)}{c + dx} dx + \frac{1}{2} \sinh\left(2a - \frac{2bc}{d}\right) \int \frac{\sinh\left(\frac{2bc}{d} + 2bx\right)}{c + dx} dx \\ &= \frac{\cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{2d} + \frac{\log(c + dx)}{2d} + \frac{\sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{2d} \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 64, normalized size = 0.82

$$\frac{\cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2b(c+dx)}{d}\right) + \sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2b(c+dx)}{d}\right) + \log(c + dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x), x]
```

```
[Out] (Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*(c + d*x))/d] + Log[c + d*x] + Sinh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/(2*d)
```

**fricas [A]** time = 0.47, size = 104, normalized size = 1.33

$$\frac{\left(\text{Ei}\left(\frac{2(bdx+bc)}{d}\right) + \text{Ei}\left(-\frac{2(bdx+bc)}{d}\right)\right) \cosh\left(-\frac{2(bc-ad)}{d}\right) + \left(\text{Ei}\left(\frac{2(bdx+bc)}{d}\right) - \text{Ei}\left(-\frac{2(bdx+bc)}{d}\right)\right) \sinh\left(-\frac{2(bc-ad)}{d}\right) + 2 \log(c + dx)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c), x, algorithm="fricas")
```

```
[Out] 1/4*((Ei(2*(b*d*x + b*c)/d) + Ei(-2*(b*d*x + b*c)/d))*cosh(-2*(b*c - a*d)/d) + (Ei(2*(b*d*x + b*c)/d) - Ei(-2*(b*d*x + b*c)/d))*sinh(-2*(b*c - a*d)/d) + 2*log(d*x + c))/d
```

**giac** [A] time = 0.12, size = 68, normalized size = 0.87

$$\frac{\operatorname{Ei}\left(\frac{2(bdx+bc)}{d}\right)e^{\left(2a-\frac{2bc}{d}\right)} + \operatorname{Ei}\left(-\frac{2(bdx+bc)}{d}\right)e^{\left(-2a+\frac{2bc}{d}\right)} + 2\log(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c),x, algorithm="giac")

[Out] 1/4\*(Ei(2\*(b\*d\*x + b\*c)/d)\*e^(2\*a - 2\*b\*c/d) + Ei(-2\*(b\*d\*x + b\*c)/d)\*e^(-2\*a + 2\*b\*c/d) + 2\*log(d\*x + c))/d

**maple** [A] time = 0.25, size = 97, normalized size = 1.24

$$\frac{\ln(dx+c)}{2d} - \frac{e^{-\frac{2(da-cb)}{d}} \operatorname{Ei}\left(1, 2bx + 2a - \frac{2(da-cb)}{d}\right)}{4d} - \frac{e^{\frac{2da-2cb}{d}} \operatorname{Ei}\left(1, -2bx - 2a - \frac{2(-da+cb)}{d}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)^2/(d\*x+c),x)

[Out] 1/2\*ln(d\*x+c)/d-1/4/d\*exp(-2\*(a\*d-b\*c)/d)\*Ei(1,2\*b\*x+2\*a-2\*(a\*d-b\*c)/d)-1/4/d\*exp(2\*(a\*d-b\*c)/d)\*Ei(1,-2\*b\*x-2\*a-2\*(-a\*d+b\*c)/d)

**maxima** [A] time = 0.53, size = 72, normalized size = 0.92

$$-\frac{e^{\left(-2a+\frac{2bc}{d}\right)}E_1\left(\frac{2(dx+c)b}{d}\right)}{4d} - \frac{e^{\left(2a-\frac{2bc}{d}\right)}E_1\left(-\frac{2(dx+c)b}{d}\right)}{4d} + \frac{\log(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c),x, algorithm="maxima")

[Out] -1/4\*e^(-2\*a + 2\*b\*c/d)\*exp\_integral\_e(1, 2\*(d\*x + c)\*b/d)/d - 1/4\*e^(2\*a - 2\*b\*c/d)\*exp\_integral\_e(1, -2\*(d\*x + c)\*b/d)/d + 1/2\*log(d\*x + c)/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + bx)^2}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^2/(c + d\*x),x)

```
[Out] int(cosh(a + b*x)^2/(c + d*x), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\cosh^2(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c), x)
```

```
[Out] Integral(cosh(a + b*x)**2/(c + d*x), x)
```

### 3.13 $\int \frac{\cosh^2(a+bx)}{(c+dx)^2} dx$

**Optimal.** Leaf size=81

$$\frac{b \sinh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{d^2} + \frac{b \cosh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{d^2} - \frac{\cosh^2(a+bx)}{d(c+dx)}$$

[Out]  $-\cosh(b*x+a)^2/d/(d*x+c)+b*\cosh(2*a-2*b*c/d)*\text{Shi}(2*b*c/d+2*b*x)/d^2+b*\text{Chi}(2*b*c/d+2*b*x)*\sinh(2*a-2*b*c/d)/d^2$

**Rubi [A]** time = 0.15, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3313, 12, 3303, 3298, 3301}

$$\frac{b \sinh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{d^2} + \frac{b \cosh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{d^2} - \frac{\cosh^2(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

[In] `Int[Cosh[a + b*x]^2/(c + d*x)^2,x]`

[Out]  $-(\text{Cosh}[a + b*x]^2/(d*(c + d*x))) + (b*\text{CoshIntegral}[(2*b*c)/d + 2*b*x]*\text{Sinh}[2*a - (2*b*c)/d])/d^2 + (b*\text{Cosh}[2*a - (2*b*c)/d]*\text{SinhIntegral}[(2*b*c)/d + 2*b*x])/d^2$

#### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

#### Rule 3298

`Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]`

#### Rule 3301

`Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]`

#### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{\cosh^2(a + bx)}{(c + dx)^2} dx &= -\frac{\cosh^2(a + bx)}{d(c + dx)} + \frac{(2ib) \int -\frac{i \sinh(2a + 2bx)}{2(c + dx)} dx}{d} \\ &= -\frac{\cosh^2(a + bx)}{d(c + dx)} + \frac{b \int \frac{\sinh(2a + 2bx)}{c + dx} dx}{d} \\ &= -\frac{\cosh^2(a + bx)}{d(c + dx)} + \frac{\left(b \cosh\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\sinh\left(\frac{2bc}{d} + 2bx\right)}{c + dx} dx}{d} + \frac{\left(b \sinh\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\cosh\left(\frac{2bc}{d} + 2bx\right)}{c + dx} dx}{d} \\ &= -\frac{\cosh^2(a + bx)}{d(c + dx)} + \frac{b \operatorname{Chi}\left(\frac{2bc}{d} + 2bx\right) \sinh\left(2a - \frac{2bc}{d}\right)}{d^2} + \frac{b \cosh\left(2a - \frac{2bc}{d}\right) \operatorname{Shi}\left(\frac{2bc}{d} + 2bx\right)}{d^2} \end{aligned}$$

**Mathematica [A]** time = 0.41, size = 75, normalized size = 0.93

$$\frac{b \sinh\left(2a - \frac{2bc}{d}\right) \operatorname{Chi}\left(\frac{2b(c + dx)}{d}\right) + b \cosh\left(2a - \frac{2bc}{d}\right) \operatorname{Shi}\left(\frac{2b(c + dx)}{d}\right) - \frac{d \cosh^2(a + bx)}{c + dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x)^2, x]
```

```
[Out] (-((d*Cosh[a + b*x]^2)/(c + d*x)) + b*CoshIntegral[(2*b*(c + d*x))/d]*Sinh[
2*a - (2*b*c)/d] + b*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])
/d^2
```

**fricas [B]** time = 0.45, size = 164, normalized size = 2.02

$$\frac{d \cosh (bx + a)^2 + d \sinh (bx + a)^2 - \left( (bdx + bc) \operatorname{Ei} \left( \frac{2(bdx + bc)}{d} \right) - (bdx + bc) \operatorname{Ei} \left( -\frac{2(bdx + bc)}{d} \right) \right) \cosh \left( -\frac{2(bc - ad)}{d} \right) - \left( (bdx + bc) \operatorname{Ei} \left( \frac{2(bdx + bc)}{d} \right) + (bdx + bc) \operatorname{Ei} \left( -\frac{2(bdx + bc)}{d} \right) \right) \sinh \left( -\frac{2(bc - ad)}{d} \right) + d}{2(d^3x + cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^2,x, algorithm="fricas")

[Out]  $-1/2*(d*\cosh(b*x + a)^2 + d*\sinh(b*x + a)^2 - ((b*d*x + b*c)*\operatorname{Ei}(2*(b*d*x + b*c)/d) - (b*d*x + b*c)*\operatorname{Ei}(-2*(b*d*x + b*c)/d))*\cosh(-2*(b*c - a*d)/d) - ((b*d*x + b*c)*\operatorname{Ei}(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*\operatorname{Ei}(-2*(b*d*x + b*c)/d))*\sinh(-2*(b*c - a*d)/d) + d)/(d^3*x + c*d^2)$

**giac [B]** time = 0.19, size = 574, normalized size = 7.09

$$\left( 2(dx + c) \left( b - \frac{bc}{dx+c} + \frac{ad}{dx+c} \right) b^2 \operatorname{Ei} \left( -\frac{2 \left( (dx+c) \left( b - \frac{bc}{dx+c} + \frac{ad}{dx+c} \right) + bc - ad \right)}{d} \right) e^{\left( \frac{2(bc-ad)}{d} \right)} + 2b^3c \operatorname{Ei} \left( -\frac{2 \left( (dx+c) \left( b - \frac{bc}{dx+c} + \frac{ad}{dx+c} \right) + bc - ad \right)}{d} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^2,x, algorithm="giac")

[Out]  $-1/4*(2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*\operatorname{Ei}(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{2*(b*c - a*d)/d} + 2*b^3*c*\operatorname{Ei}(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{2*(b*c - a*d)/d} - 2*a*b^2*d*\operatorname{Ei}(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{2*(b*c - a*d)/d} - 2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*\operatorname{Ei}(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{-2*(b*c - a*d)/d} - 2*b^3*c*\operatorname{Ei}(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{-2*(b*c - a*d)/d} + 2*a*b^2*d*\operatorname{Ei}(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^{-2*(b*c - a*d)/d} + b^2*d*e^{2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d} + b^2*d*e^{-2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d} + 2*b^2*d*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^4 + b*c*d^4 - a*d^5)*b)$

**maple [A]** time = 0.24, size = 152, normalized size = 1.88

$$\frac{1}{2d(dx + c)} - \frac{be^{-2bx-2a}}{4(bdx + cb)d} + \frac{be^{-\frac{2(da-cb)}{d}} \operatorname{Ei} \left( 1, 2bx + 2a - \frac{2(da-cb)}{d} \right)}{2d^2} - \frac{be^{2bx+2a}}{4d^2 \left( \frac{bc}{d} + bx \right)} - \frac{be^{\frac{2da-2cb}{d}} \operatorname{Ei} \left( 1, -2bx - 2a - \frac{2(da-cb)}{d} \right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(b*x+a)^2/(d*x+c)^2,x)`

[Out] 
$$-1/2/d/(d*x+c) - 1/4*b*\exp(-2*b*x-2*a)/(b*d*x+b*c)/d + 1/2*b/d^2*\exp(-2*(a*d-b*c)/d)*Ei(1, 2*b*x+2*a-2*(a*d-b*c)/d) - 1/4*b/d^2*\exp(2*b*x+2*a)/(b*c/d+b*x) - 1/2*b/d^2*\exp(2*(a*d-b*c)/d)*Ei(1, -2*b*x-2*a-2*(-a*d+b*c)/d)$$

**maxima** [A] time = 0.38, size = 88, normalized size = 1.09

$$-\frac{e^{\left(-2a+\frac{2bc}{d}\right)}E_2\left(\frac{2(dx+c)b}{d}\right)}{4(dx+c)d} - \frac{e^{\left(2a-\frac{2bc}{d}\right)}E_2\left(-\frac{2(dx+c)b}{d}\right)}{4(dx+c)d} - \frac{1}{2(d^2x+cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")`

[Out] 
$$-1/4*e^{(-2*a + 2*b*c/d)*\exp\_integral\_e(2, 2*(d*x + c)*b/d)/((d*x + c)*d)} - 1/4*e^{(2*a - 2*b*c/d)*\exp\_integral\_e(2, -2*(d*x + c)*b/d)/((d*x + c)*d)} - 1/2/(d^2*x + c*d)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + bx)^2}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(a + b*x)^2/(c + d*x)^2,x)`

[Out] `int(cosh(a + b*x)^2/(c + d*x)^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)**2/(d*x+c)**2,x)`

[Out] `Integral(cosh(a + b*x)**2/(c + d*x)**2, x)`

$$3.14 \quad \int \frac{\cosh^2(a+bx)}{(c+dx)^3} dx$$

**Optimal.** Leaf size=112

$$\frac{b^2 \cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{d^3} + \frac{b^2 \sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b \sinh(a+bx) \cosh(a+bx)}{d^2(c+dx)} - \frac{\cosh^2(a+bx)}{2d(c+dx)}$$

[Out]  $b^2 \text{Chi}\left(\frac{2bc}{d} + 2bx\right) \cosh\left(2a - \frac{2bc}{d}\right) / d^3 - 1/2 \cosh(bx+a)^2 / d / (dx+c)^2 + b^2 \text{Shi}\left(\frac{2bc}{d} + 2bx\right) \sinh\left(2a - \frac{2bc}{d}\right) / d^3 - b \cosh(bx+a) \sinh(bx+a) / d^2 / (dx+c)$

**Rubi [A]** time = 0.19, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {3314, 31, 3312, 3303, 3298, 3301}

$$\frac{b^2 \cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{d^3} + \frac{b^2 \sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b \sinh(a+bx) \cosh(a+bx)}{d^2(c+dx)} - \frac{\cosh^2(a+bx)}{2d(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b\*x]^2/(c + d\*x)^3, x]

[Out]  $-\text{Cosh}[a + b*x]^2 / (2*d*(c + d*x)^2) + (b^2*\text{Cosh}[2*a - (2*b*c)/d]*\text{CoshIntegral}[(2*b*c)/d + 2*b*x]) / d^3 - (b*\text{Cosh}[a + b*x]*\text{Sinh}[a + b*x]) / (d^2*(c + d*x)) + (b^2*\text{Sinh}[2*a - (2*b*c)/d]*\text{SinhIntegral}[(2*b*c)/d + 2*b*x]) / d^3$

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 3298**

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

**Rule 3301**

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]



Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_.))^(m_)*sin[(e_.) + (f_.)*(x_.)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_)), x_Symbo
l] := Simp[((c + d*x)^(m + 1)*(b*Sine[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sine[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sine[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*(b*Sine[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(a + bx)}{(c + dx)^3} dx &= -\frac{\cosh^2(a + bx)}{2d(c + dx)^2} - \frac{b \cosh(a + bx) \sinh(a + bx)}{d^2(c + dx)} - \frac{b^2 \int \frac{1}{c+dx} dx}{d^2} + \frac{(2b^2) \int \frac{\cosh^2(a+bx)}{c+dx} dx}{d^2} \\
&= -\frac{\cosh^2(a + bx)}{2d(c + dx)^2} - \frac{b^2 \log(c + dx)}{d^3} - \frac{b \cosh(a + bx) \sinh(a + bx)}{d^2(c + dx)} + \frac{(2b^2) \int \left( \frac{1}{2(c+dx)} + \frac{\cosh(2a+2bx)}{c+dx} \right) dx}{d^2} \\
&= -\frac{\cosh^2(a + bx)}{2d(c + dx)^2} - \frac{b \cosh(a + bx) \sinh(a + bx)}{d^2(c + dx)} + \frac{b^2 \int \frac{\cosh(2a+2bx)}{c+dx} dx}{d^2} \\
&= -\frac{\cosh^2(a + bx)}{2d(c + dx)^2} - \frac{b \cosh(a + bx) \sinh(a + bx)}{d^2(c + dx)} + \frac{\left( b^2 \cosh\left(2a - \frac{2bc}{d}\right) \right) \int \frac{\cosh\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx}{d^2} \\
&= -\frac{\cosh^2(a + bx)}{2d(c + dx)^2} + \frac{b^2 \cosh\left(2a - \frac{2bc}{d}\right) \operatorname{Chi}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b \cosh(a + bx) \sinh(a + bx)}{d^2(c + dx)} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.88, size = 102, normalized size = 0.91

$$\frac{2b^2 \cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2b(c+dx)}{d}\right) + 2b^2 \sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2b(c+dx)}{d}\right) - \frac{d(b(c+dx) \sinh(2(a+bx)) + d \cosh^2(a+bx))}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]^2/(c + d\*x)^3,x]

[Out] (2\*b^2\*Cosh[2\*a - (2\*b\*c)/d]\*CoshIntegral[(2\*b\*(c + d\*x))/d] - (d\*(d\*Cosh[a + b\*x]^2 + b\*(c + d\*x)\*Sinh[2\*(a + b\*x)]))/(c + d\*x)^2 + 2\*b^2\*Sinh[2\*a - (2\*b\*c)/d]\*SinhIntegral[(2\*b\*(c + d\*x))/d])/(2\*d^3)

**fricas [B]** time = 0.54, size = 278, normalized size = 2.48

$$\frac{d^2 \cosh(bx + a)^2 + d^2 \sinh(bx + a)^2 + 4(bd^2x + bcd) \cosh(bx + a) \sinh(bx + a) + d^2 - 2\left(b^2d^2x^2 + 2b^2cdx + \dots\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^3,x, algorithm="fricas")

[Out] -1/4\*(d^2\*cosh(b\*x + a)^2 + d^2\*sinh(b\*x + a)^2 + 4\*(b\*d^2\*x + b\*c\*d)\*cosh(b\*x + a)\*sinh(b\*x + a) + d^2 - 2\*((b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2)\*Ei(2\*(b\*d\*x + b\*c)/d) + (b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2)\*Ei(-2\*(b\*d\*x + b\*c)/d))\*cosh(-2\*(b\*c - a\*d)/d) - 2\*((b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2)\*Ei(2\*(b\*d\*x + b\*c)/d) - (b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2)\*Ei(-2\*(b\*d\*x + b\*c)/d))\*sinh(-2\*(b\*c - a\*d)/d))/(d^5\*x^2 + 2\*c\*d^4\*x + c^2\*d^3)

**giac [B]** time = 0.13, size = 330, normalized size = 2.95

$$\frac{4b^2d^2x^2\text{Ei}\left(\frac{2(bdx+bc)}{d}\right)e^{\left(2a-\frac{2bc}{d}\right)} + 4b^2d^2x^2\text{Ei}\left(-\frac{2(bdx+bc)}{d}\right)e^{\left(-2a+\frac{2bc}{d}\right)} + 8b^2cdx\text{Ei}\left(\frac{2(bdx+bc)}{d}\right)e^{\left(2a-\frac{2bc}{d}\right)} + 8b^2cdx\text{Ei}\left(-\frac{2(bdx+bc)}{d}\right)e^{\left(-2a+\frac{2bc}{d}\right)}}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^3,x, algorithm="giac")

[Out] 1/8\*(4\*b^2\*d^2\*x^2\*Ei(2\*(b\*d\*x + b\*c)/d)\*e^(2\*a - 2\*b\*c/d) + 4\*b^2\*d^2\*x^2\*Ei(-2\*(b\*d\*x + b\*c)/d)\*e^(-2\*a + 2\*b\*c/d) + 8\*b^2\*c\*d\*x\*Ei(2\*(b\*d\*x + b\*c)/d)\*e^(2\*a - 2\*b\*c/d) + 8\*b^2\*c\*d\*x\*Ei(-2\*(b\*d\*x + b\*c)/d)\*e^(-2\*a + 2\*b\*c/d) + 4\*b^2\*c^2\*Ei(2\*(b\*d\*x + b\*c)/d)\*e^(2\*a - 2\*b\*c/d) + 4\*b^2\*c^2\*Ei(-2\*(b\*d\*x + b\*c)/d)\*e^(-2\*a + 2\*b\*c/d) - 2\*b\*d^2\*x\*e^(2\*b\*x + 2\*a) + 2\*b\*d^2\*x\*e^(-2\*b\*x - 2\*a))

$$\frac{(-2bx - 2a) - 2b^3cd e^{2bx+2a} + 2b^3cd e^{-2bx-2a} - d^2 e^{2bx+2a} - d^2 e^{-2bx-2a} - 2d^2}{(d^5 x^2 + 2c^2 d^4 x + c^2 d^3)}$$

**maple [B]** time = 0.25, size = 299, normalized size = 2.67

$$\frac{1}{4d(dx+c)^2} + \frac{b^3 e^{-2bx-2a} x}{4d(b^2 d^2 x^2 + 2b^2 cdx + c^2 b^2)} + \frac{b^3 e^{-2bx-2a} c}{4d^2(b^2 d^2 x^2 + 2b^2 cdx + c^2 b^2)} - \frac{b^2 e^{-2bx-2a}}{8d(b^2 d^2 x^2 + 2b^2 cdx + c^2 b^2)} - \frac{b^2 e^{2bx+2a}}{8d(b^2 d^2 x^2 + 2b^2 cdx + c^2 b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)^2/(d\*x+c)^3,x)

[Out]  $-\frac{1}{4} \frac{1}{d} \frac{1}{(dx+c)^2} + \frac{1}{4} \frac{b^3 \exp(-2bx-2a)}{d} \frac{1}{(b^2 d^2 x^2 + 2b^2 cdx + c^2 b^2)} x + \frac{1}{4} \frac{b^3 \exp(-2bx-2a)}{d^2} \frac{1}{(b^2 d^2 x^2 + 2b^2 cdx + c^2 b^2)} c - \frac{1}{8} \frac{b^2 \exp(-2bx-2a)}{d} \frac{1}{(b^2 d^2 x^2 + 2b^2 cdx + c^2 b^2)} - \frac{1}{2} \frac{b^2}{d^3} \exp(-2(a-d-bc)/d) \operatorname{Ei}\left(1, \frac{2bx+2a-2(a-d-bc)}{d}\right) - \frac{1}{8} \frac{b^2}{d^3} \exp(2bx+2a) \frac{1}{(bc/d+bx)^2} - \frac{1}{4} \frac{b^2}{d^3} \exp(2bx+2a) \frac{1}{(bc/d+bx)} - \frac{1}{2} \frac{b^2}{d^3} \exp(2(a-d-bc)/d) \operatorname{Ei}\left(1, -\frac{2bx-2a-2(a-d-bc)}{d}\right)$

**maxima [A]** time = 0.60, size = 99, normalized size = 0.88

$$\frac{1}{4(d^3 x^2 + 2cd^2 x + c^2 d)} - \frac{e^{\left(-2a + \frac{2bc}{d}\right)} E_3\left(\frac{2(dx+c)b}{d}\right)}{4(dx+c)^2 d} - \frac{e^{\left(2a - \frac{2bc}{d}\right)} E_3\left(-\frac{2(dx+c)b}{d}\right)}{4(dx+c)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^3,x, algorithm="maxima")

[Out]  $-\frac{1}{4} \frac{1}{(d^3 x^2 + 2c^2 d^2 x + c^2 d)} - \frac{1}{4} \frac{e^{(-2a + 2bc/d)} \operatorname{exp\_integral\_e}(3, 2(dx+c)b/d)}{((dx+c)^2 d)} - \frac{1}{4} \frac{e^{(2a - 2bc/d)} \operatorname{exp\_integral\_e}(3, -2(dx+c)b/d)}{((dx+c)^2 d)}$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + bx)^2}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^2/(c + d\*x)^3,x)

[Out] int(cosh(a + b\*x)^2/(c + d\*x)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a + bx)}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*\*2/(d\*x+c)\*\*3,x)

[Out] Integral(cosh(a + b\*x)\*\*2/(c + d\*x)\*\*3, x)

$$3.15 \quad \int \frac{\cosh^2(a+bx)}{(c+dx)^4} dx$$

**Optimal.** Leaf size=162

$$\frac{2b^3 \sinh\left(2a - \frac{2bc}{d}\right) \operatorname{Chi}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} + \frac{2b^3 \cosh\left(2a - \frac{2bc}{d}\right) \operatorname{Shi}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} - \frac{2b^2 \cosh^2(a+bx)}{3d^3(c+dx)} - \frac{b \sinh(a+bx) \operatorname{Chi}\left(\frac{2bc}{d} + 2bx\right)}{3d^2(c+dx)}$$

[Out]  $1/3*b^2/d^3/(d*x+c)-1/3*\cosh(b*x+a)^2/d/(d*x+c)^3-2/3*b^2*\cosh(b*x+a)^2/d^3/(d*x+c)+2/3*b^3*\cosh(2*a-2*b*c/d)*\operatorname{Shi}(2*b*c/d+2*b*x)/d^4+2/3*b^3*\operatorname{Chi}(2*b*c/d+2*b*x)*\sinh(2*a-2*b*c/d)/d^4-1/3*b*\cosh(b*x+a)*\sinh(b*x+a)/d^2/(d*x+c)^2$

**Rubi [A]** time = 0.18, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {3314, 32, 3313, 12, 3303, 3298, 3301}

$$\frac{2b^3 \sinh\left(2a - \frac{2bc}{d}\right) \operatorname{Chi}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} + \frac{2b^3 \cosh\left(2a - \frac{2bc}{d}\right) \operatorname{Shi}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} - \frac{2b^2 \cosh^2(a+bx)}{3d^3(c+dx)} - \frac{b \sinh(a+bx) \operatorname{Chi}\left(\frac{2bc}{d} + 2bx\right)}{3d^2(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cosh}[a + b*x]^2/(c + d*x)^4, x]$

[Out]  $b^2/(3*d^3*(c + d*x)) - \operatorname{Cosh}[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*\operatorname{Cosh}[a + b*x]^2)/(3*d^3*(c + d*x)) + (2*b^3*\operatorname{CoshIntegral}[(2*b*c)/d + 2*b*x]*\operatorname{Sinh}[2*a - (2*b*c)/d])/(3*d^4) - (b*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/(3*d^2*(c + d*x)^2) + (2*b^3*\operatorname{Cosh}[2*a - (2*b*c)/d]*\operatorname{SinhIntegral}[(2*b*c)/d + 2*b*x])/(3*d^4)$

**Rule 12**

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match}Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

**Rule 32**

$\operatorname{Int}[(a_.) + (b_.)*(x_)]^{(m_)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m + 1)}/(b*(m + 1)), x] /; \operatorname{FreeQ}\{a, b, m, x\} \ \&\& \ \operatorname{NeQ}[m, -1]$

**Rule 3298**

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[(I*\operatorname{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz, x\} \ \&\& \ \operatorname{EqQ}[d*e - c*f*fz*I, 0]$

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
LtQ[m, -1]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol]
:> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(a+bx)}{(c+dx)^4} dx &= -\frac{\cosh^2(a+bx)}{3d(c+dx)^3} - \frac{b \cosh(a+bx) \sinh(a+bx)}{3d^2(c+dx)^2} - \frac{b^2 \int \frac{1}{(c+dx)^2} dx}{3d^2} + \frac{(2b^2) \int \frac{\cosh^2(a+bx)}{(c+dx)^2} dx}{3d^2} \\
&= \frac{b^2}{3d^3(c+dx)} - \frac{\cosh^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2 \cosh^2(a+bx)}{3d^3(c+dx)} - \frac{b \cosh(a+bx) \sinh(a+bx)}{3d^2(c+dx)^2} + \frac{(4ib^3)}{3d^3(c+dx)} \\
&= \frac{b^2}{3d^3(c+dx)} - \frac{\cosh^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2 \cosh^2(a+bx)}{3d^3(c+dx)} - \frac{b \cosh(a+bx) \sinh(a+bx)}{3d^2(c+dx)^2} + \frac{(2b^3)}{3d^3(c+dx)} \\
&= \frac{b^2}{3d^3(c+dx)} - \frac{\cosh^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2 \cosh^2(a+bx)}{3d^3(c+dx)} - \frac{b \cosh(a+bx) \sinh(a+bx)}{3d^2(c+dx)^2} + \frac{(2b^3)}{3d^3(c+dx)} \\
&= \frac{b^2}{3d^3(c+dx)} - \frac{\cosh^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2 \cosh^2(a+bx)}{3d^3(c+dx)} + \frac{2b^3 \text{Chi}\left(\frac{2bc}{d} + 2bx\right) \sinh\left(2a - \frac{2bc}{d}\right)}{3d^4}
\end{aligned}$$

**Mathematica [A]** time = 0.86, size = 121, normalized size = 0.75

$$\frac{4b^3 \sinh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2b(c+dx)}{d}\right) + 4b^3 \cosh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2b(c+dx)}{d}\right) - \frac{d(\cosh(2(a+bx))(2b^2(c+dx)^2+d^2)+d(b(c+dx) \sinh(2(a+bx))))}{(c+dx)^3}}{6d^4}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]^2/(c + d\*x)^4,x]

[Out] (4\*b^3\*CoshIntegral[(2\*b\*(c + d\*x))/d]\*Sinh[2\*a - (2\*b\*c)/d] - (d\*((d^2 + 2\*b^2\*(c + d\*x)^2)\*Cosh[2\*(a + b\*x)] + d\*(d + b\*(c + d\*x)\*Sinh[2\*(a + b\*x)])))/(c + d\*x)^3 + 4\*b^3\*Cosh[2\*a - (2\*b\*c)/d]\*SinhIntegral[(2\*b\*(c + d\*x))/d])/ (6\*d^4)

**fricas [B]** time = 0.84, size = 409, normalized size = 2.52

$$\frac{d^3 + (2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d + d^3) \cosh(bx+a)^2 + 2(bd^3x + bcd^2) \cosh(bx+a) \sinh(bx+a) + (2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d + d^3) \cosh(bx+a) \sinh(bx+a)}{6d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^4,x, algorithm="fricas")

[Out] -1/6\*(d^3 + (2\*b^2\*d^3\*x^2 + 4\*b^2\*c\*d^2\*x + 2\*b^2\*c^2\*d + d^3)\*cosh(b\*x + a)^2 + 2\*(b\*d^3\*x + b\*c\*d^2)\*cosh(b\*x + a)\*sinh(b\*x + a) + (2\*b^2\*d^3\*x^2 + 4\*b^2\*c\*d^2\*x + 2\*b^2\*c^2\*d + d^3)\*cosh(b\*x + a)\*sinh(b\*x + a))/6\*d^4

$$4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*\sinh(b*x + a)^2 - 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\text{Ei}(2*(b*d*x + b*c)/d) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\text{Ei}(-2*(b*d*x + b*c)/d))*\cosh(-2*(b*c - a*d)/d) - 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\text{Ei}(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\text{Ei}(-2*(b*d*x + b*c)/d))*\sinh(-2*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)$$

**giac [B]** time = 0.13, size = 537, normalized size = 3.31

$$4 b^3 d^3 x^3 \text{Ei}\left(\frac{2(bdx+bc)}{d}\right) e^{\left(2a-\frac{2bc}{d}\right)} - 4 b^3 d^3 x^3 \text{Ei}\left(-\frac{2(bdx+bc)}{d}\right) e^{\left(-2a+\frac{2bc}{d}\right)} + 12 b^3 c d^2 x^2 \text{Ei}\left(\frac{2(bdx+bc)}{d}\right) e^{\left(2a-\frac{2bc}{d}\right)} - 12 b^3 c d^2 x^2 \text{Ei}\left(-\frac{2(bdx+bc)}{d}\right) e^{\left(-2a+\frac{2bc}{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^4,x, algorithm="giac")

[Out]  $\frac{1}{12}*(4*b^3*d^3*x^3*\text{Ei}(2*(b*d*x + b*c)/d)*e^{(2*a - 2*b*c/d)} - 4*b^3*d^3*x^3*\text{Ei}(-2*(b*d*x + b*c)/d)*e^{(-2*a + 2*b*c/d)} + 12*b^3*c*d^2*x^2*\text{Ei}(2*(b*d*x + b*c)/d)*e^{(2*a - 2*b*c/d)} - 12*b^3*c*d^2*x^2*\text{Ei}(-2*(b*d*x + b*c)/d)*e^{(-2*a + 2*b*c/d)} + 12*b^3*c^2*d*x*\text{Ei}(2*(b*d*x + b*c)/d)*e^{(2*a - 2*b*c/d)} - 12*b^3*c^2*d*x*\text{Ei}(-2*(b*d*x + b*c)/d)*e^{(-2*a + 2*b*c/d)} - 2*b^2*d^3*x^2*e^{(2*b*x + 2*a)} - 2*b^2*d^3*x^2*e^{(-2*b*x - 2*a)} + 4*b^3*c^3*\text{Ei}(2*(b*d*x + b*c)/d)*e^{(2*a - 2*b*c/d)} - 4*b^3*c^3*\text{Ei}(-2*(b*d*x + b*c)/d)*e^{(-2*a + 2*b*c/d)} - 4*b^2*c*d^2*x*e^{(2*b*x + 2*a)} - 4*b^2*c*d^2*x*e^{(-2*b*x - 2*a)} - 2*b^2*c^2*d*e^{(2*b*x + 2*a)} - b*d^3*x*e^{(2*b*x + 2*a)} - 2*b^2*c^2*d*e^{(-2*b*x - 2*a)} + b*d^3*x*e^{(-2*b*x - 2*a)} - b*c*d^2*e^{(2*b*x + 2*a)} + b*c*d^2*e^{(-2*b*x - 2*a)} - d^3*e^{(2*b*x + 2*a)} - d^3*e^{(-2*b*x - 2*a)} - 2*d^3)/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)$

**maple [B]** time = 0.25, size = 555, normalized size = 3.43

$$\frac{1}{6d(dx+c)^3} - \frac{b^5 e^{-2bx-2a} x^2}{6d(b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 dx + c^3 b^3)} - \frac{b^5 e^{-2bx-2a} cx}{3d^2(b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 dx + c^3 b^3)} - \frac{1}{6d^3(b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 dx + c^3 b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)^2/(d\*x+c)^4,x)

[Out]  $-1/6/d/(d*x+c)^3 - 1/6*b^5*\exp(-2*b*x-2*a)/d/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*x^2 - 1/3*b^5*\exp(-2*b*x-2*a)/d^2/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c*x - 1/6*b^5*\exp(-2*b*x-2*a)/d^3/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c^2 + 1/12*b^4*\exp(-2*b*x-2*a)/d/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)$



$$\frac{1}{6(d^4x^3 + 3cd^3x^2 + 3c^2d^2x + c^3d)} - \frac{e^{\left(-2a + \frac{2bc}{d}\right)} E_4\left(\frac{2(dx+c)b}{d}\right)}{4(dx+c)^3d} - \frac{e^{\left(2a - \frac{2bc}{d}\right)} E_4\left(-\frac{2(dx+c)b}{d}\right)}{4(dx+c)^3d}$$

**maxima** [A] time = 0.47, size = 110, normalized size = 0.68

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^4,x, algorithm="maxima")

[Out]  $-1/6/(d^4x^3 + 3cd^3x^2 + 3c^2d^2x + c^3d) - 1/4e^{(-2a + 2bc/d)} \exp\_integral\_e(4, 2*(d*x + c)*b/d)/((d*x + c)^3*d) - 1/4e^{(2a - 2bc/d)} \exp\_integral\_e(4, -2*(d*x + c)*b/d)/((d*x + c)^3*d)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + bx)^2}{(c + dx)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^2/(c + d\*x)^4,x)

[Out] int(cosh(a + b\*x)^2/(c + d\*x)^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a + bx)}{(c + dx)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*\*2/(d\*x+c)\*\*4,x)

[Out] Integral(cosh(a + b\*x)\*\*2/(c + d\*x)\*\*4, x)

### 3.16 $\int (c + dx)^4 \cosh^3(a + bx) dx$

**Optimal.** Leaf size=225

$$\frac{8d^4 \sinh^3(a + bx)}{81b^5} + \frac{488d^4 \sinh(a + bx)}{27b^5} - \frac{8d^3(c + dx) \cosh^3(a + bx)}{27b^4} - \frac{160d^3(c + dx) \cosh(a + bx)}{9b^4} + \frac{80d^2(c + dx)^2}{9b^3}$$

[Out]  $-160/9*d^3*(d*x+c)*\cosh(b*x+a)/b^4-8/3*d*(d*x+c)^3*\cosh(b*x+a)/b^2-8/27*d^3*(d*x+c)*\cosh(b*x+a)^3/b^4-4/9*d*(d*x+c)^3*\cosh(b*x+a)^3/b^2+488/27*d^4*\sinh(b*x+a)/b^5+80/9*d^2*(d*x+c)^2*\sinh(b*x+a)/b^3+2/3*(d*x+c)^4*\sinh(b*x+a)/b+4/9*d^2*(d*x+c)^2*\cosh(b*x+a)^2*\sinh(b*x+a)/b^3+1/3*(d*x+c)^4*\cosh(b*x+a)^2*\sinh(b*x+a)/b+8/81*d^4*\sinh(b*x+a)^3/b^5$

**Rubi [A]** time = 0.28, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3311, 3296, 2637, 2633}

$$\frac{80d^2(c + dx)^2 \sinh(a + bx)}{9b^3} - \frac{8d^3(c + dx) \cosh^3(a + bx)}{27b^4} - \frac{160d^3(c + dx) \cosh(a + bx)}{9b^4} + \frac{4d^2(c + dx)^2 \sinh(a + bx)}{9b^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^4\*Cosh[a + b\*x]^3,x]

[Out]  $(-160*d^3*(c + d*x)*\text{Cosh}[a + b*x])/(9*b^4) - (8*d*(c + d*x)^3*\text{Cosh}[a + b*x])/(3*b^2) - (8*d^3*(c + d*x)*\text{Cosh}[a + b*x]^3)/(27*b^4) - (4*d*(c + d*x)^3*\text{Cosh}[a + b*x]^3)/(9*b^2) + (488*d^4*\text{Sinh}[a + b*x])/(27*b^5) + (80*d^2*(c + d*x)^2*\text{Sinh}[a + b*x])/(9*b^3) + (2*(c + d*x)^4*\text{Sinh}[a + b*x])/(3*b) + (4*d^2*(c + d*x)^2*\text{Cosh}[a + b*x]^2*\text{Sinh}[a + b*x])/(9*b^3) + ((c + d*x)^4*\text{Cosh}[a + b*x]^2*\text{Sinh}[a + b*x])/(3*b) + (8*d^4*\text{Sinh}[a + b*x]^3)/(81*b^5)$

#### Rule 2633

Int[sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

#### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[

$e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 3311

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*((b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(d*m*(c + d*x)^{(m-1)}*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n-1))/n, \text{Int}[(c + d*x)^m*(b*\sin[e + f*x])^{(n-2)}, x], x] - \text{Dist}[(d^2*m*(m-1))/(f^2*n^2), \text{Int}[(c + d*x)^{(m-2)}*(b*\sin[e + f*x])^n, x], x] - \text{Simp}[(b*(c + d*x)^m*\cos[e + f*x]*(b*\sin[e + f*x])^{(n-1)}]/(f*n), x]) /;$   
 $\text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{GtQ}[m, 1]$

### Rubi steps

$$\begin{aligned} \int (c + dx)^4 \cosh^3(a + bx) dx &= -\frac{4d(c + dx)^3 \cosh^3(a + bx)}{9b^2} + \frac{(c + dx)^4 \cosh^2(a + bx) \sinh(a + bx)}{3b} + \frac{2}{3} \int (c + dx)^3 \cosh^2(a + bx) \sinh(a + bx) dx \\ &= -\frac{8d^3(c + dx) \cosh^3(a + bx)}{27b^4} - \frac{4d(c + dx)^3 \cosh^3(a + bx)}{9b^2} + \frac{2(c + dx)^4 \sinh(a + bx)}{3b} \\ &= -\frac{8d(c + dx)^3 \cosh(a + bx)}{3b^2} - \frac{8d^3(c + dx) \cosh^3(a + bx)}{27b^4} - \frac{4d(c + dx)^3 \cosh^3(a + bx)}{9b^2} \\ &= -\frac{16d^3(c + dx) \cosh(a + bx)}{9b^4} - \frac{8d(c + dx)^3 \cosh(a + bx)}{3b^2} - \frac{8d^3(c + dx) \cosh^3(a + bx)}{27b^4} \\ &= -\frac{160d^3(c + dx) \cosh(a + bx)}{9b^4} - \frac{8d(c + dx)^3 \cosh(a + bx)}{3b^2} - \frac{8d^3(c + dx) \cosh^3(a + bx)}{27b^4} \\ &= -\frac{160d^3(c + dx) \cosh(a + bx)}{9b^4} - \frac{8d(c + dx)^3 \cosh(a + bx)}{3b^2} - \frac{8d^3(c + dx) \cosh^3(a + bx)}{27b^4} \end{aligned}$$

**Mathematica [A]** time = 0.97, size = 385, normalized size = 1.71

$$\frac{243b^4c^4 \sinh(a + bx) + 27b^4c^4 \sinh(3(a + bx)) + 972b^4c^3dx \sinh(a + bx) + 108b^4c^3dx \sinh(3(a + bx)) + 1458b^4c^3dx \sinh(a + bx) + 1458b^4c^3dx \sinh(3(a + bx))}{1}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^4\*Cosh[a + b\*x]^3,x]

[Out]  $(-972*b*d*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*\text{Cosh}[a + b*x] - 12*b*d*(c + d*x)*(2*d^2 + 3*b^2*(c + d*x)^2)*\text{Cosh}[3*(a + b*x)] + 243*b^4*c^4*\text{Sinh}[a + b*x] + 2916*b^2*c^2*d^2*\text{Sinh}[a + b*x] + 5832*d^4*\text{Sinh}[a + b*x] + 972*b^4*c^3*d*x*\text{Sinh}[a + b*x] + 5832*b^2*c*d^3*x*\text{Sinh}[a + b*x] + 1458*b^4*c^2*d^2*x^2*\text{Sinh}[a + b*x] + 2916*b^2*d^4*x^2*\text{Sinh}[a + b*x] + 972*b^4*c*d^3*x^3*\text{Sinh}[a + b*x])$

$$\frac{b^4 x^4 + 243 b^4 d^4 x^4 \operatorname{Sinh}[a + b x] + 27 b^4 c^4 \operatorname{Sinh}[3(a + b x)] + 36 b^2 c^2 d^2 \operatorname{Sinh}[3(a + b x)] + 8 d^4 \operatorname{Sinh}[3(a + b x)] + 108 b^4 c^3 d x \operatorname{Sinh}[3(a + b x)] + 72 b^2 c d^3 x \operatorname{Sinh}[3(a + b x)] + 162 b^4 c^2 d^2 x^2 \operatorname{Sinh}[3(a + b x)] + 36 b^2 d^4 x^2 \operatorname{Sinh}[3(a + b x)] + 108 b^4 c d^3 x^3 \operatorname{Sinh}[3(a + b x)] + 27 b^4 d^4 x^4 \operatorname{Sinh}[3(a + b x)]}{(324 b^5)}$$

**fricas [B]** time = 0.52, size = 528, normalized size = 2.35

$$12(3b^3d^4x^3 + 9b^3cd^3x^2 + 3b^3c^3d + 2bcd^3 + (9b^3c^2d^2 + 2bd^4)x) \cosh(bx + a)^3 + 36(3b^3d^4x^3 + 9b^3cd^3x^2 + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^4\*cosh(b\*x+a)^3,x, algorithm="fricas")

[Out] 
$$\frac{-1/324*(12*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d + 2*b*c*d^3 + (9*b^3*c^2*d^2 + 2*b*d^4)*x)*\cosh(b*x + a)^3 + 36*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d + 2*b*c*d^3 + (9*b^3*c^2*d^2 + 2*b*d^4)*x)*\cosh(b*x + a)*\sinh(b*x + a)^2 - (27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 + 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d + 2*b^2*c*d^3)*x)*\sinh(b*x + a)^3 + 972*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d + 6*b*c*d^3 + 3*(b^3*c^2*d^2 + 2*b*d^4)*x)*\cosh(b*x + a) - 3*(81*b^4*d^4*x^4 + 324*b^4*c*d^3*x^3 + 81*b^4*c^4 + 972*b^2*c^2*d^2 + 1944*d^4 + 486*(b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + (27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 + 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d + 2*b^2*c*d^3)*x)*\cosh(b*x + a)^2 + 324*(b^4*c^3*d + 6*b^2*c*d^3)*x)*\sinh(b*x + a))/b^5$$

**giac [B]** time = 0.13, size = 654, normalized size = 2.91

$$\frac{(27b^4d^4x^4 + 108b^4cd^3x^3 + 162b^4c^2d^2x^2 - 36b^3d^4x^3 + 108b^4c^3dx - 108b^3cd^3x^2 + 27b^4c^4 - 108b^3c^2d^2x + 36b^2c^2d^2x^2 + \dots)}{648b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^4\*cosh(b\*x+a)^3,x, algorithm="giac")

[Out] 
$$\frac{1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 - 36*b^3*d^4*x^3 + 108*b^4*c^3*d*x - 108*b^3*c*d^3*x^2 + 27*b^4*c^4 - 108*b^3*c^2*d^2*x + 36*b^2*d^4*x^2 - 36*b^3*c^3*d + 72*b^2*c*d^3*x + 36*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*d^3 + 8*d^4)*e^{(3*b*x + 3*a)}/b^5 + 3/8*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x - 12*b^3*c*d^3*x^2 + b^4*c^4 - 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^2 - 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*d^3 + 24*d^4)*e^{(b*x + a)}/b^5 - 3/8*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4 + 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^2 + \dots)}$$

$$4x^2 + 4b^3c^3d + 24b^2cd^3x + 12b^2c^2d^2 + 24bd^4x + 24b^3cd^3 + 24d^4)e^{(-bx - a)}/b^5 - 1/648(27b^4d^4x^4 + 108b^4cd^3x^3 + 162b^4c^2d^2x^2 + 36b^3d^4x^3 + 108b^4c^3dx + 108b^3cd^3x^2 + 27b^4c^4 + 108b^3c^2d^2x + 36b^2d^4x^2 + 36b^3c^3d + 72b^2cd^3x + 36b^2c^2d^2 + 24bd^4x + 24b^3cd^3 + 8d^4)e^{(-3bx - 3a)}/b^5$$

**maple [B]** time = 0.17, size = 1139, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^4*cosh(b*x+a)^3,x)`

[Out] 
$$\frac{1}{b} \left( \frac{1}{b^4 d^4} \left( \frac{2}{3} (bx+a)^4 \sinh(bx+a) + \frac{1}{3} (bx+a)^4 \sinh(bx+a) \cosh(bx+a) \right) - \frac{8}{3} (bx+a)^3 \cosh(bx+a) + \frac{80}{9} (bx+a)^2 \sinh(bx+a) - \frac{160}{9} (bx+a) \cosh(bx+a) + \frac{1456}{81} \sinh(bx+a) - \frac{4}{9} (bx+a)^3 \cosh(bx+a)^3 + \frac{4}{9} (bx+a)^2 \sinh(bx+a) \cosh(bx+a)^2 - \frac{8}{27} (bx+a) \cosh(bx+a)^3 + \frac{8}{81} \cosh(bx+a)^2 \sinh(bx+a) \right) - \frac{4}{b^4 d^4} \left( \frac{2}{3} (bx+a)^3 \sinh(bx+a) + \frac{1}{3} (bx+a)^3 \sinh(bx+a) \cosh(bx+a) \right) - \frac{2}{9} (bx+a)^2 \cosh(bx+a) + \frac{40}{9} (bx+a) \sinh(bx+a) - \frac{40}{9} \cosh(bx+a) - \frac{1}{3} (bx+a)^2 \cosh(bx+a)^3 + \frac{2}{9} (bx+a) \sinh(bx+a) \cosh(bx+a)^2 - \frac{2}{27} \cosh(bx+a)^3 \right) + \frac{6}{b^4 d^4} \left( \frac{2}{3} (bx+a)^2 \sinh(bx+a) + \frac{1}{3} (bx+a)^2 \sinh(bx+a) \cosh(bx+a) \right) - \frac{4}{3} (bx+a) \cosh(bx+a) + \frac{40}{27} \sinh(bx+a) - \frac{2}{9} (bx+a) \cosh(bx+a)^3 + \frac{2}{27} \cosh(bx+a)^2 \sinh(bx+a) \right) - \frac{4}{b^4 d^4} \left( \frac{2}{3} (bx+a) \sinh(bx+a) + \frac{1}{3} (bx+a) \sinh(bx+a) \cosh(bx+a) \right) - \frac{2}{3} \cosh(bx+a) - \frac{1}{9} \cosh(bx+a)^3 \right) + \frac{1}{b^4 d^4} \left( \frac{2}{3} + \frac{1}{3} \cosh(bx+a)^2 \right) \sinh(bx+a) + \frac{4}{b^3 c d^3} \left( \frac{2}{3} (bx+a)^3 \sinh(bx+a) + \frac{1}{3} (bx+a)^3 \sinh(bx+a) \cosh(bx+a) \right) - \frac{2}{9} (bx+a)^2 \cosh(bx+a) + \frac{40}{9} (bx+a) \sinh(bx+a) - \frac{40}{9} \cosh(bx+a) - \frac{1}{3} (bx+a)^2 \cosh(bx+a)^3 + \frac{2}{9} (bx+a) \sinh(bx+a) \cosh(bx+a)^2 - \frac{2}{27} \cosh(bx+a)^3 \right) - \frac{12}{b^3 c d^3} \left( \frac{2}{3} (bx+a)^2 \sinh(bx+a) + \frac{1}{3} (bx+a)^2 \sinh(bx+a) \cosh(bx+a) \right) - \frac{4}{3} (bx+a) \cosh(bx+a) + \frac{40}{27} \sinh(bx+a) - \frac{2}{9} (bx+a) \cosh(bx+a)^3 + \frac{2}{27} \cosh(bx+a)^2 \sinh(bx+a) \right) + \frac{12}{b^3 c d^3} \left( \frac{2}{3} (bx+a) \sinh(bx+a) + \frac{1}{3} (bx+a) \sinh(bx+a) \cosh(bx+a) \right) - \frac{1}{9} \cosh(bx+a)^3 - \frac{4}{b^3 c d^3} \left( \frac{2}{3} + \frac{1}{3} \cosh(bx+a)^2 \right) \sinh(bx+a) + \frac{6}{b^2 c^2 d^2} \left( \frac{2}{3} (bx+a)^2 \sinh(bx+a) + \frac{1}{3} (bx+a)^2 \sinh(bx+a) \cosh(bx+a) \right) - \frac{4}{3} (bx+a) \cosh(bx+a) + \frac{40}{27} \sinh(bx+a) - \frac{2}{9} (bx+a) \cosh(bx+a)^3 + \frac{2}{27} \cosh(bx+a)^2 \sinh(bx+a) \right) - \frac{12}{b^2 c^2 d^2} \left( \frac{2}{3} (bx+a) \sinh(bx+a) + \frac{1}{3} (bx+a) \sinh(bx+a) \cosh(bx+a) \right) - \frac{2}{3} \cosh(bx+a) - \frac{1}{9} \cosh(bx+a)^3 \right) + \frac{6}{b^2 c^2 d^2} \left( \frac{2}{3} + \frac{1}{3} \cosh(bx+a)^2 \right) \sinh(bx+a) + \frac{4}{b c^3 d} \left( \frac{2}{3} (bx+a) \sinh(bx+a) + \frac{1}{3} (bx+a) \sinh(bx+a) \cosh(bx+a) \right) - \frac{2}{3} \cosh(bx+a) - \frac{1}{9} \cosh(bx+a)^3 \right) - \frac{4}{b c^3 d} \left( \frac{2}{3} + \frac{1}{3} \cosh(bx+a)^2 \right) \sinh(bx+a) + \frac{c^4}{b^2} \left( \frac{2}{3} + \frac{1}{3} \cosh(bx+a)^2 \right) \sinh(bx+a) \right)$$

**maxima [B]** time = 0.40, size = 644, normalized size = 2.86

$$\frac{1}{18} c^3 d \left( \frac{(3bx e^{(3a)} - e^{(3a)}) e^{(3bx)}}{b^2} + \frac{27(bx e^a - e^a) e^{(bx)}}{b^2} - \frac{27(bx+1) e^{(-bx-a)}}{b^2} - \frac{(3bx+1) e^{(-3bx-3a)}}{b^2} \right) + \frac{1}{24} c^4 \left( \frac{e^{(3bx+3a)}}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^4\*cosh(b\*x+a)^3,x, algorithm="maxima")

[Out]  $\frac{1}{18}c^3d^4\left(\frac{3bx e^{3a} - e^{3a}}{b^2} + 27\frac{bx e^a - e^a}{b^2} + 27\frac{bx + 1}{b^2}e^{-bx - a} - \frac{3bx + 1}{b^2}e^{-3bx - 3a}\right) + \frac{1}{24}c^4\left(\frac{e^{3bx + 3a}}{b} + 9\frac{e^{bx + a}}{b} - 9\frac{e^{-bx - a}}{b} - e^{-3bx - 3a}\right) + \frac{1}{36}c^2d^2\left(\frac{9b^2x^2e^{3a} - 6bx e^{3a} + 2e^{3a}}{b^3} + 81\frac{b^2x^2e^a - 2bx e^a + 2e^a}{b^3} + 81\frac{(b^2x^2 + 2bx + 2)e^{-bx - a}}{b^3} - \frac{9b^2x^2 + 6bx + 2}{b^3}e^{-3bx - 3a}\right) + \frac{1}{54}cd^3\left(\frac{9b^3x^3e^{3a} - 9b^2x^2e^{3a} + 6bx e^{3a} - 2e^{3a}}{b^4} + 81\frac{b^3x^3e^a - 3b^2x^2e^a + 6bx e^a - 6e^a}{b^4} + 81\frac{b^3x^3 + 3b^2x^2 + 6bx + 6}{b^4}e^{-bx - a}\right) - \frac{9b^3x^3 + 9b^2x^2 + 6bx + 2}{b^4}e^{-3bx - 3a} + \frac{1}{648}d^4\left(\frac{27b^4x^4e^{3a} - 36b^3x^3e^{3a} + 36b^2x^2e^{3a} - 24bx e^{3a} + 8e^{3a}}{b^5} + 243\frac{b^4x^4e^a - 4b^3x^3e^a + 12b^2x^2e^a - 24bx e^a + 24e^a}{b^5} + 243\frac{b^4x^4 + 4b^3x^3 + 12b^2x^2 + 24bx + 24}{b^5}e^{-bx - a} - \frac{27b^4x^4 + 36b^3x^3 + 36b^2x^2 + 24bx + 8}{b^5}e^{-3bx - 3a}\right)$

**mupad [B]** time = 1.35, size = 532, normalized size = 2.36

$$\frac{\cosh(a + bx)^2 \sinh(a + bx) \left(27b^4 c^4 + 252b^2 c^2 d^2 + 488d^4\right)}{27b^5} \frac{2 \sinh(a + bx)^3 \left(27b^4 c^4 + 360b^2 c^2 d^2 + 728d^4\right)}{81b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^3\*(c + d\*x)^4,x)

[Out]  $\frac{(\cosh(a + bx)^2 \sinh(a + bx) (488d^4 + 27b^4c^4 + 252b^2c^2d^2))}{(27b^5) - (2\sinh(a + bx)^3(728d^4 + 27b^4c^4 + 360b^2c^2d^2))}{(81b^5) - (4\cosh(a + bx)^3(122cd^3 + 21b^2c^3d))}{(27b^4) + (8\cosh(a + bx)\sinh(a + bx)^2(20cd^3 + 3b^2c^3d))}{(9b^4) - (28d^4x^3\cosh(a + bx)^3)}{-(9b^2) - (4xx\cosh(a + bx)^3(122d^4 + 63b^2c^2d^2))}{(27b^4) - (2d^4x^4\sinh(a + bx)^3)}{-(3b) - (8xx\sinh(a + bx)^3(20cd^3 + 3b^2c^3d))}{(9b^3) - (4x^2\sinh(a + bx)^3(20d^4 + 9b^2c^2d^2))}{(9b^3) + (2x^2\cosh(a + bx)^2\sinh(a + bx)(14d^4 + 9b^2c^2d^2))}{(3b^3) - (28cd^3x^2\cosh(a + bx)^3)}{-(3b^2) + (d^4x^4\cosh(a + bx)^2\sinh(a + bx))}{b + (8d^4x^3\cosh(a + bx)\sinh(a + bx)^2)}{-(3b^2) - (8cd^3x^3\sinh(a + bx)^3)}{-(3b) + (8xx\cosh(a + bx)\sinh(a + bx)^2(20d^4 + 9b^2c^2d^2))}{(9b^4) + (4xx\cosh(a + bx)^2\sinh(a + bx)(14cd^3 + 3b^2c^3d))}{(3b^3) + (4cd^3x^3\cosh(a + bx)^2\sinh(a + bx))}{b + (8cd^3x^2\cosh(a + bx)\sinh(a + bx)^2)}{b^2}$

sympy [A] time = 7.95, size = 772, normalized size = 3.43

$$\left\{ \begin{array}{l} -\frac{2c^4 \sinh^3(ax+bx)}{3b} + \frac{c^4 \sinh(ax+bx) \cosh^2(ax+bx)}{b} - \frac{8c^3 dx \sinh^3(ax+bx)}{3b} + \frac{4c^3 dx \sinh(ax+bx) \cosh^2(ax+bx)}{b} - \frac{4c^2 d^2 x^2 \sinh^3(ax+bx)}{b} + \frac{6c^2 d^2 x^2}{b} \\ \left( c^4 x + 2c^3 dx^2 + 2c^2 d^2 x^3 + cd^3 x^4 + \frac{d^4 x^5}{5} \right) \cosh^3(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*4\*cosh(b\*x+a)\*\*3,x)

[Out] Piecewise((-2\*c\*\*4\*sinh(a + b\*x)\*\*3/(3\*b) + c\*\*4\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/b - 8\*c\*\*3\*d\*x\*sinh(a + b\*x)\*\*3/(3\*b) + 4\*c\*\*3\*d\*x\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/b - 4\*c\*\*2\*d\*\*2\*x\*\*2\*sinh(a + b\*x)\*\*3/b + 6\*c\*\*2\*d\*\*2\*x\*\*2\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/b - 8\*c\*d\*\*3\*x\*\*3\*sinh(a + b\*x)\*\*3/(3\*b) + 4\*c\*d\*\*3\*x\*\*3\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/b - 2\*d\*\*4\*x\*\*4\*sinh(a + b\*x)\*\*3/(3\*b) + d\*\*4\*x\*\*4\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/b + 8\*c\*\*3\*d\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/(3\*b\*\*2) - 28\*c\*\*3\*d\*cosh(a + b\*x)\*\*3/(9\*b\*\*2) + 8\*c\*\*2\*d\*\*2\*x\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b\*\*2 - 28\*c\*\*2\*d\*\*2\*x\*cosh(a + b\*x)\*\*3/(3\*b\*\*2) + 8\*c\*d\*\*3\*x\*\*2\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b\*\*2 - 28\*c\*d\*\*3\*x\*\*2\*cosh(a + b\*x)\*\*3/(3\*b\*\*2) + 8\*d\*\*4\*x\*\*3\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/(3\*b\*\*2) - 28\*d\*\*4\*x\*\*3\*cosh(a + b\*x)\*\*3/(9\*b\*\*2) - 80\*c\*\*2\*d\*\*2\*sinh(a + b\*x)\*\*3/(9\*b\*\*3) + 28\*c\*\*2\*d\*\*2\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/(3\*b\*\*3) - 160\*c\*d\*\*3\*x\*sinh(a + b\*x)\*\*3/(9\*b\*\*3) + 56\*c\*d\*\*3\*x\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/(3\*b\*\*3) - 80\*d\*\*4\*x\*\*2\*sinh(a + b\*x)\*\*3/(9\*b\*\*3) + 28\*d\*\*4\*x\*\*2\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/(3\*b\*\*3) + 160\*c\*d\*\*3\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/(9\*b\*\*4) - 488\*c\*d\*\*3\*cosh(a + b\*x)\*\*3/(27\*b\*\*4) + 160\*d\*\*4\*x\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/(9\*b\*\*4) - 488\*d\*\*4\*x\*cosh(a + b\*x)\*\*3/(27\*b\*\*4) - 1456\*d\*\*4\*sinh(a + b\*x)\*\*3/(81\*b\*\*5) + 488\*d\*\*4\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/(27\*b\*\*5), Ne(b, 0)), ((c\*\*4\*x + 2\*c\*\*3\*d\*x\*\*2 + 2\*c\*\*2\*d\*\*2\*x\*\*3 + c\*d\*\*3\*x\*\*4 + d\*\*4\*x\*\*5/5)\*cosh(a)\*\*3, True))

### 3.17 $\int (c + dx)^3 \cosh^3(a + bx) dx$

**Optimal.** Leaf size=175

$$\frac{2d^3 \cosh^3(a + bx)}{27b^4} - \frac{40d^3 \cosh(a + bx)}{9b^4} + \frac{40d^2(c + dx) \sinh(a + bx)}{9b^3} + \frac{2d^2(c + dx) \sinh(a + bx) \cosh^2(a + bx)}{9b^3} - \frac{d(c + dx)^2 \cosh^3(a + bx)}{3b^2} - \frac{2d(c + dx)^2 \cosh(a + bx)}{b^2}$$

[Out]  $-40/9*d^3*\cosh(b*x+a)/b^4-2*d*(d*x+c)^2*\cosh(b*x+a)/b^2-2/27*d^3*\cosh(b*x+a)^3/b^4-1/3*d*(d*x+c)^2*\cosh(b*x+a)^3/b^2+40/9*d^2*(d*x+c)*\sinh(b*x+a)/b^3+2/3*(d*x+c)^3*\sinh(b*x+a)/b+2/9*d^2*(d*x+c)*\cosh(b*x+a)^2*\sinh(b*x+a)/b^3+1/3*(d*x+c)^3*\cosh(b*x+a)^2*\sinh(b*x+a)/b$

**Rubi [A]** time = 0.18, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3311, 3296, 2638, 3310}

$$\frac{40d^2(c + dx) \sinh(a + bx)}{9b^3} + \frac{2d^2(c + dx) \sinh(a + bx) \cosh^2(a + bx)}{9b^3} - \frac{d(c + dx)^2 \cosh^3(a + bx)}{3b^2} - \frac{2d(c + dx)^2 \cosh(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^3\*Cosh[a + b\*x]^3,x]

[Out]  $(-40*d^3*\cosh[a + b*x])/(9*b^4) - (2*d*(c + d*x)^2*\cosh[a + b*x])/b^2 - (2*d^3*\cosh[a + b*x]^3)/(27*b^4) - (d*(c + d*x)^2*\cosh[a + b*x]^3)/(3*b^2) + (40*d^2*(c + d*x)*\sinh[a + b*x])/(9*b^3) + (2*(c + d*x)^3*\sinh[a + b*x])/(3*b) + (2*d^2*(c + d*x)*\cosh[a + b*x]^2*\sinh[a + b*x])/(9*b^3) + ((c + d*x)^3*\cosh[a + b*x]^2*\sinh[a + b*x])/(3*b)$

#### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[((c + d\*x)^m\*cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m-1)\*cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*(b\*sin[e + f\*x])^n)/(f^n), x] + (Dist[(b^2\*(n-1))/n, Int[(c + d\*x)\*(b\*sin[e + f\*x])^(n-2), x], x] - Simp[(b\*(c + d\*x)\*cos[e + f\*x]\*(b\*sin[e + f\*x])^(n-1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]



]

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^(m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist
[d^2*m*(m - 1)/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \cosh^3(a + bx) dx &= -\frac{d(c + dx)^2 \cosh^3(a + bx)}{3b^2} + \frac{(c + dx)^3 \cosh^2(a + bx) \sinh(a + bx)}{3b} + \frac{2}{3} \int (c + dx)^2 \cosh^3(a + bx) dx \\
&= -\frac{2d^3 \cosh^3(a + bx)}{27b^4} - \frac{d(c + dx)^2 \cosh^3(a + bx)}{3b^2} + \frac{2(c + dx)^3 \sinh(a + bx)}{3b} + \frac{2}{3} \int (c + dx)^2 \cosh^3(a + bx) dx \\
&= -\frac{2d(c + dx)^2 \cosh(a + bx)}{b^2} - \frac{2d^3 \cosh^3(a + bx)}{27b^4} - \frac{d(c + dx)^2 \cosh^3(a + bx)}{3b^2} + \frac{2}{3} \int (c + dx)^2 \cosh^3(a + bx) dx \\
&= -\frac{4d^3 \cosh(a + bx)}{9b^4} - \frac{2d(c + dx)^2 \cosh(a + bx)}{b^2} - \frac{2d^3 \cosh^3(a + bx)}{27b^4} - \frac{d(c + dx)^2 \cosh^3(a + bx)}{3b^2} + \frac{2}{3} \int (c + dx)^2 \cosh^3(a + bx) dx \\
&= -\frac{40d^3 \cosh(a + bx)}{9b^4} - \frac{2d(c + dx)^2 \cosh(a + bx)}{b^2} - \frac{2d^3 \cosh^3(a + bx)}{27b^4} - \frac{d(c + dx)^2 \cosh^3(a + bx)}{3b^2} + \frac{2}{3} \int (c + dx)^2 \cosh^3(a + bx) dx
\end{aligned}$$

**Mathematica [A]** time = 0.96, size = 122, normalized size = 0.70

$$\frac{-486d \cosh(a + bx) (b^2(c + dx)^2 + 2d^2) - 2d \cosh(3(a + bx)) (9b^2(c + dx)^2 + 2d^2) + 12b(c + dx) \sinh(a + bx) (9b^2(c + dx)^2 + 2d^2)}{216b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3\*Cosh[a + b\*x]^3,x]

[Out] (-486\*d\*(2\*d^2 + b^2\*(c + d\*x)^2)\*Cosh[a + b\*x] - 2\*d\*(2\*d^2 + 9\*b^2\*(c + d\*x)^2)\*Cosh[3\*(a + b\*x)] + 12\*b\*(c + d\*x)\*(82\*d^2 + 15\*b^2\*(c + d\*x)^2 + (2\*d^2 + 3\*b^2\*(c + d\*x)^2)\*Cosh[2\*(a + b\*x)])\*Sinh[a + b\*x]/(216\*b^4)

**fricas [B]** time = 0.57, size = 343, normalized size = 1.96

$$\frac{(9b^2d^3x^2 + 18b^2cd^2x + 9b^2c^2d + 2d^3) \cosh(bx + a)^3 + 3(9b^2d^3x^2 + 18b^2cd^2x + 9b^2c^2d + 2d^3) \cosh(bx + a) \sinh(bx + a)^2}{216b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*cosh(b\*x+a)^3,x, algorithm="fricas")

[Out] 
$$-1/108*((9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d + 2*d^3)*\cosh(b*x + a)^3 + 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d + 2*d^3)*\cosh(b*x + a)*\sinh(b*x + a)^2 - 3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 + 2*b*c*d^2 + (9*b^3*c^2*d + 2*b*d^3)*x)*\sinh(b*x + a)^3 + 243*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + 2*d^3)*\cosh(b*x + a) - 9*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 9*b^3*c^3 + 54*b*c*d^2 + (3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 + 2*b*c*d^2 + (9*b^3*c^2*d + 2*b*d^3)*x)*\cosh(b*x + a)^2 + 27*(b^3*c^2*d + 2*b*d^3)*x)*\sinh(b*x + a))/b^4$$

**giac** [B] time = 0.13, size = 414, normalized size = 2.37

$$\frac{(9b^3d^3x^3 + 27b^3cd^2x^2 + 27b^3c^2dx - 9b^2d^3x^2 + 9b^3c^3 - 18b^2cd^2x - 9b^2c^2d + 6bd^3x + 6bcd^2 - 2d^3)e^{(3bx+3a)}}{216b^4} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*cosh(b\*x+a)^3,x, algorithm="giac")

[Out] 
$$1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 27*b^3*c^2*d*x - 9*b^2*d^3*x^2 + 9*b^3*c^3 - 18*b^2*c*d^2*x - 9*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 2*d^3)*e^{(3*b*x + 3*a)}/b^4 + 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x - 3*b^2*d^3*x^2 + b^3*c^3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 6*d^3)*e^{(b*x + a)}/b^4 - 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x - 3*b^2*d^3*x^2 + b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 6*d^3)*e^{(-b*x - a)}/b^4 - 1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 27*b^3*c^2*d*x + 9*b^2*d^3*x^2 + 9*b^3*c^3 + 18*b^2*c*d^2*x + 9*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 2*d^3)*e^{(-3*b*x - 3*a)}/b^4$$

**maple** [B] time = 0.16, size = 634, normalized size = 3.62

$$d^3 \left( \frac{2(bx+a)^3 \sinh(bx+a)}{3} + \frac{(bx+a)^3 \sinh(bx+a) (\cosh^2(bx+a))}{3} - 2(bx+a)^2 \cosh(bx+a) + \frac{40(bx+a) \sinh(bx+a)}{9} - \frac{40 \cosh(bx+a)}{9} - \frac{(bx+a)^2 (\cosh^3(bx+a))}{3} + \frac{2(bx+a) \sinh(bx+a) (\cosh^2(bx+a))}{9} \right) / b^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3\*cosh(b\*x+a)^3,x)

[Out] 
$$1/b*(1/b^3*d^3*(2/3*(b*x+a)^3*\sinh(b*x+a)+1/3*(b*x+a)^3*\sinh(b*x+a)*\cosh(b*x+a)^2-2*(b*x+a)^2*\cosh(b*x+a)+40/9*(b*x+a)*\sinh(b*x+a)-40/9*\cosh(b*x+a)-1/3*(b*x+a)^2*\cosh(b*x+a)^3+2/9*(b*x+a)*\sinh(b*x+a)*\cosh(b*x+a)^2-2/27*\cosh(b$$

$*x+a)^3)-3/b^3*d^3*a*(2/3*(b*x+a)^2*\sinh(b*x+a)+1/3*(b*x+a)^2*\sinh(b*x+a)*\cosh(b*x+a)^2-4/3*(b*x+a)*\cosh(b*x+a)+40/27*\sinh(b*x+a)-2/9*(b*x+a)*\cosh(b*x+a)^3+2/27*\cosh(b*x+a)^2*\sinh(b*x+a))+3/b^3*d^3*a^2*(2/3*(b*x+a)*\sinh(b*x+a)+1/3*(b*x+a)*\sinh(b*x+a)*\cosh(b*x+a)^2-2/3*\cosh(b*x+a)-1/9*\cosh(b*x+a)^3)-1/b^3*d^3*a^3*(2/3+1/3*\cosh(b*x+a)^2)*\sinh(b*x+a)+3/b^2*c*d^2*(2/3*(b*x+a)^2*\sinh(b*x+a)+1/3*(b*x+a)^2*\sinh(b*x+a)*\cosh(b*x+a)^2-4/3*(b*x+a)*\cosh(b*x+a)+40/27*\sinh(b*x+a)-2/9*(b*x+a)*\cosh(b*x+a)^3+2/27*\cosh(b*x+a)^2*\sinh(b*x+a))-6/b^2*c*d^2*a*(2/3*(b*x+a)*\sinh(b*x+a)+1/3*(b*x+a)*\sinh(b*x+a)*\cosh(b*x+a)^2-2/3*\cosh(b*x+a)-1/9*\cosh(b*x+a)^3)+3/b^2*c*d^2*a^2*(2/3+1/3*\cosh(b*x+a)^2)*\sinh(b*x+a)+3/b*c^2*d*(2/3*(b*x+a)*\sinh(b*x+a)+1/3*(b*x+a)*\sinh(b*x+a)*\cosh(b*x+a)^2-2/3*\cosh(b*x+a)-1/9*\cosh(b*x+a)^3)-3/b*c^2*d*a*(2/3+1/3*\cosh(b*x+a)^2)*\sinh(b*x+a)+c^3*(2/3+1/3*\cosh(b*x+a)^2)*\sinh(b*x+a)$

**maxima [B]** time = 0.47, size = 439, normalized size = 2.51

$$\frac{1}{24} c^2 d \left( \frac{(3 b x e^{(3 a)} - e^{(3 a)}) e^{(3 b x)}}{b^2} + \frac{27 (b x e^a - e^a) e^{(b x)}}{b^2} - \frac{27 (b x + 1) e^{(-b x - a)}}{b^2} - \frac{(3 b x + 1) e^{(-3 b x - 3 a)}}{b^2} \right) + \frac{1}{24} c^3 \left( \frac{e^{(3 b x + 3 a)}}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*cosh(b\*x+a)^3,x, algorithm="maxima")

[Out]  $1/24*c^2*d*((3*b*x*e^{(3*a)} - e^{(3*a)})*e^{(3*b*x)}/b^2 + 27*(b*x*e^a - e^a)*e^{(b*x)}/b^2 - 27*(b*x + 1)*e^{(-b*x - a)}/b^2 - (3*b*x + 1)*e^{(-3*b*x - 3*a)}/b^2) + 1/24*c^3*(e^{(3*b*x + 3*a)}/b + 9*e^{(b*x + a)}/b - 9*e^{(-b*x - a)}/b - e^{(-3*b*x - 3*a)}/b) + 1/72*c*d^2*((9*b^2*x^2*e^{(3*a)} - 6*b*x*e^{(3*a)} + 2*e^{(3*a)})*e^{(3*b*x)}/b^3 + 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^{(b*x)}/b^3 - 81*(b^2*x^2 + 2*b*x + 2)*e^{(-b*x - a)}/b^3 - (9*b^2*x^2 + 6*b*x + 2)*e^{(-3*b*x - 3*a)}/b^3) + 1/216*d^3*((9*b^3*x^3*e^{(3*a)} - 9*b^2*x^2*e^{(3*a)} + 6*b*x*e^{(3*a)} - 2*e^{(3*a)})*e^{(3*b*x)}/b^4 + 81*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*e^{(b*x)}/b^4 - 81*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^{(-b*x - a)}/b^4 - (9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^{(-3*b*x - 3*a)}/b^4)$

**mupad [B]** time = 1.14, size = 364, normalized size = 2.08

$$\frac{\cosh(a + b x)^2 \sinh(a + b x) (3 b^2 c^3 + 14 c d^2)}{3 b^3} - \frac{2 \sinh(a + b x)^3 (3 b^2 c^3 + 20 c d^2)}{9 b^3} - \frac{\cosh(a + b x)^3 (63 b^2 c^2 d)}{27 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^3\*(c + d\*x)^3,x)

[Out]  $(\cosh(a + b*x)^2*\sinh(a + b*x)*(14*c*d^2 + 3*b^2*c^3))/(3*b^3) - (2*\sinh(a + b*x)^3*(20*c*d^2 + 3*b^2*c^3))/(9*b^3) - (\cosh(a + b*x)^3*(122*d^3 + 63*b^2*c^2*d))/(27*b^4) + (2*\cosh(a + b*x)*\sinh(a + b*x)^2*(20*d^3 + 9*b^2*c^2*d))/(9*b^4) - (2*x*\sinh(a + b*x)^3*(20*d^3 + 9*b^2*c^2*d))/(9*b^3) - (7*d^3$

$$x^2 \cosh(a + bx)^3 / (3b^2) - (2d^3 x^3 \sinh(a + bx)^3) / (3b) - (14cd^2 x \cosh(a + bx)^3) / (3b^2) + (x \cosh(a + bx)^2 \sinh(a + bx) (14d^3 + 9b^2 c^2 d)) / (3b^3) + (d^3 x^3 \cosh(a + bx)^2 \sinh(a + bx)) / b + (2d^3 x^2 \cosh(a + bx) \sinh(a + bx)^2) / b^2 - (2cd^2 x^2 \sinh(a + bx)^3) / b + (3cd^2 x^2 \cosh(a + bx)^2 \sinh(a + bx)) / b + (4cd^2 x \cosh(a + bx) \sinh(a + bx)^2) / b^2$$

**sympy [A]** time = 4.22, size = 495, normalized size = 2.83

$$\left\{ \begin{array}{l} -\frac{2c^3 \sinh^3(a+bx)}{3b} + \frac{c^3 \sinh(a+bx) \cosh^2(a+bx)}{b} - \frac{2c^2 dx \sinh^3(a+bx)}{b} + \frac{3c^2 dx \sinh(a+bx) \cosh^2(a+bx)}{b} - \frac{2cd^2 x^2 \sinh^3(a+bx)}{b} + \frac{3cd^2 x^2 \sinh^2(a+bx) \cosh(a+bx)}{b} \\ \left( c^3 x + \frac{3c^2 dx^2}{2} + cd^2 x^3 + \frac{d^3 x^4}{4} \right) \cosh^3(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3\*cosh(b\*x+a)\*\*3,x)

[Out] Piecewise((-2\*c\*\*3\*sinh(a + b\*x)\*\*3/(3\*b) + c\*\*3\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/b - 2\*c\*\*2\*d\*x\*sinh(a + b\*x)\*\*3/b + 3\*c\*\*2\*d\*x\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/b - 2\*c\*d\*\*2\*x\*\*2\*sinh(a + b\*x)\*\*3/b + 3\*c\*d\*\*2\*x\*\*2\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/b - 2\*d\*\*3\*x\*\*3\*sinh(a + b\*x)\*\*3/(3\*b) + d\*\*3\*x\*\*3\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/b + 2\*c\*\*2\*d\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b\*\*2 - 7\*c\*\*2\*d\*cosh(a + b\*x)\*\*3/(3\*b\*\*2) + 4\*c\*d\*\*2\*x\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b\*\*2 - 14\*c\*d\*\*2\*x\*cosh(a + b\*x)\*\*3/(3\*b\*\*2) + 2\*d\*\*3\*x\*\*2\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b\*\*2 - 7\*d\*\*3\*x\*\*2\*cosh(a + b\*x)\*\*3/(3\*b\*\*2) - 40\*c\*d\*\*2\*sinh(a + b\*x)\*\*3/(9\*b\*\*3) + 14\*c\*d\*\*2\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/(3\*b\*\*3) - 40\*d\*\*3\*x\*sinh(a + b\*x)\*\*3/(9\*b\*\*3) + 14\*d\*\*3\*x\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/(3\*b\*\*3) + 40\*d\*\*3\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/(9\*b\*\*4) - 122\*d\*\*3\*cosh(a + b\*x)\*\*3/(27\*b\*\*4), Ne(b, 0)), ((c\*\*3\*x + 3\*c\*\*2\*d\*x\*\*2/2 + c\*d\*\*2\*x\*\*3 + d\*\*3\*x\*\*4/4)\*cosh(a)\*\*3, True))

### 3.18 $\int (c + dx)^2 \cosh^3(a + bx) dx$

**Optimal.** Leaf size=123

$$\frac{2d^2 \sinh^3(a + bx)}{27b^3} + \frac{14d^2 \sinh(a + bx)}{9b^3} - \frac{2d(c + dx) \cosh^3(a + bx)}{9b^2} - \frac{4d(c + dx) \cosh(a + bx)}{3b^2} + \frac{2(c + dx)^2 \sinh(a + bx)}{3b}$$

[Out]  $-4/3*d*(d*x+c)*\cosh(b*x+a)/b^2-2/9*d*(d*x+c)*\cosh(b*x+a)^3/b^2+14/9*d^2*\sinh(b*x+a)/b^3+2/3*(d*x+c)^2*\sinh(b*x+a)/b+1/3*(d*x+c)^2*\cosh(b*x+a)^2*\sinh(b*x+a)/b+2/27*d^2*\sinh(b*x+a)^3/b^3$

**Rubi [A]** time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3311, 3296, 2637, 2633}

$$\frac{2d(c + dx) \cosh^3(a + bx)}{9b^2} - \frac{4d(c + dx) \cosh(a + bx)}{3b^2} + \frac{2d^2 \sinh^3(a + bx)}{27b^3} + \frac{14d^2 \sinh(a + bx)}{9b^3} + \frac{2(c + dx)^2 \sinh(a + bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^2\*Cosh[a + b\*x]^3,x]

[Out]  $(-4*d*(c + d*x)*\text{Cosh}[a + b*x])/(3*b^2) - (2*d*(c + d*x)*\text{Cosh}[a + b*x]^3)/(9*b^2) + (14*d^2*\text{Sinh}[a + b*x])/(9*b^3) + (2*(c + d*x)^2*\text{Sinh}[a + b*x])/(3*b) + ((c + d*x)^2*\text{Cosh}[a + b*x]^2*\text{Sinh}[a + b*x])/(3*b) + (2*d^2*\text{Sinh}[a + b*x]^3)/(27*b^3)$

#### Rule 2633

Int[sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

#### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[((c + d\*x)^m\*cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

### Rubi steps

$$\begin{aligned} \int (c + dx)^2 \cosh^3(a + bx) dx &= -\frac{2d(c + dx) \cosh^3(a + bx)}{9b^2} + \frac{(c + dx)^2 \cosh^2(a + bx) \sinh(a + bx)}{3b} + \frac{2}{3} \int (c + dx) \cosh^2(a + bx) dx \\ &= -\frac{2d(c + dx) \cosh^3(a + bx)}{9b^2} + \frac{2(c + dx)^2 \sinh(a + bx)}{3b} + \frac{(c + dx)^2 \cosh^2(a + bx) \sinh(a + bx)}{3b} \\ &= -\frac{4d(c + dx) \cosh(a + bx)}{3b^2} - \frac{2d(c + dx) \cosh^3(a + bx)}{9b^2} + \frac{2d^2 \sinh(a + bx)}{9b^3} + \frac{2(c + dx)^2 \sinh(a + bx)}{3b} \\ &= -\frac{4d(c + dx) \cosh(a + bx)}{3b^2} - \frac{2d(c + dx) \cosh^3(a + bx)}{9b^2} + \frac{14d^2 \sinh(a + bx)}{9b^3} + \frac{2(c + dx)^2 \sinh(a + bx)}{3b} \end{aligned}$$

**Mathematica [A]** time = 0.55, size = 93, normalized size = 0.76

$$\frac{2 \sinh(a + bx) \left( \cosh(2(a + bx)) (9b^2(c + dx)^2 + 2d^2) + 45b^2(c + dx)^2 + 82d^2 \right) - 162bd(c + dx) \cosh(a + bx) - 6b^3 \cosh^3(a + bx)}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cosh[a + b*x]^3,x]
```

```
[Out] (-162*b*d*(c + d*x)*Cosh[a + b*x] - 6*b*d*(c + d*x)*Cosh[3*(a + b*x)] + 2*(
82*d^2 + 45*b^2*(c + d*x)^2 + (2*d^2 + 9*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)]
)*Sinh[a + b*x])/(108*b^3)
```

**fricas [A]** time = 0.54, size = 199, normalized size = 1.62

$$\frac{6(bd^2x + bcd) \cosh(bx + a)^3 + 18(bd^2x + bcd) \cosh(bx + a) \sinh(bx + a)^2 - (9b^2d^2x^2 + 18b^2cdx + 9b^2c^2 + 2d^2)}{108b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*(6*(b*d^2*x + b*c*d)*cosh(b*x + a)^3 + 18*(b*d^2*x + b*c*d)*cosh(b*x
+ a)*sinh(b*x + a)^2 - (9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 2*d^2)*
```

$$\frac{\sinh(bx+a)^3 + 162(bd^2x + bcd)\cosh(bx+a) - 3(27b^2d^2x^2 + 54b^2cdx + 27b^2c^2 + (9b^2d^2x^2 + 18b^2cdx + 9b^2c^2 + 2d^2)\cosh(bx+a)^2 + 54d^2)\sinh(bx+a)}{b^3}$$

**giac [B]** time = 0.16, size = 230, normalized size = 1.87

$$\frac{(9b^2d^2x^2 + 18b^2cdx + 9b^2c^2 - 6bd^2x - 6bcd + 2d^2)e^{(3bx+3a)}}{216b^3} + \frac{3(b^2d^2x^2 + 2b^2cdx + b^2c^2 - 2bd^2x - 2bcd + 2d^2)e^{(bx+a)}}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*cosh(b\*x+a)^3,x, algorithm="giac")

[Out]  $\frac{1}{216}(9b^2d^2x^2 + 18b^2cdx + 9b^2c^2 - 6bd^2x - 6bcd + 2d^2)e^{(3bx+3a)}/b^3 + \frac{3}{8}(b^2d^2x^2 + 2b^2cdx + b^2c^2 - 2bd^2x - 2bcd + 2d^2)e^{(bx+a)}/b^3 - \frac{3}{8}(b^2d^2x^2 + 2b^2cdx + b^2c^2 + 2bd^2x + 2bcd + 2d^2)e^{(-bx-a)}/b^3 - \frac{1}{216}(9b^2d^2x^2 + 18b^2cdx + 9b^2c^2 + 6bd^2x + 6bcd + 2d^2)e^{(-3bx-3a)}/b^3$

**maple [B]** time = 0.16, size = 302, normalized size = 2.46

$$\frac{d^2 \left( \frac{2(bx+a)^2 \sinh(bx+a)}{3} + \frac{(bx+a)^2 \sinh(bx+a) \cosh^2(bx+a)}{3} - \frac{4(bx+a) \cosh(bx+a)}{3} + \frac{40 \sinh(bx+a)}{27} - \frac{2(bx+a) \cosh^3(bx+a)}{9} + \frac{2 \cosh^2(bx+a) \sinh(bx+a)}{27} \right)}{b^2} - \frac{2d^2 a \left( \frac{2(bx+a)}{3} \right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*cosh(b\*x+a)^3,x)

[Out]  $\frac{1}{b} \left( \frac{1}{b^2} d^2 \left( \frac{2}{3} (bx+a)^2 \sinh(bx+a) + \frac{1}{3} (bx+a)^2 \sinh(bx+a) \cosh(bx+a) - \frac{4}{3} (bx+a) \cosh(bx+a) + \frac{40}{27} \sinh(bx+a) - \frac{2}{9} (bx+a) \cosh(bx+a)^3 + \frac{2}{27} \cosh(bx+a)^2 \sinh(bx+a) \right) - \frac{2}{b^2} d^2 a \left( \frac{2}{3} (bx+a) \sinh(bx+a) + \frac{1}{3} (bx+a) \sinh(bx+a) \cosh(bx+a)^2 - \frac{2}{3} \cosh(bx+a) - \frac{1}{9} \cosh(bx+a)^3 \right) + \frac{1}{b^2} d^2 a^2 \left( \frac{2}{3} + \frac{1}{3} \cosh(bx+a)^2 \right) \sinh(bx+a) + \frac{2}{b} c d \left( \frac{2}{3} (bx+a) \sinh(bx+a) + \frac{1}{3} (bx+a) \sinh(bx+a) \cosh(bx+a)^2 - \frac{2}{3} \cosh(bx+a) - \frac{1}{9} \cosh(bx+a)^3 \right) - \frac{2}{b} c d a \left( \frac{2}{3} + \frac{1}{3} \cosh(bx+a)^2 \right) \sinh(bx+a) + c^2 \left( \frac{2}{3} + \frac{1}{3} \cosh(bx+a)^2 \right) \sinh(bx+a) \right)$

**maxima [B]** time = 0.42, size = 272, normalized size = 2.21

$$\frac{1}{36} cd \left( \frac{(3bx e^{(3a)} - e^{(3a)}) e^{(3bx)}}{b^2} + \frac{27(bx e^a - e^a) e^{(bx)}}{b^2} - \frac{27(bx+1) e^{(-bx-a)}}{b^2} - \frac{(3bx+1) e^{(-3bx-3a)}}{b^2} \right) + \frac{1}{24} c^2 \left( \frac{e^{(3bx+3a)}}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*cosh(b\*x+a)^3,x, algorithm="maxima")

[Out]  $\frac{1}{36}cd((3bx^2e^{3a} - e^{3a})e^{3bx}/b^2 + 27(bxe^a - e^a)e^{bx})/b^2 - 27(bx + 1)e^{-bx - a}/b^2 - (3bx + 1)e^{-3bx - 3a}/b^2 + \frac{1}{24}c^2(e^{3bx + 3a}/b + 9e^{bx + a}/b - 9e^{-bx - a}/b - e^{-3bx - 3a}/b) + \frac{1}{216}d^2((9b^2x^2e^{3a} - 6bx^2e^{3a} + 2e^{3a})e^{3bx}/b^3 + 81(b^2x^2e^a - 2bx^2e^a + 2e^a)e^{bx})/b^3 - 81(b^2x^2 + 2bx + 2)e^{-bx - a}/b^3 - (9b^2x^2 + 6bx + 2)e^{-3bx - 3a}/b^3)$

**mupad [B]** time = 1.22, size = 183, normalized size = 1.49

$$\frac{3d^2 \sinh(ax+bx)}{2} + \frac{d^2 \sinh(3a+3bx)}{54} + \frac{3b^2c^2 \sinh(ax+bx)}{4} + \frac{b^2c^2 \sinh(3a+3bx)}{12} + \frac{3b^2d^2x^2 \sinh(ax+bx)}{4} - \frac{bcd \cosh(3a+3bx)}{18} - \frac{3bd^2x \cosh(ax+bx)}{2}$$

$b^3$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^3\*(c + d\*x)^2,x)

[Out]  $((3d^2 \sinh(ax+bx))/2 + (d^2 \sinh(3a+3bx))/54 + (3b^2c^2 \sinh(ax+bx))/4 + (b^2c^2 \sinh(3a+3bx))/12 + (3b^2d^2x^2 \sinh(ax+bx))/4 - (b^2cd \cosh(3a+3bx))/18 - (3bd^2x \cosh(ax+bx))/2 + (b^2d^2x^2 \sinh(3a+3bx))/12 - (bd^2x \cosh(3a+3bx))/18 - (3b^2cd \cosh(ax+bx))/2 + (b^2cdx \sinh(3a+3bx))/6 + (3b^2cdx \sinh(ax+bx))/2)/b^3$

**sympy [A]** time = 2.25, size = 284, normalized size = 2.31

$$\left\{ \begin{array}{l} -\frac{2c^2 \sinh^3(ax+bx)}{3b} + \frac{c^2 \sinh(ax+bx) \cosh^2(ax+bx)}{b} - \frac{4cdx \sinh^3(ax+bx)}{3b} + \frac{2cdx \sinh(ax+bx) \cosh^2(ax+bx)}{b} - \frac{2d^2x^2 \sinh^3(ax+bx)}{3b} + \frac{d^2x^2 \sinh(ax+bx) \cosh^2(ax+bx)}{b} \\ \left( c^2x + cdx^2 + \frac{d^2x^3}{3} \right) \cosh^3(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2\*cosh(b\*x+a)\*\*3,x)

[Out] Piecewise((-2c\*\*2\*sinh(a + b\*x)\*\*3/(3\*b) + c\*\*2\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/b - 4\*c\*d\*x\*sinh(a + b\*x)\*\*3/(3\*b) + 2\*c\*d\*x\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/b - 2\*d\*\*2\*x\*\*2\*sinh(a + b\*x)\*\*3/(3\*b) + d\*\*2\*x\*\*2\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/b + 4\*c\*d\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/(3\*b\*\*2) - 14\*c\*d\*cosh(a + b\*x)\*\*3/(9\*b\*\*2) + 4\*d\*\*2\*x\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/(3\*b\*\*2) - 14\*d\*\*2\*x\*cosh(a + b\*x)\*\*3/(9\*b\*\*2) - 40\*d\*\*2\*sinh(a + b\*x)\*\*3/(27\*b\*\*3) + 14\*d\*\*2\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/(9\*b\*\*3), Ne(b, 0)), ((c\*\*2\*x + c\*d\*x\*\*2 + d\*\*2\*x\*\*3/3)\*cosh(a)\*\*3, True))



### 3.19 $\int (c + dx) \cosh^3(a + bx) dx$

**Optimal.** Leaf size=75

$$\frac{d \cosh^3(a + bx)}{9b^2} - \frac{2d \cosh(a + bx)}{3b^2} + \frac{2(c + dx) \sinh(a + bx)}{3b} + \frac{(c + dx) \sinh(a + bx) \cosh^2(a + bx)}{3b}$$

[Out]  $-2/3*d*\cosh(b*x+a)/b^2-1/9*d*\cosh(b*x+a)^3/b^2+2/3*(d*x+c)*\sinh(b*x+a)/b+1/3*(d*x+c)*\cosh(b*x+a)^2*\sinh(b*x+a)/b$

**Rubi [A]** time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3310, 3296, 2638}

$$\frac{d \cosh^3(a + bx)}{9b^2} - \frac{2d \cosh(a + bx)}{3b^2} + \frac{2(c + dx) \sinh(a + bx)}{3b} + \frac{(c + dx) \sinh(a + bx) \cosh^2(a + bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)\*Cosh[a + b\*x]^3,x]

[Out]  $(-2*d*Cosh[a + b*x])/(3*b^2) - (d*Cosh[a + b*x]^3)/(9*b^2) + (2*(c + d*x)*Sinh[a + b*x])/(3*b) + ((c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b)$

#### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] := -Simp[((c + d\*x)^m\*cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_.))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] := Simp[(d\*(b\*Sine[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)\*(b\*Sine[e + f\*x])^(n - 2), x], x] - Simp[(b\*(c + d\*x)\*Cos[e + f\*x]\*(b\*Sine[e + f\*x])^(n - 1)/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

#### Rubi steps

$$\begin{aligned}
\int (c + dx) \cosh^3(a + bx) dx &= -\frac{d \cosh^3(a + bx)}{9b^2} + \frac{(c + dx) \cosh^2(a + bx) \sinh(a + bx)}{3b} + \frac{2}{3} \int (c + dx) \cosh(a + bx) dx \\
&= -\frac{d \cosh^3(a + bx)}{9b^2} + \frac{2(c + dx) \sinh(a + bx)}{3b} + \frac{(c + dx) \cosh^2(a + bx) \sinh(a + bx)}{3b} \\
&= -\frac{2d \cosh(a + bx)}{3b^2} - \frac{d \cosh^3(a + bx)}{9b^2} + \frac{2(c + dx) \sinh(a + bx)}{3b} + \frac{(c + dx) \cosh^2(a + bx) \sinh(a + bx)}{3b}
\end{aligned}$$

**Mathematica** [A] time = 0.24, size = 52, normalized size = 0.69

$$-\frac{3b(c + dx)(9 \sinh(a + bx) + \sinh(3(a + bx))) + 27d \cosh(a + bx) + d \cosh(3(a + bx))}{36b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)\*Cosh[a + b\*x]^3, x]

[Out] -1/36\*(27\*d\*Cosh[a + b\*x] + d\*Cosh[3\*(a + b\*x)] - 3\*b\*(c + d\*x)\*(9\*Sinh[a + b\*x] + Sinh[3\*(a + b\*x)]))/b^2

**fricas** [A] time = 0.47, size = 95, normalized size = 1.27

$$\frac{d \cosh(bx + a)^3 + 3d \cosh(bx + a) \sinh(bx + a)^2 - 3(bdx + bc) \sinh(bx + a)^3 + 27d \cosh(bx + a) - 9(3bdx + 3bc + d)e^{3bx+3a}}{36b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*cosh(b\*x+a)^3, x, algorithm="fricas")

[Out] -1/36\*(d\*cosh(b\*x + a)^3 + 3\*d\*cosh(b\*x + a)\*sinh(b\*x + a)^2 - 3\*(b\*d\*x + b\*c)\*sinh(b\*x + a)^3 + 27\*d\*cosh(b\*x + a) - 9\*(3\*b\*d\*x + (b\*d\*x + b\*c)\*cosh(b\*x + a)^2 + 3\*b\*c)\*sinh(b\*x + a))/b^2

**giac** [A] time = 0.12, size = 98, normalized size = 1.31

$$\frac{(3bdx + 3bc - d)e^{(3bx+3a)}}{72b^2} + \frac{3(bdx + bc - d)e^{(bx+a)}}{8b^2} - \frac{3(bdx + bc + d)e^{(-bx-a)}}{8b^2} - \frac{(3bdx + 3bc + d)e^{(-3bx-3a)}}{72b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*cosh(b\*x+a)^3, x, algorithm="giac")

[Out] 1/72\*(3\*b\*d\*x + 3\*b\*c - d)\*e^(3\*b\*x + 3\*a)/b^2 + 3/8\*(b\*d\*x + b\*c - d)\*e^(b\*x + a)/b^2 - 3/8\*(b\*d\*x + b\*c + d)\*e^(-b\*x - a)/b^2 - 1/72\*(3\*b\*d\*x + 3\*b\*c + d)\*e^(-3\*b\*x - 3\*a)/b^2

**maple [A]** time = 0.15, size = 109, normalized size = 1.45

$$\frac{d \left( \frac{2(bx+a) \sinh(bx+a)}{3} + \frac{(bx+a) \sinh(bx+a) (\cosh^2(bx+a))}{3} - \frac{2 \cosh(bx+a)}{3} - \frac{(\cosh^3(bx+a))}{9} \right)}{b} - \frac{da \left( \frac{2}{3} + \frac{(\cosh^2(bx+a))}{3} \right) \sinh(bx+a)}{b} + c \left( \frac{2}{3} + \frac{(\cosh^2(bx+a))}{3} \right) \sinh(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)\*cosh(b\*x+a)^3,x)

[Out] 1/b\*(1/b\*d\*(2/3\*(b\*x+a)\*sinh(b\*x+a)+1/3\*(b\*x+a)\*sinh(b\*x+a)\*cosh(b\*x+a)^2-2/3\*cosh(b\*x+a)-1/9\*cosh(b\*x+a)^3)-1/b\*d\*a\*(2/3+1/3\*cosh(b\*x+a)^2)\*sinh(b\*x+a)+c\*(2/3+1/3\*cosh(b\*x+a)^2)\*sinh(b\*x+a))

**maxima [B]** time = 0.38, size = 143, normalized size = 1.91

$$\frac{1}{72} d \left( \frac{(3bx e^{3a}) - e^{3a}) e^{3bx}}{b^2} + \frac{27(bx e^a - e^a) e^{bx}}{b^2} - \frac{27(bx+1) e^{(-bx-a)}}{b^2} - \frac{(3bx+1) e^{(-3bx-3a)}}{b^2} \right) + \frac{1}{24} c \left( \frac{e^{(3bx+3a)}}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*cosh(b\*x+a)^3,x, algorithm="maxima")

[Out] 1/72\*d\*((3\*b\*x\*e^(3\*a) - e^(3\*a))\*e^(3\*b\*x)/b^2 + 27\*(b\*x\*e^a - e^a)\*e^(b\*x)/b^2 - 27\*(b\*x + 1)\*e^(-b\*x - a)/b^2 - (3\*b\*x + 1)\*e^(-3\*b\*x - 3\*a)/b^2) + 1/24\*c\*(e^(3\*b\*x + 3\*a)/b + 9\*e^(b\*x + a)/b - 9\*e^(-b\*x - a)/b - e^(-3\*b\*x - 3\*a)/b)

**mupad [B]** time = 0.22, size = 77, normalized size = 1.03

$$\frac{\frac{3c \sinh(a+bx)}{4} + \frac{c \sinh(3a+3bx)}{12} + \frac{dx \sinh(3a+3bx)}{12} + \frac{3dx \sinh(a+bx)}{4}}{b} - \frac{d \cosh(3a+3bx)}{36b^2} - \frac{3d \cosh(a+bx)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^3\*(c + d\*x),x)

[Out] ((3\*c\*sinh(a + b\*x))/4 + (c\*sinh(3\*a + 3\*b\*x))/12 + (d\*x\*sinh(3\*a + 3\*b\*x))/12 + (3\*d\*x\*sinh(a + b\*x))/4)/b - (d\*cosh(3\*a + 3\*b\*x))/(36\*b^2) - (3\*d\*cosh(a + b\*x))/(4\*b^2)

**sympy [A]** time = 0.94, size = 126, normalized size = 1.68

$$\left\{ \begin{array}{l} -\frac{2c \sinh^3(a+bx)}{3b} + \frac{c \sinh(a+bx) \cosh^2(a+bx)}{b} - \frac{2dx \sinh^3(a+bx)}{3b} + \frac{dx \sinh(a+bx) \cosh^2(a+bx)}{b} + \frac{2d \sinh^2(a+bx) \cosh(a+bx)}{3b^2} - \frac{7d \cosh^3(a+bx)}{9b^2} \\ \left( cx + \frac{dx^2}{2} \right) \cosh^3(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a)**3,x)
```

```
[Out] Piecewise((-2*c*sinh(a + b*x)**3/(3*b) + c*sinh(a + b*x)*cosh(a + b*x)**2/b  
- 2*d*x*sinh(a + b*x)**3/(3*b) + d*x*sinh(a + b*x)*cosh(a + b*x)**2/b + 2*  
d*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**2) - 7*d*cosh(a + b*x)**3/(9*b**2),  
Ne(b, 0)), ((c*x + d*x**2/2)*cosh(a)**3, True))
```

$$3.20 \quad \int \frac{\cosh^3(a+bx)}{c+dx} dx$$

**Optimal.** Leaf size=121

$$\frac{3 \cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{4d} + \frac{\cosh\left(3a - \frac{3bc}{d}\right) \text{Chi}\left(\frac{3bc}{d} + 3bx\right)}{4d} + \frac{3 \sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{4d} + \frac{\sinh\left(3a - \frac{3bc}{d}\right)}{4d}$$

[Out]  $1/4*\text{Chi}(3*b*c/d+3*b*x)*\cosh(3*a-3*b*c/d)/d+3/4*\text{Chi}(b*c/d+b*x)*\cosh(a-b*c/d)/d+1/4*\text{Shi}(3*b*c/d+3*b*x)*\sinh(3*a-3*b*c/d)/d+3/4*\text{Shi}(b*c/d+b*x)*\sinh(a-b*c/d)/d$

**Rubi [A]** time = 0.24, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3312, 3303, 3298, 3301}

$$\frac{3 \cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{4d} + \frac{\cosh\left(3a - \frac{3bc}{d}\right) \text{Chi}\left(\frac{3bc}{d} + 3bx\right)}{4d} + \frac{3 \sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{4d} + \frac{\sinh\left(3a - \frac{3bc}{d}\right)}{4d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cosh}[a + b*x]^3/(c + d*x), x]$

[Out]  $(3*\text{Cosh}[a - (b*c)/d]*\text{CoshIntegral}[(b*c)/d + b*x])/(4*d) + (\text{Cosh}[3*a - (3*b*c)/d]*\text{CoshIntegral}[(3*b*c)/d + 3*b*x])/(4*d) + (3*\text{Sinh}[a - (b*c)/d]*\text{SinhIntegral}[(b*c)/d + b*x])/(4*d) + (\text{Sinh}[3*a - (3*b*c)/d]*\text{SinhIntegral}[(3*b*c)/d + 3*b*x])/(4*d)$

### Rule 3298

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \text{Simp}[(I*\text{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \&\& \text{EqQ}[d*e - c*f*fz*I, 0]$

### Rule 3301

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \text{Simp}[\text{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \&\& \text{EqQ}[d*(e - \text{Pi}/2) - c*f*fz*I, 0]$

### Rule 3303

$\text{Int}[\sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/(c + d*x), x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\&$

NeQ[d\*e - c\*f, 0]

### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rubi steps

$$\begin{aligned} \int \frac{\cosh^3(a + bx)}{c + dx} dx &= \int \left( \frac{3 \cosh(a + bx)}{4(c + dx)} + \frac{\cosh(3a + 3bx)}{4(c + dx)} \right) dx \\ &= \frac{1}{4} \int \frac{\cosh(3a + 3bx)}{c + dx} dx + \frac{3}{4} \int \frac{\cosh(a + bx)}{c + dx} dx \\ &= \frac{1}{4} \cosh\left(3a - \frac{3bc}{d}\right) \int \frac{\cosh\left(\frac{3bc}{d} + 3bx\right)}{c + dx} dx + \frac{1}{4} \left(3 \cosh\left(a - \frac{bc}{d}\right)\right) \int \frac{\cosh\left(\frac{bc}{d} + bx\right)}{c + dx} dx + \\ &= \frac{3 \cosh\left(a - \frac{bc}{d}\right) \operatorname{Chi}\left(\frac{bc}{d} + bx\right)}{4d} + \frac{\cosh\left(3a - \frac{3bc}{d}\right) \operatorname{Chi}\left(\frac{3bc}{d} + 3bx\right)}{4d} + \frac{3 \sinh\left(a - \frac{bc}{d}\right) \operatorname{Shi}\left(\frac{bc}{d} + bx\right)}{4d} \end{aligned}$$

**Mathematica** [A] time = 0.23, size = 102, normalized size = 0.84

$$\frac{3 \cosh\left(a - \frac{bc}{d}\right) \operatorname{Chi}\left(b\left(\frac{c}{d} + x\right)\right) + \cosh\left(3a - \frac{3bc}{d}\right) \operatorname{Chi}\left(\frac{3b(c+dx)}{d}\right) + 3 \sinh\left(a - \frac{bc}{d}\right) \operatorname{Shi}\left(b\left(\frac{c}{d} + x\right)\right) + \sinh\left(3a - \frac{3bc}{d}\right) \operatorname{Shi}\left(\frac{3b(c+dx)}{d}\right)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]^3/(c + d\*x), x]

[Out] (3\*Cosh[a - (b\*c)/d]\*CoshIntegral[b\*(c/d + x)] + Cosh[3\*a - (3\*b\*c)/d]\*CoshIntegral[(3\*b\*(c + d\*x))/d] + 3\*Sinh[a - (b\*c)/d]\*SinhIntegral[b\*(c/d + x)] + Sinh[3\*a - (3\*b\*c)/d]\*SinhIntegral[(3\*b\*(c + d\*x))/d])/(4\*d)

**fricas** [A] time = 0.80, size = 186, normalized size = 1.54

$$\frac{3 \left( \operatorname{Ei}\left(\frac{bdx+bc}{d}\right) + \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right) \right) \cosh\left(-\frac{bc-ad}{d}\right) + \left( \operatorname{Ei}\left(\frac{3(bdx+bc)}{d}\right) + \operatorname{Ei}\left(-\frac{3(bdx+bc)}{d}\right) \right) \cosh\left(-\frac{3(bc-ad)}{d}\right) + 3 \left( \operatorname{Ei}\left(\frac{bdx+bc}{d}\right) + \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right) \right) \sinh\left(\frac{bc-ad}{d}\right) + 3 \left( \operatorname{Ei}\left(\frac{3(bdx+bc)}{d}\right) + \operatorname{Ei}\left(-\frac{3(bdx+bc)}{d}\right) \right) \sinh\left(\frac{3(bc-ad)}{d}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c),x, algorithm="fricas")

[Out]  $\frac{1}{8} * (3 * (\text{Ei}((b*d*x + b*c)/d) + \text{Ei}(-(b*d*x + b*c)/d)) * \cosh(-(b*c - a*d)/d) + (\text{Ei}(3*(b*d*x + b*c)/d) + \text{Ei}(-3*(b*d*x + b*c)/d)) * \cosh(-3*(b*c - a*d)/d) + 3 * (\text{Ei}((b*d*x + b*c)/d) - \text{Ei}(-(b*d*x + b*c)/d)) * \sinh(-(b*c - a*d)/d) + (\text{Ei}(3*(b*d*x + b*c)/d) - \text{Ei}(-3*(b*d*x + b*c)/d)) * \sinh(-3*(b*c - a*d)/d)) / d$

**giac** [A] time = 0.14, size = 112, normalized size = 0.93

$$\frac{\text{Ei}\left(\frac{3(bdx+bc)}{d}\right) e^{\left(3a - \frac{3bc}{d}\right)} + 3 \text{Ei}\left(\frac{bdx+bc}{d}\right) e^{\left(a - \frac{bc}{d}\right)} + 3 \text{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(-a + \frac{bc}{d}\right)} + \text{Ei}\left(-\frac{3(bdx+bc)}{d}\right) e^{\left(-3a + \frac{3bc}{d}\right)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c),x, algorithm="giac")

[Out]  $\frac{1}{8} * (\text{Ei}(3*(b*d*x + b*c)/d) * e^{(3*a - 3*b*c/d)} + 3 * \text{Ei}((b*d*x + b*c)/d) * e^{(a - b*c/d)} + 3 * \text{Ei}(-(b*d*x + b*c)/d) * e^{(-a + b*c/d)} + \text{Ei}(-3*(b*d*x + b*c)/d) * e^{(-3*a + 3*b*c/d)}) / d$

**maple** [A] time = 0.30, size = 166, normalized size = 1.37

$$\frac{e^{-\frac{3(da-cb)}{d}} \text{Ei}\left(1, 3bx + 3a - \frac{3(da-cb)}{d}\right)}{8d} - \frac{3e^{-\frac{da-cb}{d}} \text{Ei}\left(1, bx + a - \frac{da-cb}{d}\right)}{8d} - \frac{3e^{\frac{da-cb}{d}} \text{Ei}\left(1, -bx - a - \frac{da-cb}{d}\right)}{8d} - \frac{e^{\frac{3da-3cb}{d}} \text{Ei}\left(1, -3bx - 3a + \frac{3(da-cb)}{d}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)^3/(d\*x+c),x)

[Out]  $-1/8/d * \exp(-3*(a*d-b*c)/d) * \text{Ei}(1, 3*b*x+3*a-3*(a*d-b*c)/d) - 3/8/d * \exp(-(a*d-b*c)/d) * \text{Ei}(1, b*x+a-(a*d-b*c)/d) - 3/8/d * \exp((a*d-b*c)/d) * \text{Ei}(1, -b*x-a-(-a*d+b*c)/d) - 1/8/d * \exp(3*(a*d-b*c)/d) * \text{Ei}(1, -3*b*x-3*a-3*(-a*d+b*c)/d)$

**maxima** [A] time = 0.41, size = 117, normalized size = 0.97

$$\frac{e^{\left(-3a + \frac{3bc}{d}\right)} E_1\left(\frac{3(dx+c)b}{d}\right)}{8d} - \frac{3e^{\left(-a + \frac{bc}{d}\right)} E_1\left(\frac{(dx+c)b}{d}\right)}{8d} - \frac{3e^{\left(a - \frac{bc}{d}\right)} E_1\left(-\frac{(dx+c)b}{d}\right)}{8d} - \frac{e^{\left(3a - \frac{3bc}{d}\right)} E_1\left(-\frac{3(dx+c)b}{d}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c),x, algorithm="maxima")

[Out]  $-1/8 * e^{(-3*a + 3*b*c/d)} * \exp\_integral\_e(1, 3*(d*x + c)*b/d)/d - 3/8 * e^{(-a + b*c/d)} * \exp\_integral\_e(1, (d*x + c)*b/d)/d - 3/8 * e^{(a - b*c/d)} * \exp\_integral\_e(1, -(d*x + c)*b/d)/d - 1/8 * e^{(3*a - 3*b*c/d)} * \exp\_integral\_e(1, -3*(d*x + c)*b/d)/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + bx)^3}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^3/(c + d\*x), x)

[Out] int(cosh(a + b\*x)^3/(c + d\*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*\*3/(d\*x+c), x)

[Out] Integral(cosh(a + b\*x)\*\*3/(c + d\*x), x)



$$3.21 \quad \int \frac{\cosh^3(a+bx)}{(c+dx)^2} dx$$

**Optimal.** Leaf size=145

$$\frac{3b \sinh\left(3a - \frac{3bc}{d}\right) \text{Chi}\left(\frac{3bc}{d} + 3bx\right)}{4d^2} + \frac{3b \sinh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{4d^2} + \frac{3b \cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{4d^2} + \frac{3b \cosh\left(3a - \frac{3bc}{d}\right) \text{Shi}\left(\frac{3bc}{d} + 3bx\right)}{4d^2}$$

[Out]  $-\cosh(b*x+a)^3/d/(d*x+c)+3/4*b*\cosh(a-b*c/d)*\text{Shi}(b*c/d+b*x)/d^2+3/4*b*\cosh(3*a-3*b*c/d)*\text{Shi}(3*b*c/d+3*b*x)/d^2+3/4*b*\text{Chi}(3*b*c/d+3*b*x)*\sinh(3*a-3*b*c/d)/d^2+3/4*b*\text{Chi}(b*c/d+b*x)*\sinh(a-b*c/d)/d^2$

**Rubi [A]** time = 0.24, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3313, 3303, 3298, 3301}

$$\frac{3b \sinh\left(3a - \frac{3bc}{d}\right) \text{Chi}\left(\frac{3bc}{d} + 3bx\right)}{4d^2} + \frac{3b \sinh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{4d^2} + \frac{3b \cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{4d^2} + \frac{3b \cosh\left(3a - \frac{3bc}{d}\right) \text{Shi}\left(\frac{3bc}{d} + 3bx\right)}{4d^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b\*x]^3/(c + d\*x)^2, x]

[Out]  $-(\text{Cosh}[a + b*x]^3/(d*(c + d*x))) + (3*b*\text{CoshIntegral}[(3*b*c)/d + 3*b*x]*\text{Sin h}[3*a - (3*b*c)/d])/(4*d^2) + (3*b*\text{CoshIntegral}[(b*c)/d + b*x]*\text{Sinh}[a - (b*c)/d])/(4*d^2) + (3*b*\text{Cosh}[a - (b*c)/d]*\text{SinhIntegral}[(b*c)/d + b*x])/(4*d^2) + (3*b*\text{Cosh}[3*a - (3*b*c)/d]*\text{SinhIntegral}[(3*b*c)/d + 3*b*x])/(4*d^2)$

**Rule 3298**

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

**Rule 3301**

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

**Rule 3303**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)

) / d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&  
NeQ[d\*e - c\*f, 0]

### Rule 3313

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] :> Si  
mp[((c + d\*x)^(m + 1)\*Sin[e + f\*x]^n)/(d\*(m + 1)), x] - Dist[(f\*n)/(d\*(m +  
1)), Int[ExpandTrigReduce[(c + d\*x)^(m + 1), Cos[e + f\*x]\*Sin[e + f\*x]^(n -  
1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&  
LtQ[m, -1]

### Rubi steps

$$\begin{aligned} \int \frac{\cosh^3(a + bx)}{(c + dx)^2} dx &= -\frac{\cosh^3(a + bx)}{d(c + dx)} + \frac{(3ib) \int \left( -\frac{i \sinh(a + bx)}{4(c + dx)} - \frac{i \sinh(3a + 3bx)}{4(c + dx)} \right) dx}{d} \\ &= -\frac{\cosh^3(a + bx)}{d(c + dx)} + \frac{(3b) \int \frac{\sinh(a + bx)}{c + dx} dx}{4d} + \frac{(3b) \int \frac{\sinh(3a + 3bx)}{c + dx} dx}{4d} \\ &= -\frac{\cosh^3(a + bx)}{d(c + dx)} + \frac{\left( 3b \cosh \left( 3a - \frac{3bc}{d} \right) \right) \int \frac{\sinh \left( \frac{3bc}{d} + 3bx \right)}{c + dx} dx}{4d} + \frac{\left( 3b \cosh \left( a - \frac{bc}{d} \right) \right) \int \frac{\sinh \left( \frac{bc}{d} + bx \right)}{c + dx} dx}{4d} \\ &= -\frac{\cosh^3(a + bx)}{d(c + dx)} + \frac{3b \operatorname{Chi} \left( \frac{3bc}{d} + 3bx \right) \sinh \left( 3a - \frac{3bc}{d} \right)}{4d^2} + \frac{3b \operatorname{Chi} \left( \frac{bc}{d} + bx \right) \sinh \left( a - \frac{bc}{d} \right)}{4d^2} + \dots \end{aligned}$$

**Mathematica** [A] time = 0.53, size = 196, normalized size = 1.35

$$\frac{3b \left( -2 \sinh \left( 3a - \frac{3bc}{d} \right) \operatorname{Chi} \left( \frac{3bc}{d} + 3bx \right) - 2 \sinh \left( a - \frac{bc}{d} \right) \operatorname{Chi} \left( \frac{bc}{d} + bx \right) - 2 \cosh \left( a - \frac{bc}{d} \right) \operatorname{Shi} \left( \frac{bc}{d} + bx \right) - 2 \cosh \left( 3a - \frac{3bc}{d} \right) \operatorname{Shi} \left( \frac{3bc}{d} + 3bx \right) \right)}{8d^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]^3/(c + d\*x)^2,x]

[Out] (-3\*Cosh[a]\*Cosh[b\*x])/(4\*d\*(c + d\*x)) - (Cosh[3\*a]\*Cosh[3\*b\*x])/(4\*d\*(c + d\*x)) - (3\*Sinh[a]\*Sinh[b\*x])/(4\*d\*(c + d\*x)) - (Sinh[3\*a]\*Sinh[3\*b\*x])/(4\*d\*(c + d\*x)) - (3\*b\*(-2\*CoshIntegral[(3\*b\*c)/d + 3\*b\*x]\*Sinh[3\*a - (3\*b\*c)/d] - 2\*CoshIntegral[(b\*c)/d + b\*x]\*Sinh[a - (b\*c)/d] - 2\*Cosh[a - (b\*c)/d]\*SinhIntegral[(b\*c)/d + b\*x] - 2\*Cosh[3\*a - (3\*b\*c)/d]\*SinhIntegral[(3\*b\*c)/d + 3\*b\*x]))/(8\*d^2)

**fricas [B]** time = 0.53, size = 305, normalized size = 2.10

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$$2 d \cosh (b x + a)^3 + 6 d \cosh (b x + a) \sinh (b x + a)^2 + 6 d \cosh (b x + a) - 3 \left( (b d x + b c) \operatorname{Ei} \left( \frac{b d x + b c}{d} \right) - (b d x + b c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")`

[Out]  $-1/8*(2*d*\cosh(b*x + a)^3 + 6*d*\cosh(b*x + a)*\sinh(b*x + a)^2 + 6*d*\cosh(b*x + a) - 3*((b*d*x + b*c)*\operatorname{Ei}((b*d*x + b*c)/d) - (b*d*x + b*c)*\operatorname{Ei}(-(b*d*x + b*c)/d))*\cosh(-(b*c - a*d)/d) - 3*((b*d*x + b*c)*\operatorname{Ei}(3*(b*d*x + b*c)/d) - (b*d*x + b*c)*\operatorname{Ei}(-3*(b*d*x + b*c)/d))*\cosh(-3*(b*c - a*d)/d) - 3*((b*d*x + b*c)*\operatorname{Ei}((b*d*x + b*c)/d) + (b*d*x + b*c)*\operatorname{Ei}(-(b*d*x + b*c)/d))*\sinh(-(b*c - a*d)/d) - 3*((b*d*x + b*c)*\operatorname{Ei}(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*\operatorname{Ei}(-3*(b*d*x + b*c)/d))*\sinh(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)$

**giac [B]** time = 0.21, size = 1075, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")`

[Out]  $-1/8*(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*\operatorname{Ei}(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + 3*b^3*c*\operatorname{Ei}(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) *e^{(3*(b*c - a*d)/d) - 3*a*b^2*d*\operatorname{Ei}(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) *e^{(3*(b*c - a*d)/d) + 3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*\operatorname{Ei}(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) *e^{((b*c - a*d)/d) + 3*b^3*c*\operatorname{Ei}(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) *e^{((b*c - a*d)/d) - 3*a*b^2*d*\operatorname{Ei}(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) *e^{((b*c - a*d)/d) - 3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*\operatorname{Ei}(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) *e^{-(b*c - a*d)/d} - 3*b^3*c*\operatorname{Ei}(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) *e^{-(b*c - a*d)/d) + 3*a*b^2*d*\operatorname{Ei}(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) *e^{-(b*c - a*d)/d} - 3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*\operatorname{Ei}(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) *e^{-3*(b*c - a*d)/d} - 3*b^3*c*\operatorname{Ei}(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) *e^{-3*(b*c - a*d)/d) + 3*a*b^2*d*\operatorname{Ei}(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) *e^{-3*(b*c - a*d)/d) + b^2*d*e^{(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)))/d} + 3*b^2*d*e^{-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)))/d} + 3*b^2*d*e^{-(d*x + c)*$

$(b - b*c/(d*x + c) + a*d/(d*x + c))/d) + b^2*d*e^{-3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d})*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)))*d^4 + b*c*d^4 - a*d^5)*b)$

**maple [A]** time = 0.30, size = 271, normalized size = 1.87

$$\frac{b e^{-3bx-3a}}{8(bdx+cb)d} + \frac{3b e^{-\frac{3(da-cb)}{d}} \operatorname{Ei}\left(1, 3bx + 3a - \frac{3(da-cb)}{d}\right)}{8d^2} - \frac{3b e^{-bx-a}}{8d(bdx+cb)} + \frac{3b e^{-\frac{da-cb}{d}} \operatorname{Ei}\left(1, bx + a - \frac{da-cb}{d}\right)}{8d^2} - \frac{3b e^{bx}}{8d^2 \left(\frac{bc}{d} - \dots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(b*x+a)^3/(d*x+c)^2,x)`

[Out]  $-1/8*b*\exp(-3*b*x-3*a)/(b*d*x+b*c)/d+3/8*b/d^2*\exp(-3*(a*d-b*c)/d)*\operatorname{Ei}(1,3*b*x+3*a-3*(a*d-b*c)/d)-3/8*b*\exp(-b*x-a)/d/(b*d*x+b*c)+3/8*b/d^2*\exp(-(a*d-b*c)/d)*\operatorname{Ei}(1,b*x+a-(a*d-b*c)/d)-3/8*b/d^2*\exp(b*x+a)/(b*c/d+b*x)-3/8*b/d^2*\exp((a*d-b*c)/d)*\operatorname{Ei}(1,-b*x-a-(a*d+b*c)/d)-1/8*b/d^2*\exp(3*b*x+3*a)/(b*c/d+b*x)-3/8*b/d^2*\exp(3*(a*d-b*c)/d)*\operatorname{Ei}(1,-3*b*x-3*a-3*(-a*d+b*c)/d)$

**maxima [A]** time = 0.46, size = 145, normalized size = 1.00

$$\frac{e^{(-3a+\frac{3bc}{d})} E_2\left(\frac{3(dx+c)b}{d}\right)}{8(dx+c)d} - \frac{3e^{(-a+\frac{bc}{d})} E_2\left(\frac{(dx+c)b}{d}\right)}{8(dx+c)d} - \frac{3e^{(a-\frac{bc}{d})} E_2\left(-\frac{(dx+c)b}{d}\right)}{8(dx+c)d} - \frac{e^{(3a-\frac{3bc}{d})} E_2\left(-\frac{3(dx+c)b}{d}\right)}{8(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")`

[Out]  $-1/8*e^{(-3*a+3*b*c/d)}*\exp\_integral\_e(2, 3*(d*x+c)*b/d)/((d*x+c)*d) - 3/8*e^{(-a+b*c/d)}*\exp\_integral\_e(2, (d*x+c)*b/d)/((d*x+c)*d) - 3/8*e^{(a-b*c/d)}*\exp\_integral\_e(2, -(d*x+c)*b/d)/((d*x+c)*d) - 1/8*e^{(3*a-3*b*c/d)}*\exp\_integral\_e(2, -3*(d*x+c)*b/d)/((d*x+c)*d)$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+bx)^3}{(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(a+b*x)^3/(c+d*x)^2,x)`

[Out] `int(cosh(a+b*x)^3/(c+d*x)^2,x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*\*3/(d\*x+c)\*\*2,x)

[Out] Integral(cosh(a + b\*x)\*\*3/(c + d\*x)\*\*2, x)

$$3.22 \quad \int \frac{\cosh^3(a+bx)}{(c+dx)^3} dx$$

**Optimal.** Leaf size=184

$$\frac{3b^2 \cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{8d^3} + \frac{9b^2 \cosh\left(3a - \frac{3bc}{d}\right) \text{Chi}\left(\frac{3bc}{d} + 3bx\right)}{8d^3} + \frac{3b^2 \sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{8d^3} + \frac{9b^2 \sinh\left(3a - \frac{3bc}{d}\right) \text{Shi}\left(\frac{3bc}{d} + 3bx\right)}{8d^3}$$

[Out]  $9/8*b^2*Chi(3*b*c/d+3*b*x)*cosh(3*a-3*b*c/d)/d^3+3/8*b^2*Chi(b*c/d+b*x)*cosh(a-b*c/d)/d^3-1/2*cosh(b*x+a)^3/d/(d*x+c)^2+9/8*b^2*Shi(3*b*c/d+3*b*x)*sinh(3*a-3*b*c/d)/d^3+3/8*b^2*Shi(b*c/d+b*x)*sinh(a-b*c/d)/d^3-3/2*b*cosh(b*x+a)^2*sinh(b*x+a)/d^2/(d*x+c)$

**Rubi [A]** time = 0.34, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3314, 3303, 3298, 3301, 3312}

$$\frac{3b^2 \cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{8d^3} + \frac{9b^2 \cosh\left(3a - \frac{3bc}{d}\right) \text{Chi}\left(\frac{3bc}{d} + 3bx\right)}{8d^3} + \frac{3b^2 \sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{8d^3} + \frac{9b^2 \sinh\left(3a - \frac{3bc}{d}\right) \text{Shi}\left(\frac{3bc}{d} + 3bx\right)}{8d^3}$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b\*x]^3/(c + d\*x)^3,x]

[Out]  $-\text{Cosh}[a + b*x]^3/(2*d*(c + d*x)^2) + (3*b^2*\text{Cosh}[a - (b*c)/d]*\text{CoshIntegral}[(b*c)/d + b*x])/(8*d^3) + (9*b^2*\text{Cosh}[3*a - (3*b*c)/d]*\text{CoshIntegral}[(3*b*c)/d + 3*b*x])/(8*d^3) - (3*b*\text{Cosh}[a + b*x]^2*\text{Sinh}[a + b*x])/(2*d^2*(c + d*x)) + (3*b^2*\text{Sinh}[a - (b*c)/d]*\text{SinhIntegral}[(b*c)/d + b*x])/(8*d^3) + (9*b^2*\text{Sinh}[3*a - (3*b*c)/d]*\text{SinhIntegral}[(3*b*c)/d + 3*b*x])/(8*d^3)$

**Rule 3298**

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

**Rule 3301**

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

**Rule 3303**

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

### Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_)), x_Symbo
l] := Simp[((c + d*x)^(m + 1)*(b*Sine[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sine[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sine[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*(b*Sine[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(a+bx)}{(c+dx)^3} dx &= -\frac{\cosh^3(a+bx)}{2d(c+dx)^2} - \frac{3b \cosh^2(a+bx) \sinh(a+bx)}{2d^2(c+dx)} - \frac{(3b^2) \int \frac{\cosh(a+bx)}{c+dx} dx}{d^2} + \frac{(9b^2) \int \frac{\cosh^3}{c+dx}}{2d^2} \\
&= -\frac{\cosh^3(a+bx)}{2d(c+dx)^2} - \frac{3b \cosh^2(a+bx) \sinh(a+bx)}{2d^2(c+dx)} + \frac{(9b^2) \int \left( \frac{3 \cosh(a+bx)}{4(c+dx)} + \frac{\cosh(3a+3bx)}{4(c+dx)} \right) dx}{2d^2} \\
&= -\frac{\cosh^3(a+bx)}{2d(c+dx)^2} - \frac{3b^2 \cosh\left(a - \frac{bc}{d}\right) \operatorname{Chi}\left(\frac{bc}{d} + bx\right)}{d^3} - \frac{3b \cosh^2(a+bx) \sinh(a+bx)}{2d^2(c+dx)} - \frac{3b^2 \cosh^2(a+bx) \sinh(a+bx)}{2d^2(c+dx)} \\
&= -\frac{\cosh^3(a+bx)}{2d(c+dx)^2} - \frac{3b^2 \cosh\left(a - \frac{bc}{d}\right) \operatorname{Chi}\left(\frac{bc}{d} + bx\right)}{d^3} - \frac{3b \cosh^2(a+bx) \sinh(a+bx)}{2d^2(c+dx)} - \frac{3b^2 \cosh^2(a+bx) \sinh(a+bx)}{2d^2(c+dx)} \\
&= -\frac{\cosh^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2 \cosh\left(a - \frac{bc}{d}\right) \operatorname{Chi}\left(\frac{bc}{d} + bx\right)}{8d^3} + \frac{9b^2 \cosh\left(3a - \frac{3bc}{d}\right) \operatorname{Chi}\left(\frac{3bc}{d} + 3bx\right)}{8d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.89, size = 218, normalized size = 1.18

$$-6b^2(c+dx)^2 \left( \cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(b\left(\frac{c}{d} + x\right)\right) + 3 \cosh\left(3a - \frac{3bc}{d}\right) \text{Chi}\left(\frac{3b(c+dx)}{d}\right) + \sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(b\left(\frac{c}{d} + x\right)\right) \right)$$


---

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]^3/(c + d\*x)^3,x]

[Out]  $-1/16*(6*d*\text{Cosh}[b*x]*(d*\text{Cosh}[a] + b*(c + d*x)*\text{Sinh}[a]) + 2*d*\text{Cosh}[3*b*x]*(d*\text{Cosh}[3*a] + 3*b*(c + d*x)*\text{Sinh}[3*a]) + 6*d*(b*(c + d*x)*\text{Cosh}[a] + d*\text{Sinh}[a])*\text{Sinh}[b*x] + 2*d*(3*b*(c + d*x)*\text{Cosh}[3*a] + d*\text{Sinh}[3*a])*\text{Sinh}[3*b*x] - 6*b^2*(c + d*x)^2*(\text{Cosh}[a - (b*c)/d]*\text{CoshIntegral}[b*(c/d + x)] + 3*\text{Cosh}[3*a - (3*b*c)/d]*\text{CoshIntegral}[(3*b*(c + d*x))/d] + \text{Sinh}[a - (b*c)/d]*\text{SinhIntegral}[b*(c/d + x)] + 3*\text{Sinh}[3*a - (3*b*c)/d]*\text{SinhIntegral}[(3*b*(c + d*x))/d]))/(d^3*(c + d*x)^2)$

**fricas [B]** time = 0.55, size = 527, normalized size = 2.86

$$2d^2 \cosh(bx+a)^3 + 6d^2 \cosh(bx+a) \sinh(bx+a)^2 + 6(bd^2x + bcd) \sinh(bx+a)^3 + 6d^2 \cosh(bx+a) - 3$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c)^3,x, algorithm="fricas")

[Out]  $-1/16*(2*d^2*\cosh(b*x + a)^3 + 6*d^2*\cosh(b*x + a)*\sinh(b*x + a)^2 + 6*(b*d^2*x + b*c*d)*\sinh(b*x + a)^3 + 6*d^2*\cosh(b*x + a) - 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\text{Ei}((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\text{Ei}(-(b*d*x + b*c)/d))*\cosh(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\text{Ei}(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\text{Ei}(-3*(b*d*x + b*c)/d))*\cosh(-3*(b*c - a*d)/d) + 6*(b*d^2*x + b*c*d + 3*(b*d^2*x + b*c*d)*\cosh(b*x + a)^2)*\sinh(b*x + a) - 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\text{Ei}((b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\text{Ei}(-(b*d*x + b*c)/d))*\sinh(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\text{Ei}(3*(b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\text{Ei}(-3*(b*d*x + b*c)/d))*\sinh(-3*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)$

**giac [B]** time = 0.13, size = 602, normalized size = 3.27

$$9b^2d^2x^2\text{Ei}\left(\frac{3(bdx+bc)}{d}\right)e^{\left(3a-\frac{3bc}{d}\right)} + 3b^2d^2x^2\text{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} + 3b^2d^2x^2\text{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)} + 9b^2d^2x^2\text{Ei}\left(-\frac{3(bdx+bc)}{d}\right)$$


---



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c)^3,x, algorithm="giac")

[Out]  $\frac{1}{16} * (9 * b^2 * d^2 * x^2 * \text{Ei}(3 * (b * d * x + b * c) / d) * e^{(3 * a - 3 * b * c / d)} + 3 * b^2 * d^2 * x^2 * \text{Ei}((b * d * x + b * c) / d) * e^{(a - b * c / d)} + 3 * b^2 * d^2 * x^2 * \text{Ei}(-(b * d * x + b * c) / d) * e^{(-a + b * c / d)} + 9 * b^2 * d^2 * x^2 * \text{Ei}(-3 * (b * d * x + b * c) / d) * e^{(-3 * a + 3 * b * c / d)} + 18 * b^2 * c * d * x * \text{Ei}(3 * (b * d * x + b * c) / d) * e^{(3 * a - 3 * b * c / d)} + 6 * b^2 * c * d * x * \text{Ei}((b * d * x + b * c) / d) * e^{(a - b * c / d)} + 6 * b^2 * c * d * x * \text{Ei}(-(b * d * x + b * c) / d) * e^{(-a + b * c / d)} + 18 * b^2 * c * d * x * \text{Ei}(-3 * (b * d * x + b * c) / d) * e^{(-3 * a + 3 * b * c / d)} + 9 * b^2 * c^2 * \text{Ei}(3 * (b * d * x + b * c) / d) * e^{(3 * a - 3 * b * c / d)} + 3 * b^2 * c^2 * \text{Ei}((b * d * x + b * c) / d) * e^{(a - b * c / d)} + 3 * b^2 * c^2 * \text{Ei}(-(b * d * x + b * c) / d) * e^{(-a + b * c / d)} + 9 * b^2 * c^2 * \text{Ei}(-3 * (b * d * x + b * c) / d) * e^{(-3 * a + 3 * b * c / d)} - 3 * b * d^2 * x * e^{(3 * b * x + 3 * a)} - 3 * b * d^2 * x * e^{(b * x + a)} + 3 * b * d^2 * x * e^{(-b * x - a)} + 3 * b * d^2 * x * e^{(-3 * b * x - 3 * a)} - 3 * b * c * d * e^{(3 * b * x + 3 * a)} - 3 * b * c * d * e^{(b * x + a)} + 3 * b * c * d * e^{(-b * x - a)} + 3 * b * c * d * e^{(-3 * b * x - 3 * a)} - d^2 * e^{(3 * b * x + 3 * a)} - 3 * d^2 * e^{(b * x + a)} - 3 * d^2 * e^{(-b * x - a)} - d^2 * e^{(-3 * b * x - 3 * a)}) / (d^5 * x^2 + 2 * c * d^4 * x + c^2 * d^3)$

**maple [B]** time = 0.32, size = 562, normalized size = 3.05

$$\frac{3b^3e^{-3bx-3a}x}{16d(b^2d^2x^2 + 2b^2cdx + c^2b^2)} + \frac{3b^3e^{-3bx-3a}c}{16d^2(b^2d^2x^2 + 2b^2cdx + c^2b^2)} - \frac{b^2e^{-3bx-3a}}{16d(b^2d^2x^2 + 2b^2cdx + c^2b^2)} - \frac{9b^2e^{-\frac{3(da-cb)}{d}} \text{Ei}\left(\right)}{16d(b^2d^2x^2 + 2b^2cdx + c^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)^3/(d\*x+c)^3,x)

[Out]  $\frac{3}{16} * b^3 * \exp(-3 * b * x - 3 * a) / d / (b^2 * d^2 * x^2 + 2 * b^2 * c * d * x + b^2 * c^2) * x + \frac{3}{16} * b^3 * \exp(-3 * b * x - 3 * a) / d^2 / (b^2 * d^2 * x^2 + 2 * b^2 * c * d * x + b^2 * c^2) * c - \frac{1}{16} * b^2 * \exp(-3 * b * x - 3 * a) / d / (b^2 * d^2 * x^2 + 2 * b^2 * c * d * x + b^2 * c^2) - \frac{9}{16} * b^2 / d^3 * \exp(-3 * (a * d - b * c) / d) * \text{Ei}\left(1, 3 * b * x + 3 * a - 3 * (a * d - b * c) / d\right) + \frac{3}{16} * b^3 * \exp(-b * x - a) / d / (b^2 * d^2 * x^2 + 2 * b^2 * c * d * x + b^2 * c^2) * x + \frac{3}{16} * b^3 * \exp(-b * x - a) / d^2 / (b^2 * d^2 * x^2 + 2 * b^2 * c * d * x + b^2 * c^2) * c - \frac{3}{16} * b^2 * \exp(-b * x - a) / d / (b^2 * d^2 * x^2 + 2 * b^2 * c * d * x + b^2 * c^2) - \frac{3}{16} * b^2 / d^3 * \exp(-(a * d - b * c) / d) * \text{Ei}\left(1, b * x + a - (a * d - b * c) / d\right) - \frac{3}{16} * b^2 / d^3 * \exp(b * x + a) / (b * c / d + b * x)^2 - \frac{3}{16} * b^2 / d^3 * \exp(b * x + a) / (b * c / d + b * x) - \frac{3}{16} * b^2 / d^3 * \exp((a * d - b * c) / d) * \text{Ei}\left(1, -b * x - a - (a * d + b * c) / d\right) - \frac{1}{16} * b^2 / d^3 * \exp(3 * b * x + 3 * a) / (b * c / d + b * x)^2 - \frac{3}{16} * b^2 / d^3 * \exp(3 * b * x + 3 * a) / (b * c / d + b * x) - \frac{9}{16} * b^2 / d^3 * \exp(3 * (a * d - b * c) / d) * \text{Ei}\left(1, -3 * b * x - 3 * a - 3 * (-a * d + b * c) / d\right)$

**maxima [A]** time = 0.45, size = 145, normalized size = 0.79

$$\frac{e^{\left(-3a + \frac{3bc}{d}\right)} E_3\left(\frac{3(dx+c)b}{d}\right)}{8(dx+c)^2d} - \frac{3e^{\left(-a + \frac{bc}{d}\right)} E_3\left(\frac{(dx+c)b}{d}\right)}{8(dx+c)^2d} - \frac{3e^{\left(a - \frac{bc}{d}\right)} E_3\left(-\frac{(dx+c)b}{d}\right)}{8(dx+c)^2d} - \frac{e^{\left(3a - \frac{3bc}{d}\right)} E_3\left(-\frac{3(dx+c)b}{d}\right)}{8(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c)^3,x, algorithm="maxima")

[Out]  $-1/8*e^{(-3*a + 3*b*c/d)*\exp\_integral\_e(3, 3*(d*x + c)*b/d)/((d*x + c)^{2*d})}$   
 $- 3/8*e^{(-a + b*c/d)*\exp\_integral\_e(3, (d*x + c)*b/d)/((d*x + c)^{2*d})} - 3/8$   
 $*e^{(a - b*c/d)*\exp\_integral\_e(3, -(d*x + c)*b/d)/((d*x + c)^{2*d})} - 1/8*e^{(3$   
 $*a - 3*b*c/d)*\exp\_integral\_e(3, -3*(d*x + c)*b/d)/((d*x + c)^{2*d})}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + bx)^3}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^3/(c + d\*x)^3,x)

[Out] int(cosh(a + b\*x)^3/(c + d\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(a + bx)}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*\*3/(d\*x+c)\*\*3,x)

[Out] Integral(cosh(a + b\*x)\*\*3/(c + d\*x)\*\*3, x)

### 3.23 $\int x^3 \cosh^4(a + bx) dx$

**Optimal.** Leaf size=172

$$\frac{3 \cosh^4(a + bx)}{128b^4} - \frac{45 \cosh^2(a + bx)}{128b^4} + \frac{3x \sinh(a + bx) \cosh^3(a + bx)}{32b^3} + \frac{45x \sinh(a + bx) \cosh(a + bx)}{64b^3} - \frac{3x^2 \cosh^4(a + bx)}{16b^2}$$

[Out]  $45/128*x^2/b^2+3/32*x^4-45/128*\cosh(b*x+a)^2/b^4-9/16*x^2*\cosh(b*x+a)^2/b^2-3/128*\cosh(b*x+a)^4/b^4-3/16*x^2*\cosh(b*x+a)^4/b^2+45/64*x*\cosh(b*x+a)*\sinh(b*x+a)/b^3+3/8*x^3*\cosh(b*x+a)*\sinh(b*x+a)/b+3/32*x*\cosh(b*x+a)^3*\sinh(b*x+a)/b^3+1/4*x^3*\cosh(b*x+a)^3*\sinh(b*x+a)/b$

**Rubi [A]** time = 0.15, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3311, 30, 3310}

$$\frac{3x^2 \cosh^4(a + bx)}{16b^2} - \frac{9x^2 \cosh^2(a + bx)}{16b^2} - \frac{3 \cosh^4(a + bx)}{128b^4} - \frac{45 \cosh^2(a + bx)}{128b^4} + \frac{3x \sinh(a + bx) \cosh^3(a + bx)}{32b^3} + \frac{45x \sinh(a + bx) \cosh(a + bx)}{64b^3} - \frac{3x^2 \cosh^4(a + bx)}{16b^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*\text{Cosh}[a + b*x]^4, x]$

[Out]  $(45*x^2)/(128*b^2) + (3*x^4)/32 - (45*\text{Cosh}[a + b*x]^2)/(128*b^4) - (9*x^2*\text{Cosh}[a + b*x]^2)/(16*b^2) - (3*\text{Cosh}[a + b*x]^4)/(128*b^4) - (3*x^2*\text{Cosh}[a + b*x]^4)/(16*b^2) + (45*x*\text{Cosh}[a + b*x]*\text{Sinh}[a + b*x])/(64*b^3) + (3*x^3*\text{Cosh}[a + b*x]*\text{Sinh}[a + b*x])/(8*b) + (3*x*\text{Cosh}[a + b*x]^3*\text{Sinh}[a + b*x])/(32*b^3) + (x^3*\text{Cosh}[a + b*x]^3*\text{Sinh}[a + b*x])/(4*b)$

#### Rule 30

$\text{Int}[(x_)^{(m_.)}, x\_Symbol] \text{ :> } \text{Simp}[x^{(m + 1)}/(m + 1), x] \text{ /; } \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 3310

$\text{Int}[((c_.) + (d_.)*(x_))*((b_.)*\sin[(e_.) + (f_.)*(x_)])^{(n_.)}, x\_Symbol] \text{ :> } \text{Simp}[(d*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n - 1))/n, \text{Int}[(c + d*x)*(b*\sin[e + f*x])^{(n - 2)}, x], x] - \text{Simp}[(b*(c + d*x)*\cos[e + f*x]*(b*\sin[e + f*x])^{(n - 1)})/(f*n), x]) \text{ /; } \text{FreeQ}\{b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[n, 1]$

#### Rule 3311

$\text{Int}[((c_.) + (d_.)*(x_))^{(m_.)}*((b_.)*\sin[(e_.) + (f_.)*(x_)])^{(n_.)}, x\_Symbol] \text{ :> } \text{Simp}[(d*m*(c + d*x)^{(m - 1)}*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}$

$[(b^2(n-1))/n, \text{Int}[(c+dx)^m(b\sin[e+fx])^{n-2}, x], x] - \text{Dist}[(d^2m(m-1))/(f^2n^2), \text{Int}[(c+dx)^{m-2}(b\sin[e+fx])^n, x], x] - \text{Simp}[(b(c+dx)^m\cos[e+fx](b\sin[e+fx])^{n-1})/(fn), x] /;$   
 $\text{FreeQ}\{b, c, d, e, f, x\} \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{GtQ}[m, 1]$

### Rubi steps

$$\begin{aligned} \int x^3 \cosh^4(a+bx) dx &= -\frac{3x^2 \cosh^4(a+bx)}{16b^2} + \frac{x^3 \cosh^3(a+bx) \sinh(a+bx)}{4b} + \frac{3}{4} \int x^3 \cosh^2(a+bx) dx + \frac{3}{4} \int x^3 \cosh^2(a+bx) dx \\ &= -\frac{9x^2 \cosh^2(a+bx)}{16b^2} - \frac{3 \cosh^4(a+bx)}{128b^4} - \frac{3x^2 \cosh^4(a+bx)}{16b^2} + \frac{3x^3 \cosh(a+bx) \sinh(a+bx)}{8b} \\ &= \frac{3x^4}{32} - \frac{45 \cosh^2(a+bx)}{128b^4} - \frac{9x^2 \cosh^2(a+bx)}{16b^2} - \frac{3 \cosh^4(a+bx)}{128b^4} - \frac{3x^2 \cosh^4(a+bx)}{16b^2} + \\ &= \frac{45x^2}{128b^2} + \frac{3x^4}{32} - \frac{45 \cosh^2(a+bx)}{128b^4} - \frac{9x^2 \cosh^2(a+bx)}{16b^2} - \frac{3 \cosh^4(a+bx)}{128b^4} - \frac{3x^2 \cosh^4(a+bx)}{16b^2} \end{aligned}$$

**Mathematica [A]** time = 0.41, size = 100, normalized size = 0.58

$$\frac{-192(2b^2x^2+1)\cosh(2(a+bx)) - 3(8b^2x^2+1)\cosh(4(a+bx)) + 4bx(32(2b^2x^2+3)\sinh(2(a+bx)) + (8b^2x^2+1)\sinh(4(a+bx)))}{1024b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Cosh[a + b\*x]^4,x]

[Out] (-192\*(1 + 2\*b^2\*x^2)\*Cosh[2\*(a + b\*x)] - 3\*(1 + 8\*b^2\*x^2)\*Cosh[4\*(a + b\*x)] + 4\*b\*x\*(24\*b^3\*x^3 + 32\*(3 + 2\*b^2\*x^2)\*Sinh[2\*(a + b\*x)] + (3 + 8\*b^2\*x^2)\*Sinh[4\*(a + b\*x)])/(1024\*b^4)

**fricas [A]** time = 0.56, size = 195, normalized size = 1.13

$$\frac{96b^4x^4 - 3(8b^2x^2+1)\cosh(bx+a)^4 + 16(8b^3x^3+3bx)\cosh(bx+a)\sinh(bx+a)^3 - 3(8b^2x^2+1)\sinh(bx+a)^4 - 192(2b^2x^2+1)\cosh(bx+a)^2 - 6(64b^2x^2+3(8b^2x^2+1)\cosh(bx+a))\sinh(bx+a)^2}{1024b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(b\*x+a)^4,x, algorithm="fricas")

[Out] 1/1024\*(96\*b^4\*x^4 - 3\*(8\*b^2\*x^2 + 1)\*cosh(b\*x + a)^4 + 16\*(8\*b^3\*x^3 + 3\*b\*x)\*cosh(b\*x + a)\*sinh(b\*x + a)^3 - 3\*(8\*b^2\*x^2 + 1)\*sinh(b\*x + a)^4 - 192\*(2\*b^2\*x^2 + 1)\*cosh(b\*x + a)^2 - 6\*(64\*b^2\*x^2 + 3\*(8\*b^2\*x^2 + 1)\*cosh(b\*x + a))\*sinh(b\*x + a)^2)

$$b^3x^3 + a)^2 + 32) \sinh(b^3x^3 + a)^2 + 16 * ((8b^3x^3 + 3b^3x) \cosh(b^3x^3 + a)^3 + 16 * (2b^3x^3 + 3b^3x) \cosh(b^3x^3 + a)) \sinh(b^3x^3 + a) / b^4$$

**giac** [A] time = 0.12, size = 150, normalized size = 0.87

$$\frac{3}{32} x^4 + \frac{(32b^3x^3 - 24b^2x^2 + 12bx - 3)e^{(4bx+4a)}}{2048b^4} + \frac{(4b^3x^3 - 6b^2x^2 + 6bx - 3)e^{(2bx+2a)}}{32b^4} - \frac{(4b^3x^3 + 6b^2x^2 + 6bx - 3)e^{(4bx+4a)}}{32b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(b\*x+a)^4,x, algorithm="giac")

[Out] 3/32\*x^4 + 1/2048\*(32\*b^3\*x^3 - 24\*b^2\*x^2 + 12\*b\*x - 3)\*e^(4\*b\*x + 4\*a)/b^4 + 1/32\*(4\*b^3\*x^3 - 6\*b^2\*x^2 + 6\*b\*x - 3)\*e^(2\*b\*x + 2\*a)/b^4 - 1/32\*(4\*b^3\*x^3 + 6\*b^2\*x^2 + 6\*b\*x + 3)\*e^(-2\*b\*x - 2\*a)/b^4 - 1/2048\*(32\*b^3\*x^3 + 24\*b^2\*x^2 + 12\*b\*x + 3)\*e^(-4\*b\*x - 4\*a)/b^4

**maple** [B] time = 0.17, size = 400, normalized size = 2.33

$$\frac{(bx+a)^3 \sinh(bx+a) (\cosh^3(bx+a))}{4} + \frac{3(bx+a)^3 \cosh(bx+a) \sinh(bx+a)}{8} + \frac{3(bx+a)^4}{32} - \frac{3(bx+a)^2 (\cosh^4(bx+a))}{16} + \frac{3(bx+a) \sinh(bx+a) (\cosh^3(bx+a))}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*cosh(b\*x+a)^4,x)

[Out] 1/b^4\*(1/4\*(b\*x+a)^3\*sinh(b\*x+a)\*cosh(b\*x+a)^3+3/8\*(b\*x+a)^3\*cosh(b\*x+a)\*sinh(b\*x+a)+3/32\*(b\*x+a)^4-3/16\*(b\*x+a)^2\*cosh(b\*x+a)^4+3/32\*(b\*x+a)\*sinh(b\*x+a)\*cosh(b\*x+a)^3+45/64\*(b\*x+a)\*cosh(b\*x+a)\*sinh(b\*x+a)+45/128\*(b\*x+a)^2-3/128\*cosh(b\*x+a)^4-45/128\*cosh(b\*x+a)^2-9/16\*(b\*x+a)^2\*cosh(b\*x+a)^2-3\*a\*(1/4\*(b\*x+a)^2\*sinh(b\*x+a)\*cosh(b\*x+a)^3+3/8\*(b\*x+a)^2\*cosh(b\*x+a)\*sinh(b\*x+a)+1/8\*(b\*x+a)^3-1/8\*(b\*x+a)\*cosh(b\*x+a)^4+1/32\*sinh(b\*x+a)\*cosh(b\*x+a)^3+15/64\*cosh(b\*x+a)\*sinh(b\*x+a)+15/64\*b\*x+15/64\*a-3/8\*(b\*x+a)\*cosh(b\*x+a)^2)+3\*a^2\*(1/4\*(b\*x+a)\*sinh(b\*x+a)\*cosh(b\*x+a)^3+3/8\*(b\*x+a)\*cosh(b\*x+a)\*sinh(b\*x+a)+3/16\*(b\*x+a)^2-1/16\*cosh(b\*x+a)^4-3/16\*cosh(b\*x+a)^2)-a^3\*((1/4\*cosh(b\*x+a)^3+3/8\*cosh(b\*x+a))\*sinh(b\*x+a)+3/8\*b\*x+3/8\*a))

**maxima** [A] time = 0.36, size = 176, normalized size = 1.02

$$\frac{3}{32} x^4 + \frac{(32b^3x^3e^{(4a)} - 24b^2x^2e^{(4a)} + 12bx e^{(4a)} - 3e^{(4a)})e^{(4bx)}}{2048b^4} + \frac{(4b^3x^3e^{(2a)} - 6b^2x^2e^{(2a)} + 6bx e^{(2a)} - 3e^{(2a)})e^{(2bx)}}{32b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(b\*x+a)^4,x, algorithm="maxima")

[Out]  $\frac{3}{32}x^4 + \frac{1}{2048}(32b^3x^3e^{(4a)} - 24b^2x^2e^{(4a)} + 12bx^1e^{(4a)} - 3e^{(4a)})e^{(4bx)}/b^4 + \frac{1}{32}(4b^3x^3e^{(2a)} - 6b^2x^2e^{(2a)} + 6bx^1e^{(2a)} - 3e^{(2a)})e^{(2bx)}/b^4 - \frac{1}{32}(4b^3x^3 + 6b^2x^2 + 6bx + 3)e^{(-2bx - 2a)}/b^4 - \frac{1}{2048}(32b^3x^3 + 24b^2x^2 + 12bx + 3)e^{(-4bx - 4a)}/b^4$

**mupad [B]** time = 0.37, size = 129, normalized size = 0.75

$$\frac{3x^4}{32} - \frac{\frac{3\cosh(2a+2bx)}{16} + \frac{3\cosh(4a+4bx)}{1024} + b^2\left(\frac{3x^2\cosh(2a+2bx)}{8} + \frac{3x^2\cosh(4a+4bx)}{128}\right) - b\left(\frac{3x\sinh(2a+2bx)}{8} + \frac{3x\sinh(4a+4bx)}{256}\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*cosh(a + b*x)^4, x)`

[Out]  $(3x^4)/32 - ((3\cosh(2a + 2bx))/16 + (3\cosh(4a + 4bx))/1024 + b^2*((3x^2\cosh(2a + 2bx))/8 + (3x^2\cosh(4a + 4bx))/128) - b*((3x\sinh(2a + 2bx))/8 + (3x\sinh(4a + 4bx))/256) - b^3*((x^3\sinh(2a + 2bx))/4 + (x^3\sinh(4a + 4bx))/32))/b^4$

**sympy [A]** time = 5.70, size = 253, normalized size = 1.47

$$\left\{ \begin{array}{l} \frac{3x^4 \sinh^4(a+bx)}{32} - \frac{3x^4 \sinh^2(a+bx) \cosh^2(a+bx)}{16} + \frac{3x^4 \cosh^4(a+bx)}{32} - \frac{3x^3 \sinh^3(a+bx) \cosh(a+bx)}{8b} + \frac{5x^3 \sinh(a+bx) \cosh^3(a+bx)}{8b} + \frac{45x^2}{4} \\ \frac{x^4 \cosh^4(a)}{4} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*cosh(b*x+a)**4, x)`

[Out] `Piecewise((3*x**4*sinh(a + b*x)**4/32 - 3*x**4*sinh(a + b*x)**2*cosh(a + b*x)**2/16 + 3*x**4*cosh(a + b*x)**4/32 - 3*x**3*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + 5*x**3*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) + 45*x**2*sinh(a + b*x)**4/(128*b**2) - 9*x**2*sinh(a + b*x)**2*cosh(a + b*x)**2/(64*b**2) - 51*x**2*cosh(a + b*x)**4/(128*b**2) - 45*x*sinh(a + b*x)**3*cosh(a + b*x)/(64*b**3) + 51*x*sinh(a + b*x)*cosh(a + b*x)**3/(64*b**3) + 45*sinh(a + b*x)**4/(256*b**4) - 51*cosh(a + b*x)**4/(256*b**4), Ne(b, 0)), (x**4*cosh(a)**4/4, True))`

### 3.24 $\int x^2 \cosh^4(a + bx) dx$

Optimal. Leaf size=134

$$\frac{\sinh(a + bx) \cosh^3(a + bx)}{32b^3} + \frac{15 \sinh(a + bx) \cosh(a + bx)}{64b^3} - \frac{x \cosh^4(a + bx)}{8b^2} - \frac{3x \cosh^2(a + bx)}{8b^2} + \frac{x^2 \sinh(a + bx)}{b}$$

[Out] 15/64\*x/b^2+1/8\*x^3-3/8\*x\*cosh(b\*x+a)^2/b^2-1/8\*x\*cosh(b\*x+a)^4/b^2+15/64\*cosh(b\*x+a)\*sinh(b\*x+a)/b^3+3/8\*x^2\*cosh(b\*x+a)\*sinh(b\*x+a)/b+1/32\*cosh(b\*x+a)^3\*sinh(b\*x+a)/b^3+1/4\*x^2\*cosh(b\*x+a)^3\*sinh(b\*x+a)/b

**Rubi [A]** time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3311, 30, 2635, 8}

$$-\frac{x \cosh^4(a + bx)}{8b^2} - \frac{3x \cosh^2(a + bx)}{8b^2} + \frac{\sinh(a + bx) \cosh^3(a + bx)}{32b^3} + \frac{15 \sinh(a + bx) \cosh(a + bx)}{64b^3} + \frac{x^2 \sinh(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Cosh[a + b\*x]^4,x]

[Out] (15\*x)/(64\*b^2) + x^3/8 - (3\*x\*Cosh[a + b\*x]^2)/(8\*b^2) - (x\*Cosh[a + b\*x]^4)/(8\*b^2) + (15\*Cosh[a + b\*x]\*Sinh[a + b\*x])/(64\*b^3) + (3\*x^2\*Cosh[a + b\*x]\*Sinh[a + b\*x])/(8\*b) + (Cosh[a + b\*x]^3\*Sinh[a + b\*x])/(32\*b^3) + (x^2\*Cosh[a + b\*x]^3\*Sinh[a + b\*x])/(4\*b)

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x])\*(b\*Sine[c + d\*x])^(n - 1)/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sine[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

### Rubi steps

$$\begin{aligned} \int x^2 \cosh^4(a + bx) dx &= -\frac{x \cosh^4(a + bx)}{8b^2} + \frac{x^2 \cosh^3(a + bx) \sinh(a + bx)}{4b} + \frac{3}{4} \int x^2 \cosh^2(a + bx) dx + \frac{\int \cosh^4(a + bx) dx}{4} \\ &= -\frac{3x \cosh^2(a + bx)}{8b^2} - \frac{x \cosh^4(a + bx)}{8b^2} + \frac{3x^2 \cosh(a + bx) \sinh(a + bx)}{8b} + \frac{\cosh^3(a + bx)}{32b} \\ &= \frac{x^3}{8} - \frac{3x \cosh^2(a + bx)}{8b^2} - \frac{x \cosh^4(a + bx)}{8b^2} + \frac{15 \cosh(a + bx) \sinh(a + bx)}{64b^3} + \frac{3x^2 \cosh(a + bx)}{8b} \\ &= \frac{15x}{64b^2} + \frac{x^3}{8} - \frac{3x \cosh^2(a + bx)}{8b^2} - \frac{x \cosh^4(a + bx)}{8b^2} + \frac{15 \cosh(a + bx) \sinh(a + bx)}{64b^3} + \frac{3x^2 \cosh(a + bx)}{8b} \end{aligned}$$

**Mathematica** [A] time = 0.16, size = 90, normalized size = 0.67

$$\frac{64b^2x^2 \sinh(2(a + bx)) + 8b^2x^2 \sinh(4(a + bx)) + 32 \sinh(2(a + bx)) + \sinh(4(a + bx)) - 64bx \cosh(2(a + bx)) - 32bx \cosh(4(a + bx))}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^4,x]
```

```
[Out] (32*b^3*x^3 - 64*b*x*Cosh[2*(a + b*x)] - 4*b*x*Cosh[4*(a + b*x)] + 32*Sinh[2*(a + b*x)] + 64*b^2*x^2*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)] + 8*b^2*x^2*Sinh[4*(a + b*x)])/(256*b^3)
```

**fricas** [A] time = 0.79, size = 147, normalized size = 1.10

$$\frac{8b^3x^3 - bx \cosh(bx + a)^4 - bx \sinh(bx + a)^4 + (8b^2x^2 + 1) \cosh(bx + a) \sinh(bx + a)^3 - 16bx \cosh(bx + a)^2 - 16bx \sinh(bx + a)^2}{256b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/64*(8*b^3*x^3 - b*x*cosh(b*x + a)^4 - b*x*sinh(b*x + a)^4 + (8*b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 - 16*b*x*cosh(b*x + a)^2 - 2*(3*b*x*cosh(b*x + a)*sinh(b*x + a)^2 - 16*b*x*sinh(b*x + a)^2))
```



$$*x + a)^2 + 8*b*x)*\sinh(b*x + a)^2 + ((8*b^2*x^2 + 1)*\cosh(b*x + a)^3 + 16*(2*b^2*x^2 + 1)*\cosh(b*x + a))*\sinh(b*x + a))/b^3$$

**giac** [A] time = 0.12, size = 118, normalized size = 0.88

$$\frac{1}{8}x^3 + \frac{(8b^2x^2 - 4bx + 1)e^{4bx+4a}}{512b^3} + \frac{(2b^2x^2 - 2bx + 1)e^{2bx+2a}}{16b^3} - \frac{(2b^2x^2 + 2bx + 1)e^{-2bx-2a}}{16b^3} - \frac{(8b^2x^2 + 4bx + 1)e^{-4bx-4a}}{512b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cosh(b\*x+a)^4,x, algorithm="giac")

[Out] 1/8\*x^3 + 1/512\*(8\*b^2\*x^2 - 4\*b\*x + 1)\*e^(4\*b\*x + 4\*a)/b^3 + 1/16\*(2\*b^2\*x^2 - 2\*b\*x + 1)\*e^(2\*b\*x + 2\*a)/b^3 - 1/16\*(2\*b^2\*x^2 + 2\*b\*x + 1)\*e^(-2\*b\*x - 2\*a)/b^3 - 1/512\*(8\*b^2\*x^2 + 4\*b\*x + 1)\*e^(-4\*b\*x - 4\*a)/b^3

**maple** [A] time = 0.16, size = 237, normalized size = 1.77

$$\frac{(bx+a)^2 \sinh(bx+a)(\cosh^3(bx+a))}{4} + \frac{3(bx+a)^2 \cosh(bx+a) \sinh(bx+a)}{8} + \frac{(bx+a)^3}{8} - \frac{(bx+a)(\cosh^4(bx+a))}{8} + \frac{\sinh(bx+a)(\cosh^3(bx+a))}{32} + \frac{15(bx+a)^2 \cosh^2(bx+a) \sinh(bx+a)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cosh(b\*x+a)^4,x)

[Out] 1/b^3\*(1/4\*(b\*x+a)^2\*sinh(b\*x+a)\*cosh(b\*x+a)^3+3/8\*(b\*x+a)^2\*cosh(b\*x+a)\*sinh(b\*x+a)+1/8\*(b\*x+a)^3-1/8\*(b\*x+a)\*cosh(b\*x+a)^4+1/32\*sinh(b\*x+a)\*cosh(b\*x+a)^3+15/64\*cosh(b\*x+a)\*sinh(b\*x+a)+15/64\*b\*x+15/64\*a-3/8\*(b\*x+a)\*cosh(b\*x+a)^2-2\*a\*(1/4\*(b\*x+a)\*sinh(b\*x+a)\*cosh(b\*x+a)^3+3/8\*(b\*x+a)\*cosh(b\*x+a)\*sinh(b\*x+a)+3/16\*(b\*x+a)^2-1/16\*cosh(b\*x+a)^4-3/16\*cosh(b\*x+a)^2)+a^2\*((1/4\*cosh(b\*x+a)^3+3/8\*cosh(b\*x+a))\*sinh(b\*x+a)+3/8\*b\*x+3/8\*a)

**maxima** [A] time = 0.38, size = 132, normalized size = 0.99

$$\frac{1}{8}x^3 + \frac{(8b^2x^2e^{4a} - 4bx e^{4a} + e^{4a})e^{4bx}}{512b^3} + \frac{(2b^2x^2e^{2a} - 2bx e^{2a} + e^{2a})e^{2bx}}{16b^3} - \frac{(2b^2x^2 + 2bx + 1)e^{-2bx-2a}}{16b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cosh(b\*x+a)^4,x, algorithm="maxima")

[Out] 1/8\*x^3 + 1/512\*(8\*b^2\*x^2\*e^(4\*a) - 4\*b\*x\*e^(4\*a) + e^(4\*a))\*e^(4\*b\*x)/b^3 + 1/16\*(2\*b^2\*x^2\*e^(2\*a) - 2\*b\*x\*e^(2\*a) + e^(2\*a))\*e^(2\*b\*x)/b^3 - 1/16\*(2\*b^2\*x^2 + 2\*b\*x + 1)\*e^(-2\*b\*x - 2\*a)/b^3 - 1/512\*(8\*b^2\*x^2 + 4\*b\*x + 1)\*e^(-4\*b\*x - 4\*a)/b^3

**mupad [B]** time = 0.25, size = 94, normalized size = 0.70

$$\frac{\frac{\sinh(2a+2bx)}{8} + \frac{\sinh(4a+4bx)}{256} - b \left( \frac{x \cosh(2a+2bx)}{4} + \frac{x \cosh(4a+4bx)}{64} \right) + b^2 \left( \frac{x^2 \sinh(2a+2bx)}{4} + \frac{x^2 \sinh(4a+4bx)}{32} \right)}{b^3} + \frac{x^3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cosh(a + b\*x)^4,x)

[Out] (sinh(2\*a + 2\*b\*x)/8 + sinh(4\*a + 4\*b\*x)/256 - b\*((x\*cosh(2\*a + 2\*b\*x))/4 + (x\*cosh(4\*a + 4\*b\*x))/64) + b^2\*((x^2\*sinh(2\*a + 2\*b\*x))/4 + (x^2\*sinh(4\*a + 4\*b\*x))/32))/b^3 + x^3/8

**sympy [A]** time = 3.27, size = 209, normalized size = 1.56

$$\left\{ \begin{array}{l} \frac{x^3 \sinh^4(a+bx)}{8} - \frac{x^3 \sinh^2(a+bx) \cosh^2(a+bx)}{4} + \frac{x^3 \cosh^4(a+bx)}{8} - \frac{3x^2 \sinh^3(a+bx) \cosh(a+bx)}{8b} + \frac{5x^2 \sinh(a+bx) \cosh^3(a+bx)}{8b} + \frac{15x \sinh^2(a+bx) \cosh^2(a+bx)}{64} \\ \frac{x^3 \cosh^4(a)}{3} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*cosh(b\*x+a)\*\*4,x)

[Out] Piecewise((x\*\*3\*sinh(a + b\*x)\*\*4/8 - x\*\*3\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)\*\*2/4 + x\*\*3\*cosh(a + b\*x)\*\*4/8 - 3\*x\*\*2\*sinh(a + b\*x)\*\*3\*cosh(a + b\*x)/(8\*b) + 5\*x\*\*2\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*3/(8\*b) + 15\*x\*sinh(a + b\*x)\*\*4/(64\*b\*\*2) - 3\*x\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)\*\*2/(32\*b\*\*2) - 17\*x\*cosh(a + b\*x)\*\*4/(64\*b\*\*2) - 15\*sinh(a + b\*x)\*\*3\*cosh(a + b\*x)/(64\*b\*\*3) + 17\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*3/(64\*b\*\*3), Ne(b, 0)), (x\*\*3\*cosh(a)\*\*4/3, True))

### 3.25 $\int x \cosh^4(a + bx) dx$

Optimal. Leaf size=80

$$-\frac{\cosh^4(a + bx)}{16b^2} - \frac{3 \cosh^2(a + bx)}{16b^2} + \frac{x \sinh(a + bx) \cosh^3(a + bx)}{4b} + \frac{3x \sinh(a + bx) \cosh(a + bx)}{8b} + \frac{3x^2}{16}$$

[Out]  $3/16*x^2-3/16*\cosh(b*x+a)^2/b^2-1/16*\cosh(b*x+a)^4/b^2+3/8*x*\cosh(b*x+a)*\sinh(b*x+a)/b+1/4*x*\cosh(b*x+a)^3*\sinh(b*x+a)/b$

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3310, 30}

$$-\frac{\cosh^4(a + bx)}{16b^2} - \frac{3 \cosh^2(a + bx)}{16b^2} + \frac{x \sinh(a + bx) \cosh^3(a + bx)}{4b} + \frac{3x \sinh(a + bx) \cosh(a + bx)}{8b} + \frac{3x^2}{16}$$

Antiderivative was successfully verified.

[In] Int[x\*Cosh[a + b\*x]^4,x]

[Out]  $(3*x^2)/16 - (3*\text{Cosh}[a + b*x]^2)/(16*b^2) - \text{Cosh}[a + b*x]^4/(16*b^2) + (3*x*\text{Cosh}[a + b*x]*\text{Sinh}[a + b*x])/(8*b) + (x*\text{Cosh}[a + b*x]^3*\text{Sinh}[a + b*x])/(4*b)$

#### Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 3310

Int[((c\_) + (d\_)\*(x\_))\*((b\_)\*sin[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*(b\*Sine[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)\*(b\*Sine[e + f\*x])^(n - 2), x], x] - Simp[(b\*(c + d\*x)\*Cos[e + f\*x]\*(b\*Sine[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

#### Rubi steps

$$\begin{aligned} \int x \cosh^4(a + bx) dx &= -\frac{\cosh^4(a + bx)}{16b^2} + \frac{x \cosh^3(a + bx) \sinh(a + bx)}{4b} + \frac{3}{4} \int x \cosh^2(a + bx) dx \\ &= -\frac{3 \cosh^2(a + bx)}{16b^2} - \frac{\cosh^4(a + bx)}{16b^2} + \frac{3x \cosh(a + bx) \sinh(a + bx)}{8b} + \frac{x \cosh^3(a + bx) \sinh(a + bx)}{4b} \\ &= \frac{3x^2}{16} - \frac{3 \cosh^2(a + bx)}{16b^2} - \frac{\cosh^4(a + bx)}{16b^2} + \frac{3x \cosh(a + bx) \sinh(a + bx)}{8b} + \frac{x \cosh^3(a + bx) \sinh(a + bx)}{4b} \end{aligned}$$

**Mathematica** [A] time = 0.18, size = 53, normalized size = 0.66

$$\frac{-4bx(8 \sinh(2(a + bx)) + \sinh(4(a + bx)) + 6bx) + 16 \cosh(2(a + bx)) + \cosh(4(a + bx))}{128b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Cosh[a + b\*x]^4, x]

[Out] -1/128\*(16\*Cosh[2\*(a + b\*x)] + Cosh[4\*(a + b\*x)] - 4\*b\*x\*(6\*b\*x + 8\*Sinh[2\*(a + b\*x)] + Sinh[4\*(a + b\*x)]))/b^2

**fricas** [A] time = 1.19, size = 114, normalized size = 1.42

$$\frac{16bx \cosh(bx + a) \sinh(bx + a)^3 + 24b^2x^2 - \cosh(bx + a)^4 - \sinh(bx + a)^4 - 2(3 \cosh(bx + a)^2 + 8) \sinh(bx + a)}{128b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(b\*x+a)^4,x, algorithm="fricas")

[Out] 1/128\*(16\*b\*x\*cosh(b\*x + a)\*sinh(b\*x + a)^3 + 24\*b^2\*x^2 - cosh(b\*x + a)^4 - sinh(b\*x + a)^4 - 2\*(3\*cosh(b\*x + a)^2 + 8)\*sinh(b\*x + a)^2 - 16\*cosh(b\*x + a)^2 + 16\*(b\*x\*cosh(b\*x + a)^3 + 4\*b\*x\*cosh(b\*x + a))\*sinh(b\*x + a))/b^2

**giac** [A] time = 0.12, size = 86, normalized size = 1.08

$$\frac{3}{16}x^2 + \frac{(4bx - 1)e^{(4bx+4a)}}{256b^2} + \frac{(2bx - 1)e^{(2bx+2a)}}{16b^2} - \frac{(2bx + 1)e^{(-2bx-2a)}}{16b^2} - \frac{(4bx + 1)e^{(-4bx-4a)}}{256b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(b\*x+a)^4,x, algorithm="giac")

[Out] 3/16\*x^2 + 1/256\*(4\*b\*x - 1)\*e^(4\*b\*x + 4\*a)/b^2 + 1/16\*(2\*b\*x - 1)\*e^(2\*b\*x + 2\*a)/b^2 - 1/16\*(2\*b\*x + 1)\*e^(-2\*b\*x - 2\*a)/b^2 - 1/256\*(4\*b\*x + 1)\*e^(-4\*b\*x - 4\*a)/b^2

**maple [A]** time = 0.16, size = 112, normalized size = 1.40

$$\frac{(bx+a) \sinh(bx+a) (\cosh^3(bx+a))}{4} + \frac{3(bx+a) \cosh(bx+a) \sinh(bx+a)}{8} + \frac{3(bx+a)^2}{16} - \frac{(\cosh^4(bx+a))}{16} - \frac{3(\cosh^2(bx+a))}{16} - a \left( \left( \frac{(\cosh^3(bx+a))}{4} + \right. \right.$$


---


$$\left. \left. \frac{3(bx+a) \cosh(bx+a) \sinh(bx+a)}{8} + \frac{3(bx+a)^2}{16} - \frac{(\cosh^4(bx+a))}{16} - \frac{3(\cosh^2(bx+a))}{16} - a \left( \frac{(\cosh^3(bx+a))}{4} + \right. \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cosh(b\*x+a)^4,x)

[Out] 1/b^2\*(1/4\*(b\*x+a)\*sinh(b\*x+a)\*cosh(b\*x+a)^3+3/8\*(b\*x+a)\*cosh(b\*x+a)\*sinh(b\*x+a)+3/16\*(b\*x+a)^2-1/16\*cosh(b\*x+a)^4-3/16\*cosh(b\*x+a)^2-a\*((1/4\*cosh(b\*x+a)^3+3/8\*cosh(b\*x+a))\*sinh(b\*x+a)+3/8\*b\*x+3/8\*a))

**maxima [A]** time = 0.39, size = 96, normalized size = 1.20

$$\frac{3}{16} x^2 + \frac{(4bx e^{4a} - e^{4a}) e^{4bx}}{256 b^2} + \frac{(2bx e^{2a} - e^{2a}) e^{2bx}}{16 b^2} - \frac{(2bx + 1) e^{-2bx - 2a}}{16 b^2} - \frac{(4bx + 1) e^{-4bx - 4a}}{256 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(b\*x+a)^4,x, algorithm="maxima")

[Out] 3/16\*x^2 + 1/256\*(4\*b\*x\*e^(4\*a) - e^(4\*a))\*e^(4\*b\*x)/b^2 + 1/16\*(2\*b\*x\*e^(2\*a) - e^(2\*a))\*e^(2\*b\*x)/b^2 - 1/16\*(2\*b\*x + 1)\*e^(-2\*b\*x - 2\*a)/b^2 - 1/256\*(4\*b\*x + 1)\*e^(-4\*b\*x - 4\*a)/b^2

**mupad [B]** time = 0.15, size = 68, normalized size = 0.85

$$\frac{3x^2}{16} - \frac{\frac{3 \cosh(a+bx)^2}{16} + \frac{\cosh(a+bx)^4}{16}}{b^2} - b \left( \frac{x \sinh(a+bx) \cosh(a+bx)^3}{4} + \frac{3x \sinh(a+bx) \cosh(a+bx)}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cosh(a + b\*x)^4,x)

[Out] (3\*x^2)/16 - ((3\*cosh(a + b\*x)^2)/16 + cosh(a + b\*x)^4/16 - b\*((x\*cosh(a + b\*x)^3\*sinh(a + b\*x))/4 + (3\*x\*cosh(a + b\*x)\*sinh(a + b\*x))/8))/b^2

**sympy [A]** time = 1.83, size = 138, normalized size = 1.72

$$\left\{ \begin{array}{l} \frac{3x^2 \sinh^4(a+bx)}{16} - \frac{3x^2 \sinh^2(a+bx) \cosh^2(a+bx)}{8} + \frac{3x^2 \cosh^4(a+bx)}{16} - \frac{3x \sinh^3(a+bx) \cosh(a+bx)}{8b} + \frac{5x \sinh(a+bx) \cosh^3(a+bx)}{8b} + \frac{3 \sinh(a+bx) \cosh^5(a+bx)}{8b^2} \\ \frac{x^2 \cosh^4(a)}{2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x**2*sinh(a + b*x)**4/16 - 3*x**2*sinh(a + b*x)**2*cosh(a + b*x)**2/8 + 3*x**2*cosh(a + b*x)**4/16 - 3*x*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + 5*x*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) + 3*sinh(a + b*x)**4/(32*b**2) - 5*cosh(a + b*x)**4/(32*b**2), Ne(b, 0)), (x**2*cosh(a)**4/2, True))
```

### 3.26 $\int (c + dx)^3 \operatorname{sech}(a + bx) dx$

**Optimal.** Leaf size=179

$$\frac{6id^3 \operatorname{Li}_4(-ie^{a+bx})}{b^4} + \frac{6id^3 \operatorname{Li}_4(ie^{a+bx})}{b^4} + \frac{6id^2(c+dx) \operatorname{Li}_3(-ie^{a+bx})}{b^3} - \frac{6id^2(c+dx) \operatorname{Li}_3(ie^{a+bx})}{b^3} - \frac{3id(c+dx)^2 \operatorname{Li}_2(-ie^{a+bx})}{b^2}$$

[Out]  $2*(d*x+c)^3*\arctan(\exp(b*x+a))/b-3*I*d*(d*x+c)^2*\operatorname{polylog}(2,-I*\exp(b*x+a))/b^2+3*I*d*(d*x+c)^2*\operatorname{polylog}(2,I*\exp(b*x+a))/b^2+6*I*d^2*(d*x+c)*\operatorname{polylog}(3,-I*\exp(b*x+a))/b^3-6*I*d^2*(d*x+c)*\operatorname{polylog}(3,I*\exp(b*x+a))/b^3-6*I*d^3*\operatorname{polylog}(4,-I*\exp(b*x+a))/b^4+6*I*d^3*\operatorname{polylog}(4,I*\exp(b*x+a))/b^4$

**Rubi [A]** time = 0.12, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {4180, 2531, 6609, 2282, 6589}

$$\frac{6id^2(c+dx)\operatorname{PolyLog}(3,-ie^{a+bx})}{b^3} - \frac{6id^2(c+dx)\operatorname{PolyLog}(3,ie^{a+bx})}{b^3} - \frac{3id(c+dx)^2\operatorname{PolyLog}(2,-ie^{a+bx})}{b^2} + \frac{3id(c+dx)^2\operatorname{PolyLog}(2,ie^{a+bx})}{b^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^3*\operatorname{Sech}[a + b*x], x]$

[Out]  $(2*(c + d*x)^3*\operatorname{ArcTan}[E^(a + b*x)])/b - ((3*I)*d*(c + d*x)^2*\operatorname{PolyLog}[2, (-I)*E^(a + b*x)])/b^2 + ((3*I)*d*(c + d*x)^2*\operatorname{PolyLog}[2, I*E^(a + b*x)])/b^2 + ((6*I)*d^2*(c + d*x)*\operatorname{PolyLog}[3, (-I)*E^(a + b*x)])/b^3 - ((6*I)*d^2*(c + d*x)*\operatorname{PolyLog}[3, I*E^(a + b*x)])/b^3 - ((6*I)*d^3*\operatorname{PolyLog}[4, (-I)*E^(a + b*x)])/b^4 + ((6*I)*d^3*\operatorname{PolyLog}[4, I*E^(a + b*x)])/b^4$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_.))^m_] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m-1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \operatorname{sech}(a + bx) dx &= \frac{2(c + dx)^3 \tan^{-1}(e^{a+bx})}{b} - \frac{(3id) \int (c + dx)^2 \log(1 - ie^{a+bx}) dx}{b} + \frac{(3id) \int (c + dx)^2 \log(1 + ie^{a+bx}) dx}{b} \\ &= \frac{2(c + dx)^3 \tan^{-1}(e^{a+bx})}{b} - \frac{3id(c + dx)^2 \operatorname{Li}_2(-ie^{a+bx})}{b^2} + \frac{3id(c + dx)^2 \operatorname{Li}_2(ie^{a+bx})}{b^2} + \frac{3id(c + dx)^2 \log(1 - ie^{a+bx})}{b} \\ &= \frac{2(c + dx)^3 \tan^{-1}(e^{a+bx})}{b} - \frac{3id(c + dx)^2 \operatorname{Li}_2(-ie^{a+bx})}{b^2} + \frac{3id(c + dx)^2 \operatorname{Li}_2(ie^{a+bx})}{b^2} + \frac{3id(c + dx)^2 \log(1 - ie^{a+bx})}{b} \\ &= \frac{2(c + dx)^3 \tan^{-1}(e^{a+bx})}{b} - \frac{3id(c + dx)^2 \operatorname{Li}_2(-ie^{a+bx})}{b^2} + \frac{3id(c + dx)^2 \operatorname{Li}_2(ie^{a+bx})}{b^2} + \frac{3id(c + dx)^2 \log(1 - ie^{a+bx})}{b} \\ &= \frac{2(c + dx)^3 \tan^{-1}(e^{a+bx})}{b} - \frac{3id(c + dx)^2 \operatorname{Li}_2(-ie^{a+bx})}{b^2} + \frac{3id(c + dx)^2 \operatorname{Li}_2(ie^{a+bx})}{b^2} + \frac{3id(c + dx)^2 \log(1 - ie^{a+bx})}{b} \end{aligned}$$

**Mathematica** [A] time = 2.65, size = 343, normalized size = 1.92

$$i(-2ib^3c^3 \tan^{-1}(e^{a+bx}) + 3b^3c^2 dx \log(1 - ie^{a+bx}) - 3b^3c^2 dx \log(1 + ie^{a+bx}) + 3b^3cd^2x^2 \log(1 - ie^{a+bx}) - 3b^3cd^2x^2 \log(1 + ie^{a+bx}))$$



Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3\*Sech[a + b\*x],x]

[Out] (I\*((-2\*I)\*b^3\*c^3\*ArcTan[E^(a + b\*x)] + 3\*b^3\*c^2\*d\*x\*Log[1 - I\*E^(a + b\*x)]) + 3\*b^3\*c\*d^2\*x^2\*Log[1 - I\*E^(a + b\*x)] + b^3\*d^3\*x^3\*Log[1 - I\*E^(a + b\*x)] - 3\*b^3\*c^2\*d\*x\*Log[1 + I\*E^(a + b\*x)] - 3\*b^3\*c\*d^2\*x^2\*Log[1 + I\*E^(a + b\*x)] - b^3\*d^3\*x^3\*Log[1 + I\*E^(a + b\*x)] - 3\*b^2\*d\*(c + d\*x)^2\*PolyLog[2, (-I)\*E^(a + b\*x)] + 3\*b^2\*d\*(c + d\*x)^2\*PolyLog[2, I\*E^(a + b\*x)] + 6\*b\*c\*d^2\*PolyLog[3, (-I)\*E^(a + b\*x)] + 6\*b\*d^3\*x\*PolyLog[3, (-I)\*E^(a + b\*x)] - 6\*b\*c\*d^2\*PolyLog[3, I\*E^(a + b\*x)] - 6\*b\*d^3\*x\*PolyLog[3, I\*E^(a + b\*x)] - 6\*d^3\*PolyLog[4, (-I)\*E^(a + b\*x)] + 6\*d^3\*PolyLog[4, I\*E^(a + b\*x)]))/b^4

**fricas** [C] time = 0.65, size = 493, normalized size = 2.75

$6i d^3 \text{polylog}(4, i \cosh(bx + a) + i \sinh(bx + a)) - 6i d^3 \text{polylog}(4, -i \cosh(bx + a) - i \sinh(bx + a)) + (3i b^2 d^3 \text{polylog}(4, i \cosh(bx + a) + i \sinh(bx + a)) - 3i b^2 d^3 \text{polylog}(4, -i \cosh(bx + a) - i \sinh(bx + a))) / b^4$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*sech(b\*x+a),x, algorithm="fricas")

[Out] (6\*I\*d^3\*polylog(4, I\*cosh(b\*x + a) + I\*sinh(b\*x + a)) - 6\*I\*d^3\*polylog(4, -I\*cosh(b\*x + a) - I\*sinh(b\*x + a)) + (3\*I\*b^2\*d^3\*x^2 + 6\*I\*b^2\*c\*d^2\*x + 3\*I\*b^2\*c^2\*d)\*dilog(I\*cosh(b\*x + a) + I\*sinh(b\*x + a)) + (-3\*I\*b^2\*d^3\*x^2 - 6\*I\*b^2\*c\*d^2\*x - 3\*I\*b^2\*c^2\*d)\*dilog(-I\*cosh(b\*x + a) - I\*sinh(b\*x + a)) + (I\*b^3\*c^3 - 3\*I\*a\*b^2\*c^2\*d + 3\*I\*a^2\*b\*c\*d^2 - I\*a^3\*d^3)\*log(cosh(b\*x + a) + sinh(b\*x + a) + I) + (-I\*b^3\*c^3 + 3\*I\*a\*b^2\*c^2\*d - 3\*I\*a^2\*b\*c\*d^2 + I\*a^3\*d^3)\*log(cosh(b\*x + a) + sinh(b\*x + a) - I) + (-I\*b^3\*d^3\*x^3 - 3\*I\*b^3\*c\*d^2\*x^2 - 3\*I\*b^3\*c^2\*d\*x - 3\*I\*a\*b^2\*c^2\*d + 3\*I\*a^2\*b\*c\*d^2 - I\*a^3\*d^3)\*log(I\*cosh(b\*x + a) + I\*sinh(b\*x + a) + 1) + (I\*b^3\*d^3\*x^3 + 3\*I\*b^3\*c\*d^2\*x^2 + 3\*I\*b^3\*c^2\*d\*x + 3\*I\*a\*b^2\*c^2\*d - 3\*I\*a^2\*b\*c\*d^2 + I\*a^3\*d^3)\*log(-I\*cosh(b\*x + a) - I\*sinh(b\*x + a) + 1) + (-6\*I\*b\*d^3\*x - 6\*I\*b\*c\*d^2)\*polylog(3, I\*cosh(b\*x + a) + I\*sinh(b\*x + a)) + (6\*I\*b\*d^3\*x + 6\*I\*b\*c\*d^2)\*polylog(3, -I\*cosh(b\*x + a) - I\*sinh(b\*x + a)))/b^4

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \operatorname{sech}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*sech(b\*x+a),x, algorithm="giac")

[Out] integrate((d\*x + c)^3\*sech(b\*x + a), x)

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \operatorname{sech}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^3*sech(b*x+a),x)`

[Out] `int((d*x+c)^3*sech(b*x+a),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2c^3 \arctan(e^{(-bx-a)})}{b} + 2 \int \frac{(d^3x^3e^a + 3cd^2x^2e^a + 3c^2dxe^a)e^{(bx)}}{e^{(2bx+2a)} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^3*sech(b*x+a),x, algorithm="maxima")`

[Out] `-2*c^3*arctan(e^(-b*x - a))/b + 2*integrate((d^3*x^3*e^a + 3*c*d^2*x^2*e^a + 3*c^2*d*x*e^a)*e^(b*x)/(e^(2*b*x + 2*a) + 1), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + dx)^3}{\cosh(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x)^3/cosh(a + b*x),x)`

[Out] `int((c + d*x)^3/cosh(a + b*x), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \operatorname{sech}(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)**3*sech(b*x+a),x)`

[Out] `Integral((c + d*x)**3*sech(a + b*x), x)`

### 3.27 $\int (c + dx)^2 \operatorname{sech}(a + bx) dx$

**Optimal.** Leaf size=119

$$\frac{2id^2\operatorname{Li}_3(-ie^{a+bx})}{b^3} - \frac{2id^2\operatorname{Li}_3(ie^{a+bx})}{b^3} - \frac{2id(c+dx)\operatorname{Li}_2(-ie^{a+bx})}{b^2} + \frac{2id(c+dx)\operatorname{Li}_2(ie^{a+bx})}{b^2} + \frac{2(c+dx)^2 \tan^{-1}(e^{a+bx})}{b}$$

[Out]  $2*(d*x+c)^2*\arctan(\exp(b*x+a))/b-2*I*d*(d*x+c)*\operatorname{polylog}(2,-I*\exp(b*x+a))/b^2+2*I*d*(d*x+c)*\operatorname{polylog}(2,I*\exp(b*x+a))/b^2+2*I*d^2*\operatorname{polylog}(3,-I*\exp(b*x+a))/b^3-2*I*d^2*\operatorname{polylog}(3,I*\exp(b*x+a))/b^3$

**Rubi [A]** time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {4180, 2531, 2282, 6589}

$$-\frac{2id(c+dx)\operatorname{PolyLog}(2,-ie^{a+bx})}{b^2} + \frac{2id(c+dx)\operatorname{PolyLog}(2,ie^{a+bx})}{b^2} + \frac{2id^2\operatorname{PolyLog}(3,-ie^{a+bx})}{b^3} - \frac{2id^2\operatorname{PolyLog}(3,ie^{a+bx})}{b^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^2*\operatorname{Sech}[a + b*x], x]$

[Out]  $(2*(c + d*x)^2*\operatorname{ArcTan}[E^{(a + b*x)}])/b - ((2*I)*d*(c + d*x)*\operatorname{PolyLog}[2, (-I)*E^{(a + b*x)}])/b^2 + ((2*I)*d*(c + d*x)*\operatorname{PolyLog}[2, I*E^{(a + b*x)}])/b^2 + ((2*I)*d^2*\operatorname{PolyLog}[3, (-I)*E^{(a + b*x)}])/b^3 - ((2*I)*d^2*\operatorname{PolyLog}[3, I*E^{(a + b*x)}])/b^3$

#### Rule 2282

$\operatorname{Int}[u_, x\_Symbol] := \operatorname{With}[\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /;$   $\operatorname{FunctionOfExponentialQ}[u, x] \ \&\& \ !\operatorname{MatchQ}[u, (w_)*((a_)*(v_)^{(n_)})^{(m_)} /;$   $\operatorname{FreeQ}[\{a, m, n\}, x] \ \&\& \ \operatorname{IntegerQ}[m*n] \ \&\& \ !\operatorname{MatchQ}[u, E^{((c_)*((a_)+(b_)*x))}*(F_)[v_]] /;$   $\operatorname{FreeQ}[\{a, b, c\}, x] \ \&\& \ \operatorname{InverseFunctionQ}[F[x]]]$

#### Rule 2531

$\operatorname{Int}[\operatorname{Log}[1 + (e_)*((F_)^{((c_)*((a_)+(b_)*(x_)))})^{(n_)}]*((f_)+(g_))*((x_))^{(m_)}, x\_Symbol] := -\operatorname{Simp}[(f + g*x)^m*\operatorname{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n])/b*c*n*\operatorname{Log}[F], x] + \operatorname{Dist}[(g*m)/(b*c*n*\operatorname{Log}[F]), \operatorname{Int}[(f + g*x)^{(m-1)}*\operatorname{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n]), x], x] /;$   $\operatorname{FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \ \&\& \ \operatorname{GtQ}[m, 0]$

#### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTan[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned} \int (c + dx)^2 \operatorname{sech}(a + bx) dx &= \frac{2(c + dx)^2 \tan^{-1}(e^{a+bx})}{b} - \frac{(2id) \int (c + dx) \log(1 - ie^{a+bx}) dx}{b} + \frac{(2id) \int (c + dx) \log(1 + ie^{a+bx}) dx}{b} \\ &= \frac{2(c + dx)^2 \tan^{-1}(e^{a+bx})}{b} - \frac{2id(c + dx) \operatorname{Li}_2(-ie^{a+bx})}{b^2} + \frac{2id(c + dx) \operatorname{Li}_2(ie^{a+bx})}{b^2} + \frac{2id(c + dx) \log(1 - ie^{a+bx})}{b} \\ &= \frac{2(c + dx)^2 \tan^{-1}(e^{a+bx})}{b} - \frac{2id(c + dx) \operatorname{Li}_2(-ie^{a+bx})}{b^2} + \frac{2id(c + dx) \operatorname{Li}_2(ie^{a+bx})}{b^2} + \frac{2id(c + dx) \log(1 - ie^{a+bx})}{b} \\ &= \frac{2(c + dx)^2 \tan^{-1}(e^{a+bx})}{b} - \frac{2id(c + dx) \operatorname{Li}_2(-ie^{a+bx})}{b^2} + \frac{2id(c + dx) \operatorname{Li}_2(ie^{a+bx})}{b^2} + \frac{2id(c + dx) \log(1 - ie^{a+bx})}{b} \end{aligned}$$

**Mathematica [A]** time = 1.52, size = 199, normalized size = 1.67

$$\frac{i(-2ib^2c^2 \tan^{-1}(e^{a+bx}) + 2b^2cdx \log(1 - ie^{a+bx}) - 2b^2cdx \log(1 + ie^{a+bx}) + b^2d^2x^2 \log(1 - ie^{a+bx}) - b^2d^2x^2 \log(1 + ie^{a+bx}))}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sech[a + b*x], x]
```

```
[Out] (I*((-2*I)*b^2*c^2*ArcTan[E^(a + b*x)] + 2*b^2*c*d*x*Log[1 - I*E^(a + b*x)] + b^2*d^2*x^2*Log[1 - I*E^(a + b*x)] - 2*b^2*c*d*x*Log[1 + I*E^(a + b*x)] - b^2*d^2*x^2*Log[1 + I*E^(a + b*x)] - 2*b*d*(c + d*x)*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*d*(c + d*x)*PolyLog[2, I*E^(a + b*x)] + 2*d^2*PolyLog[3, (-I)*E^(a + b*x)] - 2*d^2*PolyLog[3, I*E^(a + b*x)]))/b^3
```

**fricas** [C] time = 0.98, size = 303, normalized size = 2.55

$$\frac{-2i d^2 \operatorname{polylog}(3, i \cosh(bx + a) + i \sinh(bx + a)) + 2i d^2 \operatorname{polylog}(3, -i \cosh(bx + a) - i \sinh(bx + a)) + (2i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*sech(b\*x+a),x, algorithm="fricas")

[Out]  $(-2*I*d^2*\operatorname{polylog}(3, I*\cosh(b*x + a) + I*\sinh(b*x + a)) + 2*I*d^2*\operatorname{polylog}(3, -I*\cosh(b*x + a) - I*\sinh(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*\operatorname{dilog}(I*\cosh(b*x + a) + I*\sinh(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*\operatorname{dilog}(-I*\cosh(b*x + a) - I*\sinh(b*x + a)) + (I*b^2*c^2 - 2*I*a*b*c*d + I*a^2*d^2)*\log(\cosh(b*x + a) + \sinh(b*x + a) + I) + (-I*b^2*c^2 + 2*I*a*b*c*d - I*a^2*d^2)*\log(\cosh(b*x + a) + \sinh(b*x + a) - I) + (-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x - 2*I*a*b*c*d + I*a^2*d^2)*\log(I*\cosh(b*x + a) + I*\sinh(b*x + a) + 1) + (I*b^2*d^2*x^2 + 2*I*b^2*c*d*x + 2*I*a*b*c*d - I*a^2*d^2)*\log(-I*\cosh(b*x + a) - I*\sinh(b*x + a) + 1))/b^3$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \operatorname{sech}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*sech(b\*x+a),x, algorithm="giac")

[Out] integrate((d\*x + c)^2\*sech(b\*x + a), x)

**maple** [F] time = 0.40, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \operatorname{sech}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*sech(b\*x+a),x)

[Out] int((d\*x+c)^2\*sech(b\*x+a),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2c^2 \arctan(e^{-bx-a})}{b} + 2 \int \frac{(d^2x^2e^a + 2cdxe^a)e^{(bx)}}{e^{(2bx+2a)} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*sech(b\*x+a),x, algorithm="maxima")

[Out]  $-2*c^2*\arctan(e^{(-b*x - a)})/b + 2*\int (d^2*x^2*e^a + 2*c*d*x*e^a)*e^{b*x}/(e^{(2*b*x + 2*a)} + 1), x$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + dx)^2}{\cosh(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^2/cosh(a + b\*x),x)

[Out] int((c + d\*x)^2/cosh(a + b\*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \operatorname{sech}(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2\*sech(b\*x+a),x)

[Out] Integral((c + d\*x)\*\*2\*sech(a + b\*x), x)

### 3.28 $\int (c + dx) \operatorname{sech}(a + bx) dx$

Optimal. Leaf size=61

$$-\frac{id\operatorname{Li}_2(-ie^{a+bx})}{b^2} + \frac{id\operatorname{Li}_2(ie^{a+bx})}{b^2} + \frac{2(c + dx) \tan^{-1}(e^{a+bx})}{b}$$

[Out]  $2*(d*x+c)*\arctan(\exp(b*x+a))/b - I*d*\operatorname{polylog}(2, -I*\exp(b*x+a))/b^2 + I*d*\operatorname{polylog}(2, I*\exp(b*x+a))/b^2$

**Rubi [A]** time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {4180, 2279, 2391}

$$-\frac{id\operatorname{PolyLog}(2, -ie^{a+bx})}{b^2} + \frac{id\operatorname{PolyLog}(2, ie^{a+bx})}{b^2} + \frac{2(c + dx) \tan^{-1}(e^{a+bx})}{b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)*\operatorname{Sech}[a + b*x], x]$

[Out]  $(2*(c + d*x)*\operatorname{ArcTan}[E^{(a + b*x)}])/b - (I*d*\operatorname{PolyLog}[2, (-I)*E^{(a + b*x)}])/b^2 + (I*d*\operatorname{PolyLog}[2, I*E^{(a + b*x)}])/b^2$

#### Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_) + (b_)*(F_)^{((e_)*((c_) + (d_)*(x_)))^{(n_)}], x\_Symbol]$   
 $\rightarrow \operatorname{Dist}[1/(d*e*n*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x \ \&\& \ \operatorname{GtQ}[a, 0]$

#### Rule 2391

$\operatorname{Int}[\operatorname{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)}]/(x_), x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c*e*x^n)]/n, x] /;$   $\operatorname{FreeQ}\{c, d, e, n\}, x \ \&\& \ \operatorname{EqQ}[c*d, 1]$

#### Rule 4180

$\operatorname{Int}[\operatorname{csc}[(e_) + \operatorname{Pi}*(k_) + (\operatorname{Complex}[0, fz_])*(f_)*(x_)]*((c_) + (d_)*(x_))^{(m_)}, x\_Symbol] \rightarrow \operatorname{Simp}[(-2*(c + d*x)^m*\operatorname{ArcTanh}[E^{-(I*e) + f*fz*x}]/E^{(I*k*\operatorname{Pi})})/(f*fz*I), x] + (-\operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 - E^{-(I*e) + f*fz*x}]/E^{(I*k*\operatorname{Pi})}], x], x] + \operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 + E^{-(I*e) + f*fz*x}]/E^{(I*k*\operatorname{Pi})}], x], x] /;$   $\operatorname{FreeQ}\{c, d, e, f, fz\}, x \ \&\& \ \operatorname{IntegerQ}[2*k] \ \&\& \ \operatorname{IGtQ}[m, 0]$

#### Rubi steps

$$\begin{aligned}
\int (c + dx) \operatorname{sech}(a + bx) dx &= \frac{2(c + dx) \tan^{-1}(e^{a+bx})}{b} - \frac{(id) \int \log(1 - ie^{a+bx}) dx}{b} + \frac{(id) \int \log(1 + ie^{a+bx}) dx}{b} \\
&= \frac{2(c + dx) \tan^{-1}(e^{a+bx})}{b} - \frac{(id) \operatorname{Subst}\left(\int \frac{\log(1-ix)}{x} dx, x, e^{a+bx}\right)}{b^2} + \frac{(id) \operatorname{Subst}\left(\int \frac{\log(1+ix)}{x} dx, x, e^{a+bx}\right)}{b^2} \\
&= \frac{2(c + dx) \tan^{-1}(e^{a+bx})}{b} - \frac{id \operatorname{Li}_2(-ie^{a+bx})}{b^2} + \frac{id \operatorname{Li}_2(ie^{a+bx})}{b^2}
\end{aligned}$$

**Mathematica [B]** time = 0.15, size = 127, normalized size = 2.08

$$\frac{bc \tan^{-1}(\sinh(a + bx)) + \frac{1}{2}d \left( -2i \left( \operatorname{Li}_2(-ie^{a+bx}) - \operatorname{Li}_2(ie^{a+bx}) \right) - ((-2ia - 2ibx + \pi) (\log(1 - ie^{a+bx}) - \log(1 + ie^{a+bx}))) \right)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)\*Sech[a + b\*x], x]

[Out] (b\*c\*ArcTan[Sinh[a + b\*x]] + (d\*(-(((2\*I)\*a + Pi - (2\*I)\*b\*x)\*(Log[1 - I\*E^(a + b\*x)] - Log[1 + I\*E^(a + b\*x)])) + ((-2\*I)\*a + Pi)\*Log[Cot[((2\*I)\*a + Pi + (2\*I)\*b\*x)/4]] - (2\*I)\*(PolyLog[2, (-I)\*E^(a + b\*x)] - PolyLog[2, I\*E^(a + b\*x)])))/b^2

**fricas [B]** time = 0.67, size = 157, normalized size = 2.57

$$\frac{id \operatorname{Li}_2(i \cosh(bx + a) + i \sinh(bx + a)) - id \operatorname{Li}_2(-i \cosh(bx + a) - i \sinh(bx + a)) + (ibc - iad) \log(\cosh(bx + a))}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sech(b\*x+a), x, algorithm="fricas")

[Out] (I\*d\*dilog(I\*cosh(b\*x + a) + I\*sinh(b\*x + a)) - I\*d\*dilog(-I\*cosh(b\*x + a) - I\*sinh(b\*x + a)) + (I\*b\*c - I\*a\*d)\*log(cosh(b\*x + a) + sinh(b\*x + a) + I) + (-I\*b\*c + I\*a\*d)\*log(cosh(b\*x + a) + sinh(b\*x + a) - I) + (-I\*b\*d\*x - I\*a\*d)\*log(I\*cosh(b\*x + a) + I\*sinh(b\*x + a) + 1) + (I\*b\*d\*x + I\*a\*d)\*log(-I\*cosh(b\*x + a) - I\*sinh(b\*x + a) + 1))/b^2

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c) \operatorname{sech}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((d\*x+c)\*sech(b\*x+a),x, algorithm="giac")

[Out] integrate((d\*x + c)\*sech(b\*x + a), x)

**maple** [B] time = 0.02, size = 147, normalized size = 2.41

$$\frac{id \ln(1 - ie^{bx+a})x}{b} - \frac{id \ln(1 + ie^{bx+a})x}{b} + \frac{id \ln(1 - ie^{bx+a})a}{b^2} - \frac{id \ln(1 + ie^{bx+a})a}{b^2} + \frac{id \operatorname{dilog}(1 - ie^{bx+a})}{b^2} - \frac{id \operatorname{dilog}(1 + ie^{bx+a})}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)\*sech(b\*x+a),x)

[Out] I/b\*d\*ln(1-I\*exp(b\*x+a))\*x-I/b\*d\*ln(1+I\*exp(b\*x+a))\*x+I/b^2\*d\*ln(1-I\*exp(b\*x+a))\*a-I/b^2\*d\*ln(1+I\*exp(b\*x+a))\*a+I/b^2\*d\*dilog(1-I\*exp(b\*x+a))-I/b^2\*d\*dilog(1+I\*exp(b\*x+a))-2/b^2\*d\*a\*arctan(exp(b\*x+a))+2/b\*c\*arctan(exp(b\*x+a))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$2d \int \frac{xe^{(bx+a)}}{e^{(2bx+2a)} + 1} dx - \frac{2c \arctan(e^{(-bx-a)})}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sech(b\*x+a),x, algorithm="maxima")

[Out] 2\*d\*integrate(x\*e^(b\*x + a)/(e^(2\*b\*x + 2\*a) + 1), x) - 2\*c\*arctan(e^(-b\*x - a))/b

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{c + dx}{\cosh(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)/cosh(a + b\*x),x)

[Out] int((c + d\*x)/cosh(a + b\*x), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \operatorname{sech}(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sech(b\*x+a),x)

[Out] Integral((c + d\*x)\*sech(a + b\*x), x)

$$3.29 \quad \int \frac{\operatorname{sech}(a+bx)}{c+dx} dx$$

Optimal. Leaf size=17

$$\operatorname{Int}\left(\frac{\operatorname{sech}(a+bx)}{c+dx}, x\right)$$

[Out] Unintegrable(sech(b\*x+a)/(d\*x+c), x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\operatorname{sech}(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[Sech[a + b\*x]/(c + d\*x), x]

[Out] Defer[Int][Sech[a + b\*x]/(c + d\*x), x]

Rubi steps

$$\int \frac{\operatorname{sech}(a+bx)}{c+dx} dx = \int \frac{\operatorname{sech}(a+bx)}{c+dx} dx$$

Mathematica [A] time = 3.71, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sech[a + b\*x]/(c + d\*x), x]

[Out] Integrate[Sech[a + b\*x]/(c + d\*x), x]

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)/(d\*x+c), x, algorithm="fricas")

[Out] `integral(sech(b*x + a)/(d*x + c), x)`

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(b*x+a)/(d*x+c),x, algorithm="giac")`

[Out] `integrate(sech(b*x + a)/(d*x + c), x)`

**maple** [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(b*x+a)/(d*x+c),x)`

[Out] `int(sech(b*x+a)/(d*x+c),x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(b*x+a)/(d*x+c),x, algorithm="maxima")`

[Out] `integrate(sech(b*x + a)/(d*x + c), x)`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\cosh(a + bx)(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cosh(a + b*x)*(c + d*x)),x)`

[Out] `int(1/(cosh(a + b*x)*(c + d*x)), x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sech(a + b*x)/(c + d*x), x)
```

$$3.30 \quad \int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=17

$$\operatorname{Int}\left(\frac{\operatorname{sech}(a+bx)}{(c+dx)^2}, x\right)$$

[Out] Unintegrable(sech(b\*x+a)/(d\*x+c)^2, x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[Sech[a + b\*x]/(c + d\*x)^2, x]

[Out] Defer[Int][Sech[a + b\*x]/(c + d\*x)^2, x]

Rubi steps

$$\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2} dx = \int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 7.28, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sech[a + b\*x]/(c + d\*x)^2, x]

[Out] Integrate[Sech[a + b\*x]/(c + d\*x)^2, x]

fricas [A] time = 0.68, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)}{d^2x^2+2cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)/(d\*x+c)^2, x, algorithm="fricas")

[Out] integral(sech(b\*x + a)/(d^2\*x^2 + 2\*c\*d\*x + c^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)/(d\*x+c)^2,x, algorithm="giac")

[Out] integrate(sech(b\*x + a)/(d\*x + c)^2, x)

**maple** [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(b\*x+a)/(d\*x+c)^2,x)

[Out] int(sech(b\*x+a)/(d\*x+c)^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)/(d\*x+c)^2,x, algorithm="maxima")

[Out] integrate(sech(b\*x + a)/(d\*x + c)^2, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\cosh(a + bx) (c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(a + b\*x)\*(c + d\*x)^2),x)

[Out] int(1/(cosh(a + b\*x)\*(c + d\*x)^2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sech(a + b*x)/(c + d*x)**2, x)
```

### 3.31 $\int (c + dx)^3 \operatorname{sech}^2(a + bx) dx$

Optimal. Leaf size=103

$$\frac{3d^3 \operatorname{Li}_3(-e^{2(a+bx)})}{2b^4} - \frac{3d^2(c+dx) \operatorname{Li}_2(-e^{2(a+bx)})}{b^3} - \frac{3d(c+dx)^2 \log(e^{2(a+bx)} + 1)}{b^2} + \frac{(c+dx)^3 \tanh(a+bx)}{b} + \frac{(c+dx)^3}{b}$$

[Out]  $(d*x+c)^3/b-3*d*(d*x+c)^2*\ln(1+\exp(2*b*x+2*a))/b^2-3*d^2*(d*x+c)*\operatorname{polylog}(2, -\exp(2*b*x+2*a))/b^3+3/2*d^3*\operatorname{polylog}(3, -\exp(2*b*x+2*a))/b^4+(d*x+c)^3*\tanh(b*x+a)/b$

**Rubi [A]** time = 0.21, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4184, 3718, 2190, 2531, 2282, 6589}

$$-\frac{3d^2(c+dx)\operatorname{PolyLog}(2, -e^{2(a+bx)})}{b^3} + \frac{3d^3\operatorname{PolyLog}(3, -e^{2(a+bx)})}{2b^4} - \frac{3d(c+dx)^2 \log(e^{2(a+bx)} + 1)}{b^2} + \frac{(c+dx)^3 \tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] `Int[(c + d*x)^3*Sech[a + b*x]^2,x]`

[Out]  $(c + d*x)^3/b - (3*d*(c + d*x)^2*\operatorname{Log}[1 + E^{(2*(a + b*x))}])/b^2 - (3*d^2*(c + d*x)*\operatorname{PolyLog}[2, -E^{(2*(a + b*x))}])/b^3 + (3*d^3*\operatorname{PolyLog}[3, -E^{(2*(a + b*x))}])/b^4 + ((c + d*x)^3*\operatorname{Tanh}[a + b*x])/b$

#### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```



)))^n)]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 3718

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*tan[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-(I\*e) + f\*fz\*x)))/(1 + E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_.)]^2\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.)]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int (c + dx)^3 \operatorname{sech}^2(a + bx) dx &= \frac{(c + dx)^3 \tanh(a + bx)}{b} - \frac{(3d) \int (c + dx)^2 \tanh(a + bx) dx}{b} \\
 &= \frac{(c + dx)^3}{b} + \frac{(c + dx)^3 \tanh(a + bx)}{b} - \frac{(6d) \int \frac{e^{2(a+bx)}(c+dx)^2}{1+e^{2(a+bx)}} dx}{b} \\
 &= \frac{(c + dx)^3}{b} - \frac{3d(c + dx)^2 \log(1 + e^{2(a+bx)})}{b^2} + \frac{(c + dx)^3 \tanh(a + bx)}{b} + \frac{(6d^2) \int (c + dx) \operatorname{sech}^2(a + bx) dx}{b} \\
 &= \frac{(c + dx)^3}{b} - \frac{3d(c + dx)^2 \log(1 + e^{2(a+bx)})}{b^2} - \frac{3d^2(c + dx) \operatorname{Li}_2(-e^{2(a+bx)})}{b^3} + \frac{(c + dx)^3 \tanh(a + bx)}{b} \\
 &= \frac{(c + dx)^3}{b} - \frac{3d(c + dx)^2 \log(1 + e^{2(a+bx)})}{b^2} - \frac{3d^2(c + dx) \operatorname{Li}_2(-e^{2(a+bx)})}{b^3} + \frac{(c + dx)^3 \tanh(a + bx)}{b} \\
 &= \frac{(c + dx)^3}{b} - \frac{3d(c + dx)^2 \log(1 + e^{2(a+bx)})}{b^2} - \frac{3d^2(c + dx) \operatorname{Li}_2(-e^{2(a+bx)})}{b^3} + \frac{3d^3 \operatorname{Li}_3(-e^{2(a+bx)})}{b^3}
 \end{aligned}$$

**Mathematica [A]** time = 2.07, size = 145, normalized size = 1.41

$$2\operatorname{sech}(a) \sinh(bx)(c + dx)^3 \operatorname{sech}(a + bx) - \frac{e^{2a} d \left( -\frac{3(e^{-2a+1})^d (2b(c+dx) \operatorname{Li}_2(-e^{-2(a+bx)}) + d \operatorname{Li}_3(-e^{-2(a+bx)}))}{b^3} + \frac{6(e^{-2a+1})^{(c+dx)^2} \log(e^{-2(a+bx)+1})}{b} \right)}{e^{2a+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3\*Sech[a + b\*x]^2,x]

[Out] 
$$\left( -\frac{(dE^{(2a)}((4(c+dx)^3)/(dE^{(2a)})) + (6(1+E^{-2a}))(c+dx)^2 \operatorname{Log}[1+E^{-2(a+bx)}])}{b} - (3d(1+E^{-2a}))(2b(c+dx) \operatorname{PolyLog}[2, -E^{-2(a+bx)}] + d \operatorname{PolyLog}[3, -E^{-2(a+bx)}])}{b^3} \right) / (1+E^{(2a)}) + 2(c+dx)^3 \operatorname{Sech}[a] \operatorname{Sech}[a+b*x] \operatorname{Sinh}[b*x] / (2b)$$

**fricas [C]** time = 0.76, size = 1332, normalized size = 12.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*sech(b\*x+a)^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -(2b^3c^3 - 6ab^2c^2d + 6a^2b^2cd^2 - 2a^3d^3 - 2(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2d^2x + 3a^2b^2cd^2 + a^3d^3) \cosh(bx+a)^2 \\ & - 4(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2d^2x + 3a^2b^2cd^2 - 3a^2b^2cd^2 + a^3d^3) \cosh(bx+a) \sinh(bx+a) - 2(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2d^2x + 3a^2b^2cd^2 - 3a^2b^2cd^2 + a^3d^3) \sinh(bx+a)^2 \\ & + 6(b^3d^3x + b^2cd^2 + (b^3d^3x + b^2cd^2) \cosh(bx+a)^2 + 2(b^3d^3x + b^2cd^2) \cosh(bx+a) \sinh(bx+a) + (b^3d^3x + b^2cd^2) \sinh(bx+a)^2) \operatorname{dilog}(I \cosh(bx+a) + I \sinh(bx+a)) \\ & + 6(b^3d^3x + b^2cd^2 + (b^3d^3x + b^2cd^2) \cosh(bx+a)^2 + 2(b^3d^3x + b^2cd^2) \cosh(bx+a) \sinh(bx+a) + (b^3d^3x + b^2cd^2) \sinh(bx+a)^2) \operatorname{dilog}(-I \cosh(bx+a) - I \sinh(bx+a)) \\ & + 3(b^2c^2d - 2ab^2cd^2 + a^2d^3) \cosh(bx+a)^2 + 2(b^2c^2d - 2ab^2cd^2 + a^2d^3) \cosh(bx+a) \sinh(bx+a) + (b^2c^2d - 2ab^2cd^2 + a^2d^3) \sinh(bx+a)^2 \log(\cosh(bx+a) + \sinh(bx+a) + I) \\ & + 3(b^2c^2d - 2ab^2cd^2 + a^2d^3) \cosh(bx+a)^2 + 2(b^2c^2d - 2ab^2cd^2 + a^2d^3) \cosh(bx+a) \sinh(bx+a) + (b^2c^2d - 2ab^2cd^2 + a^2d^3) \sinh(bx+a)^2 \log(\cosh(bx+a) + \sinh(bx+a) - I) \\ & + 3(b^2d^3x^2 + 2b^2cd^2x + 2ab^2cd^2 - a^2d^3) \cosh(bx+a)^2 + 2(b^2d^3x^2 + 2b^2cd^2x + 2ab^2cd^2 - a^2d^3) \cosh(bx+a) \sinh(bx+a) \\ & + (b^2d^3x^2 + 2b^2cd^2x + 2ab^2cd^2 - a^2d^3) \sinh(bx+a)^2 \log(I \cosh(bx+a) + I \sinh(bx+a) + 1) + 3(b^2d^3x^2 + 2b^2 \end{aligned}$$

\*c\*d^2\*x + 2\*a\*b\*c\*d^2 - a^2\*d^3 + (b^2\*d^3\*x^2 + 2\*b^2\*c\*d^2\*x + 2\*a\*b\*c\*d^2 - a^2\*d^3)\*cosh(b\*x + a)^2 + 2\*(b^2\*d^3\*x^2 + 2\*b^2\*c\*d^2\*x + 2\*a\*b\*c\*d^2 - a^2\*d^3)\*cosh(b\*x + a)\*sinh(b\*x + a) + (b^2\*d^3\*x^2 + 2\*b^2\*c\*d^2\*x + 2\*a\*b\*c\*d^2 - a^2\*d^3)\*sinh(b\*x + a)^2\*log(-I\*cosh(b\*x + a) - I\*sinh(b\*x + a) + 1) - 6\*(d^3\*cosh(b\*x + a)^2 + 2\*d^3\*cosh(b\*x + a)\*sinh(b\*x + a) + d^3\*sinh(b\*x + a)^2 + d^3)\*polylog(3, I\*cosh(b\*x + a) + I\*sinh(b\*x + a)) - 6\*(d^3\*cosh(b\*x + a)^2 + 2\*d^3\*cosh(b\*x + a)\*sinh(b\*x + a) + d^3\*sinh(b\*x + a)^2 + d^3)\*polylog(3, -I\*cosh(b\*x + a) - I\*sinh(b\*x + a))/(b^4\*cosh(b\*x + a)^2 + 2\*b^4\*cosh(b\*x + a)\*sinh(b\*x + a) + b^4\*sinh(b\*x + a)^2 + b^4)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \operatorname{sech}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*sech(b\*x+a)^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^3\*sech(b\*x + a)^2, x)

**maple** [B] time = 0.29, size = 298, normalized size = 2.89

$$\frac{2(d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3)}{b(1 + e^{2bx+2a})} - \frac{3dc^2 \ln(1 + e^{2bx+2a})}{b^2} + \frac{6dc^2 \ln(e^{bx+a})}{b^2} + \frac{6d^3a^2 \ln(e^{bx+a})}{b^4} + \frac{2d^3x^3}{b} - \frac{6d^3a^2x}{b^3} - 4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3\*sech(b\*x+a)^2,x)

[Out] -2\*(d^3\*x^3+3\*c\*d^2\*x^2+3\*c^2\*d\*x+c^3)/b/(1+exp(2\*b\*x+2\*a))-3\*d/b^2\*c^2\*ln(1+exp(2\*b\*x+2\*a))+6\*d/b^2\*c^2\*ln(exp(b\*x+a))+6\*d^3/b^4\*a^2\*ln(exp(b\*x+a))+2\*d^3/b\*x^3-6\*d^3/b^3\*a^2\*x-4\*d^3/b^4\*a^3-3\*d^3/b^2\*ln(1+exp(2\*b\*x+2\*a))\*x^2-3\*d^3/b^3\*polylog(2,-exp(2\*b\*x+2\*a))\*x+3/2\*d^3\*polylog(3,-exp(2\*b\*x+2\*a))/b^4-12\*d^2/b^3\*c\*a\*ln(exp(b\*x+a))+6\*d^2/b\*c\*x^2+12\*d^2/b^2\*c\*a\*x+6\*d^2/b^3\*c\*a^2-6\*d^2/b^2\*c\*ln(1+exp(2\*b\*x+2\*a))\*x-3\*d^2/b^3\*c\*polylog(2,-exp(2\*b\*x+2\*a))

**maxima** [B] time = 0.86, size = 238, normalized size = 2.31

$$3c^2d \left( \frac{2xe^{(2bx+2a)}}{be^{(2bx+2a)} + b} - \frac{\log\left(\left(e^{(2bx+2a)} + 1\right)e^{(-2a)}\right)}{b^2} \right) - \frac{3(2bx \log(e^{(2bx+2a)} + 1) + \operatorname{Li}_2(-e^{(2bx+2a)}))cd^2}{b^3} + \frac{2c}{b(e^{(-2bx-2a)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*sech(b\*x+a)^2,x, algorithm="maxima")

```
[Out] 3*c^2*d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) + b) - log((e^(2*b*x + 2*a) + 1)*e^(-2*a))/b^2) - 3*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x + 2*a)))*c*d^2/b^3 + 2*c^3/(b*(e^(-2*b*x - 2*a) + 1)) - 2*(d^3*x^3 + 3*c*d^2*x^2)/(b*e^(2*b*x + 2*a) + b) - 3/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) + 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))*d^3/b^4 + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2)/b^4
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + dx)^3}{\cosh(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/cosh(a + b*x)^2,x)
```

```
[Out] int((c + d*x)^3/cosh(a + b*x)^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \operatorname{sech}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sech(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*sech(a + b*x)**2, x)
```

### 3.32 $\int (c + dx)^2 \operatorname{sech}^2(a + bx) dx$

Optimal. Leaf size=73

$$-\frac{d^2 \operatorname{Li}_2(-e^{2(a+bx)})}{b^3} - \frac{2d(c + dx) \log(e^{2(a+bx)} + 1)}{b^2} + \frac{(c + dx)^2 \tanh(a + bx)}{b} + \frac{(c + dx)^2}{b}$$

[Out]  $(d*x+c)^2/b-2*d*(d*x+c)*\ln(1+\exp(2*b*x+2*a))/b^2-d^2*\operatorname{polylog}(2,-\exp(2*b*x+2*a))/b^3+(d*x+c)^2*\tanh(b*x+a)/b$

**Rubi [A]** time = 0.13, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {4184, 3718, 2190, 2279, 2391}

$$-\frac{d^2 \operatorname{PolyLog}(2, -e^{2(a+bx)})}{b^3} - \frac{2d(c + dx) \log(e^{2(a+bx)} + 1)}{b^2} + \frac{(c + dx)^2 \tanh(a + bx)}{b} + \frac{(c + dx)^2}{b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^2*\operatorname{Sech}[a + b*x]^2, x]$

[Out]  $(c + d*x)^2/b - (2*d*(c + d*x)*\operatorname{Log}[1 + E^{2*(a + b*x)}])/b^2 - (d^2*\operatorname{PolyLog}[2, -E^{2*(a + b*x)}])/b^3 + ((c + d*x)^2*\operatorname{Tanh}[a + b*x])/b$

#### Rule 2190

$\operatorname{Int}[\frac{((F_)^{((g_)*(e_)+(f_)*(x_)))^{(n_)*((c_)+(d_)*(x_))^{(m_)}}}{((a_)+(b_)*((F_)^{((g_)*(e_)+(f_)*(x_)))^{(n_)}}), x\_Symbol]}{> \operatorname{Simp}[\frac{(c + d*x)^m*\operatorname{Log}[1 + (b*(F^{(g*(e + f*x)))^n)/a]}{b*f*g*n*\operatorname{Log}[F]}, x] - \operatorname{Dist}[\frac{d*m}{b*f*g*n*\operatorname{Log}[F]}, \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 + (b*(F^{(g*(e + f*x)))^n)/a}], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_)+(b_)*((F_)^{((e_)*((c_)+(d_)*(x_)))^{(n_)}})], x\_Symbol]}{> \operatorname{Dist}[1/(d*e*n*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x)))^n}], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \operatorname{GtQ}[a, 0]$

#### Rule 2391

$\operatorname{Int}[\operatorname{Log}[(c_)*((d_)+(e_)*(x_))^{(n_)}]/(x_), x\_Symbol]}{> -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c*e*x^n)]/n, x] /; \operatorname{FreeQ}\{c, d, e, n\}, x\} \&\& \operatorname{EqQ}[c*d, 1]$

#### Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-I*e) + f*fz*x)))/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned} \int (c + dx)^2 \operatorname{sech}^2(a + bx) dx &= \frac{(c + dx)^2 \tanh(a + bx)}{b} - \frac{(2d) \int (c + dx) \tanh(a + bx) dx}{b} \\ &= \frac{(c + dx)^2}{b} + \frac{(c + dx)^2 \tanh(a + bx)}{b} - \frac{(4d) \int \frac{e^{2(a+bx)}(c+dx)}{1+e^{2(a+bx)}} dx}{b} \\ &= \frac{(c + dx)^2}{b} - \frac{2d(c + dx) \log(1 + e^{2(a+bx)})}{b^2} + \frac{(c + dx)^2 \tanh(a + bx)}{b} + \frac{(2d^2) \int \log(\dots)}{b} \\ &= \frac{(c + dx)^2}{b} - \frac{2d(c + dx) \log(1 + e^{2(a+bx)})}{b^2} + \frac{(c + dx)^2 \tanh(a + bx)}{b} + \frac{d^2 \operatorname{Subst}\left(\int \dots\right)}{b} \\ &= \frac{(c + dx)^2}{b} - \frac{2d(c + dx) \log(1 + e^{2(a+bx)})}{b^2} - \frac{d^2 \operatorname{Li}_2(-e^{2(a+bx)})}{b^3} + \frac{(c + dx)^2 \tanh(a + bx)}{b} \end{aligned}$$

**Mathematica [C]** time = 6.30, size = 277, normalized size = 3.79

$$\frac{2cd \operatorname{sech}(a) (\cosh(a) \log(\sinh(a) \sinh(bx) + \cosh(a) \cosh(bx)) - bx \sinh(a))}{b^2 (\cosh^2(a) - \sinh^2(a))} - \frac{d^2 \operatorname{csch}(a) \operatorname{sech}(a) \left( b^2 x^2 e^{-\tanh^{-1}(c)} \right)}{b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sech[a + b*x]^2,x]
```

```
[Out] (-2*c*d*Sech[a]*(Cosh[a]*Log[Cosh[a]*Cosh[b*x] + Sinh[a]*Sinh[b*x]] - b*x*S
inh[a]))/(b^2*(Cosh[a]^2 - Sinh[a]^2)) - (d^2*Csch[a]*((b^2*x^2)/E^ArcTanh[
Coth[a]] - (I*Coth[a]*(-b*x*(-Pi + (2*I)*ArcTanh[Coth[a]])) - Pi*Log[1 + E
```

$$\frac{\begin{aligned} & \left( (2bx)^2 - 2(Ibx + I \operatorname{ArcTanh}[\operatorname{Coth}[a]]) \operatorname{Log}[1 - E^{((2I)(Ibx + I \operatorname{ArcTanh}[\operatorname{Coth}[a]])} + \operatorname{Pi} \operatorname{Log}[\operatorname{Cosh}[bx]] + (2I) \operatorname{ArcTanh}[\operatorname{Coth}[a]] \operatorname{Log}[I \operatorname{Sinh}[bx + \operatorname{ArcTanh}[\operatorname{Coth}[a]]]] + I \operatorname{PolyLog}[2, E^{((2I)(Ibx + I \operatorname{ArcTanh}[\operatorname{Coth}[a]])} \right) \right) / \sqrt{1 - \operatorname{Coth}[a]^2} \operatorname{Sech}[a] / (b^3 \sqrt{[\operatorname{CsCh}[a]^2 (-\operatorname{Cosh}[a]^2 + \operatorname{Sinh}[a]^2)] + (\operatorname{Sech}[a] \operatorname{Sech}[a + bx] (c^2 \operatorname{Sinh}[bx] + 2cdx \operatorname{Sinh}[bx] + d^2 x^2 \operatorname{Sinh}[bx])) / b \end{aligned}$$

**fricas** [C] time = 0.61, size = 715, normalized size = 9.79

$$\frac{2(b^2c^2 - 2abcd + a^2d^2 - (b^2d^2x^2 + 2b^2cdx + 2abcd - a^2d^2) \cosh(bx + a)^2 - 2(b^2d^2x^2 + 2b^2cdx + 2abcd - a^2d^2) \sinh(bx + a)^2)}{b^3 \sqrt{1 - \operatorname{Coth}[a]^2} \operatorname{Sech}[a]}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((dx+c)^2\*sech(b\*x+a)^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -2(b^2c^2 - 2ab^2cd + a^2d^2 - (b^2d^2x^2 + 2b^2cdx + 2ab^2cd - a^2d^2) \cosh(bx + a)^2 - 2(b^2d^2x^2 + 2b^2cdx + 2ab^2cd - a^2d^2) \sinh(bx + a)^2 - (b^2d^2x^2 + 2b^2cdx + 2ab^2cd - a^2d^2) \cosh(bx + a) \sinh(bx + a) - (b^2d^2x^2 + 2b^2cdx + 2ab^2cd - a^2d^2) \sinh(bx + a)^2 + (d^2 \cosh(bx + a)^2 + 2d^2 \cosh(bx + a) \sinh(bx + a) + d^2 \sinh(bx + a)^2 + d^2) \operatorname{dilog}(I \cosh(bx + a) + I \sinh(bx + a)) + (d^2 \cosh(bx + a)^2 + 2d^2 \cosh(bx + a) \sinh(bx + a) + d^2 \sinh(bx + a)^2 + d^2) \operatorname{dilog}(-I \cosh(bx + a) - I \sinh(bx + a)) + (b^2cd - a^2d^2 + (b^2cd - a^2d^2) \cosh(bx + a)^2 + 2(b^2cd - a^2d^2) \cosh(bx + a) \sinh(bx + a) + (b^2cd - a^2d^2) \sinh(bx + a)^2) \log(\cosh(bx + a) + \sinh(bx + a) + I) + (b^2cd - a^2d^2 + (b^2cd - a^2d^2) \cosh(bx + a)^2 + 2(b^2cd - a^2d^2) \cosh(bx + a) \sinh(bx + a) + (b^2cd - a^2d^2) \sinh(bx + a)^2) \log(\cosh(bx + a) + \sinh(bx + a) - I) + (b^2d^2x + a^2d^2 + (b^2d^2x + a^2d^2) \cosh(bx + a)^2 + 2(b^2d^2x + a^2d^2) \cosh(bx + a) \sinh(bx + a) + (b^2d^2x + a^2d^2) \sinh(bx + a)^2) \log(I \cosh(bx + a) + I \sinh(bx + a) + 1) + (b^2d^2x + a^2d^2 + (b^2d^2x + a^2d^2) \cosh(bx + a)^2 + 2(b^2d^2x + a^2d^2) \cosh(bx + a) \sinh(bx + a) + (b^2d^2x + a^2d^2) \sinh(bx + a)^2) \log(-I \cosh(bx + a) - I \sinh(bx + a) + 1) / (b^3 \cosh(bx + a)^2 + 2b^3 \cosh(bx + a) \sinh(bx + a) + b^3 \sinh(bx + a)^2 + b^3) \end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \operatorname{sech}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((dx+c)^2\*sech(b\*x+a)^2,x, algorithm="giac")

[Out] integrate((dx + c)^2\*sech(b\*x + a)^2, x)

**maple** [B] time = 0.26, size = 159, normalized size = 2.18

$$-\frac{2(d^2x^2 + 2cdx + c^2)}{b(1 + e^{2bx+2a})} - \frac{2dc \ln(1 + e^{2bx+2a})}{b^2} + \frac{4dc \ln(e^{bx+a})}{b^2} + \frac{2d^2x^2}{b} + \frac{4d^2ax}{b^2} + \frac{2d^2a^2}{b^3} - \frac{2d^2 \ln(1 + e^{2bx+2a})x}{b^2} - \frac{d^2}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*sech(b\*x+a)^2,x)

[Out]  $-2*(d^2*x^2+2*c*d*x+c^2)/b/(1+\exp(2*b*x+2*a))-2*d/b^2*c*\ln(1+\exp(2*b*x+2*a))+4*d/b^2*c*\ln(\exp(b*x+a))+2*d^2/b*x^2+4*d^2/b^2*a*x+2*d^2/b^3*a^2-2*d^2/b^2*\ln(1+\exp(2*b*x+2*a))*x-d^2*\text{polylog}(2,-\exp(2*b*x+2*a))/b^3-4*d^2/b^3*a*\ln(\exp(b*x+a))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-2d^2\left(\frac{x^2}{be^{2bx+2a}+b} - 2\int\frac{x}{be^{2bx+2a}+b}dx\right) + 2cd\left(\frac{2xe^{(2bx+2a)}}{be^{2bx+2a}+b} - \frac{\log\left(\left(e^{(2bx+2a)}+1\right)e^{(-2a)}\right)}{b^2}\right) + \frac{2c^2}{b\left(e^{(-2bx-2a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*sech(b\*x+a)^2,x, algorithm="maxima")

[Out]  $-2*d^2*(x^2/(b*e^{(2*b*x+2*a)}+b) - 2*\text{integrate}(x/(b*e^{(2*b*x+2*a)}+b), x)) + 2*c*d*(2*x*e^{(2*b*x+2*a)}/(b*e^{(2*b*x+2*a)}+b) - \log((e^{(2*b*x+2*a)}+1)*e^{(-2*a)})/b^2) + 2*c^2/(b*(e^{(-2*b*x-2*a)}+1))$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + dx)^2}{\cosh(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^2/cosh(a + b\*x)^2,x)

[Out] int((c + d\*x)^2/cosh(a + b\*x)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \operatorname{sech}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2\*sech(b\*x+a)\*\*2,x)

[Out] Integral((c + d\*x)\*\*2\*sech(a + b\*x)\*\*2, x)



### 3.33 $\int (c + dx) \operatorname{sech}^2(a + bx) dx$

Optimal. Leaf size=29

$$\frac{(c + dx) \tanh(a + bx)}{b} - \frac{d \log(\cosh(a + bx))}{b^2}$$

[Out]  $-d*\ln(\cosh(b*x+a))/b^2+(d*x+c)*\tanh(b*x+a)/b$

**Rubi [A]** time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {4184, 3475}

$$\frac{(c + dx) \tanh(a + bx)}{b} - \frac{d \log(\cosh(a + bx))}{b^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)*\text{Sech}[a + b*x]^2, x]$

[Out]  $-((d*\text{Log}[\text{Cosh}[a + b*x]])/b^2) + ((c + d*x)*\text{Tanh}[a + b*x])/b$

Rule 3475

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 4184

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cot}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cot}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

Rubi steps

$$\begin{aligned} \int (c + dx) \operatorname{sech}^2(a + bx) dx &= \frac{(c + dx) \tanh(a + bx)}{b} - \frac{d \int \tanh(a + bx) dx}{b} \\ &= -\frac{d \log(\cosh(a + bx))}{b^2} + \frac{(c + dx) \tanh(a + bx)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 51, normalized size = 1.76

$$-\frac{d \log(\cosh(a + bx))}{b^2} + \frac{c \tanh(a + bx)}{b} + \frac{dx \tanh(a)}{b} + \frac{dx \operatorname{sech}(a) \sinh(bx) \operatorname{sech}(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)\*Sech[a + b\*x]^2,x]

[Out] -((d\*Log[Cosh[a + b\*x]])/b^2) + (d\*x\*Sech[a]\*Sech[a + b\*x]\*Sinh[b\*x])/b + (d\*x\*Tanh[a])/b + (c\*Tanh[a + b\*x])/b

**fricas** [B] time = 0.76, size = 161, normalized size = 5.55

$$\frac{2 b d x \cosh (b x+a)^2+4 b d x \cosh (b x+a) \sinh (b x+a)+2 b d x \sinh (b x+a)^2-2 b c-\left(d \cosh (b x+a)^2+2 d \cosh (b x+a) \sinh (b x+a)+d \sinh (b x+a)^2\right)}{b^2 \cosh (b x+a)^2+2 b^2 \cosh (b x+a) \sinh (b x+a)+b^2 \sinh (b x+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sech(b\*x+a)^2,x, algorithm="fricas")

[Out] (2\*b\*d\*x\*cosh(b\*x + a)^2 + 4\*b\*d\*x\*cosh(b\*x + a)\*sinh(b\*x + a) + 2\*b\*d\*x\*sinh(b\*x + a)^2 - 2\*b\*c - (d\*cosh(b\*x + a)^2 + 2\*d\*cosh(b\*x + a)\*sinh(b\*x + a) + d\*sinh(b\*x + a)^2 + d)\*log(2\*cosh(b\*x + a)/(cosh(b\*x + a) - sinh(b\*x + a))))/(b^2\*cosh(b\*x + a)^2 + 2\*b^2\*cosh(b\*x + a)\*sinh(b\*x + a) + b^2\*sinh(b\*x + a)^2 + b^2)

**giac** [B] time = 0.14, size = 78, normalized size = 2.69

$$\frac{2 b d x e^{2 b x+2 a}-d e^{2 b x+2 a} \log \left(e^{2 b x+2 a}+1\right)-2 b c-d \log \left(e^{2 b x+2 a}+1\right)}{b^2 e^{2 b x+2 a}+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sech(b\*x+a)^2,x, algorithm="giac")

[Out] (2\*b\*d\*x\*e^(2\*b\*x + 2\*a) - d\*e^(2\*b\*x + 2\*a)\*log(e^(2\*b\*x + 2\*a) + 1) - 2\*b\*c - d\*log(e^(2\*b\*x + 2\*a) + 1))/(b^2\*e^(2\*b\*x + 2\*a) + b^2)

**maple** [A] time = 0.18, size = 57, normalized size = 1.97

$$\frac{2 d x}{b}+\frac{2 d a}{b^2}-\frac{2(d x+c)}{b\left(1+e^{2 b x+2 a}\right)}-\frac{d \ln \left(1+e^{2 b x+2 a}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)\*sech(b\*x+a)^2,x)

[Out] 2\*d/b\*x+2\*d/b^2\*a-2\*(d\*x+c)/b/(1+exp(2\*b\*x+2\*a))-d/b^2\*ln(1+exp(2\*b\*x+2\*a))

**maxima** [B] time = 0.58, size = 72, normalized size = 2.48

$$d\left(\frac{2xe^{(2bx+2a)}}{be^{(2bx+2a)}+b} - \frac{\log\left(\left(e^{(2bx+2a)}+1\right)e^{(-2a)}\right)}{b^2}\right) + \frac{2c}{b\left(e^{(-2bx-2a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sech(b\*x+a)^2,x, algorithm="maxima")

[Out] d\*(2\*x\*e^(2\*b\*x + 2\*a)/(b\*e^(2\*b\*x + 2\*a) + b) - log((e^(2\*b\*x + 2\*a) + 1)\*e^(-2\*a))/b^2) + 2\*c/(b\*(e^(-2\*b\*x - 2\*a) + 1))

**mupad** [B] time = 0.09, size = 50, normalized size = 1.72

$$\frac{2dx}{b} - \frac{2(c+dx)}{b(e^{2a+2bx}+1)} - \frac{d \ln(e^{2a}e^{2bx}+1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)/cosh(a + b\*x)^2,x)

[Out] (2\*d\*x)/b - (2\*(c + d\*x))/(b\*(exp(2\*a + 2\*b\*x) + 1)) - (d\*log(exp(2\*a)\*exp(2\*b\*x) + 1))/b^2

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \operatorname{sech}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sech(b\*x+a)\*\*2,x)

[Out] Integral((c + d\*x)\*sech(a + b\*x)\*\*2, x)

$$3.34 \quad \int \frac{\operatorname{sech}^2(a+bx)}{c+dx} dx$$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\operatorname{sech}^2(a+bx)}{c+dx}, x\right)$$

[Out] Unintegrable(sech(b\*x+a)^2/(d\*x+c), x)

**Rubi [A]** time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[Sech[a + b\*x]^2/(c + d\*x), x]

[Out] Defer[Int][Sech[a + b\*x]^2/(c + d\*x), x]

Rubi steps

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx} dx = \int \frac{\operatorname{sech}^2(a+bx)}{c+dx} dx$$

**Mathematica [A]** time = 17.78, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sech[a + b\*x]^2/(c + d\*x), x]

[Out] Integrate[Sech[a + b\*x]^2/(c + d\*x), x]

**fricas [A]** time = 0.69, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)^2}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)^2/(d\*x+c),x, algorithm="fricas")

[Out] integral(sech(b\*x + a)^2/(d\*x + c), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)^2/(d\*x+c),x, algorithm="giac")

[Out] integrate(sech(b\*x + a)^2/(d\*x + c), x)

**maple** [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^2}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(b\*x+a)^2/(d\*x+c),x)

[Out] int(sech(b\*x+a)^2/(d\*x+c),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-4d \int \frac{1}{2(bd^2x^2 + 2bcdx + bc^2 + (bd^2x^2e^{(2a)} + 2bcdxe^{(2a)} + bc^2e^{(2a)})e^{(2bx)})} dx - \frac{2}{bdx + bc + (bdxe^{(2a)} + bce^{(2a)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)^2/(d\*x+c),x, algorithm="maxima")

[Out] -4\*d\*integrate(1/2/(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2 + (b\*d^2\*x^2\*e^(2\*a) + 2\*b\*c\*d\*x\*e^(2\*a) + b\*c^2\*e^(2\*a))\*e^(2\*b\*x)), x) - 2/(b\*d\*x + b\*c + (b\*d\*x\*e^(2\*a) + b\*c\*e^(2\*a))\*e^(2\*b\*x))

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cosh(a+bx)^2(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(a + b\*x)^2\*(c + d\*x)),x)

```
[Out] int(1/(cosh(a + b*x)^2*(c + d*x)), x)
```

```
sympy [A] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\operatorname{sech}^2(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2/(d*x+c), x)
```

```
[Out] Integral(sech(a + b*x)**2/(c + d*x), x)
```

$$3.35 \quad \int \frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2}, x\right)$$

[Out] Unintegrable(sech(b\*x+a)^2/(d\*x+c)^2, x)

**Rubi** [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[Sech[a + b\*x]^2/(c + d\*x)^2, x]

[Out] Defer[Int][Sech[a + b\*x]^2/(c + d\*x)^2, x]

Rubi steps

$$\int \frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2} dx = \int \frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2} dx$$

**Mathematica** [A] time = 17.96, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sech[a + b\*x]^2/(c + d\*x)^2, x]

[Out] Integrate[Sech[a + b\*x]^2/(c + d\*x)^2, x]

**fricas** [A] time = 2.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)^2}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)^2/(d\*x+c)^2,x, algorithm="fricas")

[Out] integral(sech(b\*x + a)^2/(d^2\*x^2 + 2\*c\*d\*x + c^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)^2/(d\*x+c)^2,x, algorithm="giac")

[Out] integrate(sech(b\*x + a)^2/(d\*x + c)^2, x)

maple [A] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(b\*x+a)^2/(d\*x+c)^2,x)

[Out] int(sech(b\*x+a)^2/(d\*x+c)^2,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-4d \int \frac{1}{bd^3x^3 + 3bcd^2x^2 + 3bc^2dx + bc^3 + (bd^3x^3e^{(2a)} + 3bcd^2x^2e^{(2a)} + 3bc^2dxe^{(2a)} + bc^3e^{(2a)})e^{(2bx)}} dx - \frac{1}{bd^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)^2/(d\*x+c)^2,x, algorithm="maxima")

[Out] -4\*d\*integrate(1/(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3 + (b\*d^3\*x^3\*e^(2\*a) + 3\*b\*c\*d^2\*x^2\*e^(2\*a) + 3\*b\*c^2\*d\*x\*e^(2\*a) + b\*c^3\*e^(2\*a))\*e^(2\*b\*x)), x) - 2/(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2 + (b\*d^2\*x^2\*e^(2\*a) + 2\*b\*c\*d\*x\*e^(2\*a) + b\*c^2\*e^(2\*a))\*e^(2\*b\*x))

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cosh(a+bx)^2 (c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(1/(cosh(a + b*x)^2*(c + d*x)^2), x)`

[Out] `int(1/(cosh(a + b*x)^2*(c + d*x)^2), x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(b*x+a)**2/(d*x+c)**2, x)`

[Out] `Integral(sech(a + b*x)**2/(c + d*x)**2, x)`

### 3.36 $\int (c + dx)^3 \operatorname{sech}^3(a + bx) dx$

**Optimal.** Leaf size=296

$$\frac{3id^3\operatorname{Li}_2(-ie^{a+bx})}{b^4} - \frac{3id^3\operatorname{Li}_2(ie^{a+bx})}{b^4} - \frac{3id^3\operatorname{Li}_4(-ie^{a+bx})}{b^4} + \frac{3id^3\operatorname{Li}_4(ie^{a+bx})}{b^4} + \frac{3id^2(c+dx)\operatorname{Li}_3(-ie^{a+bx})}{b^3} - \frac{3id^2(c+dx)}{b^3}$$

[Out]  $-6*d^2*(d*x+c)*\arctan(\exp(b*x+a))/b^3+(d*x+c)^3*\arctan(\exp(b*x+a))/b+3*I*d^3*\operatorname{polylog}(2,-I*\exp(b*x+a))/b^4-3/2*I*d*(d*x+c)^2*\operatorname{polylog}(2,-I*\exp(b*x+a))/b^2-3*I*d^3*\operatorname{polylog}(2,I*\exp(b*x+a))/b^4+3/2*I*d*(d*x+c)^2*\operatorname{polylog}(2,I*\exp(b*x+a))/b^2+3*I*d^2*(d*x+c)*\operatorname{polylog}(3,-I*\exp(b*x+a))/b^3-3*I*d^2*(d*x+c)*\operatorname{polylog}(3,I*\exp(b*x+a))/b^3-3*I*d^3*\operatorname{polylog}(4,-I*\exp(b*x+a))/b^4+3*I*d^3*\operatorname{polylog}(4,I*\exp(b*x+a))/b^4+3/2*d*(d*x+c)^2*\operatorname{sech}(b*x+a)/b^2+1/2*(d*x+c)^3*\operatorname{sech}(b*x+a)*\tanh(b*x+a)/b$

**Rubi [A]** time = 0.22, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {4186, 4180, 2279, 2391, 2531, 6609, 2282, 6589}

$$\frac{3id^2(c+dx)\operatorname{PolyLog}(3,-ie^{a+bx})}{b^3} - \frac{3id^2(c+dx)\operatorname{PolyLog}(3,ie^{a+bx})}{b^3} - \frac{3id(c+dx)^2\operatorname{PolyLog}(2,-ie^{a+bx})}{2b^2} + \frac{3id(c+dx)}{b^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^3*\operatorname{Sech}[a + b*x]^3,x]$

[Out]  $(-6*d^2*(c + d*x)*\operatorname{ArcTan}[E^{(a + b*x)}])/b^3 + ((c + d*x)^3*\operatorname{ArcTan}[E^{(a + b*x)}])/b + ((3*I)*d^3*\operatorname{PolyLog}[2, (-I)*E^{(a + b*x)}])/b^4 - (((3*I)/2)*d*(c + d*x)^2*\operatorname{PolyLog}[2, (-I)*E^{(a + b*x)}])/b^2 - ((3*I)*d^3*\operatorname{PolyLog}[2, I*E^{(a + b*x)}])/b^4 + (((3*I)/2)*d*(c + d*x)^2*\operatorname{PolyLog}[2, I*E^{(a + b*x)}])/b^2 + ((3*I)*d^2*(c + d*x)*\operatorname{PolyLog}[3, (-I)*E^{(a + b*x)}])/b^3 - ((3*I)*d^2*(c + d*x)*\operatorname{PolyLog}[3, I*E^{(a + b*x)}])/b^3 - ((3*I)*d^3*\operatorname{PolyLog}[4, (-I)*E^{(a + b*x)}])/b^4 + ((3*I)*d^3*\operatorname{PolyLog}[4, I*E^{(a + b*x)}])/b^4 + (3*d*(c + d*x)^2*\operatorname{Sech}[a + b*x])/(2*b^2) + ((c + d*x)^3*\operatorname{Sech}[a + b*x]*\operatorname{Tanh}[a + b*x])/(2*b)$

**Rule 2279**

$\operatorname{Int}[\operatorname{Log}[(a_) + (b_.)*((F_)^{((e_.)*((c_.) + (d_.)*(x_)))})^{(n_.)}], x\_Symbol]$   
 $\rightarrow \operatorname{Dist}[1/(d*e*n*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, n, x\} \ \&\& \ \operatorname{GtQ}[a, 0]$

**Rule 2282**

$\operatorname{Int}[u_, x\_Symbol] \rightarrow \operatorname{With}\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /;$   $\operatorname{Funci}$

```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*(f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/((f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^n_)*((c_.) + (d_.)*(x_))^m_), x_Symbo
l] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*(a_.) + (b_.
)*(x_)))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \operatorname{sech}^3(a + bx) dx &= \frac{3d(c + dx)^2 \operatorname{sech}(a + bx)}{2b^2} + \frac{(c + dx)^3 \operatorname{sech}(a + bx) \tanh(a + bx)}{2b} + \frac{1}{2} \int (c + dx)^3 \operatorname{sech}^2(a + bx) dx \\
&= -\frac{6d^2(c + dx) \tan^{-1}(e^{a+bx})}{b^3} + \frac{(c + dx)^3 \tan^{-1}(e^{a+bx})}{b} + \frac{3d(c + dx)^2 \operatorname{sech}(a + bx)}{2b^2} + \frac{1}{2} \int (c + dx)^3 \operatorname{sech}^2(a + bx) dx \\
&= -\frac{6d^2(c + dx) \tan^{-1}(e^{a+bx})}{b^3} + \frac{(c + dx)^3 \tan^{-1}(e^{a+bx})}{b} - \frac{3id(c + dx)^2 \operatorname{Li}_2(-ie^{a+bx})}{2b^2} + \frac{1}{2} \int (c + dx)^3 \operatorname{sech}^2(a + bx) dx \\
&= -\frac{6d^2(c + dx) \tan^{-1}(e^{a+bx})}{b^3} + \frac{(c + dx)^3 \tan^{-1}(e^{a+bx})}{b} + \frac{3id^3 \operatorname{Li}_2(-ie^{a+bx})}{b^4} - \frac{3id(c + dx)^2 \operatorname{sech}(a + bx)}{2b} + \frac{1}{2} \int (c + dx)^3 \operatorname{sech}^2(a + bx) dx \\
&= -\frac{6d^2(c + dx) \tan^{-1}(e^{a+bx})}{b^3} + \frac{(c + dx)^3 \tan^{-1}(e^{a+bx})}{b} + \frac{3id^3 \operatorname{Li}_2(-ie^{a+bx})}{b^4} - \frac{3id(c + dx)^2 \operatorname{sech}(a + bx)}{2b} + \frac{1}{2} \int (c + dx)^3 \operatorname{sech}^2(a + bx) dx \\
&= -\frac{6d^2(c + dx) \tan^{-1}(e^{a+bx})}{b^3} + \frac{(c + dx)^3 \tan^{-1}(e^{a+bx})}{b} + \frac{3id^3 \operatorname{Li}_2(-ie^{a+bx})}{b^4} - \frac{3id(c + dx)^2 \operatorname{sech}(a + bx)}{2b} + \frac{1}{2} \int (c + dx)^3 \operatorname{sech}^2(a + bx) dx
\end{aligned}$$

**Mathematica [A]** time = 29.52, size = 455, normalized size = 1.54

$$\frac{b^2(c + dx)^2 \operatorname{sech}(a + bx)(b(c + dx) \tanh(a + bx) + 3d) + i(-2ib^3c^3 \tan^{-1}(e^{a+bx}) + 3b^3c^2 dx \log(1 - ie^{a+bx}) - 3b^3c^2 dx \log(1 + ie^{a+bx}))}{b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3\*Sech[a + b\*x]^3,x]

[Out] (I\*((-2\*I)\*b^3\*c^3\*ArcTan[E^(a + b\*x)] + (12\*I)\*b\*c\*d^2\*ArcTan[E^(a + b\*x)] + 3\*b^3\*c^2\*d\*x\*Log[1 - I\*E^(a + b\*x)] - 6\*b\*d^3\*x\*Log[1 - I\*E^(a + b\*x)] + 3\*b^3\*c\*d^2\*x^2\*Log[1 - I\*E^(a + b\*x)] + b^3\*d^3\*x^3\*Log[1 - I\*E^(a + b\*x)]) - 3\*b^3\*c^2\*d\*x\*Log[1 + I\*E^(a + b\*x)] + 6\*b\*d^3\*x\*Log[1 + I\*E^(a + b\*x)] - 3\*b^3\*c\*d^2\*x^2\*Log[1 + I\*E^(a + b\*x)] - b^3\*d^3\*x^3\*Log[1 + I\*E^(a + b\*x)]) - 3\*d\*(-2\*d^2 + b^2\*(c + d\*x)^2)\*PolyLog[2, (-I)\*E^(a + b\*x)] + 3\*d\*(-2\*d^2 + b^2\*(c + d\*x)^2)\*PolyLog[2, I\*E^(a + b\*x)] + 6\*b\*c\*d^2\*PolyLog[3, (-I)\*E^(a + b\*x)] + 6\*b\*d^3\*x\*PolyLog[3, (-I)\*E^(a + b\*x)] - 6\*b\*c\*d^2\*PolyLog[3, I\*E^(a + b\*x)] - 6\*b\*d^3\*x\*PolyLog[3, I\*E^(a + b\*x)] - 6\*d^3\*PolyLog[3, I\*E^(a + b\*x)]

4,  $(-I)*E^{\wedge}(a + b*x)] + 6*d^{\wedge}3*PolyLog[4, I*E^{\wedge}(a + b*x)] + b^{\wedge}2*(c + d*x)^{\wedge}2*Sech[a + b*x]*(3*d + b*(c + d*x)*Tanh[a + b*x])/(2*b^{\wedge}4)$

**fricas** [C] time = 1.22, size = 4735, normalized size = 16.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*sech(b\*x+a)^3,x, algorithm="fricas")

[Out]  $\frac{1}{2}*(2*(b^3*d^3*x^3 + b^3*c^3 + 3*b^2*c^2*d + 3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*x)*\cosh(b*x + a)^3 + 6*(b^3*d^3*x^3 + b^3*c^3 + 3*b^2*c^2*d + 3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*x)*\cosh(b*x + a)*\sinh(b*x + a)^2 + 2*(b^3*d^3*x^3 + b^3*c^3 + 3*b^2*c^2*d + 3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*x)*\sinh(b*x + a)^3 - 2*(b^3*d^3*x^3 + b^3*c^3 - 3*b^2*c^2*d + 3*(b^3*c*d^2 - b^2*d^3)*x^2 + 3*(b^3*c^2*d - 2*b^2*c*d^2)*x)*\cosh(b*x + a) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3)*\cosh(b*x + a)^4 + (12*I*b^2*d^3*x^2 + 24*I*b^2*c*d^2*x + 12*I*b^2*c^2*d - 24*I*d^3)*\cosh(b*x + a)*\sinh(b*x + a)^3 + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3)*\sinh(b*x + a)^4 - 6*I*d^3 + (6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d - 12*I*d^3)*\cosh(b*x + a)^2 + (6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d - 12*I*d^3 + (18*I*b^2*d^3*x^2 + 36*I*b^2*c*d^2*x + 18*I*b^2*c^2*d - 36*I*d^3)*\cosh(b*x + a)^2)*\sinh(b*x + a)^2 + ((12*I*b^2*d^3*x^2 + 24*I*b^2*c*d^2*x + 12*I*b^2*c^2*d - 24*I*d^3)*\cosh(b*x + a)^3 + (12*I*b^2*d^3*x^2 + 24*I*b^2*c*d^2*x + 12*I*b^2*c^2*d - 24*I*d^3)*\cosh(b*x + a))*\sinh(b*x + a))*\operatorname{dilog}(I*\cosh(b*x + a) + I*\sinh(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3)*\cosh(b*x + a)^4 + (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d + 24*I*d^3)*\cosh(b*x + a)*\sinh(b*x + a)^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3)*\sinh(b*x + a)^4 + 6*I*d^3 + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d + 12*I*d^3)*\cosh(b*x + a)^2 + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d + 12*I*d^3 + (-18*I*b^2*d^3*x^2 - 36*I*b^2*c*d^2*x - 18*I*b^2*c^2*d + 36*I*d^3)*\cosh(b*x + a)^2)*\sinh(b*x + a)^2 + ((-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d + 24*I*d^3)*\cosh(b*x + a)^3 + (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d + 24*I*d^3)*\cosh(b*x + a))*\sinh(b*x + a))*\operatorname{dilog}(-I*\cosh(b*x + a) - I*\sinh(b*x + a)) + (I*b^3*c^3 - 3*I*a*b^2*c^2*d + 3*I*(a^2 - 2)*b*c*d^2 + (I*b^3*c^3 - 3*I*a*b^2*c^2*d + 3*I*(a^2 - 2)*b*c*d^2 - I*(a^3 - 6*a)*d^3)*\cosh(b*x + a)^4 + (4*I*b^3*c^3 - 12*I*a*b^2*c^2*d + 12*I*(a^2 - 2)*b*c*d^2 - 4*I*(a^3 - 6*a)*d^3)*\cosh(b*x + a)*\sinh(b*x + a)^3 + (I*b^3*c^3 - 3*I*a*b^2*c^2*d + 3*I*(a^2 - 2)*b*c*d^2 - I*(a^3 - 6*a)*d^3)*\sinh(b*x + a)^4 - I*(a^3 - 6*a)*d^3 + (2*I*b^3*c^3 - 6*I*a*b^2*c^2*d + 6*I*(a^2 - 2)*b*c*d^2 - 2*I*(a^3 - 6*a)*d^3)*\cosh(b*x + a)^2 + (2*I*b^3*c^3 - 6*I*a*b^2*c^2*d + 6*I*(a^2 - 2)*b*c*d^2 - 2*I*(a^3 - 6*a)*d^3)*\sinh(b*x + a)^2$

$$\begin{aligned}
& - 6*a)*d^3 + (6*I*b^3*c^3 - 18*I*a*b^2*c^2*d + 18*I*(a^2 - 2)*b*c*d^2 - 6* \\
& I*(a^3 - 6*a)*d^3)*\cosh(b*x + a)^2*\sinh(b*x + a)^2 + ((4*I*b^3*c^3 - 12*I* \\
& a*b^2*c^2*d + 12*I*(a^2 - 2)*b*c*d^2 - 4*I*(a^3 - 6*a)*d^3)*\cosh(b*x + a)^3 \\
& + (4*I*b^3*c^3 - 12*I*a*b^2*c^2*d + 12*I*(a^2 - 2)*b*c*d^2 - 4*I*(a^3 - 6* \\
& a)*d^3)*\cosh(b*x + a))*\sinh(b*x + a))*\log(\cosh(b*x + a) + \sinh(b*x + a) + I \\
& ) + (-I*b^3*c^3 + 3*I*a*b^2*c^2*d - 3*I*(a^2 - 2)*b*c*d^2 + (-I*b^3*c^3 + 3 \\
& *I*a*b^2*c^2*d - 3*I*(a^2 - 2)*b*c*d^2 + I*(a^3 - 6*a)*d^3)*\cosh(b*x + a)^4 \\
& + (-4*I*b^3*c^3 + 12*I*a*b^2*c^2*d - 12*I*(a^2 - 2)*b*c*d^2 + 4*I*(a^3 - 6 \\
& *a)*d^3)*\cosh(b*x + a)*\sinh(b*x + a)^3 + (-I*b^3*c^3 + 3*I*a*b^2*c^2*d - 3* \\
& I*(a^2 - 2)*b*c*d^2 + I*(a^3 - 6*a)*d^3)*\sinh(b*x + a)^4 + I*(a^3 - 6*a)*d^ \\
& 3 + (-2*I*b^3*c^3 + 6*I*a*b^2*c^2*d - 6*I*(a^2 - 2)*b*c*d^2 + 2*I*(a^3 - 6* \\
& a)*d^3)*\cosh(b*x + a)^2 + (-2*I*b^3*c^3 + 6*I*a*b^2*c^2*d - 6*I*(a^2 - 2)*b \\
& *c*d^2 + 2*I*(a^3 - 6*a)*d^3 + (-6*I*b^3*c^3 + 18*I*a*b^2*c^2*d - 18*I*(a^2 \\
& - 2)*b*c*d^2 + 6*I*(a^3 - 6*a)*d^3)*\cosh(b*x + a)^2*\sinh(b*x + a)^2 + ((- \\
& 4*I*b^3*c^3 + 12*I*a*b^2*c^2*d - 12*I*(a^2 - 2)*b*c*d^2 + 4*I*(a^3 - 6*a)*d \\
& ^3)*\cosh(b*x + a)^3 + (-4*I*b^3*c^3 + 12*I*a*b^2*c^2*d - 12*I*(a^2 - 2)*b*c \\
& *d^2 + 4*I*(a^3 - 6*a)*d^3)*\cosh(b*x + a))*\sinh(b*x + a))*\log(\cosh(b*x + a) \\
& + \sinh(b*x + a) - I) + (-I*b^3*d^3*x^3 - 3*I*b^3*c*d^2*x^2 - 3*I*a*b^2*c^2* \\
& *d + 3*I*a^2*b*c*d^2 + (-I*b^3*d^3*x^3 - 3*I*b^3*c*d^2*x^2 - 3*I*a*b^2*c^2* \\
& d + 3*I*a^2*b*c*d^2 - I*(a^3 - 6*a)*d^3 - 3*I*(b^3*c^2*d - 2*b*d^3)*x)*\cosh \\
& (b*x + a)^4 + (-4*I*b^3*d^3*x^3 - 12*I*b^3*c*d^2*x^2 - 12*I*a*b^2*c^2*d + 1 \\
& 2*I*a^2*b*c*d^2 - 4*I*(a^3 - 6*a)*d^3 - 12*I*(b^3*c^2*d - 2*b*d^3)*x)*\cosh( \\
& b*x + a)*\sinh(b*x + a)^3 + (-I*b^3*d^3*x^3 - 3*I*b^3*c*d^2*x^2 - 3*I*a*b^2* \\
& c^2*d + 3*I*a^2*b*c*d^2 - I*(a^3 - 6*a)*d^3 - 3*I*(b^3*c^2*d - 2*b*d^3)*x)* \\
& \sinh(b*x + a)^4 - I*(a^3 - 6*a)*d^3 + (-2*I*b^3*d^3*x^3 - 6*I*b^3*c*d^2*x^2 \\
& - 6*I*a*b^2*c^2*d + 6*I*a^2*b*c*d^2 - 2*I*(a^3 - 6*a)*d^3 - 6*I*(b^3*c^2*d \\
& - 2*b*d^3)*x)*\cosh(b*x + a)^2 + (-2*I*b^3*d^3*x^3 - 6*I*b^3*c*d^2*x^2 - 6* \\
& I*a*b^2*c^2*d + 6*I*a^2*b*c*d^2 - 2*I*(a^3 - 6*a)*d^3 + (-6*I*b^3*d^3*x^3 - \\
& 18*I*b^3*c*d^2*x^2 - 18*I*a*b^2*c^2*d + 18*I*a^2*b*c*d^2 - 6*I*(a^3 - 6*a) \\
& *d^3 - 18*I*(b^3*c^2*d - 2*b*d^3)*x)*\cosh(b*x + a)^2 - 6*I*(b^3*c^2*d - 2*b \\
& *d^3)*x)*\sinh(b*x + a)^2 - 3*I*(b^3*c^2*d - 2*b*d^3)*x + ((-4*I*b^3*d^3*x^3 \\
& - 12*I*b^3*c*d^2*x^2 - 12*I*a*b^2*c^2*d + 12*I*a^2*b*c*d^2 - 4*I*(a^3 - 6* \\
& a)*d^3 - 12*I*(b^3*c^2*d - 2*b*d^3)*x)*\cosh(b*x + a)^3 + (-4*I*b^3*d^3*x^3 \\
& - 12*I*b^3*c*d^2*x^2 - 12*I*a*b^2*c^2*d + 12*I*a^2*b*c*d^2 - 4*I*(a^3 - 6*a) \\
& )*d^3 - 12*I*(b^3*c^2*d - 2*b*d^3)*x)*\cosh(b*x + a))*\sinh(b*x + a))*\log(I*c \\
& osh(b*x + a) + I*\sinh(b*x + a) + 1) + (I*b^3*d^3*x^3 + 3*I*b^3*c*d^2*x^2 + \\
& 3*I*a*b^2*c^2*d - 3*I*a^2*b*c*d^2 + (I*b^3*d^3*x^3 + 3*I*b^3*c*d^2*x^2 + 3* \\
& I*a*b^2*c^2*d - 3*I*a^2*b*c*d^2 + I*(a^3 - 6*a)*d^3 + 3*I*(b^3*c^2*d - 2*b* \\
& d^3)*x)*\cosh(b*x + a)^4 + (4*I*b^3*d^3*x^3 + 12*I*b^3*c*d^2*x^2 + 12*I*a*b^ \\
& 2*c^2*d - 12*I*a^2*b*c*d^2 + 4*I*(a^3 - 6*a)*d^3 + 12*I*(b^3*c^2*d - 2*b*d^ \\
& 3)*x)*\cosh(b*x + a)*\sinh(b*x + a)^3 + (I*b^3*d^3*x^3 + 3*I*b^3*c*d^2*x^2 + \\
& 3*I*a*b^2*c^2*d - 3*I*a^2*b*c*d^2 + I*(a^3 - 6*a)*d^3 + 3*I*(b^3*c^2*d - 2* \\
& b*d^3)*x)*\sinh(b*x + a)^4 + I*(a^3 - 6*a)*d^3 + (2*I*b^3*d^3*x^3 + 6*I*b^3* \\
& c*d^2*x^2 + 6*I*a*b^2*c^2*d - 6*I*a^2*b*c*d^2 + 2*I*(a^3 - 6*a)*d^3 + 6*I*( \\
& b^3*c^2*d - 2*b*d^3)*x)*\cosh(b*x + a)^2 + (2*I*b^3*d^3*x^3 + 6*I*b^3*c*d^2*x
\end{aligned}$$

```

x^2 + 6*I*a*b^2*c^2*d - 6*I*a^2*b*c*d^2 + 2*I*(a^3 - 6*a)*d^3 + (6*I*b^3*d^
3*x^3 + 18*I*b^3*c*d^2*x^2 + 18*I*a*b^2*c^2*d - 18*I*a^2*b*c*d^2 + 6*I*(a^3
- 6*a)*d^3 + 18*I*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^2 + 6*I*(b^3*c^2*
d - 2*b*d^3)*x)*sinh(b*x + a)^2 + 3*I*(b^3*c^2*d - 2*b*d^3)*x + ((4*I*b^3*d
^3*x^3 + 12*I*b^3*c*d^2*x^2 + 12*I*a*b^2*c^2*d - 12*I*a^2*b*c*d^2 + 4*I*(a^
3 - 6*a)*d^3 + 12*I*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^3 + (4*I*b^3*d^3
*x^3 + 12*I*b^3*c*d^2*x^2 + 12*I*a*b^2*c^2*d - 12*I*a^2*b*c*d^2 + 4*I*(a^3
- 6*a)*d^3 + 12*I*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a))*sinh(b*x + a))*lo
g(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) + (6*I*d^3*cosh(b*x + a)^4 + 24*I
*d^3*cosh(b*x + a)*sinh(b*x + a)^3 + 6*I*d^3*sinh(b*x + a)^4 + 12*I*d^3*cos
h(b*x + a)^2 + 6*I*d^3 + (36*I*d^3*cosh(b*x + a)^2 + 12*I*d^3)*sinh(b*x + a
)^2 + (24*I*d^3*cosh(b*x + a)^3 + 24*I*d^3*cosh(b*x + a))*sinh(b*x + a))*po
lylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) + (-6*I*d^3*cosh(b*x + a)^4 - 2
4*I*d^3*cosh(b*x + a)*sinh(b*x + a)^3 - 6*I*d^3*sinh(b*x + a)^4 - 12*I*d^3*cos
h(b*x + a)^2 - 6*I*d^3 + (-36*I*d^3*cosh(b*x + a)^2 - 12*I*d^3)*sinh(b*x
+ a)^2 + (-24*I*d^3*cosh(b*x + a)^3 - 24*I*d^3*cosh(b*x + a))*sinh(b*x + a
))*polylog(4, -I*cosh(b*x + a) - I*sinh(b*x + a)) + (-6*I*b*d^3*x + (-6*I*b
*d^3*x - 6*I*b*c*d^2)*cosh(b*x + a)^4 + (-24*I*b*d^3*x - 24*I*b*c*d^2)*cosh
(b*x + a)*sinh(b*x + a)^3 + (-6*I*b*d^3*x - 6*I*b*c*d^2)*sinh(b*x + a)^4 -
6*I*b*c*d^2 + (-12*I*b*d^3*x - 12*I*b*c*d^2)*cosh(b*x + a)^2 + (-12*I*b*d^3
*x - 12*I*b*c*d^2 + (-36*I*b*d^3*x - 36*I*b*c*d^2)*cosh(b*x + a)^2)*sinh(b*
x + a)^2 + ((-24*I*b*d^3*x - 24*I*b*c*d^2)*cosh(b*x + a)^3 + (-24*I*b*d^3*x
- 24*I*b*c*d^2)*cosh(b*x + a))*sinh(b*x + a))*polylog(3, I*cosh(b*x + a) +
I*sinh(b*x + a)) + (6*I*b*d^3*x + (6*I*b*d^3*x + 6*I*b*c*d^2)*cosh(b*x + a
)^4 + (24*I*b*d^3*x + 24*I*b*c*d^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (6*I*b*
d^3*x + 6*I*b*c*d^2)*sinh(b*x + a)^4 + 6*I*b*c*d^2 + (12*I*b*d^3*x + 12*I*b
*c*d^2)*cosh(b*x + a)^2 + (12*I*b*d^3*x + 12*I*b*c*d^2 + (36*I*b*d^3*x + 36
*I*b*c*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + ((24*I*b*d^3*x + 24*I*b*c*d^
2)*cosh(b*x + a)^3 + (24*I*b*d^3*x + 24*I*b*c*d^2)*cosh(b*x + a))*sinh(b*x
+ a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(b^3*d^3*x^3 + b^3
*c^3 - 3*b^2*c^2*d + 3*(b^3*c*d^2 - b^2*d^3)*x^2 - 3*(b^3*d^3*x^3 + b^3*c^3
+ 3*b^2*c^2*d + 3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*
x)*cosh(b*x + a)^2 + 3*(b^3*c^2*d - 2*b^2*c*d^2)*x)*sinh(b*x + a))/(b^4*cos
h(b*x + a)^4 + 4*b^4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 +
2*b^4*cosh(b*x + a)^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 + b^4)*sinh(b*x + a)
^2 + 4*(b^4*cosh(b*x + a)^3 + b^4*cosh(b*x + a))*sinh(b*x + a))

```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \operatorname{sech}(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*sech(b\*x+a)^3,x, algorithm="giac")

[Out] integrate((d\*x + c)^3\*sech(b\*x + a)^3, x)

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \operatorname{sech}(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3\*sech(b\*x+a)^3,x)

[Out] int((d\*x+c)^3\*sech(b\*x+a)^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2 d^3 \int \frac{x^3 e^{(bx+a)}}{b^2 e^{(2bx+2a)} + b^2} dx + 3 b^2 c d^2 \int \frac{x^2 e^{(bx+a)}}{b^2 e^{(2bx+2a)} + b^2} dx + 3 b^2 c^2 d \int \frac{x e^{(bx+a)}}{b^2 e^{(2bx+2a)} + b^2} dx - c^3 \left( \frac{\arctan\left(\frac{e^{(-bx-a)}}{b}\right)}{b} \right) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*sech(b\*x+a)^3,x, algorithm="maxima")

[Out] b^2\*d^3\*integrate(x^3\*e^(b\*x + a)/(b^2\*e^(2\*b\*x + 2\*a) + b^2), x) + 3\*b^2\*c\*d^2\*integrate(x^2\*e^(b\*x + a)/(b^2\*e^(2\*b\*x + 2\*a) + b^2), x) + 3\*b^2\*c^2\*d\*integrate(x\*e^(b\*x + a)/(b^2\*e^(2\*b\*x + 2\*a) + b^2), x) - c^3\*(arctan(e^(-b\*x - a))/b - (e^(-b\*x - a) - e^(-3\*b\*x - 3\*a))/(b\*(2\*e^(-2\*b\*x - 2\*a) + e^(-4\*b\*x - 4\*a) + 1))) - 6\*d^3\*integrate(x\*e^(b\*x + a)/(b^2\*e^(2\*b\*x + 2\*a) + b^2), x) - 6\*c\*d^2\*arctan(e^(b\*x + a))/b^3 + ((b\*d^3\*x^3\*e^(3\*a) + 3\*c^2\*d\*e^(3\*a) + 3\*(b\*c\*d^2 + d^3)\*x^2\*e^(3\*a) + 3\*(b\*c^2\*d + 2\*c\*d^2)\*x\*e^(3\*a))\*e^(3\*b\*x) - (b\*d^3\*x^3\*e^a - 3\*c^2\*d\*e^a + 3\*(b\*c\*d^2 - d^3)\*x^2\*e^a + 3\*(b\*c^2\*d - 2\*c\*d^2)\*x\*e^a)\*e^(b\*x))/(b^2\*e^(4\*b\*x + 4\*a) + 2\*b^2\*e^(2\*b\*x + 2\*a) + b^2)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + dx)^3}{\cosh(a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^3/cosh(a + b\*x)^3,x)

[Out] int((c + d\*x)^3/cosh(a + b\*x)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \operatorname{sech}^3(a + bx) dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sech(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*sech(a + b*x)**3, x)
```

### 3.37 $\int (c + dx)^2 \operatorname{sech}^3(a + bx) dx$

**Optimal.** Leaf size=175

$$\frac{id^2 \operatorname{Li}_3(-ie^{a+bx})}{b^3} - \frac{id^2 \operatorname{Li}_3(ie^{a+bx})}{b^3} - \frac{d^2 \tan^{-1}(\sinh(a + bx))}{b^3} - \frac{id(c + dx) \operatorname{Li}_2(-ie^{a+bx})}{b^2} + \frac{id(c + dx) \operatorname{Li}_2(ie^{a+bx})}{b^2} + \frac{d(c + dx)}{b}$$

[Out]  $(d*x+c)^2*\arctan(\exp(b*x+a))/b-d^2*\arctan(\sinh(b*x+a))/b^3-I*d*(d*x+c)*\operatorname{polylog}(2,-I*\exp(b*x+a))/b^2+I*d*(d*x+c)*\operatorname{polylog}(2,I*\exp(b*x+a))/b^2+I*d^2*\operatorname{polylog}(3,-I*\exp(b*x+a))/b^3-I*d^2*\operatorname{polylog}(3,I*\exp(b*x+a))/b^3+d*(d*x+c)*\operatorname{sech}(b*x+a)/b^2+1/2*(d*x+c)^2*\operatorname{sech}(b*x+a)*\tanh(b*x+a)/b$

**Rubi [A]** time = 0.13, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4186, 3770, 4180, 2531, 2282, 6589}

$$-\frac{id(c + dx)\operatorname{PolyLog}(2, -ie^{a+bx})}{b^2} + \frac{id(c + dx)\operatorname{PolyLog}(2, ie^{a+bx})}{b^2} + \frac{id^2\operatorname{PolyLog}(3, -ie^{a+bx})}{b^3} - \frac{id^2\operatorname{PolyLog}(3, ie^{a+bx})}{b^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^2*\operatorname{Sech}[a + b*x]^3, x]$

[Out]  $((c + d*x)^2*\operatorname{ArcTan}[E^{(a + b*x)}])/b - (d^2*\operatorname{ArcTan}[\operatorname{Sinh}[a + b*x]])/b^3 - (I*d*(c + d*x)*\operatorname{PolyLog}[2, (-I)*E^{(a + b*x)}])/b^2 + (I*d*(c + d*x)*\operatorname{PolyLog}[2, I*E^{(a + b*x)}])/b^2 + (I*d^2*\operatorname{PolyLog}[3, (-I)*E^{(a + b*x)}])/b^3 - (I*d^2*\operatorname{PolyLog}[3, I*E^{(a + b*x)}])/b^3 + (d*(c + d*x)*\operatorname{Sech}[a + b*x])/b^2 + ((c + d*x)^2*\operatorname{Sech}[a + b*x]*\operatorname{Tanh}[a + b*x])/(2*b)$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m-1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/ (f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^2 \operatorname{sech}^3(a + bx) dx &= \frac{d(c + dx) \operatorname{sech}(a + bx)}{b^2} + \frac{(c + dx)^2 \operatorname{sech}(a + bx) \tanh(a + bx)}{2b} + \frac{1}{2} \int (c + dx)^2 \operatorname{sech} \\
&= \frac{(c + dx)^2 \tan^{-1}(e^{a+bx})}{b} - \frac{d^2 \tan^{-1}(\sinh(a + bx))}{b^3} + \frac{d(c + dx) \operatorname{sech}(a + bx)}{b^2} + \frac{(c + dx)^2 \operatorname{sech}(a + bx) \tanh(a + bx)}{2b} \\
&= \frac{(c + dx)^2 \tan^{-1}(e^{a+bx})}{b} - \frac{d^2 \tan^{-1}(\sinh(a + bx))}{b^3} - \frac{id(c + dx) \operatorname{Li}_2(-ie^{a+bx})}{b^2} + \frac{id(c + dx)^2 \operatorname{sech}(a + bx) \tanh(a + bx)}{2b} \\
&= \frac{(c + dx)^2 \tan^{-1}(e^{a+bx})}{b} - \frac{d^2 \tan^{-1}(\sinh(a + bx))}{b^3} - \frac{id(c + dx) \operatorname{Li}_2(-ie^{a+bx})}{b^2} + \frac{id(c + dx)^2 \operatorname{sech}(a + bx) \tanh(a + bx)}{2b} \\
&= \frac{(c + dx)^2 \tan^{-1}(e^{a+bx})}{b} - \frac{d^2 \tan^{-1}(\sinh(a + bx))}{b^3} - \frac{id(c + dx) \operatorname{Li}_2(-ie^{a+bx})}{b^2} + \frac{id(c + dx)^2 \operatorname{sech}(a + bx) \tanh(a + bx)}{2b}
\end{aligned}$$

**Mathematica [A]** time = 5.52, size = 270, normalized size = 1.54

$$\frac{i(-2ib^2c^2 \tan^{-1}(e^{a+bx}) + 2b^2cdx \log(1 - ie^{a+bx}) - 2b^2cdx \log(1 + ie^{a+bx}) + b^2d^2x^2 \log(1 - ie^{a+bx}) - b^2d^2x^2 \log(1 + ie^{a+bx}))}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2\*Sech[a + b\*x]^3,x]

[Out] (I\*((-2\*I)\*b^2\*c^2\*ArcTan[E^(a + b\*x)] + (4\*I)\*d^2\*ArcTan[E^(a + b\*x)] + 2\*b^2\*c\*d\*x\*Log[1 - I\*E^(a + b\*x)] + b^2\*d^2\*x^2\*Log[1 - I\*E^(a + b\*x)] - 2\*b^2\*c\*d\*x\*Log[1 + I\*E^(a + b\*x)] - b^2\*d^2\*x^2\*Log[1 + I\*E^(a + b\*x)] - 2\*b\*d\*(c + d\*x)\*PolyLog[2, (-I)\*E^(a + b\*x)] + 2\*b\*d\*(c + d\*x)\*PolyLog[2, I\*E^(a + b\*x)] + 2\*d^2\*PolyLog[3, (-I)\*E^(a + b\*x)] - 2\*d^2\*PolyLog[3, I\*E^(a + b\*x)]) + b^2\*(c + d\*x)^2\*Sech[a]\*Sech[a + b\*x]^2\*Sinh[b\*x] + b\*(c + d\*x)\*Sech[a + b\*x]\*(2\*d + b\*(c + d\*x)\*Tanh[a]))/(2\*b^3)

**fricas [C]** time = 1.27, size = 2613, normalized size = 14.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*sech(b\*x+a)^3,x, algorithm="fricas")

[Out] 1/2\*(2\*(b^2\*d^2\*x^2 + b^2\*c^2 + 2\*b\*c\*d + 2\*(b^2\*c\*d + b\*d^2)\*x)\*cosh(b\*x + a)^3 + 6\*(b^2\*d^2\*x^2 + b^2\*c^2 + 2\*b\*c\*d + 2\*(b^2\*c\*d + b\*d^2)\*x)\*cosh(b\*x + a)\*sinh(b\*x + a)^2 + 2\*(b^2\*d^2\*x^2 + b^2\*c^2 + 2\*b\*c\*d + 2\*(b^2\*c\*d + b\*d^2)\*x)\*sinh(b\*x + a)^3 - 2\*(b^2\*d^2\*x^2 + b^2\*c^2 - 2\*b\*c\*d + 2\*(b^2\*c\*d - b\*d^2)\*x)\*cosh(b\*x + a) + ((2\*I\*b\*d^2\*x + 2\*I\*b\*c\*d)\*cosh(b\*x + a)^4 + (

$$\begin{aligned}
& 8I*b*d^2*x + 8I*b*c*d)*\cosh(b*x + a)*\sinh(b*x + a)^3 + (2I*b*d^2*x + 2I \\
& *b*c*d)*\sinh(b*x + a)^4 + 2I*b*d^2*x + 2I*b*c*d + (4I*b*d^2*x + 4I*b*c* \\
& d)*\cosh(b*x + a)^2 + (4I*b*d^2*x + 4I*b*c*d + (12I*b*d^2*x + 12I*b*c*d) \\
& *\cosh(b*x + a)^2)*\sinh(b*x + a)^2 + ((8I*b*d^2*x + 8I*b*c*d)*\cosh(b*x + a \\
& )^3 + (8I*b*d^2*x + 8I*b*c*d)*\cosh(b*x + a))*\sinh(b*x + a))*\operatorname{dilog}(I*\cosh( \\
& b*x + a) + I*\sinh(b*x + a)) + ((-2I*b*d^2*x - 2I*b*c*d)*\cosh(b*x + a)^4 + \\
& (-8I*b*d^2*x - 8I*b*c*d)*\cosh(b*x + a)*\sinh(b*x + a)^3 + (-2I*b*d^2*x - \\
& 2I*b*c*d)*\sinh(b*x + a)^4 - 2I*b*d^2*x - 2I*b*c*d + (-4I*b*d^2*x - 4I \\
& *b*c*d)*\cosh(b*x + a)^2 + (-4I*b*d^2*x - 4I*b*c*d + (-12I*b*d^2*x - 12I \\
& *b*c*d)*\cosh(b*x + a)^2)*\sinh(b*x + a)^2 + ((-8I*b*d^2*x - 8I*b*c*d)*\cosh \\
& (b*x + a)^3 + (-8I*b*d^2*x - 8I*b*c*d)*\cosh(b*x + a))*\sinh(b*x + a))*\operatorname{dilo} \\
& \operatorname{g}(-I*\cosh(b*x + a) - I*\sinh(b*x + a)) + ((I*b^2*c^2 - 2I*a*b*c*d + I*(a^2 \\
& - 2)*d^2)*\cosh(b*x + a)^4 + (4I*b^2*c^2 - 8I*a*b*c*d + 4I*(a^2 - 2)*d^2) \\
& *\cosh(b*x + a)*\sinh(b*x + a)^3 + (I*b^2*c^2 - 2I*a*b*c*d + I*(a^2 - 2)*d^2) \\
& )*\sinh(b*x + a)^4 + I*b^2*c^2 - 2I*a*b*c*d + I*(a^2 - 2)*d^2 + (2I*b^2*c^ \\
& 2 - 4I*a*b*c*d + 2I*(a^2 - 2)*d^2)*\cosh(b*x + a)^2 + (2I*b^2*c^2 - 4I*a \\
& *b*c*d + 2I*(a^2 - 2)*d^2 + (6I*b^2*c^2 - 12I*a*b*c*d + 6I*(a^2 - 2)*d^ \\
& 2)*\cosh(b*x + a)^2)*\sinh(b*x + a)^2 + ((4I*b^2*c^2 - 8I*a*b*c*d + 4I*(a^ \\
& 2 - 2)*d^2)*\cosh(b*x + a)^3 + (4I*b^2*c^2 - 8I*a*b*c*d + 4I*(a^2 - 2)*d^ \\
& 2)*\cosh(b*x + a))*\sinh(b*x + a))*\log(\cosh(b*x + a) + \sinh(b*x + a) + I) + ( \\
& (-I*b^2*c^2 + 2I*a*b*c*d - I*(a^2 - 2)*d^2)*\cosh(b*x + a)^4 + (-4I*b^2*c^ \\
& 2 + 8I*a*b*c*d - 4I*(a^2 - 2)*d^2)*\cosh(b*x + a)*\sinh(b*x + a)^3 + (-I*b^ \\
& 2*c^2 + 2I*a*b*c*d - I*(a^2 - 2)*d^2)*\sinh(b*x + a)^4 - I*b^2*c^2 + 2I*a*a \\
& b*c*d - I*(a^2 - 2)*d^2 + (-2I*b^2*c^2 + 4I*a*b*c*d - 2I*(a^2 - 2)*d^2)* \\
& \cosh(b*x + a)^2 + (-2I*b^2*c^2 + 4I*a*b*c*d - 2I*(a^2 - 2)*d^2 + (-6I*b \\
& ^2*c^2 + 12I*a*b*c*d - 6I*(a^2 - 2)*d^2)*\cosh(b*x + a)^2)*\sinh(b*x + a)^2 \\
& + ((-4I*b^2*c^2 + 8I*a*b*c*d - 4I*(a^2 - 2)*d^2)*\cosh(b*x + a)^3 + (-4I \\
& I*b^2*c^2 + 8I*a*b*c*d - 4I*(a^2 - 2)*d^2)*\cosh(b*x + a))*\sinh(b*x + a))* \\
& \log(\cosh(b*x + a) + \sinh(b*x + a) - I) + (-I*b^2*d^2*x^2 - 2I*b^2*c*d*x + \\
& (-I*b^2*d^2*x^2 - 2I*b^2*c*d*x - 2I*a*b*c*d + I*a^2*d^2)*\cosh(b*x + a)^4 \\
& + (-4I*b^2*d^2*x^2 - 8I*b^2*c*d*x - 8I*a*b*c*d + 4I*a^2*d^2)*\cosh(b*x + \\
& a)*\sinh(b*x + a)^3 + (-I*b^2*d^2*x^2 - 2I*b^2*c*d*x - 2I*a*b*c*d + I*a^2 \\
& *d^2)*\sinh(b*x + a)^4 - 2I*a*b*c*d + I*a^2*d^2 + (-2I*b^2*d^2*x^2 - 4I*b \\
& ^2*c*d*x - 4I*a*b*c*d + 2I*a^2*d^2)*\cosh(b*x + a)^2 + (-2I*b^2*d^2*x^2 - \\
& 4I*b^2*c*d*x - 4I*a*b*c*d + 2I*a^2*d^2 + (-6I*b^2*d^2*x^2 - 12I*b^2*c \\
& *d*x - 12I*a*b*c*d + 6I*a^2*d^2)*\cosh(b*x + a)^2)*\sinh(b*x + a)^2 + ((-4I \\
& I*b^2*d^2*x^2 - 8I*b^2*c*d*x - 8I*a*b*c*d + 4I*a^2*d^2)*\cosh(b*x + a)^3 \\
& + (-4I*b^2*d^2*x^2 - 8I*b^2*c*d*x - 8I*a*b*c*d + 4I*a^2*d^2)*\cosh(b*x + \\
& a))*\sinh(b*x + a))*\log(I*\cosh(b*x + a) + I*\sinh(b*x + a) + 1) + (I*b^2*d^2 \\
& *x^2 + 2I*b^2*c*d*x + (I*b^2*d^2*x^2 + 2I*b^2*c*d*x + 2I*a*b*c*d - I*a^2 \\
& *d^2)*\cosh(b*x + a)^4 + (4I*b^2*d^2*x^2 + 8I*b^2*c*d*x + 8I*a*b*c*d - 4I \\
& I*a^2*d^2)*\cosh(b*x + a)*\sinh(b*x + a)^3 + (I*b^2*d^2*x^2 + 2I*b^2*c*d*x + \\
& 2I*a*b*c*d - I*a^2*d^2)*\sinh(b*x + a)^4 + 2I*a*b*c*d - I*a^2*d^2 + (2I*I \\
& b^2*d^2*x^2 + 4I*b^2*c*d*x + 4I*a*b*c*d - 2I*a^2*d^2)*\cosh(b*x + a)^2 + \\
& (2I*b^2*d^2*x^2 + 4I*b^2*c*d*x + 4I*a*b*c*d - 2I*a^2*d^2 + (6I*b^2*d^2
\end{aligned}$$

```

*x^2 + 12*I*b^2*c*d*x + 12*I*a*b*c*d - 6*I*a^2*d^2)*cosh(b*x + a)^2)*sinh(b
*x + a)^2 + ((4*I*b^2*d^2*x^2 + 8*I*b^2*c*d*x + 8*I*a*b*c*d - 4*I*a^2*d^2)*
cosh(b*x + a)^3 + (4*I*b^2*d^2*x^2 + 8*I*b^2*c*d*x + 8*I*a*b*c*d - 4*I*a^2*
d^2)*cosh(b*x + a))*sinh(b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) +
1) + (-2*I*d^2*cosh(b*x + a)^4 - 8*I*d^2*cosh(b*x + a)*sinh(b*x + a)^3 - 2
*I*d^2*sinh(b*x + a)^4 - 4*I*d^2*cosh(b*x + a)^2 + (-12*I*d^2*cosh(b*x + a)
^2 - 4*I*d^2)*sinh(b*x + a)^2 - 2*I*d^2 + (-8*I*d^2*cosh(b*x + a)^3 - 8*I*d
^2*cosh(b*x + a))*sinh(b*x + a))*polylog(3, I*cosh(b*x + a) + I*sinh(b*x +
a)) + (2*I*d^2*cosh(b*x + a)^4 + 8*I*d^2*cosh(b*x + a)*sinh(b*x + a)^3 + 2*
I*d^2*sinh(b*x + a)^4 + 4*I*d^2*cosh(b*x + a)^2 + (12*I*d^2*cosh(b*x + a)^2
+ 4*I*d^2)*sinh(b*x + a)^2 + 2*I*d^2 + (8*I*d^2*cosh(b*x + a)^3 + 8*I*d^2*
cosh(b*x + a))*sinh(b*x + a))*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)
) - 2*(b^2*d^2*x^2 + b^2*c^2 - 2*b*c*d - 3*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d
+ 2*(b^2*c*d + b*d^2)*x)*cosh(b*x + a)^2 + 2*(b^2*c*d - b*d^2)*x)*sinh(b*x
+ a))/(b^3*cosh(b*x + a)^4 + 4*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sin
h(b*x + a)^4 + 2*b^3*cosh(b*x + a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 + b^3
)*sinh(b*x + a)^2 + 4*(b^3*cosh(b*x + a)^3 + b^3*cosh(b*x + a))*sinh(b*x +
a))

```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \operatorname{sech}(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*sech(b\*x+a)^3,x, algorithm="giac")

[Out] integrate((d\*x + c)^2\*sech(b\*x + a)^3, x)

**maple** [F] time = 0.57, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \operatorname{sech}(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*sech(b\*x+a)^3,x)

[Out] int((d\*x+c)^2\*sech(b\*x+a)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2 d^2 \int \frac{x^2 e^{(bx+a)}}{b^2 e^{(2bx+2a)} + b^2} dx + 2 b^2 c d \int \frac{x e^{(bx+a)}}{b^2 e^{(2bx+2a)} + b^2} dx - c^2 \left( \frac{\arctan\left(e^{(-bx-a)}\right)}{b} - \frac{e^{(-bx-a)} - e^{(-3bx-3a)}}{b(2e^{(-2bx-2a)} + e^{(-4bx-4a)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*sech(b\*x+a)^3,x, algorithm="maxima")

[Out]  $b^2*d^2*\int x^2*e^{(b*x+a)}/(b^2*e^{(2*b*x+2*a)}+b^2), x) + 2*b^2*c*d*\int x*e^{(b*x+a)}/(b^2*e^{(2*b*x+2*a)}+b^2), x) - c^2*(\arctan(e^{(-b*x-a)})/b - (e^{(-b*x-a)} - e^{(-3*b*x-3*a)})/(b*(2*e^{(-2*b*x-2*a)} + e^{(-4*b*x-4*a)} + 1))) + ((b*d^2*x^2*e^{(3*a)} + 2*c*d*e^{(3*a)} + 2*(b*c*d + d^2)*x*e^{(3*a)})*e^{(3*b*x)} - (b*d^2*x^2*e^a - 2*c*d*e^a + 2*(b*c*d - d^2)*x*e^a)*e^{(b*x)})/(b^2*e^{(4*b*x+4*a)} + 2*b^2*e^{(2*b*x+2*a)} + b^2) - 2*d^2*a*\arctan(e^{(b*x+a)})/b^3$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+dx)^2}{\cosh(a+bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^2/cosh(a + b\*x)^3,x)

[Out] int((c + d\*x)^2/cosh(a + b\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \operatorname{sech}^3(a+bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2\*sech(b\*x+a)\*\*3,x)

[Out] Integral((c + d\*x)\*\*2\*sech(a + b\*x)\*\*3, x)

### 3.38 $\int (c + dx)\operatorname{sech}^3(a + bx) dx$

**Optimal.** Leaf size=102

$$-\frac{id\operatorname{Li}_2(-ie^{a+bx})}{2b^2} + \frac{id\operatorname{Li}_2(ie^{a+bx})}{2b^2} + \frac{d\operatorname{sech}(a + bx)}{2b^2} + \frac{(c + dx)\tan^{-1}(e^{a+bx})}{b} + \frac{(c + dx)\tanh(a + bx)\operatorname{sech}(a + bx)}{2b}$$

[Out] (d\*x+c)\*arctan(exp(b\*x+a))/b-1/2\*I\*d\*polylog(2,-I\*exp(b\*x+a))/b^2+1/2\*I\*d\*polylog(2,I\*exp(b\*x+a))/b^2+1/2\*d\*sech(b\*x+a)/b^2+1/2\*(d\*x+c)\*sech(b\*x+a)\*tanh(b\*x+a)/b

**Rubi [A]** time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {4185, 4180, 2279, 2391}

$$-\frac{id\operatorname{PolyLog}(2, -ie^{a+bx})}{2b^2} + \frac{id\operatorname{PolyLog}(2, ie^{a+bx})}{2b^2} + \frac{d\operatorname{sech}(a + bx)}{2b^2} + \frac{(c + dx)\tan^{-1}(e^{a+bx})}{b} + \frac{(c + dx)\tanh(a + bx)\operatorname{sech}(a + bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)\*Sech[a + b\*x]^3, x]

[Out] ((c + d\*x)\*ArcTan[E^(a + b\*x)]/b - ((I/2)\*d\*PolyLog[2, (-I)\*E^(a + b\*x)])/b^2 + ((I/2)\*d\*PolyLog[2, I\*E^(a + b\*x)])/b^2 + (d\*Sech[a + b\*x])/(2\*b^2) + ((c + d\*x)\*Sech[a + b\*x]\*Tanh[a + b\*x])/(2\*b)

#### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]



Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.)), x_Symbol] :=
  -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x
], x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned} \int (c + dx) \operatorname{sech}^3(a + bx) dx &= \frac{d \operatorname{sech}(a + bx)}{2b^2} + \frac{(c + dx) \operatorname{sech}(a + bx) \tanh(a + bx)}{2b} + \frac{1}{2} \int (c + dx) \operatorname{sech}(a + bx) dx \\ &= \frac{(c + dx) \tan^{-1}(e^{a+bx})}{b} + \frac{d \operatorname{sech}(a + bx)}{2b^2} + \frac{(c + dx) \operatorname{sech}(a + bx) \tanh(a + bx)}{2b} - \frac{1}{2} \int (c + dx) \operatorname{sech}(a + bx) dx \\ &= \frac{(c + dx) \tan^{-1}(e^{a+bx})}{b} + \frac{d \operatorname{sech}(a + bx)}{2b^2} + \frac{(c + dx) \operatorname{sech}(a + bx) \tanh(a + bx)}{2b} - \frac{1}{2} \int (c + dx) \operatorname{sech}(a + bx) dx \\ &= \frac{(c + dx) \tan^{-1}(e^{a+bx})}{b} - \frac{id \operatorname{Li}_2(-ie^{a+bx})}{2b^2} + \frac{id \operatorname{Li}_2(ie^{a+bx})}{2b^2} + \frac{d \operatorname{sech}(a + bx)}{2b^2} + \frac{(c + dx) \operatorname{sech}(a + bx) \tanh(a + bx)}{2b} \end{aligned}$$

**Mathematica** [A] time = 3.02, size = 178, normalized size = 1.75

$$bc \tan^{-1}(\sinh(a + bx)) + bc \tanh(a + bx) \operatorname{sech}(a + bx) + \frac{1}{2} d \left( -2i \left( \operatorname{Li}_2(-ie^{a+bx}) - \operatorname{Li}_2(ie^{a+bx}) \right) - ((-2ia - 2ibx + \dots)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)\*Sech[a + b\*x]^3,x]

[Out] (b\*c\*ArcTan[Sinh[a + b\*x]] + (d\*(-(((2\*I)\*a + Pi - (2\*I)\*b\*x)\*(Log[1 - I\*E^(a + b\*x)] - Log[1 + I\*E^(a + b\*x)]))) + ((-2\*I)\*a + Pi)\*Log[Cot[((2\*I)\*a + Pi + (2\*I)\*b\*x)/4]] - (2\*I)\*(PolyLog[2, (-I)\*E^(a + b\*x)] - PolyLog[2, I\*E^(a + b\*x)]))/2 + b\*d\*x\*Sech[a]\*Sech[a + b\*x]^2\*Sinh[b\*x] + d\*Sech[a + b\*x]\*(1 + b\*x\*Tanh[a]) + b\*c\*Sech[a + b\*x]\*Tanh[a + b\*x])/(2\*b^2)

**fricas** [B] time = 1.21, size = 1243, normalized size = 12.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sech(b\*x+a)^3,x, algorithm="fricas")

```
[Out] 1/2*(2*(b*d*x + b*c + d)*cosh(b*x + a)^3 + 6*(b*d*x + b*c + d)*cosh(b*x + a)
)*sinh(b*x + a)^2 + 2*(b*d*x + b*c + d)*sinh(b*x + a)^3 - 2*(b*d*x + b*c -
d)*cosh(b*x + a) + (I*d*cosh(b*x + a)^4 + 4*I*d*cosh(b*x + a)*sinh(b*x + a)
^3 + I*d*sinh(b*x + a)^4 + 2*I*d*cosh(b*x + a)^2 + (6*I*d*cosh(b*x + a)^2 +
2*I*d)*sinh(b*x + a)^2 + (4*I*d*cosh(b*x + a)^3 + 4*I*d*cosh(b*x + a)*sin
h(b*x + a) + I*d)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + (-I*d*cosh(b*x
+ a)^4 - 4*I*d*cosh(b*x + a)*sinh(b*x + a)^3 - I*d*sinh(b*x + a)^4 - 2*I*d
*cosh(b*x + a)^2 + (-6*I*d*cosh(b*x + a)^2 - 2*I*d)*sinh(b*x + a)^2 + (-4*I
*d*cosh(b*x + a)^3 - 4*I*d*cosh(b*x + a))*sinh(b*x + a) - I*d)*dilog(-I*cos
h(b*x + a) - I*sinh(b*x + a)) + ((I*b*c - I*a*d)*cosh(b*x + a)^4 + (4*I*b*c
- 4*I*a*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b*c - I*a*d)*sinh(b*x + a)^4
+ (2*I*b*c - 2*I*a*d)*cosh(b*x + a)^2 + ((6*I*b*c - 6*I*a*d)*cosh(b*x + a)
^2 + 2*I*b*c - 2*I*a*d)*sinh(b*x + a)^2 + I*b*c - I*a*d + ((4*I*b*c - 4*I*a
*d)*cosh(b*x + a)^3 + (4*I*b*c - 4*I*a*d)*cosh(b*x + a))*sinh(b*x + a))*log
(cosh(b*x + a) + sinh(b*x + a) + I) + ((-I*b*c + I*a*d)*cosh(b*x + a)^4 + (
-4*I*b*c + 4*I*a*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b*c + I*a*d)*sinh(b
*x + a)^4 + (-2*I*b*c + 2*I*a*d)*cosh(b*x + a)^2 + ((-6*I*b*c + 6*I*a*d)*co
sh(b*x + a)^2 - 2*I*b*c + 2*I*a*d)*sinh(b*x + a)^2 - I*b*c + I*a*d + ((-4*I
*b*c + 4*I*a*d)*cosh(b*x + a)^3 + (-4*I*b*c + 4*I*a*d)*cosh(b*x + a))*sinh(
b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) + ((-I*b*d*x - I*a*d)*cosh
(b*x + a)^4 + (-4*I*b*d*x - 4*I*a*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b*
d*x - I*a*d)*sinh(b*x + a)^4 - I*b*d*x + (-2*I*b*d*x - 2*I*a*d)*cosh(b*x +
a)^2 + (-2*I*b*d*x + (-6*I*b*d*x - 6*I*a*d)*cosh(b*x + a)^2 - 2*I*a*d)*sinh
(b*x + a)^2 - I*a*d + ((-4*I*b*d*x - 4*I*a*d)*cosh(b*x + a)^3 + (-4*I*b*d*x
- 4*I*a*d)*cosh(b*x + a))*sinh(b*x + a))*log(I*cosh(b*x + a) + I*sinh(b*x
+ a) + 1) + ((I*b*d*x + I*a*d)*cosh(b*x + a)^4 + (4*I*b*d*x + 4*I*a*d)*cosh
(b*x + a)*sinh(b*x + a)^3 + (I*b*d*x + I*a*d)*sinh(b*x + a)^4 + I*b*d*x + (
2*I*b*d*x + 2*I*a*d)*cosh(b*x + a)^2 + (2*I*b*d*x + (6*I*b*d*x + 6*I*a*d)*c
osh(b*x + a)^2 + 2*I*a*d)*sinh(b*x + a)^2 + I*a*d + ((4*I*b*d*x + 4*I*a*d)*
cosh(b*x + a)^3 + (4*I*b*d*x + 4*I*a*d)*cosh(b*x + a))*sinh(b*x + a))*log(-
I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 2*(b*d*x - 3*(b*d*x + b*c + d)*cos
h(b*x + a)^2 + b*c - d)*sinh(b*x + a))/(b^2*cosh(b*x + a)^4 + 4*b^2*cosh(b*
x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 + 2*b^2*cosh(b*x + a)^2 + 2*(3
*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh(b*x + a)^3
+ b^2*cosh(b*x + a))*sinh(b*x + a))
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c) \operatorname{sech}(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sech(b*x + a)^3, x)
```

**maple** [B] time = 0.28, size = 216, normalized size = 2.12

$$\frac{e^{bx+a} (bdx e^{2bx+2a} + bc e^{2bx+2a} - bdx + d e^{2bx+2a} - cb + d)}{b^2 (1 + e^{2bx+2a})^2} + \frac{c \arctan(e^{bx+a})}{b} - \frac{id \ln(1 + i e^{bx+a}) x}{2b} - \frac{id \ln(1 + i e^{bx+a})}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)\*sech(b\*x+a)^3,x)

[Out] exp(b\*x+a)\*(b\*d\*x\*exp(2\*b\*x+2\*a)+b\*c\*exp(2\*b\*x+2\*a)-b\*d\*x+d\*exp(2\*b\*x+2\*a)-c\*b+d)/b^2/(1+exp(2\*b\*x+2\*a))^2+1/b\*c\*arctan(exp(b\*x+a))-1/2\*I/b\*d\*ln(1+I\*exp(b\*x+a))\*x-1/2\*I/b^2\*d\*ln(1+I\*exp(b\*x+a))\*a+1/2\*I/b\*d\*ln(1-I\*exp(b\*x+a))\*x+1/2\*I/b^2\*d\*ln(1-I\*exp(b\*x+a))\*a-1/2\*I/b^2\*d\*dilog(1+I\*exp(b\*x+a))+1/2\*I/b^2\*d\*dilog(1-I\*exp(b\*x+a))-1/b^2\*d\*a\*arctan(exp(b\*x+a))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$d \left( \frac{(bx e^{3a} + e^{3a}) e^{3bx} - (bx e^a - e^a) e^{bx}}{b^2 e^{4bx+4a} + 2 b^2 e^{2bx+2a} + b^2} + 8 \int \frac{x e^{(bx+a)}}{8 (e^{2bx+2a} + 1)} dx \right) - c \left( \frac{\arctan(e^{(-bx-a)})}{b} - \frac{e^{(-bx-a)} - e^{(-3bx-3a)}}{b(2e^{(-2bx-2a)} + e^{(-4bx-4a)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sech(b\*x+a)^3,x, algorithm="maxima")

[Out] d\*(((b\*x\*e^(3\*a) + e^(3\*a))\*e^(3\*b\*x) - (b\*x\*e^a - e^a)\*e^(b\*x))/(b^2\*e^(4\*b\*x + 4\*a) + 2\*b^2\*e^(2\*b\*x + 2\*a) + b^2) + 8\*integrate(1/8\*x\*e^(b\*x + a)/(e^(2\*b\*x + 2\*a) + 1), x)) - c\*(arctan(e^(-b\*x - a))/b - (e^(-b\*x - a) - e^(-3\*b\*x - 3\*a))/(b\*(2\*e^(-2\*b\*x - 2\*a) + e^(-4\*b\*x - 4\*a) + 1)))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c + dx}{\cosh(a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)/cosh(a + b\*x)^3,x)

[Out] int((c + d\*x)/cosh(a + b\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \operatorname{sech}^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*sech(a + b*x)**3, x)
```

$$3.39 \quad \int \frac{\operatorname{sech}^3(a+bx)}{c+dx} dx$$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\operatorname{sech}^3(a+bx)}{c+dx}, x\right)$$

[Out] Unintegrable(sech(b\*x+a)^3/(d\*x+c), x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\operatorname{sech}^3(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[Sech[a + b\*x]^3/(c + d\*x), x]

[Out] Defer[Int][Sech[a + b\*x]^3/(c + d\*x), x]

Rubi steps

$$\int \frac{\operatorname{sech}^3(a+bx)}{c+dx} dx = \int \frac{\operatorname{sech}^3(a+bx)}{c+dx} dx$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Sech[a + b\*x]^3/(c + d\*x), x]

[Out] \$Aborted

fricas [A] time = 0.86, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)^3}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)^3/(d\*x+c), x, algorithm="fricas")

[Out] `integral(sech(b*x + a)^3/(d*x + c), x)`

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx + a)^3}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(b*x+a)^3/(d*x+c),x, algorithm="giac")`

[Out] `integrate(sech(b*x + a)^3/(d*x + c), x)`

**maple** [A] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx + a)^3}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(b*x+a)^3/(d*x+c),x)`

[Out] `int(sech(b*x+a)^3/(d*x+c),x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(bdxe^{(3a)} + (bc - d)e^{(3a)})e^{(3bx)} - (bdxe^a + (bc + d)e^a)e^{(bx)}}{b^2d^2x^2 + 2b^2cdx + b^2c^2 + (b^2d^2x^2e^{(4a)} + 2b^2cdxe^{(4a)} + b^2c^2e^{(4a)})e^{(4bx)} + 2(b^2d^2x^2e^{(2a)} + 2b^2cdxe^{(2a)} + b^2c^2e^{(2a)})e^{(2bx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(b*x+a)^3/(d*x+c),x, algorithm="maxima")`

[Out] `((b*d*x*e^(3*a) + (b*c - d)*e^(3*a))*e^(3*b*x) - (b*d*x*e^a + (b*c + d)*e^a)*e^(b*x))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2*e^(4*a) + 2*b^2*c*d*x*e^(4*a) + b^2*c^2*e^(4*a))*e^(4*b*x) + 2*(b^2*d^2*x^2*e^(2*a) + 2*b^2*c*d*x*e^(2*a) + b^2*c^2*e^(2*a))*e^(2*b*x)) + 8*integrate(1/8*(b^2*d^2*x^2*e^a + 2*b^2*c*d*x*e^a + (b^2*c^2 - 2*d^2)*e^a)*e^(b*x)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3*e^(2*a) + 3*b^2*c*d^2*x^2*e^(2*a) + 3*b^2*c^2*d*x*e^(2*a) + b^2*c^3*e^(2*a))*e^(2*b*x)), x)`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cosh(a + bx)^3 (c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^3*(c + d*x)),x)
```

```
[Out] int(1/(cosh(a + b*x)^3*(c + d*x)), x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sech(a + b*x)**3/(c + d*x), x)
```

$$3.40 \quad \int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2}, x\right)$$

[Out] Unintegrable(sech(b\*x+a)^3/(d\*x+c)^2,x)

**Rubi** [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[Sech[a + b\*x]^3/(c + d\*x)^2,x]

[Out] Defer[Int][Sech[a + b\*x]^3/(c + d\*x)^2, x]

Rubi steps

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2} dx = \int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2} dx$$

**Mathematica** [F] time = 180.02, size = 0, normalized size = 0.00

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Sech[a + b\*x]^3/(c + d\*x)^2,x]

[Out] \$Aborted

**fricas** [A] time = 2.02, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)^3}{d^2x^2+2cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(sech(b\*x+a)^3/(d\*x+c)^2,x, algorithm="fricas")

[Out] integral(sech(b\*x + a)^3/(d^2\*x^2 + 2\*c\*d\*x + c^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)^3/(d\*x+c)^2,x, algorithm="giac")

[Out] integrate(sech(b\*x + a)^3/(d\*x + c)^2, x)

**maple** [A] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(b\*x+a)^3/(d\*x+c)^2,x)

[Out] int(sech(b\*x+a)^3/(d\*x+c)^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(bdxe^{(3a)} + (bc - 2d)e^{(3a)})e^{(3bx)} - (bdxe^a + (bc + 2d)e^a)e^{(4bx)}}{b^2d^3x^3 + 3b^2cd^2x^2 + 3b^2c^2dx + b^2c^3 + (b^2d^3x^3e^{(4a)} + 3b^2cd^2x^2e^{(4a)} + 3b^2c^2dxe^{(4a)} + b^2c^3e^{(4a)})e^{(4bx)} + 2(b^2d^3x^3 + 3b^2cd^2x^2 + 3b^2c^2dx + b^2c^3)e^{(4a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)^3/(d\*x+c)^2,x, algorithm="maxima")

[Out] ((b\*d\*x\*e^(3\*a) + (b\*c - 2\*d)\*e^(3\*a))\*e^(3\*b\*x) - (b\*d\*x\*e^a + (b\*c + 2\*d)\*e^a)\*e^(b\*x))/(b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3 + (b^2\*d^3\*x^3\*e^(4\*a) + 3\*b^2\*c\*d^2\*x^2\*e^(4\*a) + 3\*b^2\*c^2\*d\*x\*e^(4\*a) + b^2\*c^3\*e^(4\*a))\*e^(4\*b\*x) + 2\*(b^2\*d^3\*x^3\*e^(2\*a) + 3\*b^2\*c\*d^2\*x^2\*e^(2\*a) + 3\*b^2\*c^2\*d\*x\*e^(2\*a) + b^2\*c^3\*e^(2\*a))\*e^(2\*b\*x)) + 8\*integrate(1/8\*(b^2\*d^2\*x^2\*e^a + 2\*b^2\*c\*d\*x\*e^a + (b^2\*c^2 - 6\*d^2)\*e^a)\*e^(b\*x)/(b^2\*d^4\*x^4 + 4\*b^2\*c\*d^3\*x^3 + 6\*b^2\*c^2\*d^2\*x^2 + 4\*b^2\*c^3\*d\*x + b^2\*c^4 + (b^2\*d^4\*x^4\*e^(2\*a) + 4\*b^2\*c\*d^3\*x^3\*e^(2\*a) + 6\*b^2\*c^2\*d^2\*x^2\*e^(2\*a) + 4\*b^2\*c^3\*d\*x\*e^(2\*a) + b^2\*c^4\*e^(2\*a))\*e^(2\*b\*x)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cosh(a+bx)^3 (c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cosh(a + b*x)^3*(c + d*x)^2), x)`

[Out] `int(1/(cosh(a + b*x)^3*(c + d*x)^2), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(b*x+a)**3/(d*x+c)**2, x)`

[Out] `Integral(sech(a + b*x)**3/(c + d*x)**2, x)`

### 3.41 $\int (c + dx)^{5/2} \cosh(a + bx) dx$

**Optimal.** Leaf size=171

$$\frac{15\sqrt{\pi} d^{5/2} e^{\frac{bc}{d}-a} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} - \frac{15\sqrt{\pi} d^{5/2} e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} + \frac{15d^2 \sqrt{c+dx} \sinh(a+bx)}{4b^3} - \frac{5d(c+dx)^{3/2} \cosh(a+bx)}{2b^2}$$

[Out]  $-5/2*d*(d*x+c)^{(3/2)}*\cosh(b*x+a)/b^2+(d*x+c)^{(5/2)}*\sinh(b*x+a)/b+15/16*d^{(5/2)}*\exp(-a+b*c/d)*\operatorname{erf}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*\Pi^{(1/2)}/b^{(7/2)}-15/16*d^{(5/2)}*\exp(a-b*c/d)*\operatorname{erfi}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*\Pi^{(1/2)}/b^{(7/2)}+15/4*d^2*\sinh(b*x+a)*(d*x+c)^{(1/2)}/b^3$

**Rubi [A]** time = 0.33, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3296, 3308, 2180, 2204, 2205}

$$\frac{15\sqrt{\pi} d^{5/2} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} - \frac{15\sqrt{\pi} d^{5/2} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} + \frac{15d^2 \sqrt{c+dx} \sinh(a+bx)}{4b^3} - \frac{5d(c+dx)^{3/2} \cosh(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^{(5/2)}*\operatorname{Cosh}[a + b*x], x]$

[Out]  $(-5*d*(c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x])/(2*b^2) + (15*d^{(5/2)}*E^{(-a + (b*c)/d)}*\operatorname{Sqrt}[\Pi]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(16*b^{(7/2)}) - (15*d^{(5/2)}*E^{(a - (b*c)/d)}*\operatorname{Sqrt}[\Pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(16*b^{(7/2)}) + (15*d^2*\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[a + b*x])/(4*b^3) + ((c + d*x)^{(5/2)}*\operatorname{Sinh}[a + b*x])/b$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_) * ((e_) + (f_) * (x_)))} / \operatorname{Sqrt}[(c_) + (d_) * (x_)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \&\& !\$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_) + (b_) * ((c_) + (d_) * (x_)) ^ 2)}, x\_Symbol] :> \operatorname{Simp}[(F^a * \operatorname{Sqrt}[\Pi] * \operatorname{Erfi}[(c + d*x) * \operatorname{Rt}[b * \operatorname{Log}[F], 2]]) / (2 * d * \operatorname{Rt}[b * \operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \&\& \operatorname{PosQ}[b]$

#### Rule 2205

$\operatorname{Int}[(F_)^{((a_) + (b_) * ((c_) + (d_) * (x_)) ^ 2)}, x\_Symbol] :> \operatorname{Simp}[(F^a * \operatorname{Sqrt}[\Pi] * \operatorname{Erf}[(c + d*x) * \operatorname{Rt}[-(b * \operatorname{Log}[F]), 2]]) / (2 * d * \operatorname{Rt}[-(b * \operatorname{Log}[F]), 2]), x] /;$   $\operatorname{Fr}$

eeQ[{F, a, b, c, d}, x] && NegQ[b]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[  
((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[  
e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3308

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I  
/2, Int[(c + d\*x)^m/E^(I\*(e + f\*x)), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(  
I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

### Rubi steps

$$\begin{aligned}
 \int (c + dx)^{5/2} \cosh(a + bx) dx &= \frac{(c + dx)^{5/2} \sinh(a + bx)}{b} - \frac{(5d) \int (c + dx)^{3/2} \sinh(a + bx) dx}{2b} \\
 &= -\frac{5d(c + dx)^{3/2} \cosh(a + bx)}{2b^2} + \frac{(c + dx)^{5/2} \sinh(a + bx)}{b} + \frac{(15d^2) \int \sqrt{c + dx} \cosh(a + bx) dx}{4b^2} \\
 &= -\frac{5d(c + dx)^{3/2} \cosh(a + bx)}{2b^2} + \frac{15d^2 \sqrt{c + dx} \sinh(a + bx)}{4b^3} + \frac{(c + dx)^{5/2} \sinh(a + bx)}{b} \\
 &= -\frac{5d(c + dx)^{3/2} \cosh(a + bx)}{2b^2} + \frac{15d^2 \sqrt{c + dx} \sinh(a + bx)}{4b^3} + \frac{(c + dx)^{5/2} \sinh(a + bx)}{b} \\
 &= -\frac{5d(c + dx)^{3/2} \cosh(a + bx)}{2b^2} + \frac{15d^2 \sqrt{c + dx} \sinh(a + bx)}{4b^3} + \frac{(c + dx)^{5/2} \sinh(a + bx)}{b} \\
 &= -\frac{5d(c + dx)^{3/2} \cosh(a + bx)}{2b^2} + \frac{15d^{5/2} e^{-a + \frac{bc}{d}} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c + dx}}{\sqrt{d}}\right)}{16b^{7/2}} - \frac{15d^{5/2} e^{a - \frac{bc}{d}} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c + dx}}{\sqrt{d}}\right)}{16b^{7/2}}
 \end{aligned}$$

**Mathematica** [A] time = 0.05, size = 107, normalized size = 0.63

$$\frac{d^3 e^{-a - \frac{bc}{d}} \left( e^{2a} \sqrt{-\frac{b(c+dx)}{d}} \Gamma\left(\frac{7}{2}, -\frac{b(c+dx)}{d}\right) + e^{\frac{2bc}{d}} \sqrt{\frac{b(c+dx)}{d}} \Gamma\left(\frac{7}{2}, \frac{b(c+dx)}{d}\right) \right)}{2b^4 \sqrt{c + dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^(5/2)\*Cosh[a + b\*x], x]

[Out]  $-1/2*(d^3*E^{-(a - (b*c)/d)}*(E^{(2*a)}*Sqrt[-((b*(c + d*x))/d)]*Gamma[7/2, -((b*(c + d*x))/d)] + E^{((2*b*c)/d)}*Sqrt[(b*(c + d*x))/d]*Gamma[7/2, (b*(c + d*x))/d]))/(b^4*Sqrt[c + d*x])$

**fricas** [B] time = 0.91, size = 523, normalized size = 3.06

$$15 \sqrt{\pi} \left( d^3 \cosh(bx + a) \cosh\left(-\frac{bc-ad}{d}\right) - d^3 \cosh(bx + a) \sinh\left(-\frac{bc-ad}{d}\right) + \left( d^3 \cosh\left(-\frac{bc-ad}{d}\right) - d^3 \sinh\left(-\frac{bc-ad}{d}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(5/2)\*cosh(b\*x+a), x, algorithm="fricas")

[Out]  $1/16*(15*\sqrt{\pi}*(d^3*\cosh(b*x + a)*\cosh(-(b*c - a*d)/d) - d^3*\cosh(b*x + a)*\sinh(-(b*c - a*d)/d) + (d^3*\cosh(-(b*c - a*d)/d) - d^3*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{b/d}*\operatorname{erf}(\sqrt{d*x + c}*\sqrt{b/d}) + 15*\sqrt{\pi}*(d^3*\cosh(b*x + a)*\cosh(-(b*c - a*d)/d) + d^3*\cosh(b*x + a)*\sinh(-(b*c - a*d)/d) + (d^3*\cosh(-(b*c - a*d)/d) + d^3*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{-b/d}*\operatorname{erf}(\sqrt{d*x + c}*\sqrt{-b/d}) - 2*(4*b^3*d^2*x^2 + 4*b^3*c^2 + 10*b^2*c*d + 15*b*d^2 - (4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x))*\cosh(b*x + a)^2 - 2*(4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x))*\cosh(b*x + a)*\sinh(b*x + a) - (4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x))*\sinh(b*x + a)^2 + 2*(4*b^3*c*d + 5*b^2*d^2)*x*\sqrt{d*x + c}))/b^4*\cosh(b*x + a) + b^4*\sinh(b*x + a))$

**giac** [A] time = 0.22, size = 232, normalized size = 1.36

$$\frac{15 \sqrt{\pi} d^4 \operatorname{erf}\left(-\frac{\sqrt{bd} \sqrt{dx+c}}{d}\right) e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd} b^3} - \frac{15 \sqrt{\pi} d^4 \operatorname{erf}\left(-\frac{\sqrt{-bd} \sqrt{dx+c}}{d}\right) e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd} b^3} - \frac{2 \left(4(dx+c)^2 b^2 d - 10(dx+c)^2 b d^2 + 15 \sqrt{dx+c} d^3\right) e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b^3}$$

$16d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(5/2)\*cosh(b\*x+a), x, algorithm="giac")

[Out]  $-1/16*(15*\sqrt{\pi}*d^4*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c}/d)*e^{((b*c - a*d)/d)}/(\sqrt{b*d}*b^3) - 15*\sqrt{\pi}*d^4*\operatorname{erf}(-\sqrt{-b*d}*\sqrt{d*x + c}/d)*e^{-(b*c - a*d)/d}/(\sqrt{-b*d}*b^3) - 2*(4*(d*x + c)^{(5/2)}*b^2*d - 10*(d*x + c)^{(3/2)}*b*d^2 + 15*\sqrt{d*x + c}*d^3))*e^{((d*x + c)*b - b*c + a*d)/d}/b^3 + 2*(4*(d*x + c)^{(5/2)}*b^2*d + 10*(d*x + c)^{(3/2)}*b*d^2 + 15*\sqrt{d*x + c}*d^3))*e^{-(d*x + c)*b - b*c + a*d)/d}/b^3)/d$

**maple** [F] time = 0.15, size = 0, normalized size = 0.00

$$\int (dx + c)^{\frac{5}{2}} \cosh(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^(5/2)\*cosh(b\*x+a), x)

[Out] int((d\*x+c)^(5/2)\*cosh(b\*x+a), x)

**maxima** [B] time = 0.45, size = 308, normalized size = 1.80

$$32(dx + c)^{\frac{7}{2}} \cosh(bx + a) - \frac{\left( \frac{105 \sqrt{\pi} d^4 \operatorname{erf}\left(\sqrt{dx+c} \sqrt{\frac{b}{d}}\right) e^{\left(a-\frac{bc}{d}\right)}}{b^4 \sqrt{-\frac{b}{d}}} - \frac{105 \sqrt{\pi} d^4 \operatorname{erf}\left(\sqrt{dx+c} \sqrt{\frac{b}{d}}\right) e^{\left(-a+\frac{bc}{d}\right)}}{b^4 \sqrt{\frac{b}{d}}} + 2 \left( 8(dx+c)^{\frac{7}{2}} b^3 d e^{\left(\frac{bc}{d}\right)} + 28(dx+c)^{\frac{5}{2}} b^2 d^2 e^{\left(\frac{bc}{d}\right)} + 70(dx+c)^{\frac{3}{2}} b d^3 e^{\left(\frac{bc}{d}\right)} + 105 \sqrt{dx+c} d^4 e^{\left(\frac{bc}{d}\right)} \right) e^{\left(-a-\frac{bc}{d}\right)} \right)}{b^4 d}$$

112 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(5/2)\*cosh(b\*x+a), x, algorithm="maxima")

[Out] 1/112\*(32\*(d\*x + c)^(7/2)\*cosh(b\*x + a) - (105\*sqrt(pi)\*d^4\*erf(sqrt(d\*x + c)\*sqrt(-b/d))\*e^(a - b\*c/d)/(b^4\*sqrt(-b/d)) - 105\*sqrt(pi)\*d^4\*erf(sqrt(d\*x + c)\*sqrt(b/d))\*e^(-a + b\*c/d)/(b^4\*sqrt(b/d)) + 2\*(8\*(d\*x + c)^(7/2)\*b^3\*d\*e^(b\*c/d) + 28\*(d\*x + c)^(5/2)\*b^2\*d^2\*e^(b\*c/d) + 70\*(d\*x + c)^(3/2)\*b\*d^3\*e^(b\*c/d) + 105\*sqrt(d\*x + c)\*d^4\*e^(b\*c/d))\*e^(-a - (d\*x + c)\*b/d)/b^4 + 2\*(8\*(d\*x + c)^(7/2)\*b^3\*d\*e^a - 28\*(d\*x + c)^(5/2)\*b^2\*d^2\*e^a + 70\*(d\*x + c)^(3/2)\*b\*d^3\*e^a - 105\*sqrt(d\*x + c)\*d^4\*e^a)\*e^((d\*x + c)\*b/d - b\*c/d)/b^4)\*b/d/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a + bx) (c + dx)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)\*(c + d\*x)^(5/2), x)

[Out] int(cosh(a + b\*x)\*(c + d\*x)^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^{\frac{5}{2}} \cosh(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cosh(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(5/2)*cosh(a + b*x), x)
```

### 3.42 $\int (c + dx)^{3/2} \cosh(a + bx) dx$

**Optimal.** Leaf size=146

$$\frac{3\sqrt{\pi} d^{3/2} e^{\frac{bc}{d}-a} \operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} + \frac{3\sqrt{\pi} d^{3/2} e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} - \frac{3d\sqrt{c+dx} \cosh(a+bx)}{2b^2} + \frac{(c+dx)^{3/2} \sinh(a+bx)}{b}$$

[Out]  $(d*x+c)^{(3/2)}*\sinh(b*x+a)/b+3/8*d^{(3/2)}*\exp(-a+b*c/d)*\operatorname{erf}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*\Pi^{(1/2)}/b^{(5/2)}+3/8*d^{(3/2)}*\exp(a-b*c/d)*\operatorname{erfi}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*\Pi^{(1/2)}/b^{(5/2)}-3/2*d*\cosh(b*x+a)*(d*x+c)^{(1/2)}/b^2$

**Rubi [A]** time = 0.24, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3296, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi} d^{3/2} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} + \frac{3\sqrt{\pi} d^{3/2} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} - \frac{3d\sqrt{c+dx} \cosh(a+bx)}{2b^2} + \frac{(c+dx)^{3/2} \sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x], x]$

[Out]  $(-3*d*\operatorname{Sqrt}[c + d*x]*\operatorname{Cosh}[a + b*x])/(2*b^2) + (3*d^{(3/2)}*E^{(-a + (b*c)/d)}*\operatorname{Sqrt}[\Pi]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/\operatorname{Sqrt}[d]])/(8*b^{(5/2)}) + (3*d^{(3/2)}*E^{(a - (b*c)/d)}*\operatorname{Sqrt}[\Pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/\operatorname{Sqrt}[d]])/(8*b^{(5/2)}) + ((c + d*x)^{(3/2)}*\operatorname{Sinh}[a + b*x])/b$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*(e_.) + (f_.)*(x_))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g, x\} \ \&\amp; \ !\$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\Pi]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\amp; \ \operatorname{PosQ}[b]$

#### Rule 2205

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\Pi]*\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]])/(2*d*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\amp; \ \operatorname{NegQ}[b]$



Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> -Simp[  
 ((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[  
 e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3307

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol  
 ] :> Dist[I/2, Int[(c + d\*x)^m/(E^(I\*k\*Pi)\*E^(I\*(e + f\*x))), x], x] - Dist[  
 I/2, Int[(c + d\*x)^m\*E^(I\*k\*Pi)\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e,  
 f, m}, x] && IntegerQ[2\*k]

Rubi steps

$$\begin{aligned}
 \int (c + dx)^{3/2} \cosh(a + bx) dx &= \frac{(c + dx)^{3/2} \sinh(a + bx)}{b} - \frac{(3d) \int \sqrt{c + dx} \sinh(a + bx) dx}{2b} \\
 &= -\frac{3d\sqrt{c + dx} \cosh(a + bx)}{2b^2} + \frac{(c + dx)^{3/2} \sinh(a + bx)}{b} + \frac{(3d^2) \int \frac{\cosh(a+bx)}{\sqrt{c+dx}} dx}{4b^2} \\
 &= -\frac{3d\sqrt{c + dx} \cosh(a + bx)}{2b^2} + \frac{(c + dx)^{3/2} \sinh(a + bx)}{b} + \frac{(3d^2) \int \frac{e^{-i(a+ibx)}}{\sqrt{c+dx}} dx}{8b^2} + \dots \\
 &= -\frac{3d\sqrt{c + dx} \cosh(a + bx)}{2b^2} + \frac{(c + dx)^{3/2} \sinh(a + bx)}{b} + \frac{(3d) \text{Subst} \left( \int e^{i\left(ia - \frac{ibc}{d}\right)} dx \right)}{4b^2} \\
 &= -\frac{3d\sqrt{c + dx} \cosh(a + bx)}{2b^2} + \frac{3d^{3/2} e^{-a + \frac{bc}{d}} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{8b^{5/2}} + \frac{3d^{3/2} e^{a - \frac{bc}{d}} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{8b^{5/2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 107, normalized size = 0.73

$$\frac{d\sqrt{c + dx} e^{-a - \frac{bc}{d}} \left( -\frac{e^{2a} \Gamma\left(\frac{5}{2}, -\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}} - \frac{e^{\frac{2bc}{d}} \Gamma\left(\frac{5}{2}, \frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}} \right)}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^(3/2)\*Cosh[a + b\*x], x]

[Out]  $(d \cdot E^{-a - (b \cdot c)/d} \cdot \text{Sqrt}[c + d \cdot x] \cdot (-((E^{(2 \cdot a)} \cdot \text{Gamma}[5/2, -((b \cdot (c + d \cdot x))/d)])/\text{Sqrt}[-((b \cdot (c + d \cdot x))/d)]) - (E^{((2 \cdot b \cdot c)/d)} \cdot \text{Gamma}[5/2, (b \cdot (c + d \cdot x))/d])/\text{Sqrt}[(b \cdot (c + d \cdot x))/d]))/(2 \cdot b^2)$

**fricas** [B] time = 2.98, size = 387, normalized size = 2.65

$$\frac{3 \sqrt{\pi} \left( d^2 \cosh(bx + a) \cosh\left(-\frac{bc-ad}{d}\right) - d^2 \cosh(bx + a) \sinh\left(-\frac{bc-ad}{d}\right) + \left( d^2 \cosh\left(-\frac{bc-ad}{d}\right) - d^2 \sinh\left(-\frac{bc-ad}{d}\right) \right) \sinh(bx + a) \right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(3/2)\*cosh(b\*x+a),x, algorithm="fricas")

[Out]  $\frac{1}{8} \cdot (3 \cdot \text{sqrt}(\pi) \cdot (d^2 \cdot \cosh(b \cdot x + a) \cdot \cosh(-\frac{b \cdot c - a \cdot d}{d}) - d^2 \cdot \cosh(b \cdot x + a) \cdot \sinh(-\frac{b \cdot c - a \cdot d}{d}) + (d^2 \cdot \cosh(-\frac{b \cdot c - a \cdot d}{d}) - d^2 \cdot \sinh(-\frac{b \cdot c - a \cdot d}{d})) \cdot \sinh(b \cdot x + a)) \cdot \text{sqrt}(b/d) \cdot \text{erf}(\text{sqrt}(d \cdot x + c) \cdot \text{sqrt}(b/d)) - 3 \cdot \text{sqrt}(\pi) \cdot (d^2 \cdot \cosh(b \cdot x + a) \cdot \cosh(-\frac{b \cdot c - a \cdot d}{d}) + d^2 \cdot \cosh(b \cdot x + a) \cdot \sinh(-\frac{b \cdot c - a \cdot d}{d}) + (d^2 \cdot \cosh(-\frac{b \cdot c - a \cdot d}{d}) + d^2 \cdot \sinh(-\frac{b \cdot c - a \cdot d}{d})) \cdot \sinh(b \cdot x + a)) \cdot \text{sqrt}(-b/d) \cdot \text{erf}(\text{sqrt}(d \cdot x + c) \cdot \text{sqrt}(-b/d)) - 2 \cdot (2 \cdot b^2 \cdot d \cdot x + 2 \cdot b^2 \cdot c - (2 \cdot b^2 \cdot d \cdot x + 2 \cdot b^2 \cdot c - 3 \cdot b \cdot d) \cdot \cosh(b \cdot x + a)^2 - 2 \cdot (2 \cdot b^2 \cdot d \cdot x + 2 \cdot b^2 \cdot c - 3 \cdot b \cdot d) \cdot \cosh(b \cdot x + a) \cdot \sinh(b \cdot x + a) - (2 \cdot b^2 \cdot d \cdot x + 2 \cdot b^2 \cdot c - 3 \cdot b \cdot d) \cdot \sinh(b \cdot x + a)^2 + 3 \cdot b \cdot d) \cdot \text{sqrt}(d \cdot x + c)) / (b^3 \cdot \cosh(b \cdot x + a) + b^3 \cdot \sinh(b \cdot x + a)))$

**giac** [A] time = 0.18, size = 202, normalized size = 1.38

$$\frac{3 \sqrt{\pi} d^3 \text{erf}\left(-\frac{\sqrt{bd} \sqrt{dx+c}}{d}\right) e^{\left(\frac{bc-ad}{d}\right)} + 3 \sqrt{\pi} d^3 \text{erf}\left(-\frac{\sqrt{-bd} \sqrt{dx+c}}{d}\right) e^{\left(-\frac{bc-ad}{d}\right)} - 2 \left(2(dx+c)^2 bd - 3 \sqrt{dx+c} d^2\right) e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)} + 2 \left(2(dx+c)^2 bd + 3 \sqrt{dx+c} d^2\right) e^{\left(-\frac{(dx+c)b-bc+ad}{d}\right)}}{\sqrt{bd} b^2} + \frac{3 \sqrt{\pi} d^3 \text{erf}\left(-\frac{\sqrt{-bd} \sqrt{dx+c}}{d}\right) e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd} b^2} - \frac{2 \left(2(dx+c)^2 bd - 3 \sqrt{dx+c} d^2\right) e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b^2} + \frac{2 \left(2(dx+c)^2 bd + 3 \sqrt{dx+c} d^2\right) e^{\left(-\frac{(dx+c)b-bc+ad}{d}\right)}}{b^2}$$

8d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(3/2)\*cosh(b\*x+a),x, algorithm="giac")

[Out]  $-1/8 \cdot (3 \cdot \text{sqrt}(\pi) \cdot d^3 \cdot \text{erf}(-\text{sqrt}(b \cdot d) \cdot \text{sqrt}(d \cdot x + c)/d) \cdot e^{((b \cdot c - a \cdot d)/d)} / (\text{sqrt}(b \cdot d) \cdot b^2) + 3 \cdot \text{sqrt}(\pi) \cdot d^3 \cdot \text{erf}(-\text{sqrt}(-b \cdot d) \cdot \text{sqrt}(d \cdot x + c)/d) \cdot e^{(-(b \cdot c - a \cdot d)/d)} / (\text{sqrt}(-b \cdot d) \cdot b^2) - 2 \cdot (2 \cdot (d \cdot x + c)^{(3/2)} \cdot b \cdot d - 3 \cdot \text{sqrt}(d \cdot x + c) \cdot d^2) \cdot e^{((d \cdot x + c) \cdot b - b \cdot c + a \cdot d)/d} / b^2 + 2 \cdot (2 \cdot (d \cdot x + c)^{(3/2)} \cdot b \cdot d + 3 \cdot \text{sqrt}(d \cdot x + c) \cdot d^2) \cdot e^{(-(d \cdot x + c) \cdot b - b \cdot c + a \cdot d)/d} / b^2) / d$

**maple** [F] time = 0.16, size = 0, normalized size = 0.00

$$\int (dx + c)^{\frac{3}{2}} \cosh(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^(3/2)*cosh(b*x+a),x)`

[Out] `int((d*x+c)^(3/2)*cosh(b*x+a),x)`

**maxima** [B] time = 0.36, size = 268, normalized size = 1.84

$$16(dx+c)^{\frac{5}{2}} \cosh(bx+a) + \frac{\left( \frac{15\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} + \frac{15\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} - 2\left(4(dx+c)^{\frac{5}{2}}b^2de^{\left(\frac{bc}{d}\right)} + 10(dx+c)^{\frac{3}{2}}bd^2e^{\left(\frac{bc}{d}\right)} + 15\right)}{d} \right)}{40d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(3/2)*cosh(b*x+a),x, algorithm="maxima")`

[Out] `1/40*(16*(d*x + c)^(5/2)*cosh(b*x + a) + (15*sqrt(pi)*d^3*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^3*sqrt(-b/d)) + 15*sqrt(pi)*d^3*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^3*sqrt(b/d)) - 2*(4*(d*x + c)^(5/2)*b^2*d*e^(b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(b*c/d) + 15*sqrt(d*x + c)*d^3*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^3 - 2*(4*(d*x + c)^(5/2)*b^2*d*e^a - 10*(d*x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x + c)*b/d - b*c/d)/b^3)*b/d)/d`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a + bx) (c + dx)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(a + b*x)*(c + d*x)^(3/2),x)`

[Out] `int(cosh(a + b*x)*(c + d*x)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^{\frac{3}{2}} \cosh(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)**(3/2)*cosh(b*x+a),x)`

[Out] `Integral((c + d*x)**(3/2)*cosh(a + b*x), x)`

### 3.43 $\int \sqrt{c + dx} \cosh(a + bx) dx$

**Optimal.** Leaf size=123

$$\frac{\sqrt{\pi} \sqrt{d} e^{\frac{bc}{d}-a} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{\pi} \sqrt{d} e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{c+dx} \sinh(a+bx)}{b}$$

[Out]  $1/4*\exp(-a+b*c/d)*\operatorname{erf}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*d^{(1/2)}*\pi^{(1/2)}/b^{(3/2)}-1/4*\exp(a-b*c/d)*\operatorname{erfi}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*d^{(1/2)}*\pi^{(1/2)}/b^{(3/2)}+\sinh(b*x+a)*(d*x+c)^{(1/2)}/b$

**Rubi [A]** time = 0.18, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3296, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi} \sqrt{d} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{\pi} \sqrt{d} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{c+dx} \sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[c + d*x]*Cosh[a + b*x], x]`

[Out]  $(\operatorname{Sqrt}[d]*E^{(-a + (b*c)/d)}*\operatorname{Sqrt}[\pi]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(4*b^{(3/2)}) - (\operatorname{Sqrt}[d]*E^{(a - (b*c)/d)}*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(4*b^{(3/2)}) + (\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[a + b*x])/b$

#### Rule 2180

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True`

#### Rule 2204

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :> Simp[(F^a*Sqrt[\pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

#### Rule 2205

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :> Simp[(F^a*Sqrt[\pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]`

Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[  
 ((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[  
 e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3308

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I  
 /2, Int[(c + d\*x)^m/E^(I\*(e + f\*x)), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(  
 I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \int \sqrt{c+dx} \cosh(ax+bx) dx &= \frac{\sqrt{c+dx} \sinh(ax+bx)}{b} - \frac{d \int \frac{\sinh(ax+bx)}{\sqrt{c+dx}} dx}{2b} \\
 &= \frac{\sqrt{c+dx} \sinh(ax+bx)}{b} - \frac{d \int \frac{e^{-i(ax+bx)}}{\sqrt{c+dx}} dx}{4b} + \frac{d \int \frac{e^{i(ax+bx)}}{\sqrt{c+dx}} dx}{4b} \\
 &= \frac{\sqrt{c+dx} \sinh(ax+bx)}{b} + \frac{\text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}} dx, x, \sqrt{c+dx}\right)}{2b} - \frac{\text{Subst}\left(\int e^{-i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}} dx, x, \sqrt{c+dx}\right)}{2b} \\
 &= \frac{\sqrt{d} e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{d} e^{a-\frac{bc}{d}} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{c+dx} \sinh(ax+bx)}{b}
 \end{aligned}$$

Mathematica [A] time = 0.09, size = 105, normalized size = 0.85

$$\frac{\sqrt{c+dx} e^{-a-\frac{bc}{d}} \left( \frac{e^{2a} \Gamma\left(\frac{3}{2}, -\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}} - \frac{e^{\frac{2bc}{d}} \Gamma\left(\frac{3}{2}, \frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}} \right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d\*x]\*Cosh[a + b\*x], x]

[Out] (E^(-a - (b\*c)/d)\*Sqrt[c + d\*x]\*((E^(2\*a)\*Gamma[3/2, -((b\*(c + d\*x))/d)])/S  
 qrt[-((b\*(c + d\*x))/d)] - (E^((2\*b\*c)/d)\*Gamma[3/2, (b\*(c + d\*x))/d])/Sqrt[  
 (b\*(c + d\*x))/d]))/(2\*b)

**fricas** [B] time = 1.02, size = 302, normalized size = 2.46

$$\sqrt{\pi} \left( d \cosh(bx + a) \cosh\left(-\frac{bc-ad}{d}\right) - d \cosh(bx + a) \sinh\left(-\frac{bc-ad}{d}\right) + \left( d \cosh\left(-\frac{bc-ad}{d}\right) - d \sinh\left(-\frac{bc-ad}{d}\right) \right) \sinh(bx + a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/4\*(sqrt(pi)\*(d\*cosh(b\*x + a)\*cosh(-(b\*c - a\*d)/d) - d\*cosh(b\*x + a)\*sinh(-(b\*c - a\*d)/d) + (d\*cosh(-(b\*c - a\*d)/d) - d\*sinh(-(b\*c - a\*d)/d))\*sinh(b\*x + a))\*sqrt(b/d)\*erf(sqrt(d\*x + c)\*sqrt(b/d)) + sqrt(pi)\*(d\*cosh(b\*x + a)\*cosh(-(b\*c - a\*d)/d) + d\*cosh(b\*x + a)\*sinh(-(b\*c - a\*d)/d) + (d\*cosh(-(b\*c - a\*d)/d) + d\*sinh(-(b\*c - a\*d)/d))\*sinh(b\*x + a))\*sqrt(-b/d)\*erf(sqrt(d\*x + c)\*sqrt(-b/d)) + 2\*(b\*cosh(b\*x + a)^2 + 2\*b\*cosh(b\*x + a)\*sinh(b\*x + a) + b\*sinh(b\*x + a)^2 - b)\*sqrt(d\*x + c))/(b^2\*cosh(b\*x + a) + b^2\*sinh(b\*x + a))

**giac** [A] time = 0.16, size = 169, normalized size = 1.37

$$\frac{\sqrt{\pi} d^2 \operatorname{erf}\left(-\frac{\sqrt{bd} \sqrt{dx+c}}{d}\right) e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd} b} - \frac{\sqrt{\pi} d^2 \operatorname{erf}\left(-\frac{\sqrt{-bd} \sqrt{dx+c}}{d}\right) e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd} b} - \frac{2 \sqrt{dx+c} d e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b} + \frac{2 \sqrt{dx+c} d e^{\left(-\frac{(dx+c)b-bc+ad}{d}\right)}}{b}$$

$4d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*(d\*x+c)^(1/2),x, algorithm="giac")

[Out] -1/4\*(sqrt(pi)\*d^2\*erf(-sqrt(b\*d)\*sqrt(d\*x + c)/d)\*e^((b\*c - a\*d)/d)/(sqrt(b\*d)\*b) - sqrt(pi)\*d^2\*erf(-sqrt(-b\*d)\*sqrt(d\*x + c)/d)\*e^(-(b\*c - a\*d)/d)/(sqrt(-b\*d)\*b) - 2\*sqrt(d\*x + c)\*d\*e^(((d\*x + c)\*b - b\*c + a\*d)/d)/b + 2\*sqrt(d\*x + c)\*d\*e^(-((d\*x + c)\*b - b\*c + a\*d)/d)/b)/d

**maple** [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \cosh(bx + a) \sqrt{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)\*(d\*x+c)^(1/2),x)

[Out] int(cosh(b\*x+a)\*(d\*x+c)^(1/2),x)

**maxima** [B] time = 0.34, size = 230, normalized size = 1.87

$$8(dx+c)^{\frac{3}{2}} \cosh(bx+a) - \frac{\left( \frac{3\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} - \frac{3\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} + \frac{2\left(2(dx+c)^{\frac{3}{2}}bde^{\left(\frac{bc}{d}\right)} + 3\sqrt{dx+c}d^2e^{\left(\frac{bc}{d}\right)}\right)e^{\left(-a-\frac{(dx+c)}{d}\right)}}{b^2} \right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*(d\*x+c)^(1/2), x, algorithm="maxima")

[Out] 1/12\*(8\*(d\*x + c)^(3/2)\*cosh(b\*x + a) - (3\*sqrt(pi)\*d^2\*erf(sqrt(d\*x + c)\*sqrt(-b/d))\*e^(a - b\*c/d)/(b^2\*sqrt(-b/d)) - 3\*sqrt(pi)\*d^2\*erf(sqrt(d\*x + c)\*sqrt(b/d))\*e^(-a + b\*c/d)/(b^2\*sqrt(b/d)) + 2\*(2\*(d\*x + c)^(3/2)\*b\*d\*e^(b\*c/d) + 3\*sqrt(d\*x + c)\*d^2\*e^(b\*c/d))\*e^(-a - (d\*x + c)\*b/d)/b^2 + 2\*(2\*(d\*x + c)^(3/2)\*b\*d\*e^a - 3\*sqrt(d\*x + c)\*d^2\*e^a)\*e^((d\*x + c)\*b/d - b\*c/d)/b^2)\*b/d)/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a + bx) \sqrt{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)\*(c + d\*x)^(1/2), x)

[Out] int(cosh(a + b\*x)\*(c + d\*x)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx} \cosh(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*(d\*x+c)\*\*(1/2), x)

[Out] Integral(sqrt(c + d\*x)\*cosh(a + b\*x), x)

$$3.44 \quad \int \frac{\cosh(a+bx)}{\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=104

$$\frac{\sqrt{\pi} e^{\frac{bc}{d}-a} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b} \sqrt{d}} + \frac{\sqrt{\pi} e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b} \sqrt{d}}$$

[Out]  $1/2*\exp(-a+b*c/d)*\operatorname{erf}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*\operatorname{Pi}^{(1/2)}/b^{(1/2)}/d^{(1/2)}+1/2*\exp(a-b*c/d)*\operatorname{erfi}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*\operatorname{Pi}^{(1/2)}/b^{(1/2)}/d^{(1/2)}$

**Rubi [A]** time = 0.13, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b} \sqrt{d}} + \frac{\sqrt{\pi} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b} \sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b\*x]/Sqrt[c + d\*x], x]

[Out]  $(E^{(-a + (b*c)/d)*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]]})/(2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d]) + (E^{(a - (b*c)/d)*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]]})/(2*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d])$

**Rule 2180**

Int[(F\_)^((g\_.)\*(e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

**Rule 2204**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

**Rule 2205**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]



Rule 3307

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_.)], x\_Symbol] :> Dist[I/2, Int[(c + d\*x)^m/(E^(I\*k\*Pi)\*E^(I\*(e + f\*x))), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*k\*Pi)\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

Rubi steps

$$\begin{aligned} \int \frac{\cosh(a + bx)}{\sqrt{c + dx}} dx &= \frac{1}{2} \int \frac{e^{-i(ia+ibx)}}{\sqrt{c + dx}} dx + \frac{1}{2} \int \frac{e^{i(ia+ibx)}}{\sqrt{c + dx}} dx \\ &= \frac{\text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}} dx, x, \sqrt{c + dx}\right)}{d} + \frac{\text{Subst}\left(\int e^{-i\left(ia-\frac{ibc}{d}\right)+\frac{bx^2}{d}} dx, x, \sqrt{c + dx}\right)}{d} \\ &= \frac{e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b} \sqrt{d}} + \frac{e^{a-\frac{bc}{d}} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b} \sqrt{d}} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 105, normalized size = 1.01

$$\frac{e^{-a-\frac{bc}{d}} \left( e^{2a} \sqrt{-\frac{b(c+dx)}{d}} \Gamma\left(\frac{1}{2}, -\frac{b(c+dx)}{d}\right) - e^{\frac{2bc}{d}} \sqrt{\frac{b(c+dx)}{d}} \Gamma\left(\frac{1}{2}, \frac{b(c+dx)}{d}\right) \right)}{2b\sqrt{c + dx}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]/Sqrt[c + d\*x], x]

[Out] (E^(-a - (b\*c)/d)\*(E^(2\*a)\*Sqrt[-((b\*(c + d\*x))/d)]\*Gamma[1/2, -((b\*(c + d\*x))/d)] - E^((2\*b\*c)/d)\*Sqrt[(b\*(c + d\*x))/d]\*Gamma[1/2, (b\*(c + d\*x))/d]))/(2\*b\*Sqrt[c + d\*x])

**fricas [A]** time = 0.56, size = 123, normalized size = 1.18

$$\frac{\sqrt{\pi} \sqrt{\frac{b}{d}} \left( \cosh\left(-\frac{bc-ad}{d}\right) - \sinh\left(-\frac{bc-ad}{d}\right) \right) \operatorname{erf}\left(\sqrt{dx + c} \sqrt{\frac{b}{d}}\right) - \sqrt{\pi} \sqrt{-\frac{b}{d}} \left( \cosh\left(-\frac{bc-ad}{d}\right) + \sinh\left(-\frac{bc-ad}{d}\right) \right) \operatorname{erf}\left(\sqrt{dx + c} \sqrt{-\frac{b}{d}}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)^(1/2), x, algorithm="fricas")

[Out]  $\frac{1}{2} * (\sqrt{\pi} * \sqrt{b/d} * (\cosh(-(b*c - a*d)/d) - \sinh(-(b*c - a*d)/d)) * \operatorname{erf}(\sqrt{d*x + c} * \sqrt{b/d}) - \sqrt{\pi} * \sqrt{-b/d} * (\cosh(-(b*c - a*d)/d) + \sinh(-(b*c - a*d)/d)) * \operatorname{erf}(\sqrt{d*x + c} * \sqrt{-b/d})) / b$

**giac** [A] time = 0.13, size = 89, normalized size = 0.86

$$\frac{\left( \frac{\sqrt{\pi} d \operatorname{erf}\left(-\frac{\sqrt{bd} \sqrt{dx+c}}{d}\right) e^{\left(\frac{bc}{d}\right)}}{\sqrt{bd}} + \frac{\sqrt{\pi} d \operatorname{erf}\left(-\frac{\sqrt{-bd} \sqrt{dx+c}}{d}\right) e^{\left(-\frac{bc-2ad}{d}\right)}}{\sqrt{-bd}} \right) e^{(-a)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")`

[Out]  $-\frac{1}{2} * (\sqrt{\pi} * d * \operatorname{erf}(-\sqrt{b*d} * \sqrt{d*x + c}/d) * e^{(b*c/d)}/\sqrt{b*d} + \sqrt{\pi} * d * \operatorname{erf}(-\sqrt{-b*d} * \sqrt{d*x + c}/d) * e^{-(b*c - 2*a*d)/d}/\sqrt{-b*d}) * e^{(-a)/d}$

**maple** [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx + a)}{\sqrt{dx + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(b*x+a)/(d*x+c)^(1/2),x)`

[Out] `int(cosh(b*x+a)/(d*x+c)^(1/2),x)`

**maxima** [B] time = 0.33, size = 180, normalized size = 1.73

$$\frac{4\sqrt{dx+c} \cosh(bx+a) + \left( \frac{\sqrt{\pi} d \operatorname{erf}\left(\sqrt{dx+c} \sqrt{\frac{b}{d}}\right) e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{\frac{b}{d}}} + \frac{\sqrt{\pi} d \operatorname{erf}\left(\sqrt{dx+c} \sqrt{\frac{b}{d}}\right) e^{\left(-a+\frac{bc}{d}\right)}}{b\sqrt{\frac{b}{d}}} - \frac{2\sqrt{dx+c} d e^{\left(a+\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b} - \frac{2\sqrt{dx+c} d e^{\left(-a-\frac{(dx+c)b}{d}+\frac{bc}{d}\right)}}{b} \right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")`

[Out]  $\frac{1}{2} * (4 * \sqrt{d*x + c} * \cosh(b*x + a) + (\sqrt{\pi} * d * \operatorname{erf}(\sqrt{d*x + c} * \sqrt{-b/d}) * e^{(a - b*c/d)/(b*\sqrt{-b/d})} + \sqrt{\pi} * d * \operatorname{erf}(\sqrt{d*x + c} * \sqrt{b/d}) * e^{(-a + b*c/d)/(b*\sqrt{b/d})} - 2 * \sqrt{d*x + c} * d * e^{(a + (d*x + c)*b/d - b*c/d)/b} - 2 * \sqrt{d*x + c} * d * e^{(-a - (d*x + c)*b/d + b*c/d)/b}) * b/d / d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + bx)}{\sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(a + b*x)/(c + d*x)^(1/2), x)`

[Out] `int(cosh(a + b*x)/(c + d*x)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a + bx)}{\sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)/(d*x+c)**(1/2), x)`

[Out] `Integral(cosh(a + b*x)/sqrt(c + d*x), x)`

### 3.45 $\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}} dx$

**Optimal.** Leaf size=119

$$-\frac{\sqrt{\pi} \sqrt{b} e^{\frac{bc}{d}-a} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{\pi} \sqrt{b} e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2 \cosh(a+bx)}{d\sqrt{c+dx}}$$

[Out]  $-\exp(-a+b*c/d)*\operatorname{erf}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*b^{(1/2)}*\operatorname{Pi}^{(1/2)}/d^{(3/2)} + \exp(a-b*c/d)*\operatorname{erfi}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*b^{(1/2)}*\operatorname{Pi}^{(1/2)}/d^{(3/2)} - 2*\cosh(b*x+a)/d/(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.18, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3297, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi} \sqrt{b} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{\pi} \sqrt{b} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2 \cosh(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cosh}[a + b*x]/(c + d*x)^{(3/2)}, x]$

[Out]  $(-2*\operatorname{Cosh}[a + b*x])/(d*\operatorname{Sqrt}[c + d*x]) - (\operatorname{Sqrt}[b]*E^{(-a + (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/d^{(3/2)} + (\operatorname{Sqrt}[b]*E^{(a - (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/d^{(3/2)})$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g, x\} \&\amp; !\$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_)))^2}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}\{F, a, b, c, d, x\} \&\amp; \operatorname{PosQ}[b]$

#### Rule 2205

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_)))^2}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]])/(2*d*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]), x] /; \operatorname{FreeQ}\{F, a, b, c, d, x\} \&\amp; \operatorname{NegQ}[b]$

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh(a + bx)}{(c + dx)^{3/2}} dx &= -\frac{2 \cosh(a + bx)}{d\sqrt{c + dx}} + \frac{(2b) \int \frac{\sinh(a + bx)}{\sqrt{c + dx}} dx}{d} \\ &= -\frac{2 \cosh(a + bx)}{d\sqrt{c + dx}} + \frac{b \int \frac{e^{-i(i a + i b x)}}{\sqrt{c + dx}} dx}{d} - \frac{b \int \frac{e^{i(i a + i b x)}}{\sqrt{c + dx}} dx}{d} \\ &= -\frac{2 \cosh(a + bx)}{d\sqrt{c + dx}} - \frac{(2b) \text{Subst}\left(\int e^{i\left(i a - \frac{i b c}{d}\right) - \frac{b x^2}{d}} dx, x, \sqrt{c + dx}\right)}{d^2} + \frac{(2b) \text{Subst}\left(\int e^{-i\left(i a - \frac{i b c}{d}\right) - \frac{b x^2}{d}} dx, x, \sqrt{c + dx}\right)}{d^2} \\ &= -\frac{2 \cosh(a + bx)}{d\sqrt{c + dx}} - \frac{\sqrt{b} e^{-a + \frac{bc}{d}} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c + dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{b} e^{a - \frac{bc}{d}} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c + dx}}{\sqrt{d}}\right)}{d^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.36, size = 118, normalized size = 0.99

$$\frac{e^{-a} \left( e^{2a - \frac{bc}{d}} \sqrt{-\frac{b(c+dx)}{d}} \Gamma\left(\frac{1}{2}, -\frac{b(c+dx)}{d}\right) - e^{-bx} (e^{2(a+bx)} + 1) + e^{\frac{bc}{d}} \sqrt{\frac{b(c+dx)}{d}} \Gamma\left(\frac{1}{2}, b\left(\frac{c}{d} + x\right)\right) \right)}{d\sqrt{c + dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x)^(3/2), x]
```

```
[Out] (-((1 + E^(2*(a + b*x)))/E^(b*x)) + E^((b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, b*(c/d + x)] + E^(2*a - (b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x))/d)]/(d*E^a*Sqrt[c + d*x])
```

**fricas** [B] time = 1.05, size = 338, normalized size = 2.84

$$\frac{\sqrt{\pi} \left( (dx+c) \cosh(bx+a) \cosh\left(-\frac{bc-ad}{d}\right) - (dx+c) \cosh(bx+a) \sinh\left(-\frac{bc-ad}{d}\right) + \left( (dx+c) \cosh\left(-\frac{bc-ad}{d}\right) - (dx+c) \sinh\left(-\frac{bc-ad}{d}\right) \right) \sinh(bx+a) \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)^(3/2),x, algorithm="fricas")

[Out]  $-(\sqrt{\pi})*((d*x + c)*\cosh(b*x + a)*\cosh(-(b*c - a*d)/d) - (d*x + c)*\cosh(b*x + a)*\sinh(-(b*c - a*d)/d) + ((d*x + c)*\cosh(-(b*c - a*d)/d) - (d*x + c)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)*\sqrt{b/d}*\operatorname{erf}(\sqrt{d*x + c}*\sqrt{b/d}) + \sqrt{\pi}*((d*x + c)*\cosh(b*x + a)*\cosh(-(b*c - a*d)/d) + (d*x + c)*\cosh(b*x + a)*\sinh(-(b*c - a*d)/d) + ((d*x + c)*\cosh(-(b*c - a*d)/d) + (d*x + c)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{-b/d}*\operatorname{erf}(\sqrt{d*x + c}*\sqrt{-b/d}) + \sqrt{d*x + c}*(\cosh(b*x + a)^2 + 2*\cosh(b*x + a)*\sinh(b*x + a) + \sinh(b*x + a)^2 + 1))/((d^2*x + c*d)*\cosh(b*x + a) + (d^2*x + c*d)*\sinh(b*x + a))$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)}{(dx+c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(cosh(b\*x + a)/(d\*x + c)^(3/2), x)

**maple** [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)}{(dx+c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)/(d\*x+c)^(3/2),x)

[Out] int(cosh(b\*x+a)/(d\*x+c)^(3/2),x)

**maxima** [A] time = 0.33, size = 104, normalized size = 0.87

$$\frac{\left( \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{dx+c} \sqrt{-\frac{b}{d}}\right) e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}} - \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{dx+c} \sqrt{\frac{b}{d}}\right) e^{\left(-a+\frac{bc}{d}\right)}}{\sqrt{\frac{b}{d}}} \right) b}{d} - \frac{2 \cosh(bx+a)}{\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)^(3/2),x, algorithm="maxima")

[Out] ((sqrt(pi)\*erf(sqrt(d\*x + c)\*sqrt(-b/d))\*e^(a - b\*c/d)/sqrt(-b/d) - sqrt(pi)\*erf(sqrt(d\*x + c)\*sqrt(b/d))\*e^(-a + b\*c/d)/sqrt(b/d))\*b/d - 2\*cosh(b\*x + a)/sqrt(d\*x + c))/d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + bx)}{(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)/(c + d\*x)^(3/2),x)

[Out] int(cosh(a + b\*x)/(c + d\*x)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a + bx)}{(c + dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)\*\*(3/2),x)

[Out] Integral(cosh(a + b\*x)/(c + d\*x)\*\*(3/2), x)

### 3.46 $\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}} dx$

**Optimal.** Leaf size=149

$$\frac{2\sqrt{\pi} b^{3/2} e^{\frac{bc}{d}-a} \operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2\sqrt{\pi} b^{3/2} e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4b \sinh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2 \cosh(a+bx)}{3d(c+dx)^{3/2}}$$

[Out]  $-2/3*\cosh(b*x+a)/d/(d*x+c)^{(3/2)}+2/3*b^{(3/2)}*\exp(-a+b*c/d)*\operatorname{erf}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*\operatorname{Pi}^{(1/2)}/d^{(5/2)}+2/3*b^{(3/2)}*\exp(a-b*c/d)*\operatorname{erfi}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*\operatorname{Pi}^{(1/2)}/d^{(5/2)}-4/3*b*\sinh(b*x+a)/d^2/(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.25, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3297, 3307, 2180, 2204, 2205}

$$\frac{2\sqrt{\pi} b^{3/2} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2\sqrt{\pi} b^{3/2} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4b \sinh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2 \cosh(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cosh}[a + b*x]/(c + d*x)^{(5/2)}, x]$

[Out]  $(-2*\operatorname{Cosh}[a + b*x])/(3*d*(c + d*x)^{(3/2)}) + (2*b^{(3/2)}*E^{(-a + (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(3*d^{(5/2)}) + (2*b^{(3/2)}*E^{(a - (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(3*d^{(5/2)}) - (4*b*\operatorname{Sinh}[a + b*x])/(3*d^2*\operatorname{Sqrt}[c + d*x])$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g, x\} \ \&\amp; \ \! \$\operatorname{UseGamma} == \operatorname{True}$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x\} \ \&\amp; \ \operatorname{PosQ}[b]$

#### Rule 2205

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]])/(2*d*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]), x] /;$   $\operatorname{Fr}$



eeQ[{F, a, b, c, d}, x] && NegQ[b]

### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

### Rule 3307

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Dist[I/2, Int[(c + d\*x)^(m)/(E^(I\*k\*Pi)\*E^(I\*(e + f\*x))), x], x] - Dist[I/2, Int[(c + d\*x)^(m)\*E^(I\*k\*Pi)\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

### Rubi steps

$$\begin{aligned} \int \frac{\cosh(a + bx)}{(c + dx)^{5/2}} dx &= -\frac{2 \cosh(a + bx)}{3d(c + dx)^{3/2}} + \frac{(2b) \int \frac{\sinh(a + bx)}{(c + dx)^{3/2}} dx}{3d} \\ &= -\frac{2 \cosh(a + bx)}{3d(c + dx)^{3/2}} - \frac{4b \sinh(a + bx)}{3d^2 \sqrt{c + dx}} + \frac{(4b^2) \int \frac{\cosh(a + bx)}{\sqrt{c + dx}} dx}{3d^2} \\ &= -\frac{2 \cosh(a + bx)}{3d(c + dx)^{3/2}} - \frac{4b \sinh(a + bx)}{3d^2 \sqrt{c + dx}} + \frac{(2b^2) \int \frac{e^{-i(ia + ibx)}}{\sqrt{c + dx}} dx}{3d^2} + \frac{(2b^2) \int \frac{e^{i(ia + ibx)}}{\sqrt{c + dx}} dx}{3d^2} \\ &= -\frac{2 \cosh(a + bx)}{3d(c + dx)^{3/2}} - \frac{4b \sinh(a + bx)}{3d^2 \sqrt{c + dx}} + \frac{(4b^2) \text{Subst}\left(\int e^{i\left(ia - \frac{ibc}{d} - \frac{bx^2}{d}\right)} dx, x, \sqrt{c + dx}\right)}{3d^3} + \frac{(4b^2) \text{Subst}\left(\int e^{-i\left(ia - \frac{ibc}{d} - \frac{bx^2}{d}\right)} dx, x, \sqrt{c + dx}\right)}{3d^3} \\ &= -\frac{2 \cosh(a + bx)}{3d(c + dx)^{3/2}} + \frac{2b^{3/2} e^{-a + \frac{bc}{d}} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c + dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2b^{3/2} e^{a - \frac{bc}{d}} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c + dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4b \sinh(a + bx)}{3d^2 \sqrt{c + dx}} \end{aligned}$$

**Mathematica [A]** time = 0.73, size = 150, normalized size = 1.01

$$\frac{e^{-a} \left( -2de^{2a - \frac{bc}{d}} \left( -\frac{b(c + dx)}{d} \right)^{3/2} \Gamma\left(\frac{1}{2}, -\frac{b(c + dx)}{d}\right) - e^{-bx} \left( 2b \left( e^{2(a + bx)} - 1 \right) (c + dx) + d \left( e^{2(a + bx)} + 1 \right) + 2de^{b\left(\frac{c}{d} + x\right)} \left( \frac{b(c + dx)}{d} \right) \right)}{3d^2(c + dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]/(c + d\*x)^(5/2), x]

[Out]  $(-((d*(1 + E^{(2*(a + b*x))}) + 2*b*(-1 + E^{(2*(a + b*x))})*(c + d*x) + 2*d*E^{(b*(c/d + x))}*((b*(c + d*x))/d)^{(3/2)}*Gamma[1/2, b*(c/d + x)]/E^{(b*x)}) - 2*d*E^{(2*a - (b*c)/d)}*(-((b*(c + d*x))/d))^{(3/2)}*Gamma[1/2, -((b*(c + d*x))/d)])/(3*d^2*E^a*(c + d*x)^{(3/2)})$

**fricas** [B] time = 0.91, size = 534, normalized size = 3.58

$$2\sqrt{\pi} \left( (bd^2x^2 + 2bcdx + bc^2) \cosh(bx + a) \cosh\left(-\frac{bc-ad}{d}\right) - (bd^2x^2 + 2bcdx + bc^2) \cosh(bx + a) \sinh\left(-\frac{bc-ad}{d}\right) \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)^(5/2), x, algorithm="fricas")

[Out]  $1/3*(2*\sqrt{\pi}*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)*\cosh(-(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)*\sinh(-(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(-(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{b/d}*erf(\sqrt{d*x + c}*\sqrt{b/d}) - 2*\sqrt{\pi}*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)*\cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)*\sinh(-(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(-(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{-b/d}*erf(\sqrt{d*x + c}*\sqrt{-b/d}) + (2*b*d*x - (2*b*d*x + 2*b*c + d)*\cosh(b*x + a)^2 - 2*(2*b*d*x + 2*b*c + d)*\cosh(b*x + a)*\sinh(b*x + a) - (2*b*d*x + 2*b*c + d)*\sinh(b*x + a)^2 + 2*b*c - d)*\sqrt{d*x + c})/((d^4*x^2 + 2*c*d^3*x + c^2*d^2)*\cosh(b*x + a) + (d^4*x^2 + 2*c*d^3*x + c^2*d^2)*\sinh(b*x + a))$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx + a)}{(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)^(5/2), x, algorithm="giac")

[Out] integrate(cosh(b\*x + a)/(d\*x + c)^(5/2), x)

**maple** [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx + a)}{(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(b*x+a)/(d*x+c)^(5/2), x)`

[Out] `int(cosh(b*x+a)/(d*x+c)^(5/2), x)`

**maxima** [A] time = 0.54, size = 115, normalized size = 0.77

$$\frac{\left( \frac{\sqrt{\frac{(dx+c)b}{d}} e^{\left(-a+\frac{bc}{d}\right)} \Gamma\left(-\frac{1}{2}, \frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}} - \frac{\sqrt{-\frac{(dx+c)b}{d}} e^{\left(a-\frac{bc}{d}\right)} \Gamma\left(-\frac{1}{2}, -\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}} \right) b}{d} - \frac{2 \cosh(bx+a)}{(dx+c)^{\frac{3}{2}}}$$

$3d$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)/(d*x+c)^(5/2), x, algorithm="maxima")`

[Out] `1/3*((sqrt((d*x + c)*b/d)*e^(-a + b*c/d)*gamma(-1/2, (d*x + c)*b/d)/sqrt(d*x + c) - sqrt(-(d*x + c)*b/d)*e^(a - b*c/d)*gamma(-1/2, -(d*x + c)*b/d)/sqrt(d*x + c))*b/d - 2*cosh(b*x + a)/(d*x + c)^(3/2))/d`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + bx)}{(c + dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(a + b*x)/(c + d*x)^(5/2), x)`

[Out] `int(cosh(a + b*x)/(c + d*x)^(5/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a + bx)}{(c + dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)/(d*x+c)**(5/2), x)`

[Out] `Integral(cosh(a + b*x)/(c + d*x)**(5/2), x)`

$$3.47 \quad \int \frac{\cosh(a+bx)}{(c+dx)^{7/2}} dx$$

**Optimal.** Leaf size=174

$$-\frac{4\sqrt{\pi} b^{5/2} e^{\frac{bc}{d}-a} \operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{4\sqrt{\pi} b^{5/2} e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{8b^2 \cosh(a+bx)}{15d^3 \sqrt{c+dx}} - \frac{4b \sinh(a+bx)}{15d^2 (c+dx)^{3/2}} - \frac{2 \cosh(a+bx)}{5d(c+dx)^{5/2}}$$

[Out]  $-2/5*\cosh(b*x+a)/d/(d*x+c)^{(5/2)}-4/15*b*\sinh(b*x+a)/d^2/(d*x+c)^{(3/2)}-4/15*b^{(5/2)}*\exp(-a+b*c/d)*\operatorname{erf}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*\operatorname{Pi}^{(1/2)}/d^{(7/2)}+4/15*b^{(5/2)}*\exp(a-b*c/d)*\operatorname{erfi}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*\operatorname{Pi}^{(1/2)}/d^{(7/2)}-8/15*b^2*\cosh(b*x+a)/d^3/(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.31, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3297, 3308, 2180, 2204, 2205}

$$-\frac{4\sqrt{\pi} b^{5/2} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{4\sqrt{\pi} b^{5/2} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{8b^2 \cosh(a+bx)}{15d^3 \sqrt{c+dx}} - \frac{4b \sinh(a+bx)}{15d^2 (c+dx)^{3/2}} - \frac{2 \cosh(a+bx)}{5d(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] `Int[Cosh[a + b*x]/(c + d*x)^(7/2), x]`

[Out]  $(-2*\operatorname{Cosh}[a + b*x])/(5*d*(c + d*x)^{(5/2)}) - (8*b^2*\operatorname{Cosh}[a + b*x])/(15*d^3*\operatorname{Sqrt}[c + d*x]) - (4*b^{(5/2)}*E^{-a + (b*c)/d}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/\operatorname{Sqrt}[d]])/(15*d^{(7/2)}) + (4*b^{(5/2)}*E^{a - (b*c)/d}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/\operatorname{Sqrt}[d]])/(15*d^{(7/2)}) - (4*b*\operatorname{Sinh}[a + b*x])/(15*d^2*(c + d*x)^{(3/2)})$

#### Rule 2180

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True`

#### Rule 2204

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

#### Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

### Rule 3308

```
Int[((c_.) + (d_.)*(x_))(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I
/2, Int[(c + d*x)m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh(a+bx)}{(c+dx)^{7/2}} dx &= -\frac{2 \cosh(a+bx)}{5d(c+dx)^{5/2}} + \frac{(2b) \int \frac{\sinh(a+bx)}{(c+dx)^{5/2}} dx}{5d} \\
&= -\frac{2 \cosh(a+bx)}{5d(c+dx)^{5/2}} - \frac{4b \sinh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{(4b^2) \int \frac{\cosh(a+bx)}{(c+dx)^{3/2}} dx}{15d^2} \\
&= -\frac{2 \cosh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2 \cosh(a+bx)}{15d^3 \sqrt{c+dx}} - \frac{4b \sinh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{(8b^3) \int \frac{\sinh(a+bx)}{\sqrt{c+dx}} dx}{15d^3} \\
&= -\frac{2 \cosh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2 \cosh(a+bx)}{15d^3 \sqrt{c+dx}} - \frac{4b \sinh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{(4b^3) \int \frac{e^{-i(a+ibx)}}{\sqrt{c+dx}} dx}{15d^3} - \frac{(4b^3) \int \frac{e^{i(a+ibx)}}{\sqrt{c+dx}} dx}{15d^3} \\
&= -\frac{2 \cosh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2 \cosh(a+bx)}{15d^3 \sqrt{c+dx}} - \frac{4b \sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{(8b^3) \text{Subst} \left( \int e^{i\left(a-\frac{ibc}{d}\right)-\frac{bx^2}{d}} dx, \right)}{15d^4} \\
&= -\frac{2 \cosh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2 \cosh(a+bx)}{15d^3 \sqrt{c+dx}} - \frac{4b^{5/2} e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{15d^{7/2}} + \frac{4b^{5/2} e^{a-\frac{bc}{d}} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{15d^{7/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.42, size = 191, normalized size = 1.10

$$\frac{e^{-a} \left( 2e^{2a} \left( -2be^{-\frac{bc}{d}} (c + dx) \left( e^{b\left(\frac{c}{d} + x\right)} (2b(c + dx) + d) + 2d \left( -\frac{b(c+dx)}{d} \right)^{3/2} \Gamma \left( \frac{1}{2}, -\frac{b(c+dx)}{d} \right) \right) - 3d^2 e^{bx} \right) + e^{-bx} \left( -8b^2(c + dx) \right)}{30d^3(c + dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]/(c + d\*x)^(7/2), x]

[Out] (2\*E^(2\*a))\*(-3\*d^2\*E^(b\*x) - (2\*b\*(c + d\*x))\*(E^(b\*(c/d + x))\*(d + 2\*b\*(c + d\*x)) + 2\*d\*(-((b\*(c + d\*x))/d))^(3/2)\*Gamma[1/2, -((b\*(c + d\*x))/d)]))/E^((b\*c)/d) + (-6\*d^2 + 4\*b\*d\*(c + d\*x) - 8\*b^2\*(c + d\*x)^2 + 8\*d^2\*E^(b\*(c/d + x)))\*((b\*(c + d\*x))/d)^(5/2)\*Gamma[1/2, (b\*(c + d\*x))/d])/E^(b\*x))/(30\*d^3\*E^a\*(c + d\*x)^(5/2))

**fricas [B]** time = 0.51, size = 853, normalized size = 4.90

$$4\sqrt{\pi} \left( (b^2 d^3 x^3 + 3 b^2 c d^2 x^2 + 3 b^2 c^2 d x + b^2 c^3) \cosh(bx + a) \cosh\left(-\frac{bc-ad}{d}\right) - (b^2 d^3 x^3 + 3 b^2 c d^2 x^2 + 3 b^2 c^2 d x + b^2 c^3) \cosh(bx + a) \sinh\left(-\frac{bc-ad}{d}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)^(7/2), x, algorithm="fricas")

[Out] -1/15\*(4\*sqrt(pi))\*((b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*cosh(b\*x + a)\*cosh(-(b\*c - a\*d)/d) - (b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*cosh(b\*x + a)\*sinh(-(b\*c - a\*d)/d) + ((b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*cosh(-(b\*c - a\*d)/d) - (b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*sinh(-(b\*c - a\*d)/d))\*sinh(b\*x + a))\*sqrt(b/d)\*erf(sqrt(d\*x + c)\*sqrt(b/d)) + 4\*sqrt(pi)\*((b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*cosh(b\*x + a)\*cosh(-(b\*c - a\*d)/d) + (b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*cosh(b\*x + a)\*sinh(-(b\*c - a\*d)/d) + ((b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*cosh(-(b\*c - a\*d)/d) + (b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*sinh(-(b\*c - a\*d)/d))\*sinh(b\*x + a))\*sqrt(-b/d)\*erf(sqrt(d\*x + c)\*sqrt(-b/d)) + (4\*b^2\*d^2\*x^2 + 4\*b^2\*c^2 - 2\*b\*c\*d + (4\*b^2\*d^2\*x^2 + 4\*b^2\*c^2 + 2\*b\*c\*d + 3\*d^2 + 2\*(4\*b^2\*c\*d + b\*d^2)\*x)\*cosh(b\*x + a)^2 + 2\*(4\*b^2\*d^2\*x^2 + 4\*b^2\*c^2 + 2\*b\*c\*d + 3\*d^2 + 2\*(4\*b^2\*c\*d + b\*d^2)\*x)\*cosh(b\*x + a)\*sinh(b\*x + a) + (4\*b^2\*d^2\*x^2 + 4\*b^2\*c^2 + 2\*b\*c\*d + 3\*d^2 + 2\*(4\*b^2\*c\*d + b\*d^2)\*x)\*sinh(b\*x + a)^2 + 3\*d^2 + 2\*(4\*b^2\*c\*d - b\*d^2)\*x)\*sqrt(d\*x + c))/((d^6\*x^3 + 3\*c\*d^5\*x^2 + 3\*c^2\*d^4\*x + c^3\*d^3)\*cosh(b\*x + a) + (d^6\*x^3 + 3\*c\*d^5\*x^2 + 3\*c^2\*d^4\*x + c^3\*d^3)\*sinh(b\*x + a))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx + a)}{(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)^(7/2),x, algorithm="giac")

[Out] integrate(cosh(b\*x + a)/(d\*x + c)^(7/2), x)

**maple** [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx + a)}{(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)/(d\*x+c)^(7/2),x)

[Out] int(cosh(b\*x+a)/(d\*x+c)^(7/2),x)

**maxima** [A] time = 0.41, size = 115, normalized size = 0.66

$$\frac{\left( \frac{\left( \frac{(dx+c)b}{d} \right)^{\frac{3}{2}} e^{\left( -a + \frac{bc}{d} \right)} \Gamma\left( -\frac{3}{2}, \frac{(dx+c)b}{d} \right) - \left( -\frac{(dx+c)b}{d} \right)^{\frac{3}{2}} e^{\left( a - \frac{bc}{d} \right)} \Gamma\left( -\frac{3}{2}, -\frac{(dx+c)b}{d} \right) \right) b}{(dx+c)^{\frac{3}{2}} (dx+c)^{\frac{3}{2}}} \frac{2 \cosh(bx+a)}{d} - \frac{2 \cosh(bx+a)}{(dx+c)^{\frac{5}{2}}}$$


---


$$5d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)/(d\*x+c)^(7/2),x, algorithm="maxima")

[Out] 1/5\*(((d\*x + c)\*b/d)^(3/2)\*e^(-a + b\*c/d)\*gamma(-3/2, (d\*x + c)\*b/d)/(d\*x + c)^(3/2) - (-d\*x + c)\*b/d)^(3/2)\*e^(a - b\*c/d)\*gamma(-3/2, -(d\*x + c)\*b/d)/(d\*x + c)^(3/2))\*b/d - 2\*cosh(b\*x + a)/(d\*x + c)^(5/2))/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + bx)}{(c + dx)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)/(c + d\*x)^(7/2),x)

[Out] `int(cosh(a + b*x)/(c + d*x)^(7/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a + bx)}{(c + dx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)/(d*x+c)**(7/2), x)`

[Out] `Integral(cosh(a + b*x)/(c + d*x)**(7/2), x)`



### 3.48 $\int (c + dx)^{5/2} \cosh^2(a + bx) dx$

**Optimal.** Leaf size=239

$$\frac{15\sqrt{\frac{\pi}{2}} d^{5/2} e^{\frac{2bc}{d}-2a} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}} - \frac{15\sqrt{\frac{\pi}{2}} d^{5/2} e^{2a-\frac{2bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}} + \frac{15d^2\sqrt{c+dx} \sinh(2a+2bx)}{64b^3} - \frac{5d(c+a)}{b^2}$$

[Out]  $5/16*d*(d*x+c)^{(3/2)}/b^2+1/7*(d*x+c)^{(7/2)}/d-5/8*d*(d*x+c)^{(3/2)}*\cosh(b*x+a)^2/b^2+1/2*(d*x+c)^{(5/2)}*\cosh(b*x+a)*\sinh(b*x+a)/b+15/512*d^{(5/2)}*\exp(-2*a+2*b*c/d)*\operatorname{erf}(2^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(7/2)}-15/512*d^{(5/2)}*\exp(2*a-2*b*c/d)*\operatorname{erfi}(2^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(7/2)}+15/64*d^2*\sinh(2*b*x+2*a)*(d*x+c)^{(1/2)}/b^3$

**Rubi [A]** time = 0.40, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {3311, 32, 3312, 3296, 3308, 2180, 2204, 2205}

$$\frac{15\sqrt{\frac{\pi}{2}} d^{5/2} e^{\frac{2bc}{d}-2a} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}} - \frac{15\sqrt{\frac{\pi}{2}} d^{5/2} e^{2a-\frac{2bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}} + \frac{15d^2\sqrt{c+dx} \sinh(2a+2bx)}{64b^3} - \frac{5d(c+a)}{b^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^{(5/2)}*\operatorname{Cosh}[a + b*x]^2, x]$

[Out]  $(5*d*(c + d*x)^{(3/2)})/(16*b^2) + (c + d*x)^{(7/2)}/(7*d) - (5*d*(c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x]^2)/(8*b^2) + (15*d^{(5/2)}*E^{(-2*a + (2*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(256*b^{(7/2)}) - (15*d^{(5/2)}*E^{(2*a - (2*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(256*b^{(7/2)}) + ((c + d*x)^{(5/2)}*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/(2*b) + (15*d^2*\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[2*a + 2*b*x])/(64*b^3)$

#### Rule 32

$\operatorname{Int}[(a + b*x)^m, x] \rightarrow \operatorname{Simp}[(a + b*x)^{m+1}/(b*(m+1)), x] /;$   $\operatorname{FreeQ}\{a, b, m, x\} \ \&\amp; \ \operatorname{NeQ}[m, -1]$

#### Rule 2180

$\operatorname{Int}[(F + (g + (e + (f + (g*x^2)/d))/Sqrt[(c + d*x)]), x] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F + (g + (e - (c*f)/d) + (f*g*x^2)/d], x], x, \operatorname{Sqrt}[c + d*x]] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g, x\} \ \&\amp; \ \operatorname{!}\$UseGamma == \operatorname{True}$

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[(Fa*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

### Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[(Fa*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 3308

```
Int[((c_.) + (d_.)*(x_))(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

### Rule 3311

```
Int[((c_.) + (d_.)*(x_))(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])(n_.), x_Symbol] := Simp[(d*m*(c + d*x)(m - 1)*(b*Ssin[e + f*x])n/(f2*n), x] + (Dist[(b2*(n - 1))/n, Int[(c + d*x)m*(b*Ssin[e + f*x])(n - 2), x], x] - Dist[(d2*m*(m - 1))/(f2*n), Int[(c + d*x)(m - 2)*(b*Ssin[e + f*x])n, x], x] - Simp[(b*(c + d*x)m*Cos[e + f*x]*(b*Ssin[e + f*x])(n - 1)/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

### Rule 3312

```
Int[((c_.) + (d_.)*(x_))(m_.)*sin[(e_.) + (f_.)*(x_) ](n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)m, Sin[e + f*x]n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

### Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cosh^2(a + bx) dx &= -\frac{5d(c + dx)^{3/2} \cosh^2(a + bx)}{8b^2} + \frac{(c + dx)^{5/2} \cosh(a + bx) \sinh(a + bx)}{2b} + \frac{1}{2} \int (c + dx)^{3/2} \cosh^2(a + bx) dx \\
&= \frac{(c + dx)^{7/2}}{7d} - \frac{5d(c + dx)^{3/2} \cosh^2(a + bx)}{8b^2} + \frac{(c + dx)^{5/2} \cosh(a + bx) \sinh(a + bx)}{2b} \\
&= \frac{5d(c + dx)^{3/2}}{16b^2} + \frac{(c + dx)^{7/2}}{7d} - \frac{5d(c + dx)^{3/2} \cosh^2(a + bx)}{8b^2} + \frac{(c + dx)^{5/2} \cosh(a + bx) \sinh(a + bx)}{2b} \\
&= \frac{5d(c + dx)^{3/2}}{16b^2} + \frac{(c + dx)^{7/2}}{7d} - \frac{5d(c + dx)^{3/2} \cosh^2(a + bx)}{8b^2} + \frac{(c + dx)^{5/2} \cosh(a + bx) \sinh(a + bx)}{2b} \\
&= \frac{5d(c + dx)^{3/2}}{16b^2} + \frac{(c + dx)^{7/2}}{7d} - \frac{5d(c + dx)^{3/2} \cosh^2(a + bx)}{8b^2} + \frac{(c + dx)^{5/2} \cosh(a + bx) \sinh(a + bx)}{2b} \\
&= \frac{5d(c + dx)^{3/2}}{16b^2} + \frac{(c + dx)^{7/2}}{7d} - \frac{5d(c + dx)^{3/2} \cosh^2(a + bx)}{8b^2} + \frac{(c + dx)^{5/2} \cosh(a + bx) \sinh(a + bx)}{2b} \\
&= \frac{5d(c + dx)^{3/2}}{16b^2} + \frac{(c + dx)^{7/2}}{7d} - \frac{5d(c + dx)^{3/2} \cosh^2(a + bx)}{8b^2} + \frac{15d^{5/2} e^{-2a + \frac{2bc}{d}} \sqrt{\frac{\pi}{2}}}{256}
\end{aligned}$$

**Mathematica [A]** time = 1.20, size = 189, normalized size = 0.79

$$\frac{\sqrt{c + dx} \left( b(c + dx) \left( 7\sqrt{2} d^3 \Gamma\left(\frac{7}{2}, \frac{2b(c+dx)}{d}\right) \left( \sinh\left(2a - \frac{2bc}{d}\right) - \cosh\left(2a - \frac{2bc}{d}\right) \right) + 64b^3(c + dx)^3 \sqrt{\frac{b(c+dx)}{d}} \right) - 7\sqrt{2} d^3 \Gamma\left(\frac{7}{2}, \frac{2b(c+dx)}{d}\right) \right)}{448b^3 d^2 \left(\frac{b(c+dx)}{d}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x)^(5/2)\*Cosh[a + b\*x]^2,x]

[Out] (Sqrt[c + d\*x]\*(-7\*Sqrt[2]\*d^4\*Sqrt[-((b^2\*(c + d\*x)^2)/d^2)]\*Gamma[7/2, (-2\*b\*(c + d\*x))/d]\*(Cosh[2\*a - (2\*b\*c)/d] + Sinh[2\*a - (2\*b\*c)/d]) + b\*(c + d\*x)\*(64\*b^3\*(c + d\*x)^3\*Sqrt[(b\*(c + d\*x))/d] + 7\*Sqrt[2]\*d^3\*Gamma[7/2, (2\*b\*(c + d\*x))/d]\*(-Cosh[2\*a - (2\*b\*c)/d] + Sinh[2\*a - (2\*b\*c)/d])))/(448\*b^3\*d^2\*((b\*(c + d\*x))/d)^(3/2))

**fricas [B]** time = 0.46, size = 1001, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(5/2)\*cosh(b\*x+a)^2,x, algorithm="fricas")

[Out]  $\frac{1}{3584} \cdot (105 \sqrt{2} \sqrt{\pi}) \cdot (d^4 \cosh(bx+a)^2 \cosh(-2(bc-ad)/d) - d^4 \cosh(bx+a)^2 \sinh(-2(bc-ad)/d) + (d^4 \cosh(-2(bc-ad)/d) - d^4 \sinh(-2(bc-ad)/d)) \sinh(bx+a)^2 + 2(d^4 \cosh(bx+a) \cosh(-2(bc-ad)/d) - d^4 \cosh(bx+a) \sinh(-2(bc-ad)/d)) \sinh(bx+a) \sqrt{b/d} \operatorname{erf}(\sqrt{2} \sqrt{dx+c} \sqrt{b/d}) + 105 \sqrt{2} \sqrt{\pi} \cdot (d^4 \cosh(bx+a)^2 \cosh(-2(bc-ad)/d) + d^4 \cosh(bx+a)^2 \sinh(-2(bc-ad)/d) + (d^4 \cosh(-2(bc-ad)/d) + d^4 \sinh(-2(bc-ad)/d)) \sinh(bx+a)^2 + 2(d^4 \cosh(bx+a) \cosh(-2(bc-ad)/d) + d^4 \cosh(bx+a) \sinh(-2(bc-ad)/d)) \sinh(bx+a) \sqrt{-b/d} \operatorname{erf}(\sqrt{2} \sqrt{dx+c} \sqrt{-b/d}) - 4(112b^3d^3x^2 + 112b^3c^2d + 140b^2cd^2 - 7(16b^3d^3x^2 + 16b^3c^2d - 20b^2cd^2 + 15bd^3 + 4(8b^3cd^2 - 5b^2d^3)x) \cosh(bx+a)^4 - 28(16b^3d^3x^2 + 16b^3c^2d - 20b^2cd^2 + 15bd^3 + 4(8b^3cd^2 - 5b^2d^3)x) \cosh(bx+a) \sinh(bx+a)^3 - 7(16b^3d^3x^2 + 16b^3c^2d - 20b^2cd^2 + 15bd^3 + 4(8b^3cd^2 - 5b^2d^3)x) \sinh(bx+a)^4 + 105bd^3 - 128(b^4d^3x^3 + 3b^4cd^2x^2 + 3b^4c^2dx + b^4c^3) \cosh(bx+a)^2 - 2(64b^4d^3x^3 + 192b^4cd^2x^2 + 192b^4c^2dx + 64b^4c^3 + 21(16b^3d^3x^2 + 16b^3c^2d - 20b^2cd^2 + 15bd^3 + 4(8b^3cd^2 - 5b^2d^3)x) \cosh(bx+a)^2) \sinh(bx+a)^2 + 28(8b^3cd^2 + 5b^2d^3)x - 4(7(16b^3d^3x^2 + 16b^3c^2d - 20b^2cd^2 + 15bd^3 + 4(8b^3cd^2 - 5b^2d^3)x) \cosh(bx+a)^3 + 64(b^4d^3x^3 + 3b^4cd^2x^2 + 3b^4c^2dx + b^4c^3) \cosh(bx+a)) \sinh(bx+a) \sqrt{dx+c}) / (b^4d \cosh(bx+a))^2 + 2b^4d \cosh(bx+a) \sinh(bx+a) + b^4d \sinh(bx+a)^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^{\frac{5}{2}} \cosh^2(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(5/2)\*cosh(b\*x+a)^2,x, algorithm="giac")

[Out] integrate((d\*x+c)^(5/2)\*cosh(b\*x+a)^2, x)

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int (dx+c)^{\frac{5}{2}} (\cosh^2(bx+a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^(5/2)\*cosh(b\*x+a)^2,x)

[Out] int((d\*x+c)^(5/2)\*cosh(b\*x+a)^2,x)

**maxima** [A] time = 0.46, size = 281, normalized size = 1.18

$$512(dx+c)^{\frac{7}{2}} - \frac{105\sqrt{2}\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2a-\frac{2bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} + \frac{105\sqrt{2}\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2a+\frac{2bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} - \frac{28\left(16(dx+c)^{\frac{5}{2}}b^2de^{\left(\frac{2bc}{d}\right)}\right)}{b^3}$$

3584

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(5/2)\*cosh(b\*x+a)^2,x, algorithm="maxima")

[Out] 1/3584\*(512\*(d\*x + c)^(7/2) - 105\*sqrt(2)\*sqrt(pi)\*d^3\*erf(sqrt(2)\*sqrt(d\*x + c)\*sqrt(-b/d))\*e^(2\*a - 2\*b\*c/d)/(b^3\*sqrt(-b/d)) + 105\*sqrt(2)\*sqrt(pi)\*d^3\*erf(sqrt(2)\*sqrt(d\*x + c)\*sqrt(b/d))\*e^(-2\*a + 2\*b\*c/d)/(b^3\*sqrt(b/d)) - 28\*(16\*(d\*x + c)^(5/2)\*b^2\*d\*e^(2\*b\*c/d) + 20\*(d\*x + c)^(3/2)\*b\*d^2\*e^(2\*b\*c/d) + 15\*sqrt(d\*x + c)\*d^3\*e^(2\*b\*c/d))\*e^(-2\*a - 2\*(d\*x + c)\*b/d)/b^3 + 28\*(16\*(d\*x + c)^(5/2)\*b^2\*d\*e^(2\*a) - 20\*(d\*x + c)^(3/2)\*b\*d^2\*e^(2\*a) + 15\*sqrt(d\*x + c)\*d^3\*e^(2\*a))\*e^(2\*(d\*x + c)\*b/d - 2\*b\*c/d)/b^3/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh(a + bx)^2 (c + dx)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^2\*(c + d\*x)^(5/2), x)

[Out] int(cosh(a + b\*x)^2\*(c + d\*x)^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^{\frac{5}{2}} \cosh^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*(5/2)\*cosh(b\*x+a)\*\*2,x)

[Out] Integral((c + d\*x)\*\*(5/2)\*cosh(a + b\*x)\*\*2, x)

### 3.49 $\int (c + dx)^{3/2} \cosh^2(a + bx) dx$

**Optimal.** Leaf size=211

$$\frac{3\sqrt{\frac{\pi}{2}} d^{3/2} e^{\frac{2bc}{d}-2a} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}} + \frac{3\sqrt{\frac{\pi}{2}} d^{3/2} e^{2a-\frac{2bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}} - \frac{3d\sqrt{c+dx} \cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2} \sin\left(\frac{2a+2b\sqrt{c+dx}}{d}\right)}{2b^2}$$

[Out]  $1/5*(d*x+c)^{(5/2)}/d+1/2*(d*x+c)^{(3/2)}*\cosh(b*x+a)*\sinh(b*x+a)/b+3/128*d^{(3/2)}*\exp(-2*a+2*b*c/d)*\operatorname{erf}(2^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(5/2)}+3/128*d^{(3/2)}*\exp(2*a-2*b*c/d)*\operatorname{erfi}(2^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(5/2)}+3/16*d*(d*x+c)^{(1/2)}/b^2-3/8*d*\cosh(b*x+a)^2*(d*x+c)^{(1/2)}/b^2$

**Rubi [A]** time = 0.30, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {3311, 32, 3312, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\frac{\pi}{2}} d^{3/2} e^{\frac{2bc}{d}-2a} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}} + \frac{3\sqrt{\frac{\pi}{2}} d^{3/2} e^{2a-\frac{2bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}} - \frac{3d\sqrt{c+dx} \cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2} \sin\left(\frac{2a+2b\sqrt{c+dx}}{d}\right)}{2b^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x]^2, x]$

[Out]  $(3*d*\operatorname{Sqrt}[c + d*x])/(16*b^2) + (c + d*x)^{(5/2)}/(5*d) - (3*d*\operatorname{Sqrt}[c + d*x]*\operatorname{Cosh}[a + b*x]^2)/(8*b^2) + (3*d^{(3/2)}*E^{(-2*a + (2*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(64*b^{(5/2)}) + (3*d^{(3/2)}*E^{(2*a - (2*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(64*b^{(5/2)}) + ((c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/(2*b)$

#### Rule 32

$\operatorname{Int}[(a_.) + (b_.)*(x_.)]^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m + 1)}/(b*(m + 1)), x] /;$   $\operatorname{FreeQ}\{a, b, m\}, x \&\& \operatorname{NeQ}[m, -1]$

#### Rule 2180

$\operatorname{Int}[(F_.)^{((g_.)*((e_.) + (f_.)*(x_.)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_.)]}, x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \&\& !\$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_.)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{$

F, a, b, c, d}, x] && PosQ[b]

### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

### Rule 3307

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I/2, Int[(c + d\*x)^m/(E^(I\*k\*Pi)\*E^(I\*(e + f\*x))), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*k\*Pi)\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Ssin[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*Ssin[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*Ssin[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*Cos[e + f\*x]\*(b\*Ssin[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rubi steps

$$\begin{aligned}
\int (c+dx)^{3/2} \cosh^2(a+bx) dx &= -\frac{3d\sqrt{c+dx} \cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2} \cosh(a+bx) \sinh(a+bx)}{2b} + \frac{1}{2} \int (c+dx)^{3/2} \cosh(a+bx) \sinh(a+bx) dx \\
&= \frac{(c+dx)^{5/2}}{5d} - \frac{3d\sqrt{c+dx} \cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2} \cosh(a+bx) \sinh(a+bx)}{2b} \\
&= \frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} - \frac{3d\sqrt{c+dx} \cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2} \cosh(a+bx) \sinh(a+bx)}{2b} \\
&= \frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} - \frac{3d\sqrt{c+dx} \cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2} \cosh(a+bx) \sinh(a+bx)}{2b} \\
&= \frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} - \frac{3d\sqrt{c+dx} \cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2} \cosh(a+bx) \sinh(a+bx)}{2b} \\
&= \frac{3d\sqrt{c+dx}}{16b^2} + \frac{(c+dx)^{5/2}}{5d} - \frac{3d\sqrt{c+dx} \cosh^2(a+bx)}{8b^2} + \frac{3d^{3/2} e^{-2a+\frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{\sqrt{c+dx} \cosh(a+bx)}{\sqrt{2}}\right)}{64b^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.60, size = 163, normalized size = 0.77

$$\frac{5\sqrt{2} d^3 \sqrt{\frac{b(c+dx)}{d}} \Gamma\left(\frac{5}{2}, \frac{2b(c+dx)}{d}\right) \left(\sinh\left(2a - \frac{2bc}{d}\right) - \cosh\left(2a - \frac{2bc}{d}\right)\right) + 5\sqrt{2} d^3 \sqrt{-\frac{b(c+dx)}{d}} \Gamma\left(\frac{5}{2}, -\frac{2b(c+dx)}{d}\right) \left(\sinh\left(2a - \frac{2bc}{d}\right) + \cosh\left(2a - \frac{2bc}{d}\right)\right)}{160b^3 d \sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^(3/2)\*Cosh[a + b\*x]^2,x]

[Out] (32\*b^3\*(c + d\*x)^3 + 5\*Sqrt[2]\*d^3\*Sqrt[(b\*(c + d\*x))/d]\*Gamma[5/2, (2\*b\*(c + d\*x))/d]\*(-Cosh[2\*a - (2\*b\*c)/d] + Sinh[2\*a - (2\*b\*c)/d]) + 5\*Sqrt[2]\*d^3\*Sqrt[-(b\*(c + d\*x))/d]\*Gamma[5/2, (-2\*b\*(c + d\*x))/d]\*(Cosh[2\*a - (2\*b\*c)/d] + Sinh[2\*a - (2\*b\*c)/d]))/(160\*b^3\*d\*Sqrt[c + d\*x])

**fricas [B]** time = 0.49, size = 755, normalized size = 3.58

$$\frac{15\sqrt{2}\sqrt{\pi}\left(d^3 \cosh(bx+a)^2 \cosh\left(-\frac{2(bc-ad)}{d}\right) - d^3 \cosh(bx+a)^2 \sinh\left(-\frac{2(bc-ad)}{d}\right) + \left(d^3 \cosh\left(-\frac{2(bc-ad)}{d}\right) - d^3 \sinh\left(-\frac{2(bc-ad)}{d}\right)\right) \sqrt{c+dx}}{64b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(3/2)\*cosh(b\*x+a)^2,x, algorithm="fricas")



[Out]  $1/640*(15*\sqrt{2}*\sqrt{\pi}*(d^3*\cosh(b*x + a)^2*\cosh(-2*(b*c - a*d)/d) - d^3*\cosh(b*x + a)^2*\sinh(-2*(b*c - a*d)/d) + (d^3*\cosh(-2*(b*c - a*d)/d) - d^3*\sinh(-2*(b*c - a*d)/d))*\sinh(b*x + a)^2 + 2*(d^3*\cosh(b*x + a)*\cosh(-2*(b*c - a*d)/d) - d^3*\cosh(b*x + a)*\sinh(-2*(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{b/d}*\operatorname{erf}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/d}) - 15*\sqrt{2}*\sqrt{\pi}*(d^3*\cosh(b*x + a)^2*\cosh(-2*(b*c - a*d)/d) + d^3*\cosh(b*x + a)^2*\sinh(-2*(b*c - a*d)/d) + (d^3*\cosh(-2*(b*c - a*d)/d) + d^3*\sinh(-2*(b*c - a*d)/d))*\sinh(b*x + a)^2 + 2*(d^3*\cosh(b*x + a)*\cosh(-2*(b*c - a*d)/d) + d^3*\cosh(b*x + a)*\sinh(-2*(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{-b/d}*\operatorname{erf}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{-b/d}) - 4*(20*b^2*d^2*x - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2))*\cosh(b*x + a)^4 - 20*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2))*\cosh(b*x + a)*\sinh(b*x + a)^3 - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2))*\sinh(b*x + a)^4 + 20*b^2*c*d + 15*b*d^2 - 32*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2))*\cosh(b*x + a)^2 - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 + 15*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2))*\cosh(b*x + a)^2*\sinh(b*x + a)^2 - 4*(5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2))*\cosh(b*x + a)^3 + 16*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2))*\cosh(b*x + a))*\sinh(b*x + a))*\sqrt{d*x + c})/(b^3*d*\cosh(b*x + a)^2 + 2*b^3*d*\cosh(b*x + a)*\sinh(b*x + a) + b^3*d*\sinh(b*x + a)^2)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^{\frac{3}{2}} \cosh^2(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(3/2)*cosh(b*x+a)^2,x, algorithm="giac")`

[Out] `integrate((d*x + c)^(3/2)*cosh(b*x + a)^2, x)`

**maple** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int (dx + c)^{\frac{3}{2}} (\cosh^2(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^(3/2)*cosh(b*x+a)^2,x)`

[Out] `int((d*x+c)^(3/2)*cosh(b*x+a)^2,x)`

**maxima** [A] time = 0.59, size = 239, normalized size = 1.13

$$128(dx + c)^{\frac{5}{2}} + \frac{15\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{-b}{d}}\right)e^{\left(2a-\frac{2bc}{d}\right)}}{b^2\sqrt{\frac{-b}{d}}} + \frac{15\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2a+\frac{2bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} - \frac{20\left(4(dx+c)^{\frac{3}{2}}bde^{\left(\frac{2bc}{d}\right)}+3\sqrt{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(3/2)\*cosh(b\*x+a)^2,x, algorithm="maxima")

[Out] 1/640\*(128\*(d\*x + c)^(5/2) + 15\*sqrt(2)\*sqrt(pi)\*d^2\*erf(sqrt(2)\*sqrt(d\*x + c)\*sqrt(-b/d))\*e^(2\*a - 2\*b\*c/d)/(b^2\*sqrt(-b/d)) + 15\*sqrt(2)\*sqrt(pi)\*d^2\*erf(sqrt(2)\*sqrt(d\*x + c)\*sqrt(b/d))\*e^(-2\*a + 2\*b\*c/d)/(b^2\*sqrt(b/d)) - 20\*(4\*(d\*x + c)^(3/2)\*b\*d\*e^(2\*b\*c/d) + 3\*sqrt(d\*x + c)\*d^2\*e^(2\*b\*c/d))\*e^(-2\*a - 2\*(d\*x + c)\*b/d)/b^2 + 20\*(4\*(d\*x + c)^(3/2)\*b\*d\*e^(2\*a) - 3\*sqrt(d\*x + c)\*d^2\*e^(2\*a))\*e^(2\*(d\*x + c)\*b/d - 2\*b\*c/d)/b^2)/d

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh(a + bx)^2 (c + dx)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^2\*(c + d\*x)^(3/2), x)

[Out] int(cosh(a + b\*x)^2\*(c + d\*x)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^{3/2} \cosh^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*(3/2)\*cosh(b\*x+a)\*\*2,x)

[Out] Integral((c + d\*x)\*\*(3/2)\*cosh(a + b\*x)\*\*2, x)

### 3.50 $\int \sqrt{c + dx} \cosh^2(a + bx) dx$

**Optimal.** Leaf size=166

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} e^{\frac{2bc}{d}-2a} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} e^{2a-\frac{2bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} + \frac{\sqrt{c+dx} \sinh(2a+2bx)}{4b} + \frac{(c+dx)^{3/2}}{3d}$$

[Out]  $1/3*(d*x+c)^{(3/2)}/d+1/32*\exp(-2*a+2*b*c/d)*\operatorname{erf}(2^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*d^{(1/2)}*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(3/2)}-1/32*\exp(2*a-2*b*c/d)*\operatorname{erfi}(2^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*d^{(1/2)}*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(3/2)}+1/4*\sinh(2*b*x+2*a)*(d*x+c)^{(1/2)}/b$

**Rubi [A]** time = 0.27, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3312, 3296, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{d} e^{\frac{2bc}{d}-2a} \operatorname{Erf}\left(\frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{d} e^{2a-\frac{2bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} + \frac{\sqrt{c+dx} \sinh(2a+2bx)}{4b} + \frac{(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[c + d*x]*\operatorname{Cosh}[a + b*x]^2, x]$

[Out]  $(c + d*x)^{(3/2)}/(3*d) + (\operatorname{Sqrt}[d]*\operatorname{E}^{(-2*a + (2*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(16*b^{(3/2)}) - (\operatorname{Sqrt}[d]*\operatorname{E}^{(2*a - (2*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(16*b^{(3/2)}) + (\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[2*a + 2*b*x])/(4*b)$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \&\& !\$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \&\& \operatorname{PosQ}[b]$

#### Rule 2205

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]])/(2*d*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]), x] /;$   $\operatorname{Fr}$

eeQ[{F, a, b, c, d}, x] && NegQ[b]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[  
((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[  
e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3308

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I  
/2, Int[(c + d\*x)^m/E^(I\*(e + f\*x)), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(  
I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] := In  
t[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f  
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rubi steps

$$\begin{aligned}
 \int \sqrt{c+dx} \cosh^2(a+bx) dx &= \int \left( \frac{1}{2} \sqrt{c+dx} + \frac{1}{2} \sqrt{c+dx} \cosh(2a+2bx) \right) dx \\
 &= \frac{(c+dx)^{3/2}}{3d} + \frac{1}{2} \int \sqrt{c+dx} \cosh(2a+2bx) dx \\
 &= \frac{(c+dx)^{3/2}}{3d} + \frac{\sqrt{c+dx} \sinh(2a+2bx)}{4b} - \frac{d \int \frac{\sinh(2a+2bx)}{\sqrt{c+dx}} dx}{8b} \\
 &= \frac{(c+dx)^{3/2}}{3d} + \frac{\sqrt{c+dx} \sinh(2a+2bx)}{4b} - \frac{d \int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}} dx}{16b} + \frac{d \int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}} dx}{16b} \\
 &= \frac{(c+dx)^{3/2}}{3d} + \frac{\sqrt{c+dx} \sinh(2a+2bx)}{4b} + \frac{\text{Subst} \left( \int e^{i \left( 2ia - \frac{2ibc}{d} \right) - \frac{2bx^2}{d}} dx, x, \sqrt{c+dx} \right)}{8b} \\
 &= \frac{(c+dx)^{3/2}}{3d} + \frac{\sqrt{d} e^{-2a + \frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erf} \left( \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{16b^{3/2}} - \frac{\sqrt{d} e^{2a - \frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erfi} \left( \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{16b^{3/2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.45, size = 129, normalized size = 0.78

$$\frac{1}{48} \sqrt{c+dx} \left( \frac{3\sqrt{2} e^{2a-\frac{2bc}{d}} \Gamma\left(\frac{3}{2}, -\frac{2b(c+dx)}{d}\right)}{b\sqrt{-\frac{b(c+dx)}{d}}} - \frac{3\sqrt{2} e^{\frac{2bc}{d}-2a} \Gamma\left(\frac{3}{2}, \frac{2b(c+dx)}{d}\right)}{b\sqrt{\frac{b(c+dx)}{d}}} + \frac{16(c+dx)}{d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d\*x]\*Cosh[a + b\*x]^2, x]

[Out] (Sqrt[c + d\*x]\*((16\*(c + d\*x))/d + (3\*Sqrt[2]\*E^(2\*a - (2\*b\*c)/d)\*Gamma[3/2, (-2\*b\*(c + d\*x))/d])/(b\*Sqrt[-((b\*(c + d\*x))/d)]) - (3\*Sqrt[2]\*E^(-2\*a + (2\*b\*c)/d)\*Gamma[3/2, (2\*b\*(c + d\*x))/d])/(b\*Sqrt[(b\*(c + d\*x))/d])))/48

**fricas [B]** time = 0.54, size = 590, normalized size = 3.55

$$3\sqrt{2}\sqrt{\pi} \left( d^2 \cosh(bx+a)^2 \cosh\left(-\frac{2(bc-ad)}{d}\right) - d^2 \cosh(bx+a)^2 \sinh\left(-\frac{2(bc-ad)}{d}\right) + \left( d^2 \cosh\left(-\frac{2(bc-ad)}{d}\right) - d^2 \sinh\left(-\frac{2(bc-ad)}{d}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2\*(d\*x+c)^(1/2), x, algorithm="fricas")

[Out] 1/96\*(3\*sqrt(2)\*sqrt(pi)\*(d^2\*cosh(b\*x + a)^2\*cosh(-2\*(b\*c - a\*d)/d) - d^2\*cosh(b\*x + a)^2\*sinh(-2\*(b\*c - a\*d)/d) + (d^2\*cosh(-2\*(b\*c - a\*d)/d) - d^2\*sinh(-2\*(b\*c - a\*d)/d))\*sinh(b\*x + a)^2 + 2\*(d^2\*cosh(b\*x + a)\*cosh(-2\*(b\*c - a\*d)/d) - d^2\*cosh(b\*x + a)\*sinh(-2\*(b\*c - a\*d)/d))\*sinh(b\*x + a))\*sqrt(b/d)\*erf(sqrt(2)\*sqrt(d\*x + c)\*sqrt(b/d)) + 3\*sqrt(2)\*sqrt(pi)\*(d^2\*cosh(b\*x + a)^2\*cosh(-2\*(b\*c - a\*d)/d) + d^2\*cosh(b\*x + a)^2\*sinh(-2\*(b\*c - a\*d)/d) + (d^2\*cosh(-2\*(b\*c - a\*d)/d) + d^2\*sinh(-2\*(b\*c - a\*d)/d))\*sinh(b\*x + a)^2 + 2\*(d^2\*cosh(b\*x + a)\*cosh(-2\*(b\*c - a\*d)/d) + d^2\*cosh(b\*x + a)\*sinh(-2\*(b\*c - a\*d)/d))\*sinh(b\*x + a))\*sqrt(-b/d)\*erf(sqrt(2)\*sqrt(d\*x + c)\*sqrt(-b/d)) + 4\*(3\*b\*d\*cosh(b\*x + a)^4 + 12\*b\*d\*cosh(b\*x + a)\*sinh(b\*x + a)^3 + 3\*b\*d\*sinh(b\*x + a)^4 + 8\*(b^2\*d\*x + b^2\*c)\*cosh(b\*x + a)^2 + 2\*(4\*b^2\*d\*x + 9\*b\*d\*cosh(b\*x + a)^2 + 4\*b^2\*c)\*sinh(b\*x + a)^2 - 3\*b\*d + 4\*(3\*b\*d\*cosh(b\*x + a)^3 + 4\*(b^2\*d\*x + b^2\*c)\*cosh(b\*x + a))\*sinh(b\*x + a))\*sqrt(d\*x + c))/(b^2\*d\*cosh(b\*x + a)^2 + 2\*b^2\*d\*cosh(b\*x + a)\*sinh(b\*x + a) + b^2\*d\*sinh(b\*x + a)^2)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx+c} \cosh(bx+a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2\*(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(d\*x + c)\*cosh(b\*x + a)^2, x)

**maple** [F] time = 0.28, size = 0, normalized size = 0.00

$$\int (\cosh^2(bx + a)) \sqrt{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)^2\*(d\*x+c)^(1/2),x)

[Out] int(cosh(b\*x+a)^2\*(d\*x+c)^(1/2),x)

**maxima** [A] time = 0.42, size = 189, normalized size = 1.14

$$\frac{3\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{2a-\frac{2bc}{d}}}{b\sqrt{-\frac{b}{d}}} - \frac{3\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{(-2a+\frac{2bc}{d})}}{b\sqrt{\frac{b}{d}}} - 32(dx+c)^{\frac{3}{2}} - \frac{12\sqrt{dx+c}de^{2a+\frac{2(dx+c)b}{d}-\frac{2bc}{d}}}{b} + \dots$$

96 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2\*(d\*x+c)^(1/2),x, algorithm="maxima")

[Out]  $-1/96*(3*\sqrt{2}*\sqrt{\pi}*d*\operatorname{erf}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{-b/d})*e^{(2*a - 2*b*c/d)/(b*\sqrt{-b/d})} - 3*\sqrt{2}*\sqrt{\pi}*d*\operatorname{erf}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/d})*e^{(-2*a + 2*b*c/d)/(b*\sqrt{b/d})} - 32*(d*x + c)^{(3/2)} - 12*\sqrt{d*x + c}*d*e^{(2*a + 2*(d*x + c)*b/d - 2*b*c/d)/b} + 12*\sqrt{d*x + c}*d*e^{(-2*a - 2*(d*x + c)*b/d + 2*b*c/d)/b})/d$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a + bx)^2 \sqrt{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^2\*(c + d\*x)^(1/2),x)

[Out] int(cosh(a + b\*x)^2\*(c + d\*x)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx} \cosh^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*\*2\*(d\*x+c)\*\*(1/2),x)

[Out] Integral(sqrt(c + d\*x)\*cosh(a + b\*x)\*\*2, x)

$$3.51 \quad \int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=138

$$\frac{\sqrt{\frac{\pi}{2}} e^{\frac{2bc}{d}-2a} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}} + \frac{\sqrt{\frac{\pi}{2}} e^{2a-\frac{2bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}} + \frac{\sqrt{c+dx}}{d}$$

[Out]  $1/8*\exp(-2*a+2*b*c/d)*\operatorname{erf}(2^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*2^{(1/2)}*\pi^{(1/2)}/b^{(1/2)}/d^{(1/2)}+1/8*\exp(2*a-2*b*c/d)*\operatorname{erfi}(2^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*2^{(1/2)}*\pi^{(1/2)}/b^{(1/2)}/d^{(1/2)}+(d*x+c)^{(1/2)}/d$

**Rubi [A]** time = 0.21, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3312, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}} e^{\frac{2bc}{d}-2a} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}} + \frac{\sqrt{\frac{\pi}{2}} e^{2a-\frac{2bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}} + \frac{\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b\*x]^2/Sqrt[c + d\*x], x]

[Out]  $\operatorname{Sqrt}[c + d*x]/d + (E^{(-2*a + (2*b*c)/d)}*\operatorname{Sqrt}[\pi/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(4*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d]) + (E^{(2*a - (2*b*c)/d)}*\operatorname{Sqrt}[\pi/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(4*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d])$

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; Fr

eeQ[{F, a, b, c, d}, x] && NegQ[b]

### Rule 3307

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I/2, Int[(c + d\*x)^m/(E^(I\*k\*Pi)\*E^(I\*(e + f\*x))), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*k\*Pi)\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^2(a + bx)}{\sqrt{c + dx}} dx &= \int \left( \frac{1}{2\sqrt{c + dx}} + \frac{\cosh(2a + 2bx)}{2\sqrt{c + dx}} \right) dx \\
 &= \frac{\sqrt{c + dx}}{d} + \frac{1}{2} \int \frac{\cosh(2a + 2bx)}{\sqrt{c + dx}} dx \\
 &= \frac{\sqrt{c + dx}}{d} + \frac{1}{4} \int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c + dx}} dx + \frac{1}{4} \int \frac{e^{i(2ia+2ibx)}}{\sqrt{c + dx}} dx \\
 &= \frac{\sqrt{c + dx}}{d} + \frac{\text{Subst} \left( \int e^{i \left( 2ia - \frac{2ibc}{d} \right) - \frac{2bx^2}{d}} dx, x, \sqrt{c + dx} \right)}{2d} + \frac{\text{Subst} \left( \int e^{-i \left( 2ia - \frac{2ibc}{d} \right) + \frac{2bx^2}{d}} dx, x, \sqrt{c + dx} \right)}{2d} \\
 &= \frac{\sqrt{c + dx}}{d} + \frac{e^{-2a + \frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erf} \left( \frac{\sqrt{2} \sqrt{b} \sqrt{c + dx}}{\sqrt{d}} \right)}{4\sqrt{b} \sqrt{d}} + \frac{e^{2a - \frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erfi} \left( \frac{\sqrt{2} \sqrt{b} \sqrt{c + dx}}{\sqrt{d}} \right)}{4\sqrt{b} \sqrt{d}}
 \end{aligned}$$

**Mathematica [A]** time = 0.13, size = 141, normalized size = 1.02

$$\frac{e^{2a - \frac{2bc}{d}} \sqrt{-\frac{b(c+dx)}{d}} \Gamma \left( \frac{1}{2}, -\frac{2b(c+dx)}{d} \right)}{4\sqrt{2} b \sqrt{c + dx}} - \frac{e^{\frac{2bc}{d} - 2a} \sqrt{\frac{b(c+dx)}{d}} \Gamma \left( \frac{1}{2}, \frac{2b(c+dx)}{d} \right)}{4\sqrt{2} b \sqrt{c + dx}} + \frac{\sqrt{c + dx}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]^2/Sqrt[c + d\*x], x]



[Out]  $\text{Sqrt}[c + d*x]/d + (E^{(2*a - (2*b*c)/d)}*\text{Sqrt}[-((b*(c + d*x))/d)]*\text{Gamma}[1/2, (-2*b*(c + d*x))/d])/(4*\text{Sqrt}[2]*b*\text{Sqrt}[c + d*x]) - (E^{(-2*a + (2*b*c)/d)}*\text{Sqrt}[(b*(c + d*x))/d]*\text{Gamma}[1/2, (2*b*(c + d*x))/d])/(4*\text{Sqrt}[2]*b*\text{Sqrt}[c + d*x])$

**fricas** [A] time = 0.51, size = 155, normalized size = 1.12

$$\frac{\sqrt{2} \sqrt{\pi} \left( d \cosh\left(-\frac{2(bc-ad)}{d}\right) - d \sinh\left(-\frac{2(bc-ad)}{d}\right) \right) \sqrt{\frac{b}{d}} \operatorname{erf}\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{d}}\right) - \sqrt{2} \sqrt{\pi} \left( d \cosh\left(-\frac{2(bc-ad)}{d}\right) + d \sinh\left(-\frac{2(bc-ad)}{d}\right) \right) \sqrt{\frac{b}{d}} \operatorname{erf}\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{d}}\right)}{8bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")`

[Out]  $1/8*(\text{sqrt}(2)*\text{sqrt}(\pi)*(d*\cosh(-2*(b*c - a*d)/d) - d*\sinh(-2*(b*c - a*d)/d))*\text{sqrt}(b/d)*\text{erf}(\text{sqrt}(2)*\text{sqrt}(d*x + c)*\text{sqrt}(b/d)) - \text{sqrt}(2)*\text{sqrt}(\pi)*(d*\cosh(-2*(b*c - a*d)/d) + d*\sinh(-2*(b*c - a*d)/d))*\text{sqrt}(-b/d)*\text{erf}(\text{sqrt}(2)*\text{sqrt}(d*x + c)*\text{sqrt}(-b/d)) + 8*\text{sqrt}(d*x + c)*b/(b*d)$

**giac** [A] time = 0.17, size = 115, normalized size = 0.83

$$\frac{\left( \frac{\sqrt{2} \sqrt{\pi} d \operatorname{erf}\left(-\frac{\sqrt{2} \sqrt{bd} \sqrt{dx+c}}{d}\right) e^{\left(\frac{2bc}{d}\right)}}{\sqrt{bd}} + \frac{\sqrt{2} \sqrt{\pi} d \operatorname{erf}\left(-\frac{\sqrt{2} \sqrt{-bd} \sqrt{dx+c}}{d}\right) e^{\left(-\frac{2(bc-2ad)}{d}\right)}}{\sqrt{-bd}} - 8 \sqrt{dx+c} e^{(2a)} \right) e^{(-2a)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")`

[Out]  $-1/8*(\text{sqrt}(2)*\text{sqrt}(\pi)*d*\text{erf}(-\text{sqrt}(2)*\text{sqrt}(b*d)*\text{sqrt}(d*x + c)/d)*e^{(2*b*c/d)}/\text{sqrt}(b*d) + \text{sqrt}(2)*\text{sqrt}(\pi)*d*\text{erf}(-\text{sqrt}(2)*\text{sqrt}(-b*d)*\text{sqrt}(d*x + c)/d)*e^{(-2*(b*c - 2*a*d)/d)}/\text{sqrt}(-b*d) - 8*\text{sqrt}(d*x + c)*e^{(2*a)}*e^{(-2*a)}/d$

**maple** [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(bx+a)}{\sqrt{dx+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(b*x+a)^2/(d*x+c)^(1/2),x)`

[Out] `int(cosh(b*x+a)^2/(d*x+c)^(1/2),x)`

**maxima** [A] time = 0.50, size = 107, normalized size = 0.78

$$\frac{\sqrt{2} \sqrt{\pi} \operatorname{erf}\left(\sqrt{2} \sqrt{dx+c} \sqrt{-\frac{b}{d}}\right) e^{\left(2a-\frac{2bc}{d}\right)}}{\sqrt{-\frac{b}{d}}} + \frac{\sqrt{2} \sqrt{\pi} \operatorname{erf}\left(\sqrt{2} \sqrt{dx+c} \sqrt{\frac{b}{d}}\right) e^{\left(-2a+\frac{2bc}{d}\right)}}{\sqrt{\frac{b}{d}}} + 8 \sqrt{dx+c}$$


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$$8d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/8\*(sqrt(2)\*sqrt(pi)\*erf(sqrt(2)\*sqrt(d\*x + c)\*sqrt(-b/d))\*e^(2\*a - 2\*b\*c/d)/sqrt(-b/d) + sqrt(2)\*sqrt(pi)\*erf(sqrt(2)\*sqrt(d\*x + c)\*sqrt(b/d))\*e^(-2\*a + 2\*b\*c/d)/sqrt(b/d) + 8\*sqrt(d\*x + c))/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + bx)^2}{\sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^2/(c + d\*x)^(1/2),x)

[Out] int(cosh(a + b\*x)^2/(c + d\*x)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a + bx)}{\sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*\*2/(d\*x+c)\*\*(1/2),x)

[Out] Integral(cosh(a + b\*x)\*\*2/sqrt(c + d\*x), x)

$$3.52 \quad \int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=142

$$-\frac{\sqrt{\frac{\pi}{2}} \sqrt{b} e^{\frac{2bc}{d}-2a} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{b} e^{2a-\frac{2bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2 \cosh^2(a+bx)}{d\sqrt{c+dx}}$$

[Out]  $-1/2*\exp(-2*a+2*b*c/d)*\operatorname{erf}(2^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*b^{(1/2)}*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/d^{(3/2)}+1/2*\exp(2*a-2*b*c/d)*\operatorname{erfi}(2^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*b^{(1/2)}*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/d^{(3/2)}-2*\cosh(b*x+a)^2/d/(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.23, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3313, 12, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}} \sqrt{b} e^{\frac{2bc}{d}-2a} \operatorname{Erf}\left(\frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{b} e^{2a-\frac{2bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2 \cosh^2(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] `Int[Cosh[a + b*x]^2/(c + d*x)^(3/2), x]`

[Out]  $(-2*\operatorname{Cosh}[a + b*x]^2)/(d*\operatorname{Sqrt}[c + d*x]) - (\operatorname{Sqrt}[b]*E^{(-2*a + (2*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/d^{(3/2)} + (\operatorname{Sqrt}[b]*E^{(2*a - (2*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/d^{(3/2)})$

### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

### Rule 2180

`Int[(F_)^((g_)*((e_) + (f_)*(x_)))/Sqrt[(c_) + (d_)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True`

### Rule 2204

`Int[(F_)^((a_) + (b_)*((c_) + (d_)*(x_))^2), x_Symbol] := Simp[(F^a*Sqrt[t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]]]/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{`

F, a, b, c, d}, x] && PosQ[b]

### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

### Rule 3308

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I/2, Int[(c + d\*x)^m/E^(I\*(e + f\*x)), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

### Rule 3313

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)]<sup>(n\_)</sup>, x\_Symbol] := Simp[((c + d\*x)<sup>(m + 1)</sup>\*Sin[e + f\*x]<sup>n</sup>/(d\*(m + 1)), x] - Dist[(f\*n)/(d\*(m + 1)), Int[ExpandTrigReduce[(c + d\*x)<sup>(m + 1)</sup>, Cos[e + f\*x]\*Sin[e + f\*x]<sup>(n - 1)</sup>, x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^2(a + bx)}{(c + dx)^{3/2}} dx &= -\frac{2 \cosh^2(a + bx)}{d\sqrt{c + dx}} + \frac{(4ib) \int -\frac{i \sinh(2a + 2bx)}{2\sqrt{c + dx}} dx}{d} \\
 &= -\frac{2 \cosh^2(a + bx)}{d\sqrt{c + dx}} + \frac{(2b) \int \frac{\sinh(2a + 2bx)}{\sqrt{c + dx}} dx}{d} \\
 &= -\frac{2 \cosh^2(a + bx)}{d\sqrt{c + dx}} + \frac{b \int \frac{e^{-i(2ia + 2ibx)}}{\sqrt{c + dx}} dx}{d} - \frac{b \int \frac{e^{i(2ia + 2ibx)}}{\sqrt{c + dx}} dx}{d} \\
 &= -\frac{2 \cosh^2(a + bx)}{d\sqrt{c + dx}} - \frac{(2b) \text{Subst} \left( \int e^{i \left( 2ia - \frac{2ibc}{d} \right) - \frac{2bx^2}{d}} dx, x, \sqrt{c + dx} \right)}{d^2} + \frac{(2b) \text{Subst} \left( \int e^{-i \left( 2ia - \frac{2ibc}{d} \right) - \frac{2bx^2}{d}} dx, x, \sqrt{c + dx} \right)}{d^2} \\
 &= -\frac{2 \cosh^2(a + bx)}{d\sqrt{c + dx}} - \frac{\sqrt{b} e^{-2a + \frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erf} \left( \frac{\sqrt{2} \sqrt{b} \sqrt{c + dx}}{\sqrt{d}} \right)}{d^{3/2}} + \frac{\sqrt{b} e^{2a - \frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erfi} \left( \frac{\sqrt{2} \sqrt{b} \sqrt{c + dx}}{\sqrt{d}} \right)}{d^{3/2}}
 \end{aligned}$$

**Mathematica [B]** time = 2.97, size = 570, normalized size = 4.01

$$e^{-\frac{2b(c+dx)}{d}} \left( -\sqrt{2\pi} \sqrt{b} \cosh(2a) \sqrt{c+dx} e^{\frac{2b(c+dx)}{d}} \sinh\left(\frac{2bc}{d}\right) \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right) - \sqrt{2\pi} \sqrt{b} \cosh(2a) \sqrt{c+dx} e^{\frac{2b(c+dx)}{d}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]^2/(c + d\*x)^(3/2), x]

[Out]  $(-2\sqrt{d} E^{\frac{2b(c+dx)}{d}} - \sqrt{d} \cosh[2a] \cosh\left(\frac{2bc}{d}\right) - \sqrt{d} E^{\frac{4b(c+dx)}{d}} \cosh[2a] \cosh\left(\frac{2bc}{d}\right) + \sqrt{d} \cosh\left(\frac{2bc}{d}\right) \sinh[2a] - \sqrt{d} E^{\frac{4b(c+dx)}{d}} \cosh\left(\frac{2bc}{d}\right) \sinh[2a] + \sqrt{2} \sqrt{d} E^{\frac{2b(c+dx)}{d}} \sqrt{-\frac{b(c+dx)}{d}} \Gamma\left[\frac{1}{2}, -\frac{2b(c+dx)}{d} \left(\cosh[2a] - \frac{2bc}{d} + \cosh\left(\frac{2bc}{d}\right) \sinh[2a]\right) - \sqrt{d} \cosh[2a] \sinh\left(\frac{2bc}{d}\right) + \sqrt{d} E^{\frac{4b(c+dx)}{d}} \cosh[2a] \sinh\left(\frac{2bc}{d}\right) - \sqrt{b} E^{\frac{2b(c+dx)}{d}} \sqrt{2\pi} \sqrt{c+dx} \cosh[2a] \operatorname{Erf}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right] \sinh\left(\frac{2bc}{d}\right) - \sqrt{b} E^{\frac{2b(c+dx)}{d}} \sqrt{2\pi} \sqrt{c+dx} \cosh[2a] \operatorname{Erfi}\left[\frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right] \sinh\left(\frac{2bc}{d}\right) + \sqrt{d} \sinh[2a] \sinh\left(\frac{2bc}{d}\right) + \sqrt{2} \sqrt{d} E^{\frac{2b(c+dx)}{d}} \sqrt{\frac{b(c+dx)}{d}} \Gamma\left[\frac{1}{2}, \frac{2b(c+dx)}{d} \left(\cosh[2a] \cosh\left(\frac{2bc}{d}\right) - \sinh[2a] \left(\cosh\left(\frac{2bc}{d}\right) + \sinh\left(\frac{2bc}{d}\right)\right)\right) \right]) / (2d^{3/2} E^{\frac{2b(c+dx)}{d}} \sqrt{c+dx})$

**fricas [B]** time = 0.54, size = 569, normalized size = 4.01

$$\sqrt{2} \sqrt{\pi} \left( (dx + c) \cosh(bx + a)^2 \cosh\left(-\frac{2(bc-ad)}{d}\right) - (dx + c) \cosh(bx + a)^2 \sinh\left(-\frac{2(bc-ad)}{d}\right) + \left( (dx + c) \cosh\left(-\frac{2(bc-ad)}{d}\right) - (dx + c) \sinh\left(-\frac{2(bc-ad)}{d}\right) \right) \sqrt{b/d} \operatorname{erf}\left(\sqrt{2} \sqrt{d} \sqrt{b/d}\right) + \sqrt{2} \sqrt{\pi} \left( (dx + c) \cosh(bx + a)^2 \cosh\left(-\frac{2(bc-ad)}{d}\right) + (dx + c) \cosh(bx + a)^2 \sinh\left(-\frac{2(bc-ad)}{d}\right) + \left( (dx + c) \cosh\left(-\frac{2(bc-ad)}{d}\right) + (dx + c) \sinh\left(-\frac{2(bc-ad)}{d}\right) \right) \sqrt{-b/d} \operatorname{erf}\left(\sqrt{2} \sqrt{d} \sqrt{-b/d}\right) + (\cosh(bx + a))^4 + 4 \cosh(bx + a) \sinh(bx + a)^3 + \sinh(bx + a)^4 + 2 \cosh(bx + a) \sinh(bx + a)^2 + 2 \sinh(bx + a)^3 + \cosh(bx + a) \sinh(bx + a)^2 + \sinh(bx + a)^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^(3/2), x, algorithm="fricas")

[Out]  $-1/2 \sqrt{2} \sqrt{\pi} \left( (dx + c) \cosh(bx + a)^2 \cosh\left(-\frac{2(bc-ad)}{d}\right) - (dx + c) \cosh(bx + a)^2 \sinh\left(-\frac{2(bc-ad)}{d}\right) + \left( (dx + c) \cosh\left(-\frac{2(bc-ad)}{d}\right) - (dx + c) \sinh\left(-\frac{2(bc-ad)}{d}\right) \right) \sqrt{b/d} \operatorname{erf}\left(\sqrt{2} \sqrt{d} \sqrt{b/d}\right) + \sqrt{2} \sqrt{\pi} \left( (dx + c) \cosh(bx + a)^2 \cosh\left(-\frac{2(bc-ad)}{d}\right) + (dx + c) \cosh(bx + a)^2 \sinh\left(-\frac{2(bc-ad)}{d}\right) + \left( (dx + c) \cosh\left(-\frac{2(bc-ad)}{d}\right) + (dx + c) \sinh\left(-\frac{2(bc-ad)}{d}\right) \right) \sqrt{-b/d} \operatorname{erf}\left(\sqrt{2} \sqrt{d} \sqrt{-b/d}\right) + (\cosh(bx + a))^4 + 4 \cosh(bx + a) \sinh(bx + a)^3 + \sinh(bx + a)^4 + 2 \cosh(bx + a) \sinh(bx + a)^2 + 2 \sinh(bx + a)^3 + \cosh(bx + a) \sinh(bx + a)^2 + \sinh(bx + a)^3 \right)$

$(3*\cosh(b*x + a)^2 + 1)*\sinh(b*x + a)^2 + 2*\cosh(b*x + a)^2 + 4*(\cosh(b*x + a)^3 + \cosh(b*x + a))*\sinh(b*x + a) + 1)*\sqrt{d*x + c})/((d^2*x + c*d)*\cosh(b*x + a)^2 + 2*(d^2*x + c*d)*\cosh(b*x + a)*\sinh(b*x + a) + (d^2*x + c*d)*\sinh(b*x + a)^2)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(bx + a)}{(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(cosh(b\*x + a)^2/(d\*x + c)^(3/2), x)

**maple** [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(bx + a)}{(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)^2/(d\*x+c)^(3/2),x)

[Out] int(cosh(b\*x+a)^2/(d\*x+c)^(3/2),x)

**maxima** [A] time = 1.28, size = 116, normalized size = 0.82

$$\frac{\sqrt{2} \sqrt{\frac{(dx+c)b}{d}} e^{\left(\frac{2(bc-ad)}{d}\right)} \Gamma\left(-\frac{1}{2}, \frac{2(dx+c)b}{d}\right)}{\sqrt{dx+c}} + \frac{\sqrt{2} \sqrt{-\frac{(dx+c)b}{d}} e^{\left(-\frac{2(bc-ad)}{d}\right)} \Gamma\left(-\frac{1}{2}, -\frac{2(dx+c)b}{d}\right)}{\sqrt{dx+c}} + \frac{4}{\sqrt{dx+c}}$$

4d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^(3/2),x, algorithm="maxima")

[Out]  $-1/4*(\sqrt{2}*\sqrt{(d*x + c)*b/d})*e^{2*(b*c - a*d)/d}*\gamma(-1/2, 2*(d*x + c)*b/d)/\sqrt{d*x + c} + \sqrt{2}*\sqrt{-(d*x + c)*b/d}*e^{-2*(b*c - a*d)/d}*\gamma(-1/2, -2*(d*x + c)*b/d)/\sqrt{d*x + c} + 4/\sqrt{d*x + c})/d$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + bx)^2}{(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(a + b*x)^2/(c + d*x)^(3/2), x)`

[Out] `int(cosh(a + b*x)^2/(c + d*x)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a + bx)}{(c + dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)**2/(d*x+c)**(3/2), x)`

[Out] `Integral(cosh(a + b*x)**2/(c + d*x)**(3/2), x)`

$$3.53 \quad \int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}} dx$$

**Optimal.** Leaf size=174

$$\frac{2\sqrt{2\pi} b^{3/2} e^{\frac{2bc}{d}-2a} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2\sqrt{2\pi} b^{3/2} e^{2a-\frac{2bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{8b \sinh(a+bx) \cosh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2 \cosh^2(a+bx)}{3d(c+dx)}$$

[Out]  $-2/3*\cosh(b*x+a)^2/d/(d*x+c)^{(3/2)}+2/3*b^{(3/2)}*\exp(-2*a+2*b*c/d)*\operatorname{erf}(2^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/d^{(5/2)}+2/3*b^{(3/2)}*\exp(2*a-2*b*c/d)*\operatorname{erfi}(2^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/d^{(5/2)}-8/3*b*\cosh(b*x+a)*\sinh(b*x+a)/d^2/(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.31, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {3314, 32, 3312, 3307, 2180, 2204, 2205}

$$\frac{2\sqrt{2\pi} b^{3/2} e^{\frac{2bc}{d}-2a} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2\sqrt{2\pi} b^{3/2} e^{2a-\frac{2bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{8b \sinh(a+bx) \cosh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2 \cosh^2(a+bx)}{3d(c+dx)}$$

Antiderivative was successfully verified.

[In] `Int[Cosh[a + b*x]^2/(c + d*x)^(5/2), x]`

[Out]  $(-2*\operatorname{Cosh}[a + b*x]^2)/(3*d*(c + d*x)^{(3/2)}) + (2*b^{(3/2)}*E^{(-2*a + (2*b*c)/d)}*\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/(3*d^{(5/2)})) + (2*b^{(3/2)}*E^{(2*a - (2*b*c)/d)}*\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/(3*d^{(5/2)})) - (8*b*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/((3*d^2*\operatorname{Sqrt}[c + d*x]))$

### Rule 32

`Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]`

### Rule 2180

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True`

### Rule 2204

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2)), x_Symbol] := Simp[(F^a*Sqrt[t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{`



F, a, b, c, d}, x] && PosQ[b]

### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F<sup>a</sup>\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

### Rule 3307

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)</sup>\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I/2, Int[(c + d\*x)<sup>m</sup>/(E<sup>(I\*k\*Pi)</sup>\*E<sup>(I\*(e + f\*x))</sup>), x], x] - Dist[I/2, Int[(c + d\*x)<sup>m</sup>\*E<sup>(I\*k\*Pi)</sup>\*E<sup>(I\*(e + f\*x))</sup>, x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)]<sup>(n\_)</sup>, x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)<sup>m</sup>, Sin[e + f\*x]<sup>n</sup>, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rule 3314

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_)</sup>\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]<sup>(n\_)</sup>, x\_Symbol] := Simp[((c + d\*x)<sup>(m + 1)</sup>\*(b\*Ssin[e + f\*x]<sup>n</sup>)/(d\*(m + 1)), x] + (Dist[(b<sup>2</sup>\*f<sup>2</sup>\*n\*(n - 1))/(d<sup>2</sup>\*(m + 1)\*(m + 2)), Int[(c + d\*x)<sup>(m + 2)</sup>\*(b\*Ssin[e + f\*x]<sup>(n - 2)</sup>, x], x] - Dist[(f<sup>2</sup>\*n<sup>2</sup>)/(d<sup>2</sup>\*(m + 1)\*(m + 2)), Int[(c + d\*x)<sup>(m + 2)</sup>\*(b\*Ssin[e + f\*x]<sup>n</sup>, x], x] - Simp[(b\*f\*n\*(c + d\*x)<sup>(m + 2)</sup>\*Cos[e + f\*x]\*(b\*Ssin[e + f\*x]<sup>(n - 1)</sup>)/(d<sup>2</sup>\*(m + 1)\*(m + 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}} dx &= -\frac{2 \cosh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{3d^2 \sqrt{c+dx}} - \frac{(8b^2) \int \frac{1}{\sqrt{c+dx}} dx}{3d^2} + \frac{(16b^2) \int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}} dx}{3d^2} \\
&= -\frac{16b^2 \sqrt{c+dx}}{3d^3} - \frac{2 \cosh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{3d^2 \sqrt{c+dx}} + \frac{(16b^2) \int \left( \frac{1}{2\sqrt{c+dx}} + \frac{\cosh(2a+2bx)}{\sqrt{c+dx}} \right) dx}{3d^2} \\
&= -\frac{2 \cosh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{3d^2 \sqrt{c+dx}} + \frac{(8b^2) \int \frac{\cosh(2a+2bx)}{\sqrt{c+dx}} dx}{3d^2} \\
&= -\frac{2 \cosh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{3d^2 \sqrt{c+dx}} + \frac{(4b^2) \int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}} dx}{3d^2} + \frac{(4b^2) \int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}} dx}{3d^2} \\
&= -\frac{2 \cosh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{3d^2 \sqrt{c+dx}} + \frac{(8b^2) \text{Subst} \left( \int e^{i(2ia-\frac{2ibc}{d})-\frac{2bx^2}{d}} dx, x, \sqrt{c+dx} \right)}{3d^3} \\
&= -\frac{2 \cosh^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{2b^{3/2} e^{-2a+\frac{2bc}{d}} \sqrt{2\pi} \operatorname{erf} \left( \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{3d^{5/2}} + \frac{2b^{3/2} e^{2a-\frac{2bc}{d}} \sqrt{2\pi} \operatorname{erfi} \left( \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{3d^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 1.38, size = 156, normalized size = 0.90

$$\frac{2e^{-2\left(a+\frac{bc}{d}\right)} \left( \sqrt{2} e^{4a} d \left( -\frac{b(c+dx)}{d} \right)^{3/2} \Gamma \left( \frac{1}{2}, -\frac{2b(c+dx)}{d} \right) + e^{2\left(a+\frac{bc}{d}\right)} \left( 2b(c+dx) \sinh(2(a+bx)) + d \cosh^2(a+bx) \right) + \sqrt{2} \right)}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]^2/(c + d\*x)^(5/2), x]

[Out] (-2\*(Sqrt[2]\*d\*E^(4\*a)\*(-(b\*(c + d\*x))/d))^(3/2)\*Gamma[1/2, (-2\*b\*(c + d\*x))/d] + Sqrt[2]\*d\*E^((4\*b\*c)/d)\*((b\*(c + d\*x))/d)^(3/2)\*Gamma[1/2, (2\*b\*(c + d\*x))/d] + E^(2\*(a + (b\*c)/d))\*(d\*Cosh[a + b\*x]^2 + 2\*b\*(c + d\*x)\*Sinh[2\*(a + b\*x)])))/(3\*d^2\*E^(2\*(a + (b\*c)/d))\*(c + d\*x)^(3/2))

**fricas [B]** time = 0.55, size = 861, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^(5/2), x, algorithm="fricas")

```
[Out] 1/6*(4*sqrt(2)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-2*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 4*sqrt(2)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-2*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) - ((4*b*d*x + 4*b*c + d)*cosh(b*x + a)^4 + 4*(4*b*d*x + 4*b*c + d)*cosh(b*x + a)*sinh(b*x + a)^3 + (4*b*d*x + 4*b*c + d)*sinh(b*x + a)^4 - 4*b*d*x + 2*d*cosh(b*x + a)^2 + 2*(3*(4*b*d*x + 4*b*c + d)*cosh(b*x + a)^2 + d)*sinh(b*x + a)^2 - 4*b*c + 4*((4*b*d*x + 4*b*c + d)*cosh(b*x + a)^3 + d*cosh(b*x + a))*sinh(b*x + a) + d)*sqrt(d*x + c))/((d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)^2 + 2*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)*sinh(b*x + a) + (d^4*x^2 + 2*c*d^3*x + c^2*d^2)*sinh(b*x + a)^2)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(bx + a)}{(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2/(d*x + c)^(5/2), x)
```

**maple** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(bx + a)}{(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^(5/2),x)
```

```
[Out] int(cosh(b*x+a)^2/(d*x+c)^(5/2),x)
```

**maxima** [A] time = 0.41, size = 118, normalized size = 0.68

$$\frac{3\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}} + \frac{3\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}} + \frac{2}{(dx+c)^{\frac{3}{2}}}$$


---


$$6d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^(5/2),x, algorithm="maxima")

[Out] -1/6\*(3\*sqrt(2)\*((d\*x + c)\*b/d)^(3/2)\*e^(2\*(b\*c - a\*d)/d)\*gamma(-3/2, 2\*(d\*x + c)\*b/d)/(d\*x + c)^(3/2) + 3\*sqrt(2)\*(-(d\*x + c)\*b/d)^(3/2)\*e^(-2\*(b\*c - a\*d)/d)\*gamma(-3/2, -2\*(d\*x + c)\*b/d)/(d\*x + c)^(3/2) + 2/(d\*x + c)^(3/2)/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + bx)^2}{(c + dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^2/(c + d\*x)^(5/2),x)

[Out] int(cosh(a + b\*x)^2/(c + d\*x)^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a + bx)}{(c + dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*\*2/(d\*x+c)\*\*(5/2),x)

[Out] Integral(cosh(a + b\*x)\*\*2/(c + d\*x)\*\*(5/2), x)

$$3.54 \quad \int \frac{\cosh^2(a+bx)}{(c+dx)^{7/2}} dx$$

**Optimal.** Leaf size=220

$$\frac{8\sqrt{2\pi} b^{5/2} e^{\frac{2bc}{d}-2a} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8\sqrt{2\pi} b^{5/2} e^{2a-\frac{2bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{32b^2 \cosh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{8b \sinh(a+bx)}{15d^2(c+dx)}$$

[Out]  $-2/5*\cosh(b*x+a)^2/d/(d*x+c)^{(5/2)}-8/15*b*\cosh(b*x+a)*\sinh(b*x+a)/d^2/(d*x+c)^{(3/2)}-8/15*b^{(5/2)}*\exp(-2*a+2*b*c/d)*\operatorname{erf}(2^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*2^{(1/2)}*\pi^{(1/2)}/d^{(7/2)}+8/15*b^{(5/2)}*\exp(2*a-2*b*c/d)*\operatorname{erfi}(2^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*2^{(1/2)}*\pi^{(1/2)}/d^{(7/2)}+16/15*b^2/d^3/(d*x+c)^{(1/2)}-32/15*b^2*\cosh(b*x+a)^2/d^3/(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.31, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {3314, 32, 3313, 12, 3308, 2180, 2204, 2205}

$$\frac{8\sqrt{2\pi} b^{5/2} e^{\frac{2bc}{d}-2a} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8\sqrt{2\pi} b^{5/2} e^{2a-\frac{2bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{32b^2 \cosh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{8b \sinh(a+bx)}{15d^2(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b\*x]^2/(c + d\*x)^(7/2), x]

[Out]  $(16*b^2)/(15*d^3*\operatorname{Sqrt}[c + d*x]) - (2*\operatorname{Cosh}[a + b*x]^2)/(5*d*(c + d*x)^{(5/2)}) - (32*b^2*\operatorname{Cosh}[a + b*x]^2)/(15*d^3*\operatorname{Sqrt}[c + d*x]) - (8*b^{(5/2)}*E^{(-2*a + (2*b*c)/d)}*\operatorname{Sqrt}[2*\pi]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(15*d^{(7/2)}) + (8*b^{(5/2)}*E^{(2*a - (2*b*c)/d)}*\operatorname{Sqrt}[2*\pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(15*d^{(7/2)}) - (8*b*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/ (15*d^2*(c + d*x)^{(3/2)})$

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 32**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

**Rule 2180**

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

### Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

### Rule 3308

```
Int[((c_.) + (d_.)*(x_)^m)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

### Rule 3313

```
Int[((c_.) + (d_.)*(x_)^m)*sin[(e_.) + (f_.)*(x_)]^n, x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
LtQ[m, -1]
```

### Rule 3314

```
Int[((c_.) + (d_.)*(x_)^m)*((b_.)*sin[(e_.) + (f_.)*(x_)])^n, x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Ssin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Ssin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Ssin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*(b*Ssin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(a+bx)}{(c+dx)^{7/2}} dx &= -\frac{2 \cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{(8b^2) \int \frac{1}{(c+dx)^{3/2}} dx}{15d^2} + \frac{(16b^2) \int \frac{\cosh^2}{(c+dx)^{3/2}} dx}{15d^2} \\
&= \frac{16b^2}{15d^3 \sqrt{c+dx}} - \frac{2 \cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2 \cosh^2(a+bx)}{15d^3 \sqrt{c+dx}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{15d^2(c+dx)^{3/2}} \\
&= \frac{16b^2}{15d^3 \sqrt{c+dx}} - \frac{2 \cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2 \cosh^2(a+bx)}{15d^3 \sqrt{c+dx}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{15d^2(c+dx)^{3/2}} \\
&= \frac{16b^2}{15d^3 \sqrt{c+dx}} - \frac{2 \cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2 \cosh^2(a+bx)}{15d^3 \sqrt{c+dx}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{15d^2(c+dx)^{3/2}} \\
&= \frac{16b^2}{15d^3 \sqrt{c+dx}} - \frac{2 \cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2 \cosh^2(a+bx)}{15d^3 \sqrt{c+dx}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{15d^2(c+dx)^{3/2}} \\
&= \frac{16b^2}{15d^3 \sqrt{c+dx}} - \frac{2 \cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2 \cosh^2(a+bx)}{15d^3 \sqrt{c+dx}} - \frac{8b^5 e^{-2a+\frac{2bc}{d}} \sqrt{2\pi} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c}}{\sqrt{d}}\right)}{15d^{7/2}}
\end{aligned}$$

**Mathematica [B]** time = 3.17, size = 825, normalized size = 3.75

$$e^{-\frac{2b(c+dx)}{d}} \left( 16\sqrt{2} d^2 e^{\frac{2b(c+dx)}{d}} \Gamma\left(\frac{1}{2}, -\frac{2b(c+dx)}{d}\right) \left( \cosh\left(2a - \frac{2bc}{d}\right) + \sinh\left(2a - \frac{2bc}{d}\right) \right) \left(-\frac{b(c+dx)}{d}\right)^{5/2} - 6d^2 e^{\frac{2b(c+dx)}{d}} - 16b^2 c^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]^2/(c + d\*x)^(7/2), x]

[Out]  $(-6*d^2*E^{((2*b*(c + d*x))/d)} - 16*b^2*c^2*\operatorname{Cosh}[2*a - (2*b*c)/d] + 4*b*c*d*\operatorname{Cosh}[2*a - (2*b*c)/d] - 3*d^2*\operatorname{Cosh}[2*a - (2*b*c)/d] - 16*b^2*c^2*E^{((4*b*(c + d*x))/d)}*\operatorname{Cosh}[2*a - (2*b*c)/d] - 4*b*c*d*E^{((4*b*(c + d*x))/d)}*\operatorname{Cosh}[2*a - (2*b*c)/d] - 3*d^2*E^{((4*b*(c + d*x))/d)}*\operatorname{Cosh}[2*a - (2*b*c)/d] - 32*b^2*c*d*x*\operatorname{Cosh}[2*a - (2*b*c)/d] + 4*b*d^2*x*\operatorname{Cosh}[2*a - (2*b*c)/d] - 32*b^2*c*d*E^{((4*b*(c + d*x))/d)}*x*\operatorname{Cosh}[2*a - (2*b*c)/d] - 4*b*d^2*E^{((4*b*(c + d*x))/d)}*x*\operatorname{Cosh}[2*a - (2*b*c)/d] - 16*b^2*d^2*x^2*\operatorname{Cosh}[2*a - (2*b*c)/d] - 16*b^2*d^2*E^{((4*b*(c + d*x))/d)}*x^2*\operatorname{Cosh}[2*a - (2*b*c)/d] + 16*\operatorname{Sqrt}[2]*d^2*E^{((2*b*(c + d*x))/d)}*((b*(c + d*x))/d)^{(5/2)}*\operatorname{Gamma}[1/2, (2*b*(c + d*x))/d]*(\operatorname{Cosh}[2*a - (2*b*c)/d] - \operatorname{Sinh}[2*a - (2*b*c)/d]) + 16*b^2*c^2*\operatorname{Sinh}[2*a - (2*b*c)/d] - 4*b*c*d*\operatorname{Sinh}[2*a - (2*b*c)/d] + 3*d^2*\operatorname{Sinh}[2*a - (2*b*c)/d] - 16*b^2*c^2$

$$2 * E^{((4 * b * (c + d * x)) / d) * \operatorname{Sinh}[2 * a - (2 * b * c) / d]} - 4 * b * c * d * E^{((4 * b * (c + d * x)) / d) * \operatorname{Sinh}[2 * a - (2 * b * c) / d]} - 3 * d^2 * E^{((4 * b * (c + d * x)) / d) * \operatorname{Sinh}[2 * a - (2 * b * c) / d]} + 32 * b^2 * c * d * x * \operatorname{Sinh}[2 * a - (2 * b * c) / d] - 4 * b * d^2 * x * \operatorname{Sinh}[2 * a - (2 * b * c) / d] - 32 * b^2 * c * d * E^{((4 * b * (c + d * x)) / d) * x * \operatorname{Sinh}[2 * a - (2 * b * c) / d]} - 4 * b * d^2 * E^{((4 * b * (c + d * x)) / d) * x * \operatorname{Sinh}[2 * a - (2 * b * c) / d]} + 16 * b^2 * d^2 * x^2 * \operatorname{Sinh}[2 * a - (2 * b * c) / d] - 16 * b^2 * d^2 * E^{((4 * b * (c + d * x)) / d) * x^2 * \operatorname{Sinh}[2 * a - (2 * b * c) / d]} + 16 * \operatorname{Sqrt}[2] * d^2 * E^{((2 * b * (c + d * x)) / d) * (-((b * (c + d * x)) / d))^{5/2} * \operatorname{Gamma}[1/2, (-2 * b * (c + d * x)) / d] * (\operatorname{Cosh}[2 * a - (2 * b * c) / d] + \operatorname{Sinh}[2 * a - (2 * b * c) / d])} / (30 * d^3 * E^{((2 * b * (c + d * x)) / d) * (c + d * x)^{5/2}})$$

**fricas** [B] time = 0.63, size = 1350, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^(7/2),x, algorithm="fricas")

[Out] 
$$-1/30 * (16 * \operatorname{sqrt}(2) * \operatorname{sqrt}(\pi) * ((b^2 * d^3 * x^3 + 3 * b^2 * c * d^2 * x^2 + 3 * b^2 * c^2 * d * x + b^2 * c^3) * \operatorname{cosh}(b * x + a)^2 * \operatorname{cosh}(-2 * (b * c - a * d) / d) - (b^2 * d^3 * x^3 + 3 * b^2 * c * d^2 * x^2 + 3 * b^2 * c^2 * d * x + b^2 * c^3) * \operatorname{cosh}(b * x + a)^2 * \operatorname{sinh}(-2 * (b * c - a * d) / d) + ((b^2 * d^3 * x^3 + 3 * b^2 * c * d^2 * x^2 + 3 * b^2 * c^2 * d * x + b^2 * c^3) * \operatorname{cosh}(-2 * (b * c - a * d) / d) - (b^2 * d^3 * x^3 + 3 * b^2 * c * d^2 * x^2 + 3 * b^2 * c^2 * d * x + b^2 * c^3) * \operatorname{sinh}(-2 * (b * c - a * d) / d)) * \operatorname{sinh}(b * x + a)^2 + 2 * ((b^2 * d^3 * x^3 + 3 * b^2 * c * d^2 * x^2 + 3 * b^2 * c^2 * d * x + b^2 * c^3) * \operatorname{cosh}(b * x + a) * \operatorname{cosh}(-2 * (b * c - a * d) / d) - (b^2 * d^3 * x^3 + 3 * b^2 * c * d^2 * x^2 + 3 * b^2 * c^2 * d * x + b^2 * c^3) * \operatorname{cosh}(b * x + a) * \operatorname{sinh}(-2 * (b * c - a * d) / d)) * \operatorname{sinh}(b * x + a)) * \operatorname{sqrt}(b / d) * \operatorname{erf}(\operatorname{sqrt}(2) * \operatorname{sqrt}(d * x + c) * \operatorname{sqrt}(b / d)) + 16 * \operatorname{sqrt}(2) * \operatorname{sqrt}(\pi) * ((b^2 * d^3 * x^3 + 3 * b^2 * c * d^2 * x^2 + 3 * b^2 * c^2 * d * x + b^2 * c^3) * \operatorname{cosh}(b * x + a)^2 * \operatorname{cosh}(-2 * (b * c - a * d) / d) + (b^2 * d^3 * x^3 + 3 * b^2 * c * d^2 * x^2 + 3 * b^2 * c^2 * d * x + b^2 * c^3) * \operatorname{cosh}(b * x + a)^2 * \operatorname{sinh}(-2 * (b * c - a * d) / d) + ((b^2 * d^3 * x^3 + 3 * b^2 * c * d^2 * x^2 + 3 * b^2 * c^2 * d * x + b^2 * c^3) * \operatorname{cosh}(-2 * (b * c - a * d) / d) + (b^2 * d^3 * x^3 + 3 * b^2 * c * d^2 * x^2 + 3 * b^2 * c^2 * d * x + b^2 * c^3) * \operatorname{sinh}(-2 * (b * c - a * d) / d)) * \operatorname{sinh}(b * x + a)^2 + 2 * ((b^2 * d^3 * x^3 + 3 * b^2 * c * d^2 * x^2 + 3 * b^2 * c^2 * d * x + b^2 * c^3) * \operatorname{cosh}(b * x + a) * \operatorname{cosh}(-2 * (b * c - a * d) / d) + (b^2 * d^3 * x^3 + 3 * b^2 * c * d^2 * x^2 + 3 * b^2 * c^2 * d * x + b^2 * c^3) * \operatorname{cosh}(b * x + a) * \operatorname{sinh}(-2 * (b * c - a * d) / d)) * \operatorname{sinh}(b * x + a)) * \operatorname{sqrt}(-b / d) * \operatorname{erf}(\operatorname{sqrt}(2) * \operatorname{sqrt}(d * x + c) * \operatorname{sqrt}(-b / d)) + (16 * b^2 * d^2 * x^2 + (16 * b^2 * d^2 * x^2 + 16 * b^2 * c^2 + 4 * b * c * d + 3 * d^2 + 4 * (8 * b^2 * c * d + b * d^2) * x) * \operatorname{cosh}(b * x + a)^4 + 4 * (16 * b^2 * d^2 * x^2 + 16 * b^2 * c^2 + 4 * b * c * d + 3 * d^2 + 4 * (8 * b^2 * c * d + b * d^2) * x) * \operatorname{cosh}(b * x + a) * \operatorname{sinh}(b * x + a)^3 + (16 * b^2 * d^2 * x^2 + 16 * b^2 * c^2 + 4 * b * c * d + 3 * d^2 + 4 * (8 * b^2 * c * d + b * d^2) * x) * \operatorname{sinh}(b * x + a)^4 + 16 * b^2 * c^2 + 6 * d^2 * \operatorname{cosh}(b * x + a)^2 - 4 * b * c * d + 6 * ((16 * b^2 * d^2 * x^2 + 16 * b^2 * c^2 + 4 * b * c * d + 3 * d^2 + 4 * (8 * b^2 * c * d + b * d^2) * x) * \operatorname{cosh}(b * x + a)^2 + d^2) * \operatorname{sinh}(b * x + a)^2 + 3 * d^2 + 4 * (8 * b^2 * c * d - b * d^2) * x + 4 * ((16 * b^2 * d^2 * x^2 + 16 * b^2 * c^2 + 4 * b * c * d + 3 * d^2 + 4 * (8 * b^2 * c * d + b * d^2) * x) * \operatorname{cosh}(b * x + a)^3 + 3 * d^2 * \operatorname{cosh}(b * x + a)) * \operatorname{sinh}(b * x + a)) * \operatorname{sqrt}(d * x + c)) / ((d^6 * x^3 + 3 * c * d^5 * x^2 + 3 * c^2 * d^4 * x + c^3 * d^3) * \operatorname{cosh}(b * x + a)^2 + 2 * (d^6 * x^3 + 3 * c * d^5 * x^2 + 3 * c^2 * d^4 * x + c^3 * d^3) * \operatorname{sinh}(b * x + a))$$



$$3*d^3*\cosh(b*x + a)*\sinh(b*x + a) + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*\sinh(b*x + a)^2$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx + a)^2}{(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^(7/2),x, algorithm="giac")

[Out] integrate(cosh(b\*x + a)^2/(d\*x + c)^(7/2), x)

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(bx + a)}{(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)^2/(d\*x+c)^(7/2),x)

[Out] int(cosh(b\*x+a)^2/(d\*x+c)^(7/2),x)

maxima [A] time = 0.45, size = 116, normalized size = 0.53

$$\frac{5\sqrt{2}\left(\frac{dx+c}{d}\right)^{\frac{5}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}} + \frac{5\sqrt{2}\left(-\frac{dx+c}{d}\right)^{\frac{5}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}} + \frac{1}{(dx+c)^{\frac{5}{2}}}$$


---


$$5d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^(7/2),x, algorithm="maxima")

[Out] -1/5\*(5\*sqrt(2))\*((d\*x + c)\*b/d)^(5/2)\*e^(2\*(b\*c - a\*d)/d)\*gamma(-5/2, 2\*(d\*x + c)\*b/d)/(d\*x + c)^(5/2) + 5\*sqrt(2)\*(-(d\*x + c)\*b/d)^(5/2)\*e^(-2\*(b\*c - a\*d)/d)\*gamma(-5/2, -2\*(d\*x + c)\*b/d)/(d\*x + c)^(5/2) + 1/(d\*x + c)^(5/2)/d

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(a + bx)^2}{(c + dx)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(a + b*x)^2/(c + d*x)^(7/2), x)`

[Out] `int(cosh(a + b*x)^2/(c + d*x)^(7/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a + bx)}{(c + dx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(b*x+a)**2/(d*x+c)**(7/2), x)`

[Out] `Integral(cosh(a + b*x)**2/(c + d*x)**(7/2), x)`

$$3.55 \quad \int \frac{\cosh^2(a+bx)}{(c+dx)^{9/2}} dx$$

**Optimal.** Leaf size=251

$$\frac{32\sqrt{2\pi} b^{7/2} e^{\frac{2bc}{d}-2a} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} + \frac{32\sqrt{2\pi} b^{7/2} e^{2a-\frac{2bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{128b^3 \sinh(a+bx) \cosh(a+bx)}{105d^4 \sqrt{c+dx}} - \frac{32}{105d^4}$$

[Out]  $16/105*b^2/d^3/(d*x+c)^{(3/2)}-2/7*\cosh(b*x+a)^2/d/(d*x+c)^{(7/2)}-32/105*b^2*c$   
 $\cosh(b*x+a)^2/d^3/(d*x+c)^{(3/2)}-8/35*b*\cosh(b*x+a)*\sinh(b*x+a)/d^2/(d*x+c)^{($   
 $5/2)}+32/105*b^{(7/2)}*\exp(-2*a+2*b*c/d)*\operatorname{erf}(2^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{($   
 $1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/d^{(9/2)}+32/105*b^{(7/2)}*\exp(2*a-2*b*c/d)*\operatorname{erfi}(2^{(1/2)}$   
 $*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/d^{(9/2)}-128/105*b^3*\cosh(b$   
 $*x+a)*\sinh(b*x+a)/d^4/(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.38, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {3314, 32, 3312, 3307, 2180, 2204, 2205}

$$\frac{32\sqrt{2\pi} b^{7/2} e^{\frac{2bc}{d}-2a} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} + \frac{32\sqrt{2\pi} b^{7/2} e^{2a-\frac{2bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{32b^2 \cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{128b^3 \sinh(a+bx)}{105d^4}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cosh}[a + b*x]^2/(c + d*x)^{(9/2)}, x]$

[Out]  $(16*b^2)/(105*d^3*(c + d*x)^{(3/2)}) - (2*\operatorname{Cosh}[a + b*x]^2)/(7*d*(c + d*x)^{(7/2)}) - (32*b^2*\operatorname{Cosh}[a + b*x]^2)/(105*d^3*(c + d*x)^{(3/2)}) + (32*b^{(7/2)}*E^{(-2*a + (2*b*c)/d)*\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(105*d^{(9/2)}) + (32*b^{(7/2)}*E^{(2*a - (2*b*c)/d)*\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(105*d^{(9/2)}) - (8*b*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/(35*d^2*(c + d*x)^{(5/2)}) - (128*b^3*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/(105*d^4*\operatorname{Sqrt}[c + d*x])$

**Rule 32**

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m + 1)}/(b*(m + 1)), x] /; \operatorname{FreeQ}\{a, b, m\}, x \ \&\& \operatorname{NeQ}[m, -1]$

**Rule 2180**

$\operatorname{Int}[(F_.)^{((g_.)*((e_.) + (f_.)*(x_.)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_.)]}, x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]]]$

x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F<sup>a</sup>\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F<sup>a</sup>\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

#### Rule 3307

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)</sup>\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I/2, Int[(c + d\*x)<sup>m</sup>/(E<sup>(I\*k\*Pi)</sup>\*E<sup>(I\*(e + f\*x))</sup>), x], x] - Dist[I/2, Int[(c + d\*x)<sup>m</sup>\*E<sup>(I\*k\*Pi)</sup>\*E<sup>(I\*(e + f\*x))</sup>, x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

#### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)]<sup>(n\_)</sup>, x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)<sup>m</sup>, Sin[e + f\*x]<sup>n</sup>, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

#### Rule 3314

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_)</sup>\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]<sup>(n\_)</sup>), x\_Symbol] := Simp[((c + d\*x)<sup>(m + 1)</sup>\*(b\*Sin[e + f\*x])<sup>n</sup>/(d\*(m + 1)), x] + (Dist[(b<sup>2</sup>\*f<sup>2</sup>\*n\*(n - 1))/(d<sup>2</sup>\*(m + 1)\*(m + 2)), Int[(c + d\*x)<sup>(m + 2)</sup>\*(b\*Sin[e + f\*x])<sup>(n - 2)</sup>, x], x] - Dist[(f<sup>2</sup>\*n<sup>2</sup>)/(d<sup>2</sup>\*(m + 1)\*(m + 2)), Int[(c + d\*x)<sup>(m + 2)</sup>\*(b\*Sin[e + f\*x])<sup>n</sup>, x], x] - Simp[(b\*f\*n\*(c + d\*x)<sup>(m + 2)</sup>\*Cos[e + f\*x]\*(b\*Sin[e + f\*x])<sup>(n - 1)</sup>/(d<sup>2</sup>\*(m + 1)\*(m + 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

#### Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(a+bx)}{(c+dx)^{9/2}} dx &= -\frac{2 \cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{(8b^2) \int \frac{1}{(c+dx)^{5/2}} dx}{35d^2} + \frac{(16b^2) \int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}} dx}{35d^2} \\
&= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2 \cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2 \cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{35d^2(c+dx)^{5/2}} \\
&= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{256b^4 \sqrt{c+dx}}{105d^5} - \frac{2 \cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2 \cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{35d^2(c+dx)^{5/2}} \\
&= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2 \cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2 \cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{35d^2(c+dx)^{5/2}} \\
&= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2 \cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2 \cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{35d^2(c+dx)^{5/2}} \\
&= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2 \cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2 \cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{35d^2(c+dx)^{5/2}} \\
&= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2 \cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2 \cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{35d^2(c+dx)^{5/2}} \\
&= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2 \cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2 \cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{32b^{7/2} e^{-2a+\frac{2bc}{d}} \sqrt{2\pi} \operatorname{erf}\left(\frac{\sqrt{2}(c+dx)}{2d}\right)}{105d^{9/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.85, size = 222, normalized size = 0.88

$$2 \left( -32b^3(c+dx)^3 \sinh(2(a+bx)) + 16\sqrt{2} b^3(c+dx)^3 e^{2a-\frac{2bc}{d}} \sqrt{-\frac{b(c+dx)}{d}} \Gamma\left(\frac{1}{2}, -\frac{2b(c+dx)}{d}\right) - 16\sqrt{2} b^3(c+dx)^3 e^{\frac{2bc}{d}-2a} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]^2/(c + d\*x)^(9/2), x]

[Out] (2\*(8\*b^2\*d\*(c + d\*x)^2 - 15\*d^3\*Cosh[a + b\*x]^2 - 16\*b^2\*d\*(c + d\*x)^2\*Cos h[a + b\*x]^2 + 16\*Sqrt[2]\*b^3\*E^(2\*a - (2\*b\*c)/d)\*(c + d\*x)^3\*Sqrt[-((b\*(c + d\*x))/d)]\*Gamma[1/2, (-2\*b\*(c + d\*x))/d] - 16\*Sqrt[2]\*b^3\*E^(-2\*a + (2\*b\*c)/d)\*(c + d\*x)^3\*Sqrt[(b\*(c + d\*x))/d]\*Gamma[1/2, (2\*b\*(c + d\*x))/d] - 6\*b\*d^2\*(c + d\*x)\*Sinh[2\*(a + b\*x)] - 32\*b^3\*(c + d\*x)^3\*Sinh[2\*(a + b\*x)])/(105\*d^4\*(c + d\*x)^(7/2))

**fricas [B]** time = 0.72, size = 1825, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^(9/2),x, algorithm="fricas")

[Out]  $\frac{1}{210} \cdot (64 \cdot \sqrt{2}) \cdot \sqrt{\pi} \cdot ((b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) \cdot \cosh(b x + a)^2 \cdot \cosh(-2(b c - a d)/d) - (b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) \cdot \cosh(b x + a)^2 \cdot \sinh(-2(b c - a d)/d) + ((b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) \cdot \cosh(-2(b c - a d)/d) - (b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) \cdot \sinh(-2(b c - a d)/d)) \cdot \sinh(b x + a)^2 + 2 \cdot ((b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) \cdot \cosh(b x + a) \cdot \cosh(-2(b c - a d)/d) - (b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) \cdot \cosh(b x + a) \cdot \sinh(-2(b c - a d)/d)) \cdot \sinh(b x + a) \cdot \sqrt{b/d} \cdot \operatorname{erf}(\sqrt{2} \cdot \sqrt{d x + c} \cdot \sqrt{b/d}) - 64 \cdot \sqrt{2} \cdot \sqrt{\pi} \cdot ((b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) \cdot \cosh(b x + a)^2 \cdot \cosh(-2(b c - a d)/d) + (b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) \cdot \cosh(b x + a)^2 \cdot \sinh(-2(b c - a d)/d) + ((b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) \cdot \cosh(-2(b c - a d)/d) + (b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) \cdot \sinh(-2(b c - a d)/d)) \cdot \sinh(b x + a)^2 + 2 \cdot ((b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) \cdot \cosh(b x + a) \cdot \cosh(-2(b c - a d)/d) + (b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) \cdot \cosh(b x + a) \cdot \sinh(-2(b c - a d)/d)) \cdot \sinh(b x + a) \cdot \sqrt{-b/d} \cdot \operatorname{erf}(\sqrt{2} \cdot \sqrt{d x + c} \cdot \sqrt{-b/d}) + (64 b^3 d^3 x^3 + 64 b^3 c^3 - 16 b^2 c^2 d - 30 d^3 \cosh(b x + a)^2 - (64 b^3 d^3 x^3 + 64 b^3 c^3 + 16 b^2 c^2 d + 12 b c d^2 + 15 d^3 + 16(12 b^3 c d^2 + b^2 d^3) x^2 + 4(48 b^3 c^2 d + 8 b^2 c d^2 + 3 b d^3) x) \cdot \cosh(b x + a)^4 - 4(64 b^3 d^3 x^3 + 64 b^3 c^3 + 16 b^2 c^2 d + 12 b c d^2 + 15 d^3 + 16(12 b^3 c d^2 + b^2 d^3) x^2 + 4(48 b^3 c^2 d + 8 b^2 c d^2 + 3 b d^3) x) \cdot \cosh(b x + a) \cdot \sinh(b x + a)^3 - (64 b^3 d^3 x^3 + 64 b^3 c^3 + 16 b^2 c^2 d + 12 b c d^2 + 15 d^3 + 16(12 b^3 c d^2 + b^2 d^3) x^2 + 4(48 b^3 c^2 d + 8 b^2 c d^2 + 3 b d^3) x) \cdot \sinh(b x + a)^4 + 12 b c d^2 - 15 d^3 + 16(12 b^3 c d^2 - b^2 d^3) x^2 - 6(5 d^3 + (64 b^3 d^3 x^3 + 64 b^3 c^3 + 16 b^2 c^2 d + 12 b c d^2 + 15 d^3 + 16(12 b^3 c d^2 + b^2 d^3) x^2 + 4(48 b^3 c^2 d + 8 b^2 c d^2 + 3 b d^3) x) \cdot \cosh(b x + a)^2) \cdot \sinh(b x + a)^2 + 4(48 b^3 c^2 d - 8 b^2 c d^2 + 3 b d^3) x - 4(15 d^3 \cosh(b x + a) + (64 b^3 d^3 x^3 + 64 b^3 c^3 + 16 b^2 c^2 d + 12 b c d^2 + 15 d^3 + 16(12 b^3 c d^2 + b^2 d^3) x^2 + 4(48 b^3 c^2 d + 8 b^2 c d^2 + 3 b d^3) x) \cdot \cosh(b x + a)^3) \cdot \sinh(b x + a) \cdot \sqrt{d x + c}) / ((d^8 x^4 + 4 c d^7 x^3 + 6 c^2 d^6 x^2 + 4 c^3 d^5 x + c^4 d^4) \cdot \cosh(b x + a)^2 + 2(d^8 x^4 + 4 c d^7 x^3 + 6 c^2 d^6 x^2 + 4 c^3 d^5 x + c^4 d^4) \cdot \cosh(b x + a) \cdot \sinh(b x + a) + (d^8 x^4 + 4 c d^7 x^3 + 6 c^2 d^6 x^2 + 4 c^3 d^5 x + c^4 d^4) \cdot \sinh(b x + a)^2)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx + a)^2}{(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^(9/2),x, algorithm="giac")

[Out] integrate(cosh(b\*x + a)^2/(d\*x + c)^(9/2), x)

**maple** [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(bx + a)}{(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)^2/(d\*x+c)^(9/2),x)

[Out] int(cosh(b\*x+a)^2/(d\*x+c)^(9/2),x)

**maxima** [A] time = 1.04, size = 116, normalized size = 0.46

$$\frac{14\sqrt{2}\left(\frac{dx+c}{d}\right)^{\frac{7}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}} + \frac{14\sqrt{2}\left(-\frac{dx+c}{d}\right)^{\frac{7}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}} + \frac{1}{(dx+c)^{\frac{7}{2}}}$$


---


$$7d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^2/(d\*x+c)^(9/2),x, algorithm="maxima")

[Out] -1/7\*(14\*sqrt(2)\*((d\*x + c)\*b/d)^(7/2)\*e^(2\*(b\*c - a\*d)/d)\*gamma(-7/2, 2\*(d\*x + c)\*b/d)/(d\*x + c)^(7/2) + 14\*sqrt(2)\*(-(d\*x + c)\*b/d)^(7/2)\*e^(-2\*(b\*c - a\*d)/d)\*gamma(-7/2, -2\*(d\*x + c)\*b/d)/(d\*x + c)^(7/2) + 1/(d\*x + c)^(7/2))/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(a + bx)^2}{(c + dx)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^2/(c + d\*x)^(9/2),x)

```
[Out] int(cosh(a + b*x)^2/(c + d*x)^(9/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**(9/2), x)
```

```
[Out] Timed out
```



### 3.56 $\int (c + dx)^{5/2} \cosh^3(a + bx) dx$

**Optimal.** Leaf size=381

$$\frac{45\sqrt{\pi} d^{5/2} e^{\frac{bc}{d}-a} \operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}} + \frac{5\sqrt{\frac{\pi}{3}} d^{5/2} e^{\frac{3bc}{d}-3a} \operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} - \frac{45\sqrt{\pi} d^{5/2} e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}} - \frac{5\sqrt{\frac{\pi}{3}} d^{5/2} e^{3a}}{576b^{7/2}}$$

[Out]  $-5/3*d*(d*x+c)^{(3/2)}*\cosh(b*x+a)/b^2-5/18*d*(d*x+c)^{(3/2)}*\cosh(b*x+a)^3/b^2+2/3*(d*x+c)^{(5/2)}*\sinh(b*x+a)/b+1/3*(d*x+c)^{(5/2)}*\cosh(b*x+a)^2*\sinh(b*x+a)/b+5/1728*d^{(5/2)}*\exp(-3*a+3*b*c/d)*\operatorname{erf}(3^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(7/2)}-5/1728*d^{(5/2)}*\exp(3*a-3*b*c/d)*\operatorname{erfi}(3^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(7/2)}+45/64*d^{(5/2)}*\exp(-a+b*c/d)*\operatorname{erf}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*\operatorname{Pi}^{(1/2)}/b^{(7/2)}-45/64*d^{(5/2)}*\exp(a-b*c/d)*\operatorname{erfi}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*\operatorname{Pi}^{(1/2)}/b^{(7/2)}+45/16*d^2*\sinh(b*x+a)*(d*x+c)^{(1/2)}/b^3+5/144*d^2*\sinh(3*b*x+3*a)*(d*x+c)^{(1/2)}/b^3$

**Rubi [A]** time = 0.91, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {3311, 3296, 3308, 2180, 2204, 2205, 3312}

$$\frac{45\sqrt{\pi} d^{5/2} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}} + \frac{5\sqrt{\frac{\pi}{3}} d^{5/2} e^{\frac{3bc}{d}-3a} \operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} - \frac{45\sqrt{\pi} d^{5/2} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}} - \frac{5\sqrt{\frac{\pi}{3}} d^{5/2} e^{3a}}{576b^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^{(5/2)}*\operatorname{Cosh}[a + b*x]^3, x]$

[Out]  $(-5*d*(c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x])/(3*b^2) - (5*d*(c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x]^3)/(18*b^2) + (45*d^{(5/2)}*E^{(-a + (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(64*b^{(7/2)}) + (5*d^{(5/2)}*E^{(-3*a + (3*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erf}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(576*b^{(7/2)}) - (45*d^{(5/2)}*E^{(a - (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(64*b^{(7/2)}) - (5*d^{(5/2)}*E^{(3*a - (3*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(576*b^{(7/2)}) + (45*d^2*\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[a + b*x])/(16*b^3) + (2*(c + d*x)^{(5/2)}*\operatorname{Sinh}[a + b*x])/(3*b) + ((c + d*x)^{(5/2)}*\operatorname{Cosh}[a + b*x]^2*\operatorname{Sinh}[a + b*x])/(3*b) + (5*d^2*\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[3*a + 3*b*x])/(144*b^3)$

**Rule 2180**

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x\_Symbol] : > \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x \&\amp; !\$UseGamma === \operatorname{True}$

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[(Fa*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[(Fa*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])(n_.), x_Symbol] := Simp[(d*m*(c + d*x)(m - 1)*(b*Ssin[e + f*x])n/(f2*n), x] + (Dist[(b2*(n - 1))/n, Int[(c + d*x)m*(b*Ssin[e + f*x])(n - 2), x], x] - Dist[(d2*m*(m - 1))/(f2*n), Int[(c + d*x)(m - 2)*(b*Ssin[e + f*x])n, x], x] - Simp[(b*(c + d*x)m*Cos[e + f*x]*(b*Ssin[e + f*x])(n - 1)/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))(m_.)*sin[(e_.) + (f_.)*(x_)](n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)m, Sin[e + f*x]n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \cosh^3(a + bx) dx &= -\frac{5d(c + dx)^{3/2} \cosh^3(a + bx)}{18b^2} + \frac{(c + dx)^{5/2} \cosh^2(a + bx) \sinh(a + bx)}{3b} + \frac{2}{3} \int \\
&= -\frac{5d(c + dx)^{3/2} \cosh^3(a + bx)}{18b^2} + \frac{2(c + dx)^{5/2} \sinh(a + bx)}{3b} + \frac{(c + dx)^{5/2} \cosh^2(a + bx)}{3b} \\
&= -\frac{5d(c + dx)^{3/2} \cosh(a + bx)}{3b^2} - \frac{5d(c + dx)^{3/2} \cosh^3(a + bx)}{18b^2} + \frac{2(c + dx)^{5/2} \sinh(a + bx)}{3b} \\
&= -\frac{5d(c + dx)^{3/2} \cosh(a + bx)}{3b^2} - \frac{5d(c + dx)^{3/2} \cosh^3(a + bx)}{18b^2} + \frac{45d^2 \sqrt{c + dx} \sinh(a + bx)}{16b^3} \\
&= -\frac{5d(c + dx)^{3/2} \cosh(a + bx)}{3b^2} - \frac{5d(c + dx)^{3/2} \cosh^3(a + bx)}{18b^2} + \frac{45d^2 \sqrt{c + dx} \sinh(a + bx)}{16b^3} \\
&= -\frac{5d(c + dx)^{3/2} \cosh(a + bx)}{3b^2} - \frac{5d(c + dx)^{3/2} \cosh^3(a + bx)}{18b^2} + \frac{45d^2 \sqrt{c + dx} \sinh(a + bx)}{16b^3} \\
&= -\frac{5d(c + dx)^{3/2} \cosh(a + bx)}{3b^2} - \frac{5d(c + dx)^{3/2} \cosh^3(a + bx)}{18b^2} + \frac{45d^{5/2} e^{-a + \frac{bc}{d}} \sqrt{\pi}}{64b^3}
\end{aligned}$$

**Mathematica [A]** time = 4.00, size = 243, normalized size = 0.64

$$d^3 \left( \sqrt{3} \sqrt{-\frac{b(c+dx)}{d}} \Gamma\left(\frac{7}{2}, -\frac{3b(c+dx)}{d}\right) \left( \sinh\left(3a - \frac{3bc}{d}\right) + \cosh\left(3a - \frac{3bc}{d}\right) \right) + \left( \cosh\left(a - \frac{bc}{d}\right) - \sinh\left(a - \frac{bc}{d}\right) \right) \left( \sqrt{\frac{b(c+dx)}{d}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^(5/2)\*Cosh[a + b\*x]^3,x]

[Out] -1/648\*(d^3\*(Sqrt[3]\*Sqrt[-((b\*(c + d\*x))/d)]\*Gamma[7/2, (-3\*b\*(c + d\*x))/d])\*(Cosh[3\*a - (3\*b\*c)/d] + Sinh[3\*a - (3\*b\*c)/d]) + (Sqrt[(b\*(c + d\*x))/d]\*(243\*Gamma[7/2, (b\*(c + d\*x))/d] + Sqrt[3]\*Gamma[7/2, (3\*b\*(c + d\*x))/d]\*(Cosh[2\*a - (2\*b\*c)/d] - Sinh[2\*a - (2\*b\*c)/d])) + 243\*Sqrt[-((b\*(c + d\*x))/d)]\*Gamma[7/2, -(b\*(c + d\*x))/d]\*(Cosh[2\*a - (2\*b\*c)/d] + Sinh[2\*a - (2\*b\*c)/d]))\*(Cosh[a - (b\*c)/d] - Sinh[a - (b\*c)/d]))/(b^4\*Sqrt[c + d\*x])

**fricas [B]** time = 0.64, size = 2092, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(5/2)\*cosh(b\*x+a)^3,x, algorithm="fricas")

[Out]  $\frac{1}{1728} \cdot (5 \sqrt{3}) \sqrt{\pi} \cdot (d^3 \cosh(bx+a)^3 \cosh(-3(bc-a)/d) - d^3 \cosh(bx+a)^3 \sinh(-3(bc-a)/d) + (d^3 \cosh(-3(bc-a)/d) - d^3 \sinh(-3(bc-a)/d)) \sinh(bx+a)^3 + 3(d^3 \cosh(bx+a) \cosh(-3(bc-a)/d) - d^3 \cosh(bx+a) \sinh(-3(bc-a)/d)) \sinh(bx+a)^2 + 3(d^3 \cosh(bx+a)^2 \cosh(-3(bc-a)/d) - d^3 \cosh(bx+a)^2 \sinh(-3(bc-a)/d)) \sinh(bx+a) \sqrt{b/d} \operatorname{erf}(\sqrt{3} \sqrt{dx+c}) \sqrt{b/d}) + 5 \sqrt{3} \sqrt{\pi} \cdot (d^3 \cosh(bx+a)^3 \cosh(-3(bc-a)/d) + d^3 \cosh(bx+a)^3 \sinh(-3(bc-a)/d) + (d^3 \cosh(-3(bc-a)/d) + d^3 \sinh(-3(bc-a)/d)) \sinh(bx+a)^3 + 3(d^3 \cosh(bx+a) \cosh(-3(bc-a)/d) + d^3 \cosh(bx+a) \sinh(-3(bc-a)/d)) \sinh(bx+a)^2 + 3(d^3 \cosh(bx+a)^2 \cosh(-3(bc-a)/d) + d^3 \cosh(bx+a)^2 \sinh(-3(bc-a)/d)) \sinh(bx+a) \sqrt{-b/d} \operatorname{erf}(\sqrt{3} \sqrt{dx+c}) \sqrt{-b/d}) + 1215 \sqrt{\pi} \cdot (d^3 \cosh(bx+a)^3 \cosh(-(bc-a)/d) - d^3 \cosh(bx+a)^3 \sinh(-(bc-a)/d) + (d^3 \cosh(-(bc-a)/d) - d^3 \sinh(-(bc-a)/d)) \sinh(bx+a)^3 + 3(d^3 \cosh(bx+a) \cosh(-(bc-a)/d) - d^3 \cosh(bx+a) \sinh(-(bc-a)/d)) \sinh(bx+a)^2 + 3(d^3 \cosh(bx+a)^2 \cosh(-(bc-a)/d) - d^3 \cosh(bx+a)^2 \sinh(-(bc-a)/d)) \sinh(bx+a) \sqrt{b/d} \operatorname{erf}(\sqrt{3} \sqrt{dx+c}) \sqrt{b/d}) + 1215 \sqrt{\pi} \cdot (d^3 \cosh(bx+a)^3 \cosh(-(bc-a)/d) + d^3 \cosh(bx+a)^3 \sinh(-(bc-a)/d) + (d^3 \cosh(-(bc-a)/d) + d^3 \sinh(-(bc-a)/d)) \sinh(bx+a)^3 + 3(d^3 \cosh(bx+a) \cosh(-(bc-a)/d) + d^3 \cosh(bx+a) \sinh(-(bc-a)/d)) \sinh(bx+a)^2 + 3(d^3 \cosh(bx+a)^2 \cosh(-(bc-a)/d) + d^3 \cosh(bx+a)^2 \sinh(-(bc-a)/d)) \sinh(bx+a) \sqrt{-b/d} \operatorname{erf}(\sqrt{3} \sqrt{dx+c}) \sqrt{-b/d}) - 6 \cdot (12b^3 d^2 x^2 - (12b^3 d^2 x^2 + 12b^3 c^2 - 10b^2 c d + 5b d^2 + 2(12b^3 c d - 5b^2 d^2) x) \cosh(bx+a)^6 - 6(12b^3 d^2 x^2 + 12b^3 c^2 - 10b^2 c d + 5b d^2 + 2(12b^3 c d - 5b^2 d^2) x) \cosh(bx+a) \sinh(bx+a)^5 - (12b^3 d^2 x^2 + 12b^3 c^2 - 10b^2 c d + 5b d^2 + 2(12b^3 c d - 5b^2 d^2) x) \sinh(bx+a)^6 + 12b^3 c^2 - 27(4b^3 d^2 x^2 + 4b^3 c^2 - 10b^2 c d + 15b d^2 + 2(4b^3 c d - 5b^2 d^2) x) \cosh(bx+a)^4 - 3(36b^3 d^2 x^2 + 36b^3 c^2 - 90b^2 c d + 135b d^2 + 5(12b^3 d^2 x^2 + 12b^3 c^2 - 10b^2 c d + 5b d^2 + 2(12b^3 c d - 5b^2 d^2) x) \cosh(bx+a)^2 + 18(4b^3 c d - 5b^2 d^2) x) \sinh(bx+a)^4 + 10b^2 c d - 4(5(12b^3 d^2 x^2 + 12b^3 c^2 - 10b^2 c d + 5b d^2 + 2(12b^3 c d - 5b^2 d^2) x) \cosh(bx+a)^3 + 27(4b^3 d^2 x^2 + 4b^3 c^2 - 10b^2 c d + 15b d^2 + 2(4b^3 c d - 5b^2 d^2) x) \cosh(bx+a)) \sinh(bx+a)^3 + 5b d^2 + 27(4b^3 d^2 x^2 + 4b^3 c^2 + 10b^2 c d + 15b d^2 + 2(4b^3 c d + 5b^2 d^2) x) \cosh(bx+a)^2 + 3(36b^3 d^2 x^2 + 36b^3 c^2 - 5(12b^3 d^2 x^2 + 12b^3 c^2 - 10b^2 c d + 5b d^2 + 2(12b^3 c d - 5b^2 d^2) x) \cosh(bx+a)^4 + 90b^2 c d + 135b d^2 - 54(4b^3 d^2 x^2 + 4b^3 c^2 - 10b^2 c d + 15b d^2 + 2(4b^3 c d - 5b^2 d^2) x) \cosh(bx+a)^2 + 18(4b^3 c d + 5b^2 d^2) x) \sinh(bx+a)^2 + 2(12b^3 c d + 5b^2 d^2) x - 6((12b^3 d^2 x^2 + 12b^3 c^2 - 10b^2 c d + 5b d^2 + 2(12b^3 c d - 5b^2 d^2) x) \cosh(bx+a)^5 + 18(4b^3 d$

$$\begin{aligned} &^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*c \\ &osh(b*x + a)^3 - 9*(4*b^3*d^2*x^2 + 4*b^3*c^2 + 10*b^2*c*d + 15*b*d^2 + 2*( \\ &4*b^3*c*d + 5*b^2*d^2)*x)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^4 \\ &*cosh(b*x + a)^3 + 3*b^4*cosh(b*x + a)^2*sinh(b*x + a) + 3*b^4*cosh(b*x + a \\ &)*sinh(b*x + a)^2 + b^4*sinh(b*x + a)^3) \end{aligned}$$

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^{\frac{5}{2}} \cosh(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(5/2)\*cosh(b\*x+a)^3,x, algorithm="giac")

[Out] integrate((d\*x + c)^(5/2)\*cosh(b\*x + a)^3, x)

**maple [F]** time = 0.51, size = 0, normalized size = 0.00

$$\int (dx + c)^{\frac{5}{2}} (\cosh^3(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^(5/2)\*cosh(b\*x+a)^3,x)

[Out] int((d\*x+c)^(5/2)\*cosh(b\*x+a)^3,x)

**maxima [A]** time = 1.54, size = 513, normalized size = 1.35

$$\frac{5\sqrt{3}\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{-b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b^3\sqrt{\frac{-b}{d}}} - \frac{5\sqrt{3}\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} + \frac{1215\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{-b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{\frac{-b}{d}}} - \frac{1215\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(5/2)\*cosh(b\*x+a)^3,x, algorithm="maxima")

[Out]  $-1/1728*(5*\sqrt{3}*\sqrt{\pi}*d^3*\operatorname{erf}(\sqrt{3}*\sqrt{d*x + c}*\sqrt{-b/d}))*e^{(3*a - 3*b*c/d)/(b^3*\sqrt{-b/d})} - 5*\sqrt{3}*\sqrt{\pi}*d^3*\operatorname{erf}(\sqrt{3}*\sqrt{d*x + c}*\sqrt{b/d}))*e^{(-3*a + 3*b*c/d)/(b^3*\sqrt{b/d})} + 1215*\sqrt{\pi}*d^3*\operatorname{erf}(\sqrt{d*x + c}*\sqrt{-b/d}))*e^{(a - b*c/d)/(b^3*\sqrt{-b/d})} - 1215*\sqrt{\pi}*d^3*\operatorname{erf}(\sqrt{d*x + c}*\sqrt{b/d}))*e^{(-a + b*c/d)/(b^3*\sqrt{b/d})} + 162*(d*x + c)^{(5/2)}*b^2*d*e^{(b*c/d)} + 10*(d*x + c)^{(3/2)}*b*d^2*e^{(b*c/d)} + 15*\sqrt{d*x + c}*d^3*e^{(b*c/d)}*e^{(-a - (d*x + c)*b/d)/b^3} + 6*(12*(d*x + c)^{(5/2)}*b^2*d*e^{(3*b*c/d)} + 10*(d*x + c)^{(3/2)}*b*d^2*e^{(3*b*c/d)} + 5*\sqrt{d*x + c})$

```
*d^3*e^(3*b*c/d))*e^(-3*a - 3*(d*x + c)*b/d)/b^3 - 6*(12*(d*x + c)^(5/2)*b^
2*d*e^(3*a) - 10*(d*x + c)^(3/2)*b*d^2*e^(3*a) + 5*sqrt(d*x + c)*d^3*e^(3*a
))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^3 - 162*(4*(d*x + c)^(5/2)*b^2*d*e^a - 1
0*(d*x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x + c)*b/d -
b*c/d)/b^3)/d
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh(a + bx)^3 (c + dx)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*(c + d*x)^(5/2), x)
```

```
[Out] int(cosh(a + b*x)^3*(c + d*x)^(5/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cosh(b*x+a)**3, x)
```

```
[Out] Timed out
```

### 3.57 $\int (c + dx)^{3/2} \cosh^3(a + bx) dx$

**Optimal.** Leaf size=326

$$\frac{9\sqrt{\pi} d^{3/2} e^{\frac{bc}{d}-a} \operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}} + \frac{\sqrt{\frac{\pi}{3}} d^{3/2} e^{\frac{3bc}{d}-3a} \operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} + \frac{9\sqrt{\pi} d^{3/2} e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}} + \frac{\sqrt{\frac{\pi}{3}} d^{3/2} e^{3a-\frac{3bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}$$

[Out]  $2/3*(d*x+c)^{(3/2)}*\sinh(b*x+a)/b+1/3*(d*x+c)^{(3/2)}*\cosh(b*x+a)^2*\sinh(b*x+a)/b+1/288*d^{(3/2)}*\exp(-3*a+3*b*c/d)*\operatorname{erf}(3^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(5/2)}+1/288*d^{(3/2)}*\exp(3*a-3*b*c/d)*\operatorname{erfi}(3^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(5/2)}+9/32*d^{(3/2)}*\exp(-a+b*c/d)*\operatorname{erf}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*\operatorname{Pi}^{(1/2)}/b^{(5/2)}+9/32*d^{(3/2)}*\exp(a-b*c/d)*\operatorname{erfi}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*\operatorname{Pi}^{(1/2)}/b^{(5/2)}-d*\cosh(b*x+a)*(d*x+c)^{(1/2)}/b^2-1/6*d*\cosh(b*x+a)^3*(d*x+c)^{(1/2)}/b^2$

**Rubi [A]** time = 0.71, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {3311, 3296, 3307, 2180, 2204, 2205, 3312}

$$\frac{9\sqrt{\pi} d^{3/2} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}} + \frac{\sqrt{\frac{\pi}{3}} d^{3/2} e^{\frac{3bc}{d}-3a} \operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} + \frac{9\sqrt{\pi} d^{3/2} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}} + \frac{\sqrt{\frac{\pi}{3}} d^{3/2} e^{3a-\frac{3bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x]^3, x]$

[Out]  $-((d*\operatorname{Sqrt}[c + d*x]*\operatorname{Cosh}[a + b*x])/b^2) - (d*\operatorname{Sqrt}[c + d*x]*\operatorname{Cosh}[a + b*x]^3)/(6*b^2) + (9*d^{(3/2)}*E^{-a + (b*c)/d}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(32*b^{(5/2)}) + (d^{(3/2)}*E^{-3*a + (3*b*c)/d}*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erf}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(96*b^{(5/2)}) + (9*d^{(3/2)}*E^{a - (b*c)/d}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(32*b^{(5/2)}) + (d^{(3/2)}*E^{3*a - (3*b*c)/d}*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(96*b^{(5/2)}) + (2*(c + d*x)^{(3/2)}*\operatorname{Sinh}[a + b*x])/(3*b) + ((c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x]^2*\operatorname{Sinh}[a + b*x])/(3*b)$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x\_Symbol] : > \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g, x\} \ \&\amp; \ !\$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{$

F, a, b, c, d}, x] && PosQ[b]

### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F<sup>a</sup>\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[(c + d\*x)<sup>m</sup>\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)<sup>(m - 1)</sup>\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3307

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)</sup>\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I/2, Int[(c + d\*x)<sup>m</sup>/(E<sup>(I\*k\*Pi)</sup>\*E<sup>(I\*(e + f\*x))</sup>), x], x] - Dist[I/2, Int[(c + d\*x)<sup>m</sup>\*E<sup>(I\*k\*Pi)</sup>\*E<sup>(I\*(e + f\*x))</sup>, x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)</sup>\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])<sup>(n\_.)</sup>, x\_Symbol] := Simp[(d\*m\*(c + d\*x)<sup>(m - 1)</sup>\*(b\*Sin[e + f\*x])<sup>n</sup>/(f<sup>2</sup>\*n<sup>2</sup>), x] + (Dist[(b<sup>2</sup>\*(n - 1))/n, Int[(c + d\*x)<sup>m</sup>\*(b\*Sin[e + f\*x])<sup>(n - 2)</sup>, x], x] - Dist[(d<sup>2</sup>\*m\*(m - 1))/(f<sup>2</sup>\*n<sup>2</sup>), Int[(c + d\*x)<sup>(m - 2)</sup>\*(b\*Sin[e + f\*x])<sup>n</sup>, x], x] - Simp[(b\*(c + d\*x)<sup>m</sup>\*Cos[e + f\*x]\*(b\*Sin[e + f\*x])<sup>(n - 1)</sup>/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)</sup>\*sin[(e\_.) + (f\_.)\*(x\_) ]<sup>(n\_.)</sup>, x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)<sup>m</sup>, Sin[e + f\*x]<sup>n</sup>, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rubi steps



$$\begin{aligned}
\int (c + dx)^{3/2} \cosh^3(a + bx) dx &= -\frac{d\sqrt{c + dx} \cosh^3(a + bx)}{6b^2} + \frac{(c + dx)^{3/2} \cosh^2(a + bx) \sinh(a + bx)}{3b} + \frac{2}{3} \int (c + dx)^{3/2} \cosh^2(a + bx) dx \\
&= -\frac{d\sqrt{c + dx} \cosh^3(a + bx)}{6b^2} + \frac{2(c + dx)^{3/2} \sinh(a + bx)}{3b} + \frac{(c + dx)^{3/2} \cosh^2(a + bx)}{3b} \\
&= -\frac{d\sqrt{c + dx} \cosh(a + bx)}{b^2} - \frac{d\sqrt{c + dx} \cosh^3(a + bx)}{6b^2} + \frac{2(c + dx)^{3/2} \sinh(a + bx)}{3b} \\
&= -\frac{d\sqrt{c + dx} \cosh(a + bx)}{b^2} - \frac{d\sqrt{c + dx} \cosh^3(a + bx)}{6b^2} + \frac{2(c + dx)^{3/2} \sinh(a + bx)}{3b} \\
&= -\frac{d\sqrt{c + dx} \cosh(a + bx)}{b^2} - \frac{d\sqrt{c + dx} \cosh^3(a + bx)}{6b^2} + \frac{2(c + dx)^{3/2} \sinh(a + bx)}{3b} \\
&= -\frac{d\sqrt{c + dx} \cosh(a + bx)}{b^2} - \frac{d\sqrt{c + dx} \cosh^3(a + bx)}{6b^2} + \frac{9d^{3/2} e^{-a + \frac{bc}{d}} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{b}}{d}\right)}{32b^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 2.01, size = 243, normalized size = 0.75

$$d^2 \left( \sqrt{3} \sqrt{-\frac{b(c+dx)}{d}} \Gamma\left(\frac{5}{2}, -\frac{3b(c+dx)}{d}\right) \left( \sinh\left(3a - \frac{3bc}{d}\right) + \cosh\left(3a - \frac{3bc}{d}\right) \right) + \left( \cosh\left(a - \frac{bc}{d}\right) - \sinh\left(a - \frac{bc}{d}\right) \right) \left( 81 \sqrt{-\frac{b(c+dx)}{d}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^(3/2)\*Cosh[a + b\*x]^3,x]

[Out] (d^2\*(Sqrt[3]\*Sqrt[-((b\*(c + d\*x))/d)]\*Gamma[5/2, (-3\*b\*(c + d\*x))/d]\*(Cosh[3\*a - (3\*b\*c)/d] + Sinh[3\*a - (3\*b\*c)/d]) + (81\*Sqrt[-((b\*(c + d\*x))/d)]\*Gamma[5/2, -((b\*(c + d\*x))/d)]\*(Cosh[2\*a - (2\*b\*c)/d] + Sinh[2\*a - (2\*b\*c)/d]) + Sqrt[(b\*(c + d\*x))/d]\*(-81\*Gamma[5/2, (b\*(c + d\*x))/d] + Sqrt[3]\*Gamma[5/2, (3\*b\*(c + d\*x))/d]\*(-Cosh[2\*a - (2\*b\*c)/d] + Sinh[2\*a - (2\*b\*c)/d])))\*(Cosh[a - (b\*c)/d] - Sinh[a - (b\*c)/d]))/(216\*b^3\*Sqrt[c + d\*x])

**fricas [B]** time = 0.67, size = 1545, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(3/2)\*cosh(b\*x+a)^3,x, algorithm="fricas")

```
[Out] 1/288*(sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d^2*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) - d^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) - d^2*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^2*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d^2*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d^2*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) + d^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^2*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + d^2*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d)) + 81*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) - d^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^2*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a)*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 81*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) + d^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^2*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a)*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) + 6*((2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a)^6 + 6*(2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a)*sinh(b*x + a)^5 + (2*b^2*d*x + 2*b^2*c - b*d)*sinh(b*x + a)^6 + 9*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x + a)^4 + 3*(6*b^2*d*x + 6*b^2*c + 5*(2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a)^2 - 9*b*d)*sinh(b*x + a)^4 - 2*b^2*d*x + 4*(5*(2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a)^3 + 9*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x + a))*sinh(b*x + a)^3 - 2*b^2*c - 9*(2*b^2*d*x + 2*b^2*c + 3*b*d)*cosh(b*x + a)^2 + 3*(5*(2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a)^4 - 6*b^2*d*x - 6*b^2*c + 18*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x + a)^2 - 9*b*d)*sinh(b*x + a)^2 - b*d + 6*((2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a)^5 + 6*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x + a)^3 - 3*(2*b^2*d*x + 2*b^2*c + 3*b*d)*cosh(b*x + a))*sinh(b*x + a)*sqrt(d*x + c))/(b^3*cosh(b*x + a)^3 + 3*b^3*cosh(b*x + a)^2*sinh(b*x + a) + 3*b^3*cosh(b*x + a)*sinh(b*x + a)^2 + b^3*sinh(b*x + a)^3)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^{\frac{3}{2}} \cosh(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(3/2)\*cosh(b\*x+a)^3,x, algorithm="giac")

[Out] integrate((d\*x + c)^(3/2)\*cosh(b\*x + a)^3, x)

**maple** [F] time = 0.50, size = 0, normalized size = 0.00

$$\int (dx + c)^{\frac{3}{2}} (\cosh^3 (bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^(3/2)\*cosh(b\*x+a)^3,x)

[Out] int((d\*x+c)^(3/2)\*cosh(b\*x+a)^3,x)

**maxima** [A] time = 0.44, size = 429, normalized size = 1.32

$$\frac{\sqrt{3} \sqrt{\pi} d^2 \operatorname{erf}\left(\sqrt{3} \sqrt{dx+c} \sqrt{-\frac{b}{d}}\right) e^{\left(3a-\frac{3bc}{d}\right)}}{b^2 \sqrt{-\frac{b}{d}}} + \frac{\sqrt{3} \sqrt{\pi} d^2 \operatorname{erf}\left(\sqrt{3} \sqrt{dx+c} \sqrt{\frac{b}{d}}\right) e^{\left(-3a+\frac{3bc}{d}\right)}}{b^2 \sqrt{\frac{b}{d}}} + \frac{81 \sqrt{\pi} d^2 \operatorname{erf}\left(\sqrt{dx+c} \sqrt{-\frac{b}{d}}\right) e^{\left(a-\frac{bc}{d}\right)}}{b^2 \sqrt{-\frac{b}{d}}} + \frac{81 \sqrt{\pi} d^2 \operatorname{erf}\left(\sqrt{dx+c} \sqrt{\frac{b}{d}}\right) e^{\left(-a+\frac{bc}{d}\right)}}{b^2 \sqrt{\frac{b}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(3/2)\*cosh(b\*x+a)^3,x, algorithm="maxima")

[Out] 1/288\*(sqrt(3)\*sqrt(pi)\*d^2\*erf(sqrt(3)\*sqrt(d\*x + c)\*sqrt(-b/d))\*e^(3\*a - 3\*b\*c/d)/(b^2\*sqrt(-b/d)) + sqrt(3)\*sqrt(pi)\*d^2\*erf(sqrt(3)\*sqrt(d\*x + c)\*sqrt(b/d))\*e^(-3\*a + 3\*b\*c/d)/(b^2\*sqrt(b/d)) + 81\*sqrt(pi)\*d^2\*erf(sqrt(d\*x + c)\*sqrt(-b/d))\*e^(a - b\*c/d)/(b^2\*sqrt(-b/d)) + 81\*sqrt(pi)\*d^2\*erf(sqrt(d\*x + c)\*sqrt(b/d))\*e^(-a + b\*c/d)/(b^2\*sqrt(b/d)) - 54\*(2\*(d\*x + c)^(3/2))\*b\*d\*e^(b\*c/d) + 3\*sqrt(d\*x + c)\*d^2\*e^(b\*c/d)\*e^(-a - (d\*x + c)\*b/d)/b^2 - 6\*(2\*(d\*x + c)^(3/2))\*b\*d\*e^(3\*b\*c/d) + sqrt(d\*x + c)\*d^2\*e^(3\*b\*c/d)\*e^(-3\*a - 3\*(d\*x + c)\*b/d)/b^2 + 6\*(2\*(d\*x + c)^(3/2))\*b\*d\*e^(3\*a) - sqrt(d\*x + c)\*d^2\*e^(3\*a)\*e^(3\*(d\*x + c)\*b/d - 3\*b\*c/d)/b^2 + 54\*(2\*(d\*x + c)^(3/2))\*b\*d\*e^a - 3\*sqrt(d\*x + c)\*d^2\*e^a)\*e^((d\*x + c)\*b/d - b\*c/d)/b^2/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh (a + b x)^3 (c + d x)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^3\*(c + d\*x)^(3/2),x)

[Out] int(cosh(a + b\*x)^3\*(c + d\*x)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^{\frac{3}{2}} \cosh^3 (a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cosh(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**(3/2)*cosh(a + b*x)**3, x)
```

### 3.58 $\int \sqrt{c + dx} \cosh^3(a + bx) dx$

**Optimal.** Leaf size=275

$$\frac{3\sqrt{\pi} \sqrt{d} e^{\frac{bc}{d}-a} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} + \frac{\sqrt{\frac{\pi}{3}} \sqrt{d} e^{\frac{3bc}{d}-3a} \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} - \frac{3\sqrt{\pi} \sqrt{d} e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{\frac{\pi}{3}} \sqrt{d} e^{3a-\frac{3bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}$$

[Out]  $\frac{1}{144} \exp(-3a+3b*c/d) \operatorname{erf}\left(3^{1/2} b^{1/2} (d*x+c)^{1/2} / d^{1/2}\right) d^{1/2} 3^{1/2} \operatorname{Pi}^{1/2} / b^{3/2} - \frac{1}{144} \exp(3a-3b*c/d) \operatorname{erfi}\left(3^{1/2} b^{1/2} (d*x+c)^{1/2} / d^{1/2}\right) d^{1/2} 3^{1/2} \operatorname{Pi}^{1/2} / b^{3/2} + \frac{3}{16} \exp(-a+b*c/d) \operatorname{erf}\left(b^{1/2} (d*x+c)^{1/2} / d^{1/2}\right) d^{1/2} \operatorname{Pi}^{1/2} / b^{3/2} - \frac{3}{16} \exp(a-b*c/d) \operatorname{erfi}\left(b^{1/2} (d*x+c)^{1/2} / d^{1/2}\right) d^{1/2} \operatorname{Pi}^{1/2} / b^{3/2} + \frac{3}{4} \sinh(b*x+a) * (d*x+c)^{1/2} / b + \frac{1}{12} \sinh(3*b*x+3*a) * (d*x+c)^{1/2} / b$

**Rubi [A]** time = 0.48, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3312, 3296, 3308, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi} \sqrt{d} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} + \frac{\sqrt{\frac{\pi}{3}} \sqrt{d} e^{\frac{3bc}{d}-3a} \operatorname{Erf}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} - \frac{3\sqrt{\pi} \sqrt{d} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{\frac{\pi}{3}} \sqrt{d} e^{3a-\frac{3bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[c + d*x] * \operatorname{Cosh}[a + b*x]^3, x]$

[Out]  $\frac{(3*\operatorname{Sqrt}[d]*E^{-a + (b*c)/d}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]]) / (16*b^{3/2}) + (\operatorname{Sqrt}[d]*E^{-3*a + (3*b*c)/d}*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erf}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]]) / (48*b^{3/2}) - (3*\operatorname{Sqrt}[d]*E^{a - (b*c)/d}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]]) / (16*b^{3/2}) - (\operatorname{Sqrt}[d]*E^{3*a - (3*b*c)/d}*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]]) / (48*b^{3/2}) + (3*\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[a + b*x]) / (4*b) + (\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[3*a + 3*b*x]) / (12*b)}$

**Rule 2180**

$\operatorname{Int}[(F_)^((g_.) * ((e_.) + (f_.) * (x_))) / \operatorname{Sqrt}[(c_.) + (d_.) * (x_)], x\_Symbol] : > \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \&\amp; !\$UseGamma == True$

**Rule 2204**

$\operatorname{Int}[(F_)^((a_.) + (b_.) * ((c_.) + (d_.) * (x_)) ^ 2), x\_Symbol] :> \operatorname{Simp}[(F^a * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(c + d*x) * \operatorname{Rt}[b * \operatorname{Log}[F], 2]]) / (2 * d * \operatorname{Rt}[b * \operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \&\amp; \operatorname{PosQ}[b]$

Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[(c + d\*x)<sup>m</sup>\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)<sup>(m - 1)</sup>\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3308

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I/2, Int[(c + d\*x)<sup>m</sup>/E^(I\*(e + f\*x)), x], x] - Dist[I/2, Int[(c + d\*x)<sup>m</sup>\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)]<sup>(n\_)</sup>, x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)<sup>m</sup>, Sin[e + f\*x]<sup>n</sup>, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rubi steps

$$\begin{aligned}
 \int \sqrt{c+dx} \cosh^3(a+bx) dx &= \int \left( \frac{3}{4} \sqrt{c+dx} \cosh(a+bx) + \frac{1}{4} \sqrt{c+dx} \cosh(3a+3bx) \right) dx \\
 &= \frac{1}{4} \int \sqrt{c+dx} \cosh(3a+3bx) dx + \frac{3}{4} \int \sqrt{c+dx} \cosh(a+bx) dx \\
 &= \frac{3\sqrt{c+dx} \sinh(a+bx)}{4b} + \frac{\sqrt{c+dx} \sinh(3a+3bx)}{12b} - \frac{d \int \frac{\sinh(3a+3bx)}{\sqrt{c+dx}} dx}{24b} - \frac{(3d) \int \sqrt{c+dx} dx}{48} \\
 &= \frac{3\sqrt{c+dx} \sinh(a+bx)}{4b} + \frac{\sqrt{c+dx} \sinh(3a+3bx)}{12b} - \frac{d \int \frac{e^{-i(3a+3ibx)}}{\sqrt{c+dx}} dx}{48b} + \frac{d \int \frac{e^{i(3a+3ibx)}}{\sqrt{c+dx}} dx}{48b} \\
 &= \frac{3\sqrt{c+dx} \sinh(a+bx)}{4b} + \frac{\sqrt{c+dx} \sinh(3a+3bx)}{12b} + \frac{\text{Subst} \left( \int e^{i(3ia-\frac{3ibc}{d})-\frac{3bx^2}{d}} dx \right)}{24b} \\
 &= \frac{3\sqrt{d} e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{16b^{3/2}} + \frac{\sqrt{d} e^{-3a+\frac{3bc}{d}} \sqrt{\frac{\pi}{3}} \operatorname{erf} \left( \frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{48b^{3/2}} - \frac{3\sqrt{d} e^{a-\frac{bc}{d}} \sqrt{\pi}}{48b^{3/2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.29, size = 210, normalized size = 0.76

$$\frac{\sqrt{c+dx} e^{-3\left(a+\frac{bc}{d}\right)} \left( \sqrt{3} e^{6a} \sqrt{\frac{b(c+dx)}{d}} \Gamma\left(\frac{3}{2}, -\frac{3b(c+dx)}{d}\right) + 27 e^{4a+\frac{2bc}{d}} \sqrt{\frac{b(c+dx)}{d}} \Gamma\left(\frac{3}{2}, -\frac{b(c+dx)}{d}\right) - e^{\frac{4bc}{d}} \sqrt{-\frac{b(c+dx)}{d}} \left( 27 e^{2a} \Gamma\left(\frac{3}{2}, -\frac{b(c+dx)}{d}\right) \right) \right)}{72b \sqrt{-\frac{b^2(c+dx)^2}{d^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d\*x]\*Cosh[a + b\*x]^3, x]

[Out] (Sqrt[c + d\*x]\*(Sqrt[3]\*E^(6\*a)\*Sqrt[(b\*(c + d\*x))/d]\*Gamma[3/2, (-3\*b\*(c + d\*x))/d] + 27\*E^(4\*a + (2\*b\*c)/d)\*Sqrt[(b\*(c + d\*x))/d]\*Gamma[3/2, -((b\*(c + d\*x))/d)] - E^((4\*b\*c)/d)\*Sqrt[-((b\*(c + d\*x))/d)]\*(27\*E^(2\*a)\*Gamma[3/2, (b\*(c + d\*x))/d] + Sqrt[3]\*E^((2\*b\*c)/d)\*Gamma[3/2, (3\*b\*(c + d\*x))/d]))/(72\*b\*E^(3\*(a + (b\*c)/d))\*Sqrt[-((b^2\*(c + d\*x)^2)/d^2)])

**fricas [B]** time = 0.51, size = 1217, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3\*(d\*x+c)^(1/2), x, algorithm="fricas")

[Out] 1/144\*(sqrt(3)\*sqrt(pi)\*(d\*cosh(b\*x + a)^3\*cosh(-3\*(b\*c - a\*d)/d) - d\*cosh(b\*x + a)^3\*sinh(-3\*(b\*c - a\*d)/d) + (d\*cosh(-3\*(b\*c - a\*d)/d) - d\*sinh(-3\*(b\*c - a\*d)/d))\*sinh(b\*x + a)^3 + 3\*(d\*cosh(b\*x + a)\*cosh(-3\*(b\*c - a\*d)/d) - d\*cosh(b\*x + a)\*sinh(-3\*(b\*c - a\*d)/d))\*sinh(b\*x + a)^2 + 3\*(d\*cosh(b\*x + a)^2\*cosh(-3\*(b\*c - a\*d)/d) - d\*cosh(b\*x + a)^2\*sinh(-3\*(b\*c - a\*d)/d))\*sinh(b\*x + a)\*sqrt(b/d)\*erf(sqrt(3)\*sqrt(d\*x + c)\*sqrt(b/d)) + sqrt(3)\*sqrt(pi)\*(d\*cosh(b\*x + a)^3\*cosh(-3\*(b\*c - a\*d)/d) + d\*cosh(b\*x + a)^3\*sinh(-3\*(b\*c - a\*d)/d) + (d\*cosh(-3\*(b\*c - a\*d)/d) + d\*sinh(-3\*(b\*c - a\*d)/d))\*sinh(b\*x + a)^3 + 3\*(d\*cosh(b\*x + a)\*cosh(-3\*(b\*c - a\*d)/d) + d\*cosh(b\*x + a)\*sinh(-3\*(b\*c - a\*d)/d))\*sinh(b\*x + a)^2 + 3\*(d\*cosh(b\*x + a)^2\*cosh(-3\*(b\*c - a\*d)/d) + d\*cosh(b\*x + a)^2\*sinh(-3\*(b\*c - a\*d)/d))\*sinh(b\*x + a)\*sqrt(-b/d)\*erf(sqrt(3)\*sqrt(d\*x + c)\*sqrt(-b/d)) + 27\*sqrt(pi)\*(d\*cosh(b\*x + a)^3\*cosh(-(b\*c - a\*d)/d) - d\*cosh(b\*x + a)^3\*sinh(-(b\*c - a\*d)/d) + (d\*cosh(-(b\*c - a\*d)/d) - d\*sinh(-(b\*c - a\*d)/d))\*sinh(b\*x + a)^3 + 3\*(d\*cosh(b\*x + a)\*cosh(-(b\*c - a\*d)/d) - d\*cosh(b\*x + a)\*sinh(-(b\*c - a\*d)/d))\*sinh(b\*x + a)^2 + 3\*(d\*cosh(b\*x + a)^2\*cosh(-(b\*c - a\*d)/d) - d\*cosh(b\*x + a)^2\*sinh(-(b\*c - a\*d)/d))\*sinh(b\*x + a)\*sqrt(b/d)\*erf(sqrt(d\*x + c)\*sqrt(b/d)) + 27\*sqrt(pi)\*(d\*cosh(b\*x + a)^3\*cosh(-(b\*c - a\*d)/d) + d\*cosh(b\*x + a)^3\*sinh(-(b\*c - a\*d)/d) + (d\*cosh(-(b\*c - a\*d)/d) + d\*sinh(-(b\*c - a\*d)/d))\*sinh(b\*x + a)^3 + 3\*(d\*cosh(b\*x + a)\*cosh(-(b\*c - a\*d)/d) + d\*cosh(b\*x + a)\*sinh(-(b\*c - a\*d)/d))\*sinh(b\*x + a)^2 + 3\*(d\*cosh(b\*x + a)^2\*cosh(-(b\*c - a\*d)/d) +

$d \cosh(bx + a)^2 \sinh(-\frac{b \cdot c - a \cdot d}{d}) \sinh(bx + a) \sqrt{-b/d} \operatorname{erf}(\sqrt{d \cdot x + c} \sqrt{-b/d}) + 6 \cdot (b \cosh(bx + a)^6 + 6 \cdot b \cosh(bx + a) \sinh(bx + a)^5 + b \sinh(bx + a)^6 + 9 \cdot b \cosh(bx + a)^4 + 3 \cdot (5 \cdot b \cosh(bx + a)^2 + 3 \cdot b) \sinh(bx + a)^4 + 4 \cdot (5 \cdot b \cosh(bx + a)^3 + 9 \cdot b \cosh(bx + a)) \sinh(bx + a)^3 - 9 \cdot b \cosh(bx + a)^2 + 3 \cdot (5 \cdot b \cosh(bx + a)^4 + 18 \cdot b \cosh(bx + a)^2 - 3 \cdot b) \sinh(bx + a)^2 + 6 \cdot (b \cosh(bx + a)^5 + 6 \cdot b \cosh(bx + a)^3 - 3 \cdot b \cosh(bx + a)) \sinh(bx + a) - b \sqrt{d \cdot x + c}) / (b^2 \cosh(bx + a)^3 + 3 \cdot b^2 \cosh(bx + a)^2 \sinh(bx + a) + 3 \cdot b^2 \cosh(bx + a) \sinh(bx + a)^2 + b^2 \sinh(bx + a)^3)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx + c} \cosh(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3\*(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(d\*x + c)\*cosh(b\*x + a)^3, x)

**maple** [F] time = 0.49, size = 0, normalized size = 0.00

$$\int (\cosh^3(bx + a)) \sqrt{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)^3\*(d\*x+c)^(1/2),x)

[Out] int(cosh(b\*x+a)^3\*(d\*x+c)^(1/2),x)

**maxima** [A] time = 0.44, size = 334, normalized size = 1.21

$$\frac{\sqrt{3} \sqrt{\pi} d \operatorname{erf}\left(\sqrt{3} \sqrt{dx+c} \sqrt{-\frac{b}{d}}\right) e^{\left(3a - \frac{3bc}{d}\right)}}{b \sqrt{-\frac{b}{d}}} - \frac{\sqrt{3} \sqrt{\pi} d \operatorname{erf}\left(\sqrt{3} \sqrt{dx+c} \sqrt{\frac{b}{d}}\right) e^{\left(-3a + \frac{3bc}{d}\right)}}{b \sqrt{\frac{b}{d}}} + \frac{27 \sqrt{\pi} d \operatorname{erf}\left(\sqrt{dx+c} \sqrt{-\frac{b}{d}}\right) e^{\left(a - \frac{bc}{d}\right)}}{b \sqrt{-\frac{b}{d}}} - \frac{27 \sqrt{\pi} d \operatorname{erf}\left(\sqrt{dx+c} \sqrt{\frac{b}{d}}\right) e^{\left(-a + \frac{bc}{d}\right)}}{b \sqrt{\frac{b}{d}}}$$

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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3\*(d\*x+c)^(1/2),x, algorithm="maxima")

[Out]  $-1/144 \cdot (\sqrt{3} \sqrt{\pi}) \cdot d \cdot \operatorname{erf}(\sqrt{3} \sqrt{d \cdot x + c} \sqrt{-b/d}) \cdot e^{(3 \cdot a - 3 \cdot b \cdot c/d)} / (b \sqrt{-b/d}) - \sqrt{3} \sqrt{\pi} \cdot d \cdot \operatorname{erf}(\sqrt{3} \sqrt{d \cdot x + c} \sqrt{b/d}) \cdot e^{(-3 \cdot a + 3 \cdot b \cdot c/d)} / (b \sqrt{b/d}) + 27 \sqrt{\pi} \cdot d \cdot \operatorname{erf}(\sqrt{d \cdot x + c} \sqrt{-b/d}) \cdot e^{(a - b \cdot c/d)} / (b \sqrt{-b/d}) - 27 \sqrt{\pi} \cdot d \cdot \operatorname{erf}(\sqrt{d \cdot x + c} \sqrt{b/d}) \cdot e^{(-a + b \cdot c/d)} / (b \sqrt{b/d}) - 6 \sqrt{d \cdot x + c} \cdot d \cdot e^{(3 \cdot a + 3 \cdot (d \cdot x + c) \cdot b^2 / d^2)}$



$c)*b/d - 3*b*c/d)/b - 54*\sqrt{d*x + c}*d*e^{(a + (d*x + c)*b/d - b*c/d)/b} +$   
 $54*\sqrt{d*x + c}*d*e^{(-a - (d*x + c)*b/d + b*c/d)/b} + 6*\sqrt{d*x + c}*d*e^{$   
 $(-3*a - 3*(d*x + c)*b/d + 3*b*c/d)/b)/d$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh(a + bx)^3 \sqrt{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^3\*(c + d\*x)^(1/2), x)

[Out] int(cosh(a + b\*x)^3\*(c + d\*x)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx} \cosh^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*\*3\*(d\*x+c)\*\*(1/2), x)

[Out] Integral(sqrt(c + d\*x)\*cosh(a + b\*x)\*\*3, x)

$$3.59 \quad \int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=228

$$\frac{3\sqrt{\pi} e^{\frac{bc}{d}-a} \operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} + \frac{\sqrt{\frac{\pi}{3}} e^{\frac{3bc}{d}-3a} \operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} + \frac{3\sqrt{\pi} e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} + \frac{\sqrt{\frac{\pi}{3}} e^{3a-\frac{3bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}$$

[Out] 1/24\*exp(-3\*a+3\*b\*c/d)\*erf(3^(1/2)\*b^(1/2)\*(d\*x+c)^(1/2)/d^(1/2))\*3^(1/2)\*Pi^(1/2)/b^(1/2)/d^(1/2)+1/24\*exp(3\*a-3\*b\*c/d)\*erfi(3^(1/2)\*b^(1/2)\*(d\*x+c)^(1/2)/d^(1/2))\*3^(1/2)\*Pi^(1/2)/b^(1/2)/d^(1/2)+3/8\*exp(-a+b\*c/d)\*erf(b^(1/2)\*(d\*x+c)^(1/2)/d^(1/2))\*Pi^(1/2)/b^(1/2)/d^(1/2)+3/8\*exp(a-b\*c/d)\*erfi(b^(1/2)\*(d\*x+c)^(1/2)/d^(1/2))\*Pi^(1/2)/b^(1/2)/d^(1/2)

**Rubi [A]** time = 0.37, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3312, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} + \frac{\sqrt{\frac{\pi}{3}} e^{\frac{3bc}{d}-3a} \operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} + \frac{3\sqrt{\pi} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} + \frac{\sqrt{\frac{\pi}{3}} e^{3a-\frac{3bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[a + b\*x]^3/Sqrt[c + d\*x], x]

[Out] (3\*E^(-a + (b\*c)/d)\*Sqrt[Pi]\*Erf[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(8\*Sqrt[b]\*Sqrt[d]) + (E^(-3\*a + (3\*b\*c)/d)\*Sqrt[Pi/3]\*Erf[(Sqrt[3]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(8\*Sqrt[b]\*Sqrt[d]) + (3\*E^(a - (b\*c)/d)\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(8\*Sqrt[b]\*Sqrt[d]) + (E^(3\*a - (3\*b\*c)/d)\*Sqrt[Pi/3]\*Erfi[(Sqrt[3]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(8\*Sqrt[b]\*Sqrt[d])

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3307

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)</sup>\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I/2, Int[(c + d\*x)<sup>m</sup>/(E^(I\*k\*Pi)\*E^(I\*(e + f\*x))), x], x] - Dist[I/2, Int[(c + d\*x)<sup>m</sup>\*E^(I\*k\*Pi)\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)]<sup>(n\_.)</sup>, x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)<sup>m</sup>, Sin[e + f\*x]<sup>n</sup>, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^3(a + bx)}{\sqrt{c + dx}} dx &= \int \left( \frac{3 \cosh(a + bx)}{4\sqrt{c + dx}} + \frac{\cosh(3a + 3bx)}{4\sqrt{c + dx}} \right) dx \\
 &= \frac{1}{4} \int \frac{\cosh(3a + 3bx)}{\sqrt{c + dx}} dx + \frac{3}{4} \int \frac{\cosh(a + bx)}{\sqrt{c + dx}} dx \\
 &= \frac{1}{8} \int \frac{e^{-i(3ia+3ibx)}}{\sqrt{c + dx}} dx + \frac{1}{8} \int \frac{e^{i(3ia+3ibx)}}{\sqrt{c + dx}} dx + \frac{3}{8} \int \frac{e^{-i(ia+ibx)}}{\sqrt{c + dx}} dx + \frac{3}{8} \int \frac{e^{i(ia+ibx)}}{\sqrt{c + dx}} dx \\
 &= \frac{\text{Subst} \left( \int e^{i(3ia - \frac{3ibc}{d}) - \frac{3bx^2}{d}} dx, x, \sqrt{c + dx} \right)}{4d} + \frac{\text{Subst} \left( \int e^{-i(3ia - \frac{3ibc}{d}) + \frac{3bx^2}{d}} dx, x, \sqrt{c + dx} \right)}{4d} + \dots \\
 &= \frac{3e^{-a + \frac{bc}{d}} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{b} \sqrt{c + dx}}{\sqrt{d}} \right)}{8\sqrt{b} \sqrt{d}} + \frac{e^{-3a + \frac{3bc}{d}} \sqrt{\frac{\pi}{3}} \operatorname{erf} \left( \frac{\sqrt{3} \sqrt{b} \sqrt{c + dx}}{\sqrt{d}} \right)}{8\sqrt{b} \sqrt{d}} + \frac{3e^{a - \frac{bc}{d}} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b} \sqrt{c + dx}}{\sqrt{d}} \right)}{8\sqrt{b} \sqrt{d}} + \dots
 \end{aligned}$$

**Mathematica [A]** time = 0.20, size = 192, normalized size = 0.84

$$\frac{e^{-3\left(a + \frac{bc}{d}\right)} \left( \sqrt{3} e^{6a} \sqrt{-\frac{b(c+dx)}{d}} \Gamma\left(\frac{1}{2}, -\frac{3b(c+dx)}{d}\right) + 9e^{4a + \frac{2bc}{d}} \sqrt{-\frac{b(c+dx)}{d}} \Gamma\left(\frac{1}{2}, -\frac{b(c+dx)}{d}\right) - e^{\frac{4bc}{d}} \sqrt{\frac{b(c+dx)}{d}} \left( 9e^{2a} \Gamma\left(\frac{1}{2}, \frac{b(c+dx)}{d}\right) \right) \right)}{24b\sqrt{c + dx}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]^3/Sqrt[c + d\*x],x]

[Out] (Sqrt[3]\*E^(6\*a)\*Sqrt[-((b\*(c + d\*x))/d)]\*Gamma[1/2, (-3\*b\*(c + d\*x))/d] + 9\*E^(4\*a + (2\*b\*c)/d)\*Sqrt[-((b\*(c + d\*x))/d)]\*Gamma[1/2, -((b\*(c + d\*x))/d)] - E^((4\*b\*c)/d)\*Sqrt[(b\*(c + d\*x))/d]\*(9\*E^(2\*a)\*Gamma[1/2, (b\*(c + d\*x))/d] + Sqrt[3]\*E^((2\*b\*c)/d)\*Gamma[1/2, (3\*b\*(c + d\*x))/d])/(24\*b\*E^(3\*(a + (b\*c)/d))\*Sqrt[c + d\*x])

**fricas** [A] time = 0.56, size = 253, normalized size = 1.11

$$\sqrt{3} \sqrt{\pi} \sqrt{\frac{b}{d}} \left( \cosh\left(-\frac{3(bc-ad)}{d}\right) - \sinh\left(-\frac{3(bc-ad)}{d}\right) \right) \operatorname{erf}\left(\sqrt{3} \sqrt{dx+c} \sqrt{\frac{b}{d}}\right) - \sqrt{3} \sqrt{\pi} \sqrt{-\frac{b}{d}} \left( \cosh\left(-\frac{3(bc-ad)}{d}\right) + \sinh\left(-\frac{3(bc-ad)}{d}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/24\*(sqrt(3)\*sqrt(pi)\*sqrt(b/d)\*(cosh(-3\*(b\*c - a\*d)/d) - sinh(-3\*(b\*c - a\*d)/d))\*erf(sqrt(3)\*sqrt(d\*x + c)\*sqrt(b/d)) - sqrt(3)\*sqrt(pi)\*sqrt(-b/d)\*(cosh(-3\*(b\*c - a\*d)/d) + sinh(-3\*(b\*c - a\*d)/d))\*erf(sqrt(3)\*sqrt(d\*x + c)\*sqrt(-b/d)) + 9\*sqrt(pi)\*sqrt(b/d)\*(cosh(-(b\*c - a\*d)/d) - sinh(-(b\*c - a\*d)/d))\*erf(sqrt(d\*x + c)\*sqrt(b/d)) - 9\*sqrt(pi)\*sqrt(-b/d)\*(cosh(-(b\*c - a\*d)/d) + sinh(-(b\*c - a\*d)/d))\*erf(sqrt(d\*x + c)\*sqrt(-b/d))/b

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(bx + a)}{\sqrt{dx + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(cosh(b\*x + a)^3/sqrt(d\*x + c), x)

**maple** [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(bx + a)}{\sqrt{dx + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)^3/(d\*x+c)^(1/2),x)

[Out] int(cosh(b\*x+a)^3/(d\*x+c)^(1/2),x)

**maxima** [A] time = 0.44, size = 177, normalized size = 0.78

$$\frac{\sqrt{3} \sqrt{\pi} \operatorname{erf}\left(\sqrt{3} \sqrt{dx+c} \sqrt{-\frac{b}{d}}\right) e^{\left(3a-\frac{3bc}{d}\right)}}{\sqrt{-\frac{b}{d}}} + \frac{\sqrt{3} \sqrt{\pi} \operatorname{erf}\left(\sqrt{3} \sqrt{dx+c} \sqrt{\frac{b}{d}}\right) e^{\left(-3a+\frac{3bc}{d}\right)}}{\sqrt{\frac{b}{d}}} + \frac{9 \sqrt{\pi} \operatorname{erf}\left(\sqrt{dx+c} \sqrt{-\frac{b}{d}}\right) e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}} + \frac{9 \sqrt{\pi} \operatorname{erf}\left(\sqrt{dx+c} \sqrt{\frac{b}{d}}\right) e^{\left(-a+\frac{bc}{d}\right)}}{\sqrt{\frac{b}{d}}}$$

$24d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c)^(1/2),x, algorithm="maxima")

[Out]  $\frac{1}{24} \sqrt{3} \sqrt{\pi} \operatorname{erf}\left(\sqrt{3} \sqrt{dx+c} \sqrt{-\frac{b}{d}}\right) e^{\left(3a-\frac{3bc}{d}\right)} + \sqrt{3} \sqrt{\pi} \operatorname{erf}\left(\sqrt{3} \sqrt{dx+c} \sqrt{\frac{b}{d}}\right) e^{\left(-3a+\frac{3bc}{d}\right)} + 9 \sqrt{\pi} \operatorname{erf}\left(\sqrt{dx+c} \sqrt{-\frac{b}{d}}\right) e^{\left(a-\frac{bc}{d}\right)} + 9 \sqrt{\pi} \operatorname{erf}\left(\sqrt{dx+c} \sqrt{\frac{b}{d}}\right) e^{\left(-a+\frac{bc}{d}\right)}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(a+bx)^3}{\sqrt{c+dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a+b\*x)^3/(c+d\*x)^(1/2),x)

[Out] int(cosh(a+b\*x)^3/(c+d\*x)^(1/2),x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*\*3/(d\*x+c)\*\*(1/2),x)

[Out] Integral(cosh(a+b\*x)\*\*3/sqrt(c+d\*x),x)

$$3.60 \quad \int \frac{\cosh^3(a+bx)}{(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=246

$$\frac{3\sqrt{\pi} \sqrt{b} e^{\frac{bc}{d}-a} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{\sqrt{3\pi} \sqrt{b} e^{\frac{3bc}{d}-3a} \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{3\sqrt{\pi} \sqrt{b} e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{\sqrt{3\pi} \sqrt{b} e^{3a-\frac{3bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}$$

[Out]  $-3/4*\exp(-a+b*c/d)*\operatorname{erf}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*b^{(1/2)}*\pi^{(1/2)}/d^{(3/2)}+3/4*\exp(a-b*c/d)*\operatorname{erfi}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*b^{(1/2)}*\pi^{(1/2)}/d^{(3/2)}-1/4*\exp(-3*a+3*b*c/d)*\operatorname{erf}(3^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*b^{(1/2)}*3^{(1/2)}*\pi^{(1/2)}/d^{(3/2)}+1/4*\exp(3*a-3*b*c/d)*\operatorname{erfi}(3^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*b^{(1/2)}*3^{(1/2)}*\pi^{(1/2)}/d^{(3/2)}-2*\cosh(b*x+a)^3/d/(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.42, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3313, 3308, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi} \sqrt{b} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{\sqrt{3\pi} \sqrt{b} e^{\frac{3bc}{d}-3a} \operatorname{Erf}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{3\sqrt{\pi} \sqrt{b} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{\sqrt{3\pi} \sqrt{b} e^{3a-\frac{3bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cosh}[a + b*x]^3/(c + d*x)^{(3/2)}, x]$

[Out]  $(-2*\operatorname{Cosh}[a + b*x]^3)/(d*\operatorname{Sqrt}[c + d*x]) - (3*\operatorname{Sqrt}[b]*E^{(-a + (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/(4*d^{(3/2)})) - (\operatorname{Sqrt}[b]*E^{(-3*a + (3*b*c)/d)}*\operatorname{Sqrt}[3*\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/(4*d^{(3/2)})) + (3*\operatorname{Sqrt}[b]*E^{(a - (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/(4*d^{(3/2)})) + (\operatorname{Sqrt}[b]*E^{(3*a - (3*b*c)/d)}*\operatorname{Sqrt}[3*\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/(4*d^{(3/2)}))$

**Rule 2180**

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}[\{F, c, d, e, f, g\}, x] \&\amp; !\$UseGamma == True$

**Rule 2204**

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}[\{$

F, a, b, c, d}, x] && PosQ[b]

### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

### Rule 3308

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I/2, Int[(c + d\*x)<sup>m</sup>/E^(I\*(e + f\*x)), x], x] - Dist[I/2, Int[(c + d\*x)<sup>m</sup>\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

### Rule 3313

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)]<sup>(n\_)</sup>, x\_Symbol] := Simp[((c + d\*x)<sup>(m + 1)</sup>\*Sin[e + f\*x]<sup>(n)</sup>/(d\*(m + 1)), x] - Dist[(f\*n)/(d\*(m + 1)), Int[ExpandTrigReduce[(c + d\*x)<sup>(m + 1)</sup>, Cos[e + f\*x]\*Sin[e + f\*x]<sup>(n - 1)</sup>, x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^3(a + bx)}{(c + dx)^{3/2}} dx &= -\frac{2 \cosh^3(a + bx)}{d\sqrt{c + dx}} + \frac{(6ib) \int \left( -\frac{i \sinh(a + bx)}{4\sqrt{c + dx}} - \frac{i \sinh(3a + 3bx)}{4\sqrt{c + dx}} \right) dx}{d} \\
 &= -\frac{2 \cosh^3(a + bx)}{d\sqrt{c + dx}} + \frac{(3b) \int \frac{\sinh(a + bx)}{\sqrt{c + dx}} dx}{2d} + \frac{(3b) \int \frac{\sinh(3a + 3bx)}{\sqrt{c + dx}} dx}{2d} \\
 &= -\frac{2 \cosh^3(a + bx)}{d\sqrt{c + dx}} + \frac{(3b) \int \frac{e^{-i(ia + ibx)}}{\sqrt{c + dx}} dx}{4d} - \frac{(3b) \int \frac{e^{i(ia + ibx)}}{\sqrt{c + dx}} dx}{4d} + \frac{(3b) \int \frac{e^{-i(3ia + 3ibx)}}{\sqrt{c + dx}} dx}{4d} - \frac{(3b) \int \frac{e^{i(3ia + 3ibx)}}{\sqrt{c + dx}} dx}{4d} \\
 &= -\frac{2 \cosh^3(a + bx)}{d\sqrt{c + dx}} - \frac{(3b) \text{Subst} \left( \int e^{i \left( 3ia - \frac{3ibc}{d} \right) - \frac{3bx^2}{d}} dx, x, \sqrt{c + dx} \right)}{2d^2} - \frac{(3b) \text{Subst} \left( \int e^{i \left( ia - \frac{ibc}{d} \right) - \frac{bx^2}{d}} dx, x, \sqrt{c + dx} \right)}{2d^2} \\
 &= -\frac{2 \cosh^3(a + bx)}{d\sqrt{c + dx}} - \frac{3\sqrt{b} e^{-a + \frac{bc}{d}} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{b} \sqrt{c + dx}}{\sqrt{d}} \right)}{4d^{3/2}} - \frac{\sqrt{b} e^{-3a + \frac{3bc}{d}} \sqrt{3\pi} \operatorname{erf} \left( \frac{\sqrt{3} \sqrt{b} \sqrt{c + dx}}{\sqrt{d}} \right)}{4d^{3/2}} + \dots
 \end{aligned}$$

**Mathematica [B]** time = 2.89, size = 717, normalized size = 2.91

$$e^{-\frac{3b(c+dx)}{d}} \left( \sqrt{3\pi} \sqrt{b} \sqrt{c+dx} e^{\frac{3b(c+dx)}{d}} \sinh\left(3a - \frac{3bc}{d}\right) \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right) + \sqrt{3\pi} \sqrt{b} \sqrt{c+dx} e^{\frac{3b(c+dx)}{d}} \sinh\left(3a - \frac{3bc}{d}\right) \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]^3/(c + d\*x)^(3/2), x]

[Out] 
$$\begin{aligned} & \left( -\sqrt{d} \operatorname{Cosh}\left[3a - \frac{3bc}{d}\right] - \sqrt{d} E^{\left(\frac{6b(c+dx)}{d}\right)} \operatorname{Cosh}\left[3a - \frac{3bc}{d}\right] - 3\sqrt{d} E^{\left(\frac{2b(c+dx)}{d}\right)} \operatorname{Cosh}\left[a - \frac{bc}{d}\right] - 3\sqrt{d} E^{\left(\frac{4b(c+dx)}{d}\right)} \operatorname{Cosh}\left[a - \frac{bc}{d}\right] + \sqrt{3} \sqrt{d} E^{\left(\frac{3b(c+dx)}{d}\right)} \sqrt{-\frac{b(c+dx)}{d}} \operatorname{Cosh}\left[3a - \frac{3bc}{d}\right] \Gamma\left[\frac{1}{2}, -\frac{3b(c+dx)}{d}\right] + 3\sqrt{d} E^{\left(\frac{3b(c+dx)}{d}\right)} \sqrt{\frac{b(c+dx)}{d}} \operatorname{Cosh}\left[a - \frac{bc}{d}\right] \Gamma\left[\frac{1}{2}, \frac{b(c+dx)}{d}\right] + \sqrt{3} \sqrt{d} E^{\left(\frac{3b(c+dx)}{d}\right)} \sqrt{\frac{b(c+dx)}{d}} \operatorname{Cosh}\left[3a - \frac{3bc}{d}\right] \Gamma\left[\frac{1}{2}, \frac{3b(c+dx)}{d}\right] + \sqrt{d} \operatorname{Sinh}\left[3a - \frac{3bc}{d}\right] - \sqrt{d} E^{\left(\frac{6b(c+dx)}{d}\right)} \operatorname{Sinh}\left[3a - \frac{3bc}{d}\right] + \sqrt{b} E^{\left(\frac{3b(c+dx)}{d}\right)} \sqrt{3\pi} \sqrt{c+dx} \operatorname{Erf}\left[\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right] \operatorname{Sinh}\left[3a - \frac{3bc}{d}\right] + \sqrt{b} E^{\left(\frac{3b(c+dx)}{d}\right)} \sqrt{3\pi} \sqrt{c+dx} \operatorname{Erfi}\left[\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right] \operatorname{Sinh}\left[3a - \frac{3bc}{d}\right] + 3\sqrt{d} E^{\left(\frac{2b(c+dx)}{d}\right)} \operatorname{Sinh}\left[a - \frac{bc}{d}\right] - 3\sqrt{d} E^{\left(\frac{4b(c+dx)}{d}\right)} \operatorname{Sinh}\left[a - \frac{bc}{d}\right] - 3\sqrt{d} E^{\left(\frac{3b(c+dx)}{d}\right)} \sqrt{\frac{b(c+dx)}{d}} \Gamma\left[\frac{1}{2}, \frac{b(c+dx)}{d}\right] \operatorname{Sinh}\left[a - \frac{bc}{d}\right] + 3\sqrt{d} E^{\left(\frac{3b(c+dx)}{d}\right)} \sqrt{-\frac{b(c+dx)}{d}} \Gamma\left[\frac{1}{2}, -\frac{b(c+dx)}{d}\right] \left(\operatorname{Cosh}\left[a - \frac{bc}{d}\right] + \operatorname{Sinh}\left[a - \frac{bc}{d}\right]\right) \right) / (4d^{3/2} E^{\left(\frac{3b(c+dx)}{d}\right)} \sqrt{c+dx}) \end{aligned}$$

**fricas [B]** time = 0.55, size = 1344, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c)^(3/2), x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -\frac{1}{4} \sqrt{3} \sqrt{\pi} \left( (d*x+c) \operatorname{cosh}(b*x+a)^3 \operatorname{cosh}\left(-\frac{3(bc-ad)}{d}\right) - (d*x+c) \operatorname{cosh}(b*x+a)^3 \operatorname{sinh}\left(-\frac{3(bc-ad)}{d}\right) + ((d*x+c) \operatorname{cosh}\left(-\frac{3(bc-ad)}{d}\right) - (d*x+c) \operatorname{sinh}\left(-\frac{3(bc-ad)}{d}\right)) \operatorname{sinh}(b*x+a)^3 + 3((d*x+c) \operatorname{cosh}(b*x+a) \operatorname{cosh}\left(-\frac{3(bc-ad)}{d}\right) - (d*x+c) \operatorname{cosh}(b*x+a) \operatorname{sinh}\left(-\frac{3(bc-ad)}{d}\right)) \operatorname{sinh}(b*x+a)^2 + 3((d*x+c) \operatorname{cosh}(b*x+a)^2 \operatorname{cosh}\left(-\frac{3(bc-ad)}{d}\right) - (d*x+c) \operatorname{cosh}(b*x+a)^2 \operatorname{sinh}\left(-\frac{3(bc-ad)}{d}\right)) \operatorname{sinh}(b*x+a) \right) \sqrt{b/d} \operatorname{erf}\left(\sqrt{3} \sqrt{d*x+c} \sqrt{b/d}\right) + \sqrt{3} \sqrt{\pi} \left( (d*x+c) \operatorname{cosh}(b*x+a)^3 \operatorname{cosh}\left(-\frac{3(bc-ad)}{d}\right) + (d*x+c) \operatorname{cosh}(b*x+a)^3 \operatorname{sinh}\left(-\frac{3(bc-ad)}{d}\right) + ((d*x+c) \operatorname{cosh}\left(-\frac{3(bc-ad)}{d}\right) + (d*x+c) \operatorname{sinh}\left(-\frac{3(bc-ad)}{d}\right)) \operatorname{sinh}(b*x+a)^3 + 3((d*x+c) \operatorname{cosh}(b*x+a) \operatorname{cosh}\left(-\frac{3(bc-ad)}{d}\right) - \right. \end{aligned}$$



$a*d)/d) + (d*x + c)*\cosh(b*x + a)*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a)^2 +$   
 $3*((d*x + c)*\cosh(b*x + a)^2*\cosh(-3*(b*c - a*d)/d) + (d*x + c)*\cosh(b*x +$   
 $a)^2*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{-b/d}*erf(\sqrt{3}*\sqrt{d*x$   
 $+ c)*\sqrt{-b/d}) + 3*\sqrt{\pi}*((d*x + c)*\cosh(b*x + a)^3*\cosh(-(b*c - a*d$   
 $)/d) - (d*x + c)*\cosh(b*x + a)^3*\sinh(-(b*c - a*d)/d) + ((d*x + c)*\cosh(-(b$   
 $*c - a*d)/d) - (d*x + c)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)^3 + 3*((d*x +$   
 $c)*\cosh(b*x + a)*\cosh(-(b*c - a*d)/d) - (d*x + c)*\cosh(b*x + a)*\sinh(-(b*c$   
 $- a*d)/d))*\sinh(b*x + a)^2 + 3*((d*x + c)*\cosh(b*x + a)^2*\cosh(-(b*c - a*d)$   
 $/d) - (d*x + c)*\cosh(b*x + a)^2*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{b$   
 $/d}*erf(\sqrt{d*x + c)*\sqrt{b/d}) + 3*\sqrt{\pi}*((d*x + c)*\cosh(b*x + a)^3*\co$   
 $sh(-(b*c - a*d)/d) + (d*x + c)*\cosh(b*x + a)^3*\sinh(-(b*c - a*d)/d) + ((d*x$   
 $+ c)*\cosh(-(b*c - a*d)/d) + (d*x + c)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)^$   
 $3 + 3*((d*x + c)*\cosh(b*x + a)*\cosh(-(b*c - a*d)/d) + (d*x + c)*\cosh(b*x +$   
 $a)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)^2 + 3*((d*x + c)*\cosh(b*x + a)^2*\cos$   
 $h(-(b*c - a*d)/d) + (d*x + c)*\cosh(b*x + a)^2*\sinh(-(b*c - a*d)/d))*\sinh(b*$   
 $x + a))*\sqrt{-b/d}*erf(\sqrt{d*x + c)*\sqrt{-b/d}) + (\cosh(b*x + a)^6 + 6*\cos$   
 $h(b*x + a)*\sinh(b*x + a)^5 + \sinh(b*x + a)^6 + 3*(5*\cosh(b*x + a)^2 + 1)*\si$   
 $nh(b*x + a)^4 + 3*\cosh(b*x + a)^4 + 4*(5*\cosh(b*x + a)^3 + 3*\cosh(b*x + a))$   
 $*\sinh(b*x + a)^3 + 3*(5*\cosh(b*x + a)^4 + 6*\cosh(b*x + a)^2 + 1)*\sinh(b*x +$   
 $a)^2 + 3*\cosh(b*x + a)^2 + 6*(\cosh(b*x + a)^5 + 2*\cosh(b*x + a)^3 + \cosh(b$   
 $*x + a))*\sinh(b*x + a) + 1))*\sqrt{d*x + c}))/((d^2*x + c*d)*\cosh(b*x + a)^3 +$   
 $3*(d^2*x + c*d)*\cosh(b*x + a)^2*\sinh(b*x + a) + 3*(d^2*x + c*d)*\cosh(b*x +$   
 $a)*\sinh(b*x + a)^2 + (d^2*x + c*d)*\sinh(b*x + a)^3)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(bx + a)}{(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(cosh(b\*x + a)^3/(d\*x + c)^(3/2), x)

**maple** [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(bx + a)}{(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)^3/(d\*x+c)^(3/2),x)

[Out] int(cosh(b\*x+a)^3/(d\*x+c)^(3/2),x)

**maxima** [A] time = 0.47, size = 196, normalized size = 0.80

$$\frac{\sqrt{3} \sqrt{\frac{(dx+c)b}{d}} e^{\left(\frac{3(bc-ad)}{d}\right)} \Gamma\left(-\frac{1}{2}, \frac{3(dx+c)b}{d}\right)}{\sqrt{dx+c}} + \frac{\sqrt{3} \sqrt{-\frac{(dx+c)b}{d}} e^{\left(-\frac{3(bc-ad)}{d}\right)} \Gamma\left(-\frac{1}{2}, -\frac{3(dx+c)b}{d}\right)}{\sqrt{dx+c}} + \frac{3 \sqrt{\frac{(dx+c)b}{d}} e^{\left(-a+\frac{bc}{d}\right)} \Gamma\left(-\frac{1}{2}, \frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}} + \frac{3 \sqrt{-\frac{(dx+c)b}{d}} e^{\left(a-\frac{bc}{d}\right)} \Gamma\left(-\frac{1}{2}, -\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}$$

$8d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c)^(3/2), x, algorithm="maxima")

[Out]  $-1/8 * (\sqrt{3} * \sqrt{(d*x + c)*b/d}) * e^{(3*(b*c - a*d)/d)} * \text{gamma}(-1/2, 3*(d*x + c)*b/d) / \sqrt{d*x + c} + \sqrt{3} * \sqrt{-(d*x + c)*b/d} * e^{(-3*(b*c - a*d)/d)} * \text{gamma}(-1/2, -3*(d*x + c)*b/d) / \sqrt{d*x + c} + 3 * \sqrt{(d*x + c)*b/d} * e^{(-a + b*c/d)} * \text{gamma}(-1/2, (d*x + c)*b/d) / \sqrt{d*x + c} + 3 * \sqrt{-(d*x + c)*b/d} * e^{(a - b*c/d)} * \text{gamma}(-1/2, -(d*x + c)*b/d) / \sqrt{d*x + c} / d$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(a + bx)^3}{(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^3/(c + d\*x)^(3/2), x)

[Out] int(cosh(a + b\*x)^3/(c + d\*x)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(a + bx)}{(c + dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*\*3/(d\*x+c)\*\*(3/2), x)

[Out] Integral(cosh(a + b\*x)\*\*3/(c + d\*x)\*\*(3/2), x)

$$3.61 \quad \int \frac{\cosh^3(a+bx)}{(c+dx)^{5/2}} dx$$

**Optimal.** Leaf size=277

$$\frac{\sqrt{\pi} b^{3/2} e^{\frac{bc}{d}-a} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{\sqrt{3\pi} b^{3/2} e^{\frac{3bc}{d}-3a} \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{\sqrt{\pi} b^{3/2} e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{\sqrt{3\pi} b^{3/2} e^{3a-\frac{3bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}$$

[Out]  $-2/3 \cosh(b*x+a)^3/d/(d*x+c)^{(3/2)+1/2*b^{(3/2)*exp(-a+b*c/d)*erf(b^{(1/2)*(d*x+c)^{(1/2)/d^{(1/2)}}*Pi^{(1/2)/d^{(5/2)+1/2*b^{(3/2)*exp(a-b*c/d)*erfi(b^{(1/2)*(d*x+c)^{(1/2)/d^{(1/2)}}*Pi^{(1/2)/d^{(5/2)+1/2*b^{(3/2)*exp(-3*a+3*b*c/d)*erf(3^{(1/2)*b^{(1/2)*(d*x+c)^{(1/2)/d^{(1/2)}}*3^{(1/2)*Pi^{(1/2)/d^{(5/2)+1/2*b^{(3/2)*exp(3*a-3*b*c/d)*erfi(3^{(1/2)*b^{(1/2)*(d*x+c)^{(1/2)/d^{(1/2)}}*3^{(1/2)*Pi^{(1/2)/d^{(5/2)-4*b*cosh(b*x+a)^2*sinh(b*x+a)/d^2/(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.60, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3314, 3307, 2180, 2204, 2205, 3312}

$$\frac{\sqrt{\pi} b^{3/2} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{\sqrt{3\pi} b^{3/2} e^{\frac{3bc}{d}-3a} \operatorname{Erf}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{\sqrt{\pi} b^{3/2} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{\sqrt{3\pi} b^{3/2} e^{3a-\frac{3bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cosh}[a + b*x]^3/(c + d*x)^{(5/2)}, x]$

[Out]  $(-2*\operatorname{Cosh}[a + b*x]^3)/(3*d*(c + d*x)^{(3/2)}) + (b^{(3/2)*E^{(-a + (b*c)/d)*\operatorname{Sqrt}[Pi]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/(2*d^{(5/2)})} + (b^{(3/2)*E^{(-3*a + (3*b*c)/d)*\operatorname{Sqrt}[3*Pi]*\operatorname{Erf}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/(2*d^{(5/2)})} + (b^{(3/2)*E^{(a - (b*c)/d)*\operatorname{Sqrt}[Pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/(2*d^{(5/2)})} + (b^{(3/2)*E^{(3*a - (3*b*c)/d)*\operatorname{Sqrt}[3*Pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/(2*d^{(5/2)})} - (4*b*\operatorname{Cosh}[a + b*x]^2*\operatorname{Sinh}[a + b*x])/(d^2*\operatorname{Sqrt}[c + d*x])$

**Rule 2180**

$\operatorname{Int}[(F\_.)^{((g\_.)*((e\_.) + (f\_.)*(x\_)))/\operatorname{Sqrt}[(c\_.) + (d\_.)*(x\_)]}, x\_Symbol] : > \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \&\& !\$UseGamma == True$

**Rule 2204**

$\operatorname{Int}[(F\_.)^{((a\_.) + (b\_.)*((c\_.) + (d\_.)*(x\_))^2)}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[Pi]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{$

F, a, b, c, d}, x] && PosQ[b]

### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F<sup>a</sup>\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

### Rule 3307

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)</sup>\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I/2, Int[(c + d\*x)<sup>m</sup>/(E<sup>(I\*k\*Pi)</sup>\*E<sup>(I\*(e + f\*x))</sup>), x], x] - Dist[I/2, Int[(c + d\*x)<sup>m</sup>\*E<sup>(I\*k\*Pi)</sup>\*E<sup>(I\*(e + f\*x))</sup>, x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)]<sup>(n\_)</sup>, x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)<sup>m</sup>, Sin[e + f\*x]<sup>n</sup>, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rule 3314

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_)</sup>\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]<sup>(n\_)</sup>, x\_Symbol] := Simp[((c + d\*x)<sup>(m + 1)</sup>\*(b\*Sin[e + f\*x])<sup>n</sup>/(d\*(m + 1)), x] + (Dist[(b<sup>2</sup>\*f<sup>2</sup>\*n\*(n - 1))/(d<sup>2</sup>\*(m + 1)\*(m + 2)), Int[(c + d\*x)<sup>(m + 2)</sup>\*(b\*Sin[e + f\*x])<sup>(n - 2)</sup>, x], x] - Dist[(f<sup>2</sup>\*n<sup>2</sup>)/(d<sup>2</sup>\*(m + 1)\*(m + 2)), Int[(c + d\*x)<sup>(m + 2)</sup>\*(b\*Sin[e + f\*x])<sup>n</sup>, x], x] - Simp[(b\*f\*n\*(c + d\*x)<sup>(m + 2)</sup>\*Cos[e + f\*x]\*(b\*Sin[e + f\*x])<sup>(n - 1)</sup>/(d<sup>2</sup>\*(m + 1)\*(m + 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(a+bx)}{(c+dx)^{5/2}} dx &= -\frac{2 \cosh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b \cosh^2(a+bx) \sinh(a+bx)}{d^2 \sqrt{c+dx}} - \frac{(8b^2) \int \frac{\cosh(a+bx)}{\sqrt{c+dx}} dx}{d^2} + \frac{(12b^2) \int \frac{\cosh(a+bx)}{\sqrt{c+dx}} dx}{d^2} \\
&= -\frac{2 \cosh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b \cosh^2(a+bx) \sinh(a+bx)}{d^2 \sqrt{c+dx}} - \frac{(4b^2) \int \frac{e^{-i(i+ibx)}}{\sqrt{c+dx}} dx}{d^2} - \frac{(4b^2) \int \frac{e^{i(i+ibx)}}{\sqrt{c+dx}} dx}{d^2} \\
&= -\frac{2 \cosh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b \cosh^2(a+bx) \sinh(a+bx)}{d^2 \sqrt{c+dx}} - \frac{(8b^2) \text{Subst} \left( \int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}} dx, x, \sqrt{c+dx} \right)}{d^3} \\
&= -\frac{2 \cosh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b^{3/2} e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{d^{5/2}} - \frac{4b^{3/2} e^{a-\frac{bc}{d}} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{d^{5/2}} - \frac{4b^2 c}{d^3} \\
&= -\frac{2 \cosh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b^{3/2} e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{d^{5/2}} - \frac{4b^{3/2} e^{a-\frac{bc}{d}} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{d^{5/2}} - \frac{4b^2 c}{d^3} \\
&= -\frac{2 \cosh^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{b^{3/2} e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{2d^{5/2}} + \frac{b^{3/2} e^{-3a+\frac{3bc}{d}} \sqrt{3\pi} \operatorname{erf} \left( \frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{2d^{5/2}} + \dots
\end{aligned}$$

**Mathematica [A]** time = 2.91, size = 253, normalized size = 0.91

$$\frac{e^{-3\left(a+\frac{bc}{d}\right)} \left( -3\sqrt{3} e^{6a} d \left( -\frac{b(c+dx)}{d} \right)^{3/2} \Gamma\left(\frac{1}{2}, -\frac{3b(c+dx)}{d}\right) - 3d e^{4a+\frac{2bc}{d}} \left( -\frac{b(c+dx)}{d} \right)^{3/2} \Gamma\left(\frac{1}{2}, -\frac{b(c+dx)}{d}\right) - 3d e^{2a+\frac{4bc}{d}} \left( \frac{b(c+dx)}{d} \right)^{3/2} \right)}{6d^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]^3/(c + d\*x)^(5/2), x]

[Out]  $(-3\sqrt{3} e^{6a} d \left( -\frac{b(c+dx)}{d} \right)^{3/2} \Gamma\left(\frac{1}{2}, -\frac{3b(c+dx)}{d}\right) - 3d e^{4a+\frac{2bc}{d}} \left( -\frac{b(c+dx)}{d} \right)^{3/2} \Gamma\left(\frac{1}{2}, -\frac{b(c+dx)}{d}\right) - 3d e^{2a+\frac{4bc}{d}} \left( \frac{b(c+dx)}{d} \right)^{3/2} \Gamma\left(\frac{1}{2}, \frac{3b(c+dx)}{d}\right) - 4e^{3(a+(b*c)/d)} \operatorname{Cosh}[a+b*x]^2 (d \operatorname{Cosh}[a+b*x] + 6*b*(c+d*x) \operatorname{Sinh}[a+b*x]) / (6*d^2 * e^{3(a+(b*c)/d)} * (c+d*x)^{3/2})$

**fricas [B]** time = 0.71, size = 2058, normalized size = 7.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c)^(5/2),x, algorithm="fricas")

[Out]  $\frac{1}{12} \cdot (6 \sqrt{3}) \sqrt{\pi} \cdot ((b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a)^3 \cosh(-3(b c - a d)/d) - (b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a)^3 \sinh(-3(b c - a d)/d) + ((b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(-3(b c - a d)/d) - (b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \sinh(-3(b c - a d)/d)) \sinh(b x + a)^3 + 3 \cdot ((b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a) \cosh(-3(b c - a d)/d) - (b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a) \sinh(-3(b c - a d)/d)) \sinh(b x + a)^2 + 3 \cdot ((b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a)^2 \cosh(-3(b c - a d)/d) - (b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a)^2 \sinh(-3(b c - a d)/d)) \sinh(b x + a) \sqrt{b/d} \operatorname{erf}(\sqrt{3} \sqrt{d x + c}) \sqrt{b/d} - 6 \sqrt{3} \sqrt{\pi} \cdot ((b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a)^3 \cosh(-3(b c - a d)/d) + (b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a)^3 \sinh(-3(b c - a d)/d) + ((b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(-3(b c - a d)/d) - (b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \sinh(-3(b c - a d)/d)) \sinh(b x + a)^3 + 3 \cdot ((b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a) \cosh(-3(b c - a d)/d) + (b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a) \sinh(-3(b c - a d)/d)) \sinh(b x + a)^2 + 3 \cdot ((b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a)^2 \cosh(-3(b c - a d)/d) + (b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a)^2 \sinh(-3(b c - a d)/d)) \sinh(b x + a) \sqrt{-b/d} \operatorname{erf}(\sqrt{3} \sqrt{d x + c}) \sqrt{-b/d} + 6 \sqrt{\pi} \cdot ((b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a)^3 \cosh(-(b c - a d)/d) - (b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a)^3 \sinh(-(b c - a d)/d) + ((b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(-(b c - a d)/d) - (b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \sinh(-(b c - a d)/d)) \sinh(b x + a)^3 + 3 \cdot ((b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a) \cosh(-(b c - a d)/d) - (b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a) \sinh(-(b c - a d)/d)) \sinh(b x + a)^2 + 3 \cdot ((b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a)^2 \cosh(-(b c - a d)/d) - (b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a)^2 \sinh(-(b c - a d)/d)) \sinh(b x + a) \sqrt{b/d} \operatorname{erf}(\sqrt{d x + c}) \sqrt{b/d} - 6 \sqrt{\pi} \cdot ((b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a)^3 \cosh(-(b c - a d)/d) + (b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a)^3 \sinh(-(b c - a d)/d) + ((b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(-(b c - a d)/d) + (b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \sinh(-(b c - a d)/d)) \sinh(b x + a)^3 + 3 \cdot ((b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a) \cosh(-(b c - a d)/d) + (b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a) \sinh(-(b c - a d)/d)) \sinh(b x + a)^2 + 3 \cdot ((b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a)^2 \cosh(-(b c - a d)/d) + (b^2 d^2 x^2 + 2 b c d x + b^2 c^2) \cosh(b x + a)^2 \sinh(-(b c - a d)/d)) \sinh(b x + a) \sqrt{-b/d} \operatorname{erf}(\sqrt{d x + c}) \sqrt{-b/d} - ((6 b d x + 6 b c + d) \cosh(b x + a)^6 + 6(6 b d x + 6 b c + d) \cosh(b x + a) \sinh(b x + a)^5 + (6 b d x + 6 b c + d) \sinh(b x + a)^6 + 3(2 b d x + 2 b c + d) \cosh(b x + a)^4 + 3(2 b d x + 5(6 b d x + 6 b c + d) \cosh(b x + a)^2 + 2 b c + d) \sinh(b x + a)^4 + 4(5(6 b d x + 6 b c + d) \cosh(b x + a)^3 + 3(2 b d x + 2 b c + d) \cosh(b x + a)) \sinh(b x + a)^3 - 6 b d x - 3(2 b d x + 2 b c - d) \cosh(b x + a)^2 + 3(5(6 b d x + 6 b c +$

$d) \cosh(bx + a)^4 - 2bdx + 6(2bdx + 2bc + d) \cosh(bx + a)^2 - 2bc + d) \sinh(bx + a)^2 - 6bc + 6((6bdx + 6bc + d) \cosh(bx + a)^5 + 2(2bdx + 2bc + d) \cosh(bx + a)^3 - (2bdx + 2bc - d) \cosh(bx + a) \sinh(bx + a) + d) \sqrt{dx + c}) / ((d^4x^2 + 2cd^3x + c^2d^2) \cosh(bx + a)^3 + 3(d^4x^2 + 2cd^3x + c^2d^2) \cosh(bx + a)^2 \sinh(bx + a) + 3(d^4x^2 + 2cd^3x + c^2d^2) \cosh(bx + a) \sinh(bx + a)^2 + (d^4x^2 + 2cd^3x + c^2d^2) \sinh(bx + a)^3)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx + a)^3}{(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c)^(5/2),x, algorithm="giac")

[Out] integrate(cosh(b\*x + a)^3/(d\*x + c)^(5/2), x)

**maple** [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(bx + a)}{(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)^3/(d\*x+c)^(5/2),x)

[Out] int(cosh(b\*x+a)^3/(d\*x+c)^(5/2),x)

**maxima** [A] time = 0.47, size = 194, normalized size = 0.70

$$\frac{3 \left( \frac{\sqrt{3} \left( \frac{dx+c}{d} \right)^{\frac{3}{2}} e^{\left( \frac{3(bc-ad)}{d} \right)} \Gamma\left( -\frac{3}{2}, \frac{3(dx+c)b}{d} \right)}{(dx+c)^{\frac{3}{2}}} + \frac{\sqrt{3} \left( -\frac{dx+c}{d} \right)^{\frac{3}{2}} e^{\left( -\frac{3(bc-ad)}{d} \right)} \Gamma\left( -\frac{3}{2}, -\frac{3(dx+c)b}{d} \right)}{(dx+c)^{\frac{3}{2}}} + \frac{\left( \frac{dx+c}{d} \right)^{\frac{3}{2}} e^{\left( -a + \frac{bc}{d} \right)} \Gamma\left( -\frac{3}{2}, \frac{(dx+c)b}{d} \right)}{(dx+c)^{\frac{3}{2}}} + \frac{\left( -\frac{dx+c}{d} \right)^{\frac{3}{2}} e^{\left( a - \frac{bc}{d} \right)} \Gamma\left( -\frac{3}{2}, -\frac{(dx+c)b}{d} \right)}{(dx+c)^{\frac{3}{2}}} \right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c)^(5/2),x, algorithm="maxima")

[Out]  $-3/8 * (\sqrt{3} * ((dx + c) * b/d)^{(3/2)} * e^{(3 * (b * c - a * d)/d)} * \text{gamma}(-3/2, 3 * (dx + c) * b/d) / (dx + c)^{(3/2)} + \sqrt{3} * (- (dx + c) * b/d)^{(3/2)} * e^{(-3 * (b * c - a * d)/d)} * \text{gamma}(-3/2, -3 * (dx + c) * b/d) / (dx + c)^{(3/2)} + ((dx + c) * b/d)^{(3/2)} * e^{(-a + b * c/d)} * \text{gamma}(-3/2, (dx + c) * b/d) / (dx + c)^{(3/2)} + (- (dx + c) * b/d)^{(3/2)} * e^{(a - b * c/d)} * \text{gamma}(-3/2, - (dx + c) * b/d) / (dx + c)^{(3/2)}) / d$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(a + bx)^3}{(c + dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^3/(c + d\*x)^(5/2), x)

[Out] int(cosh(a + b\*x)^3/(c + d\*x)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(a + bx)}{(c + dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*\*3/(d\*x+c)\*\*(5/2), x)

[Out] Integral(cosh(a + b\*x)\*\*3/(c + d\*x)\*\*(5/2), x)



$$3.62 \quad \int \frac{\cosh^3(a+bx)}{(c+dx)^{7/2}} dx$$

**Optimal.** Leaf size=331

$$\frac{\sqrt{\pi} b^{5/2} e^{\frac{bc}{d}-a} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{3\sqrt{3\pi} b^{5/2} e^{\frac{3bc}{d}-3a} \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{\sqrt{\pi} b^{5/2} e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{3\sqrt{3\pi} b^{5/2} e^{3a-\frac{3b}{d}}}{5d^{7/2}}$$

[Out]  $-2/5*\cosh(b*x+a)^3/d/(d*x+c)^{(5/2)}-4/5*b*\cosh(b*x+a)^2*\sinh(b*x+a)/d^2/(d*x+c)^{(3/2)}-1/5*b^{(5/2)}*\exp(-a+b*c/d)*\operatorname{erf}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*\operatorname{Pi}^{(1/2)}/d^{(7/2)}+1/5*b^{(5/2)}*\exp(a-b*c/d)*\operatorname{erfi}(b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*\operatorname{Pi}^{(1/2)}/d^{(7/2)}-3/5*b^{(5/2)}*\exp(-3*a+3*b*c/d)*\operatorname{erf}(3^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/d^{(7/2)}+3/5*b^{(5/2)}*\exp(3*a-3*b*c/d)*\operatorname{erfi}(3^{(1/2)}*b^{(1/2)}*(d*x+c)^{(1/2)}/d^{(1/2)})*3^{(1/2)}*\operatorname{Pi}^{(1/2)}/d^{(7/2)}+16/5*b^2*\cosh(b*x+a)/d^3/(d*x+c)^{(1/2)}-24/5*b^2*\cosh(b*x+a)^3/d^3/(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.68, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {3314, 3297, 3308, 2180, 2204, 2205, 3313}

$$\frac{\sqrt{\pi} b^{5/2} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{3\sqrt{3\pi} b^{5/2} e^{\frac{3bc}{d}-3a} \operatorname{Erf}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{\sqrt{\pi} b^{5/2} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{3\sqrt{3\pi} b^{5/2} e^{3a-\frac{3b}{d}}}{5d^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cosh}[a + b*x]^3/(c + d*x)^{(7/2)}, x]$

[Out]  $(16*b^2*\operatorname{Cosh}[a + b*x])/(5*d^3*\operatorname{Sqrt}[c + d*x]) - (2*\operatorname{Cosh}[a + b*x]^3)/(5*d*(c + d*x)^{(5/2)}) - (24*b^2*\operatorname{Cosh}[a + b*x]^3)/(5*d^3*\operatorname{Sqrt}[c + d*x]) - (b^{(5/2)}*E^{(-a + (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(5*d^{(7/2)}) - (3*b^{(5/2)}*E^{(-3*a + (3*b*c)/d)}*\operatorname{Sqrt}[3*\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(5*d^{(7/2)}) + (b^{(5/2)}*E^{(a - (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(5*d^{(7/2)}) + (3*b^{(5/2)}*E^{(3*a - (3*b*c)/d)}*\operatorname{Sqrt}[3*\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(5*d^{(7/2)}) - (4*b*\operatorname{Cosh}[a + b*x]^2*\operatorname{Sinh}[a + b*x])/(5*d^2*(c + d*x)^{(3/2)})$

**Rule 2180**

$\operatorname{Int}[(F\_)^((g\_)*(e\_)+(f\_)*(x\_))]/\operatorname{Sqrt}[(c\_)+(d\_)*(x\_)], x\_Symbol] :$   
 $> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x\} \&\& \text{!}\$UseGamma == \text{True}$

**Rule 2204**

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))<sup>2</sup>), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[((c + d\*x)<sup>(m + 1)</sup>\*Sin[e + f\*x]/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)<sup>(m + 1)</sup>\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

### Rule 3308

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I/2, Int[(c + d\*x)<sup>m</sup>/E^(I\*(e + f\*x)), x], x] - Dist[I/2, Int[(c + d\*x)<sup>m</sup>\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

### Rule 3313

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)]<sup>(n\_)</sup>, x\_Symbol] := Simp[((c + d\*x)<sup>(m + 1)</sup>\*Sin[e + f\*x]<sup>n</sup>/(d\*(m + 1)), x] - Dist[(f\*n)/(d\*(m + 1)), Int[ExpandTrigReduce[(c + d\*x)<sup>(m + 1)</sup>, Cos[e + f\*x]\*Sin[e + f\*x]<sup>(n - 1)</sup>, x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

### Rule 3314

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_)</sup>\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])<sup>(n\_)</sup>, x\_Symbol] := Simp[((c + d\*x)<sup>(m + 1)</sup>\*(b\*Ssin[e + f\*x])<sup>n</sup>/(d\*(m + 1)), x] + (Dist[(b^2\*f^2\*n\*(n - 1))/(d^2\*(m + 1)\*(m + 2)), Int[(c + d\*x)<sup>(m + 2)</sup>\*(b\*Ssin[e + f\*x])<sup>(n - 2)</sup>, x], x] - Dist[(f^2\*n^2)/(d^2\*(m + 1)\*(m + 2)), Int[(c + d\*x)<sup>(m + 2)</sup>\*(b\*Ssin[e + f\*x])<sup>n</sup>, x], x] - Simp[(b\*f\*n\*(c + d\*x)<sup>(m + 2)</sup>\*Cos[e + f\*x]\*(b\*Ssin[e + f\*x])<sup>(n - 1)</sup>/(d^2\*(m + 1)\*(m + 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(a+bx)}{(c+dx)^{7/2}} dx &= -\frac{2 \cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{4b \cosh^2(a+bx) \sinh(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{(8b^2) \int \frac{\cosh(a+bx)}{(c+dx)^{3/2}} dx}{5d^2} + \frac{(12b^2) \int \frac{\cosh(a+bx)}{(c+dx)^{3/2}} dx}{5d^2} \\
&= \frac{16b^2 \cosh(a+bx)}{5d^3 \sqrt{c+dx}} - \frac{2 \cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2 \cosh^3(a+bx)}{5d^3 \sqrt{c+dx}} - \frac{4b \cosh^2(a+bx) \sinh(a+bx)}{5d^2(c+dx)^{3/2}} \\
&= \frac{16b^2 \cosh(a+bx)}{5d^3 \sqrt{c+dx}} - \frac{2 \cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2 \cosh^3(a+bx)}{5d^3 \sqrt{c+dx}} - \frac{4b \cosh^2(a+bx) \sinh(a+bx)}{5d^2(c+dx)^{3/2}} \\
&= \frac{16b^2 \cosh(a+bx)}{5d^3 \sqrt{c+dx}} - \frac{2 \cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2 \cosh^3(a+bx)}{5d^3 \sqrt{c+dx}} - \frac{4b \cosh^2(a+bx) \sinh(a+bx)}{5d^2(c+dx)^{3/2}} \\
&= \frac{16b^2 \cosh(a+bx)}{5d^3 \sqrt{c+dx}} - \frac{2 \cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2 \cosh^3(a+bx)}{5d^3 \sqrt{c+dx}} + \frac{8b^{5/2} e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} \\
&= \frac{16b^2 \cosh(a+bx)}{5d^3 \sqrt{c+dx}} - \frac{2 \cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2 \cosh^3(a+bx)}{5d^3 \sqrt{c+dx}} - \frac{b^{5/2} e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}
\end{aligned}$$

**Mathematica [B]** time = 6.35, size = 3211, normalized size = 9.70

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[Cosh[a + b\*x]^3/(c + d\*x)^(7/2), x]

[Out] (3\*(Sinh[a]\*(-1/30\*((-2\*E^((b\*(c + d\*x))/d))\*(3\*d^2 + 2\*b\*d\*(c + d\*x) + 4\*b^2\*(c + d\*x)^2) + 8\*d^2\*(-((b\*(c + d\*x))/d))^(5/2)\*Gamma[1/2, -((b\*(c + d\*x))/d)] + (-6\*d^2 + 4\*b\*d\*(c + d\*x) - 8\*b^2\*(c + d\*x)^2 + 8\*b\*d\*E^((b\*(c + d\*x))/d)\*(c + d\*x)\*((b\*(c + d\*x))/d)^(3/2)\*Gamma[1/2, (b\*(c + d\*x))/d])/E^((b\*(c + d\*x))/d))\*Sinh[(b\*c)/d])/(d^3\*(c + d\*x)^(5/2)) + (2\*Cosh[(b\*c)/d]\*(-1/2\*(b\*(c + d\*x)\*(2\*E^((b\*(c + d\*x))/d)\*(d + 2\*b\*(c + d\*x)) + 4\*d\*(-((b\*(c + d\*x))/d))^(3/2)\*Gamma[1/2, -((b\*(c + d\*x))/d)] + (2\*(d - 2\*b\*(c + d\*x) + 2\*d\*E^((b\*(c + d\*x))/d)\*((b\*(c + d\*x))/d)^(3/2)\*Gamma[1/2, (b\*(c + d\*x))/d])))/E^((b\*(c + d\*x))/d)) - 3\*d^2\*Sinh[(b\*(c + d\*x))/d])/(15\*d^3\*(c + d\*x)^(5/2)) + Cosh[a]\*((Cosh[(b\*c)/d]\*(-2\*E^((b\*(c + d\*x))/d)\*(3\*d^2 + 2\*b\*d\*(c + d\*x) + 4\*b^2\*(c + d\*x)^2) + 8\*d^2\*(-((b\*(c + d\*x))/d))^(5/2)\*Gamma[1/2, -((b\*(c + d\*x))/d)] + (-6\*d^2 + 4\*b\*d\*(c + d\*x) - 8\*b^2\*(c + d\*x)^2 + 8\*b\*d\*E^((b\*(c + d\*x))/d)\*(c + d\*x)\*((b\*(c + d\*x))/d)^(3/2)\*Gamma[1/2, (b\*(c + d\*x))/d])))/E^((b\*(c + d\*x))/d))

$$\begin{aligned}
& x))/d))/E^((b*(c + d*x))/d)))/(30*d^3*(c + d*x)^(5/2)) - (2*\text{Sinh}[(b*c)/d]*( \\
& -1/2*(b*(c + d*x)*(2*E^((b*(c + d*x))/d)*(d + 2*b*(c + d*x)) + 4*d*(-((b*(c \\
& + d*x))/d))^(3/2)*\text{Gamma}[1/2, -((b*(c + d*x))/d)] + (2*(d - 2*b*(c + d*x) + \\
& 2*d*E^((b*(c + d*x))/d)*((b*(c + d*x))/d)^(3/2)*\text{Gamma}[1/2, (b*(c + d*x))/d \\
& ]))/E^((b*(c + d*x))/d)) - 3*d^2*\text{Sinh}[(b*(c + d*x))/d]))/(15*d^3*(c + d*x) \\
& ^{(5/2))))/4 + (\text{Sinh}[3*a]*(-1/10*((1 + 2*\text{Cosh}[(2*b*c)/d])*(-2*E^((3*b*(c + \\
& d*x))/d)*(d^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*\text{Sqrt}[3]*d^2*(-(( \\
& b*(c + d*x))/d))^(5/2)*\text{Gamma}[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c \\
& + d*x) - 24*b^2*(c + d*x)^2 + 24*\text{Sqrt}[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + \\
& d*x))/d)^(5/2)*\text{Gamma}[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d))*\text{Sinh}[ \\
& (b*c)/d]/(d^3*(c + d*x)^(5/2)) - (2*\text{Cosh}[(b*c)/d]*(-1 + 2*\text{Cosh}[(2*b*c)/d]) \\
& *(-6*b^(5/2)*\text{Sqrt}[3*\text{Pi}]*(c + d*x)^(5/2)*\text{Erf}[(\text{Sqrt}[3]*\text{Sqrt}[b]*\text{Sqrt}[c + d*x]) \\
& ]/\text{Sqrt}[d]] - 6*b^(5/2)*\text{Sqrt}[3*\text{Pi}]*(c + d*x)^(5/2)*\text{Erfi}[(\text{Sqrt}[3]*\text{Sqrt}[b]*\text{Sqrt} \\
& [c + d*x])/\text{Sqrt}[d]] + \text{Sqrt}[d]*(2*b*d*(c + d*x)*\text{Cosh}[(3*b*(c + d*x))/d] + (d \\
& ^2 + 12*b^2*(c + d*x)^2)*\text{Sinh}[(3*b*(c + d*x))/d]))/(5*d^(7/2)*(c + d*x)^(5 \\
& /2))) + \text{Cosh}[3*a]*((\text{Cosh}[(b*c)/d]*(-1 + 2*\text{Cosh}[(2*b*c)/d])*(-2*E^((3*b*(c + \\
& d*x))/d)*(d^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*\text{Sqrt}[3]*d^2*(-( \\
& (b*(c + d*x))/d))^(5/2)*\text{Gamma}[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c \\
& + d*x) - 24*b^2*(c + d*x)^2 + 24*\text{Sqrt}[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c \\
& + d*x))/d)^(5/2)*\text{Gamma}[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d)))/(10 \\
& *d^3*(c + d*x)^(5/2)) + (2*(1 + 2*\text{Cosh}[(2*b*c)/d])*\text{Sinh}[(b*c)/d]*(-6*b^(5/2) \\
& )*\text{Sqrt}[3*\text{Pi}]*(c + d*x)^(5/2)*\text{Erf}[(\text{Sqrt}[3]*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]] - \\
& 6*b^(5/2)*\text{Sqrt}[3*\text{Pi}]*(c + d*x)^(5/2)*\text{Erfi}[(\text{Sqrt}[3]*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/ \\
& \text{Sqrt}[d]] + \text{Sqrt}[d]*(2*b*d*(c + d*x)*\text{Cosh}[(3*b*(c + d*x))/d] + (d^2 + 12*b^2 \\
& *(c + d*x)^2)*\text{Sinh}[(3*b*(c + d*x))/d]))/(5*d^(7/2)*(c + d*x)^(5/2))))/4 + \\
& (- (\text{Cosh}[3*a]*(-1/10*((1 + 2*\text{Cosh}[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(d^2 \\
& + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*\text{Sqrt}[3]*d^2*(-((b*(c + d*x))/ \\
& d))^(5/2)*\text{Gamma}[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b \\
& ^2*(c + d*x)^2 + 24*\text{Sqrt}[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(5/ \\
& 2)*\text{Gamma}[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d))*\text{Sinh}[(b*c)/d]/(d^ \\
& 3*(c + d*x)^(5/2)) - (2*\text{Cosh}[(b*c)/d]*(-1 + 2*\text{Cosh}[(2*b*c)/d])*(-6*b^(5/2)* \\
& \text{Sqrt}[3*\text{Pi}]*(c + d*x)^(5/2)*\text{Erf}[(\text{Sqrt}[3]*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]] - 6 \\
& *b^(5/2)*\text{Sqrt}[3*\text{Pi}]*(c + d*x)^(5/2)*\text{Erfi}[(\text{Sqrt}[3]*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/ \\
& \text{Sqrt}[d]] + \text{Sqrt}[d]*(2*b*d*(c + d*x)*\text{Cosh}[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*( \\
& c + d*x)^2)*\text{Sinh}[(3*b*(c + d*x))/d]))/(5*d^(7/2)*(c + d*x)^(5/2)))) - \text{Sinh} \\
& [3*a]*((\text{Cosh}[(b*c)/d]*(-1 + 2*\text{Cosh}[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(d \\
& ^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*\text{Sqrt}[3]*d^2*(-((b*(c + d*x) \\
& )/d))^(5/2)*\text{Gamma}[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24 \\
& *b^2*(c + d*x)^2 + 24*\text{Sqrt}[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^( \\
& 5/2)*\text{Gamma}[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d)))/(10*d^3*(c + d* \\
& x)^(5/2)) + (2*(1 + 2*\text{Cosh}[(2*b*c)/d])*\text{Sinh}[(b*c)/d]*(-6*b^(5/2)*\text{Sqrt}[3*\text{Pi}] \\
& *(c + d*x)^(5/2)*\text{Erf}[(\text{Sqrt}[3]*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]] - 6*b^(5/2)*\text{S} \\
& \text{qrt}[3*\text{Pi}]*(c + d*x)^(5/2)*\text{Erfi}[(\text{Sqrt}[3]*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/\text{Sqrt}[d]] + \text{S} \\
& \text{qrt}[d]*(2*b*d*(c + d*x)*\text{Cosh}[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2) \\
& )*\text{Sinh}[(3*b*(c + d*x))/d]))/(5*d^(7/2)*(c + d*x)^(5/2))))/8 + (\text{Cosh}[3*a]*
\end{aligned}$$

$$\begin{aligned}
& -1/10*((1 + 2*\text{Cosh}[(2*b*c)/d])*(-2*\text{E}^((3*b*(c + d*x))/d))*(d^2 + 2*b*d*(c + \\
& d*x) + 12*b^2*(c + d*x)^2) + 24*\text{Sqrt}[3]*d^2*(-((b*(c + d*x))/d))^(5/2)*\text{Gamma} \\
& a[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c + d*x)^2 \\
& + 24*\text{Sqrt}[3]*d^2*\text{E}^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(5/2)*\text{Gamma}[1/2, \\
& (3*b*(c + d*x))/d]/\text{E}^((3*b*(c + d*x))/d))*\text{Sinh}[(b*c)/d]/(d^3*(c + d*x)^(5 \\
& /2)) - (2*\text{Cosh}[(b*c)/d]*(-1 + 2*\text{Cosh}[(2*b*c)/d])*(-6*b^(5/2)*\text{Sqrt}[3*\text{Pi})*(c \\
& + d*x)^(5/2)*\text{Erf}[\text{Sqrt}[3]*\text{Sqrt}[b]*\text{Sqrt}[c + d*x]/\text{Sqrt}[d]] - 6*b^(5/2)*\text{Sqrt}[ \\
& 3*\text{Pi})*(c + d*x)^(5/2)*\text{Erfi}[\text{Sqrt}[3]*\text{Sqrt}[b]*\text{Sqrt}[c + d*x]/\text{Sqrt}[d]] + \text{Sqrt}[ \\
& d]*(2*b*d*(c + d*x)*\text{Cosh}[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2)*\text{Si} \\
& nh[(3*b*(c + d*x))/d]))/(5*d^(7/2)*(c + d*x)^(5/2))) + \text{Sinh}[3*a]*((\text{Cosh}[(b \\
& *c)/d]*(-1 + 2*\text{Cosh}[(2*b*c)/d])*(-2*\text{E}^((3*b*(c + d*x))/d))*(d^2 + 2*b*d*(c + \\
& d*x) + 12*b^2*(c + d*x)^2) + 24*\text{Sqrt}[3]*d^2*(-((b*(c + d*x))/d))^(5/2)*\text{Gamma} \\
& a[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c + d*x)^2 \\
& + 24*\text{Sqrt}[3]*d^2*\text{E}^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(5/2)*\text{Gamma}[1/2, \\
& (3*b*(c + d*x))/d]/\text{E}^((3*b*(c + d*x))/d)))/(10*d^3*(c + d*x)^(5/2)) + (2* \\
& (1 + 2*\text{Cosh}[(2*b*c)/d])*\text{Sinh}[(b*c)/d]*(-6*b^(5/2)*\text{Sqrt}[3*\text{Pi})*(c + d*x)^(5/2) \\
& )*\text{Erf}[\text{Sqrt}[3]*\text{Sqrt}[b]*\text{Sqrt}[c + d*x]/\text{Sqrt}[d]] - 6*b^(5/2)*\text{Sqrt}[3*\text{Pi})*(c + \\
& d*x)^(5/2)*\text{Erfi}[\text{Sqrt}[3]*\text{Sqrt}[b]*\text{Sqrt}[c + d*x]/\text{Sqrt}[d]] + \text{Sqrt}[d]*(2*b*d*( \\
& c + d*x)*\text{Cosh}[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2)*\text{Sinh}[(3*b*(c \\
& + d*x))/d]))/(5*d^(7/2)*(c + d*x)^(5/2))))/8
\end{aligned}$$

**fricas [B]** time = 0.62, size = 3280, normalized size = 9.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c)^(7/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned}
& -1/20*(12*\text{sqrt}(3)*\text{sqrt}(\text{pi})*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x \\
& + b^2*c^3)*\text{cosh}(b*x + a)^3*\text{cosh}(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 \\
& + 3*b^2*c^2*d*x + b^2*c^3)*\text{cosh}(b*x + a)^3*\text{sinh}(-3*(b*c - a*d)/d) + \\
& ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\text{cosh}(-3*(b*c - \\
& a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\text{sinh}(-3 \\
& *(b*c - a*d)/d))*\text{sinh}(b*x + a)^3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2 \\
& *c^2*d*x + b^2*c^3)*\text{cosh}(b*x + a)*\text{cosh}(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 + \\
& 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\text{cosh}(b*x + a)*\text{sinh}(-3*(b*c - a*d \\
& )/d))*\text{sinh}(b*x + a)^2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + \\
& b^2*c^3)*\text{cosh}(b*x + a)^2*\text{cosh}(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2 \\
& *x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\text{cosh}(b*x + a)^2*\text{sinh}(-3*(b*c - a*d)/d))*\text{si} \\
& nh(b*x + a))*\text{sqrt}(b/d)*\text{erf}(\text{sqrt}(3)*\text{sqrt}(d*x + c)*\text{sqrt}(b/d)) + 12*\text{sqrt}(3)*\text{sqrt}(\text{pi})* \\
& ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\text{cosh}(b*x \\
& + a)^3*\text{cosh}(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2 \\
& *d*x + b^2*c^3)*\text{cosh}(b*x + a)^3*\text{sinh}(-3*(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3* \\
& b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\text{cosh}(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 \\
& + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\text{sinh}(-3*(b*c - a*d)/d))*\text{si}
\end{aligned}$$

$$\begin{aligned}
& \text{nh}(b*x + a)^3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3) \\
& )*\cosh(b*x + a)*\cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3 \\
& *b^2*c^2*d*x + b^2*c^3)*\cosh(b*x + a)*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a) \\
& ^2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cosh(b*x \\
& + a)^2*\cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2* \\
& d*x + b^2*c^3)*\cosh(b*x + a)^2*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{ \\
& -b/d)*\text{erf}(\sqrt{3}*\sqrt{d*x + c}*\sqrt{-b/d}) + 4*\sqrt{\pi}*((b^2*d^3*x^3 + 3* \\
& b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cosh(b*x + a)^3*\cosh(-(b*c - a*d)/ \\
& d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cosh(b*x + a) \\
& )^3*\sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x \\
& + b^2*c^3)*\cosh(-(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^ \\
& 2*d*x + b^2*c^3)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)^3 + 3*((b^2*d^3*x^3 + \\
& 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cosh(b*x + a)*\cosh(-(b*c - a*d)/ \\
& d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cosh(b*x + a) \\
& )*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)^2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 \\
& + 3*b^2*c^2*d*x + b^2*c^3)*\cosh(b*x + a)^2*\cosh(-(b*c - a*d)/d) - (b^2*d^3 \\
& *x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cosh(b*x + a)^2*\sinh(-(b* \\
& c - a*d)/d))*\sinh(b*x + a))*\sqrt{b/d)*\text{erf}(\sqrt{d*x + c}*\sqrt{b/d}) + 4*\sqrt{ \\
& \pi}*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cosh(b*x + \\
& a)^3*\cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x \\
& + b^2*c^3)*\cosh(b*x + a)^3*\sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d \\
& ^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b \\
& ^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a) \\
& ^3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cosh(b*x \\
& + a)*\cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x \\
& + b^2*c^3)*\cosh(b*x + a)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)^2 + 3*((b^2*d^ \\
& 3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cosh(b*x + a)^2*\cosh(-(b \\
& *c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\co \\
& sh(b*x + a)^2*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{-b/d)*\text{erf}(\sqrt{d*x \\
& + c}*\sqrt{-b/d}) + ((12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^ \\
& 2*c*d + b*d^2)*x)*\cosh(b*x + a)^6 + 6*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c* \\
& d + d^2 + 2*(12*b^2*c*d + b*d^2)*x)*\cosh(b*x + a)*\sinh(b*x + a)^5 + (12*b^2 \\
& *d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*c*d + b*d^2)*x)*\sinh(b*x \\
& + a)^6 + 12*b^2*d^2*x^2 + (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2* \\
& (4*b^2*c*d + b*d^2)*x)*\cosh(b*x + a)^4 + (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c \\
& *d + 15*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*c*d + b*d^ \\
& 2)*x)*\cosh(b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*\sinh(b*x + a)^4 + \\
& 12*b^2*c^2 + 4*(5*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2* \\
& c*d + b*d^2)*x)*\cosh(b*x + a)^3 + (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3* \\
& d^2 + 2*(4*b^2*c*d + b*d^2)*x)*\cosh(b*x + a))*\sinh(b*x + a)^3 - 2*b*c*d + ( \\
& 4*b^2*d^2*x^2 + 4*b^2*c^2 - 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*\cosh \\
& (b*x + a)^2 + (4*b^2*d^2*x^2 + 15*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + \\
& d^2 + 2*(12*b^2*c*d + b*d^2)*x)*\cosh(b*x + a)^4 + 4*b^2*c^2 - 2*b*c*d + 6*( \\
& 4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*\cosh \\
& (b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*\sinh(b*x + a)^2 + d^2 + 2*(1
\end{aligned}$$

$2*b^2*c*d - b*d^2)*x + 2*(3*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*c*d + b*d^2)*x)*\cosh(b*x + a)^5 + 2*(4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*\cosh(b*x + a)^3 + (4*b^2*d^2*x^2 + 4*b^2*c^2 - 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*\cosh(b*x + a)*\sinh(b*x + a)*\sqrt{d*x + c})/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*\cosh(b*x + a)^3 + 3*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*\cosh(b*x + a)^2*\sinh(b*x + a) + 3*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*\cosh(b*x + a)*\sinh(b*x + a)^2 + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*\sinh(b*x + a)^3)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(bx + a)}{(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c)^(7/2),x, algorithm="giac")

[Out] integrate(cosh(b\*x + a)^3/(d\*x + c)^(7/2), x)

**maple** [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(bx + a)}{(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(b\*x+a)^3/(d\*x+c)^(7/2),x)

[Out] int(cosh(b\*x+a)^3/(d\*x+c)^(7/2),x)

**maxima** [A] time = 0.73, size = 196, normalized size = 0.59

$$\frac{3 \left( \frac{3 \sqrt{3} \left( \frac{(dx+c)b}{d} \right)^{\frac{5}{2}} e^{\left( \frac{3(bc-ad)}{d} \right)} \Gamma\left( -\frac{5}{2}, \frac{3(dx+c)b}{d} \right)}{(dx+c)^{\frac{5}{2}}} + \frac{3 \sqrt{3} \left( -\frac{(dx+c)b}{d} \right)^{\frac{5}{2}} e^{\left( -\frac{3(bc-ad)}{d} \right)} \Gamma\left( -\frac{5}{2}, -\frac{3(dx+c)b}{d} \right)}{(dx+c)^{\frac{5}{2}}} + \frac{\left( \frac{(dx+c)b}{d} \right)^{\frac{5}{2}} e^{\left( -a + \frac{bc}{d} \right)} \Gamma\left( -\frac{5}{2}, \frac{(dx+c)b}{d} \right)}{(dx+c)^{\frac{5}{2}}} + \frac{\left( -\frac{(dx+c)b}{d} \right)^{\frac{5}{2}} e^{\left( -a + \frac{bc}{d} \right)} \Gamma\left( -\frac{5}{2}, -\frac{(dx+c)b}{d} \right)}{(dx+c)^{\frac{5}{2}}} \right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)^3/(d\*x+c)^(7/2),x, algorithm="maxima")

[Out]  $-3/8*(3*\sqrt{3})*((d*x + c)*b/d)^{(5/2)}*e^{(3*(b*c - a*d)/d)}*\gamma(-5/2, 3*(d*x + c)*b/d)/(d*x + c)^{(5/2)} + 3*\sqrt{3}*(-(d*x + c)*b/d)^{(5/2)}*e^{(-3*(b*c -$

$a*d)/d)*\text{gamma}(-5/2, -3*(d*x + c)*b/d)/(d*x + c)^{(5/2)} + ((d*x + c)*b/d)^{(5/2)}*e^{(-a + b*c/d)*\text{gamma}(-5/2, (d*x + c)*b/d)/(d*x + c)^{(5/2)} + (-(d*x + c)*b/d)^{(5/2)}*e^{(a - b*c/d)*\text{gamma}(-5/2, -(d*x + c)*b/d)/(d*x + c)^{(5/2))}/d$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(a + bx)^3}{(c + dx)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)^3/(c + d\*x)^(7/2), x)

[Out] int(cosh(a + b\*x)^3/(c + d\*x)^(7/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(a + bx)}{(c + dx)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(b\*x+a)\*\*3/(d\*x+c)\*\*(7/2), x)

[Out] Integral(cosh(a + b\*x)\*\*3/(c + d\*x)\*\*(7/2), x)



### 3.63 $\int (dx)^{3/2} \cosh(fx) dx$

**Optimal.** Leaf size=111

$$\frac{3\sqrt{\pi} d^{3/2} \operatorname{erf}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} + \frac{3\sqrt{\pi} d^{3/2} \operatorname{erfi}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} - \frac{3d\sqrt{dx} \cosh(fx)}{2f^2} + \frac{(dx)^{3/2} \sinh(fx)}{f}$$

[Out]  $(d*x)^{(3/2)*\sinh(f*x)/f+3/8*d^{(3/2)*\operatorname{erf}(f^{(1/2)*(d*x)^{(1/2)/d^{(1/2)}})*\operatorname{Pi}^{(1/2)/f^{(5/2)+3/8*d^{(3/2)*\operatorname{erfi}(f^{(1/2)*(d*x)^{(1/2)/d^{(1/2)}})*\operatorname{Pi}^{(1/2)/f^{(5/2)-3/2*d*\cosh(f*x)*(d*x)^{(1/2)/f^2}}$

**Rubi [A]** time = 0.16, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {3296, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi} d^{3/2} \operatorname{Erf}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} + \frac{3\sqrt{\pi} d^{3/2} \operatorname{Erfi}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} - \frac{3d\sqrt{dx} \cosh(fx)}{2f^2} + \frac{(dx)^{3/2} \sinh(fx)}{f}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d*x)^{(3/2)*\operatorname{Cosh}[f*x], x]$

[Out]  $(-3*d*\operatorname{Sqrt}[d*x]*\operatorname{Cosh}[f*x])/(2*f^2) + (3*d^{(3/2)*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d*x])/\operatorname{Sqrt}[d]])/(8*f^{(5/2)}) + (3*d^{(3/2)*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d*x])/\operatorname{Sqrt}[d]])/(8*f^{(5/2)}) + ((d*x)^{(3/2)*\operatorname{Sinh}[f*x])/f$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*(e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x \ \&\amp; \ !\$UseGamma === \operatorname{True}$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \ \&\amp; \ \operatorname{PosQ}[b]$

#### Rule 2205

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]])/(2*d*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \ \&\amp; \ \operatorname{NegQ}[b]$

Rule 3296

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[  
 ((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[  
 e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rule 3307

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol  
 ] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[  
 I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,  
 f, m}, x] && IntegerQ[2*k]`

Rubi steps

$$\begin{aligned}
 \int (dx)^{3/2} \cosh(fx) dx &= \frac{(dx)^{3/2} \sinh(fx)}{f} - \frac{(3d) \int \sqrt{dx} \sinh(fx) dx}{2f} \\
 &= -\frac{3d\sqrt{dx} \cosh(fx)}{2f^2} + \frac{(dx)^{3/2} \sinh(fx)}{f} + \frac{(3d^2) \int \frac{\cosh(fx)}{\sqrt{dx}} dx}{4f^2} \\
 &= -\frac{3d\sqrt{dx} \cosh(fx)}{2f^2} + \frac{(dx)^{3/2} \sinh(fx)}{f} + \frac{(3d^2) \int \frac{e^{-fx}}{\sqrt{dx}} dx}{8f^2} + \frac{(3d^2) \int \frac{e^{fx}}{\sqrt{dx}} dx}{8f^2} \\
 &= -\frac{3d\sqrt{dx} \cosh(fx)}{2f^2} + \frac{(dx)^{3/2} \sinh(fx)}{f} + \frac{(3d) \text{Subst}\left(\int e^{-\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{4f^2} + \frac{(3d) \text{Subst}\left(\int e^{\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{4f^2} \\
 &= -\frac{3d\sqrt{dx} \cosh(fx)}{2f^2} + \frac{3d^{3/2}\sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} + \frac{3d^{3/2}\sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} + \frac{(dx)^{3/2} \sinh(fx)}{f}
 \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 51, normalized size = 0.46

$$\frac{d^2 \left( \sqrt{-fx} \Gamma\left(\frac{5}{2}, -fx\right) - \sqrt{fx} \Gamma\left(\frac{5}{2}, fx\right) \right)}{2f^3 \sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)\*Cosh[f\*x], x]

[Out] (d^2\*(Sqrt[-(f\*x)]\*Gamma[5/2, -(f\*x)] - Sqrt[f\*x]\*Gamma[5/2, f\*x])/(2\*f^3\*Sqrt[d\*x]))

**fricas [B]** time = 0.45, size = 191, normalized size = 1.72

$$\frac{3\sqrt{\pi}(d^2 \cosh(fx) + d^2 \sinh(fx))\sqrt{\frac{f}{d}} \operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right) - 3\sqrt{\pi}(d^2 \cosh(fx) + d^2 \sinh(fx))\sqrt{-\frac{f}{d}} \operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{8(f^3 \cosh(fx) + f^3 \sinh(fx))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*cosh(f\*x),x, algorithm="fricas")

[Out]  $\frac{1}{8}*(3*\sqrt{\pi}*(d^2*\cosh(f*x) + d^2*\sinh(f*x))*\sqrt{f/d}*\operatorname{erf}(\sqrt{d*x}*\sqrt{f/d}) - 3*\sqrt{\pi}*(d^2*\cosh(f*x) + d^2*\sinh(f*x))*\sqrt{-f/d}*\operatorname{erf}(\sqrt{d*x}*\sqrt{-f/d}) - 2*(2*d*f^2*x - (2*d*f^2*x - 3*d*f)*\cosh(f*x)^2 - 2*(2*d*f^2*x - 3*d*f)*\sinh(f*x) - (2*d*f^2*x - 3*d*f)*\sinh(f*x)^2 + 3*d*f)*\sqrt{d*x})/(f^3*\cosh(f*x) + f^3*\sinh(f*x))$

**giac [A]** time = 0.15, size = 145, normalized size = 1.31

$$-\frac{1}{8}d \left( \frac{3\sqrt{\pi}d^3 \operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}f^2} + \frac{2(2\sqrt{dx}d^2fx + 3\sqrt{dx}d^2)e^{(-fx)}}{f^2} + \frac{3\sqrt{\pi}d^3 \operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}f^2} - \frac{2(2\sqrt{dx}d^2fx - 3\sqrt{dx}d^2)e^{(fx)}}{f^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*cosh(f\*x),x, algorithm="giac")

[Out]  $-1/8*d*((3*\sqrt{\pi})*d^3*\operatorname{erf}(-\sqrt{d*f}*\sqrt{d*x}/d)/(\sqrt{d*f}*f^2) + 2*(2*\sqrt{d*x}*d^2*f*x + 3*\sqrt{d*x}*d^2)*e^{(-f*x)/f^2}/d^2 + (3*\sqrt{\pi})*d^3*\operatorname{erf}(-\sqrt{-d*f}*\sqrt{d*x}/d)/(\sqrt{-d*f}*f^2) - 2*(2*\sqrt{d*x}*d^2*f*x - 3*\sqrt{d*x}*d^2)*e^{(f*x)/f^2}/d^2)$

**maple [C]** time = 0.09, size = 133, normalized size = 1.20

$$\frac{2i(dx)^{\frac{3}{2}}\sqrt{2}\sqrt{\pi} \left( -\frac{\sqrt{x}\sqrt{2}(if)^{\frac{5}{2}}(10fx+15)e^{-fx}}{80\sqrt{\pi}f^2} - \frac{\sqrt{x}\sqrt{2}(if)^{\frac{5}{2}}(-10fx+15)e^{fx}}{80\sqrt{\pi}f^2} + \frac{3(if)^{\frac{5}{2}}\sqrt{2}\operatorname{erf}(\sqrt{x}\sqrt{f})}{32f^{\frac{5}{2}}} + \frac{3(if)^{\frac{5}{2}}\sqrt{2}\operatorname{erfi}(\sqrt{x}\sqrt{f})}{32f^{\frac{5}{2}}} \right)}{x^{\frac{3}{2}}(if)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)\*cosh(f\*x),x)

[Out]  $-2*I*(d*x)^{(3/2)}/x^{(3/2)}*2^{(1/2)}/(I*f)^{(3/2)}*Pi^{(1/2)}/f*(-1/80/Pi^{(1/2)}*x^{(1/2)}*2^{(1/2)}*(I*f)^{(5/2)}*(10*f*x+15)/f^2*\exp(-f*x)-1/80/Pi^{(1/2)}*x^{(1/2)}*2^{(1/2)}$

$(1/2)*(I*f)^{(5/2)*(-10*f*x+15)/f^2*\exp(f*x)+3/32*(I*f)^{(5/2)*2^{(1/2)}/f^{(5/2)}}*erf(x^{(1/2)*f^{(1/2)}})+3/32*(I*f)^{(5/2)*2^{(1/2)}/f^{(5/2)*erfi(x^{(1/2)*f^{(1/2)}}))$

**maxima** [B] time = 0.32, size = 174, normalized size = 1.57

$$16(dx)^{\frac{5}{2}} \cosh(fx) + \frac{f \left( \frac{15\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f^3\sqrt{\frac{f}{d}}} + \frac{15\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f^3\sqrt{-\frac{f}{d}}} - 2 \left( \frac{4(dx)^{\frac{5}{2}}df^2 - 10(dx)^{\frac{3}{2}}d^2f + 15\sqrt{dx}d^3 \right) e^{(fx)}}{f^3} - 2 \left( \frac{4(dx)^{\frac{5}{2}}df^2 + 10(dx)^{\frac{3}{2}}d^2f + 15\sqrt{dx}d^3 \right) e^{(-fx)}}{f^3} \right)}{40d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*cosh(f\*x),x, algorithm="maxima")

[Out]  $\frac{1}{40} * (16 * (d*x)^{(5/2)} * \cosh(f*x) + f * (15 * \sqrt{\pi} * d^3 * \operatorname{erf}(\sqrt{d*x} * \sqrt{f/d}) / (f^3 * \sqrt{f/d}) + 15 * \sqrt{\pi} * d^3 * \operatorname{erf}(\sqrt{d*x} * \sqrt{-f/d}) / (f^3 * \sqrt{-f/d}) - 2 * (4 * (d*x)^{(5/2)} * d*f^2 - 10 * (d*x)^{(3/2)} * d^2*f + 15 * \sqrt{d*x} * d^3) * e^{(f*x)} / f^3 - 2 * (4 * (d*x)^{(5/2)} * d*f^2 + 10 * (d*x)^{(3/2)} * d^2*f + 15 * \sqrt{d*x} * d^3) * e^{(-f*x)} / f^3) / d) / d$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(fx) (dx)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f\*x)\*(d\*x)^(3/2),x)

[Out] int(cosh(f\*x)\*(d\*x)^(3/2), x)

**sympy** [C] time = 18.03, size = 131, normalized size = 1.18

$$\frac{5d^{\frac{3}{2}}x^{\frac{3}{2}} \sinh(fx) \Gamma\left(\frac{5}{4}\right)}{4f \Gamma\left(\frac{9}{4}\right)} - \frac{15d^{\frac{3}{2}}\sqrt{x} \cosh(fx) \Gamma\left(\frac{5}{4}\right)}{8f^2 \Gamma\left(\frac{9}{4}\right)} + \frac{15\sqrt{2}\sqrt{\pi}d^{\frac{3}{2}}e^{-\frac{i\pi}{4}} C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right) \Gamma\left(\frac{5}{4}\right)}{16f^{\frac{5}{2}} \Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(3/2)\*cosh(f\*x),x)

[Out]  $5*d^{(3/2)}*x^{(3/2)}*\sinh(f*x)*\gamma(5/4)/(4*f*\gamma(9/4)) - 15*d^{(3/2)}*\sqrt{x}*\cosh(f*x)*\gamma(5/4)/(8*f^2*\gamma(9/4)) + 15*\sqrt{2}*\sqrt{\pi}*d^{(3/2)}*\exp(-I*\pi/4)*\operatorname{fresnelc}(\sqrt{2}*\sqrt{f}*\sqrt{x})*\exp(I*\pi/4)/\sqrt{\pi}*\gamma(5/4)/(16*f^{(5/2)}*\gamma(9/4))$

### 3.64 $\int \sqrt{dx} \cosh(fx) dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\pi} \sqrt{d} \operatorname{erf}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} - \frac{\sqrt{\pi} \sqrt{d} \operatorname{erfi}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} + \frac{\sqrt{dx} \sinh(fx)}{f}$$

[Out]  $1/4*\operatorname{erf}(f^{(1/2)}*(d*x)^{(1/2)}/d^{(1/2)})*d^{(1/2)}*\pi^{(1/2)}/f^{(3/2)}-1/4*\operatorname{erfi}(f^{(1/2)}*(d*x)^{(1/2)}/d^{(1/2)})*d^{(1/2)}*\pi^{(1/2)}/f^{(3/2)}+\sinh(f*x)*(d*x)^{(1/2)}/f$

**Rubi [A]** time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {3296, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi} \sqrt{d} \operatorname{Erf}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} - \frac{\sqrt{\pi} \sqrt{d} \operatorname{Erfi}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} + \frac{\sqrt{dx} \sinh(fx)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*x]\*Cosh[f\*x], x]

[Out]  $(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[\pi]*\operatorname{Erf}[(\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d*x])/(\operatorname{Sqrt}[d])])/(4*f^{(3/2)}) - (\operatorname{Sqrt}[d]*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d*x])/(\operatorname{Sqrt}[d])])/(4*f^{(3/2)}) + (\operatorname{Sqrt}[d*x]*\operatorname{Sinh}[f*x])/f$

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[\pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[\pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{dx} \cosh(fx) dx &= \frac{\sqrt{dx} \sinh(fx)}{f} - \frac{d \int \frac{\sinh(fx)}{\sqrt{dx}} dx}{2f} \\
&= \frac{\sqrt{dx} \sinh(fx)}{f} + \frac{d \int \frac{e^{-fx}}{\sqrt{dx}} dx}{4f} - \frac{d \int \frac{e^{fx}}{\sqrt{dx}} dx}{4f} \\
&= \frac{\sqrt{dx} \sinh(fx)}{f} + \frac{\text{Subst}\left(\int e^{-\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{2f} - \frac{\text{Subst}\left(\int e^{\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{2f} \\
&= \frac{\sqrt{d} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} - \frac{\sqrt{d} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} + \frac{\sqrt{dx} \sinh(fx)}{f}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 48, normalized size = 0.52

$$-\frac{d\left(\sqrt{-fx}\Gamma\left(\frac{3}{2}, -fx\right) + \sqrt{fx}\Gamma\left(\frac{3}{2}, fx\right)\right)}{2f^2\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*Cosh[f*x], x]
```

```
[Out] -1/2*(d*(Sqrt[-(f*x)]*Gamma[3/2, -(f*x)] + Sqrt[f*x]*Gamma[3/2, f*x]))/(f^2
*Sqrt[d*x])
```

**fricas** [B] time = 0.63, size = 138, normalized size = 1.50

$$\frac{\sqrt{\pi} (d \cosh(fx) + d \sinh(fx)) \sqrt{\frac{f}{d}} \operatorname{erf}\left(\sqrt{dx} \sqrt{\frac{f}{d}}\right) + \sqrt{\pi} (d \cosh(fx) + d \sinh(fx)) \sqrt{-\frac{f}{d}} \operatorname{erf}\left(\sqrt{dx} \sqrt{-\frac{f}{d}}\right) + 2 \sqrt{dx} \operatorname{erfi}\left(\sqrt{dx} \sqrt{\frac{f}{d}}\right) + 2 \sqrt{dx} \operatorname{erfi}\left(\sqrt{dx} \sqrt{-\frac{f}{d}}\right)}{4 (f^2 \cosh(fx) + f^2 \sinh(fx))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f\*x)\*(d\*x)^(1/2),x, algorithm="fricas")

[Out] 1/4\*(sqrt(pi)\*(d\*cosh(f\*x) + d\*sinh(f\*x))\*sqrt(f/d)\*erf(sqrt(d\*x)\*sqrt(f/d)) + sqrt(pi)\*(d\*cosh(f\*x) + d\*sinh(f\*x))\*sqrt(-f/d)\*erf(sqrt(d\*x)\*sqrt(-f/d))) + 2\*(f\*cosh(f\*x)^2 + 2\*f\*cosh(f\*x)\*sinh(f\*x) + f\*sinh(f\*x)^2 - f)\*sqrt(d\*x)/(f^2\*cosh(f\*x) + f^2\*sinh(f\*x))

**giac** [A] time = 0.15, size = 108, normalized size = 1.17

$$-\frac{\frac{\sqrt{\pi} d^2 \operatorname{erf}\left(-\frac{\sqrt{df} \sqrt{dx}}{d}\right)}{\sqrt{df} f} + \frac{2 \sqrt{dx} d e^{-fx}}{f}}{4 d} + \frac{\frac{\sqrt{\pi} d^2 \operatorname{erf}\left(-\frac{\sqrt{-df} \sqrt{dx}}{d}\right)}{\sqrt{-df} f} + \frac{2 \sqrt{dx} d e^{fx}}{f}}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f\*x)\*(d\*x)^(1/2),x, algorithm="giac")

[Out] -1/4\*(sqrt(pi)\*d^2\*erf(-sqrt(d\*f)\*sqrt(d\*x)/d)/(sqrt(d\*f)\*f) + 2\*sqrt(d\*x)\*d\*e^(-f\*x)/f)/d + 1/4\*(sqrt(pi)\*d^2\*erf(-sqrt(-d\*f)\*sqrt(d\*x)/d)/(sqrt(-d\*f)\*f) + 2\*sqrt(d\*x)\*d\*e^(f\*x)/f)/d

**maple** [C] time = 0.08, size = 121, normalized size = 1.32

$$-\frac{i\sqrt{\pi} \sqrt{dx} \sqrt{2} \left( \frac{\sqrt{x} \sqrt{2} (if)^{\frac{3}{2}} e^{fx}}{4\sqrt{\pi} f} - \frac{\sqrt{x} \sqrt{2} (if)^{\frac{3}{2}} e^{-fx}}{4\sqrt{\pi} f} + \frac{(if)^{\frac{3}{2}} \sqrt{2} \operatorname{erf}(\sqrt{x} \sqrt{f})}{8f^{\frac{3}{2}}} - \frac{(if)^{\frac{3}{2}} \sqrt{2} \operatorname{erfi}(\sqrt{x} \sqrt{f})}{8f^{\frac{3}{2}}} \right)}{\sqrt{x} \sqrt{if} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f\*x)\*(d\*x)^(1/2),x)

[Out] -I\*Pi^(1/2)\*(d\*x)^(1/2)/x^(1/2)\*2^(1/2)/(I\*f)^(1/2)/f\*(1/4/Pi^(1/2)\*x^(1/2)\*2^(1/2)\*(I\*f)^(3/2)/f\*exp(f\*x)-1/4/Pi^(1/2)\*x^(1/2)\*2^(1/2)\*(I\*f)^(3/2)/f\*exp(-f\*x)+1/8\*(I\*f)^(3/2)\*2^(1/2)/f^(3/2)\*erf(x^(1/2)\*f^(1/2))-1/8\*(I\*f)^(3/2)\*2^(1/2)/f^(3/2)\*erfi(x^(1/2)\*f^(1/2)))

**maxima** [B] time = 0.55, size = 148, normalized size = 1.61

$$8 (dx)^{\frac{3}{2}} \cosh(fx) + \frac{f \left( \frac{3 \sqrt{\pi} d^2 \operatorname{erf}\left(\sqrt{dx} \sqrt{\frac{f}{d}}\right)}{f^2 \sqrt{\frac{f}{d}}} - \frac{3 \sqrt{\pi} d^2 \operatorname{erf}\left(\sqrt{dx} \sqrt{-\frac{f}{d}}\right)}{f^2 \sqrt{-\frac{f}{d}}} - \frac{2 \left(2(dx)^{\frac{3}{2}} d f - 3 \sqrt{dx} d^2\right) e^{(fx)}}{f^2} - \frac{2 \left(2(dx)^{\frac{3}{2}} d f + 3 \sqrt{dx} d^2\right) e^{(-fx)}}{f^2} \right)}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f\*x)\*(d\*x)^(1/2),x, algorithm="maxima")

[Out] 1/12\*(8\*(d\*x)^(3/2)\*cosh(f\*x) + f\*(3\*sqrt(pi)\*d^2\*erf(sqrt(d\*x)\*sqrt(f/d))/(f^2\*sqrt(f/d)) - 3\*sqrt(pi)\*d^2\*erf(sqrt(d\*x)\*sqrt(-f/d))/(f^2\*sqrt(-f/d)) - 2\*(2\*(d\*x)^(3/2)\*d\*f - 3\*sqrt(d\*x)\*d^2)\*e^(f\*x)/f^2 - 2\*(2\*(d\*x)^(3/2)\*d\*f + 3\*sqrt(d\*x)\*d^2)\*e^(-f\*x)/f^2)/d/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(fx) \sqrt{dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f\*x)\*(d\*x)^(1/2),x)

[Out] int(cosh(f\*x)\*(d\*x)^(1/2), x)

**sympy** [C] time = 1.40, size = 100, normalized size = 1.09

$$\frac{3\sqrt{d}\sqrt{x}\sinh(fx)\Gamma\left(\frac{3}{4}\right)}{4f\Gamma\left(\frac{7}{4}\right)} - \frac{3\sqrt{2}\sqrt{\pi}\sqrt{d}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{8f^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f\*x)\*(d\*x)\*\*(1/2),x)

[Out] 3\*sqrt(d)\*sqrt(x)\*sinh(f\*x)\*gamma(3/4)/(4\*f\*gamma(7/4)) - 3\*sqrt(2)\*sqrt(pi)\*sqrt(d)\*exp(-3\*I\*pi/4)\*fresnels(sqrt(2)\*sqrt(f)\*sqrt(x)\*exp(I\*pi/4)/sqrt(pi))\*gamma(3/4)/(8\*f\*\*(3/2)\*gamma(7/4))



### 3.65 $\int \frac{\cosh(fx)}{\sqrt{dx}} dx$

**Optimal.** Leaf size=77

$$\frac{\sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d} \sqrt{f}} + \frac{\sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d} \sqrt{f}}$$

[Out]  $1/2*\operatorname{erf}(f^{(1/2)}*(d*x)^{(1/2)}/d^{(1/2)})*\operatorname{Pi}^{(1/2)}/d^{(1/2)}/f^{(1/2)}+1/2*\operatorname{erfi}(f^{(1/2)}*(d*x)^{(1/2)}/d^{(1/2)})*\operatorname{Pi}^{(1/2)}/d^{(1/2)}/f^{(1/2)}$

**Rubi [A]** time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi} \operatorname{Erf}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d} \sqrt{f}} + \frac{\sqrt{\pi} \operatorname{Erfi}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d} \sqrt{f}}$$

Antiderivative was successfully verified.

[In] `Int[Cosh[f*x]/Sqrt[d*x], x]`

[Out]  $(\operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erf}[(\operatorname{Sqrt}[f] * \operatorname{Sqrt}[d*x]) / \operatorname{Sqrt}[d]]) / (2 * \operatorname{Sqrt}[d] * \operatorname{Sqrt}[f]) + (\operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(\operatorname{Sqrt}[f] * \operatorname{Sqrt}[d*x]) / \operatorname{Sqrt}[d]]) / (2 * \operatorname{Sqrt}[d] * \operatorname{Sqrt}[f])$

#### Rule 2180

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True`

#### Rule 2204

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

#### Rule 2205

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]`

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh(fx)}{\sqrt{dx}} dx &= \frac{1}{2} \int \frac{e^{-fx}}{\sqrt{dx}} dx + \frac{1}{2} \int \frac{e^{fx}}{\sqrt{dx}} dx \\ &= \frac{\text{Subst}\left(\int e^{-\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{d} + \frac{\text{Subst}\left(\int e^{\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{d} \\ &= \frac{\sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d} \sqrt{f}} + \frac{\sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d} \sqrt{f}} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 48, normalized size = 0.62

$$\frac{\sqrt{-fx} \Gamma\left(\frac{1}{2}, -fx\right) - \sqrt{fx} \Gamma\left(\frac{1}{2}, fx\right)}{2f\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[f*x]/Sqrt[d*x], x]
```

```
[Out] (Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] - Sqrt[f*x]*Gamma[1/2, f*x])/(2*f*Sqrt[d*x])
```

**fricas** [A] time = 0.46, size = 59, normalized size = 0.77

$$\frac{\sqrt{\pi} \sqrt{\frac{f}{d}} \operatorname{erf}\left(\sqrt{dx} \sqrt{\frac{f}{d}}\right) - \sqrt{\pi} \sqrt{-\frac{f}{d}} \operatorname{erf}\left(\sqrt{dx} \sqrt{-\frac{f}{d}}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(1/2), x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(pi)*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d)) - sqrt(pi)*sqrt(-f/d)*erf(sqrt(d*x)*sqrt(-f/d)))/f
```

**giac** [A] time = 0.12, size = 60, normalized size = 0.78

$$\frac{\frac{\sqrt{\pi} d \operatorname{erf}\left(-\frac{\sqrt{df} \sqrt{dx}}{d}\right)}{\sqrt{df}} + \frac{\sqrt{\pi} d \operatorname{erf}\left(-\frac{\sqrt{-df} \sqrt{dx}}{d}\right)}{\sqrt{-df}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f\*x)/(d\*x)^(1/2),x, algorithm="giac")

[Out] -1/2\*(sqrt(pi)\*d\*erf(-sqrt(d\*f)\*sqrt(d\*x)/d)/sqrt(d\*f) + sqrt(pi)\*d\*erf(-sqrt(-d\*f)\*sqrt(d\*x)/d)/sqrt(-d\*f))/d

**maple** [C] time = 0.09, size = 72, normalized size = 0.94

$$\frac{i\sqrt{\pi} \sqrt{x} \sqrt{2} \sqrt{if} \left( \frac{\sqrt{if} \sqrt{2} \operatorname{erf}(\sqrt{x} \sqrt{f})}{2\sqrt{f}} + \frac{\sqrt{if} \sqrt{2} \operatorname{erfi}(\sqrt{x} \sqrt{f})}{2\sqrt{f}} \right)}{2\sqrt{dx} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f\*x)/(d\*x)^(1/2),x)

[Out] -1/2\*I\*Pi^(1/2)/(d\*x)^(1/2)\*x^(1/2)\*2^(1/2)\*(I\*f)^(1/2)/f\*(1/2\*(I\*f)^(1/2)\*2^(1/2)/f^(1/2)\*erf(x^(1/2)\*f^(1/2))+1/2\*(I\*f)^(1/2)\*2^(1/2)/f^(1/2)\*erfi(x^(1/2)\*f^(1/2)))

**maxima** [B] time = 0.50, size = 117, normalized size = 1.52

$$\frac{4\sqrt{dx} \cosh(fx) - \left( \frac{2\sqrt{dx} \operatorname{de}(fx)}{f} + \frac{2\sqrt{dx} \operatorname{de}(-fx)}{f} - \frac{\sqrt{\pi} d \operatorname{erf}\left(\sqrt{dx} \sqrt{\frac{f}{d}}\right)}{f\sqrt{\frac{f}{d}}} - \frac{\sqrt{\pi} d \operatorname{erf}\left(\sqrt{dx} \sqrt{-\frac{f}{d}}\right)}{f\sqrt{-\frac{f}{d}}} \right) f}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f\*x)/(d\*x)^(1/2),x, algorithm="maxima")

[Out] 1/2\*(4\*sqrt(d\*x)\*cosh(f\*x) - (2\*sqrt(d\*x)\*d\*e^(f\*x)/f + 2\*sqrt(d\*x)\*d\*e^(-f\*x)/f - sqrt(pi)\*d\*erf(sqrt(d\*x)\*sqrt(f/d))/(f\*sqrt(f/d)) - sqrt(pi)\*d\*erf(sqrt(d\*x)\*sqrt(-f/d))/(f\*sqrt(-f/d)))\*f/d)/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(fx)}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(f*x)/(d*x)^(1/2), x)`

[Out] `int(cosh(f*x)/(d*x)^(1/2), x)`

sympy [C] time = 0.96, size = 66, normalized size = 0.86

$$\frac{\sqrt{2} \sqrt{\pi} e^{-\frac{i\pi}{4}} C\left(\frac{\sqrt{2} \sqrt{f} \sqrt{x} e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right) \Gamma\left(\frac{1}{4}\right)}{4\sqrt{d} \sqrt{f} \Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x)/(d*x)**(1/2), x)`

[Out] `sqrt(2)*sqrt(pi)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/sqrt(pi))*gamma(1/4)/(4*sqrt(d)*sqrt(f)*gamma(5/4))`

$$3.66 \quad \int \frac{\cosh(fx)}{(dx)^{3/2}} dx$$

**Optimal.** Leaf size=88

$$-\frac{\sqrt{\pi} \sqrt{f} \operatorname{erf}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{\pi} \sqrt{f} \operatorname{erfi}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2 \cosh(fx)}{d\sqrt{dx}}$$

[Out]  $-\operatorname{erf}(f^{1/2}*(d*x)^{1/2}/d^{1/2})*f^{1/2}*Pi^{1/2}/d^{3/2}+\operatorname{erfi}(f^{1/2}*(d*x)^{1/2}/d^{1/2})*f^{1/2}*Pi^{1/2}/d^{3/2}-2*\cosh(f*x)/d/(d*x)^{1/2}$

**Rubi [A]** time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {3297, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi} \sqrt{f} \operatorname{Erf}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{\pi} \sqrt{f} \operatorname{Erfi}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2 \cosh(fx)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] `Int[Cosh[f*x]/(d*x)^(3/2), x]`

[Out]  $(-2*\operatorname{Cosh}[f*x])/(d*\operatorname{Sqrt}[d*x]) - (\operatorname{Sqrt}[f]*\operatorname{Sqrt}[Pi]*\operatorname{Erf}[(\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d*x])/ \operatorname{Sqrt}[d]])/d^{3/2} + (\operatorname{Sqrt}[f]*\operatorname{Sqrt}[Pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d*x])/ \operatorname{Sqrt}[d]])/d^{3/2}$

#### Rule 2180

`Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True`

#### Rule 2204

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

#### Rule 2205

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]`

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh(fx)}{(dx)^{3/2}} dx &= -\frac{2 \cosh(fx)}{d\sqrt{dx}} + \frac{(2f) \int \frac{\sinh(fx)}{\sqrt{dx}} dx}{d} \\ &= -\frac{2 \cosh(fx)}{d\sqrt{dx}} - \frac{f \int \frac{e^{-fx}}{\sqrt{dx}} dx}{d} + \frac{f \int \frac{e^{fx}}{\sqrt{dx}} dx}{d} \\ &= -\frac{2 \cosh(fx)}{d\sqrt{dx}} - \frac{(2f) \text{Subst}\left(\int e^{-\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{d^2} + \frac{(2f) \text{Subst}\left(\int e^{\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{d^2} \\ &= -\frac{2 \cosh(fx)}{d\sqrt{dx}} - \frac{\sqrt{f} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{f} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 67, normalized size = 0.76

$$\frac{xe^{-fx} \left(-e^{2fx} + e^{fx} \sqrt{-fx} \Gamma\left(\frac{1}{2}, -fx\right)\right) + e^{fx} \sqrt{fx} \Gamma\left(\frac{1}{2}, fx\right) - 1}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[f*x]/(d*x)^(3/2), x]
```

```
[Out] (x*(-1 - E^(2*f*x) + E^(f*x)*Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] + E^(f*x)*Sqrt[f*x]*Gamma[1/2, f*x]))/(E^(f*x)*(d*x)^(3/2))
```

**fricas** [B] time = 0.48, size = 136, normalized size = 1.55

$$\frac{\sqrt{\pi} (dx \cosh(fx) + dx \sinh(fx)) \sqrt{\frac{f}{d}} \operatorname{erf}\left(\sqrt{dx} \sqrt{\frac{f}{d}}\right) + \sqrt{\pi} (dx \cosh(fx) + dx \sinh(fx)) \sqrt{-\frac{f}{d}} \operatorname{erf}\left(\sqrt{dx} \sqrt{-\frac{f}{d}}\right)}{d^2 x \cosh(fx) + d^2 x \sinh(fx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f\*x)/(d\*x)^(3/2), x, algorithm="fricas")

[Out]  $-(\sqrt{\pi} * (d*x*\cosh(f*x) + d*x*\sinh(f*x)) * \sqrt{f/d} * \operatorname{erf}(\sqrt{d*x} * \sqrt{f/d})) + \sqrt{\pi} * (d*x*\cosh(f*x) + d*x*\sinh(f*x)) * \sqrt{-f/d} * \operatorname{erf}(\sqrt{d*x} * \sqrt{-f/d}) + \sqrt{d*x} * (\cosh(f*x)^2 + 2*\cosh(f*x)*\sinh(f*x) + \sinh(f*x)^2 + 1) / (d^2*x*\cosh(f*x) + d^2*x*\sinh(f*x))$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(fx)}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f\*x)/(d\*x)^(3/2), x, algorithm="giac")

[Out] integrate(cosh(f\*x)/(d\*x)^(3/2), x)

**maple** [C] time = 0.08, size = 115, normalized size = 1.31

$$\frac{i\sqrt{\pi} x^{\frac{3}{2}} \sqrt{2} (if)^{\frac{3}{2}} \left( -\frac{2\sqrt{2} e^{fx}}{\sqrt{\pi} \sqrt{x} \sqrt{if}} - \frac{2\sqrt{2} e^{-fx}}{\sqrt{\pi} \sqrt{x} \sqrt{if}} - \frac{2\sqrt{2} \sqrt{f} \operatorname{erf}(\sqrt{x} \sqrt{f})}{\sqrt{if}} + \frac{2\sqrt{2} \sqrt{f} \operatorname{erfi}(\sqrt{x} \sqrt{f})}{\sqrt{if}} \right)}{4 (dx)^{\frac{3}{2}} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f\*x)/(d\*x)^(3/2), x)

[Out]  $-1/4 * I * \pi^{1/2} / (d*x)^{3/2} * x^{3/2} * 2^{1/2} * (I*f)^{3/2} / f * (-2/\pi^{1/2} / x^{1/2} * 2^{1/2} / (I*f)^{1/2} * \exp(f*x) - 2/\pi^{1/2} / x^{1/2} * 2^{1/2} / (I*f)^{1/2} * \exp(-f*x) - 2 / (I*f)^{1/2} * 2^{1/2} * f^{1/2} * \operatorname{erf}(x^{1/2} * f^{1/2}) + 2 / (I*f)^{1/2} * 2^{1/2} * f^{1/2} * \operatorname{erfi}(x^{1/2} * f^{1/2}))$

**maxima** [A] time = 1.00, size = 76, normalized size = 0.86

$$\frac{f \left( \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{dx} \sqrt{\frac{f}{d}}\right)}{\sqrt{\frac{f}{d}}} - \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{dx} \sqrt{-\frac{f}{d}}\right)}{\sqrt{-\frac{f}{d}}} \right)}{d} + \frac{2 \cosh(fx)}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f\*x)/(d\*x)^(3/2),x, algorithm="maxima")

[Out]  $-(f*\sqrt{\pi}*\operatorname{erf}(\sqrt{d*x}*\sqrt{f/d}))/\sqrt{f/d} - \sqrt{\pi}*\operatorname{erf}(\sqrt{d*x}*\sqrt{f/d})/\sqrt{f/d} + 2*\cosh(f*x)/\sqrt{d*x})/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(fx)}{(dx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f\*x)/(d\*x)^(3/2),x)

[Out] int(cosh(f\*x)/(d\*x)^(3/2), x)

sympy [C] time = 2.79, size = 99, normalized size = 1.12

$$-\frac{\sqrt{2}\sqrt{\pi}\sqrt{f}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{1}{4}\right)}{2d^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)} + \frac{\cosh(fx)\Gamma\left(-\frac{1}{4}\right)}{2d^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f\*x)/(d\*x)\*\*(3/2),x)

[Out]  $-\sqrt{2}*\sqrt{\pi}*\sqrt{f}*\exp(-3*I*\pi/4)*\operatorname{fresnels}(\sqrt{2}*\sqrt{f}*\sqrt{x})*\exp(I*\pi/4)/\sqrt{\pi}*\gamma(-1/4)/(2*d**(3/2)*\gamma(3/4)) + \cosh(f*x)*\gamma(-1/4)/(2*d**(3/2)*\sqrt{x}*\gamma(3/4))$



### 3.67 $\int \frac{\cosh(fx)}{(dx)^{5/2}} dx$

**Optimal.** Leaf size=114

$$\frac{2\sqrt{\pi} f^{3/2} \operatorname{erf}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2\sqrt{\pi} f^{3/2} \operatorname{erfi}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4f \sinh(fx)}{3d^2 \sqrt{dx}} - \frac{2 \cosh(fx)}{3d(dx)^{3/2}}$$

[Out]  $-2/3*\cosh(f*x)/d/(d*x)^{(3/2)}+2/3*f^{(3/2)}*\operatorname{erf}(f^{(1/2)}*(d*x)^{(1/2)}/d^{(1/2)})*Pi^{(1/2)}/d^{(5/2)}+2/3*f^{(3/2)}*\operatorname{erfi}(f^{(1/2)}*(d*x)^{(1/2)}/d^{(1/2)})*Pi^{(1/2)}/d^{(5/2)}-4/3*f*\sinh(f*x)/d^2/(d*x)^{(1/2)}$

**Rubi [A]** time = 0.15, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {3297, 3307, 2180, 2204, 2205}

$$\frac{2\sqrt{\pi} f^{3/2} \operatorname{Erf}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2\sqrt{\pi} f^{3/2} \operatorname{Erfi}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4f \sinh(fx)}{3d^2 \sqrt{dx}} - \frac{2 \cosh(fx)}{3d(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cosh}[f*x]/(d*x)^{(5/2)}, x]$

[Out]  $(-2*\operatorname{Cosh}[f*x])/(3*d*(d*x)^{(3/2)}) + (2*f^{(3/2)}*\operatorname{Sqrt}[Pi]*\operatorname{Erf}[(\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d*x])/ \operatorname{Sqrt}[d]])/(3*d^{(5/2)}) + (2*f^{(3/2)}*\operatorname{Sqrt}[Pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d*x])/ \operatorname{Sqrt}[d]])/(3*d^{(5/2)}) - (4*f*\operatorname{Sinh}[f*x])/(3*d^2*\operatorname{Sqrt}[d*x])$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*(e_.) + (f_.)*(x_))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x\} \&\& \operatorname{!}\$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[Pi]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x\} \&\& \operatorname{PosQ}[b]$

#### Rule 2205

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[Pi]*\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]])/(2*d*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]), x] /; \operatorname{Fr}$

eeQ[{F, a, b, c, d}, x] && NegQ[b]

### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

### Rule 3307

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Dist[I/2, Int[(c + d\*x)^m/(E^(I\*k\*Pi)\*E^(I\*(e + f\*x))), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*k\*Pi)\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

### Rubi steps

$$\begin{aligned}
 \int \frac{\cosh(fx)}{(dx)^{5/2}} dx &= -\frac{2 \cosh(fx)}{3d(dx)^{3/2}} + \frac{(2f) \int \frac{\sinh(fx)}{(dx)^{3/2}} dx}{3d} \\
 &= -\frac{2 \cosh(fx)}{3d(dx)^{3/2}} - \frac{4f \sinh(fx)}{3d^2 \sqrt{dx}} + \frac{(4f^2) \int \frac{\cosh(fx)}{\sqrt{dx}} dx}{3d^2} \\
 &= -\frac{2 \cosh(fx)}{3d(dx)^{3/2}} - \frac{4f \sinh(fx)}{3d^2 \sqrt{dx}} + \frac{(2f^2) \int \frac{e^{-fx}}{\sqrt{dx}} dx}{3d^2} + \frac{(2f^2) \int \frac{e^{fx}}{\sqrt{dx}} dx}{3d^2} \\
 &= -\frac{2 \cosh(fx)}{3d(dx)^{3/2}} - \frac{4f \sinh(fx)}{3d^2 \sqrt{dx}} + \frac{(4f^2) \text{Subst}\left(\int e^{-\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{3d^3} + \frac{(4f^2) \text{Subst}\left(\int e^{\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{3d^3} \\
 &= -\frac{2 \cosh(fx)}{3d(dx)^{3/2}} + \frac{2f^{3/2} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2f^{3/2} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4f \sinh(fx)}{3d^2 \sqrt{dx}}
 \end{aligned}$$

**Mathematica** [A] time = 0.10, size = 78, normalized size = 0.68

$$\frac{x \left( -2e^{fx}(2fx + 1) - 4(-fx)^{3/2} \Gamma\left(\frac{1}{2}, -fx\right) + e^{-fx} \left( 4fx - 4e^{fx}(fx)^{3/2} \Gamma\left(\frac{1}{2}, fx\right) - 2 \right) \right)}{6(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[f\*x]/(d\*x)^(5/2),x]

[Out]  $(x*(-2E^{(fx)}*(1 + 2fx) - 4*(-fx)^{(3/2)}*\Gamma[1/2, -(fx)] + (-2 + 4fx - 4E^{(fx)}*(fx)^{(3/2)}*\Gamma[1/2, fx])/E^{(fx)}))/(6*(d*x)^{(5/2)})$

**fricas** [B] time = 0.54, size = 179, normalized size = 1.57

$$\frac{2\sqrt{\pi}(dfx^2 \cosh(fx) + dfx^2 \sinh(fx))\sqrt{\frac{f}{d}} \operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right) - 2\sqrt{\pi}(dfx^2 \cosh(fx) + dfx^2 \sinh(fx))\sqrt{-\frac{f}{d}} \operatorname{erfi}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{3(d^3x^2 \cosh(fx) + d^3x^2 \sinh(fx))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f\*x)/(d\*x)^(5/2),x, algorithm="fricas")

[Out]  $\frac{1}{3}*(2*\sqrt{\pi}*(d*fx^2*\cosh(f*x) + d*fx^2*\sinh(f*x))*\sqrt{f/d}*\operatorname{erf}(\sqrt{d*x}*\sqrt{f/d}) - 2*\sqrt{\pi}*(d*fx^2*\cosh(f*x) + d*fx^2*\sinh(f*x))*\sqrt{-f/d}*\operatorname{erfi}(\sqrt{d*x}*\sqrt{-f/d}) - ((2*fx + 1)*\cosh(f*x)^2 + 2*(2*fx + 1)*\cosh(f*x)*\sinh(f*x) + (2*fx + 1)*\sinh(f*x)^2 - 2*fx + 1)*\sqrt{d*x})/(d^3*x^2*\cosh(f*x) + d^3*x^2*\sinh(f*x))$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(fx)}{(dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f\*x)/(d\*x)^(5/2),x, algorithm="giac")

[Out] integrate(cosh(f\*x)/(d\*x)^(5/2), x)

**maple** [C] time = 0.08, size = 126, normalized size = 1.11

$$\frac{i\sqrt{\pi}x^{\frac{5}{2}}\sqrt{2}(if)^{\frac{5}{2}}\left(-\frac{8\sqrt{2}\left(-fx+\frac{1}{2}\right)e^{-fx}}{3\sqrt{\pi}x^{\frac{3}{2}}(if)^{\frac{3}{2}}}-\frac{8\sqrt{2}\left(fx+\frac{1}{2}\right)e^{fx}}{3\sqrt{\pi}x^{\frac{3}{2}}(if)^{\frac{3}{2}}}+\frac{8\sqrt{2}f^{\frac{3}{2}}\operatorname{erf}(\sqrt{x}\sqrt{f})}{3(if)^{\frac{3}{2}}}+\frac{8\sqrt{2}f^{\frac{3}{2}}\operatorname{erfi}(\sqrt{x}\sqrt{f})}{3(if)^{\frac{3}{2}}}\right)}{8(dx)^{\frac{5}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f\*x)/(d\*x)^(5/2),x)

[Out]  $-1/8*I*\Pi^{(1/2)}/(d*x)^{(5/2)}*x^{(5/2)}*2^{(1/2)}*(I*f)^{(5/2)}/f*(-8/3/\Pi^{(1/2)}/x^{(3/2)}*2^{(1/2)})/(I*f)^{(3/2)}*(-f*x+1/2)*\exp(-f*x)-8/3/\Pi^{(1/2)}/x^{(3/2)}*2^{(1/2)}$

$$\frac{1}{(I*f)^{(3/2)}*(f*x+1/2)*\exp(f*x)+8/3/(I*f)^{(3/2)*2^{(1/2)}*f^{(3/2)}*\operatorname{erf}(x^{(1/2)}*f^{(1/2)})+8/3/(I*f)^{(3/2)*2^{(1/2)}*f^{(3/2)}*\operatorname{erfi}(x^{(1/2)}*f^{(1/2)})}$$

**maxima** [A] time = 3.61, size = 58, normalized size = 0.51

$$\frac{f \left( \frac{\sqrt{fx} \Gamma\left(-\frac{1}{2}, fx\right)}{\sqrt{dx}} - \frac{\sqrt{-fx} \Gamma\left(-\frac{1}{2}, -fx\right)}{\sqrt{dx}} \right)}{d} - \frac{2 \cosh(fx)}{(dx)^{\frac{3}{2}}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f\*x)/(d\*x)^(5/2), x, algorithm="maxima")

[Out]  $\frac{1}{3} * (f * (\operatorname{sqrt}(f*x) * \operatorname{gamma}(-1/2, f*x) / \operatorname{sqrt}(d*x) - \operatorname{sqrt}(-f*x) * \operatorname{gamma}(-1/2, -f*x) / \operatorname{sqrt}(d*x))) / d - 2 * \cosh(f*x) / (d*x)^{(3/2)} / d$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(fx)}{(dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f\*x)/(d\*x)^(5/2), x)

[Out] int(cosh(f\*x)/(d\*x)^(5/2), x)

**sympy** [C] time = 19.88, size = 124, normalized size = 1.09

$$-\frac{\sqrt{2} \sqrt{\pi} f^{\frac{3}{2}} e^{-\frac{i\pi}{4}} C \left( \frac{\sqrt{2} \sqrt{f} \sqrt{x} e^{\frac{i\pi}{4}}}{\sqrt{\pi}} \right) \Gamma\left(-\frac{3}{4}\right)}{d^{\frac{5}{2}} \Gamma\left(\frac{1}{4}\right)} + \frac{f \sinh(fx) \Gamma\left(-\frac{3}{4}\right)}{d^{\frac{5}{2}} \sqrt{x} \Gamma\left(\frac{1}{4}\right)} + \frac{\cosh(fx) \Gamma\left(-\frac{3}{4}\right)}{2d^{\frac{5}{2}} x^{\frac{3}{2}} \Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f\*x)/(d\*x)\*\*(5/2), x)

[Out]  $-\operatorname{sqrt}(2) * \operatorname{sqrt}(\pi) * f^{(3/2)} * \exp(-I * \pi / 4) * \operatorname{fresnelc}(\operatorname{sqrt}(2) * \operatorname{sqrt}(f) * \operatorname{sqrt}(x)) * \exp(I * \pi / 4) / \operatorname{sqrt}(\pi) * \operatorname{gamma}(-3/4) / (d^{(5/2)} * \operatorname{gamma}(1/4)) + f * \sinh(f*x) * \operatorname{gamma}(-3/4) / (d^{(5/2)} * \operatorname{sqrt}(x) * \operatorname{gamma}(1/4)) + \cosh(f*x) * \operatorname{gamma}(-3/4) / (2 * d^{(5/2)} * x^{(3/2)} * \operatorname{gamma}(1/4))$

### 3.68 $\int \sqrt{c + dx} \operatorname{sech}(a + bx) dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\sqrt{c + dx} \operatorname{sech}(a + bx), x\right)$$

[Out] Unintegrable(sech(b\*x+a)\*(d\*x+c)^(1/2), x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \sqrt{c + dx} \operatorname{sech}(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[c + d\*x]\*Sech[a + b\*x], x]

[Out] Defer[Int][Sqrt[c + d\*x]\*Sech[a + b\*x], x]

Rubi steps

$$\int \sqrt{c + dx} \operatorname{sech}(a + bx) dx = \int \sqrt{c + dx} \operatorname{sech}(a + bx) dx$$

Mathematica [A] time = 11.38, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx} \operatorname{sech}(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[c + d\*x]\*Sech[a + b\*x], x]

[Out] Integrate[Sqrt[c + d\*x]\*Sech[a + b\*x], x]

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{dx + c} \operatorname{sech}(bx + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)\*(d\*x+c)^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(d\*x + c)\*sech(b\*x + a), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx + c} \operatorname{sech}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)\*(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(d\*x + c)\*sech(b\*x + a), x)

**maple** [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(bx + a) \sqrt{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(b\*x+a)\*(d\*x+c)^(1/2),x)

[Out] int(sech(b\*x+a)\*(d\*x+c)^(1/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx + c} \operatorname{sech}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)\*(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d\*x + c)\*sech(b\*x + a), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{c + dx}}{\cosh(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^(1/2)/cosh(a + b\*x),x)

[Out] int((c + d\*x)^(1/2)/cosh(a + b\*x), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx} \operatorname{sech}(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)\*(d\*x+c)\*\*(1/2),x)

[Out] Integral(sqrt(c + d\*x)\*sech(a + b\*x), x)

$$3.69 \quad \int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}} dx$$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}}, x\right)$$

[Out] Unintegrable(sech(b\*x+a)/(d\*x+c)^(1/2), x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}} dx$$

Verification is Not applicable to the result.

[In] Int[Sech[a + b\*x]/Sqrt[c + d\*x], x]

[Out] Defer[Int][Sech[a + b\*x]/Sqrt[c + d\*x], x]

Rubi steps

$$\int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}} dx = \int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}} dx$$

Mathematica [A] time = 9.89, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sech[a + b\*x]/Sqrt[c + d\*x], x]

[Out] Integrate[Sech[a + b\*x]/Sqrt[c + d\*x], x]

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{sech}(bx+a)}{\sqrt{dx+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)/(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] integral(sech(b\*x + a)/sqrt(d\*x + c), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx + a)}{\sqrt{dx + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)/(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(sech(b\*x + a)/sqrt(d\*x + c), x)

**maple** [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx + a)}{\sqrt{dx + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(b\*x+a)/(d\*x+c)^(1/2),x)

[Out] int(sech(b\*x+a)/(d\*x+c)^(1/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx + a)}{\sqrt{dx + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)/(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sech(b\*x + a)/sqrt(d\*x + c), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cosh(a + bx) \sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(a + b\*x)\*(c + d\*x)^(1/2)),x)

[Out] int(1/(cosh(a + b\*x)\*(c + d\*x)^(1/2)), x)



sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(a + bx)}{\sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(b\*x+a)/(d\*x+c)\*\*(1/2), x)

[Out] Integral(sech(a + b\*x)/sqrt(c + d\*x), x)

$$3.70 \quad \int \frac{\cosh^{\frac{3}{2}}(x)}{x^3} dx$$

**Optimal.** Leaf size=63

$$\frac{9}{8} \text{Int} \left( \frac{\cosh^{\frac{3}{2}}(x)}{x}, x \right) - \frac{3}{8} \text{Int} \left( \frac{1}{x\sqrt{\cosh(x)}}, x \right) - \frac{\cosh^{\frac{3}{2}}(x)}{2x^2} - \frac{3 \sinh(x)\sqrt{\cosh(x)}}{4x}$$

[Out]  $-1/2*\cosh(x)^{(3/2)}/x^2-3/4*\sinh(x)*\cosh(x)^{(1/2)}/x+9/8*\text{Unintegrable}(\cosh(x)^{(3/2)}/x,x)-3/8*\text{Unintegrable}(1/x/\cosh(x)^{(1/2)},x)$

**Rubi [A]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int [Cosh[x]^(3/2)/x^3, x]

[Out]  $-\text{Cosh}[x]^{(3/2)}/(2*x^2) - (3*\text{Sqrt}[\text{Cosh}[x]]*\text{Sinh}[x])/(4*x) - (3*\text{Defer}[\text{Int}][1/(x*\text{Sqrt}[\text{Cosh}[x]]), x])/8 + (9*\text{Defer}[\text{Int}][\text{Cosh}[x]^{(3/2)}/x, x])/8$

Rubi steps

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3} dx = -\frac{\cosh^{\frac{3}{2}}(x)}{2x^2} - \frac{3\sqrt{\cosh(x)} \sinh(x)}{4x} - \frac{3}{8} \int \frac{1}{x\sqrt{\cosh(x)}} dx + \frac{9}{8} \int \frac{\cosh^{\frac{3}{2}}(x)}{x} dx$$

**Mathematica [A]** time = 4.09, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[Cosh[x]^(3/2)/x^3, x]

[Out] Integrate[Cosh[x]^(3/2)/x^3, x]

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)^(3/2)/x^3,x, algorithm="fricas")`

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)^(3/2)/x^3,x, algorithm="giac")`

[Out] `integrate(cosh(x)^(3/2)/x^3, x)`

**maple** [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(x)^(3/2)/x^3,x)`

[Out] `int(cosh(x)^(3/2)/x^3,x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(x)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)^(3/2)/x^3,x, algorithm="maxima")`

[Out] `integrate(cosh(x)^(3/2)/x^3, x)`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(x)^{3/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^(3/2)/x^3,x)
```

```
[Out] int(cosh(x)^(3/2)/x^3, x)
```

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**(3/2)/x**3,x)
```

```
[Out] Integral(cosh(x)**(3/2)/x**3, x)
```

$$3.71 \quad \int \left( \frac{x}{\cosh^{\frac{3}{2}}(x)} + x\sqrt{\cosh(x)} \right) dx$$

Optimal. Leaf size=20

$$\frac{2x \sinh(x)}{\sqrt{\cosh(x)}} - 4\sqrt{\cosh(x)}$$

[Out]  $2*x*\sinh(x)/\cosh(x)^{(1/2)}-4*\cosh(x)^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {3315}

$$\frac{2x \sinh(x)}{\sqrt{\cosh(x)}} - 4\sqrt{\cosh(x)}$$

Antiderivative was successfully verified.

[In] `Int[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]`

[Out] `-4*Sqrt[Cosh[x]] + (2*x*Sinh[x])/Sqrt[Cosh[x]]`

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
  Simp[((c + d*x)*Cos[e + f*x]*(b*Sinh[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
  (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sinh[e + f*x])^(n + 2), x], x]
  ] - Simp[(d*(b*Sinh[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned} \int \left( \frac{x}{\cosh^{\frac{3}{2}}(x)} + x\sqrt{\cosh(x)} \right) dx &= \int \frac{x}{\cosh^{\frac{3}{2}}(x)} dx + \int x\sqrt{\cosh(x)} dx \\ &= -4\sqrt{\cosh(x)} + \frac{2x \sinh(x)}{\sqrt{\cosh(x)}} \end{aligned}$$

Mathematica [B] time = 0.39, size = 46, normalized size = 2.30

$$\frac{2 \sinh(x) \left( x - \frac{2 \sinh(x) \cosh(x) \sqrt{\tanh^2\left(\frac{x}{2}\right)}}{(\cosh(x)-1)^{3/2} \sqrt{\cosh(x)+1}} \right)}{\sqrt{\cosh(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

```
[Out] (2*Sinh[x]*(x - (2*Cosh[x]*Sinh[x]*Sqrt[Tanh[x/2]^2])/((-1 + Cosh[x])^(3/2)
*Sqrt[1 + Cosh[x]])))/Sqrt[Cosh[x]]
```

```
fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (has polynomial part)
```

```
giac [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int x\sqrt{\cosh(x)} + \frac{x}{\cosh(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

```
maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00
```

$$\int \frac{x}{\cosh(x)^{\frac{3}{2}}} + x\left(\sqrt{\cosh(x)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

```
[Out] int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

```
maxima [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int x\sqrt{\cosh(x)} + \frac{x}{\cosh(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="maxima")
```

[Out] integrate(x\*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)

**mupad** [B] time = 0.97, size = 39, normalized size = 1.95

$$\frac{2\sqrt{\frac{e^{-x}}{2} + \frac{e^x}{2}} (x + 2e^{2x} - xe^{2x} + 2)}{e^{2x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cosh(x)^(1/2) + x/cosh(x)^(3/2), x)

[Out] -(2\*(exp(-x)/2 + exp(x)/2)^(1/2)\*(x + 2\*exp(2\*x) - x\*exp(2\*x) + 2))/(exp(2\*x) + 1)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(\cosh^2(x) + 1)}{\cosh^{\frac{3}{2}}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/cosh(x)\*\*(3/2)+x\*cosh(x)\*\*(1/2), x)

[Out] Integral(x\*(cosh(x)\*\*2 + 1)/cosh(x)\*\*(3/2), x)

$$3.72 \quad \int \left( \frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}} \right) dx$$

Optimal. Leaf size=24

$$\frac{4}{3\sqrt{\cosh(x)}} + \frac{2x \sinh(x)}{3 \cosh^{\frac{3}{2}}(x)}$$

[Out]  $2/3*x*\sinh(x)/\cosh(x)^{(3/2)}+4/3/\cosh(x)^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$ , Rules used = {3315}

$$\frac{4}{3\sqrt{\cosh(x)}} + \frac{2x \sinh(x)}{3 \cosh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

[In] `Int[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]`

[Out]  $4/(3*\text{Sqrt}[\text{Cosh}[x]]) + (2*x*\text{Sinh}[x])/(3*\text{Cosh}[x]^{(3/2)})$

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
  Simp[((c + d*x)*Cos[e + f*x]*(b*Ssin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
  (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Ssin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Ssin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned} \int \left( \frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}} \right) dx &= -\left( \frac{1}{3} \int \frac{x}{\sqrt{\cosh(x)}} dx \right) + \int \frac{x}{\cosh^{\frac{5}{2}}(x)} dx \\ &= \frac{4}{3\sqrt{\cosh(x)}} + \frac{2x \sinh(x)}{3 \cosh^{\frac{3}{2}}(x)} \end{aligned}$$

Mathematica [A] time = 0.08, size = 16, normalized size = 0.67

$$\frac{2(x \tanh(x) + 2)}{3\sqrt{\cosh(x)}}$$



Antiderivative was successfully verified.

[In] Integrate[x/Cosh[x]^(5/2) - x/(3\*Sqrt[Cosh[x]]), x]

[Out] (2\*(2 + x\*Tanh[x]))/(3\*Sqrt[Cosh[x]])

**fricas** [B] time = 0.53, size = 109, normalized size = 4.54

$$\frac{4((x+2)\cosh(x)^3 + 3(x+2)\cosh(x)\sinh(x)^2 + (x+2)\sinh(x)^3 - (x-2)\cosh(x) + (3(x+2)\cosh(x)^2 - x + 2)\sinh(x))\sqrt{\cosh(x)}}{3(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2(3\cosh(x)^2 + 1)\sinh(x)^2 + 2\cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x)\sinh(x)^2 + \sinh(x)^3 + 1))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/cosh(x)^(5/2)-1/3\*x/cosh(x)^(1/2), x, algorithm="fricas")

[Out] 4/3\*((x + 2)\*cosh(x)^3 + 3\*(x + 2)\*cosh(x)\*sinh(x)^2 + (x + 2)\*sinh(x)^3 - (x - 2)\*cosh(x) + (3\*(x + 2)\*cosh(x)^2 - x + 2)\*sinh(x))\*sqrt(cosh(x))/(cosh(x)^4 + 4\*cosh(x)\*sinh(x)^3 + sinh(x)^4 + 2\*(3\*cosh(x)^2 + 1)\*sinh(x)^2 + 2\*cosh(x)^2 + 4\*(cosh(x)^3 + cosh(x)\*sinh(x)^2 + sinh(x)^3 + 1))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{3\sqrt{\cosh(x)}} + \frac{x}{\cosh(x)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/cosh(x)^(5/2)-1/3\*x/cosh(x)^(1/2), x, algorithm="giac")

[Out] integrate(-1/3\*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)

**maple** [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x}{\cosh(x)^{\frac{5}{2}}} - \frac{x}{3\sqrt{\cosh(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/cosh(x)^(5/2)-1/3\*x/cosh(x)^(1/2), x)

[Out] int(x/cosh(x)^(5/2)-1/3\*x/cosh(x)^(1/2), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{3\sqrt{\cosh(x)}} + \frac{x}{\cosh(x)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/cosh(x)^(5/2)-1/3\*x/cosh(x)^(1/2),x, algorithm="maxima")

[Out] integrate(-1/3\*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)

mupad [B] time = 0.94, size = 42, normalized size = 1.75

$$\frac{4e^x \sqrt{\frac{e^{-x}}{2} + \frac{e^x}{2}} (2e^{2x} - x + xe^{2x} + 2)}{3(e^{2x} + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/cosh(x)^(5/2) - x/(3\*cosh(x)^(1/2)),x)

[Out] (4\*exp(x)\*(exp(-x)/2 + exp(x)/2)^(1/2)\*(2\*exp(2\*x) - x + x\*exp(2\*x) + 2))/(3\*(exp(2\*x) + 1)^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \left( -\frac{3x}{\cosh^2(x)} \right) dx + \int \frac{x}{\sqrt{\cosh(x)}} dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/cosh(x)\*\*(5/2)-1/3\*x/cosh(x)\*\*(1/2),x)

[Out] -(Integral(-3\*x/cosh(x)\*\*(5/2), x) + Integral(x/sqrt(cosh(x)), x))/3

$$3.73 \quad \int \left( \frac{x}{\cosh^{\frac{7}{2}}(x)} + \frac{3}{5}x\sqrt{\cosh(x)} \right) dx$$

Optimal. Leaf size=47

$$\frac{4}{15 \cosh^{\frac{3}{2}}(x)} - \frac{12\sqrt{\cosh(x)}}{5} + \frac{2x \sinh(x)}{5 \cosh^{\frac{5}{2}}(x)} + \frac{6x \sinh(x)}{5\sqrt{\cosh(x)}}$$

[Out] 4/15/cosh(x)^(3/2)+2/5\*x\*sinh(x)/cosh(x)^(5/2)+6/5\*x\*sinh(x)/cosh(x)^(1/2)-12/5\*cosh(x)^(1/2)

**Rubi [A]** time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$ , Rules used = {3315}

$$\frac{4}{15 \cosh^{\frac{3}{2}}(x)} - \frac{12\sqrt{\cosh(x)}}{5} + \frac{2x \sinh(x)}{5 \cosh^{\frac{5}{2}}(x)} + \frac{6x \sinh(x)}{5\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

[In] Int[x/Cosh[x]^(7/2) + (3\*x\*Sqrt[Cosh[x]])/5,x]

[Out] 4/(15\*Cosh[x]^(3/2)) - (12\*Sqrt[Cosh[x]])/5 + (2\*x\*Sinh[x])/(5\*Cosh[x]^(5/2)) + (6\*x\*Sinh[x])/(5\*Sqrt[Cosh[x]])

Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
  Simp[((c + d*x)*Cos[e + f*x]*(b*Sinh[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
  (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sinh[e + f*x])^(n + 2), x], x]
  ] - Simp[(d*(b*Sinh[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\int \left( \frac{x}{\cosh^{\frac{7}{2}}(x)} + \frac{3}{5} x \sqrt{\cosh(x)} \right) dx &= \frac{3}{5} \int x \sqrt{\cosh(x)} dx + \int \frac{x}{\cosh^{\frac{7}{2}}(x)} dx \\
&= \frac{4}{15 \cosh^{\frac{3}{2}}(x)} + \frac{2x \sinh(x)}{5 \cosh^{\frac{5}{2}}(x)} + \frac{3}{5} \int \frac{x}{\cosh^{\frac{3}{2}}(x)} dx + \frac{3}{5} \int x \sqrt{\cosh(x)} dx \\
&= \frac{4}{15 \cosh^{\frac{3}{2}}(x)} - \frac{12 \sqrt{\cosh(x)}}{5} + \frac{2x \sinh(x)}{5 \cosh^{\frac{5}{2}}(x)} + \frac{6x \sinh(x)}{5 \sqrt{\cosh(x)}}
\end{aligned}$$

**Mathematica [A]** time = 0.64, size = 64, normalized size = 1.36

$$\frac{1}{5} \sqrt{\cosh(x)} \left( 6x \tanh(x) + \left( 2x \tanh(x) + \frac{4}{3} \right) \operatorname{sech}^2(x) - \frac{12 \sinh^2(x)}{\sqrt{\cosh(x) - 1} (\cosh(x) + 1)^{3/2} \sqrt{\tanh^2\left(\frac{x}{2}\right)}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/Cosh[x]^(7/2) + (3\*x\*Sqrt[Cosh[x]])/5, x]

[Out] (Sqrt[Cosh[x]]\*((-12\*Sinh[x]^2)/(Sqrt[-1 + Cosh[x]]\*(1 + Cosh[x])^(3/2)\*Sqrt[Tanh[x/2]^2]) + 6\*x\*Tanh[x] + Sech[x]^2\*(4/3 + 2\*x\*Tanh[x]))) / 5

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/cosh(x)^(7/2)+3/5\*x\*cosh(x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3}{5} x \sqrt{\cosh(x)} + \frac{x}{\cosh(x)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/cosh(x)^(7/2)+3/5\*x\*cosh(x)^(1/2), x, algorithm="giac")

[Out] integrate(3/5\*x\*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)

**maple** [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\cosh(x)^{\frac{7}{2}}} + \frac{3x(\sqrt{\cosh(x)})}{5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/cosh(x)^(7/2)+3/5\*x\*cosh(x)^(1/2),x)

[Out] int(x/cosh(x)^(7/2)+3/5\*x\*cosh(x)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3}{5} x \sqrt{\cosh(x)} + \frac{x}{\cosh(x)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/cosh(x)^(7/2)+3/5\*x\*cosh(x)^(1/2),x, algorithm="maxima")

[Out] integrate(3/5\*x\*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)

**mupad** [B] time = 1.08, size = 110, normalized size = 2.34

$$\frac{e^{2x} \left( \frac{8x}{5} + \frac{16}{15} \right) \sqrt{\frac{e^{-x}}{2} + \frac{e^x}{2}}}{(e^{2x} + 1)^2} - \left( \frac{6x}{5} + \frac{12}{5} \right) \sqrt{\frac{e^{-x}}{2} + \frac{e^x}{2}} + \frac{12x e^{2x} \sqrt{\frac{e^{-x}}{2} + \frac{e^x}{2}}}{5(e^{2x} + 1)} - \frac{16x e^{2x} \sqrt{\frac{e^{-x}}{2} + \frac{e^x}{2}}}{5(e^{2x} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3\*x\*cosh(x)^(1/2))/5 + x/cosh(x)^(7/2),x)

[Out] (exp(2\*x)\*((8\*x)/5 + 16/15)\*(exp(-x)/2 + exp(x)/2)^(1/2))/(exp(2\*x) + 1)^2 - ((6\*x)/5 + 12/5)\*(exp(-x)/2 + exp(x)/2)^(1/2) + (12\*x\*exp(2\*x)\*(exp(-x)/2 + exp(x)/2)^(1/2))/(5\*(exp(2\*x) + 1)) - (16\*x\*exp(2\*x)\*(exp(-x)/2 + exp(x)/2)^(1/2))/(5\*(exp(2\*x) + 1)^3)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/cosh(x)\*\*(7/2)+3/5\*x\*cosh(x)\*\*(1/2),x)

[Out] Timed out

$$3.74 \quad \int \left( \frac{x^2}{\cosh^{\frac{3}{2}}(x)} + x^2 \sqrt{\cosh(x)} \right) dx$$

Optimal. Leaf size=36

$$\frac{2x^2 \sinh(x)}{\sqrt{\cosh(x)}} - 8x\sqrt{\cosh(x)} - 16iE\left(\frac{ix}{2} \middle| 2\right)$$

[Out]  $-16*I*(\cosh(1/2*x)^2)^{(1/2)}/\cosh(1/2*x)*\text{EllipticE}(I*\sinh(1/2*x), 2^{(1/2)})+2*x^2*\sinh(x)/\cosh(x)^{(1/2)}-8*x*\cosh(x)^{(1/2)}$

**Rubi [A]** time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {3316, 2639}

$$\frac{2x^2 \sinh(x)}{\sqrt{\cosh(x)}} - 8x\sqrt{\cosh(x)} - 16iE\left(\frac{ix}{2} \middle| 2\right)$$

Antiderivative was successfully verified.

[In] Int[x^2/Cosh[x]^(3/2) + x^2\*Sqrt[Cosh[x]], x]

[Out]  $-8*x*\text{Sqrt}[\text{Cosh}[x]] - (16*I)*\text{EllipticE}[(I/2)*x, 2] + (2*x^2*\text{Sinh}[x])/\text{Sqrt}[\text{Cosh}[x]]$

### Rule 2639

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - P i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

### Rule 3316

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[((c + d\*x)^m\*Cos[e + f\*x]\*(b\*Ssin[e + f\*x])^(n + 1))/(b\*f\*(n + 1)), x] + (Dist[(n + 2)/(b^2\*(n + 1)), Int[(c + d\*x)^m\*(b\*Ssin[e + f\*x])^(n + 2), x], x] + Dist[(d^2\*m\*(m - 1))/(b^2\*f^2\*(n + 1)\*(n + 2)), Int[(c + d\*x)^(m - 2)\*(b\*Ssin[e + f\*x])^(n + 2), x], x] - Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Ssin[e + f\*x])^(n + 2))/(b^2\*f^2\*(n + 1)\*(n + 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2] && GtQ[m, 1]

### Rubi steps

$$\begin{aligned}
\int \left( \frac{x^2}{\cosh^{\frac{3}{2}}(x)} + x^2 \sqrt{\cosh(x)} \right) dx &= \int \frac{x^2}{\cosh^{\frac{3}{2}}(x)} dx + \int x^2 \sqrt{\cosh(x)} dx \\
&= -8x \sqrt{\cosh(x)} + \frac{2x^2 \sinh(x)}{\sqrt{\cosh(x)}} + 8 \int \sqrt{\cosh(x)} dx \\
&= -8x \sqrt{\cosh(x)} - 16iE \left( \frac{ix}{2} \middle| 2 \right) + \frac{2x^2 \sinh(x)}{\sqrt{\cosh(x)}}
\end{aligned}$$

**Mathematica [C]** time = 1.03, size = 76, normalized size = 2.11

$$\frac{4\sqrt{\cosh(x)} (\sinh(x) + \cosh(x)) \left( 8 {}_2F_1 \left( -\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; -e^{2x} \right) (\sinh(x) - \cosh(x)) \sqrt{\sinh(2x) + \cosh(2x) + 1} + x^2 \sinh(x) \right)}{e^{2x} + 1}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Cosh[x]^(3/2) + x^2\*Sqrt[Cosh[x]],x]

[Out] (4\*Sqrt[Cosh[x]]\*(Cosh[x] + Sinh[x])\*(-4\*(-2 + x)\*Cosh[x] + x^2\*Sinh[x] + 8\*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^(2\*x)]\*(-Cosh[x] + Sinh[x])\*Sqrt[1 + Cosh[2\*x] + Sinh[2\*x]]))/(1 + E^(2\*x))

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/cosh(x)^(3/2)+x^2\*cosh(x)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{\cosh(x)} + \frac{x^2}{\cosh(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/cosh(x)^(3/2)+x^2\*cosh(x)^(1/2),x, algorithm="giac")

[Out] integrate(x^2\*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)

**maple** [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\cosh(x)^{\frac{3}{2}}} + x^2 \left( \sqrt{\cosh(x)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/cosh(x)^(3/2)+x^2\*cosh(x)^(1/2),x)

[Out] int(x^2/cosh(x)^(3/2)+x^2\*cosh(x)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{\cosh(x)} + \frac{x^2}{\cosh(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/cosh(x)^(3/2)+x^2\*cosh(x)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2\*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^2 \sqrt{\cosh(x)} + \frac{x^2}{\cosh(x)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cosh(x)^(1/2) + x^2/cosh(x)^(3/2),x)

[Out] int(x^2\*cosh(x)^(1/2) + x^2/cosh(x)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (\cosh^2(x) + 1)}{\cosh^{\frac{3}{2}}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/cosh(x)\*\*(3/2)+x\*\*2\*cosh(x)\*\*(1/2),x)

[Out] Integral(x\*\*2\*(cosh(x)\*\*2 + 1)/cosh(x)\*\*(3/2), x)



### 3.75 $\int (c + dx)^m (b \cosh(e + fx))^n dx$

Optimal. Leaf size=21

$$\text{Int}((c + dx)^m (b \cosh(e + fx))^n, x)$$

[Out] Unintegrable((d\*x+c)^m\*(b\*cosh(f\*x+e))^n,x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (c + dx)^m (b \cosh(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m\*(b\*Cosh[e + f\*x])^n,x]

[Out] Defer[Int][(c + d\*x)^m\*(b\*Cosh[e + f\*x])^n, x]

Rubi steps

$$\int (c + dx)^m (b \cosh(e + fx))^n dx = \int (c + dx)^m (b \cosh(e + fx))^n dx$$

Mathematica [A] time = 3.19, size = 0, normalized size = 0.00

$$\int (c + dx)^m (b \cosh(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m\*(b\*Cosh[e + f\*x])^n,x]

[Out] Integrate[(c + d\*x)^m\*(b\*Cosh[e + f\*x])^n, x]

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}((dx + c)^m (b \cosh(fx + e))^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(b\*cosh(f\*x+e))^n,x, algorithm="fricas")

[Out] integral((d\*x + c)^m\*(b\*cosh(f\*x + e))^n, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m (b \cosh(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(b\*cosh(f\*x+e))^n,x, algorithm="giac")

[Out] integrate((d\*x + c)^m\*(b\*cosh(f\*x + e))^n, x)

**maple** [A] time = 0.14, size = 0, normalized size = 0.00

$$\int (dx + c)^m (b \cosh(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*(b\*cosh(f\*x+e))^n,x)

[Out] int((d\*x+c)^m\*(b\*cosh(f\*x+e))^n,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m (b \cosh(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(b\*cosh(f\*x+e))^n,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m\*(b\*cosh(f\*x + e))^n, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (b \cosh(e + fx))^n (c + dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*cosh(e + f\*x))^n\*(c + d\*x)^m,x)

[Out] int((b\*cosh(e + f\*x))^n\*(c + d\*x)^m, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cosh(e + fx))^n (c + dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m\*(b\*cosh(f\*x+e))\*\*n,x)

[Out] Integral((b\*cosh(e + f\*x))\*\*n\*(c + d\*x)\*\*m, x)

### 3.76 $\int (c + dx)^m \cosh^3(a + bx) dx$

**Optimal.** Leaf size=237

$$\frac{3^{-m-1} e^{3a - \frac{3bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{3b(c+dx)}{d}\right)}{8b} + \frac{3e^{a - \frac{bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{b(c+dx)}{d}\right)}{8b} - \frac{3e^{\frac{bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{b(c+dx)}{d}\right)}{8b}$$

[Out]  $\frac{1}{8} 3^{(-1-m)} \exp(3a - 3b*c/d) * (d*x+c)^m * \text{GAMMA}(1+m, -3*b*(d*x+c)/d) / b / ((-b*(d*x+c)/d)^m) + \frac{3}{8} \exp(a - b*c/d) * (d*x+c)^m * \text{GAMMA}(1+m, -b*(d*x+c)/d) / b / ((-b*(d*x+c)/d)^m) - \frac{3}{8} \exp(-a + b*c/d) * (d*x+c)^m * \text{GAMMA}(1+m, b*(d*x+c)/d) / b / ((b*(d*x+c)/d)^m) - \frac{1}{8} 3^{(-1-m)} \exp(-3*a + 3*b*c/d) * (d*x+c)^m * \text{GAMMA}(1+m, 3*b*(d*x+c)/d) / b / ((b*(d*x+c)/d)^m)$

**Rubi [A]** time = 0.28, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {3312, 3307, 2181}

$$\frac{3^{-m-1} e^{3a - \frac{3bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{3b(c+dx)}{d}\right)}{8b} + \frac{3e^{a - \frac{bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{b(c+dx)}{d}\right)}{8b} - \frac{3e^{\frac{bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{b(c+dx)}{d}\right)}{8b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^m * \text{Cosh}[a + b*x]^3, x]$

[Out]  $(3^{(-1-m)} * E^{(3*a - (3*b*c)/d)} * (c + d*x)^m * \text{Gamma}[1+m, (-3*b*(c + d*x))/d]) / (8*b * ((b*(c + d*x))/d)^m) + (3 * E^{(a - (b*c)/d)} * (c + d*x)^m * \text{Gamma}[1+m, -(b*(c + d*x))/d]) / (8*b * ((b*(c + d*x))/d)^m) - (3 * E^{(-a + (b*c)/d)} * (c + d*x)^m * \text{Gamma}[1+m, (b*(c + d*x))/d]) / (8*b * ((b*(c + d*x))/d)^m) - (3^{(-1-m)} * E^{(-3*a + (3*b*c)/d)} * (c + d*x)^m * \text{Gamma}[1+m, (3*b*(c + d*x))/d]) / (8*b * ((b*(c + d*x))/d)^m)$

**Rule 2181**

$\text{Int}[(F_)^((g_.) * (e_.) + (f_.) * (x_))) * ((c_.) + (d_.) * (x_))^{(m_.)}, x\_Symbol]$   
 $:\> -\text{Simp}[(F^{(g*(e - (c*f)/d)}) * (c + d*x)^{\text{FracPart}[m]} * \text{Gamma}[m+1, (-((f*g*\text{Log}[F])/d)) * (c + d*x)]) / (d * (-((f*g*\text{Log}[F])/d))^{(\text{IntPart}[m] + 1)} * (-((f*g*\text{Log}[F]) * (c + d*x))/d))^{\text{FracPart}[m]}], x] /;$  FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

**Rule 3307**

$\text{Int}(((c_.) + (d_.) * (x_))^{(m_.)} * \sin[(e_.) + \text{Pi} * (k_.) + (f_.) * (x_)], x\_Symbol]$   
 $:\> \text{Dist}[I/2, \text{Int}[(c + d*x)^m / (E^{(I*k*Pi)} * E^{(I*(e + f*x))}), x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m * E^{(I*k*Pi)} * E^{(I*(e + f*x))}, x], x] /;$  FreeQ[{c, d, e,

f, m}, x] && IntegerQ[2\*k]

### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rubi steps

$$\begin{aligned} \int (c + dx)^m \cosh^3(a + bx) dx &= \int \left( \frac{3}{4}(c + dx)^m \cosh(a + bx) + \frac{1}{4}(c + dx)^m \cosh(3a + 3bx) \right) dx \\ &= \frac{1}{4} \int (c + dx)^m \cosh(3a + 3bx) dx + \frac{3}{4} \int (c + dx)^m \cosh(a + bx) dx \\ &= \frac{1}{8} \int e^{-i(3ia+3ibx)}(c + dx)^m dx + \frac{1}{8} \int e^{i(3ia+3ibx)}(c + dx)^m dx + \frac{3}{8} \int e^{-i(ia+ibx)}(c + dx)^m dx \\ &= \frac{3^{-1-m} e^{3a - \frac{3bc}{d}} (c + dx)^m \left( -\frac{b(c+dx)}{d} \right)^{-m} \Gamma\left(1 + m, -\frac{3b(c+dx)}{d}\right)}{8b} + \frac{3e^{a - \frac{bc}{d}} (c + dx)^m \left( -\frac{b(c+dx)}{d} \right)^{-m} \Gamma\left(1 + m, -\frac{b(c+dx)}{d}\right)}{8b} \end{aligned}$$

**Mathematica** [A] time = 0.20, size = 205, normalized size = 0.86

$$\frac{3^{-m-1} e^{-3\left(a + \frac{bc}{d}\right)} (c + dx)^m \left( -\frac{b^2(c+dx)^2}{d^2} \right)^{-m} \left( e^{6a} \left( b \left( \frac{c}{d} + x \right) \right)^m \Gamma\left(m + 1, -\frac{3b(c+dx)}{d}\right) + 3^{m+2} e^{4a + \frac{2bc}{d}} \left( b \left( \frac{c}{d} + x \right) \right)^m \Gamma\left(m + 1, -\frac{b(c+dx)}{d}\right) \right)}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^m\*Cosh[a + b\*x]^3,x]

[Out] (3^(-1 - m)\*(c + d\*x)^m\*(E^(6\*a)\*(b\*(c/d + x))^m\*Gamma[1 + m, (-3\*b\*(c + d\*x))/d] + 3^(2 + m)\*E^(4\*a + (2\*b\*c)/d)\*(b\*(c/d + x))^m\*Gamma[1 + m, -(b\*(c + d\*x))/d]) - E^(((4\*b\*c)/d)\*(-(b\*(c + d\*x))/d))^m\*(3^(2 + m)\*E^(2\*a)\*Gamma[1 + m, (b\*(c + d\*x))/d] + E^(((2\*b\*c)/d)\*Gamma[1 + m, (3\*b\*(c + d\*x))/d]))/(8\*b\*E^(3\*(a + (b\*c)/d))\*(-(b^2\*(c + d\*x)^2)/d^2))^m

**fricas** [A] time = 0.55, size = 340, normalized size = 1.43

$$\frac{\cosh\left(\frac{dm \log\left(\frac{3b}{d}\right) - 3bc + 3ad}{d}\right) \Gamma\left(m + 1, \frac{3(bdx+bc)}{d}\right) + 9 \cosh\left(\frac{dm \log\left(\frac{b}{d}\right) - bc + ad}{d}\right) \Gamma\left(m + 1, \frac{bdx+bc}{d}\right) - 9 \cosh\left(\frac{dm \log\left(-\frac{b}{d}\right) + bc - ad}{d}\right) \Gamma\left(m + 1, \frac{bdx+bc}{d}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*cosh(b\*x+a)^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/24*(\cosh((d*m*\log(3*b/d) - 3*b*c + 3*a*d)/d)*\gamma(m + 1, 3*(b*d*x + b*c)/d) + 9*\cosh((d*m*\log(b/d) - b*c + a*d)/d)*\gamma(m + 1, (b*d*x + b*c)/d) - \\ & 9*\cosh((d*m*\log(-b/d) + b*c - a*d)/d)*\gamma(m + 1, -(b*d*x + b*c)/d) - \cosh((d*m*\log(-3*b/d) + 3*b*c - 3*a*d)/d)*\gamma(m + 1, -3*(b*d*x + b*c)/d) - \gamma(m + 1, 3*(b*d*x + b*c)/d)*\sinh((d*m*\log(3*b/d) - 3*b*c + 3*a*d)/d) - 9 \\ & *\gamma(m + 1, (b*d*x + b*c)/d)*\sinh((d*m*\log(b/d) - b*c + a*d)/d) + 9*\gamma(m + 1, -(b*d*x + b*c)/d)*\sinh((d*m*\log(-b/d) + b*c - a*d)/d) + \gamma(m + 1, -3*(b*d*x + b*c)/d)*\sinh((d*m*\log(-3*b/d) + 3*b*c - 3*a*d)/d))/b \end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \cosh(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*cosh(b\*x+a)^3,x, algorithm="giac")

[Out] integrate((d\*x + c)^m\*cosh(b\*x + a)^3, x)

**maple** [F] time = 0.32, size = 0, normalized size = 0.00

$$\int (dx + c)^m (\cosh^3(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*cosh(b\*x+a)^3,x)

[Out] int((d\*x+c)^m\*cosh(b\*x+a)^3,x)

**maxima** [A] time = 0.56, size = 161, normalized size = 0.68

$$\frac{(dx + c)^{m+1} e^{\left(-3a + \frac{3bc}{d}\right)} E_{-m}\left(\frac{3(dx+c)b}{d}\right)}{8d} - \frac{3(dx + c)^{m+1} e^{\left(-a + \frac{bc}{d}\right)} E_{-m}\left(\frac{(dx+c)b}{d}\right)}{8d} - \frac{3(dx + c)^{m+1} e^{\left(a - \frac{bc}{d}\right)} E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*cosh(b\*x+a)^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/8*(d*x + c)^{(m + 1)}*e^{(-3*a + 3*b*c/d)}*\exp\_integral\_e(-m, 3*(d*x + c)*b/d)/d - 3/8*(d*x + c)^{(m + 1)}*e^{(-a + b*c/d)}*\exp\_integral\_e(-m, (d*x + c)*b/d)/d - 3/8*(d*x + c)^{(m + 1)}*e^{(a - b*c/d)}*\exp\_integral\_e(-m, -(d*x + c)*b/d) \end{aligned}$$

$d)/d - 1/8*(d*x + c)^{(m + 1)}*e^{(3*a - 3*b*c/d)}*\exp\_integral\_e(-m, -3*(d*x + c)*b/d)/d$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh(a + bx)^3 (c + dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(a + b*x)^3*(c + d*x)^m, x)`

[Out] `int(cosh(a + b*x)^3*(c + d*x)^m, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \cosh^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)**m*cosh(b*x+a)**3, x)`

[Out] `Integral((c + d*x)**m*cosh(a + b*x)**3, x)`

### 3.77 $\int (c + dx)^m \cosh^2(a + bx) dx$

**Optimal.** Leaf size=144

$$\frac{2^{-m-3} e^{2a - \frac{2bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-m-3} e^{\frac{2bc}{d} - 2a} (c + dx)^m \left(\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{2b(c+dx)}{d}\right)}{b} + \dots$$

[Out]  $1/2*(d*x+c)^{(1+m)}/d/(1+m)+2^{(-3-m)*\exp(2*a-2*b*c/d)*(d*x+c)^m*\text{GAMMA}(1+m,-2*b*(d*x+c)/d)/b/((-b*(d*x+c)/d)^m)-2^{(-3-m)*\exp(-2*a+2*b*c/d)*(d*x+c)^m*\text{GAMMA}(1+m,2*b*(d*x+c)/d)/b/((b*(d*x+c)/d)^m)$

**Rubi [A]** time = 0.18, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {3312, 3307, 2181}

$$\frac{2^{-m-3} e^{2a - \frac{2bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-m-3} e^{\frac{2bc}{d} - 2a} (c + dx)^m \left(\frac{b(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, \frac{2b(c+dx)}{d}\right)}{b} + \dots$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^m * \text{Cosh}[a + b*x]^2, x]$

[Out]  $(c + d*x)^{(1 + m)}/(2*d*(1 + m)) + (2^{(-3 - m)*E^{(2*a - (2*b*c)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, (-2*b*(c + d*x))/d]}/(b*(-((b*(c + d*x))/d))^m) - (2^{(-3 - m)*E^{(-2*a + (2*b*c)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, (2*b*(c + d*x))/d]}/(b*((b*(c + d*x))/d)^m)$

#### Rule 2181

$\text{Int}[(F\_)^{((g\_)*(e\_)+(f\_)*(x\_))}*((c\_)+(d\_)*(x\_))^{(m\_)}, x\_Symbol]$   
 $\rightarrow -\text{Simp}[(F^{(g*(e - (c*f)/d)}*(c + d*x)^{\text{FracPart}[m]*\text{Gamma}[m + 1, (-((f*g*\text{Log}[F])/d))*(c + d*x)]})/(d*(-((f*g*\text{Log}[F])/d))^{(\text{IntPart}[m] + 1)*(-((f*g*\text{Log}[F])*(c + d*x))/d))^{\text{FracPart}[m]})}, x] /; \text{FreeQ}\{F, c, d, e, f, g, m\}, x] \&\& !\text{IntegerQ}[m]$

#### Rule 3307

$\text{Int}[(c + d*x)^m * \sin[(e + \text{Pi}*(k + f*x))], x\_Symbol]$   
 $\rightarrow \text{Dist}[I/2, \text{Int}[(c + d*x)^m/(E^{(I*k*\text{Pi})}*E^{(I*(e + f*x))}), x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m * E^{(I*k*\text{Pi})} * E^{(I*(e + f*x))}, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x] \&\& \text{IntegerQ}[2*k]$

#### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int
[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

### Rubi steps

$$\begin{aligned}
\int (c + dx)^m \cosh^2(a + bx) dx &= \int \left( \frac{1}{2}(c + dx)^m + \frac{1}{2}(c + dx)^m \cosh(2a + 2bx) \right) dx \\
&= \frac{(c + dx)^{1+m}}{2d(1+m)} + \frac{1}{2} \int (c + dx)^m \cosh(2a + 2bx) dx \\
&= \frac{(c + dx)^{1+m}}{2d(1+m)} + \frac{1}{4} \int e^{-i(2ia+2ibx)}(c + dx)^m dx + \frac{1}{4} \int e^{i(2ia+2ibx)}(c + dx)^m dx \\
&= \frac{(c + dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m} e^{2a - \frac{2bc}{d}} (c + dx)^m \left( -\frac{b(c+dx)}{d} \right)^{-m} \Gamma\left(1 + m, -\frac{2b(c+dx)}{d}\right) - 2^{-3-m} e^{-2a + \frac{2bc}{d}} (c + dx)^m \left( \frac{b(c+dx)}{d} \right)^{-m} \Gamma\left(m + 1, \frac{2b(c+dx)}{d}\right)}{b}
\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 132, normalized size = 0.92

$$\frac{1}{8}(c+dx)^m \left( \frac{2^{-m} e^{2a - \frac{2bc}{d}} \left( -\frac{b(c+dx)}{d} \right)^{-m} \Gamma\left(m + 1, -\frac{2b(c+dx)}{d}\right) - 2^{-m} e^{\frac{2bc}{d} - 2a} \left( \frac{b(c+dx)}{d} \right)^{-m} \Gamma\left(m + 1, \frac{2b(c+dx)}{d}\right)}{b} + \frac{4c + 4dx}{dm + d} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cosh[a + b*x]^2,x]
```

```
[Out] ((c + d*x)^m*((4*c + 4*d*x)/(d + d*m) + (E^(2*a - (2*b*c)/d)*Gamma[1 + m, (-2*b*(c + d*x))/d])/(2^m*b*(-((b*(c + d*x))/d))^m) - (E^(-2*a + (2*b*c)/d)*Gamma[1 + m, (2*b*(c + d*x))/d])/(2^m*b*((b*(c + d*x))/d)^m))/8
```

**fricas [A]** time = 0.55, size = 241, normalized size = 1.67

$$\frac{(dm + d) \cosh\left(\frac{dm \log\left(\frac{2b}{d}\right) - 2bc + 2ad}{d}\right) \Gamma\left(m + 1, \frac{2(bdx+bc)}{d}\right) - (dm + d) \cosh\left(\frac{dm \log\left(-\frac{2b}{d}\right) + 2bc - 2ad}{d}\right) \Gamma\left(m + 1, -\frac{2(bdx+bc)}{d}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a)^2,x, algorithm="fricas")
```



[Out]  $-1/8*((d*m + d)*\cosh((d*m*\log(2*b/d) - 2*b*c + 2*a*d)/d)*\gamma(m + 1, 2*(b*d*x + b*c)/d) - (d*m + d)*\cosh((d*m*\log(-2*b/d) + 2*b*c - 2*a*d)/d)*\gamma(m + 1, -2*(b*d*x + b*c)/d) - (d*m + d)*\gamma(m + 1, 2*(b*d*x + b*c)/d)*\sinh((d*m*\log(2*b/d) - 2*b*c + 2*a*d)/d) + (d*m + d)*\gamma(m + 1, -2*(b*d*x + b*c)/d)*\sinh((d*m*\log(-2*b/d) + 2*b*c - 2*a*d)/d) - 4*(b*d*x + b*c)*\cosh(m*\log(d*x + c)) - 4*(b*d*x + b*c)*\sinh(m*\log(d*x + c)))/(b*d*m + b*d)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \cosh(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*cosh(b*x+a)^2,x, algorithm="giac")`

[Out] `integrate((d*x + c)^m*cosh(b*x + a)^2, x)`

**maple** [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (dx + c)^m (\cosh^2(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^m*cosh(b*x+a)^2,x)`

[Out] `int((d*x+c)^m*cosh(b*x+a)^2,x)`

**maxima** [A] time = 1.17, size = 102, normalized size = 0.71

$$-\frac{(dx + c)^{m+1} e^{\left(-2a + \frac{2bc}{d}\right)} E_{-m}\left(\frac{2(dx+c)b}{d}\right)}{4d} - \frac{(dx + c)^{m+1} e^{\left(2a - \frac{2bc}{d}\right)} E_{-m}\left(-\frac{2(dx+c)b}{d}\right)}{4d} + \frac{(dx + c)^{m+1}}{2d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*cosh(b*x+a)^2,x, algorithm="maxima")`

[Out]  $-1/4*(d*x + c)^{(m + 1)}*e^{(-2*a + 2*b*c/d)}*\exp\_integral\_e(-m, 2*(d*x + c)*b/d)/d - 1/4*(d*x + c)^{(m + 1)}*e^{(2*a - 2*b*c/d)}*\exp\_integral\_e(-m, -2*(d*x + c)*b/d)/d + 1/2*(d*x + c)^{(m + 1)}/(d*(m + 1))$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a + bx)^2 (c + dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*(c + d*x)^m,x)
```

```
[Out] int(cosh(a + b*x)^2*(c + d*x)^m, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \cosh^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cosh(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*cosh(a + b*x)**2, x)
```

### 3.78 $\int (c + dx)^m \cosh(a + bx) dx$

**Optimal.** Leaf size=110

$$\frac{e^{a-\frac{bc}{d}}(c+dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{b(c+dx)}{d}\right)}{2b} - \frac{e^{\frac{bc}{d}-a}(c+dx)^m \left(\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{b(c+dx)}{d}\right)}{2b}$$

[Out]  $1/2*\exp(a-b*c/d)*(d*x+c)^m*\text{GAMMA}(1+m, -b*(d*x+c)/d)/b/((-b*(d*x+c)/d)^m)-1/2*\exp(-a+b*c/d)*(d*x+c)^m*\text{GAMMA}(1+m, b*(d*x+c)/d)/b/((b*(d*x+c)/d)^m)$

**Rubi [A]** time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3307, 2181}

$$\frac{e^{a-\frac{bc}{d}}(c+dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{b(c+dx)}{d}\right)}{2b} - \frac{e^{\frac{bc}{d}-a}(c+dx)^m \left(\frac{b(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, \frac{b(c+dx)}{d}\right)}{2b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^m*\text{Cosh}[a + b*x], x]$

[Out]  $(E^{(a - (b*c)/d)*(c + d*x)^m*\text{Gamma}[1 + m, -((b*(c + d*x))/d])})/(2*b*(-((b*(c + d*x))/d))^m) - (E^{(-a + (b*c)/d)*(c + d*x)^m*\text{Gamma}[1 + m, (b*(c + d*x))/d]})/(2*b*((b*(c + d*x))/d)^m)$

#### Rule 2181

```
Int[(F_)^((g_)*(e_) + (f_)*(x_))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rule 3307

```
Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol]
:> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]
```

#### Rubi steps

$$\int (c + dx)^m \cosh(a + bx) dx = \frac{1}{2} \int e^{-i(i a + i b x)} (c + dx)^m dx + \frac{1}{2} \int e^{i(i a + i b x)} (c + dx)^m dx$$

$$= \frac{e^{a - \frac{bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{b(c+dx)}{d}\right)}{2b} - \frac{e^{-a + \frac{bc}{d}} (c + dx)^m \left(\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, \frac{b(c+dx)}{d}\right)}{2b}$$

**Mathematica [A]** time = 0.06, size = 102, normalized size = 0.93

$$\frac{e^{-a - \frac{bc}{d}} (c + dx)^m \left( e^{2a} \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{b(c+dx)}{d}\right) - e^{\frac{2bc}{d}} \left(b\left(\frac{c}{d} + x\right)\right)^{-m} \Gamma\left(m + 1, \frac{b(c+dx)}{d}\right) \right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^m\*Cosh[a + b\*x], x]

[Out] (E^(-a - (b\*c)/d)\*(c + d\*x)^m\*((E^(2\*a)\*Gamma[1 + m, -((b\*(c + d\*x))/d)])/(-(b\*(c + d\*x))/d))^m - (E^((2\*b\*c)/d)\*Gamma[1 + m, (b\*(c + d\*x))/d])/(b\*(c + d\*x))^m)/(2\*b)

**fricas [A]** time = 0.46, size = 168, normalized size = 1.53

$$\frac{\cosh\left(\frac{dm \log\left(\frac{b}{d}\right) - bc + ad}{d}\right) \Gamma\left(m + 1, \frac{bdx + bc}{d}\right) - \cosh\left(\frac{dm \log\left(-\frac{b}{d}\right) + bc - ad}{d}\right) \Gamma\left(m + 1, -\frac{bdx + bc}{d}\right) - \Gamma\left(m + 1, \frac{bdx + bc}{d}\right) \sinh\left(\frac{dm \log\left(\frac{b}{d}\right) - bc + ad}{d}\right) + \Gamma\left(m + 1, -\frac{bdx + bc}{d}\right) \sinh\left(\frac{dm \log\left(-\frac{b}{d}\right) + bc - ad}{d}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*cosh(b\*x+a), x, algorithm="fricas")

[Out] -1/2\*(cosh((d\*m\*log(b/d) - b\*c + a\*d)/d)\*gamma(m + 1, (b\*d\*x + b\*c)/d) - cosh((d\*m\*log(-b/d) + b\*c - a\*d)/d)\*gamma(m + 1, -(b\*d\*x + b\*c)/d) - gamma(m + 1, (b\*d\*x + b\*c)/d)\*sinh((d\*m\*log(b/d) - b\*c + a\*d)/d) + gamma(m + 1, -(b\*d\*x + b\*c)/d)\*sinh((d\*m\*log(-b/d) + b\*c - a\*d)/d))/b

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \cosh(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*cosh(b\*x+a), x, algorithm="giac")

[Out] integrate((d\*x + c)^m\*cosh(b\*x + a), x)

**maple** [F] time = 0.15, size = 0, normalized size = 0.00

$$\int (dx + c)^m \cosh(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*cosh(b\*x+a), x)

[Out] int((d\*x+c)^m\*cosh(b\*x+a), x)

**maxima** [A] time = 2.57, size = 79, normalized size = 0.72

$$-\frac{(dx + c)^{m+1} e^{\left(-a + \frac{bc}{d}\right)} E_{-m}\left(\frac{(dx+c)b}{d}\right)}{2d} - \frac{(dx + c)^{m+1} e^{\left(a - \frac{bc}{d}\right)} E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*cosh(b\*x+a), x, algorithm="maxima")

[Out] -1/2\*(d\*x + c)^(m + 1)\*e^(-a + b\*c/d)\*exp\_integral\_e(-m, (d\*x + c)\*b/d)/d - 1/2\*(d\*x + c)^(m + 1)\*e^(a - b\*c/d)\*exp\_integral\_e(-m, -(d\*x + c)\*b/d)/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a + bx) (c + dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(a + b\*x)\*(c + d\*x)^m, x)

[Out] int(cosh(a + b\*x)\*(c + d\*x)^m, x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m\*cosh(b\*x+a), x)

[Out] Exception raised: TypeError

### 3.79 $\int (c + dx)^m \operatorname{sech}(a + bx) dx$

Optimal. Leaf size=17

$$\operatorname{Int}(\operatorname{sech}(a + bx)(c + dx)^m, x)$$

[Out] Unintegrable((d\*x+c)^m\*sech(b\*x+a), x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (c + dx)^m \operatorname{sech}(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m\*Sech[a + b\*x], x]

[Out] Defer[Int] [(c + d\*x)^m\*Sech[a + b\*x], x]

Rubi steps

$$\int (c + dx)^m \operatorname{sech}(a + bx) dx = \int (c + dx)^m \operatorname{sech}(a + bx) dx$$

Mathematica [A] time = 5.93, size = 0, normalized size = 0.00

$$\int (c + dx)^m \operatorname{sech}(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m\*Sech[a + b\*x], x]

[Out] Integrate[(c + d\*x)^m\*Sech[a + b\*x], x]

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\operatorname{integral}((dx + c)^m \operatorname{sech}(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*sech(b\*x+a), x, algorithm="fricas")

[Out] integral((d\*x + c)^m\*sech(b\*x + a), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \operatorname{sech}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*sech(b\*x+a),x, algorithm="giac")

[Out] integrate((d\*x + c)^m\*sech(b\*x + a), x)

**maple** [A] time = 0.14, size = 0, normalized size = 0.00

$$\int (dx + c)^m \operatorname{sech}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*sech(b\*x+a),x)

[Out] int((d\*x+c)^m\*sech(b\*x+a),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \operatorname{sech}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*sech(b\*x+a),x, algorithm="maxima")

[Out] integrate((d\*x + c)^m\*sech(b\*x + a), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{(c + dx)^m}{\cosh(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^m/cosh(a + b\*x),x)

[Out] int((c + d\*x)^m/cosh(a + b\*x), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \operatorname{sech}(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m\*sech(b\*x+a),x)

[Out] Integral((c + d\*x)\*\*m\*sech(a + b\*x), x)

### 3.80 $\int (c + dx)^m \operatorname{sech}^2(a + bx) dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\operatorname{sech}^2(a + bx)(c + dx)^m, x\right)$$

[Out] Unintegrable((d\*x+c)^m\*sech(b\*x+a)^2,x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (c + dx)^m \operatorname{sech}^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m\*Sech[a + b\*x]^2,x]

[Out] Defer[Int][(c + d\*x)^m\*Sech[a + b\*x]^2, x]

Rubi steps

$$\int (c + dx)^m \operatorname{sech}^2(a + bx) dx = \int (c + dx)^m \operatorname{sech}^2(a + bx) dx$$

Mathematica [A] time = 3.49, size = 0, normalized size = 0.00

$$\int (c + dx)^m \operatorname{sech}^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m\*Sech[a + b\*x]^2,x]

[Out] Integrate[(c + d\*x)^m\*Sech[a + b\*x]^2, x]

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\operatorname{integral}\left((dx + c)^m \operatorname{sech}(bx + a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*sech(b\*x+a)^2,x, algorithm="fricas")

[Out] integral((d\*x + c)^m\*sech(b\*x + a)^2, x)



**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \operatorname{sech}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*sech(b\*x+a)^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^m\*sech(b\*x + a)^2, x)

**maple** [A] time = 0.14, size = 0, normalized size = 0.00

$$\int (dx + c)^m \operatorname{sech}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*sech(b\*x+a)^2,x)

[Out] int((d\*x+c)^m\*sech(b\*x+a)^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \operatorname{sech}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*sech(b\*x+a)^2,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m\*sech(b\*x + a)^2, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(c + dx)^m}{\cosh(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^m/cosh(a + b\*x)^2,x)

[Out] int((c + d\*x)^m/cosh(a + b\*x)^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \operatorname{sech}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m\*sech(b\*x+a)\*\*2,x)

[Out] Integral((c + d\*x)\*\*m\*sech(a + b\*x)\*\*2, x)

### 3.81 $\int x^{3+m} \cosh(a + bx) dx$

Optimal. Leaf size=59

$$-\frac{e^a x^m (-bx)^{-m} \Gamma(m+4, -bx)}{2b^4} - \frac{e^{-a} x^m (bx)^{-m} \Gamma(m+4, bx)}{2b^4}$$

[Out]  $-1/2 * \exp(a) * x^m * \text{GAMMA}(4+m, -b*x) / b^4 / ((-b*x)^m) - 1/2 * x^m * \text{GAMMA}(4+m, b*x) / b^4 / \exp(a) / ((b*x)^m)$

**Rubi [A]** time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3307, 2181}

$$-\frac{e^a x^m (-bx)^{-m} \text{Gamma}(m+4, -bx)}{2b^4} - \frac{e^{-a} x^m (bx)^{-m} \text{Gamma}(m+4, bx)}{2b^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(3+m)} * \text{Cosh}[a + b*x], x]$

[Out]  $-(E^a * x^m * \text{Gamma}[4+m, -(b*x)]) / (2*b^4 * (-b*x)^m) - (x^m * \text{Gamma}[4+m, b*x]) / (2*b^4 * E^a * (b*x)^m)$

#### Rule 2181

$\text{Int}[(F_)^{((g_.) * ((e_.) + (f_.) * (x_))) * ((c_.) + (d_.) * (x_))^{(m_)}}, x\_Symbol]$   
 $:= -\text{Simp}[(F^{(g*(e - (c*f)/d)) * (c + d*x)^{\text{FracPart}[m]} * \text{Gamma}[m+1, -(f*g*\text{Log}[F])/d]) * (c + d*x)] / (d * (-(f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1} * (-(f*g*\text{Log}[F] * (c + d*x))/d)^{\text{FracPart}[m]}], x] /;$   $\text{FreeQ}\{F, c, d, e, f, g, m\}, x \ \&\& \ !\text{IntegerQ}[m]$

#### Rule 3307

$\text{Int}[(c_.) + (d_.) * (x_)]^{(m_.)} * \sin[(e_.) + \text{Pi} * (k_.) + (f_.) * (x_)], x\_Symbol]$   
 $:= \text{Dist}[I/2, \text{Int}[(c + d*x)^m / (E^{(I*k*Pi)} * E^{(I*(e + f*x))}), x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m * E^{(I*k*Pi)} * E^{(I*(e + f*x))}, x], x] /;$   $\text{FreeQ}\{c, d, e, f, m\}, x \ \&\& \ \text{IntegerQ}[2*k]$

#### Rubi steps

$$\begin{aligned} \int x^{3+m} \cosh(a + bx) dx &= \frac{1}{2} \int e^{-i(ia+ibx)} x^{3+m} dx + \frac{1}{2} \int e^{i(ia+ibx)} x^{3+m} dx \\ &= -\frac{e^a x^m (-bx)^{-m} \Gamma(4+m, -bx)}{2b^4} - \frac{e^{-a} x^m (bx)^{-m} \Gamma(4+m, bx)}{2b^4} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 54, normalized size = 0.92

$$\frac{e^a x^m (-bx)^{-m} \Gamma(m+4, -bx) + e^{-a} x^m (bx)^{-m} \Gamma(m+4, bx)}{2b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3 + m)\*Cosh[a + b\*x], x]

[Out] -1/2\*((E^a\*x^m\*Gamma[4 + m, -(b\*x)])/(-(b\*x))^m + (x^m\*Gamma[4 + m, b\*x])/(E^a\*(b\*x)^m))/b^4

**fricas [A]** time = 0.46, size = 86, normalized size = 1.46

$$\frac{\cosh((m+3)\log(b)+a)\Gamma(m+4, bx) - \cosh((m+3)\log(-b)-a)\Gamma(m+4, -bx) + \Gamma(m+4, -bx)\sinh((m+3)\log(b)+a) - \Gamma(m+4, bx)\sinh((m+3)\log(-b)-a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3+m)\*cosh(b\*x+a), x, algorithm="fricas")

[Out] -1/2\*(cosh((m+3)\*log(b)+a)\*gamma(m+4, b\*x) - cosh((m+3)\*log(-b)-a)\*gamma(m+4, -b\*x) + gamma(m+4, -b\*x)\*sinh((m+3)\*log(-b)-a) - gamma(m+4, b\*x)\*sinh((m+3)\*log(b)+a))/b

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+3} \cosh(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3+m)\*cosh(b\*x+a), x, algorithm="giac")

[Out] integrate(x^(m+3)\*cosh(b\*x+a), x)

**maple [C]** time = 0.10, size = 73, normalized size = 1.24

$$\frac{x^{4+m} \operatorname{hypergeom}\left(\left[2 + \frac{m}{2}\right], \left[\frac{1}{2}, 3 + \frac{m}{2}\right], \frac{x^2 b^2}{4}\right) \cosh(a)}{4+m} + \frac{b x^{5+m} \operatorname{hypergeom}\left(\left[\frac{5}{2} + \frac{m}{2}\right], \left[\frac{3}{2}, \frac{7}{2} + \frac{m}{2}\right], \frac{x^2 b^2}{4}\right) \sinh(a)}{5+m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3+m)\*cosh(b\*x+a), x)

[Out] 1/(4+m)\*x^(4+m)\*hypergeom([2+1/2\*m], [1/2, 3+1/2\*m], 1/4\*x^2\*b^2)\*cosh(a)+b/(5+m)\*x^(5+m)\*hypergeom([5/2+1/2\*m], [3/2, 7/2+1/2\*m], 1/4\*x^2\*b^2)\*sinh(a)

**maxima** [A] time = 1.38, size = 55, normalized size = 0.93

$$-\frac{1}{2} (bx)^{-m-4} x^{m+4} e^{(-a)} \Gamma(m+4, bx) - \frac{1}{2} (-bx)^{-m-4} x^{m+4} e^a \Gamma(m+4, -bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3+m)\*cosh(b\*x+a), x, algorithm="maxima")

[Out] -1/2\*(b\*x)^(-m - 4)\*x^(m + 4)\*e^(-a)\*gamma(m + 4, b\*x) - 1/2\*(-b\*x)^(-m - 4)\*x^(m + 4)\*e^a\*gamma(m + 4, -b\*x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m+3} \cosh(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(m + 3)\*cosh(a + b\*x), x)

[Out] int(x^(m + 3)\*cosh(a + b\*x), x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3+m)\*cosh(b\*x+a), x)

[Out] Exception raised: TypeError

### 3.82 $\int x^{2+m} \cosh(a + bx) dx$

Optimal. Leaf size=59

$$\frac{e^a x^m (-bx)^{-m} \Gamma(m+3, -bx)}{2b^3} - \frac{e^{-a} x^m (bx)^{-m} \Gamma(m+3, bx)}{2b^3}$$

[Out]  $1/2 * \exp(a) * x^m * \text{GAMMA}(3+m, -b*x) / b^3 / ((-b*x)^m) - 1/2 * x^m * \text{GAMMA}(3+m, b*x) / b^3 / \exp(a) / ((b*x)^m)$

**Rubi [A]** time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3307, 2181}

$$\frac{e^a x^m (-bx)^{-m} \text{Gamma}(m+3, -bx)}{2b^3} - \frac{e^{-a} x^m (bx)^{-m} \text{Gamma}(m+3, bx)}{2b^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(2+m)} * \text{Cosh}[a + b*x], x]$

[Out]  $(E^a * x^m * \text{Gamma}[3+m, -(b*x)]) / (2*b^3 * (-b*x)^m) - (x^m * \text{Gamma}[3+m, b*x]) / (2*b^3 * E^a * (b*x)^m)$

#### Rule 2181

$\text{Int}[(F_)^((g_.) * ((e_.) + (f_.) * (x_))) * ((c_.) + (d_.) * (x_))^{(m_)}, x\_Symbol]$   
 $\rightarrow -\text{Simp}[(F^{(g*(e - (c*f)/d)) * (c + d*x)^{\text{FracPart}[m]} * \text{Gamma}[m+1, (-((f*g*\text{Log}[F])/d)) * (c + d*x)]}) / (d * (-((f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1} * (-((f*g*\text{Log}[F]) * (c + d*x)) / d))^{\text{FracPart}[m]}], x] /;$   $\text{FreeQ}\{F, c, d, e, f, g, m\}, x \ \&\& \ !\text{IntegerQ}[m]$

#### Rule 3307

$\text{Int}[(c_.) + (d_.) * (x_)]^{(m_.)} * \sin[(e_.) + \text{Pi} * (k_.) + (f_.) * (x_)], x\_Symbol]$   
 $\rightarrow \text{Dist}[I/2, \text{Int}[(c + d*x)^m / (E^{(I*k*Pi)} * E^{(I*(e + f*x))}), x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m * E^{(I*k*Pi)} * E^{(I*(e + f*x))}, x], x] /;$   $\text{FreeQ}\{c, d, e, f, m\}, x \ \&\& \ \text{IntegerQ}[2*k]$

#### Rubi steps

$$\begin{aligned} \int x^{2+m} \cosh(a + bx) dx &= \frac{1}{2} \int e^{-i(ia+ibx)} x^{2+m} dx + \frac{1}{2} \int e^{i(ia+ibx)} x^{2+m} dx \\ &= \frac{e^a x^m (-bx)^{-m} \Gamma(3+m, -bx)}{2b^3} - \frac{e^{-a} x^m (bx)^{-m} \Gamma(3+m, bx)}{2b^3} \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 54, normalized size = 0.92

$$\frac{e^{-a}x^m \left( e^{2a}(-bx)^{-m}\Gamma(m+3, -bx) - (bx)^{-m}\Gamma(m+3, bx) \right)}{2b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^(2 + m)\*Cosh[a + b\*x], x]

[Out] (x^m\*((E^(2\*a)\*Gamma[3 + m, -(b\*x)])/(-(b\*x))^m - Gamma[3 + m, b\*x]/(b\*x)^m))/ (2\*b^3\*E^a)

**fricas** [A] time = 0.45, size = 86, normalized size = 1.46

$$\frac{\cosh((m+2)\log(b)+a)\Gamma(m+3, bx) - \cosh((m+2)\log(-b)-a)\Gamma(m+3, -bx) + \Gamma(m+3, -bx)\sinh((m+2)\log(b)+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2+m)\*cosh(b\*x+a), x, algorithm="fricas")

[Out] -1/2\*(cosh((m+2)\*log(b)+a)\*gamma(m+3, b\*x) - cosh((m+2)\*log(-b)-a)\*gamma(m+3, -b\*x) + gamma(m+3, -b\*x)\*sinh((m+2)\*log(-b)-a) - gamma(m+3, b\*x)\*sinh((m+2)\*log(b)+a))/b

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+2} \cosh(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2+m)\*cosh(b\*x+a), x, algorithm="giac")

[Out] integrate(x^(m+2)\*cosh(b\*x+a), x)

**maple** [C] time = 0.13, size = 73, normalized size = 1.24

$$\frac{x^{3+m} \operatorname{hypergeom}\left(\left[\frac{3}{2} + \frac{m}{2}\right], \left[\frac{1}{2}, \frac{5}{2} + \frac{m}{2}\right], \frac{x^2 b^2}{4}\right) \cosh(a)}{3+m} + \frac{b x^{4+m} \operatorname{hypergeom}\left(\left[2 + \frac{m}{2}\right], \left[\frac{3}{2}, 3 + \frac{m}{2}\right], \frac{x^2 b^2}{4}\right) \sinh(a)}{4+m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2+m)\*cosh(b\*x+a), x)

[Out] 1/(3+m)\*x^(3+m)\*hypergeom([3/2+1/2\*m], [1/2, 5/2+1/2\*m], 1/4\*x^2\*b^2)\*cosh(a)+ b/(4+m)\*x^(4+m)\*hypergeom([2+1/2\*m], [3/2, 3+1/2\*m], 1/4\*x^2\*b^2)\*sinh(a)

**maxima** [A] time = 1.23, size = 55, normalized size = 0.93

$$-\frac{1}{2} (bx)^{-m-3} x^{m+3} e^{(-a)} \Gamma(m+3, bx) - \frac{1}{2} (-bx)^{-m-3} x^{m+3} e^a \Gamma(m+3, -bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2+m)\*cosh(b\*x+a), x, algorithm="maxima")

[Out]  $-1/2*(b*x)^{-m-3}*x^{m+3}*e^{-a}*gamma(m+3, b*x) - 1/2*(-b*x)^{-m-3}*x^{m+3}*e^a*gamma(m+3, -b*x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m+2} \cosh(a + b x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(m+2)\*cosh(a+b\*x), x)

[Out] int(x^(m+2)\*cosh(a+b\*x), x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(2+m)\*cosh(b\*x+a), x)

[Out] Exception raised: TypeError

### 3.83 $\int x^{1+m} \cosh(a + bx) dx$

Optimal. Leaf size=59

$$-\frac{e^a x^m (-bx)^{-m} \Gamma(m+2, -bx)}{2b^2} - \frac{e^{-a} x^m (bx)^{-m} \Gamma(m+2, bx)}{2b^2}$$

[Out]  $-1/2 * \exp(a) * x^m * \text{GAMMA}(2+m, -b*x) / b^2 / ((-b*x)^m) - 1/2 * x^m * \text{GAMMA}(2+m, b*x) / b^2 / \exp(a) / ((b*x)^m)$

**Rubi [A]** time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3307, 2181}

$$-\frac{e^a x^m (-bx)^{-m} \text{Gamma}(m+2, -bx)}{2b^2} - \frac{e^{-a} x^m (bx)^{-m} \text{Gamma}(m+2, bx)}{2b^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(1+m)} * \text{Cosh}[a + b*x], x]$

[Out]  $-(E^a * x^m * \text{Gamma}[2+m, -(b*x)]) / (2*b^2 * (-b*x)^m) - (x^m * \text{Gamma}[2+m, b*x]) / (2*b^2 * E^a * (b*x)^m)$

#### Rule 2181

$\text{Int}[(F_)^{((g_.) * ((e_.) + (f_.) * (x_))) * ((c_.) + (d_.) * (x_))^{(m_)}}, x\_Symbol]$   
 $:= -\text{Simp}[(F^{(g*(e - (c*f)/d)) * (c + d*x)^{\text{FracPart}[m]} * \text{Gamma}[m+1, -(f*g*\text{Log}[F])/d]) * (c + d*x)] / (d * (-(f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1} * (-(f*g*\text{Log}[F] * (c + d*x))/d)^{\text{FracPart}[m]}], x] /;$   $\text{FreeQ}\{F, c, d, e, f, g, m\}, x \ \&\& \ !\text{IntegerQ}[m]$

#### Rule 3307

$\text{Int}[(c_.) + (d_.) * (x_)]^{(m_.)} * \sin[(e_.) + \text{Pi} * (k_.) + (f_.) * (x_)], x\_Symbol]$   
 $:= \text{Dist}[I/2, \text{Int}[(c + d*x)^m / (E^{(I*k*Pi)} * E^{(I*(e + f*x))}), x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m * E^{(I*k*Pi)} * E^{(I*(e + f*x))}, x], x] /;$   $\text{FreeQ}\{c, d, e, f, m\}, x \ \&\& \ \text{IntegerQ}[2*k]$

#### Rubi steps

$$\begin{aligned} \int x^{1+m} \cosh(a + bx) dx &= \frac{1}{2} \int e^{-i(ia+ibx)} x^{1+m} dx + \frac{1}{2} \int e^{i(ia+ibx)} x^{1+m} dx \\ &= -\frac{e^a x^m (-bx)^{-m} \Gamma(2+m, -bx)}{2b^2} - \frac{e^{-a} x^m (bx)^{-m} \Gamma(2+m, bx)}{2b^2} \end{aligned}$$



**Mathematica [A]** time = 0.03, size = 54, normalized size = 0.92

$$\frac{e^a x^m (-bx)^{-m} \Gamma(m+2, -bx) + e^{-a} x^m (bx)^{-m} \Gamma(m+2, bx)}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^(1+m)\*Cosh[a+b\*x],x]

[Out] -1/2\*((E^a\*x^m\*Gamma[2+m,-(b\*x)]/(-(b\*x))^m+(x^m\*Gamma[2+m,b\*x])/E^a\*(b\*x)^m))/b^2

**fricas [A]** time = 0.44, size = 86, normalized size = 1.46

$$\frac{\cosh((m+1)\log(b)+a)\Gamma(m+2,bx) - \cosh((m+1)\log(-b)-a)\Gamma(m+2,-bx) + \Gamma(m+2,-bx)\sinh((m+1)\log(b)+a) - \Gamma(m+2,bx)\sinh((m+1)\log(-b)-a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+m)\*cosh(b\*x+a),x, algorithm="fricas")

[Out] -1/2\*(cosh((m+1)\*log(b)+a)\*gamma(m+2,b\*x) - cosh((m+1)\*log(-b)-a)\*gamma(m+2,-b\*x) + gamma(m+2,-b\*x)\*sinh((m+1)\*log(-b)-a) - gamma(m+2,b\*x)\*sinh((m+1)\*log(b)+a))/b

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+1} \cosh(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+m)\*cosh(b\*x+a),x, algorithm="giac")

[Out] integrate(x^(m+1)\*cosh(b\*x+a),x)

**maple [C]** time = 0.12, size = 73, normalized size = 1.24

$$\frac{x^{2+m} \operatorname{hypergeom}\left(\left[1+\frac{m}{2}\right], \left[\frac{1}{2}, 2+\frac{m}{2}\right], \frac{x^2 b^2}{4}\right) \cosh(a)}{2+m} + \frac{b x^{3+m} \operatorname{hypergeom}\left(\left[\frac{3}{2}+\frac{m}{2}\right], \left[\frac{3}{2}, \frac{5}{2}+\frac{m}{2}\right], \frac{x^2 b^2}{4}\right) \sinh(a)}{3+m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1+m)\*cosh(b\*x+a),x)

[Out] 1/(2+m)\*x^(2+m)\*hypergeom([1+1/2\*m],[1/2,2+1/2\*m],1/4\*x^2\*b^2)\*cosh(a)+b/(3+m)\*x^(3+m)\*hypergeom([3/2+1/2\*m],[3/2,5/2+1/2\*m],1/4\*x^2\*b^2)\*sinh(a)

**maxima** [A] time = 0.74, size = 55, normalized size = 0.93

$$-\frac{1}{2} (bx)^{-m-2} x^{m+2} e^{(-a)} \Gamma(m+2, bx) - \frac{1}{2} (-bx)^{-m-2} x^{m+2} e^a \Gamma(m+2, -bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+m)\*cosh(b\*x+a), x, algorithm="maxima")

[Out] -1/2\*(b\*x)^(-m - 2)\*x^(m + 2)\*e^(-a)\*gamma(m + 2, b\*x) - 1/2\*(-b\*x)^(-m - 2)\*x^(m + 2)\*e^a\*gamma(m + 2, -b\*x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m+1} \cosh(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(m + 1)\*cosh(a + b\*x), x)

[Out] int(x^(m + 1)\*cosh(a + b\*x), x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1+m)\*cosh(b\*x+a), x)

[Out] Exception raised: TypeError

### 3.84 $\int x^m \cosh(a + bx) dx$

Optimal. Leaf size=59

$$\frac{e^a x^m (-bx)^{-m} \Gamma(m+1, -bx)}{2b} - \frac{e^{-a} x^m (bx)^{-m} \Gamma(m+1, bx)}{2b}$$

[Out]  $1/2 \exp(a) x^m \text{GAMMA}(1+m, -b*x) / b / ((-b*x)^m) - 1/2 x^m \text{GAMMA}(1+m, b*x) / b / \exp(a) / ((b*x)^m)$

**Rubi** [A] time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3307, 2181}

$$\frac{e^a x^m (-bx)^{-m} \text{Gamma}(m+1, -bx)}{2b} - \frac{e^{-a} x^m (bx)^{-m} \text{Gamma}(m+1, bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Cosh[a + b\*x], x]

[Out]  $(E^a x^m \text{Gamma}[1 + m, -(b*x)]) / (2*b*(-(b*x))^m) - (x^m \text{Gamma}[1 + m, b*x]) / (2*b * E^a (b*x)^m)$

#### Rule 2181

Int[(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol]  
 :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rule 3307

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*sin[(e\_) + Pi\*(k\_) + (f\_)\*(x\_)], x\_Symbol]  
 ] :> Dist[I/2, Int[(c + d\*x)^m/(E^(I\*k\*Pi)\*E^(I\*(e + f\*x))), x], x] - Dist[I/2, Int[(c + d\*x)^m \* E^(I\*k\*Pi) \* E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

#### Rubi steps

$$\begin{aligned} \int x^m \cosh(a + bx) dx &= \frac{1}{2} \int e^{-i(i a + i b x)} x^m dx + \frac{1}{2} \int e^{i(i a + i b x)} x^m dx \\ &= \frac{e^a x^m (-bx)^{-m} \Gamma(1 + m, -bx)}{2b} - \frac{e^{-a} x^m (bx)^{-m} \Gamma(1 + m, bx)}{2b} \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 54, normalized size = 0.92

$$\frac{e^{-a}x^m \left( e^{2a}(-bx)^{-m}\Gamma(m+1, -bx) - (bx)^{-m}\Gamma(m+1, bx) \right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*Cosh[a + b\*x], x]

[Out] (x^m\*((E^(2\*a)\*Gamma[1 + m, -(b\*x)])/(-(b\*x))^m - Gamma[1 + m, b\*x]/(b\*x)^m))/ (2\*b\*E^a)

**fricas** [A] time = 0.46, size = 78, normalized size = 1.32

$$\frac{\cosh(m \log(b) + a) \Gamma(m+1, bx) - \cosh(m \log(-b) - a) \Gamma(m+1, -bx) + \Gamma(m+1, -bx) \sinh(m \log(-b) - a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cosh(b\*x+a), x, algorithm="fricas")

[Out] -1/2\*(cosh(m\*log(b) + a)\*gamma(m + 1, b\*x) - cosh(m\*log(-b) - a)\*gamma(m + 1, -b\*x) + gamma(m + 1, -b\*x)\*sinh(m\*log(-b) - a) - gamma(m + 1, b\*x)\*sinh(m\*log(b) + a))/b

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cosh(b\*x+a), x, algorithm="giac")

[Out] integrate(x^m\*cosh(b\*x + a), x)

**maple** [C] time = 0.11, size = 73, normalized size = 1.24

$$\frac{x^{1+m} \operatorname{hypergeom}\left(\left[\frac{1}{2} + \frac{m}{2}\right], \left[\frac{1}{2}, \frac{3}{2} + \frac{m}{2}\right], \frac{x^2 b^2}{4}\right) \cosh(a)}{1+m} + \frac{b x^{2+m} \operatorname{hypergeom}\left(\left[1 + \frac{m}{2}\right], \left[\frac{3}{2}, 2 + \frac{m}{2}\right], \frac{x^2 b^2}{4}\right) \sinh(a)}{2+m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*cosh(b\*x+a), x)

[Out] 1/(1+m)\*x^(1+m)\*hypergeom([1/2+1/2\*m], [1/2, 3/2+1/2\*m], 1/4\*x^2\*b^2)\*cosh(a)+ b/(2+m)\*x^(2+m)\*hypergeom([1+1/2\*m], [3/2, 2+1/2\*m], 1/4\*x^2\*b^2)\*sinh(a)

**maxima** [A] time = 1.21, size = 55, normalized size = 0.93

$$-\frac{1}{2} (bx)^{-m-1} x^{m+1} e^{(-a)} \Gamma(m+1, bx) - \frac{1}{2} (-bx)^{-m-1} x^{m+1} e^a \Gamma(m+1, -bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cosh(b\*x+a),x, algorithm="maxima")

[Out]  $-1/2*(b*x)^{-m-1}*x^{m+1}*e^{-a}*gamma(m+1, b*x) - 1/2*(-b*x)^{-m-1}*x^{m+1}*e^a*gamma(m+1, -b*x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \cosh(a + b x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*cosh(a + b\*x),x)

[Out] int(x^m\*cosh(a + b\*x), x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*cosh(b\*x+a),x)

[Out] Exception raised: TypeError

### 3.85 $\int x^{-1+m} \cosh(a + bx) dx$

Optimal. Leaf size=49

$$-\frac{1}{2}e^a x^m (-bx)^{-m} \Gamma(m, -bx) - \frac{1}{2}e^{-a} x^m (bx)^{-m} \Gamma(m, bx)$$

[Out]  $-1/2*\exp(a)*x^m*\text{GAMMA}(m, -b*x)/((-b*x)^m) - 1/2*x^m*\text{GAMMA}(m, b*x)/\exp(a)/((b*x)^m)$

**Rubi [A]** time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3307, 2181}

$$-\frac{1}{2}e^a x^m (-bx)^{-m} \text{Gamma}(m, -bx) - \frac{1}{2}e^{-a} x^m (bx)^{-m} \text{Gamma}(m, bx)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(-1 + m)}*\text{Cosh}[a + b*x], x]$

[Out]  $-(E^a*x^m*\text{Gamma}[m, -(b*x)])/(2*(-(b*x))^m) - (x^m*\text{Gamma}[m, b*x])/(2*E^{-a}*(b*x)^m)$

#### Rule 2181

$\text{Int}[(F_)^((g_)*(e_) + (f_)*(x_))*((c_) + (d_)*(x_))^{(m_)}, x\_Symbol]$   
 $:= -\text{Simp}[(F^{(g*(e - (c*f)/d))}*(c + d*x)^{\text{FracPart}[m]*\text{Gamma}[m + 1, -(f*g*\text{Log}[F])/d]}*(c + d*x)]/(d*(-(f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1}*(-(f*g*\text{Log}[F]*(c + d*x)/d))^{\text{FracPart}[m]}, x] /; \text{FreeQ}\{F, c, d, e, f, g, m\}, x \ \&\amp; \ !\text{IntegerQ}[m]$

#### Rule 3307

$\text{Int}[(c_) + (d_)*(x_))^{(m_)}*\sin[(e_) + \text{Pi}*(k_) + (f_)*(x_)], x\_Symbol]$   
 $:= \text{Dist}[I/2, \text{Int}[(c + d*x)^m/(E^{I*k*Pi}*E^{I*(e + f*x)}), x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m*E^{I*k*Pi}*E^{I*(e + f*x)}, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x \ \&\amp; \ \text{IntegerQ}[2*k]$

#### Rubi steps

$$\begin{aligned} \int x^{-1+m} \cosh(a + bx) dx &= \frac{1}{2} \int e^{-i(i a + i b x)} x^{-1+m} dx + \frac{1}{2} \int e^{i(i a + i b x)} x^{-1+m} dx \\ &= -\frac{1}{2} e^a x^m (-bx)^{-m} \Gamma(m, -bx) - \frac{1}{2} e^{-a} x^m (bx)^{-m} \Gamma(m, bx) \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 49, normalized size = 1.00

$$-\frac{1}{2}e^a x^m (-bx)^{-m} \Gamma(m, -bx) - \frac{1}{2}e^{-a} x^m (bx)^{-m} \Gamma(m, bx)$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + m)\*Cosh[a + b\*x], x]

[Out] -1/2\*(E^a\*x^m\*Gamma[m, -(b\*x)])/(-(b\*x))^m - (x^m\*Gamma[m, b\*x])/(2\*E^a\*(b\*x)^m)

**fricas [A]** time = 0.44, size = 78, normalized size = 1.59

$$\frac{\cosh((m-1)\log(b)+a)\Gamma(m, bx) - \cosh((m-1)\log(-b)-a)\Gamma(m, -bx) + \Gamma(m, -bx)\sinh((m-1)\log(-b)-a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+m)\*cosh(b\*x+a), x, algorithm="fricas")

[Out] -1/2\*(cosh((m-1)\*log(b)+a)\*gamma(m, b\*x) - cosh((m-1)\*log(-b)-a)\*gamma(m, -b\*x) + gamma(m, -b\*x)\*sinh((m-1)\*log(-b)-a) - gamma(m, b\*x)\*sinh((m-1)\*log(b)+a))/b

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-1} \cosh(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+m)\*cosh(b\*x+a), x, algorithm="giac")

[Out] integrate(x^(m-1)\*cosh(b\*x+a), x)

**maple [C]** time = 0.12, size = 67, normalized size = 1.37

$$\frac{x^m \operatorname{hypergeom}\left(\left[\frac{m}{2}\right], \left[\frac{1}{2}, 1 + \frac{m}{2}\right], \frac{x^2 b^2}{4}\right) \cosh(a)}{m} + \frac{b x^{1+m} \operatorname{hypergeom}\left(\left[\frac{1}{2} + \frac{m}{2}\right], \left[\frac{3}{2}, \frac{3}{2} + \frac{m}{2}\right], \frac{x^2 b^2}{4}\right) \sinh(a)}{1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+m)\*cosh(b\*x+a), x)

[Out] 1/m\*x^m\*hypergeom([1/2\*m], [1/2, 1+1/2\*m], 1/4\*x^2\*b^2)\*cosh(a)+b/(1+m)\*x^(1+m)\*hypergeom([1/2+1/2\*m], [3/2, 3/2+1/2\*m], 1/4\*x^2\*b^2)\*sinh(a)

**maxima** [A] time = 0.77, size = 43, normalized size = 0.88

$$-\frac{x^m e^{(-a)} \Gamma(m, bx)}{2 (bx)^m} - \frac{x^m e^a \Gamma(m, -bx)}{2 (-bx)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>^</sup>(-1+m)\*cosh(b\*x+a), x, algorithm="maxima")

[Out] -1/2\*x<sup>^</sup>m\*e<sup>^</sup>(-a)\*gamma(m, b\*x)/(b\*x)<sup>^</sup>m - 1/2\*x<sup>^</sup>m\*e<sup>^</sup>a\*gamma(m, -b\*x)/(-b\*x)<sup>^</sup>m

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m-1} \cosh(a + b x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>^</sup>(m - 1)\*cosh(a + b\*x), x)

[Out] int(x<sup>^</sup>(m - 1)\*cosh(a + b\*x), x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>^</sup>(-1+m)\*cosh(b\*x+a), x)

[Out] Exception raised: TypeError



### 3.86 $\int x^{-2+m} \cosh(a + bx) dx$

Optimal. Leaf size=55

$$\frac{1}{2}e^a bx^m (-bx)^{-m} \Gamma(m-1, -bx) - \frac{1}{2}e^{-a} bx^m (bx)^{-m} \Gamma(m-1, bx)$$

[Out]  $1/2*b*\exp(a)*x^m*\text{GAMMA}(-1+m, -b*x)/((-b*x)^m) - 1/2*b*x^m*\text{GAMMA}(-1+m, b*x)/\exp(a)/((b*x)^m)$

**Rubi [A]** time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3307, 2181}

$$\frac{1}{2}e^a bx^m (-bx)^{-m} \text{Gamma}(m-1, -bx) - \frac{1}{2}e^{-a} bx^m (bx)^{-m} \text{Gamma}(m-1, bx)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(-2 + m)*\text{Cosh}[a + b*x]}, x]$

[Out]  $(b*E^a*x^m*\text{Gamma}[-1 + m, -(b*x)])/(2*(-(b*x))^m) - (b*x^m*\text{Gamma}[-1 + m, b*x])/ (2*E^a*(b*x)^m)$

#### Rule 2181

$\text{Int}[(F_)^((g_)*(e_) + (f_)*(x_))*((c_) + (d_)*(x_))^{(m_)}, x\_Symbol]$   
 $\rightarrow -\text{Simp}[(F^{(g*(e - (c*f)/d))* (c + d*x)^{\text{FracPart}[m]}*\text{Gamma}[m + 1, -(f*g*\text{Log}[F])/d])*(c + d*x)]/(d*(-(f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1}*(-(f*g*\text{Log}[F])*(c + d*x)/d)^{\text{FracPart}[m]}], x] /;$  FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rule 3307

$\text{Int}[(c_ + (d_)*(x_))^{(m_)*\sin[(e_) + \text{Pi}*(k_) + (f_)*(x_)]}, x\_Symbol]$   
 $\rightarrow \text{Dist}[I/2, \text{Int}[(c + d*x)^m/(E^{(I*k*Pi)*E^{(I*(e + f*x))}}), x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m*E^{(I*k*Pi)*E^{(I*(e + f*x))}}, x], x] /;$  FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

#### Rubi steps

$$\begin{aligned} \int x^{-2+m} \cosh(a + bx) dx &= \frac{1}{2} \int e^{-i(i+ibx)} x^{-2+m} dx + \frac{1}{2} \int e^{i(i+ibx)} x^{-2+m} dx \\ &= \frac{1}{2} b e^a x^m (-bx)^{-m} \Gamma(-1 + m, -bx) - \frac{1}{2} b e^{-a} x^m (bx)^{-m} \Gamma(-1 + m, bx) \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 52, normalized size = 0.95

$$\frac{1}{2}e^{-a}bx^m \left( e^{2a}(-bx)^{-m}\Gamma(m-1, -bx) - (bx)^{-m}\Gamma(m-1, bx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x<sup>(-2 + m)</sup>\*Cosh[a + b\*x], x]

[Out] (b\*x<sup>m</sup>\*((E<sup>(2\*a)</sup>\*Gamma[-1 + m, -(b\*x)]))/(-(b\*x))<sup>m</sup> - Gamma[-1 + m, b\*x]/(b\*x<sup>m</sup>))/(2\*E<sup>a</sup>)

**fricas** [A] time = 0.51, size = 86, normalized size = 1.56

$$\frac{\cosh((m-2)\log(b)+a)\Gamma(m-1, bx) - \cosh((m-2)\log(-b)-a)\Gamma(m-1, -bx) + \Gamma(m-1, -bx)\sinh((m-2)\log(b)+a) - \Gamma(m-1, b*x)\sinh((m-2)\log(-b)-a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-2+m)</sup>\*cosh(b\*x+a), x, algorithm="fricas")

[Out] -1/2\*(cosh((m-2)\*log(b)+a)\*gamma(m-1, b\*x) - cosh((m-2)\*log(-b)-a)\*gamma(m-1, -b\*x) + gamma(m-1, -b\*x)\*sinh((m-2)\*log(-b)-a) - gamma(m-1, b\*x)\*sinh((m-2)\*log(b)+a))/b

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-2} \cosh(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-2+m)</sup>\*cosh(b\*x+a), x, algorithm="giac")

[Out] integrate(x<sup>(m-2)</sup>\*cosh(b\*x+a), x)

**maple** [C] time = 0.13, size = 67, normalized size = 1.22

$$\frac{x^{-1+m} \operatorname{hypergeom}\left(\left[-\frac{1}{2} + \frac{m}{2}\right], \left[\frac{1}{2}, \frac{1}{2} + \frac{m}{2}\right], \frac{x^2 b^2}{4}\right) \cosh(a)}{-1+m} + \frac{b x^m \operatorname{hypergeom}\left(\left[\frac{m}{2}\right], \left[\frac{3}{2}, 1 + \frac{m}{2}\right], \frac{x^2 b^2}{4}\right) \sinh(a)}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>(-2+m)</sup>\*cosh(b\*x+a), x)

[Out] 1/(-1+m)\*x<sup>(-1+m)</sup>\*hypergeom([-1/2+1/2\*m], [1/2, 1/2+1/2\*m], 1/4\*x<sup>2</sup>\*b<sup>2</sup>)\*cosh(a)+b/m\*x<sup>m</sup>\*hypergeom([1/2\*m], [3/2, 1+1/2\*m], 1/4\*x<sup>2</sup>\*b<sup>2</sup>)\*sinh(a)

**maxima** [A] time = 1.19, size = 55, normalized size = 1.00

$$-\frac{1}{2} (bx)^{-m+1} x^{m-1} e^{(-a)} \Gamma(m-1, bx) - \frac{1}{2} (-bx)^{-m+1} x^{m-1} e^a \Gamma(m-1, -bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-2+m)</sup>\*cosh(b\*x+a), x, algorithm="maxima")

[Out] -1/2\*(b\*x)<sup>(-m + 1)</sup>\*x<sup>(m - 1)</sup>\*e<sup>(-a)</sup>\*gamma(m - 1, b\*x) - 1/2\*(-b\*x)<sup>(-m + 1)</sup>\*x<sup>(m - 1)</sup>\*e<sup>a</sup>\*gamma(m - 1, -b\*x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m-2} \cosh(a + b x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>(m - 2)</sup>\*cosh(a + b\*x), x)

[Out] int(x<sup>(m - 2)</sup>\*cosh(a + b\*x), x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-2+m)</sup>\*cosh(b\*x+a), x)

[Out] Exception raised: TypeError

### 3.87 $\int x^{-3+m} \cosh(a + bx) dx$

Optimal. Leaf size=59

$$-\frac{1}{2}e^a b^2 x^m (-bx)^{-m} \Gamma(m-2, -bx) - \frac{1}{2}e^{-a} b^2 x^m (bx)^{-m} \Gamma(m-2, bx)$$

[Out]  $-1/2*b^2*\exp(a)*x^m*\text{GAMMA}(-2+m, -b*x)/((-b*x)^m) - 1/2*b^2*x^m*\text{GAMMA}(-2+m, b*x)/\exp(a)/((b*x)^m)$

**Rubi [A]** time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3307, 2181}

$$-\frac{1}{2}e^a b^2 x^m (-bx)^{-m} \text{Gamma}(m-2, -bx) - \frac{1}{2}e^{-a} b^2 x^m (bx)^{-m} \text{Gamma}(m-2, bx)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(-3 + m)}*\text{Cosh}[a + b*x], x]$

[Out]  $-(b^2*E^a*x^m*\text{Gamma}[-2 + m, -(b*x)])/(2*(-(b*x))^m) - (b^2*x^m*\text{Gamma}[-2 + m, b*x])/(2*E^a*(b*x)^m)$

#### Rule 2181

$\text{Int}[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^{(m_)}], x\_Symbol]$   
 $:= -\text{Simp}[(F^{(g*(e - (c*f)/d))}*(c + d*x)^{\text{FracPart}[m]}*\text{Gamma}[m + 1, -(f*g*\text{Log}[F])/d])*(c + d*x)]/(d*(-(f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1}*(-(f*g*\text{Log}[F])*(c + d*x)/d)^{\text{FracPart}[m]}, x] /;$   $\text{FreeQ}\{F, c, d, e, f, g, m\}, x$  &&  $!\text{IntegerQ}[m]$

#### Rule 3307

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_.)}*\sin[(e_.) + \text{Pi}*(k_.) + (f_.)*(x_)], x\_Symbol]$   
 $:= \text{Dist}[I/2, \text{Int}[(c + d*x)^m/(E^{I*k*Pi}*E^{I*(e + f*x)}), x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m*E^{I*k*Pi}*E^{I*(e + f*x)}, x], x] /;$   $\text{FreeQ}\{c, d, e, f, m\}, x$  &&  $\text{IntegerQ}[2*k]$

#### Rubi steps

$$\begin{aligned} \int x^{-3+m} \cosh(a + bx) dx &= \frac{1}{2} \int e^{-i(i a + i b x)} x^{-3+m} dx + \frac{1}{2} \int e^{i(i a + i b x)} x^{-3+m} dx \\ &= -\frac{1}{2} b^2 e^a x^m (-bx)^{-m} \Gamma(-2 + m, -bx) - \frac{1}{2} b^2 e^{-a} x^m (bx)^{-m} \Gamma(-2 + m, bx) \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 55, normalized size = 0.93

$$\frac{1}{2}e^{-a}b^2x^m \left( -e^{2a}(-bx)^{-m}\Gamma(m-2, -bx) - (bx)^{-m}\Gamma(m-2, bx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(-3 + m)\*Cosh[a + b\*x], x]

[Out] (b^2\*x^m\*(-((E^(2\*a)\*Gamma[-2 + m, -(b\*x)])/(-(b\*x))^m) - Gamma[-2 + m, b\*x]/(b\*x)^m))/(2\*E^a)

**fricas [A]** time = 0.42, size = 86, normalized size = 1.46

$$\frac{\cosh((m-3)\log(b)+a)\Gamma(m-2, bx) - \cosh((m-3)\log(-b)-a)\Gamma(m-2, -bx) + \Gamma(m-2, -bx)\sinh((m-3)\log(b)+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-3+m)\*cosh(b\*x+a), x, algorithm="fricas")

[Out] -1/2\*(cosh((m-3)\*log(b)+a)\*gamma(m-2, b\*x) - cosh((m-3)\*log(-b)-a)\*gamma(m-2, -b\*x) + gamma(m-2, -b\*x)\*sinh((m-3)\*log(-b)-a) - gamma(m-2, b\*x)\*sinh((m-3)\*log(b)+a))/b

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-3} \cosh(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-3+m)\*cosh(b\*x+a), x, algorithm="giac")

[Out] integrate(x^(m-3)\*cosh(b\*x+a), x)

**maple [C]** time = 0.10, size = 71, normalized size = 1.20

$$\frac{x^{-2+m} \operatorname{hypergeom}\left(\left[-1+\frac{m}{2}\right], \left[\frac{1}{2}, \frac{m}{2}\right], \frac{x^2 b^2}{4}\right) \cosh(a)}{-2+m} + \frac{b x^{-1+m} \operatorname{hypergeom}\left(\left[-\frac{1}{2}+\frac{m}{2}\right], \left[\frac{3}{2}, \frac{1}{2}+\frac{m}{2}\right], \frac{x^2 b^2}{4}\right) \sinh(a)}{-1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-3+m)\*cosh(b\*x+a), x)

[Out] 1/(-2+m)\*x^(-2+m)\*hypergeom([-1+1/2\*m], [1/2, 1/2\*m], 1/4\*x^2\*b^2)\*cosh(a)+b/(-1+m)\*x^(-1+m)\*hypergeom([-1/2+1/2\*m], [3/2, 1/2+1/2\*m], 1/4\*x^2\*b^2)\*sinh(a)

**maxima** [A] time = 2.31, size = 55, normalized size = 0.93

$$-\frac{1}{2} (bx)^{-m+2} x^{m-2} e^{(-a)} \Gamma(m-2, bx) - \frac{1}{2} (-bx)^{-m+2} x^{m-2} e^a \Gamma(m-2, -bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>^</sup>(-3+m)\*cosh(b\*x+a), x, algorithm="maxima")

[Out] -1/2\*(b\*x)<sup>^</sup>(-m + 2)\*x<sup>^</sup>(m - 2)\*e<sup>^</sup>(-a)\*gamma(m - 2, b\*x) - 1/2\*(-b\*x)<sup>^</sup>(-m + 2)\*x<sup>^</sup>(m - 2)\*e<sup>^</sup>a\*gamma(m - 2, -b\*x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m-3} \cosh(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>^</sup>(m - 3)\*cosh(a + b\*x), x)

[Out] int(x<sup>^</sup>(m - 3)\*cosh(a + b\*x), x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>^</sup>(-3+m)\*cosh(b\*x+a), x)

[Out] Exception raised: TypeError

### 3.88 $\int x^{3+m} \cosh^2(a + bx) dx$

**Optimal.** Leaf size=86

$$-\frac{e^{2a}2^{-m-6}x^m(-bx)^{-m}\Gamma(m+4,-2bx)}{b^4} - \frac{e^{-2a}2^{-m-6}x^m(bx)^{-m}\Gamma(m+4,2bx)}{b^4} + \frac{x^{m+4}}{2(m+4)}$$

[Out]  $1/2*x^{(4+m)/(4+m)-2^{(-6-m)*exp(2*a)*x^m*GAMMA(4+m,-2*b*x)/b^4/((-b*x)^m)-2^{(-6-m)*x^m*GAMMA(4+m,2*b*x)/b^4/exp(2*a)/((b*x)^m)}$

**Rubi [A]** time = 0.15, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3312, 3307, 2181}

$$-\frac{e^{2a}2^{-m-6}x^m(-bx)^{-m}\Gamma(m+4,-2bx)}{b^4} - \frac{e^{-2a}2^{-m-6}x^m(bx)^{-m}\Gamma(m+4,2bx)}{b^4} + \frac{x^{m+4}}{2(m+4)}$$

Antiderivative was successfully verified.

[In] Int[x^(3 + m)\*Cosh[a + b\*x]^2,x]

[Out]  $x^{(4+m)/(2*(4+m)) - (2^{(-6-m)*E^{(2*a)*x^m*Gamma[4+m,-2*b*x]})/(b^4*(-(b*x)^m) - (2^{(-6-m)*x^m*Gamma[4+m,2*b*x]})/(b^4*E^{(2*a)*(b*x)^m})}$

#### Rule 2181

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, -(f*g*Log[F])/d])*(c + d*x)]/(d*(-(f*g*Log[F])/d)^(IntPart[m] + 1)*(-(f*g*Log[F])*(c + d*x)/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]
```

#### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol]
:> Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\int x^{3+m} \cosh^2(a + bx) dx &= \int \left( \frac{x^{3+m}}{2} + \frac{1}{2} x^{3+m} \cosh(2a + 2bx) \right) dx \\
&= \frac{x^{4+m}}{2(4+m)} + \frac{1}{2} \int x^{3+m} \cosh(2a + 2bx) dx \\
&= \frac{x^{4+m}}{2(4+m)} + \frac{1}{4} \int e^{-i(2ia+2ibx)} x^{3+m} dx + \frac{1}{4} \int e^{i(2ia+2ibx)} x^{3+m} dx \\
&= \frac{x^{4+m}}{2(4+m)} - \frac{2^{-6-m} e^{2a} x^m (-bx)^{-m} \Gamma(4+m, -2bx)}{b^4} - \frac{2^{-6-m} e^{-2a} x^m (bx)^{-m} \Gamma(4+m, 2bx)}{b^4}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 79, normalized size = 0.92

$$\frac{1}{64} x^m \left( -\frac{e^{2a} 2^{-m} (-bx)^{-m} \Gamma(m+4, -2bx)}{b^4} - \frac{e^{-2a} 2^{-m} (bx)^{-m} \Gamma(m+4, 2bx)}{b^4} + \frac{32x^4}{m+4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(3 + m)\*Cosh[a + b\*x]^2,x]

[Out] (x^m\*((32\*x^4)/(4 + m) - (E^(2\*a)\*Gamma[4 + m, -2\*b\*x])/(2^m\*b^4\*(-(b\*x))^m) - Gamma[4 + m, 2\*b\*x]/(2^m\*b^4\*E^(2\*a)\*(b\*x)^m)))/64

**fricas [A]** time = 0.51, size = 136, normalized size = 1.58

$$\frac{4bx \cosh((m+3)\log(x)) - (m+4) \cosh((m+3)\log(2b) + 2a) \Gamma(m+4, 2bx) + (m+4) \cosh((m+3)\log(-2b) - 2a) \Gamma(m+4, -2bx)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3+m)\*cosh(b\*x+a)^2,x, algorithm="fricas")

[Out] 1/8\*(4\*b\*x\*cosh((m+3)\*log(x)) - (m+4)\*cosh((m+3)\*log(2\*b) + 2\*a)\*gamma(m+4, 2\*b\*x) + (m+4)\*cosh((m+3)\*log(-2\*b) - 2\*a)\*gamma(m+4, -2\*b\*x) + (m+4)\*gamma(m+4, 2\*b\*x)\*sinh((m+3)\*log(2\*b) + 2\*a) - (m+4)\*gamma(m+4, -2\*b\*x)\*sinh((m+3)\*log(-2\*b) - 2\*a) + 4\*b\*x\*sinh((m+3)\*log(x)))/(b\*m + 4\*b)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+3} \cosh(bx + a)^2 dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3+m)\*cosh(b\*x+a)^2,x, algorithm="giac")

[Out] integrate(x^(m + 3)\*cosh(b\*x + a)^2, x)

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int x^{3+m} (\cosh^2(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3+m)\*cosh(b\*x+a)^2,x)

[Out] int(x^(3+m)\*cosh(b\*x+a)^2,x)

maxima [A] time = 1.34, size = 71, normalized size = 0.83

$$-\frac{1}{4} (2bx)^{-m-4} x^{m+4} e^{(-2a)} \Gamma(m+4, 2bx) - \frac{1}{4} (-2bx)^{-m-4} x^{m+4} e^{(2a)} \Gamma(m+4, -2bx) + \frac{x^{m+4}}{2(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3+m)\*cosh(b\*x+a)^2,x, algorithm="maxima")

[Out] -1/4\*(2\*b\*x)^(-m - 4)\*x^(m + 4)\*e^(-2\*a)\*gamma(m + 4, 2\*b\*x) - 1/4\*(-2\*b\*x)^(-m - 4)\*x^(m + 4)\*e^(2\*a)\*gamma(m + 4, -2\*b\*x) + 1/2\*x^(m + 4)/(m + 4)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+3} \cosh(a + bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(m + 3)\*cosh(a + b\*x)^2,x)

[Out] int(x^(m + 3)\*cosh(a + b\*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+3} \cosh^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3+m)\*cosh(b\*x+a)\*\*2,x)

[Out] Integral(x\*\*(m + 3)\*cosh(a + b\*x)\*\*2, x)

### 3.89 $\int x^{2+m} \cosh^2(a + bx) dx$

**Optimal.** Leaf size=85

$$\frac{e^{2a}2^{-m-5}x^m(-bx)^{-m}\Gamma(m+3,-2bx)}{b^3} - \frac{e^{-2a}2^{-m-5}x^m(bx)^{-m}\Gamma(m+3,2bx)}{b^3} + \frac{x^{m+3}}{2(m+3)}$$

[Out]  $1/2*x^{(3+m)/(3+m)+2^{(-5-m)*exp(2*a)}*x^m*\text{GAMMA}(3+m,-2*b*x)/b^3/((-b*x)^m)-2^{(-5-m)*x^m*\text{GAMMA}(3+m,2*b*x)/b^3/exp(2*a)/((b*x)^m)$

**Rubi [A]** time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3312, 3307, 2181}

$$\frac{e^{2a}2^{-m-5}x^m(-bx)^{-m}\text{Gamma}(m+3,-2bx)}{b^3} - \frac{e^{-2a}2^{-m-5}x^m(bx)^{-m}\text{Gamma}(m+3,2bx)}{b^3} + \frac{x^{m+3}}{2(m+3)}$$

Antiderivative was successfully verified.

[In] Int[x^(2 + m)\*Cosh[a + b\*x]^2, x]

[Out]  $x^{(3+m)/(2*(3+m))} + (2^{(-5-m)*E^{(2*a)}*x^m*\text{Gamma}[3+m,-2*b*x]})/(b^3*(-(b*x))^m) - (2^{(-5-m)*x^m*\text{Gamma}[3+m,2*b*x]})/(b^3*E^{(2*a)}*(b*x)^m)$

#### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, -(f*g*Log[F])/d])*(c + d*x)]/(d*(-(f*g*Log[F])/d)^(IntPart[m] + 1)*(-(f*g*Log[F])*(c + d*x)/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]
```

#### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol]
:> Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\int x^{2+m} \cosh^2(a + bx) dx &= \int \left( \frac{x^{2+m}}{2} + \frac{1}{2} x^{2+m} \cosh(2a + 2bx) \right) dx \\
&= \frac{x^{3+m}}{2(3+m)} + \frac{1}{2} \int x^{2+m} \cosh(2a + 2bx) dx \\
&= \frac{x^{3+m}}{2(3+m)} + \frac{1}{4} \int e^{-i(2ia+2ibx)} x^{2+m} dx + \frac{1}{4} \int e^{i(2ia+2ibx)} x^{2+m} dx \\
&= \frac{x^{3+m}}{2(3+m)} + \frac{2^{-5-m} e^{2a} x^m (-bx)^{-m} \Gamma(3+m, -2bx)}{b^3} - \frac{2^{-5-m} e^{-2a} x^m (bx)^{-m} \Gamma(3+m, 2bx)}{b^3}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 78, normalized size = 0.92

$$\frac{1}{32} x^m \left( \frac{e^{2a} 2^{-m} (-bx)^{-m} \Gamma(m+3, -2bx)}{b^3} - \frac{e^{-2a} 2^{-m} (bx)^{-m} \Gamma(m+3, 2bx)}{b^3} + \frac{16x^3}{m+3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(2+m)\*Cosh[a+b\*x]^2,x]

[Out] (x^m\*((16\*x^3)/(3+m) + (E^(2\*a)\*Gamma[3+m, -2\*b\*x])/(2^m\*b^3\*(-(b\*x))^m) - Gamma[3+m, 2\*b\*x]/(2^m\*b^3\*E^(2\*a)\*(b\*x)^m)))/32

**fricas [A]** time = 0.51, size = 136, normalized size = 1.60

$$\frac{4bx \cosh((m+2)\log(x)) - (m+3) \cosh((m+2)\log(2b) + 2a) \Gamma(m+3, 2bx) + (m+3) \cosh((m+2)\log(2b) - 2a) \Gamma(m+3, -2bx)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2+m)\*cosh(b\*x+a)^2,x, algorithm="fricas")

[Out] 1/8\*(4\*b\*x\*cosh((m+2)\*log(x)) - (m+3)\*cosh((m+2)\*log(2\*b) + 2\*a)\*gamma(m+3, 2\*b\*x) + (m+3)\*cosh((m+2)\*log(-2\*b) - 2\*a)\*gamma(m+3, -2\*b\*x) + (m+3)\*gamma(m+3, 2\*b\*x)\*sinh((m+2)\*log(2\*b) + 2\*a) - (m+3)\*gamma(m+3, -2\*b\*x)\*sinh((m+2)\*log(-2\*b) - 2\*a) + 4\*b\*x\*sinh((m+2)\*log(x)))/(b\*m + 3\*b)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+2} \cosh(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2+m)\*cosh(b\*x+a)^2,x, algorithm="giac")

[Out] integrate(x^(m + 2)\*cosh(b\*x + a)^2, x)

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int x^{2+m} (\cosh^2 (bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2+m)\*cosh(b\*x+a)^2,x)

[Out] int(x^(2+m)\*cosh(b\*x+a)^2,x)

maxima [A] time = 1.41, size = 71, normalized size = 0.84

$$-\frac{1}{4} (2bx)^{-m-3} x^{m+3} e^{(-2a)} \Gamma(m+3, 2bx) - \frac{1}{4} (-2bx)^{-m-3} x^{m+3} e^{(2a)} \Gamma(m+3, -2bx) + \frac{x^{m+3}}{2(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2+m)\*cosh(b\*x+a)^2,x, algorithm="maxima")

[Out] -1/4\*(2\*b\*x)^(-m - 3)\*x^(m + 3)\*e^(-2\*a)\*gamma(m + 3, 2\*b\*x) - 1/4\*(-2\*b\*x)^( -m - 3)\*x^(m + 3)\*e^(2\*a)\*gamma(m + 3, -2\*b\*x) + 1/2\*x^(m + 3)/(m + 3)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+2} \cosh(a + bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(m + 2)\*cosh(a + b\*x)^2,x)

[Out] int(x^(m + 2)\*cosh(a + b\*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+2} \cosh^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(2+m)\*cosh(b\*x+a)\*\*2,x)

[Out] Integral(x\*\*(m + 2)\*cosh(a + b\*x)\*\*2, x)

### 3.90 $\int x^{1+m} \cosh^2(a + bx) dx$

**Optimal.** Leaf size=86

$$-\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\Gamma(m+2,-2bx)}{b^2} - \frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\Gamma(m+2,2bx)}{b^2} + \frac{x^{m+2}}{2(m+2)}$$

[Out]  $1/2*x^{(2+m)}/(2+m)-2^{(-4-m)*\exp(2*a)}*x^m*\text{GAMMA}(2+m,-2*b*x)/b^2/((-b*x)^m)-2^{(-4-m)*x^m*\text{GAMMA}(2+m,2*b*x)/b^2/\exp(2*a)/((b*x)^m)$

**Rubi [A]** time = 0.13, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3312, 3307, 2181}

$$-\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\text{Gamma}(m+2,-2bx)}{b^2} - \frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\text{Gamma}(m+2,2bx)}{b^2} + \frac{x^{m+2}}{2(m+2)}$$

Antiderivative was successfully verified.

[In] Int[x^(1 + m)\*Cosh[a + b\*x]^2, x]

[Out]  $x^{(2+m)}/(2*(2+m)) - (2^{(-4-m)*E^{(2*a)}}*x^m*\text{Gamma}[2+m,-2*b*x])/(b^2*(-(b*x)^m)) - (2^{(-4-m)*x^m*\text{Gamma}[2+m,2*b*x]})/(b^2*E^{(2*a)}*(b*x)^m)$

#### Rule 2181

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, -(f*g*Log[F])/d])*(c + d*x)]/(d*(-(f*g*Log[F])/d)^(IntPart[m] + 1)*(-(f*g*Log[F])*(c + d*x)/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]
```

#### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol]
:> Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\int x^{1+m} \cosh^2(a + bx) dx &= \int \left( \frac{x^{1+m}}{2} + \frac{1}{2} x^{1+m} \cosh(2a + 2bx) \right) dx \\
&= \frac{x^{2+m}}{2(2+m)} + \frac{1}{2} \int x^{1+m} \cosh(2a + 2bx) dx \\
&= \frac{x^{2+m}}{2(2+m)} + \frac{1}{4} \int e^{-i(2ia+2ibx)} x^{1+m} dx + \frac{1}{4} \int e^{i(2ia+2ibx)} x^{1+m} dx \\
&= \frac{x^{2+m}}{2(2+m)} - \frac{2^{-4-m} e^{2a} x^m (-bx)^{-m} \Gamma(2+m, -2bx)}{b^2} - \frac{2^{-4-m} e^{-2a} x^m (bx)^{-m} \Gamma(2+m, 2bx)}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 79, normalized size = 0.92

$$\frac{1}{16} x^m \left( -\frac{e^{2a} 2^{-m} (-bx)^{-m} \Gamma(m+2, -2bx)}{b^2} - \frac{e^{-2a} 2^{-m} (bx)^{-m} \Gamma(m+2, 2bx)}{b^2} + \frac{8x^2}{m+2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(1+m)\*Cosh[a+b\*x]^2,x]

[Out] (x^m\*((8\*x^2)/(2+m) - (E^(2\*a)\*Gamma[2+m, -2\*b\*x])/(2^m\*b^2\*(-(b\*x))^m) - Gamma[2+m, 2\*b\*x]/(2^m\*b^2\*E^(2\*a)\*(b\*x)^m)))/16

**fricas [A]** time = 0.54, size = 136, normalized size = 1.58

$$\frac{4bx \cosh((m+1)\log(x)) - (m+2) \cosh((m+1)\log(2b) + 2a) \Gamma(m+2, 2bx) + (m+2) \cosh((m+1)\log(-2b) - 2a) \Gamma(m+2, -2bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+m)\*cosh(b\*x+a)^2,x, algorithm="fricas")

[Out] 1/8\*(4\*b\*x\*cosh((m+1)\*log(x)) - (m+2)\*cosh((m+1)\*log(2\*b) + 2\*a)\*gamma(m+2, 2\*b\*x) + (m+2)\*cosh((m+1)\*log(-2\*b) - 2\*a)\*gamma(m+2, -2\*b\*x) + (m+2)\*gamma(m+2, 2\*b\*x)\*sinh((m+1)\*log(2\*b) + 2\*a) - (m+2)\*gamma(m+2, -2\*b\*x)\*sinh((m+1)\*log(-2\*b) - 2\*a) + 4\*b\*x\*sinh((m+1)\*log(x)))/(b\*m + 2\*b)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+1} \cosh(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+m)\*cosh(b\*x+a)^2,x, algorithm="giac")

[Out] integrate(x^(m + 1)\*cosh(b\*x + a)^2, x)

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int x^{1+m} (\cosh^2(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1+m)\*cosh(b\*x+a)^2,x)

[Out] int(x^(1+m)\*cosh(b\*x+a)^2,x)

maxima [A] time = 2.77, size = 71, normalized size = 0.83

$$-\frac{1}{4} (2bx)^{-m-2} x^{m+2} e^{(-2a)} \Gamma(m+2, 2bx) - \frac{1}{4} (-2bx)^{-m-2} x^{m+2} e^{(2a)} \Gamma(m+2, -2bx) + \frac{x^{m+2}}{2(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+m)\*cosh(b\*x+a)^2,x, algorithm="maxima")

[Out] -1/4\*(2\*b\*x)^(-m - 2)\*x^(m + 2)\*e^(-2\*a)\*gamma(m + 2, 2\*b\*x) - 1/4\*(-2\*b\*x)^( -m - 2)\*x^(m + 2)\*e^(2\*a)\*gamma(m + 2, -2\*b\*x) + 1/2\*x^(m + 2)/(m + 2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+1} \cosh(a + bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(m + 1)\*cosh(a + b\*x)^2,x)

[Out] int(x^(m + 1)\*cosh(a + b\*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+1} \cosh^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1+m)\*cosh(b\*x+a)\*\*2,x)

[Out] Integral(x\*\*(m + 1)\*cosh(a + b\*x)\*\*2, x)

### 3.91 $\int x^m \cosh^2(a + bx) dx$

**Optimal.** Leaf size=85

$$\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b} - \frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b} + \frac{x^{m+1}}{2(m+1)}$$

[Out]  $1/2*x^{(1+m)}/(1+m)+2^{(-3-m)}*exp(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)-2^{(-3-m)}*x^m*GAMMA(1+m,2*b*x)/b/exp(2*a)/((b*x)^m)$

**Rubi [A]** time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3312, 3307, 2181}

$$\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b} - \frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b} + \frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Cosh[a + b\*x]^2,x]

[Out]  $x^{(1+m)}/(2*(1+m)) + (2^{(-3-m)}*E^{(2*a)}*x^m*\Gamma[1+m,-2*b*x])/(b*(-(b*x))^m) - (2^{(-3-m)}*x^m*\Gamma[1+m,2*b*x])/(b*E^{(2*a)}*(b*x)^m)$

#### Rule 2181

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, -(f*g*Log[F])/d])*(c + d*x)]/(d*(-(f*g*Log[F])/d)^(IntPart[m] + 1)*(-(f*g*Log[F])*(c + d*x)/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rule 3307

```
Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol]
:> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]
```

#### Rule 3312

```
Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + (f_)*(x_)]^(n_), x_Symbol]
:> Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```



Rubi steps

$$\begin{aligned}
\int x^m \cosh^2(a + bx) dx &= \int \left( \frac{x^m}{2} + \frac{1}{2} x^m \cosh(2a + 2bx) \right) dx \\
&= \frac{x^{1+m}}{2(1+m)} + \frac{1}{2} \int x^m \cosh(2a + 2bx) dx \\
&= \frac{x^{1+m}}{2(1+m)} + \frac{1}{4} \int e^{-i(2ia+2ibx)} x^m dx + \frac{1}{4} \int e^{i(2ia+2ibx)} x^m dx \\
&= \frac{x^{1+m}}{2(1+m)} + \frac{2^{-3-m} e^{2a} x^m (-bx)^{-m} \Gamma(1+m, -2bx)}{b} - \frac{2^{-3-m} e^{-2a} x^m (bx)^{-m} \Gamma(1+m, 2bx)}{b}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 76, normalized size = 0.89

$$\frac{1}{8} x^m \left( \frac{e^{2a} 2^{-m} (-bx)^{-m} \Gamma(m+1, -2bx)}{b} - \frac{e^{-2a} 2^{-m} (bx)^{-m} \Gamma(m+1, 2bx)}{b} + \frac{4x}{m+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*Cosh[a + b\*x]^2,x]

[Out] (x^m\*((4\*x)/(1+m) + (E^(2\*a)\*Gamma[1+m, -2\*b\*x])/(2^m\*b\*(-(b\*x))^m) - Gamma[1+m, 2\*b\*x]/(2^m\*b\*E^(2\*a)\*(b\*x)^m)))/8

**fricas [A]** time = 0.49, size = 122, normalized size = 1.44

$$\frac{4bx \cosh(m \log(x)) - (m+1) \cosh(m \log(2b) + 2a) \Gamma(m+1, 2bx) + (m+1) \cosh(m \log(-2b) - 2a) \Gamma(m+1, -2bx)}{b(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cosh(b\*x+a)^2,x, algorithm="fricas")

[Out] 1/8\*(4\*b\*x\*cosh(m\*log(x)) - (m+1)\*cosh(m\*log(2\*b) + 2\*a)\*gamma(m+1, 2\*b\*x) + (m+1)\*cosh(m\*log(-2\*b) - 2\*a)\*gamma(m+1, -2\*b\*x) + (m+1)\*gamma(m+1, 2\*b\*x)\*sinh(m\*log(2\*b) + 2\*a) - (m+1)\*gamma(m+1, -2\*b\*x)\*sinh(m\*log(-2\*b) - 2\*a) + 4\*b\*x\*sinh(m\*log(x)))/(b\*m + b)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cosh(b\*x+a)^2,x, algorithm="giac")

[Out] integrate(x^m\*cosh(b\*x + a)^2, x)

**maple** [F] time = 0.21, size = 0, normalized size = 0.00

$$\int x^m (\cosh^2 (bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*cosh(b\*x+a)^2,x)

[Out] int(x^m\*cosh(b\*x+a)^2,x)

**maxima** [A] time = 0.90, size = 71, normalized size = 0.84

$$-\frac{1}{4} (2bx)^{-m-1} x^{m+1} e^{(-2a)} \Gamma(m+1, 2bx) - \frac{1}{4} (-2bx)^{-m-1} x^{m+1} e^{(2a)} \Gamma(m+1, -2bx) + \frac{x^{m+1}}{2(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cosh(b\*x+a)^2,x, algorithm="maxima")

[Out] -1/4\*(2\*b\*x)^(-m - 1)\*x^(m + 1)\*e^(-2\*a)\*gamma(m + 1, 2\*b\*x) - 1/4\*(-2\*b\*x)^(m + 1)\*x^(m + 1)\*e^(2\*a)\*gamma(m + 1, -2\*b\*x) + 1/2\*x^(m + 1)/(m + 1)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \cosh(a + bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*cosh(a + b\*x)^2,x)

[Out] int(x^m\*cosh(a + b\*x)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*cosh(b\*x+a)\*\*2,x)

[Out] Integral(x\*\*m\*cosh(a + b\*x)\*\*2, x)

### 3.92 $\int x^{-1+m} \cosh^2(a + bx) dx$

Optimal. Leaf size=72

$$e^{2a} (-2^{-m-2}) x^m (-bx)^{-m} \Gamma(m, -2bx) - e^{-2a} 2^{-m-2} x^m (bx)^{-m} \Gamma(m, 2bx) + \frac{x^m}{2m}$$

[Out]  $1/2*x^m/m-2^{-(2-m)}*\exp(2*a)*x^m*\text{GAMMA}(m, -2*b*x)/((-b*x)^m)-2^{-(2-m)}*x^m*\text{GAMMA}(m, 2*b*x)/\exp(2*a)/((b*x)^m)$

**Rubi [A]** time = 0.12, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3312, 3307, 2181}

$$e^{2a} (-2^{-m-2}) x^m (-bx)^{-m} \text{Gamma}(m, -2bx) - e^{-2a} 2^{-m-2} x^m (bx)^{-m} \text{Gamma}(m, 2bx) + \frac{x^m}{2m}$$

Antiderivative was successfully verified.

[In] Int[x<sup>(-1 + m)</sup>\*Cosh[a + b\*x]<sup>2</sup>, x]

[Out]  $x^m/(2*m) - (2^{-(2 - m)}*E^{(2*a)}*x^m*\text{Gamma}[m, -2*b*x])/(-b*x)^m - (2^{-(2 - m)}*x^m*\text{Gamma}[m, 2*b*x])/(E^{(2*a)}*(b*x)^m)$

#### Rule 2181

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]
```

#### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol]
:> Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

#### Rubi steps

$$\begin{aligned}
\int x^{-1+m} \cosh^2(a + bx) dx &= \int \left( \frac{x^{-1+m}}{2} + \frac{1}{2} x^{-1+m} \cosh(2a + 2bx) \right) dx \\
&= \frac{x^m}{2m} + \frac{1}{2} \int x^{-1+m} \cosh(2a + 2bx) dx \\
&= \frac{x^m}{2m} + \frac{1}{4} \int e^{-i(2ia+2ibx)} x^{-1+m} dx + \frac{1}{4} \int e^{i(2ia+2ibx)} x^{-1+m} dx \\
&= \frac{x^m}{2m} - 2^{-2-m} e^{2a} x^m (-bx)^{-m} \Gamma(m, -2bx) - 2^{-2-m} e^{-2a} x^m (bx)^{-m} \Gamma(m, 2bx)
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 64, normalized size = 0.89

$$\frac{1}{4} x^m \left( e^{2a} (-2^{-m}) (-bx)^{-m} \Gamma(m, -2bx) - e^{-2a} 2^{-m} (bx)^{-m} \Gamma(m, 2bx) + \frac{2}{m} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + m)\*Cosh[a + b\*x]^2,x]

[Out] (x^m\*(2/m - (E^(2\*a)\*Gamma[m, -2\*b\*x]))/(2^m\*(-(b\*x))^m) - Gamma[m, 2\*b\*x]/(2^m\*E^(2\*a)\*(b\*x)^m))/4

**fricas [A]** time = 0.48, size = 117, normalized size = 1.62

$$4bx \cosh((m-1)\log(x)) - m \cosh((m-1)\log(2b) + 2a) \Gamma(m, 2bx) + m \cosh((m-1)\log(-2b) - 2a) \Gamma(m,$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+m)\*cosh(b\*x+a)^2,x, algorithm="fricas")

[Out] 1/8\*(4\*b\*x\*cosh((m-1)\*log(x)) - m\*cosh((m-1)\*log(2\*b) + 2\*a)\*gamma(m, 2\*b\*x) + m\*cosh((m-1)\*log(-2\*b) - 2\*a)\*gamma(m, -2\*b\*x) + m\*gamma(m, 2\*b\*x)\*sinh((m-1)\*log(2\*b) + 2\*a) - m\*gamma(m, -2\*b\*x)\*sinh((m-1)\*log(-2\*b) - 2\*a) + 4\*b\*x\*sinh((m-1)\*log(x)))/(b\*m)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-1} \cosh(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+m)\*cosh(b\*x+a)^2,x, algorithm="giac")

[Out] integrate(x^(m - 1)\*cosh(b\*x + a)^2, x)

**maple** [F] time = 0.25, size = 0, normalized size = 0.00

$$\int x^{-1+m} (\cosh^2(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+m)\*cosh(b\*x+a)^2,x)

[Out] int(x^(-1+m)\*cosh(b\*x+a)^2,x)

**maxima** [A] time = 0.64, size = 55, normalized size = 0.76

$$-\frac{x^m e^{(-2a)} \Gamma(m, 2bx)}{4 (2bx)^m} - \frac{x^m e^{(2a)} \Gamma(m, -2bx)}{4 (-2bx)^m} + \frac{x^m}{2m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+m)\*cosh(b\*x+a)^2,x, algorithm="maxima")

[Out] -1/4\*x^m\*e^(-2\*a)\*gamma(m, 2\*b\*x)/(2\*b\*x)^m - 1/4\*x^m\*e^(2\*a)\*gamma(m, -2\*b\*x)/(-2\*b\*x)^m + 1/2\*x^m/m

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-1} \cosh(a + bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(m - 1)\*cosh(a + b\*x)^2,x)

[Out] int(x^(m - 1)\*cosh(a + b\*x)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-1} \cosh^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(-1+m)\*cosh(b\*x+a)\*\*2,x)

[Out] Integral(x\*\*(m - 1)\*cosh(a + b\*x)\*\*2, x)

### 3.93 $\int x^{-2+m} \cosh^2(a + bx) dx$

Optimal. Leaf size=83

$$e^{2a}b2^{-m-1}x^m(-bx)^{-m}\Gamma(m-1, -2bx) - e^{-2a}b2^{-m-1}x^m(bx)^{-m}\Gamma(m-1, 2bx) - \frac{x^{m-1}}{2(1-m)}$$

[Out]  $-1/2*x^{(-1+m)/(1-m)}+2^{(-1-m)*b*\exp(2*a)}*x^m*\text{GAMMA}(-1+m, -2*b*x)/((-b*x)^m)-2^{(-1-m)*b*x^m*\text{GAMMA}(-1+m, 2*b*x)/\exp(2*a)/((b*x)^m)$

**Rubi [A]** time = 0.13, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3312, 3307, 2181}

$$e^{2a}b2^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m-1, -2bx) - e^{-2a}b2^{-m-1}x^m(bx)^{-m}\text{Gamma}(m-1, 2bx) - \frac{x^{m-1}}{2(1-m)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(-2 + m)*\text{Cosh}[a + b*x]^2, x]$

[Out]  $-x^{(-1 + m)/(2*(1 - m))} + (2^{(-1 - m)*b*E^{(2*a)}*x^m*\text{Gamma}[-1 + m, -2*b*x]}/((-b*x))^m - (2^{(-1 - m)*b*x^m*\text{Gamma}[-1 + m, 2*b*x]}/(E^{(2*a)}*(b*x)^m)$

#### Rule 2181

$\text{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))}*((c_.) + (d_.)*(x_))^{(m_)}, x\_Symbol]$   
 $:= -\text{Simp}[(F^{(g*(e - (c*f)/d))}*(c + d*x)^{\text{FracPart}[m]*\text{Gamma}[m + 1, -(f*g*\text{Log}[F])/d]}*(c + d*x)]/(d*(-(f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1}*(-(f*g*\text{Log}[F]*(c + d*x))/d)^{\text{FracPart}[m]}, x] /; \text{FreeQ}\{F, c, d, e, f, g, m\}, x \ \&\& \ !\text{IntegerQ}[m]$

#### Rule 3307

$\text{Int}(((c_.) + (d_.)*(x_))^{(m_.)}*\sin[(e_.) + \text{Pi}*(k_.) + (f_.)*(x_)], x\_Symbol]$   
 $:= \text{Dist}[I/2, \text{Int}[(c + d*x)^m/(E^{I*k*Pi}*E^{I*(e + f*x)}), x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m*E^{I*k*Pi}*E^{I*(e + f*x)}, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x \ \&\& \ \text{IntegerQ}[2*k]$

#### Rule 3312

$\text{Int}(((c_.) + (d_.)*(x_))^{(m_.)}*\sin[(e_.) + (f_.)*(x_)]^{(n_.)}, x\_Symbol]$   $:= \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[e + f*x]^n, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x \ \&\& \ \text{IGtQ}[n, 1] \ \&\& \ (!\text{RationalQ}[m] \ || \ (\text{GeQ}[m, -1] \ \&\& \ \text{LtQ}[m, 1]))$

Rubi steps

$$\begin{aligned}
\int x^{-2+m} \cosh^2(a + bx) dx &= \int \left( \frac{x^{-2+m}}{2} + \frac{1}{2} x^{-2+m} \cosh(2a + 2bx) \right) dx \\
&= -\frac{x^{-1+m}}{2(1-m)} + \frac{1}{2} \int x^{-2+m} \cosh(2a + 2bx) dx \\
&= -\frac{x^{-1+m}}{2(1-m)} + \frac{1}{4} \int e^{-i(2ia+2ibx)} x^{-2+m} dx + \frac{1}{4} \int e^{i(2ia+2ibx)} x^{-2+m} dx \\
&= -\frac{x^{-1+m}}{2(1-m)} + 2^{-1-m} b e^{2a} x^m (-bx)^{-m} \Gamma(-1+m, -2bx) - 2^{-1-m} b e^{-2a} x^m (bx)^{-m} \Gamma(-1+m, 2bx)
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 73, normalized size = 0.88

$$\frac{1}{2} x^m \left( e^{2a} b 2^{-m} (-bx)^{-m} \Gamma(m-1, -2bx) - e^{-2a} b 2^{-m} (bx)^{-m} \Gamma(m-1, 2bx) + \frac{1}{(m-1)x} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(-2 + m)\*Cosh[a + b\*x]^2,x]

[Out] (x^m\*(1/((-1 + m)\*x) + (b\*E^(2\*a)\*Gamma[-1 + m, -2\*b\*x]))/(2^m\*(-(b\*x))^m) - (b\*Gamma[-1 + m, 2\*b\*x])/(2^m\*E^(2\*a)\*(b\*x)^m))/2

**fricas [A]** time = 0.46, size = 136, normalized size = 1.64

$$4bx \cosh((m-2)\log(x)) - (m-1) \cosh((m-2)\log(2b) + 2a) \Gamma(m-1, 2bx) + (m-1) \cosh((m-2)\log(-2b) - 2a) \Gamma(m-1, -2bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-2+m)\*cosh(b\*x+a)^2,x, algorithm="fricas")

[Out] 1/8\*(4\*b\*x\*cosh((m-2)\*log(x)) - (m-1)\*cosh((m-2)\*log(2\*b) + 2\*a)\*gamma(m-1, 2\*b\*x) + (m-1)\*cosh((m-2)\*log(-2\*b) - 2\*a)\*gamma(m-1, -2\*b\*x) + (m-1)\*gamma(m-1, 2\*b\*x)\*sinh((m-2)\*log(2\*b) + 2\*a) - (m-1)\*gamma(m-1, -2\*b\*x)\*sinh((m-2)\*log(-2\*b) - 2\*a) + 4\*b\*x\*sinh((m-2)\*log(x)))/(b\*m - b)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-2} \cosh(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-2+m)</sup>\*cosh(b\*x+a)<sup>2</sup>,x, algorithm="giac")

[Out] integrate(x<sup>(m - 2)</sup>\*cosh(b\*x + a)<sup>2</sup>, x)

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int x^{-2+m} (\cosh^2 (bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>(-2+m)</sup>\*cosh(b\*x+a)<sup>2</sup>,x)

[Out] int(x<sup>(-2+m)</sup>\*cosh(b\*x+a)<sup>2</sup>,x)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-2+m)</sup>\*cosh(b\*x+a)<sup>2</sup>,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(m-2>0)', see `assume?` for more details)Is m-2 equal to -1?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-2} \cosh (a + bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>(m - 2)</sup>\*cosh(a + b\*x)<sup>2</sup>,x)

[Out] int(x<sup>(m - 2)</sup>\*cosh(a + b\*x)<sup>2</sup>, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-2} \cosh^2 (a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>\*\*(-2+m)</sup>\*cosh(b\*x+a)<sup>\*\*2</sup>,x)

[Out] Integral(x<sup>\*\* (m - 2)</sup>\*cosh(a + b\*x)<sup>\*\*2</sup>, x)



### 3.94 $\int x^{-3+m} \cosh^2(a + bx) dx$

Optimal. Leaf size=84

$$-e^{2a}b^22^{-m}x^m(-bx)^{-m}\Gamma(m-2, -2bx) - e^{-2a}b^22^{-m}x^m(bx)^{-m}\Gamma(m-2, 2bx) - \frac{x^{m-2}}{2(2-m)}$$

[Out]  $-1/2*x^{(-2+m)/(2-m)-b^2*\exp(2*a)*x^m*\text{GAMMA}(-2+m, -2*b*x)/(2^m)/((-b*x)^m)-b^2*x^m*\text{GAMMA}(-2+m, 2*b*x)/(2^m)/\exp(2*a)/((b*x)^m)$

**Rubi [A]** time = 0.14, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3312, 3307, 2181}

$$-e^{2a}b^22^{-m}x^m(-bx)^{-m}\text{Gamma}(m-2, -2bx) - e^{-2a}b^22^{-m}x^m(bx)^{-m}\text{Gamma}(m-2, 2bx) - \frac{x^{m-2}}{2(2-m)}$$

Antiderivative was successfully verified.

[In] Int[x<sup>(-3 + m)</sup>\*Cosh[a + b\*x]<sup>2</sup>, x]

[Out]  $-x^{(-2+m)/(2*(2-m))} - (b^2*E^{(2*a)}*x^m*\text{Gamma}[-2+m, -2*b*x])/(2^m*(-(b*x)^m)) - (b^2*x^m*\text{Gamma}[-2+m, 2*b*x])/(2^m*E^{(2*a)}*(b*x)^m)$

#### Rule 2181

Int[((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rule 3307

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*sin[(e\_) + Pi\*(k\_) + (f\_)\*(x\_)], x\_Symbol] :> Dist[I/2, Int[(c + d\*x)^m/(E^(I\*k\*Pi)\*E^(I\*(e + f\*x))), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*k\*Pi)\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

#### Rule 3312

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*sin[(e\_) + (f\_)\*(x\_)]^(n\_), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rubi steps

$$\begin{aligned}
\int x^{-3+m} \cosh^2(a + bx) dx &= \int \left( \frac{x^{-3+m}}{2} + \frac{1}{2} x^{-3+m} \cosh(2a + 2bx) \right) dx \\
&= -\frac{x^{-2+m}}{2(2-m)} + \frac{1}{2} \int x^{-3+m} \cosh(2a + 2bx) dx \\
&= -\frac{x^{-2+m}}{2(2-m)} + \frac{1}{4} \int e^{-i(2ia+2ibx)} x^{-3+m} dx + \frac{1}{4} \int e^{i(2ia+2ibx)} x^{-3+m} dx \\
&= -\frac{x^{-2+m}}{2(2-m)} - 2^{-m} b^2 e^{2a} x^m (-bx)^{-m} \Gamma(-2+m, -2bx) - 2^{-m} b^2 e^{-2a} x^m (bx)^{-m} \Gamma(-2+m, 2bx)
\end{aligned}$$

**Mathematica** [A] time = 0.11, size = 84, normalized size = 1.00

$$-e^{2a} b^2 2^{-m} x^m (-bx)^{-m} \Gamma(m-2, -2bx) - e^{-2a} b^2 2^{-m} x^m (bx)^{-m} \Gamma(m-2, 2bx) - \frac{x^{m-2}}{2(2-m)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-3 + m)\*Cosh[a + b\*x]^2,x]

[Out] -1/2\*x^(-2 + m)/(2 - m) - (b^2\*E^(2\*a)\*x^m\*Gamma[-2 + m, -2\*b\*x])/(2^m\*(-(b\*x))^m) - (b^2\*x^m\*Gamma[-2 + m, 2\*b\*x])/(2^m\*E^(2\*a)\*(b\*x)^m)

**fricas** [A] time = 0.44, size = 136, normalized size = 1.62

$$\frac{4bx \cosh((m-3)\log(x)) - (m-2) \cosh((m-3)\log(2b) + 2a) \Gamma(m-2, 2bx) + (m-2) \cosh((m-3)\log(-2b) - 2a) \Gamma(m-2, -2bx)}{(b^m - 2^m)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-3+m)\*cosh(b\*x+a)^2,x, algorithm="fricas")

[Out] 1/8\*(4\*b\*x\*cosh((m-3)\*log(x)) - (m-2)\*cosh((m-3)\*log(2\*b) + 2\*a)\*gamma(m-2, 2\*b\*x) + (m-2)\*cosh((m-3)\*log(-2\*b) - 2\*a)\*gamma(m-2, -2\*b\*x) + (m-2)\*gamma(m-2, 2\*b\*x)\*sinh((m-3)\*log(2\*b) + 2\*a) - (m-2)\*gamma(m-2, -2\*b\*x)\*sinh((m-3)\*log(-2\*b) - 2\*a) + 4\*b\*x\*sinh((m-3)\*log(x)))/(b^m - 2^m)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-3} \cosh(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-3+m)</sup>\*cosh(b\*x+a)<sup>2</sup>,x, algorithm="giac")

[Out] integrate(x<sup>(m - 3)</sup>\*cosh(b\*x + a)<sup>2</sup>, x)

**maple** [F] time = 0.20, size = 0, normalized size = 0.00

$$\int x^{-3+m} (\cosh^2(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>(-3+m)</sup>\*cosh(b\*x+a)<sup>2</sup>,x)

[Out] int(x<sup>(-3+m)</sup>\*cosh(b\*x+a)<sup>2</sup>,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-3+m)</sup>\*cosh(b\*x+a)<sup>2</sup>,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(m-3>0)', see `assume?` for more details)Is m-3 equal to -1?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-3} \cosh(a + bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>(m - 3)</sup>\*cosh(a + b\*x)<sup>2</sup>,x)

[Out] int(x<sup>(m - 3)</sup>\*cosh(a + b\*x)<sup>2</sup>, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-3} \cosh^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>\*\*(-3+m)</sup>\*cosh(b\*x+a)<sup>\*\*2</sup>,x)

[Out] Integral(x<sup>\*\* (m - 3)</sup>\*cosh(a + b\*x)<sup>\*\*2</sup>, x)

$$3.95 \quad \int \left( \frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{1}{3} x \sqrt{\operatorname{sech}(x)} \right) dx$$

Optimal. Leaf size=24

$$\frac{2x \sinh(x)}{3\sqrt{\operatorname{sech}(x)}} - \frac{4}{9\operatorname{sech}^{\frac{3}{2}}(x)}$$

[Out]  $-4/9/\operatorname{sech}(x)^{(3/2)}+2/3*x*\sinh(x)/\operatorname{sech}(x)^{(1/2)}$

**Rubi [A]** time = 0.09, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {4187, 4189}

$$\frac{2x \sinh(x)}{3\sqrt{\operatorname{sech}(x)}} - \frac{4}{9\operatorname{sech}^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x/\text{Sech}[x]^{(3/2)} - (x*\text{Sqrt}[\text{Sech}[x]])/3, x]$

[Out]  $-4/(9*\text{Sech}[x]^{(3/2)}) + (2*x*\text{Sinh}[x])/(3*\text{Sqrt}[\text{Sech}[x]])$

Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_)*((c_.) + (d_.)*(x_.)), x_Symbol] :=
  Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_)*((c_.) + (d_.)*(x_.))^(m_.), x_Symb
ol] := Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
+ f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \left( \frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{1}{3} x \sqrt{\operatorname{sech}(x)} \right) dx &= - \left( \frac{1}{3} \int x \sqrt{\operatorname{sech}(x)} dx \right) + \int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)} dx \\
&= - \frac{4}{9 \operatorname{sech}^{\frac{3}{2}}(x)} + \frac{2x \sinh(x)}{3 \sqrt{\operatorname{sech}(x)}} + \frac{1}{3} \int x \sqrt{\operatorname{sech}(x)} dx - \frac{1}{3} (\sqrt{\cosh(x)} \sqrt{\operatorname{sech}(x)}) \\
&= - \frac{4}{9 \operatorname{sech}^{\frac{3}{2}}(x)} + \frac{2x \sinh(x)}{3 \sqrt{\operatorname{sech}(x)}}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 17, normalized size = 0.71

$$\frac{2(3x \tanh(x) - 2)}{9 \operatorname{sech}^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sech[x]^(3/2) - (x\*Sqrt[Sech[x]])/3,x]

[Out] (2\*(-2 + 3\*x\*Tanh[x]))/(9\*Sech[x]^(3/2))

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sech(x)^(3/2)-1/3\*x\*sech(x)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{3} x \sqrt{\operatorname{sech}(x)} + \frac{x}{\operatorname{sech}(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sech(x)^(3/2)-1/3\*x\*sech(x)^(1/2),x, algorithm="giac")

[Out] integrate(-1/3\*x\*sqrt(sech(x)) + x/sech(x)^(3/2), x)

**maple** [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{sech}(x)^{\frac{3}{2}}} - \frac{x\sqrt{\operatorname{sech}(x)}}{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x)`

[Out] `int(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{3}x\sqrt{\operatorname{sech}(x)} + \frac{x}{\operatorname{sech}(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(-1/3*x*sqrt(sech(x)) + x/sech(x)^(3/2), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{x\sqrt{\frac{1}{\cosh(x)}}}{3} - \frac{x}{\left(\frac{1}{\cosh(x)}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(1/cosh(x))^(3/2) - (x*(1/cosh(x))^(1/2))/3,x)`

[Out] `-int((x*(1/cosh(x))^(1/2))/3 - x/(1/cosh(x))^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \left( -\frac{3x}{\operatorname{sech}^{\frac{3}{2}}(x)} \right) dx + \int x\sqrt{\operatorname{sech}(x)} dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/sech(x)**(3/2)-1/3*x*sech(x)**(1/2),x)`

[Out] `-(Integral(-3*x/sech(x)**(3/2), x) + Integral(x*sqrt(sech(x)), x))/3`

$$3.96 \quad \int \left( \frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)} - \frac{3x}{5\sqrt{\operatorname{sech}(x)}} \right) dx$$

Optimal. Leaf size=24

$$\frac{2x \sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{4}{25\operatorname{sech}^{\frac{5}{2}}(x)}$$

[Out]  $-4/25/\operatorname{sech}(x)^{(5/2)}+2/5*x*\sinh(x)/\operatorname{sech}(x)^{(3/2)}$

**Rubi [A]** time = 0.09, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {4187, 4189}

$$\frac{2x \sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{4}{25\operatorname{sech}^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

[In] Int[x/Sech[x]^(5/2) - (3\*x)/(5\*Sqrt[Sech[x]]),x]

[Out]  $-4/(25*\operatorname{Sech}[x]^{(5/2)}) + (2*x*\operatorname{Sinh}[x])/(5*\operatorname{Sech}[x]^{(3/2)})$

Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :=
  Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] := Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
+ f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \left( \frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)} - \frac{3x}{5\sqrt{\operatorname{sech}(x)}} \right) dx &= - \left( \frac{3}{5} \int \frac{x}{\sqrt{\operatorname{sech}(x)}} dx \right) + \int \frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)} dx \\
&= - \frac{4}{25\operatorname{sech}^{\frac{5}{2}}(x)} + \frac{2x \sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)} + \frac{3}{5} \int \frac{x}{\sqrt{\operatorname{sech}(x)}} dx - \frac{1}{5} (3\sqrt{\cosh(x)} \sqrt{\operatorname{sech}(x)}) \\
&= - \frac{4}{25\operatorname{sech}^{\frac{5}{2}}(x)} + \frac{2x \sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)}
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 17, normalized size = 0.71

$$\frac{2(5x \tanh(x) - 2)}{25\operatorname{sech}^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sech[x]^(5/2) - (3\*x)/(5\*Sqrt[Sech[x]]), x]

[Out] (2\*(-2 + 5\*x\*Tanh[x]))/(25\*Sech[x]^(5/2))

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sech(x)^(5/2)-3/5\*x/sech(x)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{3x}{5\sqrt{\operatorname{sech}(x)}} + \frac{x}{\operatorname{sech}(x)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sech(x)^(5/2)-3/5\*x/sech(x)^(1/2), x, algorithm="giac")

[Out] integrate(-3/5\*x/sqrt(sech(x)) + x/sech(x)^(5/2), x)



**maple** [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{sech}(x)^{\frac{5}{2}}} - \frac{3x}{5\sqrt{\operatorname{sech}(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x)`

[Out] `int(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{3x}{5\sqrt{\operatorname{sech}(x)}} + \frac{x}{\operatorname{sech}(x)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(-3/5*x/sqrt(sech(x)) + x/sech(x)^(5/2), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{3x}{5\sqrt{\frac{1}{\cosh(x)}}} - \frac{x}{\left(\frac{1}{\cosh(x)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(1/cosh(x))^(5/2) - (3*x)/(5*(1/cosh(x))^(1/2)),x)`

[Out] `-int((3*x)/(5*(1/cosh(x))^(1/2)) - x/(1/cosh(x))^(5/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \left( -\frac{5x}{\operatorname{sech}^{\frac{5}{2}}(x)} \right) dx + \int \frac{3x}{\sqrt{\operatorname{sech}(x)}} dx}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/sech(x)**(5/2)-3/5*x/sech(x)**(1/2),x)`

[Out] `-(Integral(-5*x/sech(x)**(5/2), x) + Integral(3*x/sqrt(sech(x)), x))/5`

$$3.97 \quad \int \left( \frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\operatorname{sech}(x)} \right) dx$$

Optimal. Leaf size=47

$$-\frac{20}{63 \operatorname{sech}^{\frac{3}{2}}(x)} - \frac{4}{49 \operatorname{sech}^{\frac{7}{2}}(x)} + \frac{2x \sinh(x)}{7 \operatorname{sech}^{\frac{5}{2}}(x)} + \frac{10x \sinh(x)}{21 \sqrt{\operatorname{sech}(x)}}$$

[Out] -4/49/sech(x)^(7/2)-20/63/sech(x)^(3/2)+2/7\*x\*sinh(x)/sech(x)^(5/2)+10/21\*x\*sinh(x)/sech(x)^(1/2)

**Rubi [A]** time = 0.10, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {4187, 4189}

$$-\frac{20}{63 \operatorname{sech}^{\frac{3}{2}}(x)} - \frac{4}{49 \operatorname{sech}^{\frac{7}{2}}(x)} + \frac{2x \sinh(x)}{7 \operatorname{sech}^{\frac{5}{2}}(x)} + \frac{10x \sinh(x)}{21 \sqrt{\operatorname{sech}(x)}}$$

Antiderivative was successfully verified.

[In] Int[x/Sech[x]^(7/2) - (5\*x\*Sqrt[Sech[x]])/21,x]

[Out] -4/(49\*Sech[x]^(7/2)) - 20/(63\*Sech[x]^(3/2)) + (2\*x\*Sinh[x])/(7\*Sech[x]^(5/2)) + (10\*x\*Sinh[x])/(21\*Sqrt[Sech[x]])

#### Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^n]*((c_.) + (d_.)*(x_)), x_Symbol] :=
  Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*Cs
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

#### Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^n]*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] := Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
+ f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]
```

#### Rubi steps

$$\begin{aligned}
\int \left( \frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\operatorname{sech}(x)} \right) dx &= - \left( \frac{5}{21} \int x \sqrt{\operatorname{sech}(x)} dx \right) + \int \frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)} dx \\
&= - \frac{4}{49 \operatorname{sech}^{\frac{7}{2}}(x)} + \frac{2x \sinh(x)}{7 \operatorname{sech}^{\frac{5}{2}}(x)} + \frac{5}{7} \int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)} dx - \frac{1}{21} (5 \sqrt{\cosh(x)} \sqrt{\operatorname{sech}(x)}) \\
&= - \frac{4}{49 \operatorname{sech}^{\frac{7}{2}}(x)} - \frac{20}{63 \operatorname{sech}^{\frac{3}{2}}(x)} + \frac{2x \sinh(x)}{7 \operatorname{sech}^{\frac{5}{2}}(x)} + \frac{10x \sinh(x)}{21 \sqrt{\operatorname{sech}(x)}} + \frac{5}{21} \int x \sqrt{\operatorname{sech}(x)} dx \\
&= - \frac{4}{49 \operatorname{sech}^{\frac{7}{2}}(x)} - \frac{20}{63 \operatorname{sech}^{\frac{3}{2}}(x)} + \frac{2x \sinh(x)}{7 \operatorname{sech}^{\frac{5}{2}}(x)} + \frac{10x \sinh(x)}{21 \sqrt{\operatorname{sech}(x)}}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 45, normalized size = 0.96

$$\sqrt{\operatorname{sech}(x)} \left( \frac{13}{42} x \sinh(2x) + \frac{1}{28} x \sinh(4x) - \frac{88}{441} \cosh(2x) - \frac{1}{98} \cosh(4x) - \frac{167}{882} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/Sech[x]^(7/2) - (5\*x\*Sqrt[Sech[x]])/21,x]

[Out] Sqrt[Sech[x]]\*(-167/882 - (88\*Cosh[2\*x])/441 - Cosh[4\*x]/98 + (13\*x\*Sinh[2\*x])/42 + (x\*Sinh[4\*x])/28)

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sech(x)^(7/2)-5/21\*x\*sech(x)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{5}{21} x \sqrt{\operatorname{sech}(x)} + \frac{x}{\operatorname{sech}(x)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sech(x)^(7/2)-5/21\*x\*sech(x)^(1/2),x, algorithm="giac")

[Out] integrate(-5/21\*x\*sqrt(sech(x)) + x/sech(x)^(7/2), x)

**maple** [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{sech}(x)^{\frac{7}{2}}} - \frac{5x\sqrt{\operatorname{sech}(x)}}{21} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/sech(x)^(7/2)-5/21\*x\*sech(x)^(1/2), x)

[Out] int(x/sech(x)^(7/2)-5/21\*x\*sech(x)^(1/2), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{5}{21} x\sqrt{\operatorname{sech}(x)} + \frac{x}{\operatorname{sech}(x)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sech(x)^(7/2)-5/21\*x\*sech(x)^(1/2), x, algorithm="maxima")

[Out] integrate(-5/21\*x\*sqrt(sech(x)) + x/sech(x)^(7/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{5x\sqrt{\frac{1}{\cosh(x)}}}{21} - \frac{x}{\left(\frac{1}{\cosh(x)}\right)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(1/cosh(x))^(7/2) - (5\*x\*(1/cosh(x))^(1/2))/21, x)

[Out] -int((5\*x\*(1/cosh(x))^(1/2))/21 - x/(1/cosh(x))^(7/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \left( -\frac{21x}{\operatorname{sech}^{\frac{7}{2}}(x)} \right) dx + \int 5x\sqrt{\operatorname{sech}(x)} dx}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sech(x)\*\*(7/2)-5/21\*x\*sech(x)\*\*(1/2), x)

[Out] -(Integral(-21\*x/sech(x)\*\*(7/2), x) + Integral(5\*x\*sqrt(sech(x)), x))/21

$$3.98 \quad \int \left( \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{1}{3}x^2 \sqrt{\operatorname{sech}(x)} \right) dx$$

**Optimal.** Leaf size=66

$$\frac{2x^2 \sinh(x)}{3\sqrt{\operatorname{sech}(x)}} - \frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)} + \frac{16 \sinh(x)}{27\sqrt{\operatorname{sech}(x)}} - \frac{16}{27}i\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}F\left(\frac{ix}{2}\middle|2\right)$$

[Out]  $-8/9*x/\operatorname{sech}(x)^{(3/2)}+16/27*\sinh(x)/\operatorname{sech}(x)^{(1/2)}+2/3*x^2*\sinh(x)/\operatorname{sech}(x)^{(1/2)}-16/27*I*(\cosh(1/2*x)^2)^{(1/2)}/\cosh(1/2*x)*\operatorname{EllipticF}(I*\sinh(1/2*x),2^{(1/2)})*\cosh(x)^{(1/2)}*\operatorname{sech}(x)^{(1/2)}$

**Rubi [A]** time = 0.17, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {4188, 4189, 3769, 3771, 2641}

$$\frac{2x^2 \sinh(x)}{3\sqrt{\operatorname{sech}(x)}} - \frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)} + \frac{16 \sinh(x)}{27\sqrt{\operatorname{sech}(x)}} - \frac{16}{27}i\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}F\left(\frac{ix}{2}\middle|2\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2/\operatorname{Sech}[x]^{(3/2)} - (x^2*\operatorname{Sqrt}[\operatorname{Sech}[x]])/3, x]$

[Out]  $(-8*x)/(9*\operatorname{Sech}[x]^{(3/2)}) - ((16*I)/27)*\operatorname{Sqrt}[\operatorname{Cosh}[x]]*\operatorname{EllipticF}[(I/2)*x, 2]*\operatorname{Sqrt}[\operatorname{Sech}[x]] + (16*\operatorname{Sinh}[x])/(27*\operatorname{Sqrt}[\operatorname{Sech}[x]]) + (2*x^2*\operatorname{Sinh}[x])/(3*\operatorname{Sqrt}[\operatorname{Sech}[x]])$

**Rule 2641**

$\operatorname{Int}[1/\operatorname{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x\_Symbol] \rightarrow \operatorname{Simp}[(2*\operatorname{EllipticF}[(1*(c - \operatorname{Pi}/2 + d*x))/2, 2])/d, x] /; \operatorname{FreeQ}[\{c, d\}, x]$

**Rule 3769**

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Cos}[c + d*x]*(b*\operatorname{Csc}[c + d*x])^{(n + 1)})/(b*d^n), x] + \operatorname{Dist}[(n + 1)/(b^2*n), \operatorname{Int}[(b*\operatorname{Csc}[c + d*x])^{(n + 2)}, x], x] /; \operatorname{FreeQ}[\{b, c, d\}, x] \&\& \operatorname{LtQ}[n, -1] \&\& \operatorname{IntegerQ}[2*n]$

**Rule 3771**

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(b*\operatorname{Csc}[c + d*x])^{(n)}*\operatorname{Sin}[c + d*x]^n, \operatorname{Int}[1/\operatorname{Sin}[c + d*x]^n, x], x] /; \operatorname{FreeQ}[\{b, c, d\}, x] \&\&$

EqQ[n^2, 1/4]

Rule 4188

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(n + 1)/(b^2*n), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n + 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^n, x], x]
+ Simp[((c + d*x)^m*Cos[e + f*x]*(b*Csc[e + f*x])^(n + 1))/(b*f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
```

Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
+ f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \left( \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{1}{3} x^2 \sqrt{\operatorname{sech}(x)} \right) dx &= - \left( \frac{1}{3} \int x^2 \sqrt{\operatorname{sech}(x)} dx \right) + \int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)} dx \\
&= -\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)} + \frac{2x^2 \sinh(x)}{3\sqrt{\operatorname{sech}(x)}} + \frac{1}{3} \int x^2 \sqrt{\operatorname{sech}(x)} dx + \frac{8}{9} \int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(x)} dx - \frac{1}{3} \\
&= -\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)} + \frac{16 \sinh(x)}{27\sqrt{\operatorname{sech}(x)}} + \frac{2x^2 \sinh(x)}{3\sqrt{\operatorname{sech}(x)}} + \frac{8}{27} \int \sqrt{\operatorname{sech}(x)} dx \\
&= -\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)} + \frac{16 \sinh(x)}{27\sqrt{\operatorname{sech}(x)}} + \frac{2x^2 \sinh(x)}{3\sqrt{\operatorname{sech}(x)}} + \frac{1}{27} (8\sqrt{\cosh(x)} \sqrt{\operatorname{sech}(x)}) \int \\
&= -\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{16}{27} i \sqrt{\cosh(x)} F\left(\frac{ix}{2} \middle| 2\right) \sqrt{\operatorname{sech}(x)} + \frac{16 \sinh(x)}{27\sqrt{\operatorname{sech}(x)}} + \frac{2x^2 \sinh(x)}{3\sqrt{\operatorname{sech}(x)}}
\end{aligned}$$

**Mathematica** [A] time = 0.11, size = 55, normalized size = 0.83

$$\frac{1}{27} \sqrt{\operatorname{sech}(x)} \left( 9x^2 \sinh(2x) - 12x + 8 \sinh(2x) - 12x \cosh(2x) - 16i \sqrt{\cosh(x)} F\left(\frac{ix}{2} \middle| 2\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sech[x]^(3/2) - (x^2\*Sqrt[Sech[x]])/3,x]

[Out] (Sqrt[Sech[x]]\*(-12\*x - 12\*x\*Cosh[2\*x] - (16\*I)\*Sqrt[Cosh[x]]\*EllipticF[(I/2)\*x, 2] + 8\*Sinh[2\*x] + 9\*x^2\*Sinh[2\*x]))/27

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/sech(x)^(3/2)-1/3\*x^2\*sech(x)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{3}x^2\sqrt{\operatorname{sech}(x)} + \frac{x^2}{\operatorname{sech}(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/sech(x)^(3/2)-1/3\*x^2\*sech(x)^(1/2),x, algorithm="giac")

[Out] integrate(-1/3\*x^2\*sqrt(sech(x)) + x^2/sech(x)^(3/2), x)

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{sech}(x)^{\frac{3}{2}}} - \frac{x^2\sqrt{\operatorname{sech}(x)}}{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/sech(x)^(3/2)-1/3\*x^2\*sech(x)^(1/2),x)

[Out] int(x^2/sech(x)^(3/2)-1/3\*x^2\*sech(x)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{3}x^2\sqrt{\operatorname{sech}(x)} + \frac{x^2}{\operatorname{sech}(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/sech(x)^(3/2)-1/3\*x^2\*sech(x)^(1/2),x, algorithm="maxima")

[Out] integrate(-1/3\*x^2\*sqrt(sech(x)) + x^2/sech(x)^(3/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^2 \sqrt{\frac{1}{\cosh(x)}}}{3} - \frac{x^2}{\left(\frac{1}{\cosh(x)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(1/cosh(x))^(3/2) - (x^2*(1/cosh(x))^(1/2))/3, x)`

[Out] `-int((x^2*(1/cosh(x))^(1/2))/3 - x^2/(1/cosh(x))^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \left( -\frac{3x^2}{\operatorname{sech}^{\frac{3}{2}}(x)} \right) dx + \int x^2 \sqrt{\operatorname{sech}(x)} dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/sech(x)**(3/2)-1/3*x**2*sech(x)**(1/2), x)`

[Out] `-(Integral(-3*x**2/sech(x)**(3/2), x) + Integral(x**2*sqrt(sech(x)), x))/3`



### 3.99 $\int (c + dx)^3 (a + a \cosh(e + fx)) dx$

**Optimal.** Leaf size=89

$$\frac{6ad^2(c + dx) \sinh(e + fx)}{f^3} - \frac{3ad(c + dx)^2 \cosh(e + fx)}{f^2} + \frac{a(c + dx)^3 \sinh(e + fx)}{f} + \frac{a(c + dx)^4}{4d} - \frac{6ad^3 \cosh(e + fx)}{f^4}$$

[Out]  $1/4*a*(d*x+c)^4/d-6*a*d^3*\cosh(f*x+e)/f^4-3*a*d*(d*x+c)^2*\cosh(f*x+e)/f^2+6*a*d^2*(d*x+c)*\sinh(f*x+e)/f^3+a*(d*x+c)^3*\sinh(f*x+e)/f$

**Rubi [A]** time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3317, 3296, 2638}

$$\frac{6ad^2(c + dx) \sinh(e + fx)}{f^3} - \frac{3ad(c + dx)^2 \cosh(e + fx)}{f^2} + \frac{a(c + dx)^3 \sinh(e + fx)}{f} + \frac{a(c + dx)^4}{4d} - \frac{6ad^3 \cosh(e + fx)}{f^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^3*(a + a*\text{Cosh}[e + f*x]), x]$

[Out]  $(a*(c + d*x)^4)/(4*d) - (6*a*d^3*\text{Cosh}[e + f*x])/f^4 - (3*a*d*(c + d*x)^2*\text{Cosh}[e + f*x])/f^2 + (6*a*d^2*(c + d*x)*\text{Sinh}[e + f*x])/f^3 + (a*(c + d*x)^3*\text{Sinh}[e + f*x])/f$

#### Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

#### Rule 3296

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

#### Rule 3317

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, (a + b*\text{Sin}[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ (\text{EqQ}[n, 1] \ || \ \text{IGtQ}[m, 0] \ || \ \text{NeQ}[a^2 - b^2, 0])$

#### Rubi steps

$$\begin{aligned}
\int (c + dx)^3 (a + a \cosh(e + fx)) dx &= \int (a(c + dx)^3 + a(c + dx)^3 \cosh(e + fx)) dx \\
&= \frac{a(c + dx)^4}{4d} + a \int (c + dx)^3 \cosh(e + fx) dx \\
&= \frac{a(c + dx)^4}{4d} + \frac{a(c + dx)^3 \sinh(e + fx)}{f} - \frac{(3ad) \int (c + dx)^2 \sinh(e + fx) dx}{f} \\
&= \frac{a(c + dx)^4}{4d} - \frac{3ad(c + dx)^2 \cosh(e + fx)}{f^2} + \frac{a(c + dx)^3 \sinh(e + fx)}{f} + \frac{(6ad^2)}{f^3} \\
&= \frac{a(c + dx)^4}{4d} - \frac{3ad(c + dx)^2 \cosh(e + fx)}{f^2} + \frac{6ad^2(c + dx) \sinh(e + fx)}{f^3} + \frac{a(c + dx)^3 \sinh(e + fx)}{f} \\
&= \frac{a(c + dx)^4}{4d} - \frac{6ad^3 \cosh(e + fx)}{f^4} - \frac{3ad(c + dx)^2 \cosh(e + fx)}{f^2} + \frac{6ad^2(c + dx) \sinh(e + fx)}{f^3} + \frac{a(c + dx)^3 \sinh(e + fx)}{f}
\end{aligned}$$

**Mathematica [A]** time = 0.56, size = 122, normalized size = 1.37

$$a \left( -\frac{3d(c^2 f^2 + 2cdf^2 x + d^2(f^2 x^2 + 2)) \cosh(e + fx)}{f^4} + \frac{(c + dx)(c^2 f^2 + 2cdf^2 x + d^2(f^2 x^2 + 6)) \sinh(e + fx)}{f^3} \right) + \dots$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3\*(a + a\*Cosh[e + f\*x]),x]

[Out] a\*((x\*(4\*c^3 + 6\*c^2\*d\*x + 4\*c\*d^2\*x^2 + d^3\*x^3))/4 - (3\*d\*(c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(2 + f^2\*x^2))\*Cosh[e + f\*x])/f^4 + ((c + d\*x)\*(c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(6 + f^2\*x^2))\*Sinh[e + f\*x])/f^3)

**fricas [A]** time = 0.65, size = 168, normalized size = 1.89

$$\frac{ad^3 f^4 x^4 + 4acd^2 f^4 x^3 + 6ac^2 d f^4 x^2 + 4ac^3 f^4 x - 12(ad^3 f^2 x^2 + 2acd^2 f^2 x + ac^2 d f^2 + 2ad^3) \cosh(fx + e) + 4(ad^3 f^2 x^2 + 2acd^2 f^2 x + ac^2 d f^2 + 2ad^3) \sinh(fx + e)}{4f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+a\*cosh(f\*x+e)),x, algorithm="fricas")

[Out] 1/4\*(a\*d^3\*f^4\*x^4 + 4\*a\*c\*d^2\*f^4\*x^3 + 6\*a\*c^2\*d\*f^4\*x^2 + 4\*a\*c^3\*f^4\*x - 12\*(a\*d^3\*f^2\*x^2 + 2\*a\*c\*d^2\*f^2\*x + a\*c^2\*d\*f^2 + 2\*a\*d^3)\*cosh(f\*x + e) + 4\*(a\*d^3\*f^3\*x^3 + 3\*a\*c\*d^2\*f^3\*x^2 + a\*c^3\*f^3 + 6\*a\*c\*d^2\*f + 3\*(a\*c^2\*d\*f^3 + 2\*a\*d^3\*f)\*x)\*sinh(f\*x + e))/f^4

giac [B] time = 0.12, size = 260, normalized size = 2.92

$$\frac{1}{4} ad^3 x^4 + acd^2 x^3 + \frac{3}{2} ac^2 dx^2 + ac^3 x + \frac{(ad^3 f^3 x^3 + 3acd^2 f^3 x^2 + 3ac^2 d f^3 x - 3ad^3 f^2 x^2 + ac^3 f^3 - 6acd^2 f^2 x - 3ac^2 d f^2 x + 3ac^3 f^2 - 3acd^2 f^2 x - 3ac^3 f^2 x + 3ac^3 f^2 x - 3acd^2 f^2 x - 3ac^3 f^2 x)}{2f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+a\*cosh(f\*x+e)),x, algorithm="giac")

[Out]  $\frac{1}{4} a d^3 x^4 + a c d^2 x^3 + \frac{3}{2} a c^2 d x^2 + a c^3 x + \frac{1}{2} (a d^3 f^3 x^3 + 3 a c d^2 f^3 x^2 + 3 a c^2 d f^3 x - 3 a d^3 f^2 x^2 + a c^3 f^3 - 6 a c d^2 f^2 x - 3 a c^3 f^2 x + 3 a c^3 f^2 x - 3 a c d^2 f^2 x - 3 a c^3 f^2 x + 3 a c^3 f^2 x - 3 a c d^2 f^2 x - 3 a c^3 f^2 x) e^{(f x + e)} / f^4 - \frac{1}{2} (a d^3 f^3 x^3 + 3 a c d^2 f^3 x^2 + 3 a c^2 d f^3 x + 3 a d^3 f^2 x^2 + a c^3 f^3 + 6 a c d^2 f^2 x + 3 a c^3 f^2 x + 6 a d^3 f^3 x + 6 a c d^2 f^3 x + 6 a c^3 f^3) e^{-(f x + e)} / f^4$

maple [B] time = 0.06, size = 482, normalized size = 5.42

$$\frac{d^3 a (f x + e)^4}{4 f^3} + \frac{d^3 a \left( (f x + e)^3 \sinh(f x + e) - 3 (f x + e)^2 \cosh(f x + e) + 6 (f x + e) \sinh(f x + e) - 6 \cosh(f x + e) \right)}{f^3} - \frac{d^3 e a (f x + e)^3}{f^3} - \frac{3 d^3 e a \left( (f x + e)^2 \sinh(f x + e) - 2 (f x + e) \cosh(f x + e) + 2 \sinh(f x + e) \right)}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3\*(a+a\*cosh(f\*x+e)),x)

[Out]  $\frac{1}{f} \left( \frac{1}{4} \frac{d^3 a (f x + e)^4}{f^3} + \frac{d^3 a \left( (f x + e)^3 \sinh(f x + e) - 3 (f x + e)^2 \cosh(f x + e) + 6 (f x + e) \sinh(f x + e) - 6 \cosh(f x + e) \right)}{f^3} - \frac{d^3 e a (f x + e)^3}{f^3} - \frac{3 d^3 e a \left( (f x + e)^2 \sinh(f x + e) - 2 (f x + e) \cosh(f x + e) + 2 \sinh(f x + e) \right)}{f^3} + \frac{1}{f^2} \frac{d^2 c a (f x + e)^3 + 3 d^2 c a (f x + e)^2 \sinh(f x + e) - 2 d^2 c a (f x + e) \cosh(f x + e) + 2 d^2 c a \sinh(f x + e)}{f^2} + \frac{3}{2} \frac{d^3 e^2 a (f x + e)^2 + 3 d^3 e^2 a (f x + e) \sinh(f x + e) - 3 d^3 e^2 a (f x + e) \cosh(f x + e) + 3 d^3 e^2 a \sinh(f x + e)}{f^2} + \frac{3}{2} \frac{d^3 e^3 a (f x + e) + 3 d^3 e^3 a \sinh(f x + e) - 3 d^3 e^3 a \cosh(f x + e) + 3 d^3 e^3 a \sinh(f x + e)}{f^2} + \frac{3 d^2 e}{f c^2} \frac{d^2 c a (f x + e) + 3 d^2 c a \sinh(f x + e) - 3 d^2 c a \cosh(f x + e) + 3 d^2 c a \sinh(f x + e)}{f^2} + \frac{3 d^2 e}{f c^2} \frac{d^2 c a \sinh(f x + e) + 3 d^2 c a \cosh(f x + e) - 3 d^2 c a \sinh(f x + e) + 3 d^2 c a \cosh(f x + e)}{f^2} \right)$

maxima [B] time = 2.31, size = 237, normalized size = 2.66

$$\frac{1}{4} ad^3 x^4 + acd^2 x^3 + \frac{3}{2} ac^2 dx^2 + ac^3 x + \frac{3}{2} ac^2 d \left( \frac{(f x e^e - e^e) e^{(f x)}}{f^2} - \frac{(f x + 1) e^{(-f x - e)}}{f^2} \right) + \frac{3}{2} acd^2 \left( \frac{(f^2 x^2 e^e - 2 f x e^e + 2 e^e)}{f^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+a\*cosh(f\*x+e)),x, algorithm="maxima")

[Out]  $\frac{1}{4}a^3d^3x^4 + ac^3d^2x^3 + \frac{3}{2}a^2c^2d^2x^2 + ac^3x + \frac{3}{2}a^2c^2d^2 \left( \frac{(f^2x^2e^e - e^e)e^{fx}}{f^2} - \frac{(f^2x^2 + 1)e^{-(fx - e)}}{f^2} \right) + \frac{3}{2}a^2c^2d^2 \left( \frac{(f^2x^2e^e - 2fx^2e^e + 2e^e)e^{fx}}{f^3} - \frac{(f^2x^2 + 2fx + 2)e^{-(fx - e)}}{f^3} \right) + \frac{1}{2}a^2d^3 \left( \frac{(f^3x^3e^e - 3f^2x^2e^e + 6fx^2e^e - 6e^e)e^{fx}}{f^4} - \frac{(f^3x^3 + 3f^2x^2 + 6fx + 6)e^{-(fx - e)}}{f^4} \right) + ac^3 \frac{\sinh(fx + e)}{f}$

**mupad [B]** time = 1.02, size = 187, normalized size = 2.10

$$\frac{\sinh(e + fx) (ac^3f^2 + 6acd^2)}{f^3} - \frac{3 \cosh(e + fx) (ac^2df^2 + 2ad^3)}{f^4} + \frac{ad^3x^4}{4} + ac^3x + \frac{3x \sinh(e + fx) (ac^2d)}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cosh(e + f*x))*(c + d*x)^3,x)`

[Out]  $(\sinh(e + fx)(ac^3f^2 + 6a^2cd^2))/f^3 - (3 \cosh(e + fx)(2a^2d^3 + a^2c^2d^2f^2))/f^4 + (a^2d^3x^4)/4 + ac^3x + (3x \sinh(e + fx)(2a^2d^3 + a^2c^2d^2f^2))/f^3 + (3a^2c^2d^2x^2)/2 + ac^3d^2x^3 - (3a^2d^3x^2 \cosh(e + fx))/f^2 + (a^2d^3x^3 \sinh(e + fx))/f - (6a^2cd^2x \cosh(e + fx))/f^2 + (3a^2cd^2x^2 \sinh(e + fx))/f$

**sympy [A]** time = 1.38, size = 264, normalized size = 2.97

$$\left\{ \begin{array}{l} ac^3x + \frac{ac^3 \sinh(e+fx)}{f} + \frac{3ac^2dx^2}{2} + \frac{3ac^2dx \sinh(e+fx)}{f} - \frac{3ac^2d \cosh(e+fx)}{f^2} + acd^2x^3 + \frac{3acd^2x^2 \sinh(e+fx)}{f} - \frac{6acd^2x \cosh(e+fx)}{f^2} + \\ (a \cosh(e) + a) \left( c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)**3*(a+a*cosh(f*x+e)),x)`

[Out] `Piecewise((a*c**3*x + a*c**3*sinh(e + f*x)/f + 3*a*c**2*d*x**2/2 + 3*a*c**2*d*x**2*sinh(e + f*x)/f - 3*a*c**2*d*cosh(e + f*x)/f**2 + a*c*d**2*x**3 + 3*a*c*d**2*x**2*sinh(e + f*x)/f - 6*a*c*d**2*x*cosh(e + f*x)/f**2 + 6*a*c*d**2*sinh(e + f*x)/f**3 + a*d**3*x**4/4 + a*d**3*x**3*sinh(e + f*x)/f - 3*a*d**3*x**2*cosh(e + f*x)/f**2 + 6*a*d**3*x*sinh(e + f*x)/f**3 - 6*a*d**3*cosh(e + f*x)/f**4, Ne(f, 0)), ((a*cosh(e) + a)*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), True))`

### 3.100 $\int (c + dx)^2 (a + a \cosh(e + fx)) dx$

**Optimal.** Leaf size=67

$$-\frac{2ad(c + dx) \cosh(e + fx)}{f^2} + \frac{a(c + dx)^2 \sinh(e + fx)}{f} + \frac{a(c + dx)^3}{3d} + \frac{2ad^2 \sinh(e + fx)}{f^3}$$

[Out]  $1/3*a*(d*x+c)^3/d-2*a*d*(d*x+c)*\cosh(f*x+e)/f^2+2*a*d^2*\sinh(f*x+e)/f^3+a*(d*x+c)^2*\sinh(f*x+e)/f$

**Rubi [A]** time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3317, 3296, 2637}

$$-\frac{2ad(c + dx) \cosh(e + fx)}{f^2} + \frac{a(c + dx)^2 \sinh(e + fx)}{f} + \frac{a(c + dx)^3}{3d} + \frac{2ad^2 \sinh(e + fx)}{f^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^2*(a + a*\text{Cosh}[e + f*x]), x]$

[Out]  $(a*(c + d*x)^3)/(3*d) - (2*a*d*(c + d*x)*\text{Cosh}[e + f*x])/f^2 + (2*a*d^2*\text{Sinh}[e + f*x])/f^3 + (a*(c + d*x)^2*\text{Sinh}[e + f*x])/f$

#### Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$   
FreeQ[{c, d}, x]

#### Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\cos[e + f*x], x], x] /;$  FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3317

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, (a + b*\sin[e + f*x])^n, x], x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

#### Rubi steps

$$\begin{aligned}
\int (c + dx)^2(a + a \cosh(e + fx)) dx &= \int (a(c + dx)^2 + a(c + dx)^2 \cosh(e + fx)) dx \\
&= \frac{a(c + dx)^3}{3d} + a \int (c + dx)^2 \cosh(e + fx) dx \\
&= \frac{a(c + dx)^3}{3d} + \frac{a(c + dx)^2 \sinh(e + fx)}{f} - \frac{(2ad) \int (c + dx) \sinh(e + fx) dx}{f} \\
&= \frac{a(c + dx)^3}{3d} - \frac{2ad(c + dx) \cosh(e + fx)}{f^2} + \frac{a(c + dx)^2 \sinh(e + fx)}{f} + \frac{(2ad^2)}{f^3} \\
&= \frac{a(c + dx)^3}{3d} - \frac{2ad(c + dx) \cosh(e + fx)}{f^2} + \frac{2ad^2 \sinh(e + fx)}{f^3} + \frac{a(c + dx)^2}{f}
\end{aligned}$$

**Mathematica [A]** time = 0.33, size = 80, normalized size = 1.19

$$a \left( \frac{(c^2 f^2 + 2cd f^2 x + d^2 (f^2 x^2 + 2)) \sinh(e + fx)}{f^3} + c^2 x - \frac{2d(c + dx) \cosh(e + fx)}{f^2} + cd x^2 + \frac{d^2 x^3}{3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2\*(a + a\*Cosh[e + f\*x]),x]

[Out] a\*(c^2\*x + c\*d\*x^2 + (d^2\*x^3)/3 - (2\*d\*(c + d\*x)\*Cosh[e + f\*x])/f^2 + ((c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(2 + f^2\*x^2))\*Sinh[e + f\*x])/f^3)

**fricas [A]** time = 0.61, size = 102, normalized size = 1.52

$$\frac{ad^2 f^3 x^3 + 3 acd f^3 x^2 + 3 ac^2 f^3 x - 6(ad^2 f x + acd f) \cosh(fx + e) + 3(ad^2 f^2 x^2 + 2 acd f^2 x + ac^2 f^2 + 2 ad^2) \sinh(fx + e)}{3 f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+a\*cosh(f\*x+e)),x, algorithm="fricas")

[Out] 1/3\*(a\*d^2\*f^3\*x^3 + 3\*a\*c\*d\*f^3\*x^2 + 3\*a\*c^2\*f^3\*x - 6\*(a\*d^2\*f\*x + a\*c\*d\*f)\*cosh(f\*x + e) + 3\*(a\*d^2\*f^2\*x^2 + 2\*a\*c\*d\*f^2\*x + a\*c^2\*f^2 + 2\*a\*d^2)\*sinh(f\*x + e))/f^3

**giac [B]** time = 0.12, size = 148, normalized size = 2.21

$$\frac{1}{3} ad^2 x^3 + acd x^2 + ac^2 x + \frac{(ad^2 f^2 x^2 + 2 acd f^2 x + ac^2 f^2 - 2 ad^2 f x - 2 acd f + 2 ad^2) e^{(fx+e)}}{2 f^3} - \frac{(ad^2 f^2 x^2 + 2 acd f^2 x + ac^2 f^2 + 2 ad^2) \sinh(fx + e)}{2 f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+a\*cosh(f\*x+e)),x, algorithm="giac")

[Out]  $\frac{1}{3}a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + \frac{1}{2}*(a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2 - 2*a*d^2*f*x - 2*a*c*d*f + 2*a*d^2)*e^{(f*x + e)}/f^3 - \frac{1}{2}*(a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2 + 2*a*d^2*f*x + 2*a*c*d*f + 2*a*d^2)*e^{(-f*x - e)}/f^3$

**maple [B]** time = 0.07, size = 240, normalized size = 3.58

$$\frac{d^2 a (f x + e)^3}{3 f^2} + \frac{d^2 a \left( (f x + e)^2 \sinh(f x + e) - 2(f x + e) \cosh(f x + e) + 2 \sinh(f x + e) \right)}{f^2} - \frac{d^2 e a (f x + e)^2}{f^2} - \frac{2 d^2 e a \left( (f x + e) \sinh(f x + e) - \cosh(f x + e) \right)}{f^2} + \frac{d c a (f x + e)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*(a+a\*cosh(f\*x+e)),x)

[Out]  $\frac{1}{f} * \left( \frac{1}{3} / f^2 * d^2 * a * (f * x + e)^3 + \frac{1}{f^2} * d^2 * a * \left( (f * x + e)^2 * \sinh(f * x + e) - 2 * (f * x + e) * \cosh(f * x + e) + 2 * \sinh(f * x + e) \right) - \frac{1}{f^2} * d^2 * e * a * (f * x + e)^2 - \frac{2}{f^2} * d^2 * e * a * \left( (f * x + e) * \sinh(f * x + e) - \cosh(f * x + e) \right) + \frac{1}{f} * d * c * a * (f * x + e)^2 + \frac{2}{f} * d * c * a * \left( (f * x + e) * \sinh(f * x + e) - \cosh(f * x + e) \right) + d^2 * e^2 / f^2 * a * (f * x + e) + d^2 * e^2 / f^2 * a * \sinh(f * x + e) - 2 * d * e / f * c * a * (f * x + e) - 2 * d * e / f * c * a * \sinh(f * x + e) + a * c^2 * (f * x + e) + a * c^2 * \sinh(f * x + e) \right)$

**maxima [B]** time = 1.05, size = 141, normalized size = 2.10

$$\frac{1}{3} a d^2 x^3 + a c d x^2 + a c^2 x + a c d \left( \frac{(f x e^e - e^e) e^{(f x)}}{f^2} - \frac{(f x + 1) e^{(-f x - e)}}{f^2} \right) + \frac{1}{2} a d^2 \left( \frac{(f^2 x^2 e^e - 2 f x e^e + 2 e^e) e^{(f x)}}{f^3} - \frac{(f^2 x^2 + 2 f x + 2) e^{(-f x - e)}}{f^3} \right) + a c^2 \sinh(f x + e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+a\*cosh(f\*x+e)),x, algorithm="maxima")

[Out]  $\frac{1}{3}a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + a*c*d*((f*x*e^e - e^e)*e^{(f*x)}/f^2 - (f*x + 1)*e^{(-f*x - e)}/f^2) + \frac{1}{2}a*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^{(f*x)}/f^3 - (f^2*x^2 + 2*f*x + 2)*e^{(-f*x - e)}/f^3) + a*c^2*\sinh(f*x + e)/f$

**mupad [B]** time = 0.14, size = 112, normalized size = 1.67

$$\frac{2 a d^2 \sinh(e + f x) - \frac{a f (6 x \cosh(e + f x) d^2 + 6 c \cosh(e + f x) d)}{3} + \frac{a f^2 (3 c^2 \sinh(e + f x) + 3 d^2 x^2 \sinh(e + f x) + 6 c d x \sinh(e + f x))}{3}}{f^3} + a (3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))*(c + d*x)^2,x)
```

```
[Out] (2*a*d^2*sinh(e + f*x) - (a*f*(6*d^2*x*cosh(e + f*x) + 6*c*d*cosh(e + f*x))
)/3 + (a*f^2*(3*c^2*sinh(e + f*x) + 3*d^2*x^2*sinh(e + f*x) + 6*c*d*x*sinh(
e + f*x)))/3)/f^3 + (a*(3*c^2*x + d^2*x^3 + 3*c*d*x^2))/3
```

**sympy [A]** time = 0.64, size = 151, normalized size = 2.25

$$\left\{ \begin{array}{l} ac^2x + \frac{ac^2 \sinh(e+fx)}{f} + acdx^2 + \frac{2acdx \sinh(e+fx)}{f} - \frac{2acd \cosh(e+fx)}{f^2} + \frac{ad^2x^3}{3} + \frac{ad^2x^2 \sinh(e+fx)}{f} - \frac{2ad^2x \cosh(e+fx)}{f^2} + \frac{2ad^2 \sinh(e+fx)}{f^3} \\ (a \cosh(e) + a) \left( c^2x + cdx^2 + \frac{d^2x^3}{3} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+a*cosh(f*x+e)),x)
```

```
[Out] Piecewise((a*c**2*x + a*c**2*sinh(e + f*x)/f + a*c*d*x**2 + 2*a*c*d*x*sinh(
e + f*x)/f - 2*a*c*d*cosh(e + f*x)/f**2 + a*d**2*x**3/3 + a*d**2*x**2*sinh(
e + f*x)/f - 2*a*d**2*x*cosh(e + f*x)/f**2 + 2*a*d**2*sinh(e + f*x)/f**3, N
e(f, 0)), ((a*cosh(e) + a)*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```



### 3.101 $\int (c + dx)(a + a \cosh(e + fx)) dx$

Optimal. Leaf size=45

$$\frac{a(c + dx) \sinh(e + fx)}{f} + \frac{a(c + dx)^2}{2d} - \frac{ad \cosh(e + fx)}{f^2}$$

[Out]  $1/2*a*(d*x+c)^2/d-a*d*\cosh(f*x+e)/f^2+a*(d*x+c)*\sinh(f*x+e)/f$

**Rubi [A]** time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {3317, 3296, 2638}

$$\frac{a(c + dx) \sinh(e + fx)}{f} + \frac{a(c + dx)^2}{2d} - \frac{ad \cosh(e + fx)}{f^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)\*(a + a\*Cosh[e + f\*x]),x]

[Out]  $(a*(c + d*x)^2)/(2*d) - (a*d*Cosh[e + f*x])/f^2 + (a*(c + d*x)*Sinh[e + f*x])/f$

#### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] := -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3317

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Sinh[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

Rubi steps

$$\begin{aligned}
\int (c + dx)(a + a \cosh(e + fx)) dx &= \int (a(c + dx) + a(c + dx) \cosh(e + fx)) dx \\
&= \frac{a(c + dx)^2}{2d} + a \int (c + dx) \cosh(e + fx) dx \\
&= \frac{a(c + dx)^2}{2d} + \frac{a(c + dx) \sinh(e + fx)}{f} - \frac{(ad) \int \sinh(e + fx) dx}{f} \\
&= \frac{a(c + dx)^2}{2d} - \frac{ad \cosh(e + fx)}{f^2} + \frac{a(c + dx) \sinh(e + fx)}{f}
\end{aligned}$$

**Mathematica** [A] time = 0.22, size = 52, normalized size = 1.16

$$\frac{a(-2(e + fx)(-2cf + de - dfx) + 4f(c + dx) \sinh(e + fx) - 4d \cosh(e + fx))}{4f^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)\*(a + a\*Cosh[e + f\*x]),x]

[Out] (a\*(-2\*(e + f\*x)\*(d\*e - 2\*c\*f - d\*f\*x) - 4\*d\*Cosh[e + f\*x] + 4\*f\*(c + d\*x)\*Sinh[e + f\*x]))/(4\*f^2)

**fricas** [A] time = 0.52, size = 51, normalized size = 1.13

$$\frac{adf^2x^2 + 2acf^2x - 2ad \cosh(fx + e) + 2(adfx + acf) \sinh(fx + e)}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+a\*cosh(f\*x+e)),x, algorithm="fricas")

[Out] 1/2\*(a\*d\*f^2\*x^2 + 2\*a\*c\*f^2\*x - 2\*a\*d\*cosh(f\*x + e) + 2\*(a\*d\*f\*x + a\*c\*f)\*sinh(f\*x + e))/f^2

**giac** [A] time = 0.12, size = 66, normalized size = 1.47

$$\frac{1}{2} adx^2 + acx + \frac{(adfx + acf - ad)e^{(fx+e)}}{2f^2} - \frac{(adfx + acf + ad)e^{(-fx-e)}}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+a\*cosh(f\*x+e)),x, algorithm="giac")

[Out]  $\frac{1}{2}ad^2x^2 + acx + \frac{1}{2}(adf^2 + acf - ad)e^{fx+e}/f^2 - \frac{1}{2}(adf^2 + acf + ad)e^{-fx-e}/f^2$

**maple** [B] time = 0.07, size = 91, normalized size = 2.02

$$\frac{\frac{da(fx+e)^2}{2f} + \frac{da((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f} - \frac{dea(fx+e)}{f} - \frac{dea\sinh(fx+e)}{f} + ca(fx+e) + ac\sinh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)*(a+a*cosh(f*x+e)),x)`

[Out]  $\frac{1}{f}(\frac{1}{2}fda(fx+e)^2 + \frac{1}{f}d^2a((fx+e)\sinh(fx+e)-\cosh(fx+e)) - d^2e/f^2a(fx+e) - d^2e/f^2a\sinh(fx+e) + c^2a(fx+e) + ac\sinh(fx+e))$

**maxima** [A] time = 0.44, size = 66, normalized size = 1.47

$$\frac{1}{2}adx^2 + acx + \frac{1}{2}ad\left(\frac{(fxe^e - e^e)e^{fx}}{f^2} - \frac{(fx+1)e^{-fx-e}}{f^2}\right) + \frac{ac\sinh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*(a+a*cosh(f*x+e)),x, algorithm="maxima")`

[Out]  $\frac{1}{2}ad^2x^2 + acx + \frac{1}{2}ad^2((fx+1)e^e - e^e)e^{fx}/f^2 - (fx+1)e^{-fx-e}/f^2 + ac\sinh(fx+e)/f$

**mupad** [B] time = 0.08, size = 53, normalized size = 1.18

$$\frac{\frac{af(2c\sinh(e+fx)+2dx\sinh(e+fx))}{2} - ad\cosh(e+fx)}{f^2} + \frac{a(dx^2+2cx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cosh(e + f*x))*(c + d*x),x)`

[Out]  $((af(2c\sinh(e+fx) + 2d^2x\sinh(e+fx)))/2 - ad^2\cosh(e+fx))/f^2 + (a(2c^2x + d^2x^2))/2$

**sympy** [A] time = 0.28, size = 68, normalized size = 1.51

$$\begin{cases} acx + \frac{ac\sinh(e+fx)}{f} + \frac{adx^2}{2} + \frac{adx\sinh(e+fx)}{f} - \frac{ad\cosh(e+fx)}{f^2} & \text{for } f \neq 0 \\ (a\cosh(e) + a)\left(cx + \frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x + a*c*sinh(e + f*x)/f + a*d*x**2/2 + a*d*x*sinh(e + f*x)/f  
- a*d*cosh(e + f*x)/f**2, Ne(f, 0)), ((a*cosh(e) + a)*(c*x + d*x**2/2), Tr  
ue))
```

$$3.102 \quad \int \frac{a+a \cosh(e+fx)}{c+dx} dx$$

Optimal. Leaf size=64

$$\frac{a \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d} + \frac{a \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d} + \frac{a \log(c + dx)}{d}$$

[Out] a\*Chi(c\*f/d+f\*x)\*cosh(-e+c\*f/d)/d+a\*ln(d\*x+c)/d-a\*Shi(c\*f/d+f\*x)\*sinh(-e+c\*f/d)/d

**Rubi [A]** time = 0.14, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3317, 3303, 3298, 3301}

$$\frac{a \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d} + \frac{a \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d} + \frac{a \log(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + a\*Cosh[e + f\*x])/(c + d\*x),x]

[Out] (a\*Cosh[e - (c\*f)/d]\*CoshIntegral[(c\*f)/d + f\*x])/d + (a\*Log[c + d\*x])/d + (a\*Sinh[e - (c\*f)/d]\*SinhIntegral[(c\*f)/d + f\*x])/d

#### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

#### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

#### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{a + a \cosh(e + fx)}{c + dx} dx &= \int \left( \frac{a}{c + dx} + \frac{a \cosh(e + fx)}{c + dx} \right) dx \\
&= \frac{a \log(c + dx)}{d} + a \int \frac{\cosh(e + fx)}{c + dx} dx \\
&= \frac{a \log(c + dx)}{d} + \left( a \cosh \left( e - \frac{cf}{d} \right) \right) \int \frac{\cosh \left( \frac{cf}{d} + fx \right)}{c + dx} dx + \left( a \sinh \left( e - \frac{cf}{d} \right) \right) \int \frac{\sinh \left( \frac{cf}{d} + fx \right)}{c + dx} dx \\
&= \frac{a \cosh \left( e - \frac{cf}{d} \right) \text{Chi} \left( \frac{cf}{d} + fx \right)}{d} + \frac{a \log(c + dx)}{d} + \frac{a \sinh \left( e - \frac{cf}{d} \right) \text{Shi} \left( \frac{cf}{d} + fx \right)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 54, normalized size = 0.84

$$\frac{a \left( \text{Chi} \left( f \left( \frac{c}{d} + x \right) \right) \cosh \left( e - \frac{cf}{d} \right) + \sinh \left( e - \frac{cf}{d} \right) \text{Shi} \left( f \left( \frac{c}{d} + x \right) \right) + \log(c + dx) \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[e + f*x])/(c + d*x),x]
```

```
[Out] (a*(Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + Log[c + d*x] + Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)])/d
```

**fricas [A]** time = 0.44, size = 111, normalized size = 1.73

$$\frac{\left( a \text{Ei} \left( \frac{dfx+cf}{d} \right) + a \text{Ei} \left( -\frac{dfx+cf}{d} \right) \right) \cosh \left( -\frac{de-cf}{d} \right) + 2 a \log(dx + c) - \left( a \text{Ei} \left( \frac{dfx+cf}{d} \right) - a \text{Ei} \left( -\frac{dfx+cf}{d} \right) \right) \sinh \left( -\frac{de-cf}{d} \right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((a*Ei((d*f*x + c*f)/d) + a*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) + 2*a*log(d*x + c) - (a*Ei((d*f*x + c*f)/d) - a*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d)/d
```

**giac** [A] time = 0.14, size = 69, normalized size = 1.08

$$\frac{a \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right) e^{\left(\frac{cf}{d}-e\right)} + a \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) e^{\left(-\frac{cf}{d}+e\right)} + 2a \log(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))/(d\*x+c),x, algorithm="giac")

[Out] 1/2\*(a\*Ei(-(d\*f\*x + c\*f)/d)\*e^(c\*f/d - e) + a\*Ei((d\*f\*x + c\*f)/d)\*e^(-c\*f/d + e) + 2\*a\*log(d\*x + c))/d

**maple** [A] time = 0.14, size = 94, normalized size = 1.47

$$\frac{a \ln(dx+c)}{d} - \frac{a e^{\frac{cf-de}{d}} \operatorname{Ei}\left(1, fx+e+\frac{cf-de}{d}\right)}{2d} - \frac{a e^{-\frac{cf-de}{d}} \operatorname{Ei}\left(1, -fx-e-\frac{cf-de}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*cosh(f\*x+e))/(d\*x+c),x)

[Out] a\*ln(d\*x+c)/d-1/2\*a/d\*exp((c\*f-d\*e)/d)\*Ei(1,f\*x+e+(c\*f-d\*e)/d)-1/2\*a/d\*exp(-(c\*f-d\*e)/d)\*Ei(1,-f\*x-e-(c\*f-d\*e)/d)

**maxima** [A] time = 0.45, size = 70, normalized size = 1.09

$$-\frac{1}{2} a \left( \frac{e^{\left(-e+\frac{cf}{d}\right)} E_1\left(\frac{(dx+c)f}{d}\right)}{d} + \frac{e^{\left(e-\frac{cf}{d}\right)} E_1\left(-\frac{(dx+c)f}{d}\right)}{d} \right) + \frac{a \log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))/(d\*x+c),x, algorithm="maxima")

[Out] -1/2\*a\*(e^(-e + c\*f/d)\*exp\_integral\_e(1, (d\*x + c)\*f/d)/d + e^(e - c\*f/d)\*exp\_integral\_e(1, -(d\*x + c)\*f/d)/d) + a\*log(d\*x + c)/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + a \cosh(e + fx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))/(c + d*x),x)
```

```
[Out] int((a + a*cosh(e + f*x))/(c + d*x), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left( \int \frac{\cosh(e + fx)}{c + dx} dx + \int \frac{1}{c + dx} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c),x)
```

```
[Out] a*(Integral(cosh(e + f*x)/(c + d*x), x) + Integral(1/(c + d*x), x))
```



$$3.103 \quad \int \frac{a+a \cosh(e+fx)}{(c+dx)^2} dx$$

**Optimal.** Leaf size=87

$$\frac{af \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{af \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^2} - \frac{a \cosh(e+fx)}{d(c+dx)} - \frac{a}{d(c+dx)}$$

[Out]  $-a/d/(d*x+c)-a*\cosh(f*x+e)/d/(d*x+c)+a*f*\cosh(-e+c*f/d)*\operatorname{Shi}(c*f/d+f*x)/d^2-a*f*\operatorname{Chi}(c*f/d+f*x)*\sinh(-e+c*f/d)/d^2$

**Rubi [A]** time = 0.17, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3317, 3297, 3303, 3298, 3301}

$$\frac{af \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{af \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^2} - \frac{a \cosh(e+fx)}{d(c+dx)} - \frac{a}{d(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + a*\operatorname{Cosh}[e + f*x])/(c + d*x)^2, x]$

[Out]  $-(a/(d*(c + d*x))) - (a*\operatorname{Cosh}[e + f*x])/(d*(c + d*x)) + (a*f*\operatorname{CoshIntegral}[(c*f)/d + f*x]*\operatorname{Sinh}[e - (c*f)/d])/d^2 + (a*f*\operatorname{Cosh}[e - (c*f)/d]*\operatorname{SinhIntegral}[(c*f)/d + f*x])/d^2$

Rule 3297

$\operatorname{Int}[(c_. + (d_.)*(x_.))^{(m_.)*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] := \operatorname{Simp}[(c + d*x)^{(m + 1)*\sin[e + f*x]}/(d*(m + 1)), x] - \operatorname{Dist}[f/(d*(m + 1)), \operatorname{Int}[(c + d*x)^{(m + 1)*\cos[e + f*x]}, x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x \&\& \operatorname{LtQ}[m, -1]$

Rule 3298

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] := \operatorname{Simp}[(I*\operatorname{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x \&\& \operatorname{EqQ}[d*e - c*f*fz*I, 0]$

Rule 3301

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] := \operatorname{Simp}[\operatorname{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x \&\& \operatorname{EqQ}[d*(e - \operatorname{Pi}/2) - c*f*fz*I, 0]$

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a + a \cosh(e + fx)}{(c + dx)^2} dx &= \int \left( \frac{a}{(c + dx)^2} + \frac{a \cosh(e + fx)}{(c + dx)^2} \right) dx \\
&= -\frac{a}{d(c + dx)} + a \int \frac{\cosh(e + fx)}{(c + dx)^2} dx \\
&= -\frac{a}{d(c + dx)} - \frac{a \cosh(e + fx)}{d(c + dx)} + \frac{(af) \int \frac{\sinh(e+fx)}{c+dx} dx}{d} \\
&= -\frac{a}{d(c + dx)} - \frac{a \cosh(e + fx)}{d(c + dx)} + \frac{\left(af \cosh\left(e - \frac{cf}{d}\right)\right) \int \frac{\sinh\left(\frac{cf}{d} + fx\right)}{c+dx} dx}{d} + \frac{\left(af \sinh\left(e - \frac{cf}{d}\right)\right) \int \frac{\cosh\left(\frac{cf}{d} + fx\right)}{c+dx} dx}{d} \\
&= -\frac{a}{d(c + dx)} - \frac{a \cosh(e + fx)}{d(c + dx)} + \frac{af \operatorname{Chi}\left(\frac{cf}{d} + fx\right) \sinh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{af \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(\frac{cf}{d} + fx\right)}{d^2}
\end{aligned}$$

**Mathematica** [A] time = 0.32, size = 68, normalized size = 0.78

$$\frac{a \left( f \operatorname{Chi}\left(f \left(\frac{c}{d} + x\right)\right) \sinh\left(e - \frac{cf}{d}\right) + f \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(f \left(\frac{c}{d} + x\right)\right) - \frac{d(\cosh(e+fx)+1)}{c+dx} \right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] (a*(-((d*(1 + Cosh[e + f*x]))/(c + d*x)) + f*CoshIntegral[f*(c/d + x)]*Sinh
[e - (c*f)/d] + f*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/d^2
```

**fricas** [A] time = 0.59, size = 162, normalized size = 1.86

$$\frac{2ad \cosh(fx + e) + 2ad - \left( (adf x + acf) \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) - (adf x + acf) \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right) \right) \cosh\left(-\frac{de-cf}{d}\right) + \left( (adf x + \dots \right)}{2(d^3x + cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))/(d\*x+c)^2,x, algorithm="fricas")

[Out]  $-1/2*(2*a*d*\cosh(f*x + e) + 2*a*d - ((a*d*f*x + a*c*f)*\operatorname{Ei}((d*f*x + c*f)/d) - (a*d*f*x + a*c*f)*\operatorname{Ei}(-(d*f*x + c*f)/d))*\cosh(-(d*e - c*f)/d) + ((a*d*f*x + a*c*f)*\operatorname{Ei}((d*f*x + c*f)/d) + (a*d*f*x + a*c*f)*\operatorname{Ei}(-(d*f*x + c*f)/d))*\sinh(-(d*e - c*f)/d))/(d^3*x + c*d^2)$

**giac** [B] time = 0.16, size = 683, normalized size = 7.85

$$-\frac{1}{2}a \left( \frac{\left( (dx+c) \left( \frac{cf}{dx+c} - f - \frac{de}{dx+c} \right) f^2 \operatorname{Ei} \left( \frac{(dx+c) \left( \frac{cf}{dx+c} - f - \frac{de}{dx+c} \right) - cf + de}{d} \right) e^{\left( \frac{cf-de}{d} \right)} - cf^3 \operatorname{Ei} \left( \frac{(dx+c) \left( \frac{cf}{dx+c} - f - \frac{de}{dx+c} \right) - cf + de}{d} \right) e^{\left( \frac{cf-de}{d} \right)}}{\left( (dx+c) d^4 \left( \frac{cf}{dx+c} - f - \frac{de}{dx+c} \right) - cd^4 f + \dots \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))/(d\*x+c)^2,x, algorithm="giac")

[Out]  $-1/2*a*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*\operatorname{Ei}(((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^{((c*f - d*e)/d)} - c*f^3*\operatorname{Ei}(((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^{((c*f - d*e)/d)} + d*f^2*\operatorname{Ei}(((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^{((c*f - d*e)/d + 1)} - d*f^2*e^{((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))/d)}*d^2/(((d*x + c)*d^4*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*d^4*f + d^5*e)*f) - ((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*\operatorname{Ei}(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^{-((c*f - d*e)/d)} - c*f^3*\operatorname{Ei}(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^{-((c*f - d*e)/d)} + d*f^2*\operatorname{Ei}(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))/d)}*d^2/(((d*x + c)*d^4*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*d^4*f + d^5*e)*f)) - a/((d*x + c)*d)$

**maple [A]** time = 0.14, size = 149, normalized size = 1.71

$$-\frac{a}{d(dx+c)} - \frac{fae^{-fx-e}}{2d(dfx+cf)} + \frac{fae^{\frac{cf-de}{d}} \operatorname{Ei}\left(1, fx+e+\frac{cf-de}{d}\right)}{2d^2} - \frac{afe^{fx+e}}{2d^2\left(\frac{cf}{d}+fx\right)} - \frac{afe^{-\frac{cf-de}{d}} \operatorname{Ei}\left(1, -fx-e-\frac{cf-de}{d}\right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*cosh(f\*x+e))/(d\*x+c)^2,x)

[Out] -a/d/(d\*x+c)-1/2\*f\*a\*exp(-f\*x-e)/d/(d\*f\*x+c\*f)+1/2\*f\*a/d^2\*exp((c\*f-d\*e)/d)\*Ei(1,f\*x+e+(c\*f-d\*e)/d)-1/2\*a\*f/d^2\*exp(f\*x+e)/(c\*f/d+f\*x)-1/2\*a\*f/d^2\*exp(-(c\*f-d\*e)/d)\*Ei(1,-f\*x-e-(c\*f-d\*e)/d)

**maxima [A]** time = 0.42, size = 87, normalized size = 1.00

$$-\frac{1}{2}a\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d} + \frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right) - \frac{a}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))/(d\*x+c)^2,x, algorithm="maxima")

[Out] -1/2\*a\*(e^(-e + c\*f/d)\*exp\_integral\_e(2, (d\*x + c)\*f/d)/((d\*x + c)\*d) + e^(e - c\*f/d)\*exp\_integral\_e(2, -(d\*x + c)\*f/d)/((d\*x + c)\*d)) - a/(d^2\*x + c\*d)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + a \cosh(e + f x)}{(c + d x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cosh(e + f\*x))/(c + d\*x)^2,x)

[Out] int((a + a\*cosh(e + f\*x))/(c + d\*x)^2, x)

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))/(d\*x+c)\*\*2,x)

[Out] Timed out

$$3.104 \quad \int \frac{a+a \cosh(e+fx)}{(c+dx)^3} dx$$

**Optimal.** Leaf size=123

$$\frac{af^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{2d^3} + \frac{af^2 \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{2d^3} - \frac{af \sinh(e+fx)}{2d^2(c+dx)} - \frac{a \cosh(e+fx)}{2d(c+dx)^2} - \frac{a}{2d(c+dx)}$$

[Out]  $-1/2*a/d/(d*x+c)^2+1/2*a*f^2*\operatorname{Chi}(c*f/d+f*x)*\cosh(-e+c*f/d)/d^3-1/2*a*\cosh(f*x+e)/d/(d*x+c)^2-1/2*a*f^2*\operatorname{Shi}(c*f/d+f*x)*\sinh(-e+c*f/d)/d^3-1/2*a*f*\sinh(f*x+e)/d^2/(d*x+c)$

**Rubi [A]** time = 0.22, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3317, 3297, 3303, 3298, 3301}

$$\frac{af^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{2d^3} + \frac{af^2 \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{2d^3} - \frac{af \sinh(e+fx)}{2d^2(c+dx)} - \frac{a \cosh(e+fx)}{2d(c+dx)^2} - \frac{a}{2d(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + a*\operatorname{Cosh}[e + f*x])/(c + d*x)^3, x]$

[Out]  $-a/(2*d*(c + d*x)^2) - (a*\operatorname{Cosh}[e + f*x])/(2*d*(c + d*x)^2) + (a*f^2*\operatorname{Cosh}[e - (c*f)/d]*\operatorname{CoshIntegral}[(c*f)/d + f*x])/(2*d^3) - (a*f*\operatorname{Sinh}[e + f*x])/(2*d^2*(c + d*x)) + (a*f^2*\operatorname{Sinh}[e - (c*f)/d]*\operatorname{SinhIntegral}[(c*f)/d + f*x])/(2*d^3)$

**Rule 3297**

$\operatorname{Int}[(c_. + (d_.)*(x_.))^(m_)*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^(m + 1)*\sin[e + f*x]/(d*(m + 1)), x] - \operatorname{Dist}[f/(d*(m + 1)), \operatorname{Int}[(c + d*x)^(m + 1)*\cos[e + f*x], x], x] /;$   $\operatorname{FreeQ}\{c, d, e, f, x\} \ \&\amp; \ \operatorname{LtQ}[m, -1]$

**Rule 3298**

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz\_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Simp}[(I*\operatorname{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /;$   $\operatorname{FreeQ}\{c, d, e, f, fz, x\} \ \&\amp; \ \operatorname{EqQ}[d*e - c*f*fz*I, 0]$

**Rule 3301**

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz\_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /;$   $\operatorname{FreeQ}\{c, d, e, f, fz, x\}$

}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

### Rule 3317

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Sin[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{a + a \cosh(e + fx)}{(c + dx)^3} dx &= \int \left( \frac{a}{(c + dx)^3} + \frac{a \cosh(e + fx)}{(c + dx)^3} \right) dx \\
 &= -\frac{a}{2d(c + dx)^2} + a \int \frac{\cosh(e + fx)}{(c + dx)^3} dx \\
 &= -\frac{a}{2d(c + dx)^2} - \frac{a \cosh(e + fx)}{2d(c + dx)^2} + \frac{(af) \int \frac{\sinh(e + fx)}{(c + dx)^2} dx}{2d} \\
 &= -\frac{a}{2d(c + dx)^2} - \frac{a \cosh(e + fx)}{2d(c + dx)^2} - \frac{af \sinh(e + fx)}{2d^2(c + dx)} + \frac{(af^2) \int \frac{\cosh(e + fx)}{c + dx} dx}{2d^2} \\
 &= -\frac{a}{2d(c + dx)^2} - \frac{a \cosh(e + fx)}{2d(c + dx)^2} - \frac{af \sinh(e + fx)}{2d^2(c + dx)} + \frac{\left( af^2 \cosh\left( e - \frac{cf}{d} \right) \right) \int \frac{\cosh\left( \frac{cf}{d} + fx \right)}{c + dx}}{2d^2} \\
 &= -\frac{a}{2d(c + dx)^2} - \frac{a \cosh(e + fx)}{2d(c + dx)^2} + \frac{af^2 \cosh\left( e - \frac{cf}{d} \right) \text{Chi}\left( \frac{cf}{d} + fx \right)}{2d^3} - \frac{af \sinh(e + fx)}{2d^2(c + dx)}
 \end{aligned}$$

**Mathematica** [A] time = 0.46, size = 90, normalized size = 0.73

$$\frac{a \left( f^2 \text{Chi}\left( f \left( \frac{c}{d} + x \right) \right) \cosh\left( e - \frac{cf}{d} \right) + f^2 \sinh\left( e - \frac{cf}{d} \right) \text{Shi}\left( f \left( \frac{c}{d} + x \right) \right) - \frac{d(f(c+dx) \sinh(e+fx) + d \cosh(e+fx) + d)}{(c+dx)^2} \right)}{2d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Cosh[e + f\*x])/(c + d\*x)^3,x]

[Out] (a\*(f^2\*Cosh[e - (c\*f)/d]\*CoshIntegral[f\*(c/d + x)] - (d\*(d + d\*Cosh[e + f\*x] + f\*(c + d\*x)\*Sinh[e + f\*x]))/(c + d\*x)^2 + f^2\*Sinh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)])/(2\*d^3)

**fricas** [B] time = 0.56, size = 274, normalized size = 2.23

$$\frac{2ad^2 \cosh(fx + e) + 2ad^2 - \left( (ad^2 f^2 x^2 + 2acdf^2 x + ac^2 f^2) \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) + (ad^2 f^2 x^2 + 2acdf^2 x + ac^2 f^2) \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right) \right)}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))/(d\*x+c)^3,x, algorithm="fricas")

[Out] -1/4\*(2\*a\*d^2\*cosh(f\*x + e) + 2\*a\*d^2 - ((a\*d^2\*f^2\*x^2 + 2\*a\*c\*d\*f^2\*x + a\*c^2\*f^2)\*Ei((d\*f\*x + c\*f)/d) + (a\*d^2\*f^2\*x^2 + 2\*a\*c\*d\*f^2\*x + a\*c^2\*f^2)\*Ei(-(d\*f\*x + c\*f)/d))\*cosh(-(d\*e - c\*f)/d) + 2\*(a\*d^2\*f\*x + a\*c\*d\*f)\*sinh(f\*x + e) + ((a\*d^2\*f^2\*x^2 + 2\*a\*c\*d\*f^2\*x + a\*c^2\*f^2)\*Ei((d\*f\*x + c\*f)/d) - (a\*d^2\*f^2\*x^2 + 2\*a\*c\*d\*f^2\*x + a\*c^2\*f^2)\*Ei(-(d\*f\*x + c\*f)/d))\*sinh(-(d\*e - c\*f)/d))/(d^5\*x^2 + 2\*c\*d^4\*x + c^2\*d^3)

**giac** [B] time = 0.12, size = 328, normalized size = 2.67

$$\frac{ad^2 f^2 x^2 \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right) e^{\left(\frac{cf}{d}-e\right)} + ad^2 f^2 x^2 \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) e^{\left(-\frac{cf}{d}+e\right)} + 2acdf^2 x \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right) e^{\left(\frac{cf}{d}-e\right)} + 2acdf^2 x \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) e^{\left(-\frac{cf}{d}+e\right)}}{d^5 x^2 + 2c d^4 x + c^2 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))/(d\*x+c)^3,x, algorithm="giac")

[Out] 1/4\*(a\*d^2\*f^2\*x^2\*Ei(-(d\*f\*x + c\*f)/d)\*e^(c\*f/d - e) + a\*d^2\*f^2\*x^2\*Ei((d\*f\*x + c\*f)/d)\*e^(-c\*f/d + e) + 2\*a\*c\*d\*f^2\*x\*Ei(-(d\*f\*x + c\*f)/d)\*e^(c\*f/d - e) + 2\*a\*c\*d\*f^2\*x\*Ei((d\*f\*x + c\*f)/d)\*e^(-c\*f/d + e) + a\*c^2\*f^2\*Ei(-(d\*f\*x + c\*f)/d)\*e^(c\*f/d - e) + a\*c^2\*f^2\*Ei((d\*f\*x + c\*f)/d)\*e^(-c\*f/d + e) - a\*d^2\*f\*x\*e^(f\*x + e) + a\*d^2\*f\*x\*e^(-f\*x - e) - a\*c\*d\*f\*e^(f\*x + e) + a\*c\*d\*f\*e^(-f\*x - e) - a\*d^2\*e^(f\*x + e) - a\*d^2\*e^(-f\*x - e) - 2\*a\*d^2)/(d^5\*x^2 + 2\*c\*d^4\*x + c^2\*d^3)

**maple** [B] time = 0.15, size = 296, normalized size = 2.41

$$\frac{a}{2d(dx+c)^2} + \frac{f^3 a e^{-fx-e} x}{4d(d^2 f^2 x^2 + 2cd f^2 x + c^2 f^2)} + \frac{f^3 a e^{-fx-e} c}{4d^2(d^2 f^2 x^2 + 2cd f^2 x + c^2 f^2)} - \frac{f^2 a e^{-fx-e}}{4d(d^2 f^2 x^2 + 2cd f^2 x + c^2 f^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*cosh(f*x+e))/(d*x+c)^3,x)`

[Out] 
$$-1/2*a/d/(d*x+c)^2+1/4*f^3*a*\exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x+1/4*f^3*a*\exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/4*f^2*a*\exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/4*f^2*a/d^3*\exp((c*f-d*e)/d)*\text{Ei}(1,f*x+e+(c*f-d*e)/d)-1/4*f^2*a/d^3*\exp(f*x+e)/(c*f/d+f*x)^2-1/4*f^2*a/d^3*\exp(f*x+e)/(c*f/d+f*x)-1/4*f^2*a/d^3*\exp(-(c*f-d*e)/d)*\text{Ei}(1,-f*x-e-(c*f-d*e)/d)$$

**maxima** [A] time = 0.57, size = 98, normalized size = 0.80

$$-\frac{1}{2}a\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right)-\frac{a}{2(d^3x^2+2cd^2x+c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cosh(f*x+e))/(d*x+c)^3,x, algorithm="maxima")`

[Out] 
$$-1/2*a*(e^{-e+c*f/d}*\exp\_integral\_e(3,(d*x+c)*f/d)/((d*x+c)^2*d)+e^{e-c*f/d}*\exp\_integral\_e(3,-(d*x+c)*f/d)/((d*x+c)^2*d))-1/2*a/(d^3*x^2+2*c*d^2*x+c^2*d)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + a \cosh(e + f x)}{(c + d x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cosh(e + f*x))/(c + d*x)^3,x)`

[Out] `int((a + a*cosh(e + f*x))/(c + d*x)^3, x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cosh(f*x+e))/(d*x+c)**3,x)`

[Out] Timed out



### 3.105 $\int (c + dx)^3 (a + a \cosh(e + fx))^2 dx$

**Optimal.** Leaf size=237

$$\frac{12a^2d^2(c + dx) \sinh(e + fx)}{f^3} + \frac{3a^2d^2(c + dx) \sinh(e + fx) \cosh(e + fx)}{4f^3} + \frac{3a^2cd^2x}{4f^2} - \frac{3a^2d(c + dx)^2 \cosh^2(e + fx)}{4f^2}$$

[Out]  $3/4*a^2*c*d^2*x/f^2+3/8*a^2*d^3*x^2/f^2+3/8*a^2*(d*x+c)^4/d-12*a^2*d^3*\cosh(f*x+e)/f^4-6*a^2*d*(d*x+c)^2*\cosh(f*x+e)/f^2-3/8*a^2*d^3*\cosh(f*x+e)^2/f^4-3/4*a^2*d*(d*x+c)^2*\cosh(f*x+e)^2/f^2+12*a^2*d^2*(d*x+c)*\sinh(f*x+e)/f^3+2*a^2*(d*x+c)^3*\sinh(f*x+e)/f+3/4*a^2*d^2*(d*x+c)*\cosh(f*x+e)*\sinh(f*x+e)/f^3+1/2*a^2*(d*x+c)^3*\cosh(f*x+e)*\sinh(f*x+e)/f$

**Rubi [A]** time = 0.27, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {3317, 3296, 2638, 3311, 32, 3310}

$$\frac{12a^2d^2(c + dx) \sinh(e + fx)}{f^3} + \frac{3a^2d^2(c + dx) \sinh(e + fx) \cosh(e + fx)}{4f^3} + \frac{3a^2cd^2x}{4f^2} - \frac{3a^2d(c + dx)^2 \cosh^2(e + fx)}{4f^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^3*(a + a*\text{Cosh}[e + f*x])^2, x]$

[Out]  $(3*a^2*c*d^2*x)/(4*f^2) + (3*a^2*d^3*x^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8*d) - (12*a^2*d^3*\text{Cosh}[e + f*x])/f^4 - (6*a^2*d*(c + d*x)^2*\text{Cosh}[e + f*x])/f^2 - (3*a^2*d^3*\text{Cosh}[e + f*x]^2)/(8*f^4) - (3*a^2*d*(c + d*x)^2*\text{Cosh}[e + f*x]^2)/(4*f^2) + (12*a^2*d^2*(c + d*x)*\text{Sinh}[e + f*x])/f^3 + (2*a^2*(c + d*x)^3*\text{Sinh}[e + f*x])/f + (3*a^2*d^2*(c + d*x)*\text{Cosh}[e + f*x]*\text{Sinh}[e + f*x])/(4*f^3) + (a^2*(c + d*x)^3*\text{Cosh}[e + f*x]*\text{Sinh}[e + f*x])/(2*f)$

#### Rule 32

$\text{Int}[(a_. + (b_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^(m + 1)/(b*(m + 1)), x] /;$   $\text{FreeQ}\{a, b, m, x\} \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 2638

$\text{Int}[\sin[(c_. + (d_.)*(x_.))], x\_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$   $\text{FreeQ}\{c, d, x\}$

#### Rule 3296

$\text{Int}[(c_. + (d_.)*(x_.))^(m_.)*\sin[(e_. + (f_.)*(x_.))], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^(m - 1)*\text{Cos}[$

$e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 3310

$\text{Int}[\{(c_.) + (d_.)*(x_.)\}*\{(b_.)*\sin[(e_.) + (f_.)*(x_.)]\}^{(n_.)}, x\_Symbol] \rightarrow$   
 $\text{Simp}[(d*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n - 1))/n, \text{Int}[(c + d*x)*(b*\sin[e + f*x])^{(n - 2)}, x], x] - \text{Simp}[(b*(c + d*x)*\cos[e + f*x]*(b*\sin[e + f*x])^{(n - 1)})/(f*n), x]) /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[n, 1]$

### Rule 3311

$\text{Int}[\{(c_.) + (d_.)*(x_.)\}^{(m_.)}*\{(b_.)*\sin[(e_.) + (f_.)*(x_.)]\}^{(n_.)}, x\_Symbol] \rightarrow$   
 $\text{Simp}[(d*m*(c + d*x)^{(m - 1)}*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n - 1))/n, \text{Int}[(c + d*x)^m*(b*\sin[e + f*x])^{(n - 2)}, x], x] - \text{Dist}[(d^2*m*(m - 1))/(f^2*n^2), \text{Int}[(c + d*x)^{(m - 2)}*(b*\sin[e + f*x])^n, x], x] - \text{Simp}[(b*(c + d*x)^m*\cos[e + f*x]*(b*\sin[e + f*x])^{(n - 1)})/(f*n), x]) /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{GtQ}[m, 1]$

### Rule 3317

$\text{Int}[\{(c_.) + (d_.)*(x_.)\}^{(m_.)}*\{(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]\}^{(n_.)}, x\_Symbol] \rightarrow$   
 $\text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, (a + b*\sin[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ (\text{EqQ}[n, 1] \ || \ \text{IGtQ}[m, 0] \ || \ \text{NeQ}[a^2 - b^2, 0])$

### Rubi steps

$$\begin{aligned} \int (c + dx)^3 (a + a \cosh(e + fx))^2 dx &= \int (a^2(c + dx)^3 + 2a^2(c + dx)^3 \cosh(e + fx) + a^2(c + dx)^3 \cosh^2(e + fx)) \\ &= \frac{a^2(c + dx)^4}{4d} + a^2 \int (c + dx)^3 \cosh^2(e + fx) dx + (2a^2) \int (c + dx)^3 \cosh(e + fx) dx \\ &= \frac{a^2(c + dx)^4}{4d} - \frac{3a^2d(c + dx)^2 \cosh^2(e + fx)}{4f^2} + \frac{2a^2(c + dx)^3 \sinh(e + fx)}{f} + \\ &= \frac{3a^2(c + dx)^4}{8d} - \frac{6a^2d(c + dx)^2 \cosh(e + fx)}{f^2} - \frac{3a^2d^3 \cosh^2(e + fx)}{8f^4} - \frac{3a^2d^3}{8f^4} \\ &= \frac{3a^2cd^2x}{4f^2} + \frac{3a^2d^3x^2}{8f^2} + \frac{3a^2(c + dx)^4}{8d} - \frac{6a^2d(c + dx)^2 \cosh(e + fx)}{f^2} - \frac{3a^2d^3}{8f^4} \\ &= \frac{3a^2cd^2x}{4f^2} + \frac{3a^2d^3x^2}{8f^2} + \frac{3a^2(c + dx)^4}{8d} - \frac{12a^2d^3 \cosh(e + fx)}{f^4} - \frac{6a^2d(c + dx)^2 \cosh(e + fx)}{f^2} - \frac{3a^2d^3}{8f^4} \end{aligned}$$

**Mathematica [A]** time = 1.49, size = 217, normalized size = 0.92

$$\frac{a^2 \left( -96d \left( c^2 f^2 + 2cdf^2 x + d^2 \left( f^2 x^2 + 2 \right) \right) \cosh(e + fx) - 3d \left( 2c^2 f^2 + 4cdf^2 x + d^2 \left( 2f^2 x^2 + 1 \right) \right) \cosh(2(e + fx)) \right)}{16f^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3\*(a + a\*Cosh[e + f\*x])^2,x]

[Out] (a^2\*(-96\*d\*(c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(2 + f^2\*x^2))\*Cosh[e + f\*x] - 3\*d\*(2\*c^2\*f^2 + 4\*c\*d\*f^2\*x + d^2\*(1 + 2\*f^2\*x^2))\*Cosh[2\*(e + f\*x)] + 2\*f\*(3\*f^3\*x\*(4\*c^3 + 6\*c^2\*d\*x + 4\*c\*d^2\*x^2 + d^3\*x^3) + 16\*(c + d\*x)\*(c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(6 + f^2\*x^2))\*Sinh[e + f\*x] + (c + d\*x)\*(2\*c^2\*f^2 + 4\*c\*d\*f^2\*x + d^2\*(3 + 2\*f^2\*x^2))\*Sinh[2\*(e + f\*x)]))/(16\*f^4)

**fricas [A]** time = 0.53, size = 395, normalized size = 1.67

$$\frac{6a^2d^3f^4x^4 + 24a^2cd^2f^4x^3 + 36a^2c^2df^4x^2 + 24a^2c^3f^4x - 3(2a^2d^3f^2x^2 + 4a^2cd^2f^2x + 2a^2c^2df^2 + a^2d^3) \cosh(e + fx)}{16f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+a\*cosh(f\*x+e))^2,x, algorithm="fricas")

[Out] 1/16\*(6\*a^2\*d^3\*f^4\*x^4 + 24\*a^2\*c\*d^2\*f^4\*x^3 + 36\*a^2\*c^2\*d\*f^4\*x^2 + 24\*a^2\*c^3\*f^4\*x - 3\*(2\*a^2\*d^3\*f^2\*x^2 + 4\*a^2\*c\*d^2\*f^2\*x + 2\*a^2\*c^2\*d\*f^2 + a^2\*d^3)\*cosh(f\*x + e)^2 - 3\*(2\*a^2\*d^3\*f^2\*x^2 + 4\*a^2\*c\*d^2\*f^2\*x + 2\*a^2\*c^2\*d\*f^2 + a^2\*d^3)\*sinh(f\*x + e)^2 - 96\*(a^2\*d^3\*f^2\*x^2 + 2\*a^2\*c\*d^2\*f^2\*x + a^2\*c^2\*d\*f^2 + 2\*a^2\*d^3)\*cosh(f\*x + e) + 4\*(8\*a^2\*d^3\*f^3\*x^3 + 24\*a^2\*c\*d^2\*f^3\*x^2 + 8\*a^2\*c^3\*f^3 + 48\*a^2\*c\*d^2\*f + 24\*(a^2\*c^2\*d\*f^3 + 2\*a^2\*d^3\*f)\*x + (2\*a^2\*d^3\*f^3\*x^3 + 6\*a^2\*c\*d^2\*f^3\*x^2 + 2\*a^2\*c^3\*f^3 + 3\*a^2\*c\*d^2\*f + 3\*(2\*a^2\*c^2\*d\*f^3 + a^2\*d^3\*f)\*x)\*cosh(f\*x + e))\*sinh(f\*x + e))/f^4

**giac [B]** time = 0.13, size = 581, normalized size = 2.45

$$\frac{3}{8}a^2d^3x^4 + \frac{3}{2}a^2cd^2x^3 + \frac{9}{4}a^2c^2dx^2 + \frac{3}{2}a^2c^3x + \frac{(4a^2d^3f^3x^3 + 12a^2cd^2f^3x^2 + 12a^2c^2df^3x - 6a^2d^3f^2x^2 + 4a^2c^3f^3 - 3(2a^2d^3f^2x^2 + 4a^2cd^2f^2x + 2a^2c^2df^2 + a^2d^3) \cosh(e + fx) + 4(8a^2d^3f^3x^3 + 24a^2cd^2f^3x^2 + 8a^2c^3f^3 + 48a^2cd^2f + 24(a^2c^2df^3 + 2a^2d^3f)x + (2a^2d^3f^3x^3 + 6a^2cd^2f^3x^2 + 2a^2c^3f^3 + 3a^2cd^2f + 3(2a^2c^2df^3 + a^2d^3f)x) \sinh(e + fx))}{32f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+a\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out] 3/8\*a^2\*d^3\*x^4 + 3/2\*a^2\*c\*d^2\*x^3 + 9/4\*a^2\*c^2\*d\*x^2 + 3/2\*a^2\*c^3\*x + 1/32\*(4\*a^2\*d^3\*f^3\*x^3 + 12\*a^2\*c\*d^2\*f^3\*x^2 + 12\*a^2\*c^2\*d\*f^3\*x - 6\*a^2\*d^3\*f^2\*x^2 + 4\*a^2\*c^3\*f^3 - 3\*(2\*a^2\*d^3\*f^2\*x^2 + 4\*a^2\*c\*d^2\*f^2\*x + 2\*a^2\*c^2\*d\*f^2 + a^2\*d^3)\*cosh(f\*x + e) + 4\*(8\*a^2\*d^3\*f^3\*x^3 + 24\*a^2\*c\*d^2\*f^3\*x^2 + 8\*a^2\*c^3\*f^3 + 48\*a^2\*c\*d^2\*f + 24\*(a^2\*c^2\*d\*f^3 + 2\*a^2\*d^3\*f)\*x + (2\*a^2\*d^3\*f^3\*x^3 + 6\*a^2\*c\*d^2\*f^3\*x^2 + 2\*a^2\*c^3\*f^3 + 3\*a^2\*c\*d^2\*f + 3\*(2\*a^2\*c^2\*d\*f^3 + a^2\*d^3\*f)\*x)\*sinh(f\*x + e)))/f^4

$$d^3f^2x^2 + 4a^2c^3f^3 - 12a^2cd^2f^2x - 6a^2c^2d^2f^2 + 6a^2d^3f^2x + 6a^2cd^2f - 3a^2d^3)e^{(2fx + 2e)}/f^4 + (a^2d^3f^3x^3 + 3a^2cd^2f^3x^2 + 3a^2c^2d^2f^3x - 3a^2d^3f^2x^2 + a^2c^3f^3 - 6a^2cd^2f^2x - 3a^2c^2d^2f^2 + 6a^2d^3fx + 6a^2cd^2f - 6a^2d^3)e^{(fx + e)}/f^4 - (a^2d^3f^3x^3 + 3a^2cd^2f^3x^2 + 3a^2c^2d^2f^3x + 3a^2d^3f^2x^2 + a^2c^3f^3 + 6a^2cd^2f^2x + 3a^2c^2d^2f^2 + 6a^2d^3fx + 6a^2cd^2f + 6a^2d^3)e^{(-fx - e)}/f^4 - 1/32(4a^2d^3f^3x^3 + 12a^2cd^2f^3x^2 + 12a^2c^2d^2f^3x + 6a^2d^3f^2x^2 + 4a^2c^3f^3 + 12a^2cd^2f^2x + 6a^2c^2d^2f^2 + 6a^2d^3fx + 6a^2cd^2f + 3a^2d^3)e^{(-2fx - 2e)}/f^4$$

**maple [B]** time = 0.08, size = 1071, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3\*(a+a\*cosh(f\*x+e))^2,x)

[Out]  $1/f*(1/4/f^3d^3a^2(f*x+e)^4+2/f^3d^3a^2((f*x+e)^3\sinh(f*x+e)-3*(f*x+e)^2\cosh(f*x+e)+6*(f*x+e)\sinh(f*x+e)-6\cosh(f*x+e))+1/f^3d^3a^2(1/2*(f*x+e)^3\cosh(f*x+e)\sinh(f*x+e)+1/8*(f*x+e)^4-3/4*(f*x+e)^2\cosh(f*x+e)^2+3/4*(f*x+e)\cosh(f*x+e)\sinh(f*x+e)+3/8*(f*x+e)^2-3/8\cosh(f*x+e)^2)+c^3a^2*(f*x+e)+2*c^3a^2\sinh(f*x+e)+c^3a^2(1/2\cosh(f*x+e)\sinh(f*x+e)+1/2fx+1/2e)+3/f^2cd^2e^2a^2(1/2\cosh(f*x+e)\sinh(f*x+e)+1/2fx+1/2e)-6/f*c^2d^2e^2a^2\sinh(f*x+e)-3/f*c^2d^2e^2a^2(1/2\cosh(f*x+e)\sinh(f*x+e)+1/2fx+1/2e)+6/f^2cd^2e^2a^2\sinh(f*x+e)+3/f^2cd^2e^2a^2(f*x+e)-3/f^2cd^2e^2a^2(f*x+e)^2-6/f^2cd^2e^2a^2(1/2(f*x+e)\cosh(f*x+e)\sinh(f*x+e)+1/4(f*x+e)^2-1/4\cosh(f*x+e)^2)-12/f^2cd^2e^2a^2((f*x+e)\sinh(f*x+e)-\cosh(f*x+e))-3/f*c^2d^2e^2a^2(f*x+e)+1/f^2cd^2a^2(f*x+e)^3+3/f^3d^3e^2a^2(1/2(f*x+e)\cosh(f*x+e)\sinh(f*x+e)+1/4(f*x+e)^2-1/4\cosh(f*x+e)^2)+6/f^2cd^2a^2((f*x+e)^2\sinh(f*x+e)-2*(f*x+e)\cosh(f*x+e)+2\sinh(f*x+e))-6/f^3d^3e^2a^2((f*x+e)^2\sinh(f*x+e)-2*(f*x+e)\cosh(f*x+e)+2\sinh(f*x+e))+6/f^3d^3e^2a^2((f*x+e)\sinh(f*x+e)-\cosh(f*x+e))+3/f^2cd^2a^2(1/2(f*x+e)^2\cosh(f*x+e)\sinh(f*x+e)+1/6(f*x+e)^3-1/2(f*x+e)\cosh(f*x+e)^2+1/4\cosh(f*x+e)\sinh(f*x+e)+1/4fx+1/4e)-3/f^3d^3e^2a^2(1/2(f*x+e)^2\cosh(f*x+e)\sinh(f*x+e)+1/6(f*x+e)^3-1/2(f*x+e)\cosh(f*x+e)^2+1/4\cosh(f*x+e)\sinh(f*x+e)+1/4fx+1/4e)+6/f*c^2d^2a^2((f*x+e)\sinh(f*x+e)-\cosh(f*x+e))+3/f*c^2d^2a^2(1/2(f*x+e)\cosh(f*x+e)\sinh(f*x+e)+1/4(f*x+e)^2-1/4\cosh(f*x+e)^2)-1/f^3d^3e^3a^2(f*x+e)+3/2/f^3d^3e^2a^2(f*x+e)^2+3/2/f*c^2d^2a^2(f*x+e)^2-1/f^3d^3e^3a^2(f*x+e)^3-2/f^3d^3e^3a^2\sinh(f*x+e)-1/f^3d^3e^3a^2(1/2\cosh(f*x+e)\sinh(f*x+e)+1/2fx+1/2e))$

**maxima [B]** time = 0.39, size = 527, normalized size = 2.22

$$\frac{1}{4} a^2 d^3 x^4 + a^2 c d^2 x^3 + \frac{3}{2} a^2 c^2 d x^2 + \frac{3}{16} \left( 4x^2 + \frac{(2fxe^{(2e)} - e^{(2e)})e^{(2fx)}}{f^2} - \frac{(2fx+1)e^{(-2fx-2e)}}{f^2} \right) a^2 c^2 d + \frac{1}{16} \left( 8x^3 + \frac{3}{16} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+a\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out]  $\frac{1}{4} a^2 d^3 x^4 + a^2 c d^2 x^3 + \frac{3}{2} a^2 c^2 d x^2 + \frac{3}{16} (4x^2 + \frac{(2fxe^{(2e)} - e^{(2e)})e^{(2fx)}}{f^2} - \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}) a^2 c^2 d + \frac{1}{16} (8x^3 + \frac{3}{16})$

**mupad [B]** time = 2.25, size = 452, normalized size = 1.91

$$16 a^2 c^3 f^3 \sinh(e + fx) - \frac{3 a^2 d^3 \cosh(2e + 2fx)}{2} - 96 a^2 d^3 \cosh(e + fx) + 12 a^2 c^3 f^4 x + 2 a^2 c^3 f^3 \sinh(2e + 2fx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cosh(e + f\*x))^2\*(c + d\*x)^3,x)

[Out]  $(16 a^2 c^3 f^3 \sinh(e + fx) - (3 a^2 d^3 \cosh(2e + 2fx)) / 2 - 96 a^2 d^3 \cosh(e + fx) + 12 a^2 c^3 f^4 x + 2 a^2 c^3 f^3 \sinh(2e + 2fx) + 3 a^2 d^3 f^4 x^4 + 96 a^2 c d^2 f \sinh(e + fx) + 96 a^2 d^3 f x \sinh(e + fx) - 3 a^2 d^3 f^2 x^2 \cosh(2e + 2fx) + 2 a^2 d^3 f^3 x^3 \sinh(2e + 2fx) - 48 a^2 c^2 d f^2 \cosh(e + fx) + 3 a^2 c d^2 f \sinh(2e + 2fx) + 3 a^2 d^3 f x \sinh(2e + 2fx) - 3 a^2 c^2 d f^2 \cosh(2e + 2fx) + 18 a^2 c^2 d f^4 x^2 + 12 a^2 c d^2 f^4 x^3 - 48 a^2 d^3 f^2 x^2 \cosh(e + fx) + 16 a^2 d^3 f^3 x^3 \sinh(e + fx) - 6 a^2 c d^2 f^2 x \cosh(2e + 2fx) + 6 a^2 c^2 d f^3 x \sinh(2e + 2fx) + 48 a^2 c d^2 f^3 x^2 \sinh(e + fx) + 6 a^2 c d^2 f^3 x^2 \sinh(2e + 2fx) - 96 a^2 c d^2 f^2 x \cosh(e + fx) + 48 a^2 c^2 d f^3 x \sinh(e + fx)) / (8 f^4)$

**sympy** [A] time = 4.08, size = 779, normalized size = 3.29

$$\left\{ \begin{array}{l} -\frac{a^2 c^3 x \sinh^2(e+fx)}{2} + \frac{a^2 c^3 x \cosh^2(e+fx)}{2} + a^2 c^3 x + \frac{a^2 c^3 \sinh(e+fx) \cosh(e+fx)}{2f} + \frac{2a^2 c^3 \sinh(e+fx)}{f} - \frac{3a^2 c^2 dx^2 \sinh^2(e+fx)}{4} + \frac{3a^2 c^2 dx^2 \cosh^2(e+fx)}{4} \\ (a \cosh(e) + a)^2 \left( c^3 x + \frac{3c^2 dx^2}{2} + cd^2 x^3 + \frac{d^3 x^4}{4} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3\*(a+a\*cosh(f\*x+e))\*\*2,x)

[Out] Piecewise((-a\*\*2\*c\*\*3\*x\*sinh(e + f\*x)\*\*2/2 + a\*\*2\*c\*\*3\*x\*cosh(e + f\*x)\*\*2/2 + a\*\*2\*c\*\*3\*x + a\*\*2\*c\*\*3\*sinh(e + f\*x)\*cosh(e + f\*x)/(2\*f) + 2\*a\*\*2\*c\*\*3\*sinh(e + f\*x)/f - 3\*a\*\*2\*c\*\*2\*d\*x\*\*2\*sinh(e + f\*x)\*\*2/4 + 3\*a\*\*2\*c\*\*2\*d\*x\*\*2\*cosh(e + f\*x)\*\*2/4 + 3\*a\*\*2\*c\*\*2\*d\*x\*\*2/2 + 3\*a\*\*2\*c\*\*2\*d\*x\*sinh(e + f\*x)\*cosh(e + f\*x)/(2\*f) + 6\*a\*\*2\*c\*\*2\*d\*x\*sinh(e + f\*x)/f - 3\*a\*\*2\*c\*\*2\*d\*sinh(e + f\*x)\*\*2/(4\*f\*\*2) - 6\*a\*\*2\*c\*\*2\*d\*cosh(e + f\*x)/f\*\*2 - a\*\*2\*c\*d\*\*2\*x\*\*3\*sinh(e + f\*x)\*\*2/2 + a\*\*2\*c\*d\*\*2\*x\*\*3\*cosh(e + f\*x)\*\*2/2 + a\*\*2\*c\*d\*\*2\*x\*\*3 + 3\*a\*\*2\*c\*d\*\*2\*x\*\*2\*sinh(e + f\*x)\*cosh(e + f\*x)/(2\*f) + 6\*a\*\*2\*c\*d\*\*2\*x\*\*2\*sinh(e + f\*x)/f - 3\*a\*\*2\*c\*d\*\*2\*x\*\*2\*sinh(e + f\*x)\*\*2/(4\*f\*\*2) - 3\*a\*\*2\*c\*d\*\*2\*x\*\*2\*cosh(e + f\*x)\*\*2/(4\*f\*\*2) - 12\*a\*\*2\*c\*d\*\*2\*x\*cosh(e + f\*x)/f\*\*2 + 3\*a\*\*2\*c\*d\*\*2\*sinh(e + f\*x)\*cosh(e + f\*x)/(4\*f\*\*3) + 12\*a\*\*2\*c\*d\*\*2\*sinh(e + f\*x)/f\*\*3 - a\*\*2\*d\*\*3\*x\*\*4\*sinh(e + f\*x)\*\*2/8 + a\*\*2\*d\*\*3\*x\*\*4\*cosh(e + f\*x)\*\*2/8 + a\*\*2\*d\*\*3\*x\*\*4/4 + a\*\*2\*d\*\*3\*x\*\*3\*sinh(e + f\*x)\*cosh(e + f\*x)/(2\*f) + 2\*a\*\*2\*d\*\*3\*x\*\*3\*sinh(e + f\*x)/f - 3\*a\*\*2\*d\*\*3\*x\*\*2\*sinh(e + f\*x)\*\*2/(8\*f\*\*2) - 3\*a\*\*2\*d\*\*3\*x\*\*2\*cosh(e + f\*x)\*\*2/(8\*f\*\*2) - 6\*a\*\*2\*d\*\*3\*x\*\*2\*cosh(e + f\*x)/f\*\*2 + 3\*a\*\*2\*d\*\*3\*x\*sinh(e + f\*x)\*cosh(e + f\*x)/(4\*f\*\*3) + 12\*a\*\*2\*d\*\*3\*x\*sinh(e + f\*x)/f\*\*3 - 3\*a\*\*2\*d\*\*3\*sinh(e + f\*x)\*\*2/(8\*f\*\*4) - 12\*a\*\*2\*d\*\*3\*cosh(e + f\*x)/f\*\*4, Ne(f, 0)), ((a\*cosh(e) + a)\*\*2\*(c\*\*3\*x + 3\*c\*\*2\*d\*x\*\*2/2 + c\*d\*\*2\*x\*\*3 + d\*\*3\*x\*\*4/4), True))

### 3.106 $\int (c + dx)^2 (a + a \cosh(e + fx))^2 dx$

**Optimal.** Leaf size=168

$$\frac{a^2 d(c + dx) \cosh^2(e + fx)}{2f^2} - \frac{4a^2 d(c + dx) \cosh(e + fx)}{f^2} + \frac{2a^2 (c + dx)^2 \sinh(e + fx)}{f} + \frac{a^2 (c + dx)^2 \sinh(e + fx)}{2f}$$

[Out]  $1/4*a^2*d^2*x/f^2+1/2*a^2*(d*x+c)^3/d-4*a^2*d*(d*x+c)*\cosh(f*x+e)/f^2-1/2*a^2*d*(d*x+c)*\cosh(f*x+e)^2/f^2+4*a^2*d^2*\sinh(f*x+e)/f^3+2*a^2*(d*x+c)^2*\sinh(f*x+e)/f+1/4*a^2*d^2*\cosh(f*x+e)*\sinh(f*x+e)/f^3+1/2*a^2*(d*x+c)^2*\cosh(f*x+e)*\sinh(f*x+e)/f$

**Rubi [A]** time = 0.19, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$ , Rules used = {3317, 3296, 2637, 3311, 32, 2635, 8}

$$\frac{a^2 d(c + dx) \cosh^2(e + fx)}{2f^2} - \frac{4a^2 d(c + dx) \cosh(e + fx)}{f^2} + \frac{2a^2 (c + dx)^2 \sinh(e + fx)}{f} + \frac{a^2 (c + dx)^2 \sinh(e + fx)}{2f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^2*(a + a*\text{Cosh}[e + f*x])^2, x]$

[Out]  $(a^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(2*d) - (4*a^2*d*(c + d*x)*\text{Cosh}[e + f*x])/f^2 - (a^2*d*(c + d*x)*\text{Cosh}[e + f*x]^2)/(2*f^2) + (4*a^2*d^2*\text{Sinh}[e + f*x])/f^3 + (2*a^2*(c + d*x)^2*\text{Sinh}[e + f*x])/f + (a^2*d^2*\text{Cosh}[e + f*x]*\text{Sinh}[e + f*x])/(4*f^3) + (a^2*(c + d*x)^2*\text{Cosh}[e + f*x]*\text{Sinh}[e + f*x])/(2*f)$

#### Rule 8

$\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

#### Rule 32

$\text{Int}[(a_. + (b_.)*(x_))^(m_), x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; \text{FreeQ}\{a, b, m\}, x \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 2635

$\text{Int}[(b_.)*\sin[(c_.) + (d_.)*(x_)]^(n_), x\_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^(n - 1)]/(d*n), x] + \text{Dist}[(b^2*(n - 1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^(n - 2), x], x] /; \text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[
(d*m*(c + d*x)^(m - 1)*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[
(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sine[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sine[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.),
x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sine[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^2 (a + a \cosh(e + fx))^2 dx &= \int (a^2(c + dx)^2 + 2a^2(c + dx)^2 \cosh(e + fx) + a^2(c + dx)^2 \cosh^2(e + fx)) dx \\
&= \frac{a^2(c + dx)^3}{3d} + a^2 \int (c + dx)^2 \cosh^2(e + fx) dx + (2a^2) \int (c + dx)^2 \cosh(e + fx) dx \\
&= \frac{a^2(c + dx)^3}{3d} - \frac{a^2 d (c + dx) \cosh^2(e + fx)}{2f^2} + \frac{2a^2(c + dx)^2 \sinh(e + fx)}{f} + \frac{a^2 d (c + dx) \cosh(e + fx)}{f} \\
&= \frac{a^2(c + dx)^3}{2d} - \frac{4a^2 d (c + dx) \cosh(e + fx)}{f^2} - \frac{a^2 d (c + dx) \cosh^2(e + fx)}{2f^2} + \frac{2a^2 d (c + dx) \sinh(e + fx)}{f} \\
&= \frac{a^2 d^2 x}{4f^2} + \frac{a^2(c + dx)^3}{2d} - \frac{4a^2 d (c + dx) \cosh(e + fx)}{f^2} - \frac{a^2 d (c + dx) \cosh^2(e + fx)}{2f^2}
\end{aligned}$$



**Mathematica [A]** time = 0.54, size = 192, normalized size = 1.14

$$\frac{a^2 (16c^2 f^2 \sinh(e + fx) + 2c^2 f^2 \sinh(2(e + fx)) + 12c^2 f^3 x + 32cdf^2 x \sinh(e + fx) + 4cdf^2 x \sinh(2(e + fx)) -$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2\*(a + a\*Cosh[e + f\*x])^2,x]

[Out] (a^2\*(12\*c^2\*f^3\*x + 12\*c\*d\*f^3\*x^2 + 4\*d^2\*f^3\*x^3 - 32\*d\*f\*(c + d\*x)\*Cosh[e + f\*x] - 2\*d\*f\*(c + d\*x)\*Cosh[2\*(e + f\*x)] + 32\*d^2\*Sinh[e + f\*x] + 16\*c^2\*f^2\*Sinh[e + f\*x] + 32\*c\*d\*f^2\*x\*Sinh[e + f\*x] + 16\*d^2\*f^2\*x^2\*Sinh[e + f\*x] + d^2\*Sinh[2\*(e + f\*x)] + 2\*c^2\*f^2\*Sinh[2\*(e + f\*x)] + 4\*c\*d\*f^2\*x\*Sinh[2\*(e + f\*x)] + 2\*d^2\*f^2\*x^2\*Sinh[2\*(e + f\*x)]))/(8\*f^3)

**fricas [A]** time = 0.51, size = 227, normalized size = 1.35

$$\frac{2a^2d^2f^3x^3 + 6a^2cdf^3x^2 + 6a^2c^2f^3x - (a^2d^2fx + a^2cdf) \cosh(fx + e)^2 - (a^2d^2fx + a^2cdf) \sinh(fx + e)^2 - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+a\*cosh(f\*x+e))^2,x, algorithm="fricas")

[Out] 1/4\*(2\*a^2\*d^2\*f^3\*x^3 + 6\*a^2\*c\*d\*f^3\*x^2 + 6\*a^2\*c^2\*f^3\*x - (a^2\*d^2\*f\*x + a^2\*c\*d\*f)\*cosh(f\*x + e)^2 - (a^2\*d^2\*f\*x + a^2\*c\*d\*f)\*sinh(f\*x + e)^2 - 16\*(a^2\*d^2\*f\*x + a^2\*c\*d\*f)\*cosh(f\*x + e) + (8\*a^2\*d^2\*f^2\*x^2 + 16\*a^2\*c\*d\*f^2\*x + 8\*a^2\*c^2\*f^2 + 16\*a^2\*d^2 + (2\*a^2\*d^2\*f^2\*x^2 + 4\*a^2\*c\*d\*f^2\*x + 2\*a^2\*c^2\*f^2 + a^2\*d^2)\*cosh(f\*x + e))\*sinh(f\*x + e))/f^3

**giac [B]** time = 0.13, size = 333, normalized size = 1.98

$$\frac{1}{2}a^2d^2x^3 + \frac{3}{2}a^2cdx^2 + \frac{3}{2}a^2c^2x + \frac{(2a^2d^2f^2x^2 + 4a^2cdf^2x + 2a^2c^2f^2 - 2a^2d^2fx - 2a^2cdf + a^2d^2)e^{(2fx+2e)}}{16f^3} + \frac{(a^2d^2fx + a^2cdf) \cosh(fx + e)^2 - (a^2d^2fx + a^2cdf) \sinh(fx + e)^2 - 16(a^2d^2fx + a^2cdf) \cosh(fx + e) + (8a^2d^2f^2x^2 + 16a^2c*d*f^2x + 8a^2c^2f^2 + 16a^2d^2 + (2a^2d^2f^2x^2 + 4a^2c*d*f^2x + 2a^2c^2f^2 + a^2d^2) \cosh(fx + e)) \sinh(fx + e)}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+a\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out] 1/2\*a^2\*d^2\*x^3 + 3/2\*a^2\*c\*d\*x^2 + 3/2\*a^2\*c^2\*x + 1/16\*(2\*a^2\*d^2\*f^2\*x^2 + 4\*a^2\*c\*d\*f^2\*x + 2\*a^2\*c^2\*f^2 - 2\*a^2\*d^2\*f\*x - 2\*a^2\*c\*d\*f + a^2\*d^2)\*e^(2\*f\*x + 2\*e)/f^3 + (a^2\*d^2\*f^2\*x^2 + 2\*a^2\*c\*d\*f^2\*x + a^2\*c^2\*f^2 - 2\*a^2\*d^2\*f\*x - 2\*a^2\*c\*d\*f + 2\*a^2\*d^2)\*e^(f\*x + e)/f^3 - (a^2\*d^2\*f^2\*x^2 + 2\*a^2\*c\*d\*f^2\*x + a^2\*c^2\*f^2 + 2\*a^2\*d^2\*f\*x + 2\*a^2\*c\*d\*f + 2\*a^2\*d^2)\*e^(-f\*x - e)/f^3 - 1/16\*(2\*a^2\*d^2\*f^2\*x^2 + 4\*a^2\*c\*d\*f^2\*x + 2\*a^2\*c^2\*f^2 + 2\*a^2\*d^2\*f\*x + 2\*a^2\*c\*d\*f + a^2\*d^2)\*e^(-2\*f\*x - 2\*e)/f^3

**maple [B]** time = 0.08, size = 541, normalized size = 3.22

$$\frac{d^2 a^2 (f x + e)^3}{3 f^2} + \frac{2 d^2 a^2 \left( (f x + e)^2 \sinh(f x + e) - 2 (f x + e) \cosh(f x + e) + 2 \sinh(f x + e) \right)}{f^2} + \frac{d^2 a^2 \left( \frac{(f x + e)^2 \cosh(f x + e) \sinh(f x + e)}{2} + \frac{(f x + e)^3}{6} - \frac{(f x + e) \cosh^2(f x + e)}{2} \right)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*(a+a\*cosh(f\*x+e))^2,x)

[Out] 1/f\*(1/3/f^2\*d^2\*a^2\*(f\*x+e)^3+2/f^2\*d^2\*a^2\*((f\*x+e)^2\*sinh(f\*x+e)-2\*(f\*x+e)\*cosh(f\*x+e)+2\*sinh(f\*x+e))+1/f^2\*d^2\*a^2\*(1/2\*(f\*x+e)^2\*cosh(f\*x+e)\*sinh(f\*x+e)+1/6\*(f\*x+e)^3-1/2\*(f\*x+e)\*cosh(f\*x+e)^2+1/4\*cosh(f\*x+e)\*sinh(f\*x+e)+1/4\*f\*x+1/4\*e)-1/f^2\*d^2\*e\*a^2\*(f\*x+e)^2-4/f^2\*d^2\*e\*a^2\*((f\*x+e)\*sinh(f\*x+e)-cosh(f\*x+e))-2/f^2\*d^2\*e\*a^2\*(1/2\*(f\*x+e)\*cosh(f\*x+e)\*sinh(f\*x+e)+1/4\*(f\*x+e)^2-1/4\*cosh(f\*x+e)^2)+1/f^2\*d^2\*e^2\*a^2\*(f\*x+e)+2/f^2\*d^2\*e^2\*a^2\*sinh(f\*x+e)+1/f^2\*d^2\*e^2\*a^2\*(1/2\*cosh(f\*x+e)\*sinh(f\*x+e)+1/2\*f\*x+1/2\*e)+1/f\*c\*d\*a^2\*(f\*x+e)^2+4/f\*c\*d\*a^2\*((f\*x+e)\*sinh(f\*x+e)-cosh(f\*x+e))+2/f\*c\*d\*a^2\*(1/2\*(f\*x+e)\*cosh(f\*x+e)\*sinh(f\*x+e)+1/4\*(f\*x+e)^2-1/4\*cosh(f\*x+e)^2)-2/f\*c\*d\*e\*a^2\*(f\*x+e)-4/f\*c\*d\*e\*a^2\*sinh(f\*x+e)-2/f\*c\*d\*e\*a^2\*(1/2\*cosh(f\*x+e)\*sinh(f\*x+e)+1/2\*f\*x+1/2\*e)+c^2\*a^2\*(f\*x+e)+2\*c^2\*a^2\*sinh(f\*x+e)+c^2\*a^2\*(1/2\*cosh(f\*x+e)\*sinh(f\*x+e)+1/2\*f\*x+1/2\*e))

**maxima [B]** time = 0.35, size = 327, normalized size = 1.95

$$\frac{1}{3} a^2 d^2 x^3 + a^2 c d x^2 + \frac{1}{8} \left( 4 x^2 + \frac{(2 f x e^{(2e)} - e^{(2e)}) e^{(2 f x)}}{f^2} - \frac{(2 f x + 1) e^{(-2 f x - 2e)}}{f^2} \right) a^2 c d + \frac{1}{48} \left( 8 x^3 + \frac{3(2 f^2 x^2 e^{(2e)} - 2 f x e^{(2e)} + e^{(2e)})}{f} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+a\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out] 1/3\*a^2\*d^2\*x^3 + a^2\*c\*d\*x^2 + 1/8\*(4\*x^2 + (2\*f\*x\*e^(2\*e) - e^(2\*e))\*e^(2\*f\*x)/f^2 - (2\*f\*x + 1)\*e^(-2\*f\*x - 2\*e)/f^2)\*a^2\*c\*d + 1/48\*(8\*x^3 + 3\*(2\*f^2\*x^2\*e^(2\*e) - 2\*f\*x\*e^(2\*e) + e^(2\*e))\*e^(2\*f\*x)/f^3 - 3\*(2\*f^2\*x^2 + 2\*f\*x + 1)\*e^(-2\*f\*x - 2\*e)/f^3)\*a^2\*d^2 + 1/8\*a^2\*c^2\*(4\*x + e^(2\*f\*x + 2\*e))/f - e^(-2\*f\*x - 2\*e)/f + a^2\*c^2\*x + 2\*a^2\*c\*d\*((f\*x\*e^e - e^e)\*e^(f\*x))/f^2 - (f\*x + 1)\*e^(-f\*x - e)/f^2 + a^2\*d^2\*((f^2\*x^2\*e^e - 2\*f\*x\*e^e + 2\*e^e)\*e^(f\*x))/f^3 - (f^2\*x^2 + 2\*f\*x + 2)\*e^(-f\*x - e)/f^3 + 2\*a^2\*c^2\*sinh(f\*x + e)/f

**mupad [B]** time = 1.29, size = 257, normalized size = 1.53

$$16 a^2 d^2 \sinh(e + f x) + \frac{a^2 d^2 \sinh(2e + 2 f x)}{2} + 8 a^2 c^2 f^2 \sinh(e + f x) + 6 a^2 c^2 f^3 x + a^2 c^2 f^2 \sinh(2e + 2 f x) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cosh(e + f*x))^2*(c + d*x)^2,x)`

[Out]  $(16*a^2*d^2*\sinh(e + f*x) + (a^2*d^2*\sinh(2*e + 2*f*x))/2 + 8*a^2*c^2*f^2*\sinh(e + f*x) + 6*a^2*c^2*f^3*x + a^2*c^2*f^2*\sinh(2*e + 2*f*x) + 2*a^2*d^2*f^3*x^3 - a^2*c*d*f*\cosh(2*e + 2*f*x) - 16*a^2*d^2*f*x*\cosh(e + f*x) + a^2*d^2*f^2*x^2*\sinh(2*e + 2*f*x) + 6*a^2*c*d*f^3*x^2 - a^2*d^2*f*x*\cosh(2*e + 2*f*x) - 16*a^2*c*d*f*\cosh(e + f*x) + 8*a^2*d^2*f^2*x^2*\sinh(e + f*x) + 16*a^2*c*d*f^2*x*\sinh(e + f*x) + 2*a^2*c*d*f^2*x*\sinh(2*e + 2*f*x))/(4*f^3)$

sympy [A] time = 1.70, size = 456, normalized size = 2.71

$$\left\{ \begin{array}{l} -\frac{a^2c^2x\sinh^2(e+fx)}{2} + \frac{a^2c^2x\cosh^2(e+fx)}{2} + a^2c^2x + \frac{a^2c^2\sinh(e+fx)\cosh(e+fx)}{2f} + \frac{2a^2c^2\sinh(e+fx)}{f} - \frac{a^2cdx^2\sinh^2(e+fx)}{2} + \frac{a^2cdx^2\cosh^2(e+fx)}{2} \\ (a\cosh(e) + a)^2 \left( c^2x + cdx^2 + \frac{d^2x^3}{3} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)**2*(a+a*cosh(f*x+e))**2,x)`

[Out] `Piecewise((-a**2*c**2*x*sinh(e + f*x)**2/2 + a**2*c**2*x*cosh(e + f*x)**2/2 + a**2*c**2*x + a**2*c**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 2*a**2*c**2*sinh(e + f*x)/f - a**2*c*d*x**2*sinh(e + f*x)**2/2 + a**2*c*d*x**2*cosh(e + f*x)**2/2 + a**2*c*d*x**2 + a**2*c*d*x*sinh(e + f*x)*cosh(e + f*x)/f + 4*a**2*c*d*x*sinh(e + f*x)/f - a**2*c*d*sinh(e + f*x)**2/(2*f**2) - 4*a**2*c*d*cosh(e + f*x)/f**2 - a**2*d**2*x**3*sinh(e + f*x)**2/6 + a**2*d**2*x**3*cosh(e + f*x)**2/6 + a**2*d**2*x**3/3 + a**2*d**2*x**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 2*a**2*d**2*x**2*sinh(e + f*x)/f - a**2*d**2*x*sinh(e + f*x)**2/(4*f**2) - a**2*d**2*x*cosh(e + f*x)**2/(4*f**2) - 4*a**2*d**2*x*cosh(e + f*x)/f**2 + a**2*d**2*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) + 4*a**2*d**2*sinh(e + f*x)/f**3, Ne(f, 0)), ((a*cosh(e) + a)**2*(c**2*x + c*d*x**2 + d**2*x**3/3), True))`

### 3.107 $\int (c + dx)(a + a \cosh(e + fx))^2 dx$

**Optimal.** Leaf size=118

$$\frac{2a^2(c + dx) \sinh(e + fx)}{f} + \frac{a^2(c + dx) \sinh(e + fx) \cosh(e + fx)}{2f} + \frac{a^2(c + dx)^2}{2d} + \frac{1}{2}a^2cx - \frac{a^2d \cosh^2(e + fx)}{4f^2} - \frac{2a^2d}{f^2}$$

[Out]  $1/2*a^2*c*x + 1/4*a^2*d*x^2 + 1/2*a^2*(d*x+c)^2/d - 2*a^2*d*\cosh(f*x+e)/f^2 - 1/4*a^2*d*\cosh(f*x+e)^2/f^2 + 2*a^2*(d*x+c)*\sinh(f*x+e)/f + 1/2*a^2*(d*x+c)*\cosh(f*x+e)*\sinh(f*x+e)/f$

**Rubi [A]** time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3317, 3296, 2638, 3310}

$$\frac{2a^2(c + dx) \sinh(e + fx)}{f} + \frac{a^2(c + dx) \sinh(e + fx) \cosh(e + fx)}{2f} + \frac{a^2(c + dx)^2}{2d} + \frac{1}{2}a^2cx - \frac{a^2d \cosh^2(e + fx)}{4f^2} - \frac{2a^2d}{f^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)\*(a + a\*Cosh[e + f\*x])^2,x]

[Out]  $(a^2*c*x)/2 + (a^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) - (2*a^2*d*Cosh[e + f*x])/f^2 - (a^2*d*Cosh[e + f*x]^2)/(4*f^2) + (2*a^2*(c + d*x)*Sinh[e + f*x])/f + (a^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(2*f)$

#### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[((c + d\*x)^m\*cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m-1)\*cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*(b\*sin[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n-1))/n, Int[(c + d\*x)\*(b\*sin[e + f\*x])^(n-2), x], x] - Simp[(b\*(c + d\*x)\*cos[e + f\*x]\*(b\*sin[e + f\*x])^(n-1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rule 3317

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned} \int (c + dx)(a + a \cosh(e + fx))^2 dx &= \int (a^2(c + dx) + 2a^2(c + dx) \cosh(e + fx) + a^2(c + dx) \cosh^2(e + fx)) dx \\ &= \frac{a^2(c + dx)^2}{2d} + a^2 \int (c + dx) \cosh^2(e + fx) dx + (2a^2) \int (c + dx) \cosh(e + fx) dx \\ &= \frac{a^2(c + dx)^2}{2d} - \frac{a^2 d \cosh^2(e + fx)}{4f^2} + \frac{2a^2(c + dx) \sinh(e + fx)}{f} + \frac{a^2(c + dx)}{4f} \\ &= \frac{1}{2} a^2 c x + \frac{1}{4} a^2 d x^2 + \frac{a^2(c + dx)^2}{2d} - \frac{2a^2 d \cosh(e + fx)}{f^2} - \frac{a^2 d \cosh^2(e + fx)}{4f^2} \end{aligned}$$

**Mathematica [A]** time = 0.54, size = 81, normalized size = 0.69

$$\frac{a^2(-6(e + fx)(d(e - fx) - 2cf) + 16f(c + dx) \sinh(e + fx) + 2f(c + dx) \sinh(2(e + fx)) - 16d \cosh(e + fx) - a^2 d \cosh^2(e + fx))}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (a^2*(-6*(e + f*x)*(-2*c*f + d*(e - f*x)) - 16*d*Cosh[e + f*x] - d*Cosh[2*(e + f*x)] + 16*f*(c + d*x)*Sinh[e + f*x] + 2*f*(c + d*x)*Sinh[2*(e + f*x)])/(8*f^2)
```

**fricas [A]** time = 0.61, size = 113, normalized size = 0.96

$$\frac{6a^2df^2x^2 + 12a^2cf^2x - a^2d \cosh^2(fx + e) - a^2d \sinh^2(fx + e) - 16a^2d \cosh(fx + e) + 4(4a^2dfx + 4a^2cf - a^2d \cosh^2(fx + e)) \sinh(fx + e)}{8f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(6*a^2*d*f^2*x^2 + 12*a^2*c*f^2*x - a^2*d*cosh(f*x + e)^2 - a^2*d*sinh(f*x + e)^2 - 16*a^2*d*cosh(f*x + e) + 4*(4*a^2*d*f*x + 4*a^2*c*f + (a^2*d*f*x + a^2*c*f)*cosh(f*x + e))*sinh(f*x + e))/f^2
```

**giac** [A] time = 0.13, size = 155, normalized size = 1.31

$$\frac{3}{4}a^2dx^2 + \frac{3}{2}a^2cx + \frac{(2a^2dfx + 2a^2cf - a^2d)e^{(2fx+2e)}}{16f^2} + \frac{(a^2dfx + a^2cf - a^2d)e^{(fx+e)}}{f^2} - \frac{(a^2dfx + a^2cf + a^2d)e^{(-fx-e)}}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+a\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out]  $\frac{3}{4}a^2d*x^2 + \frac{3}{2}a^2*c*x + \frac{1}{16}*(2*a^2*d*f*x + 2*a^2*c*f - a^2*d)*e^{(2*f*x + 2*e)}/f^2 + \frac{(a^2*d*f*x + a^2*c*f - a^2*d)*e^{(f*x + e)}}{f^2} - \frac{(a^2*d*f*x + a^2*c*f + a^2*d)*e^{(-f*x - e)}}{f^2} - \frac{1}{16}*(2*a^2*d*f*x + 2*a^2*c*f + a^2*d)*e^{(-2*f*x - 2*e)}/f^2$

**maple** [A] time = 0.07, size = 211, normalized size = 1.79

$$\frac{d^2a^2(fx+e)^2}{2f} + \frac{2da^2((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f} + \frac{da^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2} + \frac{(fx+e)^2}{4} - \frac{\cosh^2(fx+e)}{4}\right)}{f} - \frac{dea^2(fx+e)}{f} - \frac{2dea^2\sinh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)\*(a+a\*cosh(f\*x+e))^2,x)

[Out]  $\frac{1}{f}*(\frac{1}{2}/f*d*a^2*(f*x+e)^2 + \frac{2}{f}*d*a^2*((f*x+e)*\sinh(f*x+e)-\cosh(f*x+e)) + \frac{1}{f}*d*a^2*(\frac{1}{2}*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e) + \frac{1}{4}*(f*x+e)^2 - \frac{1}{4}*\cosh(f*x+e)^2) - \frac{d*e}{f}*a^2*(f*x+e) - \frac{2*d*e}{f}*a^2*\sinh(f*x+e) - \frac{d*e}{f}*a^2*(\frac{1}{2}*\cosh(f*x+e)*\sinh(f*x+e) + \frac{1}{2}*f*x + \frac{1}{2}*e) + c*a^2*(f*x+e) + 2*c*a^2*\sinh(f*x+e) + c*a^2*(\frac{1}{2}*\cosh(f*x+e)*\sinh(f*x+e) + \frac{1}{2}*f*x + \frac{1}{2}*e))$

**maxima** [A] time = 0.49, size = 167, normalized size = 1.42

$$\frac{1}{2}a^2dx^2 + \frac{1}{16}\left(4x^2 + \frac{(2fxe^{(2e)} - e^{(2e)})e^{(2fx)}}{f^2} - \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)a^2d + \frac{1}{8}a^2c\left(4x + \frac{e^{(2fx+2e)}}{f} - \frac{e^{(-2fx-2e)}}{f}\right) + a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+a\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out]  $\frac{1}{2}a^2d*x^2 + \frac{1}{16}*(4*x^2 + (2*f*x*e^{(2*e)} - e^{(2*e)})*e^{(2*f*x)}/f^2 - (2*f*x + 1)*e^{(-2*f*x - 2*e)}/f^2)*a^2d + \frac{1}{8}a^2*c*(4*x + e^{(2*f*x + 2*e)}/f - e^{(-2*f*x - 2*e)}/f) + a^2*c*x + a^2*d*((f*x*e^e - e^e)*e^{(f*x)}/f^2 - (f*x + 1)*e^{(-f*x - e)}/f^2) + 2*a^2*c*\sinh(f*x + e)/f$

**mupad [B]** time = 0.13, size = 123, normalized size = 1.04

$$\frac{3a^2 dx^2}{4} + \frac{3a^2 cx}{2} - \frac{a^2 d \cosh(e + fx)^2}{4f^2} - \frac{2a^2 d \cosh(e + fx)}{f^2} + \frac{2a^2 c \sinh(e + fx)}{f} + \frac{a^2 c \cosh(e + fx) \sinh(e + fx)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cosh(e + f*x))^2*(c + d*x), x)`

[Out]  $(3a^2 dx^2)/4 + (3a^2 cx)/2 - (a^2 d \cosh(e + fx)^2)/(4f^2) - (2a^2 d \cosh(e + fx))/f^2 + (2a^2 c \sinh(e + fx))/f + (a^2 c \cosh(e + fx) \sinh(e + fx))/(2f) + (2a^2 d x \sinh(e + fx))/f + (a^2 d x \cosh(e + fx) \sinh(e + fx))/(2f)$

**sympy [A]** time = 0.67, size = 219, normalized size = 1.86

$$\left\{ \begin{array}{l} -\frac{a^2 cx \sinh^2(e+fx)}{2} + \frac{a^2 cx \cosh^2(e+fx)}{2} + a^2 cx + \frac{a^2 c \sinh(e+fx) \cosh(e+fx)}{2f} + \frac{2a^2 c \sinh(e+fx)}{f} - \frac{a^2 dx^2 \sinh^2(e+fx)}{4} + \frac{a^2 dx^2 \cosh^2(e+fx)}{4} \\ (a \cosh(e) + a)^2 \left( cx + \frac{dx^2}{2} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*(a+a*cosh(f*x+e))**2,x)`

[Out] `Piecewise((-a**2*c*x*sinh(e + f*x)**2/2 + a**2*c*x*cosh(e + f*x)**2/2 + a**2*c*x + a**2*c*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 2*a**2*c*sinh(e + f*x)/f - a**2*d*x**2*sinh(e + f*x)**2/4 + a**2*d*x**2*cosh(e + f*x)**2/4 + a**2*d*x**2/2 + a**2*d*x*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 2*a**2*d*x*sinh(e + f*x)/f - a**2*d*sinh(e + f*x)**2/(4*f**2) - 2*a**2*d*cosh(e + f*x)/f**2, Ne(f, 0)), ((a*cosh(e) + a)**2*(c*x + d*x**2/2), True))`

$$3.108 \quad \int \frac{(a+a \cosh(e+fx))^2}{c+dx} dx$$

**Optimal.** Leaf size=145

$$\frac{2a^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d} + \frac{a^2 \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{2d} + \frac{2a^2 \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d} + \frac{a^2 \sinh\left(2e - \frac{2cf}{d}\right) \operatorname{Shi}\left(2xf + \frac{2cf}{d}\right)}{2d}$$

[Out]  $\frac{1}{2}a^2 \operatorname{Chi}\left(\frac{2cf}{d} + 2fx\right) \cosh\left(-\frac{2e + 2cf/d}{d} + 2fx\right) + \frac{a^2 \operatorname{Chi}\left(\frac{cf}{d} + fx\right) \cosh\left(-\frac{e + cf/d}{d} + 3fx\right)}{3} + \frac{3a^2 \ln(d*x+c)}{2d} - \frac{1}{2}a^2 \operatorname{Shi}\left(\frac{2cf}{d} + 2fx\right) \sinh\left(-\frac{2e + 2cf/d}{d} + 2fx\right) - \frac{a^2 \operatorname{Shi}\left(\frac{cf}{d} + fx\right) \sinh\left(-\frac{e + cf/d}{d} + fx\right)}{d}$

**Rubi [A]** time = 0.34, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3318, 3312, 3303, 3298, 3301}

$$\frac{2a^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d} + \frac{a^2 \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{2d} + \frac{2a^2 \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d} + \frac{a^2 \sinh\left(2e - \frac{2cf}{d}\right) \operatorname{Shi}\left(2xf + \frac{2cf}{d}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a + a\*Cosh[e + f\*x])^2/(c + d\*x), x]

[Out]  $(2a^2 \operatorname{Cosh}\left[e - \frac{cf}{d}\right] \operatorname{CoshIntegral}\left[\frac{cf}{d} + fx\right])/d + (a^2 \operatorname{Cosh}\left[2e - \frac{2cf}{d}\right] \operatorname{CoshIntegral}\left[\frac{2cf}{d} + 2fx\right])/(2d) + (3a^2 \operatorname{Log}[c + d*x])/(2d) + (2a^2 \operatorname{Sinh}\left[e - \frac{cf}{d}\right] \operatorname{SinhIntegral}\left[\frac{cf}{d} + fx\right])/d + (a^2 \operatorname{Sinh}\left[2e - \frac{2cf}{d}\right] \operatorname{SinhIntegral}\left[\frac{2cf}{d} + 2fx\right])/(2d)$

**Rule 3298**

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

**Rule 3301**

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

**Rule 3303**

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x] /; FreeQ[{c, d, e, f}, x] &&



NeQ[d\*e - c\*f, 0]

### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rule 3318

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] := Dist[(2\*a)^n, Int[(c + d\*x)^m\*Sin[(1\*(e + (Pi\*a)/(2\*b)))/2 + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + a \cosh(e + fx))^2}{c + dx} dx &= (4a^2) \int \frac{\sin^4\left(\frac{1}{2}(ie + \pi) + \frac{ifx}{2}\right)}{c + dx} dx \\
 &= (4a^2) \int \left( \frac{3}{8(c + dx)} + \frac{\cosh(e + fx)}{2(c + dx)} + \frac{\cosh(2e + 2fx)}{8(c + dx)} \right) dx \\
 &= \frac{3a^2 \log(c + dx)}{2d} + \frac{1}{2}a^2 \int \frac{\cosh(2e + 2fx)}{c + dx} dx + (2a^2) \int \frac{\cosh(e + fx)}{c + dx} dx \\
 &= \frac{3a^2 \log(c + dx)}{2d} + \frac{1}{2} \left( a^2 \cosh\left(2e - \frac{2cf}{d}\right) \right) \int \frac{\cosh\left(\frac{2cf}{d} + 2fx\right)}{c + dx} dx + \left( 2a^2 \cosh\left( \right. \right. \\
 &= \frac{2a^2 \cosh\left(e - \frac{cf}{d}\right) \operatorname{Chi}\left(\frac{cf}{d} + fx\right)}{d} + \frac{a^2 \cosh\left(2e - \frac{2cf}{d}\right) \operatorname{Chi}\left(\frac{2cf}{d} + 2fx\right)}{2d} + \frac{3a^2 \log}{2}
 \end{aligned}$$

**Mathematica [A]** time = 0.21, size = 113, normalized size = 0.78

$$\frac{a^2 \left( 4 \operatorname{Chi}\left(f\left(\frac{c}{d} + x\right)\right) \cosh\left(e - \frac{cf}{d}\right) + \operatorname{Chi}\left(\frac{2f(c+dx)}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right) + 4 \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(f\left(\frac{c}{d} + x\right)\right) + \sinh\left(2e - \frac{2cf}{d}\right) \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Cosh[e + f\*x])^2/(c + d\*x), x]

[Out] (a^2\*(4\*Cosh[e - (c\*f)/d]\*CoshIntegral[f\*(c/d + x)] + Cosh[2\*e - (2\*c\*f)/d]\*CoshIntegral[(2\*f\*(c + d\*x))/d] + 3\*Log[c + d\*x] + 4\*Sinh[e - (c\*f)/d]\*Sin

$\text{hIntegral}[f*(c/d + x)] + \text{Sinh}[2*e - (2*c*f)/d]*\text{SinhIntegral}[(2*f*(c + d*x))/d])/(2*d)$

**fricas** [A] time = 0.43, size = 228, normalized size = 1.57

$$6 a^2 \log(dx + c) + 4 \left( a^2 \text{Ei} \left( \frac{dfx+cf}{d} \right) + a^2 \text{Ei} \left( -\frac{dfx+cf}{d} \right) \right) \cosh \left( -\frac{de-cf}{d} \right) + \left( a^2 \text{Ei} \left( \frac{2(dfx+cf)}{d} \right) + a^2 \text{Ei} \left( -\frac{2(dfx+cf)}{d} \right) \right) \cosh \left( -\frac{de-cf}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))^2/(d\*x+c),x, algorithm="fricas")

[Out]  $\frac{1}{4} * (6 * a^2 * \log(dx + c) + 4 * (a^2 * \text{Ei}((d*f*x + c*f)/d) + a^2 * \text{Ei}(-(d*f*x + c*f)/d)) * \cosh(-(d*e - c*f)/d) + (a^2 * \text{Ei}(2*(d*f*x + c*f)/d) + a^2 * \text{Ei}(-2*(d*f*x + c*f)/d)) * \cosh(-2*(d*e - c*f)/d) - 4 * (a^2 * \text{Ei}((d*f*x + c*f)/d) - a^2 * \text{Ei}(-(d*f*x + c*f)/d)) * \sinh(-(d*e - c*f)/d) - (a^2 * \text{Ei}(2*(d*f*x + c*f)/d) - a^2 * \text{Ei}(-2*(d*f*x + c*f)/d)) * \sinh(-2*(d*e - c*f)/d)) / d$

**giac** [A] time = 0.15, size = 139, normalized size = 0.96

$$\frac{a^2 \text{Ei} \left( -\frac{2(dfx+cf)}{d} \right) e^{\left( \frac{2cf}{d} - 2e \right)} + 4 a^2 \text{Ei} \left( -\frac{dfx+cf}{d} \right) e^{\left( \frac{cf}{d} - e \right)} + 4 a^2 \text{Ei} \left( \frac{dfx+cf}{d} \right) e^{\left( -\frac{cf}{d} + e \right)} + a^2 \text{Ei} \left( \frac{2(dfx+cf)}{d} \right) e^{\left( -\frac{2cf}{d} + 2e \right)} + 6 a^2 \log(dx + c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))^2/(d\*x+c),x, algorithm="giac")

[Out]  $\frac{1}{4} * (a^2 * \text{Ei}(-2*(d*f*x + c*f)/d) * e^{(2*c*f/d - 2*e)} + 4 * a^2 * \text{Ei}(-(d*f*x + c*f)/d) * e^{(c*f/d - e)} + 4 * a^2 * \text{Ei}((d*f*x + c*f)/d) * e^{(-c*f/d + e)} + a^2 * \text{Ei}(2*(d*f*x + c*f)/d) * e^{(-2*c*f/d + 2*e)} + 6 * a^2 * \log(dx + c)) / d$

**maple** [A] time = 0.37, size = 191, normalized size = 1.32

$$\frac{a^2 e^{\frac{cf-de}{d}} \text{Ei} \left( 1, fx + e + \frac{cf-de}{d} \right)}{d} - \frac{a^2 e^{-\frac{cf-de}{d}} \text{Ei} \left( 1, -fx - e - \frac{cf-de}{d} \right)}{d} + \frac{3a^2 \ln(dx + c)}{2d} - \frac{a^2 e^{\frac{2cf-2de}{d}} \text{Ei} \left( 1, 2fx + 2e + \frac{2cf-2de}{d} \right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*cosh(f\*x+e))^2/(d\*x+c),x)

[Out]  $-a^2/d * \exp((c*f-d*e)/d) * \text{Ei}(1, fx+e+(c*f-d*e)/d) - a^2/d * \exp(-(c*f-d*e)/d) * \text{Ei}(1, -fx-e-(c*f-d*e)/d) + 3/2 * a^2 * \ln(dx+c)/d - 1/4 * a^2/d * \exp(2*(c*f-d*e)/d) * \text{Ei}(1, 2*f*x+2*e+2*(c*f-d*e)/d) - 1/4 * a^2/d * \exp(-2*(c*f-d*e)/d) * \text{Ei}(1, -2*f*x-2*e-2*(c*f-d*e)/d)$

**maxima** [A] time = 0.53, size = 149, normalized size = 1.03

$$-\frac{1}{4}a^2 \left( \frac{e^{\left(-2e + \frac{2cf}{d}\right)} E_1\left(\frac{2(dx+c)f}{d}\right)}{d} + \frac{e^{\left(2e - \frac{2cf}{d}\right)} E_1\left(-\frac{2(dx+c)f}{d}\right)}{d} - \frac{2 \log(dx+c)}{d} \right) - a^2 \left( \frac{e^{\left(-e + \frac{cf}{d}\right)} E_1\left(\frac{(dx+c)f}{d}\right)}{d} + \frac{e^{\left(e - \frac{cf}{d}\right)} E_1\left(-\frac{(dx+c)f}{d}\right)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))^2/(d\*x+c),x, algorithm="maxima")

[Out]  $-1/4*a^2*(e^{(-2*e + 2*c*f/d)*exp\_integral\_e(1, 2*(d*x + c)*f/d)/d} + e^{(2*e - 2*c*f/d)*exp\_integral\_e(1, -2*(d*x + c)*f/d)/d} - 2*\log(d*x + c)/d) - a^2*(e^{(-e + c*f/d)*exp\_integral\_e(1, (d*x + c)*f/d)/d} + e^{(e - c*f/d)*exp\_integral\_e(1, -(d*x + c)*f/d)/d}) + a^2*\log(d*x + c)/d$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \cosh(e + f x))^2}{c + d x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cosh(e + f\*x))^2/(c + d\*x),x)

[Out] int((a + a\*cosh(e + f\*x))^2/(c + d\*x), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left( \int \frac{2 \cosh(e + f x)}{c + d x} dx + \int \frac{\cosh^2(e + f x)}{c + d x} dx + \int \frac{1}{c + d x} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))\*\*2/(d\*x+c),x)

[Out]  $a**2*(Integral(2*cosh(e + f*x)/(c + d*x), x) + Integral(cosh(e + f*x)**2/(c + d*x), x) + Integral(1/(c + d*x), x))$

$$3.109 \quad \int \frac{(a+a \cosh(e+fx))^2}{(c+dx)^2} dx$$

**Optimal.** Leaf size=157

$$\frac{a^2 f \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{d^2} + \frac{2a^2 f \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{2a^2 f \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^2} + \frac{a^2 f c}{d^2}$$

[Out]  $-4*a^2*\cosh(1/2*e+1/2*f*x)^4/d/(d*x+c)+2*a^2*f*\cosh(-e+c*f/d)*\operatorname{Shi}(c*f/d+f*x)/d^2+a^2*f*\cosh(-2*e+2*c*f/d)*\operatorname{Shi}(2*c*f/d+2*f*x)/d^2-a^2*f*\operatorname{Chi}(2*c*f/d+2*f*x)*\sinh(-2*e+2*c*f/d)/d^2-2*a^2*f*\operatorname{Chi}(c*f/d+f*x)*\sinh(-e+c*f/d)/d^2$

**Rubi [A]** time = 0.33, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3318, 3313, 3303, 3298, 3301}

$$\frac{a^2 f \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{d^2} + \frac{2a^2 f \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{2a^2 f \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^2} + \frac{a^2 f c}{d^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + a*\operatorname{Cosh}[e + f*x])^2/(c + d*x)^2, x]$

[Out]  $(-4*a^2*\operatorname{Cosh}[e/2 + (f*x)/2]^4)/(d*(c + d*x)) + (a^2*f*\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x]*\operatorname{Sinh}[2*e - (2*c*f)/d])/d^2 + (2*a^2*f*\operatorname{CoshIntegral}[(c*f)/d + f*x]*\operatorname{Sinh}[e - (c*f)/d])/d^2 + (2*a^2*f*\operatorname{Cosh}[e - (c*f)/d]*\operatorname{SinhIntegral}[(c*f)/d + f*x])/d^2 + (a^2*f*\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/d^2$

### Rule 3298

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[(I*\operatorname{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \operatorname{EqQ}[d*e - c*f*fz*I, 0]$

### Rule 3301

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \operatorname{EqQ}[d*(e - \operatorname{Pi}/2) - c*f*fz*I, 0]$

### Rule 3303

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Dist}[\operatorname{Cos}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \operatorname{Dist}[\operatorname{Sin}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Cos}[(c*f)/d + f*x]/(c + d*x), x], x]$

) / d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&  
 NeQ[d\*e - c\*f, 0]

### Rule 3313

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] :> Si  
 mp[((c + d\*x)^(m + 1)\*Sin[e + f\*x]^n)/(d\*(m + 1)), x] - Dist[(f\*n)/(d\*(m +  
 1)), Int[ExpandTrigReduce[(c + d\*x)^(m + 1), Cos[e + f\*x]\*Sin[e + f\*x]^(n -  
 1), x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&  
 LtQ[m, -1]

### Rule 3318

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_.)  
 , x\_Symbol] :> Dist[(2\*a)^n, Int[(c + d\*x)^m\*Sin[(1\*(e + (Pi\*a)/(2\*b)))/2 +  
 (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2  
 , 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + a \cosh(e + fx))^2}{(c + dx)^2} dx &= (4a^2) \int \frac{\sin^4\left(\frac{1}{2}(ie + \pi) + \frac{ifx}{2}\right)}{(c + dx)^2} dx \\
 &= -\frac{4a^2 \cosh^4\left(\frac{e}{2} + \frac{fx}{2}\right)}{d(c + dx)} + \frac{(8ia^2f) \int \left(-\frac{i \sinh(e+fx)}{4(c+dx)} - \frac{i \sinh(2e+2fx)}{8(c+dx)}\right) dx}{d} \\
 &= -\frac{4a^2 \cosh^4\left(\frac{e}{2} + \frac{fx}{2}\right)}{d(c + dx)} + \frac{(a^2f) \int \frac{\sinh(2e+2fx)}{c+dx} dx}{d} + \frac{(2a^2f) \int \frac{\sinh(e+fx)}{c+dx} dx}{d} \\
 &= -\frac{4a^2 \cosh^4\left(\frac{e}{2} + \frac{fx}{2}\right)}{d(c + dx)} + \frac{\left(a^2f \cosh\left(2e - \frac{2cf}{d}\right)\right) \int \frac{\sinh\left(\frac{2cf}{d} + 2fx\right)}{c+dx} dx}{d} + \frac{(2a^2f \cosh(e + fx)) \int \frac{\sinh(e+fx)}{c+dx} dx}{d} \\
 &= -\frac{4a^2 \cosh^4\left(\frac{e}{2} + \frac{fx}{2}\right)}{d(c + dx)} + \frac{a^2f \operatorname{Chi}\left(\frac{2cf}{d} + 2fx\right) \sinh\left(2e - \frac{2cf}{d}\right)}{d^2} + \frac{2a^2f \operatorname{Chi}\left(\frac{cf}{d} + fx\right) \sinh(e + fx)}{d^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.74, size = 207, normalized size = 1.32

$$a^2 \left( 2f(c + dx) \operatorname{Chi}\left(\frac{2f(c+dx)}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right) + 4f(c + dx) \operatorname{Chi}\left(f\left(\frac{c}{d} + x\right)\right) \sinh\left(e - \frac{cf}{d}\right) + 4dfx \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(\frac{cf}{d} + fx\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (a^2*(-3*d - 4*d*Cosh[e + f*x] - d*Cosh[2*(e + f*x)] + 2*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*Sinh[2*e - (2*c*f)/d] + 4*f*(c + d*x)*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + 4*c*f*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + 4*d*f*x*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + 2*c*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] + 2*d*f*x*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d]))/(2*d^2*(c + d*x))
```

**fricas [B]** time = 0.57, size = 359, normalized size = 2.29

$$a^2 d \cosh(fx + e)^2 + a^2 d \sinh(fx + e)^2 + 4 a^2 d \cosh(fx + e) + 3 a^2 d - 2 \left( (a^2 d f x + a^2 c f) \operatorname{Ei} \left( \frac{d f x + c f}{d} \right) - (a^2 d f x \right.$$


---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*d*cosh(f*x + e)^2 + a^2*d*sinh(f*x + e)^2 + 4*a^2*d*cosh(f*x + e) + 3*a^2*d - 2*((a^2*d*f*x + a^2*c*f)*Ei((d*f*x + c*f)/d) - (a^2*d*f*x + a^2*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - ((a^2*d*f*x + a^2*c*f)*Ei(2*(d*f*x + c*f)/d) - (a^2*d*f*x + a^2*c*f)*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*(d*e - c*f)/d) + 2*((a^2*d*f*x + a^2*c*f)*Ei((d*f*x + c*f)/d) + (a^2*d*f*x + a^2*c*f)*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) + ((a^2*d*f*x + a^2*c*f)*Ei(2*(d*f*x + c*f)/d) + (a^2*d*f*x + a^2*c*f)*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^3*x + c*d^2)
```

**giac [B]** time = 0.24, size = 1226, normalized size = 7.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*(d*x + c)*a^2*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*Ei(2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^(2*(c*f - d*e)/d) - 2*a^2*c*f^3*Ei(2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^(2*(c*f - d*e)/d) + 4*(d*x + c)*a^2*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*Ei(((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^(((c*f - d*e)/d) - 4*a^2*c*f^3*Ei(((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^(((c*f - d*e)/d) - 4*(d*x + c)*a^2*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*Ei(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))
```

$$\begin{aligned}
& -c*f + d*e)/d)*e^{-(c*f - d*e)/d} + 4*a^2*c*f^3*Ei(-((d*x + c)*(c*f/(d*x + c) \\
& + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^{-(c*f - d*e)/d} - 2*(d*x + c)* \\
& a^2*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*Ei(-2*((d*x + c)*(c*f/(d*x + c) \\
& - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^{-2*(c*f - d*e)/d} + 2*a^2*c*f^3*Ei \\
& (-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^{-2*(c \\
& *f - d*e)/d} + 2*a^2*d*f^2*Ei(2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + \\
& c)) - c*f + d*e)/d)*e^{2*(c*f - d*e)/d + 1} + 4*a^2*d*f^2*Ei(((d*x + c)*(c* \\
& f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^{((c*f - d*e)/d + 1) - 4* \\
& a^2*d*f^2*Ei(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d \\
& ))*e^{-(c*f - d*e)/d + 1} - 2*a^2*d*f^2*Ei(-2*((d*x + c)*(c*f/(d*x + c) - f \\
& - d*e/(d*x + c)) - c*f + d*e)/d)*e^{-2*(c*f - d*e)/d + 1} - a^2*d*f^2*e^{2* \\
& (d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))/d} - 4*a^2*d*f^2*e^{((d*x + c) \\
& *(c*f/(d*x + c) - f - d*e/(d*x + c))/d} - 4*a^2*d*f^2*e^{-(d*x + c)*(c*f/(d \\
& *x + c) - f - d*e/(d*x + c))/d} - a^2*d*f^2*e^{-2*(d*x + c)*(c*f/(d*x + c) \\
& - f - d*e/(d*x + c))/d} - 6*a^2*d*f^2)*d^2/(((d*x + c)*d^4*(c*f/(d*x + c) - \\
& f - d*e/(d*x + c)) - c*d^4*f + d^5*e)*f)
\end{aligned}$$

**maple [A]** time = 0.38, size = 308, normalized size = 1.96

$$\frac{f a^2 e^{-f x - e}}{d (d f x + c f)} + \frac{f a^2 e^{\frac{c f - d e}{d}} \operatorname{Ei}\left(1, f x + e + \frac{c f - d e}{d}\right)}{d^2} - \frac{f a^2 e^{f x + e}}{d^2 \left(\frac{c f}{d} + f x\right)} - \frac{f a^2 e^{-\frac{c f - d e}{d}} \operatorname{Ei}\left(1, -f x - e - \frac{c f - d e}{d}\right)}{d^2} - \frac{3 a^2}{2 d (d x + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*cosh(f\*x+e))^2/(d\*x+c)^2,x)

[Out]  $-f*a^2*\exp(-f*x-e)/d/(d*f*x+c*f)+f*a^2/d^2*\exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-f*a^2/d^2*\exp(f*x+e)/(c*f/d+f*x)-f*a^2/d^2*\exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)-3/2*a^2/d/(d*x+c)-1/4*f*a^2*\exp(-2*f*x-2*e)/d/(d*f*x+c*f)+1/2*f*a^2/d^2*\exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/4*f*a^2/d^2*\exp(2*f*x+2*e)/(c*f/d+f*x)-1/2*f*a^2/d^2*\exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)$

**maxima [A]** time = 0.58, size = 182, normalized size = 1.16

$$-\frac{1}{4} a^2 \left( \frac{e^{\left(-2e + \frac{2cf}{d}\right)} E_2\left(\frac{2(dx+cf)}{d}\right)}{(dx+c)d} + \frac{e^{\left(2e - \frac{2cf}{d}\right)} E_2\left(-\frac{2(dx+cf)}{d}\right)}{(dx+c)d} + \frac{2}{d^2 x + cd} \right) - a^2 \left( \frac{e^{\left(-e + \frac{cf}{d}\right)} E_2\left(\frac{(dx+cf)}{d}\right)}{(dx+c)d} + \frac{e^{\left(e - \frac{cf}{d}\right)} E_2\left(-\frac{(dx+cf)}{d}\right)}{(dx+c)d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))^2/(d\*x+c)^2,x, algorithm="maxima")

```
[Out] -1/4*a^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)*
d) + e^(2*e - 2*c*f/d)*exp_integral_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d) +
2/(d^2*x + c*d) - a^2*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/((d
*x + c)*d) + e^(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d))
- a^2/(d^2*x + c*d)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \cosh(e + fx))^2}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))^2/(c + d*x)^2,x)
```

```
[Out] int((a + a*cosh(e + f*x))^2/(c + d*x)^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left( \int \frac{2 \cosh(e + fx)}{c^2 + 2cdx + d^2x^2} dx + \int \frac{\cosh^2(e + fx)}{c^2 + 2cdx + d^2x^2} dx + \int \frac{1}{c^2 + 2cdx + d^2x^2} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*cosh(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(
cosh(e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c**2 + 2*c*
d*x + d**2*x**2), x))
```



$$3.110 \quad \int \frac{(a+a \cosh(e+fx))^2}{(c+dx)^3} dx$$

**Optimal.** Leaf size=207

$$\frac{a^2 f^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d^3} + \frac{a^2 f^2 \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{d^3} + \frac{a^2 f^2 \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^3} + \dots$$

[Out]  $a^2 f^2 \operatorname{Chi}(2cf/d + 2fx) \cosh(-2e + 2cf/d)/d^3 + a^2 f^2 \operatorname{Chi}(cf/d + fx) \cosh(-e + cf/d)/d^3 - 2a^2 \cosh(1/2e + 1/2fx)^4/d/(dx+c)^2 - a^2 f^2 \operatorname{Shi}(2cf/d + 2fx) \sinh(-2e + 2cf/d)/d^3 - a^2 f^2 \operatorname{Shi}(cf/d + fx) \sinh(-e + cf/d)/d^3 - 4a^2 f \cosh(1/2e + 1/2fx)^3 \sinh(1/2e + 1/2fx)/d^2/(dx+c)$

**Rubi [A]** time = 0.50, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {3318, 3314, 3312, 3303, 3298, 3301}

$$\frac{a^2 f^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d^3} + \frac{a^2 f^2 \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{d^3} + \frac{a^2 f^2 \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^3} + \dots$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + a \operatorname{Cosh}[e + fx])^2/(c + dx)^3, x]$

[Out]  $(-2a^2 \operatorname{Cosh}[e/2 + (fx)/2]^4)/(d(c + dx)^2) + (a^2 f^2 \operatorname{Cosh}[e - (cf)/d] \operatorname{CoshIntegral}[(cf)/d + fx])/d^3 + (a^2 f^2 \operatorname{Cosh}[2e - (2cf)/d] \operatorname{CoshIntegral}[(2cf)/d + 2fx])/d^3 - (4a^2 f \operatorname{Cosh}[e/2 + (fx)/2]^3 \operatorname{Sinh}[e/2 + (fx)/2])/(d^2(c + dx)) + (a^2 f^2 \operatorname{Sinh}[e - (cf)/d] \operatorname{SinhIntegral}[(cf)/d + fx])/d^3 + (a^2 f^2 \operatorname{Sinh}[2e - (2cf)/d] \operatorname{SinhIntegral}[(2cf)/d + 2fx])/d^3$

### Rule 3298

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[(I \operatorname{SinhIntegral}[(cf*fz)/d + f*fz*x])/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz, x\} \ \&\& \operatorname{EqQ}[d*e - cf*fz*I, 0]$

### Rule 3301

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CoshIntegral}[(cf*fz)/d + f*fz*x]/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz, x\} \ \&\& \operatorname{EqQ}[d*(e - \operatorname{Pi}/2) - cf*fz*I, 0]$

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

### Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbo
l] := Simp[((c + d*x)^(m + 1)*(b*Ssin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Ssin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Ssin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*(b*Ssin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

### Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.)
, x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*Ssin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + a \cosh(e + fx))^2}{(c + dx)^3} dx &= (4a^2) \int \frac{\sin^4\left(\frac{1}{2}(ie + \pi) + \frac{ifx}{2}\right)}{(c + dx)^3} dx \\
&= -\frac{2a^2 \cosh^4\left(\frac{e}{2} + \frac{fx}{2}\right)}{d(c + dx)^2} - \frac{4a^2 f \cosh^3\left(\frac{e}{2} + \frac{fx}{2}\right) \sinh\left(\frac{e}{2} + \frac{fx}{2}\right)}{d^2(c + dx)} - \frac{(6a^2 f^2) \int \frac{\cosh^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{c + dx}}{d^2} \\
&= -\frac{2a^2 \cosh^4\left(\frac{e}{2} + \frac{fx}{2}\right)}{d(c + dx)^2} - \frac{4a^2 f \cosh^3\left(\frac{e}{2} + \frac{fx}{2}\right) \sinh\left(\frac{e}{2} + \frac{fx}{2}\right)}{d^2(c + dx)} - \frac{(6a^2 f^2) \int \left(\frac{1}{2(c + dx)}\right)}{d^2} \\
&= -\frac{2a^2 \cosh^4\left(\frac{e}{2} + \frac{fx}{2}\right)}{d(c + dx)^2} - \frac{4a^2 f \cosh^3\left(\frac{e}{2} + \frac{fx}{2}\right) \sinh\left(\frac{e}{2} + \frac{fx}{2}\right)}{d^2(c + dx)} + \frac{(a^2 f^2) \int \frac{\cosh(2e + 2fx)}{c + dx}}{d^2} \\
&= -\frac{2a^2 \cosh^4\left(\frac{e}{2} + \frac{fx}{2}\right)}{d(c + dx)^2} - \frac{4a^2 f \cosh^3\left(\frac{e}{2} + \frac{fx}{2}\right) \sinh\left(\frac{e}{2} + \frac{fx}{2}\right)}{d^2(c + dx)} + \frac{(a^2 f^2 \cosh\left(2e - \frac{2cf}{d}\right))}{d^2} \\
&= -\frac{2a^2 \cosh^4\left(\frac{e}{2} + \frac{fx}{2}\right)}{d(c + dx)^2} + \frac{a^2 f^2 \cosh\left(e - \frac{cf}{d}\right) \text{Chi}\left(\frac{cf}{d} + fx\right)}{d^3} + \frac{a^2 f^2 \cosh\left(2e - \frac{2cf}{d}\right)}{d^3}
\end{aligned}$$

**Mathematica [A]** time = 1.22, size = 353, normalized size = 1.71

$$a^2 \left( 4c^2 f^2 \sinh\left(e - \frac{cf}{d}\right) \text{Shi}\left(f\left(\frac{c}{d} + x\right)\right) + 4c^2 f^2 \sinh\left(2e - \frac{2cf}{d}\right) \text{Shi}\left(\frac{2f(c+dx)}{d}\right) + 4f^2(c + dx)^2 \text{Chi}\left(f\left(\frac{c}{d} + x\right)\right) \cos\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Cosh[e + f\*x])^2/(c + d\*x)^3,x]

[Out] (a^2\*(-3\*d^2 - 4\*d^2\*Cosh[e + f\*x] - d^2\*Cosh[2\*(e + f\*x)] + 4\*f^2\*(c + d\*x)^2\*Cosh[e - (c\*f)/d]\*CoshIntegral[f\*(c/d + x)] + 4\*f^2\*(c + d\*x)^2\*Cosh[2\*e - (2\*c\*f)/d]\*CoshIntegral[(2\*f\*(c + d\*x))/d] - 4\*c\*d\*f\*Sinh[e + f\*x] - 4\*d^2\*f\*x\*Sinh[e + f\*x] - 2\*c\*d\*f\*Sinh[2\*(e + f\*x)] - 2\*d^2\*f\*x\*Sinh[2\*(e + f\*x)] + 4\*c^2\*f^2\*Sinh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)] + 8\*c\*d\*f^2\*x\*Sinh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)] + 4\*d^2\*f^2\*x^2\*Sinh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)] + 4\*c^2\*f^2\*Sinh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*f\*(c + d\*x))/d] + 8\*c\*d\*f^2\*x\*Sinh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*f\*(c + d\*x))/d] + 4\*d^2\*f^2\*x^2\*Sinh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*f\*(c + d\*x))/d]))/(4\*d^3\*(c + d\*x)^2)

**fricas** [B] time = 0.50, size = 596, normalized size = 2.88

$$a^2 d^2 \cosh(fx + e)^2 + a^2 d^2 \sinh(fx + e)^2 + 4 a^2 d^2 \cosh(fx + e) + 3 a^2 d^2 - 2 \left( (a^2 d^2 f^2 x^2 + 2 a^2 c d f^2 x + a^2 c^2 f^2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))^2/(d\*x+c)^3,x, algorithm="fricas")

[Out]  $-1/4*(a^2*d^2*\cosh(f*x + e)^2 + a^2*d^2*\sinh(f*x + e)^2 + 4*a^2*d^2*\cosh(f*x + e) + 3*a^2*d^2 - 2*((a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei((d*f*x + c*f)/d) + (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*\cosh(-(d*e - c*f)/d) - 2*((a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d) + (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*\cosh(-2*(d*e - c*f)/d) + 4*(a^2*d^2*f*x + a^2*c*d*f + (a^2*d^2*f*x + a^2*c*d*f)*\cosh(f*x + e))*\sinh(f*x + e) + 2*((a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei((d*f*x + c*f)/d) - (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*\sinh(-(d*e - c*f)/d) + 2*((a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d) - (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*\sinh(-2*(d*e - c*f)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)$

**giac** [B] time = 0.14, size = 706, normalized size = 3.41

$$4 a^2 d^2 f^2 x^2 Ei\left(-\frac{2(dfxc+cf)}{d}\right) e^{\left(\frac{2cf}{d}-2e\right)} + 4 a^2 d^2 f^2 x^2 Ei\left(-\frac{dfxc+cf}{d}\right) e^{\left(\frac{cf}{d}-e\right)} + 4 a^2 d^2 f^2 x^2 Ei\left(\frac{dfxc+cf}{d}\right) e^{\left(-\frac{cf}{d}+e\right)} + 4 a^2 d^2 f^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))^2/(d\*x+c)^3,x, algorithm="giac")

[Out]  $1/8*(4*a^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^{(2*c*f/d - 2*e)} + 4*a^2*d^2*f^2*x^2*Ei(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} + 4*a^2*d^2*f^2*x^2*Ei((d*f*x + c*f)/d)*e^{(-c*f/d + e)} + 4*a^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^{(-2*c*f/d + 2*e)} + 8*a^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f)/d)*e^{(2*c*f/d - 2*e)} + 8*a^2*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} + 8*a^2*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^{(-c*f/d + e)} + 8*a^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e^{(-2*c*f/d + 2*e)} + 4*a^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^{(2*c*f/d - 2*e)} + 4*a^2*c^2*f^2*Ei(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} + 4*a^2*c^2*f^2*Ei((d*f*x + c*f)/d)*e^{(-c*f/d + e)} + 4*a^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^{(-2*c*f/d + 2*e)} - 2*a^2*d^2*f*x*e^{(2*f*x + 2*e)} - 4*a^2*d^2*f*x*e^{(f*x + e)} + 4*a^2*d^2*f*x*e^{(-f*x - e)} + 2*a^2*d^2*f*x*e^{(-2*f*x - 2*e)} - 2*a^2*c*d*f*e^{(2*f*x + 2*e)}$

) - 4\*a^2\*c\*d\*f\*e^(f\*x + e) + 4\*a^2\*c\*d\*f\*e^(-f\*x - e) + 2\*a^2\*c\*d\*f\*e^(-2\*f\*x - 2\*e) - a^2\*d^2\*e^(2\*f\*x + 2\*e) - 4\*a^2\*d^2\*e^(f\*x + e) - 4\*a^2\*d^2\*e^(-f\*x - e) - a^2\*d^2\*e^(-2\*f\*x - 2\*e) - 6\*a^2\*d^2)/(d^5\*x^2 + 2\*c\*d^4\*x + c^2\*d^3)

**maple [B]** time = 0.39, size = 618, normalized size = 2.99

$$\frac{f^3 a^2 e^{-f x - e} x}{2d(d^2 f^2 x^2 + 2cd f^2 x + c^2 f^2)} + \frac{f^3 a^2 e^{-f x - e} c}{2d^2(d^2 f^2 x^2 + 2cd f^2 x + c^2 f^2)} - \frac{f^2 a^2 e^{-f x - e}}{2d(d^2 f^2 x^2 + 2cd f^2 x + c^2 f^2)} - \frac{f^2 a^2 e^{\frac{cf - de}{d}} \text{Ei}\left(1, \frac{cf - de}{d}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*cosh(f\*x+e))^2/(d\*x+c)^3,x)

[Out] 1/2\*f^3\*a^2\*exp(-f\*x-e)/d/(d^2\*f^2\*x^2+2\*c\*d\*f^2\*x+c^2\*f^2)\*x+1/2\*f^3\*a^2\*exp(-f\*x-e)/d^2/(d^2\*f^2\*x^2+2\*c\*d\*f^2\*x+c^2\*f^2)\*c-1/2\*f^2\*a^2\*exp(-f\*x-e)/d/(d^2\*f^2\*x^2+2\*c\*d\*f^2\*x+c^2\*f^2)-1/2\*f^2\*a^2/d^3\*exp((c\*f-d\*e)/d)\*Ei(1,f\*x+e+(c\*f-d\*e)/d)-1/2\*a^2\*f^2/d^3\*exp(f\*x+e)/(c\*f/d+f\*x)^2-1/2\*a^2\*f^2/d^3\*exp(f\*x+e)/(c\*f/d+f\*x)-1/2\*a^2\*f^2/d^3\*exp(-(c\*f-d\*e)/d)\*Ei(1,-f\*x-e-(c\*f-d\*e)/d)-3/4\*a^2/d/(d\*x+c)^2+1/4\*f^3\*a^2\*exp(-2\*f\*x-2\*e)/d/(d^2\*f^2\*x^2+2\*c\*d\*f^2\*x+c^2\*f^2)\*x+1/4\*f^3\*a^2\*exp(-2\*f\*x-2\*e)/d^2/(d^2\*f^2\*x^2+2\*c\*d\*f^2\*x+c^2\*f^2)\*c-1/8\*f^2\*a^2\*exp(-2\*f\*x-2\*e)/d/(d^2\*f^2\*x^2+2\*c\*d\*f^2\*x+c^2\*f^2)-1/2\*f^2\*a^2/d^3\*exp(2\*(c\*f-d\*e)/d)\*Ei(1,2\*f\*x+2\*e+2\*(c\*f-d\*e)/d)-1/8\*f^2\*a^2/d^3\*exp(2\*f\*x+2\*e)/(c\*f/d+f\*x)^2-1/4\*f^2\*a^2/d^3\*exp(2\*f\*x+2\*e)/(c\*f/d+f\*x)-1/2\*f^2\*a^2/d^3\*exp(-2\*(c\*f-d\*e)/d)\*Ei(1,-2\*f\*x-2\*e-2\*(c\*f-d\*e)/d)

**maxima [A]** time = 0.52, size = 202, normalized size = 0.98

$$-\frac{1}{4} a^2 \left( \frac{1}{d^3 x^2 + 2cd^2 x + c^2 d} + \frac{e^{\left(-2e + \frac{2cf}{d}\right)} E_3\left(\frac{2(dx+c)f}{d}\right)}{(dx+c)^2 d} + \frac{e^{\left(2e - \frac{2cf}{d}\right)} E_3\left(-\frac{2(dx+c)f}{d}\right)}{(dx+c)^2 d} \right) - a^2 \left( \frac{e^{\left(-e + \frac{cf}{d}\right)} E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2 d} + \frac{e^{\left(e - \frac{cf}{d}\right)} E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2 d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))^2/(d\*x+c)^3,x, algorithm="maxima")

[Out] -1/4\*a^2\*(1/(d^3\*x^2 + 2\*c\*d^2\*x + c^2\*d) + e^(-2\*e + 2\*c\*f/d)\*exp\_integral\_e(3, 2\*(d\*x + c)\*f/d)/((d\*x + c)^2\*d) + e^(2\*e - 2\*c\*f/d)\*exp\_integral\_e(3, -2\*(d\*x + c)\*f/d)/((d\*x + c)^2\*d)) - a^2\*(e^(-e + c\*f/d)\*exp\_integral\_e(3, (d\*x + c)\*f/d)/((d\*x + c)^2\*d) + e^(e - c\*f/d)\*exp\_integral\_e(3, -(d\*x + c)\*f/d)/((d\*x + c)^2\*d)) - 1/2\*a^2/(d^3\*x^2 + 2\*c\*d^2\*x + c^2\*d)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + a \cosh(e + f x))^2}{(c + d x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cosh(e + f\*x))^2/(c + d\*x)^3,x)

[Out] int((a + a\*cosh(e + f\*x))^2/(c + d\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left( \int \frac{2 \cosh(e + f x)}{c^3 + 3c^2 dx + 3cd^2 x^2 + d^3 x^3} dx + \int \frac{\cosh^2(e + f x)}{c^3 + 3c^2 dx + 3cd^2 x^2 + d^3 x^3} dx + \int \frac{1}{c^3 + 3c^2 dx + 3cd^2 x^2 + d^3 x^3} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(f\*x+e))\*\*2/(d\*x+c)\*\*3,x)

[Out] a\*\*2\*(Integral(2\*cosh(e + f\*x)/(c\*\*3 + 3\*c\*\*2\*d\*x + 3\*c\*d\*\*2\*x\*\*2 + d\*\*3\*x\*\*3), x) + Integral(cosh(e + f\*x)\*\*2/(c\*\*3 + 3\*c\*\*2\*d\*x + 3\*c\*d\*\*2\*x\*\*2 + d\*\*3\*x\*\*3), x) + Integral(1/(c\*\*3 + 3\*c\*\*2\*d\*x + 3\*c\*d\*\*2\*x\*\*2 + d\*\*3\*x\*\*3), x))

$$3.111 \quad \int \frac{(c+dx)^3}{a+a \cosh(e+fx)} dx$$

Optimal. Leaf size=117

$$\frac{12d^2(c+dx)\text{Li}_2(-e^{e+fx})}{af^3} - \frac{6d(c+dx)^2 \log(e^{e+fx}+1)}{af^2} + \frac{(c+dx)^3 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} + \frac{(c+dx)^3}{af} + \frac{12d^3\text{Li}_3(-e^{e+fx})}{af^4}$$

[Out] (d\*x+c)^3/a/f-6\*d\*(d\*x+c)^2\*ln(1+exp(f\*x+e))/a/f^2-12\*d^2\*(d\*x+c)\*polylog(2, -exp(f\*x+e))/a/f^3+12\*d^3\*polylog(3, -exp(f\*x+e))/a/f^4+(d\*x+c)^3\*tanh(1/2\*e+1/2\*f\*x)/a/f

**Rubi [A]** time = 0.27, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$ , Rules used = {3318, 4184, 3718, 2190, 2531, 2282, 6589}

$$\frac{12d^2(c+dx)\text{PolyLog}(2, -e^{e+fx})}{af^3} + \frac{12d^3\text{PolyLog}(3, -e^{e+fx})}{af^4} - \frac{6d(c+dx)^2 \log(e^{e+fx}+1)}{af^2} + \frac{(c+dx)^3 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^3/(a + a\*Cosh[e + f\*x]), x]

[Out] (c + d\*x)^3/(a\*f) - (6\*d\*(c + d\*x)^2\*Log[1 + E^(e + f\*x)])/(a\*f^2) - (12\*d^2\*(c + d\*x)\*PolyLog[2, -E^(e + f\*x)])/(a\*f^3) + (12\*d^3\*PolyLog[3, -E^(e + f\*x)])/(a\*f^4) + ((c + d\*x)^3\*Tanh[e/2 + (f\*x)/2])/(a\*f)

#### Rule 2190

Int[(((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\* (F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 3318

```
Int[((c_.) + (d_.)*(x_)^(m_.))*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 3718

```
Int[((c_.) + (d_.)*(x_)^(m_.))*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_)^(m_.), x_Symbol] := -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(c+dx)^3}{a+a \cosh(e+fx)} dx &= \frac{\int (c+dx)^3 \csc^2\left(\frac{1}{2}(ie+\pi) + \frac{ifx}{2}\right) dx}{2a} \\
&= \frac{(c+dx)^3 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} - \frac{(3d) \int (c+dx)^2 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right) dx}{af} \\
&= \frac{(c+dx)^3}{af} + \frac{(c+dx)^3 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} - \frac{(6d) \int \frac{e^{2\left(\frac{e}{2} + \frac{fx}{2}\right)} (c+dx)^2}{1+e^{2\left(\frac{e}{2} + \frac{fx}{2}\right)}} dx}{af} \\
&= \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2 \log(1+e^{e+fx})}{af^2} + \frac{(c+dx)^3 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} + \frac{(12d^2) \int (c+dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right) dx}{af^2} \\
&= \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2 \log(1+e^{e+fx})}{af^2} - \frac{12d^2(c+dx) \text{Li}_2(-e^{e+fx})}{af^3} + \frac{(c+dx)^3 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} \\
&= \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2 \log(1+e^{e+fx})}{af^2} - \frac{12d^2(c+dx) \text{Li}_2(-e^{e+fx})}{af^3} + \frac{(c+dx)^3 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} \\
&= \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2 \log(1+e^{e+fx})}{af^2} - \frac{12d^2(c+dx) \text{Li}_2(-e^{e+fx})}{af^3} + \frac{12d^3 \text{Li}_3(-e^{e+fx})}{af^4}
\end{aligned}$$

**Mathematica [A]** time = 2.30, size = 154, normalized size = 1.32

$$\frac{2 \cosh\left(\frac{1}{2}(e+fx)\right) \left(2 \cosh\left(\frac{1}{2}(e+fx)\right) \left(6d^2 (f(c+dx) \text{Li}_2(-e^{-e-fx}) + d \text{Li}_3(-e^{-e-fx})) - \frac{f^3(c+dx)^3}{e^{e+1}} - 3df^2(c+dx)\right) \right)}{af^4(\cosh(e+fx)+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(c+d\*x)^3/(a+a\*Cosh[e+f\*x]),x]

[Out] (2\*Cosh[(e+f\*x)/2]\*(2\*Cosh[(e+f\*x)/2]\*(-(f^3\*(c+d\*x)^3)/(1+E^e)) - 3\*d\*f^2\*(c+d\*x)^2\*Log[1+E^(-e-f\*x)] + 6\*d^2\*(f\*(c+d\*x)\*PolyLog[2,-E^(-e-f\*x)] + d\*PolyLog[3,-E^(-e-f\*x)])) + f^3\*(c+d\*x)^3\*Sech[e/2]\*Sinh[(f\*x)/2))/(a\*f^4\*(1+Cosh[e+f\*x]))

**fricas [C]** time = 0.64, size = 438, normalized size = 3.74

$$\frac{2(d^3e^3 - 3cd^2e^2f + 3c^2def^2 - c^3f^3 + (d^3f^3x^3 + 3cd^2f^3x^2 + 3c^2df^3x + d^3e^3 - 3cd^2e^2f + 3c^2def^2) \cosh(fx))}{af^4(\cosh(e+fx)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+a\*cosh(f\*x+e)),x, algorithm="fricas")

[Out]  $2*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3 + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2)*\cosh(f*x + e) - 6*(d^3*f*x + c*d^2*f + (d^3*f*x + c*d^2*f)*\cosh(f*x + e) + (d^3*f*x + c*d^2*f)*\sinh(f*x + e))*\operatorname{dilog}(-\cosh(f*x + e) - \sinh(f*x + e)) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*\cosh(f*x + e) + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*\sinh(f*x + e))*\log(\cosh(f*x + e) + \sinh(f*x + e) + 1) + 6*(d^3*\cosh(f*x + e) + d^3*\sinh(f*x + e) + d^3)*\operatorname{polylog}(3, -\cosh(f*x + e) - \sinh(f*x + e)) + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2)*\sinh(f*x + e))/(a*f^4*\cosh(f*x + e) + a*f^4*\sinh(f*x + e) + a*f^4)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^3}{a \cosh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+a\*cosh(f\*x+e)),x, algorithm="giac")

[Out] integrate((d\*x + c)^3/(a\*cosh(f\*x + e) + a), x)

**maple** [B] time = 0.24, size = 325, normalized size = 2.78

$$\frac{2(d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3)}{fa(e^{fx+e} + 1)} - \frac{6dc^2 \ln(e^{fx+e} + 1)}{af^2} + \frac{6d \ln(e^{fx+e})c^2}{af^2} + \frac{6d^3e^2 \ln(e^{fx+e})}{af^4} + \frac{2d^3x^3}{af} - \frac{6d^3e^2x}{af^3} - \frac{4d^3e^2}{af^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3/(a+a\*cosh(f\*x+e)),x)

[Out]  $-2/f*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/a/(\exp(f*x+e)+1)-6/a/f^2*d*c^2*\ln(\exp(f*x+e)+1)+6/a/f^2*d*\ln(\exp(f*x+e))*c^2+6/a/f^4*d^3*e^2*\ln(\exp(f*x+e))+2/a/f*d^3*x^3-6/a/f^3*d^3*e^2*x-4/a/f^4*d^3*e^3-6/a/f^2*d^3*\ln(\exp(f*x+e)+1)*x^2-12/a/f^3*d^3*\operatorname{polylog}(2, -\exp(f*x+e))*x+12*d^3*\operatorname{polylog}(3, -\exp(f*x+e))/a/f^4-12/a/f^3*d^2*c*e*\ln(\exp(f*x+e))+6/a/f*d^2*c*x^2+12/a/f^2*d^2*c*e*x+6/a/f^3*d^2*c*e^2-12/a/f^2*d^2*c*\ln(\exp(f*x+e)+1)*x-12/a/f^3*d^2*c*\operatorname{polylog}(2, -\exp(f*x+e))$

**maxima** [B] time = 0.69, size = 228, normalized size = 1.95

$$6c^2d \left( \frac{xe^{(fx+e)}}{afe^{(fx+e)} + af} - \frac{\log\left(\left(e^{(fx+e)} + 1\right)e^{(-e)}\right)}{af^2} \right) + \frac{2c^3}{\left(ae^{(-fx-e)} + a\right)f} - \frac{2(d^3x^3 + 3cd^2x^2)}{afe^{(fx+e)} + af} - \frac{12\left(fx \log\left(e^{(fx+e)} + 1\right) + \dots\right)}{af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+a\*cosh(f\*x+e)),x, algorithm="maxima")

[Out]  $6*c^2*d*(x*e^{f*x+e})/(a*f*e^{f*x+e}+a*f) - \log((e^{f*x+e}+1)*e^{-e})/(a*f^2) + 2*c^3/((a*e^{-f*x-e}+a)*f) - 2*(d^3*x^3+3*c*d^2*x^2)/(a*f*e^{f*x+e}+a*f) - 12*(f*x*\log(e^{f*x+e}+1) + \operatorname{dilog}(-e^{f*x+e})) * c*d^2/(a*f^3) - 6*(f^2*x^2*\log(e^{f*x+e}+1) + 2*f*x*\operatorname{dilog}(-e^{f*x+e})) - 2*\operatorname{polylog}(3, -e^{f*x+e})) * d^3/(a*f^4) + 2*(d^3*f^3*x^3+3*c*d^2*f^3*x^2)/(a*f^4)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+dx)^3}{a+a\cosh(e+fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d\*x)^3/(a+a\*cosh(e+f\*x)),x)

[Out] int((c+d\*x)^3/(a+a\*cosh(e+f\*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^3}{\cosh(e+fx)+1} dx + \int \frac{d^3x^3}{\cosh(e+fx)+1} dx + \int \frac{3cd^2x^2}{\cosh(e+fx)+1} dx + \int \frac{3c^2dx}{\cosh(e+fx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3/(a+a\*cosh(f\*x+e)),x)

[Out]  $(\operatorname{Integral}(c**3/(\cosh(e+f*x)+1), x) + \operatorname{Integral}(d**3*x**3/(\cosh(e+f*x)+1), x) + \operatorname{Integral}(3*c*d**2*x**2/(\cosh(e+f*x)+1), x) + \operatorname{Integral}(3*c**2*d*x/(\cosh(e+f*x)+1), x))/a$

$$3.112 \quad \int \frac{(c+dx)^2}{a+a \cosh(e+fx)} dx$$

Optimal. Leaf size=88

$$-\frac{4d(c+dx) \log(e^{e+fx} + 1)}{af^2} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} + \frac{(c+dx)^2}{af} - \frac{4d^2 \text{Li}_2(-e^{e+fx})}{af^3}$$

[Out] (d\*x+c)^2/a/f-4\*d\*(d\*x+c)\*ln(1+exp(f\*x+e))/a/f^2-4\*d^2\*polylog(2,-exp(f\*x+e))/a/f^3+(d\*x+c)^2\*tanh(1/2\*e+1/2\*f\*x)/a/f

**Rubi [A]** time = 0.20, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {3318, 4184, 3718, 2190, 2279, 2391}

$$-\frac{4d^2 \text{PolyLog}\left(2, -e^{e+fx}\right)}{af^3} - \frac{4d(c+dx) \log(e^{e+fx} + 1)}{af^2} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} + \frac{(c+dx)^2}{af}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^2/(a + a\*Cosh[e + f\*x]),x]

[Out] (c + d\*x)^2/(a\*f) - (4\*d\*(c + d\*x)\*Log[1 + E^(e + f\*x)])/(a\*f^2) - (4\*d^2\*PolyLog[2, -E^(e + f\*x)])/(a\*f^3) + ((c + d\*x)^2\*Tanh[e/2 + (f\*x)/2])/(a\*f)

Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^2}{a+a \cosh(e+fx)} dx &= \frac{\int (c+dx)^2 \csc^2\left(\frac{1}{2}(ie+\pi) + \frac{ifx}{2}\right) dx}{2a} \\
&= \frac{(c+dx)^2 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} - \frac{(2d) \int (c+dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right) dx}{af} \\
&= \frac{(c+dx)^2}{af} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} - \frac{(4d) \int \frac{e^{2\left(\frac{e}{2} + \frac{fx}{2}\right)}(c+dx)}{1+e^{2\left(\frac{e}{2} + \frac{fx}{2}\right)}} dx}{af} \\
&= \frac{(c+dx)^2}{af} - \frac{4d(c+dx) \log(1+e^{e+fx})}{af^2} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} + \frac{(4d^2) \int \log\left(1 - \frac{e^{2\left(\frac{e}{2} + \frac{fx}{2}\right)}}{1+e^{2\left(\frac{e}{2} + \frac{fx}{2}\right)}}\right) dx}{af} \\
&= \frac{(c+dx)^2}{af} - \frac{4d(c+dx) \log(1+e^{e+fx})}{af^2} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} + \frac{(4d^2) \text{Subst}\left(\int \frac{\log\left(1 - \frac{e^{2u}}{1+e^{2u}}\right)}{1+e^{2u}} du\right)}{af} \\
&= \frac{(c+dx)^2}{af} - \frac{4d(c+dx) \log(1+e^{e+fx})}{af^2} - \frac{4d^2 \text{Li}_2(-e^{e+fx})}{af^3} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af}
\end{aligned}$$

**Mathematica** [C] time = 6.38, size = 472, normalized size = 5.36

$$\frac{2\operatorname{sech}\left(\frac{e}{2}\right)\cosh\left(\frac{e}{2} + \frac{fx}{2}\right)\left(c^2\sinh\left(\frac{fx}{2}\right) + 2cdx\sinh\left(\frac{fx}{2}\right) + d^2x^2\sinh\left(\frac{fx}{2}\right)\right)}{f(a\cosh(e + fx) + a)} - \frac{8cd\operatorname{sech}\left(\frac{e}{2}\right)\cosh^2\left(\frac{e}{2} + \frac{fx}{2}\right)\left(\cosh\left(\frac{e}{2} + \frac{fx}{2}\right) + \sinh\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{f^2\left(\cosh^2\left(\frac{e}{2} + \frac{fx}{2}\right) + \sinh^2\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x)^2/(a + a\*Cosh[e + f\*x]),x]

[Out] (-8\*c\*d\*Cosh[e/2 + (f\*x)/2]^2\*Sech[e/2]\*(Cosh[e/2]\*Log[Cosh[e/2]\*Cosh[(f\*x)/2] + Sinh[e/2]\*Sinh[(f\*x)/2]] - (f\*x\*Sinh[e/2])/2)/(f^2\*(a + a\*Cosh[e + f\*x]))\*(Cosh[e/2]^2 - Sinh[e/2]^2) - (8\*d^2\*Cosh[e/2 + (f\*x)/2]^2\*Csch[e/2]\*((f^2\*x^2)/(4\*E^ArcTanh[Coth[e/2]]) - (I\*Coth[e/2]\*(-1/2\*(f\*x\*(-Pi + (2\*I)\*ArcTanh[Coth[e/2]])) - Pi\*Log[1 + E^(f\*x)] - 2\*((I/2)\*f\*x + I\*ArcTanh[Coth[e/2]]))\*Log[1 - E^((2\*I)\*((I/2)\*f\*x + I\*ArcTanh[Coth[e/2]])]) + Pi\*Log[Cosh[(f\*x)/2]] + (2\*I)\*ArcTanh[Coth[e/2]]\*Log[I\*Sinh[(f\*x)/2 + ArcTanh[Coth[e/2]]]]) + I\*PolyLog[2, E^((2\*I)\*((I/2)\*f\*x + I\*ArcTanh[Coth[e/2]])]))/Sqrt[1 - Coth[e/2]^2])\*Sech[e/2])/(f^3\*(a + a\*Cosh[e + f\*x])\*Sqrt[Csch[e/2]^2\*(-Cosh[e/2]^2 + Sinh[e/2]^2)) + (2\*Cosh[e/2 + (f\*x)/2]\*Sech[e/2]\*(c^2\*Sinh[(f\*x)/2] + 2\*c\*d\*x\*Sinh[(f\*x)/2] + d^2\*x^2\*Sinh[(f\*x)/2]))/(f\*(a + a\*Cosh[e + f\*x])))

**fricas** [B] time = 0.41, size = 243, normalized size = 2.76

$$\frac{2(d^2e^2 - 2cdef + c^2f^2 - (d^2f^2x^2 + 2cdf^2x - d^2e^2 + 2cdef)\cosh(fx + e) + 2(d^2\cosh(fx + e) + d^2\sinh(fx + e))}{a^3\cosh(fx + e) + a^3\sinh(fx + e) + a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+a\*cosh(f\*x+e)),x, algorithm="fricas")

[Out] -2\*(d^2\*e^2 - 2\*c\*d\*e\*f + c^2\*f^2 - (d^2\*f^2\*x^2 + 2\*c\*d\*f^2\*x - d^2\*e^2 + 2\*c\*d\*e\*f)\*cosh(f\*x + e) + 2\*(d^2\*cosh(f\*x + e) + d^2\*sinh(f\*x + e) + d^2)\*dilog(-cosh(f\*x + e) - sinh(f\*x + e)) + 2\*(d^2\*f\*x + c\*d\*f + (d^2\*f\*x + c\*d\*f)\*cosh(f\*x + e) + (d^2\*f\*x + c\*d\*f)\*sinh(f\*x + e))\*log(cosh(f\*x + e) + sinh(f\*x + e) + 1) - (d^2\*f^2\*x^2 + 2\*c\*d\*f^2\*x - d^2\*e^2 + 2\*c\*d\*e\*f)\*sinh(f\*x + e))/(a\*f^3\*cosh(f\*x + e) + a\*f^3\*sinh(f\*x + e) + a\*f^3)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^2}{a \cosh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+a\*cosh(f\*x+e)),x, algorithm="giac")

[Out] integrate((d\*x + c)^2/(a\*cosh(f\*x + e) + a), x)

**maple [B]** time = 0.21, size = 174, normalized size = 1.98

$$\frac{2(d^2x^2 + 2cdx + c^2)}{fa(e^{fx+e} + 1)} - \frac{4dc \ln(e^{fx+e} + 1)}{af^2} + \frac{4d \ln(e^{fx+e})c}{af^2} + \frac{2d^2x^2}{af} + \frac{4d^2ex}{af^2} + \frac{2d^2e^2}{af^3} - \frac{4d^2 \ln(e^{fx+e} + 1)x}{af^2} - \frac{4d^2 \text{polylog}(2, -\exp(fx+e))}{af^3} + \frac{4d^2 \ln(\exp(fx+e))}{af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2/(a+a\*cosh(f\*x+e)),x)

[Out] -2/f\*(d^2\*x^2+2\*c\*d\*x+c^2)/a/(exp(f\*x+e)+1)-4/a/f^2\*d\*c\*ln(exp(f\*x+e)+1)+4/a/f^2\*d\*ln(exp(f\*x+e))\*c+2/a/f\*d^2\*x^2+4/a/f^2\*d^2\*e\*x+2/a/f^3\*d^2\*e^2-4/a/f^2\*d^2\*ln(exp(f\*x+e)+1)\*x-4\*d^2\*polylog(2,-exp(f\*x+e))/a/f^3-4/a/f^3\*d^2\*e\*ln(exp(f\*x+e))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-2d^2 \left( \frac{x^2}{afe^{(fx+e)} + af} - 2 \int \frac{x}{afe^{(fx+e)} + af} dx \right) + 4cd \left( \frac{xe^{(fx+e)}}{afe^{(fx+e)} + af} - \frac{\log\left(\left(e^{(fx+e)} + 1\right)e^{(-e)}\right)}{af^2} \right) + \frac{2c^2}{(ae^{(-fx-e)} + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+a\*cosh(f\*x+e)),x, algorithm="maxima")

[Out] -2\*d^2\*(x^2/(a\*f\*e^(f\*x + e) + a\*f) - 2\*integrate(x/(a\*f\*e^(f\*x + e) + a\*f), x)) + 4\*c\*d\*(x\*e^(f\*x + e)/(a\*f\*e^(f\*x + e) + a\*f) - log((e^(f\*x + e) + 1)\*e^(-e))/(a\*f^2)) + 2\*c^2/((a\*e^(-f\*x - e) + a)\*f)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + dx)^2}{a + a \cosh(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^2/(a + a\*cosh(e + f\*x)),x)

[Out] int((c + d\*x)^2/(a + a\*cosh(e + f\*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^2}{\cosh(e+fx)+1} dx + \int \frac{d^2x^2}{\cosh(e+fx)+1} dx + \int \frac{2cdx}{\cosh(e+fx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2/(a+a\*cosh(f\*x+e)),x)

[Out] (Integral(c\*\*2/(cosh(e + f\*x) + 1), x) + Integral(d\*\*2\*x\*\*2/(cosh(e + f\*x) + 1), x) + Integral(2\*c\*d\*x/(cosh(e + f\*x) + 1), x))/a



$$3.113 \quad \int \frac{c+dx}{a+a \cosh(e+fx)} dx$$

Optimal. Leaf size=49

$$\frac{(c+dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} - \frac{2d \log\left(\cosh\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{af^2}$$

[Out]  $-2*d*\ln(\cosh(1/2*e+1/2*f*x))/a/f^2+(d*x+c)*\tanh(1/2*e+1/2*f*x)/a/f$

**Rubi [A]** time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3318, 4184, 3475}

$$\frac{(c+dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} - \frac{2d \log\left(\cosh\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{af^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)/(a + a\*Cosh[e + f\*x]), x]

[Out]  $(-2*d*\text{Log}[\text{Cosh}[e/2 + (f*x)/2]])/(a*f^2) + ((c + d*x)*\text{Tanh}[e/2 + (f*x)/2])/(a*f)$

#### Rule 3318

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[(2\*a)^n, Int[(c + d\*x)^m\*Sin[(1\*(e + (Pi\*a)/(2\*b)))/2 + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])

#### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

#### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m-1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{c + dx}{a + a \cosh(e + fx)} dx &= \frac{\int (c + dx) \csc^2\left(\frac{1}{2}(ie + \pi) + \frac{ifx}{2}\right) dx}{2a} \\ &= \frac{(c + dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} - \frac{d \int \tanh\left(\frac{e}{2} + \frac{fx}{2}\right) dx}{af} \\ &= -\frac{2d \log\left(\cosh\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{af^2} + \frac{(c + dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} \end{aligned}$$

**Mathematica [A]** time = 0.27, size = 70, normalized size = 1.43

$$\frac{2 \cosh\left(\frac{1}{2}(e + fx)\right) \left(f(c + dx) \sinh\left(\frac{1}{2}(e + fx)\right) - 2d \cosh\left(\frac{1}{2}(e + fx)\right) \log\left(\cosh\left(\frac{1}{2}(e + fx)\right)\right)\right)}{af^2(\cosh(e + fx) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)/(a + a\*Cosh[e + f\*x]),x]

[Out] (2\*Cosh[(e + f\*x)/2]\*(-2\*d\*Cosh[(e + f\*x)/2]\*Log[Cosh[(e + f\*x)/2]] + f\*(c + d\*x)\*Sinh[(e + f\*x)/2]))/(a\*f^2\*(1 + Cosh[e + f\*x]))

**fricas [B]** time = 0.54, size = 92, normalized size = 1.88

$$\frac{2(dfx \cosh(fx + e) + dfx \sinh(fx + e) - cf - (d \cosh(fx + e) + d \sinh(fx + e) + d) \log(\cosh(fx + e) + \sinh(fx + e)))}{af^2 \cosh(fx + e) + af^2 \sinh(fx + e) + af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+a\*cosh(f\*x+e)),x, algorithm="fricas")

[Out] 2\*(d\*f\*x\*cosh(f\*x + e) + d\*f\*x\*sinh(f\*x + e) - c\*f - (d\*cosh(f\*x + e) + d\*sinh(f\*x + e) + d)\*log(cosh(f\*x + e) + sinh(f\*x + e) + 1))/(a\*f^2\*cosh(f\*x + e) + a\*f^2\*sinh(f\*x + e) + a\*f^2)

**giac [A]** time = 0.12, size = 71, normalized size = 1.45

$$\frac{2\left(dfxe^{(fx+e)} - de^{(fx+e)} \log\left(e^{(fx+e)} + 1\right) - cf - d \log\left(e^{(fx+e)} + 1\right)\right)}{af^2e^{(fx+e)} + af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+a\*cosh(f\*x+e)),x, algorithm="giac")

[Out]  $2*(d*f*x*e^{(f*x + e)} - d*e^{(f*x + e)}*\log(e^{(f*x + e)} + 1) - c*f - d*\log(e^{(f*x + e)} + 1))/(a*f^2*e^{(f*x + e)} + a*f^2)$

**maple [A]** time = 0.13, size = 63, normalized size = 1.29

$$\frac{2dx}{af} + \frac{2de}{af^2} - \frac{2(dx+c)}{fa(e^{fx+e}+1)} - \frac{2d \ln(e^{fx+e}+1)}{af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)/(a+a\*cosh(f\*x+e)),x)

[Out]  $2*d/a/f*x+2*d/a/f^2*e-2/f*(d*x+c)/a/(exp(f*x+e)+1)-2*d/a/f^2*\ln(exp(f*x+e)+1)$

**maxima [A]** time = 0.37, size = 71, normalized size = 1.45

$$2d \left( \frac{x e^{(fx+e)}}{a f e^{(fx+e)} + a f} - \frac{\log\left(\left(e^{(fx+e)} + 1\right) e^{(-e)}\right)}{a f^2} \right) + \frac{2c}{\left(a e^{(-fx-e)} + a\right) f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+a\*cosh(f\*x+e)),x, algorithm="maxima")

[Out]  $2*d*(x*e^{(f*x + e)}/(a*f*e^{(f*x + e)} + a*f) - \log((e^{(f*x + e)} + 1)*e^{(-e)})/(a*f^2)) + 2*c/((a*e^{(-f*x - e)} + a)*f)$

**mupad [B]** time = 0.90, size = 53, normalized size = 1.08

$$\frac{2dx}{af} - \frac{2(c+dx)}{af(e^{e+fx}+1)} - \frac{2d \ln(e^{fx}e^e+1)}{af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)/(a + a\*cosh(e + f\*x)),x)

[Out]  $(2*d*x)/(a*f) - (2*(c + d*x))/(a*f*(exp(e + f*x) + 1)) - (2*d*log(exp(f*x)*exp(e) + 1))/(a*f^2)$

sympy [A] time = 0.72, size = 76, normalized size = 1.55

$$\begin{cases} \frac{c \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} + \frac{dx \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af} - \frac{dx}{af} + \frac{2d \log\left(\tanh\left(\frac{e}{2} + \frac{fx}{2}\right) + 1\right)}{af^2} & \text{for } f \neq 0 \\ \frac{cx + \frac{dx^2}{2}}{a \cosh(e) + a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+a\*cosh(f\*x+e)),x)

[Out] Piecewise((c\*tanh(e/2 + f\*x/2)/(a\*f) + d\*x\*tanh(e/2 + f\*x/2)/(a\*f) - d\*x/(a\*f) + 2\*d\*log(tanh(e/2 + f\*x/2) + 1)/(a\*f\*\*2), Ne(f, 0)), ((c\*x + d\*x\*\*2/2)/(a\*cosh(e) + a), True))

$$3.114 \quad \int \frac{1}{(c+dx)(a+a \cosh(e+fx))} dx$$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a \cosh(e+fx)+a)}, x\right)$$

[Out] Unintegrable(1/(d\*x+c)/(a+a\*cosh(f\*x+e)), x)

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(c+dx)(a+a \cosh(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d\*x)\*(a + a\*Cosh[e + f\*x])), x]

[Out] Defer[Int][1/((c + d\*x)\*(a + a\*Cosh[e + f\*x])), x]

Rubi steps

$$\int \frac{1}{(c+dx)(a+a \cosh(e+fx))} dx = \int \frac{1}{(c+dx)(a+a \cosh(e+fx))} dx$$

Mathematica [A] time = 9.25, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+a \cosh(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d\*x)\*(a + a\*Cosh[e + f\*x])), x]

[Out] Integrate[1/((c + d\*x)\*(a + a\*Cosh[e + f\*x])), x]

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{adx+ac+(adx+ac)\cosh(fx+e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+a\*cosh(f\*x+e)),x, algorithm="fricas")

[Out] integral(1/(a\*d\*x + a\*c + (a\*d\*x + a\*c)\*cosh(f\*x + e)), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)(a \cosh(fx + e) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+a\*cosh(f\*x+e)),x, algorithm="giac")

[Out] integrate(1/((d\*x + c)\*(a\*cosh(f\*x + e) + a)), x)

**maple** [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)(a + a \cosh(fx + e))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x+c)/(a+a\*cosh(f\*x+e)),x)

[Out] int(1/(d\*x+c)/(a+a\*cosh(f\*x+e)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-2d \int \frac{1}{ad^2fx^2 + 2acdfx + ac^2f + (ad^2fx^2e^e + 2acdfxe^e + ac^2fe^e)e^{(fx)}} dx - \frac{2}{adfx + acf + (adfxe^e + acfe^e)e^{(fx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+a\*cosh(f\*x+e)),x, algorithm="maxima")

[Out] -2\*d\*integrate(1/(a\*d^2\*f\*x^2 + 2\*a\*c\*d\*f\*x + a\*c^2\*f + (a\*d^2\*f\*x^2\*e^e + 2\*a\*c\*d\*f\*x\*e^e + a\*c^2\*f\*e^e)\*e^(f\*x)), x) - 2/(a\*d\*f\*x + a\*c\*f + (a\*d\*f\*x\*e^e + a\*c\*f\*e^e)\*e^(f\*x))

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a + a \cosh(e + fx))(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a\*cosh(e + f\*x))\*(c + d\*x)),x)

[Out] `int(1/((a + a*cosh(e + f*x))*(c + d*x)), x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c \cosh(e+fx)+c+dx \cosh(e+fx)+dx} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*x+c)/(a+a*cosh(f*x+e)), x)`

[Out] `Integral(1/(c*cosh(e + f*x) + c + d*x*cosh(e + f*x) + d*x), x)/a`

$$3.115 \quad \int \frac{1}{(c+dx)^2(a+a \cosh(e+fx))} dx$$

**Optimal.** Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a \cosh(e+fx)+a)}, x\right)$$

[Out] Unintegrable(1/(d\*x+c)^2/(a+a\*cosh(f\*x+e)), x)

**Rubi [A]** time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+a \cosh(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d\*x)^2\*(a + a\*Cosh[e + f\*x])), x]

[Out] Defer[Int][1/((c + d\*x)^2\*(a + a\*Cosh[e + f\*x])), x]

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a \cosh(e+fx))} dx = \int \frac{1}{(c+dx)^2(a+a \cosh(e+fx))} dx$$

**Mathematica [A]** time = 9.61, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+a \cosh(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d\*x)^2\*(a + a\*Cosh[e + f\*x])), x]

[Out] Integrate[1/((c + d\*x)^2\*(a + a\*Cosh[e + f\*x])), x]

**fricas [A]** time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{ad^2x^2 + 2acdx + ac^2 + (ad^2x^2 + 2acdx + ac^2) \cosh(fx + e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(1/(d\*x+c)^2/(a+a\*cosh(f\*x+e)),x, algorithm="fricas")

[Out] integral(1/(a\*d^2\*x^2 + 2\*a\*c\*d\*x + a\*c^2 + (a\*d^2\*x^2 + 2\*a\*c\*d\*x + a\*c^2)\*cosh(f\*x + e)), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2(a \cosh(fx+e)+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+a\*cosh(f\*x+e)),x, algorithm="giac")

[Out] integrate(1/((d\*x + c)^2\*(a\*cosh(f\*x + e) + a)), x)

**maple** [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2(a+a \cosh(fx+e))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x+c)^2/(a+a\*cosh(f\*x+e)),x)

[Out] int(1/(d\*x+c)^2/(a+a\*cosh(f\*x+e)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-4d \int \frac{1}{ad^3fx^3 + 3acd^2fx^2 + 3ac^2dfx + ac^3f + (ad^3fx^3e + 3acd^2fx^2e + 3ac^2dfxe + ac^3fe)e^{(fx)}} dx - \frac{1}{ad^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+a\*cosh(f\*x+e)),x, algorithm="maxima")

[Out] -4\*d\*integrate(1/(a\*d^3\*f\*x^3 + 3\*a\*c\*d^2\*f\*x^2 + 3\*a\*c^2\*d\*f\*x + a\*c^3\*f + (a\*d^3\*f\*x^3\*e + 3\*a\*c\*d^2\*f\*x^2\*e + 3\*a\*c^2\*d\*f\*x\*e + a\*c^3\*f\*e)\*e^(f\*x)), x) - 2/(a\*d^2\*f\*x^2 + 2\*a\*c\*d\*f\*x + a\*c^2\*f + (a\*d^2\*f\*x^2\*e + 2\*a\*c\*d\*f\*x\*e + a\*c^2\*f\*e)\*e^(f\*x))

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a+a \cosh(e+fx))(c+dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + a*cosh(e + f*x))*(c + d*x)^2), x)`

[Out] `int(1/((a + a*cosh(e + f*x))*(c + d*x)^2), x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c^2 \cosh(e+fx)+c^2+2cdx \cosh(e+fx)+2cdx+d^2x^2 \cosh(e+fx)+d^2x^2} dx$$

$a$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*x+c)**2/(a+a*cosh(f*x+e)), x)`

[Out] `Integral(1/(c**2*cosh(e + f*x) + c**2 + 2*c*d*x*cosh(e + f*x) + 2*c*d*x + d**2*x**2*cosh(e + f*x) + d**2*x**2), x)/a`

$$3.116 \quad \int \frac{(c+dx)^3}{(a+a \cosh(e+fx))^2} dx$$

**Optimal.** Leaf size=255

$$\frac{4d^2(c+dx)\text{Li}_2(-e^{e+fx})}{a^2f^3} - \frac{2d^2(c+dx)\tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{a^2f^3} - \frac{2d(c+dx)^2\log(e^{e+fx}+1)}{a^2f^2} + \frac{d(c+dx)^2\text{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{2a^2f^2} +$$

[Out]  $1/3*(d*x+c)^3/a^2/f-2*d*(d*x+c)^2*\ln(1+\exp(f*x+e))/a^2/f^2+4*d^3*\ln(\cosh(1/2*e+1/2*f*x))/a^2/f^4-4*d^2*(d*x+c)*\text{polylog}(2,-\exp(f*x+e))/a^2/f^3+4*d^3*\text{polylog}(3,-\exp(f*x+e))/a^2/f^4+1/2*d*(d*x+c)^2*\text{sech}(1/2*e+1/2*f*x)^2/a^2/f^2-2*d^2*(d*x+c)*\tanh(1/2*e+1/2*f*x)/a^2/f^3+1/3*(d*x+c)^3*\tanh(1/2*e+1/2*f*x)/a^2/f+1/6*(d*x+c)^3*\text{sech}(1/2*e+1/2*f*x)^2*\tanh(1/2*e+1/2*f*x)/a^2/f$

**Rubi [A]** time = 0.36, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$ , Rules used = {3318, 4186, 4184, 3475, 3718, 2190, 2531, 2282, 6589}

$$\frac{4d^2(c+dx)\text{PolyLog}(2,-e^{e+fx})}{a^2f^3} + \frac{4d^3\text{PolyLog}(3,-e^{e+fx})}{a^2f^4} - \frac{2d^2(c+dx)\tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{a^2f^3} - \frac{2d(c+dx)^2\log(e^{e+fx})}{a^2f^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c+d*x)^3/(a+a*\text{Cosh}[e+f*x])^2,x]$

[Out]  $(c+d*x)^3/(3*a^2*f) - (2*d*(c+d*x)^2*\text{Log}[1+E^(e+f*x)])/(a^2*f^2) + (4*d^3*\text{Log}[\text{Cosh}[e/2+(f*x)/2]])/(a^2*f^4) - (4*d^2*(c+d*x)*\text{PolyLog}[2,-E^(e+f*x)])/(a^2*f^3) + (4*d^3*\text{PolyLog}[3,-E^(e+f*x)])/(a^2*f^4) + (d*(c+d*x)^2*\text{Sech}[e/2+(f*x)/2]^2)/(2*a^2*f^2) - (2*d^2*(c+d*x)*\text{Tanh}[e/2+(f*x)/2])/(a^2*f^3) + ((c+d*x)^3*\text{Tanh}[e/2+(f*x)/2])/(3*a^2*f) + ((c+d*x)^3*\text{Sech}[e/2+(f*x)/2]^2*\text{Tanh}[e/2+(f*x)/2])/(6*a^2*f)$

**Rule 2190**

$\text{Int}[(((F_)^(g_)*(e_)+(f_)*(x_)))^(n_)*((c_)+(d_)*(x_))^(m_)]/((a_)+(b_)*((F_)^(g_)*(e_)+(f_)*(x_)))^(n_), x\_Symbol] \rightarrow \text{Simp}[(c+d*x)^m*\text{Log}[1+(b*(F^(g*(e+f*x)))^n)/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c+d*x)^(m-1)*\text{Log}[1+(b*(F^(g*(e+f*x)))^n)/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n, x\} \&\& \text{IGtQ}[m, 0]$

**Rule 2282**

$\text{Int}[u, x\_Symbol] \rightarrow \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialFunction}[u, x] \neq 0$

```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

### Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-I*e) + f*fz*x)))/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbo
l] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
```

e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.)]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int \frac{(c + dx)^3}{(a + a \cosh(e + fx))^2} dx &= \frac{\int (c + dx)^3 \csc^4\left(\frac{1}{2}(ie + \pi) + \frac{ifx}{2}\right) dx}{4a^2} \\
 &= \frac{d(c + dx)^2 \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{2a^2 f^2} + \frac{(c + dx)^3 \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2 f} + \frac{\int (c + dx)^3 \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right) dx}{2a^2 f^2} \\
 &= \frac{d(c + dx)^2 \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{2a^2 f^2} - \frac{2d^2(c + dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{a^2 f^3} + \frac{(c + dx)^3 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2 f} \\
 &= \frac{(c + dx)^3}{3a^2 f} + \frac{4d^3 \log\left(\cosh\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{a^2 f^4} + \frac{d(c + dx)^2 \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{2a^2 f^2} - \frac{2d^2(c + dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{a^2 f^3} \\
 &= \frac{(c + dx)^3}{3a^2 f} - \frac{2d(c + dx)^2 \log(1 + e^{e+fx})}{a^2 f^2} + \frac{4d^3 \log\left(\cosh\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{a^2 f^4} + \frac{d(c + dx)^2 \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{2a^2 f^2} \\
 &= \frac{(c + dx)^3}{3a^2 f} - \frac{2d(c + dx)^2 \log(1 + e^{e+fx})}{a^2 f^2} + \frac{4d^3 \log\left(\cosh\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{a^2 f^4} - \frac{4d^2(c + dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{a^2 f^3} \\
 &= \frac{(c + dx)^3}{3a^2 f} - \frac{2d(c + dx)^2 \log(1 + e^{e+fx})}{a^2 f^2} + \frac{4d^3 \log\left(\cosh\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{a^2 f^4} - \frac{4d^2(c + dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{a^2 f^3} \\
 &= \frac{(c + dx)^3}{3a^2 f} - \frac{2d(c + dx)^2 \log(1 + e^{e+fx})}{a^2 f^2} + \frac{4d^3 \log\left(\cosh\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{a^2 f^4} - \frac{4d^2(c + dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{a^2 f^3}
 \end{aligned}$$

**Mathematica** [A] time = 3.49, size = 462, normalized size = 1.81

$$\cosh\left(\frac{1}{2}(e + fx)\right) \left( \operatorname{sech}\left(\frac{e}{2}\right) (c + dx) \left( c^2 f^2 \sinh\left(e + \frac{3fx}{2}\right) + 3c^2 f^2 \sinh\left(\frac{fx}{2}\right) + 2cdf^2 x \sinh\left(e + \frac{3fx}{2}\right) + 3df(c + dx) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3/(a + a\*Cosh[e + f\*x])^2,x]

[Out] (Cosh[(e + f\*x)/2]\*((-8\*d\*Cosh[(e + f\*x)/2]^3\*(-6\*d^2\*x + 3\*c^2\*f^2\*x + 3\*c\*d\*f^2\*x^2 + d^2\*f^2\*x^3 + 6\*c\*d\*f\*x\*Log[1 + Cosh[e + f\*x] - Sinh[e + f\*x]])\*(1 + Cosh[e] + Sinh[e]) + 3\*d^2\*f\*x^2\*Log[1 + Cosh[e + f\*x] - Sinh[e + f\*x]])\*(1 + Cosh[e] + Sinh[e]) - (3\*(-2\*d^2 + c^2\*f^2)\*(f\*x - Log[1 + Cosh[e + f\*x] + Sinh[e + f\*x]])\*(1 + Cosh[e] + Sinh[e]))/f - 6\*c\*d\*PolyLog[2, -Cosh[e + f\*x] + Sinh[e + f\*x]]\*(1 + Cosh[e] + Sinh[e]) - (6\*d^2\*(f\*x\*PolyLog[2, -Cosh[e + f\*x] + Sinh[e + f\*x]] + PolyLog[3, -Cosh[e + f\*x] + Sinh[e + f\*x]])\*(1 + Cosh[e] + Sinh[e]))/f)/(1 + Cosh[e] + Sinh[e]) + (c + d\*x)\*Sech[e/2]\*(3\*d\*f\*(c + d\*x)\*Cosh[(f\*x)/2] + 3\*d\*f\*(c + d\*x)\*Cosh[e + (f\*x)/2] - 12\*d^2\*Sinh[(f\*x)/2] + 3\*c^2\*f^2\*Sinh[(f\*x)/2] + 6\*c\*d\*f^2\*x\*Sinh[(f\*x)/2] + 3\*d^2\*f^2\*x^2\*Sinh[(f\*x)/2] + 6\*d^2\*Sinh[e + (f\*x)/2] - 6\*d^2\*Sinh[e + (3\*f\*x)/2] + c^2\*f^2\*Sinh[e + (3\*f\*x)/2] + 2\*c\*d\*f^2\*x\*Sinh[e + (3\*f\*x)/2] + d^2\*f^2\*x^2\*Sinh[e + (3\*f\*x)/2]))/(3\*a^2\*f^3\*(1 + Cosh[e + f\*x])^2)

**fricas** [C] time = 0.58, size = 1863, normalized size = 7.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+a\*cosh(f\*x+e))^2,x, algorithm="fricas")

[Out] 2/3\*(d^3\*e^3 + 3\*c^2\*d\*e\*f^2 - c^3\*f^3 - 6\*d^3\*e + (d^3\*f^3\*x^3 + 3\*c\*d^2\*f^3\*x^2 + d^3\*e^3 - 3\*c\*d^2\*e^2\*f + 3\*c^2\*d\*e\*f^2 - 6\*d^3\*e + 3\*(c^2\*d\*f^3 - 2\*d^3\*f)\*x)\*cosh(f\*x + e)^3 + (d^3\*f^3\*x^3 + 3\*c\*d^2\*f^3\*x^2 + d^3\*e^3 - 3\*c\*d^2\*e^2\*f + 3\*c^2\*d\*e\*f^2 - 6\*d^3\*e + 3\*(c^2\*d\*f^3 - 2\*d^3\*f)\*x)\*sinh(f\*x + e)^3 + 3\*(d^3\*f^3\*x^3 + d^3\*e^3 - 6\*d^3\*e + (3\*c^2\*d\*e + c^2\*d)\*f^2 + (3\*c\*d^2\*f^3 + d^3\*f^2)\*x^2 - (3\*c\*d^2\*e^2 - 2\*c\*d^2)\*f + (3\*c^2\*d\*f^3 + 2\*c\*d^2\*f^2 - 4\*d^3\*f)\*x)\*cosh(f\*x + e)^2 + 3\*(d^3\*f^3\*x^3 + d^3\*e^3 - 6\*d^3\*e + (3\*c^2\*d\*e + c^2\*d)\*f^2 + (3\*c\*d^2\*f^3 + d^3\*f^2)\*x^2 - (3\*c\*d^2\*e^2 - 2\*c\*d^2)\*f + (3\*c^2\*d\*f^3 + 2\*c\*d^2\*f^2 - 4\*d^3\*f)\*x + (d^3\*f^3\*x^3 + 3\*c\*d^2\*f^3\*x^2 + d^3\*e^3 - 3\*c\*d^2\*e^2\*f + 3\*c^2\*d\*e\*f^2 - 6\*d^3\*e + 3\*(c^2\*d\*f^3 - 2\*d^3\*f)\*x)\*cosh(f\*x + e)\*sinh(f\*x + e)^2 - 3\*(c\*d^2\*e^2 - 2\*c\*d^2)\*f + 3\*(d^3\*f^2\*x^2 + d^3\*e^3 - c^3\*f^3 - 6\*d^3\*e + (3\*c^2\*d\*e + c^2\*d)\*f^2 - (3\*c\*d^2\*e^2 - 4\*c\*d^2)\*f + 2\*(c\*d^2\*f^2 - d^3\*f)\*x)\*cosh(f\*x + e) - 6\*(d^3

```

*f*x + c*d^2*f + (d^3*f*x + c*d^2*f)*cosh(f*x + e)^3 + (d^3*f*x + c*d^2*f)*
sinh(f*x + e)^3 + 3*(d^3*f*x + c*d^2*f)*cosh(f*x + e)^2 + 3*(d^3*f*x + c*d^
2*f + (d^3*f*x + c*d^2*f)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(d^3*f*x + c*d
^2*f)*cosh(f*x + e) + 3*(d^3*f*x + c*d^2*f + (d^3*f*x + c*d^2*f)*cosh(f*x +
e)^2 + 2*(d^3*f*x + c*d^2*f)*cosh(f*x + e))*sinh(f*x + e))*dilog(-cosh(f*x
+ e) - sinh(f*x + e)) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*
f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3)*cosh(f*x + e)^3 + (d^3*f^2*x^2
+ 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3)*sinh(f*x + e)^3 - 2*d^3 + 3*(d^3*f^2*
x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3)*cosh(f*x + e)^2 + 3*(d^3*f^2*x^2 +
2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f
^2 - 2*d^3)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x
+ c^2*d*f^2 - 2*d^3)*cosh(f*x + e) + 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*
d*f^2 - 2*d^3 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3)*cosh(f*x
+ e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3)*cosh(f*x + e))
*sinh(f*x + e))*log(cosh(f*x + e) + sinh(f*x + e) + 1) + 6*(d^3*cosh(f*x +
e)^3 + d^3*sinh(f*x + e)^3 + 3*d^3*cosh(f*x + e)^2 + 3*d^3*cosh(f*x + e) +
d^3 + 3*(d^3*cosh(f*x + e) + d^3)*sinh(f*x + e)^2 + 3*(d^3*cosh(f*x + e)^2
+ 2*d^3*cosh(f*x + e) + d^3)*sinh(f*x + e))*polylog(3, -cosh(f*x + e) - sin
h(f*x + e)) + 3*(d^3*f^2*x^2 + d^3*e^3 - c^3*f^3 - 6*d^3*e + (3*c^2*d*e + c
^2*d)*f^2 + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^
2*d*e*f^2 - 6*d^3*e + 3*(c^2*d*f^3 - 2*d^3*f)*x)*cosh(f*x + e)^2 - (3*c*d^2
*e^2 - 4*c*d^2)*f + 2*(c*d^2*f^2 - d^3*f)*x + 2*(d^3*f^3*x^3 + d^3*e^3 - 6*
d^3*e + (3*c^2*d*e + c^2*d)*f^2 + (3*c*d^2*f^3 + d^3*f^2)*x^2 - (3*c*d^2*e^
2 - 2*c*d^2)*f + (3*c^2*d*f^3 + 2*c*d^2*f^2 - 4*d^3*f)*x)*cosh(f*x + e))*si
nh(f*x + e))/(a^2*f^4*cosh(f*x + e)^3 + a^2*f^4*sinh(f*x + e)^3 + 3*a^2*f^4
*cosh(f*x + e)^2 + 3*a^2*f^4*cosh(f*x + e) + a^2*f^4 + 3*(a^2*f^4*cosh(f*x
+ e) + a^2*f^4)*sinh(f*x + e)^2 + 3*(a^2*f^4*cosh(f*x + e)^2 + 2*a^2*f^4*co
sh(f*x + e) + a^2*f^4)*sinh(f*x + e))

```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^3}{(a \cosh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+a\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^3/(a\*cosh(f\*x + e) + a)^2, x)

**maple** [B] time = 0.30, size = 600, normalized size = 2.35

---


$$2(3f^2d^3x^3e^{fx+e} + 9f^2cd^2x^2e^{fx+e} + d^3f^2x^3 - 3d^3fx^2e^{2fx+2e} + 9f^2c^2dxe^{fx+e} + 3cd^2f^2x^2 - 6cd^2fxe^{2fx+2e} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^3/(a+a*cosh(f*x+e))^2,x)`

[Out] 
$$-2/3*(3*f^2*d^3*x^3*\exp(f*x+e)+9*f^2*c*d^2*x^2*\exp(f*x+e)+d^3*f^2*x^3-3*d^3*f*x^2*\exp(2*f*x+2*e)+9*f^2*c^2*d*x*\exp(f*x+e)+3*c*d^2*f^2*x^2-6*c*d^2*f*x*\exp(2*f*x+2*e)-3*f*d^3*x^2*\exp(f*x+e)+3*f^2*c^3*\exp(f*x+e)+3*c^2*d*f^2*x-3*c^2*d*f*\exp(2*f*x+2*e)-6*f*c*d^2*x*\exp(f*x+e)-6*d^3*x*\exp(2*f*x+2*e)+c^3*f^2-3*f*c^2*d*\exp(f*x+e)-6*c*d^2*\exp(2*f*x+2*e)-12*d^3*x*\exp(f*x+e)-12*c*d^2*\exp(f*x+e)-6*d^3*x-6*c*d^2)/f^3/a^2/(\exp(f*x+e)+1)^3-4/a^2/f^2*d^2*\ln(\exp(f*x+e)+1)*c*x-4/3/a^2/f^4*d^3*e^3-2/a^2/f^2*d^3*\ln(\exp(f*x+e)+1)*x^2-4/a^2/f^3*d^3*\operatorname{polylog}(2,-\exp(f*x+e))*x+2/3/a^2/f*d^3*x^3-2/a^2/f^2*d*c^2*\ln(\exp(f*x+e)+1)+2/a^2/f^2*d*c^2*\ln(\exp(f*x+e))+2/a^2/f^4*d^3*e^2*\ln(\exp(f*x+e))-4/a^2/f^3*d^2*c*\operatorname{polylog}(2,-\exp(f*x+e))-4/a^2/f^3*d^2*c*e*\ln(\exp(f*x+e))+4/a^2/f^2*d^2*c*e*x+4*d^3*\operatorname{polylog}(3,-\exp(f*x+e))/a^2/f^4+4/a^2/f^4*d^3*\ln(\exp(f*x+e)+1)-4/a^2/f^4*d^3*\ln(\exp(f*x+e))-2/a^2/f^3*d^3*e^2*x+2/a^2/f*d^2*c*x^2+2/a^2/f^3*d^2*c*e^2$$

**maxima** [B] time = 0.62, size = 610, normalized size = 2.39

$$2c^2d \left( \frac{fxe^{(3fx+3e)} + (3fxe^{(2e)} + e^{(2e)})e^{(2fx)} + e^{(fx+e)}}{a^2f^2e^{(3fx+3e)} + 3a^2f^2e^{(2fx+2e)} + 3a^2f^2e^{(fx+e)} + a^2f^2} - \frac{\log\left(\left(e^{(fx+e)} + 1\right)e^{(-e)}\right)}{a^2f^2} \right) + \frac{2}{3}c^3 \left( \frac{1}{(3a^2e^{(-fx-e)} + 3a^2e^{(-fx-e)} + 3a^2e^{(-fx-e)} + 3a^2e^{(-fx-e)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^3/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")`

[Out] 
$$2*c^2*d*((f*x*e^{(3*f*x + 3*e)} + (3*f*x*e^{(2*e)} + e^{(2*e)})*e^{(2*f*x)} + e^{(f*x + e)})/(a^2*f^2*e^{(3*f*x + 3*e)} + 3*a^2*f^2*e^{(2*f*x + 2*e)} + 3*a^2*f^2*e^{(f*x + e)} + a^2*f^2) - \log((e^{(f*x + e)} + 1)*e^{(-e)})/(a^2*f^2)) + 2/3*c^3*(3*e^{(-f*x - e)}/((3*a^2*e^{(-f*x - e)} + 3*a^2*e^{(-2*f*x - 2*e)} + a^2*e^{(-3*f*x - 3*e)} + a^2)*f) + 1/((3*a^2*e^{(-f*x - e)} + 3*a^2*e^{(-2*f*x - 2*e)} + a^2*e^{(-3*f*x - 3*e)} + a^2)*f)) - 2/3*(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 - 6*d^3*x - 6*c*d^2 - 3*(d^3*f*x^2*e^{(2*e)} + 2*c*d^2*e^{(2*e)} + 2*(c*d^2*f*e^{(2*e)} + d^3*e^{(2*e)})*x)*e^{(2*f*x)} + 3*(d^3*f^2*x^3*e^e - 4*c*d^2*e^e + (3*c*d^2*f^2*e^e - d^3*f*e^e)*x^2 - 2*(c*d^2*f*e^e + 2*d^3*e^e)*x)*e^{(f*x)})/(a^2*f^3*e^{(3*f*x + 3*e)} + 3*a^2*f^3*e^{(2*f*x + 2*e)} + 3*a^2*f^3*e^{(f*x + e)} + a^2*f^3) - 4*(f*x*log(e^{(f*x + e)} + 1) + \operatorname{dilog}(-e^{(f*x + e)}))*c*d^2/(a^2*f^3) - 4*d^3*x/(a^2*f^3) - 2*(f^2*x^2*log(e^{(f*x + e)} + 1) + 2*f*x*\operatorname{dilog}(-e^{(f*x + e)})) - 2*\operatorname{polylog}(3, -e^{(f*x + e)})*d^3/(a^2*f^4) + 4*d^3*log(e^{(f*x + e)} + 1)/(a^2*f^4) + 2/3*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2)/(a^2*f^4)$$



mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + dx)^3}{(a + a \cosh(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^3/(a + a\*cosh(e + f\*x))^2,x)

[Out] int((c + d\*x)^3/(a + a\*cosh(e + f\*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^3}{\cosh^2(e+fx)+2\cosh(e+fx)+1} dx + \int \frac{d^3x^3}{\cosh^2(e+fx)+2\cosh(e+fx)+1} dx + \int \frac{3cd^2x^2}{\cosh^2(e+fx)+2\cosh(e+fx)+1} dx + \int \frac{3c^2d}{\cosh^2(e+fx)+2\cosh(e+fx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3/(a+a\*cosh(f\*x+e))\*\*2,x)

[Out] (Integral(c\*\*3/(cosh(e + f\*x)\*\*2 + 2\*cosh(e + f\*x) + 1), x) + Integral(d\*\*3\*x\*\*3/(cosh(e + f\*x)\*\*2 + 2\*cosh(e + f\*x) + 1), x) + Integral(3\*c\*d\*\*2\*x\*\*2/(cosh(e + f\*x)\*\*2 + 2\*cosh(e + f\*x) + 1), x) + Integral(3\*c\*\*2\*d\*x/(cosh(e + f\*x)\*\*2 + 2\*cosh(e + f\*x) + 1), x))/a\*\*2

$$3.117 \quad \int \frac{(c+dx)^2}{(a+a \cosh(e+fx))^2} dx$$

Optimal. Leaf size=200

$$-\frac{4d(c+dx) \log(e^{e+fx} + 1)}{3a^2f^2} + \frac{d(c+dx) \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right) \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2f}$$

[Out] 1/3\*(d\*x+c)^2/a^2/f-4/3\*d\*(d\*x+c)\*ln(1+exp(f\*x+e))/a^2/f^2-4/3\*d^2\*polylog(2,-exp(f\*x+e))/a^2/f^3+1/3\*d\*(d\*x+c)\*sech(1/2\*e+1/2\*f\*x)^2/a^2/f^2-2/3\*d^2\*tanh(1/2\*e+1/2\*f\*x)/a^2/f^3+1/3\*(d\*x+c)^2\*tanh(1/2\*e+1/2\*f\*x)/a^2/f+1/6\*(d\*x+c)^2\*sech(1/2\*e+1/2\*f\*x)^2\*tanh(1/2\*e+1/2\*f\*x)/a^2/f

**Rubi [A]** time = 0.25, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$ , Rules used = {3318, 4186, 3767, 8, 4184, 3718, 2190, 2279, 2391}

$$-\frac{4d^2 \operatorname{PolyLog}\left(2, -e^{e+fx}\right)}{3a^2f^3} - \frac{4d(c+dx) \log(e^{e+fx} + 1)}{3a^2f^2} + \frac{d(c+dx) \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right) \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2f}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^2/(a + a\*Cosh[e + f\*x])^2,x]

[Out] (c + d\*x)^2/(3\*a^2\*f) - (4\*d\*(c + d\*x)\*Log[1 + E^(e + f\*x)])/(3\*a^2\*f^2) - (4\*d^2\*PolyLog[2, -E^(e + f\*x)])/(3\*a^2\*f^3) + (d\*(c + d\*x)\*Sech[e/2 + (f\*x)/2]^2)/(3\*a^2\*f^2) - (2\*d^2\*Tanh[e/2 + (f\*x)/2])/(3\*a^2\*f^3) + ((c + d\*x)^2\*Tanh[e/2 + (f\*x)/2])/(3\*a^2\*f) + ((c + d\*x)^2\*Sech[e/2 + (f\*x)/2]^2\*Tanh[e/2 + (f\*x)/2])/(6\*a^2\*f)

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))
]^(n), x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)])*(b_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^2}{(a+a\cosh(e+fx))^2} dx &= \frac{\int (c+dx)^2 \csc^4\left(\frac{1}{2}(ie+\pi) + \frac{ifx}{2}\right) dx}{4a^2} \\
&= \frac{d(c+dx)\operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)\tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2f} + \frac{\int (c+dx)^2\operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right) dx}{6a^2f} \\
&= \frac{d(c+dx)\operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2\operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)\tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2f} \\
&= \frac{(c+dx)^2}{3a^2f} + \frac{d(c+dx)\operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2f^2} - \frac{2d^2\tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2f^3} + \frac{(c+dx)^2\tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2f} \\
&= \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log(1+e^{e+fx})}{3a^2f^2} + \frac{d(c+dx)\operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2f^2} - \frac{2d^2\tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2f^3} \\
&= \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log(1+e^{e+fx})}{3a^2f^2} + \frac{d(c+dx)\operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2f^2} - \frac{2d^2\tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2f^3} \\
&= \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log(1+e^{e+fx})}{3a^2f^2} - \frac{4d^2\operatorname{Li}_2(-e^{e+fx})}{3a^2f^3} + \frac{d(c+dx)\operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2f^2}
\end{aligned}$$

**Mathematica** [C] time = 6.48, size = 637, normalized size = 3.18

$$\frac{\operatorname{sech}\left(\frac{e}{2}\right)\cosh\left(\frac{e}{2} + \frac{fx}{2}\right)\left(c^2f^2\sinh\left(e + \frac{3fx}{2}\right) + 3c^2f^2\sinh\left(\frac{fx}{2}\right) + 2cdf^2x\sinh\left(e + \frac{3fx}{2}\right) + 2cdf\cosh\left(e + \frac{fx}{2}\right) + 6\right)}{3a^2f^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x)^2/(a + a\*Cosh[e + f\*x])^2,x]

[Out] (-16\*c\*d\*Cosh[e/2 + (f\*x)/2]^4\*Sech[e/2]\*(Cosh[e/2]\*Log[Cosh[e/2]\*Cosh[(f\*x)/2] + Sinh[e/2]\*Sinh[(f\*x)/2]] - (f\*x\*Sinh[e/2])/2)/(3\*f^2\*(a + a\*Cosh[e

$$+ f*x))^{2*(\text{Cosh}[e/2]^2 - \text{Sinh}[e/2]^2)} - (16*d^2*\text{Cosh}[e/2 + (f*x)/2]^4*\text{Csch}[e/2]*((f^2*x^2)/(4*E^{\text{ArcTanh}[\text{Coth}[e/2]]}) - (I*\text{Coth}[e/2]*(-1/2*(f*x*(-\text{Pi} + (2*I)*\text{ArcTanh}[\text{Coth}[e/2]]))) - \text{Pi}*\text{Log}[1 + E^{(f*x)}] - 2*((I/2)*f*x + I*\text{ArcTanh}[\text{Coth}[e/2]])*\text{Log}[1 - E^{((2*I)*((I/2)*f*x + I*\text{ArcTanh}[\text{Coth}[e/2]])}])) + \text{Pi}*\text{Log}[\text{Cosh}[(f*x)/2] + (2*I)*\text{ArcTanh}[\text{Coth}[e/2]]*\text{Log}[I*\text{Sinh}[(f*x)/2 + \text{ArcTanh}[\text{Coth}[e/2]]]]) + I*\text{PolyLog}[2, E^{((2*I)*((I/2)*f*x + I*\text{ArcTanh}[\text{Coth}[e/2]])}])))/\text{Sqrt}[1 - \text{Coth}[e/2]^2]*\text{Sech}[e/2]/(3*f^3*(a + a*\text{Cosh}[e + f*x])^2*\text{Sqrt}[\text{Csch}[e/2]^2*(-\text{Cosh}[e/2]^2 + \text{Sinh}[e/2]^2)]) + (\text{Cosh}[e/2 + (f*x)/2]*\text{Sech}[e/2]*(2*c*d*f*\text{Cosh}[(f*x)/2] + 2*d^2*f*x*\text{Cosh}[(f*x)/2] + 2*c*d*f*\text{Cosh}[e + (f*x)/2] + 2*d^2*f*x*\text{Cosh}[e + (f*x)/2] - 4*d^2*\text{Sinh}[(f*x)/2] + 3*c^2*f^2*\text{Sinh}[(f*x)/2] + 6*c*d*f^2*x*\text{Sinh}[(f*x)/2] + 3*d^2*f^2*x^2*\text{Sinh}[(f*x)/2] + 2*d^2*\text{Sinh}[e + (f*x)/2] - 2*d^2*\text{Sinh}[e + (3*f*x)/2] + c^2*f^2*\text{Sinh}[e + (3*f*x)/2] + 2*c*d*f^2*x*\text{Sinh}[e + (3*f*x)/2] + d^2*f^2*x^2*\text{Sinh}[e + (3*f*x)/2]))/(3*f^3*(a + a*\text{Cosh}[e + f*x])^2)$$

**fricas [B]** time = 0.69, size = 963, normalized size = 4.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+a\*cosh(f\*x+e))^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -2/3*(d^2*e^2 - 2*c*d*e*f + c^2*f^2 - (d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*\cosh(f*x + e)^3 - (d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*\sinh(f*x + e)^3 - (3*d^2*f^2*x^2 - 3*d^2*e^2 + 2*d^2 + 2*(3*c*d*e + c*d)*f + 2*(3*c*d*f^2 + d^2*f)*x)*\cosh(f*x + e)^2 - (3*d^2*f^2*x^2 - 3*d^2*e^2 + 2*d^2 + 2*(3*c*d*e + c*d)*f + 2*(3*c*d*f^2 + d^2*f)*x + 3*(d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*\cosh(f*x + e))*\sinh(f*x + e)^2 - 2*d^2 + (3*d^2*e^2 + 3*c^2*f^2 - 2*d^2*f*x - 4*d^2 - 2*(3*c*d*e + c*d)*f)*\cosh(f*x + e) + 2*(d^2*\cosh(f*x + e)^3 + d^2*\sinh(f*x + e)^3 + 3*d^2*\cosh(f*x + e)^2 + 3*d^2*\cosh(f*x + e) + 3*(d^2*\cosh(f*x + e) + d^2)*\sinh(f*x + e)^2 + d^2 + 3*(d^2*\cosh(f*x + e)^2 + 2*d^2*\cosh(f*x + e) + d^2)*\sinh(f*x + e))*\text{dilog}(-\cosh(f*x + e) - \sinh(f*x + e)) + 2*(d^2*f*x + (d^2*f*x + c*d*f)*\cosh(f*x + e)^3 + (d^2*f*x + c*d*f)*\sinh(f*x + e)^3 + c*d*f + 3*(d^2*f*x + c*d*f)*\cosh(f*x + e)^2 + 3*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*\cosh(f*x + e))*\sinh(f*x + e)^2 + 3*(d^2*f*x + c*d*f)*\cosh(f*x + e) + 3*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*\cosh(f*x + e)^2 + 2*(d^2*f*x + c*d*f)*\cosh(f*x + e))*\sinh(f*x + e))*\log(\cosh(f*x + e) + \sinh(f*x + e) + 1) + (3*d^2*e^2 + 3*c^2*f^2 - 2*d^2*f*x - 3*(d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*\cosh(f*x + e)^2 - 4*d^2 - 2*(3*c*d*e + c*d)*f - 2*(3*d^2*f^2*x^2 - 3*d^2*e^2 + 2*d^2 + 2*(3*c*d*e + c*d)*f + 2*(3*c*d*f^2 + d^2*f)*x)*\cosh(f*x + e))*\sinh(f*x + e))/ (a^2*f^3*\cosh(f*x + e)^3 + a^2*f^3*\sinh(f*x + e)^3 + 3*a^2*f^3*\cosh(f*x + e)^2 + 3*a^2*f^3*\cosh(f*x + e) + a^2*f^3 + 3*(a^2*f^3*\cosh(f*x + e) + a^2*f^3)*\sinh(f*x + e)^2 + 3*(a^2*f^3*\cosh(f*x + e)^2 + 2*a^2*f^3*\cosh(f*x + e) + a^2*f^3)*\sinh(f*x + e)) \end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^2}{(a \cosh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+a\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^2/(a\*cosh(f\*x + e) + a)^2, x)

**maple** [A] time = 0.24, size = 313, normalized size = 1.56

$$\frac{2(3f^2d^2x^2e^{fx+e} + 6f^2cdxe^{fx+e} + d^2f^2x^2 - 2d^2fxe^{2fx+2e} + 3f^2c^2e^{fx+e} + 2cd f^2x - 2cdf e^{2fx+2e} - 2f d^2x e^{fx+e})}{3f^3a^2(e^{fx+e} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2/(a+a\*cosh(f\*x+e))^2,x)

[Out] 
$$-2/3*(3*f^2*d^2*x^2*\exp(f*x+e)+6*f^2*c*d*x*\exp(f*x+e)+d^2*f^2*x^2-2*d^2*f*x*\exp(2*f*x+2*e)+3*f^2*c^2*\exp(f*x+e)+2*c*d*f^2*x-2*c*d*f*\exp(2*f*x+2*e)-2*f*d^2*x*\exp(f*x+e)+c^2*f^2-2*f*c*d*\exp(f*x+e)-2*d^2*\exp(2*f*x+2*e)-4*d^2*\exp(f*x+e)-2*d^2)/f^3/a^2/(\exp(f*x+e)+1)^3-4/3/a^2*d/f^2*c*\ln(\exp(f*x+e)+1)+4/3/a^2*d/f^2*c*\ln(\exp(f*x+e))+2/3/a^2*d^2/f*x^2+4/3/a^2*d^2/f^2*e*x+2/3/a^2*d^2/f^3*e^2-4/3/a^2*d^2/f^2*\ln(\exp(f*x+e)+1)*x-4/3*d^2*\text{polylog}(2,-\exp(f*x+e)))/a^2/f^3-4/3/a^2*d^2/f^3*e*\ln(\exp(f*x+e))$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{3}d^2 \left( \frac{f^2x^2 - 2(fxe^{2e} + e^{2e})e^{2fx} + (3f^2x^2e^e - 2fxe^e - 4e^e)e^{fx} - 2}{a^2f^3e^{3fx+3e} + 3a^2f^3e^{2fx+2e} + 3a^2f^3e^{fx+e} + a^2f^3} - 6 \int \frac{x}{3(a^2fe^{fx+e} + a^2f)} dx \right) + \frac{4}{3}cd$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+a\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out] 
$$-2/3*d^2*((f^2*x^2 - 2*(f*x*e^{2e} + e^{2e}))*e^{2*f*x} + (3*f^2*x^2*e^e - 2*f*x*e^e - 4*e^e)*e^{f*x} - 2)/(a^2*f^3*e^{3*f*x + 3*e} + 3*a^2*f^3*e^{2*f*x + 2*e} + 3*a^2*f^3*e^{f*x + e} + a^2*f^3) - 6*\text{integrate}(1/3*x/(a^2*f*e^{f*x + e} + a^2*f), x) + 4/3*c*d*((f*x*e^{3*f*x + 3*e} + (3*f*x*e^{2e} + e^{2e}))*e^{2*f*x} + e^{f*x + e})/(a^2*f^2*e^{3*f*x + 3*e} + 3*a^2*f^2*e^{2*f*x + 2*e} + 3*a^2*f^2*e^{f*x + e} + a^2*f^2) - \log((e^{f*x + e} + 1)*e^{(-$$

$e)) / (a^2 f^2) + 2/3 c^2 (3e^{-fx - e}) / ((3a^2 e^{-fx - e} + 3a^2 e^{-2fx - 2e}) + a^2 e^{-3fx - 3e}) + a^2 (f) + 1 / ((3a^2 e^{-fx - e} + 3a^2 e^{-2fx - 2e}) + a^2 e^{-3fx - 3e}) + a^2 (f))$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + dx)^2}{(a + a \cosh(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^2/(a + a\*cosh(e + f\*x))^2,x)

[Out] int((c + d\*x)^2/(a + a\*cosh(e + f\*x))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^2}{\cosh^2(e+fx)+2\cosh(e+fx)+1} dx + \int \frac{d^2 x^2}{\cosh^2(e+fx)+2\cosh(e+fx)+1} dx + \int \frac{2cdx}{\cosh^2(e+fx)+2\cosh(e+fx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2/(a+a\*cosh(f\*x+e))\*\*2,x)

[Out] (Integral(c\*\*2/(cosh(e + f\*x)\*\*2 + 2\*cosh(e + f\*x) + 1), x) + Integral(d\*\*2\*x\*\*2/(cosh(e + f\*x)\*\*2 + 2\*cosh(e + f\*x) + 1), x) + Integral(2\*c\*d\*x/(cosh(e + f\*x)\*\*2 + 2\*cosh(e + f\*x) + 1), x))/a\*\*2

$$3.118 \quad \int \frac{c+dx}{(a+a \cosh(e+fx))^2} dx$$

**Optimal.** Leaf size=123

$$\frac{(c+dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2 f} + \frac{(c+dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right) \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2 f} + \frac{d \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2 f^2} - \frac{2d \log\left(\cosh\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{3a^2 f^2}$$

[Out]  $-2/3*d*\ln(\cosh(1/2*e+1/2*f*x))/a^2/f^2+1/6*d*\operatorname{sech}(1/2*e+1/2*f*x)^2/a^2/f^2+1/3*(d*x+c)*\tanh(1/2*e+1/2*f*x)/a^2/f+1/6*(d*x+c)*\operatorname{sech}(1/2*e+1/2*f*x)^2*\tanh(1/2*e+1/2*f*x)/a^2/f$

**Rubi [A]** time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3318, 4185, 4184, 3475}

$$\frac{(c+dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2 f} + \frac{(c+dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right) \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2 f} + \frac{d \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2 f^2} - \frac{2d \log\left(\cosh\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{3a^2 f^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)/(a + a*\text{Cosh}[e + f*x])^2, x]$

[Out]  $(-2*d*\text{Log}[\text{Cosh}[e/2 + (f*x)/2]])/(3*a^2*f^2) + (d*\text{Sech}[e/2 + (f*x)/2]^2)/(6*a^2*f^2) + ((c + d*x)*\text{Tanh}[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d*x)*\text{Sech}[e/2 + (f*x)/2]^2*\text{Tanh}[e/2 + (f*x)/2])/(6*a^2*f)$

### Rule 3318

$\text{Int}[(c + d*x)^m * \sin(e + f*x)^n, x\_Symbol] := \text{Dist}[(2*a)^n, \text{Int}[(c + d*x)^m * \sin((1*(e + (Pi*a)/(2*b)))/2 + (f*x)/2)^{2*n}], x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])

### Rule 3475

$\text{Int}[\tan(c + d*x), x\_Symbol] := -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /;$  FreeQ[{c, d}, x]

### Rule 4184

$\text{Int}[\csc(e + f*x)^2 * (c + d*x)^m, x\_Symbol] := -\text{Simp}[(c + d*x)^m * \text{Cot}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{m-1} * \text{Cot}[e + f*x], x], x] /;$  FreeQ[{c, d, e, f}, x] && GtQ[m, 0]



Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.)), x_Symbol] :=
-Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x
], x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned} \int \frac{c + dx}{(a + a \cosh(e + fx))^2} dx &= \frac{\int (c + dx) \csc^4\left(\frac{1}{2}(ie + \pi) + \frac{ifx}{2}\right) dx}{4a^2} \\ &= \frac{d \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2 f^2} + \frac{(c + dx) \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2 f} + \frac{\int (c + dx) \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right) dx}{6a^2} \\ &= \frac{d \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2 f^2} + \frac{(c + dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2 f} + \frac{(c + dx) \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2 f} \\ &= -\frac{2d \log\left(\cosh\left(\frac{e}{2} + \frac{fx}{2}\right)\right)}{3a^2 f^2} + \frac{d \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2 f^2} + \frac{(c + dx) \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2 f} + \frac{(c + dx) \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2} \end{aligned}$$

**Mathematica [A]** time = 0.46, size = 114, normalized size = 0.93

$$\frac{\cosh\left(\frac{1}{2}(e + fx)\right) \left( f(c + dx) \left( 3 \sinh\left(\frac{1}{2}(e + fx)\right) + \sinh\left(\frac{3}{2}(e + fx)\right) \right) - 2d \cosh\left(\frac{3}{2}(e + fx)\right) \log\left(\cosh\left(\frac{1}{2}(e + fx)\right)\right) \right)}{3a^2 f^2 (\cosh(e + fx) + 1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)/(a + a\*Cosh[e + f\*x])^2, x]

[Out] (Cosh[(e + f\*x)/2]\*(-2\*d\*Cosh[(3\*(e + f\*x))/2]\*Log[Cosh[(e + f\*x)/2]] + Cosh[(e + f\*x)/2]\*(2\*d - 6\*d\*Log[Cosh[(e + f\*x)/2]]) + f\*(c + d\*x)\*(3\*Sinh[(e + f\*x)/2] + Sinh[(3\*(e + f\*x))/2]))/(3\*a^2\*f^2\*(1 + Cosh[e + f\*x])^2)

**fricas [B]** time = 0.53, size = 385, normalized size = 3.13

$$\frac{2(dfx \cosh(fx + e))^3 + dfx \sinh(fx + e)^3 + (3dfx + d) \cosh(fx + e)^2 + (3dfx \cosh(fx + e) + 3dfx + d) \sinh(fx + e)}{3a^2 f^2 (\cosh(e + fx) + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+a\*cosh(f\*x+e))^2,x, algorithm="fricas")

[Out]  $\frac{2}{3}(d*f*x*cosh(f*x + e)^3 + d*f*x*sinh(f*x + e)^3 + (3*d*f*x + d)*cosh(f*x + e)^2 + (3*d*f*x*cosh(f*x + e) + 3*d*f*x + d)*sinh(f*x + e)^2 - c*f - (3*c*f - d)*cosh(f*x + e) - (d*cosh(f*x + e)^3 + d*sinh(f*x + e)^3 + 3*d*cosh(f*x + e)^2 + 3*(d*cosh(f*x + e) + d)*sinh(f*x + e)^2 + 3*d*cosh(f*x + e) + 3*(d*cosh(f*x + e)^2 + 2*d*cosh(f*x + e) + d)*sinh(f*x + e) + d)*log(cosh(f*x + e) + sinh(f*x + e) + 1) + (3*d*f*x*cosh(f*x + e)^2 - 3*c*f + 2*(3*d*f*x + d)*cosh(f*x + e) + d)*sinh(f*x + e))/(a^2*f^2*cosh(f*x + e)^3 + a^2*f^2*sinh(f*x + e)^3 + 3*a^2*f^2*cosh(f*x + e)^2 + 3*a^2*f^2*cosh(f*x + e) + a^2*f^2 + 3*(a^2*f^2*cosh(f*x + e) + a^2*f^2)*sinh(f*x + e)^2 + 3*(a^2*f^2*cosh(f*x + e)^2 + 2*a^2*f^2*cosh(f*x + e) + a^2*f^2)*sinh(f*x + e))$

**giac [B]** time = 0.13, size = 207, normalized size = 1.68

$$\frac{2 \left( d f x e^{(3 f x+3 e)} + 3 d f x e^{(2 f x+2 e)} - 3 c f e^{(f x+e)} - d e^{(3 f x+3 e)} \log \left( e^{(f x+e)} + 1 \right) - 3 d e^{(2 f x+2 e)} \log \left( e^{(f x+e)} + 1 \right) - 3 d e^{(f x+e)} \log \left( e^{(f x+e)} + 1 \right) - c f + d e^{(2 f x+2 e)} + d e^{(f x+e)} - d \log \left( e^{(f x+e)} + 1 \right) \right)}{3 \left( a^2 f^2 e^{(3 f x+3 e)} + 3 a^2 f^2 e^{(2 f x+2 e)} + 3 a^2 f^2 e^{(f x+e)} + a^2 f^2 + 3 \left( a^2 f^2 e^{(3 f x+3 e)} + a^2 f^2 e^{(2 f x+2 e)} \right) \sinh (f x+e) + 3 \left( a^2 f^2 e^{(3 f x+3 e)} + a^2 f^2 e^{(2 f x+2 e)} \right) \cosh (f x+e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+a\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out]  $\frac{2}{3}(d*f*x*e^{(3*f*x + 3*e)} + 3*d*f*x*e^{(2*f*x + 2*e)} - 3*c*f*e^{(f*x + e)} - d*e^{(3*f*x + 3*e)}*log(e^{(f*x + e)} + 1) - 3*d*e^{(2*f*x + 2*e)}*log(e^{(f*x + e)} + 1) - 3*d*e^{(f*x + e)}*log(e^{(f*x + e)} + 1) - c*f + d*e^{(2*f*x + 2*e)} + d*e^{(f*x + e)} - d*log(e^{(f*x + e)} + 1))/(a^2*f^2*e^{(3*f*x + 3*e)} + 3*a^2*f^2*e^{(2*f*x + 2*e)} + 3*a^2*f^2*e^{(f*x + e)} + a^2*f^2)$

**maple [A]** time = 0.20, size = 108, normalized size = 0.88

$$\frac{\frac{2dx}{3fa^2} + \frac{2de}{3f^2a^2} - \frac{2(3dfxe^{fx+e} + 3cfe^{fx+e} + dfx - e^{2fx+2e}d + cf - de^{fx+e})}{3f^2a^2(e^{fx+e} + 1)^3} - \frac{2d \ln(e^{fx+e} + 1)}{3a^2f^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)/(a+a\*cosh(f\*x+e))^2,x)

[Out]  $\frac{2}{3}d/f/a^2*x + \frac{2}{3}d/f^2/a^2*e - \frac{2}{3}*(3*d*f*x*exp(f*x+e) + 3*c*f*exp(f*x+e) + d*f*x - exp(2*f*x+2*e)*d + c*f - d*exp(f*x+e))/f^2/a^2/(exp(f*x+e)+1)^3 - \frac{2}{3}d/a^2/f^2 * \ln(exp(f*x+e)+1)$

**maxima [B]** time = 0.37, size = 239, normalized size = 1.94

$$\frac{2}{3}d \left( \frac{f x e^{(3fx+3e)} + (3fx e^{(2e)} + e^{(2e)})e^{(2fx)} + e^{(fx+e)}}{a^2 f^2 e^{(3fx+3e)} + 3a^2 f^2 e^{(2fx+2e)} + 3a^2 f^2 e^{(fx+e)} + a^2 f^2} - \frac{\log\left(\left(e^{(fx+e)} + 1\right)e^{(-e)}\right)}{a^2 f^2} \right) + \frac{2}{3}c \left( \frac{1}{\left(3a^2 e^{(-fx-e)} + 3a^2\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+a\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out]  $\frac{2}{3}d \cdot \left( \frac{f x e^{(3fx+3e)} + (3fx e^{(2e)} + e^{(2e)})e^{(2fx)} + e^{(fx+e)}}{a^2 f^2 e^{(3fx+3e)} + 3a^2 f^2 e^{(2fx+2e)} + 3a^2 f^2 e^{(fx+e)} + a^2 f^2} - \frac{\log\left(\left(e^{(fx+e)} + 1\right)e^{(-e)}\right)}{a^2 f^2} \right) + \frac{2}{3}c \cdot \left( \frac{1}{\left(3a^2 e^{(-fx-e)} + 3a^2\right)} \right)$

**mupad [B]** time = 0.89, size = 138, normalized size = 1.12

$$\frac{2d}{3a^2 f^2 (e^{e+fx} + 1)} - \frac{2(d + cf + dfx)}{3a^2 f^2 (2e^{e+fx} + e^{2e+2fx} + 1)} + \frac{2dx}{3a^2 f} - \frac{2d \ln(e^{fx} e^e + 1)}{3a^2 f^2} - \frac{4e^{e+fx} (c + dx)}{3a^2 f (3e^{e+fx} + 3e^{2e+2fx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)/(a + a\*cosh(e + f\*x))^2,x)

[Out]  $\frac{(2*d)}{(3*a^2*f^2*(\exp(e + f*x) + 1))} - \frac{(2*(d + c*f + d*f*x))}{(3*a^2*f^2*(2*\exp(e + f*x) + \exp(2*e + 2*f*x) + 1))} + \frac{(2*d*x)}{(3*a^2*f)} - \frac{(2*d*\log(\exp(f*x)*\exp(e) + 1))}{(3*a^2*f^2)} - \frac{(4*\exp(e + f*x)*(c + d*x))}{(3*a^2*f*(3*\exp(e + f*x) + 3*\exp(2*e + 2*f*x) + \exp(3*e + 3*f*x) + 1))}$

**sympy [A]** time = 1.26, size = 156, normalized size = 1.27

$$\left\{ \begin{array}{l} \frac{c \tanh^3\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2 f} + \frac{c \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{2a^2 f} - \frac{dx \tanh^3\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2 f} + \frac{dx \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{2a^2 f} - \frac{dx}{3a^2 f} + \frac{2d \log\left(\tanh\left(\frac{e}{2} + \frac{fx}{2}\right) + 1\right)}{3a^2 f^2} - \frac{d \tanh^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{6a^2 f^2} \\ \frac{cx + \frac{dx^2}{2}}{(a \cosh(e) + a)^2} \end{array} \right. \begin{array}{l} \text{for} \\ \text{oth} \end{array}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+a\*cosh(f\*x+e))\*\*2,x)

[Out]  $\text{Piecewise}\left(\left(-c*\tanh\left(\frac{e}{2} + \frac{f*x}{2}\right)\right)**3/(6*a**2*f) + c*\tanh\left(\frac{e}{2} + \frac{f*x}{2}\right)/(2*a**2*f) - d*x*\tanh\left(\frac{e}{2} + \frac{f*x}{2}\right)**3/(6*a**2*f) + d*x*\tanh\left(\frac{e}{2} + \frac{f*x}{2}\right)/(2*a**2*f)\right)$

```
) - d*x/(3*a**2*f) + 2*d*log(tanh(e/2 + f*x/2) + 1)/(3*a**2*f**2) - d*tanh(
e/2 + f*x/2)**2/(6*a**2*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*cosh(e) + a)
**2, True))
```

$$3.119 \quad \int \frac{1}{(c+dx)(a+a \cosh(e+fx))^2} dx$$

**Optimal.** Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a \cosh(e+fx)+a)^2}, x\right)$$

[Out] Unintegrable(1/(d\*x+c)/(a+a\*cosh(f\*x+e))^2, x)

**Rubi [A]** time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(c+dx)(a+a \cosh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d\*x)\*(a + a\*Cosh[e + f\*x]))^2, x]

[Out] Defer[Int][1/((c + d\*x)\*(a + a\*Cosh[e + f\*x]))^2, x]

Rubi steps

$$\int \frac{1}{(c+dx)(a+a \cosh(e+fx))^2} dx = \int \frac{1}{(c+dx)(a+a \cosh(e+fx))^2} dx$$

**Mathematica [A]** time = 30.76, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+a \cosh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d\*x)\*(a + a\*Cosh[e + f\*x]))^2, x]

[Out] Integrate[1/((c + d\*x)\*(a + a\*Cosh[e + f\*x]))^2, x]

**fricas [A]** time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2 dx + a^2 c + (a^2 dx + a^2 c) \cosh(fx + e)^2 + 2(a^2 dx + a^2 c) \cosh(fx + e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+a\*cosh(f\*x+e))^2,x, algorithm="fricas")

[Out] integral(1/(a^2\*d\*x + a^2\*c + (a^2\*d\*x + a^2\*c)\*cosh(f\*x + e)^2 + 2\*(a^2\*d\*x + a^2\*c)\*cosh(f\*x + e)), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)(a \cosh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+a\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate(1/((d\*x + c)\*(a\*cosh(f\*x + e) + a)^2), x)

**maple** [A] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)(a + a \cosh(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x+c)/(a+a\*cosh(f\*x+e))^2,x)

[Out] int(1/(d\*x+c)/(a+a\*cosh(f\*x+e))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$2 \left( d^2 f^2 x^2 + 2 c d f^2 x + c^2 f^2 - 2 d^2 + (d^2 f x e^{2e} + c d f^2 x e^{2e} + c^2 f^2 e^{2e}) \right)$$

---


$$3 \left( a^2 d^3 f^3 x^3 + 3 a^2 c d^2 f^3 x^2 + 3 a^2 c^2 d f^3 x + a^2 c^3 f^3 + (a^2 d^3 f^3 x^3 e^{3e} + 3 a^2 c d^2 f^3 x^2 e^{3e} + 3 a^2 c^2 d f^3 x e^{3e} + a^2 c^3 f^3 e^{3e}) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+a\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out] -2/3\*(d^2\*f^2\*x^2 + 2\*c\*d\*f^2\*x + c^2\*f^2 - 2\*d^2 + (d^2\*f\*x\*e^(2\*e) + c\*d\*f\*e^(2\*e) - 2\*d^2\*e^(2\*e))\*e^(2\*f\*x) + (3\*d^2\*f^2\*x^2\*e^e + 3\*c^2\*f^2\*e^e + c\*d\*f\*e^e - 4\*d^2\*e^e + (6\*c\*d\*f^2\*e^e + d^2\*f\*e^e)\*x)\*e^(f\*x))/(a^2\*d^3\*f^3\*x^3 + 3\*a^2\*c\*d^2\*f^3\*x^2 + 3\*a^2\*c^2\*d\*f^3\*x + a^2\*c^3\*f^3 + (a^2\*d^3\*f^3\*x^3\*e^(3\*e) + 3\*a^2\*c\*d^2\*f^3\*x^2\*e^(3\*e) + 3\*a^2\*c^2\*d\*f^3\*x\*e^(3\*e) + a^2\*c^3\*f^3\*e^(3\*e))\*e^(3\*f\*x) + 3\*(a^2\*d^3\*f^3\*x^3\*e^(2\*e) + 3\*a^2\*c\*d^2\*f^3\*x^2\*e^(2\*e) + 3\*a^2\*c^2\*d\*f^3\*x\*e^(2\*e) + a^2\*c^3\*f^3\*e^(2\*e))\*e^(2\*f\*x) + 3\*(a^2\*d^3\*f^3\*x^3\*e^e + 3\*a^2\*c\*d^2\*f^3\*x^2\*e^e + 3\*a^2\*c^2\*d\*f^3\*x\*e^e + a^2\*c^3\*f^3\*e^e)\*e^(f\*x)) - integrate(2/3\*(d^3\*f^2\*x^2 + 2\*c\*d^2\*f^2\*x + c^2\*d\*f^2 - 2\*d^2 + (d^2\*f\*x\*e^(2\*e) + c\*d\*f\*e^(2\*e) - 2\*d^2\*e^(2\*e))\*e^(2\*f\*x) + (3\*d^2\*f^2\*x^2\*e^e + 3\*c^2\*f^2\*e^e + c\*d\*f\*e^e - 4\*d^2\*e^e + (6\*c\*d\*f^2\*e^e + d^2\*f\*e^e)\*x)\*e^(f\*x)), x)

$(c^2*d*f^2 - 6*d^3)/(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4*e^e + 4*a^2*c*d^3*f^3*x^3*e^e + 6*a^2*c^2*d^2*f^3*x^2*e^e + 4*a^2*c^3*d*f^3*x*e^e + a^2*c^4*f^3*e^e)*e^{(f*x)}), x)$

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a + a \cosh(e + f x))^2 (c + d x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + a\*cosh(e + f\*x))^2\*(c + d\*x)), x)

[Out] int(1/((a + a\*cosh(e + f\*x))^2\*(c + d\*x)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c \cosh^2(e+fx)+2c \cosh(e+fx)+c+dx \cosh^2(e+fx)+2dx \cosh(e+fx)+dx} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+a\*cosh(f\*x+e))\*\*2,x)

[Out] Integral(1/(c\*cosh(e + f\*x)\*\*2 + 2\*c\*cosh(e + f\*x) + c + d\*x\*cosh(e + f\*x)\*\*2 + 2\*d\*x\*cosh(e + f\*x) + d\*x), x)/a\*\*2

$$3.120 \quad \int \frac{1}{(c+dx)^2(a+a \cosh(e+fx))^2} dx$$

**Optimal.** Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a \cosh(e+fx)+a)^2}, x\right)$$

[Out] Unintegrable(1/(d\*x+c)^2/(a+a\*cosh(f\*x+e))^2, x)

**Rubi [A]** time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+a \cosh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d\*x)^2\*(a + a\*Cosh[e + f\*x]))^2, x]

[Out] Defer[Int][1/((c + d\*x)^2\*(a + a\*Cosh[e + f\*x]))^2, x]

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a \cosh(e+fx))^2} dx = \int \frac{1}{(c+dx)^2(a+a \cosh(e+fx))^2} dx$$

**Mathematica [A]** time = 32.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+a \cosh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d\*x)^2\*(a + a\*Cosh[e + f\*x]))^2, x]

[Out] Integrate[1/((c + d\*x)^2\*(a + a\*Cosh[e + f\*x]))^2, x]

**fricas [A]** time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2d^2x^2 + 2a^2cdx + a^2c^2 + (a^2d^2x^2 + 2a^2cdx + a^2c^2) \cosh(fx + e)^2 + 2(a^2d^2x^2 + 2a^2cdx + a^2c^2) \cosh(fx + e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(1/(d\*x+c)^2/(a+a\*cosh(f\*x+e))^2,x, algorithm="fricas")

[Out] integral(1/(a^2\*d^2\*x^2 + 2\*a^2\*c\*d\*x + a^2\*c^2 + (a^2\*d^2\*x^2 + 2\*a^2\*c\*d\*x + a^2\*c^2)\*cosh(f\*x + e)^2 + 2\*(a^2\*d^2\*x^2 + 2\*a^2\*c\*d\*x + a^2\*c^2)\*cosh(f\*x + e)), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)^2 (a \cosh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+a\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate(1/((d\*x + c)^2\*(a\*cosh(f\*x + e) + a)^2), x)

**maple** [A] time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)^2 (a + a \cosh(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x+c)^2/(a+a\*cosh(f\*x+e))^2,x)

[Out] int(1/(d\*x+c)^2/(a+a\*cosh(f\*x+e))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$2(d^2 f^2 x^2 + 2 cd)$$

---


$$3(a^2 d^4 f^3 x^4 + 4 a^2 c d^3 f^3 x^3 + 6 a^2 c^2 d^2 f^3 x^2 + 4 a^2 c^3 d f^3 x + a^2 c^4 f^3 + (a^2 d^4 f^3 x^4 e^{(3e)} + 4 a^2 c d^3 f^3 x^3 e^{(3e)} + 6 a^2 c^2 d^2 f^3 x^2 e^{(3e)} + 4 a^2 c^3 d f^3 x e^{(3e)} + a^2 c^4 f^3 e^{(3e)})) e^{(2f*x)} + (3 d^2 f^2 x^2 e^{(2e)} + 3 c^2 f^2 e^{(2e)} + 2 c d f e^{(2e)} - 12 d^2 e^{(2e)} + 2(3 c d f^2 e^{(2e)} + d^2 f e^{(2e)}) x) e^{(f*x)}) / (a^2 d^4 f^3 x^4 + 4 a^2 c d^3 f^3 x^3 + 6 a^2 c^2 d^2 f^3 x^2 + 4 a^2 c^3 d f^3 x + a^2 c^4 f^3 + (a^2 d^4 f^3 x^4 e^{(3e)} + 4 a^2 c d^3 f^3 x^3 e^{(3e)} + 6 a^2 c^2 d^2 f^3 x^2 e^{(3e)} + 4 a^2 c^3 d f^3 x e^{(3e)} + a^2 c^4 f^3 e^{(3e)}) e^{(3f*x)} + 3(a^2 d^4 f^3 x^4 e^{(2e)} + 4 a^2 c d^3 f^3 x^3 e^{(2e)} + 6 a^2 c^2 d^2 f^3 x^2 e^{(2e)} + 4 a^2 c^3 d f^3 x e^{(2e)} + a^2 c^4 f^3 e^{(2e)}) e^{(2f*x)} + 2(d^2 f^2 x^2 + 2 cd) e^{(f*x)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+a\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out] -2/3\*(d^2\*f^2\*x^2 + 2\*c\*d\*f^2\*x + c^2\*f^2 - 6\*d^2 + 2\*(d^2\*f\*x\*e^(2\*e) + c\*d\*f\*e^(2\*e) - 3\*d^2\*e^(2\*e))e^(2\*f\*x) + (3\*d^2\*f^2\*x^2\*e^e + 3\*c^2\*f^2\*e^e + 2\*c\*d\*f\*e^e - 12\*d^2\*e^e + 2\*(3\*c\*d\*f^2\*e^e + d^2\*f\*e^e)\*x)\*e^(f\*x))/(a^2\*d^4\*f^3\*x^4 + 4\*a^2\*c\*d^3\*f^3\*x^3 + 6\*a^2\*c^2\*d^2\*f^3\*x^2 + 4\*a^2\*c^3\*d\*f^3\*x + a^2\*c^4\*f^3 + (a^2\*d^4\*f^3\*x^4\*e^(3\*e) + 4\*a^2\*c\*d^3\*f^3\*x^3\*e^(3\*e) + 6\*a^2\*c^2\*d^2\*f^3\*x^2\*e^(3\*e) + 4\*a^2\*c^3\*d\*f^3\*x\*e^(3\*e) + a^2\*c^4\*f^3\*e^(3\*e))\*e^(3\*f\*x) + 3\*(a^2\*d^4\*f^3\*x^4\*e^(2\*e) + 4\*a^2\*c\*d^3\*f^3\*x^3\*e^(2\*e) + 6\*a^2\*c^2\*d^2\*f^3\*x^2\*e^(2\*e) + 4\*a^2\*c^3\*d\*f^3\*x\*e^(2\*e) + a^2\*c^4\*f^3\*e^(2\*e))\*e^(2\*f\*x) + 2\*(d^2\*f^2\*x^2 + 2\*c\*d\*f^2\*x + c^2\*f^2 - 6\*d^2 + 2\*(d^2\*f\*x\*e^(2\*e) + c\*d\*f\*e^(2\*e) - 3\*d^2\*e^(2\*e))e^(2\*f\*x) + (3\*d^2\*f^2\*x^2\*e^e + 3\*c^2\*f^2\*e^e + 2\*c\*d\*f\*e^e - 12\*d^2\*e^e + 2\*(3\*c\*d\*f^2\*e^e + d^2\*f\*e^e)\*x)\*e^(f\*x))

```

3*e^(2*e))*e^(2*f*x) + 3*(a^2*d^4*f^3*x^4*e^e + 4*a^2*c*d^3*f^3*x^3*e^e + 6
*a^2*c^2*d^2*f^3*x^2*e^e + 4*a^2*c^3*d*f^3*x*e^e + a^2*c^4*f^3*e^e)*e^(f*x)
) - integrate(4/3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 12*d^3)/(a^2*d
^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*
f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3 + (a^2*d^5*f^3*x^5*e^e + 5*a^2*c*
d^4*f^3*x^4*e^e + 10*a^2*c^2*d^3*f^3*x^3*e^e + 10*a^2*c^3*d^2*f^3*x^2*e^e +
5*a^2*c^4*d*f^3*x*e^e + a^2*c^5*f^3*e^e)*e^(f*x)), x)

```

**mupad [A]** time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a + a \cosh(e + f x))^2 (c + d x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cosh(e + f*x))^2*(c + d*x)^2), x)
```

```
[Out] int(1/((a + a*cosh(e + f*x))^2*(c + d*x)^2), x)
```

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c^2 \cosh^2(e+fx) + 2c^2 \cosh(e+fx) + c^2 + 2cdx \cosh^2(e+fx) + 4cdx \cosh(e+fx) + 2cdx + d^2x^2 \cosh^2(e+fx) + 2d^2x^2 \cosh(e+fx) + d^2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+a*cosh(f*x+e))**2, x)
```

```
[Out] Integral(1/(c**2*cosh(e + f*x)**2 + 2*c**2*cosh(e + f*x) + c**2 + 2*c*d*x*c
osh(e + f*x)**2 + 4*c*d*x*cosh(e + f*x) + 2*c*d*x + d**2*x**2*cosh(e + f*x)
**2 + 2*d**2*x**2*cosh(e + f*x) + d**2*x**2), x)/a**2
```

### 3.121 $\int x^3 \sqrt{a + a \cosh(c + dx)} dx$

**Optimal.** Leaf size=110

$$-\frac{96\sqrt{a \cosh(c + dx) + a}}{d^4} + \frac{48x \tanh\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c + dx) + a}}{d^3} - \frac{12x^2 \sqrt{a \cosh(c + dx) + a}}{d^2} + \frac{2x^3 \tanh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d}$$

[Out]  $-96*(a+a*\cosh(d*x+c))^(1/2)/d^4-12*x^2*(a+a*\cosh(d*x+c))^(1/2)/d^2+48*x*(a+a*\cosh(d*x+c))^(1/2)*\tanh(1/2*d*x+1/2*c)/d^3+2*x^3*(a+a*\cosh(d*x+c))^(1/2)*\tanh(1/2*d*x+1/2*c)/d$

**Rubi [A]** time = 0.15, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3319, 3296, 2638}

$$-\frac{12x^2 \sqrt{a \cosh(c + dx) + a}}{d^2} - \frac{96\sqrt{a \cosh(c + dx) + a}}{d^4} + \frac{48x \tanh\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c + dx) + a}}{d^3} + \frac{2x^3 \tanh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d}$$

Antiderivative was successfully verified.

[In] `Int[x^3*Sqrt[a + a*Cosh[c + d*x]],x]`

[Out]  $(-96*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]])/d^4 - (12*x^2*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]])/d^2 + (48*x*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]]*\text{Tanh}[c/2 + (d*x)/2])/d^3 + (2*x^3*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]]*\text{Tanh}[c/2 + (d*x)/2])/d$

#### Rule 2638

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

#### Rule 3296

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

#### Rule 3319

`Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[((2*a)^IntPart[n]*(a + b*Ssin[e + f*x])^FracPart[n])/Sin[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Ssin[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])`

Rubi steps

$$\begin{aligned}
\int x^3 \sqrt{a + a \cosh(c + dx)} dx &= \left( \sqrt{a + a \cosh(c + dx)} \csc \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} + \frac{idx}{2} \right) \right) \int x^3 \sin \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} + \frac{idx}{2} \right) dx \\
&= \frac{2x^3 \sqrt{a + a \cosh(c + dx)} \tanh \left( \frac{c}{2} + \frac{dx}{2} \right)}{d} - \frac{\left( 6\sqrt{a + a \cosh(c + dx)} \csc \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} + \frac{idx}{2} \right) \right) \int x^2 \sin \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} + \frac{idx}{2} \right) dx}{d} \\
&= -\frac{12x^2 \sqrt{a + a \cosh(c + dx)}}{d^2} + \frac{2x^3 \sqrt{a + a \cosh(c + dx)} \tanh \left( \frac{c}{2} + \frac{dx}{2} \right)}{d} + \frac{\left( 24\sqrt{a + a \cosh(c + dx)} \csc \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} + \frac{idx}{2} \right) \right) \int x \sin \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} + \frac{idx}{2} \right) dx}{d} \\
&= -\frac{12x^2 \sqrt{a + a \cosh(c + dx)}}{d^2} + \frac{48x \sqrt{a + a \cosh(c + dx)} \tanh \left( \frac{c}{2} + \frac{dx}{2} \right)}{d^3} + \frac{2x^3 \sqrt{a + a \cosh(c + dx)}}{d^3} \\
&= -\frac{96\sqrt{a + a \cosh(c + dx)}}{d^4} - \frac{12x^2 \sqrt{a + a \cosh(c + dx)}}{d^2} + \frac{48x \sqrt{a + a \cosh(c + dx)}}{d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 53, normalized size = 0.48

$$\frac{2 \left( dx (d^2 x^2 + 24) \tanh \left( \frac{1}{2} (c + dx) \right) - 6 (d^2 x^2 + 8) \right) \sqrt{a (\cosh(c + dx) + 1)}}{d^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Sqrt[a + a\*Cosh[c + d\*x]],x]

[Out] (2\*Sqrt[a\*(1 + Cosh[c + d\*x])]\*(-6\*(8 + d^2\*x^2) + d\*x\*(24 + d^2\*x^2)\*Tanh[(c + d\*x)/2]))/d^4

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac [A]** time = 0.12, size = 147, normalized size = 1.34

$$\frac{\sqrt{2} \left( \sqrt{a} d^3 x^3 e^{\left( \frac{1}{2} dx + \frac{1}{2} c \right)} - \sqrt{a} d^3 x^3 e^{\left( -\frac{1}{2} dx - \frac{1}{2} c \right)} - 6 \sqrt{a} d^2 x^2 e^{\left( \frac{1}{2} dx + \frac{1}{2} c \right)} - 6 \sqrt{a} d^2 x^2 e^{\left( -\frac{1}{2} dx - \frac{1}{2} c \right)} + 24 \sqrt{a} dx e^{\left( \frac{1}{2} dx + \frac{1}{2} c \right)} - 24 \sqrt{a} dx e^{\left( -\frac{1}{2} dx - \frac{1}{2} c \right)} \right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="giac")

[Out] sqrt(2)\*(sqrt(a)\*d^3\*x^3\*e^(1/2\*d\*x + 1/2\*c) - sqrt(a)\*d^3\*x^3\*e^(-1/2\*d\*x - 1/2\*c) - 6\*sqrt(a)\*d^2\*x^2\*e^(1/2\*d\*x + 1/2\*c) - 6\*sqrt(a)\*d^2\*x^2\*e^(-1/2\*d\*x - 1/2\*c) + 24\*sqrt(a)\*d\*x\*e^(1/2\*d\*x + 1/2\*c) - 24\*sqrt(a)\*d\*x\*e^(-1/2\*d\*x - 1/2\*c) - 48\*sqrt(a)\*e^(1/2\*d\*x + 1/2\*c) - 48\*sqrt(a)\*e^(-1/2\*d\*x - 1/2\*c))/d^4

**maple [A]** time = 0.14, size = 108, normalized size = 0.98

$$\frac{\sqrt{2} \sqrt{a \left( e^{dx+c} + 1 \right)^2} e^{-dx-c} \left( d^3 x^3 e^{dx+c} - d^3 x^3 - 6d^2 x^2 e^{dx+c} - 6d^2 x^2 + 24dx e^{dx+c} - 24dx - 48 e^{dx+c} - 48 \right)}{\left( e^{dx+c} + 1 \right) d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+a\*cosh(d\*x+c))^(1/2),x)

[Out] 2^(1/2)\*(a\*(exp(d\*x+c)+1)^2\*exp(-d\*x-c))^(1/2)/(exp(d\*x+c)+1)\*(d^3\*x^3\*exp(d\*x+c)-d^3\*x^3-6\*d^2\*x^2\*exp(d\*x+c)-6\*d^2\*x^2+24\*d\*x\*exp(d\*x+c)-24\*d\*x-48\*exp(d\*x+c)-48)/d^4

**maxima [A]** time = 0.45, size = 120, normalized size = 1.09

$$\frac{\left( \sqrt{2} \sqrt{a} d^3 x^3 + 6 \sqrt{2} \sqrt{a} d^2 x^2 + 24 \sqrt{2} \sqrt{a} dx - \left( \sqrt{2} \sqrt{a} d^3 x^3 e^c - 6 \sqrt{2} \sqrt{a} d^2 x^2 e^c + 24 \sqrt{2} \sqrt{a} dx e^c - 48 \sqrt{2} \sqrt{a} e^c \right) \right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] -(sqrt(2)\*sqrt(a)\*d^3\*x^3 + 6\*sqrt(2)\*sqrt(a)\*d^2\*x^2 + 24\*sqrt(2)\*sqrt(a)\*d\*x - (sqrt(2)\*sqrt(a)\*d^3\*x^3\*e^c - 6\*sqrt(2)\*sqrt(a)\*d^2\*x^2\*e^c + 24\*sqrt(2)\*sqrt(a)\*d\*x\*e^c - 48\*sqrt(2)\*sqrt(a)\*e^c)\*e^(d\*x) + 48\*sqrt(2)\*sqrt(a)\*e^(-1/2\*d\*x - 1/2\*c)/d^4

**mupad [B]** time = 0.21, size = 117, normalized size = 1.06

$$\frac{\sqrt{a + a \left( \frac{e^{c+dx}}{2} + \frac{e^{-c-dx}}{2} \right)} \left( \frac{96 e^{c+dx}}{d^4} + \frac{48x}{d^3} + \frac{96}{d^4} + \frac{2x^3}{d} + \frac{12x^2}{d^2} - \frac{2x^3 e^{c+dx}}{d} + \frac{12x^2 e^{c+dx}}{d^2} - \frac{48x e^{c+dx}}{d^3} \right)}{e^{c+dx} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a + a\*cosh(c + d\*x))^(1/2),x)

```
[Out] -((a + a*(exp(c + d*x)/2 + exp(- c - d*x)/2))^(1/2)*((96*exp(c + d*x))/d^4
+ (48*x)/d^3 + 96/d^4 + (2*x^3)/d + (12*x^2)/d^2 - (2*x^3*exp(c + d*x))/d +
(12*x^2*exp(c + d*x))/d^2 - (48*x*exp(c + d*x))/d^3))/(exp(c + d*x) + 1)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int x^3 \sqrt{a(\cosh(c + dx) + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*(cosh(c + d*x) + 1)), x)
```

### 3.122 $\int x^2 \sqrt{a + a \cosh(c + dx)} dx$

**Optimal.** Leaf size=88

$$\frac{16 \tanh\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c + dx) + a}}{d^3} - \frac{8x \sqrt{a \cosh(c + dx) + a}}{d^2} + \frac{2x^2 \tanh\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c + dx) + a}}{d}$$

[Out]  $-8*x*(a+a*\cosh(d*x+c))^{(1/2)}/d^2+16*(a+a*\cosh(d*x+c))^{(1/2)}*\tanh(1/2*d*x+1/2*c)/d^3+2*x^2*(a+a*\cosh(d*x+c))^{(1/2)}*\tanh(1/2*d*x+1/2*c)/d$

**Rubi [A]** time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3319, 3296, 2637}

$$-\frac{8x \sqrt{a \cosh(c + dx) + a}}{d^2} + \frac{16 \tanh\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c + dx) + a}}{d^3} + \frac{2x^2 \tanh\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c + dx) + a}}{d}$$

Antiderivative was successfully verified.

[In] `Int[x^2*Sqrt[a + a*Cosh[c + d*x]],x]`

[Out]  $(-8*x*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]])/d^2 + (16*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]]*\text{Tanh}[c/2 + (d*x)/2])/d^3 + (2*x^2*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]]*\text{Tanh}[c/2 + (d*x)/2])/d$

#### Rule 2637

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`  
`FreeQ[{c, d}, x]`

#### Rule 3296

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[`  
`((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[`  
`e + f*x], x], x] /;` `FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

#### Rule 3319

`Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.),`  
`x_Symbol] := Dist[((2*a)^IntPart[n]*(a + b*Ssin[e + f*x])^FracPart[n])/Sin[`  
`e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Ssin[e/2 + (a`  
`*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /;` `FreeQ[{a, b, c, d, e, f, m}, x] && E`  
`qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])`

Rubi steps

$$\begin{aligned}
\int x^2 \sqrt{a + a \cosh(c + dx)} dx &= \left( \sqrt{a + a \cosh(c + dx)} \csc \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} + \frac{idx}{2} \right) \right) \int x^2 \sin \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} + \frac{idx}{2} \right) dx \\
&= \frac{2x^2 \sqrt{a + a \cosh(c + dx)} \tanh \left( \frac{c}{2} + \frac{dx}{2} \right)}{d} - \frac{\left( 4\sqrt{a + a \cosh(c + dx)} \csc \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} + \frac{idx}{2} \right) \right) dx}{d} \\
&= -\frac{8x \sqrt{a + a \cosh(c + dx)}}{d^2} + \frac{2x^2 \sqrt{a + a \cosh(c + dx)} \tanh \left( \frac{c}{2} + \frac{dx}{2} \right)}{d} + \frac{\left( 8\sqrt{a + a \cosh(c + dx)} \csc \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} + \frac{idx}{2} \right) \right) dx}{d^2} \\
&= -\frac{8x \sqrt{a + a \cosh(c + dx)}}{d^2} + \frac{16\sqrt{a + a \cosh(c + dx)} \tanh \left( \frac{c}{2} + \frac{dx}{2} \right)}{d^3} + \frac{2x^2 \sqrt{a + a \cosh(c + dx)}}{d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 44, normalized size = 0.50

$$\frac{2 \left( (d^2 x^2 + 8) \tanh \left( \frac{1}{2}(c + dx) \right) - 4dx \right) \sqrt{a(\cosh(c + dx) + 1)}}{d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Sqrt[a + a\*Cosh[c + d\*x]],x]

[Out] (2\*Sqrt[a\*(1 + Cosh[c + d\*x])]\*(-4\*d\*x + (8 + d^2\*x^2)\*Tanh[(c + d\*x)/2]))/d^3

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac [A]** time = 0.14, size = 107, normalized size = 1.22

$$\frac{\sqrt{2} \left( \sqrt{a} d^2 x^2 e^{\left( \frac{1}{2} dx + \frac{1}{2} c \right)} - \sqrt{a} d^2 x^2 e^{\left( -\frac{1}{2} dx - \frac{1}{2} c \right)} - 4 \sqrt{a} dx e^{\left( \frac{1}{2} dx + \frac{1}{2} c \right)} - 4 \sqrt{a} dx e^{\left( -\frac{1}{2} dx - \frac{1}{2} c \right)} + 8 \sqrt{a} e^{\left( \frac{1}{2} dx + \frac{1}{2} c \right)} - 8 \sqrt{a} e^{\left( -\frac{1}{2} dx - \frac{1}{2} c \right)} \right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x^2\*(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="giac")

[Out] sqrt(2)\*(sqrt(a)\*d^2\*x^2\*e^(1/2\*d\*x + 1/2\*c) - sqrt(a)\*d^2\*x^2\*e^(-1/2\*d\*x - 1/2\*c) - 4\*sqrt(a)\*d\*x\*e^(1/2\*d\*x + 1/2\*c) - 4\*sqrt(a)\*d\*x\*e^(-1/2\*d\*x - 1/2\*c) + 8\*sqrt(a)\*e^(1/2\*d\*x + 1/2\*c) - 8\*sqrt(a)\*e^(-1/2\*d\*x - 1/2\*c))/d^3

**maple** [A] time = 0.09, size = 86, normalized size = 0.98

$$\frac{\sqrt{2} \sqrt{a \left( e^{dx+c} + 1 \right)^2 e^{-dx-c} \left( d^2 x^2 e^{dx+c} - d^2 x^2 - 4dx e^{dx+c} - 4dx + 8 e^{dx+c} - 8 \right)}}{\left( e^{dx+c} + 1 \right) d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+a\*cosh(d\*x+c))^(1/2),x)

[Out] 2^(1/2)\*(a\*(exp(d\*x+c)+1)^2\*exp(-d\*x-c))^(1/2)/(exp(d\*x+c)+1)\*(d^2\*x^2\*exp(d\*x+c)-d^2\*x^2-4\*d\*x\*exp(d\*x+c)-4\*d\*x+8\*exp(d\*x+c)-8)/d^3

**maxima** [A] time = 0.45, size = 90, normalized size = 1.02

$$\frac{\left( \sqrt{2} \sqrt{a} d^2 x^2 + 4 \sqrt{2} \sqrt{a} dx - \left( \sqrt{2} \sqrt{a} d^2 x^2 e^c - 4 \sqrt{2} \sqrt{a} dx e^c + 8 \sqrt{2} \sqrt{a} e^c \right) e^{(dx)} + 8 \sqrt{2} \sqrt{a} \right) e^{\left( -\frac{1}{2} dx - \frac{1}{2} c \right)}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] -(sqrt(2)\*sqrt(a)\*d^2\*x^2 + 4\*sqrt(2)\*sqrt(a)\*d\*x - (sqrt(2)\*sqrt(a)\*d^2\*x^2\*e^c - 4\*sqrt(2)\*sqrt(a)\*d\*x\*e^c + 8\*sqrt(2)\*sqrt(a)\*e^c)\*e^(d\*x) + 8\*sqrt(2)\*sqrt(a)\*e^(-1/2\*d\*x - 1/2\*c)/d^3

**mupad** [B] time = 0.93, size = 95, normalized size = 1.08

$$\frac{\sqrt{a + a \left( \frac{e^{c+dx}}{2} + \frac{e^{-c-dx}}{2} \right)} \left( \frac{8x}{d^2} - \frac{16e^{c+dx}}{d^3} + \frac{16}{d^3} + \frac{2x^2}{d} - \frac{2x^2 e^{c+dx}}{d} + \frac{8x e^{c+dx}}{d^2} \right)}{e^{c+dx} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a + a\*cosh(c + d\*x))^(1/2),x)

[Out] -((a + a\*(exp(c + d\*x)/2 + exp(-c - d\*x)/2))^(1/2)\*((8\*x)/d^2 - (16\*exp(c + d\*x))/d^3 + 16/d^3 + (2\*x^2)/d - (2\*x^2\*exp(c + d\*x))/d + (8\*x\*exp(c + d\*x))/d^2))/(exp(c + d\*x) + 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a (\cosh(c + dx) + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+a\*cosh(d\*x+c))\*\*(1/2),x)

[Out] Integral(x\*\*2\*sqrt(a\*(cosh(c + d\*x) + 1)), x)

### 3.123 $\int x \sqrt{a + a \cosh(c + dx)} dx$

Optimal. Leaf size=53

$$\frac{2x \tanh\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c + dx) + a}}{d} - \frac{4\sqrt{a \cosh(c + dx) + a}}{d^2}$$

[Out]  $-4*(a+a*\cosh(d*x+c))^{(1/2)}/d^2+2*x*(a+a*\cosh(d*x+c))^{(1/2)}*\tanh(1/2*d*x+1/2*c)/d$

**Rubi [A]** time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {3319, 3296, 2638}

$$\frac{2x \tanh\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c + dx) + a}}{d} - \frac{4\sqrt{a \cosh(c + dx) + a}}{d^2}$$

Antiderivative was successfully verified.

[In] Int[x\*Sqrt[a + a\*Cosh[c + d\*x]], x]

[Out]  $(-4*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]])/d^2 + (2*x*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]]*\text{Tanh}[c/2 + (d*x)/2])/d$

#### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] :> -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3319

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Dist[((2\*a)^IntPart[n]\*(a + b\*Sin[e + f\*x])^FracPart[n])/Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*FracPart[n]), Int[(c + d\*x)^m\*Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

#### Rubi steps

$$\begin{aligned}
\int x\sqrt{a+a\cosh(c+dx)} dx &= \left(\sqrt{a+a\cosh(c+dx)} \csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right) \int x \sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right) dx \\
&= \frac{2x\sqrt{a+a\cosh(c+dx)} \tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(2\sqrt{a+a\cosh(c+dx)} \csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)}{d} \\
&= -\frac{4\sqrt{a+a\cosh(c+dx)}}{d^2} + \frac{2x\sqrt{a+a\cosh(c+dx)} \tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 34, normalized size = 0.64

$$\frac{2\left(dx \tanh\left(\frac{1}{2}(c+dx)\right) - 2\right)\sqrt{a(\cosh(c+dx)+1)}}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sqrt[a + a\*Cosh[c + d\*x]],x]

[Out] (2\*Sqrt[a\*(1 + Cosh[c + d\*x])]\*(-2 + d\*x\*Tanh[(c + d\*x)/2]))/d^2

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac [A]** time = 0.13, size = 67, normalized size = 1.26

$$\frac{\sqrt{2}\left(\sqrt{a} dx e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)} - \sqrt{a} dx e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)} - 2\sqrt{a} e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)} - 2\sqrt{a} e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="giac")

[Out] sqrt(2)\*(sqrt(a)\*d\*x\*e^(1/2\*d\*x + 1/2\*c) - sqrt(a)\*d\*x\*e^(-1/2\*d\*x - 1/2\*c) - 2\*sqrt(a)\*e^(1/2\*d\*x + 1/2\*c) - 2\*sqrt(a)\*e^(-1/2\*d\*x - 1/2\*c))/d^2

**maple** [A] time = 0.10, size = 64, normalized size = 1.21

$$\frac{\sqrt{2} \sqrt{a(e^{dx+c} + 1)^2 e^{-dx-c}} (dx e^{dx+c} - dx - 2 e^{dx+c} - 2)}{(e^{dx+c} + 1) d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+a\*cosh(d\*x+c))^(1/2),x)

[Out]  $2^{(1/2)} * (a * (\exp(d*x+c)+1)^2 * \exp(-d*x-c))^{(1/2)} / (\exp(d*x+c)+1) * (d*x*\exp(d*x+c) - d*x - 2*\exp(d*x+c) - 2) / d^2$

**maxima** [A] time = 0.45, size = 60, normalized size = 1.13

$$\frac{(\sqrt{2} \sqrt{a} dx - (\sqrt{2} \sqrt{a} dx e^c - 2 \sqrt{2} \sqrt{a} e^c) e^{(dx)} + 2 \sqrt{2} \sqrt{a}) e^{(-\frac{1}{2} dx - \frac{1}{2} c)}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="maxima")

[Out]  $-(\text{sqrt}(2)*\text{sqrt}(a)*d*x - (\text{sqrt}(2)*\text{sqrt}(a)*d*x*e^c - 2*\text{sqrt}(2)*\text{sqrt}(a)*e^c)*e^{(d*x)} + 2*\text{sqrt}(2)*\text{sqrt}(a)*e^{(-1/2*d*x - 1/2*c)})/d^2$

**mupad** [B] time = 0.91, size = 56, normalized size = 1.06

$$\frac{2 x \sinh\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a + a \cosh(c + dx)}}{d \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)} - \frac{4 \sqrt{a + a \cosh(c + dx)}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a + a\*cosh(c + d\*x))^(1/2),x)

[Out]  $(2*x*\sinh(c/2 + (d*x)/2)*(a + a*\cosh(c + d*x))^(1/2))/(d*\cosh(c/2 + (d*x)/2)) - (4*(a + a*\cosh(c + d*x))^(1/2))/d^2$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{a (\cosh(c + dx) + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+a\*cosh(d\*x+c))\*\*(1/2),x)

[Out] Integral(x\*sqrt(a\*(cosh(c + d\*x) + 1)), x)

$$3.124 \quad \int \frac{\sqrt{a+a \cosh(c+dx)}}{x} dx$$

**Optimal.** Leaf size=83

$$\cosh\left(\frac{c}{2}\right) \operatorname{Chi}\left(\frac{dx}{2}\right) \operatorname{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c+dx) + a} + \sinh\left(\frac{c}{2}\right) \operatorname{Shi}\left(\frac{dx}{2}\right) \operatorname{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c+dx) + a}$$

[Out] Chi(1/2\*d\*x)\*cosh(1/2\*c)\*sech(1/2\*d\*x+1/2\*c)\*(a+a\*cosh(d\*x+c))^(1/2)+sech(1/2\*d\*x+1/2\*c)\*Shi(1/2\*d\*x)\*sinh(1/2\*c)\*(a+a\*cosh(d\*x+c))^(1/2)

**Rubi [A]** time = 0.13, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3319, 3303, 3298, 3301}

$$\cosh\left(\frac{c}{2}\right) \operatorname{Chi}\left(\frac{dx}{2}\right) \operatorname{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c+dx) + a} + \sinh\left(\frac{c}{2}\right) \operatorname{Shi}\left(\frac{dx}{2}\right) \operatorname{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c+dx) + a}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a\*Cosh[c + d\*x]]/x,x]

[Out] Cosh[c/2]\*Sqrt[a + a\*Cosh[c + d\*x]]\*CoshIntegral[(d\*x)/2]\*Sech[c/2 + (d\*x)/2] + Sqrt[a + a\*Cosh[c + d\*x]]\*Sech[c/2 + (d\*x)/2]\*Sinh[c/2]\*SinhIntegral[(d\*x)/2]

#### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

#### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

#### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
  x_Symbol] :> Dist[((2*a)^(IntPart[n])*(a + b*Sin[e + f*x])^(FracPart[n]))/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + a \cosh(c + dx)}}{x} dx &= \left( \sqrt{a + a \cosh(c + dx)} \csc\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{idx}{2}\right) \right) \int \frac{\sin\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{idx}{2}\right)}{x} dx \\ &= \left( \cosh\left(\frac{c}{2}\right) \sqrt{a + a \cosh(c + dx)} \csc\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{idx}{2}\right) \right) \int \frac{\cosh\left(\frac{dx}{2}\right)}{x} dx + \dots \\ &= \cosh\left(\frac{c}{2}\right) \sqrt{a + a \cosh(c + dx)} \operatorname{Chi}\left(\frac{dx}{2}\right) \operatorname{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) + \sqrt{a + a \cosh(c + dx)} \operatorname{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \end{aligned}$$

**Mathematica** [A] time = 0.07, size = 54, normalized size = 0.65

$$\operatorname{sech}\left(\frac{1}{2}(c + dx)\right) \sqrt{a(\cosh(c + dx) + 1)} \left( \cosh\left(\frac{c}{2}\right) \operatorname{Chi}\left(\frac{dx}{2}\right) + \sinh\left(\frac{c}{2}\right) \operatorname{Shi}\left(\frac{dx}{2}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[c + d*x]]/x,x]
```

```
[Out] Sqrt[a*(1 + Cosh[c + d*x])]*Sech[(c + d*x)/2]*(Cosh[c/2]*CoshIntegral[(d*x)/2] + Sinh[c/2]*SinhIntegral[(d*x)/2])
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)
```

**giac** [A] time = 0.12, size = 32, normalized size = 0.39

$$\frac{1}{2} \sqrt{2} \left( \sqrt{a} \operatorname{Ei}\left(\frac{1}{2} dx\right) e^{\left(\frac{1}{2} c\right)} + \sqrt{a} \operatorname{Ei}\left(-\frac{1}{2} dx\right) e^{\left(-\frac{1}{2} c\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(d\*x+c))^(1/2)/x,x, algorithm="giac")

[Out] 1/2\*sqrt(2)\*(sqrt(a)\*Ei(1/2\*d\*x)\*e^(1/2\*c) + sqrt(a)\*Ei(-1/2\*d\*x)\*e^(-1/2\*c))

**maple** [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + a \cosh(dx + c)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*cosh(d\*x+c))^(1/2)/x,x)

[Out] int((a+a\*cosh(d\*x+c))^(1/2)/x,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \cosh(dx + c) + a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(d\*x+c))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(a\*cosh(d\*x + c) + a)/x, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + a \cosh(c + dx)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cosh(c + d\*x))^(1/2)/x,x)

[Out] int((a + a\*cosh(c + d\*x))^(1/2)/x, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\cosh(c + dx) + 1)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(d\*x+c))\*\*(1/2)/x,x)

[Out] Integral(sqrt(a\*(cosh(c + d\*x) + 1))/x, x)



$$3.125 \quad \int \frac{\sqrt{a+a \cosh(c+dx)}}{x^2} dx$$

**Optimal.** Leaf size=110

$$\frac{1}{2}d \sinh\left(\frac{c}{2}\right) \text{Chi}\left(\frac{dx}{2}\right) \text{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c+dx) + a} + \frac{1}{2}d \cosh\left(\frac{c}{2}\right) \text{Shi}\left(\frac{dx}{2}\right) \text{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c+dx)}$$

[Out]  $-(a+a*\cosh(d*x+c))^{(1/2)}/x+1/2*d*\cosh(1/2*c)*\text{sech}(1/2*d*x+1/2*c)*\text{Shi}(1/2*d*x)$   
 $*(a+a*\cosh(d*x+c))^{(1/2)}+1/2*d*\text{Chi}(1/2*d*x)*\text{sech}(1/2*d*x+1/2*c)*\sinh(1/2*c)$   
 $*(a+a*\cosh(d*x+c))^{(1/2)}$

**Rubi [A]** time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3319, 3297, 3303, 3298, 3301}

$$\frac{1}{2}d \sinh\left(\frac{c}{2}\right) \text{Chi}\left(\frac{dx}{2}\right) \text{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c+dx) + a} + \frac{1}{2}d \cosh\left(\frac{c}{2}\right) \text{Shi}\left(\frac{dx}{2}\right) \text{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a\*Cosh[c + d\*x]]/x^2,x]

[Out]  $-(\text{Sqrt}[a + a*\text{Cosh}[c + d*x]]/x) + (d*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]]*\text{CoshIntegral}[(d*x)/2]*\text{Sech}[c/2 + (d*x)/2]*\text{Sinh}[c/2])/2 + (d*\text{Cosh}[c/2]*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]]*\text{Sech}[c/2 + (d*x)/2]*\text{SinhIntegral}[(d*x)/2])/2$

Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.),
x_Symbol] := Dist[((2*a)^IntPart[n]*(a + b*SIN[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*SIN[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + a \cosh(c + dx)}}{x^2} dx &= \left( \sqrt{a + a \cosh(c + dx)} \csc \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} + \frac{idx}{2} \right) \right) \int \frac{\sin \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} + \frac{idx}{2} \right)}{x^2} dx \\ &= -\frac{\sqrt{a + a \cosh(c + dx)}}{x} + \frac{1}{2} \left( d \sqrt{a + a \cosh(c + dx)} \csc \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} + \frac{idx}{2} \right) \right) \int \frac{1}{x} dx \\ &= -\frac{\sqrt{a + a \cosh(c + dx)}}{x} + \frac{1}{2} \left( d \cosh \left( \frac{c}{2} \right) \sqrt{a + a \cosh(c + dx)} \csc \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} \right) \right) \int \frac{1}{x} dx \\ &= -\frac{\sqrt{a + a \cosh(c + dx)}}{x} + \frac{1}{2} d \sqrt{a + a \cosh(c + dx)} \operatorname{Chi} \left( \frac{dx}{2} \right) \operatorname{sech} \left( \frac{c}{2} + \frac{dx}{2} \right) \sinh \left( \frac{c}{2} \right) \end{aligned}$$

**Mathematica** [A] time = 0.14, size = 75, normalized size = 0.68

$$\frac{\sqrt{a(\cosh(c + dx) + 1)} \left( dx \sinh \left( \frac{c}{2} \right) \operatorname{Chi} \left( \frac{dx}{2} \right) \operatorname{sech} \left( \frac{1}{2}(c + dx) \right) + dx \cosh \left( \frac{c}{2} \right) \operatorname{Shi} \left( \frac{dx}{2} \right) \operatorname{sech} \left( \frac{1}{2}(c + dx) \right) - 2 \right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[c + d*x]]/x^2,x]
```

```
[Out] (Sqrt[a*(1 + Cosh[c + d*x])]*(-2 + d*x*CoshIntegral[(d*x)/2]*Sech[(c + d*x)/2]*Sinh[c/2] + d*x*Cosh[c/2]*Sech[(c + d*x)/2]*SinhIntegral[(d*x)/2]))/(2*x)
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(d\*x+c))^(1/2)/x^2,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac** [A] time = 0.14, size = 68, normalized size = 0.62

$$\frac{\sqrt{2} \left( \sqrt{a} dx \operatorname{Ei} \left( \frac{1}{2} dx \right) e^{\left( \frac{1}{2} c \right)} - \sqrt{a} dx \operatorname{Ei} \left( -\frac{1}{2} dx \right) e^{\left( -\frac{1}{2} c \right)} - 2 \sqrt{a} e^{\left( \frac{1}{2} dx + \frac{1}{2} c \right)} - 2 \sqrt{a} e^{\left( -\frac{1}{2} dx - \frac{1}{2} c \right)} \right)}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(d\*x+c))^(1/2)/x^2,x, algorithm="giac")

[Out] 1/4\*sqrt(2)\*(sqrt(a)\*d\*x\*Ei(1/2\*d\*x)\*e^(1/2\*c) - sqrt(a)\*d\*x\*Ei(-1/2\*d\*x)\*e^(-1/2\*c) - 2\*sqrt(a)\*e^(1/2\*d\*x + 1/2\*c) - 2\*sqrt(a)\*e^(-1/2\*d\*x - 1/2\*c)) /x

**maple** [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + a \cosh(dx + c)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*cosh(d\*x+c))^(1/2)/x^2,x)

[Out] int((a+a\*cosh(d\*x+c))^(1/2)/x^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \cosh(dx + c) + a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(d\*x+c))^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(a\*cosh(d\*x + c) + a)/x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + a \cosh(c + dx)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cosh(c + d*x))^(1/2)/x^2, x)`

[Out] `int((a + a*cosh(c + d*x))^(1/2)/x^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\cosh(c + dx) + 1)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cosh(d*x+c))**(1/2)/x**2, x)`

[Out] `Integral(sqrt(a*(cosh(c + d*x) + 1))/x**2, x)`

$$3.126 \quad \int \frac{\sqrt{a+a \cosh(c+dx)}}{x^3} dx$$

**Optimal.** Leaf size=151

$$\frac{1}{8}d^2 \cosh\left(\frac{c}{2}\right) \operatorname{Chi}\left(\frac{dx}{2}\right) \operatorname{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c+dx) + a} + \frac{1}{8}d^2 \sinh\left(\frac{c}{2}\right) \operatorname{Shi}\left(\frac{dx}{2}\right) \operatorname{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c+dx) + a}$$

[Out]  $-1/2*(a+a*\cosh(d*x+c))^{(1/2)}/x^2+1/8*d^2*\operatorname{Chi}(1/2*d*x)*\cosh(1/2*c)*\operatorname{sech}(1/2*d*x+1/2*c)*(a+a*\cosh(d*x+c))^{(1/2)}+1/8*d^2*\operatorname{sech}(1/2*d*x+1/2*c)*\operatorname{Shi}(1/2*d*x)*\sinh(1/2*c)*(a+a*\cosh(d*x+c))^{(1/2)}-1/4*d*(a+a*\cosh(d*x+c))^{(1/2)}*\tanh(1/2*d*x+1/2*c)/x$

**Rubi [A]** time = 0.17, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3319, 3297, 3303, 3298, 3301}

$$\frac{1}{8}d^2 \cosh\left(\frac{c}{2}\right) \operatorname{Chi}\left(\frac{dx}{2}\right) \operatorname{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c+dx) + a} + \frac{1}{8}d^2 \sinh\left(\frac{c}{2}\right) \operatorname{Shi}\left(\frac{dx}{2}\right) \operatorname{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh(c+dx) + a}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a\*Cosh[c + d\*x]]/x^3,x]

[Out]  $-\operatorname{Sqrt}[a + a*\operatorname{Cosh}[c + d*x]]/(2*x^2) + (d^2*\operatorname{Cosh}[c/2]*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[c + d*x]])*\operatorname{CoshIntegral}[(d*x)/2]*\operatorname{Sech}[c/2 + (d*x)/2]/8 + (d^2*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[c + d*x]])*\operatorname{Sech}[c/2 + (d*x)/2]*\operatorname{Sinh}[c/2]*\operatorname{SinhIntegral}[(d*x)/2]/8 - (d*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[c + d*x]])*\operatorname{Tanh}[c/2 + (d*x)/2]/(4*x)$

#### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

#### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)])/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

#### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x]
+ Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /;
FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.),
x_Symbol]
:> Dist[((2*a)^(IntPart[n]*(a + b*Sin[e + f*x])^(FracPart[n]))/Sin[e/2
+ (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a*Pi)
/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2
- b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + a \cosh(c + dx)}}{x^3} dx &= \left( \sqrt{a + a \cosh(c + dx)} \csc \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} + \frac{idx}{2} \right) \right) \int \frac{\sin \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} + \frac{idx}{2} \right)}{x^3} dx \\ &= -\frac{\sqrt{a + a \cosh(c + dx)}}{2x^2} + \frac{1}{4} \left( d \sqrt{a + a \cosh(c + dx)} \csc \left( \frac{1}{2} \left( ic + \frac{\pi}{2} \right) + \frac{\pi}{4} + \frac{idx}{2} \right) \right) \int \frac{1}{x^2} dx \\ &= -\frac{\sqrt{a + a \cosh(c + dx)}}{2x^2} - \frac{d \sqrt{a + a \cosh(c + dx)} \tanh \left( \frac{c}{2} + \frac{dx}{2} \right)}{4x} + \frac{1}{8} \left( d^2 \sqrt{a + a \cosh(c + dx)} \right) \int \frac{1}{x} dx \\ &= -\frac{\sqrt{a + a \cosh(c + dx)}}{2x^2} - \frac{d \sqrt{a + a \cosh(c + dx)} \tanh \left( \frac{c}{2} + \frac{dx}{2} \right)}{4x} + \frac{1}{8} \left( d^2 \cosh \left( \frac{c}{2} \right) \sqrt{a + a \cosh(c + dx)} \right) \int \frac{1}{x} dx \\ &= -\frac{\sqrt{a + a \cosh(c + dx)}}{2x^2} + \frac{1}{8} d^2 \cosh \left( \frac{c}{2} \right) \sqrt{a + a \cosh(c + dx)} \operatorname{Chi} \left( \frac{dx}{2} \right) \operatorname{sech} \left( \frac{c}{2} + \frac{dx}{2} \right) \end{aligned}$$

**Mathematica [A]** time = 0.21, size = 97, normalized size = 0.64

$$\frac{\sqrt{a(\cosh(c + dx) + 1)} \left( d^2 x^2 \cosh \left( \frac{c}{2} \right) \operatorname{Chi} \left( \frac{dx}{2} \right) \operatorname{sech} \left( \frac{1}{2}(c + dx) \right) + d^2 x^2 \sinh \left( \frac{c}{2} \right) \operatorname{Shi} \left( \frac{dx}{2} \right) \operatorname{sech} \left( \frac{1}{2}(c + dx) \right) - 2dx \right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[c + d*x]]/x^3,x]
```

```
[Out] (Sqrt[a*(1 + Cosh[c + d*x])]*(-4 + d^2*x^2*Cosh[c/2]*CoshIntegral[(d*x)/2]*
Sech[(c + d*x)/2] + d^2*x^2*Sech[(c + d*x)/2]*Sinh[c/2]*SinhIntegral[(d*x)/
2] - 2*d*x*Tanh[(c + d*x)/2]))/(8*x^2)
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (has polynomial part)
```

**giac** [A] time = 0.13, size = 107, normalized size = 0.71

$$\frac{\sqrt{2} \left( \sqrt{a} d^2 x^2 \operatorname{Ei} \left( \frac{1}{2} dx \right) e^{\left( \frac{1}{2} c \right)} + \sqrt{a} d^2 x^2 \operatorname{Ei} \left( -\frac{1}{2} dx \right) e^{\left( -\frac{1}{2} c \right)} - 2 \sqrt{a} dx e^{\left( \frac{1}{2} dx + \frac{1}{2} c \right)} + 2 \sqrt{a} dx e^{\left( -\frac{1}{2} dx - \frac{1}{2} c \right)} - 4 \sqrt{a} e^{\left( \frac{1}{2} dx + \frac{1}{2} c \right)} \right)}{16 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(sqrt(a)*d^2*x^2*Ei(1/2*d*x)*e^(1/2*c) + sqrt(a)*d^2*x^2*Ei(-1
/2*d*x)*e^(-1/2*c) - 2*sqrt(a)*d*x*e^(1/2*d*x + 1/2*c) + 2*sqrt(a)*d*x*e^(-
1/2*d*x - 1/2*c) - 4*sqrt(a)*e^(1/2*d*x + 1/2*c) - 4*sqrt(a)*e^(-1/2*d*x -
1/2*c))/x^2
```

**maple** [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + a \cosh(dx + c)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(1/2)/x^3,x)
```

```
[Out] int((a+a*cosh(d*x+c))^(1/2)/x^3,x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \cosh(dx + c) + a}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(d\*x+c))^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(a\*cosh(d\*x + c) + a)/x^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + a \cosh(c + d x)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cosh(c + d\*x))^(1/2)/x^3,x)

[Out] int((a + a\*cosh(c + d\*x))^(1/2)/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a (\cosh(c + dx) + 1)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(d\*x+c))\*\*(1/2)/x\*\*3,x)

[Out] Integral(sqrt(a\*(cosh(c + d\*x) + 1))/x\*\*3, x)



### 3.127 $\int x^3 \sqrt{a + a \cosh(x)} dx$

Optimal. Leaf size=68

$$2x^3 \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - 12x^2 \sqrt{a \cosh(x) + a} - 96 \sqrt{a \cosh(x) + a} + 48x \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a}$$

[Out]  $-96*(a+a*\cosh(x))^{(1/2)}-12*x^2*(a+a*\cosh(x))^{(1/2)}+48*x*(a+a*\cosh(x))^{(1/2)}$   
 $*\tanh(1/2*x)+2*x^3*(a+a*\cosh(x))^{(1/2)}*\tanh(1/2*x)$

**Rubi [A]** time = 0.12, antiderivative size = 68, normalized size of antiderivative = 1.00,  
 number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} =$   
 0.214, Rules used = {3319, 3296, 2638}

$$-12x^2 \sqrt{a \cosh(x) + a} + 2x^3 \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - 96 \sqrt{a \cosh(x) + a} + 48x \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a}$$

Antiderivative was successfully verified.

[In] Int[x^3\*Sqrt[a + a\*Cosh[x]],x]

[Out]  $-96*\text{Sqrt}[a + a*\text{Cosh}[x]] - 12*x^2*\text{Sqrt}[a + a*\text{Cosh}[x]] + 48*x*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Tanh}[x/2]$   
 $+ 2*x^3*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Tanh}[x/2]$

#### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] :> -Simp[  
 ((c + d\*x)^m\*cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3319

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.),  
 x\_Symbol] :> Dist[((2\*a)^IntPart[n]\*(a + b\*Sin[e + f\*x])^FracPart[n])/Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*FracPart[n]), Int[(c + d\*x)^m\*Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

#### Rubi steps

$$\begin{aligned}
\int x^3 \sqrt{a + a \cosh(x)} dx &= \left( \sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right) \right) \int x^3 \cosh\left(\frac{x}{2}\right) dx \\
&= 2x^3 \sqrt{a + a \cosh(x)} \tanh\left(\frac{x}{2}\right) - \left( 6\sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right) \right) \int x^2 \sinh\left(\frac{x}{2}\right) dx \\
&= -12x^2 \sqrt{a + a \cosh(x)} + 2x^3 \sqrt{a + a \cosh(x)} \tanh\left(\frac{x}{2}\right) + \left( 24\sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right) \right) \\
&= -12x^2 \sqrt{a + a \cosh(x)} + 48x \sqrt{a + a \cosh(x)} \tanh\left(\frac{x}{2}\right) + 2x^3 \sqrt{a + a \cosh(x)} \tanh\left(\frac{x}{2}\right) \\
&= -96\sqrt{a + a \cosh(x)} - 12x^2 \sqrt{a + a \cosh(x)} + 48x \sqrt{a + a \cosh(x)} \tanh\left(\frac{x}{2}\right) + 2x^3 \sqrt{a + a \cosh(x)} \tanh\left(\frac{x}{2}\right)
\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 33, normalized size = 0.49

$$2 \left( x \left( x^2 + 24 \right) \tanh\left(\frac{x}{2}\right) - 6 \left( x^2 + 8 \right) \right) \sqrt{a(\cosh(x) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Sqrt[a + a\*Cosh[x]],x]

[Out] 2\*Sqrt[a\*(1 + Cosh[x])]\*(-6\*(8 + x^2) + x\*(24 + x^2)\*Tanh[x/2])

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+a\*cosh(x))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \cosh(x) + a} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+a\*cosh(x))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a\*cosh(x) + a)\*x^3, x)

**maple** [A] time = 0.10, size = 62, normalized size = 0.91

$$\frac{\sqrt{2} \sqrt{a(e^x + 1)^2 e^{-x}} (x^3 e^x - x^3 - 6x^2 e^x - 6x^2 + 24x e^x - 24x - 48 e^x - 48)}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+a\*cosh(x))^(1/2), x)

[Out] 2^(1/2)\*(a\*(exp(x)+1)^2\*exp(-x))^(1/2)/(exp(x)+1)\*(x^3\*exp(x)-x^3-6\*x^2\*exp(x)-6\*x^2+24\*x\*exp(x)-24\*x-48\*exp(x)-48)

**maxima** [A] time = 0.41, size = 88, normalized size = 1.29

$$-\left(\sqrt{2} \sqrt{a} x^3 + 6 \sqrt{2} \sqrt{a} x^2 + 24 \sqrt{2} \sqrt{a} x - \left(\sqrt{2} \sqrt{a} x^3 - 6 \sqrt{2} \sqrt{a} x^2 + 24 \sqrt{2} \sqrt{a} x - 48 \sqrt{2} \sqrt{a}\right) e^x + 48 \sqrt{2} \sqrt{a}\right) e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+a\*cosh(x))^(1/2), x, algorithm="maxima")

[Out] -(sqrt(2)\*sqrt(a)\*x^3 + 6\*sqrt(2)\*sqrt(a)\*x^2 + 24\*sqrt(2)\*sqrt(a)\*x - (sqrt(2)\*sqrt(a)\*x^3 - 6\*sqrt(2)\*sqrt(a)\*x^2 + 24\*sqrt(2)\*sqrt(a)\*x - 48\*sqrt(2)\*sqrt(a))\*e^x + 48\*sqrt(2)\*sqrt(a))\*e^(-1/2\*x)

**mupad** [B] time = 0.91, size = 63, normalized size = 0.93

$$\frac{\sqrt{a + a \left( \frac{e^{-x}}{2} + \frac{e^x}{2} \right)} (48x + 96e^x + 12x^2 e^x - 2x^3 e^x - 48x e^x + 12x^2 + 2x^3 + 96)}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a + a\*cosh(x))^(1/2), x)

[Out] -((a + a\*(exp(-x)/2 + exp(x)/2))^(1/2)\*(48\*x + 96\*exp(x) + 12\*x^2\*exp(x) - 2\*x^3\*exp(x) - 48\*x\*exp(x) + 12\*x^2 + 2\*x^3 + 96))/(exp(x) + 1)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{a(\cosh(x) + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+a\*cosh(x))\*\*(1/2), x)

[Out] Integral(x\*\*3\*sqrt(a\*(cosh(x) + 1)), x)

### 3.128 $\int x^2 \sqrt{a + a \cosh(x)} dx$

Optimal. Leaf size=53

$$2x^2 \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - 8x \sqrt{a \cosh(x) + a} + 16 \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a}$$

[Out]  $-8*x*(a+a*\cosh(x))^{(1/2)}+16*(a+a*\cosh(x))^{(1/2)}*\tanh(1/2*x)+2*x^2*(a+a*\cosh(x))^{(1/2)}*\tanh(1/2*x)$

**Rubi [A]** time = 0.10, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3319, 3296, 2637}

$$2x^2 \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - 8x \sqrt{a \cosh(x) + a} + 16 \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a}$$

Antiderivative was successfully verified.

[In] `Int[x^2*Sqrt[a + a*Cosh[x]],x]`

[Out]  $-8*x*\text{Sqrt}[a + a*\text{Cosh}[x]] + 16*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Tanh}[x/2] + 2*x^2*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Tanh}[x/2]$

#### Rule 2637

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`  
`FreeQ[{c, d}, x]`

#### Rule 3296

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[`  
`((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[`  
`e + f*x], x], x] /;` `FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

#### Rule 3319

`Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),`  
`x_Symbol] := Dist[((2*a)^IntPart[n]*(a + b*Ssin[e + f*x])^FracPart[n])/Sin[`  
`e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a`  
`*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /;` `FreeQ[{a, b, c, d, e, f, m}, x] && E`  
`qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])`

#### Rubi steps

$$\begin{aligned}
\int x^2 \sqrt{a + a \cosh(x)} dx &= \left( \sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right) \right) \int x^2 \cosh\left(\frac{x}{2}\right) dx \\
&= 2x^2 \sqrt{a + a \cosh(x)} \tanh\left(\frac{x}{2}\right) - \left( 4\sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right) \right) \int x \sinh\left(\frac{x}{2}\right) dx \\
&= -8x \sqrt{a + a \cosh(x)} + 2x^2 \sqrt{a + a \cosh(x)} \tanh\left(\frac{x}{2}\right) + \left( 8\sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right) \right) \int x dx \\
&= -8x \sqrt{a + a \cosh(x)} + 16\sqrt{a + a \cosh(x)} \tanh\left(\frac{x}{2}\right) + 2x^2 \sqrt{a + a \cosh(x)} \tanh\left(\frac{x}{2}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 31, normalized size = 0.58

$$8 \left( \frac{1}{4} (x^2 + 8) \tanh\left(\frac{x}{2}\right) - x \right) \sqrt{a(\cosh(x) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Sqrt[a + a\*Cosh[x]],x]

[Out] 8\*Sqrt[a\*(1 + Cosh[x])]\*(-x + ((8 + x^2)\*Tanh[x/2])/4)

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+a\*cosh(x))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \cosh(x) + a x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+a\*cosh(x))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a\*cosh(x) + a)\*x^2, x)

**maple [A]** time = 0.08, size = 50, normalized size = 0.94

$$\frac{\sqrt{2} \sqrt{a(e^x + 1)^2 e^{-x}} (x^2 e^x - x^2 - 4x e^x - 4x + 8 e^x - 8)}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+a*cosh(x))^(1/2),x)`

[Out]  $2^{1/2}*(a*(\exp(x)+1)^2*\exp(-x))^{1/2}/(\exp(x)+1)*(x^2*\exp(x)-x^2-4*x*\exp(x)-4*x+8*\exp(x)-8)$

**maxima** [A] time = 0.42, size = 66, normalized size = 1.25

$$-\left(\sqrt{2}\sqrt{a}x^2 + 4\sqrt{2}\sqrt{a}x - \left(\sqrt{2}\sqrt{a}x^2 - 4\sqrt{2}\sqrt{a}x + 8\sqrt{2}\sqrt{a}\right)e^x + 8\sqrt{2}\sqrt{a}\right)e^{\left(-\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+a*cosh(x))^(1/2),x, algorithm="maxima")`

[Out]  $-(\sqrt{2}*\sqrt{a}*x^2 + 4*\sqrt{2}*\sqrt{a}*x - (\sqrt{2}*\sqrt{a}*x^2 - 4*\sqrt{2}*(2)*\sqrt{a}*x + 8*\sqrt{2}*\sqrt{a})*e^x + 8*\sqrt{2}*\sqrt{a})*e^{(-1/2*x)}$

**mupad** [B] time = 0.07, size = 51, normalized size = 0.96

$$\frac{\sqrt{a + a\left(\frac{e^{-x}}{2} + \frac{e^x}{2}\right)}(8x - 16e^x - 2x^2e^x + 8xe^x + 2x^2 + 16)}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + a*cosh(x))^(1/2),x)`

[Out]  $-((a + a*(\exp(-x)/2 + \exp(x)/2))^{1/2}*(8*x - 16*\exp(x) - 2*x^2*\exp(x) + 8*x*\exp(x) + 2*x^2 + 16))/(\exp(x) + 1)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a(\cosh(x)+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+a*cosh(x))**(1/2),x)`

[Out] `Integral(x**2*sqrt(a*(cosh(x) + 1)), x)`

### 3.129 $\int x\sqrt{a + a \cosh(x)} dx$

Optimal. Leaf size=32

$$2x \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - 4\sqrt{a \cosh(x) + a}$$

[Out]  $-4*(a+a*\cosh(x))^{(1/2)}+2*x*(a+a*\cosh(x))^{(1/2)}*\tanh(1/2*x)$

**Rubi [A]** time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3319, 3296, 2638}

$$2x \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - 4\sqrt{a \cosh(x) + a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*\text{Sqrt}[a + a*\text{Cosh}[x]], x]$

[Out]  $-4*\text{Sqrt}[a + a*\text{Cosh}[x]] + 2*x*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Tanh}[x/2]$

#### Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$  FreeQ[{c, d}, x]

#### Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$  FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3319

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[(2*a)^{\text{IntPart}[n]}*(a + b*\text{Sin}[e + f*x])^{\text{FracPart}[n]}/\text{Sin}[e/2 + (a*\text{Pi})/(4*b) + (f*x)/2]^{(2*\text{FracPart}[n])}, \text{Int}[(c + d*x)^m*\text{Sin}[e/2 + (a*\text{Pi})/(4*b) + (f*x)/2]^{(2*n)}, x], x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

#### Rubi steps

$$\begin{aligned}
\int x\sqrt{a+a\cosh(x)} dx &= \left(\sqrt{a+a\cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right)\right) \int x \cosh\left(\frac{x}{2}\right) dx \\
&= 2x\sqrt{a+a\cosh(x)} \tanh\left(\frac{x}{2}\right) - \left(2\sqrt{a+a\cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right)\right) \int \sinh\left(\frac{x}{2}\right) dx \\
&= -4\sqrt{a+a\cosh(x)} + 2x\sqrt{a+a\cosh(x)} \tanh\left(\frac{x}{2}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 22, normalized size = 0.69

$$2\left(x \tanh\left(\frac{x}{2}\right) - 2\right) \sqrt{a(\cosh(x) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sqrt[a + a\*Cosh[x]],x]

[Out] 2\*Sqrt[a\*(1 + Cosh[x])]\*(-2 + x\*Tanh[x/2])

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+a\*cosh(x))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \cosh(x) + a} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+a\*cosh(x))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a\*cosh(x) + a)\*x, x)

**maple [A]** time = 0.08, size = 38, normalized size = 1.19

$$\frac{\sqrt{2} \sqrt{a(e^x + 1)^2 e^{-x}} (x e^x - x - 2 e^x - 2)}{e^x + 1}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+a*cosh(x))^(1/2),x)`

[Out]  $2^{(1/2)}*(a*(\exp(x)+1)^2*\exp(-x))^{(1/2)}/(\exp(x)+1)*(x*\exp(x)-x-2*\exp(x)-2)$

**maxima** [A] time = 0.41, size = 44, normalized size = 1.38

$$-\left(\sqrt{2}\sqrt{a}x - \left(\sqrt{2}\sqrt{a}x - 2\sqrt{2}\sqrt{a}\right)e^x + 2\sqrt{2}\sqrt{a}\right)e^{\left(-\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+a*cosh(x))^(1/2),x, algorithm="maxima")`

[Out]  $-(\sqrt{2}*\sqrt{a}*x - (\sqrt{2}*\sqrt{a}*x - 2*\sqrt{2}*\sqrt{a}))*e^x + 2*\sqrt{2}*\sqrt{a})*e^{(-1/2*x)}$

**mupad** [B] time = 0.88, size = 39, normalized size = 1.22

$$\frac{\sqrt{a + a \left(\frac{e^{-x}}{2} + \frac{e^x}{2}\right)} (2x + 4e^x - 2xe^x + 4)}{e^x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + a*cosh(x))^(1/2),x)`

[Out]  $-((a + a*(\exp(-x)/2 + \exp(x)/2))^{(1/2)}*(2*x + 4*\exp(x) - 2*x*\exp(x) + 4))/(\exp(x) + 1)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a(\cosh(x) + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+a*cosh(x))**(1/2),x)`

[Out] `Integral(x*sqrt(a*(cosh(x) + 1)), x)`

$$3.130 \quad \int \frac{\sqrt{a+a \cosh(x)}}{x} dx$$

**Optimal.** Leaf size=23

$$\operatorname{Chi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a}$$

[Out] Chi(1/2\*x)\*sech(1/2\*x)\*(a+a\*cosh(x))^(1/2)

**Rubi [A]** time = 0.08, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3319, 3301}

$$\operatorname{Chi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a\*Cosh[x]]/x,x]

[Out] Sqrt[a + a\*Cosh[x]]\*CoshIntegral[x/2]\*Sech[x/2]

#### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

#### Rule 3319

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] :> Dist[((2\*a)^(IntPart[n])\*(a + b\*Sin[e + f\*x])^(FracPart[n]))/Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*FracPart[n]), Int[(c + d\*x)^m\*Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a+a \cosh(x)}}{x} dx &= \left( \sqrt{a+a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right) \right) \int \frac{\cosh\left(\frac{x}{2}\right)}{x} dx \\ &= \sqrt{a+a \cosh(x)} \operatorname{Chi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 23, normalized size = 1.00

$$\operatorname{Chi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) \sqrt{a(\cosh(x) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a\*Cosh[x]]/x,x]

[Out] Sqrt[a\*(1 + Cosh[x])]\*CoshIntegral[x/2]\*Sech[x/2]

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))^(1/2)/x,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \cosh(x) + a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))^(1/2)/x,x, algorithm="giac")

[Out] integrate(sqrt(a\*cosh(x) + a)/x, x)

**maple** [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + a \cosh(x)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*cosh(x))^(1/2)/x,x)

[Out] int((a+a\*cosh(x))^(1/2)/x,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \cosh(x) + a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(a\*cosh(x) + a)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a + a \cosh(x)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cosh(x))^(1/2)/x,x)

[Out] int((a + a\*cosh(x))^(1/2)/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\cosh(x) + 1)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))\*\*(1/2)/x,x)

[Out] Integral(sqrt(a\*(cosh(x) + 1))/x, x)

$$3.131 \quad \int \frac{\sqrt{a+a \cosh(x)}}{x^2} dx$$

**Optimal.** Leaf size=42

$$\frac{1}{2} \operatorname{Shi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - \frac{\sqrt{a \cosh(x) + a}}{x}$$

[Out]  $-(a+a \cosh(x))^{(1/2)}/x+1/2*\operatorname{sech}(1/2*x)*\operatorname{Shi}(1/2*x)*(a+a \cosh(x))^{(1/2)}$

**Rubi [A]** time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3319, 3297, 3298}

$$\frac{1}{2} \operatorname{Shi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - \frac{\sqrt{a \cosh(x) + a}}{x}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[a + a \operatorname{Cosh}[x]]/x^2, x]$

[Out]  $-(\operatorname{Sqrt}[a + a \operatorname{Cosh}[x]]/x) + (\operatorname{Sqrt}[a + a \operatorname{Cosh}[x]]*\operatorname{Sech}[x/2]*\operatorname{SinhIntegral}[x/2])/2$

#### Rule 3297

$\operatorname{Int}[(c_. + (d_.)*(x_))^{(m_)}*\sin[(e_.) + (f_.)*(x_)], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m+1)}*\sin[e + f*x]/(d*(m+1)), x] - \operatorname{Dist}[f/(d*(m+1)), \operatorname{Int}[(c + d*x)^{(m+1)}*\cos[e + f*x], x], x] /;$   $\operatorname{FreeQ}\{c, d, e, f\}, x \ \&\& \ \operatorname{LtQ}[m, -1]$

#### Rule 3298

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)]]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[(I*\operatorname{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /;$   $\operatorname{FreeQ}\{c, d, e, f, fz\}, x \ \&\& \ \operatorname{EqQ}[d*e - c*f*fz*I, 0]$

#### Rule 3319

$\operatorname{Int}[(c_. + (d_.)*(x_))^{(m_)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)])^{(n_)}, x\_Symbol] \rightarrow \operatorname{Dist}[(2*a)^{\operatorname{IntPart}[n]}*(a + b*\sin[e + f*x])^{\operatorname{FracPart}[n]}/\sin[e/2 + (a*\pi)/(4*b) + (f*x)/2]^{(2*\operatorname{FracPart}[n])}, \operatorname{Int}[(c + d*x)^m*\sin[e/2 + (a*\pi)/(4*b) + (f*x)/2]^{(2*n)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \ \operatorname{IntegerQ}[n + 1/2] \ \&\& \ (\operatorname{GtQ}[n, 0] \ || \ \operatorname{IGtQ}[m, 0])$

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + a \cosh(x)}}{x^2} dx &= \left( \sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right) \right) \int \frac{\cosh\left(\frac{x}{2}\right)}{x^2} dx \\
&= -\frac{\sqrt{a + a \cosh(x)}}{x} + \frac{1}{2} \left( \sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right) \right) \int \frac{\sinh\left(\frac{x}{2}\right)}{x} dx \\
&= -\frac{\sqrt{a + a \cosh(x)}}{x} + \frac{1}{2} \sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right) \operatorname{Shi}\left(\frac{x}{2}\right)
\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 33, normalized size = 0.79

$$\frac{\sqrt{a(\cosh(x) + 1)} \left( x \operatorname{Shi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) - 2 \right)}{2x}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a\*Cosh[x]]/x^2,x]

[Out] (Sqrt[a\*(1 + Cosh[x])]\*(-2 + x\*Sech[x/2]\*SinhIntegral[x/2]))/(2\*x)

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))^(1/2)/x^2,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \cosh(x) + a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))^(1/2)/x^2,x, algorithm="giac")

[Out] integrate(sqrt(a\*cosh(x) + a)/x^2, x)

**maple** [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + a \cosh(x)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*cosh(x))^(1/2)/x^2,x)`

[Out] `int((a+a*cosh(x))^(1/2)/x^2,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \cosh(x) + a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cosh(x))^(1/2)/x^2,x, algorithm="maxima")`

[Out] `integrate(sqrt(a*cosh(x) + a)/x^2, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a + a \cosh(x)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cosh(x))^(1/2)/x^2,x)`

[Out] `int((a + a*cosh(x))^(1/2)/x^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\cosh(x) + 1)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cosh(x))**(1/2)/x**2,x)`

[Out] `Integral(sqrt(a*(cosh(x) + 1))/x**2, x)`

$$3.132 \quad \int \frac{\sqrt{a+a \cosh(x)}}{x^3} dx$$

**Optimal.** Leaf size=67

$$\frac{1}{8} \operatorname{Chi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - \frac{\sqrt{a \cosh(x) + a}}{2x^2} - \frac{\tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a}}{4x}$$

[Out]  $-1/2*(a+a*\cosh(x))^{(1/2)}/x^2+1/8*\operatorname{Chi}(1/2*x)*\operatorname{sech}(1/2*x)*(a+a*\cosh(x))^{(1/2)}$   
 $-1/4*(a+a*\cosh(x))^{(1/2)}*\tanh(1/2*x)/x$

**Rubi [A]** time = 0.11, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3319, 3297, 3301}

$$\frac{1}{8} \operatorname{Chi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - \frac{\sqrt{a \cosh(x) + a}}{2x^2} - \frac{\tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a}}{4x}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + a*Cosh[x]]/x^3,x]`

[Out]  $-\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]]/(2*x^2) + (\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]]*\operatorname{CoshIntegral}[x/2]*\operatorname{Sech}[x/2])/8 - (\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]]*\operatorname{Tanh}[x/2])/(4*x)$

**Rule 3297**

`Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]`

**Rule 3301**

`Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]`

**Rule 3319**

`Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[((2*a)^(IntPart[n]*(a + b*Ssin[e + f*x])^FracPart[n])/Sin[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Ssin[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])`



Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + a \cosh(x)}}{x^3} dx &= \left( \sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right) \right) \int \frac{\cosh\left(\frac{x}{2}\right)}{x^3} dx \\
&= -\frac{\sqrt{a + a \cosh(x)}}{2x^2} + \frac{1}{4} \left( \sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right) \right) \int \frac{\sinh\left(\frac{x}{2}\right)}{x^2} dx \\
&= -\frac{\sqrt{a + a \cosh(x)}}{2x^2} - \frac{\sqrt{a + a \cosh(x)} \tanh\left(\frac{x}{2}\right)}{4x} + \frac{1}{8} \left( \sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right) \right) \int \frac{\cosh\left(\frac{x}{2}\right)}{x} dx \\
&= -\frac{\sqrt{a + a \cosh(x)}}{2x^2} + \frac{1}{8} \sqrt{a + a \cosh(x)} \operatorname{Chi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) - \frac{\sqrt{a + a \cosh(x)} \tanh\left(\frac{x}{2}\right)}{4x}
\end{aligned}$$

**Mathematica** [A] time = 0.07, size = 44, normalized size = 0.66

$$\frac{\sqrt{a(\cosh(x) + 1)} \left( x^2 \operatorname{Chi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) - 2x \tanh\left(\frac{x}{2}\right) - 4 \right)}{8x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a\*Cosh[x]]/x^3,x]

[Out] (Sqrt[a\*(1 + Cosh[x])]\*(-4 + x^2\*CoshIntegral[x/2]\*Sech[x/2] - 2\*x\*Tanh[x/2]))/(8\*x^2)

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))^(1/2)/x^3,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \cosh(x) + a}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))^(1/2)/x^3,x, algorithm="giac")

[Out] integrate(sqrt(a\*cosh(x) + a)/x^3, x)

**maple** [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + a \cosh(x)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*cosh(x))^(1/2)/x^3,x)

[Out] int((a+a\*cosh(x))^(1/2)/x^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \cosh(x) + a}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(a\*cosh(x) + a)/x^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + a \cosh(x)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cosh(x))^(1/2)/x^3,x)

[Out] int((a + a\*cosh(x))^(1/2)/x^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a (\cosh(x) + 1)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))\*\*(1/2)/x\*\*3,x)

[Out] Integral(sqrt(a\*(cosh(x) + 1))/x\*\*3, x)

### 3.133 $\int x^3(a + a \cosh(x))^{3/2} dx$

**Optimal.** Leaf size=185

$$\frac{4}{3}ax^3 \sinh\left(\frac{x}{2}\right) \cosh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} + \frac{8}{3}ax^3 \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - \frac{8}{3}ax^2 \cosh^2\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - 16ax^2 \sqrt{a \cosh(x) + a} + \frac{4}{3}ax^3 \sinh\left(\frac{x}{2}\right) \cosh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} + \frac{8}{3}ax^3 \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - \frac{8}{3}ax^2 \cosh^2\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - 16ax^2 \sqrt{a \cosh(x) + a}$$

[Out]  $-1280/9*a*(a+a*\cosh(x))^{(1/2)}-16*a*x^2*(a+a*\cosh(x))^{(1/2)}-64/27*a*\cosh(1/2*x)^2*(a+a*\cosh(x))^{(1/2)}-8/3*a*x^2*\cosh(1/2*x)^2*(a+a*\cosh(x))^{(1/2)}+32/9*a*x*\cosh(1/2*x)*\sinh(1/2*x)*(a+a*\cosh(x))^{(1/2)}+4/3*a*x^3*\cosh(1/2*x)*\sinh(1/2*x)*(a+a*\cosh(x))^{(1/2)}+640/9*a*x*(a+a*\cosh(x))^{(1/2)}*\tanh(1/2*x)+8/3*a*x^3*(a+a*\cosh(x))^{(1/2)}*\tanh(1/2*x)$

**Rubi [A]** time = 0.19, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {3319, 3311, 3296, 2638, 3310}

$$-\frac{8}{3}ax^2 \cosh^2\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - 16ax^2 \sqrt{a \cosh(x) + a} + \frac{4}{3}ax^3 \sinh\left(\frac{x}{2}\right) \cosh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} + \frac{8}{3}ax^3 \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + a\*Cosh[x])^(3/2), x]

[Out]  $(-1280*a*\text{Sqrt}[a + a*\text{Cosh}[x]])/9 - 16*a*x^2*\text{Sqrt}[a + a*\text{Cosh}[x]] - (64*a*\text{Cosh}[x/2]^2*\text{Sqrt}[a + a*\text{Cosh}[x]])/27 - (8*a*x^2*\text{Cosh}[x/2]^2*\text{Sqrt}[a + a*\text{Cosh}[x]])/3 + (32*a*x*\text{Cosh}[x/2]*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Sinh}[x/2])/9 + (4*a*x^3*\text{Cosh}[x/2]*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Sinh}[x/2])/3 + (640*a*x*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Tanh}[x/2])/9 + (8*a*x^3*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Tanh}[x/2])/3$

#### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] := -Simp[((c + d\*x)^m\*cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_.))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] := Simp[(d\*(b\*Sine[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)\*(b\*Sine[e + f\*x])^(n - 2), x], x] - Simp[(b\*(c + d\*x)\*Cos[e + f\*x]\*(b

`*Sin[e + f*x])^(n - 1))/(f*n), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]`

### Rule 3311

`Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sine[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sine[e + f*x])^n, x] - Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]`

### Rule 3319

`Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sine[e + f*x])^FracPart[n])/Sin[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sine[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Eqq[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])`

### Rubi steps

$$\begin{aligned}
 \int x^3(a + a \cosh(x))^{3/2} dx &= \left(2a\sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right)\right) \int x^3 \cosh^3\left(\frac{x}{2}\right) dx \\
 &= -\frac{8}{3}ax^2 \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} + \frac{4}{3}ax^3 \cosh\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} \sinh\left(\frac{x}{2}\right) + \frac{1}{3}\left(4\sqrt{a + a \cosh(x)}\right) \\
 &= -\frac{64}{27}a \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} - \frac{8}{3}ax^2 \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} + \frac{32}{9}ax \cosh\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} \\
 &= -16ax^2 \sqrt{a + a \cosh(x)} - \frac{64}{27}a \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} - \frac{8}{3}ax^2 \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} \\
 &= -\frac{128}{9}a \sqrt{a + a \cosh(x)} - 16ax^2 \sqrt{a + a \cosh(x)} - \frac{64}{27}a \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} - \frac{8}{3}ax^2 \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} \\
 &= -\frac{1280}{9}a \sqrt{a + a \cosh(x)} - 16ax^2 \sqrt{a + a \cosh(x)} - \frac{64}{27}a \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} - \frac{8}{3}ax^2 \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)}
 \end{aligned}$$

**Mathematica** [A] time = 0.29, size = 70, normalized size = 0.38

$$\frac{2}{27}a\sqrt{a(\cosh(x) + 1)} \left(-2(117x^2 + 968) + 3x(15x^2 + 328) \tanh\left(\frac{x}{2}\right) + \cosh(x) \left(3x(3x^2 + 8) \tanh\left(\frac{x}{2}\right) - 2(9x^2 - 1)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + a\*Cosh[x])^(3/2),x]

[Out] (2\*a\*Sqrt[a\*(1 + Cosh[x])]\*(-2\*(968 + 117\*x^2) + 3\*x\*(328 + 15\*x^2)\*Tanh[x/2] + Cosh[x]\*(-2\*(8 + 9\*x^2) + 3\*x\*(8 + 3\*x^2)\*Tanh[x/2]))) / 27

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+a\*cosh(x))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac** [A] time = 0.14, size = 192, normalized size = 1.04

$$-\frac{1}{54}\sqrt{2}\left(54a^{\frac{3}{2}}x^3e^{\left(-\frac{1}{2}x\right)}+9a^{\frac{3}{2}}x^3e^{\left(-\frac{3}{2}x\right)}+324a^{\frac{3}{2}}x^2e^{\left(-\frac{1}{2}x\right)}+18a^{\frac{3}{2}}x^2e^{\left(-\frac{3}{2}x\right)}+1296a^{\frac{3}{2}}xe^{\left(-\frac{1}{2}x\right)}+24a^{\frac{3}{2}}xe^{\left(-\frac{3}{2}x\right)}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+a\*cosh(x))^(3/2),x, algorithm="giac")

[Out]  $-1/54*\sqrt{2}*(54*a^{(3/2)}*x^3*e^{(-1/2*x)} + 9*a^{(3/2)}*x^3*e^{(-3/2*x)} + 324*a^{(3/2)}*x^2*e^{(-1/2*x)} + 18*a^{(3/2)}*x^2*e^{(-3/2*x)} + 1296*a^{(3/2)}*x*e^{(-1/2*x)} + 24*a^{(3/2)}*x*e^{(-3/2*x)} + 2592*a^{(3/2)}*e^{(-1/2*x)} + 16*a^{(3/2)}*e^{(-3/2*x)} - (9*a^{(3/2)}*x^3 - 18*a^{(3/2)}*x^2 + 24*a^{(3/2)}*x - 16*a^{(3/2)})*e^{(3/2*x)} - 81*(a^{(3/2)}*x^3 - 6*a^{(3/2)}*x^2 + 24*a^{(3/2)}*x - 48*a^{(3/2)})*e^{(1/2*x)} + 27*(a^{(3/2)}*x^3 + 6*a^{(3/2)}*x^2 + 24*a^{(3/2)}*x + 48*a^{(3/2)})*e^{(-1/2*x)})$

**maple** [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^3 (a + a \cosh(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+a\*cosh(x))^(3/2),x)

[Out] int(x^3\*(a+a\*cosh(x))^(3/2),x)

**maxima** [A] time = 0.45, size = 180, normalized size = 0.97

$$-\frac{1}{54}\left(9\sqrt{2}a^{\frac{3}{2}}x^3+18\sqrt{2}a^{\frac{3}{2}}x^2+24\sqrt{2}a^{\frac{3}{2}}x+16\sqrt{2}a^{\frac{3}{2}}-\left(9\sqrt{2}a^{\frac{3}{2}}x^3-18\sqrt{2}a^{\frac{3}{2}}x^2+24\sqrt{2}a^{\frac{3}{2}}x-16\sqrt{2}a^{\frac{3}{2}}\right)e^{(3x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+a\*cosh(x))^(3/2),x, algorithm="maxima")

[Out]  $-1/54*(9*\sqrt{2}*a^{(3/2)}*x^3 + 18*\sqrt{2}*a^{(3/2)}*x^2 + 24*\sqrt{2}*a^{(3/2)}*x + 16*\sqrt{2}*a^{(3/2)} - (9*\sqrt{2}*a^{(3/2)}*x^3 - 18*\sqrt{2}*a^{(3/2)}*x^2 + 24*\sqrt{2}*a^{(3/2)}*x - 16*\sqrt{2}*a^{(3/2)})*e^{(3*x)} - 81*(\sqrt{2}*a^{(3/2)}*x^3 - 6*\sqrt{2}*a^{(3/2)}*x^2 + 24*\sqrt{2}*a^{(3/2)}*x - 48*\sqrt{2}*a^{(3/2)})*e^{(2*x)} + 81*(\sqrt{2}*a^{(3/2)}*x^3 + 6*\sqrt{2}*a^{(3/2)}*x^2 + 24*\sqrt{2}*a^{(3/2)}*x + 48*\sqrt{2}*a^{(3/2)})*e^x)*e^{(-3/2*x)}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 (a + a \cosh(x))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a + a\*cosh(x))^(3/2),x)

[Out] int(x^3\*(a + a\*cosh(x))^(3/2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+a\*cosh(x))\*\*(3/2),x)

[Out] Timed out

### 3.134 $\int x^2(a + a \cosh(x))^{3/2} dx$

**Optimal.** Leaf size=145

$$\frac{4}{3}ax^2 \sinh\left(\frac{x}{2}\right) \cosh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} + \frac{8}{3}ax^2 \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - \frac{16}{9}ax \cosh^2\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - \frac{32}{3}$$

[Out]  $-32/3*a*x*(a+a*\cosh(x))^{(1/2)}-16/9*a*x*\cosh(1/2*x)^2*(a+a*\cosh(x))^{(1/2)}+4/3*a*x^2*\cosh(1/2*x)*\sinh(1/2*x)*(a+a*\cosh(x))^{(1/2)}+224/9*a*(a+a*\cosh(x))^{(1/2)}*\tanh(1/2*x)+8/3*a*x^2*(a+a*\cosh(x))^{(1/2)}*\tanh(1/2*x)+32/27*a*\sinh(1/2*x)^2*(a+a*\cosh(x))^{(1/2)}*\tanh(1/2*x)$

**Rubi [A]** time = 0.15, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {3319, 3311, 3296, 2637, 2633}

$$\frac{4}{3}ax^2 \sinh\left(\frac{x}{2}\right) \cosh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} + \frac{8}{3}ax^2 \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - \frac{16}{9}ax \cosh^2\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - \frac{32}{3}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + a\*Cosh[x])^(3/2), x]

[Out]  $(-32*a*x*\text{Sqrt}[a + a*\text{Cosh}[x]])/3 - (16*a*x*\text{Cosh}[x/2]^2*\text{Sqrt}[a + a*\text{Cosh}[x]])/9 + (4*a*x^2*\text{Cosh}[x/2]*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Sinh}[x/2])/3 + (224*a*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Tanh}[x/2])/9 + (8*a*x^2*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Tanh}[x/2])/3 + (32*a*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Sinh}[x/2]^2*\text{Tanh}[x/2])/27$

#### Rule 2633

Int[sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

#### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sine[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sine[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

### Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sine[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sine[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rubi steps

$$\begin{aligned} \int x^2(a + a \cosh(x))^{3/2} dx &= \left(2a\sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right)\right) \int x^2 \cosh^3\left(\frac{x}{2}\right) dx \\ &= -\frac{16}{9}ax \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} + \frac{4}{3}ax^2 \cosh\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} \sinh\left(\frac{x}{2}\right) + \frac{1}{3}(4 \\ &= -\frac{16}{9}ax \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} + \frac{4}{3}ax^2 \cosh\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} \sinh\left(\frac{x}{2}\right) + \frac{8}{3}ax \\ &= -\frac{32}{3}ax\sqrt{a + a \cosh(x)} - \frac{16}{9}ax \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} + \frac{4}{3}ax^2 \cosh\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} \\ &= -\frac{32}{3}ax\sqrt{a + a \cosh(x)} - \frac{16}{9}ax \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} + \frac{4}{3}ax^2 \cosh\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} \end{aligned}$$

**Mathematica** [A] time = 0.22, size = 54, normalized size = 0.37

$$\frac{2}{27}a\sqrt{a(\cosh(x) + 1)} \left( (45x^2 + 328) \tanh\left(\frac{x}{2}\right) + \cosh(x) \left( (9x^2 + 8) \tanh\left(\frac{x}{2}\right) - 12x \right) - 156x \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + a*Cosh[x])^(3/2), x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cosh[x])]*(-156*x + (328 + 45*x^2)*Tanh[x/2] + Cosh[x]*(-1
2*x + (8 + 9*x^2)*Tanh[x/2]))) / 27
```

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+a\*cosh(x))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac** [A] time = 0.13, size = 144, normalized size = 0.99

$$-\frac{1}{54}\sqrt{2}\left(54a^{\frac{3}{2}}x^2e^{\left(-\frac{1}{2}x\right)}+9a^{\frac{3}{2}}x^2e^{\left(-\frac{3}{2}x\right)}+216a^{\frac{3}{2}}xe^{\left(-\frac{1}{2}x\right)}+12a^{\frac{3}{2}}xe^{\left(-\frac{3}{2}x\right)}+432a^{\frac{3}{2}}e^{\left(-\frac{1}{2}x\right)}+8a^{\frac{3}{2}}e^{\left(-\frac{3}{2}x\right)}-\left(9a^{\frac{3}{2}}x^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+a\*cosh(x))^(3/2),x, algorithm="giac")

[Out]  $-1/54*\text{sqrt}(2)*(54*a^{(3/2)}*x^2*e^{(-1/2*x)} + 9*a^{(3/2)}*x^2*e^{(-3/2*x)} + 216*a^{(3/2)}*x*e^{(-1/2*x)} + 12*a^{(3/2)}*x*e^{(-3/2*x)} + 432*a^{(3/2)}*e^{(-1/2*x)} + 8*a^{(3/2)}*e^{(-3/2*x)} - (9*a^{(3/2)}*x^2 - 12*a^{(3/2)}*x + 8*a^{(3/2)})*e^{(3/2*x)} - 81*(a^{(3/2)}*x^2 - 4*a^{(3/2)}*x + 8*a^{(3/2)})*e^{(1/2*x)} + 27*(a^{(3/2)}*x^2 + 4*a^{(3/2)}*x + 8*a^{(3/2)})*e^{(-1/2*x)})$

**maple** [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^2 (a + a \cosh(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+a\*cosh(x))^(3/2),x)

[Out] int(x^2\*(a+a\*cosh(x))^(3/2),x)

**maxima** [A] time = 0.41, size = 136, normalized size = 0.94

$$-\frac{1}{54}\left(9\sqrt{2}a^{\frac{3}{2}}x^2+12\sqrt{2}a^{\frac{3}{2}}x+8\sqrt{2}a^{\frac{3}{2}}-\left(9\sqrt{2}a^{\frac{3}{2}}x^2-12\sqrt{2}a^{\frac{3}{2}}x+8\sqrt{2}a^{\frac{3}{2}}\right)e^{(3x)}-81\left(\sqrt{2}a^{\frac{3}{2}}x^2-4\sqrt{2}a^{\frac{3}{2}}x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+a\*cosh(x))^(3/2),x, algorithm="maxima")

[Out]  $-1/54*(9*\text{sqrt}(2)*a^{(3/2)}*x^2 + 12*\text{sqrt}(2)*a^{(3/2)}*x + 8*\text{sqrt}(2)*a^{(3/2)} - (9*\text{sqrt}(2)*a^{(3/2)}*x^2 - 12*\text{sqrt}(2)*a^{(3/2)}*x + 8*\text{sqrt}(2)*a^{(3/2)})*e^{(3*x)} - 81*(\text{sqrt}(2)*a^{(3/2)}*x^2 - 4*\text{sqrt}(2)*a^{(3/2)}*x + 8*\text{sqrt}(2)*a^{(3/2)})*e^{(2*x)} + 81*(\text{sqrt}(2)*a^{(3/2)}*x^2 + 4*\text{sqrt}(2)*a^{(3/2)}*x + 8*\text{sqrt}(2)*a^{(3/2)})*e^{(x)})*e^{(-3/2*x)}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 (a + a \cosh(x))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + a*cosh(x))^(3/2), x)`

[Out] `int(x^2*(a + a*cosh(x))^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a (\cosh(x) + 1))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+a*cosh(x))**(3/2), x)`

[Out] `Integral(x**2*(a*(cosh(x) + 1))**(3/2), x)`

### 3.135 $\int x(a + a \cosh(x))^{3/2} dx$

**Optimal.** Leaf size=89

$$-\frac{8}{9}a \cosh^2\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - \frac{16}{3}a \sqrt{a \cosh(x) + a} + \frac{4}{3}ax \sinh\left(\frac{x}{2}\right) \cosh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} + \frac{8}{3}ax \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a}$$

[Out]  $-16/3*a*(a+a*\cosh(x))^{(1/2)}-8/9*a*\cosh(1/2*x)^2*(a+a*\cosh(x))^{(1/2)}+4/3*a*x*\cosh(1/2*x)*\sinh(1/2*x)*(a+a*\cosh(x))^{(1/2)}+8/3*a*x*(a+a*\cosh(x))^{(1/2)}*\tanh(1/2*x)$

**Rubi [A]** time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3319, 3310, 3296, 2638}

$$-\frac{8}{9}a \cosh^2\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} - \frac{16}{3}a \sqrt{a \cosh(x) + a} + \frac{4}{3}ax \sinh\left(\frac{x}{2}\right) \cosh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} + \frac{8}{3}ax \tanh\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(a + a*\text{Cosh}[x])^{(3/2)}, x]$

[Out]  $(-16*a*\text{Sqrt}[a + a*\text{Cosh}[x]])/3 - (8*a*\text{Cosh}[x/2]^2*\text{Sqrt}[a + a*\text{Cosh}[x]])/9 + (4*a*x*\text{Cosh}[x/2]*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Sinh}[x/2])/3 + (8*a*x*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Tanh}[x/2])/3$

#### Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

#### Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

#### Rule 3310

$\text{Int}[((c_.) + (d_.)*(x_.))*((b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(d*(b*\text{Sin}[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n-1))/n, \text{Int}[(c + d*x)*(b*\text{Sin}[e + f*x])^{(n-2)}, x], x] - \text{Simp}[(b*(c + d*x)*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(n-1)})/(f*n), x]) /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[n, 1]$

#### Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[((2*a)^(IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
 e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
 *Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
 qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rubi steps

$$\begin{aligned} \int x(a + a \cosh(x))^{3/2} dx &= \left(2a\sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right)\right) \int x \cosh^3\left(\frac{x}{2}\right) dx \\ &= -\frac{8}{9}a \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} + \frac{4}{3}ax \cosh\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} \sinh\left(\frac{x}{2}\right) + \frac{1}{3}\left(4a\sqrt{a + a \cosh(x)}\right) \\ &= -\frac{8}{9}a \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} + \frac{4}{3}ax \cosh\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} \sinh\left(\frac{x}{2}\right) + \frac{8}{3}ax\sqrt{a + a \cosh(x)} \\ &= -\frac{16}{3}a\sqrt{a + a \cosh(x)} - \frac{8}{9}a \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} + \frac{4}{3}ax \cosh\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 56, normalized size = 0.63

$$\frac{1}{9}a \operatorname{sech}\left(\frac{x}{2}\right) \sqrt{a(\cosh(x) + 1)} \left(3x \left(9 \sinh\left(\frac{x}{2}\right) + \sinh\left(\frac{3x}{2}\right)\right) - 54 \cosh\left(\frac{x}{2}\right) - 2 \cosh\left(\frac{3x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + a*Cosh[x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cosh[x])]*Sech[x/2]*(-54*Cosh[x/2] - 2*Cosh[(3*x)/2] + 3*x*(
9*Sinh[x/2] + Sinh[(3*x)/2]))) / 9
```

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (has polynomial part)
```

**giac [A]** time = 0.14, size = 96, normalized size = 1.08

$$-\frac{1}{18}\sqrt{2}\left(18a^{\frac{3}{2}}xe^{\left(-\frac{1}{2}x\right)} + 3a^{\frac{3}{2}}xe^{\left(-\frac{3}{2}x\right)} + 36a^{\frac{3}{2}}e^{\left(-\frac{1}{2}x\right)} + 2a^{\frac{3}{2}}e^{\left(-\frac{3}{2}x\right)} - \left(3a^{\frac{3}{2}}x - 2a^{\frac{3}{2}}\right)e^{\left(\frac{3}{2}x\right)} - 27\left(a^{\frac{3}{2}}x - 2a^{\frac{3}{2}}\right)e^{\left(\frac{1}{2}x\right)} + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+a\*cosh(x))^(3/2),x, algorithm="giac")

[Out] -1/18\*sqrt(2)\*(18\*a^(3/2)\*x\*e^(-1/2\*x) + 3\*a^(3/2)\*x\*e^(-3/2\*x) + 36\*a^(3/2)\*e^(-1/2\*x) + 2\*a^(3/2)\*e^(-3/2\*x) - (3\*a^(3/2)\*x - 2\*a^(3/2))\*e^(3/2\*x) - 27\*(a^(3/2)\*x - 2\*a^(3/2))\*e^(1/2\*x) + 9\*(a^(3/2)\*x + 2\*a^(3/2))\*e^(-1/2\*x))

**maple** [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x (a + a \cosh(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+a\*cosh(x))^(3/2),x)

[Out] int(x\*(a+a\*cosh(x))^(3/2),x)

**maxima** [A] time = 0.42, size = 92, normalized size = 1.03

$$-\frac{1}{18} \left( 3 \sqrt{2} a^{\frac{3}{2}} x + 2 \sqrt{2} a^{\frac{3}{2}} - \left( 3 \sqrt{2} a^{\frac{3}{2}} x - 2 \sqrt{2} a^{\frac{3}{2}} \right) e^{(3x)} - 27 \left( \sqrt{2} a^{\frac{3}{2}} x - 2 \sqrt{2} a^{\frac{3}{2}} \right) e^{(2x)} + 27 \left( \sqrt{2} a^{\frac{3}{2}} x + 2 \sqrt{2} a^{\frac{3}{2}} \right) e^x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+a\*cosh(x))^(3/2),x, algorithm="maxima")

[Out] -1/18\*(3\*sqrt(2)\*a^(3/2)\*x + 2\*sqrt(2)\*a^(3/2) - (3\*sqrt(2)\*a^(3/2)\*x - 2\*sqrt(2)\*a^(3/2))\*e^(3\*x) - 27\*(sqrt(2)\*a^(3/2)\*x - 2\*sqrt(2)\*a^(3/2))\*e^(2\*x) + 27\*(sqrt(2)\*a^(3/2)\*x + 2\*sqrt(2)\*a^(3/2))\*e^x\*e^(-3/2\*x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x (a + a \cosh(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a + a\*cosh(x))^(3/2),x)

[Out] int(x\*(a + a\*cosh(x))^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a (\cosh(x) + 1))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))**(3/2),x)
```

```
[Out] Integral(x*(a*(cosh(x) + 1))**(3/2), x)
```

$$3.136 \quad \int \frac{(a+a \cosh(x))^{3/2}}{x} dx$$

**Optimal.** Leaf size=55

$$\frac{3}{2}a \operatorname{Chi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} + \frac{1}{2}a \operatorname{Chi}\left(\frac{3x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a}$$

[Out]  $3/2*a*\operatorname{Chi}(1/2*x)*\operatorname{sech}(1/2*x)*(a+a*\cosh(x))^{(1/2)}+1/2*a*\operatorname{Chi}(3/2*x)*\operatorname{sech}(1/2*x)*(a+a*\cosh(x))^{(1/2)}$

**Rubi [A]** time = 0.13, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3319, 3312, 3301}

$$\frac{3}{2}a \operatorname{Chi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a} + \frac{1}{2}a \operatorname{Chi}\left(\frac{3x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) \sqrt{a \cosh(x) + a}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + a*\operatorname{Cosh}[x])^{(3/2)}/x, x]$

[Out]  $(3*a*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]]*\operatorname{CoshIntegral}[x/2]*\operatorname{Sech}[x/2])/2 + (a*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]]*\operatorname{CoshIntegral}[(3*x)/2]*\operatorname{Sech}[x/2])/2$

#### Rule 3301

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /;$   $\operatorname{FreeQ}\{c, d, e, f, fz\}, x \ \&\& \ \operatorname{EqQ}[d*(e - \operatorname{Pi}/2) - c*f*fz*I, 0]$

#### Rule 3312

$\operatorname{Int}[((c_.) + (d_.)*(x_))^{(m_.)}*\sin[(e_.) + (f_.)*(x_)]^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrigReduce}[(c + d*x)^m, \sin[e + f*x]^n, x], x] /;$   $\operatorname{FreeQ}\{c, d, e, f, m\}, x \ \&\& \ \operatorname{IGtQ}[n, 1] \ \&\& \ (\operatorname{!RationalQ}[m] \ \|\ (\operatorname{GeQ}[m, -1] \ \&\& \ \operatorname{LtQ}[m, 1]))$

#### Rule 3319

$\operatorname{Int}[((c_.) + (d_.)*(x_))^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(2*a)^{\operatorname{IntPart}[n]}*(a + b*\sin[e + f*x])^{\operatorname{FracPart}[n]}/\sin[e/2 + (a*\operatorname{Pi})/(4*b) + (f*x)/2]^{(2*\operatorname{FracPart}[n])}, \operatorname{Int}[(c + d*x)^m*\sin[e/2 + (a*\operatorname{Pi})/(4*b) + (f*x)/2]^{(2*n)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \ \operatorname{IntegerQ}[n + 1/2] \ \&\& \ (\operatorname{GtQ}[n, 0] \ \|\ \operatorname{IGtQ}[m, 0])$

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + a \cosh(x))^{3/2}}{x} dx &= \left(2a\sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right)\right) \int \frac{\cosh^3\left(\frac{x}{2}\right)}{x} dx \\
&= \left(2a\sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right)\right) \int \left(\frac{3 \cosh\left(\frac{x}{2}\right)}{4x} + \frac{\cosh\left(\frac{3x}{2}\right)}{4x}\right) dx \\
&= \frac{1}{2} \left(a\sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right)\right) \int \frac{\cosh\left(\frac{3x}{2}\right)}{x} dx + \frac{1}{2} \left(3a\sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right)\right) \int \frac{\cosh\left(\frac{x}{2}\right)}{x} dx \\
&= \frac{3}{2} a\sqrt{a + a \cosh(x)} \operatorname{Chi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) + \frac{1}{2} a\sqrt{a + a \cosh(x)} \operatorname{Chi}\left(\frac{3x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 36, normalized size = 0.65

$$\frac{1}{2} a \left( 3 \operatorname{Chi}\left(\frac{x}{2}\right) + \operatorname{Chi}\left(\frac{3x}{2}\right) \right) \operatorname{sech}\left(\frac{x}{2}\right) \sqrt{a(\cosh(x) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Cosh[x])^(3/2)/x,x]

[Out] (a\*Sqrt[a\*(1 + Cosh[x])]\*(3\*CoshIntegral[x/2] + CoshIntegral[(3\*x)/2])\*Sech[x/2])/2

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))^(3/2)/x,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (has polynomial part)

**giac [A]** time = 0.15, size = 40, normalized size = 0.73

$$\frac{1}{4} \sqrt{2} \left( a^{\frac{3}{2}} \operatorname{Ei}\left(\frac{3}{2}x\right) + 3 a^{\frac{3}{2}} \operatorname{Ei}\left(\frac{1}{2}x\right) + 3 a^{\frac{3}{2}} \operatorname{Ei}\left(-\frac{1}{2}x\right) + a^{\frac{3}{2}} \operatorname{Ei}\left(-\frac{3}{2}x\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))^(3/2)/x,x, algorithm="giac")



[Out]  $\frac{1}{4}\sqrt{2}(a^{3/2})\text{Ei}(3/2*x) + 3a^{3/2}\text{Ei}(1/2*x) + 3a^{3/2}\text{Ei}(-1/2*x) + a^{3/2}\text{Ei}(-3/2*x)$

**maple** [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(a + a \cosh(x))^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*cosh(x))^(3/2)/x,x)`

[Out] `int((a+a*cosh(x))^(3/2)/x,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \cosh(x) + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cosh(x))^(3/2)/x,x, algorithm="maxima")`

[Out] `integrate((a*cosh(x) + a)^(3/2)/x, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + a \cosh(x))^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a*cosh(x))^(3/2)/x,x)`

[Out] `int((a + a*cosh(x))^(3/2)/x, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\cosh(x) + 1))^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cosh(x))**(3/2)/x,x)`

[Out] `Integral((a*(cosh(x) + 1))**(3/2)/x, x)`

$$3.137 \quad \int \frac{(a+a \cosh(x))^{3/2}}{x^2} dx$$

**Optimal.** Leaf size=79

$$\frac{3}{4}a\text{Shi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a \cosh(x) + a} + \frac{3}{4}a\text{Shi}\left(\frac{3x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a \cosh(x) + a} - \frac{2a \cosh^2\left(\frac{x}{2}\right)\sqrt{a \cosh(x) + a}}{x}$$

[Out]  $-2*a*\cosh(1/2*x)^2*(a+a*\cosh(x))^{(1/2)}/x+3/4*a*\text{sech}(1/2*x)*\text{Shi}(1/2*x)*(a+a*\cosh(x))^{(1/2)}+3/4*a*\text{sech}(1/2*x)*\text{Shi}(3/2*x)*(a+a*\cosh(x))^{(1/2)}$

**Rubi [A]** time = 0.13, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3319, 3313, 3298}

$$\frac{3}{4}a\text{Shi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a \cosh(x) + a} + \frac{3}{4}a\text{Shi}\left(\frac{3x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a \cosh(x) + a} - \frac{2a \cosh^2\left(\frac{x}{2}\right)\sqrt{a \cosh(x) + a}}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + a\*Cosh[x])^(3/2)/x^2,x]

[Out]  $(-2*a*\text{Cosh}[x/2]^2*\text{Sqrt}[a + a*\text{Cosh}[x]])/x + (3*a*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Sech}[x/2]*\text{SinhIntegral}[x/2])/4 + (3*a*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Sech}[x/2]*\text{SinhIntegral}[(3*x)/2])/4$

#### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

#### Rule 3313

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x]^n)/(d\*(m + 1)), x] - Dist[(f\*n)/(d\*(m + 1)), Int[ExpandTrigReduce[(c + d\*x)^(m + 1), Cos[e + f\*x]\*Sin[e + f\*x]^(n - 1), x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

#### Rule 3319

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] :> Dist[((2\*a)^IntPart[n]\*(a + b\*Ssin[e + f\*x])^FracPart[n])/Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*FracPart[n]), Int[(c + d\*x)^m\*Ssin[e/2 + (a

$\frac{\pi}{4b} + (fx)/2]^{(2n)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{E}$   
 $\text{qQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[n + 1/2] \ \&\& \ (\text{GtQ}[n, 0] \ || \ \text{IGtQ}[m, 0])$

### Rubi steps

$$\begin{aligned} \int \frac{(a + a \cosh(x))^{3/2}}{x^2} dx &= \left(2a\sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right)\right) \int \frac{\cosh^3\left(\frac{x}{2}\right)}{x^2} dx \\ &= -\frac{2a \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)}}{x} + \left(3ia\sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right)\right) \int \left(-\frac{i \sinh\left(\frac{x}{2}\right)}{4x} - \frac{1}{4x}\right) dx \\ &= -\frac{2a \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)}}{x} + \frac{1}{4} \left(3a\sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right)\right) \int \frac{\sinh\left(\frac{x}{2}\right)}{x} dx + \frac{1}{4} \int \frac{1}{x} dx \\ &= -\frac{2a \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)}}{x} + \frac{3}{4} a \sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right) \operatorname{Shi}\left(\frac{x}{2}\right) + \frac{3}{4} a \sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right) \ln|x| \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 53, normalized size = 0.67

$$\frac{a \operatorname{sech}\left(\frac{x}{2}\right) \sqrt{a(\cosh(x) + 1)} \left(-3x \operatorname{Shi}\left(\frac{x}{2}\right) - 3x \operatorname{Shi}\left(\frac{3x}{2}\right) + 8 \cosh^3\left(\frac{x}{2}\right)\right)}{4x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Cosh[x])^(3/2)/x^2,x]

[Out]  $-1/4*(a*\text{Sqrt}[a*(1 + \text{Cosh}[x])]*\text{Sech}[x/2]*(8*\text{Cosh}[x/2]^3 - 3*x*\text{SinhIntegral}[x/2] - 3*x*\text{SinhIntegral}[(3*x)/2]))/x$

**fricas [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))^(3/2)/x^2,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac** [A] time = 0.15, size = 112, normalized size = 1.42

$$\frac{1}{8} \sqrt{2} \left( \frac{3 a^{\frac{3}{2}} x \operatorname{Ei}\left(\frac{3}{2} x\right) + 3 a^{\frac{3}{2}} x \operatorname{Ei}\left(\frac{1}{2} x\right) - a^{\frac{3}{2}} x \operatorname{Ei}\left(-\frac{1}{2} x\right) - 2 a^{\frac{3}{2}} e^{\left(\frac{3}{2} x\right)} - 6 a^{\frac{3}{2}} e^{\left(\frac{1}{2} x\right)} - 2 a^{\frac{3}{2}} e^{\left(-\frac{1}{2} x\right)}}{x} - \frac{2 a^{\frac{3}{2}} x \operatorname{Ei}\left(-\frac{1}{2} x\right) + 3 a^{\frac{3}{2}} x \operatorname{Ei}\left(\frac{1}{2} x\right) - a^{\frac{3}{2}} x \operatorname{Ei}\left(\frac{3}{2} x\right) - 2 a^{\frac{3}{2}} e^{\left(-\frac{1}{2} x\right)} - 6 a^{\frac{3}{2}} e^{\left(\frac{1}{2} x\right)} - 2 a^{\frac{3}{2}} e^{\left(\frac{3}{2} x\right)}}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))^(3/2)/x^2,x, algorithm="giac")

[Out] 1/8\*sqrt(2)\*((3\*a^(3/2)\*x\*Ei(3/2\*x) + 3\*a^(3/2)\*x\*Ei(1/2\*x) - a^(3/2)\*x\*Ei(-1/2\*x) - 2\*a^(3/2)\*e^(3/2\*x) - 6\*a^(3/2)\*e^(1/2\*x) - 2\*a^(3/2)\*e^(-1/2\*x))/x - (2\*a^(3/2)\*x\*Ei(-1/2\*x) + 3\*a^(3/2)\*x\*Ei(-3/2\*x) + 4\*a^(3/2)\*e^(-1/2\*x) + 2\*a^(3/2)\*e^(-3/2\*x))/x)

**maple** [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(a + a \cosh(x))^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*cosh(x))^(3/2)/x^2,x)

[Out] int((a+a\*cosh(x))^(3/2)/x^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \cosh(x) + a)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))^(3/2)/x^2,x, algorithm="maxima")

[Out] integrate((a\*cosh(x) + a)^(3/2)/x^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \cosh(x))^{3/2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cosh(x))^(3/2)/x^2,x)

```
[Out] int((a + a*cosh(x))^(3/2)/x^2, x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{(a(\cosh(x) + 1))^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(3/2)/x**2,x)
```

```
[Out] Integral((a*(cosh(x) + 1))**(3/2)/x**2, x)
```

$$3.138 \quad \int \frac{(a+a \cosh(x))^{3/2}}{x^3} dx$$

**Optimal.** Leaf size=109

$$\frac{3}{16}a\text{Chi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a} + \frac{9}{16}a\text{Chi}\left(\frac{3x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a} - \frac{a\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}}{x^2} - \frac{3a}{x^2}$$

[Out]  $-a*\cosh(1/2*x)^2*(a+a*\cosh(x))^{(1/2)}/x^2+3/16*a*\text{Chi}(1/2*x)*\text{sech}(1/2*x)*(a+a*\cosh(x))^{(1/2)}+9/16*a*\text{Chi}(3/2*x)*\text{sech}(1/2*x)*(a+a*\cosh(x))^{(1/2)}-3/2*a*\cosh(1/2*x)*\sinh(1/2*x)*(a+a*\cosh(x))^{(1/2)}/x$

**Rubi [A]** time = 0.17, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3319, 3314, 3301, 3312}

$$\frac{3}{16}a\text{Chi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a} + \frac{9}{16}a\text{Chi}\left(\frac{3x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a} - \frac{a\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}}{x^2} - \frac{3a}{x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + a\*Cosh[x])^(3/2)/x^3,x]

[Out]  $-((a*\text{Cosh}[x/2]^2*\text{Sqrt}[a + a*\text{Cosh}[x]])/x^2) + (3*a*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{CoshIntegral}[x/2]*\text{Sech}[x/2])/16 + (9*a*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{CoshIntegral}[(3*x)/2]*\text{Sech}[x/2])/16 - (3*a*\text{Cosh}[x/2]*\text{Sqrt}[a + a*\text{Cosh}[x]]*\text{Sinh}[x/2])/(2*x)$

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rule 3314

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_)), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*(b\*SIN[e + f\*x])^n)/(d\*(m + 1)), x] + (Dist[(b^2\*f^2\*n\*(n - 1))/(d^2\*(m + 1)\*(m + 2)), Int[(c + d\*x)^(m + 2)\*(b\*SIN[e + f\*x])^(n - 2), x], x] - Dist[(f^2\*n^2)/(d^2\*(m + 1)\*(m + 2)), Int[(c + d\*x)

$(m + 2) * (b * \sin[e + f * x])^n, x] - \text{Simp}[(b * f * n * (c + d * x)^{(m + 2)} * \cos[e + f * x] * (b * \sin[e + f * x])^{(n - 1)}) / (d^{2 * (m + 1)} * (m + 2)), x] / ; \text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{LtQ}[m, -2]$

### Rule 3319

$\text{Int}[(c + d * x)^m * (a + b * \sin[e + f * x])^n, x\_Symbol] :> \text{Dist}[(2 * a)^{\text{IntPart}[n]} * (a + b * \sin[e + f * x])^{\text{FracPart}[n]} / \sin[e/2 + (a * \pi) / (4 * b) + (f * x) / 2]^{(2 * \text{FracPart}[n])}, \text{Int}[(c + d * x)^m * \sin[e/2 + (a * \pi) / (4 * b) + (f * x) / 2]^{(2 * n)}, x], x] / ; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[n + 1/2] \&\& (\text{GtQ}[n, 0] \parallel \text{IGtQ}[m, 0])$

### Rubi steps

$$\begin{aligned} \int \frac{(a + a \cosh(x))^{3/2}}{x^3} dx &= \left( 2a \sqrt{a + a \cosh(x)} \operatorname{sech}\left(\frac{x}{2}\right) \right) \int \frac{\cosh^3\left(\frac{x}{2}\right)}{x^3} dx \\ &= -\frac{a \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)}}{x^2} - \frac{3a \cosh\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)} \sinh\left(\frac{x}{2}\right)}{2x} - \frac{1}{2} \left( 3a \sqrt{a + a \cosh(x)} \operatorname{Chi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) - \frac{3a \cosh\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)}}{2} \right) \\ &= -\frac{a \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)}}{x^2} - \frac{3}{2} a \sqrt{a + a \cosh(x)} \operatorname{Chi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) - \frac{3a \cosh\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)}}{2} \\ &= -\frac{a \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)}}{x^2} - \frac{3}{2} a \sqrt{a + a \cosh(x)} \operatorname{Chi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) - \frac{3a \cosh\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)}}{2} \\ &= -\frac{a \cosh^2\left(\frac{x}{2}\right) \sqrt{a + a \cosh(x)}}{x^2} + \frac{3}{16} a \sqrt{a + a \cosh(x)} \operatorname{Chi}\left(\frac{x}{2}\right) \operatorname{sech}\left(\frac{x}{2}\right) + \frac{9}{16} a \sqrt{a + a \cosh(x)} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 69, normalized size = 0.63

$$\frac{(a(\cosh(x) + 1))^{3/2} \left( 3x^2 \operatorname{Chi}\left(\frac{x}{2}\right) \operatorname{sech}^3\left(\frac{x}{2}\right) + 9x^2 \operatorname{Chi}\left(\frac{3x}{2}\right) \operatorname{sech}^3\left(\frac{x}{2}\right) - 8 \left( 3x \tanh\left(\frac{x}{2}\right) + 2 \right) \right)}{32x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a\*Cosh[x])^(3/2)/x^3, x]

[Out] ((a\*(1 + Cosh[x]))^(3/2)\*(3\*x^2\*CoshIntegral[x/2]\*Sech[x/2]^3 + 9\*x^2\*CoshIntegral[(3\*x)/2]\*Sech[x/2]^3 - 8\*(2 + 3\*x\*Tanh[x/2])))/(32\*x^2)

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))^(3/2)/x^3,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ  
rate: implementation incomplete (has polynomial part)

**giac** [B] time = 0.15, size = 170, normalized size = 1.56

$$\frac{1}{32} \sqrt{2} \left( \frac{9 a^{\frac{3}{2}} x^2 \operatorname{Ei}\left(\frac{3}{2} x\right) + 3 a^{\frac{3}{2}} x^2 \operatorname{Ei}\left(\frac{1}{2} x\right) + a^{\frac{3}{2}} x^2 \operatorname{Ei}\left(-\frac{1}{2} x\right) - 6 a^{\frac{3}{2}} x e^{\left(\frac{3}{2} x\right)} - 6 a^{\frac{3}{2}} x e^{\left(\frac{1}{2} x\right)} + 2 a^{\frac{3}{2}} x e^{\left(-\frac{1}{2} x\right)} - 4 a^{\frac{3}{2}} e^{\left(\frac{3}{2} x\right)} - 1}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))^(3/2)/x^3,x, algorithm="giac")

[Out] 1/32\*sqrt(2)\*((9\*a^(3/2)\*x^2\*Ei(3/2\*x) + 3\*a^(3/2)\*x^2\*Ei(1/2\*x) + a^(3/2)\*  
x^2\*Ei(-1/2\*x) - 6\*a^(3/2)\*x\*e^(3/2\*x) - 6\*a^(3/2)\*x\*e^(1/2\*x) + 2\*a^(3/2)\*  
x\*e^(-1/2\*x) - 4\*a^(3/2)\*e^(3/2\*x) - 12\*a^(3/2)\*e^(1/2\*x) - 4\*a^(3/2)\*e^(-1  
/2\*x))/x^2 + (2\*a^(3/2)\*x^2\*Ei(-1/2\*x) + 9\*a^(3/2)\*x^2\*Ei(-3/2\*x) + 4\*a^(3/  
2)\*x\*e^(-1/2\*x) + 6\*a^(3/2)\*x\*e^(-3/2\*x) - 8\*a^(3/2)\*e^(-1/2\*x) - 4\*a^(3/2)  
\*e^(-3/2\*x))/x^2)

**maple** [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(a + a \cosh(x))^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*cosh(x))^(3/2)/x^3,x)

[Out] int((a+a\*cosh(x))^(3/2)/x^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \cosh(x) + a)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+a\*cosh(x))^(3/2)/x^3,x, algorithm="maxima")

[Out] integrate((a\*cosh(x) + a)^(3/2)/x^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \cosh(x))^{3/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cosh(x))^(3/2)/x^3,x)

[Out] int((a + a\*cosh(x))^(3/2)/x^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a (\cosh(x) + 1))^{3/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(x))\*\*(3/2)/x\*\*3,x)

[Out] Integral((a\*(cosh(x) + 1))\*\*(3/2)/x\*\*3, x)

$$3.139 \quad \int \frac{x^3}{\sqrt{a+a \cosh(c+dx)}} dx$$

**Optimal.** Leaf size=383

$$\frac{96i\text{Li}_4\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^4\sqrt{a \cosh(c+dx)+a}} + \frac{96i\text{Li}_4\left(ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^4\sqrt{a \cosh(c+dx)+a}} + \frac{48ix\text{Li}_3\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a \cosh(c+dx)+a}} - \frac{48ix\text{Li}_3\left(ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a \cosh(c+dx)+a}}$$

[Out]  $4*x^3*\arctan(\exp(1/2*d*x+1/2*c))*\cosh(1/2*d*x+1/2*c)/d/(a+a*\cosh(d*x+c))^(1/2)-12*I*x^2*\cosh(1/2*d*x+1/2*c)*\text{polylog}(2,-I*\exp(1/2*d*x+1/2*c))/d^2/(a+a*\cosh(d*x+c))^(1/2)+12*I*x^2*\cosh(1/2*d*x+1/2*c)*\text{polylog}(2,I*\exp(1/2*d*x+1/2*c))/d^2/(a+a*\cosh(d*x+c))^(1/2)+48*I*x*\cosh(1/2*d*x+1/2*c)*\text{polylog}(3,-I*\exp(1/2*d*x+1/2*c))/d^3/(a+a*\cosh(d*x+c))^(1/2)-48*I*x*\cosh(1/2*d*x+1/2*c)*\text{polylog}(3,I*\exp(1/2*d*x+1/2*c))/d^3/(a+a*\cosh(d*x+c))^(1/2)-96*I*\cosh(1/2*d*x+1/2*c)*\text{polylog}(4,-I*\exp(1/2*d*x+1/2*c))/d^4/(a+a*\cosh(d*x+c))^(1/2)+96*I*\cosh(1/2*d*x+1/2*c)*\text{polylog}(4,I*\exp(1/2*d*x+1/2*c))/d^4/(a+a*\cosh(d*x+c))^(1/2)$

**Rubi [A]** time = 0.21, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3319, 4180, 2531, 6609, 2282, 6589}

$$\frac{12ix^2 \cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \text{PolyLog}\left(2, -ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d^2\sqrt{a \cosh(c+dx)+a}} + \frac{12ix^2 \cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \text{PolyLog}\left(2, ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d^2\sqrt{a \cosh(c+dx)+a}} + \frac{48ix \cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \text{PolyLog}\left(3, -ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d^3\sqrt{a \cosh(c+dx)+a}} - \frac{48ix \cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \text{PolyLog}\left(3, ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d^3\sqrt{a \cosh(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[a + a\*Cosh[c + d\*x]],x]

[Out]  $(4*x^3*\text{ArcTan}[E^{(c/2 + (d*x)/2)}]*\text{Cosh}[c/2 + (d*x)/2])/(d*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]]) - ((12*I)*x^2*\text{Cosh}[c/2 + (d*x)/2]*\text{PolyLog}[2, (-I)*E^{(c/2 + (d*x)/2)}])/d^2*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]]) + ((12*I)*x^2*\text{Cosh}[c/2 + (d*x)/2]*\text{PolyLog}[2, I*E^{(c/2 + (d*x)/2)}])/d^2*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]]) + ((48*I)*x*\text{Cosh}[c/2 + (d*x)/2]*\text{PolyLog}[3, (-I)*E^{(c/2 + (d*x)/2)}])/d^3*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]]) - ((48*I)*x*\text{Cosh}[c/2 + (d*x)/2]*\text{PolyLog}[3, I*E^{(c/2 + (d*x)/2)}])/d^3*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]]) - ((96*I)*\text{Cosh}[c/2 + (d*x)/2]*\text{PolyLog}[4, (-I)*E^{(c/2 + (d*x)/2)}])/d^4*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]]) + ((96*I)*\text{Cosh}[c/2 + (d*x)/2]*\text{PolyLog}[4, I*E^{(c/2 + (d*x)/2)}])/d^4*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]])$

**Rule 2282**

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[

{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*  
(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)  
\*(x\_)^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))  
)^n])]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -  
1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f,  
g, n}, x] && GtQ[m, 0]

### Rule 3319

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.),  
x\_Symbol] := Dist[((2\*a)^IntPart[n]\*(a + b\*Sin[e + f\*x])^FracPart[n])/Sin[  
e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*FracPart[n]), Int[(c + d\*x)^m\*Sin[e/2 + (a  
\*Pi)/(4\*b) + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E  
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_  
))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(  
I\*k\*Pi)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1  
- E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c +  
d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c,  
d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_S  
ymbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d,  
e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.  
)\*(x\_))))^(p\_.)], x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a  
+ b\*x)))^p])]/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(  
m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c,  
d, e, f, n, p}, x] && GtQ[m, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a + a \cosh(c + dx)}} dx &= \frac{\sin\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{idx}{2}\right) \int x^3 \csc\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{idx}{2}\right) dx}{\sqrt{a + a \cosh(c + dx)}} \\
&= \frac{4x^3 \tan^{-1}\left(e^{\frac{c}{2} + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{dx}{2}\right) - \left(6i \sin\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{idx}{2}\right)\right) \int x^2 \log\left(1 - ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d\sqrt{a + a \cosh(c + dx)}} - \frac{\left(6i \sin\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{idx}{2}\right)\right) \int x^2 \log\left(1 - ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d\sqrt{a + a \cosh(c + dx)}} \\
&= \frac{4x^3 \tan^{-1}\left(e^{\frac{c}{2} + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\sqrt{a + a \cosh(c + dx)}} - \frac{12ix^2 \cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \text{Li}_2\left(-ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d^2\sqrt{a + a \cosh(c + dx)}} + \frac{12ix^2 \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d^2\sqrt{a + a \cosh(c + dx)}} \\
&= \frac{4x^3 \tan^{-1}\left(e^{\frac{c}{2} + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\sqrt{a + a \cosh(c + dx)}} - \frac{12ix^2 \cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \text{Li}_2\left(-ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d^2\sqrt{a + a \cosh(c + dx)}} + \frac{12ix^2 \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d^2\sqrt{a + a \cosh(c + dx)}} \\
&= \frac{4x^3 \tan^{-1}\left(e^{\frac{c}{2} + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\sqrt{a + a \cosh(c + dx)}} - \frac{12ix^2 \cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \text{Li}_2\left(-ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d^2\sqrt{a + a \cosh(c + dx)}} + \frac{12ix^2 \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d^2\sqrt{a + a \cosh(c + dx)}} \\
&= \frac{4x^3 \tan^{-1}\left(e^{\frac{c}{2} + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\sqrt{a + a \cosh(c + dx)}} - \frac{12ix^2 \cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \text{Li}_2\left(-ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d^2\sqrt{a + a \cosh(c + dx)}} + \frac{12ix^2 \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d^2\sqrt{a + a \cosh(c + dx)}}
\end{aligned}$$

**Mathematica [A]** time = 1.78, size = 213, normalized size = 0.56

$$\frac{2i \cosh\left(\frac{1}{2}(c + dx)\right) \left(d^3 x^3 \log\left(1 - ie^{\frac{1}{2}(c+dx)}\right) - d^3 x^3 \log\left(1 + ie^{\frac{1}{2}(c+dx)}\right) - 6d^2 x^2 \text{Li}_2\left(-ie^{\frac{1}{2}(c+dx)}\right) + 6d^2 x^2 \text{Li}_2\left(ie^{\frac{1}{2}(c+dx)}\right)\right)}{d^4 \sqrt{a(\cosh(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[a + a\*Cosh[c + d\*x]], x]

[Out] ((2\*I)\*Cosh[(c + d\*x)/2]\*(d^3\*x^3\*Log[1 - I\*E^((c + d\*x)/2)] - d^3\*x^3\*Log[1 + I\*E^((c + d\*x)/2)] - 6\*d^2\*x^2\*PolyLog[2, (-I)\*E^((c + d\*x)/2)] + 6\*d^2\*x^2\*PolyLog[2, I\*E^((c + d\*x)/2)] + 24\*d\*x\*PolyLog[3, (-I)\*E^((c + d\*x)/2)] - 24\*d\*x\*PolyLog[3, I\*E^((c + d\*x)/2)] - 48\*PolyLog[4, (-I)\*E^((c + d\*x)/2)] + 48\*PolyLog[4, I\*E^((c + d\*x)/2)]))/(d^4\*Sqrt[a\*(1 + Cosh[c + d\*x])])

**fricas [F]** time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{\sqrt{a \cosh(dx + c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(x^3/sqrt(a\*cosh(d\*x + c) + a), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a \cosh(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(x^3/sqrt(a\*cosh(d\*x + c) + a), x)

**maple** [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a + a \cosh(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+a\*cosh(d\*x+c))^(1/2),x)

[Out] int(x^3/(a+a\*cosh(d\*x+c))^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$2\sqrt{2}d^3 \int \frac{x^3 e^{\left(\frac{1}{2}dx + \frac{1}{2}c\right)}}{\sqrt{a}d^3 e^{2dx+2c} + 2\sqrt{a}d^3 e^{dx+c} + \sqrt{a}d^3} dx + 12\sqrt{2}d^2 \int \frac{x^2 e^{\left(\frac{1}{2}dx + \frac{1}{2}c\right)}}{\sqrt{a}d^3 e^{2dx+2c} + 2\sqrt{a}d^3 e^{dx+c} + \sqrt{a}d^3} dx +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] 2\*sqrt(2)\*d^3\*integrate(x^3\*e^(1/2\*d\*x + 1/2\*c)/(sqrt(a)\*d^3\*e^(2\*d\*x + 2\*c) + 2\*sqrt(a)\*d^3\*e^(d\*x + c) + sqrt(a)\*d^3), x) + 12\*sqrt(2)\*d^2\*integrate(x^2\*e^(1/2\*d\*x + 1/2\*c)/(sqrt(a)\*d^3\*e^(2\*d\*x + 2\*c) + 2\*sqrt(a)\*d^3\*e^(d\*x + c) + sqrt(a)\*d^3), x) + 48\*sqrt(2)\*d\*integrate(x\*e^(1/2\*d\*x + 1/2\*c)/(sqrt(a)\*d^3\*e^(2\*d\*x + 2\*c) + 2\*sqrt(a)\*d^3\*e^(d\*x + c) + sqrt(a)\*d^3), x) + 96\*sqrt(2)\*(e^(1/2\*d\*x + 1/2\*c)/((sqrt(a)\*d^3\*e^(d\*x + c) + sqrt(a)\*d^3)\*d) + arctan(e^(1/2\*d\*x + 1/2\*c))/(sqrt(a)\*d^4)) - 2\*(sqrt(2)\*sqrt(a)\*d^3\*x^3\*e^(1/2\*c) + 6\*sqrt(2)\*sqrt(a)\*d^2\*x^2\*e^(1/2\*c) + 24\*sqrt(2)\*sqrt(a)\*d\*x\*e^(1/2\*c) + 48\*sqrt(2)\*sqrt(a)\*e^(1/2\*c))\*e^(1/2\*d\*x)/(a\*d^4\*e^(d\*x + c) + a\*d^4)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{a + a \cosh(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + a\*cosh(c + d\*x))^(1/2), x)

[Out] int(x^3/(a + a\*cosh(c + d\*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a(\cosh(c + dx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(a+a\*cosh(d\*x+c))\*\*(1/2), x)

[Out] Integral(x\*\*3/sqrt(a\*(cosh(c + d\*x) + 1)), x)

$$3.140 \quad \int \frac{x^2}{\sqrt{a+a \cosh(c+dx)}} dx$$

**Optimal.** Leaf size=269

$$\frac{16i\text{Li}_3\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a \cosh(c+dx)+a}} - \frac{16i\text{Li}_3\left(ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a \cosh(c+dx)+a}} - \frac{8ix\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a \cosh(c+dx)+a}} + \frac{8ix\text{Li}_2\left(ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a \cosh(c+dx)+a}}$$

[Out]  $4x^2 \arctan(\exp(1/2 dx + 1/2 c)) \cosh(1/2 dx + 1/2 c) / d (a + a \cosh(dx + c))^{1/2} - 8I x \cosh(1/2 dx + 1/2 c) \text{polylog}(2, -I \exp(1/2 dx + 1/2 c)) / d^2 (a + a \cosh(dx + c))^{1/2} + 8I x \cosh(1/2 dx + 1/2 c) \text{polylog}(2, I \exp(1/2 dx + 1/2 c)) / d^2 (a + a \cosh(dx + c))^{1/2} + 16I \cosh(1/2 dx + 1/2 c) \text{polylog}(3, -I \exp(1/2 dx + 1/2 c)) / d^3 (a + a \cosh(dx + c))^{1/2} - 16I \cosh(1/2 dx + 1/2 c) \text{polylog}(3, I \exp(1/2 dx + 1/2 c)) / d^3 (a + a \cosh(dx + c))^{1/2}$

**Rubi [A]** time = 0.16, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3319, 4180, 2531, 2282, 6589}

$$-\frac{8ix \cosh\left(\frac{c}{2}+\frac{dx}{2}\right) \text{PolyLog}\left(2, -ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a \cosh(c+dx)+a}} + \frac{8ix \cosh\left(\frac{c}{2}+\frac{dx}{2}\right) \text{PolyLog}\left(2, ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a \cosh(c+dx)+a}} + \frac{16i \cosh\left(\frac{c}{2}+\frac{dx}{2}\right) \text{PolyLog}\left(3, -ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^3\sqrt{a \cosh(c+dx)+a}} - \frac{16i \cosh\left(\frac{c}{2}+\frac{dx}{2}\right) \text{PolyLog}\left(3, ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^3\sqrt{a \cosh(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[a + a\*Cosh[c + d\*x]],x]

[Out]  $(4x^2 \text{ArcTan}[E^{(c/2 + (d*x)/2)}] \text{Cosh}[c/2 + (d*x)/2]) / (d \text{Sqrt}[a + a \text{Cosh}[c + d*x]]) - ((8I) x \text{Cosh}[c/2 + (d*x)/2] \text{PolyLog}[2, (-I) E^{(c/2 + (d*x)/2)}]) / (d^2 \text{Sqrt}[a + a \text{Cosh}[c + d*x]]) + ((8I) x \text{Cosh}[c/2 + (d*x)/2] \text{PolyLog}[2, I E^{(c/2 + (d*x)/2)}]) / (d^2 \text{Sqrt}[a + a \text{Cosh}[c + d*x]]) + ((16I) \text{Cosh}[c/2 + (d*x)/2] \text{PolyLog}[3, (-I) E^{(c/2 + (d*x)/2)}]) / (d^3 \text{Sqrt}[a + a \text{Cosh}[c + d*x]]) - ((16I) \text{Cosh}[c/2 + (d*x)/2] \text{PolyLog}[3, I E^{(c/2 + (d*x)/2)}]) / (d^3 \text{Sqrt}[a + a \text{Cosh}[c + d*x]])$

**Rule 2282**

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\* (F\_)] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]

**Rule 2531**

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 3319

```
Int[((c_.) + (d_.)*(x_)^(m_.))*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] := Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi))]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps



$$\begin{aligned}
\int \frac{x^2}{\sqrt{a + a \cosh(c + dx)}} dx &= \frac{\sin\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{idx}{2}\right) \int x^2 \csc\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{idx}{2}\right) dx}{\sqrt{a + a \cosh(c + dx)}} \\
&= \frac{4x^2 \tan^{-1}\left(e^{\frac{c}{2} + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\sqrt{a + a \cosh(c + dx)}} - \frac{\left(4i \sin\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{idx}{2}\right)\right) \int x \log\left(1 - ie^{\frac{c}{2} + \frac{dx}{2}}\right) dx}{d\sqrt{a + a \cosh(c + dx)}} \\
&= \frac{4x^2 \tan^{-1}\left(e^{\frac{c}{2} + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\sqrt{a + a \cosh(c + dx)}} - \frac{8ix \cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \operatorname{Li}_2\left(-ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d^2\sqrt{a + a \cosh(c + dx)}} + \frac{8ix \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d^2\sqrt{a + a \cosh(c + dx)}} \\
&= \frac{4x^2 \tan^{-1}\left(e^{\frac{c}{2} + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\sqrt{a + a \cosh(c + dx)}} - \frac{8ix \cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \operatorname{Li}_2\left(-ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d^2\sqrt{a + a \cosh(c + dx)}} + \frac{8ix \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d^2\sqrt{a + a \cosh(c + dx)}} \\
&= \frac{4x^2 \tan^{-1}\left(e^{\frac{c}{2} + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\sqrt{a + a \cosh(c + dx)}} - \frac{8ix \cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \operatorname{Li}_2\left(-ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d^2\sqrt{a + a \cosh(c + dx)}} + \frac{8ix \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d^2\sqrt{a + a \cosh(c + dx)}}
\end{aligned}$$

**Mathematica [A]** time = 1.75, size = 163, normalized size = 0.61

$$\frac{2i \cosh\left(\frac{1}{2}(c + dx)\right) \left(d^2 x^2 \log\left(1 - ie^{\frac{1}{2}(c+dx)}\right) - d^2 x^2 \log\left(1 + ie^{\frac{1}{2}(c+dx)}\right) - 4dx \operatorname{Li}_2\left(-ie^{\frac{1}{2}(c+dx)}\right) + 4dx \operatorname{Li}_2\left(ie^{\frac{1}{2}(c+dx)}\right)\right)}{d^3 \sqrt{a(\cosh(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[a + a\*Cosh[c + d\*x]], x]

[Out] ((2\*I)\*Cosh[(c + d\*x)/2]\*(d^2\*x^2\*Log[1 - I\*E^((c + d\*x)/2)] - d^2\*x^2\*Log[1 + I\*E^((c + d\*x)/2)] - 4\*d\*x\*PolyLog[2, (-I)\*E^((c + d\*x)/2)] + 4\*d\*x\*PolyLog[2, I\*E^((c + d\*x)/2)] + 8\*PolyLog[3, (-I)\*E^((c + d\*x)/2)] - 8\*PolyLog[3, I\*E^((c + d\*x)/2)]))/(d^3\*Sqrt[a\*(1 + Cosh[c + d\*x])])

**fricas [F]** time = 0.64, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x^2}{\sqrt{a \cosh(dx + c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+a\*cosh(d\*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(x^2/sqrt(a\*cosh(d\*x + c) + a), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a \cosh(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(x^2/sqrt(a\*cosh(d\*x + c) + a), x)

**maple** [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + a \cosh(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+a\*cosh(d\*x+c))^(1/2),x)

[Out] int(x^2/(a+a\*cosh(d\*x+c))^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$2\sqrt{2}d^2 \int \frac{x^2 e^{\left(\frac{1}{2}dx + \frac{1}{2}c\right)}}{\sqrt{a}d^2 e^{(2dx+2c)} + 2\sqrt{a}d^2 e^{(dx+c)} + \sqrt{a}d^2} dx + 8\sqrt{2}d \int \frac{x e^{\left(\frac{1}{2}dx + \frac{1}{2}c\right)}}{\sqrt{a}d^2 e^{(2dx+2c)} + 2\sqrt{a}d^2 e^{(dx+c)} + \sqrt{a}d^2} dx + 16 \int \frac{1}{\sqrt{a}d^2 e^{(2dx+2c)} + 2\sqrt{a}d^2 e^{(dx+c)} + \sqrt{a}d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] 2\*sqrt(2)\*d^2\*integrate(x^2\*e^(1/2\*d\*x + 1/2\*c)/(sqrt(a)\*d^2\*e^(2\*d\*x + 2\*c) + 2\*sqrt(a)\*d^2\*e^(d\*x + c) + sqrt(a)\*d^2), x) + 8\*sqrt(2)\*d\*integrate(x\*e^(1/2\*d\*x + 1/2\*c)/(sqrt(a)\*d^2\*e^(2\*d\*x + 2\*c) + 2\*sqrt(a)\*d^2\*e^(d\*x + c) + sqrt(a)\*d^2), x) + 16\*sqrt(2)\*(e^(1/2\*d\*x + 1/2\*c)/((sqrt(a)\*d^2\*e^(d\*x + c) + sqrt(a)\*d^2)\*d) + arctan(e^(1/2\*d\*x + 1/2\*c))/(sqrt(a)\*d^3)) - 2\*(sqrt(2)\*d^2\*x^2\*e^(1/2\*c) + 4\*sqrt(2)\*d\*x\*e^(1/2\*c) + 8\*sqrt(2)\*e^(1/2\*c))\*e^(1/2\*d\*x)/(sqrt(a)\*d^3\*e^(d\*x + c) + sqrt(a)\*d^3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a + a \cosh(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a + a*cosh(c + d*x))^(1/2), x)`

[Out] `int(x^2/(a + a*cosh(c + d*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a(\cosh(c + dx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(a+a*cosh(d*x+c))**(1/2), x)`

[Out] `Integral(x**2/sqrt(a*(cosh(c + d*x) + 1)), x)`

$$3.141 \quad \int \frac{x}{\sqrt{a+a \cosh(c+dx)}} dx$$

**Optimal.** Leaf size=157

$$\frac{4i\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a \cosh(c+dx)+a}} + \frac{4i\text{Li}_2\left(ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a \cosh(c+dx)+a}} + \frac{4x \tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a \cosh(c+dx)+a}}$$

[Out]  $4*x*\arctan(\exp(1/2*d*x+1/2*c))*\cosh(1/2*d*x+1/2*c)/d/(a+a*\cosh(d*x+c))^{1/2}$   
 $-4*I*\cosh(1/2*d*x+1/2*c)*\text{polylog}(2,-I*\exp(1/2*d*x+1/2*c))/d^2/(a+a*\cosh(d*x+c))^{1/2}$   
 $+4*I*\cosh(1/2*d*x+1/2*c)*\text{polylog}(2,I*\exp(1/2*d*x+1/2*c))/d^2/(a+a*\cosh(d*x+c))^{1/2}$

**Rubi [A]** time = 0.08, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3319, 4180, 2279, 2391}

$$\frac{4i \cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \text{PolyLog}\left(2, -ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d^2\sqrt{a \cosh(c+dx)+a}} + \frac{4i \cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \text{PolyLog}\left(2, ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d^2\sqrt{a \cosh(c+dx)+a}} + \frac{4x \tan^{-1}\left(e^{\frac{c}{2} + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\sqrt{a \cosh(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a + a\*Cosh[c + d\*x]],x]

[Out]  $(4*x*\text{ArcTan}[E^{(c/2 + (d*x)/2)}]*\text{Cosh}[c/2 + (d*x)/2])/(d*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]]) - ((4*I)*\text{Cosh}[c/2 + (d*x)/2]*\text{PolyLog}[2, (-I)*E^{(c/2 + (d*x)/2)}])/(d^2*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]]) + ((4*I)*\text{Cosh}[c/2 + (d*x)/2]*\text{PolyLog}[2, I*E^{(c/2 + (d*x)/2)}])/(d^2*\text{Sqrt}[a + a*\text{Cosh}[c + d*x]])$

**Rule 2279**

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

**Rule 2391**

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

**Rule 3319**

Int[((c\_) + (d\_)\*(x\_)^(m\_))\*((a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)])^(n\_), x\_Symbol] :> Dist[((2\*a)^IntPart[n]\*(a + b\*Sin[e + f\*x])^FracPart[n])/Sin[

$e/2 + (a\pi)/(4b) + (f*x)/2)^{(2*\text{FracPart}[n])}$ , Int[(c + d\*x)^m\*Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)]\*((c\_.) + (d\_.)\*(x\_.))^m\_., x\_Symbol] :> Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)])/ (f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{a + a \cosh(c + dx)}} dx &= \frac{\sin\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{id x}{2}\right) \int x \csc\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{id x}{2}\right) dx}{\sqrt{a + a \cosh(c + dx)}} \\ &= \frac{4x \tan^{-1}\left(e^{\frac{c}{2} + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\sqrt{a + a \cosh(c + dx)}} - \frac{\left(2i \sin\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{id x}{2}\right)\right) \int \log\left(1 - ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d\sqrt{a + a \cosh(c + dx)}} \\ &= \frac{4x \tan^{-1}\left(e^{\frac{c}{2} + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\sqrt{a + a \cosh(c + dx)}} - \frac{\left(4i \sin\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{\pi}{4} + \frac{id x}{2}\right)\right) \text{Subst}\left(\int \frac{\log(1-i)}{x}\right)}{d^2\sqrt{a + a \cosh(c + dx)}} \\ &= \frac{4x \tan^{-1}\left(e^{\frac{c}{2} + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\sqrt{a + a \cosh(c + dx)}} - \frac{4i \cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \text{Li}_2\left(-ie^{\frac{c}{2} + \frac{dx}{2}}\right)}{d^2\sqrt{a + a \cosh(c + dx)}} + \frac{4i \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d^2\sqrt{a + a \cosh(c + dx)}} \end{aligned}$$

**Mathematica [A]** time = 0.71, size = 117, normalized size = 0.75

$$\frac{4 \cosh\left(\frac{1}{2}(c + dx)\right) \left(-i \text{Li}_2\left(-i \left(\cosh\left(\frac{1}{2}(c + dx)\right) + \sinh\left(\frac{1}{2}(c + dx)\right)\right)\right) + i \text{Li}_2\left(i \left(\cosh\left(\frac{1}{2}(c + dx)\right) + \sinh\left(\frac{1}{2}(c + dx)\right)\right)\right)}{d^2 \sqrt{a} (\cosh(c + dx) + 1)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/Sqrt[a + a\*Cosh[c + d\*x]], x]

[Out] (4\*Cosh[(c + d\*x)/2]\*(d\*x\*ArcTan[Cosh[(c + d\*x)/2] + Sinh[(c + d\*x)/2]] - I\*PolyLog[2, (-I)\*(Cosh[(c + d\*x)/2] + Sinh[(c + d\*x)/2])] + I\*PolyLog[2, I\*(Cosh[(c + d\*x)/2] + Sinh[(c + d\*x)/2])])/(d^2\*Sqrt[a\*(1 + Cosh[c + d\*x])])

**fricas** [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{\sqrt{a \cosh(dx + c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(x/sqrt(a\*cosh(d\*x + c) + a), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a \cosh(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(x/sqrt(a\*cosh(d\*x + c) + a), x)

**maple** [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + a \cosh(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+a\*cosh(d\*x+c))^(1/2),x)

[Out] int(x/(a+a\*cosh(d\*x+c))^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$2\sqrt{2}d \int \frac{xe^{\left(\frac{1}{2}dx + \frac{1}{2}c\right)}}{\sqrt{a}de^{(2dx+2c)} + 2\sqrt{a}de^{(dx+c)} + \sqrt{a}d} dx + 4\sqrt{2} \left( \frac{e^{\left(\frac{1}{2}dx + \frac{1}{2}c\right)}}{(\sqrt{a}de^{(dx+c)} + \sqrt{a}d)d} + \frac{\arctan\left(e^{\left(\frac{1}{2}dx + \frac{1}{2}c\right)}\right)}{\sqrt{a}d^2} \right) - \frac{2\left(\sqrt{2}\sqrt{a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] 2\*sqrt(2)\*d\*integrate(x\*e^(1/2\*d\*x + 1/2\*c)/(sqrt(a)\*d\*e^(2\*d\*x + 2\*c) + 2\*sqrt(a)\*d\*e^(d\*x + c) + sqrt(a)\*d), x) + 4\*sqrt(2)\*(e^(1/2\*d\*x + 1/2\*c)/((sqrt(a)\*d\*e^(d\*x + c) + sqrt(a)\*d)\*d) + arctan(e^(1/2\*d\*x + 1/2\*c))/(sqrt(a))

$*d^2)) - 2*(\sqrt{2}*\sqrt{a}*d*x*e^{(1/2*c)} + 2*\sqrt{2}*\sqrt{a}*e^{(1/2*c)})*e^{(1/2*d*x)/(a*d^2*e^{(d*x + c)} + a*d^2)}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a + a \cosh(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a + a*cosh(c + d*x))^(1/2), x)`

[Out] `int(x/(a + a*cosh(c + d*x))^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a(\cosh(c + dx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+a*cosh(d*x+c))**(1/2), x)`

[Out] `Integral(x/sqrt(a*(cosh(c + d*x) + 1)), x)`

$$3.142 \quad \int \frac{1}{x\sqrt{a+a \cosh(c+dx)}} dx$$

**Optimal.** Leaf size=21

$$\text{Int}\left(\frac{1}{x\sqrt{a \cosh(c+dx)+a}}, x\right)$$

[Out] Unintegrable(1/x/(a+a\*cosh(d\*x+c))^(1/2), x)

**Rubi [A]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x\sqrt{a+a \cosh(c+dx)}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*Sqrt[a + a\*Cosh[c + d\*x]]), x]

[Out] Defer[Int][1/(x\*Sqrt[a + a\*Cosh[c + d\*x]]), x]

Rubi steps

$$\int \frac{1}{x\sqrt{a+a \cosh(c+dx)}} dx = \int \frac{1}{x\sqrt{a+a \cosh(c+dx)}} dx$$

**Mathematica [A]** time = 2.99, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+a \cosh(c+dx)}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*Sqrt[a + a\*Cosh[c + d\*x]]), x]

[Out] Integrate[1/(x\*Sqrt[a + a\*Cosh[c + d\*x]]), x]

**fricas [A]** time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a \cosh(dx+c)+a}}{ax \cosh(dx+c)+ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(1/x/(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a\*cosh(d\*x + c) + a)/(a\*x\*cosh(d\*x + c) + a\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \cosh(dx + c) + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(a\*cosh(d\*x + c) + a)\*x), x)

**maple** [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a + a \cosh(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+a\*cosh(d\*x+c))^(1/2),x)

[Out] int(1/x/(a+a\*cosh(d\*x+c))^(1/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \cosh(dx + c) + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a\*cosh(d\*x + c) + a)\*x), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\sqrt{a + a \cosh(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + a\*cosh(c + d\*x))^(1/2)),x)

[Out] int(1/(x\*(a + a\*cosh(c + d\*x))^(1/2)), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a(\cosh(c + dx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+a\*cosh(d\*x+c))\*\*(1/2),x)

[Out] Integral(1/(x\*sqrt(a\*(cosh(c + d\*x) + 1))), x)

$$3.143 \quad \int \frac{1}{x^2 \sqrt{a+a \cosh(c+dx)}} dx$$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^2 \sqrt{a \cosh(c+dx) + a}}, x\right)$$

[Out] Unintegrable(1/x^2/(a+a\*cosh(d\*x+c))^(1/2), x)

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2 \sqrt{a + a \cosh(c + dx)}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*Sqrt[a + a\*Cosh[c + d\*x]]), x]

[Out] Defer[Int][1/(x^2\*Sqrt[a + a\*Cosh[c + d\*x]]), x]

Rubi steps

$$\int \frac{1}{x^2 \sqrt{a + a \cosh(c + dx)}} dx = \int \frac{1}{x^2 \sqrt{a + a \cosh(c + dx)}} dx$$

Mathematica [A] time = 1.76, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{a + a \cosh(c + dx)}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*Sqrt[a + a\*Cosh[c + d\*x]]), x]

[Out] Integrate[1/(x^2\*Sqrt[a + a\*Cosh[c + d\*x]]), x]

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a \cosh(dx + c) + a}}{ax^2 \cosh(dx + c) + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a\*cosh(d\*x + c) + a)/(a\*x^2\*cosh(d\*x + c) + a\*x^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \cosh(dx + c) + a} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(a\*cosh(d\*x + c) + a)\*x^2), x)

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{a + a \cosh(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+a\*cosh(d\*x+c))^(1/2),x)

[Out] int(1/x^2/(a+a\*cosh(d\*x+c))^(1/2),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \cosh(dx + c) + a} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+a\*cosh(d\*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a\*cosh(d\*x + c) + a)\*x^2), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2 \sqrt{a + a \cosh(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*(a + a\*cosh(c + d\*x))^(1/2)),x)

[Out] int(1/(x^2\*(a + a\*cosh(c + d\*x))^(1/2)), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{a (\cosh(c + dx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+a\*cosh(d\*x+c))\*\*(1/2), x)

[Out] Integral(1/(x\*\*2\*sqrt(a\*(cosh(c + d\*x) + 1))), x)

$$3.144 \quad \int \frac{x^3}{(a+a \cosh(x))^{3/2}} dx$$

**Optimal.** Leaf size=402

$$-\frac{3ix^2 \operatorname{Li}_2(-ie^{x/2}) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a \cosh(x) + a}} + \frac{3ix^2 \operatorname{Li}_2(ie^{x/2}) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a \cosh(x) + a}} + \frac{12ix \operatorname{Li}_3(-ie^{x/2}) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a \cosh(x) + a}} - \frac{12ix \operatorname{Li}_3(ie^{x/2}) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a \cosh(x) + a}} + \frac{24i \operatorname{Li}_2(-ie^{x/2}) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a \cosh(x) + a}}$$

[Out]  $3x^2/a/(a+a*\cosh(x))^{(1/2)}-24*x*\arctan(\exp(1/2*x))*\cosh(1/2*x)/a/(a+a*\cosh(x))^{(1/2)}+x^3*\arctan(\exp(1/2*x))*\cosh(1/2*x)/a/(a+a*\cosh(x))^{(1/2)}+24*I*\cosh(1/2*x)*\operatorname{polylog}(2,-I*\exp(1/2*x))/a/(a+a*\cosh(x))^{(1/2)}-3*I*x^2*\cosh(1/2*x)*\operatorname{polylog}(2,-I*\exp(1/2*x))/a/(a+a*\cosh(x))^{(1/2)}-24*I*\cosh(1/2*x)*\operatorname{polylog}(2,I*\exp(1/2*x))/a/(a+a*\cosh(x))^{(1/2)}+3*I*x^2*\cosh(1/2*x)*\operatorname{polylog}(2,I*\exp(1/2*x))/a/(a+a*\cosh(x))^{(1/2)}+12*I*x*\cosh(1/2*x)*\operatorname{polylog}(3,-I*\exp(1/2*x))/a/(a+a*\cosh(x))^{(1/2)}-12*I*x*\cosh(1/2*x)*\operatorname{polylog}(3,I*\exp(1/2*x))/a/(a+a*\cosh(x))^{(1/2)}-24*I*\cosh(1/2*x)*\operatorname{polylog}(4,-I*\exp(1/2*x))/a/(a+a*\cosh(x))^{(1/2)}+24*I*\cosh(1/2*x)*\operatorname{polylog}(4,I*\exp(1/2*x))/a/(a+a*\cosh(x))^{(1/2)}+1/2*x^3*\tanh(1/2*x)/a/(a+a*\cosh(x))^{(1/2)}$

**Rubi [A]** time = 0.26, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$ , Rules used = {3319, 4186, 4180, 2279, 2391, 2531, 6609, 2282, 6589}

$$-\frac{3ix^2 \cosh\left(\frac{x}{2}\right) \operatorname{PolyLog}\left(2, -ie^{x/2}\right)}{a\sqrt{a \cosh(x) + a}} + \frac{3ix^2 \cosh\left(\frac{x}{2}\right) \operatorname{PolyLog}\left(2, ie^{x/2}\right)}{a\sqrt{a \cosh(x) + a}} + \frac{12ix \cosh\left(\frac{x}{2}\right) \operatorname{PolyLog}\left(3, -ie^{x/2}\right)}{a\sqrt{a \cosh(x) + a}} - \frac{12ix \cosh\left(\frac{x}{2}\right) \operatorname{PolyLog}\left(3, ie^{x/2}\right)}{a\sqrt{a \cosh(x) + a}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + a\*Cosh[x])^(3/2), x]

[Out]  $(3x^2)/(a*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]]) - (24*x*\operatorname{ArcTan}[E^{(x/2)}]*\operatorname{Cosh}[x/2])/(a*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]]) + (x^3*\operatorname{ArcTan}[E^{(x/2)}]*\operatorname{Cosh}[x/2])/(a*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]]) + ((24*I)*\operatorname{Cosh}[x/2]*\operatorname{PolyLog}[2, (-I)*E^{(x/2)}])/(a*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]]) - ((3*I)*x^2*\operatorname{Cosh}[x/2]*\operatorname{PolyLog}[2, (-I)*E^{(x/2)}])/(a*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]]) - ((24*I)*\operatorname{Cosh}[x/2]*\operatorname{PolyLog}[2, I*E^{(x/2)}])/(a*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]]) + ((3*I)*x^2*\operatorname{Cosh}[x/2]*\operatorname{PolyLog}[2, I*E^{(x/2)}])/(a*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]]) + ((12*I)*x*\operatorname{Cosh}[x/2]*\operatorname{PolyLog}[3, (-I)*E^{(x/2)}])/(a*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]]) - ((12*I)*x*\operatorname{Cosh}[x/2]*\operatorname{PolyLog}[3, I*E^{(x/2)}])/(a*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]]) - ((24*I)*\operatorname{Cosh}[x/2]*\operatorname{PolyLog}[4, (-I)*E^{(x/2)}])/(a*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]]) + ((24*I)*\operatorname{Cosh}[x/2]*\operatorname{PolyLog}[4, I*E^{(x/2)}])/(a*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]]) + (x^3*\operatorname{Tanh}[x/2])/(2*a*\operatorname{Sqrt}[a + a*\operatorname{Cosh}[x]])$

**Rule 2279**

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_)))]^(n\_.)], x\_Symbol]  
 :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x))

)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_)^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 3319

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[((2\*a)^IntPart[n]\*(a + b\*Sin[e + f\*x])^FracPart[n])/Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*FracPart[n]), Int[(c + d\*x)^m\*Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)])/ (f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 4186

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(b^2\*(c + d\*x)^m\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*d^2\*m\*(m - 1))/(f^2\*(n - 1)\*(n - 2)), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n] && GtQ[m, 0]

```
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{(a + a \cosh(x))^{3/2}} dx &= \frac{\cosh\left(\frac{x}{2}\right) \int x^3 \operatorname{sech}^3\left(\frac{x}{2}\right) dx}{2a\sqrt{a + a \cosh(x)}} \\
 &= \frac{3x^2}{a\sqrt{a + a \cosh(x)}} + \frac{x^3 \tanh\left(\frac{x}{2}\right)}{2a\sqrt{a + a \cosh(x)}} + \frac{\cosh\left(\frac{x}{2}\right) \int x^3 \operatorname{sech}\left(\frac{x}{2}\right) dx}{4a\sqrt{a + a \cosh(x)}} - \frac{(6 \cosh\left(\frac{x}{2}\right)) \int x^2 \operatorname{sech}\left(\frac{x}{2}\right) dx}{a\sqrt{a + a \cosh(x)}} \\
 &= \frac{3x^2}{a\sqrt{a + a \cosh(x)}} - \frac{24x \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} + \frac{x^3 \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} + \frac{x^3 \tanh\left(\frac{x}{2}\right)}{2a\sqrt{a + a \cosh(x)}} \\
 &= \frac{3x^2}{a\sqrt{a + a \cosh(x)}} - \frac{24x \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} + \frac{x^3 \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} - \frac{3ix^2 \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} \\
 &= \frac{3x^2}{a\sqrt{a + a \cosh(x)}} - \frac{24x \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} + \frac{x^3 \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} + \frac{24i \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} \\
 &= \frac{3x^2}{a\sqrt{a + a \cosh(x)}} - \frac{24x \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} + \frac{x^3 \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} + \frac{24i \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} \\
 &= \frac{3x^2}{a\sqrt{a + a \cosh(x)}} - \frac{24x \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} + \frac{x^3 \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} + \frac{24i \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}}
 \end{aligned}$$



**Mathematica [A]** time = 2.97, size = 716, normalized size = 1.78

$$i \cosh\left(\frac{x}{2}\right) \left(48x^2 \operatorname{Li}_2\left(-ie^{x/2}\right) \cosh^2\left(\frac{x}{2}\right) - 48\left(-x^2 - 2i\pi x + \pi^2 + 8\right) \operatorname{Li}_2\left(-ie^{-x/2}\right) \cosh^2\left(\frac{x}{2}\right) + 96i\pi x \operatorname{Li}_2\left(ie^{x/2}\right) \cosh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + a\*Cosh[x])^(3/2), x]

[Out] 
$$\begin{aligned} &((-1/8*I)*\operatorname{Cosh}[x/2]*((48*I)*x^2*\operatorname{Cosh}[x/2] + 7*\operatorname{Pi}^4*\operatorname{Cosh}[x/2]^2 + (4*I)*\operatorname{Pi}^3 \\ &*x*\operatorname{Cosh}[x/2]^2 + 6*\operatorname{Pi}^2*x^2*\operatorname{Cosh}[x/2]^2 - (4*I)*\operatorname{Pi}*x^3*\operatorname{Cosh}[x/2]^2 - x^4*\operatorname{Cosh}[x/2]^2 \\ &- 192*x*\operatorname{Cosh}[x/2]^2*\operatorname{Log}[1 - I/E^{(x/2)}] + (8*I)*\operatorname{Pi}^3*\operatorname{Cosh}[x/2]^2*\operatorname{Log}[1 + I/E^{(x/2)}] \\ &+ 192*x*\operatorname{Cosh}[x/2]^2*\operatorname{Log}[1 + I/E^{(x/2)}] + 24*\operatorname{Pi}^2*x*\operatorname{Cosh}[x/2]^2*\operatorname{Log}[1 + I/E^{(x/2)}] \\ &- (24*I)*\operatorname{Pi}*x^2*\operatorname{Cosh}[x/2]^2*\operatorname{Log}[1 + I/E^{(x/2)}] - 8*x^3*\operatorname{Cosh}[x/2]^2*\operatorname{Log}[1 + I/E^{(x/2)}] \\ &- 24*\operatorname{Pi}^2*x*\operatorname{Cosh}[x/2]^2*\operatorname{Log}[1 - I/E^{(x/2)}] + (24*I)*\operatorname{Pi}*x^2*\operatorname{Cosh}[x/2]^2*\operatorname{Log}[1 - I/E^{(x/2)}] \\ &- (8*I)*\operatorname{Pi}^3*\operatorname{Cosh}[x/2]^2*\operatorname{Log}[1 + I/E^{(x/2)}] + 8*x^3*\operatorname{Cosh}[x/2]^2*\operatorname{Log}[1 + I/E^{(x/2)}] \\ &+ (8*I)*\operatorname{Pi}^3*\operatorname{Cosh}[x/2]^2*\operatorname{Log}[\operatorname{Tan}[(\operatorname{Pi} + I*x)/4]] - 48*(8 + \operatorname{Pi}^2 - (2*I)*\operatorname{Pi}*x - x^2)*\operatorname{Cosh}[x/2]^2 \\ &*\operatorname{PolyLog}[2, (-I)/E^{(x/2)}] + 384*\operatorname{Cosh}[x/2]^2*\operatorname{PolyLog}[2, I/E^{(x/2)}] + 48*x^2*\operatorname{Cosh}[x/2]^2 \\ &*\operatorname{PolyLog}[2, (-I)*E^{(x/2)}] - 48*\operatorname{Pi}^2*\operatorname{Cosh}[x/2]^2*\operatorname{PolyLog}[2, I/E^{(x/2)}] + (96*I)*\operatorname{Pi}*x*\operatorname{Cosh}[x/2]^2 \\ &*\operatorname{PolyLog}[2, I/E^{(x/2)}] + (192*I)*\operatorname{Pi}*\operatorname{Cosh}[x/2]^2*\operatorname{PolyLog}[3, (-I)/E^{(x/2)}] + 192*x*\operatorname{Cosh}[x/2]^2 \\ &*\operatorname{PolyLog}[3, (-I)/E^{(x/2)}] - 192*x*\operatorname{Cosh}[x/2]^2*\operatorname{PolyLog}[3, (-I)*E^{(x/2)}] - (192*I)*\operatorname{Pi}*\operatorname{Cosh}[x/2]^2 \\ &*\operatorname{PolyLog}[3, I/E^{(x/2)}] + 384*\operatorname{Cosh}[x/2]^2*\operatorname{PolyLog}[4, (-I)/E^{(x/2)}] + 384*\operatorname{Cosh}[x/2]^2 \\ &*\operatorname{PolyLog}[4, (-I)*E^{(x/2)}] + (8*I)*x^3*\operatorname{Sinh}[x/2]))/(a*(1 + \operatorname{Cosh}[x]))^{(3/2)} \end{aligned}$$

**fricas [F]** time = 0.50, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{a \cosh(x) + a} x^3}{a^2 \cosh(x)^2 + 2 a^2 \cosh(x) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+a\*cosh(x))^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(a\*cosh(x) + a)\*x^3/(a^2\*cosh(x)^2 + 2\*a^2\*cosh(x) + a^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a \cosh(x) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+a\*cosh(x))^(3/2),x, algorithm="giac")

[Out] integrate(x^3/(a\*cosh(x) + a)^(3/2), x)

**maple** [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + a \cosh(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+a\*cosh(x))^(3/2),x)

[Out] int(x^3/(a+a\*cosh(x))^(3/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{8}{27} \sqrt{2} \left( \frac{3e^{\left(\frac{5}{2}x\right)} + 8e^{\left(\frac{3}{2}x\right)} - 3e^{\left(\frac{1}{2}x\right)}}{a^{\frac{3}{2}}e^{(3x)} + 3a^{\frac{3}{2}}e^{(2x)} + 3a^{\frac{3}{2}}e^x + a^{\frac{3}{2}}} + \frac{3 \arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{a^{\frac{3}{2}}} \right) + 36 \sqrt{2} \int \frac{x^3 e^{\left(\frac{3}{2}x\right)}}{9\left(a^{\frac{3}{2}}e^{(4x)} + 4a^{\frac{3}{2}}e^{(3x)} + 6a^{\frac{3}{2}}e^{(2x)} + 4a^{\frac{3}{2}}e^x + a^{\frac{3}{2}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+a\*cosh(x))^(3/2),x, algorithm="maxima")

[Out] 8/27\*sqrt(2)\*((3\*e^(5/2\*x) + 8\*e^(3/2\*x) - 3\*e^(1/2\*x))/(a^(3/2)\*e^(3\*x) + 3\*a^(3/2)\*e^(2\*x) + 3\*a^(3/2)\*e^x + a^(3/2)) + 3\*arctan(e^(1/2\*x))/a^(3/2)) + 36\*sqrt(2)\*integrate(1/9\*x^3\*e^(3/2\*x)/(a^(3/2)\*e^(4\*x) + 4\*a^(3/2)\*e^(3\*x) + 6\*a^(3/2)\*e^(2\*x) + 4\*a^(3/2)\*e^x + a^(3/2)), x) + 72\*sqrt(2)\*integrate(1/9\*x^2\*e^(3/2\*x)/(a^(3/2)\*e^(4\*x) + 4\*a^(3/2)\*e^(3\*x) + 6\*a^(3/2)\*e^(2\*x) + 4\*a^(3/2)\*e^x + a^(3/2)), x) + 96\*sqrt(2)\*integrate(1/9\*x\*e^(3/2\*x)/(a^(3/2)\*e^(4\*x) + 4\*a^(3/2)\*e^(3\*x) + 6\*a^(3/2)\*e^(2\*x) + 4\*a^(3/2)\*e^x + a^(3/2)), x) - 4/27\*(9\*sqrt(2)\*sqrt(a)\*x^3 + 18\*sqrt(2)\*sqrt(a)\*x^2 + 24\*sqrt(2)\*sqrt(a)\*x + 16\*sqrt(2)\*sqrt(a))\*e^(3/2\*x)/(a^2\*e^(3\*x) + 3\*a^2\*e^(2\*x) + 3\*a^2\*e^x + a^2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(a + a \cosh(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + a\*cosh(x))^(3/2),x)

[Out] int(x^3/(a + a\*cosh(x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a(\cosh(x) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(a+a\*cosh(x))\*\*(3/2), x)

[Out] Integral(x\*\*3/(a\*(cosh(x) + 1))\*\*(3/2), x)

$$3.145 \quad \int \frac{x^2}{(a+a \cosh(x))^{3/2}} dx$$

**Optimal.** Leaf size=248

$$-\frac{2ix\text{Li}_2(-ie^{x/2})\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}} + \frac{2ix\text{Li}_2(ie^{x/2})\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}} + \frac{4i\text{Li}_3(-ie^{x/2})\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}} - \frac{4i\text{Li}_3(ie^{x/2})\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}} + \frac{x^2 \tan^{-1}\left(e^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}$$

[Out]  $2*x/a/(a+a*\cosh(x))^{(1/2)}+x^2*\arctan(\exp(1/2*x))*\cosh(1/2*x)/a/(a+a*\cosh(x))^{(1/2)}-4*\arctan(\sinh(1/2*x))*\cosh(1/2*x)/a/(a+a*\cosh(x))^{(1/2)}-2*I*x*\cosh(1/2*x)*\text{polylog}(2,-I*\exp(1/2*x))/a/(a+a*\cosh(x))^{(1/2)}+2*I*x*\cosh(1/2*x)*\text{polylog}(2,I*\exp(1/2*x))/a/(a+a*\cosh(x))^{(1/2)}+4*I*\cosh(1/2*x)*\text{polylog}(3,-I*\exp(1/2*x))/a/(a+a*\cosh(x))^{(1/2)}-4*I*\cosh(1/2*x)*\text{polylog}(3,I*\exp(1/2*x))/a/(a+a*\cosh(x))^{(1/2)}+1/2*x^2*\tanh(1/2*x)/a/(a+a*\cosh(x))^{(1/2)}$

**Rubi [A]** time = 0.19, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {3319, 4186, 3770, 4180, 2531, 2282, 6589}

$$-\frac{2ix \cosh\left(\frac{x}{2}\right) \text{PolyLog}\left(2, -ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}} + \frac{2ix \cosh\left(\frac{x}{2}\right) \text{PolyLog}\left(2, ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}} + \frac{4i \cosh\left(\frac{x}{2}\right) \text{PolyLog}\left(3, -ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}} - \frac{4i \cosh\left(\frac{x}{2}\right) \text{PolyLog}\left(3, ie^{x/2}\right)}{a\sqrt{a\cosh(x)+a}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + a\*Cosh[x])^(3/2), x]

[Out]  $(2*x)/(a*\text{Sqrt}[a + a*\text{Cosh}[x]]) + (x^2*\text{ArcTan}[E^{(x/2)}]*\text{Cosh}[x/2])/(a*\text{Sqrt}[a + a*\text{Cosh}[x]]) - (4*\text{ArcTan}[\text{Sinh}[x/2]]*\text{Cosh}[x/2])/(a*\text{Sqrt}[a + a*\text{Cosh}[x]]) - ((2*I)*x*\text{Cosh}[x/2]*\text{PolyLog}[2, (-I)*E^{(x/2)}])/(a*\text{Sqrt}[a + a*\text{Cosh}[x]]) + ((2*I)*x*\text{Cosh}[x/2]*\text{PolyLog}[2, I*E^{(x/2)}])/(a*\text{Sqrt}[a + a*\text{Cosh}[x]]) + ((4*I)*\text{Cosh}[x/2]*\text{PolyLog}[3, (-I)*E^{(x/2)}])/(a*\text{Sqrt}[a + a*\text{Cosh}[x]]) - ((4*I)*\text{Cosh}[x/2]*\text{PolyLog}[3, I*E^{(x/2)}])/(a*\text{Sqrt}[a + a*\text{Cosh}[x]]) + (x^2*\text{Tanh}[x/2])/(2*a*\text{Sqrt}[a + a*\text{Cosh}[x]])$

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_) /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.))]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x

)))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 3319

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[((2\*a)^IntPart[n]\*(a + b\*Sin[e + f\*x])^FracPart[n])/Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*FracPart[n]), Int[(c + d\*x)^m\*Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 4186

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(b^2\*(c + d\*x)^m\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*d^2\*m\*(m - 1))/(f^2\*(n - 1)\*(n - 2)), Int[(c + d\*x)^(m - 2)\*(b\*Csc[e + f\*x])^(n - 2), x], x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(c + d\*x)^m\*(b\*Csc[e + f\*x])^(n - 2), x], x] - Simp[(b^2\*d\*m\*(c + d\*x)^(m - 1)\*(b\*Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a + a \cosh(x))^{3/2}} dx &= \frac{\cosh\left(\frac{x}{2}\right) \int x^2 \operatorname{sech}^3\left(\frac{x}{2}\right) dx}{2a\sqrt{a + a \cosh(x)}} \\
&= \frac{2x}{a\sqrt{a + a \cosh(x)}} + \frac{x^2 \tanh\left(\frac{x}{2}\right)}{2a\sqrt{a + a \cosh(x)}} + \frac{\cosh\left(\frac{x}{2}\right) \int x^2 \operatorname{sech}\left(\frac{x}{2}\right) dx}{4a\sqrt{a + a \cosh(x)}} - \frac{(2 \cosh\left(\frac{x}{2}\right)) \int x^2 \operatorname{sech}\left(\frac{x}{2}\right) dx}{a\sqrt{a + a \cosh(x)}} \\
&= \frac{2x}{a\sqrt{a + a \cosh(x)}} + \frac{x^2 \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} - \frac{4 \tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} + \frac{x^2 \tanh\left(\frac{x}{2}\right)}{2a\sqrt{a + a \cosh(x)}} \\
&= \frac{2x}{a\sqrt{a + a \cosh(x)}} + \frac{x^2 \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} - \frac{4 \tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} - \frac{2ix \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} \\
&= \frac{2x}{a\sqrt{a + a \cosh(x)}} + \frac{x^2 \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} - \frac{4 \tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} - \frac{2ix \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} \\
&= \frac{2x}{a\sqrt{a + a \cosh(x)}} + \frac{x^2 \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} - \frac{4 \tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} - \frac{2ix \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}}
\end{aligned}$$

**Mathematica [A]** time = 0.91, size = 214, normalized size = 0.86

$$\frac{\cosh\left(\frac{x}{2}\right) \left(-4ix \cosh^2\left(\frac{x}{2}\right) \operatorname{Li}_2\left(-i\left(\cosh\left(\frac{x}{2}\right) + \sinh\left(\frac{x}{2}\right)\right)\right) + 4ix \cosh^2\left(\frac{x}{2}\right) \operatorname{Li}_2\left(i\left(\cosh\left(\frac{x}{2}\right) + \sinh\left(\frac{x}{2}\right)\right)\right) + 8i \cosh^2\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/(a + a\*Cosh[x])^(3/2),x]

[Out] (Cosh[x/2]\*(4\*x\*Cosh[x/2] - 16\*ArcTan[Cosh[x/2] + Sinh[x/2]]\*Cosh[x/2]^2 + 2\*x^2\*ArcTan[Cosh[x/2] + Sinh[x/2]]\*Cosh[x/2]^2 - (4\*I)\*x\*Cosh[x/2]^2\*PolyLog[2, (-I)\*(Cosh[x/2] + Sinh[x/2])]) + (4\*I)\*x\*Cosh[x/2]^2\*PolyLog[2, I\*(Cosh[x/2] + Sinh[x/2])]) + (8\*I)\*Cosh[x/2]^2\*PolyLog[3, (-I)\*(Cosh[x/2] + Sinh[x/2])]) - (8\*I)\*Cosh[x/2]^2\*PolyLog[3, I\*(Cosh[x/2] + Sinh[x/2])]) + x^2\*Sinh[x/2])/(a\*(1 + Cosh[x]))^(3/2)

**fricas [F]** time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{a \cosh(x) + a x^2}}{a^2 \cosh(x)^2 + 2 a^2 \cosh(x) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+a\*cosh(x))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(a\*cosh(x) + a)\*x^2/(a^2\*cosh(x)^2 + 2\*a^2\*cosh(x) + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a \cosh(x) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+a\*cosh(x))^(3/2),x, algorithm="giac")

[Out] integrate(x^2/(a\*cosh(x) + a)^(3/2), x)

**maple** [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + a \cosh(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+a\*cosh(x))^(3/2),x)

[Out] int(x^2/(a+a\*cosh(x))^(3/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4}{27} \sqrt{2} \left( \frac{3e^{\left(\frac{5}{2}x\right)} + 8e^{\left(\frac{3}{2}x\right)} - 3e^{\left(\frac{1}{2}x\right)}}{a^{\frac{3}{2}}e^{(3x)} + 3a^{\frac{3}{2}}e^{(2x)} + 3a^{\frac{3}{2}}e^x + a^{\frac{3}{2}}} + \frac{3 \arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{a^{\frac{3}{2}}} \right) + 36 \sqrt{2} \int \frac{x^2 e^{\left(\frac{3}{2}x\right)}}{9\left(a^{\frac{3}{2}}e^{(4x)} + 4a^{\frac{3}{2}}e^{(3x)} + 6a^{\frac{3}{2}}e^{(2x)} + 4a^{\frac{3}{2}}e^x + a^{\frac{3}{2}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+a\*cosh(x))^(3/2),x, algorithm="maxima")

[Out] 4/27\*sqrt(2)\*((3\*e^(5/2\*x) + 8\*e^(3/2\*x) - 3\*e^(1/2\*x))/(a^(3/2)\*e^(3\*x) + 3\*a^(3/2)\*e^(2\*x) + 3\*a^(3/2)\*e^x + a^(3/2)) + 3\*arctan(e^(1/2\*x))/a^(3/2)) + 36\*sqrt(2)\*integrate(1/9\*x^2\*e^(3/2\*x)/(a^(3/2)\*e^(4\*x) + 4\*a^(3/2)\*e^(3\*x) + 6\*a^(3/2)\*e^(2\*x) + 4\*a^(3/2)\*e^x + a^(3/2)), x) + 48\*sqrt(2)\*integrate(1/9\*x\*e^(3/2\*x)/(a^(3/2)\*e^(4\*x) + 4\*a^(3/2)\*e^(3\*x) + 6\*a^(3/2)\*e^(2\*x) + 4\*a^(3/2)\*e^x + a^(3/2)), x) - 4/27\*(9\*sqrt(2)\*x^2 + 12\*sqrt(2)\*x + 8\*sqrt(2))\*e^(3/2\*x)/(a^(3/2)\*e^(3\*x) + 3\*a^(3/2)\*e^(2\*x) + 3\*a^(3/2)\*e^x + a^(3/2))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(a + a \cosh(x))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a + a*cosh(x))^(3/2), x)`

[Out] `int(x^2/(a + a*cosh(x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a(\cosh(x) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(a+a*cosh(x))**(3/2), x)`

[Out] `Integral(x**2/(a*(cosh(x) + 1))**(3/2), x)`



$$3.146 \quad \int \frac{x}{(a+a \cosh(x))^{3/2}} dx$$

**Optimal.** Leaf size=140

$$-\frac{i\text{Li}_2(-ie^{x/2}) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a \cosh(x) + a}} + \frac{i\text{Li}_2(ie^{x/2}) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a \cosh(x) + a}} + \frac{1}{a\sqrt{a \cosh(x) + a}} + \frac{x \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a \cosh(x) + a}} + \frac{x \tanh\left(\frac{x}{2}\right)}{2a\sqrt{a \cosh(x) + a}}$$

[Out] 1/a/(a+a\*cosh(x))^(1/2)+x\*arctan(exp(1/2\*x))\*cosh(1/2\*x)/a/(a+a\*cosh(x))^(1/2)-I\*cosh(1/2\*x)\*polylog(2,-I\*exp(1/2\*x))/a/(a+a\*cosh(x))^(1/2)+I\*cosh(1/2\*x)\*polylog(2,I\*exp(1/2\*x))/a/(a+a\*cosh(x))^(1/2)+1/2\*x\*tanh(1/2\*x)/a/(a+a\*cosh(x))^(1/2)

**Rubi [A]** time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {3319, 4185, 4180, 2279, 2391}

$$-\frac{i \cosh\left(\frac{x}{2}\right) \text{PolyLog}\left(2, -ie^{x/2}\right)}{a\sqrt{a \cosh(x) + a}} + \frac{i \cosh\left(\frac{x}{2}\right) \text{PolyLog}\left(2, ie^{x/2}\right)}{a\sqrt{a \cosh(x) + a}} + \frac{1}{a\sqrt{a \cosh(x) + a}} + \frac{x \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a \cosh(x) + a}} + \frac{x \tanh\left(\frac{x}{2}\right)}{2a\sqrt{a \cosh(x) + a}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + a\*Cosh[x])^(3/2), x]

[Out] 1/(a\*Sqrt[a + a\*Cosh[x]]) + (x\*ArcTan[E^(x/2)]\*Cosh[x/2])/(a\*Sqrt[a + a\*Cosh[x]]) - (I\*Cosh[x/2]\*PolyLog[2, (-I)\*E^(x/2)])/(a\*Sqrt[a + a\*Cosh[x]]) + (I\*Cosh[x/2]\*PolyLog[2, I\*E^(x/2)])/(a\*Sqrt[a + a\*Cosh[x]]) + (x\*Tanh[x/2])/(2\*a\*Sqrt[a + a\*Cosh[x]])

**Rule 2279**

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

**Rule 2391**

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

**Rule 3319**

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Dist[((2\*a)^IntPart[n]\*(a + b\*Sin[e + f\*x])^FracPart[n])/Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*FracPart[n]), Int[(c + d\*x)^m\*Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E

qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 4185

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> -Simp[(b^2\*(c + d\*x)\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*(n - 2))/(n - 1), Int[(c + d\*x)\*(b\*Csc[e + f\*x])^(n - 2), x], x] - Simp[(b^2\*d\*(b\*Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]

### Rubi steps

$$\begin{aligned} \int \frac{x}{(a + a \cosh(x))^{3/2}} dx &= \frac{\cosh\left(\frac{x}{2}\right) \int x \operatorname{sech}^3\left(\frac{x}{2}\right) dx}{2a\sqrt{a + a \cosh(x)}} \\ &= \frac{1}{a\sqrt{a + a \cosh(x)}} + \frac{x \tanh\left(\frac{x}{2}\right)}{2a\sqrt{a + a \cosh(x)}} + \frac{\cosh\left(\frac{x}{2}\right) \int x \operatorname{sech}\left(\frac{x}{2}\right) dx}{4a\sqrt{a + a \cosh(x)}} \\ &= \frac{1}{a\sqrt{a + a \cosh(x)}} + \frac{x \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} + \frac{x \tanh\left(\frac{x}{2}\right)}{2a\sqrt{a + a \cosh(x)}} - \frac{\left(i \cosh\left(\frac{x}{2}\right)\right) \int \log}{2a\sqrt{a + a \cosh(x)}} \\ &= \frac{1}{a\sqrt{a + a \cosh(x)}} + \frac{x \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} + \frac{x \tanh\left(\frac{x}{2}\right)}{2a\sqrt{a + a \cosh(x)}} - \frac{\left(i \cosh\left(\frac{x}{2}\right)\right) \operatorname{Subst}}{a\sqrt{a + a \cosh(x)}} \\ &= \frac{1}{a\sqrt{a + a \cosh(x)}} + \frac{x \tan^{-1}\left(e^{x/2}\right) \cosh\left(\frac{x}{2}\right)}{a\sqrt{a + a \cosh(x)}} - \frac{i \cosh\left(\frac{x}{2}\right) \operatorname{Li}_2\left(-ie^{x/2}\right)}{a\sqrt{a + a \cosh(x)}} + \frac{i \cosh\left(\frac{x}{2}\right) \operatorname{Li}_2\left(ie^{x/2}\right)}{a\sqrt{a + a \cosh(x)}} \end{aligned}$$

**Mathematica** [A] time = 0.11, size = 137, normalized size = 0.98

$$\frac{2 \cosh^3\left(\frac{x}{2}\right) \left(-i \left(\operatorname{Li}_2\left(-ie^{-x/2}\right) - \operatorname{Li}_2\left(ie^{-x/2}\right)\right) - \frac{1}{2}ix \left(\log\left(1 - ie^{-x/2}\right) - \log\left(1 + ie^{-x/2}\right)\right)\right)}{\left(a(\cosh(x) + 1)\right)^{3/2}} + \frac{2 \cosh^2\left(\frac{x}{2}\right)}{\left(a(\cosh(x) + 1)\right)^{3/2}} + \frac{x \operatorname{sech}\left(\frac{x}{2}\right)}{a(\cosh(x) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + a\*Cosh[x])^(3/2),x]

[Out] (2\*Cosh[x/2]^2)/(a\*(1 + Cosh[x]))^(3/2) + (2\*Cosh[x/2]^3\*((-1/2\*I)\*x\*(Log[1 - I/E^(x/2)] - Log[1 + I/E^(x/2)]) - I\*(PolyLog[2, (-I)/E^(x/2)] - PolyLog[2, I/E^(x/2)])))/(a\*(1 + Cosh[x]))^(3/2) + (x\*Cosh[x/2]\*Sinh[x/2])/(a\*(1 + Cosh[x]))^(3/2)

**fricas** [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a \cosh(x) + a} x}{a^2 \cosh(x)^2 + 2 a^2 \cosh(x) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+a\*cosh(x))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(a\*cosh(x) + a)\*x/(a^2\*cosh(x)^2 + 2\*a^2\*cosh(x) + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a \cosh(x) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+a\*cosh(x))^(3/2),x, algorithm="giac")

[Out] integrate(x/(a\*cosh(x) + a)^(3/2), x)

**maple** [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + a \cosh(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+a\*cosh(x))^(3/2),x)

[Out] int(x/(a+a\*cosh(x))^(3/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{9} \sqrt{2} \left( \frac{3 e^{\left(\frac{5}{2}x\right)} + 8 e^{\left(\frac{3}{2}x\right)} - 3 e^{\left(\frac{1}{2}x\right)}}{a^{\frac{3}{2}} e^{(3x)} + 3 a^{\frac{3}{2}} e^{(2x)} + 3 a^{\frac{3}{2}} e^x + a^{\frac{3}{2}}} + \frac{3 \arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{a^{\frac{3}{2}}} \right) + 12 \sqrt{2} \int \frac{x e^{\left(\frac{3}{2}x\right)}}{3 \left( a^{\frac{3}{2}} e^{(4x)} + 4 a^{\frac{3}{2}} e^{(3x)} + 6 a^{\frac{3}{2}} e^{(2x)} + 4 a^{\frac{3}{2}} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+a\*cosh(x))^(3/2),x, algorithm="maxima")

[Out]  $\frac{1}{9}\sqrt{2} * ((3e^{5/2x} + 8e^{3/2x} - 3e^{1/2x}) / (a^{3/2}e^{3x} + 3a^{3/2}e^{2x} + 3a^{3/2}e^x + a^{3/2})) + 3\arctan(e^{1/2x}) / a^{3/2} + 12\sqrt{2} * \int (1/3xe^{3/2x} / (a^{3/2}e^{4x} + 4a^{3/2}e^{3x} + 6a^{3/2}e^{2x} + 4a^{3/2}e^x + a^{3/2})) dx - 4/9 * (3\sqrt{2}\sqrt{a})x + 2\sqrt{2}\sqrt{a})e^{3/2x} / (a^2e^{3x} + 3a^2e^{2x} + 3a^2e^x + a^2)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a + a \cosh(x))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + a\*cosh(x))^(3/2),x)

[Out] int(x/(a + a\*cosh(x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a(\cosh(x) + 1))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+a\*cosh(x))\*\*(3/2),x)

[Out] Integral(x/(a\*(cosh(x) + 1))\*\*(3/2), x)

$$3.147 \quad \int \frac{1}{x(a+a \cosh(x))^{3/2}} dx$$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{x(a \cosh(x) + a)^{3/2}}, x\right)$$

[Out] Unintegrable(1/x/(a+a\*cosh(x))^(3/2), x)

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a + a \cosh(x))^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + a\*Cosh[x])^(3/2)), x]

[Out] Defer[Int][1/(x\*(a + a\*Cosh[x])^(3/2)), x]

Rubi steps

$$\int \frac{1}{x(a + a \cosh(x))^{3/2}} dx = \int \frac{1}{x(a + a \cosh(x))^{3/2}} dx$$

Mathematica [A] time = 8.94, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + a \cosh(x))^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + a\*Cosh[x])^(3/2)), x]

[Out] Integrate[1/(x\*(a + a\*Cosh[x])^(3/2)), x]

fricas [A] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a \cosh(x) + a}}{a^2 x \cosh(x)^2 + 2 a^2 x \cosh(x) + a^2 x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+a\*cosh(x))^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(a\*cosh(x) + a)/(a^2\*x\*cosh(x)^2 + 2\*a^2\*x\*cosh(x) + a^2\*x), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cosh(x) + a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+a\*cosh(x))^(3/2),x, algorithm="giac")

[Out] integrate(1/((a\*cosh(x) + a)^(3/2)\*x), x)

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + a \cosh(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+a\*cosh(x))^(3/2),x)

[Out] int(1/x/(a+a\*cosh(x))^(3/2),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cosh(x) + a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+a\*cosh(x))^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a\*cosh(x) + a)^(3/2)\*x), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x(a + a \cosh(x))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + a\*cosh(x))^(3/2)),x)

[Out] int(1/(x\*(a + a\*cosh(x))^(3/2)), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x (a (\cosh(x) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+a\*cosh(x))\*\*(3/2),x)

[Out] Integral(1/(x\*(a\*(cosh(x) + 1))\*\*(3/2)), x)

$$3.148 \quad \int \frac{1}{x^2(a+a \cosh(x))^{3/2}} dx$$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{x^2(a \cosh(x) + a)^{3/2}}, x\right)$$

[Out] Unintegrable(1/x^2/(a+a\*cosh(x))^(3/2), x)

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2(a + a \cosh(x))^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*(a + a\*Cosh[x])^(3/2)), x]

[Out] Defer[Int][1/(x^2\*(a + a\*Cosh[x])^(3/2)), x]

Rubi steps

$$\int \frac{1}{x^2(a + a \cosh(x))^{3/2}} dx = \int \frac{1}{x^2(a + a \cosh(x))^{3/2}} dx$$

Mathematica [A] time = 10.68, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a + a \cosh(x))^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*(a + a\*Cosh[x])^(3/2)), x]

[Out] Integrate[1/(x^2\*(a + a\*Cosh[x])^(3/2)), x]

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a \cosh(x) + a}}{a^2 x^2 \cosh(x)^2 + 2 a^2 x^2 \cosh(x) + a^2 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+a\*cosh(x))^(3/2), x, algorithm="fricas")



[Out] integral(sqrt(a\*cosh(x) + a)/(a^2\*x^2\*cosh(x)^2 + 2\*a^2\*x^2\*cosh(x) + a^2\*x^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cosh(x) + a)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+a\*cosh(x))^(3/2), x, algorithm="giac")

[Out] integrate(1/((a\*cosh(x) + a)^(3/2)\*x^2), x)

**maple** [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + a \cosh(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+a\*cosh(x))^(3/2), x)

[Out] int(1/x^2/(a+a\*cosh(x))^(3/2), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \cosh(x) + a)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+a\*cosh(x))^(3/2), x, algorithm="maxima")

[Out] integrate(1/((a\*cosh(x) + a)^(3/2)\*x^2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x^2 (a + a \cosh(x))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*(a + a\*cosh(x))^(3/2)), x)

[Out] int(1/(x^2\*(a + a\*cosh(x))^(3/2)), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a (\cosh(x) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+a\*cosh(x))\*\*(3/2),x)

[Out] Integral(1/(x\*\*2\*(a\*(cosh(x) + 1))\*\*(3/2)), x)

$$3.149 \quad \int \frac{\sqrt[3]{a+a \cosh(c+dx)}}{x} dx$$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{\sqrt[3]{a \cosh(c+dx)+a}}{x}, x\right)$$

[Out] Unintegrable((a+a\*cosh(d\*x+c))^(1/3)/x,x)

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt[3]{a+a \cosh(c+dx)}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(a + a\*Cosh[c + d\*x])^(1/3)/x,x]

[Out] Defer[Int] [(a + a\*Cosh[c + d\*x])^(1/3)/x, x]

Rubi steps

$$\int \frac{\sqrt[3]{a+a \cosh(c+dx)}}{x} dx = \int \frac{\sqrt[3]{a+a \cosh(c+dx)}}{x} dx$$

Mathematica [A] time = 2.80, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+a \cosh(c+dx)}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + a\*Cosh[c + d\*x])^(1/3)/x,x]

[Out] Integrate[(a + a\*Cosh[c + d\*x])^(1/3)/x, x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(d\*x+c))^(1/3)/x,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \cosh(dx + c) + a)^{\frac{1}{3}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(d\*x+c))^(1/3)/x,x, algorithm="giac")

[Out] integrate((a\*cosh(d\*x + c) + a)^(1/3)/x, x)

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(a + a \cosh(dx + c))^{\frac{1}{3}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a\*cosh(d\*x+c))^(1/3)/x,x)

[Out] int((a+a\*cosh(d\*x+c))^(1/3)/x,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \cosh(dx + c) + a)^{\frac{1}{3}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(d\*x+c))^(1/3)/x,x, algorithm="maxima")

[Out] integrate((a\*cosh(d\*x + c) + a)^(1/3)/x, x)

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a + a \cosh(c + dx))^{1/3}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cosh(c + d\*x))^(1/3)/x,x)

[Out] int((a + a\*cosh(c + d\*x))^(1/3)/x, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a(\cosh(c + dx) + 1)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a\*cosh(d\*x+c))\*\*(1/3)/x,x)

[Out] Integral((a\*(cosh(c + d\*x) + 1))\*\*(1/3)/x, x)

### 3.150 $\int (c + dx)^m (a + a \cosh(e + fx))^n dx$

Optimal. Leaf size=23

$$\text{Int}((c + dx)^m (a \cosh(e + fx) + a)^n, x)$$

[Out] Unintegrable((d\*x+c)^m\*(a+a\*cosh(f\*x+e))^n,x)

**Rubi [A]** time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (c + dx)^m (a + a \cosh(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m\*(a + a\*Cosh[e + f\*x])^n,x]

[Out] Defer[Int] [(c + d\*x)^m\*(a + a\*Cosh[e + f\*x])^n, x]

Rubi steps

$$\int (c + dx)^m (a + a \cosh(e + fx))^n dx = \int (c + dx)^m (a + a \cosh(e + fx))^n dx$$

**Mathematica [A]** time = 6.52, size = 0, normalized size = 0.00

$$\int (c + dx)^m (a + a \cosh(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m\*(a + a\*Cosh[e + f\*x])^n,x]

[Out] Integrate[(c + d\*x)^m\*(a + a\*Cosh[e + f\*x])^n, x]

**fricas [A]** time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}((dx + c)^m (a \cosh(fx + e) + a)^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+a\*cosh(f\*x+e))^n,x, algorithm="fricas")

[Out] integral((d\*x + c)^m\*(a\*cosh(f\*x + e) + a)^n, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m (a \cosh(fx + e) + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+a\*cosh(f\*x+e))^n,x, algorithm="giac")

[Out] integrate((d\*x + c)^m\*(a\*cosh(f\*x + e) + a)^n, x)

**maple** [A] time = 0.17, size = 0, normalized size = 0.00

$$\int (dx + c)^m (a + a \cosh(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*(a+a\*cosh(f\*x+e))^n,x)

[Out] int((d\*x+c)^m\*(a+a\*cosh(f\*x+e))^n,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m (a \cosh(fx + e) + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+a\*cosh(f\*x+e))^n,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m\*(a\*cosh(f\*x + e) + a)^n, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (a + a \cosh(e + fx))^n (c + dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cosh(e + f\*x))^n\*(c + d\*x)^m,x)

[Out] int((a + a\*cosh(e + f\*x))^n\*(c + d\*x)^m, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m\*(a+a\*cosh(f\*x+e))\*\*n,x)

[Out] Timed out

### 3.151 $\int (c + dx)^m (a + a \cosh(e + fx))^3 dx$

**Optimal.** Leaf size=402

$$\frac{a^3 3^{-m-1} e^{3e-\frac{3cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{3f(c+dx)}{d}\right)}{8f} + \frac{3a^3 2^{-m-3} e^{2e-\frac{2cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2f(c+dx)}{d}\right)}{f}$$

[Out]  $5/2*a^3*(d*x+c)^(1+m)/d/(1+m)+1/8*3^(-1-m)*a^3*\exp(3*e-3*c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, -3*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+3*2^(-3-m)*a^3*\exp(2*e-2*c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, -2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+15/8*a^3*\exp(e-c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, -f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)-15/8*a^3*\exp(-e+c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-3*2^(-3-m)*a^3*\exp(-2*e+2*c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, 2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-1/8*3^(-1-m)*a^3*\exp(-3*e+3*c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, 3*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)$

**Rubi [A]** time = 0.55, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3318, 3312, 3307, 2181}

$$\frac{a^3 3^{-m-1} e^{3e-\frac{3cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{3f(c+dx)}{d}\right)}{8f} + \frac{3a^3 2^{-m-3} e^{2e-\frac{2cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{2f(c+dx)}{d}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^m\*(a + a\*Cosh[e + f\*x])^3, x]

[Out]  $(5*a^3*(c+d*x)^(1+m))/(2*d*(1+m)) + (3^(-1-m)*a^3*E^(3*e-(3*c*f)/d)*(c+d*x)^m*\text{Gamma}[1+m, (-3*f*(c+d*x))/d])/(8*f*(-((f*(c+d*x))/d))^m) + (3*2^(-3-m)*a^3*E^(2*e-(2*c*f)/d)*(c+d*x)^m*\text{Gamma}[1+m, (-2*f*(c+d*x))/d])/(f*(-((f*(c+d*x))/d))^m) + (15*a^3*E^(e-(c*f)/d)*(c+d*x)^m*\text{Gamma}[1+m, -((f*(c+d*x))/d)])/(8*f*(-((f*(c+d*x))/d))^m) - (15*a^3*E^(-e+(c*f)/d)*(c+d*x)^m*\text{Gamma}[1+m, (f*(c+d*x))/d])/(8*f*((f*(c+d*x))/d)^m) - (3*2^(-3-m)*a^3*E^(-2*e+(2*c*f)/d)*(c+d*x)^m*\text{Gamma}[1+m, (2*f*(c+d*x))/d])/(f*((f*(c+d*x))/d)^m) - (3^(-1-m)*a^3*E^(-3*e+(3*c*f)/d)*(c+d*x)^m*\text{Gamma}[1+m, (3*f*(c+d*x))/d])/(8*f*((f*(c+d*x))/d)^m)$

**Rule 2181**

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol]  
 := -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d))^FracPart[m]], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I



ntegerQ[m]

### Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
  := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
  I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
  f, m}, x] && IntegerQ[2*k]
```

### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f,
  m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

### Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
  (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rubi steps

$$\begin{aligned}
 \int (c + dx)^m (a + a \cosh(e + fx))^3 dx &= (8a^3) \int (c + dx)^m \sin^6 \left( \frac{1}{2}(ie + \pi) + \frac{ifx}{2} \right) dx \\
 &= (8a^3) \int \left( \frac{5}{16}(c + dx)^m + \frac{15}{32}(c + dx)^m \cosh(e + fx) + \frac{3}{16}(c + dx)^m \cosh^2(e + fx) \right) dx \\
 &= \frac{5a^3(c + dx)^{1+m}}{2d(1+m)} + \frac{1}{4}a^3 \int (c + dx)^m \cosh(3e + 3fx) dx + \frac{1}{2}(3a^3) \int (c + dx)^m \cosh^2(e + fx) dx \\
 &= \frac{5a^3(c + dx)^{1+m}}{2d(1+m)} + \frac{1}{8}a^3 \int e^{-i(3ie+3ifx)}(c + dx)^m dx + \frac{1}{8}a^3 \int e^{i(3ie+3ifx)}(c + dx)^m dx \\
 &= \frac{5a^3(c + dx)^{1+m}}{2d(1+m)} + \frac{3^{-1-m}a^3 e^{3e-\frac{3cf}{d}}(c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{3f(c+dx)}{d}\right)}{8f}
 \end{aligned}$$

**Mathematica [A]** time = 2.42, size = 429, normalized size = 1.07

$$a^3 2^{-m-6} 3^{-m-1} e^{-3\left(\frac{cf}{d}+e\right)} (c + dx)^m (\cosh(e + fx) + 1)^3 \operatorname{sech}^6\left(\frac{1}{2}(e + fx)\right) \left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m} \left(2^m e^{\frac{3cf}{d}} \left(d(m+1)e^{\frac{3cf}{d}} \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{3f(c+dx)}{d}\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^m\*(a + a\*Cosh[e + f\*x])^3,x]

[Out]  $-(2^{(-6-m)}3^{(-1-m)}a^3(c+d*x)^m(1+\cosh[e+f*x])^3(-2^m d E^{(6e)}(1+m)((f(c+d*x))/d)^m \Gamma[1+m, (-3f(c+d*x))/d] - 3^{(2+m)} d E^{(5e+(c*f)/d)}(1+m)(f(c/d+x))^m \Gamma[1+m, (-2f(c+d*x))/d] - 5*2^m 3^{(2+m)} d E^{(4e+(2*c*f)/d)}(1+m)((f(c+d*x))/d)^m \Gamma[1+m, -((f(c+d*x))/d)] + 5*2^m 3^{(2+m)} d E^{(2e+(4*c*f)/d)}(1+m)(-((f(c+d*x))/d))^m \Gamma[1+m, (f(c+d*x))/d] + 3^{(2+m)} d E^{(e+(5*c*f)/d)}(1+m)(-((f(c+d*x))/d))^m \Gamma[1+m, (2f(c+d*x))/d] + 2^m E^{((3*c*f)/d)}(-20*3^{(1+m)} E^{(3e)} f(c+d*x)(-((f^2(c+d*x)^2)/d^2))^m + d E^{((3*c*f)/d)}(1+m)(-((f(c+d*x))/d))^m \Gamma[1+m, (3f(c+d*x))/d]) * \operatorname{Sech}[(e+f*x)/2]^6 / (d E^{(3(e+(c*f)/d))} f(1+m)(-((f^2(c+d*x)^2)/d^2))^m)$

**fricas** [A] time = 0.71, size = 713, normalized size = 1.77

$$(a^3 d m + a^3 d) \cosh\left(\frac{d m \log\left(\frac{3f}{d}\right) + 3 d e - 3 c f}{d}\right) \Gamma\left(m + 1, \frac{3(d f x + c f)}{d}\right) + 9 (a^3 d m + a^3 d) \cosh\left(\frac{d m \log\left(\frac{2f}{d}\right) + 2 d e - 2 c f}{d}\right) \Gamma\left(m + 1, \frac{2(d f x + c f)}{d}\right) + 45 (a^3 d m + a^3 d) \cosh\left(\frac{d m \log(f/d) + d e - c f}{d}\right) \Gamma\left(m + 1, (d f x + c f)/d\right) - 45 (a^3 d m + a^3 d) \cosh\left(\frac{d m \log(-f/d) - d e + c f}{d}\right) \Gamma\left(m + 1, -(d f x + c f)/d\right) - 9 (a^3 d m + a^3 d) \cosh\left(\frac{d m \log(-2f/d) - 2 d e + 2 c f}{d}\right) \Gamma\left(m + 1, -2(d f x + c f)/d\right) - (a^3 d m + a^3 d) \cosh\left(\frac{d m \log(-3f/d) - 3 d e + 3 c f}{d}\right) \Gamma\left(m + 1, -3(d f x + c f)/d\right) - (a^3 d m + a^3 d) \Gamma\left(m + 1, 3(d f x + c f)/d\right) \sinh\left(\frac{d m \log(3f/d) + 3 d e - 3 c f}{d}\right) - 9 (a^3 d m + a^3 d) \Gamma\left(m + 1, 2(d f x + c f)/d\right) \sinh\left(\frac{d m \log(2f/d) + 2 d e - 2 c f}{d}\right) - 45 (a^3 d m + a^3 d) \Gamma\left(m + 1, (d f x + c f)/d\right) \sinh\left(\frac{d m \log(f/d) + d e - c f}{d}\right) + 45 (a^3 d m + a^3 d) \Gamma\left(m + 1, -(d f x + c f)/d\right) \sinh\left(\frac{d m \log(-f/d) - d e + c f}{d}\right) + 9 (a^3 d m + a^3 d) \Gamma\left(m + 1, -2(d f x + c f)/d\right) \sinh\left(\frac{d m \log(-2f/d) - 2 d e + 2 c f}{d}\right) + (a^3 d m + a^3 d) \Gamma\left(m + 1, -3(d f x + c f)/d\right) \sinh\left(\frac{d m \log(-3f/d) - 3 d e + 3 c f}{d}\right) - 60 (a^3 d f x + a^3 c f) \cosh(m \log(d x + c)) - 60 (a^3 d f x + a^3 c f) \sinh(m \log(d x + c)) / (d f m + d f)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+a\*cosh(f\*x+e))^3,x, algorithm="fricas")

[Out]  $-1/24*((a^3 d^m + a^3 d) \cosh((d m \log(3f/d) + 3 d e - 3 c f)/d) \Gamma(m + 1, 3(d f x + c f)/d) + 9(a^3 d^m + a^3 d) \cosh((d m \log(2f/d) + 2 d e - 2 c f)/d) \Gamma(m + 1, 2(d f x + c f)/d) + 45(a^3 d^m + a^3 d) \cosh((d m \log(f/d) + d e - c f)/d) \Gamma(m + 1, (d f x + c f)/d) - 45(a^3 d^m + a^3 d) \cosh((d m \log(-f/d) - d e + c f)/d) \Gamma(m + 1, -(d f x + c f)/d) - 9(a^3 d^m + a^3 d) \cosh((d m \log(-2f/d) - 2 d e + 2 c f)/d) \Gamma(m + 1, -2(d f x + c f)/d) - (a^3 d^m + a^3 d) \cosh((d m \log(-3f/d) - 3 d e + 3 c f)/d) \Gamma(m + 1, -3(d f x + c f)/d) - (a^3 d^m + a^3 d) \Gamma(m + 1, 3(d f x + c f)/d) \sinh((d m \log(3f/d) + 3 d e - 3 c f)/d) - 9(a^3 d^m + a^3 d) \Gamma(m + 1, 2(d f x + c f)/d) \sinh((d m \log(2f/d) + 2 d e - 2 c f)/d) - 45(a^3 d^m + a^3 d) \Gamma(m + 1, (d f x + c f)/d) \sinh((d m \log(f/d) + d e - c f)/d) + 45(a^3 d^m + a^3 d) \Gamma(m + 1, -(d f x + c f)/d) \sinh((d m \log(-f/d) - d e + c f)/d) + 9(a^3 d^m + a^3 d) \Gamma(m + 1, -2(d f x + c f)/d) \sinh((d m \log(-2f/d) - 2 d e + 2 c f)/d) + (a^3 d^m + a^3 d) \Gamma(m + 1, -3(d f x + c f)/d) \sinh((d m \log(-3f/d) - 3 d e + 3 c f)/d) - 60(a^3 d f x + a^3 c f) \cosh(m \log(d x + c)) - 60(a^3 d f x + a^3 c f) \sinh(m \log(d x + c)) / (d f m + d f)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cosh(fx + e) + a)^3 (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+a\*cosh(f\*x+e))^3,x, algorithm="giac")

[Out] integrate((a\*cosh(f\*x + e) + a)^3\*(d\*x + c)^m, x)

**maple** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int (dx + c)^m (a + a \cosh(fx + e))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*(a+a\*cosh(f\*x+e))^3,x)

[Out] int((d\*x+c)^m\*(a+a\*cosh(f\*x+e))^3,x)

**maxima** [A] time = 0.48, size = 373, normalized size = 0.93

$$\frac{1}{8} \left( \frac{(dx + c)^{m+1} e^{\left(-3e + \frac{3cf}{d}\right)} E_{-m}\left(\frac{3(dx+c)f}{d}\right)}{d} + \frac{3(dx + c)^{m+1} e^{\left(-e + \frac{cf}{d}\right)} E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} + \frac{3(dx + c)^{m+1} e^{\left(e - \frac{cf}{d}\right)} E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+a\*cosh(f\*x+e))^3,x, algorithm="maxima")

[Out] -1/8\*((d\*x + c)^(m + 1)\*e^(-3\*e + 3\*c\*f/d)\*exp\_integral\_e(-m, 3\*(d\*x + c)\*f/d)/d + 3\*(d\*x + c)^(m + 1)\*e^(-e + c\*f/d)\*exp\_integral\_e(-m, (d\*x + c)\*f/d)/d + 3\*(d\*x + c)^(m + 1)\*e^(e - c\*f/d)\*exp\_integral\_e(-m, -(d\*x + c)\*f/d)/d + (d\*x + c)^(m + 1)\*e^(3\*e - 3\*c\*f/d)\*exp\_integral\_e(-m, -3\*(d\*x + c)\*f/d)/d)\*a^3 - 3/4\*((d\*x + c)^(m + 1)\*e^(-2\*e + 2\*c\*f/d)\*exp\_integral\_e(-m, 2\*(d\*x + c)\*f/d)/d + (d\*x + c)^(m + 1)\*e^(2\*e - 2\*c\*f/d)\*exp\_integral\_e(-m, -2\*(d\*x + c)\*f/d)/d - 2\*(d\*x + c)^(m + 1)/(d\*(m + 1)))\*a^3 - 3/2\*((d\*x + c)^(m + 1)\*e^(-e + c\*f/d)\*exp\_integral\_e(-m, (d\*x + c)\*f/d)/d + (d\*x + c)^(m + 1)\*e^(e - c\*f/d)\*exp\_integral\_e(-m, -(d\*x + c)\*f/d)/d)\*a^3 + (d\*x + c)^(m + 1)\*a^3/(d\*(m + 1))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a \cosh(e + fx))^3 (c + dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))^3*(c + d*x)^m,x)
```

```
[Out] int((a + a*cosh(e + f*x))^3*(c + d*x)^m, x)
```

```
sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*cosh(f*x+e))**3,x)
```

```
[Out] Exception raised: TypeError
```

### 3.152 $\int (c + dx)^m (a + a \cosh(e + fx))^2 dx$

**Optimal.** Leaf size=263

$$\frac{a^2 2^{-m-3} e^{2e-\frac{2cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2f(c+dx)}{d}\right)}{f} + \frac{a^2 e^{e-\frac{cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{f(c+dx)}{d}\right)}{f}$$

[Out]  $3/2*a^2*(d*x+c)^{(1+m)/d/(1+m)+2^{(-3-m)}*a^2*\exp(2*e-2*c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, -2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+a^2*\exp(e-c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, -f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)-a^2*\exp(-e+c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-2^{(-3-m)}*a^2*\exp(-2*e+2*c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, 2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)$

**Rubi [A]** time = 0.34, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3318, 3312, 3307, 2181}

$$\frac{a^2 2^{-m-3} e^{2e-\frac{2cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{2f(c+dx)}{d}\right)}{f} + \frac{a^2 e^{e-\frac{cf}{d}} (c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{f(c+dx)}{d}\right)}{f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^m*(a + a*\text{Cosh}[e + f*x])^2, x]$

[Out]  $(3*a^2*(c + d*x)^{(1 + m)/(2*d*(1 + m))} + (2^{(-3 - m)}*a^2*E^{(2*e - (2*c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, (-2*f*(c + d*x))/d])/((f*(-((f*(c + d*x))/d))^m) + (a^2*E^{(e - (c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) - (a^2*E^{(-e + (c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, (f*(c + d*x))/d])/((f*((f*(c + d*x))/d))^m) - (2^{(-3 - m)}*a^2*E^{(-2*e + (2*c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, (2*f*(c + d*x))/d])/((f*((f*(c + d*x))/d))^m)$

#### Rule 2181

$\text{Int}[(F_)^{((g_.)*(e_.) + (f_.)*(x_))}*((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol]$   
 $:\> -\text{Simp}[(F^{(g*(e - (c*f)/d)}*(c + d*x)^{\text{FracPart}[m]}*\text{Gamma}[m + 1, (-((f*g*\text{Log}[F])/d))*(c + d*x)])/(d*(-((f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1}*(-((f*g*\text{Log}[F])*(c + d*x))/d))^{\text{FracPart}[m]})], x] /;$  FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rule 3307

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_.)}*\sin[(e_.) + \text{Pi}*(k_.) + (f_.)*(x_)], x\_Symbol]$   
 $:\> \text{Dist}[I/2, \text{Int}[(c + d*x)^m/(E^{(I*k*Pi)}*E^{(I*(e + f*x))}), x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m*E^{(I*k*Pi)}*E^{(I*(e + f*x))}, x], x] /;$  FreeQ[{c, d, e,

f, m}, x] && IntegerQ[2\*k]

### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rule 3318

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] := Dist[(2\*a)^n, Int[(c + d\*x)^m\*Sin[(1\*(e + (Pi\*a)/(2\*b)))/2 + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])

### Rubi steps

$$\begin{aligned}
 \int (c + dx)^m (a + a \cosh(e + fx))^2 dx &= (4a^2) \int (c + dx)^m \sin^4\left(\frac{1}{2}(ie + \pi) + \frac{ifx}{2}\right) dx \\
 &= (4a^2) \int \left(\frac{3}{8}(c + dx)^m + \frac{1}{2}(c + dx)^m \cosh(e + fx) + \frac{1}{8}(c + dx)^m \cosh(2e + 2fx)\right) dx \\
 &= \frac{3a^2(c + dx)^{1+m}}{2d(1+m)} + \frac{1}{2}a^2 \int (c + dx)^m \cosh(2e + 2fx) dx + (2a^2) \int (c + dx)^m \cosh(e + fx) dx \\
 &= \frac{3a^2(c + dx)^{1+m}}{2d(1+m)} + \frac{1}{4}a^2 \int e^{-i(2ie+2ifx)}(c + dx)^m dx + \frac{1}{4}a^2 \int e^{i(2ie+2ifx)}(c + dx)^m dx \\
 &= \frac{3a^2(c + dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m}a^2e^{2e-\frac{2cf}{d}}(c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{2f(c+dx)}{d}\right)}{f}
 \end{aligned}$$

**Mathematica [A]** time = 1.09, size = 302, normalized size = 1.15

$$a^2 2^{-m-5} e^{-2\left(\frac{cf}{d}+e\right)} (c + dx)^m (\cosh(e + fx) + 1)^2 \operatorname{sech}^4\left(\frac{1}{2}(e + fx)\right) \left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m} \left(-3f2^{m+2}(c + dx)e^{2\left(\frac{cf}{d}+e\right)}\left(-\frac{f^2(c+dx)^2}{d^2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^m\*(a + a\*Cosh[e + f\*x])^2,x]

[Out] -((2^(-5 - m)\*a^2\*(c + d\*x)^m\*(1 + Cosh[e + f\*x])^2\*(-3\*2^(2 + m)\*E^(2\*(e + (c\*f)/d))\*f\*(c + d\*x)\*(-(f^2\*(c + d\*x)^2)/d^2))^m - d\*E^(4\*e)\*(1 + m)\*(f\*

$$\begin{aligned} & (c/d + x)^m \Gamma[1 + m, (-2*f*(c + d*x))/d] - 2^{(3 + m)} * d * E^{(3*e + (c*f)/d)} * (1 + m) * (f*(c/d + x))^m \Gamma[1 + m, -((f*(c + d*x))/d)] + 2^{(3 + m)} * d * E^{(e + (3*c*f)/d)} * (1 + m) * (-((f*(c + d*x))/d))^m \Gamma[1 + m, (f*(c + d*x))/d] \\ & + d * E^{((4*c*f)/d)} * (1 + m) * (-((f*(c + d*x))/d))^m \Gamma[1 + m, (2*f*(c + d*x))/d] * \operatorname{Sech}[(e + f*x)/2]^4 / (d * E^{(2*(e + (c*f)/d))} * f * (1 + m) * (-((f^2*(c + d*x)^2)/d^2))^m) \end{aligned}$$

**fricas** [A] time = 0.80, size = 493, normalized size = 1.87

$$(a^2 d m + a^2 d) \cosh\left(\frac{d m \log\left(\frac{2f}{d}\right) + 2 d e - 2 c f}{d}\right) \Gamma\left(m + 1, \frac{2(d f x + c f)}{d}\right) + 8 (a^2 d m + a^2 d) \cosh\left(\frac{d m \log\left(\frac{f}{d}\right) + d e - c f}{d}\right) \Gamma(m + 1,$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+a\*cosh(f\*x+e))^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/8*((a^2*d*m + a^2*d)*\cosh((d*m*\log(2*f/d) + 2*d*e - 2*c*f)/d)*\gamma(m + 1, 2*(d*f*x + c*f)/d) + 8*(a^2*d*m + a^2*d)*\cosh((d*m*\log(f/d) + d*e - c*f)/d)*\gamma(m + 1, (d*f*x + c*f)/d) - 8*(a^2*d*m + a^2*d)*\cosh((d*m*\log(-f/d) - d*e + c*f)/d)*\gamma(m + 1, -(d*f*x + c*f)/d) - (a^2*d*m + a^2*d)*\cosh((d*m*\log(-2*f/d) - 2*d*e + 2*c*f)/d)*\gamma(m + 1, -2*(d*f*x + c*f)/d) - (a^2*d*m + a^2*d)*\gamma(m + 1, 2*(d*f*x + c*f)/d)*\sinh((d*m*\log(2*f/d) + 2*d*e - 2*c*f)/d) - 8*(a^2*d*m + a^2*d)*\gamma(m + 1, (d*f*x + c*f)/d)*\sinh((d*m*\log(f/d) + d*e - c*f)/d) + 8*(a^2*d*m + a^2*d)*\gamma(m + 1, -(d*f*x + c*f)/d)*\sinh((d*m*\log(-f/d) - d*e + c*f)/d) + (a^2*d*m + a^2*d)*\gamma(m + 1, -2*(d*f*x + c*f)/d)*\sinh((d*m*\log(-2*f/d) - 2*d*e + 2*c*f)/d) - 12*(a^2*d*f*x + a^2*c*f)*\cosh(m*\log(d*x + c)) - 12*(a^2*d*f*x + a^2*c*f)*\sinh(m*\log(d*x + c)))/(d*f*m + d*f) \end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cosh(fx + e) + a)^2 (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+a\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate((a\*cosh(f\*x + e) + a)^2\*(d\*x + c)^m, x)

**maple** [F] time = 0.32, size = 0, normalized size = 0.00

$$\int (dx + c)^m (a + a \cosh(fx + e))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*(a+a\*cosh(f\*x+e))^2,x)

[Out] int((d\*x+c)^m\*(a+a\*cosh(f\*x+e))^2,x)

**maxima** [A] time = 0.43, size = 209, normalized size = 0.79

$$-\frac{1}{4} \left( \frac{(dx+c)^{m+1} e^{\left(-2e+\frac{2cf}{d}\right)} E_{-m}\left(\frac{2(dx+c)f}{d}\right)}{d} + \frac{(dx+c)^{m+1} e^{\left(2e-\frac{2cf}{d}\right)} E_{-m}\left(-\frac{2(dx+c)f}{d}\right)}{d} - \frac{2(dx+c)^{m+1}}{d(m+1)} \right) a^2 - \left( \frac{(dx+c)^{m+1}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+a\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out] -1/4\*((d\*x + c)^(m + 1)\*e^(-2\*e + 2\*c\*f/d)\*exp\_integral\_e(-m, 2\*(d\*x + c)\*f/d)/d + (d\*x + c)^(m + 1)\*e^(2\*e - 2\*c\*f/d)\*exp\_integral\_e(-m, -2\*(d\*x + c)\*f/d)/d - 2\*(d\*x + c)^(m + 1)/(d\*(m + 1))\*a^2 - ((d\*x + c)^(m + 1)\*e^(-e + c\*f/d)\*exp\_integral\_e(-m, (d\*x + c)\*f/d)/d + (d\*x + c)^(m + 1)\*e^(e - c\*f/d)\*exp\_integral\_e(-m, -(d\*x + c)\*f/d)/d)\*a^2 + (d\*x + c)^(m + 1)\*a^2/(d\*(m + 1))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a \cosh(e + f x))^2 (c + d x)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a\*cosh(e + f\*x))^2\*(c + d\*x)^m,x)

[Out] int((a + a\*cosh(e + f\*x))^2\*(c + d\*x)^m, x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m\*(a+a\*cosh(f\*x+e))\*\*2,x)

[Out] Exception raised: TypeError



### 3.153 $\int (c + dx)^m (a + a \cosh(e + fx)) dx$

**Optimal.** Leaf size=131

$$\frac{ae^{-\frac{cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{f(c+dx)}{d}\right)}{2f} - \frac{ae^{\frac{cf}{d}-e}(c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{f(c+dx)}{d}\right)}{2f} + \frac{a(c+dx)^{m+1}}{d(m+1)}$$

[Out]  $a*(d*x+c)^{(1+m)/d}/(1+m)+1/2*a*\exp(e-c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, -f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)-1/2*a*\exp(-e+c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)$

**Rubi [A]** time = 0.15, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3317, 3307, 2181}

$$\frac{ae^{-\frac{cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{f(c+dx)}{d}\right)}{2f} - \frac{ae^{\frac{cf}{d}-e}(c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, \frac{f(c+dx)}{d}\right)}{2f} + \frac{a(c+dx)^{m+1}}{d(m+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^m*(a + a*\text{Cosh}[e + f*x]), x]$

[Out]  $(a*(c + d*x)^{(1 + m)}/(d*(1 + m)) + (a*E^{(e - (c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, -((f*(c + d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) - (a*E^{(-e + (c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))/d)^m)$

#### Rule 2181

$\text{Int}[(F_)^((g_) * ((e_) + (f_) * (x_))) * ((c_) + (d_) * (x_))^{(m_)}, x\_Symbol]$   
 $\rightarrow -\text{Simp}[(F^{(g*(e - (c*f)/d)}) * (c + d*x)^{\text{FracPart}[m]} * \text{Gamma}[m + 1, (-((f*g*\text{Log}[F])/d)) * (c + d*x)]) / (d * (-((f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1} * (-((f*g*\text{Log}[F]) * (c + d*x))/d))^{\text{FracPart}[m]}], x] /;$  FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rule 3307

$\text{Int}[(c + d*x)^m * \sin[e + \text{Pi} * k + f * x], x\_Symbol]$   
 $\rightarrow \text{Dist}[I/2, \text{Int}[(c + d*x)^m / (E^{(I*k*Pi)} * E^{(I*(e + f*x)}), x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m * E^{(I*k*Pi)} * E^{(I*(e + f*x)}), x], x] /;$  FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

#### Rule 3317

$\text{Int}[(c + d*x)^m * ((a + b * \sin[e + f * x])^n), x\_Symbol]$   
 $\rightarrow \text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, (a + b * \sin[e + f * x])^n], x]$

x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

### Rubi steps

$$\begin{aligned}
 \int (c + dx)^m (a + a \cosh(e + fx)) dx &= \int (a(c + dx)^m + a(c + dx)^m \cosh(e + fx)) dx \\
 &= \frac{a(c + dx)^{1+m}}{d(1+m)} + a \int (c + dx)^m \cosh(e + fx) dx \\
 &= \frac{a(c + dx)^{1+m}}{d(1+m)} + \frac{1}{2}a \int e^{-i(e+ifx)}(c + dx)^m dx + \frac{1}{2}a \int e^{i(e+ifx)}(c + dx)^m dx \\
 &= \frac{a(c + dx)^{1+m}}{d(1+m)} + \frac{ae^{-\frac{cf}{d}}(c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{f(c+dx)}{d}\right) - ae^{-e+\frac{cf}{d}}}{2f}
 \end{aligned}$$

**Mathematica [A]** time = 0.33, size = 189, normalized size = 1.44

$$\frac{ae^{-\frac{cf}{d}-e}(c + dx)^m (\cosh(e + fx) + 1) \operatorname{sech}^2\left(\frac{1}{2}(e + fx)\right) \left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m} \left(-2f(c + dx)e^{\frac{cf}{d}+e} \left(-\frac{f^2(c+dx)^2}{d^2}\right)^m - de^{2e}(m + 1)\right)}{4df(m + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^m\*(a + a\*Cosh[e + f\*x]),x]

[Out] -1/4\*(a\*E^(-e - (c\*f)/d)\*(c + d\*x)^m\*(1 + Cosh[e + f\*x])\*(-2\*E^(e + (c\*f)/d)\*f\*(c + d\*x)\*(-(f^2\*(c + d\*x)^2)/d^2))^m - d\*E^(2\*e)\*(1 + m)\*(f\*(c/d + x))^m\*Gamma[1 + m, -(f\*(c + d\*x))/d] + d\*E^((2\*c\*f)/d)\*(1 + m)\*(-(f\*(c + d\*x))/d))^m\*Gamma[1 + m, (f\*(c + d\*x))/d]\*Sech[(e + f\*x)/2]^2/(d\*f\*(1 + m)\*(-(f^2\*(c + d\*x)^2)/d^2))^m

**fricas [A]** time = 0.59, size = 249, normalized size = 1.90

$$\frac{(adm + ad) \cosh\left(\frac{dm \log\left(\frac{f}{d}\right) + de - cf}{d}\right) \Gamma\left(m + 1, \frac{dfx + cf}{d}\right) - (adm + ad) \cosh\left(\frac{dm \log\left(-\frac{f}{d}\right) - de + cf}{d}\right) \Gamma\left(m + 1, -\frac{dfx + cf}{d}\right) - (adm + ad) \cosh\left(\frac{dm \log\left(\frac{f}{d}\right) + de - cf}{d}\right) \Gamma\left(m + 1, \frac{dfx + cf}{d}\right) - (adm + ad) \cosh\left(\frac{dm \log\left(-\frac{f}{d}\right) - de + cf}{d}\right) \Gamma\left(m + 1, -\frac{dfx + cf}{d}\right)}{4df(m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+a\*cosh(f\*x+e)),x, algorithm="fricas")

[Out]  $-1/2*((a*d*m + a*d)*\cosh((d*m*\log(f/d) + d*e - c*f)/d)*\gamma(m + 1, (d*f*x + c*f)/d) - (a*d*m + a*d)*\cosh((d*m*\log(-f/d) - d*e + c*f)/d)*\gamma(m + 1, -(d*f*x + c*f)/d) - (a*d*m + a*d)*\gamma(m + 1, (d*f*x + c*f)/d)*\sinh((d*m*\log(f/d) + d*e - c*f)/d) + (a*d*m + a*d)*\gamma(m + 1, -(d*f*x + c*f)/d)*\sinh((d*m*\log(-f/d) - d*e + c*f)/d) - 2*(a*d*f*x + a*c*f)*\cosh(m*\log(d*x + c)) - 2*(a*d*f*x + a*c*f)*\sinh(m*\log(d*x + c)))/(d*f*m + d*f)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \cosh(fx + e) + a)(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*(a+a*cosh(f*x+e)),x, algorithm="giac")`

[Out] `integrate((a*cosh(f*x + e) + a)*(d*x + c)^m, x)`

**maple** [F] time = 0.15, size = 0, normalized size = 0.00

$$\int (dx + c)^m (a + a \cosh(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^m*(a+a*cosh(f*x+e)),x)`

[Out] `int((d*x+c)^m*(a+a*cosh(f*x+e)),x)`

**maxima** [A] time = 0.41, size = 100, normalized size = 0.76

$$-\frac{1}{2} \left( \frac{(dx + c)^{m+1} e^{\left(-e + \frac{cf}{d}\right)} E_{-m} \left( \frac{(dx+c)f}{d} \right)}{d} + \frac{(dx + c)^{m+1} e^{\left(e - \frac{cf}{d}\right)} E_{-m} \left( -\frac{(dx+c)f}{d} \right)}{d} \right) a + \frac{(dx + c)^{m+1} a}{d(m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*(a+a*cosh(f*x+e)),x, algorithm="maxima")`

[Out]  $-1/2*((d*x + c)^{(m + 1)}*e^{(-e + c*f/d)}*\exp\_integral\_e(-m, (d*x + c)*f/d)/d + (d*x + c)^{(m + 1)}*e^{(e - c*f/d)}*\exp\_integral\_e(-m, -(d*x + c)*f/d)/d)*a + (d*x + c)^{(m + 1)}*a/(d*(m + 1))$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a \cosh(e + fx)) (c + dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))*(c + d*x)^m,x)
```

```
[Out] int((a + a*cosh(e + f*x))*(c + d*x)^m, x)
```

```
sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*cosh(f*x+e)),x)
```

```
[Out] Exception raised: TypeError
```

$$3.154 \quad \int \frac{(c+dx)^m}{a+a \cosh(e+fx)} dx$$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(c+dx)^m}{a \cosh(e+fx)+a}, x\right)$$

[Out] Unintegrable((d\*x+c)^m/(a+a\*cosh(f\*x+e)), x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(c+dx)^m}{a+a \cosh(e+fx)} dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m/(a + a\*Cosh[e + f\*x]), x]

[Out] Defer[Int] [(c + d\*x)^m/(a + a\*Cosh[e + f\*x]), x]

Rubi steps

$$\int \frac{(c+dx)^m}{a+a \cosh(e+fx)} dx = \int \frac{(c+dx)^m}{a+a \cosh(e+fx)} dx$$

Mathematica [A] time = 5.25, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{a+a \cosh(e+fx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m/(a + a\*Cosh[e + f\*x]), x]

[Out] Integrate[(c + d\*x)^m/(a + a\*Cosh[e + f\*x]), x]

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx+c)^m}{a \cosh(fx+e)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+a\*cosh(f\*x+e)),x, algorithm="fricas")

[Out] integral((d\*x + c)^m/(a\*cosh(f\*x + e) + a), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{a \cosh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+a\*cosh(f\*x+e)),x, algorithm="giac")

[Out] integrate((d\*x + c)^m/(a\*cosh(f\*x + e) + a), x)

**maple** [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{a + a \cosh(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m/(a+a\*cosh(f\*x+e)),x)

[Out] int((d\*x+c)^m/(a+a\*cosh(f\*x+e)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{a \cosh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+a\*cosh(f\*x+e)),x, algorithm="maxima")

[Out] integrate((d\*x + c)^m/(a\*cosh(f\*x + e) + a), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c + dx)^m}{a + a \cosh(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^m/(a + a\*cosh(e + f\*x)),x)

[Out] int((c + d\*x)^m/(a + a\*cosh(e + f\*x)), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(c+dx)^m}{\cosh(e+fx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m/(a+a\*cosh(f\*x+e)),x)

[Out] Integral((c + d\*x)\*\*m/(cosh(e + f\*x) + 1), x)/a

$$3.155 \quad \int \frac{(c+dx)^m}{(a+a \cosh(e+fx))^2} dx$$

Optimal. Leaf size=23

$$\text{Int} \left( \frac{(c+dx)^m}{(a \cosh(e+fx) + a)^2}, x \right)$$

[Out] Unintegrable((d\*x+c)^m/(a+a\*cosh(f\*x+e))^2,x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(c+dx)^m}{(a+a \cosh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m/(a + a\*Cosh[e + f\*x])^2,x]

[Out] Defer[Int] [(c + d\*x)^m/(a + a\*Cosh[e + f\*x])^2, x]

Rubi steps

$$\int \frac{(c+dx)^m}{(a+a \cosh(e+fx))^2} dx = \int \frac{(c+dx)^m}{(a+a \cosh(e+fx))^2} dx$$

Mathematica [A] time = 9.87, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{(a+a \cosh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m/(a + a\*Cosh[e + f\*x])^2,x]

[Out] Integrate[(c + d\*x)^m/(a + a\*Cosh[e + f\*x])^2, x]

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(dx+c)^m}{a^2 \cosh^2(fx+e) + 2a^2 \cosh(fx+e) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((d\*x+c)^m/(a+a\*cosh(f\*x+e))^2,x, algorithm="fricas")

[Out] integral((d\*x + c)^m/(a^2\*cosh(f\*x + e)^2 + 2\*a^2\*cosh(f\*x + e) + a^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{(a \cosh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+a\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^m/(a\*cosh(f\*x + e) + a)^2, x)

**maple** [A] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{(a + a \cosh(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m/(a+a\*cosh(f\*x+e))^2,x)

[Out] int((d\*x+c)^m/(a+a\*cosh(f\*x+e))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{(a \cosh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+a\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m/(a\*cosh(f\*x + e) + a)^2, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c + dx)^m}{(a + a \cosh(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^m/(a + a\*cosh(e + f\*x))^2,x)

[Out] `int((c + d*x)^m/(a + a*cosh(e + f*x))^2, x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(c+dx)^m}{\cosh^2(e+fx)+2\cosh(e+fx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)**m/(a+a*cosh(f*x+e))**2,x)`

[Out] `Integral((c + d*x)**m/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x)/a**2`

### 3.156 $\int (c + dx)^3 (a + b \cosh(e + fx)) dx$

**Optimal.** Leaf size=89

$$\frac{a(c + dx)^4}{4d} + \frac{6bd^2(c + dx) \sinh(e + fx)}{f^3} - \frac{3bd(c + dx)^2 \cosh(e + fx)}{f^2} + \frac{b(c + dx)^3 \sinh(e + fx)}{f} - \frac{6bd^3 \cosh(e + fx)}{f^4}$$

[Out]  $1/4*a*(d*x+c)^4/d-6*b*d^3*\cosh(f*x+e)/f^4-3*b*d*(d*x+c)^2*\cosh(f*x+e)/f^2+6*b*d^2*(d*x+c)*\sinh(f*x+e)/f^3+b*(d*x+c)^3*\sinh(f*x+e)/f$

**Rubi [A]** time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3317, 3296, 2638}

$$\frac{a(c + dx)^4}{4d} + \frac{6bd^2(c + dx) \sinh(e + fx)}{f^3} - \frac{3bd(c + dx)^2 \cosh(e + fx)}{f^2} + \frac{b(c + dx)^3 \sinh(e + fx)}{f} - \frac{6bd^3 \cosh(e + fx)}{f^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^3*(a + b*\text{Cosh}[e + f*x]), x]$

[Out]  $(a*(c + d*x)^4)/(4*d) - (6*b*d^3*\text{Cosh}[e + f*x])/f^4 - (3*b*d*(c + d*x)^2*\text{Cosh}[e + f*x])/f^2 + (6*b*d^2*(c + d*x)*\text{Sinh}[e + f*x])/f^3 + (b*(c + d*x)^3*\text{Sinh}[e + f*x])/f$

#### Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

#### Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

#### Rule 3317

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, (a + b*\text{Sin}[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ (\text{EqQ}[n, 1] \ || \ \text{IGtQ}[m, 0] \ || \ \text{NeQ}[a^2 - b^2, 0])$

#### Rubi steps

$$\begin{aligned}
\int (c + dx)^3 (a + b \cosh(e + fx)) dx &= \int (a(c + dx)^3 + b(c + dx)^3 \cosh(e + fx)) dx \\
&= \frac{a(c + dx)^4}{4d} + b \int (c + dx)^3 \cosh(e + fx) dx \\
&= \frac{a(c + dx)^4}{4d} + \frac{b(c + dx)^3 \sinh(e + fx)}{f} - \frac{(3bd) \int (c + dx)^2 \sinh(e + fx) dx}{f} \\
&= \frac{a(c + dx)^4}{4d} - \frac{3bd(c + dx)^2 \cosh(e + fx)}{f^2} + \frac{b(c + dx)^3 \sinh(e + fx)}{f} + \frac{(6bd^2)}{f^3} \\
&= \frac{a(c + dx)^4}{4d} - \frac{3bd(c + dx)^2 \cosh(e + fx)}{f^2} + \frac{6bd^2(c + dx) \sinh(e + fx)}{f^3} + \frac{b(c + dx)^3}{f^3} \\
&= \frac{a(c + dx)^4}{4d} - \frac{6bd^3 \cosh(e + fx)}{f^4} - \frac{3bd(c + dx)^2 \cosh(e + fx)}{f^2} + \frac{6bd^2(c + dx) \sinh(e + fx)}{f^3} + \frac{b(c + dx)^3}{f^3}
\end{aligned}$$

**Mathematica [A]** time = 0.51, size = 123, normalized size = 1.38

$$\frac{1}{4}ax(4c^3 + 6c^2dx + 4cd^2x^2 + d^3x^3) - \frac{3bd(c^2f^2 + 2cdf^2x + d^2(f^2x^2 + 2)) \cosh(e + fx)}{f^4} + \frac{b(c + dx)(c^2f^2 + 2cdf^2x + d^2(f^2x^2 + 2)) \sinh(e + fx)}{f^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3\*(a + b\*Cosh[e + f\*x]), x]

[Out] (a\*x\*(4\*c^3 + 6\*c^2\*d\*x + 4\*c\*d^2\*x^2 + d^3\*x^3))/4 - (3\*b\*d\*(c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(2 + f^2\*x^2))\*Cosh[e + f\*x])/f^4 + (b\*(c + d\*x)\*(c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(6 + f^2\*x^2))\*Sinh[e + f\*x])/f^3

**fricas [A]** time = 0.58, size = 168, normalized size = 1.89

$$\frac{ad^3f^4x^4 + 4acd^2f^4x^3 + 6ac^2df^4x^2 + 4ac^3f^4x - 12(bd^3f^2x^2 + 2bcd^2f^2x + bc^2df^2 + 2bd^3) \cosh(fx + e) + 4(bd^3f^2x^2 + 2bcd^2f^2x + bc^2df^2 + 2bd^3) \sinh(fx + e)}{4f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+b\*cosh(f\*x+e)), x, algorithm="fricas")

[Out] 1/4\*(a\*d^3\*f^4\*x^4 + 4\*a\*c\*d^2\*f^4\*x^3 + 6\*a\*c^2\*d\*f^4\*x^2 + 4\*a\*c^3\*f^4\*x - 12\*(b\*d^3\*f^2\*x^2 + 2\*b\*c\*d^2\*f^2\*x + b\*c^2\*d\*f^2 + 2\*b\*d^3)\*cosh(f\*x + e) + 4\*(b\*d^3\*f^3\*x^3 + 3\*b\*c\*d^2\*f^3\*x^2 + b\*c^3\*f^3 + 6\*b\*c\*d^2\*f + 3\*(b\*c^2\*d\*f^3 + 2\*b\*d^3\*f)\*x)\*sinh(f\*x + e))/f^4

**giac** [B] time = 0.12, size = 260, normalized size = 2.92

$$\frac{1}{4} ad^3 x^4 + acd^2 x^3 + \frac{3}{2} ac^2 dx^2 + ac^3 x + \frac{(bd^3 f^3 x^3 + 3bcd^2 f^3 x^2 + 3bc^2 d f^3 x - 3bd^3 f^2 x^2 + bc^3 f^3 - 6bcd^2 f^2 x - 3bc^2 d f^2 x^2 + 3bd^3 f^2 x^2 + bc^3 f^3 - 6bcd^2 f^2 x - 3bc^2 d f^2 x^2)}{2f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+b\*cosh(f\*x+e)),x, algorithm="giac")

[Out] 1/4\*a\*d^3\*x^4 + a\*c\*d^2\*x^3 + 3/2\*a\*c^2\*d\*x^2 + a\*c^3\*x + 1/2\*(b\*d^3\*f^3\*x^3 + 3\*b\*c\*d^2\*f^3\*x^2 + 3\*b\*c^2\*d\*f^3\*x - 3\*b\*d^3\*f^2\*x^2 + b\*c^3\*f^3 - 6\*b\*c\*d^2\*f^2\*x - 3\*b\*c^2\*d\*f^2 + 6\*b\*d^3\*f\*x + 6\*b\*c\*d^2\*f - 6\*b\*d^3)\*e^(f\*x + e)/f^4 - 1/2\*(b\*d^3\*f^3\*x^3 + 3\*b\*c\*d^2\*f^3\*x^2 + 3\*b\*c^2\*d\*f^3\*x + 3\*b\*d^3\*f^2\*x^2 + b\*c^3\*f^3 + 6\*b\*c\*d^2\*f^2\*x + 3\*b\*c^2\*d\*f^2 + 6\*b\*d^3\*f\*x + 6\*b\*c\*d^2\*f + 6\*b\*d^3)\*e^(-f\*x - e)/f^4

**maple** [B] time = 0.07, size = 482, normalized size = 5.42

$$\frac{d^3 a (f x + e)^4}{4 f^3} + \frac{d^3 b \left( (f x + e)^3 \sinh(f x + e) - 3 (f x + e)^2 \cosh(f x + e) + 6 (f x + e) \sinh(f x + e) - 6 \cosh(f x + e) \right)}{f^3} - \frac{d^3 e a (f x + e)^3}{f^3} - \frac{3 d^3 e b \left( (f x + e)^2 \sinh(f x + e) - 2 (f x + e) \cosh(f x + e) + 2 \sinh(f x + e) \right)}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3\*(a+b\*cosh(f\*x+e)),x)

[Out] 1/f\*(1/4/f^3\*d^3\*a\*(f\*x+e)^4+1/f^3\*d^3\*b\*((f\*x+e)^3\*sinh(f\*x+e)-3\*(f\*x+e)^2\*cosh(f\*x+e)+6\*(f\*x+e)\*sinh(f\*x+e)-6\*cosh(f\*x+e))-1/f^3\*d^3\*e\*a\*(f\*x+e)^3-3/f^3\*d^3\*e\*b\*((f\*x+e)^2\*sinh(f\*x+e)-2\*(f\*x+e)\*cosh(f\*x+e)+2\*sinh(f\*x+e))+1/f^2\*d^2\*c\*a\*(f\*x+e)^3+3/f^2\*d^2\*c\*b\*((f\*x+e)^2\*sinh(f\*x+e)-2\*(f\*x+e)\*cosh(f\*x+e)+2\*sinh(f\*x+e))+3/2/f^3\*d^3\*e^2\*a\*(f\*x+e)^2+3/f^3\*d^3\*e^2\*b\*((f\*x+e)\*sinh(f\*x+e)-cosh(f\*x+e))-3/f^2\*d^2\*e\*c\*a\*(f\*x+e)^2-6/f^2\*d^2\*e\*c\*b\*((f\*x+e)\*sinh(f\*x+e)-cosh(f\*x+e))+3/2/f\*d\*c^2\*a\*(f\*x+e)^2+3/f\*d\*c^2\*b\*((f\*x+e)\*sinh(f\*x+e)-cosh(f\*x+e))-d^3\*e^3/f^3\*a\*(f\*x+e)-d^3\*e^3/f^3\*b\*sinh(f\*x+e)+3\*d^2\*e^2/f^2\*c\*a\*(f\*x+e)+3\*d^2\*e^2/f^2\*c\*b\*sinh(f\*x+e)-3\*d\*e/f\*c^2\*a\*(f\*x+e)-3\*d\*e/f\*c^2\*b\*sinh(f\*x+e)+c^3\*a\*(f\*x+e)+b\*c^3\*sinh(f\*x+e))

**maxima** [B] time = 0.35, size = 237, normalized size = 2.66

$$\frac{1}{4} ad^3 x^4 + acd^2 x^3 + \frac{3}{2} ac^2 dx^2 + ac^3 x + \frac{3}{2} bcd^2 d \left( \frac{(f x e^e - e^e) e^{(f x)}}{f^2} - \frac{(f x + 1) e^{(-f x - e)}}{f^2} \right) + \frac{3}{2} bcd^2 \left( \frac{(f^2 x^2 e^e - 2 f x e^e + 2 e^e)}{f^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+b\*cosh(f\*x+e)),x, algorithm="maxima")

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3/2*b*c^2*d*((f*x
*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-f*x - e)/f^2) + 3/2*b*c*d^2*((f^2*x
^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e
)/f^3) + 1/2*b*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6*f*x*e^e - 6*e^e)*e^(f*
x)/f^4 - (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x - e)/f^4) + b*c^3*sinh(f
*x + e)/f
```

**mupad [B]** time = 0.99, size = 187, normalized size = 2.10

$$\frac{\sinh(e + fx) (bc^3 f^2 + 6bcd^2)}{f^3} - \frac{3 \cosh(e + fx) (bc^2 d f^2 + 2bd^3)}{f^4} + \frac{ad^3 x^4}{4} + ac^3 x + \frac{3x \sinh(e + fx) (bc^2 d)}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))*(c + d*x)^3,x)
```

```
[Out] (sinh(e + f*x)*(b*c^3*f^2 + 6*b*c*d^2))/f^3 - (3*cosh(e + f*x)*(2*b*d^3 + b
*c^2*d*f^2))/f^4 + (a*d^3*x^4)/4 + a*c^3*x + (3*x*sinh(e + f*x)*(2*b*d^3 +
b*c^2*d*f^2))/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 - (3*b*d^3*x^2*cosh(e +
f*x))/f^2 + (b*d^3*x^3*sinh(e + f*x))/f - (6*b*c*d^2*x*cosh(e + f*x))/f^2
+ (3*b*c*d^2*x^2*sinh(e + f*x))/f
```

**sympy [A]** time = 1.41, size = 264, normalized size = 2.97

$$\left\{ \begin{array}{l} ac^3x + \frac{3ac^2dx^2}{2} + acd^2x^3 + \frac{ad^3x^4}{4} + \frac{bc^3 \sinh(e+fx)}{f} + \frac{3bc^2dx \sinh(e+fx)}{f} - \frac{3bc^2d \cosh(e+fx)}{f^2} + \frac{3bcd^2x^2 \sinh(e+fx)}{f} - \frac{6bcd^2x \cosh(e+fx)}{f^2} \\ (a + b \cosh(e)) \left( c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*cosh(f*x+e)),x)
```

```
[Out] Piecewise((a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 + b
*c**3*sinh(e + f*x)/f + 3*b*c**2*d*x*sinh(e + f*x)/f - 3*b*c**2*d*cosh(e +
f*x)/f**2 + 3*b*c*d**2*x**2*sinh(e + f*x)/f - 6*b*c*d**2*x*cosh(e + f*x)/f*
*2 + 6*b*c*d**2*sinh(e + f*x)/f**3 + b*d**3*x**3*sinh(e + f*x)/f - 3*b*d**3
*x**2*cosh(e + f*x)/f**2 + 6*b*d**3*x*sinh(e + f*x)/f**3 - 6*b*d**3*cosh(e
+ f*x)/f**4, Ne(f, 0)), ((a + b*cosh(e))*(c**3*x + 3*c**2*d*x**2/2 + c*d**2
*x**3 + d**3*x**4/4), True))
```

### 3.157 $\int (c + dx)^2 (a + b \cosh(e + fx)) dx$

**Optimal.** Leaf size=67

$$\frac{a(c + dx)^3}{3d} - \frac{2bd(c + dx) \cosh(e + fx)}{f^2} + \frac{b(c + dx)^2 \sinh(e + fx)}{f} + \frac{2bd^2 \sinh(e + fx)}{f^3}$$

[Out]  $1/3*a*(d*x+c)^3/d-2*b*d*(d*x+c)*\cosh(f*x+e)/f^2+2*b*d^2*\sinh(f*x+e)/f^3+b*(d*x+c)^2*\sinh(f*x+e)/f$

**Rubi [A]** time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3317, 3296, 2637}

$$\frac{a(c + dx)^3}{3d} - \frac{2bd(c + dx) \cosh(e + fx)}{f^2} + \frac{b(c + dx)^2 \sinh(e + fx)}{f} + \frac{2bd^2 \sinh(e + fx)}{f^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^2*(a + b*\text{Cosh}[e + f*x]), x]$

[Out]  $(a*(c + d*x)^3)/(3*d) - (2*b*d*(c + d*x)*\text{Cosh}[e + f*x])/f^2 + (2*b*d^2*\text{Sinh}[e + f*x])/f^3 + (b*(c + d*x)^2*\text{Sinh}[e + f*x])/f$

#### Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$   
FreeQ[{c, d}, x]

#### Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$  FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3317

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, (a + b*\sin[e + f*x])^n, x], x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

#### Rubi steps

$$\begin{aligned}
\int (c + dx)^2 (a + b \cosh(e + fx)) dx &= \int (a(c + dx)^2 + b(c + dx)^2 \cosh(e + fx)) dx \\
&= \frac{a(c + dx)^3}{3d} + b \int (c + dx)^2 \cosh(e + fx) dx \\
&= \frac{a(c + dx)^3}{3d} + \frac{b(c + dx)^2 \sinh(e + fx)}{f} - \frac{(2bd) \int (c + dx) \sinh(e + fx) dx}{f} \\
&= \frac{a(c + dx)^3}{3d} - \frac{2bd(c + dx) \cosh(e + fx)}{f^2} + \frac{b(c + dx)^2 \sinh(e + fx)}{f} + \frac{(2bd^2)}{f^3} \\
&= \frac{a(c + dx)^3}{3d} - \frac{2bd(c + dx) \cosh(e + fx)}{f^2} + \frac{2bd^2 \sinh(e + fx)}{f^3} + \frac{b(c + dx)^2 \sinh(e + fx)}{f}
\end{aligned}$$

**Mathematica [A]** time = 0.35, size = 83, normalized size = 1.24

$$\frac{1}{3}ax(3c^2 + 3cdx + d^2x^2) + \frac{b(c^2f^2 + 2cdf^2x + d^2(f^2x^2 + 2)) \sinh(e + fx)}{f^3} - \frac{2bd(c + dx) \cosh(e + fx)}{f^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2\*(a + b\*Cosh[e + f\*x]),x]

[Out] (a\*x\*(3\*c^2 + 3\*c\*d\*x + d^2\*x^2))/3 - (2\*b\*d\*(c + d\*x)\*Cosh[e + f\*x])/f^2 + (b\*(c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(2 + f^2\*x^2))\*Sinh[e + f\*x])/f^3

**fricas [A]** time = 0.59, size = 102, normalized size = 1.52

$$\frac{ad^2f^3x^3 + 3acdf^3x^2 + 3ac^2f^3x - 6(bd^2fx + bcdf) \cosh(fx + e) + 3(bd^2f^2x^2 + 2bcd f^2x + bc^2f^2 + 2bd^2) \sinh(fx + e)}{3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+b\*cosh(f\*x+e)),x, algorithm="fricas")

[Out] 1/3\*(a\*d^2\*f^3\*x^3 + 3\*a\*c\*d\*f^3\*x^2 + 3\*a\*c^2\*f^3\*x - 6\*(b\*d^2\*f\*x + b\*c\*d\*f)\*cosh(f\*x + e) + 3\*(b\*d^2\*f^2\*x^2 + 2\*b\*c\*d\*f^2\*x + b\*c^2\*f^2 + 2\*b\*d^2)\*sinh(f\*x + e))/f^3

**giac [B]** time = 0.14, size = 148, normalized size = 2.21

$$\frac{1}{3}ad^2x^3 + acdx^2 + ac^2x + \frac{(bd^2f^2x^2 + 2bcd f^2x + bc^2f^2 - 2bd^2fx - 2bcd f + 2bd^2)e^{(fx+e)}}{2f^3} - \frac{(bd^2f^2x^2 + 2bcd f^2x + bc^2f^2 + 2bd^2) \sinh(fx + e)}{f^3}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+b\*cosh(f\*x+e)),x, algorithm="giac")

[Out]  $\frac{1}{3}a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + \frac{1}{2}*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2 - 2*b*d^2*f*x - 2*b*c*d*f + 2*b*d^2)*e^{(f*x + e)}/f^3 - \frac{1}{2}*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2 + 2*b*d^2*f*x + 2*b*c*d*f + 2*b*d^2)*e^{(-f*x - e)}/f^3$

**maple [B]** time = 0.06, size = 240, normalized size = 3.58

$$\frac{d^2 a (f x + e)^3}{3 f^2} + \frac{d^2 b \left( (f x + e)^2 \sinh(f x + e) - 2(f x + e) \cosh(f x + e) + 2 \sinh(f x + e) \right)}{f^2} - \frac{d^2 e a (f x + e)^2}{f^2} - \frac{2 d^2 e b \left( (f x + e) \sinh(f x + e) - \cosh(f x + e) \right)}{f^2} + \frac{d c a (f x + e)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*(a+b\*cosh(f\*x+e)),x)

[Out]  $\frac{1}{f} * \left( \frac{1}{3} / f^2 * d^2 * a * (f * x + e)^3 + \frac{1}{f^2} * d^2 * b * \left( (f * x + e)^2 * \sinh(f * x + e) - 2 * (f * x + e) * \cosh(f * x + e) + 2 * \sinh(f * x + e) \right) - \frac{1}{f^2} * d^2 * e * a * (f * x + e)^2 - \frac{2}{f^2} * d^2 * e * b * \left( (f * x + e) * \sinh(f * x + e) - \cosh(f * x + e) \right) + \frac{1}{f} * d * c * a * (f * x + e)^2 + \frac{2}{f} * d * c * b * \left( (f * x + e) * \sinh(f * x + e) - \cosh(f * x + e) \right) + d^2 * e^2 / f^2 * a * (f * x + e) + d^2 * e^2 / f^2 * b * \sinh(f * x + e) - 2 * d * e / f * c * a * (f * x + e) - 2 * d * e / f * c * b * \sinh(f * x + e) + a * c^2 * (f * x + e) + b * c^2 * \sinh(f * x + e) \right)$

**maxima [B]** time = 0.33, size = 141, normalized size = 2.10

$$\frac{1}{3} a d^2 x^3 + a c d x^2 + a c^2 x + b c d \left( \frac{(f x e^e - e^e) e^{(f x)}}{f^2} - \frac{(f x + 1) e^{-(f x - e)}}{f^2} \right) + \frac{1}{2} b d^2 \left( \frac{(f^2 x^2 e^e - 2 f x e^e + 2 e^e) e^{(f x)}}{f^3} - \frac{(f^2 x^2 + 2 f x + 2) e^{-(f x - e)}}{f^3} \right) + b c^2 \sinh(f x + e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+b\*cosh(f\*x+e)),x, algorithm="maxima")

[Out]  $\frac{1}{3}a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + b*c*d*((f*x*e^e - e^e)*e^{(f*x)}/f^2 - (f*x + 1)*e^{-(f*x - e)}/f^2) + \frac{1}{2}b*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^{(f*x)}/f^3 - (f^2*x^2 + 2*f*x + 2)*e^{-(f*x - e)}/f^3) + b*c^2*\sinh(f*x + e)/f$

**mupad [B]** time = 0.95, size = 110, normalized size = 1.64

$$\frac{a d^2 x^3}{3} + \frac{\sinh(e + f x) (b c^2 f^2 + 2 b d^2)}{f^3} + a c^2 x + a c d x^2 - \frac{2 b d^2 x \cosh(e + f x)}{f^2} + \frac{b d^2 x^2 \sinh(e + f x)}{f} - \frac{2 b c d (f x + 1) e^{-(f x - e)}}{f^2} + \frac{1}{2} b d^2 \left( \frac{(f^2 x^2 e^e - 2 f x e^e + 2 e^e) e^{(f x)}}{f^3} - \frac{(f^2 x^2 + 2 f x + 2) e^{-(f x - e)}}{f^3} \right) + b c^2 \sinh(f x + e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*cosh(e + f*x))*(c + d*x)^2,x)`

[Out]  $(a*d^2*x^3)/3 + (\sinh(e + f*x)*(2*b*d^2 + b*c^2*f^2))/f^3 + a*c^2*x + a*c*d*x^2 - (2*b*d^2*x*cosh(e + f*x))/f^2 + (b*d^2*x^2*sinh(e + f*x))/f - (2*b*c*d*cosh(e + f*x))/f^2 + (2*b*c*d*x*sinh(e + f*x))/f$

sympy [A] time = 0.64, size = 151, normalized size = 2.25

$$\left\{ \begin{array}{l} ac^2x + acdx^2 + \frac{ad^2x^3}{3} + \frac{bc^2 \sinh(e+fx)}{f} + \frac{2bcdx \sinh(e+fx)}{f} - \frac{2bcd \cosh(e+fx)}{f^2} + \frac{bd^2x^2 \sinh(e+fx)}{f} - \frac{2bd^2x \cosh(e+fx)}{f^2} + \frac{2bd^2 \sinh(e+fx)}{f^3} \\ (a + b \cosh(e)) \left( c^2x + cdx^2 + \frac{d^2x^3}{3} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)**2*(a+b*cosh(f*x+e)),x)`

[Out] `Piecewise((a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 + b*c**2*sinh(e + f*x)/f + 2*b*c*d*x*sinh(e + f*x)/f - 2*b*c*d*cosh(e + f*x)/f**2 + b*d**2*x**2*sinh(e + f*x)/f - 2*b*d**2*x*cosh(e + f*x)/f**2 + 2*b*d**2*sinh(e + f*x)/f**3, N e(f, 0)), ((a + b*cosh(e))*(c**2*x + c*d*x**2 + d**2*x**3/3), True))`

### 3.158 $\int (c + dx)(a + b \cosh(e + fx)) dx$

Optimal. Leaf size=45

$$\frac{a(c + dx)^2}{2d} + \frac{b(c + dx) \sinh(e + fx)}{f} - \frac{bd \cosh(e + fx)}{f^2}$$

[Out]  $1/2*a*(d*x+c)^2/d-b*d*\cosh(f*x+e)/f^2+b*(d*x+c)*\sinh(f*x+e)/f$

**Rubi** [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {3317, 3296, 2638}

$$\frac{a(c + dx)^2}{2d} + \frac{b(c + dx) \sinh(e + fx)}{f} - \frac{bd \cosh(e + fx)}{f^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)\*(a + b\*Cosh[e + f\*x]),x]

[Out]  $(a*(c + d*x)^2)/(2*d) - (b*d*Cosh[e + f*x])/f^2 + (b*(c + d*x)*Sinh[e + f*x])/f$

#### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] := -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3317

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Sinh[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

#### Rubi steps

$$\begin{aligned}
\int (c + dx)(a + b \cosh(e + fx)) dx &= \int (a(c + dx) + b(c + dx) \cosh(e + fx)) dx \\
&= \frac{a(c + dx)^2}{2d} + b \int (c + dx) \cosh(e + fx) dx \\
&= \frac{a(c + dx)^2}{2d} + \frac{b(c + dx) \sinh(e + fx)}{f} - \frac{(bd) \int \sinh(e + fx) dx}{f} \\
&= \frac{a(c + dx)^2}{2d} - \frac{bd \cosh(e + fx)}{f^2} + \frac{b(c + dx) \sinh(e + fx)}{f}
\end{aligned}$$

**Mathematica** [A] time = 0.12, size = 46, normalized size = 1.02

$$\frac{f(afx(2c + dx) + 2b(c + dx) \sinh(e + fx)) - 2bd \cosh(e + fx)}{2f^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)\*(a + b\*Cosh[e + f\*x]),x]

[Out] (-2\*b\*d\*Cosh[e + f\*x] + f\*(a\*f\*x\*(2\*c + d\*x) + 2\*b\*(c + d\*x)\*Sinh[e + f\*x])/(2\*f^2)

**fricas** [A] time = 0.77, size = 51, normalized size = 1.13

$$\frac{adf^2x^2 + 2acf^2x - 2bd \cosh(fx + e) + 2(bdfx + bcf) \sinh(fx + e)}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+b\*cosh(f\*x+e)),x, algorithm="fricas")

[Out] 1/2\*(a\*d\*f^2\*x^2 + 2\*a\*c\*f^2\*x - 2\*b\*d\*cosh(f\*x + e) + 2\*(b\*d\*f\*x + b\*c\*f)\*sinh(f\*x + e))/f^2

**giac** [A] time = 0.12, size = 66, normalized size = 1.47

$$\frac{1}{2} adx^2 + acx + \frac{(bdfx + bcf - bd)e^{(fx+e)}}{2f^2} - \frac{(bdfx + bcf + bd)e^{(-fx-e)}}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+b\*cosh(f\*x+e)),x, algorithm="giac")

[Out]  $\frac{1}{2}a dx^2 + acx + \frac{1}{2}(bdfx + bcf - bd)e^{(fx + e)}/f^2 - \frac{1}{2}(bdfx + bcf + bd)e^{-(fx - e)}/f^2$

**maple** [B] time = 0.05, size = 91, normalized size = 2.02

$$\frac{\frac{da(fx+e)^2}{2f} + \frac{db((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f} - \frac{dea(fx+e)}{f} - \frac{deb\sinh(fx+e)}{f} + ca(fx+e) + cb\sinh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)*(a+b*cosh(f*x+e)),x)`

[Out]  $\frac{1}{f} * (\frac{1}{2} / f * d * a * (f * x + e)^2 + \frac{1}{f} * d * b * ((f * x + e) * \sinh(f * x + e) - \cosh(f * x + e)) - d * e / f * a * (f * x + e) - d * e / f * b * \sinh(f * x + e) + c * a * (f * x + e) + c * b * \sinh(f * x + e))$

**maxima** [A] time = 0.35, size = 66, normalized size = 1.47

$$\frac{1}{2} adx^2 + acx + \frac{1}{2} bd \left( \frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{-(fx-e)}}{f^2} \right) + \frac{bc \sinh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*(a+b*cosh(f*x+e)),x, algorithm="maxima")`

[Out]  $\frac{1}{2}a dx^2 + acx + \frac{1}{2}b * d * ((f * x * e^e - e^e) * e^{(f * x)} / f^2 - (f * x + 1) * e^{-(f * x - e)} / f^2) + b * c * \sinh(f * x + e) / f$

**mupad** [B] time = 0.08, size = 49, normalized size = 1.09

$$\frac{f(bc \sinh(e + fx) + bdx \sinh(e + fx)) - bd \cosh(e + fx)}{f^2} + acx + \frac{adx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*cosh(e + f*x))*(c + d*x),x)`

[Out]  $(f * (b * c * \sinh(e + f * x) + b * d * x * \sinh(e + f * x)) - b * d * \cosh(e + f * x)) / f^2 + a * c * x + (a * d * x^2) / 2$

**sympy** [A] time = 0.29, size = 68, normalized size = 1.51

$$\begin{cases} acx + \frac{adx^2}{2} + \frac{bc \sinh(e+fx)}{f} + \frac{bdx \sinh(e+fx)}{f} - \frac{bd \cosh(e+fx)}{f^2} & \text{for } f \neq 0 \\ (a + b \cosh(e)) \left( cx + \frac{dx^2}{2} \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x + a*d*x**2/2 + b*c*sinh(e + f*x)/f + b*d*x*sinh(e + f*x)/f  
- b*d*cosh(e + f*x)/f**2, Ne(f, 0)), ((a + b*cosh(e))*(c*x + d*x**2/2), Tr  
ue))
```

$$3.159 \quad \int \frac{a+b \cosh(e+fx)}{c+dx} dx$$

**Optimal.** Leaf size=64

$$\frac{a \log(c+dx)}{d} + \frac{b \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d} + \frac{b \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d}$$

[Out]  $b \operatorname{Chi}(c*f/d+f*x) * \cosh(-e+c*f/d)/d + a * \ln(d*x+c)/d - b * \operatorname{Shi}(c*f/d+f*x) * \sinh(-e+c*f/d)/d$

**Rubi [A]** time = 0.12, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3317, 3303, 3298, 3301}

$$\frac{a \log(c+dx)}{d} + \frac{b \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d} + \frac{b \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b * \operatorname{Cosh}[e + f * x]) / (c + d * x), x]$

[Out]  $(b * \operatorname{Cosh}[e - (c * f) / d] * \operatorname{CoshIntegral}[(c * f) / d + f * x]) / d + (a * \operatorname{Log}[c + d * x]) / d + (b * \operatorname{Sinh}[e - (c * f) / d] * \operatorname{SinhIntegral}[(c * f) / d + f * x]) / d$

#### Rule 3298

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_]) * (f_.) * (x_.)] / ((c_.) + (d_.) * (x_.)), x\_Symbol]$   $\rightarrow \operatorname{Simp}[(I * \operatorname{SinhIntegral}[(c * f * fz) / d + f * fz * x]) / d, x] /;$   $\operatorname{FreeQ}\{c, d, e, f, fz\}, x]$  &&  $\operatorname{EqQ}[d * e - c * f * fz * I, 0]$

#### Rule 3301

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_]) * (f_.) * (x_.)] / ((c_.) + (d_.) * (x_.)), x\_Symbol]$   $\rightarrow \operatorname{Simp}[\operatorname{CoshIntegral}[(c * f * fz) / d + f * fz * x] / d, x] /;$   $\operatorname{FreeQ}\{c, d, e, f, fz\}, x]$  &&  $\operatorname{EqQ}[d * (e - \operatorname{Pi} / 2) - c * f * fz * I, 0]$

#### Rule 3303

$\operatorname{Int}[\sin[(e_.) + (f_.) * (x_.)] / ((c_.) + (d_.) * (x_.)), x\_Symbol]$   $\rightarrow \operatorname{Dist}[\operatorname{Cos}[(d * e - c * f) / d], \operatorname{Int}[\operatorname{Sin}[(c * f) / d + f * x] / (c + d * x), x], x] + \operatorname{Dist}[\operatorname{Sin}[(d * e - c * f) / d], \operatorname{Int}[\operatorname{Cos}[(c * f) / d + f * x] / (c + d * x), x], x] /;$   $\operatorname{FreeQ}\{c, d, e, f\}, x]$  &&  $\operatorname{NeQ}[d * e - c * f, 0]$

#### Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \cosh(e + fx)}{c + dx} dx &= \int \left( \frac{a}{c + dx} + \frac{b \cosh(e + fx)}{c + dx} \right) dx \\
&= \frac{a \log(c + dx)}{d} + b \int \frac{\cosh(e + fx)}{c + dx} dx \\
&= \frac{a \log(c + dx)}{d} + \left( b \cosh \left( e - \frac{cf}{d} \right) \right) \int \frac{\cosh \left( \frac{cf}{d} + fx \right)}{c + dx} dx + \left( b \sinh \left( e - \frac{cf}{d} \right) \right) \int \frac{\sinh \left( \frac{cf}{d} + fx \right)}{c + dx} dx \\
&= \frac{b \cosh \left( e - \frac{cf}{d} \right) \text{Chi} \left( \frac{cf}{d} + fx \right)}{d} + \frac{a \log(c + dx)}{d} + \frac{b \sinh \left( e - \frac{cf}{d} \right) \text{Shi} \left( \frac{cf}{d} + fx \right)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 57, normalized size = 0.89

$$\frac{a \log(c + dx) + b \text{Chi} \left( f \left( \frac{c}{d} + x \right) \right) \cosh \left( e - \frac{cf}{d} \right) + b \sinh \left( e - \frac{cf}{d} \right) \text{Shi} \left( f \left( \frac{c}{d} + x \right) \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[e + f*x])/(c + d*x),x]
```

```
[Out] (b*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + a*Log[c + d*x] + b*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)]/d
```

**fricas [A]** time = 0.56, size = 111, normalized size = 1.73

$$\frac{\left( b \text{Ei} \left( \frac{dfx+cf}{d} \right) + b \text{Ei} \left( -\frac{dfx+cf}{d} \right) \right) \cosh \left( -\frac{de-cf}{d} \right) + 2a \log(dx + c) - \left( b \text{Ei} \left( \frac{dfx+cf}{d} \right) - b \text{Ei} \left( -\frac{dfx+cf}{d} \right) \right) \sinh \left( -\frac{de-cf}{d} \right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((b*Ei((d*f*x + c*f)/d) + b*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) + 2*a*log(d*x + c) - (b*Ei((d*f*x + c*f)/d) - b*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d))/d
```



**giac** [A] time = 0.12, size = 69, normalized size = 1.08

$$\frac{b \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right) e^{\left(\frac{cf}{d}-e\right)} + b \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) e^{\left(-\frac{cf}{d}+e\right)} + 2a \log(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cosh(f\*x+e))/(d\*x+c),x, algorithm="giac")

[Out] 1/2\*(b\*Ei(-(d\*f\*x + c\*f)/d)\*e^(c\*f/d - e) + b\*Ei((d\*f\*x + c\*f)/d)\*e^(-c\*f/d + e) + 2\*a\*log(d\*x + c))/d

**maple** [A] time = 0.10, size = 94, normalized size = 1.47

$$\frac{a \ln(dx+c)}{d} - \frac{b e^{\frac{cf-de}{d}} \operatorname{Ei}\left(1, fx + e + \frac{cf-de}{d}\right)}{2d} - \frac{b e^{-\frac{cf-de}{d}} \operatorname{Ei}\left(1, -fx - e - \frac{cf-de}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cosh(f\*x+e))/(d\*x+c),x)

[Out] a\*ln(d\*x+c)/d-1/2\*b/d\*exp((c\*f-d\*e)/d)\*Ei(1,f\*x+e+(c\*f-d\*e)/d)-1/2\*b/d\*exp(-(c\*f-d\*e)/d)\*Ei(1,-f\*x-e-(c\*f-d\*e)/d)

**maxima** [A] time = 0.39, size = 70, normalized size = 1.09

$$-\frac{1}{2}b \left( \frac{e^{\left(-e+\frac{cf}{d}\right)} E_1\left(\frac{(dx+c)f}{d}\right)}{d} + \frac{e^{\left(e-\frac{cf}{d}\right)} E_1\left(-\frac{(dx+c)f}{d}\right)}{d} \right) + \frac{a \log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cosh(f\*x+e))/(d\*x+c),x, algorithm="maxima")

[Out] -1/2\*b\*(e^(-e + c\*f/d)\*exp\_integral\_e(1, (d\*x + c)\*f/d)/d + e^(e - c\*f/d)\*exp\_integral\_e(1, -(d\*x + c)\*f/d)/d) + a\*log(d\*x + c)/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b \cosh(e + fx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))/(c + d*x),x)
```

```
[Out] int((a + b*cosh(e + f*x))/(c + d*x), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{a + b \cosh(e + fx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c),x)
```

```
[Out] Integral((a + b*cosh(e + f*x))/(c + d*x), x)
```

$$3.160 \quad \int \frac{a+b \cosh(e+fx)}{(c+dx)^2} dx$$

**Optimal.** Leaf size=87

$$-\frac{a}{d(c+dx)} + \frac{bf \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{bf \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^2} - \frac{b \cosh(e+fx)}{d(c+dx)}$$

[Out]  $-a/d/(d*x+c)-b*\cosh(f*x+e)/d/(d*x+c)+b*f*\cosh(-e+c*f/d)*\operatorname{Shi}(c*f/d+f*x)/d^2-b*f*\operatorname{Chi}(c*f/d+f*x)*\sinh(-e+c*f/d)/d^2$

**Rubi [A]** time = 0.15, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3317, 3297, 3303, 3298, 3301}

$$-\frac{a}{d(c+dx)} + \frac{bf \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{bf \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^2} - \frac{b \cosh(e+fx)}{d(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{Cosh}[e + f*x])/(c + d*x)^2, x]$

[Out]  $-(a/(d*(c + d*x))) - (b*\operatorname{Cosh}[e + f*x])/(d*(c + d*x)) + (b*f*\operatorname{CoshIntegral}[(c*f)/d + f*x]*\operatorname{Sinh}[e - (c*f)/d])/d^2 + (b*f*\operatorname{Cosh}[e - (c*f)/d]*\operatorname{SinhIntegral}[(c*f)/d + f*x])/d^2$

Rule 3297

$\operatorname{Int}[(c_. + (d_.)*(x_.))^(m_)*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^(m + 1)*\operatorname{Sin}[e + f*x]/(d*(m + 1)), x] - \operatorname{Dist}[f/(d*(m + 1)), \operatorname{Int}[(c + d*x)^(m + 1)*\operatorname{Cos}[e + f*x], x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x \&\& \operatorname{LtQ}[m, -1]$

Rule 3298

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Simp}[(I*\operatorname{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x \&\& \operatorname{EqQ}[d*e - c*f*fz*I, 0]$

Rule 3301

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x \&\& \operatorname{EqQ}[d*(e - \operatorname{Pi}/2) - c*f*fz*I, 0]$

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a + b \cosh(e + fx)}{(c + dx)^2} dx &= \int \left( \frac{a}{(c + dx)^2} + \frac{b \cosh(e + fx)}{(c + dx)^2} \right) dx \\ &= -\frac{a}{d(c + dx)} + b \int \frac{\cosh(e + fx)}{(c + dx)^2} dx \\ &= -\frac{a}{d(c + dx)} - \frac{b \cosh(e + fx)}{d(c + dx)} + \frac{(bf) \int \frac{\sinh(e + fx)}{c + dx} dx}{d} \\ &= -\frac{a}{d(c + dx)} - \frac{b \cosh(e + fx)}{d(c + dx)} + \frac{\left( bf \cosh\left(e - \frac{cf}{d}\right) \right) \int \frac{\sinh\left(\frac{cf}{d} + fx\right)}{c + dx} dx}{d} + \frac{\left( bf \sinh\left(e - \frac{cf}{d}\right) \right) \int \frac{\cosh\left(\frac{cf}{d} + fx\right)}{c + dx} dx}{d} \\ &= -\frac{a}{d(c + dx)} - \frac{b \cosh(e + fx)}{d(c + dx)} + \frac{bf \operatorname{Chi}\left(\frac{cf}{d} + fx\right) \sinh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{bf \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(\frac{cf}{d} + fx\right)}{d^2} \end{aligned}$$

**Mathematica** [A] time = 0.42, size = 71, normalized size = 0.82

$$\frac{-\frac{d(a+b \cosh(e+fx))}{c+dx} + bf \operatorname{Chi}\left(f\left(\frac{c}{d} + x\right)\right) \sinh\left(e - \frac{cf}{d}\right) + bf \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(f\left(\frac{c}{d} + x\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[e + f*x])/(c + d*x)^2, x]
```

```
[Out] (-((d*(a + b*Cosh[e + f*x]))/(c + d*x)) + b*f*CoshIntegral[f*(c/d + x)]*Sin
h[e - (c*f)/d] + b*f*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)])/d^2
```

**fricas [A]** time = 0.49, size = 162, normalized size = 1.86

$$\frac{2bd \cosh(fx + e) + 2ad - \left( (bdfx + bcf) \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) - (bdfx + bcf) \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right) \right) \cosh\left(-\frac{de-cf}{d}\right) + \left( (bdfx + bcf) \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) - (bdfx + bcf) \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right) \right) \sinh\left(-\frac{de-cf}{d}\right)}{2(d^3x + cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cosh(f\*x+e))/(d\*x+c)^2,x, algorithm="fricas")

[Out]  $-1/2*(2*b*d*cosh(f*x + e) + 2*a*d - ((b*d*f*x + b*c*f)*Ei((d*f*x + c*f)/d) - (b*d*f*x + b*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) + ((b*d*f*x + b*c*f)*Ei((d*f*x + c*f)/d) + (b*d*f*x + b*c*f)*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d)/(d^3*x + c*d^2)$

**giac [B]** time = 0.16, size = 683, normalized size = 7.85

$$-\frac{1}{2}b \left( \frac{\left( (dx+c) \left( \frac{cf}{dx+c} - f - \frac{de}{dx+c} \right) f^2 \operatorname{Ei}\left( \frac{(dx+c) \left( \frac{cf}{dx+c} - f - \frac{de}{dx+c} \right) - cf + de}{d} \right) e^{\left( \frac{cf-de}{d} \right)} - cf^3 \operatorname{Ei}\left( \frac{(dx+c) \left( \frac{cf}{dx+c} - f - \frac{de}{dx+c} \right) - cf + de}{d} \right) e^{\left( \frac{cf-de}{d} \right)}}{\left( (dx+c) d^4 \left( \frac{cf}{dx+c} - f - \frac{de}{dx+c} \right) - cd^4 f + d^5 e \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cosh(f\*x+e))/(d\*x+c)^2,x, algorithm="giac")

[Out]  $-1/2*b*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*Ei(((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^{((c*f - d*e)/d)} - c*f^3*Ei(((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^{((c*f - d*e)/d)} + d*f^2*Ei(((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^{((c*f - d*e)/d + 1)} - d*f^2*e^{((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))/d)}*d^2/(((d*x + c)*d^4*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*d^4*f + d^5*e)*f) - ((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*Ei(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^{-((c*f - d*e)/d)} - c*f^3*Ei(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^{-((c*f - d*e)/d)} + d*f^2*Ei(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^{-((c*f - d*e)/d + 1)} + d*f^2*e^{-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))/d)}*d^2/(((d*x + c)*d^4*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*d^4*f + d^5*e)*f)) - a/((d*x + c)*d)$

**maple** [A] time = 0.11, size = 149, normalized size = 1.71

$$-\frac{a}{d(dx+c)} - \frac{fb e^{-fx-e}}{2d(dfx+cf)} + \frac{fb e^{\frac{cf-de}{d}} \operatorname{Ei}\left(1, fx+e+\frac{cf-de}{d}\right)}{2d^2} - \frac{fb e^{fx+e}}{2d^2\left(\frac{cf}{d}+fx\right)} - \frac{fb e^{-\frac{cf-de}{d}} \operatorname{Ei}\left(1, -fx-e-\frac{cf-de}{d}\right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cosh(f\*x+e))/(d\*x+c)^2,x)

[Out] -a/d/(d\*x+c)-1/2\*f\*b\*exp(-f\*x-e)/d/(d\*f\*x+c\*f)+1/2\*f\*b/d^2\*exp((c\*f-d\*e)/d)\*Ei(1,f\*x+e+(c\*f-d\*e)/d)-1/2\*f\*b/d^2\*exp(f\*x+e)/(c\*f/d+f\*x)-1/2\*f\*b/d^2\*exp(-(c\*f-d\*e)/d)\*Ei(1,-f\*x-e-(c\*f-d\*e)/d)

**maxima** [A] time = 0.40, size = 87, normalized size = 1.00

$$-\frac{1}{2} b \left( \frac{e^{\left(-e+\frac{cf}{d}\right)} E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d} + \frac{e^{\left(e-\frac{cf}{d}\right)} E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d} \right) - \frac{a}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cosh(f\*x+e))/(d\*x+c)^2,x, algorithm="maxima")

[Out] -1/2\*b\*(e^(-e+c\*f/d)\*exp\_integral\_e(2, (d\*x+c)\*f/d)/((d\*x+c)\*d) + e^(e-c\*f/d)\*exp\_integral\_e(2, -(d\*x+c)\*f/d)/((d\*x+c)\*d) - a/(d^2\*x+c\*d)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \cosh(e + f x)}{(c + d x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*cosh(e + f\*x))/(c + d\*x)^2,x)

[Out] int((a + b\*cosh(e + f\*x))/(c + d\*x)^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cosh(f\*x+e))/(d\*x+c)\*\*2,x)

[Out] Timed out

$$3.161 \quad \int \frac{a+b \cosh(e+fx)}{(c+dx)^3} dx$$

**Optimal.** Leaf size=123

$$-\frac{a}{2d(c+dx)^2} + \frac{bf^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{2d^3} + \frac{bf^2 \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{2d^3} - \frac{bf \sinh(e+fx)}{2d^2(c+dx)} - \frac{b \cosh(e+fx)}{2d(c+dx)}$$

[Out]  $-1/2*a/d/(d*x+c)^2+1/2*b*f^2*Chi(c*f/d+f*x)*\cosh(-e+c*f/d)/d^3-1/2*b*\cosh(f*x+e)/d/(d*x+c)^2-1/2*b*f^2*Shi(c*f/d+f*x)*\sinh(-e+c*f/d)/d^3-1/2*b*f*\sinh(f*x+e)/d^2/(d*x+c)$

**Rubi [A]** time = 0.20, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3317, 3297, 3303, 3298, 3301}

$$-\frac{a}{2d(c+dx)^2} + \frac{bf^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{2d^3} + \frac{bf^2 \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{2d^3} - \frac{bf \sinh(e+fx)}{2d^2(c+dx)} - \frac{b \cosh(e+fx)}{2d(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{Cosh}[e + f*x])/(c + d*x)^3, x]$

[Out]  $-a/(2*d*(c + d*x)^2) - (b*\operatorname{Cosh}[e + f*x])/(2*d*(c + d*x)^2) + (b*f^2*\operatorname{Cosh}[e - (c*f)/d]*\operatorname{CoshIntegral}[(c*f)/d + f*x])/(2*d^3) - (b*f*\operatorname{Sinh}[e + f*x])/(2*d^2*(c + d*x)) + (b*f^2*\operatorname{Sinh}[e - (c*f)/d]*\operatorname{SinhIntegral}[(c*f)/d + f*x])/(2*d^3)$

**Rule 3297**

$\operatorname{Int}[(c_. + (d_.)*(x_.))^(m_)*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^(m + 1)*\operatorname{Sin}[e + f*x]/(d*(m + 1)), x] - \operatorname{Dist}[f/(d*(m + 1)), \operatorname{Int}[(c + d*x)^(m + 1)*\operatorname{Cos}[e + f*x], x], x] /;$   $\operatorname{FreeQ}\{c, d, e, f, x\} \ \&\amp; \ \operatorname{LtQ}[m, -1]$

**Rule 3298**

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Simp}[(I*\operatorname{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /;$   $\operatorname{FreeQ}\{c, d, e, f, fz, x\} \ \&\amp; \ \operatorname{EqQ}[d*e - c*f*fz*I, 0]$

**Rule 3301**

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /;$   $\operatorname{FreeQ}\{c, d, e, f, fz, x\}$

}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

### Rule 3317

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Sin[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \cosh(e + fx)}{(c + dx)^3} dx &= \int \left( \frac{a}{(c + dx)^3} + \frac{b \cosh(e + fx)}{(c + dx)^3} \right) dx \\
 &= -\frac{a}{2d(c + dx)^2} + b \int \frac{\cosh(e + fx)}{(c + dx)^3} dx \\
 &= -\frac{a}{2d(c + dx)^2} - \frac{b \cosh(e + fx)}{2d(c + dx)^2} + \frac{(bf) \int \frac{\sinh(e + fx)}{(c + dx)^2} dx}{2d} \\
 &= -\frac{a}{2d(c + dx)^2} - \frac{b \cosh(e + fx)}{2d(c + dx)^2} - \frac{bf \sinh(e + fx)}{2d^2(c + dx)} + \frac{(bf^2) \int \frac{\cosh(e + fx)}{c + dx} dx}{2d^2} \\
 &= -\frac{a}{2d(c + dx)^2} - \frac{b \cosh(e + fx)}{2d(c + dx)^2} - \frac{bf \sinh(e + fx)}{2d^2(c + dx)} + \frac{\left( bf^2 \cosh\left(e - \frac{cf}{d}\right) \right) \int \frac{\cosh\left(\frac{cf}{d} + fx\right)}{c + dx}}{2d^2} \\
 &= -\frac{a}{2d(c + dx)^2} - \frac{b \cosh(e + fx)}{2d(c + dx)^2} + \frac{bf^2 \cosh\left(e - \frac{cf}{d}\right) \text{Chi}\left(\frac{cf}{d} + fx\right)}{2d^3} - \frac{bf \sinh(e + fx)}{2d^2(c + dx)}
 \end{aligned}$$

**Mathematica [A]** time = 0.63, size = 95, normalized size = 0.77

$$\frac{-\frac{d(ad + bf(c + dx) \sinh(e + fx) + bd \cosh(e + fx))}{(c + dx)^2} + bf^2 \text{Chi}\left(f\left(\frac{c}{d} + x\right)\right) \cosh\left(e - \frac{cf}{d}\right) + bf^2 \sinh\left(e - \frac{cf}{d}\right) \text{Shi}\left(f\left(\frac{c}{d} + x\right)\right)}{2d^3}$$

Antiderivative was successfully verified.



[In] Integrate[(a + b\*Cosh[e + f\*x])/(c + d\*x)^3,x]

[Out] (b\*f^2\*Cosh[e - (c\*f)/d]\*CoshIntegral[f\*(c/d + x)] - (d\*(a\*d + b\*d\*Cosh[e + f\*x] + b\*f\*(c + d\*x)\*Sinh[e + f\*x]))/(c + d\*x)^2 + b\*f^2\*Sinh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)]/(2\*d^3)

**fricas** [B] time = 0.54, size = 274, normalized size = 2.23

$$\frac{2bd^2 \cosh(fx + e) + 2ad^2 - \left( (bd^2 f^2 x^2 + 2bcd f^2 x + bc^2 f^2) \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) + (bd^2 f^2 x^2 + 2bcd f^2 x + bc^2 f^2) \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right) \right)}{(d^5 x^2 + 2cd^4 x + c^2 d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cosh(f\*x+e))/(d\*x+c)^3,x, algorithm="fricas")

[Out] -1/4\*(2\*b\*d^2\*cosh(f\*x + e) + 2\*a\*d^2 - ((b\*d^2\*f^2\*x^2 + 2\*b\*c\*d\*f^2\*x + b\*c^2\*f^2)\*Ei((d\*f\*x + c\*f)/d) + (b\*d^2\*f^2\*x^2 + 2\*b\*c\*d\*f^2\*x + b\*c^2\*f^2)\*Ei(-(d\*f\*x + c\*f)/d))\*cosh(-(d\*e - c\*f)/d) + 2\*(b\*d^2\*f\*x + b\*c\*d\*f)\*sinh(f\*x + e) + ((b\*d^2\*f^2\*x^2 + 2\*b\*c\*d\*f^2\*x + b\*c^2\*f^2)\*Ei((d\*f\*x + c\*f)/d) - (b\*d^2\*f^2\*x^2 + 2\*b\*c\*d\*f^2\*x + b\*c^2\*f^2)\*Ei(-(d\*f\*x + c\*f)/d))\*sinh(-(d\*e - c\*f)/d)/(d^5\*x^2 + 2\*c\*d^4\*x + c^2\*d^3)

**giac** [B] time = 0.13, size = 328, normalized size = 2.67

$$\frac{bd^2 f^2 x^2 \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right) e^{\left(\frac{cf}{d}-e\right)} + bd^2 f^2 x^2 \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) e^{\left(-\frac{cf}{d}+e\right)} + 2bcd f^2 x \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right) e^{\left(\frac{cf}{d}-e\right)} + 2bcd f^2 x \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) e^{\left(-\frac{cf}{d}+e\right)}}{(d^5 x^2 + 2cd^4 x + c^2 d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cosh(f\*x+e))/(d\*x+c)^3,x, algorithm="giac")

[Out] 1/4\*(b\*d^2\*f^2\*x^2\*Ei(-(d\*f\*x + c\*f)/d)\*e^(c\*f/d - e) + b\*d^2\*f^2\*x^2\*Ei((d\*f\*x + c\*f)/d)\*e^(-c\*f/d + e) + 2\*b\*c\*d\*f^2\*x\*Ei(-(d\*f\*x + c\*f)/d)\*e^(c\*f/d - e) + 2\*b\*c\*d\*f^2\*x\*Ei((d\*f\*x + c\*f)/d)\*e^(-c\*f/d + e) + b\*c^2\*f^2\*Ei(-(d\*f\*x + c\*f)/d)\*e^(c\*f/d - e) + b\*c^2\*f^2\*Ei((d\*f\*x + c\*f)/d)\*e^(-c\*f/d + e) - b\*d^2\*f\*x\*e^(f\*x + e) + b\*d^2\*f\*x\*e^(-f\*x - e) - b\*c\*d\*f\*e^(f\*x + e) + b\*c\*d\*f\*e^(-f\*x - e) - b\*d^2\*e^(f\*x + e) - b\*d^2\*e^(-f\*x - e) - 2\*a\*d^2)/(d^5\*x^2 + 2\*c\*d^4\*x + c^2\*d^3)

**maple** [B] time = 0.12, size = 296, normalized size = 2.41

$$\frac{a}{2d(dx+c)^2} + \frac{f^3 b e^{-fx-e} x}{4d(d^2 f^2 x^2 + 2cd f^2 x + c^2 f^2)} + \frac{f^3 b e^{-fx-e} c}{4d^2(d^2 f^2 x^2 + 2cd f^2 x + c^2 f^2)} - \frac{f^2 b e^{-fx-e}}{4d(d^2 f^2 x^2 + 2cd f^2 x + c^2 f^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*cosh(f*x+e))/(d*x+c)^3,x)`

[Out] 
$$-1/2*a/d/(d*x+c)^2+1/4*f^3*b*\exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x+1/4*f^3*b*\exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/4*f^2*b*\exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/4*f^2*b/d^3*\exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/4*f^2*b/d^3*\exp(f*x+e)/(c*f/d+f*x)^2-1/4*f^2*b/d^3*\exp(f*x+e)/(c*f/d+f*x)-1/4*f^2*b/d^3*\exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)$$

**maxima** [A] time = 0.41, size = 98, normalized size = 0.80

$$-\frac{1}{2}b\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right)-\frac{a}{2(d^3x^2+2cd^2x+c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cosh(f*x+e))/(d*x+c)^3,x, algorithm="maxima")`

[Out] 
$$-1/2*b*(e^{(-e+c*f/d)}*\exp\_integral\_e(3,(d*x+c)*f/d)/((d*x+c)^2*d)+e^{(e-c*f/d)}*\exp\_integral\_e(3,-(d*x+c)*f/d)/((d*x+c)^2*d))-1/2*a/(d^3*x^2+2*c*d^2*x+c^2*d)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \cosh(e + f x)}{(c + d x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*cosh(e + f*x))/(c + d*x)^3,x)`

[Out] `int((a + b*cosh(e + f*x))/(c + d*x)^3, x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cosh(f*x+e))/(d*x+c)**3,x)`

[Out] Timed out

### 3.162 $\int (c + dx)^3 (a + b \cosh(e + fx))^2 dx$

**Optimal.** Leaf size=250

$$\frac{a^2(c + dx)^4}{4d} + \frac{12abd^2(c + dx) \sinh(e + fx)}{f^3} - \frac{6abd(c + dx)^2 \cosh(e + fx)}{f^2} + \frac{2ab(c + dx)^3 \sinh(e + fx)}{f} - \frac{12abd^3 \cos}{f}$$

[Out]  $3/4*b^2*c*d^2*x/f^2+3/8*b^2*d^3*x^2/f^2+1/4*a^2*(d*x+c)^4/d+1/8*b^2*(d*x+c)^4/d-12*a*b*d^3*\cosh(f*x+e)/f^4-6*a*b*d*(d*x+c)^2*\cosh(f*x+e)/f^2-3/8*b^2*d^3*\cosh(f*x+e)^2/f^4-3/4*b^2*d*(d*x+c)^2*\cosh(f*x+e)^2/f^2+12*a*b*d^2*(d*x+c)*\sinh(f*x+e)/f^3+2*a*b*(d*x+c)^3*\sinh(f*x+e)/f+3/4*b^2*d^2*(d*x+c)*\cosh(f*x+e)*\sinh(f*x+e)/f^3+1/2*b^2*(d*x+c)^3*\cosh(f*x+e)*\sinh(f*x+e)/f$

**Rubi [A]** time = 0.28, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {3317, 3296, 2638, 3311, 32, 3310}

$$\frac{a^2(c + dx)^4}{4d} + \frac{12abd^2(c + dx) \sinh(e + fx)}{f^3} - \frac{6abd(c + dx)^2 \cosh(e + fx)}{f^2} + \frac{2ab(c + dx)^3 \sinh(e + fx)}{f} - \frac{12abd^3 \cos}{f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^3*(a + b*\text{Cosh}[e + f*x])^2, x]$

[Out]  $(3*b^2*c*d^2*x)/(4*f^2) + (3*b^2*d^3*x^2)/(8*f^2) + (a^2*(c + d*x)^4)/(4*d) + (b^2*(c + d*x)^4)/(8*d) - (12*a*b*d^3*\text{Cosh}[e + f*x])/f^4 - (6*a*b*d*(c + d*x)^2*\text{Cosh}[e + f*x])/f^2 - (3*b^2*d^3*\text{Cosh}[e + f*x]^2)/(8*f^4) - (3*b^2*d*(c + d*x)^2*\text{Cosh}[e + f*x]^2)/(4*f^2) + (12*a*b*d^2*(c + d*x)*\text{Sinh}[e + f*x])/f^3 + (2*a*b*(c + d*x)^3*\text{Sinh}[e + f*x])/f + (3*b^2*d^2*(c + d*x)*\text{Cosh}[e + f*x]*\text{Sinh}[e + f*x])/(4*f^3) + (b^2*(c + d*x)^3*\text{Cosh}[e + f*x]*\text{Sinh}[e + f*x])/(2*f)$

#### Rule 32

$\text{Int}[(a_. + (b_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^(m + 1)/(b*(m + 1)), x] /;$   $\text{FreeQ}\{a, b, m\}, x \&\& \text{NeQ}[m, -1]$

#### Rule 2638

$\text{Int}[\sin[(c_. + (d_.)*(x_.))], x\_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /;$   $\text{FreeQ}\{c, d\}, x]$

#### Rule 3296

$\text{Int}[(c_. + (d_.)*(x_.))^(m_.)*\sin[(e_. + (f_.)*(x_.))], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^(m - 1)*\text{Cos}[$

$e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 3310

$\text{Int}[\{(c_.) + (d_.)*(x_.)\}*\{(b_.)*\sin[(e_.) + (f_.)*(x_.)]\}^{(n_.)}, x\_Symbol] \rightarrow$   
 $\text{Simp}[(d*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n - 1))/n, \text{Int}[(c + d*x)*(b*\sin[e + f*x])^{(n - 2)}, x], x] - \text{Simp}[(b*(c + d*x)*\cos[e + f*x]*(b*\sin[e + f*x])^{(n - 1)})/(f*n), x]) /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[n, 1]$

### Rule 3311

$\text{Int}[\{(c_.) + (d_.)*(x_.)\}^{(m_.)}*\{(b_.)*\sin[(e_.) + (f_.)*(x_.)]\}^{(n_.)}, x\_Symbol] \rightarrow$   
 $\text{Simp}[(d*m*(c + d*x)^{(m - 1)}*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n - 1))/n, \text{Int}[(c + d*x)^m*(b*\sin[e + f*x])^{(n - 2)}, x], x] - \text{Dist}[(d^2*m*(m - 1))/(f^2*n^2), \text{Int}[(c + d*x)^{(m - 2)}*(b*\sin[e + f*x])^n, x], x] - \text{Simp}[(b*(c + d*x)^m*\cos[e + f*x]*(b*\sin[e + f*x])^{(n - 1)})/(f*n), x]) /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{GtQ}[m, 1]$

### Rule 3317

$\text{Int}[\{(c_.) + (d_.)*(x_.)\}^{(m_.)}*\{(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]\}^{(n_.)}, x\_Symbol] \rightarrow$   
 $\text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, (a + b*\sin[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ (\text{EqQ}[n, 1] \ || \ \text{IGtQ}[m, 0] \ || \ \text{NeQ}[a^2 - b^2, 0])$

### Rubi steps

$$\begin{aligned} \int (c + dx)^3 (a + b \cosh(e + fx))^2 dx &= \int (a^2(c + dx)^3 + 2ab(c + dx)^3 \cosh(e + fx) + b^2(c + dx)^3 \cosh^2(e + fx)) \\ &= \frac{a^2(c + dx)^4}{4d} + (2ab) \int (c + dx)^3 \cosh(e + fx) dx + b^2 \int (c + dx)^3 \cosh^2(e + fx) dx \\ &= \frac{a^2(c + dx)^4}{4d} - \frac{3b^2d(c + dx)^2 \cosh^2(e + fx)}{4f^2} + \frac{2ab(c + dx)^3 \sinh(e + fx)}{f} + \frac{b^2(c + dx)^4}{8d} \\ &= \frac{a^2(c + dx)^4}{4d} + \frac{b^2(c + dx)^4}{8d} - \frac{6abd(c + dx)^2 \cosh(e + fx)}{f^2} - \frac{3b^2d^3 \cosh^2(e + fx)}{8f^4} \\ &= \frac{3b^2cd^2x}{4f^2} + \frac{3b^2d^3x^2}{8f^2} + \frac{a^2(c + dx)^4}{4d} + \frac{b^2(c + dx)^4}{8d} - \frac{6abd(c + dx)^2 \cosh(e + fx)}{f^2} \\ &= \frac{3b^2cd^2x}{4f^2} + \frac{3b^2d^3x^2}{8f^2} + \frac{a^2(c + dx)^4}{4d} + \frac{b^2(c + dx)^4}{8d} - \frac{12abd^3 \cosh(e + fx)}{f^4} \end{aligned}$$



$$2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x - 6*b^2*d^3*f^2*x^2 + 4*b^2*c^3*f^3 - 12*b^2*c*d^2*f^2*x - 6*b^2*c^2*d*f^2 + 6*b^2*d^3*f*x + 6*b^2*c*d^2*f - 3*b^2*d^3)*e^{(2*f*x + 2*e)}/f^4 + (a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3*x - 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 - 6*a*b*c*d^2*f^2*x - 3*a*b*c^2*d*f^2 + 6*a*b*d^3*f*x + 6*a*b*c*d^2*f - 6*a*b*d^3)*e^{(f*x + e)}/f^4 - (a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3*x*x + 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 + 6*a*b*c*d^2*f^2*x + 3*a*b*c^2*d*f^2 + 6*a*b*d^3*f*x + 6*a*b*c*d^2*f + 6*a*b*d^3)*e^{(-f*x - e)}/f^4 - 1/32*(4*b^2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x + 6*b^2*d^3*f^2*x^2 + 4*b^2*c^3*f^3 + 12*b^2*c*d^2*f^2*x + 6*b^2*c^2*d*f^2 + 6*b^2*d^3*f*x + 6*b^2*c*d^2*f + 3*b^2*d^3)*e^{(-2*f*x - 2*e)}/f^4$$

**maple [B]** time = 0.08, size = 1061, normalized size = 4.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^3*(a+b*cosh(f*x+e))^2,x)`

[Out]  $\frac{1}{f}(-12/f^2*c*d^2*e*a*b*((f*x+e)*\sinh(f*x+e)-\cosh(f*x+e))-6/f*c^2*d*e*a*b*\sinh(f*x+e)+6/f^2*c*d^2*e^2*a*b*\sinh(f*x+e)+1/4/f^3*d^3*a^2*(f*x+e)^4+2*c^3*a*b*\sinh(f*x+e)+1/f^3*d^3*b^2*(1/2*(f*x+e)^3*\cosh(f*x+e)*\sinh(f*x+e)+1/8*(f*x+e)^4-3/4*(f*x+e)^2*\cosh(f*x+e)^2+3/4*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)+3/8*(f*x+e)^2-3/8*\cosh(f*x+e)^2)-3/f*c^2*d*e*b^2*(1/2*\cosh(f*x+e)*\sinh(f*x+e)+1/2*f*x+1/2*e)+6/f^2*c*d^2*a*b*((f*x+e)^2*\sinh(f*x+e)-2*(f*x+e)*\cosh(f*x+e))+2*\sinh(f*x+e))-2/f^3*d^3*e^3*a*b*\sinh(f*x+e)+3/f^2*c*d^2*e^2*b^2*(1/2*\cosh(f*x+e)*\sinh(f*x+e)+1/2*f*x+1/2*e)-6/f^2*c*d^2*e*b^2*(1/2*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)+1/4*(f*x+e)^2-1/4*\cosh(f*x+e)^2)+b^2*c^3*(1/2*\cosh(f*x+e)*\sinh(f*x+e)+1/2*f*x+1/2*e)+c^3*a^2*(f*x+e)+3/f^2*c*d^2*e^2*a^2*(f*x+e)-3/f^2*c*d^2*e*a^2*(f*x+e)^2-3/f*c^2*d*e*a^2*(f*x+e)+1/f^2*c*d^2*a^2*(f*x+e)^3+3/f^2*c*d^2*b^2*(1/2*(f*x+e)^2*\cosh(f*x+e)*\sinh(f*x+e)+1/6*(f*x+e)^3-1/2*(f*x+e)*\cosh(f*x+e)^2+1/4*\cosh(f*x+e)*\sinh(f*x+e)+1/4*f*x+1/4*e)+2/f^3*d^3*a*b*((f*x+e)^3*\sinh(f*x+e)-3*(f*x+e)^2*\cosh(f*x+e)+6*(f*x+e)*\sinh(f*x+e)-6*\cosh(f*x+e))+3/f*c^2*d*b^2*(1/2*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)+1/4*(f*x+e)^2-1/4*\cosh(f*x+e)^2)+3/f^3*d^3*e^2*b^2*(1/2*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)+1/4*(f*x+e)^2-1/4*\cosh(f*x+e)^2)-1/f^3*d^3*e^3*b^2*(1/2*\cosh(f*x+e)*\sinh(f*x+e)+1/2*f*x+1/2*e)-3/f^3*d^3*e*b^2*(1/2*(f*x+e)^2*\cosh(f*x+e)*\sinh(f*x+e)+1/6*(f*x+e)^3-1/2*(f*x+e)*\cosh(f*x+e)^2+1/4*\cosh(f*x+e)*\sinh(f*x+e)+1/4*f*x+1/4*e)-1/f^3*d^3*e^3*a^2*(f*x+e)+3/2/f^3*d^3*e^2*a^2*(f*x+e)^2+3/2/f*c^2*d*a^2*(f*x+e)^2-1/f^3*d^3*e*a^2*(f*x+e)^3-6/f^3*d^3*e*a*b*((f*x+e)^2*\sinh(f*x+e)-2*(f*x+e)*\cosh(f*x+e)+2*\sinh(f*x+e))+6/f*c^2*d*a*b*((f*x+e)*\sinh(f*x+e)-\cosh(f*x+e))+6/f^3*d^3*e^2*a*b*((f*x+e)*\sinh(f*x+e)-\cosh(f*x+e)))$

**maxima [B]** time = 0.39, size = 523, normalized size = 2.09

$$\frac{1}{4} a^2 d^3 x^4 + a^2 c d^2 x^3 + \frac{3}{2} a^2 c^2 d x^2 + \frac{3}{16} \left( 4x^2 + \frac{(2fxe^{(2e)} - e^{(2e)})e^{(2fx)}}{f^2} - \frac{(2fx+1)e^{(-2fx-2e)}}{f^2} \right) b^2 c^2 d + \frac{1}{16} \left( 8x^3 + \frac{3}{16} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+b\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out]  $\frac{1}{4} a^2 d^3 x^4 + a^2 c d^2 x^3 + \frac{3}{2} a^2 c^2 d x^2 + \frac{3}{16} (4x^2 + \frac{(2fxe^{(2e)} - e^{(2e)})e^{(2fx)}}{f^2} - \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}) b^2 c^2 d + \frac{1}{16} (8x^3 + \frac{3}{16})$

**mupad [B]** time = 2.64, size = 481, normalized size = 1.92

$$a^2 c^3 x + \frac{b^2 c^3 x}{2} + \frac{a^2 d^3 x^4}{4} + \frac{b^2 d^3 x^4}{8} + \frac{3a^2 c^2 d x^2}{2} + a^2 c d^2 x^3 + \frac{3b^2 c^2 d x^2}{4} + \frac{b^2 c d^2 x^3}{2} - \frac{3b^2 d^3 \cosh(2e + 2fx)}{16f^4} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*cosh(e + f\*x))^2\*(c + d\*x)^3,x)

[Out]  $a^2 c^3 x + \frac{(b^2 c^3 x)}{2} + \frac{(a^2 d^3 x^4)}{4} + \frac{(b^2 d^3 x^4)}{8} + \frac{(3a^2 c^2 d x^2)}{2} + \frac{(a^2 c d^2 x^3)}{2} + \frac{(3b^2 c^2 d x^2)}{4} + \frac{(b^2 c d^2 x^3)}{2} - \frac{(3b^2 d^3 \cosh(2e + 2fx))}{(16f^4)} + \frac{(b^2 c^3 \sinh(2e + 2fx))}{(4f)} - \frac{(12a^2 b^2 d^3 \cosh(e + fx))}{f^4} + \frac{(2a^2 b^2 c^3 \sinh(e + fx))}{f} - \frac{(3b^2 d^3 x^2 \cosh(2e + 2fx))}{(8f^2)} + \frac{(b^2 d^3 x^3 \sinh(2e + 2fx))}{(4f)} - \frac{(3b^2 c^2 d \cosh(2e + 2fx))}{(8f^2)} + \frac{(3b^2 c^2 d^2 \sinh(2e + 2fx))}{(8f^3)} + \frac{(3b^2 d^3 x \sinh(2e + 2fx))}{(8f^3)} - \frac{(3b^2 c^2 d^2 x \cosh(2e + 2fx))}{(4f^2)} + \frac{(3b^2 c^2 d x \sinh(2e + 2fx))}{(4f)} - \frac{(6a^2 b^2 c^2 d \cosh(e + fx))}{f^2} + \frac{(12a^2 b^2 c^2 d \sinh(e + fx))}{f^3} + \frac{(12a^2 b^2 d^3 x \sinh(e + fx))}{f^3} + \frac{(3b^2 c^2 d^2 x^2 \sinh(2e + 2fx))}{(4f)} - \frac{(6a^2 b^2 d^3 x^2 \cosh(e + fx))}{f^2} + \frac{(2a^2 b^2 d^3 x^3 \sinh(e + fx))}{f} + \frac{(6a^2 b^2 c^2 d^2 x^2 \sinh(e + fx))}{f} - \frac{(12a^2 b^2 c^2 d^2 x \cosh(e + fx))}{f^2} + \frac{(6a^2 b^2 c^2 d x \sinh(e + fx))}{f}$

sympy [A] time = 3.61, size = 779, normalized size = 3.12

$$\left\{ \begin{array}{l} a^2 c^3 x + \frac{3a^2 c^2 dx^2}{2} + a^2 cd^2 x^3 + \frac{a^2 d^3 x^4}{4} + \frac{2abc^3 \sinh(e+fx)}{f} + \frac{6abc^2 dx \sinh(e+fx)}{f} - \frac{6abc^2 d \cosh(e+fx)}{f^2} + \frac{6abcd^2 x^2 \sinh(e+fx)}{f} - 12abcd^2 x \cosh(e+fx) \\ (a + b \cosh(e))^2 \left( c^3 x + \frac{3c^2 dx^2}{2} + cd^2 x^3 + \frac{d^3 x^4}{4} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3\*(a+b\*cosh(f\*x+e))\*\*2,x)

[Out] Piecewise((a\*\*2\*c\*\*3\*x + 3\*a\*\*2\*c\*\*2\*d\*x\*\*2/2 + a\*\*2\*c\*d\*\*2\*x\*\*3 + a\*\*2\*d\*\*3\*x\*\*4/4 + 2\*a\*b\*c\*\*3\*sinh(e + f\*x)/f + 6\*a\*b\*c\*\*2\*d\*x\*sinh(e + f\*x)/f - 6\*a\*b\*c\*\*2\*d\*cosh(e + f\*x)/f\*\*2 + 6\*a\*b\*c\*d\*\*2\*x\*\*2\*sinh(e + f\*x)/f - 12\*a\*b\*c\*d\*\*2\*x\*cosh(e + f\*x)/f\*\*2 + 12\*a\*b\*c\*d\*\*2\*sinh(e + f\*x)/f\*\*3 + 2\*a\*b\*d\*\*3\*x\*\*3\*sinh(e + f\*x)/f - 6\*a\*b\*d\*\*3\*x\*\*2\*cosh(e + f\*x)/f\*\*2 + 12\*a\*b\*d\*\*3\*x\*sinh(e + f\*x)/f\*\*3 - 12\*a\*b\*d\*\*3\*cosh(e + f\*x)/f\*\*4 - b\*\*2\*c\*\*3\*x\*sinh(e + f\*x)\*\*2/2 + b\*\*2\*c\*\*3\*x\*cosh(e + f\*x)\*\*2/2 + b\*\*2\*c\*\*3\*sinh(e + f\*x)\*cosh(e + f\*x)/(2\*f) - 3\*b\*\*2\*c\*\*2\*d\*x\*\*2\*sinh(e + f\*x)\*\*2/4 + 3\*b\*\*2\*c\*\*2\*d\*x\*\*2\*cosh(e + f\*x)\*\*2/4 + 3\*b\*\*2\*c\*\*2\*d\*x\*sinh(e + f\*x)\*cosh(e + f\*x)/(2\*f) - 3\*b\*\*2\*c\*\*2\*d\*sinh(e + f\*x)\*\*2/(4\*f\*\*2) - b\*\*2\*c\*d\*\*2\*x\*\*3\*sinh(e + f\*x)\*\*2/2 + b\*\*2\*c\*d\*\*2\*x\*\*3\*cosh(e + f\*x)\*\*2/2 + 3\*b\*\*2\*c\*d\*\*2\*x\*\*2\*sinh(e + f\*x)\*cosh(e + f\*x)/(2\*f) - 3\*b\*\*2\*c\*d\*\*2\*x\*sinh(e + f\*x)\*\*2/(4\*f\*\*2) - 3\*b\*\*2\*c\*d\*\*2\*x\*cosh(e + f\*x)\*\*2/(4\*f\*\*2) + 3\*b\*\*2\*c\*d\*\*2\*sinh(e + f\*x)\*cosh(e + f\*x)/(4\*f\*\*3) - b\*\*2\*d\*\*3\*x\*\*4\*sinh(e + f\*x)\*\*2/8 + b\*\*2\*d\*\*3\*x\*\*4\*cosh(e + f\*x)\*\*2/8 + b\*\*2\*d\*\*3\*x\*\*3\*sinh(e + f\*x)\*cosh(e + f\*x)/(2\*f) - 3\*b\*\*2\*d\*\*3\*x\*\*2\*sinh(e + f\*x)\*\*2/(8\*f\*\*2) - 3\*b\*\*2\*d\*\*3\*x\*\*2\*cosh(e + f\*x)\*\*2/(8\*f\*\*2) + 3\*b\*\*2\*d\*\*3\*x\*sinh(e + f\*x)\*cosh(e + f\*x)/(4\*f\*\*3) - 3\*b\*\*2\*d\*\*3\*sinh(e + f\*x)\*\*2/(8\*f\*\*4), Ne(f, 0)), ((a + b\*cosh(e))\*\*2\*(c\*\*3\*x + 3\*c\*\*2\*d\*x\*\*2/2 + c\*d\*\*2\*x\*\*3 + d\*\*3\*x\*\*4/4), True))



### 3.163 $\int (c + dx)^2 (a + b \cosh(e + fx))^2 dx$

**Optimal.** Leaf size=182

$$\frac{a^2(c + dx)^3}{3d} - \frac{4abd(c + dx) \cosh(e + fx)}{f^2} + \frac{2ab(c + dx)^2 \sinh(e + fx)}{f} + \frac{4abd^2 \sinh(e + fx)}{f^3} - \frac{b^2 d(c + dx) \cosh^2(e + fx)}{2f^2}$$

[Out]  $1/4*b^2*d^2*x/f^2+1/3*a^2*(d*x+c)^3/d+1/6*b^2*(d*x+c)^3/d-4*a*b*d*(d*x+c)*\cosh(f*x+e)/f^2-1/2*b^2*d*(d*x+c)*\cosh(f*x+e)^2/f^2+4*a*b*d^2*\sinh(f*x+e)/f^3+2*a*b*(d*x+c)^2*\sinh(f*x+e)/f+1/4*b^2*d^2*\cosh(f*x+e)*\sinh(f*x+e)/f^3+1/2*b^2*(d*x+c)^2*\cosh(f*x+e)*\sinh(f*x+e)/f$

**Rubi [A]** time = 0.19, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$ , Rules used = {3317, 3296, 2637, 3311, 32, 2635, 8}

$$\frac{a^2(c + dx)^3}{3d} - \frac{4abd(c + dx) \cosh(e + fx)}{f^2} + \frac{2ab(c + dx)^2 \sinh(e + fx)}{f} + \frac{4abd^2 \sinh(e + fx)}{f^3} - \frac{b^2 d(c + dx) \cosh^2(e + fx)}{2f^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^2*(a + b*\text{Cosh}[e + f*x])^2, x]$

[Out]  $(b^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(3*d) + (b^2*(c + d*x)^3)/(6*d) - (4*a*b*d*(c + d*x)*\text{Cosh}[e + f*x])/f^2 - (b^2*d*(c + d*x)*\text{Cosh}[e + f*x]^2)/(2*f^2) + (4*a*b*d^2*\text{Sinh}[e + f*x])/f^3 + (2*a*b*(c + d*x)^2*\text{Sinh}[e + f*x])/f + (b^2*d^2*\text{Cosh}[e + f*x]*\text{Sinh}[e + f*x])/(4*f^3) + (b^2*(c + d*x)^2*\text{Cosh}[e + f*x]*\text{Sinh}[e + f*x])/(2*f)$

#### Rule 8

$\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

#### Rule 32

$\text{Int}[(a_. + (b_.)*(x_))^(m_), x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; \text{FreeQ}\{a, b, m\}, x \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 2635

$\text{Int}[(b_.)*\sin[(c_. + (d_.)*(x_))]^(n_), x\_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^(n - 1)/(d*n), x] + \text{Dist}[(b^2*(n - 1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^(n - 2), x], x] /; \text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[
(d*m*(c + d*x)^(m - 1)*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[
(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sine[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sine[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.),
x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sine[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^2 (a + b \cosh(e + fx))^2 dx &= \int (a^2(c + dx)^2 + 2ab(c + dx)^2 \cosh(e + fx) + b^2(c + dx)^2 \cosh^2(e + fx)) \\
&= \frac{a^2(c + dx)^3}{3d} + (2ab) \int (c + dx)^2 \cosh(e + fx) dx + b^2 \int (c + dx)^2 \cosh^2(e + fx) dx \\
&= \frac{a^2(c + dx)^3}{3d} - \frac{b^2 d(c + dx) \cosh^2(e + fx)}{2f^2} + \frac{2ab(c + dx)^2 \sinh(e + fx)}{f} + \frac{b^2 d(c + dx) \cosh^2(e + fx)}{2f^2} \\
&= \frac{a^2(c + dx)^3}{3d} + \frac{b^2(c + dx)^3}{6d} - \frac{4abd(c + dx) \cosh(e + fx)}{f^2} - \frac{b^2 d(c + dx) \cosh^2(e + fx)}{2f^2} \\
&= \frac{b^2 d^2 x}{4f^2} + \frac{a^2(c + dx)^3}{3d} + \frac{b^2(c + dx)^3}{6d} - \frac{4abd(c + dx) \cosh(e + fx)}{f^2} - \frac{b^2 d(c + dx) \cosh^2(e + fx)}{2f^2}
\end{aligned}$$

**Mathematica [A]** time = 1.05, size = 252, normalized size = 1.38

$$\frac{1}{24} \left( 24a^2c^2x + 24a^2cdx^2 + 8a^2d^2x^3 + \frac{48abc^2 \sinh(e + fx)}{f} - \frac{96abd(c + dx) \cosh(e + fx)}{f^2} + \frac{96abcdx \sinh(e + fx)}{f} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2\*(a + b\*Cosh[e + f\*x])^2,x]

[Out] (24\*a^2\*c^2\*x + 12\*b^2\*c^2\*x + 24\*a^2\*c\*d\*x^2 + 12\*b^2\*c\*d\*x^2 + 8\*a^2\*d^2\*x^3 + 4\*b^2\*d^2\*x^3 - (96\*a\*b\*d\*(c + d\*x)\*Cosh[e + f\*x])/f^2 - (6\*b^2\*d\*(c + d\*x)\*Cosh[2\*(e + f\*x)]/f^2 + (96\*a\*b\*d^2\*Sinh[e + f\*x])/f^3 + (48\*a\*b\*c^2\*Sinh[e + f\*x])/f + (96\*a\*b\*c\*d\*x\*Sinh[e + f\*x])/f + (48\*a\*b\*d^2\*x^2\*Sinh[e + f\*x])/f + (3\*b^2\*d^2\*Sinh[2\*(e + f\*x)]/f^3 + (6\*b^2\*c^2\*Sinh[2\*(e + f\*x)]/f + (12\*b^2\*c\*d\*x\*Sinh[2\*(e + f\*x)]/f + (6\*b^2\*d^2\*x^2\*Sinh[2\*(e + f\*x)]/f)/24

**fricas [A]** time = 0.53, size = 240, normalized size = 1.32

$$2(2a^2 + b^2)d^2f^3x^3 + 6(2a^2 + b^2)cdf^3x^2 + 6(2a^2 + b^2)c^2f^3x - 3(b^2d^2fx + b^2cdf) \cosh(fx + e)^2 - 3(b^2d^2f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+b\*cosh(f\*x+e))^2,x, algorithm="fricas")

[Out] 1/12\*(2\*(2\*a^2 + b^2)\*d^2\*f^3\*x^3 + 6\*(2\*a^2 + b^2)\*c\*d\*f^3\*x^2 + 6\*(2\*a^2 + b^2)\*c^2\*f^3\*x - 3\*(b^2\*d^2\*f\*x + b^2\*c\*d\*f)\*cosh(f\*x + e)^2 - 3\*(b^2\*d^2\*f\*x + b^2\*c\*d\*f)\*sinh(f\*x + e)^2 - 48\*(a\*b\*d^2\*f\*x + a\*b\*c\*d\*f)\*cosh(f\*x + e) + 3\*(8\*a\*b\*d^2\*f^2\*x^2 + 16\*a\*b\*c\*d\*f^2\*x + 8\*a\*b\*c^2\*f^2 + 16\*a\*b\*d^2 + (2\*b^2\*d^2\*f^2\*x^2 + 4\*b^2\*c\*d\*f^2\*x + 2\*b^2\*c^2\*f^2 + b^2\*d^2)\*cosh(f\*x + e))\*sinh(f\*x + e))/f^3

**giac [B]** time = 0.15, size = 349, normalized size = 1.92

$$\frac{1}{3} a^2 d^2 x^3 + \frac{1}{6} b^2 d^2 x^3 + a^2 c d x^2 + \frac{1}{2} b^2 c d x^2 + a^2 c^2 x + \frac{1}{2} b^2 c^2 x + \frac{(2 b^2 d^2 f^2 x^2 + 4 b^2 c d f^2 x + 2 b^2 c^2 f^2 - 2 b^2 d^2 f x - 2 b^2 c d f^2)}{16 f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+b\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out] 1/3\*a^2\*d^2\*x^3 + 1/6\*b^2\*d^2\*x^3 + a^2\*c\*d\*x^2 + 1/2\*b^2\*c\*d\*x^2 + a^2\*c^2\*x + 1/2\*b^2\*c^2\*x + 1/16\*(2\*b^2\*d^2\*f^2\*x^2 + 4\*b^2\*c\*d\*f^2\*x + 2\*b^2\*c^2\*f^2 - 2\*b^2\*d^2\*f\*x - 2\*b^2\*c\*d\*f + b^2\*d^2)\*e^(2\*f\*x + 2\*e)/f^3 + (a\*b\*d^2\*f^2\*x^2 + 2\*a\*b\*c\*d\*f^2\*x + a\*b\*c^2\*f^2 - 2\*a\*b\*d^2\*f\*x - 2\*a\*b\*c\*d\*f + 2\*

$$a*b*d^2)*e^{(f*x + e)/f^3} - (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 + 2*a*b*d^2*f*x + 2*a*b*c*d*f + 2*a*b*d^2)*e^{(-f*x - e)/f^3} - 1/16*(2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 + 2*b^2*d^2*f*x + 2*b^2*c*d*f + b^2*d^2)*e^{(-2*f*x - 2*e)/f^3}$$

**maple** [B] time = 0.07, size = 535, normalized size = 2.94

$$\frac{d^2 a^2 (f x + e)^3}{3 f^2} + \frac{2 d^2 a b \left( (f x + e)^2 \sinh(f x + e) - 2 (f x + e) \cosh(f x + e) + 2 \sinh(f x + e) \right)}{f^2} + \frac{d^2 b^2 \left( \frac{(f x + e)^2 \cosh(f x + e) \sinh(f x + e)}{2} + \frac{(f x + e)^3}{6} - \frac{(f x + e) \cosh^2(f x + e)}{2} \right)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*(a+b\*cosh(f\*x+e))^2,x)

[Out] 1/f\*(1/3/f^2\*d^2\*a^2\*(f\*x+e)^3+2/f^2\*d^2\*a\*b\*((f\*x+e)^2\*sinh(f\*x+e)-2\*(f\*x+e)\*cosh(f\*x+e)+2\*sinh(f\*x+e))+1/f^2\*d^2\*b^2\*(1/2\*(f\*x+e)^2\*cosh(f\*x+e)\*sinh(f\*x+e)+1/6\*(f\*x+e)^3-1/2\*(f\*x+e)\*cosh(f\*x+e)^2+1/4\*cosh(f\*x+e)\*sinh(f\*x+e)+1/4\*f\*x+1/4\*e)-1/f^2\*d^2\*e\*a^2\*(f\*x+e)^2-4/f^2\*d^2\*e\*a\*b\*((f\*x+e)\*sinh(f\*x+e)-cosh(f\*x+e))-2/f^2\*d^2\*e\*b^2\*(1/2\*(f\*x+e)\*cosh(f\*x+e)\*sinh(f\*x+e)+1/4\*(f\*x+e)^2-1/4\*cosh(f\*x+e)^2)+1/f^2\*d^2\*e^2\*a^2\*(f\*x+e)+2/f^2\*d^2\*e^2\*a\*b\*sinh(f\*x+e)+1/f^2\*d^2\*e^2\*b^2\*(1/2\*cosh(f\*x+e)\*sinh(f\*x+e)+1/2\*f\*x+1/2\*e)+1/f\*c\*d\*a^2\*(f\*x+e)^2+4/f\*c\*d\*a\*b\*((f\*x+e)\*sinh(f\*x+e)-cosh(f\*x+e))+2/f\*c\*d\*b^2\*(1/2\*(f\*x+e)\*cosh(f\*x+e)\*sinh(f\*x+e)+1/4\*(f\*x+e)^2-1/4\*cosh(f\*x+e)^2)-2/f\*c\*d\*e\*a^2\*(f\*x+e)-4/f\*c\*d\*e\*a\*b\*sinh(f\*x+e)-2/f\*c\*d\*e\*b^2\*(1/2\*cosh(f\*x+e)\*sinh(f\*x+e)+1/2\*f\*x+1/2\*e)+c^2\*a^2\*(f\*x+e)+2\*c^2\*a\*b\*sinh(f\*x+e)+c^2\*b^2\*(1/2\*cosh(f\*x+e)\*sinh(f\*x+e)+1/2\*f\*x+1/2\*e))

**maxima** [A] time = 0.37, size = 324, normalized size = 1.78

$$\frac{1}{3} a^2 d^2 x^3 + a^2 c d x^2 + \frac{1}{8} \left( 4 x^2 + \frac{(2 f x e^{(2 e)} - e^{(2 e)}) e^{(2 f x)}}{f^2} - \frac{(2 f x + 1) e^{(-2 f x - 2 e)}}{f^2} \right) b^2 c d + \frac{1}{48} \left( 8 x^3 + \frac{3 (2 f^2 x^2 e^{(2 e)} - 2 f x e^{(2 e)} + e^{(2 e)})}{f^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+b\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out] 1/3\*a^2\*d^2\*x^3 + a^2\*c\*d\*x^2 + 1/8\*(4\*x^2 + (2\*f\*x\*e^(2\*e) - e^(2\*e))\*e^(2\*f\*x)/f^2 - (2\*f\*x + 1)\*e^(-2\*f\*x - 2\*e)/f^2)\*b^2\*c\*d + 1/48\*(8\*x^3 + 3\*(2\*f^2\*x^2\*e^(2\*e) - 2\*f\*x\*e^(2\*e) + e^(2\*e))\*e^(2\*f\*x)/f^3 - 3\*(2\*f^2\*x^2 + 2\*f\*x + 1)\*e^(-2\*f\*x - 2\*e)/f^3)\*b^2\*d^2 + 1/8\*b^2\*c^2\*(4\*x + e^(2\*f\*x + 2\*e))/f - e^(-2\*f\*x - 2\*e)/f + a^2\*c^2\*x + 2\*a\*b\*c\*d\*((f\*x\*e^e - e^e)\*e^(f\*x)/f^2 - (f\*x + 1)\*e^(-f\*x - e)/f^2) + a\*b\*d^2\*((f^2\*x^2\*e^e - 2\*f\*x\*e^e + 2\*e^e)\*e^(f\*x)/f^3 - (f^2\*x^2 + 2\*f\*x + 2)\*e^(-f\*x - e)/f^3) + 2\*a\*b\*c^2\*sinh(f\*x + e)/f

**mupad [B]** time = 1.33, size = 281, normalized size = 1.54

$$a^2 c^2 x + \frac{b^2 c^2 x}{2} + \frac{a^2 d^2 x^3}{3} + \frac{b^2 d^2 x^3}{6} + \frac{b^2 c^2 \sinh(2e + 2fx)}{4f} + \frac{b^2 d^2 \sinh(2e + 2fx)}{8f^3} + a^2 c d x^2 + \frac{b^2 c d x^2}{2} + \frac{2 a b c^2}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*cosh(e + f\*x))^2\*(c + d\*x)^2,x)

[Out] a^2\*c^2\*x + (b^2\*c^2\*x)/2 + (a^2\*d^2\*x^3)/3 + (b^2\*d^2\*x^3)/6 + (b^2\*c^2\*sinh(2\*e + 2\*f\*x))/(4\*f) + (b^2\*d^2\*sinh(2\*e + 2\*f\*x))/(8\*f^3) + a^2\*c\*d\*x^2 + (b^2\*c\*d\*x^2)/2 + (2\*a\*b\*c^2\*sinh(e + f\*x))/f + (4\*a\*b\*d^2\*sinh(e + f\*x))/f^3 + (b^2\*d^2\*x^2\*sinh(2\*e + 2\*f\*x))/(4\*f) - (b^2\*c\*d\*cosh(2\*e + 2\*f\*x))/(4\*f^2) - (b^2\*d^2\*x\*cosh(2\*e + 2\*f\*x))/(4\*f^2) - (4\*a\*b\*c\*d\*cosh(e + f\*x))/f^2 - (4\*a\*b\*d^2\*x\*cosh(e + f\*x))/f^2 + (2\*a\*b\*d^2\*x^2\*sinh(e + f\*x))/f + (b^2\*c\*d\*x\*sinh(2\*e + 2\*f\*x))/(2\*f) + (4\*a\*b\*c\*d\*x\*sinh(e + f\*x))/f

**sympy [A]** time = 1.65, size = 456, normalized size = 2.51

$$\left\{ \begin{array}{l} a^2 c^2 x + a^2 c d x^2 + \frac{a^2 d^2 x^3}{3} + \frac{2 a b c^2 \sinh(e + f x)}{f} + \frac{4 a b c d x \sinh(e + f x)}{f} - \frac{4 a b c d \cosh(e + f x)}{f^2} + \frac{2 a b d^2 x^2 \sinh(e + f x)}{f} - \frac{4 a b d^2 x \cosh(e + f x)}{f^2} \\ (a + b \cosh(e))^2 \left( c^2 x + c d x^2 + \frac{d^2 x^3}{3} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2\*(a+b\*cosh(f\*x+e))\*\*2,x)

[Out] Piecewise((a\*\*2\*c\*\*2\*x + a\*\*2\*c\*d\*x\*\*2 + a\*\*2\*d\*\*2\*x\*\*3/3 + 2\*a\*b\*c\*\*2\*sinh(e + f\*x)/f + 4\*a\*b\*c\*d\*x\*sinh(e + f\*x)/f - 4\*a\*b\*c\*d\*cosh(e + f\*x)/f\*\*2 + 2\*a\*b\*d\*\*2\*x\*\*2\*sinh(e + f\*x)/f - 4\*a\*b\*d\*\*2\*x\*cosh(e + f\*x)/f\*\*2 + 4\*a\*b\*d\*\*2\*sinh(e + f\*x)/f\*\*3 - b\*\*2\*c\*\*2\*x\*sinh(e + f\*x)\*\*2/2 + b\*\*2\*c\*\*2\*x\*cosh(e + f\*x)\*\*2/2 + b\*\*2\*c\*\*2\*sinh(e + f\*x)\*cosh(e + f\*x)/(2\*f) - b\*\*2\*c\*d\*x\*\*2\*sinh(e + f\*x)\*\*2/2 + b\*\*2\*c\*d\*x\*\*2\*cosh(e + f\*x)\*\*2/2 + b\*\*2\*c\*d\*x\*sinh(e + f\*x)\*cosh(e + f\*x)/f - b\*\*2\*c\*d\*sinh(e + f\*x)\*\*2/(2\*f\*\*2) - b\*\*2\*d\*\*2\*x\*\*3\*sinh(e + f\*x)\*\*2/6 + b\*\*2\*d\*\*2\*x\*\*3\*cosh(e + f\*x)\*\*2/6 + b\*\*2\*d\*\*2\*x\*\*2\*sinh(e + f\*x)\*cosh(e + f\*x)/(2\*f) - b\*\*2\*d\*\*2\*x\*sinh(e + f\*x)\*\*2/(4\*f\*\*2) - b\*\*2\*d\*\*2\*x\*cosh(e + f\*x)\*\*2/(4\*f\*\*2) + b\*\*2\*d\*\*2\*sinh(e + f\*x)\*cosh(e + f\*x)/(4\*f\*\*3), Ne(f, 0)), ((a + b\*cosh(e))\*\*2\*(c\*\*2\*x + c\*d\*x\*\*2 + d\*\*2\*x\*\*3/3), True))

### 3.164 $\int (c + dx)(a + b \cosh(e + fx))^2 dx$

**Optimal.** Leaf size=116

$$\frac{a^2(c + dx)^2}{2d} + \frac{2ab(c + dx) \sinh(e + fx)}{f} - \frac{2abd \cosh(e + fx)}{f^2} + \frac{b^2(c + dx) \sinh(e + fx) \cosh(e + fx)}{2f} + \frac{1}{2} b^2 c x - \frac{b^2 d c}{2}$$

[Out]  $\frac{1}{2} b^2 c x + \frac{1}{4} b^2 d x^2 + \frac{1}{2} a^2 (d x + c)^2 / d - 2 a b d \cosh(f x + e) / f^2 - \frac{1}{4} b^2 (c + d x)^2 \cosh(f x + e)^2 / f^2 + 2 a b (d x + c) \sinh(f x + e) / f + \frac{1}{2} b^2 (d x + c) \cosh(f x + e) \sinh(f x + e) / f$

**Rubi [A]** time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3317, 3296, 2638, 3310}

$$\frac{a^2(c + dx)^2}{2d} + \frac{2ab(c + dx) \sinh(e + fx)}{f} - \frac{2abd \cosh(e + fx)}{f^2} + \frac{b^2(c + dx) \sinh(e + fx) \cosh(e + fx)}{2f} + \frac{1}{2} b^2 c x - \frac{b^2 d c}{2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)\*(a + b\*Cosh[e + f\*x])^2,x]

[Out]  $(b^2 c x) / 2 + (b^2 d x^2) / 4 + (a^2 (c + d x)^2) / (2 d) - (2 a b d \cosh[e + f x]) / f^2 - (b^2 d \cosh[e + f x]^2) / (4 f^2) + (2 a b (c + d x) \sinh[e + f x]) / f + (b^2 (c + d x) \cosh[e + f x] \sinh[e + f x]) / (2 f)$

#### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[((c + d\*x)^m \* Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1) \* Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*(b\*Sine[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)\*(b\*Sine[e + f\*x])^(n - 2), x], x] - Simp[(b\*(c + d\*x)\*Cos[e + f\*x]\*(b\*Sine[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rule 3317

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Sin[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

Rubi steps

$$\begin{aligned} \int (c + dx)(a + b \cosh(e + fx))^2 dx &= \int (a^2(c + dx) + 2ab(c + dx) \cosh(e + fx) + b^2(c + dx) \cosh^2(e + fx)) dx \\ &= \frac{a^2(c + dx)^2}{2d} + (2ab) \int (c + dx) \cosh(e + fx) dx + b^2 \int (c + dx) \cosh^2(e + fx) dx \\ &= \frac{a^2(c + dx)^2}{2d} - \frac{b^2 d \cosh^2(e + fx)}{4f^2} + \frac{2ab(c + dx) \sinh(e + fx)}{f} + \frac{b^2(c + dx) \sinh^2(e + fx)}{4f^2} \\ &= \frac{1}{2} b^2 cx + \frac{1}{4} b^2 dx^2 + \frac{a^2(c + dx)^2}{2d} - \frac{2abd \cosh(e + fx)}{f^2} - \frac{b^2 d \cosh^2(e + fx)}{4f^2} \end{aligned}$$

**Mathematica [A]** time = 0.89, size = 96, normalized size = 0.83

$$\frac{2(2a^2 + b^2)(e + fx)(d(e - fx) - 2cf) - 16abf(c + dx) \sinh(e + fx) + 16abd \cosh(e + fx) - 2b^2f(c + dx) \sinh^2(e + fx)}{8f^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)\*(a + b\*Cosh[e + f\*x])^2, x]

[Out] -1/8\*(2\*(2\*a^2 + b^2)\*(e + f\*x)\*(-2\*c\*f + d\*(e - f\*x)) + 16\*a\*b\*d\*Cosh[e + f\*x] + b^2\*d\*Cosh[2\*(e + f\*x)] - 16\*a\*b\*f\*(c + d\*x)\*Sinh[e + f\*x] - 2\*b^2\*f\*(c + d\*x)\*Sinh[2\*(e + f\*x)])/f^2

**fricas [A]** time = 0.56, size = 122, normalized size = 1.05

$$\frac{2(2a^2 + b^2)df^2x^2 + 4(2a^2 + b^2)cf^2x - b^2d \cosh^2(fx + e) - b^2d \sinh^2(fx + e) - 16abd \cosh(fx + e) + 4(4abd \cosh(fx + e) \sinh(fx + e) - b^2d \cosh^2(fx + e) \sinh(fx + e))}{8f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+b\*cosh(f\*x+e))^2,x, algorithm="fricas")

[Out] 1/8\*(2\*(2\*a^2 + b^2)\*d\*f^2\*x^2 + 4\*(2\*a^2 + b^2)\*c\*f^2\*x - b^2\*d\*cosh(f\*x + e)^2 - b^2\*d\*sinh(f\*x + e)^2 - 16\*a\*b\*d\*cosh(f\*x + e) + 4\*(4\*a\*b\*d\*f\*x + 4\*a\*b\*c\*f + (b^2\*d\*f\*x + b^2\*c\*f)\*cosh(f\*x + e))\*sinh(f\*x + e))/f^2

**giac** [A] time = 0.14, size = 164, normalized size = 1.41

$$\frac{1}{2}a^2dx^2 + \frac{1}{4}b^2dx^2 + a^2cx + \frac{1}{2}b^2cx + \frac{(2b^2dfx + 2b^2cf - b^2d)e^{(2fx+2e)}}{16f^2} + \frac{(abdfx + abcf - abd)e^{(fx+e)}}{f^2} - \frac{(abdfx + abcf - abd)e^{(fx+e)}}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+b\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out]  $\frac{1}{2}a^2d^2x^2 + \frac{1}{4}b^2d^2x^2 + a^2c^2x + \frac{1}{2}b^2c^2x + \frac{1}{16}(2b^2d^2fx + 2b^2c^2f - b^2d^2)e^{(2fx+2e)}/f^2 + (abdfx + abcf - abd)e^{(fx+e)}/f^2 - (abdfx + abcf + abd)e^{(-fx-e)}/f^2 - \frac{1}{16}(2b^2d^2fx + 2b^2c^2f + b^2d^2)e^{(-2fx-2e)}/f^2$

**maple** [A] time = 0.07, size = 208, normalized size = 1.79

$$\frac{d^2a^2(fx+e)^2}{2f} + \frac{2dab((fx+e)\sinh(fx+e) - \cosh(fx+e))}{f} + \frac{db^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2} + \frac{(fx+e)^2}{4} - \frac{(\cosh^2(fx+e))}{4}\right)}{f} - \frac{dea^2(fx+e)}{f} - \frac{2deab\sinh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)\*(a+b\*cosh(f\*x+e))^2,x)

[Out]  $\frac{1}{f}\left(\frac{1}{2}f^2d^2a^2(fx+e)^2 + 2f^2d^2ab((fx+e)\sinh(fx+e) - \cosh(fx+e)) + \frac{1}{4}f^2d^2b^2((fx+e)\cosh(fx+e)\sinh(fx+e) + \frac{1}{4}(fx+e)^2 - \frac{1}{4}\cosh^2(fx+e)) - d^2e/f^2a^2(fx+e) - 2d^2e/f^2ab\sinh(fx+e) - d^2e/f^2b^2(\frac{1}{2}\cosh(fx+e)\sinh(fx+e) + \frac{1}{2}fx + \frac{1}{2}e) + c^2a^2(fx+e) + 2c^2ab\sinh(fx+e) + c^2b^2(\frac{1}{2}\cosh(fx+e)\sinh(fx+e) + \frac{1}{2}fx + \frac{1}{2}e)\right)$

**maxima** [A] time = 0.37, size = 165, normalized size = 1.42

$$\frac{1}{2}a^2dx^2 + \frac{1}{16}\left(4x^2 + \frac{(2fxe^{(2e)} - e^{(2e)})e^{(2fx)}}{f^2} - \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)b^2d + \frac{1}{8}b^2c\left(4x + \frac{e^{(2fx+2e)}}{f} - \frac{e^{(-2fx-2e)}}{f}\right) + a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+b\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out]  $\frac{1}{2}a^2d^2x^2 + \frac{1}{16}(4x^2 + (2fxe^{(2e)} - e^{(2e)})e^{(2fx)}/f^2 - (2fx+1)e^{(-2fx-2e)}/f^2)b^2d + \frac{1}{8}b^2c^2(4x + e^{(2fx+2e)}/f - e^{(-2fx-2e)}/f) + a^2c^2x + ab^2d^2((fx+e)e^{(fx+e)} - e^{(fx+e)})e^{(fx+e)}/f^2 - (fx+1)e^{(-fx-e)}/f^2 + 2ab^2c^2\sinh(fx+e)/f$



**mupad [B]** time = 0.15, size = 135, normalized size = 1.16

$$\frac{a^2 dx^2}{2} + \frac{b^2 dx^2}{4} + a^2 cx + \frac{b^2 cx}{2} - \frac{b^2 d \cosh(e + fx)^2}{4f^2} + \frac{b^2 c \cosh(e + fx) \sinh(e + fx)}{2f} - \frac{2abd \cosh(e + fx)}{f^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*cosh(e + f*x))^2*(c + d*x), x)`

[Out]  $(a^2 dx^2)/2 + (b^2 dx^2)/4 + a^2 cx + (b^2 cx)/2 - (b^2 d \cosh(e + fx))^2 / (4f^2) + (b^2 c \cosh(e + fx) \sinh(e + fx)) / (2f) - (2abd \cosh(e + fx)) / f^2 + (2abdc \sinh(e + fx)) / f + (2abd x \sinh(e + fx)) / f + (b^2 dx \cosh(e + fx) \sinh(e + fx)) / (2f)$

**sympy [A]** time = 0.66, size = 219, normalized size = 1.89

$$\left\{ \begin{array}{l} a^2 cx + \frac{a^2 dx^2}{2} + \frac{2abc \sinh(e+fx)}{f} + \frac{2abdx \sinh(e+fx)}{f} - \frac{2abd \cosh(e+fx)}{f^2} - \frac{b^2 cx \sinh^2(e+fx)}{2} + \frac{b^2 cx \cosh^2(e+fx)}{2} + \frac{b^2 c \sinh(e+fx)}{2f} \\ (a + b \cosh(e))^2 \left( cx + \frac{dx^2}{2} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*(a+b*cosh(f*x+e))**2,x)`

[Out] `Piecewise((a**2*c*x + a**2*d*x**2/2 + 2*a*b*c*sinh(e + f*x)/f + 2*a*b*d*x*sinh(e + f*x)/f - 2*a*b*d*cosh(e + f*x)/f**2 - b**2*c*x*sinh(e + f*x)**2/2 + b**2*c*x*cosh(e + f*x)**2/2 + b**2*c*sinh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d*x**2*sinh(e + f*x)**2/4 + b**2*d*x**2*cosh(e + f*x)**2/4 + b**2*d*x*sinh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d*sinh(e + f*x)**2/(4*f**2), Ne(f, 0)), ((a + b*cosh(e))**2*(c*x + d*x**2/2), True))`

$$3.165 \quad \int \frac{(a+b \cosh(e+fx))^2}{c+dx} dx$$

**Optimal.** Leaf size=156

$$\frac{a^2 \log(c+dx)}{d} + \frac{2ab \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d} + \frac{2ab \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d} + \frac{b^2 \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{2d}$$

[Out]  $1/2*b^2*\operatorname{Chi}(2*c*f/d+2*f*x)*\cosh(-2*e+2*c*f/d)/d+2*a*b*\operatorname{Chi}(c*f/d+f*x)*\cosh(-e+c*f/d)/d+a^2*\ln(d*x+c)/d+1/2*b^2*\ln(d*x+c)/d-1/2*b^2*\operatorname{Shi}(2*c*f/d+2*f*x)*\sinh(-2*e+2*c*f/d)/d-2*a*b*\operatorname{Shi}(c*f/d+f*x)*\sinh(-e+c*f/d)/d$

**Rubi [A]** time = 0.31, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {3317, 3303, 3298, 3301, 3312}

$$\frac{a^2 \log(c+dx)}{d} + \frac{2ab \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d} + \frac{2ab \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d} + \frac{b^2 \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{2d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{Cosh}[e + f*x])^2/(c + d*x), x]$

[Out]  $(2*a*b*\operatorname{Cosh}[e - (c*f)/d]*\operatorname{CoshIntegral}[(c*f)/d + f*x])/d + (b^2*\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x])/(2*d) + (a^2*\operatorname{Log}[c + d*x])/d + (b^2*\operatorname{Log}[c + d*x])/(2*d) + (2*a*b*\operatorname{Sinh}[e - (c*f)/d]*\operatorname{SinhIntegral}[(c*f)/d + f*x])/d + (b^2*\operatorname{Sinh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/(2*d)$

#### Rule 3298

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)]]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[(I*\operatorname{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \operatorname{EqQ}[d*e - c*f*fz*I, 0]$

#### Rule 3301

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)]]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \operatorname{EqQ}[d*(e - \operatorname{Pi}/2) - c*f*fz*I, 0]$

#### Rule 3303

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Dist}[\operatorname{Cos}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \operatorname{Dist}[\operatorname{Sin}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Cos}[(c*f)/d + f*x]/(c + d*x), x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \ \&\&$

NeQ[d\*e - c\*f, 0]

### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rule 3317

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_.)), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Sin[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \cosh(e + fx))^2}{c + dx} dx &= \int \left( \frac{a^2}{c + dx} + \frac{2ab \cosh(e + fx)}{c + dx} + \frac{b^2 \cosh^2(e + fx)}{c + dx} \right) dx \\
 &= \frac{a^2 \log(c + dx)}{d} + (2ab) \int \frac{\cosh(e + fx)}{c + dx} dx + b^2 \int \frac{\cosh^2(e + fx)}{c + dx} dx \\
 &= \frac{a^2 \log(c + dx)}{d} + b^2 \int \left( \frac{1}{2(c + dx)} + \frac{\cosh(2e + 2fx)}{2(c + dx)} \right) dx + \left( 2ab \cosh \left( e - \frac{cf}{d} \right) \right) \\
 &= \frac{2ab \cosh \left( e - \frac{cf}{d} \right) \text{Chi} \left( \frac{cf}{d} + fx \right)}{d} + \frac{a^2 \log(c + dx)}{d} + \frac{b^2 \log(c + dx)}{2d} + \frac{2ab \sinh \left( e - \frac{cf}{d} \right)}{d} \\
 &= \frac{2ab \cosh \left( e - \frac{cf}{d} \right) \text{Chi} \left( \frac{cf}{d} + fx \right)}{d} + \frac{a^2 \log(c + dx)}{d} + \frac{b^2 \log(c + dx)}{2d} + \frac{2ab \sinh \left( e - \frac{cf}{d} \right)}{d} \\
 &= \frac{2ab \cosh \left( e - \frac{cf}{d} \right) \text{Chi} \left( \frac{cf}{d} + fx \right)}{d} + \frac{b^2 \cosh \left( 2e - \frac{2cf}{d} \right) \text{Chi} \left( \frac{2cf}{d} + 2fx \right)}{2d} + \frac{a^2 \log(c + dx)}{d}
 \end{aligned}$$

**Mathematica [A]** time = 0.31, size = 133, normalized size = 0.85

$$\frac{2a^2 \log(c + dx) + 4ab \text{Chi} \left( f \left( \frac{c}{d} + x \right) \right) \cosh \left( e - \frac{cf}{d} \right) + 4ab \sinh \left( e - \frac{cf}{d} \right) \text{Shi} \left( f \left( \frac{c}{d} + x \right) \right) + b^2 \text{Chi} \left( \frac{2f(c+dx)}{d} \right) \cosh \left( 2e - \frac{2cf}{d} \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cosh[e + f\*x])^2/(c + d\*x), x]

[Out] (4\*a\*b\*Cosh[e - (c\*f)/d]\*CoshIntegral[f\*(c/d + x)] + b^2\*Cosh[2\*e - (2\*c\*f)/d]\*CoshIntegral[(2\*f\*(c + d\*x))/d] + 2\*a^2\*Log[c + d\*x] + b^2\*Log[c + d\*x] + 4\*a\*b\*Sinh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)] + b^2\*Sinh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*f\*(c + d\*x))/d])/(2\*d)

**fricas** [A] time = 0.51, size = 230, normalized size = 1.47

$$4 \left( ab \operatorname{Ei} \left( \frac{dfx+cf}{d} \right) + ab \operatorname{Ei} \left( -\frac{dfx+cf}{d} \right) \right) \cosh \left( -\frac{de-cf}{d} \right) + \left( b^2 \operatorname{Ei} \left( \frac{2(dfx+cf)}{d} \right) + b^2 \operatorname{Ei} \left( -\frac{2(dfx+cf)}{d} \right) \right) \cosh \left( -\frac{2(de-cf)}{d} \right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cosh(f\*x+e))^2/(d\*x+c), x, algorithm="fricas")

[Out] 1/4\*(4\*(a\*b\*Ei((d\*f\*x + c\*f)/d) + a\*b\*Ei(-(d\*f\*x + c\*f)/d))\*cosh(-(d\*e - c\*f)/d) + (b^2\*Ei(2\*(d\*f\*x + c\*f)/d) + b^2\*Ei(-2\*(d\*f\*x + c\*f)/d))\*cosh(-2\*(d\*e - c\*f)/d) + 2\*(2\*a^2 + b^2)\*log(d\*x + c) - 4\*(a\*b\*Ei((d\*f\*x + c\*f)/d) - a\*b\*Ei(-(d\*f\*x + c\*f)/d))\*sinh(-(d\*e - c\*f)/d) - (b^2\*Ei(2\*(d\*f\*x + c\*f)/d) - b^2\*Ei(-2\*(d\*f\*x + c\*f)/d))\*sinh(-2\*(d\*e - c\*f)/d))/d

**giac** [A] time = 0.13, size = 148, normalized size = 0.95

$$\frac{b^2 \operatorname{Ei} \left( -\frac{2(dfx+cf)}{d} \right) e^{\left( \frac{2cf}{d} - 2e \right)} + 4 ab \operatorname{Ei} \left( -\frac{dfx+cf}{d} \right) e^{\left( \frac{cf}{d} - e \right)} + 4 ab \operatorname{Ei} \left( \frac{dfx+cf}{d} \right) e^{\left( -\frac{cf}{d} + e \right)} + b^2 \operatorname{Ei} \left( \frac{2(dfx+cf)}{d} \right) e^{\left( -\frac{2cf}{d} + 2e \right)} + 4a}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cosh(f\*x+e))^2/(d\*x+c), x, algorithm="giac")

[Out] 1/4\*(b^2\*Ei(-2\*(d\*f\*x + c\*f)/d)\*e^(2\*c\*f/d - 2\*e) + 4\*a\*b\*Ei(-(d\*f\*x + c\*f)/d)\*e^(c\*f/d - e) + 4\*a\*b\*Ei((d\*f\*x + c\*f)/d)\*e^(-c\*f/d + e) + b^2\*Ei(2\*(d\*f\*x + c\*f)/d)\*e^(-2\*c\*f/d + 2\*e) + 4\*a^2\*log(d\*x + c) + 2\*b^2\*log(d\*x + c))/d

**maple** [A] time = 0.33, size = 202, normalized size = 1.29

$$\frac{ab e^{\frac{cf-de}{d}} \operatorname{Ei} \left( 1, fx + e + \frac{cf-de}{d} \right) - ab e^{-\frac{cf-de}{d}} \operatorname{Ei} \left( 1, -fx - e - \frac{cf-de}{d} \right)}{d} + \frac{a^2 \ln(dx + c)}{d} + \frac{b^2 \ln(dx + c)}{2d} - \frac{b^2 e^{\frac{2cf-2de}{d}} \operatorname{Ei} \left( 1, \dots \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cosh(f\*x+e))^2/(d\*x+c), x)

[Out]  $-a*b/d*\exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-a*b/d*\exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)+a^2*\ln(d*x+c)/d+1/2*b^2*\ln(d*x+c)/d-1/4*b^2/d*\exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/4*b^2/d*\exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)$

**maxima** [A] time = 0.43, size = 148, normalized size = 0.95

$$-\frac{1}{4}b^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_1\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{e^{\left(2e-\frac{2cf}{d}\right)}E_1\left(-\frac{2(dx+c)f}{d}\right)}{d}-\frac{2\log(dx+c)}{d}\right)-ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cosh(f*x+e))^2/(d*x+c),x, algorithm="maxima")`

[Out]  $-1/4*b^2*(e^{(-2*e+2*c*f/d)*\exp\_integral\_e(1,2*(d*x+c)*f/d)/d}+e^{(2*e-2*c*f/d)*\exp\_integral\_e(1,-2*(d*x+c)*f/d)/d}-2*\log(d*x+c)/d)-a*b*(e^{(-e+c*f/d)*\exp\_integral\_e(1,(d*x+c)*f/d)/d}+e^{(e-c*f/d)*\exp\_integral\_e(1,-(d*x+c)*f/d)/d})+a^2*\log(d*x+c)/d$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \cosh(e + f x))^2}{c + d x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*cosh(e + f*x))^2/(c + d*x),x)`

[Out] `int((a + b*cosh(e + f*x))^2/(c + d*x), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \cosh(e + f x))^2}{c + d x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cosh(f*x+e))**2/(d*x+c),x)`

[Out] `Integral((a + b*cosh(e + f*x))**2/(c + d*x), x)`

$$3.166 \quad \int \frac{(a+b \cosh(e+fx))^2}{(c+dx)^2} dx$$

**Optimal.** Leaf size=183

$$-\frac{a^2}{d(c+dx)} + \frac{2abf \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{2abf \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^2} - \frac{2ab \cosh(e+fx)}{d(c+dx)} + \frac{b^2 f \operatorname{Chi}\left(2\right)}{d^2}$$

[Out]  $-a^2/d/(d*x+c) - 2*a*b*\cosh(f*x+e)/d/(d*x+c) - b^2*\cosh(f*x+e)^2/d/(d*x+c) + 2*a*b*f*\cosh(-e+c*f/d)*\operatorname{Shi}(c*f/d+f*x)/d^2 + b^2*f*\cosh(-2*e+2*c*f/d)*\operatorname{Shi}(2*c*f/d+2*f*x)/d^2 - b^2*f*\operatorname{Chi}(2*c*f/d+2*f*x)*\sinh(-2*e+2*c*f/d)/d^2 - 2*a*b*f*\operatorname{Chi}(c*f/d+f*x)*\sinh(-e+c*f/d)/d^2$

**Rubi [A]** time = 0.34, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$ , Rules used = {3317, 3297, 3303, 3298, 3301, 3313, 12}

$$-\frac{a^2}{d(c+dx)} + \frac{2abf \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{2abf \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^2} - \frac{2ab \cosh(e+fx)}{d(c+dx)} + \frac{b^2 f \operatorname{Chi}\left(2\right)}{d^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{Cosh}[e + f*x])^2/(c + d*x)^2, x]$

[Out]  $-(a^2/(d*(c + d*x))) - (2*a*b*\operatorname{Cosh}[e + f*x])/(d*(c + d*x)) - (b^2*\operatorname{Cosh}[e + f*x]^2)/(d*(c + d*x)) + (b^2*f*\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x]*\operatorname{Sinh}[2*e - (2*c*f)/d])/d^2 + (2*a*b*f*\operatorname{CoshIntegral}[(c*f)/d + f*x]*\operatorname{Sinh}[e - (c*f)/d])/d^2 + (2*a*b*f*\operatorname{Cosh}[e - (c*f)/d]*\operatorname{SinhIntegral}[(c*f)/d + f*x])/d^2 + (b^2*f*\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/d^2$

**Rule 12**

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match}Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

**Rule 3297**

$\operatorname{Int}[(c_*) + (d_*)(x_*)^{(m_*)}*\sin[(e_*) + (f_*)(x_*)], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m+1)}*\operatorname{Sin}[e + f*x]/(d*(m+1)), x] - \operatorname{Dist}[f/(d*(m+1)), \operatorname{Int}[(c + d*x)^{(m+1)}*\operatorname{Cos}[e + f*x], x], x] /; \operatorname{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \operatorname{LtQ}[m, -1]$

**Rule 3298**

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d],
Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

### Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Simp[
((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m + 1)),
Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n - 1), x], x], x]
/; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]
```

### Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.)), x_Symbol]
:> Int[ExpandIntegrand[(c + d*x)^m, (a + b*SIN[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \cosh(e + fx))^2}{(c + dx)^2} dx &= \int \left( \frac{a^2}{(c + dx)^2} + \frac{2ab \cosh(e + fx)}{(c + dx)^2} + \frac{b^2 \cosh^2(e + fx)}{(c + dx)^2} \right) dx \\
&= -\frac{a^2}{d(c + dx)} + (2ab) \int \frac{\cosh(e + fx)}{(c + dx)^2} dx + b^2 \int \frac{\cosh^2(e + fx)}{(c + dx)^2} dx \\
&= -\frac{a^2}{d(c + dx)} - \frac{2ab \cosh(e + fx)}{d(c + dx)} - \frac{b^2 \cosh^2(e + fx)}{d(c + dx)} + \frac{(2abf) \int \frac{\sinh(e+fx)}{c+dx} dx}{d} + \frac{(2b^2f) \int \frac{\sinh(2e+2fx)}{c+dx} dx}{d} \\
&= -\frac{a^2}{d(c + dx)} - \frac{2ab \cosh(e + fx)}{d(c + dx)} - \frac{b^2 \cosh^2(e + fx)}{d(c + dx)} + \frac{(b^2 f) \int \frac{\sinh(2e+2fx)}{c+dx} dx}{d} + \frac{(2abf) \int \frac{\sinh(e+fx)}{c+dx} dx}{d} \\
&= -\frac{a^2}{d(c + dx)} - \frac{2ab \cosh(e + fx)}{d(c + dx)} - \frac{b^2 \cosh^2(e + fx)}{d(c + dx)} + \frac{2abf \operatorname{Chi}\left(\frac{cf}{d} + fx\right) \sinh\left(e - \frac{cf}{d}\right)}{d^2} \\
&= -\frac{a^2}{d(c + dx)} - \frac{2ab \cosh(e + fx)}{d(c + dx)} - \frac{b^2 \cosh^2(e + fx)}{d(c + dx)} + \frac{b^2 f \operatorname{Chi}\left(\frac{2cf}{d} + 2fx\right) \sinh\left(2e - \frac{2cf}{d}\right)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.80, size = 233, normalized size = 1.27

$$-2a^2d + 4abf(c + dx)\operatorname{Chi}\left(f\left(\frac{c}{d} + x\right)\right) \sinh\left(e - \frac{cf}{d}\right) + 4abcf \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(f\left(\frac{c}{d} + x\right)\right) + 4abdfx \cosh\left(e - \frac{cf}{d}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cosh[e + f\*x])^2/(c + d\*x)^2,x]

[Out] (-2\*a^2\*d - b^2\*d - 4\*a\*b\*d\*Cosh[e + f\*x] - b^2\*d\*Cosh[2\*(e + f\*x)] + 2\*b^2\*f\*(c + d\*x)\*CoshIntegral[(2\*f\*(c + d\*x))/d]\*Sinh[2\*e - (2\*c\*f)/d] + 4\*a\*b\*f\*(c + d\*x)\*CoshIntegral[f\*(c/d + x)]\*Sinh[e - (c\*f)/d] + 4\*a\*b\*c\*f\*Cosh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)] + 4\*a\*b\*d\*f\*x\*Cosh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)] + 2\*b^2\*c\*f\*Cosh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*f\*(c + d\*x))/d] + 2\*b^2\*d\*f\*x\*Cosh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*f\*(c + d\*x))/d])/(2\*d^2\*(c + d\*x))

**fricas [A]** time = 0.50, size = 355, normalized size = 1.94

$$b^2d \cosh(fx + e)^2 + b^2d \sinh(fx + e)^2 + 4abd \cosh(fx + e) + (2a^2 + b^2)d - 2\left((abdfx + abcf)\operatorname{Ei}\left(\frac{dfx+cf}{d}\right) - \dots\right)$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*d*cosh(f*x + e)^2 + b^2*d*sinh(f*x + e)^2 + 4*a*b*d*cosh(f*x + e)
+ (2*a^2 + b^2)*d - 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/d) - (a*b*d*
f*x + a*b*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - ((b^2*d*f*x + b
^2*c*f)*Ei(2*(d*f*x + c*f)/d) - (b^2*d*f*x + b^2*c*f)*Ei(-2*(d*f*x + c*f)/d
))*cosh(-2*(d*e - c*f)/d) + 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/d) +
(a*b*d*f*x + a*b*c*f)*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) + ((b^2*d*
f*x + b^2*c*f)*Ei(2*(d*f*x + c*f)/d) + (b^2*d*f*x + b^2*c*f)*Ei(-2*(d*f*x +
c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^3*x + c*d^2)
```

**giac** [B] time = 0.25, size = 1227, normalized size = 6.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*(d*x + c)*b^2*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*Ei(2*((d*x +
c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^(2*(c*f - d*e)/d)
- 2*b^2*c*f^3*Ei(2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d
*e)/d)*e^(2*(c*f - d*e)/d) + 4*(d*x + c)*a*b*(c*f/(d*x + c) - f - d*e/(d*x
+ c))*f^2*Ei(((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)
*e^((c*f - d*e)/d) - 4*a*b*c*f^3*Ei(((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*
x + c)) - c*f + d*e)/d)*e^((c*f - d*e)/d) - 4*(d*x + c)*a*b*(c*f/(d*x + c)
- f - d*e/(d*x + c))*f^2*Ei(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))
- c*f + d*e)/d)*e^(-(c*f - d*e)/d) + 4*a*b*c*f^3*Ei(-((d*x + c)*(c*f/(d*x
+ c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^(-(c*f - d*e)/d) - 2*(d*x + c)*
b^2*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*Ei(-2*((d*x + c)*(c*f/(d*x + c)
- f - d*e/(d*x + c)) - c*f + d*e)/d)*e^(-2*(c*f - d*e)/d) + 2*b^2*c*f^3*Ei
(-2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^(-2*(c
*f - d*e)/d) + 2*b^2*d*f^2*Ei(2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x +
c)) - c*f + d*e)/d)*e^(2*(c*f - d*e)/d + 1) + 4*a*b*d*f^2*Ei(((d*x + c)*(c*
f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^((c*f - d*e)/d + 1) - 4*
a*b*d*f^2*Ei(-((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d
)*e^(-(c*f - d*e)/d + 1) - 2*b^2*d*f^2*Ei(-2*((d*x + c)*(c*f/(d*x + c) - f
- d*e/(d*x + c)) - c*f + d*e)/d)*e^(-2*(c*f - d*e)/d + 1) - b^2*d*f^2*e^(2*
(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))/d) - 4*a*b*d*f^2*e^((d*x + c)
*(c*f/(d*x + c) - f - d*e/(d*x + c))/d) - 4*a*b*d*f^2*e^(-(d*x + c)*(c*f/(d
*x + c) - f - d*e/(d*x + c))/d) - b^2*d*f^2*e^(-2*(d*x + c)*(c*f/(d*x + c)
- f - d*e/(d*x + c))/d) - 4*a^2*d*f^2 - 2*b^2*d*f^2)*d^2/(((d*x + c)*d^4*(c
*f/(d*x + c) - f - d*e/(d*x + c)) - c*d^4*f + d^5*e)*f)
```

**maple [A]** time = 0.36, size = 319, normalized size = 1.74

$$-\frac{fab e^{-fx-e}}{d(df x + cf)} + \frac{fab e^{\frac{cf-de}{d}} \operatorname{Ei}\left(1, fx + e + \frac{cf-de}{d}\right)}{d^2} - \frac{abf e^{fx+e}}{d^2\left(\frac{cf}{d} + fx\right)} - \frac{abf e^{-\frac{cf-de}{d}} \operatorname{Ei}\left(1, -fx - e - \frac{cf-de}{d}\right)}{d^2} - \frac{a^2}{d(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*cosh(f*x+e))^2/(d*x+c)^2,x)`

[Out] `-f*a*b*exp(-f*x-e)/d/(d*f*x+c*f)+f*a*b/d^2*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-a*b*f/d^2*exp(f*x+e)/(c*f/d+f*x)-a*b*f/d^2*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)-a^2/d/(d*x+c)-1/2*b^2/d/(d*x+c)-1/4*f*b^2*exp(-2*f*x-2*e)/d/(d*f*x+c*f)+1/2*f*b^2/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/4*f*b^2/d^2*exp(2*f*x+2*e)/(c*f/d+f*x)-1/2*f*b^2/d^2*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)`

**maxima [A]** time = 0.44, size = 181, normalized size = 0.99

$$-\frac{1}{4} b^2 \left( \frac{e^{\left(-2e + \frac{2cf}{d}\right)} E_2\left(\frac{2(dx+c)f}{d}\right)}{(dx+c)d} + \frac{e^{\left(2e - \frac{2cf}{d}\right)} E_2\left(-\frac{2(dx+c)f}{d}\right)}{(dx+c)d} + \frac{2}{d^2x + cd} \right) - ab \left( \frac{e^{\left(-e + \frac{cf}{d}\right)} E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d} + \frac{e^{\left(e - \frac{cf}{d}\right)} E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")`

[Out] `-1/4*b^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)*d) + e^(2*e - 2*c*f/d)*exp_integral_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d) + 2/(d^2*x + c*d) - a*b*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/((d*x + c)*d) + e^(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d) - a^2/(d^2*x + c*d)`

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \cosh(e + f x))^2}{(c + d x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*cosh(e + f*x))^2/(c + d*x)^2,x)`

[Out] `int((a + b*cosh(e + f*x))^2/(c + d*x)^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \cosh(e + fx))^2}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cosh(f\*x+e))\*\*2/(d\*x+c)\*\*2,x)

[Out] Integral((a + b\*cosh(e + f\*x))\*\*2/(c + d\*x)\*\*2, x)

$$3.167 \quad \int \frac{(a+b \cosh(e+fx))^2}{(c+dx)^3} dx$$

**Optimal.** Leaf size=242

$$-\frac{a^2}{2d(c+dx)^2} + \frac{abf^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d^3} + \frac{abf^2 \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^3} - \frac{abf \sinh(e+fx)}{d^2(c+dx)} - \frac{ab \cosh(e+fx)}{d(c+dx)}$$

[Out]  $-1/2*a^2/d/(d*x+c)^2+b^2*f^2*\operatorname{Chi}(2*c*f/d+2*f*x)*\cosh(-2*e+2*c*f/d)/d^3+a*b*f^2*\operatorname{Chi}(c*f/d+f*x)*\cosh(-e+c*f/d)/d^3-a*b*\cosh(f*x+e)/d/(d*x+c)^2-1/2*b^2*c*\cosh(f*x+e)^2/d/(d*x+c)^2-b^2*f^2*\operatorname{Shi}(2*c*f/d+2*f*x)*\sinh(-2*e+2*c*f/d)/d^3-a*b*f^2*\operatorname{Shi}(c*f/d+f*x)*\sinh(-e+c*f/d)/d^3-a*b*f*\sinh(f*x+e)/d^2/(d*x+c)-b^2*f*\cosh(f*x+e)*\sinh(f*x+e)/d^2/(d*x+c)$

**Rubi [A]** time = 0.43, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {3317, 3297, 3303, 3298, 3301, 3314, 31, 3312}

$$-\frac{a^2}{2d(c+dx)^2} + \frac{abf^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d^3} + \frac{abf^2 \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^3} - \frac{abf \sinh(e+fx)}{d^2(c+dx)} - \frac{ab \cosh(e+fx)}{d(c+dx)}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*Cosh[e + f*x])^2/(c + d*x)^3,x]`

[Out]  $-a^2/(2*d*(c+d*x)^2) - (a*b*Cosh[e+f*x])/(d*(c+d*x)^2) - (b^2*Cosh[e+f*x]^2)/(2*d*(c+d*x)^2) + (a*b*f^2*Cosh[e-(c*f)/d]*CoshIntegral[(c*f)/d+f*x])/d^3 + (b^2*f^2*Cosh[2*e-(2*c*f)/d]*CoshIntegral[(2*c*f)/d+2*f*x])/d^3 - (a*b*f*Sinh[e+f*x])/(d^2*(c+d*x)) - (b^2*f*Cosh[e+f*x]*Sinh[e+f*x])/(d^2*(c+d*x)) + (a*b*f^2*Sinh[e-(c*f)/d]*SinhIntegral[(c*f)/d+f*x])/d^3 + (b^2*f^2*Sinh[2*e-(2*c*f)/d]*SinhIntegral[(2*c*f)/d+2*f*x])/d^3$

### Rule 31

`Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

### Rule 3297

`Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m+1)*Sin[e + f*x])/(d*(m+1)), x] - Dist[f/(d*(m+1)), Int[(c + d*x)^(m+1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]`

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f,
fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz},
x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)
]/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f,
m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol]
:> Simp[((c + d*x)^(m + 1)*(b*Ssin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Ssin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Ssin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*(b*Ssin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Ssin[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \cosh(e + fx))^2}{(c + dx)^3} dx &= \int \left( \frac{a^2}{(c + dx)^3} + \frac{2ab \cosh(e + fx)}{(c + dx)^3} + \frac{b^2 \cosh^2(e + fx)}{(c + dx)^3} \right) dx \\
&= -\frac{a^2}{2d(c + dx)^2} + (2ab) \int \frac{\cosh(e + fx)}{(c + dx)^3} dx + b^2 \int \frac{\cosh^2(e + fx)}{(c + dx)^3} dx \\
&= -\frac{a^2}{2d(c + dx)^2} - \frac{ab \cosh(e + fx)}{d(c + dx)^2} - \frac{b^2 \cosh^2(e + fx)}{2d(c + dx)^2} - \frac{b^2 f \cosh(e + fx) \sinh(e + fx)}{d^2(c + dx)} \\
&= -\frac{a^2}{2d(c + dx)^2} - \frac{ab \cosh(e + fx)}{d(c + dx)^2} - \frac{b^2 \cosh^2(e + fx)}{2d(c + dx)^2} - \frac{b^2 f^2 \log(c + dx)}{d^3} - \frac{ab f \sinh(e + fx)}{d^2(c + dx)} \\
&= -\frac{a^2}{2d(c + dx)^2} - \frac{ab \cosh(e + fx)}{d(c + dx)^2} - \frac{b^2 \cosh^2(e + fx)}{2d(c + dx)^2} - \frac{ab f \sinh(e + fx)}{d^2(c + dx)} - \frac{b^2 f \cosh(e + fx)}{d^2(c + dx)} \\
&= -\frac{a^2}{2d(c + dx)^2} - \frac{ab \cosh(e + fx)}{d(c + dx)^2} - \frac{b^2 \cosh^2(e + fx)}{2d(c + dx)^2} + \frac{ab f^2 \cosh\left(e - \frac{cf}{d}\right) \text{Chi}\left(\frac{cf}{d}\right)}{d^3} \\
&= -\frac{a^2}{2d(c + dx)^2} - \frac{ab \cosh(e + fx)}{d(c + dx)^2} - \frac{b^2 \cosh^2(e + fx)}{2d(c + dx)^2} + \frac{ab f^2 \cosh\left(e - \frac{cf}{d}\right) \text{Chi}\left(\frac{cf}{d}\right)}{d^3}
\end{aligned}$$

**Mathematica [A]** time = 1.32, size = 394, normalized size = 1.63

$$\frac{2a^2d^2 - 4abc^2f^2 \sinh\left(e - \frac{cf}{d}\right) \text{Shi}\left(f\left(\frac{c}{d} + x\right)\right) - 4abf^2(c + dx)^2 \text{Chi}\left(f\left(\frac{c}{d} + x\right)\right) \cosh\left(e - \frac{cf}{d}\right) - 4abd^2f^2x^2 \sinh\left(e - \frac{cf}{d}\right)}{d^3(c + dx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Cosh[e + f\*x])^2/(c + d\*x)^3,x]

[Out] -1/4\*(2\*a^2\*d^2 + b^2\*d^2 + 4\*a\*b\*d^2\*Cosh[e + f\*x] + b^2\*d^2\*Cosh[2\*(e + f\*x)]) - 4\*a\*b\*f^2\*(c + d\*x)^2\*Cosh[e - (c\*f)/d]\*CoshIntegral[f\*(c/d + x)] - 4\*b^2\*f^2\*(c + d\*x)^2\*Cosh[2\*e - (2\*c\*f)/d]\*CoshIntegral[(2\*f\*(c + d\*x))/d] + 4\*a\*b\*c\*d\*f\*Sinh[e + f\*x] + 4\*a\*b\*d^2\*f\*x\*Sinh[e + f\*x] + 2\*b^2\*c\*d\*f\*Sinh[2\*(e + f\*x)] + 2\*b^2\*d^2\*f\*x\*Sinh[2\*(e + f\*x)] - 4\*a\*b\*c^2\*f^2\*Sinh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)] - 8\*a\*b\*c\*d\*f^2\*x\*Sinh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)] - 4\*a\*b\*d^2\*f^2\*x^2\*Sinh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)] - 4\*b^2\*c^2\*f^2\*Sinh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*f\*(c + d\*x))/d] - 8\*b^2\*c\*d\*f^2\*x\*Sinh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*f\*(c + d\*x))/d] - 4\*b^2\*d^2\*f^2\*x^2\*Sinh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*f\*(c + d\*x))/d]/(d^3\*(c + d\*x)^2)

**fricas** [B] time = 0.52, size = 586, normalized size = 2.42

$$b^2 d^2 \cosh(fx + e)^2 + b^2 d^2 \sinh(fx + e)^2 + 4 abd^2 \cosh(fx + e) + (2a^2 + b^2)d^2 - 2 \left( (abd^2 f^2 x^2 + 2abcd f^2 x \right.$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cosh(f\*x+e))^2/(d\*x+c)^3,x, algorithm="fricas")

[Out] 
$$-1/4*(b^2*d^2*\cosh(f*x + e)^2 + b^2*d^2*\sinh(f*x + e)^2 + 4*a*b*d^2*\cosh(f*x + e) + (2*a^2 + b^2)*d^2 - 2*((a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*\text{Ei}((d*f*x + c*f)/d) + (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*\text{Ei}(-(d*f*x + c*f)/d))*\cosh(-(d*e - c*f)/d) - 2*((b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*\text{Ei}(2*(d*f*x + c*f)/d) + (b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*\text{Ei}(-2*(d*f*x + c*f)/d))*\cosh(-2*(d*e - c*f)/d) + 4*(a*b*d^2*f*x + a*b*c*d*f + (b^2*d^2*f*x + b^2*c*d*f)*\cosh(f*x + e))*\sinh(f*x + e) + 2*((a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*\text{Ei}((d*f*x + c*f)/d) - (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*\text{Ei}(-(d*f*x + c*f)/d))*\sinh(-(d*e - c*f)/d) + 2*((b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*\text{Ei}(2*(d*f*x + c*f)/d) - (b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*\text{Ei}(-2*(d*f*x + c*f)/d))*\sinh(-2*(d*e - c*f)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)$$

**giac** [B] time = 0.15, size = 702, normalized size = 2.90

$$4 b^2 d^2 f^2 x^2 \text{Ei}\left(-\frac{2(dfx+cf)}{d}\right) e^{\left(\frac{2cf}{d}-2e\right)} + 4 abd^2 f^2 x^2 \text{Ei}\left(-\frac{dfx+cf}{d}\right) e^{\left(\frac{cf}{d}-e\right)} + 4 abd^2 f^2 x^2 \text{Ei}\left(\frac{dfx+cf}{d}\right) e^{\left(-\frac{cf}{d}+e\right)} + 4 b^2 d^2 f$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cosh(f\*x+e))^2/(d\*x+c)^3,x, algorithm="giac")

[Out] 
$$1/8*(4*b^2*d^2*f^2*x^2*\text{Ei}(-2*(d*f*x + c*f)/d)*e^{(2*c*f/d - 2*e)} + 4*a*b*d^2*f^2*x^2*\text{Ei}(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} + 4*a*b*d^2*f^2*x^2*\text{Ei}((d*f*x + c*f)/d)*e^{(-c*f/d + e)} + 4*b^2*d^2*f^2*x^2*\text{Ei}(2*(d*f*x + c*f)/d)*e^{(-2*c*f/d + 2*e)} + 8*b^2*c*d*f^2*x*\text{Ei}(-2*(d*f*x + c*f)/d)*e^{(2*c*f/d - 2*e)} + 8*a*b*c*d*f^2*x*\text{Ei}(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} + 8*a*b*c*d*f^2*x*\text{Ei}((d*f*x + c*f)/d)*e^{(-c*f/d + e)} + 8*b^2*c*d*f^2*x*\text{Ei}(2*(d*f*x + c*f)/d)*e^{(-2*c*f/d + 2*e)} + 4*b^2*c^2*f^2*\text{Ei}(-2*(d*f*x + c*f)/d)*e^{(2*c*f/d - 2*e)} + 4*a*b*c^2*f^2*\text{Ei}(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} + 4*a*b*c^2*f^2*\text{Ei}((d*f*x + c*f)/d)*e^{(-c*f/d + e)} + 4*b^2*c^2*f^2*\text{Ei}(2*(d*f*x + c*f)/d)*e^{(-2*c*f/d + 2*e)} - 2*b^2*d^2*f*x*e^{(2*f*x + 2*e)} - 4*a*b*d^2*f*x*e^{(f*x + e)} + 4*a*b*d^2*f*x*e^{(-f*x - e)} + 2*b^2*d^2*f*x*e^{(-2*f*x - 2*e)} - 2*b^2*c*d*f*e^{(2*f*x + 2*e)}$$

) - 4\*a\*b\*c\*d\*f\*e^(f\*x + e) + 4\*a\*b\*c\*d\*f\*e^(-f\*x - e) + 2\*b^2\*c\*d\*f\*e^(-2\*f\*x - 2\*e) - b^2\*d^2\*e^(2\*f\*x + 2\*e) - 4\*a\*b\*d^2\*e^(f\*x + e) - 4\*a\*b\*d^2\*e^(-f\*x - e) - b^2\*d^2\*e^(-2\*f\*x - 2\*e) - 4\*a^2\*d^2 - 2\*b^2\*d^2)/(d^5\*x^2 + 2\*c\*d^4\*x + c^2\*d^3)

**maple** [B] time = 0.38, size = 626, normalized size = 2.59

$$\frac{f^3 ab e^{-fx-e} x}{2d(d^2 f^2 x^2 + 2cd f^2 x + c^2 f^2)} + \frac{f^3 ab e^{-fx-e} c}{2d^2(d^2 f^2 x^2 + 2cd f^2 x + c^2 f^2)} - \frac{f^2 ab e^{-fx-e}}{2d(d^2 f^2 x^2 + 2cd f^2 x + c^2 f^2)} - \frac{f^2 ab e^{\frac{cf-de}{d}} \text{Ei}\left(1, \frac{cf-de}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*cosh(f\*x+e))^2/(d\*x+c)^3,x)

[Out] 1/2\*f^3\*a\*b\*exp(-f\*x-e)/d/(d^2\*f^2\*x^2+2\*c\*d\*f^2\*x+c^2\*f^2)\*x+1/2\*f^3\*a\*b\*exp(-f\*x-e)/d^2/(d^2\*f^2\*x^2+2\*c\*d\*f^2\*x+c^2\*f^2)\*c-1/2\*f^2\*a\*b\*exp(-f\*x-e)/d/(d^2\*f^2\*x^2+2\*c\*d\*f^2\*x+c^2\*f^2)-1/2\*f^2\*a\*b/d^3\*exp((c\*f-d\*e)/d)\*Ei(1,f\*x+e+(c\*f-d\*e)/d)-1/2\*a\*b\*f^2/d^3\*exp(f\*x+e)/(c\*f/d+f\*x)^2-1/2\*a\*b\*f^2/d^3\*exp(f\*x+e)/(c\*f/d+f\*x)-1/2\*a\*b\*f^2/d^3\*exp(-(c\*f-d\*e)/d)\*Ei(1,-f\*x-e-(c\*f-d\*e)/d)-1/2\*a^2/d/(d\*x+c)^2-1/4\*b^2/d/(d\*x+c)^2+1/4\*f^3\*b^2\*exp(-2\*f\*x-2\*e)/d/(d^2\*f^2\*x^2+2\*c\*d\*f^2\*x+c^2\*f^2)\*x+1/4\*f^3\*b^2\*exp(-2\*f\*x-2\*e)/d^2/(d^2\*f^2\*x^2+2\*c\*d\*f^2\*x+c^2\*f^2)\*c-1/8\*f^2\*b^2\*exp(-2\*f\*x-2\*e)/d/(d^2\*f^2\*x^2+2\*c\*d\*f^2\*x+c^2\*f^2)-1/2\*f^2\*b^2/d^3\*exp(2\*(c\*f-d\*e)/d)\*Ei(1,2\*f\*x+2\*e+2\*(c\*f-d\*e)/d)-1/8\*b^2\*f^2/d^3\*exp(2\*f\*x+2\*e)/(c\*f/d+f\*x)^2-1/4\*b^2\*f^2/d^3\*exp(2\*f\*x+2\*e)/(c\*f/d+f\*x)-1/2\*b^2\*f^2/d^3\*exp(-2\*(c\*f-d\*e)/d)\*Ei(1,-2\*f\*x-2\*e-2\*(c\*f-d\*e)/d)

**maxima** [A] time = 0.45, size = 201, normalized size = 0.83

$$-\frac{1}{4} b^2 \left( \frac{1}{d^3 x^2 + 2cd^2 x + c^2 d} + \frac{e^{\left(-2e + \frac{2cf}{d}\right)} E_3\left(\frac{2(dx+c)f}{d}\right)}{(dx+c)^2 d} + \frac{e^{\left(2e - \frac{2cf}{d}\right)} E_3\left(-\frac{2(dx+c)f}{d}\right)}{(dx+c)^2 d} \right) - ab \left( \frac{e^{\left(-e + \frac{cf}{d}\right)} E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2 d} + \frac{e^{\left(e - \frac{cf}{d}\right)} E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2 d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cosh(f\*x+e))^2/(d\*x+c)^3,x, algorithm="maxima")

[Out] -1/4\*b^2\*(1/(d^3\*x^2 + 2\*c\*d^2\*x + c^2\*d) + e^(-2\*e + 2\*c\*f/d)\*exp\_integral\_e(3, 2\*(d\*x + c)\*f/d)/((d\*x + c)^2\*d) + e^(2\*e - 2\*c\*f/d)\*exp\_integral\_e(3, -2\*(d\*x + c)\*f/d)/((d\*x + c)^2\*d)) - a\*b\*(e^(-e + c\*f/d)\*exp\_integral\_e(3, (d\*x + c)\*f/d)/((d\*x + c)^2\*d) + e^(e - c\*f/d)\*exp\_integral\_e(3, -(d\*x + c)\*f/d)/((d\*x + c)^2\*d)) - 1/2\*a^2/(d^3\*x^2 + 2\*c\*d^2\*x + c^2\*d)



mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \cosh(e + fx))^2}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*cosh(e + f\*x))^2/(c + d\*x)^3, x)

[Out] int((a + b\*cosh(e + f\*x))^2/(c + d\*x)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \cosh(e + fx))^2}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*cosh(f\*x+e))\*\*2/(d\*x+c)\*\*3, x)

[Out] Integral((a + b\*cosh(e + f\*x))\*\*2/(c + d\*x)\*\*3, x)

$$3.168 \quad \int \frac{(c+dx)^3}{a+b \cosh(e+fx)} dx$$

**Optimal.** Leaf size=436

$$-\frac{6d^2(c+dx)\text{Li}_3\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}} + \frac{6d^2(c+dx)\text{Li}_3\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}} + \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}} - \frac{3d(c+dx)^2\text{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}$$

[Out]  $(d*x+c)^3*\ln(1+b*\exp(f*x+e)/(a-(a^2-b^2)^{(1/2)}))/f/(a^2-b^2)^{(1/2)}-(d*x+c)^3*\ln(1+b*\exp(f*x+e)/(a+(a^2-b^2)^{(1/2)}))/f/(a^2-b^2)^{(1/2)}+3*d*(d*x+c)^2*polylog(2,-b*\exp(f*x+e)/(a-(a^2-b^2)^{(1/2)}))/f^2/(a^2-b^2)^{(1/2)}-3*d*(d*x+c)^2*polylog(2,-b*\exp(f*x+e)/(a+(a^2-b^2)^{(1/2)}))/f^2/(a^2-b^2)^{(1/2)}-6*d^2*(d*x+c)*polylog(3,-b*\exp(f*x+e)/(a-(a^2-b^2)^{(1/2)}))/f^3/(a^2-b^2)^{(1/2)}+6*d^2*(d*x+c)*polylog(3,-b*\exp(f*x+e)/(a+(a^2-b^2)^{(1/2)}))/f^3/(a^2-b^2)^{(1/2)}+6*d^3*polylog(4,-b*\exp(f*x+e)/(a-(a^2-b^2)^{(1/2)}))/f^4/(a^2-b^2)^{(1/2)}-6*d^3*polylog(4,-b*\exp(f*x+e)/(a+(a^2-b^2)^{(1/2)}))/f^4/(a^2-b^2)^{(1/2)}$

**Rubi [A]** time = 0.82, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$ , Rules used = {3320, 2264, 2190, 2531, 6609, 2282, 6589}

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}} + \frac{6d^2(c+dx)\text{PolyLog}\left(3,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^3\sqrt{a^2-b^2}} + \frac{3d(c+dx)^2\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^3/(a + b\*Cosh[e + f\*x]),x]

[Out]  $((c+d*x)^3*\text{Log}[1+(b*E^{(e+f*x)})/(a-\text{Sqrt}[a^2-b^2])])/(a-\text{Sqrt}[a^2-b^2])*f - ((c+d*x)^3*\text{Log}[1+(b*E^{(e+f*x)})/(a+\text{Sqrt}[a^2-b^2])])/(a+\text{Sqrt}[a^2-b^2])*f + (3*d*(c+d*x)^2*\text{PolyLog}[2,-((b*E^{(e+f*x)})/(a-\text{Sqrt}[a^2-b^2]))])/(a-\text{Sqrt}[a^2-b^2])*f^2 - (3*d*(c+d*x)^2*\text{PolyLog}[2,-((b*E^{(e+f*x)})/(a+\text{Sqrt}[a^2-b^2]))])/(a+\text{Sqrt}[a^2-b^2])*f^2 - (6*d^2*(c+d*x)*\text{PolyLog}[3,-((b*E^{(e+f*x)})/(a-\text{Sqrt}[a^2-b^2]))])/(a-\text{Sqrt}[a^2-b^2])*f^3 + (6*d^2*(c+d*x)*\text{PolyLog}[3,-((b*E^{(e+f*x)})/(a+\text{Sqrt}[a^2-b^2]))])/(a+\text{Sqrt}[a^2-b^2])*f^3 + (6*d^3*\text{PolyLog}[4,-((b*E^{(e+f*x)})/(a-\text{Sqrt}[a^2-b^2]))])/(a-\text{Sqrt}[a^2-b^2])*f^4 - (6*d^3*\text{PolyLog}[4,-((b*E^{(e+f*x)})/(a+\text{Sqrt}[a^2-b^2]))])/(a+\text{Sqrt}[a^2-b^2])*f^4$

**Rule 2190**

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Di

Int[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^(c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 3320

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(E^(I\*Pi\*(k - 1/2))\*(b + (2\*a\*E^(-(I\*e) + f\*fz\*x))/E^(I\*Pi\*(k - 1/2)) - (b\*E^(2\*(-I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi)), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^(c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(p\_.)], x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a

$+ b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{(c + dx)^3}{a + b \cosh(e + fx)} dx &= 2 \int \frac{e^{e+fx}(c + dx)^3}{b + 2ae^{e+fx} + be^{2(e+fx)}} dx \\ &= \frac{(2b) \int \frac{e^{e+fx}(c+dx)^3}{2a-2\sqrt{a^2-b^2}+2be^{e+fx}} dx}{\sqrt{a^2-b^2}} - \frac{(2b) \int \frac{e^{e+fx}(c+dx)^3}{2a+2\sqrt{a^2-b^2}+2be^{e+fx}} dx}{\sqrt{a^2-b^2}} \\ &= \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a - \sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2} f} - \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a + \sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2} f} - \frac{(3d) \int (c + dx)^2 \log\left(1 + \frac{be^{e+fx}}{a - \sqrt{a^2 - b^2}}\right) dx}{\sqrt{a^2 - b^2} f^2} \\ &= \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a - \sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2} f} - \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a + \sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2} f} + \frac{3d(c + dx)^2 \text{Li}_2\left(-\frac{be^{e+fx}}{a - \sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2} f^2} \\ &= \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a - \sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2} f} - \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a + \sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2} f} + \frac{3d(c + dx)^2 \text{Li}_2\left(-\frac{be^{e+fx}}{a - \sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2} f^2} \\ &= \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a - \sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2} f} - \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a + \sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2} f} + \frac{3d(c + dx)^2 \text{Li}_2\left(-\frac{be^{e+fx}}{a - \sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2} f^2} \\ &= \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a - \sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2} f} - \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a + \sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2} f} + \frac{3d(c + dx)^2 \text{Li}_2\left(-\frac{be^{e+fx}}{a - \sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2} f^2} \end{aligned}$$

**Mathematica [A]** time = 1.62, size = 384, normalized size = 0.88

$$\frac{3d\left(f^2(c+dx)^2 \text{Li}_2\left(\frac{b(\cosh(e+fx)+\sinh(e+fx))}{\sqrt{a^2-b^2}-a}\right)\right) - 2df(c+dx) \text{Li}_3\left(\frac{b(\cosh(e+fx)+\sinh(e+fx))}{\sqrt{a^2-b^2}-a}\right) + 2d^2 \text{Li}_4\left(\frac{b(\cosh(e+fx)+\sinh(e+fx))}{\sqrt{a^2-b^2}-a}\right)}{f^3} - \frac{3d\left(f^2(c+dx)^2 \text{Li}_2\left(-\frac{b(\cosh(e+fx)+\sinh(e+fx))}{\sqrt{a^2-b^2}+a}\right)\right)}{f^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3/(a + b\*Cosh[e + f\*x]), x]

[Out] ((c + d\*x)^3\*Log[1 + (b\*(Cosh[e + f\*x] + Sinh[e + f\*x]))/(a - Sqrt[a^2 - b^2])] - (c + d\*x)^3\*Log[1 + (b\*(Cosh[e + f\*x] + Sinh[e + f\*x]))/(a + Sqrt[a^2 - b^2])]) / (a^2 - b^2)

```

2 - b^2))] + (3*d*(f^2*(c + d*x)^2*PolyLog[2, (b*(Cosh[e + f*x] + Sinh[e +
f*x]))/(-a + Sqrt[a^2 - b^2])] - 2*d*f*(c + d*x)*PolyLog[3, (b*(Cosh[e + f*
x] + Sinh[e + f*x]))/(-a + Sqrt[a^2 - b^2])] + 2*d^2*PolyLog[4, (b*(Cosh[e
+ f*x] + Sinh[e + f*x]))/(-a + Sqrt[a^2 - b^2])]))/f^3 - (3*d*(f^2*(c + d*x
)^2*PolyLog[2, -((b*(Cosh[e + f*x] + Sinh[e + f*x]))/(a + Sqrt[a^2 - b^2]))
] - 2*d*f*(c + d*x)*PolyLog[3, -((b*(Cosh[e + f*x] + Sinh[e + f*x]))/(a + S
qrt[a^2 - b^2]))] + 2*d^2*PolyLog[4, -((b*(Cosh[e + f*x] + Sinh[e + f*x]))/
(a + Sqrt[a^2 - b^2]))]))/f^3)/(Sqrt[a^2 - b^2]*f)

```

**fricas** [C] time = 0.62, size = 1042, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] (6*b*d^3*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cosh(f*x + e) + a*sinh(f*x +
e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b) - 6*b*d^
3*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cosh(f*x + e) + a*sinh(f*x + e) - (b
*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b) + 3*(b*d^3*f^2*x
^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(f
*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 -
b^2)/b^2) + b)/b + 1) - 3*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*
sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f
*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + (b*d^3*e^3 -
3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt((a^2 - b^2)/b^2)*log(2
*b*cosh(f*x + e) + 2*b*sinh(f*x + e) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - (
b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt((a^2 - b^2)
/b^2)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 - b^2)/b^2)
+ 2*a) + (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3
- 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*sqrt((a^2 - b^2)/b^2)*log((a*cosh(f*x
+ e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^
2)/b^2) + b)/b) - (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*
d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*sqrt((a^2 - b^2)/b^2)*log((a*c
osh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((
a^2 - b^2)/b^2) + b)/b) - 6*(b*d^3*f*x + b*c*d^2*f)*sqrt((a^2 - b^2)/b^2)*p
olylog(3, -(a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f
*x + e))*sqrt((a^2 - b^2)/b^2))/b) + 6*(b*d^3*f*x + b*c*d^2*f)*sqrt((a^2 -
b^2)/b^2)*polylog(3, -(a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e)
+ b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b))/((a^2 - b^2)*f^4)

```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^3}{b \cosh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*cosh(f*x + e) + a), x)
```

**maple** [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^3}{a + b \cosh(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*cosh(f*x+e)),x)
```

```
[Out] int((d*x+c)^3/(a+b*cosh(f*x+e)),x)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + dx)^3}{a + b \cosh(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*cosh(e + f*x)),x)
```

```
[Out] int((c + d*x)^3/(a + b*cosh(e + f*x)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx)^3}{a + b \cosh(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*cosh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**3/(a + b*cosh(e + f*x)), x)
```

$$3.169 \quad \int \frac{(c+dx)^2}{a+b \cosh(e+fx)} dx$$

**Optimal.** Leaf size=320

$$\frac{2d(c+dx)\operatorname{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}} - \frac{2d(c+dx)\operatorname{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}} + \frac{(c+dx)^2 \log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}} + 1\right)}{f\sqrt{a^2-b^2}} - \frac{(c+dx)^2 \log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a} + 1\right)}{f\sqrt{a^2-b^2}}$$

[Out]  $(d*x+c)^2*\ln(1+b*\exp(f*x+e)/(a-(a^2-b^2)^{(1/2)}))/f/(a^2-b^2)^{(1/2)}-(d*x+c)^2*\ln(1+b*\exp(f*x+e)/(a+(a^2-b^2)^{(1/2)}))/f/(a^2-b^2)^{(1/2)}+2*d*(d*x+c)*\operatorname{polylog}(2,-b*\exp(f*x+e)/(a-(a^2-b^2)^{(1/2)}))/f^2/(a^2-b^2)^{(1/2)}-2*d*(d*x+c)*\operatorname{polylog}(2,-b*\exp(f*x+e)/(a+(a^2-b^2)^{(1/2)}))/f^2/(a^2-b^2)^{(1/2)}-2*d^2*\operatorname{polylog}(3,-b*\exp(f*x+e)/(a-(a^2-b^2)^{(1/2)}))/f^3/(a^2-b^2)^{(1/2)}+2*d^2*\operatorname{polylog}(3,-b*\exp(f*x+e)/(a+(a^2-b^2)^{(1/2)}))/f^3/(a^2-b^2)^{(1/2)}$

**Rubi [A]** time = 0.67, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {3320, 2264, 2190, 2531, 2282, 6589}

$$\frac{2d(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}} - \frac{2d(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^2\sqrt{a^2-b^2}} - \frac{2d^2\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}} + \frac{2d^2\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^3\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^2/(a + b*\operatorname{Cosh}[e + f*x]), x]$

[Out]  $((c + d*x)^2*\operatorname{Log}[1 + (b*E^{(e + f*x)})/(a - \operatorname{Sqrt}[a^2 - b^2])])/( \operatorname{Sqrt}[a^2 - b^2]*f) - ((c + d*x)^2*\operatorname{Log}[1 + (b*E^{(e + f*x)})/(a + \operatorname{Sqrt}[a^2 - b^2])])/( \operatorname{Sqrt}[a^2 - b^2]*f) + (2*d*(c + d*x)*\operatorname{PolyLog}[2, -((b*E^{(e + f*x)})/(a - \operatorname{Sqrt}[a^2 - b^2]))])/( \operatorname{Sqrt}[a^2 - b^2]*f^2) - (2*d*(c + d*x)*\operatorname{PolyLog}[2, -((b*E^{(e + f*x)})/(a + \operatorname{Sqrt}[a^2 - b^2]))])/( \operatorname{Sqrt}[a^2 - b^2]*f^2) - (2*d^2*\operatorname{PolyLog}[3, -((b*E^{(e + f*x)})/(a - \operatorname{Sqrt}[a^2 - b^2]))])/( \operatorname{Sqrt}[a^2 - b^2]*f^3) + (2*d^2*\operatorname{PolyLog}[3, -((b*E^{(e + f*x)})/(a + \operatorname{Sqrt}[a^2 - b^2]))])/( \operatorname{Sqrt}[a^2 - b^2]*f^3)$

**Rule 2190**

$\operatorname{Int}[(((F_)^{((g_)*(e_)+(f_)*(x_)))^{(n_)*((c_)+(d_)*(x_))^{(m_))}}/((a_)+(b_)*((F_)^{((g_)*(e_)+(f_)*(x_)))^{(n_))}}), x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^m*\operatorname{Log}[1 + (b*(F^{(g*(e + f*x)))^n)/a)]/(b*f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(b*f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 + (b*(F^{(g*(e + f*x)))^n)/a]), x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \operatorname{IGtQ}[m, 0]$

**Rule 2264**



```
Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^(c_)*((a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 3320

```
Int[((c_) + (d_)*(x_))^(m_)/((a_) + (b_)*sin[(e_) + Pi*(k_) + (Comple
x[0, fz_])*(f_)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-I*e) + f*fz*x)))/E^(2*I*k*Pi))], x], x] /; FreeQ[{a, b, c,
d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^2}{a+b \cosh(e+fx)} dx &= 2 \int \frac{e^{e+fx}(c+dx)^2}{b+2ae^{e+fx}+be^{2(e+fx)}} dx \\
&= \frac{(2b) \int \frac{e^{e+fx}(c+dx)^2}{2a-2\sqrt{a^2-b^2}+2be^{e+fx}} dx}{\sqrt{a^2-b^2}} - \frac{(2b) \int \frac{e^{e+fx}(c+dx)^2}{2a+2\sqrt{a^2-b^2}+2be^{e+fx}} dx}{\sqrt{a^2-b^2}} \\
&= \frac{(c+dx)^2 \log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f} - \frac{(c+dx)^2 \log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f} - \frac{(2d) \int (c+dx) \log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right) dx}{\sqrt{a^2-b^2}} \\
&= \frac{(c+dx)^2 \log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f} - \frac{(c+dx)^2 \log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f} + \frac{2d(c+dx) \operatorname{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f^2} \\
&= \frac{(c+dx)^2 \log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f} - \frac{(c+dx)^2 \log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f} + \frac{2d(c+dx) \operatorname{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f^2} \\
&= \frac{(c+dx)^2 \log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f} - \frac{(c+dx)^2 \log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f} + \frac{2d(c+dx) \operatorname{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f^2}
\end{aligned}$$

**Mathematica [A]** time = 1.05, size = 247, normalized size = 0.77

$$\frac{2d\left(f(c+dx) \operatorname{Li}_2\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}-a}\right) - d \operatorname{Li}_3\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}-a}\right)\right)}{f^2} - \frac{2d\left(f(c+dx) \operatorname{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right) - d \operatorname{Li}_3\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)\right)}{f^2} + \frac{(c+dx)^2 \log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}} + 1\right) - (c+dx)^2 \log\left(\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}} + 1\right)}{f\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2/(a + b\*Cosh[e + f\*x]),x]

[Out] ((c + d\*x)^2\*Log[1 + (b\*E^(e + f\*x))/(a - Sqrt[a^2 - b^2])] - (c + d\*x)^2\*Log[1 + (b\*E^(e + f\*x))/(a + Sqrt[a^2 - b^2])]) + (2\*d\*(f\*(c + d\*x)\*PolyLog[2, (b\*E^(e + f\*x))/(-a + Sqrt[a^2 - b^2])] - d\*PolyLog[3, (b\*E^(e + f\*x))/(-a + Sqrt[a^2 - b^2])])/f^2 - (2\*d\*(f\*(c + d\*x)\*PolyLog[2, -(b\*E^(e + f\*x))/(a + Sqrt[a^2 - b^2])] - d\*PolyLog[3, -(b\*E^(e + f\*x))/(a + Sqrt[a^2 - b^2])])/f^2)/(Sqrt[a^2 - b^2]\*f)

**fricas** [C] time = 0.49, size = 736, normalized size = 2.30

$$2bd^2\sqrt{\frac{a^2-b^2}{b^2}}\operatorname{polylog}\left(3,-\frac{a\cosh(fx+e)+a\sinh(fx+e)+(b\cosh(fx+e)+b\sinh(fx+e))\sqrt{\frac{a^2-b^2}{b^2}}}{b}\right)-2bd^2\sqrt{\frac{a^2-b^2}{b^2}}\operatorname{polylog}\left(3,-\frac{a\cosh(fx+e)+a\sinh(fx+e)-(b\cosh(fx+e)+b\sinh(fx+e))\sqrt{\frac{a^2-b^2}{b^2}}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*cosh(f\*x+e)),x, algorithm="fricas")

[Out]  $-(2*b*d^2*\sqrt{(a^2 - b^2)/b^2}*\operatorname{polylog}(3, -(a*\cosh(f*x + e) + a*\sinh(f*x + e) + (b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{(a^2 - b^2)/b^2}))/b - 2*b*d^2*\sqrt{(a^2 - b^2)/b^2}*\operatorname{polylog}(3, -(a*\cosh(f*x + e) + a*\sinh(f*x + e) - (b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{(a^2 - b^2)/b^2}))/b - 2*(b*d^2*f*x + b*c*d*f)*\sqrt{(a^2 - b^2)/b^2}*\operatorname{dilog}(-(a*\cosh(f*x + e) + a*\sinh(f*x + e) + (b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{(a^2 - b^2)/b^2}) + b)/b + 1) + 2*(b*d^2*f*x + b*c*d*f)*\sqrt{(a^2 - b^2)/b^2}*\operatorname{dilog}(-(a*\cosh(f*x + e) + a*\sinh(f*x + e) - (b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{(a^2 - b^2)/b^2}) + b)/b + 1) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\sqrt{(a^2 - b^2)/b^2}*\log(2*b*\cosh(f*x + e) + 2*b*\sinh(f*x + e) + 2*b*\sqrt{(a^2 - b^2)/b^2} + 2*a) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\sqrt{(a^2 - b^2)/b^2}*\log(2*b*\cosh(f*x + e) + 2*b*\sinh(f*x + e) - 2*b*\sqrt{(a^2 - b^2)/b^2} + 2*a) - (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*\sqrt{(a^2 - b^2)/b^2}*\log((a*\cosh(f*x + e) + a*\sinh(f*x + e) + (b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{(a^2 - b^2)/b^2}) + b)/b + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*\sqrt{(a^2 - b^2)/b^2}*\log((a*\cosh(f*x + e) + a*\sinh(f*x + e) - (b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{(a^2 - b^2)/b^2}) + b)/b)/((a^2 - b^2)*f^3)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^2}{b \cosh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*cosh(f\*x+e)),x, algorithm="giac")

[Out] integrate((d\*x + c)^2/(b\*cosh(f\*x + e) + a), x)

**maple** [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^2}{a + b \cosh(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^2/(a+b*cosh(f*x+e)),x)`

[Out] `int((d*x+c)^2/(a+b*cosh(f*x+e)),x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see 'assume?' for more details)Is 4\*a^2-4\*b^2 positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + dx)^2}{a + b \cosh(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x)^2/(a + b*cosh(e + f*x)),x)`

[Out] `int((c + d*x)^2/(a + b*cosh(e + f*x)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx)^2}{a + b \cosh(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)**2/(a+b*cosh(f*x+e)),x)`

[Out] `Integral((c + d*x)**2/(a + b*cosh(e + f*x)), x)`

$$3.170 \quad \int \frac{c+dx}{a+b \cosh(e+fx)} dx$$

**Optimal.** Leaf size=203

$$\frac{(c+dx) \log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{f\sqrt{a^2-b^2}} - \frac{(c+dx) \log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{f\sqrt{a^2-b^2}} + \frac{d\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}} - \frac{d\text{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}$$

[Out] (d\*x+c)\*ln(1+b\*exp(f\*x+e)/(a-(a^2-b^2)^(1/2)))/f/(a^2-b^2)^(1/2)-(d\*x+c)\*ln(1+b\*exp(f\*x+e)/(a+(a^2-b^2)^(1/2)))/f/(a^2-b^2)^(1/2)+d\*polylog(2,-b\*exp(f\*x+e)/(a-(a^2-b^2)^(1/2)))/f^2/(a^2-b^2)^(1/2)-d\*polylog(2,-b\*exp(f\*x+e)/(a+(a^2-b^2)^(1/2)))/f^2/(a^2-b^2)^(1/2)

**Rubi [A]** time = 0.38, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3320, 2264, 2190, 2279, 2391}

$$\frac{d\text{PolyLog}\left(2, -\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}} - \frac{d\text{PolyLog}\left(2, -\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^2\sqrt{a^2-b^2}} + \frac{(c+dx) \log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{f\sqrt{a^2-b^2}} - \frac{(c+dx) \log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{f\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)/(a + b\*Cosh[e + f\*x]), x]

[Out] ((c + d\*x)\*Log[1 + (b\*E^(e + f\*x))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]\*f) - ((c + d\*x)\*Log[1 + (b\*E^(e + f\*x))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]\*f) + (d\*PolyLog[2, -((b\*E^(e + f\*x))/(a - Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]\*f^2) - (d\*PolyLog[2, -((b\*E^(e + f\*x))/(a + Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]\*f^2)

**Rule 2190**

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m-1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

**Rule 2264**

Int[((F\_)^(u\_)\*((f\_) + (g\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*(F\_)^(u\_)) + (c\_)\*(F\_)^(v\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u]/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u]/(b + q + 2\*c\*F^u), x], x] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,

2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_)))^(n\_.)], x\_Symbol]
 := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x))
 )^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2
 , -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 3320

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))/((a\_) + (b\_.)\*sin[(e\_.) + Pi\*(k\_.) + (Comple
 x[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) +
 f\*fz\*x))/(E^(I\*Pi\*(k - 1/2))\*(b + (2\*a\*E^(-(I\*e) + f\*fz\*x))/E^(I\*Pi\*(k - 1/
 2)) - (b\*E^(2\*(-(I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{c + dx}{a + b \cosh(e + fx)} dx &= 2 \int \frac{e^{e+fx}(c + dx)}{b + 2ae^{e+fx} + be^{2(e+fx)}} dx \\
 &= \frac{(2b) \int \frac{e^{e+fx}(c+dx)}{2a-2\sqrt{a^2-b^2}+2be^{e+fx}} dx}{\sqrt{a^2-b^2}} - \frac{(2b) \int \frac{e^{e+fx}(c+dx)}{2a+2\sqrt{a^2-b^2}+2be^{e+fx}} dx}{\sqrt{a^2-b^2}} \\
 &= \frac{(c + dx) \log\left(1 + \frac{be^{e+fx}}{a - \sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f} - \frac{(c + dx) \log\left(1 + \frac{be^{e+fx}}{a + \sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f} - \frac{d \int \log\left(1 + \frac{2be^{e+fx}}{2a-2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f} \\
 &= \frac{(c + dx) \log\left(1 + \frac{be^{e+fx}}{a - \sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f} - \frac{(c + dx) \log\left(1 + \frac{be^{e+fx}}{a + \sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f} - \frac{d \operatorname{Subst}\left(\int \frac{\log\left(1 + \frac{2bx}{2a-2\sqrt{a^2-b^2}}\right)}{x}\right)}{\sqrt{a^2-b^2} f} \\
 &= \frac{(c + dx) \log\left(1 + \frac{be^{e+fx}}{a - \sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f} - \frac{(c + dx) \log\left(1 + \frac{be^{e+fx}}{a + \sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f} + \frac{d \operatorname{Li}_2\left(-\frac{be^{e+fx}}{a - \sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f^2} - \frac{d \operatorname{Li}_2\left(-\frac{be^{e+fx}}{a + \sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2} f^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.96, size = 152, normalized size = 0.75

$$\frac{f(c + dx) \left( \log \left( \frac{be^{e+fx}}{a - \sqrt{a^2 - b^2}} + 1 \right) - \log \left( \frac{be^{e+fx}}{\sqrt{a^2 - b^2} + a} + 1 \right) \right) + d \operatorname{Li}_2 \left( \frac{be^{e+fx}}{\sqrt{a^2 - b^2} - a} \right) - d \operatorname{Li}_2 \left( -\frac{be^{e+fx}}{a + \sqrt{a^2 - b^2}} \right)}{f^2 \sqrt{a^2 - b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)/(a + b\*Cosh[e + f\*x]),x]

[Out] (f\*(c + d\*x)\*(Log[1 + (b\*E^(e + f\*x))/(a - Sqrt[a^2 - b^2]]] - Log[1 + (b\*E^(e + f\*x))/(a + Sqrt[a^2 - b^2]]]) + d\*PolyLog[2, (b\*E^(e + f\*x))/(-a + Sqrt[a^2 - b^2])] - d\*PolyLog[2, -((b\*E^(e + f\*x))/(a + Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]\*f^2)

**fricas [B]** time = 0.49, size = 473, normalized size = 2.33

$$bd\sqrt{\frac{a^2-b^2}{b^2}} \operatorname{Li}_2 \left( -\frac{a \cosh(fx+e) + a \sinh(fx+e) + (b \cosh(fx+e) + b \sinh(fx+e))\sqrt{\frac{a^2-b^2}{b^2}} + b}{b} + 1 \right) - bd\sqrt{\frac{a^2-b^2}{b^2}} \operatorname{Li}_2 \left( -\frac{a \cosh(fx+e) + a \sinh(fx+e)}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*cosh(f\*x+e)),x, algorithm="fricas")

[Out] (b\*d\*sqrt((a^2 - b^2)/b^2)\*dilog(-(a\*cosh(f\*x + e) + a\*sinh(f\*x + e) + (b\*cosh(f\*x + e) + b\*sinh(f\*x + e))\*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - b\*d\*sqrt((a^2 - b^2)/b^2)\*dilog(-(a\*cosh(f\*x + e) + a\*sinh(f\*x + e) - (b\*cosh(f\*x + e) + b\*sinh(f\*x + e))\*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + (b\*d\*e - b\*c\*f)\*sqrt((a^2 - b^2)/b^2)\*log(2\*b\*cosh(f\*x + e) + 2\*b\*sinh(f\*x + e) + 2\*b\*sqrt((a^2 - b^2)/b^2) + 2\*a) - (b\*d\*e - b\*c\*f)\*sqrt((a^2 - b^2)/b^2)\*log(2\*b\*cosh(f\*x + e) + 2\*b\*sinh(f\*x + e) - 2\*b\*sqrt((a^2 - b^2)/b^2) + 2\*a) + (b\*d\*f\*x + b\*d\*e)\*sqrt((a^2 - b^2)/b^2)\*log((a\*cosh(f\*x + e) + a\*sinh(f\*x + e) + (b\*cosh(f\*x + e) + b\*sinh(f\*x + e))\*sqrt((a^2 - b^2)/b^2) + b)/b) - (b\*d\*f\*x + b\*d\*e)\*sqrt((a^2 - b^2)/b^2)\*log((a\*cosh(f\*x + e) + a\*sinh(f\*x + e) - (b\*cosh(f\*x + e) + b\*sinh(f\*x + e))\*sqrt((a^2 - b^2)/b^2) + b)/b))/((a^2 - b^2)\*f^2)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx + c}{b \cosh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*cosh(f\*x+e)),x, algorithm="giac")

[Out] integrate((d\*x + c)/(b\*cosh(f\*x + e) + a), x)

**maple [B]** time = 0.22, size = 437, normalized size = 2.15

$$\frac{2c \arctan\left(\frac{2be^{fx+e}+2a}{2\sqrt{-a^2+b^2}}\right)}{f\sqrt{-a^2+b^2}} + \frac{d \ln\left(\frac{-be^{fx+e}+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)x}{f\sqrt{a^2-b^2}} + \frac{d \ln\left(\frac{-be^{fx+e}+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)e}{f^2\sqrt{a^2-b^2}} - \frac{d \ln\left(\frac{be^{fx+e}+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)x}{f\sqrt{a^2-b^2}} - \frac{d \ln\left(\frac{be^{fx+e}+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)e}{f^2\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)/(a+b\*cosh(f\*x+e)),x)

[Out]  $\frac{2}{f} \frac{c}{(-a^2+b^2)^{1/2}} \arctan\left(\frac{1}{2} \frac{(2b \exp(fx+e)+2a)}{(-a^2+b^2)^{1/2}}\right) + \frac{1}{f} \frac{d}{(a^2-b^2)^{1/2}} \ln\left(\frac{-b \exp(fx+e)+(a^2-b^2)^{1/2}-a}{-a+(a^2-b^2)^{1/2}}\right) + \frac{1}{f^2} \frac{d}{(a^2-b^2)^{1/2}} \ln\left(\frac{-b \exp(fx+e)+(a^2-b^2)^{1/2}-a}{-a+(a^2-b^2)^{1/2}}\right) + \frac{1}{f^2} \frac{d}{(a^2-b^2)^{1/2}} \ln\left(\frac{b \exp(fx+e)+(a^2-b^2)^{1/2}+a}{a+(a^2-b^2)^{1/2}}\right) + \frac{1}{f^2} \frac{d}{(a^2-b^2)^{1/2}} \operatorname{dilog}\left(\frac{-b \exp(fx+e)+(a^2-b^2)^{1/2}-a}{-a+(a^2-b^2)^{1/2}}\right) - \frac{1}{f^2} \frac{d}{(a^2-b^2)^{1/2}} \operatorname{dilog}\left(\frac{b \exp(fx+e)+(a^2-b^2)^{1/2}+a}{a+(a^2-b^2)^{1/2}}\right) - \frac{2}{f^2} \frac{d}{(a^2-b^2)^{1/2}} \frac{e}{(-a^2+b^2)^{1/2}} \arctan\left(\frac{1}{2} \frac{(2b \exp(fx+e)+2a)}{(-a^2+b^2)^{1/2}}\right)$

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*cosh(f\*x+e)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see 'assume?' for more details) Is 4\*a^2-4\*b^2 positive or negative?

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c + dx}{a + b \cosh(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)/(a + b\*cosh(e + f\*x)),x)

[Out] int((c + d\*x)/(a + b\*cosh(e + f\*x)), x)



sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx}{a + b \cosh(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*cosh(f\*x+e)),x)

[Out] Integral((c + d\*x)/(a + b\*cosh(e + f\*x)), x)

$$3.171 \quad \int \frac{1}{(c+dx)(a+b \cosh(e+fx))} dx$$

**Optimal.** Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+b \cosh(e+fx))}, x\right)$$

[Out] Unintegrable(1/(d\*x+c)/(a+b\*cosh(f\*x+e)), x)

**Rubi [A]** time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(c+dx)(a+b \cosh(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d\*x)\*(a + b\*Cosh[e + f\*x])), x]

[Out] Defer[Int][1/((c + d\*x)\*(a + b\*Cosh[e + f\*x])), x]

Rubi steps

$$\int \frac{1}{(c+dx)(a+b \cosh(e+fx))} dx = \int \frac{1}{(c+dx)(a+b \cosh(e+fx))} dx$$

**Mathematica [A]** time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+b \cosh(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d\*x)\*(a + b\*Cosh[e + f\*x])), x]

[Out] Integrate[1/((c + d\*x)\*(a + b\*Cosh[e + f\*x])), x]

**fricas [A]** time = 1.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{adx + ac + (bdx + bc) \cosh(fx + e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+b\*cosh(f\*x+e)),x, algorithm="fricas")

[Out] integral(1/(a\*d\*x + a\*c + (b\*d\*x + b\*c)\*cosh(f\*x + e)), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)(b \cosh(fx + e) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+b\*cosh(f\*x+e)),x, algorithm="giac")

[Out] integrate(1/((d\*x + c)\*(b\*cosh(f\*x + e) + a)), x)

**maple** [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)(a + b \cosh(fx + e))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x+c)/(a+b\*cosh(f\*x+e)),x)

[Out] int(1/(d\*x+c)/(a+b\*cosh(f\*x+e)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)(b \cosh(fx + e) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+b\*cosh(f\*x+e)),x, algorithm="maxima")

[Out] integrate(1/((d\*x + c)\*(b\*cosh(f\*x + e) + a)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a + b \cosh(e + fx))(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*cosh(e + f\*x))\*(c + d\*x)),x)

[Out] int(1/((a + b\*cosh(e + f\*x))\*(c + d\*x)), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \cosh(e + fx))(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+b\*cosh(f\*x+e)),x)

[Out] Integral(1/((a + b\*cosh(e + f\*x))\*(c + d\*x)), x)

$$3.172 \quad \int \frac{1}{(c+dx)^2(a+b \cosh(e+fx))} dx$$

**Optimal.** Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+b \cosh(e+fx))}, x\right)$$

[Out] Unintegrable(1/(d\*x+c)^2/(a+b\*cosh(f\*x+e)), x)

**Rubi [A]** time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b \cosh(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d\*x)^2\*(a + b\*Cosh[e + f\*x])), x]

[Out] Defer[Int][1/((c + d\*x)^2\*(a + b\*Cosh[e + f\*x])), x]

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b \cosh(e+fx))} dx = \int \frac{1}{(c+dx)^2(a+b \cosh(e+fx))} dx$$

**Mathematica [A]** time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+b \cosh(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d\*x)^2\*(a + b\*Cosh[e + f\*x])), x]

[Out] Integrate[1/((c + d\*x)^2\*(a + b\*Cosh[e + f\*x])), x]

**fricas [A]** time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{ad^2x^2 + 2acdx + ac^2 + (bd^2x^2 + 2bcdx + bc^2) \cosh(fx + e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+b\*cosh(f\*x+e)),x, algorithm="fricas")

[Out] integral(1/(a\*d^2\*x^2 + 2\*a\*c\*d\*x + a\*c^2 + (b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*cosh(f\*x + e)), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)^2 (b \cosh(fx + e) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+b\*cosh(f\*x+e)),x, algorithm="giac")

[Out] integrate(1/((d\*x + c)^2\*(b\*cosh(f\*x + e) + a)), x)

**maple** [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)^2 (a + b \cosh(fx + e))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x+c)^2/(a+b\*cosh(f\*x+e)),x)

[Out] int(1/(d\*x+c)^2/(a+b\*cosh(f\*x+e)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)^2 (b \cosh(fx + e) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+b\*cosh(f\*x+e)),x, algorithm="maxima")

[Out] integrate(1/((d\*x + c)^2\*(b\*cosh(f\*x + e) + a)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a + b \cosh(e + fx)) (c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*cosh(e + f\*x))\*(c + d\*x)^2),x)

[Out] `int(1/((a + b*cosh(e + f*x))*(c + d*x)^2), x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \cosh(e + fx))(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*x+c)**2/(a+b*cosh(f*x+e)), x)`

[Out] `Integral(1/((a + b*cosh(e + f*x))*(c + d*x)**2), x)`

$$3.173 \quad \int \frac{(c+dx)^3}{(a+b \cosh(e+fx))^2} dx$$

**Optimal.** Leaf size=823

$$\frac{6\text{Li}_3\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)d^3}{(a^2-b^2)f^4} - \frac{6\text{Li}_3\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)d^3}{(a^2-b^2)f^4} + \frac{6a\text{Li}_4\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)d^3}{(a^2-b^2)^{3/2}f^4} - \frac{6a\text{Li}_4\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)d^3}{(a^2-b^2)^{3/2}f^4} + \frac{6(c+dx)\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^3}$$

```
[Out] -(d*x+c)^3/(a^2-b^2)/f+3*d*(d*x+c)^2*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/
/(a^2-b^2)/f^2+a*(d*x+c)^3*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)
^(3/2)/f+3*d*(d*x+c)^2*ln(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/f^2
-a*(d*x+c)^3*ln(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f+6*d^2
*(d*x+c)*polylog(2,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/f^3+3*a*d*
(d*x+c)^2*polylog(2,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^2+6
*d^2*(d*x+c)*polylog(2,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/f^3-3*a
*d*(d*x+c)^2*polylog(2,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f
^2-6*d^3*polylog(3,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/f^4-6*a*d^2
*(d*x+c)*polylog(3,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^3-6
*d^3*polylog(3,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/f^4+6*a*d^2*(d*
x+c)*polylog(3,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^3+6*a*d
^3*polylog(4,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^4-6*a*d^3
*polylog(4,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^4-b*(d*x+c)
^3*sinh(f*x+e)/(a^2-b^2)/f/(a+b*cosh(f*x+e))
```

**Rubi [A]** time = 1.35, antiderivative size = 823, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$ , Rules used = {3324, 3320, 2264, 2190, 2531, 6609, 2282, 6589, 5562}

$$\frac{6\text{PolyLog}\left(3, -\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)d^3}{(a^2-b^2)f^4} - \frac{6\text{PolyLog}\left(3, -\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)d^3}{(a^2-b^2)f^4} + \frac{6a\text{PolyLog}\left(4, -\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)d^3}{(a^2-b^2)^{3/2}f^4} - \frac{6a\text{PolyLog}\left(4, -\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)d^3}{(a^2-b^2)^{3/2}f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Cosh[e + f*x])^2, x]
```

```
[Out] -((c + d*x)^3/((a^2 - b^2)*f)) + (3*d*(c + d*x)^2*Log[1 + (b*E^(e + f*x))]/(
a - Sqrt[a^2 - b^2]))/((a^2 - b^2)*f^2) + (a*(c + d*x)^3*Log[1 + (b*E^(e +
f*x))]/(a - Sqrt[a^2 - b^2]))/((a^2 - b^2)^(3/2)*f) + (3*d*(c + d*x)^2*Log
[1 + (b*E^(e + f*x))]/(a + Sqrt[a^2 - b^2]))/((a^2 - b^2)*f^2) - (a*(c + d*
x)^3*Log[1 + (b*E^(e + f*x))]/(a + Sqrt[a^2 - b^2]))/((a^2 - b^2)^(3/2)*f)
+ (6*d^2*(c + d*x)*PolyLog[2, -((b*E^(e + f*x)))/(a - Sqrt[a^2 - b^2])])/((
a^2 - b^2)*f^3) + (3*a*d*(c + d*x)^2*PolyLog[2, -((b*E^(e + f*x)))/(a - Sqrt
```



$$\frac{[a^2 - b^2]^{(3/2)}}{(a^2 - b^2)^{3/2} f^2} + \frac{(6d^2(c + dx) \text{PolyLog}[2, -(bE^{e+fx})/(a + \sqrt{a^2 - b^2})])}{(a^2 - b^2) f^3} - \frac{(3ad^2(c + dx)^2 \text{PolyLog}[2, -(bE^{e+fx})/(a + \sqrt{a^2 - b^2})])}{(a^2 - b^2)^{3/2} f^2} - \frac{(6d^3 \text{PolyLog}[3, -(bE^{e+fx})/(a - \sqrt{a^2 - b^2})])}{(a^2 - b^2) f^4} - \frac{(6ad^2(c + dx) \text{PolyLog}[3, -(bE^{e+fx})/(a - \sqrt{a^2 - b^2})])}{(a^2 - b^2)^{3/2} f^3} - \frac{(6d^3 \text{PolyLog}[3, -(bE^{e+fx})/(a + \sqrt{a^2 - b^2})])}{(a^2 - b^2) f^4} + \frac{(6ad^2(c + dx) \text{PolyLog}[3, -(bE^{e+fx})/(a + \sqrt{a^2 - b^2})])}{(a^2 - b^2)^{3/2} f^3} + \frac{(6ad^3 \text{PolyLog}[4, -(bE^{e+fx})/(a - \sqrt{a^2 - b^2})])}{(a^2 - b^2)^{3/2} f^4} - \frac{(6ad^3 \text{PolyLog}[4, -(bE^{e+fx})/(a + \sqrt{a^2 - b^2})])}{(a^2 - b^2)^{3/2} f^4} - \frac{(b(c + dx)^3 \text{Sinh}[e + fx])}{(a^2 - b^2) f (a + b \cos h[e + fx])}$$

### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + dx)^m*Log[1 + (b*(F^(g*(e + fx)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + dx)^(m - 1)*Log[1 + (b*(F^(g*(e + fx)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Complex[0, fz_] * (f_.)*(x_))], x_Symbol] := Dist[2, Int[((c + d*x)^m * E^(-(I*e) + f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/2))) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := Simp[(b*(c + d*x)^m * Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e + f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x], x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m * E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m * E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m * PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^3}{(a+b \cosh(e+fx))^2} dx &= -\frac{b(c+dx)^3 \sinh(e+fx)}{(a^2-b^2)f(a+b \cosh(e+fx))} + \frac{a \int \frac{(c+dx)^3}{a+b \cosh(e+fx)} dx}{a^2-b^2} + \frac{(3bd) \int \frac{(c+dx)^2 \sinh(e+fx)}{a+b \cosh(e+fx)} dx}{(a^2-b^2)f} \\
&= -\frac{(c+dx)^3}{(a^2-b^2)f} - \frac{b(c+dx)^3 \sinh(e+fx)}{(a^2-b^2)f(a+b \cosh(e+fx))} + \frac{(2a) \int \frac{e^{e+fx}(c+dx)^3}{b+2ae^{e+fx}+be^{2(e+fx)}} dx}{a^2-b^2} + \dots \\
&= -\frac{(c+dx)^3}{(a^2-b^2)f} + \frac{3d(c+dx)^2 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} + \frac{3d(c+dx)^2 \log\left(1 + \frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} \\
&= -\frac{(c+dx)^3}{(a^2-b^2)f} + \frac{3d(c+dx)^2 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} + \frac{a(c+dx)^3 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \dots \\
&= -\frac{(c+dx)^3}{(a^2-b^2)f} + \frac{3d(c+dx)^2 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} + \frac{a(c+dx)^3 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \dots \\
&= -\frac{(c+dx)^3}{(a^2-b^2)f} + \frac{3d(c+dx)^2 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} + \frac{a(c+dx)^3 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \dots \\
&= -\frac{(c+dx)^3}{(a^2-b^2)f} + \frac{3d(c+dx)^2 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} + \frac{a(c+dx)^3 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \dots \\
&= -\frac{(c+dx)^3}{(a^2-b^2)f} + \frac{3d(c+dx)^2 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} + \frac{a(c+dx)^3 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \dots
\end{aligned}$$

**Mathematica [B]** time = 27.20, size = 11178, normalized size = 13.58

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x)^3/(a + b\*Cosh[e + f\*x])^2,x]

[Out] Result too large to show

**fricas [C]** time = 0.77, size = 7116, normalized size = 8.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -(2*(a^2*b - b^3)*d^3*e^3 - 6*(a^2*b - b^3)*c*d^2*e^2*f + 6*(a^2*b - b^3)*c^2*d*e*f^2 - 2*(a^2*b - b^3)*c^3*f^3 + 2*((a^2*b - b^3)*d^3*f^3*x^3 + 3*(a^2*b - b^3)*c*d^2*f^3*x^2 + 3*(a^2*b - b^3)*c^2*d*f^3*x + (a^2*b - b^3)*d^3*e^3 - 3*(a^2*b - b^3)*c*d^2*e^2*f + 3*(a^2*b - b^3)*c^2*d*e*f^2)*cosh(f*x + e)^2 + 2*((a^2*b - b^3)*d^3*f^3*x^3 + 3*(a^2*b - b^3)*c*d^2*f^3*x^2 + 3*(a^2*b - b^3)*c^2*d*f^3*x + (a^2*b - b^3)*d^3*e^3 - 3*(a^2*b - b^3)*c*d^2*e^2*f + 3*(a^2*b - b^3)*c^2*d*e*f^2)*sinh(f*x + e)^2 - 6*(a*b^2*d^3*cosh(f*x + e)^2 + a*b^2*d^3*sinh(f*x + e)^2 + 2*a^2*b*d^3*cosh(f*x + e) + a*b^2*d^3 + 2*(a*b^2*d^3*cosh(f*x + e) + a^2*b*d^3)*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b) + 6*(a*b^2*d^3*cosh(f*x + e)^2 + a*b^2*d^3*sinh(f*x + e)^2 + 2*a^2*b*d^3*cosh(f*x + e) + a*b^2*d^3 + 2*(a*b^2*d^3*cosh(f*x + e) + a^2*b*d^3)*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b) + 2*((a^3 - a*b^2)*d^3*f^3*x^3 + 3*(a^3 - a*b^2)*c*d^2*f^3*x^2 + 3*(a^3 - a*b^2)*c^2*d*f^3*x + 2*(a^3 - a*b^2)*d^3*e^3 - 6*(a^3 - a*b^2)*c*d^2*e^2*f + 6*(a^3 - a*b^2)*c^2*d*e*f^2 - (a^3 - a*b^2)*c^3*f^3)*cosh(f*x + e) - 3*(2*(a^2*b - b^3)*d^3*f*x + 2*(a^2*b - b^3)*c*d^2*f + 2*((a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)*cosh(f*x + e)^2 + 2*((a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)*sinh(f*x + e)^2 + 4*((a^3 - a*b^2)*d^3*f*x + (a^3 - a*b^2)*c*d^2*f)*cosh(f*x + e) + 4*((a^3 - a*b^2)*d^3*f*x + (a^3 - a*b^2)*c*d^2*f + ((a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)*cosh(f*x + e))*sinh(f*x + e) + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d*f^2 + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d*f^2)*cosh(f*x + e)^2 + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d*f^2)*sinh(f*x + e)^2 + 2*(a^2*b*d^3*f^2*x^2 + 2*a^2*b*c*d^2*f^2*x + a^2*b*c^2*d*f^2)*cosh(f*x + e) + 2*(a^2*b*d^3*f^2*x^2 + 2*a^2*b*c*d^2*f^2*x + a^2*b*c^2*d*f^2 + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d*f^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))*dilog(-(a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 3*(2*(a^2*b - b^3)*d^3*f*x + 2*(a^2*b - b^3)*c*d^2*f + 2*((a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)*cosh(f*x + e)^2 + 2*((a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)*sinh(f*x + e)^2 + 4*((a^3 - a*b^2)*d^3*f*x + (a^3 - a*b^2)*c*d^2*f)*cosh(f*x + e) + 4*((a^3 - a*b^2)*d^3*f*x + (a^3 - a*b^2)*c*d^2*f + ((a^2*b - b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)*cosh(f*x + e))*sinh(f*x + e) - (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d*f^2 + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d*f^2)*cosh(f*x + e)^2 + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d*f^2)*sinh(f*x + e)^2 + 2*(a^2*b*d^3*f^2*x^2 + 2*a^2*b*c*d^2*f^2*x + a^2*b*c^2*d*f^2)*cosh(f*x + e) + 2*(a^2*b*d^3*f^2*x^2 + 2*a^2*b*c*d^2*f^2*x + a^2*b*c^2*d*f^2 + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d*f^2)
```



$$\begin{aligned}
& b^2*c*d^2*f^3*x^2 + 3*a*b^2*c^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2 \\
& *f + 3*a*b^2*c^2*d*e*f^2 + (a*b^2*d^3*f^3*x^3 + 3*a*b^2*c*d^2*f^3*x^2 + 3*a \\
& *b^2*c^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2) \\
& *cosh(f*x + e)^2 + (a*b^2*d^3*f^3*x^3 + 3*a*b^2*c*d^2*f^3*x^2 + 3*a*b^2*c \\
& ^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2)*sin \\
& h(f*x + e)^2 + 2*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*c*d^2*f^3*x^2 + 3*a^2*b*c^2*d \\
& *f^3*x + a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2)*cosh(f* \\
& x + e) + 2*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*c*d^2*f^3*x^2 + 3*a^2*b*c^2*d*f^3*x \\
& + a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 + (a*b^2*d^3*f \\
& ^3*x^3 + 3*a*b^2*c*d^2*f^3*x^2 + 3*a*b^2*c^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a* \\
& b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(( \\
& a^2 - b^2)/b^2))*log((a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) \\
& + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b) - (3*(a^2*b - b^3)*d^3*f^2 \\
& *x^2 + 6*(a^2*b - b^3)*c*d^2*f^2*x - 3*(a^2*b - b^3)*d^3*e^2 + 6*(a^2*b - b \\
& ^3)*c*d^2*e*f + 3*((a^2*b - b^3)*d^3*f^2*x^2 + 2*(a^2*b - b^3)*c*d^2*f^2*x \\
& - (a^2*b - b^3)*d^3*e^2 + 2*(a^2*b - b^3)*c*d^2*e*f)*cosh(f*x + e)^2 + 3*(( \\
& a^2*b - b^3)*d^3*f^2*x^2 + 2*(a^2*b - b^3)*c*d^2*f^2*x - (a^2*b - b^3)*d^3* \\
& e^2 + 2*(a^2*b - b^3)*c*d^2*e*f)*sinh(f*x + e)^2 + 6*((a^3 - a*b^2)*d^3*f^2 \\
& *x^2 + 2*(a^3 - a*b^2)*c*d^2*f^2*x - (a^3 - a*b^2)*d^3*e^2 + 2*(a^3 - a*b^2) \\
& )*c*d^2*e*f)*cosh(f*x + e) + 6*((a^3 - a*b^2)*d^3*f^2*x^2 + 2*(a^3 - a*b^2) \\
& *c*d^2*f^2*x - (a^3 - a*b^2)*d^3*e^2 + 2*(a^3 - a*b^2)*c*d^2*e*f + ((a^2*b \\
& - b^3)*d^3*f^2*x^2 + 2*(a^2*b - b^3)*c*d^2*f^2*x - (a^2*b - b^3)*d^3*e^2 + \\
& 2*(a^2*b - b^3)*c*d^2*e*f)*cosh(f*x + e))*sinh(f*x + e) - (a*b^2*d^3*f^3*x^ \\
& 3 + 3*a*b^2*c*d^2*f^3*x^2 + 3*a*b^2*c^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a*b^2*c \\
& *d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 + (a*b^2*d^3*f^3*x^3 + 3*a*b^2*c*d^2*f^3*x^2 + 3 \\
& *a*b^2*c^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e* \\
& f^2)*cosh(f*x + e)^2 + (a*b^2*d^3*f^3*x^3 + 3*a*b^2*c*d^2*f^3*x^2 + 3 \\
& *a*b^2*c^2*d*f^3*x + a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e* \\
& f^2)*sinh(f*x + e)^2 + 2*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*c*d^2*f^3*x^2 + 3*a^2 \\
& *b*c^2*d*f^3*x + a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2) \\
& *cosh(f*x + e) + 2*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*c*d^2*f^3*x^2 + 3*a^2*b*c^2 \\
& *d*f^3*x + a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 + (a*b \\
& ^2*d^3*f^3*x^3 + 3*a*b^2*c*d^2*f^3*x^2 + 3*a*b^2*c^2*d*f^3*x + a*b^2*d^3*e^ \\
& 3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2)*cosh(f*x + e))*sinh(f*x + e) \\
& )*sqrt((a^2 - b^2)/b^2))*log((a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f \\
& *x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b) + 6*((a^2*b - b^3) \\
& *d^3*cosh(f*x + e)^2 + (a^2*b - b^3)*d^3*sinh(f*x + e)^2 + 2*(a^3 - a*b^2)* \\
& d^3*cosh(f*x + e) + (a^2*b - b^3)*d^3 + 2*((a^2*b - b^3)*d^3*cosh(f*x + e) \\
& + (a^3 - a*b^2)*d^3)*sinh(f*x + e) + (a*b^2*d^3*f*x + a*b^2*c*d^2*f + (a*b^ \\
& 2*d^3*f*x + a*b^2*c*d^2*f)*cosh(f*x + e)^2 + (a*b^2*d^3*f*x + a*b^2*c*d^2*f \\
& )*sinh(f*x + e)^2 + 2*(a^2*b*d^3*f*x + a^2*b*c*d^2*f)*cosh(f*x + e) + 2*(a^ \\
& 2*b*d^3*f*x + a^2*b*c*d^2*f + (a*b^2*d^3*f*x + a*b^2*c*d^2*f)*cosh(f*x + e) \\
& )*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))*polylog(3, -(a*cosh(f*x + e) + a*si \\
& nh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b) \\
& + 6*((a^2*b - b^3)*d^3*cosh(f*x + e)^2 + (a^2*b - b^3)*d^3*sinh(f*x + e)^2
\end{aligned}$$

+ 2\*(a^3 - a\*b^2)\*d^3\*cosh(f\*x + e) + (a^2\*b - b^3)\*d^3 + 2\*((a^2\*b - b^3)\*d^3\*cosh(f\*x + e) + (a^3 - a\*b^2)\*d^3)\*sinh(f\*x + e) - (a\*b^2\*d^3\*f\*x + a\*b^2\*c\*d^2\*f + (a\*b^2\*d^3\*f\*x + a\*b^2\*c\*d^2\*f)\*cosh(f\*x + e)^2 + (a\*b^2\*d^3\*f\*x + a\*b^2\*c\*d^2\*f)\*sinh(f\*x + e)^2 + 2\*(a^2\*b\*d^3\*f\*x + a^2\*b\*c\*d^2\*f)\*cosh(f\*x + e) + 2\*(a^2\*b\*d^3\*f\*x + a^2\*b\*c\*d^2\*f + (a\*b^2\*d^3\*f\*x + a\*b^2\*c\*d^2\*f)\*cosh(f\*x + e))\*sinh(f\*x + e))\*sqrt((a^2 - b^2)/b^2))\*polylog(3, -(a\*cosh(f\*x + e) + a\*sinh(f\*x + e) - (b\*cosh(f\*x + e) + b\*sinh(f\*x + e))\*sqrt((a^2 - b^2)/b^2))/b) + 2\*((a^3 - a\*b^2)\*d^3\*f^3\*x^3 + 3\*(a^3 - a\*b^2)\*c\*d^2\*f^3\*x^2 + 3\*(a^3 - a\*b^2)\*c^2\*d\*f^3\*x + 2\*(a^3 - a\*b^2)\*d^3\*e^3 - 6\*(a^3 - a\*b^2)\*c\*d^2\*e^2\*f + 6\*(a^3 - a\*b^2)\*c^2\*d\*e\*f^2 - (a^3 - a\*b^2)\*c^3\*f^3 + 2\*((a^2\*b - b^3)\*d^3\*f^3\*x^3 + 3\*(a^2\*b - b^3)\*c\*d^2\*f^3\*x^2 + 3\*(a^2\*b - b^3)\*c^2\*d\*f^3\*x + (a^2\*b - b^3)\*d^3\*e^3 - 3\*(a^2\*b - b^3)\*c\*d^2\*e^2\*f + 3\*(a^2\*b - b^3)\*c^2\*d\*e\*f^2)\*cosh(f\*x + e))\*sinh(f\*x + e))/((a^4\*b - 2\*a^2\*b^3 + b^5)\*f^4\*cosh(f\*x + e)^2 + (a^4\*b - 2\*a^2\*b^3 + b^5)\*f^4\*sinh(f\*x + e)^2 + 2\*(a^5 - 2\*a^3\*b^2 + a\*b^4)\*f^4\*cosh(f\*x + e) + (a^4\*b - 2\*a^2\*b^3 + b^5)\*f^4 + 2\*((a^4\*b - 2\*a^2\*b^3 + b^5)\*f^4\*cosh(f\*x + e) + (a^5 - 2\*a^3\*b^2 + a\*b^4)\*f^4)\*sinh(f\*x + e))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^3}{(b \cosh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+b\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^3/(b\*cosh(f\*x + e) + a)^2, x)

**maple** [F] time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^3}{(a + b \cosh(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3/(a+b\*cosh(f\*x+e))^2,x)

[Out] int((d\*x+c)^3/(a+b\*cosh(f\*x+e))^2,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+b\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see `assume?` for more details)Is 4\*a^2-4\*b^2 positive or negative?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + dx)^3}{(a + b \cosh(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^3/(a + b\*cosh(e + f\*x))^2,x)

[Out] int((c + d\*x)^3/(a + b\*cosh(e + f\*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3/(a+b\*cosh(f\*x+e))\*\*2,x)

[Out] Timed out



$$3.174 \quad \int \frac{(c+dx)^2}{(a+b \cosh(e+fx))^2} dx$$

**Optimal.** Leaf size=593

$$\frac{2ad(c+dx)\operatorname{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2(a^2-b^2)^{3/2}} - \frac{2ad(c+dx)\operatorname{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^2(a^2-b^2)^{3/2}} + \frac{2d(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{f^2(a^2-b^2)} + \frac{2d(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}}\right)}{f^2(a^2-b^2)}$$

[Out]  $-(d*x+c)^2/(a^2-b^2)/f+2*d*(d*x+c)*\ln(1+b*\exp(f*x+e)/(a-(a^2-b^2)^{(1/2)}))/((a^2-b^2)/f^2+a*(d*x+c)^2*\ln(1+b*\exp(f*x+e)/(a-(a^2-b^2)^{(1/2)}))/(a^2-b^2)^{(3/2)}/f+2*d*(d*x+c)*\ln(1+b*\exp(f*x+e)/(a+(a^2-b^2)^{(1/2)}))/(a^2-b^2)/f^2-a*(d*x+c)^2*\ln(1+b*\exp(f*x+e)/(a+(a^2-b^2)^{(1/2)}))/(a^2-b^2)^{(3/2)}/f+2*d^2*\operatorname{polylog}(2,-b*\exp(f*x+e)/(a-(a^2-b^2)^{(1/2)}))/(a^2-b^2)/f^3+2*a*d*(d*x+c)*\operatorname{polylog}(2,-b*\exp(f*x+e)/(a-(a^2-b^2)^{(1/2)}))/(a^2-b^2)^{(3/2)}/f^2+2*d^2*\operatorname{polylog}(2,-b*\exp(f*x+e)/(a+(a^2-b^2)^{(1/2)}))/(a^2-b^2)/f^3-2*a*d*(d*x+c)*\operatorname{polylog}(2,-b*\exp(f*x+e)/(a+(a^2-b^2)^{(1/2)}))/(a^2-b^2)^{(3/2)}/f^2-2*a*d^2*\operatorname{polylog}(3,-b*\exp(f*x+e)/(a-(a^2-b^2)^{(1/2)}))/(a^2-b^2)^{(3/2)}/f^3+2*a*d^2*\operatorname{polylog}(3,-b*\exp(f*x+e)/(a+(a^2-b^2)^{(1/2)}))/(a^2-b^2)^{(3/2)}/f^3-b*(d*x+c)^2*\sinh(f*x+e)/(a^2-b^2)/f/(a+b*\cosh(f*x+e))$

**Rubi [A]** time = 1.03, antiderivative size = 593, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {3324, 3320, 2264, 2190, 2531, 2282, 6589, 5562, 2279, 2391}

$$\frac{2ad(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2(a^2-b^2)^{3/2}} - \frac{2ad(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^2(a^2-b^2)^{3/2}} + \frac{2d^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^3(a^2-b^2)} + \frac{2d^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^3(a^2-b^2)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c+d*x)^2/(a+b*\operatorname{Cosh}[e+f*x])^2,x]$

[Out]  $-\left(\frac{(c+d*x)^2}{(a^2-b^2)*f}\right) + \frac{(2*d*(c+d*x)*\operatorname{Log}[1+(b*E^{e+f*x})/(a-\sqrt{a^2-b^2})])}{((a^2-b^2)*f^2) + (a*(c+d*x)^2*\operatorname{Log}[1+(b*E^{e+f*x})/(a-\sqrt{a^2-b^2})])}{((a^2-b^2)^{(3/2)*f) + (2*d*(c+d*x)*\operatorname{Log}[1+(b*E^{e+f*x})/(a+\sqrt{a^2-b^2})])}{((a^2-b^2)*f^2) - (a*(c+d*x)^2*\operatorname{Log}[1+(b*E^{e+f*x})/(a+\sqrt{a^2-b^2})])}{((a^2-b^2)^{(3/2)*f) + (2*d^2*\operatorname{PolyLog}[2,-((b*E^{e+f*x})/(a-\sqrt{a^2-b^2})])}{((a^2-b^2)*f^3) + (2*a*d*(c+d*x)*\operatorname{PolyLog}[2,-((b*E^{e+f*x})/(a-\sqrt{a^2-b^2})])}{((a^2-b^2)^{(3/2)*f^2) + (2*d^2*\operatorname{PolyLog}[2,-((b*E^{e+f*x})/(a+\sqrt{a^2-b^2})])}{((a^2-b^2)*f^3) - (2*a*d*(c+d*x)*\operatorname{PolyLog}[2,-((b*E^{e+f*x})/(a+\sqrt{a^2-b^2})])}{((a^2-b^2)^{(3/2)*f^2) - (2*a*d^2*\operatorname{PolyLog}[3,-$

$$\frac{((b \cdot E^{(e + f \cdot x)}) / (a - \sqrt{a^2 - b^2}))}{((a^2 - b^2)^{3/2} \cdot f^3)} + \frac{(2 \cdot a \cdot d^2 \cdot \text{PolyLog}[3, -((b \cdot E^{(e + f \cdot x)}) / (a + \sqrt{a^2 - b^2}))]}{((a^2 - b^2)^{3/2} \cdot f^3)} - (b \cdot (c + d \cdot x)^2 \cdot \text{Sinh}[e + f \cdot x])}{((a^2 - b^2) \cdot f \cdot (a + b \cdot \text{Cosh}[e + f \cdot x]))}$$
Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
```

, g, n}, x] && GtQ[m, 0]

### Rule 3320

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_) + (b\_.)\*sin[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_] \*(f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^(m)\*E^(-I\*e) + f\*fz\*x)/(E^(I\*Pi\*(k - 1/2))\*(b + (2\*a\*E^(-I\*e) + f\*fz\*x))/E^(I\*Pi\*(k - 1/2)) - (b\*E^(2\*(-I\*e) + f\*fz\*x))/E^(2\*I\*k\*Pi))], x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 3324

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^2, x\_Symbol] := Simp[(b\*(c + d\*x)^m\*Cos[e + f\*x])/(f\*(a^2 - b^2)\*(a + b\*Sin[e + f\*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d\*x)^m/(a + b\*Sin[e + f\*x]), x], x] - Dist[(b\*d\*m)/(f\*(a^2 - b^2)), Int[((c + d\*x)^(m - 1)\*Cos[e + f\*x])/(a + b\*Sin[e + f\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 5562

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]/(Cosh[(c\_.) + (d\_.)\*(x\_)]\*(b\_.) + (a\_)), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 - b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 - b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^2}{(a+b \cosh(e+fx))^2} dx &= -\frac{b(c+dx)^2 \sinh(e+fx)}{(a^2-b^2)f(a+b \cosh(e+fx))} + \frac{a \int \frac{(c+dx)^2}{a+b \cosh(e+fx)} dx}{a^2-b^2} + \frac{(2bd) \int \frac{(c+dx) \sinh(e+fx)}{a+b \cosh(e+fx)} dx}{(a^2-b^2)f} \\
&= -\frac{(c+dx)^2}{(a^2-b^2)f} - \frac{b(c+dx)^2 \sinh(e+fx)}{(a^2-b^2)f(a+b \cosh(e+fx))} + \frac{(2a) \int \frac{e^{e+fx}(c+dx)^2}{b+2ae^{e+fx}+be^{2(e+fx)}} dx}{a^2-b^2} + \frac{(2bd) \int \frac{(c+dx) \sinh(e+fx)}{a+b \cosh(e+fx)} dx}{(a^2-b^2)f} \\
&= -\frac{(c+dx)^2}{(a^2-b^2)f} + \frac{2d(c+dx) \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} + \frac{2d(c+dx) \log\left(1 + \frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} - \frac{(2bd) \int \frac{(c+dx) \sinh(e+fx)}{a+b \cosh(e+fx)} dx}{(a^2-b^2)f} \\
&= -\frac{(c+dx)^2}{(a^2-b^2)f} + \frac{2d(c+dx) \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} + \frac{a(c+dx)^2 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \frac{(2bd) \int \frac{(c+dx) \sinh(e+fx)}{a+b \cosh(e+fx)} dx}{(a^2-b^2)f} \\
&= -\frac{(c+dx)^2}{(a^2-b^2)f} + \frac{2d(c+dx) \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} + \frac{a(c+dx)^2 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \frac{(2bd) \int \frac{(c+dx) \sinh(e+fx)}{a+b \cosh(e+fx)} dx}{(a^2-b^2)f} \\
&= -\frac{(c+dx)^2}{(a^2-b^2)f} + \frac{2d(c+dx) \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} + \frac{a(c+dx)^2 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \frac{(2bd) \int \frac{(c+dx) \sinh(e+fx)}{a+b \cosh(e+fx)} dx}{(a^2-b^2)f} \\
&= -\frac{(c+dx)^2}{(a^2-b^2)f} + \frac{2d(c+dx) \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} + \frac{a(c+dx)^2 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \frac{(2bd) \int \frac{(c+dx) \sinh(e+fx)}{a+b \cosh(e+fx)} dx}{(a^2-b^2)f}
\end{aligned}$$

**Mathematica [B]** time = 22.80, size = 6018, normalized size = 10.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2/(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

**fricas [C]** time = 0.55, size = 4105, normalized size = 6.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*cosh(f\*x+e))^2,x, algorithm="fricas")

[Out]  $(2*(a^2*b - b^3)*d^2*e^2 - 4*(a^2*b - b^3)*c*d*e*f + 2*(a^2*b - b^3)*c^2*f^2 - 2*((a^2*b - b^3)*d^2*f^2*x^2 + 2*(a^2*b - b^3)*c*d*f^2*x - (a^2*b - b^3)*d^2*e^2 + 2*(a^2*b - b^3)*c*d*e*f)*\cosh(f*x + e)^2 - 2*((a^2*b - b^3)*d^2*f^2*x^2 + 2*(a^2*b - b^3)*c*d*f^2*x - (a^2*b - b^3)*d^2*e^2 + 2*(a^2*b - b^3)*c*d*e*f)*\sinh(f*x + e)^2 - 2*(a*b^2*d^2*\cosh(f*x + e)^2 + a*b^2*d^2*\sinh(f*x + e)^2 + 2*a^2*b*d^2*\cosh(f*x + e) + a*b^2*d^2 + 2*(a*b^2*d^2*\cosh(f*x + e) + a^2*b*d^2)*\sinh(f*x + e))*\sqrt{((a^2 - b^2)/b^2)*\text{polylog}(3, -(a*\cosh(f*x + e) + a*\sinh(f*x + e) + (b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{(a^2 - b^2)/b^2}))/b} + 2*(a*b^2*d^2*\cosh(f*x + e)^2 + a*b^2*d^2*\sinh(f*x + e)^2 + 2*a^2*b*d^2*\cosh(f*x + e) + a*b^2*d^2 + 2*(a*b^2*d^2*\cosh(f*x + e) + a^2*b*d^2)*\sinh(f*x + e))*\sqrt{((a^2 - b^2)/b^2)*\text{polylog}(3, -(a*\cosh(f*x + e) + a*\sinh(f*x + e) - (b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{(a^2 - b^2)/b^2}))/b} - 2*((a^3 - a*b^2)*d^2*f^2*x^2 + 2*(a^3 - a*b^2)*c*d*f^2*x - 2*(a^3 - a*b^2)*d^2*e^2 + 4*(a^3 - a*b^2)*c*d*e*f - (a^3 - a*b^2)*c^2*f^2)*\cosh(f*x + e) + 2*((a^2*b - b^3)*d^2*\cosh(f*x + e)^2 + (a^2*b - b^3)*d^2*\sinh(f*x + e)^2 + 2*(a^3 - a*b^2)*d^2*\cosh(f*x + e) + (a^2*b - b^3)*d^2 + 2*((a^2*b - b^3)*d^2*\cosh(f*x + e) + (a^3 - a*b^2)*d^2)*\sinh(f*x + e) + (a*b^2*d^2*f*x + a*b^2*c*d*f + (a*b^2*d^2*f*x + a*b^2*c*d*f)*\cosh(f*x + e)^2 + (a*b^2*d^2*f*x + a*b^2*c*d*f)*\sinh(f*x + e)^2 + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*\cosh(f*x + e) + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f + (a*b^2*d^2*f*x + a*b^2*c*d*f)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{((a^2 - b^2)/b^2)}*\text{dilog}(-(a*\cosh(f*x + e) + a*\sinh(f*x + e) + (b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{(a^2 - b^2)/b^2}) + b)/b + 1) + 2*((a^2*b - b^3)*d^2*\cosh(f*x + e)^2 + (a^2*b - b^3)*d^2*\sinh(f*x + e)^2 + 2*(a^3 - a*b^2)*d^2*\cosh(f*x + e) + (a^2*b - b^3)*d^2 + 2*((a^2*b - b^3)*d^2*\cosh(f*x + e) + (a^3 - a*b^2)*d^2)*\sinh(f*x + e) - (a*b^2*d^2*f*x + a*b^2*c*d*f + (a*b^2*d^2*f*x + a*b^2*c*d*f)*\cosh(f*x + e)^2 + (a*b^2*d^2*f*x + a*b^2*c*d*f)*\sinh(f*x + e)^2 + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*\cosh(f*x + e) + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f + (a*b^2*d^2*f*x + a*b^2*c*d*f)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{((a^2 - b^2)/b^2)}*\text{dilog}(-(a*\cosh(f*x + e) + a*\sinh(f*x + e) - (b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{(a^2 - b^2)/b^2}) + b)/b + 1) - (2*(a^2*b - b^3)*d^2*e - 2*(a^2*b - b^3)*c*d*f + 2*((a^2*b - b^3)*d^2*e - (a^2*b - b^3)*c*d*f)*\cosh(f*x + e)^2 + 2*((a^2*b - b^3)*d^2*e - (a^2*b - b^3)*c*d*f)*\sinh(f*x + e)^2 + 4*((a^3 - a*b^2)*d^2*e - (a^3 - a*b^2)*c*d*f + ((a^2*b - b^3)*d^2*e - (a^2*b - b^3)*c*d*f)*\cosh(f*x + e))*\sinh(f*x + e) + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*\cosh(f*x + e)^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*\sinh(f*x + e)^2 + 2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{((a^2 - b^2)/b^2)}*\log(2*b*\cosh(f*x + e) + 2*b*\sinh(f*x + e) + 2*b*\sqrt{(a^2 - b^2)/b^2} + 2*a) - (2*(a^2*b - b^3)*d^2*e - 2*(a^2*b - b^3)*c*d*f + 2*((a^2*b - b^3)*d^2*e - (a^2*b - b^3)*c*d*f)*$

$$\begin{aligned}
& \cosh(f*x + e)^2 + 2*((a^2*b - b^3)*d^2*e - (a^2*b - b^3)*c*d*f)*\sinh(f*x + \\
& e)^2 + 4*((a^3 - a*b^2)*d^2*e - (a^3 - a*b^2)*c*d*f)*\cosh(f*x + e) + 4*((a^3 - a*b^2)*d^2*e - (a^3 - a*b^2)*c*d*f) \\
& + ((a^2*b - b^3)*d^2*e - (a^2*b - b^3)*c*d*f)*\cosh(f*x + e))*\sinh(f*x + e) - (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + \\
& a*b^2*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*\cosh(f*x + e)^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + \\
& a*b^2*c^2*f^2)*\sinh(f*x + e)^2 + 2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*\cosh(f*x + e) + 2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + \\
& a^2*b*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{((a^2 - b^2)/b^2)} \\
& )*\log(2*b*\cosh(f*x + e) + 2*b*\sinh(f*x + e) - 2*b*\sqrt{((a^2 - b^2)/b^2)} + 2*a) + (2*(a^2*b - b^3)*d^2*f*x + 2*(a^2*b - b^3)*d^2*e + 2*((a^2*b - b^3)*d^2*f*x + \\
& (a^2*b - b^3)*d^2*e)*\cosh(f*x + e)^2 + 2*((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*d^2*e)*\sinh(f*x + e)^2 + 4*((a^3 - a*b^2)*d^2*f*x + (a^3 - a*b^2)*d^2*e) \\
& *\cosh(f*x + e) + 4*((a^3 - a*b^2)*d^2*f*x + (a^3 - a*b^2)*d^2*e + ((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*d^2*e)*\cosh(f*x + e))*\sinh(f*x + e) \\
& ) + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f) \\
& *\cosh(f*x + e)^2 + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*\sinh(f*x + e)^2 + 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d*f^2*x - \\
& a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f)*\cosh(f*x + e) + 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d*f^2*x - a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f + (a*b^2*d^2*f^2*x^2 \\
& + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{((a^2 - b^2)/b^2)}*\log((a*\cosh(f*x + e) + a*\sinh(f*x + e) + \\
& (b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{((a^2 - b^2)/b^2)} + b)/b) + (2*(a^2*b - b^3)*d^2*f*x + 2*(a^2*b - b^3)*d^2*e + 2*((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3) \\
& *d^2*e)*\cosh(f*x + e)^2 + 2*((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*d^2*e)*\sinh(f*x + e)^2 + 4*((a^3 - a*b^2)*d^2*f*x + (a^3 - a*b^2)*d^2*e)*\cosh(f*x + e) + 4*((a^3 - a*b^2)*d^2*f*x + (a^3 - a*b^2)*d^2*e + ((a^2*b - b^3)*d^2*f*x + (a^2*b - b^3)*d^2*e)*\cosh(f*x + e))*\sinh(f*x + e) - (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*\cosh(f*x + e)^2 + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*\sinh(f*x + e)^2 + 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d*f^2*x - a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f)*\cosh(f*x + e) + 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d*f^2*x - a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{((a^2 - b^2)/b^2)}*\log((a*\cosh(f*x + e) + a*\sinh(f*x + e) - (b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{((a^2 - b^2)/b^2)} + b)/b) - 2*((a^3 - a*b^2)*d^2*f^2*x^2 + 2*(a^3 - a*b^2)*c*d*f^2*x - 2*(a^3 - a*b^2)*d^2*e^2 + 4*(a^3 - a*b^2)*c*d*e*f - (a^3 - a*b^2)*c^2*f^2 + 2*((a^2*b - b^3)*d^2*f^2*x^2 + 2*(a^2*b - b^3)*c*d*f^2*x - (a^2*b - b^3)*d^2*e^2 + 2*(a^2*b - b^3)*c*d*e*f)*\cosh(f*x + e))*\sinh(f*x + e))/((a^4*b - 2*a^2*b^3 + b^5)*f^3*\cosh(f*x + e)^2 + (a^4*b - 2*a^2*b^3 + b^5)*f^3*\sinh(f*x + e)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*f^3*\cosh(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*f^3 + 2*((a^4*b - 2*a^2*b^3 + b^5)*f^3*\cosh(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*f^3)*\sinh(f*x + e))
\end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^2}{(b \cosh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^2/(b\*cosh(f\*x + e) + a)^2, x)

**maple** [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^2}{(a + b \cosh(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2/(a+b\*cosh(f\*x+e))^2,x)

[Out] int((d\*x+c)^2/(a+b\*cosh(f\*x+e))^2,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see `assume?` for more details)Is 4\*a^2-4\*b^2 positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + dx)^2}{(a + b \cosh(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^2/(a + b\*cosh(e + f\*x))^2,x)

[Out] int((c + d\*x)^2/(a + b\*cosh(e + f\*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2/(a+b\*cosh(f\*x+e))\*\*2,x)

[Out] Timed out



$$3.175 \quad \int \frac{c+dx}{(a+b \cosh(e+fx))^2} dx$$

**Optimal.** Leaf size=274

$$\frac{a(c+dx) \log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{f(a^2-b^2)^{3/2}} - \frac{a(c+dx) \log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{f(a^2-b^2)^{3/2}} - \frac{b(c+dx) \sinh(e+fx)}{f(a^2-b^2)(a+b \cosh(e+fx))} + \frac{ad \operatorname{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2(a^2-b^2)^{3/2}}$$

[Out]  $d \cdot \ln(a+b \cdot \cosh(f \cdot x+e)) / (a^2-b^2) / f^2 + a \cdot (d \cdot x+c) \cdot \ln(1+b \cdot \exp(f \cdot x+e) / (a-(a^2-b^2)^{1/2})) / (a^2-b^2)^{3/2} / f - a \cdot (d \cdot x+c) \cdot \ln(1+b \cdot \exp(f \cdot x+e) / (a+(a^2-b^2)^{1/2})) / (a^2-b^2)^{3/2} / f + a \cdot d \cdot \operatorname{polylog}(2, -b \cdot \exp(f \cdot x+e) / (a-(a^2-b^2)^{1/2})) / (a^2-b^2)^{3/2} / f^2 - a \cdot d \cdot \operatorname{polylog}(2, -b \cdot \exp(f \cdot x+e) / (a+(a^2-b^2)^{1/2})) / (a^2-b^2)^{3/2} / f^2 - b \cdot (d \cdot x+c) \cdot \sinh(f \cdot x+e) / (a^2-b^2) / f / (a+b \cdot \cosh(f \cdot x+e))$

**Rubi [A]** time = 0.46, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {3324, 3320, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{ad \operatorname{PolyLog}\left(2, -\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2(a^2-b^2)^{3/2}} - \frac{ad \operatorname{PolyLog}\left(2, -\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f^2(a^2-b^2)^{3/2}} + \frac{a(c+dx) \log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{f(a^2-b^2)^{3/2}} - \frac{a(c+dx) \log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}\right)}{f(a^2-b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c+d \cdot x) / (a+b \cdot \operatorname{Cosh}[e+f \cdot x])^2, x]$

[Out]  $(a \cdot (c+d \cdot x) \cdot \operatorname{Log}[1+(b \cdot E^{(e+f \cdot x)}) / (a-\operatorname{Sqrt}[a^2-b^2])]) / ((a^2-b^2)^{(3/2)} \cdot f) - (a \cdot (c+d \cdot x) \cdot \operatorname{Log}[1+(b \cdot E^{(e+f \cdot x)}) / (a+\operatorname{Sqrt}[a^2-b^2])]) / ((a^2-b^2)^{(3/2)} \cdot f) + (d \cdot \operatorname{Log}[a+b \cdot \operatorname{Cosh}[e+f \cdot x]]) / ((a^2-b^2) \cdot f^2) + (a \cdot d \cdot \operatorname{PolyLog}[2, -((b \cdot E^{(e+f \cdot x)}) / (a-\operatorname{Sqrt}[a^2-b^2])]) / ((a^2-b^2)^{(3/2)} \cdot f^2) - (a \cdot d \cdot \operatorname{PolyLog}[2, -((b \cdot E^{(e+f \cdot x)}) / (a+\operatorname{Sqrt}[a^2-b^2])]) / ((a^2-b^2)^{(3/2)} \cdot f^2) - (b \cdot (c+d \cdot x) \cdot \operatorname{Sinh}[e+f \cdot x]) / ((a^2-b^2) \cdot f \cdot (a+b \cdot \operatorname{Cosh}[e+f \cdot x]))$

**Rule 31**

$\operatorname{Int}[(a_+ + (b_+)(x_+))^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a+b \cdot x, x]] / b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

**Rule 2190**

$\operatorname{Int}[(F_+)^{(g_+)} \cdot ((e_+ + (f_+)(x_+)))^{(n_+)} \cdot ((c_+ + (d_+)(x_+))^{(m_+)}) / ((a_+ + (b_+)(x_+)) \cdot (F_+)^{(g_+)} \cdot ((e_+ + (f_+)(x_+)))^{(n_+)})], x\_Symbol] \rightarrow \operatorname{Simp}[(c+d \cdot x)^m \cdot \operatorname{Log}[1+(b \cdot (F_+)^{(g_+)} \cdot (e_+ + f_+ \cdot x)) / a] / (b \cdot f \cdot g \cdot n \cdot \operatorname{Log}[F_+]), x] - \operatorname{Di}$

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))
)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))]^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m
_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^(p - 1)/
2], x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

### Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2, x_
Symbol] := Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
```

2, 0] &amp;&amp; IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{(a+b \cosh(e+fx))^2} dx &= -\frac{b(c+dx) \sinh(e+fx)}{(a^2-b^2) f(a+b \cosh(e+fx))} + \frac{a \int \frac{c+dx}{a+b \cosh(e+fx)} dx}{a^2-b^2} + \frac{(bd) \int \frac{\sinh(e+fx)}{a+b \cosh(e+fx)} dx}{(a^2-b^2) f} \\
&= -\frac{b(c+dx) \sinh(e+fx)}{(a^2-b^2) f(a+b \cosh(e+fx))} + \frac{(2a) \int \frac{e^{e+fx}(c+dx)}{b+2ae^{e+fx}+be^{2(e+fx)}} dx}{a^2-b^2} + \frac{d \operatorname{Subst}\left(\int \frac{1}{a+x} dx\right)}{(a^2-b^2) f} \\
&= \frac{d \log(a+b \cosh(e+fx))}{(a^2-b^2) f^2} - \frac{b(c+dx) \sinh(e+fx)}{(a^2-b^2) f(a+b \cosh(e+fx))} + \frac{(2ab) \int \frac{e^{e+fx}(c+dx)}{2a-2\sqrt{a^2-b^2}} dx}{(a^2-b^2) f} \\
&= \frac{a(c+dx) \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2} f} - \frac{a(c+dx) \log\left(1 + \frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2} f} + \frac{d \log(a+b \cosh(e+fx))}{(a^2-b^2) f} \\
&= \frac{a(c+dx) \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2} f} - \frac{a(c+dx) \log\left(1 + \frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2} f} + \frac{d \log(a+b \cosh(e+fx))}{(a^2-b^2) f} \\
&= \frac{a(c+dx) \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2} f} - \frac{a(c+dx) \log\left(1 + \frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2} f} + \frac{d \log(a+b \cosh(e+fx))}{(a^2-b^2) f}
\end{aligned}$$

**Mathematica [A]** time = 5.04, size = 509, normalized size = 1.86

$$(a^2-b^2) \left( 2acf \sqrt{b^2-a^2} \tanh^{-1}\left(\frac{a+be^{e+fx}}{\sqrt{a^2-b^2}}\right) - ad \sqrt{b^2-a^2} \operatorname{Li}_2\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}-a}\right) + ad \sqrt{b^2-a^2} \operatorname{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right) + d \sqrt{-(a^2-b^2)^2} (e+fx) - ad \sqrt{b^2-a^2} (e+fx) \log\left(\frac{b}{a-b}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x)/(a + b\*Cosh[e + f\*x])^2, x]

```

[Out] (((a^2 - b^2)*(Sqrt[-(a^2 - b^2)^2]*d*(e + f*x) - 2*a*Sqrt[a^2 - b^2]*d*ArcTan[(a + b*E^(e + f*x))/Sqrt[-a^2 + b^2]] - 2*a*Sqrt[-a^2 + b^2]*d*ArcTanh[(a + b*E^(e + f*x))/Sqrt[a^2 - b^2]] - 2*a*Sqrt[-a^2 + b^2]*d*e*ArcTanh[(a

```



$x + e) + (a^2*b - b^3)*d + 2*((a^2*b - b^3)*d*\cosh(f*x + e) + (a^3 - a*b^2)*d)*\sinh(f*x + e) - (a*b^2*d*e - a*b^2*c*f + (a*b^2*d*e - a*b^2*c*f)*\cosh(f*x + e))^2 + (a*b^2*d*e - a*b^2*c*f)*\sinh(f*x + e)^2 + 2*(a^2*b*d*e - a^2*b*c*f + (a*b^2*d*e - a*b^2*c*f)*\cosh(f*x + e) + 2*(a^2*b*d*e - a^2*b*c*f + (a*b^2*d*e - a*b^2*c*f)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{(a^2 - b^2)/b^2})*\log(2*b*\cosh(f*x + e) + 2*b*\sinh(f*x + e) - 2*b*\sqrt{(a^2 - b^2)/b^2} + 2*a) + 2*((a^3 - a*b^2)*d*f*x + 2*(a^3 - a*b^2)*d*e - (a^3 - a*b^2)*c*f + 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*d*e)*\cosh(f*x + e))*\sinh(f*x + e))/((a^4*b - 2*a^2*b^3 + b^5)*f^2*\cosh(f*x + e)^2 + (a^4*b - 2*a^2*b^3 + b^5)*f^2*\sinh(f*x + e)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*f^2*\cosh(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*f^2 + 2*((a^4*b - 2*a^2*b^3 + b^5)*f^2*\cosh(f*x + e) + (a^5 - 2*a^3*b^2 + a*b^4)*f^2)*\sinh(f*x + e))$

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx + c}{(b \cosh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate((d\*x + c)/(b\*cosh(f\*x + e) + a)^2, x)

**maple [B]** time = 0.30, size = 585, normalized size = 2.14

$$\frac{2(dx+c)(ae^{fx+e}+b)}{f(a^2-b^2)(be^{2fx+2e}+2ae^{fx+e}+b)} + \frac{2ac \arctan\left(\frac{2be^{fx+e}+2a}{2\sqrt{-a^2+b^2}}\right)}{f(a^2-b^2)\sqrt{-a^2+b^2}} + \frac{da \ln\left(\frac{-be^{fx+e}+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)x}{f(a^2-b^2)^{\frac{3}{2}}} + \frac{da \ln\left(\frac{-be^{fx+e}+\sqrt{a^2-b^2}}{-a+\sqrt{a^2-b^2}}\right)}{f^2(a^2-b^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)/(a+b\*cosh(f\*x+e))^2,x)

[Out]  $2*(d*x+c)*(a*\exp(f*x+e)+b)/f/(a^2-b^2)/(b*\exp(2*f*x+2*e)+2*a*\exp(f*x+e)+b)+2/f/(a^2-b^2)*a*c/(-a^2+b^2)^{(1/2)}*\arctan(1/2*(2*b*\exp(f*x+e)+2*a)/(-a^2+b^2)^{(1/2)})+1/f/(a^2-b^2)^{(3/2)}*d*a*\ln((-b*\exp(f*x+e)+(a^2-b^2)^{(1/2)}-a)/(-a+(a^2-b^2)^{(1/2)}))*x+1/f^2/(a^2-b^2)^{(3/2)}*d*a*\ln((-b*\exp(f*x+e)+(a^2-b^2)^{(1/2)}-a)/(-a+(a^2-b^2)^{(1/2)}))*e-1/f/(a^2-b^2)^{(3/2)}*d*a*\ln((b*\exp(f*x+e)+(a^2-b^2)^{(1/2)}+a)/(a+(a^2-b^2)^{(1/2)}))*x-1/f^2/(a^2-b^2)^{(3/2)}*d*a*\ln((b*\exp(f*x+e)+(a^2-b^2)^{(1/2)}+a)/(a+(a^2-b^2)^{(1/2)}))*e+1/f^2/(a^2-b^2)^{(3/2)}*d*a*\operatorname{dilog}((-b*\exp(f*x+e)+(a^2-b^2)^{(1/2)}-a)/(-a+(a^2-b^2)^{(1/2)}))-1/f^2/(a^2-b^2)^{(3/2)}*d*a*\operatorname{dilog}((b*\exp(f*x+e)+(a^2-b^2)^{(1/2)}+a)/(a+(a^2-b^2)^{(1/2)}))+1/f^2/(a^2-b^2)*d*\ln(b*\exp(2*f*x+2*e)+2*a*\exp(f*x+e)+b)-2/f^2/(a^2-b^2)*d*\ln$

$(\exp(f*x+e))^{-2}/f^2/(a^2-b^2)*a*d*e/(-a^2+b^2)^{(1/2)}*\arctan(1/2*(2*b*\exp(f*x+e)+2*a)/(-a^2+b^2)^{(1/2)})$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*a^2-4\*b^2>0)', see `assume?` for more details)Is 4\*a^2-4\*b^2 positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c + dx}{(a + b \cosh(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)/(a + b\*cosh(e + f\*x))^2,x)

[Out] int((c + d\*x)/(a + b\*cosh(e + f\*x))^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*cosh(f\*x+e))\*\*2,x)

[Out] Timed out

$$3.176 \quad \int \frac{1}{(c+dx)(a+b \cosh(e+fx))^2} dx$$

**Optimal.** Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+b \cosh(e+fx))^2}, x\right)$$

[Out] Unintegrable(1/(d\*x+c)/(a+b\*cosh(f\*x+e))^2, x)

**Rubi [A]** time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(c+dx)(a+b \cosh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d\*x)\*(a + b\*Cosh[e + f\*x])^2), x]

[Out] Defer[Int][1/((c + d\*x)\*(a + b\*Cosh[e + f\*x])^2), x]

Rubi steps

$$\int \frac{1}{(c+dx)(a+b \cosh(e+fx))^2} dx = \int \frac{1}{(c+dx)(a+b \cosh(e+fx))^2} dx$$

**Mathematica [A]** time = 51.53, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+b \cosh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d\*x)\*(a + b\*Cosh[e + f\*x])^2), x]

[Out] Integrate[1/((c + d\*x)\*(a + b\*Cosh[e + f\*x])^2), x]

**fricas [A]** time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2dx + a^2c + (b^2dx + b^2c) \cosh(fx + e)^2 + 2(abdx + abc) \cosh(fx + e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+b\*cosh(f\*x+e))^2,x, algorithm="fricas")

[Out] integral(1/(a^2\*d\*x + a^2\*c + (b^2\*d\*x + b^2\*c)\*cosh(f\*x + e)^2 + 2\*(a\*b\*d\*x + a\*b\*c)\*cosh(f\*x + e)), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)(b \cosh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+b\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate(1/((d\*x + c)\*(b\*cosh(f\*x + e) + a)^2), x)

**maple** [A] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)(a + b \cosh(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x+c)/(a+b\*cosh(f\*x+e))^2,x)

[Out] int(1/(d\*x+c)/(a+b\*cosh(f\*x+e))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$2 \left( a e^{(fx+e)} + b \right)$$

---


$$a^2bcf - b^3cf + (a^2bdf - b^3df)x + (a^2bcfe^{(2e)} - b^3cfe^{(2e)} + (a^2bdfe^{(2e)} - b^3dfe^{(2e)})x)e^{(2fx)} + 2(a^3cfe^e - ab^2cf$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+b\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out] 2\*(a\*e^(f\*x + e) + b)/(a^2\*b\*c\*f - b^3\*c\*f + (a^2\*b\*d\*f - b^3\*d\*f)\*x + (a^2\*b\*c\*f\*e^(2\*e) - b^3\*c\*f\*e^(2\*e) + (a^2\*b\*d\*f\*e^(2\*e) - b^3\*d\*f\*e^(2\*e))\*x)\*e^(2\*f\*x) + 2\*(a^3\*c\*f\*e^e - a\*b^2\*c\*f\*e^e + (a^3\*d\*f\*e^e - a\*b^2\*d\*f\*e^e)\*x)\*e^(f\*x)) + integrate(2\*(b\*d + (a\*d\*f\*x\*e^e + (c\*f\*e^e + d\*e^e)\*a)\*e^(f\*x))/(a^2\*b\*c^2\*f - b^3\*c^2\*f + (a^2\*b\*d^2\*f - b^3\*d^2\*f)\*x^2 + 2\*(a^2\*b\*c\*d\*f - b^3\*c\*d\*f)\*x + (a^2\*b\*c^2\*f\*e^(2\*e) - b^3\*c^2\*f\*e^(2\*e) + (a^2\*b\*d^2\*f\*e^(2\*e) - b^3\*d^2\*f\*e^(2\*e))\*x^2 + 2\*(a^2\*b\*c\*d\*f\*e^(2\*e) - b^3\*c\*d\*f\*e^(2\*e))\*x)\*e^(2\*f\*x) + 2\*(a^3\*c^2\*f\*e^e - a\*b^2\*c^2\*f\*e^e + (a^3\*d^2\*f\*e^e - a\*b^2\*d^2\*f\*e^e)\*x^2 + 2\*(a^3\*c\*d\*f\*e^e - a\*b^2\*c\*d\*f\*e^e)\*x)\*e^(f\*x)), x)



mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a + b \cosh(e + f x))^2 (c + d x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*cosh(e + f\*x))^2\*(c + d\*x)),x)

[Out] int(1/((a + b\*cosh(e + f\*x))^2\*(c + d\*x)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+b\*cosh(f\*x+e))\*\*2,x)

[Out] Timed out

$$3.177 \quad \int \frac{1}{(c+dx)^2(a+b \cosh(e+fx))^2} dx$$

**Optimal.** Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+b \cosh(e+fx))^2}, x\right)$$

[Out] Unintegrable(1/(d\*x+c)^2/(a+b\*cosh(f\*x+e))^2, x)

**Rubi [A]** time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b \cosh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d\*x)^2\*(a + b\*Cosh[e + f\*x]))^2, x]

[Out] Defer[Int][1/((c + d\*x)^2\*(a + b\*Cosh[e + f\*x]))^2, x]

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b \cosh(e+fx))^2} dx = \int \frac{1}{(c+dx)^2(a+b \cosh(e+fx))^2} dx$$

**Mathematica [A]** time = 54.96, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+b \cosh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d\*x)^2\*(a + b\*Cosh[e + f\*x]))^2, x]

[Out] Integrate[1/((c + d\*x)^2\*(a + b\*Cosh[e + f\*x]))^2, x]

**fricas [A]** time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2d^2x^2 + 2a^2cdx + a^2c^2 + (b^2d^2x^2 + 2b^2cdx + b^2c^2) \cosh(fx + e)^2 + 2(abd^2x^2 + 2abcdx + abc^2) \cosh(fx + e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+b\*cosh(f\*x+e))^2,x, algorithm="fricas")

[Out] integral(1/(a^2\*d^2\*x^2 + 2\*a^2\*c\*d\*x + a^2\*c^2 + (b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2)\*cosh(f\*x + e)^2 + 2\*(a\*b\*d^2\*x^2 + 2\*a\*b\*c\*d\*x + a\*b\*c^2)\*cosh(f\*x + e)), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2 (b \cosh(fx+e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+b\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate(1/((d\*x + c)^2\*(b\*cosh(f\*x + e) + a)^2), x)

**maple** [A] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2 (a + b \cosh(fx+e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x+c)^2/(a+b\*cosh(f\*x+e))^2,x)

[Out] int(1/(d\*x+c)^2/(a+b\*cosh(f\*x+e))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$a^2bc^2f - b^3c^2f + (a^2bd^2f - b^3d^2f)x^2 + 2(a^2bcd f - b^3cdf)x + (a^2bc^2fe^{(2e)} - b^3c^2fe^{(2e)} + (a^2bd^2fe^{(2e)} - b^3d^2f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+b\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out] 2\*(a\*e^(f\*x + e) + b)/(a^2\*b\*c^2\*f - b^3\*c^2\*f + (a^2\*b\*d^2\*f - b^3\*d^2\*f)\*x^2 + 2\*(a^2\*b\*c\*d\*f - b^3\*c\*d\*f)\*x + (a^2\*b\*c^2\*f\*e^(2\*e) - b^3\*c^2\*f\*e^(2\*e) + (a^2\*b\*d^2\*f\*e^(2\*e) - b^3\*d^2\*f\*e^(2\*e))\*x^2 + 2\*(a^2\*b\*c\*d\*f\*e^(2\*e) - b^3\*c\*d\*f\*e^(2\*e))\*x)\*e^(2\*f\*x) + 2\*(a^3\*c^2\*f\*e^e - a\*b^2\*c^2\*f\*e^e + (a^3\*d^2\*f\*e^e - a\*b^2\*d^2\*f\*e^e)\*x^2 + 2\*(a^3\*c\*d\*f\*e^e - a\*b^2\*c\*d\*f\*e^e)\*x)\*e^(f\*x)) + integrate(2\*(2\*b\*d + (a\*d\*f\*x\*e^e + (c\*f\*e^e + 2\*d\*e^e)\*a)\*e^(f\*x))/(a^2\*b\*c^3\*f - b^3\*c^3\*f + (a^2\*b\*d^3\*f - b^3\*d^3\*f)\*x^3 + 3\*(a^2\*b\*c\*d^2\*f - b^3\*c\*d^2\*f)\*x^2 + 3\*(a^2\*b\*c^2\*d\*f - b^3\*c^2\*d\*f)\*x + (a^2\*b\*c^2

```

3*f*e^(2*e) - b^3*c^3*f*e^(2*e) + (a^2*b*d^3*f*e^(2*e) - b^3*d^3*f*e^(2*e))
*x^3 + 3*(a^2*b*c*d^2*f*e^(2*e) - b^3*c*d^2*f*e^(2*e))*x^2 + 3*(a^2*b*c^2*d
*f*e^(2*e) - b^3*c^2*d*f*e^(2*e))*x)*e^(2*f*x) + 2*(a^3*c^3*f*e^e - a*b^2*c
^3*f*e^e + (a^3*d^3*f*e^e - a*b^2*d^3*f*e^e)*x^3 + 3*(a^3*c*d^2*f*e^e - a*b
^2*c*d^2*f*e^e)*x^2 + 3*(a^3*c^2*d*f*e^e - a*b^2*c^2*d*f*e^e)*x)*e^(f*x),
x)

```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a + b \cosh(e + f x))^2 (c + d x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*cosh(e + f*x))^2*(c + d*x)^2),x)
```

```
[Out] int(1/((a + b*cosh(e + f*x))^2*(c + d*x)^2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Timed out
```

### 3.178 $\int (c + dx)^m (a + b \cosh(e + fx))^n dx$

Optimal. Leaf size=23

$$\text{Int}((c + dx)^m (a + b \cosh(e + fx))^n, x)$$

[Out] Unintegrable((d\*x+c)^m\*(a+b\*cosh(f\*x+e))^n, x)

**Rubi** [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (c + dx)^m (a + b \cosh(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m\*(a + b\*Cosh[e + f\*x])^n, x]

[Out] Defer[Int] [(c + d\*x)^m\*(a + b\*Cosh[e + f\*x])^n, x]

Rubi steps

$$\int (c + dx)^m (a + b \cosh(e + fx))^n dx = \int (c + dx)^m (a + b \cosh(e + fx))^n dx$$

**Mathematica** [A] time = 4.53, size = 0, normalized size = 0.00

$$\int (c + dx)^m (a + b \cosh(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m\*(a + b\*Cosh[e + f\*x])^n, x]

[Out] Integrate[(c + d\*x)^m\*(a + b\*Cosh[e + f\*x])^n, x]

**fricas** [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}((dx + c)^m (b \cosh(fx + e) + a)^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*cosh(f\*x+e))^n, x, algorithm="fricas")

[Out] integral((d\*x + c)^m\*(b\*cosh(f\*x + e) + a)^n, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m (b \cosh(fx + e) + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*cosh(f\*x+e))^n,x, algorithm="giac")

[Out] integrate((d\*x + c)^m\*(b\*cosh(f\*x + e) + a)^n, x)

**maple** [A] time = 0.15, size = 0, normalized size = 0.00

$$\int (dx + c)^m (a + b \cosh(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*(a+b\*cosh(f\*x+e))^n,x)

[Out] int((d\*x+c)^m\*(a+b\*cosh(f\*x+e))^n,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m (b \cosh(fx + e) + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*cosh(f\*x+e))^n,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m\*(b\*cosh(f\*x + e) + a)^n, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (a + b \cosh(e + fx))^n (c + dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*cosh(e + f\*x))^n\*(c + d\*x)^m,x)

[Out] int((a + b\*cosh(e + f\*x))^n\*(c + d\*x)^m, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m\*(a+b\*cosh(f\*x+e))\*\*n,x)

[Out] Timed out

### 3.179 $\int (c + dx)^m (a + b \cosh(e + fx))^3 dx$

**Optimal.** Leaf size=543

$$\frac{a^3(c + dx)^{m+1}}{d(m+1)} + \frac{3a^2be^{-\frac{cf}{d}}(c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{f(c+dx)}{d}\right)}{2f} - \frac{3a^2be^{\frac{cf}{d}-e}(c + dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{f(c+dx)}{d}\right)}{2f}$$

[Out]  $a^3(d*x+c)^{(1+m)/d}/(1+m)+3/2*a*b^2*(d*x+c)^{(1+m)/d}/(1+m)+1/8*3^{(-1-m)*b^3*\exp(3*e-3*c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, -3*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+3*2^{(-3-m)*a*b^2*\exp(2*e-2*c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, -2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+3/2*a^2*b*\exp(e-c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, -f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+3/8*b^3*\exp(e-c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, -f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)-3/2*a^2*b*\exp(-e+c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-3/8*b^3*\exp(-e+c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-3*2^{(-3-m)*a*b^2*\exp(-2*e+2*c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, 2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-1/8*3^{(-1-m)*b^3*\exp(-3*e+3*c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m, 3*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)$

**Rubi [A]** time = 0.74, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3317, 3307, 2181, 3312}

$$\frac{3a^2be^{-\frac{cf}{d}}(c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{f(c+dx)}{d}\right)}{2f} - \frac{3a^2be^{\frac{cf}{d}-e}(c + dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, \frac{f(c+dx)}{d}\right)}{2f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^m*(a + b*\text{Cosh}[e + f*x])^3, x]$

[Out]  $(a^3*(c + d*x)^{(1 + m)}/(d*(1 + m)) + (3*a*b^2*(c + d*x)^{(1 + m)})/(2*d*(1 + m)) + (3^{(-1 - m)*b^3*E^{(3*e - (3*c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, (-3*f*(c + d*x))/d]}/(8*f*(-((f*(c + d*x))/d))^m) + (3*2^{(-3 - m)*a*b^2*E^{(2*e - (2*c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, (-2*f*(c + d*x))/d]}/(f*(-((f*(c + d*x))/d))^m) + (3*a^2*b*E^{(e - (c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, -((f*(c + d*x))/d)])/((2*f*(-((f*(c + d*x))/d))^m) + (3*b^3*E^{(e - (c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, -((f*(c + d*x))/d)])/((8*f*(-((f*(c + d*x))/d))^m) - (3*a^2*b*E^{(-e + (c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, (f*(c + d*x))/d]}/(2*f*((f*(c + d*x))/d))^m) - (3*b^3*E^{(-e + (c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, (f*(c + d*x))/d]}/(8*f*((f*(c + d*x))/d))^m) - (3*2^{(-3 - m)*a*b^2*E^{(-2*e + (2*c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, (2*f*(c + d*x))/d]}/(f*((f*(c + d*x))/d))^m) - (3^{(-1 - m)*b^3*E^{(-3*e + (3*c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, (3*f*(c + d*x))/d]}/(8*f*((f*(c + d*x))/d))^m)$

**Rule 2181**

```
Int[(F_)^((g_)*(e_) + (f_)*(x_))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x]))/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*
(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

### Rule 3307

```
Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol]
:> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]
```

### Rule 3312

```
Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + (f_)*(x_)]^(n_), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

### Rule 3317

```
Int[((c_) + (d_)*(x_))^(m_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^(n_)), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])
```

### Rubi steps



$$\begin{aligned}
\int (c + dx)^m (a + b \cosh(e + fx))^3 dx &= \int \left( a^3 (c + dx)^m + 3a^2 b (c + dx)^m \cosh(e + fx) + 3ab^2 (c + dx)^m \cosh^2(e + fx) + b^3 (c + dx)^m \cosh^3(e + fx) \right) dx \\
&= \frac{a^3 (c + dx)^{1+m}}{d(1+m)} + (3a^2 b) \int (c + dx)^m \cosh(e + fx) dx + (3ab^2) \int (c + dx)^m \cosh^2(e + fx) dx + b^3 \int (c + dx)^m \cosh^3(e + fx) dx \\
&= \frac{a^3 (c + dx)^{1+m}}{d(1+m)} + \frac{1}{2} (3a^2 b) \int e^{-i(i e + i f x)} (c + dx)^m dx + \frac{1}{2} (3a^2 b) \int e^{i(i e + i f x)} (c + dx)^m dx \\
&= \frac{a^3 (c + dx)^{1+m}}{d(1+m)} + \frac{3ab^2 (c + dx)^{1+m}}{2d(1+m)} + \frac{3a^2 b e^{e - \frac{cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{f(c+dx)}{d}\right)}{2f} \\
&= \frac{a^3 (c + dx)^{1+m}}{d(1+m)} + \frac{3ab^2 (c + dx)^{1+m}}{2d(1+m)} + \frac{3a^2 b e^{e - \frac{cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{f(c+dx)}{d}\right)}{2f} \\
&= \frac{a^3 (c + dx)^{1+m}}{d(1+m)} + \frac{3ab^2 (c + dx)^{1+m}}{2d(1+m)} + \frac{3^{-1-m} b^3 e^{3e - \frac{3cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{f(c+dx)}{d}\right)}{8f}
\end{aligned}$$

**Mathematica [A]** time = 1.74, size = 447, normalized size = 0.82

$$2^{-m-3} 3^{-m-1} e^{-3\left(\frac{cf}{d}+e\right)} (c+dx)^m \left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m} \left(b d 2^m 3^{m+2} (m+1) (4a^2+b^2) e^{\frac{2cf}{d}+4e} \left(\frac{f(c+dx)}{d}\right)^m \Gamma\left(m+1, -\frac{f(c+dx)}{d}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^m\*(a + b\*Cosh[e + f\*x])^3,x]

[Out] (2^(-3 - m)\*3^(-1 - m)\*(c + d\*x)^m\*(2^m\*b^3\*d\*E^(6\*e)\*(1 + m)\*((f\*(c + d\*x))/d)^m\*Gamma[1 + m, (-3\*f\*(c + d\*x))/d] + 3^(2 + m)\*a\*b^2\*d\*E^(5\*e + (c\*f)/d)\*(1 + m)\*(f\*(c/d + x))^m\*Gamma[1 + m, (-2\*f\*(c + d\*x))/d] + 2^m\*3^(2 + m)\*b\*(4\*a^2 + b^2)\*d\*E^(4\*e + (2\*c\*f)/d)\*(1 + m)\*((f\*(c + d\*x))/d)^m\*Gamma[1 + m, -((f\*(c + d\*x))/d)] - E^((3\*c\*f)/d)\*(2^m\*3^(2 + m)\*b\*(4\*a^2 + b^2)\*d\*E^(2\*e + (c\*f)/d)\*(1 + m)\*(-((f\*(c + d\*x))/d))^m\*Gamma[1 + m, (f\*(c + d\*x))/d] + 3^(2 + m)\*a\*b^2\*d\*E^(e + (2\*c\*f)/d)\*(1 + m)\*(-((f\*(c + d\*x))/d))^m\*Gamma[1 + m, (2\*f\*(c + d\*x))/d] + 2^m\*(-4\*3^(1 + m)\*a\*(2\*a^2 + 3\*b^2)\*E^(3\*e)\*f\*(c + d\*x)\*(-((f^2\*(c + d\*x)^2)/d^2))^m + b^3\*d\*E^((3\*c\*f)/d)\*(1 + m)\*(-((f\*(c + d\*x))/d))^m\*Gamma[1 + m, (3\*f\*(c + d\*x))/d]))/(d\*E^(3\*(e + (c\*f)/d))\*f\*(1 + m)\*(-((f^2\*(c + d\*x)^2)/d^2))^m)

**fricas** [A] time = 0.62, size = 813, normalized size = 1.50

$$(b^3 dm + b^3 d) \cosh\left(\frac{dm \log\left(\frac{3f}{d}\right) + 3de - 3cf}{d}\right) \Gamma\left(m + 1, \frac{3(df x + cf)}{d}\right) + 9(ab^2 dm + ab^2 d) \cosh\left(\frac{dm \log\left(\frac{2f}{d}\right) + 2de - 2cf}{d}\right) \Gamma\left(m + 1, \frac{2(df x + cf)}{d}\right) + 9((4a^2 b + b^3) dm + (4a^2 b + b^3) d) \cosh\left(\frac{dm \log(f/d) + de - cf}{d}\right) \Gamma\left(m + 1, \frac{df x + cf}{d}\right) - 9((4a^2 b + b^3) dm + (4a^2 b + b^3) d) \cosh\left(\frac{dm \log(-f/d) - de + cf}{d}\right) \Gamma\left(m + 1, -\frac{df x + cf}{d}\right) - 9(a b^2 dm + a b^2 d) \cosh\left(\frac{dm \log(-2f/d) - 2de + 2cf}{d}\right) \Gamma\left(m + 1, -\frac{2(df x + cf)}{d}\right) - (b^3 dm + b^3 d) \cosh\left(\frac{dm \log(-3f/d) - 3de + 3cf}{d}\right) \Gamma\left(m + 1, -\frac{3(df x + cf)}{d}\right) - (b^3 dm + b^3 d) \Gamma\left(m + 1, \frac{3(df x + cf)}{d}\right) \sinh\left(\frac{dm \log(3f/d) + 3de - 3cf}{d}\right) - 9(a b^2 dm + a b^2 d) \Gamma\left(m + 1, \frac{2(df x + cf)}{d}\right) \sinh\left(\frac{dm \log(2f/d) + 2de - 2cf}{d}\right) - 9((4a^2 b + b^3) dm + (4a^2 b + b^3) d) \Gamma\left(m + 1, \frac{df x + cf}{d}\right) \sinh\left(\frac{dm \log(f/d) + de - cf}{d}\right) + 9((4a^2 b + b^3) dm + (4a^2 b + b^3) d) \Gamma\left(m + 1, -\frac{df x + cf}{d}\right) \sinh\left(\frac{dm \log(-f/d) - de + cf}{d}\right) + 9(a b^2 dm + a b^2 d) \Gamma\left(m + 1, -\frac{2(df x + cf)}{d}\right) \sinh\left(\frac{dm \log(-2f/d) - 2de + 2cf}{d}\right) + (b^3 dm + b^3 d) \Gamma\left(m + 1, -\frac{3(df x + cf)}{d}\right) \sinh\left(\frac{dm \log(-3f/d) - 3de + 3cf}{d}\right) - 12((2a^3 + 3a b^2) d f x + (2a^3 + 3a b^2) c f) \cosh(m \log(dx + c)) - 12((2a^3 + 3a b^2) d f x + (2a^3 + 3a b^2) c f) \sinh(m \log(dx + c)) / (d f m + d f)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*cosh(f\*x+e))^3,x, algorithm="fricas")

[Out] -1/24\*((b^3\*d\*m + b^3\*d)\*cosh((d\*m\*log(3\*f/d) + 3\*d\*e - 3\*c\*f)/d)\*gamma(m + 1, 3\*(d\*f\*x + c\*f)/d) + 9\*(a\*b^2\*d\*m + a\*b^2\*d)\*cosh((d\*m\*log(2\*f/d) + 2\*d\*e - 2\*c\*f)/d)\*gamma(m + 1, 2\*(d\*f\*x + c\*f)/d) + 9\*((4\*a^2\*b + b^3)\*d\*m + (4\*a^2\*b + b^3)\*d)\*cosh((d\*m\*log(f/d) + d\*e - c\*f)/d)\*gamma(m + 1, (d\*f\*x + c\*f)/d) - 9\*((4\*a^2\*b + b^3)\*d\*m + (4\*a^2\*b + b^3)\*d)\*cosh((d\*m\*log(-f/d) - d\*e + c\*f)/d)\*gamma(m + 1, -(d\*f\*x + c\*f)/d) - 9\*(a\*b^2\*d\*m + a\*b^2\*d)\*cosh((d\*m\*log(-2\*f/d) - 2\*d\*e + 2\*c\*f)/d)\*gamma(m + 1, -2\*(d\*f\*x + c\*f)/d) - (b^3\*d\*m + b^3\*d)\*cosh((d\*m\*log(-3\*f/d) - 3\*d\*e + 3\*c\*f)/d)\*gamma(m + 1, -3\*(d\*f\*x + c\*f)/d) - (b^3\*d\*m + b^3\*d)\*gamma(m + 1, 3\*(d\*f\*x + c\*f)/d)\*sinh((d\*m\*log(3\*f/d) + 3\*d\*e - 3\*c\*f)/d) - 9\*(a\*b^2\*d\*m + a\*b^2\*d)\*gamma(m + 1, 2\*(d\*f\*x + c\*f)/d)\*sinh((d\*m\*log(2\*f/d) + 2\*d\*e - 2\*c\*f)/d) - 9\*((4\*a^2\*b + b^3)\*d\*m + (4\*a^2\*b + b^3)\*d)\*gamma(m + 1, (d\*f\*x + c\*f)/d)\*sinh((d\*m\*log(f/d) + d\*e - c\*f)/d) + 9\*((4\*a^2\*b + b^3)\*d\*m + (4\*a^2\*b + b^3)\*d)\*gamma(m + 1, -(d\*f\*x + c\*f)/d)\*sinh((d\*m\*log(-f/d) - d\*e + c\*f)/d) + 9\*(a\*b^2\*d\*m + a\*b^2\*d)\*gamma(m + 1, -2\*(d\*f\*x + c\*f)/d)\*sinh((d\*m\*log(-2\*f/d) - 2\*d\*e + 2\*c\*f)/d) + (b^3\*d\*m + b^3\*d)\*gamma(m + 1, -3\*(d\*f\*x + c\*f)/d)\*sinh((d\*m\*log(-3\*f/d) - 3\*d\*e + 3\*c\*f)/d) - 12\*((2\*a^3 + 3\*a\*b^2)\*d\*f\*x + (2\*a^3 + 3\*a\*b^2)\*c\*f)\*cosh(m\*log(d\*x + c)) - 12\*((2\*a^3 + 3\*a\*b^2)\*d\*f\*x + (2\*a^3 + 3\*a\*b^2)\*c\*f)\*sinh(m\*log(d\*x + c)))/(d\*f\*m + d\*f)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cosh(fx + e) + a)^3 (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*cosh(f\*x+e))^3,x, algorithm="giac")

[Out] integrate((b\*cosh(f\*x + e) + a)^3\*(d\*x + c)^m, x)

**maple** [F] time = 0.36, size = 0, normalized size = 0.00

$$\int (dx + c)^m (a + b \cosh(fx + e))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*(a+b\*cosh(f\*x+e))^3,x)

[Out] int((d\*x+c)^m\*(a+b\*cosh(f\*x+e))^3,x)

**maxima** [A] time = 0.51, size = 375, normalized size = 0.69

$$-\frac{3}{2} \left( \frac{(dx+c)^{m+1} e^{\left(-e+\frac{cf}{d}\right)} E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} + \frac{(dx+c)^{m+1} e^{\left(e-\frac{cf}{d}\right)} E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d} \right) a^2 b - \frac{3}{4} \left( \frac{(dx+c)^{m+1} e^{\left(-2e+\frac{2cf}{d}\right)} E_{-m}\left(\frac{2(dx+c)f}{d}\right)}{d} + \frac{(dx+c)^{m+1} e^{\left(2e-\frac{2cf}{d}\right)} E_{-m}\left(-\frac{2(dx+c)f}{d}\right)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*cosh(f\*x+e))^3,x, algorithm="maxima")

[Out]  $-3/2*((d*x + c)^{(m + 1)}*e^{(-e + c*f/d)}*\exp\_integral\_e(-m, (d*x + c)*f/d)/d + (d*x + c)^{(m + 1)}*e^{(e - c*f/d)}*\exp\_integral\_e(-m, -(d*x + c)*f/d)/d)*a^2 *b - 3/4*((d*x + c)^{(m + 1)}*e^{(-2*e + 2*c*f/d)}*\exp\_integral\_e(-m, 2*(d*x + c)*f/d)/d + (d*x + c)^{(m + 1)}*e^{(2*e - 2*c*f/d)}*\exp\_integral\_e(-m, -2*(d*x + c)*f/d)/d - 2*(d*x + c)^{(m + 1)}/(d*(m + 1)))*a*b^2 - 1/8*((d*x + c)^{(m + 1)}*e^{(-3*e + 3*c*f/d)}*\exp\_integral\_e(-m, 3*(d*x + c)*f/d)/d + 3*(d*x + c)^{(m + 1)}*e^{(-e + c*f/d)}*\exp\_integral\_e(-m, (d*x + c)*f/d)/d + 3*(d*x + c)^{(m + 1)}*e^{(e - c*f/d)}*\exp\_integral\_e(-m, -(d*x + c)*f/d)/d + (d*x + c)^{(m + 1)}*e^{(3*e - 3*c*f/d)}*\exp\_integral\_e(-m, -3*(d*x + c)*f/d)/d)*b^3 + (d*x + c)^{(m + 1)}*a^3/(d*(m + 1))$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \cosh(e + f x))^3 (c + d x)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*cosh(e + f\*x))^3\*(c + d\*x)^m,x)

[Out] int((a + b\*cosh(e + f\*x))^3\*(c + d\*x)^m, x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m\*(a+b\*cosh(f\*x+e))\*\*3,x)

[Out] Exception raised: TypeError

### 3.180 $\int (c + dx)^m (a + b \cosh(e + fx))^2 dx$

**Optimal.** Leaf size=282

$$\frac{a^2(c + dx)^{m+1}}{d(m + 1)} + \frac{abe^{e-\frac{cf}{d}}(c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{f(c+dx)}{d}\right)}{f} - \frac{abe^{\frac{cf}{d}-e}(c + dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, \frac{f(c+dx)}{d}\right)}{f}$$

[Out]  $a^2*(d*x+c)^{(1+m)}/d/(1+m)+1/2*b^2*(d*x+c)^{(1+m)}/d/(1+m)+2^{(-3-m)*b^2*\exp(2*e-2*c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m,-2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+a*b*\exp(e-c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)-a*b*\exp(-e+c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m,f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-2^{(-3-m)*b^2*\exp(-2*e+2*c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)$

**Rubi [A]** time = 0.36, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {3317, 3307, 2181, 3312}

$$\frac{abe^{e-\frac{cf}{d}}(c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m + 1, -\frac{f(c+dx)}{d}\right)}{f} - \frac{abe^{\frac{cf}{d}-e}(c + dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m + 1, \frac{f(c+dx)}{d}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^m\*(a + b\*Cosh[e + f\*x])^2,x]

[Out]  $(a^2*(c + d*x)^{(1 + m)}/(d*(1 + m)) + (b^2*(c + d*x)^{(1 + m)}/(2*d*(1 + m)) + (2^{(-3 - m)*b^2}*E^{(2*e - (2*c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, (-2*f*(c + d*x))/d])/f*(-((f*(c + d*x))/d))^m) + (a*b*E^{(e - (c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, -((f*(c + d*x))/d)])/f*(-((f*(c + d*x))/d))^m - (a*b*E^{(-e + (c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, (f*(c + d*x))/d])/f*((f*(c + d*x))/d)^m - (2^{(-3 - m)*b^2}*E^{(-2*e + (2*c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, (2*f*(c + d*x))/d])/f*((f*(c + d*x))/d))^m)$

#### Rule 2181

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x]))/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

#### Rule 3307

```
Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + Pi*(k_) + (f_)*(x_)], x_Symbol]
:> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

$I/2$ , Int[(c + d\*x)^m \* E^(I\*k\*Pi) \* E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rule 3317

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Sin[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

### Rubi steps

$$\begin{aligned}
 \int (c + dx)^m (a + b \cosh(e + fx))^2 dx &= \int (a^2(c + dx)^m + 2ab(c + dx)^m \cosh(e + fx) + b^2(c + dx)^m \cosh^2(e + fx)) dx \\
 &= \frac{a^2(c + dx)^{1+m}}{d(1+m)} + (2ab) \int (c + dx)^m \cosh(e + fx) dx + b^2 \int (c + dx)^m \cosh^2(e + fx) dx \\
 &= \frac{a^2(c + dx)^{1+m}}{d(1+m)} + (ab) \int e^{-i(e+ifx)} (c + dx)^m dx + (ab) \int e^{i(e+ifx)} (c + dx)^m dx \\
 &= \frac{a^2(c + dx)^{1+m}}{d(1+m)} + \frac{b^2(c + dx)^{1+m}}{2d(1+m)} + \frac{abe^{e-\frac{cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{f(c+dx)}{d}\right)}{f} \\
 &= \frac{a^2(c + dx)^{1+m}}{d(1+m)} + \frac{b^2(c + dx)^{1+m}}{2d(1+m)} + \frac{abe^{e-\frac{cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{f(c+dx)}{d}\right)}{f} \\
 &= \frac{a^2(c + dx)^{1+m}}{d(1+m)} + \frac{b^2(c + dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m} b^2 e^{2e-\frac{2cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{f(c+dx)}{d}\right)}{f}
 \end{aligned}$$

**Mathematica [A]** time = 0.80, size = 254, normalized size = 0.90

$$(c + dx)^m \left( 8a^2 f(c + dx) + 8abd(m + 1) e^{e-\frac{cf}{d}} \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{f(c+dx)}{d}\right) - 8abd(m + 1) e^{\frac{cf}{d}-e} \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, \frac{f(c+dx)}{d}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^m\*(a + b\*Cosh[e + f\*x])^2,x]

[Out]  $((c + d*x)^m*(8*a^2*f*(c + d*x) + 4*b^2*f*(c + d*x) + (b^2*d*E^{(2*e - (2*c*f)/d)}*(1 + m)*Gamma[1 + m, (-2*f*(c + d*x))/d]))/(2^m*((f*(c + d*x))/d)^m) + (8*a*b*d*E^{(e - (c*f)/d)}*(1 + m)*Gamma[1 + m, -(f*(c + d*x))/d])/(-((f*(c + d*x))/d))^m - (8*a*b*d*E^{(-e + (c*f)/d)}*(1 + m)*Gamma[1 + m, (f*(c + d*x))/d])/((f*(c + d*x))/d)^m - (b^2*d*E^{(-2*e + (2*c*f)/d)}*(1 + m)*Gamma[1 + m, (2*f*(c + d*x))/d])/(2^m*((f*(c + d*x))/d)^m))/(8*d*f*(1 + m))$

**fricas** [A] time = 0.80, size = 509, normalized size = 1.80

$$\frac{(b^2dm + b^2d) \cosh\left(\frac{dm \log\left(\frac{2f}{d}\right) + 2de - 2cf}{d}\right) \Gamma\left(m + 1, \frac{2(df x + cf)}{d}\right) + 8(abdm + abd) \cosh\left(\frac{dm \log\left(\frac{f}{d}\right) + de - cf}{d}\right) \Gamma\left(m + 1, \frac{d}{d}\right)}{8d^2f(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*cosh(f\*x+e))^2,x, algorithm="fricas")

[Out]  $-1/8*((b^2*d*m + b^2*d)*\cosh((d*m*\log(2*f/d) + 2*d*e - 2*c*f)/d)*\gamma(m + 1, 2*(d*f*x + c*f)/d) + 8*(a*b*d*m + a*b*d)*\cosh((d*m*\log(f/d) + d*e - c*f)/d)*\gamma(m + 1, (d*f*x + c*f)/d) - 8*(a*b*d*m + a*b*d)*\cosh((d*m*\log(-f/d) - d*e + c*f)/d)*\gamma(m + 1, -(d*f*x + c*f)/d) - (b^2*d*m + b^2*d)*\cosh((d*m*\log(-2*f/d) - 2*d*e + 2*c*f)/d)*\gamma(m + 1, -2*(d*f*x + c*f)/d) - (b^2*d*m + b^2*d)*\gamma(m + 1, 2*(d*f*x + c*f)/d)*\sinh((d*m*\log(2*f/d) + 2*d*e - 2*c*f)/d) - 8*(a*b*d*m + a*b*d)*\gamma(m + 1, (d*f*x + c*f)/d)*\sinh((d*m*\log(f/d) + d*e - c*f)/d) + 8*(a*b*d*m + a*b*d)*\gamma(m + 1, -(d*f*x + c*f)/d)*\sinh((d*m*\log(-f/d) - d*e + c*f)/d) + (b^2*d*m + b^2*d)*\gamma(m + 1, -2*(d*f*x + c*f)/d)*\sinh((d*m*\log(-2*f/d) - 2*d*e + 2*c*f)/d) - 4*((2*a^2 + b^2)*d*f*x + (2*a^2 + b^2)*c*f)*\cosh(m*\log(d*x + c)) - 4*((2*a^2 + b^2)*d*f*x + (2*a^2 + b^2)*c*f)*\sinh(m*\log(d*x + c)))/(d*f*m + d*f)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cosh(fx + e) + a)^2 (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate((b\*cosh(f\*x + e) + a)^2\*(d\*x + c)^m, x)

**maple** [F] time = 0.28, size = 0, normalized size = 0.00

$$\int (dx + c)^m (a + b \cosh(fx + e))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^m*(a+b*cosh(f*x+e))^2,x)`

[Out] `int((d*x+c)^m*(a+b*cosh(f*x+e))^2,x)`

**maxima** [A] time = 0.42, size = 208, normalized size = 0.74

$$-\left(\frac{(dx+c)^{m+1}e^{-\frac{e+cf}{d}}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} + \frac{(dx+c)^{m+1}e^{\frac{e-cf}{d}}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)ab - \frac{1}{4}\left(\frac{(dx+c)^{m+1}e^{-2e+\frac{2cf}{d}}E_{-m}\left(\frac{2(dx+c)f}{d}\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*(a+b*cosh(f*x+e))^2,x, algorithm="maxima")`

[Out] `-((d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d + (d*x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a*b - 1/4*((d*x + c)^(m + 1)*e^(-2*e + 2*c*f/d)*exp_integral_e(-m, 2*(d*x + c)*f/d)/d + (d*x + c)^(m + 1)*e^(2*e - 2*c*f/d)*exp_integral_e(-m, -2*(d*x + c)*f/d)/d - 2*(d*x + c)^(m + 1)/(d*(m + 1)))*b^2 + (d*x + c)^(m + 1)*a^2/(d*(m + 1))`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \cosh(e + f x))^2 (c + d x)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*cosh(e + f*x))^2*(c + d*x)^m,x)`

[Out] `int((a + b*cosh(e + f*x))^2*(c + d*x)^m, x)`

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)**m*(a+b*cosh(f*x+e))**2,x)`

[Out] Exception raised: TypeError

### 3.181 $\int (c + dx)^m (a + b \cosh(e + fx)) dx$

**Optimal.** Leaf size=131

$$\frac{a(c + dx)^{m+1}}{d(m + 1)} + \frac{be^{e-\frac{cf}{d}}(c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{f(c+dx)}{d}\right)}{2f} - \frac{be^{\frac{cf}{d}-e}(c + dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, \frac{f(c+dx)}{d}\right)}{2f}$$

[Out]  $a*(d*x+c)^{(1+m)}/d/(1+m)+1/2*b*\exp(e-c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)-1/2*b*\exp(-e+c*f/d)*(d*x+c)^m*\text{GAMMA}(1+m,f*(d*x+c)/d)/f/(f*(d*x+c)/d)^m$

**Rubi [A]** time = 0.15, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3317, 3307, 2181}

$$\frac{be^{e-\frac{cf}{d}}(c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m + 1, -\frac{f(c+dx)}{d}\right)}{2f} - \frac{be^{\frac{cf}{d}-e}(c + dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m + 1, \frac{f(c+dx)}{d}\right)}{2f} + a$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^m*(a + b*\text{Cosh}[e + f*x]), x]$

[Out]  $(a*(c + d*x)^{(1 + m)})/(d*(1 + m)) + (b*E^{(e - (c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, -((f*(c + d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) - (b*E^{(-e + (c*f)/d)}*(c + d*x)^m*\text{Gamma}[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))/d)^m)$

#### Rule 2181

$\text{Int}[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^{(m_)}, x\_Symbol]$   
 $\rightarrow -\text{Simp}[(F_)^{(g_*(e - (c*f)/d))}*(c + d*x)^{\text{FracPart}[m]}*\text{Gamma}[m + 1, (-((f*g*\text{Log}[F])/d))*c + d*x)]/(d*(-((f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1}*(-((f*g*\text{Log}[F])*(c + d*x))/d))^{\text{FracPart}[m]}, x] /; \text{FreeQ}\{F, c, d, e, f, g, m\}, x \ \&\& \ !\text{IntegerQ}[m]$

#### Rule 3307

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_.)}*\sin[(e_.) + \text{Pi}*(k_.) + (f_.)*(x_)], x\_Symbol]$   
 $\rightarrow \text{Dist}[I/2, \text{Int}[(c + d*x)^m/(E^{(I*k*Pi)}*E^{(I*(e + f*x))}), x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m*E^{(I*k*Pi)}*E^{(I*(e + f*x))}, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x \ \&\& \ \text{IntegerQ}[2*k]$

#### Rule 3317

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)])^{(n_.)}, x\_Symbol]$   
 $\rightarrow \text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, (a + b*\text{Sin}[e + f*x])^n], x],$



`x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])`

### Rubi steps

$$\begin{aligned}
 \int (c + dx)^m (a + b \cosh(e + fx)) dx &= \int (a(c + dx)^m + b(c + dx)^m \cosh(e + fx)) dx \\
 &= \frac{a(c + dx)^{1+m}}{d(1+m)} + b \int (c + dx)^m \cosh(e + fx) dx \\
 &= \frac{a(c + dx)^{1+m}}{d(1+m)} + \frac{1}{2}b \int e^{-i(i e + i f x)} (c + dx)^m dx + \frac{1}{2}b \int e^{i(i e + i f x)} (c + dx)^m dx \\
 &= \frac{a(c + dx)^{1+m}}{d(1+m)} + \frac{b e^{-\frac{c f}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{f(c+dx)}{d}\right) - b e^{-e + \frac{c f}{d}}}{2f}
 \end{aligned}$$

**Mathematica [A]** time = 0.18, size = 119, normalized size = 0.91

$$\frac{1}{2}(c+dx)^m \left( \frac{2a(c+dx)}{d(m+1)} - \frac{b e^{\frac{c f}{d} - e} \left(f \left(\frac{c}{d} + x\right)\right)^{-m} \Gamma\left(m+1, \frac{f(c+dx)}{d}\right)}{f} + \frac{b e^{-\frac{c f}{d}} \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{f(c+dx)}{d}\right)}{f} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^m\*(a + b\*Cosh[e + f\*x]),x]

[Out] ((c + d\*x)^m\*((2\*a\*(c + d\*x))/(d\*(1 + m)) + (b\*E^(e - (c\*f)/d)\*Gamma[1 + m, -((f\*(c + d\*x))/d)])/(f\*(-((f\*(c + d\*x))/d))^m) - (b\*E^(-e + (c\*f)/d)\*Gamma[a[1 + m, (f\*(c + d\*x))/d]]/(f\*(f\*(c/d + x))^m))/2

**fricas [A]** time = 0.56, size = 249, normalized size = 1.90

$$\frac{(b d m + b d) \cosh\left(\frac{d m \log\left(\frac{f}{d}\right) + d e - c f}{d}\right) \Gamma\left(m + 1, \frac{d f x + c f}{d}\right) - (b d m + b d) \cosh\left(\frac{d m \log\left(-\frac{f}{d}\right) - d e + c f}{d}\right) \Gamma\left(m + 1, -\frac{d f x + c f}{d}\right) -}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*cosh(f\*x+e)),x, algorithm="fricas")

[Out] -1/2\*((b\*d\*m + b\*d)\*cosh((d\*m\*log(f/d) + d\*e - c\*f)/d)\*gamma(m + 1, (d\*f\*x + c\*f)/d) - (b\*d\*m + b\*d)\*cosh((d\*m\*log(-f/d) - d\*e + c\*f)/d)\*gamma(m + 1,

$$-(d*f*x + c*f)/d) - (b*d*m + b*d)*\text{gamma}(m + 1, (d*f*x + c*f)/d)*\text{sinh}((d*m*\log(f/d) + d*e - c*f)/d) + (b*d*m + b*d)*\text{gamma}(m + 1, -(d*f*x + c*f)/d)*\text{sinh}((d*m*\log(-f/d) - d*e + c*f)/d) - 2*(a*d*f*x + a*c*f)*\text{cosh}(m*\log(d*x + c)) - 2*(a*d*f*x + a*c*f)*\text{sinh}(m*\log(d*x + c)))/(d*f*m + d*f)$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \cosh(fx + e) + a)(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*cosh(f\*x+e)),x, algorithm="giac")

[Out] integrate((b\*cosh(f\*x + e) + a)\*(d\*x + c)^m, x)

**maple** [F] time = 0.15, size = 0, normalized size = 0.00

$$\int (dx + c)^m (a + b \cosh(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*(a+b\*cosh(f\*x+e)),x)

[Out] int((d\*x+c)^m\*(a+b\*cosh(f\*x+e)),x)

**maxima** [A] time = 0.41, size = 100, normalized size = 0.76

$$-\frac{1}{2} \left( \frac{(dx + c)^{m+1} e^{\left(-e + \frac{cf}{d}\right)} E_{-m} \left(\frac{(dx+c)f}{d}\right)}{d} + \frac{(dx + c)^{m+1} e^{\left(e - \frac{cf}{d}\right)} E_{-m} \left(-\frac{(dx+c)f}{d}\right)}{d} \right) b + \frac{(dx + c)^{m+1} a}{d(m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*cosh(f\*x+e)),x, algorithm="maxima")

[Out] -1/2\*((d\*x + c)^(m + 1)\*e^(-e + c\*f/d)\*exp\_integral\_e(-m, (d\*x + c)\*f/d)/d + (d\*x + c)^(m + 1)\*e^(e - c\*f/d)\*exp\_integral\_e(-m, -(d\*x + c)\*f/d)/d)\*b + (d\*x + c)^(m + 1)\*a/(d\*(m + 1))

**mapad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \cosh(e + fx)) (c + dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))*(c + d*x)^m,x)
```

```
[Out] int((a + b*cosh(e + f*x))*(c + d*x)^m, x)
```

```
sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*cosh(f*x+e)),x)
```

```
[Out] Exception raised: TypeError
```

$$3.182 \quad \int \frac{(c+dx)^m}{a+b \cosh(e+fx)} dx$$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(c+dx)^m}{a+b \cosh(e+fx)}, x\right)$$

[Out] Unintegrable((d\*x+c)^m/(a+b\*cosh(f\*x+e)), x)

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(c+dx)^m}{a+b \cosh(e+fx)} dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m/(a + b\*Cosh[e + f\*x]), x]

[Out] Defer[Int] [(c + d\*x)^m/(a + b\*Cosh[e + f\*x]), x]

Rubi steps

$$\int \frac{(c+dx)^m}{a+b \cosh(e+fx)} dx = \int \frac{(c+dx)^m}{a+b \cosh(e+fx)} dx$$

Mathematica [A] time = 1.20, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{a+b \cosh(e+fx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m/(a + b\*Cosh[e + f\*x]), x]

[Out] Integrate[(c + d\*x)^m/(a + b\*Cosh[e + f\*x]), x]

fricas [A] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx+c)^m}{b \cosh(fx+e)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*cosh(f\*x+e)),x, algorithm="fricas")

[Out] integral((d\*x + c)^m/(b\*cosh(f\*x + e) + a), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{b \cosh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*cosh(f\*x+e)),x, algorithm="giac")

[Out] integrate((d\*x + c)^m/(b\*cosh(f\*x + e) + a), x)

**maple** [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{a + b \cosh(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m/(a+b\*cosh(f\*x+e)),x)

[Out] int((d\*x+c)^m/(a+b\*cosh(f\*x+e)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{b \cosh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*cosh(f\*x+e)),x, algorithm="maxima")

[Out] integrate((d\*x + c)^m/(b\*cosh(f\*x + e) + a), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c + dx)^m}{a + b \cosh(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^m/(a + b\*cosh(e + f\*x)),x)

[Out] int((c + d\*x)^m/(a + b\*cosh(e + f\*x)), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx)^m}{a + b \cosh(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m/(a+b\*cosh(f\*x+e)),x)

[Out] Integral((c + d\*x)\*\*m/(a + b\*cosh(e + f\*x)), x)

$$3.183 \quad \int \frac{(c+dx)^m}{(a+b \cosh(e+fx))^2} dx$$

Optimal. Leaf size=23

$$\text{Int} \left( \frac{(c+dx)^m}{(a+b \cosh(e+fx))^2}, x \right)$$

[Out] Unintegrable((d\*x+c)^m/(a+b\*cosh(f\*x+e))^2, x)

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(c+dx)^m}{(a+b \cosh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m/(a + b\*Cosh[e + f\*x])^2, x]

[Out] Defer[Int] [(c + d\*x)^m/(a + b\*Cosh[e + f\*x])^2, x]

Rubi steps

$$\int \frac{(c+dx)^m}{(a+b \cosh(e+fx))^2} dx = \int \frac{(c+dx)^m}{(a+b \cosh(e+fx))^2} dx$$

Mathematica [A] time = 5.58, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{(a+b \cosh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m/(a + b\*Cosh[e + f\*x])^2, x]

[Out] Integrate[(c + d\*x)^m/(a + b\*Cosh[e + f\*x])^2, x]

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(dx+c)^m}{b^2 \cosh^2(fx+e) + 2ab \cosh(fx+e) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*cosh(f\*x+e))^2,x, algorithm="fricas")

[Out] integral((d\*x + c)^m/(b^2\*cosh(f\*x + e)^2 + 2\*a\*b\*cosh(f\*x + e) + a^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{(b \cosh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*cosh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^m/(b\*cosh(f\*x + e) + a)^2, x)

**maple** [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{(a + b \cosh(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m/(a+b\*cosh(f\*x+e))^2,x)

[Out] int((d\*x+c)^m/(a+b\*cosh(f\*x+e))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^m}{(b \cosh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*cosh(f\*x+e))^2,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m/(b\*cosh(f\*x + e) + a)^2, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c + dx)^m}{(a + b \cosh(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x)^m/(a + b\*cosh(e + f\*x))^2,x)



```
[Out] int((c + d*x)^m/(a + b*cosh(e + f*x))^2, x)
```

```
sympy [A] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{(c + dx)^m}{(a + b \cosh(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)**m/(a + b*cosh(e + f*x))**2, x)
```



# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```

If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
  If[LeafCount[result]<=2*LeafCount[optimal],
    "A",
    "B"],
  "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

```

```
(* ::Text:: *)
```

```
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
```

```
(*1 = rational function*)
```

```
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
```

```
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
```

```
(*6 = appell function*)
```

```
(*7 = rootsum function*)
```

```
(*8 = integrate function*)
```

```
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
```

```
  If[AtomQ[expn],
```

```
    1,
```

```
  If[ListQ[expn],
```

```
    Max[Map[ExpnType,expn]],
```

```
  If[Head[expn]===Power,
```

```
    If[IntegerQ[expn[[2]],
```

```
      ExpnType[expn[[1]],
```

```
    If[Head[expn[[2]]]===Rational,
```

```
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
```

```
        1,
```

```
        Max[ExpnType[expn[[1]],2]],
```

```
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
```

```
  If[Head[expn]===Plus || Head[expn]===Times,
```

```
    Max[ExpnType[First[expn],ExpnType[Rest[expn]]],
```

```
  If[ElementaryFunctionQ[Head[expn]],
```

```
    Max[3,ExpnType[expn[[1]]],
```

```
  If[SpecialFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
```

```
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
```

```
  If[AppellFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```

If[Head[expn]===RootSum,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
If[Head[expn]===Integrate || Head[expn]===Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```



```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
    MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

## 4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print (">>>>Enter expnType, expn=", expn)
        print (">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
#is checked before calling the grading function that is passed.
#but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

```
#main function
```

```
def grade_antiderivative(result,optimal):
```

```

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

```

```

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```



```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```